ABSTRACT

AHLUWALIA, SINGH HARSHEE. Modeling and Optimization of a Rotary Dryer in a Manufacturing Plant. (Under the direction of James W. Leach)

The aim of this thesis is to predict the effects of reducing excess air levels and fuel burned in a rotary dryer being operated in a mining facility in North Carolina. The dryer is now being operated at very high excess combustion air levels, resulting in high stack losses and low thermal efficiency. Reducing the excess air entering the burner and the amount of fuel burned would result in energy and cost savings. However, experimental data to confirm that drying rates can be maintained at the required level by simultaneously increasing the combustion air temperature and reducing the air flow are unavailable. This work attempts to predict the dryer performance at variable air flows from a computer model. The modeling involves an analysis of the heat and mass transfer that occurs in the drying process. The model is validated by comparing predicted performance to actual performance at present operating condition. The excess air levels and the fuel burned are then adjusted to obtain the best efficiency. It is expected that the model prediction will encourage the mining facility to experiment with combustion air flow.
Modeling and Optimization of a Rotary Dryer in a Manufacturing Plant

By

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This thesis is dedicated to my family. Thank You Mom, Dad, Simi, Vicky, Tishya, Pujit, Beeji (both) and Darji for all your support and encouragement. I Love You and I could not have done this without you.
Biography

Harshdeep Singh Ahluwalia was born in Secunderabad, India on the 29th of July 1978. He went to St. Patrick’s High School in Secunderabad where he was a school Captain during his final year and also played for the school Basketball team. He attended Little Flower Junior College where he got his Intermediate Degree majoring in Math, Physics and Chemistry.

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Chapter 1

An Introduction to Drying

Industry and Drying

Most manufacturing operations, particularly in the chemical and metallurgical fields, entail one or more stages where drying in one form or another is carried out. Industrial dryers are ravenous consumers of energy, and according to a recent study [Robinson][1] they consume one quad of fossil fuel energy annually in the United States. This amounts to the entire residential usage of energy in the State of New York. A 2% reduction in U.S. dryer energy consumption would be an equivalent of savings of approximately 4 million barrels of oil per year.

The Industrial Assessment Center (IAC) at North Carolina State University has evaluated several large dryers over the past decade. A significant fraction of these dryers were inefficient and in need of repair or replacement. Very often the process or product at a plant will change, but the dryer remains the same. Sometimes a used dryer is purchased for economic reasons, even though it is not the right size or design. It is not uncommon to find dryers operating at 30% efficiency. By comparison, a typical industrial boiler would operate at 80% efficiency. The IAC experience indicates that there are significant opportunities for energy conservation in many industrial dryer operations.

The objective of the present work is to evaluate and improve the performance of a rotary dryer operating at a mining facility in North Carolina. We will evaluate
alternative methods for analyzing heat and mass transfer in the dryer and propose modifications to improve its energy efficiency. The dryer removes moisture from about two tons/hr of wet mica granules. The mica, which has a consistency similar to that of beach sand, must be dried before it is ground into a fine powder. The dryer was originally designed to dry a different product. It was purchased second hand by the mining facility, and the system was modified to dry mica by trial and error. The efficiency of the dryer was low and the plant manager wanted the IAC team to look into the matter. However, before we go about modeling and optimizing the performance of the rotary dryer, it is essential to understand the drying process.

**Drying**

The objective of drying is the removal of a liquid phase from a solid phase by means of thermal energy. It is a physical separation process where the liquid, generally water, is liberated by the process of vaporization. Drying could also mean the removal of water or other volatile liquid from another liquid or gas, or the removal of water from a suspension or solution of a solid. The above definition distinguishes thermal drying from mechanical dewatering as in the action of filters or centrifuges. It does not, however, distinguish drying from the process usually referred to as evaporation carried out in commercial evaporators. The best method of differentiating between drying and evaporation is on the basis of the type of equipment employed in the two processes, and that in evaporation much larger quantities of liquid component are removed per hour than occurs in drying operations. Moreover, the solid component is recovered from an evaporator either
in the form of a concentrated solution or suspension or as wet slurry, whereas that from a dryer is either quite dry or substantially so. Before getting into more details about drying, it is appropriate to define several terms peculiar to the drying operation. The basics of drying are well explained in [Williams-Gardner][2], [Schweitzer][3] and [Perry][4].

**Definition of Terms Employed**

**Moisture Content:**

Moisture content of a solid is the weight of moisture per unit weight of the dry or wet solid. Wet-weight basis expresses the moisture in a material as a percentage of the weight of the wet solid, whereas the dry-weight basis express the moisture in the material as a percentage of the weight of bone dry solid material. The relationship between the wet-weight and the dry-weight basis can be expressed as:

\[
W_w = \frac{m_w}{m_s + m_w} = \frac{m_w/m_s}{m_w/m_s + m_w/m_s} = \frac{W_d}{1 + W_d}
\]

- \(m_w\) = mass of moisture.
- \(m_s\) = mass of solid.
- \(W_w\) = pounds of moisture per pound of wet material.
- \(W_d\) = pounds of moisture per pound of bone-dry material.
Bound Moisture:
Water may become bound in a solid by retention in capillaries, solution in cellular structures, solution within the solid, or by chemical or physical adsorption on the surface of the solid. This water will exert a vapor pressure lower than that of pure water at the existing temperature and can only be removed from the solid under specific conditions of humidity in the surrounding atmosphere.

Unbound Moisture:
Unbound moisture in a hygroscopic material is defined as the moisture in excess of the equilibrium moisture content corresponding to the saturation humidity. All the moisture content of a non-hygroscopic material is unbound moisture.

Free Moisture Content:
It is the moisture content, which is removable at a given temperature and may include both bound and unbound moisture.

Equilibrium Moisture Content:
It is the moisture content to which the material can be dried under specified conditions of temperature and humidity of the surrounding atmosphere. It can also be defined as the amount of moisture in the solid that is in thermodynamic equilibrium with its vapor in the gas phase. For given temperature and humidity conditions, the material cannot be dried below its corresponding equilibrium moisture content.
**Constant Rate Drying Period:**
It is that period in a drying operation during which the rate of moisture per unit of drying surface is constant.

**Critical Moisture Content:**
It is the moisture content existing in the material at the termination of constant rate drying period. The critical moisture content is not a unique property of the material but is influenced by its physical shape as well as the conditions of the drying process.

**Falling Rate Drying Period:**
It is that period of the drying cycle during which the instantaneous drying rate continuously decreases and which commences at the end of the constant rate period. During this period the physical and transport properties of the material control the drying process.

**Internal Diffusion:**
It is the movement of the moisture liquid phase through the mass of the solid during the drying operation.

**Dryer Efficiency:**
It is that fraction of total heat supplied during a drying operation which is usefully used in evaporating moisture.
**Batch Drying:**
It is an intermittent drying operation where a batch of material is charged into a dryer and the drying completed before its removal from the dryer.

**Continuous Drying:**
It constitutes a continuous feed of moist material to a dryer and the continuous discharge from the dryer of the dried material.

**Counter Flow Drying:**
It occurs when the drying medium and the product move in opposite direction to one another within the dryer.

**Parallel Flow Drying:**
It occurs when the drying medium and product move in the same general direction through the dryer. It is also know as co-current drying.

**Direct Heat Dryers:**
The dryers where the drying medium is brought into direct and intimate contact with the material subjected to drying are classified as direct heat dryers.

**Indirect Heat Dryers:**
Dryers that operate without direct contact between the drying medium and moist material are classified as indirect heat dryers.
**Through-Circulation Dryers:**

The dryers where the heated drying medium is passed through a bed of moist material subjected to drying are known as through-circulation dryers.
Chapter 2

Theoretical Considerations

The Mechanism of Drying

The fundamental principles governing the performance and design of industrial drying equipment are those typical of the science of heat and mass transfer. The basic characteristics of the drying equipment may be described by an air-movement system consisting of fans or blowers and appropriate ducting, an energy source (combustion of fuel), and a system for dispersing and/or conveying the solid phase through the dryer. The purpose of the air-movement system is twofold:

1) It is the source that carries energy to the wet solid, and
2) It also carries vapors produced in the drying process away from the dryer.

The energy source (fuel) produces the heat necessary for the drying process. The dispersal or conveyance system (air) transports the solid phase through the drying medium or distributes the solid phase throughout the drying medium. In essence, the theory and design of the dryer are determined by the principles of heat and mass transfer and fluid mechanics that govern the performance of the subsystems described above.

The mechanism of the drying process consists of the transport of mass from
the interior of the solid to the surface, the vaporization of the liquid at or near the surface, and the transport of the vapor into the bulk gas phase. The vaporization of the liquid depends on external conditions of temperature, air humidity, airflow, area of exposed surface and supernatant pressure. Either of these mechanisms may be a limiting factor on the rate of drying although both proceed simultaneously throughout the drying cycle. Simultaneously, heat is transferred from the bulk gas phase to the solid phase where a portion of it provides for vaporization and the remainder accumulates in the solid as sensible heat. The overall rate by which the above sequence of steps takes place defines the drying rate and is inversely related to the drying time. Since the estimation of the drying time is generally mandatory in dryer design, it is essential to look into the analytical description of the drying mechanism.

**Internal Conditions**

As a result of heat transfer to the wet solid, a temperature gradient develops from the heated surface inwards, while evaporation occurs at the surface. This forces a migration of moisture from within the solid to the surface. The migration occurs through one or more mechanisms, namely diffusion, capillary flow, or internal pressures set up by shrinkage during drying. These phenomena may occur simultaneously or one or more may be predominant at different stages of drying, but the net result will be a moisture gradient through the thickness of the material.

