ABSTRACT

MISHRA, MUDIT. Frequency Domain Multiplexing of Pulse Mode Radiation Detectors. (Under the direction of John Mattingly.)

Multiplexing of radiation detector signals into a single channel significantly reduces the need for a large number of digitizer channels, which reduces the cost and power consumption of a data acquisition system. Frequency domain multiplexing (FDM) has been previously applied to transition-edge sensors (TESs) by amplitude modulating the carrier current flowing through each TES. Because amplitude modulation cannot be implemented for pulse mode radiation detectors, a new method of frequency domain multiplexing using convolution and deconvolution was developed for this dissertation: each detector pulse was converted to a damped sinusoid of a specific frequency which was then combined into a single channel; the combined signal was digitized and the original detector signal was recovered from the damped sinusoid by deconvolution. This multiplexing method was demonstrated for organic and inorganic scintillator pulses.

Multiplexing of two EJ-309 organic scintillators was demonstrated using a prototype FDM system. When a single EJ-309 detector pulse was multiplexed in a digitized record, the pulse was recovered precisely by deconvolution. When two detector pulses (one from each detector) were multiplexed in a digitized record and the pulses did not overlap in time, the first pulse could be recovered precisely, while the second pulse was recovered with a substantial loss in the energy and timing precision. When the two pulses overlapped, it was demonstrated both theoretically and experimentally that the part of the first pulse that did not overlap with the second could be recovered accurately (and precisely); the second pulse could not be recovered accurately and the inaccuracy depended on the degree of overlap between the two pulses. The design of the FDM system was also modified to implement 4 to 1 multiplexing of 64 EJ-204 organic scintillator pillars of the single volume scatter camera (SVSC) being developed at Sandia National Laboratories. A multiplexing pattern for the pillars was identified to ensure that a single pulse (from any four multiplexed pillars) was combined in a digitized record so that the pulse could be recovered precisely. Using the new multiplexer, multiplexing of signals from four EJ-204 fast organic scintillators cubes (5 mm in length), each coupled to a silicon photomultiplier (with $6 \times 6 \text{ mm}^2$ area), was demonstrated. Calibrations to estimate charge collected and time of arrival from pulses reconstructed by deconvolution were developed for the 4:1 SiPM multiplexer.

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Frequency Domain Multiplexing of Pulse Mode Radiation Detectors

by
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DEDICATION

This work is dedicated to my maternal grandfather Dr. Jagdish Prasad Dikshit (1935-2018) who taught me the value of discipline, hard work and compassion towards mother nature. As a former professor at G.S.V.M Medical College, Kanpur, India, and a respected medical doctor, he not only treated patients for free, but also generated employment for people through now-defunct Burhwal Sugar Mill. He will be remembered among his family and friends for his leadership and philanthropy.

I also dedicate this work to Dr. Robert Kolbas (1953-2020), professor and former head of the Department of Electrical and Computer Engineering at NC State, with whom I had the opportunity to work on this research. Bob was a wonderful scientist and an excellent teacher. True to his parting advice to me in March 2020, he indeed gave more to society than he ever took.

BIOGRAPHY

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CHAPTER

1

INTRODUCTION

1.1 Motivation

Multiplexing of signals from radiation detectors enables data acquisition using a low-channel-density waveform digitizer, especially in the case when a large number of detector outputs must be digitized. This reduces the cost per channel of the digitizer - which increases with increasing sampling speed - it also reduces the data throughput for analysis.

1.2 Background

Signal multiplexing was first applied to an array of sodium iodide detectors (gamma camera), each coupled to a photomultiplier tube, to estimate the X-Y location of a gamma-ray-emitting isotope [1]. The application of the camera ranged from imaging radiological contamination to measuring ^{123}I uptake by the thyroid gland in a human subject. With advancement in small photon-sensitive semiconductor devices like silicon photomultipliers (SiPMs), a large number of scintillator detectors can be closely packed to precisely localize an event within a small volume. Many of these photodetectors can be multiplexed to reduce the complexity of the back-end data acquisition system. Several multiplexing schemes have been applied to these highly pixelated detectors to detect annihilation photons at the opposite pixels of a positron emission tomography (PET) system (Fig. 1.1a).

Signal multiplexing can also be applied to an array of backing detectors to detect a neutron that has

previously scattered with a target detector whose light output response is to be measured [2]. Similarly, multiplexing of detectors has become necessary for the coherent neutrino-nucleus scattering experiment where data acquisition from a large number of sodium iodide (NaI) detectors is required (Fig. 1.1b).

Frequency domain multiplexing (FDM) is a technique that encodes multiple detector signals in the frequency domain and combines them into a single channel. The combined signal is digitized using a single digitizer input channel, and information about each individual detector signal can be recovered from the combined signal in the frequency domain. This information includes:

- Origin of the detector signal: the detector number that produced it
- Energy and timing of the signal
- Particle identification: the ability to distinguish a gamma event from a neutron event in some organic scintillators based on the shape of the signal pulse

(a) A 3-D position sensitive PET detector with 80 readout channels proposed by Bieniosek et al. The system reads layers of LYSO crystal coupled to SiPMs (in yellow) [3].

(b) The coherent neutrino-nucleus scattering setup. Several NaI detectors are placed together to increase the coherent scatter cross-section of neutrino by nucleons. (COHERENT collaboration, <https://sites.duke.edu/coherent/>)

Figure 1.1 Examples of radiation detection system with a large number of readout channels.

1.3 Prior work

FDM of radiation detectors has been previously implemented for transition-edge sensor (TES) calorimeters to measure the energy deposited by a photon. The TES arrays operate at sub-Kelvin temperatures, and FDM helps to reduce the heat load associated with large number of wires connecting the cryogenic hardware to room-temperature electronics. Multiplexing of transition-edge sensors is performed by modulating a carrier current flowing through each TES with its sensor signal in the time domain. The

carrier current is a sinusoid of a specific frequency ($\sin(2\pi f_c t)$) that gives a 'tag' to each sensor, and the amplitude of the carrier is modulated by the sensor signal $x(t)$ to give an output $y(t)$:

$$y(t) = x(t)\sin(2\pi f_c t) \quad (1.1)$$

The equivalent equation in the frequency domain is:

$$Y(f) = \frac{X(f - f_c) - X(f + f_c)}{2i} \quad (1.2)$$

The modulation of two signals in time is convolution between them in frequency. As a result, modulation between the sensor signal and its sinusoidal carrier signal moves the spectrum of the sensor signal from the baseband to a passband centered at frequency f_c of the carrier, shown by Eq. 1.2. All the amplitude-modulated carrier currents flowing through their respective TESs are summed into a single channel and finally demodulated to recover each individual TES signal [4–7]. This concept is analogous to the transmission of multiple analog voice signals over a single channel by shifting each signal to a unique frequency subband in a frequency division multiplexed telephone system [8]. In order to recover the sensor signal from the modulated signal, the positive frequency components of $Y(f)$ are shifted back to baseband and the inverse Fourier transform ¹ is applied:

$$x(t) = 2i\mathcal{F}^{-1}(Y(f + f_c)u(f + f_c)) \quad (1.3)$$

where u is the unit step function.

1.3.1 Multiplexing of transition-edge sensors

TESs are voltage-biased with a constant voltage for stability [9]. When each sensor to be multiplexed is voltage-biased with a sinusoidal voltage of a constant amplitude and a specific frequency, multiplexing of sensors can be achieved in the frequency domain.

Carrier currents of different frequencies enter the cryostat in a single wire where the sensors to be multiplexed are connected in parallel. Fig. 1.2 shows a frequency-domain multiplexed system to multiplex four TESs (shown as variable resistors) into a single channel; in this case, four carriers of different frequencies enter the cold electronics. A tuned bandpass filter consisting of an inductor L_i and a capacitor C_i is connected in series to each sensor. Each LC filter allows a carrier current of a specific frequencies to pass through and suppresses the other frequencies. This results in carrier currents of different frequencies flowing through the sensors in parallel.

A carrier current flowing through a TES is amplitude modulated due to change in its resistance when a photon deposits energy on it. This results in a sensor signal $x_k(t)$ corresponding to carrier

¹ $\mathcal{F} = \int_{-\infty}^{+\infty} e^{-i2\pi ft} dt$ and $\mathcal{F}^{-1} = \int_{-\infty}^{+\infty} e^{i2\pi ft} df$

Figure 1.2 Frequency domain multiplexed system to combine signals from four TESs into a single channel via modulation between the carrier and the sensor signal. The combined signal is amplified using a transimpedance amplifier. The sensor signal corresponding to a carrier can be recovered from the amplified signal by multiplying the carrier signal to the combined signal and passing the resulting signal to a low-pass filter.

frequency f_{ck} , where $k = 0, 1, 2, \dots, N - 1$. The amplitude modulated carrier currents from the sensors are combined into a single channel, and the combined signal is amplified using a transimpedance amplifier

(a current-to-voltage converter). The amplified signal is passed to the room-temperature electronics where demodulation is performed to recover the individual sensor signals. The amplified signal for a N:1 multiplexed system is given by:

$$y(t) = \sum_{k=0}^{N-1} x_k(t) \sin(2\pi f_{ck}t) \quad (1.4)$$

The equivalent equation in the frequency domain is:

$$Y(f) = \sum_{k=0}^{N-1} \frac{X_k(f - f_{ck}) - X_k(f + f_{ck})}{2i} \quad (1.5)$$

Eq. 1.5 shows that the spectrum of each sensor signal is shifted to its carrier frequency. When the carrier frequencies are chosen to be far enough from each other so that the spectrum of each sensor signal resides in a separate band in the frequency domain, the individual signals are recoverable from the multiplexed signal through demodulation.

1.3.1.1 LC filters

When several TESs are connected in parallel, each individual sensor biased with a sinusoidal voltage of a specific frequency is a series RLC circuit as shown in Fig. 1.3. The transfer function $H(\omega)$ of the circuit is given by the ratio of the voltage across the TES to the input voltage:

$$H(\omega) = \frac{V_{R_i}}{V_i} = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}} \quad (1.6)$$

where ω is the angular frequency. The transfer function shows that the LC filter offers very low impedance around the resonant frequency $f_{LC} = \frac{1}{2\pi\sqrt{LC}}$, thereby allowing the current around frequency f_{LC} to flow through the TES while blocking currents of other frequencies (Fig. 1.4). The values of L and C for each TES are chosen such that its resonant frequency f_{LC} matches one of the carrier frequencies. This ensures that each sensor chooses a carrier current of a specific frequency and rejects other off-resonance carrier currents that enter the cryostat.

Figure 1.3 The circuit diagram depicting one of the TESs in the multiplexed case. The TES only allows the carrier current whose frequency matches with the resonant frequency of the circuit f_{LC} .

Figure 1.4 The response of a series RLC filter with $L_i = 20$ uH, $C_i = 5$ nC and $R_i = 1$ ohm. The peak lies at the resonant frequency of $f_i = \frac{1}{2\pi L_i C_i} = 500$ KHz.

1.3.1.2 Demodulation

The sum of amplitude-modulated TES currents is read out by a low-noise transimpedance amplifier. The combined signal is digitized by an analog-to-digital converter (ADC) and processed further to recover

the individual sensor signals. The combined signal in discrete-time $y(n)$ is given by:

$$y(n) = \sum_{k=0, \dots, N-1} x_k(n) \cos(2\pi f_{ck}n + \phi_k) \quad (1.7)$$

where $x_k(n)$ is the individual sensor signal and ϕ_k is the phase shift in the individual carriers due to propagation delay through cables and digital processing. The digitized signal is frequency-shifted back to zero with I/Q (in-phase/quadrature) demodulators for each carrier signal.

1.3.2 Other multiplexing schemes for detectors

Anger describes a multiplexing scheme (Anger logic) to combine outputs of multiple photomultiplier tubes (PMTs) connected to a sodium iodide crystal into four channels [1]. The signals from the four channels are used to estimate the X and Y locations of a scintillation pulse. Each PMT output current is first distributed to two horizontal and two vertical branches (i.e., four branches) using four resistors. The values of the resistors for each PMT are chosen such that a particular combination of four multiplexed signals is proportional to the X and Y positions (Fig. 1.5a).

Siegel describes a discretized positioning circuit (DPC) to multiplex an 8x8 array of BGO crystals coupled to a 64 channel MAPMT (multi-anode photomultiplier tube) to estimate the X and Y locations of a scintillation pulse [10]. Each MAPMT output is connected to a node in a row of several resistors, with a column of resistors at the two ends connecting each row. An 8x8 array of 64 channels is read by 4 output channels achieving a multiplexing ratio of 16:1. The values of the resistors in each row and in each column at the two ends are again chosen so that a particular combination of the four multiplexed outputs is proportional to the interaction location (Fig. 1.5b). Siegel also demonstrated a hybrid multiplexing scheme, which was a combination of Anger and DPC multiplexing to couple 64 BGO crystals to an MAPMT. DPC has also been applied to multiplex silicon photomultiplier (SiPM) readouts where the four multiplexed signals are summed to estimate the energy deposited and the timing of an event [11, 12].

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Crosswire readout multiplexing sums the signals in each row and column of an $N \times M$ array of PMTs/SiPMs to produce $N+M$ readout channels. The multiplexed signal in any row/column is combined using an Anger circuit to estimate the location of a scintillation pulse [13]. This technique uses fewer resistors because Anger logic is only performed on the multiplexed signal of any row/column. The energy and timing are estimated from the sum of multiplexed outputs; sometimes the timing signal is independently extracted from the circuit [14].

The resistive charge-division methods described above, when applied to SiPMs, degrade the timing resolution of the detectors due to the high RC time-constant depending on the position of the detector in the resistive network. It can be overcome by using capacitive charge-division circuits, which also reduce the accumulation of dark current as well as the summed capacitive load, because fewer neighboring SiPMs are connected to the SiPM that produces the signal [15] (Fig. 1.6). Another way to minimize the effect of dark noise is to combine only a few SiPM channels to a single comparator and then combine the comparator outputs into a single channel [16]. Signal driven multiplexing (SDM) is another approach to improve timing performance where a Schottky diode pair is connected to each SiPM [17]; Schottky diodes only connect the SiPM that produces a signal to the multiplexed channel. Another variant of SDM is the pulse width modulation (PWM) technique, where detector pulses are converted to digital signals with pulse widths proportional to the energy deposited [3]. Each channel is connected to a charge-to-time converter only when it receives a detector pulse, and the pulse is integrated over a set period to maintain high signal-to-noise ratio.

Figure 1.6 Capacitive multiplexing scheme to combine 16 SiPM outputs. Each anode signal of the SiPMs is divided into one or two signals using capacitors, which are then transferred to one of the four output channels (P, Q, R, S). The summed capacitive load at the outputs is reduced because only 7 SiPMs are connected to each output channel (Courtesy: Choe et al. [15]).

Delay-grid multiplexing is a simple time-based multiplexing method to combine multiple SiPM signals into a single channel by giving a unique transit time to each SiPM signal from the SiPM channel to the readout channels [18]. This scheme multiplexes an arbitrary array of SiPMs into 4 readout channels; the unique transit times for the SiPMs encode their position information. The energy and timing information is estimated from the sum of all four multiplexed channels.

