ABSTRACT

SHENDE, KETAN VISHWANATH. Evaluation of Statistical Energy Analysis as a Design Methodology for Fluid Structure Coupling in Thin Sheet Metal Tanks by Experimental and Simulation studies. (Under the direction of Dr. Richard Keltie.)

Previous studies [1], [2] have shown that the vibration and acoustic characteristics of thin sheet metal tanks vary with the presence of enclosed volume of fluid which can lead to unacceptable impacts on the noise signature of automobiles. The usual deterministic approach of Finite Element Analysis was unable to account for the irregularities in the manufacturing processes and variability of the surface responses. Application of Finite Element Analysis in the higher frequency domain is restricted by high computational costs and low modal assurance criteria values. The purpose of this thesis is to study this fluid structure coupling in thin sheet metal tanks by experimentation and to utilize Statistical Energy Analysis to simulate this phenomenon. The experimental and simulation responses are correlated and the applicability of SEA as a simulation tool to aid the design process is discussed.

For this research project, a simple stainless steel parallelepiped tank was manufactured using standard industrial processes. The tank dimensions were determined to be sufficiently large to hold the required quantity of fluid for coupling effects to be measurable. The assumptions of the principles of reciprocity and symmetry were checked by impulse excitation using a modal impact hammer on the tank surfaces and minor deviations were observed indicating presence of geometrical irregularities. A support system was developed to simulate free-free boundary conditions and the tank was suspended with the help of elastic cords in the anechoic chamber. An electromagnetic shaker was used to excite the structure with a uniform random white noise over a wide frequency bandwidth. Spatially averaged vibration responses and far field acoustic responses were measured for different fluid levels in the tank. For
correlation purposes, parameters like damping loss factor and power input are measured experimentally and used as inputs for the simulation techniques.

Simulation of the experimental setup was carried out using the FEA and SEA modules of commercially available Vibro-Acoustics software suite developed by ESI group (ESI VA One). Finite Element Analysis code for fluid structure coupling simulation was chosen for reference due to its widespread use. Certain simplifying assumptions were made for simulation purposes and the results were compared. Comparison was made between the simulation techniques for their accuracy, computational costs and applicability to the given problem. At low frequencies, it was observed that the modal density was low, resulting in large deviations from FEA results for vibration response. It was also observed that the computation time for SEA is a fraction of that for FEA, and the computation time increases rapidly for FEA simulation as the frequency increases. The response comparison also showed that the SEA results approach the FEA results as the frequency is increased. Comparison with experimental results demonstrated the ability of SEA to simulate the fluid structure coupling effectively.

This correlation process exhibited the feasibility of SEA as a simulation tool for design process. Some applications of this simulation method to identify critical vibration paths and optimize the geometric parameters for better overall response of the component are discussed in this thesis.
Evaluation of Statistical Energy Analysis as a Design Methodology for Fluid Structure Coupling in Thin Sheet Metal Tanks by Experimental and Simulation studies

by
Ketan Vishwanath Shende

A thesis submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the Degree of Master of Science

Mechanical Engineering

Raleigh, North Carolina

2015

APPROVED BY:

_______________________________  ______________________________
Dr. Lawrence Silverberg  Dr. Yun Jing

_______________________________
Dr. Richard Keltie
Chair of Advisory Committee
BIOGRAPHY

Ketan Shende was born in Pune, India on 22nd December, 1988 to Mr. Vishwanath Shende and Mrs. Vaijayanti Shende. He completed his Bachelor of Technology in Mechanical Engineering from College of Engineering Pune, an autonomous university affiliated with Pune University India. After graduation, he worked as CAE engineer in Research & Development (R&D) Dept. of Bajaj Auto Ltd., the third largest automobile company in India, for almost three years. During this period, he worked on different projects spanning the areas of Noise Vibration Harshness (NVH), vehicle dynamics and multi body dynamics.

He was accepted to the Master of Science degree program in the Mechanical and Aerospace Engineering Department of the North Carolina State University and joined the graduate school in August 2013. During this period he was also a teaching assistant for a couple of undergraduate courses.

Apart from academics, he is an avid reader and likes to play badminton, racquetball and tennis. He is also interested in outdoor activities like camping, hiking and kayaking.
ACKNOWLEDGMENTS

I would like to thank Dr. Keltie for giving me an opportunity to work on my thesis project under his guidance and mentorship. It was a great learning experience and I appreciate your commitment and patience during this entire period. I would also like to thank Dr. Silverberg and Dr. Jing for being a part of my defense committee.

This project proposal is partly inspired by some research carried out in my previous job and I would like to thank Bajaj Auto Ltd. and my former colleagues for their frank discussions and knowledge sharing.

I am grateful for the lab facilities in the Mechanical and Aerospace Engineering Department of NC State University. I would also like to take this opportunity to thank all the faculty and the staff, especially Ms. Annie, for their help and support through this process.

I would like to thank my parents and my brother for their unwavering support and belief. It was because of all your hard work that I have been able to achieve success in my life. My siblings and my friends have always been a source of comfort and have helped my gain some priceless memories over the years. Special thanks to my roommates during graduate school for their support as well.

I would like to thank one and all, who directly or indirectly have played a role in helping me with this thesis project.
TABLE OF CONTENTS

LIST OF TABLES ............................................................................................................ vi
LIST OF FIGURES .......................................................................................................... vii
CHAPTER 1 .................................................................................................................. 1
  1.1. Introduction ....................................................................................................... 1
  1.2. Motivation ........................................................................................................ 3
CHAPTER 2 .................................................................................................................. 6
  2.1. Background ...................................................................................................... 6
  2.2. Parameter Definition ...................................................................................... 8
  2.3. Formulation ..................................................................................................... 11
  2.4. Summary ......................................................................................................... 13
CHAPTER 3 .................................................................................................................. 14
  3.1. Thesis Scope .................................................................................................... 14
  3.2. Design Parameters ......................................................................................... 15
  3.3. Preliminary Testing ......................................................................................... 17
  3.4. Summary ......................................................................................................... 23
CHAPTER 4 .................................................................................................................. 24
  4.1. Experimental Setup ......................................................................................... 24
  4.2. Experimental Apparatus ................................................................................ 25
  4.3. Signal Generation ............................................................................................ 26
  4.4. Data Acquisition Parameters ......................................................................... 28
  4.5. Experiment Description .................................................................................. 30
  4.7. Observations – Vibration Response ............................................................... 36
  4.8. Experimental Results: Acoustic Measurements ............................................ 37
  4.9. Observations – Acoustic Response ................................................................. 40
  4.10. Conclusions ................................................................................................. 40
CHAPTER 5 .................................................................................................................. 42
  5.1. Introduction .................................................................................................... 42
  5.2. SEA Simulation model ................................................................................... 42
  5.3. FEA Simulation Model .................................................................................... 44
  5.4. Assumptions .................................................................................................... 45
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5.</td>
<td>Results</td>
<td>..........................................................</td>
<td>47</td>
</tr>
<tr>
<td>5.6.</td>
<td>Observations</td>
<td>..........................................................</td>
<td>52</td>
</tr>
<tr>
<td>5.7.</td>
<td>Experimental SEA</td>
<td>..........................................................</td>
<td>53</td>
</tr>
<tr>
<td>6.1.</td>
<td>Introduction</td>
<td>..........................................................</td>
<td>57</td>
</tr>
<tr>
<td>6.2.</td>
<td>Comparison Summary</td>
<td>..........................................................</td>
<td>57</td>
</tr>
<tr>
<td>6.3.</td>
<td>Results - Vibration Measurements</td>
<td>..........................................................</td>
<td>60</td>
</tr>
<tr>
<td>6.4.</td>
<td>Observations – Vibration Measurements</td>
<td>..........................................................</td>
<td>62</td>
</tr>
<tr>
<td>6.5.</td>
<td>Results – Acoustic Response</td>
<td>..........................................................</td>
<td>65</td>
</tr>
<tr>
<td>6.6.</td>
<td>Observations - Acoustic Measurements</td>
<td>..........................................................</td>
<td>67</td>
</tr>
<tr>
<td>6.7.</td>
<td>Variance Study</td>
<td>..........................................................</td>
<td>69</td>
</tr>
<tr>
<td>Chapter 7</td>
<td>7.1.</td>
<td>Project Summary</td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>7.2.</td>
<td>Conclusions</td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>7.3.</td>
<td>Future Work</td>
<td>..........................................................</td>
</tr>
<tr>
<td>APPENDICES</td>
<td></td>
<td></td>
<td>..........................................................</td>
</tr>
<tr>
<td>Appendix A</td>
<td></td>
<td></td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>A1)</td>
<td>Verification of VA One SEA results with theoretical results</td>
<td>..........................................................</td>
</tr>
<tr>
<td>Appendix B</td>
<td></td>
<td></td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>B1)</td>
<td>Experimental Setup Component Drawings</td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>B2)</td>
<td>Transducer technical specifications</td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>B3)</td>
<td>LabVIEW data acquisition VI’s</td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>B4)</td>
<td>Signal Generation</td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>B5)</td>
<td>Frequency Averaging of Experimental Data</td>
<td>..........................................................</td>
</tr>
<tr>
<td>Appendix C</td>
<td></td>
<td></td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>C1)</td>
<td>Damping Loss Factor Calculation</td>
<td>..........................................................</td>
</tr>
<tr>
<td></td>
<td>C2)</td>
<td>Input Power Calculation</td>
<td>..........................................................</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 4.5-1: Experiment Description .................................................................31
Table 6.2-1: SEA vs Experimental Results: Comparison .........................................60
LIST OF FIGURES

Figure 1.2:1: FSI simulation: Tank Model........................................................................................................4
Figure 1.2:2: Fluid Mesh Model........................................................................................................................4
Figure 2.2:1: Plate - Mode subsystem ..................................................................................................................8
Figure 2.3:1: Three Subsystem SEA model ........................................................................................................11
Figure 3.1:1: Gas Tank (JP cycles) ....................................................................................................................14
Figure 3.2:1: CAD Tank Model .........................................................................................................................17
Figure 3.3:1: Preliminary Test Setup ................................................................................................................18
Figure 3.3:2: Reciprocity Top Side .....................................................................................................................19
Figure 3.3:3: Symmetry: Front Face: Longitudinal Direction ...........................................................................20
Figure 3.3:4: Symmetry: Front Face: Lateral Direction ....................................................................................20
Figure 3.3:5: X excitation: Top Face response ..................................................................................................21
Figure 3.3:6: Y excitation: Top Face ..................................................................................................................22
Figure 3.3:7: Z excitation: Top Face ..................................................................................................................22
Figure 4.1:1: CAD Model: Tank Structure .......................................................................................................24
Figure 4.1:2: Experimental Setup ....................................................................................................................25
Figure 4.3:1: Input Signal PSD ........................................................................................................................27
Figure 4.4:1: Near Field Experimental Setup ...................................................................................................29
Figure 4.5:1: Tank Nomenclature .....................................................................................................................30
Figure 4.5:2: Expt.: Empty tank vs 40% Filled tank: Force Response: Base .....................................................33
Figure 4.5:3: Expt.: Empty Tank vs 40% Filled Tank: Force Response: Front ....................................................33
Figure 4.5:4: Expt.: Empty tank vs 40% Filled tank: Acceleration Response: Base .......................................34
Figure 4.5:5: Expt.: Empty tank vs 40% Filled tank: Acceleration Response: Front .......................................34
Figure 4.5:6: Expt.: Empty tank vs 40% Filled tank: Acceleration Response: Right ......................................34
Figure 4.5:7: Expt.: 70% Filled tank vs 40% Filled tank: Acceleration Response: Different Fluid Levels ........................................................................................................................................35
Figure 4.8:1: Expt.: Empty Tank: Acoustic Response: Far Field: Front & Back faces ..................................37
Figure 4.8:2: Expt.: Empty tank vs 40% Filled tank: Acoustic response: Far Field: Back ...........................38
Figure 4.8:3: Expt.: 40% Filled tank: Acoustic Response: Back: near field vs far field

