ABSTRACT

SHELTON, WILSON ANDREW. Development of MEMs Micro-Bridge Mechanical Resonators Interrogated By Microcavity Interferometry. (Under the direction of Leda M. Lunardi and John F. Muth.)

Miniature resonators such as microelectromechanical systems (MEMS) cantilevers have found multiple uses including development of mass resonant sensing. In this thesis, by the microcavity interferometer technique, we investigate the properties of bulk micromachined silicon nitride membranes that are wafer bonded to a substrate to form a Fabry-Perot cavity.

Photothermal actuation is used to drive the motion of the bridges by frequency modulating a 980nm diode laser, while the shift in Fabry-Perot resonance is detected by monitoring the intensity using of a continuous wave 1550nm tunable laser. Using this technique, 100 femtometer deflections can be easily monitored.

A variety of double clamped beam structures were investigated as a function of beam width and length. Depending on the dimensions of the micro-bridges, resonance frequencies from 10kHz to 5MHz were observed. For smaller bridges of dimensions 30μm length and 10μm width, resonance frequencies near 1MHz, quality factors of Q ~ 150 were observed in air at room temperature. However in vacuum, the quality factors on the order of Q ~ 7000 were observed, verifying air damping as the main source of energy loss.

Moreover, in air at room temperature, the Brownian motion of the bridge was observable without any photothermal driving force suggesting the possibility of using this structure as an un-powered chemical sensor and demonstrating the sensitivity of the optical system to measure small deflections.
The sensitivity of the bridge to chemical exposure was not quantified, but environmental perturbations were observed to change the resonant frequency of the bridge by several kilohertz indicative of a device structure suitable for sensor applications.
BIOGRAPHY

Wilson Andrew “Andy” Shelton was born in Martinsville, Virginia on November 10, 1981 and grew up in Stoneville, North Carolina. He attended Dalton L. McMichael High School where he played football, basketball, and baseball. After graduating high school in June 2000, he attended Wake Forest University. While there, he played on the football team and participated in undergraduate research in the physics department. This research dealt with optical trapping and micromechanics, in both of which he took great interest. His undergraduate research honors thesis, entitled “Nonlinear Motion of Optically Torqued Nanorods” was published in Physical Review E in March 2005. After receiving his Bachelors of Science degree in Physics from Wake Forest in May 2004, he attended North Carolina State University to pursue graduate work in Electrical Engineering. While there, his thesis research was an optical MEMS project which was a perfect fit considering his optics and micromechanics background. He was married on July 29, 2006 to his high school sweetheart Katherine Leigh Morrison. He received his Masters degree in Electrical Engineering in the fall of 2006.
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1 OPTICAL ELECTRONIC NOSES

1.1 Introduction

All living organisms from bacteria to humans respond to chemicals in the environment. In more developed organisms such as animals and humans, chemicals are most often detected by using the taste or olfactory senses. The human sense of smell is a very sensitive system able to respond for some compounds at parts per billion (ppb) concentrations. However it is unreliable because it can vary between different noses from nil to highly sensitive\(^1\).

The need to detect deadly explosive or chemical agents has become very important for homeland security and defense applications. While explosive agents may not be detectible by the human noses, animals such as dogs and rats have been used successfully to perform the detection task, requiring sophisticated handling and training besides the short lifetime. Among the most toxic and rapidly acting of the known chemical warfare agents are nerve agents such as sarin gas, so human and animals noses would not be used to detect these agents. Another alternative is to use a mechanical olfaction system.

The generalized detection of volatile organic compounds (VOCs) is most often referred to as electronic nose technology. The electronic nose has generally been based on sensors that detect adsorption into a set of chemoselective polymers, as shown in Figure 1.1.
The first artificial nose created by Persaud and Dodd used semiconductor transducers to detect various chemicals. There are currently several electronic nose sensor technologies. Included are metal oxide and conducting polymer sensors, both of which use conductivity as the principle of operation. There are also quartz crystal microbalances and surface acoustic wave sensors which work on piezoelectricity. Metal Oxide Substrate Field Effect Transistor (MOSFET) electronic noses use capacitive charge coupling as a means of detecting gases. Gas chromatography, mass spectrometry, and light spectrum sensors rely on molecular spectra, atomic mass spectra, and transmitted light spectra respectively. The ultimate electronic nose sensor technology is the optical sensor which could work in a variety of methods. Optically based artificial noses have only been introduced in the last few years.

Much advancement in optics in the last fifty years, including lasers, laser diodes, light-emitting diodes, optical fibers, and very sensitive optical detectors have led to the ability to sense trace gases optically. An optical sensor is a device that measures change in one of the properties of light: absorbance, fluorescence, polarization, refractive index, interference, scattering, or in the reflected wavelength. Optical sensors consist of four main
components: a light source to interrogate the sensor, optics for directing the light to/from the sensor, a detector to measure the light from the sensor, and the sensor itself.}

1.2 Optical Sensing Methods

There are five methods of optical vapor sensing: luminescent methods, colorimetric methods, surface plasmon resonance methods, interference and reflection based methods, and finally mass-loading methods. Luminescence is among the most popular method of optical vapor sensing. Fluorophores are molecules that absorb light at one wavelength and emit light at another longer wavelength. Photoluminescence is a type of electromagnetic spectroscopy that uses a beam of light, usually in the ultraviolet wavelengths, to excite the electrons in molecules of certain compounds to emit light of a lower energy. This lower energy emission is often in the visible spectrum. This change in absorption and emission wavelength is referred to as a Stokes shift and the size of this shift represents the energy lost by the fluorophore during absorption, as shown in Figure 1.2. The efficiency of photon loss after absorption is referred to as the quantum yield. Luminescence is a popular means of sensing because of high quantum yields, large separation between excitation and emission wavelengths, and its intrinsic sensitivity. One example of the mechanisms by which the fluorophores work is twisted intramolecular charge transfer (TICT). These fluorophore molecules are designed to be highly polarized in the excited state. When the excited states are stabilized by the gas to be detected, red-shifts are detected in the emission spectra.
Figure 1.2 – A change in the Stokes shift denotes a change in the concentration of analyte vapors present when bound to the fluorophores.

The second method of optical vapor sensing is the colorimetric method\textsuperscript{5,6}. Colorimetric sensors measure a change in absorbance or refractive index resulting from indicator color changes. They rely on changes in the color of an organic sensing material. These can be small colored beads placed at the ends of an optical fiber bundle, or colored spots on a chip imaged by a camera. Another type of colorimetric sensor employs a multi-pass absorption cell such as in the schematic shown in Figure 1.3. Gas is pumped into the cell which is a gas-tight two mirror cell and allows multiple reflections of laser light, much like a laser cavity. During these multiple passes, certain wavelengths are absorbed by the gas molecules and a spectrometer is able to detect these absorbed wavelengths. The input beam is directed into an iris in one sealed face of the cell and after multiple reflections the beam exits from the same iris located at the bottom of the left face of the cell in Figure 1.3. From the absorbed wavelength and the intensity of the output signal, some quantification of the concentration of gas inside the chamber can be made.
The next type of sensing method is surface plasmon resonance based sensing. This technique uses the free conducting electron gas found at the surface of conductive metal films such as gold and silver. If the interface between two media of differing refractive indices is coated with a thin layer of gold, the intensity of monochromatic reflected light decreases at a precise, resonant incident angle. This surface plasmon resonance is due to the resonance energy transfer between evanescent wave and surface plasmons. The resonance conditions, specifically angles and wavelengths, are influenced by the material adsorbed onto the thin metal film. The resonant angle is dependent upon the refractive index of the dielectric medium, so any changes in refractive index at the surface, caused by the absorption of vapor molecules into the chemoselective polymer on the surface, can be monitored in real time by measuring the value of the resonant angle as shown in Figure 1.4. Some drawbacks of the surface plasmon resonance method are that it is not very sensitive and the response and recovery times are several minutes long.
Figure 1.4 - A surface plasmon resonance based sensor. With no analyte vapors present, the incoming beam experiences a reflection minima at a particular angle (solid line). The angle of minimum reflection shifts (dotted line) as the analyte vapor is captured and adheres to the sensor surface.

Interference and reflection based methods employ reflectometric interference spectroscopy (RIfS)\(^8\). Some polymeric films when exposed to specific chemical vapors experience a large change in optical thickness. The films also experience a change in refractive index, but the index change is quite small relative to the optical thickness change. This technique which uses light incident at the interface of two planar optical layers is known as reflectometric interference spectroscopy. The light reflects from both the top and bottom of the film, setting up an interference pattern that is very sensitive to changes in the optical thickness of the polymer sensing film, as shown in Figure 1.5. The adsorption of the analyte vapors onto the surface gives rise to shifts in the Fabry-Perot interference fringes, which can be detected with a spectrometer. Also, one can monitor the emission at a specific wavelength and observe the sharp decreases in intensity as the interaction with the analyte vapors takes place.
Figure 1.5 - An interference method based sensor. When a vapor analyte is adsorbed into the top layer, the optical thickness of the body changes thus producing a shift in the interference fringes.

The final method of optical vapor sensing is referred to as mass loading\textsuperscript{9,10,11,12,13,14}. This method makes use of resonating microcantilever transducers, much like what is used on an atomic force microscope (AFM) for molecular level imaging. In mass loading, the cantilever transducers are coated with a polymeric film which reacts with a specific analyte vapor. As the analyte vapors contact the polymeric film on the cantilever, the mass and/or strain of the cantilever could change. The general idea behind these sensors is that the physical, chemical, or biological stimuli are able to affect the mechanical characteristics of the resonating transducer so that the change can be detected by electrical or optical means. Many polymeric films today are able to be synthesized so that they react with a single chemical. These polymeric films remain noble until the chemicals contained within the gas to be detected come into contact with the sensor. The selectivity of these polymers removes some uncertainty in the detection process. They can be chosen to detect specific airborne hydrocarbons or discern between humidity and other similar polar molecules. The chemoselective polymer is the main means of determining the type of gas that is being
detected. This change in mass or strain as the analyte vapors are adsorbed can be detected optically by measuring the deflection of an incident laser beam, as shown in Figure 1.6. This deflection could be a static deflection in which the adsorption simply causes the cantilever to bend or the deflections could also be dynamic in scope, where the frequency at which the cantilever is oscillating is the measured quantity. As the analyte vapors adsorb onto the cantilever in this method, the added mass and variation of strain will produce a shift in the resonant frequency of the cantilever.

Figure 1.6 - Schematic showing the mass-loading technique. The analyte vapors change the mechanical properties of the cantilever which causes a shift in the resonant frequency at which the cantilever oscillates.

This optical method of detecting a change in the resonant frequency is the method used in this research. Cantilever transducers continue to improve as methods of chemical sensing because of the ever-changing and improving technology. As the size of MEMS devices continues to decrease, the ability to detect smaller masses become a definite possibility. The mass loading/resonant cantilever platform was chosen because of the continuing areas of improvement due to technological advancements in MEMS fabrication techniques.
1.3 Goals of This Research

Most chemical sensors require a great deal of signal processing and amplification at the source besides having a large area and lots of interconnects\(^1\). This project uses an optical-based chemical sensor that is a micromachined Fabry-Perot microcavity. The goal is to build sensors that are able to be remotely modulated and interrogated without any electrical connections or signal processing at the sensor level by both modulating the signal and directing that modulated light back to the detector. Remote interrogation will make this type of sensor much safer to use as a means of detecting deadly gases.

Silicon nitride is the material of choice for the cantilever in this project. This material is attractive because it has been used for many years in atomic force microscope (AFM) probes with pre-existing technology and knowledge as a resonator.

Chapter 2 includes the use of optical micro electromechanical systems (MEMS) interferometry as a technique for detecting small mechanical deflections, discussing the basics of Fabry-Perot interferometry and how small mechanical deflections are used to change the interference characteristics of the device.

In Chapter 3, the mechanical theory of the double-clamped beam is presented. This analysis includes the material properties of silicon nitride, the Euler-Bernoulli theory of motion for the double-clamped beam, and the effects that air damping has on its motion. Thermal-mechanical noise within the system is also discussed.

In Chapter 4, the steps to fabricate the devices are given in detail. From the bare double-side polished silicon wafer, the final $1\text{cm}^2$ chips contained over 800 double-clamped beams of various dimensions.
In Chapter 5, the setup for optical interrogation and data acquisition are presented. The setup includes light sources, fiber and free space optics, detection devices, network and spectrum analyzers, and data acquisition routines.

Chapter 6 is devoted to the analysis and discussion of the experimental results. Data for characterization included resonant frequency measurements as functions of bridge length and width as well as clamped and unclamped mechanical modes. The effects of various experimental conditions such as source intensity and wavelength on the resonant frequencies were studied.