1.4 Novel contribution

The work presented in this dissertation demonstrates a new technique (both in theory and its hardware implementation) to multiplex pulse mode detectors by encoding signals from each detector at a different frequency. The frequency-encoded signals from all the detectors are combined into a single channel, and the information about each individual detector signal is recovered from the multiplexed signal. The main results of this work are:

- An algorithm to multiplex/demultiplex signals from pulse-mode radiation detectors via convolution/deconvolution
- A hardware system to perform frequency domain multiplexing of signals via convolution
- Demonstration of multiplexing of two EJ-309 fast organic scintillators when one of the multiplexed detectors produces a signal in any digitized record
- Demonstration of multiplexing when both the multiplexed EJ-309 detectors produce signals in any digitized record
- Development of a pattern to multiplex 64 channels of the single volume scatter camera (SVSC) in the 8x8 configuration via frequency domain multiplexing to achieve a multiplexing ratio of 4:1 [19]
- A hardware system to multiplex 4 SiPM fast outputs (with each SiPM coupled to an EJ-204 cube of 5 mm length) to demonstrate its application to the single volume scatter camera

1.5 Organization of dissertation

In the following chapters we will explain the technique of convolution/deconvolution in detail, discuss the development of the hardware design of a prototype frequency domain multiplexer system, and present the measurement and results of the multiplexer designed to combine signals from pulse mode radiation detectors. We will also demonstrate an application of this technique to multiplex 128 channels of the single volume scatter camera, currently being developed at Sandia National Laboratories, with a multiplexing ratio of 4:1.

CHAPTER

2

FREQUENCY DOMAIN MULTIPLEXING BY CONVOLUTION AND DECONVOLUTION

Frequency domain multiplexing (FDM) has been previously implemented for applications in astronomy using amplitude modulation on radiation detectors such as transition-edge sensors. This technique of amplitude modulation of carrier frequency is not feasible with pulse mode nuclear radiation detectors because the detector signal consists of current impulses. Instead, we propose to use convolution [20] as a new way to implement FDM for pulse mode detector signals (shown in Figure 2.2). The method employs a resonator circuit whose impulse response is a damped sinusoid. The pulses from the radiation detector act as input to this circuit, and the frequency of the resonator output gives a unique tag to each detector. All the detectors to be multiplexed are connected to resonator circuits with different oscillation frequencies respectively. If one of the multiplexed detectors emits a pulse, the resulting damped sinusoid is passed to a single digitizer channel using a fan-in circuit.¹

In this chapter, we describe the method to convert the resonator output back to the original detector pulse that produced it by deconvolution. The pulse recovered through deconvolution can be used to

¹We initially used the digitized sinusoidal output to estimate the charge collected and the time-of-arrival of the original detector pulse; the amplitude and the leading edge of the sinusoid were respectively used to estimate the charge and the timing of the original detector pulse (see Appendix A for details). We found deconvolution produced more precise estimates of charge and arrival time.

estimate the charge collected and the time-of-arrival using standard charge integration and time-pickoff methods. Furthermore, for detectors that exhibit variations in pulse shape, particle identification can be implemented by applying pulse shape discrimination to the recovered pulse. This deconvolution method is intended for single-event multiplexing where the probability of more than one multiplexed detector producing signals in a single record is small. Our multiplexer system has been particularly designed to acquire data for the radiation detection measurements with moderate aggregate count rate ($\lesssim 10^4$ counts per second, see Fig. 2.1).

Figure 2.1 The Poisson probability of more than one count in a single digitizer record length of $4 \mu\text{s}$. This probability is 0.001 for count rates ≈ 12000 cps.

2.1 FDM by convolution/deconvolution

Each detector is connected to a resonator with a unique oscillation frequency. When one of the multiplexed detectors (e.g. detector 1 shown in Fig. 2.2) emits a pulse, the corresponding damped sinusoid produced by its resonator $r1$ is passed into a single digitizer input channel by a fan-in circuit. The digitized output $y(n)$ is the convolution between the impulse response $h_{r1}(n)$ of the resonator $r1$ in series with the fan-in circuit and the detector input $x(n)$ [21]:

$$y(n) = \sum_{m=0}^n x(m)h_{r1}(n-m) \quad n = 0, 1, \dots, N-1 \quad (2.1)$$

where N is the record length of the digitizer (i.e., the number of samples per record). The discrete Fourier transform of $y(n)$ is

$$Y(k) = \sum_{n=0}^{N-1} y(n) \exp\left(\frac{-j2\pi kn}{N}\right) \quad k = 0, 1, \dots, N-1 \quad (2.2)$$

In the frequency domain, the fan-in output $Y(k)$ is equivalent to the product of the impulse response $H_{r1}(k)$ and the detector pulse $X(k)$ given by

$$Y(k) = H_{r1}(k)X(k) \quad k = 0, 1, \dots, N-1 \quad (2.3)$$

The detector pulse $X(k)$ can be recovered from the damped sinusoid $Y(k)$ by deconvolution,

$$X(k) = Y(k)/H_{r1}(k) \quad (2.4)$$

such that the discrete-time detector signal $x(n)$ can finally be recovered by performing inverse Fourier transform on $X(k)$:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \exp\left(\frac{j2\pi kn}{N}\right) \quad (2.5)$$

Figure 2.2 Block diagram of a frequency-domain multiplexer system using convolution/ deconvolution.

2.2 Circuit design

The design described in this section is intended for fast organic scintillators which produce pulses having a width of about 30 ns, with a rise and fall time of about 10 and 20 ns respectively. However, it can be modified for other types of detectors (as discussed in chapter 5).

The resonator circuit, shown in Figure 2.3, is a parallel RLC circuit connected as a negative feedback to an operational amplifier in non-inverting configuration. It is a linear time-invariant (LTI) system described by a linear second-order differential equation that exhibits oscillatory behavior:

$$\frac{1}{\omega_{LC}^2} \frac{d^2 V_{out}}{dt} + \frac{1}{Q\omega_{LC}} \frac{dV_{out}}{dt} + V_{out} = \frac{1}{\omega_{LC}^2} \frac{d^2 V_{in}}{dt} + \left(\frac{1}{Q\omega_{LC}} + \frac{Z_{LC}}{R_2\omega_{LC}} \right) \frac{dV_{in}}{dt} + V_{in} \quad (2.6)$$

where $Q = R_1 \sqrt{\frac{C_1}{L_1}}$, $\omega_{LC} = \frac{1}{\sqrt{L_1 C_1}}$ and $Z_{LC} = \sqrt{\frac{L_1}{C_1}}$. The impulse response of the circuit is an under-damped sinusoid, given by [22]

$$V_{out} \propto \exp\left(\frac{-\omega_{LC} t}{2Q}\right) \sin(\omega_T t + \Phi) \quad (2.7)$$

where $\omega_T = \omega_{LC} \sqrt{1 - \frac{1}{4Q^2}}$ and Φ depends on the initial conditions (i.e., initial voltage across the capacitor). The Q factor dictates the decay time of the damped sinusoid and $f_T = \frac{\omega_T}{2\pi}$ is the frequency of oscillation.

Figure 2.3 The resonator circuit.

The amplitude of the sinusoid decays to approximately 4% of its original value in Q cycles of oscillation. A large Q results in $f_T \approx \frac{\omega_{LC}}{2\pi} = \frac{1}{2\pi\sqrt{L_1 C_1}}$; so L_1 and C_1 determine the frequency of oscillation.

The value of R_1 determines the Q factor for fixed L_1 and C_1 . The transfer function of the non-inverting amplifier, defined as the ratio of the Fourier transform of the output to the input with zero initial conditions, is [22]

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = 1 + \frac{\frac{Z_{LC}}{R_2}}{j\left(\frac{\omega}{\omega_{LC}} - \frac{\omega_{LC}}{\omega}\right) + \frac{1}{Q}} \quad (2.8)$$

The transfer function has a maximum gain of $\left(1 + \frac{R_1}{R_2}\right)$ with a phase of zero at resonant frequency $f_T \approx \frac{\omega_{LC}}{2\pi}$ due to the non-inverting configuration. The bandwidth $BW = \frac{\omega_{LC}}{2\pi Q}$, is the difference between half power frequencies. The half power occurs at the frequencies for which the amplitude of the output voltage equals $1/\sqrt{2}$ of its maximum.

Four resonator prototypes were designed and fabricated. We chose the decay time of the damped sinusoid $< 1.5 \mu s$ for each resonator, which resulted in $R_1 C_1 < 0.233 \mu s$ so that the sinusoids completely decay to zero within a digitizer record (of length $4 \mu s$). This ensured that the resonator output is completely acquired in a digitized record to be able to perform deconvolution. We chose resonant frequencies high enough (starting from 7 MHz) so that $Q > 10$ for each resonator, which led to the bandwidth $BW \ll 2 MHz$. With resonator bandwidths of less than 2 MHz, we chose a difference of about 2 MHz between adjacent resonant frequencies to be able to distinguish them from one another. We chose the values of C_1 and L_1 to keep the resonant frequencies between 7 and 15 MHz; the value of R_1 was chosen to keep the Q factor between 10 and 20 (Table 2.1). The value of R_2 was 50 ohm for all the circuits.

Each resonator has a pass-through that permits the original signal to be simultaneously digitized. The pass-through allows us to characterize the accuracy of the pulses reconstructed by deconvolution; it is not a necessary feature of the resonator. The pass-through is 50 ohm terminated.

The fan-in circuit is a summing amplifier that combines the incoming sinusoidal waveforms into a single output channel. A voltage adder circuit with a gain of 2 was used in an inverting configuration using a high bandwidth operational amplifier. The circuit shown in Figure 2.4 combines four damped sinusoids into a single channel.

The resonator uses OPA690, a high bandwidth (500 MHz for a gain of 1), low noise ($5.5 \text{ nV}/\sqrt{Hz}$) operational amplifier [23]. The fan-in design uses OPA659 [24], a high bandwidth (650 MHz for a gain of 1) operational amplifier. A double sided printed circuit board was used to make the circuits, with signal traces on the top layer and a full ground plane on the bottom. The short traces along with the bottom ground plane ensured lowest possible impedance in the current return paths to eliminate electromagnetic interference.

Figure 2.4 The fan-in circuit ($\frac{R_5}{R_1} = \frac{R_5}{R_2} = \frac{R_5}{R_3} = \frac{R_5}{R_4} = 2$).

Table 2.1 The values of the components used in the resonators.

Resonator	R_1	L_1	C_1	f_T	Q
<i>MHz</i>	<i>ohm</i>	μH	<i>pF</i>	<i>MHz</i>	<i>cycles</i>
7.0	416	1.0	560	6.7	9.8
9.0	233	0.33	1000	8.8	13.0
11.25	416	0.33	560	11.7	17.1
15.25	1000	0.68	150	15.8	23.0

(a) The resonator layout as sent to fabrication. The orange traces are in the top layer.

(b) The fan-in layout as sent to fabrication. The orange traces are in the top layer.

Figure 2.5 Resonator and fan-in PCB layouts.

2.3 FDM setup

Two 7.6 cm x 7.6 cm EJ-309 liquid organic scintillators, each coupled to an Electron Tube 9821KEB photomultiplier tube (PMT), were connected to two resonators with resonant frequencies of 7.00 MHz and 15.25 MHz, shown in Fig. 2.6. The detector signals acted as inputs to the resonator circuits. The resulting damped sinusoids from the resonator outputs were combined using the fan-in circuit. The fan-in output was connected to a CAEN DT5730B 14-bit 500 megasample/sec (MS/s) digitizer. The digitizer saved a record length of 4 μ s (2000 samples) for each channel when triggered.

Figure 2.6 The FDM system. Two EJ-309 liquid organic scintillators were connected to two resonators, and the resonator outputs were finally combined by the fan-in circuit. For scale, the connectors shown are SMA connectors.

2.4 Measurements and results

2.4.1 Impulse response

When one of the multiplexed detectors emits a pulse, the digitized damped sinusoidal output is deconvolved in the frequency domain using the impulse response of the corresponding resonator in series with the fan-in circuit. The impulse response of each resonator was measured by connecting it to the fan-in circuit and feeding it a 1 MHz, 1 Vpp noise signal from a function generator. We simultaneously digitized the noise input via the resonator pass-through and the corresponding resonator output via the fan-in circuit.

We used the cross-correlation technique to measure the impulse response of each resonator in series with the fan-in circuit. The cross-correlation between the output and the input sequence is given by [21]

$$r_{yx}(m) = \sum_{l=0}^m h(l)r_{xx}(m-l) \quad m = 0, 1, \dots, N-1 \quad (2.9)$$

where r_{yx} is the cross-correlation of the fan-in output with the noise input, r_{xx} is the autocorrelation of the noise input, and h is the impulse response; r_{yx} and r_{xx} are defined as

$$\begin{aligned}
r_{xx}(m) &= \sum_n x(n)x(n+m), \\
r_{yx}(m) &= \sum_n y(n)x(n+m)
\end{aligned}
\tag{2.10}$$

The autocorrelation of an ideal white noise signal is a delta function (which is flat in the frequency domain) and the corresponding cross-correlation output provides the impulse response; the noise signal produced by the signal generator was pink; the autocorrelation function $S_{xx}(k)$ (where S_{xx} is the Fourier transform of r_{xx}) is shown in Fig. 2.7. The corresponding relationship in the frequency domain is $S_{yx}(k) = H(k)S_{xx}(k)$, such that the impulse response can be estimated from

$$H(k) = \frac{S_{yx}(k)}{S_{xx}(k)} \quad k = 0, 1, \dots, N-1 \tag{2.11}$$

where $H(k)$, $S_{yx}(k)$ and $S_{xx}(k)$ are the discrete Fourier transforms of h , r_{yx} and r_{xx} in the discrete-time domain. With $N = 2,000$, the range $0 \leq k \leq \frac{N}{2} - 1$ corresponds to a frequency range between 0 and 250 MHz for the sampling rate of 500 MS/s.

Figure 2.7 The autocorrelation of noise signal in the frequency domain.

We first acquired 10,000 records (each with 2,000 samples) of noise input and the corresponding fan-in output. We then computed the cross-correlation and the autocorrelation sequences for each of the 10,000 records. Subsequently, the average cross-correlation (r_{yx}) and the average autocorrelation (r_{xx}) were computed by averaging the individual cross-correlation and the autocorrelation overall records. The impulse response was finally computed by dividing the Fourier transform of the average cross-correlation (S_{yx}) by the Fourier transform of the average autocorrelation (S_{xx}). The impulse response of the 7 MHz and 15.25 MHz resonator in series with the fan-in circuit is shown in Fig. 2.8.

(a) The impulse response of the 7.00 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 7 MHz.

(b) The impulse response of the 15.25 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 15.25 MHz.

Figure 2.8 Resonator/fan-in impulse response.

2.4.2 Deconvolution

Deconvolution can be performed by dividing the discrete Fourier transform of the output sequence $Y(k)$ by the impulse response $H(k)$ of the corresponding resonator in series with the fan-in circuit,

$$X(k) = \frac{Y(k)}{H(k)} \quad k = 0, 1, \dots, N-1 \quad \text{for} \quad N = 2000 \quad (2.12)$$

Ideally $Y(k) = H(k)X(k)$, but in reality the additive noise of the system $N(k)$ is summed with the fan-in output which results in $Y(k) = H(k)X(k) + N(k)$. Fig. 2.8 shows that the amplitude of the impulse response above 200 MHz is close to zero because the fan-in output signal becomes comparable to the noise of the system. Therefore, after deconvolution, $X(k) = Y(k)/H(k) + N(k)/H(k)$ which results in the amplification of noise term $N(k)/H(k)$ in the recovered signal $X(k)$ at these high frequencies [25]. This noise is filtered from the recovered signal $X(k)$ by applying an optimized fourth order Butterworth low-pass filter with a cutoff frequency of 160 MHz [26].

