ABSTRACT


The ability to succeed in Science, Technology, Engineering, and Mathematics (STEM) careers is contingent on a student’s ability to engage in mathematical problem solving. As a result, there has been increased focus on students’ ability to think critically by providing them more with problem solving experiences in the classroom. Much research has been conducted on mathematical problem solving, beginning with Polya’s (1945) seminal work. Subsequent work has extensively studied factors that influence students’ success in problem solving. However, there is a need to accumulate this research so that it can be successfully connected to practice. One way to accomplish this is to develop an overarching problem solving theory. Schoenfeld (2011) developed such a theory and attributes an individual’s goals (personal aims to achieve), resources (knowledge available), and orientations (beliefs, values) as influential factors in the decisions made during any goal-oriented activity. This theory was tested and a model created for mathematics teaching but not formally for students’ mathematical problem solving. This study fills this gap by testing and validating Schoenfeld’s theory for problem solving as a way to document and assess students’ problem solving process.

The six students that participated in this study were incoming and returning freshmen at an HBCU participating in a summer bridge program. This multi-case study investigated the problem solving process of these students. A conceptual framework was created based on Schoenfeld’s (2008) problem solving theory, along with Carlson and Bloom’s (2005) framework for problem solving process and Debellis and Goldin (2006) framework for
affect. Within this conceptual framework, the decisions students made during problem solving were attributed to their goals, knowledge, affect, and external contextual factors. To test this theory, empirical evidence was collected to: (1) provide detailed, rich descriptions about how and why students make decisions during problem solving and (2) validate the scope of the proposed problem-solving theory. Data collection included the use of individual task-based interviews followed by video-stimulated response interviews. The transcripts were transcribed and analyzed and coded for these factors. Cross-case finding for each factor was reported.

Empirical evidence validated the proposed problem solving theory. That is, students’ goals, knowledge, and affect were factors that influenced the choices students made. Although external contextual factors did impact students’ motivation, it was not prevalent for all students. Furthermore, the data indicate that affect is the driving force that moves the student through the problem-solving phases. If students do not believe that it takes exploration and patience to solve problems, if negative emotions are felt, their ability to access knowledge needed to connect mathematical ideas were hindered. However, if they hold favorable beliefs about mathematics and problem solving, then a negative emotion will be used as motivation to continue the problem-solving process. These findings confirm that affect is the factor that may be the most influential predictor for students’ decision to persevere. Therefore, the problem solving theory includes all factors (i.e., goals, knowledge, and affect) that influence the decisions students make during the problem solving process. These findings confirm how the factors are intertwined and connected to influence student’s ability to persevere.
Investigating a Proposed Problem Solving Theory in the context of Mathematical Problem Solving: A multi-case study

by
Nadia Monrose Mills

A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Mathematics Education

Raleigh, North Carolina

2015

APPROVED BY:

_______________________________
Allison W. McCulloch
Committee Chair

_______________________________
Karen Allen Keene

_______________________________
Lee V. Stiff

_______________________________
Alina Duca
DEDICATION

For my family, especially my husband, Jahmed Mills, my children Orion, Malachi, and Jacob, my aunt, Olive Roger, my parents, siblings, and all those who have contributed and sacrificed in the accomplishment of my dream.
BIOGRAPHY

Nadia Monrose Mills was born on May 19, 1979, in St. Croix, Virgin Islands. She is the daughter of Gilbert and Elizabeth Monrose, immigrants from St. Lucia. In 1997, Nadia graduated from Ivanna Eudora Kean High School in St. Thomas, Virgin Islands, and attended the University of the Virgin Islands for her undergraduate studies. She earned a B.S. in Mathematics and graduated cum laude in May 2001. Nadia then immediately enrolled in graduate school at the University of Maryland, College Park, where she earned a Master’s of Education in Curriculum and Instruction with a concentration in Mathematics Education. While completing her Master’s degree, which was earned in May 2006, Nadia taught mathematics at both public high schools in St. Thomas, Charlotte Amalie High School and the Ivanna Eudora Kean High School.

After teaching high school mathematics for six year, Nadia applied to the doctoral program for Mathematics Education at North Carolina State University. As a graduate student, Nadia worked as a research assistant on Diagnostic E-Learning Trajectories Approach (DELTA) Project, Learning Trajectories Based Instruction (LTBI) Project, and Accomplished Elementary Teachers of Mathematics and Science (ATOMS) Project. She also served as a teaching assistant for two undergraduate courses, Calculus for the Elementary School Teacher and Introduction to 21st Century Teaching.

Upon receipt of her degree, Nadia will return to St. Thomas, Virgin Islands, where she will assume a tenure-track position as Assistant Professor of Mathematics. Nadia is married to Jahmed Mills and they have three children, Orion, Malachi, and Jacob.
ACKNOWLEDGMENTS

Thanks to the chair of my dissertation committee, Dr. Allison W. McCulloch. She has motivated, encouraged, and provided her expertise, which made this all possible. Her expert feedback and understanding through all the personal obstacles during my year of writing made this dissertation possible.

Thanks to the other members of my dissertation committee, Dr. Karen Keene, Dr. Lee V. Stiff, and Dr. Alina Duca for your time and support during this process. Special thanks to Dr. Lee V. Stiff for his expertise, being a second opinion and a sounding board during the shaping of my research question, and providing support for my job talk.

Thanks to the board of directors of the Jones, Holloway and Bryan Foundation, especially Dr. Judith Grybowski, for believing in me and offering their financial support for the five years it took to complete this degree. Thanks also to the Virgin Islands Cultural and Academic Endowment (VICAE) through the Office of the Provost at the University of the Virgin Islands for their financial support in helping me make my dream a reality.

Thanks also to my wonderful classmates for your support inside and outside the classroom. A special thanks goes out to Ayanna Perry for the support she has given me even from afar.

Thanks to the students who participated in my study. Thank you for sharing your experiences and time with me. I am truly grateful for your honesty and your participation in my study.

Thanks to all those that made my family’s stay here in North Carolina pleasant. Thanks to the friends and supporters at Sha’arei Shalom for their prayers and
encouragements. To the parents and coaches that became our friends on our sons’ soccer teams.

I thank my mom, dad, siblings, relatives, and close friends that offered prayers and well wishes even when they did not understand the details of my struggles as a graduate student.

A special thank you goes to Aunty Olive, who left her home to stay with me and has been a mother, sister, child care taker, and helped in anyway that I needed her. I can never repay what you have done. This degree is as much yours as it is mine. Without you, this would be impossible.

Finally, thanks to my husband, Jahmed Mills, who has sacrificed his career to help me fulfill my dreams. Thanks also to my sons, Orion, Malachi and Jacob for being patient and understanding during this entire process.
TABLE OF CONTENTS

LIST OF TABLES .................................................................................................................. x
LIST OF FIGURES ................................................................................................................. xi
Chapter One .............................................................................................................................. 1
   Purpose of the Study and Research Question ................................................................. 3
   Definition of General Terms ............................................................................................. 4
   Problem ............................................................................................................................... 4
   Mathematical problem solving ......................................................................................... 4
Significance of the Study ........................................................................................................ 5
Overview of Methodological Approach .............................................................................. 5
Organization of the Dissertation ......................................................................................... 6
Chapter Two ........................................................................................................................... 7
   Problem Solving Process ................................................................................................. 7
      Decision-making During Problem Solving .................................................................... 10
      Knowledge of knowing ................................................................................................. 12
      Monitoring and self-regulation ..................................................................................... 13
   Goals and Problem Solving ............................................................................................. 14
   Knowledge of Content and Strategies ........................................................................... 17
   Affect and Problem Solving ......................................................................................... 20
      Local affect .................................................................................................................. 21
      Global affect .............................................................................................................. 22
   Contextual Factors and Problem Solving ...................................................................... 24
   Conceptual Framework ................................................................................................. 26
      Goals ........................................................................................................................... 28
      Knowledge .................................................................................................................. 28
      Affect .......................................................................................................................... 29
      External contextual factors ....................................................................................... 30
   Chapter Summary ........................................................................................................... 31
Chapter Three ......................................................................................................................... 33
   Study Design ................................................................................................................. 34
      Participants and Study Context .................................................................................. 36
      Student selection ...................................................................................................... 37
         Mathematical knowledge test .................................................................................. 38
         Mathematical beliefs survey ................................................................................. 39
         Last mathematics course ....................................................................................... 42
   Brief Case Descriptions ................................................................................................. 45
      Tony .............................................................................................................................. 45
Knowledge of similar problems
Knowledge of mathematical relationships
Knowledge of Traditional Problem Solving Strategies

Chapter Four
Goals
- Orienting phase
- Planning phase
- Executing phase
- Checking phase

Knowledge
- Knowledge of Traditional Problem Solving Strategies
- Knowledge of recently encountered content
- Knowledge of mathematical relationships
- Knowledge of similar problems

Summary
Appendix G: Summary Table
Appendix H: Themes ................................................................. 216
Appendix I: Students’ Written Work on Task-based Interview Problems .......... 219
LIST OF TABLES

Table 1: Problem Solving Process .............................................................................. 10
Table 2: Mathematical knowledge test results ............................................................ 39
Table 3: Part I: Beliefs about mathematics and mathematical problem solving ..... 40
Table 4: Score distribution for attitudes and beliefs about mathematics and problem solving ........................................................................................................................................ 42
Table 5: Last mathematics course and grade earned .................................................. 43
Table 6: Summary table ............................................................................................... 44
Table 7: Student pseudonyms and performance level ............................................... 45
Table 8: Problems ....................................................................................................... 54
Table 9: Nathan’s additional problems ....................................................................... 56
Table 10: Examples of memory decay ...................................................................... 60
Table 11: Sample questions from video-SR interview protocol ............................... 63
Table 12: Sample transcription along with non-verbal expressions .......................... 65
Table 13: Example passages ...................................................................................... 67
Table 14: Influential factors, descriptions, and examples ......................................... 71
Table 15: Sample summary table .............................................................................. 74
Table 16: Themes for goals ....................................................................................... 76
Table 17: Themes for local affect .............................................................................. 77
Table 18: Student performance on problems .............................................................. 88
Table G 1: Summary Table ....................................................................................... 213
Table H 1: Themes for goals ................................................................................... 216
Table H 2: Themes for content knowledge ................................................................. 217
Table H 3: Themes for local affect ......................................................................... 217
Table H 4: Themes for global affect ........................................................................ 218
Table H 5: Themes for external contextual factors .................................................. 218
LIST OF FIGURES

Figure 1: The association between goals, plans and resources (Slade, 1994, p. 71) .................. 16
Figure 2: DeBellis and Goldin’s (2006) framework for their representational perspective on affect .......................................................... 25
Figure 3: Conceptual framework of proposed study ............................................................. 28
Figure 4: Revised conceptual framework ............................................................................ 87
Figure 5: Examples of Leann’s annotations ........................................................................ 90
Figure 6: Example of Tony’s graph used in orienting phase .............................................. 91
Figure 7: Leann’s multiplication table ................................................................................ 94
Figure 8: Nathan’s written work for the Money Problem .................................................. 96
Figure 9: Dani’s written work for the Camper Problem .................................................... 97
Figure 10: Leann’s looking for a pattern to solve the Magic Square Problem ................. 104
Figure 11: Phil’s table for the Game Problem ................................................................. 105
Figure 12: Tony’s drawing of functions to solve the Points, Lines, and Planes Problem ... 105
Figure 13: Kirsten’s triangles ............................................................................................ 106
Figure 14: Tony’s written work for the Points, Lines, Planes Problem ............................. 107
Figure 15: Kirsten’s table for the Money Problem ............................................................ 108
Figure 16: Tony’s solution to part a of Points, Lines, and Planes Problem written in functional notation ........................................................................................................... 111
Figure 17: Tony’s written work during the video-SR interview for the Points, Lines, and Planes Problem ........................................................................................................................................... 112
Figure 18: Leann’s solution to the Points, Lines, and Planes Problem ............................... 112
Figure 19: Dani’s written work for the Camper Problem .................................................. 117
Figure 20: Dani’s diagram for solving the Points, Lines, and Planes Problem ................. 118
Figure 21: Tony’s list of triples that sum to 15 and their frequency .................................. 121
Figure 22: Leann’s hand computation ................................................................................ 138
Figure I 1: Tony’s written work ....................................................................................... 219
Figure I 2: Leann’s written work ...................................................................................... 226
Figure I 3: Kirsten’s written work .................................................................................... 232
Figure I 4: Dani’s written work ....................................................................................... 244
Figure I 5: Nathan’s written work .................................................................................... 256
Figure I 6: Phil’s written work ....................................................................................... 263
Chapter One

Mathematical problem solving matters. It is a substantial element of the reasoning expected in Science, Technology, Engineering, and Mathematics (STEM) college level courses. Success in STEM related degrees translates to more opportunities in today’s workforce (Pender, Marcotte, Sto Domingo, & Maton, 2010). Success in any STEM career requires proficiency in mathematics and in particular mathematical problem solving. Therefore, there is an urgency to integrate problem solving into the mathematics curriculum at the K-12 level (English & Sriraman, 2010).

The National Council of Teachers of Mathematics’ (NCTM) (2000), Principles and Standards for School Mathematics supports making problem solving an integral part of mathematics instruction because it enables students to “acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom” (p. 52). Problem solving can be used as a way for students to learn new mathematics and develop critical thinking skills which may support the development of life skills, such as persistence, curiosity, and confidence in unfamiliar situations (NCTM, 2000; Thayer et. al, 1940). The importance of the integration of the learning of mathematics and problem solving is also evident in the Common Core State Standards-Mathematics (CCSS-M) (CCSSI, 2010). CCSS-M integrates problem-solving skills into their 8 Mathematical Practices; practices students should engage in to be efficient problem solvers. For example, three of their practices indicate that students should “make sense of problems and persevere in solving them”, “reason abstractly and quantitatively”, and
“model with mathematics” (2010, pp. 6-8). These two influential documents suggest that problem solving is still relevant in curricula and should continue to be studied.

Early work in mathematical problem solving can be traced back to the 1930s, however, it is the seminal work of George Polya (1945), *How to Solve It*, that motivated the large surge of research that followed (English & Sriraman, 2010). Polya’s work focused on the problem-solving process and strategies needed for successful problem solving (Polya, 1945). Subsequent research focused on identifying the behaviors of expert problem solvers that enable them to be successful at problem solving (Carlson & Bloom, 2005). The differences between expert and novice problem-solving behavior have been studied along with the identification of the factors that directly impact problem-solving success (Lester, 1994; Schoenfeld A. H., 1985; Carlson & Bloom, 2005). These factors include individuals’ decision making as it is influenced by metacognition (monitoring and self-regulation), the ability to set pertinent goals and achieve them, content knowledge and knowledge of problem-solving strategies, affective influences (i.e., emotions, attitudes, beliefs), and other external factors (e.g., problem-solving tools, sociocultural contexts) (English & Sriraman, 2010; Carlson & Bloom, 2005). This body of research was informative and gave descriptive insight into students’ problem-solving process; however, it did not make much progress in improving students’ problem-solving skills (English & Sriraman, 2010). To connect research to practice, it has been suggested that problem-solving research should focus on theory creation (Lesh & Zawojewski, 2007; English & Sriraman, 2010).
Prominent researchers in the field have suggested that the development of a problem-solving theory should be given top priority (i.e. Lester, 1994; English & Sriraman, 2010, Schoenfeld, 2010). They state that the purpose for theory development is not to produce a “‘grand theory’ of problem solving” (p. 269). Rather, creating a theory will force researchers to collaborate and accumulate problem-solving knowledge into one depository (English & Sriraman, 2010). The hope is that the creation of a theory will allow researchers to “more reliably observe, document, and assess important mathematical developments in our students” (English & Sriraman, 2010, p. 269). Therefore, one way to advance problem-solving research is to create a theory.

**Purpose of the Study and Research Question**

The goal of this study is to contribute to the creation of a proposed problem-solving theory. Therefore, empirical evidence was collected to: (1) provide detailed, rich descriptions about how and why students make decisions during problem solving and (2) validate the proposed problem-solving theory. This required an in-depth qualitative investigation into the factors (i.e., goals, knowledge, affect, and external contextual factors) that contribute to the choices students make during problem solving. These descriptions provided detailed insight supplying the means to test the accuracy, range, and scope of the theory regarding students’ problem-solving processes (Schoenfeld A. H., 2008). Therefore, the following research question was proposed:

1. In what ways do students’ goals, knowledge, affect, and external contextual factors inform their choices during the mathematical problem-solving process?
a. How do students’ goals inform their choices during the mathematical problem-solving process?

b. How does students’ knowledge inform their choices during the mathematical problem-solving process?

c. How does students’ affect inform their choices during the mathematical problem-solving process?

d. How do external contextual factors inform students’ choices during the mathematical problem-solving process?

**Definition of General Terms**

In this section, I define terms that may be ambiguous in mathematics research literature to provide clarity in this study.

**Problem.** In this study, Schoenfeld’s (1992) definition of a problem was used. He states:

[a] problem is only a problem (as mathematicians use the word) if you don’t know how to go about solving it. A problem that has no ‘surprises’ in store, and can be solved comfortably by routine or familiar procedures (no matter how difficult!) is an exercise (Schoenfeld A. H., 1992, p. 41).

**Mathematical problem solving.** Since problem solving is a process, in this study it is defined as, “The process of interpreting a situation mathematically, which usually involves several iterative cycles of expressing, testing and revising mathematical interpretations-and
sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics” (Lesh & Zawojewski, 2007, p. 782).

**Significance of the Study**

This theory on problem solving is necessary because there is a gap in the literature that needs to be filled. I used empirical data to study individual students’ problem-solving process and created rich descriptions of how their goals, knowledge, affect, and external contextual factors influence their choices. Schoenfeld (1983) supports the notion that there is great value in studying an individual’s problem solving process in isolation. He stated “if you just keep your eyes open and take a close look at what people do when they try to solve problems, you’re almost guaranteed to see something of interest” (p. 38 as cited in Schoenfeld A. H., 2010). Therefore, the empirical data gathered from students’ problem-solving process would provide sufficient data to create these rich descriptions and validate this theory.

**Overview of Methodological Approach**

To conduct a study on students’ problem solving processes, in-depth qualitative analysis must be employed. In this section, I give a brief summary of the methodological approach for this study. A more thorough treatment of this subject is found in Chapter 3. To investigate the research question, a multi-case study was employed. There were six participants in this study. They were incoming freshmen or current freshmen STEM majors enrolled in a summer bridge program at a Historically Black University (HBCU). Data sources include individual task-based interviews followed by video-stimulated recall (video-
SR) interviews. Interviews were audio and video recorded, transcribed, and coded. Triangulation is necessary to increase the rigor, validity and reliability of results (Creswell, 2013), therefore, student artifacts and researcher journal were collected to provide additional evidence of students’ problem solving process.

**Organization of the Dissertation**

This dissertation consists of five chapters. In this chapter, I discussed the importance of problem solving skills in STEM curricula and the impact it has on the future workforce. Chapter Two begins with a review and synthesize of the literature on the problem-solving process, the factors that influence problem-solving success and Schoenfeld’s theory of problem solving. I conclude with a discussion of my initial conceptual framework that drove the design of this study. In Chapter Three I present the research question, methodology, participants, and data analysis methods used to determine my findings. In Chapter Four, I first present my revised conceptual framework and then provide my findings through detailed cross-case analysis of the factors that influenced the problem-solving process across all students. In Chapter Five, I summarize my findings to answer the research questions and connect to the literature. I then discuss the implications and limitations of the work. I end the dissertation with potential extensions for this work.
Chapter Two

In this chapter, I present a review of the literature as it pertains to problem solving. I begin with a review of the problem-solving process. I then review the literature pertaining to the factors that have been researched and proven to affect problem-solving success. These factors include the problem-solving process, students’ goals, mathematical content knowledge, knowledge of problem-solving strategies, global and local affect, context of situation, and in-the-moment decision fueled by students’ metacognition skills (i.e. monitoring and self-regulation). I conclude by presenting the conceptual framework that was used to drive my study.

Problem Solving Process

To understand the factors that contribute to success in problem solving, it is imperative to first describe research-based frameworks for the problem-solving process. George Polya, known for his seminal work How to Solve It (1945), proposed the first widely used problem-solving process framework. His framework has been understood as a linear progression (Carlson & Bloom, 2005) and consists of four phases: (1) understanding the problem, (2) planning your solution, (3) carrying out your plan, and (4) checking your solution. He suggests that after students build an understanding of the problem, they then move into the next phase where they make connections between the givens and unknowns and devise a plan. Then they carry out the plan they devise, arrive at a solution, and check the solution for validity (Polya, 1945; Carlson & Bloom, 2005).
Subsequent research found that the problem-solving process is more complex and cyclic in nature. That is, an individual may take nonlinear paths through the problem-solving process and may not even employ each phase (Garofalo & Lester, 1985). For example, one landmark framework, created by Schoenfeld (1985), identified five phases or episodes of the problem solving process. They are reading, analysis, exploration, planning/implementation, and verification. In the reading phase, students read the question and work at understanding it. During the analysis phase “an attempt is made to fully understand a problem to select an appropriate perspective and reformulate the problem in those terms, and to introduce for consideration whatever principles or mechanisms might be appropriate” (Schoenfeld A. H., 1985, p. 298). The next less structured phase is exploration where student may conduct a broad search for relevant information that can be use to solve the problem. During the planning/implementation phase, students may show evidence of creating an explicit plan and work towards achieving it. In the last phase, verification, students check their answer to ensure that it makes sense given the context of the problem. This framework dissected Polya’s understanding the problem into reading and analysis. The planning/implementation phase overlaps Polya’s planning your solution and carrying out your plan and both end with checking or verification of the solution.

Garofalo and Lester (1985) created a cognitive-metacognitive framework that describes metacognitive behaviors that may occur during each of the problem solving phases. This was needed since Polya’s (1945) and Schoenfeld’s frameworks did not acknowledge the role of metacognition explicitly and treated the problem-solving process as purely cognitive
Although Garofalo and Lester’s (1985) framework does not provide a comprehensive list of all possible cognitive and metacognitive behaviors that can occur, it does identify areas where students’ cognitive actions may be influenced by their metacognitive decision-making. These metacognitive behaviors are organized based on the problem solving phases: *orientation, organization, execution, and verification.* For example, in the organization phase students plan their behavior and choice of actions. The metacognitive behavior that may occur is the “identification of goals and sub goals, global planning, and local planning to implement global plans” (Garofalo & Lester, 1985, p. 171), which is similar to Polya’s four-phase framework.

Artzt and Armour-Thomas (1992) created a framework to study problem solving in a small group setting. This framework is a modification of Schoenfeld’s framework. This modification occurred based on the following claims: (1) Schoenfeld’s framework was limited due to the exclusion of identifying metacognitive behaviors, and (2) in Schoenfeld’s framework students were studied in a small group setting versus independently. Artzt and Armour-Thomas’ (1992) framework adds an *understanding* phase and separates the *planning/implementation* into two separate phases. The *planning/implementation* phase was separated because these may not have occurred sequentially in a group setting since a plan may be proposed and rejected before implementation can occur. Artzt and Armour-Thomas’ seven-phases process consists of reading, understanding, analysis, exploration, planning, *implementation*, and verification.
Most recently, Carlson and Bloom (2005) presented a version of the problem solving process similar to Polya’s (1945) and Garofalo and Lester’s (1985) frameworks. They claim their four phases include all phases added by previous researchers. These phases, described in Table 1, are orienting (includes reading and understanding), planning (which includes exploration, analysis), executing (which includes implementation) and checking (which includes verification) (Carlson & Bloom, 2005). The evolution of these frameworks provides evidence that problem-solving research is progressing and that problem solving consists of both cognitive and metacognitive abilities.

Table 1: Problem Solving Process

<table>
<thead>
<tr>
<th>Phase</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orienting</td>
<td>Includes reading and analyzing (Schoenfeld A. H., 1983) and refers to the preliminary actions students take to understand the problem.</td>
</tr>
<tr>
<td>Planning</td>
<td>Includes exploration and analysis. This is the phase where explore problem solving strategies, knowledge base that they have and make a plan to solve the problem.</td>
</tr>
<tr>
<td>Executing</td>
<td>Includes implementation and is the phase where students carry out their plan from the previous phase.</td>
</tr>
<tr>
<td>Checking</td>
<td>Includes verification and is the phase where students determine the reasonableness of their solution.</td>
</tr>
</tbody>
</table>

Decision-making During Problem Solving

During the problem solving process, students are constantly making decisions. The problem solver’s ability to successfully traverse the phases of the problem-solving process depends primarily on their decision-making (Schoenfeld A. H., 2010). Routine decision-making is not as interesting to study since it is relatively unproblematic and the individual
follows a familiar course of action each time (Schoenfeld A. H., 2010). For example, if a student knows how to use the division algorithm, observing them solve multiple problems will be monotonic. However, when the process is non-routine, such as in problem solving, the decision-making is more problematic and may be unpredictable. In this study, non-routine decision-making under time constraint, sometimes called in-the-moment decision-making, was studied to learn what contributes to students’ decision-making in a time-constrained environment when posed with a problem-solving task.

Research has shown that students’ metacognitive skills are important for making productive decisions during the problem-solving process (Lester, Garofalo, & Kroll, 1989; Schoenfeld A. H., 1983). During problem solving a student’s metacognition might influence decisions regarding which problem solving strategy to implement. Furthermore, with effective metacognitive skills the amount of time a student dedicates to a strategy is also managed (Schoenfeld A. H., 1983). Metacognitive behaviors include “predicting, checking, monitoring, reality testing, and coordination and control of deliberate attempts to solve problems “ (Schoenfeld A. H., 1983, p. 335). Carlson and Bloom (2005) describe metacognition as “knowledge about and monitoring of one’s thought processes and control during problem solving” (Carlson & Bloom, 2005, p. 48). In defining metacognition, it is also necessary to define monitoring and control. Monitoring is the cognitive ability to reflect on how well actions are contributing to the progression of the problem-solving task (Carlson & Bloom, 2005). According to Schoenfeld (1985) and Lester, Garofalo, and Kroll (1989) control is the decisions made about the planning, evaluating, monitoring, and regulating of
available knowledge during mathematical problem solving. Therefore, the terms metacognition, monitoring, and control overlap, are codependent, and have held similar meanings in the literature (Carlson & Bloom, 2005). Wilson and Clarke (2004) refer to metacognition as “the awareness individuals have of their own thinking: their evaluation of that thinking, and their regulation of that thinking” (p. 26). Their definition of metacognition consists of three distinct components: metacognition awareness, evaluation, and regulation. Combining the definitions, metacognition is being aware of one’s knowledge and thought processes and exhibiting control of them during the problem solving process. Therefore metacognition has two main components: (1) having knowledge of one’s own knowing and thought processes (awareness), and (2) having control (i.e. monitoring and self-regulation (evaluation and regulation)) of one’s decisions during problem solving.

**Knowledge of knowing.** Metacognition controls the ability to utilize knowledge during the problem-solving process (Lester, Garofaol, & Kroll, 1989; Kilpatrick, 1985). Research has shown the ability of expert problems solvers to activate their knowledge about their particular content causes them to solve problems differently than novice problem solvers (Polya, 1962; Carlson & Bloom, 2005; Schoenfeld A. H., 1985). This work suggests that experts spend most of their time in the understanding and planning phases while novice problem solvers spend their time in the executing phase without making much progress. Wilson and Clarke’s (2004) study on elementary students problem solving supports this conclusion. They found that elementary school students’ awareness of their knowledge and
how to implement it was the least reported metacognitive function and the most reported was evaluation (i.e. monitoring).

Research by Cai (1994) on college-aged students also supports this conclusion. He found that students who took more advanced mathematics courses displayed more metacognitive behaviors than students with less exposure to higher-level mathematics courses. That is, students with more mathematics experience were aware of what they knew and recognized how they could use that knowledge in a sequential order to get to the solution (Cai, 1994).

**Monitoring and self-regulation.** Someone that makes productive in-the-moment decisions during problem solving monitors and regulates actions based on how well the problem-solving session is going (Kilpatrick, 1985). Goos, Galbraith, and Renshaw (2000), refer to monitoring when all is going well as routine monitoring. However, they state what is more interesting is the monitoring that occurs when students encounter specific difficulties caused by non-routine tasks. These difficulties trigger metacognitive actions called red flags. Situations which trigger red flags are: (1) lack of progress, (2) error detection, and (3) anomalous results. For example, when there is a lack of progress, students have to make a decision about their next steps. They must decide whether to persist on the current path or reassess and begin using a new strategy. If students encounter error detection while working, students should check and verify their written work. If their solution contains anomalous results, this should also trigger a calculation check. When these triggers occur, the problem solver knows something is wrong and may make decisions to overcome the inconsistency.
The decisions students make that are not necessarily influenced by red flags may be made to meet a conscious or unconscious goal whether productive or counterproductive (Schoenfeld A. H., 2010; Goos, Galbraith, & Renshaw, 2000). According to Goos, Galbraith and Renshaw (2000), monitoring during problem solving is when students assess (a) knowledge, (b) understanding of the problem, (c) appropriateness of strategy, (c) progress towards a goal, (d) execution of the strategy and (e) the result for accuracy and sense. These monitoring actions are differentiated from self-regulation actions such as identify new information, reinterpret the problem, change strategy, and correct calculation errors (Goos, Galbraith, & Renshaw, 2000). According to Schoenfeld (1992) monitoring and acting in response to the results of progress are the core components of self-regulation. In a similar vein to Schoenfeld, the terms monitoring and self-regulation will be used together to represent both related actions.

Decision-making has been modeled using two different perspectives. The first and older perspective is prescriptive in nature and models decision-making as a mathematical function that is based on maximizing an expected value based on the probabilities (Slade, 1994; Schoenfeld A. H., 2010). Most recently, decision-making has been modeled as a process that is based primarily on an individual’s goals. Goals drive the decision process and in the next section, I address its role in problem solving.

**Goals and Problem Solving**

Problem solving is a goal-oriented behavior. It is prominent human behavior to formulate a goal(s), act to accomplish the goal(s), then assess whether the actual outcome is
aligned to the intended outcome (Brown, 1987; Slade, 1994). What is a goal? The goal concept is very broad and ranges in abstractness (Carver & Scheier, 1998; Slade, 1994). Carver and Scheier (1998) state that goals are hierarchical, where more abstract goals are ranked more highly and smaller-grained more concrete goals are called sub goals. For example, when given a mathematical problem-solving task, the overarching goal is to solve the problem. Then as students attack the problem, sub goals are made (e.g., read the problem, create a graph, solve for x). As problem solvers seek to achieve these sub goals, they activate their metacognitive skills to attain a solution. After a solution is reached, it is then checked to determine its validity.

According to Sweller (1988), novice and expert problem solvers differ by the goals and sub goals they set. For example, he found that novice problem solvers use the means-end strategy which means that they work backwards through the problem and as a result have to find sub goals and solve those first because a direct relationship between the goal and the givens were not identified. However, expert problem solvers have the knowledge and schema required to be able to apply a working-forward method allowing for a more straightforward process because of their ability to identify the goals and sub goals. The problems used in this study were more algebraic in nature and are more efficiently solved using the working-forward method.

According to Slade (1994), goals, plans, and resources are closely associated. Figure 1 shows their model of this association. According to this model, goals lead to the creation of plans, which in turn may involve the creation of sub goals. The execution of plans, require
the use of resources (e.g. knowledge), which may also lead to the creation of more plans and goals. In the context of mathematical problem solving, the goal that students set enables them to create a plan of action to solve the problem. This plan will require that students use mathematical knowledge to execute the plan. The resources students choose may provide an opportunity to either create a new plan or a new goal. The student will then move among goals, plans and resources during the problems solving process.

![Diagram](image)

Figure 1: The association between goals, plans and resources (Slade, 1994, p. 71)

In his study of college-aged students, Cai (1994) noted that while problem solving, students who were successful created sub-goals that directly related to the main goal of the problem. Unsuccessful students however, stated goals and created sub-goals that had weaker connections. To achieve goals, the problem solver must activate their knowledge and self-regulating skills to attain a viable solution (Zimmerman, 2000).

Self-regulation and monitoring of goals are important to problem solving. The ability to set goals and put the goals into action are indicative of an individual’s ability to self
regulate failures and monitor by comparing information about the current behavior of the problem-solving process to the goals that were set (Sheeran & Webb, 2012).

**Knowledge of Content and Strategies**

It is evident that students’ ability to solve any problem is contingent on their knowledge base (Schoenfeld A. H., 1985). Schoenfeld (2011) defines an individual’s knowledge as “the information that he or she has potentially available to bring to bear in order to solve problems, achieve goals, or perform other such tasks” (p. 25). In the context of problem solving, this knowledge will consist of mathematical facts, procedural knowledge (i.e., how to do things, use algorithms), conceptual knowledge (i.e., how and why things work), and problem-solving strategies (Schoenfeld A. H., 2010; Jonassen D., 2000). These different types of knowledge are all connected and work together during problem solving.