Finally, in Chapter 7, conclusions and future directions for the work are addressed.
1.4 References


2 OPTICAL MEMS INTERFEROMETRY

2.1 Introduction

In the past ten years, micromachined Fabry-Perot interferometers have been an area of extensive development in MEMS processing\(^{1,2,3}\). Some of these include sensing elements in the form of accelerometers\(^4\), microphones\(^5\), capacitive switches\(^6\), and pressure sensors\(^7,8,9\). They have also found their way into optical communications systems and are being used as tunable filters in wavelength division multiplexing (WDM) fiber optic systems because of their ability to be low loss, narrow linewidth, and tunable at low-voltages\(^10\). There are several advantages to using optical interferometry to characterize micromachined systems including immunity to electromagnetic interference, the non-destructive and non-contact nature of the measurement, the very high bandwidth limited only by the photodetector response, and the high precision of the measurement. This section addresses the principles of microcavity interferometry and how it can be used to detect mechanical motion.

2.2 Optical Detection of Micromechanical Deflections

One main goal of this project is to detect the frequency, amplitude, and phase at which a driven micromachined beam is vibrating. Several optical methods have been used for detecting deflections of micromachined devices but the main methods include the optical beam deflection, laser Doppler vibrometry, and interferometric techniques\(^11\). These techniques allow for interrogation of the sample with no physical contact. In the optical beam deflection technique\(^12\), which is used in atomic force microscopy, a laser beam is directed at a microcantilever with a mirrored surface. The reflected beam is directed at a detector called a position sensitive photodetector which consists of four quadrants. The photocurrent, if the spot is perfectly centered, sums to zero; if the spot is off center, the signal
from one quadrant dominates and allows the position of the cantilever to be determined. Figure 2.1 demonstrates how the incident and reflected beams are unable to be oriented along the same direction in an optical beam deflection setup.

Figure 2.1 – The optical beam deflection technique such as is used in an atomic force microscope.

The advantages of this system are the absence of any bulk electrical connections to the microcantilever and the simplicity of the design. Some disadvantages of this type of optical system are that changes in the optical properties of the surrounding material may cause unwanted refraction of the laser beam. Also, current position sensitive photodetectors have very low bandwidths on the order of a few hundred kilohertz due to their relatively large size. Also, incoming and reflected beams cannot be oriented along the same path at normal incidence when deflections are only along the z-axis. This prevents the transmitter and receiver from being near each other if interrogated at long distances.

Another vibrometry technique is laser Doppler vibrometry\textsuperscript{13} which relies on the detection of a Doppler shift in the frequency of coherent light scattered by a moving target. From this, a time-resolved measurement of the target velocity can be obtained. This
technique does allow for the incoming and outgoing beams to be aligned along the same axis, but it suffers from many drawbacks. One drawback is that the Doppler shift gives no information on the static position of the structure. Shown in Figure 2.2, the Doppler vibrometry information is very dependent on the position of the reference optics and requires that they be damped and stationary at all times. Any vibration or misalignment of the reference optics will introduce unwanted noise into the system.

Figure 2.2 - Laser Doppler vibrometry schematic.

2.3 Optical Microcavity Interferometry
The technique used for this research is an interferometric technique known as optical microcavity interferometry. Optical microcavity interferometry is a technique which uses the optical interference between two surfaces as the means of detecting motion. Microcavity interferometry allows for high-bandwidth, high-resolution measurements of the nanoscale deflections of the device under test. The interference not only allows for the incoming and reflected beam to be aligned atop each other, but requires it. The splitting of the incoming and reflected beams can be done near the source, allowing for remote interrogation without
the need for complex optics. Also any small vibrations of the reference optics have no effect on the measured properties of the device because all interference depends on the separation between the two surfaces of the cavity. The microcavity in effect becomes a Fabry-Perot interferometer which consists of a pair of parallel, partially-reflective surfaces. When light is incident upon a Fabry-Perot interferometer, the light between the two surfaces interferes and produces a very characteristic reflection or transmission spectrum. The reflected intensity for a Fabry-Perot cavity is given by the following equation:

\[
R_{\text{net}} = \left[ \frac{E_r}{E_i} \right]^2 = \frac{(\sqrt{R_1} - \sqrt{R_2})^2 + 4\sqrt{R_1}R_2 \sin^2 \left(\frac{2\pi n d}{\lambda} \right)}{(1 - \sqrt{R_1}R_2)^2 + 4\sqrt{R_1}R_2 \sin^2 \left(\frac{2\pi n d}{\lambda} \right)}
\] (2.1)

where \( E_r \) and \( E_i \) are the reflected and incident electric fields, \( R_1 \) and \( R_2 \) are the reflectances of each plate, \( n \) is the index of the material between the plates, \( d \) is the separation between the plates, and \( \lambda \) is the wavelength of the incident beam. There are many interesting details about this equation which can and must be analyzed for use in the interferometer. These include the effect each variable in the above equation has on the resulting spectrum. The effect of different reflectance values are addressed in the analysis including whether or not the reflectance values of each surface be the same and if so, how high those reflectances should be. The optimal plate separation is addressed based on the tunability of the laser source and finally, the effect a changing or oscillating plate separation has on the spectrum is also addressed.

2.4 Effects of Variables on Fabry-Perot Spectrum

The following section uses several plots to show the effect each variable within the Fabry-Perot equation has on the reflection spectrum, including the reflectances \( R_1 \) and \( R_2 \) and the plate separation \( d \). The tuning range of the laser used is 100nm at 1525nm to
1625nm. The appropriate plate separation based on the wavelength separation between each resonance of the curve is given by the following equation

\[ d = \frac{\lambda^2}{2n\Delta\lambda} \]  

(2.2)

where \( \lambda \) is the center wavelength between the peaks, \( n \) is the index of the cavity material, and \( \Delta\lambda \) is the wavelength separation between each peak. For the 100nm tuning range of the laser centered at 1575nm, the minimum plate separation needs to be 12.4\( \mu m \) in order to see at least one resonance over that tuning range. For simplicity, each of the following analyses assume a cavity separation of 15\( \mu m \). Another equation relating the plate separation to the spectral location of each resonant minima is given by the following:

\[ \lambda_j = \frac{2nd}{j+1} \]  

(2.3)

for \( j=0, 1, 2, \ldots \). Each of these resonant minima occurs when the argument of the sin function within the net reflectance equation equals any multiple of \( \pi \). The plot in Figure 2.3 shows the effect of having \( R_1 \) and \( R_2 \) differ from each other.
Figure 2.3 - Net reflectance of an ideal Fabry-Perot interferometer considering non-equal reflective surfaces. From top to bottom, the respective reflectances are \([R_1,R_2] = [0.1,0.9], [0.2,0.8], [0.3,0.7], [0.4,0.6], \) and \([0.5,0.5]\). The plate separation \(d=15\mu\text{m}\) and index \(n=1\).

When the reflectance of each plate is matched, the reflection minimum is zero. Therefore, matching the reflectance values of each surface is necessary to achieve the difference between the maximum and minimum values. The plot in Figure 2.4 demonstrates the effect of changing the reflectance values if they both are identical \((R_1=R_2)\).
Figure 2.4 - Net reflectance of an ideal Fabry-Perot interferometer with identical reflective surfaces ($R_1 = R_2$). From top to bottom, the respective reflectances are $R = 0.9, 0.8, 0.6, 0.4,$ and $0.2$. The plate separation $d = 15 \mu m$ and index $n = 1$.

As the reflectance values of each face increase, the reflection maximum gets closer to one. For very high reflectance values, the slope of the curve at each resonance increases dramatically. Having a sharp slope is very important in terms of sensor sensitivity as will be discussed later. The term characterizing the sharpness of each optical resonance is known as the finesse. Assuming the reflectances of each surface are equal, the finesse of a Fabry-Perot interferometer is given by the following equation:

$$F = \frac{\pi}{2} \sqrt{\frac{4R}{(1 - R)^2}}$$

(2.4)

where $R$ is the reflectance of each face of the etalon. There are many factors which can have a detrimental affect on the effective finesse of a cavity. These include spherical bowing, surface roughness, and departure from parallelism/tilt$^{17,18,19}$. The overall defect finesse $N_D$ is given by the relationship:
\[ N_D = \frac{1}{\sqrt{\left( \frac{2\delta_s}{\lambda} \right)^2 + \left( \frac{4.7\delta_G}{\lambda} \right)^2 + \left( \frac{\sqrt{3}\delta_p}{\lambda} \right)^2}} \]  \tag{2.5}

where \( \delta_s, \delta_G, \) and \( \delta_p \) are measures of the spherical bowing, surface roughness (RMS deviation) and tilt respectively. The resulting overall cavity finesse \( N_E \) is given by

\[ N_E = \frac{1}{\sqrt{\frac{1}{N_D^2} + \frac{1}{F^2}}} \]  \tag{2.6}

where \( F \) is the finesse derived from Equation 2.4. In the case of this research, the main defect to worry about was the tilt. To keep the tilt defect finesse factor from dominating the overall effective finesse of the cavity, the reflectance values were kept low enough so that the reflectance finesse \( F \) dictated the finesse. The tilt was often very apparent because of the effect a small change in cavity separation had on the Fabry-Perot spectrum. The spectral location of each resonance changes with the cavity separation. The plot in Figure 2.5 shows the effect of changing the cavity separation \( d \) while the other variables remain constant.
As the cavity separation increases, the spectrum makes a shift to the right. If the mode index $j$ is known from Equation 2.3, the following equation gives an exact value for the spectral shift due to a change in cavity separation $\Delta d$:

$$\Delta \lambda_j = \frac{2n}{j+1} \Delta d$$

(2.7)

The most important things learned from the above equations and plot analyses are that the reflectances of each face need to be matched and also fairly high. The reflectance cannot be made too high because a small beam deflection or misalignment would lead to the resonant minimum being moved easily off of the set interrogation laser wavelength. Though the above net reflectance equation for a Fabry-Perot interferometer is based on ideal boundary conditions and only two partially reflective surfaces, practical applications do not involve such ideal situations.
2.5 Transfer Matrix Method

Practical problems usually involve waves propagating in bounded regions in which different stratified media may be present, and require that account be taken of the complicating effects at boundary surfaces. Typical boundary surfaces lie between regions of different permittivities, conductivities, or permeabilities. When an electromagnetic wave comes into contact with a boundary, part of its energy is reflected back into the incident medium and the rest of the energy is transmitted through to the next medium. Using Maxwell’s equations with the appropriate boundary conditions, mathematical expressions for these reflected and transmitted waves can be found. For both simplicity and practicality of the application, only waves at normal incidence on planar interfaces will be considered in this analysis.

The following analyses use the matrix method to describe wave propagation in stratified media. The stratified media in this analysis are layers in which the properties are constant throughout each plane perpendicular to the optical axis of propagation. Some applications of stratified media include antireflection coatings, dielectric mirrors, chromatic filters, and beam splitters. An analysis by Born and Wolf\textsuperscript{20} uses Maxwell’s equations to derive a 2x2 matrix method for determining the net reflectance and transmission of a stack of stratified media. A 4x4 matrix method by Berreman\textsuperscript{21} would also work but is unnecessary due to the polarization independence of the stratified layers. The following describes the 2x2 transfer matrix method and in turn a wavelength-dependent reflectance of the stack of media used in this experiment is determined.

Derived from Maxwell’s equations, the characteristic matrix of a stratified medium is a convenient method of expressing solutions to those equations. Born and Wolf find a solution for both the transverse electric (TE) and transverse magnetic (TM) waves. In this
analysis, only the transverse electric waves are considered. The following equations are for a homogenous dielectric film. The refractive index $n$ is given by the following equation:

$$n = \sqrt{\varepsilon_r \mu_r}$$  \hspace{1cm} (2.8)

where $\varepsilon_r$ is the relative electric permittivity and $\mu_r$ is the relative magnetic permeability.

