ABSTRACT

Ajeet. Using the ADI-FDTD Method to Compute Currents Induced in the Human Body by HEMI Devices at Low Frequencies. (Under the direction of Dr. Gianluca Lazzi).

The traditional explicit Finite-Difference Time-Domain (FDTD) is a conditionally stable method where the time step must adhere to the Courant-Friedrichs-Lewy (CFL) limit. In problems such as those encountered in the bioelectromagnetics and VLSI circuits, the spatial resolution is dictated by the geometric detail rather than the resolution of the smallest wavelength. Thus, severe limitations are often imposed on the time step, leading to long time-domain simulations. This is particularly true for low-frequency problems which would require prohibitively large number of time steps with the explicit method.

The Alternating-Direction Implicit Finite Difference Time Domain (ADI-FDTD) is a theoretically unconditionally stable method, which allows the use of an arbitrarily large time step for the simulations. Research presented in this thesis aims to compute the induced electric field and current densities in the human body due to the contact electrodes of a Human Electro-Muscular Incapacitation (HEMI) device at frequencies below 200 KHz using the ADI-FDTD method in a D-H formulation.

In order to reduce the memory and simulation time requirements, logarithmic expanding grid technique has been used for modeling the human body. Computational model resolution of 1 mm has been used for most of the human body model, including regions proximal to the current contact points, while a progressively coarser resolution up to 5 mm is utilized according to an expanding grid scheme for body regions distant from the source, such as the lower extremities.
Discrete Fourier Transform (DFT) of the electric field has been computed at the dominant frequencies present in the source signal to find out the electric field distribution in the model due to the application of the HEMI pulse. Using quasi-static assumptions, computation of the DFT values have been done for time durations much shorter than the time periods of the different frequencies. The field values induced in the human body were then obtained as the ratios of the DFT magnitudes with respect to the source, which can be scaled depending on the magnitude of the electric field at source.

This study suggests that the ADI-FDTD method can be effectively used for the solution of low frequency bioelectromagnetic problems. When paired with quasi-static assumptions and Fourier series decomposition for the considered problem, this can lead to simulations that are four orders of magnitude faster than the computational time required with the use of a traditional FDTD method.
Using the ADI-FDTD Method to Compute Currents Induced in the Human Body by HEMI Devices at Low Frequencies

by

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DEDICATION

To my family
BIOGRAPHY

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Chapter 1

Introduction

Human Electro-Muscular Incapacitation (HEMI) devices are used to temporarily disable the neuromuscular system of a person by momentarily applying large electric pulses to the body. These devices come under the class of the Non-Lethal Weapons (NLW), which are used to control violence without causing death or great bodily injury to the targeted person. Some other examples of NLWs include pepper Spray, tear gas, stun guns and rubber bullets [1]. It is understood that an HEMI device works by stimulating the motor nerves of the subject with electric pulses which in turn causes the clonic (tetanic) muscle contractions [2]. Due to the sudden strong muscle contractions, subject may lose control and fall to the ground, which gives the law enforcing officers an opportunity to arrest the violent subject. These devices are increasingly gaining popularity among the law enforcement agencies to control violence without causing any permanent damage or fatality to the targeted person. Increasing usage has generated concern regarding the safety issues of such incapacitation devices and has led to several computational and experimental investigations in that direction [2, 3, 4].

The goal of this thesis is to compute the induced electric field and current density
in the human body due to the current injection by a typical HEMI device. The electrodes of the device were placed in contact with the torso region in the body. The Alternating Direction Implicit Finite Difference Time Domain (ADI-FDTD) method in the D-H formulation has been used for the computation of the field values. The human body model was developed from the Visible Man Project of the National Library of Medicine [5]. A 3-dimensional expanding grid technique has been employed, which allows the use of different resolution in different regions in the model, thereby reducing the total model size without causing any loss in accuracy in the computations of induced electric field or current density values. Use of the expanding grid scheme results into the reduction of memory requirement and simulation time. In order to further reduce the simulation time, techniques based on Discrete Fourier Transform (DFT) averaging of the relevant signal frequencies with the quasi-static assumption have been used to obtain electric field magnitudes at the observation points relative to the source. This chapter gives the overview of the HEMI devices and their operating mechanisms.

1.1 Background of HEMI Devices

Most of the non lethal weapons presently in use works by inflicting pain to the subject. However, these pain causing methods are generally not very effective on a strong or a large person, who may have a greater tolerance towards bearing pain. Moreover, such methods do not work well on the violent subjects under the influence of drugs and thus are more likely to cause physical damage to them. Therefore, an ideal non-lethal weapon should not be solely dependent on causing pain to the subject. In addition to that, it must also be able to be used from a safe distance so that the officers do not need to get in contact with the violent subject [6].
An HEMI device is a type of NLW which is able to satisfy all the above requirements. An HEMI device temporarily immobilizes the subject by applying electric pulses which have been specifically designed with human physiology in mind. These electric pulses are applied to the subject through two barbed darts. When the trigger is pulled, the darts are pulled towards the subject. Each dart generally weighs around 2 grams and has a 5 mm to 10 mm long tip to penetrate the clothing and insulting layer of skin. A special electrical signal waveform is transmitted through the trailing wires to the place where the darts make contacts with the body [6].

1.2 HEMI Pulse Characteristics

The applied waveform in an HEMI device generally has a large open circuit peak voltage of about 50 kV. The reason for using such a high voltage is to ionize the intervening air to provide a conductive path in case the barbs fail to make contact with the body. However, it should be noted that the body is never exposed to such a high voltage, as this is the voltage until the barbs make contact with the conductive flesh [6]. As an example, it is reported in [2] and [3] that the device X26 TASER has the pulse depicted in figure 1.1 for 50 Ω load resistance. By itself, this pulse waveform is intended to be a damped sinusoid at a frequency of 120 kHz superimposed on a decaying unipolar double exponential pulse [3]. The duration of the pulse is about 0.17 millisecond. In an HEMI device, such pulses are triggers several times in a second. The HEMI devices are current-source devices with very high output impedances. Therefore, unless the load is comparable to the HEMI output impedance, the current injected is expected to be similar. Experimental data show a reduction in peak current by only 20% when the impedance is increased from 47 to 4700 Ω [3].

An HEMI device disables the subject by causing the muscle contraction and in-
tense pain. Although, the exact mechanism by which the device causes muscle contraction is not exactly known, it is likely to be a result of either indirect or direct stimulation of the skeletal muscles. Skeletal muscles constitutes around 40% of our body mass and are responsible for all the contraction actions performed by the body such as biceps flexing, binding of fingers etc. It is understood that in the vicinity of the barbs, skeletal muscle fibers are stimulated directly, whereas in the regions at a significant distance from the barbs indirect stimulation of skeletal muscle fibers take place by way of the motor nerve innervation. This happens because the electric field generated in the body has a greater magnitude near the region of contact as compared to the regions which are far from the contact points.

The HEMI device through the indirect or direct stimulation causes the skeletal muscles to contract strongly and continuously, which causes the violent subject to experience dysreflexia, lose locomotive control and collapse to the ground [2]. The amount by which muscle contracts is dependent upon the frequency of the applied pulse.
1.3 Motivation

In order to disable a subject, the HEMI device must be able to cause the skeletal muscle contraction in a large volume of the body. Therefore, it is important that the electric Field \((E)\) produced in the body must be greater than the threshold value for the motor nerve excitation in a sufficient volume of the skeletal muscle. The threshold values required for the nerve excitation depend upon the diameter of the fiber and the type, duration and the frequency of the pulse. However, to avoid permanent damage to the tissues, the electric field and current density \((J)\) must be less than the thresholds which may cause the cell electroporation. For the given HEMI waveform, it has been estimated in [2] that the \(E\) field required to successfully activate motor nerves has to exceed 15 to 225 V/m depending upon the fiber diameter. Moreover, the \(E\) field value must be less than 45-160 kV/m to avoid causing the electroporation [2].

In addition to the electric field magnitude, excitation of the fiber also depends upon the gradient of the electric field. Gradient of electric field shows the change in electric field with respect to the position vector. Nerve stimulation can take place when the change in electric field is above the threshold required for excitation. Once again, the thresholds for excitation depend on nerve fiber width and duration and type of signal. In this research, our objective is to compute the induced electric field values, current density and the gradient of induced electric field in the human body due to the current injection by an HEMI device.

1.4 Organization of Thesis

This thesis is organized as follows:

Chapter 2 discusses the computational methods generally used for the bioelectromagnetics problems. Discussion of the explicit FDTD method and the ADI-FDTD
method in a D-H formulation has been carried out. Further, techniques based on
the DFT computation with the quasi-static assumption to allow shorter run-times
for the low frequency bioelectromagnetic simulations have been discussed. The use
of the 3-dimensional expanding grid to reduce the model size and thus memory and
simulation time requirement has also been investigated.

Chapter 3 discusses the validation of the D-H ADI-FDTD, 3-D expanding grid tech-
niques and the DFT averaging method. Discussion of errors, comparison with the
explicit Finite Difference Time Domain (FDTD) method and the assumptions used
in the simulations have been presented. The computation results for the full body
simulation are described in chapter 4, followed by the conclusion and future work in
chapter 5.
Chapter 2

Computational Methods for Bioelectromagnetic Problems

The majority of the real-life bioelectromagnetic problems involve the interaction of the time varying electromagnetic fields with different biological medias. Because of the complexity involved, obtaining closed form solution is almost impossible for such problems. Thus, computational techniques are applied in order to obtain a solution. Computational Electromagnetics (CEM) methods generally work by solving Maxwell’s coupled curl equations across the domain of the problem to compute the Electric Field (E) and the Magnetic Field (H) values. Among these CEM methods, Finite Difference Time Domain (FDTD), Finite Element Method (FEM) and Methods of Moments (MoM) are few commonly used techniques to solve complex electromagnetic problems [7].

