Computing Blocking Probabilities in Multi-Class Wavelength Routing Networks with Multicast Calls *

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TR-01-02

January 7, 2001

Abstract

We present an approximate analytical method to compute efficiently the call blocking probabilities in wavelength routing networks with both unicast and multicast calls. The model is fairly general and allows each source-destination pair to serve calls of different classes, with each call occupying one wavelength per link. Our approach involves the following steps. We first consider a linear, single-class unicast network where the arrival process of calls on multi-hop routes is modified slightly to obtain an approximate network model with a product-form solution. This method is used to solve single-class unicast wavelength routing networks with a small number of hops. Next, all classes of calls on a particular route are aggregated to give an equivalent single-class model. We then extend the path decomposition algorithms that we have developed for single-class networks to handle mesh networks with multiple classes of calls. We show how to use these path decomposition algorithms to decompose large networks with multicast paths into smaller sub-systems with only linear paths, which, in turn, are solved by the product-form approximation algorithm. We also consider a state-dependent Poisson arrival process for multicast calls which is more accurate in capturing the behavior of these calls. To the best of our knowledge, this is the first model that can be used to analyze efficiently wavelength routing networks of arbitrary mesh topologies, with multi-class service and multicast calls.

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*This work was supported by the NSF under grant ANI-9805016.

1 Introduction

A basic property of single mode optical fiber is its enormous low-loss bandwidth of several tens of Terahertz. However, due to dispersive effects and limitations in optical device technology, single channel transmission is limited to only a small fraction of the fiber capacity. To take full advantage of the potential of fiber, the use of wavelength division multiplexing (WDM) techniques has become the option of choice, and WDM networks have been a subject of research both theoretically and experimentally [3]. Optical networks have the potential of delivering an aggregate throughput in the order of Terabits per second, and they appear as a viable approach to satisfying the ever-growing demand for more bandwidth per user on a sustained, long-term basis.

The wavelength routing mesh architecture appears promising for Wide Area Networks (WAN). The architecture consists of wavelength routers interconnected by fiber links. A wavelength router is capable of switching a light signal at a given wavelength from any input port to any output port. A router may also be capable of enforcing a shift in wavelength [10], in which case a light signal may emerge from the switch at a different wavelength than the one it arrived. By appropriately configuring the routers, all-optical paths (lightpaths) may be established in the network. Lightpaths represent direct optical connections without any intermediate electronics. Because of the long propagation delays, and the time required to configure the routers, wavelength routing WANs are expected to operate in circuit-switched mode. This architecture is attractive for two reasons: the same wavelength can be used simultaneously at different parts of the network, and the signal power is channeled to the receiver and is not spread to the entire network. Hence, wavelength routing WANs can be highly scalable.

Given the installed base of fiber and the maturing of optical component technology, it appears that wavelength routing technology is poised to make the leap from promise to reality. Thus, it is not surprising that a significant amount of research effort in recent years has been devoted to understanding and addressing the set of problems and issues that arise in the design of wavelength routing networks [3]. One problem that has attracted considerable attention is that of computing call blocking probabilities in such networks. Several variations of this problem have been studied, mainly differing in the underlying assumptions regarding key elements of the wavelength routing architecture, including: the availability, type, and location of converters, the routing algorithm (which finds a path for an incoming call), the wavelength allocation policy (which assigns a free wavelength to a new call), the traffic type (unicast and/or multicast), and the arrival process (mostly Poisson, with a few exceptions). A major difficulty in analyzing wavelength routing networks is the tight coupling between routing and wavelength allocation, which is not present in conventional

circuit-switched networks, and which requires the development of new analytical techniques.

The problem of computing call blocking probabilities under static or fixed alternate routing with random wavelength allocation and with or without converters has been considered in several studies, including [1, 8, 2, 5, 12, 14]. Each work develops approximate techniques to analyze the blocking performance of wavelength routing networks. In general, these approximations are based on various independence assumptions (e.g., independence of link loads, link blocking events, or wavelength usage), and also exploit the analytical tractability of the random allocation policy. Other wavelength allocation schemes, as well as dynamic routing are harder to analyze. First-fit wavelength allocation was studied using simulation in [8], and it was shown to perform better than random allocation, while an analytical overflow model for first-fit allocation was developed in [6]. A dynamic routing algorithm that selects the least loaded path-wavelength pair was also studied in [6], and in [9] an unconstrained dynamic routing scheme with a number of wavelength allocation policies was evaluated.

Except in [12, 13], all of the above studies assume that either all or none of the wavelength routers have full wavelength conversion capabilities. The two studies in [12, 13] consider networks with *sparse* conversion, whereby only a subset of the set of nodes are equipped with converters. Specifically, [12] takes a probabilistic approach in modeling wavelength conversion by introducing the converter density, which represents the probability that a node is capable of conversion independently of other nodes in the network. The focus of [13], on the other hand, is on capacity planning, and a dynamic programming algorithm to determine the location of converters on a single path that minimizes average or maximum blocking probability is developed under the assumption of independent link loads. A different type of wavelength conversion is studied in [16, 15], where it is assumed that all nodes in the network are equipped with devices which can convert each wavelength to only a subset of the supported wavelengths (usually, to ones close to the given wavelength in the optical spectrum). This type of conversion is referred to as *limited* conversion, as opposed to the full conversion (i.e., the capability of converting a wavelength to any other supported wavelength) assumed by previously discussed works. Approximate analytical techniques are developed in [16, 15] to compute call blocking probabilities in networks with limited conversion.