2.4.3 Signal reconstruction

Deconvolution was performed on the damped sinusoids when one of the multiplexed detectors fired in a particular record of digitized data. The anode pulses induced by a Cs-137 source were simultaneously digitized via the resonator pass-through and the damped sinusoids via the fan-in output. Fig. 2.9 compares the original anode pulses from each of the EJ-309 detectors to the corresponding recovered pulses deconvolved from the damped sinusoids produced by each of the resonators. This comparison is shown for a low amplitude and a high amplitude pulse. The residual shown for each pulse exhibits a ringing corresponding to the resonant frequency in the additive noise which was not removed by deconvolution. Note the amplitude of the residuals do not change with pulse amplitude.

We also used a CeBr₃ inorganic scintillator connected to the circuits². Fig. 2.10 compares the anode pulses induced by a Cs-137 source to the corresponding recovered pulses.

²The CeBr₃ inorganic scintillator was chosen because its pulse shape is comparable to an organic scintillator so that we did not have to redesign the FDM circuits. We estimated the width of the photopeak at 662 keV using the original and recovered pulses.

(a) The comparison between the anode and the recovered pulses for the detector 1 connected to the 7.00 MHz resonator. The difference between the anode and the recovered pulses is shown on the right.

(b) The comparison between the anode and the recovered pulses for the detector 2 connected to the 15.25 MHz resonator. The difference between the anode and the recovered pulses is shown on the right.

Figure 2.9 Anode signal recovery.

Figure 2.10 Anode pulse recovery using a $CeBr_3$ detector connected to the 7.00 MHz resonator in series with the fan-in circuit. The difference between the anode and the recovered pulses is shown on the right.

2.4.4 Charge estimation using the recovered pulse

The charge collected (i.e., the area under the anode pulse) from the EJ-309 detector can be estimated from the corresponding recovered pulse. Each resonator was connected to the fan-in circuit separately, and anode pulses were induced using a Cs-137 source. The charge collected under the anode pulse acquired via the resonator pass-through was compared to the charge collected under the recovered pulse deconvolved from the corresponding damped sinusoid. Fig. 2.11a shows the scatter plot between the charge under the anode pulse and the charge under the recovered pulse. Fig. 2.11b shows the events of the scatter plot as the histogram of the difference between the anode charge and the recovered charge. The distribution exhibits 3.3 ± 0.03 keVee standard deviation in the estimate of the anode charge from the recovered charge.

(a) Scatter plot between the anode charge from EJ-309 plotted against the recovered charge using the 7.00 MHz resonator in series with the fan-in circuit.

(b) The events of the scatter plot in Fig. 2.11a shown as the histogram of the difference between the anode charge and the recovered charge with a standard deviation of 3.3 ± 0.03 keVee.

Figure 2.11 Estimation of the charge collected under the anode pulse from the recovered pulse using EJ-309.

We also used a CeBr_3 detector connected to the 7.00 MHz resonator in series with the fan-in circuit to accumulate the pulse height spectrum using a Cs-137 source. The anode pulses were again digitized via the resonator pass-through and the corresponding damped sinusoids were digitized via the fan-in output. Fig. 2.12a shows the scatter plot between the anode charge and the recovered charge, which is also shown as the histogram of the difference between the anode charge and the recovered charge in Fig. 2.12b. The standard deviation in the recovered charge is 3.2 ± 0.3 keV. Fig. 2.13 compares the pulse height spectrum using the anode pulses from the CeBr_3 detector to the spectrum using the deconvolved pulses from the corresponding damped sinusoids. The standard deviation of the photopeak at 662 keV for the spectrum using the anode pulses (Fig. A.8a) is 13.5 ± 0.4 keV while the standard deviation using the recovered pulses (Fig. A.8b) is 13.8 ± 0.4 keV. Note the area under the 662 keV photopeak is identical in both spectra. In both tests, a slight bias in the mean is due to the ringing corresponding to the resonant frequency in the additive noise; the bias tends to increase with increasing pulse amplitude.

(a) Scatter plot between the anode charge from CeBr_3 plotted against the recovered charge using the 7.00 MHz resonator in series with the fan-in circuit.

(b) The events of the scatter plot in Fig. 2.12a shown as the histogram of the difference between the anode charge and the recovered charge with a standard deviation of 3.2 ± 0.3 keV.

Figure 2.12 Estimation of the charge collected under the anode pulse from the recovered pulse using CeBr_3 .

(a) Pulse height spectrum measured from the anode pulses. The standard deviation at 662 keV is 13.5 ± 0.4 keV.

(b) Pulse height spectrum measured from the recovered pulses. The standard deviation at 662 keV is 13.8 ± 0.4 keV.

Figure 2.13 Comparison of γ -ray spectra from CeBr₃ measuring Cs-137 using the anode pulses and the recovered pulses respectively.

2.4.5 Time pick-off using the recovered pulse

Constant fraction discrimination (CFD) [27] was performed on the anode pulse and the recovered pulse respectively. The CFD time pick-off used an attenuation fraction of 0.2 with a 6.4 ns delay. Fig. 2.14a shows the scatter plot of the time-pickoff between the anode pulse and the recovered pulse. The same plot is also shown as the histogram of the difference between the anode timing and the recovered timing

in Fig. 2.14b. The uncertainty in the estimate of the anode timing from the recovered pulse is $88 \pm 2 \text{ ps}^3$; a slight bias in the mean is due to the ringing corresponding to the resonant frequency in the additive noise. The error in CFD timing remains the same as a function of pulse amplitude (see Table 2.2).

(a) Scatter plot between time pick-off of the anode pulse from EJ-309 plotted against the time pick-off of the recovered pulse using the 7.00 MHz resonator in series with the fan-in circuit.

(b) The events of the scatter plot in Fig. 2.14a shown as the histogram of the difference between the anode timing and the recovered timing with a standard deviation of $88 \pm 2 \text{ ps}$.

Figure 2.14 Estimation of the time pick-off of the anode pulse from the recovered pulse using EJ-309.

³The uncertainty include the additive noise by FDM and the timing uncertainty inherent to the digitizer. The inherent timing uncertainty of DT5730 digitizer was not estimated separately.

Table 2.2 The error in CFD timing for different energy ranges.

Energy range	Error in timing
<i>keVee</i>	<i>ps</i>
80-120	-37 ± 6
120-160	-42 ± 4
160-200	-46 ± 4
200-300	-45 ± 4
300-400	-49 ± 2
400-600	-50 ± 2

We also performed coincidence measurements using two EJ-309 detectors to measure the uncertainty in the time pick-off of the anode pulses and the recovered pulses. We placed an Na-22 source between the two detectors, which were connected to the 7.00 MHz and 15.25 MHz resonators respectively. Each resonator was connected to a separate fan-in circuit to digitize sinusoids from coincident events. The anode pulses from the detectors and the corresponding damped sinusoids from the fan-ins were digitized simultaneously. We then calculated the time-of-arrival of the coincident pulses using CFD. Fig. 2.15 shows the histogram of the difference in the time-of-arrival of the coincident events using the anode pulse (standard deviation = 603 ± 9 ps) and using the recovered pulse (standard deviation = 617 ± 13 ps). The increased timing uncertainty in Fig. 2.15b is the result of the additional uncertainty (see Fig. 2.14) introduced when the recovered pulses are used to calculate the time-of-arrival. If the inherent uncertainty in the timing t_1, t_2 of the original coincident pulses (from Fig. 2.15a) is $\sigma_t = \sigma_{t_1} = \sigma_{t_2} = \sigma_{t_1-t_2}/\sqrt{2}$, and the errors introduced to the timing of the recovered coincident pulses t_{r1}, t_{r2} by the amplified noise are e_1 and e_2 respectively, then

$$\begin{aligned}
 t_{r1} &= t_1 + e_1, & t_{r2} &= t_2 + e_2 \\
 t_{r1} - t_{r2} &= t_1 - t_2 + e_1 - e_2 \\
 \sigma_{t_{r1}-t_{r2}}^2 &= 2\sigma_t^2 + \sigma_{e_1}^2 + \sigma_{e_2}^2 - 2cov(e_1, e_2)
 \end{aligned} \tag{2.13}$$

If $\sigma_{e1}^2 = \sigma_{e2}^2 = \sigma_e^2$,

$$\begin{aligned}
 \sigma_{t_{r1}-t_{r2}}^2 &= 2\sigma_t^2 + 2\sigma_e^2 - 2cov(e_1, e_2) \\
 \sigma_{t_{r1}-t_{r2}} &= \sqrt{2(\sigma_t^2 + \sigma_e^2 - cov(e_1, e_2))}
 \end{aligned} \tag{2.14}$$

Because the correlation coefficient between e_1 and e_2 is less than 2%, the uncertainty estimates of t_{r1}, t_{r2}

were assumed to be independent:

$$\sigma_{t_{r1}-t_{r2}} = \sqrt{2(\sigma_t^2 + \sigma_e^2)} \quad (2.15)$$

(a) The histogram of the difference in the time of arrival of the coincident gamma rays using anode pulses (standard deviation = 603 ± 9 ps).

(b) The histogram of the difference in the time of arrival of the coincident gamma rays using recovered pulses (standard deviation = 617 ± 13 ps).

Figure 2.15 Coincidence measurements with Na-22.

2.4.6 Pulse-shape discrimination (PSD) using the recovered pulse

We used a Cf-252 source to induce pulses in an EJ-309 detector connected to the 7.00 MHz resonator in series with the fan-in circuit. The charge-integration method was used to discriminate the neutron pulses from the gamma pulses.

The discrimination parameter (Q_S/Q_L) was chosen as the ratio of the area under the anode pulse excluding its tail (Q_S , "S" for "short"), to the total area (Q_L , "L" for "long") under the pulse (Fig. 2.16). The start time of the pulse was fixed at six samples to the left of the pulse peak. The pulse length was kept constant at 120 samples. The stop time of the integration length for Q_S was optimized by varying it from 0 to 20 samples and selecting the integration length that maximized the figure of merit (FOM) [28]. The figure of merit (FOM) was calculated using the ratio of the difference between the means of the two peaks to the sum of the full-width at half-maxima (FWHM) of the two peaks.

$$FOM = \frac{\mu_{\text{gamma}} - \mu_{\text{neutron}}}{FWHM_{\text{gamma}} + FWHM_{\text{neutron}}} \quad (2.16)$$

where μ denotes centroid and $FWHM$ denotes full-width at half-maximum. Fig. 2.17 shows the 2D-histogram of the discrimination parameter Q_S/Q_L against the total charge Q_L of the pulse for the anode and the recovered pulses respectively. The gamma band at the top and the neutron band at the bottom for the recovered pulses (Fig. 2.17b) show equally good separation compared to the anode pulses (Fig. 2.17a) for a lower threshold of 80 keVee. Fig. 2.18 shows the distribution of the discrimination parameter when the 2D-histograms shown in Fig. 2.17 were summed onto the y-axis. The peak with lower discrimination parameter corresponds to the neutrons and the peak with the higher parameter corresponds to the gamma-rays.

Figure 2.16 The normalised anode pulse. The areas Q_S and Q_L are marked.

We obtained an FOM of 1.20 using the original anode pulses; it decreased slightly to 1.08 when recovered pulses were used; that was most likely due to amplified noise on the baseline of the recovered pulses. The amplified noise at high frequencies makes it more difficult to precisely calculate the area under the tail of the recovered pulse to determine Q_L . This reduces the sensitivity of the discrimination parameter to the shape of the pulse, which increases the FWHM, and consequently reduces the FOM.

(a) Distribution of the discrimination parameter against the total charge using the anode pulses.

(b) Distribution of the discrimination parameter against the total charge using the recovered pulses.

Figure 2.17 Pulse shape discrimination.

(a) Distribution of the discrimination parameter when anode pulses were used (FOM = 1.2).

(b) Distribution of the discrimination parameter when recovered pulses were used (FOM = 1.08).

Figure 2.18 Comparison of the figure of merit.

2.4.7 Noise analysis of the recovered signal

The residuals (shown in Fig. 2.9) for the EJ-309 pulse incident on the 7 MHz resonator remain the same as a function of the pulse height, resulting in a decrease in the relative root-mean-square error (RMSE) with an increase in the pulse height (shown in Fig. 2.19).

Figure 2.19 The relative RMSE as a function of pulse height for the EJ-309 pulse incident on the 7 MHz resonator.

2.5 Comparison between multiplexing by convolution/deconvolution and modulation/demodulation

As described in this chapter, multiplexing of the detector signals is performed by the convolution between the impulse response of a resonator with input signal in the time domain, which is equivalent to multiplication between them in the frequency domain. Multiplexing by modulation (as explained in chapter 1) is performed by the multiplication between a carrier current and detector signal in the time domain, which is equivalent to convolution between them in frequency.

The deconvolution can theoretically recover the original detector signals from the multiplexed output if the signals do not overlap in time; conversely, demodulation can theoretically recover the detector signals from the multiplexed output if the signals do not overlap in frequency.

2.6 Summary

We demonstrated frequency domain multiplexing of two EJ-309 liquid organic scintillators by convolution and deconvolution. The charge collected under the anode pulse can be estimated from the pulse recovered by deconvolution with an uncertainty of about 3.3 keVee. The time of arrival of the anode pulse can be estimated from the recovered pulse with an uncertainty of about 88 ps in addition to the inherent timing uncertainty associated with the original anode pulse. A CeBr₃ inorganic scintillator was also connected to the multiplexer to measure the Cs-137 gamma spectrum; a resolution of 4.89 ± 0.14 % was observed at 662 keV using the deconvolved pulses compared to a resolution of 4.80 ± 0.13 % when the original pulses were used. Coincidence measurements with Na-22 resulted in a timing uncertainty of 603 ± 9 ps using the original detector pulses that increased to 617 ± 13 ps using the recovered pulses.

Pulse shape discrimination performed on the recovered pulses showed a small decrease in the FOM. A FOM of 1.08 was observed when the charge integration method was applied on the recovered pulses compared to 1.2 when the original pulses were used.

CHAPTER

3

RECOVERY OF COINCIDENT FREQUENCY DOMAIN MULTIPLEXED DETECTOR PULSES

Previously, we discussed pulse recovery when one of the multiplexed detectors emits a pulse in a single digitized record. Our multiplexer was designed to acquire data for the radiation detection experiments with $\lesssim 10^4$ counts per second (cps) where the Poisson probability of occurrence of two or more events producing signals in a single digitizer record (of length $4\ \mu\text{s}$ in our case) is less than one in a thousand.

In this chapter, a new variant of the deconvolution method is demonstrated (using the same multiplexer design discussed in chapter 2) to enable the recovery of multiple signals when multiple detectors produce signals in the same digitized record. The pulses from two different detectors are recovered from the same digitizer record, and the recovered pulses are used to estimate the charge collected, timing and particle identification of the original pulses. In particular, the first pulse can be recovered with error comparable to that shown in the previous chapter, when there is only a single pulse.

3.1 FDM by convolution and deconvolution under multiple occupancy

In this section, we discuss the pulse recovery when two detectors produce signals in the same digitized record, which we refer to as multiple occupancy, for the case when the signals do not overlap in time.

The setup is shown as a block diagram in Fig. 3.2. A single 7.6 cm x 7.6 cm EJ-309 detector, coupled to an Electron Tube 9821KEB PMT, was used to produce signals using a Cs-137 source. We constructed a ‘signal copier’ circuit to generate two copies of each detector pulse. This circuit splits the detector signal to generate two copies of the pulse using two high-bandwidth operational amplifiers in a non-inverting configuration with a gain of 2 (Fig. 3.1).

Figure 3.1 The signal copier circuit ($R_1 = R_2 = R_3 = R_4$).

