

ABSTRACT

GONZALEZ, MARGGIE DENISE. Understanding the Technological, Pedagogical, and Mathematical Issues that Emerge as Secondary Mathematics Teachers Design Lessons that Integrate Technology. (Under the direction of Dr. Hollylynne Lee and Dr. Allison McCulloch).

This multiple case study examines four groups of secondary mathematics teachers engaged in a Lesson Study approach to professional development where they planned and taught lessons that integrate technology. Informed by current literature, a framework was developed to focus on the dimensions of teacher's knowledge to teach mathematics with technology that appear to influence teacher's decisions during lesson planning. These dimensions of teacher's knowledge include knowledge of mathematics, knowledge of pedagogy, knowledge of technology, and other types of knowledge that are developed by the interactions of those.

At the beginning and end of the Lesson Study, quantitative data was collected using a pre-post survey to measure changes in teacher's self-perceptions of the knowledge to teach mathematics with technology. Qualitative data was collected as the groups engaged in designing a mathematical lesson, as a volunteer teacher from each group implemented that lesson, and as a group reflected in the planning and implementation of the lesson. This includes audio recordings, video recordings, and field notes.

Results of this study demonstrate that when teachers are engaged in professional development activities that are focused on having teachers plan thoughtful and detailed lessons that integrate technology, their self-perceptions of their mathematical knowledge (MK), technological knowledge (TK), and technological, pedagogical, and mathematical knowledge (TPACK) increases significantly. Finding suggests that professional development

opportunities where teachers collaborate with colleagues could motivate the integration of technology tools into teaching, and could stimulate changes to pedagogical approaches.

Findings also suggest that when teachers exhibit appropriate mathematical knowledge, and have in-depth discussions about the pedagogical approach to take, the integration of technology seemed to flow naturally. Findings also suggest that when teachers seemed to lack mathematical understanding of the topic being taught, and are not willing or motivated to change their pedagogical approaches, they do not use technology in ways that can potentially impact students' engagement with mathematical ideas.

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Understanding the Technological, Pedagogical, and Mathematical Issues that Emerge
as Secondary Mathematics Teachers Design Lessons that Integrate Technology

by
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DEDICATION

To my husband Pedro for his unconditional support

&

To our two sons Fabián & Sebastián

BIOGRAPHY

Great teachers are inspiring and can change the life of any of their students. That was the case for Marggie D. Gonzalez, born in Arecibo, Puerto Rico. When Marggie was in high school she had the privilege of having extraordinary mathematics teachers who encouraged her to pursue a degree in mathematics. Following that encouragement and motivation, Marggie pursued a bachelor degree in Mathematics Education at the University of Puerto Rico in Mayaguez. But that degree was the beginning of something bigger.

Marggie graduated from Luis Muñoz Rivera High School with honors and began a career in mathematics education at The University of Puerto Rico in Mayaguez (UPRM), which she completed in 2002. After having a great experience in her student teaching, Marggie decided she wanted to continue her professional career in the educational field. She decided to pursue graduate education and completed a Master in Science in Mathematics with emphasis in Statistics in 2005. After completing her MS she got married and started teaching at the University of Puerto Rico in Utuado. She taught in this institution for almost two years having the opportunity to teach from elementary algebra courses to Calculus I.

In 2007, Marggie moved to Washington D.C., where she worked at the US Census Bureau as a Survey Statistician for the Economic Census of Puerto Rico and US Territories. A year later, she moved to Raleigh, NC to fulfill another chapter in her live, to continue graduate school and pursue her Ph.D. In 2010 she completed a Master in Science in Mathematics Education from North Carolina State University and continued into the Ph.D. program. In July 2013, motivated by a job offer to her husband, Marggie moved back to

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Chapter 1

Introduction

“Technology is essential in teaching and learning of mathematics; it influences the mathematics that is taught and enhances student learning” (National Council of Teachers of Mathematics, NCTM, 2000, p. 24)

Technology is a vital part of our society nowadays. Its uses have greatly influenced how we do even the most basic tasks of our daily routines. According to results from a 2013 survey published by the U.S. Census Bureau, 83.3% of households own a computer (desktop, laptop, or handheld device). In addition, 74.4% of U.S. households reported having some Internet subscription with 73.2% having high-speed Internet connection (File & Ryan, 2014). In Puerto Rico, according to the Federal Communications Commission, approximately 86% of households had access to a broadband platform by 2011, without taking into consideration mobile platforms (Puerto Rico Broadband Strategic Plan, 2012).

The constant use of technology has also influenced our teachers and the ways they teach. However, using technology tools in effective and meaningful ways in the classroom is not an easy endeavor. Many national organizations have agreed on the importance of incorporating technology into the K-12 mathematics classrooms. The Association of Mathematics Teacher Educators [AMTE] stated, “If technology is used to improve the learning of mathematics at all levels, K-12 students will be better prepared to use technology appropriately, fluently, and efficiently to do mathematics on the techno-rich environments in which they will study and work in the future” (AMTE, 2015). Yet, the type of knowledge teachers need to teach mathematics with technology in ways that can enhance students’

learning of a mathematical idea is still a topic of interest for researchers in the area. In this regard, our teachers are in special need of learning how to effectively use those tools in their classrooms in ways that are significant for student learning. But, what types of knowledge do teachers need in order to teach mathematics with technology effectively, and how do they draw upon that knowledge as they learn to design and implement technology-enabled mathematics lessons?

This study was embedded within a larger grant that offers professional development to teachers in the western region of Puerto Rico. The goal of the study was to identify and describe the issues that were emerging as secondary mathematics teachers engaged in a Lesson Study approach to professional development, which will be described in more detail later. Through the professional development activities, they were asked to plan lessons with the caveat that they had to integrated the use of technology into their teaching.

Background

Research shows that teaching with technology is not an easy endeavor, even more when it is intended to impact student learning in meaningful ways (Zbiek & Hollebrands, 2008). Technology alone will not make a difference in student learning, “it is the confluence of technological environment, teachers, curriculum, and mathematical activity that sets the stage for changes in the teaching and learning of mathematics in the context of technology” (Heid & Blume, 2008, p. 420). Thus, if we want mathematics teachers to orchestrate classroom discussions that are rich in the mathematics and that influence student learning, teachers first need to plan thoughtful and detailed lessons (Smith, Bill, & Hughes, 2008;

Smith & Stein, 2011). According to Panasuk and Todd (2005), lesson planning is “a systematic development of instructional requirements, arrangements, conditions, and materials and activities, as well as testing and evaluation of teaching and learning” (p. 215).

Although it sounds simple to achieve, in the process of helping teachers gain awareness of students’ learning, and designing a lesson that takes that into account, many issues will arise that need special attention. As teachers think about a mathematical task to use during instruction, the pedagogical approach to be used, and the technology tool that best help achieve the mathematical goals, many types of knowledge will emerge. Those types of knowledge are comprised into the knowledge teachers need to teach mathematics with technology.

Teachers’ knowledge for teaching mathematics is very complex. According to literature (Ball, Lubienski, & Mewborn, 2001), it entails not only mathematical knowledge and skills that are common in settings others than teaching, but mathematical knowledge and skills that are unique to teaching (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012). Teachers’ knowledge for teaching also demands teachers know of students’ understanding and potential misunderstandings of mathematics, curricular materials and instructional strategies (Grossman, 1989), among many others.

In the late 1900s, the uses of educational technology come to the forefront in education. Adding this new layer makes teachers’ knowledge for teaching even more complex. Researchers in the last two decades have developed theory and gained insights into the concept of teaching mathematics with technology. In 2006, Mishra and Koehler

introduced the Technological, Pedagogical, and Content Knowledge (TPACK) framework as a way to characterize the type of knowledge teachers needed to teach with technology. These types of knowledge include technological knowledge (TK), pedagogical knowledge (PK), content knowledge (CK), and the intersections between them: technological pedagogical knowledge (TPK), technological content knowledge (TCK), pedagogical content knowledge (PCK), and technological pedagogical content knowledge (TPACK). Some researchers agree that TPACK is a unique type of knowledge that needs its own development (Angeli & Valadines, 2009), while others agree that it is a combination of technology, pedagogy, and content knowledge and that it will appear if the three types of knowledge are present (Mishra & Koehler, 2006).

Professional development is a venue to offer mathematics teachers the experiences they need in order to learn how to integrate technology into their teaching, thus help in the development of their TPACK. A professional development experience that is long-term, practice-based, and collaborative, such as Lesson Study, seems to have the potential to increase teachers' knowledge to teach mathematics with technology (Darling-Hammond, ChungWei, Andree, Richardson, & Stelios-Orphanos, 2009).

Statement of the Study and Research Questions

The purpose of this study was to identify the issues that emerged as teachers were planning thoughtful and detailed lessons that integrated technology. Using a Lesson Study approach to professional development, 18 secondary mathematics teachers engaged in the design of two lesson plans that integrated technology into the teaching of a mathematical

concept. The integration of the Japanese Lesson Study approach to professional development has demonstrated to increase teacher's awareness of student learning and its importance when planning a lesson, and to develop lesson plans that are based on an understanding of students, the mathematics, and pedagogy (Lewis, Perry, Murata, 2006; Olson, White, & Sparrow, 2011; Perry & Lewis, 2008; Stigler & Hiebert, 1999).

To assist mathematics teachers in the development of their knowledge to teach mathematics with technology, it is necessary to acknowledge the issues they encounter as they plan lesson where they use technology. Insight into the breadth and depth of teacher's different types of knowledge (e.g., mathematics, pedagogy, technology) can inform changes to professional development programs.

This study aimed to describe how a group of secondary mathematics teachers engaged in a professional development focused on designing and implementing mathematics lessons that integrate technology. The study focused on how teacher's technological, pedagogical, and mathematical knowledge was used in their lesson planning and implementation. This study also seek to describe how teachers' attention to those aspects of knowledge changed as they became more experienced in designing mathematics lessons that integrate technology tools in the teaching of a mathematical idea. Specifically, the research questions the study addressed were:

1. How do a group of secondary mathematics teachers' self-perceptions of their technological knowledge (TK), mathematical knowledge (MK), and technological, mathematical and pedagogical knowledge (TPACK) change as a

result of engaging in a Lesson Study focused on designing mathematics lessons that integrate technology?

2. What issues emerged (mathematical, pedagogical, and technological) in the process of designing, implementing, and reflecting on lessons? And how do teacher's attention to any of the above issues change as they become more experienced in designing lessons that integrate technology?

These research questions are answered from different perspectives that are worth mentioning. The first research question will be answered from the teachers' self-perceptions of the different types of knowledge. On the other hand, the second research question will be answered from the researcher's perspective as I identify the issues that are emerging as teachers plan and implement the lessons, and then as they reflect on that planning and implementation.

Although research in the areas of teacher knowledge to teach mathematics and to teach with technology has been explored, limited research has looked into how teacher's technological, pedagogical, and mathematical knowledge is used as teachers design mathematics lessons that integrate technology. In addition, although Lesson Study has been widely used in settings with preservice and inservice teachers at the elementary and secondary level (e.g., Meyer & Wilkerson, 2011; Perry & Lewis, 2008), research on the development of teacher's TPACK through the use of Lesson Study for secondary mathematics teachers is very limited (e.g., Groth, Spickler, Bergner, & Bardzell, 2009; Pierce & Stacey, 2009). This study seek to contribute to the base knowledge in mathematics

education, specifically: (1) in understanding if the use of a Lesson Study approach is an appropriate venue to help teachers develop or improve their TPACK; and (2) in understanding how the technological, pedagogical and mathematical knowledge emerged as teachers make decisions on the design of mathematics lessons that integrate technology.

Overview of Methodological Approach

The purpose of this convergent parallel, multi-case study (Creswell, 2007; Stake, 1995) was to gain information regarding the types of knowledge teacher's draw upon, and what issues emerge as they are engaged in designing lessons that integrate technology. In a convergent parallel design, the quantitative and qualitative strands are kept independent during collection and analysis and are brought together during overall interpretations.

In order to measure changes in teacher's self-perception of their technological, pedagogical, and content knowledge, a survey was adapted from (Zelkowski, Gleason, Cox, & Bismarck, 2013) and validated for use in Spanish. This quantitative data was collected at the beginning and end of a Lesson Study approach to professional development. In addition, to get in-depth descriptions of the issues that were emerging as teachers engaged in the design and implementation of the lesson, qualitative data was collected in the form of audio, and video recording, respectively. Field notes were collected as teachers reflected on the planning and implementation of the lessons.

Outline of the Document

This dissertation is divided into 6 chapters. Chapter 1, the introduction, explains the purpose and significance of the study. Chapter 2 reviews the most relevant literature in the

areas of teachers' knowledge to teach mathematics, teachers' knowledge to teach mathematics with technology, the role of lesson planning in mathematics instruction, and literature on effective professional development. Chapter 3 describes the methodology used during the study including the conceptual framework, participant selection, data collection methods and analysis. Chapter 4 presents results obtained through the analysis of the quantitative data. Chapter 5 discusses in detail the results of the multiple case studies. Lastly, Chapter 6 summarizes the key findings regarding teacher's changes in their self-perceptions of their technological knowledge (TK), mathematical knowledge (MK), and technological, mathematical and pedagogical knowledge (TPACK), as well as the issues that emerged as they designed their lessons. This last chapter also discusses implications and limitations of the study, and areas of future research.

Chapter 2

Literature Review

This study aims to describe how a group of secondary mathematics teachers engaged in a professional development focused on designing and implementing mathematics lessons that integrate technology. Thus, in order to set the stage about what issues could emerge as teachers design and implement lessons, this chapter reviews literature on what constitutes the notion of teacher knowledge. Teacher knowledge embraces different types of knowledge from knowing about the content they teach, pedagogy, curriculum, and student learning, among many others. Since integration of technology was a crucial piece on the context of the study, literature on the particular complex knowledge teachers need to teach mathematics with technology is also reviewed. The Technological Pedagogical Content Knowledge (TPACK) framework, and its influences in mathematics education are discussed. Then, literature on professional development for teachers is examined, finishing with a discussion on the Lesson Study approach to professional development and its influences on teachers' mathematical knowledge for teaching.

Teacher Knowledge

Many would agree that understanding the mathematical content is of vital importance to teaching mathematics. But, what constitutes a good understanding of mathematical content or any other content area? Is there any additional piece of knowledge a teacher needs in order to be successful in teaching mathematics? These, and many other questions regarding teachers' knowledge, have been a focus of research in teacher education for more than two

decades (e.g., Ball, Lubienski, & Mewborn, 2001; Ball, Thames, & Phelps, 2008; Shulman, 1986).

Teacher's pedagogical content knowledge. In 1986, Lee Shulman published a seminal work on teachers' knowledge. He introduced the notion of pedagogical content knowledge as a special teachers' knowledge that "goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for teaching*" (Shulman, 1986, p. 9, [italics in original]). This type of knowledge, according to Shulman, includes "for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9). This notion of pedagogical content knowledge (PCK) exists at the intersection of content and pedagogy, representing how content and pedagogy blends together into an understanding of how particular aspects of subject matter are organized, adapted, and represented for instruction.

According to Shulman (1986), pedagogical content knowledge was not the only important piece of teachers' knowledge. He categorized the base of teachers' knowledge as consisting of the following: (1) content knowledge; (2) general pedagogies knowledge; (3) curriculum knowledge; (4) pedagogical content knowledge; (5) knowledge of learners and their characteristics; (6) knowledge of educational contexts; and (7) "knowledge of educational ends, purposes, and values, and their philosophical and historical grounds" (p. 8). Building from this characterization, other researchers have further investigated teachers'

knowledge (Ball & Cohen, 1999; Ball, Lubienski, & Mewborn, 2001; Ball, Thames, & Phelps, 2008; Grossman, 1989; Mishra & Koehler, 2006; Niess, Ronau, Shafer, Driskell, Harper, Johnson, & Browning, 2009). Grossman (1989), wanting to investigate the role of subject-specific components of teacher education programs in the development of PCK, found that this role consists of four central components. Thus, results from his study with six beginning English teachers, led him to propose PCK as a form of knowledge that consists of “overarching conceptions of what it means to teach a particular subject, knowledge of curricular materials and curriculum in a particular field, knowledge of students’ understanding and potential misunderstandings of a subject area, and knowledge of instructional strategies and representations for teaching particular topics” (p. 25).

Teacher’s knowledge for teaching mathematics. Ball and colleagues argued that our “understanding of the mathematical knowledge it takes to teach well” (Ball et al, 2001, p. 433) is insufficient. This insufficiency, they claimed, is being transmitted as “inadequate opportunities for teachers to develop the requisite mathematical knowledge and the ability to use it in practice” (p. 433). Ball and colleagues (Ball et al., 2001) have identified some factors that have influenced our way of teaching mathematics through the years. Some of those factors are the idea of knowledge as something fixed and not dynamic were students just store and remember the knowledge given by their teachers as facts, and the attention administrators and school boards has given to test scores, which have lead teachers to focus on skills and procedures not allowing students to appreciate, or develop an appreciation for, “the elegance and power of mathematics as a system of human thought” (p. 435), among

others. However, for students to develop an appreciation for the power of mathematics, teachers' knowledge for teaching needs to be well developed.

Although the mathematical knowledge of a teacher is an important factor for teaching mathematics, what that mathematical knowledge entails has been an area of research for many years (Ball et al., 2001; Ball, Sleep, Boerst, & Bass, 2009; Ball, Thames, & Phelps, 2008; Hill, 2010). Building on the notion of pedagogical content knowledge (Shulman, 1986), Ball, Thames, and Phelps (2008) proposed that mathematical knowledge for teaching was divided in four domains. First, teachers need *common content knowledge* (CCK), which means they must know how to solve mathematics problems correctly. Second, teachers need a more *specialized content knowledge* (SCK), which they use in a daily basis with their students. In practice, teachers need to unpack their mathematical knowledge in ways not needed in any other settings than teaching. Some of those practices are shown in Figure 1.

-
- Presenting mathematical ideas
 - Responding to students' "why" questions
 - Finding an example to make a specific mathematical point
 - Recognizing what is involved in using a particular representation
 - Linking representations to underlying ideas and to other representations
 - Connecting a topic being taught to topics from prior or future years
 - Explaining mathematical goals and purposes to parents
 - Appraising and adapting the mathematical content of textbooks
 - Modifying tasks to be either easier or harder
 - Evaluating the plausibility of students' claims (often quickly)
 - Giving or evaluating mathematical explanations
 - Choosing and developing useable definitions
 - Using mathematical notation and language and critiquing its use
 - Asking productive mathematical questions
 - Selecting representations for particular purposes
 - Inspecting equivalencies
-

Figure 1. Specialized Content Knowledge (from Ball et al. 2008)

This specialized content knowledge, according to McCrory et al. (2012) encompasses practices such as (1) recognizing differences among student's solutions, unpacking underlying concepts, and attaching meaning to symbols and algorithms; (2) make mathematics accessible to students, paying attention not to embed ideas that are correct for the current context but may lead to problems in more advanced mathematics; and (3) connect mathematical concept so that mathematics is presented as a coherent and connected endeavor. These, according to the authors, are three categories of teachers' work, trimming, bridging, and decompressing respectively that separate teachers from other users of mathematics (McCrory, et al., 2012).

Third, teachers need *knowledge of content and students* (KCS). This type of knowledge includes knowing students' conceptions and misconceptions about mathematical concepts, hearing student voices and to interpret their thinking, and to be able to anticipate students' responses when presenting with a particular task. Fourth, teachers need *knowledge of content and learning* (KCL) which represents an interaction between mathematical understanding and pedagogical issues that affects student learning. For example, the teacher must decide which representation to use when presenting students certain mathematical ideas or to decide in which order examples should be presented in order to enhance student learning of the topic. The domains for mathematical knowledge as seen and illustrated by Ball and colleagues are illustrated in Figure 2.

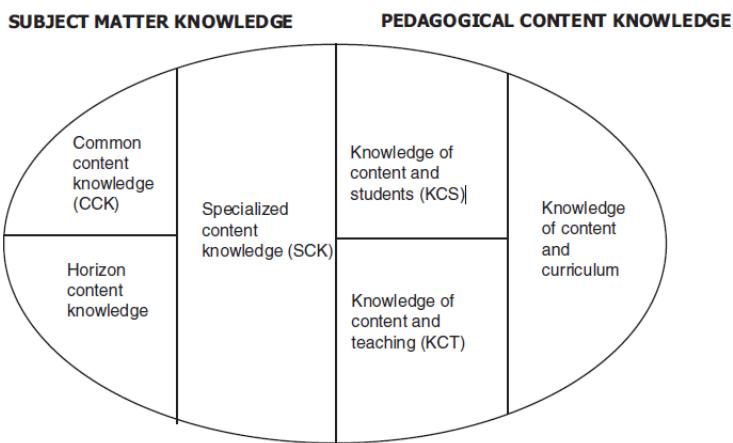


Figure 2. Domains of Mathematical Knowledge (from Ball et al., 2008)

Schoenfield and Kilpatrick (2008) provided a provisional Framework for Proficiency in Teaching Mathematics, which consists of seven dimensions the authors considered necessary for a teacher to be proficiency in teaching mathematics (Figure 3). They agreed with Ball and colleagues (Ball et al., 2001) that for a teacher to be proficient in teaching mathematics they need to know the mathematics they will teach in depth and breadth. According to the authors, the knowledge of school mathematics is broad because teachers need “multiple ways of conceptualizing the current grade-level content [and] represent it in a variety of ways, understand the key aspects of each topics, and see connection to other topics at the same time.” It is deep since teachers know the origins and organization of the mathematical content in the curriculum and “understand how the mathematical ideas flow conceptually” (Schoenfield & Kilpatrick, 2008, p. 2). In addition, teachers need to know their students as thinkers and learners, which goes parallel with KCS, a domain Ball and

colleagues discussed (Ball et al., 2001). This entails unpacking and understanding student's thinking when solving a problem as well as teachers being aware of how their own theories of learning plays out in classroom activities and their interactions with students.

-
- Knowing school mathematics in depth and breadth
 - Knowing students as thinkers
 - Knowing students as learners
 - Crafting and managing learning environments
 - Developing classroom norms and supporting classroom discourse as part of “teaching for understanding”
 - Building relationships that support learning
 - Reflecting on one’s practice
-

Figure 3. Framework for Proficiency in Teaching Mathematics
(from Schoenfield and Kilpatrick, 2008)

Proficient teachers, according to Schoenfield and Kilpatrick (2008), also need to know how to orchestrate classroom discussions in a way that encouraging learning environments for their students are created. Last, but not least, proficient teachers must reflect on their own teaching practices. Reflecting on teaching practices goes hand-by-hand with the practice-based models of professional development for teachers, which will be reviewed in more detail later in this chapter.

In summary, the knowledge teachers need to teach mathematics is very complex. During the last two decades, researchers tend to agree that teachers' knowledge for teaching mathematics must include knowledge about general pedagogy, common mathematical content, student learning, curriculum materials, pedagogy specific to teaching mathematics, mathematical content specific for teachers, among others (Ball et al., 2001; Ball, Sleep,

Boerst, and Bass, 2009; Ball, Thames, and Phelps, 2008; Hill, 2010; Schoenfield and Kilpatrick, 2008).

Research on teachers' knowledge to teach mathematics. For more than two decades, research has been examining the knowledge teachers need to teach mathematics. Although it is a very complex type of knowledge, nowadays we have information about the different types of knowledge it entails. On the other hand, for the last 15 years, researchers have tried to link teachers' mathematical knowledge with the quality of mathematics instruction (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Fennema, Franke, Carpenter, & Carey, 1993; Hill et al., 2008; Leinhardt & Smith, 1985; Lloyd & Wilson, 1998; Swafford, Jones, & Thornton, 1997; Thompson & Thompson, 1994). Results from these studies offer evidence that stronger teacher knowledge yields benefits for classroom instruction and student achievement. Research has found evidence that when teachers have weak mathematical knowledge they make significant mathematical errors and imprecisions (Hill et al., 2008; Heaton, 1992), use incomplete or inappropriate definitions (Hill et al., 2008; Stein et al., 1990), accept inaccurate guesses from students (Cohen, 1990), and use inappropriate language including conventional notation (e.g., uses of the equal sign), technical language (e.g., equation vs. expression) and general language (Bill et al., 2008), among others.

Identifying the knowledge teachers need to teach mathematics has taken many years of research. But what knowledge is needed to teach mathematics with technology? Research on teachers' knowledge has recently added a new piece to the puzzle, making teacher's knowledge an even more complex and dynamic.

Teacher's knowledge to teach with technology

In the 1990s, technology came to the forefront of educational discourse. Through the years, with the rapid evolution of new digital technology tools, teachers were challenged to learn new techniques and skills as current technologies became obsolete as a result of the appearance of new, more powerful technologies (Mishra & Koehler, 2006). Accordingly, research began to focus on the interplay between technology and Shulman's (1986) PCK. Common conclusions started to emerge. Research found that proficient use of technology at a personal level does not necessarily lead to successful integration of technology into teaching and learning of a subject matter (Keating & Evans, 2001). Others also found that integrating technology into teaching and learning in efficient and effective ways was a complex process (Margerum-Lays & Marx, 2002; Niess, 2005). Thus, studying approaches of how to prepare mathematics teachers to integrate technology successfully is a research area of interest in the mathematics education community.

The knowledge needed for teachers to teach mathematics with technology has been the focus of many researchers for more than a decade now. Pierson (2001, as cited by Voogt, Fisser, Roblin, Tondeurt, & van Braak, 2012) articulated the ideas of a more content-specific orientation to technology integration, followed by other researchers that investigated the role of educational technology in teaching practices (e.g., Keating & Evan, 2001; Margerum-Lays & Marx, 2002; Niess, 2005). The term TPCK gained popularity in 2006 when Mishra and Koehler outlined a model that described the *Technological Pedagogical Content Knowledge*

(TPCK) framework consisting of seven domains. In 2007, the acronym was changed to TPACK (Thompson & Mishra, 2007).

The TPACK framework builds on Shulman's (1986) conceptions of pedagogical content knowledge by integrating the component of technological knowledge into the model (Figure 4). Thus, the framework includes three core types of knowledge (content-, pedagogical-, and technological knowledge) and four additional types of knowledge that arise from their intersections: pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK), and technological pedagogical and content knowledge (TPACK).

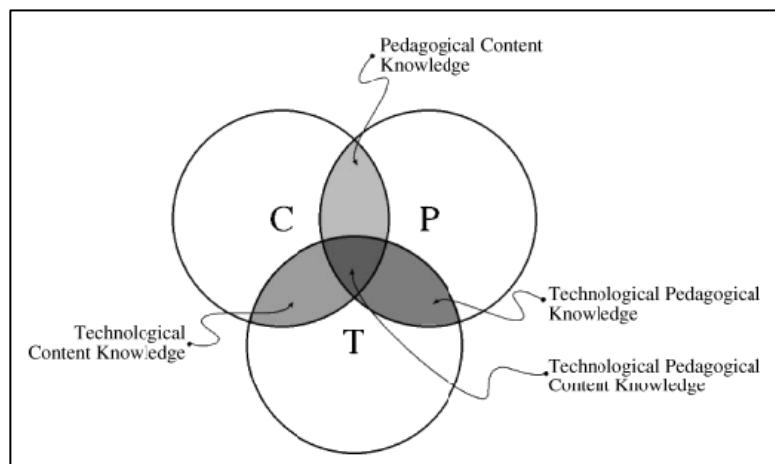


Figure 4. TPACK framework (from Mishra & Koehler, 2006, p. 1025)

Mishra and Koehler (2006) provided the following theoretical definitions for each of these domains:

- Content knowledge (CK) is the “knowledge about actual subject matter that is to be learned or taught” (Mishra & Koehler, 2006, p. 1026). Teachers must know about the

content they are going to teach and how the nature of knowledge is different for various content areas.

- Pedagogical knowledge (PK) refers to the methods and processes of teaching and includes knowledge in classroom management, methods of assessment, lesson plan development, teaching approaches, student learning, and theories about learning.
- Technological knowledge (TK) refers to the knowledge about various technologies, ranging from low-tech technologies such as pencil and paper to digital technologies such as the Internet, digital video, interactive whiteboards, to technologies more relevant to mathematics such as computer algebra systems, dynamic mathematical environments, java applets, graphing calculators, and CBLs.
- Pedagogical content knowledge (PCK) refers to the content knowledge that deals with the teaching process. It is different for various content areas, as it blends both content and pedagogy with the goal being to develop better teaching practices in the content areas.
- Technological content knowledge (TCK) refers to the knowledge of how technology can create new representations for specific content. It suggests that teachers understand that, by using a specific technology, they can change the way learners practice and understand concepts in a specific content area.
- Technological pedagogical knowledge (TPK) refers to the knowledge of how various technologies can be used in teaching, and to understanding that using technology may change the way teachers teach.

- Technological pedagogical content knowledge (TPACK) refers to the knowledge required by teachers for integrating technology into their teaching in any content area.

Teacher's knowledge to teach mathematics with technology. Learning mathematics with technology is not the same as learning how to teach mathematics with technology. Thus, to develop mathematics teachers' TPACK, teacher preparation programs and professional development efforts must consider how the three components of TPACK interact with one another. There are two types of technology that can be useful for mathematics teaching: conveyance technologies and mathematical action technologies (Dick & Hollebrands, 2008). Conveyance technologies are “used to convey, that is, to transmit and/or receive information” (p. xi). These tools have not been designed specifically to teach mathematics. Such tools include smart boards, projectors, presentation software, assessment systems, and clickers, among many others. Mathematical action technologies “can perform mathematical tasks and/or respond to the user’s actions in mathematically defined ways” (p. xii). Among the technologies that are included in this group are tools such as dynamic geometry environments, computer algebra systems, and simulators.

Over the last two decades, standards have been developed to offer guidelines for thinking about the knowledge mathematics teachers need in order to teach mathematics with technology. In the early 2000s, The International Society for Technology and Education (ISTE) released national standards for students and teachers with the goal of supporting the integration of technologies in schools. Then, the TPACK framework emerged and numerous researchers focused on studying the integration of technology, content, and pedagogy (e.g.,

Guerrero, 2010; Lee & Hollebrands, 2008; Groth et al., 2009; Niess, 2005; Ozgun-Koka, Edwards & Meagher, 2010; Ozgun-Koka, Meagher, & Edwards, 2011; Pamuk, 2012).

Yet, years before the TPACK framework emerged, the NCTM (2000) supported the integration of technology into the mathematics classroom in meaningful ways with its Technology Principle stating that “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (p. 24). Thus, technology was not expected to be used as a verification tool, or for drill and practice, but as a tool that influences the mathematics, for instance opening the possibility to teach mathematical ideas that were previously tedious or impossible. NCTM called for teacher education programs to prepare teachers to meet those new standards, thus to develop mathematics teachers’ TPACK (NCTM, 2000).

In support of the NCTM Technology Principle, the Association of Mathematics Teacher Educators (AMTE) has also released Technology Position Statements, the most recent in 2015. The statement states, “mathematics teacher preparation programs must ensure that all mathematics teachers and teacher candidates have opportunities to acquire the knowledge and experiences needed to incorporate technology in the context of teaching and learning effectively” (AMTE, 2015, p. 1).

More or less at the same time, the term TPACK emerged as a framework, Niess (2005), amending Grossman’s (1989) model and building on PCK (Shulman, 1986), proposed a technology-enhanced PCK framework. Herein, technology-enhanced PCK, was presented as, “the integration of the development of knowledge of subject matter with the

development of technology and of knowledge of teaching and learning" (Niess, 2005, p. 510) and proposed to include:

- (1) an overarching conception of what it means to teach a particular subject integrating technology in the learning; (2) knowledge of instructional strategies and representations for teaching particular topics with technology; (3) knowledge of students' understandings, thinking, and learning with technology in a particular subject; (4) knowledge of curriculum and curriculum materials that integrate technology with learning in the subject area (Niess, 2005, p. 511).

Niess, Ronau, Shafer, Driskell, Harper, Johnson, and Browning (2009) identified a five-stage developmental model to describe how teachers' technology integration might develop their TPACK. The five stages are recognizing, accepting, adapting, exploring, and advancing (see Figure 5). In the initial stage of the model teachers begin *recognizing* the technology and its alignment with mathematical content, but have not decided to integrate it in teaching and learning of mathematics. Then, teachers start forming an opinion (*accepting*) about the effectiveness of technology when teaching mathematics. In the next stage (*adapting*), teachers decide to use technology and start engaging in developing activities to be used with their students. Once the activity is implemented in the classroom, the teachers have reached an additional stage in the model (*exploring*). Last, teachers evaluate the results of integrating technology into their classrooms in ways that it enhances student learning (*advancing*). This developmental model is not meant to be hierarchical nor linear.

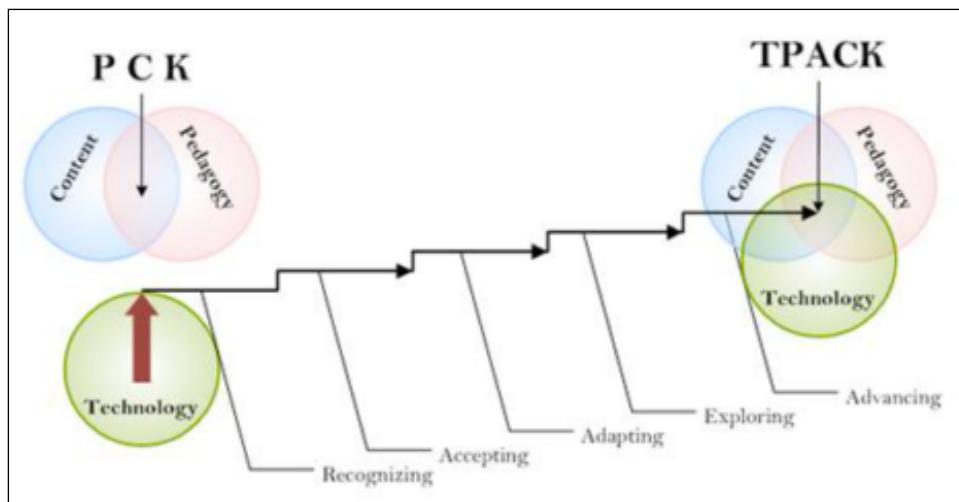


Figure 5. Five-stages developmental model (Niess et al., 2009)

Additionally, Niess et al. (2009) proposed a Mathematics Teacher TPACK developmental model, which consists of four themes: curriculum and assessment, learning, teaching, and access. The model uses the five-stages to describe the development of teachers' TPACK. As illustrated in Figure 5, the TPACK developmental model starts with a developed PCK. As teachers progress through the stages in each of the four themes, TPACK forms and expands.

More recently, Guerrero (2011), situating TPACK in the mathematics classroom, argued that there are four components of knowledge necessary for effective technology integration in mathematics:

1. Conception and use of technology: this includes ways in which a teacher can conceptualize the use of specific technologies to support teaching and learning mathematics;

2. Technology-based mathematics instruction: this includes the teacher's ability to make changes to pedagogy and recognize the need for flexibility in instruction that results from the use of technology;
3. Technology-based classroom management: this includes a range of issues relating to implementation of technology including maintaining student engagement, dealing with the physical environment and hardware issues, and dealing with behavior management; and
4. Depth and breadth of mathematics content: this component deals with the teacher's knowledge base in terms of the mathematics content and a willingness to allow students to explore mathematical content that may arise during students' investigations using technology.

These four components of knowledge are aligned with different areas of the teachers' knowledge to teach mathematics discussed by Ball and colleagues (2001) and Schoenfeld and Kilpatrick (2008), but with the complexity technology adds to it. These components, as described by Guerrero (2010) bring a different perspective on TPACK. This knowledge to teach mathematics with technology is seen as transformative (e.g., a new synthesized form of knowledge that cannot be explained by the sum of its parts). This agrees with Angeli and Valadines (2009) who claimed TPACK as a distinct body of knowledge that can be developed and assessed on its own, thus presenting a transformative view. At the same time this point of view contradicts Mishra and Koehler (2006) who proposed an integrative view

of TPACK (e.g., growth in TPACK implies growth in the three knowledge domains of the framework).

Research on Teaching Mathematics with Technology. Understanding how TPACK develops and expands has been an area of research with preservice and inservice mathematics teachers. Quantitatively, TPACK surveys have been used in mathematics and other content areas to measure teachers' perception of their knowledge and abilities to teach with technology. TPACK surveys have been used to obtain information on the effectiveness of a course (Koehler & Mishra, 2005; Schmidt et al., 2009), or professional development (Graham, Burgoyne, Cantrell, Smith, Clair, & Harris, 2009; Handal, Campbell, Cavanagh, Petocz, & Kelly, 2012) designed to develop teacher's TPACK. Research has found that offering opportunities to teachers to engage in technology-enhanced environments could enrich their TPACK knowledge and abilities (Graham, Burgoyne, Cantrell, Smith, Clair, & Harris, 2009; Handal, Campbell, Cavanagh, Petocz, & Kelly, 2012; Koehler & Mishra, 2005; Schmidt et al., 2009; Zelkowski, et al., 2013).

Qualitatively, teachers' measures of TPACK have been mostly assessed through unit or lesson plans and research has found teacher uses of technology changes significantly when they are exposed to the design and implementation of mathematical tasks (Graham, Borup, & Smith, 2012; Ozgun-Koka, Meagher & Edwards, 2010). In a study that used teacher's planning and decision-making to assess preservice teachers' TPACK, Graham et al. (2012) found that instances of instructional strategies based on TK decreased, whereas instructional strategies based on various aspects of TPK increased (e.g., classroom management, teaching

strategies, assessment, project-based learning). Harris and Hofer (2011) also found that inservice teachers were more focused on using technology to improve students' learning and not as tool for engagement by the end of a professional development.

Using lesson plans as an artifact, Ozgun-Koka et al. (2010) explored how preservice teachers' TPACK developed while designing and implementing mathematical tasks that require using the TI-Nspire handheld. They found that by the end of the course preservice teachers were using technology for developing mathematical ideas more often. Teachers also demonstrated an awareness of being teachers of mathematics, which offered them a more holistic view of how to incorporate technology, pedagogy, and content in their teaching practices. Bowers and Stephens (2011) also explored how preservice teachers enhance lesson with the use of technology. Their results showed TPACK is demonstrated when technology is used as a way to produce perturbations for the students and devises an exploration to leverage ways to confront it, and when it offer opportunities to notice mathematical relationships between objects.

Teachers' TPACK has been assessed through classroom observations while implementing a lesson. In a study with preservice teachers in Turkey, Pamuk (2012) found that when teachers demonstrated a significant lack of general pedagogy, they exhibited extreme difficulty with developing pedagogical approaches directly related to content (PCK) and a very limited use of technology when implementing their lessons (TPK) even when they were considered as having a strong technology background and no problem with the content.

This agrees with the idea of TPACK as an integrative type of knowledge, as introduced by Mishra and Koehler (2006).

In summary, research suggests the use of lesson plans offers great potential as an artifact to measure teachers' TPACK and that uses of technology are significantly influenced when teachers design and implement lessons that require the use of technology (Graham, et al., 2012; Harris & Hofer, 2011; Ozgun-Koka, et al., 2010). In addition, Graham et al. (2010) suggested that when teachers are planning mathematics lessons that integrate technology, research suggests offering teachers some guidelines with prompt and questions to reflect on and require a written rational that justify their decisions regarding the selection of the technology tool. Also, research has suggested that when teachers lack of knowledge in pedagogy, content, or technology the creation of new knowledge base for TPK, TCK, PCK and TPACK will be greatly influenced (Pamuk, 2012).

The Role of Lesson Planning in Mathematics Instruction. Lesson Planning is a psychological process of foreseeing the future, considering the learning goals and ways of achieving them (Clark & Dunn, 1991). It can be defined as “a systematic development of instructional requirements, arrangements, conditions, and materials and activities, as well as testing and evaluation of teaching and learning” (Panasuk & Todd, 2005, p. 215). NCTM (1991) suggested four key areas for classroom practices: (1) teacher needs to set goals and select/create mathematical tasks to help students achieve those goals, (2) teacher should stimulate and manage classroom discourse so that both the students and the teacher are clearer about what is being learned, (3) teacher needs to create a classroom environment to

support teaching and learning mathematics, and (4) teachers should analyze student learning, the mathematical tasks, and the environment in order to make ongoing instructional decisions. Thus, NCTM agrees with Clark and Dunn (1991) in the sense that teachers should first select a learning goal to be achieved during the planned lesson. Most recently, Smith and Stein (2011) suggested “specifying the mathematical goal for the lesson is a critical starting point for planning and teaching a lesson” (p. 13).

In 1995 Simon conducted a teaching experiment with 26 prospective elementary teachers. He focused his analysis on a sequence of three teaching situations and examined the relationship among the pedagogical deliberations [he as] the teacher makes and the activities used in the classroom. His findings were captured in the *Mathematics Teaching Cycle* (Simon, 1995), a model that illustrates the cyclical interrelationships of different domains of teacher knowledge, thinking, pedagogy, and activity (Figure 6).

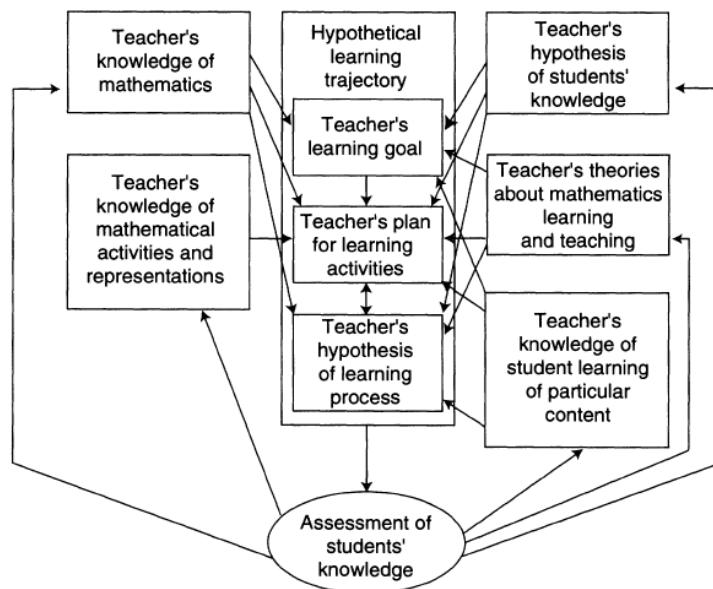


Figure 6. The Mathematics Teaching Cycle (from Simon, 1995)

Through the *Mathematics Teaching Cycle* teachers generate a hypothetical learning trajectory (hereafter HLT), which starts by selecting the learning goal. The learning goal provides direction to the HLT and is based on a hypothesis of students' prior knowledge and the teacher's mathematical knowledge. Once the learning goal has been set, the teacher needs to plan for learning activities. The learning activities must be carefully selected and sequenced to provoke the generation of powerful mathematical ideas in the students. Simultaneously with the selection and sequencing of learning activities, the teacher develops a hypothesis of the learning process regarding how the plan might materialize in the classroom. As students engage with learning activities, and the teacher observes and communicates with the students, new learning difficulties will arise among the students.

These interactions will influence the teacher's ongoing pedagogical deliberations, which at the same time will generate a new leaning goal, adding learning activities, and simultaneously modifying the hypothesis of the learning process; thus the teaching cycle starts all over again. As portrayed in Figure 7, the development of the hypothetical learning trajectory within the mathematics teaching cycle is greatly influenced by the teachers mathematical knowledge, knowledge of mathematical activities, pedagogical knowledge, as well as' knowledge of student learning. All those issues come together as teachers design lessons that are particularly focused on student learning.

In 2011, Smith and Stein proposed a pedagogical model designed specifically for orchestrating productive whole-class mathematics discussions. The five practices include: anticipating, monitoring, selecting, sequencing, and connecting student responses of the mathematical task. *Anticipating* student responses involves an a priori interpretation of how students might interpret the problem mathematically. When anticipating student's responses teachers must take into consideration not only correct answers, but also incorrect approaches as well as a previous analysis of how student's interpretations of the task and the strategies they are using to solve the task "relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn" (p. 8). *Monitoring* student responses involves teacher's conscious attention to student's mathematical thinking and strategies as they engage in the task. While students are engaged in the mathematical task, the teacher should use questioning to make student's thinking visible, to engage all students in the task, and to make students aware of particular aspects of

the task the teacher wants them to attend. As the students engage in a task, the teacher must be *selecting* the particular solutions that will be presented to the whole-class during discussion. Next, the teachers must sequence the solutions carefully. *Sequencing* must be aligned to the mathematical goals of the lesson, but the way they are put into a sequence is completely up to the teacher (e.g., from more complex to more abstract, starting with the strategy most commonly used). The sequencing practice goes hand-to-hand with *connecting* student strategies. During the connecting phase “the teacher helps students draw connections between their solutions and other students’ solution as well as the key mathematical ideas in the lesson” (p. 11).