Another value used in the analysis is $p$ given by the following equation:

$$p = \sqrt{\frac{\varepsilon_r}{\mu_r}} \cos(\theta)$$  \hspace{1cm} (2.9)

where $\theta$ is the angle of incidence for the wave with respect to the normal of the stack. For a film at normal incidence ($\theta = 0$) and relative magnetic permeability 1, the variable $p$ and the refractive index $n$ are equal. There are no magnetic materials used in this experiment so for all films in the stack, $\mu_r = 1$. Also, the wave number for a dielectric film is given by the equation

$$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda_0}$$  \hspace{1cm} (2.10)

The 2x2 characteristic matrix for a TE beam in a layer of thickness $d$ is given by the following equation.

$$M_{TE} = \begin{bmatrix}
\cos(k_0nd\cos(\theta)) & -\frac{i}{p}\sin(k_0nd\cos(\theta)) \\
-ip\sin(k_0nd\cos(\theta)) & \cos(k_0nd\cos(\theta))
\end{bmatrix}$$  \hspace{1cm} (2.11)

The characteristic matrix for the $j^{th}$ film in which the electric permittivity $\varepsilon$ and the magnetic permeability $\mu$ are assumed constant in the layer, the characteristic matrix is given by

$$M_j = \begin{bmatrix}
\cos(k_0n_jd_j\cos(\theta_j)) & -\frac{i}{p_j}\sin(k_0n_jd_j\cos(\theta_j)) \\
-ip_j\sin(k_0n_jd_j\cos(\theta_j)) & \cos(k_0n_jd_j\cos(\theta_j))
\end{bmatrix}$$  \hspace{1cm} (2.12)
The overall characteristic matrix for a number of stratified media packed together is found by multiplying the characteristic matrix of each layer together to yield the overall characteristic matrix, given by the following relationship

\[ M = \prod_{j=1}^{n} M_j \]  

(2.13)

The reflection and transmission coefficients are given by the following equations

\[ r = \frac{(m_{11} + m_{12} p_f)p_0 - (m_{21} + m_{22} p_f)}{(m_{11} + m_{12} p_f)p_0 + (m_{21} + m_{22} p_f)} \]  

(2.14)

\[ t = \frac{2p_0}{(m_{11} + m_{12} p_f)p_0 + (m_{21} + m_{22} p_f)} \]  

(2.15)

where \( m_{ij} \) is the \( ij \) element of the characteristic matrix, \( p_0 \) is the \( p \)-value for the incident medium (before the stack) and \( p_f \) is the \( p \)-value for the terminating medium (after the stack). In terms of \( r \) and \( t \), the net reflectance and transmittance for the stack is given by the following equations

\[ R = |r|^2 \]  

(2.16)

\[ T = \frac{p_f}{p_0} |t|^2 \]  

(2.17)

2.6 Non-Dielectric Media

This matrix analysis remains valid for non-dielectric media within the stratification. This matrix method can also be used with thin metallic films by considering the metal film to have a complex dielectric constant. This can simply be done by replacing the refractive index \( n \) of the dielectric with the complex refractive index for the metal.
2.7 Aluminum Optical Properties

For this experiment, aluminum was chosen as the partial reflector for a number of reasons. The metal of choice had to possess a number of qualities including a high imaginary index of refraction, low density, and ease of use. Aluminum had all of these qualities. The importance of the imaginary index is that a very thin film will produce a large reflectance. Aluminum is also very easy to apply using an electron beam evaporator. Also because aluminum is non-magnetic, the p-value for that layer can be replaced with the refractive index also because the $\mu_r$ for aluminum is very close to one. One negative to aluminum is its ease of oxidation. The electron beam deposition system used, even though pumped down to near $10^{-5}$ torr, still has enough residual oxygen and water vapor to oxidize the first several nanometers of the aluminum film. The aluminum oxide layer, a dielectric, thus reduces the reflectance of the film so the aluminum thickness before and after oxidation must be considered. The thickness measured by the crystal monitor, the physical thickness and the metal thickness are all different because of the oxidation process within the chamber. The thickness of the oxide layer can approach about 10nm if given enough time. This thickness is taken into account and the thickness of the reflective aluminum is reduced. The wavelength-dependent refractive index of aluminum was found using data from Ordal et. al\textsuperscript{22}. The plot in Figure 2.6 gives the real and imaginary indices for aluminum as a function of wavelength for wavelengths near the tuning range of the laser used.
A linear approximation for the indices in the above plot yields the following relationship for the index:

\[
n_{Al} = (0.00131012\lambda - 0.48239341) - i(0.01068789\lambda - 0.68104984)\).
\]  

(2.18)

where \(\lambda\) is the wavelength in nm. The following table gives the real and imaginary indices of aluminum at different wavelengths in the tuning range.

Table 2.1 – Real and imaginary refractive indices of aluminum for wavelengths in the tuning range of the laser.

<table>
<thead>
<tr>
<th>wavelength (nm)</th>
<th>(n)</th>
<th>(k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1525</td>
<td>1.52</td>
<td>-15.62</td>
</tr>
<tr>
<td>1550</td>
<td>1.55</td>
<td>-15.89</td>
</tr>
<tr>
<td>1575</td>
<td>1.58</td>
<td>-16.15</td>
</tr>
<tr>
<td>1600</td>
<td>1.61</td>
<td>-16.42</td>
</tr>
<tr>
<td>1625</td>
<td>1.65</td>
<td>-16.69</td>
</tr>
</tbody>
</table>
At $\lambda=1575\text{nm}$, the center wavelength of the tuning range for the laser, the refractive index for aluminum is approximated as $n_{\text{Al}} = 1.58-\text{j}16.15$. To keep computation complexity to a minimum by removing the wavelength dependence, this index at 1575nm was used as the index value over the total tuning range.

### 2.8 Transfer Matrix Theory Applied to Stacks in this Work

The drawing in Figure 2.7 shows the stacks of materials used for this experiment. Included in the stack are the glass substrate, the first aluminum film, an air-filled microcavity, the second metal film, and a silicon nitride film.

![Figure 2.7 - Each layer in the stack for the analysis. Included in the stack is a 1588$\mu$m plate glass layer, two thin films of aluminum, an air-filled microcavity, and a layer of silicon nitride. The stack begins and terminates into air.](image-url)
Table 2.2 - Table showing thickness and refractive index of each layer.

<table>
<thead>
<tr>
<th>Layer Identity</th>
<th>Thickness</th>
<th>Refractive Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Borosilicate Plate Glass</td>
<td>1588μm</td>
<td>1.472</td>
</tr>
<tr>
<td>2. Aluminum Film</td>
<td>15nm</td>
<td>1.58-j16.15</td>
</tr>
<tr>
<td>3. Aluminum Oxide</td>
<td>10nm</td>
<td>1.746</td>
</tr>
<tr>
<td>4. Optical Microcavity</td>
<td>15μm</td>
<td>1</td>
</tr>
<tr>
<td>5. Aluminum Oxide</td>
<td>10nm</td>
<td>1.746</td>
</tr>
<tr>
<td>6. Aluminum Film</td>
<td>15nm</td>
<td>1.58-j16.15</td>
</tr>
<tr>
<td>7. Silicon Nitride</td>
<td>200nm</td>
<td>2.0</td>
</tr>
</tbody>
</table>

The characteristic matrix for the stack in Figure 2.7 is given by the following equation

$$M = M_{glass} \cdot M_{Al} \cdot M_{AlO} \cdot M_{microcavity} \cdot M_{AlO} \cdot M_{Al} \cdot M_{SiN}$$  \hspace{1cm} (2.19)

Equation 2.14 was applied to find the reflection coefficient for the stack, then Equation 2.15 were used to determine the net reflectance of the stack. The plot in Figure 2.8 shows the net reflectance of the stack using the characteristic matrix analysis.

![Net Reflectance of Stack of Materials](image)

Figure 2.8 - Plot of net reflectance (using transfer matrix method) of the stack of various materials as a function of the incident wavelength.

From the plot, two sources of interference can be seen. The large dip in the plot is caused by the interference of the microcavity. This is the desired interference minimum to be
used in the experiment. The shorter period interference pattern that modulates on top of the longer period interference pattern comes from the interference of the thick plate glass substrate. One way to minimize this unwanted interference is to apply an anti-reflection coating to the bottom of the plate glass. This will reduce the magnitude of the unwanted interference and will yield only the desired reflection interference minima caused by the microcavity. An ideal anti-reflection coating would have index

\[ n = \sqrt{n_{\text{air}}n_{\text{glass}}} = \sqrt{1.472} = 1.213 \]

and thickness \( d = \frac{\lambda}{4n} = \frac{1575}{4(1.213)} = 324.6 \text{nm} \). The reflectance of the same stack as in Figure 2.8 but with an AR coating of index 1.213 and thickness 324nm is shown in the Figure 2.9.

![Net Reflectance of Stack of Materials](image)

**Figure 2.9** - Plot of the net reflectance (using transfer matrix method) of the stack of materials with an ideal AR coating on the bottom of the plate glass to prevent optical interference of the glass layer.

The curve is much smoother because interference from the glass substrate is minimized. The AR film with index closest to the desired 1.213 is MgF₂ with a refractive index of 1.346 at \( \lambda = 1575 \text{nm} \). MgF₂ is a typical AR coating for glass substrates and lenses.
because of its low refractive index. A plot of the net reflectance of the same stack with a 293nm thick AR coating of MgF₂ is shown in Figure 2.10.

![Plot of the net reflectance (using transfer matrix method) of the stack of materials with a 293nm thick AR coating of MgF₂ (n=1.346).](image)

Figure 2.10 - Plot of the net reflectance (using transfer matrix method) of the stack of materials with a 293nm thick AR coating of MgF₂ (n=1.346).

Though the AR coating is not needed, it is preferred in order to get the maximum amount of light to the cavity. Any bridge motion can still be seen but the signal may not be as clean and strong as it would if there was an AR coating. Most of these fast oscillations within the reflectance scans will not be seen due to the size of the laser spot. Even for very high quality optical flats, most of this interference will get cancelled out over the round trip within the stack.

When the interrogation laser is tuned to the wavelength of sharpest slope, small movements of the beam can be detected optically. As the microcavity separation changes, the intensity of the reflected light will change correspondingly. Any resonant motion of the bridge can be detected using a photodetector connected to a network analyzer.
2.9 References


3 MICROMECHANICAL THEORY

3.1 Introduction

The emerging field of micro-electromechanical systems (MEMS) has rapidly evolved in recent years. One of the benefits came from years of integrated circuits processing technologies such as photolithography, thin-film deposition, and chemical/plasma etching\textsuperscript{1,2}. For the most part, the characterization of MEMS consists of the mechanical properties of these devices. Following these lines, one of the primary goals of this work is to understand the mechanics of the micromachined double-clamped beam, the desired device type for the cavities. This section contains the theory of the motion of the double-clamped beam including the effect air-damping has on its motion.

It is important to note that the measurements for the resonant frequencies derived in this analysis will not be taken as the theoretical values for the devices in this work. Much of the mechanical theory is complicated by the aluminum coating on the bridges. This analysis will simply be used as a means of getting a first-order idea of the frequencies to be expected. Also, by using the estimated frequencies, this analysis can be used as a means of determining the dimensions and scale of the devices to be created.

3.2 Double-Clamped Beam

A double-clamped beam, displayed in Figure 3.1, is a microscale device which is held on its ends to create a bridge.
3.3 Euler-Bernoulli Theory

The Euler-Bernoulli theory, a simplified analysis from the theory of elasticity, can describe the dynamics of a double-clamped beam provided that the beam obeys the following properties: a) the length is significantly greater than both the width and thickness, b) the cross-sectional area of the beam is constant along the total length, c) the beam is loaded in its plane of symmetry, without any torsion, d) any deflections of the beam are small compared to the length, e) the material used for the beam is isotropic, and f) the plane sections of the beam remain plane. The motion along the z-direction $Y(x,t)$ as a function of both longitudinal displacement and time is described by the differential equation

$$IE \frac{\partial^4 Y(x,t)}{\partial x^4} + \rho tw \frac{\partial^2 Y(x,t)}{\partial t^2} = q(x,t)$$

(3.1)
where $\rho$ is the density of the beam material, $t$ is the thickness of the beam, $w$ is the width of the beam, $E$ is Young’s Modulus of Elasticity, $I = wt^3/12$ is the bending moment of inertia for a double-clamped beam, and $q(x,t)$ is the external force per unit length on the beam. The external forces include any driving forces plus any damping terms due to air resistance. The boundary conditions at $x=0$ and $x=L$ are given by the equations:

$$Y(0) = Y(L) = 0 \quad (3.2)$$

$$\dot{Y}(0) = \dot{Y}(L) = 0 \quad (3.3)$$

where the overdot denotes a time derivative. The solution of the differential equation, which assumes that there are no external forces, thus $q(x,t) = 0$, is given by the following equation:

$$Y_n(x,t) = [C_{1n} \cos k_n x - \cosh k_n x] \text{e}^{-\Omega_n t} \quad (3.4)$$

The eigenvectors $k_n$ are satisfied by the equation $\cos k_n L \cosh k_n L = 1$ after applying the boundary conditions. The first four eigenvectors are given by the following relationships:

$k_n L = 4.73004, 7.8532, 10.9956, \text{ and } 14.1372$. The solved values of $k_n$ can be substituted into the following equation to yield the angular eigenfrequencies:

$$\Omega_n = \sqrt{\frac{EI}{\rho A}} k_n^2 \quad (3.5)$$

The eigenfrequencies are given by

$$\nu_n = \frac{\Omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{EI}{\rho A}} k_n^2 \quad (3.6)$$

Solving for the fundamental frequency yields

$$\nu_1 = \frac{\Omega_1}{2\pi} = 1.027 \sqrt{\frac{E}{\rho L^2}} t \quad (3.7)$$
Table 3.1 – Various material properties used in the resonant frequency analysis

<table>
<thead>
<tr>
<th>Silicon Nitride Material Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Density</td>
<td>$\rho$</td>
<td>3290 kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>Young’s Modulus of Elasticity</td>
<td>$E$</td>
<td>3.00x10$^{11}$ N/m$^2$</td>
<td></td>
</tr>
<tr>
<td>Beam Thickness</td>
<td>$t$</td>
<td>200 nm</td>
<td></td>
</tr>
</tbody>
</table>

Substituting the silicon nitride material properties into the equation, the fundamental frequency becomes:

$$\nu_t = 1.961 \times 10^{-3} L^{-2}$$  \hspace{1cm} (3.8)

A plot of equation 3.8 for a silicon nitride beam of various lengths is shown in Figure 3.2.