The FDTD method is arguably one of the most commonly used computational method for the analysis of bioelectromagnetic problems. The method works by time stepping through the discretized form of Maxwell’s equation over the computational space. FDTD method unlike the FEM and MoM does not impose any upper bound
to the number of unknowns it can solve [8, 9, 10]. FDTD method allows the user to specify the material property at all points within the computational domain and therefore can be used to accurately model large CEM problems with inhomogeneous media. Moreover, being a time domain method, FDTD can also be used to obtain the response of the system over wide range of frequencies by simulating it with a broadband pulse such as the gaussian signal.

Since some of the bioelectromagnetic problems involve very large models, to solve them computations are generally performed on fast computers with larger memory or parallel computers and high performance clusters. However, for very large problems these methods can take excessively long time to simulate even on the fastest workstations. Moreover, a large model size leads to a large memory requirement, which may not be available on the workstations. Research done in the field of CEM, has resulted in the development of alternate computational methods which reduces the memory and simulation time requirements. One such method to reduce simulation time is the Alternating Direction Implicit Finite Difference Time Domain (ADI-FDTD) method. The ADI-FDTD method allows for the use of an arbitrarily large time step value for the simulation. The first attempt towards using the ADI method to solve the Maxwell’s equations was done in the 1980s [11]. However, the ADI-FDTD in its present form was first demonstrated by Namiki [12] and Zheng et al [13]. The following sections discuss the theory and the implementation of the FDTD and the ADI-FDTD methods and other computational techniques adopted to compute the induced electric field and current density in the human body due to the current injection by contact electrode of an HEMI device.
2.1 Finite-Difference Time-Domain Method for Maxwell Equations

2.1.1 Theory

The FDTD method, first proposed by Yee in 1966, works by discretizing the differential form of Maxwell’s curl equations in both time and space with second order central difference approximations [14]. The time dependent Maxwell’s equation in free space is given as

\[
\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0} \nabla \times H \\
\frac{\partial H}{\partial t} = -\frac{1}{\mu_0} \nabla \times E
\]  

(2.1) 

(2.2)

The Yee algorithm solves for both electric (E) and magnetic field (H) in time and space. Yee scheme uses two different grids for the E and H field components, which are shifted with each other in time and space by half time step and half cell size respectively. Thus, every E-field component is surrounded by four circulating H field components and vice versa. Fig. 2.1 provides a pictorial representation of the Yee lattice.

The computation of the E and H fields are done using a “leapfrog” scheme [8]. First, the E-field values are computed using the previously stored H-field values in the space, then these recently computed E-field values are used to compute the H-field values. In the next time-step the computation of the E and H fields is done again and the process is repeated till the end of simulation. One advantage of this time marching scheme is that it can be parallelized over parallel high performance clusters.

The theory behind the FDTD method can be explained using a one-dimensional problem [15]. For a plane wave propagating in x-direction, Maxwell’s equations in one-dimension can be written as
After applying central difference approximations to equations 2.3 and 2.4 and discretizing in time and space, we get

\[ \frac{\Delta t}{H_z^{n+1/2}(k+1/2) - H_z^n(k + 1/2)} = \frac{\Delta x}{E_y n+1/2(k + 1) - E_y n+1/2(k)} \] (2.5)

Here \( E_y n+1/2(k) \) denotes the electric field value at time-step \( n+1/2 \) and at the position \( k \) and \( H_z^n(k+1/2) \) denotes the magnetic field at time-step \( n \) and at the position \( k+1/2 \). \( \Delta x \) and \( \Delta t \) are grid width and time-step value respectively. It should be noted that
$E$ and $H$ fields are staggered in time and space by half time-step and half-cell width respectively.

The field component $E_y$ at a given time step can be computed using the values of field components $E_y$ and $H_z$ at the previous time-step. After that $H_z$ is computed using the knowledge of $H_z$ at the previous time-step and recently computed $E_y$ values. The spatial discretization ($\Delta x$) is decided by the fine geometric detail of the modeled object or by the minimum wavelength present in the source signal. Similarly, the simulation time is discretized in terms of time-step ($\Delta t$), value of which depends on the grid width used in the simulation. This formulation can be extended to the 3-dimensional problems.

### 2.1.2 D-H FDTD formulation

The equations 2.5 and 2.6 can be modified to the D-H FDTD formulation, which gives the relationship between the Electric Displacement field ($D$) and the $H$ field. Normalized Maxwell’s equation in the D-H FDTD formulations are written as [16]

\[
\begin{align*}
  j\omega \vec{D} &= c_0.\nabla \times \vec{H} \\
  \vec{D}(\omega) &= \epsilon_r(\omega)\vec{E}(\omega) \\
  j\omega \vec{H} &= -c_0.\nabla \times \vec{E}
\end{align*}
\]

The following equations show the 3-dimensional FDTD expressions of $D$ and $H$ fields in free space[16]

\[
D_{x|_{i+1/2,j,k}}^{n+1/2} = D_{x|_{i+1/2,j,k}}^{n-1/2} + \frac{\Delta t}{\Delta y \sqrt{\epsilon_0 \mu_0}} \left( H_z |_{i+1/2,j+1/2,k}^n - H_z |_{i+1/2,j-1/2,k}^n \right) \\
- \frac{\Delta t}{\Delta z \sqrt{\epsilon_0 \mu_0}} \left( H_y |_{i+1/2,j,k+1/2}^n - H_y |_{i+1/2,j,k-1/2}^n \right)
\]
\[ D_y^{n+1/2} = D_y^{n-1/2} + \frac{\Delta t}{\Delta z \sqrt{\varepsilon_0 \mu_0}} (H_x^n_{i,j+1/2,k+1/2} - H_x^n_{i,j+1/2,k-1/2}) - \frac{\Delta t}{\Delta x \sqrt{\varepsilon_0 \mu_0}} (H_z^n_{i+1/2,j+1/2,k} - H_z^n_{i-1/2,j+1/2,k}) \] (2.11)

\[ D_z^{n+1/2} = D_z^{n-1/2} + \frac{\Delta t}{\Delta x \sqrt{\varepsilon_0 \mu_0}} (H_y^n_{i+1/2,j,k+1/2} - H_y^n_{i-1/2,j,k+1/2}) - \frac{\Delta t}{\Delta y \sqrt{\varepsilon_0 \mu_0}} (H_x^n_{i,j+1/2,k+1/2} - H_x^n_{i,j-1/2,k+1/2}) \] (2.12)

\[ H_x^{n+1}_{i,j+1/2,k+1/2} = H_x^n_{i,j+1/2,k+1/2} - \frac{\Delta t}{\Delta y \sqrt{\varepsilon_0 \mu_0}} (E_z^n_{i,j+1/2,k+1/2} - E_z^n_{i,j,k+1/2}) + \frac{\Delta t}{\Delta z \sqrt{\varepsilon_0 \mu_0}} (E_y^n_{i,j+1/2,k+1/2} - E_y^n_{i,j,k+1/2}) \] (2.13)

\[ H_y^{n+1}_{i+1/2,j,k+1/2} = H_y^n_{i+1/2,j,k+1/2} - \frac{\Delta t}{\Delta z \sqrt{\varepsilon_0 \mu_0}} (E_x^n_{i+1/2,j,k+1/2} - E_x^n_{i,j,k+1/2}) + \frac{\Delta t}{\Delta x \sqrt{\varepsilon_0 \mu_0}} (E_z^n_{i+1,j,k+1/2} - E_z^n_{i,j,k+1/2}) \] (2.14)

\[ H_z^{n+1}_{i+1/2,j+1/2,k} = H_z^n_{i+1/2,j+1/2,k} - \frac{\Delta t}{\Delta x \sqrt{\varepsilon_0 \mu_0}} (E_y^n_{i+1/2,j+1/2,k} - E_y^n_{i,j+1/2,k}) + \frac{\Delta t}{\Delta y \sqrt{\varepsilon_0 \mu_0}} (E_x^n_{i+1,j+1/2,k} - E_x^n_{i,j+1/2,k}) \] (2.15)

The E-field can be obtained from the D-field using the equation 2.8.