According to the Smith and Stein (2011), “lesson plans have traditionally been seen as directions for executing particular lesson with an emphasis on procedures and structures, with limited attention to how the lesson will help develop understanding of key disciplinary ideas” (p. 76). Other researchers have found lesson plans are rather sketchy (Reiser, 1994), brief (Reiser & Mory, 1991) and cryptic shorthand (Kagan & Tipping, 1992). The five practices could be used to help teachers pay more attention to how the lesson will help students develop the understanding of the mathematical ideas in the lesson. The authors suggest putting the five practices in the context of lesson planning. First, although not included as practices within the model, there are two essential aspects of planning that must happen before the implementation of the five practices discussed above. Before starting with the planning process teachers must set the mathematical goal of the lesson and select an appropriate mathematical task that has the potential to help students achieve the

mathematical goals (Simon, 1995; Smith & Stein, 2011). Within the lesson plan, the teacher must anticipate the possible ways the task can be solve and which of these solutions student might use. The teacher must also be aware of the misconceptions students might have and the most common errors students might make.

When selecting and sequencing, within the lesson plan, teachers could focus on the questions they will use to elucidate students' thinking, and to help them advance their understanding of the mathematical idea. Although a teacher will not be able to predict all of the student responses while planning the lesson, the teacher could create a very comprehensive list of possible responses so that recognizing them during monitoring is easily. As teachers develop that comprehensive list, teachers could reason about the mathematics represented in each solution and, in a way, develop a plan to use when selecting, sequencing, and connecting student solutions. Indeed, as stated by the authors, "selecting, sequencing, and connecting, in turn, built on effective monitoring [which] will yield the substance for a discussion that builds on student thinking" (Smith & Stein, 2011, p. 12). In order for teachers to develop thoughtful and though lesson plan with the details expose above, teachers must develop a strong knowledge for teaching (Hill et al., 2008).

Teachers could develop thoughtful and through lesson plans using the Thinking Through a Lesson Protocol [TLLP] developed by Smith, Bill, and Hughes (2008). The protocol "scaffold teachers' work in planning a lesson by providing a set of questions organized around three key activities: (1) selecting and setting up a mathematical task, (2) supporting students' exploration of the task, and (3) sharing and discussing the task" (Smith

& Stein, 2011, p. 78). The protocol is intended to promote careful and detailed lesson planning. Through the use of the TTLP teachers articulate the goal for the lesson, anticipate student responses to the task, and create questions that assess and advance students' thinking.

Role of Technology in Mathematics Instruction. In 2007, under the auspices of the *President's Committee of Advisors on Science and Technology* (PCAST), the *Panel on Educational Technology* (PET) found that the way teachers use technology was still very limited to teaching students about the technology itself or for drill and practice to acquire basic skills. The findings guided the panel to give specific recommendations related to aspects we should pay attention to when teachers are using technology for educational purposes: (1) focus on learning with technology, not about technology; (2) emphasize content and pedagogy, and not just hardware, and (3) give special attention to professional development. First, although the panel considers computer-related skills are unquestionably important for students and teachers in the 21st Century, they recommend is even more important for technology to be integrated throughout the K-12 curriculum in a way that facilitates learning about any subject area. Second, "particular attention should be given to the potential role of technology in achieving the goals of current educational reform efforts through the use of new pedagogic methods focusing on the development of higher-order reasoning and problem-solving skills" (p. 8). Third, K-12 teachers should be "provided with ongoing mentoring and consultative support, and with the time required to familiarize themselves with available software and content, to incorporate technology into their lesson plans, and to discuss technology use with other teachers" (p. 8).

Research has also found that teachers have mostly use technology for administrative responsibilities such as preparation, accommodation, delivery, email, and grading (Palak & Walls, 2009; Rusell, Bebell, O'Dwyer, & O'Connor, 2003). Russell et al. (2003) examined the technology uses of 2,894 preservice and inservice teachers of different content areas. They defined six specific categories of instructional use of technology: use for preparation, for delivery, for e-mail, for recording grades, teacher-directed student use of technology, and teacher use of technology for special education and accommodation. Teachers in the study mostly used technology outside of the classroom, particularly for preparation and professional communication via email. Palak and Walls (2009) found similar results as they examined how teachers' belief related to their instructional technology practices. They used data from 138 teachers from West Virginia and found that teacher used technology most frequently for outside of the classroom practices (e.g., administrative purposes). Also, use of technology in student-centered manner was rare even though teachers were located in technology-rich schools. In a research study with three inservice mathematics teachers, Clayton (2012) found contrasting results when all mathematics teachers that participated in her study used the technology more frequently as a student-centered manner.

When technology is used for educational purposes, research has suggested several theoretical perspectives that could be used in technology-related research within the mathematics education community (Drijvers et al., 2010). One of those theories was re-elaborated by Pea (1987). A cognitive technology, according to Pea (1987) "is a medium that helps transcend the limitations of the mind in thinking, learning, and problem-solving

activities” (p.91). He proposed two types of functions by which information technologies can promote the development of mathematical thinking skills: *purpose functions* and *process functions*. Purpose functions engage students to think mathematically, participating in and owning what is learned. In purpose functions students construct ownership by owning the solution of the problem, construct self-worth, and the mathematical knowledge and skills students acquire have an impact in their lives outside of schools. Purpose functions “EMPOWER children to understand or do something better than they could prior to its acquisition” (p. 102, [emphasis in original]). Process functions provide important cognitive support, helping students understand and use the different mental activities involved in mathematical thinking. Process functions include tools for developing conceptual fluency, mathematical exploration, integrating different mathematical representations, and learning problem-solving methods.

In summary, although it seems obvious to relate teachers, or the act of teaching, with planning instruction, research has demonstrated teachers lesson planning is rather sketchy and brief. However, research suggests that when lessons are planed thoughtfully, student learning is greatly influenced (Smith & Stein, 2011; Stigler and Hiebert, 1999). Thoughtful and deliberate lesson planning draws a line that divides effective teaching from the rest. If mathematics teachers want their students to learn, student learning must be in the forefront of their lesson planning (Smith & Stein, 2011). Thus, setting goals, short- and long-term, for their students became essential. Those goals will determine the mathematical task to be

implemented, whether the use of a technology tool is appropriate, and the type of assessment to be used during the lesson.

Now, in order to help teachers' knowledge for teaching to develop, and to help teachers' design lesson plans that are focused on student thinking and learning we need to learn about professional development. Specially, we need information about which approaches have shown to be effective in helping teachers develop their knowledge to teach mathematics and in helping teachers gain experience on designing lesson plans that focus on student learning. In the next section we will discuss research on professional development for teachers.

Professional Development for Mathematics Teachers

With time we have gained important knowledge regarding effective professional development experiences for teacher. That knowledge has helped professional development designers to re-design the existing models in models that engage teachers in activities more in accordance with their daily practices. Matos, Powell, and Sztajn (2009) have summarized it as a movement from a 'training model' to a 'practice-based model' of professional development. During many years, professional development for teachers was focused on developing teacher's repertoire of classroom practice where teachers often engaged in activities that aimed at increasing their own knowledge of mathematics, or to acquire new techniques for mathematics instruction. It follows a training model, where teachers were expected to be observant and to acquire some sort of knowledge. More recently, professional development offers teachers opportunities to be participants in the teaching and learning

process, working collaboratively with peers on activities closely related to the context of teaching, namely lesson planning, tasks development, student learning, teaching practices, among others. When designing professional development activities for teachers, it is useful to think about helping teachers create communities of practice.

According to Wegner, McDermott and Snyder (as cited by Matos et al., 2009) a community of practice should have three key structural elements: domain, community, and practice. The domain refers to creating the common ground among participants and the practice refers to setting the tools, information, and documents the community members will share. The community is the central element, where teachers, facilitator, and stakeholders interact, learn together, and develop a sense of mutual engagement and belonging. For a professional development to be based on practice does not mean it has to happen within the classroom setting (Matos et al., 2009). Rather, the experiences provided by the professional development should be focused on the activities teachers engage in on a daily basis, such as teaching practices, student's learning, lesson planning, developing of materials, among many others.

Throughout the years, research on professional development has identified certain characteristics that have proven to be effective on improving teachers' knowledge for teaching and their instructional practices (Borko, 2004; Darling-Hammond et al., 2009; Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007). Research agrees that for a professional development to be effective it has to provide spaces and opportunities for teachers to enhance their knowledge and to develop new instructional practices (Borko,

2004). Professional development should be intensive, ongoing, collaborative, and connected to practice; focused on student learning and it should address the teaching of specific content. It should be a coherent and consistent system that follows a predetermined curriculum. In addition, it should offer teachers opportunities to apply their knowledge to planning and instruction.

Research has also demonstrated that it is often useful for teachers to be exposed to the same material they intend to teach to their own students and to reflect on their knowledge and on the pedagogical approaches they use (Darling-Hammond et al., 2009). Professional development should be aligned with school goals and should develop strong collaboration among teachers. Although frequent collaboration among teachers is not a practice commonly seen in the US, research suggest teacher should observe each other's teaching and provide constructive feedback in order to increase such practice (Darling-Hammond et al., 2009).

Many of these characteristics of professional development are seen in the Professional Learning Tasks (PLTs). PLTs are “complex tasks that create opportunities for teachers to ponder pedagogical problems and their potential solutions through processes of reflection, knowledge sharing, and knowledge building” (Silver et al., 2007, p. 262). Using a multi-year practice-based professional development with 12 middle school mathematics teachers, Silver et al. (2007) examined how PLTs “made available *opportunities* for teachers to *work on* and *learn about* mathematical ideas” (Silver et al., 2007, p. 261, [italics in original]). They found that PLTs can “provide opportunities for teachers to rethink and reorganize the mathematics

that they encounter in their practice, ... allowing them to render their mathematical knowledge more useful and usable" (p. 276).

When technology is added to the picture, professional development facilitators need to incorporate additional characteristic when designing technology professional development opportunities for teachers. To provide teachers with professional development opportunities focused on increasing the technological knowledge needed to teach mathematics with technology in ways that are beneficial for their students is a great challenge. According to literature, for technology integration to be beneficial for students learning, teachers need to experience some sort of change in any or all of the following: content knowledge, instructional practices, and beliefs and attitudes towards their pedagogical ideologies (Ertmen, & Ottenbreit-Leftwich, 2010). Those changes are produced with time, as teachers gain experience and get more comfortable with the technology itself. Thus, research agrees that brief exposure to technology use will not support the kind of change teachers need to experience so that they feel comfortable and ready to use technology (Schrum, 1999). For a technology professional development to be effective, it should (1) be voluntarily; (2) expose teachers to the use of the new technology a considerable amount of time; (3) provide access to the technology at home so teachers have extensive practice and build comfort; (4) provide access to the technology at school so teachers have the opportunity to try it in their classroom; (5) offer teachers authentic reasons to why technology is essential for teaching; (6) offer individualized attention to teachers; and (7) provide ongoing support (Ertmen, & Ottenbreit-Leftwich, 2010; Schrum, 1999).

Literature on practice-based and technology professional development agrees on the recommendation of Lesson Study as an approach to professional development; an approach that suggest helping teachers develop the confidence they need with the technology itself and the encouragement they need to integrate technology tools in their teaching practices (Ertmen, & Ottenbreit-Leftwich, 2010; Matos et al., 2009). Lesson Study fits nicely with the characteristics described above for both, a practice-based professional development and technology professional development.

Lesson Study. Lesson Study is the most common form of professional development for teachers of many content areas in Japan. It was introduced to U.S in the late 1990s with the work of Lewis and Tsuchida (1997), Yoshida (1999), and with the publication of *The Teaching Gap* (Stigler & Hiebert, 1999), among others. Since then, researchers have been encouraged to try out Lesson Study as a way to build professional knowledge for teaching and improve teaching and learning (Stigler & Hiebert, 1999; Yoshida, 1999). The Lesson Study approach to professional development has overcome culture barriers demonstrating it could be effectively adapted and used with teachers in the U.S., independently of the strong cultural differences teachers from both countries have.

Lesson Study is a cycle of instructional improvement in which teachers collaboratively study curriculum and formulate goals, plan a lesson, implement it, and reflect on it. The process of a Lesson Study has been described elsewhere (e.g., Lewis, Perry, and Murata, 2011; Meng & Sam, 2013; Perry & Lewis, 2008; Stigler and Hiebert, 1999). It encompasses a Four-Phase cycle (Figure 7), starting with *Phase 1: Focusing the lesson*

where teachers study the curriculum and formulate long-term goals for their student learning and identify a topic of interest. Then during *Phase 2: Planning the research lesson*, teachers design and write a detailed research lesson that includes long-term goals, anticipated student thinking, and a plan for data collection. *Phase 3: Teach and discuss the research lesson* entails the implementation of the lesson in an actual classroom by a volunteer teacher while other members of the group observe and take notes. To complete a cycle of Lesson Study, during *Phase 4: Reflect and re-teach* all members of a group meet to share their data collected from the lesson and to have a discussion around issues they feel need to be discussed. The lesson is revised, if needed, and a new cycle begins with the formulation of goals and the selection of a new topic of interest. Lesson Study as a professional development is sustained, coherent, cohesive, intense, collaborative, and practice-based.

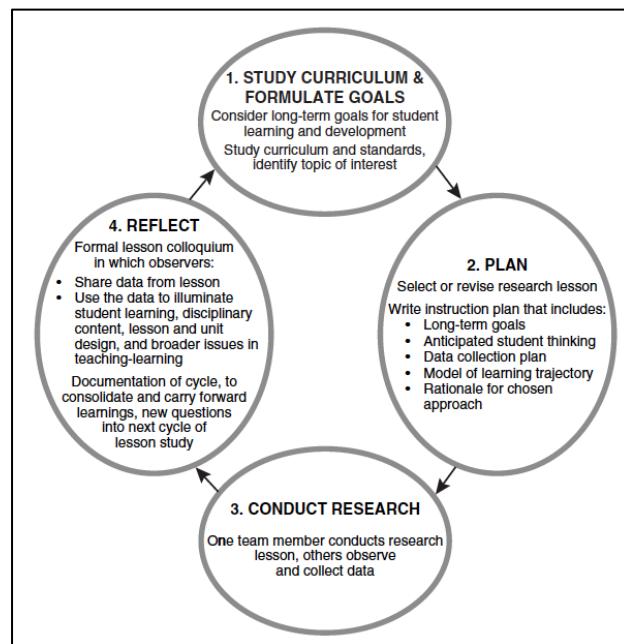


Figure 7. Lesson Study Cycle (from Lewis, Perry, & Murata, 2006)

Perry and Lewis (2008) reported on a Lesson Study effort with one K-8 school in the US. The school engaged in Lesson Study for over four years. Interviews conducted throughout the years (2001-2004) revealed teachers' changes in instruction as well as changes in collaboration because of their participation in Lesson Study. Among the changes in instruction reported are an increased use of tasks that elicit student thinking and support student interaction; increased emphasis on anticipating student thinking; increased awareness of discussing student solutions, even the incorrect solutions; and increased use of student data to inform instruction. Changes in collaboration included an increased communication with colleagues including discussion of student thinking; and increased interest in observing and have discussion around each other teaching. This agrees with results from Olson, White, and Sparrow (2011) who conducted a Lesson Study with ten elementary teachers and found Lesson Study influences the pedagogical practices of teachers engaged in the Lesson Study cycle in ways Perry and Lewis described.

Cady, Hopkins and Hodges (2008) also reported an increase in their own pedagogical mathematical knowledge as they engage in Lesson Study to develop a lesson about place value for preservice and inservice teachers. Following the Lesson Study cycle they taught-revise-taught the lesson a total of eight times. The authors created a base-five system to help teachers better understand the place value relationships in the base-ten system. They created cognitive dissonances that forced teachers to examine those relationships in a system that was not familiar for them, a base-five system. The authors stated how engaging in a Lesson

Study, and the reflections and observations inherent to it, increased their pedagogical content knowledge.

In a study with 24 middle school inservice mathematics teachers, Meyer and Wilkerson (2011) examined the opportunities Lesson Study provides to improve teachers' knowledge for teaching mathematics. The authors identified three important factors that may help or hinder those opportunities: the lesson plan or task to be implemented, teachers' discussion while planning the lesson; and the level to which teachers anticipated students' responses/questions. Teachers who demonstrate to have improved their knowledge for teaching mathematics either created a new lesson or made major changes to an existing lesson plan, and spent a significant amount of time discussing their understanding of the mathematics and discussing and predicting student questions and responses.

Meng and Sam (2013) studied changes in pre-service secondary teachers' self-perception of TPACK. During the study, 46 pre-service mathematics secondary teachers, engaged in a cycle of Lesson Study as they were enrolled in a teaching methods course in a Malaysian public university. For the study they validated a survey adapted from the Survey of Teachers' Knowledge of Teaching and Technology (Schmidt et al., 2009). The survey contained seven subscales (TK, CK, PK, PCK, TCK, TPK, TPACK) with 47 "self-reported items that assessed the participants' TPACK for teaching mathematics with [the Geometer Sketchpad (GSP)]" (p. 3). During the course, pre-service teachers were divided into groups of two and were asked to complete a Lesson Study cycle as they planned, taught, reflected, re-teach, and reflect on a lesson they had planned. Results show that pre-service secondary

teacher self-perceptions of their TK, CK, PK, PCK, TCK, TPK, and TPACK improved significantly.

Thus, a Lesson Study approach to professional development can create communities of practices among teachers. The approach has demonstrated to help teachers develop their mathematical knowledge as well as their pedagogical content knowledge (Lewis, Perry, Murata, 2006; Perry & Lewis, 2008; Stigler & Hiebert, 1999). The reflection, observation, and collaboration inherent to the process itself have shown to be effective practices for teachers to increase their knowledge for teaching. Lesson Study has been used in different settings with preservice and inservice teachers at the elementary and secondary level. Most of the research has been focused on how the use of Lesson Study as a community of learning has influenced teacher's knowledge for teaching (Meyer & Wilkerson, 2011; Olson, White & Sparrow, 2011).

When the use of educational technology in teaching mathematics is brought into the picture, research on the development of teachers' TPACK through the use of Lesson Study is very limited. Pierce and Stacey (2009) conducted a Lesson Study with two Australian secondary schools to examine the design of lessons when teachers used the TI-Nspire (CAS). At the time of the study, Lesson Study was totally new for teachers in Australia. Results demonstrated that Lesson Study as a professional development approach helped mathematics teachers to improve the quality of technology use. Teachers in the study also became more open to having colleagues observing their teaching. The authors suggest that for Lesson Study to be established in a school, teachers need to experience more than one cycle.

Groth, Spickler, Bergner, and Bardzell (2009) proposed a model to qualitatively assess teachers' development of TPACK. Using Lesson Study groups (LSG) teachers collaboratively designed and implemented lessons that integrated technology. The proposed model was named the Lesson Study TPACK (LS-TPACK) assessment model. This model consists of Lesson Study cycles where "a group of teachers collaboratively construct a lesson on a shared learning goal for students, implement it, observe the implementation, and then debrief on the strengths and weaknesses of the lesson" (Groth et al., 2009). The most salient findings illustrate how teachers' inexperience on how to appropriately use the technology affects the mathematics that teachers present to their students. In general, the technology was not used to its fullest to efficiently demonstrate multiple mathematical representations and on a different occasion it was used as an answer-producing machine, missing teachable moments that could help students understanding the relation between the output offered by the graphing calculator and what they already know mathematically.

Conceptual Framework

Ball and colleagues (2001), Schoenfield and Kilpatrick (2008), and Guerrero (2010) have identified the many types of knowledge teachers need to teach mathematics with technology. Although they have used different names to refer to the distinct types of knowledge, they all agree mathematics teachers need to know the mathematics they teach (knowledge of mathematics), as well as the technology they are using (knowledge of technology). But teachers also need those types of knowledge that are more specific to teaching mathematics, such as knowledge of math and students, knowledge of math and

learning, and knowledge of math and curriculum. When the mathematics, the pedagogy, and the technology are integrated a particular type of knowledge is needed; technology-based mathematics instruction knowledge. This type of knowledge refers to the teachers' ability to integrate the technology into their classroom and use it effectively to teach the mathematics. All these types of knowledge influence the decisions teachers make when planning a lesson. All these different types of knowledge are illustrated with the blue circles in Figure 8. At the center of the diagram are the different phases of the modified Lesson Study approach to professional development followed during this study. What this framework highlights is that as teachers collaboratively engage in designing lessons with peers, their existing knowledge to teach mathematics with technology emerges and influences decisions made by a group of teachers. At the same time, those discussions with peers regarding pedagogical approaches, the technology tool to be used, and how to present the mathematical content, among others, is expected to influence teachers' own knowledge. This perspective is illustrated in Figure 8 with the two-ways arrows.

According to research, the quality of instruction is greatly influenced by the teachers' mathematical knowledge for teaching (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Fennema, Franke, Carpenter, & Carey, 1993; Hill et al., 2008; Leinhardt & Smith, 1985; Lloyd & Wilson, 1998; Swafford, Jones, & Thornton, 1997; Thompson & Thompson, 1994). When teachers have a weak mathematical knowledge for teaching, research have demonstrated several issues emerge as teachers plan and enact mathematics lessons. The mathematical content issues may include the presence of mathematical errors (Hill et al.,

2008), use of incomplete definitions (Hill et al., 2008, Stein et al., 1990), absence of mathematical explanations, absence of multiple representations, and inappropriate use of language, among others. The pedagogical issues refer to the general pedagogical issues such as classroom management, the pedagogical issues specific to the teaching of mathematics such as knowledge of curriculum organization or how to manage students' misunderstandings, and the pedagogical issues specific to teaching with technology such as how to manage a hardware issues. Technological issues encompass issues related to the technology itself, and issues with its use in the mathematics classroom. This includes issues with the basis of the technological tool as well as issues connecting the technological tool with the mathematics.

The framework in Figure 8 is used in this study to guide data collection and analysis. As a group of teachers made decisions and scripted detailed lesson plans of mathematics classes, it was expected that all these different types of knowledge would interact with one another and emerge as they discussed the mathematical activities to include in the plan, the sequence of events within the lesson, the problems to include and the order those should be discussed (e.g., simple to more complicated). During analysis, teachers' discussions when planning were carefully analyzed to identify similar issues in the different areas identified above, but also other issues related to their knowledge to teach mathematics with technology. In addition, those different types of knowledge guided the field notes taken during classroom implementation, as well as during the reflection meetings. All the issues identified through the literature, which are portrayed in the framework, served as the primary "theory codes"

during the analysis of the data. In addition, a pre- and post- TPACK survey was used to describe the changes in teachers' self-perception of their knowledge to teach mathematics with technology.

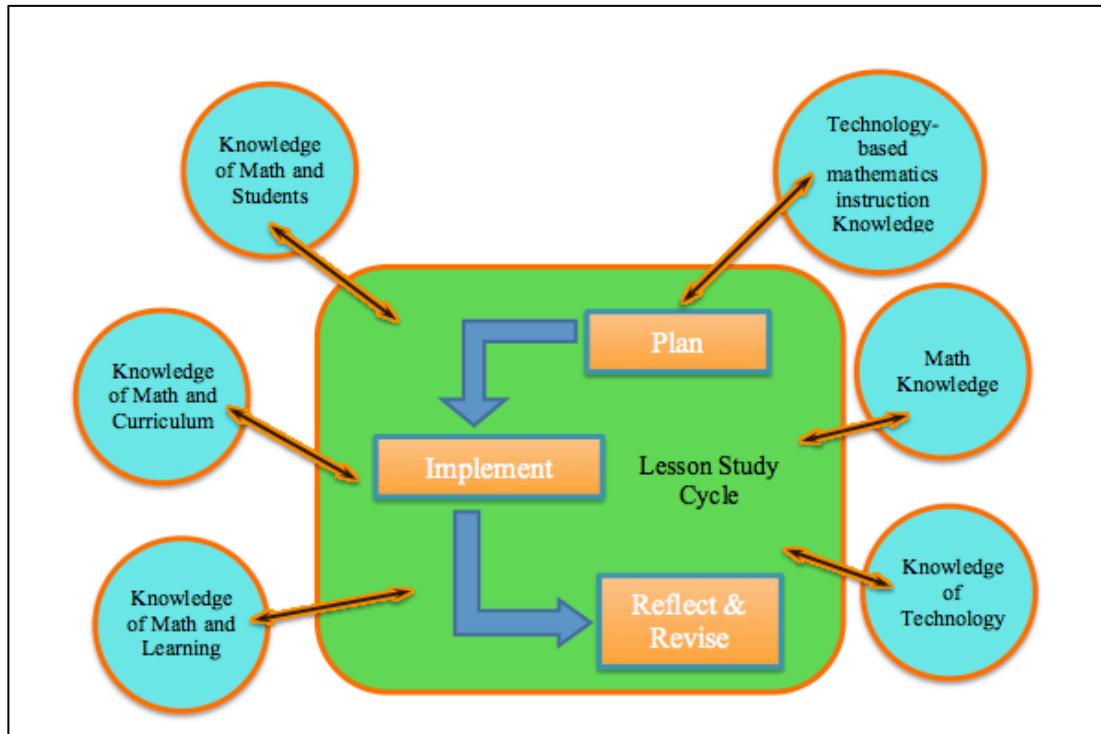


Figure 8. Dimensions of Teacher's Knowledge to Teach Mathematics with Technology as they Influence Teacher's Decisions during Lesson Planning

Summary of Chapter 2

In this chapter I have identified the important pieces of teachers' knowledge for teaching and have presented TPACK as a theoretical framework to describe the different constructs one need to consider when teaching with technology. I have presented results from research where Lesson Study has been used and how it has greatly influenced teachers' mathematical knowledge for teaching. Still, a gap in literature exists in investigating how

teachers' knowledge to teach mathematics with technology (TPACK) develops when teachers are engaged in a Lesson Study approach to professional development.

Acknowledging the positive results research has demonstrated through the use of Lesson Study, I investigated the issues that emerge as secondary mathematics teachers were engaged in a professional development focused on designing mathematical lessons that integrated technology. Results from this study could bring fresh and important information on the development of in-service secondary mathematics teachers' knowledge to teach mathematics with technology (TPACK) as they collaboratively designed and taught lesson plans that integrate technology.

Chapter 3

Methods

The purpose of this study is to identify the issues that arise when secondary mathematics teachers collaboratively plan a lesson where technology tools should be incorporated as a way to enhance student learning. The goals of this study are: (1) to describe the technological, pedagogical, and mathematical issues that emerge in the process of designing and implementing the lesson, and (2) to describe changes in those issues as teachers engaged in the process of designing and implementing a lesson for a second time.

To fulfill the aforementioned goals, three research questions were investigated within the context of secondary mathematics teachers engaged in a Lesson Study based professional development focused on designing mathematical lessons that integrate technology:

1. How do a group of secondary mathematics teachers' self-perceptions of their technological knowledge (TK), mathematical knowledge (MK), and technological, mathematical and pedagogical knowledge (TPACK) change as a result of engaging in a Lesson Study focused on designing mathematics lessons that integrate technology?
2. What issues emerged (mathematical, pedagogical, and technological) in the process of designing, implementing, and reflecting on lessons? And how do teacher's attention to any of the above issues change as they become more experienced in designing lessons that integrate technology?

To carefully study the emerging issues secondary mathematics teachers encountered while planning lessons that integrate technology, a convergent parallel, multi-case study design was used (Creswell, 2007, Stake, 1995). A case is a group of teachers, thus there were four cases in the study. The case study was situated within the context of the *Mathematics and Science Partnership* project, described below. In this chapter, I will first describe the *Mathematics and Science Partnership* project, its context, goals, strategies, and some critical issues one must consider up front before continuing reading. Then, the research design is explained including the study layout, the conceptual framework, participant selection, data collection and analysis. Lastly, internal validity, reliability, external validity, as well as ethical issues are considered.

Mathematics and Science Partnership

AFAMaC-Mathematics (short for *Alianza para el Fortalecimiento en el Aprendizaje de las Matemáticas*) is a project funded through the Department of Education of Puerto Rico (hereafter DEPR) since 2003. The project headquarters are located at the Department of Mathematical Sciences at the University of Puerto Rico Mayagüez (UPRM). At the time of the study I was an instructor of mathematics in the same department and had previously worked as a resource for the project, offering several workshops on mathematical content to previous participants of the project (2013-2014 school year). As a graduate student at the Department of Mathematical Sciences at UPRM from 2003 to 2005 I was actively involved with AFAMaC, thus I had some previous knowledge of the project itself.

Context of AFAMaC-Mathematics. A total of twenty-nine (29) public and private rural schools in the western part of Puerto Rico were impacted through *AFAMaC-Mathematics* in 2014-2015. That included a total of thirty-two teachers (9-elementary, 10-middle school, 13-high school) located in the municipalities shown in Table 1 (boldface indicates municipalities of participant teachers). The DEPR divides the island into seven Educational Regions, with Mayagüez being one of them. The Mayagüez Educational Region is made up of four school districts. *AFAMaC-Mathematics* aims to improve the mathematics achievement of students in fourth to twelfth grade enrolled in schools from the school districts of Aguadilla, San Sebastián, Mayagüez, and Cabo Rojo.

Table 1. Municipalities in the Mayaguez Educational Region

Mayagüez Educational Region			
<i>Aguadilla District</i>	<i>San Sebastián District</i>	<i>Mayagüez District</i>	<i>Cabo Rojo District</i>
Aguadilla	San Sebastián	Maricao	San Germán
Aguada	Moca	Mayagüez	Cabo Rojo
Rincón	Isabela	Las Marías	Lajas
Añasco		Hormigueros	Sabana Grande

Project Goals. With an intensive program of support, mentoring and professional development *AFAMaC-Mathematics* aims to increase the academic preparation of teachers of mathematics, and to create a cohort of teachers that will serve as mentors to other teachers. The objectives of the project, as they appear in the grant proposal, are:

- To professionally improve mathematics teachers;
- To support teacher's extracurricular efforts with human and material resources;

- To establish a system to recognize and reward mathematics teachers;
- To create a network of mathematics teachers highly motivated and qualified; and
- To improve the mathematical achievement of the students of mathematics teachers impacted through the project.

Project Strategies. The strategies used by the project included a two-week Summer Institute, six Saturday Academies, and one *AFAMaC*-Residential. During the summer of 2014 teachers participated in a two-week non-consecutive Summer Institute (Week 1 on-campus, Week 2 at a hotel). During the Summer Institute teachers attended workshops on different topics. During the fall semester teachers participated in six Saturday Academies. During the academies, teachers received workshops from various faculty members of the Department of Mathematical Sciences in topics such as Normal Distribution, Exponential and Logarithmic Functions, and Patterns, Sequences and Series, among others. Each Saturday Academy lasted five hours and was on-campus. In addition, teachers attended the *AFAMaC*-Residential, which took place on a Saturday and lasted 7 hours. During this Residential, teachers planned a lesson.

Critical Issues. Several critical issues need to be mentioned before continuing. First, the professional development focused on the technological, pedagogical and mathematical issues that emerged as secondary mathematics teachers were designing lessons. At the same time, workshops focused on mathematical content occurred during the Saturday Academies as part of the *AFAMaC-Mathematics* activities. For the purpose of designing the lesson, each group of teachers chose a mathematical content as the focus of their lesson. Although some

overlapping of mathematical content occurred, the focus of each group depended on their goals for students' learning.

Second, I was the facilitator of the professional development for all the activities related to the Lesson Study. At the very beginning of the professional development activities, many of the teachers attended a series of workshops, which were offered by me. Attendance of teachers varied by day or time of the day. Once the cycles of Lesson Study began, I acted as a resource for the teachers as they designed their lessons. I actively observed the interactions between the teachers taking notes of the issues that were emerging as teachers made decisions regarding the design of their lessons. If there were particular instances where I needed to intervene, my role was mainly to guide the teachers in the search of a solution, but no solution was offered. In the event that teachers did not know what to do and had exhaustively searched for responses on their own, then I offered some concrete suggestions to the group of teachers.

Third, participation in the research study was voluntary, and no monetary compensation was offered in addition to the stipends participants received for their participation in *AFAMaC-Mathematics*. Participants designed lessons in collaboration with peers, thus an effort was made to group teachers accordingly to municipality, grade level, and schools mainly to facilitate the scheduling of additional meetings, if necessary.

Forth, at the beginning of the study teachers encountered a very difficult situation with the standards. A new set of standards was released late summer of 2014, the Puerto Rico Core Standards (PRCS, 2014). Although the activities related to the professional

development itself started in June of 2014, data collection did not start until July when teachers started planning their first lesson. Thus, at the beginning many teachers were disoriented regarding the release and implementation of the new set of standards. They were required to start implementing those standards in August 2014, thus the participants were encouraged to design their lessons based on those newly published standards. This situation raised a big issue for participants as they familiarized themselves with the new format and language of standards. By the time they started thinking about the topics for their first lesson, the curricular maps were not released and many of them had no idea which topics they were supposed to be teaching around the time of the implementation of the first lesson (around the 2nd or 3rd week of classes). Also, the format of the standards was new, a format pretty similar to the Common Core State Standards (CCSS, 2010). Thus, the high school teachers did not have the standards by grade as they were used to, but by strands with no guide on the order the topics were to be covered in the grades they were going to be teaching in August.

The Professional Development

The study of focus here occurred within the context of *AFAMaC-Mathematics* during the summer and fall semester of 2014. The purpose of the professional development was to engage teachers in activities where they collaborated with peers and designed mathematics lessons that integrate technology. The goals of the professional development were to enrich teacher's knowledge for teaching mathematics with technology by engaging in two cycles of a lesson Study, where teachers conducted research on a mathematical idea of interest and designed lessons to be implemented in their classroom. When conducting research, teachers

needed to focused on students' understanding of the mathematical idea, misconceptions students might have, technological approaches that best fit the teacher's goal for the lesson, and pedagogical approaches that best fit the teaching and learning of the mathematical idea. By the end of the professional development all teachers had the experience of designing mathematics lessons that integrate technology and had carefully designed two mathematics lessons they could implement in their classrooms.

During the summer of 2014, a total of eighteen secondary teachers volunteered to participate in the professional development opportunity for the fall semester of 2014. Since the professional development followed the structure of a Lesson Study, some of the strategies of *AFAMaC-Mathematics* were accommodated in order to facilitate the structure for the current study. For example, some of the activities related to the Lesson Study occurred during the Summer Institute, Saturday Academy #3, and the *AFAMaC-Residential*. Those are indicated with a green box in Figure 9. Others activities of the Lesson Study occurred throughout the academic year, but did not overlap with those of *AFAMaC-Mathematics*. Figure 9 shows a timeline to illustrate the different events that occurred, and how the activities of the Lesson Study were intertwined with the activities of the *AFAMaC-Mathematics* project. The blue shapes represent the Saturday Academies, where teachers meet on-campus for 5 hours to participate on math-focused workshops. During the fall semester of 2014 the topic for those Saturday Academies were, in chronological order: Sequences and Series; Solutions and Applications of Logarithmic and Exponential Functions;

Statistics in Practice Part I (lasted only 3 hours); Statistics in Practice Part II; and Normal Distributions and its Applications.

During the Summer Institute Week 1 in June, I offered workshops for secondary mathematics teachers on Uses of Technology to Teach Mathematics, the Lesson Study approach, and the Five Practices for Orchestrating Classroom Discourse for a total of 10 contact hours. The formal invitation to participate in the study happened during the last day of this week. Teachers participated in additional math-focused workshops during that week. During the Summer Institute Week 2 in July, teachers completed Phase 1 (Focusing the Lesson) and Phase 2 (Planning the Lesson) of Cycle 1, for a total of 10 contact hours. In addition, teachers participated in math-focused workshops during that week as well.

As the semester progressed, Phase 3 (Implementation of the Lesson) for all groups took place in their respective schools. Volunteer teachers were observed while implementing the lesson they had designed with their groups. After implementation, we completed Phase 4 (Reflection Meeting) as each group individually met with the PI of *AFAMaC-Mathematics* and myself to reflect on various aspects of the design and implantation of the lessons. These meeting happened on-campus, during after school hours. As observe in Figure 9, Cycle 1 was completed during the month of October. During the Saturday Academy #3, I used two hours to engage the teachers in Phase 1 (Focusing the Lesson) for Cycle 2. Then, during the *AFAMaC-Residential*, Phase 2 (Planning the Lesson) for Cycle 2 took place. This planning occurred on-campus and lasted around 7 hours. This was followed by Implementation of the

Lesson and Reflection Meeting for Cycle 2, which occurred during November and December of 2014.

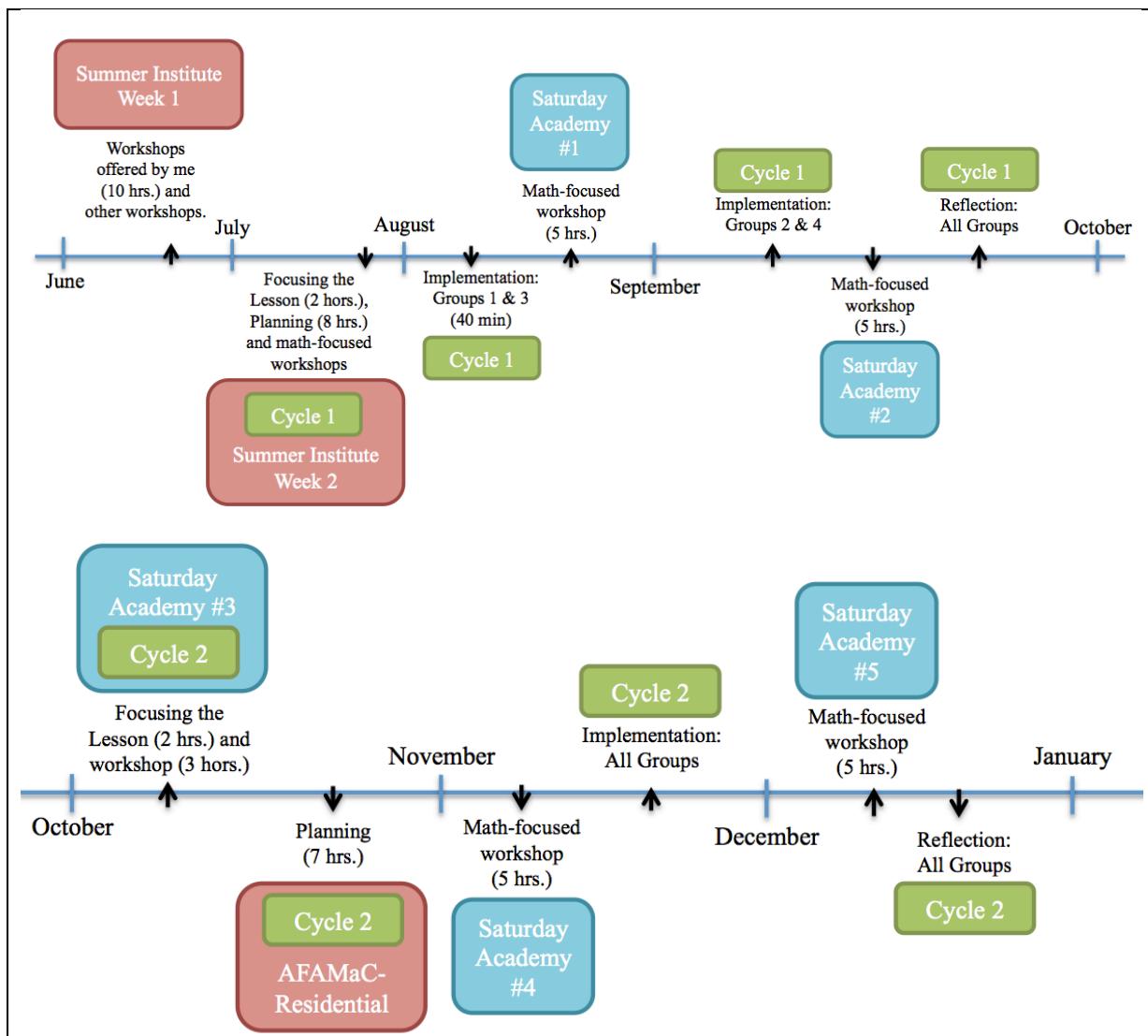


Figure 9. Timeline of Professional Development Sessions within the Context of *AFAMaC-Mathematics*

During the planning sessions for both cycles, teachers had access to computers with Internet access. To facilitate the process of planning a research lesson, I discussed and shared the following material resources for the teachers. During Summer Institute Week 1, I shared with the teachers (1) a Lesson Study flowchart for teachers to use as a guide in the process of planning their lesson (Appendix E); (2) the Thinking Through the Lesson Protocol (translated into Spanish) for teachers to follow as they wrote their detailed lesson plan to help obtain a rich, detailed description of the lesson (Appendix F); and (3) a document containing a list of useful resources for teachers, including links to websites where teachers could find ideas of mathematical activities to implement, names of databases they could access to find literature on student's learning, and links to free online journals (Appendix G). All of these documents were written in Spanish for the benefit of the participants.

Research Design

Layout of the Study. A mixed methods research study is defined as “a type of research in which a researcher or team of researchers combines elements of qualitative and quantitative research approaches for the broad purpose of breadth and depth of understanding and corroboration” (Johnson, Onwuegbuzie, & Tuner, 2007, p. 123). This study followed a fixed, qualitative-dominant mixed methods design to answer the research questions. The mixed methods design is fixed in the sense that the use of quantitative and qualitative methods is predetermined and planned since the beginning of the research process and not emergent to fulfill inadequacies found for using one method (Creswell & Plano-Clark, 2011).

A qualitative dominant mixed methods research “relies on a qualitative, constructivist-poststructuralist-critical view of the research process” (Johnson et al., 2007, p. 124).

A multi-case, convergent parallel design was used to guide the process of data collection and data analysis. A convergent parallel mixed method design occurs when the researcher uses concurrent timing to implement the quantitative and qualitative strands during the same phase of the research process (Creswell & Plano-Clark, 2011). Although in a typical convergent parallel design equal priority is given to both strands, this study prioritizes the qualitative strand in data collection and analysis (Figure 10). A convergent parallel design fits this study because quantitative and qualitative strands are kept independent during collection and analysis and are mixed during the overall interpretation.

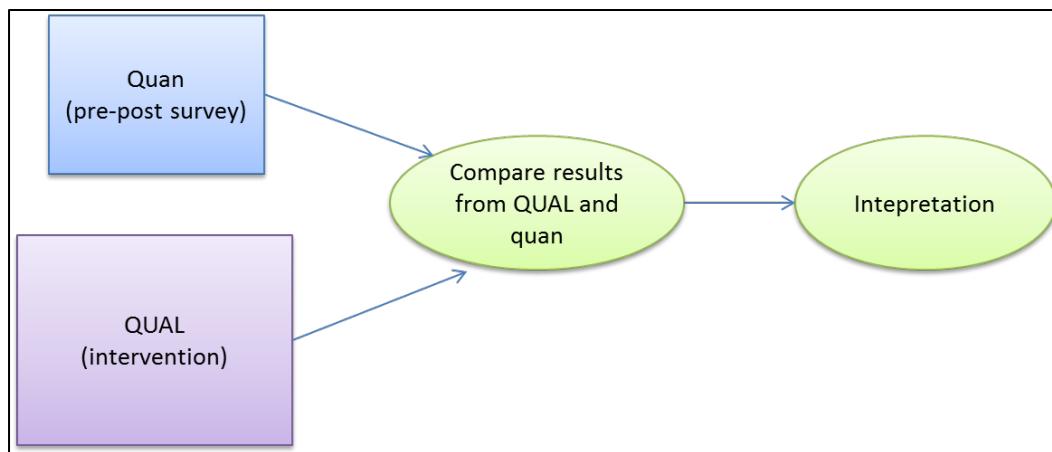


Figure 10. Convergent Parallel Method Design

In the qualitative strand (QUAL), the study followed a multiple case study strategy to collect and analyze data. A multiple case study research involves the study of an issue explored through several cases within a bounded system (Creswell, p. 73). In the study a case

was a group of mathematics teachers. There were a total of four cases in the study, two groups of middle school teachers and two groups of high school teachers. Data sources included audiotape of planning meetings, videotape of classroom implementations, audiotape of debrief meetings, as well as artifacts from classroom observations, and journal entries. Field notes were taken from classroom observations, planning and debrief meetings, and any other meeting. Details about the sources of data are described later in this chapter.

In the quantitative strand (quan), a survey was used as a measurement instrument. The survey was an adaptation of the TPACK (Technological Pedagogical and Content Knowledge) instrument for Secondary Mathematics Preservice Teachers developed by Zelkowski, J., Gleason, J., Cox, D. C., and Bismarck, S. (2013). Participants completed the survey before the Lesson Study professional development began and once it was finalized. The survey is described in detail later and then in Chapter 4.

