Abstract

WALTER, CATRINA ANN. Behavior of Slender, Precast L-Shaped Spandrel Beams.
(Under the direction of Dr. Sami Rizkalla.)

The objective of this research is to study the behavior of slender, precast L-shaped spandrel beams under combined loading conditions. These beams are commonly used in parking structures to transfer vertical loads from deck members to columns. They typically have unsymmetrical cross-sections and are often subjected to heavy, eccentric loading. These factors induce a complex structural response including significant out-of-plane behavior. Traditionally, slender spandrel beams have been reinforced using the torsion and shear provisions of ACI-318 which require closely spaced closed reinforcement to resist spiral cracking typically reported for members subjected to combined shear and torsion.

This thesis describes a research program including loading four full-scale slender L-shaped spandrel beams to failure. A three-dimensional non-linear finite element analytical model was developed to study the behavior of these types of beams at different limit states. The main variables considered in the study are the use of open reinforcement versus traditional closed stirrups and reinforced concrete versus prestressed concrete for L-shaped spandrel beams.

Research findings indicate that out-of-plane bending may control end region behavior for these types of beams. Behavior of beams reinforced with open and closed stirrups were identical up to service loading levels. Based on the results of the experimental data and the analytical model, a simple and rational design methodology, using open reinforcement, is proposed to help practitioners design these types of beams.
Behavior of Slender, Precast L-Shaped Spandrel Beams

by

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A thesis submitted to the Graduate Faculty of North Carolina State University In partial fulfillment of the Requirements for the degree of Master of Science

Civil Engineering

Raleigh, North Carolina

2008

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Biography

Catrina Walter began her college education in August, 2000 at Central Piedmont Community College in Charlotte, North Carolina. After receiving her Associate in Science in May, 2003, Catrina transferred into the undergraduate department in the Civil Engineering program at North Carolina State University. While an undergraduate at NC State, Catrina gained engineering research experience at the university’s Constructed Facilities Laboratory. She graduated Magna Cum Laude in May, 2006 and enrolled in the graduate program at North Carolina State University to pursue a Master of Science degree in Civil Engineering with a emphasis on structures and engineering mechanics. Upon completion of her Masters, Catrina will begin working for an engineering firm based in Federal Way, WA.
Acknowledgements

Completing graduate school could not have occurred without the day to day support of family and friends. Thanks to the Frankls, Griffins and Wades for looking in on me and making me laugh. Thanks to my Mum for inspiring and encouraging me every day.

I would like to thank the Precast/Prestressed Concrete Institute for funding this research, and members of the R&D Committee for sharing their insights and knowledge. Thanks to all the industry partners who provided support in time, donations and funding. Special thanks to Gary Klein of Wiss, Janney and Elstner for providing project funding and additional contributions to the research effort. Further thanks to Harry Gleich of Metromont for supplying the specimens tested in this thesis and to the Metromont crew at the Charlotte, NC plant for fabricating them. Special thanks to Dr. Paul Zia and Dr. Sami Rizkalla for their unwavering guidance and support; the knowledge you have imparted goes beyond that just found in text books. Thanks to Dr. Rudi Seracino and Dr. Vernon Matzen for joining my advisory committee and to all my professors who played such important roles in my life during this time of change and growth.

The testing schedule would not have been kept without the help of some great undergraduate assistants: Kurtis, Chad, Kent, Bradley and Jacob. The CFL staff, who make research projects so successful here at NCSU, deserve a round of applause for all they do: Jerry Atkinson, Bill Dunleavy, Lee Nelson, Amy Yonai and Diana Lotito.

