

## ABSTRACT

EDGINGTON, CYNTHIA PAGE. Kindergarten Teachers' Mathematics Teaching Cycle: Attending to Issues of Culture and Student Understanding. (Under the direction of Dr. Allison McCulloch).

The purpose of this study is to examine the mathematics teaching cycle of two kindergarten teachers who took part in a professional development project that promoted culturally relevant pedagogy and teaching mathematics for understanding. The study aims to address the lack of research with respect to how teaching mathematics for understanding and attending to students' cultural backgrounds can effectively be incorporated into teachers' lesson planning practices. The present study also examines if and how the teachers' enacted math lessons are consistent with the ideologies associated with culturally relevant pedagogy and teaching for understanding.

The participants for this study were two kindergarten teachers in North Carolina who participated for one year in a three-year professional development project called Nurturing Mathematics Dreamkeepers. The data consisted of a lesson planning interview, a lesson planning observation, video-taped math lessons, and a post-lesson reflective session.

The conceptual framework for this study considers Simon's (1995) mathematics teaching cycle as a way to describe the planning and teaching process. Within the mathematics teaching cycle, Ladson-Billings' (1995a) tenets of culturally relevant pedagogy and Hiebert, et al.'s (1997) dimensions of classrooms that support teaching for understanding are both used as a lens to examine the participants' teaching cycles.

The findings from this study suggest that the teachers attend to many things during their lesson planning, including the learning objective, classroom activities and their students' backgrounds. Some aspects of their enacted lessons were consistent with the

ideologies associated with culturally relevant pedagogy and teaching for understanding. The teachers exhibited high academic expectations for all students and provided contexts that are meaningful for their students. Although the teachers encouraged their students to develop their own strategies for solving problems, they did not value all of the strategies suggested by their students. Overall, some aspects of CRP and teaching for understanding were evident in the teachers' lesson planning observation and in their enacted lessons. If a goal of mathematics instruction is to increase student understanding in a learning environment that is accessible to all students and where academic success is experienced by all students, the mathematics education community can learn from studies such as this how to make this goal a reality.

Kindergarten Teachers' Mathematics Teaching Cycle: Attending to Issues of Culture and  
Student Understanding

by  
Cynthia Page Edgington

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APPROVED BY:

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Dr. Allison McCulloch  
Committee Chair

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Dr. Hollylynne Stohl Lee

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Dr. Ronald Fulp

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Dr. Patricia Marshall

## DEDICATION

To David, Zakary, and Emilie.

## BIOGRAPHY

Cynthia Page Whitehouse was born November, 14, 1969 in Fair Haven, New Jersey. As an Army brat, Cyndi lived in many different parts of the country growing up. Upon graduating from Mater Dei High School in Santa Ana, California in 1988, Cyndi enrolled in North Carolina State University. She graduated Magna Cum Laude with a Bachelor of Science in Mathematics Education in 1992. Cyndi immediately began teaching high school math in Wake County, NC.

Cyndi spent the majority of her early teaching career at Garner Senior High School where she taught Geometry, Algebra I and Pre-Calculus. In 1998, Cyndi married David Edgington, also an NCSU alumnus. She continued teaching in Garner as the lead Geometry teacher and class sponsor for the Class of 2000. At the end of the 1999-2000 school year, Cyndi decided to leave the classroom to stay home with her family. Once her son started kindergarten, she began working on her Master's Degree on a part-time basis and enrolled full-time when her daughter started school two years later. After receiving her Master's Degree, Cyndi plans to pursue a Ph.D. in Mathematics Education at North Carolina State University.

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## CHAPTER 1

### INTRODUCTION

Despite reform movements concerning the teaching and learning of mathematics, instruction in the typical mathematics classroom has not significantly changed in the last 100 years (Hiebert, 2008). Simon and Tzur (1999) recognize that reform efforts to improve mathematics education require change in teachers' views on the teaching and learning of mathematics and that one means to this end is through professional development. Hiebert, Gallimore, and Stigler (2002) claim that to improve mathematics teaching in a lasting way, teachers need to take part in professional development that is long term, collaborative in nature, and emphasizes students' learning. Regardless of how the transformation takes place, teachers are at the core of creating change in the mathematics classroom.

One of the overarching themes of NCTM's (2000) *Principles and Standards for School Mathematics* is that of equity. The Equity Principle emphasizes that "all students should have access to an excellent and equitable mathematics program that provides solid support for their learning and is responsive to their prior knowledge, intellectual strengths, and personal interests" (NCTM, 2000, p. 12). However, reports such as the National Assessment of Educational Progress (2007) continue to show discrepancies between achievement in mathematics by white American students and students of color, particularly African Americans and Hispanic Americans. This achievement gap has driven many researchers to explore issues related to culture in education (Gutstein, Lipman, Hernandez, & de los Reyes, 1997; Howard, 2001; Ladson-Billings, 1994; Leonard & Guha, 2002; Malloy & Malloy, 1998; Tate, 1995; Villegas & Lucas, 2002).

One explanation for the achievement gap is that students who are not from the dominant culture often experience a disconnection between their home culture and the culture of the traditional classroom (Howard, 2001; Ladson-Billings, 1995; Leonard & Guha, 2002; Malloy & Malloy, 1998). Research has emerged to examine the pedagogy of teachers that are successful at teaching students of color without disregarding their home culture. Ladson-Billings (1994) has coined the term culturally relevant pedagogy (CRP) and defines it as “a pedagogy that empowers students intellectually, socially, emotionally, and politically by using cultural referents to impart knowledge, skills, and attitudes” (p. 17-18).

As a research assistant, this researcher was involved in a study associated with the professional development project entitled Nurturing Mathematics Dreamkeepers (NMD), funded by the National Science Foundation. One goal of the project was to lessen the achievement gap in mathematics for African American students of the kindergarten, first and second grade teachers who participated in the project. In addition, the project aimed to acquire a knowledge base for how, if at all, teachers develop a culturally relevant pedagogy and adjust their instruction toward teaching mathematics for understanding as a result of taking part in the professional development activities. Part of this researcher’s participation in the project involved video-taping project teachers’ math lessons and facilitating discussions on the effectiveness of the teachers’ lessons and how culture may have played a part in the lessons. After observing several math lessons, the researcher began to contemplate how the project teachers were planning for their lessons and how the interventions they participated in during NMD retreats impacted the construction of their mathematics lessons. What do teachers attend to in constructing their math lessons once they

have been exposed to the tenets of culturally relevant pedagogy and what it means to teach for understanding?

### Statement of the Problem

Literature on the nature of teacher planning in light of reform efforts to change the teaching and learning of mathematics is sparse (John, 2006; Simon, 1995; Simon & Tzur, 1999). The majority of research on teacher planning is not specific to mathematics nor does it address how teachers attend to the cultural aspects of teaching and learning or what teachers need to do to promote learning mathematics for understanding (Decker & Ware, 2001; Leinhardt, 1989; McCutcheon, 1980; Yinger, 1980; Zahorik, 1975). Additionally, research has not addressed how teaching mathematics for understanding and attending to students' cultural backgrounds can effectively be incorporated into teachers' lesson planning practices (Eisenhart, et al., 1993; Gutstein, Lipman, Hernandez & de los Reyes, 1997; Ladson-Billings, 1995b; Putnam, Heaton, Prawat & Remillard, 1992; Putnam & Reineke, 1993).

This thesis intends to fill this gap in research on teacher planning. The present study will examine specifically what two kindergarten teachers attend to throughout the teaching cycle as well as the ways in which their attention to culture and teaching for understanding impact their lesson planning and teaching processes.

### Definitions of Terms

At this point, it is important to define several terms that will be used throughout the study. First, since a main focus of this research has to do with culturally relevant pedagogy, it is necessary to clarify what is meant by *culture*. As a principal investigator with the NMD project, Dr. Patricia Marshall used the following definition as a basis for her work with the project participants. Marshall (2002) defines culture as “the consistent ways in which people experience, interpret, and respond to the world around them” (p. 8). Culture includes attitudes about food, music, religion, and human nature in general.

With respect to teacher planning, the term *objective* is defined as a specific behavior that students should be able to accomplish after a lesson. Explicit student learning is characterized in such a way so that it can somehow be measured (Yinger, 1980). The term *learning goal* is synonymous with objective. In the context of this study, the term *activity* describes the learning experiences that the students and teacher engage in throughout the course of a lesson. It includes instructional processes and strategies and is influenced by content and materials (Yinger, 1980, Zahorik, 1975). For this study, the term activity is defined as something different from a *task*. According to Heibert, et al (1997), *mathematical tasks* should offer problematic mathematics that requires students to reflect on and communicate ideas about the mathematics involved. In addition, tasks should permit students to utilize various tools to solve problems and offer students opportunities to build understanding that they can take with them. The term *content* refers to the specific subject matter attended to in a lesson.

### Organization of Paper

The next chapter contains a review of the literature on the three aspects of the current study: culturally relevant pedagogy, teaching mathematics for understanding and finally, teacher planning. The chapter concludes with a description of the conceptual framework used for the current study and the specific research questions. Next, the methodology is described, followed by how the data was analyzed and a discussion of the findings from this research. The paper concludes with a summary and implications for further research.

## CHAPTER 2

### BACKGROUND

Because the nature of teaching is so complex, it is important to examine how teachers go about planning their instruction (Clark & Lampert, 1986; Fernandez & Cannon, 2005; Leinhardt, 1989; Leinhardt & Greeno, 1986; Leinhardt, 1989; Yinger, 1980). Specifically considering math lessons, there are many aspects teachers can attend to, including but not limited to, what the learning goals are for the lesson, how instruction can be relevant and accessible to all students, and what is necessary in order to teach mathematics for understanding. In light of the intervention the teachers took part in as a part the Nurturing Mathematics Dreamkeepers project, literature will be reviewed with respect to culturally relevant pedagogy, teaching for understanding, and finally teacher planning. Because the conceptual framework for this study is tied closely to the literature, it is presented after the literature review. The specific research questions explored in the present study are listed at the conclusion of this chapter.

#### Culturally Relevant Pedagogy

Research has recently developed to describe the pedagogy of teachers that are successful at teaching students of color without disregarding their home culture known as culturally relevant pedagogy (CRP). CRP does not adhere to an explicit list of behaviors that if performed will ensure the success of all students, but rather is an ideology that may look different in different classrooms. CRP is supported by three general tenets: (1) Academic success is experienced by all students; (2) Students nurture and sustain cultural competence;

and (3) Students develop critical sociopolitical conscientiousness (Ladson-Billings, 1995a).

Each tenet will be discussed in the review of the literature that follows.

### *High Academic Success*

Several studies that examine the practices of culturally responsive teachers note that these teachers expect high academic achievement from all students (Gutstein, et al, 1997; Howard, 2001; Ladson-Billings, 1994; Ladson-Billings, 1995a; Ladson-Billings, 1995b; Tate, 1995). In her 1994 study, Ladson-Billings describes the teaching philosophies and practices of eight teachers that she deems as successful teachers of African American students. Ladson-Billings (1995b) found that all eight of the teachers drew on the individual strengths of their students to help develop academic skills and saw student excellence as something that is not restricted to standardized test scores. Ladson-Billings (1995a) emphasizes that culturally relevant pedagogy is not simply encouraging students to feel good about themselves, but it uses contexts that are meaningful to students to motivate them to want to excel and take pride in their academic achievement. In addition, CRP uses students' cultural experiences and out of school knowledge to further their learning and provide opportunities for students to experience excellence in the school setting (Gutstein, et al., 1997; Howard, 2001; Ladson-Billings, 1994; Tate, 1995).

### *Cultural Competence*

The second tenet of CRP refers to cultural competence which means maintaining the identity of one's own culture while at the same time having the ability to effectively interact with individuals of other cultures (Ladson-Billings, 1995a; Ladson-Billings, 1995b). By using students' cultural experiences as a spring board to learning, teachers are also validating

students' cultural identities and encouraging students to become culturally competent.

Ladson-Billings (1995b) described how one teacher used a student's cultural competence in his language and pride in his heritage to become a school leader both socially and academically. The student was encouraged to use the language and dress he was comfortable with while at the same time experiencing scholastic achievement, becoming an academic role model for students who related to him culturally. Similarly, by involving parents and individuals from the community in classroom activities, the teachers in Ladson-Billings' (1995a) study acknowledged the importance of the cultural knowledge that these individuals hold. One teacher in particular initiated a program in which parents and relatives were invited into the classroom to teach or demonstrate a skill that the teacher would later build upon to explore different academic areas such as science and mathematics (Ladson-Billings, 1995a).

Through a three year study of teachers from an urban public school whose student population is ninety-nine percent Latino, Gutstein, et al. (1997) described the orientation effective teachers of Mexican Americans have towards their students' culture and experience which promotes cultural competence in their students. The authors illustrated how a teacher can be informed of a student's culture but see it as a limitation as opposed to using the student's culture to draw from and build upon. When teachers use their knowledge about students' culture to empower and encourage students, they are seen as "in solidarity" (Gutstein, et al., 1997, p. 728) with the students. Teachers in their study were described as having shared aims with their students and their students' families. That is, the teachers believed they were equals with parents, whom they can learn from and work together with

for positive change, as opposed to thinking they were superior to parents, who along with their children were victims of society (Gutstein, et al., 1997). By creating an empowering orientation towards students' cultures, teachers helped students build on the strengths of their culture while at the same time acknowledging the culture's imperfections.

Another means by which CRP encourages cultural competence is through language. Research has shown that students experience less discontinuity between their home culture and the classroom culture when the forms of communication used in the classroom are consistent with students' cultural ways of expression (Gutstein, et al., 1997; Howard, 2001; Ladson-Billings, 1995a; Ladson-Billings, 1995b; Malloy & Malloy, 1998). The teachers in Gutstein, et al.'s 1997 study consistently used both English and Spanish (the native language of the majority of their students), using informal means of communicating with students, but also encouraging proper translations and correct use of mathematical terms. In his study of four elementary school teachers, Howard (2001) described teachers who have open conversations with their students about the use of Ebonics and Black English Vernacular whereby teachers do not degrade students for using such language, but help them to understand its appropriate and contextual use. In addition, teachers explained how the use of the standard code of English can help students navigate social, political, and financial opportunities (Howard, 2001).

### *Sociopolitical Consciousness*

Not only does CRP validate students' cultural identities, but it also encourages students to develop critical sociopolitical consciousness. That is, teachers "must help students to recognize, understand, and critique current social inequities" (Ladson-Billings,

1995b, p. 476). Howard's 2001 study illustrated the "holistic instructional strategies" (p. 186) of effective teachers of African American students whereby teachers emphasize students' social, emotional and ethical development as well as their academic growth. The teachers in his study were concerned with helping African American students realize their ability to use knowledge as power to address inequities they see in their communities (Howard, 2001). One teacher in particular used trips to soup kitchens, homeless shelters, and nursing homes to teach her students about character, values and citizenship (Howard, 2001).