Participant Selection. Subjects selected to participate in this study were teachers participating in the *AFAMaC-Mathematics* project. All participants were secondary mathematics teachers from the Mayaguez Educational Region (refer to Table 1). During May 2014 all secondary mathematics teachers in those 15 municipalities were invited to submit an application as part of the recruitment process. The application form asked demographic information, as well as information about the school they were teaching during the 2014-2015 school year.

By the end of the month of May, 32 elementary and secondary teachers that met a set of a priori criterion were selected by the PIs of the project. In general, the PIs gave high-

priority to those teachers from municipalities considered high necessity municipalities. Those municipalities are: Aguada, Rincón, San Sebastian, Moca, Mayaguez and Hormigueros. The project's PIs took into account teacher's depth of content knowledge (since they participate in workshops focused on mathematical content), and teacher's school level/grade (to have all secondary grades represented) among other characteristics. From the 32 teachers that were recruited to participate in the AFAMaC project, a total of 23 were secondary mathematics teachers. The workshops and lectures offered by the project during the academic year were open for participants to attend. Each time they participated in a workshop, teachers received a stipend. If for any reason a teacher decided not to attend a workshop, they did not receive the stipend for that particular day. Thus, attendance to all workshops was not required.

During the first week of the Summer Institute in June, I offered workshops on diverse topics. At the end of the week, I introduced the research study and described the professional development activities in detail, including the data I was going to collect from them. Participation was voluntary and it was made very clear that their participation in the study did not affect their participation in *AFAMaC-Mathematics*. Once the details of the study were clarified, those interested in being part of the study signed an Informed Consent Form (Appendix A). Twenty secondary teachers attended the first week of the Summer Institute and nineteen showed their interest in the study by signing the consent form.

One teacher who signed the consent form during the first week of the Summer Institute did not attend the second week of the Summer Institute, and thus did not participate in the planning of the lesson. Moreover, she did not participate in most of the Saturday Academies

offered through the semester. Thus, she was not assigned to a group and therefore no data was collected from her. In addition, there were three teachers that did not participate in the first week of the Summer Institute, but participated in the second week. Those teachers were not invited to participate in the study given that they did not attend the sessions offered during the first week in June, which were essential in understanding the professional development approach and the data that was going to be collected. Those three teachers, plus the teacher that did not sign the consent form constituted Group 5. A new member was added to Group 5 during the planning of the lesson in Cycle 2. Those teachers participated in the professional development, but no data was collected as they participated in the different activities related to the Lesson Study.

Four groups were formed with the teachers that attended the first week and signed the informed consent form. There were two groups consisting of five high school teachers each, and two groups consisting of four middle school teachers each. Through the course of the study, two teachers did not attend most of the Saturday Academies and the Residential and were dropped from the study. One of the teachers completed his participation in Cycle 1, with all data sources collected. Thus his data for Cycle 1 is included in the analysis. He participated in Phase 1: Focusing the Lesson on Cycle 2, but he did not participate in any of the professional development activities thereafter. The second teacher participated in the planning of the lesson during Cycle 1, but he did not participate in any activities related to the professional development thereafter. Both teachers are included in the participant description that follows.

Participant Description. At the beginning of the study there were eighteen (18) teachers participating, with eight (8) middle school teachers and ten (10) high school teachers. Teachers were from eight middle schools (2 private, 6 public) and eight high schools (1 private, 7 public). There were 11 female teachers and seven male teachers participating in the study. The number of years of teaching experience ranged from one to 18. Regarding highest academic degree achieved, seven teachers reported having a Bachelor degree, and 11 teachers reported having a Master's degree. At the moment of the study, there were two teachers pursuing a Master's degree and another teacher had just started pursuing a PhD degree.

Data Collection Methods. Qualitative studies use rich, thick descriptions of the phenomenon under study that is obtained from multiple sources (Creswell, 2007; Merriam, 1998). The primary sources of data from the study include a pre-post survey, digital audio recordings from all planning sessions, video recordings of the implementation of lessons, field notes from reflection meetings, journal entries, and detailed written lesson plans for each group. Data collection started in June 2014 and was completed by February 2015. The research questions, sources of data used to answer those questions, and how the data was analyzed is summarized into Table 2, and described in detail in the following paragraphs.

Table 2. Summary of the data sources and analysis

Research Question	Sources of Data	Data Analysis
1. How do a group of secondary mathematics teachers' self-perceptions of their technological knowledge (TK), mathematical knowledge (MK) and technological, mathematical and pedagogical knowledge (TPACK) change as a result of engaging in a Lesson Study focused on designing mathematics lessons that integrate technology?	<ul style="list-style-type: none"> • Pre-Survey (EMMES-CTPC) • Post-Survey (EMMES-CTPC) 	<ul style="list-style-type: none"> • Quantitative analysis, pair t-test
2. What issues emerged (mathematical, pedagogical, and technological) in the process of designing, implementing, and reflecting on lessons? And how do teachers' attention to any of the above issues change as they become more experienced in designing lessons that integrate technology?	<ul style="list-style-type: none"> • Audio recording from planning sessions from both cycles • Video recording from implementations from both cycles • Field notes from reflection meetings from both cycles • Journal entry of individual teachers 	<ul style="list-style-type: none"> • Coding using Transana • Cross-case analysis of all four groups per cycle • Within-case analysis of each group

TPACK survey. At the beginning and at the end of the Lesson Study professional development all participants were asked to complete a survey that assessed self-perceptions of their knowledge to teach mathematics with technology. As a pre-test, the survey was administered physically at the end of the first week of the Summer Institute. For the post-test, the survey was administered in person at the moment each group met to reflect on the implementation of the lesson during the second cycle. For those teachers that were unable to participate in those meetings, the survey was collected by email or postal mail. Still others completed the survey in February, during the first Saturday Academy in Spring 2015.

The survey is an adaptation of the *TPACK (Technological Pedagogical and Content Knowledge) instrument for Secondary Mathematics Preservice Teachers* that was published in 2013 by Jeremy Zelkowski, Jim Gleason, Dana C. Cox, and Stephen Bismarck in the Journal of Research on Technology in Education (Appendix B). The survey was translated into Spanish (Appendix C) to get more trustworthy data since the primary language of the study participants was Spanish. In order to assure validity of the results, the survey was validated at the University of Puerto Rico at Mayaguez in May of 2014. The validated survey consisted of 18 items divided into four (3) constructs: technological knowledge (TK), mathematical content knowledge (CK) and technological pedagogical and mathematical content knowledge (TPCK) (Appendix D). More details of the validation process are reported in Chapter 4.

Planning session data. The first and second phases of a Lesson Study were focused on the planning of a lesson. During Phase 1, teachers set their goals (i.e. goals specific to the lesson, goals specific to the unit, goals specific to the subject-matter, and goals specific to student development) and selected the mathematical content to focus their lesson on. During Phase 2, the four (4) groups of teachers worked concurrently on designing a mathematics lesson plan. As teachers discussed the mathematical content, the technology tools available, the pedagogical approaches, and how to use all that information for the design of a mathematics lesson I paid attention to the issues that were emerging from those discussions. All planning sessions were audio recorded, and when possible field notes were taken to record those issues.

Journal Entry Data. Journal entries were collected on several occasions during the Lesson Study. The purpose of the journal entries was to offer teachers opportunities for individual reflection on the process of designing a lesson that integrates technology. In the journal entries teachers were asked to reflect on the discussion the group members had around the mathematical content, students' difficulties and misconceptions, students' learning of the mathematical content, technology tools available to teach that particular mathematical content, the pedagogical approaches the group considered using, and any other topic they felt was relevant. To get data that was rich and offered detailed descriptions, teachers were given a set of prompts for their journal entries (Appendix H). Teachers were asked to write their individual journal entries twice during a cycle, for a total of 4 journal entries: one after completing the planning of a lesson (Phase 2) and another one after the reflection meeting (Phase 4). They were asked to share those entries with me electronically via email. Teachers that were not with the group at the time of planning or reflecting were asked not to write a reflection. Thus, not all teachers had four journal entries, depending on their presence at the planning or reflection meetings. A total of 53 journal entries were collected; 32 journal entries were collected during Cycle 1, and 21 during Cycle 2.

Detailed lesson plans and additional classroom artifacts. A paper copy of the detailed lesson plan, as well as any additional classroom artifacts teachers were using during the implementation of the lesson was collected before the implementation day. The purpose of collecting those in advance was to have a hard copy of the lesson to follow the implementation as it occurred in the classroom. At the end of each cycle each group of

teachers was required to share the final version of the lesson plan. The final version of the lesson plan includes revisions made after their group reflection meeting. A total of seven lessons with revisions were collected. An example of a detailed lesson plan is included in Appendix J, and an example of a sparse lesson plan is included in Appendix K. These are both the original lessons designed by the groups of teachers, thus they are written in Spanish.

Classroom observation data. Classroom observation data includes a videotape of the implementation of the lesson and, when possible, field notes were taken. Implementation of the lessons was Phase 3 of the Lesson Study professional development and it took place in August for Cycle 1 and November for Cycle 2. During the implementation of the lesson, paper copies of all documents (lesson plan and additional artifacts) were used to follow the teacher as the lesson was being implemented, and to make observation notes to be later discussed in the reflection meeting. During the implementation, the video camera followed the teacher and missed student-student interactions. Since I was managing the video camera, when possible, I took field notes during the implementation of the lesson to capture important moments of student-student interaction. Field notes were also used to annotate any particular instance of teacher-student interaction that the video camera did not capture. Those instances were focused on teacher knowledge to teach mathematics with technology, and in particular how they are related to some of the issues that emerged as teachers designed the lesson. It was of particular interest as well to observe how the teacher resolved those issues. A total of eight lesson implementations were observed, one per group of teachers per cycle.

Reflection meeting data. The reflection meeting was Phase 4 of the Lesson Study professional development and occurred as soon as possible after each implementation. All members of the group, the mathematician, and I participated in the meetings. There were a total of eight reflection meetings, one per group of teachers per cycle. During each meeting, I introduced the group, mentioned the teacher who implemented the lesson, and the mathematical topic the lesson was focused on. A hard copy of the lesson and all additional artifacts were given to each teacher in the meeting, and the mathematician. I had my own copy as well. Then, we watched the video of the implementation of the lesson without pausing it at any moment. Teachers were asked to take notes as they watched the video and to reflect back on what the lesson plan was about, to follow the written plan they had in front of them and to use that hard copy to write memos of everything that came to their minds as they watched the implementation. Once we all watched the video, I asked the teacher who implemented the lesson to be the first to reflect on the implementation. Then, other teachers in the group had the opportunity to reflect on the implementation. All teachers were encouraged to suggest changes to the lesson plan as well as to reflect on the mathematics that was taught, the pedagogy that was employed, and the technology used during the lesson. Next, the mathematician offered some feedback, mostly around the mathematics being taught in the lesson. Last, I offered some additional feedback mainly focused on the uses of technology as well as pedagogical approaches. I closed the meeting by summarizing the main points of the meeting, which teachers later used to revise the lesson.

If a particular lesson needed to be revised, that group of teachers needed to plan accordingly to make changes to their lesson using their observation notes and the feedback received from the mathematician and myself during the reflection meeting. The mathematician and I did not participate in these revision meetings. However, teachers were required to share the final version of the written lesson plan with me before starting a new cycle. A total of seven revised lesson plans were collected, four during Cycle 1 and three during Cycle 2.

Data Analysis Methods. In accordance with a convergent parallel mixed method design quantitative and qualitative data are kept separately and mixed during the overall interpretation. Thus, analysis of quantitative and qualitative data was conducted separately until overall interpretations were made. Figure 11 shows how the data analysis occurred. The analysis of the quantitative data was completed first. Then, the analysis of the qualitative data was conducted. Lastly, overall interpretations of the data were made using a combination of both the quantitative and qualitative data results.

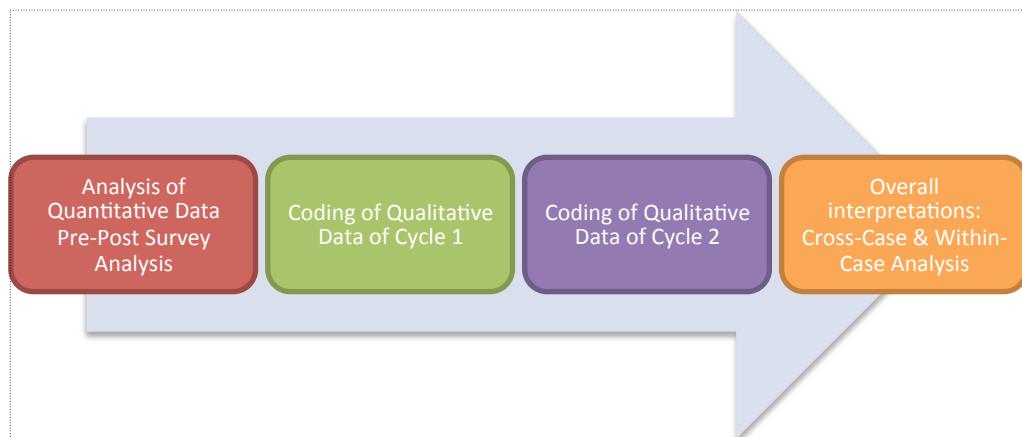


Figure 11. Process of Data Analysis

Analysis of Quantitative Data. Data from the *Encuesta para Maestros de Matematica de Escuela Secundaria: Conocimiento Tecnologico, Pedagogico, Matematico* (EMMES-CTPM) paper survey was first recorded and organized into an Excel spreadsheet, and saved as a csv file. The spreadsheet only included data from the teachers that completed both, the pre and post survey ($n = 14$). Then, the data was exported to the statistical software package *RStudio* (RStudio, 2012). First, a descriptive analysis of data was conducted. The descriptive analysis included a comparison of the mean scores per construct for the pre and post survey. Individual scores per construct resulted from adding teachers' scores in all items within each construct. A visual representation of pre and post mean scores per construct was used to determine whether numerical differences existed between samples, within each construct. The data was further analyzed to determine if mean score differences were statistically significant using a repeated measures (paired) t-test. A paired t-test was appropriate since repeated measures were collected from the same individuals before and after the intervention.

The normality assumption of the paired t-test was reasonable because each construct is the sum of multiple items in the survey. However, the data was tested for normality using the Shapiro Test function in *RStudio*. All constructs passed the normality test.

Analysis of Qualitative Data. In this analysis, each group of teachers represents a case. The qualitative data analyzed included audio recording of the planning sessions, video recording of the implementation of the lesson (n=8), notes from the reflection meetings (n=8), journal entries (n=34), and a final reflection some teachers voluntarily wrote (n= 10) at the end of the professional development. The qualitative data was analyzed following the chronological order in which event in the professional development occurred. For the analysis I used the software *Transana* (Woods & Fassnacht, 2015).

For the analysis, I started with data from Cycle 1, and following the same order in which data was collected, completed the analysis for all four groups and then moved to Cycle 2. I first listened to the audio recordings of the planning sessions. Using a list of theory-driven codes, included in the conceptual framework in the Chapter 2 (see Figure 8), I added codes to the data. Second, I watched the video of the implementation and coded it using the same theory-driven codes. As I coded the planning and the implementation, I searched for other important instances where issues with the mathematical content, pedagogy or technology emerged. If some of the issues on the planning influenced how the lesson was implemented, it was also noted. Lastly, I used the field notes I took during the reflection meeting and searched for evidence of our conversation around the issues that emerged during planning and/or implementation. Since it was not necessary to identify instances by creating

a new set of codes, only theory-driven codes were used (see Appendix I). The journal entries were used to support or contrast claims of particular issues found during the planning, or implementation of the lessons.

After the coding was completed for the planning of each group, a Keyword Map was created. The Keyword Map allows observing the codes as they were applied, chronologically (refer to the Figures in Chapter 5 at the beginning of each planning session). The timing that appears at the top of each Keyword Map is in minutes. Each colored rectangle represents a code that was applied for the amount of time represented by the rectangle's width. The total of minutes on the Keyword Map do not represent the total time of the planning session. As teachers planned the lesson, they had conversations around many other topics not related to the planning of the lesson. In occasions, they had long periods of silence as teachers within each group worked individually on tasks related to the planning of the lesson. For Cycle 1, all planning sessions were loaded into a single *Transana* database. For Cycle 2, each group was loaded into an individual database to avoid the issue with the timing in the Keyword Map. When applying the codes during Cycle 1, even when the analysis was done individually per group, *Transana* created the Keyword Map using consecutive times: Group 1 (0:00:00-0:16:20); Group 2 (0:16:20-0:30:00); Group 3 (0:30:00-0:53:30); and Group 4 (0:53:30-1:38:34).

Within-case analysis. For a within case analysis, the issues that emerged during Cycle 1 were compared to those that emerged during Cycle 2 within each group of teachers. For each particular group of teachers, pre and post mean scores per construct on the

EMMES-CTPC for individual teachers were examined. Changes in a particular construct were further examined and explained using the analysis of the qualitative data. In this sense, the qualitative and the quantitative results were used to complement each other.

Cross-case analysis. As a last step in the data analysis, in order to respond to my research questions, a cross-case analysis was done. In the context of this study, each group of teachers represented a case, thus there are four cases. For a cross-case analysis, the issues that emerged were compared and contrasted across the groups of teachers using the codes that were applied to the data: knowledge of math and curriculum, knowledge of math and students, knowledge of pedagogy, knowledge of math and teaching, knowledge of technology, and knowledge of mathematics. Data was analyzed to compare and contrast how these groups of teachers used these types of knowledge as they planned and implemented the lessons during both cycles. A discussion of these similarities and differences is in Chapter 6 as I summarize findings for the second research question this study aimed answering.

Internal Validity, Reliability, and External Reliability

Reporting the validity and reliability of a qualitative research study is crucial in order to inform readers how the data collected was used to write results that are credible, trustworthiness, and logical. What follows is a description of the strategies used in this study to support the strategies used to establish the validity and reliability of the study.

Internal Validity. The term validity, according to Creswell and Miller (2000), refers to how accurately the results and interpretations represent participant's realities and how credible it is to them. In order to establish internal validity this study employs triangulation

and a researcher discloses assumptions, beliefs, and biases through a subjectivity statement.

Triangulation is a process where researchers search for convergence using multiple sources of data to form themes or categories in the study. Researchers disclosing of assumptions, beliefs, and biases is the process where researchers report on their personal beliefs, values, and biases to allow readers to understand their positions.

Reliability. Reliability, according to Merriam (2002) refers to how logical the results are based on the data collected. Two strategies to establish reliability are triangulation, and researcher disclosure of assumptions, beliefs, and biases. Again, both strategies were employed in this study.

External Validity. To establish external validity this study employed the use of rich and thick description. These rich descriptions will allow readers to decide how applicable the findings of this study might be to other settings or similar context they are familiar with (Creswell & Miller, 2000). This study has provided a detailed description of the context of the study and will provide rich and thick descriptions of the outcomes as well.

Subjectivity Statement

I have been a student for most of my life. I always excelled in mathematics classes, but there was a point at which those courses became very boring. As an undergraduate in Mathematics Education I wanted to make a difference and started implementing innovative ways of teaching even during my student teaching year. After graduation I worked as a mathematics instructor for about 2 and a half years. Being a college instructor of mathematics in a small university in the center of Puerto Rico gave some opportunities to do

so, but with limited resources I needed to learn more about ways of teaching mathematics that were encouraging even for college students. That desire to become better and help others be better as well inspired me to pursue a doctoral degree in mathematics education. My experiences as a student in mathematics classrooms will influence what I pay attention to during the lesson planning sessions.

During my years as a graduate student I learned about how technology can be used to teach mathematics, and learned about a variety of technology tools that are available to that end. I worked with a grant project focused on developing materials to help teachers develop the knowledge they need in order to teach mathematics with technology integrating the mathematical content, the pedagogical knowledge and the technology. My experiences through the development of those materials, including technology files to be used in the classroom, students' worksheets for recording their work, as well as the focus on the integration of technology, pedagogy, and the mathematics will influence the ways I served as a resource to the participant teachers of this study while planning their lessons.

Ethical Issues

The University of Puerto Rico Mayaguez Institutional Review Board (UPRM-IRB) as well as the North Carolina State University Institutional Review Board granted the necessary consents to conduct this study. Each participant was asked to sign a legal Consent Form before participating in the study. This Consent Form was written in Spanish since participants were all Spanish speakers (Appendix A). Permissions from teachers to go into their classrooms and video record the lesson were included in the consent form. This study

was conducted under the context of the *AFAMaC-Mathematics* project, thus permissions to go into the schools, visit teachers classrooms, and to video record them while teaching was granted though the project's proposal. While videotaping, the camera followed the teacher at all times. In the case it was requested by the teachers, a personalized Film Permit was sent to the principal of the school for their records. There was no need to ask the parents of the students for permissions since many schools have parents sign a general document that included permission to film at the beginning of the school year.

There was minimal risk anticipated for participation in the study. All participants were given a Teacher ID and all documents collected from them were immediately blinded. When verbatim expressions from the audio recordings are used in the analysis, the Teacher ID is always used to identify the teachers. Written documents were scanned. All digital data were stored in my personal computer, which is password protected. Backs up files were kept in my work computer, which is also password protected, and in an external hard drive that was securely stored. Hard copies of written documents were locked in a cabinet in my office.

Summary of Chapter 3

This chapter offers extensive details of the methodology of the study. I have presented the research questions that will guide the study, followed by a description of the context in which this case study was situated. The research design was discussed in detail, which includes the conceptual framework, and participant selection. Data collection methods and analysis were also specified. In addition, I have considered validity, reliability, and ethical issues.

Results from the quantitative analysis will be presented first (Chapter 4), followed by qualitative rich descriptions of each case (Chapter 5). Findings in the qualitative strand are organized by the order the activities occurred: planning, implementation, and reflection for Cycle 1, and planning, implementation, and reflection for Cycle2 for each group of teachers.

Chapter 4

Survey Validation and Pre-Post Analysis

This multi-case study used a convergent parallel design, as described in Chapter 3. This design comprises quantitative and qualitative data. The quantitative component consisted of analyzing data from 14 teachers pre- and post-survey.

Secondary Mathematics Teachers TPACK survey

In order to investigate changes in teachers TPACK self-perceptions, teachers responded to a survey before and after engaging in the professional development activities of planning two lessons that integrate technology using a modified Lesson Study approach. The survey needed to be translated to Spanish and validated before being used. This section includes results from the validation study first followed by a discussion of the results obtained from a paired t-test statistical analysis of the pre and post survey.

Survey Validation

The Instrument. The survey used in this study was translated into Spanish and adapted from a survey developed by Zelkowski, Gleason, Cox, and Bismarck (2013). The survey was intended to measure pre-service teachers' self-perceptions of their knowledge to teach mathematics with technology. Through a validation process (details in Zelkowski et al., 2013), it was found that the survey successfully identified four constructs from the TPACK framework (CK, TK, PK, and TPACK). The original survey was developed for, and validated with secondary mathematics pre-service teachers.

The participants in the current study were all Spanish speakers, Spanish being their mother tongue. For that reason there was a need for the original survey to be translated into Spanish. In addition, the participants in the current study were in-service teachers whereas Zelkowski et al. (2013) used pre-service teachers as participants. These two main differences took away the validation of the original TPACK survey and created a need to go through an additional validation process for the new survey in Spanish. Given the researcher's fluency in both English and Spanish, she translated the survey. Before the survey was administered to the in-service teachers, the researcher asked a specialist in linguistics to verify that the translation from English to Spanish was accurate.

Participants and Context. The survey was administered to forty-nine secondary mathematics in-service teachers at the University of Puerto Rico Mayagüez in May 2014. A secondary school in Puerto Rico goes from seventh to twelfth grade. Thirty-six of these teachers (73%) participated in a yearlong professional development that focused, primarily, on mathematical content. Through the 2013-14 academic year they attended workshops on different mathematical content including algebra, geometry, and statistics and probability. Thirteen of the teachers (27%) participated as a control group in the project. As a control group they did not participate in any of the professional development activities the project provided during the year. The survey was administered during the closure activities of the project for that particular academic year.

The group of 49 teachers included 30 females (61%) and 18 males (18%), one person did not indicate gender. Thirteen teachers (27%) identified themselves as middle school

teachers (7th to 9th grade), thirty-three (67%) as high school teachers (10th to 12th grade), and three (6%) as secondary teachers (7th to 12th grade). Regarding their years of experience, 45% had less than 10 years of experience teaching, 27% between 10 and 20 years of experience, and 28% more than 20 years of experience. The teacher with the least experience has been in the classroom for four years while the teacher with the most experience has been in the classroom for 29 years. From the total of 49 teachers that participated in the study, 76% have obtained a master's degree and 24% a bachelor's degree.

Data Analysis. The purpose of the analysis was to test the internal reliability and validity of the *Encuesta para Maestros de Matematica de Escuela Secundaria: Conocimiento Tecnologico, Pedagogico, Matematico* (hereafter EMMES-CTPM). In order to verify that the items in the EMMES-CTPM survey followed the a priori structure of having four constructs (TK, CK, PK, and TPACK), or any other structure, an exploratory factor analysis (EFA) was conducted. The open-source, statistical package *RStudio* was used to analyze the data (RStudio, 2012). Following the recommendation offered by Costello and Osborne (2005), factor analysis with principal axial factoring (PAF) as the extraction method was used. To help in the decision about the number of factors to retain during the EFA, information from a “scree test” as well as the *a priori* structure was used. A “scree test” is a graphical representation of the eigenvalues. The number of factors to retain is suggested by the data points above the line $y = 1$. Through the EFA, factors with eigenvalues greater than one, and items loading greater than 0.32, were selected. In addition, items with low loadings or cross loadings were removed. A cross-loading item has a loading of 0.32 or higher in two or more

factors (Costello & Osborne, 2005). Several goodness-of-fit indices were used in order to test how well the proposed model fitted the data.

Findings of exploratory factor analysis. Deciding the number of factors to include in the analysis was not trivial. Although *a priori* information from Zelkowski et al. (2013) suggested there might be four factors, the “scree test” suggested there might be three factors (Figure 12). Following the suggestions of Costello and Osborne (2005) the data was run with three and four factors. Because of the small sample size in this study ($N=49$), Costello and Osborne suggested three conditions must be met in order to achieve “strong data” in factor analysis. The first was to use a rule of thumb of 0.32 when it comes to the loading of an item. Second was to have factors with five or more strongly loading items (items loading more than 0.50). Third was to have low to moderate item communalities (0.40 to 0.70).

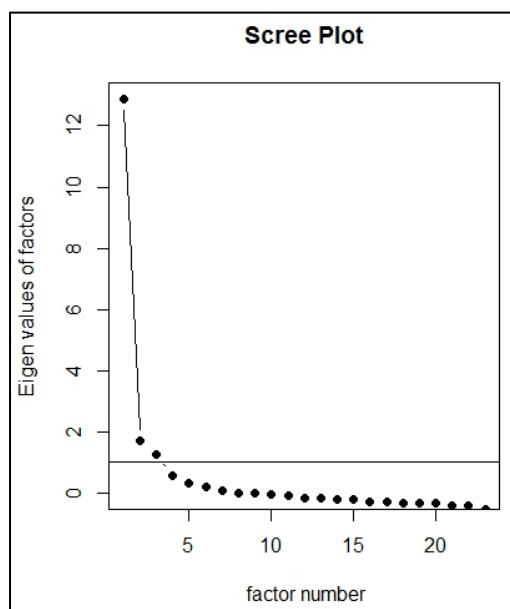


Figure 12. Scree Test to decide number of factors to retain

Table 3. Factor Loadings when four factors are extracted

Item	Factor 1	Factor 2	Factor 3	Factor 4
TK1			.40	
TK2			.75	
TK3	.33		.55	
TK4		.41		
TK5		.40	.56	
TK6			.88	
CK7		.98		
CK8		.87		
CK9		.50		.41
CK10		.83		
CK11		.52		
PK12	.74			
PK13	.55			.34
PK14	.43	.32		.36
PK15	.51			.35
PK16	.65			
TPACK17	.82			
TPACK18	.72			
TPACK19	.99			
TPACK20	.97			
TPACK21	.81			
TPACK22	.90			

Items with loadings ≥ 0.32 , boldface indicates cross-loading items.

EFA with Oblimin rotation and PAF extraction method

Using an Oblimin rotation and a PAF extraction method, four factors were extracted (see Table 3). Although four items loaded onto Factor 4, they were all cross-loading items and needed to be removed from the data (CK9, PK13, PK14, and PK15). Items TK3 and TK5 were cross-loading items and needed to be removed as well. These items were considered

cross-loadings given that each loaded onto more than one factor with an item loading greater than 0.32.

In order to decide if cross-loading items should be removed the researcher took into consideration two things: (1) items loading onto Factor 4 were all cross-loading items, and there was not a particular structure within that factor, and (2) after removing those items and running the EFA, the remaining three factors were considered weak and unstable since they had less than five items each and the conditions to achieve “strong data” in factor analysis were not met (Costello & Osborne, 2005).

When three factors were extracted, using Oblimin rotation and PAF, the loadings of the items were clearer (see Table 4). Again, 0.32 was used as a rule of thumb for minimum loading of an item. As observed in Table 2, items TK5, PK13, PK14, and TPACK18 were cross-loading items. These four items were: “*I know about a lot of different technologies*” (TK5), “*I can adapt my teaching based upon what students currently understand or do not understand*” (PK13), “*I can adapt my teaching style to different learners*” (PK14), and “*I can choose technologies that enhance the mathematics for a lesson*” (TPCK18). If these items were removed, this three-factor model would meet the condition of each factor having five or more strongly loading items (.50 or better). Once the items TK5, PK13, PK14, and TPACK18 were removed an EFA was completed extracting 3 factors using an Oblimin rotation and the PAF extraction method. This three-factor model matched the data in a better way, explaining 70% of the cumulative variances, with PK and TPACK loading onto Factor 1, TK loading onto Factor 2 and CK loading onto Factor 3 (see Table 5).

Table 4. Factor Loadings when three factors are extracted

Item	Factor 1	Factor 2	Factor 3
TK1			.69
TK2			.64
TK3			.71
TK4			.61
TK5		.41	.54
TK6			.70
CK7		.86	
CK8		.89	
CK9		.57	
CK10		.83	
CK11		.52	
PK12	.65		
PK13	.66	.33	
PK14	.55	.39	
PK15	.61		
PK16	.70		
TPACK17	.83		
TPACK18	.65	.42	
TPACK19	.97		
TPACK20	.94		
TPACK21	.78		
TPACK22	.88		

*Items with loadings ≥ 0.32 , boldface indicates cross-loading items.
EFA with Oblimin rotation and PAF extraction method*

Table 5. Resulting Factors

Item	Factor 1	Factor 2	Factor 3
TK1			.69
TK2			.64
TK3			.71
TK4			.61
TK6			.70
CK7		.86	
CK8		.89	
CK9		.57	
CK10		.83	
CK11		.52	
PK12	.65		
PK15	.61		
PK16	.70		
TPACK17	.83		
TPACK19	.97		
TPACK20	.94		
TPACK21	.78		
TPACK22	.88		
	Factor 1	Factor 2	Factor 3
SS loadings	5.93	3.75	2.85
Proportion Variance	0.33	0.21	0.16
Cumulative Variance	0.33	0.54	0.70
Factor Correlation			
Factor 1	1.00	0.60	0.52
Factor 2	0.60	1.00	0.36
Factor 3	0.52	0.36	1.00

According to Costello and Osborne (2005), item communality, which is “essentially correlation coefficients” (Costello & Osborne, 2005, p. 4), was also a good measure to decide

whether an item loads into an appropriate factor. In the social sciences, item communalities are considered acceptable if they are between .40 and .70. Item communalities for items in the proposed 3-factors model were all between the suggested boundaries (see Table 6). Moreover, eight of the items had communalities greater than or equal to 0.80, which indicated a high communality, thus they were highly correlated with the other items within the factor.

Table 6. Item Communalities

TK1	TK2	TK3	TK4	TK6	CK7	CK8	CK9	CK10	CK11	PK12
0.59	0.65	0.80	0.52	0.48	0.84	0.89	0.41	0.85	0.52	0.68
PK15	PK16		TPACK17	TPACK19	TPACK20	TPACK21	TPACK22			
0.54	0.52		0.72		0.88		0.92		0.83	0.90

Goodness-of-fit indices. In order to test the goodness-of-fit of the model obtained through EFA, a Confirmatory Factor Analysis (CFA) is usually used. However, in the case of this study, a CFA could not be used given that these procedures are for large data sets. Thus, in order to test the goodness of fit of this three-factor model, I considered several indices of goodness of fit suggested by the literature (Cole, 1987). These indices include the goodness-of-fit index (GFI), the adjusted GFI, and the root-mean-square error of the residuals (RMS) index. According to Cole for a model to be considered acceptable, the GFI index should be greater than 0.90, the adjusted GFI index indicates a good fit if greater than .8, and an RMS index less than .10 is sufficient, while a residual greater than .10 calls into question the quality of the model. For the proposed three-factor model the GFI index was 0.972, the

adjusted GFI index was 0.991 and the RMS index was .052. This indicates the proposed three-factor model seems to fit the EMMES-CTPM survey data in an acceptable way.

Internal validity. Another important aspect when validating an instrument is the internal validity of the instrument. Internal validity means “scores received from participants are meaningful indicators of the construct being measured” (Creswell & Plano-Clark, 2011, p. 210). In order to test the internal validity of the EMMES-CTPM survey, Cronbach’s alpha per construct was examined. For this part of the analysis the removed items TK5, PK13, PK14, and TPACK18 were not considered. Cronbach’s alphas for the TK, CK, and TPACK constructs were .87, .91, and .95 respectively. A Cronbach’s alpha of .70 or greater is considered to be acceptable.

In summary, through EFA and by using Cronbach’s alphas I have verified the internal structure and the internal validity of a Spanish version of the TPACK survey. The validated survey consists of three strong factors, meaning they each consist of 5 or more strongly loading items. The validated survey, which will now be called EMMES-CTPM, consisted of a total of 18 items, five within the TK construct, five within the CK construct and eight within the TPACK construct (Appendix D). In addition, item communalities were within the recommended boundaries to be considered acceptable, meaning items loading onto a factor were highly correlated. Moreover, Cronbach’s alphas were also considered to be acceptable being greater than .70. All these results indicate this EMMES-CTPM survey is a valid and reliable survey that can be used with Spanish speakers’ secondary mathematics in-service

teachers. Thus, the validated survey could be used to measure Spanish-speaking teachers' self-perceptions of their knowledge on these three constructs: TK, CK, and TPACK.

Pre and Post Survey Analysis

The EMMES-CTPM was used to investigate changes in teachers' self-perceptions of their technological knowledge (TK), mathematical knowledge (CK), and technological, pedagogical, and mathematical knowledge (TPACK). The survey was administered at the beginning and at the end of the professional development, which took place during the 2014-2015 academic year. Although 18 secondary teachers participated in the study, the analysis of the quantitative data excludes data from four teachers. Three of these teachers completed the pre-survey but not the post-survey and a fourth teacher did not complete either the pre or the post survey since she was absent the days the data from the survey was collected. Thus, data from fourteen teachers ($n=14$) was used in this part of the analysis.

The survey was administered face-to-face using paper and pencil. Once the survey had been administered, the data was entered into a spreadsheet, and surveys were scanned and securely stored. There was a missing value from one teacher in the CK sub-scale. In that case, the missing value was replaced by the mode of the items in the CK sub-scale of that particular teacher (Downey & King, 1998; Hawthorne & Elliott, 2005). Then, the database was loaded into *RStudio* (2012) for analysis. Using the sum of the scores within each construct per teacher, new variables were created for each construct: TK, CK, and TPACK. To obtain a descriptive comparison of the total scores of the pre- and the post-survey within

each construct, box plots were created. As observed in Figure 13, the total scores within all three constructs tended to increase in the post-survey.

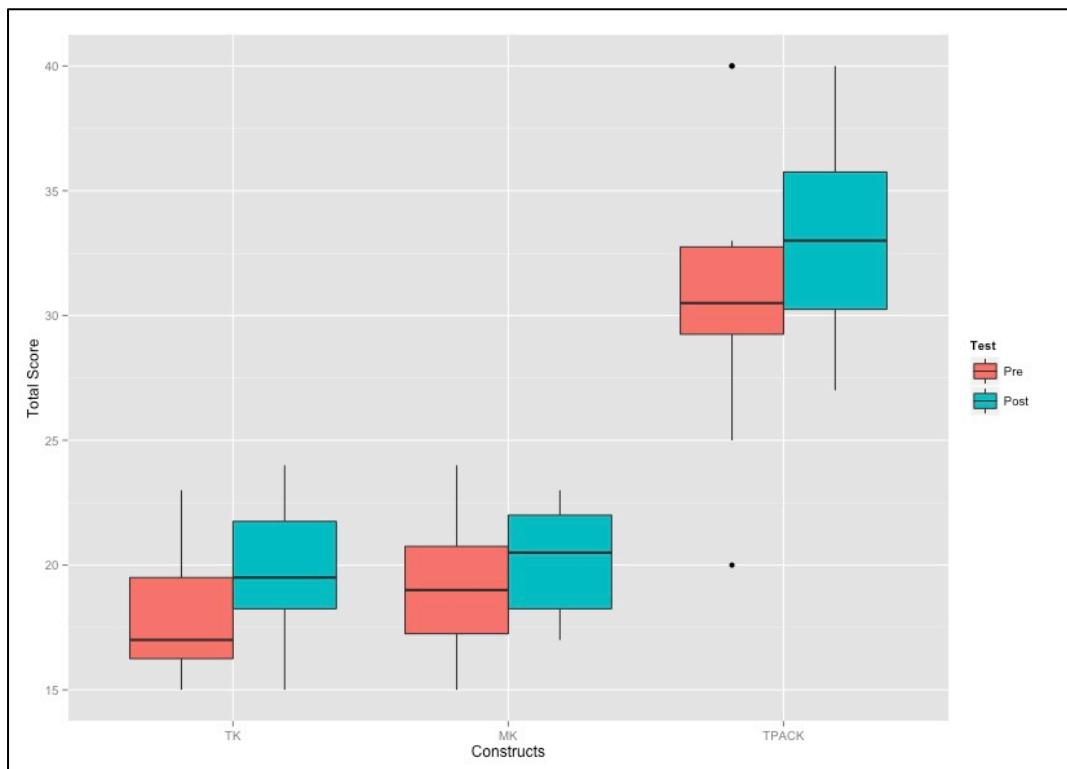


Figure 13. Descriptive Analysis of Test Scores

In order to test if there were significant differences, a paired t-test was performed for each one of the three constructs. The test is a paired t-test since the total scores from the pre- and total scores from the post- are dependent; they were collected from the same individual across time. First, it was necessary to assess if the score differences (individual total post minus total pre) were normally distributed. For that, a Shapiro test for normality was used. The Shapiro test is a non-parametric test restricted to sample size of less than 50. The values of the test statistic (W) lie between zero and one, with values close to one indicating the

normality assumption of the data is reasonable (Razali & Wah, 2011). The null-hypothesis for the Shapiro test is that the data is normally distributed. As shown in Table 7 all p-values were larger than $\alpha=0.05$, thus, the null hypothesis cannot be rejected. This means that the differences between scores of the pre-survey and post-survey for each construct can be assumed to follow a normal distribution.

Table 7. Shapiro Test Results

Construct	Test Statistic (W)	p-value
TK	0.9461	0.5021
MK	0.908	0.1476
TPACK	0.9423	0.4482

A paired t-test was then used to test for mean differences between total pre and total post survey for each construct. The alternate hypothesis of the t-test was that the mean score for the post-survey was greater than the mean score for the pre-survey. As shown in Table 8, there were significant differences for TK and TPACK at a 5% significance level, as well as for MK at a 10% significance level.

Table 8. Paired t-test Results

Construct	Mean Difference	Test Statistics (t)	p-value
TK	1.8571	3.0907	0.0043*
MK	1.1429	1.7489	0.0519*
TPACK	2.5	1.9233	0.0383*

*Statistically significant at a 10% level

These results show that teacher's self-perception of their technological knowledge (TK), mathematical knowledge (MK), and their technological, pedagogical, and mathematical knowledge (TPACK) significantly changed after engaging in the activities of

the professional development with an increment of 1.86, 1.14, and 2.5 in the mean total score for each construct, respectively. This increment in their TK self-perception is absolutely positive. After engaging in the professional development teachers perceived themselves as more knowledgeable in the technological construct. One reason for this increment might be that many of the technology tools the teachers were interested in using for their lessons were new for most of them (e.g., GeoGebra, Fathom). This means teachers had to acquire some knowledge on how to use the technology itself before using it in their classrooms. It is reasonable that after discussing with colleagues, and having some time to practice with the technology they feel they have gained some knowledge about how to use some of the technology tools.

Teacher's self-perception of their TPACK also increased significantly. With a mean difference of 2.5, teachers in the study perceived themselves as more knowledgeable in applying different teaching strategies combined with uses of technology in order to optimize their teaching in different areas of mathematics. Through the professional development, teachers had to work together to thoughtfully design a lesson, incorporating the use of a technology tool. Teachers were asked to take different pedagogical approaches to the ones they usually used, challenging them to try new things in their classroom. Using new technology tools and new teaching strategies might allow them to also think about the mathematical content in a different way. Perhaps, since the emphasis of the professional development was the integration of pedagogy, technology and the content as they planned a detailed lesson, it is reasonable to think they may have gained some knowledge in how to

integrate these three aspects of teaching. This approach was new to most of the participant teachers, which truly focused them on the learning of the technology tools and the mathematics they were teaching in each particular lesson and how to incorporate them in ways that were meaningful to their students.

Teacher's self-perception of their mathematical knowledge (MK) also increased significantly, with a mean difference of 1.1429. This change in mean scores for the Mathematical Knowledge construct means teachers self-perception of their content knowledge increased after being engaged in the professional development. This increment could mean they feel more knowledgeable about the mathematics they teach after they engaged in the activities of the professional development. These activities included a series of math-focused workshops during the Saturday Academies as well as detailed design of mathematical lessons.

Chapter 5

Multiple-Case Analysis

To better understand how teacher's technological, pedagogical, and mathematical knowledge was used in their lesson planning and implementation, all qualitative sources of data were analyzed. In this multi-case study, each group of teachers represents a case. First, I carefully listened to the audio file of the planning sessions per group for Cycle 1 and coded the data using a list of theory-driven codes, included in the conceptual framework in the Chapter 2 (see Figure 8). Second, I watched the video of the implementation and coded it using the same theory-driven codes. In addition, I searched for other important moments where issues with the mathematical content, pedagogy or technology emerged. Since it was not necessary to identify moments by creating a new set of codes, only theory-driven codes were used (see Appendix I).

This chapter is divided into four sections, one for each case: Case 1 (HS), Case 2 (MS), Case 3 (HS), and Case 4 (MS). Following the order of the activities the teachers were engaged in, each section includes a detailed description of the issues that emerged as the group planned their lesson, then as the lesson was implemented, and lastly as the group reflected on the strengths and weakness of the lesson. Each section includes the analyses for both cycles. At the end of a section, after the analysis of all data sources of a group, a within-case analysis was done.

Case 1

Case 1 consisted of a group of five high school mathematics teachers, all from public schools. This group had a teacher from the Special Education program. There were four female and one male. Their years of teaching experience range from seven to 13 years. The teachers that implemented the lessons for Cycle 1 and Cycle 2 had six and nine years of teaching experience, respectively. Teachers are from two different school districts. This group of teachers showed evidence of a well-established knowledge of general pedagogy. They also exhibited a good grasp of the mathematical knowledge they teach. Some of the teachers in this group are known in the project for being excellent teachers and have a strong mathematical background. On the other hand, this group demonstrated having issues with the knowledge of technological tools that are aligned with the teaching of mathematics.

Cycle 1. For Cycle 1 this group chose to design a lesson about *distance between two points in the Cartesian plane*. The lesson was implemented with a group of 10th graders in a regular high school. For the lesson the teacher used a laptop and a projector. As the technology tool, the group used GeoGebra.

Planning Session for Cycle 1. At the beginning of the planning session, evidence regarding this group's knowledge about the curriculum, students' understanding, abilities and misconceptions, pedagogy, technology, and content were found. As teachers were discussing and developing their lesson, teacher's discussions were mostly focused on how to teach the concept of distance between points, what pedagogical approaches to use, decisions regarding the types of question they will ask to their students, the technology tool as it relates to the

mathematical concept they were going to teach, and anticipation of students responses to the questions they were planning on posing. Figure 14 shows a chronological view of the codes that emerged during the planning session for this group.