Last but not least, I would have been lost without the friendship and guidance of Gregory Lucier, not only in completing this research and thesis but surviving life in general.
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1. Background
Precast concrete spandrel beams, commonly known as spandrels, are typically used in parking structures to transfer vertical loads from deck members to columns. Typical spandrels are simply supported by column haunches or corbels. In many cases, a continuous ledge runs along the bottom edge on one side of the beam, resulting in what is known as an L-shaped spandrel. In other cases discrete haunches are used in place of the continuous ledge creating what is known as a corbelled spandrel. Since either the ledge or corbels are used to provide bearing for the deck sections, the typical precast spandrel beam is therefore subjected to a series of discrete eccentric loads. Eccentrically loading the slender cross-section results in a complex structural behavior which can require complicated reinforcement details. Frequently steel is heavily congested in critical zones such as the end regions where prestressing strands and reinforcing bars have to be woven through numerous closed stirrups that are closely spaced as required by the ACI-318 Code.

The ends of these spandrels are also connected to the columns to prevent out-of-plane rotation about the longitudinal axis. In addition, the deck sections are often connected to the spandrel web inner face at discrete locations, providing lateral restraint along the span. The eccentric location of the applied loads with respect to the unsymmetrical L-shaped spandrel cross-section causes vertical displacement in addition to significant lateral displacement and rotation. The maximum torsional and shear effects occur in the end regions. Figure 1.1 depicts a typical slender L-shaped spandrel. This particular spandrel supports four double-tees and one centered single-tee. Point loads from the deck sections are shown along the
ledge, column reactions are shown at each end, and lateral restrain forces from the spandrel-deck connection are shown along the inner face of the web.

![Figure 1.1: Loads and reactions typically imposed on a precast slender spandrel.](image)

The current procedure used by the Industry to design precast and prestressed spandrel beams is based on test data taken from reinforced concrete beams having compact cross-sections. When applied to the slender cross-sections of typical L-shaped spandrel beams, the resulting closed reinforcement is usually tightly congested and difficult to install. While the current design practice results in safe, reliable beams, observed behavior and limited experimental data have prompted many within the Industry to question the need for heavy, closed reinforcement in the end regions of slender spandrels. It has been suggested that closed stirrups could be eliminated from the design of slender spandrels in favor of a more economical and production-friendly open reinforcement approach.
1.1. Objective
The objective of the ongoing research program is to study the behavior of precast, concrete L-shaped slender spandrel beams and to develop a rational design methodology for said beams loaded eccentrically along their bottom edge. It is intended that any underlying assumptions of the proposed methodology closely reflect the observed response and failure modes of slender spandrel members. In addition, the proposed design methodology should allow for simplified detailing of reinforcement when compared to current practice. The methodology itself must be straightforward enough to be used in everyday practice.

1.2. Scope
To complete the primary objective, the scope of the investigation included the following:

1. A comprehensive literature review of the development of torsion design for precast concrete members. The review included published reports of field observations of slender spandrel beams in service.

2. An experimental program including testing full-scale slender L-shaped spandrel beams with various reinforcing schemes.

3. Develop and calibrate a three-dimensional, nonlinear finite element model (FEM) to predict the behavior of precast concrete slender spandrel beams.

4. Use the model to study the effect of various parameters believed to affect the behavior of these types of beams.

5. Validation based on a thorough understanding of the analytical model and comparing proposed equations to experimental results.
2. Literature Review

2.1. Overview
The literature presented in this chapter includes published experimental work, analytical studies and field observations of precast spandrel beams. This chapter provides a summary of the survey including: 1) basic concept of torsion design for reinforced and prestressed concrete members, 2) the current practice for designing slender spandrel beams, 3) challenges to the current practice 4) previous research and observed behavior of slender spandrel beams, 5) the observed skew-bending failure mode of slender spandrel beams and 6) a preliminary proposed approach for designing slender precast spandrel beams.