The D-H FDTD formulation enables an easy implementation of the Perfectly Matched Layer (PML) as the absorbing boundary condition, which is discussed in the next section. The D-H FDTD formulation also enables modeling of the frequency-dependent materials [16].
2.1.3 Perfectly Matched Layer implementation

In an FDTD simulation, computational space is bounded by the available resources. Therefore, Absorbing Boundary Conditions (ABCs) must be employed in order to prevent the outgoing E and H fields from being reflected back to the main computational space. Perfectly Matched Layer (PML) developed by Berenger is one of the most efficient and easy implementation of the absorbing boundary conditions [17]. This method works by matching the impedance of the main computational space with that of the PML region in order to prevent reflection at the interface of the two mediums. The PML region is made lossy so that the wave traveling into the PML region attenuates before it reaches again the simulation space of interest. This is done by making the permittivity ($\epsilon$) and permeability ($\mu$) complex, as the imaginary term is responsible for the decay [16]. Moreover it is shown in [18] that the relative permittivity and permeability in the direction perpendicular to the boundary is made inverse of those in the other directions. The D-H FDTD method allows for the PML condition to be independent of the background material used in the FDTD grid [16]. Therefore, the FDTD equation with such PML in a 3-dimensional space can be derived from

$$j\omega D_x \left(1 + \frac{\sigma_{x}^{PML}(x)}{j\omega\epsilon_0}\right)^{-1} \left(1 + \frac{\sigma_{y}^{PML}(y)}{j\omega\epsilon_0}\right) \left(1 + \frac{\sigma_{z}^{PML}(z)}{j\omega\epsilon_0}\right) = c_0 \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)$$

(2.16)

for the $D_x$ component and similar equations for the remaining components. In equation 2.16, $\sigma_i(i)$ is the conductivity profile of PML in x, y or z direction.
2.1.4 Stability Criterion

An important issue in an FDTD simulation is the choice of the cell size ($\Delta x$) and the time-step size ($\Delta t$). In the case of a sinusoidal excitation, simulation needs to be performed for at least a few periods of the sinusoid to compute the field values. Therefore, to reduce the simulation time, it is desirable to keep $\Delta t$ as large as possible, as that will result in less number of time-steps required for the simulation. However for computational stability, the maximum value of the time step which can be used for simulation is bounded by the cell size. This bound is known as Courant-Friedrichs-Lewy (CFL) stability bound. For a 3-dimension wave propagation, this bound is given by [8]-

$$\Delta t \leq \frac{1}{c_0 \sqrt{\frac{1}{\Delta x_{\text{min}}} + \frac{1}{\Delta y_{\text{min}}} + \frac{1}{\Delta z_{\text{min}}}}} \quad (2.17)$$

where, $\Delta x_{\text{min}}$, $\Delta y_{\text{min}}$, $\Delta z_{\text{min}}$ are the minimum cell sizes in x, y and z directions respectively. It has been shown in [8] that any value of $\Delta t$ greater than the limit in equation 2.17 leads to instability.

It can be seen from equation 2.17 that the time-step size is dependent on the cell size. The choice of the cell size to be used for the simulation depends on the maximum frequency of interest and the required geometrical detail of the model. In order to be able to sufficiently resolve the signal maximum cell size should be less than one tenth of the minimum wavelength present in the source excitation [16].
Table 2.1: Simulation parameter for low frequency Bioelectromagnetic and VLSI problems

<table>
<thead>
<tr>
<th>Problem Class</th>
<th>$\Delta x$</th>
<th>Frequency</th>
<th>$\Delta t$</th>
<th>$T_{sim}$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-frequency bioelectromagnetics</td>
<td>1 mm</td>
<td>100 kHz</td>
<td>1.9 ps</td>
<td>10$\mu$s</td>
<td>5.2×10$^6$</td>
</tr>
<tr>
<td>VLSI digital logic</td>
<td>45 nm</td>
<td>10 GHz</td>
<td>0.08 fs</td>
<td>125ps</td>
<td>1.25×10$^6$</td>
</tr>
</tbody>
</table>

2.2 Alternating Direction Finite-Difference Time-Domain Method

2.2.1 Theory

In a large number of electromagnetic problems, such as problems involving bioelectromagnetic interactions or digital VLSI circuits, fine geometric details put a restriction on the maximum cell size which can be used for the simulation. Since the maximum time-step size which can be used in an FDTD simulation is bounded by the Courant’s stability condition, an FDTD simulation would require prohibitively large number of time steps to obtain the solution in such problems. Table 2.1 shows the simulation parameters for the VLSI and low frequency bioelectromagnetic simulations. An FDTD simulation of a bioelectromagnetic problem with a 100 kHz sinusoidal excitation and at 1nm resolution will require around 5 million time steps for the completion of one cycle of sinusoid, which is an intractable problem if one considers a large model such as the model used in this work.

In such cases, the Alternating-Direction-Implicit Finite-Difference Time-Domain (ADI-FDTD) method offers a computationally very efficient alternative. This method allows for the use of time steps size ($\Delta t$) much larger than the limit set by the Courant stability condition [8] [12] [13] [19]. It has been shown in [8] and [13] that the method
is unconditionally stable for an arbitrarily large value of time step \((\Delta t)\). Similar to the explicit FDTD method, the ADI-FDTD method also uses a staggered grid formulation for the E and H field components. However, unlike the explicit FDTD method where E and H field components are staggered in time, in the ADI-FDTD they are computed at the same time. The computation is performed in two sub-iterations. In the first sub-iteration, computation is done for the time step \(n\) and time step is advanced from \(n\) to \(n+1/2\), whereas in the second sub-iteration, computation is done for the time step \(n+1/2\) and the time-step is advanced from \(n+1/2\) to \(n+1\). Since the field components are co-located, the H field values are used to update the E field values. These equations yield a tri-diagonal matrix system, which can be easily solved by matrix inversion. Equations 2.18 through 2.23 below show the 3-dimensional ADI-FDTD expressions for \(E\) and \(H\) fields in the first half time-step [8].

\[
\begin{align*}
1 + \left[ \frac{(\Delta t)^2}{2\mu \epsilon (\Delta y)^2} \right] E_x^{n+1/2}_{i+1/2,j,k} &- \left[ \frac{(\Delta t)^2}{4\mu \epsilon (\Delta y)^2} \right] \left( E_x^{n+1/2}_{i+1/2,j-1,k} + E_x^{n+1/2}_{i+1/2,j+1,k} \right) \\
&= E_x^n_{i+1/2,j,k} + \left( \frac{\Delta t}{2\epsilon \Delta y} \right) \left( H_x^n_{i+1/2,j+1/2,k} - H_x^n_{i+1/2,j-1/2,k} \right) \\
&\quad - \left( \frac{\Delta t}{2\epsilon \Delta z} \right) \left( H_y^n_{i+1/2,j,k+1/2} - H_y^n_{i+1/2,j,k-1/2} \right) \\
- \left[ \frac{(\Delta t)^2}{4\mu \epsilon \Delta y \Delta x} \right] \left( E_y^n_{i,j+1/2,k} - E_y^n_{i,j+1/2,k} - E_y^n_{i,j-1/2,k} + E_y^n_{i,j-1/2,k} \right) & (2.18)
\end{align*}
\]

\[
\begin{align*}
1 + \left[ \frac{(\Delta t)^2}{2\mu \epsilon (\Delta z)^2} \right] E_y^{n+1/2}_{i,j+1/2,k} &- \left[ \frac{(\Delta t)^2}{4\mu \epsilon (\Delta z)^2} \right] \left( E_y^{n+1/2}_{i,j+1/2,k-1} + E_y^{n+1/2}_{i,j+1/2,k+1} \right) \\
&= E_y^n_{i,j+1/2,k} + \left( \frac{\Delta t}{2\epsilon \Delta z} \right) \left( H_x^n_{i,j+1/2,k+1/2} - H_x^n_{i,j+1/2,k-1/2} \right) \\
&\quad - \left( \frac{\Delta t}{2\epsilon \Delta x} \right) \left( H_z^n_{i+1/2,j+1/2,k} - H_z^n_{i-1/2,j+1/2,k} \right) \\
- \left[ \frac{(\Delta t)^2}{4\mu \epsilon \Delta y \Delta z} \right] \left( E_z^n_{i,j+1,k+1/2} - E_z^n_{i,j,k+1/2} - E_z^n_{i,j+1,k-1/2} + E_z^n_{i,j,k-1/2} \right) & (2.19)
\end{align*}
\]
\[ 1 + \frac{(\Delta t)^2}{2\mu \epsilon (\Delta x)^2} E_z^{n+1/2}_{i,j,k+1/2} - \left( \frac{(\Delta t)^2}{4\mu \epsilon (\Delta x)^2} \right) \left( E_z^{n+1/2}_{i-1,j,k+1/2} + E_z^{n+1/2}_{i+1,j,k+1/2} \right) \\
= E_z^n_{i,j,k+1/2} + \frac{\Delta t}{2\epsilon \Delta x} (H_y^n_{i+1/2,j,k+1/2} - H_y^n_{i-1/2,j,k+1/2}) \\
- \frac{\Delta t}{2\mu \epsilon (\Delta y)^2} (H_x^n_{i+1/2,j,k+1/2} - H_x^n_{i-1/2,j,k+1/2}) \] (2.20)

\[ H_x^{n+1/2}_{i,j+1/2,k+1/2} = H_x^n_{i,j+1/2,k+1/2} + \frac{\Delta t}{2\mu \epsilon \Delta z} (E_z^{n+1/2}_{i+1,j,k+1/2} - E_z^{n+1/2}_{i,j,k+1/2}) \\
- \frac{\Delta t}{2\mu \epsilon (\Delta x)^2} (E_x^{n+1/2}_{i,j+1/2,k+1/2} - E_x^{n+1/2}_{i,j,k+1/2}) \] (2.21)

\[ H_y^{n+1/2}_{i+1/2,j,k+1/2} = H_y^n_{i+1/2,j,k+1/2} + \frac{\Delta t}{2\mu \epsilon \Delta x} (E_z^{n+1/2}_{i+1,j,k+1/2} - E_z^{n+1/2}_{i,j,k+1/2}) \\
- \frac{\Delta t}{2\mu \epsilon (\Delta z)^2} (E_y^{n+1/2}_{i+1/2,j+1/2,k} - E_y^{n+1/2}_{i+1/2,j,k}) \] (2.22)

\[ H_x^{n+1/2}_{i+1/2,j+1/2,k} = H_x^n_{i+1/2,j+1/2,k} + \frac{\Delta t}{2\mu \epsilon \Delta y} (E_x^{n+1/2}_{i+1/2,j+1/2,k} - E_x^{n+1/2}_{i+1/2,j,k}) \\
- \frac{\Delta t}{2\mu \epsilon (\Delta x)^2} (E_y^{n+1/2}_{i+1/2,j+1/2,k} - E_y^{n+1/2}_{i+1/2,j,k}) \] (2.23)