Each output signal is a copy of the input signal with the same shape and amplitude. The first copy $x_1(n)$ of the detector pulse was input to a 7 MHz resonator. The second copy $x_2(n)$ was delayed by 220 ns using a passive delay line and then input to a 9 MHz resonator. The resonator outputs were combined using the fan-in circuit, and the combined signal was digitized using a CAEN DT5730B 500 MS/s digitizer. The two pulses digitized using their respective resonator pass-throughs are shown in Fig. 3.3. The second pulse $x_2(n)$ exhibited $\sim 35\%$ attenuation and bandwidth reduction after passing through the passive delay line, which is clearly visible in Fig. 3.3. The digitized fan-in output, which is the sum of two damped sinusoids of frequencies 7 and 9 MHz, is shown in Fig. 3.4a. The corresponding spectrum shown in Fig. 3.4b has two peaks at 7 and 9 MHz respectively, clearly revealing that the signals were produced by the detectors connected to these resonators. The pulse that arrives first can be identified by comparing the phases of the two peaks (i.e., ϕ/f).

Figure 3.2 The double occupancy setup. An EJ-309 detector is connected to the signal copier circuit that produces two copies of a detector pulse. The first copy is input to the 7 MHz resonator, and the second copy (after being delayed by 220 ns) is input to the 9 MHz resonator. The resonator outputs are then combined by the fan-in circuit and the fan-in output goes to a single digitizer input channel.

Figure 3.3 The original pulses $x_1(n)$ and $x_2(n)$ shown together. The second pulse arrives after the first pulse has decayed to zero. The second pulse $x_2(n)$ undergoes $\sim 35\%$ attenuation and bandwidth reduction after passing through the passive delay line.

(a) The fan-in output when two pulses arrive together in the same digitized record. The damped sinusoids from the 7 MHz and 9 MHz resonators are combined together.

(b) The spectrum of the fan-in output shown in Fig. 3.4a. The peaks reveal the resonators that produced the signals, which in turn reveal the detector numbers that produced the signals. The bumps on the right of the 9 MHz peak correspond to the sudden change in the fan-in output with the arrival of the second pulse. In our case, the signal sources were the two copies of a detector pulse produced by the signal copier circuit.

Figure 3.4 The fan-in output in case of double occupancy.

3.1.1 Sequential deconvolution method

The fan-in output in the discrete-time domain in case of double occupancy is given by

$$y(n) = \sum_{m=0}^n x_1(m)h_{r1}(n-m) + \sum_{m=0}^n x_2(m)h_{r2}(n-m) \quad n = 0, 1, \dots, N-1 \quad (3.1)$$

where h_{r1} , h_{r2} are the impulse responses of the 7 MHz and the 9 MHz resonators, $x_1(n)$, $x_2(n)$ are the inputs to the respective resonators, and $N = 2000$ is the record length of the digitizer. The discrete Fourier transform of $y(n)$ is given by

$$Y(k) = \sum_{n=0}^{N-1} y(n) \exp\left(\frac{-j2\pi kn}{N}\right) \quad k = 0, 1, \dots, N-1 \quad (3.2)$$

In the frequency domain, the fan-in output $Y(k)$ is equivalent to the sum of the products of $H_{r1}(k)$ with $X_1(k)$ and $H_{r2}(k)$ with $X_2(k)$:

$$Y(k) = H_{r1}(k)X_1(k) + H_{r2}(k)X_2(k) \quad k = 0, 1, \dots, N-1 \quad (3.3)$$

Both the signals can be recovered when $x_1(n)$ and $x_2(n)$ are separable in time. Since $x_1(n)$ arrives first in time, deconvolution gives

$$Y_1(k) = \frac{Y(k)}{H_{r1}(k)} = X_1(k) + \frac{H_{r2}(k)X_2(k)}{H_{r1}(k)} \quad (3.4)$$

Since $x_1(n)$ and $x_2(n)$ are separable in time, the two terms on the right hand side of Eq. 3.4 are also separable in the time domain. Taking the inverse Fourier transform of Eq. 3.4:

$$\begin{aligned} y_1(n) &= \mathcal{F}^{-1}(X_1(k)) + \mathcal{F}^{-1}\left(\frac{H_{r2}(k)X_2(k)}{H_{r1}(k)}\right) \\ &= x_1(n) + \mathcal{F}^{-1}\left(\frac{H_{r2}(k)X_2(k)}{H_{r1}(k)}\right) \quad n = 0, 1, \dots, N-1 \end{aligned} \quad (3.5)$$

Fig. 3.5a shows the deconvolved signal $y_1(n)$ in the time domain. The pulse $x_1(n)$ lies between sample numbers 300 and 430, and the second term $\mathcal{F}^{-1}\left(\frac{H_{r2}(k)X_2(k)}{H_{r1}(k)}\right)$ appears later as seen in Fig. 3.5. The second term is a function of the pulse $X_2(k)$ in the frequency domain, which appears in $y_1(n)$ when the second pulse $x_2(n)$ arrives. The pulse $x_1(n)$ is recovered from $y_1(n)$ as shown in Fig. 3.5b. Fig. 3.6 shows the comparison between the original and the recovered pulse $x_1(n)$.

(a) The deconvolved signal $y_1(n)$ shows the pulse $x_1(n)$ between sample numbers 300 and 430, followed by the second term $\mathcal{F}^{-1} \left(\frac{H_{r2}(k)X_2(k)}{H_{r1}(k)} \right)$.

(b) The signal $y_1(n)$ shown around the region where the pulse $x_1(n)$ is clearly visible. $x_1(n)$ is recovered from this region between sample numbers 300 and 430. The black dotted line shows sample number where the second pulse $x_2(n)$ arrives. The sinusoidal signal after the black dotted line is the second term $\mathcal{F}^{-1} \left(\frac{H_{r2}(k)X_2(k)}{H_{r1}(k)} \right)$. The original pulse $x_1(n)$ is also shown in orange for reference.

Figure 3.5 The deconvolved signal $y_1(n)$.

Figure 3.6 Comparison of the original pulse to the recovered pulse $x_1(n)$.

The recovered signal $x_1(n)$ is then subtracted from the deconvolved signal in the time domain and the resulting signal is shown in Fig. 3.7. The Fourier transform is a linear operator, so the subtraction of $x_1(n)$ from $y_1(n)$ is equivalent to the subtraction of $X_1(k)$ from $Y_1(k)$. Converting $y_1(n) - x_1(n)$ in Fig. 3.7 to the frequency domain and using Eq. 3.4:

$$Y_1(k) - X_1(k) = \frac{Y(k)}{H_{r1}(k)} - X_1(k) = \frac{H_{r2}(k)X_2(k)}{H_{r1}} \quad (3.6)$$

Figure 3.7 The subtracted deconvolved signal $y_1(n) - x_1(n)$. In the frequency domain, it is equivalent to $Y_1(k) - X_1(k)$ given by Eq. 3.6.

To recover $x_2(n)$, the terms in Eq.3.6 are rearranged as shown below:

$$X_2(k) = \frac{H_{r1}(k)}{H_{r2}(k)} (Y_1(k) - X_1(k)) \quad (3.7)$$

The discrete-time detector signal $x_2(n) = \mathcal{F}^{-1}(X_2(k))$ can finally be recovered by performing the inverse Fourier transform. Fig. 3.8 shows the comparison between the original and the recovered pulse $x_2(n)$.

Figure 3.8 Comparison of the original pulse to the recovered pulse $x_2(n)$.

3.2 Pulse recovery

The corresponding recovered signals for pulses $x_1(n)$ and $x_2(n)$ show slight deviation from their original counterparts. The comparison between the original and the recovered pulses is shown in Fig. 3.9 for a low amplitude and a high amplitude pulse. The residuals shown for both the pulses exhibit ringing corresponding to the resonant frequency in the additive noise which was not removed by deconvolution.

(a) The comparison between the anode and the recovered pulse $x_1(n)$ incident on the 7 MHz resonator. The difference between the anode and the recovered pulses is shown on the right.

(b) The comparison between the anode and the recovered pulse $x_2(n)$ incident on the 9 MHz resonator. The difference between the anode and the recovered pulses is shown on the right.

Figure 3.9 Signal recovery.

3.2.1 Charge estimation using the recovered pulse

The charge collected under the first pulse $x_1(n)$ can be estimated from the corresponding recovered pulse. Fig. 3.10a shows a scatter plot between the charge under the original first pulse and the charge under the recovered pulse. The uncertainty in the estimate of the charge from the first recovered pulse using the histogram in Fig. 3.10b is 3.6 ± 0.05 keVee. Similarly, the uncertainty in the estimate of the charge from the second recovered pulse using the histogram in Fig. 3.11 is 7.1 ± 0.06 keVee. The increase in uncertainty in the estimate of charge of the second pulse is due to $\sim 35\%$ attenuation and bandwidth reduction encountered by the second pulse while passing through the passive delay line; the amplified noise at high frequencies makes it difficult to precisely calculate the area under the tail of the recovered pulse, particularly because the pulse is attenuated and its tail is elongated.

(a) Scatter plot between the charge from the first pulse incident on the 7.00 MHz resonator plotted against the recovered charge.

(b) The events in Fig. 3.10a shown as the distribution of the difference between the charge from the first pulse and the recovered charge with $\sigma = 3.6 \pm 0.05$ keVee. A slight bias in the mean is due to the ringing corresponding to the resonant frequency in the additive noise.

Figure 3.10 Charge estimation (or area) under the first pulse $x_1(n)$ from the recovered pulse.

(a) Scatter plot between the charge from the second pulse incident on the 9.00 MHz resonator plotted against the recovered charge.

(b) The events in Fig. 3.11a shown as the distribution of the difference between the charge from the second pulse and the recovered charge with $\sigma = 7.1 \pm 0.06$ keVee. A slight bias in the mean is due to the ringing corresponding to the resonant frequency in the additive noise.

Figure 3.11 Charge estimation (or area) under the second pulse $x_2(n)$ from the recovered pulse.

3.2.2 Time pick-off using the recovered pulse

Constant fraction discrimination (CFD) was performed on both the pulses and their recovered counterparts [27]. The CFD time pick-off used an attenuation fraction of 0.2 with a 6.4 ns delay for the first pulse. Fig. 3.12a shows the scatter plot of the time pick-off between the first pulse and the recovered pulse. The uncertainty in the estimate of the timing of the first pulse from the recovered pulse using the histogram in Fig. 3.12b is 87 ± 1.3 ps.

We used an attenuation fraction of 0.3 with a 6.8 ns delay for the second pulse to account for its attenuation and bandwidth reduction.¹ The uncertainty in the estimate of the timing of the second pulse from the recovered pulse using the histogram in Fig. 3.13 is 131 ± 2.9 ps.

(a) Scatter plot between time pick-off of the first pulse incident on the 7.00 MHz resonator plotted against the time pick-off of the recovered pulse.

(b) The events in Fig. 3.12a shown as the distribution of the difference between the timing of the first pulse and the recovered timing with $\sigma = 87 \pm 1.3$ ps. A small bias in the mean is due to the ringing corresponding to the resonant frequency in the additive noise.

Figure 3.12 Timing estimation of the first pulse $x_1(n)$ from the recovered pulse.

¹Under normal circumstances, the CFD parameters for both the pulses would be the same; however, we changed the attenuation fraction and the delay for the second pulse to account for the change in the pulse shape after passing through the passive delay line.

(a) Scatter plot between time pick-off of the second pulse incident on the 9.00 MHz resonator plotted against the time pick-off of the recovered pulse.

(b) The events in Fig. 3.13a shown as the distribution of the difference between the timing of the second pulse and the recovered timing with $\sigma = 131 \pm 2.9$ ps. A small bias in the mean is due to the ringing corresponding to the resonant frequency in the additive noise.

Figure 3.13 Timing estimation of the second pulse $x_2(n)$ from the recovered pulse.

3.2.3 Pulse-shape discrimination (PSD) using the recovered pulse

We used a Cf-252 source to induce pulses in an EJ-309 detector using the same setup in Fig. 3.2. The two copies ($x_1(n)$ and $x_2(n)$) of the detector signal generated using the signal copier were recovered from the combined signal using the sequential deconvolution method.

The charge-integration method was used to discriminate neutron pulses from gamma pulses. The discrimination parameter was defined as the ratio of the area under the pulse excluding its tail (Q_S), to the total area (Q_L) under the pulse. We plotted a 2D-histogram of the discrimination parameter $\frac{Q_S}{Q_L}$ against the total charge (area) Q_L under the pulse for the original and the recovered pulses for a threshold of 80 keVee (Fig. 3.14, Fig. 3.16). For the first pulse $x_1(n)$, the pulse length was kept constant at 102 samples. The start time of the integration length for Q_S was fixed at six samples to the left of the pulse peak, and the stop time was optimized by selecting the integration length that maximized the figure of merit $FOM = (\mu_{\text{gamma}} - \mu_{\text{neutron}})/(FWHM_{\text{gamma}} + FWHM_{\text{neutron}})$, where μ denotes the mean and $FWHM$ denotes the full width at half-maximum [28]. Using the distribution of the discrimination parameter shown in Fig. 3.15, an FOM of 1.16 was obtained using the original first pulse incident on the 7 MHz resonator compared to 1.04 when its recovered counterpart was used. For the second pulse $x_2(n)$, the pulse length was kept constant at 162 samples. The start time of the integration length for Q_S was fixed at eight samples to the left of the pulse peak, and the stop time was optimized to maximize the FOM. An FOM of 1.2 was obtained using the original second pulse incident on the 9 MHz resonator compared to 1.09 when its recovered counterpart was used (Fig. 3.17). The reduction in FOM for the recovered pulses is due to the ringing corresponding to the resonant frequency in the additive noise (see Fig. 3.9). The error in the discrimination parameter $\frac{Q_S}{Q_L}$ tends to increase with decreasing pulse amplitude.

(a) Distribution of the discrimination parameter against the total charge Q_L using the first pulse.

(b) Distribution of the discrimination parameter against the total charge Q_L using the recovered pulse.

Figure 3.14 Pulse shape discrimination for the first pulse.

(a) Distribution of the discrimination parameter for the first pulse (FOM = 1.16).

(b) Distribution of the discrimination parameter for the recovered pulse (FOM = 1.04).

Figure 3.15 Comparison of the figure of merit for the first pulse incident on the 7 MHz resonator.

(a) Distribution of the discrimination parameter against the total charge Q_L using the second pulse.

(b) Distribution of the discrimination parameter against the total charge Q_L using the recovered pulse.

Figure 3.16 Pulse shape discrimination for the second pulse.

(a) Distribution of the discrimination parameter for the second pulse. (FOM = 1.20).

(b) Distribution of the discrimination parameter for the recovered pulse (FOM = 1.09). A reduced FOM for the recovered second pulse is attributed to the ringing corresponding to the resonant frequency in the additive noise, which becomes significant due to the attenuation and bandwidth reduction encountered by $x_2(n)$.

Figure 3.17 Comparison of the figure of merit for the second pulse incident on the 9 MHz resonator.

3.3 Overlapping pulses

We used the same setup shown in Fig. 2 to generate two copies of a EJ-309 pulse with the signal copier circuit. The delay in the second copy $x_2(n)$ was chosen to be 30 ns so that it overlapped with the tail of the first pulse $x_1(n)$ as shown in Fig. 3.18. The fan-in output in case the pulses overlap is shown in Fig. 3.19.

Figure 3.18 The original pulses $x_1(n)$ and $x_2(n)$ shown together. The second pulse arrives 30 ns after the first pulse.

(a) The fan-in output when two pulses that overlap in time arrive together in the same digitized record. The damped sinusoids from the 7 MHz and the 9 MHz resonators are combined together.