Figure 5.2:1: SEA Model Setup

Figure 5.3:1: FEA Model Setup

Figure 5.5:1: FEA vs SEA: Empty Tank

Figure 5.5:2: FEA vs SEA: Filled Tank

Figure 5.5:3: FEA: Empty Tank vs Filled Tank

Figure 5.5:4: SEA: Empty Tank vs Filled Tank

Figure 5.7:1: Damping Loss Factor Plot

Figure 5.7:2: Input Power

Figure 6.1:1: Comparison: SEA vs Expt.: Base Model

Figure 6.1:2: SEA: Refined model

Figure 6.3:1: SEA vs Expt.: Empty Tank: Front: Acc. Response

Figure 6.3:2: SEA vs Expt.: 40% Filled Tank: Front

Figure 6.3:3: SEA vs Expt.: Empty Tank: Base

Figure 6.3:4: SEA vs Expt.: 40% Filled Tank: Base

Figure 6.3:5: SEA: Empty vs 40% Filled Tank: Front

Figure 6.3:6: SEA: Completely Filled Tank vs 40% Filled Tank: Front

Figure 6.5:1: SEA vs Expt.: Far Field: Empty tank: Back face SPL

Figure 6.5:2: SEA vs Expt.: Far Field: 40% Filled Tank: Back face SPL

Figure 6.5:3: SEA vs Expt.: Empty Tank: Near Field: Back face SPL

Figure 6.5:4: SEA: Empty tank: Far Field vs Near Field: Back face

Figure 6.7:1: Variance study: Empty Tank: Base

Figure 6.7:2: Variance study: Filled Tank: Base

Figure A1:1: Two Plate System

Figure A1:2: Modal density Comparison: VA One & Theoretical Results

Figure A1:3: Coupling Loss Factor Comparison: VA One vs Theoretical Results

Figure A1:4: RMS Velocity Comparison: VA One vs Theoretical Results

Figure B1:1: Expt. Setup: Assembly

Figure B1:2: Support Base: Expt. setup
Figure B1:3: Support mount: Expt. setup ................................................................. 86
Figure B1:4: Elastic cord support: Expt. Setup .......................................................... 86
Figure B1:5: Base support bracket: Expt. Setup ......................................................... 87
Figure B1:6: Stinger & Connector: Expt. Setup ........................................................... 87
Figure B2:1: Accelerometer Specification ................................................................... 88
Figure B2:2: Microphone & Preamplifier specifications ................................................. 89
Figure B2:3: Electromagnetic shaker specifications ...................................................... 89
Figure B3:1: 4 channel DAQ VI: Block Diagram ......................................................... 90
Figure B3:2: 4 channel DAQ: Front Panel ................................................................. 91
Figure B4:1: white noise VI: Block Diagram ............................................................... 92
Figure B4:2: white noise VI: Front Panel .................................................................... 93
Figure C1:1: Damping Calculation: Curve Fit .............................................................. 97
Figure C2:1: Input Power Calculation ........................................................................... 98
CHAPTER 1

1.1. Introduction

The interaction of solids with the interior or exterior fluid media plays a significant role in many engineering applications from civil [3] to aerospace [4] to automobile engineering [5]. For many such fluid structure coupling applications the closed form or analytical solutions are highly complex and difficult to obtain which led to greater emphasis on experimental testing and development of numerical techniques. The realistic limitations of testing and advancement of computational capabilities have contributed to considerable development of these simulation techniques.

The type of coupling between the solid and the fluid media determines the applicability of the numerical technique to simulate that coupling[6], [7]. If the fluid motion causes a significant deformation of the solid body leading to a change of boundary conditions for the fluid, then this coupling is called as a strong coupling or a 2 way coupling. A monolithic formulation in which a single system equation is solved can be applied for such type of coupling. If only the solid body vibrations cause a change in the fluid behavior then it is called as a weak coupling or a 1 way coupling for which a partitioned approach in which the structural and fluid system equations are solved separately can be applied.

One such area of interest considered in this thesis is the influence of fluid structure coupling on the frequency response of partially filled fluid containers such as liquid storage tanks used in water distribution systems[2], rockets and satellites for aerospace industry and gas tanks and oil pans for automobiles. While the vibrations of shells in air are similar to those in vacuum, the presence of liquid changes the frequency response of the shells.[8]
For fluids enclosed within a container, the strength of coupling depends on the frequency range of interest along with fluid characteristics. The strong coupling between fluid and structure at low frequencies causes the phenomenon of ‘sloshing’ which can induce considerable dynamic stresses on the structure and can lead to component failures. Considerable research has been carried out to develop analytical and numerical techniques to model sloshing and has been experimentally verified to some extent as well [2], [8], [9].

At high frequencies this coupling is weak in nature, and affects only the surface vibrations of the structure. As a result, a partitioned approach can be used to compute the surface vibration response. The numerical techniques discussed above are deterministic in nature (i.e. for a given system, there will always be only one solution available). Thus, to account for geometric irregularities, a design of experiment approach is required to obtain an envelope of possible responses. This approach becomes cumbersome for high frequency applications as the computation time is dependent on the mesh quality. The modal assurance criteria (MAC) reduces with the increase in frequency and the contribution of the off diagonal terms increase significantly[10]. A study for experimental correlation and optimization by Marburg et al[11] showed that FEA process could be reliably used only up to a certain frequency limit.

Another approach available for high frequency applications is Statistical Energy Analysis which uses energy transfers to calculate the average responses on the surfaces. This formulation has the ability to disregard minor geometric details and thus can cope with the irregularities of the manufacturing process.

The focus of this thesis is to test the feasibility of Statistical Energy Analysis as a numerical method to aid the design process by performing quick and accurate computations.
1.2. Motivation

This phenomenon of fluid structure coupling was encountered during my stint as a computer aided engineering (CAE) analyst, in the Noise, Vibration and Harshness (NVH) group of the Research & Development (R&D) Dept. of Bajaj Auto Ltd. an automotive company in India. One of the persistent field complaints was regarding the jarring/rattling noise heard from the motorcycle during high speed use. Some standard tests were carried out such as the Pass-By-Noise (PBN) and Noise Source Identification (NSI) and the tank region was highlighted as the problem region. The gas tank of the motorcycle is a thin sheet metal structure with thickness approximately in the range of 0.8mm-1mm as shown in Figure 1.2:1. This tank was manufactured by the stamping process which led to thinning of the material near the curvatures. Seam welding along the flange and manual spot welding for mounting the required accessories resulted in a component with varying thickness and geometric irregularities. The gas tank weighed around 2-3 kg and has a capacity to contain 10-14 L of gasoline or approximately 8.5-12 kg of fuel with specific gravity of 0.8, depending on the make and model of the motorcycle. Static load tests revealed the ability of the gas tank to carry the weight of the fluid. However, impact tests and frequency response measurements with varying levels of liquid showed the variation of the tank surface response due to the fluid present in the tank. The FEA simulation which was carried out using the partitioned approach is shown in Figure 1.2:2. For this simulation, the fluid chosen was water as its specific density is close to the specific density of gas.
It was decided to study this phenomena in greater detail using experimental testing and Statistical Energy Analysis simulation technique for this thesis project. First, a brief description of Statistical Energy Analysis and its formulation in standard form is shown. The Vibro-Acoustics software suite of ESI Group (ESI VA One) is used for simulation and its usage is
shown. To verify the appropriate use of the software, a simple simulation of two plates connected perpendicular to each other is carried out and the results are compared with the theoretical calculations in Appendix A. Then, the problem statement is defined concisely and the experimental setup and data acquisition system is described in detail. Some preliminary results and measurement of input parameters is discussed in that chapter. SEA and FEA modules of VA One software are used to simulate the experimental setup after some simplifying assumptions. In the succeeding chapter, results from simulation techniques and experimental measurements are compared for correlation purposes. Computational parameters are also compared to prove viability of SEA as a design tool.
CHAPTER 2

2.1. Background

The development of Statistical Energy Analysis is credited to independent research on resonator vibrations carried out by R.H Lyon and P. W. Smith in the early 1960’s[12]. This method developed from the need of an approach which used as little system information as possible to predict a response which would be consistent with the results with some degree of variation, especially for problems which deterministic approaches couldn’t handle efficiently and effectively at that time. Statistical Energy Analysis is a method to calculate the flow and storage of dynamical energy in subsystems of a complex structure. Energy is provided to the system by external sources like acoustic pressure or mechanical excitation which is then dissipated in the components and transferred between different storage subsystems. As relative phase and amplitude of frequency response at individual points is unpredictable in nature, it was considered as a random variable, and hence the statistical response variables as the ensemble mean square value over space and frequency were considered[13]. Thus, the statistical nature of the input and response variables meant an undetailed model could be used for this analysis. There are different methods to formulate the energy transfer equations of Statistical Energy Analysis[12], [13] which have been briefly described below:

1. Mode Based Approach:

The exchange of energy between different subsystems in the analysis can be expressed in terms of the interaction between modes of the uncoupled subsystems. The selection of the possible modes is dependent on the boundary conditions. Thus, the power transferred[12] between the subsystems is proportional to the difference of the modal energies of the two systems.
\[ P_{12} = g \times [E_1 - E_2] \] (1)

This method requires the strength of coupling to be weak, so that there is negligible intra subsystem interaction due to the other subsystem and hence is suited for vibro-acoustic problems which involve acoustic interactions between structures and enclosed fluid volumes.

2. Wave Based Approach:

In this method, the subsystems are expressed in terms of the superposition of travelling waves, and the interaction between the subsystems is evaluated from the wave transmission and reflection considerations at the junctions. Thus, the power transferred is a function of the wave impedances of the uncoupled systems at the junction.

This approach is used when there is a greater modal overlap causing many waves to contribute to the response at a frequency. As the waves are independent of the boundary conditions and depend only on the wave carrying medium, this method is used in case of strong coupling as those between structures especially if they are not isotropic.

While the approaches are different, the concept of mode-wave duality ensures the equivalency of the methods and hence at least theoretically it is possible to arrive at the same results using either of the methods mentioned above. The SEA module of ESI VA-One software suite makes use of the wave approach to calculate SEA parameters.

Over the years, the development of Statistical Energy Analysis split into two parts. One was to develop the theoretical processes to make SEA work with other systems, define the restrictions of this method and quantify the uncertainties of this process. The other was to clarify the assumptions and work on correlating experimental results with the SEA calculations. This
resulted in the development of experimental SEA[14], hybrid SEA (a combination of FEA and SEA)[15] and many parameter testing methods[16]–[19].

2.2. Parameter Definition

The basic principles of Statistical Energy Analysis are explained briefly in this section and the general form of the system equation is derived.

- **Subsystem**

A subsystem is an energy storing element which is a set of ‘similar’ energy storage modes based on the type of the component. The number and type of the subsystem depends on the system under consideration. The selection of the subsystem is critical as one assumption of SEA is that all modes in a subsystem are equally excited by the input power sources and hence have equal energies. A plate can be considered as a system and is composed of three distinct subsystems – flexural modes, extensional modes and the shear modes (face and thickness shear) as shown in Figure 2.2:1.

*Figure 2.2:1: Plate - Mode subsystem*
• **Mode Count**

The mode count represents the number of resonant modes available to exchange energy between interacting subsystems and is dependent on the geometric and material properties of the system. It can be expressed as:

- Number of modes in a frequency range \( (\Delta N) \)

- Modal density \( n(\omega) = \frac{dN}{d\omega} \text{ rad/s} \) or

- Average frequency spacing between modes \( \delta f = \frac{1}{\{2\pi n(\omega)\}} \)

• **Subsystem Energy** \( E_i \)

The total energy associated with all the modes in the given bandwidth of a subsystem is called as the energy of that subsystem. Hence, for a subsystem mass \( m_i \) and mean square surface velocity, \( \langle v_i^2 \rangle \) the total subsystem energy is related to the spatially averaged surfaces responses as shown in equation (2)

\[
E_i = m_i \langle v_i^2 \rangle
\]  

(2)

The assumption of equal distribution of energy among all the modes of the subsystem means that the total energy of the subsystem is the product of number of modes of the subsystem and the energy associated with each mode.

\[
E_i = N_i \ast \epsilon
\]  

(3)

• **Damping Loss Factor** \( \eta_{ii} \)

The damping loss factor indicates the energy dissipated in the subsystem. It is an input parameter which is material and noise treatment specific and is related to other damping parameters like critical damping ratio or the quality factor.
The power dissipated in the subsystem is proportional to the average energy lost to the mechanical vibrations in the subsystem and is only dependent on the modal energy of that subsystem.

For an $i^{th}$ subsystem with damping loss factor ($\eta_{ii}$) the total modal energy ($E_i$) the power dissipated is given as follows.

$$\pi_{i,\text{diss}} = 2\pi f \eta_{ii} E_i$$  \hspace{1cm} (4)

- **Coupling Loss Factor** ($\eta_{ij}$)

The coupling loss factor is associated with the average power flow between the coupled subsystems and thus is proportional to the difference in the modal energies of the subsystems. The CLF is dependent on the two coupled media as well as the geometric description of coupling (i.e. point, line or area coupled).

The power transferred between the $i^{th}$ and $j^{th}$ subsystem with coupling loss factors $\eta_{ij}$ and $\eta_{ji}$ is given as

$$\pi_{ij} = 2\pi f (\eta_{ij} E_i - \eta_{ji} E_j)$$  \hspace{1cm} (5)

As the total energy of a subsystem is just the mode count times the modal energy, the coupling loss factor is subject to the reciprocity relation.

$$\eta_{ij} * N_i = \eta_{ji} * N_j$$  \hspace{1cm} (6)

- **External Excitation** ($\Pi_i$)

As SEA is essentially a power flow method, the measured responses are proportional to the external excitations. External inputs can be acoustic pressures, point or distributed mechanical excitations, turbulent boundary layer to name a few.
2.3. Formulation

As statistical energy analysis is based on the theory of power flow and energy transfer, a common representation of SEA model can be shown as in the following figure.