Internal movement of moisture is not a big issue for substances that do not absorb moisture internally, but becomes important when it is a controlling factor, as
occurs after critical moisture content in a drying operation carried out to very low final moisture contents. Here variables such as air quantities, which enhance the rate of surface evaporation become of decreasing importance except that they promote higher heat transfer rates. Similarly in the case of materials like ceramics and timber, where considerable shrinkage occurs, excessive surface evaporation sets up high moisture gradients from the interior towards the surface, which is liable to cause over drying, excessive shrinkage and consequently high tension resulting in cracking or warping. In such cases it is essential not to incur too high moisture gradients by retarding surface evaporation through the employment of high air humidities while maintaining the highest safe rate of internal moisture movement by virtue of heat transmission. The temperature gradient set up in the solid also creates a pressure gradient, which in turn result in moisture vapor diffusion to the surface and this frequently occurs simultaneously with liquid moisture movement.

**External Conditions**

The essential external variables in the drying process are temperature, humidity, rate and direction of airflow, the physical form of the solid, the desirability of agitation and the method of supporting the wet solid during the drying operation. In the initial stages of drying materials of high moisture content attaining the highest possible rate of surface evaporation is critical. Surface evaporation is essentially the diffusion of vapor from the surface of the solid to the surrounding atmosphere through a relatively stationary film of air in contact with its surface. The air acts as a
resistance to vapor and heat flow. As the velocity of air or gas (drying medium) increases, the thickness of the film in contact with the wet solid decreases. The air film in contact with the wet solid remains saturated with vapor as long as the solid possesses free surface moisture. This results in a vapor pressure gradient existing through the film from the wetted solid surface to the outer air and with large air movements, the rate of moisture diffusion through the air film will be considerable.

The rate of evaporation of the moisture is directly proportional to the exposed surface area of the solid and the difference in the vapor pressure of the moisture at the temperature of the inner air film surface and the partial pressure of water vapor in the surrounding air. The rate of evaporation is inversely proportional to the film thickness. Thus, large exposed solid surface area, high air velocities relative to the solid surface and low relative humidity of the drying air or gas are favorable conditions for high surface evaporation. High air velocities also aid by increasing the heat transfer coefficient and hence improving conditions for evaporation.

It should be noted that, since the layer of air film in contact with the wetted solid undergoing drying remains saturated at the temperature of the area of contact, the temperature of the solid surface which still possess free moisture will lie very close to the wet bulb temperature of the air.

The Periods of Drying

The drying characteristics of wet solids are best described by plotting the average moisture content of the material (usually defined on the bone-dry basis) against elapsed time measured from the beginning of the drying process. Most wet
solids exhibit a drying time curve similar to that of Figure 1. The curve indicated that the drying rate varies considerably during the course of drying and is in no way constant. Most materials, which are surface wet, show three distinct stages in the drying rate curve, namely, section AB where the wet material is warming up, section BC where the rate of drying remains substantially constant and section CD where the rate of drying continuously. Point C at which the constant rate ends and the falling rate commences is termed the point of critical moisture content.

![Figure 1. Drying Curve](image)

**Constant Rate Period**

During this period, which exists for materials having free surface moisture, evaporation is taking place from the surface of the solid and as long as this remains surface-wet, drying is independent of internal mechanisms within the solid. The rate
of drying is essentially that of evaporation of the liquid component. The temperature of the liquid film and that of the adjacent solid surface will remain substantially constant and will be approximately that of the wet-bulb temperature of the moving air assuming radiation and conduction effects are negligible. However, these two effects are unlikely to be wholly absent and may result in a higher surface temperature than the wet-bulb temperature and may produce a higher constant drying rate. Summarizing the above, it is evident that the rate of drying during the constant rate period depends on three externally operating factors, namely, the heat or mass transfer coefficient, the area of solid exposed to the drying medium and the difference between the temperature and humidity of the drying medium and that of the wet solid. Increase in any of these factors results in increased drying rates.

**Heat and Mass Transfer**

The rate of heat transfer from the heating medium to the wet solid in a commercial convection dryer will depend upon the velocity of the air or gases used to apply heat. High air velocities increase the heat and mass transfer coefficients. The mass transfer coefficients are related to the heat transfer coefficient by the Lewis factor and this is discussed in a later chapter of this thesis.

**Falling Rate Period**

The constant rate period is followed by a period during which the rate of drying progressively decreases, the transmission from one period to another taking place at the point of critical moisture content of the solid. If the original feedstock
has such an initial moisture content and the product may be required with such high residual moisture content that all the drying will take place during the constant rate period. More usually, both the phenomena will be present and with very slow drying materials most of the drying operation may exist in the falling rate period. However, if the moisture pertaining to the wet solid is just surface moisture the drying rate will be that of constant rate period.

During the constant rate drying period, all the surfaces of the exposed solid are completely wetted and at the change to the falling rate period some of the solid surfaces will still be wet and some dry depending largely on the material being dried. The rate of evaporation of the less moist surfaces will be lower than that of the completely wetted portions, the net result being a fall in the rate of drying when compared with the rate during constant rate period. The rate of drying in this section of the drying curve will still be affected by factors which influence the constant rate drying period, as discussed above. Once the surface of the solid is dry, the rate of drying will be a function of the rate at which moisture or moisture vapor can move physically by diffusion and capillarity from within the solid to its surface.

Evidently, where the initial moisture content of a feedstock is relatively low and the final moisture content required is extremely low, the falling rate period becomes very important. Drying times become long and the variables affecting the drying rate in the constant rate period cease to be that important. Air velocities become important only to the extent to which they enhance heat transfer rates. The factors that affect drying to a greater extent are air temperature, humidity, material thickness and bed depth. When the rate of diffusion is the controlling factor,
particularly when long drying times are required to attain low final moisture contents, the rate of drying during the falling rate period varies as the square of the material thickness, which indicates the desirability of granulating the feedstock. Even so, extremely low final moisture contents for materials showing predominantly falling rate characteristics usually demand considerable drying times, and hence higher capital investment in the installed dryer. Therefore it is important, at the outset, in considering a drying problem to establish what final moisture content is permissible and acceptable. Where a product attains equilibrium with the relative humidity of the surrounding atmosphere in storage or in packages at a certain moisture content, the specifying of a final moisture content of the product discharged from the dryer below this figure is rarely justified in view of the extra cost of the dryer which would be required.

**Equilibrium Moisture Content**

As pointed out earlier, while considering a drying problem it is important to establish at the earliest stages, the final or residual moisture content of the product that can be accepted. This is important, as many materials are to some extent hygroscopic and if dried below certain moisture content will absorb or regain moisture from the surrounding atmosphere, depending upon its temperature and humidity. The material will establish a condition in equilibrium with the atmosphere and the moisture content of the material under this condition is termed as equilibrium moisture content. Equilibrium moisture content is not greatly affected at the lower end of atmospheric scale, but as this temperature increases the equilibrium moisture
content decreases, which explains why materials can in fact be dried in the presence of superheated moisture vapor. Drying medium temperatures and humidities assume considerable importance in the operation of direct dryers, e.g. during the drying of certain inorganic salts to a specified content of water of crystallization, as does the particular design of the dryer employed: continuous parallel flow pneumatic and rotary dryers facilitate control of the temperature and humidity of the atmosphere in contact with the product during the course of drying and hence the equilibrium conditions between the product and drying medium. In the more common drying operations, the equilibrium moisture content of a material is important as drying may be carried out unnecessarily far, resulting in the reduction of capacity of a given drying installation and an unjustifiable high cost of drying. The phenomenon and theoretical considerations of the drying process are described in detail in [Williams-Gardner][2], [Schweitzer][3] and [Perry][4].
Chapter 3

Classification of Industrial Dryers

A wide variety of dryer designs have been evolved over the years with a view to carry out the unit operation of drying. Industrial dryers may be classified according to:

(1) Mode of operation.
(2) Physical properties of the material to be dried.
(3) Method of conveyance or dispersal of the material through the dryer.
(4) Method of supplying energy to the material undergoing drying.

Mode of Operation

This category refers to the nature of the production schedule. For large-scale production the appropriate dryer is of the continuous type with continuous flow of the material into and out of the dryer. Conversely, for small production requirements, batch-type operation is generally desirable.

Physical Properties of the Material

The physical state of the feed is probably the most important factor in the selection of the dryer type. The wet feed may vary from a liquid solution, slurry, a paste, or filter cake to free-flowing powders, granulations, and fibrous and non-fibrous solids.
The design of the dryer is greatly influenced by the properties of the feed; thus dryers handling similar feeds have many design characteristics in common. A comprehensive classification of commercial dryers based on materials handled is given in [Perry’s]^{[4]}.

**Method of Conveyance**

In many cases, the physical state of the feed dictates the method of conveyance of the material through the dryer; however, when the feed is capable of being preformed, the handling characteristics of the feed may be modified so that the method of conveyance can be selected with greater flexibility. Generally, the mode of conveyance correlates with the physical properties of the feed.

**Method of Energy Supply**

Heat must be transferred to the wet material to promote the drying operation. Based on the method of heat transfer industrial dryers are classified as Convection dryers, Conduction dryers and Radiation dryers.

**Convection Dryers:** The dryers where the heating medium, usually air or the products of combustion, are in direct contact with the wet material are classified as convection dryers. Some examples of convection dryers are continuous through-circulation or band dryers, rotary dryers, fluidized bed dryers, spray dryers and flash dryers.
**Conduction Dryers:** If the heat is transmitted indirectly by contact of the wet material with a heating surface, then those dryers are classified as conduction dryers. Some examples of conduction dryers are vacuum tray dryers, vacuum double cone dryers, trough dryers, pan dryers and rotary dryers for vacuum operation.