Specifically with respect to mathematical ability, Tate (1995) highlighted the importance of supporting African American students to "use mathematics as an agent to change their out-of-school realities" (p. 169). In his 1995 study, Tate described how a teacher led her classes through a project to relocate liquor stores that were in proximity to their school. The students used research on local laws and the economics of their community and compared maps to actual measurements to make their case against the locations of the liquor stores. They used fractions, decimals and percents to communicate their findings to local newspapers and government agencies, resulting in real change in their community. Similarly, Ladson-Billings (1994, 1995a) gave an example of teachers who used out-dated textbooks as a springboard to encourage students to critique the textbook material and also exposed them to other perceptions of the topics being covered (other than those in the out-dated textbooks). In addition, the teachers took these opportunities to discuss the disproportion in education that allow some students to have more current textbooks while others do not (Ladson-Billings, 1995a). Both examples show how teachers who practice

CRP use the classroom to have open, honest discussions about inequities in the students' communities and how students can use their abilities and knowledge to illicit change in their society.

The current research aims to examine how kindergarten teachers who have been exposed to the tenets of CRP through a professional development project plan for instruction. A critical component of CRP is that the teacher has knowledge of the students' culture and the culture of the community (Gutstein, et al., 1997; Howard, 2001; Leonard & Guha, 2002; Malloy & Malloy, 1998). The teacher is willing and able to use this knowledge to increase the academic achievements of students and at the same time facilitate students' development and maintenance of strong cultural identities. The research discussed above describes the pedagogy and practices of teachers who are previously known to be effective teachers of students of color. However, there is a lack of research examining how exposing teachers to the principles of CRP influences their knowledge, lesson planning and instruction.

#### Teaching for Understanding

In addition to CRP, the NMD professional development that the participants in this study took part in also promoted teaching mathematics for understanding. Recent developments in mathematics education stress the importance of teaching mathematics for understanding. NCTM's *Principles and Standards for School Mathematics* (2000) highlight the importance of conceptual understanding in addition to factual knowledge and proficiency with procedures when it comes to increasing students' mathematical sense making. Nevertheless, describing what it means to teach for understanding or learn mathematics with understanding can be a complex task (Eisenhart, et al., 1993; Putnam, Heaton, Prawat, &

Remillard, 1992; Putnam & Reineke, 1993). As often is the case with teaching computational algorithms, it is uncomplicated to assess whether students have learned the algorithms or not; when the goal of instruction is to increase students' understanding, however, the evidence of what students have learned may not be as clear (Hiebert & Grouws, 2007; Putnam & Reineke, 1993). As we learn more about how students think and learn, we are better able to express what it means to understand mathematics. Hiebert, et al. (1997) state that understanding is formed when we are able to make sense of new knowledge by relating it and connecting it to what we already know.

### *Making Connections*

Research exists in support of the idea that making mathematical connections has a positive impact on increasing understanding (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Carpenter & Lehrer, 1999; Hiebert & Grouws, 2007; Smith, 2000; Wearne & Hiebert, 1989). Useful connections can be between new knowledge and prior knowledge as well as between various mathematical representations. In their synthesis of research on the effects of classroom instruction on students' learning, Hiebert and Grouws (2007) found that despite the form of instruction (expository or discovery), students' understanding can develop when mathematical connections are explicitly made. One study is specifically noted where clear connections were made between physical objects and written representations which resulted in higher student performance (Hiebert & Grouws, 2007). In this study, different instructional strategies such as teacher demonstration and small group work were used to teach whole number addition and subtraction utilizing base-10 blocks. It was concluded that the high levels of achievement found across the sample of students in the study were due to

the clear connections made between the manipulatives and the written representations and not by the instructional strategies used (Hiebert & Grouws, 2007).

Likewise, in order to facilitate students' understanding of written symbols associated with decimal numbers, Wearne and Hiebert (1989) designed studies to aid students in connecting written decimal symbols to concrete manipulatives. Fourth grade students were engaged in what the researchers call Conceptually Based Instruction whereby students developed connections between base-10 blocks and written decimal symbols and then were able to use the manipulatives to create procedures for adding and subtracting decimal numbers. Students were encouraged to use written representations for the actions on the base-10 blocks. The results of the studies showed that students of all ability levels were able to make connections between concrete manipulatives and written decimal symbols and use that meaning to develop procedures for operating with decimals (Wearne & Hiebert, 1989).

In an effort to develop a pedagogical framework that promotes mathematical understanding, Smith (2000) studied practices of six primary grades classroom teachers whose focus was to teach for understanding. An element of classroom instruction found throughout the study was teachers having information about students' prior knowledge and subsequently connecting that to new concepts. Through their research on how students develop mathematical meaning, Carpenter and Lehrer (1999) noted how young children have already begun to develop informal mathematical relationships prior to formal schooling. By building on these experiences, teachers can help students develop more sophisticated mathematical concepts (Carpenter, Fennema, & Franke, 1996; Carpenter & Lehrer, 1999). Teachers can help foster their students' mathematical understanding by allowing students to

make connections to prior knowledge and experiences as well as encouraging connections between mathematical representations.

### *Reflection and Communication*

The literature on teaching mathematics for understanding identifies two processes that influence our ability to make connections: reflection and communication (Carpenter & Lehrer, 1999; Hiebert, et al., 1997; Simon, 1995; Smith, 2000). Reflection is identified as a cognitive process whereby one is consciously thinking about experiences and carefully examining the “why” of those experiences. Communication, a social process, incorporates verbal, non-verbal, and written forms of interaction that allow us to share ideas and listen to ideas of others (Hiebert, et al., 1997). Effective communication encourages us to think about our ideas more thoroughly in order to give clear explanations or justifications to others. By reflecting and communicating on and about mathematics, relationships may be created between new and prior knowledge, forming connections which have the potential to increase understanding (Carpenter & Lehrer, 1999; Hiebert, et al., 1997).

In considering a constructivist viewpoint of learning, Simon (1995) promoted the use of problems and tasks that encourage reflection on the part of the learner in order to change or build upon current conceptions of knowledge. Through exploration and reflective inquiry, students are able to become active participants in their construction of knowledge.

Thompson, Philipp, Thompson, and Boyd (1994) contrasted the teaching orientations of two middle school math teachers. They described teachers that teach for understanding as having a conceptual orientation to teaching; that is, they focus on relationships among ideas and representation as well as conceptual explanations as opposed to only focusing on procedures

and explanations of algorithms. Their findings showed that a teacher with a conceptual orientation to teaching more consistently encouraged classroom discourse where students often communicate about each others' ideas and strategies. Also, a teacher with this type of orientation aimed to provide students with opportunities to reflect on their reasoning as a goal of instruction (Thompson, et al., 1994).

By using pre- and post-lesson interviews with teachers in her project, Smith (2000) characterized the instruction of six primary grades math teachers. Patterns found among all of the teachers in the study included fostering student communication both verbally and in writing as well as providing opportunities for students to self-assess and reflect on their own thinking. The social nature of the classrooms observed permitted students to verbalize their thoughts and “students often self-corrected themselves as they explained and monitored their solutions” (Smith, 2000, p. 12). Carpenter and Lehrer (1999) described how reflecting on and communication about mathematics contribute to students' learning with understanding. By reflecting on their learning, students can examine how new knowledge relates to what they already know. This reflection, too, is necessary in order to communicate ideas effectively in that it helps students identify the meaningful aspects of an activity or task that need to be conveyed (Carpenter & Lehrer, 1999).

### *Teacher Knowledge*

To teach for understanding, it is essential to understand the role that making connections through reflection and communication plays in the mathematics classroom. It is also important to think about what knowledge is necessary on the part of the teacher in order to effectively provide instruction that promote these practices in the classroom. Shulman

(1986) describes three types of teacher knowledge: content knowledge, pedagogical content knowledge and curricular knowledge. Content knowledge describes a teacher's knowledge and understanding of a subject such that the teacher can explain why a subject is worthy of being studied and how it relates to other areas. Pedagogical content knowledge includes the methods a teacher uses to make content knowledge understandable by others. Too, it encompasses knowing conceptions and misconceptions students have and knowing successful strategies to further a student's understanding. Curricular knowledge is described as knowledge of different instructional materials and curricula available for instruction (Shulman, 1986).

By studying the beliefs and practices of four elementary school teachers, Putnam, et al. (1992) found that although the teachers in their study attempted to teach for understanding, their lack of mathematical content knowledge limited their facility to provide students with rich mathematical experiences. Too, the teachers believed students could not learn for understanding until they had mastered procedural skills. Although content knowledge is certainly necessary, research shows that mathematical content knowledge is not enough to effectively teach for understanding (Ball, 1993; Eisenhart, et al., 1993; Putnam & Reineke, 1993).

Putnam & Reineke (1993) examined the practices of a teacher whom they identify as a teacher with a deep understanding of the mathematical content she teaches. They found that although she had a strong content knowledge, it was "not clear how accessible this content was to all students" (Putnam & Reineke, 1993, p. 31). Instead of focusing on allowing students to make sense of the concepts being studied, the teacher was more focused

on making sure the correct mathematics was being covered. Additionally, Eisenhart, et al (1993) conducted a study of 8 preservice teachers to examine the practices for teaching procedural knowledge compared to teaching conceptual knowledge. Through interviews and classroom observations of one preservice teacher, the researchers concluded that the teacher's lack of content and pedagogical knowledge restricted her ability to communicate how she would teach for understanding and that because she was more confident in her procedural knowledge, she actually taught with that type of orientation most often (Eisenhart, et al, 1993).

Along with content knowledge, Thompson, et al. (1994) also recognized the importance of teacher knowledge about how students think about mathematics (pedagogical content knowledge) in order to facilitate understanding. This is not just knowing multiple solution methods that students may come up with in solving problems, but it requires having a thorough understanding of the situation in order to “orient student thinking in productive ways” (Thompson, et al., 1994, p. 13).

In what ways can teachers' content knowledge, pedagogical content knowledge and curricular knowledge provide students with opportunities to communicate and reflect, therefore increasing understanding? Cognitively Guided Instruction (CGI) is a program that aims to answer this question. CGI, based on the idea that students learn by constructing their own knowledge, is a framework to help teachers understand how students develop mathematical ideas and how teachers can use that knowledge to inform instruction (Carpenter, et al., 1989). In a study that compared classrooms that used CGI to classrooms that did not, Carpenter, et al., (1989) found that “providing teachers access to explicit

knowledge derived from research on children's thinking did influence their instruction and their students' achievement" (p. 529). In addition, the teachers who experienced the CGI training were able to regularly assess student thinking in order to plan for future instruction. One of the goals of CGI is "to help the teachers understand the ways students intuitively solve problems, so that they can help students build on that knowledge" (Carpenter, et al., 1996, p. 15). In this way, CGI focuses not only on teachers' content knowledge, but also on knowledge of representations and explanations and knowledge of students' thinking.

#### *Dimensions of Classrooms that Support Teaching for Understanding*

In order to help the field come to a consensus on how to effectively teach mathematics for understanding, Hiebert, et al. (1997) characterized the "essential features of classrooms that are designed to support students' understanding" (p. 13). The book, *Making Sense: Teaching and learning mathematics with understanding* (1997) draws on the research of four large projects that focus on students' conceptions of multidigit addition and subtraction: Cognitively Guided Instruction, Conceptually Based Instruction, Problem Centered Learning, and Supporting Ten-Structured Thinking. By examining commonalities found in the classrooms across all four studies, Hiebert, et al., (1997) were able to characterize classrooms that use reflection and communication as a means to make connections which assist in promoting students' mathematical understanding. Hiebert, et al. (1997) calls these critical characteristics *dimensions* of classrooms that support mathematical understanding.

There are five dimensions identified to describe classroom instruction, each with a subset of core features that highlight the important aspects of each dimension. The first

dimension is the *nature of classroom tasks*. According to Hiebert, et al. (1997), to teach for understanding, teachers should choose tasks that allow students to communicate about meaningful mathematics. Tasks should engage students in such a way that they are interested in solving a mathematical problem as well as making connections and reflecting on the importance of the mathematics involved. In addition, tasks should “leave behind important residue” (Hiebert, et al, 1997, p. 22). That is, the mathematical understanding that students take with them by experiencing classroom tasks should be something of value to students.

The second dimension of Hiebert, et al.’s (1997) framework is the *role of the teacher*. The teacher should select tasks based on students’ current knowledge in addition to what the teacher knows about student learning and the mathematical goals intended for the students. Teachers should provide pertinent information that encourages students to solve problems as long as it does not deter students from reflecting on and communicating about the mathematics involved. When students communicate what they already know both verbally and in writing, teachers can share mathematical conventions such as symbols and terminology to help students become more efficient in their ability to solve problems. In addition, teachers can create a classroom that supports reflection and communication by focusing on strategies for solving problems and allowing students to share their methods.

The *social culture of the classroom* characterizes the third dimension of classrooms that promote teaching and learning with understanding. A mathematical community where all students work towards a common goal of solving problems and understanding methods encourages collaboration and hence requires communication. In the context described by Hiebert, et al (1997), the culture of the classroom values students methods and ideas as well

as provides opportunities for students to choose methods that are meaningful to them. Additionally, students' mistakes should be expected and used as opportunities for reflection and growth. Lastly, correctness should reside in the logic of the mathematical argument and not just because the teacher said it was so. Teachers can step back from the position of authority on knowledge to allow students to reflect on the soundness of their own justifications and those of their classmates.

The fourth dimension is *mathematical tools as learning supports*. Tools are defined to include “oral language, physical materials, written symbols, and skills students have already acquired” (Hiebert, et al., 1997, p. 53) that are used to solve problems. Tools become valuable aids in understanding mathematics when students are able to construct their own meaning for tools and use them to communicate about mathematics. When words, symbols or physical materials are used purposefully to solve problems and to record and communicate ideas, important connections can be made between them which can lead to understanding.

The fifth and last dimension is *equity and accessibility*. According to Hiebert et al. (1997), classrooms that promote teaching for understanding ensure that each student grows in his or her understanding of mathematics. These classrooms build an environment where the individuality of each child is valued and where all students share in the responsibility of communicating and explaining their ideas. By posing tasks that are contextual in nature and relevant to students' interests or backgrounds, teachers can make tasks accessible to all students.

Connections can be made between the dimensions of classrooms that support teaching for understanding and CRP. For example, choosing tasks that are relevant to

students' lives supports the idea of making connections to students' cultures in the classroom. In addition, creating a classroom culture that is equitable and accessible can promote academic success among all students. This thesis intends to fill the gap in research related to how teachers incorporate the ideologies associated with teaching for understanding and CRP throughout the teaching cycle.

### Teacher Planning

The literature in this section describes what we know about what teachers attend to during lesson planning. Research that has investigated teacher planning ranges from studies that examine how teachers spend their planning time (Decker & Ware, 2001; Yinger, 1980) to studies that examine what teachers attend to in their lesson planning (Clark & Lampert, 1986; Fernandez & Cannon, 2005; John, 2006; McCutcheon, 1980; Yinger 1980; Zahorik, 1975). Because planning is a topic that is often emphasized in pre-service teacher education programs, much of the research examines planning models that teachers can follow as they think about designing lessons (John, 2006; McCutcheon, 1980; Yinger, 1980).