![Figure 2.3: Three Subsystem SEA model](image)

The input power is an external user defined excitation for each subsystem of the model and thus for a ‘N’ subsystem model, it is a N x 1 column matrix

$$\pi_{in} = [\pi_1 \ \pi_2 ... \pi_N]^T$$  \hspace{1cm} (7)

The expressions for power dissipated by damping and the power transferred between subsystems were described above. Thus the power distribution equation for the i\textsuperscript{th} subsystem is given as shown in equation (8)
\[ \pi_i = 2\pi f \eta_i E_i + 2\pi f \sum_{j=1 \atop j \neq i}^N \eta_{ij} E_i - \eta_{ji} E_j \]  

(8)

For the N=3 subsystem model shown, the above system of equations can be expressed in the matrix form as shown in equation (9)

\[
\begin{pmatrix}
\pi_1 \\
\pi_2 \\
\pi_3
\end{pmatrix} = 2\pi f \begin{bmatrix}
\eta_{11} + \eta_{12} + \eta_{13} & -\eta_{21} & -\eta_{31} \\
-\eta_{12} & \eta_{21} + \eta_{22} + \eta_{23} & -\eta_{32} \\
-\eta_{13} & -\eta_{23} & \eta_{31} + \eta_{32} + \eta_{33}
\end{bmatrix} \begin{pmatrix}
E_1 \\
E_2 \\
E_3
\end{pmatrix}
\]

(9)

Or in simplified form

\[
\{\pi_{in}\} = 2\pi f [T]\{E\}
\]

(10)

\([T]\) is called as the loss factor matrix as it is composed of the damping and coupling loss factors.

This formulation is known as the unsymmetrical form of equations as \(T\) is not symmetrical.

The \(T\) matrix can be made symmetrical using the reciprocity relation and the symmetric form, which is used for experimental SEA as well as for VA One SEA formulation. The symmetric system of equations is given as shown in equation (11)

\[
\begin{pmatrix}
\pi_1 \\
\pi_2 \\
\pi_3
\end{pmatrix} = 2\pi f \begin{bmatrix}
(\eta_{11} + \eta_{12} + \eta_{13})N_1 & -\eta_{21}N_1 & -\eta_{31}N_1 \\
-\eta_{12}N_2 & (\eta_{21} + \eta_{22} + \eta_{23})N_2 & -\eta_{32}N_2 \\
-\eta_{13}N_3 & -\eta_{23}N_3 & (\eta_{31} + \eta_{32} + \eta_{33})N_3
\end{bmatrix} \begin{pmatrix}
E_1 \\
E_2 \\
E_3
\end{pmatrix}
\]

(11)

With

\[
\eta_{12}N_1 = \eta_{21}N_2 \atop \eta_{13}N_1 = \eta_{31}N_3 \atop \eta_{23}N_2 = \eta_{32}N_3
\]

(12)

The spatially averaged surface responses are related to the modal energies as defined. In this way, the method of Statistical Energy Analysis can be used to calculate the responses when the input excitations and the loss factors are known.
2.4. Summary

To summarize, in this chapter, the formulation of Statistical Energy Analysis system of equation is studied and the significant model parameters are described. SEA uses a statistical approach to dynamical system and hence this method requires only a coarse description of the system to provide an initial idea about the trend of the system response.

As it is a power flow method, it provides the designer an idea about power transmission through the system and thus the contribution of different components to the system response can be identified.

As with any statistical approach, there is some uncertainty associated with the averaged responses, especially when the modal density is low or the bandwidth of excitation frequency is too narrow.
CHAPTER 3

3.1. Thesis Scope

The scope and extent of this thesis project is defined in this chapter after reviewing the basic concepts earlier. As stated, the study of noise and vibration characteristics of the motorcycle gas tank was the motivation for this project. While the shape and capacity of the tank depends on the automobile type, it is usual for motorcycles to have a fuel capacity of 4-5 US gallons with an additional reserve of 0.6-1 gallon. The specific gravity of gasoline is in the range of 0.71-0.77 which means approximately 14-15kg of fuel mass. The shape of the tank is highly dependent on the packaging constraints and aesthetic requirements as shown in Figure 3.1:1.

![Gas Tank (JP cycles)](image)

*Figure 3.1:1: Gas Tank (JP cycles)*

The gas tank is made from thin sheet metal or strengthened plastic material which affects the tank response characteristics.
3.2. Design Parameters

In this project, the tank dimensions were dependent on the design requirements, manufacturing process and the experimental testing limitations.

Design Requirements

While the tank used in real applications has curvatures and complicated shapes, the design was simplified to produce a simple parallelepiped tank model. It was decided to make one dimension of the tank significantly larger than other similar to the actual gas tank. Water was used as fluid instead of gasoline to avoid any potential conflagration issues. The acoustic pressure modes for water were dependent on the tank dimensions. After some initial calculations of fluid volume, the tank dimensions were chosen to be 18 in X 8 in X 8 in with a thickness of 0.036 in.

Manufacturing Constraints

The simple tank design and decision to manufacture the tank using standard welding technique of gas welding reduced the manufacturing time. However, it was found that there was too much distortion during the welding process for sheet metal with thickness 0.036 in. In order to this thickness of sheet metal, it would have been necessary to use dedicated fixtures for supporting tank surfaces during welding.

Experimental Testing Constraints

The vibrations and acoustic measurements were to be conducted in the NC State anechoic chamber. There were three lifting hooks attached to the roof of the chamber and each was capable of supporting up to 100 lb. according to the documentation provided.
As the thickness of the tank increases, the excitation frequency required for fluid structure interaction increases. The accelerometers used have an operable frequency range of 5-10000 Hz which though lowest among all the testing equipment used was still sufficient for testing purposes. As Statistical Energy Analysis assumes that resonant frequencies and modal behavior is unknown, simply increasing the frequency range of testing could lead to measuring the fluid structure interaction effects on the tank surfaces. To avoid the necessity to use some lifting mechanism for tank testing, the dimensions of the tank were not altered significantly. Instead, it was decided to increase the frequency of excitation up to 10,000 Hz which is the limit of the test setup.

It was decided to increase the thickness of sheet metal from 0.036 in to 0.06 in and the tank dimensions were modified. The final tank dimensions are 16 in X10 in X8 in with sheet metal thickness of 0.06 in. The mounts are L shaped brackets with a 3 in X3 in cross section and a length of 8 in and thickness 0.12 in which are welded to the sides as shown. With this design, the tank could hold up to 6 gallons of water which would weigh around 21 kg.
3.3. Preliminary Testing

After manufacturing the tank described above, some preliminary impact hammer tests were carried out to test the behavior of the tank. The implicit assumptions in simulation techniques were checked to see the conformity of the tank responses to the assumptions which are discussed in this section. The tank was clamped to the table at the mounting locations and responses were measured using accelerometers. A grid was marked on the tank surfaces and measurements were taken at the grid points as shown in Figure 3.3:1. The top face has dimension of 16 in X 8 in, front face is 16 in X 10 in and the right face is 10 in X 8 in. A correlation coefficient was used to measure the similarity of the test responses and assign a numerical value to the correlation of the test responses.
Reciprocity theorem

This theorem states that a response measured at point ‘i’ when excited at point ‘j’ is the same as the response measured at point ‘j’ when excited at point ‘i’. This follows from the assumption that the frequency response function for linear, isotropic systems is symmetric in nature. To verify this, for each face, the response was measured at one location when excited from another point on the same face, and then vice versa. The correlation in the frequency domain for these signals is plotted as shown in Figure 3.3:2. The equal bandwidth averaged frequency response is shown with the solid lines to highlight the similarities and differences in the response behavior of the two signals. If the assumption of reciprocity was true, then the responses would match each other and the correlation coefficient would be $R=1$. However, as can be seen the correlation coefficient is close but isn’t equal to 1. By visual inspection, it could be seen that some surfaces had geometric irregularities which could affect the measurements.
Symmetry

This assumption states that the response measured at locations which are equidistant from the excitation location and the surface boundaries should show the same response. This is important as the eigenvectors and mode shapes are symmetric by theoretical calculations. Each tank surface was excited at the center of the face, and responses were measured at symmetric points along the longitudinal and lateral directions of the plate faces to ensure in plane symmetry of response. As before, correlation coefficient was calculated for these responses. It can be seen from Figure 3.3:3 and Figure 3.3:4, that there is a significant correlation between the responses, but it isn’t exactly equal to 1 which meant presence of some geometric irregularities in the surface. The longitudinal and lateral directions are along the perpendicular directions in the plane of the surface.
Figure 3.3.3: Symmetry: Front Face: Longitudinal Direction

Figure 3.3.4: Symmetry: Front Face: Lateral Direction
**In Plane Vibrations:**

This measurement was carried out to check the contribution of different modal subsystems to the vibration response of the plate. Theoretically, it is proven that the flexural modes dominate the surface responses. A tri axial accelerometer was used and each face was excited in the three orthogonal directions. The accelerometer was mounted in such a way that the Z direction was always perpendicular to the plane of the face. For excitation along the in plane directions, the impact was made at the boundary normal to that direction. As can be seen from Figure 3.3.5 and Figure 3.3.6, the flexural response does dominate the vibration response of the plate even if the excitation is provided to the in plane vibration modes. This is useful, as further measurements can be made with a uniaxial accelerometer which can measure only out of plane responses.

*Figure 3.3.5: X excitation: Top Face response*
The response for excitation normal to the plane surface also shows the flexural response to be significantly higher than the in-plane response as seen in Figure 3.3:7.
3.4. Summary

In this chapter the scope of this thesis project was discussed. It was decided to experimentally determine the surface vibration and acoustic responses of the tank and use the FEA and SEA software modules of the ESI Vibro-Acoustics software suite to simulate the experiment. The design and manufacturing considerations for determination of the tank dimensions were explained. To mimic the application, it was decided to excite the tank only from its mounting points. It was decided to measure responses over a frequency range from 0 Hz – 11220 Hz using one-third octave frequency bands to see the effects of fluid structure interactions. The testing procedures and results are explained in detail in the next chapter.

The results of some preliminary tests carried out were discussed in this chapter to highlight the difference of the real world applications from the idealized assumptions of the simulation environment. The presence of this difference indicated the need to obtain simulation responses over a range of variations of geometrical parameters in order to correlate the experimental and simulation responses. Variance studies were carried out to account for this reason in this thesis. As correlation requires measurement of some input parameters like damping and power input, these are calculated from testing and are used as inputs for the simulation tools. This is described in detail in Chapter 5 along with an explanation of the modelling methods and simulation techniques used. The results from SEA and FEA are compared and some observations are made regarding the efficiency and accuracy of these methods. In Chapter 6, the experimental results are compared with SEA for checking the correlation of testing and simulation techniques. Some application of SEA for analysis led design process are discussed in that chapter.
CHAPTER 4

4.1. Experimental Setup

The experimental setup and testing procedures are explained and the results are discussed in this chapter. It was decided to replicate the boundary conditions and the excitation experienced during the real life use of the gas tank. To achieve this, a base structure was designed from Aluminum T shaped and L shaped channels and the tank was mounted on this structure as shown in Figure 4.1:1. The details of this model are attached in Appendix B. This arrangement allowed for simulation of the tank excitation via its connections to the vehicle frame.

![Figure 4.1:1: CAD Model: Tank Structure](image)

The approach of mounting the tank structure to the shaker table to provide base excitation required the availability of a shaker table with the necessary loading capacity. Another approach is to use the free-free modal analysis conditions wherein either the test component or the electrodynamic shaker is suspended using elastic cords. As the excitation to the tank during operating conditions is primarily normal to the base, it was decided to suspend the tank using elastic cords and use the electrodynamic shaker to provide normal excitation from the bottom. In order to measure the acoustic and vibration response, the anechoic chamber in NC State University MAE Dept. was used. This experimental setup is shown in Figure 4.1:2.
4.2. Experimental Apparatus

In this thesis, vibration and acoustic measurements were carried out to measure the tank responses for varying liquid levels. The following transducers were used for experimentation.

**Accelerometers**

PCB ICP uniaxial shear accelerometers of the model type 352C33 are used for measuring the flexural vibrations of the tank surfaces. With a sensitivity of 100mV/g and a frequency range of 0.5Hz to 10,000Hz, these accelerometers were suitable for the testing purposes.

**Microphones & Pre-amplifiers**

The B&K 4188 A ½ inch free field microphone with a preamplifier with a sensitivity of 31.5mV/Pa and a frequency range of 20 Hz-12,500 Hz was chosen for measuring the acoustic response of the tank.
Electromagnetic Shaker & Amplifier

B&K LDS V203 low force range shaker with a frequency range of 5-13,000Hz was used along with PA 25E power amplifier to excite the test setup.