**Radiation Dryers:** The dryers where heat is transmitted directly and solely from the heated body to the wet material are classified as radiation dryers. A few examples of radiation dryers are infra red dryers and microwave dryers.
Chapter 4

Rotary Dryers

Rotary dryers are widely used in industry when compared to other types of dryers. If the design and operation of these dryers is correct, then they can be very efficient. They require little or no labor to operate and if properly maintained, particularly as regards to lubrication, they can be operated over lengthy periods under automatic control with occasional supervision.

A rotary dryer operated at atmospheric pressure consists of a cylindrical shell, ranging in length from four to ten times its diameter, into which wet charge is fed continuously at one end and is discharged at the other. The movement of the material is usually due to the combined effect of inclination of the shell to the horizontal and the action of internal lifting flights during the rotation of the shell. The shell is usually provided with machined girth rings supported on suitably designed bearings. Rotation of the shell is secured by the action of suitable gearing or in some cases a chain drive.

Direct Rotary dryers

If it is suitable to dry the wet material by bring it in direct contact with combustion gases or hot air, direct rotary dryers are used. Depending on the direction of flow of hot air direct rotary dryers are further classified as parallel or counter flow. When flowing parallel with the charge, the drawing medium will, to
greater or lesser extent, assist the flow of materials through the dryer. In some designs flow may be both parallel and counter, and in both indirect and direct contact with the material. This type is generally referred to as an indirect-direct rotary dryer.

A typical direct rotary dryer (parallel flow) is shown in fig3. It consists essentially of a connection for the introduction of the heating and drying medium, a feed connection for wet material, the main cylinder or shell provided with machined girth rings riding on a suitable bearing, girth gear ring, pinion and reduction gear drive, a discharge hood from which dry product and humid gases are discharged and internal flights. Flights at the material feed end are usually spirally arranged to propel the feedstock fairly rapidly from the feed point into the dryer, followed by additional flights running longitudinally down the length of the dryer. Suitable seals, both dust and air tight are provided between the stationary sections and the rotating shell.

A counter current direct rotary dryer is constructed on similar lines except that feedstock and hot gas enter at opposite ends. The gas exit hood is provided with a gas exhausting duct connected to dust recovery equipment as this type of dryer is best operated under a mild suction to an exhausting fan. If the dryer is operated under balanced pressure throughout a fan at both inlet and exhaust ends is required. Typical arrangements of direct parallel flow, direct counter flow and double shell indirect-direct rotary dryers are shown in the figure. In operation, the shell rotates at 4 to 5 r/min and the retention time of the product varies from 5 min to 2 h. The gas velocity throughout the cylinder varies from 4.9 to 9.8 ft/s.
Feeding arrangements for the wet feedstock depends on the physical form of the wet material. Free flowing feedstock enters by means of an inclined chute fed from a rotary valve, a vibrating feeder or rotary table or a belt conveyor. Discharge of dry product is secured either through a counter balanced flap valve or rotary valve or a screw conveyor located at the base of discharge hood. The most effective of rotary dryer shell seals employs heat and wear resisting friction material, such as brake lining material riveted to a continuous or two section steel ring and spring loaded to maintain a close rubbing contact with a true faced ring welded to the stationary hood.

**Flights**

The effectiveness of a direct rotary dryer, apart from the quantity and temperature of the in-going heating medium depends upon the extent and uniformity...
of the feedstock contact with the hot gases and the residence time of the material in the dryer. The effectiveness is a function of the number, size and design of the lifting flights. During the rotation of the shell the lifting flights pick up wet material and shower it as a curtain in the path of hot gases, thus drying it. The depth and design of the flights determines the holdup in the dryer and hence the residence time. For free-flowing granular materials, a radial flight with a 90° lip is effective, whereas, for sticky materials, a straight radial flight is satisfactory. Flights may be arranged straight through or staggered. The number of flights is usually between two and four times the diameter measured in feet, while the depth of flight is between 1/12th to 1/8th of the diameter.

Hold up in the dryer depends on the design and number of flights used, the speed of rotation and the angle of inclination of the dryer and these are arranged to suit the drying characteristics of the material. Holdup usually lies between 3-12% of the volume of the dryer, 7-8% being a very usual figure. Holdup and residence times in parallel flow rotary dryers tend to be less than in counter current dryers and with some material this effect is such as to require a negative slope of the dryer in order to attain the required residence time.

Retention Time

The effect of each of the above factors on the residence time is discussed qualitatively by [Williams-Gardner][2]. Empirical expressions, based on experimental work, for the residence time are:

\[ t = \left( \frac{KL}{nDS} \right) + Yv \]
where $t =$ retention time, min

$L =$ effective length of dryer, ft

$N =$ rotations/min

$D =$ diameter of shell, ft

$S =$ slope of shell, in/ft

$v =$ air velocity, ft/min

$K$ and $Y =$ constants

The constants $K$ and $Y$ depend on certain design characteristics such as number of flights and flight design, size and density of particle, and method of operating the dryer.

For co-current dryers the residence time is given by[4]:

$$t_R = \frac{1.2L}{NDS}$$

The drying of material in a direct rotary dryer can be considered to take place in three stages:

(1) Heating the wet material and its moisture content to the constant rate drying temperature, which will be approximately that of the wet-bulb temperature of the drying medium.

(2) Drying of the material substantially at this temperature.

(3) Heating up of the material to its discharge temperature and evaporation of the moisture remaining at the end of stage (2).
Each stage entails its own heat transfer effect and if treated as an overall heat transfer operation, drying can be covered by the expression:

\[ Q_t = U_a V (\Delta T)_m \]

where \( Q_t \) = total heat transferred, Btu/hr.

\( U_a \) = volumetric heat transfer coefficient in Btu/hr-ft\(^3\).

\( V \) = dryer volume, ft\(^3\).

\( (\Delta T)_m \) = overall mean temperature difference between materials and gases, \( ^\circ F \).

Determination of temperature difference between material and gases at the end of the three stages of drying is difficult, so that an overall treatment of the heat transfer mechanism covered by the above expression is more reasonable. \( (\Delta T)_m \) cab be approximated to the logarithmic mean between the wet bulb depression of the inlet and outlet gases.

The general empirical form of the volumetric heat transfer coefficient is:

\[ U_a = KG^n / D \]

where \( K \) = a constant.

\( G \) = gas mass velocity in lb/hr-ft\(^2\) dryer cross-sectional area

\( D \) = shell diameter, ft

\( n \) = a constant

The value of \( n = 0.67 \) is probably the most representative of commercial equipment.
A more detailed discussion regarding the heat and mass transfer equations and other empirical equations pertaining to rotary dryers is presented in the coming chapters.
Chapter 5

Preliminary Analyses of Rotary Dryers

Preliminary Analyses

This chapter contains information from an IAC report to Oglebay Norton Corp., a mica ore extraction and processing plant in Kings Mountain, North Carolina. The report documents the existing operating conditions of three dryers operating at the facility, and proposes modifications to improve the energy efficiency. The recommendations are based on empirical heat and mass transfer correlations [Perry]^{4}, [Williams-Gardener]^{2}. The correlations from the two references give substantially different results. The IAC recommendations to Oglebay Norton are based on the more conservative results. The present work develops a computer model of one of the dryers, described in chapter 6.

Production data for the three dryers are listed in the table below. The production rates and moisture content listed in the table were provided by the plant engineer. The exhaust temperatures were measured by the IAC team. The final mica temperatures were taken from a dryer consultant’s report. Two airflow rates are listed in the table. The first value is taken from the consultant’s report. It was estimated by the consultant from measurements of the oxygen content of the flue gases. The IAC team, from an energy balance, established the second value.
### Table 1. Rotary Dryer Data

<table>
<thead>
<tr>
<th></th>
<th>Moss Plant</th>
<th>Patterson Plant</th>
<th>Battleground Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Rate, tons/hr</td>
<td>1.12</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>Moisture removed, tons/hr</td>
<td>0.8</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>% Moisture</td>
<td>43</td>
<td>35</td>
<td>26</td>
</tr>
<tr>
<td>Dryer exhaust temperature, °F</td>
<td>302</td>
<td>325</td>
<td>210</td>
</tr>
<tr>
<td>Mica leaving temperature, °F</td>
<td>270</td>
<td>260</td>
<td>168</td>
</tr>
<tr>
<td>% Excess air</td>
<td>980</td>
<td>690</td>
<td>1020</td>
</tr>
<tr>
<td>Flow rate of air, excess O₂, lb/hr</td>
<td>40,000</td>
<td>29,337</td>
<td>23,229</td>
</tr>
<tr>
<td>Air flow rate, energy balance, lb/hr</td>
<td>45,700</td>
<td>40,700</td>
<td>24,000</td>
</tr>
<tr>
<td>Firing rate, MMBTU/hr</td>
<td>5.0</td>
<td>5.0</td>
<td>2.8</td>
</tr>
</tbody>
</table>

![Figure 3. Moss Plant Dryer](image)
The Moss plant dryer is 5’ x 40’ long parallel flow rotary dryer. Calculations for the heat loads of the Moss dryer are presented below in detail. Heat loads for the other two dryers are similar and are summarized in the table below. The sensible heat absorbed by the mica is:

\[
Q_{\text{mica}} = m C_p \Delta T
\]

\[
= 1.12 \text{ tons/hr} \times 2000 \text{ lbs/ton} \times 0.21 \text{ BTU/lb-}^{\circ}\text{F} \times (270 - 60^{\circ}\text{F})
\]

\[
= 98,784 \text{ BTU/hr}
\]

The heat required to evaporate the water in the mica is:

\[
Q_{\text{water}} = m \Delta h
\]

\[
= 0.8 \text{ tons/hr} \times 2000 \text{ lb/ton} \times (1,156 \text{ BTU/lb water})
\]

\[
= 1,850,000 \text{ BTU/hr}
\]

