ABSTRACT

HE, JIAZE. Time-reversal Based Array Damage Imaging in Structural Health Monitoring. (Under the direction of Dr. Fuh-Gwo Yuan).

Composite materials are receiving increasing attention and broadly used in aerospace industry due to their superior strength-to-weight ratio, corrosion resistance and design flexibility. The need for rapid nondestructive evaluation (NDE) techniques for composites is growing rapidly as the complexity and dimensions of the structures are increasing significantly. Structural health monitoring (SHM) has been attracting much attention as a means of providing in-service and in-situ monitoring of various critical structures. Due to their capability of long-range and through-the-thickness interrogation of the structures for small defects, guided waves have been studied extensively in damage detection for plate-like structures.

However, a few challenges exist when Lamb wave-based SHM/NDE techniques are employed. For example, the dispersion effect decreases the accuracy of many damage imaging algorithms; damage severity quantification is always a difficult problem. To provide possible solutions to above challenges, two damage imaging algorithms were developed and utilized for Lamb-wave based damage imaging.

The first algorithm is reverse-time migration (RTM), which was first used in geophysics to provide proper solutions to complex wave phenomena. The traditional imaging condition utilized in SHM is called excitation-time imaging condition, which used ray tracing and group velocity corresponding to the center frequency of the input signal. Due to the dispersion effect, the time-of-flight (ToF) estimation cannot always be accurate, especially for the situations that the Lamb waves propagate for a long distance. In this thesis, new imaging conditions are proposed to form enhanced zero-lag cross-correlation reverse-time
migration (E-CCRTM) techniques. The proposed damage imaging technique takes into account the amplitude, phase, and all the frequency content of the Lamb waves propagating in the plate; thus, the severity of multiple sites of damage can be non-biasedly imaged regardless of the damage locations in comparison with using existing imaging conditions.

The other imaging algorithm is called ‘DORT-MUSIC’. A Lamb wave-based, subwavelength imaging algorithm is developed for damage imaging in large-scale, plate-like structures based on a decomposition of the time-reversal operator (DORT) method combined with the multiple signal classification (MUSIC) algorithm in the space-frequency domain. The physics of wave propagation, reflection, and scattering that underlies the response matrix in the DORT method is mathematically formulated in the context of guided waves. Singular value decomposition (SVD) is then employed to decompose the experimentally measured response matrix into three matrices, detailing the incident wave propagation from the linear actuator array, reflection from the damage, and followed by scattering waves toward the linear sensing array for each small damage. The SVD and MUSIC-based imaging condition enable quantifying the damage severity by a ‘reflectivity’ parameter and super-resolution imaging.

The two algorithms were also integrated with a hybrid system mainly comprised piezoelectric actuators mounted onto the structure and a laser Doppler vibrometer (LDV) for reception. The flexibility of the proposed system was used for inspection of various plate-like structures. The experimental results show that the 2-D E-CCRTM has robust performance to image and quantify multiple sites of damage in large area of the plate using a single PZT actuator with a nearby areal scan using LDV, and the DORT-MUSIC (TR-MUSIC) imaging
technique can provide rapid, highly accurate imaging results as well as damage quantification with unknown material properties.
Time-reversal Based Array Damage Imaging in Structural Health Monitoring

by
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DEDICATION

To My Parents

&

My Lovely Wife Tingting
BIOGRAPHY

Jiaze He was born on September 28, 1988 in Shuangyashan, Heilongjiang Province in China. In 2011, he obtained his B.S. degree in Theoretical and Applied Mechanics from Jilin University, Changchun, China. Starting from 2011, he worked as a research assistant in the Mechanical and Aerospace Engineering department at North Carolina State University, where he completed his M.S. in 2013. In the summer of 2014, he worked as an intern at BOSE Cooperation. Then he started to work at the National Institute of Aerospace (NIA) as a graduate research assistant.
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Chapter 1

Introduction
1.1 Research motivation

Guided wave-based damage detection techniques for plate-like structures using transducer array have been receiving increasing attention in structural health monitoring (SHM) and non-destructive inspection (NDI). Damage imaging based on Lamb waves enables fast non-destructive assessment to plate-like structures with large areas. Array configuration provides higher signal-to-noise (SNR) ratio and beam focusing directivity compared with conventional distributed sensors. Especially for quantitative damage imaging, the array also exhibits more stable and reliable capability (Ambroziński, 2014).

Dispersion effect of Lamb waves is that wave velocities are frequency-dependent, which causes waveform change during propagation and introduces difficulties in many damage imaging algorithms. Damage severity qualification is also a tough problem in the area. To provide possible solutions to above challenges, the objective of this thesis is to propose two time-reversal based array damage imaging algorithms and to integrate the two algorithms with a hybrid scanning system using lead zirconate titanate (PZT) actuators and a laser Doppler vibrometer (LDV) for reception.

Due to time-reversal symmetry of the wave equation, the dispersion effect can be automatically compensated by the first imaging technique, reverse-time migration (RTM), without extra efforts. With the proposed proper imaging condition, damage reflectivity can be possibly quantified using RTM technique.

The other method studied in this research is called DORT-MUSIC. DORT is short for “decomposition of the time-reversal operator”, which has the ability of imaging multiple sites of small (relative to wavelength) damage separately. Multiple signal classification (MUSIC)
was firstly used in active source localization. Due to damage acts as secondary sources in ultrasonic inspection, the MUSIC algorithm, thus, can be integrated with DORT method for damage imaging. The damage imaging process is performed in frequency-spatial domain such that the influence of the dispersion effect is minimized. Through the study of the underlying physics, damage reflectivity can be quantified through the process of singular value decomposition (SVD).

The advantages of the two imaging techniques were verified through integration with the hybrid PZT/LDV system. The inspection structures are both in isotropic and composite structures.

1.2 Damage detection techniques in NDE/SHM

1.2.1 General damage detection techniques

Common flaw types in composite materials (Bossi, 2006; Kassapoglou, 2013; Ibrahim, 2014; Mallick, 2014; Cawley, 2006)[1,2,3,4,5] include but not limited to porosity, subsurface fiber waviness, delamination, foreign object debris (FOD), tow gap and overlap, fiber breaking, and microcracking.

Conventional NDE techniques have been developed to localize and quantify various flaws in composites induced during manufacturing and service cycles. X-ray has been used for damage imaging in composite materials as radiograph (Jones, 1988; Tan, 2011) and computed tomography (CT) (Aroush, 2006; Tsao, 2005), and more recently three-dimensional (3-D) reconstructions have also been built for straightforward illustration.
Schilling, 2005; Wright, 2008; Bull, 2013). Ultrasonic testing techniques using bulk waves also have demonstrated their capabilities in damage imaging evolved from A-scan (Olympus) to more commonly used C-Scan (Scarponi, 2000; Ruzek, 2006; Hasiotis, 2011). Terahertz spectroscopy is a relatively new technology to use electromagnetic waves with a frequency range in order of $1 \text{THz}$ ($10^{12} \text{Hz}$) to generate high resolution images (Stoik, 2010; Chady, 2012; Hsu, 2011). All the methods can characterize the details of the composite materials such as fiber breaking and microcracking, but the application of those methods is time-consuming and labor-intensive.

1.2.2 Thermography, Shearography and Ultrasonics for rapid inspection

Infrared (IR) thermography (Muzia, 2007; Roemer, 2013; Goidescu, 2013; Grammatikos, 2014) measures the temporal evolution of the temperature field generated by a heat source and detected by IR camera. The heat ‘wavefront’ is blocked by defects such as delamination or porosity with higher thermal impedance, which will be shown on the surface of the composite materials in the IR camera. The steepness of the temperature gradient provides the contrast between defective and non-defective areas (Cawley, 2006). However, the IR camera can only map the radiation in the infrared range of the electromagnetic spectrum on the surface of the composites. Thus, the sensitivity will decrease as damage locations become further away from the surface. Also the detectability of defects is strongly dependent on the materials and the defect types in thermography.

Shearography (Santos, 2004; Taillade, 2011; De Angelis, 2012; De Angelis, 2013; Ryley, 2007) can also image a large area within a short period of time. The surface of the damaged and undamaged region of the test object display different deformations when subject to
proper loading, such as vacuum pressing (Newman, 1990; Newman, 1991), thermal excitation (Hofmann, 2008) and mechanical excitation (Fernandez, 2000). A laser source illuminates the testing object and creates the speckle patterns on the surface after the laser beam splits into two beams through an optical shearing device. The difference between two speckle patterns of both undeformed and deformed structures can be collected by an image sensor and forms an absolute deformation image or a gradient deformation image (Dantec Dynamics). The major disadvantages of shearography are with low lateral resolution (Ryley, 2007) and the loading to the structures needs to be large enough to cause significant change on the surface, leading to the sensitivity of the method dropping for deeper damage.

Ultrasonics for NDE has evolved from under water immersion inspection to contact probes (GE Measurement & Control; Olympus)[34, 35]. However, even combined with automated robotics, the conventional ultrasonic inspection for large structures can still be time-consuming, and couplants between the testing object and the transducers is still needed. Lamb waves were intensely studied and applied for large-area NDE and structural health monitoring (SHM) due to their superior capabilities in long-distance propagation and through-thickness interrogation (Park, 2014; Park, 2014; Hall, 2014; Yu, 2008; Ambroziński, 2014). Laser ultrasonics were applied to excite bulk waves in isotropic materials (Murray, 1996). In recent years, as a high-frequency laser reception technique, laser Doppler vibrometer (LDV), become more commercially available, fast fully noncontact laser ultrasonic inspection for composites based on Lamb waves (Park, 2014; Park, 2014) receives more attention. Conventional arrays for Lamb wave application based on piezoelectric wafers are commonly used in distributed (Hall, 2014), linear (Yu, 2008), and areal (Ambroziński,
2014) patterns. To minimize the scanning time for large complex composite structures and remains detectability of the damage with relative large sizes or severities, noncontact laser ultrasonics can be embedded with the Lamb wave based array technology when only scanning a limited line or a small area to access a much larger area. Park and Sohn (Park, 2014; Park, 2014) demonstrated the effectiveness of the combination of the noncontact YAG laser and LDV through the inspection of a CFRP aircraft wing segment using Lamb waves. Most of the commonly used Lamb wave based NDE techniques can be applied with laser ultrasonics, and the resolution concerns associated with dimension of the transducers are eliminated (Ambroziński, 2014).

1.2.3 Guided wave-based NDE/SHM in large scale plate-like structures

Guided waves involve two dominant types: passive sensing and active sensing. The most commonly used passive sensing method is acoustic emission (AE), where stress wave signals are emitted from sudden energy release (e.g. crack propagation) and passively received by sensors. AE has been applied to complex aerospace metallic structures (Pavlopoulou, 2011; Gangadharan, 2009), composite materials (De Oliveira, 2008), and civil infrastructures (Nair, 2010; Christian, 2006). In active sensing, however, the piezoelectric (PZT) transducers can serve as both actuators and sensors. Numerous methods fall into this category, and various parameters can be defined and examined to perform damage detection, such as E/M impedance (Gresil, 2012), time-of-flight (Ramadas, 2011), amplitude (Croxford, 2007), wave energy (Liu, 2008) and damage index (Park, 2007; Ihn, 2008; Gresil, 2012; Soutis, 2009).

A number of researchers have attempted to locate the impact loading and damage area using a distributed sensor array (Chen, 2013; Michaels, 2008). Spatially distributed ultrasonic sensors
were used to excite and receive broadband Lamb wave signals considering temperature change and optimal baseline selection (Michaels, 2008). The damage images were formed at each selected frequency by the use of delay-and-sum techniques; the images were then fused together. However, these methods, in general, could only roughly evaluate the size of damage.

1.3 Guided wave based array damage imaging

One-dimensional (1-D) array, namely, linear array imaging based on Lamb waves, will generate a mirrored ghost image along with the original one (Giurgiutiu, 2004). This is due to the geometry of the array is symmetrical, and no elements distribute in the perpendicular direction of the linear array such that only from the signals received, the array cannot differentiate the waves coming from which side of the array. This limitation could be overcome by designing the array into two-dimensional geometric configuration. Yu and Giurgiutiu (Yu, 2008) designed a 2-D phased using PWAS. Ambroziński (Ambroziński, 2014) proposed a system to design the array into various 2-D topologies efficiently with the comparison of the beam patterns for each topology.

1.3.1 Phased array techniques

Phased array was first conceptually introduced in 1905 by Karl Ferdinand Braun with demonstrated transmission of radio waves in a single direction (Braun, 1909). For meeting various requirements using radar for detection, phased array has been developed significantly. In ultrasonic imaging for medical diagnoses, early developed phased array systems weighed about 800 lbs (Szabo, 2004). Nowadays, commercially available systems are smaller than the
size of personal computers (Scampini, 2010). The developments in phased array technologies in all areas are highly dependent on and parallel with the advancement of electronics, transducers, signal and image processing as well as the computational speed and capacities. By manipulating wave interference constructively using elements and electronic time delays, phased array can focus energy like a beam to the location of interest. The focused beam can scan a large area and yield much better precision on the focal path than merely using unfocused wave fields. Therefore, phased array acquires the ability to quickly examine large structures with high resolution. For example, a linear array can be employed in a sector scan mode where the incident angle is electronically steered. The most general means of acquiring data is referred to as full matrix capture (FMC), which means that signals are stored in a matrix with each component presenting signals obtained from an actuator-sensor (or transmitter-receiver) pair.

Phased array technology has demonstrated the ability of locating and imaging cracks and corrosions in bulk materials for nondestructive evaluation (NDE) usage (Dube, 2007). Three beam scanning patterns are generally used for flaw inspection, which are electronic scanning (E-scans), sectorial scanning (S-scans) and dynamic depth forcing (DDF). In E-scans, the angle of the focused beam is constant and same time delay is multiplexed across the actuator group along the phased array probe. Then the beam can be swept along a straight line. In S-scans, the angle of the focused beam changes in a range and the beam can be swept through an angular range with certain focal depth. In DDF, the angle of the focused beam is fixed and different focal depths can be chosen to perform scanning. Based on the above mentioned scanning strategies, beam steering and beam focusing are identified as two principal methods.
for applying time delay to each element of the phased array. Figure 1.1(a) and Fig. 1.1(b) schematically demonstrate the beam steering and beam focusing. In steering, for a cross-section, the wave fronts form a line and the energy is directed into a certain angle. In focusing, however, the wave front from each element arrives at a point simultaneously, which is called focal point.

Figure 1.1 The schematics of (a) beam steering and (b) beam focusing.

The phased array technology usually is based on a critical assumption that the wave velocities in the air (radar), human body (medical) and structures (NDE) are constant. Under
this assumption, the time delay for each element can then be calculated from the distance difference, the steering angle and the focal length for the array can then be determined. Finally, the location of concerned objects can be found by converting the signals in time domain to the positions in space based on time-of-flight. Most of the ultrasonic inspections are performed through the thickness direction. However, for thin-walled structures, the bulk wave based phased array requires frequency range, usually greater than 1 MHz, and an extremely long inspection time, involving moving the array from one location to another. To overcome these limitations and increase the inspection efficiency, guided wave based phased array technology is introduced to plate-like structures for rapid scanning over large areas. The wave velocity will not only be influenced by its frequency and modes, but also could be dependent on the direction of propagation due to the anisotropic behavior of composite material. The complexity in guided wave velocity leads to challenges in application of the phased array technology.

Strategies for applying guided wave based phased array technology have been proposed lately for damage imaging using piezoelectric wafers (Yu, 2005; Yu, 2007; Yu, 2008). Yu and Giurgiutiu (Yu, 2007; Yu, 2008) derived the mathematical expression for one-dimensional (1-D) linear phased array and experimentally demonstrated the beam steering capacity. An adaptive self-focusing technique was developed by Beardsley et al. (Beardsley, 1995) to obtain the optimal delays for the array elements based on the unfocused scattered wave signals. For anisotropic materials, wave velocity becomes angle-dependent such that beamforming becomes difficult to achieve. Yan and Rose (Yan, 2007) carefully chosen the quasi-isotropic mode in the dispersion curves of a composite plate and redefined the delay for
each element by considering the direction varying wave velocity.

More details can be seen in Appendix B.

1.3.2 Synthetic aperture focusing technique

The synthetic aperture (SA) technique is firstly developed in radar technology. The aperture is synthesized using the measurements at different locations from a radar device on a moving airplane. Similar concepts are applied in medical ultrasound and non-destructive testing (NDT). The key idea of post-data processing for SA is to achieve a full dynamic focusing through the virtual delay-and-sum (DAS) based on the time-of-flight (TOF). When the SNR is high for both PA and SA, SA provides higher resolution compared with PA since it provides the focusing at every pixel in the image both in transmitting and receiving mode (Tanabe, 2011). On the other hand, PA has a stronger ability in obtaining a higher SNR with the same level of the input signals with SA due to PA physically transmitting and focusing the energy into the structures. From the perspective of data generation and storage, different modes such as single-input and single-output (SISO) and multiple-input and multiple-output (MIMO) could be defined. When all the elements in the array are used to both transmit and receive signals, this special case of MIMO is often called full matrix capture (FMC) and the corresponding imaging algorithm could be called as total focusing method (TFM) (Holmes, 2005). In damage detection for plate-like structures, piezoelectric wafer active sensors (PWAS) are commonly used. Due to the inexpensiveness and good dynamic performance, the piezoelectric (PZT) wafers are suitable for in-situ structural health monitoring, in which they are permanently mounted onto the structures and could not move. The name of SAFT is used as long as the key part of imaging utilizes virtual delay-and-sum for post data
processing, no matter if the array is actually moving during inspection (Stepinski, 2010). The similar changes are also found in medical ultrasounds (Tanabe, 2011; Jensen, 2006).