One focus of research on teacher planning has identified different levels of planning: yearly planning, unit planning, weekly planning, and daily planning (Yinger, 1980). McCutcheon (1980) noted that because many teachers use textbooks as a source of long term sequencing of concepts, they do not engage in long range planning. In addition, teachers perceived long range planning as inflexible and therefore not a productive way to spend planning time. Much of teachers' day-to-day planning is done mentally or as notes jotted down in a plan book with textbook pages listed (John, 2006; McCutcheon, 1980). With respect to how specific planning time is spent, a study by Decker and Ware (2001) revealed

that very little time is “actually devoted to the tasks that enhance the form of teacher delivery of instruction” (p. 8) and that specific planning time during the school day is more likely to be spent doing other tasks such as photocopying, running errands, or interacting with other teachers. Although levels of planning and time are important factors when considering teacher planning, this thesis will focus more specifically on what teachers attend to in their lesson planning.

A common practice in any teacher preparation program is to exercise the craft of writing detailed lesson plans (John, 2006; McCutcheon, 1980). However, such a craft is not always easy to teach or to learn. Since the 1950’s, a model for planning that is commonly taught to preservice teachers and used universally is the rational model, or objectives-first model (McCutcheon, 1980; Yinger 1980; Zahorik, 1975). This model is expressed as a linear model where planning begins with formulating an objective that describes a desired student behavior. Next, the teacher selects learning activities, organizes the lesson around these activities, and finally chooses some form of assessment (John, 2006; Yinger, 1980; Zahorik, 1975).

What little research exists on teacher planning has shown that most teachers do not follow the dominant model of planning; that is, they do not specifically attend to behavioral objectives first, then activities and finally assessment (Clark & Lampert, 1986; John, 2006; McCutcheon, 1980; Yinger, 1980; Zahorik, 1975). Studies on what teachers attend to in planning their lessons indicate that teachers focus on ideas such as content, activities or tasks, materials, textbooks, routines, as well as students’ needs and backgrounds (Clark & Lampert, 1986; Fernandez & Cannon, 2005; McCutcheon, 1980; Yinger, 1980; Zahorik, 1975). In his

1975 study of teacher planning, Zahorik surveyed 194 teachers from various fields to determine what teachers attended to most often in their lesson planning. The teachers were asked to list what they focused on before teaching a lesson. Their responses were classified as to what they attended to such as objectives, content, activities, materials, and assessment, as well as the order in which they attended to each item. The results of the study showed teachers attended to content more often and attended to content first more often than objectives. Also, although they were not always attended to first, activities were thought about by the majority of teachers in the study (Zahorik, 1975).

Similarly, in a 1980 case study, Yinger found that instructional activities were the predominant feature of lesson planning for one teacher. By studying the lesson planning practices of one elementary school teacher for five months, Yinger (1980) concluded that in addition to instructional activities, the teacher also attended to content and materials as they aided the teacher in clearly defining activities that form lessons. As well, it is noted that students' "background characteristics" (Yinger, 1980, p. 124) were attended to as a way to guide the teacher's planning processes.

In a study contrasting U.S. and Japanese teacher construction of math lessons, Fernandez and Cannon (2005) interviewed 61 Japanese and U.S. teachers to analyze what these teachers thought about when constructing lessons. The results highlighted that although both U.S. and Japanese teachers attended to mathematical content, Japanese teachers were more likely to attend to how students learn such content. The researchers suggested that this was due to the fact that Japanese teachers were more likely to allow students to guide the direction of a lesson, typically through discovery, and therefore were

more interested in how students think about the mathematical concepts. Conversely, the U.S. teachers in the study were more focused on constructing lessons that teach the content successfully and less focused on how the students learned the content. Moreover, the Japanese teachers saw the planning process as more complex and involved than U.S. teachers (Fernandez & Cannon, 2005).

Though it has been shown that an objectives first model for planning is the most prominent model taught in teacher education programs (John, 2006; McCutcheon, 1980; Yinger 1980), none of the aforementioned studies support the notion that teachers first attend to forming objectives in their lesson planning. One reason for this could be that many teachers have district or state objectives that are already in place and teachers form activities around these already present objectives (Yinger, 1980). Similarly, it is often noted that many teachers do not attend to aspects of assessment in their lesson planning (Yinger, 1980; Zahorik, 1975). It is suggested that one reason for this is that often a teacher's assessment procedures are routinized in the form of graded homework or in-class work, for example (Yinger, 1980).

In light of recent reform movements to improve the teaching and learning of mathematics (NCTM, 2000), one may question what *should* be the focus of planning for a teacher who aims to teach mathematics for understanding and adhere to culturally relevant pedagogy. Specifically with respect to mathematics lesson planning, Ainley and Pratt (2006) state, "If teachers plan from tightly focused learning objectives, the tasks they set are likely to be unrewarding for the pupils, and mathematically impoverished. If teaching is planned around engaging tasks the pupils' activity may be far richer but it is likely to be less focused

and learning may be difficult to assess” (p. 24). This quote describes a complex planning paradox that teachers must learn to resolve in order to teach for understanding. This thesis aims to fill the gap in research on teacher planning specific to mathematics instruction and how teaching for understanding and CRP can be incorporated into teachers’ planning practices.

#### Conceptual Framework for Examining the Mathematics Teaching Cycle

The conceptual framework for this study comes directly from the previously discussed research. Due to the nature of the NMD interventions, this study is framed theoretically by tenets of CRP and Hiebert et al.’s (1997) dimensions of teaching for understanding. Since the focus is on teacher planning in addition to classroom practices, Simon’s (1995) mathematics teaching cycle will be used as a lens through which to view this phenomenon. The mathematics teaching cycle was chosen because it is grounded in the constructivists’ view of learning, and as such, is consistent with CRP and teaching for understanding. This section will describe how the mathematics teaching cycle together with CRP and teaching for understanding frames the design of this study.

Simon (1995) offers a framework for teaching, referred to as the mathematics teaching cycle that supports the constructivists’ view of learning. Constructivists believe that students learn when they are able to construct their own knowledge. Therefore, activities where students practice problems and then discuss them are not adequate for promoting learning mathematics with understanding (Simon, 1995). Instead, planning for mathematical activities or tasks should be informed by how students think about mathematics. In other

words, it is not enough for teachers to attend solely to objectives, content, activities, or assessment; student thinking is an equally important component.

Simon (1995) suggests that the mathematical teaching cycle (see Figure 1) can describe the teacher decision-making process whereby teacher's knowledge of learning goals and activities or tasks, as well as their predictions about how students learn should all be a part of the lesson planning process. Using pedagogical knowledge and knowledge of how students construct knowledge of concepts, the teacher can choose learning goals that inform the plans for a lesson. This begins what Simon (1995) refers to as the hypothetical learning trajectory (HLT); that is, anchored in the chosen goals, the teacher selects learning activities or tasks based on how the teacher best believes learning will proceed. The teacher uses information about students' current knowledge, the specific content, and pedagogical content knowledge on how students' best learn the content in designing a lesson (Simon & Tzur, 2004). This requires the teacher to have the ability to identify classroom tasks that can lead to the intended concepts.

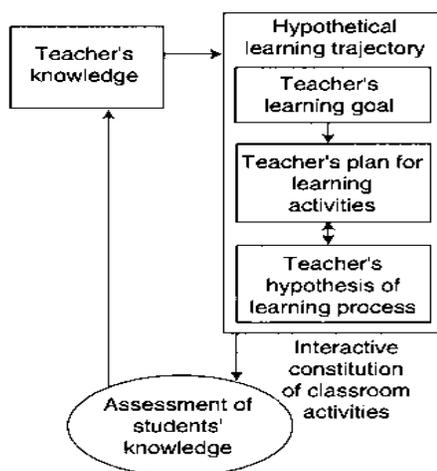


Figure 1: Mathematics teaching cycle (Simon, 1995)

Once the lesson is enacted, the classroom experience modifies the teacher's knowledge in several ways. First, in the mathematics teaching cycle, student knowledge is continually assessed. Therefore, a completed lesson will influence a teacher's knowledge of students' mathematical understanding. In addition, the classroom experience adds to the teachers' knowledge of student learning as well as the teacher's own knowledge of mathematics. These domains of knowledge then, in turn, allow the teacher to create new learning goals and trajectories for learning. As the teacher's knowledge is enhanced, modifications to a lesson design will take place either in the goals, the activities, or the hypothesized learning.

The framework design for this study utilizes the three major components of the mathematics teaching cycle: planning (drawing on teacher knowledge and the creation of a HLT), teaching (the enactment of classroom activities) and assessment (formative). In each of these components, the researcher will use tenets of CRP and dimensions of teaching for understanding as the lens through which to analyze the data (see Table 1 and Figure 2). Teachers whose beliefs are consistent with CRP and teaching for understanding will draw on these constructs during each phase of the mathematics teaching cycle. For example, during the planning phase, one might expect that a teacher would draw on students' prior knowledge, students' out of school knowledge as well as the teachers' own mathematical knowledge. During the teaching phase, one might expect to see a classroom culture where students are encouraged to choose and share their own methods for solving problems and where those methods are valued. In addition, there would be evidence of high academic success by all students and students would be communicating about mathematics in ways

that are consistent with their cultural ways of expression. During the assessment phase, one might expect to see teachers' attend to how students are communicating and reflecting on mathematics and how student learning informs the teachers' own knowledge. This framework was used to drive the design of the study as well as direct the analysis of the data to answer the research questions.

Table 1: Features of CRP and Teaching for Understanding

Features of CRP and Teaching for Understanding (TU) (adapted from Ladson-Billings (1994) and Hiebert, et al. (1997) respectively)	
<b>CRP:</b>	<ul style="list-style-type: none"> <li>• High academic achievement for all students</li> <li>• Cultural competence</li> <li>• Sociopolitical consciousness</li> </ul>
<b>Teaching for Understanding (TU):</b>	<ul style="list-style-type: none"> <li>• Nature of classroom tasks</li> <li>• Role of the teacher</li> <li>• Social culture of the classroom</li> <li>• Mathematical tools as learning supports</li> <li>• Equity and accessibility</li> </ul>

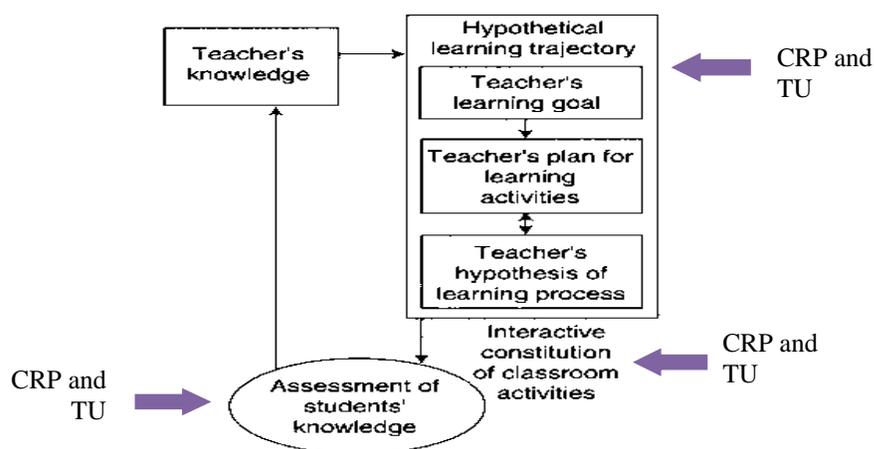


Figure 2: Conceptual Framework

### Research Questions

This study investigates two kindergarten teachers' mathematics teaching cycles and if there is evidence of CRP and teaching for understanding in any parts of each teacher's cycles. Ladson-Billings' (1994) tenets of CRP and Hiebert, et al.'s (1997) dimensions of classrooms that promote teaching for understanding will be used as a lens for analyzing the teachers' mathematics teaching cycle using Simon's (1995) model. In the context of the NMD professional development project that promoted teaching for understanding and CRP, these questions are of interest:

- 1) What does a kindergarten teacher attend to in each phase of the mathematics teaching cycle?
- 2) Is there evidence of the ideologies associated with CRP and teaching for understanding within each phase of the teaching cycle?

## CHAPTER 3

### METHODOLOGY

This study is a case study of two teachers who were involved in a professional development project in North Carolina. The purpose of this section is to describe the context for the study, the participants, the sources of data, as well as the methods of analyzing the data.

#### Context for the study

##### *Nurturing Mathematics Dreamkeepers*

Participants in the current study are involved in the Nurturing Mathematics Dreamkeepers (NMD) project, a local professional development project for kindergarten through second grade elementary school teachers. Funded by the National Science Foundation, the NMD project spans five years and involves over 50 teachers. Motivated by the nation's achievement gap between White American students and African American students as well as the No Child Left Behind legislation, the project's goals are to study how early elementary teachers develop culturally relevant pedagogy that promotes conceptual understanding of early number concepts in their classrooms. Three cohorts of teachers from six local elementary schools were chosen to participate in the project. Schools were selected based in part on their Adequate Yearly Progress and the fact that their student populations were at least 30% African American and 30% White American.

Each cohort of teachers attended professional development retreats for varying lengths of time (cohort I for three years, cohort II for two years, and cohort III for one year). Each retreat lasted either one or two days and took place throughout the school year with two

retreats held in the fall and two in the spring. During the retreats, teachers took part in various mathematical tasks aimed at improving not only their content knowledge and own mathematical understanding, but also their pedagogical content knowledge. In addition, teachers participated in a variety of activities where they were asked to engage in critical reflection on issues related to culture in the teaching-learning process. The overall themes for each retreat that the participants took part in appear in Table 2. For data collection purposes, teachers were organized in groups called “buddy pairs” where teachers from the same school and often the same grade, observed each others’ math lessons. Additionally, buddy pairs participated in a video-taped post-lesson “reflective session” where they were asked to reflect on their math lessons with respect to culture as well as the ways in which their mathematics instruction might have promoted or hindered their students’ understanding.