![Resonant Frequency vs Length for Silicon Nitride Double Clamped Beams](image)

Figure 3.2 – Double log scale plot of the theoretical fundamental resonant frequency of a double-clamped silicon nitride beam of length 10µm to 1000µm.
The frequencies of the higher order modes are given by \( \nu_2 = 2.756\nu_1 \), \( \nu_3 = 5.404\nu_1 \), \( \nu_4 = 8.933\nu_1 \). Each of these frequencies plus the eigenvectors and constants \( C_{1n} \) and \( C_{2n} \) are given in the following table for each of the first four eigenvalues.

<table>
<thead>
<tr>
<th>Table 3.2 - Various constants associated with the first four transverse modes of a double-clamped beam.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_n L )</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>( n = 1 )</td>
</tr>
<tr>
<td>( n = 2 )</td>
</tr>
<tr>
<td>( n = 3 )</td>
</tr>
<tr>
<td>( n = 4 )</td>
</tr>
</tbody>
</table>

### 3.4 Mechanical Quality Factor

A figure of merit describing the motion and more importantly the mechanical energy dissipation mechanism of the microbeam is the mechanical quality factor. The mechanical quality factor \( Q \) is given by the following equation:

\[
Q = \frac{f_0}{BW_{-3dB}}
\]  

(3.9)

where \( f_0 \) is the resonant frequency and \( BW \) is the -3dB bandwidth of the resonance. The sensitivity of a mechanical oscillator depends on the ratio of the smallest resolvable resonant frequency shift divided by the resonant frequency. The mechanical Q-factor is important because it is a way of quantifying the sensitivity; the higher the Q-factor, the lower the resolvable frequency shift, thus the higher the sensitivity. Because of the dissipation of energy in a micromechanical resonator, the mechanical Q-factor has a finite magnitude. Complexities of the motion of the microbeam are due to the length scale dependence of the resonant frequency and the mechanical quality factor. As the lengths of the beams decrease the resonant frequencies at which they operate increase. The increase in frequency leads to
an increase in the air-damping effects, thus decreasing the mechanical quality factor. This particular double-clamped beam design has two main sources of energy loss: e.g., first is the intrinsic loss in the beam material which is given by Zener’s model for anelastic solids and the second and more dominant source of loss is the air damping force exerted on the resonator. These damping terms must be included in the Euler-Bernoulli theory above, thus changing the effective resonant frequencies and causing the Q-factor to decrease in value. In this analysis, only the more dominant air damping force is considered.

3.5 Air Damping Effects

The magnitude of the air damping depends on various physical parameters of the beam. The type of treatment to be used depends on the separation between the beam and the underlying substrate. If the separation $h$, shown in Figure 3.3, is smaller than the width of the beam, then the type of damping is referred to as squeeze-film damping$^{4,5,6}$. For any cavity separations larger than $w$, an infinite gas medium treatment is more appropriate, where the drag force shifts from squeeze film damping being the dominant force to the simple air-damping above and below the beam. The infinite gas limit is based on the Oseen solution for the drag force on a long cylinder moving in an incompressible fluid$^7$. Here, only the squeeze-film damping approach is examined since the cavity separation $h$ is smaller than the width of the beams used.
The squeeze film damping effect on a beam resonator, caused by the thin gas layer between the structure and the supporting substrate, is analyzed on the basis of the coupled elastic beam theory and the Reynolds equation for isothermal incompressible gas films. Zhang et al.\textsuperscript{4,7} described a set of equations using coupled solid deformation and fluid flow equations which depend only on various beam and fluid properties to solve for the mechanical Q factor and the effective shift of the resonant frequency. The analysis proceeds with the pressure distribution of a squeezed-film determined by the Reynolds equation for an incompressible fluid:

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 12\mu \frac{\partial (ph)}{\partial t}$$

(3.10)

where $p(x, y, z)$ is the position dependent pressure, $\mu$ is the gas dynamic viscosity, and $h$ is the separation between the beam and the substrate. The boundary conditions are:

$$p(x, w/2, t) = p(x, -w/2, t) = p_a$$

(3.11)

$$\frac{\partial p(0, y, t)}{\partial x} = \frac{\partial p(L, y, t)}{\partial x} = 0$$

(3.12)
where \( p_a \) is the ambient pressure. These boundary conditions assume that the pressure on the edges of the beam is equal to the ambient pressure and the pressure at the ends of the beam remains constant. The fluid properties of air are given in the following table:

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas Dynamic Viscosity of Air</td>
<td>1.81x10(^{-5})</td>
<td>kg·m/s</td>
</tr>
<tr>
<td>Ambient Air Pressure</td>
<td>1.013x10(^{5})</td>
<td>N/m(^{2})</td>
</tr>
</tbody>
</table>

Another important term is the squeeze number, \( \sigma \), given by the equation:

\[
\sigma = \frac{12 \mu \omega L^2}{p_a h_0^2}
\]

(3.13)

where \( \omega \) is the excitation frequency of the beam and all other constants are as previously defined. The squeeze number is a measure of the compressibility of the fluid in the gap. A plot of the squeeze number \( \sigma \) as a function of the squeeze film thickness \( h_0 \) for a beam of length \( L=100 \mu\text{m} \) and frequency 161 kHz in air is given in Figure 3.4.
Figure 3.4 - Double log scale plot of the squeeze number as a function of the squeeze film thickness for a beam in air of length 100 μm oscillating at frequency 161kHz.

Combining the Reynolds and Euler-Bernoulli equations modified with both damping and driving terms, yields a set of complex nonlinear equations. When it is assumed that the beam is undergoing small oscillations \( u \ll h_0 \), the set of nonlinear equations can be linearized into a much simpler set of equations. The important terms solved for are the squeeze inertia number \( m_a \) and the squeeze damping number \( c_a \). Both of these values are without units and are used to factor the squeeze film damping into the mechanical quality factor and the resonant frequency. The linearly approximated values of these terms are given by the following equations:

\[
m_a = 1.1977 \frac{\mu^2 w^4}{p_a \rho h_0^3}
\]
\[ c_a = 0.1525 \frac{\mu l^3 w^2}{\sqrt{\rho E h_0^3 t^2}} \]  

(3.15)

In these equations, both the widths and thicknesses of the beams are introduced into the damping analysis. Before these equations, the thickness was used only to solve for the resonant eigenfrequency and the width had yet to be used. The plots in Figure 3.5 show the accuracy of the approximations made for the squeeze inertia number and the squeeze damping number, especially for low squeeze numbers. For squeeze numbers less than 500, the approximations are very close to the actual values derived from the set of nonlinear equations. For the squeeze film thicknesses close to 15\(\mu\)m, which produces squeeze numbers in the order of 0.1, the linear approximation is nearly identical to the actual values.

![Figure 3.5 - Squeeze inertia number and squeeze damping number as functions of the squeeze number. The dotted line represents the simple approximation made for small squeeze numbers and solid line represents actual solution. (Images taken from Zhang\(^4\),\(^7\) with permission)](image)

The new quality factor and resonant frequency that take squeeze-film damping into account by including the squeeze inertia number and the squeeze damping number are given by equations 16 and 17.
\[ Q = \frac{\sqrt{1 - m_a}}{c_a} \]  
\[ \omega_r = \frac{\sqrt{1 - ma - 0.5c_a^2}}{1 - ma} \omega_0 \]  

Plots of the quality factor and relative frequency as a function of the squeeze film thickness for a silicon nitride beam of thickness \( t = 200 \text{nm} \), width \( w = 40 \mu \text{m} \), and length \( l = 100 \mu \text{m} \) are shown in Figure 3.6 and Figure 3.7.

![Mechanical Q vs. Squeeze Film Thickness](image)

**Figure 3.6** - Plot of the Q factor as a function of the squeeze film thickness for a silicon nitride beam of thickness 200 nm, width 40 μm, and length 100 μm in air.
The analysis of a silicon nitride beam of the above dimensions with a squeeze film thickness of 15μm yields a Q-factor of 96 and a relative resonant frequency $\omega/\omega_0$ of 0.9999828139. At squeeze film thicknesses of 15μm and higher, the cavity separations used in this work, the resonant frequency remains virtually unchanged after factoring the effects of the squeeze-film. The Q-factors continue to scale up exponentially as the cavity separation is increased until the infinite gas limit is reached.

The only other way to further increase the Q-factor without changing the properties of the device would be to alter the properties of the fluid in which the beam is immersed. If the pressure of the air around the beam were decreased, the motion would eventually be removed from the squeeze film limits and the damping effect would disappear.
3.6 Noise in Double-Clamped Beam Micromechanical Systems

The fluctuation-dissipation theorem states that any system that dissipates energy is a source of noise\(^3\). This theorem for micromechanical bodies is equivalent to the Nyquist-Johnson theorem for electrical circuits. The Nyquist-Johnson theorem states that there will be fluctuation of the voltage (or equivalently the current) across (or through) an electrical impedance. A plot of the voltage across an impedance would have a peak value at the ideal \(V=I*Z\) value but would vary slightly by having noise sidebands about that point. The fluctuation-dissipation theorem equivalently applies to micromechanical resonators with finite quality factors (Q).

The main type of noise which affects the quality factor is the dissipation-induced amplitude noise\(^3\) \((S_{na}(\omega))\). With no noise, the oscillator would undergo harmonic motion at the carrier frequency \(\omega_c\). With the noninfinite Q and nonzero temperature \(T\), thermal-mechanical noise will exist in a system. The main source of energy dissipation for the double-clamped beam is also the main source of the thermal-mechanical noise\(^8\). Since the main source of energy dissipation is the squeeze-film damping, the thermal-mechanical noise is due to the interaction of the air molecules in the cavity with the bridge. With no outside driving forces present, the mean energy of each mode \(n\) of the resonator is given by

\[
\langle E_n \rangle = k_b T \tag{3.18}
\]

where \(k_b\) is Boltzmann’s constant and \(T\) is the temperature of the resonator\(^3\). Although each mode has the same amount of energy, the amplitude at which each mode oscillates will not be constant. The spectral density of the thermally driven amplitude of each mode \(n\) is given by the following equation\(^3\):
where $M$ is the bridge mass, $L$ is the bridge length and $Q$ is the mechanical quality factor.

On resonance ($\omega = \Omega_n$), the dissipation-induced amplitude noise reaches a maximum and simplifies to the following relationship:

$$S_{a_n}(\omega) = \frac{\Omega_n}{(\Omega_n^2 - \omega^2)^{\frac{3}{2}} + (\Omega_n^2 / Q)^{\frac{3}{2}}} \frac{2k_BT}{\pi ML^2 Q}$$

(3.19)

Assuming constant variables for each mode except for the frequency $\Omega_n$, this equation indicates an inverse relationship between the amplitude noise and the resonant frequency. Higher order modes have higher resonant frequencies and thus have a lower distribution of the noise when compared to the fundamental resonant frequency. The following plot of the log of the dissipation-induced amplitude noise $S_{a_n}(\omega)$ gives the general form of the amplitude noise peak at the fundamental resonant frequency and the noise sidebands. The noise sidebands narrow and the amplitude at the resonant frequency increases as the quality factor increases.
Figure 3.8 – Frequency spectrum of dissipation-induced amplitude noise showing center peak at the fundamental resonant frequency and the noise sidebands.

When the amplitude noise of the higher order modes are added to the fundamental mode noise, the result is the overall noise distribution. The higher order resonances will have peaks at the resonant frequencies but lower in amplitude due to the distribution between the various modes. In theory, given no driving source, the thermally-induced motion from the noise allows the resonant frequencies to be detected, but the low deflection amplitudes make this very difficult.
3.7 References


4 Fabrication of the Silicon Nitride Microcavities

4.1 Advantages and Characteristics of Silicon Nitride

Silicon nitride ($\text{Si}_3\text{N}_4$) films have been used for several applications including pressure sensors\(^1\), micromesh bolometers\(^2\), interferometers\(^3,4\), in microscopy as TEM support grids\(^5\), and mechanical resonators and cantilevers\(^6\). The processing steps for silicon nitride were developed in the 1960’s while researching new materials to strengthen metals\(^7\). Silicon nitride has high temperature strength, good thermal shock resistance\(^8\), mechanical wear resistance, high fracture toughness, mechanical fatigue resistance\(^9\), and oxidation resistance. Above all, it is compatible to semiconductor processing.