A slender cylindrical steel rod of 12 in was manufactured to connect the shaker to the test component. The purpose of this rod called ‘stinger’ is to transmit only the axial force from the shaker to the tank and to decouple the shaker mass from the lateral and transverse fluctuations of the tank.

Force Transducer

PCB ICP force transducer of model type 221A05 with a sensitivity of 2.25 mV/N was used to measure the force. The force transducer is connected between the stinger and tank and is used to measure the net dynamic force on the tank.

The complete specifications of these instruments are attached in Appendix B.

4.3. Signal Generation

There are various deterministic or random signals[20] which can be used to excite the test structure for the measurement of the vibration responses. Deterministic signals like swept sine, saw tooth waveform or sine chirp can be described by mathematical functions. Sinusoidal or sine sweep excitation can provide a very good response spectrum and is used for modal analysis. However, it is the slowest technique for signal excitation, as each spectral frequency is excited individually and there is some delay time for system stability.

Random signals such as Uniform White Noise and Pink Noise among others cannot be described by mathematical functions but by statistical characteristics. White noise is characterized by a uniform power spectral density while for pink noise, the power spectral
density varies inversely with frequency. Random signals excite the structure with varying amplitudes and frequencies and tends to average out the responses over time. While there are some leakages due to side lobes of window functions, this type of input signal characterizes most of the real life excitations and hence, a uniform white random noise was selected for system excitation.

National Instruments (NI) LabVIEW was used to generate the uniform white random noise in different frequency bandwidth. The Figure 4.3:1 shows the power spectral density (PSD) (ref 1V^2/Hz) of a uniform white random signal within a frequency range of 0-11360 Hz with a maximum amplitude of 0.5V rms.

![Figure 4.3:1: Input Signal PSD](image)

An Agilent U8903A audio analyzer was used to transmit the NI LabVIEW generated random signal to the electromagnetic shaker. The process to generate the required signal after
accounting for the difference in sampling frequencies of these two systems is explained in Appendix B.

It was decided to excite the tank over a large frequency range as the modal information of the system was unknown. Hence, input signals up to 11,360 Hz were generated in different frequency bandwidths which just reached the limit of the functional range of the accelerometers.

4.4. Data Acquisition Parameters

General purpose coaxial cables were used to connect the transducers to BNC cables. No separate signal conditioning unit was required as the transducers used were integrated piezoelectric transducers. National Instruments PXI chassis 1031 with a 4 channel 24 bit analog input data acquisition card PXI 4462 was used for acquiring and processing the response signals. The inbuilt analog to digital convertor of NI was used for processing the data. NI LabVIEW, a system design platform and development environment using graphical language was used for specifying the important data acquisition parameters and carrying out data manipulation and post processing.

The sampling frequency for data acquisition was chosen as 40,000 Hz, so that the Nyquist frequency cut off of 20,000 Hz was significantly greater than the excited frequency range. Hanning window was used to reduce the side lobe leakages and 32 RMS averages were computed for each response. The relevant LabVIEW program codes called ‘VI’s’ are attached for reference in Appendix B.

As Statistical Energy Analysis calculates the spatially averaged surface responses, it was decided to take multiple responses on the tank surfaces. A survey of some previous
experimental studies [14] mention measuring responses at 3-5 locations for good spatial averaging. In order to account for the geometric irregularities, it was decided to measure the 15 responses each on 16 in X 10 in side and 16 in X 8 in side. The measured acceleration response was spatially averaged and its power spectral density with ref to $\frac{1g^2}{Hz}$ was calculated. In order to study the influence of fluid on the overall tank response, it was decided to measure the sound pressure level around the tank in the anechoic chamber. For the acoustic response measurement, the microphones were mounted along the normal direction to the tank surface. Near field acoustic response (~ 2 in away) and Far field acoustic response (~ 40 in away) was measured for the four vertical sides for both the experimental conditions. The near field experiment setup is shown in Figure 4.4:1.

![Near Field Experimental Setup](image)

**Figure 4.4:1: Near Field Experimental Setup**

It was decided to measure the responses for different fluid levels and as such experiments were conducted with 40% water filled and 70% water filled tank along with an empty tank setup.
4.5. Experiment Description

In this section the results of the experiments are explained and some observations are made regarding the behavior of the fluid tank system. The tank surfaces are named as shown in Figure 4.5:1. The L shaped mounts are welded to the left and the right face of the tank. These mounts are used for suspending the tank in the anechoic chamber.

![Tank Nomenclature](image)

Figure 4.5:1: Tank Nomenclature

The vibration and acoustic measurements are performed on both the empty and fluid filled tank. While many experiments were carried out, some of the representative tests are discussed and the results plotted in this chapter.

The experiments conducted for testing the fluid structure coupling in the tank and measured parameters are tabulated below.
Table 4.5-1: Experiment Description

<table>
<thead>
<tr>
<th>Experiment Type</th>
<th>Tank Setup Description</th>
<th>Transducer mounting</th>
<th>Measurement Location</th>
<th>Measurement Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration</td>
<td>Empty Tank</td>
<td>On Tank</td>
<td>Front Face Base Face Right Face</td>
<td>Acceleration Force</td>
</tr>
<tr>
<td></td>
<td>40% Filled Tank</td>
<td>On Tank</td>
<td>Front Face Base Face Right Face</td>
<td>Acceleration Force</td>
</tr>
<tr>
<td></td>
<td>70% Filled Tank</td>
<td>On Tank</td>
<td>Front Face Base Face Right Face</td>
<td>Acceleration Force</td>
</tr>
<tr>
<td>Acoustic</td>
<td>Empty Tank</td>
<td>Near Field (~2’’)</td>
<td>Front Face Back Face Right Face Left Face</td>
<td>Sound Pressure Level</td>
</tr>
<tr>
<td></td>
<td>40% Filled Tank</td>
<td>Far Field (~40’’)</td>
<td>Front Face Back Face Right Face Left Face</td>
<td>Sound Pressure Level</td>
</tr>
</tbody>
</table>

Based on the sampling rate and the number of samples, the frequency step size in the power spectral density plot is almost 20Hz. This data was averaged over the one-third octave frequency bandwidths and the raw PSD data along with the bandwidth averaged PSD data is plotted in this section. This is done as the bandwidth average data would be enable a better understanding of the vibration trends of the tank. The raw PSD data is plotted with dotted line and its one-third frequency bandwidth averaged response curve is plotted as a bold dashed line in the following plots for comparison.

In order to study the effect of fluid on the vibration response, it was decided to first compare the power spectral density of the net force dynamic response for the empty tank and 40% filled
tank experiment to verify the similarity of the input for both testing conditions as shown for the base in Figure 4.5:2 and the front face in Figure 4.5:3.

The power spectral density of the acceleration response for the tank faces for the different setup conditions is compared to show the influence of the fluid on the vibration response of the surface. The response for base (Figure 4.5:4), front (Figure 4.5:5) and the right face (Figure 4.5:6) are attached. The response for the front face for different fluid levels is also studied as shown in Figure 4.5:7.

For the acoustic measurements the sound pressure level was measured at different locations and different test conditions for different faces. To show the similarity of responses from the symmetrical face pairs like front and back or the left and right face, their responses are plotted for comparison in Figure 4.8:1. Due to this similarity, only the responses of the back face are chosen for plotting in this section. The difference in the acoustic response for the back face for different tank setups and for far field condition is shown in Figure 4.8:2. The near and far field response for the back surface is plotted in Figure 4.8:3.
Experimental Results: Vibration Testing

Figure 4.5:2: Expt.: Empty tank vs 40% Filled tank: Force Response: Base

Figure 4.5:3: Expt.: Empty Tank vs 40% Filled Tank: Force Response: Front
Acceleration Response

Figure 4.5.4: Expt.: Empty tank vs 40% Filled tank: Acceleration Response: Base

Figure 4.5.5: Expt.: Empty tank vs 40% Filled tank: Acceleration Response: Front
Acceleration Response

Figure 4.5.6: Expt.: Empty tank vs 40% Filled tank: Acceleration Response: Right

Figure 4.5.7: Expt.: 70% Filled tank vs 40% Filled tank: Acceleration Response: Different Fluid Levels
4.7. Observations – Vibration Response

- The comparison of net dynamic input forces for the empty tank and 40% fluid filled tank in Figure 4.5:2 and Figure 4.5:3 show a good similarity. The fluctuations can be attributed to the additional inertial mass of fluid and the change of the elastic cord stiffness. This fluctuation isn’t large enough to cause non-linear or indirect coupling and hence the two experimental conditions shouldn’t violate the basic assumption of small displacements.

- The general effect of addition of fluid has been to reduce the amplitude of the vibration response especially in the high frequency region. This is seen explicitly in the response of the front face (Figure 4.5:5), while the base face response (Figure 4.5:4) shows some interactions at lower frequencies as well. It is expected that the base response would be affected the most by the addition of fluid, due to the inertial effects and the damping of the fluid.

- The acceleration response of the right side doesn’t indicate such a significant fluctuation in Figure 4.5:6. This could be due to the additional stiffness of the face by welding of the thicker mount. This could reduce the influence of the fluid on the vibration response of the face as the face isn’t a thin enough sheet metal surface.

- The front face response due to varying fluid volume in the tank shown in Figure 4.5:7 indicates almost no change due to the additional fluid volume. This is quite possible as the major influence of fluid on this face is by the dissipative coupling which wouldn’t change much due to the volume change of the fluid.
4.8. Experimental Results: Acoustic Measurements

Figure 4.8.1: Expt.: Empty Tank: Acoustic Response: Far Field: Front & Back faces
Sound Pressure Level Response

Figure 4.8.2: Expt.: Empty tank vs 40% Filled tank: Acoustic response: Far Field: Back
Figure 4.8:3: Expt.: 40% Filled tank: Acoustic Response: Back: near field vs far field
4.9. Observations – Acoustic Response

- The far field acoustic response for the front and back faces of the tank is similar as shown in Figure 4.8:1. This similarity is expected due to similar dimensions and properties of the faces. The small variations can be attributed to the geometric irregularities and the effect of the surrounding structures on the measured response. Due to this type of behavior, only the responses of the back surfaces were shown in this chapter. The observations about the experimental trends should be valid for other faces.

- The far field acoustic response for the back face for empty and 40% fluid filled tank (Figure 4.8:2) shows the reduction in acoustic sound pressure level at high frequencies due to the presence of fluids. This is consistent with the acceleration response of the faces which reduce due to addition of fluid volume. The variation of acoustic response in the low range up to 3000 Hz was unexpected and may be due to acoustic inputs from the base structures and the surrounding bodies in the anechoic chamber.

- The near field and far field comparison of the acoustic response (Figure 4.8:3) indicates that the near field sound pressure is higher than the far field response. This is expected, as the sound pressure decreases with the increase in distance from the sound source.

4.10. Conclusions

It can be seen from all the plots that the frequency bandwidth averaged response plots are a good approximation of the response signals and can be used for identifying the data trends and make the necessary observations.

The acoustic and vibration responses are highly dependent on the input force characteristics. The vibration responses indicate that the predominant effect of addition of the fluid volume to
the empty tank is to reduce the amplitude of the response. Increase in the fluid volume from 40% to 70% of the tank volume did not have a significant effect on the vibration response. The experimental results show influence of some external parameters as seen from the fluctuations in the input force responses and acoustic responses. The acoustic responses measure the contribution of all the participating components and hence is more sensitive to the experimental setup conditions.

These responses are compared with the Statistical Energy Analysis (SEA) response in the later chapters for correlation.
CHAPTER 5

5.1. Introduction

It was found that the commercial use of Statistical Energy Analysis as a simulation tool was not as widespread as other Finite Element Analysis formulation codes. While there were some basic standalone software codes for SEA modeling and simulation, only a couple of well-established software modules were found. The Vibro-Acoustics software suite of ESI group contained both the SEA and FEA modules for simulation purposes. The advantage of using the same software suite for both simulation methods is in the compatibility of the modeling and system parameters. It was easy to define the entire model for SEA simulation and simply convert it into a FEA model without redefining a lot of the system parameters. The same NASTRAN solver was used and hence the solver parameters and solver efficiency were the same for both the modules.

In this chapter, modelling and simulation of the test setup is described with an explanation of the underlying assumptions and the results are compared for the simulation methods. The appropriate use of the SEA module for a simple two plate system is verified by comparison with theoretical results in Appendix A.

5.2. SEA Simulation model

The tank system was modeled in the SEA module as shown in Figure 5.2:1. This simplified model is used for comparison with FEA as the number of subsystems are less. A significant difference was found when results of this model were compared with the experimental studies. As a result, a more detailed model was created which is explained in detail in the next chapter.
The tank faces are assigned a steel material with thickness 0.06 in and the L mounts are assigned the same steel material and have a thickness 0.12 in. The seam welding along the face edges is defined by a line junction between the faces. Line mass is added to the junction to account for additional filler material mass. The welding connection between the L mount and tank surface is along the vertical edge as shown and not across the upper edge of the plate. In order to model welding along the same edges as the manufactured tank, the coincident node technique was used. Hence, two separate nodes were created close to each other and were defined in the face subsystem and the mount subsystem. Thus, it was possible to define the line connection along this edge, without specifying any other contact region.