1.3.3 Tomography

Tomography might be the most straightforward guided wave imaging algorithm using Lamb waves, since it requires only the feature changes caused by damage while the feature changes could be defined in multiple ways (Prasad, 2004; Zhao, 2011; Belanger, 2009; Yan, 2010). Tomography is with 2-D array topologies in general. As the algorithms evolve over the years, more complex and effective imaging algorithms are proposed (Zhao, 2011; Yan, 2010). To maximize the ray path coverage, most commonly used geometry arrays are circular, rectangular and parallel linear array (Yu, 2008; Yan, 2010). One of the biggest limitations is that the tomography requires the damage to be in the enclosure of the array and on the direct path of element pairs. The array arrangement also leads to more difficult of separation of the scattered wave signals because the direct arrivals is not received much earlier than the scatter wave signals hence overlapping happens easily. For most scenarios, tomography requires a baseline.

1.4 Time-reversal based array damage imaging

To obtain a high resolution image of damage, the method to form focusing is critical. Besides using phased array, self-focusing could be achieved by physically retransmitting the time-reversed received signals into the structures (Deutsch, 1997). This iterative focusing method requires hardware with programmable signal transmitting abilities, which is more costly than phased array system. On the other hand, if the SNR is good enough, the focusing process is
preferable to be performed in the post data processing. SAFT can be considered falling into this category. Due to the wave dispersion, the performance of the time-of-flight based imaging methods for plate-like structures including phased array (PA) and synthetic aperture (SA) is lower than in non-dispersive materials.

### 1.4.1 DORT-MUSIC Imaging

The time-reversal mirror (TRM) and the decomposition of the time reversal operator (DORT) have been widely used in various applications for non-destructive testing (NDT) (Kerbrat, 2002), acoustics (Fink, 2000), guided waves (Ing, 1998), electromagnetic fields (Yavuz, 2006), microwaves (Yavuz, 2009) and nonlinear waves (Barbieri, 2010). Fink, Wu et al. (Fink, 1992; Wu, 1992) proposed the TRM concept in 1992, which can be used for target detection. Based on the reciprocity theorem, the TRM algorithm has ability to, after the iteration process, focus the wave energy back to the strongest target in the field. Prada and Fink (Prada, 1994) illustrated the use of the DORT algorithm to focus the wave energy on multiple targets, even for those that are neither point-like nor perfectly resolved. The whole DORT process is described in detail in (Prada, 1996). The TRM requires a programmable transducer array to synthesize and transmit the time-reversed signals simultaneously. Due to the cost and complexity of the hardware system, the application of TRM is limited.

For small, point-like damage, whose dimensions are small compared to the input wavelength, each eigenvalue typically will correspond to each distinct scatterer. One of the major advantages of the DORT algorithm is that it has the capability of individually imaging these scatterers by back propagating the singular vectors obtained from eigenvalue decomposition (EVD) of the time-reversal operator (TRO), providing separate information about each
scatterer. However, when the scatterers are relatively large compared to the input wavelength, a single scatterer might generate multiple significant eigenvalues. In this case, even though the back propagation of the eigenvectors can provide a certain amount of information about the relatively large scatterers, DORT has difficulties in providing quantitative information about each scatterer. A similar difficulty occurs when the multiple scattering effects are strong (Gruber, 2004); that is, each of the finite sized damages will be seen as a combination of multiple scatterers by the DORT method.

Lehman and Devaney (Lehman, 2003) developed a combined DORT and multiple signal classification (MUSIC) method to image multiple buried cylinders in the application of seismo-acoustics. This combined method has been noted as DORT-MUSIC or time-reversal MUSIC (TR-MUSIC). The MUSIC method was originally proposed by Schmidt in 1986 (Schmidt, 1986) and is widely used in various disciplines (Meng, 2012; Odendaal, 1994; Koles, 1998). The MUSIC algorithm is capable of providing the location or the directions of arrival (DOA) of the active sources in the field. For the damage detection problem, according to Huygens-Fresnel principles, damage acts as a secondary source during either transmission or reflection of the waves at the boundaries of the damage. Thus, the MUSIC algorithm can also be used for damage imaging if the damage is considered to be a source. Labyed and Huang (Labyed, 2013) applied the TR-MUSIC algorithm in imaging phantoms utilizing a frequency band and applying phase compensation due to the response lag of the transducer. Fan and Drinkwater (Fan, 2014) conducted a comparison between DORT-MUSIC imaging and conventional ultrasonic array beamforming for NDT, and the results showed that the TR-
MUSIC exhibits superior performance over the total focusing method (TFM) for resolving closely spaced scatterers when the signal-to-noise ratio (SNR) is greater than 20 dB. Despite the broad application of the DORT algorithm, the MUSIC algorithm or a combination of these two in various areas, the application to damage imaging using Lamb wave-based on DORT-MUSIC is limited. Due to the dispersive nature of Lamb waves and the complexity of multi-mode behavior during propagation, the accurate representation of the background Green’s function is difficult to attain. Engholm and Stepinski (Engholm, 2011) implemented a circular array using MUSIC to estimate the direction-of-arrival (DOA) of Lamb waves; however, this technique gives no clear indication on the distance of the source. Qu (Qu, 2013) and Ambrozinski (Ambrozinski, 2014) applied the DORT algorithm for damage localization on an aluminum plate, but it does not provide an accurate damage image. Zhong and Yuan (Zhong, 2014) utilized the MUSIC algorithm to find the DOA and the distance of the damages in composite plates. However, due to the difficulty of finding a theoretical expression for the tone-burst excitation in plate-like structures, these studies only provide for rough damage localization and no quantification results are presented. One of the most important capabilities of DORT is multiple damage separation and localization, which has not yet been illustrated for plate-like structures. To the authors’ knowledge, the combination of DORT and MUSIC (TR-MUSIC) has also not been discussed and evaluated in Lamb wave-based damage imaging.
1.4.2 Reverse-time migration technique

Reverse-time migration (RTM), originally designed as an imaging method for interpreting seismic data has become a promising technique in SHM with the capability of quantitative damage identification and imaging as well as simple hardware requirements. Liu et al. (Liu, 1996) first introduced the migration method into the non-destructive evaluation (NDE) field by imaging surface-breaking concrete cracks. Experimental studies using RTM for damage identification have been presented in (Lin, 2005), where an arc-shaped damage in an aluminum plate was successfully imaged. Although imaging damage in composite plates is significantly more difficult than metal plates due to the anisotropy of these structures and significant signal attenuation at higher frequencies, RTM has proven to be a powerful technique (Wang, 2005). Rodriguez et al. (Rodriguez, 2014) applied similar principles using a linear array with 128 elements to excite plane waves vertically with respect to boundaries of isotropic materials. Though the topological imaging results are good and the process is quite fast, the cost of the system and the difficulty of performing in-situ monitoring or inspection with the specific experimental setup limit its application. However, laser based non-contact technology (Park, 2014; Tian, 2014) could be a solution to a large number of transducer requirements for guided wave applications.

To date, migration techniques for damage imaging in composites have merely utilized the signal group velocity at the center frequency of the narrowband tone-burst signal. Although the signal is a narrow-banded signal, there are still other frequency contents other than the center frequency of the signal that can be used for imaging.
1.5 Objective and scope

The objective of this thesis is to develop two time-reversal based damage imaging algorithms and to integrate the algorithms with the hybrid PZT/LDV system. The first algorithm is called reverse-time migration (RTM). The other one is referred to as DORT-MUSIC.

The thesis consists of five chapters.

Chapter 1 is the introduction. After the statement of the motivation of this research, damage detection techniques in NDE/SHM, especially for large-area structures, are introduced. Then several guided wave based array damage imaging methods are summarized and compared. Time-reversal operation compensates the dispersion effect because of time-reversal symmetry of the wave equation such that the two time-reversal based methods are suitable for Lamb wave-based damage imaging. The two methods introduced in later part of this chapter is RTM and DORT-MUSIC.

Chapter 2 summarizes the theory of reverse-time migration (RTM) and the steps of RTM application. New imaging conditions are proposed to enhance the damage imaging results. Then the numerical simulation results are presented from using a finite difference algorithm for solving the equations in Mindlin plate theory. Both isotropic materials and composites are studied through numerical simulations. Comparison is made between different imaging conditions. The results from one-dimensional (1-D) linear scan are also compared with those from two-dimensional (2-D) areal scan.

Chapter 3 introduces the experimental setups and results for reverse-time migration technique in both composite and aluminum plates. First, a linear piezoelectric wafer array was used with a zero-lag cross-correlation (ZLCC) imaging condition, which utilizes all the
frequency components of the input signals. Then Enhanced-ZLCC (E-ZLCC) imaging conditions are introduced to compensate the attenuation effects caused by wave propagation. 2-D areal scan was also performed using the LDV and combined with RTM with E-ZLCC imaging condition such that multiple sites of damage can be imaged in both isotropic and composite materials.

Chapter 4 organizes the theoretical framework of the combination of decomposition of the time-reversal operator (DORT) method with the multiple signal classification (MUSIC) algorithm. First, the theoretical expressions have been derived in the circumstances of Lamb-wave damage detection, with detailed explanations on the underlying physics. Then the numerical simulation results are presented to verify the abilities of this technique in subwavelength damage imaging, damage reflectivity quantification and multiple sites of damage imaging.

Chapter 5 illustrates the capability of DORT-MUSIC technique in subwavelength imaging through experimental study for isotropic materials. The setup of the hybrid PZT and LDV system is introduced. A linear piezoelectric wafer array was used as excitation sources while the scanning laser Doppler vibrometer was utilized to scan a line as the reception. Since the underlying physics of the singular value decomposition (SVD) of the transfer matrix $K$ has been researched in details in the last chapter, the proposed experimental setup provided a large degree of flexibility in scan patterns. Multiple sites of damage with the dimensions smaller than the input wavelength were well imaged. The advantage of the DORT-MUSIC in future application is also addressed.
Chapter 2

Reverse-time Migration Theory and Simulation
Reverse-time migration (RTM), originally designed as an imaging method for interpreting seismic data has become a promising technique in SHM with the capability of quantitative damage identification and imaging together simple hardware requirements. First, theory and procedures are presented for damage imaging for composite plates using a zero-lag cross-correlation (ZLCC) imaging condition for RTM, briefly called CCRTM. The ZLCC were calculated between the forward wavefield and the backward wavefield. The forward wavefield is formed by the excitation from the actuator using a finite difference (FD) method, and the backward wavefield is generated by back-propagating the time-reversed scattered wavefield using the same FD method. Simulation studies were first examined to verify the capability of using the proposed ZLCC imaging condition to image single and multiple sites of damage.

Then the Enhance-ZLCC (E-ZLCC) was proposed to compensate the attenuation effects during Lamb wave propagation. The proposed damage imaging technique takes into account the amplitude, phase, and all the frequency content of the Lamb waves propagating in the plate; thus, the severity of multiple sites of damage can be unbiasedly imaged regardless of the damage locations in comparison with using existing imaging conditions. Comparisons were made in terms of damage imaging quality between 2-D areal scans and linear scans as well as between the proposed and existing imaging conditions.

2.1 Finite difference based wave propagation

Fromme (Fromme, 2009) studied the scattering pattern of guided waves using 3D finite element analysis to solve the complexity introduced by a stiffener on a steel plate. Bostron et al.
(Bostron, 2013) studied interface waves travelling at the boundary between a soft and stiff solid using analytical and FEM methods. Shen and Giurgiutiu (Shen, 2014) developed a thorough analytical tool to simulate Lamb wave propagation and interaction with damage including the consideration of mode conversion, transmission, reflection and nonlinear higher harmonics. Mindlin plate theory has been used to model transient waves in isotropic plates, and have been extended to composite plates by Lih and Mal (Lih, 1995; Lih, 1995) for the fundamental mode of flexural waves, taking into account the effects of transverse shear deformation and rotary inertia. Kirchhoff’s theory can work only in lower frequency range compared with Mindlin plate theory. The sensitivity of the selected wave modes to the damage is satisfactory only if the wavelength is on the same order as the dimensions of the damage. Therefore, in this research, Mindlin plate theory rather than the Kirchhoff’s theory is applied to model the transient wave behavior.

The stress resultants $Q_x$, $Q_y$, $M_x$, $M_y$, and $M_{xy}$, the plate displacement components $w$, $\psi_x$ and $\psi_y$, and the external transverse loading on the plate $q$ are used to define two vectors:

$$ u = [w, \psi_x, \psi_y, Q_x, Q_y, M_x, M_y, M_{xy}]^T, \quad q = [q, 0, 0, 0, 0, 0, 0, 0]^T. $$

The revised governing equation can then be rewritten as

$$ \frac{\partial U}{\partial t} = A_i \frac{\partial U}{\partial x} + B_i \frac{\partial U}{\partial y} + C_i U + q $$ (2.1)

where $U = E_0 u$, $A_i = A_0 E_0^{-1}$, $B_i = B_0 E_0^{-1}$, $C_i = C_0 E_0^{-1}$.

Detailed expressions for the variables and matrices such as $E_0$, $A_0$, $B_0$, $C_0$ are provided in (Wang, 2005), and, thus, are not repeated here. The bending and transverse shear stiffness matrix of the entire composite laminate are obtained through the integration (in thickness direction) of each
layer’s transformed reduced stiffness matrix, which can be routinely calculated from the lamina properties and coordinate transformations (Jones, 1999). Attenuation effects due to geometric spreading and wave dispersion are modeled using the finite difference algorithm. The attenuation caused by internal friction and other thermodynamic effects or by wave scattering from the fibers or other inhomogeneities in the material are not considered by this algorithm. An $N \times N$ mesh will be superimposed on the composite plate. A forward wavefield is obtained by exciting from an actuator using the 2-6-order finite difference algorithm, which is developed from the 2-4-order MacCormack algorithm (Chang, 1986). After the scattered wave signals are extracted from the sensing data, the same finite difference algorithm is implemented to back-propagate the time-reversed scattered wave signals. Details of the finite difference algorithm used in this study are described in (Wang, 2005).

### 2.2 Imaging damage using RTM with new imaging conditions

One actuator on the plate can serve as a source to generate a forward wavefield and a sensor array could receive the scattered signals from the damage. The E-CCRTM has three steps: forward-time extrapolation from the actuator, reverse-time extrapolation of the time-reversed received scattered wavefield at the sensor array, and application of the new imaging condition. Both extrapolations in the first two steps need to be done through the finite difference algorithm. For the second extrapolation, however, the received scattered wavefield may be obtained at sensor locations either through experiments or simulation on structures with damage. The imaging condition is defined based on the concept that if both incident waves and the scattered waves are extrapolated separately, the damage boundaries exist at the places where these two
waves are time-coincident (in phase) to each other. In this study, an imaging condition is formulated by performing zero-lag cross-correlation (ZLCC) between the two wavefields for every grid at all times. The cross-correlation value in the damage area will be much larger than that of the remaining areas. Based on this observation, the image of the damage can be presented by showing the ZLCC results at each grid. The following three steps are taken to image the damage size.

The wavefields are extrapolated through a finite difference algorithm, which is based on solving the governing equations of quasi-flexural waves ($A_0$ mode Lamb waves). The accuracy of this algorithm has been verified in previous studies (Lin, 2001; Wang, 2005). Attenuation effects due to geometric spreading and wave dispersion are modeled using the finite difference algorithm. The attenuation caused by internal friction and other thermodynamic effects in the materials, which are insignificant in metallic materials, are not considered by this algorithm. A narrow band tone-burst signal with the ability of suppressing the dispersion effect is applied to the actuator as the excitation function. In finite difference calculation, the excitations from the designated source locations serve as boundary conditions varying with time at the corresponding grids to simulate the wave propagation problem.

2.2.1 The forward-time extrapolation – Step One

Since all the material properties and structural information are known a priori, using the excitation signal from the actuator as the boundary conditions at each time-step, the finite difference algorithm could generate and record the wavefields at all the grids for each time step, which is called forward wavefield. The actuator is modeled as a grid in finite difference algorithm corresponding to the center of the piezoelectric wafer in the experiments. The
forward wavefield contains an out-of-plane velocity signals with respect to the variation of
time at all the grids. In this way, all the frequency content is preserved as well. The
extrapolated forward wavefields will be used in Step Three.

2.2.2 Time-reversed scattered wavefield extrapolation – Step Two

The scattered signals could be obtained through either experiments for real damage detection
or simulation for numerical study. The received scattered signals are reversed in time and are
extrapolated simultaneously as the boundary conditions at all the sensor locations in the finite
difference algorithm. This extrapolation will also generate a wavefield containing out-of-
plane velocity signals with respect to the variation of time at all the grids, which is called
backward wavefield. The backward wavefield also is saved and will be applied to imaging
conditions in Step Three.

2.2.3 An enhanced zero-lag cross-correlation imaging condition – Step Three

The physical process of the wave excitation and receiving could be described as follows: the
waves are excited from the actuator, propagate in the plate-like structures, interact with the
boundaries of the damage and the reflected signals are received at the sensor array. Due to
the reciprocity theorem, if the received scattered signals are reversed in time and extrapolated
into the field from the same sensor locations, the wavefronts will be converging toward the
damage. The dispersion effect would be automatically compensated during this process.

The total time for one measurement is assumed to be $T$. For one ideal point on the boundary
of the damage, the time for the incident wave to propagate to it is assumed to be $T_1$ and the
rest of time $T_2 = T - T_1$. This applies to every point on the boundaries of the damage. The
time for the wavefronts in the backward wavefields to arrive at the boundaries of the damage is exactly the same with the difference obtained by the subtraction of the time for the incident wave signals in the forward wavefield to travel to the boundaries of the damage from the total propagation time. This observation lays the foundation of reverse time migration. Imaging conditions determine when and where to image while the time-reversed received signals are being extrapolated.