Based on these many years of semiconductor development and research, silicon nitride layers are routinely produced using either Plasma Enhanced Chemical Vapor Deposition (PECVD) or Low Pressure Chemical Vapor Deposition (LPCVD) techniques. Using a vapor mixture of silane ($\text{SiH}_4$) and ammonia ($\text{NH}_3$), the chemical composition of the layers can be made either silicon or nitrogen rich. The PECVD process is used to easily control many characteristics of the layers including film thickness, stress, and composition. For the membranes used in this work, low-stress layers will prevent buckling once freed from the underlying silicon. The mechanical and thermal properties of silicon nitride also make it a good candidate for our use as a membrane. Using bulge tests, the Young’s modulus of LPCVD thin-film silicon nitride has been measured at $255\pm5\text{GPa}^{10}$. Also, the coefficient of thermal expansion is comparable to that of silicon. This allows the layers that are deposited on top of silicon to undergo changes in temperature without worry of breaking the membrane.
Silicon nitride in recent years has become a very useful tool in MEMS device fabrication. When wet chemical etching by KOH, silicon nitride can be used as a very effective etch stop layer because the etch rate of silicon nitride is many orders of magnitude slower than bulk silicon. Silicon nitride etches at a rate slower than one nanometer per hour where as bulk silicon etches at a rate of up to several hundred microns per hour. The etching of silicon in KOH is very anisotropic: the \{100\} planes and \{110\} planes are selectively etched, while the etch rate in the \{111\} direction is much lower. As a result, when etching a square membrane in a heated KOH bath, sidewalls are formed at an angle of 54.74° with respect to the surface, as shown in Figure 4.1.

Figure 4.1 - Image of a piece of silicon which has been anisotropically etched to the backside silicon nitride layer.
4.2 Optical Characteristics of Silicon Nitride

The following plots show the optical transmission characteristics of silicon nitride as a function of wavelength which were obtained using a Perkin-Elmer spectrophotometer.

Figure 4.2 shows the transmittance for the wavelength range $\lambda=200\text{nm}-2\mu\text{m}$. Figure 4.3 shows the transmittance for the wavelength range $\lambda=2\mu\text{m}-15\mu\text{m}$. Although, the end application of the silicon nitride was used in a reflective application, many of our early Fabry-Perot tests were performed in transmission mode.

![200nm Silicon Nitride Transmittance](image)

**Figure 4.2 - Visible and Near Infrared Transmittance of a 200nm thick layer of Silicon Nitride obtained using a Perkin-Elmer spectrophotometer.**

The first two peaks at 450nm and 800nm are functions of the interference of the 200nm thick silicon nitride layer. Below 400nm, the transmission falls off due to the UV absorption of silicon nitride.
At the desired testing wavelengths in the C-band, the 200nm thin film has a transmittance of around 0.55 which means that this thin film can be used in either a reflection or transmission application.

### 4.3 Fabrication of Silicon Nitride Membranes

Fabricating quality silicon nitride membranes is a time-intensive, multi-step process which starts with a high quality double-side polished silicon wafer. The silicon wafers used were 300μm thick 4 inch wafers. These wafers were then shipped out of house and the silicon nitride layer was deposited by the CVD company Strataglass, Inc.. A LPCVD process was used by them to deposit 200nm of low-stress silicon-rich silicon nitride. The remaining steps in the fabrication of the membranes were performed at the NCSU clean room facility. First the wafers were cleaved into quarters so that the pieces would fit into the various instruments.
needed for fabrication. Also, the areas needed were small enough that whole wafers were not needed.

The fabrication steps to create the silicon nitride membranes from the coated silicon wafer is described below. The series of images in Figure 4.4 demonstrate the process of creating a silicon nitride membrane from the silicon nitride coated silicon wafer. A diagram of a silicon wafer coated is shown in Figure 4.4a. As a first step, a thin layer of photoresist was spun on the quarter wafer pieces, as shown in Figure 4.4b. The photoresist used was AZ-5214E and the thickness of the layer was 1.4um. This type of resist was used because of its ease of use and its ability to be used as either a positive or negative photoresist. In the next step, a specially designed mask was used to create an array of membranes, with enough margin left in the mask design because of the etch angles of the silicon. Using the desired membrane width, the silicon wafer thickness, and the angle of the anisotropic etch, the dimensions of the membrane patterns on the mask were slightly larger than the desired membrane dimensions. The dimensions of the mask feature size were based on the angle of the etch (54.74°) and the thickness of the wafer (300μm). Once exposed in a Karl Suss MJB3 photoaligner, the wafer was developed in MF-319 photoresist developer. After developing, cleaning, and drying the wafer, the selected areas were void of the photoresist, shown in Figure 4.4c. The next step was a Reactive Ion Etch (RIE) of the wafer to remove the now exposed silicon nitride. This was done using a trifluoromethane (CHF₃) and oxygen (O₂) plasma mixture. As shown in Figure 4.4d, the photoresist acted as a mask to prevent the mixture from reaching unwanted parts of the wafer. After the RIE process was complete, the remaining photoresist was stripped using acetone and methanol. With the bulk silicon now exposed in the areas to be etched, the next step was the etching through the silicon layer with
a heated KOH solution. The average etch rate scales up exponentially with temperature. The temperature and KOH concentrations were kept under control so that the etch rate was just under 100μm/hr, or about 4 hours for the total etch time. The silicon nitride layer on the opposite side acts as an etchant stop and the reaction finishes once the KOH reaches that layer. Once the etching process was finished, the membrane was free from the bulk silicon layer and was ready for use, as shown in Figure 4.4e.
Figure 4.4 – a) The membrane creation process starts with a 300μm thick double side polished silicon wafer coated on both sides with a 200nm thick layer of silicon nitride. b) A thin layer of photoresist is spun on one side of the wafer. c) Using a photomask, the area to be etched is exposed using a photoaligner and the exposed photoresist is developed away. d) A reactive ion etch removes the exposed silicon nitride layer. e) A heated KOH bath anisotropically etches the exposed silicon along the \{100\} and \{110\} planes until it reaches the opposite silicon nitride layer which acts as an etch stop.
Figure 4.5 shows a top-view of a 1500μm wide membrane created by an anisotropic KOH wet etch.

![Figure 4.5 - A top-view of a silicon nitride membrane. The yellow area in the center is the silicon nitride membrane, the black rim around it is the silicon sidewalls and the blue area on the outside is the silicon nitride layer on top.](image)

### 4.4 Fabrication of Silicon Nitride Bridges

For the fabrication of silicon nitride double-clamped beams, first the silicon nitride membranes must be created. AZ-5214E photoresist was spun on the membrane side of the wafer, and then patterned so that strips in the shape of the bridges wanted were void of any photoresist. Aluminum was then deposited onto the wafer to cover the areas that were void of the photoresist. To remove the unwanted aluminum, a liftoff was performed by soaking the wafer in acetone. The acetone dissolved the photoresist under the aluminum, lifting away the aluminum in areas where none was wanted. After cleaning, only aluminum strips
remained in areas that were to become double-clamped beams, as shown in Figure 4.6. The width of these aluminum strips becomes the width of the bridges and the width of the membrane underneath becomes the length of the bridges once freed.

![Figure 4.6 - A schematic of the silicon nitride membrane after putting strips of aluminum onto the membrane in the shape of the double-clamped beams.](image)

The thickness of the aluminum was chosen to be the thickness of the Fabry-Perot reflector. The aluminum layer had to be thick enough to be reflective but not too thick as to complicate the optics. For this application, 20-25nm of aluminum was deposited onto the membrane. This amount, which is unusually thick for a partial aluminum reflector, takes into account the effect of the oxidation of the aluminum after being exposed to air. Once the aluminum strips are in place, the wafer was then placed in the reactive ion etching system to remove the exposed silicon nitride. Just as in the membrane creation process, a trifluoromethane/oxygen mixture was used to free the beams, as shown in Figure 4.7.
Figure 4.7 - Schematic of the wafer after the RIE process removes the silicon nitride, thus creating the freed double-clamped beams.

An actual image of the silicon nitride double-clamped beams created using this process is shown in Figure 4.8.

Figure 4.8 - Image of 1000μm long silicon nitride double-clamped beams of varying widths. From left to right, the widths of each group is 50μm, 40μm, 30μm, 20μm, 10μm, and 5μm.

4.5 Fabrication of the Microcavity

To create the microcavity, first a glass substrate had to be mirrored. For a substrate, a 4” diameter x 1/16” thick piece of Schott Borofloat® plate glass was used. This particular type of glass was chosen because of its high optical quality: the plate glass used was both flat and had a high percent transmission throughout the visible and NIR wavelengths, as shown in
Figure 4.9. The flatness was checked by looking at the interference pattern created with an optical flat.

![Graph showing percent transmission for 2mm thick Borofloat® plate glass.](image)

**Figure 4.9 - Plot of the percent transmission for 2mm thick Borofloat® plate glass.**

The metal type, thickness, and amount used to create the partially reflecting surface was the same as was used to create the double-clamped beams. This was to ensure that the reflectance values of both the substrate and the beam were as closely matched as possible. The metal layer was applied using the e-beam evaporator. Once the reflective layer was applied, a technique to bond the now reflective substrate to the wafer containing the reflective beams had to be determined. First, the method of bonding had to be patternable so as to make contact with certain parts of the wafer but not actually be within the optical microcavity. Second, the bonding method had to be quick and easy to do. This required a low-temperature method of bonding. Pan et al. described a bonding method for MEMS processing that is both low-temperature and used patternable materials as the bonding layer\(^{11}\). Plots of the bonding strengths of some patternable materials are shown in Figure 4.10.
Several common photoresists were tested for their effectiveness as bonding layers. Included in these common photoresists was AZ-5214E and SPR220-7.0, but both of these resists had the same common problems. First, the layer could not be spun to the thickness to create the needed microcavity separation of 15μm. Secondly, these photoresists would deform and outgas during the bonding process which would hurt the optical quality and bring unwanted variation into the microcavity separation. The deformation of the bonding layer is shown in Figure 4.11.

To overcome these problems, the photo-definable epoxy SU-8 was used as the bonding layer. SU-8 is a photodefinable epoxy used extensively in MEMS processing. In
the plots in Figure 4.10, SU-8 exhibits the strongest bonding strength both at high temperatures and high bonding pressures. SU-8 also exhibits many positive characteristics such as the inability to be deformed and it can be spun to fairly large thicknesses. SU-8-5 was the particular type used and could easily be spun to a thickness of 15\(\mu\)m. It also stayed firm and did not “flow” when being pressed or heated during the bonding process. The importance of these characteristics is the consistency in knowing the Fabry-Perot cavity will stay at a constant separation throughout the processing. A schematic of the SU-8 bonding pad is shown in Figure 4.12.

Figure 4.12 – Shape of bonding pad created with the SU-8. The wafer containing the arrays of bridges was just slightly smaller than the size of the pad for alignment purposes. Also, all of the bridges were contained within the area void of any SU-8.

After aligning the wafer onto the bonding pad, the two were pressed together using a simple improvised tool utilizing a large screw to tighten the pieces together. The device, now in the vise, was then baked in an oven at 110°C for thirty minutes. The high temperature further cross-links the SU-8 and strengthens the bond. After being allowed to cool, the device with its many microcavities created by the arrays of double-clamped beams were ready for testing. A photograph of a device is shown in Figure 4.13.
Figure 4.13 – Image of a completed device. The slight outline around the wafer is the SU-8 bonding pad. This particular wafer contains over 800 bridges that can be analyzed.
References


5. DuraSiN™, Protochips, Inc.


5  **OPTICAL INTERROGATION SETUP AND METHODS**

5.1  **Overview**

Since the goal of the work is to optically test the mechanical properties of the devices, an optical setup capable of doing this had to be constructed. The optical setup consists of the four main components of any optical sensor: a light source to interrogate the sensor, optics to direct the beam, the sensor itself, and a detector for the modulated light. An optical driving source (a 980nm laser) coupled with the interrogation source was also used to drive the motion of the membrane.

5.2  **C-band Interrogation Laser**

A Hewlett-Packard 8168F tunable laser source, shown in Figure 5.1, was used as the interrogation source. The wavelength tuning range of this laser was between 1525nm and 1625nm with an output power of up to 5mW. The source puts the light into a single mode optical fiber by means of an APC connection.

