ABSTRACT

WAGHELA, RAJMOHAN. Characterization of Aerodynamic Sails Designed For Guiding High-Altitude Balloons. (Under the direction of Dr. Ashok Gopalarathnam.)

Scientific high-altitude ballooning has been an important platform for astrophysicists and earth scientists for observations and measurements. High-altitude balloons have evolved over time to increase payload and mission endurance. A further boost in capabilities can be achieved with the ability to guide high-altitude balloons. An aerodynamic sail is a wing-tail aerodynamic structures that is attached to the balloon with a long tether. The atmospheric wind gradients create a relative wind with respect to the sail which can be used to generate aerodynamic forces to guide a high-altitude balloon in the desired direction. The focus of this study was to characterize the aerodynamic sail. A scale analysis of the acceleration of the balloon showed that Coriolis, tangential, and centripetal accelerations may be ignored for instantaneous analyses. Further, the balloon-sail system was decoupled owing to the large difference in their masses. This simplification allowed for closed-form expressions for maximum sail guidance force and static pitch stability. The maximum sail guidance force depended on the mass of the sail and a ratio of spanwise locations of center of mass and center of lift. The static stability criterion relates the location of center of mass, neutral point, and tether location with an inequality. Next, a fully non-linear equilibrium solution for the sail was developed to determine the key variables for the design of aerodynamic sails. This solution assumed a mass-less tether and a linear variation of wind with altitude. Using this solution, parametric analyses were conducted. The metric of performance was the velocity of the balloon due to the sail perpendicular to the winds surrounding the balloon. The analysis showed that induced drag adversely affects performance by increasing the angle of attack required to achieve a given performance. The location of tether attachment changes the trim configuration by altering the elevator deflection required to trim. The resulting change in drag can have a marginal effect on the performance at high coefficients of lift. The mass of the sail and wing area are directly related. The analysis showed that a sail design is not only sensitive to the mass-to-area ratio but to the extent of the those parameters as well. Both larger mass and larger area are required to improve performance but with only diminishing returns. Finally, the spanwise location of center of mass can have a impact on the sail performance, particularly for light-weight sail designs.
Characterization of Aerodynamic Sails Designed For Guiding High-Altitude Balloons

by

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DEDICATION

To my family for their endless love, support, and encouragement.
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Chapter 1

Introduction

1.1 Background

In 1783, human flight with hot-air balloon was mastered by Joseph-Michel Montgolfier and Jaques-Étienne Montgolfier in France [1, 8]. Ever since, balloons have been used for recreational purposes and by scientists for in-situ measurements of atmospheric temperatures and pressures.

With the progress of balloon technology, increasingly sophisticated measurements and scientific discoveries were facilitated. In 1912, Victor Hess conducted experiments to understand the radiation changes with altitude. These experiments lead to the discovery of cosmic rays for which Hess was later awarded the Nobel prize in 1936 [9].

More recently, balloon borne observations have led to breakthroughs in astrophysics. In 1997, the BOOMERanG (Balloon Observation Of Millimetric Extragalatic Radiation ANd Geophysics) experiment measured the cosmic microwave background radiation of a small part of the sky using a high-altitude balloon flown over Antarctica. This experiment laid the foundation for later experimental work conducted with satellites and eventually led to the 2006 Balzan Prize for Astronomy and Physics [2]. More recently, observations regarding high-energy cosmic ray electrons and a negative search for annihilation signatures of dark matter in the anti-proton channel have been conducted using high-altitude balloon based platforms [9]. High-altitude balloons are suitable for research areas such as cosmic microwave background, hard x-ray astronomy, gamma-ray astrophysics, and cosmic ray investigations. High-altitude balloons can allow sensors to be above 99.5 percent of Earth’s atmosphere, which is essentially as good as being in space[1].

High-altitude balloons can be used for atmospheric or earth science measurements as well. These balloon can provide in-situ and higher fidelity data compared to satellites or other means. In Earth-science measurements, such as earth radiation measurements conducted with satellites, can be validated with in-situ stratospheric measurements. In-situ measurements sidestep errors
and assumptions inherent in remote-sensing algorithms. High-altitude balloons also provide an excellent vehicle for measuring magnetic field of the Earth’s crust. Understanding of the local magnetic field led to the theory of plate tectonics and would provide further understanding of the physics of geomagnetic field changes. Aircraft lack the range to cover vast parts of the globe whereas the magnetic field is often too weak for satellites to resolve. Measuring local magnetic field with ships is usually expensive and is a time-consuming process. In addition, high-altitude balloons can also be used for measuring the atmospheric chemistry in the upper stratosphere and lower troposphere [10].

The majority of the high-altitude balloon research is lead by NASA. In the 2001 Science News Metrics, 8.3 percent of the world’s discoveries and technologies were attributed to NASA and 83 percent of these discoveries were space related. Of the latter, 25 percent were attributed to the Hubble Space Telescope, 19 percent to Chandra mission, 9 percent to Mars Global Surveyor, 9 percent to balloons, 6 percent to Near Earth Asteroid Rendezvous missions, 5 percent to astro-biology, 5 percent to the International Solar Terrestrial Program, and 16 percent to other categories. This metric demonstrates the importance and significance of balloon-based observations on new discoveries and technologies [11].

High-altitude balloons have significant advantage over satellites. The cost of a satellite, including the launch cost, can be easily over $200 million. The estimated cost of a balloon flight is between $500,000 and few million depending on the cost of the science payload [10]. The BOOMERanG experiment is an excellent example of how high-altitude balloons facilitated initial observations, which were then completed by a dedicated satellite. The balloon’s payload can be recovered and used for repeat flights several times. Under the current NASA program, the payloads are recovered, refurbished, and re-flown several times, with successive flights providing higher quality measurements [9].

Often, different students and young researchers are involved in different flights. High-altitude balloons provide excellent opportunities for a large number of young scientists, engineers, and technicians to gain valuable hands-on experience with cutting-edge space research and technology development. The low cost of the balloon missions offers realistic opportunities to involve researchers from across the world.

The evolution of high-altitude balloon has allowed improvements in observational capabilities. Large balloons capable of reaching high altitudes were first introduced in the 1920s. These large rubberized balloons were capable of reaching altitudes as high as 20 km in the atmosphere.
In the 1950, Otto C. Winzen patented the modern-day, naturally-shaped balloons (pictured in Fig. 1.1). These polyethylene balloons had integral load tapes that enabled heavier payloads and larger envelopes. These balloons could reach almost 30 km altitude. Since the 1950s, balloon design has essentially remained unchanged but with gradual improvements the range of possible missions has vastly increased. The conventional zero-pressure polyethylene balloons used by NASA for Long-Duration Balloon (LDB) can fly for 10-20 days over the Antarctic and Arctic regions. These flights can carry suspended payloads of about 2500 kg with scientific instruments in excess of 1000 kg to altitudes approaching 40 km. The LDB balloons have limited endurance, as maintaining altitude requires expending ballast and buoyant gases to counter the changes in temperature during the diurnal cycle. The extended operating time is achieved in the polar regions during the respective summers as the solar radiation is relatively constant [12].

In a parallel effort to extend the balloon mission duration, NASA has been developing super-pressure balloons termed Ultra-Long-Duration balloon (ULDB). Super-pressure balloons are essentially constant-volume systems which require that the strength of the balloon skin be sufficient to withstand the increase in pressure created by solar radiation heating of the gas during the day, and still remain pressurized at night after the gas has cooled. The amount of strain that the balloon envelope undergoes has some effect on the balloon altitude, but ballast is
not required for nominal operation. These balloon are designed to operate for 60-100 days and even operate at more moderate latitudes. Unfortunately, NASA has suffered repeated test flight failures with the largest balloons envisioned under this program. However, relatively smaller balloons have been deployed successfully. In December 2008, NASA successfully launched a ULDB with an approximate volume of 200,000 m$^3$. Figure 1.2 shows a picture of ULDB over Antarctica at an altitude of 33 km.

As ballooning technology improves to facilitate increases in payload and endurance, there is a desire to be able to control the trajectory of the balloon. In 2008, a super-pressure balloon similar to the one pictured in Fig. 1.2 was being test flown over Antarctica. The balloon remained aloft for 54 days and circumnavigated Antarctica 3 times before the flight had to be terminated as the flight path was tending to go off the continent [3].

Both passive and active methods for controlling the trajectory have been suggested. The active methods employ a propeller powered by either fuel in chemical form or by batteries. These options can potentially provide greater control over the balloon trajectory and can even provide limited ability to station-keep. However, due to the large mass footprint of the system and severe limitation on flight endurance, active systems are not desirable. The passive technologies usually employ an aerodynamic surface, similar to a sail, attached with a tether several kilometers long such that it can gently nudge the balloon towards a desirable direction [3, 13, 10]. Figure 1.3 shows a depiction of a balloon with a aerodynamic sail. Such sails are also being proposed for extra-terrestrial exploration of Mars and Venus [14].

The fundamental operating principle behind the sail is that of atmospheric wind gradi-
Winds relative to the earth’s surface at higher altitude are faster compared to those at lower altitude. This trend is corroborated with numerous US National Center for Atmospheric Research (NCAR) reports including NCAR TN-336 [6]. A consequence of this wind-velocity gradient is that there is a relative wind on the aerodynamic sail, which can then be used to produce aerodynamic lift.

Under contract from NASA’s Small Business Innovation Research program, the Global Aerospace Corporation has conducted initial proof-of-concept research studies, modeling, and experimental testing of a scaled model. Researchers have outlined the structural details of balloon-sail systems. This research provides valuable insight into its weight budget, geometry, and operational mechanics. Such a qualitative treatment is useful to understanding the intricate physics behind its motion. However, there is still paucity in quantitative analysis which would represent the realistic dynamics of such tethered systems. The purpose of this work is to expand on the dynamics of a tethered sail to bridge the gap between the physical system and its actual behavior in the atmosphere.
1.2 Research Objectives

This research effort is geared towards improving the understanding of the dynamics of the aerodynamic sail used to guide high-altitude balloons. Concomitantly, improved understanding of what a given sail can achieve in steady-state conditions is desirable for design and future analysis of the balloon-sail system.

Initially, the focus of this research was on identifying the dominant inertial effects i.e., the inertial accelerations that directly affect the system behavior. Such an analysis would lay a foundation for further study by providing a cogent argument for directing the effects that need to be modeled.

The behavior of such a system can be gained through analytical solutions or numerical modeling. In addition, closed-form expressions are sought that characterize the performance and stability of a sail. Then, a high-fidelity full-numerical solution methodology is sought to conduct a parametric study to assess and identify key parameters for the design of more efficient sails.

1.3 Thesis Outline

In Chapter 2, a scale analysis is conducted by systematically evaluating inertial-acceleration terms and identifying the dominant terms. This chapter provides the simplifications that lay the foundation for further analyses. In Chapter 3, the sail characteristics are isolated with the assumption of small perturbations. This chapter provides analytical expressions that provide insight in the guidance ability of the sail and an expression for static stability of the sail. Next, in Chapter 4 a non-linear equilibrium solution for the balloon-sail system is developed. This solution involves sophisticated aerodynamic modeling of the sail with vortex-lattice methods for aerodynamic forces and stability derivatives. A sample solution is shown in this chapter to highlight the intricacies of the system. Chapter 5 uses the equilibrium solution and presents the parametric analyses to identify and showcase the key parameters to design of the sail. Finally, Chapter 6 concludes the findings of this research effort.
Chapter 2

Acceleration Scale Analysis

The purpose of this analysis is to identify the acceleration terms that are significant and the terms that may be ignored in order to simplify further analysis and modeling. As the balloon is the largest mass in the system, acceleration scale analysis is focused on the balloon. The results of this analysis can then be extended to the sail.

In the most general form, the acceleration of the balloon center of mass may be expressed as:

\[ \vec{a}_{cm/o} = a_{CM/o} + 2 \omega \times \vec{V}_{cm/o} + \omega \times \vec{r}_{cm/o} + (\omega \times \omega \times \vec{r}_{cm/o}) \]

(2.1)

In Eq. 2.1, there are four different types of accelerations - relative, Coriolis, tangential, and centripetal. Each acceleration has a unique effect on the system behavior. If this analysis shows that the Coriolis, tangential, and centripetal terms are negligible, i.e.:

\[ \vec{a}_{cm/o} \approx a_{CM/o} \]

(2.2)

then, further analysis can be substantially simplified.

2.1 Simplifying The Expression For Acceleration

A convenient set of frames is established to calculate the expression for acceleration. Fig. 2.1 shows an inertial frame of reference placed at the center of the earth and a second frame at the center of mass of the balloon. The frame located at the center of mass of the balloon rotates such that \( \hat{k}_{B} \) is always perpendicular to the surface of the earth. As a consequence, the position of the balloon with respect to the center of earth can be written with two angles and a distance that is analogous to altitude.
Figure 2.1: Inertial($\Omega$) Frame and Balloon-Body($\mathcal{B}$) Frame

It is preferable to express the acceleration terms in a certain manner such that the magnitude of each of the terms can be most easily assessed. As the $\mathcal{B}$-frame is located at the center of mass of the balloon. The acceleration can be expressed as:

$$\mathcal{B}\mathbf{a}_{cm/o} = \mathcal{B}\mathbf{a}_{cm/o} + 2 \mathcal{B}\mathbf{\omega}_B \times \mathcal{B}\mathbf{V}_{cm/o} + \mathcal{B}\mathbf{\omega}_B \times \mathcal{B}\mathbf{r}_{cm/o} + \mathcal{B}\mathbf{\omega}_B \times (\mathcal{B}\mathbf{\omega}_B \times \mathcal{B}\mathbf{r}_{cm/o})$$  \hspace{1cm} (2.3)

If term 1 is significantly larger compared to term 2 identified in Eq. 2.3, then term 2 can be ignored for further analysis.