According to Jonassen (2000), domain knowledge is a strong predictor of problem-solving ability. Domain knowledge includes mathematical facts and procedural knowledge. Jonassen claims that to improve problem solving, domain knowledge is not enough; it must be integrated with the “knowledge of how concepts within a domain are interrelated” (p. 69). He refers to this integrated knowledge as structural knowledge or cognitive structure and observed that the stronger it is integrated with domain knowledge, the better the students’ problem solving ability. This structural knowledge can also be considered conceptual knowledge.

Mayer (1998) noted that teaching domain-specific skill is not enough to guarantee improvement in problem solving because it is not sufficient to guarantee transfer. That is,
students do not benefit from learning domain-specific skills only but need guided instruction and training that focuses on the metacognitive skills required to successfully complete the task (Mayer, 2000; English & Sriraman, 2010; Lester, 1994; Schoenfeld A. H., 1985).

Early research in mathematical problem solving focused on problem-solving strategies students need to know to be successful problem solvers. In Polya’s (1945) seminal work, *How to Solve It*, problem-solving strategies of successful problem solvers were investigated and summarized. They include strategies such as (a) *making a diagram*, (b) *making a table*, (c) *working backwards*, and (d) *rereading the question* (Polya, 1945). Polya suggested that if students are taught these strategies and heuristics, they will be successful at problem solving. However, subsequent research found that teaching students strategies and heuristics alone did not make significant improvements in their problem solving abilities (Lesh & Zawojewski, 2007; Schoenfeld A. H., 1992; Gick, 1986). One explanation is that Polya’s problem solving strategies are too broad, too numerous, and more complex than anticipated (Schoenfeld A. H., 1992; English & Sriraman, 2010).

Although having knowledge of problem-solving strategies are not solely sufficient to significantly improve a student’s problem solving ability, research on the characteristics of expert and novice problem solvers show that expert problem solvers have a large repertoire of problem solving strategies (Lester, 1994). That is, they have the knowledge base necessary to solve the problem successfully. It is very difficult to teach students problem-solving strategies expecting it to transfer to actual problem solving; though, students can master these strategies it is more complex and more time consuming than originally thought by Polya
(Schoenfeld A. H., 1985). To complicate things further, knowing these strategies is not enough, students must also know when to employ which strategy to ensure the right one is chosen. Schoenfeld (1985) also noted that knowing problem solving strategies does not imply that one knows how to solve problems, deep content knowledge is also required.

Expert problem solvers also possess a large schema for problem solving. Gick (1986) defines schema as “a cluster of knowledge related to a problem type” (p. 102). This cluster of knowledge includes the goals that are typical to the problem, the constraints, and solution procedures. However, novice problem solvers do not have a large schema, therefore, they will need to search more for a solution. The necessity to search requires the usage of problem-solving strategies and may include breaking the problem down into smaller problems or subgoals (Gick, 1986). Research has shown that individuals who are conscious about the use of problem-solving strategies are more successful at problem solving (Gick, 1986). Experts do not rely solely on mean-ends analysis as their general search strategy to solve non-routine problems but use a sophisticated repertoire of information-gathering search strategies (p. 116). Problem-solving strategies may be schema driven (i.e., envoked by a schema), mostly in routine problem solving, or search based when there is no schema, mostly in non-routine problem solving (Gick, 1986).

In summary, research has shown that having any type of knowledge in isolation does not guarantee problem-solving success. Content knowledge and knowledge of problem solving strategies must be integrated and invoked by metacognition to have an impact on problem-solving ability.
Affect and Problem Solving

Previously, research on the influential factors (i.e. problem solving process, metacognition, goals and knowledge) on decision-making during mathematical problem solving has been reviewed. This body of research has provided evidence that decision-making during problem solving is not influenced by any one factor. In this section, I will review literature on the role of affect in decision-making during problem solving.

McLeod (1989) found that problem solvers who were aware of their emotional reactions to problem solving were more successful. The realization that cognitive processes along with non-cognitive processes contribute to problem-solving success motivated a new area of research on affect and problem solving.

What is affect? Affect is a general term that includes beliefs, attitudes, and emotional states (DeBellis & Goldin, 2006). Therefore, to understand affect a working definition of emotions, attitudes, and beliefs is warranted. “Emotions describe rapidly-changing states of feeling experienced consciously or occurring preconsciously or unconsciously during mathematical (or other) activity” (p. 135). “Attitudes describe orientations or predispositions toward certain sets of emotional feelings (positive or negative) in particular (mathematical) contexts” (p.135). “Beliefs involve the attribution of some sort of external truth or validity to systems of propositions or other cognitive configurations” (p. 135). From these definitions, one can conclude that affect is complex, mostly non-cognitive, and may be stable or unstable, making it a difficult construct to measure and observe. Therefore, I will use the framework
by DeBellis and Goldin (2006) that categorizes affect into local affect and global affect to organize my review of the literature.

**Local affect.** In problem solving, affect can be categorized into local affect and global affect (DeBellis & Goldin, 2006). Local affect is short-termed and changes frequently (i.e., unstable) during the problem solving process. Local affect is based on emotions, which is described as a short-term feeling. Local affect is more easily observed since emotions such as happiness, frustration, and confusion, are often captured by a person’s facial expressions or body language. DeBellis and Goldin (2006) suggested a representational and structural concept of affect. To view affect as representational is to suggest that the emotions “carry meaning for the individual” (DeBellis & Goldin, 2006, p. 133) and those emotions dictate the interaction between content knowledge and metacognition impacting the problem solving process. Viewing affect as representational also suggests that there are physical cues that allow affect to be communicated during interactions between oneself and others (DeBellis & Goldin, 2006). For example, “intonation, eye movements, facial expressions, ‘body language’, laughter, tears, noises, exclamations” are all physical cues of local affect (DeBellis & Goldin, 2006, p. 133). Op 'T Eynde, de Corte, & Verschaffel (2006) defined emotions from a socio-constructivist perspective as having two characteristics: 1) “emotion is a process in which appraisal processes play a central role” and 2) “emotions are social in nature and situated in a specific socio-historical context and positioned.” (p. 195). In this study, they investigated the relationship between emotions and problem solving behavior inside the classroom found that students in general experienced negative emotions when their
problem solving was not going as expected. This negative emotion either caused students to give up or persevere in looking for other strategies to solve the problem. This provides evidence that emotions are an integral component in problem-solving success.

**Global affect.** Global affect is longer-termed, influenced by local affect, and impressionable (DeBellis & Goldin, 2006). That is, global affect is made up primarily of attitudes and beliefs. Research by Carlson (1999) found that students with favorable beliefs about mathematics and problem solving were more successful. The beliefs students have about mathematics cause them to become frustrated or excited depending on if they believe mathematical problem solving should take some time and not all solutions come quickly.

The Indiana Mathematics Beliefs Scale was developed to measure the beliefs of high school and college-level students on problem solving. It focused on five beliefs about problem solving that impact students’ motivation to solve problems (Kloosterman & Stage, 1992). These beliefs are (a) *I can solve time-consuming mathematics problems*, (b) *There are word problems that cannot be solved with simple step-by-step procedures*, (c) *Understanding concepts are important in mathematics*, (d) *Word problems are important in mathematics*, (e) *Effort can increase mathematical ability* (Kloosterman & Stage, 1992). In contrast, Schoenfeld (1992) compiled a list of beliefs that negatively impact students’ motivation to solve problems. They include (a) *Mathematics problems have one and only one right answer*, (b) *Ordinary students are not expected to understand mathematics*, (c) *Students who understand mathematics can/will be able to solve a problem in five minutes or less* (Schoenfeld A. H., 1992).
Local affect and global affect can be described as empowering or disempowering (DeBellis & Goldin, 2006). Empowering affect allows students to persevere in problem solving, try new strategies, and spend more time understanding the problem. For example, a student that experiences curiosity and bewilderment when posed with a problem may be motivated to understand the problem and employ strategies to help him/her do so. Gaining insight into the problem and making progress will turn bewilderment into pleasure. In contrast, the same feeling can be disempowering and hinder performance by creating a cognitive block so that information cannot be retrieved and applied in a productive manner (DeBellis & Goldin, 2006). For example, a student may begin solving a problem with curiosity and bewilderment but may choose one set strategy that may not work. This then creates frustration, anxiety, and despair in the student, which may cause the student to stop exploring alternate strategies and leave the problem unsolved.

Research on affect has concluded that it is complex and impacts students’ problem solving ability (McLeod, 1989; Schoenfeld A. H., 1983; Lester, 1994; Garofalo & Lester, 1985; Mayer, 2000; DeBellis & Goldin, 2006). Students’ affect changes throughout the problem solving process and will continue to change depending on the heuristic strategy used and path taken. That is, students’ affect influences decision-making during problem solving (Schoenfeld A. H., 2010). During the problem solving process, students experience short spans of intense affective reactions (McLeod, 1989). This emotional process can be periodic where students will go between feelings of positive emotions when progress is being made to negative emotions when there is a hindrance in the progress (McLeod, 1989). These
emotions can cause students to either give up or continue to persevere depending on their beliefs about mathematics and problem solving (Kloosterman & Stage, 1992). Students must experience mathematics as sense making to create attitudes and beliefs about mathematics that will empower them to engage in problem solving productively.

**Contextual Factors and Problem Solving**

It is important to note that there are external factors, which I will refer to as contextual factors, which impact students’ decision-making during problem solving. According to Jonassen (2000), these external factors are cultural, social, and organizational (Jonassen D., 2011). DeBellis and Goldin (2006), characterize external factors that influence the affective domain as the beliefs, attitudes, emotions, and values of others (i.e. peers, school authority), which in turn are influenced by social and cultural conditions and external contextual factors (see Figure 2). According to Lave and Wagner (1991), the social context and culture shared by students, school, and society determines the type of knowledge and practices that students construct. External factors include the physical surroundings, time constraints, working individually versus in a group, and the ability to use technological tools (DeBellis & Goldin, 2006; Jonassen D., 2000).

The design of a study may also be an external factor that affects students’ performance. For example, in a laboratory environment, students may not feel comfortable explaining their problem-solving thinking and process that may affect their cognitive ability to solve the problem (Silver & Metzger, 1989; Wilson & Clarke, 2004). Time constraint is another external factor that may add additional stress and anxiety to students (Onwuegbuzie
Onwuegbuzie and Seaman (1995) conducted a study on students’ performance on statistics assessments that were timed and untimed. They found that all students’ performance improved on the untimed test. Therefore, to comprehensively study student decision-making during the problem solving process, it is imperative that contextual factors are included. That is, external factors will have an influence on students’ decision making during the problem solving process.

Figure 2: DeBellis and Goldin’s (2006) framework for their representational perspective on affect.

The literature reviewed so far discusses the factors that influence decision-making and success in mathematical problem solving. In the following section, these heavily researched factors along with a proposed problem solving theory will be used as a foundation for modeling students’ problem solving.
Conceptual Framework

Schoenfeld (2011), proposed a theory of goal-oriented decision-making, which attributes an individual’s goals (personal aims to achieve), resources (knowledge available), and orientations (beliefs, values) as being the influential factors to the decisions made during any goal-oriented activity. Schoenfeld (2011) proposed this theory for any goal-oriented activity such as cooking, car repair, mathematical problem solving, or teaching. Before Schoenfeld summarized this theory into general terms, he tested the theory on science and mathematics teaching. In this work, Schoenfeld (2008) developed a theory of teaching-in-context and provided a model and theoretical account of how and why teachers decide on which moves to make during active teaching. In teaching, this theory claims that teachers’ knowledge, goals, and beliefs are factors that influence how and why they make decisions in the classroom (Schoenfeld A. H., 2008).

Research has shown that students’ goals, knowledge, affect, and decision-making ability fueled by metacognition (monitoring and self-regulation) are important factors that determine problem-solving success. Schoenfeld supports the application of this theory explicitly to mathematical problem solving when he states: “I found that the following were major determinants of problem-solving success or failure: the knowledge base, heuristic strategies, metacognition (specifically, monitoring and self-regulation), beliefs, and practices” (Schoenfeld A. H., 2010, p. 107). Therefore, it is logical that problem solving can be studied using Schoenfeld’s goal-oriented decision-making theory.
To frame this study, I drew on Schoenfeld’s work (1985, 2008 & 2011), the mathematical problem solving process, and factors discussed in the review of the literature that have been shown to influence problems solving success.

My conceptual framework (see Figure 3) begins with the student entering the problem solving process after being presented with a problem. I used the Multidimensional Problem-Solving Framework proposed by Carlson and Bloom (2005) to describe the problem solving process. A description of this framework is found in Chapter Two in Table 1. This framework recognizes that the problem solving is cyclic and that expert problem solvers cycle through the planning, executing, and checking phases represented by the bold arrow on the left side of the conceptual framework diagram. Since students do not solve problems like experts do, my framework includes all possible paths they may take. The arrows on the right side of the diagram represent these paths.

As students begin problem solving, they are constantly making decisions. The decisions they make are contingent on their metacognitive skills, in particular monitoring and self-regulation. The shaded box that contains the problem solving process represents this. These in-the-moment decisions are influenced by their goals, content knowledge, knowledge of problem solving strategies, local affect, global affect, and other external factors labeled context of the situation. Below I provide a brief description of each factor.
Goals. In Schoenfeld’s theory, he suggests that goals and goal-oriented behavior are almost omnipresent. He defines a goal as “something that an individual wants to achieve, even if simply in the service of other goals” (p. 20). These goals may be conscious or unconscious and are created by the problem solver to engage in the process. In my framework, goals were defined and used in alignment with Schoenfeld’s goal-oriented decision-making theory.

Knowledge. Schoenfeld (2010) defines an individual’s knowledge as “the information that he or she has potentially available to bring to bear in order to solve
problems, achieve goals, or perform other such tasks” (p. 25). In this definition, he also includes conceptions with misconceptions and notes that to determine the correctness of solutions the focus will be on the student’s correct mathematical thinking. He also recognizes that students use different types of knowledge, content knowledge, and knowledge of problem-solving strategies, when problem solving (Schoenfeld A. H., 2010). In the context of mathematical problem solving, this knowledge will consist of mathematical facts, procedural knowledge (how to do things, using algorithms), conceptual knowledge (how and why things work), and problem solving strategies and heuristics (Schoenfeld A. H., 2010). These different types of mathematical knowledge are all connected and work together during problem solving.

In my conceptual framework, mathematical facts, procedural knowledge and conceptual knowledge were referred to as content knowledge. Knowledge of problem solving strategies was separated into its own factor. To understand and explain students’ choices as a result of their knowledge, misconceptions were also taken into consideration (Schoenfeld, 2010).

Affect. Schoenfeld (2011) used the term orientations to include all terms related to “dispositions, beliefs, values, tastes, and preferences” (p. 29), and simplified this label to beliefs in his study on teaching-in-context. Research supports the claim that students’ emotions, attitudes and beliefs all affect their problem solving. I used DeBellis and Goldin’s local and global affect to characterize affect. Local affect represents less stable and
constantly changing affect (i.e., emotions) and global affect more stable affect (i.e., attitudes and beliefs).

**External contextual factors.** There are other factors that are not included in Schoenfeld’s theory that research has shown may also have an affect on students’ problem-solving success. They include social, cultural and physical factors. Provisions to capture these contexts and others unique to this study were made. Restating this theory to reflect my conceptual framework, a students’ in-the-moment decision-making can be attributed to their goals, knowledge, affect, and external contextual factors. My conceptual framework implies that when students encounter a non-routine problem-solving task, their in-the-moment decisions are goal-oriented and managed by their ability to monitor and self-regulate achieving these goals. Hence monitoring and self-regulation will not be treated as an external factor. The in-the-moment decisions, which are aligned with their goals, will be influenced by their content knowledge, knowledge of problem solving strategies, global affect, local affect, and external contextual factors.

To validate any theory, it is necessary to provide empirical evidence and rich descriptions of the data gathered with which a model can be created. The empirical evidence serves to validate the theory and the model tests the scope and range of the theory (Schoenfeld A. H., 2008). In Schoenfeld’s (2008) study on teaching, he claimed that a teacher’s actions in the classroom could be attributed to his/her knowledge, goals, beliefs, and decision-making mechanism. To support his claim, empirical evidence (episodes of teachers with various teaching experience) was collected, coded and analyzed to create a
detailed description of the lesson and models of each teaching episode. These models represented what teachers did and why they did them during teaching. He found that a productive teaching routine that can be applied to all teachers and could also be used as a means to help other teachers learn this routine. This study collected empirical evidence that informed detailed descriptions of each student’s problem solving process. However only the cross-case findings is reported. These findings allowed for the research questions to be answered along with the validation of the theory.

**Chapter Summary**

In this chapter the literature related to the problem solving process and factors that influence decision-making was reviewed. In particular, the research on students’ goals, knowledge, affect, and metacognition (monitoring and self-regulation) was reviewed. In summary, problem solving is complex and its success is codependent on students’ cognitive and metacognitive abilities. That is, knowledge alone does not guarantee problem-solving success; success also depends on the productive interplay among knowledge, the awareness of that knowledge and how and when to use it along with the beliefs and emotions that occur during the process.

Primarily, these studies provided frameworks that described how a construct (e.g., problem solving strategies) impacts a student’s ability to successfully solve a problem. However, these frameworks are not prescriptive and haven’t allowed for the ability to generalize what students attribute to their in-the-moment decision-making during problem solving and how those decisions are made in a structured way (Schoenfeld A. H., 2010).
Hence the need for a problem solving theory that can provide a mechanism for modeling students’ in-the-moment decision making during problem solving. Schoenfeld (2011), proposed such a theory that has been used to create models of teaching. The goal of this study is to validate this goal-oriented decision-making theory with empirical data and rich descriptions of students’ mathematical problem solving process.

To conduct this research, I created a conceptual framework that combined research in problem solving with Schoenfeld’s goal-oriented decision-making theory. To ensure the validity of this theory, empirical evidence must be collected. Therefore, this multi-case study collected empirical data in the form of task-based interviews, video-stimulated recall (video-SR) interview, and field notes. These data will be transcribed and analyze to provide rich descriptions across cases. A more thorough treatment of my methods will be given in Chapter Three.
This study seeks to contribute to the creation of a theory for problem solving. This problem-solving theory, proposed by Schoenfeld (2011), states that an individual’s goals, knowledge, affect and his/her decision-making mechanisms are influential factors in the decisions made during problem solving. The creation of a problem-solving theory is important because it will add to existing mathematical problem-solving research by providing a lens through which one can understand how and why students make decisions during the problem-solving process. For this proposed theory to be widely accepted, it must be tested to determine its range and scope. Therefore, this study has two goals, (1) to provide detailed, rich descriptions about how and why students make decisions during problem solving and (2) to validate the proposed problem-solving theory. According to Eisenhardt (1989), a case study is one methodology that can be used to provide detailed, rich descriptions and to test or generate theory. Therefore, to accomplish these goals, empirical evidence captured by qualitative methods about an individual’s goals, knowledge, affect, and external contextual factors during problem solving is needed. Based on the aforementioned goals, the following research question is investigated.

1. In what ways do students’ goals, knowledge, affect, and external contextual factors inform their choices during the mathematical problem solving process?
   
a. How do students’ goals inform their choices during the mathematical problem solving process?
b. How does students’ knowledge inform their choices during the mathematical problem solving process?

c. How does students’ affect inform their choices during the mathematical problem solving process?

d. How do external contextual factors inform students’ choices during the mathematical problem solving process?

In the next few paragraphs, I discuss the features of multi-case study methodology employed to carefully study this phenomenon. This is followed by a description of the study design, case selection methods, data collection methods, and analysis methods. Lastly, validity and reliability issues are addressed.

**Study Design**

Because I wanted to know how goals, knowledge, affect, and external contextual factors influence the choices students make during problem solving, a qualitative, case study design was necessary (Yin, 2009). Qualitative research is diverse, but includes certain features to be characterized as such. Bogdan and Biklen (2007) describe five features that qualitative studies may possess: that the study occurs in its natural setting, is descriptive, motivated by process, inductive, and concerned with participant perspectives. The goals and structure of this study are in alignment with three of those features. This qualitative research study is descriptive and focused on process and participant perspectives. This study is descriptive because it uses “multiple sources of information (e.g., observations, interviews, audiovisual material, and documents and reports), and reports a case description and case
themes” (Creswell, 2013, p. 97). These multiple sources of information are required to ensure a detailed and in-depth analysis (Eisenhardt, 1989; Creswell, 2013; Stake, 1995; Yin, 2009). Therefore in this study task-based interviews, video-stimulated recall (video-SR) interviews, student artifacts, and the researcher’s journal were used as data sources. According to Creswell (2013), these multiple sources of data are needed to give a robust description of each case.

This study is focused on process because I am interested in how students go about solving problems rather than whether they successfully complete the task. And lastly, this study is concerned with participant perspectives. In this study, the participant perspective is represented by students’ affect (i.e., emotions, beliefs, and attitudes) during their problem solving experience. This study does not occur in its ideal naturalistic setting. Ways in which I compensated for this will be discussed later in this section. Also, since a goal of this research is to validate theory it is not absolutely inductive. However, as indicated by my conceptual framework, I allowed for some induction to occur with the inclusion of the external contextual factors component requiring me to look beyond the theory’s factors in my analysis.

In Creswell’s (2013) definition of a case study, the researcher “explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection” (Creswell, 2013, p. 97). In this multi-case study, students represent the real-life contemporary bounded systems. The students were purposefully selected so that a range of cases could be explored. According to Borman,
Clarke, Cotner, and Lee (2006), a range of cases is necessary to allow for generalization across cases. The within-case analysis was used to support the across-case analysis in providing detailed descriptions of students’ problem solving process. These descriptions were then used to validate the theory. The number of cases in multiple case studies is typically between four and five (Creswell, 2013); however, in this study, six cases were identified to provide a more in-depth description and analysis of students with varying backgrounds. In the upcoming sections, I provide a detailed description of the case selection methods and context for this study.

**Participants and Study Context**

The participants in this study were a subset of students enrolled in a summer bridge program at a Historically Black University (HBCU) in the Caribbean. This HBCU has been hosting this program for the past several years at no cost to the participants. The program is based on the belief that improving mathematics competency, particularly in calculus concepts, is a major component for success in STEM majors (UVI, 2013). In addition to a course in mathematics, courses in computer science, scientific writing and reading, and seminars focused on the transition to college life were offered. Each day, the students were required to attend either a bridge to pre-calculus or a bridge to calculus course immediately followed by the seminar courses. They would then have scientific writing and reading in the evenings and their computer science course on Friday mornings only. The rest of the evenings were devoted to completing homework. They were assigned two teacher assistants
who stayed with them in the dormitories and helped them with their homework and other assignments. This residential program ran for six weeks, June 23, 2014 to August 1, 2014.

The students enrolled in the summer bridge program were incoming or current freshmen interested in pursuing a STEM degree, with special considerations given to students who planned to attend the HBCU. The 19 total students, 16-19 years of age, were asked to participate in this study during the application phase. The consent form (see Appendix A) was included in the documentation that the program mailed to accepted applicants. There were 14 students who agreed to participate in the study. Out of the 14 students who volunteered, 6 were purposefully selected to represent variability in the influential factors (i.e., goals, knowledge, affect, and external contextual factors) of problem solving. In the next section, I discuss the methods used to purposefully select the students.

**Student selection.** Students were purposefully selected to represent a range of content knowledge, beliefs about mathematics, and problem solving, and also to represent polar types (Eisenhardt, 1989; Creswell, 2013). The data that supported this purposeful sampling were the results of a mathematical knowledge test and a survey on mathematics and problem solving beliefs. The result of any summative test alone is not always the best indication of concept-ownership; therefore, the last mathematics course taken and the grade earned in that course were also used to select students. Before I gathered the test results, I arranged the students’ first names in alphabetical order and then assigned numbers (S1-S14) to minimize bias. In the next section, I discuss how the mathematical knowledge test was used in student selection.
Mathematical knowledge test. Prior to instruction, participants took a test (see Appendix B) on their mathematics knowledge using the instrument mandated by the program. This test focused on pre-calculus skills and included topics such as: solving linear and quadratic equations, simplifying expressions, properties of exponents and logarithms, graphing linear equations, properties of functions and trigonometry. The test was scored out of 31 points with scores ranging from 3 points to 26 points, with a mean score of 13.7 points and median score of 16 points. Students were then assigned a performance level. High performing students scored between 20-31 points, average performing students 10-19 points, and low performing students 9 points and below. Table 2 shows the results of the mathematical knowledge test. With the results of the test organized without name identifiers, my next course of action was to analyze the mathematical beliefs survey.
Table 2: Mathematical knowledge test results

<table>
<thead>
<tr>
<th>Student</th>
<th>Score (graded out of 31)</th>
<th>Performance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>18</td>
<td>Average</td>
</tr>
<tr>
<td>S2</td>
<td>22</td>
<td>High</td>
</tr>
<tr>
<td>S3</td>
<td>21</td>
<td>High</td>
</tr>
<tr>
<td>S4</td>
<td>6</td>
<td>Low</td>
</tr>
<tr>
<td>S5</td>
<td>3</td>
<td>Low</td>
</tr>
<tr>
<td>S6</td>
<td>14</td>
<td>Average</td>
</tr>
<tr>
<td>S7</td>
<td>10</td>
<td>Average</td>
</tr>
<tr>
<td>S8</td>
<td>16</td>
<td>Average</td>
</tr>
<tr>
<td>S9</td>
<td>5</td>
<td>Low</td>
</tr>
<tr>
<td>S10</td>
<td>16</td>
<td>Average</td>
</tr>
<tr>
<td>S11</td>
<td>3</td>
<td>Low</td>
</tr>
<tr>
<td>S12</td>
<td>16</td>
<td>Average</td>
</tr>
<tr>
<td>S13</td>
<td>26</td>
<td>High</td>
</tr>
<tr>
<td>S14</td>
<td>16</td>
<td>Average</td>
</tr>
<tr>
<td>Mean Score</td>
<td>13.7</td>
<td>Average</td>
</tr>
<tr>
<td>Median Score</td>
<td>16</td>
<td>Average</td>
</tr>
</tbody>
</table>

**Mathematical beliefs survey.** Self-reports of mathematics beliefs and attitudes are reliable (Cifarelli, Goodson-Epsy, & Chae, 2010) and used extensively in mathematics education research to measure these constructs. This survey instrument was adapted from the Mathematical Beliefs Survey created by Yackel (1984) and additional questions from the Indiana Mathematics Beliefs Scale (Kloosterman & Stage, 1992), were added to increase the number of questions that addressed mathematical problem solving. The revised survey is found in Appendix C. The Mathematics Beliefs Survey was originally created to measure undergraduate students’ beliefs about mathematics, fitting for this group of students. This instrument was created using Skemp’s framework (1976) on relational and instrumental understanding of mathematics. Within this framework, a student that holds a relational
understanding of mathematics has a reasoning orientation towards mathematics and views mathematics as sense making. In contrast, one who holds an instrumental understanding possesses a rule following orientation towards mathematics (Quillen, 2004).

Past studies that have utilized this instrument (Quillen, 2004; Cifarelli, Goodson-Epsy, & Chae, 2010) used a numbered scale on a continuum from 1 to 5, with 1 being a strong instructional view and 5 being a strong relational view. This numbered continuum is used in conjunction with a 5-point Likert scale. Students respond to a statement regarding their beliefs about mathematics and problem solving, using Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), and Strongly Agree (SA). An answer of Undecided receives no points. The choice that represents a relational view of mathematics and problem solving received a higher point value. To increase the survey’s validity, the most favorable response on a question that will receive the maximum value of 5 points could be either SD or SA. Table 3 shows two questions from Part I of the survey. In the first question, the most favorable response is SD, whereas in the second question, the most favorable response is SA.

Table 3: Part I: Beliefs about mathematics and mathematical problem solving

<table>
<thead>
<tr>
<th>Question</th>
<th>SD</th>
<th>D</th>
<th>U</th>
<th>A</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Getting the right answer is the most important part of mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. I usually try to understand the reasoning behind all of the rules I use in mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
I adopted the coding scheme as presented in Cifarelli, Goodson-Epsy and Chae (2010). To find the overall survey score, they found the average value and classified the scores as follows: an overall survey score of 1.0-2.0 is instrumental, 2.1-3.0 is somewhat instrumental, 3.1-4.0 is somewhat relational and 4.1-5.0 as relational.

Part II of the Mathematical Beliefs Survey measured attitude towards mathematics and mathematical problem solving. Similar to Part I, the most favorable response on a question will receive the maximum value of five points but could be either SD or SA, to increase the survey’s validity. A more positive attitude corresponds to a higher point value. The coding scheme as presented in Cifarelli, Goodson-Epsy and Chae (2010) was also used for Part II. The scale is as follows: 1.0-2.0 as a negative attitude, 2.1-3.0 as a somewhat negative attitude, 3.1-4.0 as somewhat positive attitude, and a 4.1-5.0 as a positive attitude.

All participants in the summer bridge program took the survey but only the participants who gave consent were analyzed. The survey scores were calculated by hand and then entered into Microsoft Excel to calculate descriptive statistics. The mean score for Part I, beliefs about mathematics and mathematical problem solving was 2.9. That is, as an aggregate, the students held a somewhat instrumental view of mathematics. There was one student who held an instrumental view of mathematics, eight that held a somewhat instrumental view, and 5 with a somewhat relational view of mathematics and problem solving. No student scored in the relational view range. The distribution of the scores is shown in Table 4.
For Part II, attitudes towards mathematics and problem solving, the mean score was 3.6. That is, as an aggregate, the students held a somewhat positive attitude towards mathematics and mathematical problem solving. Three students received a 4.1-5.0, a positive attitude, nine students scored a 3.1-4.0, a somewhat positive attitude, two students scored 2.1-3.0, a somewhat negative attitude, or 1.0-2.0, a negative attitude. The distribution of these scores is also in Table 4.

Table 4: Score distribution for attitudes and beliefs about mathematics and problem solving

<table>
<thead>
<tr>
<th>Student</th>
<th>Beliefs Score</th>
<th>Beliefs Scale</th>
<th>Attitudes Score</th>
<th>Attitudes Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>3.2</td>
<td>Somewhat relational</td>
<td>4.6</td>
<td>Positive attitude</td>
</tr>
<tr>
<td>S2</td>
<td>2.9</td>
<td>Somewhat instrumental</td>
<td>3.1</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>S3</td>
<td>4</td>
<td>Somewhat relational</td>
<td>4.7</td>
<td>Positive attitude</td>
</tr>
<tr>
<td>S4</td>
<td>3.1</td>
<td>Somewhat relational</td>
<td>3.4</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>S5</td>
<td>2.3</td>
<td>Somewhat instrumental</td>
<td>3.1</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>S6</td>
<td>3</td>
<td>Somewhat instrumental</td>
<td>3.9</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>S7</td>
<td>3.3</td>
<td>Somewhat relational</td>
<td>3.7</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>S8</td>
<td>2.6</td>
<td>Somewhat instrumental</td>
<td>3.9</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>S9</td>
<td>1.9</td>
<td>Instrumental</td>
<td>2.8</td>
<td>Somewhat negative</td>
</tr>
<tr>
<td>S10</td>
<td>3</td>
<td>Somewhat instrumental</td>
<td>3.4</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>S11</td>
<td>2.8</td>
<td>Somewhat instrumental</td>
<td>3.1</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>S12</td>
<td>2.7</td>
<td>Somewhat instrumental</td>
<td>3</td>
<td>Somewhat negative</td>
</tr>
<tr>
<td>S13</td>
<td>3.2</td>
<td>Somewhat relational</td>
<td>4.7</td>
<td>Positive attitude</td>
</tr>
<tr>
<td>S14</td>
<td>2.2</td>
<td>Somewhat instrumental</td>
<td>3.1</td>
<td>Somewhat positive</td>
</tr>
<tr>
<td>Mean</td>
<td>2.9</td>
<td>Somewhat instrumental</td>
<td>3.6</td>
<td>Somewhat positive</td>
</tr>
</tbody>
</table>

Last mathematics course. I also considered the last mathematics course taken and grade earned to determine student selection. A student whose last mathematics course was Calculus or AP Calculus was labeled as high, Precalculus was labeled average, and any course below
Precalculus (e.g. Trigonometry and Advanced Algebra) was labeled low. Table 5 shows this information.

