ABSTRACT

WILCOX, ANDREW GORDON. Systemic Risk and the Variability of SRISK. (Under the direction of Peter Bloomfield.)

In the wake of the 2007-2009 financial crisis, many government and academic researchers have turned their focus towards defining and measuring systemic risk. They aim to ensure the soundness of the entire financial system and eliminate the need to bail out individual financial institutions. While there are a variety of systemic risk measures that identify and rank these Systemically Important Financial Institutions (SIFIs), few have had the impact of the SRISK index created by Brownlees and Engle (2011). SRISK’s dependence on publicly available data, its similarities to stress testing, and its straightforward interpretation have made it a leading metric for measuring an individual firm’s systemic risk contribution.

While much of the research around SRISK has focused on its ability to monitor systemic risk, the validity of its SIFI rankings, and possible regulatory responses, there has been far less focus on the variability present in SRISK’s estimation procedure. The computation of SRISK is neither simple nor straightforward, but instead, relies on a bivariate dynamic process and a simulation procedure to estimate a firm’s Long Run Marginal Expected Shortfall (LRMES). By definition, a firm’s LRMES is the percentage of equity that firm will lose, conditionally on the market falling into a crisis. In this dissertation, we explore how changes to the various simulation settings and statistical assumptions required to compute LRMES affect its variability. Specifically, we demonstrate that the use of a leptokurtic working likelihood in the GJR-GARCH model produces LRMES differences on the order of 20% for certain firm and date combinations. While these differences vary by firm and date, the typical leptokurtic LRMES estimate is smaller than its Gaussian counterpart, indicating a reduced systemic risk contribution. A study of asymmetric working likelihoods shows that while many firm and date combinations exhibit asymmetry, the corresponding differences in LRMES are negligible.

In order to further comment on LRMES variability, we also propose a new block bootstrapping methodology that allows for the propagation of DCC-GARCH parameter estimation error through the LRMES simulation procedure. Our Block Bootstrap for Estimating Equations (BBEE) methodology is
unique in its approach to estimating DCC-GARCH parameter estimation error. Under typical LRMES settings, the BBEE methodology often outperforms asymptotic standard error approximations. Additionally, the use of our BBEE methodology allows us to better quantify the full amount of error present in estimating LRMES. The amount of LRMES variability due to DCC-GARCH parameter estimation is often larger than the LRMES variability due to model selection, and has the potential for billion dollar changes to SRISK estimates. By addressing the variability in LRMES that comes from both model selection and parameter estimation, we provide a better understanding of the SRISK index and systemic risk as a whole.
Systemic Risk and the Variability of SRISK

by
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APPROVED BY:

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DEDICATION

To my family and friends for their continual support over the last five years.

In loving memory of Douglas Bethel, Jane Bethel, and Sandy Wilcox.
Andrew Gordon Wilcox was born on February 15, 1988 to parents Dan and Gwen Wilcox. He grew up just outside of Baltimore, MD in the town of Elkridge, where his love of baseball and the Baltimore Orioles helped develop a growing interest in math and statistics. During his childhood, Andrew was often more interested in the back of his baseball cards than the picture on the front, and would use dice to simulate fictional baseball leagues. In 2006, he graduated from Chapelgate Christian Academy and left the state of Maryland to continue his education at the College of William and Mary.

While at college, Andrew majored in Mathematics and Economics and continued to play baseball for the William and Mary Club Baseball team. In his two years as president, Andrew acquired important leadership and communication skills that would later prove valuable in both classroom and workplace settings. It was also at William and Mary where Andrew was introduced to academic research for the first time. He spent two summers researching within the mathematics department and was fortunate enough to travel with his advisor to present research findings at multiple universities in Hong Kong. While Andrew found mathematical research interesting, he ultimately wanted to work on problems with a more applied focus. For his undergraduate thesis, Andrew extended his mathematics research to include statistical methods and was quickly hooked by the field.

With statistics proving to be the right fit for Andrew’s interests, he enrolled in the statistics PhD program at North Carolina State University after graduating from William and Mary in 2010. En route to his PhD, Andrew earned a Masters of Statistics in 2012. While working on his dissertation, Andrew served as the CALS statistical consultant and was reminded how much he loved using statistics to solve applied problems. Andrew also got to try his hand at using statistics and data visualization to solve business problems in a graduate internship at Red Hat. These positive experiences confirmed to Andrew that he was in the right discipline and would be pursuing a career in industry after graduation. Under the direction of Dr. Peter Bloomfield, Andrew was able to marry his economics and statistical knowledge and is set to earn his PhD in the fall of 2015. After graduation, Andrew will begin working for Andrew Davidson and Company where he will build risk models for mortgage backed securities.
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backgrounds. Joy, you gave me the confidence to tackle complex analyses and to trust my statistical instincts, even when the client’s design was far from perfect. Keith, you taught me so much about how to find, address, and communicate meaningful results in real world business problems. I truly enjoyed coming to Red Hat every day because of your leadership, and am proud to say we helped move the business forward and established a more data-driven culture.

I am also thankful for the incredible education I received at the College of William and Mary. My undergraduate thesis advisors, Dr. Chi-Kwong Li and Dr. Tanujit Dey, were instrumental in my growth as a statistician, as well as my decision to pursue a PhD at NC State.

While they aren’t typically found in a classroom, I honestly don’t know where I would be without the amazing staff within the NC State Statistics department. Allison, you truly are like a North Carolina mom to me, and there was no greater comfort than knowing you would always be there for a hug, a kind word, or simply a break from work. I am also convinced that my code would still be running if it were not for the incredible support of Chris Waddell. Your help making my code more efficient, loading R packages onto the cluster, and making sure I fully utilized our computing resources was invaluable to my graduate school experience. I am also thankful to Lanakila Alexander and Adrian Blue for their help regarding graduate school issues and funding.

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Part I

Introduction
CHAPTER 1

SYSTEMIC RISK

1.1 Overview

The financial crisis of 2007-2009 saw the collapse of large financial institutions, which in turn challenged the stability of the financial markets as a whole. American International Group (AIG), Bear Stearns, Citigroup, Fannie Mae, and Freddie Mac were just a few of the failed financial institutions that required large scale intervention from the United States (US) government. Total government intervention was on the scale of trillions of dollars. These failures had an enormous impact on the global economy. From late 2008 through 2009 stock markets fell by 42% in the US, 49% in Europe, 35% in Japan, and 50% in Latin America on a dollar-adjusted basis (Acharya et al., 2012).

It is now widely accepted that the models used to capture the effects of major disturbances to the financial system proved inadequate. As a result there has been a substantial increase in government and academic research regarding the concept of systemic risk, the risk inherent to the entire market. In 2010 the US Congress passed the Dodd Frank Act as a comprehensive financial reform bill in response to the financial crisis. The starting point for all of the bill’s directives was the accurate and timely measurement
of systemic risk (Bisias et al., 2012). The Act also established the Financial Stability Oversight Council and charged them with the task of identifying systemic risks and recommending policies to help minimize these risks (Acharya et al., 2013). In the academic world, the number of publications containing the phrases “systemic risk” and “financial system” jumped from approximately 500 publications in 2006 to over 2,000 publications in 2010 (Markeloff et al., 2012). This research has been wide in scope, with methods that rely on probability distributions, macroeconomic variables, network analysis, stress-testing, illiquidity, and defaults (Bisias et al., 2012). The financial crisis has created a new focus on trying to define, measure, and regulate systemic risk.

With such a widespread focus on systemic risk, there are multiple working definitions for the term. Hendricks et al. (2007) defines systemic risk as: “the risk that an event will trigger a loss of economic value or confidence in, and attendant increases in uncertainty about, a substantial portion of the financial system that is serious enough to quite probably have significant adverse effects on the real economy.” Potential events that fit this definition include market crashes, wars, and electronic security breaches. However, a large focus has been placed on a different type of event, the failure of large financial institutions. The financial crisis of 2007-2009 provided multiple examples of how the failure of these large financial institutions could be a mechanism for systemic risk. This caused Federal Reserve Board Governor Daniel Tarullo to state, “financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy” (Brownlees and Engle, 2011). Thus much of the recent research regarding systemic risk has been focused on Systemically Important Financial Insitutions (SIFIs). Their goal is to be able to identify and rank these SIFIs in a way that could potentially lead to regulation, and hence reduce a market’s systemic risk.

While many researchers have created their own systemic risk measure that can be used to identify and rank SIFIs, few have had the impact of the SRISK measure created by Brownlees and Engle (2011). SRISK uses publicly available data to calculate the capital shortage a firm would be faced with if there was a significant decline in the market. Additionally the SRISK calculation is done in such a way that the contribution of each individual firm can be aggregated to obtain the systemic risk level of the entire
financial system. Thus, firms with high SRISK have the largest potential for big losses during a market decline and as such, pose the largest threat to the financial system as a whole (Acharya et al., 2013).

The SRISK calculation depends on a financial institution’s size and leverage, which can be represented by the firm’s debt and equity values. However, the SRISK calculation hinges on estimating the amount of equity a firm would lose conditional upon a substantial market decline (Brownlees and Engle, 2011). This quantity is known as a firm’s Marginal Expected Shortfall (MES) and it is based on the firm’s volatility as well as its correlation with the market. Brownlees and Engle use a dynamic bivariate process of the financial institution and a broad market index to reflect the market’s view of that institution’s systemic risk (Acharya et al., 2013). Specifically, the process simulates market and firm returns into the future by decomposing return behavior into volatility, correlation, and tail behavior for both the market and the firm. All of these quantities are time varying and dependent on the econometric model used for their estimation.

Much of the research to date has studied the validity of SRISK and other systemic risk metrics, including possible regulatory responses based on these metrics and how these methods compare in their ranking of SIFIs. However, far less work has focused on the econometric methods and estimation procedures that drive these systemic risk metrics. Throughout this dissertation our goal is to study how changes to the econometric methods and estimation procedures impact the estimation of MES and SRISK. We provide insight regarding the variability of MES and SRISK that comes from both model selection and parameter estimation. The remainder of this chapter gives a brief history of the academic research regarding systemic risk metrics and how SRISK fits into the overall landscape.

1.2 Literature Review

Since the financial crisis of 2007-2009 there has been considerable interest in systemic risk analysis. There are now a plethora of models that emphasize different aspects of systemic risk and use a wide array of statistical methods in their implementation (Bisias et al., 2012). As such it is not feasible to provide an exhaustive review of the literature that deals with the subject of systemic risk. However in this section we attempt to provide an overview of systemic risk measures that are most often compared to the
SRISK procedure (Brownlees and Engle, 2011). More complete summaries of the academic literature regarding systemic risk can be found in the works of Bisias et al. (2012), Brunnermeier and Oehmke (2012), and Markeloff et al. (2012). In their taxonomy of systemic risk measures, Bisias et al. have labeled the majority of the methods that are often compared with SRISK as cross-sectional measures due to the fact that “such an approach aims to examine the co-dependence of institutions on each other’s ‘health’.” Additionally, many of these measures are linked because they rely on probability distribution methods in their calculation of systemic risk (Bisias et al., 2012).

The SRISK methodology of Brownlees and Engle (2011) is rooted in the Systemic Expected Shortfall (SES) method first introduced by Acharya et al. (2010). The creation of SES was motivated by the fact that the regulations put forth in Basel I and II were only designed to limit a financial institution’s risk in isolation. Thus Acharya et al. felt that there was not a sufficient focus on the systemic risk of a financial institution and proposed SES as a simple model of systemic risk that could be used in regulatory policy. Like Brownlees and Engle, Acharya et al. view systemic risk as a firm’s likelihood to be undercapitalized when the market is also undercapitalized. Thus, as a theoretical construct, SES attempts to capture the expected systemic cost of a financial institution by multiplying the probability of a systemic crisis by the conditional loss of the firm in such a crisis. Mathematically this is formulated as

\[ SES^i = E[\mathbb{1}(za^i - w^i_1)] = P(\mathbb{1}) \times E[(za^i - w^i_1) | \mathbb{1}] \tag{1.1} \]

where \( \mathbb{1} \) is an indicator of a systemic crisis, \( w^i_1 \) is the equity for firm \( i \), and \( za^i \) is the target level of equity denoted as a fraction, \( z \), of assets, \( a^i \), for firm \( i \).

According to Brownlees and Engle (2011), “the main shortcoming of SES, however, is that it cannot be used for ex-ante systemic risk measurement: their approach requires data from the actual financial crisis for estimation. Thus it is unclear whether it would have been possible to compute SES before 2007-2009.” In their work Acharya et al. (2010) attempted to develop some leading indicators of systemic risk in their own form of a Marginal Expected Shortfall (MES) calculation and a calculation for a firm’s leverage. However, these indicators were not used in the calculation of SES but instead were simply
regressed on their SES calculation to see if the indicators had any predictive power.

The success of Brownlees and Engle (2011) is to take the theoretical ideas presented in Acharya et al. (2010) and use them in a dynamic process that is able to compute current levels of systemic risk, without the need for actual financial crisis data. The SRISK measure uses data on a given firm’s equity and debt that can easily be measured from public databases, like the Securities and Exchange Commission’s (SEC) 10-Q Database. The only additional data SRISK requires is a given firm’s daily returns from the stock market. However, the heart of the SRISK calculation is still rooted in the vision of Acharya et al. to determine how the undercapitalization of a firm imposes a cost on the economy and market as a whole. As Brownlees and Engle simply stated “a shortage of capital is dangerous for the firm and its bondholders, but it is dangerous for the whole economy if it occurs just when the rest of the sector is undercapitalized.”

Work has recently been completed to extend the SRISK framework. The new Component Expected Shortfall (CES) was designed by Banulescu and Dumitrescu (2015) to use firm level variables within their calculation of MES. CES still relies upon the same bivariate process used by Brownlees and Engle (2011) to calculate MES, but CES directly weights MES in accordance with the size of the firm during the MES calculation. Thus CES is a component based approach to systemic risk measurement, as opposed to the marginal approach taken by SRISK. In the computation of SRISK, a layer of firm level characteristics is added after the computation of MES. One of the proposed benefits of CES is the ability to truly produce daily estimates of systemic risk, since it does not rely on debt and equity data that is not publicly available on a daily basis. Additionally, just like SRISK, the new CES measure decomposes the systemic risk of the market into the systemic risk of each individual firm, making it easily interpretable. In terms of SIFI rankings, CES has been shown to give similar but not identical results to SRISK. This is an example of how a change in the estimation procedure of SRISK can produce differences in SIFI rankings.

The contemporary systemic risk measure that is most often compared to SRISK is the CoVaR methodology introduced by Adrian and Brunnermeier (2011). In their work they view a firm’s contribution to systemic risk as the risk to the market conditional on that financial institution being in a state of distress. Note that this is the reverse conditioning of the approach taken by Acharya et al. (2010) and Brownlees and Engle (2011). The reason for this reverse conditioning is twofold. First, Adrian and Brunnermeier
wanted a risk measure that could identify the risk to the system from both large and small institutions. They argue that there is the potential for systemic risk in large institutions because of their size and their interconnectedness that can cause a negative risk spillover effect on other firms. However, there is also the potential for systemic risk in small institutions whose actions can mirror one another and hence contribute to the systemic risk present in the herd of smaller institutions. Second, Adrian and Brunnermeier wanted to capture the risk that can build up in the background during times when the market is strong, but only materializes during a crisis. This reasoning causes Adrian and Brunnermeier to criticize the expected shortfall methods of Acharya et al. and Brownlees and Engle due to the fact that their methods have the opposite conditioning and “do not address the stylized fact that risk is building up in the background during boom phases characterized by low volatility and materializes only in crisis times.”

By definition \( \text{CoVaR}_{ij}^q \) is defined as the \( q \) quantile VaR of institution \( j \) conditional on some event \( C(X^i) \) of institution \( i \)

\[
P(X^i \leq \text{CoVaR}_{ij}^q | C(X^i)) = q. \tag{1.2}
\]

In Adrian and Brunnermeier (2011), the primary case is to have \( j \) be the entire financial system and the conditional event to be \( C(X^i) = (X^i = \text{VaR}_i^q) \). The marginal contribution of financial institution \( i \) to the entire financial system is then defined as

\[
\Delta \text{CoVaR}_{ij}^q = \text{CoVaR}_{ij}^{X_i=\text{VaR}_i^q} - \text{CoVaR}_{ij}^{X_i=\text{Median}_i^q}. \tag{1.3}
\]

Thus the systemic risk measure \( \Delta \text{CoVaR}_{ij}^q \) is measuring the difference between the financial system’s VaR when institution \( i \) is in distress, and the financial system’s VaR when institution \( i \) is at its median state. While there are multiple computation methods for CoVaR, the primary method focused on by Adrian and Brunnermeier is to use quantile regression. By regressing the system on institution \( i \), the predicted values from the quantile regression yield the VaR of the entire financial system conditional on the conditioning event \( X^i \).

A more generalized approach to the CoVaR method was later put forth by Girardi and Tolga Ergün (2013). The major change in this new work was to modify the definition of the conditioning event from
having losses being exactly equal to the firm’s VaR, to having losses being at most equal to the firm’s VaR. Thus for their new systemic risk measure $\Delta CoVaR^\leq$, the conditioning event is defined as

$$C(X^i) = (X^i \leq VaR^i_q).$$  \hfill (1.4)

where $X^i$ and $VaR^i_q$ are negative to represent firm losses. This slight change has major implications for the systemic risk measure. First the new measure can now consider even more severe distress events from institution $i$. Previously, any changes in the distribution of CoVaR past the firm’s VaR were ignored by $\Delta CoVaR$. Thus if the losses of a firm exceeded the value of its VaR, the systemic risk impact of the additional losses was not accounted for. Second, changing the conditioning event allows for strictly monotonic behavior between $\Delta CoVaR^\leq$ and the dependence parameter. Under the previous conditioning event CoVaR would decrease once it passed a threshold of correlation between the institution and the financial system (Mainik and Schananning, 2014). Thus it was failing to detect additional systemic risk in cases where there is an even higher degree of dependence between the firm and the financial system. Finally, by changing the conditioning event, Girardi and Tolga Ergün were able to estimate the CoVaR of an institution as time-varying, which was not the case in Adrian and Brunnermeier (2011). Computation of $\Delta CoVaR^\leq$ is now done via a three step procedure that includes GARCH methods to estimate a firm’s VaR, and DCC methods to estimate the joint distribution of the firm and the financial system.

$\Delta CoVaR$ and its modification $\Delta CoVaR^\leq$ are seen along with SRISK as the leading return-based metrics for systemic risk. As such, works such as Benoit et al. (2013) and Jiang (2012) have developed a common framework for both metrics in order to compare them. The work of Benoit et al. focuses on the reasoning behind differences in the SIFI rankings of these two metrics. In an empirical comparison they found that the top 10 SIFI rankings according to $\Delta CoVaR$ and SRISK averaged just 9.9% concordant pairs. This led them to derive the conditions for which the rankings for both metrics would converge. As expected, given the empirical results, they found that for a given institution and time point, the ratio between a firms $\Delta CoVaR$ and its MES (and hence SRISK) is firm-dependent. Thus they derived the condition under which both SIFI rankings are convergent. When comparing two firms with equivalent
levels of liabilities, firm $i$ is riskier than firm $j$ according to both SRISK and CoVaR if

$$\rho_{it} \geq \rho_{jt} \quad \text{and} \quad W_{it} \leq W_{jt} \times \exp[18 \times ES_{mt}(\alpha) \times (\beta_{jt} - \beta_{it})]$$

(1.5)

where $\rho$ is the correlation between the market and the firm, $W$ is the daily market capitalization for the firm, $ES_{mt}$ is the expected shortfall of the market for the $\alpha$ level VaR, and $\beta_{it} = \rho_{it} \frac{\sigma_{it}}{\sigma_{mt}}$. Thus for SRISK and $\Delta\text{CoVaR}$ to be convergent in their SIFI rankings, the riskier firm needs to have a higher correlation and a lower market capitalization (higher leverage). Benoit et al. also question whether or not these risk measures are capturing the complex nature of systemic risk by showing that MES, SRISK, and CoVaR are all strongly related to a single factor. According to Benoit et al. strong positive relationships exist between MES and $\beta_{it}$, SRISK and liabilities, and $\Delta\text{CoVaR}$ and VaR.

The work by Jiang (2012) also develops a common framework for SRISK and $\Delta\text{CoVaR}$ but does so based on a copula model to study tail dependence. The thought is that using only the correlation coefficient, $\rho_{it}$, to measure co-movement of the market and the firm during a crisis is not sufficient. In their findings, they first note that estimating the original $\Delta\text{CoVaR}$ (Adrian and Brunnermeier, 2011) via quantile regression will underestimate the systemic risk when there is lower tail dependence. Second, they concur with the findings of Mainik and Schaanning (2014) that $\Delta\text{CoVaR}$ is in conflict with dependence measures, but this is fixed by $\Delta\text{CoVaR}^\leq$ (Girardi and Tolga Ergün, 2013). Finally, while $\Delta\text{CoVaR}^\leq$ is strongly correlated with tail dependence, SRISK is more connected with firm level characteristics. This implies that $\Delta\text{CoVaR}^\leq$ is more in line with the commonly used “too interconnected to fail” paradigm, while SRISK is more in line with the “too big to fail” paradigm (Jiang, 2012).

While there is quite a lot of attention being paid to SRISK and CoVaR, there has also been a focus on using Credit Default Swap (CDS) data to more accurately measure systemic risk. Chan-Lau et al. (2009) point out that the co-movements in risk of financial institutions do not always follow a linear pattern. Thus they introduce the idea of Co-Risk, which extends the quantile regression estimation framework of CoVaR (Adrian and Brunnermeier, 2011) to be based on CDS data. The use of CDS data in the quantile regression allows for a more accurate estimation of the co-movement in risk factors when the market
is under duress. Similarly, Huang et al. (2012) use CDS data under an Expected Shortfall framework to create the Distress Insurance Premium (DIP) for measuring systemic risk. To compute a systemic risk indicator, Huang et al. create sub-portfolios for each firm from the full portfolio of the market. This market portfolio can then be used to measure the price of insurance against a systemic event in the market. The DIP is simply the risk-neutral expected loss of the firm’s portfolio given that the losses in the market portfolio exceed a given threshold. Thus it is similar to the concepts put forth by Acharya et al. (2010) and Brownlees and Engle (2011) in computing MES, but differs in the fact that it uses risk-neutral expectations in the definition of a systemic event for the market.

There have also been important contributions to the area of systemic risk that are not cross-sectional by nature, and as such, use different statistical techniques. For example, Billio et al. (2010) introduces 5 different measures of systemic risk to capture its various components. In addition to using correlations, these methods rely on cross-autocorrelations, principal component analysis (PCA), regime-switching models, and Granger causality tests. Billio et al. (2012) extends this work, focusing specifically on their PCA and Granger causality test methods. Kritzman et al. (2011) also uses PCA to create their systemic risk measure, known as the absorption ratio. The absorption ratio measures how unified markets are, with the belief that when financial markets are tightly coupled they are more fragile to a negative shock, and hence have higher systemic risk. Other approaches include Zhou (2010) and Balla et al. (2012) who both use extreme value theory in their approach to measure systemic risk. The existence of this diversified research in systemic risk points to the importance of the field as a whole, and the challenge of trying to define such a complex issue.

The rest of the dissertation is structured as follows. Chapter 2 presents the basic statistical modeling tools that are prevalent and often used in systemic risk metrics. In Chapter 3, we formally define SRISK and thoroughly discuss its computation and the simulation procedure required for computing MES. Also included in this chapter is a discussion regarding importance sampling and how it can be used to produce a more efficient simulation method. Chapter 4 discusses how changes to a distributional assumption in the initial DCC-GARCH model can affect the estimation of MES. Thus, in essence, we are quantifying the variability present in MES due to model selection. In order to discuss the variability present in MES
due to parameter estimation, Chapters 5 introduces our Block Bootstrap for Estimating Equation (BBEE) methodology. Through simulations we are able to show that our new methodology adequately captures the DCC-GARCH parameter error, and allows for its propagation through the MES simulation procedure. Chapter 6 then uses our new BBEE methodology to analyze the variability in MES due to parameter error. Finally, Chapter 7 provides a comparison of the two different sources of variability, and identifies areas for future research.
2.1 GARCH Modeling

Volatility has always played an important role in financial modeling, especially models focused on risk management. The only way to understand the magnitude of a portfolio’s potential losses is to accurately estimate the volatility or variation in each of the financial instruments. The leading models for measuring volatility are the Autoregressive Conditional Heteroskedasticity (ARCH) model and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, introduced by Engle (1982) and Bollerslev (1986) respectively. The ARCH model estimates the variance of the conditional distribution by formulating it as a linear function of squared past residual values. The GARCH model extended this idea by adding past variance estimates to the linear function. Today these models are still widely used and considered the standard for volatility estimation. One reason for these models’ unparalleled success is their ability to capture the volatility clustering phenomenon that is often present in financial time series. Volatility clustering is the tendency for large changes in volatility to be followed by more large changes in volatility, regardless of sign (Mandelbrot, 1963). Similarly small changes in
volatility tend to lead to more small changes in volatility.

The GARCH model as originally put forth by Bollerslev (1986) can be noted as

\[ r_t = \sigma_t \epsilon_t \]

\[ \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i r_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \]  (2.1)

where \( \omega > 0, \alpha_i \geq 0, \) and \( \beta_j \geq 0. \) Here \( r_t \) is a series of log-returns such that for an asset price \( P_t \) the return is measured as \( r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \). The error distribution for the innovations \( \epsilon_t \) is often taken to be Gaussian with zero mean and unit variance. However, this error distribution is able to take other forms in order to better capture distinct features within the error distribution. For example, in distributions where the residuals exhibit longer tails leptokurtic error distributions such as the Student-\( t \) or Generalized Error Distribution are often used.

The amount of volatility clustering that is present within a GARCH model is often measured by its persistence (Ghalanos, 2014) defined as

\[ \hat{P} = \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j. \]  (2.2)

The higher the persistence the longer it takes for the model to return to the unconditional variance, \( \hat{\sigma} \), of the process. The unconditional variance is defined as

\[ \hat{\sigma} = \frac{\omega}{1 - \hat{P}}. \]  (2.3)

It is widely accepted that GARCH models where \( p = q = 1 \) provide an adequate fit for stock market return data and have even been shown to capture nonlinear serial dependence (Ashley and Patterson, 2010). For the \( p = q = 1 \) case the volatility is stable in the sense that it will continue to fluctuate around \( \hat{\sigma} \) as long as

\[ \alpha_1 + \beta_1 < 1. \]  (2.4)
Subsequent research has suggested many variations to the original ARCH and GARCH models in an attempt to better capture the behavior of specific time series. A recent review of the field found that there are more than 100 variations of the original ARCH and GARCH models published in academic literature (Bollerslev, 2008). The variation that is most important for our work is the GJR-GARCH model\(^1\) (Glosten et al., 1993). This variation extends the standard GARCH framework by allowing for asymmetry between positive and negative shocks. In its \(p = q = 1\) version the GJR-GARCH model has the parametric form

\[
\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \gamma r_{t-1}^2 \mathbb{I}(r_{t-1} < 0) + \beta \sigma_{t-1}^2 \tag{2.5}
\]

where once again \(\omega > 0, \alpha \geq 0, \beta \geq 0,\) and \(\mathbb{I}\) is a standard indicator function. While it is commonly assumed that \(\gamma \geq 0\) such that negative shocks lead to larger volatilities, there is nothing in the estimation of the GJR-GARCH model that requires \(\gamma \geq 0\). Additionally while the standardized residuals, \(r_t / \sigma_t\), are often taken to be standard normal, other error distributions can be used as well.

More generally the GJR-GARCH model fits into a family of GARCH models that can be described using a more general formalization of equation (2.1) and whose stationarity conditions have been derived by Ling and McAleer (2002). The specific stationarity condition for the GJR-GARCH model with \(p = q = 1\) is defined as

\[
\alpha + \beta + \kappa \gamma < 1 \tag{2.6}
\]

where \(\kappa = E[\mathbb{I}(\epsilon_t)\epsilon_t^2]\). Thus the persistence for the GJR-GARCH model is defined as

\[
\hat{P} = \alpha + \beta + \kappa \gamma \tag{2.7}
\]

where \(\kappa\) is once again the expected probability of a negative return. Since it is typically assumed that \(\epsilon_t\) follows a symmetric distribution, \(\kappa\) is often assumed to be one-half. Thus the standard assumption for GJR-GARCH stationarity is that the process reverts back to an unconditional volatility as long as

\[
\alpha + \beta + \frac{1}{2} \gamma < 1. \tag{2.8}
\]

---

\(^1\)This model is closely related to and often referred to as the TGARCH model (Rabemananjara and Zakoïan, 1993)
We highlight the GJR-GARCH model because it is the GARCH specification used by Brownlees and Engle (2011) in their calculation of SRISK. Additionally Brownlees et al. (2012) champion the GJR-GARCH model as the best GARCH specification in terms of volatility forecasting. However, other authors have championed different GARCH specifications and even other types of volatility models for their systemic risk metrics. We hope that further research will continue to check the robustnesses of SRISK and other systemic risk measures to different GARCH specifications and alternative volatility models.