The heat required to evaporate water formed by the reaction of hydrogen in natural gas is:

\[
Q_{\text{comb water}} = 90 \text{ lb/MMBTU} \times 5 \text{ MMBTU/hr} \times 1,000 \text{ BTU/lb}
\]

\[
= 450,000 \text{ BTU/hr}
\]
The heat loss from the insulated and un-insulated surfaces of the kiln shell was determined from measurements made at various locations on the shell. Note that about two-thirds of the Moss dryer was insulated. The heat losses are the sum of the heat losses from the insulated and un-insulated surfaces. The heat transfer coefficients, \( h \), were determined using tabulated values in [Marks’ Handbook for Mechanical Engineers][5].

\[
Q_{\text{shell}} = Q_{\text{insulated surface}} + Q_{\text{uninsulated surface}} \\
= h A (T_{\text{scf}} - T_{\text{room}}) + h A (T_{\text{scf}} - T_{\text{room}}) \\
= 92,225 \text{ BTU/hr}
\]

The airflow rate can now be calculated from the burner-firing rate as shown below.

\[
5,000,000 \text{ BTU/hr} = 1,850,000 + 92,225 + 98,784 + 450,000 + Q_{\text{air}}
\]

Solving the energy relation above, the heat added to the air is:

\[
Q_{\text{air}} = 2,508,991 \text{ BTU/hr}
\]

Since the leaving air temperature is 302\(^\circ\)F and the entering air temperature is 77\(^\circ\)F, the airflow rate is:

\[
m_{\text{air}} = \frac{2,508,991 \text{ BTU/hr}}{0.244 \text{ BTU/lb-}^\circ\text{F} / (302 - 77^\circ\text{F})}
\]
The efficiency of the Moss dryer is computed by comparing the heat used to remove moisture from the mica to the total heat required.

Efficiency = \( \frac{1,850,000}{5,000,000} \) BTU/hr

= 37.0%

Heat balances for the other two dryers were done in the same way and the results are given in the table below:

**Table 2. Heat Balance for the Dryers**

<table>
<thead>
<tr>
<th>Type of Loss</th>
<th>Moss plant Btu/hr</th>
<th>Patterson plant Btu/hr</th>
<th>Battleground Btu/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat absorbed by mica</td>
<td>98,784</td>
<td>100,000</td>
<td>92,000</td>
</tr>
<tr>
<td>Heat absorbed by moisture</td>
<td>1,850,000</td>
<td>1,850,000</td>
<td>1,550,000</td>
</tr>
<tr>
<td>Heat absorbed by air</td>
<td>2,508,991</td>
<td>2,400,000</td>
<td>798,000</td>
</tr>
<tr>
<td>Heat in combustion water</td>
<td>450,000</td>
<td>450,000</td>
<td>256,607</td>
</tr>
<tr>
<td>Drum loss</td>
<td>92,225</td>
<td>192,250</td>
<td>153,153</td>
</tr>
<tr>
<td>Total heat required</td>
<td>5,000,000</td>
<td>5,000,000</td>
<td>2,850,000</td>
</tr>
<tr>
<td>Efficiency</td>
<td>37%</td>
<td>37%</td>
<td>54%</td>
</tr>
</tbody>
</table>

The table shows that about half of the energy provided by the fuel leaves the dryers with the exhaust gases. Lowering the airflow rates can reduce the stack losses. Dryers of this size are capable of much higher heat transfer rates than are required in this application. It appears that the burners have been downsized, but the airflows have not been adjusted for efficient operation. The existing airflows are much higher than are needed to support combustion.
The consultant that analyzed the dryers measured excess air levels for the dryers at the Moss plant, Patterson plant and the Battleground plant to be 980%, 690%, and 1,020% respectively. He proposed to lower the excess air levels to the range of 30-50%, and predicted that fuel consumption would be lowered by about 50%. However, the consultant’s analysis does not show how heat transfer rates would be affected by lower air mass velocities. Also, the high combustion temperatures that would result from the proposed air / fuel ratio would probably require that refractory liners be installed in the dryers. Below we present the result of preliminary parametric analyses to show how the dryer performance depends on combustion airflow. A better and more detailed analysis of the ideal operating conditions of the rotary dryer will be discussed in the coming chapters. This preliminary analysis is based on a couple of empirical relations for rotary dryers.

**Anticipated Savings**

Table 3 presents the results of heat transfer analyses to show how the fuel consumption and flame temperatures in the Moss plant dryer depend on airflow rate. The first row in the table gives the existing operating conditions for each airflow. The fuel flow has been adjusted until the flame temperature is high enough to dry the moisture from the mica. The heat transfer from the exhaust gases to the wet mica is computed \([\text{Perry}]^{[4]}, \text{[Williams-Gardener]}^{[2]}\).

\[
Q = U \times V \times \Delta T_m
\]

where,
\[ Q = Q_{mica} + Q_{water} + Q_{shell} \]

\[ U = \text{volumetric heat transfer coefficient} \]

\[ V = \text{dryer volume} \]

\[ \Delta T_m = \text{mean temperature difference} \]

Since most of the heat transfer is required to evaporate water, the mean temperature difference is given approximately by:

\[ \Delta T_m = \frac{T_1 - T_2}{\ln \left( \frac{T_1 - T_{wb}}{T_2 - T_{wb}} \right)} \]

where,

\[ T_1 = \text{entering gas temperature} \]

\[ T_2 = \text{exit gas temperature} \]

\[ T_{wb} = \text{entering gas wet-bulb temperature} \]

The temperatures \( T_1 \) and \( T_{wb} \) can be determined from analyses of the combustion process, whereas \( T_2 \) can be determined from an energy balance for the exhaust gases.

Reference [4] discusses correlations from the literature for estimating the volumetric heat transfer coefficient. All of the published relationships can be reduced to the form:
\[ U \times V = k \times L \times D \times G^N \]

where \( k \) and \( N \) are constants, \( L \) is the length of the dryer, \( D \) is the diameter, and \( G \) is the gas mass velocity, given by:

\[ G = \frac{4\dot{m}_{\text{gas}}}{\pi D^2} \]

If the constant, \( N \), is specified, then \( k \) can be determined from the measured performance of the dryer at operating conditions. Reference [4] recommends \( N=0.67 \), whereas Reference [2] recommends \( N=0.16 \). Sample calculations showed that the value from Reference [4] is the most conservative. It predicts lower savings resulting from reductions in airflow through the dryer. Therefore the calculations in the tables below are based on \( N=0.67 \). From the existing production rates and temperatures for the Moss plant dryer, the corresponding values of the second constant, \( k \), is calculated to be \( k=0.217 \).

<table>
<thead>
<tr>
<th>( m_{\text{air}} ) lb/hr</th>
<th>( m_{\text{fuel}} ) lb/hr</th>
<th>( U \times V ) BTU/hr-°F</th>
<th>( T_1 ) °F</th>
<th>( T_2 ) °F</th>
<th>Efficiency %</th>
</tr>
</thead>
<tbody>
<tr>
<td>45,700</td>
<td>210.0</td>
<td>7,864</td>
<td>481</td>
<td>302</td>
<td>36.8</td>
</tr>
<tr>
<td>35,700</td>
<td>199.9</td>
<td>6,669</td>
<td>568</td>
<td>336</td>
<td>39.1</td>
</tr>
<tr>
<td>25,700</td>
<td>188.5</td>
<td>5,357</td>
<td>718</td>
<td>389</td>
<td>42.3</td>
</tr>
<tr>
<td>15,700</td>
<td>175.2</td>
<td>3,860</td>
<td>1,042</td>
<td>485</td>
<td>47.1</td>
</tr>
<tr>
<td>10,700</td>
<td>168.0</td>
<td>2,974</td>
<td>1,418</td>
<td>577</td>
<td>50.8</td>
</tr>
<tr>
<td>5,700</td>
<td>162.5</td>
<td>1,976</td>
<td>2,431</td>
<td>776</td>
<td>56.3</td>
</tr>
</tbody>
</table>
The results in Table 3 show that the fuel consumption drops from 210 lb/hr to 168 lb/hr when the combustion airflow is reduced from the existing value of 45,700 lb/hr, to 10,700 lb/hr. This represents savings of 20%, or about 1 MMBTU/hr. Annually, this represents 3,560 MMBTU/yr of energy savings at a projected cost savings of $18,723/yr.

The savings would be 48%, or $44,954/yr, if the volumetric heat transfer coefficient actually changes according to the correlation from Reference [2], N=0.16. Reference [2] predicts lower values of T2 than are given in the table above, and lower stack losses.

At an airflow rate of 10,700 lb/hr, the excess air is 270%. Lower air flows and greater fuel savings will be possible if the uninsulated dryer walls can tolerate the higher combustion temperature. The maximum safe wall temperature for carbon steel is about 750°F. The wall temperature should be significantly lower than the entering gas temperature in Table 3 because the solids in the dryer will be at a temperature near the wet bulb temperature, which is about 160°F.

Experimentation will be needed to determine the actual wall temperature. Also, safe operating temperatures will have to be established to insure that the burner does not operate for extended periods without wet solids in the dryer.

Overheating of the exhaust system at low airflow rates could also be a problem if the predicted exhaust gas temperatures in Table 3 are correct. However,
the heat transfer correlations from Reference [2] predict that $T_2$ will decrease below the existing value of 302°F, when the airflow is reduced. At $m_{\text{air}} = 10,700$ lb/hr, the correlation from Reference [2] predicts $T_2=228°F$. The actual values of $T_2$, and the minimum acceptable values of the airflow rate will have to be determined by experiment. However, we believe that the values in Table 3 are conservative in every respect.
Chapter 6

Computer Model of Rotary Dryers

As seen in the previous chapter, the rotary dryers at the mining facility are inefficient. Empirical heat and mass transfer correlations from the literature indicate that the combustion air flow can be adjusted to conserve fuel. The empirical equations used in the previous chapter, which are typical of rotary dryers, do not give consistent results. A better and more direct method is needed. The present model is based on a method of analysis discussed in recent papers by [Kemp and Oakley][6], [Kiranoudis][7] and [Papadakis][8]. In addition it is based on the following assumptions.