Table 5: Last mathematics course and grade earned

<table>
<thead>
<tr>
<th>Student</th>
<th>Last Mathematics Course Taken</th>
<th>Grade Earned (out of 100%)</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>AP Calculus</td>
<td>96</td>
<td>High</td>
</tr>
<tr>
<td>S2</td>
<td>AP Calculus</td>
<td>95</td>
<td>High</td>
</tr>
<tr>
<td>S3</td>
<td>AP Calculus</td>
<td>98</td>
<td>High</td>
</tr>
<tr>
<td>S4</td>
<td>Precalculus</td>
<td>65</td>
<td>Average</td>
</tr>
<tr>
<td>S5</td>
<td>International Student</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>S6</td>
<td>MAT 024</td>
<td>P</td>
<td>Low</td>
</tr>
<tr>
<td>S7</td>
<td>Precalculus</td>
<td>94</td>
<td>Average</td>
</tr>
<tr>
<td>S8</td>
<td>MAT 024</td>
<td>P</td>
<td>Low</td>
</tr>
<tr>
<td>S9</td>
<td>Algebra II</td>
<td>80</td>
<td>Low</td>
</tr>
<tr>
<td>S10</td>
<td>Calculus</td>
<td>98</td>
<td>High</td>
</tr>
<tr>
<td>S11</td>
<td>Trigonometry</td>
<td>88</td>
<td>Low</td>
</tr>
<tr>
<td>S12</td>
<td>Precalculus</td>
<td>90</td>
<td>Low</td>
</tr>
<tr>
<td>S13</td>
<td>AP Calculus</td>
<td>105</td>
<td>High</td>
</tr>
<tr>
<td>S14</td>
<td>AP Calculus</td>
<td>96</td>
<td>High</td>
</tr>
</tbody>
</table>

With information from these sources, I selected participants for the study. I created a spreadsheet using Microsoft Excel to summarize the students’ scores on the mathematics knowledge test, belief survey, and last mathematics course taken. Table 6 shows this information.
Table 6: Summary table

<table>
<thead>
<tr>
<th>Levels</th>
<th>Mathematics Knowledge Test</th>
<th>Last math Course</th>
<th>Beliefs</th>
<th>Attitudes</th>
<th>Selected Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>High/ Relational/ Positive attitude</td>
<td>S13, S2, S3</td>
<td>S1, S2, S3, S13, S14</td>
<td>N/A</td>
<td>S1, S3, S13</td>
<td>S13, S2</td>
</tr>
<tr>
<td>Average/ Somewhat Relational/ Somewhat Positive</td>
<td>S6, S8, S10, S12, S14</td>
<td>S4, S7</td>
<td>S1, S3, S4, S7, S13, S14</td>
<td>S2, S4, S5, S6, S7, S8</td>
<td>S6, S4</td>
</tr>
<tr>
<td>Low/ Somewhat Instrumental/ Somewhat Negative</td>
<td>S4, S5, S9, S11</td>
<td>S5, S6, S8, S9, S10, S11, S12</td>
<td>S2, S5, S6, S8, S9, S10, S11, S12, S14</td>
<td>S9, S12</td>
<td>S9, S12</td>
</tr>
</tbody>
</table>

To ensure that a range of students was represented, two students represented by each level were selected. The following students were chosen, S2 and S13 who fell mostly in the high/relational/positive attitude categories, S4 and S6 who fell mostly in the average/somewhat relational/somewhat positive categories, and S9 and S12 who fell mostly in the low/somewhat instrumental/somewhat negative categories. After the students were selected, I replaced the student number with their name and later on with their pseudonym. Table 7 shows the student number, aligned with their pseudonym and their performance level. In the next section, I briefly describe each student selected.
Table 7: Student pseudonyms and performance level

<table>
<thead>
<tr>
<th>Student Number</th>
<th>Pseudonym</th>
<th>Performance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>S13</td>
<td>Tony</td>
<td>High</td>
</tr>
<tr>
<td>S2</td>
<td>Leann</td>
<td>High</td>
</tr>
<tr>
<td>S4</td>
<td>Kirsten</td>
<td>Average</td>
</tr>
<tr>
<td>S6</td>
<td>Dani</td>
<td>Average</td>
</tr>
<tr>
<td>S9</td>
<td>Nathan</td>
<td>Low</td>
</tr>
<tr>
<td>S12</td>
<td>Phil</td>
<td>Low</td>
</tr>
</tbody>
</table>

**Brief Case Descriptions**

**Tony.** At the time of the study, Tony was a seventeen-year-old male freshman entering college. He attended public high school and was enrolled in AP Calculus AB during his senior year. Although his major was undeclared, he was interested in pursuing a career in engineering. Tony was selected to represent a high performing student because of his strong mathematics background and his favorable orientation towards mathematics and problem solving.

**Leann.** At the time of the study, Leann was a seventeen-year-old female freshman entering college. She attended public high school and was enrolled in AP Calculus AB during her senior year. She declared biology as her intended major and plans to pursue a career as a research scientist. Leann was selected to represent a high performing student because she scored high on the components pertaining to her mathematical knowledge and had an average score on her attitudes towards mathematics and problem solving, though her beliefs about mathematics and problem solving was low.
Kirsten. At the time of the study, Kirsten was an eighteen-year-old female freshman entering college. She attended a parochial private high school and was enrolled in pre-calculus during her senior year. She declared biology as her intended major and plans to pursue a career in veterinary medicine. Kirsten was selected to represent an average performing student because although she scored low and average on the components pertaining to her mathematical knowledge, she scored average on her beliefs and attitudes towards mathematics and problem solving.

Dani. At the time of the study, Dani was an eighteen-year-old female and a returning freshman at this HBCU. Although her last mathematics course taken at a parochial private high school was calculus, she was placed into a non-credit developmental mathematics course at the university. She declared physics as her intended major and plans to pursue a career as an astrophysicist. Dani was selected to represent an average performing student because of the variability in her mathematical knowledge components (average) and beliefs (low) and attitudes (average) towards mathematics and problem solving.

Nathan. At the time of the study, Nathan was a nineteen-year-old male freshman entering college. He attended public high school and was enrolled in Algebra II during his senior year. Nathan was returning to school after taking a year off. He declared computer science as his intended major and plans to pursue a career in computer science. Nathan was selected to represent a low performing student because of his low performance on all components.
Phil. At the time of the study, Phil was an eighteen-year-old male freshman entering college. He attended a parochial private high school and was enrolled in pre-calculus during his senior year. He declared computer engineering technology as his intended major and plans to pursue a career as an engineer. Phil was selected to represent a low performing student because of his low performance on all components except the mathematics knowledge test, on which he earned an average score.

After purposefully selecting students based on their mathematical knowledge and beliefs about mathematics and problem solving, empirical data was collected to provide a basis for the detailed descriptions. In the next section, I share the data sources.

Data Collection Methods

To provide detailed descriptions of students’ problem solving based on their decision-making, it is necessary to observe them while engaged in problem solving and to interpret what was done (Goldin, 2000). Task-based interviews are used in mathematics education research to study mathematical behavior and require interaction among the student, interviewer, and the task. The analysis of the behavior in a task-based interview allows for inferences to be made by the researcher about the students’ problem solving. According to Goldin (2000), these inferences are necessary for research where the purpose is providing descriptions, testing of hypotheses, and assessing the validity of models of problem solving. These requirements are aligned with the goals of my study and the requirements for case study methodology. The primary data sources for this study were task-based interviews and video-stimulated recall interviews (video-SR) using researcher-designed protocols. This
includes the student artifacts (i.e., student work and calculator screen capture). A researcher journal was used as the secondary data source. More details about these data sources are shared in the next several sections.

**Task-based interviews**

Task-based interviews are used in qualitative studies to observe and interpret mathematical behavior (Goldin, 2000). In this study unstructured individual, task-based interviews were conducted. Students were interviewed with no “substantial assistance that would facilitate a solution” from the researcher (Goldin, 2000, p. 519). According to Goldin (2000), *free* or unstructured problem solving should be an integral component of task-based interviews. That is,

subjects should engage in free problem solving during the interview to the maximum extent possible, in order to allow observation of their spontaneous behaviors and their reasons given for spontaneous choices … This technique permits exploration of the subjects’ freely chosen strategies, representations, and so forth, maximizing the information gained (2000, p. 542).

In light of Goldin’s recommendations, the researcher did not interact with the participants during the task-based interview to limit the influence on students’ in-the-moment decision-making and problem solving process. This was an attempt to ensure that the study occurred in the most naturalistic conditions possible as recommended for qualitative research by Bogdan and Biklen (2007).
Ideally, students would not be working on problems in isolation; however, observing students in small groups would not allow me to understand their individual problem solving process. Therefore, I chose a more laboratory style setting to conduct the interviews. This laboratory style setting is a qualitative research method that has been used extensively to gather evidence related to how subjects think (Schoenfeld A. H., 2010).

There were three task-based interviews per student. The first task-based interview occurred during the first or second week of the program, the second during the third or fourth week, and the third interview during the final two weeks of the program. These interviews occurred mostly in unoccupied classrooms in the same building their mathematics course was taught in. These interviews occurred during the lunch break, which occurred from 12:00 pm to 1:30 pm daily. Students had the choice of interviewing either at the beginning or towards the end of the lunch break. However, interviews that occurred on July 3, 2015 and July 4, 2015 were held at alternate locations due to university closings. On July 3rd, Leann’s video-SR interview was conducted in the dorm’s common area and Dani’s in the cafeteria. Dani’s video-SR interview occurred the next day in a classroom close to the students’ dorms. The task-based interviews lasted for as little as seven minutes and as long as 75 minutes. However, most interviews lasted between 20 to 30 minutes.

During each task-based interview, the student worked independently to solve one or more non-routine mathematics problems. These sessions were video recorded by me so that physiological factors such as facial expressions, body movements, and changes in tone were captured to inform the verbal responses (McLeod, 1989; Goldin, 2000). The video recordings
were also used to capture student written work and any tool use. Two cameras were used, one focused on the student’s face and the other on the student’s work. Audio recordings were also captured as a support to the video recordings.

Goldin (2000) suggests that in task-based interviews, students should be allowed to interact with external representations so that they can tangibly show their thinking. The external representations that were available to students were paper with one printed problem, additional blank paper for scratch work, a felt tip black pen, and a graphing calculator. Students were asked not to erase their written work so that a complete record of their written problem solving process would be available. Making mistakes and taking less efficient routes are all important characteristics of the problem solving process that needed to be captured. If students decided to use the graphing calculator, this working was also captured. I used a calculator screen capture method (c.f. McCulloch, 2009) to record this data. While recording the task-based interviews, I collected field notes and was a silent observer. These notes were collected to identify crucial moments that I needed to explore during the video-SR interviews. These notes were recorded on blank sheets of paper, on the interview protocol, or in my researcher journal. I also reflected on the interviews and the data collection process in my researcher journal. Next, I discuss the problems presented during the task-based interviews.

**Problem-solving items.** An important component to task-based interviews is the task or problem. The problem should be truly problematic but still accessible to the student; therefore, six problems of varying difficulty were chosen. According to Goldin (2000), the
problems chosen should have the following features: (1) non-routine, (2) evoke complex problem solving strategies processes requiring multiple representations, (3) encourage verbal discussion, and (4) induce affective responses. It is only through these types of problems that the influential factors of students’ in-the-moment decision-making can be authentically studied (Borkowski, 1989; Goldin, 2000). To accomplish the goals of this study, which is to study how goals, knowledge, affect, and external contextual factors influence the choices students make during problem solving, all of these features were necessary.

To ensure the problems were non-routine, I followed a similar vein to Silver and Metzger (1989) and did not include classic problem or ones that have well-known solutions. The problems selected for this study is shown in Table 9. Some of these tasks have been used in previous research and covered topics in geometry, algebra, and number theory. To ensure the problems required the use of complex problem-solving strategies along with multiple representations was challenging for several reasons. First, I did not know the mathematical capability of the students. However, I did know that they must have completed Algebra II (sometimes referred to as Advanced Algebra) since it is required to graduate from the local high schools. Second, because of the varying mathematical strengths I predicted the students might have, I wanted to ensure that the students would be able to engage with the problems. Therefore, problems were chosen that could be solved using algebraic and geometric reasoning but also could be solved using basic properties and operations of numbers.

Students could also use multiple representations to solve the problems. Some problems required the use of tables (e.g., the Game Problem) and drawings (e.g., the Points,
Lines, and Planes Problem); however students could solve them less efficiently without the use of these representations. The problems were also chosen so that students could use several problem-solving strategies. For example, students could create equations to solve the Money Problem, draw a diagram to solve the Points, Lines, and Planes Problem, and work backwards to solve the Game Problem. Strategic guess and check could be used to solve most of the problems (e.g., the Money Problem and the Magic Square Problem). These types of problems were accessible to the students but at the same time challenging, using content knowledge and problem solving strategies that should be readily known to a high school graduate.

The third feature, encouraging verbal discussion, was not as evident since students were working individually. However, during the task-based interviews, students were asked to speak aloud and share their thinking. Also, during the video-SR interviews, students had the opportunity to discuss aspects of their problem solving related to their goals, knowledge, and affect. The last feature, which states that the problems should induce affective response, was met. Since the problems were non-routine and the students were relatively novice problem solvers, affective responses were inevitable.

After ensuring that the problem set had the features suggested by Borkowski (1989) and Goldin (2000), I piloted them to further determine if they were appropriate for the participants. I first tested the problems on an eleventh grader who had completed pre-calculus. She was given all the problems shown in Appendix E. She was not able to solve any of the problems correctly but was able to engage with most. The problems were also
piloted on other high school students who had just completed pre-calculus and calculus. Each student in the second round of pilot testing was given one problem. There were some who had success on the Magic Square Problem and the Triangle Problem. Although students were not able to correctly answer the other problems, they were able to engage with the problems in a meaningful way. The suggestions from these students also allowed me to reword some of the problems for clarity. After piloting the problems, six potential problems remained. The final set of problems that were brought to the research site is shown in Table 8, all other problems are presented in Appendix E.
Table 8: Problems

<table>
<thead>
<tr>
<th>Problem Name</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Money Problem</td>
<td>Divide five dollars amongst eighteen children such that each girl gets</td>
</tr>
<tr>
<td></td>
<td>two cents less than each boy.</td>
</tr>
<tr>
<td>The Game Problem</td>
<td>There are three friends, Joe, Mary and Helen who are playing a game.</td>
</tr>
<tr>
<td></td>
<td>There is one loser and two winners at the end of each round of the</td>
</tr>
<tr>
<td></td>
<td>game. The loser pays to each winner the amount of money that person</td>
</tr>
<tr>
<td></td>
<td>had at the beginning of the round.</td>
</tr>
<tr>
<td></td>
<td>Round</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Points, Lines, and</td>
<td>Find a formula that will determine:</td>
</tr>
<tr>
<td>Planes Problem</td>
<td>a. At most how many regions on a line are determined by n points on</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>The Magic Square</td>
<td>Place the positive integers 1-9 in a 3 x 3 grid. Each integer should</td>
</tr>
<tr>
<td>Problem</td>
<td>be used only once and only one integer should be placed in a grid.</td>
</tr>
<tr>
<td></td>
<td>Arrange the integers such that the sum for each row, column and</td>
</tr>
<tr>
<td></td>
<td>diagonal is the same.</td>
</tr>
<tr>
<td>The Camper Problem</td>
<td>We have 3 campers, A, B and C. They can combine supplies for dinner.</td>
</tr>
<tr>
<td></td>
<td>C has no rice, A has 500 grams of rice and B has 300 grams of rice.</td>
</tr>
<tr>
<td></td>
<td>They combined all the rice and shared it so that all three campers</td>
</tr>
<tr>
<td></td>
<td>got the same share. C pays $8 for the meal to A and B. How much</td>
</tr>
<tr>
<td></td>
<td>should Camper C give Campers A and B so that they are paid</td>
</tr>
<tr>
<td></td>
<td>according to how much they contributed towards Camper C’s meal?</td>
</tr>
</tbody>
</table>

The problems in Table 8 are presented in the intended order of presentation to the students. During the first task-based interview, the students should have received the Money Problem first, followed by the Game Problem. For the second task-based interview, the Points, Lines, and Planes Problem followed by the Magic Square Problem. On the third task-based
interview, the Camper Problem should have been presented. However, there were several
students who did not solve the problems in the order that was intended. For example, during
Kirsten’s first task-based interview, she was given the Money Problem first. However, she
was having difficulty with the problem and began to get so frustrated, I decided to give her a
different problem for her second problem. She was given the following problem (Appendix
E, problem 7): Find all triangles with integral lengths for sides and whose perimeter is 12
inches (Krulik & Rudnick, 1996, p. 70). She was then given the problems in the original
order and instead solved two problems during her final task-based interview. Phil also
received the problems in the same order but at different times. He was only able to complete
the Money Problem during his first task-based interview due to scheduling. Therefore, he had
to solve two problems each session for the remaining task-based interviews.

Nathan required the most modifications. Nathan did not engage with the problems
presented. He gave his best effort in solving the first problem, the Money Problem.
Afterwards, he read the others but did not attempt to solve them. As a result, we had time to
conduct the video-SR interview immediately afterwards. After consulting with my advisor,
we decided to give him two extra problems. The problems created required solving systems
of equations, content he had just completed in his bridge to pre-calculus course. One problem
was given along with an exercise to solve a system of equations. The problems he was
presented are shown in Table 9. The problems were created as parallels to problems solved in
class. I had hoped to see him experience some success in problem solving and also witness
his problem solving process. However, he did not engage with the problem but he was able to find the value of $x$ for the systems of equations.

Table 9: Nathan’s additional problems

| Question 1 | Jamal and Kebo are selling fruit for the summer. Customers can buy boxes of genips and boxes of mangoes. Jamal sold 3 boxes of genips and 14 boxes of mangoes for a total of $203. Kebo sold 11 boxes of genips and 11 boxes of mangoes for a total of $220.

Mikey wants to also sell fruit. However, to be competitive he needs to know how much Jamal and Kebo are selling a box of genips and a box of mangoes. Jamal and Kebo will not tell him, they only gave him the information above.

How much should Mikey sell each box of genips and mangoes? Why? |
|---|---|
| Question 2 | Solve.

\[
\begin{align*}
-4x + y &= 6 \\
-5x - y &= 21
\end{align*}
\]

After solving these additional problems, we met once more for the Camper Problem. His final video-SR interview was conducted immediately afterwards.

The problems chosen for data collection were purposefully selected. Care was taken to ensure that the problems as a collection satisfied the four features necessary to study student cognition and capture the goals of the study: (1) non-routine, (2) evoke complex problem solving strategies processes requiring multiple representations, (3) encourage verbal discussion, and (4) induce affective responses. The problems were given to most students in
a specified order but due to special circumstances, there were students who did not follow this procedure.

In an attempt to promote a naturalistic problem-solving environment students were not interrupted and had the choice to verbalize their thinking process during the task-based interviews. Research by Silver and Metzger (1989) and Wilson and Clarke (2004) found that there are students who prefer not to verbalize their cognitive process during problem solving because it proved burdensome. However, most students did verbalize their thinking during the task-based interviews. The most verbal were Tony, Kirsten, Dani, and Phil. Nathan and Leann did not provide much insight into their thinking during the task-based interview making the video-SR interview even more valuable. In the next section, I discuss the video-SR interviews.

**Video-stimulated recall interviews**

Video-SR interviews refer to a qualitative research method that stimulates the memory of the interviewee through viewing previously captured actions (Lyle, 2003; Henry & Fetters, 2012). They are recommended for use when cognitive processes are of interest, which aligns with the goals of this research (Lyle, 2003; Goos, Galbraith, & Renshaw, 2000). Similar to the task-based interviews, each student participated in a total of three video-SR interviews. These interviews occurred within one day of the task-based interview. The video-SR interviews also occurred mostly in the same unoccupied classrooms as the task-based interviews during the students’ lunch break. The video-SR interviews were on average longer than the task-based interviews. They ranged from about 15 minutes to 90 minutes long.
Prior to the video-SR interview, when needed I viewed all or parts of a student’s task-based interview the same day it was taken or the morning before the video-SR interview. Viewing the task-based interview prior to the interview allowed me to take further notes in addition to the ones taken during the task-based interviews to help me choose points to discuss. The entire task-based interview was viewed during the video-SR interview in an attempt to recreate the experience for the student. These interviews were also conducted individually. The video-SR interviews occurred during the same time of day and location as the task-based interviews. There were three task-based interviews per student. The first task-based interview occurred during the first or second week of the program, the second during the third or fourth week, and the third interview during the final two weeks of the program. The task-based interviews lasted for as little as seven minutes to as much as 75 minutes. However, most interviews lasted between 20 to 30 minutes. The student and I sat side by side to view the video on my laptop computer. There were also given the opportunity to view their written work along with the calculator screen capture. Similar to structured task-based interviews, preplanned questions were asked to discover the reasons students made their decisions. Questions such as “What were you feeling at that moment?” were asked. Other sample questions can be found in Appendix D. Students were also free to comment on the decisions made and why they made those decisions without prompting. In addition, episodes of interest were identified prior to the interview. One example of how these episodes were identified is when students encountered difficulties caused by red flags (Goos, Galbraith, & Renshaw, 2000). These red flags most likely occurred when students expressed lack of
progress, detected an error, or got an anomalous result (Goos, Galbraith, & Renshaw, 2000). These red flags influenced local affect and were sometimes easily detected by students’ facial expressions, voice tone, and physical gestures (Goos, Galbraith, & Renshaw, 2000; Thompson & Thompson, 1989).

As with any other qualitative research methods, there are concerns about the validity and reliability of video-SR interviews. Some concerns about this research method include the participant’s ability to truthfully and accurately recall, due to the length of time between the initial interview and the video-SR interview (Lyle, 2003) and to differentiate between reflection and recall (Gass, 2001). In this research, attempts were made to minimize the impact of these issues. Lyle (2003) suggests that minimizing the time between the initial interview and the SR interview will reduce the likelihood of memory decay. Therefore, in my study, the interviews were conducted within one day with only one occurring two days after the initial task-based interview due to scheduling issues. Another difficulty in ensuring the validity and reliability of video-SR interviews is being able to differentiate between reflection and recall. To minimize this, attempts were made to include both recall and reflective responses.

Even though these efforts were made, there were instances of minor memory loss concerning what took place during the task-based interview. Table 10 includes examples from the data where memory decay occurred.
Table 10: Examples of memory decay

| Example 1 | Interviewer: So you would have \( n \) is equal to \( n \) but here you have \( n-1 \) so how did you get that you think?  
Tony: I’m trying to remember how I had do [sic] it. |
| Example 2 | Interviewer: so you said a possible formula could be \( Y \) equals \( X \) squared that should show an infinite number of regions for a line. So if you thought about regions as a coordinate plane how would you show an infinite number of regions for a line?  
Leann: I don’t know. I don’t even remember why I wrote that  
Interviewer: so you think you were just writing stuff down?  
Leann: I was probably just writing stuff down. |

Students were also asked to recall emotions felt during the interview and what caused them to feel that way. In agreement with Lyle (2003), students were asked questions about the problem-solving process that were reflective. For example, students were asked to think of another strategy they could have used to solve the problem. The response to this question provided information concerning the knowledge (i.e., content and problem solving strategies) available to the student. Students were also given the opportunity to continue solving the problem during the video-SR interview. For example, during one of Tony’s video-SR interviews, he completed a problem he did not complete during the task-based interview.
Even with its challenges, video-SR “provides a vehicle for accessing cognitive processes when care is taken to reduce memory decay and make the questions/prompts consonant with the process being investigated” (Lyle, 2003, p. 875). Similar to the task-based interviews, the video-SR interviews were audio and video-recorded and transcribed verbatim. In the next section, I discuss my data analysis methods.

**Data Analysis Methods**

“Data that have been analyzed while still being collected are both parsimonious and illuminating” (Merriam, 2009, p. 171). Therefore, I ensured that my data analysis began during data collection. This early analysis served to identify emerging themes and further refine my interview protocol resulting in improved data collection processes. This type of analysis that occurs during data collection is called ongoing (Cobb, Confrey, diSessa, Lehrer, & Schaubb, 2003). After my data were transcribed and coded, I began retrospective analysis (Cobb, Confrey, diSessa, Lehrer, & Schaubb, 2003). During retrospective analysis, the researcher undergoes a “complex process that involves moving back and forth between concrete bits of data and abstract concepts, between inductive and deductive reasoning, between description and interpretation” (Merriam, 2009, p. 176). In the following sections, I discuss the details of my ongoing and retrospective analysis.

**Ongoing Analysis.** Analysis during data collection was useful on many accounts. Bogdan and Biklen (2007) reported ten suggestions for why it is helpful to analyze data during data collection. The four suggestions used explicitly in my analysis were: 1) record comments which can stimulate critical thinking about what is seen, 2) write memos to reflect
on issues that may impact the goals of the study, 3) develop analytic questions after reflecting on data collected on the field, and 4) try out ideas and themes that may emerged as you gather data.

I began my first round of data collection on June 30, 2014. During these interviews, I enacted two of Bogdan and Bilken’s (2007) suggestions: record observer comments and write memos. For example, I recorded comments about the types of problem-solving strategies Tony used. This information gave me points to discuss during the video-SR interviews. It also allowed me to begin comparing and contrasting problem-solving strategies used by the students and writing memos that allowed me to reflect on any themes that I might have noticed. One emergent theme from my reflections is perseverance. I noticed that the students whose goals were to impress the researcher spent more time on problems. I then had something to look for in subsequent interviews. On July 10, 2014, I completed my first round of data collection. This first round included all task-based interviews along with their corresponding video-SR interviews for all six students. I took that day and the next to reflect on memos I wrote concerning my data collection process. I reflected on the challenges during the first several interviews. One such challenge was the attempt to sync the recorded task-based interview with the calculator screen capture. This challenge was resolved quickly and did not affect the long-term data collection process. I improvised by viewing both the interview and the calculator capture recordings in separate programs on my computer screen. Because of this, there would be several seconds delay between the video and the calculator screen capture as the student and I viewed their interview. The students used the information
from the calculator screen capture sparingly since the primary data were the video recording of their task-based interview and their written work. Therefore, the time it took to get the calculator screen capture to catch up with the video recording was negligible.

I also reflected on the data collected. This allowed me to develop analytic questions (Bogdan & Biklen, 2007) that led to revising the interview protocol. I included more structured questions specific to the influential factors (i.e., goals, knowledge, affect) that may have impacted their decisions during the problem solving sessions. As a result, the new protocol was used in the second and third round of interviews. This revised protocol increased my focus during the video-SR interviews and ensured that the questions asked would evoke the responses needed to answer my research questions. The questions that guided the questions about local affect are shown in Table 11. The original protocol can be seen in Appendix D and the revised protocol in Appendix F.

Table 11: Sample questions from video-SR interview protocol

<table>
<thead>
<tr>
<th>Local Affect (Emotions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Tell me how you felt about the entire problem solving process?</td>
</tr>
<tr>
<td>• Do you feel like this each time you are presented with a mathematical problem-solving task?</td>
</tr>
<tr>
<td>• How did you feel at this instant?</td>
</tr>
<tr>
<td>• How did this feeling affect what you decided to do next?</td>
</tr>
</tbody>
</table>

My ongoing analysis also allowed me to try out ideas on the students (Bogdan & Biklen, 2007). During data collection, I had an idea to modify and add problems to certain
students in order to get more information about their problem solving process. As discussed earlier in the chapter, Kirsten was given an additional problem because her frustration level was high. Therefore, for her second problem, I thought it was a good idea to forego the originally planned problem (i.e., the Game Problem) and give her another problem that did not use the guess and check strategy. Also, Nathan did not experience any success with the problems given, so I created problems aligned to the content recently covered in his Bridge to Pre-calculus course. These added problems allowed me to get a closer look at their problem solving process. This concluded my ongoing analysis.

**Retrospective Analysis.** The next phase of my analysis occurred during the transcribing of the interviews. All interviews were transcribed verbatim along with a description of all physical cues such as facial expressions, gestures and body language. Being able to capture these physical cues provided further evidence of the emotions experienced during the task-based interviews. Table 12 gives examples of instances when the transcript of the task-based interview includes non-verbal expression that helped me remember and understand the students’ emotions.
Table 12: Sample transcription along with non-verbal expressions

<table>
<thead>
<tr>
<th>Transcript</th>
</tr>
</thead>
<tbody>
<tr>
<td>[slams down pen]</td>
</tr>
<tr>
<td>What did I do wrong?</td>
</tr>
<tr>
<td>[3:12-3:25] [With a confused face, lips twisted to a side, he looks at his written work and on the calculator screen and pauses from speaking and writing]</td>
</tr>
<tr>
<td>What am I thinking right now? I don’t know what to do.</td>
</tr>
<tr>
<td>[Breathes hard, shakes head, continues to tap pen and put into mouth 5:23, shows signs of frustration]</td>
</tr>
<tr>
<td>Oh my gosh why did I do that?</td>
</tr>
<tr>
<td>[Shakes head, frustration level increasing, starts to play with hair]</td>
</tr>
<tr>
<td>[6:14]</td>
</tr>
<tr>
<td>Okay well right now I really want to give up but I want to finish this problem but I don’t know where to go from here. [Exhales loudly]</td>
</tr>
<tr>
<td>[Starts to get frustrated. Puts finger in mouth 17:57]</td>
</tr>
<tr>
<td>Hmm. If Maria lost the 2nd round and she ends up with $12, how much did she give to them? So I’m thinking, that don’t make sense.</td>
</tr>
<tr>
<td>[Starts to show signs of frustration and confusion, scratches head]</td>
</tr>
</tbody>
</table>

**Coding Scheme.** Next in my analysis was the coding of the primary documents, the task-based interviews and video-SR interviews. These documents were coded based on predetermined codes informed by my conceptual framework described in Chapter 2. All coding and organization of data were done using the ATLAS.ti, qualitative software that analyzes large quantities of textual, graphical, audio and video data.

The phases of problem solving (i.e., orienting, planning, executing and checking) were coded for during the first round of coding. The students’ transcripts were coded in the following order, Tony, Kirsten, Nathan, Leann, Phil, and then Dani. This ordering was based on their availability. They were interviewed in this order and their transcripts were coded in this order with the exception of Phil and Dani’s transcripts.
The data were coded for problem solving phases beginning with Tony’s first task-based interview and continued in chronological order. For example, Tony’s first task-based interview was first followed by its corresponding video-SR interview. I then proceeded to the second task-based interview followed by the second video-SR interview and so forth. After completing Tony’s interviews, I then moved on to Kirsten, Nathan, Leann, Phil, and Dani. Each phase was coded as they appeared in the transcript. Table 13 shows examples of passages coded for each of the problem solving phases from the task-based interview and the corresponding passage from the video-SR interview.
Table 13: Example passages

<table>
<thead>
<tr>
<th>Problem-Solving Phases</th>
<th>Description</th>
<th>Sample Quotation from Task-based Interview</th>
<th>Sample Quotation from video-SR Interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orienting</td>
<td>The predominant behavior of sense making, organizing where students may define unknowns, sketch a graph, draw a table, etc.</td>
<td>Kirsten: [3:51] It’s a weird question it's a really weird question, I have no clue where to begin. Wouldn’t all the regions be determined by the points. I think so, yeah. I see what you’re saying. Non-parallel lines what’s that have to do with it. [4:43 yawns] Kirsten: So if they are non-parallel they might be perpendicular but be four regions on a plane. Yeah, think so.</td>
<td>Kirsten: I figured that all the regions would be determined by n because the points on the line I thought it would determine the regions, all the regions. Interviewer: What do you mean by points on the line and the regions. Kirsten: Because that’s what it asked for how many regions on the line is determined by the points on the line, by n points on the line and I figured all the points would affect and determine the regions. Interviewer: Yeah. So what does that mean? Kirsten: Because I think how the points are on the line and the graph would affect what the regions are and how they are determined. Interviewer: So when you thought about lines, you thought about a graph.</td>
</tr>
</tbody>
</table>
### Table 13 Continued

| Planning | Students may devise conjectures about a viable solution approach, imagine the playing out of each approach, imaging how the solution approach would play out, and evaluate the viability of the conjectured approach. | [looks perplexed. Smile is gone for a bit, eyebrows creased. Smiles again and continued]  
Tony: So I will pretend that there’s an even amount of boys and girls so 9 girls and 9 boys.  
[Writes the information down]  
Interviewer: okay. And what happened afterwards, what were you doing here?  
Tony: That’s when I assumed that it had an even amount of boys and girls  
Interviewer: okay  
Tony: And, then I plug into the calculator for the nine and the nine and I realize it went above 5. Then I was like wait, what happened? |
| --- | --- | --- |
| Executing | Engaged in behaviors that involve making constructions and carrying out computations. This may include writing logically connected mathematical statements, accessing resources. | Tony: 5 over 8. I know 1 over 8 equals 0.125 when you times that by 5 you get something.  
[uses the calculator to find 5*0.125]  
Tony: 0.625. But then you want each girl gets 2 cents less than each boy?  
Interviewer: So what was your strategy when you began working this out?  
Tony: You should ignore here  
Interviewer: Why should I ignore here? What were you doing here?  
Tony: Because I put 8 instead of 18  
Interviewer: Oh, so you were thinking here that it was 8 kids instead of 18 in total?  
Tony: yeah |
Table 13 Continued

| Checking | Students shift to verification behaviors, assess the correctness of their computations, included spoken reflections about the reasonableness of the solution and written computation. During this phase, they can accept the result or reject and cycle back | Phil: I think that's it. Let me run it through and see. So, they initially had, Joey had 39, Maria had 21, Helen had 12. Joey lost the first round gave her twice as much, gave her twice as much, gave her 21, 39 - 21 = 18, and gave her twice as much, which is 12, uh huh, 12 and gave her 24 and end up with 6. Yeah, I think that's it. | Interviewer: So what did you do after you found the answer right there? Phil: I think I went to go check it Interviewer: Hmm you checked it Phil: I was going to check the whole thing but I said, “Too much!” Interviewer: You’re already confident that that was it Phil: Yeah |
Coding the data into problem-solving phases allowed one level of organization of the data. To further organize the data, I began coding for the influential factors in the following order, knowledge (content knowledge then problem solving strategies), affect (global affect first then local affect), goals, and finally external contextual factors. This order was chosen based on convenience. I was still getting acquainted with my data; therefore, I coded content knowledge first because it is most prevalent during problem solving. I then coded for problem solving strategies, local affect, global affect, goals, and finally external contextual factors. I will discuss how they were later refined in the upcoming sections.