Table 2: NMD Retreat Interventions

NMD Retreat Interventions – Year 3: Topics covered during the third year of project retreats with respect to CRP and Teaching for Understanding		
	<b>CRP</b>	<b>Teaching for Understanding</b>
<b>Retreat I</b>	<ul style="list-style-type: none"> <li>Understanding culture</li> </ul>	<ul style="list-style-type: none"> <li>Constructing counting systems</li> <li>Base 10 and base 4 number systems</li> </ul>
<b>Retreat II</b>	<ul style="list-style-type: none"> <li>Professionalism: the Dreamkeeper Identity</li> <li>Defining CRP</li> <li>High academic achievement</li> <li>Cultural competence</li> </ul>	<ul style="list-style-type: none"> <li>Mathematical models and tools</li> <li>Teaching mathematics for understanding</li> <li>Learning trajectories for addition and subtraction</li> </ul>
<b>Retreat III</b>	<ul style="list-style-type: none"> <li>Promoting academic achievement</li> <li>Language and cultural identity</li> </ul>	<ul style="list-style-type: none"> <li>Algebraic thinking</li> <li>NCTM Standards</li> <li>Patterning</li> </ul>
<b>Retreat IV</b>	<ul style="list-style-type: none"> <li>Cultural competence</li> <li>Enthoracial identity</li> <li>Sociopolitical consciousness</li> </ul>	<ul style="list-style-type: none"> <li>Early rational number sense</li> <li>Fair shares</li> </ul>

### *Participants*

This thesis is a study of two kindergarten teachers who were participants in the third cohort of teachers. As a research assistant for the professional development project, this researcher was assigned to eight teachers for data collection purposes. Two of these teachers were asked to participate in the current research on lesson planning and each was given a pseudonym. Sarah is a white female in her twenty-ninth year of teaching kindergarten and pre-kindergarten. Pamela is a white female who has spent both of her two years of experience teaching kindergarten. Sarah and Pamela worked together as a buddy pair during the 2007-2008 school year. Both teachers hold undergraduate degrees in elementary education and have not received any specific training in mathematics instruction other than the curriculum training that is required by the county in which they teach. The school where they teach is a magnet school with a Center for Spanish Language where students receive Spanish instruction daily and the core curriculum is emphasized and broadened in the second language.

Since the teachers in this study taught in the state of North Carolina, they were provided with a Standard Course of Study (NC DPI, 2003) that guided the curriculum they taught. The objectives for kindergarten were divided among four quarters where each quarter lasted approximately nine weeks. The teachers were supplied with math cards that list the objectives that should be covered each quarter. In addition, the county where they teach adopted the *Math Trailblazers* curriculum based on the notion that this program is aligned with the North Carolina Standard Course of Study and provides students with both problem solving and computational practice.

### *Sources of Data*

For this study, four sources of data were collected: a lesson planning interview, an observation of a lesson planning session, video-recordings of two consecutive math lessons, and a post-lesson reflective session. Each source of data will be described and data protocol can be referenced in the appendix.

The lesson planning interviews (see Appendix A for the interview protocol) as well as the lesson planning observation were audio recorded and transcribed. The lesson planning interview questions asked the teachers to describe and discuss what they attend to when planning their mathematics lessons. Questions were specific to content, types of activities, resources, assessment (both formative and summative), and student knowledge. The teachers were also asked if they planned alone or in collaboration with other teachers and how far in advance they planned (i.e. daily, weekly, monthly, etc). During the lesson planning observation, the researcher was able to observe the teachers' planning and then immediately ask questions regarding motivation and clarification of planning decisions. The researcher was then able to video tape the math lessons that were planned during the lesson planning observation.

The math lessons and post-lesson reflective session were video recorded. The math lessons were described in five minute increments and portions were transcribed. The post-lesson reflective session was transcribed verbatim. The videotaped math lessons took place in the teachers' classrooms during their regularly scheduled math times for two consecutive days. On the first day, Sarah's math lesson was recorded first while Pamela observed, and then Pamela's math lesson was recorded while Sarah observed. On the second day, Pamela's

math lesson was recorded while Sarah observed and then Sarah's math lesson was recorded while Pamela observed. The reflective session (see Appendix B for reflective session protocol, which is a modification of the questions used in the larger NMD study) took place four days after the second videotaped lesson. In the reflective session, the teachers discussed how their lessons supported or hindered their students' conceptual understanding of the mathematics and were asked questions regarding their students' out-of-school and in-school knowledge as well as if their enacted lessons followed the lessons they had planned.

### Analysis of Data

The data was analyzed to describe what the teachers attended to in their lesson planning and what parts, if any, of their lesson planning and enacted lessons were consistent with culturally relevant pedagogy and teaching for understanding. In addition, the enacted lessons were compared to the lesson planning observation to determine if the lessons were consistent with what was planned. The four sources data were analyzed in three separate phases. The phases are described below.

#### *Phase 1: Teacher planning*

First, the three transcribed sources of data (interviews, observation, and reflective session) were organized with respect to Simon's (1995) Mathematics Teaching Cycle and each piece of data was coded for instances of teacher knowledge, HLT, and assessment. Any parts of the data relating to teacher knowledge were then combined into one document. Similarly, new documents were created for parts of the data pertaining to HLT and assessment. The researcher then analyzed and coded each new document for what each

teacher focused on in their lesson planning such as objectives, content, activities, materials, etc.

*Phase 2: Teaching for Understanding and CRP*

During the second phase, each new document was analyzed for teaching for understanding and CRP. First, the documents for teacher knowledge, HLT and assessment were coded for evidence of teaching for understanding. Table 2 describes the codes used with respect to Hiebert et al.'s dimensions of teaching for understanding.

Table 3: Codes for Teaching for Understanding (adapted from Heibert, et al., 1997)

<b>Code</b>	<b>Description</b>	<b>Example</b>
<b>NCT</b>	Nature of the Classroom Task <ul style="list-style-type: none"> <li>- Makes mathematics problematic</li> <li>- Connects with where students are</li> </ul>	“How can we share these cookies fairly?”
<b>RT</b>	Role of the Teacher <ul style="list-style-type: none"> <li>- Selects task with goal in mind</li> <li>- Shares essential information</li> <li>- Establishes classroom culture</li> </ul>	Teachers use knowledge of equality and students’ prior knowledge to choose task based on the goal of sharing fairly.
<b>SCC</b>	Social Culture of the Classroom <ul style="list-style-type: none"> <li>- Ideas and methods are valued</li> <li>- Students choose and share methods</li> <li>- Correctness resides in mathematical argument</li> </ul>	“Who has an idea how we could figure out how to share it fairly?”
<b>MT</b>	Mathematical Tools as Learning Supports <ul style="list-style-type: none"> <li>- Meaning of tools is constructed by learner</li> <li>- Tools are used to solve problems</li> <li>- Used for recording, communicating and thinking</li> </ul>	“So I’m gonna count out ten cookies from the bag and as a partner, me and Naque are going to figure out how to share these fairly. When we think we’ve shared them fairly, I need you to draw a picture of what your plate looks like and what you’re partner’s plate looks like.”
<b>EA</b>	Equity and Accessibility <ul style="list-style-type: none"> <li>- Tasks are accessible to all students</li> <li>- Every student is heard</li> <li>- Every student contributes</li> </ul>	By highlighting the strategy of passing out one cookie at a time, the teacher believes she has made the task accessible to all students.

In order to analyze the data for evidence of CRP, the researcher initially coded with respect to the three tenets of CRP: high academic achievement, cultural competence, and sociopolitical consciousness. Once the data were coded, the initial codes did little to answer the research questions. More specific codes were created as sub-codes of the three tenets of CRP. Table 3 provides a description of these codes. The sub-codes were adapted from a rubric developed to code NMD project data by the principle investigators of the project, Dr Patricia Marshall, Dr. Jessica DeCuir-Gunby, and Dr. Allison McCulloch. These particular codes were chosen because they helped to identify aspects of CRP that were otherwise not visible in the lessons. Since CRP is an ideology, it may not be readily observable in a kindergarten classroom.

Table 4: Codes for Culturally Relevant Pedagogy (CRP)

<b>Code</b>	<b>Description</b>	<b>Example</b>
<b>AA</b>	High Academic Achievement	“There was some level of every child that was able to come up, either tell you what the number was or draw [it] out...they seem to be overall very successful with it.”
<b>CC</b>	Cultural Competence <ul style="list-style-type: none"> <li>- Cultural connecting</li> <li>- Language Matching</li> </ul>	Student: We can cut ‘em in pizza halves?  Teacher: You could cut ‘em in pizza halves. So would that be smaller pieces?
<b>SC</b>	Sociopolitical Consciousness	Discussions of how students can use mathematics to create positive change in their communities.

### *Phase 3: Video-taped Lessons*

In addition to the documents, the video-taped lessons were also coded for teaching for understanding and CRP. According to Powell, Francisco, and Maher (2003), one method for video analysis involves seven, nonlinear, interactive phases. The phases are as follows: viewing the video attentively, describing the video data, identifying critical events, transcribing, coding, constructing a storyline, and composing a narrative. This model was adapted by the researcher to analyze the video-taped lessons for the current study.

First, the four video-taped lessons were transferred to DVD. The lessons were viewed several times and then they were described in five minute increments. The lessons were viewed again and critical events were identified and transcribed. Then, the lessons were coded for evidence of teaching for understanding and CRP. The transcriptions included dialogue by the teacher and students as well as a description of any actions that were visible on the video recording. Lastly, the video-taped lessons were compared to the lesson planning observation to determine if the enacted lessons were consistent with what was planned.

## CHAPTER 4

### FINDINGS

This chapter describes the findings from the data analysis. As noted in Chapter 3, there are four sources of data: lesson planning interviews, lesson planning observation, video-taped lessons, and reflective session. The chapter will be organized according to Simon's (1995) mathematics teaching cycle: planning, teaching, and assessment. First, a description of what the teachers in this study attend to during the mathematics lesson planning is presented. Next, the enacted lessons will be described and then compared to the lesson planning observation. Then, what the teachers attend to in terms of assessment will be presented. Finally, the planning and lessons will be described with respect to CRP and teaching for understanding.

As stated at the end of Chapter 2, this study attempts to answer the following research questions:

- 1) What does a kindergarten teacher attend to in each phase of the mathematics teaching cycle?
- 2) Is there evidence of the ideologies associated with CRP and teaching for understanding with in each phase of the teaching cycle?

#### Lesson Planning

A review of the mathematics teaching cycle of the teachers in this study will begin with their lesson planning. First, generalities of each teacher's lesson planning will be characterized individually. Following will be what they attended to in the specific lesson planning session that was observed including deciding on an activity, focusing on students'

knowledge and how they will approach the chosen activities, the tools they will use as well as pedagogical decisions such as student pairings and classroom set up.

#### *Overview of Sarah's Lesson Planning*

Sarah was a seasoned teacher whose goal was to prepare lessons that meet the needs of all of her students. Due to the wide range of students' abilities and backgrounds in her class, Sarah struggled to plan lessons that she felt reached all of her students. She found that using literature and manipulatives helps her students engage in mathematical thinking and develop mathematical ideas. Sarah indicated that she uses several teacher idea books as resources, such as Mailbox Magazine, and pulls from Trailblazers "a little bit." She stated that "Trailblazers doesn't go as advanced as what our real curriculum is, I don't think. It's almost like we've finished Trailblazers before Christmas."

Since Sarah taught in the state of North Carolina, she was provided with a Standard Course of Study (NC DPI, 2003) that guides the curriculum that she taught. She indicated that she begins planning her math lessons based on specific objectives from the North Carolina Standard Course of Study. She also specified that she will teach concepts that are not included in the Standards if she feels like her students are interested and capable, for example, telling time. In developing a classroom activity focused on a specific objective, Sarah attended to many things. When asked how she decides what specific activities or tasks she uses in her classroom, Sarah stated, "A variety of things. I like things that use manipulatives. I like using literature if I can find something that kind of ties into the concept like *The Grouchy Ladybug that Tells Time.*"

When thinking about what type of classroom activities to use, Sarah also attended to what appeals to her students. She stated that she tries to use contexts that are familiar to her students culturally and that connect to their home life. Sarah used home visits to learn more about her students' home lives and encourage communication with parents. Sarah indicated that because she has some students whom English is not their first language and students who have not been exposed to formal schooling prior to kindergarten, she tries to use manipulatives to "make it more concrete" and using literature "exposes them in a context" that she believes her students will be familiar with. In this way, Sarah also focused on the types of materials she uses in her lessons, manipulatives and literature for example.

#### *Overview of Pamela's Lesson Planning*

Pamela was a novice teacher who valued creativity in her lesson designs and believed that students learn best when activities are hands-on. She strove to use contexts in her lessons that she believed her students were interested in such as sports. Increasing her students' self-esteem and helping her students have a positive attitude were important to Pamela. She also respected and appreciated the ideas her more experienced colleagues shared with her. When speaking of what resources she draws from, Pamela states "I use Trailblazers some, but we pick and choose from that...I just use my team of teachers, they've all been teaching a lot longer than I have...and so I use them I would say the most."

As is the case for Sarah, Pamela taught in North Carolina and as such was provided with a Standard Course of Study (NC DPI, 2003) which guided the curriculum that she taught. The mathematical content she focused on in her math lessons was based on the Standard Course of Study. In developing a classroom activity for her students Pamela first

attended to the objective, then she considered activities based on that objective. When asked how she decided what activities or tasks she uses in her classroom she stated, “I tend to be drawn to the things that are real hands-on. Um and that the kids will have fun with and engage in...like manipulatives and color and things that are different than actual just number and paper.” With respect to Trailblazers, the county-adopted curriculum, Pamela said, “my problem is that Trailblazers is way ahead of where we are or way beneath where we are. I just have to read it and assess whether it’s going to fit or not.”

Pamela emphasized her inclusion of manipulatives as a way to keep her students interest and encourage a positive attitude towards mathematics. She asserted, “I think when they’re having fun they’re more tuned in to what’s happening and I want them to be excited about the math things that we’re doing...So I just think their attitude changes when it’s manipulatives and hands-on. And I think the manipulative part is good for their learning to know how to do things and how it looks and works...” Pamela also drew on her students’ interests in her lesson planning. She indicated that she attempts to use examples that are related to sports or team names to spark her students’ interest.

In thinking about planning classroom activities, Pamela also thought about her students’ mathematical knowledge. She identified that students’ prior knowledge was important when she said, “I could think they don’t know anything and they could know a lot more and if I know that already, then I can jump into the things they don’t [know] and focus on that.” How she determined her students’ prior knowledge was not clear.

*Planning for “Fair Shares”*

Sarah and Pamela did not plan collaboratively on a regular basis; however, because of the nature of data collection for the NMD project and the fact that they worked together in a buddy pair, they chose to plan their video-taped lessons together. The content they chose came directly from the North Carolina Standard Course of Study (NC DPI, 2003) for kindergarten: “The learner will share equally (divide) between two people; explain.” This concept was also referred to by the teachers as *sharing fairly*. The teachers chose this content based on the fact that it was listed as an objective from the Standard Course of Study for the current quarter.

During the lesson planning session, the teachers began by stating the objective they wanted to cover. Sarah referred to a previous activity where her students struggled with the concept of equality and states, “So that’s why I feel like we really need to do this.” Sarah conveyed to Pamela the story, *The Doorbell Rang*, by Pat Hutchins.

The story is about a mother who offers her two children a plate of 12 cookies to share between themselves. After the children decide they will get 6 cookies each, the doorbell rings and in comes two more children. Now they must decide how to share the 12 cookies between four children. The story continues this way until each person has only one cookie and the doorbell rings one last time. Thankfully, it is Grandma with more cookies.