### 2.2 Calculating Acceleration Expression

The relation between the inertial frame and the balloon-body frame can be achieved by using Euler-angle transformations. This transformation is completed with two rotations, $\phi$ and $\lambda$. No-
tice from Fig. 2.1 that $\lambda$ is a negative rotation. The Euler angle transformation can be described as:

$$
\overline{O}_{[c]}^{B} = \begin{bmatrix}
\cos(\phi) & 0 & \sin(\phi) \\
0 & 1 & 0 \\
-\sin(\phi) & 0 & \cos(\phi)
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\lambda) & \sin(\lambda) \\
0 & -\sin(\lambda) & \cos(\lambda)
\end{bmatrix}
$$

(2.4)

The transformation in Eq. 2.4 is useful for calculation of angular velocity, which is required for calculating the acceleration terms identified in the previous section. The angular velocity vector components can be defined as:

$$
\{\overline{O}_{\omega}^{B}\} = \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix}
$$

(2.5)

Using Eq. 2.4, the angular velocity components can be easily calculated using the definition of angular velocity. The angular velocity of the balloon is:

$$
\omega_x = \begin{bmatrix}
0 & 0 & 1
\end{bmatrix}^{B} [c] \overline{O}_{[c]}^{B} \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} = -\dot{\lambda}
$$

(2.6)

$$
\omega_y = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}^{B} [c] \overline{O}_{[c]}^{B} \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} = \cos(\lambda)\dot{\phi}
$$

(2.7)

$$
\omega_z = \begin{bmatrix}
0 & 1 & 0
\end{bmatrix}^{B} [c] \overline{O}_{[c]}^{B} \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} = \sin(\lambda)\dot{\phi}
$$

(2.8)

In the angular velocity terms, $\dot{\lambda}$ is analogous to motion along a longitude. Similarly, the $\dot{\phi}$ terms are analogous to motion along the latitudes. $\dot{\phi}$ includes contributions due to the rotation of the earth and due to the velocity relative to the earth. Only $\lambda$ has an effect on the angular velocity; the angular velocity is independent of $\phi$, as may be expected.

The position vector for the balloon can be described as:

$$
\tilde{r}_{CM/o} = x\hat{i} + y\hat{j} + z\hat{k} = R_{BLN}\hat{k}_{B}
$$

(2.9)
Using the position vector in Eq. 2.9, the relative velocity of the balloon can be defined as:

\[
\begin{align*}
\{\vec{V}_{cm/o}\}_B &= \begin{bmatrix} 0 \\ 0 \\ \dot{R}_{BLN} \end{bmatrix} 
\end{align*}
\]

Using the relative velocity vector in Eq. 2.10, term 1 of Eq. 2.3 can be easily calculated with simple differentiation:

\[
\begin{align*}
\{\vec{a}_{cm/o}\}_B &= \begin{bmatrix} 0 \\ 0 \\ \ddot{R}_{BLN} \end{bmatrix} 
\end{align*}
\]

In the \(B\)-frame, term 1 of Eq. 2.3 is simply the acceleration of the balloon altitude.

The angular velocity can now be used to compute term 2 of the acceleration as identified in Eq. 2.3:

\[
\begin{align*}
\left\{ \vec{a}_{cm/o} \right\}_B &= \begin{bmatrix} 0 \\ 0 \\ \ddot{R}_{BLN} \end{bmatrix} 
\end{align*}
\]

Term 2 in Eq. 2.12 contains accelerations due to motion along the horizon i.e., no change in altitude. Each component is scaled with altitude of the balloon.

### 2.3 Approximating Orders of Magnitude

The magnitudes of \(R_{BLN}, \dot{R}_{BLN}, \ddot{R}_{BLN}, \dot{\phi}, \ddot{\phi}, \dot{\lambda}, \) and \(\ddot{\lambda}\) need to be identified for a comparison of terms 1 and 2 of Eq. 2.3. Some parameters can be identified from the balloon-sail parameters, whereas other parameters require additional analysis.

**Magnitude of \(R_{BLN}\)**

\(R_{BLN}\) is simply the altitude of the balloon. As the system has to be designed such that the balloon maintains an altitude of 35,000 km, the magnitude of \(R_{BLN}\) can simply be assumed to be the sum of above-mean-sea-level (MSL) altitude and the radius of the earth, which is approximately 6,406,000 m or \(6.406 \times 10^6\). \(R_{BLN}\) has an order of magnitude of 6 or \(O(6)\).
Magnitude of $\dot{R}_{BLN}$

The most likely cause of the balloon to change altitude is the due to a change in the weight of the gondola due release of a dropsonde or due to change in the weight of the sail that is supported by the balloon. Such a change would occur if the sail is configured to produce a large guiding force. As the velocity of the balloon is damped by drag, the magnitude of the balloon altitude change would of the order of magnitude is $-1$ or $\mathcal{O}(-1)$.

Magnitude of $\ddot{R}_{BLN}$

The acceleration along the altitude would be caused by the same factors listed for $\dot{R}_{BLN}$. In such a situation, a force imbalance would cause the balloon to accelerate to transition to a equilibrium condition. Fig. 2.2 shows the magnitude of the acceleration that would be achieved instantaneously if a net force acted on the balloon. Due to the large mass of the balloon and the added mass effect, the acceleration is relatively small. Typically, the acceleration can be said to have an acceleration between 0.02 and 0.04m/s$^2$. The order of magnitude of acceleration is $-2$ or $\mathcal{O}(-2)$. Additionally, such an acceleration occurs only very briefly.

Figure 2.2: Balloon Acceleration Due Force
Magnitude of $\dot{\phi}$

The magnitude of $\dot{\phi}$ has two components. The rotation of earth and the velocity relative to earth contribute to $\dot{\phi}$. The component due to the velocity relative to earth is dependent on the latitudinal position of the balloon. The latitudinal dependency must be accounted for since the balloon-sail system is expected to operate near the poles.

The two components can be expressed as:

$$\dot{\phi} = \frac{u}{R_{BLN}\cos(\lambda)} + \Omega$$  \hspace{1cm} (2.13)

The velocity $u$ is dependent on the zonal winds. Zonal and meridional winds are meteorological terms for winds parallel to latitudes and longitudes, respectively. The axis system established in Fig.2.1 is such that velocity $u$ is zonal winds. NCAR TN 336 [6] provides monthly averages of zonal winds at various pressure altitudes at latitudes from $-85^\circ$ to $85^\circ$. Using this wind information, the approximate maximum zonal wind velocity can be identified. This technical report shows that the maximum zonal winds can be as high as 50 to 70 m/s, especially in the higher latitudes. As a result, the most winds can contribute is approximately $\approx 1.0 \times 10^{-5}$ rad/s.

The contribution due to the rotation of the earth, $\Omega$, is fixed at $7.27 \times 10^{-5}$ rad/s.

As $\dot{\phi}$ is the sum of the two components, $1.0 \times 10^{-5}$ rad/s + $7.27 \times 10^{-5}$ rad/s is $8.27 \times 10^{-5}$ rad/s. Therefore, the order of magnitude of $\dot{\phi}$ is $-5$ or $O(-5)$.

Magnitude of $\dot{\lambda}$

Unlike $\dot{\phi}$, $\dot{\lambda}$ has only one component. Motion along the longitudes can occur only due to winds relative to the spinning earth. Meridional winds are perturbation to the mean and are more difficult to characterize. However, for the purposes of this analysis, the worst-case scenario can be borrowed from zonal winds data, i.e., wind velocity of 50 to 70 m/s can be used for approximation. The axis system established in Fig. 2.1 is such that velocity $v$ is meridional winds. Meridional winds can be related to $\dot{\lambda}$ as:

$$\dot{\lambda} = \frac{v}{R_{BLN}}$$  \hspace{1cm} (2.14)

The worst case assumption provides that $\dot{\lambda} \approx 1.0 \times 10^{-5}$ rad/s. Therefore, the order of magnitude of $\dot{\lambda}$ is $-5$ or $O(-5)$.
Magnitude of $\ddot{\phi}$

$\dot{\phi}$ has two component but $\ddot{\phi}$ only has a single component. The rotation of earth is fixed and does not contribute to the $\ddot{\phi}$. The balloon can accelerate with respect to the spinning earth due to gusts or other atmospheric disturbances. Figure 2.3 shows the acceleration of the balloon due to relative wind. The acceleration is caused by drag force acting on the balloon. Drag force is not a function of velocity with respect to spinning earth but the velocity of air with respect to the balloon.

![Figure 2.3: Balloon Acceleration Due To Relative Wind](image)

The drag force quadruples if the relative wind speed is doubled. However, relatively large gusts cause an acceleration with an order of magnitude of $-1$ or $O(-1)$. Assuming the gusts and disturbances do not cause a change in altitude, the order of magnitude can be approximated to be $-8$ or $O(-8)$. 


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Magnitude of $\ddot{\lambda}$

The order of magnitude of $\ddot{\lambda}$ can be approximated similarly to $\dot{\phi}$. Based on Fig. 2.3, the magnitude of acceleration can be approximated to an order of magnitude of $-1$ or $O(-1)$. Again, assuming that gusts and disturbances do not cause a change in altitude, the order of magnitude can be approximated to be $-9$ or $O(-9)$.

Table 2.1: Approximate Orders of Magnitude

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$O$</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ddot{R}_{BLN}$</td>
<td>6</td>
<td>m</td>
</tr>
<tr>
<td>$\dot{R}_{BLN}$</td>
<td>$-1$</td>
<td>m/s</td>
</tr>
<tr>
<td>$\ddot{R}_{BLN}$</td>
<td>$-2$</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>$\dot{\phi}$</td>
<td>$-5$</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\dot{\lambda}$</td>
<td>$-5$</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\ddot{\phi}$</td>
<td>$-8$</td>
<td>rad/s$^2$</td>
</tr>
<tr>
<td>$\ddot{\lambda}$</td>
<td>$-9$</td>
<td>rad/s$^2$</td>
</tr>
</tbody>
</table>

In summary, the orders of magnitude are listed in Table 2.1. These approximations can now be applied to the equations developed in Section 2.2.

2.4 Comparing Acceleration Terms

The two acceleration terms identified in Eq. 2.3 can now be compared. Term 1 only contains $\ddot{R}_{BLN}$ as shown in Eq. 2.11. From Table 2.1, $\ddot{R}_{BLN}$ has an approximate order of magnitude of $-2$ or $O(-2)$.

Term 2 is more complex as seen in Eq 2.12. Each component can be evaluated individually. The $\hat{i}_B$ component is $2\dot{R}_{BLN}\cos(\lambda)\dot{\phi} + R_{BLN}\left(\dot{\phi}\cos(\lambda) - 2\dot{\phi}\dot{\lambda}\sin(\lambda)\right)$. The net order of magnitude is $-3$ or $O(-3)$. The $\hat{j}_B$ component is $2\dot{R}_{BLN}\dot{\lambda} + R_{BLN}\left(\ddot{\lambda} + \dot{\phi}^2\cos(\lambda)\sin(\lambda)\right)$. The net order of magnitude is $-3$ or $O(-3)$. The $\hat{k}_B$ component is $-R_{BLN}\left(\dot{\phi}^2\cos(\lambda)^2 + \dot{\lambda}^2\right)$. The net order of magnitude is $-4$ or $O(-4)$.

Two components of term 2 have $O(-3)$ and one component has $O(-4)$. A simple 2-norm would not be representative comparison as the orders of magnitude represent the worst-case conditions. In addition, a norm would not be representative as those conditions cannot concurrently occur. However, since each component of term 2 is at least one order of magnitude smaller, an argument can be constructed that for instantaneous analysis pertaining to equilibrium, stability, and performance can be conducted without the inclusion of term 2. Simulations
for extended periods of time would require that term 2 of Eq. 2.3 be included.

2.5 Summary

A scale analysis of acceleration terms is conducted in this section. Acceleration can comprise of various terms, often some being significantly larger than other, which add to the complexity of further analysis. If the acceleration expression can be simplified, further analysis can also be simplified.

In Chapter 2.1, a set of frames was established such that the general acceleration expression was simplified and broken down into two terms - term 1 and term 2. A relative comparison of the two provides an insight to which terms dominate the acceleration expression. In Chapter 2.2, terms 1 and 2 were explicitly calculated. Once these expressions were derived, the key variables were identified and the order of magnitudes of these variables can be approximated. In Chapter 2.3, the orders of magnitude of the variables identified in the previous section are evaluated for a worst-case scenario using system parameters and wind information available in literature. Finally, in Chapter 2.4, the orders of magnitude of terms 1 and 2 were compared. All components of term 2 are at least one order of magnitude smaller compared to term 1. As a result, term 2 may be ignored for instantaneous analyses dealing with equilibrium, stability, and performance. Analyses for extended period of time would require that the full expression of acceleration be used.
Chapter 3

Sail Characterization

The balloon-sail system has complex interactions between the balloon and the sail. A fully non-linear set of equations describing these interactions would be too complex to provide an intuitive understanding of the behavior of either.

The balloon is two orders of magnitude more massive compared to the sail and, in addition, the acceleration of the balloon is further languished by the added mass effect. In [13], Nock et al. approximate balloon and gondola mass to be almost 4100 kg whereas the sail is expected to be around 90 kg, but for extra-terrestrial observations estimates are as low as 12.5 kg [15]. The added-mass effect for the balloon essentially increases the mass of the balloon by 50% [16] as the air around the balloon has to be displaced to allow acceleration of the balloon. At this point, a reasonable assumption would be that small perturbation to the sail due to atmospheric disturbances do not have any effect on the balloon. This assumption would enable a simplified analysis of the sail.

The coordinate frames used in this chapter are defined in detail in Chapter 4.1. These coordinate frames are adopted from aircraft literature for compatibility with vortex-lattice methods.

This chapter presents two analyses for an isolated sail from first principles. The first concerns the aerodynamic design and performance of the sail in terms of guidance force that a sail could theoretically provide. The second analysis concerns the static stability condition for the sail.