It can be seen that equation 2.18 leads to tridiagonal system of equations for each y-cut which can be solved to obtain the field component \( E_x^{n+1/2} \). Similarly, 2.19 and 2.20 can be solved to obtain the \( E_y^{n+1/2} \) and \( E_z^{n+1/2} \) field components. H-field components can then be computed explicitly from the knowledge of the past values of H field and recently computed E-field. Expression for \( E \) and \( H \) field components for seconds half step is given by
\[
\begin{align*}
1 + \frac{(\Delta t)^2}{2\mu\epsilon(\Delta z)^2} E_x^{n+1}_{i+1/2,j,k} &= \left[ \frac{(\Delta t)^2}{4\mu\epsilon(\Delta z)^2} \right] \left( E_x^{n+1}_{i+1/2,j,k-1} + E_x^{n+1}_{i+1/2,j,k+1} \right) \\
&- \frac{(\Delta t)}{2\epsilon\Delta y} \left( H_z^{n+1/2}_{i+1/2,j+1/2,k} - H_z^{n+1/2}_{i+1/2,j-1/2,k} \right) \\
&- \frac{(\Delta t)}{2\epsilon\Delta z} \left( H_y^{n+1/2}_{i+1/2,j,k+1/2} - H_y^{n+1/2}_{i+1/2,j,k-1/2} \right)
\end{align*}
\]

\[
\begin{align*}
1 + \frac{(\Delta t)^2}{2\mu\epsilon(\Delta x)^2} E_y^{n+1}_{i,j+1/2,k} &= \left[ \frac{(\Delta t)^2}{4\mu\epsilon(\Delta x)^2} \right] \left( E_y^{n+1}_{i-1,j+1/2,k} + E_y^{n+1}_{i+1,j+1/2,k} \right) \\
&- \frac{(\Delta t)}{2\epsilon\Delta x} \left( H_z^{n+1/2}_{i+1/2,j,k+1/2} - H_z^{n+1/2}_{i-1,j,k+1/2} \right) \\
&- \frac{(\Delta t)}{2\epsilon\Delta y} \left( H_x^{n+1/2}_{i+1,j+1/2,k+1} - H_x^{n+1/2}_{i-1,j+1/2,k} \right)
\end{align*}
\]

\[
\begin{align*}
1 + \frac{(\Delta t)^2}{2\mu\epsilon(\Delta y)^2} E_z^{n+1}_{i,j,k+1/2} &= \left[ \frac{(\Delta t)^2}{4\mu\epsilon(\Delta y)^2} \right] \left( E_z^{n+1}_{i,j-1,k+1/2} + E_z^{n+1}_{i,j+1,k+1/2} \right) \\
&- \frac{(\Delta t)}{2\epsilon\Delta x} \left( H_y^{n+1/2}_{i+1,j+1/2,k+1} - H_y^{n+1/2}_{i-1,j+1/2,k} \right) \\
&- \frac{(\Delta t)}{2\epsilon\Delta y} \left( H_x^{n+1/2}_{i,j+1,k+1/2} - H_x^{n+1/2}_{i,j-1,k+1/2} \right)
\end{align*}
\]

\[
\begin{align*}
H_z^{n+1}_{i,j+1/2,k+1/2} &= H_x^{n+1/2}_{i,j+1/2,k+1/2} + \frac{(\Delta t)}{2\mu\Delta z} \left( E_y^{n+1/2}_{i,j+1/2,k+1} - E_y^{n+1/2}_{i,j+1/2,k} \right) \\
&- \frac{(\Delta t)}{2\mu\Delta y} \left( E_z^{n+1}_{i,j+1,k+1/2} - E_z^{n+1}_{i,j+1,k+1/2} \right)
\end{align*}
\]
\[
H_y|_{i+1/2,j,k+1/2}^{n+1} = H_y|_{i+1/2,j,k+1/2}^{n+1/2} + \frac{(\Delta t)}{2\mu \Delta x} \left( E_z|_{i+1,j,k+1/2}^{n+1/2} - E_z|_{i,j,k+1/2}^{n+1} \right) - \frac{(\Delta t)}{2\mu \Delta z} \left( E_x|_{i+1/2,j,k+1}^{n+1/2} - E_x|_{i+1/2,j,k}^{n+1} \right) \tag{2.28}
\]

\[
H_z|_{i+1/2,j+1/2,k}^{n+1} = H_z|_{i+1/2,j+1/2,k}^{n+1/2} + \frac{(\Delta t)}{2\mu \Delta y} \left( E_x|_{i+1/2,j+1,k}^{n+1/2} - E_x|_{i+1/2,j,k}^{n+1} \right) - \frac{(\Delta t)}{2\mu \Delta x} \left( E_y|_{i+1,j+1/2,k}^{n+1/2} - E_y|_{i,j+1/2,k}^{n+1} \right) \tag{2.29}
\]

Equation 2.24 leads to another tridiagonal system of equations for each z-cut to obtain the field component \(E_x^{n+1}\). Similarly, the field components \(E_y^{n+1}\) and \(E_z^{n+1}\) can also be obtained.

In general, time step size is represented in terms of the CFL Number or ADI factor which represents the ratio of the time-step with respect to the limit imposed by the Courant’s condition. The CFL number is given by \(\frac{c \Delta t}{\Delta x}\). For an FDTD simulation this number must be smaller than 1. However, in an ADI-FDTD simulation this number can be larger than 1 as the ADI-FDTD method does not need to conform to the Courant’s condition. For example, a CFL number (or ADI factor) of 4 means that the time step used for simulation is 4 times larger than the maximum time-step as per the Courant condition.

### 2.2.2 D-H ADI FDTD Method

Liu et al. proposed an implementation of PML for the ADI-FDTD method, which was based on the split field formulation of Berenger [17][20]. However, in order to truncate large models, it is often necessary to extend the dielectric material into the absorbing boundary conditions. Thus, it is desirable to have a PML implementation which is independent of the background material being used in the grid. As discussed
in section 2.1.2, the D-H formulation enable an easy and efficient implementation of the Perfectly Matched Layer (PML) as the absorbing boundary condition by allowing the PML condition independent of the background material. In [21], the ADI-FDTD method extended to a D-H formulation with unsplit field components PML absorbing boundary conditions has been proposed which allows immersing dielectric into the PML. However, it was observed that late-time error induced by the corner cells of the absorbing boundary conditions were not negligible and were increasing with the ADI factor. In [22] and [23], this implementation of the D-H ADI-FDTD method has been extended to reduce the reflection errors. The method involves implicit computation of D-field using tridiagonal matrix inversion. Then, E-field is computed explicitly from D-field using equation 2.8 and after that the H-field is updated explicitly using the E-field values. The same process repeats for the second half step. As discussed in section 2.1.3, the initial equation for the D-H ADI-FDTD formulation with PML absorbing boundary conditions is given as [21]

\[
j\omega D_x \left(1 + \frac{\sigma_{PML}(x)}{j\omega \varepsilon_0} \right)^{-1} \left(1 + \frac{\sigma_{PML}(y)}{j\omega \varepsilon_0} \right) \left(1 + \frac{\sigma_{PML}(z)}{j\omega \varepsilon_0} \right) = c_0 \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)
\]

(2.30)

Solving this equation using the ADI scheme, the expressions for the D, E and H fields can be obtained. The equation for the field component \( D_x \) in the D-H ADI FDTD formulation with PML as absorbing boundary conditions for first half step is given as [22][24]:

\[
D_x^{n+1/2} = \frac{P_{y}^{N1} P_{z}^{N1}}{P_{y}^{D} P_{z}^{D}} D_x^n - 4 \frac{P_{y}^{N2} P_{z}^{N2}}{P_{y}^{D} P_{z}^{D}} \sum_{s=1/2}^n D_s^s + c_0 \Delta t
\]

\[
\left[ \frac{P_{x}^{N3}}{P_{y}^{D} P_{z}^{D}} \frac{\partial H_{y}^{n+1/2}}{\partial y} - \frac{P_{x}^{N4}}{P_{y}^{D} P_{z}^{D}} \frac{\partial H_{y}^{n}}{\partial z} + 2 \frac{P_{x}^{N5}}{P_{y}^{D} P_{z}^{D}} \sum_{s=1/2}^n \left( \frac{\partial H_{y}^{s}}{\partial y} - \frac{\partial H_{y}^{s}}{\partial z} \right) \right]
\]

(2.31)
and for the second half-step

\[ D_{x}^{n+1} = \frac{P_{N1}^{y} P_{N1}^{z}}{P_{D}^{y} P_{D}^{z}} D_{x}^{n+1/2} - 4 \frac{P_{N2}^{y} P_{N2}^{z}}{P_{D}^{y} P_{D}^{z}} \sum_{s=1/2}^{n+1/2} D_{x}^{s} + c_{0} \Delta t \]

\[ \left[ \frac{P_{N4}^{x}}{P_{D}^{x} P_{D}^{z}} \frac{\partial H_{z}^{n+1/2}}{\partial y} - \frac{P_{N3}^{x}}{P_{D}^{x} P_{D}^{z}} \frac{\partial H_{y}^{n}}{\partial z} + 2 \frac{P_{N5}^{x}}{P_{D}^{x} P_{D}^{z}} \sum_{s=1/2}^{n+1/2} \left( \frac{\partial H_{z}^{s}}{\partial y} - \frac{\partial H_{y}^{s}}{\partial z} \right) \right] \] (2.32)