The teachers decided to use this story. Their reasoning is evident in the following excerpt from the lesson planning observation:

Sarah: I just like teaching from stories and I think it adds a lot more depth language-wise as well as concept-wise. And it’s demonstrating a family

which they're all from different kinds of families and they've probably had to share cookies or something.

Pamela: Yeah, and I feel like especially in kindergarten they have a hard time drawing up their own idea of "how do you share at home?" without kind of being prompted. Because they don't necessarily quite know how to put it into words so that story would prompt their brain to think about, connect to their home life and how they share. It's a good visual connection.

Sarah: A home connection is good because that story is about a family.

Next they discussed the possibility of having pretend cookies at the students' tables in order for the students to practice sharing fairly. Pamela then brought up the idea of modeling sharing fairly for the class:

Pamela: If you had two plates and one child came up and divided out and we saw how many were on one plate and how many and was that equal, equally divided, and then maybe put three plates out and see if they're equally divided. But I think it would be also good to see if what the kids did to let them try to figure out a dilemma if there were two plates and an odd number of cookies, you know, and let them have a chance to figure out different ways that they would solve that before we walked them through the process. Give them a chance to explore it a little bit first.

They decided to act out the story using pretend plates and cookies on the first day and then, depending on time, the students would have a chance to practice sharing fairly on their own, as is evident from the following excerpt:

Pamela: Um, so, if we're going with this idea, which that is a cute story, I know that story, that's really directly applicable. So if you read the story, um...and then...I'm kind of thinking like would you want to be acting it out as the story was being read? Do you know, with a set of cookies? Or just read it first.

Sarah: I think I'll just read it the first time and then maybe read it a second time and act it out. And then if this is the first day that we're doing fair share, we'll go to the tables and just do some simple fair sharing, I don't know how much time we'd have.

Pamela: Yeah, simple and then throw in the...since we haven't done a lot of fair sharing either so that might be good to just do that first.

Sarah: Because I think the emphasis is going to have to be the equal part because like I said I know I have some kids that equal is shaky. Which I thought we knew equals because we write equals on our problem solving, but when we did that hats thing they didn't get equals.

Sarah emphasized her students' knowledge of equality as a key factor in this lesson. In this way, she has attended to her pedagogical content knowledge; specifically, what she knew about her students' mathematical knowledge. She identified that students' knowledge of equality was necessary in order to understand fair shares.

Once the teachers decided that on the first day, they would read the story and act it out as a demonstration of sharing fairly, Pamela introduced a dilemma. She had already

mentioned once having two plates and an odd number of cookies. This is the discussion that followed:

Pamela: I'm wondering...would you like brainstorm at the carpet about, like, what happens if you have five cookies and two people or would you...or do you think it would be better to let them have that kind of situation at the table? Or are you not wanting to throw that in yet?

Sarah: I think I would just do even numbers that day.

Pamela: Even numbers first and then build on that.

Sarah: And build on it and um...I know in the past when I've tried it, the one with the cookies, things will say things like "break it in half," "we could each take a bite out of it." I've had them do that when you're talking about cookies.

Pamela: Some people say throw it away.

Sarah: Or give it to the dog. Um, but I'm not sure how many will have it day two to throw in the odd one.

Pamela: True.

Sarah: I mean, your advanced kids probably will, but, the average kid...

Pamela struggled with when to introduce a situation where there would be a leftover cookie.

When the researcher asked why they chose to only use even numbers, Pamela said, "I think I was kind of throwing that out there as a question too..." and she looked to Sarah for the answer:

Sarah: I wasn't sure about having the one that makes it unequal what that would do with the whole concept of equal or not equal. If, when they divide it

correctly everyone has the same, it emphasizes that equal. And then we could throw in the next day “oops, what happened? Now are they both the same? What can we do to make it the same?”

Pamela: Yeah, and I think it’s just one of those dilemmas with teaching because the, my high, my kids who are really getting math they love the challenge and just eat it up and think of all these ideas. But then I have just as many or more that are, that would kind of struggle and I think it would confuse them and it would just be overload and they would get nothing out of it.

Here, the teachers hypothesized about how students learn sharing fairly. They thought about explanations students would give for a left over cookie and how they could emphasize the idea of equality. In addition, they thought about their students’ ability levels and what would be the most appropriate for the entire class. When asked what they knew about students’ learning of fair share, they responded:

Sarah: My thing that I notice is that they have to understand that concept of equal. If they don’t understand what equal means, that sharing fair means equal and they have to know that four on your plate and four on my plate are equal. If they don’t have that then it’s not going to work.

Pamela: But I think what they bring to the table, they know that, they don’t necessarily understand, couldn’t give you a definition of “fair” but they know when someone is not being fair.

Sarah: And cookies are something they all know.

Pamela: So “that’s not fair, he got more than me,” you know.

Both teachers attended to their students’ background and home life in addition to their understanding of what it means to be equal. When asked how they thought their students would think about the concept of fair shares, Sarah hypothesized that some students will automatically know what the result will be for equally dividing numbers that are less than or equal to 10 as evident by the following excerpt:

Sarah: ...they’ll be somebody there, maybe Abby that will say “Six? Oh that’s three each. Eight? That’s four each.” I have some that will do that. Because she tends to be that way with adding. She’ll have a number and be like “Oh, two and two? That’s four.” Whether she’ll apply it to this, I don’t know.

Pamela: Yeah, and the challenge there, in some ways my initial instinct, or my thought would be, give them higher numbers, but it’s not really higher level thinking, it’s just more counting. More time...so...yeah.

Sarah: I guess if those kids fly through this you could just say “What happens if you have 7?”

Pamela: Yeah, that’s true. Count out 7 and see what happens. So that could be an on-the-fly adaptation for the high. And then some of my low kids, um, I think they’ll be inconsistent and won’t really understand exactly what the dividing [is]...

Again, the teachers focused on their students' prior knowledge and how they would think about sharing fairly. They also thought about how to extend the activity for students who they believed could understand how to share an odd number of cookies.

Next, the teachers moved on to discuss the activity for the second day. They decided to continue with the theme of the story and use paper cookies and plates in order for the students to practice sharing fairly. Pamela introduced the idea of using a worksheet for students to record their results. They created a chart that the students would fill in as they practice sharing fairly. The chart told the students how many cookies to share and then the students would fill out how many cookies each person gets by drawing the number of cookies and writing the number next to the drawing. When asked why they chose to use the chart, they responded:

Pamela: I think initially it was kind of organization, like how are you gonna, like Sarah had said "Ok, now count out six" or whatever, just a way to do that in a more controlled, and let them do it at their own pace, so if they've already done the four, pick a new number. So I think that was my initial mindset. Then when I was thinking about it, just thinking, this will be a good visual representation of what they do.

Sarah: The good thing about this time of year is even my low kids have numerals pretty much to ten. So they can be independent and go on to the next number without us having to try to see where 22 kids are and what number to give next.

The teachers focused on pedagogical decisions such as having students work in pairs, and allowing them to work at their own pace, focusing on keeping students engaged throughout the lesson.

The two teachers discussed the logistics of how they would have the students practice and decided that it might be best to have students work in pairs so that they are “more involved.” They discussed modeling how students will work with their partner to complete the chart before they let students work on their own:

Pamela: I would maybe just have like 2 [cookies] and 4 [cookies] up at the carpet and [model] 2 and 4 up there together and show them how.

Sarah: Well you would act it out with the plates and cookies...

Pamela: Uh-huh.

Sarah: ...and then show them how to represent it.

Pamela: Right. And do like 2 together and give them their own paper. And I would probably just give them their paper on clip board with their thing of cookies and send them to their seat and let them record. Or are you thinking have one for each person?

Sarah: I think having a paper for each person. I think I would have [teacher assistant] go ahead and have the paper and the plates and cookies already at the table while we're at the carpet.

Pamela: And they just come sit down.

Sarah: Because handing all that out takes up so much time of your thirty minutes.

Pamela: That's true. Good point.

Sarah: So we need a chart at the carpet...

Pamela: Make it on Word? Or how are you going to do it? Just get a piece of paper and do it that way?

Sarah: I think I might just grab a piece of chart paper and (inaudible)

Pamela: But for their individual papers?

Sarah: Their individual paper maybe make it in Word. How many numbers are we going to give them?

Pamela: Well, I think it's not necessary for them to finish but I think there need to be enough for the kids who can do it more quickly. So maybe like, well, if we had 10 cookies... if we're not doing odd numbers, 10, 8, 6, 4, 2.

Sarah: I think we should mix up the order.

Pamela: I do too. 6, 10, 2, 8, and 4. How about that?

Sarah: Ok. And we'll see how they do with it and then we can talk about...you could extend it with throwing in odd numbers.

They decided to have the students share only even numbers of cookies and use 6, 10, 2, 8, and 4 cookies in that order to share between two people. However, the teachers did not explain why they wanted to mix up the numbers. They indicated that using more than 10 cookies could take too long to count out and they would rather the students' focus on the "sharing" part of the activity and not counting out cookies.

The teachers ended the lesson planning session by discussing what activities they could do in subsequent lessons to reinforce and practice the idea of fair shares. Although the

teachers made notes during the lesson planning session, neither teacher wrote out a detailed lesson plan describing the objectives, materials needed, or specific activities for each day.

### Enacted Lessons

In continuing with the mathematics teaching cycle, the following section provides a description of each teacher's enacted lessons. The enacted lessons show how the teachers implemented the activities discussed during their planning as well as how the pedagogical decisions they made regarding tools and student understanding of fair shares were played out in the classroom.

#### *Sarah's Lesson Day 1*

Sarah began the first lesson by introducing the idea of sharing fairly. She showed the class the story they would be reading, *The Doorbell Rang*, and asked the students to listen to how the children in the story share fairly. She tried to get her students to remember what they have talked about with respect to sharing as evident in the following excerpt:

Sarah: In the story you're going to be looking and listening for how the children share fairly. Yesterday when we were doing our writing on integrity, do you remember what we said about being fair? Who remembers something about being fair? Alyssa, what does that mean?

Alyssa: Um, you can share.

Sarah: Ok, but if you're sharing fairly, what does that tell you?

Adair: It means equal things.

Sarah: You used the word equal. What does equal mean? If they're sharing fairly and they have equal, what do they have, Tameo? What does equal mean?

Tameo: If you're friend has one toy and he has four more that would be five equals five.

S: Ok, that's still not telling me what sharing fairly would mean. What do you think it means, John?

John: It means...you and someone else has the same amount.

S: The same amount. So equal means the same amount? Does equal mean the same amount? So if I have 5 hot dogs, then the person with me needs to have...

Some students: Five!

S:...if it's going to be equal or fair. If I had one baby doll, my friend would need to have [pause] one baby doll in order for it to be...

Student: Equal.

S: Equal, or the same...

Student: Fair.

S: Fair, or the same amount.

Sarah began to read the book, *The Doorbell Rang* to her class and had some students act out the story as they read using a play set of plates and cookies. Sarah continually asked the students for ideas on how they can share the cookies fairly and the students were very willing to share their ideas. In the following excerpt, Sarah has called two students, Danny

and Leah, to the front to be the two characters in the story, Sam and Victoria. They each were given a plate and the cookies were in a pile in the middle. Click on the screen shot for this first video clip where Sarah's class discussed how to share 12 cookies among two people:



[Video Clip 1: Sarah's Lesson Part 1](#)

In the beginning, a student started out by guessing a number that he thought would work (Jack guessed three). When they point out that there are still some left over, another students suggested five instead of three. Sarah assumed the student meant five *more* cookies in addition to the three already there; it is possible the student meant to start with five cookies each. Sarah did not clarify the student's intentions. Sarah listened to the ideas offered and acted of some of them, but she was not content until a student suggested passing out one cookie at a time. Once Will, John Levi and Alyssa discussed giving three of the remaining six cookies to each person, the problem was solved; the students could have determined there were six cookies on each plate. However, Sarah was determined to use the strategy of passing out one cookie at a time. Knowing that "three and three is six" was not enough for

her. Once the strategy of passing out one cookie at a time was offered, she was willing to continue on with the story. In addition, Sarah did not ask her students to reflect on any of the ideas offered by their classmates.

In the next part of the story, two more children are added to share the 12 cookies. In the demonstration, two students (Leah and Danny) already had six cookies on their plates and Sarah called two more students, Leo and Abby, to come up to the front. Leo and Abby were each given a plate. Click on the screen shot to view how Sarah continued to focus on getting the students to pass out one cookie at a time in this second video clip:



[Video Clip 2: Sarah's Lesson Part 2](#)

Again, a student, Charlotte, suggested a strategy whereby she was able to count the cookies and determine how many each person should get. Towards the end of the excerpt, Sarah explained what she *believed* to be Charlotte's thinking. She did not attempt to have Charlotte explain her strategy nor did she ask the class what they thought of this strategy. She continued to question the students and even put all the cookies back in the middle, until Zemiell suggested giving one cookie to each person.

At the conclusion of the story, Sarah asks what would happen if more people came to the table and there weren't enough cookies. She called on only one student who suggested that they could break the cookies in half to give to the other people. She reviewed the idea of sharing fairly and then gave some background for the activity they will do for the next lesson.

*Sarah's Lesson Day 2*

On the second day, Sarah reviews what it means to share fairly or equally. She has a volunteer come to the front in order to model the day's activity where the students will be working with partners to practice sharing cookies fairly. Using paper cut-outs of cookies and paper plates, Sarah models with the student volunteer how to share 6 cookies fairly. Then, using a large piece of chart paper that is identical to the students' worksheet, she shows the students how to fill out the chart to record their results. She draws 3 circles in each of two columns (one for each person in the pair) to represent the three cookies each person gets after sharing 6 cookies and writes the numeral 3 next to the drawings.

Next Sarah dismisses the students to their tables and assigns partners. Each pair of students gets a bag of 10 paper cookies and two paper plates. Each student has their own worksheet to fill out. The students begin working on sharing different numbers of cookies and filling out the chart while Sarah walks around to monitor the students' progress. Due to the nature of the activity and the limitations of the video camera, the recording was not able to pick up all of Sarah's conversations. She moves from table to table to make sure students are completing the worksheet correctly as is evident in the following excerpt:

Sarah: [Sitting with two students, one White male and one African American female, she addresses the female] how many do you have?

Female student: Three.

Sarah: How many does he have?

Female student: Three.

Sarah: Is that fair?

Female student: Yes.

Sarah: So what do you need to do here (points to paper)?

Male student: Draw three cookies here.

Sarah: Ok.

Sarah did not ask the students to explain their strategy or how they knew they had shared fairly. She mainly focused on making sure the students were filling out the worksheet correctly. However, later on Sarah worked with students to determine their understanding of sharing fairly by watching students share the cookies and asking questions. The following excerpt provides an example of how Sarah interacted with her students:

Sarah: I want you to show me how you do it. Show me what you do. I didn't see you do it. Show me how you did it. How did you figure it out? What did you count out on the table?

Annabelle: [inaudible]

Sarah: Ok, what's this number [pointing to 10 on the worksheet]? Danny, I need you to count out 10 cookies. [She takes the bag of cookies from Danny and hands it to Annabelle]. Show me what you did.

