ABSTRACT

ISUKAPATI, ISAAC KUMAR. Intersection Control as a Shared Decision Process. (Under the direction of Dr. George F. List).

This research introduces a novel way to develop a signal control strategy in which drivers (figuratively and literally) play an active role in making control decisions. A bid-based control strategy is proposed to explore these ideas.

In the proposed framework there are high and low value-of-time drivers that interact with movement managers by paying compulsory and voluntary fees to reduce their delays. The movement managers, one for each turning movement, place bids for control of the intersection. A municipality receives the bids and decides which movement managers win. First-price bidding is employed. The thesis describes what these players do, how they interact, influence of data availability on how they behave, and how their decisions influence the resulting signal timings.

To facilitate the examination of these ideas, an agent-based simulation model of the strategy is created in Python. The simulation model involves an intersection involving two single-lane one-way streets with a total intersection volume held constant at 1500 veh/hr/lane. The analysis for three traffic flow combinations is presented. For a given input volume on the facility, the program creates a sequence of arrival headways for both approaches. The arrival headway distribution is generated from a shifted negative exponential distribution with a minimum headway of 1.5 seconds and an average headway consistent with the arrival flow rate. A Python-based model of actuated control is also developed for the purposes of benchmarking the analysis. These two simulation models exist within the same analysis program.

Creation of the simulation model illustrates that fact that signal control strategies can be created by using auctioning principles. The Preliminary findings from the simulation are encouraging and suggest that there is value in viewing traffic signal control as a shared decision process.

Intersection Control as a Shared Decision Process

by
Isaac Kumar Isukapati

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Civil Engineering

Raleigh, North Carolina

2014

APPROVED BY:

Dr. Alain L. Kornhauser
(Princeton University)

Dr. Alan F. Karr
(UNC Chapel Hill)

Dr. Billy M. Williams

Dr. Nagui M. Rouphail

Dr. George F. List
Chair of Advisory Committee

DEDICATION

Were the whole realm of nature mine;

That were an offering far too small;

Love so amazing, so divine;

Demands my soul, my life, my all….

I dedicate this thesis first to the Glory of my maker and savior Lord Jesus Christ, and then to my late mom… I love you mom!

BIOGRAPHY

Isaac Kumar was born in India and received Bachelors of Engineering from Andhara University. He started graduate studies at North Carolina State University in fall of 2006.

ACKNOWLEDGMENTS

- Thanks to my parents and extended family: especially want to thank my dad, late mom, my dear brother Philip, and his wife Shivani for supporting me throughout the process
- Thanks to Dr. List for being a caring and insightful mentor. I am deeply indebted to him for shaping me the researcher that I am today. You are epitome of what an ideal advisor
- Thanks to my committee members for their continued guidance. I specially want to thank Dr. Kornhauser, and Dr. Karr for their insights
- A special thanks to Dr. Hoon Hong for his wise counsel whenever I needed it
- Thanks to Raja aunty & Uncle John for their continued support. Both of you are very special to me
- Thanks to Andy & Erica Owens for the care that you showed towards me when I was in the hospital
- Thanks to Dr. Richard Chulie for caring and praying for me when I was in the hospital. I am deeply indebted to you
- Thanks to my friends in graduate school Naresh, Sashi, Shalini, Venu, Senganal, Fatemeh, Sarah & Yuriy; a special thanks to Sashi, and Yuriy for the long drives and philosophical conversations at 1 AM
- Thanks to Ajay & Prasanna for your support and the beautiful fellowship that we share
- Thanks to Ajay Kumar for showing me how amazing computer science actually is
- Thanks to John & Sharyn Gibson – both of you are very special. John thank you for the conversations that I had with you (music, theology, technology, engineering, history just to name a few).
- Thanks to Danielle, Anna and Vasek for all our nerdy conversations.
- Thanks to IBS family: Mrs. McGee, Kosobucki's, Whites, Stevens, Jenkins (Chase & Laura both of you are amazing)
- Thanks to Jenn & Ani for all the good times we had
- Thanks to FBC church family for the wonderful fellowship that I had there
- Thanks to Steve Edwards for his constant encouragement, exhortation, and other nerdy conversations. You are special dear brother!
- Thanks to Emmy, Teddy, Amy & Ivan Kandilov for the love that you showered upon me. Both of you are very special to me; a special thanks to my little buddy "Emmy" for playing with me whenever I needed a break.
- Thanks to Justin, Matt & Sijy Abraham for your wonderful friendship. Three of you are very, very special. Thank you for your care and support; Justin I want to extend special thanks to you for proofreading all my documents.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

1. Introduction

The role of economics is instrumental in transportation decision making. However, only in recent years, have competitive market ideas been proposed as a paradigm for controlling urban road traffic systems (68-70). In this research, auction based principles and a paradigm akin to a game (but not derived from game theory) is used to create a signal control strategy. That is, instead of using the standard techniques of minimum greens, maximum greens, and gaps to control the signal indications, an economically based game structure is employed. The intersection's space is viewed as a scarce commodity whose use is determined through a bidding process. Movement Managers oversee the vehicle departures for specific turning movements. Arriving motorists pay the Movement Managers to arrange times of entry for them. Movement Managers submit bids for use of the intersection's space and the highest bidders win. Distributed processing and connected vehicle technology (1-8) are the mechanism by which implementation would be feasible. The value of these ideas is that one can study the economics that underlie the control.

1.1. Problem Motivation

Doctoral research focuses on new ideas, and finding creative solutions to important societal problems. One such problem is traffic signal control. The evolution of this field in the last several decades has been significant. It has progressed from simple fixed-time control to actuated control and distributed control. Control and systems engineers have advanced the state-of-the-art and state-of-the-practice based on their domain knowledge of instrumentation and control.

An interest in both agent-based modeling and traffic signal control motivated this research to study signal control from the perspective of economic theory. While the topic can clearly be pursued from the perspective of intellectual curiosity, it is possible today to picture how it might be implemented. Vehicle to infrastructure (V2I) communication has reached a stage

where it is reasonable to think about signals talking to vehicles and vice versa. An extensive body of research that focuses on use of these technologies is enhancing both observability and the safety of transportation networks (1- 8).

Philosophers share a common interest with economists in the sense of maximizing human welfare. In addition, philosophers are concerned with logical justification of actions with respect to their expected outcomes. The thesis that underpins this proposed research is that a control strategy created based on auction theory principles would provide interesting insights into the costs of delay at a signalized intersections and aide our understanding of how to make investment decisions.

1.2. Problem Formulation

The processing space of an intersection is viewed as a scarce commodity. In that sense, one can look at that commodity from an economic perspective and determine how to allocate its use, but to the best of this author's knowledge, this has not been done. Rather, the use of this commodity in time and space has been studied through system engineering concepts related to control theory: determining what movements should be allowed to move when, for how long, to optimize performance measures such as delay, queues, stops etc. The advent of V2I makes it possible to consider the control problem from an economic perspective: having the users pay to get responsive treatment. The expectation is that such ideas will produce solutions that are different from those obtained today; solutions whose economics is understood. So, this research proposes to investigate the ways of creating signal control strategies using auction theory principles.

1.3. Research Contributions

This research includes following contributions.

1.3.1. Bid Based Control Strategy

This research has introduced a novel way to develop a signal control strategy that is based on economic considerations. The control strategy creates signal timing plans whose economics can be understood.

1.3.2. Agent-based control

An agent-based model of intersection control is presented. Using a game-like setting, movement managers (computer applications) bid for green time on behalf of vehicles associated with specific turning movements. The movement managers also collect fees from arriving vehicles, determine what those fees should be, and strive to minimize the delays for their constituents while not charging more than the minimum required to do so. In each turn of the game, winning movement managers discharge a single vehicle from the front of their queues. An additional entity (e.g., a municipality) oversees the game and manages the bidding process.

1.4. Document Organization

Chapter 2 reviews related literature focusing on resource allocation as well as isolated intersection control. Chapter 3 provides overview of the methodology. Chapter 4 presents a detailed discussion on how to create a realization of the game. Chapter 5 presents details on simulation experiments and analysis; Chapter 6 provides concluding remarks and future direction of the research.

CHAPTER 2: RELATED RESEARCH

2. Introduction

Resource allocation problems deal with the assignment of resources to activities and the scheduling of those activities. For example, resource allocation pertains to the allocation of landing slots at airports, workstations in job shops, CPU time slices for computers, and beds and medical personnel in healthcare facilities. Moreover, simulation-based games offer a way to study these problems.

The allocation of intersection space through signal control is a problem to which resource allocation techniques can be applied. The signal control can be seen as a process that assigns intersection space (the resource) to vehicles. It does this by separating the intersecting vehicle movements in space and time. These separations are manifest in the phase sequences and switching times. (Here, a phase is considered a combination of movement greens that occur simultaneously, as would be the case with simultaneous eastbound and westbound left turns.) Ostensibly, the switching strategy is not only safe but efficient. Not only can intersection control be viewed as a resource allocation problem, but it can also be seen as a game in which a shared decision process among various players determines the phase sequence and switching times.

Ideas from competitive markets and multi-agent systems can be used to create the models of such control systems. Hence, this review of related research focuses on instances in which market-inspired approaches or multi-agent systems have been applied to traffic signal control. Furthermore, the research's focus on isolated intersection control makes it necessary to review prior work in that area, but not for network control. Readers interested in network signal timing strategies can find comprehensive reviews in (9-32).

While the agent-based simulation model created in this doctoral work is not technically consistent with or derived from formal game theory, it is useful to provide a brief review of

work in this domain. These efforts helped shape the ideas upon which the simulation model is based. Game theory is the branch of mathematics that focuses on studying strategic interactions among various players. The players often want to maximize their own success in response to the strategies used by the other players in the game. Game theory pertains to many situations. Resource allocation is one of them. For example, Grether et al. (72) use game theory to study the allocation of landing rights. Adler (73) presents a two-stage game-theoretic framework to solve the landing slot-allocation problem. Chang et al. (74) use cooperative and non-cooperative game mechanisms to study the bandwidth allocation problem in wireless networks. Ahmad et al (75) use game theory for scheduling tasks on multicore processors. McMillan (76) use an auction-based game to study the sale of radio spectrum rights. Zhou et al. (77) propose a game theory-based approach to study job scheduling in networked manufacturing. Kutanoglu, Wu (78) apply a combinatorial auction to solve a distributed resource scheduling problem. Li et al. (79) use cooperative gaming to automate the process of planning and scheduling.

2.1. Market-inspired ideas applied to traffic signal control

In recent years, researchers have begun applying agent-based techniques to traffic control. Agent-based control involves using distributed intelligence, often autonomous, to develop problem solutions. For example, Choy et al. (80) present a cooperative, hierarchical, multiagent system for real-time traffic signal control. The control problem is divided into sub-problems, and each sub-problem is handled by an intelligent agent that makes decisions using principles from fuzzy-neural networks. The decisions made by lower agents are mediated by higher level agents. This means the multi-agent system is hierarchical.

Agent-based systems often use learning techniques to adapt to the evolving traffic conditions. For example, Steingrover et al. (81), Weiring (82) employ a reinforcement learning technique to minimize the overall waiting time of the vehicles. Here the learning task is represented as a feedback loop focused on the aggregated waiting times for individual vehicles.

Some researchers suggested adapting market-based ideas to traffic signal control. For example, Dias et al. (83) suggest a market-based coordination mechanism for signal coordination. These mechanisms are analogous to the functioning of commodity markets. The agents that regulate the infrastructure act as a team to achieve a desirable solution. Schepperle and Bohm (84) proposed an auction-based policy for intersection control. Here, the intersection control agent starts an auction for the earliest departure time slot among the vehicles that are approaching the intersection on each lane. Only the lead vehicle in each queue is allowed to participate in the auction.

Similarly, Vasirani and Ossowski (69, 70) propose a multiagent approach to design a competitive computational market for the distributed allocation of an urban road network. In specific, they propose intersection manager – driver model in conjunction with cooperative learning techniques to coordinate the prices of individual intersections. This framework uses walrasian auction system for selling intersection schedule slots. Basically walrasian auction involves a set of buyers and a set of suppliers. At a given time, each buyer in the set of buyers notifies the suppliers the quantity of resources he/she going to buy at a preset published price. Intersection manager in turn uses this information to compute total demand and excess demand. Using this information the model updates intersection reserve prices (reserve prices are upward adjusted in case of excess demand and downward adjusted in case of excess supply). Drivers choose routes based on their own preferences between time and cost, participating in intersections auctions as long as they are willing to meet the reserve price. So, in that sense the intersections that drivers pass through are a function of their willingness to pay the preset price for using those intersections. Although the research work conducted in this thesis bears some similarity to this approach, there are some key distinctions. Firstly, the game-like framework proposed in this research does not constrain drivers from using the intersection space by setting undesirable costs of service. It rather imposes an initial fee on all drivers (this is to cover marginal costs of operating the signal) and enables drivers to make decisions about voluntary contributions to help expedite their travel times. Secondly, this doctoral work views traffic signal control as a shared decision

system, i.e., every driver that passes through the intersection influences the performance of the signal. Thirdly, this doctoral work generalizes the notion of auctions subjected to constraints set by traditional signal control theory (use of minimum greens, gaps, clearance intervals) whereas intersection managers proposed by Vasirani et al., control the decision nodes of an O-D in an urban network. Therefore in that sense these managers do not directly influence the operations at a signalized intersection.

2.2. Classic control strategies for isolated intersections

Inasmuch as the focus of this research is on signal timing for intersections, it is important to review more classic control strategies. These methods can be placed in three categories: 1) fixed-time control, 2) actuated control, and 3) adaptive control. A brief discussion of each is presented below.

2.2.1. Fixed-Time Control

In fixed-time control a pre-determined phase sequence is combined with fixed green times (and inter-green times). The same signal timings repeat from one cycle to the next. Obviously, fixed-time control is unresponsive to any variations in demand that emerge from cycle to cycle. The signal timings may vary across the course of a day, but that is because a sequence of fixed timing plans is selected from a family of predetermined plans that are developed off-line on the basis of historical traffic data (37).

2.2.2. Actuated Control

Actuated control uses detector inputs to determine the green time durations. Movement sequences run in parallel, called rings, and they resolve the spatial conflicts. The ring structure ensures that movements end simultaneously to ensure that spatial conflicts do not arise. Green time durations are determined by minimum greens, gap timers, and maximum greens. The resulting phase patterns (movement combinations) and switching times are superior to fixed-time control (38).

Actuated control can further be classified into fully-actuated control and semi-actuated control (39). We discuss fully-actuated control here. Semi-actuated control pertains to actuated signals in coordinated networks, and this paper does not focus on that domain.

In fully-actuated control, detectors are placed on all of the movements, upstream of the stop-bar. The detectors identify the passage or presence of vehicles. The detector inputs enable the signal controller to create phase sequences (movement combinations) and switching times that are response to the traffic streams. Typically, a minimum green is followed by gap timing; and if a gap timer reaches a maximum, which means a minimum headway has been exceeded, the green time ends. In some cases, the minimum green time is variable. Time is added if vehicles arrive while the signal is red. In volume-density control, the maximum allowable gap reduces linearly after a prescribed amount of green time has elapsed. The green also ends if a maximum green is reached.

Although actuated control works well, it has two major drawbacks. The first is that the phase sequence options are restricted by the ring definitions. The controller does not have the option to switch without restraint from one movement combination to another. The second is that queue length does not influence decisions about whether or not to stop the extension of green. The green time decisions are made strictly on the basis of gap sizes and minimum and maximum greens. For example, at an intersection where the flow of traffic on various approaches is significantly different, vehicles on the approaches with lower flows may experience high delays. This is due to lack of opportunities for gap-out on approaches with higher flows. Thus, it can be inferred that when traffic demand is heavy on all approaches, actuated control may not produce the best performance (38).

2.2.3. Adaptive Control

Adaptive control is a hybrid strategy. The phase sequences and the switching times are based on fluctuations in the traffic flows. The state of the entire intersection is taken into account in

deciding whether to continue green for the current phase or to switch to a different phase. However, the detectors are not always linked directly to the movements, so the green times are not necessarily determined by minimum greens, gaps, and maximum greens. The phase durations can vary from one cycle to the next. Because the phase sequences can be changed, adaptive control has an ability to outperform both pre-timed and actuated control. Examples of adaptive control include MOVA: Microprocessor Optimized Vehicle Actuations (40-44), CRONOS: ContROl of Networks by Optimization of Switchovers (45, 46) and SPPORT: Signal Priority Procedure for Optimization in Real Time (47-49), OPAC: Optimized Policies for Adaptive Control (50-53), COP: Controlled Optimization of Phases (54), PRODYN: Programmation Dynamique (55-57), SCOOT (58), and SCATS(59). Descriptions of each of these can be found in Appendix B.

2.3. Conclusions

This chapter has presented an overview on family of resource allocation problems and provided a few references on prior attempts to apply agent-based modeling ideas to explore such problems. Furthermore, since the focus of this research is traffic signal control, an overview of existing approaches to signal control at isolated intersections and/or agent-based control. Most of the control system strategies for isolated intersections are based on systems engineering concepts that pertain to control theory, but not economic theory. However, in recent years, competitive market ideas have been proposed for controlling urban road traffic systems (68-70), but these are not applied to fundamental problem of traffic signal control.

CHAPTER 3: METHODOLOGY

3. Intersection Control in a Game-Like Structure

As mentioned earlier, the intersection control problem is treated here in a game-like fashion where there are movement managers (figuratively and literally) that negotiate for use of the intersection on behalf of the drivers making a specific turning movement. This means at a typical four-leg intersection there would be eight movement managers: four for the through-and-right movements; and four for the left turns. The outcome is a sequence of wins, for specific movement managers, which translates into vehicle discharges by movement. In effect, playing the game creates the signal timings.

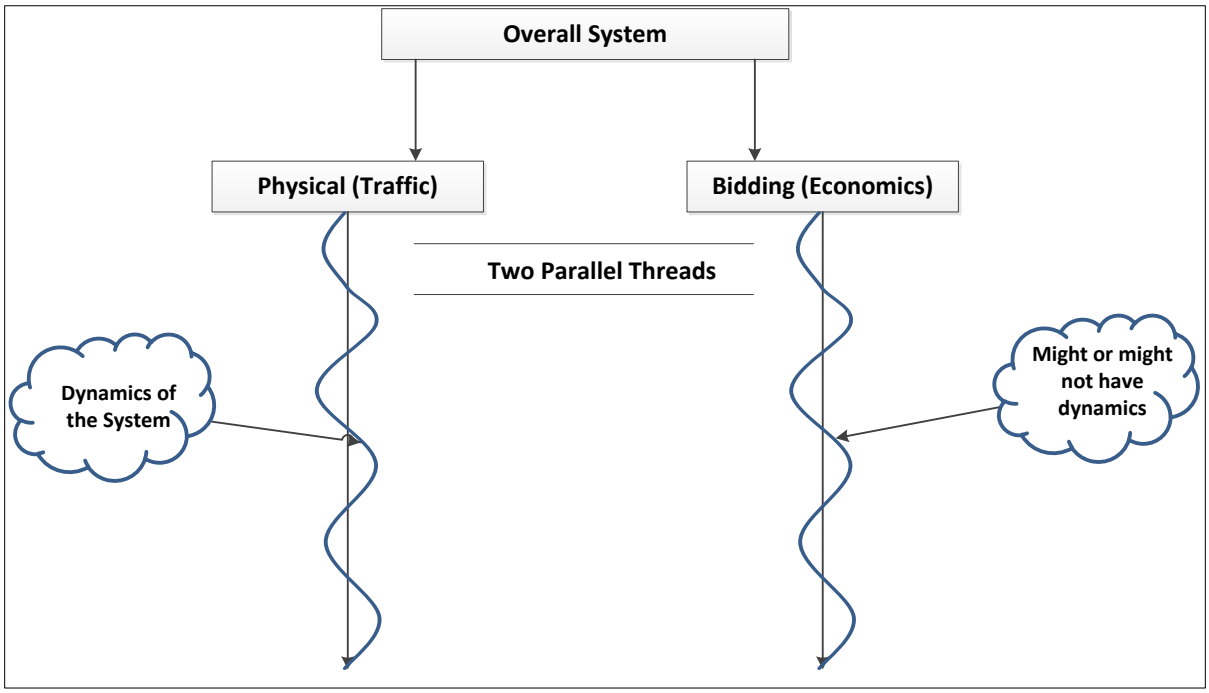


Figure 1: Representation of the overall system

As presented in Fig. 1 the game-based formulation has two worlds that evolve in parallel. The first is the physical system. The second is the game. The game outcome affects the

physical system and vice versa. Game results determine what vehicles get to be discharged, and those discharge events affect the bidding decisions of the movement managers.

3.1. Description of the players

The players in the game are as follows:

Drivers: Drivers are players who travel through the network. They arrive at the intersection because it is in their path. They want to pass through the intersection in minimum time. Drivers can be either active or passive players in the game. If they are passive, then they pay the movement manager to arrange a time for them to go and then they wait for their appointed entry time. On the other hand, if they are active players, they constantly evaluate the system, and take actions to expedite their service.

Movement Managers: Movement managers are players who bid against one another for use of the intersection. When they win, they release motorists from their waiting queue. They develop bidding strategies that maximize the likelihood of their winning. The bidding strategies can be developed either from principles of probability theory or state-observer systems. They collect fees from their drivers and then pay the municipality for the use of the intersection. Movement managers have “bank accounts” into which they deposit payments from the drivers and from which they pay the municipality.

The Municipality: The municipality is the player that oversees the game. It manages the bidding process and determines which movement manager(s) win. The municipality also ensures that scarce societal resources are utilized optimally. In addition, the municipality creates the game and its rules.

3.2. Game Terminology

As with any game, rules govern how the game is played, and the relationships among the players. These rules are described below, in the context of the signal control game.

A ***time step*** is the increment by which the game proceeds. In the context of this discussion, the time step is 0.1 seconds. The state of the game is updated for every 0.1 seconds.

A ***discharge slot*** is an increment of time allocated for discharging a single vehicle; presently, it is 2.1 seconds. This value is chosen because it is typical saturation headway between vehicles being discharged at the stop bar.

An ***initial fee*** is the amount paid by arriving motorists to their respective movement mangers upon their arrival.

A ***nominal fee*** is the minimum cost for using the intersection.

Enfranchised manger is the movement manager that currently holds the intersection control.

Disenfranchised manager is the movement manager that doesn't hold intersection control currently, but is competing for it.

Delay is the additional time that drivers incur if their movement manager loses the current bid event.

Marginal cost of delay is the marginal cost associated with the delay.

Financial transactions occur between the movement managers and the travelers or movement managers and the municipality. The movement managers charge an ***initial fee*** to the travelers. Movement managers pay the municipality for discharging their vehicles – either what they bid – if they win a bid – or a ***nominal fee*** if no competitive bid takes place (because no other movement manager had vehicles wanting to use the facility in a particular discharge slot). The financial transactions also occur during each turn, subsequent to the bidding process, and prior to completion of the vehicle discharges. Funds are transferred

between the movement managers and the travelers and movement managers and the municipality.

Each movement manager has a bank ***account*** into which payments by the drivers are placed and from which payments to the municipality are made. Each movement manager monitors the balance in her/his account to ensure that it remains solvent.

Events are the actions that take place during the game. Three events take place in each ***time step***. The first is the bid. The bidding determines how the intersection is used in the next discharge slot. The second is one or more financial transactions. Effectively, time "stands still" while these two events take place. The third event is the "vehicle release". It takes place once discharge slot(s) have been assigned to specific movement manger(s). The vehicles then pass through the intersection. As indicated above, for any given movement, one vehicle per lane can be released on any given movement during a single discharge slot.

A ***turn of the game*** encompasses these three events: the bidding followed by the financial transactions, and then the vehicle discharge(s). Each turn ends when the "business" of the turn is completed: the transactions between the winning movement manager(s) and municipality are finished, the winning movement manager(s) have discharged their first vehicles in queue; the system information is updated, and the players are ready to participate in the next turn of the game. This process continues until the game ends (which is at the end of the analysis period).

A ***performance metric*** is a measure used to assess the quality of the signal control. Consistent with conventional practice, delay is used. Early ideas about delay were developed by Wardrop, (33) and analytic expressions for the delays were later presented by Webster (34, 35) and Newell (36) provides additional thoughts about the manner in which delay can be studied.

3.3. Realizations of the game

The concepts presented above must be translated into a specific realization of the game in order for the game to be played. A given realization is defined by: 1) the rules by which the game is played, 2) the bidding strategies, 3) the objectives being pursued by the players, and 4) the information being shared.