3.1 Sail Performance Analysis

In equilibrium, i.e., when the sum of forces and moments is zero, the forces and moments on the sail can be easily represented on a free-body diagram as shown in Fig. 3.1. The figure shows the sail from a front-view. The relative wind goes into the page. There are three primary forces - tension from the tether, aerodynamic lift force, and gravitational force.
Tension is split into two components along the $\hat{j}_\sigma$ and $\hat{k}_\sigma$. There is a third component of tension which comes out of the page but is not considered here as this is only a two-dimensional analysis. The aerodynamic force can be considered perpendicular to the sail as traditionally lift force is defined perpendicular to the flow rather than perpendicular to the aerodynamic panel producing the force. Lift is always accompanied by drag. However, similar to the third component of tension, drag may be ignored for this two-dimensional analysis.

Now, Newton’s second law of motion can be applied to sum the forces:

$$\sum F_{z\sigma} = -L \cos(\theta_x) + T_{z\sigma} = 0 \quad (3.1)$$

$$\sum F_{y\sigma} = L \sin(\theta_x) + T_{y\sigma} - m_{SL}g = 0 \quad (3.2)$$
Similarly, the sum of moments is:

\[ \sum M_{x\sigma} = y_l L - m_{SL} g y_{cm} \sin(\theta_x) = 0 \]  

(3.3)

Note that the moments in Eq. 3.3 are summed about the tether location. It can be easily shown that the sum of moment about any arbitrary point must equal zero for equilibrium. Simplifying Eqs. 3.1-3.3 and solving for tension force components, the following expressions can be easily derived:

\[ T_{z\sigma} = L \sqrt{1 - \left( \frac{L}{m_{SL} g} \right)^2 \left( \frac{y_l}{y_{cm}} \right)^2} \]  

(3.4)

\[ T_{y\sigma} = m_{SL} g - \left( \frac{L^2}{m_{SL} g y_{cm}} \frac{y_l}{y_{cm}} \right) \]  

(3.5)

The tension force component along the \( \hat{j} \) direction, \( T_{y\sigma} \), is approximately \(^1\) the force that helps guide the balloon in the desired direction. This force has a strong dependency on the ratio of the distances \( y_l \) and \( y_{cm} \). The sail performance improves as the distance \( y_l \) decreases and distance \( y_{cm} \) increases.

A simple maximization of \( T_{z\sigma} \) with respect to lift can be used to calculate the lift force that would translate into the highest possible guiding force, \( T_{z\sigma} \):

\[ L)_{T_{z\sigma MAX}} = \frac{1}{\sqrt{2}} \left( \frac{y_{cm}}{y_l} \right) m_{SL} g \]  

(3.6)

Substituting Eq. 3.6 in Eq. 3.4 provides an expression for the maximum guiding force a sail can generate:

\[ T_{z\sigma MAX} = \frac{1}{2} \left( \frac{y_{cm}}{y_l} \right) m_{SL} g \]  

(3.7)

The maximum guiding force that a sail can generate is dependent on the ratio of the distance to the center of mass from the tether point and the distance to the lift force centroid from the tether point, and the mass of the sail. A well designed sail would minimize the amount of mass required by maximizing the aforementioned ratio.

Finally, some of the assumptions of this analysis require further discussion. Lift force depends on three parameters - coefficient of lift, dynamic pressure, and a reference area:

\[ L = C_L q S \]  

(3.8)

\(^1\)Read discussion regarding the change in relative wind due to drifting of the balloon due to the sail at the end of Chapter 3.1

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The reference area, $S$, is projected area of the sail wing and is fixed. Dynamic pressure, $q$, is a function of relative wind velocity, and ambient air density. The coefficient of lift can be set with user input or an on-board controller by deflecting the elevator section on the tail surface of the sail. The coefficient of lift can only have a narrow band of values. The maximum value of the coefficient of lift can depend on the airfoil section and the Reynolds number of the wing section. Reference [13] provides a typical value of the Reynolds number of the sail at approximately 70,000. At such Reynolds number, the maximum coefficient of lift may be limited to at most 1 or 1.2. As a consequence, the lift required to provide the guidance force as shown by Eq. 3.6 may never be achieved. The sail would undergo aerodynamic stall if the required coefficient of lift exceeds the maximum coefficient of lift.

When the sail is producing significant amount of lift, tension on the tether pulls the balloon. Eventually, the balloon stops accelerating and maintains a constant velocity with respect to the wind. The sail also moves along with the balloon. As a result, there is an additional component of wind that causes an additional rotation of the sail. Figure 4.3 shows the aforementioned rotation.

The net result of the relative wind rotation is that the guiding force, $T_{zo}$, is in effect lower compared to the derivation shown above.

3.2 Sail Stability Analysis

A definition for static stability can be adopted from aircraft literature. Static stability can be defined as the initial tendency of the sail, following a perturbation from a steady-state condition, to develop aerodynamic forces or moments that are in a direction to return the sail to the steady-state condition [17].

The most accessible explanation of the concept of stability can be illustrated with a marble in a bowl as shown in Fig. 3.2. If in each case, the marble is initially at rest or in equilibrium, then direction of the resulting force that develops after the marble is perturbed from equilibrium determines the static stability of the system. In the first instance, if the marble is perturbed then the resulting forces tend to return it to equilibrium or exhibits positive static stability. In the second instance, if the marble is perturbed then no restoring forces arise. The marble has neutral static stability. Lastly, if the marble is perturbed then the resulting force causes the ball to further diverge from the original equilibrium or exhibit negative static stable.

The primary focus of this section is on pitch stability. A perturbation in the pitch of the sail would change the angle of attack and result in different value of the lift force. As lift force is one of the most significant force in the system, pitch stability is essential for sail design.

A static stability analysis requires expressions for applied forces and moments that act on the sail. Fig. 3.3 shows the forces and moments. Note that Figs. 3.1 and 3.3 are not identical.
Fig. 3.2: Examples of Positive (Top), Neutral (Middle), and Negative (Bottom) Static Stability

Fig. 3.3 has tension forces due to the tether oriented to simplify the expressions. The force and moment expression are as shown in Eq. 3.9-3.12:

\[ \sum F_x = T_x - \text{Drag} = 0 \] (3.9)
\[ \sum F_y = T_y - m_{sl}g \cos(\theta_x) = 0 \] (3.10)
\[ \sum F_z = T_z - \text{Lift} + m_{sl}g \sin(\theta_x) = 0 \] (3.11)
\[ \sum M_T = y_L L - m_{sl}g \sin(\theta_x) y_{cm} = 0 \] (3.12)

If Eq. 3.12 is used to solve for \( \sin(\theta_x) \) and substituted in Eq. 3.11, then the tension force component can be written in terms of the lift force and the parameters \( y_L \) and \( y_{cm} \):

\[ T_z = \frac{L}{y_L} \left( 1 - \frac{y_L}{y_{cm}} \right) \] (3.13)

Eq. 3.13 provides an important intermediate result that will be used later in this analysis.

The aerodynamic forces and moments warrant further discussion. The lift vector in Fig. 3.3
Figure 3.3: Equilibrium Sail Free-Body Diagram

is placed at the intersection of the spanwise center of pressure and the chordwise position of the neutral point. Each location has a unique significance. The center of pressure is a location about which the lift force does not produce a moment. Placing the lift vector at the spanwise location of center of pressure ensures that no rolling moment is generated due to the lift. Neutral point is a chordwise location where the pitching moment is not zero but is invariant with angle of attack. In general, the aerodynamic influences can be described with a lift vector and a constant pitching moment.

The moments for this analysis must be balanced about the center of mass. Otherwise, complex correction factors are introduced that will lead to more complex final result. Based on Fig. 3.3, the moment balance can be expressed as:

$$\sum \vec{M}_{cm} = \vec{M}_{aero} + \vec{M}_L + \vec{M}_T$$  \hspace{1cm} (3.14)

Now the moments due to lift and tether tension force need to be evaluated. The moment
arm need to be evaluated in the coordinate frame shown in Fig. 3.3. As mentioned earlier, lift is always accompanied with drag. However, drag is usually much smaller compared to lift. As a simplification, the moment due to drag is ignored. Not only drag is \( \frac{1}{6} \) or \( \frac{1}{3} \) of lift but the moment arm of drag for the pitching moment is extremely small compared to moment arm of lift for the pitching moment. The moment due to lift can be written as:

\[
\vec{M}_L = \vec{r}_{np/cm} \times \vec{L} = \begin{pmatrix} x_{cm} - x_{np} \\ y_{cm} - y_l \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -L \end{pmatrix} = \begin{pmatrix} L(y_l - y_{cm}) \\ L(x_{cm} - x_{np}) \\ 0 \end{pmatrix}
\]  

(3.15)

Similarly, the moment due to tether tension can be written as:

\[
\vec{M}_T = \vec{r}_{T/cm} \times \vec{T} = \begin{pmatrix} x_{cm} - x_T \\ y_{cm} \\ 0 \end{pmatrix} \times \begin{pmatrix} T_x \vec{S} \\ T_y \vec{S} \\ T_z \vec{S} \end{pmatrix} = \begin{pmatrix} T_z \vec{S} y_{cm} \\ T_z \vec{S} (x_T - x_{cm}) \\ T_y \vec{S} (x_{cm} - x_T) - y_{cm} T_z \vec{S} \end{pmatrix}
\]  

(3.16)

Based on Eq. 3.15 and Eq. 3.16, the pitch axis moment expression is:

\[
M_{y\vec{S}} = M_0 + L(x_{cm} - x_{np}) + T_z \vec{S} (x_T - x_{cm})
\]  

(3.17)

\[
M_{y\vec{S}} = M_0 + L(x_{cm} - x_{np}) + L \left(1 - \frac{y_l}{y_{cm}}\right)(x_T - x_{cm})
\]  

(3.18)

Note in Eq. 3.18, \( T_z \vec{S} \) has been substituted with Eq. 3.13. Now this expression can be converted to aerodynamic coefficient form by substituting in Eq. 3.8 for lift force and Eq. 3.19 for moments:

\[
M = C_m q S \vec{c}
\]  

(3.19)

The resulting equation can be simplified to calculate pitching moment:

\[
C_m = C_{m0} + C_L \left(\frac{x_{cm} - x_{np}}{c}\right) + C_L \left(1 - \frac{y_l}{y_{cm}}\right) \frac{(x_T - x_{cm})}{c}
\]  

(3.20)

The pitching moment expression in Eq. 3.20 can have one additional term for the elevator deflection. However, this term can be lumped with \( C_{m0} \) in the interest of simplicity.

The definition for static stability at the beginning of this section translates into the mathematical condition:

\[
C_{m0} < 0
\]  

(3.21)

If this constraint is applied to Eq. 3.20, a stability condition can be derived. First, Eq. 3.20 is partially differentiated with respect to angle of attack, \( \alpha \), and then simplified to have \( x_{cm} \) as
the subject of the expression:

\[
C_{m,\alpha} = \frac{\partial C_m}{\partial \alpha} = \frac{C_{L,\alpha}}{c} \left( x_T - x_{np} + \left( \frac{y_l}{y_{cm}} \right) (x_{cm} - x_T) \right) < 0 \tag{3.22}
\]

\[
x_{cm} < \left( \frac{y_l}{y_{cm}} \right) (x_{np} - x_T) + x_T \tag{3.23}
\]

Eq. 3.23 provides a stability condition reminiscent of static stability condition for aircraft, \(x_{cm} < x_{np}\). The tether introduces more complexity. Eq. 3.23 can be deconstructed in three important cases:

1. \(x_{np} > x_T\): In this case, the neutral point is behind the tether location. As a result, the quantity \(x_T - x_{np}\) is positive. The center of mass can be marginally aft of the tether location without the onset of instability.

2. \(x_{np} = x_T\): In this case, the neutral point is aligned with the tether location. The inequality in Eq. 3.23 simplifies to \(x_{cm} < x_{np}\). The center of mass is required to be forward of the tether location/neutral point. The sail parameter \(y_l/y_{cm}\) will have no bearing on the stability.

3. \(x_{np} < x_T\): In this case, the tether location is behind the neutral point. As a result, the quantity \(x_T - x_{np}\) is negative. The center of mass would be required to be forward of the tether location.

The expression in this analysis were developed in the stability frame. Stability frame simplifies aerodynamic influences but impose the assumption of small angles. The quantities \(x_{cm}\), \(x_{np}\), and \(x_T\) are most easily expressed in the sail body frame. The parameters \(y_l\) and \(y_{cm}\) are unaffected by this coordinate transformation. The difference between the stability frame and the sail body frame is a single negative rotation by the angle of attack. As long as the angle of attack is small, this analysis will provide reasonable predictions.

### 3.3 Summary

A simplified analysis of the sail was conducted in this section. This analysis assumed that small perturbations to the sail due to atmospheric disturbances have negligible effect on the balloon and that sail dynamics can be studied in isolation.

In Chapter 3.1 the ability of the sail to generate a guiding force was investigated. The component of the tension force responsible for guiding the balloon depends on the mass of the sail, and the ratio of two distances - \(y_l\) and \(y_{cm}\). \(y_l\) is the distance from the tethered end of the sail to the spanwise centroid of lift and \(y_{cm}\) is the distance from the tethered end of the sail
to the spanwise center-of-mass location. The ratio $\frac{y_{cm}}{y_l}$ must be maximized to improve the performance of the sail. In Section 3.2, the pitch stability of the sail was investigated. Using a unique arrangement of aerodynamic forces and moments, an expression for the pitching moment about the center of mass of the sail can be derived. Using this expression along with stability condition requiring $C_{m\alpha} < 0$, a condition for the position of center of mass can be derived. The inequality prescribing the location of center of mass is a function of $x_{np}, x_T, y_l$ and $y_{cm}$. 
Chapter 4

Balloon-Sail Equilibrium Solution

In this chapter, the equations required for the equilibrium solution for the balloon-sail system are derived. The derivation uses basic principles of physics and borrows other models such as the standard atmospheric model and vortex-lattice aerodynamic models to fully capture the behavior of the system. Also, linear variation of winds is assumed with altitude.

The equilibrium model is useful for performance analysis of the system. Parameters such as guiding force, drift velocity, etc. are useful for determining the effectiveness of the system and provide important feedback to improve the overall design. In addition, for conducting eigenvalue-based stability analysis, a set of equations of motion linearized about an equilibrium is required.