Different imaging conditions could be used to image damage based on this phenomenon. The zero-lag cross-correlation (multiplication) imaging condition is firstly proposed for Lamb wave application in (He, 2015) and result in better damage imaging results than using the traditional excitation-time imaging condition (Wang, 2005).

The time needed for the incident waves in the forward wavefield to propagate from the actuator to the damage boundaries ($T_1$) plus the time needed for the wavefronts in the backward wavefield to propagate from all the sensor locations to the damage boundaries ($T_2$) equals to the total time ($T$). The excitation-time imaging condition uses the group velocity of the chosen Lamb wave mode at the center frequency of the input signal to estimate the grids to be imaged at each time step for the backward wavefield. The estimation is only corresponding to the center frequency and other frequency content has yet not been utilized.

On the other hand, the zero-lag cross-correlation (ZLCC) imaging condition based on underling physics that not only the arrival time ($T_1$) for both the time-reversed backward wavefields and the forward wavefields at the boundaries is the same, but also the two wavefields are exactly in phase at the damage boundaries. If the forward wavefield is reversed in time, the wavefronts are going to shrink from a circular or near-circular shape to
the location of the actuator (Lin, 2001) depending on whether the material is isotropic. The zero-lag cross-correlation between these two fields uses all the frequency content to calculate the value for each location \((x, y)\) on the image. Mathematically, the image value for grid \((x, y)\) is defined as:

\[
I(x, y) = \int_{0}^{\tau} W_f(x, y, \tau)W_b(x, y, T-\tau)d\tau
\]  

(2.2)

where \(W_f(x, y, t)\) is the forward wavefield and \(W_b(x, y, t)\) stands for the backward wavefield, \(T\) is the time duration in a single migration experiment. Both \(W_f\) and \(W_b\) contain real signals. This method is referred to as CCRTM.

However, this image, with an amplitude-squared unit, does not reflect the accurate relative damage strengths if there are multiple sites of damage with different distances to the scan area and the actuator. The CCRTM does not consider the amplitude change caused by attenuation. Attenuation on plate-like structures can be attributed to three major factors: geometric spreading, dispersion and dissipation from the material itself. The dissipation is caused by the conversion from mechanical energy into heat. For composites, dissipation and scattering from the fibers and possible porosities are strong. In aluminum plates, the dissipation is relatively weak and could be ignored in imaging. The finite difference algorithm can simulate the physical wave attenuation induced by geometric spreading and dispersion. During the reverse-time migration, the amplitude change caused by dispersion is automatically restored in the backward wavefields, which is a significant advantage of this method. However, the CCRTM did not consider the amplitude change caused by geometric spreading. For with the same size and severity but different locations, the further away the damage is from the source, the weaker the image of the damage in CCRTM.
Zero-lag cross-correlation between the forward wavefield and the backward wavefield is taken for every grid. Thus, the influence of geometric spreading in ZLCC will be compensated in the new imaging condition. When the actuator and the scan area center are the same, the intensity in the image can be normalized by the square of the source illumination strength (Kaelin, 2006; Chattopadhyay, 2008) Mathematically, the imaging condition could be expressed as:

\[
I(x, y) = \frac{\int_0^T W_f(x, y, \tau)W_b(x, y, T-\tau)d\tau}{\int_0^T W_f(x, y, \tau)^2 d\tau}
\]  

(2.3)

where \(I\) represents imaging intensity of the position in the field. This source-normalized ZLCC reverse time migration could be referred to as SCRTM. From a point source, the flexural wave amplitude change from the geometric spreading is inverse proportion to \(\sqrt{r}\), where \(r\) is the distance of the propagation. If the center of the scan area is not at the same position with the actuator, the imaging condition could be further improved as

\[
I(x, y) = \frac{\int_0^T W_f(x, y, \tau)W_b(x, y, T-\tau)d\tau}{\int_0^T |W_f(x, y, \tau)|d\tau} \cdot \sqrt{r(x, y)}
\]  

(2.4)

where \(r(x, y)\) is the distance from each \((x, y)\) to the center of the sensor array. The new imaging method will be referred to as enhanced zero-lag cross-correlation reverse time migration (E-CCRTM).

### 2.3 Numerical simulation 1-D CCRTM for composites

This section compares the damage imaging results of the prestack reverse-time migration and the proposed CCRTM techniques through simulation. The material properties for each lamina are given in Table 2.1. The plate dimensions are 800 mm×800 mm×2 mm and the layup of the
composite panel is [45/0/90/45]_{2s}, both of which are identical to those used in the experiments (section 3.1.1). In this research, twenty-one wafers are used, illuminating the whole plate area with a 400×400 finite difference mesh. The origin of the coordinate system is set at the center of the plate. The PZTs are located from (-80, 0) mm to (80, 0) mm, with spacing of 8 mm. The center of the damage is located at (-30, 150) mm, modeled as a circular area with one eighth of the original bending stiffness. The diameter of the damage is 18 mm to be comparable with the experiments, and, for the finite difference algorithm, all of the Cartesian mesh grids within the circle were modeled with reduced bending stiffness.

Table 2.1 Material properties of AS4/8552-2 composite lamina

<table>
<thead>
<tr>
<th></th>
<th>$E_1$ (GPa)</th>
<th>$E_2$ (GPa)</th>
<th>$E_3$ (GPa)</th>
<th>$G_{12}$ (GPa)</th>
<th>$G_{13}$ (GPa)</th>
<th>$G_{23}$ (GPa)</th>
<th>$\nu_{12}$</th>
<th>$\nu_{13}$</th>
<th>$\nu_{23}$</th>
<th>$\rho$ (×10^3 kg/m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>142.2</td>
<td>10.5</td>
<td>10.5</td>
<td>4.90</td>
<td>4.90</td>
<td>3.67</td>
<td>0.39</td>
<td>0.39</td>
<td>0.11</td>
<td>1.600</td>
</tr>
</tbody>
</table>

During each test, one wafer serves as an actuator and the remaining serve as sensors to collect the reflected signal from the damage. The time-reversed scattered wavefield will be extrapolated at each sensor location as new boundary conditions at each step. One image is formed by using the ZLCC from a single test by actuation from (0, 0) mm. The stacked image obtained from twenty-one RTM tests are also generated for comparison. The red circle indicates the location and shape of the damage.

The wafer array consists of twenty-one elements distributed uniformly and horizontally at the center of the plate. The backward wavefield is extrapolated at all the sensor locations. The backward wavefield is in phase with the forward wavefield when propagating to the damage. To clearly illustrate the relationship between the forward and the backward wavefields, the forward wavefield is also displayed with decreasing time in Figure 2.1, producing an equivalent
realization of eq. (3) in which the time is increasing. In this simulation, the entire time period is 300 μs. Thus, the back-propagating field is calculated from 300 μs to 0. Since the illustration is reversed in time, the forward wavefield, actuated by the single actuator at the center of the plate, will be displayed as a nearly circular-shaped field converging to the actuator location. From the snapshots in Figure 2.1, the convergence of the two wavefields can be observed at the damaged area.

Figure 2.1 Snapshots of the back-propagating wavefields for damage imaging at varying reversed times in simulation study for actuator located at the center of the plate at: (a) 280 μs; (b) 176 μs; (c) 140 μs; (d) 116 μs.
In Figure 2.2, wave signals at different locations for both the forward and backward wavefield are shown. For the grid point at (-30, 150) mm, which is at the center of the damage area, the forward and backward signals are shown in Figure 2.2(a). Figures 2.2(b)-(d) illustrate the forward and backward signals for locations at (-30, 200), (-30, 280) and (200, 200) mm, which are far away from the damage area. The two wave signals at (-30, 150) mm are much more similar than those of other points shown in Figure 2.2. Thus, the ZLCC value for (-30, 150) mm is much larger than others. An interesting observation is that although signals obtained from forward and backward wavefields are in phase only for (-30, 150) mm, the amplitude of the backward signals at (-30, 200) and (-30, 280) mm is much larger than that of (200, 200) mm. The reason is that the two former points are on the converging path of the backward wavefield, while point (200, 200) mm is not. Thus, from Figure 2.1, the converging path of the backward wavefield can be spotted. The ZLCC imaging condition will ensure large values to be obtained at the damage.
Figure 2.2 Forward and backward wave signals at different locations. For the location: (a) (-30, 150) mm (true damage center location); (b) (-30, 200) mm; (c) (-30, 280) mm; (d) (200, 200) mm. The signals from backward wavefield have been amplified twenty times to be comparable to the signal from forward wavefield.

In Figure 2.3, simulation results using the pre-stack RTM and CCRTM imaging techniques for one circular damage located at (-30,150) mm is shown; the damage area is displayed as a red circle. The image is normalized by the maximum value in the results. In the current research, a linear piezoelectric wafer array is used, which cannot differentiate the propagating direction of the wave coming from either side of the linear array. The damage images in this paper display the top half of the plate. Both pre-stack RTM and CCRTM have been used here. The stacked image from 21 tests using RTM is shown in Figure 2.3(a), which only utilizes a single center frequency. The image from a single CCRTM test using all frequencies with only one actuation
at the center of the plate \((0, 0)\) mm is shown in Figure 2.3(b). The image from a single CCRTM is much better than that of the traditional RTM where the images are stacked by exciting all 21 actuators.

Figure 2.3 Images of the damage obtained from the simulation using (a) prestack RTM; (b) CCRTM.

Figure 2.4 illustrates the capability of the CCRTM technique in detecting multiple instances of damage. One circular damage center is located at \((-90, 150)\) mm. The other circular damage center is located at \((110, 90)\) mm. Both of the damages are with a diameter of 18 mm. The image of the farther damage is less distinct. The reason is that signals scattered from the farther damage are much weaker compared with the signals from the closer damage due to severe attenuation in composite plates. For the damage further away from the actuator, its stacked image from RTM is very weak while its image from CCRTM reveals much higher resolution.
Figure 2.4 Images of the multiple damages from the simulation using (a) prestack RTM; (b) CCRTM.

2.4 Simulation results – 2-D E-CCRTM imaging

In the simulation, only $A_0$ mode Lamb waves are used. For all the simulation results, the center frequency of the five-peaked tone-burst signal is 100 kHz, corresponding to a wavelength of 12 mm. 1-D array imaging based on Lamb waves will generate a mirrored ghost image along with the original damage. By designing the array into 2-D geometric configuration, this limitation could be overcome. With the LDV scanning system going to be introduced in the next section, the difficulties and the expense of mounting traditional PZT wafers onto the structures could be minimized. Moreover, since the LDV pointing area on the structures for data acquisition is with a diameter of 30 μm.
2.4.1 Comparison between CCRTM and E-CCRTM

The 2-D areal scan configuration is shown in Figure 2.5 using both CCRTM and E-CCRTM through the numerical study. The plate area is $600 \text{ mm} \times 600 \text{ mm}$. The thickness of the plate is 2 mm. The plate is modeled as aluminum T6061 and the material properties listed in Table 2.2. The only actuator is located at $(0 \text{ mm}, 0 \text{ mm})$. The areal scan $(26 \times 26)$ is with a uniform spacing of 2 mm. The areal scan configuration in this section is comparable with the LDV scanning points in the next experiment section. The areal scan covers a range of $(-25 \text{ mm}, 25 \text{ mm})$ in $x$ direction and $(-25 \text{ mm}, 25 \text{ mm})$ also in $y$ direction. Ambrozinski (Ambrozinski, 2014) proposed a system to design the array into several 2-D topologies efficiently. RTM should work with all the common topologies. Here, without losing generality, 2-D areal scan is formed with a rectangular shape.

![Figure 2.5 2-D areal scan configuration for imaging multiple sites of damage.](image-url)
In the finite difference algorithm, the plate is illuminated by a $601 \times 601$ finite difference mesh. The grid is $1\, mm \times 1\, mm$. The actuator and the 2-D scan points are modeled as the grids on the corresponding locations in finite difference. Two sites of damage are modeled as circular areas with the same diameter of $20\, mm$. The center of damage A is located at $(140\, mm, 60\, mm)$. The center of damage B is at $(70\, mm, -30\, mm)$. The distances to the center of the scanning area from the two damage centers are $152.3\, mm$ and $76.2\, mm$ respectively. The bending stiffness of the damaged area is modeled as 1/20 of that of the healthy area. All the Cartesian mesh grids within the circle were modeled with the reduced stiffness.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Density ($kg, m^{-3}$)</th>
<th>Young’s modulus (Gpa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>2700</td>
<td>68.9</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The imaging results using CCRTM and E-CCRTM are shown in Figure 2.6(a-b). The real damage shapes and locations are presented using either white or red circles in the figure depending on the local contrast. Both sites of the damage could be imaged using both methods. But the intensity of the further damage by CCRTM is much weaker than the closer one. This is due to attenuation caused by different radius of geometric spreading. In E-CCRTM, the wave attenuation is compensated by the incident waves and the distance to the scan area center, resulting in larger values around the further damage in the image. The image values represent intensity. Fifty percent of the intensity value corresponds to -$3\, dB$. The artifacts can be reduced to a very low level by applying a -$3\, dB$ threshold. In Figure 2.6 (c-d), all the values under the -$3\, dB$ threshold are set to 0 in the images from CCRTM and E-CCRTM. For CCRTM, the further damage area is all under -$3\, dB$, which is the reason that the further damage did not even displayed in the image in Figure 2.6(c). In the E-CCRTM, a
longer distance is compensated with a larger value. In Figure 2.6(d), both of the damage sites are imaged with high resolution using E-CCRTM. The threshold selection is certainly critical for damage quantification, and using statistical models such as probability of detection (POD) a more rigorous threshold can be selected but was not pursued in this paper.

Figure 2.6 Damage imaging results using 2-D areal scan. Original images from: (a) CCRTM; (b) E-CCRTM and images with -3 dB threshold from: (c) CCRTM; (d) E-CCRTM. The circles represent the damage shapes and locations.
2.4.2 1-D and 2-D scan comparison

The objective of this section is to compare the imaging results using 1-D and 2-D scan based on both CCRTM and E-CCRTM with various damage locations. The 2-D areal scan covers a range of (-50 mm, 50 mm) in the x direction and (-50 mm, 50 mm) also in the y direction. The 2-D areal scan has a number of 21 × 21. For comparison, the imaging results using a 1-D linear scan with the same aperture in x direction are also presented. The actuator for the 1-D scan is also located at (0 mm, 0 mm). The 1-D data acquisition points are located on the x-axis from -25 mm to 25 mm with the same spacing of 5 mm. A larger spacing is selected in this simulation section compared with that in the experiments on purpose to show less data acquisition points also work. Again, this paper is not devoted to the 2-D areal scan optimization.

In each simulation, one circular damage with a diameter of 20 mm is simulated with 1/20 the bending stiffness of the healthy area at different locations. The damage locations and the 2-D areal scan configuration are shown in Figure 2.7. The damage center locations are varying in each simulation and summarized in Table 2.3. The angle is defined starting counterclockwise from the positive x-axis.

<table>
<thead>
<tr>
<th>Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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</thead>
<tbody>
<tr>
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<td>-20</td>
<td>-40</td>
<td>-60</td>
<td>-80</td>
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<td>-100</td>
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</tr>
<tr>
<td>y (mm)</td>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>80</td>
<td>60</td>
<td>40</td>
<td>20</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Distance (mm)</td>
<td>140</td>
<td>102</td>
<td>107.7</td>
<td>116.6</td>
<td>128.1</td>
<td>141.4</td>
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<td>Angle (Degree)</td>
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<td>135</td>
<td>141.34</td>
<td>149.04</td>
<td>158.20</td>
<td>168.69</td>
<td>180</td>
</tr>
</tbody>
</table>
A few images using CCRTM and E-CCRTM are selected and shown for both 1-D and 2-D scan in Figure 2.8 with the damage locations. As expected, the 1-D CCRTM and E-CCRTM images all have a mirrored damage displayed. As the angle between the damage center and the $y$-axis becomes larger, the ghost image and the real image of the damage are moving closer to each other. In Figure 2.8(i-j), the mirrored one and the true one merged together. Compared with 1-D scan, the 2-D scan configuration eliminates the mirrored image and has
more stable imaging performance as the angle of the damage increasing. For 1-D CCRTM and 1-D E-CCRTM, the area behind the damage has relative large values too and can be referred to as tailed area. This is because the aperture of the scanning area is limited and some scattering waves from the back side of the circular damage are not fully acquired. As it can be seen from the Figure 2.8, designing the scan points into a 2-D shape can suppress the tailed imaging region. 1-D E-CCRTM has larger values in the tailed area than that of the 1-D CCRTM, which is due to the intensity compensation factors for further distances are larger in E-CCRTM.

![Figure 2.8 Selected damage images from CCRTM and E-CCRTM for both 1-D and 2-D configuration. Images for the damage at: (a-d) (0 mm, 140 mm); (e-h) (-100 mm, 40 mm); (i-l) (-100 mm, 0 mm).](image-url)
To quantify and evaluate the performance of 1-D and 2-D scan imaging using both E-CCRTM and CCRTM, three evaluation parameters are proposed. A few variables to be used are defined first. As mentioned earlier, the actuator is located at (0 mm, 0 mm). A distance from point \((x, y)\) to the actuator is defined as \(R = \sqrt{x^2 + y^2}\). For the damage located at \((x_t, y_t)\), the true damage center-to-actuator distance is noted as \(R_t (x_t, y_t)\). The angle between positive x-axis and a point-to-actuator segment is noted as \(\theta\). The true damage-to-actuator angle is \(\theta_t\). For a single damage case, the distance and angle of the maximum value on the image can be regarded as imaged damage center, that is \((R_c, \theta_c)\) in polar system and \((x_c, y_c)\) in Cartesian coordinates. The area of the true damage is \(A_t\). The imaged damage area can be measured as the preserved area in the -3 dB image and noted as \(A_c\).