2.2 DCC Modeling

In many applications modeling univariate volatility via a GARCH specification is not enough. Financial institutions interact with one another and with entire financial markets. Thus there is an obvious need to capture how different firms and markets correlate with one another. Any work that relies on a combination of firms and markets requires a multivariate method that estimates both volatility and correlation. Popular multivariate methods that focus on this volatility-correlation decomposition include the Constant Conditional Correlation (CCC) model (Bollerslev, 1990), the Dynamic Conditional Correlation (DCC) model (Engle, 2002), the Varying Correlation model (Tse and Tsui, 2002), and the Regime Switching Dynamic Correlation model (Pelletier, 2006). All of these models have their unique advantages, but here we restrict our focus to the DCC model. Once again this is in keeping with the choice of Brownlees and Engle (2011) for their SRISK methodology.

In estimating SRISK Brownlees and Engle (2011) view systemic risk as a firm’s capital shortfall conditional on the market falling below a certain threshold. Thus the correlation between the market and the financial institution of interest is a necessary component in their bivariate simulation process. Additionally, this correlation between the market and the financial institution does not need to stay constant across time. The DCC model (Engle, 2002) is based on first computing univariate conditional variances and then using the resulting standardized returns in order to provide a time-varying correlation estimate. Notationally for a bivariate time series the DCC model estimates $P_t$, a time-varying $2 \times 2$ correlation matrix between the market and firm $i$. Thus the estimate of the conditional correlation between
the market and the firm is simply the off-diagonal entry of $P_{it}$. Using matrix notation this can be seen as

\[
\text{Var}_{t-1} \begin{pmatrix} r_{mt} \\ r_{it} \end{pmatrix} = D_{it} P_{it} D_{it} = \begin{bmatrix} \sigma_{mt} & 0 \\ 0 & \sigma_{it} \end{bmatrix} \begin{bmatrix} 1 & \rho_{mt} \\ \rho_{it} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{mt} & 0 \\ 0 & \sigma_{it} \end{bmatrix}.
\]

However, $P_{it}$ needs to be a positive definite correlation matrix at every time point. As such the DCC method does not model $P_{it}$ directly, but instead defines the matrix $Q_{it}$ as a proxy for $P_{it}$. The transformation that assures $P_{it}$ is a positive definite correlation matrix at every time point is

\[
P_{it} = \text{diag}(Q_{it})^{-1/2} Q_{it} \text{diag}(Q_{it})^{-1/2}.
\] (2.9)

Here the $\text{diag}(A)$ operator denotes a diagonal matrix with the same elements as $A$ on the diagonal and zeros everywhere else. The specification of the proxy correlation matrix, $Q_{it}$, is itself based on a GARCH-type dynamic. Let $z_t$ contain the standardized residuals from both univariate GARCH fits for the conditional variance. Then $Q_{it}$ is defined as

\[
Q_{it} = (1 - \alpha_C - \beta_C) S_i + \alpha_C z_{it-1} z_{it-1} + \beta_C Q_{it-1}.
\] (2.10)

Here $S_i$ is the unconditional correlation matrix for the market and firm returns. The necessary conditions for $Q_{it}$ being positive definite mirror the stationarity conditions for a standard GARCH model. That is $S_i$ must be positive definite, $\alpha_C \geq 0$, $\beta_C \geq 0$, and $\alpha_C + \beta_C < 1$. When these conditions are met, $Q_{it}$ is guaranteed to be positive definite which in turn guarantees $P_{it}$ is positive definite correlation matrix.

The choice of Engle (2002) to model $Q_{it}$ using equation (2.10) is due to its relation to the familiar Multivariate GARCH process (MGARCH). The MGARCH model simply attempts to provide the matrix generalization to univariate GARCH models. Thus multiple univariate volatilities $\sigma_i$ are included in a
volatility matrix $H_t$ that has GARCH dynamics in vector form (Bollerslev et al., 1988). That is

$$\text{vech}(H_t) = s + \text{Aveh}(r_{t-1}r_{t-1}') + B\text{vech}(H_{t-1})$$

(2.11)

where vech() creates a vector out of the unique elements of its matrix counterpart. As seen in equation (2.10), the dynamic used to solve for $Q_t$ is GARCH by nature and as such $Q_t$ is often treated as a MGARCH process (Ding and Engle, 2001).

However, as Aielli (2006) pointed out, treating $Q_t$ as a true MGARCH process is not exactly correct. Aielli notes that $Q_t$ does not strictly follow an MGARCH process because the true conditional covariance matrix of $z_t$ is $P_t$ not $Q_t$. Aielli conjectured and later proved (Aielli, 2013) that treating $S_t$ as the second moment of $z_t$ and estimating it via the sample second moment of the estimated standardized returns yields an inconsistent estimator. The corrected DCC (cDCC) model (Aielli, 2006) instead defines

$$Q_t = (1 - \alpha_C - \beta_C)S_t + \alpha_C z_{it-1}^*z_{it-1}' + \beta_C Q_{t-1}$$

(2.12)

where $z_{it}^* = \text{diag}(Q_{1/2}^{1/2}z_{it})$. In this way $z_{it}^*$ follows a linear MGARCH process and $S_t$ is the sample second moment of $z_{it}^*$.

Simulation results from Aielli (2013) show that the DCC estimator can produce biased correlation estimates under certain settings. This bias is most pronounced in the cases where the true correlation is strong in magnitude, $\alpha_C^0$ is high, and the true persistence $\alpha_C^0 + \beta_C^0$ is close to 1. However, Aielli (2013) noted that “for typical values of the dynamic parameters ($\alpha_C^0 + \beta_C^0 > .8$ and $\alpha_C^0 \leq .04$) the bias is negligible.” In our work the majority of our estimates fit the definition of typical values noted by Aielli. As such, while Brownlees and Engle (2011) implement the cDCC model for their estimation of SRISK, we continue to implement the standard DCC model for computational ease and efficiency.
CHAPTER 3

SRISK

3.1 Definition

The premise first set forth by Acharya et al. (2010) and echoed by Brownlees and Engle (2011) is that while a shortage of capital is dangerous for a financial institution at all times, it becomes a threat to the entire economy if the rest of the market is undercapitalized as well. If a financial institution’s shortage of capital turns into a bankruptcy during a stable market, other institutions will be in a position to purchase the failing firm. Thus the risk imposed on the market by any single firm is relatively small. However, when the market is undercapitalized, even strong financial institutions will have less capital and will be less willing to take on more debt. Thus systemic risk can be realized in cases where the market as a whole has become undercapitalized.

The derivation of the SRISK measure starts by defining a capital buffer for a financial institution $i$ at a given timepoint $t$

$$CB_{it} = W_{it} - k(D_{it} + W_{it}).$$  \hfill (3.1)

Here $D_{it}$ is the book value of a firm’s debt at time $t$, $W_{it}$ is the market value of the firm’s equity at time $t$.\hfill (3.1)
and $k$ is the fraction of the firm’s assets that should be maintained as equity. Thus $k$ can be thought of as a prudential capital fraction. It is evident from equation (3.1) that a properly functioning firm should look to have a positive $CB$. That is, at any given time a firm should hold at least as much equity as what is thought to be prudent. Any firm with a negative $CB$ has the potential to burden the entire economy if the market falls into distress. In Brownlees and Engle (2011) the distress event is defined as a drop of the market index below a given threshold $C$ over a time horizon $h$. Using this definition of a distress event, the expected capital shortfall for any single firm is defined as

$$CS_{it+h|t} = -E_t[CB_{it+h}|R_{mt:t+h} < C]$$

$$= kE_t[D_{it+h}|R_{mt:t+h} < C] - (1-k)E_t[W_{it+h}|R_{mt:t+h} < C]$$  

where $R_{mt:t+h}$ is the cumulative market return between time periods $t$ and $t+h$. By assuming that debt can not be renegotiated in the case of a systemic event Brownlees and Engle are able to simplify equation (3.2) to

$$CS_{it+h|t} = kD_{it} - (1-k)E_t[W_{it+h}|R_{mt:t+h} < C]$$

$$= kD_{it} - (1-k)W_{it}(1 + E_t[R_{it:t+h}|R_{mt:t+h} < C])$$

$$= kD_{it} - (1-k)W_{it}(1 - MES_{it+h|t}(C)).$$  

(3.3)

Here a firm’s Marginal Expected Shortfall (MES) is defined to be the expected percentage loss in equity between time $t$ and $t+h$ conditional on the distress event. The SRISK index used to measure systemic risk is then

$$SRISK_{it} = \max(0, CS_{it+h}).$$  

(3.4)

Thus firms are considered to be without systemic risk if their estimated capital shortfall is negative when the market is in distress.

The strengths of SRISK as a systemic risk metric lie in its ability to include the size and leverage of a firm in its calculation, its ability be aggregated, and its use of only publicly available data. These features
lead to a risk measure that is not only inexpensive to implement, but also provides an easy interpretations with regards to a firm’s risk level. The SRISK of a single firm is simply the expected capital shortfall, in standard currency amounts, conditional on a distress event. Thus aggregate SRISK is simply

\[ SRISK_t = \sum_{i=1}^{I} SRISK_{it} \]

and it can be interpreted as the total amount of capital needed to bailout the entire financial system during a crisis event. Aggregate SRISK allows for economy wide systemic risk to be measured over time and can be used as a warning for potential distress events in the real economy. Brownlees and Engle (2011) showed that, when applied retroactively, aggregate SRISK did rapidly increase starting in 2007 and that the level of aggregate SRISK had tripled by January 2008. Additionally, the computation of aggregate SRISK allows for the calculation of an SRISK percentage for a particular firm. This percentage captures the proportion of bailout money that would be required by a particular firm in a crisis event. These percentages allow for a straightforward ranking of firms based on the stress they impose on the economy as a whole.

### 3.2 Computation

The computation of SRISK for a given firm requires three inputs: the firm’s book value of debt, the firm’s market value of equity, and an estimate for the firm’s Marginal Expected Shortfall (MES). Since firm debt and equity values can be gathered from publicly available data, the only computation required is the estimation of a firm’s MES. Thus it is important to study the statistical models and assumptions behind MES estimation. Large scale changes in MES carry the potential for large scale changes in SRISK, which in turn could affect SIFI rankings and potential regulatory responses.

Recall from equation (3.3) that MES is defined to be the firm’s expected cumulative return conditional on the market falling below a given threshold. The model proposed by Brownlees and Engle (2011) to compute returns for both the market and the firm is a dynamic bivariate process that relies on volatility and correlation estimates. By letting \( r_{mt} \) and \( r_{it} \) be the market and firm log-return on day \( t \), the bivariate
process used to simulate returns over time is

\[ r_{mt} = \sigma_{mt} \varepsilon_{mt} \]  \hspace{1cm} (3.6)

\[ r_{it} = \sigma_{it} \left( \rho_{it} \varepsilon_{mt} + \xi_{it} \sqrt{1 - \rho_{it}^2} \right) \]  \hspace{1cm} (3.7)

\[ (\varepsilon_{mt}, \xi_{it}) \sim F. \]  \hspace{1cm} (3.8)

Here \( \sigma_{mt} \) and \( \sigma_{it} \) are the conditional standard deviations for the market and firm returns, \( \rho_{it} \) is the conditional correlation between the market and the firm, and \( (\varepsilon_{mt}, \xi_{it}) \) are the market and firm shocks that help drive the system. The shocks, or innovations, are constrained to have zero mean, unit variance, and zero covariance while being identically distributed over time. However, since it is reasonable to think that systemically risky firms could be affected by extreme shocks in similar ways to the market, the innovations are not required to be independent.

The bivariate process in equation (3.6) and equation (3.7) allows for considerable flexibility in the specification of the volatility and correlation. Additionally there is no specification needed for the distribution of \( F \). Thus while the process is not dependent on any one particular choice of volatility or correlation model, the estimation of MES is clearly dependent on the chosen modeling framework. Ideally MES will be robust to the choice of modeling framework, but the potential for MES variability due to model selection should not be ignored. In their work, Brownlees and Engle (2011) champion a multi-stage modeling approach that is specified as follows. First, the two conditional standard deviations are estimated using the GJR-GARCH model presented in equation (2.5). Then estimates of the conditional correlation are obtained by using the DCC estimation procedure presented in Section 2.2. Finally inference regarding the innovations in \( F \) can be based on the residuals obtained from the DCC-GARCH model.

In addition to the specifications required for modeling volatility and correlations, the SRISK computation is dependent on multiple tuning parameters. The choice of the prudential ratio, \( k \), the threshold, \( C \), and the time horizon, \( h \), all play a role in the final value of SRISK. Recall from equation (3.1) that \( k \) denotes the assumed fraction of a firm’s assets that are to be maintained as equity. Clearly \( k \) is typically tied to regulatory procedures for a set of firms and can be changed to match any changes in regulations.
Brownlees and Engle discuss the potential to change $k$ for different institutions or assets, but rely on fixing $k = 8\%$ for their SRISK calculations. The distress event or crisis definition is controlled by the time horizon, $h$, and the market threshold, $C$. Recall from equation (3.2) that SRISK is calculated using the conditional event of market returns falling below $C$ for a given time period $h$. This allows SRISK to be flexible to different crisis definitions and allows for different SIFI rankings based on different crisis definitions. For their original SRISK calculations Brownlees and Engle (2011) defined a crisis as a market drop of 40% over a 6-month window. However, in the latest update to their paper, Brownlees and Engle (2015) focus on a market drop of 10% in a 1-month window. The reasoning given for this change is “to compare more naturally our methodology with other monthly frequency indicators of distress.” For the main results of our work we will continue to use the original crisis definition of a 40% market drop over a 6-month window. This not only is consistent with Brownlees and Engle original idea, but also is consistent with their website (NYU Stern Volatility Institute, 2014). A brief look at how our results change for this change in crisis definition is given in Appendix A.

In their work Brownlees and Engle provide a small assessment regarding the sensitivity of SRISK to the choice of $k$ and $C$. As expected, an increase in $k$, which is equivalent to imposing stricter capital requirements, creates larger SRISK estimates for an identical crisis definition. Similarly, larger thresholds for the systemic event also increase the capital shortage of a firm. However, the profiles of the SRISK estimate over time are unaffected by the choice of $k$ and $C$. In terms of the SRISK ranking of SIFIs, there is little change to the top 10 firms, but there is the potential for substantial movement lower in the rankings. Brownlees and Engle did not provide a comparison based on the time horizon $h$, always using either a 6-month (Brownlees and Engle, 2011) or 1-month (Brownlees and Engle, 2015) time horizon.

Regardless of the model specification or tuning parameters, over a single day time period there is a closed form solution to MES. This solution is derived in Brownlees and Engle (2011) as

$$
MES_{t-1}(C) = E_{t-1}(r_{it}|r_{mt} < C)
= \sigma_{it}E_{t-1}(\rho_{it}\epsilon_{mt} + \xi_{it}\sqrt{1 - \rho_{it}^2}|\epsilon_{mt} < C/\sigma_{mt})
= \sigma_{it}\rho_{it}E_{t-1}(\epsilon_{mt}|\epsilon_{mt} < C/\sigma_{mt}) + \sigma_{it}\sqrt{1 - \rho_{it}^2}E_{t-1}(\xi_{it}|\epsilon_{mt} < C/\sigma_{mt}).
$$

(3.9)
From equation (3.9) it is evident that a firm’s MES increases as a firm’s volatility increases. This makes intuitive sense as firms that are more volatile have the potential for more loss, hence a larger expected shortfall when the market is in distress. Additionally we see that the correlation between the market and the firm acts as a weight. When the market and firm are more correlated, the tail expectation of the market residuals plays a larger role. In contrast, when the market and firm are less correlated then the tail expectation of the firm’s residuals plays the larger role.

However, there is no closed form solution when calculating MES over a multiple time periods. The benefit of using a multi-period MES estimate is that it captures the exposure of a firm over a period of time instead of being limited to the reaction of the firm in a single day. For their SRISK calculations Brownlees and Engle rely on a MES that is calculated daily over a 6-month period using a simulation procedure. For clarity calculations of MES over a longer time horizon that require a simulation procedure are referred to as Long Run Marginal Expected Shortfall (LRMES).

### 3.3 Simulation Procedure for LRMES

Since Long Run Marginal Expected Shortfall (LRMES) estimates cannot be found via a closed form solution, a simulation procedure is used to forecast market and firm returns. The goal of a $h$-period ahead LRMES starting at time $t$ is to simulate $J$ paths of forecasted returns

$$
\begin{align*}
\left\{ r^j_{mt+\tau-1} \right\}_{\tau=1}^{h} & \quad j = 1, \ldots, J. 
\end{align*}
$$

(3.10)

The first step in this simulation procedure is to obtain current estimates for the conditional volatility and conditional correlation. This is done by applying the DCC process, introduced in Section 2.2, to a set of market and firm returns collected from time $(t-1-w)$ to time $(t-1)$. Thus the initial values and parameter estimates are based on a historical window that includes the previous $w$ trading days. Within the DCC process the GJR-GARCH model, introduced in Section 2.1, is used instead of a standard GARCH model for computing univariate volatility estimates. The empirical distribution of the standardized
residuals from the DCC-GARCH fit are then saved as \( \hat{F} \), which estimates the true innovations distribution, \( F \), via an empirical cumulative distribution function.

Once the current levels of volatility and correlation have been saved, the simulation proceeds to compute market and firm returns on each of the \( h \)-days for every path. Note that as the simulation procedure moves forward in time the parameter estimates \( (\omega, \alpha, \beta, \gamma, \alpha_C, \beta_C, S_i) \) are fixed allowing for recursive updates to \( \sigma_{mt}, \sigma_{it} \) and \( \rho_{it} \) along each path. Additionally, samples are taken with replacement from \( \hat{F} \) such that

\[
\left( \varepsilon^{j}_{mt+h-1}, \varepsilon^{j}_{it+h-1} \right)_{\tau=1}^{h} \sim \hat{F} \quad \text{for} \quad j = 1, \ldots, J. \tag{3.11}
\]

The dynamic volatility and correlation estimates for each path are combined with the samples from \( \hat{F} \) using equation (3.6) and equation (3.7) to obtain the full path of market and firm returns. For each path the firm’s expected return is calculated as

\[
R^{j}_{it+h-1} = \exp\left\{ \sum_{\tau=1}^{h} r^{j}_{it+\tau-1} \right\} - 1 \tag{3.12}
\]

with an analogous calculation for the market return \( R^{j}_{mt+h-1} \).

The LRMES is then calculated as the average firm loss conditional on the market falling below the threshold, \( C \). As such in Brownlees and Engle (2011) LRMES is defined as

\[
\text{LRMES}(C) = \frac{\sum_{j=1}^{J} R^{j}_{it+h-1} I(R^{j}_{it+h-1} < C)}{\sum_{j=1}^{J} I(R^{j}_{mt+h-1} < C)} \tag{3.13}
\]

where \( I \) is an indicator function that takes the value 1 when the condition is true and 0 otherwise. However, there can be confusion when the term “firm loss” is used. This confusion is based on whether a gross firm loss, which only includes the negative values of \( R^{j}_{it+h-1} \), or a net firm loss, which averages over all values of \( R^{j}_{it+h-1} \), should be used. To clarify we define LRMES as

\[
\text{LRMES}(C) = \frac{\sum_{j=1}^{J} R^{j}_{it+h-1} I(R^{j}_{it+h-1} < 0) I(R^{j}_{mt+h-1} < C)}{\sum_{j=1}^{J} I(R^{j}_{mt+h-1} < C)} \tag{3.14}
\]

thus clearly stating that a gross expected firm loss is being used.
Brownlees and Engle note that this simulation process is similar in nature to the risk management practice of scenario analysis. Thus its format and functionality should be familiar to regulators who are comfortable with stress test type calculations. Additionally, the simulation procedure also provides an estimate of the conditional probability of a systemic event as

\[
P_t(R_{mt+t+h-1}^j < C) = \frac{1}{J} \sum_{j=1}^{J} \mathbb{I}(R_{mt+t+h-1}^j < C).
\]

(3.15)

This in and of itself can be a useful calculation in that it defines the likelihood of a particular crisis scenario using current market conditions.

When using LRMES over an extended time horizon as opposed to the single period MES, there is an additional computational burden. Fortunately, the LRMES algorithm lends itself to a few shortcuts that can drastically reduce computation time. First, as Brownlees and Engle (2011) noted, the conditional event required in equation (3.14) is only dependent on market parameters. As such we can simulate all \( J \) market paths first in order to check and see which paths meet the crisis event criteria. For each path we can save the sequence of draws from the innovations distribution and then only construct firm return trajectories when the market is deemed in crisis. This saves us from having to compute all \( J \) firm paths and then discarding paths where the market is not in crisis.

An additional speedup that was not explicitly discussed in Brownlees and Engle (2011) is the use of an importance sampler to create more scenarios where the market is in a crisis state. Recall that importance sampling can be used in the generic Monte Carlo integration problem

\[
E_f[h(X)] = \int_X h(x)f(x)dx.
\]

(3.16)

Importance sampling is essentially a weighted sampling method that is used where there is a benefit from sampling from a distribution \( g \) instead of a distribution \( f \) (Robert and Casella, 2004). By definition, importance sampling can be represented as

\[
E_f[h(X)] = \int_X h(x)\frac{f(x)}{g(x)}g(x)dx
\]

(3.17)
and hence approximates the expectation as

\[ E_f[h(X)] \approx \frac{1}{m} \sum_{j=1}^{m} f(X_j) h(X_j). \]  

(3.18)

As noted in Robert and Casella, while the convergence of equation (3.18) to equation (3.16) is assured, the variance is not constrained to be finite. As such an alternative approach is to divide by the sum of the weights so that

\[ E_f[h(X)] \approx \frac{\sum_{j=1}^{m} h(X_j)f(X_j)/g(X_j)}{\sum_{j=1}^{m} f(X_j)/g(X_j)}. \]  

(3.19)

This approximation also converges to equation (3.16) while adding a small amount of bias to improve the variance of the estimator. In practice equation (3.19) is generally preferred to equation (3.18) (Robert and Casella, 2004), but in our case the calculation of LRMES requires the estimation of a conditional expectation. This means that there is integration in both the numerator and denominator of equation (3.14) and thus the choice of equation (3.18) or equation (3.19) is irrelevant. The additional denominator component of equation (3.19) is canceled out in the computation of LRMES.

Our LRMES simulation procedure relies on importance sampling so that we can draw more extreme innovations which leads to more crisis scenarios. While the use of importance sampling is less necessary when obtaining LRMES estimates during times when economic conditions lend themselves to systemic behavior, it is a powerful speedup in non-crisis times when the market is more calm. Recall from equation (3.11) that we need to sample innovation pairs from the empirical cumulative density function of estimated residuals, \( \hat{F} \). This sampling is typically done with equal weighting so that every innovation pair has the same probability of being sampled. For our importance sampling we define \( g \) as weighted sampling from \( \hat{F} \) so that the probability of being sampled is equal to

\[ \frac{\exp\{-\lambda \epsilon_{mt} - \mu \xi_{it}\}}{\sum_{t=1}^{h} \exp\{-\lambda \epsilon_{mt} - \mu \xi_{it}\}}. \]  

(3.20)

Thus when \( \lambda \) is large there is a higher probability to sample pairs with large negative market innovations and when \( \mu \) is large there is a higher probability to sample pairs with large negative firm innovations.
In the case where both \( \lambda = 0 \) and \( \mu = 0 \) all of the weights are equal, which is equivalent to not using importance sampling.

The choice of \( \lambda \) and \( \mu \) should increase the probability of a crisis occurring in our simulations while also helping to minimize the variance of our LRMES estimate. As such we studied these effects over a range of \( \lambda \) and \( \mu \) values. In this study we simulated \( J = 25,000 \) market paths and found the percentage of times the market fell by more than 40%. From Table 3.1 we see that in historically calm times before the start of the financial crisis it is extremely hard to produce a crisis scenario without importance sampling. As an example our simulation estimating LRMES without importance sampling on January 31, 2007 produced only 1 crisis scenario in 25,000 market paths. This makes it impossible to compute LRMES standard errors and is extremely inefficient requiring us to ignore 99.99% of our simulated market paths. Table 3.1 also shows that even during the financial crisis of 2007-2009 the percentage of paths in which the market dropped below 40% is fairly low without importance sampling. During this time the equally weighted random draws from our innovation distribution still require us to ignore 97% of our market paths. From Table 3.1 it is also evident that increasing \( \lambda \) has a larger impact on increasing the percentage of crisis iterations. This is due to the fact that \( \lambda \) is the direct weight on the market innovations whereas \( \mu \) is the direct weight on the firm’s innovations. Since the innovations \((\epsilon_{mt}, \xi_{it})\) are not independent, there is reason to believe that choosing extreme firm innovations could also lead to choosing extreme market innovations. However, Table 3.1 shows that this effect is negligible when compared to directly choosing more extreme market innovations.

But how do the values of \( \lambda \) and \( \mu \) affect the standard error of LRMES estimate? Table 3.2 illustrates how the LRMES standard error changes for JP Morgan Chase on January 30, 2009. Note that in Table 3.2 we also considered negative values of \( \mu \). By using negative value of \( \mu \) we are biasing the sampling to include pairs of innovations where the firm innovation is largely positive. The use of negative \( \mu \) values could possibly lead to more crisis iterations if firm innovations are typically of the opposite sign of market innovations. The highlighted result of Table 3.2 is that LRMES standard error is minimized when \( \lambda = .15 \) and \( \mu = 0 \). Cross referencing this result with Table 3.1 shows that this corresponds to the case where approximately 40% of our simulations were considered to be a crisis scenario. While it would be
Table 3.1: The percentage of simulations where the market dropped more than 40% for varying importance sampling weights.

<table>
<thead>
<tr>
<th></th>
<th>( \mu = 0 )</th>
<th>( \mu = 0.05 )</th>
<th>( \mu = 0.10 )</th>
<th>( \mu = 0.15 )</th>
<th>( \mu = 0.20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/31/2007</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda = 0 )</td>
<td>0.004%</td>
<td>0.008%</td>
<td>0%</td>
<td>0.004%</td>
<td>0%</td>
</tr>
<tr>
<td>( \lambda = 0.05 )</td>
<td>0.03%</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
</tr>
<tr>
<td>( \lambda = 0.10 )</td>
<td>0.16%</td>
<td>0.13%</td>
<td>0.12%</td>
<td>0.08%</td>
<td>0.07%</td>
</tr>
<tr>
<td>( \lambda = 0.15 )</td>
<td>0.60%</td>
<td>0.66%</td>
<td>0.57%</td>
<td>0.58%</td>
<td>0.61%</td>
</tr>
<tr>
<td>( \lambda = 0.20 )</td>
<td>2.43%</td>
<td>2.35%</td>
<td>2.32%</td>
<td>2.28%</td>
<td>2.18%</td>
</tr>
<tr>
<td>1/30/2009</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda = 0 )</td>
<td>2.95%</td>
<td>2.90%</td>
<td>2.92%</td>
<td>2.92%</td>
<td>3.03%</td>
</tr>
<tr>
<td>( \lambda = 0.05 )</td>
<td>8.58%</td>
<td>8.54%</td>
<td>8.67%</td>
<td>8.79%</td>
<td>9.00%</td>
</tr>
<tr>
<td>( \lambda = 0.10 )</td>
<td>21.34%</td>
<td>21.04%</td>
<td>20.94%</td>
<td>20.89%</td>
<td>21.08%</td>
</tr>
<tr>
<td>( \lambda = 0.15 )</td>
<td>40.28%</td>
<td>40.03%</td>
<td>39.73%</td>
<td>39.87%</td>
<td>40.02%</td>
</tr>
<tr>
<td>( \lambda = 0.20 )</td>
<td>62.88%</td>
<td>62.38%</td>
<td>61.84%</td>
<td>61.80%</td>
<td>61.84%</td>
</tr>
</tbody>
</table>

unwise to simply conclude that \( \mu = 0 \) produces the best results from a single firm and date combination, additional analysis confirms that setting \( \mu = 0 \) consistently produces the lowest LRMES standard errors.

Paired with our previous result that \( \mu \) has a negligible effect on the percentage of crisis scenarios, we set \( \mu = 0 \) for the remainder of the dissertation.