A major simplification to this analysis is the assumption of mass-less tether. The tether mass is small compared to the balloon, but is of the same order of magnitude as the sail. The tether dynamics are an important aspect of the system dynamics but are too complex for this analysis.

This section first presents various frames used to derive the equilibrium solution and the transformation matrices that provide a relationship between these frames. As the atmosphere pays a vital role in the system, the standard atmospheric model is briefly discussed for its implications. Next, the force and moment equations are developed for the balloon and the sail using free-body diagrams. The equilibrium conditions are discussed next to reflect the intricacies of the system. Using these equilibrium conditions and system constraints, the constraint equations are developed to form a coherent set of equations. Finally, the solution methodology is discussed using a flow chart.
4.1 Frames Of Reference

Newton’s laws of motion are valid in an inertial frame of reference but expressing forces, moments, velocities, and distances is often simpler in non-inertial frames. In some instances additional frames of references can help better understand the phenomenon being studied. As a guiding principle, the frames are oriented such that aerodynamic forces and moments from aircraft conventions can be most easily incorporated.

In this analysis, a total of 5 frames are used, namely:

1. Inertial Frame: $\mathbf{i} = \{\hat{\mathbf{i}}_o, \hat{\mathbf{j}}_o, \hat{\mathbf{k}}_o\}$

   Based on the scale analysis presented in Chapter 2, this analysis can exclude the rotation of the earth. The inertial frame is oriented such that unit vector $\hat{\mathbf{j}}_o$ points upwards (i.e. the direction of altitude), and vector $\hat{\mathbf{i}}_o$ is oriented along the winds at the balloon altitude. The direction of wind at sail altitude is defined relative to the wind at balloon altitude. The vector $\hat{\mathbf{k}}_o$ is defined as per the right-hand rule.

2. Balloon-Body Frame: $\mathbf{B} = \{\hat{\mathbf{i}}_B, \hat{\mathbf{j}}_B, \hat{\mathbf{k}}_B\}$

   The balloon-body frame is located at the center of mass of the balloon-gondola system and this system is treated as a point mass to simplify the analysis. Chapter 4.3 discusses the simplification further. As the balloon-gondola system is treated as a point mass, there is no rotation of the balloon-body frame with respect to the inertial frame.

3. Sail-Wind Frame: $\mathbf{W} = \{\hat{\mathbf{i}}_W, \hat{\mathbf{j}}_W, \hat{\mathbf{k}}_W\}$

   The sail-wind frame is required for two reasons. The winds at balloon altitude and the winds at sail altitude, in general, are not aligned. A consequence of the sail guiding the balloon is a change in relative wind direction for the sail. The sail-wind frame provides a convenient coordinate system for capturing this effect. In addition, sail parameters are defined with respect to the relative wind or sail-wind frame.

   The sail-wind frame is a simple rotation with respect to the balloon-body frame about the vector $\hat{\mathbf{j}}_B$ with the angle $\psi$. The Euler angle transformation is:

   \[
   \overline{B}_{[c]} \overline{W} = \begin{bmatrix}
   \cos(\psi) & 0 & \sin(\psi) \\
   0 & 1 & 0 \\
   -\sin(\psi) & 0 & \cos(\psi)
   \end{bmatrix}
   \]  

(4.1)
4. Sail-Body Frame: \( \mathbf{F} = \{ \hat{i}_F, \hat{j}_F, \hat{k}_F \} \)

The sail-body frame moves and rotates with the sail. The sail-body frame is useful for expressing the physical dimensions of the sail for moment arms, etc. The body-frame of the sail also helps in determining the rotations that the sail undergoes to achieve equilibrium.

The sail-body frame rotates with respect to the sail-wind frame. In general, all three rotations are possible - \( \theta_x, \theta_y \) and \( \theta_z \). The Euler angle transformation using the conventional 3-2-1 rotation sequence is:

\[
W \begin{bmatrix} c \end{bmatrix} F = \begin{bmatrix}
\cos(\theta_z) & -\sin(\theta_z) & 0 \\
\sin(\theta_z) & \cos(\theta_z) & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos(\theta_y) & 0 & \sin(\theta_y) \\
0 & 1 & 0 \\
-\sin(\theta_y) & 0 & \cos(\theta_y)
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\theta_y) & -\sin(\theta_y) \\
0 & \sin(\theta_y) & \cos(\theta_y)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos(\theta_y) \cos(\theta_x) & \sin(\theta_y) \cos(\theta_x) \sin(\theta_z) - \cos(\theta_x) \sin(\theta_z) & \sin(\theta_z) \sin(\theta_x) + \sin(\theta_y) \cos(\theta_x) \cos(\theta_z) \\
\cos(\theta_y) \sin(\theta_x) & \cos(\theta_x) \cos(\theta_z) + \sin(\theta_y) \sin(\theta_x) \sin(\theta_z) & \sin(\theta_x) \cos(\theta_y) \sin(\theta_z) - \cos(\theta_x) \sin(\theta_z) \\
-\sin(\theta_y) & \cos(\theta_y) \sin(\theta_x) & \cos(\theta_y) \cos(\theta_x)
\end{bmatrix}
\]

(4.2)

5. Sail-Stability Frame: \( \mathbf{S} = \{ \hat{i}_S, \hat{j}_S, \hat{k}_S \} \)

The aerodynamic forces and moments can be most easily expressed in the sail-stability frame. Aerodynamic forces are defined with respect to relative wind velocity and the sail-stability axis aligns with the relative wind.

The sail-stability frame is a simple negative rotation about the sail-body axis \( \hat{j}_F \) vector by the angle of attack, \( \alpha \). The Euler angle transformation is:

\[
W \begin{bmatrix} c \end{bmatrix} S = \begin{bmatrix}
\cos(\alpha) & 0 & -\sin(\alpha) \\
0 & 1 & 0 \\
\sin(\alpha) & 0 & \cos(\alpha)
\end{bmatrix}
\]

(4.3)

Using these various frames the problem of equilibrium solution can be simplified into smaller and more manageable tasks.

4.2 Standard Atmospheric Model

The atmosphere plays a vital role in modeling the balloon-sail system. The ambient density along with the displacement of the balloon determines the equilibrium altitude. As for the sail, the guidance force is directly related to the dynamic pressure, which is a function of the
atmospheric density. The standard atmosphere provides a tool to coherently model atmospheric parameters.

The U.S. Standard Atmosphere 1976 model [18] is based on the averaged temperature profiles for the atmosphere. Fig. 4.1 shows the variation of temperature as a function of altitude. As seen in Fig. 4.1, there are isothermal and gradient regions. In isothermal regions, the temperature does not change and in gradient regions, the temperature changes linearly with altitude. For each region, expressions for density can be evaluated using the equation of state and the hydro-static equation under the ideal gas assumption.

The sail is typically placed at 20 km of geometric altitude. Based on Fig. 4.1, the sail can be modeled in the gradient region. When the sail is producing lift, there is a change in position of the sail which places it well within the gradient region. Similarly, the balloon is placed at 35 km of geometric altitude and based on Fig. 4.1 the balloon also belongs to another gradient region.

The first step to determining the density at a desired altitude in gradient region is calculating the temperature. As the temperature varies linearly, the expression for interpolating temperature is quite simple:

\[ T = T_1 + a (h - h_1) \] (4.4)

The temperature lapse rate, \( a \), along with base altitude, \( h_1 \), and temperature, \( T_1 \), for a region are defined as part of the atmospheric model. Using this temperature, the expression for density at an altitude within a gradient region can be easily derived to be:

\[ \frac{\rho}{\rho_1} = \left( \frac{T}{T_1} \right)^{-[\frac{g_0}{3R}+1]} \] (4.5)

Similarly, the base density, \( \rho_1 \), is defined as part of the atmospheric model.

If the altitude change is too large, a standard atmospheric model computer function can be developed to model the density changes experienced by the balloon and the sail. Such a function can easily cover the entire model.

### 4.3 Balloon Force And Moment Equations

The balloon experiences four major forces: gravitational, buoyancy, aerodynamic drag, and tension from the tether:

\[ \sum \vec{F}_{BLN} = \vec{F}_g + \vec{F}_b + \vec{F}_{drag} + \vec{T} \] (4.6)

The gravitational, buoyancy, and aerodynamic drag forces act at approximately the center of the balloon. Tension force due to the tether has a moment arm of approximately one balloon radius. Now, if the balloon is modeled as a sphere, there is no relation between the forces
Figure 4.1: U.S. Standard Atmosphere 1976 [4]
that a balloon experiences and the precise orientation of the balloon. As a result, modeling
the balloon as point mass simplifies the non-linear system of equations required to solve for
equilibrium solution.

Each force acting on the balloon requires some discussion for developing a suitable expres-
sion:

**Gravitational Force**

At an altitude of 35 km, the acceleration due to gravity is notably different compared to 9.81
m/s\(^2\) used for sea-level analysis. Instead of using a fixed value evaluated at 35 km, the general
expression is used to enable flexibility with the range of altitudes:

\[
g_{BLN} = G \frac{M_e}{(R_e + h_{BLN})^2} \tag{4.7}
\]

The gravitational force on the balloon-gondola can be expressed as:

\[
\vec{F}_g = m_{BLN} g_{BLN} = m_{BLN} G \frac{M_e}{(R_e + h_{BLN})^2} \hat{j} \tag{4.8}
\]

**Buoyancy Force**

Archimedes of Syracuse discovered that “any object, wholly or partially immersed in a fluid,
is buoyed up by a force equal to the weight of the fluid displaced by the object” [19]. Using
this principle, the buoyancy force can be approximated using the volume of the balloon, the
ambient density, and the gravitational acceleration:

\[
\vec{F}_b = \rho_{BLN} g_{BLN} V_{BLN} \hat{j} \tag{4.9}
\]

The density, \(\rho_{BLN}\), at the balloon altitude can be calculated from the standard atmosphere
model and the gravitational acceleration, \(g_{BLN}\) can be calculated from Newton’s law of gravi-
tation as discussed earlier. The volume of the balloon can be more challenging to calculate.

In general, diurnal changes in temperature cause a change in the volume of the gasses inside
the balloon. These effects are magnified due to the greenhouse effect inside the balloon. However,
balloons used for long-term high-altitude mission are superpressure balloons. In Ref. [9], Jones
has proposed the use of such balloons developed under the NASA’s Ultra Long Duration Balloon
(ULDB) program. These balloons have an inelastic skin that helps keep the volume of the
balloon relatively constant. The volume of the balloon can be simply estimated with balloon
specifications. The variables of ambient temperature and balloon history are eliminated from
the equation.
Aerodynamic Drag

A balloon is a bluff body and creates symmetric curvature of streamlines; i.e., does not produce any lift. As a result, the balloon can only produce aerodynamic drag when exposed to relative wind. If the balloon did not have a sail connected, the balloon would eventually reach the wind velocity due to the aerodynamic drag; the drag force would ensure that the balloon will travel with the local wind. Since the balloon has a sail attached, there is an external force acting on the balloon and the only mechanism for balancing this external force is drag, i.e., due to the sail, the balloon does not travel with the wind but maintains a steady relative velocity with respect to the wind. The aerodynamic force is directly related to the relative wind velocity vector.

The general expression for drag is:

\[
\vec{F}_d = -C_D(Re) \left( \frac{1}{2} \rho_{\infty} \parallel \vec{V}_{BLN-Rel} \parallel^2 \right) \left( \pi R_{BLN}^2 \right) \frac{\vec{V}_{BLN-Rel}}{\parallel \vec{V}_{BLN-Rel} \parallel} \tag{4.10}
\]

The coefficient of drag is a function of Reynolds number. Fig. 4.2 [5] shows the variation of \( C_D \) with Reynolds number. Fig. 4.2 shows that drag coefficient is approximately constant between Reynolds numbers of \( 10^3 \) to \( 10^5 \). At an altitude of 35 km, a 50-m diameter balloon has the Reynolds number expression \( Re = 5.54 \times 10^4 V_{\infty} \). Small relative wind velocities to significantly large relative wind velocities, \( (1 - 3 m/s) \), lead to a Reynolds number where the coefficient of drag is approximately constant. As a result, a constant drag coefficient of 0.47 can be used for modeling the aerodynamic drag on the balloon. The drag on the gondola is assumed to be negligible.

![Figure 10. Experimental drag coefficients of the sphere as a function of Reynolds number.](image)

Figure 4.2: Change Coefficient of Drag due to Reynolds Number [5]
The simplified expression for drag force is:

\[
\vec{F}_d = -\frac{1}{2} \pi C_D \rho \infty R_{BLN}^2 \vec{V}_{BLN-Rel} \parallel \vec{V}_{BLN-Rel} - \vec{V}_{BLN-Rel} \parallel
\]  \hspace{1cm} (4.11)

Note that drag vector can have all three components. The negative sign for aerodynamic drag force is required based on the definition of relative wind velocity.

**Tension Due To Tether**

The tension acting on the balloon is an unknown force. In general, the tension depends on the sail configuration and the winds around the sail. For the balloon sum of forces, tension can be represented as set of unknowns in the required frame of reference.

Summation of forces on the balloon provide 3 equations towards the larger system of equations required to solve for the equilibrium of the balloon and sail system.