(b) The spectrum of the fan-in output shown in Fig. 3.4a. The peaks reveal the resonators that produced the signals, which in turn reveal the detector numbers that produced the signals.

Figure 3.19 The fan-in output in case the pulses overlap.

Fig. 3.20a shows the deconvolved signal $y_1(n)$ in the time domain. The pulse $x_1(n)$ lying between sample numbers 300 and 430 overlaps the second term in Eq. 3.5 that arrives on the top of the tail of $x_1(n)$ as shown in Fig. 3.20. Therefore, $x_1(n)$ is accurately recoverable only until the arrival of the second pulse. The partial recovery of the pulse $x_1(n)$ from $y_1(n)$ is shown in Fig. 3.20b. Fig. 3.21 shows the comparison between the original pulse $x_1(n)$ and the partially recovered pulse $\tilde{x}_1(n)$.

(a) The deconvolved signal $y_1(n)$ shows the pulse $x_1(n)$ starting near sample number 320 with the second term $\mathcal{F}^{-1}\left(\frac{H_{r2}(k)X_2(k)}{H_{r1}(k)}\right)$ overlapping its tail.

(b) The signal $y_1(n)$ shown around the region where the pulse $x_1(n)$ is clearly visible. $x_1(n)$ is recovered from this region from when it arrives near sample number 320 until the arrival of the second pulse 30 ns later. The black dotted line shows sample number from where the second pulse $x_2(n)$ arrives. The sinusoidal signal after the black dotted line is the second term $\mathcal{F}^{-1}\left(\frac{H_{r2}(k)X_2(k)}{H_{r1}(k)}\right)$ contaminated by the additive tail of the first pulse. The original pulse $x_1(n)$ is also shown in orange for reference.

Figure 3.20 The deconvolved signal $y_1(n)$.

Figure 3.21 The comparison between the original pulse $x_1(n)$ and the partially recovered pulse $\tilde{x}_1(n)$.

The partially recovered signal $\tilde{x}_1(n)$ is then subtracted from the deconvolved signal in the time domain and the resulting signal $y_1(n) - \tilde{x}_1(n)$ is shown in Fig. 3.22. Converting $y_1(n) - \tilde{x}_1(n)$ in Fig. 3.22 to the frequency domain and using Eq. 3.4:

$$Y_1(k) - \tilde{X}_1(k) = \frac{Y(k)}{H_{r1}(k)} - \tilde{X}_1(k) = \frac{H_{r2}(k)X_2(k)}{H_{r1}(k)} + \mathcal{F}(\text{tail of } x_1(n)) \quad (3.8)$$

Figure 3.22 The subtracted deconvolved signal $y_1(n) - \tilde{x}_1(n)$. In the frequency domain, it is equivalent to $Y_1(k) - \tilde{X}_1(k)$ given by Eq. 3.8.

In this case, when Eq. 3.8 is rearranged as shown below, $x_2(n)$ cannot be accurately recovered due to the error introduced by the addition of the tail of the first pulse to the second term $\frac{H_{r2}(k)X_2(k)}{H_{r1}(k)}$:

$$\tilde{X}_2(k) = X_2(k) + \frac{H_{r1}(k)}{H_{r2}(k)} \mathcal{F}(\text{tail of } x_1(n)) = \frac{H_{r1}(k)}{H_{r2}(k)} (Y_1(k) - \tilde{X}_1(k)) \quad (3.9)$$

The error term $\left(\frac{H_{r1}(k)}{H_{r2}(k)} \mathcal{F}(\text{tail of } x_1(n)) \right)$ increases when $x_1(n)$ and $x_2(n)$ are closer together in time and vanishes when the two pulses cease to overlap in time. The discrete-time detector signal $\tilde{x}_2(n) = x_2(n) + \text{error term} = \mathcal{F}^{-1}(\tilde{X}_2(k))$ can be recovered by performing the inverse Fourier transform. Fig. 3.23 shows the recovered pulse $\tilde{x}_2(n)$ does not accurately follow the original pulse $x_2(n)$, especially on the rising edge.

Figure 3.23 The comparison between the original pulse $x_2(n)$ and the recovered pulse $\tilde{x}_2(n)$.

The energy and timing of the original pulses $x_1(n)$ and $x_2(n)$ were again estimated using their recovered counterparts $\tilde{x}_1(n)$ and $\tilde{x}_2(n)$; however, the charge collected under $x_1(n)$ could not be estimated from the partially recovered $\tilde{x}_1(n)$. The charge collected under $x_2(n)$ was estimated from the inaccurately recovered pulse $\tilde{x}_2(n)$ with an uncertainty of 9.5 ± 0.1 keVee shown in Fig. 3.24.

(a) Scatter plot between the charge from the second pulse incident on the 9.00 MHz resonator plotted against the recovered charge.

(b) The events in Fig. 3.24a shown as the distribution of the difference between the charge from the second pulse and the recovered charge with $\sigma = 9.5 \pm 0.1$ keVee. The error term $\left(\frac{H_{r1}(k)}{H_{r2}(k)} \mathcal{P}(\text{tail of } x_1(n)) \right)$ introduces a bias in the distribution with the mean at -20 keVee.

Figure 3.24 Charge estimation under the second pulse $x_2(n)$ from the recovered pulse $\tilde{x}_2(n)$.

Fig. 3.25a shows the scatter plot of the time pick-off between the first pulse $x_1(n)$ and the recovered pulse. The uncertainty in the estimate of the timing of the first pulse from the recovered pulse using the histogram in Fig. 3.25b is 89 ± 2.1 ps. The timing of the second pulse $x_2(n)$ was estimated using the recovered pulse with an uncertainty of 133 ± 1.9 ps as shown in Fig. 3.26.

(a) Scatter plot between time pick-off of the first pulse incident on the 7.00 MHz resonator plotted against the time pick-off of the recovered pulse.

(b) The events in Fig. 3.25a shown as the distribution of the difference between the timing of the first pulse and the recovered timing with $\sigma = 89 \pm 2.1$ ps. The timing was computed by applying CFD with an attenuation fraction of 0.2 and a delay of 6.4 ns. A small bias in the mean is due to the ringing corresponding to the resonant frequency in the additive noise.

Figure 3.25 Timing estimation of the first pulse $x_1(n)$ from the recovered pulse $\tilde{x}_1(n)$.

(a) Scatter plot between time pick-off of the second pulse incident on the 9.00 MHz resonator plotted against the time pick-off of the recovered pulse.

(b) The events in Fig. 3.26a shown as the histogram of the difference between the timing of the second pulse and the recovered timing with $\sigma = 133 \pm 1.9$ ps. The error term $\left(\frac{H_{r1}(k)}{H_{r2}(k)} \mathcal{F}(\text{tail of } x_1(n)) \right)$ introduces a positive bias in the distribution with the mean at 177 ps. The timing was computed by applying CFD with an attenuation fraction of 0.3 and a delay of 5.6 ns to account for the attenuation and bandwidth reduction of the pulse due to the passive delay line.

Figure 3.26 Timing estimation of the second pulse $x_2(n)$ from the recovered pulse $\tilde{x}_2(n)$.

3.4 Noise analysis of the recovered signal

The residuals (shown in Fig. 3.9a) for the pulse $x_1(n)$ incident on the 7 MHz resonator remain the same as a function of the pulse height, resulting in a decrease in the relative root-mean-square error (RMSE) with an increase in the pulse height (shown in Fig. 3.27a). The residuals (shown in Fig. 3.9b) for the pulse $x_2(n)$ incident on the 9 MHz resonator increase as a function of the pulse height, resulting in a nearly flat relative RMSE as a function of the pulse height (shown in Fig. 3.27b); the RMSE increases with the pulse height because as the pulse height increases, the decay time of the pulse (to the baseline) also increases resulting in a slight overlap between the two pulses $x_1(n)$ and $x_2(n)$ in time. The overlap increases with pulse height, which also increases the error term (explained in section 3.3).

(a) The relative RMSE as a function of pulse height for the first pulse incident on the 7 MHz resonator.

(b) The relative RMSE as a function of pulse height for the second pulse incident on the 9 MHz resonator.

Figure 3.27 The relative RMSE as a function of pulse height.

3.5 Conclusions

We demonstrated frequency domain multiplexing of two EJ-309 organic scintillator detectors by convolution and deconvolution when both the detectors produce signals in the same digitizer record. The anode charge and timing of the first pulse were estimated from its recovered counterpart with an uncertainty of about 3.6 keVee and, 87 ps respectively. The anode charge and timing of the second pulse were estimated from the corresponding recovered pulse with an uncertainty of about 7.1 keVee and 131 ps respectively². Pulse shape discrimination using charge integration performed on both the recovered pulses showed a small decrease in the FOM.

²The uncertainties depend on the experimental setup used in the measurement: two copies of a EJ-309 pulse from the signal copier acted as inputs to two resonators, where one of the copies was subjected to $\sim 35\%$ attenuation and bandwidth reduction after passing through a passive delay line to delay it by 220 ns.

The reduction in precision in the charge, timing and FOM for the second pulse is attributed to: (1) the attenuation and bandwidth reduction undergone by the second pulse due to the passive delay line; (2) increase in the relative RMSE between the original and the recovered second pulse as a function of the pulse height.

When the two detector pulses overlap in time, the first pulse can be partially recovered up to the time when the second pulse arrives. The second pulse cannot be recovered accurately; the inaccuracy depends on the error introduced by the portion of the first pulse overlapping the second pulse.

CHAPTER

4

APPLICATION OF FDM TO THE SINGLE VOLUME SCATTER CAMERA

Sandia National Laboratories (SNL) is developing a prototype Single Volume Scatter Camera (SVSC) to estimate the direction of the incoming neutrons over a 4π space to localize and image neutron sources. The camera works on the principle of scatter kinematics between a neutron and a hydrogen-1 nucleus in an organic scintillator [19]. When a neutron scatters twice with hydrogen-1 nuclei in an organic scintillator, a cone of possible neutron directions can be found by estimating the: (1) location of both scatters; (2) the time of both scatters; and (3) the energy deposited in the first scatter. The opening angle of the cone whose axis lies on the line connecting the two scatter locations is given by:

$$\theta = \tan^{-1} \sqrt{\frac{E_p}{E'_n}} \quad (4.1)$$

where E_p is the energy deposited in the first scatter (i.e., the recoil proton energy) and E'_n is the energy of the neutron after the first scatter. In one SVSC design, the volume of the organic scintillator is subdivided into optically isolated pillars to constrain the transport of light along the length of the pillar (Fig 4.1). E_p can be estimated using the light output of the photodetectors connected to the pillar that encounters the first scatter. E'_n can be estimated by the time of flight between the two scatters (using the locations

and the times of two scatters)¹. The pillar which produces light due to a neutron scatter gives the (x,y) position of the scatter location; the scatter location along the z-direction (along the length of the pillar) is estimated from the relative amplitude and/or timing of the pulses produced by the photodetectors at each end of the pillar. The prototype design has 64 EJ-204 scintillator pillars of dimension 5 mm x 5 mm x 200 mm in an 8x8 array. As a consequence, the prototype will have 128 photodetector output channels (64 on each side). One of the design challenges for the camera is to be able to resolve scattering events at 2 cm separation and 2 ns apart so that its efficiency is improved by an order of magnitude compared to conventional dual-plane scatter cameras [29].

Figure 4.1 The Single Volume Scatter Camera (SVSC) design.

¹In this case, the counts are correlated (as in multiple scatters), so the aggregate count rate of the camera does not matter because of which the simple Poisson probability of Fig. 2.1 no longer applies.

4.1 Frequency domain multiplexing of SVSC channels

Frequency domain multiplexing of the channels of the SVSC reduces the need to digitize all 128 channels. We plan to divide 64 channels on each side of the camera into 8 to 16 groups of 4 to 8 channels each to multiplex signals of each group into a single channel, thereby achieving a multiplexing ratio of 4:1 or 8:1. Since the deconvolution method for reconstructing multiple detector pulses that arrive during the same digitizer record is not able to resolve coincident pulses that are 2 ns apart, the channels were divided into groups so that the probability of more than one pulse arriving in the multiplexed readout of any group is minimized. Simulated annealing - which has been previously used to optimize functions of combinatorial variables like the traveling salesman problem - was used to identify an optimal multiplexing pattern [30].

To illustrate this approach, we first ran a test case to find the multiplexing pattern for a 3x3 configuration of EJ-204 pillars of dimension 5 mm x 5 mm x 200 mm. A 50 cm spherical surface source of neutrons surrounding a 3 x 3 configuration of EJ-204 pillars was simulated using Geant4 (see Fig. 4.2) [31]. The neutrons were modeled using the Cf-252 spontaneous fission neutron spectrum:

$$N(E) \propto \exp(-0.88E) \sinh(\sqrt{2E}) \quad 0.2 < E(\text{MeV}) < 7 \quad (4.2)$$

where $N(E)$ is the number of neutrons of energy E (in MeV) per unit energy interval. A spherical surface source was chosen to ensure that the optimal multiplexing pattern was agnostic to the direction of the incoming neutrons. The neutron source locations were sampled uniformly on the surface of the sphere, and the neutron directions for each source location were sampled uniformly in 4π . We simulated 10^8 neutron histories, and recorded the hit locations for neutrons that underwent two scatters inside any two of the 64 pillars.

(a) EJ-204 pillars of dimension 5 mm x 5 mm x 200 mm in 3x3 configuration simulated using Geant4. The inset figure shows the cross-section with the pillar numbers.

(b) A spherical surface source of Cf-252 with radius 50 cm was simulated with the camera kept at the center. The neutron source locations were sampled uniformly on the surface of the sphere along with its direction in a uniform 4π space. The green lines are the neutron trajectories which begin at the surface of the sphere.

Figure 4.2 Multiplexing pattern study for EJ-204 pillars in the 3x3 configuration.

We divided the pillars into 5 groups: 4 groups had two pillars each and the last group had a single pillar. The cost function was defined as:

$$f = \frac{1}{M} \sum_{i=1}^M \sum_{j=1}^N \text{number of multiple hits by neutron } n_i \text{ in group } j \quad (4.3)$$

where N = number of groups and M = number of neutron histories. The solution space S is given by:

$$S = \text{all permutations of SVSC pillars divided into } N \text{ groups} \quad (4.4)$$

For 3x3 configuration of pillars with $N = 4$, $|S| = \frac{{}^9C_2 \cdot {}^7C_2 \cdot {}^5C_2 \cdot {}^3C_2}{4!} = 945$. Using $M = 90,000$ neutron histories that underwent two scatters inside any two pillars, we found the optimized pattern of pillars that minimized the occurrence of more than one neutron scatter in each group using simulated annealing [32,33]. Starting from any point in the solution space, a move to a random neighbor is always accepted if it improves the solution (reduces the cost function); otherwise, the neighbor can also be selected with probability drawn from the Boltzmann distribution $\exp(\frac{-\Delta E}{T})$, analogous to the annealing of metal. Similar to the metallic annealing, the term T in the probability distribution is analogous to the temperature, which is initially kept high and gradually decreased according to an annealing schedule. The neighbors are selected iteratively till the global minimum is reached near a temperature close to zero. We used geometric decay cooling schedule $T = T_0 r^t$, which is frequently used annealing schedule for the traveling salesman problem, with the initial temperature $T_0 = 1000$, and $r = 0.972$ (t is the iteration number); r is usually chosen close to 1 for slow cooling. The algorithm is described below:

- Step 1: Randomly generate a starting point S_0 and initial temperature $T_0 = 1000$.
- Step 2: With previous neighbor as S_i , randomly generate new neighbor S_j and compute $\Delta E = f(S_j) - f(S_i)$; neighbor is generated by randomly selecting one pillar each from any two groups, and interchanging them.
- Step 3: Compute new temperature T_i .
- Step 4: If $\Delta E < 0$, go to Step 6, else go to Step 5.
- Step 5: If $\exp(-\Delta E/T_i) > \text{uniform}(0,1)$, then go to Step 6, else go to Step 7.
- Step 6: Accept the new path: $S_i = S_j$.
- Step 7: If $T_i < 10^{-5}$, then STOP, else go to Step 2.