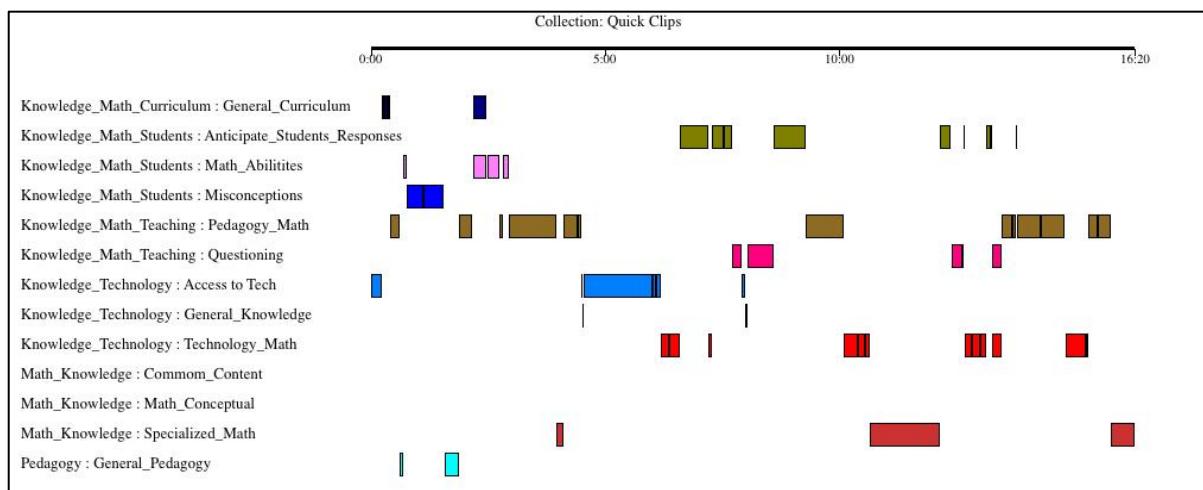


Figure 14. Chronological codes applied to Planning session Case 1, Cycle 1.

Most of the teachers (three out of five) in this group felt comfortable with the topic they were working with. Thus, it was not surprising teachers had little, to no, issues with their *knowledge of mathematical content* on the distance between two points in the Cartesian plane. This was confirmed in their journal entries when one teacher wrote “I understand that based on the selected topic I possessed enough experience, which makes me feel comfortable and secure when teaching the skill” (Teacher 2). A second teacher also wrote “According to my mathematical knowledge, both in my own learning and my teaching experience, I understand I am prepared to work with the topic we have selected” (Teacher 3).

Their *knowledge of mathematics* was also evident through their planning as no mathematical errors were identified while they planned the lesson. There were only few occasions where they used incorrect language. For example, when they were talking about point A and B, they did not refer to the coordinates of the point in the plane correctly. When writing their lesson plan, one teacher said “we substitute point A for x_1 ” (Transcript_Group1_Cycle1) referring to the x –coordinate, instead of saying to substitute point A for (x_1, y_1) as an ordered pair.

Some issues that emerged at the beginning of the planning, and that took quite some time to get solved, were related to their *knowledge of the curriculum*. As mentioned in Chapter 3, a new set of standards was just released that summer of 2014. Teachers were unfamiliar with the format of the standards and had a hard time figuring out what topic to choose for their lesson since they had no information about the order the topics were going to be distributed across the grade levels in high school. From the time they started planning, it took them around two hours to find out the topics a teacher in 10th grade must be teaching at the beginning of the school year and which standard better portrayed what they wanted to teach. The timing was important since they would be observed implementing the lesson during the first two weeks of classes in August.

Once the standard was chosen, issues about their *knowledge of technology* started to emerge. Regarding access to technology, three out of five teachers had access to a projector and a computer, but only two had smart boards in their classrooms. One teacher had a smart board and an iPad that she could connect to the smart board via wireless, but she had no

knowledge on how to use it since it was new in her school. Their access to and general knowledge of technology were two things the group took into consideration to decide who was going to implement the lesson. The teacher with access to a projector and his computer, but no access to a smart board implemented the lesson mainly because of his knowledge and comfort level on managing the technology itself.

The issues around technology as it related to mathematics, in this case the use of GeoGebra, appeared more toward the middle-end of the planning phase. As teachers made decisions of how to integrate the technology, their individual *technological knowledge* and *technological mathematical knowledge* started to emerge and to influence one another. For example, one teacher had some previous experiences using GeoGebra, whereas the teacher that was going to implement the lesson had no previous experiences with the tool (“I am weak with GeoGebra” Transcript_Group1_Cycle1). Since the beginning of planning, the teacher to implement the lesson (Teacher 2) clearly stated that he needed help in understanding how the tool was going to be used during the lesson and that he needed to learn how to use the tool before implementing the lesson. The other teachers in the group had little to no experience using GeoGebra. During the discussions, there were times where they were explaining to one another how to draw points, calculate distance, and draw segments with GeoGebra. Teachers also spent time trying to figure out how to fix points to the plane so that the horizontal and vertical distances stayed fixed. Moreover, they wanted the distances to be Pythagorean triple numbers, so that the distance of the diagonal was also a whole number.

This shows evidence of their *specialized content knowledge*. They used tools such as YouTube and Google to find out how to use GeoGebra.

As it can be observed in Figure 14, issues related to teachers' *knowledge of math and teaching* are the most prevalent in, and spread throughout, the planning sessions during Cycle 1. This type of knowledge is concerned with the interaction between the mathematical understanding and the pedagogical issues that affects student learning. During planning, teachers had a hard time organizing the lesson. At first they wanted to provide the distance formula and have students apply it with coordinates in the plane. It was difficult for them to get the idea that, using appropriate scaffolding, students could reason and deduct a formula. In several occasions they said "our students do not have the abilities to deduct a formula by themselves" (Transcript_Case1_Cycle1). However, I encouraged them to search for a different way of approaching that topic, perhaps helping the students to deduct the formula using previous knowledge. Planning a lesson using this researcher-suggested change in pedagogy was difficult for the teachers. The way they were planning on teaching the concept was new for all of them. Thus, they spent some time trying to come up with an activity where students could deduct the distance formula while the teacher guided a discussion by questioning the students.

Using a picture of a man and a lightning that strikes at a certain distance from the man, they had planned for students to use their previous knowledge of the Pythagorean theorem to deduct a formula to find the distance from the men to the top of the lightning. The idea of using this picture to introduce distance between points was brought forth by the

teacher that was going to implement the lesson. He had used the picture before when introducing this mathematical idea to his students, but this time he was going to use the picture differently. The picture was inserted in GeoGebra and with a Cartesian plane on top of it (Figure 15).

In their planning, teachers started with a horizontal line from point A to point B and a vertical line from point B to point C. Point A was the position of the men, point B was the position where the lightning stroke the ground, and point C was a second point in the lightning (Figure 15). Evidence of their *knowledge of math and students* emerged as they anticipated students were going to count the squares to measure the length of segments a and b . They also anticipated there might be students that were going to subtract to get the length of the segments. Then they drew a segment from point A to point C, which is a diagonal. Since they intended to start the lesson by measuring lengths of segments in GeoGebra, it was hard to move from there to the formula. They spent some time deciding how to make an easier transition from the triangle they had on hand to the Pythagorean theorem, and then to the distance formula. To this matter, after they formed the triangle they named the sides using the letters a , b and c to use a more familiar notation for the students. Then they added coordinates to point A and C and substitute lengths a , and b using the coordinates of those point; (x_1, y_1) and (x_2, y_2) . Then, using the Pythagorean theorem they solve for c (distance from A to C) and renamed it d .

Given that the students would not have access to GeoGebra during the lesson, the group planned on having a mini whiteboards, markers and an eraser for each student. They

expected the students to follow the discussion, and to work all problems on the whiteboard and not in their individual notebooks.

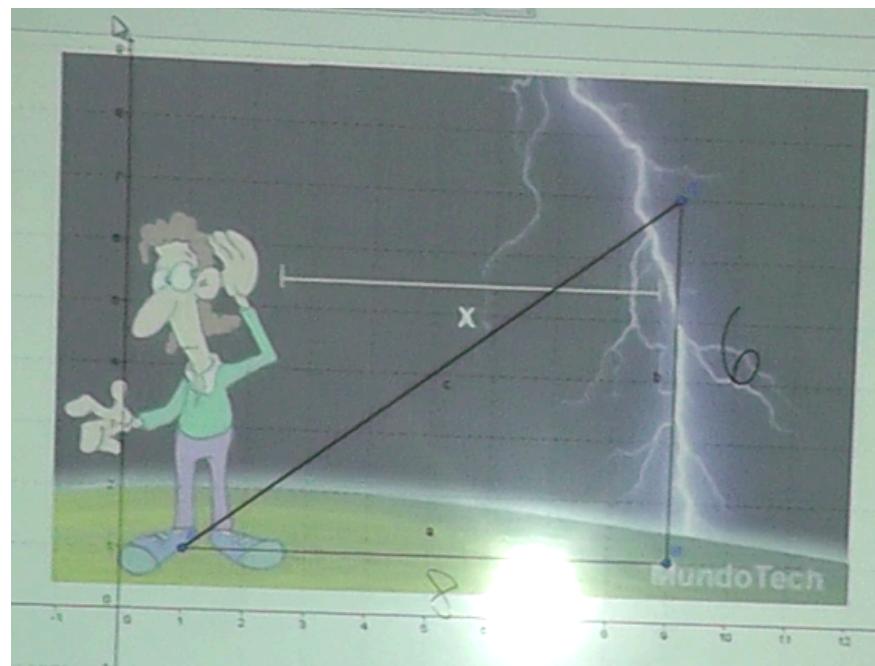


Figure 15. Image to calculate distance between two points used by Case 1

Implementation of the Lesson for Cycle 1. The lesson was implemented with a 10th grade class. The teacher started the class by summarizing what they had talked about distance the day before. Then, the teacher, using the projector, showed the picture of the man and the lightning. He spent some time talking about the fact that when lightning strikes near a water park, people are asked to get out of the water. He explained some physics behind it and gave some importance to the topic of how to find the distance between the man and the lightning. Then he put the figure on top of a Cartesian plane using GeoGebra.

In general, the teacher followed the lesson plan, but he forgot some details with regard to tool use. First, the teachers had planned for students to use mini white boards with a Cartesian plane drawn on it so that students, following the discussion in class, would draw the points and segments, and use it to calculate the distances. However, the teacher did not use these in the lesson. Second, the teachers had planned for GeoGebra to be used to measure the lengths of the segments so that the students compared their results with the ones given by the tool. The teacher did not use the tool in that manner. Thus, two tool uses meant to enhance students' engagement with the mathematics were left out of the lesson.

As mentioned before, the teacher felt comfortable with the topic he was teaching. As an evidence of that confidence, the class flowed without any *mathematical issue*. As he guided the students, asked questions, and wrote on the board no errors in the mathematics were found. He even used the correct language throughout the implementation of the lesson.

Regarding *technological issues*, even though the teacher was not familiar with GeoGebra, he used the tool with ease and there was no evidence of his inexperience with the tool during the implementation. He knew how to draw points and segments. However, he did not use any other advanced feature of the tool, like measuring. Even though the group of teachers used a mathematical action tool, it was not used as a way to enhance students understanding of the topic. They could have calculated the distance between the points with the tool to demonstrate that counting squares or subtracting was equivalent to the distance showed by the tool. In the same way, they could have used the tool to find the distance

between points A and C to demonstrate that counting was not a good strategy in the case of a diagonal.

As mentioned in the previous section, issues regarding their *knowledge of pedagogy*, as it relates directly to the mathematics, such as the teacher's *knowledge of math and teaching* were evident during planning the lesson as teachers organized the lesson. Since teachers spent time during the planning deciding how to organize the lesson in a way that was easy for the students to move from the segments, to the ordered pairs, to use the Pythagorean theorem to deduct the distance formula, it seemed to flow smooth during the implementation of the lesson. Teachers' anticipation of students' responses was accurate. Both responses (count the squares, and subtract) showed up during the discussion of distance between points that were horizontally or vertically aligned, showing teachers' *knowledge of mathematics and students*. In another instance the teacher drew the segment between A and C, and then asked how can they know the distance between those two points. This time the line was a diagonal, neither horizontal nor vertical. They anticipated that some students would respond they would count the squares, in a similar way they did with the horizontal and vertical segments. But they also anticipated some students might recognize the right triangle and remember the Pythagorean theorem. When the teacher asked, the discussion in class was as follows:

- S: you flip it; you make it horizontal or vertical
- T: I flip it
- S: in the same way it is, you count
- S: a squared plus b squared equals c squared.
- T: and who is that which you are talking about there?
- S: Pythagoras

T: the Pythagorean theorem. Okay, perfect. So, you are saying that the Pythagorean theorem works?

S: yes

T: why?

S: because it has a right angle

Reflection Meeting for Cycle 1. The reflection meeting for this group was very short.

We all watched the implementation of the lesson, which lasted about 40 minutes. All of them agreed the lesson was “great”, and that the teacher that implemented the lesson was fantastic, and offered very little to no critical discussion. The teacher implementing the lesson did notice some of the issues I mentioned before during the implementation. For example, he did notice he never used the mini white boards as they had planned for in the lesson. While watching the implementation, the teacher noticed he responded to questions very quickly without allowing student enough time to think and respond. This observation made the teacher reflect on his *pedagogy*. An observation all teachers agreed on was on the way the lesson flowed and how students actively participated during the lesson. During the planning they mentioned in several occasions how their students did not have the abilities to reason about the mathematics, or to even deduct a formula. After watching the video they were all impressed on how students followed the teacher and seemed to understand where the distance formula came from. This observation may have changed teachers’ *knowledge of math and teaching*. Reflecting on how well students did during the implementation, teachers in this group may be more enthusiastic about making changes to their pedagogical approaches when teaching mathematics.

Regarding their *knowledge of technology*, we all agreed he did a good job in illustrating the concept of the distance between two points using the image of the man and the lightning. I only suggested using the measurement feature of GeoGebra to get the length of the segments, and to also try adding some of the dynamic features that GeoGebra affords, such as dynamic texts. The teacher that implemented the lesson mentioned how he felt more comfortable using Power Point, and how he has used Power Point to teach that lesson in the past.

Cycle 2. For Cycle 2 this group of teachers designed a lesson about *Addition of Vectors*. As technology tools they used an animated Power Point, a Smart board, and GeoGebra. The lesson was implemented with a group of 11th graders in a vocational public school¹. This group remained with five members to the end of Cycle 2, although two of the teachers did not participate in the planning. The volunteer teacher who implemented the lesson for Cycle 2 was different than the volunteer teacher for Cycle 1.

Planning Session for Cycle 2. During this cycle, this group quickly decided which teacher was going to implement the lesson. There was one teacher absent during the planning session, and another teacher that arrived late. Thus, there were only three teachers at the beginning of the planning session: the teacher that implemented the lesson for Cycle 1 (Teacher 2), a teacher of the Special Education program (Teacher 5), and a teacher of a vocational school (Teacher 1). Given her interest in implementing the lesson, Teacher 1 was

¹ A vocational school is a secondary school that offers short, career-focused programs that prepare students for the workforce.

the volunteer to implement the lesson for this cycle. There was no objection from the rest of the teachers in the group.

As it can be observed in Figure 16, teachers showed evidence of their knowledge related to the math curriculum, technology, teaching, and content. For this group, issues about technology were most predominant during the planning of Cycle 2 as teachers used and compared GeoGebra and Power Point.

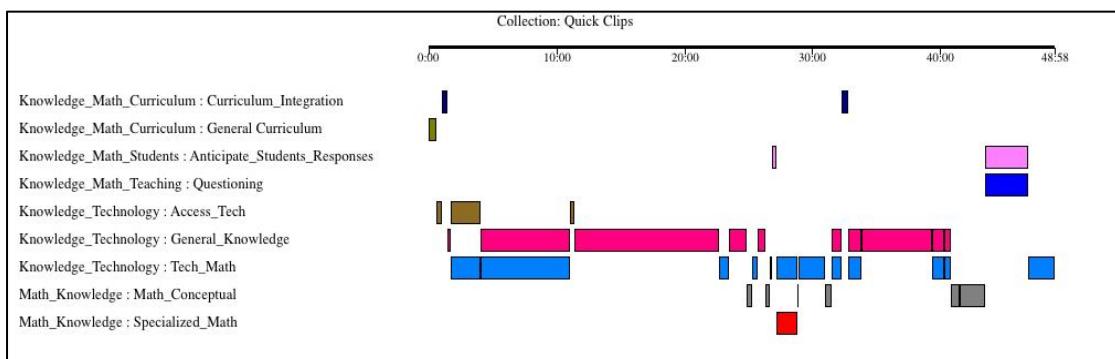


Figure 16. Chronological codes applied to Planning session Case 1, Cycle 2.

Their *knowledge of the curriculum* was evident during the planning session. When the group started planning their lesson, they had already thought about introduction of vectors as a possible topic for their lesson. This planning session occurred in the middle of the semester, so they were familiarized with the new set of standards and the curricular maps for the grade they were teaching. Knowing the topics that were left during the semester really helped them to quickly decide which mathematical topic to focus on during this cycle's planning session.

Even though they initially thought they were going to focus the lesson on introducing vectors, and talking about the components of the vector and its magnitude, they finally

decided to focused the lesson on the topic that followed that introduction; addition of vectors. However, as their journals reflected, most teachers in this group have had not experiences on the teaching of vectors. Teachers expressions such as, “[I] do not have experience teaching vectors,” (Journal entry #3, Teacher 4) and “... the selected topic is one I have never taught because the grade I teach do not require it... In terms of previous experiences I have never work with vectors” (Journal entry #3, Teacher 2) showed evidence of their lack of experience teaching the topic. Only the teacher to implement the lesson, Teacher 1, has previous experience teaching the topic as she wrote “[I] feel pretty secure on this topic, because I have been teaching it for two years” (Journal entry #3, Teacher 1). Not only Teacher 2 noted little experience with teaching vectors, it also became apparent during the planning that his knowledge of the topic was not strong. This was evident in the questions he asked during the planning session:

T2: when you talked about the components of the vector, you are talking about points?

T1: the horizontal and vertical movement

T2: and when you talked about magnitude?

T1: that's the length...

...

T2: from the tail of the vector to the head...

T1: that is a copy-paste from the curricular map

T2: it says that... head and tail?

T1: yes, in the curricular map... that is the point of origin and the point of... to the arrow

T2: okay...

From the conversation, it is evident Teacher 2 had some issues with the mathematical content, specifically the vocabulary used when teaching vectors. First, it seems he cannot identify the component and the magnitude of a vector. Second, he did not recognize the

initial point is called the head of the vector while the terminal point is called the tail of the vector. Teacher 2, is not only lacking the experience teaching vectors, but has also issues with the mathematical knowledge about that topic. On the other hand, Teacher 1 seems to be familiar with the vocabulary and knew what the components and magnitude are as she responded to his questions.

In class, the day before the lesson of focus here, the teacher planned to introduce vectors and talked about magnitude and components of a vector. For the lesson of focus here, they had planned to use GeoGebra, with a map of Puerto Rico as the background (Figure 17) and use vectors to represent the displacement of an airplane as it travels to different towns to introduce addition of vectors. Students would be given a worksheet with a map of Puerto Rico, as shown in Figure 17, that included detail instructions for the students to follow and questions students were asked to complete. In addition, each student would be given a ruler and three pencils of different colors to draw the vectors.

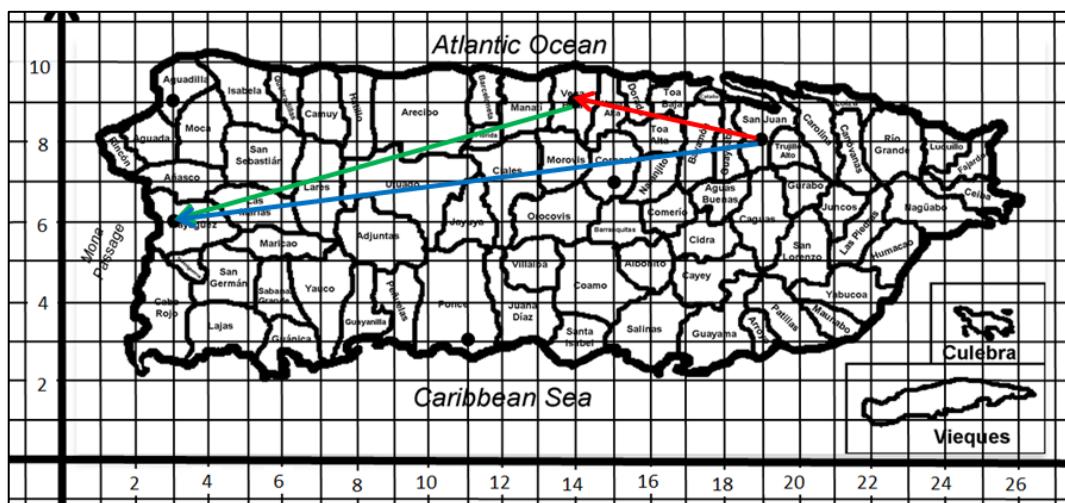


Figure 17. Map of Puerto Rico over a Cartesian plane used by Case 1 to teach vectors

They had planned to ask students to mark how they would travel from one town to another, making a stop in another town. The students were asked to draw vectors to show the displacements during a trip. For example, for the first activity students had to travel from San Juan to Mayaguez, making a stop in Vega Baja. Thus, the students had to draw a vector from San Juan to Vega Baja and a vector from Vega Baja to Mayaguez, red and green vector in Figure 17, respectively. Then, students were asked to draw a vector from San Juan (head) to Mayaguez (tail). The task also asked students to find the horizontal and vertical components of each vector, completing the table that was included in the worksheet. The new vector (blue vector in Figure 17) was the sum of the two vectors they drew first. They expected the students to notice that the horizontal and vertical components of the resulting vector were just the sum of the horizontal and vertical components of the two vectors. Students would use pencils of colors and a ruler to draw the vectors.

As it can be observed in Figure 17, the teachers had added points on the towns the students were going to be using (specifically (19,8), (14,9), (3,6), (15,7), (11,3), and (3,9) in Figure 17). They did this so that all students would be working with the same vectors. Thus, they all had the same initial and terminal point for their vectors, which meant they also had the same magnitude. This was a pedagogical decision they made in advance; anticipating students would start or end at any point within the towns, and, as a consequence, produce different vectors. They carefully located those points so that both coordinates were integers. This was done to ensure the components of the vectors were all integers so that students

could notice the relationship between the components of the vectors with the resulting vector.

All these pedagogical decisions evidence their *knowledge of math and teaching*.

During their planning there was no conversation regarding the questions they planned to ask their students and little to no conversation about anticipation of students' responses to the task. Towards the end of the planning session, Teacher 2 was reading the written lesson plan to make suggestions to Teacher 1. There was some additional evidence of *knowledge of students and teaching* as he read some questions Teacher 1 had included in the lesson plan. They did not include some of the wrong responses they might anticipate from their students for those questions. As they were planning, Teacher 2 said they only needed to include the correct answer, "because that was discussed the previous day" thus students will know the answers. In this case their anticipation of student's responses was limited to the correct answer for each question.

There were many conversations during the planning session regarding the uses of, and their *knowledge of technology*. For this lesson they had planned to use GeoGebra with the map of Puerto Rico as a background. Their intention was to add vectors for the different parts of the trip so that when they were discussing it in class they only needed to show the vectors instead of drawing them. Thus, they intended to use GeoGebra to draw the vectors, there were no intentions of using a dynamic feature of the tool. However, Teacher 2 seemed not to be convinced of using GeoGebra. He has plenty of experience using Power Point in his teaching and far less with GeoGebra. During the planning he constantly compared Power Point with GeoGebra, how he felt very comfortable playing around with the animations

options provided by Power Point and how his knowledge about GeoGebra was very limited.

At some point, as Teacher 1 thought about the goal for the lesson, and started the lesson plan, Teacher 2 worked on GeoGebra trying to figure out how to add the map as a background, and how to draw the vectors. It took him around an hour to add the map to GeoGebra. He added the points to be used to draw the vectors, and figured out how to draw a vector in GeoGebra using the input bar, realizing it was different than using the option to draw a ray. Once he had the map on the background he showed it to Teacher 1. Then he said, “all that I did [in GeoGebra] in like an hour... in Power Point I do it quickly. And I add the airplane...” Thus, he opened Power Point, and in just a few minutes he had the map, the points, and the vectors.

As the conversation continued between Teacher 1 and Teacher 2 the following emerged:

T2: I found an angry bird [activity in GeoGebra] to teach parabolas, but when I tried to create it myself it was easier with Power Point... honestly GeoGebra is very complicated... In addition, I wanted that when [the students] finished, I could push a button for the birds to move it. I did it with Power Point.

T1: GeoGebra is because the lack of knowledge. Perhaps it can [be use to] do a lot of things that [you] do not know... Is like... I do all my works in Excel, I do not use Word as much because Excel is the one I dominate and in Excel I do a lot of things that in Words I cannot”

This conversation shows evidence of how they struggled with the selection of technology because of their *knowledge of technology*. They felt they could teach mathematics with Power Point more easily than using a mathematical action tool. This perhaps they had more experience using the tool in their teaching. As they were not using the dynamic features of GeoGebra, they saw Power Point as useful for illustration and animation as GeoGebra to teach the topic they were working on.

Implementation of the Lesson for Cycle 2. This lesson was implemented in a vocational public school with an 11th grade class and lasted about 40 minutes. The teacher (Teacher 1) started by reviewing the definition of a vector, and its geometrical and algebraic representations, which she had discussed the previous class. She started by stating that all vectors have magnitude and direction. Then she showed a vector on the Cartesian plane, and asked how they could get the algebraic representation of that vector. Many students responded they needed the horizontal and vertical displacement. She used animations on the Power Point presentation to add a horizontal vector, then a vertical vector representing the components, respectively. She then wrote the algebraic representation of the vector on the board. She asked if the notation $\vec{u} = < 6,6 >$ was the same as an ordered pair, to which a student correctly responded it was not. She showed the same vector in different places around the Cartesian plane to demonstrate that the vector $< 6,6 >$ remained the same, thus it was not an ordered pair. Figure 18 shows the smart board with the Power Point diagrams, and in it the same vector in different places on the Cartesian plane. She explicitly said it was the same vector since they all have the same horizontal and vertical displacement. On the white board the algebraic representation of that vector can be seen.

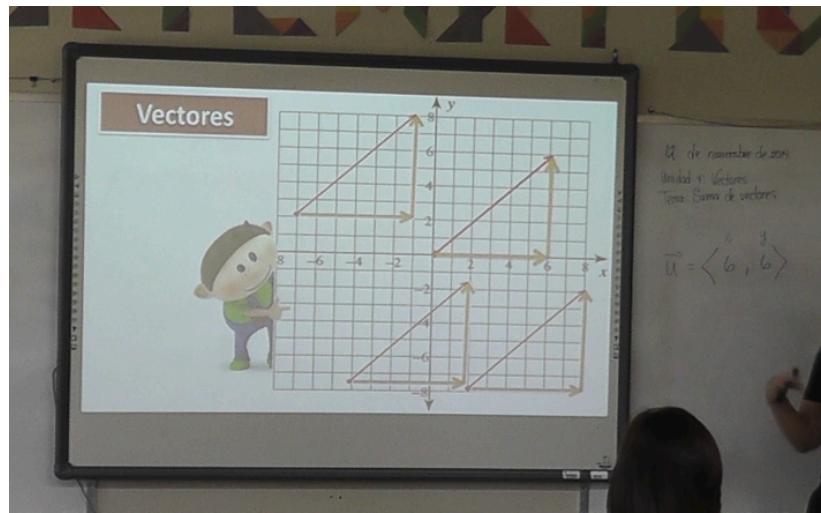


Figure 18. Teacher's review of previous class, Case 1, Cycle 2

Then she moved on to the activities the group had planned for the day. She showed the map of Puerto Rico over a Cartesian plane within the same Power Point file, and distributed worksheets, rulers, and color pencils. For the first activity, students were asked to complete a trip from San Juan to Mayaguez, with a stop in Vega Baja. She animated an airplane that moves from San Juan to Vega Baja and then to Mayaguez (Figure 19). As the airplane flew to the different destinations according to the trips the students were doing, she was asking the components of each vector. The students that participated out loud all offered the correct answers. She asked “why” questions when a component was negative. Again, students responded correctly, a negative component means the vector’s head is looking towards the west (horizontally) or to the south (vertically), respectively.

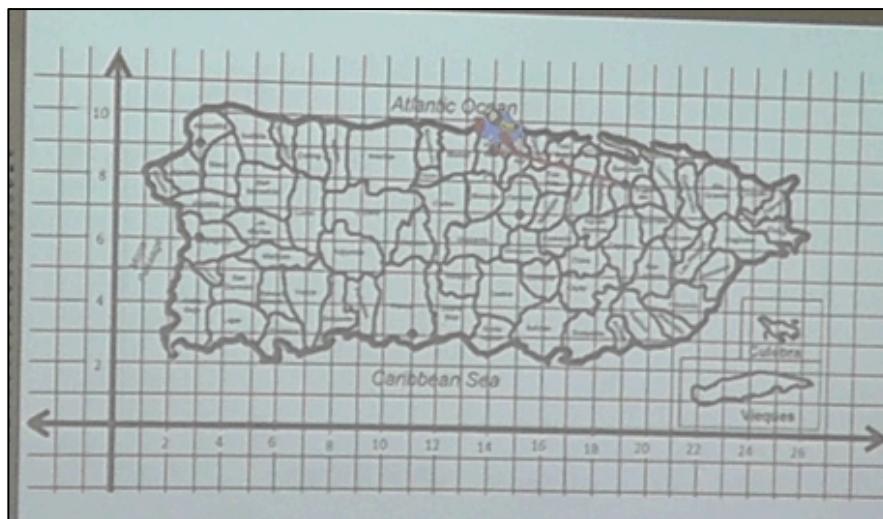


Figure 19. Animation of an airplane to show the displacement of a vector

As the students worked on the activities, drew the vectors, and found its components the teacher wrote the components on the board. When they found the components of the resulting vector (from San Juan to Mayaguez) the teacher asked “Looking at the information you have there of the components of each vector, what do you see in comparison of vector 1 and 2 with vector 3? What do you see there?” One student quickly responded, “that if we add the vectors of the first two we get the third.” The teacher then asked “we add what?” and the student responded “the x with the x and the y with the y .” Even though the student is using x and y as it were an ordered pair it is clear he noticed the relationship between the two vectors and the vector that resulted by adding the components. The teacher continued explaining the sum of the components and did nothing with the use of x and y , even though at the beginning of the class she explicitly said a vector was not an ordered pair. The students then worked on Activity 2, where they completed another trip. They traveled from Aguadilla to Ponce,

making a stop at Corozal. This discussion and interaction between the teacher and the students shows evidence of the teacher's *mathematical knowledge*, as well as her *knowledge of math and teaching*. During the implementation of the lesson she used the correct language most of the time. However, she lacked consistency with respect of using horizontal and vertical component. On occasions she let her students talk about the components as if they were part of an ordered pair, referring to x and y . That said, there were not mathematical errors during the lesson.

During the implementation there were no issues regarding her *knowledge of technology*. For example, there were no technological issues as she used the smart board. Her experience with the tool was noticeable as she managed the board without being worried of making an error. During implementation there was not used of GeoGebra and Power Point, as the group had planned. Instead, Teacher 1 only used an animated Power Point presentation during the implementation. During planning they had mentioned their comfort level about using Power Point to teach, and their lack of knowledge with GeoGebra. They felt more secure by implementing the lesson only using Power Point with the animations Teacher 2 said he felt more comfortable with. Thus, they used an image of the map of Puerto Rico over the Cartesian Plan, and using animations, the vectors were added to that picture as the teacher discussed the activity with the students, resulting in Figure 17. After completing the two trips (Activity 1 & Activity 2), the teacher ended the lesson by summarizing what they had learned during that lesson.

Reflection Meeting for Cycle 2. The reflection meeting for this cycle was very short. After watching the video with the implementation all teachers agreed the teacher did a great job with the implementation of the lesson. They offered no suggestions and made no observations for the teacher, perhaps because they were not familiarized with the content knowledge of the lesson as evident during the planning sessions.

The *knowledge of mathematics* for Teacher 1 was recognized by the mathematician, as there were no mathematical errors observed during the implementation. The use of correct vocabulary was consistent, and even though she refer to the components of the vector as similar to x and y on an ordered pair, she was also clear that those were the horizontal and vertical displacement of the vector. Thus, he only mentioned the lack of consistency with the use of vocabulary. She said she felt a little insecure during the implementation because, even though she had taught the topic before, it has not been a lot as this is just the third year she covered the topic in that grade level. In addition, a suggestion was made to add more complex examples to the lesson plan.

When it comes to their *knowledge of technology*, I had to ask about why they chose to only use Power Point if they had mentioned to use GeoGebra. They said they felt more comfortable with the tool. They wanted the airplane to “fly” from one town to the next, but they wanted that to be transparent for the students. For example, they did not want to have to move a slider to be able to move the airplane along the vectors to complete the trip. This could be easily done using animations in Power Point and the airplane will move from one town to the next by pressing a bottom on the remote control. They thought that, the animation

of the airplane and the appearance of the vectors, were enough to demonstrate to the students what they wanted. In the manner the technology tool was used, it would not make a difference to use GeoGebra if the group was not taking advantages of the dynamic features of the tool. Their issues with their *knowledge of technology*, when it comes to the use of GeoGebra, did not allow them to try that technology tool during this cycle.

Within-Case Analysis: Case 1

Overall, Case 1 showed evidence on the importance of having a good understanding of the mathematics being taught and some knowledge of the pedagogy that is specific to the teaching of mathematics before planning on integrating technology into teaching. As a group they exhibited knowledge of both content and pedagogy. They were trying to use the technology to demonstrate the concepts of distance between point and sum of vectors, but not feeling comfortable in using the technology to do mathematics actions (e.g., measuring, computing) was a struggle they carried over through both cycles.

The mathematical knowledge of distance between points, and the general pedagogical knowledge this group had during Cycle 1 facilitated the process of designing the lesson. The pedagogical knowledge specific to the teaching of mathematics was an indispensable piece within the design of the lesson, as the group took into consideration student misconceptions, mathematical abilities, and previous knowledge. This group lacked the knowledge of technology, necessary to fulfill the requirements of the planning. However, being comfortable with the mathematics, and having discussed in detail the pedagogical approach really helped this group succeed when the technology was integrated into the lesson.

Regarding uses of the technology, they could have used GeoGebra in a more dynamic way during Cycle 1. For example, they could have inserted a static text showing the distance formula. Then a dynamic text being filled up with coordinates from the endpoints of a diagonal of a right triangle. These are examples of ways the teacher *could have* done more examples by dragging the endpoints around the Cartesian plane.

During Cycle 2 this group encountered a difficult situation when only one member of the group seemed to have a strong grasp of the mathematical content, and experience teaching the topic of focus. As a group, they had encountered big issues with the mathematics, there was no evidence of the group's pedagogical knowledge, and there were still issues with their general knowledge of using technology to engage in mathematical actions, especially GeoGebra. Given that the interactions between the teachers were very limited, conversations around content knowledge, teaching experiences, ideas for uses of technology, students' common misconceptions, among many other aspects did not happen. As a consequence of the lack of communication in this group, although they had planned to use GeoGebra, at implementation only Power Point was used. As the teacher that implemented the lesson wrote, "At the end [of the planning], we decided to use GeoGebra as the technological tool, because it was the one recommended by the professor and we wrote the lesson plan with all its parts. When the time came to present the topic and my lack of experience with GeoGebra, I decided to use Power Point as technological tool" (Teacher 1, Final Reflection). The technology integration into this class went very well. This perhaps

because Teacher 1 felt very comfortable with the mathematics, the pedagogical approach, and the technology tool she decided to use.

Figure 20 shows the pre and post mean score per teacher, per construct for Case 1. Teachers' self-perceptions of their knowledge of the mathematics they teach stayed the same for the two teachers that implemented the lessons (T1 and T2). This could reflect their comfort level with the mathematics they teach, specially the mathematics they focused their lessons on for both cycles. T5 increment seems surprising given that her participation during planning sessions and reflection meetings was not frequent. As a group, their mean score for the CK construct did not change significantly.

Regarding their self-perception of their technological knowledge, while T1 and T4 self-perceptions showed some increment, T2 and T5 self-perceptions stayed the same after the professional development. During both cycles, T2 demonstrated feeling comfortable with the use of technology to teach mathematics showing his confidence on the use of conveyance tools to illustrate mathematics concepts. For T1, the increment could reflect how she perceived herself more prepared to use the animation feature of Power Point to illustrate mathematics concepts as she did in the implementation of Cycle 2. As a group, their means score does not seem to increase, but it does for the teachers that implemented one of the lessons.

With respect to their self-perception of their technological, pedagogical, and mathematical knowledge, two teachers seemed to be more self-aware of what it means to teach mathematics using technology after the professional development. This is surprising

given these two teachers were not as engaged in the conversations as the other teachers. But, for the two teachers that implemented the lessons, not showing an increment in their self-perceptions could mean they feel comfortable with how they integrate technology into their teaching of mathematics using those conveyance tools they feel comfortable with (e.g., Power Point).

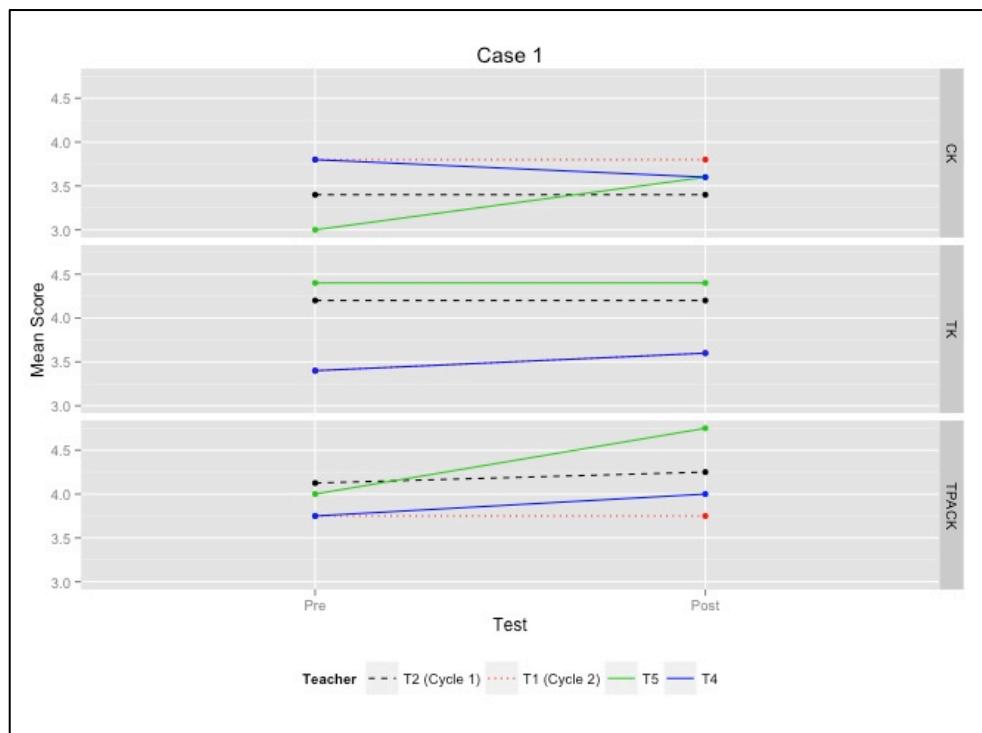


Figure 20. Pre and Post mean score per sub-scale construct per teacher for Case 1

Note: for the TK sub-scale construct, the mean scores for T1 and T4 for the pre and post survey were the same, thus their lines overlap. Dashed and dotted lines represent the teachers that implemented the lessons.

Case 2

Case 2 originally consisted of four middle school mathematics teachers, all from public schools. There were two female and two male and there were from two different school districts. Their years of teaching experience range from seven to nine years. The teacher that implemented both lessons had seven years of teaching experience. After the planning sessions of Cycle 1, one teacher got accepted into graduate school and did not return to the professional development activities. This group exhibited being unsure of some of the mathematics they teach. The group demonstrated having a good grasp of general pedagogical knowledge, but had issues with the knowledge of technology tools that are specifically designed to teach mathematics.

Cycle 1. For this cycle, they chose to design a lesson on *introduction to functions*. Planning a lesson around this topic was very challenging for the group. The volunteer teacher implemented the lesson with a group of 8th graders in a middle school that is known for the discipline problems of its students. The teacher used a smart board and students had laptops they used to work in groups of two or three. As the technology tool for the mathematics, they used GeoGebra.

Planning Session for Cycle 1. At the beginning of the planning session, issues around the curriculum, their access to technology, and their specialized mathematical knowledge about functions emerged. As illustrated in Figure 21, as the discussion and planning progressed, issues started to focus on the pedagogical approach to take, the questions they were going to ask, and anticipation of students' responses to those questions. At the end,

issues were more around their conceptual knowledge of the mathematics as two teachers engaged in a discussion about the activity they chose to introduce the lesson.

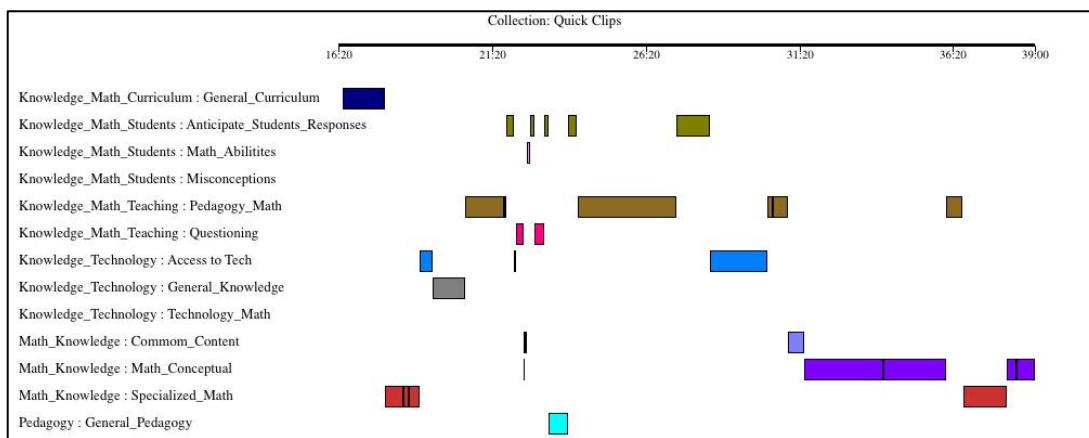


Figure 21. Chronological codes applied to Planning Session Case 2, Cycle 1

At the beginning of the planning, this group of teachers spent some time reading the standards to decide which mathematical topic to choose. Their *curriculum knowledge* was exhibited through the discussions the group had during the beginning of the planning session. Three of them had some previous contact with the recently published standards before coming into the planning session and had the curricular maps for the grade levels they teach. In addition, since standards for middle school were divided by grade level, the standards were somewhat easier to interpret for them.

Once they chose the topic, their *knowledge of technology* started to emerge. As they decided which teacher was going to implement the lesson, they also discussed the technology tools they had access to in their classrooms or schools. They all had smart boards in their classrooms, but only one had a desktop computer connected to it. The rest of the teachers had

laptops they connect to their smart boards. Two of them mentioned they had access to a laptop cart from the school and, when discussing the assessment to use at the end of the lesson, they also mentioned they had access to clickers. Taking into consideration their access to, and general knowledge about the technology, they decided Teacher 1 was going to implement the lesson. He had a desktop computer connected to a smart board in his classroom and had access to a laptop cart in his school. He volunteered to implement the lesson mainly because the rest of the teachers did not show interest in doing it.

Evidence regarding their *mathematical content knowledge* emerged throughout the planning sessions. In their journals they consistently expressed feeling confident with the content of functions. For example, Teacher 2 wrote that “[he] feel comfortable with the topic” (Journal entry #1), and Teacher 1 expressed that “[he] feel comfortable with the mathematical content, [he has] discussed the topic with an 8th grade in the past” (Journal entry #1). However, while planning the lesson the following conversation emerged as they typed the definition of function from the Power Point presentation to the lesson plan:

T3: a set X called domain and a set Y called codomain. Are you going to leave it like that?
Domain, codomain...
T4: don't write codomain
T1: it sounds “ugly”
T4: write range instead
T3: range, exactly, its more common
T4: every time it says codomain, change it to range so that it looks better.

But codomain and range are not the same, and none of the teachers seemed to be aware of the difference. Codomain is the set of all possible outcomes of a relation, whereas range is the set of the actual outcomes of a relation. Searching the 2014 PR Core Standards the word

codomain appeared once, as a synonym of range. Thus, teacher's misuse of the word codomain might be from the wording in the standard itself and not solely their misunderstanding of the mathematical concept.

