BARRETT, PAUL RYAN. Unified Constitutive Modeling for High Temperature Fatigue-Creep and Creep Responses of Haynes 230. (Under the direction of Dr. Tasnim Hassan).

The reliability of structural components in high temperature systems experiencing cyclic thermo-mechanical loading is an important problem, especially for airplane turbo engine components. To investigate the failure mechanisms of these components, advanced material models are required that can describe the complex material phenomena and accurately predict the deformation and damage behavior. These material models should be developed and validated against a broad set of material responses for a range of loading histories. These experimental loading histories strive to replicate in-service conditions of structural components. As a result, this dissertation has undertaken both the experimental analysis and model development for a nickel-base superalloy, Haynes 230. Haynes 230 is used as combustor liner material in airplane turbine engines, where the temperatures can be as high as 982°C (1800°F).

The ability to understand and engineer more robust and complex systems using high temperature materials, such as Haynes 230, has required innovation and maturation of material models that are capable of predicting broader ranges of phenomena. The unified constitutive model developed in this work has the capability of simulating fatigue, fatigue-creep, creep, and thermo-mechanical fatigue responses. However, this level of modeling comes at a cost, in terms of the amount of parameters. Large parameterized models are often avoided, due to the challenge alone in finding an optimal set of parameters. Therefore, it was also imperative that an efficient parameter identification algorithm be developed, such that a robust set of parameters can be found. The material parameters of the advanced constitutive model have been systematically categorized for robust estimation using both local and global optimization methods, with a hybrid genetic algorithm and other classical optimization methods, like gradient-based solvers.
Unified Constitutive Modeling for High Temperature Fatigue-Creep and Creep Responses of Haynes 230

by
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DEDICATION

All praises to God for giving me the opportunity to live, love and grow each and every day alive. When you start to connect the dots in your life – you will smile when you realize it was never a coincidence. To my parents, who have always sacrificed and loved to build the legacy we have today. For my grandparents, who dared to dream for not only today, but for a brighter tomorrow. To my closest friends, who pushed me in directions I could have never imagined possible. And most importantly to my wife Kateia and son Michael who have blessed me with unconditional love and support to share in this lifetime. There is nothing more rewarding in life, besides life itself, than raising a family with the one you love.
BIOGRAPHY

Paul Ryan Barrett is from Durham, North Carolina. He received his Bachelors of Science in Civil Engineering with a specialization in Structural Engineering and Mechanics in December of 2009 at North Carolina State University. Also, as an undergraduate, he studied a minor in Applied Mathematics. Since then he has received his Master of Civil Engineering in Structural Engineering and Mechanics, December of 2012 at North Carolina State University, while actively pursuing a Ph.D. His Ph.D. research has been closely collaborated with Honeywell Aerospace in the development of an advanced constitutive model for simulation of stress-strain responses of Haynes 230 under thermo-mechanical cyclic loading with application to gas turbine engines. His work also extends to evolutionary computation and the development of global and local optimization algorithms for automated parameter determination. His personal objective is to serve our society as a Civil Engineer, with a passion for helping people, by providing essential service to the community. His personal philosophy for engineering is that, as Engineers, we have a societal commitment and possess the power to be doctors of society. A person who pursues engineering must understand the extraordinary responsibility that he or she has earned. These responsibilities entail a sense of serviceability for society that should always be beneficial through an ethical means. Paul strongly believes that through both compassion and positive energy one can stimulate the mind of anyone and spawn seeds of hope and desire into the hearts of others for them to bring it to fruition.
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CHAPTER 1: INTRODUCTION

1. Background and motivation

Service-conditions of nickel-base superalloy components in the aerospace and nuclear power industries experience start-up and shut-down cycles that induce repeated thermo-mechanical stresses. Under these high temperature thermo-mechanical cycles, damage accumulates eventually leading to a component failure. The type of damage seen in these high temperature service zones can include out-of-phase thermo-mechanical fatigue, creep-fatigue damage accumulation among many other factors [1-3]. Accurate TMF life prediction of components is of paramount importance to facilitate improved design-by-analysis methodology for maximizing the resistance of high temperature components and understand the failure mechanisms. In order to achieve a desired level of fidelity with reasonable accuracy without guess work, one should be able to identify all the phenomena of the material in these harsh environments. It is essential to understand these complex material responses, under realistic loading conditions, in order to substantially improve current design methodologies. Current industry standards use life prediction methods based on either simple elastic analyses or simplified inelasticity models using commercial finite element analysis (FEA) software and thus lack fidelity in the service life predictions. This calls for the development of a robust constitutive model capable of describing the interaction of different time and temperature-dependent phenomena, which occur as a result of very high temperature operation. The overall big picture process flow is shown in Fig. 1 for the development of a unified viscoplastic model needed for improved stress-strain fidelity in finite element simulation and fatigue design of a gas turbine engine.

The first step towards the development of a robust constitutive model in Fig. 1 (step I) is a comprehensive understanding of material behavior under various loading conditions and across the entire temperature spectrum encompassing the operating service conditions. Macroscopic constitutive models require a breadth of experimental loading histories for verification and validation. Hence, an experimental program has therefore been undertaken to
generate the necessary responses of Haynes 230 under the full scope of service conditions seen in the combustor liner for an experimentally validated constitutive model. The Haynes 230 material in Fig. 1 was tested under a wide range of loading histories representative of service conditions. One such loading condition is shown in Fig. 1 for a cyclic isothermal strain-controlled low cycle fatigue test at an isothermal temperature of 427°C. Once the experimental responses are well understood, the unified constitutive model is developed in a hierarchical superposition, such that all mechanical features can be modeled.

Fig. 1: Big picture process flow for the development of the unified viscoplastic model for improved stress-strain fidelity in finite element simulation and fatigue design of a gas turbine engine
The modeling challenge was to develop a constitutive model capable of simulating the isothermal fatigue, creep-fatigue, and creep responses, as well as the thermo-mechanical creep-fatigue responses acceptably well for Haynes 230. A Chaboche-based constitutive model with several added features has been chosen. The constitutive model has the features of rate-dependence, static recovery, kinematic hardening evolution, strain range dependence, and mean stress evolution. These features are essential to adequately describe the broader ranges of material responses. The constitutive model development of Haynes 230 is demonstrated in steps II and III of Fig. 1 for simulating a complex in-service condition, like thermo-mechanical fatigue. The capability of the unified viscoplastic model to simulate out-of-phase thermo-mechanical fatigue is shown in Fig. 1 for a temperature cycling going from a minimum temperature of 316°C to a maximum temperature of 971°C that is out-of-phase to the strain cycling of a minimum strain of -0.4% (compression) to a maximum strain of 0%.

Complex macroscopic constitutive models, like the one developed herein are essential in enhancing the accuracy and reliability of analysis and design of various engineering applications. But just as important is an automated parameter identification algorithm that can robustly determine an optimum solution set for the advanced material model. This research addressed this need by providing a reliable, user-friendly parameter identification program, that engineers can effectively take advantage of the enhanced simulation fidelity of a more complex model as shown in step III of Fig. 1. An automated parameter determination scheme is presented for the developed unified constitutive model. Our research work will hopefully contribute enough knowledge and effort towards incorporating more complicated analysis methods into current design methodologies that will only ensure public safety and facilitate the much needed revolution in sustainably designing systems as demonstrated by the finite element model example of a gas turbine engine of step IV in Fig. 1.
2. Scope and organization

The findings of this study are presented through the five chapters that follow written in journal paper format. Chapter 1 is the current chapter introducing the dissertation through a motivation and organization section. Chapters 2 to 6 are the five journal papers describing the various findings in detail. Finally, chapter 7 is the concluding chapter discussing the major conclusions and findings from this work and includes recommendations for future research.

Chapter 2 is a journal paper to be submitted to the International Journal of Plasticity on the isothermal fatigue and creep-fatigue responses of Haynes 230. Chapter 3 is a journal paper to be submitted to the International Journal of Solids and Structures on thermo-mechanical fatigue-creep responses of Haynes 230. These two chapters are the experimental program undertaken in this study needed for constitutive model development. The unified constitutive model for simulation of the fatigue, fatigue-creep, and creep responses of Haynes 230 are presented in the next two chapters, Chapters 4 and 5. Chapter 4 is a journal paper to be submitted to the International Journal of Solids and Structures on the simulation of isothermal low cycle fatigue and creep-fatigue responses of Haynes 230. The challenge was the development of a robust constitutive model capable of simulating a broad set of fatigue and creep-fatigue experiments across seven different temperatures ranging from 24 to 982°C (75 to 1800°F). The features of the constitutive model were dictated by the material response of Haynes 230 in Chapters 2 and 3. The modeling and the parameter determination were done in such a manner that the thermo-mechanical fatigue simulations were also successful. The developed Chaboche-based constitutive model of Chapter 4 includes features of rate-dependence, static recovery, kinematic hardening evolution, strain range dependence, and mean stress evolution.

Chapter 5 is a journal paper to be submitted to the Journal of the Mechanics and Physics of Solids on the simulation of fatigue-creep interactions of Haynes 230. The modeling challenge was to develop a constitutive model capable of simulating a broad range of behavior that not only includes strain-controlled (fatigue-creep) behavior, but also stress-controlled (creep) behavior and their interactions. In order to achieve fidelity in both domains
that are classically separated, novel ways of material identification of creep and fatigue-creep were essential and an isotropic damage theory was implemented for improving creep simulations. The gap between strain-controlled fatigue-type cyclic behavior and stress-controlled creep behavior is one that must be resolved for design. This research shows the ability to simulate both fatigue and creep using a unified viscoplastic constitutive model that has been missing in the literature.

Chapter 6 is a journal paper to be submitted to the International Journal of Computers and Structures on the implementation of a robust parameter determination for the unified constitutive model using both local and global optimization methods. Systematic calibration of material parameters was outlined and a user-developed hybrid genetic algorithm is shown. The benefits of a genetic algorithm were combined with the underlining physics of the material model in the initial estimation of parameters to guide an intelligent search. This hybrid simultaneous GA conducts both an exploitative and explorative search to find optimal parameter sets of the modified Chaboche model for unified viscoplasticity.

3. References


CHAPTER 2: AN EXPERIMENTAL STUDY ON HIGH TEMPERATURE LOW-CYCLE FATIGUE AND FATIGUE-CREEP RESPONSES OF HAYNES 230

Abstract

The reliability of structural components in high temperature systems experiencing cyclic thermo-mechanical loading is an important problem, especially for airplane turbo engine components. To investigate the failure mechanisms of these components, material models are required that can describe the complex material phenomena and accurately predict the deformation and damage behavior. These material models should be developed and validated against a broad set of material responses developed by prescribing a range of loading histories. These loading histories strive to replicate in-service conditions of structural components. In the present study, a comprehensive experimental database of Haynes 230, a nickel-based superalloy, for isothermal low cycle fatigue and creep-fatigue experiments has been critically examined with the intention of testing the predictive capabilities of an advanced material model. The experimental database encompasses a broad set of low cycle fatigue, symmetric, uniaxial strain-controlled loading histories which include isothermal with and without peak hold times, with and without a mean strain, at temperatures ranging from 24 to 982°C (75 to 1800°F). Haynes 230 is a complex material that shows unique creep, oxidation, dynamic strain aging, creep-fatigue, thermo-mechanical fatigue, strain rate sensitivity, and strain range dependent properties. Strain range dependence and temperature dependent material phenomena of Haynes 230 show the unique material behavior that present challenges in modeling. Dynamic strain aging has been found to occur in the intermediate temperature domain, 427-760°C (800-1400°F). Isothermal experiments at different strain rates show that the Haynes 230 can be considered to be rate-independent below and at 760°C. However, while the material is considered to be time-independent below 760°C, isothermal peak strain hold experiments still show viscoplastic stress relaxation at 649°C (1200°F) and 760°C (1400°F). Stress relaxation during fatigue-creep peak strain dwells experiments show both temperature and hold-time dependence. Macroscopic material
responses are correlated to justifiable microscopic evidence. Fatigue lives of isothermal creep-fatigue experiments have been closely examined to support ‘lifing’ models. It is essential to understand these complex material phenomena, under broad loading conditions and harsh environments, in order to substantially improve current design methodologies through developing experimentally validated advanced constitutive models.

Keywords: Haynes 230, creep-fatigue, fatigue life, dynamic strain aging, high temperature LCF

1. Introduction

The design of components and systems that experience thermo-mechanical fatigue at very high temperatures is extremely complex involving many failure mechanisms. Service components in the aerospace, nuclear power, chemical and automobile industry are some of these unique high-temperature systems that are rich in material complexities. Due to start-up and shut-down cycles, repeated thermo-mechanical stresses are induced that gradually degrade the life of a machine and its components. In the present study, the component of interest is the combustor liner in gas turbine engines, which are fabricated from sheets of Haynes 230. Haynes 230 is a Ni-Cr-W-Mo solid-solution strengthened superalloy, which possesses excellent high temperature strength and outstanding resistance to oxidation in deleterious environments. During turbine engine operation, the combustor components are subjected to thermo-mechanical fatigue (TMF) conditions, with temperature fluctuating between room temperature to as high as 982°C (1800°F). Consequently, some locations in the engine are subjected to strain or stress cycles of varying amplitudes, with temperature cycling either in-phase or out-of-phase to the mechanical strain cycle.

During a typical airplane flight, on-load periods occur where maximum operating temperature remains steady along with maximum compressive strains or stresses, during which creep deformation becomes prominent. The cyclic nature of the loading given by repeated airplane flights combined with the dwell periods at high temperatures results in a complex creep-fatigue interaction. This creep-fatigue interaction is further complicated by high temperature exposures across a thermal gradient resulting in thermo-mechanical fatigue.
Steep thermal gradients caused by the temperature fluctuations lead to the creation of “hot spots” surrounding the effusion and dilution holes of the combustor liner (see Fig. 1). These critical spots initiate low-cycle fatigue cracks as shown in the inset of Fig. 1, which often reduces the design life of combustor liners from 10,000 hours to as low as 2,000 hours. Accurate TMF life prediction of components is of paramount importance to facilitate improved design-by-analysis methodology for maximizing the resistance of high temperature components. Material responses of these critical components may involve time-dependent processes such as creep, oxidation, dynamic strain aging, creep-fatigue, thermo-mechanical fatigue and cyclic creep or ratcheting that influence the integrity of the component [1-4].

![Combustor liner with cracking due to TMF](Image)

**Fig. 1: Combustor liner with cracking due to TMF (Courtesy of Honeywell Aerospace)**

It is essential to understand these complex material responses, under realistic loading conditions and harsh environments, in order to substantially improve current design methodologies. Current industry standards use life prediction methods based on either simple elastic analyses or simplified inelasticity models using commercial finite element analysis (FEA) software and thus lack fidelity in the service life predictions. High temperature components should function both efficiently and economically over their expected lifetime, which requires accurate prediction of service life fatigue responses through FEA. An accurate description of thermo-mechanical cyclic stress-strain responses during service is
essential for the development of reliable life prediction techniques for critical components in aerospace, as well as automobile and power generation industries. This calls for the development of an appropriate robust constitutive model capable of describing the interaction of different time and temperature-dependent phenomena which occur as a result of very high temperature operation.

The first step towards the development of a robust constitutive model is a comprehensive understanding of material behavior under various loading conditions and across the entire temperature spectrum encompassing the operating service conditions. Despite the use of high temperature resistant superalloys, such as Haynes 230, combustor liners prematurely show cracking, which significantly increases the operation and maintenance expenses of jet engines. Hence, Haynes 230 must be understood under the full scope of service conditions seen in the combustor liner. Researchers have characterized the low-cycle fatigue behavior of various austenitic carbide precipitating (ACP) alloys, such as: IN617, HA188, Nimonic PE-16, 316L (N) stainless steel, and modified 9Cr-1Mo steel under high temperatures for different isothermal temperatures, strain rates, waveforms, peak hold positions, hold durations, and strain range parameters [4-26]. Most of these works primarily focused on discussing the micromechanical features of the fatigue responses from a limited number of experiments. Their studies of the mechanical behavior for these ACP alloys under uniaxial cyclic loading exposed to high temperatures revealed that generally increasing the temperature, decreasing the strain rate, or introducing a hold time causes a reduction in fatigue life [4,8-9]. The micromechanical evidence was provided in these studies to reveal the time-dependent damage mechanisms, such as dynamic strain aging, oxidation, creep, and the creep-fatigue crack initiation processes.

Experimental study of ACP alloys beyond fatigue life has been analyzed for hardening-softening behavior [9,21-22], dynamic strain aging [9,13], strain rate sensitivity [9,13-15,23], and creep-fatigue stress relaxation for various dwell times [8,24,26]. Rao et al. [8] presented creep-fatigue stress relaxation of Inconel 617 at 950°C for different dwell times (up to a maximum of 120 minutes) with hold conditions at peak strains including tension-only, compression-only, and finally both tension and compression holds. Also, in the same study
positive strain rate sensitivity of Inconel (Alloy) 617 at 950°C was ascertained. Again, much of their experimental study focused on the creep-fatigue crack initiation processes for the low-cycle fatigue tests with dwell times all at a mechanical constant strain range of 0.6%. In another study by Rao et al. [9], the influence of strain rate (10^{-6} to 10^{-3} s^{-1}) and temperature (750, 850, 950°C) for Alloy 617 were investigated. Dynamic strain aging characterized by serrated flow was also shown at 750°C and at 850°C for higher strain rates. The influence of temperature across their temperature domain showed a complex cyclic stress response with varying material responses of either cyclically hardening or softening type behavior and a combination of the two.

Haynes 188 was studied [15,17] across a larger spectrum of temperatures encompassing the operating service conditions (25°C up to 1000°C) for low cycle fatigue using a symmetric triangular waveform at a strain range of 0.8%. Haynes 188 showed a dynamic strain aging temperature regime between 300-750°C. Dynamic strain aging causes the alloy to cyclically harden significantly while also showing negative strain rate sensitivity [13]. Negative strain sensitivity results in greater cyclic stress responses for slower strain rates. Hasselqvist reported [22] the austenitic carbide precipitating (ACP) mechanism of various alloys showing cyclic responses of Haynes 188 [17], 316L [19], and Haynes HR-120 [20] across a wide range of temperatures for constant strain ranges and strain rates. More recently, Lu et al. [24] and Chen et al. [26] have investigated Haynes 230 for different temperatures, strain ranges, and dwell times. The influence of hold time (0, 2, 10, and 60 minutes) was presented for 816 and 927°C experiments in [24] showing the effect on cyclic hardening/softening behavior as well as the fatigue life. Inconel 617 and Haynes 230 were compared by Chen et al. [26] for creep-fatigue responses at 850°C showing that for all creep-fatigue tests, Haynes 230 exhibited longer life than Inconel 617. Also, in this same study two creep-fatigue life prediction methods, linear damage summation and frequency-modified tensile hysteresis energy modeling, were evaluated. In addition, few TMF experimental studies of ACP alloys, in particular nickel base superalloys, have been carried out [29-35]. Experimental data of Haynes 230 is still very limited with respect to the service-replicated conditions and this experimental study undertook the challenge of generating a broad set of test data that would
be required in the development of a robust constitutive model for high temperature design applications.

To develop a constitutive model capable of simulating a broad range of material responses, one must plan an experimental study that replicates the service-conditions as closely as possible. An experimental program of Haynes 230 for low-cycle fatigue and fatigue-creep are presented in this study, while thermo-mechanical fatigue has been shown in a sister paper (Ahmed et al. [36]). These experiments were done with the aim of characterizing the scope of material behavior for the combustor liner service-like conditions as closely as possible. For the case of the airplane a typical flight consists of a start-up phase which leads to temperature rise, a steady phase coinciding with the flight time (0.5-14 hours), which keeps the high temperatures constant, and finally a shut-down phase which leads to temperature drop. The thermal nature of the loading and the geometry of the structure shown in Fig. 1 results in constraints which lead to compressive strains being induced in the vicinity of hot spots that try to expand. Thus, in combustor liners out-of-phase (OP) thermo-mechanical fatigue (TMF) is the phenomenon of primary interest, which is the decrease in axial strain (compressive strain) with an increase in temperature.

Due to the limitations in economy of trying to reproduce an ‘exact’ test representative of the service conditions, one must shorten the hold duration of the flight for best economy. However, by decreasing the hold duration at peak strains, fatigue-creep deformation is more likely to be induced than a creep-fatigue mechanism. This limitation of economy in performing longer hold duration tests is usually not overcome and researchers perform hold durations in orders of magnitude (seconds/minutes) less than the actual service conditions (hours). However, to develop a creep perspective of the material, separate creep tests can be performed, which are much more economically attractive but lack fidelity compared to the service thermo-mechanical creep-fatigue cycle. The developed constitutive model is thus challenged by not only low cycle fatigue-creep responses, but also pure creep material responses. Our experimental program includes low-cycle fatigue, fatigue-creep, pure creep, and thermo-mechanical fatigue tests for developing a unified constitutive model by studying the material response under service-like loading conditions in an integrated manner. Again,
the challenge is to develop experimental data in a short period of time to imitate the relevant damage mechanisms, like creep, fatigue, TMF, and their interactions in several high temperature applications that can be used for robust constitutive model development. The modeling of thermo-mechanical creep-fatigue interaction is a challenge that requires a well-designed experimental program that includes isothermal LCF, fatigue-creep, pure creep, and anisothermal TMF tests.

To analyze engine operating conditions in the most efficient economic way, material testing was carried out on ASTM standard coupons of Haynes 230 for isothermal low-cycle fatigue, fatigue-creep, pure creep, and anisothermal thermo-mechanical fatigue tests. The experimental database encompassed a broad set of low cycle fatigue (LCF), symmetric, axial strain-controlled loading histories which included isothermal, with and without a mean strain, at temperatures ranging from 24 to 982°C (75 to 1800°F) for varying strain ranges. The experimental scope also investigated creep-fatigue responses for temperatures ranging from 649 to 982°C (1200 to 1800°F) with the introduction of dwell periods (hold times) at the peak compressive strain. Thermo-mechanical fatigue (TMF) experiments with phase angles between strain and temperature corresponding to in-phase and out-of-phase were also conducted on Haynes 230. In-phase TMF experiments corresponded to an increase in axial tensile strain with an increase in temperature, while out-of-phase experiments corresponded to a decrease in axial strain (i.e. increase in axial compressive strain) with an increase in temperature. Service-conditions were replicated as closely as possible for test parameters including but not limited to strain ranges, strain rates, and hold durations. For reproducibility and material reliability, over 120 isothermal low-cycle fatigue and fatigue-creep experiments at seven different temperatures ranging from 24 to 982°C (75 to 1800°F) were carried out.

Preliminary finite element analysis has shown that the combustor liner of gas turbine engines experiences stress states that are not purely uniaxial, but rather multiaxial. As a result, the multiaxial responses of Haynes 230 have been undertaken, in addition to the uniaxial investigation. These multiaxial stress states can be idealized in the context of mechanical behavior of materials as a mixed combination of strain or stress controlled loading. Ratcheting or cyclic creep of a material under multiaxial loading can occur if at least
one component of stress is prescribed in a multiaxial cyclic loading history involving some plastic deformation. The phenomenon of ratcheting refers to the progressive accumulation of deformation under cyclic loading, which has been of much interest in the plasticity community for many years. However, few experimental studies have been done for nickel-based superalloys like Haynes 230. Multiaxial tests have been conducted using axial-torsional tests. In the first phase of the multiaxial experiments, the influence of multiaxiality on ratcheting-creep-fatigue of Haynes 230 is investigated through prescribing symmetric shear-strain cycle at steady axial stress at various temperatures between 24 to 982°C (75-1800°F). The equivalent mean or steady axial stress, $\sigma_m$, in the stress-controlled experiments is taken as $0.1\sigma_y$, where $\sigma_y$ is the 0.2% offset yield stress of the monotonic tensile behavior of Haynes 230. Also, the equivalent shear strain amplitude, $\varepsilon_a$, is kept constant and the shear strain cycles are conducted at a strain rate of 20 cpm. These tests are taken to failure of the specimen. The next phase of the multiaxial experiments investigated the cyclic hardening-softening features of Haynes 230 under strain-controlled cycling of proportional triangular, axial cycling followed by the highest degree of non-proportionality, 90° out-of-phase sinusoidal out-of-phase cycling at the same equivalent strain amplitude, $\varepsilon_a$, at temperatures ranging from 24 to 982°C (75-1800°F). Multiaxial features of Haynes 230 will be presented in a future paper.

The complexity and features of a constitutive model are dictated by the experimental responses of the material. The experimental characterization of isothermal, low cycle fatigue of Haynes 230 over a wide scope of test parameters - temperature, strain range, strain rate, and dwell (hold) times representative of the service-conditions - are presented in order to accurately validate the robustness of the developed material model. The present study will focus on the material responses of Haynes 230 shown in the isothermal experiments for low cycle fatigue and fatigue-creep only. The anisothermal behavior of Haynes 230 has been discussed for convenience in a sister paper (Ahmed et al. [36]). Out-of-phase thermo-mechanical fatigue tests were chosen as they replicate the operating conditions most closely. In-phase thermo-mechanical fatigue tests were also conducted to enhance the robustness of the developed constitutive model.
2. Experimental Study

2.1. Experimental program

All testing was performed by an outside test vendor under close coordination with the investigators of the study. LCF tests with axial strain-controlled loading histories were performed at isothermal states with and without hold times, with and without a mean strain, as well as varying strain rates and strain ranges, at temperatures ranging from 24 to 982°C (75 to 1800°F). Continuous strain-controlled axial cycling had a triangular waveform with cyclic frequencies of 0.2, 2, and 20 cycles per minute (cpm) and imposed strain ranges varying from 0.30-1.60%. In order to explore the influence of test parameters, experiments were divided into four groups as shown in Table 1. Isothermal LCF tests under strain control conducted at a constant frequency of 20 cpm, with a strain ratio (min/max) of -1.0, and temperatures ranging from 24 to 982°C (75 to 1800°F), comprised Group 1. It is noted that some of these tests were conducted with sinusoidal control waveforms along with a different cyclic frequency in order to avoid instabilities experienced during these tests.

The isothermal, continuous LCF tests of Group 1 allow for the investigation of the effect of temperature and strain range on the hysteresis curve and Bauschinger effect, including the cyclic stress behavior as well as the isothermal LCF lifing. The Group 2 tests were conducted to explore the rate effects of Haynes 230 under cyclic frequencies of 0.2, 2, and 20 cpm. These cyclic frequencies were chosen to investigate the possible presence of rate effects. The plastic strain range can be calculated as shown in Fig. 2a, and roughly corresponds to the width of the hysteresis loop at zero stress. The tangent lines shown in Fig. 2a have slope equal to the elastic modulus for that particular temperature. Due to plasticity, with increasing temperature, the plastic strain range tends to increase faster as the material becomes viscous, as seen in Fig. 2b with the highest linear slope given by 982°C. As a result, it was important to keep a constant plastic strain range, such that a prescribed total strain range can be selected for each temperature. A fixed plastic strain was decided, as shown in Fig. 2b, by plotting the measured plastic strain range versus the total strain range at half-life for temperatures tested in Group 1. From the plot of plastic strain range vs. total strain range for all temperatures, in Fig. 2b, a baseline level of 0.2% ‘fixed’ plastic strain range, $\Delta e_p^f$, was decided to ensure that
all temperatures were considered in the study of the rate effects. With a fixed plastic strain range of 0.2%, the various total strain ranges can be decided on for Group 2 testing to fully understand the rate dependency of HA230. Group 3 tests were performed under continuous, isothermal LCF with symmetric, axial strain-controlled loading histories on the tension side at a strain ratio (min/max), R= 0. The total strain ranges and temperatures are similar to many of the Group 1 experiments; however, the difference is manifested in the positive mean strain. The responses of the positive mean strain tests (Group 3) will demonstrate the effect of the mean strain along with temperature and strain range on the hysteresis curve and lifing.

**Table 1**

LCF tests for Groups 1-4 with appropriate temperature and strain history over a scope of test parameters

<table>
<thead>
<tr>
<th>Description of Tests</th>
<th>Temperature History</th>
<th>Strain History</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Groups 1 &amp; 2:</strong> Isothermal LCF by prescribing symmetric, axial strain ($\varepsilon_x$) cycle at various temperatures (T) between 24-982°C with strain ratio, R = -1.0; <strong>Group 1:</strong> six strain ranges at a cyclic frequency, f = 20 cpm, and <strong>Group 2:</strong> three strain rates with cyclic frequencies, f = 0.2, 2 and 20 cpm, for a ‘fixed’ plastic strain range, $\Delta\varepsilon_x^p = 0.20%$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>$\varepsilon_x$</td>
<td>$t$</td>
</tr>
<tr>
<td><strong>Group 3:</strong> Isothermal LCF by prescribing axial strain ($\varepsilon_x$) cycle at various temperatures (T) between 24-982°C, with strain ratio, R = 0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>$\varepsilon_x$</td>
<td>$t$</td>
</tr>
<tr>
<td><strong>Group 4 (G4):</strong> Isothermal LCF by prescribing symmetric, axial strain ($\varepsilon_x$) cycle with compression peak holds, $t_H = 60$ and 120 seconds, at various temperatures (T) between 649-982°C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>$\varepsilon_x$</td>
<td>$t$</td>
</tr>
</tbody>
</table>
All of these tests, Groups 1-3, had strain ranges that were appropriately selected, given the test temperature, to obtain failure lives, $N_f$, between 2,000-100,000 cycles. The reduction in the prescribed strain range with increased temperatures is needed to maintain reasonable plastic strain ranges as the material becomes more viscous. An increasing viscosity results in a softer material that is easily damaged; thus plastic strain ranges are larger. To avoid this problem, total strain ranges were reduced and as a result comparable fatigue lives were obtained. For all of these tests, the number of cycles to crack initiation $N_i$, and to failure $N_f$, were defined as the cycle count where the amplitude stresses dropped to a value of approximately 5 and 30% respectively below the stabilized saturated stresses on the cyclic
hardening/softening curves. This is a common technique used in LCF testing in determining what constitutes crack initiation and failure.

A series of Group 4 tests investigated the creep-fatigue responses of HA230 by prescribing, isothermal LCF with hold times imposed at the peak compressive strain. Dwell periods during a strain hold results in creep-type stress relaxation of the material. All Group 4 test strain ranges were decided based on the constant-plastic-strain-range-level 0.2% method of Fig. 2 and a fixed loading rate (ramping time) of 20 cpm. The temperatures ranged from 649°C to 982°C (1200°F to 1800°F). Hold periods, t_H, of 60 and 120 seconds were imposed to investigate the effect of various hold times. Operating temperatures were in the range where creep deformation occurs, so that the interaction between LCF and creep can be investigated. The microstructural characterization as well as the development of creep-fatigue interaction tests at elevated temperatures was pioneered by Rao and Rodriguez [4, 8]. Their contributions were made in highlighting the phenomenological effects of various strain-time loading histories on LCF and creep-fatigue interaction life as well as the mechanistic aspects of these macroscopic observations, specifically the nucleation and growth of cracks and cavities. In our present investigation, the hold periods were imposed at the compressive peak strain to imitate the flight periods of the gas turbine engines. Unlike, the previous experiments, Groups 1-3, these particular tests, Group 4, were not taken to failure because of the experimental costs.

2.2. Quantification of responses

Figure 3 shows a typical stress versus strain hysteresis loop response of Haynes 230 for the Group 4 experiments in Table 1. Group 4 was chosen to define all the necessary parameters for discussion purposes. In Fig. 3, the 500th cycle is shown for an experiment with an isothermal temperature of 871°C, strain range of 0.53%, and a hold time of 120 seconds. The figure shows the different measures used to quantify the cyclic response. The maximum stress (σ_{max}) and minimum stress (σ_{min}) values correspond to the extreme points on the hysteresis cycle. The stress range is Δσ. The stress amplitude and mean stress can be calculated as shown in Eqs. 1 and 2. Point B corresponds to the time just before hold and
point A corresponds to the time at the end of the hold. The difference between the two is the stress relaxation denoted by $\Delta \sigma_{xr}$.

$$\sigma_{xr} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \equiv \frac{\Delta \sigma}{2}$$  \hspace{1cm} (1)$$

$$\sigma_{\text{mm}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}$$ \hspace{1cm} (2)

![Hysteresis loop diagram](image)

**Fig. 3.** Typical creep-fatigue hysteresis loop showing measurements made to quantify cyclic response

### 2.3. LCF and creep-fatigue testing procedure

The geometry of the specimens were solid circular with a gage length of 0.63 in and inner diameter of 0.250 in. as shown in Fig. 4. The specimen fabrication is done through machine processing that transforms the bulk material rods into a precise, useable specimen. After the material is blanked it must be rough machined from a rectangular geometry to a rounded, cylindrical shape through a low-stress grind technique. Once the shank and gage sections of the specimen are machined, Sheffield threads are ground in the LCF specimen providing extra friction in the grips of the load machine. Axial and circumferential polishing is done before the final quality control inspection is performed.
After manufacturing, low cycle fatigue tests were performed on universal, servo hydraulic testing machines. The frames are outfitted with commercially available software used to control the test and collect data. All of the calibrations are done for each frame whether it is the load cell, alignment, extensometry, or general computer calibrations. For elevated temperature testing, heating is achieved through an induction heating system with thermocouples aligned for temperature uniformity. LCF tests were conducted in accordance to ASTM E606-04 standards.

2.4. Test material and metallography

The Haynes 230 alloy was supplied by Haynes International Inc. and received as bulk rods in solution annealed conditions. The as-received microstructure is presented in Fig. 5a. Microstructural grains are manifested as non-straight boundaries, whereas inside these boundaries annealing twins characterized by straight boundaries develop internally, as shown in Fig. 5b. The Haynes 230 alloy also contains precipitates which manifest themselves as particles inside the matrix (Fig. 5b). The sample was electrolytically etched in a solution containing hydrochloric acid (HCl) and hydrogen peroxide (H₂O₂) to reveal the microstructural features. The precipitates were tungsten-rich primary carbides of the stoichiometric composition M₆C type, where M denotes the metallic atom and C represents the carbon contribution. Grain size measurements were performed on the heat-treated specimen in which microstructural images at 100x magnification were compared with the standard ASTM plates, according to ASTM E112-10. The average grain size was found to be around 60 μm. The nominal chemical composition of the alloy is presented in Table 2.
fracture surfaces of the LCF specimens were examined by a scanning electron microscope (SEM) to ascertain the crack initiation and propagation modes.

![Typical microstructure of Haynes 230 alloy showing (a) rolling texture morphology of the primary carbides, M₆C, and (b) annealing twins.](image)

**Fig.5.** Typical microstructure of Haynes 230 alloy showing (a) rolling texture morphology of the primary carbides, M₆C, and (b) annealing twins.

### Table 2

Nominal chemical composition of Haynes 230 alloy, in wt. %

<table>
<thead>
<tr>
<th>Element</th>
<th>Ni</th>
<th>Cr</th>
<th>W</th>
<th>Mo</th>
<th>Fe</th>
<th>Mn</th>
<th>Si</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co</td>
<td>60.344</td>
<td>21.76</td>
<td>13.99</td>
<td>1.32</td>
<td>1.07</td>
<td>0.49</td>
<td>0.40</td>
<td>0.32</td>
</tr>
<tr>
<td>Co</td>
<td>0.13</td>
<td>0.10</td>
<td>0.04</td>
<td>0.015</td>
<td>0.01</td>
<td>0.005</td>
<td>0.004</td>
<td>0.002</td>
</tr>
</tbody>
</table>

^a As balance  
^b Maximum

### 3. Results

**3.1. Cyclic stress-strain responses from Groups 1-3**

**3.1.1. General hysteresis characteristics and the effect of mean strain**

Hysteresis curves at different temperatures and the two strain ratios of Groups 1 and 3 can be compared in understanding the cyclic stress-strain subtleties that may exist for different test parameters. In general, the effect of a mean strain seems to be negligible as demonstrated by comparing both the stress-strain hysteresis evolution, and amplitude and mean stress cyclic evolution as a function of cycles for the various temperatures (Figs. 6-9). The hysteresis curves for all cycles recorded are plotted for 24°C in Figs. 6 a-b, which do not depict clearly the initial cyclic hardening followed by cyclic softening as demonstrated by the stress amplitude/mean plot shown in Fig. 6c. Hence, for other experiments the hysteresis
responses and stress amplitude/mean evolutions are plotted for intermittent cycles, Figs. 7-9. The loading rates of the tests are also shown in the figures. Despite the difference in the strain ratio (R) of the prescribed strain-cycles in Group 1 and Group 3 experiments, a key observation made was that the hysteresis loops, and amplitude and mean responses from the two groups for temperatures up to 760°C (1400°F) are similar as shown in Figs. 6-7. For temperature 760°C and above, some differences between the responses are observed, which are magnified in the stress amplitude responses of Figs. 8-9. With a positive mean strain (R=0), the stress amplitude responses for both 760°C (Fig. 8c) and 982°C (Fig. 9c) are lower when compared to the symmetric Group 1 experiments without a mean strain (R=-1). The mean stresses from the positive mean strain Group 3 experiments behaved quite similarly to Group 1 experiments without a mean strain, as can be seen in the mean stress evolutions of Figs. 6-9. For both Group 1 and 3 experiments, small negative mean stresses develop indicating that hysteresis cycles are asymmetric with respect to the strain axis for all temperatures. The asymmetry of the hysteresis cycle develops due to the material developing larger stresses in compression than in tension. For high temperature experiments 427-982°C (800-1800°F), controlling the strains in a stable manner was difficult, and hence prescribed loading histories were changed from saw-tooth to sinusoidal and the desired strain range was often arrived at incrementally (Figs. 7b, 8, 9b). Sinusoidal loading history at high temperatures yields rounded hysteresis loop peaks as shown in Fig. 9b. Another interesting phenomenon is the time-dependent nature of the hysteresis loop specifically seen at 982°C (Fig. 9a) even for a saw-tooth loading history. Viscous effects of the material can cause the hysteresis loops to round at the stress peaks and valleys. This rounding effect was also seen in stainless steel [37]; as a result, the hysteresis cycles become rounded and the peak stresses do not match peak strains.

The hardening/softening evolutionary behavior of the Haynes 230 alloy can be characterized by analyzing the stress amplitude responses in Figs. 6-9. The results indicate that cyclic deformation and the developed stresses are highly sensitive to temperature under low cycle fatigue conditions. At room temperature, Fig. 6, HA230 cyclically hardens initially followed by cyclic softening. Whereas, at 204-649°C (400-1200°F), the material continues to
cyclically harden without any sign of stabilization as can be seen in Fig. 7, for which the cycles continuously cyclically harden (thinning of the hysteresis loop widths) until failure. For 649-760°C (1200-1400°F), the rate of cyclic hardening gradually reduced, Fig. 8c, when compared to 204-649°C such as Fig. 7c. At 871°C (1600°F), the material uniquely shows either cyclic hardening or cyclic softening (fattening of hysteresis loop widths) depending on the strain rate. The material shows cyclic softening at 982°C for all strain rates as shown in Fig. 9. The rapid hardening evolution found between 204-760°C (400-1400°F) seems to be a direct connection to the time-dependent effects of dynamic strain aging, which strengthens the strain hardening mechanisms. Dynamic strain aging will be an important topic extensively studied in Section 3.2.
Fig. 6. HA230 responses under LCF, strain range 1.4%, 24°C for (a) Hysteresis loops from symmetric, saw-tooth, strain-controlled cycles ($R = -1$), (b) Hysteresis loops from tensile, saw-tooth, strain-controlled cycles ($R = 0$), and (c) stress amplitudes and means as a function of cycles from experiments (1MPa=0.145ksi).

Fig. 7. HA230 responses under LCF, strain range 1.0%, 427°C for (a) Hysteresis loops from symmetric, saw-tooth, strain-controlled cycles ($R = -1$), (b) Hysteresis loops from tensile, saw-tooth, strain-controlled cycles ($R = 0$), and (c) stress amplitudes and means as a function of cycles from experiments (1MPa=0.145ksi).
Fig. 8. HA230 responses under LCF, strain range 0.6%, 760°C for (a) Hysteresis loops from symmetric, sinusoidal, strain-controlled cycles (R = -1), (b) Hysteresis loops from tensile, sinusoidal, strain-controlled cycles (R = 0), and (c) stress amplitudes and means as a function of cycles from experiments (1MPa=0.145ksi)

Fig. 9. HA230 responses under LCF, strain range 0.39%, 982°C for (a) Hysteresis loops from symmetric, saw-tooth, strain-controlled cycles (R = -1), (b) Hysteresis loops from tensile, sinusoidal, strain-controlled cycles (R = 0), and (c) stress amplitudes and means as a function of cycles from experiments (1MPa=0.145ksi)
3.1.2. Cyclic hardening-softening evolution

Three distinct cyclic hardening-softening regions are reinforced further by exploring both the temperature and strain range dependence of the cyclic stress evolution. Stress amplitudes and means for a given strain range, but at different temperatures for Group 1 experiments in order to capture the temperature dependence of the material, are plotted in Figs. 10a-10d. Softening of the material with increasing temperature is observed at the very first cycle in each of the Figs. 10a-10d. In Fig. 10a, this softening of the material is shown for the first cycle whereby from 24°C to 427°C the stress amplitude response is nearly 140 MPa less. The stress amplitude responses in these figures show complex phenomena of cyclic hardening-softening, continued cyclic hardening or cyclic softening dependent on both temperature and strain range. Also, in Fig. 10a 427°C shows a completely different cyclic hardening nature as compared to 24 and 204°C. At 427°C in Fig. 10a the material continuously cyclically hardens in a two-slope linear manner when looking at the responses in semi-log. However, for the material responses at 24 and 204°C for a particular strain range, like 1.6% in Fig. 10a, cyclically hardening to a peak followed by a degree of subsequent softening is observed. This unique material change in the evolutionary response of Haynes 230, from a cyclically hardening-softening (24-204°C) behavior to a continuous cyclically hardening (427°C) as a function of temperature, presents a challenge in constitutive modeling. For temperatures in the range 427-649°C (800-1200°F), the rate of cyclic hardening is observed to be increasing with increase in temperature (Figs. 10a-10c). In contrast, above 649°C (1200°F) the rate of cyclic hardening gradually decreases with increase in temperature (Figs. 10b-10d). In Fig. 10d, at 982°C (1800°F) Haynes 230 shows cyclic softening. In regard to the mean stresses, a small but nonzero mean stress is developed for all temperatures; however, these mean stresses do not differ in magnitude as a function of temperature, as shown by the overlapping responses in Figs. 10a-10d.

Previously, it was shown that Haynes 230 exhibits a unique temperature dependence for a given strain range. The effect of strain range is also explored in a similar manner by examining stress amplitudes and means for a given temperature at various strain ranges for Group 1 experiments shown in Figs. 11a-11d. In Figs. 11a-11c, for temperatures spanning
24-871°C, an increased rate of cyclic hardening with an increase in strain range develops showing steeper slopes in the response. For example, in Fig. 11b, at 427°C the highest strain range tested of 1.6% has a steeper slope compared to the lowest strain range tested of 0.6%. However, at 982°C, Haynes 230 exhibits cyclic softening and shows an increase in rate of softening, comparing the highest strain range 0.4% to the lowest strain range 0.3% in Fig. 11d. The cyclically hardening and subsequent softening for temperatures 24°C to 204°C that was shown in Fig. 10a tends to diminish with decreasing strain ranges as shown in Fig. 11a. This subsequent cyclic softening rate, when observed, reduces with decrease in strain range, whereby at a 0.6% strain range the material shows a slow cyclic hardening nature to a stable response, in contrast to the highest strain range of 1.6% where cyclic hardening followed by subsequent softening is pronounced (Fig. 11a).