An acoustic cavity is created within the enclosed volume of the tank. For an empty tank, the cavity is considered to be filled with air. The acoustic cavity is water in the case of a fluid filled tank. The acoustic cavity is connected to tank faces by area junctions.
To measure the sound pressure level at a fixed distance from the model, a semi-infinite fluid region was created at distances specified from the experimental testing. This acted like an energy sink, and stored the energy transmitted from tank without reflecting it back or interacting with the system.

A NASTRAN internal solver is used for calculation and the frequency was chosen from 100 Hz to 16000 Hz in one-third octave frequency bandwidths and acceleration responses were computed along with other parameters for comparison.

5.3. FEA Simulation Model

The SEA model described above was converted easily into a Finite Element Analysis model. The material property parameters and the damping factors are transferred directly between the simulation modules. Each tank face was defined as a separate FE subsystem and was meshed with at least 4 elements per wavelength. As the frequency increases, the wavelength decreases, and hence the total number of elements in the FE system increase as the geometric size of the system remains the same. Thus, the frequency range determined the mesh size as well the computation time for FEA simulation. The acoustic cavity was also meshed and the FEA simulation model is shown in Figure 5.3:1.
The response was measured in the normal direction to the tank faces and in order to compare the FEA responses with the spatially averaged SEA responses, approximately 30 locations were used for response measurement in the simulation and the averaged response was used.

5.4. Assumptions

- As explained in the previous chapter, the free-free boundary condition is used to isolate the tank from the surroundings. This boundary condition can be simulated by not applying any constraints to the model.

- For experimental testing, a base structure was created to excite the tank from the mounts with the help of the electromagnetic shaker placed at an equal distance from the mounts. For the ease of modelling the system, and considering the relative stiffness of the base structure and the tank it was initially decided to consider the base structure to be rigid so that there is a direct transfer of the input power to the tank. However, at
high frequencies this assumption may not hold and hence, as will be explained in the next chapter, a more detailed model of the test setup was created for correlation purposes.

- As SEA responses are not affected significantly by minor geometric irregularities of holes and plate warping, it was decided to model the tank faces as simple rectangular plates. As the L mounts are bolted to the T beams, the influence of mounting holes on vibration response is reduced, and hence L mounts are also modeled without the mounting holes.

- While the damping loss factor for the tank surfaces was calculated, the damping loss factor for water was assumed to be 20%. [21] The damping loss factor of air is taken as 1%.

- The theoretically calculated modal density and coupling loss factors were used for simulation as these are not user dependent but geometry and material dependent.

- For a simple plate, the out of plane flexural vibrations were observed to be more dominant than the in-plane shear or extensional modes during the preliminary testing. As a result, uniaxial accelerometers were used for measuring the vibration response for the tank surfaces, and hence it was decided to exclude the extensional and shear modes from the SEA formulation. This was done for convenience and it was seen that the presence of the plane modes doesn’t have any significant effect on the plate response.

- The basic assumptions of material isotropy, homogeneity and linearity are used.
5.5. Results

With these assumptions in place, it was decided to compare the responses of the SEA and FEA simulation models for a unit force excitation and a constant damping loss factor of 0.1% as these parameters are unknown and assumed constant during the initial design process. Usually in such cases, simulation tools are used to compute the response trend lines and study the behavior of the system. For this reason, the results of both the models for an empty tank and a completely filled fluid tank were compared for initial estimation of computational time and accuracy. The flexural or out of plane RMS velocity responses of three sides of the tank are plotted for the above mentioned input conditions. A large frequency range was chosen for this study and the responses are calculated in one-third octave frequency bandwidths.

It was decided to plot the responses for the three different sized surfaces of the tank as these would be representative of the vibration response of the tank. These plots are compared in the next section, and some observations are made regarding the nature of solutions of SEA and FEA as simulation tools.
RMS velocity response: SEA vs FEA for empty tank

Figure 5.5: FEA vs SEA: Empty Tank

RMS Velocity Response: Filled tank: SEA vs FEA
Figure 5.5.2: FEA vs SEA: Filled Tank
RMS Velocity Response: FEA: Empty Tank vs Filled Tank

Figure 5.5.3: FEA: Empty Tank vs Filled Tank
RMS Velocity Response: SEA: Completely Filled vs Empty Tank

Figure 5.5: SEA Spatially Averaged Response

Figure 5.5:4: SEA: Empty Tank vs Filled Tank
5.6. Observations

- It can be seen from Figure 5.5:1 and Figure 5.5:2 that at low frequencies the SEA velocity RMS response is significantly larger than the FEA velocity response. As the frequency increases, the FEA and SEA responses start to converge. One of the possible reasons for this could be the increase of the number of modes participating in the energy transfer with frequency. At low modal densities the SEA assumption of equal distribution of energies for all the subsystem modes might be violated and hence, the response might be a bit on the higher side.

- The FEA response comparison for a completely filled tank vs an empty tank exhibits more fluctuations over the frequency range than the SEA response comparison for the same case as seen in Figure 5.5:3 and Figure 5.5:4. This is to be expected as SEA is an averaged response over the entire surface and while the FEA response is measured at 20 locations, it is still not ideally spatially averaged.

- It can be seen that the fluid filled tank response is higher than the empty tank response in some frequency bandwidths for FEA simulation method (Figure 5.5:3) while the response decreases almost uniformly for all the surfaces for the SEA method. (Figure 5.5:4). One of the reasons for this could be the difference in the method used for the computation of the fluid structure coupling for these simulation tools.

- The computational time for FEA method was greatly dependent on the frequency range of interest. As FEA required creation of global stiffness and mass matrix for Eigenvector and Eigenvalue calculations, it took a long time to finish the intermediate computational processes. For the computationally more intensive simulation of fluid
filled tank, it took approximately 3.5 hrs. for the software to complete all calculations for FEA simulation. In comparison, SEA computation took only a couple of minutes to complete its calculations. All the simulations were carried out on the same DELL Precision T5610 workstation with no programs running in the background.

- The power flow plots for SEA method provided insights into the contributions of different components to the overall response of a surface similar to the transfer path analysis approach for FEA studies. However, this being an inherent part of computation required no extra setup which was an added advantage.

Thus, SEA is a valid alternative to FEA for simulation purposes, especially in the high modal density applications. With this validity established, it was decided to check the correlation of SEA results with experimental data in the next chapter.

5.7. Experimental SEA

As explained before, out of the four critical parameters of SEA methodology, the specification of the damping loss factor of the components and power input to the system are user dependent, while computation of modal densities and coupling loss factors are geometry and material dependent.

**Damping Loss Factor**

There are several methods to measure the damping factor of lightly damped materials such as the half power method and decay method[12], [14], [16]. In this project, the decay rate method was used for experimental measurement of the damping loss factor.

After an initial transient excitation, the response of the surface decays exponentially and is dependent on a constant as shown below
Thus, once the peaks in the response are identified, a curve fitting method can be used to fit an exponential curve with the constant $A$ and the damping loss factor $\eta$ as the variables.

A transient excitation was applied with a modal impact hammer and the acceleration frequency response was measured on the tank face at 2 response points. The damping loss factor was computed over the entire frequency range as well as for one third octave bandwidth limited frequency range.

A MATLAB code was developed to identify the peaks in the response and use an exponential curve fitting function and is attached in Appendix C for reference. The value of frequency $\omega$ was chosen as the center frequency of the frequency band and the variation of the damping loss factor with frequency is plotted in Figure 5.7.1.

![Damping Loss Factor Plot](image)

*Figure 5.7.1: Damping Loss Factor Plot*
The average damping loss factor over the entire frequency range is \( \eta_i \sim 0.14\% \) which is consistent with the defined values of damping for steel. The damping loss factor was measured for two faces and as it is a function of the material type the same damping loss factor was used as an input for the tank faces for both the simulation techniques.

**Input Power**

Input power is usually computed with an impedance head using the mobility and point force data to compute the power. However, it is possible to derive the input power using accelerometer and force transducer data\[22\], mounted at different locations if the data acquisition starts simultaneously for good phase matching. The input power can be computed as the cross correlation at \( \tau = 0 \) of the force and accelerometer signals by the formula below.

\[
R(f, v, 0) = - \int_{-\infty}^{\infty} \text{Imag} \left\{ \frac{G(f, a, \omega)}{\omega} \right\} d\omega \quad [22]
\]

Where \( G(f, a, \omega) \) is the cross spectral density of force and acceleration.

A MATLAB code was developed to compute the input power over one-third octave frequency bandwidths and is attached for reference in Appendix C. The input signal to the shaker was limited to 11,360 Hz while data was acquired up to 20k Hz. The computed input power is shown in Figure 5.7.2. As the accelerometer data is accurate up to 10,000 Hz that is the maximum range of Input Power shown.
The modal density of a plate is a property of the geometrical dimensions and material properties. Considerable work has already been carried out to experimentally verify the modal density \[18\], \[23\] by measuring the frequency response function for the plates.

Coupling loss factor is another critical parameter for SEA simulation. For the initial design, the loss factors cannot be experimentally determined and hence it is expected that the designer would have to rely on the theoretically calculated coupling loss factors. There are methods for experimental verification of the coupling loss factors such as the power injection method and the power transmission coefficient method. The coupling loss factors for the plates welded perpendicular to each other have been verified \[16\], \[18\], \[19\] and hence in this thesis, it was decided to use the theoretical values of the modal density and coupling loss factors for the SEA simulation. With these inputs, the SEA model simulation is carried out and the results are compared with the experimental testing data in the next chapter.

\[Figure\ 5.7:2:\ Input\ Power\]
CHAPTER 6

6.1. Introduction

In the previous chapter, the comparison of SEA and FEA simulation techniques highlighted the advantages of using SEA for quick initial design considerations. The tank model developed in the last chapter was used to compute the simulation results and the SEA and experimental results for the spatially and frequency averaged responses were compared for correlation studies in this chapter. In case of acoustical measurements, a semi-infinite fluid was used to record the sound pressure levels at the distances specified from experimental studies from the tank body. A semi-infinite fluid is an energy sink, which measures the incident pressure without causing any reverberation thus modelling the behavior of the anechoic chamber.

The comparison of the acceleration response for the front face of an empty tank for SEA simulation and experimental testing is shown in Figure 6.1.1.

![Acceleration PSD response](image)

*Figure 6.1.1: Comparison: SEA vs Expt.: Base Model*
A significant difference in the amplitudes of the SEA response and the experimental results can be seen. A similarity in the trend of the vibration response between the two methods is seen as well. This difference is observed in the vibration responses for the other tank faces as well. Thus, it can be said that this response behavior is characteristic of the SEA simulation model and hence factors which could affect the entire tank responses were considered for refinement of the SEA model. As a result, minor details like holes on the top and bottom face for water inlets and outlets, or on the L mounts for bolts are not considered significant.

This difference could be attributed to several structural factors which could affect the results in varying degrees. The coupling loss factors used are theoretically derived and there may be some variation in the behavior of the experimental coupling loss factors especially with varying modal densities. However, this has been verified to some extent [16], [17] and as explained before, as this should be a geometrical factor, it was decided to use the theoretical values. The use of elastic cords to simulate free-free conditions is an approximation which isn’t considered during the SEA simulation. This assumption is shown to affect the lower frequencies much more[24] and hence isn’t considered for the model refinement.

Another obvious reason for the higher response for the simulation results for all the tank surfaces was the assumption of rigid support structure which cannot act as a vibration sink or as an energy transfer subsystem. Modelling the support structure would provide additional paths for energy transfer and energy dissipation between the participating subsystems. To test this hypothesis, the SEA simulation model was refined to include additional details as shown in Figure 6.1:2. The mass and material properties were defined for the additional components according to the design specifications.
Some simplifying assumptions were made when this refinement was carried out to reduce the SEA modelling time. The L mounts of the tank were bolted to the support structure. As the L mounts were bolted at 3 locations over the surface, this bolting connected was modelled as a line junction in SEA simulation. Similarly, the bolting connections between the different brackets were also modelled as line junctions, with the lines extending for approximately the projected bolt head length. The brackets connecting the tank structure to the elastic cords were not modelled as these don’t affect the tank vibration response directly.

As the tank support structure is now modelled, the input power was calculated and applied at the excitation point, which is approximately the center of the structure as seen in the Figure 6.1:2. It was expected that as the accelerometer is mounted almost exactly on top of the force transducer, the input power calculation would be more accurate as compared to the power input calculated at the L mounts.
The vibration and acoustic responses for the SEA simulation model were computed again after these refinements and are used for comparison with the experimental results as shown in the next section.