Issues related to their *knowledge of math and teaching* were prevalent throughout the planning. This group of teachers had a very hard time organizing the lesson and finding the activities to use during the lesson. This group of teachers first intended to introduce the lesson by writing the definition of functions on the board, discuss some examples, and then have students work on some problems. As I encouraged them to try a different teaching approach, they decided to start with an activity about dependent and independent variables, followed by a discussion about what a relation means, and then an introduction to the concept of function.

To introduce the lesson they used a pre-constructed GeoGebra activity about dependent and independent variables. I suggested the teachers to consider this activity, after they had spent a lot of time searching for activities to introduce the lesson. The activity is called "Who's related to whom?" In the activity, as students drag points, they are asked to list the independent variables with the variables that depend on them (see Figure 22).

Who's related to whom? Drag the points to figure it out.

List the independent variables. For each independent variable, name the dependent variable(s) that depend on it, and describe how they are related.

Adapted from Task Created by Scott Steketee and Daniel Scher:
Steketee, S., & Scher, D. (2011). A geometric path to the concept of function.
Mathematics Teaching in the Middle School, 17(1), 48-55.

McCallum, A., Lewis, H. S., & Mcdonald, K. F. (in progress). *Learning to Teach Mathematics with Technology: An Integrated Approach to Algebra*.

Figure 22. Introductory activity used by Case 2

In this activity, it is important to recognize that each point represents an independent (domain) or dependent (range) variable of a relation. This is a different, and new representation of a relation for many teachers. In order to understand what is happening with the points, a strong mathematical knowledge about relations and functions is needed. In the activity, if the point represents a dependent variable then it is not going to move when dragged. Only points that represent independent variables will move since they are free. When point V is dragged, it moves, but no other point named with a letter moves with it. This means that the dependent and independent variable for that relation is represented by point V. It is independent because it moves when dragged, thus it takes values freely. Its dependent variable is represented by point V as well, which means that for each input value, the output value is the same (as the function $y = x$). Point A also represents an independent

variable, and point Y is its dependent variable. Thus, when A is dragged, Y moves along with it.

As a classroom activity they created a table for the students to complete. As teachers were answering the table to be included on the lesson plan, an interesting discussion emerged between Teacher 1 and Teacher 2 regarding the movement of letter V and its meaning. At first the column headers read “Point chosen,” and “Points that move” instead of “Independent variable” and “Dependent variable.” Teacher 1 seemed confused with the meaning of the headings:

T1: ...I'm seeing that you have “Point chosen” A [from the table]

T4: that was the point I chose

T1: yes, it is the one we chose to move. Then, “Points that move”

T4: the one it moves

T1: okay, that moves, you are answering Y here and that is right (A moves Y). But you are saying that V moves V. Then if V moves V, A is also moving A... so here [Points that move column] you have to include A and in every variable you have to include them because they also move.

It seemed Teacher 1 was interpreting the activity as movements of points and was not seeing each point as a representation of a variable in a relation. Perhaps, he was confused with the wording used in the headings of the table they were using. He was claiming that if the point chosen was V and the point that it moved was also V, then when point A was chosen it moves points A and Y and the same was true for all points that moved when dragged. This might be an issue with his mathematical knowledge for Teacher 1, or just a wrong interpretation of the activity itself. As the conversation continued, Teacher 2 showed some evidence to have a clearer understanding of what the GeoGebra file represented. It

seems like Teacher 2 is seeing each point as an input and output of a relation, thus helping him interpret the GeoGebra file correctly:

T2: ... you have to look at it like this, I go into A and where did I get out? at Y. I go into V and I get out where? At V

T1: no, no because I have nothing

T2: of course you do, $y=x$ (referring to the function)

T2: ... look I am at A. I move three steps to the right, where did I end up? Up here at Y. Now V, I am here. I moved three steps to the right, where did I end up? At the same place [where V is]...

T1: okay, you moved A three steps

T2: yes, where did I end up? At Y

Implementation of the Lesson Cycle 1. The teacher started the class being specific with the students that laptops were not going to be open until he indicated to do so. In doing so he shows some of his *technological pedagogical knowledge*, avoiding that his students' attention was deviated from what he had to say to initiate the class and the activity. He spent some time at the beginning mentioning how the lesson was created, and giving credit to the other teachers that participated in the planning of the lesson. Then he mentioned the objectives of the lesson, and the standard he will cover during the lesson.

The teacher used the technology tool as the group had planned and there were no technological issues during the implementation of the lesson. Once he mentioned the objectives of the lesson, the teacher opened the GeoGebra using the smart board. The use of the smart board was done flawlessly, showing his familiarity with the tool and his *technological knowledge* with respect to that tool. The use of GeoGebra was basically dragging points to notice relationships between them, which the teacher managed without any problem.

Issues about *knowledge of math and teaching* emerged as Teacher 1 implemented the lesson using a pedagogical approach that was completely new to him. The first activity was exploratory and its goal was for students to generate definitions for independent and dependent variables. When introducing the activity, as the file was showing in the smart board, the teacher explicitly asked what does independent mean. One student responded, and the conversation that followed took away the exploration embedded in the activity:

T: What is independent?

S: a variable that moves by itself without depending on the movements of other variables or other points.

T: okay, independent is definitively what he is saying, it is a variable... anyone... and you move it. If the point with the letter moves then he does not depend on anyone. If you move [a point] and that is what you will see there, and another point moves, what does that mean? What will happen with that other point?

S: it is dependent

T: It is dependent because what happened... [dragged a student in his chair] your movement depended on my movement.

All students knowing the definition of independent variable, which was the main goal of the activity, focused students' attention on the identification of independent and dependent variables instead of identifying the different characteristics between them in order to write, in their own words, definitions for independent and dependent variable.

However, not all students understood what an independent and dependent variable meant and it was difficult for some to create the list on their notebooks. For example, one student was still confused and raised his hand. The teacher tried to explain using our planetary system as an example. He tried to explain that in our planetary system all planets depend on the Sun to move around the Milky Way. This example could be very complex if the student does not have the knowledge about the Milky Way or the physics behind the

movement of the planets around the Sun. This issue of his *knowledge of math and teaching* could have been avoided if the group had worked anticipating that some students might have issues understanding what a dependent and independent variable was. They could have planned on simpler examples to explain to the students in case such a question arise during the implementation.

Another issue with his *mathematical knowledge* that emerged was the consistent misuse of the words relation and correspondence throughout the lesson. When discussing the second activity (see Figure 23) he said that each number in the set on the left had a relation with the number on the set on the right, following the arrows (e.g., number 1 has a relation with number 4). However, the relation exists between the variables (sets), not between the elements in the sets. The correct vocabulary to use would be that number 1 has a correspondence with number 4 using the relation (rule) that exists between the variables (sets).

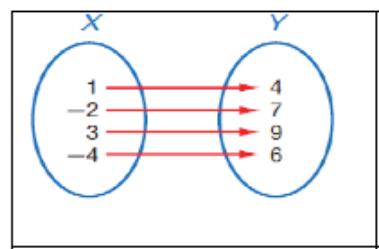


Figure 23. Example shown in class, Case 2

Issues regarding his *pedagogical knowledge* were observed throughout the implementation of the lesson. The lesson was unorganized and lacked connection with the mathematical concept the group was trying to teach. After students worked on their own with

the first activity, the teacher showed a slide that reads “a relation is a correspondence of a first set, called domain, with a second set, called range, so that each element in the domain corresponds to one or more elements in the range.” This definition introduced new vocabulary (domain and range) and was not using the independent and dependent vocabulary to make the connection with the previous activity. He tried to make the connection between independent (domain) and dependent (range) variables, but it is not clear if students were able to make the connection with the activity they were working with. Then, using the smart board he dragged the points and asked the students which point was independent and dependent, something he was supposed to do before defining relations. He showed the definitions of independent and dependent variables afterwards. He could have done so in a different order: discuss the activity using the smart board by dragging points, show the dependent and independent variables definitions, and then the definition of relations without introducing new vocabulary.

Evidence of his *knowledge of pedagogy*, in general, included great class management, and use of a tone on his voice appropriate for the classroom size. He showed issues with his *knowledge of pedagogy* as they asked questions to his students, but did not leave enough time for his students to think and respond.

Reflection Meeting for Cycle 1. For this group, the reflection meeting lasted an hour and 40 minutes. First, we all watch the implementation of the class, which lasted about 40 minutes. For this meeting there were three teachers, the forth teacher only participated in the planning session of Cycle 1. The group reflected on how the teacher did not follow what the

group had planned in some of the activities because it was not specified in the plan. They all agreed the plan was vague and it lacked details. The teacher implementing the lesson noticed he talked too fast while teaching. He also noticed that, when asking questions to his students, he responded too quickly not allowing time to his students to think and answer. He even noticed that he ignored some students as they offered correct answers, something he had not realize before. Through these observations the teacher reflected on his *pedagogy*.

Most time during this meeting was spent talking about the need for adding more examples, the constant use of incorrect vocabulary, poor time management, and the lack of organization of the lesson. All these issues regarding his *mathematical knowledge* and his *knowledge of math and learning* emerged during implementation. Throughout the planning and implementation of the lesson I noted most of the observations noted by the mathematician. For example, the mathematician suggested adding more examples to the lesson. He observed there were not examples where the codomain and range were different. This goes back with their conversation during planning where they all seemed to agree that codomain and range were different names for the same set, which is not true. By missing the fact that range and codomain were not the same, they lacked examples that were more challenging for their students where not all elements in the second set of the function were reached by elements of the domain. They all seemed to lack that *mathematical knowledge*.

Regarding the incorrect use of vocabulary, Teacher 1 was unaware of his issues with the use of relation and correspondence. In his second journal entry, he reflected about this as he wrote: “The misuse of the concepts of relation and correspondence was new to me. During

my years as a teacher I have used it erroneously... After the discussion of the class with my colleagues and the professors I still have issues with the concept because I am used to talk about a relation [between elements] and not a correspondence.” With this in mind he might use the correct vocabulary in the future while teaching functions.

Cycle 2. For Cycle 2 this group of teachers designed a lesson about *transformations of functions*. As technology tools they used a smart board, graphing calculators, and GeoGebra. The lesson was implemented with the same group of 8th graders that the lesson for Cycle 1 was implemented. For this cycle there were three teachers in this group as one teacher left after the planning sessions for Cycle 1. The volunteer teacher for Cycle 2 was the same volunteer teacher as for Cycle 1.

Planning Session for Cycle 2. During the planning session for Cycle 2, this group of teachers showed evidence of their knowledge of curriculum, knowledge of mathematics, and knowledge of technology. As can be observed in Figure 24, toward the beginning of the planning evidence of their curriculum knowledge emerged. As the discussion progressed, some evidence of knowledge of their students’ abilities, technology, and math emerged. Also, evidence of their knowledge of math and students, knowledge of math and teaching, and knowledge of general pedagogy were very limited. It is important to note that this group of teachers remained silent for long periods of time during planning as teachers worked individually in different pieces of the planning. Teacher 1 took charge of creating the GeoGebra file to be used during the lesson. Meanwhile, Teacher 4 took care of planning the lesson adding all the details to the lesson plan.

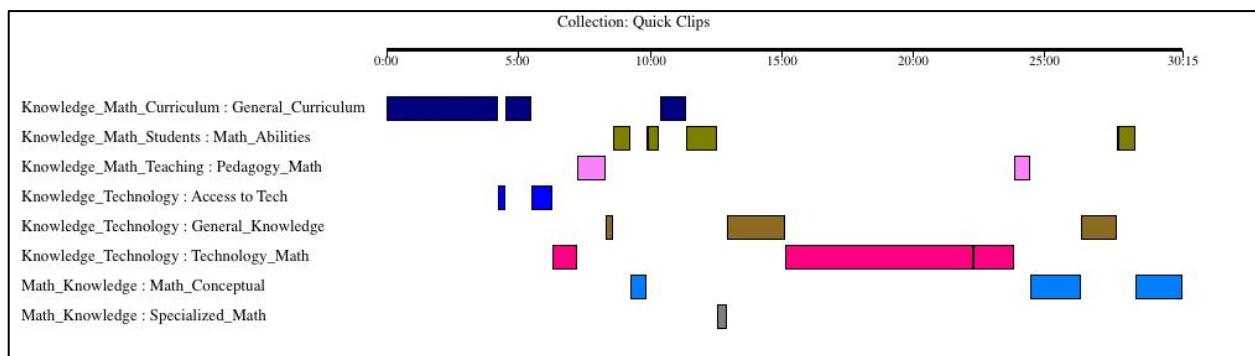


Figure 24. Chronological codes applied to Planning Session Case 2, Cycle 2

They had first decided the teacher that would implement the lesson. Two teachers (Teacher 1 and Teacher 4) were willing to implement the lesson, but eventually they chose the teacher that had implemented the lesson for Cycle 1 (Teacher 1). The reasons were not clear through the discussion. Then, they decided the topic accordingly to the curricular map of the grade and course he was teaching. They decided to design a lesson on translations of functions, which Teacher 1 and Teacher 4 had experience teaching. In their journals, Teacher 1 explicitly wrote, “I have offered this topic in previous years for 9th grade.” On the other hand, Teacher 3 wrote, “The topic chosen, I have never taught it before.” Teacher 4 was not explicit about this in her journal, but during planning she made some suggestions based on her experience teaching this topic. Perhaps lacking that teaching experience was one reason why Teacher 3’s participation in the conversations with the group was minimal.

Their *knowledge of curriculum* was evident at the beginning of the planning as they discussed the topic to focus the lesson on. Regarding their comfort with the content knowledge, there was not explicit comment about it in the journal entries. They only

mentioned the topic was easier than the one they chose for Cycle 1. Teacher 1 wrote, “this

topic is way simpler to work with my students than the topic of the [class for Cycle 1].”

During planning, there were minimal conversations where teachers discussed the

mathematical content, which makes it difficult to identify and describe any conceptual

understandings these teachers might have.

His *knowledge of math and students* was evident as Teacher 1 insisted on the weakness of his student’s mathematical abilities. He was worried about what the other teachers in the group were expecting his students to accomplish during the lesson. According to him, his students were “very lazy and were not able to deduct a definition by observing a behavior.” This discussion came after the teachers had planned on doing an activity where the students would observe the behavior of the graph as they change the value of the parameters s , and t in the function $f(x) = \sqrt{x+s} + t$, individually. As a consequence, the group spent a long time deciding how to design the lesson to include an introduction that helped his students to accomplish the activities they had in mind. Thus, they decided to begin the lesson by graphing the square root function. This decision came after they had decided to start the lesson with horizontal shift. As they discussed how to design the worksheet the students were going to be working on, Teacher 1 mentioned his concern. He was basically worried his students were not going to understand the translations of a graph if they have not seen the basic function constructed before. It is important to note that the square root function does not appear into the mathematics curriculum until high school.

Some evidence of their *technological knowledge* emerged through that discussion as they plotted points and graphed the function using the input bar in GeoGebra. They had planned to design an activity in GeoGebra where they would use sliders for the students to change the values of the parameters s , and t . They did not want both shifts (vertical and horizontal) to show at the same time, thus in the sketch they used the Hide/Show option in GeoGebra to show one slider at a time. In addition, the sketch included the parent function so the students could compare both graphs and observe the behavior caused by changing the values of the parameters. This showed evidence of their *knowledge of general technology*, *knowledge of technology* as it relates to the mathematics, as well as *knowledge of math and teaching* as they decided how to do so.

There were several codes that were not used for this group for Cycle 2. For example, regarding their *knowledge of math and students*, this group did not anticipate students' responses and did not mention specific misconceptions their students might bring as they learn the new topic. In addition, concerning their *knowledge of math and teaching*, although they exhibited having that knowledge regarding pedagogy specific to mathematics, they did not discuss the possible questions to ask students during the discussion of the topic. This might be a consequence of the way they worked during this planning session. During planning, Teacher 4 was mostly responsible for writing the lesson plan, having no discussions with the rest of the teachers in the group. It seemed Teacher 1 were focused on designing the GeoGebra activity that was going to be used. Thus, even though the detailed lesson plan included a set of questions the students were going to work on and the responses

to it, this was never discussed during planning. At some point, Teacher 1 read the questions on the worksheet to himself, and agreed with the content Teacher 4 had included. Given that Teacher 3 had no experience teaching the topic, her contributions to the lesson were minimal.

Implementation of the Lesson for Cycle 2. This lesson was implemented with an 8th grade class. The teacher had been using GeoGebra with his students since Cycle 1 so they were quite familiar with the tool. The students were paired up, and each pair had a laptop, but laptops were not open until the teacher instructed to do so before starting Activity 1 later on.

To start the lesson, the teacher discussed the graph of the square root function. His *mathematical knowledge* was evident as he graphed the function. He spent a lot of time constructing that graph. This was added during planning mainly because the teacher was afraid his students were not going to be able to proceed and work with the activities they had planned for if they had not graphed the basic function of square root before seeing any transformation. He was also afraid his students would not know how to graph it, demonstrating his *knowledge of math and teaching*. He created a table of values on the board and, using calculators, he asked his students to find the square root of some perfect squares, as shown in Figure 25. Then he plotted the ordered pairs using GeoGebra and graphed the function. This showed some evidence of his *technological knowledge* as he plotted the points using the input bar, and then typed $f(x) = \sqrt{x}$ to graph the function. By doing this he wanted to show how the ordered pairs they had found using the table of values were aligned with the curve of the function. He also wanted to review some of the graphing skills his students

might forget before continuing into transformations. This was a pedagogical decision he took as the group planned the lesson. Student's laptops remained closed at this point.

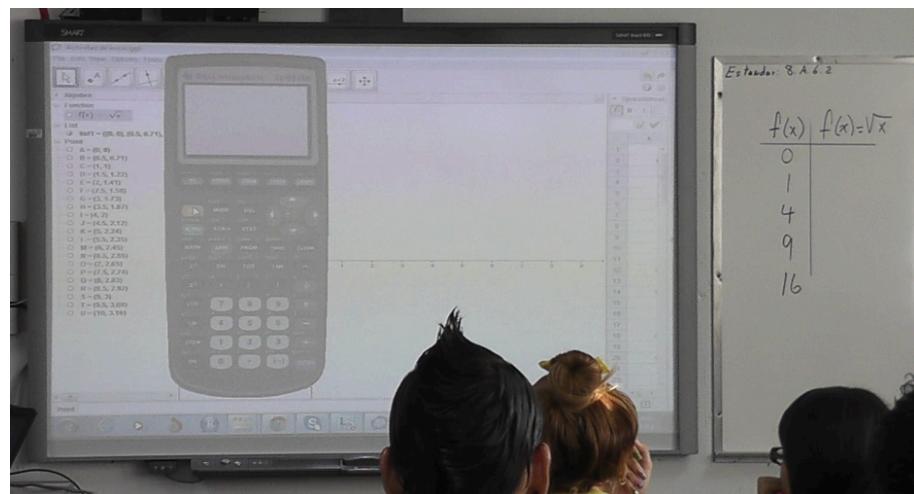


Figure 25. Case 2 use of a graphing calculator to graph the square root function

Once they had discussed the graph, the teacher moved on to the activities the group had planned for the day. The goal for all activities was for the students to describe the transformations they observed as they changed values of parameters for a function using sliders in GeoGebra. Activity 1 was about vertical and horizontal shifts, and Activity 2 about vertical shrink and stretch. For both activities, students worked in pairs and respond to 12 questions as they moved the sliders. The last question in the worksheet was, "What type of movement do you observe as you move point s (or t) from -10 to 10?" Thus, the goal was for the student to identify the translation by observing the behavior provoked as sliders were moved.

Regarding his *pedagogical knowledge*, this teacher used his class management abilities to capture his students' attention to give instructions for the next activities, where

students were going to work in pairs using the laptops. He gave very detailed instructions, helping his students open the GeoGebra file and understand the expectations the teacher had. The activity included vertical and horizontal shift, which they called Movement 1 and Movement 2 to avoid using the names of the transformation. He demonstrated how to move the sliders so that students could observe that a new graph was constructed. He emphasized that the new graph was created from the graph of $f(x)$ that they constructed at the beginning. He also wanted the students to observe how the original graph was affected by changing the values of s (Movement 1), where s produced a horizontal shift and t (Movement 2), where t produced a vertical shift.

These teachers were not used to letting students work on their own, even if they create a very detailed worksheet with questions that would guide students to make observations and to make conclusions to deduct a mathematical behavior. This pedagogical approach of having students working on their own to describe or to make their own conclusions was new to the teacher. He has only done it during Cycle 1. His *knowledge of pedagogy* was shown as he implemented this lesson, as he guided the students to complete the worksheet, and as he encouraged them to write conclusions from their observations. This was all new to him, but that was not noticeable as he implemented the lesson.

His *mathematical knowledge* with respect to transformations was also evident through the implementation of the lesson as he used the correct language when referring to the transformations. There was only one mathematical error observed on the table of values that he wrote on the board. As he discussed it, he mentioned the values of x , but as it can be

seeing in Figure 25 he wrote $f(x)$ on both sides of the table. He seems to know what he was doing, and did not notice the mathematical error on the board. When discussing questions regarding Movement 1, the following conversation emerged:

T: [the worksheet] says to move the point until it says $s = 2$ (the teacher changed the slider to $s = 2$)... it is important that we can see what is happening in that function... (opening the Algebra window) look in here, in red, what do we have now that we didn't have before?
 S: plus 2

T: and what happened?

S: it shifts to the negatives

T: there was a movement of the graph, right... it is important that you see that the whole graph shifted. Perhaps it could be a little difficult to observe that because of [the vertical] space between the graphs... what happens is that this point here [point (4,2)] shift to here [point (2,2)]... thus if we look, all point on the graphs moved two units to the left.

S: question... why... if we are adding... why does it moves to the left?

T: okay, there is a behavior and that is what we are going to see now... and that is what I want you to see. We have a plus 2... we are getting ahead... and the behavior of the function is? It shifts to the left two units.

From there he continued discussing the questions on the worksheet and missed the opportunity to explain why it moves to the left if you add a number inside the square root.

Having a short amount of time left to finish the lesson, the teacher quickly discussed Movement 2, which was the vertical shift. This is easier for the students to understand and there were no questions from the students. Activity 2 was about vertical shrink and stretch. Again, the students seemed to understand this transformation as they correctly answered all the questions, and there were no questions for the teacher.

Not explaining the reasons why those horizontal shifts behave as they do could be a teachers decision to save the response for later in the lesson, or that the teacher did not had the *conceptual mathematical knowledge* to answer that question. Perhaps, this could be a consequence of not having that conceptual conversation during planning with the rest of the

teachers. If they had anticipated students might ask why adding a positive number to the input resulted on a graph shifting to the left, they could have also plan on how to respond it and have examples at hand to explain. For example, they could have use table of values for $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x + 2}$, and have students compare the values of the independent variable as one keeps the dependent variable the same.

Issues with *pedagogy* were evident as well. He spent a lot of time graphing the square root function at the beginning of the lesson, and then allowed the students to work on Activity 1 longer than they group had planned. These two events resulted in no time to let students work in pairs on Activity 2, thus he discussed both activities in a hurry, ignoring some of his students' questions about the horizontal shift. Also, he had no time to complete the closure activity where students were going to be asked to match equations and graph that were transformed using one the function transformations discussed in class. He used some time he had left at the end to review the new concepts of the day. He asked what happened to the graph of $f(x) = \sqrt{x + 5}$ to which students answered, "shift to the left." He then asked the transformation that occurred on $f(x) = \sqrt{x} + 5$ and students responded, "move to the negatives." He ignored that response and noted the student that said it shift the graph up 5 units. It is evident not all students understood the topic being taught.

Reflection Meeting for Cycle 2. After we all watched the implementation of the lesson, each teacher had the opportunity to reflect on how the implementation of the lesson they had planned went. Teacher 1, who implemented the lesson felt satisfied with what he had done. He said, "I feel I did a better job in this lesson than the previous one." The rest of

the teachers agreed he had followed the lesson plan as they had planned, with the exception of the time he let the students work in pairs during Activity 1, which jeopardized the quality of the discussion afterwards and the closure of the lesson. Teacher 1 also reflected on his pedagogy as he realized he had ignored an important question his student asked. The other two teachers offered no more observations after watching the implementation.

The mathematician made just a few observations. First, he noticed the lost time at the beginning when graphing the square root function. He suggested graphing the function in GeoGebra directly, and omitting the use of the table of values. Second, he noticed he ignored the student that asked about the horizontal shift and suggested not ignoring his students.

When my turn came, I was also very pleased with the lesson. They used more features of the technology, and used it for the students to explore a mathematical idea. They created the exploration for the students, which showed their specialized content knowledge. The technology files were also created by the teachers, thus their *knowledge of the technology* was evident. During the implementation, I noticed he mentioned he was “getting ahead” when the student asked about the horizontal shift. I used the reflection meeting to remind them that a lesson plan was mainly a plan to follow, but that they needed to be open and prepared to switch gears if students ask questions that made our plans to felt apart.

It was also important to have a discussion about the horizontal shift and the question asked by his student. I mentioned the importance about anticipating students’ questions and also anticipate some responses to those questions. I suggested that it would be beneficial if the teacher connected that behavior to inputs and outputs in the function in order to help

students better understand horizontal and vertical shifts. All horizontal shifts occur because we are adding or subtracting something to the x , the input of the function. The change in behavior will be observed in the x-axis, thus horizontally. Similarly with the vertical shift, but instead we are affecting the outputs of the function. As my field notes reflect, a deeper mathematical conversation about why that behavior is observed does not occur during the reflection.

Within-Case Analysis: Case2

Case 2 took real advantages of the opportunities they had to learn how technology could be integrated into their teaching. Although they had some issues with the mathematics they were teaching, they integrated a technology tool that was new to them using a pedagogical approach they had never tried before. Something particular to this group was that the same teacher implemented the lessons for both cycles.

During Cycle 1, although the group seemed to be lacking some deeper mathematical knowledge, their use of technology was acceptable given it was the first time the group was planning a lesson using GeoGebra. The pedagogical approach was also new to them as they are used to offering all definitions first and then work on problems. The unfamiliarity with the pedagogical approach, and the issues with the mathematical knowledge in part jeopardized the main goal of the lesson the group had planned.

During Cycle 2, the use of the same technological tool, the same pedagogical approach, and the same teacher benefited the group as a whole. The experience and knowledge the group gained during Cycle 1 helped them succeed during planning and implementation of

Cycle 2. It was evident the teacher that implemented the lesson felt comfortable using the technology tool. It was also evident he felt more confident with the pedagogy when asking questions to his students and responding to his students' questions. Even though there were some issues with the mathematical knowledge, the experiences with the technology and the pedagogy eased the process of planning for the group, and the implementation of the lesson for the teacher, when the use of technology was required.

As I compare the results from the EMMES-CTPC pre- and post-survey (Figure 26) with the changes I have observed in this group over the course of the professional development, it should not be a surprise they perceived themselves more knowledgeable with respect to the mathematics, technology, and TPACK. With respect to technology knowledge, as mentioned above, this group did not only learn how to use the technology tool during planning, but the tool was used in the classroom with ease. Regarding mathematical knowledge, the increment could be related to their experiences during the two cycles as well. They had some issues with their mathematical knowledge in both cycles, and the observations and feedback received during the reflections meetings could have impacted their mathematical knowledge positively.

The increment on their self-perception of their TPACK should not be surprising given this group's attitude toward learning how to integrate technology into their mathematics' teaching practices. In the case of T4, she seemed to come into the professional development perceiving herself knowledgeable about how to integrate technology into her teaching. The

experiences she had during the professional development seemed to impact that perception and make her realized teaching mathematics with technology was not as she had perceived it.

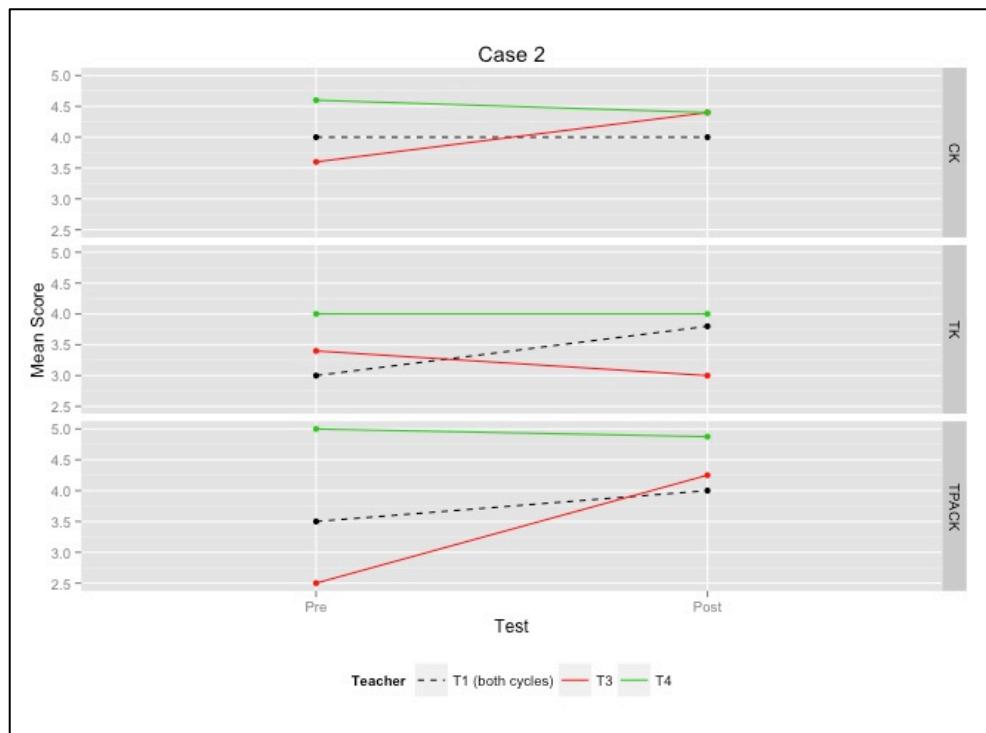


Figure 26. Pre and Post mean score per sub-scale construct per teacher for Case 2. Dashed and dotted lines represent the teachers that implemented the lessons.

Case 3

Case 3 originally consisted of five high school mathematics teachers, 4 teachers from public school and one from a private school. There were four female and one male. The years of experience teaching for the teachers in this group range from four to 18 years. The teachers that implemented the lessons had 6 and 8 years experience teaching, respectively. Teachers were from two different school districts. The teacher from the private school, the only male of the group, that has 18 years of teaching experience, left the study after the

culmination of cycle 1. This group had issues with their knowledge of the mathematics they teach. They also exhibited issues with the knowledge of technology tools that are designed to be used to teach mathematics. Their implementation of technology was vague at first, but it was later improved as they used an appropriate technological tool for the topic of focus.

Cycle 1. For this cycle they chose to design a lesson on *domain and range of functions*. The lesson was implemented with a group of 11th graders, in a vocational public school. For the lesson, the teachers decided to project a Power Point presentation on a smart board, using a desktop computer. They also used GeoGebra.

Planning Session for Cycle 1. At the beginning of the planning session, there was some evidence of their knowledge of curriculum, and their access to technology. As illustrated in Figure 27, as the planning sessions progressed, discussions were focused on students' abilities, teachers' technological knowledge, access to technology, and their mathematical knowledge about relations, function, and domain and range.

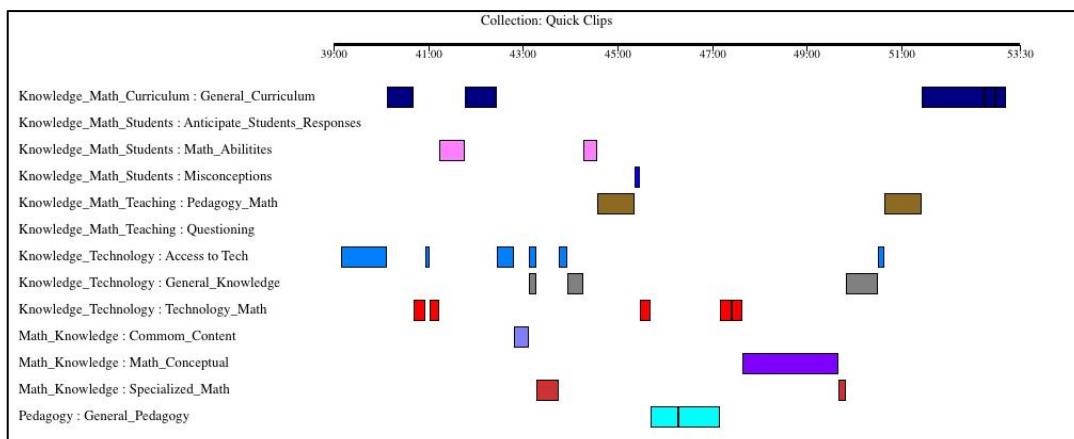


Figure 27. Chronological codes applied to Planning Session Case 3, Cycle 1

Regarding their *knowledge of curriculum*, teachers in this group seemed to be aware of the curricular sequence for teaching functions. Using their *knowledge of math and students*, they decided to leave graphical representations of functions to the end since “it is difficult [for students] to see domain and range from a graph because they do not know how to graph” (Transcript_Group3_Cycle1). Thus, their examples only included relations/functions represented by table of values, sets, and ordered pairs. At some point, as they looked through the new set of standards they noticed that introduction of relations, functions, and domain and range of functions were not standards for high school anymore, but for middle school. After spending some time searching for the appropriate standards, they decided to use the standard from the previous set of standards given that the new standards were not going to be applied to their grade level (11th grade) that academic year. They had received instructions that the new set of standards was going to only be applied to grade 10 during the 2014-2015 academic year.

Issues regarding their *knowledge of technology* emerged during their planning as they talked about their access to, and general knowledge of, technology. Regarding their access to technology, only one teacher had a smart board and a desktop computer in her classroom, one had access to an iPad and a projector, and the other three had no access to technology in their classroom. From those with no access to technology, one could borrow a computer lab at her school. Because most of the members of this group lacked access to technology, issues around it kept emerging throughout the planning. The teacher with access to a smart board implemented the lesson mainly because of her comfort level with respect to managing the technology itself. She also volunteered to implement the lesson because this group started their planning using a previous Power Point presentation about relations, functions, domain, and range that she owned.

As the planning progressed, I pushed them to think about different technologies that could be integrated into the lesson, something they struggled with for several hours. At first their only technology tool was Power Point, and their objective was that the students “will learn what a function is by reading and following the animation of the power point slides” (Transcript_Group3_Cycle1). However, not all teachers in the group were satisfied with that objective because it was lacking a more active role from the students and it was not measurable. Searching for options they questioned if it was possible to create a video while using GeoGebra and then uploading it to the Power Point presentation. Throughout the discussion the group mentioned tools such as GeoGebra, Mathway, Wolfram Alpha, and Illumination. However, none of these teachers felt they had the general technological

knowledge needed to use any of these tools. All teachers in this group did not know that it was not necessary to have Internet connection in their classrooms to be able to use GeoGebra. Thus, once they learned that GeoGebra could be installed in their computers and used without Internet connection they chose to use GeoGebra

When it comes to their *mathematical knowledge* teachers in this group felt comfortable with the topic they chose. However, they showed having some misunderstanding when it comes to domain and range of functions. For example, at some point the group decided that the lesson I was going to observe was focused on domain and range, and not the introduction of relations and functions. In the discussion one teacher said that “[in order to] find the domain and range they needed a function” because if it was not a function they cannot find domain and range (Transcript_Group3_Cycle1). This is not true since the domain and range are sets of values for the inputs and outputs of any relation. None of the teachers in the group reacted to what the teacher said about needing a function to be able to talk about domain and range.

Their *mathematical knowledge* was also evident as they used specialized mathematics vocabulary throughout their discussion. However, there were some issues of inappropriate use of this specialized vocabulary, evident throughout the planning. For example, their definition of relations was: “an ordered pair is a relation when the first component of the first set corresponds to one or more second components of the second set” (Transcript_Group3_Cycle1). There are several issues with this definition. First, a relation occurs between two sets, not between two numbers in an ordered pair; and second, the

language used could be very confusing for the students. Using their definition, in the ordered pairs $(2,4)$ and $(3,5)$ we have that number 2 is the domain and number 4 is the range and that number 3 is the domain and number 5 is the range. Instead, it should be stated that numbers 2 and 3 are elements on the domain, whereas numbers 4 and 5 are elements on the range of that particular relation. The group read this definition out loud during planning and none of them reacted to it. A clearer definition could be that a relation exists between sets A and B if each element in set A has a correspondence with one or more elements in set B. They could show examples using multiple ordered pairs.

As they introduced domain and range, their definitions for these two concepts did not include the appropriate vocabulary. Domain was defined as “the first number of each set (x). Ex: $(-1, 3)$ ” and they identified -1 as the domain. Range was defined as “the second number of each set (y). Ex: $(-1, 3)$ ” and 3 was identified as the range. Again, it is not that -1 is the domain and 3 is the range, but that -1 belongs to the domain and 3 belongs to the range. For one of the activities for this lesson, this group had planned for students to use their previous knowledge of the Cartesian plane to plot an ordered pair, given by the teacher, in GeoGebra using the smart board. From there each student was asked to identify the domain and range in each ordered pair, going back to the same issue with the vocabulary. At the end there was a conversation between two teachers where they kept mentioning, “to find the domain and range of letter H” where H was a point in the Cartesian plane. They are lacking the correct vocabulary to define these concepts.

Their *pedagogical knowledge* emerged during the planning as they discussed how to assign a more active role to the students during the lesson. They wanted to start the lesson with a brainstorming activity where students had to mention what they thought a relation was. There are some issues with such an open question. First, a relation in mathematics involves two sets that need to be previously defined, which is not necessarily true for a relation outside of mathematics. Second, in order to have a successful mathematical discourse teachers must: (1) anticipate their student's responses, (2) think of a way they should sequence those responses, and (3) how to connect them. And last but not least, they need to think in advance how they will connect that to the definition of a mathematical relation. This anticipation, including the questions they might ask their students did not happen during planning. Codes regarding their *knowledge of math and students*, and their *knowledge of math and teaching* were minimal during planning as they missed having conversations around all this anticipation.

Implementation of the Lesson for Cycle 1. This lesson was implemented in a vocational public school with an 11th grade class and lasted about 40 minutes. The teacher started by collecting homework and calling attendance of students. She turned her desktop computer and smart board on and started the lesson without any problem, showing her confidence in managing the technology tools she was using. Given that she had not much time for the lesson, she started right away.

Regarding her *knowledge of math and teaching*, the lesson flowed without any problem and the teacher followed the lesson plan as the group had planned. For the most part,

the role of the student was passive. The teacher was quiet for long periods of time while the students copied the material in their notebooks. The teacher showed each Power Point slide, read its content, and paused while the students copied what was in each slide. The pedagogical approach taken by the teachers for this lesson was very traditional. First, the mathematical idea was discussed and definitions were given. As she discussed the material, she showed and solved some examples. Then, the students worked individually on a set of problems. Because of lack of time, the teacher could not discuss the problems on the board to close the lesson. This traditional way of teaching could be a consequence of lacking knowledge about innovative pedagogical approaches they could use to teach mathematics. Similarly, as they lacked knowledge of, and access to, technology they might lack motivation to try those new tools and approaches to teach mathematical concepts.

The lesson started with the question, “What is a relation?” on the smart board. Using a brainstorming activity, they had planned to leave it to the students to define and give examples of relations. Some of the examples the students mentioned were: friendship, a couple, mom and daughter, and two things that are together. The teacher wrote those answers on the board and without any transition she moved on to the next slide in the presentation, which was the definition of relation. Without a connection with the mathematical idea, it was not clear whether or not students saw any relevance to that question. Using the definition mentioned before, she defined relation using just the components of an ordered pair and gave some examples. When defining domain and range, the teacher used the same vocabulary they

used during the planning session. She used the ordered pair (-1, 3) and said that -1 was the domain and 3 the range. The same occurred throughout the implementation of the lesson.

After an additional example was discussed, she opened GeoGebra in the smart board. As it is observed in Figure 28, the group had previously chosen the point they used during the activity. However, for this activity the group had planned for the students to be active participants and being called to the board to draw the points in the Cartesian plane and identify domain and range. The teacher may have forgotten about this and drew some of the points herself. GeoGebra was opened for about a minute and a half while the teacher showed three points in the Cartesian plane. However, it was used without difficulty.

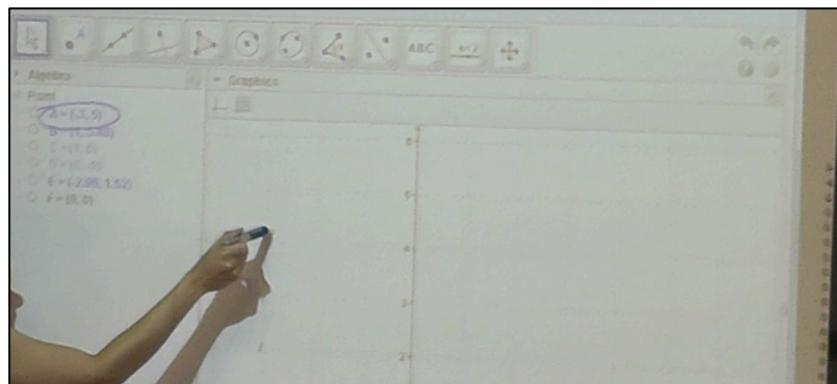


Figure 28. Teacher using GeoGebra during implementation of Case 3, Cycle 1

The examples the group chose while introducing domain and range for relations were all single ordered pairs (e.g., (-1,3)). Even though incorrect language was used, it was evident the students understood that -1 was the domain and 3 was the range. After working with relations, the teacher defined functions as “a rule that produces a correspondence that to each value of the first set it corresponds only one value of the second set.” Following the

definition, some examples represented as sets (or arrow diagrams) were shown. For these examples, the teacher emphasized that for that relation to be a function the first number could not be repeated. Thus, looking at the arrows in the diagram they cannot have two arrows going out the same number. Next, examples of relations and functions were represented as a collection of ordered pairs. She presented the example $\{(1,4), (2,3), (3,2), (4,3), (5,4)\}$ and asked the students to identify the domain. She did not get a response at first and students seemed confused. Once she started circling the first coordinate in each pair some students mentioned the domain was 1, 2, 3, 4, and 5. As she continued showing more examples students followed her and responded to all of her questions correctly.

Several *mathematical issues* emerged during this implementation that were carried over from the planning. First, the definitions used incorrect language. Second, students were directed to only find domain and range for those relations that were functions. This appeared during the planning, and even though this might not confuse students it might communicate an incorrect message. It may be the case that the students believe domain and range only exist for functions and carried that over to their future math classes. Even though the vocabulary was used incorrectly, the teacher used the correct notation when answering the problems on the board as she wrote the domain and range set.

Reflection Meeting for Cycle 1. The reflection meeting for this group lasted one hour and 10 minutes. All teachers in the group participated of the reflection meeting. After we watched the implementation of the lesson, the teacher that implemented the lesson said she was satisfied with her class. She did not reflect critically on how the lesson was implemented

and only mentioned that, “the class was good, but at the end time betrayed [her].” All teachers in the group agreed that the teacher followed the lesson plan and made suggestions of things that could be added to the lesson as well as others that needed improvement.

The first observation was made by one of the teachers. She said that the teacher “in occasions answered the questions she asked the students” (Teacher 5). Again, the justification was the lack of time. Another teacher suggested adding examples that were not numeric. For example, he said that, “since that [was] a vocational school where there are mechanical workshops... [she] could use makes and models of cars, which is a function depending on [how you select the sets]” (Teacher 4). There is a relation between these two sets, and depending on which set we use as the domain, the relation could be a function. Another suggestion was to at least mention to the students other names that are used to refer to the range. In Spanish there are multiple ways of referring to the range of a function. In secondary school they are used to name it “alcance,” but in college math classes “rango” is more common. When Teacher2 made the suggestion, the teacher that implemented the lesson said that, “those words do not appear in the standardized test” (Teacher 1) and that was why she did not use them. The other teachers agreed they needed to mention those other names to their students so they get familiarized with the terms commonly used in college. As teachers mentioned the different names used to refer to the range, one teacher commented “and codomain” (Teacher 1). The mathematician and myself reacted to her comment saying that codomain and range were not the same. The mathematician said he was going to comment further on that later on.

Another teacher mentioned that the responses of students in the open question at the beginning of the lesson needed to be connected with what was next in the class because it was left on the air. All teachers in the group, the mathematician, and myself agreed that a connection was needed before continuing on with the definition of relation. Teacher 3 found a moment during the implementation where one student had said, “a relation [was] something about two people.” She continued saying that “... perhaps [the teacher] had taken advantage of that and go deeper there. I don’t know... that their answers were not left there and connect them with domain and range (Teacher 3).” The mathematician also said that the teacher missed that opportunity to connect that response with ordered pairs and moved to the definition of relation.