The results in Figs. 10-11 indicate that cyclic deformation and the developed stresses are highly sensitive to both temperature and strain range under low cycle fatigue conditions. At room temperature (24°C), HA230 cyclically hardens initially followed by a degree of cyclic softening depending on the strain range (Fig. 11a). Whereas at 204-649°C (400-1200°F), the material continues to cyclically harden without any sign of stabilization (Fig. 11b). For 649-871°C (1200-1600°F), the rate of cyclic hardening is gradually reduced (Fig. 11c). The rapid hardening evolution found for temperatures 204-649°C (400-1200°F) seems to be related to the time-dependent effects of dynamic strain aging, which strengthens the strain hardening mechanisms [21-22]. At 982°C (1800°F), cyclic softening is demonstrated by HA230 as shown in Fig. 11d, where loading rate effect is also observed.
Fig 10. HA230 temperature dependence upon cyclic stress evolution, with constant strain ranges: (a) $\Delta\varepsilon=1.6\%$, (b) $\Delta\varepsilon=0.8\%$ (c) $\Delta\varepsilon=0.6\%$, and (d) $\Delta\varepsilon=0.4\%$ at several temperatures (1MPa=0.145ksi)
Fig 11. Haynes 230 strain range dependence upon cyclic stress evolution, with stress amplitude and mean responses for Group 1 experiments with constant isothermal temperatures: (a) 24°C (75°F), (b) 427°C (800°F), (c) 760°C (1400°F), and (d) 982°C (1800°F) at several strain ranges (1MPa=0.145ksi)

3.1.3. Effect of strain rate

The rate dependence of loading on the HA230 responses is characterized through Group 2 experiments. In general, viscoplastic materials exhibit loading rate effects as well as creep
and/or stress relaxation. Rate effects can occur due to the time-dependent nature of permanent deformation. Positive rate dependent behavior of a material has been shown for austenitic stainless steels [10,38] and is indicated by a higher stress amplitude for faster loading strain rates. The stress amplitudes, $\sigma_{xa}$, and mean stresses, $\sigma_{xm}$, against number of cycles from symmetric, strain-controlled experiments for loading rates of 0.2, 2, and 20 cpm for temperatures 24-982°C (75-1800°F) are depicted in Figs. 12-13. The loading rates of 0.2, 2 and 20 cpm correspond to strain rates in the order of $10^{-5}$, $10^{-4}$ and $10^{-3}$ per second respectively. At the lower temperatures, 24°C and 204°C (75°F, 400°F) (Figs. 12a-12b), the stress amplitude responses are essentially superimposed indicating the effect of loading (strain) rate is not present.

In contrast, in the temperature regime 427-649°C (800-1200°F) (Figs. 12c-12d), the stress amplitudes for lower loading rates (i.e. lower strain rates) are greater in most cases. At 427°C (800°F), in Fig. 12c the stress response is higher at 0.2 cpm as compared to the fastest loading rate of 20 cpm for the entire cyclic life. At 649°C (1200°F), in Fig. 12d, the material response shows almost rate-independent behavior for cycles less than 25; however, in the latter cycles like cycle 100, the material exhibits noticeable negative strain rate sensitivity where the stress amplitude response for the highest strain rate of 20 cpm is of the lowest value while the stress amplitude for the lowest strain rate of 0.2 cpm is of the highest value.

Negative strain rate sensitivity is known to be one of the manifestations of dynamic strain aging [10,13]. The material is also not perfectly rate-independent at 760°C (1400°F). In Fig. 13a we observe that with >100 cycles the stress amplitude responses show some positive rate-dependence where the stress amplitude is greater for faster loading strain rates. Also at 760°C (1400°F), in Fig. 13a, it can also be argued that there is some (albeit very little) negative strain rate sensitivity observed for cycles less than 25 where the stress amplitude responses for the lower strain rates are higher than those for the higher strain rates. In essence, over the temperature domain 427-760°C (800-1400°F), negative rate sensitivity of the stress response is believed to be caused by the effect of dynamic strain aging [13-15]. In modeling, for practical purposes the stress amplitude responses can be considered to be rate-independent for temperatures below and including 760°C (1400°F). Positive rate-dependent
responses can be observed for both 871°C (1600°F) and 982°C (1800°F), Figs. 13b-13c, where stress amplitude responses are higher for higher strain rates. However, interestingly at 871°C (1600°F), while there is a clear positive rate-dependence going from 0.2 to 2 cpm, there is almost a saturation of stress amplitude responses going from 2 to 20 cpm. In Fig. 13c, at 982°C (1800°F), we observe the typical positive strain rate dependence found in viscous materials. Yaguchi et al. [27] also found a nickel-base alloy IN792 to be rate-independent at lower temperatures (<700°C) and rate-dependent (800°C) at the higher temperatures.

Fig. 12. Haynes 230 strain rate dependence upon cyclic stress evolution, with stress amplitude and mean responses for Group 2 experiments with constant isothermal temperatures: (a) T = 24°C (75°F), (b) T = 204°C (400°F), (c) T = 427°C (800°F), and (d) T = 649°C (1200°F) at various loading rates (0.2, 2, and 20 cpm) (1MPa=0.145ksi)
3.2. Role of Dynamic Strain Aging (DSA)

Dynamic strain aging has been found to occur in the intermediate temperature domain for austenitic carbide precipitating (ACP) alloys, which encompass Ni base superalloys, like Haynes 230, Co base superalloys, like Haynes 188, FeNi base super stainless steels, like Haynes HR-120, and ferritic steels, like 9Cr-1Mo. All of these alloys are austenitic, solution strengthed that develop similar physical mechanisms and time dependent processes [21-22]. While many researchers understand the creep, oxidation, and metallurgical instabilities that arise because of the time dependent nature caused by elevated temperature, few have tried to understand the phenomenon of dynamic strain aging [13-15]. The overall characterization of the effects of microstructure and various testing variables on the macroscopic
phenomenological manifestations including the microscopic evidence has been studied by Rao et al. [8] and Mannan et al. [13]. Their research has provided invaluable contributions in developing a deeper understanding of the various time and temperature dependent processes influencing low cycle fatigue behavior of these ACP alloys. Some of the macroscopic manifestations of DSA include strong cyclic hardening, negative strain rate dependency, and serrated yielding [13].

Microscopically these DSA effects are associated with the tendency of mobile solutes to interact with the quasi-viscous dislocation movement [39]. This causes a solute drag exerted on the dislocations inducing macroscopically unstable stress responses (serrations), hence causing problems in strain controlled LCF testing. In essence, dislocations are being produced, marked by an increase in dislocation density, for which these newly produced dislocations are subsequently being immobilized by solute locking until stress is high enough to break away the new dislocations [21]. As a result, the viscoplastic strain rate increases abruptly and the servo system tries to compensate the rate increase by decreasing the stress, thus unavoidably leading to negative strain rates with abrupt stress drops. In effect, the strain-controlled response of the system leads to serrated unstable response due to this negative feedback caused by the solute drag, as can be seen in Fig. 14a. Once initiated, the strain hardening process is affected to a maximum extent by dislocation immobilization.

This microscopic response links the unstable plastic flow in the stress-strain hysteresis loops, known as the serrated yielding along with marked cyclic hardening. The serrated yielding can be clearly seen in the initial cycle shown in Fig. 14c for 427°C. However, interestingly the serrations in the DSA regime usually disappeared after a certain amount of cycling, as is the case in Fig. 14d where by the 200th cycle the serrations have disappeared. Another unique observation found was that serrated yielding was dependent on the imposed strain rate. In Figs. 14a-d, the imposed strain rate was at 2 cpm which is in the order of $10^{-4}$/s, a considerably low strain rate causing the serrated flow to disappear after 200 cycles. As a result, the serrated flow seemingly disappears after a certain amount of cycling at these lower strain rates, which is consistent with Rao et al. [9]. For strain rates in the order of $10^{-3}$/s for 20 cpm, serrations remained throughout the entire cyclic life due to the faster loading rate.
The occurrence of unstable plastic flow, serrated yielding, usually appears within the temperature regime of DSA, 427-760°C (800-1400°F).

Further macroscopic evidence of DSA includes the presence of strong cyclic hardening over all strain ranges as seen in Fig. 11b for 427°C (800°F). Another macroscopic manifestation of DSA is the presence of negative strain rate sensitivity in the cyclic stress evolution. Again, this negative strain rate sensitivity was seen in the cyclic stress evolution from 427-760°C (800-1400°F) in Figs. 12c-d and Fig. 13a. Negative strain rate sensitivity (NSRS) of the cyclic stress evolution occurs when an increase in the stress amplitude corresponds to a decrease in strain rate for deformation at an isothermal state. Consequently, in the domain of occurrence of the DSA 427-760°C (800-1400°F), a continuous decrease in fatigue life associated with either an increase in temperature or decrease in strain rate characterizes the deleterious effects imposed by DSA as a consequence of higher stress levels being developed.

3.3. Haynes 230 Elastic Characterization (Groups 1-3)

The elastic domain of Haynes 230 can be precisely identified experimentally using the cyclic hysteresis curves of initial cycles for all temperatures and strain ranges. The elastic modulus or Young’s modulus is a well-known property; however, the identification of this elastic modulus and understanding the yield domain of a material should be done from the first cyclic reversal and not the monotonic sequence. In our case, the monotonic path of the loading history was conducted in compression followed by cyclic reversals in triangular, trapezoidal, or sinusoidal form (Table 1). One such experiment clearly showing this identification of the elastic nature of the material is given at room temperature 24°C (75°F) with a triangular loading protocol of Group 1 at a cyclic frequency of 20 cpm and a strain range of 1.6%, as shown in Figure 15. In this particular case, the first full cycle is shown where the onset of yielding and the elastic domain is clearly defined by the asymptotic slope of Fig. 15a. This asymptotic slope can be further justified by plotting the total mechanical strain versus the plastic strain as shown in Figure 15b for the same experimental conditions of Figure 15a. Once again, upon each reversal, an asymptotic yielding occurs defining the
elastic domain of the material. The elastic yield domain of the material results in only elastic strain in terms of total mechanical strain; thus the plastic strain, does not accumulate.

Fig. 14. The effect of dynamic strain aging (DSA) on the hysteresis responses at T = 427°C: (a) Negative feedback in strain-controlled wave due to solute drag (DSA) for the initial cycle, (b) DSA disappears after 200 cycles, (c) DSA appearance in initial hysteresis cycle with unstable plastic flow in both compressive and tensile portions of the hysteresis, and (d) the absence of DSA at cycle 200 (1MPa=0.145ksi)
Fig. 15. HA230 initial plastic hysteresis cycle at room temperature (24°C) showing yielding, \(\sigma_0\) and (b) yielding at same testing conditions showing asymptotic fixed plastic strain during yield.

The elastic characterization of Haynes 230 across different temperatures is consistent with monotonic tensile tests, where 0.2% yield and ultimate tensile strength is shown in Figure 16a. Haynes 230 is an excellent high temperature strength material with outstanding resistance to oxidation damage in combustion environments. The shape of the yield and ultimate strength is of the utmost importance in understanding how the elastic nature of the material behaves. In our cyclic tests, the elastic domain and stiffness given by the parameters of yield stress, \(\sigma_o\), and the elastic modulus, \(E\), respectively follows the same general trend as shown in Figs. 16a-b. The yield stress, \(\sigma_o\), denotes the elastic domain of linearity of the hysteresis cycle while the elastic modulus, \(E\), represents the initial stiffness. The elastic nature of Haynes 230 is temperature dependent, meaning that the yield stress along with the elastic modulus of the material progressively diminishes with increasing temperature. Another important feature of Haynes 230 is the almost non-existent yield domain seen at higher temperatures due to the dominating viscoplastic response of the material as can be seen by the monotonic tensile properties of Fig. 16a.
Fig. 16. Haynes 230 temperature dependence of the material for (a) monotonic tensile properties of 0.2% yield stress and ultimate tensile strength, and (b) cyclic elastic modulus.

Fig. 17. (a) HA 230 plastic hysteresis cycle (982°C) showing a physically defined viscous stress $\sigma_v$, and (b) plastic strain accumulation corresponding to the creep overstress viscous rounding.
3.4. Haynes 230 Viscous Characterization (Groups 1-3)

At higher temperatures, the viscoplastic nature of Haynes 230 becomes progressively important. Not only do we see strain rate sensitivity of the material (Section 3.1.3), but also we see dominated viscous creep behavior of the material that influences the underlining damage mechanisms. At temperatures above and at 871°C (1600°F), Haynes 230 shows significant viscous creep behavior whereby hysteresis peaks begin to round as well as show a defined viscous domain that is significant for modeling. One such example is at 982°C (1800°F), for an experiment conducted at 20 cpm at a strain range of 0.4%. The hysteresis behavior of the cycles shows viscous creep as seen in Figure 17a, where the peaks begin to round significantly. Viscoplastic hysteretic cycle, Fig. 17a, shows the absence of an elastic yield domain and the unified deformation of plasticity and creep. In Fig. 17b, total mechanical strain vs. plastic strain can be shown to see the absence of an elastic, yield domain indicating that the material at and above 871°C (1600°F) is fully characterized by viscoplasticity. The viscous domain of the material can be defined by looking at Figs. 17a and 17b where the point A \( \left( \varepsilon^p, \sigma_p \right) \) on Fig. 17a corresponds to point A' \( \left( \varepsilon^p, \varepsilon_{\text{max}} \right) \) where the total strain is at a maximum before the cyclic reversal shown in Fig. 17b. The next set of points needed to define the viscous domain is point B \( \left( \varepsilon^p_{\text{max}}, \sigma_{\varepsilon^p_{\text{max}}} \right) \) on Fig. 17a, where the plastic strain has reached a maximum while the total strain has decreased due to the cyclic reversal corresponding to point B' \( \left( \varepsilon^p_{\text{max}}, \varepsilon \right) \) on Fig. 17b. The creep overstress defining the viscous rounding is the stress difference between point A on Fig. 17a, where the total strain is a maximum, and point B on Fig. 17a where the plastic strain has reached a maximum.

This phenomenon was first observed in Pritchard and Hassan [40] for physically defining the viscous domain of Alloy 617. The viscous domain can be physically quantified experimentally, given that the plastic strain steadily accumulates upon a reversal to a maximum even though the total mechanical strain is decreasing in magnitude. This plastic strain accumulation to a maximum peak for decreasing total strain is the inherent time-dependent viscous nature the material undergoes. As a result, this directly corresponds to the viscous rounding manifested in the hysteresis cycles, shown in Figs. 18a-b. In terms of
modeling, this identification of the viscous domain enables one to precisely link the parameter determination of a unified constitutive model describing viscoplastic flow for plasticity and creep. Another advantage of this novel identification procedure is that in terms of the elastic domain the yield stress should be physically representative of the material because, in modeling for creep, standard creep tests at a particular temperature and stress state might show creep; however, if the yield domain and the viscous domain are not properly identified, then stresses below an improper yield domain would not produce creep in terms of modeling for a unified constitutive model. The physical understanding of the material enables better modeling and proper identification of damage mechanisms.

3.5. Effect of hold time on hysteresis responses (Group 4)

3.5.1. Creep-fatigue interaction background

The introduction of a hold time in a strain controlled, low cycle fatigue test causes stress relaxation for viscous materials which can lead to creep-fatigue interaction [8, 16]. This creep-fatigue interaction occurs because a dwell period at a strain peak for viscous materials can cause a material to inelastically relax. The cycling of strain inherently gives fatigue-type damage; however, with the introduction of a hold time, like Group 4 experiments of Table 1, creep-type damage can be induced in the material leading to cavities in the microstructure as grain boundary cavitation and oxidation is initiated [8]. The inelastic relaxation during a strain dwell can lead to severe cavitation damage indicative of creep for long dwell periods. As a result, creep-fatigue interaction occurs when the rate of damage accumulation under a complex loading, like the strain-controlled cycling with dwell periods (Group 4 of Table 1) at strain peaks that leads to a combination of damage from both creep and fatigue. The simultaneous interaction of creep and fatigue and the influence of viscosity at different temperatures was a problem we wanted to understand through our Group 4 experiments. Isothermal stress relaxation experiments were vital in the development of a robust constitutive model as well as provide a lead-up to the more complex thermo-mechanical experiments shown in Ahmed et al. [36].
Group 4 experiments for creep-fatigue interaction were not conducted until failure, but rather to around 600 cycles, due to the saturation of responses shown in Figs. 13a-13c and for economy. While, creep-fatigue interaction experiments conducted in Group 4 were not carried out to failure, it is established that, compared to continuous cycling, the imposition of a hold time at the peak strain, compressive or tensile, decreases the fatigue life [4]. Rodriguez et al. [4] showed that microstructurally creep and plasticity interactions happen because of similar dislocation mechanisms leading to vacancy diffusion and dislocation glide. It was also shown that during a creep-fatigue experiment (strain-dwells) time-recovery effects occur in the material corresponding to a slow restoration of the crystalline structure. As a result, this static recovery (slow restoration) nature of the microstructure can influence the material’s viscosity and macroscopically lead to stress relaxation for creep-fatigue tests. Static recovery in constitutive modeling is typically done through the kinematic hardening rule [38]. The quantification of stress relaxation under strain holds is critical in terms of creep-fatigue damage analysis of the actual component. To investigate the creep-fatigue damage, Group 4 experiments were conducted by imposing compressive strain holds (Table 1). Compressive dwells were chosen because service conditions of combustor liners cause the liner material to be subject to thermally induced compressive strains with dwell periods. Finally, temperatures for the isothermal creep-fatigue experiments were in the range 649-982°C (1200-1800°F). These temperatures were selected as this was the most relevant regime for investigating the viscosity of Haynes 230 and understanding the creep-fatigue interaction.

3.5.2. Creep-fatigue hysteresis characteristics

Figs. 18-19 show cyclic responses for the isothermal creep-fatigue experiments for two different hold times (60, 120 seconds) at temperatures 649-982°C. The examination of the hysteresis responses for each temperature at different hold time uncovers some unique observations. First, a degree of stress relaxation is seen at all temperatures tested in Group 4 (649-982°C), despite previously being determined in Section 3.1.3 that the material behaves rate-independently (viscously minimal) for temperatures below and including 760°C with respect to the loading rate. However, in the creep-fatigue testing at 649°C in Figs. 18a-b and
at 760°C in Figs. 18c-d the material viscously responds and stress relaxation develops during the strain-dwells for both dwell periods (60, 120 seconds). Stress relaxation in the cyclic response for a compressive strain hold is manifested as an upward differential stress at the imposition of the strain hold. However, in comparison, Figs. 18a-b at 649°C shows significantly less stress relaxation at the strain peaks for both hold times than Figs. 18c-d at 760°C. The appearance of serrations indicative of dynamic strain aging, in the temperature domain of occurrence, can be observed in the initial cycles shown in the hysteresis responses of Figs. 18; however, as in continuous LCF testing, these serrations disappear at higher cycles even for creep-fatigue testing. In Figs. 18a-b, at 649°C Haynes 230 shows significant cyclic hardening. The stress relaxation also increases with cycling in Figs. 18a-b at 649°C as evident by the minimal stress relaxation in the initial cycle of Fig. 18a; but by the 600th cycle considerably more stress relaxation has developed. In Figs. 18c-d, at 760°C Haynes 230 again shows cyclic hardening, but much less compared to 649°C, which is consistent with Figs. 10b-c.

The stress relaxation of 760°C for both hold times shows an increase with cycles in Figs. 18c-d. The cyclic creep-fatigue responses for both hold times are shown for the rate-dependent temperatures 871°C and 982°C in Figs 19a-d. In the rate-dependent temperature regime stress relaxation magnitudes increase significantly. In general, from Figs. 18-19 the stress relaxation nature of Haynes 230 increases with temperature; however, the magnitudes of stress relaxation for the rate-dependent temperatures (871 and 982°C) of Figs. 19 are significantly larger than the rate-independent temperatures (649 and 760°C) of Figs. 18. In Figs. 19a-b, at 871°C Haynes 230 again shows a degree of cyclic hardening consistent with Fig. 13b for higher strain rates. In Figs. 19c-d, at 982°C where cyclic softening ensues, one can discover the viscous nature caused by the elevated temperature. The viscosity of the material is reflected in the rounding of the hysteresis before the strain reversal. At 982°C, in Figs. 19c-d a tensile overshoot is also seen in the hysteresis loops. A tensile overshoot manifests itself in the hysteresis curve as a viscous peak rounding followed by a subsequent softening seen in cycle 500 of Fig 19c. This tensile overshoot happens after the strain hold, but during the increase in strain corresponding to the upgoing portion of the hysteresis cycle.
before the final strain reversal. However, at 982°C, as the material softens and cycles progress, the viscous rounding at the tensile peak becomes less pronounced and the tensile peak becomes more or less flat for a few strain increments before the strain reversal, which can be seen by cycle 200 of Fig 19c. This flattening out of the tensile peak with a constant stress value is indicative of a creep type experiment whereby the stress is held constant and the strain accumulates.

3.5.3. Creep-fatigue cyclic evolution

The cyclic evolution for the isothermal creep-fatigue experiments for both hold times (60 and 120 seconds) are compared to the LCF experiments without a hold time for the temperatures 649-982°C in Figs. 20a-d. In Figs. 20a-d, the cyclic stress amplitude and mean stress are plotted at a particular temperature for various hold times; including the LCF experiments of Group 2 at the same strain range with a zero hold time (continuous cycling). Aside from the difference in cyclic stress amplitude response at 649°C of Fig. 20a, all other temperatures in Figs. 20b-d showed similar cyclic evolution for stress amplitudes when comparing the effect of a hold time. In Fig. 20a, at 649°C the presence of a strain dwell (hold time) at the compressive peak resulted in a higher cyclic stress amplitude evolution. However, the difference between the stress response in Fig. 20a, once a hold time is considered for creep-fatigue cycling, results in similar magnitudes in stress amplitude irrespective of the hold time (60 or 120 seconds). It is interesting to notice however that a non-zero mean stress evolves with cycles for the creep-fatigue tests of Group 4 with the introduction of a hold time in Figs. 20a-d. On the contrary, LCF tests of Group 2 experiments without a hold time show nearly zero mean stresses. As a result, the introduction of a hold time for creep-fatigue testing introduces a nonzero mean stress in the cyclic response. This suggests that mean stress evolution is a time-dependent phenomenon. The evolutionary behavior of mean stress with cycles was reported by Yaguchi et al. [27], who observed the evolution in mean stress for isothermal experiments with both compressive and tensile holds of a nickel-based superalloy IN738LC.
Fig. 18. Haynes 230 hold time dependence upon cyclic stress evolution, specifically hysteretic behavior of the cycles as well as the manifestations of the stress relaxation at a strain range and temperature corresponding to (a) T: 649°C, $\Delta \varepsilon=0.76\%$ with a hold duration of $t_H = 60$ seconds, (b) T: 649°C, $\Delta \varepsilon=0.76\%$ with a hold duration of $t_H = 120$ seconds, (c) T: 760°C, $\Delta \varepsilon=0.64\%$ with a hold duration of $t_H = 60$ seconds, and (d) T: 760°C, $\Delta \varepsilon=0.64\%$ with a hold duration of $t_H = 120$ seconds (1MPa=0.145ksi)
Fig. 19. Haynes 230 hold time dependence upon cyclic stress evolution, specifically hysteretic behavior of the cycles as well as the manifestations of the stress relaxation at a strain range and temperature corresponding to (a) T: 871°C, $\Delta\varepsilon=0.53\%$ with a hold duration of $t_H = 60$ seconds, (b) T: 871°C, $\Delta\varepsilon=0.53\%$ with a hold duration of $t_H = 120$ seconds, (c) T: 982°C, $\Delta\varepsilon=0.39\%$ with a hold duration of $t_H = 60$ seconds, and (d) T: 982°C, $\Delta\varepsilon=0.39\%$ with a hold duration of $t_H = 120$ seconds (1MPa=0.145ksi).

Another unique observation in Figs. 20a-d is that the mean stress is generally slightly larger for the 120s hold compared to the 60s hold experiment. The development of mean
stresses as a time-dependent phenomenon showing up for strain-dwells with varying magnitude, depending on the hold time, will be critical in thermo-mechanical fatigue responses of Haynes 230 presented in our sister paper Ahmed et al. [36].

3.5.4. Creep-fatigue stress relaxation

The Haynes 230 material, which experiences cyclic hardening for temperatures below 871°C and cyclic softening at 982°C, undergoes increased relaxed stresses as the cycles evolve for all temperatures. Naturally, one would assume that an increase in the relaxed stress would be directly correlated to the temperature of the creep-fatigue interaction; however, it can be shown that the cyclic hardening and/or softening evolutions of the material state also influences the relaxed stresses during cyclic deformation. Figure 21 shows the evolution of Haynes 230 stress relaxation with cycles during compressive hold of 60 and 120s at four different temperatures ranging from 649 to 982°C (1200 to 1800°F). The relaxed stresses have been determined by calculating the change in stress values during the compressive hold for each cycle discussed in the quantification Section 2.2. Figure 21 shows that the magnitude of stress relaxation increases with increase in temperature from 649 to 871°C (1200 to 1600°F), but decreases with increase in temperature from 871 to 982°C (1600 to 1800°F). The increase in stress relaxation from 649 to 871°C (1200 to 1600°F) is caused by the increase in viscosity with temperature. However, the decrease in stress relaxation from 871 to 982°C (1600 to 1800°F) can be explained by the lower stress amplitudes at 982°C (1800°F) due to the cyclic softening behavior seen in Fig. 20d of 982°C. The introduction of a longer hold time from 60s to 120s does cause higher stress relaxation for all temperatures (649-982°C) with the largest differences seen for the rate-independent temperatures (649°C and 760°C) compared to the rate-dependent temperatures (871 and 982°C) in Fig. 21.
Fig. 20. Haynes 230 hold time dependence upon cyclic stress evolution, with stress amplitude and stress mean responses for Group 4 experiments and Group 2, as a function of cycles: (a) T: 649°C, $\Delta\varepsilon=0.76\%$, (b) T: 760°C, $\Delta\varepsilon=0.64\%$, (c) T: 871°C, $\Delta\varepsilon=0.53\%$, and (d) T: 982°C, $\Delta\varepsilon=0.39\%$ for various hold times $t_h = 0, 60, 120$ seconds (1MPa=0.145ksi)

Another interesting observation in the relaxed stresses for various hold times (60, 120s) at a particular temperature shows a cyclically evolving nature similar to the type of cyclic nature seen in Figs. 20a-d. In particular, the rate-independent temperatures (649°C and
760°C) of Fig. 21 show that the relaxed stresses nonlinearly evolve similarly to the cyclic hardening nature of Figs. 20a-b. In addition, relaxed stresses of the rate-dependent temperatures (871 and 982°C) also evolve in a consistent manner with respect to the type of cyclic nature seen in Figs. 20c-d respectively. The general trend of the stress relaxation in Fig. 21 for each temperature can be seen to increase with cycles in a similar manner to the cyclic nature of the stress response of Figs 20a-d. At each temperature the stress relaxation can be seen to evolve and finally reach a saturated value. In order to quantify the stress relaxation further for various temperatures tested, we have calculated a normalized relaxation stress at each cycle. The normalized relaxed stresses is quantified by scaling the relaxed stress $\Delta \sigma_{x_r}$ by the stress amplitude of each cycle to further illustrate the effect of temperature on the viscous behavior, as shown in Fig. 22. Figure 22 shows that with increase in temperature there is an increase in the normalized stress relaxation. In Fig. 21, we saw that the relaxed stresses of 982°C were lower than both 760°C and 871°C. However, when one normalizes the effect of the cyclic evolution for either cyclic hardening (649, 760, 871°C) or cyclic softening (982°C), the viscosity of the material shows a consistent trend with respect to temperature. Figure 22 shows that the viscosity of the material increases with increase in temperature from 649 to 982°C (1200 to 1800°F). The effect of the hold time is again seen in Fig. 22, whereby an increasing dwell time results in higher relaxed stresses.

The evolution of the relaxed stresses as a function of time at hold is of great significance due to the influence of stress relaxation in the creep-fatigue life of actual in-service components. The relaxation behavior directly gives us important information regarding the viscoplastic or time-dependent nature of the material in question. In Figs. 23a-c, the general trend for the relaxation curves for temperatures 649-871°C suggests that the relaxation rates and magnitudes increase with increasing cycles. As a result, as shown in Figs. 23a-c, the first cycle has the least relaxation, while the final cycle has the most. At 649°C, in Fig. 23a, the relaxation rate and magnitudes for all cycles are lower than compared to Figs. 23b-c. In Fig. 23d, at 982°C, where Haynes 230 exhibits cyclic softening, the relaxation curves show that the highest relaxation occurs at the first cycle and the subsequent cycles have lower stress relaxation. In Figs. 23a-d, relaxation curves are shown for hold times of 120 seconds; the
same general trends develop for the lower hold time of 60 seconds. In summary, at the higher temperatures (871-982°C) for which we know the material behaves in a viscous manner, as expected we see significant stress relaxation with hold times. At the lower temperatures of 649 and 760°C, despite the material response being mostly rate-independent with respect to different loading strain rates, we still see a considerable amount of stress relaxation. These experiments reveal some of the material complexities of Haynes 230 even at isothermal temperatures. The combined effects of time-dependent processes such as creep, fatigue, dynamic strain aging, and cyclic relaxation inherently control the structural responses a combustor liner would incur.

Fig. 21: Isothermal creep-fatigue stress relaxation ($\Delta \sigma_{xr}$) responses of Haynes 230 at four temperatures (T) of 649, 760, 871 & 982°C, and two strain holds ($t_H$) of 60 & 120s ($1\text{MPa}=0.145\text{ksi}$)
3.6. Low cycle fatigue life (Groups 1-3)

3.6.1. Effect of mean strain

Figure 24, which expresses the fatigue life as a function of the total strain range, can be used to understand how temperature, total strain range, and the imposed strain ratio influence the fatigue life. Figure 24a shows fatigue lives for various temperatures and strain ranges at a strain ratio, $R = -1$, corresponding to a zero mean strain of Group 1 experiments in Table 2. Figure 24b shows fatigue lives for various temperatures and strain ranges at a strain ratio, $R = 0$, corresponding to a non-zero mean strain of Group 3 experiments in Table 2. In general, the fatigue life decreased with increasing temperature for both strain ratios in Figures 24a-b. Another commonality between the two groups ($R = -1$) and ($R=0$) is that, due to the exponential decay nature of the fatigue lives, a two-slope behavior is seen at a particular temperature as shown for 649°C in Figs. 24a-b. In lifing analysis, the two-slope behavior is an indicator of the plastic regime mostly associated with low-cycle fatigue (LCF) and an elastic regime often attributed to high-cycle fatigue (HCF) behavior.
Fig. 23. Evolution of stress relaxation ($\Delta \sigma_{xr}$) with time at hold ($t_H$) for isothermal Group 4 creep-fatigue experiments of Haynes 230: (a) $T$: 649°C, $\Delta \varepsilon = 0.76\%$, $t_H = 120$ s, (b) $T$: 760°C, $\Delta \varepsilon = 0.64\%$, $t_H = 120$ s, (c) $T$: 871°C, $\Delta \varepsilon = 0.53\%$, $t_H = 120$ s, and (d) $T$: 982°C, $\Delta \varepsilon = 0.39\%$, $t_H = 120$ s (1MPa=0.145ksi)
The high negative slope corresponds to the plasticity regime of LCF, while the more stabilized (flat) slope corresponds to the elasticity regime of HCF. As a result, above a certain strain range and at a particular temperature domain, lives differed minimally as a result of the high negative slope for the plasticity regime of LCF. On the contrary, below this strain range and at a particular temperature domain an asymptotic, stabilized small slope accounts for larger differences in life when comparing one strain range to the next for the elasticity regime of HCF. For example, at 649°C at a strain range of 1.0% in the LCF plasticity regime of the fatigue curve, the life for either strain ratio of Figs. 24a-b was in the range of 2500 cycles. However, at a strain range of 0.4%, again for 649°C, where now HCF elasticity is more representative of the fatigue, the life for either strain ratio of Figs. 24a-b was in the range of 250,000 cycles. This large difference in life (2,500 LCF to 250,000 HCF regimes) from one strain range to the next demonstrates the damaging effects of plasticity, whereby significant irreversible hysteresis for LCF is much more detrimental than the elastic and minimal hysteresis of HCF.

Fig. 24. Haynes 230 fatigue life as a function of the total strain range at different temperatures for a strain ratio, R = -1 (zero mean strain), and (b) a strain of R = 0 (non-zero mean strain).
The transition between LCF to HCF can be established by characterizing the exponential nature of the curve through a strain range threshold. For both Group 1 (zero mean strain) and Group 3 (non-zero mean strain), at temperatures between 24-427°C (75-800°F), this strain range threshold occurs around $\Delta \varepsilon = 1.0\%$. For temperatures above 427°C (800°F) and below 871°C (1600°F) this strain range threshold occurs around $\Delta \varepsilon = 0.60\%$, while at temperatures between 871-982°C (1600-1800°F) it occurs at about $\Delta \varepsilon = 0.40\%$. By comparing the smooth trend patterns of both groups in Figs 24 a-b, it is clear that no noticeable differences seem to manifest themselves, suggesting that a nonzero mean strain has a negligible effect on the fatigue life. This is consistent with the hysteresis responses examined earlier for which the mean strain posits minimal variation for the Haynes 230 material.

### 3.6.2. Effect of strain rate

In order to assess the influence of strain rate on fatigue life, one can characterize the rate effects of the material by exploring the tests conducted in Group 2. Sensitivity of the strain rate and the temperatures is illustrated in Fig 25, in which the fatigue life as a function of the temperature for different cyclic frequencies is shown. Figure 25 suggests that a continuous decrease in cyclic frequency, i.e. strain rate, at an isothermal temperature results in the reduction of fatigue life. For example, at room temperature (24°C), from the highest cyclic frequency (20 cpm) tested to the lowest cyclic frequency (0.2 cpm), the fatigue life reduces from around 26,000 cycles to 1,200. This steady decrease in fatigue life as the loading (strain) rate is increased occurs for all temperatures (24-982°C). For the lowest cyclic frequency (0.2 cpm) the fatigue life more or less shows a stabilized response, indicating that irrespective of temperature the tests resulted in similar fatigue lives, thereby showing the importance of the strain rate.

Another unique observation relates to the temperature dependence between 24-204°C (75-400°F), where the fatigue life increases with increasing temperature for all three rates tested. For example, in Fig. 25, going from 24°C to 204°C the fatigue life increases from
around 16,000 cycles to 25,000 cycles for the 2 cpm tests. However, between 204-649°C (400-1200°F), an increase in temperature resulted in a steady monotonic decrease in fatigue life for all three strain rates. Eventually, the fatigue life had relative magnitudes for the temperature domain 649-982°C (1200-1800°F) where the variation in fatigue life from one temperature to the next is minimal for all three strain rates.

![Fig. 25. Haynes 230 fatigue life as a function of temperature for various strain rates (cyclic frequencies)](image)

### 3.6.3. Fatigue damage

The fatigue damage can be quantified further than the traditional lifing plot of Fig. 24 by computing the energy dissipated during the hysteretic cycle. The energy dissipated during the cycle mathematically correlates to the area enclosed under the hysteresis curve. This method was first suggested by Morrow [41], whereby the fatigue damage was quantified for the area under a stress versus plastic strain curve of each hysteresis cycle. The energy can be computed per hysteresis cycle and accumulated to obtain the total amount of energy dissipated up to a particular cycle. This cumulative area enclosed within the hysteresis loops gives a measure of the total energy absorbed.

Different temperatures for a particular strain range were organized in order to understand the effect of temperature on the calculated total cumulative energy dissipated during the low cycle fatigue experiments shown in Figure 26. There, for comparison purposes, the amount of cycles shown are to 1000 rather than to failure, since increasing the temperature
significantly reduced the fatigue life. The amount of total cumulative energy dissipated during the low cycle fatigue is closely related to the cyclic stress evolutions and the types of hardening and/or softening responses reflected in the hysteresis curves of Figs. 6-9, 10, 11. It was shown in Section 3.1.2, for the various cyclic hardening-softening evolutions across different temperatures, that the hysteretic behavior of the low cycle fatigue experiments changes. In Figure 26, the variation in the nearly linear cumulative energy lines for various temperatures shows some of these consistent trends regarding the cyclic hardening nature of the material across different temperatures. The highest bounded energy line in Fig. 26 is room temperature (24°C) as the material shows hardening to a peak followed by saturation in Fig. 10b, where the majority of the cyclic life is spent at or near the peak hardening response. The cyclic hardening evolutions, such as those in Fig. 10b, influence both the widths and stress peaks of the hysteresis cycles at each temperature. In addition, the widths of the hysteresis cycles as well as the peaks also influence the area calculation for energy. As a result, the cumulative energy absorbed for the low cycle fatigue life is dependent on the cyclic stress evolution patterns. For 760°C (1400°F) the cyclic hardening behavior was relatively slower compared to the lower temperatures, such as 427°C (800°F) or 649°C (1200°F), which experienced similar cyclic hardening behavior shown in Fig. 10b. As a result the hysteresis loop widths for 760°C were much larger than for 427°C (800°F) or 649°C (1200°F), such that the cumulative energy line for 760°C is greater than both 427°C and 649°C.
Fig. 26. Haynes 230 fatigue damage as a function of temperature, $\Delta \varepsilon = 0.8\%$ of Group 1

In terms of the effect of the strain range, Fig. 27 shows that for larger strain ranges more energy is dissipated, since larger inelastic strain ranges are being produced correlating to higher cumulative energy. Also in Fig. 27, the effect of a hold time is shown comparing 60 seconds to 120 seconds compressive holds of Group 4 experiments. In Fig. 27 no consistent trend develops for the cumulative energy lines. This lack of trend between hold times for a particular temperature and strain range suggests that, irrespective of the hold time, similar hysteresis shapes were produced as was seen in Figs. 18-19. In Figs. 18-19, the amount of stress relaxation did increase with increasing hold time; however, the overall shape of the hysteresis curve did not change much at each temperature and thus in the calculation of the cumulative area for each hold time (60 or 120 seconds) not much differences develop in Fig. 27.
4. Conclusions

Mechanical testing is an irreplaceable, economic medium that allows for constitutive model development, which is integral in the design-by-analysis of high temperature systems ranging from aeronautical applications and land-based power plants to the automotive industry. In our paper, we have looked at service-like conditions of a combustor liner fabricated from sheets of Haynes 230 used in airplane gas-turbine engines. The reduction of actual real service conditions experienced by the combustor liner to the mechanical testing level of Haynes 230 is a challenge. However, we have tried to address the service-like conditions as closely as possible to ensure robust constitutive model development. A large set of isothermal and anisothermal experiments were carried out on Haynes 230, in an effort to understand its high temperature fatigue constitutive response. The responses from isothermal experiments, Groups 1-4 in Table 2, were presented in this paper, while the anisothermal experiments are presented in the sister paper Ahmed et al. [36]. In Groups 1-4, Haynes 230 was tested for low-cycle fatigue and creep-fatigue across a range of temperatures, strain ranges, and strain rates.
The material behavior of Haynes 230 was complex. The isothermal cyclic stress evolution of the material was both temperature and strain range dependent. A degree of cyclic hardening-softening behavior was seen for temperatures 24-204°C depending on the imposed strain range. Strong cyclic hardening of the material without any sign of stabilization was seen for temperatures 204-649°C. For 649-871°C, the rate of cyclic hardening is gradually reduced; however, again the material shows a lack of stabilization until failure. Above 871°C, the material shows a cyclic softening behavior, where loading rate effect is also observed. Strain rate sensitivity of Haynes 230 was shown to be rate-independent below 760°C (1400°F), with rate-dependent behavior seen above. Negative strain rate sensitivity was observed from 427-760°C (800-1400°F); however, for practical purposes the material was assumed to be rate-independent. Also, interestingly enough stress relaxation experiments (Group 4) for strain dwells show stress relaxation at temperatures 649°C and 760°C (1200 and 1400°F) where the material was assumed to be rate-independent due to the presence of negative strain rate sensitivity. The presence of stress relaxation at these temperatures (649°C and 760°C) suggests the need of static recovery in a constitutive model. The stress relaxation behavior of the material is highly temperature dependent, with increasing viscosity shown for increasing temperatures.

The phenomena of dynamic strain aging manifests itself macroscopically as strong cyclic hardening, negative strain rate dependency, and serrated yielding from 427-760°C (800-1400°F). This broad temperature band adds material complexity and modeling challenges. The elastic and viscous nature of the material as a function of temperature shows consistent trends with monotonic tensile tests. The proper identification of the elastic and viscous domain is critically important for robustness in the model. The viscoplasticity nature of Haynes 230 is emphasized by showing the creep-type behavior seen in the hysteresis responses of creep-fatigue experiments. Novel ways of analyzing this behavior are shown in physically understanding the plasticity and creep interactions of Haynes 230. Some of these novel characteristics included normalized stress relaxation with cycles along with relaxation rate curves with time for various cycles.
The introduction of a dwell period at the compressive strain peak introduces not only a stress relaxation, but a nonzero mean stress in the cyclic response. This suggests that mean stress evolution is a time-dependent phenomenon. The presence of a mean stress is of paramount importance in the sister paper of anisothermal experiments presented in Ahmed et al. [36].

In terms of life prediction, traditional lifing curves have been shown. The lifing curves for Groups 1 and 3 were shown characterizing both the plasticity regime (LCF) and elasticity regime (HCF) of the fatigue plot. The total energy dissipated given by the accumulated hysteresis area shows the linear summation type behavior of damage across different temperatures. The life prediction of fatigue requires fidelity in stress and strain responses. This paper has tried to understand a complex material, Haynes 230, under broad loading conditions, with the aim of constitutive model development. The experimental validation of an advanced constitutive model should help in substantially improving current design-by-analysis methodologies.

5. **Acknowledgements**

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Turbo Expo*.


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Hold time effects on low cycle fatigue behavior of HAYNES 230® superalloy at high


CHAPTER 3: OUT-OF-PHASE AND IN-PHASE HIGH TEMPERATURE THERMO-MECHANICAL FATIGUE-CREEP OF HAYNES 230

Abstract

Service temperatures of airplane gas turbine engine combustor components can be as high as 982°C (1800°F). This induces a thermo-mechanical fatigue (TMF) loading, which as a result of dwell periods and repetitions eventually leads to failure of the components via creep-fatigue processes. A large set of isothermal and anisothermal experiments have been carried out on Haynes 230, in an effort to understand its high temperature fatigue responses. Isothermal experiments at different loading strain rates show that the material can be considered to be rate-independent below and at 760°C (1400°F). However, isothermal strain hold experiments show stress relaxations at 649 and 760°C (1200 and 1400°F). TMF experiments were conducted to replicate actual service conditions. The out-of-phase strain-controlled TMF experiments with a compressive dwell period show a positive mean stress evolution response, while the in-phase TMF experiments with a tensile dwell-period strain show a negative mean stress evolution response. The cyclic hardening response was found to depend on the maximum temperature in the loading cycle. In the calculation of the inelastic strain in the TMF experiments, it was found that the time derivative of the elastic modulus needed to be considered to prevent anomalous shifting of the hysteresis loops. The stress relaxation for thermo-mechanical fatigue experiments in general decreased with cycles, while that for isothermal experiments with dwell periods increased with cycles. The inelastic strain rate behavior during the relaxation phase indicated that traditional flow rules such as Norton’s power law may be used in a chosen constitutive model. The fatigue lives of TMF experiments were adversely affected by higher maximum temperatures and longer hold times.

Keywords: Haynes 230, thermo-mechanical fatigue, stress relaxation, fatigue life
1. Introduction

Nickel-base superalloys have been favored in the high temperature service zones of gas turbine engines owing to their excellent mechanical properties at elevated temperatures. The particular nickel-base superalloy of interest in the current research is Haynes 230, which is used to manufacture combustor liners of airplane gas turbine engines. Turbine engine operation can cause combustor components to be subject to thermo-mechanical cyclic loading with temperature fluctuating between ambient temperatures to as high as 982°C (1800°F). Service conditions lead to the creation of “hot spots”, which are areas of considerably higher temperature than surrounding areas. The geometry of the structure results in constraints that lead to compressive strains in the vicinity of the hot spot as the hot spot tries to expand. Thus, in combustor liners out-of-phase (OP) thermo-mechanical fatigue (TMF) is the phenomenon of primary interest, which is the increase in compressive strain with increase in temperature. Strain-controlled TMF experiments are known to represent the service conditions of combustor liners most closely [1]. Service conditions in liners lead to temperature gradients that result in an essentially strain–controlled load cycle [2, 3]. Airplane flight causes the turbine engine materials to be repeatedly subject to dwell periods at the compressive peak of the thermo-mechanical fatigue loading. This leads to phenomena such as creep-fatigue interaction, which is a life-limiting factor through processes of crack initiation, propagation and failure.

High temperature components in the aerospace industry have been known to fail earlier than the predicted life. As a result there is a need to investigate the behavior of the high temperature application materials under service-like conditions. A comprehensive understanding of the cyclic stress-strain responses during service is essential for the development of a reliable robust constitutive model to predict the same. A reliable robust constitutive model is in turn essential to develop reliable life prediction techniques for critical high temperature components in the aerospace, nuclear power, chemical and automobile industries.