Assumptions

1. All the moisture is assumed to be on the surface of the mica particles. The internal resistance to mass transfer is assumed to be negligible because of the small particle diameter and the relatively impervious structure of mica.
2. It is assumed that the heat and mass transfer takes place only when the mica particles are falling from the flights.
3. The mica particles are assumed to be spherical with a diameter of 0.00042 m.
4. The surface area of the liquid film is assumed to be the same as the surface area of the dry solid particle.
5. Average values of specific heats are taken.
6. The entering atmospheric air is assumed to have a humidity of 50% at 25°C.

7. It is assumed that mica occupies 20% of the cross sectional area of the dryer.

8. The heat loss through the shell of the dryer is assumed to be constant and uniform over the length of the dryer.

9. The dryer is assumed to be revolving at 5 rpm and its slope is 1/24.

Model

Consider the elemental section of the dryer shown in figure 4.

The mass flow rate, absolute humidity, temperature and velocity of air and mica are given by $m^*_a$, $Y$, $T_a$, $V_a$ and $m^*_s$, $X$, $T_s$, $U_s$ respectively.

![Figure 4. Illustration of Incremental Model](image-url)
Mica absorbs heat from the hot drying air. The heat absorbed by mica results in increase in its temperature and removal of some moisture. The temperature of the solid in the control volume increases by $\Delta T_s$ and its moisture content decreases by $m_s^* \Delta X$ as it passes through the control volume. Note that $\Delta X$ is a negative number since the moisture content of mica decreases. The steady state heat balance equation is:

$$q = m_s^* [C_{ps} + X C_{pl}] \Delta T_s - m_s^* \Delta X h_{fg}$$

where $m_s^*$ = mass flow rate of mica (Kg/s).

$C_{ps}$ = Specific heat of mica.

$C_{pl}$ = Specific heat of water.

$h_{fg}$ = Latent heat of vaporization of water.
q = heat transfer between air and mica.

The mass flow rate of the solid is:

\[ m_s^* = \rho A U_s = N m_p A U_s/A \Delta z \]

\[ = N m_p U_s/\Delta z \]

where \( m_p \) = mass of particle.

\( U_s \) = velocity of mica in the dryer.

The heat absorbed by an individual mica particle \( (Q_p) \) that is falling through the air is given by:

\[ Q_p = h A_p (T_a-T_s)f \]

where \( h \) = heat transfer coefficient.

\( A_p \) = surface area of mica particle.

\( f \) = fraction of time that an individual mica particle is airborne.

The heat absorbed from air is the same as that absorbed by \( N \) particles. Therefore

\[ q = N Q_p. \]

This gives

\[ N Q_p = N m_p U_s/\Delta z \left[ \left( C_{ps} + X C_{pl} \right) \Delta T_s - \Delta X h_{fg} \right] \]

Or

\[ \Delta T_s = \frac{Q_p \Delta z + \Delta X \times h_{fg}}{m_p U_s / C_{ps} + X C_{pl}} \]

\[ \text{----------(1)} \]
The mass transfer equation for the moisture movement from the wet mica to the drying air is given by:

\[ m^* \Delta X = -N K_m A_p f \left( \frac{P_{sat}}{RT_s} - \frac{PY}{(Y + 0.62)R_1T_A} \right) \]

where \( K_m \) = mass transfer coefficient.

\( P_{sat} \) = Saturation pressure (K Pascals).

\( P \) = atmospheric pressure.

\( R \) = gas constant = 0.4614 kPa. m\(^3\)/kg/K

\( R_1 \) = gas constant = 0.287 kPa. m\(^3\)/Kg/K

Substituting for \( m^* \) we get:

\[ \Delta X = -K_m A_p \left( \frac{P_{sat}}{RT_s} - \frac{PY}{(Y + 0.62)R_1T_A} \right) \frac{f\Delta z}{m_p U_s} \] \hspace{1cm} \text{(2)}

where \( P_{sat} \) = Saturation pressure.

\( P \) = atmospheric pressure.

An approximate equation for the saturation pressure of water is:

\[ P_{sat} = \exp(16.7609 - \frac{4090}{T_s + 237}) \]

\( T_s \) = temperature of mica in degrees Celsius.
The control volume for air is shown in figure 6.

\[ \Delta Q_w \]

\[ m^* a (1 + Y) \]

\[ m^* a (1 + Y + \Delta Y) \]

\[ m^* s \Delta X \]

\[ q \]

**Figure 6. Control Volume for Air**

The drying air gives heat to the wet mica and absorbs moisture from it. The temperature of the drying air reduces by \( \Delta T_a \) and humidity increases by \( m^* a \Delta Y \). The drying air also loses heat to the dryer walls (\( \Delta Q_w \)). The mass balance for the drying air is given by:

\[ m^* s \Delta X = -m^* a \Delta Y \] \[ (3) \]

The energy balance for drying air is given by:

\[ -\Delta Q_w - q = m^* a (C_{pa} + YC_{pv}) \Delta T_a + m^* a \Delta Y C_{pv} (T_a - T_s) \]

Substituting for \( q \) we get
Equations (1) - (4) represent the mass and energy balance of the mica and air. These four equations can be solved at an incremental distance step along the dryer length for X, Y, T_s and T_a provided the initial conditions are known. The Crank-Nicolson\(^{[11]}\) method is employed to avoid numerical instabilities.

We start by making equation (1) implicit in the first term.

\[
\left(C_{ps} + XC_{pl}\right) \Delta T_s = \frac{f \Delta x}{m_p U_s} h A_p \left(\frac{T_a - T_s + T'_a - T'_s}{2}\right) + \Delta H_f g
\]  

Equation (4) is of the form:

\[
T'_a = T_a + A_1 + B_1 \Delta T_s
\]

and

\[
T'_s = T_s + \Delta T_s
\]

Subtracting the two equations above.

\[
T'_a - T'_s = (T_a - T_s) + A_1 + (B_1 - 1) \Delta T_s
\]

Substituting this in equation (5) we get:

\[
\left(C_{ps} + XC_{pl}\right) \Delta T_s = \frac{f \Delta x}{m_p U_s} h A_p \left(\frac{2(T_a - T_s) + A_1 + (B_1 - 1) \Delta T_s}{2}\right) + \Delta H_f g
\]

Or

\[
\left(C_{ps} + XC_{pl} - \frac{\Delta x}{2m_p U_s} h A_p (B_1 - 1)\right) \Delta T_s = \frac{f \Delta x}{m_p U_s} h A_p \left(\frac{2(T_a - T_s) + A_1}{2}\right) + \Delta H_f g \]  

-----(6)
Now we have to make equation (1) implicit with respect to the second term. $\Delta X$ is given by:

$$\Delta X = -K_m A_p \left( \frac{P_{sat}}{RT_s} - \frac{PY}{(Y + 0.62)R_i T_A} \right) \frac{f\Delta z}{m_p U_s}$$

To make it implicit we write it as:

$$\Delta X = -K_m A_p \left( \frac{P_{sat}(T'_s)}{2RT_s} + \frac{P_{sat}(T_s)}{2RT_s} - \frac{PY}{(Y + 0.62)R_i T_A} \right) \frac{f\Delta z}{m_p U_s}$$

For small step sizes, 

$$P_{sat}(T'_s) = P_{sat}(T_s) + \frac{dP_{sar}}{dT_s} \Delta T_s$$

Substituting the above two equations in equation (6) and solving for $\Delta T_s$ we get:

$$\Delta T_s = \left[ \frac{hA_p (T_a - T_s + A_1 / 2) - h_{fg} K_m A_p \left( \frac{P_{sat}}{RT_s} - \frac{PY}{R_i T_A (Y + 0.62)} \right)}{m_p U_s (C_{ps} + XC_{pl}) - \frac{hA_p (B_1 - 1)}{2} + \frac{h_{fg} K_m A_p}{2RT_s} \frac{dP_{sat}}{dT_s}} \right] \frac{f\Delta z}{f\Delta z} \left( C_{ps} + XC_{pl} \right) - \frac{hA_p (B_1 - 1)}{2} + \frac{h_{fg} K_m A_p}{2RT_s} \frac{dP_{sat}}{dT_s}$$

Equation (7) is used instead of equation (1) in modeling the rotary dryer.

To solve the heat and mass transfer equations, we first need to find the values of the parameters required in those equations. Some of the parameters like flow rates are already known and the others like adiabatic flame temperature have to
be evaluated using thermodynamic considerations or empirical equations. Below all the parameters required for the Moss plant dryer are evaluated and listed.

For the initial conditions, the temperature and absolute humidity of mica and the drying air are required. The values of temperature and humidity for the drying air will have to be calculated after it goes through the burner. For the existing operating conditions of the Moss plant dryer,

\[ m_s^* = \text{Flow rate of mica} \quad \text{db} = 1.12 \text{ tons/hr} = 1,018 \text{ kg/hr}. \]
\[ = 0.2828 \text{ kg/s} \]

\[ X = \text{absolute humidity of mica, db}. \]
\[ = 0.8 \text{ tons of moisture/1.12 tons of mica}. \]
\[ = 0.7143 \]

\[ m_{a}^* = \text{Existing flow rate of drying air, db}. \]
\[ = 45,700 \text{ lbs/hr} \]
\[ = 5.77 \text{ kg/s} \]

The absolute humidity of air is calculated as follows. The gas burned is assumed to be primarily methane (CH₄). The chemical reaction for complete combustion of one mole of methane is:

\[ \text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} \]

Assume that \( n_1 \) moles of air react with 1 mole of methane. The chemical equation that represents this reaction is:
$CH_4 + n_1 O_2 + n_1 \times 3.76 \, N_2 + n_2 \, H_2O \rightarrow CO_2 + (2 + n_2) \, H_2O + (n_1 - 2) \, O_2 + n_1 \times 3.76 \, N_2$ 

$n_2$ moles of water on the reactant side of the equation are from a typical value of 50% relative humidity at 77°F. From psychrometric charts, the humidity amounts to 0.01 lbs of moisture/lb of dry air. Using this value, the number of moles of moisture present in one mole of air is calculated as below:

$$0.01 = \frac{n_2 \times 18}{n_1(32 + 3.76 \times 28)}$$

This gives $n_2 = 0.07627n_1$.