Where,

\[ P_{D}^{x} = P_{N3}^{x} = 1 + \frac{\sigma^{\text{PML}} x \Delta t}{2 \epsilon_{0}} = 1 + X_{n}(i) \]

\[ P_{x}^{N1} = P_{x}^{N4} = 1 \frac{\sigma^{\text{PML}} x \Delta t}{2 \epsilon_{0}} = 1 - X_{n}(i) \]

\[ P_{x}^{N2} = P_{x}^{N5} = \frac{\sigma^{\text{PML}} x \Delta t}{2 \epsilon_{0}} = X_{n}(i) \] (2.33)

\[ H \text{ field components are derived explicitly. The equation for } H_{z} \text{ for second half step is given by } [22] : \]

\[ H_{z}^{n+1} = \frac{P_{N1}^{y} P_{N1}^{z}}{P_{D}^{y} P_{D}^{z}} H_{z}^{n+1/2} - 4 \frac{P_{N2}^{y} P_{N2}^{z}}{P_{D}^{y} P_{D}^{z}} \sum_{s=1/2}^{n+1/2} H_{z}^{s} + c_{0} \Delta t \]

\[ \left[ \frac{P_{N4}^{x}}{P_{D}^{x} P_{D}^{y}} \frac{\partial E_{x} n + 1/2}{\partial y} - \frac{P_{N3}^{x}}{P_{D}^{x} P_{D}^{y}} \frac{\partial E_{y} n + 1}{\partial x} + 2 \frac{P_{N5}^{x}}{P_{D}^{x} P_{D}^{y}} \sum_{s=1/2}^{n+1/2} \left( \frac{\partial E_{x}^{s}}{\partial y} - \frac{\partial E_{y}^{s}}{\partial x} \right) \right] \] (2.34)

The electric field can be computed from the D field using the finite difference equation, for the y-component this is given in equation 2.35:

\[ E_{y}^{n+1} = \frac{D_{y}^{n+1} - \frac{\sigma_{y} \Delta t}{\epsilon_{0}} \sum_{s=1/2}^{n+1/2} E_{y}^{s}}{\epsilon_{r,y} + \frac{\sigma_{y} \Delta t}{\epsilon_{0}}} \] (2.35)

The E-field values from equation 2.35 is substituted into 2.34 to obtain the H-fields and then H-fields values are substituted into 2.32, to obtain the trigonal system of equations that can be solved implicitly to get the \( D_{x}^{n+1/2} \) along the y-axis for the first time-step. Similarly, other components for D-field can also be obtained.
2.3 Computational Techniques for Low Frequency Simulations

As discussed in previous sections, the ADI-FDTD method is an unconditionally stable method and it does not need to conform to the Courant-Friedrich-Lewy (CFL) condition. This makes it especially useful for over-resolved problems, where the spatial resolution is much smaller than that required to resolve the smallest signal wavelength. However, there are two basic problems with the ADI-FDTD method. First, it is computationally more intensive than the explicit FDTD method, implying that simulation time gains are possible only if the acceleration factor (ADI factor) is sufficiently high. Second, the dispersion error is larger than that of the FDTD method and increases with the ADI factor [25]. Furthermore, the truncation error which results from discretization, also increases with the time-step [26]. Therefore, it appears that the ADI-FDTD method, with a high ADI factor, would be more useful in applications with a higher error tolerance, especially where the quantities of interest are averaged. One such area is bioelectromagnetics, where averaged quantities such as the peak 1-g SAR and current densities are generally of prime importance.

However, for some bio-electromagnetic problems such as the computation of induced current by HEMI devices at low frequencies, even the ADI-FDTD method will require a large number of steps. For example, for a 120 KHz signal, simulation with the ADI FDTD method with ADI factor equal to 16 will require about 300,000 steps for the completion of a full sinusoid which is still an intractable problem if one considers a full body model at 1 mm resolution (about 370 million computational cells). Since, simulation with larger ADI factor may result in increasing errors, other computation techniques or approximations must be employed to reduce the number of simulation steps.
Over the last few years, a number of methods and techniques have been used to solve for the induced fields and currents in the human body due to exposure to the low-frequency electric and magnetic fields. In [27], FDTD simulations were performed at 10 MHz and then scaled down to 60 Hz. However, for this method, one will need to ensure that the quasi-static approximations are valid for the higher frequency for the model under consideration. The impedance method [28] and the Scalar Potential Finite-Difference (SPFD) method [29] have been successfully used to compute induced current densities due to magnetic fields. A hybrid method using the SPFD and the traditional FDTD was also developed which allowed for the usage of the SPFD method to compute induced fields due to incident electric fields [30]. For the FDTD simulations, a well known method to obtain field values for sinusoidal excitations using reduced run-times is the two-equation two-unknown method [31]. However, in the problem at hand, at points far away from the source, numerical noise will likely dominate the actual signal values and therefore, two-equations two unknown method may not be a suitable method for the simulations. A review of the numerical techniques used for 60 Hz computations can be found in [32].

In this work, DFT averaging method has been used in conjunction with the ADI-FDTD method. It has been shown that the simulation time required using the above techniques is much smaller compared to that required by the explicit method. Expanding grid method has been used to reduce the model size and thus amount of computations required for each time-step thereby reducing the memory and simulation time requirements for the model. The following sections describe the expanding grid and DFT averaging methods in detail.
2.3.1 Expanding Grid Technique

One of the disadvantages of using the ADI-FDTD method is that the memory requirement is much larger as compared to the explicit FDTD method. This is particularly true for the D-H formulation of the ADI-FDTD method. Moreover, the large number of computational cells needed for the considered problem leads to a very large model size, which requires a very large memory. For example, the full body man model used in this work at 1 mm uniform resolution requires approximately 370 million cells to model the computational space. This would require around 75 GB of memory when the problem is solved using the ADI-FDTD method. Such large memory requirement makes the problem unsolvable even on a computer with a large memory. Moreover, large model size drastically increases the simulation time as in each time-step computations have to be performed at more number of points.

One approach to circumvent this problem is to use a logarithmic expanding grid algorithm [33]. The main idea behind the expanding grid technique is to use a variable grid width profile, where in the region of interest, a fine resolution is employed, whereas the resolution is decreased gradually in regions which are further from the points of interest. In the problems, where grid width is determined by the minimum wavelength present in the source, its profile is decided based upon the dielectric material. By modeling the regions of lesser importance with coarser resolution, expanding grid technique helps in reducing the model size, thereby reducing the memory requirement. Moreover, accurate modeling of the region of interest is also possible, where a finer grid is used. In the considered problems, the model size with the use of expanding grid model can be easily reduced by a factor of 50, and thus the problem which was unsolvable with traditional methods and resources can be solved on a commercial system with several gigabytes of memory.

The generation of the expanding grid is done by selecting the minimum ($\Delta_{\text{min}}$) and
the maximum grid sizes ($\Delta_{\text{max}}$) and the cell expansion factors ($\alpha$). The grid width at any point in the space can be specified in terms of these parameters. Material allocation in the model can be done by clustering together the required number of cells. For the expanding grid, the equations describing the E and H field relationship differ slightly from the original FDTD equations. The detailed discretized differential equations for the explicit FDTD in the E-H formulation are given in [33].

2.3.2 DFT Averaging

Discrete Fourier Transform (DFT) of the signal at relevant signal frequencies with the quasi-static assumption can be used to obtain the electric field magnitudes at the observation points relative to the source. Using this method, electric field values can be obtained by running the simulation much shorter than the full cycle of the waveform. In an FDTD simulation, we need to compute the amplitudes and phases of the field values after they have reached the steady state sinusoidal behavior, which typically requires a simulation time of a few periods. By using the quasi-static assumption, we take advantage of the fact that the phases of the field values are known and therefore, the E-field values can be computed by obtaining the amplitudes of the fields alone. As the phases are same everywhere in the model, the E-field values will simply scale with the source location and therefore, the solution can be obtained in a much smaller time steps. For quasi static assumptions to be valid [34],

1) The conduction current should dominate the displacement current
2) The size of the model should be much smaller than the incident field wavelength

In the problem at hand, the minimum wavelength present in the source is around 15 meters, which is much larger than the model size of 1.8 meter. Also, since $\sigma >> \omega \varepsilon$ for most of the tissues, conduction current dominates the displacement current. Therefore, quasi static assumption can be applied in our work. In this research,
Discrete Fourier Transform (DFT) based on the quasi static assumptions has been used to compute the induced electric field for the HEMI pulse. This method works by decomposing the source waveform into dominant frequencies present in the signal. DFT of the E-field is then computed for the desired frequencies for time periods much smaller than the full cycle of the respective sinusoid. The excitation function should be a broadband input signal like the Gaussian pulse with a frequency spectrum width similar to the original source waveform. After the initial transient, using the ratios of the magnitudes of DFT of E-field at all points in the model with respect to the source, the E-field values normalized to the source location can be estimated. The normalization of the DFT of E-field values is performed as follows

\[
F(i, j, k) = \frac{\sum T_{E_{i,j,k}}(t)e^{-j\omega t}}{\sum T_{E_{source}}(t)e^{-j\omega t}}
\]

(2.36)

where \( F(i, j, k) \) is the ratio of the field value at an observation point, \( T \) is the time up to which the DFT of was obtained, \( \omega \) is the angular frequency under consideration, and \( E \) is the electric field.

These ratios provide the field values in the model with respect to the E-field at the source location. Once these ratios are known for the desired frequencies, the electric field values can be scaled with the source, and the signal can be re-constructed. This is done by computing the current flowing through the electrodes for the normalized E-field of 1 V/m at the source location and then scaling it to the actual current flowing through the current. This will provide the actual E-field values at all points in the model for the steady state condition.
Chapter 3

Validation and Simulation Methods

In this work, the ADI-FDTD method in a D-H formulation with expanding grid and DFT averaging technique has been used to compute the induced electric field and current density due to the contact electrodes of a Human Electro-Muscular Incapacitation Devices (HEMI) device placed at the torso region of the human body. In order to evaluate the accuracy of these techniques numerical validation has been performed by simulating smaller models and using higher frequencies. The following sections discuss the simulation methods and validation results.