All of the above studies describe techniques to analyze networks in which calls are point-to-point and arrivals are Poisson. A study of call blocking under non-Poisson input traffic is presented in [14], where the assumption is made that link loads are statistically independent. Finally, the problem of routing and wavelength assignment when the network carries multicast calls is addressed in [11].

Most of the approximate analytical techniques developed for computing blocking probabilities

in wavelength routing networks and discussed above, amount to the well-known *link decomposition* approach [4]. Further, the development of some of the techniques is based on additional assumptions, for instance, that statistics of link loads are mutually independent, link blocking events are independent, or that wavelength use on each link is characterized by a fixed probability independent of other wavelengths and links. Link decomposition has been extensively used in conventional circuit-switched networks where there is no requirement for the *same* wavelength to be used on successive links of the path taken by a call. The accuracy of these underlying approximations also depends on the traffic load, the network topology, and the routing and wavelength allocation schemes employed. While link decomposition techniques make it possible to study the qualitative behavior of wavelength routing networks, more accurate analytical tools are needed to evaluate the performance of these networks efficiently, as well as to tackle complex network design problems, such as selecting the nodes where to employ converters.

The authors have developed an iterative *path decomposition* algorithm [21, 19] for analyzing arbitrary wavelength routing network topologies with sparse conversion. Specifically, we analyze a given network with point-to-point calls by decomposing it into a number of path sub-systems. These sub-systems are analyzed in isolation using our approximation algorithm for computing blocking probabilities in a single path of a network [18]. The individual solutions are appropriately combined to form a solution for the overall network, and the process repeats until the blocking probabilities converge. Our approach accounts for the correlation of both link loads and link blocking events, giving accurate results for a wide range of loads and network topologies where only a fixed but arbitrary subset of nodes are capable of wavelength conversion. Therefore, our algorithm can be an important tool in the development and evaluation of converter placement strategies. Also, in [20] we studied the first-fit and most-used wavelength allocation policies, and we showed that they have almost identical performance in terms of blocking probability for all calls in the network. We also demonstrated that the blocking probabilities under the random wavelength allocation policy with no converters and with converters at all nodes provide upper and lower bounds for the values of the blocking probabilities under the first-fit and most-used policies.

In this paper we demonstrate how powerful the path decomposition approach can be by extending the decomposition algorithm in [21] to evaluate the blocking performance of optical networks with multiple classes of calls and multicast traffic. While multicast calls were considered in [11], all previous studies on call blocking probabilities have mostly focused on single-class wavelength routing networks. However, future networks will be utilized by a wide range of applications with varying characteristics in terms of their arrival rates and call holding times. To the best of our knowledge, this is the first time that multi-class wavelength routing networks with multicast calls

Figure 1: A k -hop path

have been analyzed. Further, we analyze a state-dependent Poisson arrival process for multicast calls which is more accurate than a Poisson model in capturing the behavior of these calls.

The development of our approximate iterative decomposition techniques involves the following steps. The wavelength routing network is first decomposed into several smaller sub-systems. The arrival rates of calls on each route in the smaller sub-systems is adjusted to reflect the arrival of calls on longer routes in the original network, as well as the arrival of multicast calls using the route. Each sub-system is then solved as follows. The arrival process of calls on multi-hop routes is modified slightly to obtain a *modified multi-class network model*. Next, all classes of calls on a particular route are aggregated to give an *equivalent single-class model*. This equivalent model has the same call blocking probability on any given route as the modified multi-class network, and can be solved using our previous algorithms.

In Section 2, we describe the network under study. In Section 3, we explain how the modified multi-class model and the equivalent single-class model are obtained for a single path of a network. In Section 4, we describe a decomposition algorithm for mesh networks. In Section 5, we extend this decomposition algorithm for networks with multicast traffic. Section 6 presents numerical results, and we conclude the paper in Section 7.

2 The Multi-Class Wavelength Routing Network

We consider a wavelength routing network with an arbitrary topology. Each link in the network supports exactly W wavelengths, and each node is capable of transmitting and receiving on any of these W wavelengths. Call requests between a source and a destination node arrive at the source according to a Poisson process with a rate that depends on the source-destination pair. If the request can be satisfied, an optical circuit is established between the source and destination for the duration of the call. In addition, there are multicast calls between a source and a set of destination nodes. The arrival of multicast calls is modeled by a state dependent Poisson process. Further, calls (multicast calls or unicast calls between any source-destination pair) may be of several classes. Without any loss of generality, we may assume that an arriving call can belong to one of R . Call

holding times are assumed to be exponentially distributed, with a mean that depends on the class of the call.

In our model, we allow some of the nodes in the network to employ wavelength converters. These nodes can switch an incoming wavelength to an arbitrary outgoing wavelength. (When there are converters at all nodes, the situation is identical to that in classical circuit-switching networks, a special case of the more general scenario discussed here.) If no wavelength converters are employed in the path between the source and the destination, a unicast call can only be established if the *same* wavelength is free on all the links used by the call. If there exists no such wavelength, the call is blocked. Similarly, a multicast call can only be established in a network without converters if the same wavelength is free on all links of the multicast tree connecting the source to the destinations. This is known as the *wavelength continuity* requirement (see [21]), and it increases the probability of call blocking. On the other hand, if a call can be accommodated, it is randomly assigned one of the wavelengths that are available on the links used by the call¹. Thus, we only consider the random wavelength assignment policy in this paper.