The ***rules*** determine how the bidding process takes place, the manner in which the movement managers can interact with one another, the amount of information shared with the bidders. For instance, the municipality gets to decide whether the bidding process is single-stage or two-stage. In addition, the municipality decides how much the winning movement managers pay: two examples are their bid amount (first price sealed bidding) or the amount bid by the second highest bidder (second price sealed bidding).

The ***bidding strategies*** determine how the movement managers develop their bids and what information they have when they do that. The movement managers might have the same or different objectives.

The ***objectives*** determine what the players are trying to do. The drivers might be trying to minimize travel times or delays. The movement managers might be interested in minimizing average delays for their drivers at as low a cost as possible. The municipality might be interested in equity or it might be interested in receiving a revenue stream that pays for the cost of the intersection and its maintenance.

Information exchanges also heavily influence how the game is played. For example, drivers might or might not want to reveal where they are going, what turning moves they want to make, when they will arrive at the intersection, how much they are willing to pay to be serviced, and whether they are interacting with other travelers or not. The movement managers might or might not know what bids were submitted by the other movement managers. The movement managers might or might not be able to share information.

3.4. Bidding strategies

The bidding strategies can be developed either by using principles from state-observer systems or from probability theory. The next two sub-sections present these details.

3.4.1. State-observer systems

In this class of bidding systems, the aim is to blend engineering attributes (length of dynamic queue, number of turns since last win, which is analogous to delay) with economic attributes (the account balances for the movement managers).

The variables in the bidding equation capture attributes of the system ('system variables'). Examples include queue length, account balance, and the numbers of turns since last win. The bidding systems based on these principles have two essential features: First, the movement managers use values of system variables that exist at the end of every turn of the game. These values are inputs for computing a bid for the next turn of the game. Second, a bid is computed using estimates of these system parameters from the previous turn, and known set of input parameters. The bidding equation based on these principles takes the following form:

$$bid_i^k = f\left\{Q_i^k, bb_i^k, w_i^k \mid \alpha_i, \beta_i, \gamma_i\right\} \quad \textbf{(1)}$$

Thus in this instance, the bid submitted by the movement manager for movement i in the kth turn (bid_i^k) is a function of the queue length on approach i (Q_i^k), the account balance for manager i (bb_i^k), and the number of turns since last win (w_i^k); the input parameters $\alpha_i, \beta_i, \gamma_i$ are either preset values or adjusted dynamically using the historic data. Finally, the bid to be submitted is obtained by substituting the values of input parameters and state based estimates in a pre-specified bid equation.

3.4.2. Probabilistic systems

In this class of bidding systems, movement managers make use of historical data (if available) in determining the optimal bid. For example, they can keep track of win/loss bids associated with every queue length that they see on their respective approaches. Using the historical information, they can develop the probability density function (PDF) associated with the winning bids for each possible queue length. Movement managers can then compute the bid to submit in a specific turn of the game in one of the following three ways: 1) maximum a posteriori estimate of PDF of winning bids 2) maximum likelihood estimate of PDF of winning bids 3) median or some other percentile of PDF of winning bids.

3.5. Summary

In summary, this chapter provided a detailed discussion on framework to develop game-based control strategies. Any realization of the game is constructed using the concepts presented in this chapter. A detailed discussion on one such realization of the game is presented in chapter 4

CHAPTER 4: A REALIZATION OF THE GAME

4. Introduction

A realization of the model has been created using an agent-based simulation model created in Python. The model includes modules for generating the vehicle inputs, playing the game, and creating output files. To keep the model simple, an intersection involving two single-lane, one-way streets (one eastbound and the other northbound) is studied. FIFO is preserved on the approaches. Shifted negative exponential distributions are used for generating the arrival headways. A uniform density function ranging from 1.5 to 2.6 seconds is used for generating the stop-bar discharge headways.

4.1. Player Details

Consistent with Chapter 3, this realization has three types of players: movement managers, drivers, and a municipality. Details about each of these players, as well as the nature of their interactions, are presented in the material that follows. In short, drivers interact with movement managers to receive information about their position in queue and make payments for service. Movement managers interact with the drivers, prepare bids, and communicate with the municipality (regarding financial transactions and the bidding process). The municipality receives bids, makes decisions about control assignment, and passes decisions back to the movement managers. The municipality also conducts financial transactions with movement managers. Clearly, this realization assumes the existence of V2I and I2I communication.

4.1.1. Movement managers

Movement managers are the players who bid against one another for use of the intersection. When movement managers win, they release motorists from their waiting queues. Movement managers develop bidding strategies that increase their chances of winning. They collect initial fees from their drivers upon arrival and any additional voluntary monetary contributions from the drivers. Movement managers cannot force drivers to pay more than

they are willing. Thus, movement managers are interested in providing a good quality of service to their drivers, subject to a constraint of financial solvency.

In the realization presented here, the movement managers only have access to information about vehicle arrival patterns in their respective approaches. They are unaware of vehicle arrival patterns in other approaches, as well as the bids submitted by other movement managers. They do know what they bid and whether or not the bid won. They do expect that higher bids increase the probability of winning. When they win, they pay what they bid (first-price bidding). The movement managers strive to maximize the chances of winning, subject to remaining solvent. Therefore, movement managers submit bids that are as high as possible, while ensuring that they have sufficient funds to discharge the rest of the vehicles in queue at a nominal fee. Consequently, the movement managers continue to learn how to negotiate on behalf of arriving drivers for the game's full duration.

Movement managers make use of historical data (if available) in determining their bids (in this context the historical data include data for the entire simulation period). In the current game realization, they keep track of win/loss bids associated with every queue length that they see on their respective approaches. Based on this historical information, for each possible queue length they develop:

a) The probability density function (PDFs) associated with the winning bids;
b) The average winning bid;
c) The odds ratio (i.e., the ratio of number of winning bids to the number of losing bids). If this ration is greater than '1', they infer that they are doing a good job of managing their queue.

As Figure 2 indicates, movement managers compute three candidate bids and select the largest.

Inputs:

q_k^i (length of queue on approach 'i', during turn 'k')

Let B be the set of all bids

Let B_w be the set of winning bids

Let B_l be the set of losing bids

Let $\{\{\mathrm{B}_w\}, \{\mathrm{B}_l\}\}$ be partion sets of $\mathrm{B} \ni \mathrm{B}_w \cup \mathrm{B}_l = \mathrm{B}$

Let $\mathrm{B}_w = \left\{\left\{\mathrm{B}_{w_1}\right\} \cup \left\{\mathrm{B}_{w_2}\right\} \cup \left\{\mathrm{B}_{w_3}\right\} \ldots\ldots\right\}$

Let $\mathrm{B}_l = \left\{\left\{\mathrm{B}_{l_1}\right\} \cup \left\{\mathrm{B}_{l_2}\right\} \cup \left\{\mathrm{B}_{l_3}\right\} \ldots\ldots\right\}$

Let $\bar{\mathbf{b}} = \left(\bar{\mathbf{b}}_{\mathbf{w}_1}, \bar{\mathbf{b}}_{\mathbf{w}_2}, \bar{\mathbf{b}}_{\mathbf{w}_3} \ldots\right)$ be the vector of average winning bids associated with each queue length

Output:

Selected bid = max (c^1, c^2, c^3)

Candidate bids:

Let c^1, c^2, c^3 be three potential candidate bids computed by the movement manager

$c^1 = P_{\min} \times (1 + \% x \mid x \in [0,100]) \ni (bb_k^i - c^1) - (q_k^i - 1) \times P_{\min} \geq 0$

$c^2 = P_{\min} + (bb_k^i - q_k^i \times P_{\min}) / q_k^i$

$c^3 = \bar{\mathbf{b}}_{\mathbf{w_i}} * \varphi$ Under the condition that: $\bar{\mathbf{b}}_{\mathbf{w_i}} \in \bar{\mathbf{b}}$. Else: $c^3 = 0$

Computation of φ

Let $O(w^{q_k^i})$ be odds of winning for a length of queue on approach 'i', during turn 'k'

Let $P(w^{q_k^i})$ be the probability of winning a bid for a queue on approach 'i' during turn 'k'

Let $P(l^{q_k^i})$ be the probability of losing a bid for a queue on approach 'i' during turn 'k'

Compute the odds of winning: $O(w^{q_k^i}) = P(w^{q_k^i}) / P(l^{q_k^i})$. And then if:

If $O(w^{q_k^i})$ < 1: $\varphi = (1 \pm \% x \mid x \in [0,100]) \ni (bb_k^i - c^3) - (q_k^i - 1) \times P_{\min} \geq 0$

If $O(w^{q_k^i})$ = 1: $\varphi = (1 + \% x \mid x \in [0, 5]) \ni (bb_k^i - c^3) - (q_k^i - 1) \times P_{\min} \geq 0$

If $O(w^{q_k^i})$ > 1 and $(bb_k^i - c^3) - (q_k^i - 1) \times P_{\min} \geq 0$ Then: $\varphi = 1$ Else: $\varphi = (1 - \% x \mid x \in [0,100])$

Figure 2: Computing the selected bid

The first candidate bid is the highest possible amount (x% above nominal fee) that the manager can bid, given a constraint that at least a nominal fee can still be paid to discharge each of the remaining vehicles in queue. The second candidate is the highest bid possible if the current funds are equally distributed among all drivers in queue. The third candidate bid is the average winning bid for this queue length, adjusted downward if necessary to ensure solvency. To determine this bid possibility, the manager makes use of knowledge about

win/loss bids associated with the current queue 'i'. The odds of winning $O(w_i)$ are computed, as well as the average winning bid $\bar{\mathbf{b}}_{\mathbf{w_i}}$ associated with it (of course one can chose other measures such as the median or 75th percentile bid as possible candidates instead of the average bid). If necessary, this bid is adjusted to ensure that the movement manager remains solvent. The details of this process can be found in Figure 2.

4.1.2. Drivers

Drivers make payments to the movement managers: first a fixed fee, and then voluntary contributions. They learn about how much to pay as their short time in the game unfolds. They have information about their queue position, and the win/loss record of their movement manager from the time they join the queue. They make contribution decisions based on the movement manager's performance, their value of time, and the delay they have incurred. Since drivers are transient players, they learn all this while in queue. Drivers do not pass on their knowledge to other drivers.

A driver's main objective is to transit the intersection in minimum time. When drivers arrive at a given intersection, they have a desired delay $\hat{d}_j$. The value of the desired delay is a function of their initial perception on movement manager's ability to submit a winning bid; the parameter ρ_j captures this aspect of their behavior. If driver j has an initial position of x_j^0 in the queue, and $\bar{h}_s$ is the saturation headway, then $\hat{d}_j = x_j^0 \times \rho_j \times \bar{h}_s$. If $\rho_j = 1$, then the drivers perspective is that the movement manager submits a winning bid every bidding cycle. On the other hand, if $\rho_j = 2.0$, the assumption is that the movement manager submits a winning bid every other bidding cycle. Therefore in this sense, the desired delay drivers want to achieve influences the monetary contributions that they make voluntarily (please read rest of this section to see why this is true). The value of ρ_j was set to 2.0 in the model realization used here.

Drivers constantly estimate the delay they anticipate incurring, d_j^k, where k = 1,2,3,…. is the number of bidding cycles since joining the queue. Every bidding cycle, they estimate a new

d_j^k to that reflects the movement manager's win ratio and their position in queue. They upward adjust the estimate if the movement manager loses (more likely to lose in the future), and downward adjust it if the movement manager wins (increased chances of winning). The estimate of d_j^{k+1} is computed using the following formula:

$$d_j^{k+1} = d_j^k + x_j^k \times (1 + p_j^{k+1}(l) / D_j^k) \times \overline{h}_s \quad \textbf{(1)}$$

Where:

d_j^{k+1} = estimated delay of vehicle 'j' in queue in '$(k+1)^{th}$' bidding cycle

d_j^k = actual delay of vehicle 'j' in queue at the end of 'k^{th}' bidding cycle

x_j^k = position of vehicle 'j' in queue at the end of 'k^{th}' bidding cycle

$p_j^{k+1}(l) / D_j^k$ = probability that the movement manager loses in 'k+1th' turn

Drivers compute $p^{k+1}(l)$ based on bidding outcome data since the time they joined until the end of the kth turn (D_j^k). The update procedure described in equation (1) is a modeling decision created by the author. The idea is that as $p_j^{k+1}(l) \rightarrow 1$ the estimate of anticipated delay increases, and as $p_j^{k+1}(l) \rightarrow 0$ the estimate of d_j^{k+1} goes down. Subsequent material presented in this section provides more details on probabilistic reasoning used by the drivers.

Here the fundamental uncertainty lies in driver perceptions about movement manager's ability to submit a winning bid. The drivers take different actions (voluntary monetary contributions, updating estimate of anticipated delay) after they update their perceptions of the movement manager's performance. Drivers use Bayesian inference to update their belief about movement manager's ability to submit a winning bid. Figure 3 presents schematic of driver *j*'s decision network.

Every bidding cycle, the movement manager either submits a winning bid (outcome = *W*) or not (outcome = L). $P(W) = \theta$ is the probability that the movement manager will win, and

$P(L) = 1-\theta$ is the probability that the movement manager will lose. Here the distribution of θ is modeled as beta distribution with parameters α and β as shown in equation (2)

$$\theta|(\alpha,\beta) \sim Beta(\alpha,\ \beta)$$
$$P(\theta \setminus \alpha,\ \beta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\beta(\alpha,\beta)} \quad \textbf{(2)}$$

Since arriving drivers do not have any historical knowledge regarding their movement manager's ability to submit a winning bid, they assign equal probabilities to the movement manager's success/failure in the next bid event; this is achieved by setting $\alpha=1;\beta=1.$ in equation (2). s_j^0 in the Figure 3 represents the state when driver j, joins the queue.

Let w_i denote the outcome of ith bidding cycle:

$$w_i = \begin{cases} 1 \ if\ the\ outcome\ is\ W \\ 0 \ if\ the\ outcome\ is\ L \end{cases} \quad \textbf{(3)}$$

It is clear from Figure 3, that there will be (n+1) states at the end of nth bidding cycle and it is easy to see the update procedure for computing posterior probability. Suppose a driver waiting in queue observes n successive bidding cycles. Among these, k outcomes were winning bids and (n-k) outcomes were losing bids. The posterior distribution is then given by:

$$\theta|D_j^n \sim Beta(\alpha+k,\ \beta+n-k)$$
$$P(\theta|D_j^n) \propto \theta^{\alpha+k-1}(1-\theta)^{\beta+n-k-1} \quad \textbf{(4)}$$

Drivers use this new information from the current bidding cycle to update their assessment of whether their movement manager will win or lose subsequent bids. They compute $P(H \setminus D_j^n)$

(Here 'H' is the hypothesis that the movement manger submits a winning bid) and $P(\tilde{H}|D_j^n)$ using equation (2), and compute Bayes factor (which is the ratio of posterior odds to prior odds) using this information as shown in equation (5):

$$\frac{O(H \mid D_j^n)}{O(H)} = \frac{P(D_j^n \mid H)}{P(D_j^n \mid H)} = K \qquad \textbf{(5)}$$

The left-hand side of equation (5) is the ratio of the posterior and prior odds, whereas the right-hand side is the likelihood ratio, also known as 'Bayes factor', *K*. For the model presented here, *K* is the win to loss ratio at the end of a given bidding cycle. If $K > 1$, then D_j^n is more likely under *H* than under $\tilde{H}$, if $K = 1$, then D_j^n is equally likely under either hypothesis, and if $K < 1$, then D_j^n is more likely under $\tilde{H}$ than under *H*.

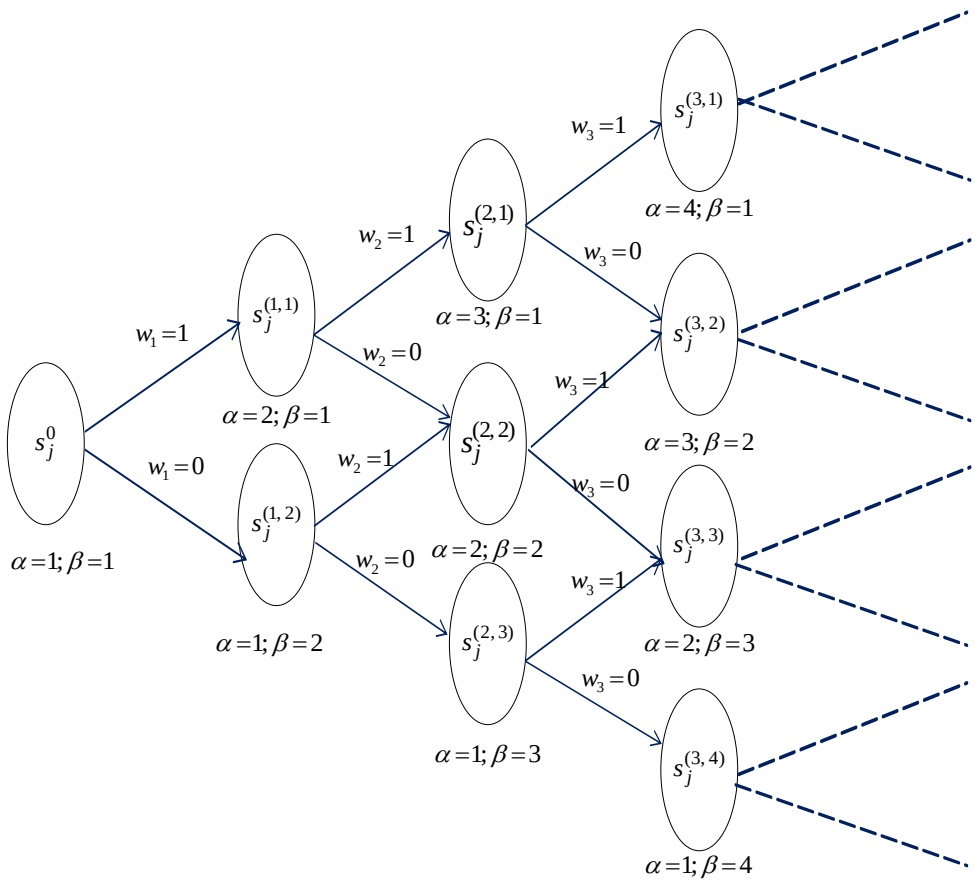


Figure 3: Schematic of driver j's decision network

For every bidding cycle, drivers compute: 1) their Bayes Factor (*K*), that is, the right-hand side of equation (5), and 2) their expected delay until departure using equation (1).

In every bidding cycle, the driver decides whether to make a voluntarily contribution. Table 1 shows the logic and the possible contributions.

Table 1: Table of possible voluntary contributions

Bayes Factor (K)	*Test*	Voluntary Contributions	
		$\tau_j^i = 1$	$\tau_j^i = 0$
$K < 1$	1	0.5	0.2
$K = 1$	1	0.4	0.15
$K > 1$	1	0.3	0.1

The driver first determines the K as described previously. Then, the outcome of *Test* is determined. The variable *Test* equals 1 if $d_j^k > \hat{d}_j$, and it equals 0 if $d_j^k \leq \hat{d}_j$. Depending on the values of K and *Test*, various possible contribution amounts pertain. For example, if $K > 1$, *Test* = 1, and the driver has a high *VOT*, then the amount is 0.50. If the driver's *VOT* is low, then it is 0.20. (The numerical values of the voluntary contributions have been chosen by the author for illustrative purposes. They are logical, but not based on any empirical data.)

Then, there is a probability that the driver will actually make the contribution indicated in Table 1. That probability is determined by the function shown in equation (6).

$$p(cont) = 1/(1+e^{10*((K/(K+1)-0.5)}) \quad \textbf{(6)}$$

To illustrate, if K is 1.0, i.e., the movement manager is as likely to win as to lose, then p(cont) = 0.5. In other words, the probability that the driver will make a contribution is 50%. As K → 0, i.e., the movement manager becomes very unlikely to win, and p(cont) → 1: the driver will almost always make a contribution. As K → ∞, i.e., the movement is very likely to win, p(cont) → 0: the driver is very unlikely to make a contribution.

4.1.3. Municipality

As mentioned earlier, rules govern how the game is played. The municipality defines these rules and makes decisions about how to assign control among the movement managers. The municipality also controls the times associated with green, yellow, and all-red durations, as well as pauses in the bidding process. The municipality acquires information on queue lengths for each approach from their respective movement managers. On the basis of this information, the municipality tells the movement managers when to submit bids.

As the reader might imagine, the municipality is interested in an equitable and efficient allocation of the intersection's processing capacity. The municipality uses the bidding process to allocate intersection capacity to different managers at different points in time. These allocations are manifest in phase sequences and switching times. Furthermore, the delays experienced by drivers are determined by the switching times, and there are costs associated with those delays. The municipality can take these costs into consideration when making decisions concerning switching times. The term for describing costs is 'Marginal cost of incremental delay (MCID)'.

The following discussion helps describe these ideas further. Let $g_{\min}$ and c be the durations of the minimum green and clearance interval respectively, and let be the cost of delay. It is useful to recall that at the end of minimum green ($g_{\min}$), both managers resubmit bids for intersection control, if necessary. If the enfranchised manager loses, then the next earliest time that manager is able to compete for intersection control is $g_{\min} + 2c$, so each vehicle on that manager's approach incurs an additional delay equivalent to $g_{\min} + 2c$. On the other hand, if the disenfranchised manager loses the bid, then the next earliest time that manager can compete for the intersection control is equal to g_{ext}, so each vehicle on that manager's approach incurs an additional delay equivalent to g_{ext} . Figure 4 presents a schematic for determining the cost of these delays. There are at least two ways to compute this cost: 1)

System Incremental Delay (SID), and 2) Subject Approach Incremental Delay (SAID). It is important to recognize that each method views the associated cost of delay differently.

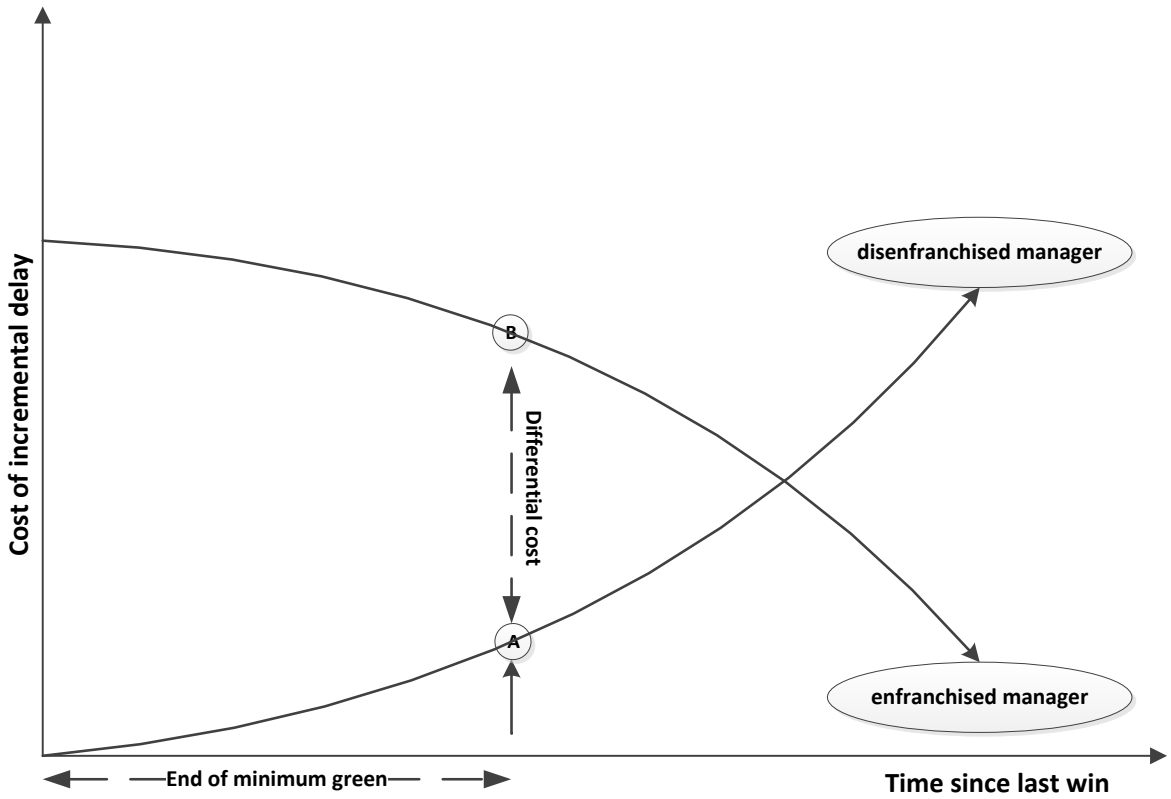


Figure 4: Schematic showing cost of incremental delay

System Incremental Delay (SID): Equation (12) shows how to compute the SID-based cost. It is the difference between delays that would be experienced by drivers on the enfranchised versus disenfranchised approaches. This difference is multiplied by the cost of delay.

$$C_{SID} = (Q_{enf} \times d_{enf} - Q_{dis} \times d_{dis}) * c_{del} \quad \textbf{(12)}$$

Subject Approach Incremental Delay (SAID) – 2: Equation (13) shows how to compute the SAID-based cost. It is the delay that would be experienced by drivers on the enfranchised movement manager's approach, multiplied by the cost of delay.

$$C_{SAID} = Q_{enf} * d_{enf} * c_{del} \quad \textbf{(13)}$$

Where,

$C_{SID} =$ Marginal cost of system incremental delay

$C_{SAID} =$ Marginal cost of system approach incremental delay

$Q_{enf} =$ Length of queue on enfranchised manager's approach

$d_{enf} =$ Future delay that would be experienced by drivers on the enfranchised Manager's approach due to a shift in control

$Q_{dis} =$ Length of queue on the disenfranchised manager's approach

$d_{dis} =$ Possible delay that would be experienced by drivers on the disenfranchised manager's approach

$c_{del} =$ Cost of delay per second experienced by drivers in queue

Based on one of these two these evaluations, the municipality adds an increment to the minimum bid the disenfranchised manager has to submit to be assigned intersection control. In the current realization, that minimum bid is the sum of the MCID (which is either C_{SID} or C_{SAID}) and the bid submitted by enfranchised manager.

$$bid_{\min} = MCID + bid_{enf} \quad \textbf{(14)}$$

If the disenfranchised manager submits a bid higher than $bid_{\min}$, then that manager is assigned control; otherwise control continues with the enfranchised manager.