From Table 3.1 it is clear that equivalent \( \lambda \) values do not produce an equivalent percentage of crisis iterations on different dates. So the question then becomes: is the LRMES standard error minimized by a fixed value of \( \lambda \) or by a fixed percentage of simulations with crisis scenarios? Studying the additional analysis of Appendix B shows that LRMES standard error is minimized at different values of \( \lambda \) on different dates. So in order to test whether or not LRMES standard errors are minimized for a fixed percentage of crisis scenarios, we wrote our own recursive search method that iterates over \( \lambda \) until the desired crisis probability is produced. This search method starts with \( \lambda = .15 \) and runs a search step that simulates the market return over 6 months 5,000 times. The method then computes the percentage of paths that ended in crisis from our search step and compares this percentage to the desired goal. If
Table 3.2: LRMES standard error estimates via importance sampling for JPM on 1/30/2009.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>-0.2</th>
<th>-0.15</th>
<th>-0.10</th>
<th>-0.05</th>
<th>0</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.148</td>
<td>8.231</td>
<td>1.879</td>
<td>1.245</td>
<td>1.137</td>
<td>1.614</td>
<td>2.948</td>
<td>4.316</td>
<td>3.652</td>
</tr>
<tr>
<td>0.05</td>
<td>2.781</td>
<td>2.760</td>
<td>1.097</td>
<td>0.747</td>
<td>0.734</td>
<td>1.000</td>
<td>1.819</td>
<td>3.451</td>
<td>3.608</td>
</tr>
<tr>
<td>0.10</td>
<td>2.317</td>
<td>1.603</td>
<td>0.882</td>
<td>0.532</td>
<td>0.535</td>
<td>0.798</td>
<td>1.857</td>
<td>3.610</td>
<td>2.817</td>
</tr>
<tr>
<td>0.15</td>
<td>2.458</td>
<td>3.943</td>
<td>0.819</td>
<td>0.509</td>
<td><strong>0.508</strong></td>
<td>0.727</td>
<td>1.458</td>
<td>2.851</td>
<td>5.539</td>
</tr>
<tr>
<td>0.20</td>
<td>2.636</td>
<td>2.549</td>
<td>0.871</td>
<td>0.557</td>
<td>0.572</td>
<td>0.786</td>
<td>1.699</td>
<td>2.596</td>
<td>1.905</td>
</tr>
</tbody>
</table>

the difference is within 0.5% then $\lambda$ is fixed and our standard LRMES simulation under importance sampling begins. If the difference is larger than 0.5% then $\lambda$ is adjusted based on the magnitude of the difference and the search step is repeated until the difference is within 0.5% or until this search step has been repeated 25 times.

Using this recursive search method we produced 50 LRMES standard error estimates over a sequence of crisis iteration percentages that increased from 15% to 40% by 5%. The crisis percentage that minimized the average LRMES standard error is shown in Table 3.3 for 3 different firms and 4 different dates. The $\lambda$ values that correspond to these results as well as the numerical standard errors can be found in Appendix B. From Table 3.3 it is clear that the LRMES standard error is often minimized when $\lambda$ is chosen to produce crisis scenarios 35% of the time. The 3 cases in Table 3.3 where the LRMES standard error is not minimized at 35%, do not show statistically significant differences in the LRMES standard error at the reported percentage and the LRMES standard error at 35%. As such for the remainder of the dissertation our first step in estimating LRMES is to first search for a $\lambda$ that will produce crisis scenarios 35% of the time. The rest of the simulation procedure is then carried out using the fixed value of $\lambda$ identified by the search.
Table 3.3: The crisis percentage in our iterative search for \( \lambda \) where the average LRMES standard error is minimized.

<table>
<thead>
<tr>
<th>Date</th>
<th>JPM</th>
<th>BAC</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/31/03</td>
<td>35%</td>
<td>35%</td>
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</tr>
<tr>
<td>1/31/05</td>
<td>35%</td>
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<tr>
<td>1/31/07</td>
<td>25%</td>
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<tr>
<td>1/30/09</td>
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</table>
Part II

LRMES Variability via Model Selection
CHAPTER 4

THE EFFECT OF NON-GAUSSIAN WORKING LIKELIHOODS

Most current research regarding systemic risk focuses on defining new systemic risk metrics or comparing how existing metrics perform in how they rank Systemically Important Financial Institutions (SIFIs). Their goal is to determine how these metrics can be used to help prevent future financial crises. Systemic risk metrics need to be able to accurately rank and identify the firms that contribute most to systemic risk. This is especially important if the Financial Stability Oversight Council, the Basel Committee on Banking Supervision, or some other regulatory body plans to monitor and regulate firm contributions to systemic risk. In this regulatory setting systemic risk metrics need to not only be accurate, but also consistent in the sense that they cannot be “gamed” by any firm. The current research has provided a much better understanding of systemic risk as a whole and there are numerous ongoing discussions regarding how to best identify SIFIs and the differences between various rankings.

However, none of this research has studied the impact of changes to the statistical models that these systemic risk metrics rely on. Metrics such as SRISK (Brownlees and Engle, 2011) do not rely on simple
or straightforward calculations. Instead they rely on statistical models and their assumptions to produce a measure of systemic risk. So what happens to a systemic risk metric if its underlying model or model assumptions change? Are systemic risk metrics robust to model specification? If not can we conclude that a given model or set of model assumptions is superior and give recommendations regarding the different choices? In this chapter we pursue the answer to these questions regarding the statistical model behind Long Run Marginal Expected Shortfall (LRMES), which is the estimated quantity that drives the SRISK index. Recall from Section 3.2 that LRMES does not have a closed form solution. When the marginal expected shortfall is calculated over a multi-day time horizon a simulation procedure is needed to produce LRMES estimates. While this simulation procedure appears complex and computationally burdensome, at its core it is simply a function of volatility, correlation, and innovations. By studying how these inputs change, we can determine their overall effect on LRMES.

All of these inputs are dependent on the statistical model used to generate their estimates. In their original work Brownlees and Engle (2011) use a DCC-GARCH model to obtain volatility, correlation, and innovation estimates and they further defend this model choice in their updated work (Brownlees and Engle, 2015). In their simulation procedure for LRMES initial values of volatility and correlation are first obtained by fitting a DCC-GARCH model to a $w$-day window of historical market and firm returns. The residuals from this model fit are then used to estimate the innovation distribution and the parameter estimates are used to recursively update volatility and correlation values within the $h$-step simulation. Here the volatility updates are dependent on the univariate GARCH specification while the correlation updates are dependent on the multivariate DCC specification. The univariate GARCH specification put forth by Brownlees and Engle is the GJR-GARCH model (Glosten et al., 1993) where the conditional distribution for the returns is modeled as Gaussian. The authors justify this specification by highlighting the model’s ability to capture asymmetric volatility changes based on the sign of the shock and the model’s strong forecasting ability. In a different work, Brownlees et al. (2012) show that the GJR-GARCH model has the best forecasting performance when compared to a variety of other GARCH methods.

What Brownlees and Engle do not discuss is their choice of a Gaussian distribution for the GJR-
GARCH likelihood function. Other authors have shown that financial returns are typically non-normal and have suggested the use of asymmetric and leptokurtic distributions in GARCH likelihood functions (Wilhelmsson, 2006; Verhoeven and McAleer, 2004). While there is a discussion in Brownlees et al. (2012) regarding the effect of modeling the conditional distribution for the returns as Student-$t$, which is a leptokurtic distribution, they conclude that it does not provide any improvement in volatility forecasting. However, even if the conditional distribution for the returns does not significantly alter the volatility forecast, it will change the estimated parameters and residuals. These changes have the potential to be magnified in LRMES where the parameter estimates and residuals are being used in multiple calculations over the $h$-step simulation procedure. As such it is important to understand how a change to the conditional distribution assumption in the GJR-GARCH model affects LRMES estimates.

In this chapter we explore how changing the conditional distribution of the GJR-GARCH model affects LRMES estimates. We start by analyzing whether or not there is additional kurtosis in the standardized residuals and whether or not it is related to differences in LRMES. Then we examine the relationships between LRMES and the estimated DCC-GARCH parameters and between LRMES and the estimated innovation distribution, $\hat{F}$. Finally we repeat this same approach with regards to asymmetry instead of kurtosis. Throughout the chapter we will use the phrase “working likelihood” to clarify that the distribution or type of distribution being discussed is being used as the distributional assumption in the GJR-GARCH likelihood function.

## 4.1 Data and Simulation Settings

For our analysis we use a consistent panel of 12 financial institutions. These firms are chosen due to the fact that they had a large market capitalization at the beginning of the financial crisis and due to the fact that they are all a part of the larger panel of firms studied by Brownlees and Engle (2011). Additionally these firms span the 4 different industry groupings discussed by Brownlees and Engle. These groupings keep with the Standard Industrial Classification (SIC) which is produced by the United States Department of Labor. Table 4.1 lists the 12 specific firms in our panel by industry grouping along with their ticker, which is used throughout the dissertation to refer to individual financial institutions. Here we follow the
decision of Brownlees and Engle to label Goldman Sachs (GS) as a Broker-Dealer instead of using their standard SIC classification of Other.

The daily return data for these firms ranges from July 1, 1999\textsuperscript{1} through December 31, 2013. This is slightly different from the data set used by Brownlees and Engle (2011), in which they use data that begins July 3, 2000 and ends June 30, 2010. Our decision to use a different data set is based on a desire to give results that are more in line with the current financial landscape while still including times before and after the financial crisis. In keeping with Brownlees and Engle, our daily market and firm return data is obtained from the Center for Research in Security Prices database (CRSP) maintained by the University of Chicago. The market returns are a value weighted index provided by the CRSP and all of the firm returns are without dividends.

There are a wide variety of settings that need to be set before computing LRMES via the simulation procedure described in Section 3.3. For all of these settings we attempt to follow the suggestions put forth by Brownlees and Engle (2011) as much as possible, but there are settings that are not explicitly discussed in their original work. First, we define a crisis event to be a drop in the market by 40\% over a 6-month time span. Thus the number of steps in our simulation is set as $h = 130$ in order to conservatively estimate of the number of trading days in 6 months. Next we need to fix the number of Monte Carlo simulations, $J$, for which we compute these 130-day paths. Since LRMES is the expected firm loss for paths where the market dropped more than 40\%, the choice of $J$ impacts the Monte Carlo simulation error in computing LRMES. Recall that by using the importance sampler discussed in Section 3.3, our procedure aims to produce 35\% crisis iterations. Thus the number of Monte Carlo simulations can stay fixed across time allowing the weight of our importance sampler, $\lambda$, to adjust for different firm and date combinations. Our choice of $J = 25,000$ Monte Carlo simulations, which produces approximately 8,750 crisis iterations, strikes a good balance of small Monte Carlo simulation error and feasible computational time. While these Monte Carlo simulation errors vary by firm and date, they are almost always less than 1.

Another major simulation setting is the length of the historical data window used to fit the DCC-

\begin{footnote}{\textsuperscript{1}By using July instead of January we are able to use a full $w$-day financial history for every combination of firm and date in our dataset. This includes Goldman Sachs whose Initial Public Offering was in May of 1999.}

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GARCH model. Recall that the DCC-GARCH model uses historical data to compute current levels of volatility and correlation and estimate the model parameters used throughout the LRMES simulation procedure. Additionally the standardized residuals from this DCC-GARCH fit are used to estimate the innovation distribution, $\hat{F}$. The advice put forth in Brownlees et al. (2012) is to use the longest available window of data for the best forecasting results. However, we do not want the size of our estimated innovation distribution changing over time. As such, we always use a history of $w = 1,000$ trading days when fitting our DCC-GARCH models. This produces a relatively long series, approximately 4 years, that gives us results that are similar to those on the authors’ VLAB website (NYU Stern Volatility Institute, 2014).

Finally, while this chapter focuses on the distributional choice for the working likelihood of the univariate GJR-GARCH model, there is also the distributional choice for the DCC likelihood function. That is, a multivariate distribution is needed for the maximum likelihood estimation of the correlation between the market and the firm. For simplicity we assume a multivariate normal distribution throughout the dissertation. However, the choice of multivariate distribution for the DCC likelihood function is another potential source of LRMES variability that we would like to study in a future work.
Table 4.1: The 12 financial institutions chosen for our panel, grouped by their SIC.

<table>
<thead>
<tr>
<th>Depositories</th>
<th>Insurance</th>
<th>Broker-Dealers</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of America (BAC)</td>
<td>American International Group (AIG)</td>
<td>Goldman Sachs (GS)</td>
<td>TD Ameritrade (AMTD)</td>
</tr>
<tr>
<td>Citigroup (C)</td>
<td>Allstate (ALL)</td>
<td>Morgan Stanley (MS)</td>
<td>Capital One Financial (COF)</td>
</tr>
<tr>
<td>JP Morgan Chase (JPM)</td>
<td>Humana (HUM)</td>
<td>Charles Schwab (SCHW)</td>
<td>Fifth Third Bankcorp (FITB)</td>
</tr>
</tbody>
</table>
4.2 Leptokurtic Working Likelihoods

In this section we focus on two working likelihoods that use distributions with more kurtosis and fatter tails than the Gaussian distribution. The first working likelihood uses the Student-\(t\) distribution which is parameterized by its shape parameter \(\nu\). For its standardization we specify \(a\) as the location parameter and \(b\) as the scale parameter so that the distribution is specified as

\[
f(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{b}\nu\pi\Gamma(\frac{\nu}{2})} \left(1 + \frac{(x-a)^2}{b\nu}\right)^{-\left(\frac{\nu+1}{2}\right)}.
\]

(4.1)

This distribution has an implied kurtosis of \(k = \frac{6}{\nu^2} + 3\) and is thus leptokurtic for \(4 < \nu \leq \infty\). In cases where \(\nu \leq 4\) the distribution exhibits infinite kurtosis.

The second working likelihood we focus on uses the Generalized Error Distribution (GED). This distribution is also parameterized by a shape parameter \(\nu\) and its special cases include the Laplace Distribution (\(\nu = 1\)) and the Gaussian Distribution (\(\nu = 2\)). The GED is leptokurtic for \(1 < \nu < 2\) and has infinite kurtosis for \(\nu \leq 1\). Once again we let \(a\) be the location parameter and \(b\) the scale parameter in the distribution specification

\[
f(x) = \frac{\nu e^{-0.5\frac{|x-a|^\nu}{\nu}}}{2^{1+\nu^{-1}}b\Gamma(\nu^{-1})}.
\]

(4.2)

All likelihood evaluations for our simulations are done in R via the rugarch package (Ghalanos, 2014). This package contains a univariate GJR-GARCH likelihood function with options to use both the Student-\(t\) distribution and the GED.

4.2.1 Shape Parameter Estimation

The LRMES estimates for both leptokurtic working likelihoods are computed on the last trading day of each month between July 2003 and December 2013 for all 12 firms in Table 4.1. Thus, for both working likelihoods there are 1,512 firm and date combinations used in our analysis. Our first step is to determine the estimated kurtosis in each working likelihood by analyzing the shape parameter, \(\nu\). In general \(\nu\) has large variation across firm and date combinations for both working likelihoods. However, the amount of
kurtosis is substantial as 65% of the Student-$t$ shape parameter estimates correspond to a distribution with an excess kurtosis of more than 2. Averaged over all date and firm combinations, the harmonic mean of the Student-$t$ shape parameter estimate is $\nu = 6.2$ while the arithmetic mean of the GED shape parameter estimate is $\nu = 1.3$. Here we prefer the harmonic mean to describe the Student-$t$ shape parameter since we use the transformation, $1 - 1/\nu$, as the shape parameter representation in our analysis. This scaling keeps large values of $\nu$ from dominating our visualizations. Additionally the scale’s interpretation is intuitive with a Normal distribution being represented as, $1 - 1/\nu = 1$, and with smaller values of the transformation indicating additional kurtosis.

Figure 4.1 visualizes the average amount of kurtosis seen in our data. All of the distributions in Figure 4.1 are standardized to have zero mean and unit variance. Thus, the horizontal axis represents the number of standard deviations away from the mean. When compared to the Normal distribution, the leptokurtic distributions have less density between one and two standard deviations, higher peaks, and fatter tails. In our data the average GED has the highest peak and slightly fatter tails when compared to the Normal distribution. The average Student-$t$ distribution is not as peaked as the GED, but has more density in the tails of the distribution.

Figure 4.2 and Figure 4.3 use box-plots to visualize the estimated values of $\nu$ for a given firm. Here
Figure 4.2: Box-plots of the estimated Student-$t$ shape parameter, broken down by firm.

Table 4.2: Average shape parameters by industry grouping for both leptokurtic working likelihoods.

<table>
<thead>
<tr>
<th>Working Likelihood</th>
<th>Depositories</th>
<th>Insurance</th>
<th>Broker-Dealers</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student-$t$</td>
<td>0.85</td>
<td>0.80</td>
<td>0.87</td>
<td>0.83</td>
</tr>
<tr>
<td>GED</td>
<td>1.37</td>
<td>1.22</td>
<td>1.46</td>
<td>1.31</td>
</tr>
</tbody>
</table>

there is variation within a firm from the 126 different dates where LRMES is estimated. Additionally these box-plots show that the variability and median estimated shape parameter vary by firm. For example both HUM and JPM show relatively small amounts of variation in their estimated shape parameter, but HUM shows considerably more kurtosis than JPM. Interestingly the firms that show the smallest and largest amount of kurtosis share the same industry grouping as given by Table 4.1. The insurance firms (AIG, ALL, HUM) have the smallest median shape parameters and hence the highest kurtosis. In fact when using the Student-$t$ working likelihood both AIG and ALL had times where, $1 - 1/\nu \leq 0.75$, indicating infinite kurtosis ($\nu \leq 4$). In contrast, the Broker-Dealer firms (GS, MS, SCHW) have relatively high shape parameter values corresponding to very little kurtosis. The average shape parameter estimate for the four industry groupings is recorded in Table 4.2. These averages are evidence that industries may not be equivalent in terms of their estimated kurtosis.

The effect of the financial crisis on kurtosis levels is clearly evident in Figure 4.4 where we plot the
shape parameter for the Student-$t$ distribution averaged over all 12 firms. Recall that to compute LRMES the GJR-GARCH model is fit to 4 years worth of historical data to obtain current volatility estimates. Thus the dates on the X-axis of Figure 4.4 correspond to the last day of each month where LRMES is being estimated. As such the minimum in early 2009 corresponds to the shape parameter estimate from a Student-$t$ GJR-GARCH model fit to historical data from 2005-2009. This implies the conditional distribution for the GJR-GARCH model is estimated to have its heaviest tails when the 4 year historical window includes the worst years of the financial crisis. At this minimum the average shape parameter is 0.805 corresponding to an excess kurtosis of more than 5. Figure 4.4 also illustrates that the distribution based on historical data from 2009-2013 is still longer tailed than the distribution based on data from 2002-2006. In other words, we are still seeing more kurtosis in July 2013 than we did before the start of the financial crisis. The same general pattern for the shape parameter is mirrored when replacing the Student-$t$ working likelihood with a GED working likelihood. Additionally the same pattern is also seen across the different industry groupings. These results highlight the long tailed nature of returns during the financial crisis.
4.2.2 LRMES Differences

Now that it is evident that a change in the working likelihood allows for additional kurtosis, we want to examine how this change affects LRMES estimates. Figure 4.5 plots the relationship between the differences in LRMES estimates when using a Student-\( t \) working likelihood and the corresponding shape parameter estimate. For easier visualization, the color shading in Figure 4.5 coincides with the industry grouping of each firm. Shades of red represent Depositories, shades of green represent Insurance firms, shades of yellow represent Broker-Dealers, and shades of blue represent firms categorized as Other. The first thing we highlight from Figure 4.5 is the magnitude of the LRMES differences. While the large majority of the differences are within \( \pm 5 \), there are still more than 250 firm and date combinations where the magnitude of the difference between the estimates is greater than 5. This illustrates that a change in working likelihood can substantially impact LRMES estimation. Figure 4.5 also shows that the majority of the differences are less than zero. This indicates that the Student-\( t \) working likelihood often produces smaller LRMES values which are indicative of less systemic risk. In fact, 69\% of the differences in Figure 4.5 are less than zero demonstrating that firms will typically be seen as less systemically risky if a
Figure 4.5: LRMES differences between the Student-$t$ and Gaussian working likelihoods.

Student-$t$ working likelihood is used.

The differences with the largest magnitude in Figure 4.5 belong to two firms. First, for multiple dates COF is estimated to have a substantially smaller LRMES when using the Student-$t$ working likelihood. The largest observed difference for COF is -26.8 and there are 27 dates where the magnitude of the difference is larger than 10. However, the LRMES estimates for COF are not uniformly smaller when using a Student-$t$ working likelihood instead of a Gaussian working likelihood. In fact, there are 3 dates where the magnitude of the difference is larger than 10 and the Student-$t$ working likelihood produces a larger LRMES estimate than the Gaussian working likelihood. The other firm with multiple large magnitude differences is BAC. For BAC the Student-$t$ working likelihood tends to produce larger estimates of LRMES. The largest difference for BAC is +25.4 and there are 12 dates where the magnitude of the difference is larger than 10. Once again the sign of the difference is not uniformly positive, even when focusing on just large magnitude differences. These results illustrate LRMES variability due to model selection is highly dependent on the specific firm and date combination.

Figure 4.6 shows the exact same plot as Figure 4.5, but uses LRMES differences from comparing the GED working likelihood with the Gaussian working likelihood. In general, Figure 4.6 mirrors the observed results of Figure 4.5 in that the vast majority of LRMES differences are small in magnitude, but large magnitude differences do exist, especially for COF and BAC. Once again 69% of the observed
LRMES Differences Versus Estimated Shape Parameter

Figure 4.6: LRMES differences between the GED and Gaussian working likelihoods.

differences are less than zero, indicating a tendency to estimate less systemic risk when using a GED working likelihood. An important observation is that the shading of the points is nearly identical in the two graphs. This indicates consistency in both the estimated kurtosis and the relative magnitude of LRMES estimates for the two different leptokurtic working likelihoods. However, there is one major difference between Figure 4.5 and Figure 4.6. For the same firm and date, the differences in Figure 4.6 tend to be smaller in magnitude than those in Figure 4.5. This is demonstrated by the fact that using a Student-\(t\) working likelihood produces differences that are within ±3 65% of the time, while using a GED working likelihood produces differences that are within ±3 83% of the time. In terms of the median magnitude of the differences, the LRMES estimates under the Student-\(t\) working likelihood differ from the LRMES estimates under the Gaussian working likelihood by 1.99 while the GED working likelihood LRMES estimates differ by 1.21. Thus in general, the GED working likelihood produces LRMES estimates that are closer to the LRMES estimates from the Gaussian working likelihood. We suspect that this is due to the fact, highlighted in Figure 4.1, that on average the GED has slimmer tails than the Student-\(t\) distribution.

From both Figure 4.5 and Figure 4.6 is is clear that a monotonic relationship between differences in LRMES and the estimated amount of kurtosis does not exist. The largest differences in LRMES occur at relatively average levels of kurtosis and the highest levels of kurtosis produce relatively small
Table 4.3: The percentage of firm and date combinations where there is a negative LRMES difference.

<table>
<thead>
<tr>
<th>Working Likelihood</th>
<th>Depositories</th>
<th>Insurance</th>
<th>Broker-Dealers</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student-(t)</td>
<td>77%</td>
<td>65%</td>
<td>77%</td>
<td>57%</td>
</tr>
<tr>
<td>GED</td>
<td>75%</td>
<td>66%</td>
<td>74%</td>
<td>57%</td>
</tr>
</tbody>
</table>

changes in LRMES. Additionally while the industry effect on the estimated kurtosis can be visualized along the horizontal axis, it is hard to see any vertical grouping by color. However, a closer look at the percentage of firm and date combinations with a negative LRMES difference is given by Table 4.3. From Table 4.3 it is more evident that there is variability in the percentage of negative LRMES differences by industry grouping. Depositories have the highest tendency to produce smaller LRMES estimates despite the fact that the largest positive differences in LRMES are from BAC. Similarly, firms classified as Other have the smallest tendency to produce smaller LRMES estimates even though COF had the largest negative differences. The conclusion of Table 4.3 is that the use of a leptokurtic working likelihood will more frequently estimate a smaller amount of systemic risk in Depositories and Broker-Dealers. Firms classified as Other are estimated to have less systemic risk just over half the time when a leptokurtic working likelihood is used. While there is an obvious need to expand these results further by adding additional firms, these results highlight the possibility of differences in LRMES estimates and variability by industry grouping. Brownlees and Engle (2011) have already shown differences in estimated volatility and correlation by industry grouping, so a natural extension would be to suspect possible differences in LRMES estimates and variability by industry grouping as well.

Over our firm and date combinations, COF and BAC have the largest differences in LRMES when using a leptokurtic working likelihood. So when do these large differences occur? Figure 4.7 and Figure 4.8 plot the LRMES estimates for all 3 working likelihoods over time. From the graphs we see that for both the largest differences mostly occur between June 2004 and June 2006. Thus, for these two firms the effect of using a leptokurtic working likelihood is largest during the years leading up to the financial crisis. During the financial crisis a change in the working likelihood has a very small effect on the LRMES estimates for BAC. However, the LRMES estimates for COF differ by more than 5 when
using a Student-$t$ working likelihood from June 2008 through June 2009. During this time though, the use of a leptokurtic working likelihood produces larger LRMES estimates that those produced by a Gaussian working likelihood. This implies that COF would have seemed risker during the financial crisis if a leptokurtic working likelihood had been used. The time plots for all 12 firms are given in the Appendix C. When looking over all firms we see a wide variety of dates where there is a noticeable difference in LRMES, but many firms do show discernible differences in LRMES during the financial crisis. This suggests that SRISK values during the crisis could have been different if a leptokurtic working likelihood was used in the computation of LRMES.

### 4.2.3 GJR-GARCH Parameter Estimation

Since differences in LRMES are not fully explained by the shape parameter alone, we continue to examine how a change in the working likelihood affects other LRMES inputs. Recall from equation (2.5) that the GJR-GARCH model is dependent on the parameters $\omega$, $\alpha$, $\beta$, and $\gamma$. Additionally since all of our working likelihoods are based on symmetric distributions, the persistence of the GJR-GARCH model is defined according to equation (2.7) where $\kappa = 0.5$. In this section we explore how changes to the working likelihood affect the GJR-GARCH parameters and the model’s persistence.
Analyzing the individual GJR-GARCH parameter estimates show that the only general tendency is for the leptokurtic working likelihoods to give higher values of $\beta$. This result is depicted in Figure 4.9 which gives side by side boxplots of the estimated $\beta$ for the different working likelihoods broken down by industry. In all industries the Student-$t$ working likelihood estimates the highest median value of $\beta$. Additionally the GED working likelihood shows a higher $\beta$ value than the Gaussian working likelihood for three of the four industry groupings. This implies that the use of a leptokurtic working likelihood produces a stronger dependence on the previous volatility value and a weaker dependence on the previous value of the return. Across industry groupings the largest shift in the median of $\beta$ comes from firms categorized as Other, while the smallest shift comes from Insurance firms.

It should be noted that while the difference in $\beta$ is consistent across industry grouping, there is a considerable amount of variability across individual firms. For example the Student-$t$ working likelihood produces $\beta$ estimates that were larger than the Gaussian working likelihood $\beta$ estimates 97% of the time for MS, but only 56% of the time for SCHW. All firms except AIG had a larger $\beta$ estimates from the Student-$t$ working likelihood more than 50% of the time. The estimates of $\alpha$ and $\gamma$ did not show any distinguishable patterns based on working likelihood or industry grouping.

The persistence of the GJR-GARCH model measures the rate at which volatility decays after a shock,
the amount of volatility clustering, and may be a better measure of how the working likelihood affects the GJR-GARCH parameters. A higher persistence should lead to stronger volatility clustering which in turn impacts the estimated firm returns over the full simulation path. Over all firm and date combinations in our data there is a tendency for the leptokurtic working likelihoods to model a higher amount of persistence. In fact, 66% of the firm and date combinations had a higher persistence when LRMES was estimated using a Student-\( t \) working likelihood as compared to a Gaussian working likelihood. This is illustrated in Figure 4.10 where once again the median persistence is higher for the Student-\( t \) working likelihood across all industries.

Figure 4.11 and Figure 4.12 further illustrate the deviation in persistence estimates across different working likelihoods. For COF the persistence modeled by leptokurtic working likelihoods is smaller than the persistence modeled by the Gaussian working likelihood between June 2004 and June 2006. Recall from Figure 4.7 that this is the same time frame with the largest magnitude differences in LRMES. Similarly the crossover to smaller persistence estimates under leptokurtic working likelihoods is in July
2005 for BAC, one month before the corresponding large differences in LRMES. Despite the timing similarities, Figure 4.13 demonstrates that the differences in persistence does directly correspond to the observed changes in LRMES.

While there is not a direct relationship between a model’s persistence and LRMES variability due to a change in the working likelihood, Figure 4.11 and Figure 4.12 convey a striking feature regarding persistence in the GJR-GARCH model. In both graphs there is a sharp decline in persistence during mid-2007 which is immediately before the traditional start of the financial crisis. Recall that this corresponds to the GJR-GARCH model being fit to historical data that ranges from mid-2003 to mid-2007. While the decline varies in magnitude and exact date across the different firms, a decline in persistence is present for all of the firms in our data set. This consistency across firms and its timing relative to the financial crisis make this an intriguing feature of the GRJ-GARCH model. However, since the persistence does not show any direct relationship to the observed differences in LRMES, we leave further exploration of GJR-GARCH model persistence and for future work.
Figure 4.11: Time series of the estimated GJR-GARCH persistence for COF using various working likelihoods.