Since our model is an extension of a model developed for a single path, we introduce some relevant notation for a linear, unicast wavelength routing network. A k -hop path (see Figure 1) consists of $k + 1$ nodes. Nodes $(i - 1)$ and i , $1 \leq i \leq k$, are said to be connected by link (hop) i . Calls originating at node $i - 1$ and terminating at node j use hops i through j , $j \geq i \geq 1$, which we shall denote by the pair (i, j) . Calls between these two nodes may belong to one of R classes, and these calls are said to use *route* (i, j) . We also define the following parameters.

- $\lambda_{ij}^{(r)}$, $j \geq i$, $1 \leq r \leq R$, is the Poisson arrival rate of calls of class r that originate at node $(i - 1)$ and terminate at node j .
- $1/\mu_{ij}^{(r)}$ is the mean of the exponentially distributed service time of calls of class r that originate at node $(i - 1)$ and terminate at node j . We also let $\rho_{ij}^{(r)} = \lambda_{ij}^{(r)} / \mu_{ij}^{(r)}$.
- $N_{ij}^{(r)}(t)$ is the number of active calls at time t on segment (i, j) belonging to class r .
- $F_{ij}(t)$ is the number of wavelengths that are free on all hops of segment (i, j) at time t . A call that arrives at time t and uses route (i, j) is blocked if $F_{ij}(t) = 0$.

¹In a network with wavelength converters, a wavelength is randomly assigned within each segment of a path whose starting and ending nodes are equipped with converters.

3 Blocking Probabilities in A Single Path of a Network

3.1 The Single-Class Case

In this section we briefly review some of our previous results for a path of a single-class wavelength routing network. These results were published earlier in [18, 21], and are included here for completeness. Consider the k -hop path shown in Figure 1. Let the state of this system at time t be described by the k^2 -dimensional process:

$$\underline{X}_k(t) = (N_{11}(t), N_{12}(t), \dots, N_{kk}(t), F_{12}(t), \dots, F_{1k}(t), F_{23}(t), \dots, F_{(k-1)k}(t)) \quad (1)$$

A closer examination of the process $\underline{X}_k(t)$ reveals that it is not time-reversible (see [18]). This result is true in general, when $k \geq 2$ and $W \geq 2$.

Since the number of states of process $\underline{X}_k(t)$ grows very fast with the number k of hops in the path and the number W of wavelengths, it is not possible to obtain the call blocking probabilities directly from the above process. Consequently, an approximate model was constructed in [18] to analyze a single-class, k -hop path of a wavelength routing network. The approximation consists of modifying the call arrival process to obtain a time-reversible Markov process that has a closed-form solution. To illustrate our approach, let us consider the Markov process corresponding to a 2-hop path:

$$\underline{X}_2(t) = (N_{11}(t), N_{12}(t), N_{22}(t), F_{12}(t)) \quad (2)$$

We now modify the arrival process of calls that use both hops (a Poisson process with rate λ_{12} in the exact model) to a state-dependent Poisson process with rate Λ_{12} given by:

$$\Lambda_{12}(n_{11}, n_{12}, n_{22}, f_{12}) = \lambda_{12} \frac{f_{12}(W - n_{12})}{f_{11}f_{12}} \quad (3)$$

The arrival process of other calls remain as in the original model. As a result, we obtain a new Markov process $\underline{X}'_2(t)$ with the same state space and the same state transitions as process $\underline{X}_2(t)$, but which differs from the latter in some of the state transition rates.

We made the observation in [18] that under the new arrival process (3) for calls using both hops, the Markov process $\underline{X}'_2(t)$ is time-reversible and the stationary vector π is given by:

$$\pi(n_{11}, n_{12}, n_{22}, f_{12}) = \frac{1}{G_2(W)} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}} \rho_{22}^{n_{22}}}{n_{11}! n_{12}! n_{22}!} \times \frac{\begin{pmatrix} f_{11} \\ f_{12} \end{pmatrix} \begin{pmatrix} n_{11} \\ W - n_{12} - n_{22} - f_{12} \end{pmatrix}}{\begin{pmatrix} W - n_{12} \\ W - n_{12} - n_{22} \end{pmatrix}} \quad (4)$$

where $G_k(W)$ is the normalizing constant for a k -hop path with W wavelengths.

Let $P(n_{11}, n_{12}, n_{22})$ be the marginal distribution over the states for which $N_{ij}(t) = n_{ij}$, $1 \leq i \leq j \leq 2$. It can be verified [18] that

$$P(n_{11}, n_{12}, n_{22}) = \frac{1}{G_2(W)} \frac{\rho_{11}^{n_{11}}}{n_{11}!} \frac{\rho_{12}^{n_{12}}}{n_{12}!} \frac{\rho_{22}^{n_{22}}}{n_{22}!} \quad (5)$$

Likewise, for a k -hop path, $k \geq 2$, with the modified state-dependent Poisson arrival process, the marginal distribution over the states for which $N_{ij}(t) = n_{ij}$, $1 \leq i \leq j \leq k$, is given by:

$$P(n_{11}, n_{12}, \dots, n_{kk}) = \frac{1}{G_k(W)} \prod_{\{(i,j)|1 \leq i \leq j \leq k\}} \frac{\rho_{ij}^{n_{ij}}}{n_{ij}!} \quad (6)$$

It is easily seen that this distribution is the same as in the case of a network with wavelength converters at each node. An interesting feature of having wavelength converters at every node is that the network has a product-form solution even when there are multiple classes of calls on each route, as long as call arrivals are Poisson, and holding times are exponential [7, 4]. Further, when calls of all classes are assigned wavelengths from the same set, we can aggregate classes to get an equivalent single-class model with the same steady-state probability distribution over the aggregated states, as we show next.