(a) **Beam Pattern.**

The angular resolution could be evaluated using beam pattern (Ambrozinski, 2012) Since the incident wavefront is cylindrical in isotropic materials, the values on a circle centered at the actuation location with a radius of \(R_t\) can be subtracted from the images and be plotted in decibel verses angles. The values at \((R_c, \theta)\) are plotted where \(\theta\) starts from 0° to 360° with an increment of 1°. In this way, the imaging width compared with the true damage width can be evaluated and sidelobes generated by the arrays also could be displayed.

(b) **Damage Center Distance Error**

To evaluate the damage localization accuracy, a damage Center Distance Error (CDE) could be defined as:

\[
CDE = \frac{\sqrt{(x_c - x_t)^2 + (y_c - y_t)^2}}{R_t} \times 100\% 
\]  
(2.5)
(c) **Area Estimation Ratio**

To evaluate the damage quantification accuracy, Area Estimation Ratio (AER) could be defined as:

$$AER = \frac{|A_t - A_c|}{A_t}$$  \hspace{1cm} (2.6)

In 1-D scan application, the whole image is symmetrical along x-axis. Thus, $A_c$ for AER in 1-D scan imaging is taken as half of the area above -3 $dB$.

The Beam Pattern plots of the three selected cases in Figure 2.8 are shown in Figure 2.9. In Figure 2.9(a), the damage is located at 90°. All four imaging methods exhibit good beam width for 90° damage. 1-D scan imaging has a mirrored beam, and 2-D scan imaging suppresses the mirrored beam successfully. As the angle becomes larger, one beam width of 1-D scan becomes wider and the real beam and the mirrored beam are getting closer to each other. When the damage is located at 180°, the two beams from 1-D scan merge into a much wider beam than the true damage width. From the 2-D scan results, there is always one single main beam as the angle varies. The Beam Patterns from CCRTM and E-CCRTM can be compared for both 1-D and 2-D scan. The main beams are similar but the side-lobes from E-CCRTM are slightly smaller than those from CCRTM.
The CDE and the AER are plotted for the eleven sites of damage verses angles in Figure 2.10. For CDE, the CCRTM has good performance for both 1-D and 2-D scan, and the CDE is under 10%. The E-CCRTM considers the attenuation compensation which leads to even better performance in localization for both 1-D and 2-D scan, controlling the CDE under 5%. For the 2-D areal scan, the area estimation is very good using both CCRTM and E-CCRTM, and the AER is within 0.3. As it can be also seen from Figure 2.8, the imaged areas are larger than the true damage area using 1-D scan, which leads to the AER values for 1-D scan are high.

Figure 2.9 Beam Patterns for damage at: (a) (0 mm, 140 mm); (b) (-100 mm, 40 mm); (c) (-100 mm, 0 mm).

Figure 2.10 Quantitative imaging performance parameters for the 11 damage sites: (a) damage center distance error (CDE); (b) area estimation ratio (AER).
In summary, the performance of the 1-D scan is good for the areas close to the perpendicular bisector of the scanning line and becomes less reliable for the areas away from the perpendicular bisector. 2-D scan shows reliable performance for all the three evaluation parameters and no mirrored ghost images of the damage are present. E-CCRTM has certain degree of improvement in comparison with CCRTM in damage localization and suppression of side-lobes. In summary, 2-D E-CCRTM exhibits stable omnidirectivity in imaging and quantification.
Chapter 3

RTM Experiments
The chapter presents a two-dimensional (2-D) non-contact areal scan system to image and quantify multiple sites of damage in isotropic plates using an enhanced zero-lag cross-correlation reverse-time migration (E-CCRTM) technique. The system comprises a single piezoelectric actuator mounted onto the structure and a laser Doppler vibrometer (LDV) for scanning a region for capturing the scattered wavefield in the vicinity of the PZT. The experimental results show that the 2-D E-CCRTM has robust performance to image and quantify multiple sites of damage in large area of the plate using a single PZT actuator with a nearby areal scan using LDV.

3.1 Experimental damage imaging using 1-D CCRTM in a composite plate

3.1.1 Experimental setup for 1-D CCRTM for composites

The experimental setup is shown in Figure 3.1. The data acquisition system consisted of a function/arbitrary waveform generator (Hewlett Packard 33120A), a wideband amplifier (Krohn-Hite 7602), and a digital phosphor oscilloscope (Tektronix DPO 2024B). The linear piezoelectric array is surfaced-mounted on the composite laminate. The piezoelectric wafers (Steiner & Martins, Inc.) are 7 mm in diameter and 0.3 mm in thickness with a resonant frequency in the radial direction of 300 kHz ±10 kHz. The adhesive layer between the PZT wafer and the composite plate is superglue with brand name ‘Gorilla’. Each PZT ceramic can serve either as an actuator or a sensor. The composite plate has a dimension of 800 mm × 800 mm × 2 mm. The composite material used in this study is AS4/8552-2 graphite/epoxy, and the
layup of the composite panel is \([45/0/90/45]_2s\). Material properties of one lamina are shown in
Table 3.1, which was provided by NASA Langley Research Center.

![Figure 3.1 Experimental setup of the zero-lag cross-correlation reverse-time migration (CCRTM) system.](image)

<p>| | | | | | | | | |</p>
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<tr>
<td></td>
<td>(E_1)</td>
<td>(E_2)</td>
<td>(E_3)</td>
<td>(G_{12})</td>
<td>(G_{13})</td>
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</tr>
<tr>
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<td>3.67</td>
<td>0.39</td>
<td>0.39</td>
<td>0.11</td>
</tr>
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A five-peaked Hanning-windowed tone burst is generated by the waveform generator and
amplified prior to being sent to the actuator. The advantage of this transient loading is that it has
a compact form in the time domain and its frequency components concentrate at a center
frequency \(f_c\); thus, the excited wave propagates in a highly non-dispersive manner. The peak-to-
peak voltage of the input signal to the PZT is 40 V for all the experiments. For each excitation,
the sampling frequency is 25 \( MHz \) and 16 measurements were taken and averaged for all the experiments.

### 3.1.1.1 Frequency tuning

In this research, the lowest mode quasi-flexural wave (\( A_0 \) mode) is used in both the simulations and experiments. Thus, it is desirable to select a frequency for which the amplitude of the \( S_0 \) mode is much smaller than that of the \( A_0 \) mode. A theoretical framework for frequency tuning was built by Santoni et al. (Santoni, 2007).

An experiment on frequency tuning is performed prior to the damage imaging experiments. In this test, one piezoelectric transducer served as an actuator while a second served as a sensor. The center frequency \( f_c \) of the actuation signals ranges from 20 \( kHz \) to 200 \( kHz \) in each test with a step size of 20 \( kHz \). The distance between the two PZT wafers was 114.3 \( mm \) (4.5 in), and the line segment between the two PZT wafers is parallel to the y-axis in the predefined coordinate system. The signals received by the oscilloscope are shown in Figure 3.2. Based on the dispersion curve, the group velocity of the \( S_0 \) mode is much larger than that of the \( A_0 \) mode. Thus, from the test results, for center frequency ranges from 20 \( kHz \) to 60 \( kHz \), there is almost no \( S_0 \) mode signal observed. From 80 \( kHz \) to 160 \( kHz \), the \( S_0 \) mode amplitude is less than or similar to that of \( A_0 \) mode signal. Beyond 180 \( kHz \), the \( S_0 \) mode signal becomes larger than the \( A_0 \) mode signal. Since the \( A_0 \) mode signal is the signal of interest in this study, \( f_c \) is chosen as 60 \( kHz \), with a corresponding wavelength of 18.3 \( mm \).
3.1.1.2 Experimental setup – linear piezoelectric array

For damage imaging using the linear PZT array, 21 piezoelectric wafers have been used to scan an 800 mm×800 mm area. Each actuator excites the fundamental mode of the transient incident wave into the test panel. The origin of the coordinate system is set at the center of the plate, and
the PZTs are located from (-80, 0) mm to (80, 0) mm, with a spacing of 8 mm. All remaining sensors are used to collect scattered waves. The damage is simulated by a pair of magnets with a diameter of 18 mm, located at (-30, 150) mm on the top and bottom of the plate. For the experiments, the same data acquisition system from the frequency tuning test is utilized where only PZT wafers are used for actuation and sensing.

3.1.1.3 Experimental setup – hybrid PZT/LDV system

A few challenges remain when using piezoelectric wafers to perform damage identification. The process of attaching and connecting piezoelectric wafers can be labor intensive and time consuming, and it requires precise positioning of the linear transducer array. If there are not enough channels for acquiring signals, switching between a limited number of acquisition channels is needed. In practice, inspections with permanently mounted sensors are not always permissible. The above concerns could be overcome if the inspection can be done without contacting the structure. To this end, a laser Doppler vibrometer (LDV) based non-contact structural health monitoring system is used to apply both RTM and CCRTM.

The experimental setup is illustrated in Figure 3.3. A data management system (DMS - an Industrial PC), a vibrometer controller (Polytec OFV-5000), and a vibrometer single point sensor head (Polytec OFV-505) are used as the LDV data acquisition system. Due to the Doppler effect, the transverse velocity of a vibrating point generates a frequency modulation, using which the velocity information could be recovered through a velocity decoder (VD-09) from the measurement. Similar to using a piezoelectric wafer array, a PZT ceramic wafer still functions as the actuator, excited by the same function generator with the same input signal and with the same amplification rate. For receiving signals, however, the laser head, which points to
one location perpendicular to the plate surface, will send and receive the laser signals and act as a movable sensor. Ten scanning points on each side of the actuator are set up with an adjacent distance of 8 mm for comparison with the piezoelectric wafer results. For future study, more scanning points with a closer distance could be used to apply higher frequency signals and, hence, obtain a higher resolution image. As shown in Figure 3.3, the velocity will be obtained through the vibrometer controllers. To accurately control the location of the data acquisition point, a 2-Axis translation stage is built. Once the laser head is well adjusted, it will be fixed the exact location for the entire RTM or CCRTM experiment. Mounted onto the 2-D scanning station, the laser head can be moved electronically such that the laser will point to the next data acquisition point accordingly with precision better than 0.1 mm.

![Figure 3.3 Hybrid experimental setup using PZT/LDV.](image)

The piezoelectric actuator, LDV data acquisition points, and the simulated damage have the same relative positions compared with the example in the last Section. The more reflective the
inspected surface, the better the signal to noise ratio (SNR). The surface of the composite plate in this study is attached to two strips of highly reflective tape from Polytec Inc. on both sides of the PZT wafer to enhance the laser signal strength. The relative position between the scanning points and the damage is consistent with that discussed in the Section concerning the piezoelectric linear array.

3.1.2 Experimental results using 1-D CCRTM in a composite plate

This section presents the experimental damage imaging results using RTM and CCRTM. First, the prestack RTM results are obtained using piezoelectric array only, and the 11th element in the array is selected as the actuator for CCRTM. In the PZT/LDV experiment, one actuation was used and all the data acquisition is accomplished using the LDV for both RTM and CCRTM. The spacing and the number of elements in the linear array could be optimized, but, in general, as long as the spacing is smaller than half of the wavelength corresponding to the highest frequency component, the spatial Nyquist rate will be satisfied and the array will have reliable performance. The optimization of the array is not the main purpose of this paper, and thus it will not be pursued.

To quantify the damage localization accuracy for the two methods, an evaluation parameter for the imaged damage center can be defined. Under the coordinate system described in the experimental setup section, the true damage location can be noted as \((x_t, y_t)\). After the damage imaging is obtained, for a single damage case, the maximum value on the image can be taken as the imaged damage center \((x_c, y_c)\). The distance from the true damage center to the origin is defined as \(R = \sqrt{x_t^2 + y_t^2} \).
The damage center distance error (CDE) can then defined as:

\[
CDE = \frac{\sqrt{(x_e - x_t)^2 + (y_e - y_t)^2}}{R_t} \times 100\%
\]  \hspace{1cm} (3.1)

3.1.2.1 Experimental results – linear piezoelectric array

Both prestack RTM and CCRTM were used to interpret the experimental data in Figure 3.4. For CCRTM, results are obtained with the actuator located at (0, 0) \( mm \). From both figures, values on the image at the vicinity of the damage are relatively large and are displayed clearly. The CDE for Figure 3.4 is 5.8 % for RTM and 8.5 % for CCRTM. As can be seen from the figures, the apparent damage image is close to the front boundary (with respect to the actuator) of the damage. Since CCRTM uses all the frequency content while RTM only uses the center frequency, the CCRTM images have a larger area. It is understandable that the most in-phase part of the forward and backward wavefield are at the boundaries of the damage. Also the damage might possess strong reflectivity, and the scattered wave from the back damage boundary might be very weak. The accuracy of the area estimation has been enhanced by using CCRTM with only one actuator such that pre-stacking of images in conventional RTM is not needed.
3.1.2.2 Experimental results – hybrid PZT/LDV system

Although sixteen measurements were taken for each signal and averaging is performed for the piezoelectric array, the signal to noise ratio (SNR) of the wave signals collected by the piezoelectric wafers is not quite satisfactory, which might be the reason that the area of the damage indicated in the figures is not considerably accurate. Better SNR and imaging results can be obtained using the Laser Doppler Vibrometer (LDV) system.

RTM and CCRTM images of the damage are obtained by the LDV based non-contact structural health monitoring system and presented in Figure 3.5. The noise region in the figure is significantly reduced compared with the results from piezoelectric linear array. The CDE using the LDV for receiving is 14.3 % and 13.1% for RTM and CCRTM, respectively. The CDE values in this section are larger than those obtained when using the linear piezoelectric array. This discrepancy may be caused by the imperfect alignment of the plate on the 2-D scanning station or possibly by slight errors in the dispersion calculation using the finite difference
method. The CCRTM results provide more accurate area imaging about the damage area than RTM. Furthermore, the data acquisition spacing could be smaller with this system which would allow for much higher frequencies to be used.

![Images of the damage from the hybrid PZT/LDV experiments using (a) RTM; (b) CCRTM.](image)

**Figure 3.5** Images of the damage from the hybrid PZT/LDV experiments using (a) RTM; (b) CCRTM.

### 3.2 Experimental damage imaging using 2-D E-CCRTM in isotropic plates

#### 3.2.1 Experimental setup

The experiments are carried out using the hybrid LDV/PZT scanning system (Figure 3.6). The system consists of an arbitrary function generator (Tektronix AFG3000 Series), a wideband power amplifier (Krohn-Hite 7602), a data management system (DMS - an Industrial PC), a vibrometer controller (Polytec OFV-5000), a vibrometer single point sensor head (Polytec OFV-505), a network controller (IAI ROBO NET) and a 2-D translational stage (IAI ROBO Cylinder).
Figure 3.6 Experimental setup for the hybrid PZT/LDV scanning system for imaging two sites of damage.

$A_0$ mode of Lamb waves is used in this test. A five-peaked tone-burst signal is predefined in the computer and transferred to the AFG through the ArbExpress waveform editing tool in advance. The DMS also contains a data acquisition board and an internal function generator board. The Polytech software can send a trigger signal through the internal function generator board to the AFG, and the AFG will send the five-peaked signal simultaneously to both the power amplifier then to the piezoelectric wafer and a reference signal back to the DMS to start the measurement. The laser head is mounted on the 2-D translational stage controlled by the ROBO NET to move freely. The laser is pointing to the plate perpendicularly to measure the out-of-plane velocity of a single point. Under our usage, the point is a small circle with a
diameter around 30 $\mu m$. Due to the Doppler effect, the velocity of a vibrating point generates a frequency modulation, using which the velocity information could be recovered through a velocity decoder (VD-09) from the measurement. For each point, 40 measurements were taken and averaged in the experiments to increase the SNR ratio.

A 1.6 $mm$ thick T-6061 aluminum panel was prepared as the test specimen. The area of the plate to inspect is 600 $\times$ 600 $mm$. The actuator is located at the center (0, 0) of the inspection area. The actuator is a piezoelectric wafer (Steiner & Martins, Inc.) with dimensions of 7 $mm$ in diameter and 0.3 $mm$ in thickness. The radial resonance frequency is 300 $kHz$. The LDV scans a rectangular area of 50 $mm$ $\times$ 50 $mm$. The 2-D areal scan covers a range of (-65 $mm$, -15 $mm$) in $x$ direction and (5 $mm$, 55 $mm$) in $y$ direction. The reason of changing the areal scan relative to the actuator compared with simulation is simply because the piezoelectric wafer has a dimension of 7 $mm$, which is larger than the spacing (2 $mm$) between two adjacent scanning points. Each of the scanning point records a signal and all the scattered signals will be reversed in time and extrapolated in the finite difference algorithm. No focusing needs to be achieved physically through the hardware. The dispersion effect will be compensated automatically in the algorithm such that no extra efforts are needed. The actuator and the LDV scanning region arrangement remains the same for the two experiments. The peak-to-peak voltage for the amplifier is 2 $V$ for all the experiments. After the amplification, the input voltage signal to the piezoelectric wafer is with a peak-to-peak voltage of 60 $V$. 
3.2.2 **Imaging a single site of damage**

A five-peaked tone-burst signal with a center frequency of 120 kHz is selected as the input signal. The wavelength of $A_0$ mode corresponding to the center frequency is 10.7 mm. The selected frequencies in this research are all under the cutoff frequency. At this frequency, $A_0$ mode Lamb waves generated by PZT are dominant in amplitude compared with $S_0$ mode, especially when the LDV only measures the out-of-plane velocity of the plate. This property of the system leads to easy separation of the $A_0$ and $S_0$ mode of Lamb waves.