The first step towards the development of a robust constitutive model is a comprehensive understanding of material behavior under various loading conditions and across the entire
temperature spectrum encompassing the operating service conditions. This necessitates the existence of a broad experimental database to characterize the material response and ensure experimental validation of any chosen constitutive model. Many researchers have investigated the isothermal fatigue response of nickel-base alloys. Experimental study has been conducted on the fatigue life [4, 5, 6], hardening-softening [6, 7, 8], viscosity [3, 7], dynamic strain aging [9], and negative strain rate sensitivity [7, 10, 11, 12, 13]. For the particular case of Haynes 230, Lu et al. [8] studied the effect of hold times (dwell periods) on the fatigue lives at high temperatures. Hasselqvist [14, 15] attributed the significant cyclic hardening of Haynes 230 at higher temperatures to the precipitation of secondary carbides on dislocations at temperatures higher than 550°C. However, only a few researchers have investigated the thermo-mechanical fatigue response of nickel base alloys [2, 16, 17, 18, 19, 20, 21]. The maximum temperatures of these thermo-mechanical fatigue studies ranged from 800 to 1200°C under varying conditions of out-of-phase and in-phase types of loading. All of these were pure thermo-mechanical fatigue experiments with no dwell periods. Of these only Affeldt et al. [2] investigated Haynes 230 for a single in-phase TMF test with minimum and maximum temperatures of 300 and 850°C respectively. Thus, a broad experimental database needed to characterize the behavior of Haynes 230 under service-like conditions does not exist.

In the present study, with the goal of developing a broad experimental database, a large number of experiments have been carried out. A sister paper, Barrett et al. [22], presented results from over 120 isothermal fatigue & creep-fatigue experiments at seven different temperatures ranging from 24 to 982°C (75 to 1800°F). The isothermal experiments were necessary to accurately validate the robustness of a developed material model. Service conditions of the combustor liner, however, lead to thermo-mechanical loading conditions with hold times at the maximum temperature of operation. In the present paper results from 24 thermo-mechanical fatigue experiments, with both out-of-phase and in-phase type loading at varying maximum temperatures ranging from 760 to 982°C (1400 to 1800°F), are presented. The minimum temperature for the thermo-mechanical fatigue experiments was 316°C (600°F). Out-of-phase thermo-mechanical fatigue tests with compressive strain
conditions were chosen as they replicate the operating conditions most closely. In-phase thermo-mechanical fatigue tests with tensile strain conditions were also conducted to enhance the robustness of a developed constitutive model.

Experimental evidence shows that the maximum temperature in the loading cycle dominated thermo-mechanical fatigue response [16, 23, 24]. Ohno et al. [23] concluded that the internal change associated with the higher temperature prevails under thermo-mechanical fatigue loading for 304 stainless steel. The internal change of a material occurs more markedly at the higher temperatures, and the internal change is not recovered markedly at lower temperatures. In the study of nickel-base alloy NiCr_{22}Co_{12}Mo_{9}, Kleinpass et al. [16] supported these findings and came to the conclusion that the microstructural changes can be different under isothermal and thermo-mechanical fatigue.

Lu et al. [8], in their microstructural study of the isothermal low cycle fatigue behavior of Haynes 230 at two temperatures of 816 and 982°C, concluded that an increase of hold time and/or increase in temperature tends to change the fracture mode from transgranular to mixed, and then to intergranular. Intergranular cracking is known to be a manifestation of creep damage. For the nickel-base superalloy René 80, Bhattachar and Stouffer [25] proposed different mechanisms of deformation at different temperature ranges. At temperatures below 700°C the inelastic deformation was proposed to take place primarily by movement of dislocations in slip planes, while between 700 and 1000°C dislocation climb was the proposed mechanism.

The evolution of mean stress with cycles under anisothermal loading conditions has been reported explicitly only in a few studies [1, 16, 21, 24]. All the reported studies are for purely thermo-mechanical fatigue loading without any strain holds. Out-of-phase loading leads to tensile mean stresses, while in-phase loading leads to compressive mean stresses. Kleinpass et al. [16] and Zauter et al. [24] reported steady mean stress for thermo-mechanical fatigue loading in a nickel-base alloy NiCr_{22}Co_{12}Mo_{9} and stainless steel 304L respectively. Zhang et al. [1] observed the progressive evolution of mean stress in the tensile direction for out-of-phase compressive thermo-mechanical fatigue loading in two martensitic steels. Yaguchi et
al. [21] reported the progressive evolution of mean stress for both symmetric out-of-phase and in-phase thermo-mechanical fatigue loading for the nickel-base alloy IN738LC.

Hong et al. [26] explained that out-of-phase thermo-mechanical fatigue experiments result in lower fatigue lives compared to isothermal low cycle fatigue experiments as the maximum tensile loading was attained during the minimum temperature. This resulted in accelerated surface crack initiation and propagation compared to low cycle fatigue conditions. Higher maximum temperatures in the thermal cycle lead to lower fatigue lives [27]. There have been studies showing that actual service components have shorter lives under thermo-mechanical fatigue loading compared to low cycle fatigue loading at comparable temperatures and strain ranges [28, 29].

A broad set of experiments on Haynes 230 (HA230) coupons were carried out as a part of this study. Isothermal experiments at different loading strain rates and compressive hold times have been presented in a sister paper, Barrett et al. [22]. The thermo-mechanical experiments, both out-of-phase and in-phase, are discussed in this paper. The effect of the maximum temperature, strain range, hold time and loading rates on the TMF response is investigated. The thermo-mechanical fatigue lives and comparison with low cycle fatigue lives is also shown.

2. Experimental Study

Strain-controlled experiments have been performed with strain and temperature ranges similar to those estimated in combustor liners during service conditions. Temperature gradients that exist in service conditions of combustor liners cause non-uniform thermal expansion (hot spots) which results in an essentially strain-controlled load cycle [2, 3].

2.1. Thermo-mechanical fatigue test experimental procedure and loading histories

The nickel-base polycrystalline superalloy Haynes 230 was received as bulk rods in solution annealed conditions. The chemical composition of the material is summarized in Table 1. The specimens were machined to a dog-bone shape with a gage length of 0.63 in. and diameter of 0.25 in. at the gage location using a low-stress grinding technique.
Table 1: Nominal chemical composition of Haynes 230 in wt. %

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<tr>
<th></th>
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<sup>a</sup>As Balance  
<sup>b</sup>Maximum

Uniaxial tests have been conducted as combustor liners in service experience primarily uniaxial loading conditions. However finite element analysis has shown that some areas of the liner may experience some amount of multiaxial loading and for that reason a number of multiaxial tests have also been planned and are currently underway.

The thermo-mechanical fatigue (TMF) tests were performed according to ASTM E2368-10 by a test vendor using universal, servo-hydraulic fatigue testing machines. The test parameters and loading sequences were closely coordinated by the investigators. Heating was achieved through an induction heating system with thermocouples aligned for temperature uniformity. Cooling was achieved through an airflow system to ensure the correct temperature gradient.

Both out-of-phase and in-phase thermo-mechanical fatigue tests were carried out. The prescribed thermo-mechanical loading histories are shown in Fig. 1. The strain histories prescribed are the mechanical strains. Thermal strains were considered as follows. The specimen was first held in force control at zero force and the thermal waveform (i.e. temperature vs. time) was imposed. The strain history recorded using the extensometer was the thermal strain history for the temperature cycle. This thermal strain history was superimposed on the prescribed mechanical strain history to determine the total strain to be imposed to the specimen for TMF testing.

The thermal cycle had a minimum temperature of 316°C (600°F) and a maximum temperature between 760 and 982°C (1400°F to 1800°F). The cycle times of the TMF experiments consisted of ramp-up time or heating time (t<sub>H</sub>), a hold time (t<sub>H</sub>), and finally a ramp-down time or cooling time (t<sub>C</sub>) as shown in Fig. 1b. The heating time varied from 55 to 105s; the hold time was either 120 or 1200s; and the cooling time was 90s. Hold times of 120 or 1200s were imposed at the maximum temperature, which coincided with the peak
compressive or tensile strain. The imposed mechanical strain ranges were varied from 0.25 to 0.7 percent.

Fig. 1: Temperature (T) vs. time (t) and strain ($\varepsilon_x$) vs. time (t) thermo-mechanical fatigue loading histories prescribed in the experiments: (a) Out-of-phase and (b) In-phase

2.2. Quantification of responses

Figure 2 shows a typical stress versus strain (mechanical) hysteresis loop response of Haynes 230. The 2nd cycle for an experiment with temperature range of 316 to 927°C (600 to 1700°F), strain range of 0.3% and hold time of 120s is shown. The heating and cooling times are 65 and 90s respectively. The path DCB on the figure corresponds to the heating time shown in Fig. 1a. The path BA corresponds to the hold time and the path AD corresponds to the cooling time. The figure shows the different measures used to quantify the cyclic response. The maximum stress ($\sigma_{max}$) and minimum stress ($\sigma_{min}$) values correspond to the extreme points D and C respectively. The stress range is $\Delta\sigma$. The stress amplitude and mean stress can be calculated as shown in Eqs. 1 and 2. Point B corresponds to the time just before hold and point A corresponds to the time at the end of the hold. The difference between the two is the stress relaxation denoted by $\Delta\sigma_{sr}$. The drop in stress value from the peak stress point C to the stress just before hold (point B) is the thermo-mechanical stress drop denoted by $\Delta\sigma_{sd}^{TM}$. 

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2.3. Thermo-mechanical fatigue experiments conducted

The loading parameters of the thermo-mechanical fatigue experiments conducted are presented in Tables 2 and 3. The loading parameters consist of the minimum temperature ($T_{\text{min}}$), maximum temperature ($T_{\text{max}}$), strain range ($\Delta \varepsilon$), heating time, hold time ($t_{\text{H}}$) and cooling time. Expt. Nos. 1 to 13 are the out-of-phase TMF experiments, while Expt. No. 14 to 17 are the in-phase TMF experiments. Experiments 5 and 6 correspond to two out-of-phase TMF experiments with the same loading parameters, with the exception of the heating time.

\[
\sigma_{xa} = \frac{(\sigma_{\text{max}} - \sigma_{\text{min}})}{2} \equiv \Delta \sigma / 2
\]
\[
\sigma_{xm} = \frac{(\sigma_{\text{max}} + \sigma_{\text{min}})}{2}
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### 3. Isothermal Strain Rate-dependence and Stress Relaxation

The isothermal low cycle fatigue (LCF) experiments have been presented in detail in a sister paper, Barrett et al. [22]. Relevant isothermal experimental results have been briefly presented in this paper to facilitate understanding of the thermo-mechanical fatigue results. Isothermal low cycle fatigue tests with strain controlled loading histories were performed at various strain rates and then with 60 or 120s hold times (dwell periods) at a constant strain rate. The loading waveform was triangular for experiments without hold times (Fig. 3a), and trapezoidal for experiments with hold times (Fig. 3b). Three loading strain rates of 0.2 cpm
(cycles per minute), 2 cpm and 20 cpm were prescribed in the experiments at temperatures ranging from 24 to 982°C (75 to 1800°F). The imposed strain ranges varied from 0.3 to 1.6%.

3.1. *Isothermal low cycle fatigue (LCF) strain rate effect*

Responses from isothermal uniaxial cyclic strain-controlled low cycle fatigue experiments at different loading strain rates were used to investigate the effect of viscosity at each temperature in the range 24 to 982°C (75 to 1800°F). The prescribed loading rates of 0.2, 2 and 20 cpm correspond to strain rates in the order of $10^{-5}$, $10^{-4}$ and $10^{-3}$ per second respectively.

![Fig. 3: Isothermal low cycle fatigue with symmetric axial strain cycling, (a) without hold, (b) with compressive hold time of $t_H$](image)

The experimental responses of axial stress amplitudes at four temperatures from 649 to 982°C (1200 to 1800°F) and three loading rates at each temperature are plotted in Fig. 4. The experimental responses at temperatures lower than 649°C (1200°F) have been presented in Barrett et al. [22]. From these experimental responses it was concluded that for temperatures up to and including 760°C (1400°F) the material behavior can be assumed to be rate-independent as the different loading strain rates do not have much of an effect on the stress amplitude responses. At 649°C (1200°F) the material response is not totally rate-independent, particularly at cycles greater than 25 (Fig. 4a). At cycle 100, the material exhibits noticeable negative strain rate sensitivity, where the stress amplitude response for the highest strain rate of 20 cpm is of the lowest value and the stress amplitude for the lowest strain rate of 0.2 cpm is of the highest value. The negative strain rate sensitivity at 427°C (800°F) has been shown in Barrett et al. [22]. Negative strain rate sensitivity is known to be
one of the manifestations of dynamic strain aging [9]. Slavik and Cook [30] observed negative strain rate sensitivity in the intermediate temperature range (760°C) for a nickel-base alloy. The material is also not perfectly rate-independent at 760°C (1400°F) as shown in Fig. 4b. We observe that with cycles >100 the stress amplitude responses show some positive rate-dependence where the stress amplitude is greater for faster loading strain rates. Also at 760°C (1400°F) it can be argued that there is some (albeit very little) negative strain rate sensitivity observed for cycles less than 25, where the stress amplitude responses for the lower strain rates are higher than those for the higher strain rates. However, for practical purposes the stress amplitude responses can be considered to be rate-independent at both 649 and 760°C (1200 and 1400°F).

![Isothermal LCF stress amplitude (Δσ_{xa}) responses of Haynes 230 at strain rates of 0.2, 2 & 20 cpm, and temperatures (T): (a) 649°C, (b) 760°C, (c) 871°C, (d) 982°C (1MPa=0.145ksi)](image-url)
Figures 4c and 4d indicate Haynes 230 to behave rate-dependently at temperatures greater than and including 871°C (1600°F). It should be noted however that at 871°C (1600°F), while there is a clear positive rate-dependence going from 0.2 to 2 cpm, there is almost a saturation of stress amplitude responses going from 2 to 20 cpm. At 982°C (1800°F) we observe the typical positive strain rate dependence found in viscous materials. Yaguchi et al. [7] also found a nickel-base alloy IN792 to be rate-independent at lower temperatures (<700°C) and rate-dependent (800°C) at the higher temperatures. Dynamic strain aging is known to reduce the viscous properties of materials [9], and this is reflected in the rate-independence of the lower temperature ranges (≤760°C) and negative strain rate sensitivity in the intermediate temperatures (427-760°F) of the experiments in the current research.

3.2. Isothermal stress relaxation

Isothermal experiments with strain holds (dwell periods) at the maximum compressive peak were carried out to serve a two-fold purpose. First, it was to investigate the viscosity of the material at the different temperatures. And second, it served as a lead-up to the more complex thermo-mechanical experiments which followed, where the temperature was also varied along with the strain controlled path. The isothermal strain hold experiments were not carried out to failure but rather to around 600 cycles, by which Haynes 230 responses saturated as presented later.

Modeling of the stress relaxation behavior at half-life under strain controlled tensile hold condition is stated as one of the most important deformation behaviors in terms of creep-fatigue damage analysis of the actual service components [7]. The experiments in the current research were conducted by imposing compressive strain holds. This is because service conditions of combustor liners cause the liner material to be subject to thermally induced compressive strains with dwell periods.

Figure 5 shows the hysteresis loops at different cycles for the isothermal experiments with compressive hold times at 649 and 982°C (1200 and 1800°F). From Fig. 5a it is immediately evident that there is little stress relaxation at 649°C (1200°F) in the initial cycle and the stress relaxation increases with cycles. At 649°C (1200°F) there is considerable
hardening of the material. 649°C (1200°F) is within the dynamic strain aging temperature regime (427-760°C) and serrations can be seen in the initial cycle which disappear at higher cycles [22]. Figure 5b shows that at 982°C (1800°F) there is significant stress relaxation at all cycles starting from the initial cycle to the end cycle of 570. Also, unlike the substantial hardening at 649°C, there is no noticeable cyclic hardening at 982°C.

![Graph showing cyclic response for isothermal low cycle fatigue experiments](image)

**Fig. 5:** Isothermal LCF stress ($\sigma_{xa}$) vs. strain ($\varepsilon$) hysteresis loops at various cycles of Haynes 230 for 120s compressive hold at two temperatures: (a) 649°C and (b) 982°C (1MPa=0.145ksi)

The typical cyclic response for the isothermal low cycle fatigue experiments with different compressive hold times of 60 and 120s at 871°C (1600°F) is shown in Fig. 6. This figure shows that the difference in hold times does not have any noticeable effect on the cyclic response. The cyclic responses at other temperatures follow this trend as shown in Barrett et al. [22]. It is interesting to notice however that a non-zero mean stress evolves with cycles. The mean stress is slightly larger for the 120s hold compared to the 60s hold experiment as shown in the figure. Similar observations were made at all other temperatures as shown in Barrett et al. [22]. For isothermal experiments with no hold times [22] there was no mean stress evolution. This suggests that mean stress evolution is a time-dependent...
phenomenon. The evolutionary behavior of mean stress with cycles was reported by Yaguchi et al. [7], who observed the evolution in mean stress for isothermal experiments with both compressive and tensile holds.

Figure 7 shows the evolution of Haynes 230 stress relaxation with cycles during compressive holds of 60 and 120s at four different temperatures ranging from 649 to 982°C (1200 to 1800°F). The stress relaxation has been determined by calculating the change in stress values during the compressive hold for each cycle (as shown in Fig. 2). The strain ranges for these isothermal experiments have been chosen to give approximately equal plastic strain ranges at half-life of the material, as presented in Barrett et al. [22]. The magnitude of stress relaxation increases with increase in temperature from 649 to 871°C (1200 to 1600°F), but decreases with increase in temperature from 871 to 982°C (1600 to 1800°F). The increase in stress relaxation from 649 to 871°C (1200 to 1600°F) is caused by the increase in viscosity with temperature. The decrease in stress relaxation from 871 to 982°C (1600 to 1800°F) can be explained by the lower stress amplitudes at 982°C (1800°F). It is also observed in Fig. 7 that greater hold times of 120s produce greater stress relaxation compared to hold times of 60s. At 871°C (1600°F), however, the relaxation responses at 60 and 120s hold times are very close to each other, implying that a 60s hold time at that
temperature is sufficient for saturation of stress relaxation. The general trend of the stress relaxation can be seen to increase with cycles at each temperature towards reaching a saturated value.

The stress relaxation is normalized with respect to the stress amplitude of each cycle to further illustrate the effect of temperature on the viscous behavior as shown in Fig. 8. The figure shows that with increase in temperature there is an increase in the normalized stress relaxation. This indicates that the viscosity of the material increases with increase in temperature from 649 to 982°C (1200 to 1800°F).

Further scrutiny at the different temperatures reveals interesting observations. At the higher temperatures (871-982°C) for which we know the material behaves in a viscous manner, as expected we see significant stress relaxation with hold times. At the lower temperatures of 649 and 760°C, despite the material response being mostly rate-independent with respect to different loading strain rates, we still see a considerable amount of stress relaxation. This type of behavior has been observed by other researchers as well [3].

Fig. 7: Isothermal LCF stress relaxation ($\Delta\sigma_{xr}$) responses of Haynes 230 at four temperatures (T) of 649, 760, 871 & 982°C, and two strain holds ($t_h$) of 60 & 120s (1MPa=0.145ksi)
Fig. 8: Isothermal LCF normalized stress relaxation (Normalized $\Delta\sigma_{xr}$) responses of Haynes 230 at four temperatures ($T$) of 649, 760, 871 & 982°C, and two strain holds ($t_H$) of 60 & 120s (1MPa=0.145ksi)

4. Thermo-mechanical response

The thermo-mechanical fatigue test loading cycle was designed to replicate combustor engine operating conditions (i.e. temperature and strain excursions at critical locations in the components) as closely as possible. Service operating conditions essentially result in thermally induced compressive strains with hold times (dwell periods) at the maximum temperature, which coincides with the maximum compressive strain. Thus, out-of-phase loading conditions are most relevant to the problem in hand and these experiments have been carried out to failure. In-phase TMF experiments have also been conducted to better understand the material behavior towards developing and validating an advanced constitutive model. In the in-phase TMF experiments the tensile strains increase as temperature increases; and there are dwell periods at the maximum temperature, which coincides with the maximum tensile strain. The in-phase experiments have been carried out to a maximum of 1500 cycles with the assumption that sufficient cycles have been allowed to achieve saturation in the material response.

The ramp cycle times (i.e. the cycle time excluding the hold time) of the thermo-mechanical fatigue experiments were in the range of 145 to 195s. On the other hand the ramp cycle times for the isothermal hold experiments were 3s. The thermo-mechanical
experiments were conducted at a much slower rate than the isothermal experiments primarily due to practical reasons. The longer cycle times were dictated by equipment heating and cooling rates. While the isothermal hold experiments have ramp loading cycle times of 3s, other isothermal experiments without hold times have been conducted at loading cycle times of 3, 30 and 300s corresponding to loading rates of 20, 2 and 0.2 cpm [22]. The constitutive model parameter determination will be performed by primarily considering the isothermal experiments, while the validation of the constitutive model will be performed by simulating thermo-mechanical fatigue experiments. Hence, slower loading-rate isothermal experimental responses will be useful in determining parameters to be used in TMF simulations.

4.1. Thermo-mechanical fatigue test hysteresis loops and mean stress evolution

Figure 9 shows the hysteresis loops at different cycles for typical out-of-phase thermo-mechanical fatigue experiments with various maximum temperatures of 816, 871, 927 and 982°C (1500, 1600, 1700 and 1800°F respectively). The minimum cycling temperature was 316°C (600°F) for all out-of-phase and in-phase thermo-mechanical fatigue experiments conducted. Hysteresis loops have been shown up to the half-life cycle of each experiment. For the experiment with maximum temperature of 982°C (Fig. 9d), the half-life cycle (cycle 2500) was not available; thus, the plot has been shown up to cycle 1400. Stress relaxation occurs at the maximum compressive strain hold at maximum temperature (Fig. 9). For all the experimental responses the hysteresis loops are observed shifting in stress space in the tensile direction.
Fig. 9: Out-of-phase TMF stress ($\sigma_x$) vs. strain ($\varepsilon_x$) hysteresis loops of Haynes 230 at various cycles up to half-life: (a) 316-816°C, (b) 316-871°C (Rate I), (c) 316-927°C and (d) 316-982°C (1MPa=0.145ksi)

Figure 10 shows the hysteresis loops at different cycles for in-phase thermo-mechanical fatigue experiments with maximum temperatures of 871 and 927°C (1600 and 1700°F). The
in-phase thermo-mechanical fatigue experiments were conducted up to 1400-1500 cycles, by which time the material response was stable. The stress relaxation for these experiments occurs at maximum tensile strain hold at maximum temperature. The hysteresis loops in case of the in-phase thermo-mechanical fatigue experiments shift in stress space in the compressive direction (Fig. 10). Thus, Figs. 9 and 10 demonstrate that for thermo-mechanical fatigue experiments of Haynes 230 the mean stresses evolve in the direction opposite to that of the hold. Such evolution of mean stress with cycles for TMF without any dwell period was reported by Yaguchi et al. [7, 21]. They also observed evolution of mean stress for isothermal experiments with holds, which was also observed, albeit of very little magnitude, for Haynes 230 as presented in Barrett et al. [22] and shown in Fig. 6. In the present study the mean stress evolution phenomenon has been observed for a combination of anisothermal conditions and hold times.

**Fig. 10:** In-phase TMF stress ($\sigma_x$) vs. strain ($\varepsilon_x$) hysteresis loops of Haynes 230 at various cycles: (a) 316-871°C and (b) 316-927°C (1MPa=0.145ksi)
Figure 11 shows evolution of mean and amplitude stresses with cycles for the out-of-phase thermo-mechanical fatigue experiments shown in Fig. 9. As discussed, the mean stresses evolve in the tensile direction for out-of-phase thermo-mechanical fatigue experiments. It is interesting to observe in this figure that, despite the difference in stress amplitudes due to different maximum cycling temperatures and strain ranges, the mean stress evolutions are remarkably similar. Though there is some variation in the mean stress responses for the different maximum temperatures, the responses lie in a relatively narrow band for all experiments. With the exception of the experiment with maximum temperature of 816°C, for the other three maximum temperatures there is a general trend in slight decrease in mean stress response with increase in maximum temperature.

![Fig. 11: Out-of-phase TMF stress amplitudes (Δσ_{xa}) and mean stresses (Δσ_{xm}) of Haynes 230 as a function of the number of cycles (N) for four different maximum cycling temperatures (1MPa=0.145ksi)](image)

Figure 12 shows the mean and amplitude stresses for out-of-phase thermo-mechanical fatigue experiments with different strain ranges and maximum cycling temperatures of 816, 871 and 927°C (1500, 1600 and 1700°F). Once again it is evident that despite the difference in stress amplitudes, due to different strain ranges and dwell periods, the mean stresses are very similar to one another or within a relatively narrow band. The mean stress responses seem to be dependent on the strain range, particularly for experiments with maximum temperature of 871°C (1600°F), as shown in Fig. 12b. There seems to be a consistent slight
decrease in mean stress response with decrease in strain range. However, an anomaly occurs in Fig. 12c for maximum temperature of 927°C (1700°F), where the greatest mean stress response occurs for the lowest strain range of 0.27%. Also, in case of 871°C (1600°F) maximum temperature (Fig. 12b) mean stresses for larger strain ranges keep evolving with a small rate, whereas for smaller strain ranges the mean stresses seem to reach a stable value.

For the in-phase thermo-mechanical fatigue experiments the mean stresses evolve in the compressive direction as shown in Fig. 13. Once again, akin to the out-of-phase experiments, the similarity in mean stresses from different experiments stands out.

Longer hold times had a small but noticeable effect on the mean stress response. For longer hold times the mean stress evolutions in thermo-mechanical fatigue experiments were lesser. From Fig. 12b for maximum temperature of 871°C (1600°F) it is evident that, for the two experiments carried out at strain range of 0.3%, the mean stresses are smaller for the 1200s hold experiment compared to the 120s hold one. This was also observed to be true at maximum temperatures of 927°C (1700°F) as shown in Fig. 12c for experiments carried at strain range of 0.3%. The influence of hold time on the mean stress response was found to be different under isothermal loading conditions as presented in Barrett et al. [22] and shown in Fig. 6 for the temperature 871°C (1600°F). Longer hold times in isothermal experiments led to a higher, albeit by a small amount, mean stress response. The mean stress evolution in isothermal experiments was always much smaller in magnitude than those in thermo-mechanical fatigue experiments.
Fig. 12: Out-of-phase TMF stress amplitudes ($\Delta \sigma_{xa}$) and mean stresses ($\Delta \sigma_{xm}$) of Haynes 230 as a function of the number of cycles (N): (a) 316-816°C, (b) 316-871°C and (c) 316-927°C (1MPa=0.145ksi)
4.2. Thermo-mechanical fatigue cyclic hardening

The nature of the cyclic hardening response for thermo-mechanical fatigue tests is not immediately apparent from looking at the hysteresis loops (Figs. 9 and 10). However, if the amplitude stresses for the experiments in Fig. 9 are plotted up to their respective half-lives as shown in Fig. 11, noticeable cyclic hardening is observed for the maximum cycling temperatures of 816 and 871°C (1500 and 1600°F), whereas the response is fairly stable for maximum cycling temperatures of 927 and 982°C (1700 and 1800°F). In fact, for the maximum cycling temperatures of 927 and 982°C (1700 and 1800°F) there is actually a sizeable amount of cyclic softening going from the first cycle to the second, after which the response is in essence stable. For the thermo-mechanical fatigue experiments notable cyclic hardening is observed only for maximum cycling temperatures of 816 and 871°C (1500 and 1600°F) for strain ranges of 0.4% and greater, as seen in Figs. 12a and 12b. Stronger cyclic hardening associated with larger strain ranges has been observed by other authors as well [31]. For maximum cycling temperatures of 816 and 871°C (1500 and 1600°F) with lower strain ranges (<0.4%) and maximum cycling temperatures of 927°C (1700°F) and higher, the stress amplitude response is mostly stable if the first cycle amplitude is ignored (Fig. 12). In certain cases we observe cyclic softening for smaller strain ranges such as 0.3% strain range, 120s hold time and maximum cycling temperature of 871°C (1600°F) (Fig. 12b) and 0.3%
strain range, 1200s hold time and maximum cycling temperature of 927°C (1700°F) (Fig. 12c). For in-phase thermo-mechanical fatigue experiments noticeable cyclic hardening is observed only for a maximum cycling temperature of 760°C (1400°F), while the response is essentially stable for maximum cycling temperatures of 871°C (1600°F) and higher as seen in Fig. 13.

Different hold times do not seem to have a significant effect on the stress amplitude responses for out-of-phase thermo-mechanical fatigue experiments. Rather, the stress amplitude responses are driven by the loading rates. From Fig. 12b, for maximum cycling temperature of 871°C (1600°F) and strain range of 0.3%, the stress amplitudes for hold times of 120 and 1200s are very similar. However, the stress amplitude for the hold time of 1200s eventually is slightly larger with cycles owing to its faster loading rate compared to the 120s hold time experiment (heating time of 75s vs. 90s respectively). From Fig. 12c, for maximum cycling temperature of 927°C (1700°F) and strain range of 0.3%, the stress amplitude response in the case of the 120s hold time is greater owing to its faster loading rate compared to the 1200s hold time experiment (heating time of 65s vs. 75s respectively).

Fig. 14: Isothermal low cycle fatigue stress amplitude ($\Delta\sigma_{xa}$) and mean stress ($\Delta\sigma_{xm}$) responses of Haynes 230 at different temperatures (T) for strain ranges ($\Delta\varepsilon$): (a) 0.6% and (b) 0.4%
The cyclic hardening response in thermo-mechanical fatigue experiments is dominated by the maximum temperature in the loading cycle. From isothermal experiments substantial cyclic hardening was observed at temperatures up to 760°C (1400°F) and very little cyclic hardening at 871°C (1600°F) and higher temperatures, as shown in Fig. 14 and presented in Barrett et al. [22]. At temperatures 427 to 760°C (800 to 1400°F) there is continuous cyclic hardening till failure with no sign of saturation. For thermo-mechanical fatigue experiments there is temperature cycling from 316°C (600°F) to maximum cycling temperatures of 760°C (1400°F) and higher. Thus for each cycle, whether it be an out-of-phase or in-phase thermo-mechanical fatigue experiment, the material is subject to variable temperatures between the minimum and maximum. Thus, despite cycling through the lower temperatures at which we observed significant cyclic hardening for isothermal experiments, very little cyclic hardening is observed for the thermo-mechanical fatigue experiments, particularly at lower strain ranges and highest maximum temperatures (Fig. 12). For thermo-mechanical fatigue experiments the degree of cyclic hardening is dependent on the maximum cycling temperature rather than intermediate temperatures. Ohno et al. [23] discussed the internal change of the material occurring more significantly at higher temperatures, and the internal change once having occurred not recovering markedly at lower temperatures. This means that the material microstructure relevant to the higher temperature dominates the response during thermo-mechanical fatigue loading. In the present set of experiments there is a dwell period of 120 or 1200s at the maximum cycling temperature, meaning the material is subjected to the highest temperature for the greatest amount of time and thus has more time to attain the microstructure at that temperature. As a result we see the cyclic hardening response associated with the maximum cycling temperature being exhibited in the thermo-mechanical fatigue experimental responses.

Figure 15 shows a comparison between out-of-phase and in-phase thermo-mechanical fatigue stress amplitudes for different maximum temperatures. For the experiments at maximum cycling temperatures of 982°C (1800°F) and strain range of 0.25%, the loading rates are identical and the stress amplitudes are superimposed on one another. For experiments at maximum cycling temperatures of 927°C (1700°F), the stress amplitudes are
similar in the out-of-phase and in-phase thermo-mechanical fatigue experiments despite the faster loading rate in the out-of-phase experiment (heating time of 65s vs. 75s in the in-phase case). For experiments at maximum cycling temperatures of 871°C (1600°F) and strain range of 0.3%, the stress amplitude response for the out-of-phase thermo-mechanical fatigue experiment is greater than that of the in-phase thermo-mechanical fatigue despite the former having a slower loading rate than in the in-phase thermo-mechanical fatigue experiment (heating times of 90s vs. 75s respectively).

![Stress Amplitude Response](image)

**Fig. 15:** Comparison between stress amplitudes ($\Delta\sigma_{xa}$) of same strain range out-of-phase (OP) and in-phase (IP) TMF experiments of Haynes 230 for three maximum temperatures (1MPa=0.145ksi)

### 4.3. Thermo-mechanical fatigue rate-dependence

In Fig. 12b, responses of two out-of-phase thermo-mechanical fatigue experiments conducted at maximum cycling temperatures of 871°C (1600°F) demonstrate the rate-dependent response of Haynes 230. The differences between the two are the loading rates. Both experiments have the same hold times of 120s and the same cooling times of 90s. Rate I has a heating time of 75s, while Rate II is a faster rate with a shorter heating time of 55s. From the stress amplitude response it can be seen that the stress amplitude is greater for Rate II, which means the material is clearly exhibiting positive rate-dependence.
4.4. *Thermo-mechanical fatigue hysteresis loop shape evolution*

The evolution of the shape of the hysteresis loop is of great interest particularly as it directly reflects information regarding the microstructure of the material. All the hysteresis loops, both out-of-phase and in-phase, exhibit roundedness in their shape as the temperature increases towards the maximum cycling temperature (Figs. 9 and 10). For all cases, as seen in the figures, the roundedness is most prominent in the initial monotonic path. For each experiment the roundedness gradually disappears with cycles. From Fig. 9 it can be seen that the roundedness appears to increase in prominence as the maximum temperature of the experiment increases; i.e. for out-of-phase experiments the roundedness is least for the maximum temperature of 816°C (Fig. 9a) and greatest for the maximum temperature of 982°C (Fig. 9d). In Fig 9d, in the monotonic path along which the temperature is increasing, the compressive stress reaches a maximum of about 210MPa at around 775°C, after which there is a drop in the stress response with increase in strain and temperature. This change in shape in the monotonic response with temperature along the monotonic path is related to the material having different material properties at different temperatures. Isothermal monotonic tensile tests, as shown in Fig. 16a, show that the material holds its yield stress very well from 316 to 816°C (600 to 1500°F) in the range of around 275MPa. At 871°C (1600°F) the yield stress drops below 200MPa and at 982°C (1800°F) it drops below 140MPa. In essence, the material becomes noticeably softer at very high temperatures. This contributes to the drop in the stress magnitude along the monotonic response as the temperature becomes higher than 800°C. A similar drop in the stress magnitude occurs in the monotonic path in the in-phase thermo-mechanical fatigue experiments in Fig. 10.
As seen in Fig. 9 the roundedness disappears for the out-of-phase thermo-mechanical fatigue experiments by the time the half-life hysteresis loop is reached. The in-phase experiments were not carried out to failure but rather to 1400-1500 cycles, which is less than the expected half-life. As a result, for the in-phase experiments, particularly in Fig. 10b for the maximum temperature experiment of 927°C (1700°F), the roundedness does not disappear in the last cycle (cycle 1400). The decreasing degree in roundedness and its eventual disappearance can be explained in terms of the material attaining and then retaining most of the microstructure associated with the maximum cycling temperature at all intermediate temperatures. Ohno et al. [23] discussed, for 304 stainless steel, the phenomenon of the internal change associated with the higher temperature prevailing under thermo-mechanical fatigue loading. Due to the strain hold at the maximum temperature the material spends most time at the highest temperature and thus also has time to acquire the microstructure associated with that of the highest temperature. The influence of maximum temperature increases with cycles as more time is spent at the highest temperature with cycles.

It is also interesting to note that the elastic region stays almost linear, particularly in the monotonic path, despite the material undergoing temperature change. This is in spite of the isothermal elastic modulus showing a decreasing trend as shown in Fig. 16b.
The evolution of the degree of roundedness in the hysteresis loops can be quantified by the thermo-mechanical stress drop ($\Delta \sigma_{\text{TM}}$) for each recorded loop, as shown in Fig. 2. Figure 17 shows the thermo-mechanical stress drop for different out-of-phase thermo-mechanical fatigue experiments with strain ranges of 0.25 to 0.3% plotted as function of the number of cycles. In general it can be said that the thermo-mechanical stress drop gradually decreases with cycles and approaches zero, as was already observed from the shapes of the hysteresis loops in Figs. 9 and 10.

![Graph with data points and labels](image)

**Fig. 17:** Thermo-mechanical stress drop ($\Delta \sigma_{\text{TM}}$) with cycles (N) for out-of-phase TMF experiments of Haynes 230 for different maximum cycling temperatures (T), strain ranges ($\Delta \varepsilon$) and hold times ($t_H$) (1MPa=0.145ksi)

### 4.5. Thermo-mechanical inelastic and elastic strain calculation

Thermo-mechanical fatigue experiments display an accumulation of inelastic strain with cycles as reported by Zhang et al. [1] for a martensitic steel. Figure 18a shows the hysteresis loops for the first 20 cycles of a thermo-mechanical fatigue test in a plot of stress against mechanical strain. The test is an out-of-phase thermo-mechanical fatigue with compressive holds. The control temperature is varied from 316 to 927°C (600 to 1700°F) at a total mechanical strain range of 0.3%. Since the test is strain-controlled there is no shifting of the
hysteresis loops in total mechanical strain direction. However, when the same plots are made with respect to inelastic strain (Fig. 18b) and elastic strain (Fig. 18c) we see hysteresis loops shifting in strain direction. The elastic \( \Delta \varepsilon^e \) and inelastic strains \( \varepsilon^{in} \) can be computed using the expressions for the elastic \( \Delta \varepsilon^e \) and inelastic strain \( \Delta \varepsilon^{in} \) increments shown in Eqs. 3 and 4,

\[
\Delta \varepsilon^e = \frac{\Delta \sigma}{E} \tag{3}
\]

\[
\Delta \varepsilon^{in} = \Delta \varepsilon - \Delta \varepsilon^e \tag{4}
\]

where, the elastic modulus, \( E \), varies with temperature as shown in Fig. 16 and is interpolated at each intermediate temperature.

The inelastic strain accumulates in the compressive direction while the elastic strain accumulates in the tensile direction for out-of-phase thermo-mechanical fatigue tests. This means that the sum of the elastic and inelastic strains is still the prescribed mechanical strain as shown in Fig. 19. The underlying cause for the inelastic and elastic strain accumulation with cycles is the change in material properties, in particular, the elastic modulus with temperature (Fig. 16). The changing elastic modulus is also apparent in the changing slope of the stress vs. elastic strain plot in Fig. 18c.
Fig. 18: Evolution of hysteresis loops for the first 20 cycles of a TMF test of Haynes 230 in: (a) mechanical strain direction \( \epsilon_x \), (b) inelastic strain direction \( \epsilon_x^{in} \) and (c) elastic strain direction \( \epsilon_x^e \)

Fig. 19: Evolution of mechanical \( \epsilon_x \), inelastic \( \epsilon_x^{in} \) and elastic strains \( \epsilon_x^e \) with time (t) for the first 20 cycles of a TMF test of Haynes 230
In the computation of elastic and inelastic strains, however, the variation of the elastic modulus, $E$, with time should also be taken into account [32]. Starting from the expression for stress (Eq. 5), the time derivative can be taken (Eq. 6). In taking the time derivative, the fact that $E$ is not a constant with respect to time is taken into consideration. Applying the strain decomposition ($\varepsilon = \varepsilon^e + \varepsilon^{in}$) we obtain Eq. 7. Expressing Eq. 7 in discretized form and replacing $\varepsilon^{in}$ with $\varepsilon^{in}_{n} + \Delta \varepsilon^{in}$ we obtain Eq. 8. The quantities without any subscripts are assumed to be at time-step $n+1$, while those with subscript $n$ are at time-step $n$. Finally by rearranging the terms an expression for $\Delta \varepsilon^{in}$ can be obtained (Eq. 9). The elastic strains can be calculated by subtracting the inelastic strain from the mechanical strain.

$$\sigma = E\varepsilon^e$$  \hspace{1cm} (5)  

$$\dot{\sigma} = E\dot{\varepsilon}^e + \dot{E}\varepsilon^e$$  \hspace{1cm} (6)  

$$\ddot{\sigma} = E(\dot{\varepsilon} - \dot{\varepsilon}^{in}) + \dot{E}(\varepsilon - \varepsilon^{in})$$  \hspace{1cm} (7)  

$$\Delta \sigma = E(\Delta \varepsilon - \Delta \varepsilon^{in}) + \Delta E(\varepsilon - \varepsilon^{in} - \varepsilon^{in}_{n})$$  \hspace{1cm} (8)  

$$\Delta \varepsilon^{in} = \frac{E\Delta \varepsilon + \Delta E(\varepsilon - \varepsilon^{in}_{n}) - \Delta \sigma}{E + \Delta E}$$  \hspace{1cm} (9)

Calculating the inelastic strain in this manner, i.e. by taking into consideration the time derivative of the elastic modulus, leads to no accumulation of inelastic strain with cycles as shown in Fig. 20. This figure shows that the hysteresis loops are no longer shifting in inelastic strain direction. Figure 21 shows that the mechanical strain maintains the control waveform and neither the inelastic nor the elastic strain is accumulating in any direction.

The significance of having stable hysteresis loops in a stress vs. inelastic strain plot can best be explained from a modeling point of view. Haynes 230, like many other materials, displays material behavior dependent on strain range [22]. In modeling such behavior it is customary to introduce a strain memory surface [34] which essentially keeps track of the largest plastic strain in loading history. Thus, from a modeling point of view a certain maximum plastic strain corresponds to a certain total strain range and represents the material properties for that particular strain range. However, upon neglect of the time derivative of the elastic modulus if the hysteresis loops shift in the inelastic strain direction (Fig. 18b), then
the maximum plastic strain increases indefinitely (Fig. 19). This prevents the maximum plastic strain being representative of any particular strain range and as a consequence hinders strain range dependent modeling.

![Graph showing hysteresis loops](image1)

**Fig. 20:** Evolution of hysteresis loops for the first 20 cycles of a TMF test of Haynes 230 with consideration the time derivative of elastic modulus (1MPa=0.145ksi)

![Graph showing strains](image2)

**Fig. 21:** Evolution of mechanical ($\varepsilon_x$), inelastic ($\varepsilon_x^{in}$) and elastic strains ($\varepsilon_x^e$) with time (t) for the first 20 cycles of a TMF test of Haynes 230 with consideration of the time derivative of elastic modulus

4.6. Thermo-mechanical fatigue stress relaxation

Hold times (dwell periods) in the loading cycles lead to stress relaxation, as already seen from the hysteresis loop plots of out-of-phase and in-phase thermo-mechanical fatigue experiments (Figs. 9 and 10). Hold times lead to complex creep-fatigue interactions; thus,
understanding the stress relaxation responses is of utmost importance. The stress relaxation versus cycles for the different maximum cycling temperatures are shown in Fig. 22. The plots show that the stress relaxation tends to decrease with cycles for all the tests.

The decrease in stress relaxation with cycles is directly related to the mean stress evolution in thermo-mechanical fatigue experiments (Figs. 12 and 13). Out-of-phase thermo-mechanical fatigue experiments exhibit positive mean stress evolution, which means that the hysteresis loops shift in the tensile direction with cycles as shown in Fig. 9. As a consequence, the absolute value of the peak compressive stress decreases with cycles. This results in the magnitude of stress relaxation also decreasing with cycles. In the case of out-of-phase thermo-mechanical fatigue experiments the hysteresis loops shift in the compressive direction (Fig. 10) and the peak tensile stress decreases with cycles. Thus in a manner similar to the out-of-phase case, the magnitude of stress relaxation also decreases with cycles as shown in Fig. 23. The evolution of the stress relaxation with cycles for thermo-mechanical fatigue experiments is in direct contrast to that of isothermal experiments with dwell periods. In case of the isothermal experiments, as already discussed, the general trend is the increase in stress relaxation with cycles (Fig. 7).

Figure 22 also reveals that the stress relaxation tends to increase with increased loading strain range, as the stress amplitude increases with strain range. From Fig. 22b, observing the two experiments conducted at a strain range of 0.3 percent, we can see that the one with the greater hold time of 1200s causes greater stress relaxation as expected. However, from Fig. 22c, looking at the two experiments at a strain range of 0.3 percent, the effect of the greater hold time is not immediately apparent. One possible explanation may be that for a maximum cycling temperature of 927°C (1700°F) a 120s hold time causes near saturation of the stress relaxation response, and as a result a greater hold time of 1200s causes little further stress relaxation.

Normalizing the out-of-phase thermo-mechanical fatigue stress relaxation allows for comparison over different temperatures and strain ranges as shown in Fig. 24. The stress relaxation has been normalized with respect to the half-life cycle stress amplitude for each experiment. From the figure we can say that in general the normalized stress relaxation
increases with increase in the loading strain range. It is also interesting to see that the normalized stress relaxation of experiments having the same loading strain ranges clustered together irrespective of the maximum cycling temperature.