3.2 The Multi-Class Case

Let us now consider a k -hop path with wavelength converters at all nodes, and with R classes of calls. If $\lambda_{ij}^{(r)}$, $1 \leq i \leq j \leq k$, $1 \leq r \leq R$, is the arrival rate of calls of class r on route (i, j) , and $1/\mu_{ij}^{(r)}$, $1 \leq i \leq j \leq k$, $1 \leq r \leq R$, is the mean of the exponential holding time of calls of class r , the probability of being in state $\underline{n} = (n_{11}^{(1)}, n_{11}^{(2)}, \dots, n_{11}^{(R)}, n_{12}^{(1)}, \dots, \dots, n_{kk}^{(R)})$ is given by:

$$P(\underline{n}) = \frac{1}{G_k(W)} \left(\prod_{\{(i,j)|1 \leq i \leq j \leq k\}} \prod_{r=1}^R \frac{(\rho_{ij}^{(r)})^{n_{ij}^{(r)}}}{n_{ij}^{(r)}!} \right) \quad (7)$$

Let $\sigma_{ij} = \sum_r \rho_{ij}^{(r)}$ and $s_{ij} = \sum_r n_{ij}^{(r)}$. As defined, s_{ij} is the total number of calls of all classes that use segment (i, j) of the path, and σ_{ij} is the total offered load of these calls. Taking the summation of (7) over all states such that $\sum_r n_{ij}^{(r)} = s_{ij}$, $1 \leq i \leq j \leq k$, we obtain:

$$P'(s_{11}, s_{12}, \dots, s_{kk}) = \sum_{\{\underline{n} | \sum_r n_{ij}^{(r)} = s_{ij}\}} P(\underline{n}) = \frac{1}{G_k(W)} \prod_{\{(i,j)|1 \leq i \leq j \leq k\}} \frac{\sigma_{ij}^{s_{ij}}}{s_{ij}!} \quad (8)$$

Observe that this is identical to the solution (6) for the single-class case obtained by substituting σ_{ij} by ρ_{ij} and s_{ij} by n_{ij} in (8).

Based on the above results, we conclude that by employing class aggregation on a multi-class path with converters at all nodes, we obtain a system equivalent to a single-class path with converters. In Section 3.1, we showed that the modified single-class wavelength routing network without converters has a steady-state marginal distribution similar to the exact single-class network with converters. We now show that a modified multi-class network without wavelength converters can also be subjected to class aggregation that results in an equivalent single-class model. The modification applied to the arrival process of calls is similar to the single-class case, and it is given by:

$$\Lambda_{ij}^{(r)}(\underline{x}) = \lambda_{ij}^{(r)} \frac{f_{ij} \left(\sum_{l=1}^i s_{li} + f_{ii} \right) \left(\sum_{l=1}^{i+1} s_{l(i+1)} + f_{(i+1)(i+1)} \right) \cdots \sum_{l=1}^{j-1} s_{l(j-1)} + f_{(m-1)(m-1)}}{f_{ii} f_{(i+1)(i+1)} \cdots f_{jj}} \quad (9)$$

Then, the probability that the equivalent single-class network without converters is in state $S_{\underline{x}} = (s_{11}, s_{12}, \dots, s_{1k}, s_{22}, \dots, s_{kk}, f_{12}, f_{13}, \dots, f_{(k-1)k})$ is given by:

$$\begin{aligned} \pi(S_{\underline{x}}) &= \left(\prod_{i,j} \frac{s_{ij}!}{\sigma_{ij}^{s_{ij}}} \right) \\ &\times \left(\frac{\prod_{l=2}^k \left(\frac{f_{1(l-1)}}{f_{1l}} \right) \left\{ \prod_{m=2}^{l-1} \left(\frac{f_{m(l-1)} - f_{(m-1)(l-1)}}{f_{ml} - f_{(m-1)l}} \right) \right\} \left(\frac{f_{ll} + n_{ll} - f_{(l-1)(l-1)}}{f_{ll} - f_{(l-1)l}} \right)}{\left(\frac{f_{ll} + n_{ll}}{f_{ll}} \right)} \right) \quad (10) \end{aligned}$$

Once again, the parameters of the single-class model are given by:

$$s_{ij} = \sum_{r=1}^R n_{ij}^{(r)} \quad 1 \leq i < j \leq k, \quad \sigma_{ij} = \sum_{r=1}^R \rho_{ij}^{(r)} \quad 1 \leq i < j \leq k \quad (11)$$

3.3 Blocking Probabilities in the Multi-Class Case

Since the arrival rate of calls of each class on each route is Poisson, the blocking probability, $Q_{ij}^{(r)}$, of a call of class r using route (i, j) is just the fraction of time that there are no wavelengths that are free on all hops along route (i, j) (see the *PASTA* theorem in [17]). Thus, we have:

$$Q_{ij}^{(r)} = \lim_{\tau \rightarrow \infty} \frac{\int_{t=0}^{\tau} I_{\{F_{ij}(t)=0\}} dt}{\tau}, \quad \text{where} \quad I_{\{F_{ij}(\tau)=0\}} = \begin{cases} 1, & \text{if } F_{ij}(\tau) = 0 \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

As can be seen, the blocking probability is class-independent.