In this experiment, damage A is simulated by a pair of circular magnets with a diameter of 25.4 mm. The center of the damage A is located at (120 mm, 40 mm). The usage of magnet pairs to simulate the damage is legitimate, since the reflected waves from real damage, i.e. damage induced by a large thickness change, are often stronger than from magnet pairs. The reflection is caused by the phase velocity difference hence the acoustic impedance mismatch of the damaged and undamaged areas. The E-CCRTM imaging results are shown in Figure 3.7. The damage area is well imaged. The other area in the image might be due to the environmental noise and side-lobes of the areal scan, which disappears in the -3 dB image. The tailed area of the damage image might be due to some scattered waves missing from the limited aperture at this frequency. The CDE is 1.28 % and the AER is 1.97 for the experiment.
Figure 3.7 Experimental damage imaging results using 2-D E-CCRTM for damage A at (-12 cm, 4 cm): (a) original image; (b) image with -3 dB threshold.

3.2.3 Imaging multiple sites of damage

The other experiment designed and performed using E-CCRTM is with two shapes of damage (Figure 3.6). The damage A is at the same location in previous case. Damage B is a rectangular with a length of 19.05 mm and a width of 12.7 mm. The center of the damage B is at (136.0 mm, 71.5 mm). The width of the rectangular has an angle of 45° to the x-axis. A five-peaked tone-burst signal with a center frequency of 100 kHz is chosen as the input signal. The corresponding wavelength is 13.2 mm.

The two sites of damage are imaged in Figure 3.8. Both of the damage areas are imaged. The locations of the two damage centers are listed in Figure 3.8(b). Damage A is imaged with a very good accuracy for both localization and area sizing. The change of the frequency from 120 kHz to 100 kHz clearly reduced the tailed area of damage A in Figure 3.7. For damage B, the localization results are good but the shape of the damage is not rectangular. In ultrasound
imaging, the corners are always difficult to be imaged, requiring a considerably small wavelength. In the radial direction towards the LDV scanning region center, the damage is elongated compared with the true damage. This might be because some wave energy is transmitted through the first length of the damage, reflected back at the second length and also received by the LDV scanning region. There might be more in-phase wave packets between the forward wavefield and the backward wavefield during RTM imaging along the center-to-actuator direction for damage B.

Figure 3.8 Experimental multiple damage imaging results using 2-D E-CCRTM for damage A and B: (a) original image; (b) image with -3 dB threshold.

3.3 Experimental damage imaging using 2-D E-CCRTM in composite plates

The same data acquisition system as in the previous section has been utilized for composite damage imaging. The composite plate has a dimension of 685 mm × 685 mm × 2.28 mm. The composite material used in this study is AS4/3502 graphite/epoxy, and the layup of the composite panel is [±45/0]_2s. Impacted damage was generated by a drop ball-test. Material
properties of one lamina are shown in Table 3.2, which was provided by NASA Langley Research Center.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$E_1$ (GPa)</th>
<th>$E_2$ (GPa)</th>
<th>$E_3$ (GPa)</th>
<th>$G_{12}$ (GPa)</th>
<th>$G_{13}$ (GPa)</th>
<th>$G_{23}$ (GPa)</th>
<th>$v_{12}$</th>
<th>$v_{13}$</th>
<th>$v_{23}$</th>
<th>$\rho$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS4/3502</td>
<td>127.6</td>
<td>11.3</td>
<td>11.3</td>
<td>5.97</td>
<td>5.97</td>
<td>3.75</td>
<td>0.30</td>
<td>0.30</td>
<td>0.34</td>
<td>1578</td>
</tr>
</tbody>
</table>

One actuator was mounted onto the center of the plate. The 2-D areal scan covers a range of (-50 mm, 50 mm) in the $x$ direction and (-50 mm, 50 mm) also in the $y$ direction in the backside of the plate. The 2-D areal scan has a number of 26 × 26. The experimental system was shown along with the C-scan image of the damage.

Figure 3.9 Hybrid PZT/LDV scanning system with C-scan of the barely visible impact damage (BVID).
Different frequencies of Lamb waves will lead to various scattering patterns when interacting with damage. Three frequencies (50, 100, 200 kHz) were employed to the 5-cycle Hanning-windowed toneburst excitation. The damage imaging results were shown with the C-Scan and B-scan results in Figure 3.10.

Figure 3.10 (a) C-scan and B-scan of the BVID. 2D E-CCRTM using a center frequency: (b) 50 kHz; (c) 100 kHz; (d) 150 kHz.

Zoomed-in images of the imaging results with the comparison with the C-scan image are shown in Figure 3.11.
As can be seen from the figure, the damage locations were identified. The CDE of 50 kHz, 100 kHz and 150 kHz are 1.57%, 2.38% and 2.95% respectively. As the frequencies increase, the damage area was better quantified, though the detailed characteristics of the delamination were not shown in the used lower ultrasonic frequency range. 200 kHz signal was also used where the reflected waves interacts with piezo causing strong scattering in this frequency.
Thus, the imaging results had strong artifacts and were not shown. With a small areal scan and Lamb waves, the 2D E-CCRTM provide fast and stable damage imaging performance.
Chapter 4

DORT-MUSIC Theory and Simulation
A Lamb wave-based, subwavelength imaging algorithm is developed for damage imaging in large-scale, plate-like structures based on a decomposition of the time-reversal operator (DORT) method combined with the multiple signal classification (MUSIC) algorithm in the space-frequency domain. In this study, a rapid, hybrid non-contact scanning system was proposed to image an aluminum plate using a piezoelectric linear array for actuation and a laser Doppler vibrometer (LDV) line-scan for sensing. The physics of wave propagation, reflection, and scattering that underlies the response matrix in the DORT method is mathematically formulated in the context of guided waves. Singular value decomposition (SVD) is then employed to decompose the experimentally measured response matrix into three matrices, detailing the incident wave propagation from the linear actuator array, reflection from the damage, and followed by scattering waves toward the linear sensing array for each small damage. The SVD and MUSIC-based imaging condition enable quantifying the damage severity by a ‘reflectivity’ parameter and super-resolution imaging. Mathematical models were formulated and the underlying physics of the DORT method were explored in the context of guided waves.

4.1 DORT and MUSIC theory

This section summarizes the theoretical basics of the DORT-MUSIC technique and the mathematics of the underlining physics. The description is specific to the application of Lamb waves in plate-like structures, but the process is similar to the physical problems in acoustics, bulk waves in solids and electromagnetic waves. Back propagation in DORT uses the eigenvectors associated with the signal space to image each damage instance, while
MUSIC-based imaging condition uses singular vectors corresponding to the complementary noise space to illuminate multiple sites of damage with super-resolution. Diffraction limits seen in traditional damage imaging can be overcome by the MUSIC imaging algorithm.

4.1.1 Formation of the transfer matrix

A linear actuator array located at $x_i^a$, $i=1, 2, \ldots, M$ and the sensors at $x_j^s$, $j=1, 2, \ldots, N$ are considered here as shown in Figure 4.1. $K$ sites of damage are assumed to exist in the field. For Lamb wave application, damage is assumed to be without variation in the plate thickness direction such that the damage can be characterized with the boundary $\partial \tau$ of the damage containing the two dimensional coordinates, $x^v$.

Figure 4.1 Schematic showing the wave propagation with damage location as well as the corresponding actuator and sensor array placement.
The DORT-MUSIC algorithm fundamentally relies on SVD of the measured transfer matrix \( K \) to be introduced. It is well-known that the waveform at a location can be obtained by integration of the input signal with the Green’s function between initial and ending locations in the time domain, or equivalently, multiplication between the input signal and the corresponding Green’s function in the frequency domain. All the data and signal processing is performed in the frequency domain and is associated with a single frequency \( \omega \). Since all the expressions are associated with the given frequency \( \omega \), \( \omega \) will be omitted from herein.

For instance, the received signal at the \( j \)th sensor with the input signal \( e_i \) at the \( i \)th actuator can be expressed as:

\[
 r_{ji}(x_j^s) = K_{ji}(x_j^s; x_i^a)e_i(x_i^a)
\]

where the transfer function involving scattering between \( j \)th sensor and the \( i \)th actuator can be written as:

\[
 K_{ji} = \int_{\partial \tau} g(x_j^s; x^v) V(x^v) g(x_i^a; x_i^s) dx^v
\]

where \( g(a; b) \) is the Green’s function from location \( b \) to \( a \), \( V(x) \) is the scattering potential (Devaney, 2012), relating the scattering caused by damage as the secondary sources with the total wavefield. When multiple sites of damage are distinct, i.e., not too close to each other, the interaction between different sites of the damage are weak and can be classified as well-resolved targets (Gruber, 2004). For small, well-resolved damage, the scattering potential can be expressed as:

\[
 V(x) = \sum_{k=1}^{K} v_k(x_i^s) \delta(x - x_i^s)
\]
where $\delta$ is the Kronecker delta, $x_i^r$ is the small damage centers, $k=1, 2, \ldots, K$ ($K \leq \min\{M, N\}$), and $v_k$ represents the apparent reflectivity at the damage location $x_i^v$. Then the Equation (4.2) can be rewritten as:

$$K_{ji} = \sum_{k=1}^{K} g(x_j^r; x_k^r) v_k(x_i^v) g(x_i^r; x_k^a)$$

(4.4)

The above equation can be written in matrix format:

$$K = G^r(x^r; x^v) V(x^v) G^a(x^v; x^a)$$

(4.5)

where $x^a, x^r, x^s$ represent the locations of actuators, damage and sensors respectively, and the Green’s function matrices, $G^a$ and $G^s$, associated with the locations of the actuators, damage and sensors can be also related to their vector form:

$$G^a = [g_1^a, \ldots, g_{\min\{M,N\}}^a]$$

(4.6a)

$$g_k^a = [g(x_k^r; x_k^a), \ldots, g(x_k^v; x_N^a)]^T \text{ for } k=1, \ldots, K$$

(4.6b)

$$G^s = [g_1^s, \ldots, g_{\min\{M,N\}}^s]$$

(4.6c)

$$g_k^s = [g(x_1^r; x_k^r), \ldots, g(x_M^v; x_k^r)]^T \text{ for } k=1, \ldots, K$$

(4.6d)

where the number of vectors in $G^a$ and $G^s$ is tacitly assumed to be $\min\{M, N\}$ to include the noise subspace to be introduced; the $L_2$ norm of the vectors can be denoted as $\| \|$; superscript $T$ denotes the transpose operation. From the above discussions, a multiple-input and multiple-output (MIMO) data acquisition method is used and can be described as the propagation of the wave signals from the actuators, the interaction with the damage, and, finally, the propagation from the damage to the sensors. To simplify the expressions, the electromechanical responses of the transducers are not included in the expressions. Transfer functions for both excitation and reception could be added to describe these two processes. In
practice, the electromechanical responses do exist and might cause a small phase lag compared with the input signal.

The well-known time-reversal operators (TRO) are defined as

\[
TRO = K^H K
\]

\[
TRO_2 = KK^H
\]

where \( H \) denotes the transpose conjugation operation.

Prada and Fink (Prada, 1994) proved that the complex conjugation of the column vector \( \bar{g}_k^a = [g(x_k^1; x_1^a), \ldots, g(x_k^N; x_N^a)]^H, k = 1, \ldots, K \) in \( G^a \) is exactly one of the eigenvectors of the \( TRO \).

That is,

\[
K^H K \bar{g}_k^a = |v_k|^2 \| \bar{g}_k^a \|^2 \| g_k^a \|^2 \bar{g}_k^a \quad \text{for } k = 1, \ldots, K
\]

Similarly, the eigenvectors of \( TRO_2 \) can be mathematically proven to be one of the predefined Green’s functions in vector form:

\[
KK^H g_k^a = |v_k|^2 \| g_k^a \|^2 \| g_k^a \|^2 g_k^a \quad \text{for } k = 1, \ldots, K
\]

The two \( TROs \) are Hermitian and positive-semidefinite such that their eigenvalues are either positive or zero. The rank of the \( TROs \) is \( K \). The non-zero eigenvalues of the \( TRO \) and \( TRO_2 \) are identical:

\[
\lambda_k = |v_k|^2 \| g_k^a \|^2 \| g_k^a \|^2 \quad \text{for } k = 1, \ldots, K
\]

Lehman and Devaney (Lehman, 2003) demonstrated the EVD of the two \( TROs \) can be simplified as the SVD of the transfer matrix \( K \):

\[
K = \hat{G} \cdot \hat{V} \hat{G}^a^H
\]

where \( \hat{G} = [\hat{g}_1^a, \ldots, \hat{g}_{\min(M,N)}^a] \) and \( \hat{G}^a = [\hat{g}_1^a, \ldots, \hat{g}_{\min(M,N)}^a] \) are the left and right singular vectors in the matrix form, \( \hat{V} \) is a diagonal matrix with the non-zero singular value \( \sigma_k \) on the diagonal.
(for $k=1, \ldots, K$), associated with the signal subspace. The remaining singular values are zero when $k = K+1, \ldots, \min\{M, N\}$. The corresponding remaining singular vectors are associated with the noise subspace and are orthogonal to the singular vectors with the signal subspace.

The relationship between the SVD of $K$ and the EVD of the two $TRO$s are

$$
\hat{g}_k = \frac{\tilde{g}_k}{\|\tilde{g}_k\|} \text{ for } k=1, \ldots, K \quad (4.11a)
$$

$$
\hat{g}'_k = \frac{g'_k}{\|g'_k\|} \text{ for } k=1, \ldots, K \quad (4.11b)
$$

From above equations, it can be easily seen that the singular vectors of $K$ are just a normalized version of the eigenvectors of the $TRO$s. The singular values are the square root of the corresponding eigenvalues.

$$
\sigma_k = \|v_k\| \frac{\|g_k\|}{\|\tilde{g}_k\|} \text{ for } k=1, \ldots, K \quad (4.12)
$$

The apparent reflectivity of the $k^{th}$ small damage can then be reversely calculated using the singular values:

$$
|v_k| = \sigma_k \frac{\|g_k\|}{\|\tilde{g}_k\|} \text{ for } k=1, \ldots, K \quad (4.13)
$$

$|v_k|$ is a critical index for damage severity.

### 4.1.2 Underlying physics of the SVD of the transfer matrix $K$

From Equation (4.8a), the underlying physics of $TRO$’s eigenvectors can be described as the transfer functions in vector form from one small damage to all the actuators or, equivalently, the time-reversed version of the transfer functions in vector form, from all the actuators to one small damage at the given frequency, $\omega$. This is referred to as the ‘actuators-to-damage process’. Similarly, the underlying physics of $TRO_2$’s eigenvectors can be described as the transfer functions in vector form, from one small damage to all receiver locations. This will be referred to as the “damage-to-sensors process” in this paper.
From Equations (4.11a-b), the left and right singular vectors are the normalized Green’s functions in vector forms. Based on this, the DORT-MUSIC damage imaging technique only requires the SVD of the transfer matrix $K$, as will be shown in the next section. From Equation (4.13), the reflectivity coefficient $|v_k|$ is just the singular values divided by the $L_2$ norms of the Green’s function vectors. In this way, the reflectivity $|v_k|$ can be calculated with the SVD of the transfer matrix $K$ such that the severity of the damage can also be estimated.

4.1.3 DORT-MUSIC imaging method

From Equation (4.11a-b), the left and right singular vectors are the normalized Green’s functions associated with the locations of the actuators, the damage, and the sensors, all in vector forms. In the case of the actuation and sensing performed by two different arrays, the two propagating processes can be separated through SVD and, accordingly, the damage imaging can be conducted separately. Finally, both images can be combined together into a final image representing the damage.

The background Green’s functions for pristine structures in vector form, starting from the actuators and sensors and ending at an arbitrary location, $x$, in the field are, respectively:

$$g^a(x) = [g(x; x_1^a), \ldots, g(x; x_N^a)]^T$$

$$g^s(x) = [g(x; x_1^s), \ldots, g(x; x_M^s)]^T$$

From the orthogonal property of eigenvectors (Lehman, 2003), the back-propagation imaging conditions for DORT can be obtained as:

$$\hat{g}_k^a \cdot g^a(x) = \begin{cases} \|g^a_k\| & x = x_k \text{ for } k = 1, \ldots, K \\ 0 & x \neq x_k \end{cases}$$

(4.15a)
\[
\hat{g}^t \cdot g^s(x) = \begin{cases} \|g^t\| & x = x_i \text{ for } k=1, \ldots, K \\ 0 & x \neq x_i \end{cases}
\] (4.15b)

Since the singular vectors associated with the singular values beyond the \(k^{th}\) value are all assumed to be generated from noise (Lehman, 2003). The MUSIC-based imaging condition can then be formulated using the noise-space signals, utilizing the randomness of the noise space to achieve subwavelength imaging. From the derivations in the last section, the imaging can be achieved separately with the actuator-to-damage and the damage-to-sensors processes respectively as:

\[
I^a(x) = \frac{1}{\sum_{k=K+1}^{\min[M,N]} \hat{g}_k^a \cdot g^a(x)}
\] (4.16a)

\[
I^s(x) = \frac{1}{\sum_{k=K+1}^{\min[M,N]} \hat{g}_k^s \cdot g^s(x)}
\] (4.16b)

To use the superior cross-range resolution of both of the arrays, the final image can be obtained by the product of Equations (4.16a) and (4.16b):

\[
I(x) = \frac{1}{\sum_{k_1=K+1}^{\min[M,N]} \hat{g}_{k_1}^a \cdot g^a(x) \sum_{k_2=K+1}^{\min[M,N]} \hat{g}_{k_2}^s \cdot g^s(x)}
\] (4.17)

### 4.2 Numerical simulation using DORT-MUSIC for isotropic plates

In this section, the physical implication of the singular value decomposition (SVD) of the transfer matrix \(K\) is verified through numerical simulation. The difference between back-propagation and MUSIC imaging conditions are discussed for the DORT method using Lamb waves.