---

Figure 5.1 - The HP 8168F C-band tunable laser used as the optical interrogation source. The output wavelength and power were able to be controlled either at the source or remotely using a GPIB interface. The light is coupled into the yellow single-mode fiber on the bottom right.
5.3 980nm Driver Laser

A Princetel 980nm pigtail laser diode module was coupled with the tunable laser light to drive the beam motion via localized heating. The 980nm wavelength was chosen for several reasons including the ability to have a high intensity output plus the beam is visible with a CCD camera which proved very useful in the alignment process. The intensity of the 980nm light was modulated using a bias-T connected to a network analyzer. A fiber coupler was used to couple both the tunable source and 980nm light into a single fiber. The 980nm driver source and fiber coupler are shown in Figure 5.2.

Figure 5.2 - The fiber coupler plus the 980nm pigtail laser source and driver used for driving the membrane motion. The driver is contained within the silver box. The BNC connection at the bottom left connects to a bias T within the driver box which allows the output to be modulated. The red device connected to the driver is the actual 980nm Princetel pigtail laser with single-mode output. The blue box underneath the driver holds the fiber coupler for the two laser sources. The yellow fiber on the left is connected to the tunable source and the white fiber on the left is from the 980nm laser. The fiber on the right side is the output fiber from the coupler which goes to the optical setup.
5.4 Optical Interrogation Setup

A simplified schematic of the optical interrogation setup is shown in Figure 5.3. It shows the four main components of any optical sensor including the source, directional optics, the sensor (optical microcavity) and the detector.

![Figure 5.3 - Schematic of the optical interrogation setup. The light is directed through an objective which focuses the light onto the sensor. Light reflected from the sensor is directed back along the same beam path and into a high speed InGaAs photodetector.](image)

The actual optical setup described below is shown in Figure 5.4. From the fiber coupler, the single fiber is connected to a fiber collimator whose purpose is to collimate the wavelengths in the beam into free space. From this collimator, the beam propagates through two irises used for alignment purposes. The irises ensure that the beam is located along the same axis at each use. From the irises, the beam reflects off a 50/50 beamsplitter which directs the beam to the “sensor cage.” At the bottom of the sensor cage is a 45° mirror which directs the beam vertically. The beam then goes through an objective which focuses the light onto the sensor. The sensor consists of the Fabry-Perot microcavity made with the silicon nitride membrane. The actual sensor is bonded to a 2 in. wafer which fits on a 2 in. mirror holder. This stage sits atop both a manual-controlled XYZ translation stage and an ORIEL 18011
Encoder Mike Controller, which is able to control movement with sub-micron precision. The XYZ translation stage allows for coarse adjustment of the stage while the Mike controller allows for finer stage adjustments. The incoming beam is reflected by the sensor back through the objective which recollimates the beam. The beam is then reflected again by the 45° mirror, transmitted through the beamsplitter, reflected off a flip-mirror and directed to a Thorlabs High Speed InGaAs detector. The voltage reading from the InGaAs detector was measured with an Agilent multimeter. The purpose of the flip-mirror is to allow imaging of the system with a CCD video camera module. The camera is used for alignment of the membrane to ensure that the interrogating beam is actually hitting the sensor, which at the micron scale is absolutely necessary. Once the beam is aligned correctly, the mirror is flipped up into position to reflect the beam to the sensor. The 980nm light can be seen with the CCD camera whereas the wavelengths from the tunable source cannot. Since the two beams are co-located, it is assumed that the tunable source light is at the same location along the z-axis as the 980nm light. The flip mirror also has fine pitch and roll control to ensure the reflected beam hits the center of the InGaAs detector.
Figure 5.4 - Image of the optical interrogation setup. Each of the main components are denoted with the letters A-K. The components are as follows: A - optical fiber collimator, B - alignment irises, C - 50/50 beamsplitter, D - 45° mirror, E – microscope objective, F – sample holder with pitch and roll control, G – XYZ translation stage, H – XY ORIEL remote controlled translation stages, I – flip mirror, J – CCD camera, K – high-speed InGaAs photodetector.

5.5 Obtaining Fabry-Perot Interference Pattern

In order to take the reflected spectral data and see the Fabry Perot pattern, the ability to simultaneously sweep through the wavelengths while recording the voltage from the multimeter was necessary. This was made possible using National Instruments LabView instrument control software. NI LabView is a software tool for designing test, measurement, and control systems. A program was written in LabView that was used to tune the laser, read the corresponding multimeter value from the photodetector, plot, and record the data to a file. In the LabView program, the user enters a starting wavelength, an ending wavelength, and the wavelength step in nanometers between subsequent data points. After pressing run, the
program takes control of the tunable laser by means of a GPIB card, tunes the laser to the starting wavelength, stores the multimeter reading, and increases the wavelength by the specified step value. This cycle continues until the laser reaches the specified final wavelength value. The program has a real-time plot function where the user can see the emerging spectrum as the laser is being tuned. Once the scan and read is finished, the data can then be saved for further analysis. Figure 5.5 is a screenshot of the LabView front end which tunes the HP laser and simultaneously reads the photodetector output.

![Screenshot of the front end of the LabView code used to simultaneously control the tunable laser source while reading the value on the multimeter. From these, the Fabry-Perot interference spectra is plotted and stored.](image)

Figure 5.5 - Screenshot of the front end of the LabView code used to simultaneously control the tunable laser source while reading the value on the multimeter. From these, the Fabry-Perot interference spectra is plotted and stored.
5.6 Checking Driver Laser Waveform

To check the waveform of the 980nm driving output, a silicon detector connected to a Tektronix oscilloscope, as shown in Figure 5.6, was used. This ensured that the output signal maintained its sinusoidal waveform and that the laser diode stayed above threshold.

![Tektronix oscilloscope](image)

Figure 5.6 - Image of the Tektronix oscilloscope used to monitor the output waveform of the 980nm driver laser. The signal was detected using a silicon photodetector.

5.7 Obtaining Bridge Frequency Response Using Network Analyzer

To see the motion of the membrane, the interrogation laser was tuned to the wavelength of maximum slope in the Fabry-Perot reflectance spectrum. Once the interrogation laser was tuned to the slope, the 980nm pigtail laser diode used to pump the membrane motion was turned on and driven by the network analyzer output. The frequency response of the sensor was found using an Agilent E5100A network analyzer, shown in Figure 5.7. This particular network analyzer was chosen because of its wide tuning range (10kHz – 300MHz) and high output power (-58dBm to +22dBm). The 980nm laser was connected to the output of the network analyzer and the InGaAs photodetector was connected to the input of the network analyzer. By observing the magnitude and phase of the frequency response of the membrane motion, any resonances were detected.
Figure 5.7 - Image of the Agilent E5100A network analyzer and monitor used to image the beams. The monitor, connected to the CCD camera, was used for alignment purposes. The two BNC cables in the network analyzer are the output and input signals. The output cable (left) connects to the 980nm laser bias T which modulates the frequency and amplitude of the driver laser signal. The input cable (right) connects to the high speed photodetector.

The E5100A network analyzer was also GPIB enabled and another LabView code was used to save the magnitude and phase data. Figure 5.8 is a screenshot of the LabView front end which was used to read the magnitude and phase on the network analyzer.
Figure 5.8 - Screenshot of the front end of the LabView code used to obtain the magnitude (left) and phase (right) data from the Agilent E5100A network analyzer.
6 EXPERIMENTAL RESULTS

6.1 Introduction

For eventual use in sensing applications it is important to thoroughly understand how to control the mechanical and optical characteristics of the bridges and Fabry-Perot cavities. Primarily, it is important to know how to predict before fabrication the precise resonant frequency. The second issue of major importance is to know how to maximize the amplitude of the resonance. In this section, the mechanical frequency responses of bridges of various dimensions are obtained and analyzed to help find any optimal designs.

The previous chapters presented the basic mechanical theory of a double-clamped beam and optical theory of Fabry-Perot cavities assuming ideal characteristics, but obviously fabricated devices will deviate from the theoretical in several ways. To first order we expect the deviations from theory to arise from what might be considered errors in fabrication. For example, bias errors can result in the bridge being longer or narrower than intended due to KOH and RIE etching variables. However, there are other considerations that are not as clearly identified. The bridge itself for example consists of two layers: the silicon nitride and the metal (usually aluminum) that was used to serve the dual purpose of etch mask and partial reflective layer. This thin metal layer is also used to absorb the heat of the pumping laser. Since the two layers have different coefficients of thermal expansion, the two layers might act as a bi-metallic strip (such as a thermostat coil) which can amplify the motion of the bridge. Similarly it has been observed that depending on the fabrication process that the deposited aluminum may have different optical characteristics, or that photoresist residue may still be on the bridge. Other issues exist, such as the bridge not necessarily being flat,
but curved, and even in some cases buckled. The stiffness of buckled beams for instance can be expected to be very different than that of a flat beam.

The experimental conditions also can strongly influence the measurements. In addition to the surrounding temperature and pressure around the bridge, the resonant frequency was found to be dependent on experimental conditions such as the driver laser power, interrogation laser power, and interrogation laser wavelength. For example, illuminating the sample with an external light source can also heat the bridges, thus changing the material properties and causing a shift in the resonant frequency.

While to first order the simple theory provides substantial insight into the behavior of these structures, it is not surprising to find more complicated behaviors upon further investigation. Thus the purpose of this chapter is to present the observed characteristics of fabricated bridges and to note where future work can focus to optimize the structures.

6.2 Obtaining Experimental Data

In general the experimental data was obtained in the following sequence:

2. Placement of the sample within the sample holder.
3. Alignment of the optical system such that the laser beam and bridge are in focus and visible on the video monitor.
4. A scan varying the interrogating laser wavelength to obtain the Fabry-Perot spectrum.
5. Tuning the laser to the appropriate wavelength based on the Fabry Perot spectrum.
6. Setting the frequency range and output power on the network analyzer.
7. Measuring the frequency and phase of the resulting signals.
8. Obtaining the peak frequency and amplitude, and Q-factor as appropriate.
6.3 Devices Tested

The experimental data presented in this chapter comes from two 1cm x 1cm wafers containing bridges of a variety of dimensions. The mask design used to make each of these devices is shown in the appendix. The first sample, shown in Figure 6.1a, contained short bridges, 10\(\mu\)m to 100\(\mu\)m long in 10\(\mu\)m increments of widths 5\(\mu\)m, 10\(\mu\)m, 20\(\mu\)m, 30\(\mu\)m, 40\(\mu\)m, and 50\(\mu\)m. The second sample, shown in Figure 6.1b, contained longer bridges, 100\(\mu\)m to 1000\(\mu\)m long in 100\(\mu\)m increments of widths 5\(\mu\)m, 10\(\mu\)m, 20\(\mu\)m, 30\(\mu\)m, 40\(\mu\)m, and 50\(\mu\)m. The purpose of varying the dimensions is to understand how the length to width ratio changes the frequency characteristics of the bridge and to see if there is an optimal dimension.

![Figure 6.1 – Devices tested. The device at left contains over 800 bridges of lengths up to 100\(\mu\)m and the device at right contains over 400 bridges of lengths up to 1mm.](image)

Various SEM images of some bridges within the devices tested are shown in Figure 6.2 and Figure 6.3.
Figure 6.2 – SEM image of 30μm long, 5μm wide bridges and sidewalls from anisotropic etch.

Figure 6.3 – SEM images of bridges. Left - 80μm long, 40μm wide bridges. Right - 100μm long, 10&20μm wide bridges. There is some image distortion due to 60Hz noise within the SEM which may make the bridges look curved but in actuality they are flat. (Each scale bar on the lower right represents 10μm.)

6.4 Frequency Response Waveforms of Bridges of Various Widths

Figure 6.4 shows a series of four frequency response scans. In each plot, the lengths are kept constant and the widths are varied. Four different lengths were tested: 50μm, 70μm, 90μm, 120μm.
Figure 6.4 - Scans at center of 50μm long bridges of various widths. (Data points for 90μm long, 30μm wide bridge missing due to broken membrane.)
In these scans, several interesting patterns can be noticed. First, the lowest frequency resonance, the fundamental transverse resonance, remains at the same frequency even as the width is varied just as the theory suggests. The fundamental resonance is always the lowest frequency resonance but is not always the “strongest” resonance. For the 5μm wide bridges, the fundamental resonance is easily seen but as the widths increase, the fundamental resonance becomes overshadowed by stronger width dependent modes.

These width-dependent modes are seen in each plot, especially for the 30μm, 40μm, and 50μm widths. Later analysis will discuss these width dependent resonances.