2.2. Basic Concept of Torsion Design
The first design provisions for torsion in reinforced concrete were published in the ACI Building Code 318-71, however, they were not applicable for prestressed concrete. Zia and McGee introduced the first design methodology for beams subjected to bending, shear and torsion in 1974 (Zia et al. 1974). The proposed method provided equations that were used to determine the induced nominal shear and torsional stresses based for a given cross-section and reinforcing material properties. The method provided also an evaluation of the concrete contribution to the member’s overall capacity, in addition to web reinforcement required for shear and torsion resistance. In a later study (Zia et al 1976), the researchers determined that the minimum reinforcement required by ACI 318-71 for flexural shear was inadequate for a prestressed member subjected to combined loading conditions. In 1977, the ACI Building Code was changed to be based on forces and moments instead of stresses. Zia and Hsu presented updated equations from the original Zia and McGee paper at a 1978 ASCE
convention for the torsion design of prestressed spandrel in the new terms (Zia et al 2004). At this same time research at the University of Miami, supervised by Hsu, was introduced to determine the minimum torsional web reinforcement for prestressed concrete. These equations were adopted by Zia and Hsu and incorporated into their design guidelines for prestressed members subjected to combined loadings, which would later become the basis for the current PCI guidelines for torsion in concrete members.

2.3. Current Practice
The current practice recommended by the American Concrete Institute for proportioning reinforcement to resist shear and torsion within a concrete member is based on a space truss analogy (ACI 318-08). Longitudinal steel and closed stirrups are provided to resist torsional stresses which are assumed to develop and spiral along the length of a member. Well distributed longitudinal steel and closed ties serve to maintain the integrity of the concrete core enclosed within the stirrups, allowing inclined compression struts to develop and resist the applied forces. The ACI approach assumes that later stage member response will be characterized by spalling of the concrete face shell outside the stirrups. Researchers have recommended detailing such as 135-degree stirrup hooks to maintain the integrity of the concrete core after spalling (Mitchell et al 1976). These detailing requirements often require tightly congested, interwoven reinforcement, especially in the end regions. It is important to mention that within the torsion provisions of ACI-318-08, there is a stipulation allowing for alternative approaches to the torsion design of solid sections having an aspect ratio of three or greater.
The most current guidelines for designing beams subjected to combined loading is outlined in the 6th edition PCI Design Handbook, and similar to the ACI approach, makes assumptions of face shell spalling and spiral cracking. The design method presented by Zia and Hsu (Zia et al 2004) provides the basis for the following design equations. The method can be used for non-prestressed and prestressed members and is based on the sectional torsional strength of compact rectangular specimens, ie aspect ratio less than 3. Currently, this procedure is also commonly used to proportion shear and torsion reinforcement in slender spandrel beams.

1. Determine the design ultimate shear, $V_u$, and the design ultimate torsional moment, $T_u$, at the critical section for shear and torsion. The critical section, determined per ACI 318, is “d” from the face of the support for non-prestressed members and “h/2” for prestressed members; “d” is to be taken from the point of load application for a spandrel beam loaded along the ledge.

2. Determine if torsion can be neglected based on specimen cross section and concrete and prestressing material properties.

$$T_u = \Phi \gamma \lambda f'_c (\sum x^2 y) \gamma$$

Equation 2-1

Where:
- $T_u =$ Factored torsional moment, lb-in
- $\Phi = 0.75$
- $\lambda =$ Conversion factor for lightweight concrete
- $f'_c =$ Concrete compressive strength, psi
- $x, y =$ Short side and long side, respectively, of a component rectangle, inch
- $\gamma =$ A factor dependent on the level of prestress
  - $\gamma = \sqrt{1 + 10 \frac{f_{pc}}{f'_c}}$
  - $\gamma = 1.0$ for non-prestressed sections
- $f_{pc} =$ Average prestress after losses
If $T_u \leq T_u(\text{min})$ no torsion reinforcement is needed and design is complete.