### 4.4 Sail Force And Moment Equations

Unlike the balloon, the orientation of the sail is has a direct impact on the forces being produced and the equilibrium solution. The forces and moments equations for the sail are required to solve for equilibrium. The sail experiences three major forces: gravitational, tension due to tether, and aerodynamic:

\[
\sum \vec{F}_{SL} = \vec{F}_g - \vec{T} + \vec{F}_{aero}
\]  \hspace{1cm} (4.12)

If the moments are summed about the center of mass (assumed to be co-incident with center of gravity), the sail experiences moments due to aerodynamic influences, and due to the tension force due to the tether:

\[
\sum \vec{M}_{cm} = \vec{M}_{aero} + \vec{M}_T
\]

\[
\sum \vec{M}_{cm} = \vec{M}_{aero} + \vec{r}_{T/cm} \times -\vec{T}
\]  \hspace{1cm} (4.13)

Each force and moment acting on the balloon requires some discussion for developing a suitable expression. The most convenient frame of reference to describe all the forces and moments acting on the sail is the sail-body frame, \(F\).
Gravitational Force

The expression for gravitational force acting on the sail is similar to the expression for gravitational force acting on the balloon:

\[
\begin{bmatrix}
\vec{F}_g \\
\end{bmatrix}_\sigma =
\begin{bmatrix}
0 \\
-mSLgSL \\
0
\end{bmatrix}
\]  
(4.14)

The gravitational force must be expressed in the sail-body frame. The correct expression can be achieved by using the rotation matrices defined in Chapter 4.1:

\[
\begin{bmatrix}
\vec{F}_g \\
\end{bmatrix}_F = \tilde{F} [c] W [c] \tilde{w} \begin{bmatrix}
\vec{F}_g \\
\end{bmatrix}_\sigma
= \frac{GM_e m_{SL}}{(R_e + h_{SL})^2} \begin{bmatrix}
-\cos(\theta_y) \sin(\theta_z) \\
-\cos(\theta_x) \cos(\theta_z) - \sin(\theta_x) \sin(\theta_z) \sin(\theta_z) \\
\cos(\theta_z) \sin(\theta_x) - \sin(\theta_y) \cos(\theta_x) \sin(\theta_z)
\end{bmatrix}
\]  
(4.15)

The expression in Eq. 4.15, as may be expected, is independent of the direction angle \(\psi\).

Tension Due To Tether

The tension due to the tether connects the balloon and the sail. For the balloon, this force can be expressed as:

\[
\begin{bmatrix}
\vec{T} \\
\end{bmatrix}_\sigma = \begin{bmatrix}
T_x \sigma \\
T_y \sigma \\
T_z \sigma
\end{bmatrix}
\]  
(4.16)

For expressing the these components in sail force and moment balance, a coordinate transformation is required. In the interest of brevity, the following dummy variables can be assumed:

\[
\begin{bmatrix}
\vec{T} \\
\end{bmatrix}_F = \tilde{F} [c] W [c] \tilde{w} \begin{bmatrix}
T_x \sigma \\
T_y \sigma \\
T_z \sigma
\end{bmatrix} = \begin{bmatrix}
T_x \sigma \\
T_y \sigma \\
T_z \sigma
\end{bmatrix}
\]  
(4.17)

Lastly, if \(\vec{T}\) acts on the balloon, then by Newton’s third law, the force acting on the sail must be \(-\vec{T}\). Note that this force is negative is Eq. 4.12.
Aerodynamic Forces

The aerodynamic forces are best expressed in the stability frame, $\mathcal{S}$. In the stability frame, the aerodynamic forces are simply lift, $L$, Drag, $D$, and sideforce, $A_y$ [17]:

$$\begin{bmatrix} F_{aero} \end{bmatrix} = \begin{bmatrix} -D \\ A_y \\ -L \end{bmatrix} = \begin{bmatrix} -D \cos(\alpha) + L \sin(\alpha) \\ A_y \\ -D \sin(\alpha) - L \cos(\alpha) \end{bmatrix} \quad (4.18)$$

Note that lift and drag are negative by convention. A positive angle of attack causes a negative lift force. Each component of the aerodynamic force can be further broken down into contributing elements in a Taylor series expansion form:

**Lift, $L$**

has three coefficient contributions that are scaled by the dynamic pressure and a reference area:

$$L = qSLS \left( CL_0 + CL_\alpha \alpha + CL_\delta \delta \right) \quad (4.19)$$

A contribution due to aerodynamic slide-slip would normally be included but as the sail does not have a an aerodynamic surface perpendicular to the main wing section, this contribution is negligible.

**Drag, $D$**

has contributions due to skin-friction, lift, and the elevator used to trim the sail in a desired configuration:

$$D = qSLS \left( CD_0 + kCL^2 + CD_\delta \delta \right) \quad (4.20)$$

where the coefficient of lift expression can simply be borrowed from Eq. 4.19:

$$CL = CL_0 + CL_\alpha \alpha + CL_\delta \delta \quad (4.21)$$

**Side Force, $A_y$**

The aerodynamic side force also has three contributions:

$$A_y = qSLS \left( Cy_0 + Cy_\beta \beta + Cy_\delta \delta \right) \quad (4.22)$$

The side force would be much smaller compared to lift or drag.

The aerodynamic coefficients for forces, in general, are functions of Reynolds number, Mach number, and geometry. See Chapter 4.8 for details on how these coefficients are estimated.
Aerodynamic Moments

The aerodynamic moments are best expressed in the stability frame, $\mathcal{S}$. In the stability frame, the aerodynamic moments are simply rolling, $l$, pitching, $m$, and yawing moments, $n$:

$$\left\{ M_{aero} \right\}_F = F \begin{bmatrix} l \\ m \\ n \end{bmatrix} = \begin{bmatrix} l \cos(\alpha) - n \sin(\alpha) \\ m \\ l \sin(\alpha) + n \cos(\alpha) \end{bmatrix}$$  \hspace{1cm} (4.23)

Aerodynamic moments are sensitive to the location of evaluation. In this instance, all aerodynamic moments are evaluated at the center of mass to avoid correction factors. Each component of the aerodynamic moment can be further broken down into contributing elements in a Taylor series expansion form:

**Rolling Moment, $l$**

The rolling moment has three contributions that are scaled by dynamic pressure, reference area, and reference span:

$$l = qSLb \left( Cl_0 + Cl_\beta \beta + Cl_\delta \delta \right)$$  \hspace{1cm} (4.24)

In the case of the sail, all three component change significantly with angle of attack. As Chapter 3.1 showed that the distance between the center of lift and center of mass must be large for an efficient design, the rolling moment is significant in magnitude.

**Pitching Moment, $m$**

The pitching moment is crucial for configuring the sail to produce the desired amount of lift. The pitching moment has three significant contributions:

$$m = qSL \bar{c} \left( C_{m_0} + C_{m_\alpha} \alpha + C_{m_\delta} \delta \right)$$  \hspace{1cm} (4.25)

$\delta$ is adjusted to achieve equilibrium in the desired angle of attack condition.

**Yawing Moment, $n$**

The yawing moment has three contributions:

$$n = qSLb \left( C_{n_0} + C_{n_\beta} \beta + C_{n_\delta} \delta \right)$$  \hspace{1cm} (4.26)

The coefficient contributing to yawing moment change significantly with angle of attack.

The aerodynamic coefficients for moments, in general, are functions of Reynolds numbers, Mach number, and geometry. See Chapter 4.8 for details on how these coefficients are estimated.
Moment Due to Tether Attachment

The moments due to tether attachment can be evaluated using cross products. Eq. 4.17 provides an expression for tension in the sail-body frame. The moment arm can be simplified without loss of generality as:

\[
\begin{bmatrix}
\vec{F}_{T/cm} \\
\vec{y} \\
0
\end{bmatrix}_T = \begin{bmatrix}
x_{TF} \\
y_{TF} \\
0
\end{bmatrix}
\]  

(4.27)

As a result, the expression for moment due to tether attachment is:

\[
\begin{bmatrix}
\vec{M}_T \\
\end{bmatrix}_F = \begin{bmatrix}
\vec{r}_{cm} \times -\vec{T}
\end{bmatrix}_F = -\begin{bmatrix}
T_{yF}y_{TF} \\
-T_{zF}x_{TF} \\
T_{yF}x_{TF} - T_{zF}y_{TF}
\end{bmatrix}
\]  

(4.28)

In general, a tether can exert a moment on the sail at the attachment location. This moment is generated by twisting of the tether about its axis [20]. However, since the tether in this problem is extremely long, this moment can be assumed to be negligible.

Summation of forces and moments on the sail provide six equations towards the larger system of equations required to solve for the equilibrium of the balloon and sail system. Eq. 4.17 provides an additional three equations required express the tension force in the correct frame of reference.

4.5 Equilibrium Conditions

In the conventional sense, equilibrium requires that sum of forces and moments is zero. In the strictest sense of this requirement, a particle or body being analyzed can be in equilibrium while in motion with constant translational and angular velocities. These conditions have to be modified for the balloon-sail system to achieve an equilibrium solution. Additional conditions are imposed on both the balloon and the sail.

In equilibrium, the balloon translates with the wind. However, the velocity along the direction of altitude must be identically zero for equilibrium. If the balloon is changing altitude, the forces of gravity and buoyancy create a net force on the balloon. However, the balloon can sustain a relative wind in the direction of altitude - the equilibrium altitude would be different if the relative wind was absent. Mathematically, this condition is:

\[
V_{BLNzO} = 0 \quad \text{or} \quad V_{BWzO} = -V_{BLN-RelyO}
\]  

(4.29)
If the inertial velocity of the balloon in the altitude direction is zero, then the balloon relative
wind velocity and balloon wind velocity in the altitude direction are equal in magnitude.

The next condition pertains to the sail. When the sail is rotating, with respect to the relative
wind around the sail, aerodynamic forces and moments are present by the virtue of rotation.
These forces and moments can either dampen or further energize the sail motion. In either case,
the sail is accelerating and therefore, not in equilibrium. As a result, equilibrium can only be
achieved if the angular velocity of the sail is identically zero. Mathematically, this condition is:

$$ \vec{\omega}^F = \vec{0} $$

(4.30)

The assumption of Eq. 4.30 are built-in in the aerodynamic force and moment expressions
developed in Chapter 4.4.

These conditions developed in this section modify existing equations. No new equations are
introduced towards the larger system of equations required to solve for the equilibrium of the
balloon and sail system.

4.6 Constraint Equations

Constraint equations are relations that describe the system but are not based on Newton’s laws
of motion. These equations help form a consistent set of equations, i.e. a system with equal
number of known and unknown variables. This section presents all the constraint equations
required to solve the balloon-sail equilibrium problem.

Balloon Wind Conditions

The velocity of the balloon, the relative wind velocity of the balloon, and the wind velocity
around the balloon are related as follows:

$$ \vec{V}^{\text{BLN/\omega}} = \vec{V}^{\text{BLN-Rel}} + \vec{V}^{\text{BW}} $$

(4.31)

The constraint equation, shown in Eq. 4.31, is true in general but requires modification to
satisfy the equilibrium condition discussed in Section 4.5. The modified set of equations is:

$$ \begin{cases}
V_{BLNx\sigma} \\
0 \\
V_{BLNz\sigma}
\end{cases}
= \begin{cases}
V_{BLN-Relx\sigma} \\
V_{BLN-Rely\sigma}
\end{cases} + \begin{cases}
V_{BWx\sigma} \\
V_{BWy\sigma} \\
V_{BWz\sigma}
\end{cases} $$

(4.32)
Eq. 4.32 provides three equations towards the larger system of equations required to solve for the equilibrium of the balloon and sail system.

**Sail Wind Conditions**

In equilibrium, the sail has a non-zero constant velocity with respect to inertial space. However, with respect to the balloon, the sail does not move. This assumption is valid as long as the tether is taut. The assumption of taut tether is reasonable and valid for low to medium lift force. At high-lift-force conditions, the weight supported by the tether decreases. This condition provides a mathematical condition for the velocity of the sail:

\[
\vec{V}_{BLN/o} = \vec{V}_{SL/o}
\]  

(4.33)

Using Eq. 4.33, a similar relationship to Eq. 4.31 can be constructed for the sail:

\[
\vec{V}_{BLN/o} = \vec{V}_{SL-Rel} + \vec{V}_{SW} = \vec{V}_{SL/o}
\]  

(4.34)

Eq. 4.34 is true in general, but has to be modified to reflect the equilibrium condition discussed in Chapter 4.5. The modified set of equations is:

\[
\begin{bmatrix}
V_{BLN\hat{x}} \\
0 \\
V_{BLN\hat{z}}
\end{bmatrix} = \begin{bmatrix}
V_{SL-Rel\hat{x}} \\
V_{SL-Rel\hat{y}} \\
V_{SL-Rel\hat{z}}
\end{bmatrix} + \begin{bmatrix}
V_{SW\hat{x}} \\
V_{SW\hat{y}} \\
V_{SW\hat{z}}
\end{bmatrix}
\]  

(4.35)

Eq. 4.35 provides three equations towards the larger system of equations required to solve for the equilibrium of the balloon and sail system.

**Sail Wind Correction**

The direction of relative wind with respect to \( \hat{\sigma} \) on the \( xz \)-plane determines the orientation and provides a reference for measuring the angle of attack. This angle can be calculated using dot products, but for numerical stability and ease of determining the right quadrant, a tangent function is more suitable. Fig. 4.3 provides an illustration of the angle being measured. The mathematical expression is simply:

\[
\psi = -\tan^{-1}\left(\frac{V_{SW-Rel\hat{z}}}{V_{SW-Rel\hat{x}}}\right)
\]  

(4.36)

The negative sign is a consequence of how relative wind velocity is defined.

Eq. 4.36 provides an equation towards the larger system of equations required to solve for the equilibrium of the balloon and sail system.
Figure 4.3: Sail Relative Wind Angle
Tether Constraints

The vector connecting the balloon and sail is constrained by several factors. Let this vector have the following components:

\[
\{\vec{r}_{SL/BLN}\}_n = \begin{cases} 
R_{x\sigma} \\
R_{y\sigma} \\
R_{z\sigma}
\end{cases}
\]  

(4.37)

The length of the tether is constrained:

\[
l_T = \sqrt{R_{x\sigma}^2 + R_{y\sigma}^2 + R_{z\sigma}^2}
\]

(4.38)

However, the length of the tether is not fixed. The tension through the causes the tether to extend. A linear model can be easily adopted to account of stretching of the tether:

\[
\sqrt{T_{x\sigma}^2 + T_{y\sigma}^2 + T_{z\sigma}^2} = EA (l_T - l_0)
\]

(4.39)

As tether has negligible resistance to bending, the tension must also be along the direction of the tether. As a result, three additional constraints are put on the balloon and sail system:

\[
\frac{R_{x\sigma}}{\sqrt{R_{x\sigma}^2 + R_{y\sigma}^2 + R_{z\sigma}^2}} = \frac{T_{x\sigma}}{\sqrt{T_{x\sigma}^2 + T_{y\sigma}^2 + T_{z\sigma}^2}}
\]

(4.40)

\[
\frac{R_{y\sigma}}{\sqrt{R_{x\sigma}^2 + R_{y\sigma}^2 + R_{z\sigma}^2}} = \frac{T_{y\sigma}}{\sqrt{T_{x\sigma}^2 + T_{y\sigma}^2 + T_{z\sigma}^2}}
\]

(4.41)

\[
\frac{R_{z\sigma}}{\sqrt{R_{x\sigma}^2 + R_{y\sigma}^2 + R_{z\sigma}^2}} = \frac{T_{z\sigma}}{\sqrt{T_{x\sigma}^2 + T_{y\sigma}^2 + T_{z\sigma}^2}}
\]

(4.42)

The tether provides five equations towards the larger system of equations required to solve for the equilibrium of balloon and sail system.