Figure 25 shows the comparison between out-of-phase and in-phase thermo-mechanical fatigue stress relaxation for two cases. In each comparison the maximum cycling temperature and the strain range are the same. To facilitate the comparison the absolute values of the stress relaxation for the in-phase thermo-mechanical fatigue experiments have been taken. It is observed that for both maximum cycling temperatures the stress relaxation of the out-of-phase and in-phase experiments are comparable.

![Graphs showing stress relaxation](image)

**Fig. 22:** Out-of-phase TMF stress relaxation ($\Delta \sigma_{xr}$) responses of Haynes 230 with cycles (N) for different strain ranges ($\Delta \varepsilon$) and hold times ($t_H$): (a) 316-816°C, (b) 316-871°C, (c) 316-927°C and (d) 316-982°C
Fig. 23: In-phase TMF stress relaxation ($\Delta\sigma_{xr}$) responses of Haynes 230 with cycles (N) for different maximum temperatures and strain ranges ($\Delta\varepsilon$) (1MPa=0.145ksi)

Fig. 24: Out-of-phase TMF normalized stress relaxation (Normalized $\Delta\sigma_{xr}$) responses of Haynes 230 with cycles (N) for different maximum temperatures, strain ranges ($\Delta\varepsilon$) and hold times ($t_H$)
4.7. Evolution of stress relaxation with time during the relaxation phase

The evolution of stress relaxation as a function of time at hold during the relaxation phase is of great significance due to the influence of stress relaxation in the creep-fatigue life of actual service components. The relaxation behavior directly gives us important information regarding the viscoplastic or time-dependent nature of the material in question. The relaxation curves for various cycles up to the half-life cycle, for two experiments with 120 and 1200s hold times having maximum temperatures of 871°C (1600°F), are shown in Fig. 26. The general trend of the relaxation curves for out-of-phase thermo-mechanical fatigue experiments is that the highest relaxation occurs at the first cycle and the subsequent cycles have lower stress relaxation. From Fig. 26a we can see that a 120s compressive hold time is not sufficient for saturation of the stress relaxation response, particular in the initial cycle. On the other hand, Fig. 26b shows that a 1200s hold time results in a relatively more saturated response. It is also interesting to note in Fig. 26 the shape of the relaxation curves going from the 1st cycle to the half-life cycle. Particularly for the 120s hold experiment in Fig. 26a, the shape of the half-life cycle relaxation curve suggests greater degree of saturation compared to the 1st cycle. The degree of saturation progressively increases with cycles as the magnitude of total stress relaxation decreases.
It is interesting to compare the relaxation curve trends of out-of-phase thermo-mechanical fatigue experiments of Fig. 26 with that of isothermal compressive hold experiments shown in Fig. 27. The isothermal hold experiments, presented in Barrett et al. [22], were conducted to about 600 cycles. The hold times of the isothermal experiments were 60 or 120s. Also, the isothermal hold experiments were conducted at a much faster strain loading ramp rate in the order of $10^{-3}$ per second, compared to the strain loading ramp rates in the order of $10^{-5}$ per second for the thermo-mechanical fatigue experiments. For isothermal experiments conducted at temperatures 649 to 871°C (1200 to 1600°F) the relaxation curves’ trends were different compared to that of the thermo-mechanical fatigue experiments. For the 760°C (1400°F) experiment shown in Fig. 27a, it is observed that highest stress relaxation occurs in the final cycle of 600 and the least in the first cycle. However, similarly to the thermo-mechanical fatigue experiments, the degree of saturation increased with decrease in stress relaxation magnitude (compare cycle 1 with cycle 600). For the experiments carried out at 982°C (1800°F), however, a trend similar to that of the thermo-mechanical fatigue experiments was seen. As shown in Fig. 27b, in this case the maximum stress relaxation occurs in the initial cycle with progressive decrease in stress relaxation with cycles.

Fig. 26: Evolution of stress relaxation ($\Delta\sigma_{xr}$) with time at hold ($t_h$) for out-of-phase TMF experiments of Haynes 230: (a) T: 316-871°C, $\Delta\varepsilon=0.4\%$, $t_h=120s$ and (b) T: 316-871°C, $\Delta\varepsilon=0.3\%$, $t_h=1200s$

(1MPa=0.145ksi)
4.8. Evolution of the inelastic strain rate with stress during the relaxation phase

The evolutionary behavior of the inelastic strain rate during the relaxation phase is important as it determines the mathematical form to be used for the flow rule of a constitutive model. The isothermal low cycle fatigue experiments with compressive dwell periods carried out at temperatures between 760 and 982°C (1400 and 1800°F) help us understand the viscoplastic behavior of Haynes 230 [22]. The inelastic strain rate $\dot{e}^{in}$ during the dwell period for a particular cycle can be determined considering the decomposition of strain, and that the total strain rate $\dot{e}$ during the dwell period is zero:

$$\dot{e}^{in} = \dot{e} - \dot{e}^{el} = -\frac{\dot{\sigma}}{E}$$  \hspace{1cm} (10)

Figure 28 shows that in a semi-logarithmic diagram $\dot{e}^{in}$ follows a concave downward shape when plotted against the stress during relaxation. In the figure two plots for the last cycle of isothermal hold experiments conducted at 871 and 982°C (1600 and 1800°F) are shown. We remind ourselves that the isothermal hold experiments were conducted to around 600 cycles, which is well within the expected half-life. It has been shown that conventional flow rules such as the Norton’s rule and the sine hyperbolic rule mathematically describe a similar trend in a semi-logarithmic diagram [33].
As thermo-mechanical experiments are of greater interest because they represent combustor liner service conditions, similar plots of inelastic strain rates versus stress during stress relaxation were made for the half-life cycle of thermo-mechanical fatigue experiments (Fig. 29). Figure 29 shows that for both hold times of 120 and 1200s and different maximum cycling temperatures of 816 and 871°C (1500 and 1600°F), respectively, the trend of the inelastic strain rate with stress for the half-life cycle is the same as that for isothermal hold experiments, i.e. concave downwards. These experimental results demonstrate that conventional flow rules may be adequate in describing the viscoplastic behavior of Haynes 230.

Fig. 28: Inelastic strain rate $\dot{\epsilon}_{\text{in}}$ versus stress during relaxation ($\sigma_x$) of isothermal LCF experiments of Haynes 230 at temperatures: (a) 871°C and (b) 982°C (1MPa=0.145ksi)
5. Fatigue lives

Analysis of fatigue lives under service mimicking conditions provides great insight into the factors that influence fatigue failure of components under operating conditions. Figure 30a shows the fatigue lives of different out-of-phase thermo-mechanical fatigue experiments at different maximum cycling temperatures and dwell periods. Analysis of this figure demonstrates that increasing the maximum cycling temperature decreases the fatigue lives for all strain ranges, as seen from the experiments having 120s hold times. This illustrates the damaging effect of the higher temperature on the material response. For the experiments with maximum temperature of 927°C (1700°F) and strain range of 0.3%, a longer hold time of 1200s caused a slight decrease in the fatigue life compared to the 120s hold time experiment. Larger dwell periods imply more time spent at the higher temperature, and a decrease in fatigue life is thus expected. The figure also shows that in general, for a particular maximum cycling temperature and dwell period, the fatigue life increases with decrease in strain range.

It is interesting to compare the fatigue lives from thermo-mechanical fatigue experiments with those of comparable isothermal low cycle fatigue experiments. In Fig. 30b the out-of-phase thermo-mechanical fatigue experiments have temperature cycling between 316-871°C (600-1600°F). The low cycle fatigue experiments are isothermal experiments at 871°C.

Fig. 29: Inelastic strain rate \( \Delta \dot{e}^\text{in}_{xx} \) versus stress during relaxation (\( \sigma_x \)) of out-of-phase TMF experiments of Haynes 230: (a) T: 316-816°C and (b) 316-871°C (1MPa=0.145ksi)
(1600°F). It should also be noted that the low cycle fatigue experiments have faster loading strain rates in the order of $10^{-3}$ per second compared to that of $10^{-5}$ per second for out-of-phase thermo-mechanical fatigue experiments. We observe that the out-of-phase thermo-mechanical fatigue experiments have lower fatigue lives compared to the isothermal experiments as can be expected. The difference in fatigue lives seems to be greater when the strain range is smaller.

![Fig. 30: Fatigue lives of out-of-phase TMF experiments of Haynes 230: (a) At different maximum temperatures and hold times ($t_H$), (b) Comparison with isothermal LCF fatigue lives](image)

The area enclosed within each hysteresis loop gives a measure of the energy absorbed by the material for that cycle [35]. The cumulative area enclosed within the hysteresis loops gives a measure of the total energy absorbed. Figure 31a shows the cumulative energy plotted as function of the number of cycles for different out-of-phase thermo-mechanical fatigue experiments with different maximum cycling temperatures. It is interesting to see that experiments with similar loading strain ranges tend to be clustered together irrespective of the maximum cycling temperature. Also in Fig. 31a are shown the cumulative energies for in-phase thermo-mechanical fatigue experiments and isothermal experiments with dwell periods. These experiments were not conducted to failure but rather up to specified cycles. Figure 31b shows a close-up of Fig. 31a up to cycle 2000 of the same experiments. Once

[103]
again we see that experiments with similar strain ranges are clustered together irrespective of them being out-of-phase, in-phase or isothermal hold experiments.

**Fig. 31**: Energy absorption with cycles for out-of-phase TMF, in-phase TMF and isothermal LCF tests of Haynes 230: (a) Up to half-life cycle, (b) Close-up of first 2000 cycles (1MPa=0.145ksi)
6. Conclusions

A large set of isothermal and anisothermal experiments were carried out on Haynes 230 in an effort to understand its high temperature fatigue constitutive response. Isothermal low cycle fatigue experiments at different loading strain rates show that the material can be considered to be rate-independent below and at 760°C (1400°F). However, isothermal strain hold experiments show stress relaxations at 649 and 760°C (1200 and 1400°F).

Thermo-mechanical fatigue (TMF) experiments were conducted with chosen temperature and strain ranges which replicate actual service combustor liner conditions closely. The TMF experiments revealed an evolution of mean stress with cycles. The out-of-phase strain-controlled TMF experiments with a compressive dwell period show a positive mean stress evolution response, while the in-phase TMF experiments with a tensile dwell period show a negative mean stress evolution response.

Cyclic hardening for the TMF experiments was found to be dependent largely on the maximum temperature in the loading cycle. The maximum temperature in the loading cycle appeared to influence the material properties at lower temperatures. This influence seemed to increase with cycles as reflected in the evolution of the hysteresis loop shapes.

In the analysis of thermo-mechanical fatigue experiments, the variation of the elastic modulus with temperature needs to be considered to achieve stable inelastic and elastic strains with cycles. Disregarding this variation results in the accumulation of inelastic and elastic strains in compressive and tensile directions, respectively, for out-of-phase thermo-mechanical fatigue tests.

The stress relaxation for thermo-mechanical fatigue experiments in general decreases with cycles, while that for isothermal experiments with dwell periods increases with cycles. The decrease in stress relaxation with cycles for the thermo-mechanical fatigue experiments can be directly attributed to the mean stress evolution.

The inelastic strain rate behavior during the relaxation phase suggests traditional flow rules may be used in a chosen constitutive model.

Fatigue lives of the thermo-mechanical fatigue experiments reflect the damaging influence of higher maximum temperatures and longer hold times. The thermo-mechanical
fatigue experiments were also found to have shorter fatigue lives compared to isothermal low cycle fatigue experiments conducted at the maximum temperature of the thermo-mechanical fatigue experiment.

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8. References


CHAPTER 4: CONSTITUTIVE MODELING FOR HIGH TEMPERATURE ISOTHERMAL LOW CYCLE FATIGUE AND FATIGUE-CREEP RESPONSES OF HAYNES 230

Abstract

This research develops a robust cyclic viscoplasticity model for simulating a broad set of high temperature isothermal, low cycle fatigue and fatigue-creep responses of Haynes 230. Structural components, such as an airplane gas-turbine engine combustor liner, must be designed considering the underlying nonlinear material complexities induced by thermo-mechanical fatigue. A novel unified viscoplastic model based on nonlinear kinematic hardening (Chaboche type), with several added features of strain range dependence, rate-dependence, temperature-dependence, static recovery, and for mean stress evolution, is evaluated for Haynes 230 (HA 230), a nickel-based superalloy used in manufacturing combustor liners. The robustness of the constitutive model is demonstrated showing the strengths of the model to accurately predict interdependent damage mechanisms, such as creep-fatigue interactions under uniaxial strain-controlled loading cycles. Challenges in parameter determination are discussed showing the importance of a well thought-out material database for experimental validation and properly linking the experimental results for physically-meaningful parameters. An effort has been made to develop an improved constitutive model and new knowledge towards incorporating more complicated analysis methods into effective design methodologies for high temperature components.

Keywords: Haynes 230, Viscoplasticity, Constitutive Modeling, Isothermal Fatigue, Creep-Fatigue, Nonlinear Structural Analysis

1. Introduction

Critical structural systems must perform safely, reliably, and predictably. To reach this level of performance, simulation-driven design processes are an invaluable tool. However, more importantly, the underlining physics inside these simulation-driven programs must be
properly modeled. Non-linear stress analysis for cyclic viscoplasticity is increasingly becoming an important modeling framework for many industries. Simplified analyses are demonstrated to be insufficient in accurately predicting the life of a critical component. For example, combustor liners of an airplane gas turbine engine are found to initiate cracks at as low as one-fifth of the estimated life. High temperature component failures are primarily induced by out-of-phase thermo-mechanical, creep-fatigue damage accumulation, among many other factors [1-4]. Due to start-up and shut-down cycles, repeated thermo-mechanical stresses are induced that gradually degrade the life of a machine and its components. These material complexities in high temperature components are manifested as time-dependent processes such as creep, oxidation, dynamic strain aging, creep-fatigue, thermo-mechanical fatigue and cyclic creep or ratcheting. As a result, cyclic viscoplastic advanced constitutive models for inelastic design-by-analysis are essential for design of components experiencing thermally induced stresses. In the cyclic plasticity context, inelastic modeling for non-linear kinematic hardening has been developed since the initiation of the Armstrong-Frederick model in 1967 [5]. Since then numerous extensions of the non-linear kinematic hardening rule have been made over the past five decades, with much emphasis given to the simulation of cyclic hardening, softening, and ratcheting under uniaxial and multiaxial stress states [6-25]. Another class of models for cyclic plasticity response has been based on multi-surface or bounding surface theories [26-34]. Chaboche discussed [35] models that are well developed for cyclic plasticity responses, with evolution equations of the non-linear kinematic hardening rule modified to account for many complex phenomena.

The present paper is concerned with developing an advanced viscoplastic constitutive model for simulating high temperature (as high as 982°C or 1800°F) isothermal fatigue and fatigue-creep responses of Haynes 230 under uniaxial loading cycles. The introduction of a hold time in a strain controlled, low cycle fatigue test causes stress relaxation for viscous materials which can lead to either fatigue-creep or creep-fatigue interactions. Fatigue-creep is one that is predominantly fatigue damage with an interacting creep damage component, while creep-fatigue shows more creep damage in the material microstructure with an interacting fatigue damage component. The cycling of strain inherently gives fatigue-type
damage; however, with the introduction of a hold time creep-type damage can be induced. The amount of damage for fatigue and creep and their combined effects determine the dominant damage mechanism of either fatigue-creep or creep-fatigue. Microscopically this interaction of fatigue and creep shows up in the crack initiation and propagation modes as either transgranular (fatigue-creep) or intergranular (creep-fatigue), as suggested by Rao et al., 1988 [58] and Rodriguez et al., 1995 [59].

Constitutive model development for high temperature cyclic viscoplasticity can be classified into two broad groups: unified and non-unified theories. In the context of unified constitutive modeling, the inelastic strain is modeled considering creep and plasticity to be a unified quantity commonly referred to as the viscoplastic strain [11,35,60,61]. Non-unified models [62,63], which consider the partition between plastic and creep strains, have been found to be incapable of predicting acceptably certain deformation features such as cyclic creep, also known as ratcheting, and creep-plasticity interaction [42-44]. Moreover, Zienkiewicz and Cormeau [44] argued that the phenomenon of creep and plasticity cannot be treated separately as only the combined effect is measurable. The experiments of Ikegami and Niitsu [64] show a coupling between plastic strain and creep strain, by way of the associated hardening effects. This has led to many researchers adopting unified theories where all aspects of inelastic deformation such as plasticity, creep and stress relaxation are represented by a single inelastic strain.

Under the unified modeling framework, cyclic viscoplasticity has been characterized by different researchers [9-11,16, 21,23,46,53-54] for various steels and nickel-base alloys. The use of non-linear kinematic hardening with features of dynamic recovery, rate-dependence, and static recovery was included to simulate a wide range of inelastic material behaviors such as cyclic hardening/softening and stress relaxation. The modeling concept of static recovery was adopted for cyclic viscoplasticity by building off research in creep modeling [45, 57]. In the unified framework for viscoplasticity, the choice of the viscosity function which dictates the viscoplastic strain rate and its dependency on the viscous stress was studied extensively by Chaboche [35]; and more recent studies have shown promising results as well for modeling the interactions of creep and plasticity [36,47-48]. Szmytka et al., 2010 [36]
showed that the relation between stress and the plastic strain rate norm is usually a highly nonlinear relationship that requires typically a power-function form like the classical Norton’s law for secondary creep [49], hyperbolic sine-function form [50], or lastly an exponential form [51] to capture the time-dependent viscous nature of the material.

Advanced constitutive models, of the type studied in the present study, have not been previously validated against a broad scope of tests, such as those presented in Barrett et al., 2013 [38] and Ahmed et al., 2013 [39], for capturing different features of strain rate sensitivity, strain range dependence, fatigue-creep stress relaxation (strain-dwell), and thermo-mechanical fatigue, across a wide temperature range (25-982°C). After proper identification of the unique material responses of Haynes 230, an advanced constitutive model was developed in this study to robustly simulate the complex material phenomena both isothermally and anisothermally for a range of testing parameters. As shown in Barrett et al., 2013 [38] a test matrix and all the relevant testing parameters were decided on to replicate the actual in-service conditions of a combustor liner of a gas turbine engine. For gas turbine engines the nature of the loading induces repeated thermomechanical stresses that gradually degrade the life. The components may experience repeated temperature cycles between ambient to 982°C (1800°F), which inherently changes the material structure and behavior of the component characterizing the complexity of material behavior. Before understanding the thermo-mechanical fatigue material behavior of Haynes 230, a broad set of isothermal low cycle fatigue (LCF) tests prescribing axial strain-controlled loading histories, included with and without hold times (strain dwells), with and without a mean strain, at temperatures ranging from 25-982°C were performed. The importance of isothermal experiments for low cycle fatigue and fatigue-creep over a scope of test parameters - temperature, strain range, strain rate, and hold times - was that it allowed for one to properly develop and validate the model. A sister paper develops the viscoplastic model for simulating anisothermal thermo-mechanical fatigue (TMF) responses of Haynes 230 [37]. Haynes 230 is a complex material that shows unique creep, oxidation, dynamic strain aging, creep-fatigue, thermomechanical fatigue, strain rate sensitivity, and strain range dependent properties.
In order to model the unique material behavior of Haynes 230 several modeling features needed to be implemented. The framework of modeling the unified cyclic viscoplasticity of Haynes 230 is of the Chaboche-type [8, 9, 35]. The adopted flow rule for the viscoplasticity is the Norton type power law adopted by Chaboche (1989) [9]. The Norton flow rule was used as it yielded reasonably good simulations for the material response in both isothermal and anisothermal experiments. Another reason why the Norton type power law was adopted was that, as discussed by Szmytka et al., 2010 [36], the evolutionary behavior of the inelastic strain rate during the relaxation phase is important as it determines the mathematical form to be used for the flow rule of a constitutive model. The concave downward shape of the semi-logarithmic plot of inelastic strain rate vs. stress during a strain dwell experiment was shown in Ahmed et al., 2013 [39], giving justification for conventional flow rules such as Norton’s power law. Limitations that were found with the Norton’s power law for viscous rate-dependent behavior included the inability to simulate saturation of rate at high strain tests as well as the exclusively positive rate-dependent behavior. Negative strain rate sensitivity, a common phenomenon of dynamic strain aging reported in similar superalloys, cannot be simulated with conventional flow rules. However, these limitations did not adversely affect simulations in anisothermal loading histories, as shown in a sister paper by Ahmed and Hassan, 2013 [37]. Nonetheless, the limitations discussed above and the modeling of creep is a future undertaking to enhance the robustness of the cyclic viscoplasticity model.

The fatigue-creep interaction caused by strain-controlled experiments with strain dwells shows cyclic stress relaxation during compressive or tensile strain holds [52]. The stress relaxation during a strain hold is a time-dependent phenomenon as a result of the viscous nature of the material. Analysis of the experimental responses [38-39] shows that Haynes 230 behaves rate-independently under cyclic loading at temperatures including and below 760°C (1400°F). However, the cyclic responses are rate-dependent at 871°C (1600°F) and higher. The isothermal stress relaxation experiments revealed that despite being overall cyclically rate-independent below and at 760°C, the material shows stress relaxation (viscous behavior) at a strain peak hold. This stress relaxation can be modeled using a static recovery term in the kinematic hardening rule, as demonstrated by the pioneering work of Chaboche and
Nouailhas, 1989 [10]. At temperatures including and greater than 871°C, the stress relaxation of Haynes 230 was modeled using a combination of rate-dependence and static recovery. Static recovery is also essential in non-linear cyclic viscoplasticity, since one also desires to simulate creep responses of a material for viscoplastic overstress, defined as the stress over yielding. Additional creep can be modeled through static recovery. Simulations of creep are not shown in this work; however, the extent of creep was shown through fatigue-creep responses of Haynes 230 during a cyclic strain-controlled experiment with strain dwells.

Cyclic fatigue-creep tests presented in Barrett et al., 2013 [38] and Ahmed et al., 2013 [39] have a significant non-zero mean stress that evolves with cycles. This suggests that mean stress evolution is a time-dependent phenomenon that must be properly accounted for in the unified constitutive model. The phenomenon of mean stress was first modeled by Yaguchi et al., 2002 [53-54] using one back stress in the kinematic hardening rule for another nickel-based polycrystalline superalloy, IN738LC at 850°C. The change in the dynamic recovery term of the kinematic hardening was introduced by Chaboche and Nouailhas [10] with the modification tensor evolution proposed by Yaguchi et al., 2002 [53-54]. The development of mean stresses as a time-dependent phenomenon and showing up for strain-dwells with varying magnitude, depending on the hold time, is more critical in the thermo-mechanical fatigue responses shown in Ahmed et al., 2013 [39]; however, this phenomena is still observed in isothermal conditions as shown in Barrett et al., 2013 [38].

As constitutive modeling for cyclic viscoplasticity incorporates various modeling features in simulating material responses for cyclic hardening/softening, rate-dependence, temperature-dependence, fatigue-creep interactions, and cyclic mean stress evolution, the parameter determination becomes complex. This is one of the primary reasons why advanced constitutive models are yet to be widely used in the industry for effective thermo-mechanical fatigue design development. Challenges in parameter determination are discussed showing the importance of a well thought-out material database for experimental validation and properly linking the experimental results for physically-meaningful parameters. The desired robustness of the advanced material model comes at a cost and that cost is usually more related to the number of material parameters. In order to maximize the attractiveness of such
an advanced material model, an automated parameter determination scheme that can be used to properly identify material behavior is developed and the necessary experiments needed to successfully find the parameters of the model are identified. Methods have also been provided to successfully determine physically meaningful parameters by linking the desired parameters for a particular model feature with an experimental response.

The model development in this study started with the existing unified viscoplastic model [8, 9, 35] in the Chaboche framework. Due to the material complexity of Haynes 230 [38-39], several features needed to be added in order to properly simulate both isothermal and anisothermal material responses. Cyclic hardening through the kinematic rule proposed by Krishna et al., 2009 [25] combined with the plastic strain memory surface theories for strain range dependence proposed by Chaboche et al., 1979 [6] were needed for Haynes 230. This cyclic hardening of a material can be properly identified as either kinematic, isotropic, or a combination of the two. Cyclic hardening through kinematic hardening may occur without isotropic hardening and vice versa. Haynes 230 isothermal cyclic stress evolution was both temperature and strain range dependent, and was kinematic in nature rather than isotropic. The main ingredients of cyclic viscoplasticity for rate-dependence and static recovery adopted are based on Chaboche, 1989 [9]. However, due to the presence of a non-zero cyclic mean stress for fatigue-creep experiments with strain dwells, the non-linear kinematic hardening rule [9] was developed further for four backstresses based on the concepts presented by Yaguchi et al., 2002 [53]. In addition, the mean stress modeling allows for better description of the evolution of the relaxed stresses seen in fatigue-creep (strain-dwell) experiments. Various features in the model for development are uniquely identified for Haynes 230 in this paper. The experimental validation of isothermal tests in the development of an advanced constitutive model is necessary before undertaking the thermo-mechanical fatigue responses. Linear interpolation of parameters for intermediate temperatures is often used for model evaluation of anisothermal material responses where temperatures change due to temperature cycling [65-67]. The use of a specific functional form for the temperature-dependent parameters is not necessary, and linear interpolation ensures that the model can best simulate the responses at isothermal temperatures.
2. Summary of Isothermal Experimental Results

The development of a robust constitutive model required a large number of experiments for Haynes 230 under a broad set of loading conditions. Before understanding the thermo-mechanical creep-fatigue interactions, the intermediate tests of isothermal low cycle fatigue (LCF) and fatigue-creep were conducted at seven different temperatures ranging from 24 to 982°C (75 to 1800°F). The types of loading histories for isothermal tests are presented in Figure 1. Figure 1a shows a typical axial strain-controlled LCF test that can be conducted at varying strain range, strain rate, and temperature to develop the constitutive model for strain range dependence, strain rate dependence, and temperature dependence. Figure 1b shows a fatigue-creep test with strain dwells at the compressive peak. In Fig. 1b, tests can be varied by strain dwell times (hold times) in addition to by strain range, strain rate, and temperature. Figure 1b allows one to characterize the isothermal fatigue-creep interactions and the viscosity of the material needed for constitutive model development.

![Fig. 1: Isothermal LCF, symmetric axial strain cycling (a) without holds, and (b) with strain holds](image)

Isothermal experiments reveal that Haynes 230 is quite a complex material with varying behavior across the temperature spectrum [38-39]. For the lower temperature ranges of 24 to 204°C (75 to 400°F) there is initial cyclic hardening followed by cyclic softening. In the intermediate temperature ranges of 427 to 760°C (800 to 1400°F) there is cyclic hardening without any stabilization. At high temperatures, i.e. greater or equal to 871°C (1600°F), there is either very little hardening or softening followed by saturation. Experiments with varying strain rates revealed that the material is rate-independent at temperatures equal to and lower than 760°C (1400°F). For temperatures equal to and greater than 871°C (1600°F) the material is rate-dependent. Experiments with strain holds (dwell periods) produce stress...
relaxation at temperatures in the rate-dependent regime, i.e. temperatures greater than or equal to 871°C (1600°F). However, at the lower temperatures of 649 and 760°C (1200 and 1400°F), though the material is mostly rate-independent with respect to different loading rates, there is still considerable stress relaxation.

3. Viscoplastic Constitutive Model

The modeling framework for the cyclic non-linear behavior chosen is a viscoplastic one. The viscoplasticity framework has been done considering a unified theory of inelasticity as shown in Fig. 2. The primary features of the unified viscoplastic Chaboche model [8-9] are presented below.

3.1 Classical viscoplasticity theory

The viscoplastic constitutive model is presented in tensor notation. The stress tensor \( \sigma \) is related to the elastic strain tensor \( \varepsilon^e \) by Hooke’s law,

\[
\sigma = E : \varepsilon^e, \tag{1}
\]

where the stress and elastic strain are second order tensors and the elasticity tensor \( E \) is of fourth order. The operation “:” is the double contracted tensor product. The additive decomposition of total strain \( \varepsilon \) into elastic and inelastic strains is considered. The sum of the elastic and inelastic strains constitutes the mechanical strain. The elastic and inelastic strain are denoted with superscript “e” and “vp” respectively,

\[
\varepsilon = \varepsilon^e + \varepsilon^{vp}. \tag{2}
\]

The elastic part obeys Hooke’s law of linear elasticity,

\[
\varepsilon^e = \frac{1+\nu}{E} \sigma - \frac{\nu}{E} (tr\sigma) I, \tag{3}
\]

where \( E \) and \( \nu \) indicate Young’s modulus and Poisson’s ratio, respectively, \( I \) is the second-order unit tensor and \( tr \) is the trace.

The major difference between viscoplasticity (time-dependent) and plasticity (time-independent) theory is that in the former, stress states are admissible in excess of the yield surface. In the Chaboche model [9], the stress in excess of the yield surface is referred to as
an “overstress” or viscous stress, which forms the basic quantity to define the viscoplastic potential. The viscoplastic potential adopted by Chaboche [9] was originally proposed by Malinin and Khadjinsky, 1972 [57] defining the power law relationship,

\[ \Omega = \frac{K}{n+1} \left( \frac{J(\sigma - a) - \sigma_o - R}{K} \right)^{n+1}, \]  

(4)

where \( \sigma_o \) is the initial size of the yield stress and \( R \) is the drag resistance (initial value zero for virgin material), which represents the isotropic hardening variable. \( K \) and \( n \) are time-dependent parameters representing the viscosity of the material.

The viscoplastic or inelastic strain can be obtained from the viscoplastic potential:

\[ \ddot{\varepsilon} = \frac{\partial \Omega}{\partial \sigma} = \frac{3}{2} \frac{\dot{p}}{J(\sigma - a)} (s - a), \]  

(5)

where \( p \) is the accumulated equivalent plastic strain, and \( s \) and \( a \) are the deviators of the stress and back stress respectively. The back stress \( a \) is the center of the yield surface. \( J(\sigma - a) \) is the von-Mises stress invariant expressed as,

\[ J(\sigma - a) = \left[ \frac{3}{2} (s - a) : (s - a) \right]^{\frac{1}{2}}. \]  

(6)

The von-Mises yield criterion used is:

\[ f(\sigma - a) = J(\sigma - a) - \sigma_o - R = 0. \]  

(7)

3.2 Flow Rule

In the framework of viscoplasticity, a viscoplastic potential surface \( \Omega \) defined by Eq. (4) (see Fig. 2) is needed for inelastic strain (viscoplastic strain). From Fig. 2 one can see that the viscoplastic potential surface lies concentric to the von-Mises yield surface with the distance separating them given by the viscous overstress, \( \sigma_v \). This excess stress or overstress is the viscous stress of the material. As a result, for rate-dependent plasticity the traditional von-Mises yield criterion Eq. (7) does not necessarily obey the classical consistency condition as stress states are admissible in excess of the elastic domain, \( f(\sigma - a) > 0 \). The viscoplastic
potential surface $\Omega$ of Eq. (4) is thermodynamically consistent with irreversible processes [9]. This viscoplastic potential defines the plastic strain rate norm, given associative viscoplasticity of the stress gradient as in Eq. (5). The form of the viscoplastic potential in Eq. (4) leads to the classical Norton’s power law

$$\dot{p} = \left( \frac{J(\sigma - \alpha) - \sigma_o - R}{K} \right)^n. \quad (8)$$

The plastic strain rate norm $\dot{p}$ of Eq. (8) is dependent on the choice of the viscosity function or viscoplastic potential $\Omega$, such as the one defined in Eq. (4). The plastic strain rate norm $\dot{p}$ of Eq. (8) is commonly expressed as

$$\dot{p} = \left( \frac{\sigma_v}{K} \right)^n, \quad (9)$$

showing the viscous stress $\sigma_v$. The effect of plastic strain and creep strain is collectively incorporated in the viscoplastic strain of Eq. (5), making it a unified approach to obtain total strain. Algebraically the viscous stress $\sigma_v$, can be determined from Eq. (9):

$$\sigma_v = K \dot{p}^n. \quad (10)$$

Fig. 2: Viscoplasticity framework in the mathematical $\pi$-plane for 3-D loading
The viscous stress of Eq. (10) allows one to understand the transition between rate-independent plasticity and rate-dependent viscoplasticity. The viscous stress accounts for the rate-dependent loading. Rate-independence can be achieved by having the viscous stress approach zero. Hence, the rate-independent model can be considered as the limiting case of rate-dependent model [35]. The form of the viscous stress of Norton’s power law (Eq. 10) makes the transition between rate-independent plasticity and rate-dependent viscoplasticity numerically easy. The limiting case of rate-independent plasticity, i.e. $\sigma_v \rightarrow 0$, can be found using $K \rightarrow 0$ and/or $n \rightarrow \infty$, as shown by Chaboche in [9, 35]. In the case of anisothermal loading presented in Ahmed and Hassan [37], the interpolation between temperatures can cause numerical difficulties when transitioning between rate-independent and rate-dependent. As a result, in order to overcome these challenges the magnitude of the viscous stress can be more suitably controlled only through the drag stress rate-dependent parameter $K$ while keeping the Norton creep exponent $n$ constant across temperatures. The case $K \rightarrow 0$ can be used to satisfy the transition between rate-independent plasticity and rate-dependent viscoplasticity.

### 3.3 Kinematic hardening and isotropic hardening rules

The Chaboche model [9] for non-linear kinematic hardening in cyclic viscoplasticity includes: (1) inelastic linear strain hardening based on Prager, 1949 [68], (2) nonlinear dynamic recovery based on Armstrong-Frederick, 1967 [5], and (3) static recovery based on creep modeling of Malinin et al., 1972 [57]. The rate-independent Chaboche kinematic hardening rule [8] is a superposition of several Armstrong-Frederick nonlinear kinematic hardening rules [5]. In our present case, four superimposed rules have been adopted to represent various regions of the back stress based on Bari and Hassan, 2000 [18]: the initial high modulus at the onset of yielding that is nonlinear but stabilizes quickly, the transient nonlinear segment given by two kinematic contributions that stabilizes at a slower rate, and finally a constant modulus segment at higher inelastic strain ranges that is linear.

The kinematic hardening evolution equation shown in Eq. (11) has the three different terms for creep-fatigue interactions of viscoplasticity: (1) inelastic linear strain hardening, (2)
nonlinear dynamic recovery, and (3) static recovery. Static recovery provides creep and thermal recovery for low strain rates and high temperatures [69]:

\[
\dot{a} = \sum_{i=1}^{4} \dot{a}_i, \\
\dot{a}_i = \frac{2}{3} C_i \varepsilon_i^{vp} - \gamma_i a_i \dot{\rho} - b_i J_{a,j}^r a_i.
\]

The second invariant of the back stress \( J_{a,j} \) is expressed as

\[
J_{a,j} = \sqrt{\frac{3}{2}} a_i : a_i.
\]

The classical isotropic hardening rule used by Zaverl and Lee, 1978 [70] and adopted by Chaboche, 1986 [8] defines the growth of the yield surface as a function of the plastic strain rate norm:

\[
\dot{R} = b (R_s - R) \dot{\rho},
\]

where \( b \) and \( R_s \) are material constants representing the rate of isotropic hardening and the total isotropic saturation size of the yield surface, respectively.

4. Various Features of the Model Development

The basic Chaboche model for uniaxial simulation of viscoplasticity was presented in Section 3. In this section, unique material characterization of Haynes 230 is provided, along with any necessary additions to the model for isothermal cyclic response.

4.1 Classical viscoplasticity theory

Due to increasing viscosity with increasing temperature summarized in Barrett et al., 2013 [38], the elastic and viscoplastic characterization of Haynes 230 shows unique material behavior that must be physically identified. In order to quantify the elastic behavior of Haynes 230 the hysteresis cycle of a strain-controlled cycle must be carefully examined. The linear response of a hysteresis cycle usually correlates to the elastic response of the material. However, the use of a linear response determination in total stress versus total strain is not the best way to determine the uniaxial yield domain, once the material becomes increasingly
viscous. This elastic characterization can be done more precisely by understanding the cyclic hysteresis loop of a typical experiment shown in Figs. 3a-b. The linear elastic response of a material is known as the elastic domain of the material given by the yield stress $\sigma_0$. In Figs. 3a-b, this elastic domain defining the yield stress can be more distinctly seen by the asymptotic slope (vertical plastic modulus) in both stress versus plastic strain (Fig. 3a) as well as strain versus plastic strain (Fig. 3b), respectively. Haynes 230 has a distinct yield stress size $\sigma_0$ for temperatures between 25-760°C. However, temperatures above 760°C, Haynes 230 have become dominantly viscous without a defined yield surface size (elastic domain). For temperatures between 25-760°C, where an elastic domain is defined in Fig. 3b, the plastic strain stays constant giving another indication that the material has a well-defined yield surface size $\sigma_0$. On the contrary, temperatures above 760°C where an elastic domain does not exist, a constant plastic strain period is absent and a viscous rounding is seen instead.

4.2 Flow rule and viscoplastic characterization

Norton’s rule defining the plastic strain accumulation for inelasticity, Eq. (9), defined by the rate-dependent parameters $K$ and $n$, is capable of simulating strain rate sensitivity and stress relaxation for fatigue-creep (strain-dwell) tests. The time dependent nature of the deformation is controlled by the viscous drag stress, $K$, and the rate of secondary creep exponent, $n$. The viscous drag stress $K$ controls the magnitude of viscous overstress in the material (Eq. 10), so for rate-independent material behavior the limiting case results in the viscous drag stress as zero. The rate of secondary creep exponent $n$ typically controls the nonlinearity of the power law. Chaboche [35] showed that the secondary creep exponent $n$ depends on the material, on the strain rate domain considered and on the temperature. It was also reported that $n=1$ corresponds to the limiting case of diffusional creep with higher creep exponent values for a material’s low viscosity range.

Another advantage of the Norton’s flow rule is that algebraically the determination of the rate-dependent parameters can be straightforward if one were able to link the viscous stress
σv physically to the hysteresis cycles of viscoplasticity. Above 760°C, Haynes 230 has become dominantly viscous without a defined yield (elastic domain). In Figs. 4a-b, similar plots to Figs. 2a-b are shown for Haynes 230 at 982°C, a rate-dependent viscous temperature, where the material does not show an elastic domain, but rather a dominating viscous response. This viscous response is characterized by a rounding of the hysteresis loop (Fig. 4a). The viscous domain of the material can be defined by understanding that the plastic strain steadily accumulates upon a cyclic reversal, even though the total mechanical strain is decreasing in magnitude during this reversal (Fig. 4b). The viscous domain of the material can be defined by looking at Figs. 4a and 4b where the point A (εp, σv) on Fig. 4a corresponds to point A’ (εp, εmax) on Fig. 4b, where the total strain is at a maximum before the cyclic reversal shown in Fig. 4b.

Fig. 3: (a) HA230 initial plastic hysteresis cycle at 24°C showing yielding and (b) yielding at same testing conditions showing asymptotic fixed plastic strain during yield
The next set of points needed to define the viscous domain is point B \((\varepsilon_{p_{\text{max}}}^p, \sigma_{p_{\text{max}}}^v)\) on Fig. 4a, where the plastic strain has reached a maximum and the total strain has decreased due to the cyclic reversal corresponding to point B’ \((\varepsilon_{p_{\text{max}}}^p, \varepsilon)\) on Fig. 4b, even though the plastic strain has reached a maximum and/or is still increasing. The creep overstress defining the viscous rounding is the stress difference between point A on Fig. 4a, where the total strain is a maximum, and point B on Fig. 4a where the plastic strain has reached a maximum. The physical definition of the viscous stress (overstress) was first reported by Pritchard and Hassan [71] for Inconel 617. Now that the viscous overstress has been physically quantified the simulation of viscous responses are significantly improved.
4.3 Cyclic hardening through the kinematic hardening rule

Haynes 230 shows cyclic strain range dependence that requires unique identification of the cyclic hardening/softening behavior of the material. The ability to differentiate cyclic hardening/softening of the material through kinematic hardening, isotropic hardening, or a combination of the two is something that is not that well understood in the literature. Krishna et al., 2009 [25] showed that while modeling through isotropic hardening can simulate peak stresses reasonably well, it can also fail to simulate the hysteresis curve shapes reasonably if the hysteresis curves experience change in shape with cycles. This change in shape with hysteresis cycles is manifested by the increase in the plastic modulus associated with kinematic hardening. In order to quantify the cyclic strain range dependence of the material, an initial hysteresis cycle can be chosen and compared to a stable hysteresis cycle. Figure 5a shows one such example, whereby the hysteresis cycles of Haynes 230 at 204°C, $\Delta \varepsilon_x = 1.20\%$ for an initial ($N = 1$), and stable cycle ($N = 5000$) are plotted to see the cyclic hardening behavior of the material for a LCF strain-controlled experiment. The cyclic evolution of the material as either isotropic, kinematic or a combination of both can be determined by taking the upgoing curves of each hysteresis cycle and shifting them to the origin in a stress vs. plastic strain plot, as seen in Fig. 5b. In Fig. 5b, it can be seen that the yielding (defining the elastic domain) for all hysteresis cycles begin more or less at the same point, indicating that the material does not isotropically harden or soften. Isotropic hardening is associated with a growth of the yield surface (linear elastic range) with cycles. The analysis of Haynes 230 similar to Figs. 5a-b for temperatures 24-982°C, shown in Barrett et al. [38], shows the absence of isotropic hardening/softening.
On the contrary, in Fig. 5b while isotropic hardening is not seen, cyclic hardening of Haynes 230 is manifested through kinematic hardening. This kinematic hardening of Fig. 5b shows an increase in the plastic modulus with hysteresis curve stiffening. The hysteresis stiffening can be represented through evolutionary equations of the $\gamma_i$ and/or $C_i$ kinematic hardening rule parameters of Eq. (11) proposed by Krishna et al., 2009 [25]. In essence, the hardening/softening response of a material can occur via the change in the hysteresis curve shape independently of isotropic hardening (growth of the linear elastic range). However, before this evolutionary equation is defined one needs to properly couple the cyclic hardening or softening nature of the material kinematically via a strain range memorization framework.

Strain range dependent modeling was introduced by Chaboche et al. [6] for simulating cyclic hardening/softening of multiple amplitude experiments. Strain range dependence is modeled by considering a strain memory surface which memorizes the prior largest plastic strain range. The memory surface equation is given by Eq. (14), with the evolution equations
of radius q and center of the strain memory surface \( \zeta \) given by Eqs. (15) and (16) respectively. \( \eta \) is a material parameter which dictates the rate of evolution to a steady memory surface, and \( H(\quad) \) is the Heaviside step function; such that \( H(g_M) = 1 \) for \( g_M > 0 \), and \( H(g_M) = 0 \) for \( g_M \leq 0 \). \( \mathbf{n}_i \) and \( \mathbf{n}^* \) are the normal vectors to the yield and memory surface respectively.

\[
g_M \left( \mathbf{e}^p - \zeta \right) = J \left( \mathbf{e}^p - \zeta \right) - q = \left[ \frac{2}{3} \left( \mathbf{e}^p - \zeta \right) : \left( \mathbf{e}^p - \zeta \right) \right]^{\frac{1}{2}} - q , \tag{14}
\]

\[
\dot{q} = \eta H(g_M) \left\langle \mathbf{n}_y : \mathbf{n}^* \right\rangle \dot{\mathbf{p}} , \tag{15}
\]

\[
\dot{\zeta} = \sqrt{\frac{3}{2}} (1 - \eta) H(g_M) \left\langle \mathbf{n}_y : \mathbf{n}^* \right\rangle \mathbf{n}^* \dot{\mathbf{p}} , \tag{16}
\]

\[
\mathbf{n}_y = \frac{3}{\sqrt{2}} \frac{\mathbf{s} - \mathbf{a}}{J(\mathbf{\sigma} - \mathbf{a})} , \tag{17}
\]

\[
\mathbf{n}^* = \frac{2}{\sqrt{3}} \frac{\mathbf{e}^p - \zeta}{J(\mathbf{e}^p - \zeta)} . \tag{18}
\]

The importance of capturing the shape of the hysteresis loops as closely as possible has been shown by researchers [25, 35] to have an impact in the overall simulation quality. Also, as shown earlier the cyclic hardening/softening responses of Haynes 230 should be modeled through only kinematic hardening as the material does not show isotropic hardening across all temperatures tested (24-982°C); however, for coupling the isotropic hardening of the model through the plastic strain memory surface, readers are referred to Krishna et al., 2009 [25]. As a result, considering cyclic hardening of Haynes 230 through the kinematic hardening rule of Eq. (11), the dynamic recovery parameters \( \gamma_i \) of Eq. (11) are varied as follows:

\[
\dot{\gamma}_i = D_{ji} \left( \gamma_i^o - \gamma_i \right) \dot{\mathbf{p}} , \tag{19}
\]

\[
\gamma_i^o = a_{ji} + b_{ji} e^{-\epsilon_i} , \tag{20}
\]
where $D_\gamma$ is a rate parameter controlling the rate of evolution of $\gamma_i$ with increment $p$ of plastic strain norm. $\gamma^\circ_i$ is the saturation value of $\gamma_i$ as a function of the strain memory surface $q$ through Eq. (20), where $a_\gamma$, $b_\gamma$, and $c_\gamma$ are material parameters. It should be noted that Hassan et al. [24] reported that the cyclic hardening/softening simulation can also be obtained through evolving the $C_i$ parameters of Eq. (11); however, as shown in Krishna et al. [25], keeping the initial plastic moduli $C_i$ constant reduces the amount of model parameters without sacrificing the quality of hysteresis loop simulation. Hence, the evolution of $C_i$ is not included in this study.