I began coding for each factor with Tony’s interviews. I coded all six of his interviews for content knowledge only. For example, I coded for content knowledge beginning with his first task-based interview, followed by its corresponding video-SR interview, then his second task-based interview followed by his second video-SR interview, and so forth. After coding all Tony’s interviews for content knowledge, I then coded only for the next factor, problem-solving strategies in the same order. I coded for problem solving strategies beginning with his first task-based and video-SR interview, followed by his second set of interviews, and then his third set. I then continued coding using this same format for global affect, local affect, goals, and contextual factors. After Tony’s interviews were all coded for the factors, I began coding Kirsten’s interviews following the same format. The coding for the remaining students, Nathan, Leann, Phil, and Dani followed next using the same format. A description of these codes along with examples from the data is found in Table 14.
### Table 14: Influential factors, descriptions, and examples

<table>
<thead>
<tr>
<th>Influential Factors for Problem Solving</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content Knowledge</td>
<td>This code refers to the mathematical knowledge of the student.</td>
<td>So it can be $2n$, $n$ being the number of lines and $r$ being the number of regions produced when they intersect but it can also be $2n + (n-2)$, $2n + (n-2)$, which is $3n-2$, when $n$ is greater than 2. But all $n$ is equal to $2n$ for all $n$ so there can be two cases.</td>
</tr>
<tr>
<td>Problem Solving Strategies</td>
<td>This will refer to any specific problem solving strategies that the student may be engaged in. This includes, guess and check, creating tables, working backwards, etc.</td>
<td>Interviewer: When you said that was your trial and error period, when you problem solve do you go through specific periods? Tony: yeah, the trial and error period, the period when I take all the information I got to make a solution to go to contribute to the answer.</td>
</tr>
<tr>
<td>Goals</td>
<td>These are conscious or unconscious goals that are created by the problem solver. It is something that the student wants to achieve.</td>
<td>Not randomly, but I used 15 and I’m going to start in the corner and then add numbers, add numbers until I could find a way to get all of them to equal the same</td>
</tr>
<tr>
<td>Local Affect</td>
<td>This code will be used to represent the less stable and constantly changing affect such as emotions. This will include and is not limited to frustration, happiness, and confusion.</td>
<td>Interviewer: Or did you feel something before everything went blank? Like the whole process Nathan: Anxiety, nervous, scared. Interviewer: Scared. Scared of what? What were you afraid of? Nathan: The problem itself is what I’m scared of.</td>
</tr>
</tbody>
</table>
Table 14 Continued

<table>
<thead>
<tr>
<th>Global Affect</th>
<th>These are the more stable affect such as attitudes and beliefs about mathematics and mathematical problem solving.</th>
<th>Interviewer: so you attributed your ability to solve this problem on what? Tony: It’s just natural.</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Contextual Factors</td>
<td>Other factors not included in Schoenfeld's theory. This will include calculator use and teacher influences</td>
<td>At this point I am annoyed and irritated because first I, I want to, I want to do well on your, on your uhm research. I want to be able to be the person that solve the entire problem. And now, I’m struggling and I’m not getting it, and I’m really not good with word problems, I don’t want you to think I’m stupid, so now it’s like “oh my goodness.”</td>
</tr>
</tbody>
</table>
While coding for influential factors, I made additional memos and observer comments in my researcher journal concerning themes that emerged for each factor. However, with this large data set, another representation was needed. I then created tables, which I referred to as summary tables, to organize the data. These tables allowed me to look within and across students’ data to identify themes about each factor. Information in the tables was organized by problems as the columns and factors as the rows. I created a separate table for each student and organized the factors in the table in the order in which they were coded. However, it became evident that this order was not efficient for noticing connections among the factors. Therefore, I changed the order to: problem solving strategies, goals, content knowledge, local affect, global affect, and external contextual factors. For these tables, problem-solving phases were not included.

I then summarized the data for each factor across problems. For example, I looked for the task-based interview that corresponded to the first problem, the Money Problem. I then looked at the data coded problem-solving strategies, and then summarized the strategies used to solve the Money Problem. I then looked at the data coded goals and summarized the goals used to solve the Money Problem. This was repeated for content knowledge, local affect, global affect, and external contextual factors. I then repeated that order using the students’ video-SR interviews this time. For example, I summarized their problem-solving strategies discussed in the video-SR interviews for the Money Problem, followed by goals, content knowledge, etc. In the summary table, TB precedes the summary from data gathered from the task-based interview data and video-SR precedes data from the video-SR interviews. This
procedure was then completed for the rest of the problems. Table 15 shows a portion of one student’s table. The full table is presented in Appendix G.

Table 15: Sample summary table

<table>
<thead>
<tr>
<th>Problem solving strategies</th>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
<th>Problem 4</th>
<th>Problem 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB &amp; V-SR: Read and rereads problem</td>
<td>None</td>
<td>TB: None</td>
<td>TB: Attempts to create a formula, reread problem multiple times for clarity, does not</td>
<td>Simplifying, making conclusions</td>
<td></td>
</tr>
<tr>
<td>Ratios (5/18), subtraction (.27-.02), V-SR: Wanted to solve algebraically but could not create a formula</td>
<td>TB: Did not know the meaning of plane, wanted numbers to work with</td>
<td>TB: Wanted to create a formula but could not, V-SR: Did not understand $3 \times 3$, only knew $n \times n$ in terms of pant size, cannot draw a picture for math</td>
<td>Comparing quantities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TB: Frustrated V-SR: not confident, dumb, doubting self, confused, not happy, agitated (needed to calm self down by whistling, rubbing head)</td>
<td>TB: Frustration V-SR: Embarrassed, frustrated, anxiety</td>
<td>TB: Frustration (‘this is already getting to me), scared (of content, language, terms)</td>
<td>TB: Frustrated, defeated, no confidence, V-SR: has anxiety when solving math problems (‘everything goes blank’), nervous, scared (didn’t know what to put in the box)</td>
<td>TB: frustration, defeat V-SR: Anxiety, doubting self, indecisiveness</td>
<td></td>
</tr>
</tbody>
</table>

The summary tables gave me an organized representation of the data that was used to identify emerging themes within the problem solving factors.

**Themes.** After revisiting the literature on goals-oriented activities by Schoenfeld (2010), I treated goals as omnipresent. This interpretation is viable because problem solving
is a goal-oriented activity and goals were set and achieved continuously throughout problem solving. Therefore, goals were set during each phase of the problem solving process. Influenced by Carlson and Bloom’s (2005) work on the problem-solving phases, Schoenfeld’s description of goals, and preliminary findings from my summary tables, goals were organized by the problem solving phases.

Carlson and Bloom’s (2005) description of the problem-solving cycle along with emergent themes influenced the themes for goals. According to Carlson and Bloom (2005), during the **orienting** phase, the student may be engaged in reading and rereading the problem, organizing the givens in the problem, and making general sense of the problem by drawing tables, graphs, diagrams, or recording text. Therefore, from my summary tables, I noticed that when students set goals to orienting themselves with the problem, they engaged in behaviors such as *reading the problem*, *organizing*, *connecting*, and *defining*. These behaviors were the themes that emerged from the summary tables.

In contrast, themes related to the other factors (i.e., knowledge, affect, and external contextual factors) did not rely on the literature. These themes emerged from my analysis of the data in the summary tables. This “involves consolidating, reducing, and interpreting what people have said and what the researcher have seen and read” (Merriam, 2009, p. 176). Table 16 is an abbreviated example of the themes and descriptors for goals. Table 17 shows an example for local affect. The complete collection of themes is found in Appendix H.
Table 16: Themes for goals

<table>
<thead>
<tr>
<th>Goals</th>
<th>Themes</th>
<th>Description</th>
</tr>
</thead>
</table>
| Orienting   | Reading the problem         | • Effort is made to read and reread to understand the problem.  
• Restates givens                                                                 |
| Organizing  |                             | • Effort is made to organize the problem by creating a table, graph, diagram, text, or expressed verbally  
• Effort is made for sense-making in general |
| Connecting  |                             | • Effort is made to connect problem to one familiar                                                                                 |
| Defining    |                             | • Terms are defined                                                                                                                  |
| Planning    | Choose a problem solving strategy | • A strategy is determined                                                                                                           |
|             | Choose an alternate problem solving strategy | • An alternate strategy is identified.                                                                                              |
| Executing   | Carries out computations    | • Calculations are carried out                                                                                                       |
|             | Implements a problem solving strategy | • *Guess and check* and other strategies are worked out                                                                             |
| Checking    | Testing results             | • Results are tested for their reasonableness  
• Decides to accept solution (correct or incorrect)  
• Gives up without solution |
Table 17: Themes for local affect

<table>
<thead>
<tr>
<th>Themes: Local affect</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anxiety</td>
<td>Expresses anxiety, nervousness, scared</td>
</tr>
<tr>
<td>Confusion</td>
<td>Expresses confusion about problem. Also feels hopeless, lost, clueless, incompetent</td>
</tr>
<tr>
<td>Frustration</td>
<td>Expresses anger, frustration or impatience, agitated, embarrassed, dumb, annoyed, sad, upset, stupid</td>
</tr>
<tr>
<td>Pleasure</td>
<td>Expresses positive emotions. Can also include feeling accomplished, happy, confident, comfortable, amused, enlightened, excitement, triumph</td>
</tr>
</tbody>
</table>

After analyzing the summary tables that generated these themes, I conducted a third round of coding. The themes related to goals, local affect, global affect, external contextual factors, content knowledge, and problem-solving strategies were coded in that order. Examples of the data that provide evidence of these themes will be shared along with the analysis in Chapter Four. At this point in my research, the data were sufficiently analyzed to look within and across cases to create rich descriptions of the cases, looking specifically for similarities and differences to answer my research questions and to validate the problem-solving theory.

In multiple case studies, it is common to provide both a within-case analysis and a cross-case analysis (Creswell, 2013; Merriam, 2009). In a within-case analysis, each case is treated as if it were a single case study where the data gathered are used to understand each case in-depth (Merriam, 2009). In a cross-case analysis, the researcher looks for similarities and differences among the cases in order to “build abstraction across cases” to create general explanations pertinent to each case (Merriam, 2009, p. 204). I analyzed the data to create rich
descriptions of each case. I reviewed all my documents, (i.e. transcripts, student work, researcher journal) and described each case according to goals, knowledge, affect, and external contextual factors. These rich descriptions are not included in the document because the cross-case descriptions answered the research question. However, the individual case descriptions were vital in the cross-case analysis.

The chronology of my within-case and across-case analyses was cyclic in nature. My analysis is organized by the factors that influence the decisions students make during problem solving (i.e. goals, knowledge, affect, external contextual factors). Within each factor, I provide a rich description of how that factor influenced the choices that each student made during problem solving. After describing each student’s data regarding that factor, I conducted a cross-case analysis of all the students. For example, for goals, I created a rich description for each student that explains in details using all forms of data how their goals influence their decision making during problem solving. After completing each student’s rich description, I then conducted a cross-case analysis on goals, where I looked for abstractions across cases. This procedure was repeated for each factor. My cross-case findings are reported in Chapter Four.

**Reliability and Validity**

In qualitative research, rigorous measures must be implemented to prove the credibility of results (Creswell, 2013). The data-driven findings from qualitative study depend on the level of rigor of the process that generated the data (Cobb, Confrey, diSessa,
Lehrer, & Schaubb, 2003; Creswell, 2013). In this study, I refer to Merriam’s (2002) three types of validity; reliability, internal validity and external validity.

**Reliability**

In qualitative research, reliability is important because it forces transparency in data collection and consistency between the data and findings (Merriam, 1998). In the previous section, I described my analysis procedures so that, if warranted, replication could be done. The issue of reliability was addressed at the initial design of my study. I used triangulation, using different forms of data sources such as task-based interviews, video-SR interviews, student artifacts, and the researcher journal to address this issue. These data sources along with a detailed and research-supported conceptual framework were outlined in Chapter 2. Transparency in subject selection along with purposefully selected students that represented the subject pool also addressed reliability. Finally, articulating my biases towards the end of this chapter will also support the reliability of my study.

**Validity**

Internal and external validity are important components in qualitative research (Merriam, 2009). They are both necessary to increase the trustworthiness of the research findings. I discuss below how I addressed the issues of internal and external validity in my research.

**Internal Validity.** To increase the internal validity of qualitative research findings, at least two of the eight recommended procedures for validating qualitative research must be employed (Creswell, 2013). Of the eight procedures, triangulation, prolonged observations,
peer review, rich descriptions, and a researcher bias statement are included in this study (Creswell, 2013). This study includes triangulation, which was discussed previously and also used to increase reliability. I observed the students over a prolonged period of 6 weeks and conducted interviews with them approximately 6 times during that time period. I was able to build a relationship with the students with these frequent meetings. There were elements of my data sources that were peer reviewed. For example, prior to data collection, my dissertation chairperson and my committee members reviewed all interview protocols and problems during my proposal defense. Before data collection, the problems were also tested and reviewed during two separate mini-pilot studies on the problems. These reviews served to clarify the problems and ensure they were accessible to the students. Throughout the analyzing phase, there were regular meetings with my advisor to discuss any questions I had regarding how to conduct my analysis and how to articulate appropriate findings to answer my research questions. I include rich descriptions of each case in the next chapter as part of my findings. Lastly, I include a bias statement at the end of this section.

**External validity.** To promote external validity, Merriam (2002) suggests that detailed rich descriptions must be used. These rich descriptions are part of my findings. To further improve external validity, it is also necessary to have multiple cases so that the likelihood of generalizability is high (Creswell, 2013). In this study, I included six cases that represented varying mathematical knowledge and beliefs and attitudes about mathematics and problem solving so that the possibility of representing different types of students is
possible. I conducted cross-case analysis (reported) and within-case analysis (not reported) to further address any issues of external validity.

**Bias Disclosure**

**Background of the Researcher.** Mathematics always came easier for me than other subjects in school. It was the only subject that kept my attention. When it was time to attend college, it was a natural decision to major in mathematics. I did not begin my undergraduate career with the dream of becoming a teacher. In fact, I was on the track to pursue a research career. After attending an undergraduate research experience in mathematical biology, I knew that this type of research was not my passion. I enjoyed learning and working with others so I considered a career in teaching. Having no education courses during my undergraduate studies, I decided to immediately attend graduate school to study mathematics education.

After the completion of my graduate degree, I taught high school mathematics for six years. During that time, I encountered a wide range of students, those who had great difficulty in mathematics and those who grasped concepts quickly. I became interested in how students come to learn mathematics. I then began the journey of earning my PhD in mathematics education. During this time, my perceptions of mathematics teaching and learning changed. I now see the importance of having students see and view math as sense making and for them to engage in mathematics in context to the world around them. I became more of a constructivist and began to see the importance of students working collaboratively and building on their prior knowledge.
Problem solving is one of my interests. I believe that when students are exposed to higher-level mathematics, they become more confident in their abilities in mathematics and feel comfortable seeing mathematics in new contexts.

As a mathematics educator, I believe that all students should be given the opportunity to engage in meaningful mathematics. As a teacher, I have seen how certain groups of students were not given the same opportunities to learn as others. It is my desire to decrease this occurrence in our schools. It is my newfound desire to be able to teach prospective and practicing teachers the tools needed to allow their students to do meaningful mathematics.

**Role of the Researcher.** It was truly a pleasure for me to work with these students during the summer. They were very generous with their time and honest with their responses. The program directors and instructors were accommodating allowing me to collect the data needed. When I first met the team, the program was in need of someone to facilitate a seminar course on how to improve their intelligence that half of the students would take. After discussing potential ethical conflicts with my dissertation chair I volunteered to teach the course. There was limited conflict of interest because it was not a mathematics course. There were two students, Tony and Dani, who were enrolled in the course and were also part of my research. This relationship outside of the research may have affected their problem solving.

I am connected to the HBCU that hosted the summer bridge program. I previously lived in that community, completed my undergraduate degree at the institution, and taught there as an adjunct faculty. This may cause me to be bias with the results. However, the
careful design of the study to address reliability, internal and external validity as previously discussed will minimize these biases.

**Ethical Issues**

I was given consent to conduct this research prior to its start date through the Institutional Review Board (IRB) at North Carolina State University, and also the University of the Virgin Islands. Potential participants were invited to participate in the study via a consent letter sent by the program in their mailed correspondences. There was minimal risk to the student for participating in my study because their participation did not affect their participation in the program. They did not miss any substantial amount of instructional time because interviews were conducted during their lunch break or when they did not have a course. Pseudonyms were created for the students and their names were removed from all documents that could be linked back to them. Data was stored on the researcher’s password protected computer, on the university’s Google drive, and on a password protected external hard drive. Paper data was stored in the researcher’s office that was kept locked. Adhering to the guidelines described in the IRB from both institutions served to reduce any ethical issues that may have surfaced.

**Limitations**

As with qualitative studies of this nature, the ability to generalize the findings beyond the students that participated in my study is not likely. However, with the rich descriptions of each student, there is much to learn about students that share similar characteristics. Another limitation to the study is the inability to include problems whose rigor was due to variability
in content and problem-solving strategies. I wanted the students to be able to engage with the problems and thought it best to make the mathematics as common as possible. However, the problems that were presented provided relevant data because most students were able to engage with the problems even if not successful in getting the correct answer. Memory decay was also a limitation of the study. However, as discussed previously, most video-SR interviews occurred within one day of the task-based interview to help minimize its effect.

**Chapter Summary**

This study has two goals, (1) to provide detailed, rich descriptions about how and why students make decisions during problem solving and (2) to validate the proposed problem-solving theory. Therefore, the following overarching research question is proposed:

In what ways do students’ goals, knowledge, affect, and external contextual factors inform their choices during the mathematical problem solving process?

To answer this question, a multi-case methodology was used since students’ cognitive and affective processes while problem solving are measured best using qualitative measures (Goldin, 2000). Student volunteers accepted into a summer bridge program hosted by an HBCU were the cases in this study. These cases were used to conduct a cross-case analysis of how each factor influences the choices students make during the problem solving process.

The conceptual framework I created for this study influenced data collection. Data from task-based interviews, video-SR interviews, student artifacts, and researcher field notes were gathered and analyzed. All interviews were audio and video recorded and transcribed verbatim before being analyzed and coded specifically for the factors (i.e., goals, knowledge,
affect, and external contextual factors) that influence problem solving success and any other emergent themes.

As with any research, there are challenges that may arise as a result of the methodology, researcher bias, and/or ethical concerns. However, the careful design of the study addressed reliability and validity concerns. The following chapter contains the findings from this research study. I present rich, detailed descriptions of my cross-case analysis organized by the main factors: goals, knowledge, affect, and external contextual factors.
Chapter Four

The purpose of this chapter is to provide rich descriptions of how the factors identified in the conceptual framework influenced the choices the students made during their problem-solving sessions. During the creation of the codebook for this study I realized there were aspects of the conceptual framework that needed to be revised. As described in Chapter Three, goals were set and achieved throughout each stage of the problem solving process. Therefore, I moved goals from outside to inside to represent its omnipresence in the problem solving process. In addition, since knowledge of content and problem solving strategies is very much intertwined, it was helpful to think about their influence on decision making together rather than separately. The same was true for local and global affect. These revisions are indicated in the revised conceptual framework in Figure 4.
This chapter is organized into four main sections according to the revised conceptual framework. They are goals, knowledge, affect, and external contextual factors. For each section I present findings from the cross-case analysis. Within each factor, I present themes that emerged from the data along with examples as evidence of those themes. Examples were taken from task-based interviews, video-SR interviews, and students’ written work. It is important to note that all students are not represented in all themes. The examples chosen throughout this chapter are intended to be exemplars of the themes and to help provide a rich description of the ways in which each influenced the choices these students made during their
problem solving sessions. Since the study’s focus was *how* the factors identified influenced the students’ decision making during problem solving, the correctness of solutions was not of primary importance. However, Table 18 provides information regarding the performance of each student. Student written work are provided in Appendix I.

Table 18: Student performance on problems

<table>
<thead>
<tr>
<th></th>
<th>Money Problem a</th>
<th>Game Problem</th>
<th>Points, Lines, and Planes Problem b</th>
<th>Magic Square Problem</th>
<th>Camper Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tony</td>
<td>1. ✓</td>
<td></td>
<td>a.</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td></td>
<td>b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leann</td>
<td>1.</td>
<td></td>
<td>a.</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td></td>
<td>b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kirsten</td>
<td>1.</td>
<td></td>
<td>a.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td></td>
<td>b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dani</td>
<td>1. ✓</td>
<td>✓</td>
<td>a. ✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td></td>
<td>b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nathan</td>
<td>1.</td>
<td></td>
<td>a.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td></td>
<td>b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phil</td>
<td>1. ✓</td>
<td></td>
<td>a.</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td></td>
<td>b.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. A check mark (✓) corresponds to a correct solution for a student on the corresponding problem.

a The Money Problem has two solutions. b The Points, Lines, and Planes Problem has two parts (a and b).

Goals

In this study, goals, either conscious or unconscious, are created by the problem solver and represent what the solver wants to achieve. Mathematical problem solving is a goal-oriented activity; therefore, goals were numerous and occurred before the student formally began solving the problem and during each phase of the problem solving process.
As explained in Chapter Three, the themes that emerged for goals were behaviors exhibited by the students during each problem solving phase (i.e., orienting, planning, executing, and checking). When students set goals to orient, they engaged in reading, rereading, restating the information provided in the problem, organizing, or defining terms. Goals set to plan, led to students engaged in choosing a problem-solving strategy, and choosing an alternate problem-solving strategy. Students who set goals to execute, engaged in behaviors such as carrying out computations, implementing problem-solving strategies, and validating conjectures. Finally, goals to check the solution resulted in students testing the correctness of their results. In this section, I present cross-case findings for goals:

**Goals: Orienting.** Goals set to orient were related to reading, organizing, and defining. To achieve this goal, most students engaged in reading the problem. Reading the problem includes reading the problem initially, rereading the problem after attempting to solve the problem, and restating information aloud provided in the problem.

Of course, all students read the problem initially. However, most students also reread the problem after attempting to solve it. They all claimed that rereading was done to keep them focused on the details, for comprehension, and as a way to regroup after getting stuck. For example, while solving the Money Problem, Dani read the problem several times and attempted to begin when she said, “What am I thinking right now? I don’t know what to do.” After thinking for a bit, she paraphrased the problem, “Split five dollars among 18 children so that each girl gets two cents less than each boy. Split five dollars that’s each $2.50 boys and girls, but girls are getting two cents less.” Still having difficulty, she
paraphrased the problem again, “Okay divide five dollars amongst 18 children and everyone has to get some so each girl gets two cents less than each boy.” For some students, rereading was intentional. For example, Leann reread to make annotations. That is, she underlined phrases she deemed as important. When asked if this was something she did for every math problem, she answered, “I would say any problem it’s not just limited to math.” See Figure 5 for an example of her annotations.

Place the positive integers 1-9 in a 3 x 3 grid. Each integer should be used only once and only one integer should be placed in a grid. Arrange the integers such that the sum for each row, column and diagonal is the same.

Figure 5: Examples of Leann’s annotations

To help get oriented to the problem, some students not only reread the problem, but also restated the problem in their own words. Some times this was done by just repeating portions of the question, and sometimes by rephrasing the problem using more familiar language. The example given above for Dani shows an example of her rereading and paraphrasing the problem when she got stuck. After this, she was able to continue implementing the guess and check strategy by changing her initial guess for the amount of money each boy received.

Students who only engaged in rereading the problem did not always accomplish their goal of getting oriented with the problem. For example, Nathan reread most of the problems several times. However, his rereading did not result in understanding the problem so he did
not set any further goals, but instead stopped altogether. For example, after reading the Game Problem several times, he said, “This will take a while. Hmm. I can’t process. Oh mehn. [exhales loudly] oh my gosh. You say you can call it quits at anytime right?” After indicating that he could, he said, “I call it quits right now mehn.” He was conscious of his decision to quit after reading and not making any progress.

Sometimes rereading led to students creating goals within other phases of their problem-solving process as well. For example, when Phil initially read the Game Problem he assumed that each player began with the same amount. However, after implementing this assumption with the guess and check strategy, he revised his assumption. He began by rereading the problem which led to him changing his original assumptions.

To orient themselves to the problem students engaged in behaviors that I refer to as organizing. When students were organizing, they created tables, graphs, and diagrams to help them make sense of the information presented. Figure 6 shows an example of diagrams Tony drew during while solving the Points, Lines, and Planes Problem.

Figure 6. Example of Tony’s graph
In addition to written organizational tools, students also used verbal statements to represent the mental organizing they were doing. For example, Nathan did not provide any written work for the Camper Problem. Nevertheless, there was evidence that he was organizing information from the problem. When solving the Camper Problem he stopped working. When asked if he had decided to stop working on the problem, he explained, “I’m trying, I’m really trying to comprehend this like 5 grams, how much they get paid, how much they contribute, how much they should get for contributing from $8, who gets more, who gets less from the $8.” Nathan was trying to organized the information verbally to make sense of the information provided in the problem, but did not attempt to write the organizational scheme on paper.

Behaviors related to defining terms were witnessed less frequently when students set goals to orient. Half of the times defining was identified in this study it was initiated by Tony. He defined terms in order to make sense of the problems. For example, while solving the Points, Lines, and Planes Problem, he deemed it necessary to define regions. For part $a$, he defined regions as curves, and for part $b$, regions were defined as quadrants. The defining of these terms influenced his choice to use algebraic concepts to solve the problem. Kirsten also defined terms while solving the Points, Lines, and Planes Problem. She explained, “So it said non-parallel so I was thinking perpendicular. So then I was thinking there would be 4 regions. Because I was thinking about a graph again and thought it was split it into 4 regions.” To make sense of the problem, she defined non-parallel lines as perpendicular lines.
with which she made connections to the coordinate plane and to define regions as quadrants. She then proceeded to also use algebraic concepts to solve the problem.

When students set goals to orient themselves with the problem, they displayed behaviors related to reading, rereading, organizing, and defining. Their success in achieving these goals influenced their next steps. They had to decide if they would persevere and if so, which knowledge (problem solving and content) they would access.

**Goal: Planning.** Students who were able to achieve goals in orienting moved to set goals related to planning. This was manifested when they engaged in identifying a problem-solving strategy along with identifying alternate problem-solving strategies. Note that details pertaining to the specific strategies that were selected will be reported with the goals set to execute. During the task-based interviews, most students did not explicitly state their plan to implement a specific problem solving strategy. However, this information could be inferred from their written work and gathered through their video-SR interview. Behaviors to choose a problem solving strategy were usually observed first, followed by implementing a problem solving strategy and lastly, to find an alternate problem-solving strategy. For example, when solving the Magic Square Problem, Leann choose a problem-solving strategy. She began by looking for a pattern and began implementation. Shortly after, she abandoned that strategy and began looking for an alternate strategy. She then implemented the guess and check strategy. When that did not work, she then began looking for an alternate problem-solving strategy. This time, she spent more time exploring alternate problem-solving strategies.
Exploring different strategies resulted in her writing down multiplication facts as shown in Figure 7.

![Leann's multiplication table](image)

Figure 7: Leann’s multiplication table

When asked why she decided to try that path, she responded, “I have no idea why, why I put times tables”. When asked, “What do you think you were hoping to find?” she replied, “I don’t know. I was just trying something. I try different things.”

There is also evidence that Phil also set goals to plan by engaging in behaviors to choose a problem-solving strategy and then cycled back to explore an alternate strategy. For example, while solving the Money Problem, he selected the guess and check strategy at first. He then cycled back to find an alternate strategy which led to him to explore formulating an equation. When asked if his plan ever changed, he said, “Kind of, yeah. When I came down here I was trying to see if I could create a formula that would help me to find at least how much girls would get a certain amount of coins.” In contrast, there were also times where students choose a problem-solving strategy and did not return to the planning phase. For
example, to solve the Money Problem, in setting goals to plan, most students looked for a problem solving strategy and implemented the *guess and check* strategy. This strategy was not changed and most times led to students not persevering in solving the problems. Goals to plan were achieved by students choosing a problem-solving strategy or choosing an alternate problem-solving strategy. Students ultimately implemented the strategy that they chose. If implementation failed, they may choose an alternate problem-solving strategy.

**Goals: Executing.** Students set goals to execute their plan. The behaviors students engaged in to achieve this goal were related to implementing a problem-solving strategy and carrying out a computation. All of the students who were able to choose a problem-solving strategy were able to implement this strategy. Phil demonstrated an example of this process as he solved the Game Problem. He identified that he will be using the *guess and check* strategy and proceeded to describe the implementation of this strategy. The following is an excerpt from his task-based interview.

So. Okay, so I going to write down their names. I’m thinking, I'm going in a route of guesswork. So, they all had $24 at the end of each, of 3 rounds. Each lost one time, so, hmmm. So I just divided from the $24 by the 3 rounds, I got 8. I'm not too sure what I would do with that, but I'm thinking if they had to pay each other, hmmm. 'Cause how I was thinking about it at first, I was saying all of them equally had $24. But then in the same round 1, Joey lost. He would have to pay $24 to both Maria and Helen, but he doesn't have 2 sets of $24 to give. So, my next thought would be,
probably one had more than the next, or one had more than the other and the other had still had more than the other, I'm not too sure. I think that may be it. Umm.

This excerpt shows how Phil made an initial guess of $8.00 and is checking the validity of the guess within the problem’s constraints.

Students carrying out a computation to achieve the goal of executing typically carried out arithmetic computations. For example, Nathan used arithmetic to solve the Money Problem. Figure 7 shows his written work.

Figure 7: Nathan’s written work for the Money Problem

However, not all computations were arithmetic in nature, some were algebraic. The Camper Problem required algebraic computations using ratios and proportions. For example, Dani solved this problem correctly using proportions. Her written work is shown in Figure 8. This figure shows her written work where she first determined the amount of rice Campers A and B each contributed to Camper C. She then confirmed that both shares summed to the amount each camper should receive, 266.66 grams of rice. She then created proportions comparing the amount of rice contributed to Camper C to the amount of rice kept, which she
set equal to the amount of money they should be compensated to the total amount of money. She solved the proportions to arrive at her solution.

\[
\frac{500}{266.66} = \frac{200}{33.33} \\
\frac{23.33}{266.66} + \frac{x}{8} = \frac{23.33}{266.66} \\
\frac{33.33}{266.66} = \frac{x}{8} \\
\frac{23.33}{266.66} = \frac{x}{8} \\
\frac{23.33}{266.66} = \frac{x}{8} \\
\]

Figure 9: Dani’s written work for the Camper Problem.

The ability for students to successfully carry out a computation was contingent on their ability to work within the mathematical knowledge required (To be discussed in further detail in a later section.). When students set goals to execute they mainly chose to implement a problem solving strategy and carry out a computation. The strategy they implemented and the computations they were able to execute were all contingent on their knowledge of problem-solving strategies and content.

**Goals: Checking.** The goals students set to check led to them checking their solutions for correctness. Most students (Tony, Leann, Dani, and Phil) purposefully checked their solutions when they felt they reached a solution to a problem. They gave several reasons for checking their solutions. They included (a) the answer did not feel right, (b) to double check their solution, and (c) an anomaly in calculations. For example, Tony reflected on his
actions after giving his final solution to the Game Problem said, “I think I was checking over. I had still feel like it wasn’t right.” When asked what he did since he was unsatisfied with his solution, he said “Ignore it.” When asked why, he responded “Because it made sense after I checked it over. Or the way I interpreted the question. After he checked his solution, although it did not feel right, it satisfied his interpretation of the problem.

Phil provides an example of double-checking a solution. He checked his solution to the Game Problem after using the guess and check and working backwards strategies. He said,

I think that's it. Let me run it through and see. So, they initially had, Joey had 39, Maria had 21, Helen had 12. Joey lost the first round gave her twice as much, gave her twice as much, gave her 21, 39-21=18, and gave her twice as much, which is 12, uh huh, 12 and gave her 24 and end up with 6. Yeah, I think that's it.