Next, we focus on the call blocking probabilities in the modified model. The arrival process of calls of class r on route (i, j) is a state-dependent Poisson process whose rate at time τ , $A_{ij}^{(r)}(\tau)$ is

a function of the state $\underline{X}(\tau)$ of the process, and is given by:

$$A_{ij}^{(r)}(\tau)\Lambda_{ij}^{(r)}(\underline{X}(\tau)) = \lambda_{ij} \frac{F_{ij}(\tau) \prod_{k=i}^{j-1} \left(\sum_{l=1}^k N_{lk}(\tau) + F_{kk}(\tau) \right)}{F_{ii}F_{i+1,i+1} \cdots F_{jj}} \quad (13)$$

Note that the modified arrival process satisfies the criterion:

$$\frac{\Lambda_{ij}^{(r_1)}(\underline{x})}{\Lambda_{ij}^{(r_2)}(\underline{x})} = \frac{\lambda_{ij}^{(r_1)}}{\lambda_{ij}^{(r_2)}} \quad 1 \leq r_1, r_2 \leq r \quad (14)$$

By applying the *PASTA* theorem conditioned on being in state $\underline{x}$, the conditional call blocking probability, $\mathcal{P}_{ij}^{(r)}(\underline{x})$, of calls of class r on route (i, j) is given by the fraction of time spent in state $\underline{x}$ in which there are no wavelengths that are free on all hops of route (i, j) . Therefore:

$$\mathcal{P}_{ij}^{(r)}(\underline{x}) = \lim_{\tau \rightarrow \infty} \frac{\int_{t=0}^{\tau} I_{\{F_{ij}(t)=0, \underline{X}(t)=\underline{x}\}} dt}{\int_{t=0}^{\tau} I_{\{\underline{X}(t)=\underline{x}\}} dt} = \begin{cases} 1, & \text{if } f_{ij} = 0 \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

Let $P_{ij}^{(r)}$ be the unconditional probability that a call of class r , on route (i, j) gets blocked in the modified multi-class model. This is given by:

$$P_{ij}^{(r)} = \frac{\sum_{\underline{x}} \Lambda_{ij}^{(r)}(\underline{x}) \pi(\underline{x}) \mathcal{P}_{ij}^{(r)}(\underline{x})}{\sum_{\underline{x}} \Lambda_{ij}^{(r)}(\underline{x}) \pi(\underline{x})} = \frac{\sum_{\{\underline{x} | f_{ij}=0\}} \Lambda_{ij}^{(r)}(\underline{x}) \pi(\underline{x})}{\sum_{\underline{x}} \Lambda_{ij}^{(r)}(\underline{x}) \pi(\underline{x})} \quad (16)$$

and can also be seen to be independent of the class r . Thus, by computing the blocking probability on the equivalent single-class path, we can obtain the solution to the multi-class path.

4 Blocking Probabilities in Mesh Topologies

The solution to single-class networks with a mesh topology, wavelength converters at an arbitrary subset of nodes, and static or fixed alternate routing, has been presented in [21, 19]. This solution involves decomposition of the network into short path segments with two or three hops, and analyzing these approximately using expression (4), or the corresponding expression for a 3-hop path which can be found in [21]. The solutions to individual segments are appropriately combined to obtain a value for the blocking probability of calls that traverse more than one segment. The effect of the wavelength continuity requirement is captured by an approximate *continuity factor* that is used to increase the blocking probability of calls continuing to the next segment to account for the possible lack of common free wavelengths in the two segments. The process repeats until the blocking probabilities converge. By applying the transformations in (11), the same algorithms may be used to calculate blocking probabilities for multi-class networks. Specifically, we use these steps for a network with R classes of calls:

1. **Path decomposition:** Decompose the multi-class mesh network topology into L single-path sub-systems using the algorithm in [19].
2. **Time-reversible process approximation:** For each single-path sub-system, modify the arrival process as given by expression (9) to obtain an approximate time-reversible Markov process for the path.
3. **Class aggregation:** For each sub-system, apply the transformations in (11) to obtain an equivalent single-class path sub-system.
4. **Calculation of blocking probabilities:** For each path sub-system, obtain the blocking probabilities as follows. If the path is at most three hops long, use expressions (16) and (10) directly. If the path sub-system is longer than three hops, analyze it by decomposing it into 2- or 3-hop paths which are solved in isolation, and combine the individual solutions to obtain the blocking probabilities along the original longer path (see [18]).
5. **Convergence:** Repeat Steps 2 to 4, after appropriately modifying the original arrival rates to each single-path sub-system to account for the new values of the blocking probabilities obtained in Step 4 (see [19]), until the blocking probabilities converge within a certain tolerance.

5 Evaluating Call Blocking Probabilities with Multicast Calls

In this section, we examine how the path decomposition technique can be used to solve for call blocking probabilities in multi-class wavelength routing networks with multicast calls. We first assume that multicast call arrivals are also Poisson. Then, we solve for state-dependent Poisson arrivals using an approximation algorithm.