### 4.4 Importance of static recovery

At the lower temperatures of 649 and 760°C (1200 and 1400°F), though the material is mostly rate-independent with respect to different loading rates, there is still considerable stress relaxation for fatigue-creep experiments. The rate-dependent parameters $K$ and $n$ for 649 and 760°C (1200 and 1400°F) were made in the limiting case to be rate-independent, given that the experimental results showed mostly rate-independent responses. The limiting case of rate-independence is such that the viscous drag stress $K$ approaches zero. As a result, with a $K$ value that is rate-independent the viscous stress relaxation during the compressive strain hold will not occur in the simulations. This is shown in Fig. 6 where, for both 649 and 760°C, stress relaxation at the compressive strain dwell does not occur unless one considers using the static recovery terms of Eq. (11). The static recovery terms allows one to simulate any additional viscosity (stress relaxation) and creep in the experimental responses. The real model simulations will use static recovery terms of Eq. (11) for not only the rate-independent temperatures 649 and 760°C, but also for the rate-dependent temperatures 871 and 982°C. However, since temperatures 871 and 982°C are rate-dependent, the rate-dependent parameters are no longer the limiting case values of rate-independence; as a result the viscous drag stress $K$ is non-zero with a value of the creep exponent $n$. The other deficiencies in Fig. 6 are related to the experimental mean stresses that develop with cycles. As a result, the hysteresis curves suffer in stress space. The modeling of mean stress evolution will allow us
to overcome deficiencies in the simulations. Mean stress will not only improve where the hysteresis curve lies in stress space, but will also help the stress relaxation responses during the dwells, as shown in later sections.

4.5 Cyclic mean stress evolution

The cyclic evolution of Haynes 230 for fatigue-creep experiments (strain-dwells) shows non-zero mean stresses that evolve with cycles when compared to tests without strain dwells, as reported in Barrett et al., 2013 [38] and Ahmed et al., 2013 [39]. Figures 7a-d shows that with a hold time (strain-dwell) the mean stresses are non-zero and evolve with cycles. In order to show the importance of mean stress modeling, experimental responses for isothermal experiments of fatigue-creep with varying hold times (t_H = 0, 60, 120s) and their subsequent simulations are shown in Figs. 7a-d, both without mean stress modeling (Fig. 7a,c) and with mean stress modeling (Fig. 7b,d) at a particular temperature and strain range. Figures 7a-b compare the viscoplastic model without mean stress modeling (Fig. 7a) to simulations with mean stress modeling (Fig. 7b) and their fidelity for simulating experimental responses at
Figure 7a shows that for non-zero strain-dwells, without considering mean stress modeling the mean stresses are both under-predicted and lack evolutionary behavior with cycles. However, in Fig. 7b the mean stresses for non-zero strain dwells are evolutionary and simulate the experimental responses much better. Figure 7b simulates the responses better, as compared to Fig. 7a, because the modeling concept of mean stress evolution has been used. In both cases, Fig. 7a and Fig. 7b for the zero strain-dwell, the responses are identical because for mean stress modeling the evolutionary behavior is time-dependent. As a result, the simulations of mean stress modeling for a zero strain-dwell, which is a pure LCF test (without strain holds) will be nearly identical to the simulations without mean stress considered. Similar results are shown in Figs. 7c-d, where the difference between considering mean stress modeling and without it is magnified even further. Figure 7c shows the mean stress simulations at 760°C without considering mean stress modeling, while Fig. 7d shows the simulations with considering mean stress modeling. Again, the simulations for mean stresses are improved when mean stress modeling is considered even though they are unpredicted in both cases (with and without mean stress modeling).

The modeling of mean stresses for Haynes 230 was achieved by modifying the kinematic hardening rule of Eq. (11) by introducing a modification to the dynamic recovery term through a second-order tensor $\mathbf{Y}$ as follows:

$$
\mathbf{a} = \sum_{i=1}^{4} \mathbf{a}_i,
$$

$$
\dot{\mathbf{a}}_i = \frac{2}{3} C_i \dot{\varepsilon}^{ap} - \gamma_i (\mathbf{a}_i - \mathbf{Y}_i) \hat{p} - b_i J_a^{-1} \mathbf{a}_i,
$$

$$
\dot{\mathbf{Y}}_i = -\alpha_{b,i} \left( \frac{3}{2} \mathbf{Y}_{a,i} J_a^{-1} \mathbf{a}_i + \mathbf{Y}_i \right) J_a^{-1}.
$$

This form of the dynamic recovery term in the kinematic hardening rule was introduced by Chaboche and Nouailhas, 1989 [10] and later adopted by Yaguchi et al., 2002 [53-54]. Yaguchi et al., 2002 had considered one kinematic hardening rule in their study and as a result required only one modification tensor $\mathbf{Y}$. In the present study this has been extended.
to multiple kinematic hardening rules with one $Y_j$ for each back stress. Yaguchi et al., 2002 introduced an evolution equation for $Y_j$ as shown in Eq. (22).

Fig. 7. Haynes 230 hold time dependence upon cyclic stress evolution, with stress amplitude and stress mean responses as a function of cycles for various hold times $t_H = 0, 60, 120$ seconds at $\Delta \varepsilon = 0.76\%$, $T = 649^\circ C$: (a) without mean stress modeling, and (b) with mean stress modeling also at $\Delta \varepsilon = 0.64\%$, $T = 760^\circ C$: (c) without mean stress modeling, and (d) with mean stress modeling
The driving force of $\mathbf{Y}_i$ was assumed to be rate/time-dependent deformation as the dislocation networks generally form under creep conditions. In Eq. (22) $J_{a,i}$ represents time-dependent deformation. $Y_{a,i}$ represents the saturation value for the mean stress tensor $\mathbf{Y}_i$, and $\alpha_{b,i}$ is a rate parameter controlling the rate of evolution of the mean stress tensor to saturation values.

Another important aspect of mean stress modeling is that the relaxation behavior for fatigue-creep experiments is improved. Due to the time-dependent nature of the mean stress evolution, the mean stress modeling allows for better simulations for stress relaxation magnitudes as well as the rates of relaxation for varying hold times and temperatures. At 649°C, without mean stress evolution, the magnitudes of stress relaxation and the rate during the hold with time is significantly underpredicted, as seen in Fig. 8a. However, in Fig. 8a with mean stress evolution, the relaxation response during the hold in time is simulated much better. At 760°C, the difference between with and without mean stress is not much; however, the simulation with mean stress evolution is slightly better in Fig. 8b. Overall, the mean stress evolution improves the relaxation responses (magnitude and rate) during the strain dwell, while also simulating the cyclic stress responses for mean stresses.

4.6 Numerical implementation of the constitutive model

The numerical implementation of the unified viscoplasticity model presented above has been shown by Ahmed and Hassan [55]. The numerical scheme presented is the implicit radial return with emphasis placed on solving the nonlinear system of equations of the material model in conjunction with the derivation of the consistent viscoplastic tangent modulus needed in finite element analysis (FEA). A standalone implementation is also presented, which requires modifications to the strain driven approach of the algorithm known as stress return mapping. The standalone implementation has been used for various loading conditions to simulate the material responses in this paper.
5. Parameter Determination

The unified constitutive model is validated by predicting a broad set of stress and plastic strain responses presented in Barrett et al. [38]. The experimental responses needed to determine the full set of parameters of the model were developed using the loading histories in Fig. 1. The first loading history in Fig. 1a is a symmetric axial strain cycling for isothermal low-cycle fatigue (LCF), which allows one to determine at each temperature the effect of strain range and strain rate sensitivity. The other loading history needed in the development and validation of the model is an isothermal axial strain cycling with strain dwells, Fig. 1b. At each temperature for isothermal simulation a total of 35 parameters is needed to fully capture all modeling features.

The elastic parameters $E$ and $\sigma_0$ are determined for each temperature from an initial cycle in Fig. 1a that is not the monotonic path. The elastic parameters are fixed for a temperature and are independent of strain range and strain rate. The next sets of parameters

![Diagram showing stress relaxation with and without mean stress evolution for Haynes 230 at 649°C and 760°C.](image-url)
determined are the basic kinematic parameters \( C_i \) and \( \gamma_i \) of the Chaboche model for nonlinear kinematic hardening at each temperature for the loading history in Fig. 1a. \( C_i \) and \( \gamma_i \) parameters were determined for the highest strain range, highest strain rate, and the initial cycle similar to Fig. 9a, using the upgoing or downgoing portion of this initial cycle. An initial cycle should be chosen that is representative of the cyclic evolution to ensure stable simulations for anisothermal loading histories. The rate-dependent parameters \( K \) and \( n \) have been physically linked to the overstress \( \sigma_i \) from Figs. 4a-b. Since the viscous stress is physically defined and using the algebraic form of Eq. (10) for Norton’s rule one can determine estimates of the rate-dependent drag stress \( K \) and rate parameter \( n \). These initial estimates of the rate-dependent parameters \( K \) and \( n \) should then be combined with the basic Chaboche parameters \( C_i \) and \( \gamma_i \) to perform optimization of the cyclic stress amplitude of LCF strain-controlled tests of Fig. 1a at different strain rates. The basic Chaboche parameters \( C_i \) and \( \gamma_i \) found are sufficient for rate independent temperatures (25-760°C); however, for rate-dependent temperatures (760-982°C) the basic kinematic Chaboche parameters \( C_i \) and \( \gamma_i \) along with the rate-dependent parameters \( K \) and \( n \) should be found using two separate experiments that differ only by the strain rate, as shown in Figs. 12-13.

The strain range dependent parameters of Eq. (20), \( a_{\gamma_i}, b_{\gamma_i}, c_{\gamma_i} \), require a minimum of three strain ranges in the experimental database, such as those presented in Barrett et al. [38] for a loading history of Fig. 1a at each temperature. The exponential equation of Eq. (20) requires for each strain range the saturated cycle to determine the stabilized kinematic parameters \( \gamma_i^o \), giving a nonlinear regression for the material parameters \( a_{\gamma_i}, b_{\gamma_i}, c_{\gamma_i} \). The stabilized kinematic parameters \( \gamma_i^o \) at each strain range are found by keeping \( C_i \) parameters fixed, while optimizing for the dynamic recovery parameters of Eq. (11) for the stable cycle. Figure 5b shows how this process is done by simulating the stable cycle with \( C_i \) parameters fixed based on the initial cycle, while changing the initial dynamic recovery parameters \( \gamma_i \) to the optimized stabilized \( \gamma_i^o \) parameters representing the stable cycle. The size of the plastic
strain memory surface \( q \) in Eq. (20) can be found experimentally from the initial cyclic plastic hysteresis width for hardening behavior. The radius of the plastic strain memory surface for each strain range is half the width of the largest plastic strain range in the cyclic history, which for cyclically hardening behavior is typically the initial cycle as the hysteresis loop widths are ‘fatter’ since, as the material strengthens, the hysteresis widths become thin. The rate of cyclic hardening or softening is controlled by \( D_{\gamma} \) of Eq. (19). This rate parameter controlling the evolutionary behavior with accumulated plastic strain (cycles) can be found from the highest strain range, similarly to Fig. 9b. The optimization of \( D_{\gamma} \) is done using the cyclic stress evolution response of stress amplitudes with cycles through a nonlinear gradient-based optimization procedure linked to the full cyclic model, whereby the fitness measure is the deviation from the stress amplitude responses for all cycles.

Static recovery parameters for stress relaxation are linked to an experimental loading history for a fatigue-creep test, such as Fig. 1b. The basic kinematic hardening parameters \( C_i \) and \( \gamma_i \), along with the rate-dependent parameters \( K \) and \( n \), must be found simultaneously. In contrast, the strain range dependent and static recovery parameters can be found independently. In this approach, the static recovery parameters \( b_i \) and \( r_i \) are found using a similar optimization procedure for the \( D_{\gamma} \) rate parameter, where a nonlinear gradient-based optimization procedure is formulated with the full cyclic viscoplasticity model for minimizing the norm of a least squares error for stress relaxation with cycles.

By following this procedure, one can successfully link most of the model parameters to an experimental response. The simulation capability of the model without the mean stress evolution has been critically evaluated over the years and has proven to be an attractive model; however, the breadth of material parameters has been a concern in terms of the applicability of the model. In order to overcome these challenges, a parameter determination scheme is presented that builds off the work done by Rahman et al. [56] for using a hybridized genetic algorithm for parameter determination to simultaneously find rate-independent and rate-dependent parameters of \( C_i, \gamma_i, K, n \). The strain range dependent
parameters $a_{ji}, b_{ji}, c_{ji}, D_{ji}$ along with the static recovery parameters $b_i, r_i$ have been found using a local nonlinear gradient-based optimization. The mean stress evolution parameters $Y_{a,i}$ and $\alpha_{b,i}$, have been obtained from thermo-mechanical fatigue responses and have been presented in Ahmed and Hassan [37].

6. Model Simulations

The Chaboche model introduced in Section 3, with the additional features of the model developed in Section 4, is evaluated against a broad set of isothermal, uniaxial LCF, with and without a mean strain, for multiple strain ranges, strain rates, and creep-fatigue (strain dwell) across a broad temperature range (25-982°C) as presented below. The parameters used in the simulations have not been disclosed due to restrictions placed by the project sponsor.

6.1 Strain Range Dependence

The cyclic deformation of Haynes 230 is highly sensitive to temperature under low cycle fatigue conditions. One such way of understanding the temperature dependence of Haynes 230 is through the cyclic stress evolution. The cyclic stress evolution of Haynes 230 shows that, from room temperature up to 204°C (400°F), the material uniquely shows a fast cyclic hardening response wherein the strength of the material is achieved within the first 50-100 cycles, followed by a degree of cyclic softening and/or stabilization until failure (Figs. 9-10). Modeling this two stage material behavior through ordinary kinematic hardening theory is a unique challenge. The modeling of subsequent softening is not undertaken in this study; however, for temperatures greater than 204°C (Figs. 11-12) the fidelity of model simulations is still generally good. Another unique cyclic stress evolution for Haynes 230 is the slow cyclic hardening evolution, whereby hardening is achieved throughout the entire cyclic life characterized by continuous hardening, without any sign of stabilization for temperatures above 204°C to as high as 760°C (Figs. 11-12). The steep cyclic hardening nature owes to the strengthening mechanism of dynamic strain aging (DSA) along with the primary M₆C carbide precipitation [38]. At 871°C the material cyclically responds to hardening mechanisms for faster rates, while slower rates show softening behavior. At the highest
temperature tested in our experimental program, 982°C, the material cyclically softens. The highest two temperatures, 871°C and 982°C, show more or less stable responses in Fig. 17 for all strain ranges tested. As a result, the modeling for strain range dependence and the cyclic evolution, Eqs. (14)-(20), was not needed unlike temperatures between 25-760°C.

While the nature of the cyclic response during a broad range of temperatures is uniquely different, the modified Chaboche model is capable of simulating the variability in material responses. The cyclic stress behavior for hardening or softening was done through the cyclic evolution of the kinematic hardening recovery parameters $\gamma_i$ according to Eq. (19), combined with the plastic strain range memory surface modeling equations for strain range dependence, Eqs. (14)-(20). Accurately capturing the cyclic hysteretic shape along with the peak stresses is achieved through this unique evolutionary equation for multiple kinematic parameters in the modified Chaboche nonlinear kinematic hardening rule of Eq. (11). The simulation fidelity of the model is not only tested for different temperatures with unique material behavior (Figs. 9-12), but also for its capability to simulate across various strain ranges, Figs. 13a-d. Figures 13a-d show the cyclic stress evolutionary responses for peak stresses as a function of various strain ranges and a particular isothermal temperature. The hysteresis loop simulations (Figs. 9a, 10a, 11a, 12a) perform well, across strain ranges, since the dynamic recovery parameter ($\gamma_i$) of the nonlinear kinematic hardening rule evolves for each strain range according to Eq. (19). An initial set of Chaboche parameters are needed for each temperature, and the chosen strain range for parameter determination in all cases was the highest. Isothermal, strain-controlled LCF conditions for various strain ranges and temperatures have been shown to be simulated well with the modified Chaboche model.
Fig. 9: Isothermal, symmetric LCF strain-controlled simulation at 24°C for HA230 (a) hysteresis cycles and (b) cyclic stress response (hardening) showing stress amplitude and mean.

Fig. 10: Isothermal, symmetric LCF strain-controlled simulation at 204°C for HA230 (a) hysteresis cycles and (b) cyclic stress response (hardening) showing stress amplitude and mean.
Fig. 11: Isothermal, symmetric LCF strain-controlled simulation at 427°C for HA230 (a) hysteresis cycles and (b) cyclic stress response (hardening) showing stress amplitude and mean.

Fig. 12: Isothermal, symmetric LCF strain-controlled simulation at 649°C for HA230 (a) hysteresis cycles and (b) cyclic stress response (hardening) showing stress amplitude and mean.
6.2 Strain Rate Sensitivity

The strain rate dependence of loading on Haynes 230 was summarized in Barrett et al., 2013 [38] and Ahmed et al., 2013 [39]. The viscosity of a material causes loading rate effects...
along with other mechanisms like stress relaxation. This time-dependent nature of the cyclic deformation is why a viscoplasticity modeling framework is needed. In terms of modeling the strain rate sensitivity, the choice of the viscosity function is critical in simulations. The plastic strain rate norm of the flow rule; such as Eq. (8), determines the viscoplastic flow of the material. As mentioned earlier, the relation between this viscous flow or stress and the rate of viscoplastic (unified) strain is usually highly nonlinear. In our present study, the viscosity function chosen was the classical Norton’s equation for secondary creep, a power law relationship with two rate dependent parameters $K$ and $n$.

Haynes 230 shows positive rate dependent behavior above 760°C; however, below this temperature between 427-760°C the material shows a small amount of negative strain rate dependence as a result of the operating temperature domain for DSA [38-39]. The modeling of a viscoplasticity framework with a proper choice of a viscosity function allows the transition between rate-independent and rate-dependent behavior of positive rate dependence; however, the negative rate dependence has not been considered in this study. As a result, the modeling of rate independence is assumed for all temperatures below 760°C, whereas above 760°C rate-dependence has been modeled. The importance of this transition (rate-independent and rate-dependent) is reiterated due to the numerical difficulties if not done properly. In order to overcome these challenges, this transition can be more suitably controlled through the variation of the drag stress rate-dependent parameter $K$. As a result, rate-independent material behavior, as seen by HA230 below 760°C, is controlled by making the drag stress rate-dependent parameter $K$ approach zero. The magnitude of the viscoplastic flow, the viscous stress, is controlled by $K$. 
Fig. 14: HA230 simulation of strain rate dependence for three strain rates showing a saturated hysteresis cycle at 871°C (a) 0.2 cpm, (b) 2 cpm, and (c) 20 cpm

Fig. 15: HA230 simulation of strain rate dependence for three strain rates showing a saturated hysteresis cycle at 982°C (a) 0.2 cpm, (b) 2 cpm, and (c) 20 cpm
The simulation of positive strain rate dependent behavior for three different cyclic frequencies is shown in Figs. 14-17 for both 871°C and 982°C. The cyclic hysteresis loop simulations for a saturated cycle at three different strain rates are shown for 871°C in Fig. 14 and at 982°C in Fig. 15. Figures 14-15 show how, with one set of parameters at each temperature, the model is capable of simulating three different strain rates with a different cyclic stress evolution as a result of the positive rate dependent behavior. In both cases, the strain ranges were kept constant at each temperature, while the only testing variable that is different is the cyclic frequency (strain rate) of loading. Another unique observation found in this study, already presented, is the notion of a physically defined overstress. As shown earlier, this overstress (viscous) flow can be physically defined, Figs. 4a-b. The viscous rounding of the hysteresis loops results in inelastic strain accumulation to a peak, while the total strain is decreasing in magnitude, as is the case for an extended time during a strain reversal. The simulation capability of the model to properly simulate this viscous rounding is shown in Figs. 16a-b for a saturated plastic hysteresis cycle for two different temperatures and rates.

The hysteresis loop shapes during the entire cycle, including the viscous rounding, were simulated well for both temperatures across all rates. However, the stress peak responses for stress amplitude shown in Figs. 17a-b shows some limitations to the adopted flow rule. In Fig. 17a at 871°C the cyclic stress evolution for positive rate dependent behavior shows a saturation of rate between 2 and 20 cpm as the stress responses are almost superimposed. As a result, simulations suffer in peak stresses for the highest strain rate tested. Chaboche [35] reported the inability of the Norton’s flow rule to simulate rate saturation and presented two alternatives in the literature related to a sine hyperbolic and exponential form of the Norton’s power law. These flow rules were not studied in this present study; however, future work on this and the challenges related to creep simulation, while maintaining fatigue fidelity, is underway. In contrast, 982°C in Fig. 17b shows clear positive rate dependent behavior, and as a result the Norton’s flow rule for modeling this strain rate sensitivity is sufficient.
**Fig. 16:** HA230 simulation capability of viscous rounding for a saturated plastic hysteresis cycle for two different temperatures and rates (a) 871°C (0.2 cpm), and (b) 982°C (2 cpm)

**Fig. 17:** HA230 simulation of strain rate dependence for three strain rates (cyclic frequencies) showing the cyclic stress amplitude responses at (a) 871°C, and (b) 982°C
6.3 Fatigue-Creep Interactions

The introduction of a dwell, either compressive or tensile, in a strain-controlled, low cycle fatigue test causes stress relaxation for a viscous material. The inelastic deformation during a strain dwell is directly linked to a material’s viscosity. The time recovery nature of the material can be modeled through both the viscous nature of the adopted flow rule, such as Eq. (8), and the nonlinear kinematic rule of Eq. (11) through the addition of a static recovery term. As summarized in Barrett et al. and Ahmed et al. [38-39], Haynes 230 viscously responds to a strain dwell for temperatures including 649°C and above. As a result, in terms of modeling, it was previously shown that the rate-dependence of the material was found to occur above 760°C. Therefore the viscous response of stress relaxation during creep-fatigue tests below 760°C must be modeled through the static recovery term of Eq. (11) only. However, for temperatures above 760°C the viscous stress relaxation seen in creep-fatigue tests can be modeled through both the flow rule rate-dependent parameters $K$ and $n$ and static recovery parameters $b$ and $r$. The static recovery term of Eq. (11) has parameters $b$ and $r$ for each hardening rule. The rate-dependence parameters $K$ and $n$ can simulate the stress relaxation during a strain hold, because during the dwell period the viscous overstress relaxes towards the yield surface, so stress points outside the yield domain gradually relax back to the yield domain. However, once the viscous stress has completely relaxed to the yield surface, additional recovery is not possible without some other modeling feature. On the other hand, for temperatures at which the material behavior is overall rate-independent, but still shows stress relaxation for strain hold experiments, the stress relaxation can only be modeled through the static recovery terms.

The simulations of the fatigue-creep experiments were for the most part successful. One important modeling feature that was needed for anisothermal loading histories of thermomechanical fatigue experiments was mean stress evolution. The modeling of mean stress evolution is especially important for anisothermal loading histories, as shown by Ahmed et al., 2013 [37]. During a strain hold, the mean stress modeling must be accounted for even for isothermal loading histories. While the mean stresses of the isothermal fatigue-creep experiments [38] did not evolve as pronouncedly as the anisothermal loading histories
[39], the mean stresses were modeled correctly for most experimental responses, Figs. 18-21. The simulations of hysteresis loop shape for a strain dwell, fatigue-creep experiment varied by temperature. For the most part, the hysteresis loop shapes were simulated well aside from 649°C. The 649°C simulation of Fig. 18 does fairly well initially for the first cycle; however, once cycles evolve the hysteresis simulations do not cyclically harden as much as the experimental response, as seen in Figs. 18 and 22. Nonetheless, the isothermal simulations without a strain hold at 649°C (Figs. 12, 13b) do perform well; however, the simulations suffer some during the strain dwell. While the stress amplitude responses are underpredicted at 649°C, the stress relaxation responses are overpredicted, Figs. 23 and 24a.

While the stress relaxation is overpredicted at 649°C, the rate of relaxation during the strain dwell of a cycle does much better (Fig. 24a). Although the 649°C simulation was not the best, the simulations did perform much better at the other temperatures, indicating the attractiveness of the modified Chaboche model to simulate fatigue and creep-fatigue interactions in cyclic viscoplasticity. The simulation of fatigue-creep interactions for 760°C shows how the cyclic hysteresis responses for shape, peak stresses, and relaxed stresses for both magnitude and rate can be simulated well (Figs. 19, 22-24). The rate of relaxation (relaxed stress vs. time) during a hold for 760°C does much better than the lowest temperature 649°C for viscous fatigue-creep responses (Fig. 24b). While the initial cycle suffers in most cases, the cycles in between and the saturated cycle perform better (Figs. 19-21).

The amount of stress relaxation can be quantified by finding the stress difference between the stress before and the stress after a strain dwell. However, this stress relaxation $\Delta \sigma_{sr}$ is not enough to reveal some interesting trends for the various temperatures tested due to the hardening and/or softening behavior of the Haynes 230 material. In order to properly quantify the amount of stress relaxation and understand the effect of a hold time for various temperatures tested, we have calculated a normalized relaxation stress. The normalized relaxed stresses are quantified by scaling the relaxed stress $\Delta \sigma_{sr}$ by the stress amplitude of each cycle.
Fig. 18: HA230 simulation of fatigue-creep (strain dwell) cyclic responses at 649°C for a compressive strain dwell of 120 seconds showing an (a) initial cycle, and (b) saturated cycle

Fig. 19: HA230 simulation of fatigue-creep (strain dwell) cyclic responses at 760°C for a compressive strain dwell of 120 seconds showing an (a) initial cycle, and (b) saturated cycle
Fig. 20: HA230 simulation of fatigue-creep (strain dwell) cyclic responses at 871°C for a compressive strain dwell of 120 seconds showing an (a) initial cycle, and (b) saturated cycle.

Fig. 21: HA230 simulation of fatigue-creep (strain dwell) cyclic responses at 982°C for a compressive strain dwell of 120 seconds showing an (a) initial cycle, and (b) saturated cycle.
The relaxation behavior across various temperatures reveals an interesting trend, and the simulations of this normalized stress relaxation are shown in Fig. 23 for various hold times. Both hold times reveal that higher normalized stresses occur with increasing temperature, and the simulations support this same trend. The simulations for stress relaxation for both 649°C and 760°C were done using only the static recovery feature of modeling for reasons mentioned above. The two highest temperatures, 871°C and 982°C, simulated stress relaxation through the rate-dependent viscous flow rule as well as the static recovery feature of modeling. The simulations of 871°C for shape, stress peaks, and stress relaxation again were done reasonably well (Figs. 20, 22-24). Although the initial cycle suffers, as was seen with 760°C, the rest of the cyclic history is simulated with precision, as shown by the cyclic response of Fig. 20 and the stress peak responses for stress amplitude of Fig. 22. The stress relaxation rate and magnitude were captured exceptionally well for 871°C, as seen in Figs. 20, 23, and 24c. The highest temperature 982°C shows the best simulations for all important features, Figs. 21-24. The relaxation rate of stress is especially captured extremely well (Fig. 24d).
Fig. 22: HA230 simulation of stress amplitude responses for fatigue-creep (strain dwell) experiments at various temperatures (649-982°C) with a compressive dwell of (a) 60 seconds, and (b) 120 seconds.

Fig. 23: HA230 simulation of normalized stress relaxation responses for fatigue-creep (strain dwell) experiments at various temperatures (649-982°C) with a compressive dwell of (a) 60 seconds, and (b) 120 seconds.
Fig. 24: HA230 simulation of stress relaxation rates during a strain dwell of 120 seconds for various saturated cycles at temperatures (a) 649°C, (b) 760°C, (c) 871°C, and (d) 982°C.
7. Conclusions

A unified viscoplastic model of the Chaboche type has been presented for Haynes 230 in simulating a broad range of behavior. The complexity of HA230 caused us to develop new ways of understanding the material response for cyclic viscoplasticity. Novel ways of material identification of macroscopic responses for cyclic hardening/softening through isotropic or kinematic hardening have been presented for an expansive strain-range-dependent material. In addition, a physical definition of the viscous overstress has been explained, showing the physics of the underlying mechanisms macroscopically. This physical definition of the viscous overstress facilitates stable parameters in the parameter determination. As a result, simulations of the strain rate sensitivity of HA230 were done successfully for multiple strain rates during the rate-dependent regime of modeling. The ability to transition between rate-independent and rate-dependent behavior was numerically presented for the Norton’s flow rule to produce stable simulations. The ability to characterize the viscous nature of a material has been improved through these methods and helped in the understanding of fatigue-creep interactions across several temperatures. Another measure that helped in uniquely identifying the material during a fatigue-creep is through a normalization procedure of stress relaxation.

The modeling challenge was to develop a constitutive model capable of simulating the isothermal fatigue and creep-fatigue responses, as well as the thermo-mechanical creep-fatigue responses acceptably well. A Chaboche-based constitutive model with several added features has been chosen. The constitutive model has the features of rate-dependence, static recovery, kinematic hardening evolution, strain range dependence, and mean stress evolution. These features are essential to adequately describe the isothermal material responses. Additional modeling features are discussed in Ahmed et al., 2013 [37] for overcoming the challenges in thermo-mechanical fatigue simulation. The constitutive model with the mean stress evolution modeling feature was required and used for isothermal simulations. However, this effect of mean stress evolution is more significant in the simulations of thermo-mechanical fatigue.
A precise parameter determination procedure was presented using these unique ways of understanding the material, which physically gave meaning to most of the parameters. In contrast, the static recovery parameters seem to be much more unstable in terms of parameter determination than all the other parameters presented in the model. One such reason for this instability may be due to the lack of true creep information. As a result, in our future work we hope to understand more clearly how creep responses can be used to understand the material response better, such that the interactions between fatigue and creep are properly captured. This is critical in lifing analysis wherein both plasticity and creep should ideally be captured.

Simulations have been shown for a broad set of uniaxial, isothermal experimental responses exhibiting cyclic strain range dependence, strain rate dependence, and stress relaxation during a strain dwell. The modeling of isothermal material responses must be done in such a way that the validations of anisothermal thermo-mechanical fatigue responses are also successful. Ahmed et al. [37], a sister paper, has shown the fidelity of the model for anisothermal thermo-mechanical fatigue responses that were successfully modeled. Much of the success of modeling anisothermal thermo-mechanical fatigue responses depends on quality isothermal simulations, such as the ones presented in this paper. However, as shown in Ahmed et al. [37], additional modeling features for anisothermal loading are required to overcome the deficiencies in the existing model. Our future work will focus on the challenges of truly simulating stress-controlled (creep-type) experimental histories while still maintaining fidelity in strain-controlled simulations.

8. Acknowledgements

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9. References


CHAPTER 5: A UNIFIED VISCOPLASTIC MODEL WITH DAMAGE FEATURES FOR HIGH TEMPERATURE FATIGUE-CREEP AND CREEP RESPONSES OF HAYNES 230

Abstract

A Chaboche-based unified viscoplastic constitutive model, including features of strain range dependence, strain rate-dependence, static recovery, and mean stress evolution, is evaluated against the responses of Haynes 230, a nickel-based superalloy used in high temperature components. This study undertakes the development of a constitutive model to simulate not only strain-controlled fatigue and fatigue-creep responses of Haynes 230, but also stress-controlled creep responses. To simulate the tertiary regime of creep, an isotropic damage model has been used. The isotropic damage model works well for stress-controlled loading histories of creep in simulating creep rupture failure; however, some drawbacks are seen when used for strain-controlled loading histories. Also, in this study, various flow rules for the inelastic strain rate have been evaluated. An exponential Norton flow rule is capable of simulating rate saturation for strain-controlled fatigue responses. However, the choice of flow rule has minimal effect on the stress-controlled creep responses. In contrast, the model feature of static recovery has been found to be essential in the simulation of both strain-controlled fatigue-creep responses and stress-controlled creep responses. As a result, the parameter determination for these static recovery parameters requires a robust optimization algorithm. The proposed unified constitutive model can adequately simulate fatigue-creep interactions found between strain controlled loading histories of fatigue with strain dwells and stress controlled loading histories of creep.

Keywords: Haynes 230, Viscoplasticity, Constitutive Modeling, Creep, Creep-Fatigue, Damage Modeling
1. Introduction

Service-conditions of nickel-base superalloy components in the aerospace and nuclear power industries experience start-up and shut-down cycles that induce repeated thermo-mechanical stresses. Under these high temperature thermo-mechanical cycles, damage accumulates eventually leading to a component failure. The type of damage seen in these high temperature service zones can include out-of-phase thermo-mechanical fatigue, creep-fatigue damage accumulation among many other factors [1-2]. One significant factor that influences the type of damage accumulation is the length of on-load dwell periods seen at the maximum operating temperatures. Shorter dwell periods followed by cycling lead to fatigue-creep damage mechanisms; whereby, fatigue is the primary damage mechanism and creep being a secondary mechanism. However, the longer the dwell period, the more creep deformation becomes prominent in a component causing creep to be the primary damage mechanism [3-4]. The cycling of strain inherently gives fatigue-type damage; however, with the introduction of a hold time creep-type damage can be induced. The amount of damage for fatigue and creep and their combined effects determine the dominant damage mechanism of either fatigue-creep or creep-fatigue. Microscopically this interaction of fatigue and creep shows up in the crack initiation and propagation modes as either transgranular (fatigue-creep) or intergranular (creep-fatigue), as suggested by Rao et al., 1988 [5] and Rodriguez et al., 1995 [6].

In order to understand this fatigue-creep interaction an experimental study on the nickel-based superalloy must include a range of dwell periods that differs in orders of magnitude (seconds/minutes to hours/days to months/years); such that, service-conditions are replicated as closely as possible. However, as the dwell period increases, performing such experiments become expensive. As a result, an experimental study for Haynes 230 that replicates the service-conditions as closely as possible and is cost effective has been outlined and shown in Barrett et al., 2014 [7] and Ahmed et al., 2014 [8]. The experimental program of Haynes 230 included isothermal low cycle fatigue, fatigue-creep and anisothermal thermo-mechanical fatigue tests, as well as, stress-controlled pure creep tests over a range of testing parameters. Isothermal strain-controlled tests with strain dwells of 1 and 2 minutes gave insight to
fatigue-creep damage for the shorter dwell periods. In contrast, the isothermal stress-controlled tests for constant creep stress amplitudes gave insight to creep damage of Haynes 230. As a result, stress-controlled creep tests were performed until ruptured giving a dwell time scale of hours/days/months. This approach of defining two extremes of the fatigue-creep interaction with one extreme being strain-controlled fatigue experiments with and without dwell times on the order of seconds/minutes and the second extreme being stress-controlled creep experiments with dwell times on the order of hours/days/months is not only useful for model development, but also is much more cost effective. The fatigue-creep interactions as the dwell time increases to 2 years or longer as seen in nuclear power plant operations are not known or data is not available.

The constitutive modeling of creep has traditionally been done separately from the cyclic viscoplasticity model based on equations motivated by the various phases seen in creep responses. As a result, creep equations have been developed that are exclusively for creep responses that are not applicable for cyclic viscoplasticity. Some of the more simplified creep models in the literature consist of using creep rupture data of high temperature alloys and modeling each stage of creep through separate creep equations that are mathematically motivated by the shape of the creep curve [9]. At low enough stresses and temperatures, most materials may exhibit an asymptotic primary creep that resembles the bounded creep on linear viscoelasticity, for which secondary and tertiary creep never appears. For stresses high enough; however, this primary creep is usually modeled by some logarithmic form \( \varepsilon \propto \ln(t) \) or even a power law form \( \varepsilon \propto t^\alpha \) \( 0 \leq \alpha \leq 1 \) that is given by the classical Andrade’s law for primary creep \( \alpha = 1/3 \).

During the secondary creep phase, a steady-state creep strain rate is typically approximated using a Norton type power law form \( \dot{\varepsilon}_{ss} = B\sigma^n \). At higher stresses, an exponential function is sometimes used for the steady-state creep strain rate [9]. The rate processes of creep is usually thermally activated and modeled using the Arrhenius type temperature dependence [10]. Other creep models also focus on each stage of creep; however, the models are physically enriched by the underlining mechanisms of each stage
Creep analysis of a nickel-base superalloy, Nimonic 263, for various stresses and temperatures were investigated by Maldin et al., 2007 [13]. The integrated constitutive equations for the creep strains are of the sine hyperbolic form, which is made a function of various microstructural parameters, like dislocation density.

In all cases, these models again are done only considering the creep responses of a material. As a result, the creep equations are developed without considering the interactions of viscoplastiscity and their constitutive framework is not applicable for cyclic response simulations. By consequence, in order to try and simulate cyclic responses, a non-unified theory is imposed which considers the partition between plastic (cyclic) and creep strains [14-15]. Non-unified models of rate-independent plasticity (low cycle fatigue) and rate-dependent inelastic deformation (creep) have been found to be incapable of predicting certain deformation features such as cyclic creep, also known as ratcheting, and creep-plasticity interaction [16]. Moreover, Zienkiewicz and Cormeau [17] argued that the phenomenon of creep and plasticity cannot be treated separately as only the combined effect is measurable. The experiments of Ikegami and Niitsu [18] show a coupling between plastic strain and creep strain, by way of the associated hardening effects. This has led to researchers adopting unified theories [19-22] where all aspects of inelastic deformation such as plasticity, creep and stress relaxation are represented by a single inelastic strain.

The gap between strain-controlled fatigue-type cyclic behavior and stress-controlled creep behavior is one that must be resolved for design. This research shows the ability to simulate both fatigue and creep using a unified viscoplastic constitutive model that has been missing in the literature. A unified viscoplastic constitutive model has been shown in Barrett et al., 2014 [23] and Ahmed et al., 2014 [24] to simulate well strain-controlled responses of isothermal fatigue, fatigue-creep, and thermo-mechanical fatigue responses. However, the simulation of stress-controlled creep responses has yet to be investigated. The unified constitutive model developed in [23]-[24] shows promising results for stress-controlled creep simulations while also maintaining the high fidelity of simulation for strain-controlled responses.
In addition, as part of this study a comprehensive evaluation of various flow rules for rate saturation is presented for strain-controlled loading histories. In the unified framework for viscoplasticity, the choice of the flow rule which dictates the viscoplastic strain rate and its dependency on the viscous stress was studied qualitatively by Chaboche [22]. Szmytka et al., 2010 [25] showed that the relation between stress and the plastic strain rate norm is usually a highly nonlinear relationship that requires typically a power-function form like the classical Norton’s law for secondary creep [26], hyperbolic sine-function form [27], or lastly an exponential form [28] to capture the time-dependent viscous nature of the material. Rate saturation requires the viscous stress to saturate with increasing viscoplastic strain rate. For Haynes 230, the exponential Norton showed the best rate saturation for strain-controlled responses. However, in regards to the performance of stress-controlled creep simulations, all flow rules were comparable given the minimal effect it has on the total inelastic strain accumulation.

The unified viscoplastic constitutive model presented in this paper to simulate the stress-controlled responses of creep was also modified using principles of continuum damage mechanics for effective stress [29-30] to model an isotropic scalar damage variable of the Kachanov creep type [31]. The definition of a mechanical damage variable using the effective stress concept allows one to apply strain equivalence, such that, the stress calculated over a section area that effectively resists the forces is given by a damage variable $D$ that is the relative corrected area of cracks and cavities [29]. As a result, this damage variable $D$ represents a surface density of discontinuities in the material and for isotropic damage; cracks and cavities are uniformly distributed in all directions. The principle of strain equivalence allows one to write a simple relationship between the Cauchy stress $\sigma$ and the effective stress $\tilde{\sigma}$ such that all deformation is only affected by damage in the form $\tilde{\sigma} = \sigma / (1 - D)$. The isotropic scalar damage variable $D$ is an evolutionary internal variable that is coupled with cyclic damage through this effective stress relationship. The coupled damage model allows one to model better tertiary creep for stress controlled loading histories. However, the strain-controlled low cycle fatigue simulations are affected by this damage variable, which causes some limitations. The unified viscoplastic model is presented
in Section 2. The various challenges for the development of the constitutive model for Haynes 230 are presented in detailed in Section 3. After thoroughly discussing all the challenges for model development, comprehensive model simulations are presented in Section 4 that show the strengths and weaknesses of the unified viscoplastic model with and without isotropic scalar damage for simulation of strain-controlled histories of fatigue-creep (strain dwell) and stress-controlled histories of creep.

2. **Viscoplastic Constitutive Model**

A unified viscoplastic constitutive model has been developed to simulate the wide range of experimental phenomena observed for strain-controlled isothermal and anisothermal loading histories of Haynes 230 in Barrett et al., 2014 [7] and Ahmed et al., 2014 [8] respectively. A description of the current state-of-the-art constitutive model has been given in Barrett et al., 2014 [23] for simulating a broad set of uniaxial, isothermal strain-controlled experimental responses of Haynes 230 exhibiting cyclic strain range dependence, strain rate dependence, and stress relaxation during a strain dwell. Anisothermal loading histories presented several other challenges to the model, whereby additional modeling features of mean stress evolution and maximum temperature influence developed by Ahmed et al., 2014 [24] proved to be invaluable. The unified constitutive model is capable of simulating the wide range of out-of-phase and in-phase thermo-mechanical fatigue hysteresis loops, stress amplitude and mean stress, and stress relaxation responses.

The primary features of the unified viscoplastic modified Chaboche model [22, 32-33] that was developed and extended by Barrett et al., 2014 [23] and Ahmed et al., 2014 [24] are first summarized below in Section 2.1 for convenience of discussion. The study of various flow rules are presented in Section 2.2 showing the importance of rate saturation. Finally, the unified constitutive model with the isotropic scalar damage variable is presented in Section 2.3.
2.1 Unified viscoplastic model

The unified viscoplastic constitutive model requires that the additive decomposition of total strain $\varepsilon$ into elastic and inelastic strains is considered. The sum of the elastic and inelastic strains constitutes the mechanical strain. The elastic and inelastic strain are denoted with superscript “e” and “vp” respectively,

$$\varepsilon = \varepsilon^e + \varepsilon^{vp}.$$  (1)

The major difference between viscoplasticity (time-dependent) and plasticity (time-independent) theory is that for viscoplasticity stress states are admissible in excess of the yield surface. This can be best understood by looking at the mathematical $\pi$-plane for 3D loading as shown in Figure 1. In Fig. 1, a viscoplastic potential surface $\Omega$ lies concentric to the von-Mises yield surface.

![Viscoplasticity framework in the mathematical $\pi$-plane for 3-D loading](image)

In the Chaboche model [33], the stress in excess of the yield surface is referred to as an “overstress” or viscous stress, which forms the basic quantity to define the viscoplastic potential. The viscoplastic or inelastic strain can be obtained from the viscoplastic potential:

$$\varepsilon^{vp} = \frac{\partial \Omega}{\partial \sigma} = \frac{3}{2} \dot{p} J \frac{\mathbf{s} - \mathbf{a}}{(\sigma - \alpha)} ,$$  (2)

where $\dot{p}$ is the pressure derivative, $J$ is the determinant of the Jacobian, and $\alpha$ is the deviatoric stress tensor.
where \( p \) is the accumulated equivalent plastic strain, and \( s \) and \( a \) are the deviators of the stress and back stress respectively. The back stress \( \alpha \) is the center of the yield surface.  

\[ J(\sigma - \alpha) \] is the von-Mises stress invariant expressed as,

\[
J(\sigma - \alpha) = \left[ \frac{3}{2} (s - a) : (s - a) \right]^{\frac{1}{2}}. 
\]

(3)

The von-Mises yield criterion used is:

\[
f(\sigma - \alpha) = J(\sigma - \alpha) - \sigma_o - R = 0. 
\]

(4)

The back stress \( \alpha \) has a rate equation expressed in its stress deviator form \( a \) that models the center of the yield surface. The translational evolution of the center of the yield surface is known as kinematic hardening. The kinematic hardening evolution equation shown in Eq. (5) has the four different terms for creep-fatigue interactions of viscoplasticity: (1) inelastic linear strain hardening, (2) nonlinear dynamic recovery with mean stress evolution, (3) static recovery, and (4) anisothermal temperature rate terms

\[
\dot{\alpha} = \sum_{i=1}^{4} \dot{a}_i, 
\]

\[
\dot{a}_i = \frac{2}{3} C_i \epsilon^v - \gamma_i (a_i - Y_i) \dot{p} - b_i J_{a,i} \eta - a_i + \frac{1}{C_i} \frac{\partial C_i}{\partial T} \dot{T} a_i. 
\]

(5)

With modeling for the mean stress evolution driven by

\[
\dot{Y}_i = -\alpha_{b,i} \left( \frac{3}{2} Y_{st,i} \frac{a_i}{J_{a,i}} + Y_i \right) J_{a,i} \eta. 
\]

(6)

In Eq. (6) \( J_{a,i} \) represents time-dependent deformation, \( Y_{st,i} \) represents the saturation value for the mean stress tensor \( Y_i \), and \( \alpha_{b,i} \) is a rate parameter controlling the rate of evolution of the mean stress tensor to saturation values.