3. If torsion cannot be neglected, check that required nominal torsional moment and shear strengths are at appropriate limits so that potential compression failures, due to over-reinforcing, do not occur.

$$T_{n(\text{max})} = \frac{\left(\frac{1}{3}K_t\lambda \sqrt{f_c' \sum x^2 y}\right)}{\sqrt{1 + \left(\frac{K_tV_u}{30C_tT_u}\right)^2}} \geq \frac{T_u}{\varphi} \quad \text{Equation 2-2}$$

$$V_{n(\text{max})} = \frac{10\lambda \sqrt{f_c'b_w d}}{\sqrt{1 + \left(\frac{30C_tT_u}{K_tV_u}\right)^2}} \geq \frac{V_u}{\varphi} \quad \text{Equation 2-3}$$

Where:

$$K_t = \gamma \left(12 - 10 \frac{f_{pc}}{f_c'}\right)$$

$V_u$ = Factored shear force, lb

$C_t = \frac{b_w d}{\sum x^2 y}$

$b_w$ = Web width of member, inch

d = Effective depth of member, inch

4. The shear and torsion interaction has long been represented by a circular curve. When the requirements in Step 3 are met, calculate the nominal torsional moment and shear strength provided by the concrete.

$$T_c = \frac{T_{c'}}{\sqrt{1 + \left(\frac{T_{c'}}{T_u}\right)^2} + \frac{V_{c'}}{V_u}} \quad \text{Equation 2-4}$$
\[
V_c = \frac{V'_c}{\sqrt{1 + \left( \frac{V'_c}{T'_c} \right)^2}}
\]

Equation 2-5

Where:
- \( T_c \) = Nominal torsional moment strength of concrete under combined shear and torsion
- \( V_c \) = Nominal shear strength of concrete under combined shear and torsion
- \( V'_c \) = \( 0.6\sqrt{f'_c + 700\frac{V_c d}{T_U}} \) = Nominal shear strength of concrete under pure torsion
- \( T'_c \) = \( 0.8\lambda\sqrt{f'_c \sum x^2 \gamma(2.5\gamma - 1.5)} \) = Nominal torsional moment strength of concrete under pure torsion

5. Provide stirrups if the torsional moment is greater than that carried by the concrete.

These stirrups are in addition to those required for shear.

\[
A_t = \left( \frac{T_u - T_c}{\phi} \right) s
\]

Equation 2-6

To ensure reasonable member ductility, a minimum area of closed stirrups should be determined:

\[
(A_v + 2A_t)_{\text{min}} = 50 \frac{b_w s}{f_y}(\gamma)^2 \leq 200 \frac{b_w s}{f_y}
\]

Equation 2-7

Where:
- \( A_t \) = Required area of one leg of closed tie, in\(^2\)
- \( x_1 \) = Short side of closed tie, inch
- \( y_1 \) = Long side of closed tie, inch
- \( s \leq (x_1+y_1)/4 \) or \( 12 = \) tie spacing, inch
- \( \alpha_t \) = \([0.66 + 0.33 \frac{y_1}{x_1}] < 1.5 = \) torsion coefficient
- \( f_y \) = Yield strength of closed tie, psi
- \( A_v \) = Area of shear reinforcement, inch
6. Provide longitudinal reinforcement to resist the longitudinal component of the diagonal tension induced by torsion. This longitudinal steel is in addition to that calculated for flexure.

\[ A_l = \frac{2A_i(x_i + y_i)}{s} \]  \hspace{1cm} \text{Equation 2-8}

Or

\[ A_i = \left[ \frac{400x}{f_y} \left( \frac{T_u}{T_u + V_y/3C_t} \right) - \frac{2A_i}{s} \right] (x_i + y_i) \]  \hspace{1cm} \text{Equation 2-9}

The value of \( A_l \), calculated in Equation 2-9, should not exceed that obtained when substituting:

\[ \frac{50b_w}{f_y} \left( 1 + \frac{12f_{pc}}{f_{c}} \right) \leq \frac{200b_w}{f_y} \text{ for } \frac{2A_i}{s} \]

The current PCI handbook does address out-of-plane bending in L-shaped spandrel beam end regions. An equation is given for determining the amount of vertical \( A_{wl} \) and longitudinal \( A_{wl} \) reinforcement on the inner face. This steel is to be distributed across a height and width equal to the distance between the two lateral equilibrium reactions.