Below are the rules by which the current realization of the game is played.

1) Movement managers collect an initial fee from the drivers; any additional monetary contributions from the drivers are voluntary.
2) Assignment of intersection control is determined at the end of each discharge headway (if vehicles are discharging), or at the end of a gap period if no subsequent vehicle arrives first.

3) Assignment of intersection control pauses during discharge headways, minimum gaps (if no subsequent vehicle arrives), clearance intervals, and minimum greens (effectively for the duration of the clearance interval and the minimum green).
4) There are no maximum greens.
5) If only one movement manager has a queue, then control is assigned to that manager. The lead vehicle in queue is discharged and the manager pays the municipality a nominal fee.
6) If more than one movement manager has a non-zero queue, then assignment of control is resolved through the submission of bids.
7) When bids are submitted, the process followed is single-stage, first-price sealed bidding. That is, both movement managers submit bids. The winner is assigned control, becomes the enfranchised manager, discharges the lead vehicle in queue, and pays the municipality an amount equal to the bid submitted (i.e., the winning bid).
8) If the winner is already the enfranchised manager, bids pause for the minimum of the discharge headway (if another vehicle arrives) or the gap time. If the winner is the disenfranchised manager, then the duration of the pause is the change interval plus the minimum green.
9) The municipality can elect to consider the marginal cost of incremental delays (MCID) to make decisions about the assignment of intersection control.
10) If MCIDs are considered, the municipality can set a minimum for the bid ($bid_{\min}$), which the disenfranchised manager has to submit to take control of the intersection.
11) If the disenfranchised manager submits a bid higher than $bid_{\min}$, then control is assigned to that manager. Otherwise, control continues to be assigned to the enfranchised manager.

4.2. Flowchart for the Game Logic

The turn-by-turn pseudo code for the model logic is presented in Figure 4. The initial conditions are first established and then turn-by-turn play commences. The fundamental time

step is 0.1 seconds and bidding events take place either at the end of vehicle discharge headway or at the end of 3 seconds, whichever occurs first. What transpires during the bidding event depends upon whether there are vehicles waiting to be serviced on both approaches, just one approach or neither approach. Bidding is suspended during the change interval and during the minimum green.

To incorporate clearance intervals, minimum greens and gaps, the following actions are taken. For the clearance intervals and minimum greens, if control shifts from one manager to another as a result of the bidding process, bid submission ceases for a clearance interval plus a minimum green. At the end of this time period, managers with non-zero queues again submit bids for use of the intersection space. For the gaps, bidding is suspended until the end of the gap time or the next vehicle arrival, whichever occurs first.

During the bidding event, if only one movement manager has a service queue that manager is an automatic winner; he is allowed to use the intersection space for a duration equivalent to the minimum green (if control shifts) or the minimum of the 3-second gap or the headway to the next arriving vehicle. For every vehicle discharged in this manner, the manager pays a nominal fee to the municipality.

If both movement managers have service queues, then both movement managers submit bids (refer to the previous section for details about how the bids are computed); the winning bidder is selected by the municipality, and the winner is allowed to use the intersection space for the same duration described earlier; the winning movement manager pays the municipality what was bid (first-price bidding); and discharges the first-in-queue vehicle at saturation headway. All movement managers then update their win/loss PDFs using the results of the bid. Remaining drivers update their belief about their movement manager's likelihood of winning, they re-compute their d_j^k, and decide whether or not they want to make voluntary contributions.

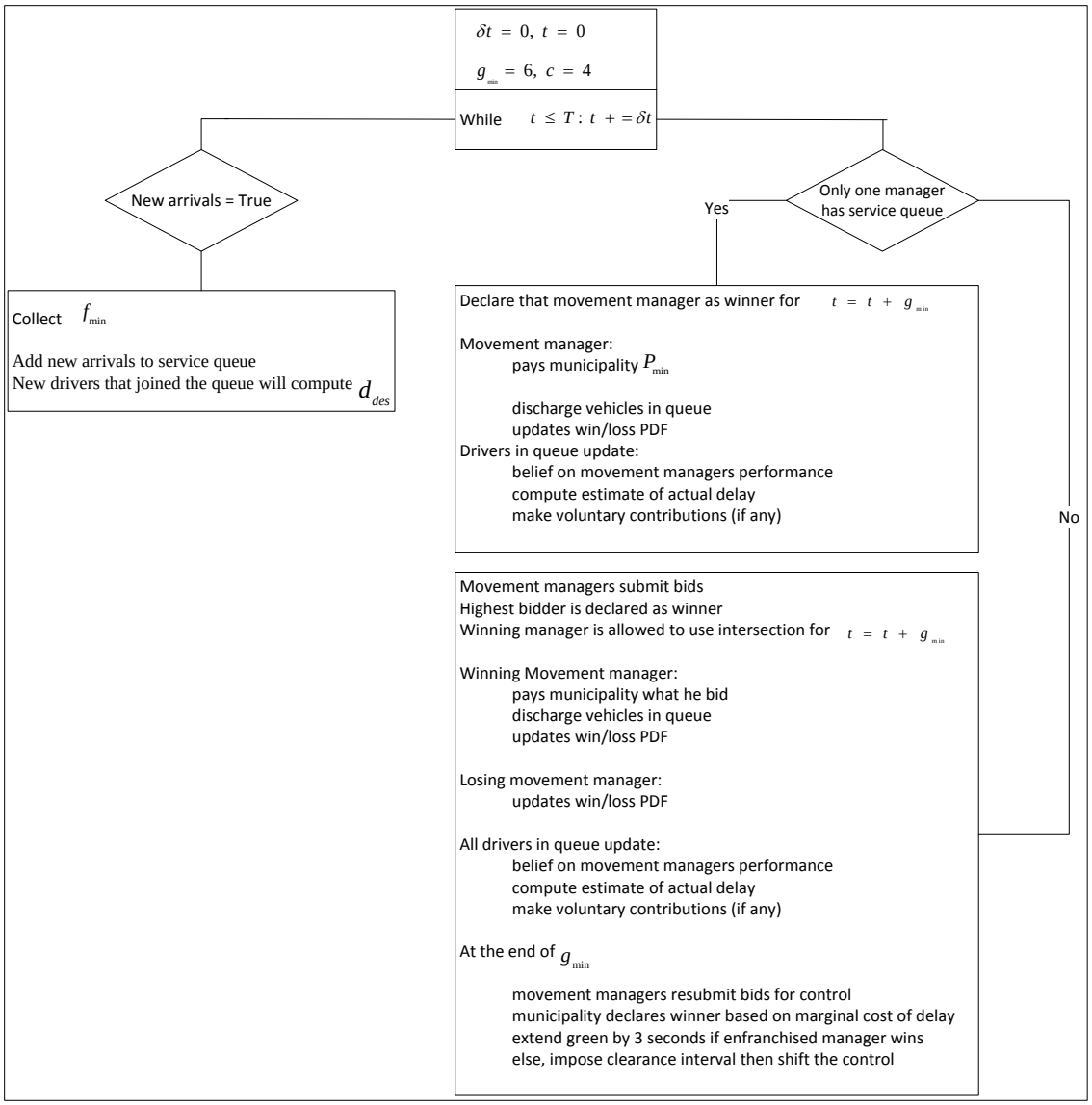


Figure 5: Flowchart for the control structure of the simulation model

4.3. Actuated Control Model

To provide benchmarks against which to compare the results from the bid-based control, a model of an actuated controller is employed. The traffic stream generator used for the bid-based control model is also used for the actuated control model to ensure that the arriving headway sequences are the same for any comparative evaluation. Detectors are placed at the stop-bar. The minimum green is set to the same value used in the bid-based control. The gap

is set to the same maximum pause duration used in the bid-based control and it is held constant (i.e., no volume-density control). Maximum greens are sometimes used and sometimes they are omitted (since the bid-based control has no maximum greens). When imposed, the maximum greens are set through a parametric analysis that matched the actuated control delays with the bid-based control delays. The purpose of not imposing maximum greens is to see if actuated control can match the bid-based control without the use of maximum greens. Appendix A has a detailed description of the actuated control model.

CHAPTER 5: SIMULATION EXPERIMENTS AND ANALYSES

5. Introduction

This chapter presents outputs from the bid-based control simulations, and compares them with various actuated control models.

As indicated in the previous chapter, these comparisons include situations where maximum greens are imposed in actuated control and conditions where they are not. The reason for studying actuated control with and without maximum greens is as follows. A traffic signal controller fundamentally makes decisions about when to shift the green from one phase to another. In the context of bid-based control, this is done through the bidding process. No maximum greens are imposed. Decisions about whether to shift the green occur at the end of every saturation headway or gap time. In actuated control, however, this is not the case. Maximum greens are employed as well. In principal, actuated control can decide whether or not to shift green at the end of each gap interval and not make use of maximum greens at all. Whether or not this produces results similar to bid-based control is a research issue. Therefore, actuated control scenarios with and without maximum greens were examined.

5.1. Simulation experiments

This analysis focuses on an intersection involving two single-lane one-way streets. The total intersecting volume is held constant at 1500 veh/hr/lane.

Three traffic flow combinations are examined (scenarios):

- *Scenario – 1*: Equal volumes on the two approaches (v_N = 750, v_E = 750)
- *Scenario – 2*: Slight imbalance (v_N = 900, v_E = 600)
- *Scenario – 3*: Significant imbalance (v_N = 1200, v_E = 300)

The bid-based control experiment employs the logic described in Chapter 4. Movement managers, drivers, and a municipality are involved. First-price bidding is employed. The

drivers fall into one of two classes based on their ‘value-of-time’: some have a high value-of-time and the others have a low value-of-time.

Two bid-based control options are considered:

b1) Gaps, minimum greens, and change intervals;

b2) Gaps, no minimum greens, and no change intervals, to reflect a more automated control condition. The purpose for creating this option is to explore maximum throughput issues, not to suggest practical options. Traffic engineers often overlook the fact that the minimum greens and clearance intervals consume what would otherwise be available vehicle processing time. There are good safety reasons for doing so, but these times interfere with intersection productivity. Hence, it is useful to see what the productivity of the intersection might be if it is controlled by an automated procedure.

Four options are considered for actuated control:

a1) Gaps, minimum greens, and clearance intervals, but no maximum greens. This allows explorations of how actuated control performs if no maximums are imposed; this parallels bid-based option #1.

a2) Gaps, minimum greens, clearance intervals, and maximum greens. This allows an analysis of the impact of including maximum greens. It is effectively a simple realization of standard practice today.

a3) Gaps, no minimum greens, no clearance intervals, and no maximum greens. This parallels the bid-based option #2.

a4) Gaps, no minimum greens, no clearance intervals, but maximum greens. This again allows an examination of the impacts of maximum greens in an option akin to bid-based control option #2.

An agent-based simulation model of the bid-based control was developed in Python consistent with the game rules previously presented. A Python-based model of actuated

control as also developed, consistent with the description in Chapter 4 and Appendix A. These two simulation models exist within the same analysis program.

For a given input volume on the facility, the program creates a sequence of arrival headways for each scenario for both approaches. A shifted negative exponential headway distribution is employed with a minimum headway of 1.5 seconds and an average headway consistent with the arrival flow rate. These arrival patterns are used both by the bid-based control model and the actuated control mode. Simulations are 9,000 seconds long. The saturation headway is set to be uniform between 1.5 to 2.6 seconds with an average of 2.1 seconds (1,714 vehicles/hour) – see below. The other simulation parameters are a nominal fee = $1 and an initial fee = $1.

5.2. Setting the saturation headway distribution

Typically, the processing capacity of a signalized intersection is about 1500-2000 vehicles per hour per lane, in terms of the sum of the critical volumes (85). This corresponds to saturation headways ranging from 1.8 to 2.4 seconds. The highway capacity manual (85) suggests ideal saturation headway of 1900 vehicles per hour of green which is then downward adjusted to account for local conditions. A value of 1700 vehicles per hour of green is common. This corresponds to average saturation headway of 2.1 seconds.

For purposes of this analysis, based on an analysis conducted by List (86), a uniform headway between 1.5 and 2.6 seconds was employed. (It has an average value of 2.1 seconds.) This results in an average saturation flow rate of 1714 vehicles per lane per hour of green. Hence, with the assumed total intersecting flow of 1500 vehicles per hour, the three scenario conditions have a volume to capacity (v/c) ratio of about 0.88.

List (86) used VISSIM to find a distribution of saturation headways that would be consistent with the 2.1 second average headway. The model employed a minimum green of 6 seconds a constant gap of 3 seconds. It used arriving flow rates sufficient to ensure constant queues on

both approaches. Interestingly, he found that the average saturation headway was significantly less than 2.1 seconds. Hence an adjustment to the VISSIM model parameters was needed to achieve parity. VISSIM does not provide a way to specify the saturation headway distribution directly. It is adjusted through the driver behavior (car following) parameter settings. The safe distance parameter was upward adjusted from its nominal value of "3" to a value of "12" to produce average saturation headway of 2.1 seconds. The resulting probability density function was then examined to determine what distribution of saturation flow rates VISSIM produced. That analysis suggested that a uniform headway distribution between 1.5 and 2.6 seconds would be appropriate.

List (86) subsequently examined comparisons of the VISSIM results with those produced by the Python-based actuated control model. For the same input volumes, minimum greens, gaps, and maximum greens, it was found that the distribution of cycle lengths and the distributions of the approach-specific delays were equivalent. This provided evidence that the Python-based model was capable of producing simulation results consistent with a commercially accepted modeling platform.

5.3.Setting the maximum greens

Inasmuch as maximum greens are employed in two of the actuated control options, a procedure was needed to determine what values to employ. It seemed most appropriate to identify combinations of maximum greens that would ensure that the actuated control delays matched those from the bid-based control. Exhaustive searches were conducted to find these values. For each approach, a range of possible maximum greens was identified, and then all combinations of these values were examined to find the best combination. An illustration of the results from this search process is provided in Table 2 for the Scenario-2 flow conditions (NB = 900 vph; EB = 600 vph). The table shows that the maximum green combinations that come closest to matching the bid-based delays. The first two columns present the average delay in seconds produced by bid-based control on the NB and EB approaches, respectively (a single value for each approach). The other columns present results for the maximum green

combinations. The values in the third and fourth columns are the maximum greens on the NB and EB approaches. Columns five and six are the average delay in seconds on the NB and EB approaches when only both minimum and maximum greens are employed. It can be seen that for this situation the maximum green combination of (55, 40) produces average delays that closely match those of bid-based control. A conclusion from this analysis is that it is possible to find a combination of max-greens for a given flow condition that allow the actuated controller to match the performance (at least in terms of delays) of bid-based control.

Table 2: Average delay (sec) for different combinations of maxgreen for Scenario-2 flow condition

Bid Based Control (Option # 1)		Actuated Control			
delay in Sec (NB)	delay in Sec (EB)	Maxgreen (NB)	Maxgreen (EB)	delay in Sec (NB)	delay in Sec (EB)
22	28	50	40	27	27
22	28	50	45	33	26
22	28	50	50	35	27
22	28	55	30	18	64
22	28	55	35	20	41
22	28	55	40	**23**	**30**
22	28	55	45	27	27
22	28	55	50	29	28
22	28	55	55	29	27
22	28	60	30	18	66
22	28	60	35	20	44
22	28	60	40	21	31

For the three scenarios considered, the combinations of maximum greens employed were as follows:

- *Scenario – 1*: (750-750) 45 seconds northbound, 45 seconds eastbound
- *Scenario – 2*: (900,600) 55 seconds northbound, 40 seconds eastbound
- *Scenario – 3*: (1200,300) 80 seconds northbound, 30 seconds eastbound

5.4. Consistency among Simulations

In principle, the system examined here should be ergodic. A system is ergodic if the statistical properties derived from a time series analysis should be the same as those from a cross-sectional analysis (across multiple simulation runs). The reason is that the distributions of the stochastic elements (arrival headways and saturation headways) are time-invariant. To check that the system actually is ergodic, the following analysis was performed. A number of Monte Carlo simulations (each 9000 seconds long) were performed and the cumulative distributions of delays among these simulations were compared.

Figure 6 presents the cumulative distribution functions (CDFs) for the delays experienced by individual vehicles on the NB and EB approaches among 10 simulation runs. It contains four subplots; each subplot presents CDFs for individual vehicle delays in each of the ten simulation runs. Subplots a and b present these results for the NB and EB approaches, respectively, for bid-based control option 'b1', whereas subplots c and d present similar results for actuated control option 'a2'.

A close examination of these CDFs suggests that the variability in vehicle delays between simulations is not significant. Kolmogorov-Smirnov (KS) tests were performed to check for any statistical differences in these distributions. Please note there has been no adjustment done for multiple testing. Table 3 summarizes the test results. There are nine columns in each table: values in column-2 represent Kolmogorov-Smirnov statistic; values in column-3 represent asymptotic Kolmogorov-Smirnov Statistic; values in column-4 represent maximum absolute deviation between Empirical Distribution Function (EDF) of two class levels; values in column-5 list the p-values for the test, i.e., the probability that D is greater than the observed value 'd' under the null hypothesis of no difference between two class levels; values in columns six to nine represent one sided test statistics.

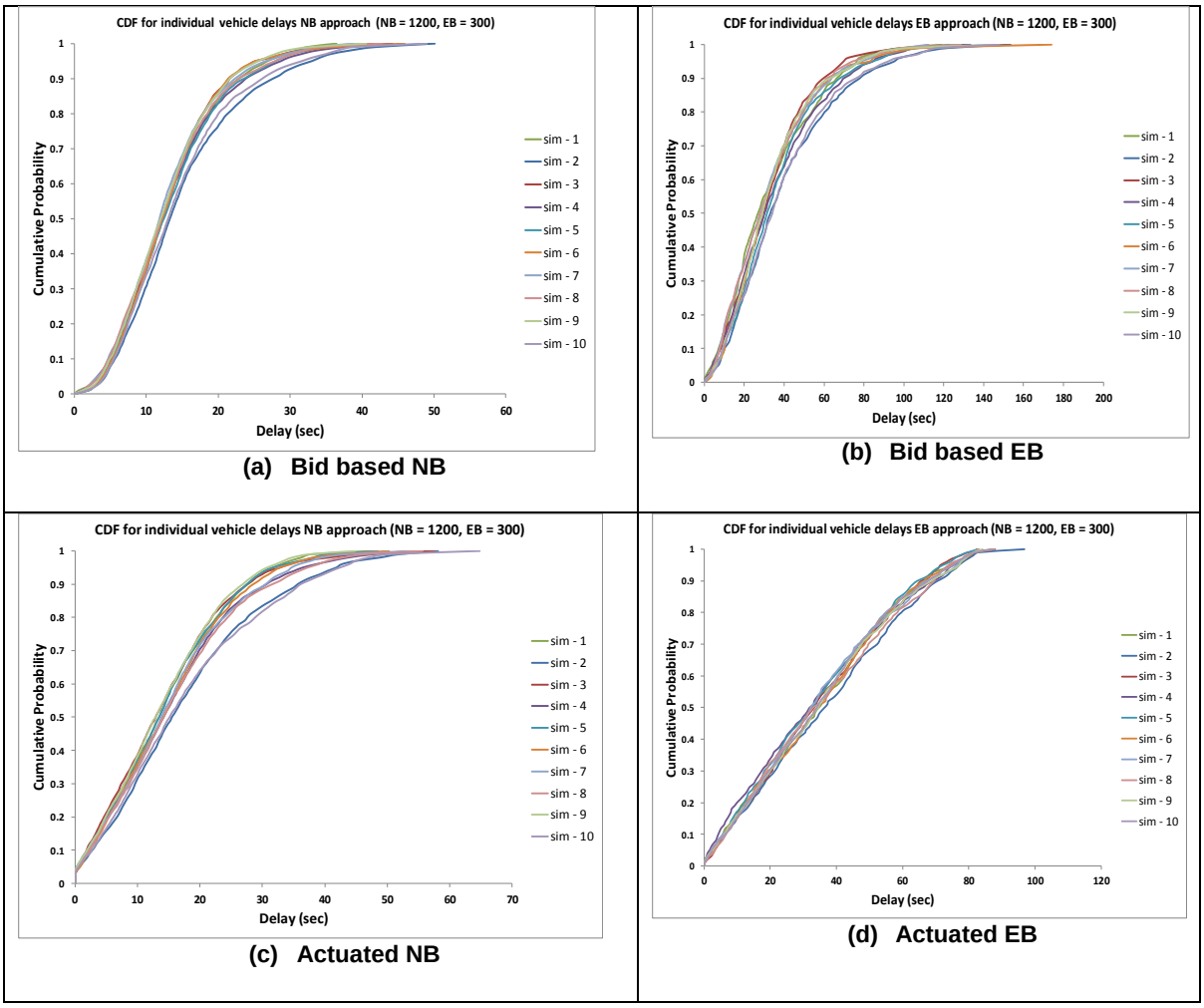


Figure 6: CDFs of individual vehicle delays in ten simulation runs

One cannot reject the null hypothesis, i.e., these distributions are similar as long as p-values (values in column 5) are greater than 0.05, and test results demonstrate that individual vehicle delay distributions produced by each simulation are statistically similar. Thus, it can be inferred that a single simulation 9,000 seconds long is sufficient to capture all the dynamics of the system.

The implication of this analysis is that single, long simulations can be performed to develop the comparisons among the various control strategies. Multiple runs are not necessary.