Phil’s statement, to “run it through and see”, is evidence of him checking the problem. After meeting this goal, he proceeded to submit the answer as his solution. Dani also double-checked her solution to the Points, Lines, and Planes Problem. After creating her formula, she checked the validity of the formula by substituting five for the number of points. She stated, “Okay. So let’s say we have a line and there are 5 points, 5+1 that’ll be 6. I doubt that but let’s try it anyway. 4, 5, 2, 3, 4, 5, 6 Aha! I conquered you!” She was elated that she had arrived at the correct solution.

There were also students who noticed anomalies in their calculations and proceeded to check their answer. This checking did not involve the final solution since these
calculations were needed to arrive at the final solution. For example, Tony, Phil, and Dani all experienced anomalies in their calculations and rechecked their written work. One example is when Dani was solving the Money Problem. She assumed the number of girls and boys were the same and said that 18 divided by 2 is 6. Checking her calculations she encountered the error and said,

Oh crap. Look what I did. Six plus six is not 18. 6+6 is 12. Lord have mercy. Okay let’s try again. 18 children. 9+9 oh gosh. How would I make it 6? 9 boys 9 girls. I’m busting my brain wondering why this ain’t working.

After she found her error, she continued solving the problem, using the correct values in implementing her guess and check strategy.

One student, Leann, did not check her solution using much rigor. For example, Leann checked her solution by just rereading the question again. The following conversation occurred during her video-SR interview about her checking methods while solving the Game Problem.

Leann: I was just there saying that it seems correct I don’t really see it being a different number anymore. So I said let me just read it over one more time to make sure.

Interviewer: So is that the way you check your answer the problem by reading the question?

Leann: Uh huh (yes) just to make sure I read it correctly in the first place.

Interviewer: So did you think about checking it another way?
Leann: Not really, I just went over the math again in my head. There was no written evidence of her checking and she submitted the answer that she arrived at as her solution to the problem.

Students set goals to check their solution based on whether they were confident in their answer. For example, Phil checked his solution to the Game Problem and the Magic Square Problem, two problems that he solved correctly. He struggled with the Points, Lines, and Planes Problem and did not check his solution. Also, Leann did not check her solution when she knew it was incorrect. She said, “I did not check it. I already knew it was wrong. I did not have to check it.”

Knowledge

The most prevalent factor in problem solving sessions is the knowledge students possess to engage in the task. As discussed earlier, my conceptual framework was revised to reflect the notion that knowledge of problem-solving strategies and content knowledge is intertwined, therefore how they influence students’ decisions occurs simultaneously. In this study, students made decisions related to their knowledge based on the following themes: knowledge of traditional problem-solving strategies, knowledge of recently encountered content, knowledge of similar problems, and knowledge of mathematical relationships. Students used content knowledge relating to algebra, arithmetic, geometry, and number sense/reasoning but were most proficient in arithmetic ideas. A full description of these themes is found in Chapter Three and Appendix H. In this section, I address how students’ decisions were influenced by each theme separately.
Knowledge of Traditional Problem Solving Strategies

Students used their knowledge of traditional problem-solving strategies to decide how to solve the problems. I will refer to traditional problem-solving strategies in this section as strategies. As covered in the literature, students are taught to employ different strategies to solve the problems. These strategies were first documented with Polya and soon gained popularity; and many were revised and added over the years. However, in this study, the themes based on students’ strategies include: *guess and check*, *look for a pattern*, *make a drawing*, and *working backwards*. I refer to these strategies as traditional since they appeared on the early lists of problem-solving strategies and are frequently mentioned and taught. I organize this section according to the problem-solving strategies themes and discuss how these strategies along with their mathematics content knowledge affect how they solved the problems.

**Guess and check.** The *guess and check* problem-solving strategy was the most commonly used among students. This may be because all of the problems could be solved using this strategy and the students were all familiar with this strategy. Several students referred to *guess and check* as ‘trial and error’. This strategy requires students to make a guess based on credible assumptions and conjectures, check the results of the guess, and then revise the guess based on results from the check. Although most students used this strategy, those who were strategic, organized, and stronger in mathematics content knowledge spent more time meaningfully engaged with the problems. For example, Tony used *guess and check* to solve the Magic Square Problem, however, it was his mathematics content
knowledge that influenced how he organized his guesses. When asked about his strategy, he said,

So like 2 odd numbers will make an even number and I’m trying to find an odd number which is 15 and 3 odd numbers so I was thinking put the odd numbers at the corners and put the even numbers in the middle so the structure would be uniform.

His knowledge of even and odd numbers allowed him to make crucial connections between the integers, their position in the grid, and the sums to find his solution. However, not all students were able to use the guess and check strategy with such precision. Their inability to make connections between their content knowledge and the requirements of the problem caused them to make unstructured, inefficient guesses. They referred to these instances in various ways. When asked about these guesses made when the students were stuck and were not clear about their next steps, Tony referred to it as “deviate from the norm”, Phil as “just trying stuff”, Kirsten as “trying a bunch of random numbers”, and Leann as “trying stuff” and “throwing out stupid answers”.

The guess and check strategy was also the strategy students tended to use when they did not know of another way to solve the problem. For example, Leann used guess and check to solve the Points, Lines, and Planes Problem, even though this strategy was not appropriate. This problem required students to create a formula so when asked if she understood part b of the question, she responded, “no…this whole section was guesswork” and “I do not know what is going on”. Leann also inappropriately used guess and check to solve the Money Problem. When asked why, she stated, “I couldn’t come up with an exact thing to use,
formula to use, so I was just trying random numbers checking them and seeing and hopefully it would work.”

When students understood the mathematics content and used *guess and check*, most made simplifying assumptions to structure their guesses. The most common simplifying assumption was to set quantities equal when there were multiple co-varying variables. For example, in the Money Problem, students assumed that the number of boys and girls were equal then used *guess and check* to find the amount of money to allot to each. In the Game Problem, they assumed that all players began with the same amount of money. These were all viable simplifications that could cause students to advance towards the solution. Two students were able to make the adjustments to this original simplifying assumption to arrive at the solution. However, two could not and gave up after trying for several minutes.

**Look for a pattern.** Students primarily used the *look for a pattern* strategy to solve problems requiring the creation of a formula. Hence, students who were able to solve the Points, Lines, and Planes Problem used this strategy. For example, Tony looked for a pattern between the number of lines and the number of regions to create his formula and Dani found a pattern between the number of points and the number of regions formed on the line to create hers. However, some students used the *look for a pattern* strategy when they were unsure of a strategy to use. For example, Leann explored using this strategy when she first attempted to solve the Magic Square Problem. She tried to determine where to place the integers into the grid by placing them in a numerical pattern. Figure 9 shows her written work of the patterns she was trying to make.
However, she did not make progress with this strategy and completed the problem using the *guess and check* strategy. Students did not use this strategy in isolation, but combined it with others to solve the problems. This will be discussed later in the multiple strategies section.

**Make a drawing.** When students decided to make a drawing to solve the problems, they primarily chose to draw tables and graphs. For example, all students made drawings to solve at least one of their problems except Nathan. Students chose tables most frequently to organize information. Students mostly used tables in organizing the Game Problem. Four students created tables; an exemplar is Phil’s table shown in Figure 10.
Students with stronger mathematics content knowledge attempted to draw graphs of functions. There were several students who used an algebraic interpretation of the Points, Lines, and Planes Problem, but only one student drew the graphs of the functions to solve the problem. An example of Tony’s graphs is shown in Figure 11.

![Figure 11: Phil’s table for the Game Problem](image)

Students made drawings in their attempt to understand the problem. One example is Kirsten. She was given an additional problem to determine the possible integral lengths of sides of a triangle with perimeter 12. She used the *make a drawing* strategy to solve the problem. When asked about the triangles, she explained that she drew the triangles “thinking
that if I could get a visual representation of it, it will help me think about it harder. But it worked a bit but not completely.” Kirsten attempted to draw the triangles with differing side lengths without using a ruler, straight edge, or other measuring tool. When questioned about how she knew the sides actually created a triangle, she said, “By the lengths of the sides. I tried to draw it as accurately as I could.” She did not have a tool to measure the lengths precisely therefore they all created triangles. The ability to use the drawing to solve the problem correctly was hindered by her content knowledge. The diagrams she drew are found in Figure 12.

![Figure 13: Kirsten’s triangles](image)

Nathan was the only student who did not make a drawing to solve the problems. However, he was aware of this strategy’s existence. He stated that as a visual learner, he would rather draw pictures or solve problems with diagrams. However, when asked if he could draw a picture for the problems, he said, “No, I can’t draw a picture for math. That kind of seems impossible for math.” His limited content knowledge prevented him from even trying to make a drawing. He believes that he does not have the ability to do so. Students also
used this strategy in combination with other strategies, which will be discussed in the next section.

**Multiple strategies and other strategies.** The students in this study were knowledgeable about traditional problem-solving strategies. As a result, many students chose to implement multiple strategies to solve a single problem. As noted in the findings for the planning phase, students would go through cycles where they chose a problem solving strategy, implement the strategy, then explore and choose alternate problem solving strategies. Most alternate problems solving strategies were chosen to support the strategy being implemented. For example, to solve the Points, Lines, and Planes Problem, it was necessary for two students to *make a drawing* in order to *look for a pattern* to create a formula. Figure 13 shows Tony’s drawings and written work for this problem.

![Figure 14: Tony’s written work for the Points, Lines, Planes Problem](image)
Since most of the problems were solved using the *guess and check* strategy, this strategy influenced them to also make a table as a way to organize their guesses. Kirsten used a table format to help facilitate her *guess and check* strategy to solve the Money Problem, however, she was having difficulty keeping track of her ‘tries’. She said the following referring to her written work shown in Figure 14, “Yeah, because how I had the paper was like this, to make sure I didn’t do the same thing twice. Which I think I still did a few times but it was more out of accident the first thing that popped in my head like this might work and then remembering I already did it.”

```
<table>
<thead>
<tr>
<th>2.48</th>
<th>2.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>.28</td>
<td>.28</td>
</tr>
<tr>
<td>2.52</td>
<td>2.52</td>
</tr>
<tr>
<td>.28</td>
<td>.28</td>
</tr>
</tbody>
</table>
```

Figure 15: Kirsten’s table for the Money Problem

The disorganization of her tables may have caused her frustration in implementing her *guess and check* strategy.

Most students used the *look for a pattern* strategy along with *making a drawing* of the grid to support *guess and check* in solving the Magic Square Problem. One example, Leann’s drawing, is shown in Figure 9. Not all students implemented strategies intentionally. One
student, Phil, stumbled upon the working backwards strategy in solving the Game Problem. Phil began solving this problem similar to the other students by assuming all players began with the same amount of money. His knowledge and comfort with the guess and check strategy influenced his decision to begin there. He created a table and through exploration he stumbled upon the working backwards strategy which ultimately led to the solution. He described what he did as working “from the bottom up”. When asked if he ever solved a problem this way before he said, “To me it’s just getting the answer. Whatever it takes to get the answer. Sideways? I’ll do that. Yeah.” He did not choose to solve the problem by working backwards intentionally.

There were two students who treated creating a formula as a problem solving strategy. For example, reflecting about solving the Money Problem Kirsten said, “I couldn’t come up with an exact thing to use, formula to use, so I was just trying random numbers checking them and seeing and hopefully it would work.” Dani also expressed a need to create a formula. After spending over 20 minutes using guess and check to solve the Magic Square Problem he stated, “This is ridiculous. There should be a set formula for this, this should not be a trial and error process.” These students wanted to create a formula to solve the problem but did not possess the knowledge to do so.

In summary, the students most commonly used problem-solving strategy is guess and check. Implementing this strategy required students to be organized, strategic, and knowledgeable about other traditional problem-solving strategies. Therefore, students chose to use other problem-solving strategies to support their guess and check. They most
commonly looked for patterns and made drawings. Their ability to engage with the mathematics content along with the strategies determined their level of engagement with the problems. Students who did not understand how to solve a problem used guess and check, and sometimes inappropriately. Students also wanted to create formulas instead of using guess and check but did not possess the knowledge to do so.

**Knowledge of recently encountered content**

The theme *knowledge of recently encountered content* refers to content knowledge the students accessed that was directly influenced by the mathematics presented at the summer bridge program. For example, the students covered mostly algebraic topics such as solving systems of linear equations and polynomial functions. Therefore, it made sense that there was a trend for students to solve the Points, Lines, and Planes Problem, using algebra concepts rather than geometry concepts. Four students, who used algebra content, interpreted a line as a function, and a region as the coordinate plane. This can be seen in Tony’s work. He interpreted a line as a polynomial function and regions as sections of the graph of the function to solve part a. The knowledge he accessed next was influenced by this interpretation. With this definition of lines and regions, he made the connection that the number of points corresponded to the degree of the polynomial and the number of regions formed as one less than its degree. This knowledge came directly from what he was learning in the bridge to calculus course. During the task-based interview, he referred to a formula and when asked which formula he was referring to he said,
I know what it is. We had just learned it in class… Yeah, if a polynomial is to a degree \( n \), the most what’s the word, I won’t say turning points but the most turns it would have is 5. So if it's a quadratic, it only turns once.

His solution is shown in Figure 15. During the video-SR interview, he recognized that this formula did not satisfy the requirements of the problem. He then decided to create a new formula, still accessing the algebra content recently covered in the summer course.

\[
\begin{align*}
2 \cdot \text{degree } n & \quad \text{regions} = \text{curves} \\
R(n) &= n - 1
\end{align*}
\]

Figure 16: Tony’s solution to part a of Points, Lines, and Planes Problem written in functional notation

He then redefined points as the inflection points of a polynomial function and the regions as “branches of the curve” or the section divided by the inflection points. Although this is not how the problem was originally conceived, he was able to make this alternate conception work. Figure 16 shows his written work.
The linear function has no inflection points so the number of regions is 0 + 1 = 1. The quadratic has one inflection point so the number of regions is 1 + 1 = 2, etc.,

The other students who attempted to use algebra context recently encountered in their summer course were not as successful. For example, Leann’s solution, was the formula is $y = x^2$. She neither defined her variables nor provided any additional written work other than the solution shown in Figure 17.

a) A possible formula could be $y = -x^2$, which should show an infinite amount of regions for a line, determined by n points.

Figure 17: Tony’s written work during the video-SR interview for the Points, Lines, and Planes Problem.

Figure 18: Leann’s solution to the Points, Lines, and Planes Problem
Leann interpreted regions as “the coordinate plane” and a line as a function. When asked to explain her formula, she responded, “I don’t know. I don’t even remember why I wrote that”. However, she commented that she did not agree with her answer and is thinking of it in a different way. She said, “I don’t know why I put $x^2$, should have just left it as $x$.” When asked why, she said, “. Because you can plug anything into $x$ and get a result without any limitations.” Her revised solution did not satisfy the requirements of the problem. Leann knew the difference between the range of the quadratic parent function and the linear parent function but was unable to articulate how it related to the problem. Further analysis of her interview proved that she defined regions as the range of the functions, and the points in the problem as the $x$-values. With this reasoning, it makes sense that the number of regions is infinite, using the formula $y = x$. The context recently encountered in the summer course helped her to situate the problem but she could not continue because she did not make connections throughout the problem.

Kirsten also decided to use algebra concepts to solve this problem. When asked what influenced her to solve the problem this way, she said, “I think I was still thinking on graphs because of math…Because that’s what we were doing in class. So I was like, oh ok, this is in my head.” One student did not attempt the problem and the other two students did not attribute their interpretation of the problem to the program. One student made geometry connections and solved part $a$ as intended and the other student’s formula was an inequality.

Students chose to use the mathematical content knowledge that they recently encountered in their summer bridge courses. This was especially evident in the Point, Lines,
and Planes problem. They used algebraic concepts of linear functions, polynomial functions, and properties about the coordinate plane when the problem called for a geometric interpretation of lines, points, and planes. One student was able to make this alternate interpretation work, whereas others had difficulty connecting the ideas.

**Knowledge of mathematical relationships**

The theme *knowledge of mathematical relationships* refers to the students’ ability to interpret the problem and make connections to the mathematics content. One trend that emerged was that students used arithmetic content to solve problems requiring algebraic concepts (i.e. ratios, proportions). For example, most students had difficulty recognizing the rational reasoning needed to solve the Camper Problem. Instead of using ratios and proportions, they provided the solution aligned with the common misconception that the two campers received $3 and $5. In the case of Kirsten, she did not recognize the proportional reasoning needed while solving the problem or during the video-SR interview. She was confident in her interpretation of her solution. She stated,

> Because it was asking how much the people should be paid according to how much they contributed. So it really had nothing to do with how much rice each person got. It was out of $8 how much should each, how much should A and B get out of it because of how much they gave. So that had nothing to with any working out anything whatsoever. Because I mean it was $8 and it was 800 grams of rice so camper A gave 500 so they will get $5 and then the other $3 so to B that gave 300. So
it really wasn’t anything there to think about. It was straightforward. I over complicated everything.

In the case of Phil, he recognized the proportional reasoning required but was not able to follow through. He began by calculating the amount of rice each camper would get if they each had a fair share. His next step was to find the amount Campers A and B should receive. He stated,

So, since camper C meal would be 266 2/3 of a gram, you would have to find. So, let me see...8/5. So, let me see something. $8 for the meal. So I'm saying they have the 800g of rice and then, let's see, he had to, since C had to pay camper A how much of his meal he contributed. If he do the whole thing, which is 800g and since A had 500g, 8/5 is 1.6, and then umm, that should be the amount of. Hmmm.

He could not articulate what the 1.6 represented because of a deficiency in his content knowledge. This led him to find “the percentage that A contributed to C and how much B contributed to C”, since he made a connection to percentages. However, he could not continue calculating the solution using percentages and at the end, he decided to “switch it up in my head, to, to like money instead.” That is, he changed all grams into dollars. The following dialogue occurred during his video-SR interview when discussing his computations.

Interviewer: You say to subtract 2.6 from 1.6, where are those numbers coming from?
Phil: I think that came from, ‘cause remember I had switched it up to dollars or money I should say, so I kinda move the place, I mean the point here over to the 2.6 and I had already had this 1.6 from before when I found out 8 divided by 5.

Interviewer: But how you could just move 266.6 grams and just move the decimal point?

Phil: Because just like how I had do it here, I had 800 grams and I...I had 800g and I kinda convert it to $8 so I have to move the decimal point from here, 1, 2, to $8. So I basically did the same thing.

His inability to make sense of the ratios he created caused him to make more mathematical errors of converting grams to money. Even after discussing his workings, Phil did not recognize his mathematical errors. In contrast, Leann recognized her misconception during the video-SR and attempted to solve but could not. She then stated that she needed to know “exactly how much A contributed to C” and “B contributed to C” but it could not be found from the information given.

Dani and Tony were able to recognize the proportional reasoning. During the video-SR interview, Dani defined ratio.

When I think about ratios I think about how to find ratios and one of my favorite ways to find ratios has to do with percent. When you find the percent of something you put the amount that you have over the total equals how much percent you want over 100. That’s the way I always remember how to find percent and when I’m thinking ratios my mind immediately jumps to this formula.
Since she associates a percentage with ratios, she found a percentage but did not know what it represented or how it could be used. Realizing that this percentage would not advance her towards a solution, her knowledge about proportions allowed her to find another way. Dani, then created the proportions shown in Figure 18.

![Figure 19: Dani’s written work for the Camper Problem.]

She explained her written work,

Well in words, 233.34 is how much the total of rice that he contributed to her, well when I was doing this problem that’s what I was thinking, and 266.68 is the amount that she had in total. And then I’m trying to find out the proportion of how much out of $8 I should give her. So $x$ is the amount that I need to find out and 8 is the total amount of money I have.

Most students had difficulty interpreting the proportional reasoning required to solve the Camper Problem. When they recognized this relationship but got stuck, they resorted to
using arithmetic by dividing values, changing units, and making connections between numbers that looked similar (e.g., 800 g and $8.00).

Students’ ability to connect mathematical relationships was tested on the Points, Lines, and Planes Problem. The problem required them to create a formula. Two students were able to make significant progress on this problem. Those that were successful, used diagrams to look for a pattern, and then interpreted that pattern in algebraic form. Figure 19 shows Dani’s written work.

![Diagram of Dani's written work](image)

**Figure 20:** Dani’s diagram for solving the Points, Lines, and Planes Problem.

There were students who did not use any diagrams. They produced formulas that were not well connected to the constraints of the problem. For example Phil’s formula for the Points, Lines, and Planes Problem was the inequality, $s \geq 2$, where $s$ is the number of regions. His formula did not depend on the number of points and he did not know how to fix this. When reflecting about his inability to create the formula, he said, “I think I did have it in
the back of my mind, like I have to find a formula but what would this formula be? I didn’t really know.” He knew he needed to find the formula but the strategy to create one was not accessible to him. Another example is Kirsten. She provided the formula, \( r = 2n \), where \( n \) is the number of points, and \( r \) is the number of regions. When asked to explain how she arrived at this formula she said, “the number 2 was in my head for whatever reason, I don’t know. The number 2 is usually my go to number when I’m trying to do something.” She did not have a mathematical reason for having 2 in the formula.

Geometry concepts were also accessed to solve the Points, Lines, and Planes problem. For example, of all the students who attempted part \( a \), only one student, Dani, interpreted a line and points as intended. Her work is shown in Figure 19. Most students used some or all of the following geometric terms to make sense of the problem: lines, points, planes, parallel lines, non-parallel lines, and perpendicular lines. However, they did not draw diagrams and could not connect what they knew about lines, points and planes with the requirements of the problem in their attempt of the problem. Students decided not to draw diagrams because they could not connect the mathematics they knew to what was required to solve problem. For example, when Phil was asked if drawing a picture would have helped him create a formula, he said, “I don’t know… I can see all of that in my head… and I had no clue.” Phil did not see how drawing a picture would have helped him advance towards a solution. Another example is Kirsten who was given an additional problem to solve. She did not know the meaning of integral lengths because she said, “I mean there could have been ones like down to the decimal and stuff that I wouldn’t have the patience to actually sit down
and try to guess and approximate that close.” This influenced her decision to quit. Nathan’s lack of understanding, due to him not being able to make mathematical connection, also impacted his ability to solve the problems. When he attempted the Points, Lines, and Planes Problem, he stated, “I don’t really know what a plane is”, he continued, “Just words alone and endpoints and stuff that’s something I can't really do.”

Students most often used **guess and check** to solve the problems. To continue to make progress using this strategy, students needed to make mathematical connections between the results from one guess and make informed revisions for the next guess. The choices made between guesses were crucial. There were some students who were able to articulate these connections but were unable to follow through. For example, Kirsten used her reasoning skills to solve the Money Problem. She stated the following regarding her progress, “Yeah, it was getting closer to working. I just had to figure out my limits, the highest and lowest I can go without going completely overboard or too low.” However, she was unable to fix the bounds on the unknowns so that her guesses could be more focused. As a result, her guesses became less organized by “just trying random numbers checking them and seeing and hopefully it would work.”

In contrast, Tony used his knowledge of arithmetic and number sense/reasoning to solve the Magic Square Problem. He did not complete the problem but his written work (see Figure 20) shows evidence of him making mathematical connections. This work includes the combinations of triples he wrote down that added to 15. Next to each number, he indicated the number of times it occurred across the sums. During the video-SR interview, he realized
that he had nearly solved the problem. He completed the problem during the video-SR interview. He stated, “When I put all the stuff we talked about it looked like that worked.”

![Table of numbers](image)

Figure 21: Tony’s list of triples that sum to 15 and their frequency

Phil’s number sense/reasoning caused him to make a vital connection in the Game Problem that led to him advancement towards the solution. While solving the problem, he recognized his guesses were not working so he decided to regroup. When asked why he decided to change direction, he said

Because I think I read it again, that’s why I was quiet there, I read it again and it said all of them had $24, so I was like that means that overall, I need to find the overall amount of money that they would have together, like if they combined their money together. That’s where I came up with the 72.”

After he realized that after each round, the total amount of money should be $72, he knew he “had to split up this 72 instead of 24.”
Students used arithmetic to solve problems containing ratios and proportions. Some did not recognize the relationship while others who did could not follow through. Students also had difficulty connecting and organizing information to create a formula. For students who were successful in creating a formula, they drew diagrams and clearly defined their variables. Those who were less successful, did not draw diagrams and made decisions about their formula based on “go to numbers” or made formulas that were not aligned with the requirements of the problem. Students also applied their knowledge of geometry and used it cohesively to solve the Points, Lines, and Planes Problem. Most knew the vocabulary but it did not compel them to draw diagrams and make meaningful connections. It was also necessary for students to apply mathematical reasoning to organize their knowledge in facilitating the guess and check strategy. The students who were able to make connections were those who were able to be critical of their process, step back, and make informed decisions based on their mathematical reasoning skills.

**Knowledge of similar problems**

Students wanted to use their knowledge of past and similar problems to help them with this set of problems. In some cases, it was successful. For example, students made connections to the Magic Square Problem to the game Sudoku. Sudoku is:

A puzzle printed on a square grid of nine large squares each subdivided into nine smaller squares, the object of which is to fill in each of the 81 squares so that each column, row, and large square contains every number from 1 to 9 (dictionary.reference.com).
His connection to Sudoku made it possible for them to implement their *guess and check* strategy because they were familiar with the goals of the problem. For example, when Phil was asked how connecting the problem to Sudoku affected his ability to solve the problem, he said, “It kind of boost confidence level in a sense, because you know you have to order the numbers in a certain way to get a certain sum and, so, yeah, I was like, I think I could figure this out.”

However, there were times when it was necessary for students to connect the problems to a problem they have done previously. When this could not be done, the student quit solving the problem. For example, Nathan attempted one problem. One reason he gave for not being able to solve the Points, Lines, and Planes Problems was, “I’ve never seen, I don’t remember, I don’t even recall ever learning about planes or segments.” When given a problem similar to what he was exposed to in the summer bridge program, he said, “Ok this, I kind of could solve but I can’t remember what formula I used from just like a week ago. We did this same type of problem but I just can’t recall which kind of formula I used to do this one before.” He also attributes his inability to solve the problems to his lack of exposure to the problem types. He stated the following about the Game Problem, “Because I have like Ok, let’s see. I guess I never had, I never saw that problem and you want me to just work with it like that? I never had the real proper learning to approach this problem.”

Phil also expressed the need to connect the problem to something done in the past while solving the Points, Lines, and Planes Problem. Part b of that problem was the only item Phil
did not attempt out of all the problems. Before he gave up, he said, “Okay, I'm really not sure what to do for (part) b, because I did those things probably like 2 years ago, so, yeah.”

When students are solving problems, there is a desire to connect the present problem to a problem they have done in the past. When this can be done successfully, students use this knowledge to help them solve the problem. In the case of Nathan, not making a connection to a prior problem, lead to him not attempting to solve the problem. Phil on the other hand, if he did not know the content, he attributes it to knowledge forgotten.

Summary

In this section, I discussed the different types of knowledge students used to solve the problems and how it impacted their next steps. There were four main themes of knowledge that emerged: knowledge of problem-solving strategies, knowledge of recently encountered content, knowledge of mathematical relationships, and knowledge of similar problems. These themes for knowledge are all connected and were accessed during the problem-solving process. Students who were able to make connections among these types of knowledge were more likely to be engaged with the problems in meaningful ways. In the next section, I discuss affect and how it influenced the decisions students made during the problem-solving process.

Affect

In this study, affect is used to represent a person’s beliefs, attitudes, and emotional states and will be categorized as local affect and global affect (DeBellis & Goldin, 2006). Local affect will be used to refer to emotions, and global affect refers to attitudes and beliefs
(DeBellis & Goldin, 2006). Affect influences students’ choices during their problem-solving process and is a determining factor when students decide to quit or persevere in problem solving. In this section, I present cross-case findings for local affect followed by cross-case findings for global affect.

**Local affect**

Local affect refers to the emotions students experience during problem solving. The emotions that occurred regularly for students in my study were *anxiety, confusion, frustration, and pleasure*. As noted earlier, emotions sometimes have positive or negative connotations. For example, frustration is often considered a negative emotion. However, when considering how these emotions influence decisions during problem solving rather than labeling them positive or negative it is more helpful to consider whether or not they were productive emotions. Did they result in a decision that moved the problem-solving process forward, or keep it at a stand still? With this in mind, the emotions were coded productive if they led to continued engagement with the problem and unproductive if the student stopped making progress. In this section, I report cross-case findings related to local affect while engaged in the problem-solving process.

**Anxiety.** Anxiety was the least expressed emotion. Feelings described as anxious, nervous, scared or panic were all represented by the theme *anxiety*. Students experienced anxiety most times while they were waiting to receive the problem or during the orienting phase of the problem solving process. For example, Leann said, “I think I looked a little nervous”, referring to before she was given the problem. She explained that this nervousness
was not unique to problem solving but is consistent with her emotions during any testing situation. She was nervous because she did not know what to expect. Her anxiety did not end after she received the problem. When asked if she was still nervous while reading and annotating, she said, “Just a little bit but it was just to make sure that I fully understood the problem.” The nervousness she experienced at the beginning of the problem still allowed her to continue engaging with the problem, therefore her nervousness was productive.

Two students, Nathan and Dani, expressed being scared and panicking during their problem-solving process. Dani stated, “I tend to overthink things because I am too busy panicking”. She states that this panicking causes her not to “see what’s obviously staring in front of me.” Explained this way, her anxiety is not productive because it prevents her from making progress during problem solving. While solving the Magic Square Problem, Nathan said that “everything went blank” and that he felt “anxiety, nervous, scared” when it happened. I considered these emotions as being extreme anxiety and they proved to be unproductive in both situations.

Although anxiety can be considered a negative emotion, it was shown to be either productive or unproductive during problem solving. It is dependent on the student and how the other influential factors all contribute to the situation.

**Confusion.** When students’ emotions were labeled confusion, it also referred to feeling hopeless, lost, clueless, and incompetent. Confusion was captured during the task-based interviews based primarily on the students’ nonverbal cues. These nonverbal cues included creased eyebrows, scratching of the head, tapping of a pen, looking intently at the
paper, or making unhappy faces. In most cases, students confirmed these instances of researcher-identified confusion. For example, while Kirsten was solving the Money Problem she began shaking her head from side to side and was asked to describe her feelings. She said, “I felt so clueless. I’m like how am I suppose to do this? Does this work? Is there an actual answer to this or are they messing with me? I had no clue.” Shortly afterwards, Kirsten decided to quit solving the problem without giving a solution.

Confusion was experienced most frequently during the orienting phase when students were reading and organizing the information for general sense-making. When students felt confused during this phase, they mostly reread the question. For example, Leann stated, “It was confusing so I just read over the question again” when asked about her feelings while solving the Money Problem. There were also many instances where students were confused during the executing phase when using the implementation of a problem solving strategy. For example, Kirsten explained the cause of her confusion while solving the Money Problem. She said, “I was getting confused. Ok, so I said one more boy and one less girl, trying to balance it out a bit, balance the number out. But that didn’t work either. Well I got close. I was about 22 cents off when I tried that, the 8 boys and 10 girls.” Confusion during this stage usually results in students discontinued engagement with the problem.

Confusion is an emotion that can be construed as negative but can cause productivity during problem solving. There were many instances that confusion was coded as being productive. For example, Phil was confused during the Game Problem and was motivated by his confusion to continue. He said, “I believe that the confusion was pushing me to try figure
out what would Maria have if she had to give money to both of them. Yeah.” Therefore, confusion can contribute to a student’s decision to persevere in solving the problem.

There were fewer instances where confusion led to unproductive behaviors. This example is from Nathan solving the Points, Lines, and Planes Problem. After less than one minute into the problem, he said,

Ah ha, this is already getting to me. I really don’t understand (part) a at all. At most how many segments are determined by n points on the line. (mumbles) Let’s see. Wow. Right now nothing is in my head. I’m ready to give up. Yeah, its not the right thing to do but I’m sorry Ms. Monrose I can’t do it.

Nathan commented that “nothing is in my head” which I interpreted as he does not know how to solve the problem so he is confused. This confusion led him to immediately end the problem-solving session. Confusion most times lead to productive behaviors while problem solving. However, if a student is confusion about the mathematics content, they have no ideas how to begin, which leads to the inability to continue solving the problem.

**Frustration.** Students experienced frustration more frequently than the other emotions. The emotions categorized as *frustration* were described as frustration, anger, impatience, agitation, and embarrassment. Students also felt dumb, annoyed, sad, upset, and stupid. Frustration is an emotion that was easier to recognize during the task-based interviews from the students’ nonverbal and verbal language. Some of the nonverbal language the students displayed were: slamming down pencil, placing hands on various locations on the
face (chin, forehead, nose), nail biting, putting pen on lips, fidgeting in the seat, tapping/shaking pen. Some verbal cues include exhaling loudly and tone of voice.