6.2. Comparison Summary

In this section, the SEA simulation and experimental testing results are compared for different tests as mentioned in Table 6.2-1.

Table 6.2-1: SEA vs Experimental Results: Comparison

<table>
<thead>
<tr>
<th>Experiment Type</th>
<th>Tank Setup Condition</th>
<th>Results Plotted</th>
<th>Measurement Location</th>
<th>Measurement Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration</td>
<td>Empty Tank</td>
<td>SEA Expt.</td>
<td>Front Face</td>
<td>Base Face</td>
</tr>
<tr>
<td></td>
<td>Filled Tank</td>
<td>SEA Expt.</td>
<td>Front Face</td>
<td>Base Face</td>
</tr>
<tr>
<td></td>
<td>Empty Tank Filled</td>
<td>SEA</td>
<td>Front Face</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40% Filled Tank</td>
<td>SEA</td>
<td>Front Face</td>
<td></td>
</tr>
<tr>
<td>Acoustic</td>
<td>Empty Tank Far Field</td>
<td>SEA Expt.</td>
<td>Back Face</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(~40 in)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Filled Tank Far Field</td>
<td>SEA Expt.</td>
<td>Back Face</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(~40 in)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Empty Tank Near Field</td>
<td>SEA Expt.</td>
<td>Back Face</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(~2 in)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Empty Tank Near Field</td>
<td>SEA</td>
<td>Back Face</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Far Field</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Vibration Response:

The spatially and frequency averaged acceleration response for the front face is shown in Figure 6.3:1 for the empty tank case to study the effect of model refinement on the simulation results as compared with the experimental test results. The response for the filled tank is also compared with the test results for the front face in Figure 6.3:2. Similar comparison for the bottom surface of the tank is made in Figure 6.3:3 and Figure 6.3:4 to highlight the similarity of the response trends for SEA and experimental correlation. The SEA response for the empty tank and the fluid filled tank is plotted for the front face to show the influence of the fluid structure coupling on the SEA simulation results. The SEA simulation results for the front face with different fluid levels is shown in Figure 6.3:6. These results are compared with the similar plot for experimental testing and some conclusions are made regarding the validity of observations made from the simulation responses.

Acoustic Response:

As shown in Chapter 4, the acoustic responses for the front and the back panel match each other by SEA simulation due to the symmetry with respect to the boundary conditions and the excitation source. As such, in this section, the sound pressure level computed from the SEA simulation response is compared with the experimental testing response for the back surface for the empty tank and the filled tank condition in Figure 6.5:1 and. The SEA near field response for the empty tank case for the back surface is plotted in Figure 6.5:3. The SEA near field and far field responses for the back face are compared in Figure 6.5:4 and the observations are discussed in the next chapter.
6.3. Results - Vibration Measurements

SEA vs Experimental Results: Front Face

Figure 6.3:1: SEA vs Expt.: Empty Tank: Front: Acc. Response

Figure 6.3:2: SEA vs Expt.: 40% Filled Tank: Front
SEA vs Experimental Results: Base Face

Figure 6.3:3: SEA vs Expt.: Empty Tank: Base

Figure 6.3:4: SEA vs Expt.: 40% Filled Tank: Base
SEA Results: Front Face

Figure 6.3:5: SEA: Empty vs 40% Filled Tank: Front

Figure 6.3:6: SEA: Completely Filled Tank vs 40% Filled Tank: Front
6.4. Observations – Vibration Measurements

It can be seen from Figure 6.3:1 that the SEA model refinement has increased the accuracy of the model to simulate the experimental results as compared to the results shown in Figure 6.1:1. As with any correlation case, some differences are expected between the test data and the simulation data which is visible in these plots.

The comparison of the acceleration response for the front face empty tank in Figure 6.3:1 indicates that the SEA response is higher than the experimental results in the low frequency region. This is expected as the modal density is low in this frequency region which would cause violation of the assumption of the equipartition of energy in every frequency bandwidth. This was observed in the previous chapter when the SEA and FEA responses were compared over similar frequency ranges. A better correlation is seen between 3000 Hz – 6000 Hz with some deviation after that frequency. As the high frequency vibrations are sensitive to the accelerometer mounting and setup conditions, there is some uncertainty associated with the measured test data as well.

The response comparison for the front face indicates greater correlation for the empty tank case than the 40% filled condition as seen from Figure 6.3:1 and Figure 6.3:2 which may be due to the influence of the fluid structure coupling effect. As the tank is partially filled during experimental testing, the coupling during experimental testing for the front face is complex and may not be captured accurately by the SEA simulation method.

Similar comparison of acceleration response for the test and simulation results was carried out for the base face as shown in Figure 6.3:3 and Figure 6.3:4. The trend of SEA response observed in the low frequency region for the base plate was similar to that of the front face. Thus, this
behavior can be attributed to the simulation parameters rather than some local factors. In case of the base face, it can be seen that the acceleration response from SEA simulations for the empty tank case as well as the filled tank case correlates well with the test data. This is expected as the entire base face is the wetted area for the fluid structure coupling. Thus, the entire flexural subsystem of the base plate participates in the energy transfer with the fluid body and hence, this effect is simulated by SEA much more effectively.

The SEA simulation trend lines of the spatially averaged surface responses for the empty tank and filled tank condition in Figure 6.3:5 indicate negligible influence of the fluid on the response at low frequencies, but considerable damping at high frequencies. It indicates the increasing influence of the fluid coupling on the surface vibration response which is due to the increase in the participating fluid modes. Comparison with experimental testing for the same conditions show similar effect of the fluid coupling. However, the experimental testing results indicated a fluctuation in the vibration response in the low frequency range (below 2000 Hz) due the fluid structure coupling which is not seen in the SEA simulation.

The SEA acceleration response for the completely filled tank and a 40% filled tank for the front face is shown in Figure 6.3:6. It can be seen that there is very small difference in the vibration response which is similar to the experimental results for the acceleration response for varying fluid levels. This indicates that the front face isn’t affected as much by the inertia of the fluid volume as by the fluid damping.

It can be seen that the SEA simulation responses were greatly dependent on the input power fluctuations which need to be measured accurately for better correlation. In the next section, the SEA simulation and experimental results are compared for the acoustic responses.
6.5. Results – Acoustic Response

**Figure 6.5:1** SEA vs Expt.: Far Field: Empty tank: Back face SPL

**Figure 6.5:2** SEA vs Expt.: Far Field: 40% Filled Tank: Back face SPL
Sound Pressure Level Response

Figure 6.5.3: SEA vs Expt.: Empty Tank: Near Field: Back face SPL

Figure 6.5.4: SEA: Empty tank: Far Field vs Near Field: Back face
6.6. Observations - Acoustic Measurements

It can be observed that there is a greater deviation for the acoustic responses as compared to the vibration responses. The SEA acoustic response plotted in this Chapter is only due to the individual tank surface normal to that location. Thus, this deviation is expected as the SEA simulation tool can measure the sound pressure level by a single panel while the sound pressure level measured during the experimental testing will have contributions from all the setup components.

The Far field sound pressure level response for the back face is shown in Figure 6.5:1 for the empty tank condition. It can be seen that the acoustic response trend line is similar to the acceleration response trend line shown in Figure 6.3:1 for the front surface for the SEA simulation response. This indicates that the acoustic sound measured at some distance from the tank is dependent on the vibration of the tank surface. This is consistent with the line of thought that the variation in the vibration response due to the fluid structure coupling can cause a change in the measured acoustic sound.

This behavior is observed for the fluid filled tank condition as well as indicated in Figure 6.5:2. The correlation between the test data and simulation results is much better for the fluid filled case than the empty tank case. While the trend for the SEA simulation remains the same, it can be seen that there is some change in the trend of the experimental results. As explained before, the addition of fluid volume could have had some influence on the structural stiffness of the experimental setup during experimental testing. Thus, there could have be some attenuation of noise resulting in an improved response measurement.
The near field acoustic response for an empty tank is shown in Figure 6.5:4 indicates very little correlation with the test data. Near field response is greatly influenced by the edge effects and is very sensitive to the measurement parameters. In addition, the near field response isn’t particularly applicable to the problem inspiration as the noise was heard by the rider and in the Pass by Noise test, which measures the sound pressures at a distance from the vehicle. Hence, the representation of the near field noise is for the sake of completeness and this lack of correlation wasn’t investigated further.

The far field and near field SEA response for the empty tank back surface as shown in Figure 6.5:4 indicates the same trend as the experimental results for the same conditions. This is expected as the response should reduce with an increase in the distance from the sound source. Thus, the vibration and acoustic responses for the SEA simulations and experimental testing were compared and a good correlation was found between these responses. In the next section, the variance studies are carried out to account for the response variations due to the coarse modelling of the experimental setup.

6.7. Variance Study

As Statistical Energy Analysis provides an averaged response by overlooking the details of the model, variance studies can be carried out to understand the maximum fluctuation of this response due to variation in the model parameters. The Variance module of ESI VA One software suite was used to highlight this type of variation study. The variation of the acceleration response of the base for different experimental conditions for a 90% confidence level is plotted in Figure 6.7:1 and Figure 6.7:2. The confidence level is a measure of the exactness of the SEA simulation model with respect to the experimental setup. As the SEA
model doesn’t account for some of the details like bolting locations, holes and additional masses a lower confidence level can be used. It can be observed from the plots that the trend of the SEA response remains the same irrespective of the confidence level. It can be seen that by accounting for the variance of the setup parameters, the SEA response can match the experimental testing response for a large frequency range of operation. The variance of the SEA response is greater in the lower frequency range to account for the low modal density and it tapers off as the frequency increases.

![Acceleration PSD response](Image)

*Figure 6.7.1: Variance study: Empty Tank: Base*
Thus, in this chapter the SEA and experimental results were compared and a good level of correlation was observed. In the next chapter, the findings of this thesis project are discussed and some avenues for further research in this field are mentioned.
Chapter 7

7.1. Project Summary

In this thesis project, the applicability of Statistical Energy Analysis (SEA) as a design tool for quick initial computations was studied by investigating the influence of fluid structure coupling on the vibration behavior of thin sheet metal structures. An experimental setup was manufactured and vibration and acoustic responses were measured in the anechoic chamber of NC State University as described in Chapter 4. The ESI VA One simulation modules of Finite Element Analysis (FEA) and Statistical Energy Analysis (SEA) were used to simulate the experimental setup and the results of SEA were compared with the FEA results for accuracy in Chapter 5. After some refinement of the SEA simulation model, the results of the experimental testing were compared with SEA responses in Chapter 6. Based on these comparisons, the following conclusions were reached in this thesis

7.2. Conclusions

- The restrictions of using FEA techniques for high frequency simulation and the limitations of deterministic methods to handle the inherent variations of the manufacturing process were highlighted. The FEA techniques can be used for the low frequency vibrations but the modal correlation reduces at high frequencies.

- The results from simulation techniques were compared in Chapter 5 and it was observed that SEA and FEA results converge at high frequencies. The computation time for SEA was a small fraction of that for FEA indicating its usefulness for quick initial design calculations.
• The influence of modal density on the SEA responses was seen at low frequencies as the SEA responses were considerably higher than the FEA responses and the experimental responses as well.

• Experimental testing of the fluid structure coupling was carried out in the anechoic chamber and the influence of this coupling on the vibration and acoustic response was studied.

• It could be seen that there is a change in the vibration behavior of the tank surfaces due to the addition of the fluid volume. This vibration change affects the acoustic response of the tank and thus the noise signature of the vehicle could be modified.

• The preliminary impact hammer tests indicated the irregularities in the experimental setup which proved the viability of SEA to model such real systems.

• Some of the test parameters were used as inputs to the SEA simulation method for correlation purposes. It was seen that the vibration and acoustic response of SEA is greatly dependent on the measurement of input power.

• Comparison of the experimental results with SEA simulation was done to refine SEA model and a good correlation was seen for the vibration and acoustic responses. It was seen that acoustic responses are more susceptible to external influences which affected the correlation.

• The trend line predictions of the system responses by SEA were similar to the experimental results. The quick computation of SEA provided more flexibility in modelling and model refinement wasn’t a time consuming process.
- Variance studies used to account for irregularities of surfaces and coarse modelling of the experimental setup indicated that the SEA response is within the range of the experimental results.

Thus, the feasibility of SEA as a simulation tool for design purposes, especially in the high frequency region was shown in this thesis project. In the next section, some of the possible research directions are discussed.

### 7.3. Future Work

For initial stages of design process, optimization of geometric parameters within the design constraints is of significance especially in the highly competitive automobile sector or aerospace applications. The shape and size of the sheet metal panels determine the vibration and acoustic response of the system and hence, these parameters can be optimized for the required response.

The critical parameters of Statistical Energy Analysis such as modal density and coupling loss factors are dependent on the geometric dimensions and material properties and hence these basic dimensions can be used as design variables for optimization. The Design of Experiments approach to study parameter sensitivity can provide results very quickly when used with SEA rather than FEA simulation.