Sail Altitude

The altitude of the sail is a useful parameter for several calculations such as density and wind conditions calculations around the sail. Vectorially, the position of the sail is simply:

\[
\vec{r}_{SL/0} = \vec{r}_{BLN/0} + \vec{r}_{SL/BLN}
\]

(4.43)

In the inertial frame, the \(j_\sigma\) components are essentially altitudes. As a result, the expression for the sail altitude is:
The sail altitude relation provides one equation towards the larger system of equations required to solve for the equilibrium of balloon and sail system.

Wind Variation With Altitude

Atmospheric variation in wind is one of the key basis for the balloon-sail system to operate. A wind variation model is required for the equilibrium solution model as a the sail can have significant change in altitude due to a change in coefficient of lift and relative wind velocity. Fig. 4.4 shows wind speeds as a function of altitude. Notice how the wind velocities between altitudes of 35 km and 20 km have an approximately linear variation. As a result, a linear wind variation model is adopted to account for the change in altitude of sail.

Mathematically, the wind expression can be written as:

\[
\begin{align*}
V_{SW,x} &= V_{35k,x} + \left( \frac{V_{35k,x} - V_{20k,x}}{15,000} \right) (35,000 - h_{SL}) \quad (4.45) \\
V_{SW,z} &= V_{35k,z} + \left( \frac{V_{35k,z} - V_{20k,z}}{15,000} \right) (35,000 - h_{SL}) \quad (4.46)
\end{align*}
\]

As a convention, \(V_{35k,z}\) is always set as zero to simplify the process of interpreting the effect of the sail on the balloon. The wind along the altitude, \(\hat{\jmath}_z\) component, is assumed to be zero.

\[
h_{SL} = h_{BLN} + R_y\theta
\]  

(4.44)
for the sail. If the such winds are required to be incorporated in the solution, a more general
method of determining the angle of attack and aerodynamic side-slip would be required.

If the balloon altitude changes significantly, equations similar to sail wind conditions can
be adopted for the balloon as well.

The sail wind variation relations provide two equations towards the larger system of equa-
tions required to solve for the equilibrium of balloon and sail system.

**Aerodynamic Constraints**

The angle of attack is synonymous with the rotation $\theta_y$ or mathematically:

$$\theta_y = \alpha$$ (4.47)

This equality holds as long as the wind component along the altitude is zero around the sail.
The balloon can have a velocity component along the altitude.

Similarly, the aerodynamic side-slip angle is synonymous with the rotation $\theta_z$ or mathemat-
ically:

$$\theta_z = \beta$$ (4.48)

This equality holds as long as the wind component along the altitude is zero.

The aerodynamic constraint relations provide two equations towards the larger system of
equations required to solve for the equilibrium of balloon and sail system.

Using the force and moment equations along with the constraint equations, a coherent
system of equations can be formed to obtain the equilibrium solution of the balloon and sail
system.

### 4.7 Unknowns Variables, System Parameters and Inputs

The purpose of this section is to clearly identify the known and unknown variables along with
providing a short description of the variables. First the unknown variables are listed. The
unknown variables are:
Table 4.1: Table Of Unknown Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Consistent Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{BLN}$</td>
<td>$m$</td>
<td>Balloon above mean sea-level altitude</td>
</tr>
<tr>
<td>$h_{SL}$</td>
<td>$m$</td>
<td>Sail above mean sea-level altitude</td>
</tr>
<tr>
<td>$\rho_{BLN}$</td>
<td>$kg/m^3$</td>
<td>Density at balloon altitude</td>
</tr>
<tr>
<td>$\rho_{SL}$</td>
<td>$kg/m^3$</td>
<td>Density at sail altitude</td>
</tr>
<tr>
<td>$T_x\bar{\sigma}$</td>
<td>$N$</td>
<td>Tether tension component along $\hat{i}_{\bar{\sigma}}$ in $\bar{\sigma}$–frame</td>
</tr>
<tr>
<td>$T_y\bar{\sigma}$</td>
<td>$N$</td>
<td>Tether tension component along $\hat{j}_{\bar{\sigma}}$ in $\bar{\sigma}$–frame</td>
</tr>
<tr>
<td>$T_z\bar{\sigma}$</td>
<td>$N$</td>
<td>Tether tension component along $\hat{k}_{\bar{\sigma}}$ in $\bar{\sigma}$–frame</td>
</tr>
<tr>
<td>$T_xF$</td>
<td>$N$</td>
<td>Tether tension component along $\hat{i}_F$ in $F$–frame</td>
</tr>
<tr>
<td>$T_yF$</td>
<td>$N$</td>
<td>Tether tension component along $\hat{j}_F$ in $F$–frame</td>
</tr>
<tr>
<td>$T_zF$</td>
<td>$N$</td>
<td>Tether tension component along $\hat{k}_F$ in $F$–frame</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>$rad$</td>
<td>Euler rotation angle from $\mathcal{W}$–frame to $F$–frame</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>$rad$</td>
<td>Euler rotation angle from $\mathcal{W}$–frame to $F$–frame</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>$rad$</td>
<td>Euler rotation angle from $\mathcal{W}$–frame to $F$–frame</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>$deg$</td>
<td>Elevator deflection angle required to trim the sail at desired angle of attack. (AVL stability derivatives dependent on elevator deflection are expressed in $deg$)</td>
</tr>
<tr>
<td>$V_{BLN-Rel\bar{\sigma}}$</td>
<td>$m/s$</td>
<td>Relative wind velocity component of the balloon along $\hat{i}_{\bar{\sigma}}$ in $\bar{\sigma}$–frame</td>
</tr>
<tr>
<td>$V_{BLN-Rely\bar{\sigma}}$</td>
<td>$m/s$</td>
<td>Relative wind velocity component of the balloon along $\hat{j}_{\bar{\sigma}}$ in $\bar{\sigma}$–frame</td>
</tr>
<tr>
<td>$V_{BLN-Relz\bar{\sigma}}$</td>
<td>$m/s$</td>
<td>Relative wind velocity component of the balloon along $\hat{k}_{\bar{\sigma}}$ in $\bar{\sigma}$–frame</td>
</tr>
</tbody>
</table>

*Continued on next page*
<table>
<thead>
<tr>
<th>Variable</th>
<th>Consistent Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{BLN_x}$</td>
<td>m/s</td>
<td>Velocity component of the balloon along $\hat{i}_\sigma$ in $\sigma$–frame</td>
</tr>
<tr>
<td>$V_{BLN_z}$</td>
<td>m/s</td>
<td>Velocity component of the balloon along $\hat{k}_\sigma$ in $\sigma$–frame</td>
</tr>
<tr>
<td>$\psi$</td>
<td>rad</td>
<td>Sail relative wind direction w.r.t $\hat{i}_{xz}$ on $xz$–plane</td>
</tr>
<tr>
<td>$\beta$</td>
<td>rad</td>
<td>Aerodynamic side-slip angle</td>
</tr>
<tr>
<td>$l_T$</td>
<td>m</td>
<td>Length of tether</td>
</tr>
<tr>
<td>$R_{x}$</td>
<td>m</td>
<td>Position of sail w.r.t balloon along $\hat{i}_\sigma$ in $\sigma$–frame</td>
</tr>
<tr>
<td>$R_{y}$</td>
<td>m</td>
<td>Position of sail w.r.t balloon along $\hat{j}_\sigma$ in $\sigma$–frame</td>
</tr>
<tr>
<td>$R_{z}$</td>
<td>m</td>
<td>Position of sail w.r.t balloon along $\hat{k}_\sigma$ in $\sigma$–frame</td>
</tr>
<tr>
<td>$V_{SW_x}$</td>
<td>m/s</td>
<td>Wind velocity component around sail along the $\hat{i}_\sigma$ in $\sigma$–frame</td>
</tr>
<tr>
<td>$V_{SW_z}$</td>
<td>m/s</td>
<td>Wind velocity component around sail along the $\hat{j}_\sigma$ in $\sigma$–frame</td>
</tr>
</tbody>
</table>

In summation, there are 27 unknown variables. The net number of equations can be reduced if some simple substitutions are used. However, substitutions have been avoided in order to limit the length of the equations.

Now, the system parameters can be listed. System parameters are the variables that define the extent of the system. The system parameters are:
Table 4.2: Table Of System Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Consistent Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{BLN}$</td>
<td>$m$</td>
<td>Radius of the fully inflated balloon</td>
</tr>
<tr>
<td>$m_{BLN}$</td>
<td>$kg$</td>
<td>Mass of the balloon</td>
</tr>
<tr>
<td>$C_D$</td>
<td>--</td>
<td>Coefficient of drag for the balloon</td>
</tr>
<tr>
<td>$m_{SL}$</td>
<td>$kg$</td>
<td>Mass of the sail</td>
</tr>
<tr>
<td>$C_{D0}$</td>
<td>--</td>
<td>Coefficient of skin-friction drag of the sail. More accurately $C_{D0} = C_{D0}(Re)$</td>
</tr>
<tr>
<td>$S$</td>
<td>$m^2$</td>
<td>Reference area of the sail</td>
</tr>
<tr>
<td>$b$</td>
<td>$m^2$</td>
<td>Reference span of the sail</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>$m$</td>
<td>Mean aerodynamic chord of the sail</td>
</tr>
<tr>
<td>$C_{L0}$</td>
<td>--</td>
<td>Sail lift coefficient at zero angle of attack. Generated with AVL</td>
</tr>
<tr>
<td>$C_{La}$</td>
<td>--</td>
<td>$\partial C_L / \partial \alpha$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{La_e}$</td>
<td>1/deg</td>
<td>$\partial C_L / \partial \delta_e$, Generated with AVL</td>
</tr>
<tr>
<td>$k$</td>
<td>--</td>
<td>$\frac{1}{\pi AR\epsilon}$, $\epsilon$ may be generated with AVL</td>
</tr>
<tr>
<td>$C_{D0_e}$</td>
<td>1/deg</td>
<td>$\partial C_D / \partial \delta_e$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{y0}$</td>
<td>--</td>
<td>Sail aerodynamic side-force coefficient at zero angle of attack. Generated with AVL</td>
</tr>
<tr>
<td>$C_{y3}$</td>
<td>--</td>
<td>$\partial C_y / \partial \beta$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{y3_e}$</td>
<td>1/deg</td>
<td>$\partial C_y / \partial \delta_e$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{l0}$</td>
<td>--</td>
<td>Sail aerodynamic rolling moment coefficient at zero angle of attack. Generated with AVL</td>
</tr>
</tbody>
</table>

Continued on next page
Table 4.2 – Continued from previous page

<table>
<thead>
<tr>
<th>Variable</th>
<th>Consistent Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{l_\beta}$</td>
<td>--</td>
<td>$\frac{\partial C_l}{\partial \beta}$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{l_{\delta e}}$</td>
<td>1/deg</td>
<td>$\frac{\partial C_l}{\partial \delta_e}$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{m_0}$</td>
<td>--</td>
<td>Sail aerodynamic pitching moment coefficient at zero angle of attack. Generated with AVL</td>
</tr>
<tr>
<td>$C_{m_\alpha}$</td>
<td>--</td>
<td>$\frac{\partial C_m}{\partial \alpha}$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{m_{\delta e}}$</td>
<td>1/deg</td>
<td>$\frac{\partial C_m}{\partial \delta_e}$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{n_0}$</td>
<td>--</td>
<td>Sail aerodynamic yawing moment coefficient at zero angle of attack. Generated with AVL</td>
</tr>
<tr>
<td>$C_{n_\beta}$</td>
<td>--</td>
<td>$\frac{\partial C_n}{\partial \beta}$, Generated with AVL</td>
</tr>
<tr>
<td>$C_{n_{\delta e}}$</td>
<td>1/deg</td>
<td>$\frac{\partial C_n}{\partial \delta_e}$, Generated with AVL</td>
</tr>
<tr>
<td>$x_{TF}$</td>
<td>m</td>
<td>Distance from center of mass of sail to tether location along $\vec{i}_F$ in the $\mathcal{F}$–frame.</td>
</tr>
<tr>
<td>$y_{TF}$</td>
<td>m</td>
<td>Distance from center of mass of sail to tether location along $\vec{j}_F$ in the $\mathcal{F}$–frame.</td>
</tr>
<tr>
<td>$l_0$</td>
<td>m</td>
<td>Unstretched tether length</td>
</tr>
<tr>
<td>$EA$</td>
<td>N/m</td>
<td>Tether stiffness parameter</td>
</tr>
</tbody>
</table>

Once all the system parameters are calculated, only the system inputs are required to populate the system of equations. Input variables define the conditions encountered by the balloon and the sail. In the case of aerodynamic derivatives, the input variables are required for calculation. The input variables are:
An alternative approach would be to set up the system of equations such that coefficient of lift is an input rather than angle of attack. However, with such an approach, the solution process would be iterative and computationally much more expensive.