Figure 4.12: Time series of the estimated GJR-GARCH persistence for BAC using various working likelihoods.
LRMES Differences Versus Persistence Differences

Persistence Difference (log(1 + (Student t − Gaussian)))

LRMES Difference (Student t − Gaussian)

AIG
ALL
AMTD
BAC
C
COF
FITB
GS
HUM
JPM
MS
SCHW

Figure 4.13: LRMES differences between the Student-t and Gaussian working likelihoods, plotted against the differences in GJR-GARCH persistence estimates.

4.2.4 Innovation Distribution

Changes to the GJR-GARCH parameters produce different sets of DCC-GARCH residuals that are used in estimating the innovation distribution, \( \hat{F} \). The LRMES simulation procedure samples with replacement from the estimated innovation distribution to obtain the innovation pair \((\epsilon_{m}, \xi_{i})\). Thus in order to continue to study LRMES variability due to a change in working likelihood, in this sub-section we explore how the marginal distributions of \( \epsilon_{m} \) and \( \xi_{i} \) change with the use of a leptokurtic working likelihood. While the importance sampler takes biased samples from \( \hat{F} \), the LMRES estimates are computed after removing the bias and are a function of the original distribution. Thus changes to the underlying distribution have the potential to affect LRMES estimates.

Our results show that in general the measures of center for the marginal distributions are not affected by a change in the working likelihood. This is especially true for the marginal distribution of market residuals, \( \epsilon_{m} \). Over all dates, the median of the market innovation distribution did not change by more than .003 when comparing estimates from the Student-t and Gaussian working likelihoods. There is only slightly more variation when comparing the marginal distributions for the firm residuals. The largest differences are once again for BAC and COF, where the center of the distribution differed by .03. In comparing this change with differences in LRMES, leptokurtic working likelihoods for COF produce
large negative LRMES differences when the center of the $\xi_i$ distribution is more positive. Conversely for BAC the leptokurtic working likelihoods produce large positive LRMES differences when the center of the $\xi_i$ distribution is more negative. The remaining firms show very little shift in the center of their marginal innovation distribution.

Since LRMES is conditional on a 40% drop in the market by definition, looking at changes to the tails of the marginal innovation distributions may be more appropriate. As a start we explore changes to the minimum and maximum values. The use of leptokurtic working likelihoods does create variation in these extreme values, and as expected the residual distributions tend to have longer tails. Specifically the minimum value of the marginal distribution of $\xi_i$ seems to be much smaller for most firms. In the case of BAC and COF, the largest differences in the minimum value when using a Student-$t$ working likelihood occur between 2003 and 2007 which corresponds to the same time window where LRMES estimates for BAC and COF are most different. For COF the extension of both tails is significant as the maximum value is also larger for many of the same dates. However, for BAC the magnitude of the maximum value is similar regardless of the working likelihood. A change in working likelihood also creates a slightly longer left tail for the market innovations, $\varepsilon_m$, but the magnitude of the difference is smaller than the differences in the firm innovations.

From our study it is apparent that using a single number summary to describe shifts in an entire distribution is not sufficient. As such we define a metric that looks to capture the magnitude of the positive and negative innovations within a marginal distribution. Specifically we define

$$PN = \frac{\sum_{j=1}^{1000} \varepsilon_{m,j}^2 \mathbb{I}_{\varepsilon_{m,j} < 0}}{\sum_{j=1}^{1000} \varepsilon_{m,j}^2 \mathbb{I}_{\varepsilon_{m,j} > 0}}$$

(4.3)

where once again $\mathbb{I}$ is simply an indicator function for the sign of the innovation. To compute this metric for the firm innovations, $\varepsilon_{m,j}$ is simply replaced by $\xi_{ij}$. $PN$ is constructed such that values greater than one indicate a marginal distribution with more large negative innovations while values less than one indicate a distribution with more large positive innovations.

In Figure 4.14, Figure 4.15, and Figure 4.16 we plot $PN$ over time for different working likelihoods.
Figure 4.14: Time series of $PN$ for the marginal market innovation distribution using various working likelihoods.

Figure 4.14 demonstrates that the choice of working likelihood has very little effect on $PN$ for the marginal distribution of the market. All of the working likelihoods follow the same pattern of becoming more positive until late 2006, after which the market innovation distribution becomes more negative throughout the financial crisis. However, it is evident from Figure 4.15 and Figure 4.16 that the working likelihood does affect $PN$ for the marginal distribution of $\xi_i$. Once again the largest differences in $PN$ occur for COF and BAC and these differences occur between 2004 and mid-2006, which overlaps the time frame of large LRMES differences. However, in Figure 4.15 and Figure 4.16 the differences in PN are in the same direction for both firms. Specifically, both firm innovation distributions have a more negatively weighted distribution when a leptokurtic working likelihood is used even though their differences in LRMES were of opposite signs. So while $PN$ helps give a more complete picture of the innovation distributions and suggests that the use of a leptokurtic working likelihood creates a more negatively weighted firm innovation distribution, we have yet to fully isolate the relationship between innovation distribution shifts and changes in LRMES.
Figure 4.15: Time series of $PN$ for the marginal firm innovation distribution for COF using various working likelihoods.

Figure 4.16: Time series of $PN$ for the marginal firm innovation distribution for BAC using various working likelihoods.
4.2.5 Isolating the Effect of Individual Inputs

Previous results show that the use of different working likelihoods change both the estimated GJR-GARCH parameters and the joint innovation distribution. Additionally these changes often occur during the same times in which there are large deviations in LRMES. Naturally this leads us to question how the changes in these inputs relate to the LRMES differences. Are the large deviations in LRMES due to changes in the GJR-GARCH parameters, the innovations distribution, or a combination of the two? In an attempt to better answer this question we fit the initial GJR-GARCH model twice, once to get parameter estimates and once to get the innovation distribution. This allows for parameter estimates to be computed using a Gaussian working likelihood and the innovation distribution to be computed using a leptokurtic working likelihood and vice versa. This is not something we recommend for obtaining LRMES estimates in practice since it requires an extra model fit and there is no reason as to why this would improve LRMES estimation. However, in this sub-section fitting the initial GJR-GARCH model in different ways allows us to isolate the effect of an individual input.

We start by isolating the effect of a leptokurtic working likelihood in GJR-GARCH parameter estimation. Throughout this process the previous results regarding the shape parameter (Section 4.2.1) and the GJR-GARCH parameters (Section 4.2.3) stay the same. The only change occurs in the LRMES estimates since the innovation distribution is now estimated using a Gaussian working likelihood instead of the corresponding leptokurtic working likelihood. Figure 4.17 once again plots LRMES differences against the estimated shape parameter of the Student-\(t\) working likelihood to mirror Figure 4.5. Visually, the results of Figure 4.17 show a very similar pattern to the one seen in Figure 4.5. However, there are a few differences between the two plots. First, the differences in LRMES for COF are much larger when the Student-\(t\) working likelihood is only used in parameter estimation. The cluster of differences for COF resides between -20 and -30 in Figure 4.17 whereas the same cluster is between -10 and -20 in Figure 4.5. Second, the large positive differences for BAC seem to be smaller in magnitude for Figure 4.17. The largest differences are now right at 20 instead of being greater than 20, and a large cluster that used to stand out from the other firms has now disapperaed into the big cluster of estimates around 0. Finally, there is a shift in the LRMES estimates for AIG when the shape parameter is small. Previously these
In terms of the sign of the differences, using a Student-$t$ working likelihood only for the parameter estimation causes 75% of the differences to be less than 0. This is an increased tendency for smaller LRMES estimates and is indicative of estimating smaller amounts of systemic risk. Additionally, the magnitude of the LRMES differences is larger than our previous result with a median difference of 2.34. Recall that when the Student-$t$ working likelihood is used throughout the entire simulation procedure, the median magnitude of LRMES differences is 1.99. These results are also mirrored when the GED working likelihood is used only in the parameter estimation.

In addition to plotting the LRMES differences, we also examine the percentages of differences that are statistically significant by firm. Here the statistical significance is in reference to our Monte Carlo standard error for LRMES. Table 4.4 shows that using a Student-$t$ working likelihood produces differences in LRMES that are significantly different from zero over half the time for every firm except SCHW. This illustrates that many of the differences in the large cloud seen in Figure 4.17 are significantly different from zero. Additionally, Table 4.4 mirrors our previous conclusion that using a GED working likelihood produces LRMES estimates that are closer to the standard LRMES estimates that use a Gaussian working likelihood. Moreover, we also see the industry grouping highlighted as the Depositories (BAC, C, JPM)
Figure 4.18: LRMES differences between the Student-$t$ and Gaussian working likelihood, where the Student-$t$ working likelihood is only used in estimating the innovation distribution.

have a larger percentage of significant differences, compared to all other industry grouping.

When only using a leptokurtic working likelihood in the parameter estimation the overall pattern remains the same when compared to using the Student-$t$ working likelihood for the entire GJR-GARCH fit. This indicates that the parameter estimates by themselves have a significant role in determining LRMES. However, there are some distinct differences in the results which is indicative of the innovation distribution also helping to determine LRMES estimates. In order to further explore this relationship we only use leptokurtic working likelihoods to estimate the innovation distribution and plot the LRMES differences in Figure 4.18. Visual inspection of Figure 4.18 show that there are obvious discrepancies from our original results in Figure 4.5. First, the large differences in LRMES estimates for COF and BAC have disappeared. In fact, COF now has larger LRMES estimates using a Student-$t$ working likelihood leading to positive differences instead of large negative ones. Additionally there are large differences coming from a variety of new firms, but they only happen a few times over our 10 year time frame. Finally the large cloud of estimates is narrower with more differences close to zero. In Figure 4.18 the median difference is just 0.94 which implies that the use of a leptokurtic working likelihood in estimating the innovation distribution is having a smaller effect on LRMES estimates.

This smaller effect for the innovation distribution is also seen in the number of differences that are
statistically different from zero. Over all firm and date combinations, the Student-\( t \) working likelihood produces statistically significant differences only 35\% of the time. Table 4.4 breaks down the percentage of significant differences by firm and when compared to Table 4.4 it is clear that the GJR-GARCH parameters are having a larger effect. However, the tables also show that there is firm dependence. For example JPM experience large drops in the percentage of significant differences while other firms, like COF, see a much smaller drop. So while the overall pattern is a smaller effect from changing just the innovation distribution, there is evidence that LRMES differences are a result of both parameter estimates and the innovation distribution.
Table 4.4: Percentage of LRMES differences that are statistically significant when the non-Gaussian working likelihood is only used in parameter estimation.

<table>
<thead>
<tr>
<th></th>
<th>AIG</th>
<th>ALL</th>
<th>AMTD</th>
<th>BAC</th>
<th>C</th>
<th>COF</th>
<th>FITB</th>
<th>GS</th>
<th>HUM</th>
<th>JPM</th>
<th>MS</th>
<th>SCHW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student-t</td>
<td>79.6%</td>
<td>63.0%</td>
<td>55.6%</td>
<td>80.6%</td>
<td>68.5%</td>
<td>72.2%</td>
<td>63.9%</td>
<td>70.4%</td>
<td>60.2%</td>
<td>85.2%</td>
<td>57.4%</td>
<td>45.4%</td>
</tr>
<tr>
<td>GED</td>
<td>62.0%</td>
<td>50.0%</td>
<td>30.6%</td>
<td>76.9%</td>
<td>57.4%</td>
<td>58.3%</td>
<td>38.9%</td>
<td>52.8%</td>
<td>51.9%</td>
<td>66.7%</td>
<td>49.1%</td>
<td>28.7%</td>
</tr>
</tbody>
</table>

Table 4.5: Percentage of LRMES differences that are statistically significant when the non-Gaussian working likelihood is only used in estimating the innovation distribution.

<table>
<thead>
<tr>
<th></th>
<th>AIG</th>
<th>ALL</th>
<th>AMTD</th>
<th>BAC</th>
<th>C</th>
<th>COF</th>
<th>FITB</th>
<th>GS</th>
<th>HUM</th>
<th>JPM</th>
<th>MS</th>
<th>SCHW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student-T</td>
<td>59.3%</td>
<td>26.9%</td>
<td>36.1%</td>
<td>42.6%</td>
<td>21.3%</td>
<td>59.3%</td>
<td>36.1%</td>
<td>34.3%</td>
<td>37.0%</td>
<td>25.9%</td>
<td>35.2%</td>
<td>10.2%</td>
</tr>
<tr>
<td>GED</td>
<td>33.3%</td>
<td>26.9%</td>
<td>12.0%</td>
<td>25.9%</td>
<td>11.1%</td>
<td>53.7%</td>
<td>22.2%</td>
<td>29.6%</td>
<td>35.2%</td>
<td>6.5%</td>
<td>7.4%</td>
<td>1.9%</td>
</tr>
</tbody>
</table>
4.3 Non-Symmetric Working Likelihoods

There are typically two characteristics that stand out in the discussion of whether or not the conditional distribution of financial returns follows a Gaussian distribution. The first is the tail length and whether or not the Gaussian distribution adequately captures the extreme values often observed. The second is the assumption that the returns are symmetric, that the tails of the distribution are of equal length. So far all of our discussion has been based around symmetric distributions. In the following section we discuss the addition of asymmetry to the working likelihood and its affect on LRMES.

Recall from Figure 4.1 that all three of our previously studied distributions are unimodal and symmetric. However, the Normal, Student-$t$, and GED distributions can all be parameterized to allow for asymmetry. Asymmetry can be introduced into any symmetric distribution by using inverse scale factors (Fernández and Steel, 1998). Given a standardized random variable $z$ and a skew parameter $\zeta$, the skewed version of a unimodal, symmetric distribution is

$$f(z|\zeta) = \frac{2}{\zeta + \zeta^{-1}} \left[ f(\zeta z)H(-z) + f(\zeta^{-1} z)H(z) \right]$$

(4.4)

where $H(\cdot)$ is the Heaviside function. Recall that the Heaviside function is simply the unit step function that is 0 for a negative value and 1 for a positive value. Thus $\zeta < 1$ will produce a distribution with negative skew and $\zeta > 1$ will produce a distribution with positive skew. When $\zeta = 1$ the distribution is symmetric.

Using this parameterization for asymmetry we explore the effect of using an asymmetric working likelihood in the computation of LRMES. Once again all of the likelihood evaluations are done in R via the rugarch package (Ghalanos, 2014). The use of a Skew-Normal working likelihood will provide a direct comparison for how the addition of asymmetry affects LRMES estimates since it does not allow for excess kurtosis. However, we will also use the results from the Skewed Student-$t$ and the Skewed GED working likelihoods to see how the addition of both kurtosis and skewness affect LRMES estimates.
4.3.1 Skewness Parameter Estimation

We begin by studying the skewness parameter estimate, $\zeta$, for the different working likelihoods. The Skew-Normal working likelihood shows a tendency for negative skewness for our data. Over all firm and date combinations, $\zeta_{\text{Norm}}$ is estimated to be less than one 68% of the time. This is echoed at a smaller magnitude for the Skewed GED working likelihood as $\zeta_{\text{GED}}$ is less than one 54.6% of the time. When using the Skewed Student-$t$ distribution there is a slight tendency for positive skewness as $\zeta_T$ is less than one just 45.1% of the time.

Figure 4.19 gives a more detailed view of the estimated skewness parameter for various asymmetric working likelihoods. The box-plots illustrate that for all firms the median skewness parameter is closer to one for the Skewed Student-$t$ and Skewed GED working likelihoods. Thus when the working likelihood allows for both kurtosis and asymmetry, there is a decrease in the amount of estimated asymmetry. This is evidence that the data contains excess kurtosis and that the Skew Normal working likelihood is estimating additional asymmetry since it can not account for the excess kurtosis. Figure 4.19 also illustrates that the amount of estimated asymmetry is firm dependent. AMTD, BAC, and SCHW all have large variability in the $\zeta$ parameter and medians that are far away from one for all three working likelihoods. These three firms are a part of different industry groupings and there does not seem to be a connection between the estimated asymmetry and a firm’s industry classification. In fact, the median skewness parameter for each of the industry groupings are within 0.05 of one another when using the Skew-Normal working likelihood and within 0.02 when using the Skewed Student-$t$ working likelihood. Further analysis shows that the largest estimated asymmetries all occur before the start of the financial crisis in 2007. Recall that this overlaps the same time frame where there are large LRMES differences possibly indicating a relationship between the asymmetry parameter and LRMES differences.

4.3.2 LRMES Differences

So does the addition of asymmetry affect our LRMES estimates? We start by focusing on the Skew-Normal working likelihood since it allows for a direct comparison of how the addition of asymmetry affects LRMES estimates. Figure 4.20 plots the LRMES differences between the Skew-Normal and
Figure 4.19: Box-plots of the estimated asymmetry parameter for various working likelihoods, broken down by firm.

Gaussian working likelihoods against the estimated skewness parameter, $\zeta$. From Figure 4.20 it is evident that majority of LRMES differences are small in magnitude except in a few cases where the skewness parameter is far away from one. The largest differences belong to BAC where a large amount of negative skewness at times produces large differences in LRMES. However, other BAC estimates with an equivalent amount of skewness show only small LRMES differences. This inconsistent relationship is also observed for AMTD and large amounts positive skewness. What should not be lost in these few extreme cases, however, is the large narrow cloud of LRMES differences centered around 0. The median LRMES difference in Figure 4.20 is -0.28 and only 10.8% of the differences in Figure 4.20 are statistically different from zero. Thus the addition of asymmetry to the working likelihood fails to produce a large number of significant differences in LRMES.

We also looked at the LRMES differences for working likelihoods that are both asymmetric and leptokurtic. Figure 4.21 demonstrates that the differences in LRMES when using a Skewed Student-$t$ working likelihood are remarkably similar to what has already been seen in Figure 4.5 where a symmetric Student-$t$ working likelihood is used. While the horizontal axis is different for the two graphs, the
firms that produce the largest LRMES differences and the magnitude of these differences are the same. Figure 4.21 also highlights that large differences in COF occur near $\zeta = 1$, implying that these large LRMES differences are strictly due to the leptokurtic nature of the working likelihood. In contrast, the large differences in BAC occur when $\zeta < 0.9$ implying that both the asymmetric and leptokurtic nature of the working likelihood is playing a role for these estimates. There are no large LRMES differences that are new to Figure 4.21, implying that the addition of asymmetry is not introducing any large differences on its own. To further illustrate this point we plot the LRMES differences between the Skewed Student-$t$ working likelihood and the symmetric Student-$t$ working likelihood in Figure 4.22. It is clearly evident from Figure 4.22 that the addition of asymmetry produces only produces small changes in LRMES. The median difference in Figure 4.22 is -0.16 and only 3.5% of the differences are statistically different from zero.

Using the skewed GED working likelihood produces similar results to the skewed Student-$t$, but with one noticeable difference. Once again Figure 4.23 shows LRMES differences that are similar to what has already been seen when using a symmetric GED in Figure 4.6. In fact when directly comparing the LRMES estimates from the skewed GED with those from the symmetric GED the median difference is -0.29 and only 4.9% of the differences are statistically different from zero. Once again the addition of asymmetry does not significantly change LRMES estimates. However, Figure 4.23 highlights a substantial
Figure 4.21: LRMES differences between the Skewed Student-\(t\) and Gaussian working likelihoods.

Figure 4.22: LRMES differences between the Skewed Student-\(t\) and Student-\(t\) working likelihoods.
difference in the estimation of $\zeta$ for COF when compared to using a Skewed Student-$t$ working likelihood. In Figure 4.21 there is no estimated asymmetry as $\zeta = 0$, but in Figure 4.23 the LRMES differences for COF have shifted to the left indicating negative skew when the Skewed GED working likelihood is used. This illustrates that there are distributional differences between the Skewed Student-$t$ and Skewed GED that have the potential to alter the estimated GJR-GARCH parameters.

Overall we conclude that asymmetric working likelihoods have a negligible effect on LRMES estimates when compared to symmetric and leptokurtic working likelihoods. The firms with the most asymmetry when using a Skew-Normal distribution (BAC, AIG) show very small changes in LRMES when comparing estimates from an asymmetric and leptokurtic working likelihood with estimates from a symmetric and leptokurtic working likelihood. This suggests that the Skew-Normal working likelihood is estimating additional asymmetry to better fit the standardized residuals. Further evidence of this is seen in the estimates of the $\zeta$ parameter getting closer to one for asymmetric and leptokurtic working likelihoods. While certain firms do exhibit asymmetry in our data, the difference in LRMES is negligible.
4.4 Summary

In this Chapter we set out to determine if changes to the working likelihood of the GJR-GARCH model would produce substantial changes in LRMES. Specifically we changed the distributional assumption to include both leptocurtic and asymmetric distributions. While the magnitude of LRMES differences varies by both firm and date, overall the use of a lepokurtic distribution in the working likelihood does produce significant changes in LRMES. In fact, there are 64 firm and date combinations where the difference between the LRMES estimate under the Student-\(t\) working likelihood and the LRMES estimate under the Gaussian working likelihood is at least 10. The use of asymmetric working likelihoods produces significant estimates of asymmetry for certain firm and date combinations, but does not produce large changes in LRMES.

In terms of statistical significance at a 95% level, using Monte Carlo error, 59% of the LRMES estimates from the Student-\(t\) working likelihood and 38% of LRMES estimates from the GED working likelihood are statistically different from their corresponding LRMES estimate using a Gaussian working likelihood. The median magnitude of the difference is 1.92 for the Student-\(t\) working likelihood and 1.11 for the GED working likelihood. The differences with the largest magnitude occur between June 2004 and June 2006 for both BAC and COF. However, large changes in LRMES are not isolated to these firms or dates. In fact when using a Student-\(t\) working likelihood all of the firms in our panel saw at least one LRMES difference of at least 7. The majority of the differences between leptokurtic working likelihoods and the Gaussian working likelihood are negative, indicating that leptokurtic working likelihoods tend to estimate reduced levels of systemic risk. The percentage of negative differences is nearly constant across the two different leptokurtic working likelihoods.

After observing significant differences in LRMES estimates when using leptokurtic working likelihoods, we further explore how different LRMES inputs contribute to the overall change in LRMES. Figure 4.5 and Figure 4.6 show that a monotonic relationship between the amount of estimated kurtosis and the changes in LRMES does not exist. While all of the firm and date combinations used in our study show kurtosis, large changes in LRMES occur for distributions with various amounts of excess kurtosis. These figures also highlight the fact that differences in LRMES are typically larger when using
a Student-\(t\) working likelihood as compared to a GED working likelihood. Thus, the differences in the shape of these two distributions have an impact on LRMES estimation. While it does not directly relate to changes in LRMES, in our small panel Insurance firms have more kurtosis when compared to the other industry classifications. Since our panel is small we hesitate to draw major conclusions from this result, but this result does provide guidance for exploration of this phenomenon moving forward.

Our study of how leptokurtic working likelihoods affect the estimated GJR-GARCH parameters shows a tendency for an increased weight on the previous volatility. That is, the \(\beta\) parameter in the GJR-GARCH model is consistently higher when estimated using a leptokurtic working likelihood. The Student-\(t\) working likelihood tends to produce larger values of \(\beta\) than the GED working likelihood, but both tend to produce larger values of \(\beta\) when compared to the Gaussian working likelihood. These higher \(\beta\) values also correspond to higher amounts of persistence in the leptokurtic working likelihoods. This means that the model for firm volatility is closer to non-stationarity and thus takes longer to return to its unconditional variance after a shock. However, as shown by Figure 4.13, this increased persistence cannot explain the differences in LRMES on its own. In fact, many of our largest differences in LRMES occur when the persistence of the leptokurtic working likelihood is less than the corresponding persistence in the Gaussian working likelihood.

We also explore the working likelihood’s affect on the standardized residuals. These residuals are used to estimate the innovations distribution, which is sampled from throughout the LRMES simulation procedure. Overall there is a tendency for leptokurtic working likelihoods to produce more negatively weighted firm innovation distributions. While the dates for largest changes in LRMES correspond to the same dates where we saw the biggest change in the estimated innovation distribution, this effect is correlated with the previously discussed change in parameter estimates. In an attempt to remove this correlation we use different working likelihoods to obtain the parameter estimates and the estimated innovation distribution separately. This exercise shows that a change in only the innovation distribution has a smaller effect on changing LRMES. Estimating the innovation distribution from a leptokurtic working likelihood can produce significant LRMES changes on its own, but the frequency of large magnitude changes is greatly reduced. When the working likelihood is only changed for the parameter
estimation, the differences in LRMES are typically larger in magnitude than the differences obtained when the working likelihood is changed for both the parameter and innovation distribution estimation.

Overall we have shown that LRMES, and hence SRISK, are not robust to distributional assumptions in the GJR-GARCH model. The LRMES differences due to a change in the working likelihood are of large enough magnitude to impact estimated SRISK values and potentially alter SIFI rankings. The change in the estimated parameters play the largest role in changing LRMES, but they also interact with changes to the standardized residuals. The simulation procedure associated with LRMES makes it hard to isolate the exact reason behind the differences, but it is still important to recognize that LRMES estimates change when different working likelihoods are used.
Part III

LRMES Variability via Parameter Estimation
The financial crisis of 2007-2009 has brought a new focus on trying to define, measure, and regulate systemic risk. Both government and academic researchers are focusing on creating and evaluating metrics that capture how large individual firms can impact the soundness of the entire financial system. From this research SRISK (Brownlees and Engle, 2011) has emerged as a leading metric for quantifying systemic risk. By definition SRISK estimates the capital shortage of an individual firm conditional on a significant drop in the market as a whole. Thus, Brownlees and Engle are able to quantify the individual contributions to systemic risk and provide an overall systemic risk level by aggregating these individual contributions.

The calculation of SRISK relies on estimating a given firm’s Long Run Marginal Expected Shortfall (LRMES). By definition LRMES is an estimate of a firm’s percentage loss in equity given that the market is in crisis. In Chapter 4 we establish that the estimation of LRMES, and hence SRISK, is sensitive to the choice of the working likelihood. Specifically the use of a leptokurtic working likelihood can produce
differences in LRMES on the order of 20% for certain firm and date combinations. These differences have already been shown to be significantly larger than the Monte Carlo error associated with the simulation procedure used in their computation, however we recognize that this simulation error does not capture the full amount of error associated with estimating LRMES.

The estimation of LRMES relies on computing time varying volatilities and correlations between the market and an individual firm. The suggestion of Brownlees and Engle (2011) is to use a DCC-GARCH model to obtain these values. Inherently the DCC-GARCH model has error in estimating its various model parameters and this error needs to be accounted for when discussing the error associated with LRMES. In this Chapter we introduce a new bootstrapping methodology that enables us to propagate the estimation error present in the DCC-GARCH model into LRMES estimates. In Section 5.1 we start by reviewing the DCC-GARCH likelihood function and the standard two-stage approach to parameter estimation. In Section 5.2 we propose our new bootstrapping methodology that extends the work of Gonçalves and White (2004) by resampling blocks of the estimating equations instead of the original data or blocks of the likelihood function. Additionally within Section 5.2 we provide the results of our new methodology for both empirical and simulated data in order to address its performance and choose an appropriate block length. Finally in Section 5.3 we preview how we would like to extend this research and provide our recommendation for the choice of block length.