5.1 Multicast calls with Poisson arrivals

Consider a single-class wavelength routing network with only unicast calls. To this network, let us add a single-class of multicast calls which are routed using static or fixed alternate routing. Under static routing, a single multicast tree is available for a call between a given source and destination set. If the call cannot be established over the tree due to lack of free wavelengths, then the call is blocked. With fixed alternate routing, a (typically small) number of multicast trees are available for each multicast call. Upon a call arrival, the corresponding trees are considered in a fixed order for carrying the call. In this case, the call is blocked if no free wavelength is found in any of the trees.

Consider a multicast call from source node i to a set of destination nodes $V = \{j_1, j_2, \dots, j_v\}$. There are two possibilities. First, the source node i , and all the destination nodes may lie on a single linear path. Let j_m and j_n be the nodes at the extremities of this path (j_m and j_n can be any of the nodes in $\{i\} \cup V$). For purposes of evaluating call blocking probabilities, this multicast session is equivalent to a unicast call between node j_m and j_n . Let λ_{j_m, j_n} be the Poisson arrival rate of unicast calls between nodes j_m and j_n , $\frac{1}{\mu_{j_m, j_n}}$ be the mean of their exponential holding time, and $\rho_{j_m, j_n} = \frac{\lambda_{j_m, j_n}}{\mu_{j_m, j_n}}$. Let $\lambda_{i, V}$ be the Poisson arrival rate of multicast calls from source node i to a set of destination nodes V , $\frac{1}{\mu_{i, V}}$ be the mean of their exponential holding time, and $\rho_{i, V} = \frac{\lambda_{i, V}}{\mu_{i, V}}$. Then, the given multicast network can be analyzed as a unicast network with the following simple modification:

$$\rho_{j_m, j_n}^{(new)} = \rho_{j_m, j_n} + \rho_{i, V} \quad (17)$$

The resulting network may be analyzed using the path decomposition algorithm in [21].

In general, however, the source and destination nodes of a multicast call need not lie on a single linear path. But even in this case, the multicast tree may be broken up into several linear segments, where each segment starts at the source (or a branch point in the tree) and terminates at a branch point (or a destination node). This network can be also analyzed using the path decomposition algorithm in [21] by considering each segment of the multicast tree as a sub-system. In turn, each resulting sub-system may be analyzed as a unicast network after applying the modification shown in (17) for each multicast group using the sub-system. The blocking probability of a multicast call can then be expressed in terms of the blocking probability on the various segments to which its multicast tree has been decomposed. Note that, since the decomposition algorithm in [21] can handle fixed alternate routing for unicast calls, it can be extended in a straightforward manner to handle fixed alternate routing for multicast calls.

To analyze a multi-class network, we first construct an equivalent single-class network (including both unicast and multicast calls) in the manner described in Section 3.2. Then, the modification in (17) is applied to this equivalent single-class network to obtain a single-class network with only unicast calls. Finally, the decomposition algorithm in [21] is applied to the resulting network.

5.2 Multicast Calls with State-Dependent Poisson Arrivals

Our discussion in the previous section assumed that multicast calls arrive according to a Poisson process. In practice, however, it is unlikely that there will be several concurrently active multicast sessions among the same set of source and destination nodes. To account for this fact, we now consider and analyze approximately a state-dependent Poisson arrival model for multicast calls.

Let us assume that there may be at most one active multicast session for each multicast group, defined by the ordered pair $G_i = (i, V)$, where i is the source node and V is the set of destination nodes. The ordered pair G_i may also be used to represent the route of the multicast group. Likewise, the route of a unicast call from node i to node j is denoted by the ordered pair (i, j) . Let $\underline{R}$ represent the total number of routes in the network representing both unicast and multicast calls.

We first consider the single-class case. Let $\lambda_{(i,j)}$ be the Poisson arrival rate of calls on a unicast route. Let $\frac{1}{\mu_{(i,j)}}$ be the mean of the exponential holding time. Let $\frac{1}{\lambda_{G_i}}$ be the mean of the exponentially distributed time until the arrival of a multicast call to group G_i , conditioned on there being no active sessions. The arrival process of multicast calls is, therefore, state-dependent Poisson. Let $\frac{1}{\mu_{G_i}}$ be the mean of the exponential holding time. We use a Poisson approximation to obtain the blocking probabilities of this type of network, which we now proceed to describe.

Suppose the wavelength routing network has an unlimited number of wavelengths. Then, the state of the network may just be defined by the number of active calls on each route, including multicast routes. Let $N_{\underline{r}}(t)$ be the number of active calls on route $\underline{r}$ at time t . The state of the system at time t may, therefore, be represented by $S(t) = (N_1(t), N_2(t), \dots, N_{\underline{R}}(t))$. Let $\pi(\underline{n})$ represent the probability $P(S(t) = \underline{n})$ in steady state. When there are an unlimited number of wavelengths, the Markov chain $S(t)$ has a product-form solution, and $\pi(\underline{n})$ may be written as:

$$\pi(n_1, n_2, \dots, n_{\underline{R}}) = \prod_{\underline{r}=1}^{\underline{R}} P_{\underline{r}}(n_{\underline{r}})$$

where $P_{\underline{r}}(n_{\underline{r}})$ is the marginal probability that the number of active calls on route $\underline{r}$ is $n_{\underline{r}}$ in steady state. If $\underline{r}$ is a unicast route, $P_{\underline{r}}(n_{\underline{r}})$ is given by:

$$P_{\underline{r}}(n_{\underline{r}}) = \frac{1}{n_{\underline{r}}!} \times \left(\frac{\lambda_{\underline{r}}}{\mu_{\underline{r}}} \right)^{n_{\underline{r}}} \times \exp \left(-\frac{\lambda_{\underline{r}}}{\mu_{\underline{r}}} \right)$$