Annabelle: [inaudible] [she places two cookies on the table]

Sarah: Ok, how many cookies did you count out?

Annabelle: [inaudible] [She is putting the cookies in to piles, alternating one at a time]

Sarah: [Picks up all the cookies Annabelle has counted and gets Danny to count the cookies. He counts 8]. That's 8 cookies. How many are you supposed to have? [Annabelle puts two more cookies out to make 10] Now how do you figure out what's fair?

[Annabelle passes out the cookies onto each plate one at a time.]

Sarah: Ok, so how many does he have [points to Danny]?

Annabelle: Five.

Sarah: How many do you have?

Annabelle: Five.

Sarah: Is that fair?

Annabelle: [nods her head yes]

Sarah: Is that equal?

Annabelle: [nods her head yes]

Sarah: How do you know that's equal?

Annabelle: [inaudible]

Sarah: Oh, the same amount.

Focusing on Annabelle, Sarah observed how Annabelle shared the cookies and asked her how she knew they were equal amounts.

*Pamela's Lesson Day 1*

Pamela began her first lesson by asking her students what it means to share fairly. Two students share what they know about sharing fairly in the following excerpt:

Pamela: Isabelle?

Isabelle: It means you both get the same amount of time to play with one toy.

Pamela: You get the same amount of time. So is that fair?

Isabelle: Mm-hmm.

Pamela: Ok. Does anyone else have an idea of what it means to share fairly or to share equally? Darren?

Darren: I seen this book before.

Pamela: Good. Listen while we read. Does anyone else have an idea of what it means to share fairly or to share equally? Avery?

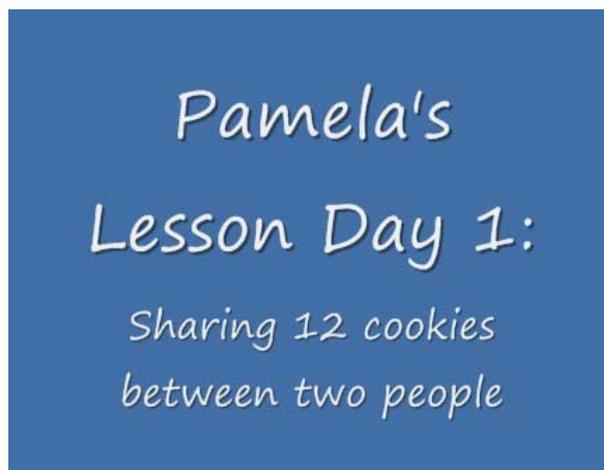
Avery: Um, to have the same amount of like candy.

Pamela: Same amount of like candy. So Isabelle said to have the same amount of time to play with a toy. That would be fair or equal. And Avery said to have the same amount of candy or something like that.

Pamela then began to read the story *The Doorbell Rang* by Pat Hutchins and had student volunteers come up to act out the story as they read using a play set of cookies and plates. Just as Sarah did, Pamela continually asked her students how they can share the cookies fairly. As they discuss how to share 12 cookies among two people, a student looked at the pile and guessed six cookies. Another student went to the pile of cookies and physically “slices it down the middle” to give some to one person and some to the other

person. After these two strategies were suggested, Pamela responded, “Is there another way we could do it without just kind of guessing that we would *know* that each person had the same amount?” Then, one student suggested giving one cookie to each person. Click on the screen shot to see how the teacher continued with this suggestion in the following video clip:

[There are two students, Julia and Brendan sitting in the front of the class. Each student has a plate with one cookie on it. Pamela is leading the discussion on what to do next in order to share the remaining cookies fairly]

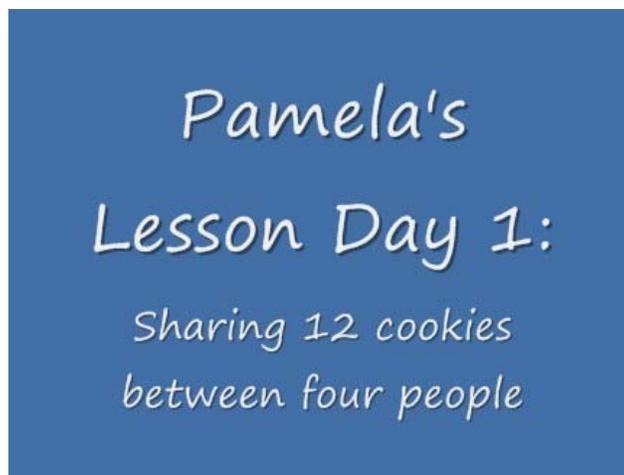


[Video Clip 3: Pamela's Lesson Part 1](#)

A student began with the idea to pass out one cookie to each person. Then, when there were 8 cookies left to share, Russell suggested giving each person two cookies. Towards the end of the excerpt, when Emma suggested giving each person two of the remaining four cookies, Pamela described what she *believed* to be Emma's thinking. She did not ask Emma to clarify her thinking nor did she ask the class to reflect on the strategies that were suggested.

In the next part of the story, two more people are added to share the 12 cookies. In the demonstration, two students (Julia and Brendan) already had six cookies on their plates

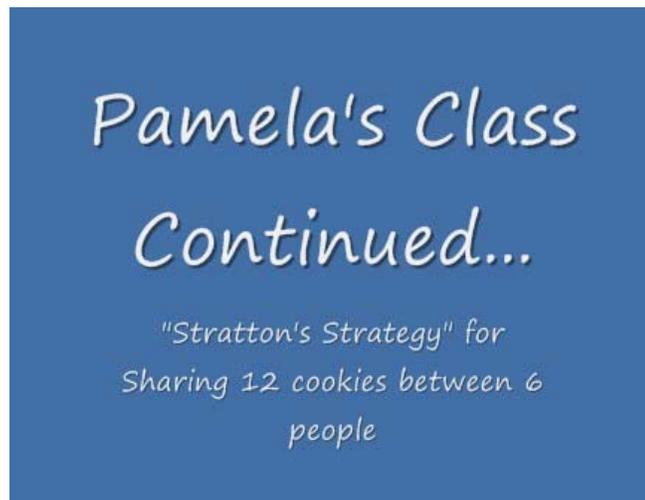
and Pamela called Ally and Adarian up to the front. Ally and Adarian were each given a plate. The class discussed how to share the 12 cookies between four people in the following excerpt. Click on the screen shot to view this teaching episode:



[Video Clip 4: Pamela's Lesson Part 2](#)

Here, Noah has recognized that if he counted the number of cookies, he could determine what half was. Again, Pamela did not ask her students what they thought of this strategy. She did have Noah explain his thinking when he says “We could give Adarian three and Julia would have three and Brendan would have three and Ally would have three” and yet how he came up with his answer was still unclear.

The last video clip shows that Pamela was looking for the strategy of passing out one cookie at a time. Click on the screen shot to view the lesson when two more people have been added to the demonstration, Alice and Caliel, so that there are a total of six people to share the 12 cookies.



[Video Clip 5: Pamela's Lesson Part 3](#)

When Stratton suggested giving one cookie to each person, Pamela immediately took up all the cookies and put them in a pile in the middle. At the end of the excerpt, she made this part of Stratton's strategy even though he only suggested giving one cookie to each person, not putting them all in the middle first. In addition, she gave value to what she was calling Stratton's strategy by saying "I like that" instead of asking the class what they thought of Stratton's idea.

As they finish the story, Pamela asked what would happen if more people came in to share the cookies. Most of the students were given the opportunity to share their ideas as seen in the following excerpt:

Pamela: You guys were right when you predicted that Grandma came in with more cookies to share. But what would have happened if we had one more person or a couple more people come. What would we have needed to do? What could we do? Would we be able to share?

Some students: No!

Some students: Yes, yes we would!

Pamela: If you think we would be able to share, how? What would we do?

Peyton?

Peyton: We would split apart...apart and we could share our plates.

Pamela: And you could share....so you would like take one and break it in half?

Peyton: Yeah and we can share our plates.

Pamela: What, oh, share your plate to eat it on? That's a good idea. If we broke one apart we could do that and that would share that cookie equally. Do you have another idea, Julia?

Julia: The mom can bake more.

Pamela: The mom can bake more. That's a good idea. Isabelle?

Isabelle: If they just...if what Peyton said, it wouldn't work out for them two because they would get less than the other people. So they all could split it in half.

Pamela: Ohhh. So Isabelle, ok, so Isabelle's saying if we break this one cookie in half, so if Rolando and I broke this cookie in half we'd each get a half. Would that be fair?

Some students: No.

Pamela: Isabelle's saying it wouldn't be fair because everybody else would get a whole cookie and we only get a half. So what was your idea again?

Isabelle: So, um, we would all split, we all split our um cookies and share.

Pamela: Ok, so everybody would split their cookies...

Isabelle: And share.

Pamela: Ok, would you split it, like just break one part? How would you split it?

Isabelle: In the middle.

Pamela: In the middle. Ok so there was two parts and they would be equal.

That's a good idea. Avery do you have another idea?

Avery: Um, but then the people that, like, if more people came in, there would be like...there would be none plate. They would have to make a plate or something.

Pamela: You're right. If we didn't, maybe we would...Miss Gabbert only had this amount of plates, but hopefully in their kitchen they would have more plates. So you're right that would be a problem. But Peyton said that maybe you could share plates. So that could be an idea. Darren?

Darren: We can all break stuff apart and put it in the middle and eat it. Eat it together.

Pamela: All what?

Darren: All the cookies and break it in half.

Pamela: Break all of 'em in half and put 'em in the middle and then everybody get one? That's a good idea, mm-hmm. That's a good way to split 'em up I think, once we break 'em in half. Russell?

Russell: We can cut 'em in pizza halves?

Pamela: You could cut 'em in pizza halves. So would that be smaller pieces?

Russell: Yeah.

Pamela: Ok, smaller pieces. Uh-huh. Audrey?

Audrey: You could like break every one apart and then keep breaking and if Grandma came after another person came we could break all those and then we could all eat 'em.

Pamela: You mean break all these too? [Points to the picture of the cookies the Grandma in the story has brought]

Audrey: Yeah, but if Grandma came after the kids who didn't have any.

Pamela: Oh, so you mean...

Audrey: We could break 'em all apart and we could all have two.

Pamela: Oh. Okay, that's a good idea. Alice?

Alice: Um, if Grandma came in before anymore children came in, then we could use some of the cookies that Grandma [inaudible].

Pamela: And do what with them?

Alice: You could keep them in whole because then everybody would have their own and there would be enough for everybody.

Pamela: You'd keep them in whole if she came before everybody started splitting their cookies? Mm-hmm. Now did they have to split their cookies in here [points to the book]?

Some students: No.

Pamela: No they didn't have to. Only if more children had come in would they have run out of cookies. Very good.

Pamela was able to have many students share their ideas and also got them thinking about what would happen if there were not enough cookies to share fairly. Pamela then concluded the lesson for the first day.

### *Pamela's Lesson Day 2*

On the second day, Pamela began the lesson by instructing the students on the day's activity. She asked a volunteer come up to the front to demonstrate the activity where students would be working with a partner to practice sharing cookies fairly. Using paper cut-outs of cookies and paper plates, Pamela modeled with the student volunteer how to share 6 cookies fairly. Because she did not want to give away the answer, she told the students to pretend one plate had 2 cookies and one plate had 4 cookies. Again similar to Sarah's lesson, Pamela used a large piece of chart paper that was identical to the students' worksheet to model how to fill out the chart. She drew 2 circles in one column and 4 circles in the other column to represent the number of cookies each person got after sharing the 6 cookies. A student told her that that was not fair and she reiterated that she did that on purpose so as to not give students the answer.

Next, Pamela dismissed students to their tables with their partners. Each pair of students had a baggie of 10 paper cookies and two paper plates. Each student had their own worksheet. The students began working on sharing different numbers of cookies and filling out the chart while Pamela walked around to monitor the students' progress. Again, due to the nature of the activity and the limitations of the video camera, not all of Pamela's

conversations were able to be recorded. She mainly checked to see that students were completing the activity correctly and did not ask many questions to check their understanding, such as “How do you know that is fair?” The following excerpt shows her interactions with two students:

[Pamela is working with Julia and Darren to share 6 cookies]

Pamela: You have to count out six cookies. Where’s your Ziploc [baggie]?

Darren: [takes a pile of cookies and counts out six] One, two, three, four, five, six.

Pamela: Ok, then Julia you need to put those and that [the extra cookies] up in the Ziploc bag. We’re only dividing equally between six. Ok, how could you share those equally?

Darren: Write it down?

Pamela: No, don’t write it down. You want to do it first on your plates.

[Julia takes up two cookies and Darren picks up the rest. He puts two on his plate]

Darren: I got two, so I supposed to...

Pamela: How many do you have?

Darren: [puts the remainder of the cookies on his plate and counts] One, two, three, four.

Pamela: Julia, how many do you have?

Julia: Two.

Pamela: Is that equal or fair?

Darren: It's not fair.

Pamela: What do we need to do to make it fair?

[Darren puts one of his cookies on Julia's plate]

Pamela: Ok, now how many do you have?

Darren: Three.

Pamela: And how many do you have?

Julia: Three.

Pamela: Is that fair?

Julia and Darren: Yes.

Pamela: Ok, then draw a picture of Darren's plate right here [points to Julia's worksheet]. So Darren has three.

Darren: So you got three [draws on his worksheet].

Pamela's students were completing the activity but they were not discussing their strategies or explaining how they knew three cookies each was fair or equal. The students continued to work on the activity. After most students have completed the chart, the teacher ends the activity.

### *Comparison of Lesson Planning Observation and Enacted Lessons*

Both teachers' enacted lessons were consistent with what was planned. From the lesson planning observation, the teachers indicated they would read the story and then act out the story with a group of students. This was what occurred during the first day of their video-taped lessons, however, due to time constraints, each teacher was only able to read the story once and act it out as it was read. Secondly, the teachers planned to use a worksheet

and paper cookies in order for the students to practice sharing fairly in pairs. This occurred on the second day of the teachers' video-taped lessons.

The lesson planning session did not include all of the details of how the lesson would be enacted, but included a discussion of the objective to be covered and the main activities used to teach that objective. The teachers never produced a written lesson plan, only jotted down notes. Both teachers indicated they mainly kept an outline in a planning book, so it was not surprising that they did not write detailed lessons. McCutcheon (1980) describes this as “mental planning”, a rich form of planning where teachers reflect and mentally rehearse a lesson. What was observed during the lesson planning observation could be considered a form of mental planning where the teachers discussed how they wanted the lessons to flow and made decisions about the types of tools they would use as well as how many cookies they would have the students practice sharing fairly with.