5.1 DCC-GARCH Likelihood

We start by establishing notation for the bivariate DCC-GARCH model. Our focus is restricted to the bivariate case since LRMES uses a bivariate process in its computation. The goal of the DCC-GARCH model is to estimate a time varying covariance matrix for a collection of returns. Thus notationally at time \( t \), the time varying covariance matrix, \( H_t \), can be decomposed into a diagonal standard deviation
matrix, \( D_t \), and a correlation matrix, \( P_t \),

\[
\text{Var} \begin{pmatrix} r_{mt} \\ r_{it} \end{pmatrix} = H_t = D_t P_t D_t = \begin{bmatrix} \sigma_{mt} & 0 \\ 0 & \sigma_{it} \end{bmatrix} \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix} \begin{bmatrix} \sigma_{mt} & 0 \\ 0 & \sigma_{it} \end{bmatrix}.
\]

(5.1)

However, the DCC-GARCH approach does not attempt to model the correlation matrix, \( P_t \), directly. Instead it uses a transformation to impose a dynamic correlation structure and ensure that \( P_t \) will be a positive definite correlation matrix. The standard correlation structure for the DCC-GARCH model mirrors a GARCH dynamic such that

\[
Q_t = (1 - \alpha_C - \beta_C)S + \alpha_C z_{t-1} z_{t-1}' + \beta_C Q_{t-1} \quad \text{ (5.2)}
\]

\[
P_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2} \quad \text{ (5.3)}
\]

where \( S \) represents the unconditional correlation matrix for the market and firm returns. Thus the model in equation (5.2) is simply a special case of the more general MGARCH framework. The computation of the pseudo-correlation matrix, \( Q_t \), is dependent on estimating a vector of standardized residuals, \( z_t \). These standardized residuals rely on the volatility computation for both the market and the firm. In our work we keep with the suggestion of Brownlees and Engle (2011) in using a GJR-GARCH(1,1) model for these volatilities such that

\[
\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \gamma r_{t-1}^2 \mathbb{I}(r_{t-1} < 0) + \beta \sigma_{t-1}^2.
\]

(5.4)

Thus there are four parameters that arise from each univariate volatility model. The diagonal of the standard deviation matrix \( D_t \) contains the square root of the time varying volatilities computed via
equation (5.4). In matrix form we define $D_t$ as the square root of the diagonal matrix

$$D_t^2 = \text{diag}(\omega_m, \omega_i) + \left\{ \text{diag}(\alpha_m, \alpha_i) + \text{diag}(\gamma_m I[r_{m,t-1} < 0], \gamma_i I[r_{i,t-1} < 0]) \right\} \circ \mathbf{r}_{t-1}' \mathbf{r}_{t-1} + \text{diag}(\beta_m, \beta_i) \circ D_{t-1}^2 \quad (5.5)$$

where $\circ$ stands for the Hadamard product for element wise multiplication and $\mathbf{r}_{t-1}$ is a vector containing both the market and firm return at time $t - 1$. This matrix form is then used to define the vector $\mathbf{z}_t$ which contains the standardized residuals for both the market and the firm

$$\mathbf{z}_t = D_t^{-1} \mathbf{r}_t. \quad (5.6)$$

An assumption of multivariate normality for the collection of returns allows for parameter estimation via maximum likelihood. However, even without the assumption of multivariate normality the estimator is considered to be a quasi-maximum-likelihood estimator (QMLE). Thus in cases where the distributional assumption of multivariate normality is not correct the estimator will be inefficient but consistent (Engle, 2002, 2009). The established notation allows for the DCC-GARCH log-likelihood function to be expressed as

$$L = -\frac{1}{2} \sum_{t=1}^{T} \left( 2 \log(2\pi) + \log |H_t| + \mathbf{r}_t' H_t^{-1} \mathbf{r}_t \right)$$

$$= -\frac{1}{2} \sum_{t=1}^{T} \left( 2 \log(2\pi) + \log |D_t D_t^{-1} P_t^{-1} D_t^{-1} \mathbf{r}_t| \right)$$

$$= -\frac{1}{2} \sum_{t=1}^{T} \left( 2 \log(2\pi) + 2 \log |D_t| + \log |P_t| + \mathbf{z}_t' P_t^{-1} \mathbf{z}_t \right)$$

$$= -\frac{1}{2} \sum_{t=1}^{T} \left( 2 \log(2\pi) + 2 \log |D_t| + \mathbf{r}_t' D_t^{-1} D_t^{-1} \mathbf{r}_t - \mathbf{z}_t' \mathbf{z}_t + \log |P_t| + \mathbf{z}_t' P_t^{-1} \mathbf{z}_t \right). \quad (5.7)$$

While it is possible to maximize equation (5.7) directly and solve for all of the parameters simultaneously, the recommendation of Engle (2002) is to use a 2-stage estimating approach. The 2-stage approach relies on the fact that the log-likelihood for univariate GARCH models is simply the log probability density...
As such, the full log-likelihood function in equation (5.7) can be further simplified by grouping parameters and viewing the function as a sum of 3 individual log-likelihood components. Let \( \theta_m = (\omega_m, \alpha_m, \gamma_m, \beta_m) \) and \( \theta_i = (\omega_i, \alpha_i, \gamma_i, \beta_i) \) be the market and firm GARCH parameters needed to estimate the market and firm volatility. Additionally let \( \phi = (\alpha_C, \beta_C) \) be the DCC parameters needed to estimate the correlation between the market and the firm. The log-likelihood function in equation (5.7) can then be further simplified as

\[
L(\theta_m, \theta_i, \phi) = -\frac{1}{2} \sum_{t=1}^{T} \left( 2 \log(2\pi) + 2 \log |D_t| + r_t D_t^{-1} D_t^{-1} r_t - z_t' z_t + \log |P_t| + z_t' P_t^{-1} z_t \right)
\]

\[
= -\frac{1}{2} \sum_{t=1}^{T} \left( 2 \log(2\pi) + 2 \log(\sigma_{mt} \sigma_{it}) + \frac{r_{mt}^2}{\sigma_{mt}^2} + \frac{r_{it}^2}{\sigma_{it}^2} - z_t' z_t + \log(1 - \rho_t^2) + z_t' z_t + 2\rho_t \frac{r_{mt} r_{it}}{\sigma_{mt} \sigma_{it}} \right)
\]

\[
= -\frac{1}{2} \sum_{t=1}^{T} \left( \log(2\pi) + 2 \log(\sigma_{mt}) + \frac{r_{mt}^2}{\sigma_{mt}^2} + \log(2\pi) + 2 \log(\sigma_{it}) + \frac{r_{it}^2}{\sigma_{it}^2} + \log(1 - \rho_t^2) + 2\rho_t \frac{r_{mt} r_{it}}{\sigma_{mt} \sigma_{it}} \right)
\]

\[
= \sum_{t=1}^{T} \left( l(\sigma_{mt}(\theta_m)|r_{mt}) + l(\sigma_{it}(\theta_i)|r_{it}) - \frac{1}{2} \left( 2 \log(1 - \rho_t^2) + 2\rho_t \frac{r_{mt} r_{it}}{\sigma_{mt} \sigma_{it}} \right) \right)
\]

\[
= \sum_{t=1}^{T} \left( l(\sigma_{mt}(\theta_m)|r_{mt}) + l(\sigma_{it}(\theta_i)|r_{it}) + l^C(\sigma_{it}(\theta_i), \sigma_{mt}(\theta_m), \rho_t(\phi)|r_{mt}, r_{it}) \right)
\]

\[
= L_{GARCH}(\theta_m|r_m) + L_{GARCH}(\theta_i|r_i) + L_{DCC}(\theta_m, \theta_i, \phi|r_m, r_i)
\] (5.9)

where \( r_m \) and \( r_i \) represent the entire history of market and firm returns.

In the 2-stage estimation approach, each of the individual likelihood components in equation (5.9) are maximized separately. The parameter estimates from this two-stage approach are similar to a QMLE in that they are inefficient but consistent. The main reason behind using this two-stage approach is that it allows for computationally feasible parameter estimates even when the correlation matrix is of high dimension (Engle, 2002). However, additional research by LaBarr (2010) has shown that even in low dimensions the estimation of \( \alpha_C \) and \( \beta_C \) can be unstable when using equation (5.7) directly for parameter
estimation. As such, we continue to use the two-stage approach for estimating parameters despite the fact that we focus on a bivariate DCC-GARCH model. Formally the two-stage estimation procedure is as follows:

Step 1: Maximize the univariate volatility models in order to estimate the GARCH parameters:

\[
\hat{\theta}_m = \arg \max_{\theta_m} L_{GARCH}(\theta_m | r_m) \quad \hat{\theta}_i = \arg \max_{\theta_i} L_{GARCH}(\theta_i | r_i).
\] (5.10)

Step 2: Hold the GARCH parameters fixed in order to maximize the correlation likelihood component:

\[
\hat{\phi} = \arg \max_{\phi} L_{DCC}(\phi | \hat{\theta}_m, \hat{\theta}_i; r_m, r_i)
\] (5.11)

Engle and Sheppard (2001) provide a sufficient set of assumptions for establishing the consistency and asymptotic normality of this two-stage DCC-GARCH estimator. Their proofs for consistency and asymptotic normality follow closely the proofs laid out by Newey and McFadden (1994) for the asymptotic distribution of two-stage GMM estimators. Consistent standard errors for \( \theta_m \) and \( \theta_i \) rely on the typical robust standard errors for GARCH models as provided by Bollerslev and Wooldridge (1992). The consistent standard errors for \( \phi \) require a modification based on the fact that we are using a two-stage approach. Engle and Sheppard (2001) provide the formula for the consistent standard errors for \( \phi \) which relies on the cross partial derivatives of the likelihood component given in equation (5.11) with respect to \( \theta_m, \theta_i, \) and \( \phi \).

### 5.2 Block Bootstrap for Estimating Equations Methodology

#### 5.2.1 Definition

Our goal is to propagate the estimation error present in the DCC-GARCH model through the LRMES simulation procedure. While Section 5.1 establishes that there are asymptotic variance estimates for the DCC-GARCH parameters when estimated via a two-stage approach, the simulation procedure used
to compute LRMES creates a complicated and difficult to derive theoretical distribution for LRMES. While it is still possible propagate the asymptotic variance through the LRMES simulation procedure via Monte Carlo methods, this would require an explicit distributional assumption. In order to provide a more non-parametric approach we instead propose a new bootstrap procedure for propagating the DCC-GARCH estimation error through the LRMES simulation procedure.

Bootstrap procedures have provided a powerful and widely used method for estimating standard errors and performing statistical inference since the original work of Efron (1979). However, special care is needed when working with time series models due to the inherent dependence in the data. Lahiri (2003) reviews in detail different bootstrap methods for dependent data and their theoretical properties, but in general there are two approaches to bootstrapping dependent data. The first is to resample residuals from a fitted model. Examples of this approach are seen in the work of Efron and Tibshirani (1986) for autoregressive moving average (ARMA) models and Liu et al. (1988) for regression with dependent data. More recently this approach has been used by Pascual et al. (2004) to provide a residual based bootstrap approach for linear autoregressive integrated moving average (ARIMA) models and by Pascual et al. (2006) for GARCH models. The second approach is to resample blocks of original data such that the dependence structure of the data can be preserved. Examples of this second approach include the moving block bootstrap (Kunsch, 1989; Liu and Singh, 1992), the circular block bootstrap (Politis and Romano, 1992), and the stationary bootstrap (Politis and Romano, 1994). Many of these block bootstrapping methods are compared by Lahiri (1999) who shows that overlapping blocks with a fixed block length are preferred.

Residual based bootstrapping methods can be thought of as a parametric approach to estimating standard errors. This is due to the fact that the residual based approaches make direct use of a specific model structure. Thus it only seems natural to use these methods when it is plausible that the true data generating process (DGP) follows the same structure as the chosen model. On the other hand, block bootstrapping approaches are more non-parametric in nature since they use the observed data to generate a dependence structure. In the context of SRISK and LRMES, the use of a DCC-GARCH model is simply a modeling strategy. Brownlees and Engle (2015) discuss other models that could be used in computing
LRMES and it seems naive to think that the true DGP exactly follows a DCC-GARCH structure. As such we turn our focus to block bootstrapping procedures to provide suitable standard error estimates for the DCC-GARCH parameters.

In equation (5.7) and equation (5.9) we see that the contribution of each observation to the DCC-GARCH likelihood function is dependent on the entire history of past observables up to that point. As pointed out by Gonçalves and White (2004) and Corradi and Iglesias (2008), in this situation resampling blocks of the original data is not the same as resampling blocks of the likelihood function. Thus in order to obtain standard error estimates for the DCC-GARCH parameters, we must resample from the likelihood function instead of the original data. However, as discussed in Section 5.1, the DCC-GARCH parameters are typically not estimated by maximizing the log-likelihood function, but are instead estimated via a two-stage approach. Thus, in order to propagate the error of the DCC-GARCH parameters into the LRMES simulation procedure, we propose the Block Bootstrap for Estimating Equation (BBEE) methodology. Our BBEE methodology extends the work of Gonçalves and White (2004) who resampled blocks of the likelihood function. Our extension is to resample blocks of the estimating equations in the DCC-GARCH two-stage approach instead of resampling blocks of the DCC-GARCH likelihood function.

By resampling from the likelihood function, which includes the entire history of past observables, Gonçalves and White (2004) are able to account for the dependence structure present in the observed data. The choice of a block bootstrapping procedure within the likelihood based approach controls for additional serial dependence that may enter due to model misspecification. So while the focus of Gonçalves and White (2004) is on block bootstrapping methods, their results regarding asymptotic validity and consistent standard errors are general in that they cover any type of bootstrap method. Within our BBEE methodology we choose to use a block bootstrap method as well because we do not believe that the DCC-GARCH model is always correctly specified. Specifically we use the moving block bootstrap (MBB) procedure of Kunsch (1989) and Liu and Singh (1992) to resample blocks of the estimating equations. The choice of the MBB follows the work of Gonçalves and White (2004) and keeps with the results of Lahiri (1999) that overlapping blocks with a fixed block length are preferred.

For our BBEE methodology we sample from the estimating equations $b$ blocks of length $\ell$ such that
The notation for the market volatility likelihood components in equation (5.12) helps show that each
individual correlation log-likelihood components are
\[
I(\sigma_{ml_t+1}(\theta_m)|r_{ml_t+1}), I(\sigma_{ml_t+2}(\theta_m)|r_{ml_t+2}), \ldots, I(\sigma_{ml_t+\ell}(\theta_m)|r_{ml_t+\ell}),
\]
\[
I(\sigma_{ml_t+1}(\theta_m)|r_{ml_t+1}), \ldots, I(\sigma_{ml_t+\ell}(\theta_m)|r_{ml_t+\ell}), \ldots, I(\sigma_{ml_b+\ell}(\theta_m)|r_{ml_b+\ell}). \tag{5.12}
\]

The notation for the market volatility likelihood components in equation (5.12) helps show that each
random draw, \(I_j = k\), carries with it the entire history of past returns up to time \(k\). Thus each bootstrap
sample, \(B\), is defined by its \(b\) random draws of \(I_j\). For a single bootstrap sample the corollary to \(L_{GARCH}\)
in equation (5.10) is given by

\[
L^{(B)}_{GARCH}(\theta_m|r_m) = \sum_{j=1}^{b} \sum_{l=1}^{\ell} I(\sigma_{ml_j+l}(\theta_m)|r_{ml_j+l}). \tag{5.13}
\]

In order to estimate the firm GARCH parameters, the individual log-likelihood components in equation (5.12) and the bootstrap corollary in equation (5.13) are identical except for replacing the market subscript \(m\) with the firm subscript \(i\).

In order to bootstrap the second stage, as given by equation (5.11), the correlation log-likelihood
components treat the estimated bootstrapped volatility parameters, \((\hat{\theta}_m^{(B)}, \hat{\theta}_i^{(B)})\), as fixed. As such the
individual correlation log-likelihood components are

\[
I^C(\rho_{l_t+1}(\phi)|\hat{\theta}_m^{(B)}, \hat{\theta}_i^{(B)}; r_{ml_t+1}, r_{il_t+1}), \ldots, I^C(\rho_{l_t+\ell}(\phi)|\hat{\theta}_m^{(B)}, \hat{\theta}_i^{(B)}; r_{ml_t+\ell}, r_{il_t+\ell}),
\]
\[
I^C(\rho_{l_t+1}(\phi)|\hat{\theta}_m^{(B)}, \hat{\theta}_i^{(B)}; r_{ml_t+1}, r_{il_t+1}), \ldots, I^C(\rho_{l_t+\ell}(\phi)|\hat{\theta}_m^{(B)}, \hat{\theta}_i^{(B)}; r_{ml_t+\ell}, r_{il_t+\ell}), \ldots,
\]
\[
\ldots, I^C(\rho_{l_b+\ell}(\phi)|\hat{\theta}_m^{(B)}, \hat{\theta}_i^{(B)}; r_{ml_b+\ell}, r_{il_b+\ell}). \tag{5.14}
\]

Here the random draws, \(I_j = k\), used to define the bootstrap sample, \(B\), are the same random draws used
in equation (5.12) and equation (5.13). Once again we can define the bootstrap corollary to \(L_{DCC}\) by using
the individual correlation log-likelihood components from equation (5.14)

\[ L_{DCC}(\phi|\hat{\theta}_m, \hat{\theta}_i; r_m, r_I) = \sum_{j=1}^{b} \sum_{t=1}^{\ell} \ell^C(\rho_{Ij+t}(\phi)|\hat{\theta}_{m}^{(B)}, \hat{\theta}_{i}^{(B)}; r_{mIj+t}, r_{Ij+t}). \]  

(5.15)

A naive approach to parameter estimation for our BBEE methodology is to simply replace \( L_{GARCH} \) and \( L_{DCC} \) with their bootstrap counterparts \( L_{GARCH}^{(B)} \) and \( L_{DCC}^{(B)} \). However, this would not preserve the higher-order properties of bootstrap sampling that are typically present in bootstrap estimators. Both Hall and Horowitz (1996) and Andrews (2002) discuss the need to recenter the function used in bootstrap estimation when the MBB is used. In this case recentering is needed to ensure a proper first moment condition. The fact that our blocks are overlapping implies that while each block has an equivalent probability of being sampled, for \( \ell > 1 \) each time point does not. This is clearly seen for the earliest time-point since by definition it can only be a part of the block \( I_j = 1 \). In contrast, the second earliest time-point would be a part of both the \( I_j = 1 \) and the \( I_j = 2 \) blocks.

The need for recentering when using the MBB was acknowledged by Gonçalves and White (2004), but they chose not to implement recentering since it was unnecessary for establishing first-order properties. Corradi and Iglesias (2008) did implement recentering for resampling the average GARCH log-likelihood as

\[ \hat{\theta}_T^* = \arg \max_{\theta} \frac{1}{T} \sum_{t=1}^{T} \left( L_T^*(\theta) - \left( E^* \left[ \nabla L_T^*(\hat{\theta}_T) \right] \right)' \theta \right) \]  

(5.16)

where \( E^* \) represents the expectation relative to the distribution of the bootstrap sample. The recentering term in equation (5.16) ensures the average score function is zero when evaluated at the optimal parameters for the original data, \( \hat{\theta} \). Without this recentering term, the bootstrap moment condition does not hold for the population of bootstrap samples. We note that equation (5.16) is written in a way that emphasizes the adjustment of each term in the log-likelihood. In this way it is more obvious that its \( E^* \) average value is zero. However, the computation of this adjustment can be simplified by noting that

\[ \sum_{t=1}^{T} E^* \left[ \nabla L_T^*(\hat{\theta}_T) \right] = E^* \left[ \nabla \sum_{t=1}^{T} L_T^*(\hat{\theta}_T) \right] = E^* \left[ \nabla L_T^*(\hat{\theta}_T) \right]. \]  

(5.17)
Thus for computational purposes equation (5.16) can be rewritten as

\[ \hat{\theta}^*_T = \arg \max_{\theta} \frac{1}{T} \left[ \left( \sum_{t=1}^{T} L^*_t(\theta) \right) - E^* \left[ \nabla L^*_T(\hat{\theta}_T) \right]' \theta \right]. \tag{5.18} \]

This emphasizes that there is a single average gradient for each bootstrap sample.

Our BBEE methodology implements the recentering of the estimating equations for the DCC-GARCH model. Notationally we define \( E^{(B)} \) to be the expectation relative to the distribution of the bootstrap sample. Then we use the likelihood components given in equation (5.12) to define our first stage estimators as

\[ \hat{\theta}^{(B)}_m = \arg \max_{\theta_m} \left( L^{(B)}_{GARCH}(\theta_m | r_m) - E^{(B)} \left[ \nabla L^{(B)}_{GARCH}(\theta_m | r_m) \right]' \theta_m \right) \]
\[ \hat{\theta}^{(B)}_i = \arg \max_{\theta_i} \left( L^{(B)}_{GARCH}(\theta_i | r_i) - E^{(B)} \left[ \nabla L^{(B)}_{GARCH}(\hat{\theta}_i | r_i) \right]' \theta_i \right) \tag{5.19} \]

The estimates \( \hat{\theta}^{(B)}_m \) and \( \hat{\theta}^{(B)}_i \) are then used as the fixed parameters in the second step of our maximization procedure. Recall that for a given bootstrap sample we use identical random draws to those used in computing \( \hat{\theta}^{(B)}_m \) and \( \hat{\theta}^{(B)}_i \). Thus the correlation parameters are estimated via

\[ \hat{\phi}^{(B)} = \arg \max_{\phi} \left( L^{(B)}_{DCC}(\phi | \hat{\theta}^{(B)}_m, \hat{\theta}^{(B)}_i; r_m, r_i) - E^{(B)} \left[ L^{(B)}_{DCC}(\phi | \hat{\theta}^{(B)}_m, \hat{\theta}^{(B)}_i; r_m, r_i) \right]' \phi \right) \tag{5.20} \]

Each bootstrap sample \( B = 1, \ldots, N_b \) provides its own set of parameter values \( (\hat{\theta}^{(B)}_m, \hat{\theta}^{(B)}_i, \hat{\phi}^{(B)}) \) and our BBEE estimates of the DCC-GARCH parameter standard errors are simply the standard deviation of the \( N_b \) bootstrap parameter values. Moreover the different sets of parameter values can also be used in our simulation procedure for estimating LRMES. Then the standard deviation of the \( N_b \) LRMES estimates provide an estimate of the full error associated with estimating LRMES for a given firm and date combination. In this way the BBEE methodology provides a way for the parameter estimation error to be propagated through the LRMES simulation procedure.

An obvious question that has yet to be addressed is how to choose the block length, \( \ell \), in our BBEE methodology. In the academic literature for the choice of optimal block length we find that Hall et al.

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(1995) show that the choice of block length depends on the context of the problem. Their suggestion in the case of variance and bias estimation is to use $N^{1/3}$. Alternatively, Bühlmann and Künsch (1999) propose using a block length equal to the inverse of the bandwidth in the spectral domain while Politis and White (2004) provide a practical way of choosing block length based on the optimal MSE of the block bootstrapping procedure. However, we would not expect any of these results to hold for our BBEE methodology since we are not resampling from the original data. Gonçalves and White (2004) choose to take the integer part of Andrews (1991) automatic procedure for computing a data driven bandwidth as their block length, but they acknowledge that this method may not be optimal. Moreover we are still unsure that using Andrews (1991) method will produce good results for our extension. As such we look for guidance in choosing a block length from examining the performance of our BBEE methodology for both empirical and simulated data.

5.2.2 Empirical Results

Our first step in exploring the performance our BBEE methodology and our choice of block length is to apply the methodology to data that has already been used in Section 4 for computing LRMES. Recall that these data sets are defined by a firm and date combination which produce a bivariate time-series of length 1,000 for the market and the specified firm. We apply our BBEE methodology to the data in order to obtain $N_b$ vectors of DCC-GARCH parameter values. The average parameter value is used as our BBEE parameter estimate and the standard deviation of these values is used as our BBEE estimate of the parameter standard error. These BBEE estimates are then compared with the results of fitting the original data with our standard DCC-GARCH fit using the rmgarch package (Ghalanos, 2015). Recall that the package uses the 2-stage estimating equation approach presented in equation (5.10) and equation (5.11) for parameter estimation and provides asymptotic standard error (ASE) approximations for its reported standard errors. Ideally the average BBEE parameter estimates will not show any bias when compared to the parameter estimates of the rmgarch package. Additionally while we expect the BBEE standard errors to change with the block length, ideally they will converge to the corresponding parameter’s ASE approximation.
The computation of our BBEE methodology relies on finding the parameters that maximize equation (5.19) and equation (5.20) for each bootstrap sample. This can be computationally demanding, especially for a large number of bootstrap samples. One way to ease this computation is to only take a few Newton steps from the parameter estimates found via the two-stage estimation of the original data \((\hat{\theta}_m, \hat{\theta}_i, \hat{\phi})\). By only taking a few steps for each bootstrap sample we are able to reduce our computation time while still providing the desired variability in the bootstrapped parameters. The idea of using only a few Newton steps in bootstrap resampling is motivated by Davidson and MacKinnon (1999) who only used a few steps to provide bootstrap p-values for likelihood ratio, Lagrange multiplier, and Wald tests. Moreover Gonçalves and White (2004) show that using a one-step estimator is first-order asymptotically equivalent to using the fully optimized block bootstrap estimator for their bootstrapped QML estimators.

In order to determine the number of steps that are needed for our BBEE methodology we fit the empirical data for COF on January 30, 2004 using \(N_b = 10,000\) bootstrap samples and various numbers of Newton steps. The bootstrap distribution of the parameter estimates for different step lengths are overlaid in Figure 5.1. Here we see that taking a single Newton step generally leads to narrower parameter distributions. So while Gonçalves and White (2004) show that it is asymptotically valid to only take 1-step, in practice our BBEE methodology tends to underestimate the variability in the DCC-GARCH parameter estimates. Similarly while the 2-step estimator is more reasonable, it also underestimates the variability of certain parameters. Figure 5.1 demonstrates that the distribution of the 4-step estimator is nearly identical to the distribution of the the 10-step estimator for all parameters. As such our computation of the BBEE methodology uses 4 Newton steps in order to give adequate parameter variability while still easing computation.

Next we look to make sure that the parameter estimates of our BBEE methodology are unbiased when compared to using the 2-stage estimating equations on the original data. Again we use the empirical data for COF on January 30, 2004 and continue to use 4 Newton steps for the maximization within our BBEE methodology. All of the following results are computed using \(N_b = 10,000\) bootstrap samples at block lengths of \(\ell = 1, 2, 4, 5, 8, 10, 20, 25, 40, 50\) and 100. It is evident from Figure 5.2 that there is very little bias in the average BBEE parameter estimate as the majority of the estimates are close to the
Figure 5.1: Bootstrap densities for all DCC-GARCH parameter estimate using various numbers of Newton steps within the BBEE methodology.
parameter estimates provided by the \texttt{rmgarch} package. The largest amount of bias occurs in $\beta_C$, but this bias is still less than 30% for all block lengths and is contained within the 95% confidence interval based on ASE approximations. In fact all of the average BBEE parameter estimates are within the 95% confidence interval based on ASE approximations. Additionally Figure 5.2 illustrates that the parameter estimates are relatively stable across block lengths for $\ell \leq 10$. For longer block lengths the average BBEE parameter estimate tends to drift away from the $\ell = 1$ estimate, but the differences are relatively small compared to the estimated standard error.

The wide confidence intervals in Figure 5.2 are an indication that the ASE approximation can at times be quite large, especially for $\gamma_m, \gamma_i$ and $\beta_C$. Nonetheless we want to compare the standard error estimates of our BBEE methodology to the ASE approximations as computed by the \texttt{rmgarch} package. Since longer block lengths attempt to capture additional dependence introduced by model misspecification, our hope is that the BBEE estimates for parameter estimation error will converge to the ASE approximation as the block length increases, regardless of whether or not the empirical data truly follows our standard DCC-GARCH specification. From Figure 5.3 is is clear that our BBEE standard error is quite different from the ASE approximations and does not converge at longer block lengths. The standard error estimates are closest for $\alpha_i$ and $\alpha_C$, but are substantially different for the other parameters. For the market parameters and $\gamma$ the BBEE estimates are lower than the ASE approximations while for $\omega_i$, $\beta_i$, and $\beta_C$ the BBEE estimates are higher. This highlights that there is not systematic error in our BBEE methodology. However, Figure 5.3 demonstrates that our BBEE methodology struggles with the $\alpha_m$ parameter, which for this firm and date combination is nearly zero. In examining this result we find that all of the bootstrap estimates for $\alpha_m$ are close to zero. This consistency causes the BBEE methodology to fail in estimating the standard error when the corresponding parameter estimate is nearly zero.

Other firm and date combinations show similar results to Figure 5.2 and Figure 5.3. Specifically, there is little to no bias in the mean BBEE parameter estimates and these estimates are always within the 95% confidence interval based on the ASE approximations. Additionally, for all but a few exceptions, the BBEE standard error estimates are different from and do not converge to their corresponding
Figure 5.2: BBEE parameter estimates by block length for COF on 1/30/2004. The purple line shows the \texttt{rmgarch} package parameter estimate. The dashed gray lines show the 95\% confidence interval based on the ASE approximation.
ASE approximations. In looking at multiple firm and date combinations we fail to find any systematic
differences between the BBEE standard error estimates and the ASE approximations that could potentially
highlight problems in our methodology. The only exception is that BBEE standard error estimates for $\beta_C$
tend to be higher than their corresponding ASE approximations. However, this exception still does not lead
to an obvious reason as to why our BBEE standard error estimates differ from the ASE approximations.
The differences between these two methodologies require additional exploration to determine whether or
not the BBEE methodology can adequately propagate parameter estimation error through the LRMES
procedure.

5.2.3 Asymptotic Standard Error Estimates

The initial results from applying our BBEE methodology to empirical data leads us to question whether
or not the asymptotic standard error (ASE) approximations produced by the \texttt{rmgarch} package accurately
represent the true amount of parameter variability. In general ASE approximations may fail to provide
an accurate representation of true parameter variability if the sample size is too small or if the numeric
evaluations of the Hessian are not close enough to their corresponding analytical values. It is reasonable
to believe that either one of these problems may exist in our empirical data. In general this data is highly
persistent, and while the 1,000 data points seems large enough to accurately estimate the volatility and
correlation dynamics, it may not be large enough for asymptotic theory to hold. As such we use this
section to test the accuracy of the ASE approximations before abandoning our BBEE methodology based
on the observed differences with the ASE approximations.