The set of simulations was designed and each simulation was performed to study the underlying physics of the singular values associated with the transfer matrix. A finite
difference algorithm was employed to simulate the quasi-flexural wave propagation based on Mindlin plate theory. The Mindlin plate theory had also been extended to composite structures (Lih, 1995; Lih, 1996). Details about the finite difference algorithm can be found in (Lin, 2001). The material properties used in the simulation are the same as those listed in Table 2.2 and the same plate thickness of 1.6 mm is used. The material properties were assigned to every grid in the finite difference algorithm. The damage was modeled as a region of inhomogeneity containing grids possessing with different material properties than those of the pristine structure.

In the simulation, the plate was covered by a 601 × 601 finite difference mesh. The grid is 1 mm × 1 mm. A 21 element linear array was used with a spacing of 5 mm from -50 mm to 50 mm on the x axis. Each element served both as an actuator and a sensor, and was represented by a single grid at the locations discussed above. A 5-cyclic Hanning windowed toneburst was adopted as the excitation signal with a center frequency of 100 kHz, corresponding to a wavelength of 12 mm. Since the actuators-to-damage and the damage-to-sensors processes will be identical, all the images generated in this section are only from the damage-to-sensors processes; that is, merely using Equation (4.15b) and (4.16b) for damage imaging.

4.2.1 Damage reflectivity quantification

The physical meaning of the singular values in the DORT-MUSIC algorithm is verified in this section through simulation. Since damage shape and size influence the scattering patterns when the Lamb waves interact with the damage, a single grid with a fixed location was modeled as one site of small damage with degraded material properties. Since in Equation (4.1) the input signals and the received signals are known after each simulation, $K$ can be
calculated inversely. The small damage was located at (0, -100) mm, and the retained percentage of its Young’s modulus $E$, referred to as ‘$\gamma$’, was varied as shown in Table 4.1. Twelve cases were studied. The acoustic impedance associated with the $A_0$ phase velocity of the pristine material is

$$Z_1 = \rho c_p (\gamma = 1)$$  \hspace{1cm} (4.18)

The acoustic impedance associated with the $A_0$ phase velocity of the damaged region is defined as

$$Z_2 = \rho c_p (\gamma)$$  \hspace{1cm} (4.19)

The corresponding reflectivity coefficients at normal incidence for large interface (compared with the input wavelengths) is defined as

$$R = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$  \hspace{1cm} (4.20)

The absolute values of the corresponding reflectivity coefficients $|R|$ are also listed in Table 4.1. The comparison results will be presented in discussion section.

### Table 4.1 The percentages of Young’s modulus $E$ of the damage, $\gamma$, and the corresponding reflectivity coefficients for the twelve test cases

<table>
<thead>
<tr>
<th>Case number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>1%</td>
<td>5%</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
<td>50%</td>
<td>60%</td>
<td>70%</td>
<td>80%</td>
<td>90%</td>
<td>95%</td>
</tr>
<tr>
<td>Reflectivity</td>
<td>0.66</td>
<td>0.45</td>
<td>0.34</td>
<td>0.24</td>
<td>0.18</td>
<td>0.13</td>
<td>0.11</td>
<td>0.08</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>

An image of the damage for Case 8 is presented in Figure 4.2 as a DORT-MUSIC imaging example, where the Young’s modulus of the damage is 60% percent of the pristine structure. The small damage at (0, -100) mm was highlighted with excellent accuracy. A ghost damage image was also generated at (0, 100) mm, because the array was linear and the array cannot discern which side of the array the scattering waves are coming from (Ambroziński, 2014).
Figure 4.2 A single small damage DORT-MUSIC imaging example. (Case 8, damage located at (0,-100) mm, $\gamma=60\%$).

Figure 4.3 shows the obtained twenty one singular values for Case 8 as an example of the 12 cases. Only one significant singular value is significant, corresponding to the single damage in the field. Similar phenomenon was seen in all other eleven cases. Thus, in all twelve cases, the noise space is associated with the singular values other than the largest one. The first singular values from all of the twelve cases are plotted in Figure 4.4. The first singular value decreases as $\gamma$ increases. This makes sense because, as $\gamma$ approaches 100\%, there is less distinct difference between the damaged and pristine regions. Therefore, the singular values can potentially be related to damage severity.
Figure 4.3 Singular values obtained for Case 8. The one significant value corresponds to the signal small damage, and the presence of only one significant singular value was typical for all twelve cases.

Figure 4.4 The first singular values from all the twelve cases for a single damage site. The small damage locations remained at the same location (0, 100) mm. The Young’s modules percentages ($\gamma_2$) of the pristine structures in the damage region are summarized in Table 4.1.
In Equation (4.13), the numerator is the singular value and the denominator is a constant only related to the background Green’s function vector norms. Once the damage is located, the denominator in Equation (4.13) can be estimated and hence the reflectivity \( |v_k| \) can be calculated using the singular values obtained from SVD. In theory, the reflectivity \( |v_k| \) represent the ratio between the amplitude of the backscattered waves and the amplitude of the incident waves at the damage boundary, both of which can be easily extracted from the simulation. The amplitude ratio in twenty-one ray paths from the twenty-one transducers to the damage and then backscattered to each of the transducer is extracted and averaged as the reflectivity values from theory. The reflectivity values of the twelve cases calculated from the theory and SVD match well in Figure 4.5, which serves as validation for the damage reflectivity calculation using singular values.

![Figure 4.5 Comparison between the simulated and theoretical reflectivity values for the twelve simulation cases. Simulated values were calculated using SVD (Equation (4.13)).](image_url)
4.2.2 Multiple sites of damage imaging

Through simulation, this section explores the difference between back propagation and MUSIC in imaging multiple sites of damage. Two sites of small damage (compared with the input wavelength) with different material properties were used. The independence between singular values corresponding to different sites of damage was also studied.

In this series of simulations, two sites of square damage with dimensions of $2 \text{ mm} \times 2 \text{ mm}$ were studied. The wavelength was, again, 12 mm such that the damage dimension was 1/6 of the wavelength. One damage center was located at (20, -80) mm with a constant $\gamma_1$ of 60%. The center of the other damage was located at (-20, -80) mm with a varying $\gamma$ in each simulation, called ‘$\gamma_2$’. The values of $\gamma_1$ and $\gamma_2$ in the five cases are summarized in Table 4.2.

Table 4.2 The values of $\gamma_1$ and $\gamma_2$ corresponding, respectively, to the two sites of damage for the multi-site simulation

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>60%</td>
<td>60%</td>
<td>60%</td>
<td>60%</td>
<td>60%</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>10%</td>
<td>20%</td>
<td>40%</td>
<td>60%</td>
<td>80%</td>
</tr>
</tbody>
</table>

The singular values from Case 1 are plotted in Figure 4.6, in which two significant singular values are shown, as expected. The back-propagation of the singular vector associated with the largest singular value using Equation (4.15b) is shown in Figure 4.7(a); it can be seen that the largest singular value corresponds to Damage 2. Similarly, the back-propagation of the second largest singular value is shown in Figure 4.7(b); the image corresponds to Damage 1.

The two sites of damage were then imaged using DORT-MUSIC, as shown in Figure 4.8(b). A reverse-time migration using a zero-lag cross-correlation imaging condition (CCRTM) technique (He, 2015) was also used for comparison in Figure 4.8(a). A regular imaging
algorithm, such as CCRTM without subwavelength imaging capability, generates a rough estimation of the damage location with an over-estimated damage area because the wavelength (12 mm) employed is much larger than the damage dimension (2 mm). The subwavelength imaging ability of DORT-MUSIC was thus verified in this example. The first two singular values from the five cases are plotted in Figure 4.9, from which it can be seen that the two singular values are independent of each other and the corresponding reflectivity obtained from the singular values is an absolute quantity other than a relative quantity. Therefore, damage severity can be quantified using the calculated reflectivity.

Figure 4.6 The singular values for Case I with two sites of square damage: Damage 1 is located at (20, -80) mm with $\gamma_1$ of 60%; Damage 2 is located at (-20, -80) mm with $\gamma_2$ of 10%.
Figure 4.7 DORT imaging of Case 1 using back-propagation of: (a) the first singular vector; (b) the second singular vector. Damage 1 is located at (20, -80) mm with $\gamma_1 = 60\%$; Damage 2 is located at (-20, -80) mm with $\gamma_2 = 10\%$.

Figure 4.8 Comparison between (a) an imaging algorithm without subwavelength imaging ability and (b) DORT-MUSIC. Specifically, the left image utilized reverse-time migration using zero-lag cross-correlation imaging condition (CCRTM). Damage 1 is located at (20, -8) mm with $\gamma_2 = 10\%$. 
Figure 4.9 The first two singular values from all the five cases for the simulations of two damage sites: Damage 1 was fixed at (20, -80) mm with a constant $\gamma_1$ of 60%; Damage 2 was located at (-20, -80) mm with various values of $\gamma_2$ (summarized in Table 4.2).

It can be seen from the simulations conducted in this study that, as the damage area’s stiffness was reduced, the singular values increased accordingly. The apparent reflectivity of each small damage site was calculated using the singular values in Figure 4.5. The math foundation of this paper is under the assumption that the damage is extremely small compared with the wavelength. Another extreme would be the damage boundary is much larger compared with the wavelength. In SHM or NDE application, most situations seem to lie in between. The Lamb wave interactions with finite-sized holes (Grahn, 2003) and cracks (Lowe, 2002) and the scattering patterns of their combinations have also been studied (Koles, 1998), whereas the reflectivity coefficients are both angle and frequency dependent. However, to the authors’ knowledge, no theory on small damage scattering and reflection for
Lamb waves has been built. Thus, in Figure 4.10, the reflectivity calculated from SVD in the twelve simulation cases were compared with reflectivity coefficients of the normal instance (Equation (4.20)). A strong linear dependence is observed. Clearly, greater damage severity will cause higher reflectivity, which can be quantified using DORT. However, the severity of the damage, in reality, might be caused by various factors, such as damage size, Young’s modulus change, density change, thickness change, or a combination effect for metals. In composites, these factors would also include fiber waviness, delamination and porosity. In other words, reflectivity cannot uniquely determine the damage type and severity. Since the finite difference algorithm in this research only solves flexural wave equations, the mode conversion phenomenon that would prevail in physical experiments was not considered. Thus, for actual engineering applications, the calculation and use of damage reflectivity as an estimation of damage severity must be approached with caution.
Figure 4.10 The dependence of the normal instance reflection coefficient on small damage reflectivity.

Despite all of the above concerns, Lamb wave-based damage detection algorithms with the capability of damage severity quantification are very limited. Therefore, it is believed that the reflectivity obtained using the proposed DORT-MUSIC technique is a legitimate indicator of the damage severity. The relationship between the reflectivity from SVD and various damage factors such as damage size, material properties and thickness change should be examined systematically in future work.
Chapter 5

DORT-MUSIC Experiments
In last chapter, the theory and simulation results of DORT-MUSIC have been presented. In this chapter, a rapid, hybrid non-contact scanning system was proposed to image aluminum and composite plate using a piezoelectric linear array for actuation and a laser Doppler vibrometer (LDV) line-scan for sensing. With the flexibility of this scanning system, a considerably large area can be imaged using lower frequency Lamb waves with limited line-scans. The experimental results showed that the hardware system with a signal processing tool such as the DORT-MUSIC (TR-MUSIC) imaging technique can provide rapid, highly accurate imaging results as well as damage quantification with unknown material properties. Since the DORT-MUSIC (TR-MUSIC) technique allows for imaging damage smaller than the excitation wavelength (subwavelength), relatively low ultrasonic frequencies can be used for damage interrogation. The application of the low frequency signals will alleviate the attenuation effect in plate-like structures, especially in composite structures.

5.1 DORT and MUSIC experiments in isotropic materials

5.1.1 Experimental setup for damage imaging

The experiments are carried out using the same hybrid LDV/PZT scanning system as presented in section 3.2.1 (Figure 5.1).
Figure 5.1 Experimental setup for the hybrid PZT/LDV scanning system.

A 1.6 mm thick T-6061 aluminum panel was prepared as the test specimen. The inspection area on the plate was 400 × 400 mm. The material properties are listed in Table 5.1. The origin of the system was located at the center of the inspection area. The actuators used in this research were piezoelectric wafers (Steiner & Martins, Inc.) with dimensions of 7 mm in diameter and 0.3 mm in thickness. Eleven actuators were located on the line of x = -80 mm and ranged from -50 mm to 100 mm in the y direction with a spacing of 15 mm. The LDV scanning points ranges from -50 mm to 100 mm in the x direction and were located along the line y = - 55 mm with a spacing of 5 mm. Each actuator was excited sequentially. When using one actuator, the LDV head was moved horizontally over the designated path shown in
Figure 5.1 and the out-of-plane velocity was measured at each scanning point with 40 measurements to increase the SNR ratio.

The actuators and the LDV line-scan arrangement remains the same for all experiments. The $A_0$ Lamb wave mode was used along with a 5-cycle Hanning windowed toneburst signal (a center frequency of 100 kHz, corresponding to a wavelength of 12 mm). The peak-to-peak input voltage for the amplifier is 2 V for all experiments. After the amplification, the output peak-to-peak voltage sent to the piezoelectric wafers was 80 V. To measure the Green’s functions through experiments, a piezoelectric wafer was mounted onto the plate and a series of LDV scan points were used to measure the responses. The distances between the LDV scan points to the actuator center ranged from 5 mm to 305 mm, and the Fourier transform of the responses were used as the background Green’s functions with the same corresponding distances between two points in the DORT-MUSIC imaging.

<table>
<thead>
<tr>
<th>Table 5.1 Material properties of T6061 aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ($kg \ m^{-3}$)</td>
</tr>
<tr>
<td>2700</td>
</tr>
</tbody>
</table>

Circular magnet pairs with a diameter of 6.35 mm were used to simulate small damage. Three cases of damage distribution with different numbers of the damage sites were studied. The damage locations in the three cases are illustrated in the first picture (a) of Figure 5.2-5.4.

5.1.2 Experimental damage imaging results

To simulate damage, the center of a circular magnetic pair (attached to the front and back of the plate) with a diameter of 6.35 mm was located at (40, 70) mm. The singular values obtained through SVD of the experimentally obtained transfer matrix $K$ are plotted in Figure
5.2(b). The first singular value is much larger than the rest of the singular values such that all the singular vectors except for the first one were considered as associated with noise space and were used for imaging in Case I. The DORT-MUSIC image from the actuators-to-damage process using Equation (4.16a) is shown in Figure 5.2(c), and the DORT-MUSIC image from the damage-to-LDV process using Equation (4.16b) is shown in Figure 5.2(d). DORT-MUSIC has superior cross-range resolution but its range resolution suffers; this can be seen in Figure 5.2(c-d), and other studies on DORT-MUSIC (Yavuz, 2006; Yavuz, 2009; Zhong, 2014; Gruber, 2004). To use the superior cross-range resolution of both the actuator array and the LDV line-scan, Equation (4.17) was applied to obtain the final image of the damage seen in Figure 5.2(e). A zoomed-in image is shown in Figure 5.2(f) to better illustrate the localization performance.
Figure 5.2 A single site of damage imaging. (a) Experimental setup, (b) singular values, (c) DORT-MUSIC image based on the actuators-to-damage process, (d) DORT-MUSIC image based on the damage-to-LDV process, (e) final image, and (f) a zoomed-in view of the final image. The damage was located at (40, 70) mm as indicated by the red circle.
To evaluate the damage localization accuracy, a damage center distance error (CDE) can be defined as:

\[
CDE = \frac{\sqrt{(x_t - x_c)^2 + (y_t - y_c)^2}}{R_t} \times 100\%
\]  

(5.1)

where \((x_t, y_t)\) is the actual location of the damage center, \((x_c, y_c)\) is the calculated damage center (defined as the maximum value in the image), and \(R_t\) is the actual damage center to actuator array center distance. Using Equation (5.1), the CDE of Case I can be calculated as 0.78%.

In Case II, two sites of damage are present. One damage center was located at \((40, 70)\) mm and the other one at \((70, 27)\) mm. As expected, two significant singular values were found and the signal space dimension was, therefore, determined as two. The zoom-in DORT-MUSIC image of the two damage sites is obtained using Equation (4.17) and shown in Figure 5.3(b). The CDE of the first and second damage was 1.7% and 0.94%, respectively.

![Figure 5.3 Two sites of damage imaging. (a) Experimental setup, and (b) a zoomed-in view of the final image for two damage sites. The damage locations are \((40, 70)\) mm and \((70, 27)\) mm.](image)
In Case III, a third damage site was added to Case II at (70, 95) mm. Three significant singular values were found, corresponding to the three sites of damage. The zoom-in DORT-MUSIC image of the three damage sites is shown in Figure 5.4(b). The CDE of the three sites of damages are 1.1%, 0.67% and 2.7%, respectively. The first two sites of damage were well-imaged while the third damage had a slight shift of about 4 mm in the y direction, although it was still very accurate in the x direction. This discrepancy may be a result of the third damage site being almost out of the actuator array range in the y direction but was still in the range of the line-scan in x the direction.