The size change of the von-Mises yield surface can be modeled through an isotropic rate equation. Isotropic hardening is associated with a growth of the yield surface (linear elastic
range) with cycles. Equation (7) defines the growth of the yield surface as a function of the plastic strain rate norm:

\[ \dot{R} = b(R_s - R) \dot{\rho}, \]  

(7)

where b and R_s are material constants representing the rate of isotropic hardening and the total isotropic saturation size of the yield surface, respectively.

2.2 Flow rule investigation

Chaboche, 2008 [22] showed that the relationship between the plastic strain rate norm \( \dot{\rho} \) and the viscous stress is usually highly nonlinear. For certain materials, such as Haynes 230 it has been shown in Barrett et al., 2014 [7] that a saturation of rate effect ensues for high strain rates. The simulation of Haynes 230 positive rate dependent behavior at 871°C (1600°F) suffers for the highest strain rate as shown in [23] when the classical Norton’s power law is used. The classical Norton’s power law has the form

\[ \dot{\rho} = \left( \frac{J(\sigma - \alpha) - \sigma_v - R}{K} \right)^n. \]  

(8)

The plastic strain rate norm \( \dot{\rho} \) of Eq. (8) is commonly expressed as

\[ \dot{\rho} = \left( \frac{\sigma_v}{K} \right)^n, \]  

(9)

showing the viscous stress \( \sigma_v \). The effect of plastic strain and creep strain is collectively incorporated in the viscoplastic strain of Eq. (2), making it a unified approach to obtain total strain. In order to achieve saturation of rate, the viscous stress must saturate with increasing plastic strain rate, i.e. rate. One such variation of the Norton’s power law relationship is a sine hyperbolic function expressed as

\[ \dot{\rho} = A \sinh \left( \left( \frac{\sigma_v}{K} \right)^n \right). \]  

(10)
The sine hyperbolic function form of Norton’s equation was first proposed mathematically by Sellars et al. [27], however, the use of this particular flow rule has yet to be used for modeling a material specifically for saturation of rate effect. Szmytka et al., 2010 [25] showed the hyperbolic sine function form of Eq. (10) in relation to the importance of the evolution of the inelastic strain rate with stress during the relaxation phase of a strain-dwell experiment. Ahmed et al., 2014 [8] showed that for Hayne 230 a concave downward evolution of the inelastic strain rate with stress manifests suggesting that the Norton’s rule of Eq. (8) and the sine hyperbolic function of Norton of Eq. (10) is suitable for simulation. In Eq. (10) the saturation parameter $A$ along with the sine hyperbolic function is intended to saturate the viscous stress at the high strain rate regime.

Another variation of the Norton’s power law relationship investigated in this study is the exponential form of Norton’s equation proposed by Kocks et al. [28], which is expressed as

$$\dot{p} = \left(\frac{\sigma_v}{K}\right)^n e^{\alpha (\frac{\sigma_v}{\kappa})^n}.$$  \hspace{1cm} (11)

Again, like the sine hyperbolic function form of Eq. (10), the exponential form of Norton’s equation has also not been used for saturation of rate effect for a specific material. In Eq. (11), the Norton’s equation is multiplied by an exponential with a power of the Norton equation scaled by a saturation parameter $\alpha$. However, the sine hyperbolic function form of Norton of Eq. (10) along with the exponential form of Norton of Eq. (11) was qualitatively investigated by Chaboche, 2008 [22] for possible saturation effects. A similar qualitative mathematical investigation is performed first before investigating the detailed simulations of the rate saturation for HA230.
All three flow rules, equations (9-11) are plotted in Fig. 2 to show the possible effect of rate saturation. Figure 2 shows that the exponential form of Norton’s equation Eq. (11) gives the best possible saturation of rate. The Norton’s equation Eq. (9) essentially gives a linear relationship for the viscous stress in relation to the plastic strain rate, while the sine hyperbolic form (Eq. 10) gives a slightly non-linear relationship. Nonetheless, the exponential form of Norton’s equation Eq. (11) gives a highly non-linear relationship for the viscous stress in relation to the plastic strain rate. The amount of non-linearity for the viscous stress is important because it implies that with increasing plastic strain rate the viscous stress begins to asymptotically saturate, which is what is desired in the high strain rate regime.

The saturation of rate effect is revisited from Barrett et al., 2014 [23] for Haynes 230 at 871°C (1600°F) using the exponential Norton of Eq. (11) and comparing it to the traditional Norton’s equation of Eq. (9). The isothermal strain-controlled experiments of 871°C at a constant strain range of 0.53% at varying strain rates were presented in Barrett et al., 2014 [23]. In Figs. 3a-b, Haynes 230 exhibits a rate saturation going from 2 cycles per

![Fig. 2: Rate saturation of Norton, sine hyperbolic Norton, and exponential Norton flow rules](image-url)
minute (cpm) to 20 cpm where the stress amplitude response superimposes on another with peak stresses of similar magnitude. The simulation of the stress peak responses for stress amplitude are shown below for the Norton equation Eq. (9) in Fig. 3a and the exponential Norton Eq. (11) in Fig. 3b. In both cases, the strain ranges were kept constant at each temperature, while the only testing variable that is different is the cyclic frequency (strain rate) of loading. The loading rates of 0.2, 2 and 20 cycles per minute (cpm) correspond to strain rates in the order of $10^{-5}$, $10^{-4}$ and $10^{-3}$ per second respectively. The exponential Norton (Eq. 11) flow rule simulates the saturation of rate effects correctly as shown in Fig. 3b, where the Norton flow (Eq. 9) failed to do so as shown in Fig. 3a.

Fig. 3: HA230 simulation of strain rate dependence for three strain rates (cyclic frequencies) showing the cyclic stress amplitude responses at 871°C for (a) Norton’s equation, and (b) exponential Norton

The flow rule used determines how much viscous stress is developed for the loading cycle. This viscous stress contributes to the total stress of the cycle. As a result, the amount of saturation of the viscous stress in the high strain rate regime effectively controls if
The viscous stress of the saturated cycles for all cyclic frequencies (0.2, 2, and 20 cpm) are compared for the Norton’s equation Eq. (9) and the exponential Norton Eq. (11) in Fig. 4. Figure 4 shows that for the highest cyclic frequency (strain rate) 20 cpm the amount of viscous stress for the exponential Norton is much less than the traditional Norton. However, the viscous stresses of the lower two cyclic frequencies (rates) of 2 and 0.2 cpm are essentially superimposed in Fig. 4. As a result, the simulation of the saturation of rate effect in Fig. 3b for the exponential Norton Eq. (11) is achieved.

The sine hyperbolic function form of Norton with $A=1$ Eq. (10) performs similar to Norton’s in the rate saturation for this particular temperature and material. Other saturated values $A$ of the sine hyperbolic function along with varying rate dependent parameters $K$
and $n$ were explored; however, no better parameter set was found to achieve the saturation at this temperature 871°C for Haynes 230. The sine hyperbolic function form does not provide rate saturation, due to the near linear relationship for the viscous stress in relation to the plastic strain rate as seen in Fig. 2. Nonetheless, it can be clearly seen from Figs. 2, 3b, and 4 that the exponential Norton does provide saturation of the rate effect.

2.3 Isotropic damage modeling

Chaboche, 1988 [29-30] showed that a scalar isotropic damage model can be coupled with cyclic damage through the effective stress relationship where the Cauchy stress $\sigma$ and the effective stress $\tilde{\sigma}$ are related through the following relationship,

$$\tilde{\sigma} = \sigma / (1 - D).$$

As a result, the effective stress is always greater than or equal to the Cauchy stress $\tilde{\sigma} \geq \sigma$. The isotropic scalar damage variable $D$ is an evolutionary internal variable that is coupled with cyclic damage through this effective stress relationship. The isotropic scalar damage variable $D$ varies from $D = 0$ where the material is non-damaged to $D = 1$ where at the moment of fracture the volume element representing the material breaks into two parts. Consequently, the effective stress varies from $\tilde{\sigma} = \sigma$ to $\tilde{\sigma} \rightarrow \infty$ at these two respective limits. This allows one to write the constitutive laws in terms of the effective stress, such that Hooke’s law for the Cauchy stress tensor $\sigma$ now becomes,

$$\sigma = E : \varepsilon (1 - D),$$

where the Cauchy stress and elastic strain are second order tensors and the elasticity tensor $E$ is of fourth order. The viscoplastic or inelastic strain of Eq. (2) in Section 2.1 is still valid; however, the plastic strain rate norm $\dot{\varepsilon}$ defined from the viscoplastic potential surface is modified based on the isotropic scalar variable $D$. The plastic strain rate norms for the three investigated flow rules of Section 2.2: Norton’s equation, sine hyperbolic Norton, and exponential Norton can be expressed in the damage form respectively as

$$\dot{\varepsilon} = \left( \frac{1}{1 - D} \right) \left( \frac{\sigma_v}{(1 - D)K} \right)^n,$$

where $\dot{\varepsilon}$ is the plastic strain rate, $\sigma_v$ is the yield stress, and $K$ is the hardening parameter.
\[
\dot{p} = A \sinh \left( \frac{1}{1-D} \right) \left( \frac{\sigma_v}{(1-D)K} \right)^n, \tag{15}
\]

\[
\dot{p} = \left( \frac{1}{1-D} \right) \left( \frac{\sigma_v}{(1-D)K} \right)^{\frac{1}{r_{\text{dam}}}} e^{\frac{1}{(1-D)K}}. \tag{16}
\]

Finally, the isotropic scalar damage variable \( D \) is an evolutionary internal variable of the Kachanov creep type [31] that is represented by the rate equation,

\[
\dot{D} = \frac{J_2(\sigma)}{(1-D)A_{\text{dam}}}^{r_{\text{dam}}}. \tag{17}
\]

\( J_2(\sigma) \) is the von-Mises stress invariant expressed as,

\[
J_2(\sigma) = \left[ \frac{3}{2} s : s \right]^{\frac{1}{2}}. \tag{18}
\]

The Kachanov creep type damage of Eq. (17) depends on the state of stress given by \( J_2(\sigma) \), however, since it is coupled with the cyclic damage stresses, strains, and the isotropic damage are calculated simultaneously. Equation (17) is numerically integrated explicitly, since the damage is updated once the unknown stress state is found in the radial return algorithm. The damage variables \( A_{\text{dam}} \) and \( r_{\text{dam}} \) control the damage evolution, whereby increasing values of either damage variable results in slower damage evolution. It should be noted that the exponent \( r_{\text{dam}} \) must be smaller than the rate dependent parameter \( n \) giving \( r_{\text{dam}} \leq n \) for numerical stability. Damage can effectively be made non-existent in the model by taking the drag stress damage variable \( A_{\text{dam}} \rightarrow \infty \) as seen in Eq. (17). By making the drag stress damage variable significantly large, the damage never evolves and thus the damage variable stays at the non-damaged state of \( D = 0 \).
3. Model Development for Simulation of Fatigue, Fatigue-Creep, and Creep Responses

The unified viscoplastic modified Chaboche model for uniaxial simulation was presented in Section 2. Section 2.1 showed the primary ingredients of the model that was used to simulate all the uniaxial strain-controlled isothermal experiments of Barrett et al., 2014 [7] and presented in Barrett et al., 2014 [23]. Section 2.1 provided the essential viscoplastic ingredients of strain decomposition, unified viscoplastic strain rate and von-Mises yield criterion. Section 2.1 also showed the complete kinematic hardening rule with features of mean stress, static recovery, and temperature rate terms developed fully in Barrett et al., 2014 [23] and Ahmed et al., 2014 [24]. However, for features related to strain range dependence and cyclic hardening through kinematic evolution readers are referred to Barrett et al., 2014 [23]. In addition, readers are referred to Ahmed et al., 2014 [24] for the novel features of maximum temperature influence on the cyclic hardening and the complete anisothermal influence on the model development. Section 2.2 revealed that for isothermal strain-controlled experiments saturation of rate effect is possible through an exponential Norton rule. Finally, the coupled isotropic damage model for viscoplasticity was presented in Section 2.3. In the following, unique material characterization of Haynes 230 is provided, along with the various challenges to successfully model both isothermal stress-controlled loading histories of creep and strain-controlled histories of fatigue and fatigue-creep.

3.1 Creep simulation without damage modeling and parameters based on strain-controlled responses

The unified viscoplastic model summarized in Section 2.1 and presented fully in Barrett et al., 2014 [23] had at each temperature for isothermal simulation a total of 38 parameters needed to fully capture all strain-controlled responses. The parameter set consists of 2 elastic parameters $E$ and $\sigma_o$, 8 rate-independent kinematic parameters $C_i$ and $\gamma_i$, 2 rate-dependent parameters $K$ and $n$, 8 static recovery parameters of $b_i$ and $r_i$, 6 mean stress parameters $Y_{st,i}$ and $\alpha_{b,i}$, and finally 12 strain range dependent parameters $a_{\gamma i}, b_{\gamma i}, c_{\gamma i}, D_{\gamma i}$. The elastic
parameters of $E$ and $\sigma_o$ can be found using a simple regression analysis. Out of the 36 remaining parameters, only 18 of those parameters comprise the main core of the parameter determination, whereby simultaneous evaluation of the parameters should be done. The other 18 parameters: $(a_{ji}, b_{ji}, c_{ji}, D_{ji})$ and $(Y_{is,i}, \alpha_{b,i})$ all can be found independently once the previous core parameters are found.

These 18 core parameters consist of the following: $(C_i, \gamma_i)$, $(K, n)$, and $(b_i, r_i)$. The rate-independent parameters $C_i$ and $\gamma_i$ along with the rate-dependent parameters $K$ and $n$ were found simultaneously using a hybridized genetic algorithm that builds off the work done by Rahman et al. [34]. However, the other 8 core parameters related to static recovery $b_i$ and $r_i$ were found independently using a local nonlinear gradient-based optimization after the $(C_i, \gamma_i)$ and $(K, n)$ were finalized. The static recovery parameters presented in Barrett et al., 2014 [23] were found from isothermal strain-controlled experiments with strain dwells, where the fitness measure was the stress relaxation with cycles. The full set of parameters used for simulation in Barrett et al., 2014 [23] have been used to simulate the creep strain responses of Haynes 230. Haynes 230 was tested at creep temperatures of 649°C, 760°C, 871°C, and 982°C. The creep stresses at a given temperature were tested incrementally at stresses above the elastic domain of the material.

The creep responses of Haynes 230 at 871°C considering the parameter determination outlined in Barrett et al., 2014 [23] and for which the static recovery parameters were found from fatigue-creep experiments are presented in Fig. 5. Figure 5 shows stress levels of 55, 62, and 69 MPa for creep strain responses that exhibit the primary parabolic creep behavior in the initial creep loading. Figure 5 shows that the creep simulation is overpredicted for all creep stresses. To understand why over prediction occurs for all creep stress levels, one can examine the state variables during the creep loading with and without certain modeling features that drives the creep accumulation.
The state variables of interest include the total creep strain, total strain, elastic strain, inelastic (viscoplastic) strain, total stress, viscous stress, and kinematic backstress. The creep stress level of 55 MPa at 871°C has been chosen for analysis to understand which modeling features in the unified viscoplastic model contribute to the creep accumulation. Figures 6-8 show three different cases for the unified viscoplastic model presented in Barrett et al., 2014 [23]. The three different cases are creep simulations of Haynes 230 at 55 MPa and 871°C with certain modeling features of the unified constitutive model included for simulation. The unified viscoplastic model for isothermal modeling of Barrett et al., 2014 [23] and summarized herein has the primary features of a flow rule (Eq. 2), mean stress evolution (Eq. 6), and static recovery of the kinematic backstress (Eq. 5) which all contribute to the creep accumulation.

Fig. 5: Creep simulation of Haynes 230 for various creep stresses at 871°C using strain-controlled parameter determination of static recovery parameters
Fig. 6: Creep simulation of Haynes 230 at 55 MPa and 871°C using strain-controlled parameter determination parameters Case I (primary features included: flow rule, static recovery, and mean stress evolution) showing (a) total creep strain, (b) strain decomposition, and (c) stress decomposition.
Fig. 7: Creep simulation of Haynes 230 at 55 MPa and 871°C using strain-controlled parameter determination parameters Case II (primary features included: flow rule and mean stress evolution) showing (a) total creep strain, (b) strain decomposition, and (c) stress decomposition.
Fig. 8: Creep simulation of Haynes 230 at 55 MPa and 871°C using strain-controlled parameter determination parameters *Case III* (primary features included: Norton flow rule) showing (a) total creep strain, (b) strain decomposition, and (c) stress decomposition.
These primary features of a flow rule, mean stress evolution, and static recovery all contribute to the creep strain accumulation in different ways. Norton’s flow rule of Eq. (5) gives the power law form of the inelastic strain rate. The mean stress evolution of Eq. (6) is integrated in the constitutive equation for the kinematic backstress accounting for nonlinear dynamic recovery (second term in Eq. 6), which helps determine the center of the yield domain accounting for mean stress effects. In addition, static recovery effects can be considered in the modeling of the kinematic backstress given by the third term of Eq. (5). An accurate representation of the center of the yield domain is important as it helps determine the viscous (overstress) from Fig. 1. This viscous stress is needed for the calculation of the inelastic strain rate norm of Eq. (8), which is used in the calculation of the total inelastic strain of Eq. (2) and directly influences the amount of creep strain accumulation possible.

The first case (Case I) is the complete unified viscoplastic constitutive model with all the primary features of a Norton’s flow rule Eq. (2), mean stress evolution Eq. (6), and static recovery Eq. (5) included for simulation of the creep experiment. The second case (Case II) is the unified constitutive model without considering the effects of static recovery in the kinematic backstress (Eq. 5), but having the Norton flow rule Eq. (2) defined and including mean stress evolution Eq. (6) in the kinematic backstress. The third case (Case III) for comparison is the unified constitutive model without static recovery (Eq. 5) and mean stress (Eq. 6) in the kinematic backstress, but of course defining the inelastic strain rate via the Norton flow rule. As a result, for Case III, the only modeling feature driving the creep strain accumulation is the flow rule as it relates to the inelastic strain (Eqs. 1-2) while the kinematic backstress used for simulation is the classical time-independent Chaboche form [32] without features of static recovery and mean stress evolution. For the three cases, the strain-controlled parameters of the unified viscoplastic model have been used to simulate the creep responses. The first case is shown in Figs. 6 whereby all primary features (flow rule, static recovery, and mean stress evolution) of the unified constitutive model of Barrett et al., 2014 [23] have been used to simulate the creep responses. Each feature activated in the model for simulation of Haynes 230 adds to the creep strain accumulation, as a result, Case I (Figs. 6) shows the highest creep accumulation of all the cases. In comparison, Case II in Figs. 7
include the primary modeling features of a unified viscoplastic flow rule and the modified
Chaboche model form for the kinematic backstress of Eq. (5) with mean stress evolution, but
without static recovery (third term). For Case II, static recovery is effectively ignored in Eq.
(5) by making the static recovery parameters $b_i = 0$. Therefore, unlike Case I (Figs. 6), Case
II (Figs. 7) does not include the static recovery of the kinematic backstress. As a result,
Figure 7a shows significantly less creep accumulation (maximum creep strain of 0.06%) com-
pared to Fig. 6a (maximum creep strain of 75%) when the static recovery features is not
included. The large difference in creep accumulation is directly linked to the fact that Case I
(Figs. 6) includes static recovery in the calculation of the kinematic backstress Eq. (5), while
Case II (Figs. 7) does not.

Strain decomposition of Eq. 1 comprises the creep strain accumulation for the unified
model and is shown in Figs. 6b and 7b for Case I and Case II respectively. The elastic
component of the strain is equivalent for both cases as it is temperature dependent and is
controlled through classical Hooke’s law. However, the viscoplastic strain accumulation for
both cases differs. As seen in Fig. 6b, the viscoplastic strain is almost linearly driven with
great magnitude when static recovery is included (Case I). In contrast, for Case II in Fig. 7b
where static recovery is not included, the viscoplastic strain is parabolically (transiently)
driven with minimal magnitude. These two phenomena can be further explained by looking
at the stress responses of Figs. 6c (Case I) and 7c (Case II). The total uniaxial stress is plotted
along with the viscous stress and the total kinematic backstress in Figs. 6c (Case I), 7c (Case
II), and 8c (Case III). For all three cases, the total uniaxial stress is the same, since it is a
stress controlled creep experiment. In each case, the creep stress is ramped up to desired
stress level (55 MPa) in seconds, before the creep stress is held for the desired simulation
time (100 hours). In viscoplasticity, the total stress is an additive linear superposition of the
yield stress, kinematic backstress, viscous stress, and isotropic hardening of the material as
seen in Fig. 1. For uniaxial stress states, the total uniaxial stress is simply the linear addition
of all these components, $\sigma_x = \sigma_0 + \alpha_x + \sigma_v + R$. For all three cases, the temperature tested
was $871^\circ$C, which is a rate-dependent temperature for Haynes 230 as shown in Barrett et al.,
and Ahmed et al., 2014 [8], showed no noticeable elastic region, thus giving a zero yield surface size and does not exhibit isotropic hardening. As a result, the total uniaxial stress for the rate dependent temperatures is comprised of only the kinematic back stress and the viscous stress.

Figures 6c (Case I), 7c (Case II), and 8c (Case III) has the evolution of the viscous stress and the total kinematic backstress plotted in order to better understand how viscoplastic strain is accumulated. In Fig. 6c, the total kinematic backstress shows relaxation, while for Fig. 7c the backstress does not relax. The total kinematic backstress relaxes in Fig. 6c, since static recovery is included in Case I giving the kinematic backstress static recovery effects during a stress dwell. In contrast, the total kinematic backstress for Case II in Fig. 7c does not relax, since static recovery was not included. Also, comparing the viscous stresses of Figs. 6c and 7c, it is clear that the viscous stress is prevented from fully relaxing to the yield surface in Fig. 6c (Case I), while in Fig. 7c (Case II) the viscous stress fully relaxes back to the yield surface. Figure 6c (Case I) shows an upward increasing rate for the viscous stress that is identical in magnitude to the kinematic backstress linear decreasing rate caused by the static recovery. In contrast, Fig. 7c (Case II) shows the full viscous stress relaxation back to the yield surface, that is at the yield point (zero yield surface size) for the rate-dependent temperatures. The static recovery feature included in the kinematic backstress (Eq. 5) for Case I results in a higher inelastic strain as compared to Case II (without static recovery), since the net effect of the relaxation of the kinematic backstress (center of the yield surface) from the viscoplastic potential surface of Fig. 1 with a subsequent increase in the viscous domain is an almost linear plastic flow. The increase in the viscous domain away from the yield surface allows for additional creep strain accumulation. As a result, the static recovery nature of the unified viscoplastic model provides an almost linear influence on the inelastic strain accumulation.

Meanwhile, in looking at Case III in Figs. 8, which only includes the unified viscoplastic flow rule with the basic time-independent Chaboche model for the kinematic backstress (without static recovery and mean stress evolution), the total kinematic backstress in Fig. 8c is constant over the stress dwell (creep), while the viscous stress asymptotically approaches
zero (yield point) as was the case for Case II (Fig. 7c), since both Case II and Case III do not have static recovery. The absence of time recovery effects (static recovery) in Cases II and III causes the creep strain accumulation to be solely dependent on the amount of viscous stress from the yield surface. This is often referred to as an overstress, since the yield surface is concentrically offset to the viscoplastic potential surface by this viscous stress as seen in Fig. 1. As a result, the viscoplastic potential surface of Fig. 1 will reduce back to the yield surface as indicated by the exponential drop of the viscous stress in Figs. 7c (Case II) and 8c (Case III) during a stress dwell. Also, from Figs. 7b and 8b one can see that if the flow rule for the viscous stress is the sole driver of the inelastic strain accumulation then the response is very limited in terms of the amount of creep strain possible. Without static recovery, the viscous domain will time-dependently relax back down to the yield surface, which is zero for this temperature considered, and once there the inelastic deformation will plateau and asymptotically be constant as was the case for Figs. 7a-b and 8a-b. The influence of mean stress is also seen here, but very limited for this particular case. The influence of mean stress can be seen comparing Fig. 7b (Case II with mean stress) and Fig. 8b (Case III without mean stress). In comparing Figs. 7b and 8b, one can see that Fig. 7b has a higher amount of inelastic strain than Fig. 8b since Fig. 7b corresponds to modeling with mean stress evolution (Case II) while Fig. 8b is without mean stress evolution (Case III).

These three cases have allowed us to better understand the importance of the primary isothermal model features (flow rule, mean stress evolution, and static recovery) for simulation of the stress-controlled loading history of creep. The creep strain accumulation is significantly dependent on the inclusion of static recovery in the Chaboche model for the kinematic backstress. While the choice of the flow rule does have an influence on the strain rate dependent responses of a material as summarized in Section 2.2, the flow rule choice is insensitive to the nature of the creep response for the unified viscoplastic model. As a result, for all three flow rules in Section 2.2, the influence on the creep strain accumulation (minimally parabolic in nature) is nearly identical as the power of various flow rules are only manifested in the strain-controlled responses for rate behavior Figs. 2-4.
Another invaluable modeling aspect of static recovery is that it is capable of predicting creep responses and stress relaxation even for rate-independent temperatures as explained in Barrett et al., 2014 [23] and Ahmed et al., 2014 [24]. While the static recovery modeling feature is invaluable in simulating creep strain responses, it must be used with caution as discussed with respect to Figs. 5-8. The caution in the static recovery is that unlike the rate-independent kinematic parameters $C_i$ and $\gamma_i$ along with the rate-dependent flow rule parameters $K$ and $n$, the static recovery parameters of $b_i$ and $r_i$ are difficult in giving physically meaningful initial estimates. As a result, a gradient based optimization searching a large parameter set is always limited by some local minima for a multi-modal objective function in minimizing the total root mean squared error for strain-controlled and stress-controlled experiments. In addition, the static recovery parameters presented in this section have been physically linked to a fatigue-creep experimental loading history, but as seen by Fig. 5 this does not ensure that creep experiments are properly being modeled.

3.2 Creep and Fatigue-Creep simulation without damage considering stress-controlled responses for static recovery parameter determination (gradient-based)

The static recovery parameters that were found from isothermal strain-controlled fatigue-creep (strain dwell) experiments and used to simulate all the strain-controlled isothermal responses of Haynes 230 in Barrett et al., 2014 [23] and anisothermal responses in Ahmed et al., 2014 [24] have proven to be physically consistent for simulating the unique stress relaxation responses seen during strain-controlled dwells (fatigue-creep) and thermo-mechanical fatigue responses. However, as seen in Section 3.1, stress-controlled experiments, like the classical creep strains are prone to over prediction with the unified viscoplastic model if the static recovery parameters are physically linked to an isothermal strain-controlled (fatigue-creep) experiment. In order to explore the model further and find the appropriate physical mechanism, parameter determination of the static recovery parameters were found from only stress-controlled creep experiments at rate-independent temperatures ($649^\circ$C, $760^\circ$C) and rate-dependent temperatures ($871^\circ$C, $982^\circ$C).
Fig. 9: Creep simulation of Haynes 230 at 871°C for various creep stresses using stress-controlled parameter determination of static recovery parameters

Fig. 10: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses using stress-controlled parameter determination of static recovery parameters at 871°C for a compressive strain dwell of 120 seconds showing an (a) initial cycle, and (b) saturated cycle
Each of these creep experiments were tested under a steady stress and a specified temperature. For evaluating the static recovery parameters the lowest creep stress level was used. The creep temperature of 871°C for all creep stresses has been chosen for analysis to better understand how calibrating only the static recovery parameters from a single creep test, while keeping all other parameters fixed based on analysis in [23-24] can improve the simulations significantly for stress-controlled responses, while also maintaining acceptable degree of accuracy for fatigue-creep strain-controlled responses.

At 871°C the lowest creep stress tested was 55 MPa and thus the static recovery parameter determination was found from this stress-controlled test through a nonlinear gradient-based optimization procedure linked to the full cyclic model, where the fitness measure was the creep strain accumulated. Inequality constraints were introduced in the optimization procedure by ensuring that a maximum and minimum bound of 25% for the secondary (linear) creep rate is achieved. This particular constraint was chosen since the nature of the static recovery modeling feature in the kinematic backstress has found to be capable of predicting secondary (linear) creep behavior (Figs. 6).

Subsequently, the simulations of stress-controlled creep responses at 871°C are presented in Fig. 9 for two creep stresses 62 MPa and 69 MPa, using the stress-controlled parameter determination of the static recovery parameters at a creep stress of 55 MPa. Figure 9 shows that the simulation of the creep responses improves significantly compared to Fig. 5 where before all creep simulations were grossly over predicted. This makes sense as the static recovery parameters have now been found directly from the creep stress-controlled loading history. However, what makes this approach advantageous is that not only are the creep simulations improved, but the simulation of strain-controlled fatigue-creep (strain dwell) cyclic responses at 871°C are still reasonably accurate as seen in Figs. 10 for a compressive strain dwell of 120 seconds. The stress-controlled parameter determination of the static recovery parameters leads to loss in fidelity compared to the strain-controlled parameter determination of the static recovery parameters indicated in Figs. 10. However, while the simulation of the strain dwell experiments do suffer some for the stress-controlled parameter determination of the static recovery parameters as seen in Figs. 10, it is not of the same
extent when static recovery parameters are found from the fatigue-creep (strain dwell) strain-controlled experiments and simulating the creep strains indicated in Fig. 5 with the significant overprediction in the creep strains. To improve the simulation further, both a gradient-based optimization procedure for both the creep stress-controlled experiments and fatigue-creep (strain-dwell) strain-controlled experiments as well as a hybridized genetic algorithm building off the work of Rahman et al. [34] and shown in Barrett et al. [35] have been evaluated. The genetic algorithm has shown to be advantageous for large parameter sets and the ability to avoid initial estimates of the solution set that is so critical for a gradient-based optimization.

3.3 Creep and Fatigue-Creep simulation without damage considering both strain and stress-controlled responses for static recovery parameter determination (genetic algorithm)

In Section 3.2 we saw that physically linking the static recovery parameters to the stress-controlled creep experiments was much more advantageous for the unified viscoplastic model, since not only was fidelity achieved for creep simulations Fig. 9, but also reasonable accuracy was seen for fatigue-creep strain controlled experiments Figs. 10. A hybridized genetic algorithm presented in Barrett et al. [35] has been leveraged to achieve better accuracy for both mechanisms creep (stress-controlled) and fatigue-creep (strain-controlled). This hybridized genetic algorithm utilizes an initial local gradient optimization guided by physically linked parameters followed by the systematic stochastic search classically seen in a genetic algorithm. Two constraints were imposed on the optimization that included the creep constraint for secondary (linear) creep rate and a stress relaxation magnitude constraint for the final total stress relaxation given by the stress difference from before and after the hold. While the genetic algorithm is classically an unconstrained global optimization, these constraints were imposed by linear penalties on the objective function.

Figure 11 shows the performance of the unified viscoplastic model for creep simulation at 871°C when this parameter determination methodology is adopted. In comparing Fig. 9 and 11, it is clear that the creep simulation of Fig. 11 is significantly better. The secondary (linear) creep rates for all three creep stress levels are consistent with the experiments in Fig.
11. The tertiary (nonlinear increasing slope) creep regime in Fig. 11 is underpredicted due to the linear steady-state secondary creep nature of the response that is dominated by the model. This secondary (linear) creep nature of the model is the capability of the model when damage is not considered. Tertiary (accelerating) creep is generally regarded as resulting from structural changes leading to a loss of strength and ultimate creep fracture [9]. Cavities and grain boundary cracks can form during creep due to plastic strain incompatibilities. Plastic strain incompatibilities at larger strains will lead to creep rupture fracture [11-13]. Due to the physical nature of the tertiary (accelerating) creep regime, a damage modeling feature must be introduced into the unified viscoplastic model that tries to simulate the damage mechanisms that ensues leading up to creep rupture. The unified viscoplastic model for cyclic viscoplasticity has thus been modified considering a scalar isotropic damage approach outline by Chaboche [29-30] to try and achieve some accuracy in the tertiary regime. The introduction of the scalar isotropic damage modeling in the unified viscoplastic model, introduces two additional parameters, $A_{\text{dam}}$ and $r_{\text{dam}}$ in Eq. (17) at the rate-dependent temperatures. The damage parameters are found manually; since the values can be guided by the rate-dependent parameters, where $r_{\text{dam}}$ must be smaller than the rate dependent parameter $n$ giving $r_{\text{dam}} \leq n$ and $A_{\text{dam}}$ can be of similar magnitude of the rate-dependent $K$. The fatigue-creep (strain dwell) strain-controlled simulation at 871°C for an initial and saturated cycle are shown in Figs. 12. The fatigue-creep simulations in Figs. 12 for cyclic shape, stress peaks, and stress relaxation are done exceptionally well compared to the simulations of Figs. 10. The stress relaxation hysteretic cycles are improved due to the increasing effect of the static recovery on the kinematic backstress evolution and by consequence the total stress hysteretic cycle. In Figs. 10 one can see that the hysteretic cycles are shifted in the tensile direction and the peaks are indicative of hardening with positive slopes. In contrast, the cyclic hysteresis loops of Figs. 12 exhibit recovery of hardening indicative of static recovery at the stress peaks. As a result, the cyclic hysteresis loops saturate in hardening with increasing strain and begin to relax as seen by the tensile stress peak of Fig. 12b.
Fig. 11: Creep simulation of Haynes 230 at 871°C for various creep stresses using both stress-controlled and strain-controlled responses in a genetic algorithm for static recovery parameters.

Fig. 12: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses using both stress-controlled and strain-controlled responses in a genetic algorithm for static recovery parameters at 871°C for a compressive strain dwell of 120 seconds showing an (a) initial cycle, and (b) saturated cycle.
3.4 Creep and Fatigue-Creep simulation with isotropic damage

The unified viscoplastic constitutive model for isotropic scalar damage presented in Section 2.3 has been developed for Haynes 230 for simulation of the stress-controlled responses of creep and strain-controlled responses of fatigue-creep. The isotropic scalar damage $D$ is an evolutionary internal variable that is coupled with cyclic damage through the effective stress relationship Eq. (12). The isotropic damage variable also modifies the plastic strain rate norm $\dot{p}$ given by the adopted flow rule. The plastic strain rate norms for the three investigated flow rules of Section 2.2: Norton’s equation, sine hyperbolic Norton, and exponential Norton can be expressed in the damage form respectively as Eqs. (14-16). The isotropic scalar damage is that of a Kachanov creep type [31] that is represented by the evolutionary Eq. (17). The damage variables $A_{\text{dam}}$ and $r_{\text{dam}}$ control the damage evolution, whereby increasing values of either damage variable results in slower damage evolution.

The stress-controlled responses of creep are once again revisited considering the continuum damage principles outlined in Section 2.3. The creep stress level of 55 MPa at 871°C has been chosen for analysis again to compare the effects of with and without damage modeling. The flow rule used for this particular study was Norton’s Eq. (14). Figure 13 shows the creep simulation with and without the isotropic Kachanov creep damage. In Fig. 13, one can see that the simulation of creep is still reasonable without damage as it is capable of predicting the secondary (steady-state) creep rate as was already shown in Fig. 11 and explained in Section 3.3. However, in order to achieve the amount of damage required for tertiary (nonlinear accelerating) creep, additional creep damage is required besides the cyclic damage. As a result, with Kachanov creep damage integrated in the unified viscoplastic model one is capable of modeling better the tertiary creep for stress controlled loading histories. Figure 13 shows the performance of the unified viscoplastic model with damage as it is capable of predicting not only the primary and secondary regimes of creep, but also the tertiary regime leading up to creep rupture.
Fig. 13: Creep simulation of Haynes 230 at 55 MPa and 871°C comparing the unified viscoplastic model with and without isotropic Kachanov creep damage modeling

Fig. 14: Creep simulation of Haynes 230 at 55 MPa and 871°C showing (a) isotropic damage evolution, and (b) the accumulated Δp (%) influence with and without damage
Figures 14a and 14b displays why the unified viscoplastic model with isotropic scalar damage is capable of predicting all three phases of creep. The isotropic damage evolution for the Kachanov creep type Eq. (17) is plotted as a history variable in Fig. 14a during the creep loading to show the driving force behind the creep accumulation. The Kachanov creep damage evolution is a power law that includes an isotropic drag stress damage variable $A_{dam}$ and a creep damage exponent $r_{dam}$. It is very similar to the Norton’s power law Eq. (9) for the plastic strain rate norm, however, it also includes a layer of recursion for the evolutionary variable as the damage rate $\dot{D}$ of Eq. (17) is inversely dependent on the damage difference $(1-D)$. As a result, the nature of the damage evolution is highly nonlinear as seen in Fig. 14a. The state variable that the damage variable $D$ directly influences in the unified viscoplastic model is the plastic strain increment $\Delta p$ that produces the creep strain accumulation in Eq. (17) for stress-controlled loading. The accumulated plastic strain can be investigated to show the direct correlation of the isotropic damage variable to the plastic strain increment as shown in Fig. 14b. Without the Kachanov creep isotropic damage, the plastic strain increment at each time step for the Norton’s equation is that of Eq. (9). By consequence, when the full unified viscoplastic model (including static recovery and mean stress) is used without the damage the nature of the accumulated plastic strain increment in Fig. 14b is primarily dominated by the secondary creep response. However, once the damage is introduced into the unified viscoplastic model the damage evolution of Fig. 14a drives the plastic strain increment in a similar manner to give the final stage of tertiary creep.

One limitation of the isotropic damage model for stress-controlled creep loading histories is that it is insensitive for rate-independent temperatures. The insensitivity of the rate-independent temperatures to the coupled damage evolution is related to how the plastic strain norm $\dot{p}$ that determines the plastic strain increment is handled numerically for rate-independent and rate-dependent temperatures. This transition numerically and the limits for rate-independent modeling was explained in Barrett et al. [23], however, it will be revisited for understanding why the rate-independent temperatures are insensitive to the isotropic damage model. At rate-independent temperatures, the rate-dependent isotropic drag stress
parameter $K$ is made nearly zero ($K \approx 0$) but not zero to avoid singularity in Eqs. (9-11). At the same time, for the isotropic damage cases Eqs. (14-16), the damage difference $(1 - D)$ is always approaching zero as the isotropic damage variable $D$ evolves towards $D = 1$ indicating a fully damaged material according to Eq. (17). The isotropic damage variable is a temporal evolution that will always approach this limit with any choice of $A_{\text{dam}}$ that is not an undamaged limit ($A_{\text{dam}} < \infty$). As a result, at fully rate-independent temperatures where $K \approx 0$ the plastic strain increments will never be modified, such as the rate-dependent case shown in Fig. 14b, for any permissible selection of the damage parameters $0 < A_{\text{dam}} < \infty$ and $r_{\text{dam}} > 0$ except at the point of complete damage $D = 1$ where the unified viscoplastic model indicates creep rupture/fracture and the strain approaches infinity.

The fatigue-creep strain-controlled experiments with strain dwells are also investigated for the coupled isotropic damage model of viscoplasticity. Figures 15a and 15b shows the comparison of the model with and without the isotropic Kachanov creep damage of the fatigue-creep cycles for simulation at 871°C and a 120 second compressive strain dwell. At first glance, it may seem like a minimal difference comparing the unified model with and without damage, however there are some differences. For the initial cycle Fig. 15a the damage evolution has just started to evolve temporally and has no significant effect on the hysteresis stress relaxation cycle. However, as seen in Fig. 15b, the saturated cycle in the experiment for damage has begun to soften the hysteresis loop. The softening of the hysteretic cycle comes from the concept of an effective stress that was explained in Section 2.3 corresponding to Eqs. (12-13). The cyclic stress, which is the Cauchy stress, will always be reduced by this damage difference $(1 - D)$ given in Eq. (13). The nature of this damage is isotropic and manifests itself in the strain-controlled loading histories, such as the fatigue-creep (strain dwell) experiments, as a softening (damage) mechanism. In essence, the Cauchy cyclic stress is effectively reduced by the isotropic damage, hence softening. At this particular rate-dependent temperature of Haynes 230 at 871°C, the isotropic damage model has proven to be physically consistent in phenomenologically capturing the material response.
Fig. 15: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses for the unified viscoplastic model with isotropic Kachanov creep damage at 871°C for a compressive strain dwell of 120 seconds showing an (a) initial cycle, and (b) saturated cycle.

Both loading histories of strain-controlled fatigue-creep and stress-controlled creep have been successfully modeled using the coupled isotropic damage model for unified viscoplasticity of Haynes 230 at 871°C (Figs. 13-15). However, additional limitations do exist for the unified viscoplastic with isotropic Kachanov creep damage. One such limitation is that while the damaged hysteretic cycles of Figs. 15 did not show a fully damaged cycle, the possibility is still there as the evolutionary rate equation for the Kachanov creep damage in Eq. (17) will always evolve toward $D = 1$ for any permissible selection of the damage parameters $0 < A_{dam} < \infty$ and $r_{dam} > 0$. In Fig. 15b, the final experimental cycle recorded for this fatigue-creep test was cycle 597 and is noted in [23] as the saturation cycle. For simulation purposes, the unified viscoplastic model with isotropic Kachanov creep damage can be ran out until failure for the fatigue-creep loading history beyond the experimentally saturated cycle 597 to show that the hysteretic cycles will eventually approach zero as they isotropically soften (damage) as will be shown in Section 4.2.
4. Model Simulations

The unified viscoplastic Chaboche model introduced in Sections 2, with features of the model, such as static recovery, mean stress evolution, and strain range dependence have been coupled with and without an isotropic Kachanov creep damage in order to successfully capture the wide range of responses seen in strain-controlled histories, but also stress-controlled histories of creep. Section 3 tried to convey the various challenges encountered in attempting to capture similar, but different physics involving fatigue, creep, and fatigue-creep interaction. Sections 3.1-3.3 explained this discontinuity between the two extremes for high temperature design considering fatigue and creep as respective limits. In the unified viscoplastic Chaboche model for the kinematic backstress the static recovery feature of Eq. (5) in Section 2.1 has been proven to be an invaluable modeling feature that is capable of achieving fidelity in both extremes of modeling considering fatigue, creep, and fatigue-creep interactions if properly calibrated. Parameter determination of these static recovery parameters have shown to be a significant challenge that requires both a robust optimization algorithm as well as the right linking of physics.

Sections 3.1-3.2 explained how static recovery parameters seem to be more physically consistent with parameter determination against stress-controlled loading histories of creep, while all other parameters included in the model that have been found previously from the required strain-controlled responses presented in [23-24] are held constant for reasonable accuracy of both stress-controlled and strain-controlled simulations. However, to improve simulation fidelity further static recovery parameters can be simultaneously evaluated by both creep and fatigue-creep (strain-dwell) experiments as shown in Section 3.3. In addition, Section 3.3 concluded that a hybridized genetic algorithm should be leveraged for the large parameter set and minimizing the amount of iterations required in the parameter determination process usually associated with a gradient-based optimization routine due to the limitation of the initial starting guess and the breadth of parameters needed for searching [35].

Finally, Section 3.4 showed specifically the capability of the unified viscoplastic model when isotropic damage is coupled. Model simulations against a broad set of isothermal,
uniaxial fatigue-creep (strain dwell) and stress-controlled creep loading histories across a broad temperature range (649-982°C) are presented below for the unified viscoplastic constitutive model with and without isotropic damage. The parameters used in the simulations have not been disclosed following the contract with the project sponsor.