In order to assess the accuracy of the ASE approximations we design Monte Carlo simulations that
incorporate the correct DCC-GARCH specification. The Monte Carlo simulations use a fixed data
generating process (DGP) to generate $M = 50,000$ bivariate series of length $w = 1,000$. In our first
set of Monte Carlo experiments the data is simulated using our standard DCC-GARCH model and
is fit using the exact same specification. From each fit the resulting parameter estimates and ASE
approximation are saved. The standard deviation of the parameter estimates provide us with a Monte
Carlo approximation to the true parameter variability. This can then be compared to the corresponding

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Figure 5.3: BBEE estimates of parameter estimation error by block length for COF on 1/30/2004. The purple line shows the ASE approximation.
average ASE approximation.

The first Monte Carlo simulation fixes the DCC-GARCH parameter values to be equivalent to the parameter estimates resulting from fitting the empirical data for COF on January 30, 2004. Thus we are directly testing whether or not the ASE approximations shown in Figure 5.3 accurately represent the true parameter variability. The results of Table 5.1 show that the mean ASE approximation is greater than the standard deviation of the parameter estimates by more than what is expected due to Monte Carlo error for all parameters except $\alpha_C$. Within these significant differences, the mean ASE for the firm parameters are still relatively close to the true parameter variability. However, the mean ASE for the market parameters all show at least a 100% difference from the true parameter variability. Digging deeper into the full distribution of the ASE approximations we find that most distributions are skewed to the right. In fact most of the parameters have at least one instance where the ASE is approximated to be larger than 1. As an indicator of how skewed these distributions are, we also include the median ASE in Table 5.1. These median ASE are often much closer to the true parameter variability than the mean ASE.

Due to the parameter constraints of the DCC-GARCH model an ASE approximation of 1 or larger is equivalent to saying that the parameter could vary over the entire feasible parameter space. Obviously this type of result is not useful when trying to understand the true parameter variability. The median ASE from Table 5.1 shows that the typical ASE approximations are often close to the true parameter variability, but for a single data set it is impossible to know where in the distribution a single ASE approximation lies. As such we do not believe our BBEE standard error estimates should use the corresponding ASE approximation as a comparison. In fact, recall from Figure 5.3 that $\alpha_C$ was the parameter whose BBEE standard error estimate was closest to the corresponding ASE approximation. In Table 5.1 we see that the only parameter whose mean ASE captured the true parameter variability is $\alpha_C$. This highlights the potential for our BBEE standard error estimates to still be reflecting the true parameter estimation error despite showing differences with the ASE approximations.
Table 5.1: Monte Carlo simulation results for parameter values equivalent to the fit of COF on 1/30/2004.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_m$</th>
<th>$\alpha_m$</th>
<th>$\beta_m$</th>
<th>$\gamma_m$</th>
<th>$\omega_i$</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
<th>$\gamma_i$</th>
<th>$\alpha_C$</th>
<th>$\beta_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>$1.2 \times 10^{-6}$</td>
<td>0.0091</td>
<td>0.0184</td>
<td>0.0294</td>
<td>$1.8 \times 10^{-5}$</td>
<td>0.0186</td>
<td>0.0221</td>
<td>0.0375</td>
<td>0.0379</td>
<td>0.2765</td>
</tr>
<tr>
<td>(3.7 $\times 10^{-9}$)</td>
<td>(0.00003)</td>
<td>(0.00006)</td>
<td>(0.00009)</td>
<td>(5.7 $\times 10^{-8}$)</td>
<td>(0.00006)</td>
<td>(0.00007)</td>
<td>(0.00012)</td>
<td>(0.00012)</td>
<td>(0.00087)</td>
<td></td>
</tr>
<tr>
<td>Mean ASE</td>
<td>$1.0 \times 10^{-5}$</td>
<td>0.0741</td>
<td>0.0564</td>
<td>0.1033</td>
<td>$1.7 \times 10^{-5}$</td>
<td>0.0209</td>
<td>0.0230</td>
<td>0.0388</td>
<td>0.0373</td>
<td>0.3983</td>
</tr>
<tr>
<td>(1.4 $\times 10^{-6}$)</td>
<td>(0.01006)</td>
<td>(0.00530)</td>
<td>(0.01276)</td>
<td>(3.6 $\times 10^{-8}$)</td>
<td>(0.00007)</td>
<td>(0.00001)</td>
<td>(0.00005)</td>
<td>(0.00416)</td>
<td>(0.06377)</td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>$1.9 \times 10^{-6}$</td>
<td>0.0171</td>
<td>0.0153</td>
<td>0.0339</td>
<td>$1.6 \times 10^{-5}$</td>
<td>0.0201</td>
<td>0.0216</td>
<td>0.0382</td>
<td>0.0306</td>
<td>0.1715</td>
</tr>
</tbody>
</table>
Table 5.2: The fixed parameter values used in the DCC(1,1)-GARCH(2,1) misspecification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_m$</td>
<td>$4.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha_{m1}$</td>
<td>.015</td>
</tr>
<tr>
<td>$\alpha_{m2}$</td>
<td>.038</td>
</tr>
<tr>
<td>$\beta_m$</td>
<td>.775</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>$4.8 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\alpha_{i1}$</td>
<td>.076</td>
</tr>
<tr>
<td>$\alpha_{i2}$</td>
<td>.046</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>.685</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>.078</td>
</tr>
<tr>
<td>$\beta_C$</td>
<td>.373</td>
</tr>
</tbody>
</table>

Simulations under identical settings for other firm and date combinations yield similar results. The average ASE is always higher than the standard deviation of parameter estimates due to outliers where the ASE approximation is larger than 1. As a result the median ASE tends to be much closer to the true parameter standard error. For all firm and date combinations the $\beta_C$ correlation parameter has some of the largest percentage differences between the median ASE and the standard deviation of the parameter estimates.

For completeness we also use Monte Carlo simulations to address how the ASE approximations perform under misspecification. Here we use 5 different DGPs to create the misspecifications. The first DGP uses a DCC(1,1)-GARCH(2,1) model with normal errors and a standard GARCH volatility specification. The data is then fit using our standard DCC(1,1)-GARCH(1,1) model with normal errors and a GJR-GARCH volatility specification. The fixed parameters in the DGP replicate a fit of the DCC(1,1)-GARCH(2,1) model on the empirical data for COF on January 30, 2004. The values for these fixed parameters are given in Table 5.2.

The next two misspecified DGPs start by generating two independent univariate time series from the GARCH(2,1) model defined by the coefficients in Table 5.2. However, these two independent time series are then correlated using a fixed dynamic correlation process instead of the DCC model. Here we follow the work of Engle (2002) and define a sine wave and a fast sine wave correlation function. Specifically the correlation is defined by

$$
\rho_t^\dagger = .5 + .4\cos(2\pi t/200) \quad \text{Sine}
$$

$$
\rho_t^\ddagger = .5 + .4\cos(2\pi t/20) \quad \text{Fast Sine.}
$$

Thus the fast sine correlation exhibits rapid changes in the correlation between the market and the firm.
while the sine correlation is much more gradual. The two independent univariate time series, \((y_1, y_2)\) are correlated via

\[
\begin{align*}
    r^*_{it} &= y_{1t} \\
    r^*_{it} &= \rho^*_{it} y_{1t} + \sqrt{1 - \rho^*_{it}^2} y_{2t}.
\end{align*}
\]

where \(\rho^*_{it}\) is defined by one of the specifications in equation (5.21). Notice that in this specification the simulated market data will exhibit true GARCH(2,1) behavior, but the simulated firm data will not due to the correlation adjustments in equation (5.23).

Finally our last two misspecified DGPs replace the GARCH(2,1) model for univariate volatilities with a stochastic volatility model. Specifically we use the model

\[
\begin{align*}
y_t &= h_t \varepsilon_t \\
\log h_t^2 &= -8.5 + 0.97(\log h_{t-1}^2 + 8.5) + 0.2 \eta_t
\end{align*}
\]

where both \(\varepsilon_t\) and \(\eta_t\) are random draws from a standard normal distribution. The fixed parameters in equation (5.24) are motivated by univariate stochastic volatility fits to log-returns of the market and COF on 1/30/2004. Once the two independent univariate time series, \((y_1, y_2)\), have been generated from equation (5.24) the two series are correlated via equation (5.23) where \(\rho^*_{it}\) is defined by one of the specifications in equation (5.21). These misspecifications are designed to be a more radical departure from the true DCC(1,1)-GARCH(1,1) model. We recognize that these 5 misspecifications do not provide exhaustive coverage for all possible misspecifications, but believe that they are different enough to provide an adequate view of how the ASE approximations perform when the model is misspecified.

A comparison of the true parameter variability with the ASE approximations for the DCC(1,1)-GARCH(2,1) misspecification is shown in Table 5.3. The results are similar to what has already been seen for correctly specified data in Table 5.1. Specifically the average ASE for the market GARCH parameters are much higher than the true parameter variability. Once again this is due to a right skewed distribution where many of the ASE approximations are larger than 1. As a result the median ASE
is a better representation of the true parameter variability for these parameters. The firm GARCH parameters in Table 5.3 adequately represent the parameter standard deviation, though the differences are still statistically significant due to the small amount of Monte Carlo error. For these parameters the distribution is slightly skewed to the left as illustrated by the smaller median ASE. Finally the ASE approximation for $\alpha_C$ is a fair representation of the true parameter estimation error, but there are large outliers for $\beta_C$ which cause the mean ASE approximation to overestimate an already large amount of parameter estimation error. All of these results point to the fact that the model is only slightly misspecified in this case.

The results of the sine wave misspecifications with GARCH(2,1) univariate volatilities are shown in Table 5.4 and Table 5.5. As expected, the market GARCH parameters for these misspecifications show the same overestimation due to large outliers previously seen in Table 5.3. The altering of the GARCH dynamic for the firm GARCH parameters causes the mean ASE approximation to drift further from the Monte Carlo standard deviation, but overall the ASE approximations still adequately represent the true parameter variability. Interestingly the ASE approximations typically underestimate the DCC parameter standard deviation for both the sine and fast sine waves, despite cases where the ASE approximation for $\alpha_C$ and $\beta_C$ is larger than one. Thus the outliers are improving the mean ASE when compared to the median ASE. So while the mean ASE is relatively close to the true parameter variability, a typical ASE approximation will underestimate the variability in the DCC parameters by a larger amount.

Finally the results of the misspecifications using stochastic volatility and sine wave correlations are shown in Table 5.6 and Table 5.7. In these misspecifications the ASE approximation underestimates the true parameter variability for both the market and the firm GARCH parameters. Notice that the median ASE is once again smaller than the mean ASE indicating a skewed right distribution and ASE approximations larger than one. Thus a typical ASE approximation for a single data set will have a high likelihood of underestimating the true amount of GARCH parameter estimation error. In contrast to Table 5.4 and Table 5.5, the DCC parameter estimation error is better estimated by ASE approximations when there is a stochastic volatility specification. For the sine wave results in Table 5.6, $\alpha_C$ and $\beta_C$ have mean and median ASE approximations that are only slightly higher than the true standard deviation.
The corresponding ASE approximations for the fast sine wave in Table 5.7 are underestimates, but not drastically so. All of the misspecifications continue to show that in general the DCC-GARCH ASE approximation does not capture the true amount of parameter estimation error for bivariate time-series of length 1,000. The propensity to misrepresent the true amount of error, especially in the GARCH parameters, highlights the need to refrain from measuring the accuracy of our BBEE standard error estimates with ASE approximations.
Table 5.3: Monte Carlo simulation results for the DCC(1,1)-GARCH(2,1) misspecification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Deviation</th>
<th>Mean ASE</th>
<th>Median ASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_m$</td>
<td>$1.0 \times 10^{-6}$</td>
<td>$2.6 \times 10^{-5}$</td>
<td>$2.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>$0.0052$</td>
<td>$0.188$</td>
<td>$0.01679$</td>
</tr>
<tr>
<td>$\beta_m$</td>
<td>$0.0154$</td>
<td>$0.117$</td>
<td>$0.0145$</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>$0.0255$</td>
<td>$0.341$</td>
<td>$0.0309$</td>
</tr>
<tr>
<td>$\omega_l$</td>
<td>$1.5 \times 10^{-5}$</td>
<td>$1.6 \times 10^{-5}$</td>
<td>$1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>$0.0208$</td>
<td>$0.0217$</td>
<td>$0.0207$</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>$0.0219$</td>
<td>$0.0214$</td>
<td>$0.0201$</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>$0.0338$</td>
<td>$0.0346$</td>
<td>$0.0342$</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>$0.0401$</td>
<td>$0.0346$</td>
<td>$0.0342$</td>
</tr>
<tr>
<td>$\beta_C$</td>
<td>$0.2851$</td>
<td>$0.379$</td>
<td>$0.1750$</td>
</tr>
</tbody>
</table>

Table 5.4: Monte Carlo simulation results for the GARCH(2,1) volatility with sine wave correlation misspecification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Deviation</th>
<th>Mean ASE</th>
<th>Median ASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_m$</td>
<td>$1.0 \times 10^{-6}$</td>
<td>$2.1 \times 10^{-5}$</td>
<td>$3.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>$0.0043$</td>
<td>$0.1621$</td>
<td>$0.0239$</td>
</tr>
<tr>
<td>$\beta_m$</td>
<td>$0.0151$</td>
<td>$0.1455$</td>
<td>$0.0160$</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>$0.0250$</td>
<td>$0.1481$</td>
<td>$0.0356$</td>
</tr>
<tr>
<td>$\omega_l$</td>
<td>$1.3 \times 10^{-5}$</td>
<td>$1.2 \times 10^{-5}$</td>
<td>$1.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>$0.0208$</td>
<td>$0.0330$</td>
<td>$0.0307$</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>$0.0212$</td>
<td>$0.0207$</td>
<td>$0.0193$</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>$0.0343$</td>
<td>$0.0383$</td>
<td>$0.0371$</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>$0.0646$</td>
<td>$0.0474$</td>
<td>$0.0357$</td>
</tr>
<tr>
<td>$\beta_C$</td>
<td>$0.0916$</td>
<td>$0.0602$</td>
<td>$0.0471$</td>
</tr>
</tbody>
</table>

Table 5.5: Monte Carlo simulation results for the GARCH(2,1) volatility with fast sine wave correlation misspecification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Deviation</th>
<th>Mean ASE</th>
<th>Median ASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_m$</td>
<td>$1.0 \times 10^{-6}$</td>
<td>$2.2 \times 10^{-5}$</td>
<td>$3.7 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>$0.0042$</td>
<td>$0.1576$</td>
<td>$0.03095$</td>
</tr>
<tr>
<td>$\beta_m$</td>
<td>$0.0151$</td>
<td>$0.1343$</td>
<td>$0.0131$</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>$0.0250$</td>
<td>$0.1835$</td>
<td>$0.0323$</td>
</tr>
<tr>
<td>$\omega_l$</td>
<td>$1.8 \times 10^{-5}$</td>
<td>$1.8 \times 10^{-5}$</td>
<td>$1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>$0.0222$</td>
<td>$0.0367$</td>
<td>$0.0337$</td>
</tr>
<tr>
<td>$\beta_l$</td>
<td>$0.0262$</td>
<td>$0.0320$</td>
<td>$0.0284$</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>$0.0352$</td>
<td>$0.0368$</td>
<td>$0.0356$</td>
</tr>
<tr>
<td>$\alpha_C$</td>
<td>$0.1059$</td>
<td>$0.0847$</td>
<td>$0.0563$</td>
</tr>
<tr>
<td>$\beta_C$</td>
<td>$0.1934$</td>
<td>$0.1714$</td>
<td>$0.0979$</td>
</tr>
</tbody>
</table>
Table 5.6: Monte Carlo simulation results for the stochastic volatility with sine wave correlation misspecification.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_m$</th>
<th>$\alpha_m$</th>
<th>$\beta_m$</th>
<th>$\gamma_m$</th>
<th>$\omega_i$</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
<th>$\gamma_i$</th>
<th>$\alpha_C$</th>
<th>$\beta_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>$1.5 \times 10^{-5}$</td>
<td>0.0169</td>
<td>0.0823</td>
<td>0.0204</td>
<td>$1.5 \times 10^{-5}$</td>
<td>0.0147</td>
<td>0.0804</td>
<td>0.0189</td>
<td>0.0091</td>
<td>0.0102</td>
</tr>
<tr>
<td></td>
<td>$(4.6 \times 10^{-8})$</td>
<td>0.00005</td>
<td>0.00026</td>
<td>0.00006</td>
<td>$(4.6 \times 10^{-8})$</td>
<td>0.00005</td>
<td>0.00025</td>
<td>0.00006</td>
<td>0.00003</td>
<td>0.00003</td>
</tr>
<tr>
<td>Mean ASE</td>
<td>$4.8 \times 10^{-6}$</td>
<td>0.0085</td>
<td>0.0223</td>
<td>0.0134</td>
<td>$1.9 \times 10^{-5}$</td>
<td>0.0073</td>
<td>0.0209</td>
<td>0.0229</td>
<td>0.0104</td>
<td>0.0117</td>
</tr>
<tr>
<td></td>
<td>$(5.8 \times 10^{-7})$</td>
<td>0.00052</td>
<td>0.00275</td>
<td>0.00056</td>
<td>$(1.4 \times 10^{-5})$</td>
<td>0.00047</td>
<td>0.00185</td>
<td>0.01161</td>
<td>0.00006</td>
<td>0.00003</td>
</tr>
<tr>
<td>Median ASE</td>
<td>$8.5 \times 10^{-8}$</td>
<td>0.0041</td>
<td>0.0003</td>
<td>0.0084</td>
<td>$6.1 \times 10^{-8}$</td>
<td>0.0014</td>
<td>0.0001</td>
<td>0.0032</td>
<td>0.0101</td>
<td>0.0113</td>
</tr>
</tbody>
</table>

Table 5.7: Monte Carlo simulation results for the stochastic volatility and fast sine wave correlation misspecification.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_m$</th>
<th>$\alpha_m$</th>
<th>$\beta_m$</th>
<th>$\gamma_m$</th>
<th>$\omega_i$</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
<th>$\gamma_i$</th>
<th>$\alpha_C$</th>
<th>$\beta_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>$1.5 \times 10^{-5}$</td>
<td>0.0171</td>
<td>0.0844</td>
<td>0.0206</td>
<td>$1.5 \times 10^{-5}$</td>
<td>0.0140</td>
<td>0.0838</td>
<td>0.0186</td>
<td>0.0412</td>
<td>0.0986</td>
</tr>
<tr>
<td></td>
<td>$(4.8 \times 10^{-8})$</td>
<td>0.00005</td>
<td>0.00027</td>
<td>0.00007</td>
<td>$(4.9 \times 10^{-8})$</td>
<td>0.00004</td>
<td>0.00027</td>
<td>0.00006</td>
<td>0.00013</td>
<td>0.00031</td>
</tr>
<tr>
<td>Mean ASE</td>
<td>$4.2 \times 10^{-6}$</td>
<td>0.0077</td>
<td>0.0211</td>
<td>0.0126</td>
<td>$8.3 \times 10^{-6}$</td>
<td>0.0109</td>
<td>0.0433</td>
<td>0.0120</td>
<td>0.0338</td>
<td>0.0768</td>
</tr>
<tr>
<td></td>
<td>$(2.9 \times 10^{-7})$</td>
<td>0.00018</td>
<td>0.00155</td>
<td>0.00023</td>
<td>$(2.4 \times 10^{-6})$</td>
<td>0.00244</td>
<td>0.01299</td>
<td>0.00113</td>
<td>0.00044</td>
<td>0.00659</td>
</tr>
<tr>
<td>Median ASE</td>
<td>$8.6 \times 10^{-8}$</td>
<td>0.0041</td>
<td>0.0003</td>
<td>0.0083</td>
<td>$5.8 \times 10^{-8}$</td>
<td>0.0007</td>
<td>0.0001</td>
<td>0.0019</td>
<td>0.0335</td>
<td>0.0591</td>
</tr>
</tbody>
</table>
5.2.4 Simulation Results

With the Asymptotic Standard Error (ASE) approximations failing to capture the true amount of parameter estimation error, we need to directly compare our BBEE standard errors with the Monte Carlo estimates of the true parameter variability. The results of Section 5.2.2 show that for empirical data the estimated standard errors from our BBEE methodology often differ from the ASE approximations. Yet what is still unaddressed is the question of whether or not our BBEE methodology accurately captures the true amount of parameter estimation error. In order to address this question we use the identical correct specification and misspecification settings as in Section 5.2.3 to compare our BBEE standard error estimates with the Monte Carlo approximations to the true parameter variability. While it is computationally infeasible to perform our BBEE methodology on all $M = 50,000$ simulated data sets, our hope is that BBEE standard error estimates obtained from a single data set will provide adequate results. The reason for this is that our BBEE methodology is already using a large number of bootstrap samples to construct a robust standard error estimate. As such we generate one additional data set from each specification and apply our BBEE methodology to the newly generated data. Moreover, we will continue to use multiple choices of $\ell$ within our BBEE methodology to help give guidance around the choice of block length.

We start by analyzing BBEE standard error estimates for correctly specified data. The BBEE standard error estimates are obtained from $N_b = 10,000$ bootstrap samples and compared to the simulation variability obtained from the Monte Carlo simulations using identical parameter estimates. In Figure 5.4 we see that the BBEE standard error estimates are very close to the Monte Carlos simulation variability for all but 3 parameters, $\omega_m$, $\alpha_m$ and $\beta_C$. The differences between the BBEE estimates and the Monte Carlo simulation variability for $\omega_m$ is still less than 100% and is comparable to the difference seen for the ASE approximation in Table 5.1. Similarly the differences in the $\beta_C$ estimates is less than 65% for all block lengths. As previously discussed, our BBEE methodology still struggles to estimate the standard error of $\alpha_m$ because its initial parameter estimate is nearly zero. The failure of the BBEE methodology is not being able to provide a long enough tail of non-zero values that would lead to a larger standard deviation.

The other seven parameters in Figure 5.4 are all well estimated by the BBEE methodology at smaller
block lengths. This includes differences of less than 20% for \( \beta_m \) and \( \gamma_m \), which are the same parameters that the mean ASE approximation differed by more than 200%. In terms of the choice of block length, the BBEE standard error estimates are fairly consistent for \( \ell \leq 10 \). This consistency is to be expected in the case of a correctly specified model since there is no additional dependence that needs to be accounted for by resampling in blocks. The resampling of estimating equations is enough to capture the dependence in the original data since the model is correctly specified. The drifting away from the \( \ell = 1 \) estimate for many of the parameters in Figure 5.4 could be due to the additional recentering that is needed at higher block lengths, or could be the result of bias in higher ordered moments.

The BBEE standard error estimates for the GARCH(2,1)-DCC(1,1) misspecified data is given in Figure 5.5. Here we see that this type of misspecification does very little to change the BBEE methodology’s ability to estimate the true parameter standard error. Once again the two parameters with the largest differences are \( \alpha_m \) and \( \beta_C \). For all of the other parameters the BBEE standard error estimate is within 45% of the Monte Carlo simulation variability for small \( \ell \). The pattern across block lengths is also very similar to what was already seen in Figure 5.4 indicating that this model misspecification introduces very little additional dependence.

The BBEE standard error estimates for the GARCH(2,1) volatility and the sine wave correlations are given in Figure 5.6, and Figure 5.7. For the sine correlation in Figure 5.6 the BBEE methodology does well for most of the GARCH parameters, but underestimates the variability in the DCC parameters. Recall from Table 5.4 that the ASE approximations also underestimates the true variability of the DCC parameters for the sine correlation. So while there is a failure of the the BBEE methodology to capture the true variability for \( \alpha_C \) and \( \beta_C \) for a fixed sine wave correlation pattern, it is not worse than the ASE approximations for this particular misspecification. In Figure 5.6 the BBEE methodology is applied to a data set where the initial estimate for \( \alpha_m \) is .007. Since this initial estimate of \( \alpha_m \) is further away from zero, the BBEE methodology is able to estimate a larger amount of parameter estimation error than what has previously been seen when the initial estimate of \( \alpha_m \) is closer to zero.

For the fast sine misspecification in Figure 5.7 we see an overestimate in the firm GARCH parameter variability, especially for small values of \( \ell \). Notice that in both Figure 5.6 and Figure 5.7 we see more
Figure 5.4: BBEE estimates of parameter estimation error by block length for a correctly specified data set mirroring COF on 1/30/2004. The red line shows the Monte Carlo simulation variability.
Figure 5.5: BBEE estimates of parameter estimation error by block length for a misspecified data set using the GARCH(2,1)-DCC(1,1) model. The red line shows the Monte Carlo simulation variability.
variation in the estimates across the choice of $\ell$ and there are parameters in both graphs where the BBEE parameter estimation error estimates are improved for larger values of $\ell$. The fact that $\ell = 1$ does not always produce the best estimate helps illustrate how longer block lengths can improve our BBEE estimates when the model is misspecified. Also notice that in Figure 5.7 the parameter estimation error for $\beta_C$ is well estimated by the BBEE methodology. This implies that the more rapidly changing correlation pattern leads to a wider spread of $\beta_C$ which is better accounted for using our BBEE methodology when compared to the tighter spread of $\beta_C$ when a slower sine wave is used.

Finally, the BBEE standard error estimates for the stochastic volatility and the sine wave correlations are given in Figure 5.8, and Figure 5.9. Here we see that the use of a stochastic volatility specification leads to an underestimate of parameter estimation error in the GARCH parameters for the market. The firm GARCH parameters are also underestimated when the sine wave correlation is used, but are well estimated for the fast sine wave correlation. For both DCC parameters the BBEE methodology does well, especially at shorter block lengths. All of these results are consistent with the ASE approximation for this type of misspecification in Table 5.6 and Table 5.7. In fact the BBEE methodology gives comparatively closer estimates to the true parameter variability when compared with the mean ASE approximation for most of the parameters. This illustrates that even in a more radical misspecification the BBEE methodology is able to produce reasonable standard error estimates when compared to its asymptotic counterpart.

5.3 Summary

From these results we feel that the BBEE methodology provides an adequate approach for capturing DCC-GARCH parameter estimation error. Moreover the BBEE methodology non-parametrically provides various sets of parameter values $(\hat{\theta}_m^{(B)}, \hat{\theta}_l^{(B)}, \hat{\phi}^{(B)})$ that can then be used in our LRMES simulation procedure to help provide a better estimate of the full amount of variability associated with LRMES. Even if the asymptotic standard error approximations could accurately capture the true parameter estimation error, a parametric assumption would have been needed to propagate that error through LRMES simulation procedure. The fact that the asymptotic standard error approximations struggle to capture the true
Figure 5.6: BBEE estimates of parameter estimation error by block length for a misspecified data set using GARCH(2,1) volatility and sine wave correlation. The red line shows the Monte Carlo simulation variability.
Figure 5.7: BBEE estimates of parameter estimation error by block length for a misspecified data set using GARCH(2,1) volatility and fast sine wave correlation. The red line shows the Monte Carlo simulation variability.
Figure 5.8: BBEE estimates of parameter estimation error by block length for a misspecified data set using stochastic volatility and sine wave correlation. The red line shows the Monte Carlo simulation variability.
Figure 5.9: BBEE estimates of parameter estimation error by block length for a misspecified data set using stochastic volatility and fast sine wave correlation. The red line shows the Monte Carlo simulation variability.
parameter variability for this type of data only furthers our belief that the BBEE methodology is needed for this work.

While the overall choice of $\ell$ is still not easily apparent, there is good guidance to be gained from the simulation results. When the data is correctly specified there is consistency in the standard error estimates for block lengths of $\ell \leq 10$. However, some of the parameters drift away from the $\ell = 1$ estimate for $\ell > 10$. Thus our recommendation would be to use block lengths of 10 or less to avoid this drift that may be caused by additional recentering or bias in higher ordered moments. Further guidance is obtained from the misspecification results where the use of block bootstrapping helps control for the additional dependence introduced by model misspecification. In terms of visual inspection, the results of Figure 5.5 show that the GARCH(2,1)-DCC(1,1) model is not highly misspecified and the standard errors follow similar patterns to the correctly specified data. The results of the GARCH(2,1) misspecification with the sine wave correlation in Figure 5.6 show continual improvement in the GARCH parameters as $\ell$ increases up to 25. In Figure 5.7 the faster sine wave correlation creates an overestimation in the firm GARCH parameters that is continually reduced by increasing block lengths. Finally the stochastic volatility misspecifications in Figure 5.8 and Figure 5.9 show overestimation of the DCC parameter estimation errors for $\ell > 10$.

Since there is no consistent pattern across the misspecifications or even the parameters within a misspecification, we aggregate the absolute percentage error across all parameters for the five misspecifications in Table 5.8. While there are different minimum values for the different misspecifications, the average absolute percentage error is lowest for $\ell = 5$, 8, and 10. This is also where the GARCH(2,1)-DCC(1,1) misspecification, which is the least radical misspecification, is minimized. These block lengths also follow the previous guidance from the correctly specified data to choose a block length that is no larger than 10. As a result our suggestion is to use a block length of $\ell = 8$ to protect against misspecification while propagating the DCC-GARCH estimation error through the LRMES simulation procedure.