3.1 Simulation method for the ADI-FDTD method

The ADI-FDTD method in the D-H formulation [21] was used for the simulations as described in [22]. The electrodes were modeled as conductors with a single cell finite-gap current source. Since the electric displacement (D) field is computed implicitly in the D-H ADI-FDTD method, the source excitation should also be done implicitly. Therefore, the source excitation function was incorporated in the known column vector on the right side of the tri-diagonal matrix system [35]. It has been shown in [36] that this implementation is more accurate as compared to the explicit
source excitation.

The implementation of the FDTD with PML for the $D_x$ field component is based on the discretization of the following equation

$$
\begin{align}
    j\omega D_x \left(1 + \frac{\sigma_{x}^{PML}(x)}{j\omega\varepsilon_0}\right)^{-1} & \left(1 + \frac{\sigma_{y}^{PML}(y)}{j\omega\varepsilon_0}\right) \left(1 + \frac{\sigma_{z}^{PML}(z)}{j\omega\varepsilon_0}\right) = \\
    c_0 \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)
\end{align}
$$

(3.1)

However, it was observed that this implementation allowed some high frequency evanescent waves to propagate. Therefore, a modified PML similar to that described by Gedney has been implemented to attenuate the spurious evanescent waves in the PML [37]. An additional loss term ($\kappa$) was incorporated in equation 3.1 to suppress the evanescent waves. The modified Maxwell’s equation for the $D_x$ field component is given by:

$$
\begin{align}
    j\omega D_x \left(\kappa + \frac{\sigma_{x}^{PML}(x)}{j\omega\varepsilon_0}\right)^{-1} & \left(\kappa + \frac{\sigma_{y}^{PML}(y)}{j\omega\varepsilon_0}\right) \left(\kappa + \frac{\sigma_{z}^{PML}(z)}{j\omega\varepsilon_0}\right) = \\
    c_0 \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)
\end{align}
$$

(3.2)

While increasing the CFL number accelerates the onset of high frequency oscillations, it was found that increasing $\kappa$ suppressed this late-time behavior. Although, a higher $\kappa$ leads to larger errors in the propagating volume, the largest errors were found near the PML interface. The reason for this is the small magnitude of the E-field near the PML region and therefore, numerical noise is more likely to dominate. Errors found in the regions of maximum induced fields near the source were small. Therefore, for the ADI-FDTD simulations with higher ADI number (for example ADI number=16), higher value of $\kappa$ ($\kappa=80$) has been used. The thickness of the PML layer was chosen as 16 cells as that was sufficient to prevent the reflected waves to re-enter the main space. For the PML conductivity-profile, a polynomial grading
$$X_n = X_{n,\text{max}} \left( \frac{i}{n_{\text{pml}}} \right)^p, \quad i = 1, 2, \ldots, n_{\text{pml}}$$ (3.3)

with $p=3$, $n_{\text{pml}}=16$ and $X_{n,\text{max}}=0.33$ was used. These parameters were determined empirically for the stable implementation of the PML. The frequency of the source stimulation was chosen as 10 MHz. The reason for choosing a frequency higher than the average frequency used for the HEMI device was to reduce the number of time-steps required for the simulation.

Variation of the voltage and current at source and electric field magnitude at several observation points with time was stored. Electric field magnitude at every point in the model was stored at every 1/8th of the cycle of the sinusoid. These values were compared against those obtained from the simulation with the explicit FDTD method. The validation model and simulation results are discussed in subsequent sections.

### 3.1.1 Validation Model for the D-H ADI-FDTD scheme

The validation of the computational method used and the analysis of error was done first on smaller models containing only the human torso. Once these methods were verified, computation of the induced electric field and current density was done in the full body model. A model of just the torso region of dimensions $110 \times 109 \times 85$ at 5mm uniform resolution was extracted, which was used for the validation of the ADI-FDTD method. Fig. 3.1 shows the sagittal view of the validation model with the contact electrodes placed in the torso region. Electrode separation was 5 cm and electrode penetration inside the body was kept as 5 mm. Simulation were performed on this model using the ADI-FDTD method with different CFL numbers. The comparison of the E-field magnitudes, voltages and currents at different points in the model were done with the values obtained using the explicit FDTD method. The
DFT Averaging method was also verified on the this torso model.

3.1.2 Simulation Results

Fig. 3.2 and Fig. 3.3 shows the time domain plot of current and voltage at the source location for the ADI-FDTD with ADI number equal to 1 and explicit FDTD methods respectively. Voltages and currents at the source are almost identical for both the FDTD and ADI-FDTD methods. Fig. 3.4 gives a time domain plot of the electric field magnitude variations at four sampling points in the tissues for ADI-FDTD method with ADI number=1 and explicit FDTD method. Two sampling points were close to the electrode contact points whereas, two other points were near the PML region. It can be seen from the plots that electric field values at the monitoring points are identical for both the methods.

The E-field magnitude in the model after every 1/8th of the time steps for the ADI-FDTD simulation are compared against those obtained from the explicit FDTD method. Maximum single-voxel errors for ADI number equal to 1 is found to be less
Figure 3.2: Current at source for the ADI-FDTD (ADI Number=1) and explicit FDTD methods.

Further simulations were done by varying the ADI numbers in the ADI-FDTD method. Table 3.1 summarizes the maximum error in the E-field and time steps required for different ADI numbers. Although the number of time-steps required for higher ADI numbers is much smaller as compared to the explicit FDTD method, simulation with higher ADI number degrades the accuracy of the method. Moreover, since the ADI-FDTD method is computationally more intensive, the simulation time benefit is seen only when the ADI number \( \geq 5 \).

Even though the single-voxel error is relatively large for higher ADI number, the overall error is small in the computation of induced current density in a tissue as the errors are averaged over many cells. Because of the very large size of the model and low frequency stimulation, explicit FDTD or the ADI-FDTD simulation with lower ADI factor will take prohibitively long to complete the simulation. Therefore, ADI number of 16 was chosen to compute the induced electric field and current density in the body.

Fig. 3.7 gives the E-field pattern relative to the source location at a cross section.
Figure 3.3: Voltage at source for the ADI-FDTD (ADI Number=1) and explicit FDTD methods

Figure 3.4: The electric field magnitude at four monitoring point in the validation for ADI FDTD with ADI Number=1 and explicit FDTD method
Table 3.1: Summary of simulation error and time for ADI-FDTD method with different ADI numbers; cell width is 5mm and frequency is 10 MHz

<table>
<thead>
<tr>
<th>ADI number</th>
<th>Number of steps</th>
<th>Maximum relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10392</td>
<td>0.0011</td>
</tr>
<tr>
<td>2</td>
<td>5196</td>
<td>0.0059</td>
</tr>
<tr>
<td>4</td>
<td>2598</td>
<td>0.0209</td>
</tr>
<tr>
<td>8</td>
<td>1299</td>
<td>0.0692</td>
</tr>
<tr>
<td>16</td>
<td>649</td>
<td>0.201</td>
</tr>
<tr>
<td>25</td>
<td>415</td>
<td>0.356</td>
</tr>
</tbody>
</table>

Figure 3.5: Current at source for the ADI-FDTD (ADI number=16)

Figure 3.6: Voltage at source for the ADI-FDTD (ADI number=16)
Figure 3.7: The E-field magnitude (V/m) with source normalized at 1V/m at a cross section for ADI-FDTD (ADI number=16)

for the ADI-FDTD method with ADI number equal to 16. As expected, maximum electric field distribution is seen at the location where electrodes are touching the body. At these locations, direct stimulation of the nerve fibers is likely to take place. The E-field magnitude reduces with the distance from the source location. Single-voxel error in the induced electric field magnitudes for the ADI FDTD method with ADI number=16 relative to the explicit FDTD method in the cross section shown in fig. 3.1 is shown in Fig. 3.8. The errors in the induced E-field averaged over 8-voxels is shown in Fig. 3.8. It can be seen that the averaged errors are much smaller than the single-voxel error. In the problem at hand, since we are interested in knowing the current density in different tissues, which are averaged over many voxels, impact of errors due to simulation with high ADI numbers is much reduced.
Figure 3.8: An illustration of the single-voxel errors in the induced electric field magnitudes of the ADI-FDTD method (ADI=16) relative to the explicit FDTD method.

Figure 3.9: An illustration of the averaged errors in the induced electric field magnitudes of the ADI-FDTD method (ADI=16) relative to the explicit FDTD method.
3.2 Validation of the expanding grid scheme

3.2.1 Validation Model for the expanding grid scheme

Validation of the expanding grid scheme was done by comparing the E-field values in the 1 mm uniform resolution model with those in the logarithmic expanding grid model. A model of just the torso region was extracted from the full body model at 1 mm resolution. This model had the dimensions of $280 \times 300 \times 400$. An expanding grid model of the torso region was generated with resolution varying from 1 mm to 3 mm in x and y directions and 1 mm to 5 mm in z direction, with a cell to cell expansion factor of 1.075. The resultant expanding grid model had the dimensions of $144 \times 152 \times 153$, around 10% of the size of the original uniform 1 mm torso model. Fig. 3.10 shows the grid scheme used to generate the expanding grid model for the torso region. Fig. 3.11 and Fig. 3.12 show the torso model in 1mm uniform grid and expanding grid schemes respectively.