### 4.8 Solution Methodology

In this section, the solution methodology is discussed. A flow chart of the solution methodology is shown in Fig. 4.5.
The solution process starts with user defined set of non-AVL generated system parameters. These include all parameters defined in Table 4.2 except the ones generated by AVL, but additionally includes the geometric definition of the sail. Using the geometry of the sail, a geometry input file can be constructed in a format that is readily readable using AVL. The geometry file also defines the type of paneling used for the aerodynamic model. In addition to the geometry file, a run file is also required for AVL. The run file contains commands to run AVL without human interference and also to set the critical parameter - angle of attack. A new set of stability derivatives is required at each angle of attack. If the side-slip angle is large, the
side-slip angle must also be accounted for in the run file. Finally, AVL can be used to solve for the stability derivatives of the sail. The stability derivatives are directly a function of sail geometry and angle of attack.

Once the sail stability derivatives are available, the system of non-linear equations can be solved. In this instance, the system of non-linear equations is solved using FSOLVE in MATLAB. FSOLVE requires an initial guess to start the solution and a good starting point can save significant amount of CPU time. Typically, each case takes between 30-45 seconds to solve.

Lastly, the solution must be post-processed to derive the desired parameters. Some of the parameters that may be desirable for understanding of the system may not be directly solved for. The solution of the system of non-linear system of equations can be used for such calculations.

4.9 Limitations of Equilibrium Solution

In this section the limitation of the equilibrium solution are identified. The limitations are:

- AVL is an inviscid aerodynamic analysis tool. As a result, the non-linearity of aerodynamic forces and moments is not captured by AVL. This simplification affects the modeling of aerodynamic stall. AVL cannot predict when aerodynamic stall will occur or how the sail will behave at post-stall angles of attack.

- The set of equations developed for the equilibrium solution assumes that the velocity of the wind around the sail in the direction of altitude is zero:

\[ V_{SWy} = 0 \quad (4.49) \]

This is a reasonable assumption as up drafts or down drafts are not constants but perturbations. In exchange, the mechanism of setting the angle of attack is greatly simplified. However, the solution can be easily modified to generate a valid solution for updrafts and down drafts around the balloon.

4.10 Summary

In this Chapter, a method of calculating the equilibrium solution is discussed. The solution methods starts with discussing the frames of reference in Chapter 4.1 used for expressing different vectors describing the system parameters. As the atmosphere plays a vital role in modeling the system behavior, Chapter 4.2 discusses the U.S. Standard Atmosphere of 1976. Next, the force and moment equations are developed for the balloon and the sail in Chapters 4.3 and 4.4,
respectively. Due to the nature of system, additional equilibrium conditions are required which are defined in Chapter 4.5. Then, to form a consistent set of equations, constraint equations are developed in Chapter 4.6. As the system of equations is large and involves several variables, chapter 4.7 tabulates the unknown and known variables. Chapter 4.8 brings all elements together to provide a method to develop all the variables and solve the equilibrium problem. Finally, Chapter 4.9 discusses the limitations of the equilibrium solution.
Chapter 5

Balloon-Sail System Parametric Analyses

In this chapter, the solution defined in Chapter 4 is used to identify the key variables to better understand the balloon-sail system.

The analysis in this chapter focuses on the effects of aerodynamic drag, tether location, sail mass and reference area, and spanwise center of mass on sail performance. In order to conduct a systematic parametric study, a standard sail is first defined.

5.1 Reference System Parameters

A reference sail can be defined using parameters found in literature [13] and using rules-of-thumb where applicable. Then, parametric analysis can be conducted by systematically varying a variable of interest. In order to make the analysis as flexible as possible, some parameters are defined in terms of other variables.

Figure 5.1 shows the geometric relationships of the sail. The tail section is located 3.5 mean aerodynamic chords behind the leading edge of the wing and has an aspect ratio of 60 percent that of the main wing panel. The area of the elevator panel is 20 percent of that of the main wing. The chordwise center of mass is placed at 50 percent mean aerodynamic chord from the leading edge. Sweep, though easy to model, is not investigated. The spanwise location of center of mass is at the spanwise centroid of the wing and elevator panels. Lastly, the tether location, $r_{T/Ax}$, is located at a distance of 60 percent mean aerodynamic chord from point A. The complete list of balloon-sail system parameters is shown in Table 5.1.
$A_R e = 0.6A_R w$

$\frac{r_T}{A x_f} = 0.6 \bar{c}$

$y_{CM}$ located at spanwise centroid of wing and elevator panels

$x_{CM} = 0.5 \bar{c}$

$S_e = 0.2S_{ref}$

Figure 5.1: Sail Geometric Definition
Table 5.1: Balloon-Sail System Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{BLN}$</td>
<td>m</td>
<td>50</td>
</tr>
<tr>
<td>$m_{BLN}$</td>
<td>kg</td>
<td>4180</td>
</tr>
<tr>
<td>$C_{D_{BLN}}$</td>
<td>---</td>
<td>0.47</td>
</tr>
<tr>
<td>$C_{D_0}$</td>
<td>---</td>
<td>0.0250</td>
</tr>
<tr>
<td>$m_{SL}$</td>
<td>kg</td>
<td>50</td>
</tr>
<tr>
<td>$S$</td>
<td>$m^2$</td>
<td>5</td>
</tr>
<tr>
<td>$AR$</td>
<td>---</td>
<td>5</td>
</tr>
<tr>
<td>$e$</td>
<td>---</td>
<td>0.5</td>
</tr>
<tr>
<td>$l_0$</td>
<td>m</td>
<td>15,000</td>
</tr>
<tr>
<td>$EA$</td>
<td>$N/m$</td>
<td>25</td>
</tr>
<tr>
<td>$V_{35kx\hat{\sigma}}$</td>
<td>m/s</td>
<td>50</td>
</tr>
<tr>
<td>$V_{35k\hat{\sigma}}$</td>
<td>m/s</td>
<td>0</td>
</tr>
<tr>
<td>$V_{35kz\hat{\sigma}}$</td>
<td>m/s</td>
<td>0</td>
</tr>
<tr>
<td>$V_{20kx\hat{\sigma}}$</td>
<td>m/s</td>
<td>30</td>
</tr>
<tr>
<td>$V_{20k\hat{\sigma}}$</td>
<td>m/s</td>
<td>0</td>
</tr>
<tr>
<td>$C_{L_{max}}$</td>
<td>---</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Unless otherwise indicated, parameters shown in Table 5.1, are used for the parametric analysis. The parameter $C_{L_{max}}$ is listed in Table 5.1 as some plots require values evaluated at maximum coefficient of lift.

5.2 Metric Of Performance

To assess the ability of the sail and to have a relative comparison, a consistent metric is required. For the balloon-sail system, one metric ideally suited is the relative wind velocity of the balloon along the $\hat{k}_\sigma$ vector. This velocity component represents the velocity induced by the balloon in the direction perpendicular to the winds at the balloon altitude. Throughout the remainder of this section, this metric will be used to assess how different parameters affect this value. This parameter is referred to as the relative wind velocity because it is also the velocity of
the balloon with respect to the winds around the balloon. A higher magnitude is indicative of better performance.

The magnitude of the relative wind velocity along \( \hat{k} \) is typically 2 m/s. Though this velocity is small, over time it is sufficient to change the balloon path significantly. For example, at 2 m/s, the balloon can be moved in the desired direction by 173 km over a period of 24 hours.

A velocity component associated aforementioned metric is the relative wind velocity of the balloon along \( \hat{i} \). A smaller magnitude of this velocity component is desirable.

### 5.3 Aerodynamic Drag

In actively propelled systems, magnitude of aerodynamic drag has an enormous impact on design and functionality. The balloon-sail system is a passive system, i.e., there are no active components propelling the balloon or the sail. Aerodynamic drag can still have some effect on the performance of the sail as discussed by Nock et. al [13]. However, their discussion does not provide any insight towards how much or what performance improvements can be achieved by reducing drag.

Aerodynamic drag is sensitive to the geometry of the wing and elevator. Kroo [21] and others have shown that lift-dependent drag is a function of span loading; traditionally span loading is defined as the ratio of weight to the span of the wing but in this instance the definition can be modified to the ratio of lift produced to the span of the main wing panel. In order to generate a significant amount of guiding force, the sail would be required to operate at high coefficients of lift - a condition where drag is primarily composed of induced drag. With these considerations, a parametric sweep of aspect ratio can provide a family of sails that have similar properties except that drag will differ within the family.

Mathematically, aspect ratio can be directly related to induced drag using the basic equation from Prandtl’s lifting-line theory. Equation 4.20 shows the relationship for total drag. The parameter \( k \) in this equation can be expanded as:

\[
k = \frac{1}{\pi e \ AR}
\]  

A lower aspect ratio would increase drag whereas a higher aspect ratio will decrease drag.

One of the most significant impacts of aspect-ratio change can be seen in the lift-curve slope. Figure 5.2 shows the change is lift-curve slopes. Next, Fig. 5.3 shows the lift and drag relationship for different aspect ratios. Even though the coefficient of drag is identical at low coefficients of lift, the benefit of higher aspect ratios is clear at high coefficients of lift. Note that coefficients shown in Figs. 5.2 and 5.3 are for trimmed conditions.

Looking at coefficients is advantageous for comparison of two different configurations. How-
Figure 5.2: Effect of Aspect Ratio on Lift Curve Slope

Figure 5.3: Effect of Aspect Ratio on Drag Coefficient
ever, in the case of the aerodynamic sails, coefficients do not provide the complete picture. The geometry of the sail has a direct impact on the equilibrium solution. For instance, the altitude at which the sail achieves equilibrium can be different due to change in drag. Figure 5.4 shows the equilibrium altitude of the sail with different aspect ratios. For a given coefficient of lift, the equilibrium altitude is different or for a given equilibrium altitude, the coefficient of lift is different. This phenomenon is further exacerbated due to the asymmetry of the sail. A change in elevator deflection changes the drag produced by the sail. The elevator deflection required to trim changes significantly with aspect ratio as shown in Figure 5.5.

The altitude change comes with a change in density and a change in relative wind velocity, which in turn, are the parameters that determine the dynamic pressure. Figure 5.6 shows the changes in dynamic pressure. Dynamic pressure scales the aerodynamic coefficients for conversion to forces and moments.

Looking at the forces that the sail exerts on the balloon can be more representative. Figures 5.7 and 5.8 show the forces on the plane parallel to the horizon \(^1\). The forces in Fig. 5.7 are primarily due to drag and are along the direction of the wind around the balloon. The forces in Fig. 5.8 are primarily due to lift and are perpendicular to the wind around the balloon.

As Fig. 5.7 shows that despite the reduction in dynamic pressure, the total pull on the

\(^1\)Note that Figs. 5.7 and 5.8 show the absolute values. By convention, a positive coefficient of lift will produce negative values for \(T_{x\pi}\), and both positive and negative values of coefficient of lift will produce a negative value of \(T_{z\pi}\). The absolute values help make the discussion easier as the magnitudes drive the characteristics and not the direction of these forces, per se.
Figure 5.5: Effect of Aspect Ratio on Elevator Deflection

Figure 5.6: Effect of Aspect Ratio on Dynamic Pressure
Figure 5.7: Effect of Aspect Ratio on $|T_{x\pi}|$

Figure 5.8: Effect of Aspect Ratio on $|T_{z\pi}|$
balloon by the lower aspect ratio sail is greater. However, as Fig. 5.8 shows, the guiding force lies on essentially the same line. The sail with lower aspect ratio requires a higher coefficient of lift to achieve the same magnitude of guiding force.

These two forces are balanced with a drag force. A small relative wind velocity is established for the balloon which scales the magnitude of the tension force acting on the balloon. The velocities created by the tension acting on the balloon due to the tether is shown \(^2\) in Figs. 5.9 and 5.10. The tension component along the \(\hat{i}_o\) has small variations due to aspect ratio. In principle, an increase in this relative wind velocity component will lead to an increase in the amount of force required along the \(\hat{k}_o\) component. However, the magnitude of drag relative to other forces is small and does not have a significant impact on the overall performance of the system. Figure 5.10 shows that relative wind velocity along \(\hat{k}_o\) is essentially unaltered due to change in aspect ratio.

As aspect ratio lowers the angle of attack required to achieve a certain performance goal. This may be expected as lift curve slope increases with aspect ratio.

\(^2\) Note that Figs. 5.9 and 5.10 show the absolute values. By convention, a positive coefficient of lift will produce negative values for \(V_{BLN-Relz}\), and both positive and negative values of coefficient of lift will produce a negative value of \(V_{BLN-Relz}\). The absolute values help make the discussion easier as the magnitudes drive the characteristics and not the direction of these forces, per se.
Figure 5.10: Balloon Relative Wind Velocity Along $\hat{k}_\pi$

5.4 Tether Location

The location where the tether is attached to the sail has implications on the stability of the sail and on the overall performance of the sail as well. The tether effectively creates a pitching moment which can, depending on the attachment location, alter the trim characteristics.

As the tether creates a pitching moment, the most significant impact of tether location is on elevator deflection required to trim. Figure 5.11 show how the elevator deflections compare for different tether locations.

The range of tether locations investigated in this analyses were selected such that the equilibrium aerodynamic side-slip was within a reasonable range of values. As side-slip increases beyond 10-15 degs, the validity of the linear-aerodynamic model becomes increasingly questionable. A larger range of tether locations can be implemented if a dual-attachment setup is adopted. This setup will have the additional benefit of reducing bending moment caused by the tether.

As the tether location moves aft, the moment that needs to be countered by the elevator increases, which in turn requires an increase in the elevator deflection required to trim. In one regard, an increased deflection may be of use. With a forward tether location, the trim deflection is small for the complete range of angles of attack. An actual implementation would require an actuator with very high resolution. If an aft tether location is used, the amount of deflection is significantly higher and a lower resolution actuator can be used.
A larger elevator deflection increases drag. There is a drag associated with elevator deflection - commonly referred to as trim drag in aircraft literature. This drag increase causes an increase in relative wind velocity of the balloon along the \( \hat{i} \) vector as shown in Fig 5.12.

Unlike with the change in aspect ratio, a change in tether location does have a measurable impact on the desirable relative wind velocity of the balloon. Figure 5.13 shows that there is an incremental advantage, at least at high coefficients of lift, to a forward tether location compared to an aft tether location. This effect is amplified for lighter sails.