Now that our bootstrapping procedure has been defined, we can move forward in applying the BBEE methodology within our LRMES simulation procedure and analyze the full amount of error
associated with LRMES. To the best of our knowledge this will be the first attempt to quantify this type of variability in LRMES and SRISK. In their more recent work Brownlees and Engle (2015) discuss the use of prediction intervals for LRMES. However, these prediction intervals rely solely on the inverse distribution function of the cumulative firm returns. As such they do not incorporate the uncertainty of the DCC-GARCH parameter estimates in their measure of variability. Since our BBEE methodology is the first attempt to quantify the full amount of variability in LRMES, there is a need to explore how this variability changes across different firm and date combinations. Additionally we would like to address how the magnitude of the parameter estimation error in LRMES via the BBEE methodology compares with the model error previously seen in the use of leptokurtic working likelihoods. Hopefully the use of our BBEE methodology will increase our knowledge regarding the variability LRMES and the uncertainty in the SRISK point estimates.
Table 5.8: The aggregated absolute percentage error for different block lengths and misspecifications.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\ell = 1$</th>
<th>$\ell = 2$</th>
<th>$\ell = 4$</th>
<th>$\ell = 8$</th>
<th>$\ell = 10$</th>
<th>$\ell = 20$</th>
<th>$\ell = 25$</th>
<th>$\ell = 40$</th>
<th>$\ell = 50$</th>
<th>$\ell = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH(2,1)-DCC(1,1)</td>
<td>300%</td>
<td>290%</td>
<td>295%</td>
<td>290%</td>
<td><strong>275%</strong></td>
<td>281%</td>
<td>293%</td>
<td>304%</td>
<td>559%</td>
<td>497%</td>
</tr>
<tr>
<td>GARCH(2,1) - Sine</td>
<td>459%</td>
<td>460%</td>
<td>449%</td>
<td>432%</td>
<td>420%</td>
<td>406%</td>
<td><strong>352%</strong></td>
<td><strong>352%</strong></td>
<td>384%</td>
<td>397%</td>
</tr>
<tr>
<td>GARCH(2,1) - Fast Sine</td>
<td>634%</td>
<td>622%</td>
<td>603%</td>
<td>593%</td>
<td>575%</td>
<td>575%</td>
<td>478%</td>
<td>431%</td>
<td>413%</td>
<td>386%</td>
</tr>
<tr>
<td>SV - Sine</td>
<td>406%</td>
<td>409%</td>
<td>413%</td>
<td><strong>407%</strong></td>
<td>429%</td>
<td>439%</td>
<td>634%</td>
<td>894%</td>
<td>1049%</td>
<td>1006%</td>
</tr>
<tr>
<td>SV - Fast Sine</td>
<td>360%</td>
<td><strong>359%</strong></td>
<td>371%</td>
<td>373%</td>
<td>387%</td>
<td>386%</td>
<td>380%</td>
<td>378%</td>
<td>428%</td>
<td>461%</td>
</tr>
<tr>
<td>Average</td>
<td>432.04%</td>
<td>428.12%</td>
<td>426.18%</td>
<td>418.99%</td>
<td><strong>417.16%</strong></td>
<td>417.18%</td>
<td>427.09%</td>
<td>471.62%</td>
<td>566.48%</td>
<td>549.38%</td>
</tr>
</tbody>
</table>
CHAPTER 6

THE EFFECT OF PARAMETER ESTIMATION ERROR

The estimation of LRMES, and hence SRISK, allows Brownlees and Engle (2011) to quantify the systemic risk contribution of individual firms. By using ideas that are conceptually similar to stress testing, they are able to estimate the capital shortage of each individual firm conditional on the market falling into crisis. Moreover, the authors have shown that their metric provides a consistent interpretation to the actual events of the 2007-2009 financial crisis. As such, the estimation of LRMES helps provide an initial understanding to the complex concept of systemic risk. However, like any estimate, LRMES is not without error. In order to deepen the initial understanding of systemic risk, as measured by LRMES and SRISK, there needs to be additional discussion regarding the error associated with LRMES estimates. Our work in Chapter 4 discusses one particular type of error that comes from a distributional assumption within the estimation procedure. In this Chapter we use our newly defined Block Bootstrap for Estimating Equation (BBEE) methodology to discuss the LRMES error due to uncertainty in the estimated DCC-GARCH parameters, which are used throughout the LRMES simulation procedure.
For this analysis we condense our initial panel of 12 financial institutions down to 6 firms spread across the industry groupings presented in Table 4.1. Specifically we chose to use American International Group (AIG), Bank of America (BAC), Capital One Financial (COF), Goldman Sachs (GS), J.P. Morgan Chase (JPM), and Morgan Stanley (MS). This reduction still allows for diversity while keeping the computation time reasonable. In keeping with Section 4.1 the daily return data for these firms range from July 1, 1999 through December 31, 2013 and we define a crisis event to be a drop in the market by 40% over a $h = 130$ trading day time span. Additionally we continue to use a fixed history of $w = 1,000$ trading days to produce relatively long time series from which the DCC-GARCH parameters are estimated. In order to balance precise estimates with computation time, we use $B = 500$ bootstrap simulations each with $J = 10,000$ Monte Carlo simulations. Since our importance sampler strives for a 35% rate of crisis iterations, each bootstrap sample is using approximately 3,500 crisis iterations for its computation of LRMES. All told, the BBEE methodology produces a bootstrap sampling distribution of 500 LRMES estimates. The LRMES error is defined as the standard error of the bootstrap sampling distribution. The estimated LRMES error via the BBEE methodology captures both the error due to uncertainty in the DCC-GARCH parameters and the Monte Carlo error that comes from using a finite number of crisis iterations to estimate LRMES. When the choice of $J$ is relatively large, the Monte Carlo error should be a small percentage of the overall LRMES variability. For our analysis with $J = 10,000$, the average amount of Monte Carlo error accounts for 5% of the variance in LRMES. There are only a few firm and date combinations where the Monte Carlo error accounts for more than 10% of the total variability in LRMES, and the Monte Carlo error never accounts for more than 20% of the total variability in LRMES. Of the firm and date combinations where the Monte Carlo error accounts for more than 10% of the total variability in LRMES, 8 of the 12 occurred during 2007. Recall from Table 3.1 that during this time period typical values of $\lambda$ produce low percentages of crisis iterations in our importance sampler. This is a result of the volatility and correlation updates being based on a relatively stable historical time series. Thus, in order to get our desired 35% crisis iterations our importance sampler uses the large weight of $\lambda = .355$ on January 31, 2007. The larger sampling weight results in an alternative sampling distribution that is further away from the optimal sampling distribution and we do not achieve the same amount of
variance reduction present for other dates in our time frame.

Since the Monte Carlo error is typically a small percentage of the overall LRMES variability, the effect of the DCC-GARCH parameter estimation error should be well proxied by the full LRMES error. The use of a proxy is necessary in our analysis because an exact calculation of the effect of the DCC-GARCH parameter estimation error is impossible without the covariance between the parameter and Monte Carlo error. Additionally, our results are more relevant to the discussion of systemic risk as a whole if we analyze the full amount of LRMES error. Thus, the following analysis obtains the full LRMES error via the BBEE methodology for the previously named 6 firms on the last trading day of January and July from 2004 to 2013.

Recall that by definition LRMES is the expected percentage equity loss for a firm conditional on the market going into crisis. Thus estimates of LRMES range between 0 to 100 and can be interpreted on a percentage scale. Over the 120 firm and date combinations in our data, the average LRMES error is 5.0 ranging between 2.1 to 12.5. The average LRMES error highlights that small changes to the DCC-GARCH parameters can produce substantial changes LRMES. As an example, Figure 6.1 shows the LRMES bootstrap densities obtained by our BBEE methodology for both BAC and COF on January 31, 2004. For both firms there is substantial variability in LRMES, with a larger estimate of LRMES error for COF due to the longer left tail in its bootstrap sampling distribution. For both of these firms the width of the percentile bootstrap confidence interval is larger than 10 at an $\alpha = .05$ level. Figure 6.1 helps illustrate that the initial uncertainty in the DCC-GARCH parameter estimates produces a substantial amount of variability in LRMES.

From the range of the LRMES error values it seems unlikely that LRMES error stays constant across different firm and date combinations. In order to deepen our understanding of LRMES error, Section 6.1 explores how LRMES error changes across both firms and dates. Additionally, since we have already seen that leptokurtic working likelihood can change LRMES point estimates, Section 6.2 examines how LRMES error estimates change when LRMES is computed using a leptokurtic working likelihood. Finally, Section 6.3 provides a summary of the important results and discusses the importance of computing LRMES error when discussing systemic risk contributions via SRISK.
6.1 Differences in LRMES Error Across Firms and Dates

The BBEE methodology provides different sets of DCC-GARCH parameter values which are then used in the LRMES simulation procedure to provide a bootstrap sampling distribution for LRMES. Since the firm and date combination determines the historical time series from which the initial DCC-GARCH parameters are estimated, it is not surprising to see varying levels of LRMES error for different firm and date combinations. However, since there are 10 different parameters and the LRMES simulation procedure is not straightforward, it is unclear exactly how the LRMES error is affected. In this section we analyze the LRMES error estimates for our 6 different firms on last trading day of January and July from 2004 to 2013.

In an attempt to isolate the firm effect on LRMES error, Figure 6.2 shows the distribution of each firm’s LRMES error estimates across all dates. In general these distributions appear very similar, with the most visible difference being BAC. From Figure 6.2 it is evident that BAC has a lowest median LRMES error and has a narrower Inter Quartile Range (IQR) than the other firms. However, the narrower IQR for BAC is buoyed by 3 larger LRMES error estimates that are considered outliers by typical box-plot definitions. Thus, the variability in the LRMES error estimates is not all that dissimilar for BAC despite the compressed IQR. While the firms achieve various maximum LRMES error estimates in Figure 6.2, a closer look at the data shows that they all occur in July 2007 or January 2008. This highlights the
possibility of a much stronger time effect.

While there are only slight variations in the firm distributions of Figure 6.2, we also note that larger LRMES error values often correspond with larger LRMES point estimates. Moreover, since the LRMES point estimates also vary by firm and date it may be more appropriate to use a more standardized measure of dispersion. Figure 6.3 gives the distribution of the LRMES coefficient of variation (CV) by firm. For our data Figure 6.3 shows that the differences in the LRMES error distributions by firm are further reduced when the CV is used as our measure of LRMES variability. Overall there does not seem to be a large firm effect on our BBEE methodology’s estimate of LRMES error.

Aggregating over all 6 firms to investigate the time effect on LRMES error shows much more discernible differences. In Figure 6.4 and Figure 6.5 we plot the LRMES error and LRMES CV over time. In both graphs the black line represents the overall average for all 6 firms while the light blue lines give the profiles for each individual firm. It is evident from both Figure 6.4 and Figure 6.5 that there is a noticeable rise in LRMES error at the start of the financial crisis. Additionally, both plots show a significant drop in LRMES error at the start of 2009 as the crisis was coming to an end. This drop is more pronounced for the CV in Figure 6.5 since during this time frame the LRMES point estimates continue to stay at higher levels, indicating larger contributions to systemic risk. The blue profile lines help show
that while there are differences between the firms, the mean well characterizes the time trend of all 6 firms. All of the firms exhibit a spike in LRMES error during the financial crisis and have returned to a lower and more stable level of error since 2009. However, Figure 6.5 does show that at the end of our time window these firms are showing an increased LRMES CV due to a slight up tick in LRMES error and LRMES point estimates that are returning to pre-crisis levels.

So why are there fluctuations in LRMES error over time? A natural first step is to look at how the DCC-GARCH parameter estimation error, as estimated by our BBEE methodology, relates to these observed changes in the overall LRMES error. Our thought is that additional variability in the DCC-GARCH parameter estimates may lead to larger LRMES error estimates. However, since there are 10 different parameters being estimated, we first compare the relative variability of all 10 parameters. It is evident from Figure 6.6 that the magnitude of the typical parameter estimation error for $\beta_C$ dwarfs the magnitude of the typical parameter estimation error for all other DCC-GARCH parameters. Thus, any aggregate measure of parameter estimation error will be dominated by $\beta_C$.

So does the parameter estimation error of $\beta_C$ contain any signal in terms of predicting the overall LRMES error? Figure 6.7 illustrates that the error in $\beta_C$ alone has very little predictive power for the full LRMES error. Even when combined with firm and date information, the $\beta_C$ parameter estimation error is
Figure 6.4: Time series of LRMES error. The black line gives the average LRMES error over all firms. The light blue lines show the profiles for the individual firms.

Figure 6.5: Time series of the LRMES coefficient of variation. The black line gives the average LRMES CV over all firms. The light blue lines show the profiles for the individual firms.
non-informative with regards to LRMES error. However, ignoring the correlation parameters and solely focusing on the volatility parameter estimation error leads to a more substantial relationship with the overall LRMES error. In Figure 6.8 there is a statistically significant positive correlation between the aggregate volatility parameter estimation error and the overall LRMES error. Additionally, Figure 6.9 shows that there is a spike in 2007 and an increase at the end of our time frame that mirrors our LRMES error time profiles shown in Figure 6.4 and Figure 6.5. In a linear model, the aggregate GARCH parameter estimation error and firm and date indicators combine to explain 70% of the variability in the estimated LRMES error and 84% of the variability in the LRMES CV. This highlights the importance of the volatility parameters when estimating LRMES.

Finally, it is interesting to note the relationship between the LRMES error and the persistence of the firm volatility parameters. Figure 6.10 shows that the persistence in the estimated firm GJR-GARCH model drops significantly at the start of the financial crisis. Additionally, from 2009 to 2011 the model reaches a highly persistent state and at times has no reversion back to the mean. A comparison with the LRMES error in Figure 6.4 shows a discernible negative correlation between the persistence in the firm GJR-GARCH model and the amount of LRMES error. This is most likely due to the fact that the optimization is constrained by the GJR-GARCH model assumption to ensure a persistence of less than 1. Thus at times when the model is highly persistent the parameters show less variability across the bootstrap samples. However, when the persistence is lower, like in 2007, the parameters have the potential
Figure 6.7: Comparison of the full LRMES error with the BBEE parameter estimation error of $\beta_c$.

Figure 6.8: Comparison of the full LRMES error with the aggregate BBEE parameter estimation error for volatility parameters.
for more variability across bootstrap samples allowing for a more variable LRMES estimate.

6.2 Differences in LRMES Error Across Working Likelihoods

The results of Chapter 4 show that the use of a leptokurtic working likelihood can produce changes in LRMES estimates when compared to a Gaussian working likelihood. Thus it is reasonable to suspect that a change in the working likelihood could lead to different amounts of LRMES error as well. For this analysis we use a Student-\(t\) working likelihood within the BBEE methodology to compute various sets of DCC-GARCH parameter estimates. The resulting LRMES error estimates are then compared to the LRMES error estimates from Section 6.1 which are computed using a Gaussian working likelihood. All of the data and simulation settings presented in Section 6.1 remain the same for this analysis.

In general, the LRMES error from the Student-\(t\) working likelihood is much lower than the LRMES error from the Gaussian working likelihood. Averaged over all firm and date combinations, the mean LRMES error is smaller by 1.1 and the mean LRMES CV is smaller by .021 when using a Student-\(t\)
Figure 6.10: Time series of the estimated GJR-GARCH persistence. The black line gives the average estimated GJR-GARCH persistence over all firms. The light blue lines show the profiles for the individual firms.

working likelihood. These smaller summary statistics are a result of the Student-t LRMES error estimate being less than the Gaussian LRMES error estimate for 87% of the firm and date combinations in our data. Of the 16 combinations where the Student-t LRMES error was larger, 10 occurred between 2012 and 2013.

In terms of firm and date contributions to the overall error in LRMES, the Student-t working likelihood is consistent with the Gaussian working likelihood. Figure 6.11 once again shows only slight differences in the distributions of the LRMES CV by firm. Additionally time profiles of the LRMES error and LRMES CV for the Student-t working likelihood, given in Figure 6.12 and Figure 6.13 respectively, show remarkably similar patterns over time to what has already been seen in Figure 6.4 and Figure 6.5. The most noticeable difference is that the overall estimates and the magnitude of the spike in 2007 are much smaller when using the Student-t working likelihood.

Since the firm and date contributions are seemingly constant across the two different working likelihoods, we would expect the difference to be coming from the individual parameter estimation errors. Indeed, when using a Student-t working likelihood 70% of the individual parameter estimation errors
Figure 6.11: Box-plots of the LRMES coefficient of variation when using a Student-$t$ working likelihood, broken down by firm.

Figure 6.12: Time series of the LRMES error using a Student-$t$ working likelihood. The black line gives the average LRMES error over all firms. The light blue lines show the profiles for the individual firms.
Figure 6.13: Time series of the LRMES coefficient of variation using a Student-t working likelihood. The black line gives the average LRMES CV over all firms. The light blue lines show the profiles for the individual firms.

are smaller than their Gaussian counterpart and in 90% of firm and date combinations the aggregated GARCH parameter estimation error is smaller for the Student-t working likelihood. Thus, the reduced variability in the estimated DCC-GARCH parameters is manifested in a reduced amount of LRMES error. But is this smaller parameter estimation error for cases using a Student-t working likelihood simply an artifact of our BBEE methodology, or is this a general result as well? To explore this question we ran Monte Carlo simulations under the correct data specification to obtain Monte Carlo approximations to the true parameter estimation error. The data generating process used the initial fit to a specific firm and date combination to generate 50,000 bivariate time series of length $w = 1,000$. Each simulated time series is then fit using both a Gaussian and Student-t working likelihood. The standard deviation of the 50,000 parameter estimates is our approximation to the true parameter estimation error for a single firm and date combination. The aggregated GARCH parameter estimation error for 4 different firm and date combinations is shown in Table 6.1, and in 3 of the 4 combinations the Student-t working likelihood gives a smaller amount of summed volatility parameter estimation error. Even in the combination where the Student-t working likelihood exhibits more parameter estimation error, the difference between the two working likelihoods is relatively small. The results shown in Table 6.1 are not intended to serve as a
Table 6.1: Aggregated Monte Carlo GARCH parameter estimation error for both the Gaussian and Student-\(t\) working likelihoods for various firm and date combinations.

<table>
<thead>
<tr>
<th>Date</th>
<th>Firm</th>
<th>Gaussian</th>
<th>Student-(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/30/2004</td>
<td>COF</td>
<td>.135</td>
<td>.138</td>
</tr>
<tr>
<td></td>
<td>BAC</td>
<td>.166</td>
<td>.144</td>
</tr>
<tr>
<td>1/30/2009</td>
<td>COF</td>
<td>.154</td>
<td>.128</td>
</tr>
<tr>
<td></td>
<td>BAC</td>
<td>.141</td>
<td>.138</td>
</tr>
</tbody>
</table>

proof that a Student-\(t\) working likelihood typically produces a smaller amount of parameter estimation error than the Gaussian working likelihood, we leave that work for future study. However, the results of Table 6.1 do give evidence that this is a common phenomenon for our particular data setting and is not simply an artifact of the BBEE methodology.

### 6.3 Summary

By obtaining various sets of bootstrapped parameter values from our BBEE Methodology, we are able to obtain the bootstrap sampling distribution of LRMES and can use this distribution to better understand the error associated with LRMES. While some of this error is due to only being able to produce a finite number of crisis iterations in the LRMES simulation procedure, the majority of this error comes from the uncertainty in the DCC-GARCH parameter estimates. Our results show that this uncertainty contributes to LRMES errors of around 5 for a typical firm and date combination when using a Gaussian working likelihood. When using a Student-\(t\) working likelihood the average amount of LRMES error is reduced to 3.8. This reduction in LRMES error is due to smaller amounts of uncertainty in the DCC-GARCH parameters when the Student-\(t\) working likelihood is used. Moreover, the reduced amount of parameter uncertainty is not an artifact of our BBEE methodology, but is present in Monte Carlo simulations as well.

Since the DCC-GARCH parameter estimates define the volatility and correlation updates within the LRMES simulation procedure, it is unsurprising that the magnitude of their uncertainty is positively correlated with LRMES error. However, we did find that while the uncertainty in the correlation parameter
\( \beta_C \) is much higher than any of the GARCH parameters or \( \alpha_C \), it provides no predictive power in estimating the LRMES error. This is an indication that the volatility parameters have a larger role in determining the overall LRMES estimate compared to the correlation parameters. It would be interesting to explore this idea further by comparing LRMES error estimates from additional correlation models, but we leave this for future work.

It is also important to note that the uncertainty of the DCC-GARCH parameter estimates is not alone in contributing to LRMES error. For both the Gaussian and Student-\( t \) working likelihoods we observe small fluctuations in LRMES error for different firms, but observe much larger fluctuations over time. Recall that the date determines the historical time series on which the innovations distribution and the initial parameters, volatility, and correlation are based on. The results of our analysis show that these factors must also play a role in determining the magnitude of LRMES error. However, we leave the work of isolating each individual contribution for a future time.

Finally, since LRMES is a major component of SRISK, it is important to understand that there is a substantial amount of variability in the LRMES estimate due to uncertainty in the DCC-GARCH parameter estimates. Ignoring this variability and treating LRMES and SRISK as point estimates can create a misunderstanding of the relative systemic risk contribution of different firms. The inclusion of LRMES error will not change the overall ranking of the financial institutions, but does provide helpful information for making direct comparisons. This is especially vital if the SRISK estimates are to be used in regulatory framework for systemic risk. Our work regarding the full LRMES error provides a better understanding of the estimated systemic risk contributions for individual firms.
Part IV

Conclusions
In the wake of the financial crisis government and academic researchers have made the measurement and potential regulation of systemic risk a focal point. They aim to ensure the soundness of the entire financial system and eliminate the need to bail out individual financial institutions that impose large amounts of systemic risk on the market. Within the vast amount of research regarding systemic risk, the SRISK index (Brownlees and Engle, 2011) has drawn considerable attention. Its strict dependence on publicly available data, its similarities to stress testing, and its straightforward interpretation have made it a leading metric for measuring an individual firm’s systemic risk contribution. However, the computation of SRISK is neither simple nor straightforward. Instead, it relies on a bivariate dynamic process and a simulation procedure to estimate a firm’s Long Run Marginal Expected Shortfall (LRMES). A firm’s LRMES is then combined with balance sheet information to produce the SRISK index. The computation of LRMES has various simulation settings and statistical assumptions that can alter its estimate to various degrees. Additionally, the simulation procedure relies on a set of estimated parameters from a DCC-GARCH model. The inherent uncertainty in these parameters produce yet another source of variability in LRMES. Before addressing the role of SRISK in the evaluation of systemic risk, it is crucial to understand the
affect of LRMES variability.

In their original work, Brownlees and Engle (2011) justify and discuss the impact of many of their simulation settings. This includes a study regarding LRMES sensitivity to both the choice of a crisis threshold, $C$, and the prudential ratio, $k$. However, due to the large number of simulation settings and assumptions needed in computing LRMES, its sensitivity has not been fully explored. Ideally any metric attempting to capture systemic risk should be robust to the statistical assumptions made in its estimation. If the model error associated with SRISK is high, then firms will be able to dispute SRISK’s evaluation of the firm’s contribution to systemic risk. Additionally, when comparing firms, it is important to know not only their SRISK point estimates, but also the variability associated with these estimates.

In this dissertation, Chapter 4 studies the effect of using non-Gaussian working likelihoods in the estimation of LRMES. The results show that a change in the working likelihood for the GJR-GARCH model can produce differences in LRMES of 10 or higher for certain firm and date combinations. Many of these large changes in LRMES occur between 2004 and 2006 for leptokurtic working likelihoods. The typical firm and date combination produces smaller LRMES estimates when using a leptokurtic working likelihood, with a typical difference of -2 when compared to the Gaussian working likelihood. However, there are firm and date combinations where the leptokurtic working likelihood actually produces a larger LRMES estimate, and extreme cases where the magnitude of the difference is larger than 25. These results highlight that at times there is substantial variability in LRMES due to model selection.

In order to further comment on LRMES variability as a whole, it is important to understand how much variability comes from its estimation procedure. Error is present in the LRMES estimate due to the fact that its estimation is based on a finite number of simulated crisis scenarios, and due to the fact that the DCC-GARCH parameters are estimated quantities themselves. Traditionally, the propagation of the asymptotic standard error of the DCC-GARCH parameters through the LRMES simulation procedure requires explicit distributional assumptions. In order to avoid additional assumptions, Chapter 5 introduces a new bootstrapping procedure that allows for a non-parametric propagation of the DCC-GARCH parameter estimation error. The Block Bootstrap for Estimating Equation (BBEE) methodology is unique in its approach to estimating DCC-GARCH parameter estimation error. Simulation results show
that under typical LRMES settings our BBEE methodology often gives standard error estimates that better approximate the true parameter variability when compared to asymptotic standard error estimates.

Chapter 6 uses our newly introduced BBEE methodology to compute the full amount of LRMES error for different firm and date combinations. Here the simulation settings are chosen such that the Monte Carlo error is only a small part of the total LRMES variability. In fact, the typical Monte Carlo error is less than 5% of the overall variability in LRMES, and is never more than 20%. This ensures that the large majority of the LRMES error is due to the variability of the DCC-GARCH parameter estimates. Our results show that the amount of LRMES error varies for different firm and date combinations, but typically has a magnitude around 5 when using a Gaussian working likelihood. Moreover, in 87% of our firm and date combinations the LRMES error is smaller when using the Student-$t$ working likelihood. The average amount of LRMES error when using a Student-$t$ working likelihood is around 4. For both working likelihoods the LRMES error is fairly stable across firms, but has large fluctuations across dates.

Overall this dissertation demonstrates that LRMES estimates are affected by both model selection and parameter estimation error. While there are still other LRMES model assumptions and simulation settings to explore, this work gives an in depth analysis into two major areas of LRMES variability. In Section 7.1 we extend our results further by directly comparing the LRMES variability due to model selection with the LRMES variability due to parameter estimation error. Additionally, Section 7.2 extends the comparison to include the SRISK index. Recall that the SRISK index combines LRMES estimates with firm balance sheet information to produce an interpretable measure of systemic risk. Finally, in Section 7.3, we discuss potential future research ideas that look to extend our results and deepen our understanding of systemic risk.

### 7.1 Comparison of Model and Parameter Estimation Error

Previous chapters establish that both model selection and parameter uncertainty create variability in LRMES estimates. However, these chapters study individual sources of LRMES variability in isolation. That is, within a given chapter there is no discussion regarding any other source of LRMES variability. In this section we combine the previous results in ways that allow for direct comparison between LRMES
variability due to model selection and LRMES variability due to parameter uncertainty. This creates discussion regarding the relative magnitude and potential relationships between these variability types.

In general, the average shift of an LRMES estimate due to a change in the working likelihood is smaller than the average LRMES error as computed by our BBEE methodology. Recall that using a Student-t working likelihood instead of a Gaussian working likelihood creates an average decrease in LRMES of 2, while the average LRMES error when using a Gaussian working likelihood is 5. This implies that the LRMES differences due to a change in the working likelihood may not be statistically significant due to the inherent uncertainty of the DCC-GARCH parameter estimates. However, recall that both sources of LRMES variability are highly dependent on the firm and date combination. Thus, it is important to compare the results for individual firm and date combinations and ensure that these general results still hold.

We start by comparing LRMES differences due to a change in the working likelihood with the LRMES standard error (SE) as measured by our BBEE methodology. Table 7.1 and Table 7.2 give a side by side comparison of these two types of LRMES variability for COF and BAC respectively. Our initial focus is restricted to these two firms since they show the largest LRMES differences when using a Student-t working likelihood instead of a Gaussian working likelihood. It is clear from Table 7.1 and Table 7.2 that the large LRMES differences in 2005 and early 2006 coincide with above average LRMES SE. Thus, it is possible that the large LRMES differences are enhanced by larger LRMES error values for the same firm and date combination. However, LRMES SE is also large during the financial crisis, and during these times the LRMES differences are extremely small. Similarly the large LRMES difference for COF in July 2009 coincides with a LRMES SE estimate that is above average. So while it is important to note that largest LRMES differences occur during times of above average LRMES SE, they do not seem to be driving forces for one another. In fact, the correlation between the LRMES differences and the LRMES SE is not statistically significant for any combination of firm and working likelihood.

Table 7.1 and Table 7.2 also bring insight to the discussion regarding which working likelihood should be used in practice. In comparing the LRMES SE for both distributions, it is clear that the use of a Student-t working likelihood generally reduces the error in LRMES due to DCC-GARCH parameter
uncertainty. However, it is important to note that these two firms are not representative of the fact that 69% of all LRMES differences in our study are negative, indicating that the Student-t working likelihood produces smaller estimates of systemic risk. To highlight this fact, Table 7.3 shows the same values for JPM. In Table 7.3, the LRMES difference is always negative, implying that the use of a Student-t working likelihood causes JPM to be seen as contributing less to systemic risk. This highlights the fact that while the choice of a working likelihood is an important and interesting question, there is not a straightforward answer. Without additional crisis data, it is challenging to know which working likelihood provides a better representation the true LRMES distribution. So in the absence of validation, is it better to minimize the LRMES error and get a more consistent LRMES point estimate, or is it more important to provide a conservative assessment regarding a firm’s systemic risk level? It seems likely that the context regarding how LRMES estimates are being used will play a large role in determining the choice of working likelihood.