\[ A_{wl} = A_{wl} = \frac{V_u e}{2\varphi f_y d_w} \]  \hspace{1cm} \text{Equation 2-10}

Where:
- \( V_u \) = Factored shear force at critical section
- \( e \) = Eccentricity, distance between ledge load and main vertical reaction
- \( \varphi \) = 0.75
- \( f_y \) = Yield strength of reinforcement
- \( d_w \) = Depth of \( A_{wl} \) and \( A_{wl} \) reinforcement from outside face of beam
2.4. Challenges to Current Practice
The approaches to torsion design advocated by ACI and PCI are widely accepted, and are routinely applied to the design of both compact and slender members. While the appropriateness of these approaches is well documented for compact sections, many have challenged the validity of applying the same design methodology to slender precast members. Decades of field observations, limited experimental testing and observation of slender section behavior have supported the argument that design approaches relying on assumptions of spiral cracking and face shell spalling are inappropriate for application to slender members. If torsional distress is not generated by eccentric loading of slender precast members, then classical torsion design procedures and detailing requirements cannot be justified.

Field Observations
Field observations of precast slender spandrel behavior have been discussed among precast producers and engineers for over six decades. However, the in-service behaviors of precast slender spandrel beams were not well documented until 1984 when a report was published on in-field spandrel beam behavior and design (Raths 1984). A substantial portion of the report describes field observations of structural distress and failures of parking garages. This document, perhaps the most thorough account of precast spandrel behavior to date, contains no evidence of a precast slender spandrel developing internal torsional distress. Instead, extensive out-of-plane bending and web face cracking were observed to resist what is referred to as “beam end torsion,” the end couple acting to restrain the beam from rolling inward due to the eccentrically applied loads.
Available Reported Research

Previous research on slender precast spandrel beams was conducted in response to the in-field observed behavior that indicated these particular beams were not failing due to typical torsion behavior which is characterized by spalling of the concrete cover. Instead, significant plate-bending effects were being observed in the end regions up to failure. Selected load tests on precast L-shaped spandrel beams have been documented since the early 1960s. Informal testing of this nature was commonly carried out by precast producers to investigate design issues which had not been formally documented at the time.

A test conducted in 1961 was unable to generate either torsional rotation or distress in a precast L-shaped spandrel subjected to eccentric vertical loading (Logan 2007). The spandrel was 20 feet long with a web aspect ratio of 2.4. The beam experienced diagonal cracking at roughly 45 degrees in the end region on the inside face (ledge side) that began to flatten out towards midspan. The outside face experienced vertical cracking near midspan that extended upwards from the bottom of the spandrel. The failure mode of the spandrels was an out-of-plane fracture in the end region. The author concluded that the torsional distress needed to cause the classic symptoms of concrete spalling and spiral cracking was not possible in the tested beam as the web was incapable of distributing the internal torsion, caused by the boundary conditions and eccentric loading. The necessity of providing complex torsion reinforcement, ie closed web stirrups, in slender L-shaped precast members was also questioned.

Formal research into the behavior of slender spandrel beams was published in 1986 (Klein 1986). This research, funded by the Precast/Prestressed Concrete Institute, included full scale
testing of two L-shaped spandrel beams loaded eccentrically through the ledge. The specimens were 28 feet long and had an aspect ratio of 9. The shear and torsion design was based on the method by Zia-Hsu published in the current PCI Design Handbook. The spandrels were loaded by deck sections consisting of double-tees and a single-tee. The spandrel inner face to deck connection was modified so that it could be removed or added when required. The beams were first loaded to service without the connection, where no cracks were observed, and then unloaded. The connections were put in place and the beams reloaded to service and then unloaded. The beams were then reloaded to failure without the deck connection. The results confirmed that neither spiral cracking nor face shell spalling occurred in any of the tested beams. Instead, a rainbow cracking pattern was observed on the inner faces and flexural cracking on the outer faces. The inclined cracks in the end region of the inner faces were at approximately 45 degree angles before flattening out towards midspan. The two L-shaped specimens failed either due to ledge separation or ledge punching. It was observed that the connection between the deck section and spandrel inner face restrained the lateral displacement of the spandrel beams. These results were also confirmed in a finite element study performed by Klein.