After each teacher had the opportunity to reflect on the implementation of the lesson, and the lesson plan they had designed, the mathematician and myself made our observations or suggestions, mostly focused on the mathematics. . Given that I noticed many mathematical issues with the incorrect use of vocabulary and some conceptual flaws during the implementation, I left the mathematicians to make his observations first. He was very honest with the teachers and said, “even though the teacher followed the lesson plan as the group had planned, the lesson had many deficiencies as it was.” He noted several things: the responses to the open question were not connected to the definition of relation; the definition of relation, domain, and range were incorrect; there was a continuous incorrect use of vocabulary; the lesson was lacking conceptually challenging examples of functions; and it was correct to find the domain and range of a relation.

During the discussion, many of the teachers agreed on the feedback given. As the discussion about the deficiencies in the definitions emerged, the teachers had no reactions or comments. It seems they were not aware of the incorrect use of their mathematical language. For example, the mathematician noted that the teacher said “the pair (4,5), which is the domain?” and he followed, “that is mathematically incorrect as one cannot talk about domain of an ordered pair. One talks about domain of a relation, and a relation is a set... it is a technicality.” Again, teachers had no reaction or comments to this observation. He went further and said to the teacher that implemented the lesson, “it seems that you think of a relation as a pair... and not... the relation is made by many pairs.” He also said, “you kept repeating that the domain does not repeat... and not... what cannot be repeated is the first element... and you have to be careful with that as well.” He showed the example $\{(1,2), (3, 5), (1,2)\}$ and said that for her students that might not be a function because she emphasized that the first number could not be repeated. She made no comments.

When discussing the types of examples that were included in the lesson plan, the mathematician mentioned they were lacking an example where the range and the codomain were not the same. Using an arrow diagram, he wrote an example on the board where the domain was $\{2, 3, 4\}$ and the codomain was $\{1, 6, 8, 5\}$. The relations existed between 2 and 1, 3 and 6, and 4 and 8. Number 5 was not reached by any element on the domain. In this example the range was $\{1, 6, 8\}$, whereas the codomain was $\{1, 6, 8, 5\}$. In other words all their examples included only surjective functions (the range and codomain are the same), or bijective (one-to-one). They lacked examples of injective functions (the range and codomain

are not the same). He used this discussion to help teachers understand the difference between range and codomain, which they thought were the same. As we discussed this, the teachers were all paying attention as if the information were new to them. They also did not include examples such as $\{(1,2), (3, 5), (1,2)\}$ where the first element is repeated, but related to the same second element. The uses of technology were not discussed during the reflection meeting.

Cycle 2. For Cycle 2 this group of teachers designed a lesson about *line of best fit*. As technology tools they projected a Prezi presentation through a laptop using a VGA projector. As the technology tool for the mathematics, they used GeoGebra and Fathom. The lesson was implemented with a group of 11th graders in a public high school. For this cycle there were four teachers in this group. One teacher excused himself from the planning session, and did not come back for the rest of Cycle 2. The volunteer teacher, who implemented the lesson for Cycle 2 was different than the volunteer teacher for Cycle 1, and the school setting was also different.

Planning Session for Cycle 2. Figure 29 shows a chronological view of the codes that were used as evidence of teacher's discussions as this group of teachers planned the lesson. As it can be observed, evidence regarding their knowledge of the curriculum and the technology emerged at the beginning of the planning. As the planning progressed, discussions were more focused on the technology, pedagogy, and mathematics. Towards the end of the planning, discussions were more centered on the mathematics and the technology.

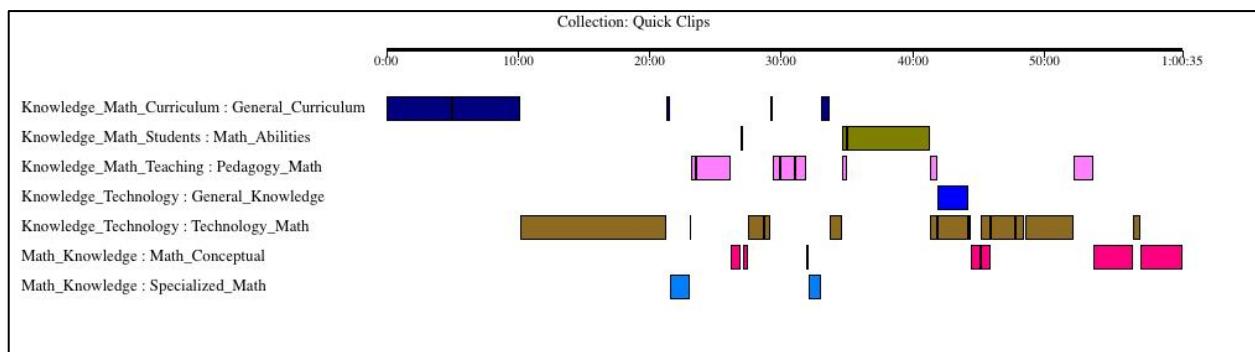


Figure 29. Chronological codes applied to Planning Session Case 3, Cycle 2

At the beginning of the planning session there were only three teachers in this group. One teacher had excused himself, and a second teacher arrived late. The group started talking about the curricular maps for grades 10th and 11th, and the standards they could work from. They had already decided Teacher 5 was going to implement the lesson, thus a decision regarding the topic had to be in accordance with the schedule of topics for the weeks after the planning session.

Evidence of their *content knowledge* emerged as they discussed the topics they considered for the lesson. They first considered designing a lesson about the properties of family of functions. They talked about what that meant, and searched for the standards that could apply to it. From the conversation one teacher asked, “odd functions are symmetrical?,” to which another teacher responded, “even functions are symmetrical with respect to the y axis, all of the exponents 2, 4, 6, 8, 10... and odd functions are symmetrical with respect to the origin... you have symmetry with respect to the x axis as well, but those are not functions.” This shows evidence of that teacher’s knowledge of the different

symmetries. After a long discussion about the topic and how to design a lesson around it, they changed their mind and started searching for a different topic. The principal reason for the change was that Teacher 5 was going to teach family of functions very soon after the planning session, which was outside of the boundaries I had given them to implement the lesson.

They got interested in a topic from the statistics and probability standards, which Teacher 5 would teach more towards the end of the semester. Searching for standards, they came across linear regression and the correlation coefficient. They thought it was a great topic to design a lesson because they could see how the use of technology could help their students better understand those concepts. Additionally, they had little to no experience teaching linear regression and the correlation coefficient and they could take advantage of my knowledge on the mathematical content and the technology. For example, Teacher 1 wrote in her journal “I felt good although [linear regression] is a new topic for me because I had never gotten to teach it.”

As they planned the activities for this lesson, their *knowledge of math and students* was evident. They had planned to discuss the correlation coefficient on a class previously to the lesson of focus here. To start the current lesson, the teacher wanted to introduce the idea of what a scatter plot was. The data was about the number of members in a family and the number of cleaning products they had. While planning this activity, Teacher 2 asked Teacher 3, “you have to think about the students... if you use many points they are not going to understand... and I want small numbers. Do you want to use decimals?” They were using

their *knowledge of students and math* to decide which numbers to use on the ordered pairs for this activity.

Their idea, initially, was to use Excel to show what a scatter plot was. They did not talk about this for long, as they considered using GeoGebra for this instead. As they discussed how to use GeoGebra, they showed evidence of their *technological knowledge* as they talked about how to plot points, and how to draw a line that could be moved around. In doing so they wanted to show how different lines could be used to model a linear relation between two variables, without mentioning correlation or linear regression, just a visual understanding of what it meant to have a linear relationship. As they discussed these pedagogical decisions, they showed evidence of their *knowledge of math and teaching* as well.

Through the process of planning the activities for the lesson, I suggested they could use an activity using the technology tool Fathom. The goal of the activity was to find the least squares line for a data set containing information regarding vehicles (e.g., make, model, mpg for city, mpg for highway, weight). This technological tool was new to the group of teachers, thus they needed some time to explore the activity themselves.

Issues with their *knowledge of technology* emerged as they used Fathom, a technology tool that was new to all of them. Since it was a new tool, I let them play around with the tool for a while. I showed Teacher 2 the basic features of the tool, such as the different menu options they had available, how to view the data set as a table, and how to create a graph, among others. I also showed Teacher 2 what they needed to teach the concept of line of best

fit, such as how to add a moveable line, or the least square line, and how to show/hide the squares. After noticing the tool was not complicated to use, they incorporated the tool into the design of their lesson. During the planning there are long periods of silence as Teacher 2 played with the tool, practicing what I have showed her. As she was getting comfortable with the tool, she shared with the rest of the group that she was able to add the squares without my help. She showed the computer screen to Teacher 3 and she said, “oh but it has the squares,” to what Teacher 2 proudly responded, “because I learned how to do it.” She learned how to open the database on a table and how to drag variables into the Graph Window. She also showed the others how to add the moveable line with the squares, and how to add the least square line to the scatter plot. She demonstrated all these to the other teachers with ease. This could show, at least, her abilities for learning how to use new technology tools, and her enthusiasm in learning them.

As they planned the lesson, they had many issues with their knowledge of the mathematics. For example, Teacher 5 seemed to be confusing the correlation coefficient with the sum of squares. She engaged in this conversation with Teacher 2:

T5: as I move this line and this number is getting smaller it means I am getting closer to that line of best fit?
T2: exactly (as she moves the line)
T2: and you see... it started to get bigger again
T5: Okay, then it has to be a number between -1 and 1
T2: no
T5: I see here 1.4
T2: that 1.4 is the intercept... you have to look at this number here 4433 (the sum of squares of the line they were working with)

From this conversation it seems that Teacher 5 expected the sum of squares to be between -1 and 1, when this range belongs to the correlation coefficient. Teacher 2 followed then explaining that when the moveable line is used, it only shows the sum of squares. The coefficient of determination (r^2) as well as the sum of squares only appear when the computed least square line was shown. Figure 30 shows both lines in Fathom, and the information each one offered at the bottom.

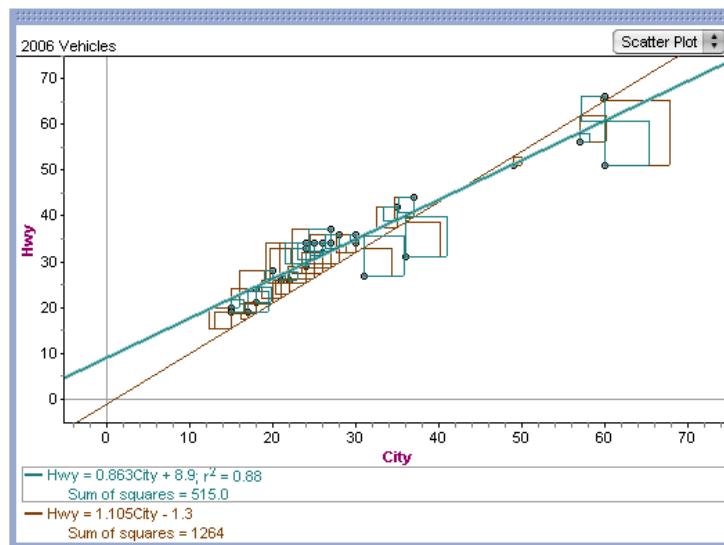


Figure 30. Moveable line and least square line, screenshot from Fathom

In addition, during the planning I asked the group some conceptual questions to get a sense about what they knew about the topic. For example, I asked them about the importance of linear regression and what were their uses in real life. I got no answers from the group. More issues with their content knowledge emerged toward the end of the planning when Teacher 5 asked, “how am I going to tell the students that it is not just because... I mean just because the program [Fathom] is saying that is the line that best fit?” Teacher 2 said, “I guess

so because you are not using the formula." They were talking with me, so I talked with them about the squares that were created using the observed and the adjusted data points, and that we add the areas of those squares. The least square regression line is the line which sum of those areas is the smallest possible and they seemed not to be aware of that.

Implementation of the Lesson for Cycle 2. For the implementation, Teacher 5 was able to find a laboratory with desktop computers for each student. Previous to the class, she had installed GeoGebra and Fathom to all the computers. She had also saved the files she would need her students to access during the lesson. Before students entered the lab, she had turned all desktop computers on, and was ready with her laptop and projector set and the Prezi presentation showing (using Prezi was not even mentioned during planning). This helped her to start the lesson on time, and to maintain control as all students entered the room and logged in into their individual computers. The teacher started the class by distributing the worksheet with the activities for the class. The worksheet contained a series of questions the students were asked to complete as they engaged with the tool. The beginning of the lesson and how the teacher managed it demonstrated her *knowledge of pedagogy*.

The teacher started the lesson without introducing the topic. Rather, she instructed her students to open a GeoGebra file to work with a scatter plot that was already created (Figure 31). The scatter plot represented family size versus number of cleaning products and the teacher wanted the students to draw a line and to locate it where they believe the line of best fit was. Issues with her *knowledge of math and teaching* were evident here, as the teacher did not review the concepts from the previous class. It seemed they had discussed some of the

concepts the previous class, but it was not clear for me as I observed the implementation of the lesson.

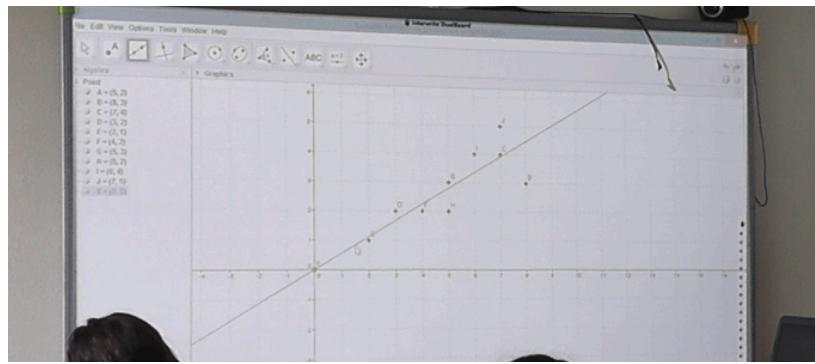


Figure 31. Scatter plot: family size vs. cleaning product, Case 3, Cycle 2

After all students had located a line, the teachers asked if there was or was not correlation between family size and number of cleaning products and if there was, which type of correlation it was. There were issues with the mathematical content, since she should ask that question differently. First, students needed to observe if the variables were linearly related. If so, then she should ask if they seemed to be positively or negatively correlated. What she wanted with that question was for the students to observe that a line could be used to model the situation. Then, she asked her students to interpret the slope and the y-intercept of the lines they each found, within the context of the situation.

As the discussion of the different equations emerged, one student said her equation was $0.86x - 1.66y = 0.66$. She had the equation in standard form, so she could not interpret the slope and intercept. The teacher stepped in and said they could change it to slope-intercept form within GeoGebra. She went back to GeoGebra and said “you can right-click

over the equation and you will see it says equation $y = mx + b$." This shows evidence of their *technological knowledge* when it comes to the use of GeoGebra. She continued her discussion, and after some equations were written on the board she emphasized they all might have different equations. Many students affirmed their equations were different, thus the teacher asked "how do we know which line is the best fit for the data?" Instead of showing the line in GeoGebra, she moved back to Prezi, and showed a definition for line of best fit.

Within the definition, she had the equation $y = a + bx$. Issues with her *content knowledge* emerge as she tried to explain the definition when she said "remember that we discussed this yesterday, and we said that a represents the height and bx was the intercept [of the line]." She continued saying "this intercept is going to tell me if there is a direct or inverse relation..." These two statements are incorrect. First, b would represent the steepness of the line while a represent the y-intercept. Second, the slope tells me if the relationship is direct or inverse, not the intercept. However, it is possible that she might have just inverted her thought as she discussed the definition. She seemed to be nervous as the video camera was following her.

As she moved to the next activity, the students had a detailed worksheet with steps that included all the instructions to complete the activity using Fathom. She guided the students as they worked on the activity, following the worksheet. They opened the file that has the database already saved. The database had information of different models of vehicles, and the students were asked to find the line that best fit the scatter plot with vehicles' city

mpg on the x-axis and vehicles' highway mpg on the y-axis (Figure 32). Similar to what they did in Activity 1, students were to use a movable line and locate it where they thought it best fit the data. Then they had a whole class discussion where they observed they had found different lines, and used those lines to make predictions. The teacher asked "What does the graph tells us?" and most of the students responded there "was a direct relation" and "a positive correlation." As the teacher discussed this activity she showed evidence of her *content knowledge*, and *technological knowledge*.

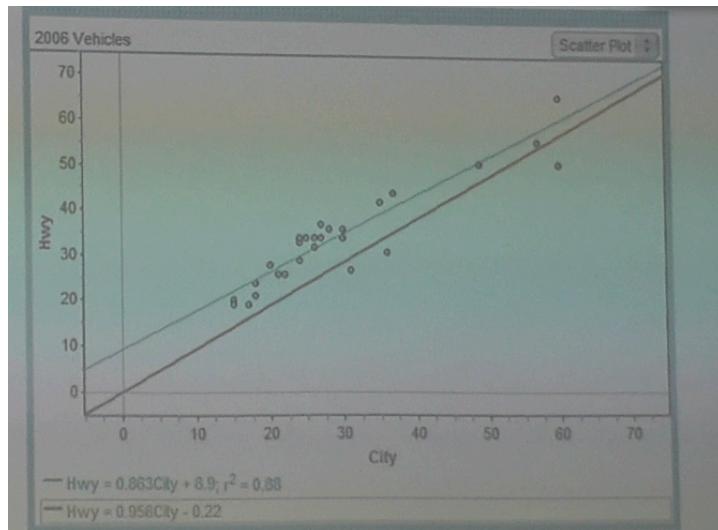


Figure 32. Scatter plot, moveable line, and line of best fit

To finalize the activity, the teacher demonstrated how they could find the least squares line and asked her students to follow her demonstration and do the same in their computers. She asked several students what was the equation of the line they just have found. They all realized the line was the same for the teacher, and in each of their computers. She then instructed her students to show the squares of both lines in their computers. She said to

observe and compare the values of the r squared. With her help they said one r squared was smaller than the other. However, the teacher did not discuss how those squares were created, and their importance in calculating the r-squared value. Understanding the role of those squares is an essential piece in understanding how the least squares line of best fit is obtained. Again, this is evidence of her *content knowledge*, moreover her *specialized content knowledge*. The lesson was concluded by an activity where students were asked to answer a series of question were they reflected about what they have learn in class, and what they thought was still confusing. There were not closure remarks to summarize the most salient points from the lesson.

Reflection Meeting for Cycle 2. Two teachers in this group did not participate in the reflection meeting. After watching the implementation of the lesson, the teachers in the group made some observations/suggestions. Teacher 2, reflecting on her *specialized content knowledge* observed that the use of the squares at the end of the lesson was left in the air. She felt there was no connection between the line of best fit, the data on the scatter plot, and the squares. As Teacher 3 reflected on her knowledge of pedagogy she observed that at the beginning the lesson was slow, and that this time could have been used at the end to summarize the big ideas of the class.

Next, I made a few observations. First, I suggested that she needed to be more careful with the use of the vocabulary. As she discussed the activities during class she talked about a line, an equation, and a line of best fit to refer to the same object. This might be a consequence of her issues with her *knowledge of the mathematics* combined with her novice

experience with the technology tool. In addition, she talked about R squared as a correlation coefficient. It is important to notice the correlation coefficient is R, not R squared and she never mentioned in class. Moreover, during the meeting she looked surprised to learn that. This goes back to the issues she had exhibited regarding her *knowledge of the mathematics*.

I also observed that the use of the squares and the discussion of the R squared should happen before, perhaps in the discussion of Activity 1. They needed to make a clear connection between the squares, the points on the scatter plot, and the least squares line. A longer discussion emerged when I asked the group of teachers if they knew what the squares represented. I got no response from the teachers. Thus, I took advantages of the meeting and explained to them where do the squares came from, and its relation to the formula to calculate the correlation coefficient. All members of the group were lacking this important conceptual *mathematical knowledge*.

Within-Case Analysis: Case 3

Overall, Case 3 showed how having issues with understanding mathematics could lead to lessons that are vague and repetitive, even when technology is not in the picture. (Cycle 1). When technology is brought into the picture, and there are issues with the technological knowledge and the mathematical knowledge, even if there is evidence of knowledge of general pedagogy, the technology is not used in significant ways to enhance student's learning.

One of the biggest issues this group had during Cycle 1 in terms of the mathematics and the pedagogy was starting from an already designed lesson. This action did not allow the

group to have important conversations around the mathematics being taught, and which pedagogical approach was best for the topic. As a consequence of the evident struggles they had with the mathematical content, this group found it difficult to find a way to integrate the technology into the teaching of domain and range of functions. Not having a deep mathematical knowledge, and lacking interest in trying new pedagogical approaches were gigantic barriers for this group as they tried to use technology to teach a topic they thought was “easy to teach” (Teacher 1, Transcript_Group3_Cycle1). In this case, technology integration was vague and lacked significance and pertinence to the lesson.

During Cycle 2, this group planned a lesson around a mathematical topic they found was complex, a technological tool they had never seen before, and a pedagogical approach that was new to them. Lacking that mathematical knowledge did not allow the group to take advantage of the representations and direct manipulations available in the technology, and the fact that each student in the classroom had their own computer to explore. Even though they could have general pedagogical knowledge, the group lacked the pedagogical knowledge specific to teach that particular topic. In this particular situation, not having a strong mathematical knowledge seemed to be a major barrier to the group to use technology in meaningful ways.

Taking into consideration these teachers mean score per construct from the EMMES-CTPM pre- and post-survey, there are some observations that are worth mentioning (see Figure 33). These teachers planned a lesson on a mathematical idea they seemed to have difficulties with during planning of Cycle 2. Observing the mean scores for the CK construct

for each teacher, only T2 perceived herself more knowledgeable about the mathematics. The teacher that implemented the lesson seemed to be aware of the content knowledge needed to teach mathematics, more than what she perceived knowing when the professional development started.

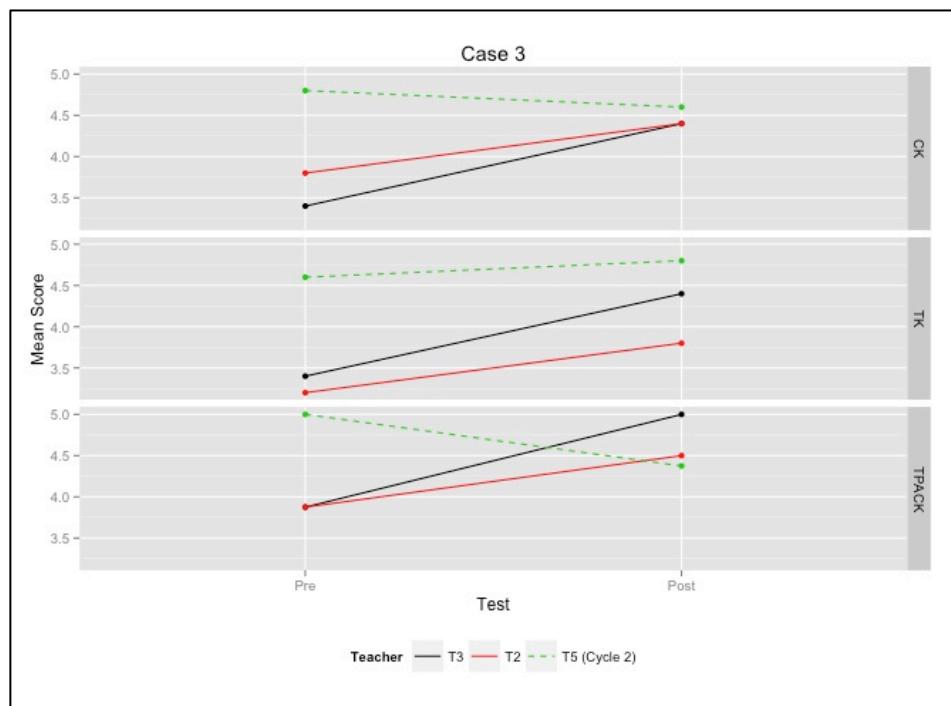


Figure 33. Pre and Post mean score per sub-scale construct per teacher for Case 3.
Dashed and dotted lines represent the teachers that implemented the lessons.

Their self-perception of their knowledge of technology was surprising, as it does not reflect their experiences during the professional development as they learned how to use some technological tools during the cycles. During planning, T2 received a one-to-one training on how to use Fathom. She learned very quickly and showed T5 how to use the tool to perform what she needed for the implementation. Thus, that reduction of T2 and T5 self-

perceptions on their TK is surprising. Perhaps, the reduction could reflect how those teachers perceived their knowledge of technology after the professional development.

This group engaged in a very challenging class for Cycle 2, which might increase their awareness on the importance of a deep conceptual understanding, new pedagogical approaches appropriate to teach with technology, and the knowledge of those tools specific to teach mathematics. That engagement could have had a positive impact in this group's self-perception of their TPACK. For the teacher that implemented the lesson, the decrease could mean her self-perceptions of her TPACK is more realistic after engaging in the professional development activities, specially the design and implementation of the lesson during Cycle 2.

Case 4

Case 4 consisted of four middle school mathematics teachers, two from public schools and two from private schools. There were one female and three males. The years of experience teaching range from one to 12 years. The teachers that implemented the lessons had 10 and 12 years of experience teaching, respectively. Teacher were from two different school districts. All four teachers participated through the end of the professional development. This group showed having a good grasp of the mathematics they teach. In addition, they exhibited evidence of their knowledge of general pedagogical. But this group demonstrated having issues with their knowledge of technology, as it concerns with their knowledge of tools specific for the teaching of mathematics.

Cycle 1. For this cycle, they chose to design a lesson on *slope of linear functions*. The volunteer teacher implemented the lesson with a group of 8th graders. The teacher used a

laptop computer projected on a smart board. He used Power Point and GeoGebra on the smart board and each student had graphing paper and a ruler.

Planning Session Cycle 1. Very early in their planning, this group of teachers decided to focus their lesson on the concept of slope of linear functions. They started the detailed lesson plan right away and, as they typed the lesson, discussions around the types of examples to include, student's previous knowledge, curriculum, student's math abilities and misconceptions, among others, emerged. Figure 34 shows a chronological view of the issues as they emerged during planning.

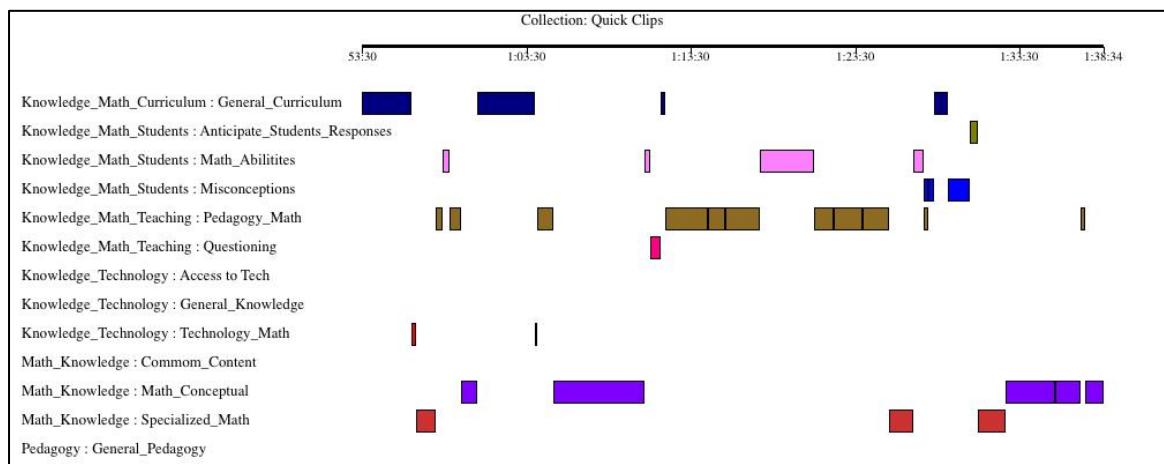


Figure 34. Chronological codes applied to Planning Session Case 4, Cycle 1

At the beginning of planning, Teacher 1 and Teacher 2 made it clear to the rest of the group they were not willing to be observed implementing any of the lessons, perhaps because they were shy and were not comfortable being videotaped. Thus, Teacher 3 and Teacher 4 decided that each of them would implement one of the lessons. Teacher 3 would implement the lesson for Cycle 1, and he only teaches 8th grade. They planned a 4-day lesson on slope of

linear functions using different representations, but he would be observed implementing only one of the lessons. For Day 1 they focused on finding slopes when the function was represented as a table of values, Day 2 when represented as a graph, and Day 3 when represented as an equation. For Day 4 they were planning on combining all representations as a review and having students determine which linear function had the greatest rate of change. This 4th lesson was the one observed during the implementation phase. For this 4th lesson they had planned to use GeoGebra.

Issues with *curriculum knowledge* were very limited. At first only one of the teachers had some previous contact with the new set of standards. She shared the standards with the rest of the group and they all started searching for an appropriate standard for grade 8. Since the standards were separated by grade level for middle school, it was not difficult to find it. Other conversations around curriculum happened throughout the planning session as teachers discussed the previous knowledge their students needed in order to complete their expectations for these lessons. This included, for example, that their students should know how to plot points on a Cartesian plane, and they should know how to solve two-step linear equations.

Evidence of their *knowledge of math and students* emerged as they planned these lessons. Through the discussions they talked about their students' math abilities and misconceptions. Students' misconceptions included how students confuse the axes on the Cartesian plane when plotting a point. For example, when plotting the point $(2, -5)$ they locate 2 in the y-axis and -5 in the x-axis. Also, they noted how it is difficult for the students

to plot axial points, those that have a coordinate equal to zero and are located in the axes such as (6,0). This is a crucial skill needed for graphing. In addition, as they debated whether or not to include lines with an undefined slope, they talked about students' misconceptions of dividing by zero. They all agreed that dividing by zero was a difficult concept for their students and decided to leave vertical lines out of the lesson. This knowledge is also evident in their journals as they all agreed students have issues with the rules of signs for operations with integers, solving an equation for a variable, and graphing.

Instances of their *knowledge of math and teaching* were evident throughout the entire planning session. Their *knowledge of pedagogy* with respect to mathematics took place in many occasions. For example, they considered the order they should discuss the different representations to help students better understand the concept of slope. They decided they should start with the table of values so that students had to use the formula for slope with ordered pairs. Then, using the table of values, they were going to ask the students to graph the line. Using the idea of rise over run, they would connect the value of the slope they found before with the one found using the graph. Last, they would use equations to demonstrate that the slope of the line is given by the coefficient of x . As mentioned before, they decided to leave out examples of linear functions with undefined slopes. They also discussed how they should start with simpler examples and leave those more complex to the end. Another pedagogical decision they made was to discuss increasing, decreasing, and zero slope on Day 2 when discussing the graphs because it was more visual and thus easier for the students.

Their questioning was not evident in their planning. There was one instance where one teacher said, “you could ask, in the table of values, what does that first ordered pair represents in the Cartesian plane? Just to make sure they know what a point is.” However there were not any other instances where they referred to the questions they would ask their students during the implementation of the lesson.

As it can be observed in Figure 34, there was minimal evidence of teachers’ *technological knowledge*. Since there was only one teacher willing to volunteer to implement the lesson, they did not talk about their access to technology in their classroom or school. The teacher that implemented the lesson had a smart board in his classroom, but there was not a conversation about the technology tools each had access to. Their technology integration, as they mentioned during planning was to use GeoGebra to graph the linear functions, and use it to draw the triangles to show that for a line, rise over run was always the same. Then they would use those lines to show which one had the greatest rate of change. Their *technological knowledge*, as it relates to the mathematics, was evident as they talked about using GeoGebra to draw the graphs. However, in the written lesson plan they did not mention they would use GeoGebra.

Regarding their *mathematical knowledge*, this group used the correct vocabulary all the time during planning. They referred to the slope of the line, the y-intercept, the coefficient of x, independent and dependent variable, ordered pairs, slope-intercept form, and standard form and used the terms appropriately throughout their conversations. This is a crucial part of their specialized mathematical knowledge, to be able to use the vocabulary

correctly. However, they showed some flaws in their conceptual mathematical knowledge.

For Day 4 they wanted to focus the discussion around rates of change and have the students identify the line with the greatest rate of change. As they discussed this, one teacher said that a slope of -5 had a greater rate of change than a slope of 2. Through the discussion one teacher said, “you have to see its steepness” (Teacher 3), and later on during the discussion they talked about seeing rate of change as the absolute value of the slope, which is not mathematically correct. They talked about a slope being constant for a line and how they will demonstrate and emphasize that during the lesson. This is an important characteristic of linear functions and teachers were aware about it and made constant reference to it during the planning sessions.

Something very particular of this group was how they organized their planning. These teachers first focused on the big picture of the lesson plans and then went to fill up the details. For example, they first divided the plan into introduction, body, and closure. In there they put bullets with very limited instructions of what is supposed to be happening. This included sentences or phrases such as, “[the] teacher will give students a linear equation and students will assign values for the independent variable to obtain values for the dependent variable,” and “complete additional examples.” However, they did not think of what equations they might include, or which additional examples they should incorporate into the lesson as they think of the order that topics needed to be discussed. Once they had the bullets completed and had structured the lessons, they started thinking about the examples and practice problems. They might have felt confident on organizing their planning in that way

because of their teaching experience, and the comfort level they had with the topic. This experience and confidence were evident in the teacher's journals for this cycle. For example, their journals had sentences such, "I feel good because I have 7 years teaching the same grade or subject" (Teacher 2), "this concept of slope I have worked with my students in the past years" (Teacher 4), "I have had experience and felt confident focusing the lesson of slopes" (Teacher 1), and "so far is a material that I understand well and it is the level that I teach" (Teacher 3).

Implementation of the Lesson for Cycle 1. For this implementation the teacher is using the 4th lesson they planned. They have discussed the other three lessons previously. The teacher started the class by reviewing what they have been learning about during the last three classes. He talked about slope and how they can find it either from two ordered pairs using the formula, or by drawing a triangle and finding rise over run, or leaving the equation in slope-intercept form and observe the coefficient of the x . Using the smart board, he projected the practice problems so that students could work on them (Figure 35). During this lesson, students worked individually on a series of problems, practicing the different skills they had learned for how to find the slope of a line.

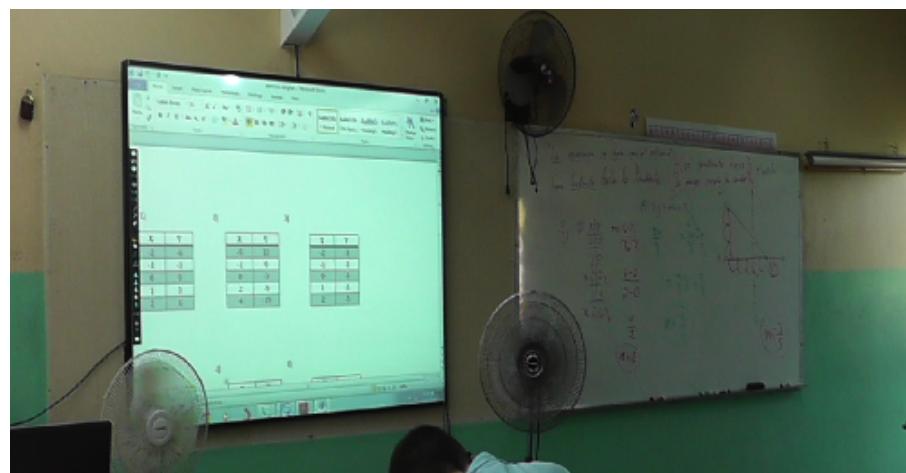


Figure 35. Practice problems using table of values projected over the smart board

Overall, the teacher implemented the lesson as the group had planned. He started with a short review, and explained to the students that the slope represented a rate of change. Then students worked individually, and the problems were then discussed on the board. Their closure was the discussion about greatest rate of change among the problems they have worked. The lesson was very passive and traditional and lasted 60 minutes. There were 5 minutes used during the opening of the lesson. Students worked individually for about 30 minutes. Then, the teacher sent students to the board to work on the problems, which lasted about 20 minutes. For the closure, since he did not have much time left he mentioned that the lines with slopes of 3 and -3 had the greatest rate of change because, as an absolute value, they were the same and the largest numbers. As mentioned before, there is an issue with this teacher's *mathematical knowledge* when it comes to their understanding of the concept of rate of change. In the same manner, there is an issue with his *knowledge of math and students* as this is a common misconception he should be aware of.

While the students worked individually on the problems, the teacher walked around answering students' doubts and difficulties. They worked on the problems, and participated in class. As the teacher asked questions to the whole class, many of them answered simultaneously the correct response. It was clear from the implementation that the students know how to compute the slope of a linear function.

Even though the group did not discuss access to, or uses of technology during the planning, this teacher had no issues with his *technological knowledge* while implementing the lesson. He used the smart board and had no problems managing it, writing, erasing, or moving from one document to the next. However, they did not use GeoGebra as they had mentioned during planning. Thus, their only technology tool was the use of the smart board as a display.

Slope is a mathematical concept that can be connected to real situations and this group did not take advantages of this. All their examples were numeric, using tables of values, equations, or graph of any particular linear function. But they could have created an example using velocity, a concept that the students use in their daily routines and represents a rate of change. Since they are discussing linear functions they could tell a story where they see slope in a situation they can relate with. For example, a velocity of 34 miles/hour means that a car moves 34 miles in one hour. The slope is a ratio between two variables, in this case distance and time. The lesson lacked a discussion about what a rate of change is, and examples of its meaning in practice.

Reflection Meeting for Cycle 1. The reflection meeting for this group was very short. The time they had to implement the lesson was the longest, 60 minutes, and the reflection meeting lasted an 80 minutes. We first watch the video of the implementation. Then the teacher that implemented the lesson had his turn reflecting on his teaching. He was not critical in his reflection, and stated the implementation was good. The rest of the teachers did not find problems with the implementation either.

When the mathematician started to make observations on the implementation, the teachers reacted and started to reflect on things they do differently and what they would change to the plan. Regarding issues with *pedagogical knowledge*, for example, they noticed that when finding slopes using tables of values the students applied the negative and change the signs in one step (Figure 36, left). In addition, when an equation has the term with the y alone in one side, he divided by its coefficient in the original equation instead of adding a step in the solution (Figure 36, right). They all agreed this could create confusion to many of the students when they are studying later from their notebooks.

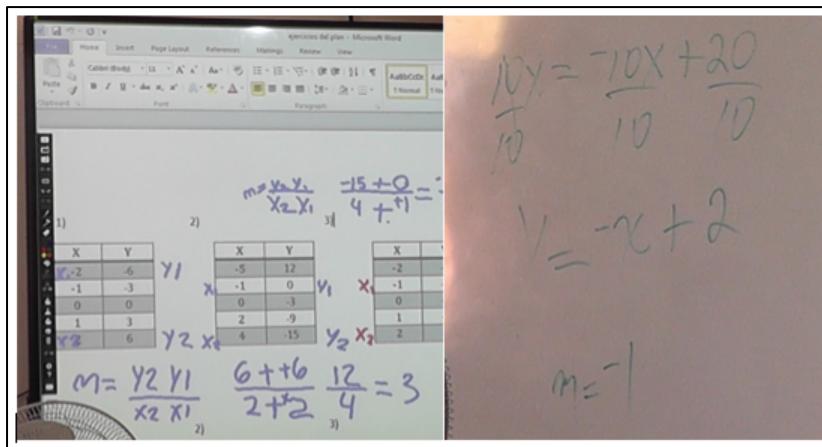


Figure 36. Student's (left) and teacher's (right) work on the board, Case 4, Cycle 1

They also mentioned to have students talk out loud when solving problems on the board. This helps the teacher better understand the student's reasoning and to make the class less passive. Another observation the mathematician pointed was to have more than one student on the board at a time to help save time during the lesson. During the implementation a single student solved a problem on the board, followed by the teacher discussing it. If the student is not asked to explain what they are doing as he solve the problem on the board, it would be more beneficial to have multiple students on the board at a time and then the teacher can discussed them. The mathematician also pointed that in various occasions he ignored the responses of the students. For example, in one problem he asked the value of slope to which some students responded $2x$. He did not use the occasion to emphasize that the slope is only the coefficient, a numerical value, and that it cannot contain any variable.

Regarding his *mathematical knowledge*, during the implementation he was consistent with the correct use of vocabulary. The mathematician noticed his misconception about equal

rate of change for $m = 3$ and $m = -3$ and explained they cannot be the same because one is increasing while the other is decreasing. All teachers agreed they have learned something new and will not teach that again erroneously.

When I offered my suggestions and observation to the lesson, I mentioned the lack of connection between the different representations during Day 4. The room we used for the meeting had a smart board connected to a desktop computer that had GeoGebra installed on it. Thus, I used it to demonstrate to the group how they could show that connection dynamically using GeoGebra. I wrote an equation of a linear function in standard form and its graph showed simultaneously. I showed how the equation could be changed to its slope-intercept form. I drew a right triangle using the line as the hypotenuse, and measured the legs to form a ratio. That ratio was the slope. I then used dynamic text to show how the slope remains the same as I moved the legs of the right triangle over the line. The teachers had a positive reaction to this demonstration and commented about how it could help their visual students to better comprehend what the slope is.

At the end of the reflection meeting one teacher mentioned the task of examples based on real situations. He suggested thinking about a situation that could be modeled using a linear function. They talked, for example, about prices of parking lots. Since the price increase by hour and it is constant, they could have used that as an example.

Cycle 2. This group remained the same during Cycle 2. This lesson was implemented with an 11th grade. For this cycle the group of teachers designed a lesson about *estimation of pi*. For this lesson the teachers used scientific calculators, PVC pipes of different diameters,

and measuring tape (Figure 37). The teacher that implemented the lesson during this cycle was different than the teacher that implemented the lesson during Cycle 1.



Figure 37. PVC pipes and measuring tape, Case 4, Cycle 2

Planning Session for Cycle 2. The issues that emerged as this group of teachers designed the lesson were very limited. At the beginning of the planning, their conversations were focused on the standards. Through out the planning they showed extensive evidence of their knowledge of pedagogy, as it relates to the teaching of mathematics, and pedagogy in general. Figure 38 shows a chronological view of the codes that were applied to the discussion this group of teachers had during the planning session. This group designed a lesson on estimation of pi.

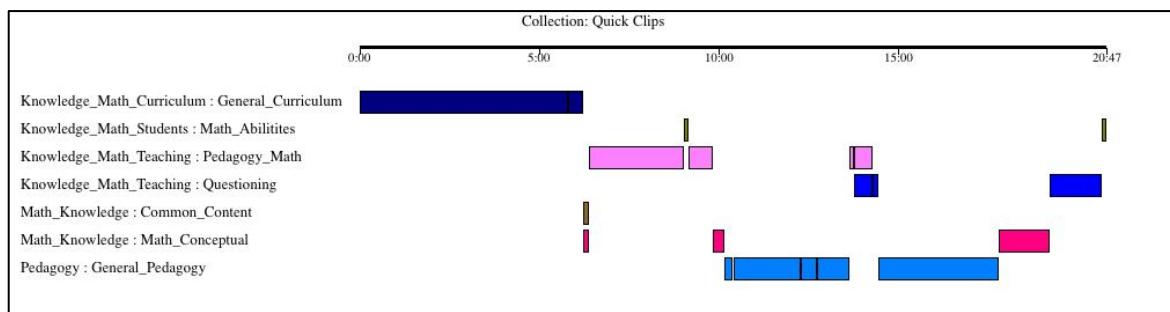


Figure 38. Chronological codes applied to Planning Session Case 4, Cycle 2

For this lesson they did not used a mathematical action technology tool that required the group to have discussions about the technology tools they had available. Thus, instances where they exhibited their *knowlede of tecnology* were very limited. The classoom of the teacher to implement the lesson was not equiped with a smart board. Even more, this teacher had no access to borrow a projector at his school. He only had a laptop computer he used to keep his student's academic records, among other administative tasks. In this case, the group had no opportunity to try a new or different technology tool that requiered the use of a computer. Thus, the only conversation they had around technology tools that were approparite to be used was mentioning the use of scientific calculators so that the students were able to find the circumference to diameter ratio. For these reasons, very limited codes of *knowledge of technology* were applied.

The idea for the lesson was brought by Teacher 4, the teacher that implemented the lesson. It was a lesson he has previously taught in his classroom. However, since the teacher did not have a previous lesson plan or power point presentation the group had to discuss the mathematics, the use of the PVC pipes, and had to make pedagogical decicions as they planned

the lesson. Teacher 4 started by explained what the activity was about, his expectations for the lesson, and how he usually teaches it. All teachers in the group agreed on how Teacher 4 originally teaches the lesson, but as the discussion progressed, they added to what Teacher 4 has done in the past.