#### *Cross-Case Comparison of the Enacted Lessons*

Given that the two teachers in this study planned together, there are some similarities and differences in how they carried out their plans. During the first lesson, Sarah was very focused on having her students see the strategy of passing out one cookie at a time until the cookies were gone, then counting to determine if they were equal amounts. Pamela, too, valued this strategy as shown by her comments on what she called “Stratton’s strategy.” However, Pamela was more willing to follow the lead of her students even if no one suggested passing out one cookie at a time. Sarah was unwilling to move on until this strategy had been used each time. Neither teacher asked her students to evaluate or reflect on any of the strategies or ideas suggested by the students.

Both teachers made assumptions about what their students were thinking. When Sarah's student suggested giving each person five cookies, Sarah assumed her student wanted to add five cookies to the three that had already been counted. In Pamela's class, Pamela assumed Stratton wanted to put all the cookies back in the middle before giving one to each person. Their reasoning behind their actions was not clear. The researcher hypothesized that Sarah wanted to point out that making a guess was not an effective method and that Pamela thought it would be easier to share the cookies if they were all in the middle instead of on some of the plates.

On the second day, both teachers modeled what they wanted their students to do to complete the day's activity. Pamela chose to not give the answer to sharing 6 cookies fairly where as Sarah worked through the example to get the correct answer. Both teachers monitored their students' work by walking around and talking with different pairs of students. From the dialogue that was audible, Sarah was heard asking her students to show how they were sharing and explain how they knew two amounts were equal. Pamela was only heard checking to see if her students were completing the activity and filling out the worksheet correctly. She did not ask for explanations. The following section will explore the teachers' lessons further by examining them with respect to teaching for understanding and CRP.

### Assessment

The final component of the mathematics teaching cycle is assessment. This section presents the findings of how the teachers attended to assessment both during their lesson planning as well as throughout their enacted lessons. With respect to assessment, Sarah

pointed out that she mainly considers assessment only in a formal way at the conclusion of teaching objectives over a period of time. She articulated, “I don’t assess them at every activity that we do other than ‘ah, they’re struggling with this’, or ‘this activity didn’t work, we need to do it a different way’. But with kindergarten, I find that I’m more successful assessing them one-on-one where I pull them and I use manipulatives to get them to demonstrate their number concept”.

Pamela, like Sarah, did not consider assessment in her lesson planning and stated “I don’t consider assessment when I’m trying to teach something.” Nor did she plan for informal assessment. For example, she stated “Sometimes I’ll keep a class list up front so like when we were doing days of the week and that’s one of the math objectives during calendar, I just pulled out the sheet and kids were raising their hands to tell me the days of the week. I’ll just do it like that informally.” Pamela also specified that she preferred to do one-on-one assessments with her students.

During the lesson planning observation, neither teacher considered planning any sort of informal assessment during the lesson. Both teachers specified that they would assess their students at a later time, one-on-one, with manipulatives:

Sarah: I always assess them one on one. So I’ll probably use manipulatives.

Researcher: So at a later time?

Pamela: Uh-huh.

Sarah: [nods head yes]

Pamela: I think I would want to do it a couple of more times, at least one other extension especially with odd numbers and things and then...and do some center activities with it and let them practice with it.

Although neither teacher planned for any formative assessment during their lessons, both Sarah and Pamela informally assessed their students' knowledge throughout the lessons and used what they learned to inform their instruction. For example, Sarah continually questioned her students on how they knew the cookies were shared fairly. When she thought some students were guessing or not using a specific strategy, she continued to emphasize passing out one cookie at a time. Pamela also questioned her students and asked them to explain their thinking. In addition, although neither teacher consider this a form of assessment, the chart the students filled out on the second day could be considered a written form of assessment that not only helped the students to organize the information, but served as a way for the teachers to quickly assess whether the students were performing the activity correctly.

During the reflective session, both teachers indicated that they presented their students with the situation of having an odd number of cookies to share between two people. They stated that they felt like their students were successful with the activities in which they were sharing an even number of cookies and wanted to see how they would handle an odd number. In this way, the teachers were using what they learned about their students' understanding to inform their instruction. In addition, both teachers planned to do more activities involving fair shares in the future.

### Teaching for Understanding and CRP

Because the two teachers in this study planned their lessons together and enacted similar lessons, the evidence of teaching for understanding and CRP in the data is very analogous in for both teachers. Therefore, to not be redundant, teaching for understanding and CRP for both teachers will be described simultaneously here and any differences between the two teachers will be noted and explained.

#### *Teaching for Understanding*

The learning experiences Sarah and Pamela chose to use in their lessons more closely resemble *activities* as opposed to *mathematical tasks*, as defined by Hiebert, et al. (1997). Although some students engaged in reflection and communication with respect to sharing equally, it was not *required* of all students as they participated in the classroom activities. It is not evident in the video-taped lessons whether each student participated in communicating about how they shared fairly. In addition, the tools students used throughout the lesson were chosen by the teacher; students did not choose tools that were meaningful to them in order to solve the problem of sharing the cookies fairly. Nonetheless, there are aspects of the teachers' lessons that are consistent with Hiebert, et al.'s (1997) dimensions for teaching for understanding. Each of the five dimensions (nature of the mathematical task, role of the teacher, social culture of the classroom, mathematical tools as learning supports, and equity and accessibility) will be examined with respect to Sarah's and Pamela's lessons.

Although the learning experiences chosen by the teachers are activities and not mathematical tasks, the activities do attend to the core features of *the nature of mathematical tasks* as described by Hiebert, et al. (1997) in some ways. The nature of the activity

presented a mathematical problem that potentially had value for the students: how can they share cookies so that each person gets a fair share? By presenting the concept in the context of a family sharing cookies, both teachers attempted to connect to experiences they believe the students were familiar with. It can certainly be argued that the mathematics the students were engaged in is of value since the concept of sharing fairly is a basis for understanding rational numbers and division.

Regarding the *role of the teacher*, Sarah and Pamela selected the learning experiences to match the goal from the North Carolina Standard Course of Study. In particular, Sarah used what she knew about students' prior knowledge of equality and how students learn about fair shares when choosing the activities as well as in making the decision to start out with only even numbers. Pamela asked her students to share their own ideas of what it means to share fairly in order to get them thinking about their own experiences.

Both Sarah and Pamela created a classroom culture where the focus was on students' methods for determining fair shares and not just the answer. Sarah and Pamela emphasized wanting to know students' strategies, frequently asking their students "What can we do to make it fair?" and "How did you know...?" Throughout Sarah's and Pamela's lessons, some students are continually communicating about their ideas of fair share and dividing equally. However, neither teacher encourages her students to reflect upon chosen strategies or how they are thinking about the mathematics involved.

For example, when Sarah described what she believes to be Charlotte's strategy of counting and then figuring out what would be fair, she did not ask Charlotte to describe what she was thinking nor did she ask the class what they thought of this strategy. Similarly, in

Pamela's class, she does not explore Russell's idea of giving each person two cookies. She did not ask him how he came up with that number. Nor did she ask students to think about the various approaches that students were using to share the cookies.

With respect to the *social culture of the classroom*, the students in both Sarah's and Pamela's classes are encouraged to come up with their own strategies of sharing fairly and some students have the opportunity to share their strategies with the class during the first lesson. However, in Sarah's classroom, it is apparent that she did not value all of these strategies as she continually emphasized (and tried to get her students to emphasize) passing out one cookie at a time until they were all gone, and then counting the cookies to determine how many each person had and if the amounts were equal. She compared the strategy of one student who counted the cookies and determined how much each person should have and the strategy of passing out one at a time and even refers to this later strategy as "easier". Pamela, too, encouraged her students to come up with and share their own strategies, yet she emphasized passing out one cookie at a time as well. In fact, "Stratton's strategy" was the only approach given by her students that Pamela explicitly describes. Despite this, Pamela was more accepting of other strategies offered by her students than Sarah. Pamela was willing to act on students' strategies where as Sarah did not act on all of her students' suggestions and was unwilling to move on until they approached the problem by passing out one cookie at a time.

Additionally, regarding the social culture of the classroom, by asking students to explain how they know or why a situation exhibits a fair share, both teachers were attempting to encourage their students to see that the accuracy of the mathematics lies in the logic of the

argument that supports it. On the first day, the teachers focused the lesson on *how* the students were determining the correct answer when a certain number of cookies were divided equally as opposed to simply knowing the right answer. But, contrary to Hiebert, et al. (1997), neither teacher asked the *students* to determine the correctness or value of the strategies offered. Hence, the teacher was still seen as the sole authority in the classroom.

In addition, on the second day, although the students were engaged in the activity, they were not directed to explain their strategies or thinking to each other in either classroom. Sarah did ask some students to explain their thinking, but she did not talk with all students. Pamela mainly focused on whether the activity was being completed and if the worksheet was being filled out correctly, not on her students communicating about the mathematics involved.

In thinking about the *mathematical tools* used during Sarah's and Pamela's lessons, the manipulatives (play cookies) and the worksheet used to record students' results were both used to solve the problem of determining how many cookies each person would get when the cookies were shared fairly. Students were able to communicate orally as they acted out the story on the first day as well as when they worked with their partners on the second day. The worksheet allowed the students to use drawings and numerals to communicate the results when they shared different numbers of cookies on the second day. However, the meaning of both the objects used to share fairly and the worksheet used to record student work was constructed by the teacher and not the students which are in contrast to the core features of mathematical tools as described by Hiebert et al. (1997). Sarah and Pamela gave specific

meaning to the cookies as well as how students should use the worksheet to record their work.

Throughout their lessons, Sarah and Pamela attempted to make the learning experiences *accessible and equitable* to all of their students by using a context that was familiar to their students. In addition, by highlighting the strategy of passing out one cookie at a time until none are left, they were attempting to provide an explicit strategy for students who they thought might struggle with finding a successful strategy when they are engaged in sharing fairly on their own. Although both teachers encouraged their students to communicate about the mathematics involved in sharing fairly and what strategies the students wanted to use, in reality, they only called on and heard about two-thirds of students during the whole class discussion on the first day and a smaller number during the pair work on the second day.

#### *Culturally Relevant Pedagogy*

In line with the first tenet of CRP discussed in the literature review, both Sarah and Pamela exhibited high expectations for all of their students. They each expected all students to participate in the learning experiences and anticipated that all of their students were capable of completing the activities. During the post-lesson reflection, the following exchange showed how the teachers thought about the success of their students:

Sarah: Well, they all remembered the story very well. The second day, no problem remembering the story. We let kids come up and draw it out on the chart and they all wanted to participate. There was some of every level of

child that was able to come up, either tell you what the number was or draw out what...they seem to be overall really successful with it.

Pamela: Yeah, mine seem to pick...yeah, they overall seemed really successful so we ended up saying “Ok, well what if we had, you know, an odd number.” And we kind of went over that again, an odd number of plates or an odd number of cookies, what would we do. And [they] came up with lots of different ways.

Despite the fact that they did not use odd numbers on the second day, both teachers revealed that they discussed what would happen with an odd number in a subsequent lesson. In agreement with Ladson-Billings (1995a), by using a story about a family and cookies, these teachers use a context that they believed was meaningful to their students to motivate them throughout the lessons.

Because of the nature of cultural competence and sociopolitical consciousness, these two tenets of CRP may not be visible in a kindergarten classroom. One aspect of cultural competence that we may expect to see at this level is a teacher connecting a home or community experience with the mathematics being learned. By using the story of a family sharing cookies, the teachers were attempting to help students make a connection between something they may have experienced at home (sharing food) and the mathematical concept of sharing fairly. In addition, during their lessons, both teachers exhibited instances of communication with their students known as “language matching” whereby they matched the informal language their students used as they explain their strategies and mathematical

thinking. An example of Sarah's use of language matching occurred in the following excerpt:

Sarah: Jack, what do you think is going to happen?

Jack: A lot more people are going to come in.

Sarah: A lot more people are going to come in...If a lot more people come in, Jack, how could you share fairly?

Pamela uses language matching much more frequently than Sarah; language matching occurs 15 times in Pamela's lessons and only 6 times in Sarah's lessons. An example of Pamela using language matching is given in the following excerpt:

Pamela: Naque, do you have an idea?

Naque: We could split 'em.

Pamela: Split 'em. Ok. What do you mean by split 'em?

Evidence of cultural competence may also be seen in a kindergarten classroom in the form of communalizing; that is, encouraging students to think of their success (or failure) as linked to that of the entire class. If one person succeeds, the whole group succeeds and students should work toward the betterment of the entire group. Although Sarah and Pamela have their students working in groups, neither teacher made any reference towards communalizing. The students were instructed to work together, but the teachers did not give specific prompts to be helpful to each other or that their success (or failure) was in any way linked to their partners or the entire class.

## CHAPTER 5

### DISCUSSION

The purpose of this study is to examine the mathematics teaching cycle of two kindergarten teachers who participated in a year-long professional development project that promoted teaching mathematics for understanding and culturally relevant pedagogy. The rationale for the study is based on several ideas. First, as a research assistant working with teachers in the NMD project, the researcher became interested in what the teachers attended to in planning their math lessons and how, if at all, exposure to the tenets of CRP and teaching for understanding impacted their lesson planning practices. Secondly, literature on the nature of teacher planning in light of reform efforts to change the teaching and learning of mathematics is sparse (John, 2006; Simon, 1995; Simon & Tzur, 1999). Additionally, research has not addressed how teaching mathematics for understanding and attending to students' cultural backgrounds can effectively be incorporated into teachers' lesson planning practices (Eisenhart, et al., 1993; Gutstein, et al., 1997; Ladson-Billings, 1995b; Putnam, et al., 1992; Putnam & Reineke, 1993).

This chapter is divided into four sections: limitations, addressing the research questions, implications, and recommendations. First, limitations of this study will be described. Next, conclusions with respect to the research questions will be presented. This will be followed by implications for teachers and professional development. Finally, recommendations for future research will be discussed.

### Limitations

One limitation to this study is that it is based on case study research. Therefore, any generalities based on the findings are limited. The researcher recognizes that the teachers took part in the NMD project, but it is unclear if the findings from the current research are a result of the teachers' participation in NMD. However, we do see evidence that these teachers have incorporated some aspects of CRP and teaching for understanding in their lesson planning practices.

Another limitation to this study was the effectiveness of the lesson planning interview protocol in producing pertinent data. Specifically with respect to assessment, the questions did not distinguish between formative and summative assessment which did not require the teachers to think about assessment in each form. The researcher assumed the teachers would answer the question for both informal and formal means of assessment. Despite this, the teachers were observed using informal means to assess their students' knowledge during their video-taped lessons. During the post-lesson reflective session, both teachers stated that they were able to use what they learned about their students' knowledge during the lessons to inform future instruction.

The data included only one lesson planning observation where the teachers planned together which was atypical of the two teachers in this study. Although conclusions were still able to be made about these teachers' lesson planning practices, multiple observations of each teacher's lesson planning could provide more details and consistencies in relation to their practices. The fact that they planned together may have skewed the way in which they would have approached the content than if they were planning alone. As well, the lessons

that they did end up planning were not very detailed nor did the teachers produce any sort of written lesson plans as a result of the lesson planning session.