The damage localization accuracy of the proposed method is considerably high relative to algorithms without subwavelength imaging ability. In Figure 5.2(c) and (d), the DORT-MUSIC imaging algorithm demonstrates high resolution in the cross-range direction but lower resolution in the range direction with respect to each array. The elongation of the damage image in the range direction was spotted in almost every study of DORT and TR-
MUSIC (Kerbrat, 2002; Yavuz, 2006; Yavuz, 2009; Gruber, 2004) and especially for Lamb waves (Engholm, 2011; Qu, 2013; Zhong, 2014). This might be due to a combination of the noise effects and phase lag due to the transducer’s acousto-electrical response (Labyed, 2013). In this research, the background Green’s function was measured through experimentation, with piezoelectric wafers as actuators and the LDV as the sensors. Thus, the measured Green’s function is an over-compensation of the possible phase-lag of both the actuator-to-damage and the damage-to-sensors processes. However, the images obtained in this study show less elongation in comparison with other Lamb wave applications (Engholm, 2011; Qu, 2013; Zhong, 2014) without the compensation of the phase lag effect from the transducer. Therefore, in general, the measured Green’s functions form a better estimation than a direct theoretical approach.

To reduce the artificial elongation in the images and to provide a better localization capability using DORT-MUSIC, this paper proposed a perpendicular setup of the actuation and reception array. As can be seen from Figure 5.2-5.4, superior localization performance is achieved in the situation where the wavelength is much larger than the damage dimension. This subwavelength imaging capability offers a critical advantage over conventional imaging algorithms for Lamb wave-based non-destructive inspection; that is, it provides greater freedom in selecting the frequency range of operation.

It is well-known for Lamb wave propagation, especially in composites materials, that attenuation of the propagating waves accrues significantly as the frequency of the input signal increases due to the micro-scale heterogeneities in the composites. Furthermore, for $A_0$ mode Lamb waves in the lower ultrasonic frequency range, the ‘steepness’ of the dispersion
relation is larger than in the higher frequency range such that the waves are more sensitive to the material property or the thickness change of the damaged area (Belanger, 2009). Thus, the proposed system using DORT-MUSIC is potentially suited to large-scale plate-like structures if the lower ultrasonic frequency range is acceptable for the application.
Chapter 6

Conclusions and Future Work
The theory, simulations and experiments using both reverse-time migration and DORT-MUSIC techniques were achieved in this research.

6.1 Reverse-time migration

6.1.1 Contributions beyond previous work
• Two new imaging conditions were used in Lamb-wave based damage detection for the first time. One is zero-lag cross-correlation (ZLCC), and the corresponding imaging technique is referred to as CCRTM. The other one is an enhanced zero-lag cross-correlation (E-ZLCC) imaging condition, resulting in the imaging technique E-CCRTM.
• 2-D areal scan was used to replace traditional 1-D line scan for RTM imaging.
• Not only isotropic materials but also composite materials were studied through experiments.

6.1.2 Conclusions and future work
In contrast to the well-known classical prestack reverse-time migration (RTM) using excitation-time imaging condition based on only a single center frequency, the CCRTM technique takes into account all frequency contents for imaging by cross-correlating forward and backward propagating wavefields. The CCRTM showed that the damage image quality using a single actuator is comparable to the prestack RTM where each actuator is sequentially excited and the resulting images are stacked to form the resulting image. By introducing the zero-lag cross-correlation (ZLCC) imaging condition, the number of migration tests and data acquisition volume required for imaging the damage can be
decreased dramatically. Meanwhile, the requirement for both hardware operation and computational and time costs was also reduced considerably.

From simulations and experiments, it demonstrated that the 2-D areal scan enables the E-CCRTM algorithm to provide robust performance for imaging damage at various locations and distances from the actuator and its nearby scanning region. Omnidirectivity is achieved using the 2-D areal scan and the mirrored ghost images using the 1-D linear scan are eliminated. The E-CCRTM technique provides much better localization results when compared with CCRTM. More importantly, the 2D E-CCRTM images represent damage reflectivity which might be able to serve as an indicator of the damage severity in the future.

In fact, most array configurations will be well-suited for use with E-CCRTM. With the development of non-contact sources, such as pulse-laser and air-coupled transducers, a fully non-contact damage imaging system can be designed and E-CCRTM is certainly a suitable imaging algorithm to adopt with other data acquisition systems. Different transducers and hardware system integration will allow for a broad application of E-CCRTM in various areas with greater engineering significance. A study of environmental effects, such as noise, electromagnetic interference and temperature variation, is recommended as possible future work for this method.

6.2 DORT-MUSIC technique

6.2.1 Contributions beyond previous work

- The combination of DORT and MUSIC methods was discussed in the context of
Lamb wave-based damage detection.

- A Lamb wave-based, subwavelength imaging technique based on a perpendicular arrangement of a linear actuator array and a LDV line-scan was proposed, allowing for separate imaging from both the actuator-to-damage and the sensor-to-damage processes using the DORT-MUSIC algorithm in the space-frequency domain.
- The underlying physics behind the DORT process is illustrated through theoretical expressions and was verified through numerical simulations.
- The singular values of the transfer matrix $K$ can be related to the reflectivity of a corresponding site of small damage, which might be used as a worthy indicator of damage severity.

6.2.2 Conclusions and future work

The perpendicular arrangement between the actuator array and the LDV line-scan, took advantage of the superior cross-range resolution of both the array and line-scan, allowing for super-resolution images to be developed. The background Green’s functions for pristine structures were measured through experiments such that neither the material properties nor the plate thickness need to be known a priori. The experimentally measured Green’s functions also provide accurate phase estimation within the input frequency band.

The back-propagation of the singular vectors demonstrated that multiple sites of damage could be imaged separately through the DORT method, but the MUSIC algorithm possesses better imaging accuracy and quality.
The subwavelength imaging ability of the proposed Lamb wave-based DORT-MUSIC is potentially capable of using limited data acquisition points and scanning an ultra large area with lower frequencies because of less attenuation.

The proposed non-destructive inspection system has DORT-MUSIC’s subwavelength imaging characteristics, potential damage quantification capability, and the separate imaging processes for both actuation and sensing. Lower frequency range with the larger flexibility of the scanning patterns can be applied to Lamb wave-based damage detection for considerably large plates with enhanced speed. Future work should focus researching the noise effect in Lamb wave-based DORT-MUSIC and to apply the method to composite structures.
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Appendix A - Signal Energy and Root Mean Square (RMS) Bandwidth of N-peaked Tone-burst

1. Signals of N-peaked tone-burst

A complex N-Peaked Tone-Burst narrowband signal is defined as

\[
f(t) = [H(t) - H(t - N_p / f_c)](1 - \cos \omega_c t / N_p)e^{i\omega_c t}
\]  \hspace{1cm} (A.1)

where \(H(t)\) is the Heaviside step function, \(N_p\) is the number of peaks of the loading, \(f_c\) the center frequency and \(\omega_c = 2\pi f_c\).

Since only the real signal can be used in practice and the tone-burst signal takes the following form:

\[
f(t) = [H(t) - H(t - N_p / f_c)](1 - \cos \omega_c t / N_p)\sin \omega_c t
\]  \hspace{1cm} (A.2)

An alternative expression for real tone-burst is

\[
f(t) = [H(t) - H(t - N_p / f_c)]\text{Re}[ie^{-i\omega_1 t} - \frac{i}{2}(e^{-i\omega_2 t} + e^{-i\omega_3 t})]
\]  \hspace{1cm} (A.3)

where \(\omega_1 = \omega_c (1 - \frac{1}{N_p})\), and \(\omega_2 = \omega_c (1 + \frac{1}{N_p})\).

Because
\[ \text{Re}[ie^{-i\omega t}] = \frac{i}{2}(e^{-i\omega t} + e^{-i\omega t}) \]
\[ = \text{Re}[i \cos(\omega_1 t) + \sin(\omega_1 t) - \frac{1}{2}(i \cos(\omega_1 t) + \sin(\omega_1 t) + i \cos(\omega_2 t) + \sin(\omega_2 t)))] \]
\[ = \sin(\omega_1 t) - \frac{1}{2}(\sin(\omega_1 t) + \sin(\omega_2 t)) \]
\[ = \sin(\omega_1 t) - \sin(\omega_1 t) \cos\left(\frac{\omega_1}{N_p} t\right) \]
\[ = (1 - \cos(\omega_1 t / N_p)) \sin(\omega_1 t) \]

The expression using exponential function will accelerate the Fourier Transform derivation.

In Figure A.1, plots of the real part from these three expressions are shown:

![Figure A.1 Plots for the complex and real tone-burst (a) the real part of the two signals. (b) The amplitude of the two signals.](image)

Apparently, the average absolute values from Equation (A.1) are larger than those of the other two signals.
2. FT, DFT and FFT of tone-burst and MATLAB realization

2.1 Fourier transform (FT).

From Equation (A.1), the complex expression of the five-peak signal, its Fourier transform is given by

\[ F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt \]
\[ = \int_{0}^{N_p/\nu_f} (1 - \cos \omega_c t/N_p)e^{-j(\omega - \omega_c) t} dt \]
\[ = \int_{0}^{N_p/\nu_f} e^{j(\omega_c - \omega)t} dt - \int_{0}^{N_p/\nu_f} \cos \omega_c t/N_p e^{j(\omega - \omega_c)t} dt \]
\[ = \frac{e^{at}}{a} \bigg|_{0}^{N_p/\nu_f} - \int_{0}^{N_p/\nu_f} e^{at} \cos bt dt \]

Where \( a = i(\omega_c - \omega) \) and \( b = \omega_c/N_p \)

Using the formula

\[ \int_{0}^{N_p/\nu_f} e^{at} \cos bt dt = \frac{1}{a^2 + b^2} (a \cos bt + b \sin bt) e^{at} \bigg|_{0}^{N_p/\nu_f} \]
\[ = \frac{a}{a^2 + b^2} (e^{aN_p/\nu_f} - 1) \] (A.6)

Equation (A.5) can be rewritten as

\[ F(\omega) = (e^{aN_p/\nu_f} - 1) \left( \frac{1}{a} - \frac{a}{a^2 + b^2} \right) \]
\[ = (e^{aN_p/\nu_f} - 1) \left( \frac{b^2}{a^3 + ab^2} \right) \] (A.7)

Substituting \( a \) and \( b \) back into the above expression

\[ F(\omega) = \frac{i\omega_c^2}{N_p^2(\omega_c - \omega)^3 - (\omega_c - \omega)\omega_c^2} [e^{i(\omega - \omega_c)N_p/\nu_f} - 1] \] (A.8)
The magnitude of $F(\omega)$ can be obtained as

$$|F(\omega)| = \sqrt{\frac{\omega_1^2}{N_p^2(\omega_c - \omega)^3 - \omega_c^2(\omega_c - \omega)}} \sqrt{\cos(\omega_c - \omega)N_p/f_c - 1}^2 + \sin^2(\omega_c - \omega)N_p/f_c$$

$$= \frac{\omega_c^2}{N_p^2(\omega_c - \omega)^3 - \omega_c^2(\omega_c - \omega)} \sqrt{2 - 2\cos(\omega_c - \omega)N_p/f_c}$$

$$= \frac{2\omega_c^2}{N_p^2(\omega_c - \omega)^3 - \omega_c^2(\omega_c - \omega)} \sqrt{1 - \cos(\omega_c - \omega)N_p/f_c}$$

$$= \frac{2\omega_c^2}{N_p^2(\omega_c - \omega)^3 - \omega_c^2(\omega_c - \omega)} \sin(\omega_c - \omega) \frac{N_p}{2f_c}$$

(A.9)

From the expression above, $|F(\omega)|$ is not an even function. The reason is that the original signal in time domain is complex.

From Equation (A.2), the real expression of the N-peaked signal, its Fourier transform is given by

$$F(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} [H(t) - H(t - N_p/f_c)] \text{Re}[ie^{-j\omega t} - \frac{i}{2}(e^{-j\omega t} + e^{-j\omega t})] dt$$

$$= (1 - e^{-j\omega N_p/f_c})\left[\frac{\omega_c}{\omega_c^2 - \omega^2} - \frac{1}{2} \left(\frac{\omega_1}{\omega_1^2 - \omega^2} + \frac{\omega_2}{\omega_2^2 - \omega^2}\right)\right]$$

(A.10)

Then, $|F(\omega)|$ can be written as:

$$|F(\omega)| = \left|1 - \cos(\omega N_p/f_c)\right|\left[\frac{\omega_c}{\omega_c^2 - \omega^2} - \frac{1}{2} \left(\frac{\omega_1}{\omega_1^2 - \omega^2} + \frac{\omega_2}{\omega_2^2 - \omega^2}\right)\right]$$

(A.11)

From the real expression of the tone-burst signal, the absolute value of the frequency spectrum is an even function. For five-peak tone burst with center frequency of 60 kHz, the plots of the spectrums is shown in Figure (A.2)
2.2 Discrete Fourier transform (DFT) and fast Fourier transform (FFT) for tone-burst

In experiments, digital signals are most often used. With discrete expressions for both signals in time and its spectrum in frequency domain with finite lengths, numerical verification can be utilized in software such as MATLAB.

The Discrete Fourier Transform of can be expressed as:

\[ X(k) = \sum_{n=0}^{N-1} x(n) \exp(-j \frac{2\pi}{N} nk) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \]  \hspace{1cm} (A.12)

where \( x(n) \) is sampled signal sequences from time domain, \( X(k) \) is sampled signal sequences from frequency domain, \( N \) is the length of the sequences, and \( W_N = \exp(-j \frac{2\pi}{N}) \).
In FT, both time and frequency signals are continuous and infinite while in DFT both signals are discrete and finite. Thus, DFT is an approximation for FT. Details discussion can be found in most of digital processing books. Here we only list several things related.

The sampling frequency for time domain signal has to be at least two times than the largest frequency component in the signal. That is the Nyquist frequency (half of sampling frequency) has to be larger than the largest frequency content. For the tone-burst signal, the Nyquist frequency needs to be much higher than center frequency of the signal, for the existence of the frequency components higher than the center frequency.

Due to the sampling for the signal, the amplitude of its spectrum will be changed $F_s$ times [1].

$F_s$ is the sampling frequency to obtain the time domain signal. Correspondently, $\Delta T = 1/F_s$ is the sampling interval. From inverse Continuous-Time Fourier Transform (iTFT),

$$x_a(t) = \int_{-\infty}^{\infty} X_a(F)e^{j2\pi Ft} dF$$ \hspace{1cm} (A.13)

Where $X_a(F)$ (continuous), is the spectrum obtained from the analog signal.

Sampling $x_a(t)$ at $t=n\Delta T$ yields,

$$x(n) = \int_{-\infty}^{\infty} X_a(F)e^{j2\pi Fn/F_s} dF$$ \hspace{1cm} (A.14)

Using the relationship for the inverse Discrete-Time Fourier Transform (iTFT), $x(n)$ can also be written as

$$x(n) = \frac{1}{F_s} \int_{-F_s/2}^{F_s/2} X(F)e^{j2\pi Fn/F_s} dF$$ \hspace{1cm} (A.15)

where $X(F)$ (Continuous) is the spectrum obtained from the digital signal.
The relationship between $X_a(F)$ and $X(F)$ can be obtained by comparing Equation (A.14) with Equation (A.15)

$$
X_a(F) = \begin{cases} 
\frac{1}{F_s} X(F), & |F| \leq \frac{F_s}{2} \\
0, & |F| > \frac{F_s}{2}
\end{cases}
$$

Thus, to obtain the true spectrum from DFT, a factor of $1/F_s$ or $\Delta T$ needs to be used. FFT result is exactly the same with that of DFT. FFT is a just fast calculation algorithm, which can be realized in MATLAB.

When FFT is performed, zeros could be added at the end of time domain sequences. The resolution of the spectrum will be increased. However, the shape of the true spectrum, which relates to sampling frequency and the number of sampling points, would not be changed by padding zeros. Another thing worth to notice is that how to recover the frequency for the FFT results. In MATLAB, for N-point FFT, a sequence with the length of N will be generated. The N-point data describes the spectrum from 0 to $2\pi$. Due to the periodicity of the spectrum of DFDT, the spectrum of FFT can be obtained by shifting the content from $\pi$ to $2\pi$, to from $-\pi$ to $\pi$ as needed. Because of the symmetry of the spectrum of real signal, half of the content is enough to represent its whole spectrum. But it is not the case for complex signal.

To recover the frequency range on x axis when displaying FFT results, the interval between two points is $F_s/N$. With the above considerations, the spectrum of FT can be compared with
that of FFT. The factor of $1/F_s$ is used for both the complex and real signals in FFT calculation. Plots are in Figure (A.3). The FT and FFT matches very well.

![Figure A.3 Spectrum from FT and FFT](image)

3. **Energy of N-peaked tone-burst**

The energy $E_t$ contained in signal $f(t)$ can be calculated in time domain[2]:

$$E_t = \|f(t)\|^2 = \int_{-\infty}^{\infty} |f(t)|^2 dt \quad (A.17)$$

If the spectrum of the signal $F(\omega)$ is obtained, the energy $E_f$ contained in the signal can be also calculated in frequency domain.

$$E_f = \|F(\omega)\|^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \quad (A.18)$$

For the complex five-peak signal from Equation (A.1), the energy in time domain $E_{t1}$ is
Similarly, the energy $E_{i2}$ the real signal from Equation (A.2) can be calculated as

$$E_{i2} = \frac{3}{4} \frac{N_p}{f_c}$$  \hspace{1cm} (A.20)

Obvious, $E_{i1} = 2E_{i2}$. This is another proof for more energy is contained by the complex signals.