4.1 Creep simulation with and without isotropic damage

Haynes 230 has shown to creep at temperatures as low as 649°C, which corresponds to roughly 50% of the melting point. The creep stress levels at a given temperature are determined by the elastic domain of the material. As a result, creep stresses are always above the yield limit of a material. The cyclic yield domains of Haynes 230 have been presented in Barrett et al., 2014 [7]. The creep stresses presented here are all above these limits. Haynes 230 was tested at creep temperatures of 649°C, 760°C, 871°C, and 982°C. It has been shown in the strain-controlled analysis for rate dependency [7] that temperatures 649°C and 760°C are considered to be rate-independent, while temperatures 871°C and 982°C are rate-dependent.

The simulations of rate-dependent temperatures are presented first for the various creep stress levels tested at each temperature. Figures 16-17 show the capability of the unified viscoplastic model without and with damage respectively for simulating the creep strains at 871°C. Both simulation sets in Figs. 16-17 achieve high fidelity for all three creep stresses. The damaged case in Fig. 17 improve on the simulations without damage in Fig. 16 by capturing not only the primary (transient) and secondary (steady-state) creep regimes, but also the tertiary (accelerating) regimes. The phenomenological reason of why the tertiary creep is captured better with the isotropic Kachanov creep damage modeling is that tertiary creep is predominantly associated with a damaged representative volume element of the material, which the isotropic damage tries to capture in conjunction with the cyclic viscoplasticity [29].
Fig. 16: Creep simulation of Haynes 230 at 871°C without isotropic damage evolution

Fig. 17: Creep simulation of Haynes 230 at 871°C with isotropic damage evolution
The simulated creep fracture for the damage case occurs at 1400 hours for a creep stress level of 55 MPa, while the experiment failed in creep at 1475 hours. In Fig. 17, the damage model causes the creep simulation to eventually reach creep rupture as \( D \to 1 \) at which the creep shows an asymptotic infinite slope indicating failure. As for the unified viscoplastic model \textit{without} damage in Fig. 16, the simulations can be run out further than the creep fracture times of the experiments, by which the creep strain accumulation would extrapolate out at about the same specified secondary creep rate that was seen in the experimental data region. One potential advantage for the unified viscoplastic model \textit{without} the Kachanov creep damage is the ability to extrapolate out the simulation data further than the domain that was tested experimentally. This could be a potential advantage over the damaged model, since one set of damage parameters are used for all creep stresses at each temperature and as seen by the creep stress level of 55 MPa in Fig. 17, the asymptotic creep failure can occur earlier than expected. However, nonetheless if the tertiary creep regime is a prominent material response the secondary (linear) creep rate characteristic of the unified viscoplastic model without damage would under predict the accelerating (nonlinear) tertiary creep phase and a damage model should be considered.

The simulations of creep strains for the highest temperature in our material database, 982°C, once again shows the power of the unified viscoplastic model without and with isotropic damage. In Fig. 18, the unified viscoplastic model simulates well the primary and secondary creep regimes at 982°C for the various creep stresses. As was the case with 871°C, the tertiary creep regime is underpredicted with the unified constitutive model \textit{without} any damage features. Figure 19 on the other hand shows the unified constitutive model with isotropic damage and a similar conclusion can be made compared to 871°C in that the tertiary creep regime is captured along with the other stages of creep reasonably well. However, like 871°C, at 982°C a few of the creep stress levels show early prediction of creep fracture. Overall, both simulation sets for creep at the rate-dependent temperatures in considering without and with isotropic damage modeling perform exceptionally well.
Fig. 18: Creep simulation of Haynes 230 at the rate-dependent temperature of 982°C without isotropic damage evolution.

Fig. 19: Creep simulation of Haynes 230 at the rate-dependent temperature of 982°C with isotropic damage evolution.
The rate-independent temperatures of 649°C and 761°C have been evaluated for Haynes 230 and are presented in Figs. 20-21 respectively. For these temperatures, damage modeling was not considered, since the results would be superimposed on one another up until the point of fracture, whereby stress singularity would eventually occur (infinite asymptotic slope) as explained in Section 3.4. For rate-independent temperatures, damage modeling does not influence the creep strains, because the inelastic strain increment Eqs. (9-11) is never modified for any permissible selection of damage parameters with a rate-independent $K \approx 0$.

The creep simulations of 649°C are by far the worst amongst all temperatures as seen in Fig. 20. At 649°C, the responses are overpredicted and are physically inconsistent to the other temperatures in predicting the steady-state secondary creep. The reason for the inconsistency at 649°C is directly related to the static recovery parameters. At 649°C, the static recovery parameters approached the limits $b_i \rightarrow 0$ and $r_i \rightarrow 1$ in the optimization process, due to the minimal amount of stress relaxation that develops in the fatigue-creep compressive strain dwell experiments. Therefore, the fatigue-creep responses were able to maintain better fidelity over the creep responses at 649°C as will be shown in Section 4.2.

The distinct parabolic shape of the creep simulation of 649°C in Fig. 20 is thus directly linked to the fact that only two modeling features are effectively being used in the viscoplastic simulation, which are the flow rule and the kinematic backstress with mean stress evolution as discussed in Section 3.1, Case II of Figs. 7, since the static recovery parameters approach the limits where static recovery is effectively turned off in the simulation. By consequence, as was explained in Section 3.1, without the influence of static recovery the simulation considering only the flow rule and mean stress as the primary drivers of the creep accumulation will always result in a parabolic shape. The magnitude of this parabolic shape is related to the amount of creep stress level applied in the simulation, with increasing distinct parabolic effect given an increasing creep stress. Nonetheless, the rate-independent temperature of 760°C performs with similar fidelity to the rate-dependent temperatures as seen in Fig. 21. The linear secondary creep rates are predicted for a range of creep stresses in Fig. 21 at 761°C.
Fig. 20: Creep simulation of Haynes 230 at the rate-independent temperature of 649°C

Fig. 21: Creep simulation of Haynes 230 at the rate-independent temperature of 760°C
4.2 Fatigue-Creep simulation with and without isotropic damage

The ability to predict the creep response of a material is an invaluable tool for design of hot section components of high temperature materials, such as, Haynes 230. In addition, to predicting the stress-controlled creep phenomena at various temperatures and stress levels, it is important to characterize the cyclic fatigue behavior and the fatigue-creep interaction experiments characteristic of strain-controlled histories with compressive or tensile strain dwells. The unified viscoplastic model without and with isotropic damage is capable of maintaining such fidelity across two domains (creep and fatigue) that are classically separated in modeling.

The fatigue-creep simulations without isotropic damage are presented first considering the methodology outlined in Section 3.3. The rate-independent temperatures of 649°C and 760°C are presented first for stable cycles at one and two minute holds. Figures 22-23 show stable cycles for the rate-independent temperatures at the one and two minute holds. The unified viscoplastic model shows that it can also predict the strain-controlled responses of fatigue-creep. The prediction of the stress relaxation, hysteresis shape and size, as well as the stress peaks for the fatigue-creep experiments have been simulated well for the rate-independent temperatures (Figs. 22-23), while also maintaining fidelity in the creep (stress-controlled) simulations (Figs. 20-21) with the same set of parameters. Rate-dependent temperatures of 871°C and 982°C are also presented for stable cycles at one and two minute strain holds. Once again, the ability to predict the prominent features of fatigue-creep for stress relaxation, hysteresis shape and size, as well as stress peaks is shown in Figs. 24-25 for the rate-dependent temperatures respectively.

The isotropic damage model coupled with the unified viscoplastic model has shown to be invaluable in predicting tertiary (accelerating) creep for rate-dependent temperatures (Figs. 16-19) in Section 4.1. However, as discussed in Sections 3.4 and 4.1, rate-independent temperatures are insensitive to the isotropic damage model, due to how the plastic strain norm \( \hat{p} \) is handled numerically for rate-independent temperatures. As a result, as was the case for creep (stress-controlled) loading histories at rate-independent temperatures, the simulations of fatigue-creep (strain-controlled) loading histories with and without the
isotropic damage model are identical, since $K \approx 0$ and the plastic strain increment is never modified, for any permissible selection of the damage parameters $0 < A_{\text{dam}} < \infty$ and $r_{\text{dam}} > 0$ to cause a difference in the simulation, so in Figs. 22-23 only the simulations without isotropic damage are shown.

Nonetheless, the rate-dependent temperature fatigue-creep (strain-controlled) loading histories are affected by the isotropic damage model. The nature of this damage is isotropic and manifests itself in the strain-controlled loading histories, such as the fatigue-creep (strain dwell) experiments, as a softening (damage) mechanism. As discussed in Section 3.4 there are some limitations to this damage (softening) mechanism in the hysteresis cycles. Figures 26a-b compares the loading history of a fatigue-creep experiment with 60 seconds compressive holds (Fig. 26a) to the simulation of that experiment with the isotropic damage model (Fig. 26b) at 871°C. The initial cycle is underpredicted, since for simulation the material is modeled as a stable material with isotropic damage (softening). In contrast, the simulation of the run-out stable cycle (cycle 600) is done with high accuracy. One should also notice that the simulated cycles are all nearly superimposed (Fig. 26b). As a result, the simulated cycles have barely begun to isotropically soften even for the experimental run-out cycle (last recorded cycle), due to the slow evolutionary nature of the isotropic scalar damage variable. In this particular case, because of this slow evolutionary nature of the isotropic damage variable, the fidelity of the fatigue-creep simulation does not suffer with the isotropic damage model.

However, at the highest temperature, 982°C the limitations of the fatigue-creep simulation with the isotropic (softening) damage mechanism is shown in Figures 27a-b. Figures 27a-b compares the loading history of a fatigue-creep experiment with 60 seconds compressive holds (Fig. 27a) to the simulation of that experiment with the isotropic damage model (Fig. 27b) at 982°C. Unlike, Figs. 26a-b for the 871°C experiment, the 982°C in Figs 27a-b shows some significant softening damage in the hysteresis cycles. While softening is seen in the experiment at 982°C as seen in Fig. 27a, it is much less than what is predicted by the model with isotropic damage (Fig. 27b). The amount of isotropic damage is linked to the evolutionary rate of the isotropic scalar variable of Eq. (17). The amount of damage that
evolves for the fatigue-creep experiment with the 60 seconds compressive holds for both rate-dependent temperatures are shown in Figs. 28a-b. The maximum amount of damage that evolves for the 871°C fatigue-creep (60 seconds dwell) experiment is 0.06 (Fig. 28a), while the maximum amount of damage for 982°C is 0.45 (Fig. 28b). As a result, the difference in magnitudes between the isotropic scalar damage is directly manifested in the hysteresis cycles of Figs. 26b and 27b. This amount of damage for both temperatures was controlled by the creep responses of Figs. 17 and 19 in Section 4.1. Thus, the isotropic Kachanov damage parameters of Eq. (17) are inherently biased towards the creep responses, however, as shown by Fig. 26 this bias can still give favorable fatigue-creep simulations, but it must be used with caution (Fig. 27).

**Fig. 22**: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses for the unified viscoplastic model without isotropic damage at 649°C for a compressive strain dwell of (a) 60 seconds, and (b) 120 seconds.
Fig. 23: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses for the unified viscoplastic model without isotropic damage at 760°C for a compressive strain dwell of (a) 60 seconds, and (b) 120 seconds.

Fig. 24: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses for the unified viscoplastic model without isotropic damage at 871°C for a compressive strain dwell of (a) 60 seconds, and (b) 120 seconds.
Fig. 25: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses for the unified viscoplastic model without isotropic damage at 982°C for a compressive strain dwell of (a) 60 seconds, and (b) 120 seconds.

Fig. 26: Haynes 230 fatigue-creep loading history at 871°C with 60 seconds compressive holds (a) experiment with cycles shown from initial to the saturated cycle, and (b) the simulation with isotropic damage.
Fig. 27: Haynes 230 fatigue-creep loading history at 982°C with 60 seconds compressive holds (a) experiment with cycles shown from initial to the saturated cycle, and (b) the simulation with isotropic damage

Fig. 28: Haynes 230 isotropic Kachanov creep damage evolution for fatigue-creep loading histories with 60 seconds compressive holds at (a) 871°C (max damage – 0.06), and (b) 982°C (max damage – 0.45)
5. Conclusions

A unified viscoplastic model of the Chaboche type has been presented for Haynes 230 in simulating a broad range of behavior that not only includes strain-controlled (fatigue-creep) behavior, but also stress-controlled (creep) behavior and their interactions. While much research has been done on the two separately, regarding fatigue and creep, few studies have tried to comprehensively evaluate both phenomena against a broad set of responses. Novel ways of material identification of creep and fatigue-creep have proven to be invaluable in overcoming the challenges for modeling both domains that are classically modeled separately.

The unified viscoplastic model was developed for Haynes 230 by considering the robustness of certain modeling features to simulate both strain-controlled and stress-controlled loading histories. In this study, various flow rules for the inelastic strain rate have been evaluated. An exponential Norton flow rule is capable of simulating rate saturation for strain-controlled fatigue responses. However, the choice of flow rule has minimal effect on the stress-controlled creep responses as a result all simulations regarding stress-controlled creep loading histories have been simulated with the classical Norton’s power law form. In contrast, the model feature of static recovery in the kinematic backstress has been found to be essential in the simulation of both strain-controlled fatigue-creep responses and stress-controlled creep responses. It was important in finding the appropriate physical mechanism of either stress-controlled creep responses or strain-controlled fatigue-creep responses to calibrate these parameters from. It was shown that the creep responses should be favored over the fatigue-creep responses when it comes to initially calibrating the static recovery parameters, as both creep and fatigue-creep are simulated reasonably well. However, due to the limitations of a starting solution and the amount of parameters a hybridized genetic algorithm was leveraged for finding robust parameters that simulated both creep and fatigue-creep with better accuracy. As a result, we have shown that being able to understand the creep responses can be used to understand the material response better, such that the interactions between fatigue and creep are properly captured. This is critical in lifing analysis wherein both plasticity and creep should ideally be captured.
In addition, the unified viscoplastic model was investigated with an isotropic scalar damage framework. In this framework for continuum damage, the isotropic damage variable was of the Kachanov type that inversely scales the Cauchy stress by \((1 - D)\) to get an effective stress. The isotropic damage model of the Kachanov type with the unified viscoplastic model has shown to be effective in modeling creep responses better over the unified viscoplastic model without damage. One advantage the unified viscoplastic model with damage has over a non-damaged one is that the tertiary (accelerating) creep stage of the creep domain is capable of being captured. However, the unified viscoplastic constitutive model with damage has shown some limitations for fatigue-creep responses with isotropic softening in the hysteresis cycles. Also, in the rate-independent regime of the material, limitations at 649°C persisted in the simulation of creep strains with a concave response envelope for the simulation, while the experimental creep strains showed an exponentially increasing convex envelope. Nonetheless, the unified viscoplastic constitutive model with or without damage has shown to be able to capture both stress-controlled (creep-type) experimental histories, while still maintaining high fidelity in strain-controlled simulations.

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7. References


CHAPTER 6: PARAMETER OPTIMIZATION OF A CHABOCHE-BASED UNIFIED VISCOPLASTIC MODEL USING BOTH LOCAL AND GLOBAL OPTIMIZATION METHODS

Abstract

Advanced material models provide accurate estimation of stresses and strains at failure locations in a finite element structural analysis for various structural components subjected to creep-fatigue interactions. One such material model includes the modified Chaboche model, which is a unified viscoplastic model based on nonlinear kinematic hardening with several added features, such as strain range dependence, rate dependence, temperature dependence, static recovery, and a mean stress shift. The modified Chaboche model is validated by predicting stress and strain responses for a broad set of loading histories representative of local structural responses. The simulation capability of the model has been critically evaluated over the years and has proven to be an attractive model; however, the breadth of material parameters has been a concern in terms of the applicability of the model for Engineers. The first part of this study strives to introduce the complexity of the model, while also providing methods of parameter estimation from cyclic characteristic tests. In addition, the algorithms used to solve the optimization problem are explored. The material parameters have been succinctly categorized for robust estimation using both local and global optimization methods.

Keywords: Viscoplasticity, Constitutive Modeling, Creep-Fatigue, Parameter Identification, Global Optimization, Genetic Algorithm

1. Introduction

The ability to understand and engineer more robust and complex systems using high temperature materials, such as Haynes 230, has required innovation and maturation of material models that are capable of predicting broader ranges of phenomena, including plasticity, temperature dependence, rate dependence, and thermo-mechanical fatigue. One
superposition of these phenomena is the broad theory of viscoplasticity. The viscoplasticity model of the Chaboche type [1-2] has been developed for unified theories for coupled simulation of both the creep and plasticity responses. Over the years, this model has grown in complexity and most recently advanced by Barrett et al., 2014 [3-4] and Ahmed et al., 2014 [5] for thermo-mechanical and creep-fatigue interactions of Haynes 230. However, with increasing complexity, models tend to get larger, as the number of parameters increases. As a result, the application of advanced unified viscoplastic models for structural analysis and design has been limited, because of the difficulty in determining a larger set of parameters. The ability to succinctly find parameters of an advanced material model has been a grand challenge for many modelers. To overcome this challenge, this research tries to better understand how an efficient process flow for the model parameters, combined with unique optimization tools of both local and global methods can robustly determine a reliable set of parameters for an advanced material model.

The modified Chaboche-based unified viscoplastic model nicely builds a hierarchical structure for modeling that can lead to a systematic material calibration [1-2]. For discussion purposes the essential constitutive equations of the unified constitutive model are summarized herein which include strain decomposition, flow rule, kinematic hardening rule, and cyclic hardening through strain range dependent kinematic hardening and/or isotropic hardening rule. The additive strain decomposition of total strain $\varepsilon$ into elastic and inelastic strains is considered. The sum of the elastic and inelastic strains constitutes the mechanical strain. The elastic and inelastic strain are denoted with superscript “e” and “vp” respectively,

$$\varepsilon = \varepsilon^e + \varepsilon^{vp}. \quad (1)$$

The elastic part obeys Hooke’s law of linear elasticity,

$$\varepsilon^e = \frac{1+\nu}{E}\sigma - \frac{\nu}{E}(tr\sigma)I, \quad (2)$$

where $E$ and $\nu$ indicate Young’s modulus and Poisson’s ratio, respectively, $I$ is the second-order unit tensor and $tr$ is the trace. The viscoplastic or inelastic strain can be obtained from the viscoplastic potential:
where \( p \) is the accumulated equivalent plastic strain, and \( s \) and \( a \) are the deviators of the stress and back stress respectively. The back stress \( a \) is the center of the yield surface. The plastic strain rate norm \( \dot{p} \) of Eq. (3) is commonly expressed as a Norton’s power law [2] as

\[
\dot{p} = \left( \frac{\sigma_v}{K} \right)^n,
\]

showing the viscous stress \( \sigma_v \). The effect of plastic strain and creep strain is collectively incorporated in the viscoplastic strain of Eq. (1), making it a unified approach to obtain total strain. The back stress \( a \) has a rate equation expressed in its stress deviator form \( \dot{a} \) that models the center of the yield surface. This modeling of the yield surface center is commonly referred to as the kinematic hardening rule, which for the Chaboche type [1-2] has been developed and modified by [3-5] to incorporate four different terms for creep-fatigue interactions of viscoplasticity: (1) inelastic linear strain hardening, (2) nonlinear dynamic recovery with mean stress evolution, (3) static recovery, and (4) anisothermal temperature rate terms

\[
\dot{a} = \sum_{i=1}^{4} \dot{a}_i,
\]

\[
\dot{a}_i = \frac{2}{3} C_i \dot{\varepsilon}_{vp} - \gamma_i (a_i - Y_i) \dot{p} - b_j J_{a,j}^{r_j} \dot{a}_j + \frac{1}{C_i} \frac{\partial C_i}{\partial T} \dot{T} a_i.
\]

With modeling for the mean stress evolution driven by

\[
\dot{Y}_i = -\alpha_{b,i} \left( \frac{3}{2} Y_{st,j} \frac{a_i}{J_{a,i}} + Y_i \right) J_{a,i}^{r_i}.
\]

Cyclic strain range hardening of a material is considered through the kinematic hardening rule of Eq. (5) by evolving the dynamic recovery parameters \( \gamma_i \) of Eq. (5) as follows

\[
\dot{\gamma}_i = D_{\gamma_i} \left( \gamma_i^{o} - \gamma_i \right) \dot{p}.
\]
\[ \gamma_i^p = a_{\gamma_i} + b_{\gamma_i} e^{-c_{\gamma_i} q}, \]  

(8)

where \( D_{\gamma_i} \) is a rate parameter controlling the rate of evolution of \( \gamma_i \) with increment \( p \) of plastic strain norm. \( \gamma_i^p \) is the saturation value of \( \gamma_i \) with material parameters \( a_{\gamma_i}, b_{\gamma_i}, c_{\gamma_i} \) [11].

The size change of the von-Mises yield surface can be modeled through an isotropic rate equation [1]. Isotropic hardening is associated with a growth of the yield surface (linear elastic range) with cycles. Equation (9) defines the growth of the yield surface as a function of the plastic strain rate norm:

\[ \dot{R} = b (R_s - R) \dot{\rho}, \]  

(9)

where \( b \) and \( R_s \) are material constants representing the rate of isotropic hardening and the total isotropic saturation size of the yield surface, respectively.

The hierarchical structure of the unified viscoplastic model [1-7] results in constitutive equations that can be systematically identified with the right process flow. A robust process flow is essential for engineers to easily identify the large set of material parameters often associated with an advanced material model. This research provides this process flow for parameter estimation of the modified Chaboche model for viscoplasticity and uniquely builds a parameter determination scheme that utilizes this process flow for local and global optimization methods.

The systematic process flow for parameter determination of an advanced material model is built from cyclic characteristic tests that give experimentally consistent parameters. These cyclic characteristic tests are directly tied to the hierarchy of the model and the level of complexity needed in the analysis. As a result, constitutive features of strain range dependence, strain rate dependence, temperature dependence, and mean stress evolution for kinematic, isotropic, and/or thermal-recovery hardening can be systematically superimposed. This superposition nature of the unified viscoplastic model is a direct consequence of the hierarchical nature of the model that allows one to systematically identify material parameters for associated material responses seen in a cyclic characteristic test.
Inverse analysis is carried out to identify the breadth of material parameters, by minimizing with respect to them a non-linear least squares norm quantifying the discrepancy between measured and predicted quantities. While the prediction of observations is a forward problem, the use of actual observations to infer the properties of a model is an inverse problem. Inverse problems are difficult because they may not have a unique solution [8]. Several studies have been done with the Chaboche model ranging from FEM applications for railway wheel steel through tension-torsion tests [9] to thick component applications like AISI1046 medium carbon steel and sheet metal forming applications, like high strength steels Dual phase DP600, transformed induced plasticity TRIP700, and AISI325 stainless steel [10]. For the application of railway wheel steel, Fedele et al., 2005 [9] developed a parameter optimization routine through a deterministic, batch (non-sequential) inverse analysis in two stages (genetic and first-order algorithms) with various constraints in the minimization process. A hybrid method was also adopted to approach an optimal solution whereby a genetic algorithm was used until convergence slowed to which a local gradient-based algorithm was used. This hybrid approach proved to be essential in achieving an optimal solution set.

For unified viscoplastic modeling, a Chaboche-based model was developed and optimized for by Zhang et al., 2002 [11] for anisothermal cyclic plasticity modeling of martensitic steels commonly used in the forging through a deterministic inverse approach. Other unified viscoplastic models undertaking the scope of material responses presented by Barrett et al., 2014 [3-4] and Ahmed et al., 2014 [5] for thermo-mechanical and creep-fatigue interactions of Haynes 230 are the studies done by Yaguchi et al., 2005 [12-13] on modified 9Cr-1Mo steel and Zhan et al., 2007 [14-15] on Alloy X a nickel-based superalloy. Yaguchi et al., 2005 [12-13] showed promising results with a modified Chaboche model developed for mean stress evolution by Yaguchi, et al., 2002 [14-15] for simulating uniaxial and multiaxial ratcheting tests at temperatures between 200 and 600°C. As a result, effects of strain and stress-controlled were explored with uniaxial ratcheting tests being conducted by prescribing unsymmetric, axial force-controlled cycles and multiaxial ratcheting tests achieved by prescribing a constant tensile stress with cyclically controlled shear strain. Zhan
et al., 2007 [16-17] showed uniaxial strain-controlled fatigue-creep simulations for Alloy X a nickel-based superalloy tested at 650°C using the mean stress evolution developed by Yaguchi et al., 2002 [14-15]. Both studies [12-13] and [16-17] used an inverse analysis that consists of a nonlinear least squares gradient-based deterministic solver for parameter optimization.

A similar modeling effort to [11-17] for unified viscoplasticity was undertaken by Barrett et al., 2014 [3-4] and Ahmed et al., 2014 [5] for thermo-mechanical and creep-fatigue interactions of Haynes 230. The characteristic cyclic tests needed for model development in [3-5] depended on the intended application for the candidate material and an understanding of the actual in-service conditions. Haynes 230 was studied extensively for gas turbine engines with in-service conditions of thermo-mechanical fatigue, creep-fatigue, and fatigue interactions [6-7]. The characteristic cyclic tests needed for replicating the in-service conditions were presented in Table 1 of Barrett et al., 2014 [6]. The characteristic tests needed for material identification include strain-controlled, symmetric triangular isothermal tests at various strain rates and strain ranges (Groups 1 and 2), strain-controlled trapezoidal cycles with compressive strain dwells for fatigue-creep at various strains dwells (Group 4). The thermo-mechanical fatigue tests of Ahmed et al., 2014 [7], both out-of-phase and in-phase, are used for experimental validation of the constitutive model. In addition, Barrett et al, 2014 [4] showed that for fidelity in fatigue, fatigue-creep, and creep simulations, stress-controlled creep experiments are needed at various creep stresses.

The minimum experiments for complete identification of Haynes 230 for the advanced material model presented in for each group in Table 1 of Barrett et al., 2014 [6] are two different strain ranges at various temperatures for a fixed strain rate (Group 1), an additional strain rate with a fixed strain range of the two tested in Group 1 (Group 2) for various temperatures (more importance for higher temperatures), and a strain hold at either compressive or tensile peaks (Group 4). Also, a stress-controlled creep test is also needed for proper identification of static recovery parameters [4]. As a result, a minimum of five isothermal experiments at each desired temperature is necessary for parameter identification. The unified viscoplastic model is temperature dependent that must include at least the
minimum, average, and maximum temperatures seen in-service, with other temperatures
determined via linear interpolation.

The parameter estimation from these cyclic characteristic tests are integral in the
systematic process flow for parameter determination as it allowed us to solve the breadth of
material parameters of the nonlinear model using a combination of optimization methods.
The algorithms used to solve the optimization problem include a user-defined hybrid genetic
algorithm (global) and classical gradient-based (local) methods.

2. Process Flow of the Unified Viscoplastic Constitutive Model

A unified viscoplastic constitutive model has been developed to simulate the wide range
of experimental phenomena observed for strain-controlled isothermal and anisothermal
respectively. A description of the current state-of-the-art constitutive model has been given in
Barrett et al., 2014 [3-4] and Ahmed et al., 2014 [5] for simulating a broad set of uniaxial,
isoothermal strain-controlled experimental responses of Haynes 230 exhibiting cyclic strain
range dependence, strain rate dependence, and stress relaxation during a strain dwell. This
model was developed further for fatigue creep interactions and stress-controlled loading
loading histories presented several other challenges to the model, whereby additional
modeling features of mean stress evolution and maximum temperature influence developed
by Ahmed et al., 2014 [5] proved to be invaluable. The unified constitutive model is capable
of simulating the wide range of out-of-phase and in-phase thermo-mechanical fatigue
hysteresis loops, stress amplitude and mean stress, and stress relaxation responses.

The primary and secondary features of the unified viscoplastic modified Chaboche model
for material behavior of Haynes 230 are modeled in the hierarchical structure for
superposition. As a result, a process flow is presented in Fig. 1 for determining the primary
(level 1) parameters systematically. The systematic grouping of parameters allows one to
determine parameters sequentially, thus minimizing the inter-dependency of the parameters.
The first group of parameters needed in the unified viscoplastic model is the elastic
parameters $E$ and $\sigma_o$ at each temperature, using an initial cyclic loading response of an
isothermal triangular low cycle fatigue (LCF) strain-controlled experiment tested at any
strain range available in the material database. After the determination of the elastic
parameters, in Fig. 1, the material must be identified as either rate-independent or rate-
dependent [6-7] to determine the next set of parameters. In order to decide if a material is
rate-independent or rate-dependent, stress amplitude responses from an isothermal triangular
LCF strain-controlled experiment for a minimum of two strain rates must be compared as
shown in Barrett et al., 2014 [6] and Ahmed et al., 2014 [7]. If the material is rate-
independent, then the second group of parameters is the rate-independent kinematic
parameters of the basic Chaboche model $\gamma_i$ and $C_i$ (Eq. 5) for an initial cycle at the highest
strain range available in the material database. On the other hand, if the material is rate-
dependent, then the third group of parameters is the kinematic parameters of the basic
Chaboche model $\gamma_i$ and $C_i$ for an initial cycle at the highest strain range available and the
rate-dependent flow rule parameters $K$ and $n$ Eq. (4) from isothermal triangular LCF strain-
controlled experiments for a minimum of two strain rates. The kinematic parameters $C_i$ and
$\gamma_i$ are the same modeling parameters used in the basic Chaboche for nonlinear kinematic
hardening, however, the only distinction is the temperature, which determines whether or not
a material is rate-independent or rate-dependent. The flow rule parameters $K$ and $n$ are
found by simulating the experimental viscous stress rounding seen in the hysteresis loops at
rate-dependent temperatures Barrett et al., 2014 [6]. The flow rule parameters $K$ and $n$ at
rate-independent temperatures can be set to the mathematical limits of $K \approx 0$ and $n$ can be
fixed based off the lowest temperature at which rate-dependency was determined, which in
the case of Haynes 230 was 871°C [3-5].

The next decision step in Fig. 1 is that of determining if the isothermal temperature is a
creep temperature. If so, then two sets of experiments are needed to achieve full fidelity of
the model, which include a strain-controlled fatigue-creep experiment and a stress-controlled
creep test [4]. From these two characteristic tests, one can determine the fourth group, the
The static recovery parameters $b_i$ and $r_i$ (Eq. 5). The static recovery parameters $b_i$ and $r_i$ are found simultaneously from both an isothermal fatigue-creep strain controlled experiment at any dwell time available (compressive or tensile dwells) and a creep stress controlled experiment at the lowest creep stress available. For the fatigue-creep strain controlled experiments with strain dwells the fitness (quality of fit) measure is the stress relaxation with cycles, while for the creep stress-controlled experiments, the fitness measure is the creep strain with time during the creep stress dwell. In addition, from the strain-controlled fatigue-creep test one can determine if the responses exhibit mean stress evolution [3, 5, 6-7]. If so, then mean stress evolution parameters make up the fifth group $Y_{st,i}$ and $\alpha_{b,i}$. The primary (level I) parameters for the modified Chaboche model can be done for any number of Chaboche rules, as given by the index $i$ in each parameter; however, in our study four rules were used to simulate the wide range of material responses seen for isothermal fatigue, fatigue-creep, and creep as well anisothermal thermo-mechanical fatigue of Haynes 230 [3-5]. However, the fourth rule in our modified Chaboche model is a linear Prager rule [18] with the fourth kinematic dynamic recovery parameter $\gamma_4$ set to zero for all temperatures except the highest temperature of 982°C tested in our material database for Haynes 230 [6-7]. As a result, only three of the four ($i = 1-3$) modified Chaboche rules have mean stress evolution in Eq. (5). In total, the primary (level I) parameters set includes: 2 elastic parameters $E$ and $\sigma_o$, 8 rate-independent kinematic parameters $C_i$ and $\gamma_i$, 2 rate-dependent parameters $K$ and $n$, 8 static recovery parameters of $b_i$ and $r_i$, and 6 mean stress parameters $Y_{st,i}$ and $\alpha_{b,i}$.

Once the primary (level I) parameters in Fig. 1 are systematically organized and estimated, the other model parameters of Fig. 2 for secondary (level II) parameters can be found independently without affecting those in Fig. 1. This is the advantage of the modified Chaboche-based unified viscoplastic model is that it allows for one to systematically calibrate parameters without having to do an all-at-once optimization. The secondary (level II) parameters of Fig. 2 require one to identify if the material is kinematic or isotropic in nature. Kinematic hardening is associated with shape change in the hysteresis loop with
cycles accounting for either hardening or softening of the material. The importance of capturing the shape of the hysteresis loops as closely as possible has been shown by Krishna et al., 2009 [19] to have an impact in the overall simulation quality of numerous responses. In contrast, isotropic hardening is associated with a growth of the linear elastic domain while maintaining a constant hysteretic shape evolution for all cycles at strains beyond yield [1]. As a result, two groups of parameters are possible depending on the type of hardening seen in the material, multi-cycle kinematic parameters for stable kinematic parameters $\gamma_i^0$ and the kinematic rate parameter $D_{\gamma_i}$ (Eq. 7) as well as the multi-cycle isotropic parameters for isotropic saturated stress $R_\delta$ and rate evolution $b$ (Eq. 9). The stable cycle for Haynes 230 was chosen as the half life cycle in terms of fatigue for isothermal triangular LCF strain-controlled experiments given the application of the nickel base superalloy in gas turbine engines, where life prediction is based on half-life. $D_{\gamma_i}$ of Eq. (7) is a rate parameter controlling the rate of evolution of $\gamma_i$ with increment $p$ of plastic strain norm (cycles) while $b$ is a rate evolution parameter also with cycles, but for the isotropic hardening variable $R$ of Eq. (9). Haynes 230 showed exclusively kinematic hardening evolution for all temperatures [3,6]. It should be noted that Hassan et al., 2008 [20] reported that the cyclic hardening/softening simulation can also be obtained through evolving the $C_i$ parameters of Eq. (5); however, however, as shown in Krishna et al., 2009 [19] keeping the initial plastic moduli $C_i$ constant reduces the amount of model parameters without sacrificing the quality of hysteresis loop simulation. Hence, the evolution of $C_i$ is not included in this study.

Finally, if the material exhibits strain range dependence then these must be properly modeled. Strain range dependence can be modeled for either kinematic or isotropic hardening as shown by Krishna et al., 2009 [19] through a plastic strain memory surface. In this study, the kinematic coupling of the plastic strain memory surface with the kinematic hardening rule is shown for strain range dependence with parameters $a_{\gamma_i}, b_{\gamma_i}, c_{\gamma_i}$ of Eq. (8). As was the case for the mean stress evolution parameters $Y_{st,i}$ and $\alpha_{b,i}$ in Fig. 1, the strain range dependent parameters $a_{\gamma_i}, b_{\gamma_i}, c_{\gamma_i}, D_{\gamma_i}$ (Eqs. 7-8) coupled with the kinematic hardening
rule of Eq. (5) are only needed for three of the four \((i = 1-3)\) modified Chaboche rules. As a result, in total 12 strain range dependent parameters \(a_{\gamma_i}, b_{\gamma_i}, c_{\gamma_i}, D_{\gamma_i}\) are needed at each temperature. The number of half-life cycles needed to properly model strain range dependence is either a single multi-range triangular LCF test with at least three ramped strain ranges in the cyclic loading path or three separate LCF strain-controlled tests at three different strain ranges. For kinematic hardening, using the half-life cycles (stable cycle) for different strain ranges, one finds the stable (half-life) kinematic parameters \(\gamma'_i\) at each strain range, while keeping the initial plastic moduli \(C_i\) constant, which were found in Fig. 1 for the primary (level I) parameters. These stable (half-life) kinematic parameters \(\gamma'_i\) are used to calibrate the material parameters \(a_{\gamma_i}, b_{\gamma_i}, c_{\gamma_i}\) of Eq. (8), such that for any strain range the correct stable (half-life) cycle parameters are used in the prediction of the kinematic parameters \(\gamma_i\) that evolves with cycles based on Eq. (7). As a result, the kinematic parameters are evolving from the initial cycle kinematic parameter \(\gamma'_i\) of Fig. 1 (primary level 1 parameters) to the half-life (stable) cycle kinematic parameters \(\gamma''_i\) based on Eq. (7), which is also a function of different strain ranges based on Eq. (8). This evolutionary behavior of the kinematic hardening parameters \(\gamma_i\) gives the cyclic hardening or softening by evolving with accumulated plastic strain (cycles) until the half-life (stable) cycle is reached. The rate parameter \(D_{\gamma_i}\) should be found from the highest strain range by using the cyclic stress evolution response of stress amplitudes with cycles for the highest strain ranges’ triangular LCF strain-controlled test.

If the material exhibits isotropic hardening/softening, the isotropic hardening/softening follows an evolutionary rate equation of Eq. (9) that can be used to solve for the rate parameter \(b\) by quantifying the isotropic change in the yield stress (growth or decay) with cycles until a saturated value \(R_s\). Once again it is important to reiterate the advantageous independence of the primary (level I) parameters and the secondary (level II) parameters in terms of optimization of the complete set of unified viscoplastic model parameters.
Fig. 1: Process flow for parameter determination of the unified viscoplastic model, level I (primary features) parameters
Fig. 2: Process flow for parameter determination of the unified viscoplastic model, level II (secondary features) parameters
3. Initial Parameter Estimation

In Section 2, the necessary process flow is established, such that accurate initial estimates of the parameters can be made. The process flow outline in Figs. 1 and 2 gives a user a better understanding of how material behavior relates to certain cyclic characteristic tests and how to properly categorize it within the hierarchical superposition structure of the modified Chaboche-based unified viscoplastic model.

3.1 Elastic and rate-independent kinematic parameters

The process flow outline in Section 2 shows that the first group of the primary features in Fig. 1 are the elastic parameters $E$ and $\sigma_o$. The elastic parameters should be found from the initial cycle hysteresis and not monotonic response, such that the linear elastic domain is properly modeled [3,6]. Also, with increasing viscosity, the elastic parameters are increasingly difficult to estimate, on which readers are referred to Barrett et al., 2014 [3] for the complete methodology. Barrett et al., 2014 [3] discusses the need to examine the hysteresis cycles in total uniaxial stress vs. plastic strain for best estimation of the yield stress. Finally, the elastic parameters can be finalized via regression techniques without a complex optimization framework.

The next set of parameters that are most important for the modified Chaboche model is the basic kinematic hardening parameters $C_i, \gamma_i$. The isothermal, rate-independent kinematic parameters are the second group in Fig. 1. The parameters $C_i$ are the initial slopes of the total backstresses $\alpha_i$ vs. $\varepsilon^p$ which together estimates the plastic modulus and has units of stress and is the main contribution of linear inelastic strain hardening Prager, 1949 [18]. The parameters $\gamma_i$ are dimensionless and controls the rate at which the plastic modulus stabilizes. These determine the nonlinearity of the hysteresis curve through the dynamic recovery form that was outlined by Armstrong and Frederick, 1967 [21]. For rate-independent material behavior or for isothermal condition, the basic kinematic parameters can be found using the methods outlined by Chaboche, 1986 [1], Bari and Hassan, 2000 [22], and Shree et al., 2009 [19]. The methods outlined in these papers explain how to decompose the hysteresis cycle for
the various kinematic contributions $\alpha_i$ in total backstress (in total stress space). The model for the kinematic hardening rule is a superposition of several Armstrong-Frederick nonlinear rules as pioneered by Chaboche et al., 1979 [23].

A stable hysteresis curve can be divided into critical segments to represent each of the superposed backstresses [18,22,24]. In our present case, four rules are adopted to represent the backstress: the initial high modulus at the onset of yielding that is nonlinear but stabilizes quickly $\alpha_1$, the transient nonlinear segment given by two kinematic contributions $\alpha_2, \alpha_3$ that stabilizes slower and a constant modulus segment at higher inelastic strain ranges $\alpha_4$ that is linear [1,22]. The functional behavior of the nonlinear $\alpha_{nl,i}$ ($i=1-3$) and linear $\alpha_{li,i}$ ($i=4$) backstresses are shown in Figure 3. The unified viscoplastic modified model for the kinematic hardening in its complete form [3-5] with all features of static recovery, mean stress evolution, and temperature dependence cannot be solved analytically for a uniaxial closed form. However, the optimization procedure uses directly the unified viscoplastic modified Chaboche constitutive model that is numerically implemented via the radial return method [13]. The unified constitutive model is nested inside a hybrid genetic algorithm that relaxes the unconstrained global algorithm of a genetic algorithm (GA) with fundamental governing plasticity constraints.

The unconstrained global optimization is modified by introducing initial parameter determination via a gradient-based nonlinear regression analysis for parameter estimation, following the work of Rahman et al., 2005 [24]. Once initial estimates are obtained the stochastic search of a GA is used to guide an explorative search in parameter space. A novel approach evaluated in this study is that, the traditional GA is modified to ensure particular plasticity constraints are satisfied through the use of linear penalty functions on the chosen fitness criterion. The hybrid GA consists of a local optimization step for initial parameter estimation as outlined in Section 3, followed by the traditional GA sequence: population initialization, fitness evaluation, selection, and population generation via crossover, mutation, and elitism. The details of the hybrid GA optimization procedure are presented in Section 4.
Fig. 3: Nonlinear and linear backstresses of the kinematic hardening [1,22]

The initial estimates of the $C_i$ and $\gamma_i$ parameters can be found based on the hysteresis cycle divided into four inelastic strain segments $\Delta \varepsilon_i^p$ ($i=1$-$4$) each of which is predominantly represented by one of the superposition kinematic hardening rules (Figure 4) [22]. The division of the hysteresis cycle into various inelastic strain segments gives the associated parameters of that kinematic rule. These inelastic strain segments for typical hysteresis cycles are defined in Table 1, which was studied extensively for different materials by Rahman et al., 2005 [24]. In Fig. 4, the initial estimates of the plastic modulus values $C_i$ can be found from the initial slope of each region. The initial estimates of the rate parameters $\gamma_i$ of the dynamic recovery terms are found using a uniaxial closed form expression (ignoring static recovery and anisothermal temperature terms) for a hysteresis curve with a plastic strain range such that all the hardening rules except the last rule stabilizes within this plastic strain range and thereby,

$$\sum_{i=1}^{3} 2 \left( \frac{C_i}{\gamma_i} \right) = \left( \sigma_x^+ - \sigma_x^- \right) - 2\sigma_o - \sigma_v - 2C_4\varepsilon_L^p. \quad (10)$$

Everything on the right hand side of Eq. (10) is either known or has been approximated. The positive stress peak $\sigma^+$ and negative stress peak $\sigma^-$ of the up-going hysteresis curve is known from the experiment. The yield stress $\sigma_o$ was determined earlier in Section 3.1 and in
particular it should be known that for rate-independent temperatures the viscous stress ($\sigma_v$) is zero, while for rate-dependent temperatures the viscous stress can be found using the methods outlined in Barrett et al., 2014 [3] and will be briefly explained in Section 3.2. As a result, the left hand side of Eq. (10) can be used to find initial estimates of the rate parameters as a percentage of the right hand side of the equation. To obtain reasonable optimized rate parameter $\gamma_i$ ($i=1-3$) contributions of 10%, 50%, and 40% for the three hardening rules respectively [24].

**Table 1**
Four segments of the kinematic decomposition

<table>
<thead>
<tr>
<th>Segment:</th>
<th>$\Delta \varepsilon_1^p$</th>
<th>$\Delta \varepsilon_2^p$</th>
<th>$\Delta \varepsilon_3^p$</th>
<th>$\Delta \varepsilon_4^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of (% of $2\varepsilon_L^p$)</td>
<td>1-2</td>
<td>15-20</td>
<td>20-35</td>
<td>Rest</td>
</tr>
</tbody>
</table>

![Diagram of kinematic decomposition](image)

**Fig. 4:** Kinematic decomposition of a hysteresis cycle (up-going portion) with initial estimates of $C_i$
Once initial estimates of the parameters are known, the genetic algorithm proceeds to find an optimized parameter set. If the last kinematic rule is to be nonlinear the contribution percentages of the kinematic rule must be adjusted to obtain estimates of the rate parameters such that Eq. (10) would no longer have the linear rule on the right-hand side, but rather have another nonlinear contribution in the summation on the left-hand side.