Table 3: KS Test results to check consistency among simulations

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(1, 2)	0.039604	0.562878	0.0792079	0.9093	0.0792079	0.5306	0	1
(1, 3)	0.0148515	0.2110793	0.029703	1	0.019802	0.9612	0.029703	0.9147
(1, 4)	0.009901	0.1407195	0.019802	1	0.019802	0.9612	0.009901	0.9901
(1, 5)	0.019802	0.281439	0.039604	1	0.039604	0.8535	0	1
(1, 6)	0.0148515	0.2110793	0.029703	1	0.029703	0.9147	0.029703	0.9147
(1, 7)	0.0148515	0.2110793	0.029703	1	0.009901	0.9901	0.029703	0.9147
(1, 8)	0.0148515	0.2110793	0.029703	1	0.009901	0.9901	0.029703	0.9147
(1, 9)	0.0148515	0.2110793	0.029703	1	0.019802	0.9612	0.029703	0.9147
(1, 10)	0.0346535	0.4925183	0.0693069	0.9685	0.0693069	0.6156	0	1

(a) NB bid-based option # b1

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(1, 2)	0.0693069	0.9850366	0.1386139	0.2864	0.1386139	0.1436	0	1
(1, 3)	0.0346535	0.4925183	0.0693069	0.9685	0.0693069	0.6156	0.0693069	0.6156
(1, 4)	0.0346535	0.4925183	0.0693069	0.9685	0.0693069	0.6156	0.009901	0.9901
(1, 5)	0.0594059	0.8443171	0.1188119	0.474	0.1188119	0.2403	0.029703	0.9147
(1, 6)	0.0445545	0.6332378	0.0891089	0.8175	0.0891089	0.4484	0.039604	0.8535
(1, 7)	0.019802	0.281439	0.039604	1	0.029703	0.9147	0.039604	0.8535
(1, 8)	0.029703	0.4221585	0.0594059	0.9941	0.039604	0.8535	0.0594059	0.7002
(1, 9)	0.039604	0.562878	0.0792079	0.9093	0.0792079	0.5306	0.0594059	0.7002
(1, 10)	0.0643564	0.9146768	0.1287129	0.3728	0.1287129	0.1876	0.009901	0.9901

(b) EB bid-based option # b1

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(1, 2)	0.0544554	0.7739573	0.1089109	0.587	0.1089109	0.3018	0.009901	0.9901
(1, 3)	0.0148515	0.2110793	0.029703	1	0.019802	0.9612	0.029703	0.9147
(1, 4)	0.0247525	0.3517988	0.049505	0.9997	0.049505	0.7807	0.009901	0.9901
(1, 5)	0.009901	0.1407195	0.019802	1	0.019802	0.9612	0.009901	0.9901
(1, 6)	0.019802	0.281439	0.039604	1	0.039604	0.8535	0.009901	0.9901
(1, 7)	0.0247525	0.3517988	0.049505	0.9997	0.049505	0.7807	0.009901	0.9901
(1, 8)	0.0247525	0.3517988	0.049505	0.9997	0.049505	0.7807	0.009901	0.9901
(1, 9)	0.0148515	0.2110793	0.029703	1	0.009901	0.9901	0.029703	0.9147
(1, 10)	0.0594059	0.8443171	0.1188119	0.474	0.1188119	0.2403	0.009901	0.9901

(c) NB actuated option # a2

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(1, 2)	0.0346535	0.4925183	0.0693069	0.9685	0.0693069	0.6156	0.009901	0.9901
(1, 3)	0.0148515	0.2110793	0.029703	1	0.019802	0.9612	0.029703	0.9147
(1, 4)	0.0247525	0.3517988	0.049505	0.9997	0.029703	0.9147	0.049505	0.7807
(1, 5)	0.0247525	0.3517988	0.049505	0.9997	0.019802	0.9612	0.049505	0.7807
(1, 6)	0.009901	0.1407195	0.019802	1	0.019802	0.9612	0.019802	0.9612
(1, 7)	0.0247525	0.3517988	0.049505	0.9997	0.029703	0.9147	0.049505	0.7807
(1, 8)	0.0247525	0.3517988	0.049505	0.9997	0.049505	0.7807	0.039604	0.8535
(1, 9)	0.019802	0.281439	0.039604	1	0.029703	0.9147	0.039604	0.8535
(1, 10)	0.0148515	0.2110793	0.029703	1	0.029703	0.9147	0.029703	0.9147

(d) EB actuated option # a2

5.5. Evaluation and analysis

Five metrics are employed to compare and contrast the results from the various scenarios and

signal control options:

1) Individual vehicle delays;
2) Cycle lengths;
3) Distribution of driver payments
4) Impact of queue length on bids
5) Total income stream received by the municipality

The results presented in this section are for the driver behavior previously described. A subsequent section considers the impacts of changing the driver behavior. For each of these analyses, results are presented for the six simulation scenarios described in the previous section.

5.5.1. CDFs of individual vehicle delays

Figures 7 presents the cumulative distribution functions (CDFs) for individual vehicle delays for the bid-based control options (dashed lines) and the actuated control options (solid lines). It contains six subplots; the three subplots to the left (three graphs in first column) represent results for the NB approach for each flow combination; three subplots to the right (three graphs in the second column) represent similar results, but for the EB approach.

The best performance is reflected by the curves furthest to the left. Those distributions have the smallest delay values. It is easy to see that the CDFs for ‘b2’ and ‘a4’ have the best distribution of delays. ‘a3’ is the next best, followed by ‘b1’, ‘a1’, and ‘a2’. In addition, with actuated control, vehicles on the NB approach, with the larger volumes, always experience higher delays than the EB approach, while the differences between the NB and EB approaches are less significant for bid-based control.

Table 4: KS Test results for delay distributions for NB

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(b2, a3)	0.1638253	10.037556	0.3276505	<.0001*	0.0586042	0.0016*	0.3276505	<.0001*
(b2, a4)	0.0501871	3.0753707	0.1003743	<.0001*	0.057491	0.0020*	0.1003743	<.0001*
(b1, a1)	0.0564731	3.4601006	0.1129462	<.0001*	0.1129462	<.0001*	0.0980288	<.0001*
(b1, a2)	0.0636654	3.9007738	0.1273308	<.0001*	0.1273308	<.0001*	0.0554076	0.0031*

(a) Scenario – 1 (NB)

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(b2, a3)	0.1573536	10.525062	0.3147072	<.0001*	0.0585606	0.0005*	0.3147072	<.0001*
(b2, a4)	0.0853822	5.7110422	0.1707644	<.0001*	0.0621368	0.0002*	0.1707644	<.0001*
(b1, a1)	0.0621368	4.1562035	0.1242736	<.0001*	0.1198033	<.0001*	0.1242736	<.0001*
(b1, a2)	0.0567725	3.7973946	0.1135449	<.0001*	0.0862763	<.0001*	0.1135449	<.0001*

(b) Scenario – 2 (NB)

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(b2, a3)	0.1242555	9.660013	0.2485109	<.0001*	0.1211118	<.0001*	0.2485109	<.0001*
(b2, a4)	0.1130046	8.785338	0.2260093	<.0001*	0.2260093	<.0001*	0.010589	0.7126
(b1, a1)	0.0727995	5.6596614	0.1455989	<.0001*	0.1455989	<.0001*	0.1207809	<.0001*
(b1, a2)	0.0746195	5.8011529	0.1492389	<.0001*	0.1141628	<.0001*	0.1492389	<.0001*

(c) Scenario – 3 (NB)

Table 5: KS Test results for delay distributions for EB

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(b2, a3)	0.1527996	9.4464255	0.3055992	<.0001*	0.0570382	0.0020*	0.3055992	<.0001*
(b2, a4)	0.0720976	4.4156488	0.1442211	<.0001*	0.0370205	0.0766	0.1442211	<.0001*
(b1, a1)	0.0693354	4.2864773	0.1386709	<.0001*	0.0900052	<.0001*	0.1386709	<.0001*
(b1, a2)	0.0570382	3.5262342	0.1140764	<.0001*	0.0884354	<.0001*	0.1140764	<.0001*

(a) Scenario – 1 (EB)

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(b2, a3)	0.183149	10.154451	0.366298	<.0001*	0.0559532	0.0081*	0.366298	<.0001*
(b2, a4)	0.0304803	1.6852626	0.0609616	0.0068*	0.0609616	0.0034*	0.0497971	0.0226*
(b1, a1)	0.0764476	4.2385365	0.1528953	<.0001*	0.102147	<.0001*	0.1528953	<.0001*
(b1, a2)	0.0614834	3.4088656	0.1229668	<.0001*	0.0897853	<.0001*	0.1229668	<.0001*

(b) Scenario – 2 (EB)

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(b2, a3)	0.2270233	8.6686054	0.4540466	<.0001*	0.0205761	0.7344	0.4540466	<.0001*
(b2, a4)	0.1014992	3.8139935	0.2031062	<.0001*	0.2031062	<.0001*	0.0518832	0.1498
(b1, a1)	0.0912209	3.4831556	0.1824417	<.0001*	0.0288066	0.5461	0.1824417	<.0001*
(b1, a2)	0.058299	2.2260769	0.1165981	<.0001*	0.0466392	0.2048	0.1165981	<.0001*

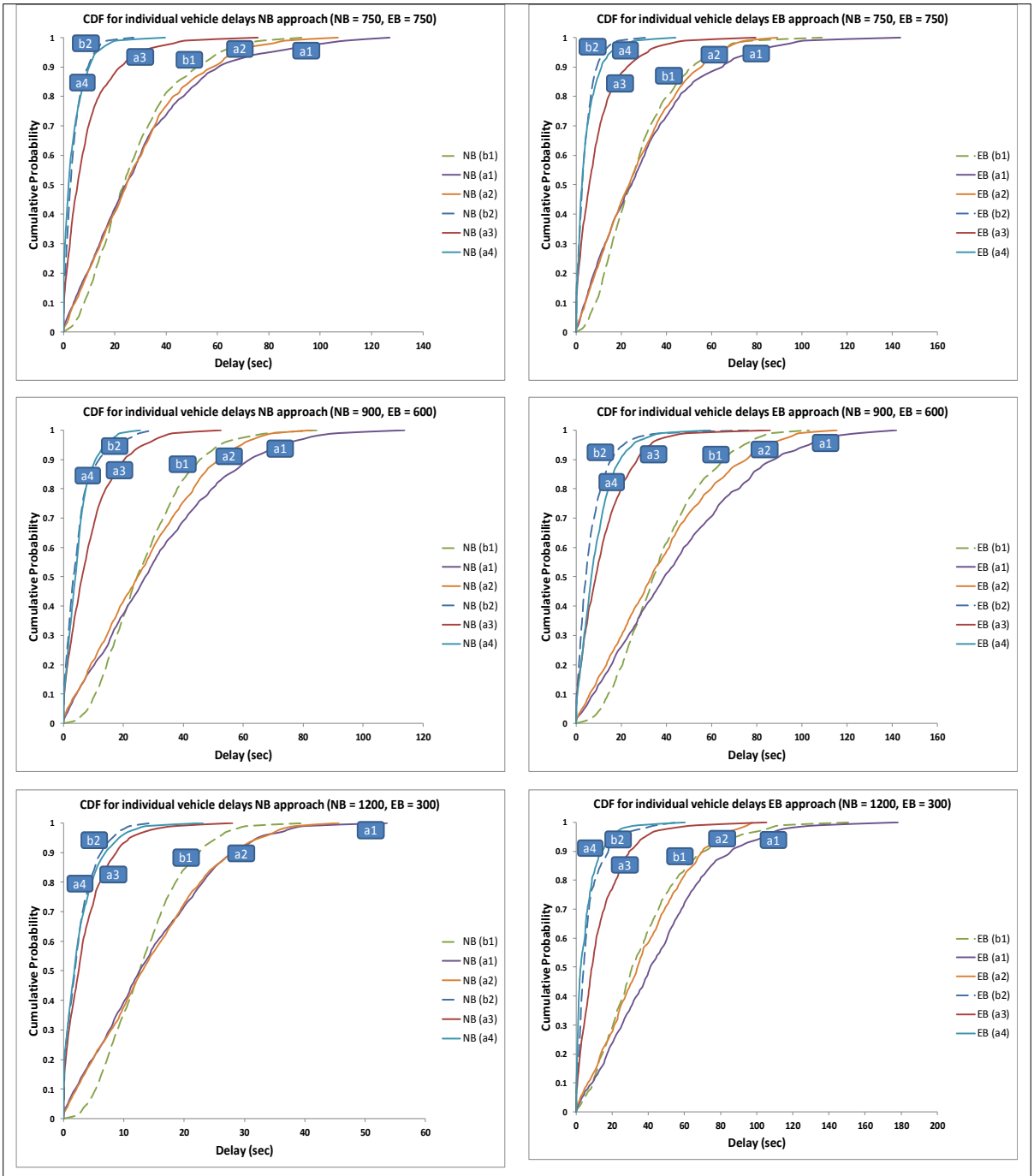
(c) Scenario – 3 (EB)

Please recall that 'a3' and 'a4' are actuated control realizations that parallel bid-based option 'b2', whereas 'a1' and 'a2' are realizations that parallel bid-based option 'b1'. Kolmogorov-Smirnov tests were conducted to check the statistical differences between actuated and bid-

based control options. Table 4 and Table 5 contain a summary of test results for NB and EB approaches, respectively. Results suggest a statistically significant difference in delay distributions for the three flow combinations and both the NB and EB approaches.

Figure 7 allows a comparison of the delays for bid-based control and actuated control. And while average delay is the main focus, the outcomes do have an economic interpretation. Smaller delays result in smaller delay costs for the drivers. Especially when minimum greens, and clearance intervals are imposed the bid-based control strategy produces driver costs that are smaller than or equal to those produced by actuated control and the results presented in Figure 7 also corroborate that inference. Secondly, the further left the delay distribution, the closer it is to the efficient frontier. That is, the combinations of average delay between the two approaches are non-dominated. There is no other control option that produces less delay. (This is an observation that will hold true among all of the volume combinations.)

Lastly, Table 6 presents various delay statistics for six simulation options. In a way these results summarize some of the statistical properties of each of the distributions presented in Figure 7; even though the differences between bid-based control and four actuated control options seem not significant at first, KS test results test suggest the distributions are statistically different.



CDFs of individual vehicle delays in each of 3 Flow Combinations (North Bound)

CDFs of individual vehicle delays in each of 3 Flow Combinations (East Bound)

Figure 7: Results (delays) for six simulation scenarios (NB)

Table 6: Various delay statistics for six simulation options

Metric	b1	a1	a2	b2	a3	a4
Mean delay (sec)	26.9	29.5	27.9	3.9	8.8	3.8
Stdev delay (sec)	17	24.1	20.3	4	10.5	4.7
15th perc delay (sec)	10.3	6.4	7.1	0.2	0.6	0.1
50th perc delay (sec)	23.4	24.3	24.5	2.7	5.5	2
75th perc delay (sec)	35.8	41	38.4	5.5	11.5	5.4
95th perc delay (sec)	60.8	80.2	67.3	12.2	28.5	12.8

(a) NB Scenario - 1

Metric	b1	a1	a2	b2	a3	a4
Mean delay (sec)	27.2	29.2	26.6	4	9.1	4.6
Stdev delay (sec)	17.1	24	19	4.1	10.6	5.6
15th perc delay (sec)	10.9	6	6.3	0.2	0.5	0.4
50th perc delay (sec)	23.9	24.6	23.5	2.7	5.8	2.6
75th perc delay (sec)	36.4	41.5	38.8	5.7	12.4	6
95th perc delay (sec)	61.6	77	62.9	11.9	31.6	15.3

(b) EB Scenario - 1

Metric	b1	a1	a2	b2	a3	a4
Mean delay (sec)	26.9	30.9	26.7	5	8.6	4.8
Stdev delay (sec)	14.1	22.2	17.9	5.4	8.7	4.3
15th perc delay (sec)	12.7	7.1	7.2	0.4	0.7	0.6
50th perc delay (sec)	24.6	27.3	24.8	3.3	6.2	4
75th perc delay (sec)	35.6	44.8	39.5	6.4	12.1	6.6
95th perc delay (sec)	52.2	73.6	59.4	16.8	26.7	13.7

(c) NB Scenario - 2

Metric	b1	a1	a2	b2	a3	a4
Mean delay (sec)	37.3	44.2	37.2	6.8	11.7	8.9
Stdev delay (sec)	19.3	30.7	25.4	8.2	11.7	8.3
15th perc delay (sec)	17.7	11.7	9.5	0.8	1.1	1.4
50th perc delay (sec)	34.3	39.1	33.4	4.4	8.7	6.8
75th perc delay (sec)	48.7	63.7	54.2	9.3	17.2	12.6
95th perc delay (sec)	74.3	103	85.3	21	32.9	25.1

(d) EB Scenario - 2

Metric	b1	a1	a2	b2	a3	a4
Mean delay (sec)	13.3	14.3	14.3	2.6	3.6	2.7
Stdev delay (sec)	6.6	10	9.7	2.6	4	3.1
15th perc delay (sec)	6.3	3.3	3.5	0.2	0.1	0.1
50th perc delay (sec)	12.7	12.6	13.1	1.9	2.5	1.8
75th perc delay (sec)	17.3	21.4	20.7	3.7	5.2	4
95th perc delay (sec)	25.5	32.2	32.6	8.1	11	9.2

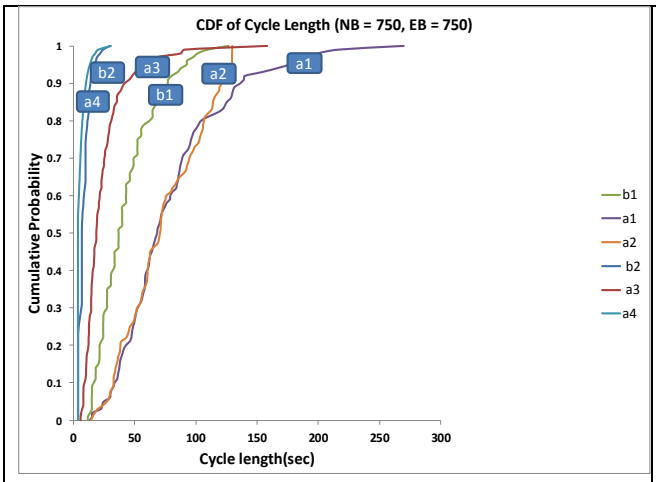
(e) NB Scenario - 3

Metric	b1	a1	a2	b2	a3	a4
Mean delay (sec)	35.1	44.9	36.3	6.8	12.6	5.6
Stdev delay (sec)	23.2	30.5	23.7	8.1	12.9	7.7
15th perc delay (sec)	11.8	13.5	10.9	1	1.4	0.4
50th perc delay (sec)	30.7	41	33.7	4.1	8.7	2.5
75th perc delay (sec)	48.4	62.3	52.8	8	17.6	7.5
95th perc delay (sec)	80.6	104	79.9	23.3	37	19.6

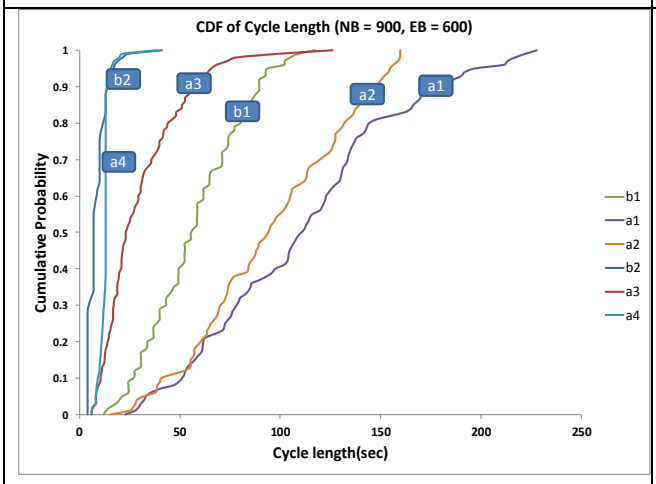
(f) EB Scenario - 3

5.5.2. CDFs of cycle lengths

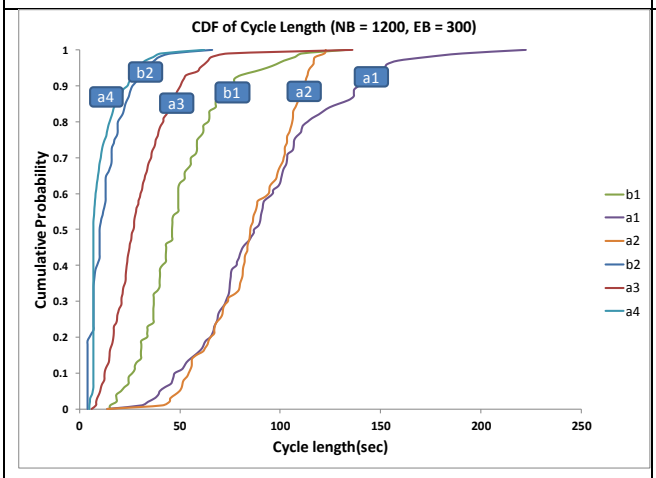
Figure 8 presents CDFs for cycle lengths and summary statistics associated of those CDFs in the three combinations of flows. Each subplot in the left column contains CDFs for the cycle lengths of both the bid-based control and actuated control options. As was the case for average delays, the best performance is reflected by the curve furthest to the left. When the constraints on minimum greens and clearance intervals are imposed the CDFs produced by bid-based control have distributions with the lowest cycle lengths (‘b1’). When maximum greens are imposed with actuated control, the distributions of cycle lengths can be made comparable to those produced by bid-based control (‘a2’, and ‘a4’). For example, the 90th percentile cycle length produced by bid-based control for option #1 is around 75-85 seconds, whereas it is around 150-180 seconds for ‘a1’.



Metric	b1	a1	a2	b2	a3	a4
Mean CL (sec)	42.7	79.5	73.2	8.6	23.8	5.8
Stdev CL (sec)	23.7	46.3	32.9	4.5	18.3	3.6
15th perc CL (sec)	20.4	37.9	35.4	4	10.6	3.8
50th perc CL (sec)	37	68.5	70.8	7	18.8	3.8
75th perc CL (sec)	52.5	96.2	103	10.2	28.3	6.5
95th perc CL (sec)	92.6	177	130	16.8	58.7	13.6



Metric	b1	a1	a2	b2	a3	a4
Mean CL (sec)	57.1	111	94.8	8.5	29.9	12.5
Stdev CL (sec)	23.8	49.8	38.1	4.7	19.8	2.7
15th perc CL (sec)	30.6	57.5	55.4	4	12.5	10.2
50th perc CL (sec)	55.6	111	94	7	23	13
75th perc CL (sec)	74.1	137	127	10	39.8	13
95th perc CL (sec)	93.7	197	155	17	65.6	15.6



Metric	b1	a1	a2	b2	a3	a4
Mean CL (sec)	49.4	91.1	85.1	13.4	30.1	11.3
Stdev CL (sec)	21.2	35.8	22	9.7	16.8	8.2
15th perc CL (sec)	30.6	57.5	58.2	4	15	6.9
50th perc CL (sec)	46.1	87	85.1	10	26.9	6.9
75th perc CL (sec)	58.7	107	103	17.4	38	12.6
95th perc CL (sec)	93.4	152	115	33.1	60	29.5

CDFs of Cycle Lengths in each of 3 Flow Combinations

Summary Statistics of Cycle Lengths in each of 3 Flow Combinations

Figure 8: Results (cycle lengths) for six simulation scenarios

Also, when maximum greens are imposed, actuated control and bid based control have very similar performance especially when constraints on minimum greens, and clearance interval are removed. Three tables in the right column present various statistics associated with cycle length for six simulation options. In a way these results summarize some of the statistical properties of each of the distributions presented in graphs in the left column.

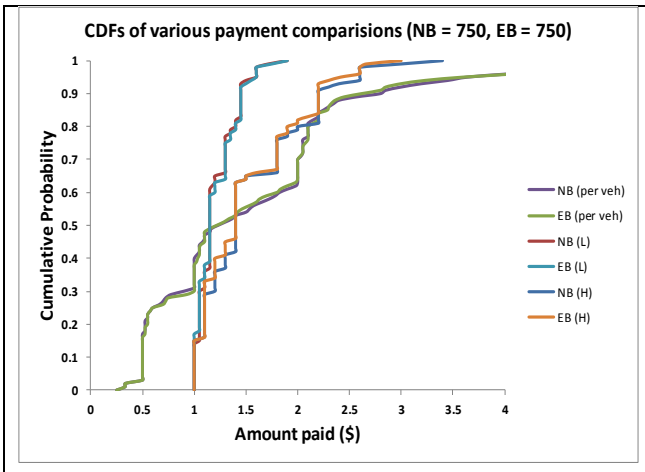
5.5.3. Distribution of driver payments

Drivers pay an initial fee of $1 followed by subsequent voluntary contributions. The question is: what additional contributions do they make, and how different are those contributions for the drivers with low and high values of time? Results from bid-based control option # b1 are presented here.

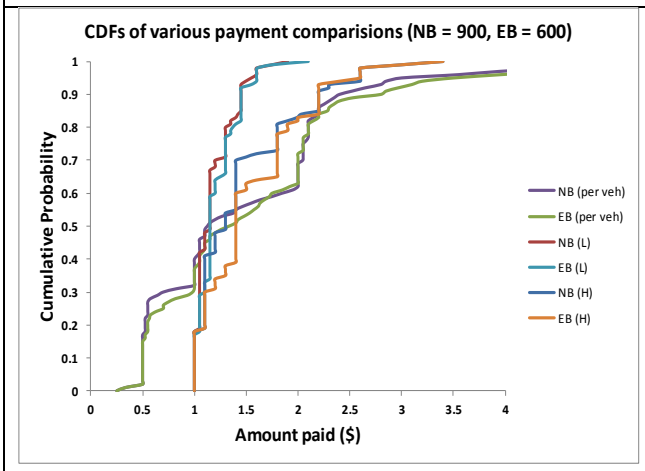
Contrasts in the contribution amounts can be seen in Figure 8. The figure presents CDFs for total amounts contributed by the drivers with both high and low values-of-time. The CDFs in light blue and red (graphs in the far-left) represent amounts paid by drivers with a low value-of-time on the EB and NB approaches, respectively. Likewise, CDFs in orange and blue represent similar results, but for drivers with a high value-of-time. Also presented are the distributions of average payments made by the movement managers to the municipality. CDFs in green and deep blue represent payments by the EB and NB movement managers, respectively.