6.5 Frequency Response Waveforms of Bridges of Various Lengths

The following series of plots show the same scans as in the previous section but makes side-by-side comparisons of bridges of equal width and varied length rather than equal length and varied width. Figure 6.5 shows the scans of bridges of widths 5μm, 10μm, and 20μm. The scans of the bridges of widths 30μm, 40μm, and 40μm are shown in Figure 6.6.
Figure 6.5 – Plots of the frequency response of 5μm, 10μm, and 20μm wide bridges comparing the resonances of various lengths up to 5MHz.

The dependence of the fundamental resonant frequency on length is easily seen in these plots. As the bridges increase in length, the resonant frequencies decrease. The noise of the signals in these curves has a couple of possible sources. The first is that the bridge itself is smaller than the laser spot size which results in a weaker returned optical signal. The
second is the very high length to width aspect ratio. This very large aspect ratio severely diminishes the ability of the bridge to be driven optically, thus resulting in a smaller beam deflections and a smaller signal to noise ratio. As the widths increase, the signal to noise ratio increases and the mode structure complicates also.

Figure 6.6 – Plots of the frequency response of 30μm, 40μm, and 50μm wide bridges comparing the resonances of various lengths up to 5MHz. (Data points for 90μm long, 30μm wide bridge missing due to broken membrane.)
In the plot of the 30μm wide bridges, the width dependent resonance drops low enough in frequency to be seen near the fundamental resonant frequency. The amplitude of this peak though is several orders of magnitude higher than the fundamental resonance. Though this peak is width dependent this plot shows that it is also length dependent. It decreases in frequency as both the length and width increase.

6.6 Investigation of Resonant Modes By Varying Location Of Excitation

One problem was to determine if the strong width dependent resonance was a torsional or transverse mode. One way to determine the mode structure of a resonance is to shift the location of the interrogation/driving laser spot on the bridge to more finely control the location of the excitation. It can be determined from the spectra whether the resonance excitation is at a node where the response is weaker or at an antinode where the response is stronger. Ideally it would be preferred to move the excitation spot and the pump spot independently, however in this discussion the pumping laser beam and the interrogating laser beam are co-linear. The following figure shows a series of dotted lines atop a bridge which represent the lines where scans were taken.
First, a higher magnification objective (20x) was chosen to get a smaller spot size. By scanning across the width of the bridge, it could be determined whether or not the very strong width dependent resonance was a torsional or transverse mode. If the resonance is torsional, the bridge would pivot along the center of the bridge and torque back and forth. There would therefore be no deflection along the center and the signal would be strongest along the edges. Figure 6.8 shows a series of numbers, 0-9, along the center of a 120μm long 50μm wide bridge. These ten spots, each separated by 5μm, represent the spot location for the ten curves in Figure 6.9.
Figure 6.8 – Schematic of a 120μm long 50μm wide bridge with ten different interrogation spots located along width of the bridge at the center. Each spot is separated by 5μm.

Figure 6.9 – Series of ten scans along width of 120μm long 50μm bridge. Each curve represents a laser spot, each separated by 5μm.

The fundamental transverse mode is seen on each curve around the 900kHz frequency. The strong mode at 1.2MHz decreases in magnitude near the edges. From this series of curves, it can be easily seen that the strong resonance located at 1.2MHz is not a
torsional mode but rather a transverse mode along the unclamped dimension, the width. A drawing of this mode is shown in Figure 6.10.

![Drawing of the transverse fundamental mode along the width rather than length. The pivot point would not occur at the edges due to the bridge not being clamped along the width.](image)

Similarly the bridge was scanned along the length of the bridge in the center (line d in Figure 6.7) with the results presented in Figure 6.11.

![Series of scans along length of 120μm long 50μm wide bridge.](image)

### 6.7 Resonant Frequency Scaling for Constant Width and Varied Length

The motivation of this experiment was twofold, first to verify that the resonance frequency dependence varies with the geometry of the beam as expected, and second as a
means of finding what might be the optimal dimensions of the bridge in terms of response.

The first tests on the devices were to find how the fundamental transverse resonant frequency scaled with length. To ensure that the mode measured was the transverse mode, the length was kept much larger than the width. The bridge width used for the short bridge sample was 10μm and the bridge width used for the long bridge samples was 50μm. If the length/width aspect ratio is too large, the ability to optically drive the bridge motion diminishes.

The plot in Figure 6.12 is of the fundamental resonant frequency for bridges of length 30μm to 120μm and width 10μm. The oversized length dimensions were due to a 20μm overetch of the sample.

![Fundamental Resonant Frequency of Al Coated SiN Bridges of Width 10μm](image)

**Figure 6.12 – Fundamental Resonant Frequency of short 10μm wide aluminum coated silicon nitride bridges as a function of bridge length. Error bars represent one standard deviation for a set of identical bridges.**

The fundamental resonant frequency definitely scales down as the length goes up.

The variation of the resonant frequency between different bridges of the same dimension may be due to several factors including variations between the length of each bridge (due to
KOH etch unevenness), particulates that may have settled on the bridges (either during KOH or reactive ion etch), or variations in the metal coating which can have a direct effect on the bridge strain. The plot in Figure 6.13 shows the fundamental resonant frequency for bridges of length 100μm to 1000μm of width 50μm.

![Fundamental Resonant Frequency of Al Coated SiN Bridges of Width 50μm](image)

**Figure 6.13 - Fundamental Resonant Frequency of long 50μm wide aluminum coated silicon nitride bridges as a function of bridge length. Error bars represent one standard deviation for a set of identical bridges.**

Again, the fundamental transverse resonant frequency decreases as the bridge length increases. For this set of points, the 50μm width was chosen so that the length to width aspect ratio would not be so large that optical driving would not be possible.

### 6.8 Resonant Frequency Scaling for Constant Length and Varied Width

As the width of the bridges change, the fundamental resonant frequency in theory should not change. This provides a good check of whether the simple theory applies well to
fabricated structures. Figure 6.14 shows the fundamental resonant frequency of 120μm long bridges of various widths. The resonant frequency remains fairly constant over that range of widths as expected, however there is a definite upward trend with increasing widths that merits further investigation.

![Fundamental Resonant Frequency of 120μm Long Bridge vs Bridge Width](image)

Figure 6.14 – Fundamental Resonant Frequency of 120μm Long Bridge as a function of bridge width. The frequency does not change significantly over the range of widths.

### 6.9 Strong Width Dependent Resonant Mode

As the width increases, a strong resonant mode becomes more dominant while decreasing in frequency. Below widths of 20μm, these modes are so small that they are undetectable, even though they exist. Above 20μm, these modes start to increase in magnitude as the fundamental transverse mode decreases in magnitude. At 50μm, this width dependent mode is so dominant that the first longitudinal transverse mode is nearly undetectable. The following graph displays these dominant modes on a linear scale.
Figure 6.15 – Linear scale frequency response of 120μm long bridges of various width. The fundamental transverse resonant frequency is the small peak located around 800kHz.

From the series of scans along the length and width and by looking at the dependence of this mode on the width, we have concluded that this mode is a transverse mode but in the unclamped direction. Figure 6.16 shows how the strong width dependent resonance is several orders of magnitude higher in amplitude than the fundamental resonant frequency. The amplitude of the resonance increases as the width increases.
As the bridge width increases, not only does the unclamped transverse resonance decrease in frequency and increase in amplitude, but the variation between bridges of the same dimensions goes down.

### 6.10 Resonant Frequency Shift with Interrogation Laser Power

The C-band wavelength interrogation laser power was varied to see the effect on the resonant frequency. Figure 6.17 shows the resonant frequency scan of a 110μm long 50μm wide bridge. The laser power was varied between 10μW and 5.3mW (the maximum power for the laser at the selected wavelength). As the power of the laser is increased, the resonant frequency decreases and the amplitude of the signal increases.
The decrease in the resonant frequency as a function of the interrogation laser power is linear, as shown in Figure 6.18. The average decrease of the resonant frequency is around 5.27Hz/uW.
Figure 6.18 – Plot showing linear relationship between the resonant frequency and interrogation laser power.

The exponential growth of the amplitude of the resonance as a function of the interrogation laser power is shown in Figure 6.19. The curve has a nearly quadratic dependence.

Figure 6.19 - Plot showing the quadratic dependence of the signal amplitude on the interrogation laser power.
6.11 Resonant Frequency Shift with Driving Laser Power

The resonant frequency is also highly dependent on the power of the driver laser. As the laser diode current on the 980nm laser is increased, the resonant frequency shifts to the left and the signal amplitude increases. This increase in the laser power does not increase the amplitude at which the light is oscillating but increases the DC intensity of the light only.

Figure 6.20 - Plot showing the shift in the resonance as the driver laser diode current is varied.

The resonant frequency experiences a linear shift to the left as the laser driver current is increased. The slope of the curve is 550Hz/mA laser diode current.
The amplitude of the resonance increases exponentially as the 980nm laser diode current is increased, as shown in Figure 6.22.

Figure 6.21 - Plot showing the linear relationship between the resonant frequency and the 980nm driver laser diode current.

Figure 6.22 - Plot showing the exponential relationship between the signal amplitude and the 980nm driver laser diode current.
6.12 Resonant Frequency Shift with Interrogation Laser Wavelength

The resonant frequency dependence on interrogation laser wavelength was also tested. Because the frequency response signal is strong along a sloped part of the Fabry-Perot curve, a return resonance can be seen anywhere along the downward slope of the curve. The signal is a little stronger at a higher voltage part of the curve though.

![Figure 6.23](image)

Figure 6.23 – At left, the Fabry-Perot interference pattern created by a 100μm long 60μm wide bridge. At right, the curve of the resonant frequency as a function of the wavelength shows similarity in shape to the optical interference pattern.

The resonant frequency along the steepest part of the curve decreases at a rate of over 1.5kHz/nm wavelength shift. The resulting resonant frequency vs. wavelength curve in effect reproduces the slope created by the optical interference pattern.

6.13 Observing Resonant Frequency Shifts With Time

A LabView program which performs a Lorentzian fit to the curves from the network analyzer to accurately find resonant frequency was used to gather temporal frequency data. This allows one to observe the response time of the system as well as observe any trends that
may occur over time. As previously mentioned, the bridge system is highly sensitive to incident light. The data shown in Figure 6.24 is of the resonant frequency of a bridge over time as an illumination lamp is powered off and on.

![Resonant Frequency Change With Illumination Lamp Power](image)

**Figure 6.24 – Plot of resonant frequency vs. time as an illumination lamp is powered off and on.**

The heating of the bridge by the lamp clearly has an effect on the resonant frequency. As the bridge is heated, the strain decreases as does the resonant frequency. The very sharp increase or decrease of about 3.5kHz upon the change of lamp power occurs in less than five seconds, illustrating a very short system response time. This type of temporal resonant frequency data is the same that will be used with chemical analysis to observe resonant frequency shifts with chemical concentration changes.

### 6.14 Using Pressure to Verify Air Damping Effects

A pressure chamber was built and connected to a dry pump to create vacuum conditions within which to test the behavior of the bridges, as shown in Figure 6.25. Some of
the mechanical properties that were to be explored were the resonant frequency and the mechanical quality factor as functions of the chamber pressure.

Figure 6.25 – Image of the pressure chamber which is connected to a dry vacuum pump to create a partial vacuum at the sample.

The amount of air damping within the system could be tested by removing most of the air within the chamber. To test this, air was pumped out of the chamber to a pressure of 28mTorr. A resonance measurement was made at this pressure and then the pump was turned off. The two plots in Figure 6.26 compare the resonance structure in air and in vacuum.
Figure 6.26 – Plots showing the clear difference between the resonance in air (left) and vacuum (right).

Not only is the width of the resonance much narrower in vacuum (higher quality factor), the amplitude is over 100 times higher in vacuum than in air. As air was slowly leaked back into the chamber, the quality factor at various pressures were taken, as shown in Figure 6.27. This data clearly shows that the damping from the air is the dominant source of energy loss in the system.

Figure 6.27 – Mechanical quality factor as a function of chamber pressure.
6.15 Measurement of Thermal-Mechanical Noise of Bridge

The thermal-mechanical noise of the bridge was also measured. This was done by first getting the Fabry-Perot spectrum with the tunable source. After getting the spectrum, the laser was tuned to the wavelength of maximum slope, just as before. After finding the frequency of the mechanical mode with the network analyzer, the 980nm driver laser is turned off, thus in effect becoming an un-powered, un-driven device. The photodetector was then connected to a spectrum analyzer so that any mechanical motion due to the noise could be observed. Because the resonant frequency was known, the spectrum analyzer could be tuned to the appropriate frequency. The thermal mechanical noise of a 110μm long, 50μm wide observed with the network analyzer is shown in Figure 6.28.