More recently, a group of precast producers partnered with North Carolina State University to conduct preliminary research and failure testing on full-scale precast slender spandrel beams (Lucier et al 2007). Four beams were designed neglecting conventional torsion procedures. The main objective of the authors’ tests was to determine if closed stirrups could be eliminated from slender L-shaped spandrel design; this was done by using a combination of L-bars, U-bars, C-bars and welded-wire reinforcement (WWR) as web reinforcement.
Four L-spandrels were tested, two with spans of 30 feet and two with 45.5 feet, all with an aspect ratio of 7.5. The beams consisted of virtually no closed reinforcement, relying on an open reinforcing scheme to resist flexure, shear and out-of-plane plate-bending. Specimen reinforcement was enhanced to prevent flexural failure and localized failure modes in order to study end-region behavior. As there were no set design guidelines for using open reinforcement in slender L-shaped spandrel beams, the authors described a general design approach they followed. Reinforcing requirements were met for traditional hanger reinforcement and vertical shear reinforcement. Ignoring torsion as the controlling failure mode, the authors instead reinforced the beam for out-of-plane bending; further details of this design approach will be discussed in Section 2.5. All four specimens were able to sustain to meet the recovery criteria set forth by ACI 318 and achieve an ultimate load carrying capacity beyond the full factored design load. While not all four beams ultimately failed due to out of plane bending in the end regions, localized failures along the spandrel ledge controlled for the 30 foot spans, the observed cracking pattern was identical for all four beams. The results from this study demonstrated that out of plane behavior was observed in slender spandrel beams loaded along the ledge and confirmed the absence of classical torsional distress associated with the current design.

A non-linear finite element modeling (FEM) approach has been proposed for the modeling of L-shaped, precast, prestressed concrete spandrels (Hassan et al 2007). The analytical model was calibrated using data from a previous experimental program (Lucier et al 2007) and then used to further analyze the behavior of slender L-shaped spandrel beams as well as that for more compact L-shape spandrel beams. Results from the analytical study indicate that the
model was able to predict the overall behavior of the experimental results for two slender L-shaped spandrel beams, including the skew-bending failure plane seen experimentally in the end regions. Several boundary conditions were studied, including the influence of the inner face spandrel to deck welded connection. Springs were used to model the properties and effects of these connections. It was concluded that these welded connections greatly affected the lateral displacement of the spandrel as it bent about its weak axis but did not greatly influence the ultimate load carrying capacity.

2.5. Skewed Failure Plane
It has been well documented that the failure mechanism for slender reinforced concrete sections subjected to combined flexure, shear, and torsion will be in the form of skew bending (Klein 1986, Logan 2007, Lucier et al 2007, Raths 1984).

The skewed failure plane may be idealized by four edges, each inclined at 45-degrees with respect to the next, forming a distorted surface. Previously published research (Hsu 84 and Walsh et al 1966, 1967) has defined three possible skew bending failure modes, depicted in Figure 2.1. All three modes are characterized by a cracked tensile zone, extending along three exterior faces that form a rectangular helix with a compression zone on the fourth face.
A Mode 1 failure, as shown in Figure 2.1a, occurs when the compression zone crosses the top surface of the beam and a Mode 3 failure, as shown in Figure 2.1c, occurs when the compression zone forms along the bottom surface. Mode 3 failures were observed in beams designed primarily for flexure, specimens were reinforced with more bottom longitudinal steel than top, and then tested under predominately torsional loads. It was determined that increasing the bending moment could help to prevent yielding of the top longitudinal steel and therefore increase the torsional capacity (or strength) of the beam. If the bending moment was too large then the tension in the bottom longitudinal steel was increased, leading to yielding of the steel and a Mode 1 failure.
A Mode 2 failure, as shown in Figure 2.1b, occurred when the compression zone, ie failure plane, occurred on the side of the specimens. The compression zone formed on the side of the beam where the shear and torsional stresses were subtractive. In the case of L-shaped spandrel beams loaded along the ledge, the compression zone occurred on the non-ledge side, as shown in Figure 2.2.