The contributions of the drivers all have a lower bound of $1; the initial payment they make. However, the distribution of payments made by the movement managers extends to lower values, because in some instances when the payment is for use of the minimum green, the average payment per vehicle can be less than $1 per vehicle discharged. It is evident the more the variations in NB, EB traffic, the higher the variation in the payment amounts across drivers. This is especially true for drivers with high value-of-time. It is evident from the graphs in this figure that drivers with high VOT on the minor approach (EB) are paying higher amounts than those by drivers with high VOT on the major approach (NB). This is

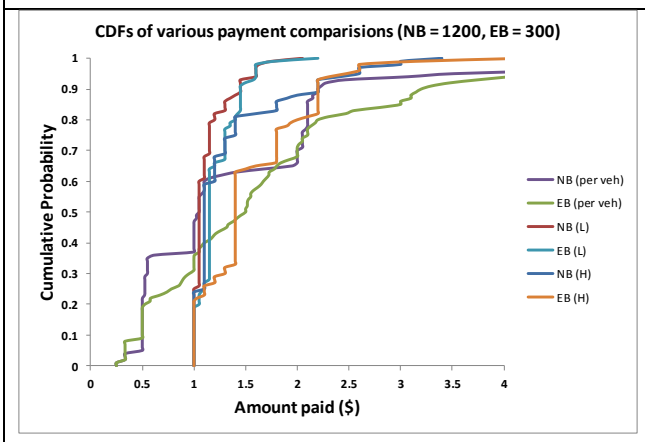
because drivers on minor approach experience higher delays and drivers likely to make higher payments in terms of voluntary contributions to reduce the delays.



Metric	Vehicle payments				Cost of service per vehicle	
	NB (L)	NB (H)	EB (L)	EB (H)	NB	EB
Mean ($)	1.2	1.5	1.2	1.5	1.6	1.6
Stdev ($)	0.2	0.5	0.2	0.5	1.1	1.1
15th perc ($)	1.1	1	1	1	0.5	0.5
50th perc ($)	1.2	1.4	1.2	1.4	1.2	1.2
75th perc ($)	1.3	1.8	1.3	1.8	2.1	2.1
95th perc ($)	1.6	2.6	1.6	2.4	3.6	3.6



Metric	Vehicle payments				Cost of service per vehicle	
	NB (L)	NB (H)	EB (L)	EB (H)	NB	EB
Mean ($)	1.2	1.5	1.2	1.5	1.5	1.6
Stdev ($)	0.2	0.5	0.2	0.5	1	1.1
15th perc ($)	1	1	1	1	0.5	0.5
50th perc ($)	1.2	1.3	1.2	1.4	1.1	1.3
75th perc ($)	1.3	1.8	1.3	1.8	2.1	2.1
95th perc ($)	1.6	2.6	1.6	2.6	3	3.5



Metric	Vehicle payments				Cost of service per vehicle	
	NB (L)	NB (H)	EB (L)	EB (H)	NB	EB
Mean ($)	1.1	1.3	1.2	1.5	1.5	1.7
Stdev ($)	0.2	0.5	0.2	0.5	1.5	1.3
15th perc ($)	1	1	1	1	0.5	0.5
50th perc ($)	1.1	1.1	1.2	1.4	1.1	1.5
75th perc ($)	1.2	1.4	1.3	1.8	2.1	2.1
95th perc ($)	1.6	2.6	1.6	2.5	3.5	4.3

CDFs of various payments in each of 3 Flow Combinations

Summary Statistics of various payments in each of 3 Flow Combinations

Figure 9: Cumulative density functions of various payments

Kolmogorov-Smirnov tests are done to evaluate statistical differences in the distribution of average payments made by NB and EB managers. Table 7 summarizes the test results. Test results suggest that one cannot reject the null hypothesis, that these distributions are similar in the case of equal flow conditions, whereas the null hypothesis can be rejected in the other two flow conditions.

Table 7: Summary of KS Test results for distribution of average payments

Scenario	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
sce -1	0.014007	0.654284	0.028017	**0.7853**	0.028017	0.4248	0.0226976	0.5701
sce - 2	0.045462	2.159346	0.093074	**0.0002***	0.0269924	0.4564	0.093074	<.0001*
sce - 3	0.072965	3.419226	0.2036817	**<.0001***	0.0444309	0.3287	0.2036817	<.0001*

In Figure 9 it is easy to see that the drivers with a high value-of-time (VOT) contribute higher amounts than those with a low VOT, while both classes of drivers experience similar quality of service (please refer to summary statistics of delays for two classes of drivers presented in Table 8).

Table 8: Summary statistics of delays for two classes of drivers

Metric	Scenario - 1				Scenario - 2				Scenario - 3			
	NB (H)	NB (L)	EB (H)	EB(L)	NB (H)	NB (L)	EB (H)	EB(L)	NB (H)	NB (L)	EB (H)	EB(L)
Mean delay (sec)	29.9	29	28.3	29	26	26.2	36.1	36.1	13.1	13.3	35.2	37
Stdev delay (sec)	17.8	17.1	16.5	16.6	14.2	14.1	20.6	20.4	6.9	7.1	24.8	25.3
15th perc delay (sec)	12.5	11.5	12	12.5	11.6	11.6	16.4	15.6	6.8	6.5	11.1	11
50th perc delay (sec)	25.8	26.4	26.1	25.8	23.1	24.1	32.5	33.1	12.3	12.8	29.9	31.1
75th perc delay (sec)	39.7	38.8	37.9	38.5	34.8	34.9	47.6	48	16.2	16.9	48	51.2
95th perc delay (sec)	66.5	62.4	58.2	61.8	52.9	52.9	78.7	76.1	25.1	25.8	85.2	89.4

Table 9 summarizes driver contributions or payments two classes of drivers made to their respective movement managers and costs incurred by movement managers to discharge them. There are 10 columns in this table. Columns 1 & 2 represent the scenario and approach these results pertain to; values in columns 3-5 represent cost of servicing two classes of

drivers and the total costs that movements managers incurred for servicing vehicles on their approach. Values in columns 6-8 represent payments made by two classes of drivers towards service, and total income movement managers received from their respective drivers. Values in column 9 represent the ratio of payments made by drivers with low value-of-time to cost of servicing them; the values in column 10 are similar but for driver with high value-of-time. Based on the values presented in the table it appears that the high-VOT drivers are contributing proportionally more income than their percentage of traffic stream (50%).

Table 9: summary of driver payments vs cost of service

Scenario	Approach	cost of service ($)			driver payments ($)			payment/cost of service	
		low VOT	high VOT	combined	low VOT	high VOT	combined	low VOT	high VOT
Sce-1	1	1315	1301	2616	1175	1441	2616	**0.45**	**0.55**
	2	1312	1280	2592	1142	1450	2592	**0.44**	**0.56**
Sce-2	1	1538	1517	3055	1387	1668	3055	**0.45**	**0.55**
	2	1001	1058	2059	902	1157	2059	**0.44**	**0.56**
Sce-3	1	1855	1833	3688	1721	1967	3688	**0.47**	**0.53**
	2	490	497	987	427	560	987	**0.43**	**0.57**

Last question to answer here is if the average cost to the movement manager of servicing two classes of vehicles is the same. For the purposes of this analysis ten Monte Carlo simulation for scenario – 2 flow conditions were conducted. Table 10 summarizes these results. This table has eight columns. Values in columns 1-2 represent simulation number and the approach that these results pertain to; values in columns 3-5 represent average cost of servicing two classes of drivers and average cost of service (irrespective of driver class). Values in columns 6-8 represent average driver payments for two classes of drivers and average driver payments (irrespective of driver class). The values associated with average cost of service for two classes of drivers (values in columns 3 & 4) in Table 10 suggest that movement managers are paying similar amount for discharging two classes of drivers. Two-Sample t-Test was conducted to check if the differences in average cost of service for two classes of drivers is zero. Table 11 summarizes test results. Since p-value of the test (0.3534)

is greater than α (0.05) one should not reject the null hypothesis that the difference in average cost of service for two classes of drivers is zero.

Table 10: summary of average driver payments vs average cost of service

Simulation #	Approach	Average cost of service ($)			Average driver payments ($)		
		Low VOT	high VOT	combined	Low VOT	high VOT	combined
1	1	1.31	1.33	1.32	1.18	1.46	1.32
	2	1.36	1.38	1.37	1.21	1.53	1.37
2	1	1.36	1.34	1.35	1.19	1.5	1.35
	2	1.42	1.35	1.38	1.21	1.57	1.38
3	1	1.3	1.36	1.33	1.19	1.47	1.33
	2	1.35	1.39	1.37	1.22	1.52	1.37
4	1	1.3	1.33	1.31	1.18	1.46	1.31
	2	1.4	1.37	1.38	1.2	1.56	1.38
5	1	1.28	1.32	1.3	1.18	1.43	1.3
	2	1.37	1.37	1.37	1.21	1.54	1.37
6	1	1.31	1.34	1.33	1.19	1.47	1.33
	2	1.35	1.39	1.37	1.21	1.53	1.37
7	1	1.3	1.34	1.32	1.19	1.46	1.32
	2	1.38	1.36	1.37	1.22	1.53	1.37
8	1	1.34	1.35	1.34	1.19	1.5	1.34
	2	1.39	1.38	1.39	1.2	1.57	1.39
9	1	1.31	1.33	1.32	1.18	1.47	1.32
	2	1.39	1.34	1.37	1.2	1.54	1.37
10	1	1.31	1.31	1.31	1.17	1.44	1.31
	2	1.33	1.4	1.36	1.2	1.54	1.36

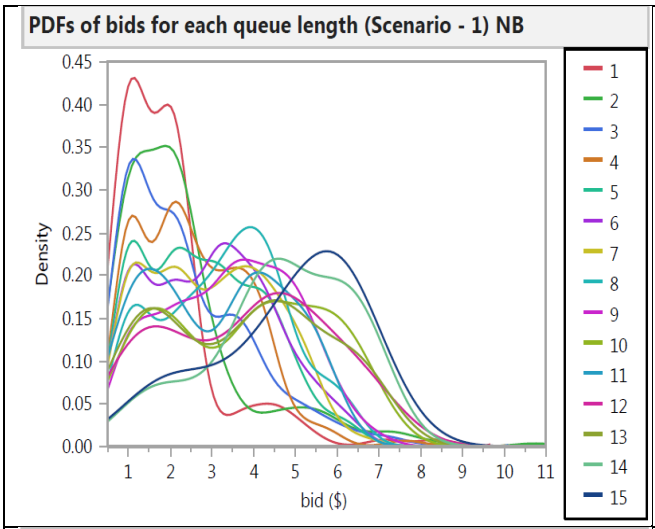
Table 11: Results of t-Test to check differences in cost of service

t-Test: Two-Sample Assuming Equal Variances			
Difference	-0.00988	t Ratio	-0.9356
Std Err Dif	0.01056	DF	38
Upper CL Dif	0.0115	Prob > |t|	0.3554
Lower CL Dif	-0.03126	Prob > t	0.8223
Confidence	0.95	Prob < t	0.1777

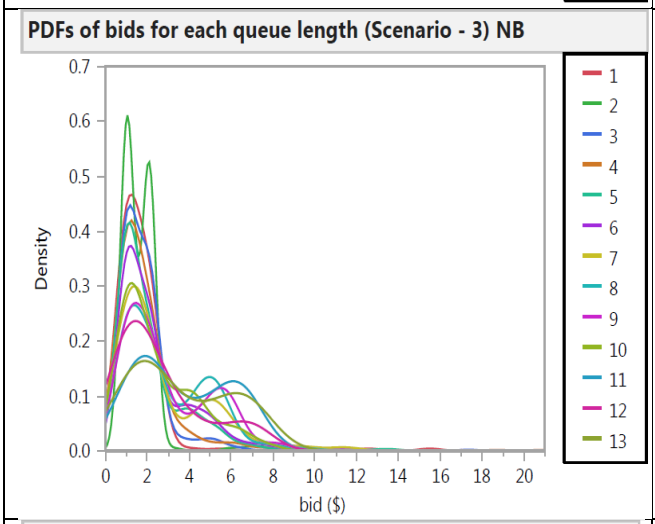
5.5.4. Impact of queue length on bids

This section analyzes the impact of queue length on bids submitted by movement managers. The simulation model was programmed to keep track of each movement manager's bid submissions associated with every queue length. These bid amounts included both winning and losing bids. Figure 10 presents the distributions of the submitted bids for all of the queue lengths that had at least 10 bid submissions.

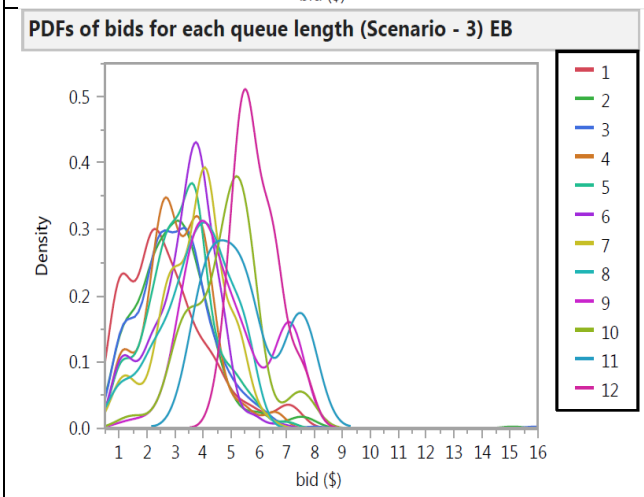
The three subplots to the left present PDFs of bids associated with each queue length and while the quantile tables to the right summarize corresponding statistics associated with each PDF. While it is difficult to see specific patterns in PDFs; the quantile tables show that the median bid increases with queue length (a few exceptions are queue lengths 11 & 13 in the case of scenario-1 NB; queue lengths 9, 10, 12 & 13 in the case of scenario – 3 NB; queue length 8 in the case of scenario - 3 EB). The general trend of increased bids with increased queue length indicates that queue length does have an impact on bids. However, because there are distributions associated with each queue length, and the quantiles vary from one queue length to another, queue length is not the only variable influencing the bid amounts.



Queue Length	Submitted bid ($) Scenario - 1, NB						
	Minimum	10%	25%	Median	75%	90%	Maximum
1	1	1	1	2	2.1	3.74	8.4
2	1	1	1.003333	2	2.55	4.91	10.75
3	1	1	1.003333	2.05	3.45	4.71	8.8
4	1	1	1.3375	2.375	3.6	4.25	8.9
5	1	1	1.7	2.75	3.95	4.8	6.65
6	1	1	2	3.075	4.05	5.12	6.45
7	1	1	2	3	4.1375	5.075	6.85
8	1	1.05	2.1	3.475	4.35	5.325	6.4
9	1	1.095	2.1	3.55	4.625	5.28	6.8
10	1	1.15	2.1	4.075	5.45	6.2	6.95
11	1	1.000002	1.625	3.5	4.35	5.3	5.75
12	1	1.05	2.075	4.2	5.3	6.35	7.25
13	1.05	1.105	1.6125	4.15	5.2	6.55	6.65
14	1.15	1.79	3.85	4.55	5.95	6.67	6.85
15	1.4	1.61	3.3625	5.3	5.95	6.81	7.05



Queue Length	Submitted bids ($), Scenario - 3, NB						
	Minimum	10%	25%	Median	75%	90%	Maximum
1	1	1	1	1.05	2.05	2.105	15.5
2	1	1	1.00005	1.1	2.09	2.1	10.15
3	1	1	1	1.1	2.1	2.776	17.35
4	1	1	1	1.1	2.1	3.65	20.85
5	1	1	1	1.1	2.2	4.55	13.75
6	1	1	1.05	2	3	4.83	11.4
7	1	1	1.05	2.05	4.5375	5.7	11.75
8	1	1	1.05	2.15	4.55	5.53	9.65
9	1	1	1.1	2.1	4.85	5.85	8.55
10	1	1	1.05	2.075	3.8	5.25	7.8
11	1	1.015007	2.0375	3.075	6	6.725	6.95
12	1	1	1	1.8	3.5625	6.82	7.1
13	1	1.08	1.45	2.5	6.225	6.98	7



Queue Length	Submitted bid ($) Scenario - 3, EB						
	Minimum	10%	25%	Median	75%	90%	Maximum
1	1	1	1.950117	2.4	3.501	4.65	7.301589
2	1	1.34	2.1425	2.95	3.6	4.45	15.1
3	1	1.1931	2.175	3.1	3.75	4.65	15.9
4	1	1.4486	2.45	3.175	3.9	4.4	6.8
5	1	1.756	2.6	3.25	3.9	4.95	7
6	1	1.4	2.6	3.65	4	4.7	6.75
7	1	2.01	3	4	4.2	5.27	6.3
8	1	1.95	3	3.92735	4.6	5.5	5.55
9	1.4	3.3	4	4.3	5.5	7.1	7.1
10	1.45	3.22	3.7375	4.95	5.5	5.65	7.5
11	4	4	4.1125	5.375	7.5	7.5	7.5
12	4.85	4.915	5.5	5.5	6.45	7.395	7.5

Bid submission for various queue lengths

Quantile table of bid submissions for various queue lengths

Figure 10: Probability Density Functions (PDFs) of bid amount for various queue lengths

5.5.5. Income stream received by the municipality

The total income received by the municipality is another economic consideration. Table 12 summarizes the payments received by the municipalities for each of three flow combinations. It is important to recognize that the nominal fee paid by the drivers influences the result. The nominal fee provides a minimum income stream. It is always $1,500 in these scenarios. Hence, the values in Table 12 can be contrasted with this value to show incremental above-minimum amounts received by the municipality. It is also useful to recognize that in Scenario – 1, the minimum income streams from the approaches are both $750, in Scenario – 2, they are $900 for the northbound approach and $600 for the eastbound approach, and in Scenario – 3 they are $1,200 for the northbound approach and $300 for the eastbound approach. In principle, one could set the nominal fee so that it covers the long-run average costs of the intersection. The additional income would then reflect the value that the drivers place on their delays.

Table 12: Total funds ($) transferred to the municipality per hour

Total income transferred	NB	EB
Sce - 1	$ 1,028	$ 1,013
Sce - 2	$ 1,179	$ 822
Sce - 3	$ 1,476	$ 414

5.6. Variations in the driver contributions

As one might expect, there are three parameters that influence the bids submitted by movement managers: 1) the initial fee paid by drivers; 2) the mix of driver classes in the traffic stream; and 3) voluntary monetary contributions made by the drivers. The purpose of this section is to investigate the influence of each parameter on the game outcome. The following seven cases explore these ideas in more detail.

c1) In this case, both NB and EB drivers pay an initial fee of $1 upon their arrival, 50 percent of drivers in the traffic stream have high value-of-time, and drivers make voluntary monetary contributions as per the logic described in Table 1. This is the default case in the sense that the output from rest of the six cases is compared and contrasted against this case. Simulation parameters on NB are left unchanged in rest of the six cases, while one simulation parameter on EB was changed in each of the six cases.

c2) In this case, initial fee paid by the drivers on EB is set to $0.5. Please recall, movement managers are allowed to collect only the initial fee from the drivers, any additional income they receive is strictly voluntary contributions made by drivers. Here EB manager is receiving a guaranteed income of $300 instead of $600. Given the fact that the nominal fee to use the intersection is still $1, it will be interesting to see the impact of this change on the economics of the system.

c3) In this case, EB drivers pay an initial fee of $2 upon arrival. Technically, the EB movement manager is receiving a guaranteed income stream of $1200 instead of $600. So, in principle, the EB movement manager has the ability to submit much higher bids. This might be both good and bad: good in the sense that the EB manager has the ability to submit higher bids, and bad if control is too snappy as a consequence. Only further analysis will reveal whether this change has an overall positive or negative impact on the system.

c4) In this case, the voluntary contributions of EB drivers with a high value-of-time are doubled. For example, previously they may have voluntarily contributed $0.5 when $K < 1$ and Test = 1 (please refer to Table 1), but now they voluntarily contribute $1 under the same circumstances. In effect, this change will increase the additional voluntary income the EB manager receives. As a result, the EB manager has the ability to submit higher bids. Whether this change has any significant impact or not is a question for future research.

c5) In this case, the voluntary contributions of EB drivers with a low value-of-time are halved. For example, previously they may have voluntarily contributed \$0.2 when K < 1 and Test = 1 (please refer to Table 1), but now they voluntarily contribute \$0.1 under similar circumstances. As one might notice, drivers with a low value-of-time already contribute fewer amounts than those contributed by drivers with a high value-of-time. Therefore, this change might have little or no impact on the system.

c6) In this case, the percentage of drivers with a high value-of-time in the EB approach is changed from 50 to 30. In other words, only a small percent of drivers make aggressive voluntary contributions. As a result, the EB manager might be constrained from submitting higher bids. It will be interesting to see if this change hampers EB drivers from receiving better service.

c7) In this case, the percentage of drivers with high value-of-time on EB is changed from 50 to 70. Contrary to the previous case, a high percentage of drivers making aggressive voluntary contributions enable the EB manager to submit higher bids. It will be interesting to see the impact this change has on the income stream received by the EB movement manager.

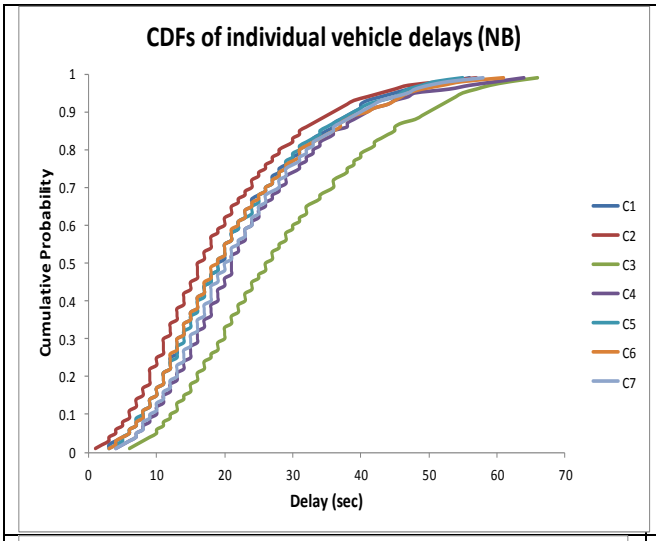
5.6.1. Analysis of Results

The Scenario – 2 flow condition (v_N = 900, v_E = 600) is used to explore the seven cases described above. For comparative purposes, the same vehicular arrival headway sequence is used in all seven cases. The following five performance metrics are used to evaluate the simulation outputs:

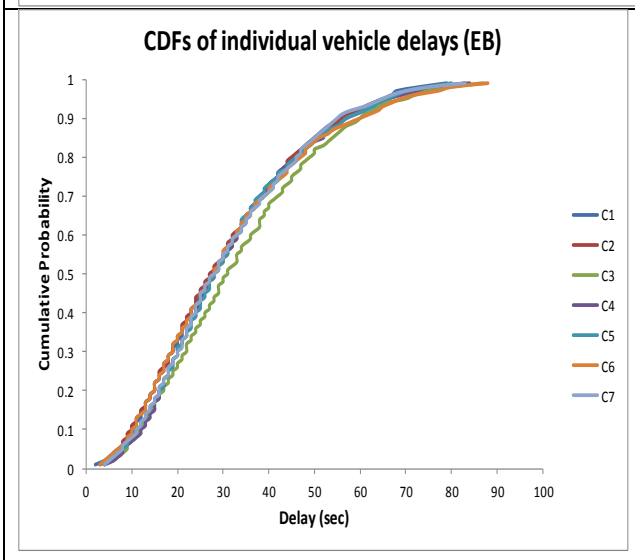
a) Individual vehicle delay distributions
b) Differences in voluntary contributions made by the two classes of drivers
c) Individual vehicle payment distributions
d) Winning bids submitted by movement managers
e) Total income transferred to the municipality

a) Individual vehicle delay distributions

This analysis compares the cumulative probability densities of delays and summary statistics associated with the NB and EB approaches among seven cases. The results are presented in Figure 11. Each subplot in the left column contains seven CDFs (one for each case), while the tables in the right column represent summary statistics of CDFs presented in the left column.