The optimized pattern is shown in shown in Fig. 4.3; each color-coded group has pillars that are physically distant from each other, with the middle pillar left alone. The scatter cross-section of a neutron with an H-1 nucleus increases after the first scatter due to the loss of about half of the neutron's initial energy (on average) during the first scatter. Consequently, the most probable location for the second scatter is near the first one. Therefore, simulated annealing chose pillars that are far from each other for each group, thereby minimizing the frequency of more than one hit of a neutron in the pillars of any group. The solution converged within 150 iterations as shown in Fig. 4.4.

Figure 4.3 The color-coded optimized multiplexing pattern for the pillars in 3x3 configuration. Simulated annealing identified the pillars that were physically away from each other for each group, with the middle pillar left alone.

Figure 4.4 Cost function as a function of iterations. The global minimum is achieved within 150 iterations.

After running the test case for the 3x3 configuration, we then simulated EJ-204 pillars in the 8x8 configuration with a 50 cm radius spherical surface source of fission spectrum neutrons. We divided the pillars into $N = 16$ groups of 4 pillars each, and used $M = 60,000$ neutron histories to get an optimized multiplexing pattern using simulated annealing with the initial temperature $T_0 = 1000$ and $r = 0.997$. The solution space ($|S| = \frac{{}^{64}C_4 \cdot {}^{60}C_4 \cdot {}^{56}C_4 \dots {}^4C_4}{16!} \approx 5^{53}$) is vast, so r was chosen very close to 1 so the solution space was extensively explored to reach the global minimum. An optimal configuration was again identified in which pillars in each group were physically distant from each other to minimize multiple hits encountered by a neutron in the pillars of the same group (Fig. 4.5). The solution converges within 7,000 iterations shown in Fig. 4.6.

Figure 4.5 The color-coded optimized multiplexing pattern for the pillars in 8x8 configuration. Simulated annealing again identified the pillars that were physically distant from each other for each group.

Figure 4.6 Cost function as a function of iterations. The global minimum is achieved within 7000 iterations.

We also calculated the geometric mean of the distance between any two pillars in a group as a function of iterations (Fig. 4.7); the geometric mean increases with the iterations and converges to a maximum confirming that the optimized solution maximizes the distance between the pillars in each group.

Figure 4.7 The geometric mean of the distance between any two pillars of a group as a function of iterations. The geometric mean of the distance between the pillars of a group for the optimized pattern was 24 mm.

4.2 Locating a point Cf-252 source using SVSC

We simulated a point Cf-252 source and placed it 50 cm away from the camera (along the Y-axis) with 64 pillars of dimension 5 mm x 5 mm x 200 mm in the 8x8 configuration using Geant4 shown in Fig 4.8. We accumulated the locations of the two scatters, the time of both scatters, and the energy deposited in the first scatter for the neutron histories that underwent multiple scatters. The x,y locations of the two scatters were assigned an uncertainty $\sqrt{\frac{5^2}{12}} = 1.44$ mm - the standard deviation of a uniform distribution over 5 mm. The uncertainty in the z location and the timing of the two scatters were chosen to be 10 mm and 140 ps respectively as reported for experiments conducted by Sweany et al. [34], where the events with energy deposition between 300–400 keV were included². The uncertainty in the energy deposited in the first scatter was chosen to be 50 keV, as reported by Weinfurther et al. [35]. This information was then used to estimate a cone of incident neutron directions for each neutron history that underwent double scatter. These cones were back-projected to get the image of the neutron source towards the direction where the cones overlapped the most.

Figure 4.8 A Cf-252 point source kept 50 cm away from the camera along the positive Y-axis was simulated using Geant4.

²We assumed the same uncertainties for all interactions irrespective of the energy deposition. In reality, the uncertainties increase as the energy deposition decreases.

We used 20,000 neutron histories that underwent double scatters in the pillars to create the images. Fig. 4.9a shows image of the neutron source along the direction of the positive Y-axis ($\theta = \phi = 90^\circ$) using these 20,000 neutron trajectories. Fig. 4.9b shows the image of the source when the pillars were multiplexed together into 16 groups. The two scatters were accepted when they happened in two different groups, but the multiple scatters in the same group of pillars were rejected. The multiplexing pattern in Fig. 4.5 was able to accept 97.6% of the 20,000 neutron histories originally used.

Figure 4.9 Back-projected images for the non-multiplexed pillars compared to the multiplexed scenario.

We also analyzed the point spread in the images by taking a slice through the maximum bin value along both the polar and the azimuthal directions to create distributions of the azimuthal and the polar

angles at $\theta = 90^\circ$ and $\phi = 90^\circ$ respectively. Fig. 4.10 shows these distribution along the azimuthal and the polar angles. The full-width-at-half-maximum (FWHM) for both the azimuthal and the polar angles when the pillars were multiplexed into 16 groups was found to be slightly larger than the non-multiplexed case. The spread was greater for the multiplexed case because the events associated with two scatters in distant pillars were rejected, which removed the cones pointing in the right direction; these rejected events where two scatters occur far away from each other reduce the uncertainty in the estimate of source direction using the estimates of energy deposited, time and locations of the two scatters [36].

(a) The distributions of the azimuthal angle at $\theta = 90^\circ$ when the pillars were multiplexed into 16 groups (FWHM = 1.14 radians) compared to the non-multiplexed case (FWHM = 1.07 radians).

(b) The distributions of the polar angle at $\phi = 90^\circ$ when the pillars were multiplexed into 16 groups (FWHM = 1.34 radians) compared to the non-multiplexed case (FWHM = 1.33 radians).

Figure 4.10 Distributions of the azimuthal and the polar angles along the maximum bin value of the images.

4.3 Summary

We developed a pattern to multiplex 64 channels of the SVSC via frequency-domain multiplexing. The 64 channels were divided into 16 groups of 4 channels each to multiplex the signals of each group into a single channel, thereby achieving a multiplexing ratio of 4:1. The deconvolution method is unable to reconstruct coincident pulses, so the channels were divided into groups such that the probability of more than one pulse arriving in the multiplexed readout of any group was minimized. Simulated annealing was used to identify an optimal multiplexing pattern that grouped channels that were physically distant.

CHAPTER

5

MULTIPLEXER DESIGN FOR THE SINGLE VOLUME SCATTER CAMERA

In Chapter 4, we identified a multiplexing pattern for the 64 photodetector output signals (on each side) of the single volume scatter camera that minimized the probability of more than one detector signal being combined into a single multiplexer channel; the pattern consisted of 16 groups of 4 channels each.

In this chapter, we describe the design of a new multiplexer that houses four resonators and a fan-in circuit in a single printed circuit board (PCB) to enable 4 to 1 multiplexing. The signals in each group of the multiplexing pattern can then be combined to ensure that a single signal occurs in a digitized record. We will discuss the accuracy and precision in the recovered pulses when four EJ-204 detectors, each coupled to a silicon photomultiplier, are multiplexed into a single channel using the multiplexer.

5.1 Introduction

The prototype Single Volume Scatter Camera (SVSC) being developed at Sandia National Laboratories (SNL) will have 128 photodetector output channels (64 on each side) which require 128 digitizer input channels for data acquisition. The new multiplexer reduces the digitizer input channels to 32, thereby reducing the need for a high-channel-density data acquisition system.

The main design constraint for the camera is to be able to resolve two pulses - that are 2 ns apart - produced by a neutron when it undergoes two hydrogen scatters in any two pillars of the camera.

When two scattering events are 2 ns apart, the signals overlap in time and the deconvolution method will be unable to recover both detector signals. Because combining the channels of each group of the multiplexing pattern maximizes the likelihood of the two scattering events occurring in two pillars of two different groups, the deconvolution method can be applied to recover the signals. The two pulses can then be used to estimate the direction of the incoming neutron.

The prototype scatter camera uses 64 EJ-204 organic scintillator pillars of dimension 200 mm x 5 mm x 5 mm, each coupled to two silicon photomultipliers (SiPMs) on either end; each pillar is wrapped in teflon to prevent light leakage to neighboring pillars and to improve the light collection efficiency at the photodetectors. We will demonstrate multiplexing by combining signals from 4 EJ-204 cubes (5mm x 5mm x 5mm), each coupled to a SIPM using the new multiplexer board.

5.2 Circuit design

The prototype SVSC uses the DRS4 digitizer which acquires signals at high sampling speeds (> 700 megasamples per second (MS/s)) with a short record length (1024 samples) [37]. In order to achieve a multiplexing ratio of 4:1, we fabricated a new multiplexer that includes resonators of four different oscillation frequencies and a fan-in circuit on a single PCB, compatible with DRS4 digitizer sampling at 1 GS/s with 1024 sample record length; the circuit diagrams of a resonator and a fan-in are shown in Figs. 2.3 and 2.4. We chose the decay time of the sinusoid for each resonator to be < 250 ns so that it decays to zero within the record length of the digitizer. We also chose the resonant frequencies to be > 50 MHz ($Q > 10$) to ensure the BW of each resonator is < 5 MHz. With the BW < 5 MHz, we chose the resonant frequencies 5 MHz apart from each other, i.e., 50 MHz, 55 MHz, 60 MHz and 65 MHz, to be able to distinguish between the peaks of two damped sinusoids of adjacent resonant frequencies in the frequency domain. Keeping the value of $L_1 = 100$ nF for each resonator, we chose the values of C_1 to keep the resonant frequencies between 50 and 65 MHz that resulted in $Q > 10$; the value of R_1 was chosen to keep the decay time < 250 ns (Table 5.1). The value of R_2 was chosen to keep the gain at the resonant frequency $(1 + \frac{R_1}{R_2})$ between 4 and 6. Each of the four resonators has a pass-through (same as the original design explained in Chapter 2) that permits the original signal to be simultaneously digitized.

The resonator design uses OPA690 [23], a high bandwidth operational amplifier. The fan-in circuit uses OPA659 [24], a high bandwidth, low noise operational amplifier. A four sided printed circuit board was used to house four resonators and a fan-in circuit together, with the top plane as signal plane, the second one as ground, and the third and fourth as power planes.

Table 5.1 The values of the components used in the resonators.

Resonator	R_1	L_1	C_1	R_2	Q
<i>MHz</i>	<i>ohm</i>	μH	<i>pF</i>	<i>ohm</i>	<i>cycles</i>
50.0	326	100.0	100	109	10.3
55.0	398	100.0	82	99	11.4
60.0	480	100.0	68	120	12.5
65.0	583	100.0	56	117	13.8

5.3 Setup

Four EJ-204 fast plastic scintillator cubes (5 mm on each side) were connected to four SensL J-Series SiPMs (6 x 6 mm² area, manufactured by On Semiconductor) using EJ-550 optical gel. Fig. 5.1 shows four EJ-204 scintillators attached to four SiPM evaluation boards with a Cs-137 source placed in the middle; each board has three SMA connectors for the positive bias voltage, the standard anode output (SOUT), and the fast output signal (FOUT). The SiPMs were biased at 30 V (overvoltage of 6 V) using a standard linear DC power supply. The FOUT signals were used to estimate both timing and energy; the FOUT terminal has a very small series capacitance resulting in a ultra-fast pulse, which gives a very precise timing mark for the signal.

Figure 5.1 Four EJ-204 scintillators attached to four SiPM evaluation boards using optical gel. The bias voltage was 30 V. The fast output signals from four SiPM boards were connected to the multiplexer using 50 ohm cables.

The four FOUT signals were then connected to four resonator inputs; Fig. 5.2 shows the new multiplexer board. Each resonator has two input connectors: one as a signal input and the other as a pass-through to digitize the original input signal simultaneously with the multiplexed output. The input signal was compared to the signal recovered by performing deconvolution on the multiplexed signal.

Figure 5.2 The 4-layered multiplexer board housing four resonators and a fan-in circuit; each resonator has two input connectors acting as a signal input and a pass-through to digitize it. The four input signals (via the pass-through connectors) and the fan-in output were digitized using the DRS4 digitizer.

The four detector signals (via the pass-through connectors) and the multiplexed signal were connected to five input channels of two DRS4 digitizers, sampling at 1 GS/s with a fixed record length of 1024 samples (1024 ns). In a DRS4 digitizer, a domino wave runs continuously in a circular manner through a series of 1024 cells until a trigger signal decouples the write switches; the time interval between adjacent samples is not equidistant because of the mismatch among the transistors (acting as voltage controlled resistors) of the 1024 cells resulting in variable propagation delay of the domino wave. For each acquisition, the time sequence for the records of the five channels were calculated (using the trigger cell) and aligned with respect to $t=0$ of the first channel. This alignment results in samples with negative time that were dropped from the record by truncating each record to 1000 samples each.

When one of the detectors produces a signal, the corresponding resonator converts it to a damped sinusoid which is passed into a single digitizer input through the fan-in circuit. The examples of the multiplexed signals (one for each resonator) are shown in Fig. 5.3a; the signals are also shown in the frequency domain (Fig. 5.3b). The channel/resonator producing the signal can be identified by the frequency of the peak of the signal.

(a) Examples of resonator outputs (one for each resonator) when a detector pulse arrives at their inputs.

(b) The four multiplexed signals (shown in Fig. 5.3a) in the frequency domain. The peak frequency identifies the channel that produces the multiplexed signal.

Figure 5.3 Examples of the multiplexed signals (one for each resonator).

5.4 Measurements and results

5.4.1 Impulse response

The impulse response was again estimated separately by connecting each resonator to a noise source; a 1 MHz, 1 Vpp noise signal from a function generator was used as input to each resonator. Using the cross-correlation technique, the mean cross-correlation between the output and the noise input was divided by the mean auto-correlation of noise input in the frequency domain to estimate the impulse response; 20,000 records of noise input and the corresponding output (each with 1000 samples) were used (see Chapter 2, Section 1.4.1 for more details). The impulse response of the four resonators are shown in Fig. 5.5 ; the peak occurs at the resonant frequency. The noise signal used to estimate the impulse response nearly vanishes above 280 MHz (Fig. 5.4); therefore, the impulse response above 280 MHz cannot be estimated. This results in the amplification of the frequency components above 280 MHz in the recovered signal after deconvolution due to division by nearly zero. This noise was filtered from the recovered signal by applying an optimized fifth-order Butterworth low-pass filter with a cutoff frequency of 275 MHz.

We also tried to estimate the impulse response of the resonator using the ultra-fast pulse of the J-Series SiPM connected to an EJ-204 scintillator, which has significant high-frequency components above 280 MHz. The details are provided in Appendix B.

Figure 5.4 The noise signal used to estimate the impulse response in the frequency domain; the signal goes to zero above 280 MHz shown in the inset plot. The peaks in the noise signal are due to nearby radio frequency interference.

(a) The impulse response of the 50 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 50 MHz.

(b) The impulse response of the 55 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 55 MHz.

(c) The impulse response of the 60 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 60 MHz.

(d) The impulse response of the 65 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 65 MHz.

Figure 5.5 Resonator impulse response.