The change in sail performance can be better understood by looking at the parameter \( \frac{y_{cm}}{y_l} \). A higher value of this parameter indicates a more efficient sail. Figure 5.14 shows how this parameter varies with coefficient of lift and tether location. As the elevator deflection required to trim the sail is negative, the location of spanwise lift moves closer to the tether attachment. This results in a relatively poorer performing sail.

### 5.5 Mass and Wing Area

The intrinsic ability of a sail to provide a force guiding the balloon in a desired direction is dependent on the mass. Equation 3.7 provides a mathematical relation to calculate the maximum force a sail can potentially generate to guide a balloon. In reality, the ability of the sail to achieve this maximum is subject to the dynamic pressure, maximum coefficient of lift, and the wing area. The average dynamic pressure can be adjusted to some extent by increasing
Figure 5.12: Balloon Relative Wind Velocity Along $\hat{i}_T$ For Different Tether Locations

Figure 5.13: Balloon Relative Wind Velocity Along $\hat{k}_T$ For Different Tether Locations
Improvements in light-weight composite technologies have enabled construction of large-lightweight aerodynamic panels. For instance, the Helios prototype (shown in Fig. 5.15) had a span of 75.3 m but only weighed about 928 kg. This mass included propulsion systems, instrumentation, ballast, and solar panels [22]. As construction of such large wings is possible, the design space available for the sail is large.

To understand how mass and wing area impact the system through the range of angles of attack, the carefully selected cases can be evaluated and plotted. These cases are:

1. Mass = 30 kg, S = 2 m
2. Mass = 130 kg, S = 2 m
3. Mass = 30 kg, S = 10 m
4. Mass = 130 kg, S = 10 m

Other parameters of the sail are as described in Table 5.1 and Fig. 5.1.

For a given sail geometry, when the weight changes, the amount of rotation that a sail must undergo to achieve equilibrium also increases. As a result, the equilibrium altitude increases and

Figure 5.14: Ratio Of Spanwise Location Of Center Of Mass With Spanwise Center of Lift For Different Tether Locations
available dynamic pressure decreases. Figure 5.16 shows the equilibrium altitude of each sail for a range of coefficients of lift. At zero coefficient of lift, the equilibrium altitude is different based on the weight of the sail. A heavier sail causes the balloon altitude to decrease and in addition, the heavier sail causes more elongation of the tether to further lower sail altitude. The largest altitude change is experienced by the configuration with largest wing area but the smallest weight. This configuration has the ability to generate the most aerodynamic force relative to its weight. However, due to this altitude change, the change in dynamic pressure is also significant. Figure 5.17 shows the dynamic pressure variation for the four identified cases.

The rotation angle plays a vital role in the altitude change. Figure 5.18 shows the rotation about the $\hat{F}$ axis. The lift is increasingly causing an altitude change rather than guiding the balloon. As rotation angle increases, a smaller portion of the weight is supported by the balloon. Concomitantly, the equilibrium position of the sail with respect to the balloon also moves laterally.

The effects of altitude change, relative wind velocity change, and rotation of the sail are presented in Figures 5.19 and 5.20. These figures show the component of the tension force guiding the balloon and the desirable velocity achieved as a result of the sail, respectively. For small wing area configurations, varying the mass significantly does not have any bearing on the system. If the lower mass system is constructed with a large wing area, the performance of the system.

Note that Figs. 5.19 and 5.20 show the absolute values. By convention, a positive coefficient of lift will produce negative values for $T_\sigma$ and $V_{BLN-Rel}\sigma$. The absolute values help make the discussion easier as the magnitudes drive the characteristics and not the direction of these forces and velocities, per se.
Figure 5.16: Equilibrium Altitude of Corner Sail Configurations

Figure 5.17: Equilibrium Dynamic Pressure of Corner Sail Configurations
Figure 5.18: Equilibrium Rotation Angle, $\theta_x$, of Corner Sail Configurations

Figure 5.19: Guidance Force $T_{zo}$ of Corner Sail Configurations
system is improved significantly. However, the highest performance is achieved by a sail that has both a large mass and a large wing area.

An attempt can be made to correlate these results with a parameter such as mass per wing area. Using such a parameter on the abscissa instead shows that sail performance is not only connected to the ratio but to the extent of sail, i.e. the mass, wing area, and even dynamic pressure. However, parametric study of mass and wing area variables can provide some insight in the characteristics of the sail.

Figures 5.21 and 5.22 show the component of tension guiding the balloon and the resultant component of balloon velocity. In these figures, mass of the sail varies from 10 kg to 150 kg and wing area from 2 m$^2$ to 10 m$^2$. There is a direct correlation between improving the system performance and increasing both the mass and wing area of the system. Increasing the wing area of low mass system shows increasingly marginal improvements. However, increasing the wing area of a more massive system provides significant improvement.

This analysis shows that the maximum tension force component guiding the balloon, $T_{z\pi}$, occurs at coefficient of lift lower than $C_{L_{\text{max}}}$ only if the mass of the sail is really low. Otherwise, $C_{L_{\text{max}}}$ and dynamic pressure limit the performance of the sail.

\[ |V_{\text{BLN} - \text{Relz}}| = \frac{0}{0} \quad |V_{\text{BLN} - \text{Relz}}| = \frac{0}{0} \quad |V_{\text{BLN} - \text{Relz}}| = \frac{0}{0} \quad |V_{\text{BLN} - \text{Relz}}| = \frac{0}{0} \]

\[ m_{\text{SL}} = 30 \text{ kg}, S = 2 \text{ m}^2 \quad m_{\text{SL}} = 130 \text{ kg}, S = 2 \text{ m}^2 \quad m_{\text{SL}} = 30 \text{ kg}, S = 10 \text{ m}^2 \quad m_{\text{SL}} = 130 \text{ kg}, S = 10 \text{ m}^2 \]

Figure 5.20: Balloon Relative Velocity Component $V_{\text{BLN} - \text{Relz}}$ For Corner Sail Configurations

\[ |V_{\text{BLN} - \text{Relz}}| = \frac{0}{0} \quad |V_{\text{BLN} - \text{Relz}}| = \frac{0}{0} \quad |V_{\text{BLN} - \text{Relz}}| = \frac{0}{0} \quad |V_{\text{BLN} - \text{Relz}}| = \frac{0}{0} \]

Note that Figs. 5.21 and 5.22 show the absolute values. By convention, a positive coefficient of lift will produce negative values for $T_{z\pi}$ and $V_{\text{BLN} - \text{Relz}}$. The absolute values help make the discussion easier as the magnitudes drive the characteristics and not the direction of these forces and velocities, per se.
Figure 5.21: Guidance Force Component $T_{zo}$ For Various Mass and Wing Area When Wind Velocity Difference Between 35,000 and 20,000 km Altitude is 20 m/s and $C_{L_{max}}$ is 1.0

Figure 5.22: Balloon Relative Wind Velocity Component $V_{BLN-Relz}$ For Various Mass and Wing Area When Wind Velocity Difference Between 35,000 and 20,000 km Altitude is 20 m/s and $C_{L_{max}}$ is 1.0
5.6 Spanwise Center Of Mass Position

Equation 3.7 showed that one of the variables that can significantly affect the performance of the sail is the spanwise position of the center of mass. A change in spanwise location of center of mass can be achieved by re-positioning the instruments or by using ballast; the sail performance is augmented by increasing the mass provided the balloon can support the addition weight.

The center of mass in previous analyses was placed at the spanwise centroid. In this analysis, the spanwise location of center of mass is varied. To preserve generality, the position of the spanwise center of mass has been expressed in terms span, b. The distance of the center of mass is measured from the shaft connecting the wing and elevator section as shown in Fig. 5.1.

The spanwise center of mass essentially changes rotation $\theta_x$ about $\hat{i}_F$ axis. As result, the mass of the sail plays a vital role on how much performance gain can be achieved. Figures 5.23 and 5.24 show the rotation angle for sails weighing 10 kg and 50 kg, respectively. The lighter sail undergoes significantly more rotation compared to the heavier sail.

As a lighter sail would rotate more compared to a heavier sail, the benefits of moving the center of mass for a lighter sail would be more significant. Figures 5.25 and 5.26 show a relative component of the relative wind velocity of the balloon along the $\hat{k}_F$ vector. The lighter sail sees a small increase in the relative velocity of the balloon whereas the heavier sail shows no tangible improvements.
Figure 5.24: Rotation Angle $\theta_x$ For A 50-kg Sail With Different Spanwise Positions Of Center Of Mass

Figure 5.25: Balloon Relative Wind Velocity Along $\hat{k}_\tau$ Due To A 10-kg Sail For Different Spanwise Center Of Mass Locations
5.7 Summary

In Chapter 5, the equilibrium solution developed in the previous chapter is used to identify the effects of design parameters. First, a reference sail was defined in Chapter 5.1. The following analysis modified this sail to parametrically study the effect of each variable. A performance metric was identified for comparison in Chapter 5.2 to structure the discussion of sail behavior. Chapter 5.3 investigates the effect of induced drag yielded that higher induced drag increases the coefficient of lift required to achieve same performance. In Chapter 5.4, the tether location was varied and the results show that a forward tether location is preferred due to lower trim drag. Next, Chapter 5.5 investigates the effects of sail mass and wing area on performance. The analysis showed that an both mass and wing area were necessary to improve overall performance of the system. Finally, the effects of spanwise center of masses investigated in Chapter 5.6 and the results showed that performance of the sail can be improved by increasing the distance $y_{cm}$ but the performance gains diminished with increasing mass.
Chapter 6

Conclusions And Recommendations

Scientific ballooning has been a valuable resource to the study of the earth’s atmosphere and to astrophysics. As the balloon technologies have improved, scientists have been able to perform increasingly complex experiments and observations. With the advent of super-pressure balloons under NASA’s ultra long duration balloon (ULDB) program, missions that provide operational capability above 99.5 percent of the earth’s atmosphere for more than 50 days are possible. To further enhance the capability of high-altitude balloons and to extend mission duration, technologies are sought that could help guide the balloon in the desired direction. This thesis investigates the use of an aerodynamic sail to guide a high-altitude balloon.

The aerodynamic sail is a passive device that is extended from the balloon using a long tether. The sail produces aerodynamic forces that can be used to guide the balloon. A balloon by itself would travel with the wind and have zero relative wind. Using a long tether between the balloon and the sail allows the sail to have relative wind due atmospheric wind gradients. Winds relative to the earth’s surface at higher altitude are faster than those at lower altitudes. The aim of this thesis is to characterize the behavior of aerodynamic sails using dynamic models.

In order to develop a simple model, a scale analysis was conducted to quantify magnitude of the relative, Coriolis, tangential, and centripetal accelerations. These accelerations, in combination, lead to a complex expression for the acceleration of the balloon. However, if the Coriolis, tangential, and centripetal accelerations are small compared to the relative acceleration, these terms may be ignored. The scale analysis showed that relative acceleration was at least one order of magnitude larger compared to other accelerations even in the worst-case scenario. As a result, simplified analysis, which only accounts for the relative acceleration, can be used for instantaneous analyses conducted for equilibrium, performance, and stability.

As the sail was the focus of this study, some insight was gained by decoupling the balloon-sail problem, i.e., by assuming that minor perturbations of the sail have no effect on the balloon. This assumption is justified as the mass of the balloon is two orders of magnitude larger compared
to that of the sail. Under this assumption, the performance limit of the sail can be identified in a closed-form expression. This expression shows that the maximum guidance force that a sail can impart on a balloon is directly proportional to the mass of the sail and the parameter $y_{cm}/y_l$. This result is contingent on the sail having sufficient dynamic pressure and/or wing area to generate the required lift without experiencing aerodynamic stall.

Further analysis was conducted regarding the static stability of the sail using the same decoupled system approach. Static stability was shown to be achieved if the location of center of mass, neutral point, and tether attachment are located such that $C_{m\alpha} < 0$ is true. An inequality was derived that relates the aforementioned parameters. This inequality can be used as a guide towards locating the tether such that the sail is aerodynamically stable. However, this condition does not eliminate the possibilities of overall system instability.

Next, an equilibrium solution method was developed to investigate the characteristics of the balloon-sail system. Additionally, this solution was used to assess the performance of sail using parametric analysis. The solution is a fully non-linear one and requires a non-linear solver. The major assumptions for this solution are that of a mass-less tether and linear variation of wind velocity with altitude.

Using this model, parametric analysis was conducted to gain insight into the characteristics of the sail. Mass of the sail almost always played a vital role in the performance. The performance was characterized by the velocity of the balloon generated by the sail in the direction perpendicular to the winds around the balloon.

The first case study was based on induced drag. Induced drag scales with aspect ratio of the sail wing panel which would be an important design parameter. The analysis showed that induced drag of the sail negligible effects on performance. A lower aspect ratio sail requires a higher angle of attack to achieve a given performance benchmark. The parametric study of varying the tether location showed a change in trim characteristics, i.e., the elevator deflection required to trim the sail changed with location of tether attachment. Due to the asymmetry of the sail and increased drag from trim deflections, a forward location of the tether near the neutral point is preferred. The effect of mass and wing area are coupled for the sail. A parametric study shows that a sail with a large wing area or a large mass performs relatively poorly. An ideal sail has a large mass and a large wing area. However, the performance does not correlate with a parameter such as mass per unit area as the actual extent of the sail play a role in determining the performance bounds. Finally, the role of spanwise location of center of mass was investigated. The location of spanwise center of mass provides marginal improvements but with decreasing significance as the mass of the sail increases.

Future work on the balloon-sail system can focus on further developing the equilibrium solution to include the effects of a massive tether and even quasi-steady solutions wherein the sail periodically oscillates with respect to the balloon. The non-linear model of the system can then
be linearized about an equilibrium solution for eigenvalue stability analysis. Eigenvalue stability analysis could show the existence of aerodynamic gradient instability - a phenomenon linked to density changes in the atmosphere that has been documented by Onada and Naoyuki [23], and by Beletsky and Levin [20] for low orbiting tethered satellite systems.
REFERENCES