This research is based on only two video-taped math lessons from each teacher. This was just a small glimpse into what occurred in their classrooms over the course of a year. However, because the data collection took place late in the school year, many routines were already in place and therefore, what went on in the classrooms during the video-taped lessons were likely to be consistent with what happened on a regular basis.

#### Addressing the Research Questions

This section will address the two research questions from this study.

*Research Question 1: What does a kindergarten teacher attend to in each phase of the mathematics teaching cycle?*

In contrast to research on teacher planning (Clark & Lampert, 1986; John, 2006; McCutcheon, 1980; Yinger, 1980; Zahorik, 1975), the teachers in this study did attend to the learning objective first in planning their math lessons. However, because Pamela and Sarah are provided with objectives from the NC SCOS, they were not writing their own objectives, merely choosing which objective they wanted to cover. In this way, they could also be characterized as attending to the content they wished to teach. In their lesson planning observation, the teachers identified the mathematical topic (fair shares) they wanted to teach by stating the NC SCOS objective that matched that topic.

In addition to focusing on the objective and content, the teachers spent most of their lesson planning time thinking about and discussing the learning activities they would present

their students and the logistics of carrying out the learning activities. Sarah presented the idea of using literature as a way to introduce the topic that might be familiar to their students while Pamela thought of acting out the story as a way to model sharing fairly for their students and as a way to keep their students involved throughout the first day's lesson. During their discussion on the activity for the second day, the teachers thought about what types of tools they would use (paper cookies, plates and the student worksheet) as well as how to appropriately structure the activity for their students. Both teachers believed the use of manipulatives was important to help make concepts concrete for their students while also keeping them engaged in the mathematics.

The teachers also used what they knew about their students' backgrounds and their students' prior knowledge as they planned their lessons. They considered how to make the lessons accessible to all of their students when they chose the context of a family sharing cookies. Sarah in particular focused on her students' prior work with equality and their number conceptions through ten when she suggested using only even numbers of cookies for the second day's activity.

During the enacted lessons, the teachers attended to the strategies their students were using to share fairly, emphasizing the strategy of dealing by ones as a way to make the activity accessible to all levels of students. Both teachers attempted to help their students make connections between sharing fairly and their real life experiences, such as sharing toys and what it means to be fair, by using a context that they believed to be familiar to their students. On the second day, the teachers focused more on ensuring that the students were

completing the activity correctly and less on explaining their strategies for sharing fairly or how they knew a certain number of cookies was shared equally. Pamela engaged her students in a discussion of what they could do if they did not have enough cookies to share fairly and both teachers eventually introduced their classes to the situation where there were an odd number of cookies to share between two people. In this way, the teachers thought about how to extend the activity to enrich their students' understanding.

According to Simon's (1995) mathematics teaching cycle, teachers can continually assess their students' thinking which in turn can modify teachers' content and pedagogical content knowledge and inform their instruction. The teachers who participated in this study stated that they only planned for formal assessment and did not think about assessment when planning, in agreement with the findings by Yinger (1980) and Zahorik (1975). However, there is evidence to suggest they did assess their students' knowledge throughout their lessons. Both teachers continually questioned their students, asking them to explain their thinking, or to show how they were sharing fairly. Because the researcher did not probe this issue with the teachers, it is unclear why the teachers did not consider this a type of assessment. In addition, the teachers thought about how to extend the activity of sharing an even number of cookies between two people to sharing an odd number of cookies.

Both teachers expressed that they were able to use what they learned about their students' knowledge to plan future lessons on sharing fairly. When asked what they would do differently or the same the next time they teach this topic, Sarah struggled with allowing her students to work with both even and odd numbers and whether to group students by same ability or mixed ability. But she did plan another activity where students would determine

how to share different amounts of “treasure” fairly. Pamela, too, thought about continuing their study of fair shares by doing a similar activity with different objects and then working with same ability groups in centers in order to challenge those students that were ready and work on reinforcing concepts with those that were not.

*Research Question 2: Is there evidence of the ideologies associated with CRP and teaching for understanding with in each phase of the mathematics teaching cycle?*

The participants’ planning and lessons showed evidence of some of the dimensions for classrooms that support teaching for understanding as outlined by Hiebert, et al (1997). The teachers were purposeful in choosing activities that were set in a context that made the mathematics problematic for their students and that brought about the intended learning goals: How could the students share the cookies fairly? The students were interested in the problem and communicated about the mathematics involved in determining fair shares. The teachers considered their students’ backgrounds when choosing the context and believed it to be one that was meaningful to their students.

The teachers wanted their students to come up with strategies to solve problems and to explain and communicate their ideas. During the planning phase, the teachers hypothesized about the importance of understanding equality and how their students might approach the chosen activities. Throughout the first lesson, both teachers encouraged their students to share their ideas and at times, both teachers acted on their students’ suggestions about how to share the cookies. However, the teachers were not open to the idea that all of their students strategies, both correct and incorrect, had value in the classroom. Both Sarah and Pamela focused on dealing out one cookie at a time when their students were in fact

using more sophisticated strategies such as dealing out by two's or three's. Although it is unclear why the teachers did not choose to focus on the more sophisticated strategies their students came up with, during the post-lesson reflection they did state that they wanted to emphasize passing out one cookie at a time in order to explicitly offer a strategy for students who struggled with determining fair shares.

In Sarah's case, as the students demonstrated the story, she did not continue on with the story until a student had suggested passing out one cookie at a time and they demonstrated that as a class. On several occasions, she would not act on students' suggestions because it was not the one strategy she was looking for (dealing by ones). However, she did validate Charlotte's strategy when she verbally explained Charlotte's thinking and compared it to passing out one cookie at a time.

Pamela, on the other hand, did act on students' strategies (as long as they were correct) even if they weren't dealing out by ones. Her students were able to use strategies of dealing out by two or three at a time in order to share the cookies fairly in the story and the class demonstrated that. Her students did not suggest passing out one cookie at a time until they were sharing the 12 cookies between 6 people. However, this is the only strategy (which Pamela calls "Stratton's strategy") that Pamela verbalized and stated that she "liked" that strategy. Although both teachers valued the approach of dealing out by ones, the way they went about exploring the strategies looked different in the two classrooms.

On the second day, the focus was less on the strategy that students were using and more on determining the correct answer when a certain number of cookies were shared fairly and filling out the worksheet correctly. Although both Pamela and Sarah did work with

individual groups of students to make sure they were sharing correctly, as students worked in their pairs, they used whatever strategy was meaningful to them and did not communicate about their strategies, only the answers.

Along with the sharing of students' strategies, the use of tools is also an important element of a classroom that supports learning mathematics for understanding. Hiebert, et al. (1997) states that tools should be used with meaning to keep records, to communicate, and to think. In this way, tools can be symbols on paper or manipulatives that students use to solve problems. In the context of this study, the tools chosen by the teachers for their students' use were the paper cookies and the worksheets the students completed on the second day. The students in both Sarah and Pamela's classes were able to use the paper cookies to determine fair shares for certain numbers of cookies. They were able to use the worksheet to keep record of their solutions and to represent their solutions with pictures and numbers.

However, the teachers in this study did not allow their students' to use tools that were meaningful to them or to create their own meaning for the tools the teachers had provided them. The teachers considered the paper cookies to be manipulatives and explicitly described how they wanted their students to use the cookies and the worksheet to represent their answers. Hiebert et al. (1997) emphasize that tools are more than just physical materials that are being manipulated and that they should support students' understanding by helping them to make certain connections. The students were able to use the tools to communicate about their *answers* to sharing fairly problems, but not to explain how they were thinking or to reflect on chosen strategies. The evidence from the video-taped lessons showed the teachers interacting with the students using the tools only in the ways prescribed

by the teachers: the cookies were to be shared between the two students and the worksheets were to be used to represent and record students' answers. It is unclear whether how the students were engaged with the specific tools the teachers gave them allowed them to make any mathematical connections.

Just as there were some aspects of the teachers' planning and lessons that were consistent with teaching for understanding, there were also some aspects that were consistent with the ideologies associated with CRP. First, it is important to note that CRP is not something that is easily seen in a kindergarten classroom. Nonetheless, both teachers did show evidence of two out of the three general tenets of CRP as described by Ladson-Billings (1995a): High academic achievement and encouraging cultural competence.

The teachers exhibited high expectations for all of their students. They each expected all students to participate in the learning experiences and anticipated that all of their students were capable of completing the activities. According to the teachers, their students were successful with the activities. In subsequent lessons, the students were able to continue to come up with strategies for sharing an even number of items fairly and had ideas for what to do with an odd number of items. Additionally, in line with Ladson-Billings (1995a), the teachers chose a context that they believed to be meaningful to their students in order to motivate them want to participate in and excel at the chosen activities.

Ladson-Billings (1995a) describes the second tenet of CRP as helping students to develop cultural competence and the teachers' lessons supported their students' development of this characteristic in two ways. First, both teachers exhibited language matching of the students' informal language they used to explain their strategies and thinking. In doing so,

the teachers can help in-class communications remain consistent with students' cultural ways of communicating. Secondly, the teachers chose a context that they hoped would connect with students' home or community experiences, thereby validating these experiences for their students.

The third tenet pertaining to sociopolitical consciousness was not apparent in these teachers' classrooms. One would not expect a kindergarten class to exhibit some of the examples of sociopolitical consciousness that Ladson-Billings (1995a) describes, such as discussions about political or social inequities, but there could be evidence of communalizing whereby the teacher encourages the class to work together for the good of the group. During the video-taped math lessons, the teachers did not give any directive to their students to help each other out or that their success was at all dependent on each other. While they did not promote any competition between or among the students in their classes, the teachers generally did not encourage communalizing in their classrooms.

Although CRP was not central to these teachers' lessons, it was apparent the teachers in this study valued knowing about their students' cultures and using their students' cultures and interests as springboards for learning. Sarah indicated she made home visits with her students to learn more about their home life and to increase communication with parents. Pamela had a holistic attitude where she aimed to help her students have a positive attitude about school in general.

### Implications

Based on the findings of the study, suggestions can be made for both teachers and for teacher education. First, the mathematics teaching cycle can be added to preservice methods

courses and inservice professional development to provide teachers with a way to think about the decisions they make in planning mathematics lessons as well as during classroom teaching. The mathematics teaching cycle highlights the importance of teachers making predictions about how their students will best learn the chosen content. In addition, information about students' cultural backgrounds as well as their current mathematical knowledge should be taken into account when choosing classroom activities. This implies that teachers must have knowledge about their students' cultural backgrounds and their students' as individuals. This can be challenging in an area where many diverse cultures are represented in one classroom.

It is important for teachers to have knowledge about their students' mathematical understanding. If teachers are using the mathematics teaching cycle, they will pay attention to assessment, both formative and summative, thereby continually learning about their students' knowledge. By frequently assessing students knowledge (through questioning and listening to students communicate and reflect about the mathematics as well as through formal assessments), teachers can use that knowledge to inform future instruction. Teachers can build on students' chosen strategies and use mistakes and misconceptions as learning sites for all students. Too, teachers can add to their existing body of knowledge on how students learn and how to structure lessons to increase student understanding.

Teachers can use literature on how students learn to inform their instruction. For example, in an article by Sally Roberts (2003) from the NCTM journal, *Teaching Children Mathematics*, teachers are offered ideas on how to teach fair shares and what strategies are typical of kindergarten students such as estimating, dealing out by ones, or grouping by twos

or threes. With respect to the current study, it can be hypothesized that if the teachers had been more aware of the different types of sharing strategies appropriate for their students, they first may have been able to identify strategies other than dealing out by one at a time, and then secondly, may have been more accepting of those strategies.

Through preservice courses and inservice professional development, it is important to bring to teachers' attention how culture can play a role in the teaching and learning process. By attending to students' cultural backgrounds in lesson planning and choosing contexts that are meaningful to students, teachers can provide students with opportunities to engage in important mathematics. Similarly, teachers should be exposed to what it means to teach for understanding and learn how they can incorporate Hiebert, et al.'s (1997) dimensions of classrooms that promote teaching for understanding into their own classrooms. It is beneficial for teachers to be aware of what it means to have students share strategies and how to value students' chosen strategies and build upon them to increase students' understanding. Teachers need to know how students create meaning for tools and how students can use tools to solve problems.

Teachers can choose appropriate classroom activities or tasks when they have information about how students may think about concepts and how those concepts can be developed over time. Through preservice methods courses and inservice professional development, teachers can gain insight into how students learn mathematics in order to accurately develop lessons that are meaningful and accessible to all students and that increase students' mathematical understanding.

### Recommendations for Future Research

Because this research was based on case studies, any generalities based on the findings are limited. Therefore, it is suggested that further research take place to examine consistencies across cases in the way that teachers incorporate teaching for understanding and CRP into their lesson planning practices as well as into their enacted lessons. For example, some future research questions might include:

- How can teachers use both students' in-school and out-of-school knowledge to inform instruction?
- How can teachers use literature on how students learn to inform the ways that they plan lessons?
- How do teachers use different strategies suggested by students as learning sites for all students and to increase students' understanding?

### Conclusion

Despite the limitations of this study, much was learned about the lesson planning practices of the two kindergarten teachers in this study and how their lessons compared to the ideologies associated with CRP and teaching for understanding. Aspects of CRP and teaching for understanding were evident in the teachers' lesson planning observation and enacted lessons. Although it is unclear whether their participation in the NMD project influenced their lesson planning and instruction, they did in fact incorporate some of these ideas into their lesson planning as well as into their enacted lessons. If a goal of mathematics instruction is to increase student understanding in a learning environment that is accessible to

all students and where academic success is experienced by all students, the mathematics education community can learn from studies such as this how to make this goal a reality.

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APPENDICES

## Appendix A

### Pre-Lesson Planning Interview Protocol:

- How do you decide what mathematical content/concepts to focus on in your math lessons? Where does the content come from?
- Why is this content important to teach? How is useful to your students?
- How do you decide what specific activities or tasks to use in your math lessons (used in the past, found in a magazine, textbook or curriculum resource, ideas from other teachers, etc)?
- What knowledge about your students do you draw on in your lesson planning?
- Do you think it is necessary to draw on student's knowledge when planning?  
What are the benefits and or drawbacks in doing so?
- Do you consider assessment in your planning? Why or why not? If yes, how?
- How much does assessment affect your lesson planning?
- What resources are available to you to use in your planning?
- Do you plan collaboratively? If so, how? If not, why not?

## Appendix B

### Post-lesson Reflective Session Protocol

- Do you believe your lesson followed your plan? What changes did you have to make, if any, and why?
- What aspects of your lesson support your students' conceptual understanding of the concepts you covered? What evidence do you have?
- What way(s) might the students' cultures have impacted their understanding of the mathematical ideas they were supposed to learn?
- What would you do differently/the same the next time you teach this concept?
- What out of school knowledge, what in school knowledge did your students draw on?
- What types of connections did your students make and how do you know?
- What were you doing that promoted or hindered the learning sought?
- Ask about specific students- did anyone surprise you? Why?
- What would you teach/do in the next lesson(s)? Why?