5.4.2 Signal recovery

The multiplexed signals digitized by the DRS4 were separated into four groups corresponding to the resonators that produced them; the resonators were identified by the peak frequency of the damped sinusoids. The multiplexed signals corresponding to each resonator/detector were deconvolved to recover the detector pulses that produced them.

Fig. 5.6 compares the original pulses from the first EJ-204 (connected to the 50 MHz resonator) to the corresponding recovered pulses deconvolved from the damped sinusoidal output. This comparison is shown for a low amplitude and a high amplitude pulse. The amplified noise in the recovered pulse

corresponds to the additive noise from the other channels that was not removed by deconvolution.

The root-mean-square error (RMSE) calculated from the residuals (shown in Fig. 5.6) remains the same as a function of pulse height, resulting in a decrease in the relative root-mean-square error (RMSE) with an increase in the pulse height (shown in Fig. 5.7). This is because the noise in the recovered signal has constant amplitude.

Figure 5.6 The comparison between the original and the recovered pulses for the detector 1 connected to the 50 MHz resonator; the inset plots show the pulses around their peaks. The difference between the anode and the recovered pulse is shown on the right.

Figure 5.7 The relative RMSE as a function of pulse height for the pulses incident on the 50 MHz resonator.

5.4.3 Energy estimation using the recovered pulse

The FOUT signal of a J-Series SiPM evaluation board is the derivative of the SOUT; consequently, undershoot occurs in the FOUT signal. The undershoot does not allow charge integration (to estimate energy deposited) because the area within the undershoot cancels the area under the rest of the pulse. So the amplitude of the recovered pulse (which increases linearly with energy deposited) was used to estimate the energy deposited by a photon.

Fig. 5.8 shows a 2D histogram of the error in energy (or pulse height) as a function of the recovered energy. The mean in the error in energy as a function of the recovered energy exhibits a curved shape. At low energies, the amplified noise dominates and the recovered pulse amplitude is larger than the original pulse; at high energies, the recovered pulse amplitude tends to diminish due to the loss of high frequency components in the recovered signal, which increase with pulse amplitude (see inset plots in Fig. 5.6). The uncertainty in the error in energy is small for the smaller pulses compared to the larger pulses because the impulse response is not able to recover high frequency components above 280 MHz. The pulses were divided into energy ranges between 90 and 600 keV to show the histograms of the error in energy for different pulse heights (Fig. 5.9); the recovered energy within an energy range can be corrected by adding the mean of the corresponding histogram to it.

Figure 5.8 The recovered energy plotted against the difference between the original and recovered energy. The mean of the error in energy as a function of recovered energy exhibits a curved shape, most likely due to the increase in the loss of high frequency components in the recovered signal.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure 5.9 The histograms of the error in energy between the original and the recovered pulse for the energy ranges between 90 to 600 keV.

5.4.4 Timing estimation using the recovered pulse

The time-of-arrival was estimated by computing the timing mark on the leading edge of the recovered pulse at 50 % of the pulse amplitude. Fig. 5.10 shows a 2D histogram of the error in timing as a function of recovered energy; the mean of the error in timing shifts to the right as energy increases, most likely due to the the amplified noise in the recovered pulse, which results in higher uncertainty in the error in timing for the smaller pulses compared to the larger pulses. The pulses were divided into energy ranges between 90 and 600 keV to show the histograms of the error in timing for different pulse heights (Fig. 5.11); the recovered timing within an energy range can be corrected by adding the mean of the corresponding histogram to it.

The details about the RMSE and the energy and timing resolutions for the signals incident on the other three channels are provided in Appendix C.

Figure 5.10 The recovered energy plotted against the difference between the original and recovered timing. The mean of the error in timing has a positive slope, most likely due to the increase in the loss of high frequency components in the recovered signal.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure 5.11 Timing estimation using the recovered pulse corresponding to the 50 MHz resonator. The amplified noise in the recovered pulse results in higher uncertainty in the error in timing for smaller pulses compared to larger pulses.

5.5 Conclusion and future work

The frequency domain multiplexing of the fast output signals from four EJ-204 fast organic scintillators cubes (5 mm on each side), each coupled to a SiPM evaluation board, was demonstrated using the new multiplexer board. The low amplitude pulses recovered by deconvolution exhibit a better energy resolution of 3.2 keV but a poor timing resolution of 101 ps; high amplitude pulses exhibit a poor energy resolution of 5.4 keV but a better timing resolution of 39 ps¹. The reconstruction RMSE appears to remain roughly constant vs. pulse amplitude.

In the future, we plan to implement FDM on the prototype single volume scatter camera being developed at Sandia National Laboratories (SNL). We also plan to use a noise source with a much higher bandwidth to be able to estimate the impulse response of each resonator for frequencies > 280 MHz; this would allow us to accurately and precisely recover very high amplitude pulses which have significant high-frequency components in them.

¹The energy and timing resolutions refer to the uncertainty in the estimate of energy and timing using the recovered pulse. These resolutions would have to be added to the inherent energy/timing resolutions of the measurement system to get the overall resolutions.

CHAPTER

6

CONCLUSIONS AND RECOMMENDATIONS

The work presented in this dissertation describes a new method to perform frequency domain multiplexing of pulse mode radiation detectors by convolution and deconvolution, in order to reduce the need for a large number of digitizer input channels for data acquisition. The deconvolution method, used to recover the detector pulse from the multiplexed output, is meant for the radiation counting experiments with low count rate ($\lesssim 10^4$ counts/sec) where a single event occurs in any digitized record. We demonstrated multiplexing of two EJ-309 liquid organic scintillators, and showed that the recovered pulse can precisely estimate the charge, timing, and particle ID of the original pulse in the case of single-event multiplexing. In case of double occupancy, when two detector pulses arrive in the same digitizer record but do not overlap in time, both the pulses can be recovered using the sequential deconvolution method. We found that the charge, arrival time, and particle type can be estimated fairly precisely for the first pulse, but the second pulse exhibits substantial degradation in the precision of the estimated charge and arrival time. When the pulses overlap in time, the part of the first pulse that does not overlap with the second can be recovered accurately, so the arrival time and amplitude of the first pulse can be estimated fairly precisely, but not the charge or particle type. None of these quantities can be estimated precisely for the second pulse when the two pulses overlap.

The multiplexer design was modified to perform 4 to 1 multiplexing of 64 EJ-204 pillars of the single volume scatter camera (SVSC) being developed at the Sandia National Laboratories. Because

the sequential deconvolution method is not able to resolve the pulses that are 2 ns apart - which is a necessary design constraint for the SVSC - we used simulated annealing to develop a multiplexing pattern where the pillars were divided into 16 groups of 4 channels each, to enable single-event multiplexing; the multiplexing pattern was found to be efficient for the high-energy deposition scatter events. A proof-of-concept 4 to 1 multiplexing of four EJ-204 detectors, each coupled to a silicon photomultiplier (SiPM), was demonstrated, where the energy and timing resolutions were found to be dependent on pulse height.

In this chapter, we recommend a few modifications to the multiplexer design and the analysis method to improve the recovery of the original signal from the multiplexed output.

6.1 Future work

6.1.1 Multiplexer modifications

As mentioned in Chapter 5, the impulse response estimated for each resonator using the noise signal (with non-zero frequency components < 280 MHz) or the fast SiPM signal (with non-zero frequency components < 300 MHz) goes to zero at high frequencies above 280 MHz; therefore, the high frequency components of an input signal, especially for the signals with large amplitude (which have large frequency components above 280 MHz), are not recovered due to the division by nearly zero after deconvolution. So, to estimate the impulse response at high frequencies (especially to recover fast SiPM pulses), a noise signal with non-zero frequency components up to at least 500 MHz should be used.

Each resonator was designed using the OPA690 operational amplifier with a bandwidth of 500 MHz, and the fan-in circuit was designed using OPA659 with a bandwidth of 650 MHz for a gain of 1 [23, 24]. This results in the bandwidth of $\frac{1}{\sqrt{1/500^2 + 1/650^2}} \approx 400$ MHz for the multiplexed signal at the fan-in output for each resonator. The bandwidth of the multiplexer can be increased by choosing a different operational amplifier with higher bandwidth (i.e. OPA820) so that the high frequency components of the multiplexed signal are not attenuated by the bandwidth of the cascaded operational amplifiers.

6.1.2 Noise considerations

Using the setup in Fig. 5.1 discussed in chapter 5, the baseline noise produced by a single EJ-204 connected to a J-series SiPM evaluation board is compared to the baseline noise of the multiplexed signal shown in Fig. 6.1. The baseline noise of the multiplexed signal (blue) is the sum of the noise produced by the four SiPMs, each convolved with the impulse response of its corresponding resonator; the noise produced by a single SiPM is shown in orange. The frequency domain representation of the multiplexed noise shows high amplitudes at the four resonant frequencies (i.e. 50, 55, 60, 65 MHz) because the SiPM noise signals were amplified at the resonant frequencies by their respective resonators. The multiplexed noise does not increase linearly with the number of input channels because the impulse responses of the resonators only amplify the frequency components of noise inputs around the resonant frequencies and

attenuate the noise at all other frequencies. This enables us to add more resonators to the multiplexer board to implement 8 to 1 multiplexing or even higher.

If we decrease the bandwidth of the damped sinusoid by increasing its decay time and reduce the gain at the resonant frequency for each resonator, the baseline SiPM noise for each input channel would be amplified to a lesser extent around the resonant frequency. This will help increase the channel density of the multiplexer even more.

(a) The baseline noise produced by a single SiPM is compared to that of the multiplexed output. The baseline noise of the multiplexed signal is the sum of the noise produced by the four SiPMs convolved with the impulse responses of the four resonators.

(b) The two signals (shown in Fig. 6.1a) in the frequency domain.

Figure 6.1 A comparison between the baseline noise produced by a single EJ-204 connected to a J-series SiPM evaluation board and the baseline noise of the multiplexed signal.

6.1.3 Higher-density multiplexing

In the future, 4 to 1 multiplexing of the 64 pillars of the first prototype of the SVSC will be tested. We have recently fabricated another 4 to 1 multiplexer board (Fig. 6.2) designed to multiplex signals from four sodium iodide (NaI) inorganic scintillators, each coupled to a photomultiplier tube. Using this multiplexer board, 4 to 1 multiplexing of signals will be performed for the coherent neutrino-nucleus scattering experiment, currently being conducted at Duke University, where data acquisition from a large number of NaI detectors is required at low event rate.

(a) A new multiplexer design with four resonators (i.e., 5, 7, 9, 11 MHz) to multiplex four NaI signals.

(b) An oscilloscope output showing an original NaI signal (yellow) connected to the 9 MHz resonator and the corresponding multiplexed signal (cyan).

Figure 6.2 4 to 1 multiplexing for NaI signals.

In the case when two pulses that arrive at two different channels of the multiplexer overlap in time, the correction factors as a function of the degree of overlap to be applied to the energy or timing estimates of both the pulses (which are inaccurately recovered by deconvolution) should be estimated.

A simulation should be performed where the resonators in the multiplexer circuit should be increased from 4 to higher values, with each resonator excited by a noise source to analyze the effect of the increase in noise in the multiplexed channel on the signal recovery as a function of the number of resonators or multiplexing ratio. The model would help choose a higher multiplexing ratio for a specific type of detector signal which would allow the signal recovery within an acceptable precision; the multiplexer with an increased channel density can then be fabricated and tested.

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APPENDICES

APPENDIX

A

ESTIMATION OF CHARGE AND TIMING FROM THE SINUSOIDAL OUTPUT

The charge and the timing of the FDM-encoded signal was initially estimated directly from the corresponding damped sinusoidal output itself using the first version of the resonators and the fan-in circuit, the details of which are explained in Mishra et al. [38].

A.1 Measurements and results

A.1.1 FDM measurement setup

The data acquisition system is shown in Figure A.1. Four EJ-309 detectors, each of 7.6 cm diameter and 7.6 cm length, were connected to four resonators with resonant frequencies of 6.25 MHz, 8.25 MHz, 11.00 MHz and 13.25 MHz respectively. A Cs-137 source was used to induce signals in the detectors, and these signals were sent to the resonator circuits. The outputs from the resonators were combined using the fan-in circuit, whose output was connected to a CAEN DT5730B digitizer. The DT5730B is a 14-bit, 500 MS/s digitizer. The digitizer was set to acquire a record length of 4 μ s for each channel whenever it was triggered. Figure A.2a shows four digitized events, one for each resonator circuit.

Figure A.1 The FDM system. Four EJ-309 detectors were connected to four resonator prototypes whose signals were combined using the fan-in circuit.

The power density spectrum of a discrete-time signal $x(n)$ is the squared magnitude of its N-point discrete Fourier transform $X(k)$, given by

$$P(k) = \frac{1}{N} |X(k)|^2 \quad k = 0, 1, \dots, N-1 \quad (\text{A.1})$$

where $X(k) = \sum_{n=0}^{N-1} x(n) \exp(-j2\pi kn/N)$ and $N = 2000$. With a sampling frequency of 500 MS/s, the range $0 \leq k \leq \frac{N}{2} - 1$ corresponds to a frequency range between 0 and 250 MHz. The corresponding power density spectra (Figure A.2b) of the digitized fan-in outputs show peaks at the resonant frequencies. The detector number that produced the damped sinusoid can be identified by the frequency of the peak of the power spectrum.

A.1.2 Relationships between anode outputs and resonator outputs

We measured the relationship between the PMT anode pulse (from the EJ-309 detector) and the corresponding sinusoid (from the fan-in circuit) to estimate the charge collected and the time-of-arrival of the original anode pulse from the damped sinusoid. We simultaneously digitized the anode pulses via the resonator pass-through and the corresponding sinusoids via the fan-in output by connecting the 6.25 MHz resonator circuit to the fan-in circuit separately (Figure A.3). A Cs-137 source was used to generate anode pulses.

(a) The digitized damped sinusoidal outputs from the fan-in circuit in four different records of digitized data.

(b) The corresponding power density spectra of the digitized fan-in outputs shown in Figure A.2a.

Figure A.2 The discrete-time fan-in outputs and their respective power density spectra.

The charge collected under the anode current pulse is proportional to the amplitude of the damped sinusoid (i.e., square root of the peak value of the power density spectrum). Figure A.4 shows the charge collected under the anode pulses compared to the amplitude of the sinusoids from their power density spectrum. The charge collected can be estimated from the power spectrum with a relative uncertainty

Figure A.3 The setup for developing relationships between the EJ-309 anode pulse and the corresponding damped sinusoid for the 6.25 MHz resonator connected to the fan-in circuit.

of about 2% using a linear fit. In addition, the time-of-arrival of the anode pulse is related to the phase of the corresponding sinusoid (i.e., $\frac{\phi}{f}$ corresponding to the peak of the power density spectrum). The time-of-arrival of the anode pulse was calculated using constant fraction discrimination (CFD) [27]. We performed digital CFD by subtracting each anode pulse delayed by 4 ns from a copy of the original pulse attenuated to 30% and then calculating the first zero crossing of the difference. Figure A.5 shows the relationship between the time-of-arrival of the anode pulse and the phase of the damped sinusoid. The time-of-arrival can be estimated from a linear fit to the phase with an uncertainty of about 850 ps.

We also measured the relationship between the CFD on the anode pulse and the CFD on the leading edge of the corresponding damped sinusoid. The CFD parameters used for the anode pulse are the same as previously described. We tested several delay and attenuation values for the sinusoid and selected optimal values of 20 ns delay and 40% attenuation to obtain the best timing resolution. Figure A.6 shows the relationship between the CFD on the anode pulse and the CFD on the damped sinusoid. The time-of-arrival of the anode pulse can be estimated from a linear fit to the CFD on the damped sinusoid with an uncertainty of about 450 ps. The uncertainties reported here are the average uncertainties over all the events.