For multicast routes, we have:

$$\begin{aligned} P_{\underline{r}}(0) &= \frac{\mu_{\underline{r}}}{\mu_{\underline{r}} + \lambda_{\underline{r}}} \\ P_{\underline{r}}(1) &= \frac{\lambda_{\underline{r}}}{\mu_{\underline{r}} + \lambda_{\underline{r}}} \\ P_{\underline{r}}(n_{\underline{r}}) &= 0 \quad \text{if } n_{\underline{r}} > 1 \end{aligned}$$

Thus, the mean arrival rate of calls on multicast route $\underline{r}$ is given by $\frac{\lambda_{\underline{r}}\mu_{\underline{r}}}{\mu_{\underline{r}} + \lambda_{\underline{r}}}$. We substitute this to obtain an approximate state-independent Poisson arrival model for multicast calls. Thus, the arrival rate of calls to multicast route $\underline{r}$ is assumed Poisson with mean $\Lambda_{\underline{r}} = \frac{\lambda_{\underline{r}}\mu_{\underline{r}}}{\mu_{\underline{r}} + \lambda_{\underline{r}}}$. Given Poisson

arrivals, we can apply (17) discussed in Section 5.1 to obtain an equivalent single-class network. With a multi-class network, we first apply the Poisson approximation to each class of multicast calls with state-dependent Poisson arrivals. From this, an equivalent single-class network may be constructed in the same way as shown in Section 3.2. Then, the modification in (17) is applied to this single-class network.

6 Numerical Results

In this section, we validate the approximate method described in Section 4 by comparing the blocking probabilities for each route as obtained from the approximate method with those obtained through simulation of the exact model.

6.1 Multi-Class Networks with Unicast Traffic

We first provide results for 3-hop paths which are the basic blocks of our decomposition algorithm. In Table 1 we show the arrival and service rates for calls on each route (i, j) , $1 \leq j \leq 3$, of a 3-hop path. There are $R = 3$ classes of calls for each route. In Figure 2, we plot the blocking probability against the number W of wavelengths for three (out of the six) types of calls in this path: calls using Route $(1, 1)$, i.e., the first hop of the path, calls on Route $(2, 3)$, that is, those using the last two hops, and calls on Route $(1, 3)$ using all three hops of the path. As we can see, the blocking probability decreases as W increases, as expected. We also observe that calls on Route $(1, 3)$ (i.e., calls using all three hops of the path) experience the highest blocking probability, again as expected. Most importantly, however, we can see that there is good agreement between the values of the blocking probabilities obtained through our analytical technique and those obtained through simulation. Similar results have been obtained for different values for the arrival and service parameters and for different number of classes, indicating that our approximate method is accurate over a wide range of network characteristics.

Next, we consider a network with a topology similar to that of the NSF network, shown in Figure 3. There are 16 nodes, and 240 uni-directional routes. There are three classes of calls on each route. The arrival and service rates of calls of a particular class are the same on each route, and are shown in Table 2. The blocking probabilities are plotted in Figure 4 for four routes, as a function of the number of wavelengths on each link. Route A is a single-hop route from nodes 1 to 5. Route B has two hops, connecting node 1 to node 3 via node 2. Route C has three hops, connecting node 1 to node 4 via nodes 2 and 3. Route D has four hops, connecting node 4 to node 5

Table 1: Arrival and service parameters for a 3-hop path

Route (i, j)	Class 1		Class 2		Class 3	
	λ	μ	λ	μ	λ	μ
(1,1)	3.0	3.0	1.0	3.0	1.0	3.0
(1,2)	1.0	6.0	2.0	6.0	1.0	6.0
(1,3)	1.0	2.0	1.0	2.0	2.0	2.0
(2,2)	1.0	3.0	1.0	3.0	2.0	3.0
(2,3)	1.0	2.0	1.0	2.0	3.0	2.0
(3,3)	3.0	6.0	1.0	6.0	1.0	6.0

Figure 2: Blocking probabilities for a 3-hop path and the parameters shown in Table 1

via nodes 3, 2 and 1.

From Figure 4 we can see that the length of the path used by a call considerably affects the blocking probability experienced by the call, an observation that is consistent with all our previous results in this section. Specifically, for a given number W of wavelengths, the blocking probability increases with the number of hops in a route, such that calls on Route A (a single-hop path) have the lowest blocking probability while calls on Route D (a four-hop path) the highest. Further, the results indicate that our approximation method can be used to estimate accurately the blocking probabilities for all calls in the network.

6.2 Multi-class Networks with Multicast Traffic

In this section, we compare the results of the approximate algorithm for networks with multicast calls to simulation results. First, we consider a network with the simple “T” topology shown in Figure 5. The network is composed of five nodes and two linear paths. In addition to each of the twenty possible unicast routes, we assume the following sets of links to carry multicast calls: (i) Multicast tree $\{1, 2, 3\}$ and (ii) Multicast tree $\{1, 2, 3, 4\}$.