3.2 Rate dependent parameters

Unified viscoplastic constitutive models, like the advanced modified Chaboche model [1-5] accounts for the interaction between creep and plasticity, giving viscoplasticity. The basic theory used to model the interaction of rate and creep is most commonly expressed by the classical Norton’s equation [26]. Norton’s equation is a power law that includes two rate-dependent parameters, the viscous drag stress, $K$ and the rate of secondary creep exponent, $n$. The Haynes 230 material exhibits loading rate behavior for temperatures above and including 760°C [6-7]. The viscous drag stress $K$ controls the magnitude of viscous overstress in the material, while the rate of secondary creep exponent $n$ controls the nonlinearity of the power law. The Norton’s flow rule algebraically makes the determination of the viscous stress rather easy by manipulating Eq. (4) as,

$$
\sigma_v = K \dot{\varepsilon}^{\frac{1}{n}}.
$$

Figure 5 shows a rate-dependent viscous cycle with zero yield stress and a viscous stress domain $\sigma_v$. The viscous stress domain is defined by the stress difference from point A on Fig. 5, the stress $\sigma_{\varepsilon_{\text{min}}}$ at the minimum plastic strain $\varepsilon_{\text{min}}^p$ to the stress $\sigma_{\varepsilon_{\text{min}}}$ at the minimum total strain $\varepsilon_{\text{min}}$ point B as developed by Graham et al., 2013 [27] for Inconel 617. Therefore, the viscous stress is calculated experimentally as $\sigma_v = \sigma_{\varepsilon_{\text{min}}^p} - \sigma_{\varepsilon_{\text{min}}}$. For rate-dependence, the viscous stress domain will change for different loading rates. Higher loading rates have higher viscous stresses then the slower loading rates and this variation is linear in nature as explained in Barrett et al., 2014 [4].
In summary, the viscous domain $\sigma_v$ can be approximated directly from the hysteresis curve as shown in Fig. 5. Also, from Fig. 5, the plastic strain rate norm $\dot{p}$ can be approximated using the plastic strain values $\varepsilon^p$ of the chosen cycle and computing the increment of plastic strain $dp$ and the time step $dt$ of the experimental data. The increment of plastic strain $dp$ used to find the plastic strain rate norm $\dot{p}$ at point A in Fig. 5 of Eq (11) is the plastic strain increment going from the minimum total strain $\varepsilon_{e,min}$ and the previous point before this. The corresponding recorded times at these points would be used to find the increment of time $dt$. In the experimental data, recorded data should include the time, strain, and stress which allows one to approximate the plastic strain rate norm. The plastic strain rate norm $\dot{p}$ at point A in Fig. 5 is then calculated based on $\dot{p} = dp / dt$. Once the plastic strain rate norm $\dot{p}$ of Eq. (11) is found at point A in Fig. 5, values of the rate dependent parameters $K$ and $n$ can be approximated. By using Eq. (11) one can algebraically estimate the rate-dependent viscous drag stress $K$, which controls the magnitude of viscous stress, by fixing the rate of secondary creep exponent $n$ to the minimum bound and the other variables $\sigma_v$ and $\dot{p}$ of Eq. (11) are now known based on the above discussion.

Fig. 5: Rate-dependent viscous cycle with zero yield stress of an isothermal, strain-controlled cyclic test
This procedure of looking at the viscous rounding in the hysteresis loop of an experimental curve can be applied to any isothermal strain controlled hysteresis cycle, such that one can determine if a viscous domain exists for any strain rate available at a particular temperature. If a viscous stress $\sigma_v$ exists at a particular temperature, such as the one seen in Fig. 5, then it is sufficient to get initial estimates of the rate-dependent parameters $K$ and $n$ from just an initial cycle of any isothermal strain controlled loading history at any strain rate available. However, if multiple strain rates are available as in Barrett et al., 2014 [6], then the optimization procedure requires the use of one or more strain rates in order to fully optimize for the rate dependent parameters. In summary, the rate dependent parameters $K$ and $n$ of Fig. 1 can be uniquely estimated for rate-dependent materials by using the novel identification of viscous stresses experimentally identifiable in a hysteresis cycle as seen in Fig. 5.

3.3 Static recovery and mean stress evolution parameters

The static recovery parameters of the modified Chaboche model are needed in order to simulate additional stress relaxation and creep strains at high temperatures. The static recovery modeling can be adopted in either the kinematic hardening rule or the isotropic hardening rule as shown in Chaboche, 1989 [2]. It is suggested that depending on the material behavior, one of the types of hardening (kinematic or isotropic) will inevitably be more influential than the other and whichever is will need to have the static recovery terms. Haynes 230 exhibits exclusively kinematic hardening, as a result, the static recovery modeling is done only through the third term $-b_i J (a_i)^{n-1} a_i$ of Eq (5) for the kinematic backstress. In the kinematic hardening rule, the first two terms of Eq. (5) are dominant under relatively high strain rates; however, for lower strain rates the hardening is lowered due to the opposing effects of recovery which is the third term. As a result, for strain hold experiments depending on the hold time complete recovery is possible. The static recovery term is thus static in nature and provides creep and thermal recovery for low strain rates. This static
The static recovery parameters, $b_i$ and $r_i$, in the process flow for parameter estimation of the primary parameters (level I) of Fig. 1 must be determined from both a strain-controlled fatigue-creep and stress-controlled creep loading histories at each temperature for which the material creeps. One challenge in the parameter estimation of the static recovery parameters is that the rate-dependent flow rule parameters of Section 3.2 can simulate some stress relaxation for strain hold experiments. Therefore, for temperatures above or at the rate-dependent temperature domain, the stress relaxation $\Delta \sigma_r$ of Figure 6 is simulated through both rate-dependent flow rule parameters as well as the static recovery parameters. The rate dependence flow rule parameters $K$ and $n$ can simulate the thermal (static) recovery during a strain hold, because at the dwell period the viscous overstress relaxes towards the yield surface. On the other hand, for temperatures at which the material behavior is overall rate-independent, but still shows stress relaxation for strain hold experiments, the stress relaxation can only be modeled through the static recovery parameters [3].
Due to the implicit nature of the kinematic hardening rule of Eq. (5) with all primary features for unified viscoplastic modeling, the initial estimation of both the static recovery (Eq. 5) and mean stress evolutionary parameters (Eq. 6) are less confident than the previous grouped parameters of Fig. 1, since closed form uniaxial expressions, like Eq. (10) cannot be approximated. However, initial estimates can be approximated. For the static recovery parameters, it was found best that the initial estimates are scaled 10% over the lower bounds to help guide the search. The lower and upper bounds for the static recovery parameters were set between $0 \leq b_i \leq 1$ and $1 \leq r_i \leq 30$. The lower limiting bounds represent where static recovery modeling is not considered. Due to the sensitivity of these parameters for simulation of both strain-controlled responses of fatigue-creep and stress-controlled responses of creep [4], it was important to consider an alternative optimization method, such as a global genetic algorithm to overcome the pitfalls of starting point values in the search for parameters. As a result, this lack of confidence in the initial estimation of static recovery parameters made a hybrid genetic algorithm a necessity as presented in Section 4.

Mean stress evolution parameters of Eq. (4) make up the fifth and final group of the process flow for parameter estimation. Like, the static recovery parameters, the mean stress evolutionary parameters were also a challenge in finding initial estimates. This is again due to the implicit nature of the kinematic backstress rule of Eq. (3). However, unlike the static recovery parameters a few of the mean stress parameters can be physically-linked immediately for estimation, in the process flow of Fig. 1. The saturated mean stress parameters $Y_{s,i}$ are estimated from the saturated stable cycle of the experiment, whereby its mean stress value corresponds to these saturated values [5]. However, the rate parameters $\alpha_{b,i}$ are less confident in that an initial estimate of 1% of the static recovery rate parameters $r_i$ was implemented by Ahmed et al., 2014 [5] for guiding the optimization.
4. **Hybrid Genetic Algorithm for Local-Global Analysis**

From Sections 2 and 3 we have established the necessary process flow and initial parameter estimation, such that physically meaningful robust parameters are found. The systematic calibration of material parameters shown in Figures 1 and 2 has allowed us to implement a hybrid genetic algorithm that utilizes an initial local optimization associated with gradient-based methods combined with the traditional genetic algorithm sequence for global search. The hybridized GA is a systematic stochastic search procedure that uses random search as a tool to guide a highly exploitative search in parameter space with initial physically meaningful parameters given by the initial local optimization step. The unconstrained global optimization is modified by introducing initial parameter determination via a gradient based nonlinear regression. This initial local optimization allows for one to satisfy initial constraints and obtain physically meaningful parameters. Once initial estimates are obtained the stochastic search of a GA is used to guide an explorative search in parameter space. Furthermore, the traditional GA is modified even further to ensure particular plasticity constraints are satisfied through the use of linear penalty functions on the chosen fitness criterion.

Section 2 explained a process flow for successful parameter determination that included several grouped parameters. These groups were decided based on the material behavior as the unified viscoplastic model hierarchically builds complexity through superposition. From Figures 1 and 2 we can systematically identify the interdependent and independent parameters within those groups. The interdependent parameters of the modified Chaboche model for rate-independent plasticity, rate-dependence, and static recovery include the following: $C_i, \gamma_i, K, n, b_i, r_i$ $(i=1-4)$. These eighteen core parameters are the basis of the unified viscoplastic model and the parameters of interest in the hybrid genetic algorithm. The parameters associate with cyclic hardening or softening either through isotropic or kinematic hardening evolutionary rules of Fig. 2 along with the strain range dependence parameters $a_{\gamma}, b_\gamma, c_\gamma, D_\gamma$ can be post-processed after the eighteen core parameters have been found and thus inherently independent. In addition, mean stress evolution parameters have also been made uniquely independent in the process flow for parameter determination. As a result, the
interdependent core parameters $C_i, \gamma_i, K, n, b_i, r_i$ are found from a user-defined hybrid genetic algorithm, while the independent parameters: 12 strain range dependent parameters $a_{ni}, b_{ni}, c_{ni}, D_{ni}$ and 6 mean stress parameters $Y_{a_i,j}$ and $\alpha_{b_i,j}$ are found from a local gradient-based solver in MATLAB, fmincon.

4.1 Conventional GA

The modified Chaboche model of [3-5] has a total of eighteen, $C_i, \gamma_i, K, n, b_i, r_i$ interdependent parameters with two elastic parameters $E, \sigma_o$ for each temperature. Parameter determination by means of trial and error iterations is intractable and time consuming. As a result, the use of probabilistic heuristic search techniques, like a genetic algorithm is more than necessary to automate the parameter determination and increase the attractiveness of such an advanced material model [9-10, 24, 28]. A conventional genetic algorithm (GA) uses a random generation of sets of real numbers as possible parameter values to conduct a probabilistic search procedure in finding an optimal solution set. The flow chart for conventional GA is shown in Fig. 7 with the dashed box surrounding the flow chart. For the conventional GA, it is seen in Fig. 7 that the first step involves population generation for parameter sets known as individuals in the population. The next step in the algorithm is determining the fitness for each parameter set based on how well it simulates observed responses. The fitness value of an individual is determined based on the root mean square deviation of distance between the experimental and model simulation responses as follows:

$$\text{Fitness measure } f_i = \sqrt{\frac{\sum_{j=1}^{N} (Y_{\text{exp},j} - Y_{m,j})^2}{N}}, \quad (12)$$

$$\text{Total fitness measure } f_i = \sum_{i=1}^{m} f_i, \quad (13)$$

In Eq. (5), the fitness measure is evaluated for the number of data points $N$, which can vary from experiment to experiment. To reduce the risk of having one experimental response
dominate the calculation of the fitness measure a root mean square error deviation is used along with making sure that for each used in the parameter determination has near equal number of data by filtering down the number of points. The total fitness measure of Eq. (6) is the total linear sum of all fitness values for \( m \) experiments used in the parameter determination.

**Fig. 7:** Flow chart for the conventional genetic algorithm (dashed box) and the hybrid genetic algorithm (solid box) with local optimization step and plasticity constraints for parameter determination
The objective function to be minimized is the least square distance measure between the experimental data and model simulations Eqs. (5-6). A nonlinear least-squares minimization is done by the GA parameter optimization, such that fitness values of the individual sets are determined and used as a metric to find the best solution set (elite) amongst the population of individuals. For the modified Chaboche model for viscoplasticity, the total fitness measure is evaluated for the three necessary cyclic tests shown in Fig. 1. A conventional GA is unconstrained with parameter searching done anywhere in the parametric space of possible solutions. For a conventional GA, parameter solutions may exist that fit a given set of data, but these particular parameters may not always have the physical significance required in the plasticity model to be used for simulation application. As a result, advanced material models have fundamental governing plasticity constraints that must be satisfied by the parameters. Some constraints relate to the algebraic consistency of parameters whereby both the plastic modulus $C_i$ and kinematic rate parameter $\gamma_i$ of the dynamic recovery term must be descending in order, e.g. $C_i \geq C_{i+1}$. To enforce these algebraic constraints the unconstrained global algorithm of the GA is constrained by adding linear penalty to the fitness value based on the violation of constraint. As a result, the parameter set that violates the constraint is penalized by adding linear penalty to the fitness value and will have higher fitness function than those that satisfies the algebraic constraints. Therefore, as the objective in the optimization scheme is to minimize the fitness function value, parameter sets violating the constraints are less likely to be accepted as the optimized parameter set. Similarly, in hysteresis curve simulation, it is critical to simulate closely the end stress which is most represented by the fourth decomposed hardening region of the hysteretic curve at the highest strain range. As a result, the best possible fourth plastic modulus ($C_4$) is achieved by applying linear penalty on the fitness functions based on the quality of simulation for the end stress and the plastic modulus, In addition, linear penalty is added to a solution set if the order of magnitudes for the kinematic hardening parameters is not physically consistent.

Once the fitness values for each individual in the population are calculated and linear penalties are imposed on any solution set that violates the governing plasticity constraints, the individual with the best fitness value is selected and saved as an elite individual. In the
following steps, GA attempts to acquire improved sets of individuals through using the Darwinian principle of “survival of the fittest” with several biologically-inspired mechanisms. This requires operations of selection (Fig. 8), crossover (Fig. 9), mutation (Fig. 10), and elitism to produce a better offspring population from the current population. All of the operations intricately work towards a global solution by using a population of solutions and improving these solutions through iterative generations of solution mutation, crossover, and elitism. The basic idea involves in these operations is that two better parents might produce a better offspring. In selection operation, binary tournament selection strategy is used, which picks the better individual from randomly chosen two individuals from the population. The process is continued until all parameter set individuals has gone through the selection operation for once and thus the better half of the population is selected. The binary selection process is repeated on the same population so that each selected individual undergoes selection criteria twice. Thus, the selection operation assures that the individuals with better fitness values would be copied in the next generation. The binary process of selection forms the mating pool for which the next two operations are used to form a new offspring population. The mating pool consists of several parents that can be used for reproduction to produce offspring solutions as shown in Figure 8.
Reproduction in the GA is carried out by a crossover operator to the selected mating population. A common crossover operator which is used for the advanced material model parameters is an arithmetic crossover. Randomly, two individuals are selected and their parameters exchange their information in order to generate two offspring in between two parents. This arithmetic crossover is done to generate offspring from two parents using the arithmetic equation shown in Figure 9. These arithmetic crossover equations generate two new parameter sets in between parent parameter sets. With some random probability, parameters are also generated outside these two parents but within the parameter range. The next operation in the generation of new parameter sets is that from the new offspring population formed by the binary tournament selection and the arithmetic crossover these new parameter sets undergoes a mutation operation. In mutation, the new offspring parameter sets are randomly selected and altered. The mutation process requires offspring individuals to be arranged in a long string for which random alterations of different parameters on the genetic sequence are done, such that diversity of solutions is maintained as shown in Figure 10.
After the generation of offspring population through selection, crossover and mutation, the parent individuals are discarded. In the flow chart of Figure 7, the new population must be evaluated for fitness. Once the fitness evaluations are known for the new population the last operation of elitism must be performed before optimization is evaluated for convergence. The elitism operation searches for the individual in the population with the best (minimum) fitness value. Another intricacy involved in this elitism operation is that the fitness value of the elite individual from the current generation is compared to that of the elite from the previous generation and the better one is saved as elite. However, in some cases the elite set of the previous generation may be better than the elite set of the current generation and as a result to ensure robustness the elite set from previous generation replaces the worst individual of the current generation. Overall, this elitism replacement operation ensures the survival of the best parameter set through generations. Finally, the elite set is scrutinized.
against the termination criteria to be determined if optimality is reached for the best parameter set. If not, the search continues with selection, crossover, and mutation for generating better offspring following the steps outlined in Figure 7.

4.2 Hybrid GA

The conventional GA of Section 4.1 is a class of evolutionary algorithms that copies the behavior of natural evolution and treats solution candidates as individuals that compete in a virtual numerical environment. This stochastic search is done systematically using a heuristic procedure, in which, solutions are essentially evolved towards global optimality through experienced-based techniques. The conventional GA without initial estimates and parameter ranges as shown in Figure 7 is inefficient for determining an optimized parameter sets for the modified Chaboche model as discussed by Rahman et al., 2005 [24]. The time of
convergence for a conventional GA can be very large and in most cases then not the optimized best individual will not always be a reasonable set of parameters in terms of the physical meaning of the parameters. While this may be the case, a small modification to the conventional GA enables significant improvement in the algorithm. The modification requires the use of initial random parameter generation as described in Section 4.1 and guide that process by initial parameter estimation highlighted in Section 3, for the physical meaning of the parameters, such that the GA search technique converges quickly to a very promising set of parameters. This modified procedure is referred as the hybrid simultaneous GA as seen in Figure 7. This hybrid simultaneous GA achieves the exploitative nature of a gradient-based optimization by finding initial physically meaningful parameters, through both gradient-based optimization and regular regression estimation given by the methods outline Section 3, while also maintaining the explorative nature of the conventional GA of Section 4.1. Thus, this hybrid simultaneous GA is capable of finding new candidate solutions in the search space, while also improving currently known solutions through exploitative measures like gradient computations.

Once the initial parameters are determined by the local optimization step as outlined in Section 3, it is necessary to establish ranges of parametric searching that can be used to guide the random generation of population. The parameters that have more physical meaning that can be determined using principles of viscoplasticity outlined in Section 3 have less variance in terms of the available parametric search space. However, parameters that have less physical meaning in the context of the parameter determination more often than not can search the entire possible domain of solutions. In the modified Chaboche model the basic kinematic parameters $C_i, \gamma_i$ have definite physical meaning in terms of the representative backstresses of the stable hysteresis curve where $C_i, \gamma_i$ are the plastic modulus and rate of dynamic recovery of each backstress term respectively. As a result, the basic kinematic parameters $C_i, \gamma_i$ can vary between $\pm(10-50\%)$ of the initial estimate given by the procedures of Section 3.1. This is one particular aspect of the genetic algorithm that requires some parametric tuning and can be varied by the user for different solutions of optimality.
The rate dependent parameters include the viscous drag stress, $K$ and the rate of secondary creep exponent, $n$ for the viscous stress of the model. Once these parameters are estimated the ranges used in the genetic algorithm to conduct the stochastic search are $\pm (10 - 20\%)$ for the secondary creep exponent, $n$ and $\pm (10 - 50\%)$ for the viscous drag stress, $K$. The final eight interdependent parameters of the modified Chaboche model for creep-fatigue simulations are the static recovery parameters $b_i, r_i$ that are less physically known. The previous ten parameters $C_i, \gamma_i, K, n$ had precise, effective ways of finding approximate estimates of the parameters. The lack of estimation of the static recovery parameters is a result of the form of the static recovery, for which a closed form of the rate equation of the kinematic rule cannot be found. As mentioned before, due to the less physically meaningfulness of these parameters the parametric search space includes the entire domain. However, it is known from these equations that some fundamental bounds are given by these parameters $0 \leq b_i \leq 1$ and $1 \leq r_i \leq 30$.

The hybrid GA continues to iterate the GA sequence based on the incorporation of the physical meaning of these parameters and the restricted search spaces. Again, the objective function to be minimized for each parameter set in the population is the least square distance measure between the experimental data and model simulations (Eqs. 5-6). A nonlinear least-squares optimization is done by the GA parameter optimization, such that fitness values of the individual sets are determined and used as a metric to find the best solution set (elite) amongst the population of individuals. For the modified Chaboche model for viscoplasticity, the total fitness measure is evaluated for three necessary cyclic tests at each temperature needed to span the minimum and maximum temperatures seen in the application component. For Haynes 230, which is used in the combustor liners of gas turbine engines, temperatures were incremented from room temperature (24°C) the minimum bound to a maximum temperature of 982°C. For the hybrid GA, eighteen interdependent parameters are divided into three constituent groups for parameter determination that define the material behavior. In the first constituency grouping, the basic kinematic parameters $C_i, \gamma_i$ for rate-independent plasticity requires the use of a uniaxial isothermal LCF strain-controlled cycling at a fixed
strain range and temperature for a particular strain rate. The second constituency grouping includes the rate dependent parameters $K$ and $n$, which can also be found from the same LCF strain-controlled cycling used to find the first constituency of parameters $C_i, \gamma_i$. For better quality of fits in the rate dependent parameters $K$ and $n$, another LCF strain-controlled cycling at a different strain rate can be included in the parameter determination, but not necessary based on the ability to determine $K$ and $n$ from the viscous stress identification discussed in Section 3.2. The third constituency grouping includes the static recovery parameters $b_i$ and $r_i$, which requires a fatigue-creep isothermal strain-controlled cycling with strain holds as well as a stress-controlled creep test.

In summary, the full eighteen interdependent parameter sets $C_i, \gamma_i, K, n, b_i, r_i$ will be evaluated for simulation fidelity against three different experiments that are known as the necessary cyclic tests. Therefore, the total fitness measure will evaluate how the full set of eighteen parameters performs for the three different unique tests. The other nuances of the hybrid simultaneous GA are essentially identical for the remainder of the algorithm sequence which includes: selection, reproduction, crossover, mutation, and elitism as shown in Figure 7. The hybrid simultaneous GA produces optimal parameter sets for different strain ranges and temperatures both efficiently and accurately. The parameters used in the simulations have not been disclosed due to restrictions placed by the project sponsor.

5. Comparison of Optimization Methods: Local vs. Global

The nonlinear least squares inverse problem for an optimal set of model parameters of the unified viscoplastic model has been studied for both local and global methods. The local optimization method used to solve the inverse problem was implemented from the MATLAB built-in function $fmincon$ in conjunction with the unified viscoplastic constitutive model. The following steps are required to implement the local optimization solver $fmincon$ with the unified constitutive model: main driver for initialization and calling of the built-in solver $fmincon$, residual function to call the unified constitutive model and calculate the $l_2$-norm of the total residual, constraint function in checking both equality and inequality constraints on
model outputs, and finally the unified constitutive model to run each loading path required for analysis. Default options for the calculation of the gradient (1\textsuperscript{st} order partials) and hessian (2\textsuperscript{nd} order partials) of the nonlinear multivariate function were used for searching. Also, parameter bounds (lower and upper) were set for all parameters based on the same bounds set in the genetic algorithm of Section 4 for direct comparison.

Simultaneous evaluation of a strain-controlled fatigue-creep experiment with 60 seconds strain holds at a strain range of 0.53% and a stress controlled creep experiment with a creep stress of 55 MPa at 871°C were used in the parameter determination of static recovery parameters for both the built-in gradient-based solver \texttt{fmincon} and the user-defined hybrid genetic algorithm of Section 4.2. For the gradient-based local solver of \texttt{fmincon}, an equality constraint was defined for the strain-controlled fatigue-creep experiment, such that the stress relaxation is matched best. In addition, an inequality constraint was defined for the stress-controlled creep experiment by ensuring that a maximum and minimum bound of 25% for the secondary (linear) creep rate is achieved. With the systematic calibration of material parameters offered by the process flow of Figure 1, the 8 static recovery parameters were optimized independently of all the other parameters, with the other 10 parameters of the 18 core parameters fixed for both optimization methods. The maximum number of iterations was set to 1000 for both the local gradient method (\texttt{fmincon}) and the user-defined global method of a hybrid genetic algorithm. For the hybrid genetic algorithm, 100 parameter sets in each generation was set and for the nested architecture of the main genetic algorithm sequence a maximum number of 2 first initial population generations in the outer loop was set along with for a maximum of 5 total new generations beyond the first initial population set within the inner loop via selection, crossover, and mutation. Thus, for the hybrid genetic algorithm a maximum number of function evaluations/iterations was set to 1000 as the product of the total number of parameter sets in each generation and the generation counts for each loop (inner and outer) gives the total max possible iterations.

Due to the dilemma of the initial starting point in a gradient-based optimization, it was necessary to go through several separate runs using converged inputs on the previous run output as the initial starting point. This is often the case for gradient-based solvers as it has a
tendency for premature convergence. In addition, for the unified viscoplastic constitutive model, the static recovery parameters are the least accurate in the initial estimation of those parameters needed for a starting point solution as explained in Section 3.3. As a result, the combined challenges of the static recovery parameters necessitated multiple runs for each temperature before an optimal set of parameters was found. One such iteration on the runs for informed initial starting solutions based on previous converged values is shown in Figures 11-12. The methodology used for the parameter determination of the static recovery parameters was discussed in Section 3.3 and extensively studied by Barrett et al., 2014 [4] in ensuring robust static recovery parameters for simulation of the strain-controlled fatigue-creep and stress-controlled creep responses. This methodology of using strain-controlled fatigue-creep and stress-controlled creep responses for robust simulation of material responses was used by both optimization solvers, the gradient-based (fmincon) solver and hybrid genetic algorithm. As a result, through several run iterations of the gradient-based solver for parameter determination of the static recovery parameters a converged solution set is shown in Figures 11-12 with a fair degree of accuracy. It is evident that from Fig. 11, the prediction of the fatigue-creep strain-controlled loading history with 120 second compressive strain dwells is done with reasonable fidelity for the gradient-based solver. However, the prediction of the creep responses suffer in simulation for creep stresses 62 MPa and 69 MPa as seen in Fig. 12, whereby the only response predicted with high fidelity is the response used for parameter determination (55 MPa).

On the other hand, the hybrid genetic algorithm can search a parametric space without an initial guess by simply using the lower and upper bounds of the parameters and letting the genetic stochastic search explore possible solutions much more effectively than the gradient-based solver. The optimal solution for the hybrid genetic algorithm is shown in Figures 13-14. In comparing the two optimal sets for fidelity, one can ascertain that the hybrid genetic algorithm shows higher fidelity over the gradient-based solution. The creep simulation of the local gradient-based method over predicts the two highest creep stresses (Figure 12), while maintaining very good fidelity in the fatigue-creep simulation (Figure 11). The hybrid genetic algorithm shows high quality simulations for both Figures 13 and 14.
Fig. 11: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses using a local gradient-based method (*fmincon*) for optimization of static recovery parameters

Fig. 12: Creep simulation of Haynes 230 at 871°C for various creep stresses using a local gradient-based method (*fmincon*) for optimization of static recovery parameters
Fig. 13: Haynes 230 simulation of fatigue-creep (strain dwell) cyclic responses using a global method of a hybrid genetic algorithm for optimization of static recovery parameters

Fig. 14: Creep simulation of Haynes 230 at 871°C for various creep stresses using a global method of a hybrid genetic algorithm for optimization of static recovery parameters
One could argue that not enough iteration of the converged runs for different starting points was taken to achieve a similar optimization. However, in this particular case a total of 5 runs were used indicating 5 different starting points, where each starting point was based off the previous runs converged values. The run times for each optimization run averaged out to be 5.5 hours/run (rounded) for the max iteration limit of 1000 iterations for the gradient-based solver of \textit{fmincon}. On the other hand, the hybrid genetic algorithm did undergo multiple runs, but only to tune the GA parameters associated with the mutation rate and obtaining good lower and upper bounds for searching. The run times of the hybrid genetic algorithm varied, but on average were about 4 hours/run for the max function evaluations limit. The simulation run times are comparable, but the process of determining parameters using the local gradient-based method is longer as more runs are required due to the importance of having a good starting point solution. Nonetheless, the local gradient-based method is still a powerful tool to use even with the sensitivity to the initial starting solution and we have used it in determining the mean stress parameters and strain range dependent parameters of Haynes 230 [3].

6. **Conclusions**

A unified constitutive model that can reasonably predict stresses and strains for a wide range of coupling phenomena is a major scientific issue motivated by industry demand. Complex macroscopic constitutive models are essential in enhancing the accuracy and reliability of analysis and design of various engineering applications. Nonetheless, however these mechanistic-based models come at a cost. In order to theoretically describe the phenomenological behavior of a material, one must increase the complexity of the model. Thus, increasing complexity, results in models that are parameterized by intricate, interdependent variables. Alternative methods; such as, fundamental heuristic methods, like trial and error or manual determination, become overtly cumbersome and highly impractical in an industrial context.

An efficient parameter identification algorithm that can robustly determine a reliable set of parameters is essential to enhance the attractiveness of such advanced material models. In
this study, we have outlined an automated parameter determination methodology and algorithm. We were able to use the benefits of a genetic algorithm by meshing some of the underlining physics of the material model in the initial estimation of parameters. This hybrid simultaneous GA conducts both an exploitative and explorative search to find optimal parameter sets of the modified Chaboche model for unified viscoplasticity. For inverse analysis of model parameters, several tools are available to perform the parameter optimization. However, it is important to include all the necessary physics required to satisfy the model which include constraints, both linear and nonlinear in nature along with proper bounding of model parameters consistent with the model to help guide the parametric search.

In addition, due to the large number of parameters in a robust unified constitutive model for viscoplastic responses requires an efficient process flow. For the unified viscoplastic model presented herein this included the development of unique identification of viscous responses for both strain-controlled and stress-controlled loading histories, specifically for the identification of the rate dependent parameters and static recovery parameters. Furthermore, with respect to the optimization solvers, a hybrid genetic algorithm was developed by taking the conventional genetic algorithm and constraining the search of parameters with linear penalty on fitness values violating various plasticity constraints. Likewise, the hybrid genetic algorithm was modified for initial local optimization of the model parameters before the genetic evolution occurred. The inclusion of these techniques improved the robustness of the parameter optimization for physically consistent parameters needed for simulation application.

A gradient-based solver in MATLAB (fmincon) was compared to the user developed genetic algorithm for efficiency and capability. The gradient-based solver fmincon works really well with the right parametric bounds and initial starting solution. However, like most gradient solvers an initial starting solution is often the limiting factor in finding an optimal solution. Therefore, several iterations on converged runs for the gradient-based solver were explored to investigate different starting points. This method of using different starting points is essential in obtaining a good solution set for gradient-based solvers. In contrast, to minimize the number of iterations and overcome the initial starting solution dilemma, a
hybrid genetic algorithm was used in parallel to find suitable solutions. It was important to leverage all the developed tools for parameter optimization in finding an optimal solution set for the unified constitutive model.

7. References


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CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

1. Conclusions

In this dissertation, simulation-driven design processes for the combustor liner material, Haynes 230 of a gas turbine engine has been the primary focus for the development of a robust unified viscoplastic constitutive model. Critical structures, like an airplane gas turbine engine or a nuclear power plant, must perform safely, reliably, and predictably. To reach this level of performance, simulation-driven design processes, like finite element analysis are an invaluable tool. However, more importantly, the underlining physics inside these simulation-driven programs must be properly modeled. In order to develop a robust constitutive model for Haynes 230 an extensive experimental program was undertaken. An initial goal of this research was to characterize the material response of Haynes 230 through a broad set of experiments, both isothermal and anisothermal, under loading conditions that replicated operating conditions. The experimental program revealed many complex features unique to Haynes 230 [1-2]. A constitutive model was then developed in a hierarchical superposition after careful analysis of the material experimental response [3-5]. The unified viscoplastic model is a macroscopic model that is characterized by material tests. The advanced material model started from the state-of-the-art modeling available in the literature and added novel features to describe the unique material behavior of Haynes 230 across the broad temperature spectrum. The unified viscoplastic constitutive model was experimentally validated against the broad set of isothermal fatigue, creep-fatigue, and creep responses.

Chapters 2 and 3 describe the broad set of isothermal fatigue and creep-fatigue, and thermo-mechanical creep-fatigue experiments carried out to characterize the material response of Haynes 230. Chapters 4 and 5 discuss the development of the unified constitutive model and the simulations for isothermal fatigue, fatigue-creep, and creep responses. This research shows the ability to simulate both fatigue (strain-controlled) and creep (stress-controlled) and their interactions using a unified viscoplastic constitutive model that has been missing in the literature. The developed Chaboche-based constitutive model includes features
of rate-dependence, static recovery, kinematic hardening evolution, strain range dependence, mean stress evolution, and isotropic scalar damage for creep. Novel modeling features were introduced to describe the experimentally exhibited influence of creep and fatigue. Overall, the proposed unified viscoplastic constitutive model with or without damage has shown to be able to capture both stress-controlled (creep-type) experimental histories, while still maintaining high fidelity in strain-controlled simulations (fatigue-type).

The ability of the unified constitutive model to simulate anisothermal thermo-mechanical fatigue is predicated on the quality of simulations seen at various isothermal temperatures. The experimental validation of isothermal tests in the development of an advanced constitutive model is necessary before undertaking the thermo-mechanical fatigue responses. Linear interpolation of parameters for intermediate temperatures is often used for model evaluation of anisothermal material responses where temperatures change due to temperature cycling. The use of a specific functional form for the temperature-dependent parameters is not necessary, and linear interpolation ensures that the model can best simulate the responses at isothermal temperatures, while also simulating the anisothermal conditions. The unified constitutive model is capable of simulating anisothermal thermo-mechanical fatigue with high fidelity. One such simulation is shown in Fig. which demonstrates the strength of the unified viscoplastic model to predict cyclic responses up to a half life cycle for thermo-mechanical fatigue needed for fatigue design of a gas-turbine engine. In Fig. the unified viscoplastic model demonstrates the ability to predict the hysteresis loops, stress amplitude and mean stress, and stress relaxation responses for a temperature cycling going from a minimum temperature of 316°C to a maximum temperature of 927°C that is out-of-phase to the strain cycling of a minimum strain of -0.3% (compression) to a maximum strain of 0%. For simulation of anisothermal responses spanning from room temperature (24°C) to the maximum temperature seen in the combustor liner at 982°C, a total of seven temperature levels were used and analyzed for isothermal responses. For complete analysis and simulation of anisothermal responses readers are referred to Ahmed et al. [5].
Fig. 1: Out-of-phase TMF stress ($\sigma_x$) vs. strain ($\varepsilon_x$) hysteresis loops of Haynes 230 at various cycles up to half-life for $T$: 316-927°C, $\Delta\varepsilon=0.4\%$, heating-hold-cooling=65-120-90s:- (a) Experiment, (b) Simulation

Chapter 6 discusses the numerical implementation of a user-defined hybrid genetic algorithm along with other optimization methods, like gradient-based solvers commercially available in MATLAB. The implementation of the optimization algorithms requires a nested architecture to solve the nonlinear least-squares inverse problem with the unified constitutive model. The inverse problem requires the optimization routines to call the full integrated unified viscoplastic model in determining a set of optimal parameters that replicates the behavior of the material in the characteristic cyclic tests. To ensure physically meaningful parameters and ease the parameter determination, a systematic process flow was given for initial parameter estimation. This systematic calibration allows for a stepped rather than an all-at-once evaluation of the full set of model parameters, which ensures optimum.
2. Recommendations for future research

The unified viscoplastic constitutive model developed herein shows the predictive capability of the model to describe broader ranges of phenomena, like fatigue, creep, and fatigue-creep of nickel-base alloys. While the model showed much strength in capturing the unique behavior of Haynes 230 for uniaxial loading histories it is imperative to explore other loading paths, like multiaxiality in addressing the robustness of the model. In addition, uncertainty quantification of the material parameters can be addressed. From the current study, one can build off this work to explore these challenges and such the following recommendations for future research work may be made.

2.1. Multiaxiality and FEA of idealized structural components

The unified constitutive model developed in this study demonstrated several strengths in simulating a combustor liner material, like Haynes 230 seen in an airplane gas turbine engine for fatigue, fatigue-creep, and thermo-mechanical. However, all the aforementioned material development was done considering only uniaxial data. Structural components, like the combustor liner or an elbow pipe in a nuclear power plant often experience stress states that are not purely uniaxial, but rather multiaxial. Multiaxial stress states can be idealized in the context of mechanical behavior of materials as a mixed combination of strain or stress controlled loading. Multiaxial tests for creep-fatigue and creep-ratcheting experiments can be carried out (which are outside the scope of the present study) to identify any potential phenomena and subsequent modeling features that are needed for constitutive robustness.

Furthermore, much can be learned from idealized structural component simulations in a finite element analysis. To test the robustness of the developed model, idealized structural simulations may be carried out. One such example would be the combustor liner. To simulate the non-uniform combustion process of the combustor liner seen in thermo-mechanical fatigue loading of the gas turbine engine, a slice of the liner modeled as a plate with a hole can be analyzed. Another example would be an elbow pipe in an intermediate heat exchanger of a nuclear power plant. Sub modeling of these structural components are invaluable for engineering safe systems.
2.2. Parameter correlation analysis

The material analyst doing constitutive model development or characterization in a large scale finite element analysis requires the analyst to make a set of assumptions, that may include the calibration of the parameter bounds, optimization methods to use for solving the inverse problem, or even considering which experimental data to use in determining an optimal set of material parameters. The uncertainty of each of these assumptions made by the analyst combined with the complex nature of the material model results in a large amount of uncertainty in the modeling and design-by-analysis process. In essence, there is always uncertainty in the uncertainty. However, a subset of the aforementioned uncertainty can be quantified and that relates to uncertainty quantification in the parameter estimation.

This requires the computation of a parameter sensitivity matrix that relates the sensitivity of the parameters to the observation data. This sensitivity matrix can reveal if parametric dependence exists in the modeling that may cause problems in the optimization study. The null space of the parameter sensitivity matrix can be computed to determine if a well-posed inverse problem exists. The null space is computed by taking the algebraic difference between the number of parameters and the rank of the matrix, where the rank of the matrix is the number of linearly independent columns/rows. A well-posed inverse problem requires the null space to not exist. If the null space exists, then one or more parameters should ideally be eliminated from the study using prior information, such that a feasible optimal solution can be found. The next thing that should be computed would be the fisher information matrix (forward sensitivity) that measures how if one parameter changes how does it affect the simulated observation response. In order to compute the covariance matrix (inverse sensitivity) one must calculate the inverse of the fisher information matrix, which requires that it be non-singular (full rank). For the covariance matrix, smaller values are desired as this implies confidence in your parameters, while large covariance implies huge statistical uncertainty. Finally, the parameter correlation matrix allows one to measure correlation among parameters to observations. Off-diagonal terms of the correlation matrix determine the redundancy of the parameters. As a result, a singular value decomposition of the
sensitivity matrix should be used for highly correlated parameters to determine, which parameters are most redundant and to eliminate those parameter from the analysis.

The sensitivity matrix for the unified constitutive model can be computed analytically for subsets of parameters for which analytical closed form solutions exists. However, for the full modeling scope of the unified viscoplastic model a finite difference approximation method, such as a forward (1st order) or central (2nd order) method is recommend for approximating the first order partials or gradients of the observations with respect to the parameter set. Due to the large differences in magnitudes between parameters, this finite difference approximation may be a challenge to avoid a singular matrix that is badly scaled and as a high condition number. Nonetheless, if a sensitivity matrix is computed and it is invertible, confidence intervals can be established assuming that the parameters are normally distributed and using either a student t-distribution or normal distribution. Since the unified viscoplastic model is built in hierarchy and consists of an integrated superposition of modeling features that corresponds to different material tests, it may be best to quantify different levels of uncertainty.

3. References


APPENDICES
A.1. – Lambert W function for the exponential Norton’s flow rule

The exponential Norton’s flow rule for rate saturation requires one to solve the plastic strain rate norm using the classical Lambert W function. The exponential Norton rule can be expressed as,

$$\dot{p} = \left( \frac{\sigma_v}{K} \right)^n e^{a \left( \frac{\sigma_v}{\bar{\sigma}} \right)^n}, \quad (A1-1)$$

where the rate saturation parameter is $\alpha$ proposed by Kocks et al., 1975 [1]. The numerical implementation of the unified viscoplastic constitutive requires one to algebraically solve for the viscous stress, such that the unknown plastic strain increment $\Delta p$ can be determined through a Newton-Raphson iteration scheme. In order to analytically solve for the viscous stress in Eq. (A1-1), the Lambert W function must be used. The Lambert W function [2] is defined as the inverse function of the form,

$$z = W(z) e^{W(z)}. \quad (A1-2)$$

As a result, using the Lambert W function of Eq. (A1-2) the viscous stress in Eq. (A1-1) can be solved analytically as,

$$\sigma_v = K \left[ \frac{W(\dot{p}a)}{\alpha} \right]^{\frac{1}{n}}. \quad (A1-3)$$

In the implicit radial return scheme presented by Ahmed et al., 2013 [3], five nonlinear equations of the unified viscoplastic model exist corresponding to five unknowns that must be solved for each iteration. These five unknowns are the plastic strain increment $\Delta p$ and the invariants of the kinematic backstresses $J_{a,d}$.

The algebraic system of five nonlinear equations is a determined system that can be used to solve for the five unknowns. A nonlinear scalar system of equations with a vector function of $n$ functions and a vector of $n$ variables $x = \{x_1, x_2, \ldots, x_n\}$ expressed as $F(x) = 0$ is commonly solved using a Newton-Raphson method. The Newton-Raphson method expands the nonlinear system of equations as a multi-dimensional Taylor series and neglects all
higher order terms with \( x^{k+1} = x^k + \Delta x^k \). The Newton-Raphson method solves updated unknowns with,

\[
F\left(x^{k+1}\right) = F\left(x^k\right) + F'(x^k)\Delta x^k, \tag{A 1-4}
\]

By setting \( F\left(x^{k+1}\right) = 0 \) we get,

\[
\Delta x^k = -\left[F'(x^k)\right]^{-1} F(x^k), \tag{A 1-5}
\]

\[
x^{k+1} = x^k + \Delta x^k. \tag{A 1-6}
\]

A Jacobian matrix relating the first order terms of the multi-dimensional Taylor series is constructed to give the following update form,

\[
\Delta x^k = -\left[J\left(x^k\right)\right]^{-1} F(x^k). \tag{A 1-7}
\]

This Newton-Raphson numerical solver for the system of nonlinear equations requires an initial guess \( x^0 \) to start the algorithm and also the construction of the Jacobian matrix \( J \). This Jacobian matrix generalized for the system of \( n \) equations is expressed as,

\[
J(x) = \begin{bmatrix}
\frac{\partial f_i}{\partial x_1} & \ldots & \frac{\partial f_i}{\partial x_j} & \ldots & \frac{\partial f_i}{\partial x_n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \ldots & \frac{\partial f_n}{\partial x_j} & \ldots & \frac{\partial f_n}{\partial x_n}
\end{bmatrix}. \tag{A 1-8}
\]

The Jacobian matrix can be calculated by either finite differences or analytically. The analytical first order partials of the Jacobian matrix are found analytically in the numerical implementation for the five nonlinear scalar equations \( L(\Delta p_{n+1}^k, J_{a_i}^k), g_i\left(\Delta p_{n+1}^k, J_{a_i}^k\right) \) such that the Jacobian matrix is expressed as
To avoid the inversion of the (5x5) Jacobian in the Newton-Raphson update, a nested architecture using only the diagonals of the Jacobian has been constructed for numerical efficiency [3]. As a result, one such analytical derivative of the Jacobian matrix Eq. (A 1-9) needed for update is the derivative of the nonlinear scalar equation \( L(\Delta p^k_{n+1}, J_{a_i}^k) = 0 \). For the various flow rules, two equations defining this nonlinear scalar equation \( L(\Delta p^k_{n+1}, J_{a_i}^k) = 0 \) must be updated, which are \( J(\sigma - \alpha) \) expressed as \( h \) and the derivative \( \partial h/\partial (\Delta p) \) [3].

For the exponential Norton of Eq. (A 1-1), the Lambert W function is used to solve for the viscous stress, which is calculated in the expression of \( h \),

\[
h = \sigma_v + \sigma_0. \tag{A 1-10}
\]

The viscous stress for the exponential Norton of Eq. (A 1-3) can be substituted in Eq. (A 1-10) to give the following expression:

\[
h = K \left[ \frac{W(\dot{\alpha})}{\alpha} \right]^n + \sigma_0. \tag{A 1-11}
\]

Finally, the analytical derivative of (A 1-11) is as follows,

\[
\frac{\partial h}{\partial (\Delta p)} = \frac{K}{n\Delta t} \left[ \frac{W(\dot{\alpha})}{\alpha} \right]^{(n-1)/n} \left[ \frac{W(\dot{\alpha})}{1+W(\dot{\alpha})} \right] \dot{\alpha}.
\tag{A 1-12}
\]
A.2. – References