Figure A.4 The linear relationship between the charge collected under the anode pulse and the amplitude of the corresponding sinusoid using the 6.25 MHz resonator with the fan-in circuit. The charge collected can be estimated from the sinusoidal amplitude with a relative uncertainty of about 2%.

Figure A.5 The linear relationship between the time-of-arrival of the anode pulse and the phase of the corresponding sinusoid using the 6.25 MHz resonator with the fan-in circuit. The time-of-arrival can be estimated from the phase with an uncertainty of about 850 ps.

Figure A.6 The linear relationship between the time-of-arrival of the anode pulse and the CFD on the sinusoidal waveform using the 6.25 MHz resonator with the fan-in circuit. The time-of-arrival can be estimated from the CFD on the damped sinusoid with an uncertainty of about 450 ps.

We also measured these linear relationships for other resonators with frequencies of 8.25 MHz, 11.00 MHz and 13.25 MHz respectively. The uncertainties with which the charge collected and the time-of-arrival of the original anode pulse can be estimated from the damped sinusoid remained nearly the same across all the resonators ¹.

A.1.3 Gamma-ray spectroscopy using a multiplexed CeBr₃ inorganic scintillator

We also wanted to assess the ability of our FDM system to perform γ -ray spectroscopy with multiplexed inorganic scintillators. Using our system, we simultaneously digitized the pulses from a CeBr₃ detector via a pass-through and the corresponding sinusoids from the 6.25 MHz resonator via the fan-in output. Cs-137 was again used to induce detector pulses. We measured the relationship between the CeBr₃ PMT anode pulses and the corresponding sinusoids from the fan-in circuit to estimate the charge collected under the original anode pulse using the linear fit as shown in Figure A.7. The charge collected under a CeBr₃ pulse can be estimated from the sinusoidal amplitude with a relative uncertainty of about 1.5%.

Figure A.8a shows the γ -ray spectrum using the anode pulses. The energy resolution at 662 keV is 4.4%. Figure A.8b shows the γ -ray spectrum by estimating the area under anode pulse from the amplitude of the sinusoid; the energy resolution at 662 keV in this case increased to 5.1%.

¹The uncertainties reported in the estimates of the charge collected and the time-of-arrival using the damped sinusoids are valid for the sampling frequency of 500 MS/s.

Figure A.7 The charge collected is compared to the amplitude of the sinusoidal output for CeBr_3 pulses with the 6.25 MHz resonator and fan-in circuit. The charge collected under the CeBr_3 pulse can be estimated from the sinusoid with a relative uncertainty of about 1.5%.

(a) γ -ray spectrum calculated directly from the anode pulses. The energy resolution at 662 keV is 4.4%.

(b) The estimated γ -ray spectrum from the amplitude of the damped sinusoids. The energy resolution at 662 keV is 5.1%.

Figure A.8 γ -ray spectra from a CeBr_3 detector.

A.1.4 Coincidence measurements using a Na-22 source

Two resonators with frequencies of 6.25 MHz and 13.25 MHz were connected to two EJ-309 organic scintillator detectors. Each resonator was connected to a separate fan-in circuit to digitize coincident events. The detectors were placed facing each other with a Na-22 source between them. The time-of-arrival of coincident gamma rays was measured using CFD applied to the original anode pulses and using CFD applied to their corresponding sinusoids. Figure A.9 shows the distribution of difference in the time-of-arrival of coincident gamma rays using anode pulses ($FWHM = 1.21$ ns) and using sinusoids ($FWHM = 1.37$ ns). There is a small increase of 160 ps in the FWHM using the resonator circuits.

(a) The distribution of differences in time-of-arrival of coincident gamma rays using anode pulses ($FWHM = 1.21$ ns/ $\sigma = 0.514$ ns).

(b) The distribution of differences in time-of-arrival using damped sinusoids ($FWHM = 1.37$ ns/ $\sigma = 0.583$ ns).

Figure A.9 γ - γ correlation functions for two EJ-309 detectors.

A.2 Summary

We demonstrated frequency-domain multiplexing of four EJ-309 organic scintillator detectors. The charge collected from the anode can be estimated from the amplitude of the damped sinusoid with a relative uncertainty of about 2%. The time of arrival can be best estimated from CFD on the leading edge of the damped sinusoid with an uncertainty of about 450 ps. We were able to reproduce the energy spectrum recorded by a $CeBr_3$ inorganic scintillator with our FDM circuits with an energy resolution of 5.1% at 662 keV. Coincidence measurements with Na-22 showed that there was a small, 160 ps increase in timing resolution when damped sinusoids were used to estimate timing compared to actual anode pulses.

APPENDIX

B

SIGNAL RECOVERY BY ESTIMATING THE IMPULSE RESPONSE USING THE SIPM FAST-OUTPUT SIGNALS

In chapter 5, we discussed the estimation of the impulse response of the resonators using a noise source (from a signal generator) whose frequency components went to zero above 280 MHz. Consequently, the impulse response, estimated using the cross-correlation technique, nearly vanishes above 280 MHz resulting in the amplification of the high frequency components in the recovered signal after deconvolution due to division by nearly zero. This noise was filtered from the recovered signal by applying an optimized fifth-order Butterworth low-pass filter with a cut-off frequency of 275 MHz.

This appendix provides the details about an alternative method where the impulse response (of each resonator) is estimated using the ultra-fast pulse of the J-Series SiPM connected to an EJ-204 scintillator, which has significant high-frequency components above 280 MHz (Fig. B.1). The response of each resonator to the high frequency components of the SiPM fast output results in non-zero frequency components in the impulse response above 280 MHz. This facilitates the recovery of the high frequency components above 280 MHz in the deconvolved signal. The signal recovery using the impulse response estimated from the ultra-fast pulse of the J-Series SiPM for the signals incident on 50 MHz resonator will be discussed.

B.1 Impulse response

The impulse response was estimated using the SiPM ultra-fast signals as input to each resonator individually. Using the cross-correlation technique, the mean cross-correlation between the output and the SiPM input signal was divided by the mean auto-correlation of the SiPM input in the frequency domain to estimate the impulse response; 40000 records of SiPM input and the corresponding output (each with 1000 samples) were used (see Chapter 2, Section 2.4.1 for more details). The impulse responses of the four resonators are shown in Fig. B.2; the peak occurs at the resonant frequency. The SiPM FOUT signal used to estimate the impulse response flattens to zero above 300 MHz (Fig. B.1); therefore, the impulse response could only be estimated to 300 MHz. This results in the amplification of the frequency components above 300 MHz in the recovered signal after deconvolution. This noise is filtered from the recovered signal by applying an optimized fifth-order Butterworth low-pass filter with a cutoff frequency of 300 MHz. The SiPM fast output signal is not smooth at high frequencies and exhibits very small peaks (see inset plot in Fig. B.1) which results in valleys in the impulse response of each resonator (in Fig. B.2) at those frequencies.

Figure B.1 The SiPM fast-output signal used to estimate the impulse response in the frequency domain; the signal goes to zero above 300 MHz shown in the inset plot.

(a) The impulse response of the 50 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 50 MHz.

(b) The impulse response of the 55 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 55 MHz.

(c) The impulse response of the 60 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 60 MHz.

(d) The impulse response of the 65 MHz resonator in series with the fan-in circuit in the frequency domain. The inset plot shows the peak at 65 MHz.

Figure B.2 Resonator impulse response using SiPM fast-output signal.

B.2 Signal recovery

Using the setup in Fig. 5.1, the multiplexed signals digitized by the DRS4 were separated into four groups corresponding to the resonators that produced them; the resonators were identified by the peak frequency of the damped sinusoids. The multiplexed signals corresponding to each resonator/detector were deconvolved to recover the detector pulses that produced them.

Fig. B.3 compares the original pulses from the first EJ-204 (connected to the 50 MHz resonator) to the corresponding recovered pulses deconvolved from the damped sinusoidal output. This comparison is

shown for a low and a high amplitude pulse. The residuals shown do not change with pulse height; this results in a decrease in the relative RMSE with an increase in the pulse height (Fig. B.4).

Figure B.3 The comparison between the original and the recovered pulses for the detector 1 connected to the 50 MHz resonator. The difference between the anode and the recovered pulse is shown on the right.

Figure B.4 The relative RMSE as a function of pulse height. The relative RMSE decreases as the pulse height increases.

B.3 Energy and timing estimation

Fig. B.5 shows 2D histograms of the error in energy (or pulse height) and the error in timing as a function of the recovered energy for the signals incident on the 50 MHz resonator. The mean in the error in energy as a function of the recovered energy exhibits a curved shape; the mean of the error in timing shifts to the right as energy increases, same as shown in Chapter 5 Figs. 5.8 and 5.10.

Figs. B.6 and B.7 show the histograms of the error in energy and timing for different pulse height; the recovered energy and timing within an energy range can be corrected by adding the mean of the corresponding histogram to it.

The uncertainties in the error in energy and timing (for different recovered energy ranges) remain the same when the impulse response was estimated using the SiPM fast-output pulses compared to when it was estimated using the noise signal (explained in chapter 5); the comparison is shown in Table B.1.

(a) The recovered energy plotted against the difference between the original and recovered energy.

(b) The recovered energy plotted against the difference between the original and recovered timing.

Figure B.5 The error in energy and timing for the pulses from detector 1 incident on the 50 MHz resonator.

B.4 Summary

The detector signal recovered from the damped sinusoid using the impulse response estimated from the SiPM fast-output pulses was compared to the signal recovered when the impulse response was estimated using the noise signal. It was found that the energy and the timing resolutions remain the same for both the cases: these resolutions lie within the experiment uncertainties of the measurement.

Table B.1 The new uncertainties in the error in energy and timing (for different recovered energy ranges) when the impulse response was estimated using the SiPM fast-output pulses compared to when it was estimated using the noise signal.

Energy range (keV)	New		Old	
	σ_e (keV)	σ_t (ps)	σ_e (keV)	σ_t (ps)
90 - 120	3.3 ± 0.08	103.6 ± 1.40	3.2 ± 0.06	101.3 ± 0.85
120 - 150	3.3 ± 0.05	84.8 ± 1.20	3.3 ± 0.05	82.6 ± 1.13
150 - 200	3.5 ± 0.07	68.1 ± 1.04	3.4 ± 0.05	67.8 ± 1.16
200 - 300	3.7 ± 0.1	54.8 ± 1.05	3.7 ± 0.05	55.4 ± 0.84
300 - 400	4.4 ± 0.09	44.6 ± 0.77	4.4 ± 0.08	44.7 ± 0.68
400 - 600	5.4 ± 0.07	40.2 ± 0.95	5.4 ± 0.12	39.0 ± 0.57

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure B.6 Energy estimation using the recovered pulse corresponding to the 50 MHz resonator.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure B.7 Timing estimation using the recovered pulse corresponding to the 50 MHz resonator.

APPENDIX

C

SIGNAL RECOVERY FOR OTHER CHANNELS OF THE SILICON PHOTOMULTIPLIER MULTIPLEXER

This appendix provides the details of the recovery of the detector signals from the damped sinusoids produced by the 55, 60 and 65 MHz resonators. The root-mean-squared error (RMSE) and the energy and timing resolutions for the recovered signals incident on the three resonators will be discussed.

C.1 Signal recovery

Using the setup described in section 5.3, the original EJ-204 detector pulses input to the four resonators were digitized along with the multiplexed output using the DRS4 digitizer. The multiplexed output was deconvolved using the impulse response of the resonator that produced it. Figs. C.1, C.2 and C.3 compare the original pulses produced by the second, third and the fourth EJ-204 to the corresponding recovered pulses for a low and a high amplitude pulse. The residuals shown do not change with pulse height; this results in a decrease in the relative RMSE with an increase in the pulse height (Fig. C.4). The relative RMSE is shown for the pulse heights between 90 keV to 550 keV. In Fig. C.4, note that detectors 2 and 4 generated smaller pulses compared to the detectors 1 and 3 for the same energy deposited. It is most likely due to lower light collection efficiency for detectors 2 and 4 (due to fewer teflon layers wrapped

around them).

Figure C.1 A comparison between the original and the recovered pulses for detector 2 connected to 55 MHz resonator. The difference between the anode and the recovered pulse is shown on the right.

Detector 3, 60 MHz resonator

Figure C.2 A comparison between the original and the recovered pulses for detector 3 connected to 60 MHz resonator. The difference between the anode and the recovered pulse is shown on the right.

Detector 4, 65 MHz resonator

Figure C.3 A comparison between the original and the recovered pulses for detector 4 connected to 65 MHz resonator. The difference between the anode and the recovered pulse is shown on the right.

Figure C.4 The RMSE as a function of pulse height. Because the residuals remain constant as a function of the pulse height, the RMSE decreases as the pulse height increases.

C.2 Energy and timing precision

Figs. C.5, C.6, C.7 show 2D histograms of the error in energy (pulse height) and the error in timing as a function of the recovered energy. The mean in the error in energy as a function of the recovered energy exhibits a curved shape. Because the pulses produced by detectors 2 and 4 are smaller compared to detectors 1 and 3 for the same energy deposited, the curve appears truncated in Figs. C.5a and C.7a compared to Fig. C.6a. The mean of the error in timing shifts to the right as energy increases (Figs. C.5b, C.6b and C.7b), same as shown in Chapter 5 Fig. 5.10.

(a) The recovered energy plotted against the difference between the original and recovered energy.

(b) The recovered energy plotted against the difference between the original and recovered timing.

Figure C.5 The error in energy and timing for the pulses from detector 2 incident on 55 MHz resonator.

(a) The recovered energy plotted against the difference between the original and recovered energy.

(b) The recovered energy plotted against the difference between the original and recovered timing.

Figure C.6 The error in energy and timing for the pulses from detector 3 incident on 60 MHz resonator.

Figure C.7 The error in energy and timing for the pulses from detector 4 incident on 65 MHz resonator.

The events in the 2D histograms for each resonator were divided into energy ranges between 90 and 600 keV to show the histograms of the error in energy and timing for different pulse heights. The energy and timing resolutions for the recovered pulses incident on 55 MHz (Fig. C.8, C.9) and 65 MHz resonator (Fig. C.12, C.13) are poor compared to 60 MHz resonator (Fig. C.10, C.11) due to the lower light collection efficiency of detectors 2 and 4.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure C.8 Energy estimation using the recovered pulse corresponding to the 55 MHz resonator.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure C.9 Timing estimation using the recovered pulse corresponding to the 55 MHz resonator.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure C.10 Energy estimation using the recovered pulse corresponding to the 60 MHz resonator.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure C.11 Timing estimation using the recovered pulse corresponding to the 60 MHz resonator.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure C.12 Energy estimation using the recovered pulse corresponding to the 65 MHz resonator.

(a) Energy range: 90 - 120 keV

(b) Energy range: 120 - 150 keV

(c) Energy range: 150 - 200 keV

(d) Energy range: 200 - 300 keV

(e) Energy range: 300 - 400 keV

(f) Energy range: 400 - 600 keV

Figure C.13 Timing estimation using the recovered pulse corresponding to the 65 MHz resonator.

C.3 Summary

The detector signal reconstructed from the damped sinusoid by deconvolution was compared to the original signal incident on 55, 60 and 65 MHz resonators. Poorer energy resolution was observed for the large amplitude pulses because the impulse response is not able to recover high frequency components above 280 MHz (see Section 5.4.3); while poorer timing resolution was observed for the small amplitude pulses because of the amplified noise in the recovered pulse (see Section 5.4.4).