The power flow method can be used to identify the contribution of different paths to the vibration response of the component. The influence of variation of any contributing paths by changing the connection type or component characteristics can be studied to identify the critical components. The power flow tools could be used to study the sensitivity of a component response to other components and hence could be used to make design decisions.
In this thesis project, a good level of correlation was obtained between test and simulation. However, certain parameters such as the fluid structure coupling loss factor, welding and bolting loss factors were not measured and correlated. Experimental measurement of these loss factors and its prerequisite modal density would help to achieve a better correlation.
REFERENCES


Appendix A  
A1) Verification of VA One SEA results with theoretical results

To verify the appropriate representation of the data from the VA One software, a simple two plate system connected to each other at right angles was considered as shown in Figure A1:1. The plate dimensions are same as the tank surface dimensions and only the flexural or the out of plane mode subsystem was considered for energy interaction. A constant input power of 1W is applied to one plate and the plate damping loss factor is considered as 1% constant.

![Figure A1:1: Two Plate System](image)

MATLAB was used to simulate this SEA model using theoretical calculations from [12], [13].

\[
E = \text{Elastic Modulus} \quad \nu = \text{Poisson's Ratio} \quad \rho = \text{Material Density}
\]

\[
h = \text{thickness} \quad S = \text{Area}
\]

Modal Density:

\[
n(\omega) = \frac{S}{4\pi \kappa c_L}; \quad \kappa = \frac{h}{\sqrt{12}}; \quad c_L = \frac{E}{\sqrt{\rho(1 - \nu^2)}}
\]  

(15)
Line Coupling Loss Factor:

\[ \eta_{12} = \frac{2c_B \tau_{12}}{\pi \omega S_1}; \quad c_B = \sqrt{\omega \kappa c_L} \]  (16)

The MATLAB code is attached at the end for reference. The comparison of critical parameters like modal density (Figure A1:2), coupling loss factor (Figure A1:3) and the spatially averaged root mean square velocity response for the plates is compared in Figure A1:4.

It can be seen that the modal density, the coupling loss factor and the spatially averaged root mean square velocity response for the VA One Software and the theoretical calculations match very well. Thus, the software usage is verified and the method of representation of the results is accurate.
Figure A1.3: Coupling Loss Factor Comparison: VA One vs Theoretical Results

Figure A1.4: RMS Velocity Comparison: VA One vs Theoretical Results
Matlab code: Plate-Plate Flexural Coupling
%SEA Validation
%Plate - Flexure Only
clear all
clc
% Material Properties
% FPS Units
E= 30.46e6;  G=11.6e6;  v=0.31;  r=0.0007299;
% Plate Dimensions
% FPS Units
l=16;   b=10;   w=8;    t=0.06;
% Loss Factor:
n1=0.01;    n2=0.01;
Ar1=l*b;    Ar2=l*w; P=2*(l+b);  k=t/(12^0.5);
c1=(E/(r*(1-v^2)))^0.5;
M1=r*Ar1*t;
M2=r*Ar2*t;
% Modal Density
ns1=Ar1/(4*pi*k*c1);
ns2=Ar2/(4*pi*k*c1);
% Frequency Domain
f0=1000;
n=3;                  %Power of octave band 2^(1/n)
m=(-7*n:1:4*n);         %Frequency Range
fc=2.^(m/n)*f0;
f1=2^(-1/2*n).*fc;
fu=2^(1/2*n).*fc;
wfc=2*pi.*fc;
df=fu-f1;
% Coupling Loss Factor Calculation
cb=(wfc*k*c1).^0.5;
r1=r;   c1=c1; t1=t;
r2=r;   c2=c2; t2=t;
phi=r1*(c1^1.5)*(t1^2.5)/(r2*(c2^1.5)*(t2^2.5));
A1=t1/t2;
B=2.754*A1/(1+3.24*A1);
tau=2*B*((phi^0.5+phi^-0.5)^-2);
n12=2*cb*l*tau./(pi*wfc*Ar1);
N1=ns1*df;  N2=ns2*df;
n21=N1.*n12./N2;
% Input Power: Constant 1W for Plate 1
P1=1;
P2=0;
Pin=[P1 P2]';
for i=1:length(wfc)
    % Damping Matrix
    w=wfc(i);
    X=w*[n1+n12(1,i) -n21(1,i);
         -n12(1,i) n21(1,i)+n2];
    Y(:,i)=inv(X)*Pin;
    % Response Calculation
    v_MS_fc(1,i)=Y(1,i)/M1;
    v_MS_fc(2,i)=Y(2,i)/M2;
\[ v_{RMS}(i,:) = (v_{MS,fc}(i,:) \cdot 0.5; \]

end

% Comparison with SEA from VA One
file=importdata('Plate_plate_SEA_software.xlsx');

% Modal Density: ns1
figure
nwp1=ns1*ones(length(fc))';
nwp2=ns2*ones(length(fc))';
semilogx(fc(1,:),nwp1(1,:),'ro-',fc(1,:),nwp2(1,:),'r*-',...
file.data(:,1),file.data(:,4),'bo-',...
file.data(:,1),file.data(:,5),'b*-');
title('Modal Density','Fontsize',14);
l=legend('Plate 1 Theoretical','Plate 2 Theoretical',...
'VA One Plate 1 SEA','VA One Plate 2 SEA');
set(l,'Fontsize',14);
xlabel('Frequency','Fontsize',14);
ylabel('Modal Density (modes/rad/s)','Fontsize',14);
grid on

% Engineering Units: Velocity RMS
figure
loglog(fc,v_RMS(1,:),'ro-',fc,v_RMS(2,:),'r*-',...
file.data(:,1),file.data(:,2),'bo-',...
file.data(:,1),file.data(:,3),'b*-');
title('Engineering Units','Fontsize',14);
l=legend('Plate 1 Theoretical','Plate 2 Theoretical',...
'VA One Plate 1 SEA','VA One Plate 2 SEA');
set(l,'Fontsize',14);
xlabel('Frequency','Fontsize',14);
ylabel('RMS velocity in/s','Fontsize',14);
grid on

% Coupling Loss Factors:
figure
loglog(fc,n12,'ro-',fc,n21,'r*-',...
file.data(:,1),file.data(:,6),'bo-',...
file.data(:,1),file.data(:,7),'b*-');
title('Coupling Loss Factors','Fontsize',14);
l=legend('Theoretical n12','Theoretical n21',...
'VA One n12','VA One n21');
set(l,'Fontsize',14);
xlabel('Frequency','Fontsize',14);
ylabel('Coupling Loss Factor','Fontsize',14);
grid on
Appendix B

B1) Experimental Setup Component Drawings

The assembly drawing is shown in Figure B1:1. In this section, the component drawings are attached for reference.

Figure B1:1: Expt. Setup: Assembly

Figure B1:2: Support Base: Expt. setup
Figure B1:3: Support mount: Expt. setup

Figure B1:4: Elastic cord support: Expt. Setup
Figure B1.5: Base support bracket: Expt. Setup

Figure B1.6: Stinger & Connector: Expt. Setup
B2) Transducer technical specifications

In this section, the technical specification sheets for the accelerometer, shaker and the microphone & preamplifier are attached for reference.

### ICP® ACCELEROMETER

<table>
<thead>
<tr>
<th>Model Number</th>
<th>ENGLISH</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>362C33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Performance**
- Sensitivity (10%) 100 mV/g ± 50 mV/g ± 50 mV/g
- Measurement Range 10.3 m/s² ± 50 m/s² ± 50 m/s²
- Frequency Range (±10% Band Width) 0.5 to 16000 Hz 0.5 to 16000 Hz
- Dynamic Range 50 dB 50 dB
- Non-Linearity ± 1% ± 1% ± 1%
- Transverse Sensitivity ≤ ± 6% ≤ ± 6% ≤ ± 6%

**Environmental**
- Overload Limit (Shock) ± 5000 g ± 49000 m/s² ± 49000 m/s²
- Temperature Range (Operating) ± 60°C to +125°C ± 60°C to +125°C
- Base Skin Sensitivity 0.003 g/s ± 0.005 g/s ± 0.005 g/s

**Electrical**
- Excitation Voltage 18 to 30 VDC 18 to 30 VDC
- Constant Current Excitation 2 to 20 mA 2 to 20 mA
- Output Impedance < 200 ohms < 200 ohms
- Output Bias Voltage 7 to 12 VDC 7 to 12 VDC
- Discharge Time Constant 1.0 to 2.5 sec 1.0 to 2.5 sec
- Sooting Time (at 10% of bias) < 10 sec < 10 sec
- Spectral Noise (1 Hz) 39 µV±10% 39 µV±10%
- Spectral Noise (10 Hz) 11 µV±10% 11 µV±10%
- Spectral Noise (100 Hz) 0.94 µV±10% 0.94 µV±10%
- Spectral Noise (1 kHz) 1.4 µV±10% 1.4 µV±10%

**Physical**
- Sensing Element Ceramic Ceramic
- Sensing Geometry Shear Shear
- Housing Material Titanium Titanium
- Sealing Hermetic Hermetic
- Size (Max x Height) 0.46 x 0.62 in 11.2 mm x 15.7 mm
- Weight 0.20 oz 5.6 g
- Electrical Connector 10-32 Coarse Thread 10-32 Coarse Thread
- Electrical Conn. Terminal Side Side
- Mounting Thread 10-32 Fine Thread 10-32 Fine Thread
- Mounting Torque 10 to 20 in-lb 10 to 20 in-lb

**Sensitivity Deviation**

![Sensitivity Deviation Graph]

All specifications are at room temperature unless otherwise specified.

*ICP® is a registered trademark of PCB Group, Inc.*

### OPTIONAL VERSIONS

<table>
<thead>
<tr>
<th>Version</th>
<th>ENGLISH</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>J - Gruen (Gruen)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
- Frequency Range (±10% Band Width) 0.5 to 16000 Hz 0.5 to 16000 Hz
- Spectral Noise (1 Hz) 10 µV ± 5% 10 µV ± 5%
- Spectral Noise (10 Hz) 32 µV ± 5% 32 µV ± 5%
- Spectral Noise (100 Hz) 100 µV ± 5% 100 µV ± 5%
- Spectral Noise (1 kHz) 300 µV ± 5% 300 µV ± 5%

**Supplied Accessories**
- Model G5A0.5G Single Axis Accelerometer
- Model G5A0.5G Single Axis Accelerometer
- Model G5A0.5G Single Axis Accelerometer
- Model G5A0.5G Single Axis Accelerometer

Figure B2:1: Accelerometer Specification
Figure B2:2: Microphone & Preamplifier specifications

- **4188-A-021 - ½-inch free-field microphone with Type 2671 preamplifier, 20 Hz to 12.5 kHz, prepolarized**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>1/2 inch</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>31.6 mV/Pa</td>
</tr>
<tr>
<td>Standards</td>
<td>TEDS UTC</td>
</tr>
<tr>
<td>Temperature Coefficient</td>
<td>0.005 dB/°C</td>
</tr>
<tr>
<td>Temperature Range</td>
<td>-30 - 100 °C</td>
</tr>
<tr>
<td>Venting</td>
<td>Rear</td>
</tr>
<tr>
<td>Input Type</td>
<td>CCLD IE/IE</td>
</tr>
<tr>
<td>Pressure Coefficient</td>
<td>-0.031 dB/kPa</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>31.6 mV/Pa</td>
</tr>
<tr>
<td>Standards</td>
<td>TEDS UTC</td>
</tr>
<tr>
<td>Temperature Coefficient</td>
<td>0.005 dB/°C</td>
</tr>
<tr>
<td>Temperature Range</td>
<td>-30 - 100 °C</td>
</tr>
<tr>
<td>Venting</td>
<td>Rear</td>
</tr>
<tr>
<td>Input Type</td>
<td>CCLD IE/IE</td>
</tr>
</tbody>
</table>

Figure B2:3: Electromagnetic shaker specifications

- **LDS V201 - Permanent Magnet Shaker**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine force peak</td>
<td>17.8 N</td>
</tr>
<tr>
<td>Max random force rms</td>
<td>91  g</td>
</tr>
<tr>
<td>Velocity sine peak</td>
<td>1.49 m/s</td>
</tr>
<tr>
<td>Displacement continuous pk-pk</td>
<td>5 mm</td>
</tr>
<tr>
<td>Moving element mass</td>
<td>0.02 kg</td>
</tr>
<tr>
<td>Usable frequency range</td>
<td>5-13,000 Hz</td>
</tr>
</tbody>
</table>
B3) LabVIEW data acquisition VI’s
The front panel and the block diagram of a 4 channel DAQ VI created to measure 4 analog input time signals, process their power spectral densities and log the data in an excel or tdms file is attached for reference in Figure B3:1 and Figure B3:2.