3.2.2 Simulation result

Our simulation results showed an average relative error of only 1.9% in the region of contact. Although the relative errors in the far region were relatively large, still gave a reasonable estimate of the actual E-field magnitude. Memory requirement with the expanding grid technique was around 10% of the original model size. Also, the simulation time in the case of the expanding grid model was less than the 1/8th of what is required in the case of the original uniform grid model. Given the benefits in the simulation time and required memory, expanding grid appears a very efficient method for the simulations involving very large models and low frequency source. Fig. 3.13 shows the plot of E-field values at a monitoring point near the electrodes in expanding and uniform resolution models, further demonstrating the robustness of
Figure 3.10: The expanding grid profiles for torso model (a) along x-axis (b) along y-axis (c) along z-axis

Figure 3.11: The uniform grid torso model at 1mm resolution
the expanding grid scheme for the D-H-ADI method.

\section*{3.3 DFT Averaging scheme}

As discussed in chapter 2, the X26 HEMI pulse waveform is a damped sinusoid at a frequency of 120 kHz superimposed on a decaying exponential pulse \cite{3}. From the fourier decomposition of the waveform, it was obtained that the signal could be represented with good accuracy by 27 low frequency harmonics, with a fundamental frequency of 5.8 kHz. Over this frequency range (5.8 kHz - 156.6 kHz), the conductivity of most tissues increases by less than 5\%. Also, since displacement currents can be neglected for these frequencies, to simplify computations, frequency dependence of the properties has not been considered. Instead, constant dielectric properties at an intermediate frequency of 120 kHz have been used. Fig. 3.14 gives the HEMI voltage pulse for a 100 \( \Omega \) load and reconstructed waveform with only 27 harmonics.
Figure 3.13: The electric field comparison at an observation point for the uniform and expanding grid methods.

Figure 3.14: Measured waveform and approximated waveform using the DC + 27 harmonics super-imposed on the original.
3.3.1 Validation of the DFT Averaging scheme

Validation of the DFT averaging scheme was done by computing the E-field magnitudes at a given frequency using the DFT averaging scheme and then comparing it against the E-field values obtained from a sinusoidal simulation of same frequency. As explained in section 2.3.2, in the DFT averaging scheme normalized DFT values were computed using the gaussian source input, which were then scaled with the E-field magnitude at the source location. All simulations were done using the ADI-FDTD method with ADI number equal to 16 and the frequency of the source stimulation was chosen as 500 kHz. Once again, the reason for choosing a frequency higher than the average frequency used in HEMI device was to reduce the number of time-steps required for simulation. This method is verified by observing the electric field magnitudes at several points in the model. Fig. 3.15 shows the E-field at four sampling points due to the simulation with both the methods. These sampling points were chosen in the regions close to the source electrode location, close to the PML region and in the heart region in the model. It can be seen from the figure that the E-field magnitudes are identical for both the methods in the regions proximal to the source electrode location. Since these simulations were performed at a frequency higher than the average frequency used in the HEMI device, phase errors are seen in the regions which are at a significant distance from the source location. Therefore, some difference is seen in the E-field values obtained from the two simulations in these regions. However, as the maximum frequency present in the HEMI waveform is less than 200 kHz, effect of these errors is very small when simulated for the HEMI waveform.

As explained in section 2.3.2, using quasi-static assumption induced E-field values can be obtained by running the simulation for a much smaller number of steps than a full cycle of the waveform. Table 3.2 shows the magnitudes of the DFT of E-field at several observation points in the model after different number of time-steps. It
can be seen from the table that the simulation results obtained after 1/8th of the cycle provide a fairly accurate solution for the E-field values. Obtaining the DFT magnitudes without simulating it for a full cycle results into drastic saving in the time required for the simulation.

Since over the frequency range of the simulation (5.8 kHz - 156.6 kHz) the conductivities of most tissues do not change significantly, induced fields are identical in this frequency range. Therefore, for the computation of the induced electric field and current density in the full body model, DFT computation at only one frequency can be considered. Once again, this is done to reduce the simulation time and the memory required to store the DFT values at different frequencies. Fig. 3.16 shows the DFT of E-field magnitudes in the model at 4 different harmonics. It can be seen from the figure that magnitudes of the DFT of E-field are identical for all the frequencies.
Table 3.2: Summary of the DFT magnitude at different sampling points after different number of time steps

<table>
<thead>
<tr>
<th>Observation point</th>
<th>after T/8</th>
<th>after T/4</th>
<th>after T/2</th>
<th>after T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.321</td>
<td>0.3388</td>
<td>0.3456</td>
<td>0.3255</td>
</tr>
<tr>
<td>2</td>
<td>0.0019</td>
<td>0.0024</td>
<td>0.0023</td>
<td>0.0020</td>
</tr>
<tr>
<td>3</td>
<td>0.0052</td>
<td>0.0065</td>
<td>0.0064</td>
<td>0.0061</td>
</tr>
<tr>
<td>4</td>
<td>0.00086</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0010</td>
</tr>
<tr>
<td>5</td>
<td>0.0041</td>
<td>0.0048</td>
<td>0.0048</td>
<td>0.0044</td>
</tr>
</tbody>
</table>

T=Number of Time steps required for the simulation of full cycle

Figure 3.16: DFT magnitudes of E-field in V/m at a)1st b)10th c)20th and d)25th harmonics
Chapter 4

Computational Results

4.1 Computational Model

The anatomical computational model used in this work was developed from the full body model taken from the *Visible Human Project* of the National Library of Medicine [5]. The full body model at 1-mm uniform resolution was of dimensions $586 \times 340 \times 1878$. This model contained approximately 374 million cell and thus required a large amount of memory when solved using the FDTD or the ADI-FDTD method. Therefore, the model was re-discretized in an expanding grid scheme (Fig. 4.1) resulting in a model of dimensions $251 \times 178 \times 687$, a reduction in voxel count by more than 90%. Figure 4.2 illustrates the human body model with the location of the electrodes in the torso region. Fig. 4.3 shows the axial view of man model in uniform and expanding grids. The source was a finite-gap current source, injecting current into the human body via metal electrodes. The vertical gap between the points where electrodes touched the body was about 20 cm, and the electrode cross-sectional area was $1 \text{ mm}^2$. The electrodes penetrated the body by a distance of 10 mm. As illustrated in Fig. 4.1, the volume near the electrode contacts was at a
uniform resolution of 1 mm. Then, using an expansion factor of 1.095, the grid-width was increased to 5 mm above the neck and below the pelvis region. The expansion factor was chosen such that to minimize the dispersion errors caused by the cell to cell grid-width transition while significantly reducing the model size. Similarly in other directions too, using the same expansion factor, the regions near the source was discretized at 1mm resolution and grid width was increased gradually in the regions far from the source location. The dielectric properties of different tissues were obtained from [38].

The ADI-FDTD method with ADI number equal to 16 was used to compute the
Figure 4.2: The human body model with the contact electrodes. The vertical distance between the electrode contacts is 20 cm

Figure 4.3: (a) Left: Slice in the xz plane at uniform 1mm resolution (b) Right: Slice at same location of expanding grid model with grid showing. The blackened portion corresponds to the region of 1mm resolution
induced electric field and current density due to a finite gap current source. DFT averaging scheme has also been employed to obtain results in lesser number of time-steps.

4.2 Induced Electric Field

The electric field values were obtained as ratios with respect to the E-field value at the source, which is the field at the finite-gap. In other words, the source was normalized to 1 V/m. The field at the source was subsequently scaled depending on the low-frequency current scaling factor, which is the current flowing in the electrode corresponding to the normalized E-field at the source. For example, a low-frequency current of 0.99 $\mu$A was found flowing in the electrodes corresponding to an electric field of 1 V/m at the source. Therefore, for a peak current of 3 A flowing through electrode, the maximum instantaneous induced electric fields is obtained by scaling up the ratios by a factor of $\frac{3}{0.99 \times 10^{-6}} = 3.03 \times 10^6$.

The plot showing the electric field values in the model is shown in Fig. 4.4b. The E-field magnitudes shown in the plot are relative to the source position.

Fig. 4.5a illustrates the single-voxel E-field along a straight line from the electrode contact points to the spinal cord. Fig. 4.6 shows the E-field distribution in 3 different slices of the body for a peak current of 3 A flowing into body. The maximum instantaneous value of E-field is found to be 450 kV/m. As expected, this E-field value is at the tip of the electrodes. The E-field value decreases rapidly with distance from the source electrodes. The E-field gets attenuated by around 30 times in the fat layer. Fig. 4.7a illustrates the electric field averaged over slices of thickness 1 mm along the length of the human body. The peak E-field values are near the locations were electrodes touches the body. It can be seen from the figures that the E-field values
Figure 4.4: (a) The sagittal cross-section consisting of the electrodes (b) The relative electric field values expressed as the ratios to that at the source point (V/m) (c) The current densities expressed as the ratio to the source location (A/m²) (d) An enlarged view of just the torso region showing the single-voxel current densities (A/m²) (e) An enlarged region near the top electrode showing the averaged current densities $J_{avg}(y)$ (A/m²)
in the brain and the heart regions are relatively less. Near the heart, the maximum instantaneous E-field value is about 700 V/m, whereas in the brain it is 114 V/m for a peak current of 3A flowing through the electrode.

Apart from the E-field magnitudes, motor nerve excitation also depends upon the gradient of the E-field. E-field gradients were computed by taking the first order derivative of E-field in x, y and z directions. For example, gradient in x-direction is computed as

\[
\frac{\partial E}{\partial x} = \frac{E(i, j, k) - E(i - 1, j, k)}{\Delta x}
\]  

Gradient of the electric field value in the torso region is shown in fig. 4.8. Once again, high values of gradients are seen near the source electrode location.