![Figure 6.28 – Noise spectrum of an un-driven bridge (980nm laser power off) as taken on a spectrum analyzer.](image)

This ability to see motion without thermally driving the motion opens up the possibility for an un-powered sensor that requires optical interrogation only.
7 CONCLUSIONS & FUTURE WORK

7.1 Conclusions

A variety of double-clamped beam structures were investigated as a function of both beam width and length. The 30μm long short bridges exhibited fundamental transverse resonant frequencies near 3.5MHz. Although the fundamental transverse mode (along the clamped dimension) was expected to have the largest deflection amplitude, a width-dependent mode had the strongest frequency response. As the bridge width values increased, this width dependent resonant mode decreased in frequency but increased in amplitude. This mode, a fundamental transverse mode along the un-clamped dimension or width, had a stronger frequency response and also had a mechanical quality factor over 100.

Due to air damping within the system, mechanical quality factors on the order of ~150 were observed for most bridges in air at room temperature. However in partial vacuum (28mTorr), quality factors increased to values as high as 7000, indicative that air damping is the main source of energy loss in the system. The resonant frequency was also shown to be highly sensitive to heat and laser illumination power. One way to minimize environmental effects in sensing applications would be to employ multiple bridges. By taking differential measurements simultaneously between the two bridges, one with the chemically selective layer and one without the environmental variables could be practically eliminated.

As the measurements in the present work have indicated, the resonant motion can still be present with extremely low powers of the interrogation laser (down to 10μW), demonstrating that remote interrogation of the device is a definite possibility. Furthermore the ability for observation of mechanical resonances with an un-driven device using only
thermal-mechanical noise opens up the possibility of an un-powered sensor. The detection of these small deflections demonstrates the sensitivity of the optical detection system and microcavity interferometry technique.

Despite the fact that the sensitivity of these bridges to chemical exposure was not quantified, the effect of various environmental conditions on the mechanical motion of the bridge was observed. Some of these variables included heat from a light source and laser source intensities. The shift in the resonant frequency of several kilohertz is indicative of a device that is suitable for chemical sensor applications.

### 7.2 Future Work

One area of this project for further technological development is to increase the deflection amplitude of the bridges, by depositing more metal in order to get a stronger response due to the bi-layer effect. The metal on the tested devices were one-tenth the thickness of the silicon nitride layer. If more metal is deposited onto the bridge, the change in thermal expansion of the two layers could be amplified leading to larger deflections.

A high quality factor is necessary to keep a high sensitivity by detecting smaller shifts in the resonant frequency. By continuing to decrease the air-damping effects, the mechanical quality factor would continue to improve as well. One key to increasing the quality factor would be to use the fundamental resonant frequency along the length so that reducing the width of the bridge would reduce the air damping effects.

Another improvement could be to simplify the optics by elimination of the magnifying objective at the sensor level. One possibility is to employ a ball lens retroreflector into the device. By combining the appropriate ball lens size with the correct
cavity dimensions, a simple device and cavity could be created. If the bottom of the ball lens was coated with a small amount of metal, the ball could act as one of the partially reflecting surfaces. With the underlying membrane coated with metal also, a simple Fabry-Perot cavity would be created. The ball lens would replace both the reflective substrate and also replace any collimating optics, such as a microscope objective. Any alignment concerns would be negated because the ball would rest on the sidewalls, centering the sphere’s vertical axis atop the underlying membrane. A laser beam shined on the sphere would focus the laser light down onto the membrane and the modulated light intensity would be recollimated and send back towards the source.

Figure 7.1 – Employing ball lens to create microcavity.

However the ball’s spherical shape could decrease the cavity finesse. The incoming beam might still be susceptible to environmental conditions; nevertheless this could be avoided by employing a strictly fiber-based system.

The above design could easily be modified to use an optical fiber to eliminate variables caused by the environment. A fiber with an exposed core could be rounded off at the end using an arc fusion splicer to in effect create a ball lens on the end of it. The following image displays a schematic of a fiber.ball lens system.
Figure 7.2 – Employing a fiber-based system to create Fabry-Perot microcavity. The fiber/ball lens combination would fit into the etched cavity.

This fiber-based system would also simplify the optics by inherently providing a means of directing the reflected beam but lacks some of the advantages of free-space remote interrogation.

Finally, this system could be tested as a chemical sensor. By applying the chemically selective layer to a silicon nitride beam and testing with a stimulant chemical analyte vapor. The chemically selective layer could be placed onto bridges using an ink-jet head while the coated devices placed into a chamber where concentration levels of a gas/chemical can be carefully controlled. While temporal data of the resonant frequency vary with the concentration level, the detection parameters can be taken. The sensitivity measurements would be in units of change in resonant frequency per change in concentration (Hz/ppm).
Appendix
8 Appendix

8.1 Mask Layouts for Membrane and Bridge Fabrication

<table>
<thead>
<tr>
<th>Small Membranes</th>
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Figure 8.1 - Mask used to create the membrane arrays. The mask design at left was used to create membranes 10\(\mu\)m to 100\(\mu\)m wide and the mask at right was used to create membranes 100\(\mu\)m to 1000\(\mu\)m wide. The mask features are oversized slightly to compensate for the anisotropic etch.

Figure 8.2 – Mask used to create the bridges. The mask is used to place the aluminum strips atop the membranes created from the masks in Figure 8.1.
8.2 Transfer Matrix Maple Worksheet

The following is the Maple code used to find the net reflectance of a stack of materials using the transfer matrix method:

Transfer Matrix to Determine Net Reflectance of a Stack of Materials. The stack begins and terminates in air. The first layer of the stack is a 1.59mm thick plate glass (n=1.472). The second layer is a thin aluminum film. The third is an air cavity of thickness 15um. The fourth layer is another thin aluminum film. The final layer is 200nm of SiN (n=2.4).

\[ \text{restart;[...]} \]
\[ \text{with(LinearAlgebra);[...]} \]

The incident angle (\(\theta\)) is 0 for the stack, \(n\) and \(p\) are identically the refractive index of each material (for films at normal incidence and relative magnetic permeability 1), \(d\) is the thickness of each layer, and for simplicity, \(x\) is the argument of the sine and cosine within each matrix.

> \[ \text{theta:}=0; \]
> \[ \theta := 0 \]

The incident angle (\(\theta\)) is 0 for the stack, \(n\) and \(p\) are identically the refractive index of each material (for films at normal incidence and relative magnetic permeability 1), \(d\) is the thickness of each layer, and for simplicity, \(x\) is the argument of the sine and cosine within each matrix.

> \[ \text{theta:}=0; \]
> \[ \theta := 0 \]
> n_glass := 1.472; n_Al := 1.61 + 16.2*I; n_cavity := 1; n_SiN := 2.4;
n_AlOx := 1.5;

    n_glass := 1.472
    n_Al := 1.61 + 16.2 I
    n_cavity := 1
    n_SiN := 2.4
    n_AlOx := 1.5

> p_glass := n_glass; p_Al := n_Al; p_cavity := n_cavity;
p_SiN := n_SiN; p_AlOx := n_AlOx;

    p_glass := 1.472
    p_Al := 1.61 + 16.2 I
    p_cavity := 1
    p_SiN := 2.4
    p_AlOx := 1.5

> d_glass := 1.5875e-3; d_Al := 5e-9; d_cavity := 15e-6; d_SiN := 200e-9;
d_AlOx := 10e-9;

    d_glass := 0.0015875
    d_Al := 0.5 \times 10^{-8}
    d_cavity := 0.000015
    d_SiN := 0.200 \times 10^{-6}
    d_AlOx := 0.10 \times 10^{-7}

> x_glass := 2*Pi*n_glass*d_glass*cos(theta)/lambda;
x_Al := 2*Pi*n_Al*d_Al*cos(theta)/lambda;
x_cavity := 2*Pi*n_cavity*d_cavity*cos(theta)/lambda;
x_SiN := 2*Pi*n_SiN*d_SiN*cos(theta)/lambda;
x_AlOx := 2*Pi*n_AlOx*d_AlOx*cos(theta)/lambda;

\[
x_{\text{glass}} := \frac{0.0046736000 \pi}{\lambda}
\]
\[
x_{\text{Al}} := \frac{(0.1610 \times 10^{-7} + 0.1620 \times 10^{-6} I) \pi}{\lambda}
\]
\[
x_{\text{cavity}} := \frac{0.000030 \pi}{\lambda}
\]
\[
x_{\text{SiN}} := \frac{0.9600 \times 10^{-5} \pi}{\lambda}
\]
The characteristic matrix for each layer is calculated below.

\[ x_{AlOx} := \frac{0.300 \times 10^{-7} \pi}{\lambda} \]

\[
MTE_{\text{glass}} := \begin{bmatrix}
\cos\left(\frac{0.0046736000 \pi}{\lambda}\right) & -0.6793478261 I \sin\left(\frac{0.0046736000 \pi}{\lambda}\right) \\
-1.472 I \sin\left(\frac{0.0046736000 \pi}{\lambda}\right) & \cos\left(\frac{0.0046736000 \pi}{\lambda}\right)
\end{bmatrix}
\]

\[
MTE_{\text{Al}} := \begin{bmatrix}
\cos\left(\frac{(0.1610 \times 10^{-7} + 0.1620 \times 10^{-6} I) \pi}{\lambda}\right), \\
(-0.06112467131 - 0.006074735853 I) \sin\left(\frac{(0.1610 \times 10^{-7} + 0.1620 \times 10^{-6} I) \pi}{\lambda}\right) \\
(16.2 - 1.61 I) \sin\left(\frac{(0.1610 \times 10^{-7} + 0.1620 \times 10^{-6} I) \pi}{\lambda}\right), \\
\cos\left(\frac{(0.1610 \times 10^{-7} + 0.1620 \times 10^{-6} I) \pi}{\lambda}\right)
\end{bmatrix}
\]

\[
MTE_{\text{cavity}} := \begin{bmatrix}
\cos\left(\frac{0.9600 \times 10^{-6} \pi}{\lambda}\right) & -0.4166666667 I \sin\left(\frac{0.9600 \times 10^{-6} \pi}{\lambda}\right) \\
-2.4 I \sin\left(\frac{0.9600 \times 10^{-6} \pi}{\lambda}\right) & \cos\left(\frac{0.9600 \times 10^{-6} \pi}{\lambda}\right)
\end{bmatrix}
\]

\[
MTE_{\text{SiN}} := \begin{bmatrix}
\cos\left(\frac{0.9600 \times 10^{-6} \pi}{\lambda}\right) & -0.4166666667 I \sin\left(\frac{0.9600 \times 10^{-6} \pi}{\lambda}\right) \\
-2.4 I \sin\left(\frac{0.9600 \times 10^{-6} \pi}{\lambda}\right) & \cos\left(\frac{0.9600 \times 10^{-6} \pi}{\lambda}\right)
\end{bmatrix}
\]
\[
M_{TE_{AlOx}} := \begin{bmatrix}
\cos\left(\frac{0.300 \times 10^{-7} \pi}{\lambda}\right) & -0.6666666667 I \sin\left(\frac{0.300 \times 10^{-7} \pi}{\lambda}\right) \\
-1.5 I \sin\left(\frac{0.300 \times 10^{-7} \pi}{\lambda}\right) & \cos\left(\frac{0.300 \times 10^{-7} \pi}{\lambda}\right)
\end{bmatrix}
\]

The overall characteristic matrix for the stack is found by multiplying the individual characteristic matrices for each layer together.

\[ M_{\text{system}} := M_{\text{TE glass}}.M_{\text{TE Al}}.M_{\text{TE AlOx}}.M_{\text{TE cavity}}.M_{\text{TE AlOx}}.M_{\text{TE Al}}.M_{\text{TE SiN}}; \]

The p-values for the beginning and terminating layers (air) are each 1.

\[ p_{\text{first}} := 1; \quad p_{\text{last}} := 1; \]

The reflection coefficient is given with the following relationship.

\[ r_{\text{TE}} := \frac{(M_{\text{system}}[1,1]+M_{\text{system}}[1,2]*p_{\text{last}})*p_{\text{first}}-(M_{\text{system}}[2,1]+M_{\text{system}}[2,2]*p_{\text{last}))}/((M_{\text{system}}[1,1]+M_{\text{system}}[1,2]*p_{\text{last}})*p_{\text{first}}+(M_{\text{system}}[2,1]+M_{\text{system}}[2,2]*p_{\text{last}})); \]

The total reflectance for the stack is found by multiplying the reflection coefficient by its complex conjugate.

\[ R_{\text{TE}} := r_{\text{TE}}*\text{conjugate}(r_{\text{TE}}); \]

A plot of the net reflectance of the stack over the wavelengths within the range of the laser (1525nm to 1625nm)

\[ \text{plot}(R_{\text{TE}}, \lambda=1525e-9..1625e-9, y=0..1, \text{numpoints}=300, \text{labels}=["wavelength (nm)", "Rnet"], \text{title} = "Net Reflectance of Stack of Materials" ); \]