Since Teacher 4 had no access to any digital technology tool, they decided to use a hands-on activity where students measured the circumference and diameter of circular objects using a measuring tape. The circular objects were all PVC pipes of different diameters. The goal of the lesson, according to the teachers, was for the students to deduct what the ratio between the circumference and the diameter of any circular object, independently of its size, would produce. One important aspect of this lesson that Teacher 4 explicitly mentioned was measurement errors. As Teacher 4 said, “to be able to get the most precise measurement of the objects is crucial to know how to use the measurement tool we are using.” Here he showed evidence of his *mathematical knowledge* and their *knowledge of math and students* as they recognized students’ difficulties with the topic they were discussing. However, they did not discuss what that meant, or how they would address measurement error during the implementation of the lesson, showing issues with their *knowledge of math and teaching*.

Instances of their *pedagogical knowledge* were the most predominant during the planning. As they planned the lesson all teachers in the group were making pedagogical decisions as they helped Teacher 4 organize the lesson so that, during implementation, it flew in an organized manner. They discussed the materials they were going to use, how the lesson

was going to be introduced, how the groups of students were going to be formed, and how to close the lesson, among others. As Teacher 3 and Teacher 4 discussed the details of the lesson they said that, “students were randomly assigned to groups at the beginning of the semester” (Teacher 4). They said, “each student in the group was in charge of measuring one of the circular objects... so that they all have similar experiences during the activity” (Teacher 3). At the end, the teacher was going to “ask each group to share their approximation of pi out loud” (Teacher 4) to elicit whole group discussion.

When it comes to their *knowledge of math and students*, this group talked about their student’s mathematical abilities, but did not anticipate students responses to their questions, and did not talk about possible student misconceptions. There were few occasions where the teachers mentioned how students forget what they learn in previous years, thus even though they “should have learned that in 9th grade” (Teacher 3), referring to solving equations for a given variable, they decided to review that concept previous to implementing the lesson. Reviewing that concept showed evidence of their *knowledge of math and teaching*. During planning, they did not anticipate students’ responses, perhaps because they did not have the PVC pipes with them during the planning of the lesson so they could measure the diameter and circumference of each object.

Regarding their *mathematical knowledge*, they talked about the circumference formula and how they would use it to approximate pi. Through the conversation Teacher 4 said, “we have to specify that is a circular object, because if it is a square they are not going to see it” referring to students who were not going to be able to find the relation of circumference and

diameter, but squares do not have circumference nor diameters. They also mentioned that, “students have to find the average of the six approximations they found” (Teacher 4). They would then share that average with the rest of the group during the closure of the lesson.

There were several instances their *knowledge of math and teaching* emerged. When it comes to questioning, only two instances where they refer to questions they might ask their students occurred. In one occasion, for example, Teacher 1 said, “we should ask them what they think they determined with that, what do they observed?” An answer for this question would be that what they found were values close to pi, or that they approximate pi by using the circumference and diameter of the circular objects.

Implementation of the Lesson for Cycle 2. This lesson was implemented with a 9th grade in a public middle school. This classroom had five tables, and for the activity students worked in groups of five or six. Before the class began, the teacher distributed all the materials the students would need to complete the activity of the lesson, leaving them on the table each group was going to work. The materials included six PVC pipes of different diameters, a measuring tape, a calculator, and the worksheet students had to complete. In addition, he had written the instructions for the activity on the white board so his students had them at all times. Most of these details, drawn from his *pedagogical knowledge* were not discussed during planning.

To start the lesson, the teacher reviewed what he had discussed the previous class about solving equations for a specific variable. That previous knowledge was needed to complete the activity the group had planned for the day. This decision may be drawn upon

the teacher's *knowledge of math and students*, as he anticipated student might have difficulties solving equations by a given variable. During the first ten minutes of class the teacher gave very specific instructions of what he expected his students to be working on during class. Since students were going to be measuring the diameter and circumference of several PVC pipes he emphasized some aspects about the measuring. First, the measuring tapes the students were going to use were not marked with consecutive numbers. After each multiple of 10, the numbers were restarted to one indicating there were one more centimeter, but that number one represented numbers such as 11, 21, 31, etcetera. Thus, the teacher briefly explained how to interpret the measuring tape they were using (Figure 39). Drawing upon his *knowledge of math and students*, again he made this clear for his students because previous experiences have taught him how students struggle interpreting the measuring tape. Again he had drawn a picture on the white board before the students entered the room. Second, in order to avoid significant measuring error he carefully explained how to measure the circumference, and to be cautious when measuring the diameter since it was not trivial to find it. All these instructions were not specified during the planning session, but it saved him time during the lesson.

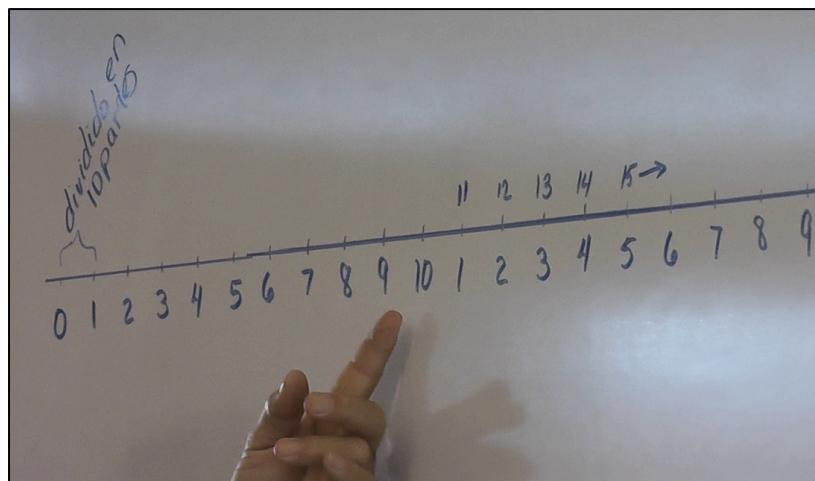


Figure 39. Explanation of the measuring tape, Case 4, Cycle 2

His *knowledge of pedagogy* showed again as he explicitly stated that each student in the group had to measure one object. In the worksheet he added a column to note the name of the students that did the measurements for that particular object. He said, “it is important that each student in the group measures one of the objects, you have to work as a team.”

For most of the class, the students were measuring and calculating the circumference to diameter ratio for all the PVC pipes in their group’s table. The teacher was walking around the classroom, asking questions to his students, responding to his students’ doubts, and making sure the measures they were obtaining were reasonable. Since he has used the activity before, he had a sense of the measures of the circumferences and diameters of the objects they used in the lesson. When he observed a measure was not reasonable, he asked the students that measured that particular object to measure it again in front of him. In doing so he could better understand why the group was getting that measure. This anticipation was

not approached during the planning since the group did not have the PVC pipes with them to do the measurements and complete the table they were going to ask the students to complete.

For the whole class discussion, the teacher asked each group to share the approximation they got. Through this short discussion his *mathematical knowledge* played an important role. He knew those approximations had to be as near as possible to 3.14. After all groups had shared their approximations, he closed the lesson by stating that, “they had approximated the value of pi using the circumference and the diameter of the PVC pipes.” This might not be a surprise for many of the students that already knew the formula to find the circumference. In addition, the teacher started the lesson by writing the formula $C = \pi d$ on the board. On the worksheet, it was explicit to measure the circumference of the object and its diameter. They were then asked to divide those numbers. Given that they had learned to solve equations, they could have solved $C = \pi d$ to get $\pi = \frac{C}{d}$. Issues with his *knowledge of math and teaching* were evident here as he started the lesson by giving a formula they could easily solve for pi. Thus, instead of approximating pi, through that lesson students were verifying that the circumference to diameter ratio was indeed pi.

Reflection Meeting for Cycle 2. The reflection meeting for this group was short. We watched the implementation and then each teacher had the opportunity to reflect on the lesson they had designed. Teacher 4 said he felt comfortable teaching the lesson and that he thought he had accomplished the lesson’s main goal. The rest of the teachers had no observations for Teacher 4 and all agreed the lesson was implemented successfully.

The mathematician was satisfied with the lesson, and mentioned how students were engaged the whole period as they measured, annotated, and used the calculator to find their approximations. His *mathematical knowledge*, as well as his *pedagogical knowledge* were evident as he justified the decisions they took as a group. The mathematician's main suggestion was to change the objective of the lesson. According to him the students were not estimating pi, they were just verifying that the ratio of circumference to diameter for any circular object was pi. Given the teacher did not show evidence of having issues with his *knowledge of the mathematics*, either at the planning or implementation of the lesson, there was no need to go over any conceptual gaps or flaws.

Another suggestion was to make some changes to the activity so it could be used for the students to discover the formula to find the circumference of a circle. I suggested not using the words circumference and diameter, and instead using distance around and distance across. This wording could distract student's attention to a formula they might hear in the past, and work with a different terminology they could relate to the vocabulary they already know later on.

We also suggested making changes to the table the students used to enter their measurements. They had a column named "Formula" and it was not clear what they meant by that. What they expected the students to write was, for example, 11.15/3.51, where 11.15 was the measure for the circumference, and 3.51 the measure for the diameter. Then, a last column named "Answer" which meant to execute the division and to write the final result, in

this case 3.18. We suggested to remove that column “Formula” and to just have a column named “Find C over d.”

Within-Case Analysis: Case 4

This group was very particular in the sense they had very limited access to technology in their classrooms and schools, thus their technological knowledge as a group was minimal. But, in terms of their mathematical knowledge and pedagogical knowledge, this group demonstrated their strength during both cycles.

During Cycle 1, this group exhibited a strong mathematical knowledge regarding slopes of linear functions. It is evident they draw on their previous experiences teaching this topic. As they felt so confident on their mathematical knowledge about the topic, they did not focus on making the lesson more challenging to their students. They demonstrated a solid knowledge of general pedagogy as well. Their pedagogical knowledge specific to mathematics was not evident, as they taught the lesson in a very traditional way. This group did not try a different pedagogical approach, and technology use was limited to a smart board to project a word document with the problems students were working on.

During Cycle 1, this group could make a better use of their time during this 4th lesson. They could have used what they had explained, and worked with, for the past three lessons and make extended connection between the multiple representations of functions. For this closure they could have used GeoGebra to demonstrate, in a dynamic manner, how the slope is seen in the equation, the formula using two ordered pairs, and the graph using the right triangle. This could bring a more visual connection for the students.

For Cycle 2, this group did not use a digital technological tool aside for a calculator to compute a ratio. Instead they used a hands-on activity. This group's mathematical knowledge, combined with their general pedagogical knowledge, was adequate to portray a lesson where students were actively engaged. This lesson demonstrated that a digital technology tool is not the only way to motivate students in a math classroom. The uses of hands-on activities are excellent resources for teachers that lack access to technology. The importance is having that mathematical, and pedagogical knowledge needed to design a lesson that is meaningful for students' learning.

When the results from the EMMES-CTPM pre- and post-survey (Figure 40) are compared with the descriptions of this group's planning-implementing-reflecting, some of the changes in the different constructs were surprising. Changes of this group's self-perception of their mathematical knowledge might be a result of the positive feedback they received during the reflection meetings as no mathematical issues were found. However, when observing the teachers individually, the reduction on T1 perception of CK is quite surprising, as she seemed to have a good grasp of the mathematical concepts the group focused their lessons on.

As observed in Figure 40, there was an important increment in the self-perception of their technological knowledge for all teachers in this group. As the professional development progressed, and even though they decided not to integrate technology tools into their teaching, perhaps because of the lack of access or lack of interest, the group was exposed to GeoGebra. This exposure was very minimal and only occurred during the reflection meeting

for Cycle 1. The increment is surprising given this group did not attempt learning about any technology tools during the professional development activities.

In addition, although two teachers seemed to increase their perceptions of TPACK, that increment is of about .25 points and the other two showed a reduction. This is not surprising either because this group did not integrate technology into their mathematics teaching, thus not impacting their knowledge to teach mathematics with technology.

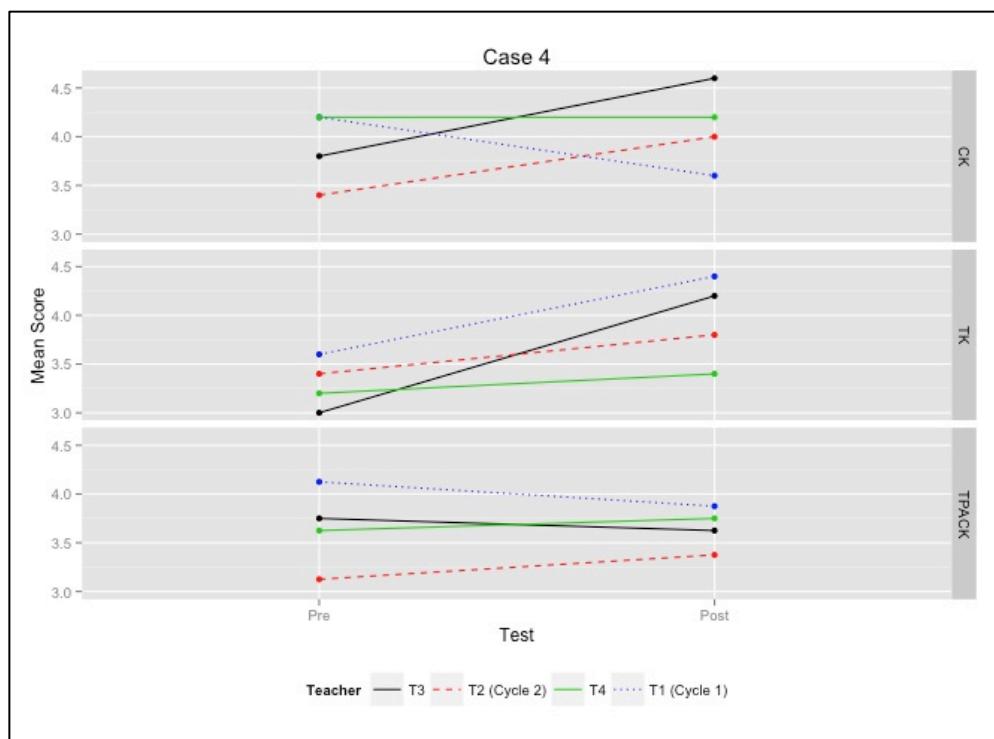


Figure 40. Pre and Post mean score per sub-scale construct per teacher for Case 4.
Dashed and dotted lines represent the teachers that implemented the lessons.

Chapter 6

Conclusions and Recommendations

This final chapter will present a discussion of the results presented in Chapter 4 and Chapter 5, to succinctly respond to the research questions this investigation aimed to answer. Using a multi-case, convergent parallel design, I aimed to investigate the issues that emerged as four groups of secondary mathematics teachers carefully designed lessons that were to integrate the use of technology. First, this chapter reviews the research questions and uses quantitative and qualitative data to answer those questions, making explicit connection to existing literature. The chapter continues with a description of the limitations of the study as well as the implications for classroom mathematics teachers, teacher educators, and providers of professional development. The chapter concludes with recommendations for future research.

Summary of Findings for Research Question 1

How do a group of secondary mathematics teachers' self-perceptions of their technological knowledge (TK), mathematical knowledge (MK), and technological, mathematical and pedagogical knowledge (TPACK) change as a result of engaging in a Lesson Study focused on designing mathematics lessons that integrate technology?

Understanding how TPACK develops and expands has been an area of research with preservice and inservice mathematics teachers for the past decade (Guerrero, 2010; Lee & Hollebrands, 2008; Mishra & Koehler, 2006; Niess, 2005). Through all this time, limited research has been done where surveys have been used to measure changes in teachers' self-

perceptions of their TPACK. Most of the research has been done with the intention of validating a survey or with a population different than inservice secondary mathematics teachers (Graham et al., 2009; Handal et al., 2012).

Secondary mathematics teachers' self-perceptions of their technological knowledge (TK), mathematical knowledge (MK), and technological, pedagogical and mathematical knowledge (TPACK) increased significantly as a result of engaging in a Lesson Study focused on designing mathematics lessons that integrate technology. Results from the EMMES-CTPC pre and post survey, discussed in Chapter 4, suggests that engaging inservice secondary mathematics teachers in cycles of Lesson Study can make a positive impact in teacher's knowledge for teaching mathematics with technology. According to these findings, having teachers collaboratively design mathematical lessons that integrate some technological tool, go into their actual classroom and implement it, and then reflect on that planning and implementation with peers, might provide the structure teachers need to increase their self-perceptions of these particular knowledge: TK, MK, and TPACK.

An increment on the average score of teachers' self-perception of their technological knowledge (TK) suggests teachers perceive they are more knowledgeable about the technology in general, and its uses to teach mathematics. Perhaps the Lesson Study provided opportunities for teachers to learn about the existence of some mathematical action tools (e.g., GeoGebra, Fathom), to better understand how those tools could be used in the classroom, and eventually see some of those tools in practice as their lessons were implemented, or during reflection sessions. Research supports the idea of using a Lesson

Study to help teachers develop their knowledge of technology and their TPACK. In a study with two Australian secondary schools, Pierce and Stacey (2009) used a modified version of a Lesson Study to design a lesson to explore the affordances of connecting multiple representations of the TI-Nspire, which was a new technology at the time. They found that Lesson Study helped mathematics teachers improve the quality of technology use. Meng and Sam (2013) also found that when pre-service secondary mathematics teachers were engaged in a Lesson Study as part of a methods course, teacher's self-perceptions of the TK increased significantly increased. During the method course, pre-service teachers were required to design a lesson using The Geometer Sketchpad.

When it comes to the mathematics, results indicate teachers' self-perceptions of their mathematical knowledge (MK) also increased. This does not mean they know more mathematics after the professional development, but that they perceived themselves more knowledgeable about the mathematics they teach. This increment might be caused by the experiences these teachers had as they went through the two cycles of the Lesson Study. Those experiences might include the conversations around the mathematics teachers had as they planned the lesson, and the observations and feedback they received during the reflections meetings. Perhaps, this increment could also be attributed to the Saturday Academies these teachers attended that were focused on the mathematics. Results suggest these experiences have impacted their perceived knowledge of the mathematics positively. This finding is supported by research done by Cady, Hopkins, and Hodges (2008). They reported how their mathematical knowledge about the place value relationships in the base-

ten system increased after they engaged in a Lesson Study to develop a lesson about place value for their preservice and inservice teachers. They created a base-five system to help their students' better understand the place value relationships in our base-ten system. As they created the base-five system using cycles of Lesson Study, their own mathematical knowledge about place value increased.

With respect to their self-perception of their technological, pedagogical, and mathematical knowledge (TPACK), this statistically significant increment could reflect their self-awareness of what it means to teach mathematics using technology in ways that are different than what they thought at the beginning of the professional development. As teachers engage in the design of challenging lessons, they might be more aware of the different knowledge that needs to come together if they want to teach their students meaningful mathematics using technology. Through the Lesson Study cycles, teachers might also perceived themselves more knowledgeable in applying different teaching strategies combined with uses of technology in order to optimize their teaching in different areas of the mathematics. As Meng and Sam (2013) concluded, "by incorporating [Lesson Study] into the mathematics teaching methods course, the pre-service secondary teacher's TPACK for teaching mathematics with GSP had improved significantly" (p. 6). Also, Groth, Spickler, Berfner, and Bardzell (2009) found that teachers' inexperience on how to appropriately use the technology affects the mathematics that is taught in ways that hinders students' understanding.

Summary of Findings for Research Question 2

What issues emerged (mathematical, pedagogical, and technological) in the process of designing the lesson? And how do teachers' attention to any of the above issues change as they become more experienced in designing lessons that integrate technology?

The dimensions of teacher's knowledge to teach mathematics with technology, as presented in Figure 8, offer an overview of the types of knowledge that previous research has identified as pieces of the knowledge teachers need to teach mathematics with technology. Those types of knowledge are the different aspects teachers draw upon when engaged in the process of designing a lesson when the use of technology is expected. These include not only their knowledge of mathematics, pedagogy, and technology, but also the natural interactions between those types of knowledge. These interactions include the knowledge of pedagogical approaches specifics for mathematics, knowledge of technological tools to teach mathematics, and the pedagogy necessary to implement technology into teaching, among others.

Looking across the four groups of teachers as they engaged in the process of designing, and implementing a lesson, and then as they reflect on that planning and implementation, issues with all of those dimensions were observed. The next paragraphs illustrate the issues these groups of teachers encounter along the planning, implementation, and reflection cycles of the Lesson Study using the five main codes I used throughout the data analysis phase. Those codes are: knowledge of pedagogy, knowledge of technology, knowledge of mathematics, knowledge of math and curriculum, knowledge of math and

students, and knowledge of math and teaching. Then, the changes that were observed as the group of teachers attended to the issues observed during Cycle 1 will be discussed.

Knowledge of Pedagogy. This knowledge, according to Mishra and Koehler (2006) refers to the methods and process of teaching, which includes classroom management, methods of assessment, and lesson plan development, among others. Results of this study suggest that teachers' knowledge of general pedagogy was adequate. All teachers in the study demonstrated having a strong classroom management. Some teachers demonstrated using an adequate tone of their voice during implementation. However, their wait time was not appropriate, not leaving enough time for the students to respond (Rowe, 1986), and they often talked too fast while teaching.

Knowledge of Technology. This type of knowledge encompasses general knowledge of the technology, as well as knowledge of technology tools specific to the teaching of mathematics. For this study, the use of technology was expected to be an integral part of these planning sessions since it was required to fulfill the purpose of this professional development. Findings suggest that many mathematics teachers have access to a computer and projector, or to a smart board in their classrooms. Having that access to technology might encourage teachers to use the technology on a daily basis and integrate it into their teaching (Cubukcuoglu, 2013). Results of this study also suggest teachers that have those technology tools in their classrooms might feel knowledgeable about how to use those tools for teaching in general.

However, the project teachers exhibited severe issues with their knowledge of technology as it relates to the teaching of mathematics. The most important factor to consider is the lack of knowledge about the tools that are available for them to use to teach mathematics. Results from this study suggest that when teachers have issues with their knowledge about technology tools specific to the teaching of mathematics, even when they have some access to technology in their classrooms, it becomes very challenging to think of ways of incorporating technology into their lessons. Margerum-Lays and Marx (2002) agree in that integrating technology into teaching and learning is a complex process.

Some teachers have the knowledge about technology tools to teach mathematics. However, they have no knowledge about its uses, affordances, and constraints. This suggests that even when a strong mathematical knowledge exists, if teachers do not know how to effectively connect the use of technology with the mathematics being taught, students' learning does not benefit from the integration of technology.

Knowledge of Mathematics. Ball and colleagues (2008) divided teachers' mathematical knowledge onto two groups: common mathematical knowledge and specialized mathematical knowledge. Results suggest that when a group of teachers lack a deeper conceptual understanding of the mathematics, they constantly used incorrect vocabulary, and inappropriate definitions. Lacking a deeper knowledge of the mathematics did not allow for the inclusion of more challenging examples in their lessons. In addition, the implementation of the lesson was disorganized and there was a lack connection between the different aspects of the mathematics being taught.

When a group showed minimal to no issues with their mathematical knowledge, they seemed to have a deeper understanding of the concepts being discussed, which helped them design a more organized lesson. For these groups, the implementations flowed without any issues, there was no incorrect use of language, and no mathematical errors were found on the board.

These issues with mathematical knowledge are consistent with findings from previous research. For example, Hill et al. (2008), Heaton (1992) and Stein et al. (1990) found that when teachers lack that mathematical knowledge, they do not only use inappropriate definitions and technical language, but teachers also make significant mathematical errors and imprecisions, and accept inaccurate guesses from their students. When teachers feel comfortable with the mathematics they are teaching, and they have a strong mathematical knowledge, classroom instruction and student achievement benefits.

Knowledge of Math and Curriculum. Curriculum knowledge was categorized by Shulman (1986) as one of the bases of teachers' knowledge. Ball and colleagues (2008) agreed with Shulman and included knowledge of mathematics and curriculum as one of the teacher's domains of mathematical knowledge, which is the vertical and horizontal knowledge of mathematics content and curriculum.

Regarding teacher's curriculum knowledge, results from this study were very particular to the situation the participants in the study encountered with the publication of a new set of standards right at the beginning of this study as explain in Chapter 3. Thus, even though teachers showed evidence of having that knowledge of how the mathematical topics

are related they struggled understanding the new format of the standards. They also had some issues with the curriculum because, even though they seemed to understand how the mathematical ideas flow throughout the years, the order in which the topics were taught has also change. Results from Chapter 5 report how teachers, particularly those from high school, struggled understanding the new curriculum as topics they usually teach in 11th grade where now moved to 10th grade.

Even though the standards were new, all groups demonstrated a strong understanding of how the math curriculum flows through the years in middle and high school. As they designed their lessons they took into consideration what students were supposed to bring from previous years and how learning that particular topic they were focusing their lesson on would help students in their future math courses, showing their horizon content knowledge, thus connecting the topic being taught to topics from prior and future years (Ball et al., 2008).

Knowledge of Math and Students. This type of knowledge refers to teachers' abilities to interpret students' thinking, to be able to anticipate students' responses, and knowing students' mathematical conceptions and misconceptions (Ball et al., 2008). All of these are crucial aspects of the teaching-learning process. Results of this study suggest that when teachers designed a lesson from scratch, they considered students misconceptions and are thoughtful about anticipating students' responses to the task. When a pre-designed lesson was used, they missed those conversations, not taking into account students mathematical misconceptions or the anticipation of students' responses. This agrees with Meyer and

Wilkerson (2011) regarding important factors that help or hinder teacher's opportunities to improve their knowledge to teach mathematics. Results from this study confirm the importance of designing lessons that are new to the teachers, and having extended conversations where they predict students' questions and analyze ways of responding to those questions. When teachers used pre-existing lessons, they only focused on complementing the presentation with math problems and did not have discussions about anticipating students' responses.

Knowledge of Math and Teaching. This type of knowledge encompasses "everything that teachers must do to support the learning [of mathematics] of their students" (Ball et al., 2008, p. 395). It is the kind of knowledge that allows a teacher to choose a particular representation for learning a mathematical concept or procedure, and which allows them to select examples or choose a textbook. It integrates the knowledge of mathematics and the knowledge of pedagogy (Carrillo, Climent, Contreras, Muñoz-Catalán, 2013).

Following the suggestions offered by Carrillo et al. (2013), findings of the study suggest that when teachers exhibit a good grasp of the mathematical knowledge (CK) and the pedagogical knowledge (PK), these teachers are more strategic when planning their lessons. Their lessons seemed to be more organized as they carefully planned how to sequence the content for instruction taking into consideration how students learn the mathematics concept. When evidence of CK and PK are exhibited, teachers also appeared to be open to create a more student-centered lesson. Chandra (2015) investigated the pedagogical content knowledge of five secondary school mathematics teachers as they taught a geometry class.

After analyzing the teaching episode and an individual interview, she found that content, and pedagogical knowledge are equally important for effective teaching.

Results also suggest that engaging teachers in a Lesson Study might create an environment for teachers to find the most appropriate ways of explaining procedures, illustrating a concept in ways that seemed clear for the students, sequencing the content for instruction taking into consideration the relations among the topics within a particular unit, and using useful form of representations during their teaching of a mathematical idea. Within a Lesson Study, Perry an Lewis (2008) found teacher's changes in instruction includes the use of tasks that elicit student thinking and support student interaction.

Many changes were observed as these four groups of teachers engaged in a second cycle of the Lesson Study. For this second cycle, the four groups of teachers remained the same and it was required all groups chose a new mathematical topic for the lesson. Furthermore, most groups used a different technological tool, and a different teacher implemented the lesson. Thus, the changes discussed here are those of the group, taking into consideration that different teachers implemented the lessons. The following paragraphs discuss the issues that were observed during Cycle 1, and how the teachers seemed to be attending to them, as they become more experience through the Lesson Study process.

At the beginning of the study all groups had issues with their knowledge of curriculum. As the semester, and the study, progressed teachers became more knowledgeable about the new set of standards. This perhaps, since the curricular maps were published for

each group's benefit. Thus, issues with curriculum seemed to be solved with ease, and during Cycle 2 all groups appeared to feel more knowledgeable regarding the curriculum.

All groups seemed to lack the knowledge of appropriate technology tools to teach the content they intended to teach. As they chose new mathematical topics for the second cycle, the same issues emerged, suggesting teachers need to learn about those technology tools before engaging in designing lessons that integrate technology. Results suggest teachers attend to this issue by searching for a technology tool within their comfort zone, even if it is not considered a mathematical action tool.

During Cycle 1, there were long conversations where teachers anticipated students' responses and purposefully planned how the mathematics was going to be presented to the students to enhance their learning of the topic. Observed changes within this group include the lack of such discussions during Cycle 2. Results suggest that for those conversations to emerge during planning, each member of the group should have the mathematical knowledge necessary to teach that concept, as well as some experiences teaching the concept. In addition, results suggest that those conversations are more feasible when teachers start their planning from scratch, not using a pre-designed lesson plan.

As teachers become more experienced, they noticed the importance of designing lessons that were not derived from pre-designed lessons. Again, results are suggesting that when a pre-designed lesson is used to start the planning of a new lesson, teachers neglect the opportunities of thinking about the mathematics, and how the principal ideas of the lesson are going to be carried over during implementation.

When the same teacher implemented the lessons, the same technological tool was used. This gave the group, and the teacher, the opportunity to gain even more experience integrating that particular tool into his teaching. When this occurred, results suggest they feel more empowered to try new pedagogical approaches, and to create more student-centered lessons.

A significant change occurred to two of the groups, as they allowed me to intervene and suggest technology tools I thought were appropriate for the teaching of the mathematical idea they were planning on designing their lessons. The use of a technology tool such as Fathom, which was a tool new to the teachers, would not be possible without my intervention with the group. In addition, as I helped another group to integrate the hide/show feature and the use of sliders in GeoGebra, they were able to use the technology in more meaningful ways for their students. Lesson Study is collaborative, not only in the sense that teachers plan a lesson in collaboration with peers, but also in the sense of having the support and collaboration of a more knowledgeable one. Receiving that support from a specialist, as I acted as a mentor with my technological expertise, was an important factor to consider in this change. This speaks on the importance of having that support during a Lesson Study if the goal is to help teachers develop their TPACK (Meng & Sam, 2013; Shafer, 2008). This agrees with results found by Meng and Sam (2013) as they engaged a group of pre-service teachers in a Lesson Study during a methods course in Malaysia. They concluded by saying that, “incorporating [Lesson Study] into the mathematics teaching methods course could be a potential means of developing pre-service secondary teachers’ TPACK for teaching

mathematics with GSP because it provided them guidance, help, and support when needed” (p. 6). In a different setting, Shafer (2008) used an apprenticeship model that paired a researcher with a classroom teacher with the goal to effectively integrate the use of GSP into the classroom teacher’s practices. The researcher acted as a technology specialist and served as a mentor offering her technology expertise to the teachers.

Limitations

This study has many limitations to take into account. One limitation of this study is sample size. Such a limitation implies that results from this study cannot be generalized to other populations. Results should be used to inform which issues are the most predominant when teachers planned their lessons, in general terms.

A second limitation is the quality of the audiotapes during the planning sessions. In many occasions, it was very challenging to understand what teachers were having conversations about because of the quality of the audio. The way teachers were arranged into the classroom while planning was a crucial factor to take into account in future research. Given all four groups were having such conversations in a single room, the audio recorders picked discussions from neighbor groups, harming the quality of analysis of the audio for the group of interest.

A third limitation of the study is time. Data collection occurred in a period of five months (an academic semester). To talk about teachers becoming more experienced in designing lessons that integrated technology is limited to the experiences these teachers gained through the two cycles of this Lesson Study.

A fourth limitation is that the professional development was not focused on a specific mathematical content, pedagogical approach, or technology tool. Thus, teachers' knowledge about the mathematics for example, is limited to the mathematical knowledge they were discussing during planning. The same is true for their knowledge of technology, as it is limited to the tools they chose to use for their lessons.

A fifth limitation of this study is that while planning was done collaboratively by a group of teachers, only one teacher in a group did the implementation. Thus, the evidences of particular types of knowledge that emerged during implementation could be specific to the teacher that implemented the lesson and not necessarily those of the group of teachers.

Implications

Results from this study provide implications for classroom mathematics teachers, teacher educators, and providers of professional development. First, results from this study provide an instrument to measure inservice, Spanish speakers, secondary mathematics teachers' self-perception of their technological knowledge (TK), mathematical knowledge (MK), and technological, pedagogical, and mathematical knowledge (TPACK). This valid and reliable instrument offers opportunities to conduct research in the area of teaching mathematics with technology with teachers in countries where they only speak Spanish. It is interesting that in this study with Spanish-speaking practicing teachers, the validation process resulted in 3 constructs, while the original English version used with preservice teachers resulted in 4 constructs.

Lesson planning is an essential task every classroom mathematics teacher must engage in on a daily basis. But, writing lesson plans that are rich in details, and that take into consideration all the aspects of teaching mathematics with technology included in the theoretical framework on Figure 8 is a challenge for many teachers. Classroom mathematics teachers could use the results from this study to reflect on their teaching practices about lesson planning, and to better understand what the purpose of writing detailed lesson plans really is. They could use the framework to inform their teaching with respect to the different types of knowledge they have to draw upon when integrating technology into their teaching of mathematics. When a teacher sees the purpose of planning a lesson in advance, they may understand the importance of paying attention to (1) students' most common mistakes, (2) the misconceptions they bring to class, and (3) the many responses students could have for a given task.

Results from this study also have implications for teacher educators. Teacher educators could utilize results from this study in their courses with pre-service secondary mathematics teachers. Teaching mathematics with technology is not an easy endeavor and pre-teachers need a lot of training before heading into their classroom. Teacher educators could create a course to support future teachers 1) knowledge of specialized mathematical content, 2) knowledge of mathematical action technologies, and 3) knowledge of pedagogy that is particular to the teaching of mathematics. One could envision this course happening right before the student-teaching experience where pre-service teachers, drawing upon those different types of knowledge, are required to 1) design a mathematical lesson that use

mathematical action tools, 2) implement the task with real students, and 3) reflect upon the implementation of the task.

For providers of professional development, results from this study could guide a creation of a professional development focused on teaching mathematics with technology. The findings of this study provide information about the different types of knowledge teachers draw upon when planning lessons. Providers of professional development should use the results from this study to better understand how the Lesson Study could be set in order to get the most benefits for the teachers. Results of this study suggest that in order to have an impact on teachers' TPACK, it is necessary to obtain the support from a specialist that help teachers in the search of technology tools to teach a particular topic, and to assist the teacher in designing a lesson where students learning is impacted by using pedagogical approaches that afford students explorations and discoveries of the mathematics. Having that support would allow teachers to get trained on different technology tools that could be used to teach a particular mathematical topic. It is also necessary to obtain support on the mathematics, thus the designed lessons exhibit a stronger mathematical content.

Results from this study also suggest that some adaptations could be made to the current Lesson Study. Professional development providers should consider having the same teacher implementing the lessons if multiple cycles of a Lesson Study are completed. Another consideration would be having the teacher to re-teach the lesson after the reflection meeting. This would encourage teachers to think of ways of improving the lesson, using the feedback and observations received during the meeting. When forming groups, results of the

study suggest making an effort to organize groups by the mathematics course they teach. This would ensure, to some extend, that all teachers in the group have some mathematical knowledge of the topics that are taught in that course as well as some experience teaching those topics.

Recommendations for Future Research

Taking into consideration the limitations of the current study, this research could be expanded in several ways. Future research could follow the design in the current study, but at the implementation phase one could have all teachers in the group implementing the lesson. This brings issues of scheduling school visits and more data to analyze, but the information about teachers' knowledge to teach mathematics with technology could be richer as results are obtained from different teachers in the same group. Another variant for future research could be to have the teacher to re-teach the lesson after the reflection meeting, as done in the original Lesson Study approach. This could bring more information regarding how the teacher resolved some of the issues that emerged during the first implementation. In the analysis phase, perhaps one could examine issues with the different pieces of knowledge during planning for those teachers that implemented the lesson versus the other teachers in the group.

In addition, to alleviate the fact that teachers had many issues with their knowledge of technology, a similar study could take place with a different group of teachers where an intensive course of how to use a specific technology tool (e.g., GeoGebra) is provided first.

Then, using the Lesson Study approach, have groups of teachers completing a cycle where they have to design a lesson using the technology tool they just learned.

A different venue for research could be to purposefully select some of the teachers in the current study, and conduct a new study using an instructional coaching approach where those teachers receive a one-to-one training on using technology to teach mathematics. Using cycles of mini teaching one could go into these teachers' classrooms and make the professional development activities even more centered in their instructional practices. This type of coaching might be time consuming, but for the teacher to receive this one-to-one type of training should be meaningful. As teachers seemed to lack that knowledge of the technology to teach mathematics, it would be beneficial to have the time to train those teachers that demonstrate a real interest in integrating technology into their teaching practices. Some of these teachers could benefit enormously from the support they could receive from the mini teaching cycles as well as the activities related to instructional coaching.

Teaching mathematics with technology could represent different thing to different people. For us, mathematics teacher educators, it means to use the affordances and constraints of a mathematical action technology tool to engage students in activities where they can see the mathematics play a more active role. It is to move from a show and tell type of lesson (lectured) to a more student-centered lesson where students are able to manipulate objects, observing many examples using a dynamic environment, or using a computer simulator to collect data on a particular event without leaving the classroom. But for teachers to be able

to understand how powerful it could be to teach mathematics with technology, results of this study suggest teachers need to be engaged in activities where they first experience what it is to learn mathematics with technology. Teachers also need some previous experience with the technology tool itself before engaging in the design of lessons or mathematical tasks that require the use of technology.

We still need to identify a professional development design that helps teachers learn what it means to use technology to enhance the mathematics they teach. Results from this study show that using a Lesson Study approach helped teachers increase their self-perception of their technological, pedagogical, and content knowledge (TPACK). This could be a starting point to design a professional development approach that uses mathematical action tools, new pedagogical approaches, and that helps teacher see the mathematics they teach in a new and innovative way.

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Appendices

Appendix A: Informed Consent Form (Spanish)

Universidad de Puerto Rico
 Recinto Universitario de Mayagüez
 Colegio de Artes y Ciencias
 Departamento de Ciencias Matemáticas

HOJA DE CONSENTIMIENTO INFORMADO

Título: Entendiendo las dificultades tecnológicas, pedagógicas y matemáticas que emergen mientras maestros de nivel secundario diseñan lecciones que integran tecnología
(“Understanding the technological, pedagogical, and mathematical issues that emerged as secondary mathematics teachers design lessons that integrate technology”)

Investigador Principal: Marggie D. González-Toledo

Propósito del Estudio

Como participante en el proyecto AFAMaC-Matemáticas (Alianza para el Fortalecimiento en el Aprendizaje de las Matemáticas), usted ha sido invitado para participar en un proyecto de investigación. El propósito de este proyecto es conocer cuáles son las dificultades que emergen mientras maestros de matemáticas diseñan lecciones que integran el uso de tecnología para enseñar matemáticas. Usted contribuirá a esta investigación asistiendo al Instituto de Verano 2014 y a las Academias Sabatinas de AFAMaC-Matemáticas para diseñar planes de lecciones de matemáticas en colaboración con otros maestros y permitiendo que el investigador tenga acceso a cualquier reunión que su grupo de maestros tenga y a cualquier documento que su grupo diseñe como parte de sus lecciones.

Si usted está de acuerdo en participar en este estudio de investigación, usted deberá:

- Contestar un pre-cuestionario y post-cuestionario como avalúo de su auto-percepción en cuanto a su conocimiento para enseñar matemáticas con tecnología.
- Llevar a cabo investigación y escribir planes de lecciones detalladas en colaboración con colegas.
- Completar un diario donde usted estará reflexionando en conversaciones y discusiones que su grupo ha tenido con referencia a contenido matemático, prácticas de enseñanza, conocimiento y conceptos erróneos de los estudiantes, herramientas tecnológicas, entre otras.
- Permitir que las reuniones de los grupos sean audio grabadas y que las implementaciones de las lecciones en la sala de clase sean video grabadas.

Además, algunos maestros serán observados en su aula escolar mientras implementan la lección diseñada por su grupo. La observación será documentada mediante notas de campo y videograbación. Las visitas al aula escolar serán planificadas con anterioridad y el maestro a ser observado voluntariamente consentirá la observación y la videograbación.

Riesgos

Su participación en este estudio no presenta riesgos físicos. Colaboración será una parte crítica en la participación de este estudio. Por tanto, de usted participar en este estudio, el único riesgo que el investigador prevé es que usted sienta ansiedad debido a su colaboración y discusiones continuas con colegas maestros mientras diseñan sus lecciones.

Beneficios

Su participación es voluntaria y su participación en el proyecto AFAMaC-Matemáticas no se verá afectada si usted decide no participar del estudio. Este estudio durará aproximadamente 7 meses, y su participación en el estudio implicará alrededor de 35 horas en ese periodo de tiempo. El conocimiento que obtengamos mediante sus experiencias añadirá al conocimiento que ya se tiene como base en el área de educación matemática, en especial con respecto a las dificultades tecnológicas, pedagógicas, y matemáticas que emergen mientras maestros de nivel secundario toman decisiones en el diseño de lecciones de matemáticas que integran tecnología. Toda la información recopilada mediante su participación en este estudio de investigación se mantendrá estrictamente confidencial. Toda la información que pueda enlazar su nombre con la data será removida y remplazada por un seudónimo. Los datos serán guardados en un archivo con llave y en una computadora protegida por una contraseña. No se hará ninguna referencia oral o por escrito que pueda enlazarlo a usted con el estudio. Si usted interesa conocer los resultados de este estudio puede comunicarse con la investigadora a su correo electrónico, número telefónico, o dirección física indicados en el siguiente párrafo.

Usted es libre de retirarse en cualquier momento del estudio sin incurrir en penalidad alguna; sin embargo, usted seguirá participando de todas las actividades requeridas por el proyecto AFAMaC-Matemáticas. Si en cualquier momento durante su participación usted tiene alguna pregunta acerca del estudio o de los procedimientos, usted puede comunicarse directamente con el investigador, Marggie D. González-Toledo al (787) 832-4040 extensión 3293 o al correo electrónico marggie.gonzalez@upr.edu. Ella está ubicada en la Universidad de Puerto Rico, Departamento de Ciencias Matemáticas, Edificio Monzón, Oficina 307. Si usted siente que no ha sido tratado de acuerdo con las descripciones en esta hoja, o que sus derechos como participante en investigación han sido violados en el curso de este proyecto, puede dirigirse al Comité para la Protección de Seres Humanos en la Investigación (CPSHI) del Recinto Universitario de Mayagüez ubicados en el edificio Celis, oficina 108 y/o el número telefónico (787) 832-4040 ext. 6277.

CONSENTIMIENTO PARA PARTICIPAR

- ____ “Estoy de acuerdo en participar en este estudio de investigación con el entendimiento que puedo escoger no participar o detener mi participación del mismo en cualquier momento sin penalidades. He leído y entendido la información antes expuesta y he recibido una copia de esta forma. En adición:
- ____ Estoy de acuerdo en participar de todas las actividades expresadas en esta hoja;
- ____ Estoy de acuerdo que mis conversaciones sean audio grabadas y;
- ____ En el caso de ser voluntario a implementar la lección, estoy de acuerdo a que mi implementación en la sala de clase sea video grabada.”

“No estoy de acuerdo en participar en este estudio de investigación.”

Nombre del Participante (en letra de molde): _____

Firma del Participante: _____ Fecha _____

Firma del Investigador: _____ Fecha _____

Appendix B: Orginal TPACK Survey (22 items)

SURVEY

(Zelkowski, Gleason, Cox, & Bismarck, 2013)

Thank you for taking time to complete this survey. Please answer each question to the best of your knowledge. You should answer demographic information first, then read each item and choose your first belief. You need not spend any lengthy time on any one item. You should be finished in about 15 minutes. Your thoughtfulness and candid responses will be greatly appreciated. Your confidentiality will not be compromised and your name will not, at any time, be associated with your responses. Your responses will be kept completely confidential.

Demographic Information

Name: _____

Age range:

- Under 19
- 19–22
- 23–26
- 27–30
- 30+

Gender:

- Female
- Male

Years of teaching experience: _____

Highest degree earned: _____

In which school level you teach:

- Middle school
- High school

Technology is a broad concept that can mean a lot of different things. For the purpose of this questionnaire, technology is referring to digital technology/technologies—that is, the digital tools we use, such as computers, laptops, iPods, handhelds, interactive whiteboards, computer software programs, graphing calculators, etc. Please answer all of the questions, and if you are uncertain of or neutral about your response, you may always select “Neither agree nor disagree.”