Metric	c1	c2	c3	c4	c5	c6	c7
Mean delay (sec)	21.4	19.1	28.8	23.7	21.5	21.7	22.8
Stdev delay (sec)	12.2	12	14.3	12.8	12	12.9	11.9
15th perc delay (sec)	9.6	7.6	14.1	11.4	9.6	9.7	11
50th perc delay (sec)	19.3	16.5	26.2	21.1	19.1	18.8	20.4
75th perc delay (sec)	28.5	25.5	38.2	30.7	28.7	28.6	29.5
95th perc delay (sec)	45.3	43	55	48.4	46	47.5	46.6



Metric	c1	c2	c3	c4	c5	c6	c7
Mean delay (sec)	30.7	30.7	33.8	31.8	31.5	31.6	31.3
Stdev delay (sec)	17.9	18.3	18.7	18	17.7	19.5	17.7
15th perc delay (sec)	12.2	12.1	14.4	14.6	13.8	12.5	13.8
50th perc delay (sec)	27.8	27.2	30.6	28.2	28.3	28	27.9
75th perc delay (sec)	42.1	41.8	45.4	41.9	41.7	43	42.5
95th perc delay (sec)	64.6	65.6	70	65.7	66	69.4	65.4

CDFs of delays in each of seven cases

Summary Statistics of delays in each of seven cases

Figure 11: CDFs of individual vehicle delays in seven cases

One can see that changing the initial fee paid by drivers on eastbound approach has the most significant impact on northbound delay distributions (variances in EB distributions are less obvious). The case in which the initial fee was set to $0.5 (‘c2’) produced the best delay distribution, whereas the case in which it was set to $2 (‘c3’) produced the worst delay distribution on NB. Intuitively this makes sense; in the former case (‘c2’), the EB manager is receiving a guaranteed income of $300 instead of $600, constraining him from submitting higher bids; whereas in the latter case (‘c3’), he is receiving a guaranteed income of $1200 instead of $600, enabling him to submit higher bids. In fact Table 17 presents total income stream received by movement managers in each of the seven cases.

Kolmogorov-Smirnov tests were performed to see if these distributions are statistically similar. As mentioned earlier, c1 is the default case, so the delay distributions for the other six cases were compared against c1. Table 13 (a) presents a summary of results for the NB approach. Test results suggest that delay distributions in ‘c2’, ‘c3’, ‘c4’, and ‘c7’ are statistically different than those in ‘c1’. However, one can reject the hypothesis that they are different in ‘c5’ and ‘c6’. Please recall that in ‘c5’, the amounts contributed by EB drivers with a low value-of-time was changed. As expected, it didn’t have a significant impact on the delay distributions of NB drivers. In ‘c6’, the percentage of drivers on EB approach with a high value-of-time was changed from 50 to 30. Apparently, that change did not have any significant impact on NB driver delay distributions. Table 13 (b) presents similar results for the EB approach. Test results suggest that only delay distributions ‘c3’ and ‘c4’ are statistically different than those in ‘c1’.

Table 13: Summary of KS test results (delay)

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.04877	3.290424	0.0975395	**<.0001***	0.0008787	0.9982	0.0975395	<.0001*
(c1, c3)	0.112917	7.618369	0.2258348	**<.0001***	0.2258348	<.0001*	0	1
(c1, c4)	0.049868	3.364533	0.0997364	**<.0001***	0.0997364	<.0001*	0	1
(c1, c5)	0.014279	0.963412	0.0285589	**0.3113**	0.0285589	0.1562	0.0118629	0.7259
(c1, c6)	0.013181	0.889304	0.026362	**0.4077**	0.026362	0.2056	0.016696	0.5302
(c1, c7)	0.039104	2.638268	0.0782074	**<.0001***	0.0782074	<.0001*	0.004833	0.9482

(a) North Bound

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.010774	0.587181	0.0215488	**0.8808**	0.0127946	0.7842	0.0215488	0.5018
(c1, c3)	0.03771	2.055132	0.0754209	**0.0004***	0.0754209	0.0002*	0	1
(c1, c4)	0.027946	1.523	0.0558923	**0.0193***	0.0558923	0.0097*	0.0121212	0.804
(c1, c5)	0.022222	1.21106	0.0444444	**0.1064**	0.0444444	0.0532	0.013468	0.7639
(c1, c6)	0.013805	0.752325	0.0276094	**0.6233**	0.0276094	0.3224	0.0148148	0.7219
(c1, c7)	0.021886	1.192711	0.043771	**0.1162**	0.043771	0.0581	0.0127946	0.7842

(b) East Bound

b) Differences in voluntary contributions made by the two classes of drivers

This analysis investigates the impact of changing various simulation parameters in voluntary monetary contributions made by the two classes of drivers. The question is: how different are those contributions for the drivers with low and high values of time among cases (c2-c7) when compared to 'c1'? Contrasts in the contributions made in each of the seven cases can be seen in Figure 12. This figure contains four subplots, and each plot contains seven CDFs (one for each case). The two plots in first column represent results for the two classes of drivers on northbound approach, whereas plots in the second column represent similar results for eastbound approach. A close examination of these plots suggests that a high percent of NB drivers (both classes) made the least voluntary contributions in 'c2', and somewhat higher contributions in 'c3'. Again, these are the two cases in which the initial fee paid by EB drivers was changed. Two CDFs standout as far as EB results are concerned: 'c5' for drivers with a low value-of-time, and 'c4' for drivers with a high value-of-time. This is not a surprise, given the fact that in both cases, the nature of voluntary contributions made by that

class of drivers was changed. Table 14 presents summary of voluntary contribution statistics for seven cases. These results summarize some of the statistical properties of each of the distributions presented in Figure 12.

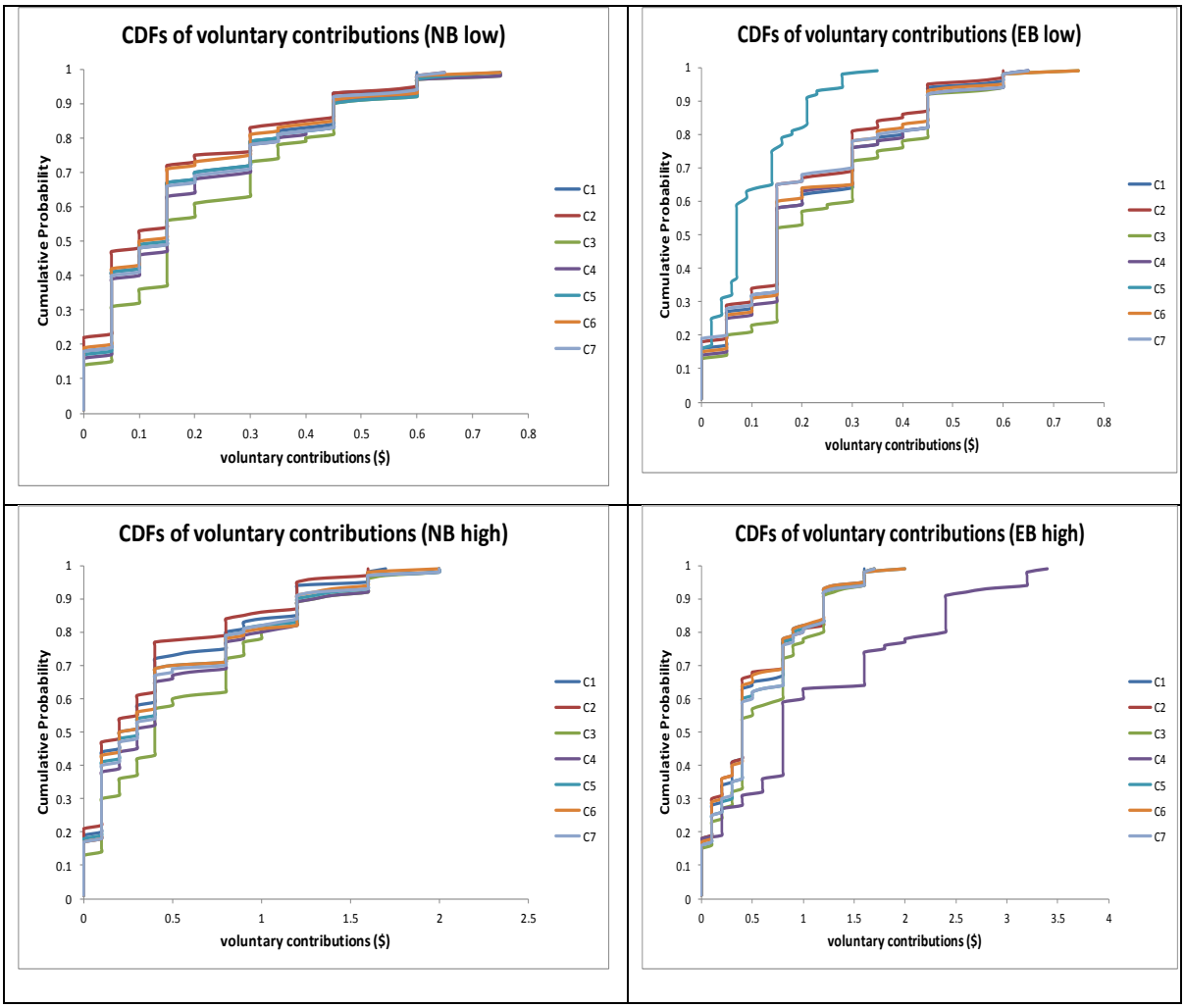


Figure 12: CDFs of voluntary contributions

Kolmogorov-Smirnov tests were performed to check the statistical differences in these distributions. Again, c1 is the default case, so the distributions of the other six cases are compared against c1. Table 15 (a) presents a summary of results for NB drivers with a low value-of-time. Test results suggest that the distribution of voluntary contributions in ‘c2’ and ‘c3’ are statistically different than those in ‘c1’. Again, these are the two cases in which the initial fee paid by EB drivers was changed. Table 15 (b) presents similar results for EB

drivers with a low value-of-time. Results suggest that these distributions in 'c3' and 'c5' are statistically different than those in 'c1'.

Table 14: Voluntary contribution statistics for seven cases

Metric	c1	c2	c3	c4	c5	c6	c7
Mean ($)	0.7	0.7	0.7	0.7	0.7	0.7	0.7
Stdev ($)	0.5	0.5	0.5	0.5	0.5	0.5	0.5
15th perc ($)	0.1	0.1	0.1	0.1	0.1	0.1	0.1
50th perc ($)	1	1	1	1	1	1	1
75th perc ($)	1.2	1.1	1.2	1.2	1.2	1.1	1.2
95th perc ($)	1.5	1.5	1.5	1.5	1.5	1.5	1.5

(a) Low VOT (NB)

Metric	c1	c2	c3	c4	c5	c6	c7
Mean ($)	0.7	0.4	1.2	0.7	0.6	0.7	0.7
Stdev ($)	0.5	0.3	1	0.5	0.5	0.5	0.5
15th perc ($)	0.1	0.1	0.2	0.2	0	0.1	0.1
50th perc ($)	1	0.5	1.5	1	0.7	1	0.9
75th perc ($)	1.2	0.7	2.2	1.2	1.1	1.2	1.2
95th perc ($)	1.5	1	2.5	1.5	1.2	1.5	1.5

(b) Low VOT (EB)

Metric	c1	c2	c3	c4	c5	c6	c7
Mean ($)	0.9	0.9	1.1	1	1	1	1
Stdev ($)	0.7	0.7	0.7	0.7	0.7	0.7	0.7
15th perc ($)	0.1	0.1	0.1	0.1	0.1	0.1	0.1
50th perc ($)	1	1	1.1	1.1	1	1.1	1
75th perc ($)	1.3	1.3	1.5	1.4	1.4	1.4	1.4
95th perc ($)	2.2	2.2	2.4	2.4	2.2	2.2	2.2

(c) **High VOT (NB)**

Metric	c1	c2	c3	c4	c5	c6	c7
Mean ($)	1	0.8	1.6	1.6	1.1	1	1.1
Stdev ($)	0.7	0.6	1.1	1.1	0.7	0.7	0.7
15th perc ($)	0.2	0.1	0.3	0.4	0.3	0.2	0.2
50th perc ($)	1	0.7	2	1.6	1.1	1.1	1.1
75th perc ($)	1.4	1.2	2.4	2.4	1.4	1.4	1.4
95th perc ($)	2.2	1.7	3.2	3.4	2.2	2.2	2.2

(d) High VOT (EB)

Table 15: Voluntary contributions made by drivers with a low value-of-time

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.028671	1.375597	0.0573414	**0.0454***	0.0026064	0.9922	0.0573414	0.0227*
(c1, c3)	0.057341	2.751194	0.1146829	**<.0001***	0.1146829	<.0001*	0	1
(c1, c4)	0.019983	0.95875	0.0399652	**0.3169**	0.0399652	0.1591	0	1
(c1, c5)	0.015204	0.729483	0.0304083	**0.6618**	0.0304083	0.345	0.0060817	0.9583
(c1, c6)	0.017376	0.833695	0.0347524	**0.4904**	0.0156386	0.7547	0.0347524	0.2491
(c1, c7)	0.006516	0.312636	0.0130321	**1**	0.0130321	0.8224	0	1

(a) North Bound

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.032432	1.247701	0.0648649	**0.0889**	0	1	0.0648649	0.0444*
(c1, c3)	0.041892	1.611613	0.0837838	**0.0111***	0.0837838	0.0055*	0	1
(c1, c4)	0.014189	0.545869	0.0283784	**0.9269**	0.0283784	0.551	0.0094595	0.9359
(c1, c5)	0.216892	8.343998	0.4337838	**<.0001***	0	1	0.4337838	<.0001*
(c1, c6)	0.011835	0.529399	0.0245151	**0.942**	0.0092205	0.9238	0.0245151	0.5709
(c1, c7)	0.027962	0.866834	0.0664486	**0.4401**	0.0174025	0.9021	0.0664486	0.2225

(b) East Bound

Table 16: Voluntary contributions made by drivers with a high value-of-time

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.023556	1.117338	0.0471111	**0.1646**	0	1	0.0471111	0.0823
(c1, c3)	0.077333	3.668242	0.1546667	**<.0001***	0.1546667	<.0001*	0.0008889	0.9991
(c1, c4)	0.035111	1.665466	0.0702222	**0.0078***	0.0702222	0.0039*	0	1
(c1, c5)	0.019556	0.927601	0.0391111	**0.3558**	0.0391111	0.1789	0	1
(c1, c6)	0.018222	0.864356	0.0364444	**0.4438**	0.0364444	0.2244	0.0008889	0.9991
(c1, c7)	0.026667	1.264911	0.0533333	**0.0815**	0.0533333	0.0408*	0.0008889	0.9991

(a) North Bound

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.019463	0.751285	0.0389262	**0.625**	0.0187919	0.7687	0.0389262	0.3234
(c1, c3)	0.043624	1.683915	0.0872483	**0.0069***	0.0872483	0.0034*	0.0026846	0.9946
(c1, c4)	0.17047	6.580223	0.3409396	**<.0001***	0.3409396	<.0001*	0.0053691	0.9788
(c1, c5)	0.026846	1.036256	0.0536913	**0.2331**	0.0536913	0.1168	0.0026846	0.9946
(c1, c6)	0.008791	0.273662	0.0208533	**1**	0.0084432	0.9757	0.0208533	0.8609
(c1, c7)	0.024257	1.087246	0.0502188	**0.1879**	0.0502188	0.094	0.0003111	0.9999

(b) East Bound

Tables 16 (a) and 16 (b) present similar results, but for drivers with a high value-of-time on northbound and eastbound approaches, respectively. The results suggest that the distribution of voluntary contributions in ‘c3’ and ‘c4’ are statistically different than those in ‘c1’.

There are a few more observations to be made concerning voluntary contributions made by the two classes of drivers: Firstly, setting the initial fee paid by eastbound drivers to $2 affected the nature of voluntary contributions (at least statistically) made by both classes of drivers on both approaches. Secondly, doubling the voluntary contributions of EB drivers with a high value-of-time also changed the distribution of voluntary contributions of the same class of drivers on the northbound approach. A possible reason for this phenomenon can be found in Table 17, which shows the total additional income contributed by the two classes of drivers on both the northbound and eastbound approaches. In ‘c4’, EB high value-of-time drivers voluntarily contributed a total of $335 (twice more than in ‘c1’), and NB low value-of-time drivers contributed amounts similar to what they did in ‘c1’. As a result, high value-of-time drivers ended up contributing more. Thirdly, notice the amounts contributed by both classes of drivers on the eastbound approach in ‘c6’ and ‘c7’ (please recall ‘c6’ has 30

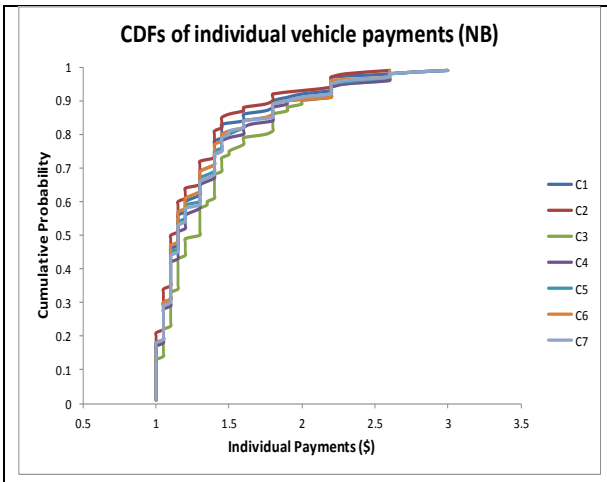
percent high value-of-time drivers and ‘c7’ has 70). The ratio of these values for high and low value-of-time drivers is about 0.46 in ‘c6’, whereas it is 15.6 in ‘c7’. These values clearly suggest movement managers receive more additional income from high value-of-time drivers than from their counterparts.

Table 17: Additional income contributed by two classes of drivers

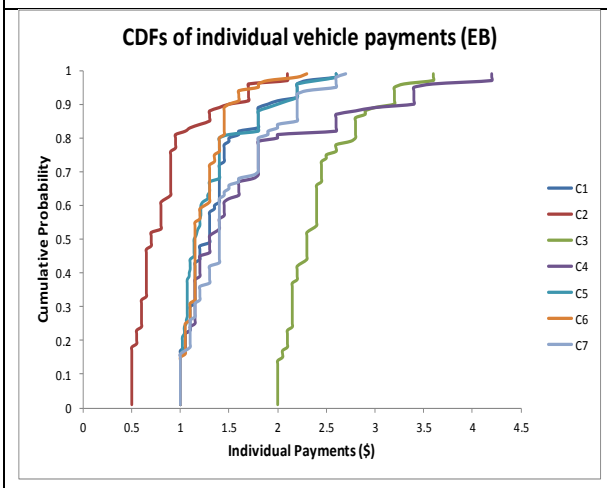
Case	total voluntary contributions by drivers ($) per hour						delay (85th percentile)	
	NB (High VOT)	NB (Low VOT)	NB (Tot)	EB (High VOT)	EB (Low VOT)	EB (Tot)	NB (sec)	EB (sec)
c1	$192	$83	$275	$155	$62	$217	34.6	49.9
c2	$171	$74	$245	$153	$56	$209	31.1	49.9
c3	$260	$102	$362	$178	$70	$248	44.8	54.7
c4	$231	$90	$321	$335	$64	$399	36.5	51.5
c5	$214	$86	$300	$165	$29	$194	34.2	51.1
c6	$210	$80	$290	$46	$105	$151	35.7	51.4
c7	$217	$85	$302	$281	$18	$299	35.5	49.7

c) Individual vehicle payment distributions

This analysis compares the cumulative probability densities of total amounts paid by drivers for the service and summary statistics associated with those CDFs for NB and EB approaches among seven cases. Please note that individual payments include the initial fee paid plus any voluntary monetary contributions that drivers make and cost of service is the amount movement manager pays to the municipality to discharge vehicles on his approach. The results are presented in Figure 13. Each subplot in the left column contains seven CDFs (one for each case), while the tables in the right column represent summary statistics of the CDFs. The total amount contributed by EB drivers seems to vary significantly among the cases, while the variance is less pronounced among NB drivers.



Metric	c1	c2	c3	c4	c5	c6	c7
Mean ($)	1.3	1.3	1.4	1.3	1.3	1.3	1.3
Stdev ($)	0.4	0.3	0.4	0.4	0.4	0.4	0.4
15th perc ($)	1	1	1.1	1	1	1	1
50th perc ($)	1.2	1.1	1.3	1.2	1.2	1.2	1.2
75th perc ($)	1.4	1.4	1.5	1.4	1.4	1.4	1.4
95th perc ($)	2.2	2.2	2.2	2.2	2.2	2.2	2.2



CDFs of individual vehicle payments in each of seven cases

Metric	c1	c2	c3	c4	c5	c6	c7
Mean ($)	1.4	0.8	2.4	1.7	1.3	1.2	1.5
Stdev ($)	0.4	0.4	0.4	0.8	0.4	0.2	0.5
15th perc ($)	1	0.5	2.1	1	1	1	1
50th perc ($)	1.3	0.7	2.3	1.3	1.1	1.2	1.4
75th perc ($)	1.5	0.9	2.5	1.8	1.4	1.4	1.8
95th perc ($)	2.2	1.7	3.2	3.4	2.2	1.6	2.3

Summary Statistics of individual vehicle payments in each of seven cases

Figure 13: CDFs of individual vehicle payments

Again, KS Tests were conducted to check for statistical differences in these distributions. Tables 18(a) and 18(b) present summaries of the test results for the NB and EB, respectively. NB test results suggest that the distribution of total amounts in ‘c2’, ‘c3’, and ‘c4’ are statistically different than those in ‘c1’. This makes sense because in all of the three cases, the EB manager’s income stream is significantly affected. In some instances, he has a higher income stream (‘c3’ and ‘c4’), while in others he has a lower income stream (‘c2’). Therefore, in ‘c2’, NB drivers were able to get the same or better service by paying less,

whereas in ‘c3’ and ‘c4’ they are forced to pay more. EB test results suggest that the distribution of total amounts in all six cases (c2-c7) are statistically different than those in ‘c1’.

Table 18: Summary of KS Test results individual payments

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.021749	1.467351	0.0434974	**0.0270***	0	1	0.0434974	0.0135*
(c1, c3)	0.063928	4.313123	0.1278559	**<.0001***	0.1278559	<.0001*	0.0004394	0.9996
(c1, c4)	0.024385	1.645212	0.0487698	**0.0089***	0.0487698	0.0045*	0	1
(c1, c5)	0.016257	1.096808	0.0325132	0.1802	0.0325132	0.0902	0	1
(c1, c6)	0.011204	0.755908	0.0224077	0.6172	0.0224077	0.3189	0.013181	0.6734
(c1, c7)	0.015817	1.067165	0.0316344	0.2048	0.0316344	0.1025	0.0004394	0.9996

(a) North Bound

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.405387	22.09267	0.8107744	**<.0001***	0	1	0.8107744	<.0001*
(c1, c3)	0.453199	24.69829	0.9063973	**<.0001***	0.9063973	<.0001*	0	1
(c1, c4)	0.089562	4.880939	0.1791246	**<.0001***	0.1791246	<.0001*	0	1
(c1, c5)	0.09899	5.394722	0.1979798	**<.0001***	0.0121212	0.804	0.1979798	<.0001*
(c1, c6)	0.076768	4.183662	0.1535354	**<.0001***	0.0121212	0.804	0.1535354	<.0001*
(c1, c7)	0.087542	4.770843	0.1750842	**<.0001***	0.1750842	<.0001*	0	1

(b) East Bound

Lastly, Table 19 presents the averages of driver payments. For example, values in column ‘4’ can be computed as ratio of sum of individual driver payments for drivers with high VOT on NB and number of drivers with high VOT on NB. The values in the Table suggest that both northbound and eastbound high value-of-time drivers contributed more than the average, while those with a low value-of-time contributed less than the average.