Various approaches can be applied to prove the validation of the derivations above. This summary is useful for comparison of signal from theory and simulation in different situations.

a). Direct integration using continuous time domain signal defined in software like MATLAB. Denote as $E_{ct1}$ and $E_{ct2}$ for five-peak signals.

b). Direct integration using continuous frequency domain signal. The frequency signal is from the derived of Fourier transform. Details can be found in Equations (A.8) and (A.10). Denote as $E_{cf1}$ and $E_{cf2}$ for five-peak signals.

c). Using sampled signals in time domain $x(n)$, and calculated the energy $E_n$ from the discrete time signals.
\[ E_{nt} = \sum_{n=1}^{L} |x(n)|^2 \cdot \Delta t \]  
\hspace{1cm} (A.21)

in which \( \Delta t \) is the sampling interval, \( L \) is the total number of samples.

Denote as \( E_{d1} \) and \( E_{d2} \) for N-peaked signals.

d) Similar to c), discrete frequency signal can be used to calculate the energy \( E_{nf} \).

\[ E_{nf} = \frac{1}{2\pi} \sum_{n=1}^{N} |X(n)|^2 \cdot \frac{2\pi F_s}{N} = \sum_{n=1}^{N} |X(n)|^2 \cdot \frac{F_s}{N} \]  
\hspace{1cm} (A.22)

where \( F_s \) is the sampling frequency, \( N \) is the total number of samples, \( X(n) \) is the discrete signal in frequency domain, obtained either from sampled FT (denote as \( E_{nf1} \) and \( E_{nf2} \)) or FFT (denote as \( E_{fft1} \) and \( E_{fft2} \)).

For five-peak single with center frequency \( f_c = 60 \text{ kHz} \), energy calculations from both time and frequency domain, with both continuous and discrete signals, for both complex and real signal is shown in Table A.1 and Table A.2.

<table>
<thead>
<tr>
<th>Energy from different approach</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{t1} )</td>
<td>1.250000000000000e-04</td>
</tr>
<tr>
<td>( E_{ct1} )</td>
<td>1.25000054801723e-04</td>
</tr>
<tr>
<td>( E_{of1} )</td>
<td>1.250000260626390e-04</td>
</tr>
<tr>
<td>( E_{d1} )</td>
<td>1.249999999999999e-04</td>
</tr>
<tr>
<td>( E_{nf1} )</td>
<td>1.249997222690896e-04</td>
</tr>
<tr>
<td>( E_{fft1} )</td>
<td>1.24999999999614e-04</td>
</tr>
</tbody>
</table>
Table A.2 energy for real tone-burst signal from Equation (A.3)

<table>
<thead>
<tr>
<th>Energy from different approach</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{t_2}$</td>
<td>6.250000000000000e-05</td>
</tr>
<tr>
<td>$E_{nt_2}$</td>
<td>6.250000000603473e-05</td>
</tr>
<tr>
<td>$E_{cf_2}$</td>
<td>6.250000000323804e-05</td>
</tr>
<tr>
<td>$E_{dt_2}$</td>
<td>6.250000000000004e-05</td>
</tr>
<tr>
<td>$E_{nft_2}$</td>
<td>6.249999999999522e-05</td>
</tr>
<tr>
<td>$E_{gt_2}$</td>
<td>6.249999999999514e-05</td>
</tr>
</tbody>
</table>

4. Root mean square (RMS) Bandwidth

Various definitions of bandwidths can be applied for signal processing: there are 3-dB, zeros crossing, equivalent noise, and RMS et al. In the following sections, the RMS bandwidth will be introduced and applied to the N-peaked tone-burst signal.

4.1 Mean frequency

The general expression for the mean frequency:

$$<\omega> = \frac{1}{2\pi E} \int_{-\infty}^{\infty} \omega |F(\omega)|^2 d\omega = \omega_c$$  \hspace{1cm} (A.23)

For the complex signal with center frequency of 60 kHz is calculated numerically as:

$$<f> = \frac{1}{2\pi} <\omega> = 6.001907348629216e+04Hz$$

For the spectrum of real signal, due to its symmetry, mean frequency is zero. Only consider the positive frequency range,

$$<\omega> = \frac{1}{2\pi (1/2E)} \int_{0}^{\infty} \omega |F(\omega)|^2 d\omega = \omega_c$$  \hspace{1cm} (A.24)
For the real signal with center frequency of 60 kHz is calculated numerically as:

\[ <f> = \frac{1}{2\pi} <\omega> = 5.99973695355797\times10^4 \text{Hz} \]

### 4.2 RMS Bandwidth

One-sided width of the frequency window, which is half of the RMS Bandwidth can be expressed as [3]

\[ \Delta\omega = \left( \frac{1}{2\pi E} \int_{-\infty}^{\infty} (\omega - <\omega>)^2 |F(\omega)|^2 d\omega \right)^{1/2} \]  
(A.25)

For the complex tone-burst signal, from Figure (A.3), its spectrum in negative frequency range essentially equals to zero. Thus, for this complex tone-burst signal, Equation (A.25) can be written as

\[ \Delta\omega = \left( \frac{1}{2\pi E} \int_{0}^{\infty} (\omega - <\omega>)^2 |F(\omega)|^2 d\omega \right)^{1/2} \]  
(A.26)

From the theoretical expression of \(|F(\omega)|\) in Equation (A.9), Equation (A.26) can be rewritten as
\[(\Delta \omega)^2 = \frac{1}{2\pi} \int_{0}^{\infty} (\omega < \omega^2) d\omega \frac{|F(\omega)|^2}{E} d\omega \]

\[= \frac{1}{2\pi E} \int_{0}^{\infty} (\omega - \omega^2)^2 |F(\omega)|^2 d\omega \]

\[= \frac{f_c}{3\pi N_p} \int_{0}^{\infty} (\omega - \omega^2)^2 \frac{2\omega^4 [1 - \cos(\omega - \omega) N_p / f_c]}{[N_p^2 (\omega - \omega)^3 - \omega^2 (\omega - \omega)]^2} d\omega \]

\[= \frac{2f_c \omega^2}{3\pi N_p} \int_{0}^{\infty} \frac{1 - \cos(\omega - \omega) N_p / f_c}{[N_p^2 (\omega - \omega)^2 - \omega^2]^2} d\omega \]

\[= \frac{2f_c (2\pi f_c)^4}{3\pi N_p} \int_{0}^{\infty} \frac{1 - \cos(\omega - \omega) N_p / f_c}{[(\frac{N_p}{f_c})^2 (\omega - \omega)^2 - (\frac{\omega}{f_c})^2]^2} d\omega \]  

(A.27)

Let \( \frac{N_p}{f_c} (\omega - \omega) = z \) and \( d\omega = -\frac{f_c}{N_p} dz \)

\[(\Delta \omega)^2 = \frac{32\pi^3 f_c^2}{3N_p} \int_{-\infty}^{\infty} \frac{1 - \cos z}{[z^2 - (2\pi)^2]^{\frac{1}{2}}} \left(-\frac{f_c}{N_p}\right) dz \]

\[= \frac{32\pi^3 f_c^2}{3N_p^2} \int_{-\infty}^{2N_p \pi} \frac{1 - \cos z}{(z - 2\pi)(z + 2\pi)^2} dz \]

\[= \frac{8\pi \omega^2}{3N_p^2} \int_{-\infty}^{2N_p \pi} \frac{1 - \cos z}{(z - 2\pi)(z + 2\pi)^2} dz \]  

(A.28)

Knowing the following result

\[\int_{-\infty}^{2N_p \pi} \frac{1 - \cos z}{(z - 2\pi)(z + 2\pi)^2} dz = \frac{1}{8\pi} \]

(A.29)

the root-mean-square bandwidth for the complex tone-burst signal can be finally obtained
\[ B = 2\Delta \omega = \frac{2 \omega_c}{\sqrt{3} N_p} \text{ (unit: rad/sec)} \]  

(A.30)

For real signal, its spectrum is symmetric. The one-sided width of the frequency window, which equals to half of the RMS bandwidth, can be expressed as:

\[ \Delta \omega = \left( \frac{1}{2\pi(1/2E)} \int_0^\infty (\omega - <\omega>)^2 |F(\omega)|^2 d\omega \right)^{1/2} \]  

(A.31)

In the above equations, only half of the whole energy needs to be divided, for the reason that only half energy distributed in the positive frequency range due to the symmetry.

Similarly, from Equation (A.10) or Equation (A.11), and Equation (A.31), the root-mean-square bandwidth for the complex tone-burst signal can be also obtained as

\[ B = 2\Delta \omega = \frac{2 \omega_c}{\sqrt{3} N_p} \text{ (unit: rad/sec)} \]  

(A.32)

Although, the amplitude and energy of the complex and real signals are different, their bandwidths are the same. For five-peak and frequency centered at

Numerical verification can be done by calculating Equation (A.25) and Equation (A.31) directly. Then doubling the values can lead to the numerical results of RMS bandwidth for the complex and real tone-burst signals. The values are \(8.705027034522078e+04 \text{ Hz}\) and \(8.70999158475123e+04 \text{ Hz}\). From Equation (A.30) or Equation (A.32), the value of theoretical RMS bandwidth is \(8.706236948324247e+04\).

With the theoretical expression in Equation (A.32), RMS bandwidth of N-peaked tone-burst is no need to be calculated numerically anymore.
Reference


Appendix B - Phased Array for Lamb-wave Application

Strategies for applying guided wave based phased array technology have been proposed lately for damage imaging using piezoelectric wafers [1-3].

1. Time-delay Calculation

1.1 Beam steering

The principle of the time-delay calculation for beam steering can be shown in Figure B.1. The distance between two adjacent piezoelements is $d$. The wave velocity of transmitting the excitation signal is $c$. The steering angle is defined as $\theta$. The time-delay between the two adjacent elements can be easily calculated from the figure as

$$\Delta \tau = \frac{d}{c} \sin \theta$$  \hspace{1cm} (B.1)

By changing the uniform delay between each two adjacent elements, different steering angles can be simply calculated by $\theta = \sin^{-1}\left(\frac{\Delta \tau}{d}\right)$. 
1.2 Beam focusing

The principle of the time-delay calculation for beam focusing is shown in Figure B.2. In most of the time-delay calculations for phased array, a sufficiently large constant delay is pre-applied to every element in order to keep all the delays positive [4], which, sometimes, can be very troublesome. Thus, a time-delay expression without requirement of any constant is developed.

In Figure B.2, the distance between the focal point $P$ and the origin $O$ is $F$. The angle between $OP$ and $y$ axis is $\theta$. The linear phased array consists of $N$ elements with $d$ being the spacing between neighboring elements. Due to the uniform distribution of the linear array, $D$, the largest distance between the focal point and the element of the array, can be obtained by comparing the distances from the first or last element to the focal point.
B.2, $D$ is defined as the distance between $S_l$ and $P$. The following geometric relationship can be obtained from the blue triangle $\Delta PGS_j$ (marked blue):

$$(F \cos \theta)^2 + \left\{ F \sin \theta - \left[ (j-1)d - \frac{N-1}{2}d \right] \right\}^2 = \left[ F - (ct_j - ct_0) \right]^2$$

(B.2)

where $t_0 = (D-F)/c$ and $t_j$ is the delay for element $j$. Rearranging the terms in Equation (1b), the delay for each element can be obtained as

$$t_j = \frac{F}{c} \left[ 1 - \sqrt{1 + \frac{d^2 \left( j - \frac{N+1}{2} \right)^2}{F^2}} - \frac{2d F \left( j - \frac{N+1}{2} \right) \sin \theta}{F^2} \right] + t_0$$

(B.3)

Figure B.2 The principle of the time delay calculation for beam focusing

2. Beamforming Directivity for Guided Waves

A series of studies to optimize the directivity of beamforming for bulk waves have been proposed by Wooh and Shi [4-5]. Similar derivation can be performed for guided waves. If a wave is generated from a single element with infinitely small size at the origin of a polar
coordinate system, the wave field at position $p(r, \theta)$ can be expressed as

$$f(r, \theta, t) = \frac{A}{\sqrt{r}} \exp\left[i(kr - \omega t)\right] \quad (B.4)$$

Replacing the single element with a linear phased array consisting of $N$ active elements, as shown in Figure B.3, the distance between the $j^{th}$ element of the array and the position $P$ can be derived from the trigonometric relationship in the triangle $\Delta POS_j$ (marked in blue) as

$$r_j = \sqrt{r^2 + \left[(j-1)d - \frac{(N-1)d}{2}\right]^2 - 2r \left[(j-1)d - \frac{(N-1)d}{2}\right] \cos\left(\frac{\pi}{2} - \theta\right)} \quad (B.5)$$

where $d$ is the spacing between two adjacent elements, $r$ and $\theta$ are coordinates of point $p$ in the polar coordinate system. Equation (B.5) can be rewritten as

$$r_j = r \sqrt{1 + \left[\frac{(j-1)d - \frac{(N-1)d}{2}}{r}\right]^2 - 2 \left[\frac{(j-1)d - \frac{(N-1)d}{2}}{r}\right] \sin \theta} \quad (B.6)$$

For far field ($d >> r$), $[(j-1)d-(N-1)d/2]/r >> 1$. Thus, Taylor expansion can be used and the second-order terms can be omitted. Then, Equation (B.6) can be further approximated as

$$r_j \approx r + \frac{r}{2} \left[\frac{(j-1)d - \frac{(N-1)d}{2}}{r}\right]^2 - \left[\frac{(j-1)d - \frac{(N-1)d}{2}}{2}\right] \sin \theta$$

$$\approx r - \frac{2j - (N+1)}{2} d \sin \theta \quad (B.7)$$
Fig. B.3 Wave field formation in beam steering

Based on the discussion in Section 1.1, a uniform time delay between two adjacent elements can be given as $\Delta \tau$ to achieve beam steering. Then, the contribution of the $j^{th}$ element to the wave field at position $p(r, \theta)$ at time $t$ can be expressed as

$$f_j(r_j, \theta, t) = \frac{A}{\sqrt{f_j}} \exp\left\{ i \left[ kr_j - \omega(t - \frac{2j - (N + 1)}{2} \Delta \tau) \right] \right\}$$  \hspace{1cm} (B.8)

Substituting Equation (B.7) into the above equation, the wave field from the $j^{th}$ element can be expressed as

$$f_j(r, \theta, t) = \frac{A \exp[i(kr - \omega t)]}{\sqrt{r - \frac{2j - (N + 1)}{2} d \sin \theta}} \exp\left[ i \frac{2j - (N + 1)}{2} \left( \omega \Delta \tau - kd \sin \theta \right) \right]$$  \hspace{1cm} (B.9)
Summing up the wave field from all the elements, the total field can be expressed as

$$ f(r, \theta, t) = \sum_{j=1}^{N} w_j f_j(r, \theta, t) $$  \hspace{1cm} (B.10) 

where \( w_j \) is the weighting factor, also called the shading of the array, which can be used to enhance the beam’s shape and reduce side lobe levels [4]. The choice of weighting factors can be a much involved topic. Thus, detailed discussion on weighting is not included.

For far field, the average distance from all the elements to position \( p(r, \theta) \) is very close to the distance from the center of the array to \( p \), which is \( r \). Therefore, the total field with unit weighting factor can be obtained from Equation (B.10) as

$$ f(r, \theta, t) = \sum_{j=1}^{N} f_j(r, \theta, t) $$  \hspace{1cm} (B.11) 

$$ = \frac{A}{\sqrt{r}} \sum_{j=1}^{N} \exp \left[ i \frac{2j - (N + 1)}{2} (\omega \Delta \tau - kd \sin \theta) \right] \exp[ikr - \omega t] $$ 

Denote that \( \phi = \omega \Delta \tau - kd \sin \theta \), the above equation can be rewritten as

$$ f(r, \theta, t) = \frac{A}{\sqrt{r}} \exp \left[ i \frac{1 - N}{2} \phi \right] \exp \left[ i \frac{1 + N}{2} \phi \right] \exp[ikr - \omega t] $$ 

$$ = \frac{A}{\sqrt{r}} \exp[1/(2i\phi)] - \exp[1/(2i\phi)] \exp[ikr - \omega t] $$  \hspace{1cm} (B.12) 

$$ = \frac{A \sin(N\phi/2)}{\sqrt{r}} \exp[i(kr - \omega t)] $$ 

The beam directivity is defined as the wave field \( f(r, \theta, t) \) at any arbitrary angle \( \theta \) with the normalization by the wave field \( f(r, \theta_s, t) \) at steering angle \( \theta_s \)

$$ H(\theta) = \frac{f(r, \theta, t)}{f(r, \theta_s, t)} $$  \hspace{1cm} (B.13)
Using the equations \( k = \omega / c \), \( \varphi = \omega \Delta \tau - kd \sin \theta \) and \( \sin \theta_s = c \Delta \tau / d \), Equation (B.13) can be organized as

\[
H(\theta) = \frac{\sin[N(\varphi - kd \sin \theta) / 2]}{N \sin[(\varphi - kd \sin \theta) / 2]} \tag{B.14}
\]

Combining with the relationship \( k = \omega / c = 2\pi / \lambda \), finally gives

\[
H(\theta) = \frac{\sin[N\pi (d / \lambda)(\sin \theta_s - \sin \theta)]}{N \sin[\pi (d / \lambda)(\sin \theta_s - \sin \theta)]} \tag{B.15}
\]

It can be found that the beam directivity for any steering angle \( \theta_s \) is only related to the ratio of \( d / \lambda \), which means for certain frequency, the distance between two adjacent elements is critical for the directivity hence the resolution of phased array.
Figure B.4 The beam directivity in 30 degree steering angle for 16-element phased array with different ratios $d/\lambda$: (a) 0.2, (b) 0.4, (c) 0.8, (d) 2.

Figure B.4 shows the influence of the ratio $d/\lambda$ on the beam directivity in 30 degree steering angle for a 16-element phased array. A critical dependence between $d/\lambda$ and the beam directivity can be spotted. As the ratio $d/\lambda$ increases, grating lobes begin to appear. In general, cutting elements into smaller elements and optimizing elements spacing will reduce the grating lobes [5].

References