Mode 2 was observed when the applied torsional moment and shear force were much greater than the applied flexural moment. At failure the two ends of the beam, separated at the failure surface, rotated against each other about a neutral axis along the compression zone.

The observed skewed failure mechanism does not exhibit the characteristic spiral cracking and face shell spalling assumed as the basis of the current ACI and PCI approaches for torsion design. In compact rectangular sections where this failure mode and spiral cracking may be observed, the failure plane is crossed effectively on all four faces by closed ties. For slender sections, however, the value of the shorter legs of the ties is questionable because the projection of the failure plane crossing the narrow face of the member is often less than the longitudinal spacing of such ties, as shown in Figure 2.3. As the short legs of the closed
stirrups have such a minimal effect in strengthening slender member it has been suggested that they could be replaced with combinations of straight bars, sheets of WWR and L-shaped bars (Lucier et al 2007). Considering the slender spandrel as an example, the tie legs crossing the top or bottom edge of the web probably have minimal effect on the overall torsional resistance of the member. Rather, it is the vertical legs of such closed ties which are providing the bulk of torsional resistance, since these legs cross the failure plane along the much longer inner and outer faces of the web. For a typical web thickness (8 inch) of a slender spandrel, the contribution of the short legs of stirrups spaced at more than 4 inches is questionable but such tight stirrup spacing is uncommon for long, slender members. If the assumed skewed plane is at 45 degrees across all four sides of the member, the short legs of any closed ties would only effectively contribute to torsional resistance of the member if tie spacing were maintained at one-half of the web thickness. Tie spacing greater than the web thickness creates a situation in which the top edge of the failure surface would likely pass between adjacent ties, as shown in Figure 2.3. Therefore the geometry of the skewed failure surface itself seems to indicate that the short legs of closed ties do not significantly contribute to torsional resistance in slender spandrels.
Figure 2.3: Project Length of Failure Planes for Compact and Slender Sections
Preliminary Design Approach
For many years, the precast prestressed concrete industry has suggested a need for an alternative procedure for the current torsion design for end region reinforcement in slender spandrel beams. The new design procedure would design end region reinforcement assuming skew-bending failure instead of classical failure mechanisms associated with torsion.

The forces that cause a skew-bending failure are due to induced torsion in the spandrel web. The applied torque, $T_u$, is caused by ledge loading and boundary conditions and is taken along the web’s centroid. The torque can be analyzed into two components along the assumed 45-degree failure plane, as given in Figure 2.4. The component that causes the spandrel to bend about the skewed failure plane, $T_{ub}$, is taken along the failure plane. The “twist” component, $T_{ut}$, to be discussed later, is taken normal to the failure plane.

![Figure 2.4: Components of Torque](image-url)
The web is bent about the upper connection along a diagonal, as shown in Figure 2.5. The inner face concrete along the diagonal is put in tension and the outer face in compression. The result is a skew-bending failure mode (i.e., Mode 2 failure).

As no formal approach currently exists for designing a slender spandrel beam without closed reinforcement, the following preliminary guidelines were followed to design two of the experimental beams described in this thesis with open reinforcement. The general approach followed in the design was taken from previously published research of precast, slender L-shaped spandrel beams (Lucier et al. 2007) with identical cross-sections to the experimental spandrel beams described in this thesis. The approach uses an open reinforcement configuration and is designed to prevent plate-bending failure in the end regions. It is assumed in all calculations that the eccentricity of the load, \( e \), is taken from the spandrel web’s centerline. A phi factor of 0.9 was assumed for all calculations.
1. Ignore traditional torsion provisions but follow conventional methods for:
   a. Flexural reinforcement
   b. Vertical shear reinforcement (assuming e=0)
   c. Ledge design (hanger steel, ledge punching, etc.)