Figure B3:1: 4 channel DAQ VI: Block Diagram
For the structure excitation, a uniform white random noise was generated using LabVIEW and an Agilent U830A audio analyzer was used to pass this signal to the shaker. The front panel and block diagram for this VI is shown in Figure B4:1 and Figure B4:2.
Figure B4.1: white noise VI: Block Diagram
Front Panel

Figure B4.2: white noise VI: Front Panel
In LabVIEW, two types of white noise – uniform random or Gaussian random can be generated as shown. The power spectral density of both the signals is constant, however, the distribution of the signal amplitude is different as shown.

This signal is passed to the audio analyzer which reads 32768 samples at a fixed frequency of 312.5e3 Hz. As the frequency at which data is read is significantly higher than the frequency at which data was created, it results in a shift in the frequency of the LabVIEW signal to the higher side. In order to compensate for this change, the frequency range of the LabVIEW signal should be lowered.

Let $f_s$ be the sampling rate at which the LabVIEW signal is generated. As maximum of 32768 data points can be read by the analyzer, let the number of points be fixed at $N = 32768$.

If $f_o$ is the maximum output frequency desired from the Agilent audio analyzer, then the maximum frequency which should be generated from LabVIEW is given as

$$f_m = \frac{f_s f_o}{312.5e3} \text{Hz}$$

(17)

Using this formulation, the lower and upper cut off can be decided for the LabVIEW signal generation.

**B5) Frequency Averaging of Experimental Data**

In this section, the MATLAB code for frequency averaging of the experimental data is attached for reference. This code was used to plot the graphs shown in Chapter 3.

```matlab
% Frequency Averaging: Experimental Data
clear all
clc

% Read the input file
sheet='base';
file=xlsread('expt_matlab_filtered',sheet);
freq=file(:,1);
f_emp=file(:,3); acc_emp=file(:,2);
f_40=file(:,5); acc_40=file(:,4);
```
% Frequency Domain
f0=1000;
n=3;               %Power of octave band 2^(1/n)
m=(-6*n:1:4*n)';    %Frequency Range
fc=2.^((m/n).*f0);
fl=2.^((-1/(2*n)).*fc);
fu=2.^((1/(2*n)).*fc);
wfc=2*pi.*fc;
% Filtering PSD based on Octave Band
for i=1:length(fc)
    temp_f_emp=zeros(size(f_emp));
    temp_f_40=zeros(size(f_40));
    temp_acc_emp=zeros(size(acc_emp));
    temp_acc_40=zeros(size(acc_40));
    count=1;
    for j=1:length(freq)-1
        if freq(j,1)>fl(i,1) && freq(j,1)<fu(i,1)
            temp_f_emp(i,1)=(temp_f_emp(i,:)+f_emp(j,1));
            temp_f_40(i,1)=(temp_f_40(i,:)+f_40(j,1));
            temp_acc_emp(i,1)=temp_acc_emp(i,:)+acc_emp(j,1);
            temp_acc_40(i,1)=temp_acc_40(i,:)+acc_40(j,1);
            count=count+1;
        end
    end
    if count==1
        temp_f_emp(i,1)=temp_f_emp(i,1);
        temp_f_40(i,1)=temp_f_40(i,1);
        temp_acc_emp(i,1)=temp_acc_emp(i,1);
        temp_acc_40(i,1)=temp_acc_40(i,1);
    else
        temp_f_emp(i,1)=temp_f_emp(i,1)/(count-1);
        temp_f_40(i,1)=temp_f_40(i,1)/(count-1);
        temp_acc_emp(i,1)=temp_acc_emp(i,1)/(count-1);
        temp_acc_40(i,1)=temp_acc_40(i,1)/(count-1);
    end
    f_emp_fil(i,1)=temp_f_emp(i,1);
    f_40_fil(i,1)=temp_f_40(i,1);
    acc_emp_fil(i,1)=temp_acc_emp(i,1);
    acc_40_fil(i,1)=temp_acc_40(i,1);
end
figure()
h=plot(freq,acc_emp,'b--',f_emp('i',i),'bo-',...
        freq,acc_40,'r--',f_40('i',i),'ro-');
set (h(2),'linewidth',2);
set (h(4),'linewidth',2);
axis([0 10000 -50 10]);
legend(sprintf('Expt-empty-raw-%s',sheet),...
        sprintf('Expt-empty-fil-%s',sheet),...
        sprintf('Expt-40%% filled-raw-%s',sheet),...
        sprintf('Expt-40%% filled-fil-%s',sheet));
title('Acceleration Response')
xlabel('Frequency Hz'); ylabel('dB scale ref 1g^2/Hz');
grid on
figure()
g=plot(freq,f_emp,'b--',fc,f_emp_fil(:,1),'bo-',...
    freq,f_40,'r--',fc,f_40_fil(:,1),'ro-');
set (g(2), 'linewidth', 2);
set (g(4), 'linewidth', 2);
axis([0 10000 -50 10]);
legend(sprintf('Expt-empty-raw-%s',sheet), ... 
    sprintf('Expt-empty-fil-%s',sheet), ... 
    sprintf('Expt-40% filled-raw-%s',sheet), ... 
    sprintf('Expt-40% filled-fil-%s',sheet))
title('Net Dynamic Force Response')
xlabel('Frequency Hz'); ylabel('dB scale ref 1N^2/Hz');
grid on
Appendix C

In this appendix, the MATLAB codes used for calculating the damping loss factor and the input power are attached for reference.

C1) **Damping Loss Factor Calculation**

The accelerometer amplitude in the time domain can be approximated as shown in the following equations.

\[
a = A e^\sigma; \quad \sigma = \zeta \omega t
\]  

Taking the natural log of both the sides, we get

\[
\ln a = \ln A + \sigma
\]

In the MATLAB code, the peaks of each sinusoid were identified and a least square error fit was carried out to obtain the constants A and \( \sigma \). The curve fit of the data points is shown in Figure C1:1.

*Figure C1:1: Damping Calculation: Curve Fit*
Based on this the critical damping ratio and damping loss factor is calculated as shown in the following equation

\[ \zeta = \frac{\sigma}{\omega t} \text{ and } \eta = 2\zeta \]  

(20)

The value of \(\omega\) is taken as the center frequency for each one-third octave bandwidth and the damping loss factor thus calculated is shown in Chapter 5.

**C2) Input Power Calculation**

Input power is calculated according to the methods suggested by F.J Fahy [22] by measuring the force and accelerometer signals as shown in Figure C2:1.

![Figure C2:1: Input Power Calculation](image)

The MATLAB code used to compute the Input power by taking the cross spectral power density of the force and accelerometer signal is attached for reference. The input power plot is shown in Chapter 5.

```matlab
% Damping Loss Factor Calculation
clear all
cic

% Import Data
filename='octave_filtered_sf2.xlsx';
```
for sheet=1:12
    file.data=xlsread(filename,sheet);
    % Trim Data to keep tmax near the peak response
    if sheet>6
        tmax=0.2;
    else
        tmax=0.5;
    end
    % file.data=file1.data.Untitled7;
dt=file.data(2,1)-file.data(1,1);
    [maxval maxInd]=max(file.data(:,2));
    file.data(1:maxInd-10,:)=[];
    n=tmax/dt;
    file.data(n:end,:)=[];
    file.data(end,:)=[];
    % Find the x,y coordinates for the oscillation peaks
    xData=file.data(:,1);
    yData1=file.data(:,2);
    yData2=file.data(:,3);
    upOrDown1 = sign(diff(yData1));
    upOrDown2 = sign(diff(yData2));
    maxFlags1 = [upOrDown1(1)<0 ; diff(upOrDown1)<0 ; upOrDown1(end)>0];
    maxFlags2 = [upOrDown2(1)<0 ; diff(upOrDown2)<0 ; upOrDown2(end)>0];
    maxIndices1 = find(maxFlags1);
    maxIndices2 = find(maxFlags2);
    xCoords1b = []; yCoords1b = [];
    xCoords2b = []; yCoords2b = [];
    for ii = 1:length(maxIndices1)
        xCoords1b = [xCoords1b xData(maxIndices1(ii))];
        yCoords1b = [yCoords1b yData1(maxIndices1(ii))];
    end
    for ii = 1:length(maxIndices2)
        xCoords2b = [xCoords2b xData(maxIndices2(ii))];
        yCoords2b = [yCoords2b yData2(maxIndices2(ii))];
    end
    % Resample the Data Points for better fit
    j=1;
    if sheet>6
        ds=5;
    else
        ds=1;
    end
    for i=1:length(xCoords1b)
        if yCoords1b(1,i)>0
            x1temp(1,j)=xCoords1b(1,i);
            y1temp(1,j)=yCoords1b(1,i);
            j=j+1;
        end
    end
    j=1;
    for i=1:length(xCoords2b)
        if yCoords2b(1,i)>0
            x2temp(1,j)=xCoords2b(1,i);
y2temp(1,j)=yCoords2b(1,i);
j=j+1;
end
end
xCoords1=x1temp(1:ds:end);
yCoords1=y1temp(1:ds:end);
xCoords2=x2temp(1:ds:end);
yCoords2=y2temp(1:ds:end);
% Calculate the optimal values of a and b
A1 = ones(length(xCoords1),2);
Y1 = ones(length(xCoords1),1);
A2 = ones(length(xCoords2),2);
Y2 = ones(length(xCoords2),1);
% We take the log of the actual data (yCoords)
for ii = 1:length(xCoords1)
    A1(ii,2) = xCoords1(ii);
    Y1(ii,1) = log(yCoords1(ii));
end
for ii = 1:length(xCoords2)
    A2(ii,2) = xCoords2(ii);
    Y2(ii,1) = log(yCoords2(ii));
end
% After finding A and B, we know that a = exp(A) and b = B
p1 = A1\Y1;
a1 = exp(p1(1));
b1 = p1(2);
p2 = A2\Y2;
a2 = exp(p2(1));
b2 = p2(2);
z(:,sheet)=[b1 b2]';
% We plug them into their function and plot away. Note that b < 0
curvePlot1 = a1*exp(b1*xData);
curvePlot2 = a2*exp(b2*xData);
subplot(3,4,sheet)
plot(xData,yData1,'-b',xCoords1,yCoords1,'or',... 
    xData,curvePlot1,'--g');
legend('Data','Points','Curve');
xlabel('Time');ylabel('Amplitude');
grid on;hold on;
end

% Input Power Calculation
clear all
clic
% Read Time Data
file=xlsread('time_data_ch3_LH','Sheet1');
time=file(:,1);
f_emp=file(:,2); acc_emp=file(:,3);
f_40=file(:,4); acc_40=file(:,5);
% Cross Spectral Power Density
Pxy1=cpsd(f_emp,acc_emp,[],[],2048);
Pxy2=cpsd(f_40,acc_40,[],[],2048);
f=(0:40000/2048:20000)';
w=2*pi*f;
dw=w(2:end,1)-w(1:end-1,1);
C1=real(Pxy1); Q1=imag(Pxy1);
C2=real(Pxy2); Q2=imag(Pxy2);
R1=-(Q1(2:end,1)/w(2:end,1)).*dw(1,1);
R2=-(Q2(2:end,1)/w(2:end,1)).*dw(1,1);
% Plot the raw Input power
plot(f(2:end,1),10*log10(abs(R1)),'r--', ... 
    f(2:end,1),10*log10(abs(R2)),'b--');

% Frequency Domain
f0=1000;
n=3; %Power of octave band 2^(1/n)
m=(-4*n:1:4*n)'; %Frequency Range
fc=2.^((m/n)*f0);
fl=2.^(-1/(2*n)).*fc;
fu=2.^((1/(2*n)).*fc;
wfc=2*pi.*fc;
%
% Filtering CPSD based on Octave Band
for i=1:length(fc)
temp_R1=zeros(size(R1));
temp_R2=zeros(size(R2));
count=1;
for j=1:length(f)-1
    if f(j,1)>fl(i,1) && f(j,1)<fu(i,1)
        temp_R1(i,1)=(temp_R1(i,:)+R1(j,1));
        temp_R2(i,1)=(temp_R2(i,:)+R2(j,1));
        count=count+1;
    end
end
if count==1
    temp_R1(i,1)=temp_R1(i,1);
    temp_R2(i,1)=temp_R2(i,1);
else
    temp_R1(i,1)=temp_R1(i,1)/(count-1);
    temp_R2(i,1)=temp_R2(i,1)/(count-1);
end
R1fil(i,:)=abs(temp_R1(i,1));
R2fil(i,:)=abs(temp_R2(i,1));
end
hold on
% Plot the Frequency Averaged CPDS
plot(fc,10*log10(abs(R1fil)),'ro-', ... 
    fc,10*log10(abs(R2fil)),'bo-');
title('Power Input-Center')
legend('Input Power Empty Tank-Raw','Input Power 40% Filled Tank-Raw', ... 
    'Input Power Empty Tank-Avg','Input Power 40% Filled Tank-Avg');
xlabel('Frequency Hz'); ylabel('dB scale ref 1W/Hz');
axis([0 10000 -100 0])
grid on