Item	Strongly Disagree	Disagree	Neither agree nor disagree	Agree	Strongly Agree
I know how to solve my own technical problems.					
I can learn technology easily.					
I keep up with important new technologies.					
I frequently play around with the technology.					
I know about a lot of different technologies.					
I have the technical skills I need to use technology.					
I have sufficient knowledge about mathematics.					
I have various strategies for developing my understanding of mathematics.					
I know about various examples of how mathematics applies in the real world.					
I have a deep and wide understanding of algebra.					
I have a deep and wide understanding of geometry.					
I know how to assess student performance in the classroom.					
I can adapt my teaching based upon what students currently understand or do not understand.					
I can adapt my teaching style to different learners.					
I can assess student learning in multiple ways.					
I can use a wide range of teaching approaches in a classroom setting.					
I can use strategies that combine mathematics, technologies, and teaching approaches that I learned about in my coursework in my classroom.					
I can choose technologies that enhance the mathematics for a lesson.					
I can select technologies to use in my classroom that enhance what I teach, how I teach it, and what students learn.					
I can teach lessons that appropriately combine mathematics, technologies, and teaching approaches.					

Item	Strongly Disagree	Disagree	Neither agree nor disagree	Agree	Strongly Agree
I can teach lesson that appropriately combine algebra, technologies, and teaching approaches.					
I can teach lesson that appropriately combine geometry, technologies, and teaching approaches					

Appendix C: Original Survey in Spanish (22 items)

CUESTIONARIO

Gracias por utilizar de su tiempo para completar este cuestionario.

Usted necesitará aproximadamente 25 minutos para completar este cuestionario. Su reflexión y respuestas sinceras serán apreciadas grandemente. Su confidencialidad no será comprometida y su nombre no será asociado con sus respuestas en ningún momento. Sus respuestas se mantendrán completamente confidenciales.

Por favor conteste las siguientes preguntas de información demográfica.

Información Demográfica

Identificación: (genere una identificación de la siguiente manera: los dos dígitos correspondientes a su mes de nacimiento, las primeras tres letras del nombre de su mama, , y los dos dígitos correspondientes al día en que usted nació. Ejemplo: si su mama se llama JUANA y usted nació el 26 de julio entonces su identificación es **07JUA26**): _____

Edad:

- 20-30
- 30-40
- 40-50
- 50+

Género:

- Femenina
- Masculino

Años de experiencia en la enseñanza: _____

Mayor grado académico completado (circule SOLO una): BS, BA, MS, MA, PhD

Indique los cursos de matemáticas que estará enseñando en agosto:

- _____
- _____
- _____
- _____

¿Qué es Tecnología? Tecnología es un concepto amplio que puede significar diferentes cosas. Para el propósito de este cuestionario tecnología se refiere a tecnologías digitales, esto es, las herramientas

tecnológicas que utilizamos, tales como computadoras, portátiles, iPods, whiteboards interactivas, programas de computadora, calculadoras gráficas, programas de acción matemáticas, etc. A continuación usted encontrará una tabla con 22 ítems. Lea cada premisa detenidamente y escoja la respuesta que mejor represente su convicción. Si usted se siente inseguro o neutral acerca de alguna respuesta, siempre puede seleccionar “Ni en acuerdo ni en desacuerdo”.

Ítem	Completamente en Desacuerdo	En Desacuerdo	Neutral	En Acuerdo	Completamente de Acuerdo
Sé cómo resolver mis propios problemas técnicos.					
Puedo aprender tecnología con facilidad.					
Me mantengo al día con tecnologías nuevas e importantes.					
Suelo jugar con tecnología con frecuencia.					
Tengo conocimiento sobre muchas herramientas tecnológicas.					
Tengo las destrezas técnicas necesarias para usar tecnología.					
Tengo suficiente conocimiento sobre las matemáticas.					
Tengo varias estrategias para desarrollar mi entendimiento de matemáticas.					
Tengo conocimiento sobre varios ejemplos de cómo matemáticas aplica a la vida real.					
Tengo un entendimiento amplio y profundo de álgebra.					
Tengo un entendimiento amplio y profundo de geometría.					
Sé cómo medir el rendimiento de los estudiantes en el aula escolar.					
Puedo adaptar mi enseñanza basándome en lo que los estudiantes comprenden y no comprenden en el momento.					

Ítem	Completamente en Desacuerdo	En Desacuerdo	Neutral	En Acuerdo	Completamente en Acuerdo
Puedo adaptar mi estilo de enseñanza a diferentes tipos de aprendizaje.					
Puedo evaluar el aprendizaje de los estudiantes en diferentes maneras.					
Puedo utilizar una amplia variedad de métodos de enseñanza en el salón de clases.					
Puedo incorporar en mi salón de clases estrategias que combinan matemáticas, herramientas tecnológicas, y métodos de enseñanza que he aprendido en diferentes actividades de desarrollo profesional.					
Puedo seleccionar herramientas tecnológicas que optimizan la matemática en la lección.					
Puedo seleccionar herramientas tecnológicas para usar en mi salón de clases que optimizan lo que enseño, cómo lo enseño, y cómo los estudiantes aprenden.					
Puedo enseñar lecciones que apropiadamente combinan matemáticas, herramientas tecnológicas, y métodos de enseñanza.					
Puedo enseñar lecciones que apropiadamente combinan álgebra, herramientas tecnológicas, y métodos de enseñanza.					
Puedo enseñar lecciones que apropiadamente combinan geometría, herramientas tecnológicas, y métodos de enseñanza.					

Appendix D: Validated Survey in Spanish (18 items, SMT-TPACK-Spanish)

CUESTIONARIO

Gracias por utilizar de su tiempo para completar este cuestionario.

Usted necesitará aproximadamente 25 minutos para completar este cuestionario. Su reflexión y respuestas sinceras serán apreciadas grandemente. Su confidencialidad no será comprometida y su nombre no será asociado con sus respuestas en ningún momento. Sus respuestas se mantendrán completamente confidenciales. Por favor conteste las siguientes preguntas de información demográfica.

Información Demográfica

Identificación: (genere una identificación de la siguiente manera: los dos dígitos correspondientes a su mes de nacimiento, las primeras tres letras del nombre de su mama, , y los dos dígitos correspondientes al día en que usted nació. Ejemplo: si su mama se llama JUANA y usted nació el 26 de julio entonces su identificación es **07JUA26**): _____

Edad:

- 20-30
- 30-40
- 40-50
- 50+

Género:

- Femenina
- Masculino

Años de experiencia en la enseñanza: _____

Mayor grado académico completado (circule SOLO una): BS, BA, MS, MA, PhD

Indique los cursos de matemáticas que estará enseñando en agosto:

- _____
- _____
- _____

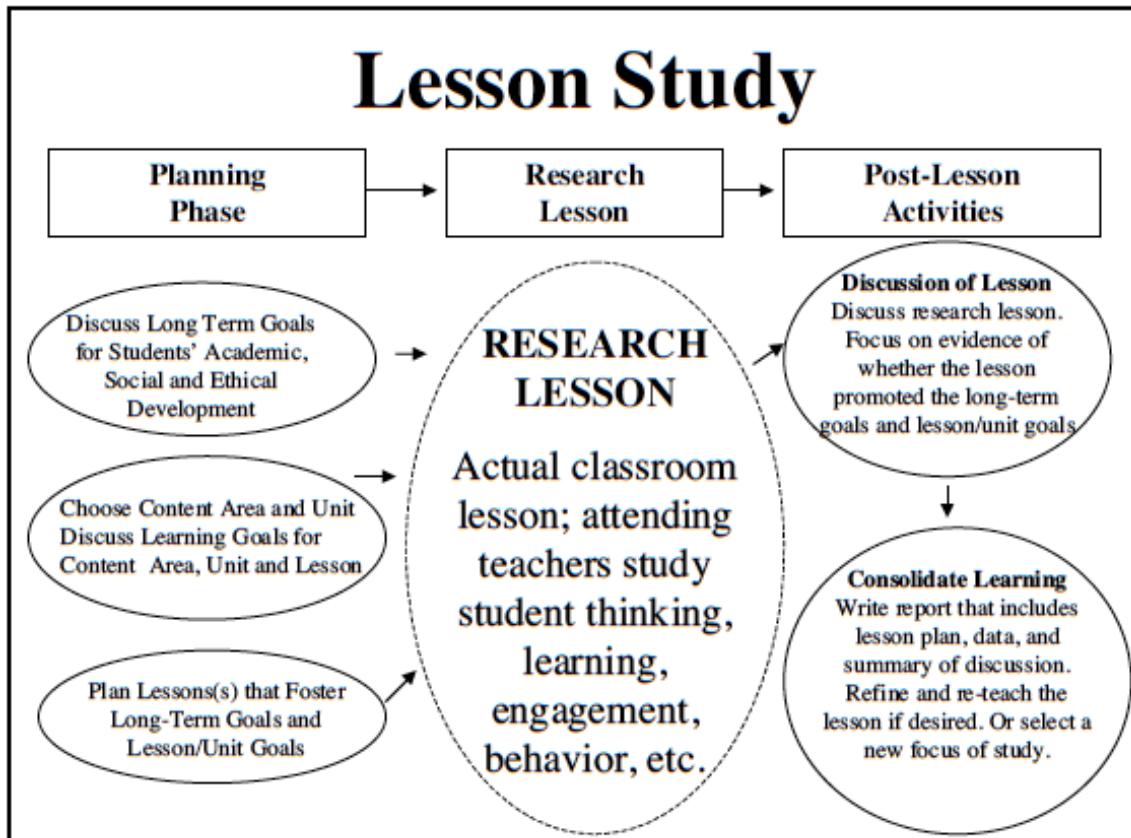
¿Qué es Tecnología? Tecnología es un concepto amplio que puede significar diferentes cosas. Para el propósito de este cuestionario tecnología se refiere a tecnologías digitales, esto es, las herramientas tecnológicas que utilizamos, tales como computadoras, portátiles, iPods, whiteboards interactivas, programas de computadora, calculadoras gráficas, programas de acción matemáticas, etc.

A continuación usted encontrará una tabla con 22 ítems. Lea cada premisa detenidamente y escoja la respuesta que mejor represente su convicción. Si usted se siente inseguro o neutral acerca de alguna respuesta, siempre puede seleccionar “Ni en acuerdo ni en desacuerdo”.

Ítem	Completamente en Desacuerdo	En Desacuerdo	Neutral	En Acuerdo	Completamente de Acuerdo
Sé cómo resolver mis propios problemas técnicos.					
Puedo aprender tecnología con facilidad.					
Me mantengo al día con tecnologías nuevas e importantes.					
Suelo jugar con tecnología con frecuencia.					
Tengo las destrezas técnicas necesarias para usar tecnología.					
Tengo suficiente conocimiento sobre las matemáticas.					
Tengo varias estrategias para desarrollar mi entendimiento de matemáticas.					
Tengo conocimiento sobre varios ejemplos de cómo matemáticas aplica a la vida real.					
Tengo un entendimiento amplio y profundo de álgebra.					
Tengo un entendimiento amplio y profundo de geometría.					
Sé cómo medir el rendimiento de los estudiantes en el aula escolar.					
Puedo evaluar el aprendizaje de los estudiantes en diferentes maneras.					
Puedo utilizar una amplia variedad de métodos de enseñanza en el salón de clases.					

Ítem	Completamente en Desacuerdo	En Desacuerdo	Neutral	En Acuerdo	Completamente de Acuerdo
Puedo enseñar lecciones que apropiadamente combinan matemáticas, herramientas tecnológicas, y métodos de enseñanza.					
Puedo enseñar lecciones que apropiadamente combinan álgebra, herramientas tecnológicas, y métodos de enseñanza.					
Puedo enseñar lecciones que apropiadamente combinan geometría, herramientas tecnológicas, y métodos de enseñanza.					

Appendix E. Lesson Study Flowchart



From: <http://www.lessonresearch.net/lessonstudycycle.pdf>

Appendix F. Thinking Through a Lesson Protocol (Spanish)

Protocolo: Pensando a través de una Lección

El propósito principal de este protocolo es estimular pensamiento profundo acerca de una lección en específico que usted estará enseñando y que está basada en una actividad matemática que es cognitivamente retadora.

Parte 1: Seleccionando y elaborando una actividad matemática	
Metas de aprendizaje / Estándar ¿Qué se espera que los estudiantes entiendan al culminar esta lección?	Evidencia ¿Qué se espera que los estudiantes digan, hagan, produzcan, etc. que proveerá evidencia de su entendimiento?
Actividad ¿Cuál es la actividad principal en la cual los estudiantes estarán trabajando durante esta lección?	Apoyo a la Instrucción – herramientas, recursos ¿Qué herramientas o recursos tendrán que usar los estudiantes durante su trabajo que los ayudara a razonar a través de la actividad?
Ejecución de la Actividad ¿Cuáles son las diferentes maneras en la cuales los estudiantes podrían resolver la actividad?	Apoyo a la Instrucción – maestro ¿Qué preguntas usted le hará a los estudiantes de tal manera que apoye la exploración de la actividad y que a su vez provea una conexión entre su ejecución y los que se espera que ellos hagan?
Parte 2: Apoyando la exploración de la actividad por parte de los estudiantes	
¿Qué preguntas hará a los estudiantes para ayudarlos a comenzar con la actividad? Para tener más claridad con respecto a lo que los estudiantes han hecho, comience con preguntas de evaluó tales como: ¿Qué hiciste? ¿Cómo obtuviste ese resultado? ¿Qué significa esto? Una vez se tenga una mejor idea de que los estudiantes entienden, entonces las preguntas pueden ser más enfocadas a la actividad. ¿Cómo usted asegurará que los estudiantes se mantengan comprometidos y enfocados en la actividad? ¿Cómo usted ayudará a un estudiante/par/grupo que está frustrado con la actividad? ¿Qué harán los estudiantes que terminen temprano?	
Parte 3: Compartiendo y discutiendo la Actividad	
Seleccionando y Ordenando ¿Cuáles soluciones desea compartir durante la lección? ¿En qué orden? ¿Porque?	Conectando Respuestas ¿Qué preguntas en específico usted hará a los estudiantes de tal manera que ayude a los estudiantes a razonar sobre las ideas matemáticas que se espera aprendan y que ayude a hacer conexiones entre las diferentes estrategias o soluciones presentadas?

Protocolo: Pensando a través de una Lección

Preguntas guías:

Parte 1: Seleccionando y elaborando una actividad matemática

- ✓ ¿Cuáles son las metas matemáticas de la lección (o sea, que usted desea que los estudiantes sepan y entiendan acerca de las matemáticas como resultado de esta lección)?
- ✓ ¿En qué maneras la actividad se basa en el conocimiento previo de los estudiantes? ¿Qué definiciones, conceptos, o ideas los estudiantes necesitan saber para comenzar a trabajar en la actividad?
- ✓ ¿Cuáles son todas las maneras en las cuales la actividad puede ser resuelta?
 - ¿Cuáles de esos métodos usted piensa sus estudiantes usarán?
 - ¿Qué concepciones erróneas usted piensa sus estudiantes tendrán?
 - ¿Qué errores usted cree sus estudiantes cometerán?
- ✓ ¿Qué usted espera de sus estudiantes mientras trabajan y completan esta actividad?
 - ¿Qué herramientas o recursos los estudiantes tendrá que usar en su trabajo?
 - ¿Cómo los estudiantes trabajarán – individual, en grupos pequeños, en pares – para explorar la actividad? ¿Qué tanto tiempo trabajarán individualmente o en grupos/pares? ¿Los estudiantes serán colocados en grupos/pares en alguna manera en específico? De ser así, en qué forma?
 - ¿Cómo los estudiantes documentaran y reportaran su trabajo?
- ✓ ¿Cómo los estudiantes serán introducidos a la actividad de tal manera que las demandas de la actividad no sean comprometidas? ¿Qué comentarios evidenciarán que los estudiantes entendieron la actividad?

Parte 2: Apoyando a los estudiantes en la exploración de la actividad

- ✓ Mientras los estudiantes trabajan individualmente o en grupos pequeños:
 - ¿Qué preguntas le hará a sus estudiantes para enfocar su razonamiento?
 - ¿Qué usted espera ver o escuchar que le hará saber cómo los estudiantes en la clase están razonando acerca de las ideas matemáticas?
 - ¿Qué preguntas le hará a sus estudiantes para evaluar el entendimiento de los estudiantes de importantes ideas matemáticas, estrategias de solución de problemas, o de representaciones?
 - ¿Qué preguntas le hará a sus estudiantes para promover el entendimiento de los estudiantes de las ideas matemáticas?
 - ¿Qué preguntas le hará a sus estudiantes para estimular a los estudiantes a compartir su pensamiento/razonamiento con otros o para evaluar sus interpretaciones acerca de las ideas de sus compañeros?
- ✓ ¿Cómo se asegurará que sus estudiantes se mantengan enfocados en la actividad?

- ¿Qué usted hará si un estudiante no sabe cómo comenzar a resolver la actividad?
- ¿Qué usted hará si un estudiante termina la actividad case inmediatamente y se aburre o comienza a distraer a sus compañeros?
- ¿Qué usted hará si algunos estudiantes se enfocan en aspectos no-matemáticos (por ejemplo, utilizan la mayor parte del tiempo embelleciendo la solución que será presentada a sus compañeros)?

Parte 3: Compartiendo y discutiendo la actividad

✓ ¿Cómo usted facilitará la discusión de la clase de tal manera que las metas matemáticas establecidas sean logradas? Específicamente:

- ¿Qué soluciones usted quiere que sean compartidas durante la discusión de la clase? ¿En qué orden serán presentadas las soluciones? ¿Por qué?
- ¿Cómo el orden en el cual se presenten las soluciones ayudara a los estudiantes a desarrollar su entendimiento de las ideas matemáticas que se han enfocado durante la lección?
- ¿Qué preguntas en específico usted hará para que sus estudiantes:
 - hagan sentido de las ideas matemáticas que usted desea que ellos aprendan?
 - expandan, debatan, y se cuestionen las soluciones que se presenten?
 - busquen patrones?
 - comiencen a formar generalizaciones?
- ¿Qué usted espera ver o escuchar que le hará saber que los estudiantes en la clase entendieron las ideas matemáticas que usted quería que ellos aprendieran?
- ¿Qué hará usted mañana que esté construido en esta lección?

Appendix G. List of Useful Resources (Spanish)

Páginas web donde puede conseguir actividades y/o recursos para planificar sus clases:

1. NCTM Illuminations (<http://illuminations.nctm.org/>)
2. Shodor Interactive Activities (<http://www.shodor.org/>)
3. GeogebraTube (<http://www.geogebratube.org/>)
4. Rice Virtual Lab on Statistics (<http://onlinestatbook.com/rvls.html>)
5. Biblioteca Nacional de Manipulativos Virtuales
(<http://nlvm.usu.edu/es/nav/siteinfo.html>)

Enlaces a journals gratuitos:

1. Journal of Educational Research (<http://www.j-e-r-o.com/index.php/jero>)
2. Journal of Education and Learning (<http://www.ccsenet.org/journal/index.php/jel>)
3. Journal of Technology Education (<http://scholar.lib.vt.edu/ejournals/JTE/>)
4. The Australian Journal of Teacher Education (<http://ro.ecu.edu.au/ajte/>)

Appendix H. Journal Entry Prompts

Journal Entries

Teacher's name: _____

Group Number: _____

Journal entry # _____

When writing your journal entries please reflect on the conversation your group members have had. Reflect on your discussions about the mathematical content, students' difficulties and misconceptions, students' learning of the mathematical content, technology tools available for you to use to teach that particular mathematical content, the pedagogical approaches your group is considering to use, and any other topic you feel is relevant. Although not inclusive, you can use the following prompts:

1. Regarding mathematical content
 - a. Discuss your comfort with the mathematical content your group is focusing the lesson on.
 - b. Discuss your previous experiences with learning and teaching the mathematical content your group is focusing the lesson on. (Reflect on what you know and what you think you don't understand well mathematically)
 - c. How your comfort level and previous experiences with the mathematical content played into the design of the lesson?
2. Regarding students' thinking
 - a. Before reading and having discussions with your group members, what did you know about students' difficulties and misunderstandings of the mathematical content your group is focusing the lesson plan on? Where you surprised by the content of the readings regarding student thinking and/or student common misconceptions?
 - b. From the reading and discussion with your group members, did you learn something new about students' difficulties and/or misunderstandings of the mathematical content you are planning to teach?
 - c. With respect to lesson planning, how beneficial was it to read literature on, and have discussions with your peers about, students' thinking and learning of that particular mathematical concept?
3. Regarding technology tools
 - a. When planning your lesson, which technology tools did your group consider using? Why?
 - b. At the end, which technology tool(s) has your group agreed upon using? Explain why you chose to use that particular technology over the other technology tools your group considered.

** Please keep a record of your journal entries in a Google document. When indicated, please share that document with marggie.gonzalez@upr.edu.

Appendix I. Codebook

Category Name	Definition
Knowledge of Math and Curriculum	<ul style="list-style-type: none"> • Knowledge of the mathematics curriculum in general
Knowledge of Math and Students	<ul style="list-style-type: none"> • Knowledge of students misconceptions • Knowledge of students' mathematical abilities. • Anticipate students responses
Knowledge of Math and Teaching	<ul style="list-style-type: none"> • Pedagogy_Math: Represents the interaction between mathematical understanding and pedagogical issues • Questioning: takes into consideration which are the most appropriate questions to ask the students while planning a lesson
Knowledge of Technology	<ul style="list-style-type: none"> • Access to Tech: which technologies they have access to in their classrooms or schools • General_Knowledge: knowledge of technology in general • Technology_Math: knowledge of technology specially designed to teach mathematics or knowledge of how the technology can be used to teach mathematics
Mathematical Knowledge	<ul style="list-style-type: none"> • Common_Content: Knows how to solve problems correctly • Specialized_Content: unpacking math knowledge in ways not needed outside of teaching • Math_Conceptual: depth and breadth of mathematical knowledge
Technology-based mathematics instruction knowledge	<ul style="list-style-type: none"> • Teacher's abilities to make changes to pedagogy • Recognize the need for flexibility in instruction that results from the use of technology

Appendix J. Example of Detailed Lesson Plan (Group 2, Cycle 1)

Estado Libre Asociado de Puerto Rico
 Departamento de Educación
 Programa de Matemáticas

PLAN DIARIO DE MATEMÁTICAS

Curso: Álgebra Grado: Octavo Grupo: 8 – 1 Fecha: _____

Tema: **Funciones**

Estrategia de base Científica: Integración de la tecnología

Descripción del proceso: El estudiante pasará a la pizarra electrónica y moverá algunos puntos. Se utilizará el programa de geogebra. El documento que se utilizará se encuentra en la siguiente página de Internet:

<http://tube.geogebra.org/student/c4212/m40009/ylyy#material/40009>

Estándar y Expectativa

Álgebra – El estudiante es capaz de realizar y representar operaciones numéricas que incluyen relaciones de cantidad, funciones, análisis de cambios, al emplear números, variables y signos para resolver problemas.

2.0 Identifica funciones al basarse en el comportamiento de su gráfica y su razón de cambio, y describe funciones al usar la notación y terminología apropiada.

8.A.2.1 Reconoce que una función de un conjunto (llamado dominio) a otro conjunto (llamado rango) le asigna a cada elemento del dominio exactamente un elemento del rango. Si f es una función y x un elemento de su dominio, entonces $f(x)$ denota la salida de la ecuación $y = f(x)$. Determina si una relación es una función a partir de su gráfica y su descripción verbal.

Objetivo:

1. Con la ayuda de la discusión el estudiante deducirá las definiciones de: variable dependiente, variable independiente, relación y función.
2. Luego de la discusión el estudiante:
 - a. determinará si las relaciones dadas son funciones.
 - b. describe conjuntos y crea relaciones.

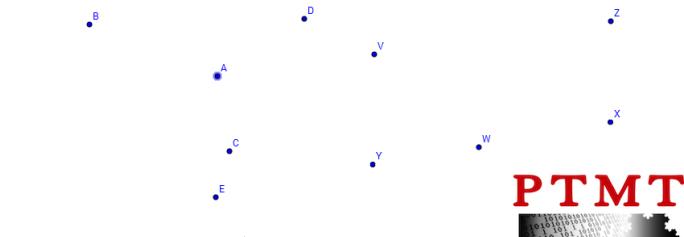
Inicio:

Actividad #1: *Determinar variable dependiente e independiente (5 minutos)*

Materiales:

- 1) Una computadora para cada estudiante
- 2) Libreta
- 3) La actividad creada por Steketee y Scher (2011) en geogebra va a estar instalado en la computadora para fácil acceso. La misma se puede obtener en la siguiente dirección:
<http://tube.geogebra.org/student/c4212/m40009/ylyy#material/40009>

Who's related to whom? Drag the points to figure it out.
 List the independent variables. For each independent variable,
 name the dependent variable(s) that depend on it,
 and describe how they are related.



Objetivo de la actividad: A través de la actividad el estudiante analizará diferentes relaciones en donde identificará la variable dependiente e independiente.

Instrucciones de la actividad: Utilizando una computadora el estudiante debe mover los puntos y escribir en su libreta que variable mueve a quien. Luego realiza una lista de las variables independientes. Por cada variable independiente menciona una variable dependiente que depende de ella y describe como es la relación.

Discusión de la actividad: (5 minutos)

Antes de llenar la tabla se va a preguntar ¿Qué sucede al mover un punto? **No se mueve ningún punto, Se mueve un punto**

Se mueve más de un punto.

Esta actividad se discutirá utilizando la pizarra electrónica y el programa en Geogebra. Se escogerán varios estudiantes al azar para que pasen al frente y se comience la discusión. Los estudiantes van a ir llenando la tabla en la pizarra electrónica. Mientras se va llenando la tabla se le va a ir preguntando a los estudiantes quien depende de quién. Una vez llenada la tabla se les preguntará a los estudiantes que digan en sus propias palabras que significa variable dependiente e independiente. Se debe copiar en la pizarra todas las ideas que los estudiantes mencionen y luego de eso presentarles las definiciones.

Variable Independiente	Variable Dependiente	Descripción de la relación
A	Y	Si, A se mueve a la derecha, Y se mueve diagonalmente a la izquierda.
C	D	Para donde se mueva C, D se mueve
V	V	V se mueve sola
W	X, E	Si W se mueve a la derecha, X se mueve a la derecha pero lento y E se mueve a la derecha pero con más velocidad
Z	B	Movimientos opuestos

Definición:

- 1) Variable Dependiente – es aquella variable que necesita de otra
- 2) Variable Independiente – es aquella variable que no necesita de ninguna otra

Desarrollo: (15 minutos)

- ✓ Al investigar un problema es común preguntarse cuál es la relación que existe entre dos cantidades descritas por variables.
- ✓ Primero nos aseguramos de asociar a cada variable un conjunto que contiene los posibles valores que ésta asume o nos interesa estudiar.
- ✓ La relación entre las variables se representa haciendo corresponder o parear los valores en los conjuntos de cada una de ellas.
- ✓ Los conjuntos pueden ser numéricos o de otra índole, dependiendo de cada aplicación particular.

Definiciones:

- 1) Relación – correspondencia entre los elementos de dos conjuntos que forman parejas ordenadas, en otras palabras es una comparación entre dos cantidades
 - a. Otra definición: Podemos definir la relación como **la correspondencia que hay entre TODOS o ALGUNOS del primer conjunto con UNO o MÁS del segundo conjunto.**
- 2) ¿Qué es un conjunto? Bueno, por decirlo de una manera simple es **una colección**. Primero eliges una propiedad común a unas "cosas" (esto lo definiremos luego) y después reúnes las "cosas" que tienen esa propiedad.



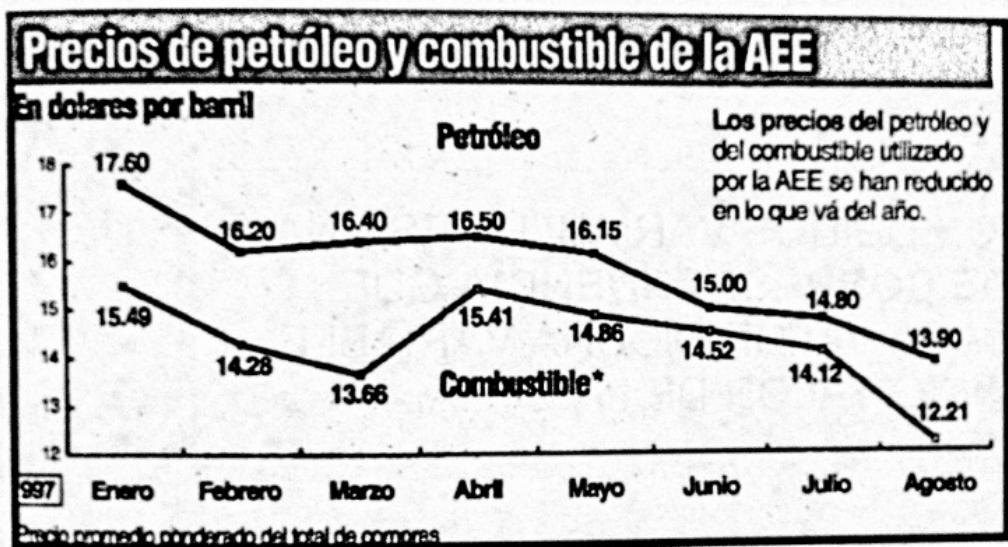
Por ejemplo, la ropa que llevas: podrían ser zapatos, medias, sombrero, camisa, pantalones y otras cosas.

Otro ejemplo sería tipos de dedos, este conjunto tendría pulgar, índice, medio, corazón y meñique

Puede haber conjuntos infinitos y conjuntos finitos. Por ejemplo: la ropa que nos podemos poner tiene una infinidad de posibles combinaciones, mientras que los tipos de dedos tiene solamente cinco posibilidades.

Para el siguiente ejemplo menciona el conjunto utilizado en la siguiente situación.

Ejemplo #1: *La autoridad de Energía Eléctrica utiliza petróleo crudo y combustible derivados del petróleo para la producción de energía eléctrica en Puerto Rico. La grafica muestra el precio del barril de petróleo y del barril de combustible durante los primeros 8 meses del año 1998.*

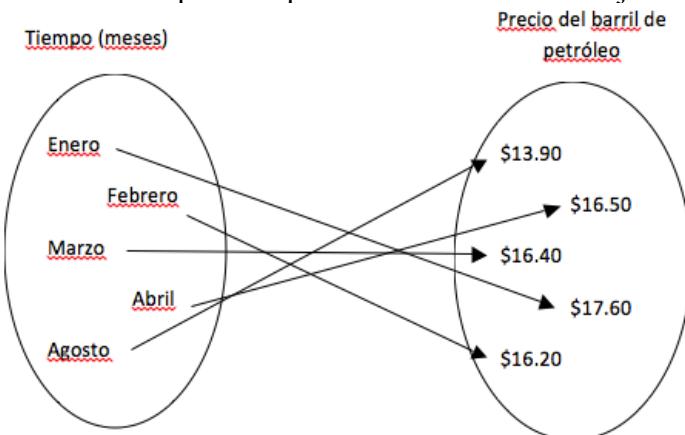


Periódico El Nuevo Día, septiembre 1998

Luego de presentar el primer ejemplo el maestro debe preguntarle a los estudiantes:

- 1) ¿Qué muestra la grafica? El estudiante puede mencionar que según los meses pasan el precio del combustible y del petróleo bajan. El mes de enero fue donde el precio del petróleo y del combustible era más alto. En agosto se redujo el precio del combustible y del petróleo.
- 2) ¿Cuál es la relación que muestra la grafica? El estudiante debe mencionar que los meses se relacionan con el precio del combustible y el precio del petróleo. Una relación puede asociar a cada mes el precio del barril de petróleo y otra el precio del barril de combustible.

Esta información se puede representar en forma de conjunto.



Ejemplo #2: *Conjunto A: Estudiantes y el Conjunto B: Su pueblo de nacimiento*

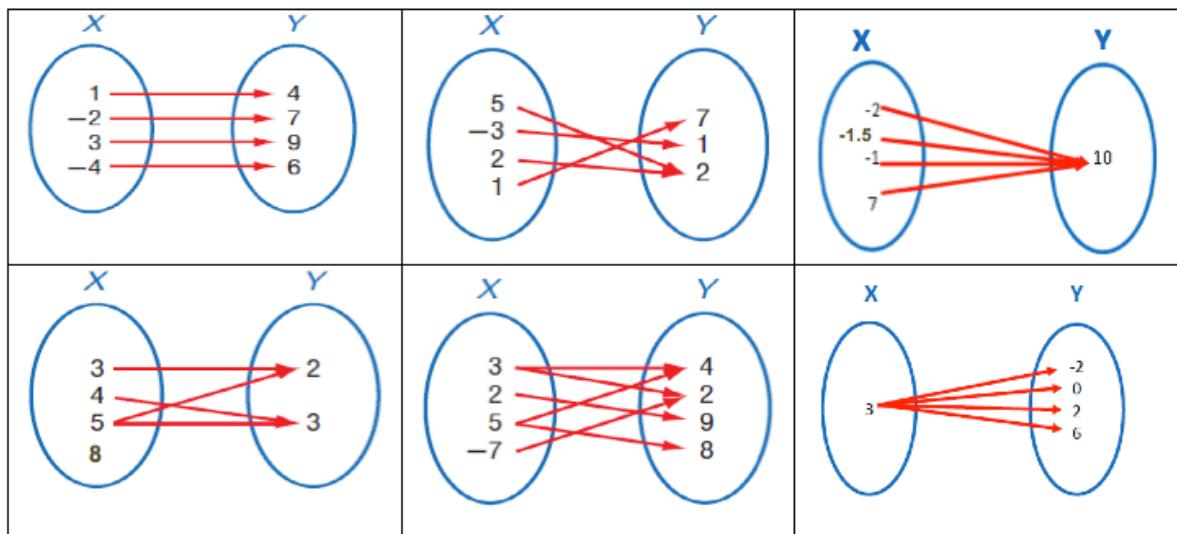
Para este ejemplo se van a escoger 5 estudiantes al azar para que pasen al frente y escriban su nombre y el pueblo en donde nacieron. Si el pueblo en donde nacieron se repite solo tienen que hacer una flecha hacia el pueblo que se repita.

En la pizarra electrónica (Power Point) el estudiante debe pasar al frente y ya van a estar los conjuntos dibujados, y el estudiante sólo debe escribir la información en el espacio provisto.

Ejemplo #3 – *Menciona un conjunto y describe la relación entre los dos conjuntos.*

Actividad #2: *Funciones* (10 minutos)

Esta segunda actividad se discutirá haciendo uso de la pizarra electrónica. Se le indicará a cada estudiante cuál de los siguientes conjuntos representa una función y los que no. Se espera que el estudiante mencione algunas características de los conjuntos que son funciones y de los que no lo son. Con esta información se espera que el estudiante construya la definición de función. Se va a escribir en la pizarra la información suministrada por los estudiantes.



Contestaciones:

- 1) Función 2) Función 3) Función
- 4) No Función 5) No Función 6) No Función

Definición:

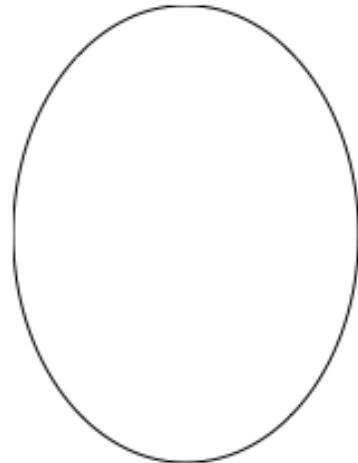
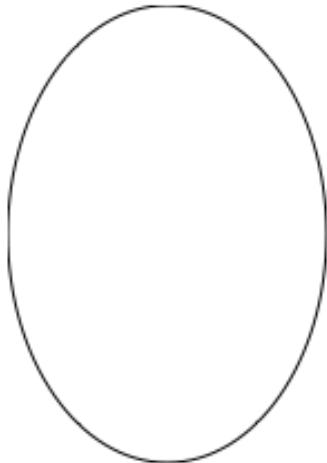
- 1) Función – Es una relación entre un conjunto dado x (variable independiente) y otro conjunto de elementos y (variable dependiente) de forma que a cada elemento en x le corresponde un único elemento en y .

Aplicaciones de la vida diaria (5 minutos)

Ejemplo #1:

Conjunto A: Marca de Automóviles

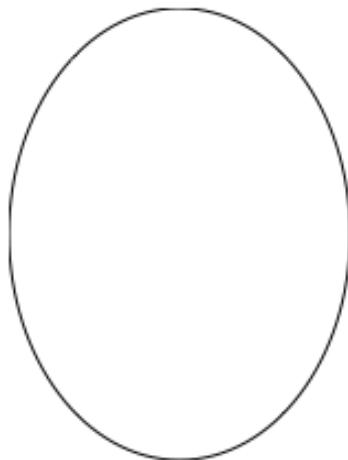
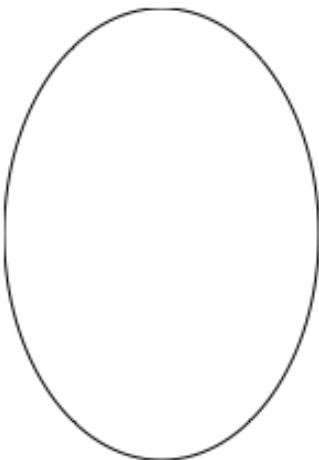
Conjunto B: Color del automóvil



Ejemplo #2:

Conjunto A: Pueblos de Puerto Rico

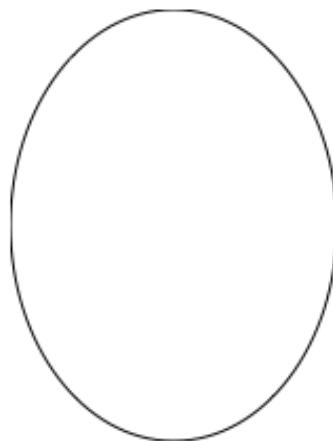
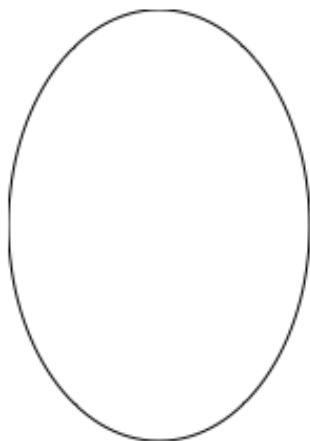
Conjunto B: Festivales



Ejemplo 3:

Conjunto A: Animales

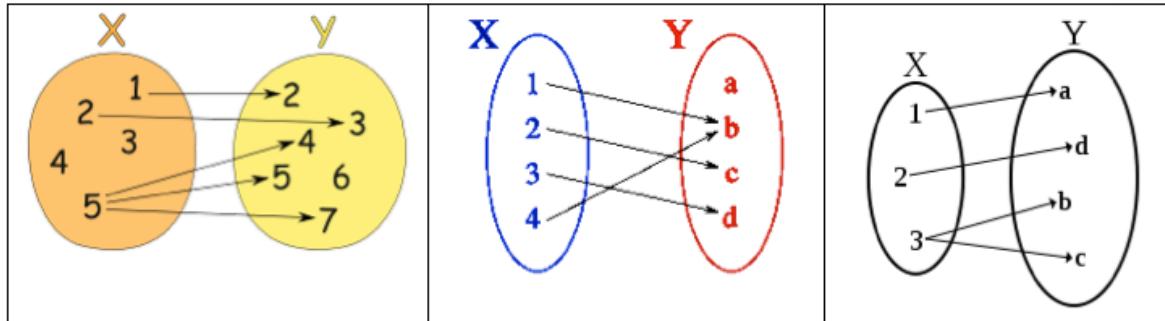
Conjunto B: Número de patas



Cierre:

Actividad #3: (10 minutos)

Parte I: *Determina si la relación entre los siguientes conjuntos representan una función.*



Parte II: *Contesta cada pregunta.*

- Define (invente) dos conjuntos
- Describe la relación entre los dos conjuntos.
- Describe una relación que sea función y otra que no es función.

Referencias

- Relación matemática -
http://enciclopedia.us.es/index.php/Relaci%C3%B3n_matem%C3%A1tica

- Disfruta las matemáticas -
<http://www.disfrutalasmaticas.com/conjuntos/conjuntos-introduccion.html>
- Torres, W.A., (2003). “PreCálculo, El estudio de las funciones matemáticas para describir procesos de cambio.” Publicaciones Puertorriqueñas.

Appendix K. Example of Sparse Lesson Plan (Group 3, Cycle 2)

Plan Regresión Lineal Plan Detallado

Grado: 10

Curso: Algebra II

Unidad II: Funciones lineales de dos variables y la regresión lineal

Actividades:

Inicio: Introducción al tema del día.

Desarrollo: Desarrollo de destrezas. Actividad #1 Ecuación del diagrama de dispersión.

Cierre: Reflexión de lo aprendido

Materiales: Programas tecnológico Geogebra y Fathom.

Presentación en PREZI

Actividad de Inicio (tiempo 5 minutos)

- 1) Saludos
- 2) Pasar Asistencia
- 3) Presentación / anuncio de visita en la sala de clases
- 4) Se distribuyen hojas de trabajo a los estudiantes
- 5) Instrucciones programas a usar

Actividad de desarrollo (tiempo 40 minutos)

- I. Se inicia la clase con la Actividad #1 Tamaño Familiar vs Productos Limpieza donde los estudiantes siguen las instrucciones para trazar la línea que mejor considere en diagrama presentado.

Actividad #1 Tamaño Familiar vs Productos Limpieza

Instrucciones:

- 1) Inicia el programa de Geogebra.
- 2) Abre el documento de la Actividad Tamaño Familiar vs Productos Limpieza
 - a) Observe el diagrama de dispersión. ¿Existe o no correlación?, de existir ¿qué tipo de correlación es?
- 3) Trace la línea en el diagrama de dispersión (selecciona el ícono de línea y deslizar en el diagrama donde crees que va la línea).
 - a) Escribe la ecuación que muestra la gráfica. _____
 - b) Identifica la pendiente e intercepto. _____

- II. La maestra establece dinámica para que los estudiantes comparan las dos líneas escribe en la pizarra las ecuaciones encontradas por los estudiantes y aprovecha para discutir los hallazgos contestando las siguientes preguntas; ¿cuál crees que es la mejor?, ¿cómo sabemos que es la mejor?
- III. La maestra procede a definir la línea de mejor ajuste.
- IV. Los estudiantes realizan la Actividad #2 Rendimiento de gasolina. Realizarán diagrama dispersión y determinarán la línea de mejor ajuste utilizando el programa tecnológico Fathom.

Actividad #2 Rendimiento de gasolina

Instrucciones:

- 1) Inicia el programa de Fathom.
- 2) Abre el documento Vehículos 2006.
- 3) Selecciona el ícono de gráfica (graph).
- 4) Sombrea la columna de Ciudad (City), arrastra y suelte al eje de x.
- 5) Sombrea la columna de Autopista (Hwy) , arrástralos y suelte al eje Y
- 6) Oprima lado derecho del ratón (mouse) para seleccionar la línea móvil (Add Movable line). Acomodas la línea en el lugar que piensas que debe ir.
- 7) Identifica la ecuación de la línea.

Conteste:

Ecuación de la línea (movible line) E_1 _____

Para cada valor de x, ¿cómo se relaciona con y? _____

¿Cuántas son las millas por galón (mpg) en el recorrido inicial por la autopista?

- 8) Oprima lado derecho del ratón (mouse) para seleccionar la línea de mejor ajuste- (Least – Square Line).

Identifica la ecuación resultante y conteste:

Ecuación de línea ajustada E_2 _____

¿Es o no igual a la suya? _____

¿Cree que se ajusta mejor a los datos? _____

- 9) La maestra define el Coeficiente de Correlación

- 10) Para el resultado del coeficiente de correlación; selecciona “Show squares” en la línea inicial que hizo y luego para la otra línea de mejor ajuste.

Suma de los cuadrados E_1 _____ E_2 _____

¿Qué podemos concluir de la suma de los cuadrados?

Compara los resultados de esa suma de cuadrados. ¿Qué opinas?

- V. Maestra aplica lo aprendido para que los estudiantes puedan predecir los resultados basados en la línea de mejor ajuste
Cuando en la ciudad las millas por galón son 13 (mpg), predecir cuantas millas por galón son en la autopista _____

Actividad de Cierre (tiempo 5 minutos)

Reflexión;

- a) En la clase de hoy aprendí _____
b) Hoy estuve confundido con _____