Most students showed frustration during the execution phase while implementing the guess and check problem solving strategy. For example, Tony got frustrated because his efforts did not produce the solution to the Money Problem. In the excerpt below, he described the face he was making during the task-based interview when he had almost found the solution. I asked him about the values (2.4 and 2.6) he was adding using the calculator and he commented on the results. He said, “but I got like 4 cents over and that’s the frustrated face.” I then asked if he was frustrated because it didn’t to 5.00 and he confirmed. This frustration did not prevent him from persevering in solving the problem. Instead he was able to change his problem solving strategy and solved the problem shortly afterwards.

From the previous example, Tony’s frustration caused productive behaviors. More than half of local affect identified as frustration was productive. For example, when Dani became “a little frustrated” solving the Camper Problem because she had “hit a real block in the road”, she decided to use percentages to solve the problem. Her frustration allowed her to change her strategy and she accessed and connected other mathematics content to solve the problem. She did not ultimately solve the problem using percentages but it kept her engaged with the problem. There were also times that Kirsten’s frustration was productive. For example, while solving the Money Problem, Kirsten said she was “getting fed up”. However, with this heightened frustration, she decided to “calm down, try to think of closer numbers that were a little lower to see if [she] can bring the number down.” Kirsten’s frustration
caused her to reflect on her problem-solving strategy and refine her guesses to look for the solution. Tony’s frustration was productive and propelled him to “just keep searching for the perfect two numbers”.

Students also seemed to experience more intense frustration when spending extended time solving a problem. For example, while solving the Magic Square Problem, Dani began to show extreme frustration by exhaling loudly and making an intense facial expression. She said, she was “irritated”, and stated, “I am upset, a bit upset with myself and with this problem because I can’t find a proper way to finish it.” On the other hand, there were times when frustration caused the student to prematurely quit solving the problem. For example, Tony also got frustrated while solving the Magic Square Problem. When he was asked if he felt frustrated. He replied, “Yes, because I hate Sudoku.” His frustration with this problem was intensified by his hatred of Sudoku. He did not complete the problem when he was so close to finding the solution.

Frustration is the most common emotion experienced by students while problem solving. This frustration occurs at all phases of the problem-solving process and is mainly productive. The leading cause for frustration leading to unproductivity is the amount of time spent working on the problem.

**Pleasure.** Pleasure was the only “positive” emotion captured in this study. Emotions identified as *pleasure* were described by the students as feeling happy, accomplished, confident, comfortable, amused, enlightened, excitement, calm, fun, and triumphant. *Pleasure* was easy to detect during the task-based interview because it resulted most times
with a smile, snapping of fingers, or even one time with a little dance. Students felt pleasure most frequently during the executing and checking problem-solving phases. For example, Tony stated that he felt “calm and nonchalant” the entire time he solved the Camper Problem. He attributed this feeling to helping him solve the problem correctly. He stated,

Yeah like I could look at this problem like its difficult like oh gosh here’s another complicated problem like the Sudoku you gave me and that could have thrown me off. I could have end up looking the wrong way and getting like $5 and $3. But instead I thought it through. I made sure I understood what they were asking me for and how it’s worded and that’s how I was able to find what I needed.

Kirsten also experienced moments of pleasure during her problem solving sessions. She described her overall experience solving the Points, Lines, and Planes Problem, “It was fun. I liked it.” “I mean it confused me a little bit but it was fun.” Dani celebrated each time she got a correct answer or made significant progress on a problem. She said phrases such as “Yes, I’m going to be smart today”, “I felt like the one guy who finally got the boulder to get up the hill”, and “I’m feeling pretty happy” to express her pleasure. Tony also did a little dance in his seat when he solved the first question correctly. Most time students felt pleasure during problem solving when they experienced success in solving the problem. Therefore, most instances where pleasure was observed, it was productive.

Summary. The students experienced many emotions while problem solving. These emotions were categorized as anxiety, confusion, frustration, and pleasure. Although the
majority of the emotions were negative, these emotions were mostly productive to their problem-solving process.

**Global affect**

Global affect is described as more stable affect and includes students’ beliefs about mathematics, mathematical problem solving, and their learning ability. Six themes emerged on global affect; *schooling influence*, *beliefs about own learning*, the belief that *mathematics involves relating different ideas*, the belief that *problem solving requires exploration and patience*, and *calculator use beliefs*. In this section, I report cross-case findings about the students’ global affect while engaged in the problem-solving process.

**Schooling Influence.** The theme *schooling influence* refers to the beliefs held by students that can be traced back to influences from their schooling. One major theme that was prevalent among students was the belief that they should have been taught to solve problems similar to the one presented or the problems should resemble those covered in school. For example, Nathan said he could not solve the problems because he “didn’t have the proper training.” Phil reflected about his inability to solve part B of the Points, Lines, and Planes Problem and said, “I think for b, I don’t remember being taught anything about this so that’s why I was like, “okay, what to do?” Kirsten could not solve the Points, Lines and Planes problem because she expected numerical values similar to problems experienced before. She said, ‘Because it was just weird for it not to have any numbers because with the points, the regions I was hoping they would give me a little something to work with, they didn't give me anything.” These beliefs were not productive because they led to students stop
working to solve the problem because they already made up their minds that they do not possess the knowledge or the problem does not contain the information needed to solve the problem.

There is evidence that students have been exposed in school to problems that may contain unnecessary information with the purpose of tricking the student. For example, while discussing Leann’s solution to the Camper Problem, she recognized that her answer of $5 and $3 was incorrect and that she needed to use all the information given. When asked why she felt comfortable not using all the information given, she said, “Sometimes it’s there to trip you up a little bit. Sometimes I think it’s there to make you do a little extra work for no reason.” Dani also expressed this belief, she stated, “they’re going to tell you stuff, and they use stuff to confuse you and I already is [sic] not good with word problems”. Students who held this belief did not put forth enough effort to access the problem-solving strategies and content knowledge needed to solve the problems.

**Beliefs about own learning.** During their problem solving, students also made statements that exposed beliefs about their mathematics and problem solving learning abilities. For example, there were many students who doubted their solution but did not challenge it mathematically, they said that they were just “over thinking” it. Over thinking was described by Leann as the act of having a “problem [that] might be extremely simple” and making it “more complex than it needs to be.” This belief is unproductive and prevents the student from challenging their answers mathematically, from looking at alternate problem-solving strategies, and from accessing other related mathematics content. Kirsten
attributed ‘overthinking’ to her inability to solve the Game Problem correctly. She said, “I was trying to figure out if I actually did it right. But then I think I realized I did something wrong, but I was just like I don’t know. I’m probably just over thinking it”. ‘Over thinking’ prevents the students from further explorations.

Another theme that was prevalent was students’ lack of confidence in their ability to solve the problems. For example, when Dani correctly solved the Camper Problem, she was surprised that her efforts produced the answer. She stated, “I didn’t expect it to work. I never think that I’ll actually get it right. To me when I get it right it’s like a miracle. I solved the problem! It worked!” In Dani’s case, the lack of confidence in her solutions was productive. It did not prevent her from engaging with the problem and also influenced her check her solutions. On the other hand, Nathan did not believe that he could solve the problems because he “didn’t have the proper training” and because his learning style prevents him from solving problems easily. This belief was unproductive because it prevented him from attempting to solve the problems. However, Tony was confident in his problem-solving ability. While discussing his solution to the Camper Problem, he recalled his calm and focused state of mind and said that his ability to do math was “natural”.

**Mathematics involves relating ideas.** The students expressed the belief that mathematics involves relating many different ideas. Students with this belief were able to connect mathematics concepts to solve a problem. For example, Dani was able to interpret a geometry problem and use algebraic ideas to create a formula to represent the pattern she uncovered from her drawing. Students not only connected mathematical ideas to solve the
problems; they also made connections to games they have played before. For example, most students made a connection between the Magic Square Problem and Sudoku. For some, this resulted in them being interested in the problem and persevering in solving it. For example, Phil liked Sudoku and the similarity between the two gave him a “boost in confidence” and he persevered in solving it. Conversely, Tony also made the connection to Sudoku, but does not like the game. He attributed his frustration with the problem to his hatred for Sudoku. This caused him to prematurely quit solving the problem even when he was making significant progress in toward the solution. His disdain for Sudoku prevented him from connecting all the information he had written down into a cohesive plan to solve the problem.

**Problem solving requires exploration and patience.** This theme refers to the belief that the problem solving process may take time and the problem solving strategy needed to solve the problem may not be readily recognized. Students with this belief about problem solving will not let their frustration or confusion at the beginning of solving the problem influence them to quit immediately. Phil displayed patience while solving the Money Problem. He said,

I guess the confusion at the start was because I just started it and when you just start a problem you might not get it done but you still have a momentum, I guess you could say, that you could solve the problem…

Phil understood that it was normal to get confused when beginning a problem but he is still motivated to continue. The extent to which students held the belief that problem solving requires exploration was tested more on the problems they solved using the *guess*
and check strategy. For example, Kirsten solved the Money Problem exploring different combinations for the number of boys and girls looking for a solution. She explored possible solutions when the number of girls and boys were equal and combinations of 8 and 10, and 7 and 11. Dani commented on her ability to explore when problem solving. She stated the following during the review of her solving the Game Problem, “And if it was just me, in another setting, I’d keep going. I’d try every number until I have to go eat or something, and then I’ll come back and keep trying.” On the other hand, Nathan did not see the benefit in exploring during problem solving. When asked why he did not write down anything in an attempt to solve the problems he stated, “Because I have never seen something like that in my life before,” and that he “don’t really think there is no [sic] value in that”. Students who believe that exploring is natural in problem solving make a conscience decision to access knowledge of problem-solving strategies and content knowledge to remain engaged in the problem-solving process.

There were students who displayed patience when problem solving. For example, when Tony was asked how he was feeling after working on a problem for 21 minutes, he said “tired” but not emotional. Tony did not allow the amount of time spent on the problem to influence him to quit. In contrast, Nathan did not believe that solving problems should take a long. He said the following after discussing why he did not spend more time trying to solve the Game Problem, “but just stay there like for a half an hour maybe 45 with just one problem. I don’t think that’s right.” When asked how it makes him feel when it takes him a long time to solve a problem, he said “It kind of make me feel like you kinda dumb.” He
measures the ability to do mathematics by the amount of time spent on a problem. Leann also experienced times during her problem-solving process where her conception of time was counter-productive. She expressed several times during her interviews that she did not like spending too much time solving a problem. During the video-SR interview of the Money problem, she thought of another way to solve the problem. When asked why she didn’t explore that option during the problem solving session, she said, “Because I was focused on getting on to the next one (laughs)” and she “thought it was going to have more problems” and “didn’t want to stick on one for too long.” She also expressed that her experience with testing instructions from her past mathematics teachers saying “don’t stick on a problem too long if its too confusing”.

The students who believed that problem solving requires exploration and patience were engaged in the problem for longer periods of time. They did not allow time spent solving the problem, initial feelings of frustration, or confusion to prevent them from persevering.

**Beliefs about calculator use.** In this study, students had the option of using a calculator to aid in solving the problems. One theme that emerged is that students held beliefs about when calculators should be used. For example, when Dani was asked why she didn’t use the calculator while checking the sums of her rows, columns, and diagonals for the Magic Square problem, she said, “I’m just adding and adding. I don’t want to use a calculator ‘cause I’ll seem like I don’t know how to add…I don’t feel like it’s necessary actually.” She chose not to use the calculator to solve the problem because she was concerned about how
her mathematics ability would be perceived by others because the calculations were simply addition. When asked about her calculator use when solving problems, Kirsten said, sometimes I use the calculator, sometimes I try to figure it out in my head. Sometimes I use the calculator it depends on the problem itself. If it was something I could have just do scratch work for myself I would have done that but that would be a lot of scratch work so just use the calculator. If I screw up with the calculator it makes it easier on myself.

Kirsten chose to use the calculator because it proved to be easier for her at that time.

While solving the Camper Problem, Leann did not use her calculator to find the amount of rice each person would receive if shared fairly. This stood out because all the other students used the calculator and the quotient was not an integer value. When asked about her choice not to use the calculator, she said, “I kind of forget [sic]” and that “It didn’t look too hard to do.” When calculations do not seem difficult, she chose to compute by hand.

Figure 21 shows her calculations.

![Figure 21: Kirsten’s hand computation](image)

Figure 22: Leann’s hand computation
The decision to use the calculator is mostly based on how difficult the student perceived the problem to be. However, students also used the calculator to help organize their written work and others were concerned that its usage would result in judgment of their mathematical ability.

**Summary**

Students’ affect influences their decision to persevere and to access knowledge in problem solving. During problem solving, the students in this study experienced local affect as anxiety, confusion, frustration, and pleasure. These emotions caused productive and unproductive behaviors during the problem solving process, depending on their beliefs (i.e. global affect) about mathematics and problem solving. Themes the emerged pertaining to their global affect were, *schooling influence, beliefs about own learning, mathematics involves relating different ideas, problem solving requires exploration and patience, and beliefs about calculator use*. Students who held beliefs that were favorable to mathematics and problem solving persevered and spent more time actively engaged with the problems.

**External contextual factors**

My conceptual framework included external contextual factors to allow myself to look for additional themes not included in Schoenfeld’s theory. *Physical limitations* and *saving face* were the themes that emerged from the data. *Physical limitation* refers to any physiological influences that had an effect on the students’ problem solving process. *Saving face* refers to the need for the student to believe that the researcher held a positive view of their mathematical ability. *Saving face* could have also been discussed as affect since it
manifests itself as local affect. However, since it was driven by the role of the researcher and this work, it will be addressed here. I will discuss physical limitations first, followed by saving face.

**Physical limitations**

There were physical limitations that impeded the progress students made during problem solving. These physical limitations include feeling tired, sleepy, and hungry. There were even students who complained of having a headache. For example, after solving the Points, Lines, and Planes Problem for about 21 minutes, when asked, Tony said that felt “tired”. Kirsten also stated explicitly and by nonverbal cues (yawning on several occasions) that she was tired during the interviews. She attributes the tiredness to staying up late, sleeping irregular hours, and also participating in the research after a full day of mathematics. She stated,

I couldn’t fall asleep for some reason I don’t know. I had just come from like almost 6 hours of math so I was just like my brain ready to shut off. Six hours of math, oh my gosh. And then going to do this and its not even something I’m used to so that confused me even more. And I’m just like, what to do with this? Am I even suppose [sic] to know what to do?

Dani also stated that she was tired while solving the problems. It was not explicit if she was tired from her lack of sleep or got tired over the duration of solving the problem. Dani’s performance was also affected by the location of the interview. Her first task-based interview was conducted in the cafeteria before lunch when there were no patrons. However, she was
still affected by being there. She stated, “I knew it wasn’t time consuming, but I took a long time. I was also hungry. I wanted to eat. Plus, we’re in the cafeteria which makes me want to eat more.”

Students experienced physical limitations that affected their ability to continue the problem-solving process. Students who were tired, sleepy, and hungry were aware that these physical needs impacted their problem-solving abilities.

**Saving face**

*Saving face* refers to the need of the student to be viewed positively by the researcher. Only Nathan and Dani provided evidence of *saving face*. Both students believed that I was judging them. For example, Nathan believed that I was “judging his intellect”. It could have impacted his decision not to engage with any other problems that were difficult for him. However, Dani’s response to her believing that the researcher was judging her was different. She stated the following when the researcher asked why she felt that she was being judged.

> Not judging me the way that…well, a little bit because I don’t like being not good at math and this is this program, you’re supposed to be good at math. So when I say I don’t know what to do, I usually like I’m done, I’m finished, please take…but I didn’t want to give up, I want to finish it. Say okay I’m going to buckle down and try this again and finish this.

This is evidence that her need to *save face* for the researcher motivated her to persevere despite her desire to quit. Another example, occurred while solving the Money Problem, she said,
At this point I am annoyed and irritated because first I, I want to, I want to do well on your, on your uhm research. I want to be able to be the person that solves the entire problem. And now, I’m struggling and I’m not getting it, and I’m really not good with word problems, I don’t want you to think I’m stupid, so now it’s like “oh my goodness, why…”

Dani was motivated because she wanted to help me with my research and she wanted me to think highly of her mathematics ability. She stated she did not want me to think she was “stupid.” She was also struggling with her identity as a mathematics student from the program. She saw herself as one of the weaker students and would constantly compare her performance to theirs. She not only wanted to save face for the researcher but also for her classmates. She stated the following when asked why she decided to check a solution to a problem.

Dani: Because I wanted it to be right, I didn’t want to have it like you know, well she just did it and she didn’t check it because she knows she’s going to be wrong and that’s how lazy she is.

Interviewer: Oh so you have been…so it’s like you think that I’m judging you in a sense?

Dani: Yeah, I feel like a lot of people judge me when it comes to math. Especially here, you know, because of all these flippin’ math geniuses that take calculus and pre-calc, AP calc. I’ve never had calculus in my life, the most I had was trig in 10th
grade and that was it. I don’t remember anything that had to do with math. So I feel out-classed, like so out-classed!

Another example of how her need to *save face* motivated her problem-solving process;

Dani: Well that there. As much as I want to continue, I know I can’t do anymore. I don’t have anything else to put into this paper. I’m just looking at this paper and nothing is clicking.

I don’t want to quit. I don’t want to disappoint you. I want them to say, at least I solved one problem.

Although the need to *save face* did not occur explicitly with all students, when it occurred it impacted the students’ ability to problem solve. Nathan dealt with this by giving up quickly and not engaging with the problem, whereas Dani was motivated and spent more time solving the problems.

**Chapter Summary**

In this chapter I discussed cross-case findings of the influential factors of problem solving: goals, knowledge, affect, and external contextual factors. First, the findings for goals were presented. The goals students set occurred during all phases of the problem-solving process. Success in achieving goals during one phase allowed them to gain access to another phase. However, for students to achieve this success, their goals had to be connected.

Second, I discussed the findings for knowledge. Students accessed knowledge pertaining to mathematical content knowledge and knowledge of problem-solving strategies, which were discussed together since they are very much intertwined. The data suggests that
students made choices pertaining to knowledge based on their ability to access this knowledge. Thirdly, I discussed findings for affect. Local and global affect influenced the students’ decision to persevere in problem solving. Students were able to be productive even while experiencing emotions that were negative (e.g. confusion and frustration). However, if students did not have favorable beliefs about mathematics and problem solving, they were not able to engage with the problems in meaningful ways. Lastly, findings on external contextual factors were presented. The data suggests that physiological issues such as hunger and tiredness affect students’ problem solving ability. Also, students’ motivation was influenced by their need to save face.

In the next chapter, I discuss the findings as they relate to the overarching research question: In what ways do students’ goals, knowledge, affect, and external contextual factors inform their choices during the mathematical problem solving process? I make connections to the literature, note the limitations of the study, discuss important implications to mathematics education, and areas for future research.
Chapter Five

The purpose of this study was to contribute to the creation of a problem-solving theory. Schoenfeld (2011) proposed a theory of goal-oriented decision-making, which claims an individual’s goals (personal aims to achieve), resources (knowledge available), and orientations (beliefs, values) are the influential factors in the decisions made during any goal-oriented activity. Schoenfeld (2011) proposed this theory for any goal-oriented activity including mathematical problem solving. To connect research to practice, researchers have recommended that a theory for problem solving is needed to “more reliably observe, document, and assess important mathematical developments in our students” (English & Sriraman, 2010, p. 269). To test this theory, empirical evidence was collected to: (1) provide detailed, rich descriptions about how and why students make decisions during problem solving and (2) validate the scope of the proposed problem-solving theory. This study design includes, the conceptual framework I created influenced by Schoenfeld’s theory (2011), Carlson and Bloom (2005), and Debellis and Goldin (2006) in which the decisions students made during problem solving were attributed to their goals, knowledge, affect, and external contextual factors. The participants in this study were six freshmen STEM majors attending a mathematics summer bridge program at an HBCU. Students were chosen from this program because the goal of the program is to improve mathematical competency as a way to improve participants’ success in their future STEM careers. Students were chosen to represent varying levels of mathematical competency and beliefs and attitudes about mathematics and problem solving. Their problem-solving process was investigated to answer the overarching research
question: *In what ways do students’ goals, knowledge, affect, and external contextual factors inform their choices during the mathematical problem solving process?*

In the previous chapter, I provided detailed descriptions of the findings as they related to each factor (i.e., goals, knowledge, affect, and external contextual factors) represented in the conceptual framework. I begin this chapter by discussing the findings with respect to the research questions with connections to existing research. I then summarize findings across all factors to answer the overarching question. This chapter concludes by discussing implications of this work as it relates to the validity of this proposed problem-solving theory along with how it can be used to observe, document and assess problem-solving performance. Finally, I will discuss the limitations of this study and plans for future research.

**Answering the Research Questions**

In this section, I answer each research questions and connect it to existing research.

*Question a: How do students’ goals inform their choices during the mathematical problem solving process?*

Analysis of the data revealed that students’ goals occurred throughout the phases of the problem-solving process. Therefore, the answer to this question will be addressed beginning with goals when they first encountered the problem through the identification of solutions.

When students’ goals were to get oriented, they typically engaged in rereading the problem and organizing information identified in the problem using a diagram of some kind. We saw this with Dani when she drew a line and placed points on it to ultimately create a
formula. Furthermore, students whose goals included drawing a diagram were more likely to make connections that allowed them to move towards a solution. The findings in Cai’s (1994) study of college-aged students supports this idea. He found that when students who are more mathematically experienced solved problems their smaller goals were all closely related to each other and to the global goal (e.g. attaining the solution). Also, Driscoll (1999), and the Common Core State Standards (CCSSI, 2010) support the notion that students that can link and move between multiple representations have more conceptual understanding. Therefore, it makes sense that the students in my study, Dani and Tony, who drew a figure to solve the Points, Lines, and Planes Problem, were able to create subsequent goals and ultimately solve the problem.

When students’ goals were to plan, they typically explored solution strategies. When exploring solution strategies, students would at times consider options that seemed unconventional. For example, Leann, while solving the Magic Square Problem, got stuck and began to write multiples of the numbers 1-9. When questioned, she said, “I have no ideas why, why I put times tables…I was just trying something.” After trying this, she was able to make a connection among the numbers, the sum, and their placement in the grid. Shortly after, she made the connection that 5 had to be placed in the center of the grid and later made advances towards a solution. There were also times when a coordinated exploration did not lead to advances. For example, while Tony was solving the Magic Square Problem, he wrote down what he called “trios”, all combinations of the integers from 1 through 9 whose sum was 15. However, he did not make that final connection to solve the problem. Not all
unrelated explorations led to further engagement nor did all related explorations end in a solution. However, what was consistent was that students who explored remained engaged with the problem-solving process increasing their chances of making further connections to implement a problem solving strategy.

This finding is consistent with the literature on experienced versus novice problem solvers, which supports the notion that expert problem solvers spend most of their time understanding and planning for the solution and less time doing calculations (Cai, 1994; Carlson & Bloom, 2005; Schoenfeld A. H., 1992). However, the students in this study displayed some attributes of expert problem solvers by spending time planning. However, similar to findings from Cai (1994), these students formulated a plan and executed it immediately, utilizing many useless calculations along the way. It took students cycles of carrying out calculations and planning to make advances towards the solution.

When students’ goals were to execute, they typically implemented a problem solving strategy. This is not surprising since the name was intended to capture this behavior. Students spend most of their time setting and achieving goals related to executing. In prior research projects, it was found that novice problem solvers spent most of their time in the executing phase of the problem solving process (Schoenfeld A. H., 1992; Carlson & Bloom, 2005; Cai, 1994). Most often students chose to implement the guess and check problem-solving strategy. The implementation of this strategy prolonged engagement if it was organized and strategic. I saw this when observing Phil, who was able to solve the Game Problem beginning with the guess that each player begins with the same amount of money.
He then used a table to organizing his guesses using this assumption. From the table, he was able to notice that at the end of each round, the total amount of money had to remain constant at $72 leading to the conclusion that his guesses were too small. As his engagement continued, his guesses evolved until he discovered the working backwards strategy to solve the problem.

Another typical choice was to *carry out computations* using arithmetic even when algebraic computations using ratios and proportions were required. Effective engagement with problems occurred when students were able to use appropriate content knowledge and make connections among ideas to arrive at a solution. For example, on the Points, Lines, and Planes Problem, Dani accessed her knowledge of geometry to draw a line and points on the line, looked for a pattern, and then used that pattern to create an algebraic formula.

Most students ended their problem-solving process in the *executing* phase. This occurred because students would quit solving the problem after carrying out computations, or they would decide not to check their answer. Some students did not check their solution because they knew their answer was incorrect. For example, Leann was asked if she checked her answer and why not. She replied, “I already knew it was wrong. I did not have to check it.”

However, there were students who set goals to *check* their solution. For example, Dani always checked her solutions. When asked about checking her solution to the Camper Problem, she said, “I hoped that it would but I’m pretty skeptical when it comes to my math. I never think it’s really going to work until it does.” There were other students who only
checked solutions in which they had confidence. For example, Phil only checked the problems he solved correctly. For students who set goals to check their solution, if the solution were correct, they would end their problem solving session. However, if their final solution were incorrect, some students would proceed to rationalize the solution. For example, when Tony checked his solution to the Game Problem, he stated, “I like it wasn’t right” but he “ignored” the feeling and rationalized his incorrect answers by redefining the parameters of the problem. Similarly, Kirsten when discussing her solution to the Points, Lines, and Planes Problem said, “I tried to think of it other ways but it didn’t make sense to me. At least this made some sense.” She knew the answer was incorrect but it was the only answer that fit based on the knowledge she possessed. This finding is consistent with research. For example, Cai (1994) found that low-experience problem solvers “did not evaluate their final results logically and accurately (p. 179).” Some students were consistent in checking their solutions. Others only checked solutions they were confident were right. If students did enter the checking phase with an incorrect answer and chose to check it, they would rationalize the solution to fit their knowledge scheme.

Based on these findings, when students’ goals were connected from one problem-solving phase to another, they typically remained engaged with the problem longer. This finding is consistent with Schoenfeld’s findings for his study on college level students’ problem solving (Schoenfeld A. H., 1985). He found that in general, novice problem solvers enter the problem solving process and create goals in reading then make quick decisions about choosing a plan of action, then pursue that plan. They usually stick to that plan until
they get frustrated and give up. Only when a student can reconsider their initial decisions and make new and focused plans are they more likely to succeed (Schoenfeld A. H., 1992).

**Question b:** How does students’ knowledge inform their choices during the mathematical problem solving process?

Students in this study used two types of knowledge to solve problems; their knowledge of problem solving strategies and their knowledge of mathematics content. These two knowledge sources are dependent on each other but will be discussed here separately. Students in this study use the following problem-solving strategies: *look for a pattern, guess and check, make a drawing, and working backwards*. They also combined *multiple strategies* to solve one problem. The mathematical content knowledge students accessed while solving these problems were related to: algebra, arithmetic, geometry, and number sense/reasoning. As stated before, just describing *what* content knowledge they possessed was not the goal. The goal was to understand *why* a specific content knowledge was accessed and *how* it influenced their next steps. I begin discussing the answer to this question with knowledge of problem solving strategies.

When students drew upon their knowledge of problem-solving strategies, they typically chose to implement the *guess and check* strategy. This makes sense because of the nature of the problems given. Most problems could be solved using the *guess and check* strategy. Also, research has shown that students are frequently taught in mathematics courses to *guess* when they approach roadblocks in problem solving (Elia, Heuvel-Panhuizen, & Kolovou, 2009). Students who made significant advances were strategic, organized, and
combined other problem solving strategies, such as *make a drawing*, to inform their *guess and check*. Students were typically able to engage with the problems for longer times using *guess and check*. For example, Phil solved the Game Problem using primarily *guess and check* but his ability to organize his written work by *making a drawing* (i.e., a table) resulted in him stumbling upon the *working backwards* strategy that ultimately led to a solution.

When students considered their content knowledge, they typically chose mathematics content requiring **knowledge of recently encountered content, knowledge of mathematical relationships, and knowledge of similar problems**. There was one problem for which this theme was especially apparent, the Points, Lines, and Planes Problem. Students used algebra content instead of the intended geometry content to interpret and solve the problem. For example, they interpreted a line as a polynomial function and regions as the coordinate plane. Only one student, Dani, interpreted the problem using geometry. However, this was the problem where students had the most difficulty interpreting the mathematical terms. Most students were not able to make clear connections between the algebra they accessed and the requirements for the problem. This is consistent with prior research that suggests that not understanding a problem regardless of the content knowledge accessed hinders student success (Elia, Heuvel-Panhuizen, & Kolovou, 2009).

The mathematics students used to solve the problem displayed their **knowledge of mathematical relationships**. It was challenging for students to connect their mathematics content to the specifics of the problem. For example, many students used arithmetic to solve problems requiring algebraic concepts. Evidence of this was seen as students attempted the
Camper problem. Examples were given in Chapter Four where Leann, Kirsten, and Phil all used algebra to solve this problem. Their misconceptions about ratios and proportions were illuminated. Only Dani and Tony were able to recognize the proportional reasoning necessary to solve the problem. Students were also not able to connect their knowledge of problem solving strategies to their content and use both to solve problems. For example, the Points, Lines, and Planes Problem required students to create a formula. To create this formula, students needed to look for a pattern that would come from a drawing. Their inability to go between their content knowledge and their knowledge of problem-solving strategies resulted in them choose to quit solving the problem.

When students encountered a problem, they chose to access their knowledge of similar problems. For example, many students made reference to Sudoku when working on the Magic Square Problem. However, there were students where the inability to access knowledge of similar problems caused them to quit solving the problem. An example of this is Nathan, who did not attempt most of the problems. He gave reasons such as “I’ve never seen” or “I don’t remember” or “I can’t recall” some concept or problem as to why he chose to quit solving the problem.

Based on these findings, when students considered their knowledge of problem-solving strategies, they typically chose a familiar strategy and used content knowledge they felt they knew the most. This makes sense since research supports the notion that success in problem solving is contingent on how students use their problem-solving strategies (Cai, 2003). However, the content that they choose to access also affects their problem solving
process. Overall, students who were unsure of what content to access made their decision based on *knowledge of recently encountered content, knowledge of mathematical relationships, and knowledge of similar problems*. Their expertise in the content that they chose depended also on the connections they were able to make to the problem. For example, Phil did not know how to create a proportion to solve the Camper Problem, although he recognized the proportional relationship. Since there was no connection between his recognition of the need to use proportions and creating a proportion, he chose instead to inappropriately change units and gave an incorrect solution.

These findings are all consistent with general assertions found in the literature. We know that knowledge is fundamental to problem solving, that knowledge and memory are associative, and that knowledge is retrieved by an individual’s categorization and experience (Schoenfeld A. H., 1983; Schoenfeld A. H., 2010). Therefore, it is not surprising that students chose to access the knowledge they just learned in their bridge to calculus course. Also, since knowledge is associative it is also natural for students to look for past similar problems. However, the notion that knowledge is associative can be more complex. The assumption is that when students access algebra, they also access connections to algebraic concepts such as creating drawings, looking for patterns, and creating formulas, which may not be the case. Students who were unable to access all of the necessary and connected information when solving a problem made choices inconsistent with the requirements of the problem. For example, Kirsten did not know the property for the length of sides of a triangle
so she attempted to draw the triangles estimating their sides to determine what side lengths created triangles.

*Question c: How does students’ affect inform their choices during the mathematical problem solving process?*

As described in my conceptual framework, affect will be separated into two sections, local affect and global affect. Students made choices based on local affect within the themes: *anxiety, confusion, frustration,* and *pleasure.* Although most emotions were negative, they were mostly productive to the problem-solving process. Global affect is described as more stable affect and includes students’ attitudes and beliefs about mathematics, mathematical problem solving, and their learning ability. Students made choices based on their global affect related to the themes: *schooling influence, beliefs about their own learning,* *mathematics involves relating different ideas,* *problem solving requires exploration and patience,* and *beliefs about calculator use.* I discuss findings for local affect first.

When students reacted to affect it either resulted in a productive or unproductive response. DeBellis and Goldin (2006) described this affect as empowering or disempowering. *Anxiety, confusion,* and *frustration,* are typically considered to be negative emotions, however when they occur during problem solving they often bring out positive reactions. Students who felt *anxiety* usually experienced this emotion before the problem was given and during the *orienting* phase. For example, Leann described her emotions at the beginning of her problem-solving sessions as nervousness. However, she was always able to further her engagement with the problems therefore it resulted in a productive reaction.
However, there were times when anxiety caused the student to end the problem solving session. These were instances where the student described being scared and panicking. I refer to these cases as extreme anxiety and they were always followed with unproductive behaviors. For example, Dani stated that her panicking causes her not to “see what’s obviously staring in front of me.” This reaction to her extreme anxiety supports the findings that unproductive emotion can prevent a student from exploring strategies to solve problems (Op’T Eynde, de Corte, & Verschaffel, 2006), and that it creates a cognitive block so that information cannot be retrieved (DeBellis & Goldin, 2006).