The enthalpy of combustion of methane is $-55,496 \, kJ/kg \, \text{of fuel} = -52,602.8 \, \text{Btu/kg of fuel}$. The Moss plant dryer has a 5 MMBTU/hr burner and so the mass of fuel required to produce 5 MMBTU/hr is 95.05 kg/hr.

From the reaction equation it is known that 1 mole of CH$_4$ reacts with air to produce 19.6 moles of O$_2$, 1 mole of CO$_2$, 2.076 moles of H$_2$O and 81.2 moles of N$_2$.

For mass of fuel having a value of $m_f$ we can calculate the following.

Mass of O$_2$ produced due to combustion = 19.6 x 32 x $m_f$ /16

Mass of N$_2$ produced due to combustion = 81.2 x 28 x $m_f$ /16

Mass of CO$_2$ produced due to combustion = 1 x 44 x $m_f$ /16
The total mass of dry air is given by.

\[ m^*_a = m_f x n_1 x (32+3.76 x 28)/16 \]

Amount of moisture per kg of dry air after combustion is given by:

\[ Y = \frac{(2 + n_2) \times 18}{44 + 32(n - 2) + 3.76n_1 \times 28} \]

The next step is to find out the temperature of air coming out of the burner. The adiabatic flame temperature is calculated as shown below.

\[ \text{CH}_4 + n_1 \text{O}_2 + n_1 x3.76 \text{N}_2 + n_2 \text{H}_2\text{O} \rightarrow \text{CO}_2 + (2+n_2) \text{H}_2\text{O} + (n_1 - 2) \text{O}_2 + n_1 x3.76 \text{N}_2 \]

Heat of reactants \((H_R)\) = Heat of products \((H_P)\)

\[ H_P = \Sigma n_e [h_f^0 + \Delta h]_e \]

Where \(n_e\) = number of moles.

\(h_f^0\) = heat of formation.

\(\Delta h\) = change in enthalpy

For the reactants side \(h_f^0\) and \(\Delta h\) of \(\text{N}_2\), \(\text{O}_2\) and \(\text{H}_2\text{O}\) is zero since they are at 25\(^\circ\)C.

\[ H_R = (h_f^0)_{\text{CH}_4} = -74,873 \text{ kJ/kmol}. \]

\[ H_P = (h_f^0 + \Delta h)_{\text{CO}_2} + 2(h_f^0 + \Delta h)_{\text{H}_2\text{O}} + n_2 (\Delta h)_{\text{H}_2\text{O}} + (n_1 - 2)(\Delta h)_{\text{O}_2} + 3.76 n_1 (\Delta h)_{\text{N}_2} \]
\[ HP = (-393,522 + \Delta h_{CO2} + 2(-241,872 + \Delta h_{H2O}) + n_2 (\Delta h_{H2O}) + (n_1 - 2) (\Delta h_{O2}) + 3.76 n_1 (\Delta h_{N2}) \]

Assuming \( T_{a2}=600K \)
\[ \Delta h_{CO2} = 12,916 \text{ kJ/kmol.} \]
\[ \Delta h_{H2O} = 10,498 \text{ kJ/kmol.} \]
\[ \Delta h_{O2} = 9,247 \text{ kJ/kmol.} \]
\[ \Delta h_{N2} = 8,891 \text{ kJ/kmol.} \]
\[ HP = 77,126 \text{ kJ/kmol.} \]

Assuming \( T_{a1}=1000K \)
\[ \Delta h_{CO2} = 33,405 \text{ kJ/kmol.} \]
\[ \Delta h_{H2O} = 25,978 \text{ kJ/kmol.} \]
\[ \Delta h_{O2} = 22,707 \text{ kJ/kmol.} \]
\[ \Delta h_{N2} = 21,460 \text{ kJ/kmol.} \]
\[ HP = -244,335 \text{ kJ/kmol.} \]

But \( H_R = -74,873 \text{ kJ/kmol.} \) By linear interpolation we get:
\[ T_a = T_{a2} - (T_{a2} - T_{a1}) \frac{H_{p2} - H_R}{H_{p2} - H_{p1}} \]

The adiabatic flame temperature calculated above is in Kelvin.

\[ Q_{shell} = Q \text{ insulated surface} + Q \text{ uninsulated surface} \]
\[ \Delta Q_w = Q_{shell} \times \Delta z/L = 2,216.3 \Delta z \text{ W}. \]

The heat transfer coefficient is determined using the following equation [Ranz and Marshall][9].

\[
Nu = \frac{h_d p_{kg}}{2 + 0.6Re^{0.5}Pr^{0.33}}
\]

\( Nu \) = Nusselt number.
\( h \) = heat transfer coefficient.
\( k_g \) = coefficient of thermal conductivity for air.
\( Re \) = Reynolds number = \( V_a \rho a D/\mu a \)
\( Pr \) = Prandalt number.
\( V_a \) = Velocity of drying air

The mass transfer coefficient and the heat transfer coefficient are connected by the following equation [Incropera][10].

\[
\frac{h}{K_m} = \rho C_p \left( \frac{\alpha}{D} \right)^{2/3}
\]

where \( h \) = heat transfer coefficient.
\( K_m \) = Mass transfer coefficient.
\( \rho \) = density of air.
\( C_p \) = specific heat of air.
\( \alpha \) = Coefficient of thermal expansion.
D = coefficient of diffusivity.

\[
D = 435.7 \frac{T_a^{3/2}}{P(V_A^{1/3} + V_B^{1/3})^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}
\]

where D = coefficient of diffusivity (cm²/s).

\[V_A = \text{molecular volume of air} = 29.9\]

\[V_B = \text{molecular volume of water vapor} = 18.8\]

\[M_A = \text{molecular weight of air} = 29\]

\[M_B = \text{molecular weight of water vapor} = 18\]

\[P = \text{Total system pressure} = 101325 \text{ pascals.}\]

The residence time of mica in the dryer \(\tau\) can be calculated using the equation given by [Papadakis]\(^8\).

\[
\tau = \frac{L(N_{t_a} + \delta)}{N_{t_a} (U_{p1} + aD\delta\tan\alpha / t_a)}
\]

where \(t_a\) = average time of fall of airborne phase.

\[U_{p1} = \text{velocity of mica in airborne phase}.\]

Using the above equation the velocity of mica through the dryer can be calculated.

\[U_s = \frac{L}{\tau}\]

The effective velocity \(U_{\text{eff}}\) is given by:

\[U_{\text{eff}} = \frac{U_s}{f} \]
The velocity of the solid particles in the airborne phase is given by [Papadakis][8].

\[ U_{p1} = U_{p1a} + U_{p1d} \]

where

\[ U_{p1a} = \text{axial velocity of airborne phase.} \]

\[ U_{p1d} = \text{drag velocity of airborne phase.} \]

\[ U_{p1d} = 7.45 \times 10^{-4} \text{Re}^{2.21} \left( \frac{\mu V_{a} t_{a}}{\rho d_{p}^2} \right) \]

where \( t_{a} \) = average time of fall of airborne phase.

\[ U_{p1a} = 0.5gt_{a} \sin \theta \]

where \( g \) = acceleration due to gravity.

\[ \theta = \text{slope of dryer.} \]

The velocity of the solids in the dense phase is given by [Papadakis][8].

\[ U_{p2} = 2.4 N D \tan \alpha \]

The model contains an unknown parameter, \( f \), which is the fraction of the time that the individual mica particles are directly exposed to the hot gases. This fraction can be roughly estimated from the diameter of the dryer and the rotation speed. Since the dryer diameter is 5 feet, the particles fall an average distance of about 3 feet. The time required to fall 3 feet is about 0.43 seconds. The time required to lift
the particles at a rotational speed of 5 rpm is about 6 seconds. From the approximate numbers we obtain:

\[ f = \frac{0.43}{6.43} = 0.067 \]

The actual value of \( f \) is expected to be smaller because some of the mica particles will clump together near the entrance of the dryer, and because there are no lifting flights near the exit of the dryer. Therefore we expect that \( f \approx 0.06 \). This value is somewhat lower than the corresponding value calculated from the phase velocities, as explained below.

The average velocity of the solids, \( U_s \), is given by:

\[ U_s = f U_{p1} + (1-f) U_{p2} \]

Substituting values of \( U_s \), \( U_{p1} \) and \( U_{p2} \) from the equation above for the existing operating conditions, we obtain

\[ 0.10383 = 0.9683 f + 0.0127 (1-f) \]

or \( f = 0.095 \). The results from this last calculation are very sensitive to the value of \( U_{p2} \), which cannot be accurately predicted. The lower value, \( f = 0.06 \), appears to be more reasonable would give a more conservative estimate of the dryer performance. Therefore, The value of \( f \) under the existing conditions is assumed to be 0.06. In chapter 7 we systematically change the air flow to improve the dryer efficiency. When the air velocity is reduced, the value of \( U_{p1} \) decreases. As a result the residence time \( \tau \) increases and \( U_s \) decreases. Since the solids enter the dryer at a
constant rate and \( \tau \) is increased, a larger fraction of the dryer volume is occupied by solids. The fraction \( f \) also decreases, as the particles have to fall a small height.