The low-frequency resistance between the two contact points is obtained as 1.01 kΩ for 1 mm² electrode contact area. Note that the human body resistance is inversely
Figure 4.6: Electric-field values in 3 different slices in the body (V/m)
Figure 4.7: Electric-field and current density averaged over 1 mm thick slices from the top to the bottom of the body.

Figure 4.8: Gradient of E-field in x-component (V/m²)
proportional to the contact area with the metal electrodes. Further simulation were
done with larger cross section area of the electrode. Low frequency resistance for 16
\( \text{mm}^2 \) cross section area of electrodes was approximately 700 \( \Omega \).

4.3 Induced Current Density

The induced current density in the body, computed using \( J(r) = \sigma(r)E(r) \), is shown
in Fig. 4.4c. Fig. 4.5b illustrates the single-voxel current density values along a
straight line from the electrode contact points to the spinal cord. Fig. 4.7b illustrates
the current densities averaged over slices of thickness 1 mm along the length of the
human body. Since both the electrode contacts are in the same side of the body
and within 20 cm from each other, Fig. 4.7 shows the large induced currents in the
immediate vicinity of the contacts.

Dosimetric measures were calculated for some specific tissues and organs using
the following equations [39]:

\[
J_{\text{avg}} = \frac{1}{V} \sum_{i} \left| J_i \right| \Delta v \tag{4.2}
\]

\[
J_{\text{rms}} = \sqrt{\frac{1}{V} \sum_{i} J_i^2 \Delta v} \tag{4.3}
\]

where \( J_{\text{avg}} \) is the average current density, \( J_{\text{rms}} \) is the RMS current density. \( \Delta v \) is the
incremental volume, \( J_i \) is the current density in the incremental volume and \( V \) is the
total volume of the tissue/organ/bone under consideration.

Table 4.1 shows the average, RMS and maximum induced current densities for
some tissues. For muscle, the averaging was done over a 7cm-by-6cm-by-28cm volume
(similar to the red box shown in Fig. 4.4) around the contact points. \( J_{\text{max}} \) is the
single-voxel maximum, while \( J_{\text{avg}} \) and \( J_{\text{rms}} \) are averaged over the entire tissue type.
Table 4.1: Averaged induced current densities in some tissues and organs for a current of 3.0 A flowing through the electrodes.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>Cond. S/m</th>
<th>$J_{avg}$ (Am$^{-2}$)</th>
<th>$J_{rms}$ (Am$^{-2}$)</th>
<th>$J_{max}$ (Am$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cerebellum</td>
<td>0.16</td>
<td>5.60</td>
<td>5.74</td>
<td>18.92</td>
</tr>
<tr>
<td>Grey Matter</td>
<td>0.14</td>
<td>3.61</td>
<td>3.67</td>
<td>16.34</td>
</tr>
<tr>
<td>Heart</td>
<td>0.221</td>
<td>64.60</td>
<td>66.22</td>
<td>168.56</td>
</tr>
<tr>
<td>Bladder</td>
<td>0.22</td>
<td>38.02</td>
<td>44.77</td>
<td>345.50</td>
</tr>
<tr>
<td>Pancreas</td>
<td>0.538</td>
<td>80.95</td>
<td>100.96</td>
<td>912.54</td>
</tr>
<tr>
<td>Intestines</td>
<td>0.599</td>
<td>326.87</td>
<td>365.79</td>
<td>1987.8</td>
</tr>
<tr>
<td>Kidneys</td>
<td>0.175</td>
<td>51.175</td>
<td>52.76</td>
<td>140.29</td>
</tr>
<tr>
<td>Muscle (around the contact points)</td>
<td>0.366</td>
<td>270.21</td>
<td>534.30</td>
<td>138220</td>
</tr>
</tbody>
</table>

It can be seen from the table 4.1 that large values of induced current density are seen in the muscle layer around the contact points. This layer attenuates the E-field values and thus current density values seen in the inner tissues are much less. Average induced current density seen in heart for a peak current of 3.0 A is 66.22 Am$^{-2}$.

Table 4.2 gives estimates of memory and simulation times for the different schemes, given the considered HEMI-device waveform. All simulations were performed on a quad-core Sun workstation with 16 GB memory. For a full time-domain simulation, while the ADI-16 simulation provides roughly a 5-fold speed improvement, it is still not a reasonable option. However, with the expanding grid (Model B) method and quasi-static assumptions (DFT ratios), the problem can be solved on desktop computers.
Table 4.2: Summary of simulation times

<table>
<thead>
<tr>
<th>Method</th>
<th>Model</th>
<th>Memory ~GB</th>
<th>Time Step (ps)</th>
<th>No. of Steps</th>
<th>Estimated Simulation time (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FDTD</td>
<td>A</td>
<td>50</td>
<td>1.92</td>
<td>$\sim 5 \times 10^6$</td>
<td>$\sim 25000$</td>
</tr>
<tr>
<td>ADI-1</td>
<td>A</td>
<td>75</td>
<td>1.92</td>
<td>$\sim 5 \times 10^6$</td>
<td>$\sim 75000$</td>
</tr>
<tr>
<td>ADI-16</td>
<td>A</td>
<td>75</td>
<td>30.1</td>
<td>$\sim 3 \times 10^5$</td>
<td>$\sim 4600$</td>
</tr>
<tr>
<td>ADI-16</td>
<td>A</td>
<td>278</td>
<td>30.1</td>
<td>$\sim 2 \times 10^4$</td>
<td>$\sim 300$</td>
</tr>
<tr>
<td>27-DFT</td>
<td>A</td>
<td>75</td>
<td>30.1</td>
<td>$\sim 2 \times 10^4$</td>
<td>$\sim 12$</td>
</tr>
<tr>
<td>ADI-16</td>
<td>A</td>
<td>75</td>
<td>30.1</td>
<td>$\sim 2 \times 10^4$</td>
<td>$\sim 110$</td>
</tr>
<tr>
<td>1-DFT</td>
<td>B</td>
<td>6.5</td>
<td>30.1</td>
<td>$\sim 2 \times 10^4$</td>
<td>$\sim 12$</td>
</tr>
<tr>
<td>ADI-16</td>
<td>1-DFT</td>
<td>B</td>
<td>6.5</td>
<td>30.1</td>
<td>$\sim 2 \times 10^4$</td>
</tr>
</tbody>
</table>

A: Uniform resolution full body model (586 $\times$ 340 $\times$ 1878)
B: Expanding grid full body model (251 $\times$ 178 $\times$ 687)
ADI-n: ADI-FDTD method with ADI factor ='n'
n-DFT: DFT summations for ‘n’ frequencies
4.4 Analysis of Errors

In this work, we have used the ADI-FDTD method in conjunction with the DFT averaging and expanding grid schemes. As shown in the table 3.1, use of the ADI-FDTD method with ADI number=16 leads to some error in the computation of the E-field values. However, these errors are mostly seen in the regions away from the source. The errors in the regions close to the source electrode position are relatively small (around 15% for ADI number=16). The modified PML formulation includes a loss term ($\kappa$) to suppress the high frequency oscillations, which also leads to relatively larger error in the propagating volume. These errors, however, are found near the PML boundary interface and are not a concern in the problem of interest.

Use of the expanding grid introduces some dispersion errors in the regions with larger grid width. Further, the DFT averaging scheme based on the quasi static assumption may lead to some phase error in the regions away from the source such as legs and feet at higher frequencies. However, since the maximum frequency present in the source signal is less than 200 kHz, effect of these errors will be very small.

In summary, our scheme results in some errors particularly near the PML boundary interface. In these regions, the field magnitudes are very small and therefore, the numerical noise dominates the actual E-field values and this leads to large relative errors. In the regions of more importance the errors are due to high ADI number only. Moreover, averaging the field magnitudes to a great extent reduces the impact of these errors.
Chapter 5

Conclusion and Future Work

In this work, induced electric field and current densities have been computed in the human body due to the low frequency current injection from the contact electrodes of an HEMI device. An otherwise intractable problem is solved using the ADI-FDTD method in conjunction with the expanding grid technique and quasi-static assumptions.

DFT summations were performed at the desired frequencies for periods much smaller than the full time period of the sinusoid, and the electric field values were obtained as ratios of the DFTs with respect to that at the source. By using a gaussian input, the response of the human body can be reconstructed for any pulse shape, as long as the dominant constituent frequencies are small enough for the quasi-static assumptions.

It was obtained that for a current magnitude of 3.0A in the contact electrodes, average current density $J_{avg, rms}$ in the heart was about 66.22 $\text{Am}^{-2}$. The low-frequency human body resistance between the contact points of the electrodes was found to be 1.01 $\text{K}\Omega$. These results serve to provide trends in the induced electric field and current relative to the electrode contact points depending on the provided tissue structure.
From figure 4.7 and figure 4.4, it is clear that the physique of the person in question, the contact points, and the penetration of the electrodes will lead to noticeable variations in the induced field quantities. For example, a deeper penetration of the electrodes, or a thinner fat layer will induce larger currents in muscle tissue.

The expanding grid method with resolutions varying between 1 mm and 5 mm have been used to reduce the model size by about 90%. The ADI-FDTD method with a ADI number 16 reduces the simulation runtime by five times as compared to the traditional explicit FDTD method. Higher ADI numbers and explicit parallelization of the ADI-FDTD code, especially the matrix inversions, should further reduce simulation times.

Future work may include understanding the variation in the induced E-field and current density due to the variation in size, penetration and the location of the electrodes. In this research, a finite gap current source excitation has been considered, future work may also include other types of source excitation. Besides, research may also be done in order to improve the accuracy of these methods used for the computation. Further, frequency dependence of dielectric tissues may also be incorporated.
Bibliography