Confusion and frustration are emotions that students most commonly felt during the problem-solving process. Students exhibited physical cues such as: slamming the pen on the desk, fidgeting in their seat, and exhaling loudly to represent these emotions. This supports DeBellis and Goldin’s (2006) view of affect as representational. These physical cues allowed me to identify moments when their affect could influence their next decisions. For example, while solving the Money Problem, Tony slammed his pen on the desk. When asked about his feelings at that time, he responded, “Frustration for sure”. He then commented that the frustration propelled him to “Just keep searching for the perfect two numbers.” Another example is Leann. She was asked about her emotions while reading the Money Problem, she replied “It was confusing so I just read over the problem.” In these two examples, their confusion and frustration influenced a productive behavior. Phil provided another example. When asked about his feelings while implementing guess and check, he stated, “I believe that the confusion was pushing me to try figure out what would Maria have if she had to give
money to both of them.” He clearly articulated how his confusion influenced his next steps. However, not all students had productive responses to confusion. For example, Nathan’s confusion during the orienting phase contributed to his decision to give up.

Most times, pleasure was experienced at the end of the problem-solving session. Among the physical cues of pleasure were smiles, snapping of fingers, and even a dance at one time. Students expressed feeling “accomplished, “calm and nonchalant”, having “fun” and “feeling pretty happy. This is consistent to findings from DeBellis and Goldin (2006) which stated that students might begin problem solving with negative emotions, such as confusion that may make them curious and cause bewilderment. These emotions motivate students to engage with the problem in meaningful ways. For example, they might be motivated to understand the problem and make efforts to do so by employing problem-solving strategies (DeBellis & Goldin, 2006). They then experience more positive emotions as they encounter small successes along the way.

According to Kloosterman and Stage (1992), emotions influence a student’s decision to give up or persevere depending on their beliefs (i.e., global affect) about mathematics and problem solving. Therefore, it is important to also consider students’ beliefs about mathematics and its influence on their choices. Students’ beliefs are influenced by their experiences in school. In this study, I refer to these beliefs as schooling beliefs. One theme that emerged is that students believe they should have been taught to solve problems similar to the ones presented or that problems should resemble those covered in school. Students who held this belief, prematurely quit solving the problems, most times without even trying.
For example, Phil, was able to engage with all the problems except part b of the Points, Lines, and Planes Problem when he said, “I think for b, I don’t remember being taught anything about this so that’s why I was like, “okay, what to do?” Nathan also held this belief and he did not engage with most of the problems presented.

Students also held the belief that problems contained unnecessary information with the sole purpose to “trip you up a bit” and “use stuff to confuse you” that can be linked to their schooling experience. These beliefs did not produce productive responses. They resulted in students being more likely not to use the all the information given and to choose a less rigorous approach to solve the problem. Another belief that can be attributed directly to schooling is that problems should be solved quickly. This belief surfaced constantly during Nathan’s interviews. Nathan said, “I don’t think that’s right” when he considered spending twenty minutes on a problem. He also shared that it makes him feel “dumb” when it takes him a long time to solve a problem. This finding is consistent to research on students’ beliefs. Schoenfeld (1988) found that students’ exposure to exercises where mastery is indicated by the finding solutions quickly contributes to this belief. Also, Schoenfeld (1992) states that the belief that students who understand mathematics will be able to solve a problem in five minutes or less negatively affects students’ motivation.

Students also held beliefs about their own learning that influenced the choices they made during problem solving. Several students contributed their inability to solving the problems to over thinking, which was described by three students as the ability to complicate problems. This belief was unproductive because when students realized they were not
solving the problem correctly, they attributed it to over thinking. This would cause them to stop looking for alternate strategies or accessing other content knowledge. The amount of confidence a student had in their mathematical ability also influenced their choices. Students who were confident in their ability were engaged with the problem and experienced fewer negative emotions. For example, Tony was confident in solving the Camper Problem and described his experience as “calm” and “nonchalant”. However, most students who were not confident did not engage with the problem for long periods of time. For example, Nathan did not believe he “had the proper training” or was naturally capable of solving problems and as a result, did not attempt most of the problems. This belief is consistent with findings from Schoenfeld (1992) show a negative impact on motivation during problem solving when students believe that “ordinary students cannot expect to understand mathematics”.

Dani, however, did not conform to the notion that high confidence leads to success in problem solving. She experienced the most success on the first try for solving the problems and she was not confident in her mathematical ability. For example, she said, “I didn’t expect it to work. I never think that I’ll actually get it right,” after solving the Camper Problem correctly. This behavior is also not consistent with research by Malloy and Jones (1998), which found that African-American students were confident in their mathematics ability regardless of their success in problem solving.

Students who believed that mathematics involves relating ideas and problem solving requires exploration and patience were more likely to engage with the problems. Students that held the belief that mathematics involves relating ideas made connections between the
problems and everyday experiences outside the classroom along with connections between their mathematics content and the problem. Students who were able to make connections between mathematical ideas engaged with the problem longer because they were also able to implement multiple problem-solving strategies. For example, the connectedness of Dani’s knowledge allowed her to used geometry and algebra along with *guess and check*, *make a drawing*, and *look for a pattern* problem solving strategies to solve the Points, Lines, and Planes Problem. This finding is consistent with research by Carlson (1999). He found that students with favorable beliefs about mathematics and problem solving were more successful at problem solving.

Students who believed that *problem solving requires exploration and patience* were more likely to persevere regardless of the negative emotions experienced during the problem solving process. For example, Phil expressed that confusion at the beginning of solving a problem was expected but it gave him momentum to continue. To explore during problem solving takes patience, which Kloosterman and Stage (1992) suggest affects students’ motivation to continue engaging in the process.

Based on these findings, student’s affect influenced their choice to persevere in problem solving. This is consistent with findings from the affect literature that emotions, beliefs, and attitudes heavily influence the behavior of someone engaged in problem solving (DeBellis & Goldin, 2006). According to Schoenfeld (2011), in research a person’s behavior is consistent with their belief, regardless if they consciously agree with the attribution. Schoenfeld (1998) also suggests that problem solving is not purely cognitive and that affect
determines the likelihood of students being able to access the knowledge necessary to solve the problem. In this study, students’ local affect (i.e., anxiety, confusion, frustration, and pleasure) influenced their decision to quit or continue the problem-solving process. This was contingent on their beliefs about mathematics and problem solving. If they felt anxiety, confusion, or frustration and held a favorable belief, they were more likely to persevere. When students who held a less favorable belief about mathematics, which could have been influenced by years of schooling or their lack of confidence in their mathematical ability, they were more likely quit engaging in the process.

Question d: How do external contextual factors inform students’ choices during the mathematical problem solving process?

There were two themes that emerged within external contextual factors; physical limitations and saving face. Students who complained about feeling tired, sleepy, hungry, and having a headache during their problem solving were less likely to continue engaging with the problem. The students were also aware of this. For example, Kirsten commented on being sleepy during her problem solving session. She said that solving these problems after having a long day of math “confused me even more.” Dani also attributes her lack of performance to her being hungry and inside of a cafeteria. These physical factors correspond to the literature, which suggests that external factors such as physical surroundings and conditions affect students’ decision-making during the problem solving process (Jonassen D., 2000; DeBellis & Goldin, 2006). In this study, these physical conditions limited students’ capacity to engage with the problems.
There were two students who were concerned about how they were perceived by the researcher, which I categorized as saving face. These students felt as if I was judging their mathematical ability but it influenced them to make different choices. Nathan did not attempt to solve the problems and was even embarrassed watching his task-based interview. Contrary to Nathan, Dani was motivated to solve the problems because she wanted “to do well” on the research and she also wanted “to be the person that solves the entire problem.” She also didn’t want me to think of her as being ‘stupid’ or ‘lazy.’ Her need to have a good impression may have been influenced by the researcher’s role in the program. I was also the facilitator of her seminar course. Nathan however, was not in this course. These students were influenced by an external factor, the researcher, and it influenced their perseverance. According to Silver and Metzger (1989), the design of the study is also an external factor that affects students’ performance. Therefore, having the researcher as the interviewer and facilitator of a summer program seminar did have an affect on the students’ problem-solving process.

One goal of this study was to provide detailed descriptions of how students’ goals, knowledge, affect, and external contextual factors inform their choices during the problem solving process. The ability to attribute these factors to the decisions students make by collecting empirical data served as a validation of Schoenfeld’s problem-solving theory. In Chapter Four and the beginning of the current chapter, I gave detailed descriptions of how these factors influenced next steps students took while problem solving. Every decision that was made during the problem-solving process can be attributed to one of these factors. The extent to which a factor influenced students’ decisions varies. However, in the next section, I
discuss how students made decisions based on my findings across the six students in my study.

**Overarching research question**

*In what ways do students’ goals, knowledge, affect, and external contextual factors inform their choices during the mathematical problem solving process?*

Although each individual student goes through the problem-solving process in a different manner, how students as an aggregate make choices can be generalized. Below I provide a narrative of how a student might go through the problem-solving process given their goals, knowledge, affect and external contextual factors in light of my empirical data.

**Narrative.** Students enter the problem solving space with the overarching goal of solving the problem. During this time, they may feel anxiety but will persevere and begin the *orienting* phase of the problem solving process. They will *read* the problem initially. If they are confused, they will then *reread* the problem and begin accessing their *knowledge of problem solving strategies*. These strategies may include, *making a drawing* or creating tables. Regardless of their understanding of the problem they may begin solving the problem using the *guess and check* strategy. However, the connectedness of their goals and the extent of their content knowledge, will determine how organized and strategic their *guess and check* may be. They will also employ other problem-solving strategies and may even stumble across new ones. If they believe that problem solving requires exploration, they will continue to cycle between setting goals in the *executing* and *planning* phases. If they are implementing a problem-solving strategy without making any major progress they may continue to engage
because they believe that *problem solving requires exploration and patience*. However, if they are getting extremely frustrated and not making any progress, or there are physiological issues such as hunger or tiredness, they will likely quit. If they do continue and are able to arrive at a solution, they will set goals to check the solution if they are confident in the answer. They will most likely experience pleasure after solving the problem.

From this narrative, all factors (i.e., goals, knowledge, and affect) influenced the choices students made. Although external contextual factors did impact students’ motivation, it was not prevalent in all students. Students set goals throughout the problem-solving process that were mostly dependent on their knowledge. However, the ability for the student to access this knowledge is dependent mostly on their local affect (i.e., their emotions). Their local affect is mostly derived from their beliefs about mathematics and problem solving. If they hold favorable beliefs about mathematics and problem solving, in particular that *problem solving requires exploration and patience*, then a negative emotion will be used as motivation to continue the problem-solving process. They will then be able to access the knowledge that they do possess and begin to set goals as they cycle through the problem-solving process. However, if they do not have favorable beliefs about mathematics and problem solving, they may quit at that point or may not be able to access the knowledge necessary to solve the problem. In that case, failure is inevitable.

Goals, knowledge, and affect are intertwined and all contribute to the choices the students make. However, the data indicate that affect is the driving force that moves the student through the problem-solving phases. If students do not believe that it takes
exploration and patience to solve problems, if negative emotions are felt, their ability to access knowledge needed to connect mathematical ideas will be hindered. Eventually, they will discontinue the problem solving process. This is consistent with Schoenfeld’s proposed theory. He states that every decision that is made can be attributed to their goals, knowledge, and beliefs, which were explicitly shown from the empirical evidence and my narrative.

**Implications**

**Implications for mathematics educators**

There was a call to create a problem-solving theory so that accumulation of problem solving research may connect research to practice (English & Sriraman, 2010; Lester, 1994; Lesh & Zawojewski, 2007). This research has added to the creating of a problem-solving theory by validating Schoenfeld’s (2011) proposed theory’s relevance to mathematical problem solving using empirical evidence. Since the there is evidence that the theory holds for mathematical problem solving specifically, mathematics educators may now use this research to build trajectories of problem solving. These trajectories have the potential to provide a means to create curricula and assess students in an organized way.

These trajectories also have the potential to be used in pedagogical development of teachers. Teachers need to experience problem solving along with guidance in the careful selection of tasks that will promote critical thinking in students. De Corte (2000) makes the argument that to make changes in instruction and learning, there must be an agenda to change the belief system of teachers concerning the “goals of education, good teaching and productive learning” (De Corte, 2000, p. 1). Therefore, any initiative to change the beliefs of
students regarding mathematics and problem solving must begin with a change in the beliefs of teachers. Findings from this research suggest that both global and local affect influence students’ problem solving decisions. Therefore, changes in teacher beliefs about mathematics and problem solving along with training on the role of local affect on student learning will be needed. Teacher preparatory courses should address local affect in the classroom and ways to help students turn negative emotions into productive outcomes.

As noted by English & Sriraman (2010), this theory can provide a means to “reliably observe, document, and assess important mathematical developments in our students” (p. 269). This study provides evidence that a student’s problem-solving process can be analyzed and each factor observed individually. Future research should chart changes in students’ goals, knowledge, and affect and show how those changes improves their problem solving process. This theory can be used to organize the documentation of student’s problem-solving development by only looking at those factors and charting its changes over time. This can be used as formative assessment on student’s problem-solving ability.

**Implications for mathematics teachers K-16**

This study has validated that students’ goals, knowledge, and affect are influential factors in the decisions students make during problem solving. Therefore, Schoenfeld’s theory of problem solving may be used to inform decisions teachers make in the classroom. This indicates that mathematics teachers need to provide experiences that will shape their students’ belief about mathematics and problem solving. These experiences may begin with varying the types of problems students encounter and the amount of time students spend on
them (Schoenfeld A. H., 1992). In problem solving, task selection will be vital, in that teachers should select problems that are high demand. These problems should not always represent the present mathematics topic but also problems on content that are not expected and chosen to build problem solving ability.

Students also need confirmation of their emotional states in the classroom and how to use these emotions to yield productive results. Findings from this study show that students’ local affect contributes greatly to their decision to persevere in problem solving. The teacher must address these personal emotions along with the inevitable emotions that may surface as a result of the nature of interactions in a classroom. Therefore, teachers must be aware and prepared for these challenges by creating an emotionally safe environment so students can engage in mathematically challenging problems (Schorr, Epstein, Warner, & Arias, 2010).

**Limitations**

As with any study, there are limitations to this dissertation. First, a novice researcher conducted this research, which is a limitation because of the large qualitative data set that had to be managed and analyzed. This is a limitation because this was the first time the researcher designed and implemented a study of this magnitude. I was also the only person that coded and analyzed the data; therefore issues concerning reliability are warranted.

Another limitation is that the participants of the study are the result of a convenience sample. The participants were all part of a summer bridge program and were interested in STEM fields whether or not they felt comfortable with the content, which suggests this sample is not representative of the problem-solving process of all entering college freshmen.
However, this may be a representative sample of the STEM students who are attending the HBCU that held the program.

Another limitation to this study is that students solved problems in a laboratory-like setting. Therefore, their true problem solving process may not have been observed. There were measures put into place that attempted to minimize the effect of the surroundings such as making talk aloud optional, but the students were probably still impacted by the surroundings. Also, problem solving in the classroom is mostly done in groups. Therefore, not allowing students to work in groups also limited the authenticity of problem solving and in doing the results yielded by these students.

This research was also limited because I was not able to capture students’ metacognitive skills. Research has shown that these skills are an important component for making productive decisions during the problem-solving process (Lester, Garofalo, & Kroll, 1989; Schoenfeld A. H., 1983). Students’ metacognition manages, predicts, checks, monitors, and controls how well the problem-solving process is going and also which problem-solving strategies to implement (Schoenfeld A. H., 1983). As a result of not explicitly analyzing the effect of students’ metacognition, these decisions were only recognized as being influenced by their goals, knowledge, or affect.

**Future Research**

The goal of this study was to contribute to the creation of a problem-solving theory by using empirical data to validate it. Empirical evidence was collected to satisfy two goals: (1) provide detailed, rich descriptions about how and why students make decisions during
problem solving and (2) validate the scope of the proposed problem-solving theory. The first area of research can also work in providing a more thorough detailed, rich description of one student’s problem solving process. Concentrating on only one student will allow the researcher to study their decision-making on a finer grain size. New data will have to be collected so that a more thorough video-SR interview can occur. This interview will put added focus on the student’s beliefs and their metacognitive activities, which were minimally studied in this research.

Another potential area of research is to duplicate this study using another group of six students. However, in addition to collecting data from individual task-based interviews, data from the students’ problem solving process while working together will also be collected. Using this research design, the scope of the theory will be tested to include the problem solving process in a more realistic classroom setting. Conducting research within the classroom setting is valuable because the data will be more authentic and the findings will have the potential of being directly applicable to classroom practice. These findings may include the possibility of other factors that may influence students’ problem solving process that are only visible in a group setting. The scope of the theory will be tested. This research design is worth exploring because the ability to capture how groups of students’ problem solve within the dynamic nature of the classroom will be beneficial in connecting research to practice.

The importance of a student’s metacognition was addressed in my conceptual framework as being present and fueling all decisions made during the problem solving
process. However, to be more closely aligned to Schoenfeld’s (2008) use of this theory to model teaching, goals, knowledge, and affect were the focus. The inclusion of additional student beliefs and their metacognition will allow for a more rigorous and comprehensive testing of this theory and make the potential for the development of a model more likely.

Another area for future research is to study problem solving in the environment that it occurs, the classroom. Interaction among students and teachers creates a rich and dynamic environment where learning occurs (Confrey & Maloney, 2007). This environment will allow teachers and student to engage in mathematical discourse and exploration that may include disagreements, incorrect answers, and thoughtful questions (Schorr, Epstein, Warner, & Arias, 2010). As a result of the nature of these interactions, affective consequences may also surface. Therefore, research on students’ problem solving process in the classroom will result in more external contextual factors due to interactions among students and teachers. Data collection over time can also be collected to determine the changes that occur in students’ problem solving process individually and also collectively. This work can also be extended to include the use of culturally relevant problems that are directly related to the students lived experiences. Using problems in contexts that are culturally relevant to students has the potential to determine whether or not personal connection to problem contexts influences students’ engagement and ability to persist in problem solving.

A final area of future research is to expand this theory of mathematical problem solving to include mathematical modeling. The CCSS-M defines mathematical modeling as, “the process of choosing and using appropriate mathematics and statistics to analyze
empirical situations, to understand them better, and to improve decisions” (National Governors Association Center for Best Practices; Council of Chief State School Officers, 2010, p. 72). With this definition of mathematical modeling, the ability for a student to access and connect their mathematics knowledge to their knowledge of the real-world context will be of special interest. Zawojewski (2002), along with other researchers (e.g. Leavitt & Ahn, 2010), believe that modeling begins with an appropriate task that cannot be reduced to problem solving. This task must be created so that interest in the real-world problem is “represented by a mathematical system – which will simplify some things, delete others, maintain some features, and distort other aspects. The modeling process will also have to be considered along with the characteristics of modeling that differs from traditional problem solving. A redefining of goals, knowledge, and affect specific to mathematical modeling will be required. This endeavor will test the scope of this problem-solving theory.

**Conclusion**

Students must be able to problem solve to thrive in STEM careers; careers that are vital to our 21st century economy. A problem solving theory has the potential to accumulate existing research along with directly impacting curricula and classroom practices. This study provided empirical evidence to validate such a theory proposed by Schoenfeld (2011). The choices that students make during problem solving can be explained by their goals, knowledge and affect. This study confirms that affect is the factor that may be the most influential predictor for students’ decision to persevere. Therefore, students need to be exposed to classroom environments where emotions are validated and where experiences that
promote favorable beliefs about mathematics and problem solving. When students have favorable beliefs about mathematics and problem solving, then they are more likely to engage in the problem solving process.
References


http://dictionary.reference.com/browse/sudoku


Modeling students' mathematical modeling competencies (pp. 327-339). New York: Springer.


Proceedings from Topic Study Group 21 at the 11th International Congress on Mathematical Education, (pp. 197-206). Monterry, Mexico.


Quillen, M. A. (2004). *Relationships among prospective elementary teachers' belief about mathematics, mathematics content knowledge, and previous mathematics course experiences*. Virginia Polytechnic Institute and State University, Blacksburg, VA.


APPENDICES
Appendix A: Informed Consent

North Carolina State University
University of the Virgin Islands

INFORMED CONSENT FORM for RESEARCH

This form is valid from June 23, 2014 through June 23, 2014
Title: The Effect of a Summer Bridge Program on Students’ Problem Solving Skills and their Preparation for STEM College Curriculum
Principle Investigator: Nadia Monrose Mills

What are some general things you should know about research studies?
You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate or to stop participating at any time without penalty. The purpose of research studies is to gain a better understanding of a certain topic or issue. You are not guaranteed any personal benefits from being in a study. Research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher named above.

What is the purpose of this study?
The goal of this study is to document the effects of the Mathematics Behind the Science summer bridge program on their participants’ problem solving abilities and preparation for STEM college coursework.

What will happen if you take part in the study?
If you agree to participate in this study, you will be asked to participate in the following:
• Complete a pre/post survey on beliefs and attitudes towards mathematics
• Participate in three (3) interviews conducted by the researcher which will be audio and video recorded.

Risks
There are no physical or emotional risks associated with participation in this study.

Benefits
There is a potential for you to further your mathematical problem solving knowledge.

Confidentiality
The information in the study records will be kept confidential to the full extent allowed by law. Both videotaped and audiotaped files will be destroyed following transcription. All digital files, including transcriptions and summary documents created from the video and audio files will be stored securely by the principle investigator. Pseudonyms will be used in oral or written reports to avoid linking you to the study.

Compensation
There is no financial compensation for participation in this study. Participation in this study, will not affect your performance in the courses offered at the Mathematics Behind the Science program.

What if you have questions about this study?
If you have questions at any time about the study or the procedures, you may contact Mrs. Nadia Mills at (340) 514-9754 or at nmonros@ncsu.edu
What if you have questions about your rights as a research participant?
If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919-515-4514).

What if you are a minor (under 18 years of age)?
If you are a minor (under 18 years of age), you will need consent from a parent or legal guardian. Your parent or legal guardians must sign below allowing you to participate.

Consent To Participate
“\textit{I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to participate or to stop participating at any time without penalty or loss of benefits to which I am otherwise entitled.}”

Please check the appropriate box

- [ ] I agree to participate in all aspects of this study including audio and video recording of the pre/post reflection session meetings and the reflection sessions themselves.

- [ ] I choose not to participate in this study.

Participant’s signature_______________________________________ Date _________________
Parent/Legal Guardian signature _____________________________ Date _________________
Investigator’s signature______________________________________ Date _________________

Consent for Videotape Use in Presentations
“\textit{I consent for short excerpts of video recording from this project, as judged useful by the researcher, to be used in professional research presentations and teacher professional development materials or meetings, as long as I am not identified by name in such presentations. I agree to have the videos in which I appear shared, with the understanding that I may choose to have certain video segments excluded from the set of videos that can be used in professional research presentations and teacher professional development materials at any time and without penalties.}”

Participant’s signature_______________________________________ Date _________________
Parent/Legal Guardian signature _____________________________ Date _________________
Investigator’s signature______________________________________ Date _________________
Appendix B: Mathematical Knowledge Test

University of the Virgin Islands
The Mathematics Behind the Science bridge program
Pre-test: Pre-calculus skills

Student ____________________________

Instructions and Advice
To gain partial credit where earned, you must show your work.
State answers clearly.
Calculators are not permitted on this test.
Some of these questions are of the "multiple choice" variety. For these, show your work just as you do for
other questions, then circle the letter of the one answer you think is the best choice. Showing your work for
multiple choice answers and/or giving reasons for your answers will result in partial credit where appropriate.

1. Let \( y = 2x^3 + x^2 - x - 2 \). If \( x = -1 \), find the value of \( y \).

2. Find the value of \( a \) if \( 2(a - 3) = a - 9 \)

3. Simplify \( (x - 3)(x + 4) - (x - 10) \)

4. Give the value of each of the following:
   a. \( 2^{-3} \)
   b. \( (-3)^2 \)
   c. \( 5^0 \)
   d. \( 16\% \)
5. Graph the line $y = \frac{2}{3}x - 1$

6. Find the slope of the line that passes through the points (-3, -8) and (5, -1).

7. Find all the solutions to the equation $3x^2 + 2x - 10 = x(2x - 1)$

8. Write an equation for the line that passes through the points (1,1) and (2,5).
9. Write an equation for the line shown on this graph.

10. Simplify the expression \(5x^3 + 6x - 6 + 3x^2 - x\)

11. Simplify the expression \(\frac{5x^3 + 2x^2 + x}{x}\)

12. The prism in the diagram below is a wooden block with square ends that are 5 cm on a side. The block is y cm long. Which of the following expressions represents the surface area of the block?
   a. 10 + y
   b. 50 + 4y
   c. 50 + 20y
   d. 25y
   e. 25y^2
13. Which of the following expressions represents the volume of the block described in question 12? (See previous page.)
   a. 10 + y
   b. 50 + 4y
   c. 50 + 20y
   d. 25y
   e. 25y^3

14. Where y = f(x) is a function whose graph is the parabola shown below, f(x) > 0 when
   a. x < -2 or x > 1
   b. x < 0
   c. -2 < x < 1
   d. x < -4
   e. x > -4

15. If the equations 3x – y = 11 and 3x + y = 1 are graphed on the same coordinate axis system, which of the following statements is true?
   a. The graphs are parallel lines; they do not intersect.
   b. The graphs intersect at a point where the y-coordinate is -5.
   c. The graphs intersect at a point where the y-coordinate is 0.
   d. The graphs intersect at a point where the y-coordinate is 2.
   e. The graphs intersect at a point where the y-coordinate is 4.
16. Which of the following resembles the graph of $f(x) = \frac{2}{3} x^3 + 4x + 1$?

a.  

b.  

c.  

d.  

e.  

17. If $\log_{10}(x + 1) = 2$, then the value of $x$ is

a.  0  
b.  9  
c.  99  
d.  999  
e.  none of these  

18. Which one of the following pairs of curves best resemble the graphs of $y = 2^x$ and $y = \log_2(x)$?

a.  

b.  

c.  

d.  

19. If \( \frac{(3x - 5)(x + 2)}{x - 4} = 0 \), then the value of \( x \) is
   a. 4
   b. -2 or 4
   c. -2 or 5/3
   d. -2 or 5/3 or 4
   e. 0 or -2 or 5/3 or 4

20. Find the distance between the points A(-1, 2) and C(4, 14).

21. If \( f(x) = 4x - 7 \), then \( f(a-1) = \)
   a. 4a - 11
   b. a - 7
   c. 4a - 7
   d. 4a - 8
   e. 7

22. \( 1 - \cos^2(\theta) = \)
   a. \( \cos(2\theta) \)
   b. \( \sin(2\theta) \)
   c. \( \sin^2(\theta) \)
   d. \( \sin(\theta) \)
   e. \( -\sin^2(\theta) \)
23. Which of the following best represents the graph of \( y = \cos(x) \)?

a. 

b. 

c. 

d. 

e. 

24. If \( f(x) = \sin(2x) \), then \( f\left(\frac{\pi}{4}\right) = \)

a. \( \frac{\sqrt{2}}{2} \)

b. \( \frac{1}{2} \)

c. 0

d. 1

e. \( -\frac{\sqrt{2}}{2} \)

25. For which of the following values of \( x \) is \( \tan(x) \) not defined?

a. \( \frac{3\pi}{2} \)

b. \( 3\pi \)

c. \( \frac{\pi}{3} \)

d. \( \frac{3}{\pi} \)

e. 0
26. \( \cos(\theta) \tan(\theta) = \)
   a. 1
   b. \( \sin(\theta) \)
   c. \( \cos^2(\theta) \)
   d. \( \cot(\theta) \)
   e. \( \csc(\theta) \)

27. Find the exact value of \( \sin\left(\frac{88\pi}{3}\right) \).

28. Which of the following pairs of graphed functions appear to be inverses of each other? (That is, if one curve on the graph is \( y = f(x) \), then the other is \( y = f^{-1}(x) \).)

   a. 
   b. 
   c. 
   d. 

Appendix C: Revised Survey

*Name:*

*The Mathematical Beliefs Survey* (Yackel, 1984)

All individual responses on this survey will be kept strictly confidential. Your responses will be used to study the relationships between student beliefs about mathematics, teaching methods used, and certain other variables such as mathematics background.

For each item, circle the response that indicates how you feel about the item as indicated below.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
</tr>
</tbody>
</table>

**Part 1.**

3. **Doing mathematics consists mainly of using rules.**
   
4. **Learning mathematics mainly involves memorizing procedures and formulas.**
   
5. **Mathematics involves relating many different ideas.**
   
6. **Getting the right answer is the most important part of mathematics.**
   
7. **In mathematics it is impossible to do a problem unless you’ve first been taught how to do one like it.**
   
8. **One reason learning mathematics is so much work is that you need to learn a different method for each new class of problems.**
   
9. **Getting good grades in mathematics is more of a motivation than is the satisfaction of learning new mathematics content.**
   
10. **When I learn something new in mathematics I often continue exploring and developing it on my own.**

11. **I usually try to understand the reasoning behind all of the rules I use in mathematics.**

12. **Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works.**

13. **A common difficulty when taking quizzes and exams in mathematics is that if you forget**
relevant formulas and rules you are lost.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>14. It is difficult to talk about mathematical ideas because all you can really do is explain how to do specific problems.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>15. Solving mathematics problems frequently involves exploration.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>16. Most mathematics problems are best solved by deciding on the type of problem and then using a previously learned solution method for that type.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>17. I forget most of the mathematics I learn in a course soon after the course is over.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>18. Mathematics consists of many unrelated topics.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>19. Mathematics is a rigid, uncreative subject.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>20. In mathematics there is always a rule to follow.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>21. I get frustrated if I don’t understand what I am studying in mathematics.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>22. The most important part of mathematics is computation.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
</tbody>
</table>

Part 2.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I usually enjoy mathematics.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>2. Mathematics is boring.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>3. Working on difficult mathematics problems makes me frustrated.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>4. When I read newspaper and magazine articles I skip over numbers and numerical material.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>5. I only take mathematics courses because they are required.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>6. I think math is fun and is a challenge to learn.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>7. Mathematics has no connection to the real world.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>Statement</td>
<td>Response</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---------------------------------------------------------------------------</td>
<td>----------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Mathematics is a subject that some people can do and others just can’t.</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>My overall feeling towards mathematics is positive.</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Mathematics is used on a daily basis in many jobs.</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Mathematics is easy for me.</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>I like to work on hard mathematics problems.</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Most mathematics courses go to fast for me</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Mathematics is a subject that men do better in than women.</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>I would like to learn more about mathematics (statistics).</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>I rate my mathematics ability as above average</td>
<td>SD</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix D: Original Video- Stimulated Recall Interview Protocol

Participant: ___________________________  Date: __________________

Interview start time: __________  Interview end time: __________

Say the following before recording

- Thank the interviewee for their time and participation in my study
- Introduce myself as a PhD student in mathematics education who is conducting this interview for my dissertation.
- Explain that the information collected from this interview will be held confidential, I will be the only person able to connect the interview to the student
- Explain that neither the interview nor the findings will have any affect on their participation in the program.
- Inform interviewee that the conversation will be recorded so that the interview data can be transcribed and analyzed.
- Inform interviewee that they may stop at anytime during the interview.

Interview:

Say: We are going to view the video of your problem solving session that you completed yesterday. I am going to ask you questions pertaining to the mathematical decisions you made while solving the problem along with the emotions you were experiencing during the interview.

Sample Questions:

- Tell me how you felt about the entire problem solving process?
- Do you feel like this each time you are presented with a mathematical problem-solving task?
- Why did you decide to use a table, calculator, etc. to organize/solve the problem?
- How did you know when you were making progress and could move on?
- Were there any other choices/methods you could have used here but didn’t? If yes, which one?
- What were you feeling at that moment?
- How did this feeling influence your next decision?
Appendix E: Piloted Problems

Problems

1. Money Problem: Divide five dollars amongst eighteen children such that each girl gets two cents less than each boy.
2. Let P be the product of all the positive divisors of 50,000. Show that \( P = (50,000)^{15} \). If possible, generalize.
3. Place the positive integers 1-9 in a 3 x 3 grid. Each integer should be used only once and only one integer should be placed in a grid. Arrange the integers such that the sum for each row, column and diagonal is the same.
4. We have 3 campers, A, B and C. They can combine supplies for dinner. C has no rice, A has 500 grams of rice and B has 300 grams of rice. They combined all the rice and shared it so that all three campers got the same share. C pays $8 for the meal to A and B. How much should Camper C give Campers A and B so that they are paid according to how much they contributed towards Camper C’s meal?
5. Find and justify a recursive formula that will determine:
   a. At most how many regions on a line are determined by \( n \) points on the line?
   b. At most how many regions in a plane are determined by \( n \) lines in the plane?
6. There are three friends, Joe, Mary and Helen who are playing a game. There is one loser and two winners at the end of each round of the game. The loser pays to each winner the amount of money that person had at the beginning of the round.