Since both \( f \) and \( U_s \) decrease, the effective velocity is assumed to remain constant for cases with lower air velocities. However, this is not exactly true as \( f \) does not decrease in proportion with \( U_s \). It is expected that \( f \) will decrease but at a lesser rate than \( U_s \). The assumption that the effective velocity remains constant as the air velocity decreases is a conservative one as reducing its value will make the solid dry out faster.

The equations described above are incorporated in a computer model listed in appendix A.
CHAPTER 7

Results

The model described in chapter 6 was applied to predict the effects of lowering the flow of combustion air. The predicted results are discussed below. The existing operating conditions are simulated first to validate the model. The temperature of the entering combustion gases are calculated from the known fuel flow and the known burner combustion air flow. An adiabatic combustion temperature of 245°C is obtained for the high excess air level, which is about 1000%. The exhaust gas exit temperature predicted by the model is 152 °C, which compares with the measured temperature of 150 °C. This confirms that the overall energy balance is correct.

The drying rates predicted from the heat and mass transfer correlations are illustrated in the graphs below. The results will show that at the existing operating conditions the water is evaporated from the mica by the time that it travels about 7 meters along the axis of the dryer. The overall length of the dryer is 12 meters, which allows for some design uncertainty.

The model predicts that the mica will still dry in 11 meters when the fuel flow is reduced from 210 lb/hr to 157 lb/h. However, this leaves little margin for error.

The results will show that reducing the combustion air flow from the high existing level will result in significant fuel savings. Results from two sets of calculations are presented. The first set of calculations requires that the moisture be
evaporated from the mica at a length of 7 meters. The second set determines the limiting case fuel consumption, which would dry the mica in 11 meters.

The model is initially run for the current operating conditions of 45,700 lbs/hr of air and 210 lbs/hr of fuel. The resulting temperature and moisture profiles are shown below.

Figure 7. Mica Temperature in dryer (Operating condition)
Figure 8. Air Temperature in dryer (Operating condition)

Figure 9. Moisture content of air (Operating condition)
As can be seen from figure 10, mica is dry at a length of about 7 m from the entrance to the dryer. The mica moisture content curve is almost linear. This is because we have assumed that the moisture is on the surface of mica and is not absorbed internally by it. Also, because of the high air flow, the mole fraction of water in air does not get high enough to slow down the drying. This explains the moisture content profile of mica and the absence of a falling rate period as stated in the earlier chapters.

The humidity ratio of air increases until all the moisture from the mica is evaporated (figure 9). The outlet temperature of mica according to the model is predicted to be 152 °C, whereas the actual mica outlet temperature from the dryer is
The exit air temperature predicted by the model is 152°C and that from the dryer is 150°C. The model predicts the actual operating conditions of the dryer and is reasonably accurate.

From figure 7 we observe that the mica temperature increases as soon as it comes in contact with the hot air. It increases from 25°C to 52 °C, which is the wet bulb temperature of air at the entrance. Mica temperature almost remains constant until all the moisture is removed. Once all the moisture is removed, the mica temperature suddenly increases to that of the air. This is because all the heat from the hot air is going towards increasing mica temperature and not evaporating moisture. This explains the sharp rise in mica temperature. In reality, as soon as the free surface moisture is evaporated, the drying process slows down since lesser free moisture is available. In turn the heat from the air makes the temperature of mica rise. This in turn causes moisture to diffuse to the surface of mica and then evaporate consequently.

As seen from the above plots the mica dries at a length of 7 m. This implies that the quantity of fuel burned can be reduced. In the program the fuel burned was reduced until the mica dried at a length of 11m. This occurred for a fuel flow of 157 kg/hr. The temperature and moisture profiles for mica and air are shown below.
Figure 11. Mica Temperature in dryer

Figure 12. Air Temperature in dryer
Figure 13. Moisture content of air

Figure 14. Moisture content of mica
When the amount of fuel burned is reduced to 157 lb/hr from 210 lb/hr and the flow rate of air is kept the same (45,700 lb/hr), it takes longer for the mica to dry, as expected. In this case it dries at 11m from the entrance as compared to 7 m in the previous case. The exit temperature of air and mica are predicted to be 106°C. The characteristics of the temperature and moisture content profiles are similar to the previous case. This confirms that the amount of fuel burned can be reduced and mica can still be dried. Reducing the fuel burned also increases the energy efficiency. The Energy and cost savings are shown below:

\[
\text{Gas Energy Savings} = (210-157 \text{ lb/hr}) \times 23,800 \text{ BTU/lb} \times 3,561 \text{ hrs/yr.}
\]
\[
= 4,492 \text{ MMBTU/yr}
\]
\[
\text{Energy Cost Savings} = 3,390 \text{ MMBTU/yr} \times 5.26/\text{MMBTU}
\]
\[
= 23,627/\text{yr}
\]

In the previous case we had only reduced the amount of fuel burned. We could also reduce the amount of air being drawn by the burners from an excess air percentage of about 1000. For the next case we reduce the amount of air being drawn from 45,700 lbs/hr to 15,700 lbs/hr and the amount of fuel burned is adjusted to 140 lbs/hr. The temperature and moisture content profiles are shown below.
Figure 15. Mica Temperature in dryer

Figure 16. Air Temperature in dryer
Figure 17. Moisture content of air

Figure 18. Moisture content of mica
For this case mica dries at a distance of 7 m from the entrance of the dryer. The temperature and moisture profiles are similar to the previous cases.

The fuel flow can be further reduced to make the mica dry at 11 m instead of 7 m. However, the former is more conservative and leaves a greater margin for error. Mica drying at 11 m is a limiting case and for an air flow of 15,700 the fuel flow can be reduced to 117 lb/hr. The resulting temperature and moisture profiles are shown below.

Figure 19. Mica Temperature in dryer
Figure 20. Air Temperature in dryer

Figure 21. Moisture content of air
The profiles predict the mica to dry at a distance of 11 m from the dryer entrance. The moisture content profile of mica (figure 22) is not linear and tends to slow down as expected in drying. It has a more prominent falling rate period when compared to the cases discussed earlier in this chapter. The reason for the falling rate drying period is that the mole fraction of water in air increases as the amount of air being drawn is reduced and this reduces the gradient for mass transfer. The final humidity ratio of air is less than 0.13 and it is much lower that the saturation value which is about 0.2. The exit temperature of moisture and air is 106°C for an air flow rate of 15,700 lbs/hr. This low exit temperature is a clear indication of how much the stack loss is reduced. Under the operating conditions of the dryer the exit temperature of
The results obtained by the computer model of the rotary dryer are compared with those obtained from empirical correlations [Perry’s][4] and [Williams-Gardener][2]. Table 4 and 5 list the results obtained from the computer model, [Perry’s][4] and [Williams-Gardener][2] respectively for mica drying at 11m and 7m from the entrance of the dryer.

<table>
<thead>
<tr>
<th>Air flow rate (lb/hr)</th>
<th>Fuel (lb/hr)</th>
<th>Fuel (lb/hr)</th>
<th>Fuel (lb/hr)</th>
<th>Final temperature of air (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45,700</td>
<td>157</td>
<td>157</td>
<td>157</td>
<td>106</td>
</tr>
<tr>
<td>35,700</td>
<td>145</td>
<td>139</td>
<td>150</td>
<td>108</td>
</tr>
<tr>
<td>25,700</td>
<td>132</td>
<td>122</td>
<td>142</td>
<td>109</td>
</tr>
<tr>
<td>15,700</td>
<td>117</td>
<td>105</td>
<td>132</td>
<td>106</td>
</tr>
<tr>
<td>10,700</td>
<td>109</td>
<td>98</td>
<td>126</td>
<td>103</td>
</tr>
</tbody>
</table>
Table 5. Comparison of Results (7m case)

<table>
<thead>
<tr>
<th>Air flow rate (lb/hr)</th>
<th>Fuel (lb/hr)</th>
<th>Fuel (lb/hr)</th>
<th>Fuel (lb/hr)</th>
<th>Final temperature of air (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45,700</td>
<td>210</td>
<td>210</td>
<td>210</td>
<td>152 150 150</td>
</tr>
<tr>
<td>35,700</td>
<td>187</td>
<td>182</td>
<td>200</td>
<td>153 146 169</td>
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<tr>
<td>25,700</td>
<td>167</td>
<td>153</td>
<td>189</td>
<td>159 139 198</td>
</tr>
<tr>
<td>15,700</td>
<td>140</td>
<td>123</td>
<td>175</td>
<td>155 123 251</td>
</tr>
<tr>
<td>10,700</td>
<td>125</td>
<td>108</td>
<td>168</td>
<td>148 108 303</td>
</tr>
</tbody>
</table>

As can be seen from the result comparison, the model predicts fuel flow values between those predicted by [Williams-Gardener][2] and [Perry’s][4]. A plot of the predicted amount of fuel burned from the three sources for the limiting case is shown in figure 23. It should be noted that the results obtained from [Perry’s][4] are the most conservative. However, the model is also conservative as a constant (higher) value of effective velocity is used.
Figure 23. Comparison of Fuel burned (limiting case)

Figure 24. Comparison of Fuel burned (7 m case)
The energy and cost savings as a result of reducing the amount of fuel being burned and the amount of air being drawn in by the rotary dryer are shown in table 5.