An additional way to analyze the differences in LRMES variability and highlight the lack of consistency in LRMES point estimates is to compare the bootstrap densities created by the BBEE methodology. As an example, Figure 7.1 gives the LRMES bootstrap distribution for COF on January 30, 2009. From the center of the distributions we can see that for this particular firm and date combination, a typical LRMES estimate from the Student-t working likelihood will be higher than a typical estimate from the Gaussian working likelihood. This holds true for our LRMES point estimates, which are 68.7 for the Gaussian working likelihood and 74.1 for the Student-t working likelihood. Additionally, comparing the width of the distributions illustrates that for this firm and date combination the LRMES error is much larger for the Gaussian working likelihood. However, the large range of possible LRMES estimates for both working likelihoods conveys the danger of taking a single point estimate when there is a significant amount of error. The bootstrap quantiles show that the Inter Quartile Range of the Gaussian distribution is 8.8 and that there is a 40% chance of seeing a Student-t LRMES estimate greater than 74.1.

Additionally, Figure 7.1 brings a different dimension to the discussion regarding which working likelihood should be used in practice. Here the distributions for LRMES estimates using Gaussian and Student-t working likelihoods overlap. This means that there is a chance for either working likelihood
Table 7.1: Comparison of variability in LRMES due to model selection with LRMES error for COF over time.

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Table 7.2: Comparison of variability in LRMES due to model selection with LRMES error for BAC over time.

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Table 7.3: Comparison of variability in LRMES due to model selection with LRMES error for JPM over time.

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</table>
to produce a larger, more conservative, point estimate of LRMES. While in Figure 7.1 there is a higher probability that the Student-\(t\) working likelihood will produce a larger LRMES point estimate, in Figure 7.2 we use a different firm to illustrate that at times it is possible for the Gaussian working likelihood to have a higher probability of a larger LRMES point estimate. Thus, if the goal is to find a conservative LRMES estimate, the bootstrap distributions of both working likelihoods need to be considered. One suggestion is to take the \(95^{\text{th}}\) quantile of both distributions and use the maximum of those two values as the single point estimate for LRMES. This is similar to taking the maximum 5\% Value At Risk (VAR) calculation of two loss distributions. By taking the \(95^{\text{th}}\) quantile of both distributions and using only the maximum value ensures a highly conservative estimate that incorporates the bootstrap distribution of both working likelihoods. For the firm and date combination shown in Figure 7.1 the maximum 5\% VAR LRMES estimate is 78.0, and for the firm and date combination shown in Figure 7.2 the maximum 5\% VAR LRMES estimate is 67.5.

So are any of the LRMES differences based on a change in the working likelihood statistically significant, given our BBEE calculation of LRMES error? Figure 7.3 plots the standard 95\% confidence intervals for COF over time using both the Gaussian and Student-\(t\) working likelihoods. Given that the large majority of the differences due to a change in working likelihood are smaller than the estimated LRMES error, it is not surprising to see the confidence intervals overlapping the majority of the time.
Figure 7.2: Bootstrap densities for GS on 1/30/2009 using the Gaussian and Student-$t$ working likelihoods.

Even some of the larger differences, like the difference of 10.7 in July 2009, show overlapping confidence intervals due to the comparatively large LRMES errors. However, for COF the most extreme differences between April 2005 and April 2006 do appear to be statistically significant. These differences ranged from -16.9 to -26.8, with an average difference of -21.1. Yet despite these large magnitude differences, there are still 2 months during this time frame where the confidence intervals overlapped. These months have Gaussian LRMES error values of 10.6 and 7.6.

Recall that BAC also shows extreme LRMES differences based on a change in working likelihood. Between August 2005 and April 2006 the differences for BAC ranged between 12.4 and 25.4 with an average difference of 16.5. However, Figure 7.4 shows that the confidence intervals overlap for this time frame and throughout the entire data set. From the relative width of the confidence intervals we can see that in the case of BAC, the Student-$t$ LRMES error is nearly as large as the Gaussian LRMES error. This helps to create overlapping confidence intervals even in the case of extreme LRMES differences. In contrast, the Student-$t$ LRMES error for COF over this same time period was typically half the magnitude of the Gaussian LRMES error. For discussion regarding overlapping confidence intervals, it should be noted that while non-overlapping confidence intervals imply a statistically significant difference between the LRMES point estimates, the converse is not necessarily true. So while we can’t definitively say whether or not the large LRMES differences for BAC are statistically significant based on the observed...
7.2 SRISK Variability

How does variability in LRMES affect the SRISK index? Throughout this dissertation our discussion has been fully focused on LRMES estimates. This is due to the fact that LRMES is the only estimated quantity in the calculation of SRISK, and hence, is the only quantity affected by statistical assumptions and parameter estimation error. Thus a discussion regarding the variability in LRMES without extending the results to the SRISK index still allows for ample discussion and understanding. However, in order to give additional perspective around the observed LRMES variability and provide more context with regards to the regulatory decisions, this section extends our variability discussion to the SRISK level.

Recall that SRISK combines the LRMES estimate with firm level debt and equity values to obtain a monetary capital shortfall (CS) for a given firm and date combination. Specifically

\[ CS_{it} = kD_{it} - (1 - k)W_{it}(1 - \text{LRMES}_{it}) \]

\[ \text{SRISK}_{it} = \max (0, CS_{it}) \] (7.1)
where $D_{it}$ is the book value of debt, $W_{it}$ is the market value of equity, $k$ is a prudential capital ratio, and $\text{LRMES}_{it}$ is the percent equity loss conditional on the market falling into crisis. For our calculations we obtain quarterly debt and equity values from the COMPUSTAT database. The book value of debt is taken to be the firm’s total assets (ATQ) minus the firm’s total liabilities (LTQ), while the market value of equity is taken to be the product of a firm’s common shares outstanding (CHSOQ) and the quarterly share price (PRCCQ). In keeping with Brownlees and Engle (2011) and the default settings of NYU Stern Volatility Institute (2014), we set the prudential capital ratio to be $k = 8\%$.

First, we compare the changes in SRISK for BAC and COF on August 31, 2005. Recall from Figure 7.3 and Figure 7.4 that on this date there are large differences in the point estimate of LRMES based on the choice of working likelihood. For COF the standard 95% confidence intervals did not overlap, indicating that the difference of -26.8 is statistically significant. For BAC the difference of +25.4 still has an overlapping confidence interval, but is the largest observed difference for the firm. Table 7.4 and Table 7.5 show that these large differences in LRMES do not impact the overall SRISK index because during this time neither of these firms have a positive CS. The large changes in LRMES do have a significant impact on the estimated CS for each of the firms, but their large equity values keep them from being considered systemically risky on August 31, 2005. The financial position of COF
Table 7.4: SRISK inputs, in millions of dollars, for various COF LRMES point estimates on 8/31/05. The difference from the Gaussian point estimate is shown in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>LRMES</th>
<th>CS</th>
<th>SRISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian Point Estimate</td>
<td>.478</td>
<td>-$6,231</td>
<td>$0</td>
</tr>
<tr>
<td>Student-t Point Estimate</td>
<td>.210</td>
<td>-$11,458</td>
<td>$0</td>
</tr>
<tr>
<td>(-.268)</td>
<td>(-$5,227)</td>
<td>($0)</td>
<td></td>
</tr>
<tr>
<td>Max 5% VAR</td>
<td>.492</td>
<td>-$5,957</td>
<td>$0</td>
</tr>
<tr>
<td>(+.014)</td>
<td>(+$273)</td>
<td>($0)</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.5: SRISK inputs, in millions of dollars, for various BAC LRMES point estimates on 8/31/05. The difference from the Gaussian point estimate is shown in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>LRMES</th>
<th>CS</th>
<th>SRISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian Point Estimate</td>
<td>.146</td>
<td>-$40,661</td>
<td>$0</td>
</tr>
<tr>
<td>Student-t Point Estimate</td>
<td>.400</td>
<td>-$1,180</td>
<td>$0</td>
</tr>
<tr>
<td>(+.254)</td>
<td>(+$39,480)</td>
<td>($0)</td>
<td></td>
</tr>
<tr>
<td>Max 5% VAR</td>
<td>.503</td>
<td>$14,829</td>
<td>$14,829</td>
</tr>
<tr>
<td>(+.357)</td>
<td>(+$55,490)</td>
<td>(+$14,829)</td>
<td></td>
</tr>
</tbody>
</table>

is such that even if we use the maximum 5% VAR calculation, discussed in Section 7.1, to ensure an extremely conservative systemic risk estimate, they still are seen as systemically riskless by the SRISK index. However, when the same calculation is applied to BAC, we see that the LRMES estimate increases to a point where the capital shortfall is positive and there is a non-zero SRISK estimate. In fact, the maximum 5% VAR calculation estimates the systemic risk of BAC at $14.8B which, according to the rankings of the NYU Stern Volatility Institute (2014), would have made BAC the 9th riskiest firm on August 31, 2005. Clearly, SRISK estimates for the other firms in this ranking would also change if their SRISK indices are estimated using a maximum 5% VAR calculation, but this still highlights how large variability in LRMES can substantially impact SRISK estimates and rankings.

To further demonstrate the impact of LRMES variability, consider the SRISK estimates right at the height of the financial crisis in January 2008. Recall from Chapter 6 that the highest LRMES error estimates occur during 2007 and 2008. In fact, the average LRMES error for a Gaussian working likelihood during this time is 6.6, and peaked for COF at 9.7. So how do SRISK estimates change during
Table 7.6: SRISK estimates, in millions of dollars, by firm for various LRMES point estimates on 1/31/08. The difference from the Gaussian SRISK is shown in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>AIG</th>
<th>BAC</th>
<th>COF</th>
<th>GS</th>
<th>JPM</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian SRISK</td>
<td>$36,814</td>
<td>$46,125</td>
<td>$288</td>
<td>$56,996</td>
<td>$65,208</td>
<td>$65,807</td>
</tr>
<tr>
<td>One SE Increase</td>
<td>$44,163</td>
<td>$57,772</td>
<td>$1,934</td>
<td>$61,333</td>
<td>$73,942</td>
<td>$68,805</td>
</tr>
<tr>
<td></td>
<td>(+$7,349)</td>
<td>(+$11,648)</td>
<td>(+$1,646)</td>
<td>(+$4,338)</td>
<td>(+$8,735)</td>
<td>(+$2,998)</td>
</tr>
<tr>
<td>Max 5% VAR</td>
<td>$47,639</td>
<td>$60,568</td>
<td>$3,393</td>
<td>$63,402</td>
<td>$76,496</td>
<td>$71,075</td>
</tr>
<tr>
<td></td>
<td>(+$10,825)</td>
<td>(+$14,443)</td>
<td>(+$3,105)</td>
<td>(+$6,406)</td>
<td>(+$11,288)</td>
<td>(+$5,268)</td>
</tr>
</tbody>
</table>

this time of high LRMES variability? Table 7.6 gives the standard SRISK value based on our Gaussian LRMES point estimate, along with new SRISK values based on a one Gaussian standard error increase in LRMES, and the maximum 5% VAR type calculation previously discussed. From Table 7.6 we see that even a one standard error increase in the LRMES estimate can produce SRISK estimates that differ by billions of dollars. Additionally, if we were to use the more conservative maximum 5% VAR calculation for estimating LRMES, half of our firms would have seen their SRISK estimate increase by more than $10B. Also notice that even within our 6 firm panel, a change in the LRMES point estimate has the ability to change SRISK rankings. When using the standard Gaussian LRMES point estimate, MS is the most risky firm in our panel followed closely by JPM. However if all of the LRMES estimates are increased by one standard error, JPM would have been risker than MS by a $5B margin. This flip-flop in ranking also holds true for the maximum 5% VAR calculation of LRMES.

In summary, our examples show that the variability of LRMES has the potential for billion dollar changes on the overall SRISK estimate. Thus systemic risk regulation based on the magnitude of the SRISK estimate needs to be keenly aware of how LRMES varies for the particular firm and date combination of interest. Even systemic risk regulation based on the ranking of a panel of firms needs to acknowledge the fact that the rankings can shift as a result of LRMES variability. This is one reason why we highlight the use of a conservative maximum 5% VAR type approach for estimating LRMES. By first observing the entire LRMES distribution for multiple working likelihoods and then computing SRISK based on a conservative LRMES estimate can help ensure that we do not underestimate the true systemic risk level for individual firms or the economy as a whole.
7.3 Future Work

This dissertation provides a first step in discussing and quantifying the variability of LRMES, and hence SRISK. However, as we have already discussed, there are still assumptions being made in the LRMES simulation procedure that have yet to be studied. For example, while Chapter 4 discusses changes to the working likelihood for the univariate GJR-GARCH model, it does not discuss how changes to the assumption of a multivariate normal distribution in the DCC-GARCH likelihood could affect LRMES. Additionally, there is also the choice of the DCC-GARCH methodology as a whole. Since the LRMES bivariate process and simulation procedure only relies on updating volatility and correlation values, the univariate GARCH specification could be changed, stochastic volatility models could be used to obtain univariate volatility, or other multivariate models could be used to estimate a time-varying correlation.

This type of SRISK sensitivity was briefly discussed in Brownlees and Engle (2015), but their analysis focuses solely on the SRISK rankings for a panel of firms, and only analyzes two alternative models. Other assumptions that have only been briefly discussed by Brownlees and Engle include the sensitivity of SRISK to a change in the crisis threshold, $C$, and the prudential capital ratio, $k$. It seems plausible that simultaneous changes to both the threshold, $C$, and the time step, $h$, could help speak to the robustness of SRISK as a measure of systemic risk. Does the risk level for firms change when faced with a quick steep market drop as opposed to a longer more sustained market drop?

Throughout the dissertation we explore the question of which working likelihood should be used in the computation of SRISK. Discussions around this question show that while the Student-$t$ working likelihood typically produces smaller LRMES error, as measured by the BBEE methodology, it also typically gives smaller LRMES estimates, which correspond to less systemic risk. Thus, while this is an important and interesting question, it seemingly does not have a simple answer. In addition to the paradox highlighted in our work, one of problems in determining the best working likelihood is the difficulty of validating LRMES. Since LRMES is conditional on the market dropping 40% in 6 months, the only recent historical data where this is observed is the financial crisis of 2007-2009. Additionally, since the computation of LRMES requires a 6 month window, there are very few non-overlapping windows in which this validation can be done. Thus, we would not only be validating on a very small amount of data,
but we would also only be validating over a single type of financial crisis. Since conclusions about which working likelihood performs better could only be based on the performance of LRMES estimates during the recent financial crisis, the results may not be consistent for future financial crises. Alternatively if we choose to simulate different types of crisis scenarios, there are many decisions regarding how the each crisis should be simulated. Thus we are creating a brand new set of assumptions in an attempt to justify which working likelihood should be used in computing LRMES. Another possible alternative is to look at which distribution gives the better fit to the marginal GJR-GARCH residuals. Using goodness of fit criteria could provide us with a justification regarding which working likelihood should be used in estimating LRMES. However, once again this could be sensitive to the choice of the historical window length, $w$, which is yet another parameter choice not discussed by Brownlees and Engle.

In working on the dissertation we also observed an interesting results with regards to the GJR-GARCH model. The stationarity condition of the GJR-GARCH model imposed during the evaluation of the likelihood function is $\alpha + \beta + \kappa \gamma < 1$. For our results we always imposed $\kappa = 0.5$ due to the symmetric nature of the Gaussian, Student-$t$, and GED distributions. This is also the standard approach used by software packages that estimate a GJR-GARCH model. However, for our data the residuals produced by the GJR-GARCH model are not symmetric, implying that the standard assumption for $\kappa$ may not be valid. In our results we often saw more negative values which suggests that $\kappa > 0.5$ may be needed to impose stationarity in practice. Thus, we are interested in testing for a more practical stationarity condition for the GJR-GARCH model, and determining how a change in the stationarity condition would affect the parameter estimates. If this does change the GJR-GARCH parameter estimates, it may also impact LRMES variability, and could be another extension of our dissertation.

Another natural extension to our dissertation is a more robust validation of our Block Bootstrap for Estimating Equation (BBEE) methodology. The original intent of our BBEE methodology is simply to provide an accurate method for non-parametrically propagating the DCC-GARCH parameter estimation error through the LRMES simulation procedure. However, our results show that the asymptotic standard error estimates are often not accurate under typical LRMES settings. Exploring other settings where the asymptotic standard error estimates for the DCC-GARCH model fail to provide an accurate view of the
parameter estimation error, and seeing how our BBEE methodology compares in those settings could further the usefulness of our methodology. Additionally, we would like to establish theoretical properties for our BBEE methodology to further validate it as a way to capture parameter estimation error in the case of 2-stage likelihood estimation.

Finally, our research began with the desire to look at the effect of data frequency on systemic risk metrics. In any time series, data can be collected at various time intervals. While daily data has become typical for GARCH type volatility models, we wonder if the signals for systemic risk are best seen at this frequency. Would the use of longer time intervals in SRISK and other systemic risk metrics detect something that is not captured at a daily frequency? Would a change in frequency produce the same view regarding the level of systemic risk for the market as a whole and for individual firms? Systemic risk is complex, and more research is needed to produce a deeper understanding of the risk that plays a vital role in the health of our financial system.
REFERENCES


APPENDIX A

ALTERNATIVE CRISIS DEFINITION

By definition, Long Run Marginal Expected Shortfall (LRMES) is conditional on a market decline greater than a threshold, $C$, over a time horizon $h$. As such, the choice of these parameters impact the LRMES estimate for a specific firm and date combination. In their original work Brownlees and Engle (2011) define a crisis as a 40% drop over 6-months, stating that this definition is chosen “to approximately match the magnitude of the market correction in the crisis.” Throughout this dissertation we also use a 40% drop over 6-months as our conditional event in an attempt to mirror the simulation settings in Brownlees and Engle (2011). However, in their most recent work Brownlees and Engle (2015) switch to using a 10% drop over 1 month as their crisis definition. In the paper they make no reference to the previous setting, only noting that “we set the horizon to a month in order to compare more naturally our methodology with other monthly frequency indicators of distress.” Due to the recency of this change, we do not address this alternative crisis definition in our main body of work. Instead we use this Appendix to review the main points of our dissertation using the newly proposed crisis definition.

In theory the choice of $C$ and $h$ can be any threshold and time horizon given that the time horizon is long enough to reflect a firm’s conditional shortfall. This flexibility can be seen as a strength of LRMES
and SRISK, as it allows for systemic risk calculations under different crisis scenarios. However, this could also be seen as a weakness if the index is highly sensitive to a change in crisis definition. Brownlees and Engle address the sensitivity of the SRISK index to a change in crisis definition in both of their works. In Brownlees and Engle (2011), the SRISK rankings are shown to be fairly stable at the top positions, but more sensitive in the lower rankings for a change of $C = -40\%$ to $C = -30\%$ over a six month time horizon. Similarly, Brownlees and Engle (2015) show that there is a high rank correlation between rankings using $C = -10\%$ and $C = -20\%$ over a one month time horizon. However, both works restrict their sensitivity analysis to a single time horizon.

From equation (3.3) it is clear that an increase in $-C$ will lead to larger capital shortfall estimates over an equivalent time horizon. In other words, when the crisis definition uses a more extreme threshold, the expected losses for a given firm should be higher. However, even when the time horizon changes, it also seems likely that a more extreme threshold will lead to larger capital shortfalls. This holds true for our firm and date combinations. The median LRMES using $C = -40\%$ and $h = 130$ is 44.2, while the median LRMES using $C = -10\%$ and $h = 22$ is 15.3. Moreover, 99.9% of our firm and date combinations are estimated to have larger LRMES estimates using a 40% drop over 6 month crisis definition when compared to using a 10% drop over 1 month crisis definition.

While the magnitude of LRMES is reduced by using the alternative crisis definition, many of our results regarding LRMES variability still hold. First, it is clear from Figure A.1 that while the magnitude of the LRMES differences between the Student-$t$ and Gaussian working likelihood is reduced by changing crisis definitions, the pattern by firm and industry grouping remains the same. Ignoring the Y-axis, the clustering of LRMES differences in the top graph of Figure A.1 is nearly identical to the clustering of LRMES differences in the bottom graph of Figure A.1. In both plots COF and BAC show the largest differences with the other firms showing a few differences outside of the cloud centered at 0. The LRMES differences in the bottom graph are smaller in magnitude because, as we have previously discussed, the LRMES estimates are also smaller. Putting the differences on a percentage scale demonstrates that the median differences are similar in magnitude. Under the initial crisis definition the median difference is -2.8% while under the alternative crisis definition the median difference is -2.0%. Comparing Figure A.2
Figure A.1: LRMES differences between the Student-$t$ and Gaussian working likelihoods. The top graph uses the standard crisis definition of a 40% drop in 6 months. The bottom graph uses the alternative crisis definition of a 10% market drop in 1 month.

and Figure 4.6 confirms that the LRMES differences between a GED and Gaussian working likelihood are also nearly identical under the alternative crisis definition. Once again the magnitude of the LRMES differences is reduced, but the clustering and relative magnitudes are nearly identical.

Using an alternative crisis definition also does not change the time series profile for a given firm’s LRMES estimates. Once again the LRMES estimates in Figure A.3 are smaller than those in Figure 4.7, but the profile of LRMES estimates for COF remains the same. Both graphs show that the largest LRMES differences occur between mid-2004 and mid-2006 and there is a rapid increase in LRMES in early 2009.
Figure A.2: LRMES differences between the GED and Gaussian working likelihoods using the alternative crisis definition of a 10% market drop in 1 month.

The comparison of Figure A.4 with Figure 4.8 shows that the similarities in time series profiles are not restricted to a single firm. These results highlight that it is the LRMES inputs, not the simulation settings, that create LRMES variability due to model selection. It is the alteration of the GJR-GARCH parameters and the innovation distribution that produces variability in LRMES estimates, regardless of the crisis definition.

By using the coefficient of variation (CV) as our measure of LRMES error we can control for the smaller LRMES estimates when using the alternative crisis definition. A comparison of the LRMES CV by firm in Figure A.5 and Figure 6.3 shows that the distributions are mostly similar. The IQR seems fairly stable across crisis definitions and the median LRMES CV is around 0.1 for all firms in both graphs. The largest discrepancy is in the maximum value of the LRMES CV. In Figure 6.3 all firms achieve a maximum CV greater than 0.2 while in Figure A.5 only COF has a maximum LRMES CV greater than 0.2. A comparison of the Figure A.6 with Figure 6.5 shows that the reduced maximum values is a result of a smaller spike at the start of the financial crisis. The spike under the original crisis definition is much larger in magnitude and is a focal point of Figure 6.5. On the other hand while the LRMES CV is higher in Figure A.6 it is not drastically so. However, both graphs show the same decrease in CV in 2009, an increase at the end of our time period, and generally higher levels of CV before the financial crisis. This
Figure A.3: Time series of COF LRMES estimates for various working likelihoods and the alternative crisis definition of a 10% market drop in 1 month.

Figure A.4: Time series of BAC LRMES estimates for various working likelihoods and the alternative crisis definition of a 10% market drop in 1 month.
Figure A.5: Box-plots of estimated LRMES coefficient of variation for the alternative crisis definition of a 10% market drop in 1 month, broken down by firm.

demonstrates that the relative amount of LRMES error is still similar under both crisis definitions.

In addition to showing that there is a substantial amount of error from propagating the GJR-GARCH parameter error through the LRMES simulation procedure, Chapter 6 also highlights that the use of a Student-$t$ working likelihood produces less LRMES error than a Gaussian working likelihood. Under the alternative crisis definition, this result also holds true. The LRMES CV profile in Figure A.7 is almost always below the profile in Figure A.6. In fact, under the alternative crisis definition 92% of the firm and date combinations have a smaller amount of LRMES error when using the Student-$t$ working likelihood instead of the Gaussian working likelihood. Figure A.7 again demonstrates that the main results of our dissertation are not an artifact of a specific crisis definition, but for both crisis definitions suggested by Brownlees and Engle.
Figure A.6: Time series of the LRMES coefficient of variation for the alternative crisis definition of a 10% market drop in 1 month. The black line gives the average LRMES CV over all firms. The light blue lines show the profiles for the individual firms.

Figure A.7: Time series of the LRMES coefficient of variation using a Student-$t$ working likelihood for the alternative crisis definition of a 10% market drop in 1 month. The black line gives the average LRMES CV over all firms. The light blue lines show the profiles for the individual firms.
APPENDIX B

IMPORTANCE SAMPLER PARAMETER DETERMINATION

Chapter 3 discusses the choice of $\lambda$ and $\mu$ in our importance sampling methods. This appendix provides additional tables that helped lead us to our final decision of searching for a $\lambda$ value that produces crisis scenarios 35% of the time. First, Table B.1 shows the $\lambda$ values that minimized the average LRMES standard error. Thus these are the lambda values produced by the goal crisis percentages reported in Table 3.3.

Table B.1: Corresponding $\lambda$ values where the average LRMES standard error is minimized for various firm and date combinations.

<table>
<thead>
<tr>
<th>Date</th>
<th>JPM</th>
<th>BAC</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/31/03</td>
<td>.15</td>
<td>.15</td>
<td>.15</td>
</tr>
<tr>
<td>1/31/05</td>
<td>.245</td>
<td>.23</td>
<td>.245</td>
</tr>
<tr>
<td>1/31/07</td>
<td>.32</td>
<td>.32</td>
<td>.345</td>
</tr>
</tbody>
</table>
Additionally, Table B.2 provides the average standard errors over 50 Monte Carlo runs that were used to determine the ideal crisis percentage in Table 3.3. The minimum average LRMES standard error is boldface for easy visualization.

Table B.2: Average LRMES standard error averaged for various importance sampler crisis percentages, firms, and dates.

<table>
<thead>
<tr>
<th>Date</th>
<th>Ideal Crisis Probability</th>
<th>JPM</th>
<th>BAC</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/31/03</td>
<td>15%</td>
<td>0.301</td>
<td>0.368</td>
<td>0.351</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>0.283</td>
<td>0.343</td>
<td>0.327</td>
</tr>
<tr>
<td></td>
<td>25%</td>
<td><strong>0.279</strong></td>
<td>0.331</td>
<td>0.316</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>0.280</td>
<td>0.324</td>
<td>0.312</td>
</tr>
<tr>
<td></td>
<td>35%</td>
<td><strong>0.279</strong></td>
<td><strong>0.323</strong></td>
<td><strong>0.311</strong></td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>0.286</td>
<td>0.329</td>
<td>0.317</td>
</tr>
<tr>
<td>1/31/05</td>
<td>15%</td>
<td>0.483</td>
<td>0.421</td>
<td>0.445</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>0.474</td>
<td>0.430</td>
<td>0.425</td>
</tr>
<tr>
<td></td>
<td>25%</td>
<td>0.459</td>
<td>0.412</td>
<td>0.418</td>
</tr>
<tr>
<td></td>
<td>30%</td>
<td>0.453</td>
<td><strong>0.398</strong></td>
<td>0.419</td>
</tr>
<tr>
<td></td>
<td>35%</td>
<td><strong>0.449</strong></td>
<td>0.400</td>
<td><strong>0.407</strong></td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>0.460</td>
<td>0.437</td>
<td>0.434</td>
</tr>
<tr>
<td>1/31/07</td>
<td>15%</td>
<td>0.818</td>
<td>0.418</td>
<td>0.852</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>0.804</td>
<td>0.443</td>
<td>0.975</td>
</tr>
<tr>
<td></td>
<td>25%</td>
<td><strong>0.712</strong></td>
<td><strong>0.335</strong></td>
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Chapter 4 discusses how LRMES estimates change with regards to different working likelihoods. In the body of the dissertation we plot the LRMES estimates for three different working likelihoods over time for 2 specific firms. This appendix provides identical plots for the remaining 10 firms in our panel. These plots once again show that the largest differences in LRMES based on the choice of working likelihood tend to occur between 2005 and 2006 or during the height of the financial crisis.
Figure C.1: Time series of AIG LRMES estimates for various working likelihoods.

Figure C.2: Time series of ALL LRMES estimates for various working likelihoods.
Figure C.3: Time series of AMTD LRMES estimates for various working likelihoods.

Figure C.4: Time series of C LRMES estimates for various working likelihoods.
Figure C.5: Time series of FITB LRMES estimates for various working likelihoods.

Figure C.6: Time series of GS LRMES estimates for various working likelihoods.
Figure C.7: Time series of HUM LRMES estimates for various working likelihoods.

Figure C.8: Time series of JPM LRMES estimates for various working likelihoods.
Figure C.9: Time series of MS LRMES estimates for various working likelihoods.

Figure C.10: Time series of SCHW LRMES estimates for various working likelihoods.