For the results presented here, we assumed that each of the above multicast trees can carry at most two concurrently active multicast sessions. For instance, each multicast tree is used by two multicast groups, each of which allows at most one active multicast session at any given time. We also assumed three classes of calls on each (unicast or multicast) route. Table 3 shows the values of the arrival and service rate parameters assumed. As we can see, this multi-class network is very general since the arrival and service parameters depend not only on the class of a call, but also on the route taken (i.e., the source and destination(s) of the call). This example illustrates the potential of our path decomposition technique which can be used to analyze wavelength routing networks with multiple classes of traffic, and each class consisting of calls with a wide range of behavior.

In Figure 6, we plot the blocking probability values against the number of wavelength per link for a few of the routes. Two sets of results are shown, one set obtained by simulation and one obtained by approximate analysis. We again observe the familiar pattern of higher blocking probability for routes consisting of larger number of links. In particular, multicast calls experience higher probability than unicast calls, as expected. (*Note:* Figure 6 plots the probability of *full* blocking [11], whereby a multicast call is blocked when it is impossible to establish the call between the source and all the destination nodes. It is, however, straightforward to compute the probability of *partial* blocking, whereby a multicast call proceeds with a subset of the destinations whenever

Figure 3: The NSF network

Table 2: Arrival and service parameters for the NSF Network

	Class 1	Class 2	Class 3
λ	0.1	0.2	0.15
μ	3.0	6.0	4.0

Figure 4: Blocking probabilities for the NSF network

free wavelengths are not available in some links of the multicast tree.) As in previous figures, we again observe the good agreement between analysis and simulation over a range of call blocking probability values which spans four orders of magnitude.

Next, we consider a 3×3 torus network with multicast calls. The network topology is shown in Figure 7. There are 18 one-hop unicast routes and 18 two-hop as shown in the figure. There are six multicast trees, each of which may be listed by the set of links they consist of: $\{1, 5, 6\}$, $\{2, 4, 7\}$, $\{3, 1, 10\}$, $\{3, 10, 12\}$, $\{6, 10, 13, 14\}$, and $\{8, 13, 15\}$. We assume these constitute routes 37 to 42. Each multicast route is assumed to carry at most two concurrent multicast calls.

There are three classes of calls (unicast or multicast) on each route. The arrival rate of calls of Class 1 on each unicast route was assumed the same and set to 0.4 (calls per time unit). The arrival rate of Class 1 calls on each multicast route, conditioned on there being no existing multicast calls was assumed equal to 0.4. The mean holding time was assumed the same on all routes and equal to $1/3$ time units. The arrival rate of calls of Class 2 on each unicast route was assumed the same and set to 0.6 (calls per time unit). The arrival rate of Class 2 calls on each multicast route, conditioned on there being no existing multicast calls was assumed equal to 0.6. The mean holding time was assumed the same on all routes and equal to $1/5$ time units. The arrival rate of calls of Class 3 on each unicast route was assumed the same and set to 0.5 (calls per time unit). The arrival rate of Class 3 calls on each multicast route, conditioned on there being no existing multicast calls was assumed equal to 0.5. The holding time was assumed the same on all routes and equal to $1/6$ time units.

In Figure 8, we plot the call blocking probabilities obtained through the approximation method and through simulation, assuming random wavelength allocation. We also plot two additional sets of simulation results: one set assuming first-fit allocation and no wavelength conversion, and one set for a network with converters at every node (in which case wavelength allocation is not an issue). It may be observed that the results obtained by approximate analysis are closer to the results for the first fit simulation for the unicast route 3 which has a single hop. However, for multiple hops and for multicast routes, the simulation and analytical results with random wavelength allocation are in closer agreement.

6.3 Discussion

Results similar to the ones presented in Figures 2,4,6, and 8 have been obtained for a wide range of traffic loads and different classes of calls, and for other network topologies. Our main conclusion is that our approximate analytical technique can be applied to compute the call blocking probabil-

Figure 5: A simple multicast network topology

Table 3: Arrival and service parameters for the simple multicast network

Route		(0, 1)	(0, 2)	(0, 3)	(0, 4)	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 4)	(3, 4)	{1, 2, 3}	{1, 2, 3, 4}
Class 1	λ	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0
	μ	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
Class 2	λ	0.5	1.5	0.5	1.5	0.5	1.5	0.5	1.5	0.5	1.5	0.5	1.5
	μ	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0
Class 3	λ	0.5	0.5	1.0	0.5	0.5	0.5	1.0	0.5	0.5	0.5	1.0	0.5
	μ	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0

Figure 6: Call blocking probability in simple multicast network

Figure 7: The 3×3 torus network

Figure 8: Call blocking probability in torus multicast network

ities in wavelength routing networks of realistic size and topology. Furthermore, the approximate technique affords a significant reduction in the time for computation of blocking probabilities. For instance, the simulation program takes approximately 100 minutes while running on a Sun Ultra 10 workstation, to compute call blocking probabilities for the network with a topology similar to the NSF network. The approximation method, on the other hand, takes less than a minute for the same network. In addition, to obtain statistically significant results, the simulation program runs until there are at least 10^5 call arrivals of every class. For call blocking probabilities less than 10^{-4} , the simulation program must consider more than 10^6 call arrivals of each class, resulting in an even longer computation time.

7 Concluding Remarks

We have considered the problem of computing call blocking probabilities in multi-class multicast wavelength routing networks which employ the random wavelength allocation policy. Our approach consists of modifying the call arrival process to obtain an approximate multi-class network model, using class aggregation to map this to an equivalent single-class network, and employing path decomposition algorithms on the latter to determine the call blocking probabilities.

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