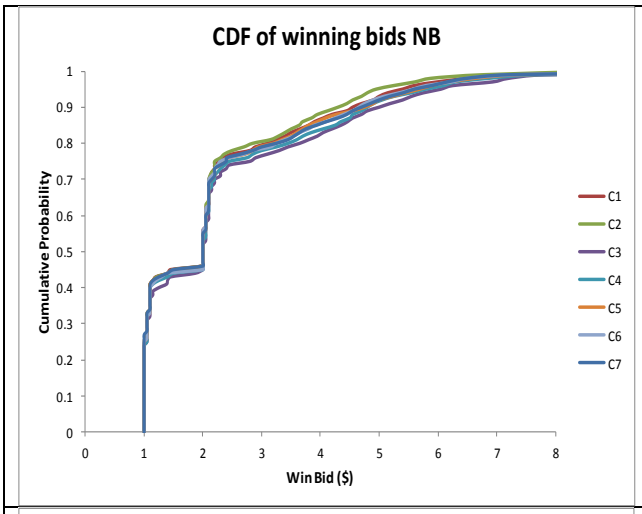
Table 19: Average cost of service

Case	Avg. cost of service		Avg. amounts contributed by drivers			
	NB	EB	NB (High VOT)	NB (Low VOT)	EB (High VOT)	EB (Low VOT)
c1	$1.30	$1.36	**$1.43**	$1.18	**$1.52**	$1.21
c2	$1.27	$0.85	**$1.38**	$1.16	**$1.01**	$0.69
c3	$1.39	$2.42	**$1.58**	$1.22	**$2.60**	$2.24
c4	$1.35	$1.67	**$1.51**	$1.19	**$2.12**	$1.21
c5	$1.33	$1.33	**$1.47**	$1.19	**$1.55**	$1.09
c6	$1.32	$1.25	**$1.47**	$1.17	**$1.51**	$1.21
c7	$1.33	$1.50	**$1.48**	$1.18	**$1.56**	$1.20

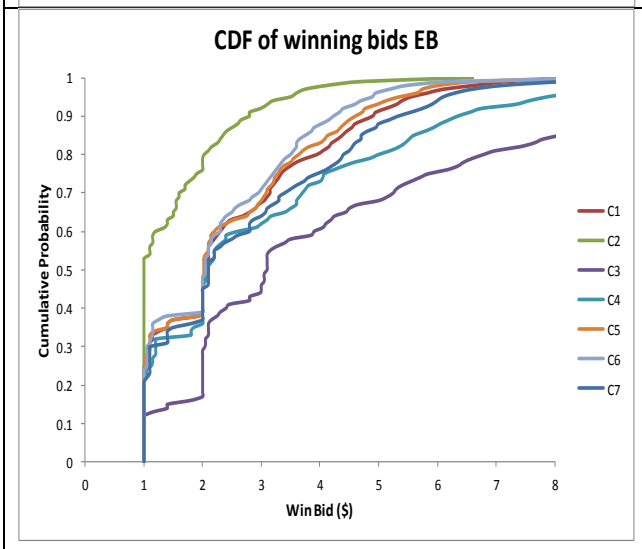
d) Winning bids submitted by movement managers

This analysis compares the cumulative probability densities of winning bids submitted by movement managers on the NB and EB approaches among seven cases. The results are presented in Figure 13. Each subplot in the left column contains seven CDFs (one for each case), while the tables in the right column present summary statistics associated with the CDFs.

Winning bids submitted by the EB movement manager seem to vary significantly among all seven cases, whereas the differences are less pronounced in the case of the NB manager. As expected, CDFs ‘c2’ and ‘c3’ stand out as far as EB distributions are concerned. The EB manager was submitting much lower bids in ‘c2’, and significantly higher bids in ‘c3’. At this point, it is safe to say that the parameters with the most significant impact on the system are the initial fee followed by voluntary contributions made by high value-of-time drivers.



Metric	c1	c2	c3	c4	c5	c6	c7
Mean ($)	2.2	2.1	2.4	2.3	2.3	2.3	2.3
Stdev ($)	1.8	1.5	2	1.9	1.7	1.7	1.7
15th perc ($)	1	1	1	1	1	1	1
50th perc ($)	2	2	2	2	2	2	2
75th perc ($)	2.3	2.2	2.8	2.5	2.4	2.3	2.4
95th perc ($)	5.4	5	6.1	5.8	5.7	5.6	5.6



CDFs of winning bids in each of seven cases

Metric	c1	c2	c3	c4	c5	c6	c7
Mean ($)	2.5	1.6	4.5	3.1	2.5	2.3	2.8
Stdev ($)	1.6	1	3.8	2.4	1.6	1.5	1.9
15th perc ($)	1	1	1.4	1	1	1	1
50th perc ($)	2.1	1	3.1	2.1	2	2.1	2.1
75th perc ($)	3.3	1.9	5.9	4.1	3.3	3.2	4
95th perc ($)	5.6	3.5	12.6	7.9	5.4	4.9	6.1

Summary Statistics of winning bids in each of seven cases

Figure 14: CDFs of winning bids

KS Tests were conducted to check for statistical differences in these distributions. Tables 20(a) and 20(b) present a summary of test results for NB and EB movement managers, respectively. NB test results suggest that the distributions in ‘c2’, ‘c6’, and ‘c7’ are statistically different than those in ‘c1’. The parameters that impacted NB win bids are a reduced initial fee and a mix of two classes of drivers on the EB approach. EB test results

suggest that the distributions in ‘c2’, ‘c3’, ‘c4’, and ‘c6’ are statistically different than those in ‘c1’.

Table 20: Summary of KS Test results win bids

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.036787	1.919651	0.0735758	**0.0013***	0.0735758	0.0006*	0.0264563	0.3856
(c1, c3)	0.023854	1.240193	0.0477086	**0.0923**	0.0477086	0.0461*	0.0244084	0.447
(c1, c4)	0.025808	1.340747	0.0516152	**0.0549**	0.0352759	0.1865	0.0516152	0.0275*
(c1, c5)	0.014083	0.734225	0.0281668	**0.6538**	0.0229341	0.4893	0.0281668	0.3402
(c1, c6)	0.035875	1.865142	0.0717496	**0.0019***	0.0172785	0.668	0.0717496	0.0010*
(c1, c7)	0.054137	2.821869	0.1082741	**<.0001***	0.0209493	0.5509	0.1082741	<.0001*

(a) North Bound

Case	KS	KSa	D=max|F1-F2|	Prob > D	D+=max(F1-F2)	Prob > D+	D-=max(F2-F1)	Prob > D-
(c1, c2)	0.190224	7.675241	0.3804522	**<.0001***	0	1	0.3804522	<.0001*
(c1, c3)	0.117878	4.740106	0.2357566	**<.0001***	0.2357566	<.0001*	0	1
(c1, c4)	0.059147	2.38137	0.1182947	**<.0001***	0.1182947	<.0001*	0.0170313	0.7905
(c1, c5)	0.032716	1.316796	0.0654321	**0.0624**	0.0296296	0.4911	0.0654321	0.0312*
(c1, c6)	0.036071	1.449575	0.0721417	**0.0299***	0.0058815	0.9725	0.0721417	0.0150*
(c1, c7)	0.032034	1.290912	0.0640671	**0.0714**	0.0640671	0.0357*	0	1

(b) East Bound

e) Total income transferred

The total income received by the municipality is another economic consideration. Table 21 summarizes the payments received by the municipalities in each of the seven cases. Values in the table make it clear that each simulation parameter influenced the overall income stream the municipality receives (whether the nature of change is statistically significant or not is a secondary question). It is also useful to recognize that among the seven cases, the total income stream received is lowest in c2, whereas it is highest in c3. Please recall that the municipality receives a guaranteed income of $900 from northbound drivers, whereas it is $600 for eastbound drivers in all cases except ‘c2’ ($300) and ‘c3’ ($1200). Additional income contributed by EB drivers is highest in ‘c4’ ($332), followed by ‘c7’ ($293). In both

cases, parameters related to high value-of-time drivers were changed, validating the assertion that this class of drivers place high value on their delays.

Table 21: Total income transferred in per hour ($) in seven cases

Total income transferred	NB	EB
c1	$1,185	$811
c2	$1,156	$506
c3	$1,272	$1,436
c4	$1,231	$992
c5	$1,210	$788
c6	$1,200	$745
c7	$1,212	$893

5.7. Summary

This chapter presents simulation results for a realization of the game. The results demonstrate the potential usefulness of incorporating economic ideas when creating signal control. The following are some crucial findings: The general trend of increased bids with increased queue length indicates that queue length does have an impact on bid submissions. However, because there are distributions associated with each queue length, and the quantiles vary from one queue length to another, queue length is not the only variable influencing the bid amounts. Secondly, high value-of-time drivers contributed more than the average, while those with a low value-of-time contributed less than the average. This is the evidence that that the high-VOT drivers are contributing proportionally more income than their percentage of traffic stream. Thirdly, sensitivity analysis was done to investigate the influence of each simulation parameter on the system. The results suggest that each parameter influences the economics of the system. In some instances, the impact was statistically significant. Fourthly, changing the initial fee paid by drivers on eastbound approach has the most significant impact on northbound delay distributions. When the initial fee was set to $2, it statistically

affected the nature of voluntary contributions made by both classes of drivers on both approaches.

In conclusion, there is value in incorporating economic principles into traffic signal control. It opens up a novel way to create signal control strategies whose economics is understood

.

CHAPTER 6: CONCLUSIONS AND FUTURE WORK

6. Concluding Remarks

This research has introduced a novel way to develop a signal control strategy that is based on economic considerations. The control strategy creates signal timing plans whose economics can be understood.

To facilitate the examination of how economics can play a role, a realization of the game was created. In this formulation of the game drivers and turning movement-specific movement managers play a simulated signal control game. The game is described in terms of what the players do, how they interact, the influence of data availability on how they behave, and how their decisions influence the outcome of the game. For example, every driver in the game has two attributes: 1) desired delay ($\hat{d}_j$) 2) given value of the time (τ_j^i). Every bidding cycle, drivers update their estimate of the anticipated delay (d_j^k) they will experience based on the movement manager's performance, and if they perceive that d_j^k is growing unacceptably, they can choose to voluntarily contribute additional funds so that the movement manager can submit larger bids. The drivers use Bayesian inference to determine when to make those voluntary contributions, and what should be the amount that they should contribute. Similarly, the movement managers develop bidding strategies that maximize the likelihood of winning. They collect initial fees from their drivers upon their arrival, and receive the voluntary contributions from the drivers; and pay the municipality for use of the intersection (by their drivers). Thus movement managers have at least one objective to achieve (increase likelihood of submitting a winning bid, discharge drivers in minimum time) and a constraint (remain solvent). The municipality is third type of player in the game; its role is to oversee the bidding process, and make decisions concerning when to shift control from movement manager to another in such a way that it minimizes marginal cost of incremental delay.

To provide benchmarks against which to compare the results from the bid-based control, a model of an actuated controller was developed. The same traffic stream generator used for

the bid-based control model was used for the actuated control model to ensure that the arriving headway sequences were the same for any comparative evaluation. Detectors were placed at the stop-bar. The minimum green was set to the same value used in the bid-based control. The gap was set to the same maximum pause duration used in the bid-based control and it was held constant (i.e., no volume-density control). Maximum greens were sometimes used and sometimes they were omitted (since the bid-based control has no maximum greens). When imposed, the maximum greens were set through a parametric analysis that matches the actuated control delays with the bid-based control delays. Following are some of the observations made in the analysis section: Firstly, with actuated control, vehicles on the NB approach, with the larger volumes, always experience higher delays than the EB approach, while the differences between the NB and EB approaches are less significant for bid-based control. Secondly, bid-based control strategy produces driver costs that are smaller than or equal to those produced by actuated control. Hence, drivers incur less costs due to incremental delay had this strategy be implemented. Thirdly, the relative delays between the two approaches are closer to being equal, so the equity between approaches is better insofar as the average delay costs are concerned. Fourthly, cycle length distributions produced by bid-based distributions are shorter, similar cycle length distributions can be achieved with actuated control by imposing maximum greens.

The results of the test case are very encouraging. That having been said, the ideas presented in this research can be further explored in future. The following sections provide brief descriptions of those ideas.

6.1.Bayesian Game

In the context of the game presented in this research, the game element is the one in which there are multiple entities that are interacting with one another and each attempting to extremize their own objective function. There isn’t a justifiable way to convert this multi-objective game into a single objective one. It is possible to represent the current realization of the game is a Bayesian game involving a large number of players. However, at a given point

in time, the game is played by the drivers in queue on both approaches, two movement managers, and the municipality. In that sense, the drivers are transient players, they learn about playing while they are in queue; on the other hand, the movement managers, continue to learn for the game's duration, because they continue to negotiate for their arriving drivers. It would be worth exploring equilibrium concepts for this Bayesian game. It is possible to formally represent this realization of the game as a mathematical game. Games of incomplete information are often modeled as Bayesian games. This section provides a discussion about why the current realization is a Bayesian game.

Before, going into the details, it is worth making some preliminary remarks about Bayesian games. In these games, each player is randomly assigned a type. Each player knows his or her type, but not anything about the other players. Composition profiles map each player to a specific type and the possible choices of actions that he/she can take. Each player in the game takes actions based on assessing the probable consequences of those actions (71).

A Bayesian game can mathematically be represented in the following way. Let $\boldsymbol{\mathcal{A}}$ denote finite set of actions; let $\boldsymbol{\mathcal{T}}$ denote the finite set of types; let $\boldsymbol{\mathcal{C}} \equiv \boldsymbol{\mathcal{A}} \times \boldsymbol{\mathcal{T}}$ denote set of possible player compositions. A Bayesian game is given by the tuple $(\boldsymbol{\mathcal{N}}, \boldsymbol{\mathcal{T}}, \boldsymbol{p}, \boldsymbol{\mathcal{A}}, \boldsymbol{u})$, where: $\boldsymbol{\mathcal{N}} = \{\mathbf{1}, \mathbf{2}, \ldots\ldots, \boldsymbol{n}\}$ represents finite player set; $\boldsymbol{\mathcal{T}} \equiv \boldsymbol{\mathcal{T}}^{\boldsymbol{\mathcal{N}}}$ is a set of type profiles; $\boldsymbol{p}: \boldsymbol{\mathcal{T}} \rightarrow [\mathbf{0}, \mathbf{1}]$ is prior probability function ɔ **p(t)** is probability of type profile **t** ϵ $\boldsymbol{\mathcal{T}}$; **A** $\equiv \boldsymbol{\mathcal{A}}^{\boldsymbol{\mathcal{N}}}$ is a set of action profiles; and $\boldsymbol{U} = \{\boldsymbol{u_1}, \boldsymbol{u_2}, \ldots\ldots, \boldsymbol{u_n}\}$ represents the utility functions for the players. If $\boldsymbol{\mathcal{C}} \equiv \boldsymbol{\mathcal{C}}^{\boldsymbol{\mathcal{N}}}$is a set of feasible composition profiles, each $\boldsymbol{u_i}$ is taken from $\boldsymbol{u_i}: \boldsymbol{C} \rightarrow [\mathbf{0}, \boldsymbol{D}]$

In the realization of the game discussed here, there are three types of players: drivers, movement managers, and the municipality. Furthermore, there are two classes of drivers: 1) drivers with a high value of time and 2) drivers with a low value of time. Drivers randomly are assigned a type. Each driver is informed only of his/her type but not that of the other

drivers. Moreover, each driver and movement manager, and the municipality, has possible actions that can be taken. The movement managers can determine what bids to submit and the drivers can decide what voluntary monetary contributions to make. Each player has a strategy set to resolve uncertainties before taking actions: for example, the drivers use a beta-Bernoulli process to evaluate the system, and make decisions concerning when to make voluntary contributions to expedite the service; on the other hand, movement managers make use of historic data to increase the likelihood of submitting a winning bid. Both drivers and movement managers are selfish (they are interested only in achieving their personal objective subjected to some constraint set). The municipality, as a player is interested in optimal use of scarce resource. It uses the marginal cost of incremental delay (MCID) to determine which bids win.

Based on this characterization, one can infer that the current realization of game may indeed be a Bayesian game. However, there are some important points to keep in mind. The number of players in a typical game theory-based model is small. But the agent-based simulation model presented here has a large number of players: 1503 to be specific (1500 drivers, 2 movement managers, and the municipality). However, even though there are many players in total, at a given point in time only a few players are involved. The number varies but it is determined by the drivers in queue on the two approaches plus the two movement managers plus the municipality. Moreover, because the drivers are transient players, they learn about playing while they are in queue. On the other hand, the movement managers continue to learn for the game's duration because they continue to negotiate for their arriving drivers. These thoughts are important because they raise questions about whether equilibrium can be achieved.

6.2. Explore auction theory concepts

Typically auctions differ in the methods used for the submission of bids and for the determination of the final price paid by the winner (57). Bids submitted in an auction can either be open outcry or sealed bid. In the former, bidders call out or in other words make

their bids; therefore everyone who is participating in the bidding process has perfect information about bid price submitted by every other bidder. In the latter, bidding is done privately, and bidders do not have information about bids submitted by other bidders. In most of the cases, only the winning bid is announced. In the problem solved for Test Case, even the winning bid amount was not revealed to agents. They only know what their outcome was in the bid process.

Also, the outcome of an auction is influenced by the number of opportunities each bidder has to submit a bid. For example, in Dutch auctions (viz. A form of open outcry) even though each bidder can submit multiple bids, only the highest among the submitted bids is relevant to the auction's outcome. On the other hand, in sealed bid auctions, a bidder can submit multiple bids before the end of the bid period. For example, in the case of on-line bidding, which falls in the category of sealed bid auctions, once a bidder gets to know that his bid is not the highest bid, he can submit a higher bid amount before the end of auction time period. Different methods used for submission of bids can certainly be explored.

Lastly, auctions differ in the final price paid by the winner. At least two methods exist. In the first, the winner pays what was bid. In the second, the highest bidder pays a price equivalent to what the second highest bidder bid. Auctions that follow the first method for collecting the final price of the bid are called 'first-price sealed-bid auctions'; whereas auctions that follow the second method are called 'second-price sealed-bid auctions'. As one might notice, first-price-sealed-bid-auction was used in the simulation model used to test the frame work. It would be interesting to explore these concepts in future research; as a consequence, it would be interesting to compare how different/similar control solutions generated will be.

6.3. Explore information theory concepts

The bidding strategies used, and in general the decisions made by each intersection agent, are influenced by and large by the information made available. For example, the bidding prices submitted by an agent will be different if it has information about the following: which agent

won the most recent bid? What was the winning bid amount? In the absence of information, an agent can only ‘guess’ what these answers are. In the future research, the value added by allowing agents at the intersection to share information and thereby effect on system dynamics will be explored.

6.4. Explore other classes of bidding systems

Section 4.4 presented at least two different frameworks (state-observer systems, and probabilistic systems) to develop the bidding strategies. In this research only probabilistic system was explore. It would be worth exploring each avenue in the future.

6.5. Develop a game for more approaches

The framework described in this paper has only been implemented so far for an intersection that involves two one-way streets. In future research, this example will be expanded to encompass more approaches and more turning movements.

REFERENCES

1. Al Azri, Abdullah, Chilamkurti, Naveen. Priority-Based Vehicle-to-Infrastructure Communication (PV2I) in UMTS 2011

2. Bohm, Annette, Jonsson, Magnus. Position-based data traffic prioritization in safety-critical, real-time vehicle-to-infrastructure Communication 2009.

3. Brahmi, N., Boukhatem, L., Boukhatem, N., et al. End-to-end routing through a hybrid ad hoc architecture for V2V and V2I communications 2010.

4. El Ali, Farah, Ducourthial, Bertrand. A light architecture for opportunistic vehicle-to-infrastructure communications 2010.

5. Jarupan, Boangoat, Ekici, Eylem. Location- and delay-aware cross-layer communication in V2I multihop vehicular networks 2009.

6. Miller, Jeffrey. Vehicle-to-vehicle-to-infrastructure (V2V2I) intelligent transportation system architecture 2008.

7. Msadaa, Ikbal C., Cataldi, Pasquale and Filali, Fethi. A comparative study between 802.11p and MOBILE WiMAX-based V2I communication networks 2010.

8. Priemer, Christian, Friedrich, Bernhard. A decentralized adaptive traffic signal control using V2I communication data 2009.

9. E.C.P. Chang, S.L. Cohen, C. Liu, N.A. Chaudbury, and C. Messer. MAXBAND-86: Program for optimizing left-turn phase sequence in multiarterial closed networks. Transportation Research Record, 1181:61–67, 1988.

10. R. K. Saraf, "Adaptive traffic control using neural networks," Ph. D. Dissertation, Departme J.D.C. Little, M.D. Kelson, and N.H. Gartner. MAXBAND: A program for setting signals on arteries and triangular networks. Transportation Research Record, 796:40–46, 1981nt of Civil and Environmental Engineering, Vanderbilt University, Nashville, TN, 1994.

11. N.H. Gartner, S.F. Assmann, F. Lasaga, and D.L. Hout. MULTIBAND—a variable bandwidth arterial progression scheme. Transportation Research Record, 1287:212–222, 1990.

12. N.H. Gartner, S.F. Assmann, F. Lasaga, and D.L. Hout. A multi-band approach to arterial traffic signal optimization. Transportation Research Part B, 25:55–74, 1991.

13. N.H. Gartner, J.D.C. Little, and H. Gabbay. Simultaneous optimization of offsets, splits, and cycle time. Transportation Research Record, 596:6–15, 1976.

14. D.K. Hale. Traffic Network Study Tool, TRANSYT-7F, United States Version. Mc-Trans Center, University of Florida, Gamesville, Florida 32611-6585, January 2006.

15. Texas Transportation Institute. PASSERTMIII-98 Application and User's Guide, 1998.

16. S. Venglar, P. Koonce, and T. Urbanik II. PASSER V. Texas Transportation Institute, 2000.

17. S.R. Sunkari, R.J. Engelbrecht, and K.N. Balke. Evaluation of advanced coordination features in traffic signal controllers. FHWA/TX-05/0-4657-1, FHWA, September 2004.

18. M.C. Bell and R. D. Bretherton. Ageing of fixed-time traffic signal plans. In Proc. IEE 2nd International Conference on Road Traffic Control, London, 1986.

19. K. Fehon. Adaptive traffic signals are we missing the boat. ITE District 6 Annual Meeting, 2004.

20. P.R. Lowrie. The Sidney Co-ordinated Adaptive Traffic System: Principles, methodology, and algorithms. In Proc. IEE Conference on Road Traffic Signaling, 207, pages 67–70, 1982.

21. R.D. Bretherton. SCOOT urban traffic control system: Philosophy and evaluation. In Proc. 6th IFAC/IFIP/IFORS Symposium on Control, Computers, and Communications in Transportation, pages 237–239, September, 1989.

22. P.B. Hunt, D.I. Robertson, R.D. Bretherton, and R.I. Winton. SCOOT - a traffic responsive method of coordinating signals. TRRL Laboratory Report 1014, TRL, 1981.

23. D.I. Robertson and R.D. Bretherton. Optimal control of an intersection for any known sequence of vehicle arrivals. In Proc. 2nd IFAC-IFIP-IFORS Symposium of Traffic Control and Transport Systems, 1974.

24. D.I. Robertson and R.D. Bretherton. Optimizing networks of traffic signals in real time—The SCOOT method. IEEE Transactions on Vehicular Technology, 40(1):11–15, February 1991.

25. C. Bielefeldt and F. Busch. MOTION - a new on-line traffic signal network control system. In Proc. 7th International Conference on Road Traffic Monitoring and Control, pages 55–59, April 1994.

26. F. Busch and G. Kruse. MOTION for SITRAFFIC - a modern approach to urban traffic control. In Proc. Intelligent Transportation Systems, pages 61–64, Oakland, CA, USA, August 2001.

27. C. Bielefeldt, H. Condie, E. Kosmatopoulos, A. Richards, M. McDonald, J. M¨uck, and A. Hanitzsch. SMART NETS final report. Napier University, 2004.

28. V. Dinopoulou, C. Diakaki, and M. Papageorgiou. Applications of the urban traffic control strategy TUC. European Journal of Operational Research, 175:1652–1665, 2006.

29. M. Papageorgiou. An introduction to signal traffic control strategy TUC. In Proc. SMART NETS Final Conference – Deliverable 25, 2003.

30. M. Papageorgiou. TUC implementation: Lessons learned and future steps. In Proc. SMART NETS Final Conference – Deliverable 25, 2003.

31. V. Mauro and C. Di Taranto. UTOPIA. In Proc. 2nd IFAC-IFIP-IFORS Symposium on Traffic Control and Transportation Systems, pages 575–597, 1989.

32. V. Mauro and C. Di Taranto. UTOPIA. In Proc. 6th IFAC/IFIP/IFORS Symposium on Control, Computers, and Communications in Transportation, September 1989.

33. J.G. Wardrop. “Some theoretical aspects of road traffic research," Proc. Institute of Civil Engineering, Part III, Vol.1, 325-378, 1952.

34. F.V. Webster. Traffic signal settings. Road Research, 39, 1958. Her Majesty’s Stationery Office, London, England.

35. F.V. Webster and B.M. Cobbe. Traffic signals. Road Research, 56, 1966. Her Majesty’s Stationery Office, London, England.

36. Gordon F. Newell, Theory of Highway Traffic Signals (Institute of Transportation Studies, 1988)

37. Lan, C. J. New Optimal Cycle Length Formulation for Pretimed Signals at Isolated Intersections. Journal of Transportation Engineering, Vol. 130, No. 5, 2004, pp. 637-647.

38. Abbas, M. M. A Real Time Offset Transitioning Algorithm for Coordinating Traffic Signals. Ph.D. Dissertation. Department of Civil Engineering, Purdue University, West Lafayette, Indiana, USA, 2001.

39. Roess, R. P., E. S. Prassas, and W. R. McShane. Traffic Engineering (3rd Edition). Pearson Education, Inc., Upper Saddle River, New Jersey, 2004.