### Table 6. Energy and Cost Savings (Model, limiting case)

<table>
<thead>
<tr>
<th>Air Flow rate lb/hr</th>
<th>Fuel flow rate lb/hr</th>
<th>Fuel used by dryer lb/hr</th>
<th>Energy Savings MMBTU/yr</th>
<th>Cost Savings $/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>45,700</td>
<td>157</td>
<td>210</td>
<td>4,492</td>
<td>23,627</td>
</tr>
<tr>
<td>35,700</td>
<td>145</td>
<td>210</td>
<td>5,509</td>
<td>28,977</td>
</tr>
<tr>
<td>25,700</td>
<td>132</td>
<td>210</td>
<td>6,611</td>
<td>34,772</td>
</tr>
<tr>
<td>15,700</td>
<td>117</td>
<td>210</td>
<td>7,882</td>
<td>41,459</td>
</tr>
<tr>
<td>10,700</td>
<td>109</td>
<td>210</td>
<td>8,560</td>
<td>45,025</td>
</tr>
</tbody>
</table>

### Table 7. Energy and Cost Savings (Model, 7m case)

<table>
<thead>
<tr>
<th>Air Flow rate lb/hr</th>
<th>Fuel flow rate lb/hr</th>
<th>Fuel used by dryer lb/hr</th>
<th>Energy Savings MMBTU/yr</th>
<th>Cost Savings $/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>45,700</td>
<td>210</td>
<td>210</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>35,700</td>
<td>187</td>
<td>210</td>
<td>1,949</td>
<td>10,253</td>
</tr>
<tr>
<td>25,700</td>
<td>167</td>
<td>210</td>
<td>3,644</td>
<td>19,169</td>
</tr>
<tr>
<td>15,700</td>
<td>140</td>
<td>210</td>
<td>5,933</td>
<td>31,206</td>
</tr>
<tr>
<td>10,700</td>
<td>125</td>
<td>210</td>
<td>7,204</td>
<td>37,893</td>
</tr>
</tbody>
</table>
Conclusions

A model that simulates drying in a rotary dryer is developed based on heat and mass transfer equations. The model is able to predict the exit temperatures of air and mica fairly accurately. On reducing the fuel and air used by the dryer it is predicted that mica can still be dried. This results in Energy and Cost Savings. By reducing the fuel and air used by the dryer, the efficiency of the dryer also increases.

The results obtained from the model are in close agreement with those obtained using the correlation from [Williams-Gardener][2] and [Perry’s][4]. The prediction of the model is conservative, at the same time is better than that of [Perry’s][4]. The model is also able to predict the temperature and moisture content profiles of air and mica along the length of the dryer, which cannot be done using [Perry’s][4] and [Williams-Gardener][2]. The temperature and moisture content profiles obtained are characteristic to that of the drying process.
References


Appendix A

Appendix A gives the Matlab code for the computer model proposed for Rotary Dryers in the thesis.

Matlab Code

% Assigning values%
ms=1018.18/3600;
mf=210/(3600*2.2);
ma=45700/(3600*2.2);

% Calculating Humidity of combustion products and other parameters %
ta=274.56*mf/16;
xsa=ma/ta-1;
n1=16*ma/(137.28*mf);
n2=0.07627*n1;
Y(1)=0.01+(2*18*mf)/(16*ma);

% Adiabatic Flame temperature %
Hr=-74873;

hco2(1)=12916;
hh2o(1)=10498;
ho2(1)=9247;
hn2(1)=8891;

hco2(2)=33405;
hh2o(2)=25978;
ho2(2)=22707;
hn2(2)=21460;

Hp(1)=(-393522+hco2(1))+ 2*(-241872+hh2o(1)) + n2*hh2o(1) + (n1-2)*ho2(1) + 3.76*n1*hn2(1);
Hp(2)=(-393522+hco2(2))+ 2*(-241872+hh2o(2)) + n2*hh2o(2) + (n1-2)*ho2(2) + 3.76*n1*hn2(2);

T1=600;
T2=1000;
Tad=T2-(T2-T1)*(Hp(2)-Hr)/(Hp(2)-Hp(1));
% Other Parameters 

\[ X(1)=0.7143; \]
\[ fd=0.2; \]
\[ rho=2900; \]
\[ dp=0.00042; \]
\[ Ap=3.14159*dp^2; \]
\[ mp=4/3*rho*3.14159*(dp/2)^3; \]
\[ f=1; \]
\[ Ta(1)=Tad-273; \]
\[ Ts(1)=25; \]
\[ Xse=0; \]
\[ hfg=2323*10^3; \]
\[ dia=5*0.3048; \]
\[ L=40*0.3048; \]
\[ Cps=879; \]
\[ Cpl=4196.4; \]
\[ Cpa=1030; \]
\[ Cpv=1984; \]
\[ R=0.4614; \]
\[ mua=270*10^{-7}; \]
\[ Pr=0.685; \]
\[ k=40.7*10^{-3}; \]
\[ alpha=56.7*10^{-6}; \]
\[ pct=1; \]
\[ Tf=0.4; \]
\[ T=0.005*Tf; \]
\[ length=[0:T:12.192]; \]
\[ s=size(length); \]
\[ Re=4*(ma*pct)*dp/(3.14159*(dia)^2*mua*(1-fd)); \]
\[ Va=ma/(0.628*3.14159*dia^2/4*(1-fd)); \]
\[ Qw=2216*T; \]
% Calculating Velocities
\[ ta = 0.414; \]
\[ N = \frac{1}{12}; \]
\[ del = 0.3166; \]
\[ Up1d = 7.45 \times 10^{-4} Re^{2.2} \left( \mu_a V_a \frac{ta}{\rho \cdot dp^2} \right); \]
\[ Up1a = 0.5 \times 9.8 \cdot ta / 24; \]
\[ Up1 = Up1d + Up1a; \]
\[ USP = N \cdot ta \cdot (Up1 + 2.4 \cdot dia \cdot del / (ta \cdot 24)) / (N \cdot ta + del); \]
\[ frac = 0.06; \]
\[ US = USP / frac; \]
% US is the same as Ueff %

% Loop %

\[
\text{for } i = 2 : s(2) \]
\[ \rho_a(i-1) = 0.628 \times 552.8 / (Ta(i-1) + 273); \]
\[ P_{sat}(i-1) = \exp(16.7609 - 4090 / (Ts(i-1) + 237.05)); \]
\[ \text{diffP}(i-1) = \exp(16.7609 - 4090 / (Ts(i-1) + 237.05)) \times 4090 / (Ts(i-1) + 237.05)^2; \]
\[ h = (2 + 0.6 \cdot Re^{0.5} \cdot Pr^{0.33}) \cdot k / dp; \]
\[ Dm(i-1) = 3.936 \times (10^9) \times ((273 + Ta(i-1))^{1.5}); \]
\[ \text{ratio}(i-1) = alpha / Dm(i-1); \]
\[ K_m(i-1) = h / (\rho_a(i-1) \cdot C_{pa} \cdot \text{ratio}(i-1))^{2/3}; \]
\[ Q_p(i-1) = h \cdot Ap \cdot (Ta(i-1) - Ts(i-1)) / \text{frac}; \]

\[
\text{X}(i) = \text{X}(i-1) - f \cdot K_m(i-1) \cdot Ap \cdot ((P_{sat}(i-1) / (R \cdot (Ts(i-1) + 273))) - 353.1 \cdot Y(i-1) / ((Y(i-1) + 0.62) \cdot (Ta(i-1) + 273))) \cdot T / (mp \cdot US); \]
\[ \text{if } X(i) \leq 0 \]
\[ \text{X}(i) = 0; \]
\[ f = 0; \]
\[ \text{else} \]
\[ f = 1; \]
\[ \text{end} \]

\[
\text{Y}(i) = \text{Y}(i-1) + ms / ma \cdot (K_m(i-1) \cdot Ap \cdot ((P_{sat}(i-1) / (R \cdot (Ts(i-1) + 273))) - 353.1 \cdot Y(i-1) / ((Y(i-1) + 0.62) \cdot (Ta(i-1) + 273))) \cdot T / (mp \cdot US)) \cdot f; \]

\[
B_1 = ms \cdot (C_{ps} \cdot X(i-1) \cdot C_{pl}) / (ma \cdot (C_{pa} \cdot Y(i-1) \cdot C_{pv})); \]
\[ A_1 = -(ma \cdot (Y(i-1) - Y(i-1)) \cdot (hfg + C_{pv} \cdot (Ta(i-1) - Ts(i-1))) + Q_w) / (ma \cdot (C_{pa} + Y(i-1) \cdot C_{pv})); \]

75
Ts(i)=Ts(i-1)+(h*Ap*((Ta(i-1)-Ts(i-1))+A1/2)-f*hfg*Km(i-1)*Ap*(((Psat(i-1))/(R*(Ts(i-1)+273))-353.1*Y(i-1)/((Y(i-1)+0.62)*(Ta(i-1)+273)))))/(mp*US*(Cps+X(i-1)*Cpl)-h*Ap*(B1-1)/2+f*hfg*Km(i-1)*Ap*diffP(i-1)/(2*R*Ts(i-1)));

Ta(i)=Ta(i-1)-(ms*(Ts(i)-Ts(i-1))*(Cps+X(i-1)*Cpl)+ma*(Y(i)-Y(i-1))*(hfg+Cpv*(Ta(i-1)-Ts(i-1)))+Qw)/(ma*(Cpa+Y(i-1)*Cpv));

end;

figure(1);
plot(length,Ts,'k-');
title('Mica temperature');
xlabel('Distance from Entrance (m)');
ylabel('Tempertaure (Celsius)');

figure(2);
plot(length,Ta,'k-');
title('Air temperature');
xlabel('Distance from Entrance (m)');
ylabel('Tempertaure (Celsius)');

figure(3);
plot(length,X,'k-');
title('Mica moisture content');
xlabel('Distance from Entrance (m)');
ylabel('Moisture content (kg/kg)');

figure(4);
plot(length,Y,'k-');
title('Air moisture content');
xlabel('Distance from Entrance (m)');
ylabel('Moisture content (kg/kg)');