<table>
<thead>
<tr>
<th>Round</th>
<th>Player Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Joe</td>
</tr>
<tr>
<td>2</td>
<td>Mary</td>
</tr>
<tr>
<td>3</td>
<td>Helen</td>
</tr>
</tbody>
</table>

For example, if before round 1 Mary had $10 and Helen $16. Since Joe lost, then Joe will have to give Mary $10 and Helen $16. At the end of 3 rounds, they all have $24. How much money did each player begin with?
7. Find all triangles with integral lengths for sides and whose perimeter is 12 inches (Krulik & Rudnick, 1996, p. 70).
8. In a sports center in the Philippines, Florentino Anonuevo Jr. polishes a pair of shoes. They are, according to the Guinness Book of Records, the world’s biggest, with a width of 7.78 feet and a length of 17.36 feet. Approximately how tall would a giant be for these shoes to fit? Explain your solution. (Blum & Ferri, Mathematical modelling: Can it be taught and learnt?, 2009, p. 45)
9. Consider a Ferris wheel with a radius of 36 feet that takes 1.2 minutes to complete a full rotation. April boards the Ferris wheel and begins a continuous ride on the Ferris wheel. If the platform to board the Ferris wheel is 8 feet off the ground, sketch a
graph that relates the total distance traveled by April and her vertical distance from the ground (Moore, Carlson, & Oehrtman, 2009).
Appendix F: Revised Video- Stimulated Recall Interview Protocol

Participant: _______________________________ Date: _____________________

Interview start time: ___________ Interview end time: ___________

Say the following before recording

- Thank the interviewee for their time and participation in my study
- Introduce myself as a PhD student in mathematics education who is conducting this interview for my dissertation.
- Explain that the information collected from this interview will be held confidential, I will be the only person able to connect the interview to the student
- Explain that neither the interview nor the findings will have any affect on their participation in the program.
- Inform interviewee that the conversation will be recorded so that the interview data can be transcribed and analyzed.
- Inform interviewee that they may stop at anytime during the interview.
- Do not share the content of this interview or the problems with anyone.

Interview:
Say: We are going to view the video of your problem solving session that you completed yesterday. I am going to ask you questions pertaining to the mathematical decisions you made while solving the problem along with the emotions you were experiencing during the interview.

Goals
- What motivated you to continue working on the problem?
- What caused you to stop working on the problem?
- What motivates you desire to engage in solving this problem?

Content Knowledge
- Could you have solved this question another way?
- If the student makes a computational error, explore this error.
- Were there any other choices/methods you could have used here but didn’t? If yes, which one?
- Did you try something new throughout the process?
- If yes, what did you try that was new and what did your hope it would produce that would bring you closer to the solution of the problem?

Knowledge of PS Strategies
- Why did you decide to use a table, calculator, etc. to organize/solve the problem?
• Can you think of another way this problem could have been solved or organized?
• Were there any other choices/methods you could have used here but didn’t? If yes, which one?

Local Affect (Emotions)
• Tell me how you felt about the entire problem solving process?
• Do you feel like this each time you are presented with a mathematical problem solving task?
• How did you feel at this instant?
• How did this feeling affect what you decided to do next?

Global Affect (attitudes/beliefs)
• Do you believe that this problem should have been solved faster?
• Do you believe that all math problems should be solved in a short space of time?
• Why do you believe that time plays a factor in whether the problem can be solved or not?

Monitoring/Self-regulation
• How did you know when you were making progress and could move on?
• Did you check your final answer? Why or why not?

Other Contextual Factors
Confidence
• How confident were you in your ability to solve this problem?
• What happened to your level of confidence throughout solving the problem?

Time
• Did you feel pressured by time? Why?
• Do you think time played a part in your problem solving process?

Group/Individual Work
• Do you believe you would be able to solve this problem if you worked in a group setting? Why/why not?
• Do you prefer to work by yourself, with a partner, or with a group?
Appendix G: Summary Table

Table G 1: Summary Table

<table>
<thead>
<tr>
<th>Problem-solving strategies</th>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
<th>Problem 4</th>
<th>Problem 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>TB &amp; V-SR: Read and rereads problem</td>
<td>None</td>
<td>TB: None</td>
<td>TB: Attempts to create a formula, reread problem multiple times for clarity, does not</td>
<td>Simplifying, making conclusions</td>
</tr>
<tr>
<td>Problem 2</td>
<td>None</td>
<td>TB: None</td>
<td>TB: Read problem for understanding, determine the meaning of plane</td>
<td>TB: None</td>
<td>TB: Spent time trying to solve the problem, gave a genuine effort; determine how much they get paid, how much they contribute, how much they should get for contributing from $8, A has to get more than B; try to determine a formula (steps to take to find the solution)</td>
</tr>
<tr>
<td>Problem 3</td>
<td>V-SR: Understand the problem, divide 5 and 18, determine # of girls (unsuccessful), write down solution</td>
<td>V-SR: Reading through problem for understanding, thinking of how problem is similar to one previously worked on, finish in a short amount of time</td>
<td>TB: Read problem for understanding, determine the meaning of plane</td>
<td>TB: None</td>
<td>TB: Spent time trying to solve the problem, gave a genuine effort; determine how much they get paid, how much they contribute, how much they should get for contributing from $8, A has to get more than B; try to determine a formula (steps to take to find the solution)</td>
</tr>
<tr>
<td>Problem 4</td>
<td>TB: Determine # of girls and boys, calc $ per person with no bias</td>
<td>TB: None</td>
<td>V-SR: Did not know the meaning of plane, wanted numbers to work with</td>
<td>TB: Wanted to create a formula but could not, V-SR: Did not understand 3 X 3, only knew n X n in terms of pant size, cannot draw a picture for math</td>
<td>Comparing quantities</td>
</tr>
<tr>
<td>Problem 5</td>
<td>V-SR: Understand the problem, divide 5 and 18, determine # of girls (unsuccessful), write down solution</td>
<td>V-SR: Reading through problem for understanding, thinking of how problem is similar to one previously worked on, finish in a short amount of time</td>
<td>TB: Read problem for understanding, determine the meaning of plane</td>
<td>TB: None</td>
<td>TB: Spent time trying to solve the problem, gave a genuine effort; determine how much they get paid, how much they contribute, how much they should get for contributing from $8, A has to get more than B; try to determine a formula (steps to take to find the solution)</td>
</tr>
<tr>
<td>Content knowledge</td>
<td>Ratios (5/18), subtraction (.27-.02),</td>
<td>TB: None</td>
<td>TB: Did not know the meaning of plane, wanted numbers to work with</td>
<td>TB: Wanted to create a formula but could not, V-SR: Did not understand 3 X 3, only knew n X n in terms of pant size, cannot draw a picture for math</td>
<td>Comparing quantities</td>
</tr>
<tr>
<td>Local affect</td>
<td>TB: Frustrated, not confident, dumb, doubting self, confused, not happy, agitated (needed to calm self down by whistling, rubbing head)</td>
<td>TB: Frustration V-SR: Embarrassed, frustrated, anxiety</td>
<td>TB: Frustration (*this is already getting to me), scared (of content, language, terms)</td>
<td>TB: Frustrated, defeated, no confidence, V-SR: has anxiety when solving math problems (“everything goes blank”), nervous, scared (didn’t know what to put in the box)</td>
<td>TB: frustration, defeat V-SR: Anxiety, doubting self, indecisiveness</td>
</tr>
</tbody>
</table>
### Table G 1 Continued

| Global affect | TB: Assumed problem should be easy, hate word problems, overthinking, V-SR: Believes his learning style affects his ability to solve problems using formulas (artist type, will do better at geometry), don’t think that PS should take a long time, the first try yield solution, does not pursue difficult problems, has gut feelings about solutions but does not act on them | TB: don’t like to attempt problems that will take a lot of time. V-SR: Believes that questions are presented in order of difficulty, believes only experience with similar problems will help solve other problems. Not creative in using a different approach, since he is a visual learner, he cannot solve problems without diagrams or formulas. Believes problems should be solved quickly, if a problem is taking too long quit solving, learned from schooling that it should not take a long time to solve problems, experiences anxiety (other local affect from Prob. 1 & 2) when solving problems | TB: Gives up easily on word problems if a clear path isn’t recognized immediately, assumes that problems come in order of difficulty (“If I can’t do A, I can’t do B) | T: problems come in order of difficulty (“I can’t even get the first one …), believes that he has to be exposed to or learn how to do a particular problem to be successful, does not see value in brainstorming/exploring ideas, Believes studying for math is not possible for him/does not understand, believes one can only learn math through lecture in a class setting or tutorial where one person teaches the other | If problem looks similar then he should be able to solve (maybe that’s why he spent a little more time on this one), should know all steps to solving a problem before writing anything down. |
| External contextual factors | Was taught that his ‘artist type’ learning styles makes him better at geometry, asking questions in classroom environment uncomfortable, uses gut feelings to determine correctness of answers, | Embarrassed, school training about timed tests, managing time based on years of schooling and personal habits (being on time always, don’t waste too much time on a problem); put time pressures on self even if given unlimited time, thinks interviewer judging his intellect | Did not have the “proper learning”, didn’t want to “waste time” Just solved formulas never had to create a formula, believes he cannot do math from small, math exploration is time consuming (time not well spent), difficult for him to understand algebra (letters with numbers); his experience in schooling has him to believe that math is not about exploration and there must be a “teacher” to instruct on how to do problems. | Stated a saying: “Think before you talk, think before you write anything down” may be schooling that gave him this idea; time did not play a factor |
## Appendix H: Themes

Table H 1: Themes for goals

<table>
<thead>
<tr>
<th>Problem-solving phase</th>
<th>Themes</th>
<th>Description</th>
</tr>
</thead>
</table>
| Orienting             | Reading the problem | • Effort is made to read and reread to understand the problem.  
                         |         | • Restates givens |
|                       | Organizing | • Effort is made to organize the problem by creating a table, graph, diagram, text, or expressed verbally  
                         |         | • Effort is made for sense-making in general |
|                       | Connecting | • Effort is made to connect problem to one familiar |
|                       | Defining   | • Terms are defined |
| Planning              | Choose a problem solving strategy | • A strategy is determined |
|                       | Choose an alternate problem solving strategy | • An alternate strategy is identified. |
| Executing             | Carries out computations | • Calculations are carried out |
|                       | Implements a problem solving strategy | • **Guess and check** and other strategies are worked out |
| Checking              | Testing results | • Results are tested for their reasonableness  
                         |         | • Decides to accept solution (correct or incorrect)  
                         |         | • Gives up without solution |
### Table H 2: Themes for content knowledge

<table>
<thead>
<tr>
<th>Themes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge of traditional problem solving strategies</td>
<td>Implementing the following problem solving strategies: look for a pattern, guess and check, make a drawing, and working backwards</td>
</tr>
<tr>
<td>Knowledge of recently encountered content</td>
<td>Content knowledge influenced by the summer bridge program</td>
</tr>
<tr>
<td>Knowledge of mathematical relationship</td>
<td>The ability to interpret the problem and make connections to the mathematics content</td>
</tr>
<tr>
<td>Knowledge of similar problems</td>
<td>Using knowledge of past problems solved or similar problems to solve</td>
</tr>
</tbody>
</table>

### Table H 3: Themes for local affect

<table>
<thead>
<tr>
<th>Themes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anxiety</td>
<td>Expresses anxiety, nervousness, scared</td>
</tr>
<tr>
<td>Confusion</td>
<td>Expresses confusion about problem. Also feels hopeless, lost, clueless, incompetent</td>
</tr>
<tr>
<td>Frustration</td>
<td>Expresses anger, frustration or impatience, agitated, embarrassed, dumb, annoyed, sad, upset, stupid</td>
</tr>
<tr>
<td>Pleasure</td>
<td>Expresses positive emotions. Can also include feeling accomplished, happy, confident, comfortable, amused, enlightened, excitement, triumph</td>
</tr>
</tbody>
</table>
Table H 4: Themes for global affect

<table>
<thead>
<tr>
<th>Theme</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schooling influence</td>
<td>Beliefs the student holds which can be attributed directly to experiences in school</td>
</tr>
<tr>
<td>Beliefs about own learning</td>
<td>Beliefs students hold about their own learning and ability to do mathematics and solve problems</td>
</tr>
<tr>
<td>Mathematics involve relating different ideas</td>
<td>Evidence that shows students’ belief about mathematics</td>
</tr>
<tr>
<td>Problem solving requires exploration and patience</td>
<td>Evidence that shows students’ belief about problem solving</td>
</tr>
<tr>
<td>Calculator beliefs</td>
<td>Evidence that shows students’ belief about the use of calculators</td>
</tr>
</tbody>
</table>

Table H 5: Themes for external contextual factors

<table>
<thead>
<tr>
<th>Theme</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical Limitations</td>
<td>Physical factors that influence students’ problem solving such as sleepiness, hunger, pains, tiredness</td>
</tr>
<tr>
<td>Saving Face</td>
<td>The need to impress the researcher or compares mathematics ability with others in the summer bridge program</td>
</tr>
</tbody>
</table>
Appendix I: Students’ Written Work on Task-based Interview Problems

Figure I 1: Tony’s written work
1. Money Problem: Divide five dollars among eighteen children such that each girl gets two cents less than each boy.

\[
\frac{5}{8} = 0.625 = 0.63
\
9 \text{ girls} \quad 5
\
9 \text{ boys}
\
\frac{5.00}{18} = 0.278 \quad 0.28
\
2.5 \quad 2.3 \times 9 = 20.7
\
0.26 \quad 0.27
\
0.29
\
0.268 \quad 2.
\
0.298
\
0.28 \times 18 = 5.04 \quad 2 \text{ girls}
\
0.26 \times 2 = 0.52
\
0.28 \times 10 = 4.48 \quad \frac{5.00}{5.00}
\
2 \text{ girls} \quad \$0.26 \text{ each}
\
16 \text{ boys} \quad \$0.28 \text{ each}
1. There are three friends, Joey, Maria and Helen who are playing a game. There is one loser and two winners at the end of each round of the game. The loser pays to each winner the amount of money that person had at the beginning of the round.

<table>
<thead>
<tr>
<th>Round</th>
<th>Player Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Joey</td>
</tr>
<tr>
<td>2</td>
<td>Maria</td>
</tr>
<tr>
<td>3</td>
<td>Helen</td>
</tr>
</tbody>
</table>

At the end of 3 rounds, they all have $24. How much money did each player begin with?

\[
24 \times 3 = 72
\]

\[
3\sqrt{24} = 2.884
\]

<table>
<thead>
<tr>
<th></th>
<th>J</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>

Each player began with $6.
Find and justify a formula that will determine:

a. At most how many regions on a line are determined by \( n \) points on the line?
   \[ R(n) = n - 1 \]

b. At most how many regions in a plane are determined by \( n \) non-parallel lines in the plane?
   \[ R(n) = 2n, \text{if } n \leq 2 \]
   \[ 2n + (n - 2), \text{if } n > 2 \]

\[ 2n + (n - n - 1) \]
Place the positive integers 1-9 in a 3 x 3 grid. Each integer should be used only once and only one integer should be placed in a grid. Arrange the integers such that the sum for each row, column and diagonal is the same.
$n$ = turning point

$R(n) =$ regions = branches of the curve

$R(n) = n + 1$

$1 \ 2 \ 3 \ 4 \ \boxed{5} \ 6 \ 7 \ 8 \ 9$

$\begin{array}{ccc}
2 & 7 & 6 \\
9 & 5 & 1 \\
4 & 3 & 8
\end{array} = 15$

$15 \ 15 \ 15 \ 15 \ 15$
We have 3 campers, A, B and C. They can combine supplies for dinner. C has no rice, A has 500 grams of rice and B has 300 grams of rice. They combined all the rice and shared it so that all three campers got the same share. C pays $8 for the meal to A and B. How much should Camper C give Campers A and B so that they are paid according to how much they contributed towards Camper C’s meal?

\[ A = 500 \]
\[ B = 300 \]
\[ C = D \]

\[ \frac{800}{3} \]
\[ \frac{300}{3} \]

$8.00

$0.01 per gram of rice

\[ \frac{266 \frac{2}{3}}{3} \]

\[ A : B \]

7 : 1

\[ A = 233 \frac{1}{3} \]
\[ B = 33 \frac{1}{3} \]

A = $7.00
B = $1.00
Figure I 2: Leann’s written work
1. Money Problem: Divide five dollars among eighteen children such that each girl gets two cents less than each boy.

\[
\text{Boys: } \frac{\$5}{9} = 56 \text{ cents each}
\]
\[
90 \times 9 = 18 \text{¢}
\]

Boys: $3.62 each
Girls: $3.58 each
1. There are three friends, Joey, Maria and Helen who are playing a game. There is one loser and two winners at the end of each round of the game. The loser pays to each winner the amount of money that person had at the beginning of the round.

<table>
<thead>
<tr>
<th>Round</th>
<th>Player Lost</th>
<th>Lost Once</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Joey</td>
<td>lost once</td>
</tr>
<tr>
<td>2</td>
<td>Maria</td>
<td>lost once</td>
</tr>
<tr>
<td>3</td>
<td>Helen</td>
<td>lost once</td>
</tr>
</tbody>
</table>

At the end of 3 rounds, they all have $24. How much money did each player begin with?

Assume that players have unlimited funds.

\[
\text{\$12 is the initial amount}
\]

Each player was a winner twice.

\[
\text{\$12 + \$12 = \$24}
\]
Find and justify a formula that will determine:

a. At most how many regions on a line are determined by \( n \) points on the line?

b. At most how many regions in a plane are determined by \( n \) non-parallel lines in the plane?

a) A possible formula could be \( \binom{n}{2} \), which should show an infinite amount of regions for a line determined by \( n \) points.

b) I do not believe that there is any \( n \) non-parallel lines in a plane, due to the fact that there can be an infinite amount of lines and it is possible to have lines that are parallel to each other.
Place the positive integers 1-9 in a 3 x 3 grid. Each integer should be used only once and only one integer should be placed in a grid. Arrange the integers such that the sum for each row, column and diagonal is the same.

<table>
<thead>
<tr>
<th>8</th>
<th>1</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>
We have 3 campers, A, B and C. They can combine supplies for dinner. C has no rice, A has 500 grams of rice and B has 300 grams of rice. They combined all the rice and shared it so that all three campers got the same share. C pays $8 for the meal to A and B. How much should Camper C give Campers A and B so that they are paid according to how much they contributed towards Camper C's meal?

\[
500 + 300 = 800
\]

\[
\frac{2}{5} \times \frac{800}{6}\]

\[
\frac{40}{80} \times \frac{5}{20} = \frac{1}{20}
\]

Each camper got approximately 2/20 of 800g of rice.

If camper C has $8 and distributes the money according to how much Campers A and B contributed to the meal, then Camper A will receive \(\frac{5}{600} \times 8\) and Camper B will receive \(\frac{3}{600} \times 8\).
Figure I 3: Kirsten’s written work
1. Money Problem: Divide five dollars among eighteen children such that each girl gets two cents less than each boy.

\[
\begin{array}{c|cccc}
\text{Girls} & 9 & \text{Boys} & 9 & \text{Total} \\
\text{Amount} & \$2.50 & \$2.50 & \$2.50 & \$7.50 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{Girls} & 2.28 & \text{Boys} & 2.28 & \text{Total} \\
\text{Amount} & \$2.25 & \$2.25 & \$2.25 & \$6.75 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{Girls} & 2.34 & \text{Boys} & 2.34 & \text{Total} \\
\text{Amount} & \$2.26 & \$2.26 & \$2.26 & \$6.78 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{Girls} & 2.52 & \text{Boys} & 2.52 & \text{Total} \\
\text{Amount} & \$2.52 & \$2.52 & \$2.52 & \$7.56 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{Girls} & 2.90 & \text{Boys} & 2.90 & \text{Total} \\
\text{Amount} & \$2.90 & \$2.90 & \$2.90 & \$8.70 \\
\end{array}
\]
2.48 g'rl
≈ .28

2.52

2.52

5.84

.28

7 boys 11 girls
≈ .28
≈ .23

2.59
2.53

1.40 4.40 2.94
3.60
1.40

8 boys 10 girls
.31 .23 .23 .25

2.48 .81 .30
2.86

4.78

2.2

4.34

.38 .36

≈ .22 per kid

9 boys 9 girls
1.08 15 1.60

.18

5.00

2.28

9.38

3.38

12 boys

6 girls

.50

.36

.34 .7

.38

.31 .33

2.31

2.58 g'rls 2.78 g'ys

2.60

2.322 4.86
4.822

1.18
Find all triangles with integral lengths for sides and whose perimeter is 12 inches.
Kirsten  July 2
Find and justify a formula that will determine:

a. At most how many regions on a line are determined by \( n \) points on the line?
b. At most how many regions in a plane are determined by \( n \) non-parallel lines in the plane?

a) All regions are determined by \( n \). \( r = n \)

b) Four regions because non-parallel lines, most likely they could be perpendicular and separate the plane into four quadrants or regions. \( r = 2n \)
Place the positive integers 1-9 in a 3 x 3 grid. Each integer should be used only once and only one integer should be placed in a grid. Arrange the integers such that the sum for each row, column and diagonal is the same.

```
6 8 1
2 4 9
7 3 5
```
Kirsten July 15
Name: Kristen

Time Start: 1:32
Time End: 1:37

Date: July 29

1. There are three friends, Joey, Maria and Helen who are playing a game. There is one loser and two winners at the end of each round of the game. The loser pays to each winner the amount of money that person had at the beginning of the round.

<table>
<thead>
<tr>
<th>Round</th>
<th>Player Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Joey</td>
</tr>
<tr>
<td>2</td>
<td>Maria</td>
</tr>
<tr>
<td>3</td>
<td>Helen</td>
</tr>
</tbody>
</table>

At the end of 3 rounds, they all have $24. How much money did each player begin with? $8, $12.
Round one 2 winners

<table>
<thead>
<tr>
<th>Round</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round 1</td>
<td>$16</td>
</tr>
<tr>
<td>Round 2</td>
<td>$12</td>
</tr>
<tr>
<td>Round 3</td>
<td>$24</td>
</tr>
</tbody>
</table>
We have 3 campers, A, B and C. They can combine supplies for dinner. C has no rice, A has 500 grams of rice and B has 300 grams of rice. They combined all the rice and shared it so that all three campers got the same share. C pays $8 for the meal to A and B. How much should Camper C give Campers A and B so that they are paid according to how much they contributed towards Camper C’s meal?

Camper A - $5
Camper B - $3
\[
\frac{\text{no rice}}{A \text{ 500 grams}} + \frac{\text{500 grams}}{B \text{ 500 grams}} \approx 100 \text{ grams of rice} \\
\approx 251 \text{ grams per camper}
\]
Figure I 4: Dani’s written work
1. Money Problem: Divide five dollars among eighteen children such that each girl gets two cents less than each boy.

\[
\begin{align*}
\text{10¢} & \quad \text{6 girls} & \quad 60\\
\text{6 boys} & \quad & \\
\text{5.18} & \quad & \\
\text{2.77} & \quad & \text{27} \quad .29\\
\text{2.70} & \quad \text{2.30} & \quad \text{2.62} \quad \text{2.38} \\
\text{2 boys} & \quad 10 girls & \quad \text{6 boys} \\
\text{10} & \quad \text{1.23} & \quad \text{.34} \\
\text{3.3} & \quad & \\
\text{.43} & \quad & \text{per boy} \\
\text{.43} & \quad & \text{per girl} \\
\text{2.68} & \quad \text{3.22} \\
\text{9 boys} & \quad \text{9 girls} & \\
\text{.29 / .30} & \quad \text{.25 / .26} \\
\text{11 boys} & \quad \text{7 girls} \\
\text{7 boys} & \quad \text{5 girls} \\
\text{.24} & \quad \text{.32 / .33}
\end{align*}
\]
1. There are three friends, Joey, Maria and Helen who are playing a game. There is one loser and two winners at the end of each round of the game. The loser pays to each winner the amount of money that person had at the beginning of the round.

<table>
<thead>
<tr>
<th>Round</th>
<th>Player Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Joey</td>
</tr>
<tr>
<td>2</td>
<td>Maria</td>
</tr>
<tr>
<td>3</td>
<td>Helen</td>
</tr>
</tbody>
</table>

At the end of 3 rounds, they all have $24. How much money did each player begin with?
Find and justify a formula that will determine:

a. At most how many segments on a line are determined by \( n \) points on the line?

b. At most how many regions in a plane are determined by \( n \) lines in the plane?
a. \( M = x \)

\[
\begin{align*}
\text{Horizontal + Vertical} & \quad y = x + (x + 1) \\
\text{y} & = 4 + (5) \\
\text{9} & \\
\end{align*}
\]

b. 

\[
\begin{align*}
\text{Horizontal} & \quad y = x + 1 \\
\text{Vertical} & \quad y = x + 1 \\
\end{align*}
\]
Place the positive integers 1-9 in a 3 x 3 grid. Each integer should be used only once and only one integer should be placed in a grid. Arrange the integers such that the sum for each row, column and diagonal is the same.
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

1 | 4 | 7 |
---|---|---|
2 | 3 | 8 |
3 | 6 | 9 |
6 | 15 | 24 |

9 | 6 | 5 |
---|---|---|
1 | 3 | 6 |
2 | 4 | 7 |

1 | 8 | 3 | X |
---|---|---|---|
2 | 5 |
4 |

4 | 6 | 5 |
---|---|---|
7 | 1 | 8 |
3 | 9 | 2 |

X
Name: Dani
Date: July 28

Time Start: 1:14  Time End: 1:38

We have 3 campers, A, B and C. They can combine supplies for dinner. C has no rice, A has 500 grams of rice and B has 300 grams of rice. They combined all the rice and shared it so that all three campers got the same share. C pays $8 for the meal to A and B. How much should Camper C give Campers A and B so that they are paid according to how much they contributed towards Camper C’s meal?
Davi

Allonzio 500g
Beth 800g

Catherine $8

\[ 3\sqrt{800} \]
266.67g

\[ \frac{266.67 + 266.67}{2} = 266.66 \]

Allonzio $5
Beth $3

\[ \frac{800}{5} = 160 \]
\[ 533.34 \]

266.66 - x
300 - 100 = 53.33%

500(2.5) = 2666

300
\[ A + B = 533.34 \text{ g} \]

\[ C = 266.66 \text{ g} \]

\[
\begin{array}{c}
\text{500} \\
266.66 \\
\hline
233.34
\end{array}
\quad
\begin{array}{c}
\text{200} \\
-266.66 \\
\hline
-33.34
\end{array}
\]

\[ \text{266.68} \]

\[ \frac{233.34}{266.68} \times = \frac{33.34}{266.68} = \frac{x}{8} \]

\[ \frac{266.68(x)}{266.68} = \frac{266.68(0.8)}{266.68} = \frac{426.72}{266.68} \]

\[ x = 6.99 \quad x = 1.00 \]

\[ x = 7.00 \quad x = 1.00 \]

\[ C \text{ pays A$7.00 and pays B$1.00} \]
Figure I 5: Nathan’s written work
1. Money Problem: Divide five dollars among eighteen children such that each girl gets two cents less than each boy.

\[
\frac{5.00}{18} = 0.27
\]

**boy:** 17¢

**girl:** 25¢
1. There are three friends, Joey, Maria and Helen who are playing a game. There is one loser and two winners at the end of each round of the game. The loser pays to each winner the amount of money that person had at the beginning of the round.

<table>
<thead>
<tr>
<th>Round</th>
<th>Player Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Joey</td>
</tr>
<tr>
<td>2</td>
<td>Maria</td>
</tr>
<tr>
<td>3</td>
<td>Helen</td>
</tr>
</tbody>
</table>

At the end of 3 rounds, they all have $24. How much money did each player begin with?
Find and justify a formula that will determine:

a. At most how many segments on a line are determined by n points on the line?

b. At most how many regions in a plane are determined by n lines in the plane?
1. Jamal and Kebo are selling fruit for the summer. Customers can buy boxes of genips and boxes of mangoes. Jamal sold 3 boxes of genips and 14 boxes of mangoes for a total of $203. Kebo sold 11 boxes of genips and 11 boxes of mangoes for a total of $220.

Mikey wants to also sell fruit. However, to be competitive he needs to know how much Jamal and Kebo are selling a box of genips and a box of mangoes. Jamal and Kebo will not tell him, they only gave him the information above.

How much should Mikey sell each box of his genips and mangoes? Why?
Solve.

\[
\begin{align*}
\text{Elimination} & \\
-4x + y &= 6 \\
4x - 5y &= 21
\end{align*}
\]

\[
\begin{align*}
-9y &= 27 \\
-9
\end{align*}
\]

\[
\begin{align*}
x &= -3
\end{align*}
\]

\[
\begin{align*}
\text{Substitution} & \\
-5x - (4x + 6) &= 21 \\
-9x - 6 &= 21
\end{align*}
\]

\[
\begin{align*}
-9x &= 27 \\
-9
\end{align*}
\]

\[
\begin{align*}
x &= -3
\end{align*}
\]
We have 3 campers, A, B and C. They can combine supplies for dinner. C has no rice, A has 500 grams of rice and B has 300 grams of rice. They combined all the rice and shared it so that all three campers got the same share. C pays $8 for the meal to A and B. How much should Camper C give Campers A and B so that they are paid according to how much they contributed towards Camper C’s meal?
Figure I 6: Phil’s written work
1. Money Problem: Divide five dollars among eighteen children such that each girl gets two cents less than each boy.

\[ \frac{5}{18} = .28 \] $/ct

\[ 2b = 2 \text{ cents less} \]

\[ .26 \rightarrow .28 \]
\[ .25 \rightarrow .27 \]
\[ .24 \rightarrow .26 \]

\[ G_1 = \frac{5}{18} - .02 \]
1. There are three friends, Joey, Maria and Helen who are playing a game. There is one loser and two winners at the end of each round of the game. The loser pays to each winner the amount of money that person had at the beginning of the round.

<table>
<thead>
<tr>
<th>Round</th>
<th>Player Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Joey</td>
</tr>
<tr>
<td>2</td>
<td>Maria</td>
</tr>
<tr>
<td>3</td>
<td>Helen</td>
</tr>
</tbody>
</table>

At the end of 3 rounds, they all have $24. How much money did each player begin with?

<table>
<thead>
<tr>
<th>Joey</th>
<th>Maria</th>
<th>Helen</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>

\[ \text{Joey: } 12 \times 4 = 48 \]
\[ \text{Maria: } 8 \times 12 = 96 \]
\[ \text{Helen: } 4 \times 12 = 48 \]
<table>
<thead>
<tr>
<th></th>
<th>Joey</th>
<th></th>
<th>Hania</th>
<th></th>
<th>Helen</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td></td>
<td>21</td>
<td>42</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td></td>
<td>12</td>
<td></td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td></td>
<td>24</td>
<td></td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

July 23 Phil
Find and justify a formula that will determine:

a. At most how many segments on a line are determined by \( n \) points on the line?

b. At most how many regions in a plane are determined by \( n \) lines in the plane?

A) \( s \geq 2 \)
We have 3 campers, A, B and C. They can combine supplies for dinner. C has no rice, A has 500 grams of rice and B has 300 grams of rice. They combined all the rice and shared it so that all three campers got the same share. C pays $8 for the meal to A and B. How much should Camper C give Campers A and B so that they are paid according to how much they contributed towards Camper C’s meal?
Phil

800 grams of rice

1 person = $266\frac{2}{3}$ grams

$266.67$ grams

A = 5
B = 3

\$ 8

1. 6

C gave A \$1.60
Based on their contribution to his meal

C gave B \$1.00
Name: Phil                                      Date: July 30
Time Start: 2:32                             Time End: 2:46

Place the positive integers 1-9 in a 3 x 3 grid. Each integer should be used only once and only one integer should be placed in a grid. Arrange the integers such that the sum for each row, column and diagonal is the same.

<table>
<thead>
<tr>
<th>4</th>
<th>a</th>
<th>2</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

9 2 4 4 9 2
5 7 3 3 5 7
1 6 8 8 1 6
\[
\begin{array}{c}
123 \\
456 \\
789 \\
\hline
121518
\end{array}
\begin{array}{c}
123 \\
456 \\
987 \\
\hline
141516
\end{array}
\begin{array}{c}
123 \\
567 \\
987 \\
\hline
161514
\end{array}
\begin{array}{c}
132 \\
546 \\
987 \\
\hline
151515
\end{array}
\begin{array}{c}
15 \\
15 \\
15
\end{array}
\begin{array}{c}
942 \quad 15 \\
537 \quad 15 \\
186 \quad 15 \\
\hline
151515
\end{array}
\]