40. M.R. Crabtree and I.R. Henderson. MOVA Traffic Control Manual, B edition, May 2006.

41. Kronborg, P., Davidsson, F. MOVA and LHOVRA. Traffic signal control for isolated intersections 1993.

42. Peirce, J. R., Vincent, R. A. Repeated-stage signal control using MOVA 1989.

43. Vincent, R. A., Young, C. P. Self-optimising traffic signal control using microprocessors-the TRRL `MOVA' strategy for isolated intersections 1986.

44. A.J.Miller. A computer control system for traffic network. In Proc. 2nd International Symposium on Theory of Road Traffic Flow, pages 201–220, 1963.

45. F. Boillot, S. Midenet, and J.C. Pierrel´ee. Real-life CRONOS evaluation. In Proc. 10th International Conference on Road Traffic Information and Control, pages 182–186, London, April 2000.

46. F. Boillot. Group-based safety constraint description of an intersection. In Proc. 9th Mini-EURO Conference on Handling Uncertainty in the Analysis of Traffic and Transportation Systems, Budva, Yugoslavia, June 2002.

47. F. Dion and B. Hellinga. A methodology for obtaining signal coordination within a distributed real- time network signal control system with transit priority. In Proc. 80th Annual Meeting of the Tranportation Research Board (TRB'01), May 2001.

48. F. Dion and B. Hellinga. A rule-based real-time traffic responsive signal control system with transit priority: Application to an isolated intersection. Transportation Research Part B, 36:325–343, May 2002.

49. M. Conrad, F. Dion, and S. Yagar. Real-time traffic signal optimization with transit priority: Recent advances in the SPPORT model. In Proc. 77th Annual Meeting of the Transportation Research Board, 1998.

50. N.H. Gartner. OPAC: A demand-responsive strategy for traffic signal control. Transportation Research Record, 906:75–81, 1983.

51. N.H. Gartner, F.J. Pooran, and C.M. Andrews. Implementation of the opac adaptive control strategy in a traffic signal network. In Proc. IEEE Intelligent Transportation Systems Conference, August 2001.

52. N.H. Gartner, C. Stamatiadis, and P.J. Tarnoff. Development of advanced traffic signal control strategies for intelligent transportation systems: Multilevel design. Transportation Research Record, 1494:98–105, 1995.

53. N.H. Gartner, P.J. Tarnoff, and C.M. Andrews. Evaluation of the optimized policies for adaptive signal control strategy. Transportation Research Record, 1683:105–114, 1999.

54. Suvrajeet Sen, Larry Head. Controlled Optimization of Phases at an intersection. Transportation Science, v 31, n 1, p 5-17, Feb 1997.

55. J.L. Farges, I. Khoudour, and J.B. Lesort. PRODYN: On site evaluation. In Proc. 3rd Int. Conference on Road Traffic Control, London, England, 1990.

56. J.J. Henry and J.L. Farges. PRODYN. In Proc. 6th IFAC/IFIP/IFORS Symposium on Control, Computers, and Communications in Transportation, Paris, France, September 1989.

57. J.J. Henry, J.L. Farges, and J. Tuffal. The PRODYN real time traffic algorithm. In Proc. 4th IFAC/IFIP/IFORMS Symposium on Control in Transportation Systems, pages 307–312, Baden-Baden, Germany, April 1983.

58. Genovese, J. A. SCOOT in the USA. Institute of Transportation Engineers 1994 Compendium of Technical Papers, Washington, D.C., 1994

59. Lowrie, P. R. SCATS, Sydney Coordinated Adaptive Traffic System: A Traffic Responsive Method of Controlling Urban Traffic. Road and Traffic Authority of New South Wales, Sydney, Australia, 1991.

60. A. Bazzan. Opportunities for Multiagent Systems and Multiagent Reinforcement Learning in Traffic Control Autonomous Agents and Multiagent Systems, Springer, 2008.

61. B. Chen and H. H. Cheng A Review of the Applications of Agent Technology in Traffic and Transportation Systems IEEE Transactions on Intelligent Transportation Systems, vol. 11, no. 2, pp. 485-497, 2010.

62. B. P. Gokulan and D. Srinivasan Distributed Geometric Fuzzy Multiagent Urban Traffic Signal Control. IEEE Transactions on Intelligent Transportation Systems, vol. 11, no. 3, pp. 714-727, 2010.

63. M. Vasirani and S. Ossowski Exploring the potential of multiagent learning for autonomous intersection management Proceedings of the 5th Workshop on Agents in Traffic and Transportation, in 7th International Joint Conference on Autonomous Agents and Multiagent Systems, 2008.

64. K. Dresner and P. Stone, "A multiagent approach to autonomous intersection management", Journal of Artificial Intelligence Research, 2008

65. M. A. Wiering, 2000, "Multi-agent reinforcement learning for traffic light control",

66. S.Box and B.Waterson., An automated signalized junction controller that learns strategies from a human expert. Engineering Applications of Artificial Intelligence. 2011

67. S.Box and B.Waterson, An automated signalized junction controller that learns strategies by temporal difference reinforcement learning.. Engineering Applications of Artificial Intelligence. 2012

68. E. T. Verhoef, 2008, "Pricing in road transport: a multi-disciplinary perspective”

69. M. Vasirani and S. Ossowski, "A market-inspired approach for intersection management in urban road traffic networks", Journal of Artificial Intelligence Research, 2012

70. M. Vasirani and S. Ossowski, "A Computational Market for Distributed Control of Urban Road Traffic Systems", IEEE Transactions on Intelligent Transportation Systems 12(2), 2011

71. Avinash Dixit, Barry Nalebuff: Thinking Strategically: The Competitive Edge in Business, Politics, and Everyday Life.

72. Grether, David M., R. Mark Isaac, and Charles R. Plott. "The allocation of landing rights by unanimity among competitors." The American Economic Review (1981): 166-171.

73. Adler, Nicole. "Competition in a deregulated air transportation market." European Journal of Operational Research 129.2 (2001): 337-345.

74. Chang, Chung-Ju, Tsung-Li Tsai, and Yung-Han Chen. "Utility and game-theory based network selection scheme in heterogeneous wireless networks." Wireless Communications and Networking Conference, 2009. WCNC 2009. IEEE. IEEE, 2009.

75. Ahmad, Ishfaq, Sanjay Ranka, and Samee Ullah Khan. "Using game theory for scheduling tasks on multi-core processors for simultaneous optimization of performance and energy." Parallel and Distributed Processing, 2008. IPDPS 2008. IEEE International Symposium on. IEEE, 2008.

76. McMillan, John. "Selling spectrum rights." The Journal of Economic Perspectives (1994): 145-162.

77. Zhou, Guanghui, Pingyu Jiang, and George Q. Huang. "A game-theory approach for job scheduling in networked manufacturing." The International Journal of Advanced Manufacturing Technology 41.9-10 (2009): 972-985.

78. Kutanoglu, Erhan, and S. David Wu. "On combinatorial auction and Lagrangean relaxation for distributed resource scheduling." IIE transactions 31.9 (1999): 813-826.

79. Li, W. D., et al. "Game theory-based cooperation of process planning and scheduling." Computer Supported Cooperative Work in Design, 2008. CSCWD 2008. 12th International Conference on. IEEE, 2008.

80. Choy, Min Chee, Dipti Srinivasan, and Ruey Long Cheu. "Cooperative, hybrid agent architecture for real-time traffic signal control." Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on 33.5 (2003): 597-607.

81. Steingrover, Merlijn, et al. "Reinforcement Learning of Traffic Light Controllers Adapting to Traffic Congestion." BNAIC. 2005.

82. Wiering, Marco, et al. "Intelligent traffic light control." Institute of Information and Computing Sciences. Utrecht University (2004).

83. Gil Jones, E., et al. "Dynamically formed heterogeneous robot teams performing tightly-coordinated tasks." Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on. IEEE, 2006.

84. Schepperle, Heiko, Klemens Böhm, and Simone Forster. "Towards valuation-aware agent-based traffic control." Proceedings of the 6th international joint conference on Autonomous agents and multiagent systems. ACM, 2007.

85. Highway Capacity Manual 2010. TRB, National Research Council, Washington, DC., 2010.

86. List (2013), Analysis of VISSIM saturation headways

APPENDICES

APPENDIX A

Introduction

The purpose of documenting actuated control is two-fold: Firstly, this control realization is programmed in Python rather than using pre-written code. Secondly, researchers who want to replicate the results will have an ability to do so. Next section describes control structure in detail.

Control Logic

The heart of the procedure focuses on figuring out switching sequences and switching times; here the duration for which a subject approach stays green is referred as phase green duration; these durations are determined by minimum greens, gap timers, and maximum greens; to ensure safe operations, a clearance interval is imposed when control shifts from one approach to another. Furthermore, the control logic presented here assumes only stop-bar detection. In other words, it does not account for the placement of advanced loops like the ones used in volume-density control.

Figure 1 presents pseudo code for actuated control. Every time step ($\delta t = 0.1$ seconds), the control logic checks to see if there are any new arrivals. If there are, then it adds the vehicles to their respective First-in-First-out (FIFO) queues. Once the intersection control is allocated to a specific approach, vehicles on that approach are serviced for a duration equivalent to minimum green. During the minimum green time vehicles on the subject approach are discharged at saturation headway.

Control Structure:

```
δt = 0.1 ( initialize time step)
t = 0  (simulation time)
t_gap = 3.0  (maximum gap between two consecutive vehicles)
h_d = 2.1 (saturation headway)
g_min = 6.0 (minimum green)
g_min  (maximum green)
c = 4.0  (clearance interval)
s = 1 (start NB approach in green)
g_s = 0.0 (current green duration for approach s)

While t ≤ T :
    t += δt
    If < new arrivals = True >:
        Add new arrivals to service queue

    If < s == i >:
        If g_s ≤ g_min :
            g_s += δt
            Discharge lead-vehicle/s in the queue on approach 'i' at a constant headway of h_d
        If g_s > g_min and g_s ≤ g_max :
            g_s += δt
                If <gap between two successive arrivals on approach 'i' is less than t_gap > :
                Discharge lead-vehicle in the queue on approach 'i'

            Else:
                If <length of conflicting queue is non-zero>:
                    Impose a clearance interval of 4 seconds
                    Shift control to next approach
        If g_s > g_max :
            If <length of conflicting queue is non-zero>:
                Impose a clearance interval of 4 seconds
                Shift control to next approach
            Else:
                Stay in green on approach i
                Discharge new vehicle arrivals (if any) at saturation headway
    End
End
```

Figure 1: Actuated control logic

Starting at the end of minimum green, decisions are made concerning extending green on the subject approach; the display of green on subject approach can be terminated in one of the two possible ways: 1) gap-out; 2) max-out.

1) *gap-out*: In the case of a gap-out termination, a gap-timer reaches zero (from some specified initial value) before the next vehicle is detected. The purpose of setting a gap-timer is to measure the time interval between successive vehicle arrivals (vehicle headways). Green extensions on the subject approach continue until the vehicle headways are greater than the pre-specified passage times on the stop-bar detector (or until maximum green is reached). If a gap out occurs and there is a non-zero queue on other conflicting approach, then display of green is terminated on the subject approach and a clearance interval of 4 seconds is imposed before shifting control to the approach with non-zero queue.

2) *max-out*: In the case of a max-out, the duration of green reaches a maximum allowable value. For this to happen, the stop-bar detector on the subject approach never gaps-out (meaning the passage time is always less than or equal to gap-timer). If this scenario persists even at the end of maximum green and if there is a non-zero service queue on the other approach, then display of green is terminated on the subject approach and a clearance interval of 4 seconds is imposed before shifting control to the approach with non-zero queue.

Code validation

Validity of the Python code was accomplished by comparing average delays against a realization of actuated control created in VISSIM, for each of the three flow conditions discussed in section 5.1. Table 1 summarizes these results. In principle the average delays generated by the realization in Python and VISSIM should match, however, as one might quickly realize, actuated control model realization created in VISSIM produced average delays that were significantly shorter than those generated by the model created in Python.

NB Flow (Vph)	EB Flow (Vph)	Average Delay (sec)	
		Python	VISSIM (default)
750	750	26	17.8
900	600	26	17.6
1200	300	26.5	13.8

Table 1: Average delay comparisons

Given the fact that the actuated control models in Python as well as VISSIM use similar logic for switching phases, an explanation for the differences was needed. A possible reason for this disparity relates to the discharge headway distributions. The actuated control model created in Python uses a uniform density function for the departure headways and it uses 2.1 seconds as the minimum value. Hence, to check for consistency/variability in discharge headways, cumulative density functions (CDFs) of discharge headways were created.

Figure 2 presents CDFs for discharge headways of individual vehicles (both for VISSIM and the Python model) in the three combinations of flows. This figure contains results for two scenarios: the subplots to the left (three graphs in the first column) represent results for discharge headways generated by Python model, whereas the subplot to the right (three graphs in the second column) represent results for discharge headways generated by VISSIM model.

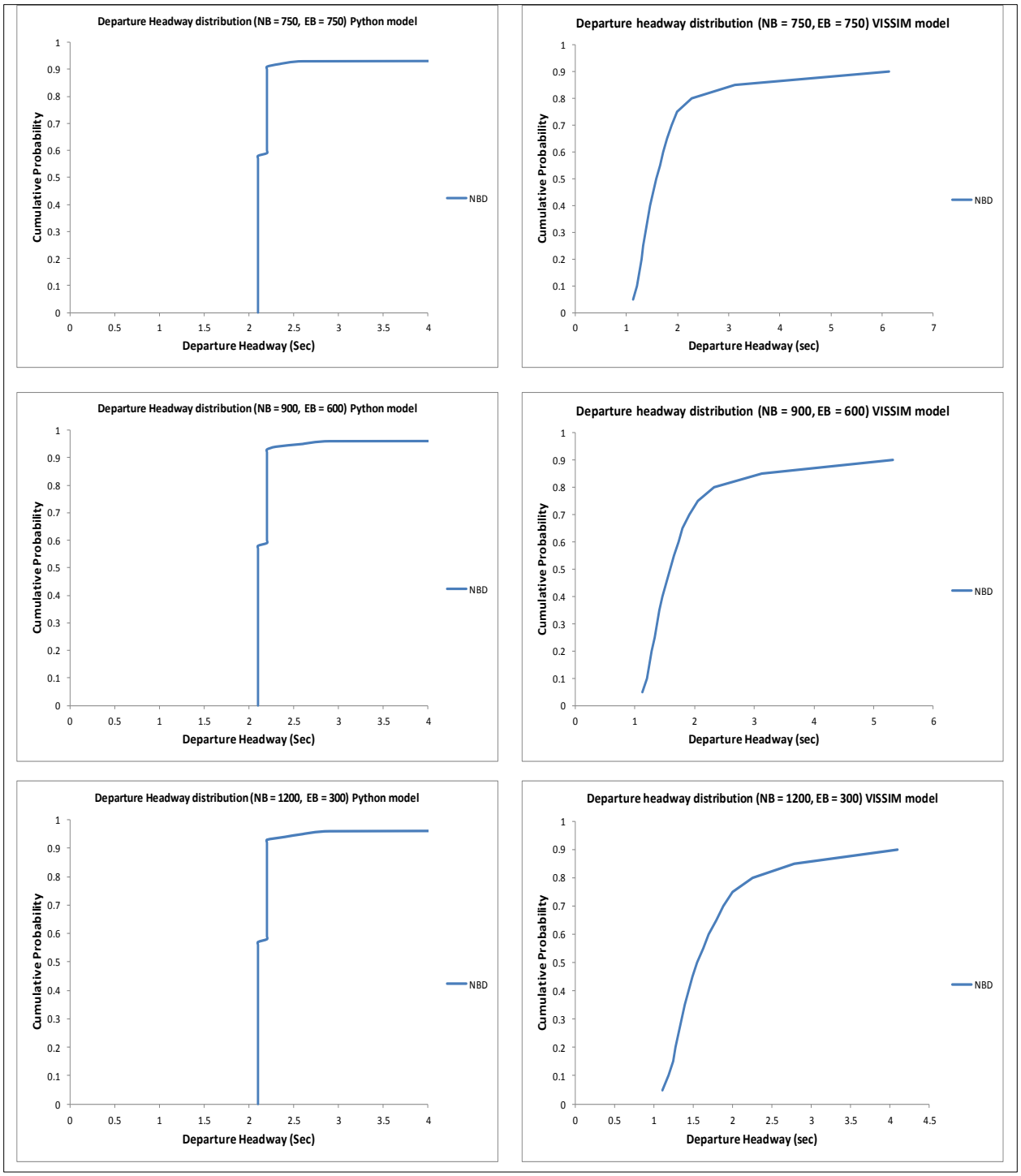


Figure 2: CDF of Headway distribution

So, the next task is to match discharge headway distributions of VISSIM with those of Python; this can be done by changing safe distance parameter in Wiedemann 74 car following model. The default value that VISSIM uses for safe distance parameter is 3.0; the value of this parameter was adjusted so as to match the departure headway distribution. Lower percentiles of departure headway distributions for Scenario-1 & Scenario-2 flow conditions match when safe distance parameter value was set to 12.0; for Scenario-3, this parameter value was set to 16.0 in order for departure headway distributions to match.

Figure 3 presents CDFs for new discharge headway distributions for three flow conditions and adjusted safe distance parameter. As one might quickly notice, 15th percentile departure headway was around 2.1 seconds.

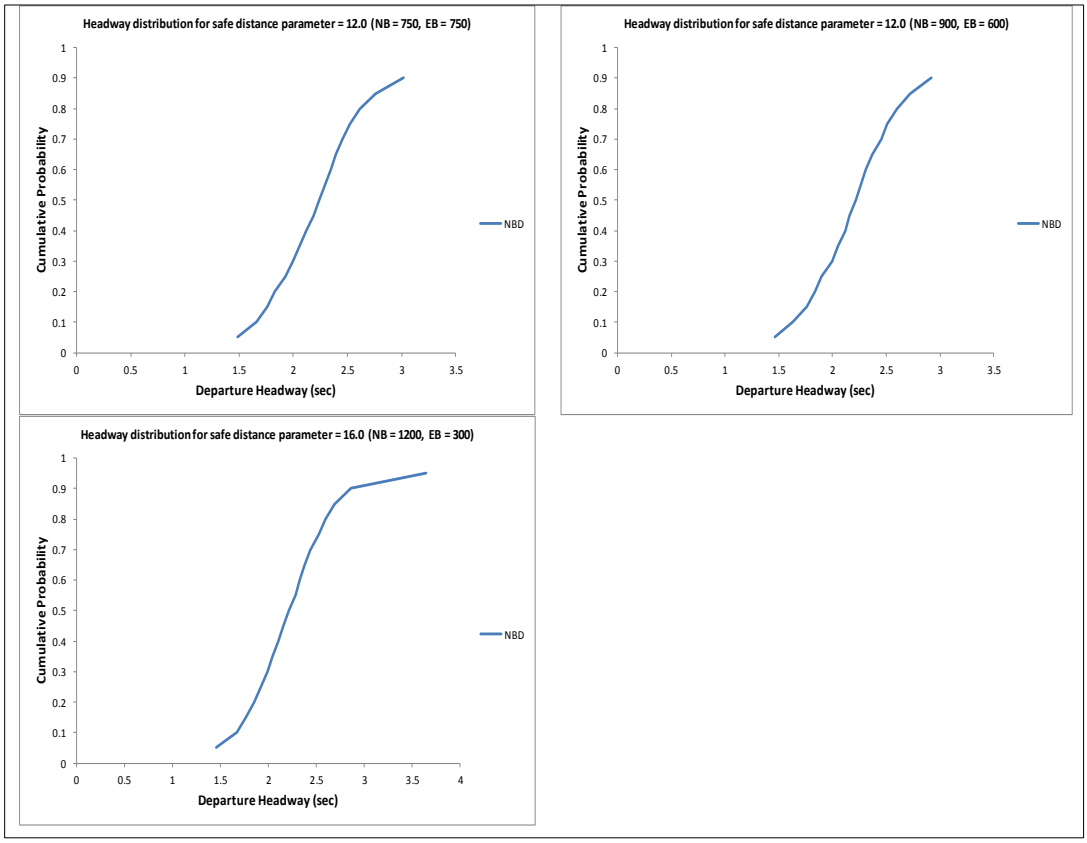


Figure 3: CDF of discharge headway distribution

Now since the discharge headway distributions both in VISSIM and Python are comparable, both models should produce average delays that are comparable. Table 2 summarizes these average delay results. As one might notice, the differences in average delays generated by these models were insignificant.

NB Flow (Vph)	EB Flow (Vph)	Average Delay (sec)	
		Python	VISSIM (adjusted)
750	750	26	30.9
900	600	26	28.3
1200	300	26.5	28.2

Table 2: Average delay comparisons

Conclusion

In this section, the control logic used for creating actuated control was described; the validity of the Python code was checked against a realization of the actuated control created in VISSIM for each of the three flow conditions; average delay was the performance metric that was used for comparison. It was noted that the actuated control model in VISSIM generated average delays that are significantly shorter than those generated by model realization in Python. The disparity in results was due to inherently different departure headway distributions used by these two models. Safe distance parameter in Wiedemann 74 car following model was adjusted to match headway distributions and in turn average delays.

APPENDIX B

This appendix provides a brief review of the more advanced traffic adaptive control strategies that have been developed in recent years.

1. MOVA

MOVA is developed in mid-eighties by the Transportation Research Laboratories (TRL) (40-43). This control strategy operates in two modes: one for uncongested conditions, and the other for congested conditions. Under uncongested mode, the control strategy seeks to clear queues on an approach that build during red, then based on the algorithm due to Miller (44), it computes for every half a second, the time gains and losses caused on all approaches if the decision to switch to a different phase is postponed. If overall losses by extending the green are higher than overall gains, then the switch takes place immediately, else the decision is postponed until next time-step.

Under congested conditions, MOVA bases its control decisions on a capacity- maximizing routine. The optimum cycle in this routine is defined as a function of intersection geometry, lost time, flows and turning movements. MOVA continually monitors conditions during oversaturated conditions, and when appropriate, select and enforce a cycle time that maximizes capacity of the intersection.

2. CRONOS

CRONOS is a real-time adaptive control strategy developed by CERT (45, 46). This algorithm is specifically designed for video detection. Using traffic measurements obtained by real-time image processing of video cameras, coupled with historic information; the traffic prediction model inside CORNOS forecasts, for a given time horizon, future spatial extension of queues on each controlled link. This prediction is based on rolling averages. Then a heuristic based optimization module computes the sequence of traffic signal states that minimizes total delay over the time horizon. Once this sequence is found, the

corresponding traffic signal states are applied on the intersection in the next time step, and the process repeats one time step later.

3. SPPORT

SPPORT was developed primarily in response to concerns that exhaustive optimization procedures such as dynamic or linear programming may be too demanding computationally for real-time applications in networks especially with highly variable demands (47, 48). SPPORT uses a heuristic rule-based optimization procedure to make switching decisions. Based on user defined prioritized events, projected vehicle arrival information obtained from traffic detectors installed at strategic locations, control logic selects the best switching combination that minimizes a predefined cost function. Predictions are made with the aid of discrete-event microscopic simulation model that was exclusive designed for SPPORT (49).

4. OPAC

Optimized Policies for Adaptive Control (OPAC) is a real-time demand-responsive traffic signal timing optimization algorithm that was originally developed for isolated intersections. It was developed by Gartner in early 80s at University of Lowell under the sponsorship of U.S. Department of Transportation.

OPAC is unique in the sense that the concept of signal cycle-was completely dropped. The practice till then was to use parametric models that optimize parameters such as cycle time, splits, and offsets. In contrast, OPAC takes actual arrival data at the intersection as input, uses a non-parametric model (a discrete-time optimization model that enumerates a few alternate solutions using a heuristic) which provides guidance in deciding the best time to switch between phases, making this model more relevant for real-time traffic-adaptive signal control. Field tests suggest that OPAC is capable of reducing delays at an intersection. (50-53).

5. COP

Controlled Optimization of Phases (COP) is another real-time demand-responsive control algorithm developed by Larry Head and Suvrajeet Sen. COP is similar to OPAC in the sense it makes phase switching decisions based on arrival data at the intersection. In addition COP provides a computational scheme that is efficient enough to be implemented in real-time. Unlike OPAC (which uses a limited heuristic that enumerates a few alternatives), COP's algorithmic process is completely based on dynamic programming. The algorithm in COP solves intersection control problems using only one state variable for optimization, resulting in efficient computational time (54).

6. PRODYN

PRODYN (Programmation Dynamique) is an adaptive traffic control algorithm (dynamic programming based), developed by Henry et al., at CERT, France. Similar to other dynamic programming based control algorithms, PRODYN does not have fixed phase sequence, phase duration, and cycle lengths. At isolated intersection level, the optimization model aims at minimizing delay by using improved forward dynamic programming with constraints on maximum and minimum greens (55-57).