2. Ignoring the torque generated by the eccentric dead load of the ledge itself, determine the applied torque, $T_u$, and lateral reactions at the column connections:
   a. $T_u = \text{Applied Ledge Load} \times e$
   b. Lateral Reactions = $T_u / \text{Distance Between Reactions}$ (assume top and bottom lateral reactions are equal and opposite)
3. Assume failure plane along a 45-degree line, extending upward from center of the bottom connection.

4. Determine the moment, $T_{ub}$, acting along the assumed failure plane.
   a. $T_{ub} = R1 \times$ Top Lateral Reaction

5. Select inner face vertical steel to satisfy either:
   a. $\frac{1}{2}$ the vertical shear requirement
   b. AND the component of plate bending ($T_{ub}$) in the end region
   c. OR hanger steel requirements per current codes and guidelines

6. Select outer face vertical steel to satisfy:
   a. $\frac{1}{2}$ the vertical shear requirement

7. Select horizontal steel within the end region to satisfy:
   a. Plate bending component ($T_{ub}$) on the inner face and
   b. Temperature, shrinkage, etc. on the outer face
A combination of welded wire reinforcement (WWR) and L-shaped mild-steel reinforcing bars may be used to meet the overall inner and outer face vertical steel requirements. In the end regions, the L-shaped reinforcing bars and WWR can be supplemented with longitudinal U-shaped mild-steel reinforcing bars to meet horizontal reinforcing requirements.

**Twist Component of Torsion**
There are two components of torsion acting action at the 45-degree failure surface: plate bending and twist. The plate bending component is reflected in the design equations shown above. The twist component is ignored, but the influence of the twist component is evident in previous experimental testing (Klein 1986, Lucier et al 2007). A possible model for the resistance to the twist is currently being developed.
3. Experimental Program
The experimental program consisted of testing full scale slender, precast L-shaped spandrel beams to study various limit state behaviors, including at service and factored loading, and failure modes. The experimental program simulated typical field conditions in terms of size, span, loading and connection details.

3.1. Specimen Design
In this research program, slender continuous ledge spandrel beams with an aspect ratio of 7.5 were tested. All of the precast, L-shaped spandrels had identical cross sections with the web dimensions of 60 by 8 inches and the ledge with dimensions of 8 by 8 inches. The specimens were 45.5 feet long and had a 43.5 foot long continuous ledge; the ledge was cut back 12 inches on either end of the spandrel to facilitate the typical web to column bearing connection. Typically, a spandrel web would never be shorter than 60 inches or have a small depth than 8 inches. All specimens were designed to resist typical dead, live, and snow loadings for parking structures. The design assumed that each spandrel supported half of a 60 foot span continuous deck. Views of a typical slender specimen are shown in Figure 3.1. For the remainder of this thesis the ledge side of the specimen will be called the inner face and the non-ledge side will be called the outer face.
Figure 3.1: Typical Specimen
The test matrix for the experimental program is given in Table 3.1. The experimental program consisted of four precast concrete slender L-shaped spandrel beams tested to failure. Two beams were prestressed and the other two were reinforced with conventional reinforcement. Within each of the two reinforcing types, the end regions of one member was designed following the Zia-Hsu procedure in the current PCI Design Handbook (SP11 and SP13) using closed stirrups and the other designed using a preliminary design approach briefly described in Section 2.5 (SP10 and SP12) with open reinforcement. Outside of the assumed failure planes, specimens were designed per current standards but spandrels SP11 and SP13 used typical detailing while spandrels SP10 and SP12 used an open reinforcing scheme.

Table 3.1: Test Matrix

<table>
<thead>
<tr>
<th>Designation</th>
<th>Concrete</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reinforced</td>
<td>Prestressed</td>
</tr>
<tr>
<td>SP10</td>
<td>♦</td>
<td>♦</td>
</tr>
<tr>
<td>SP11</td>
<td>♦</td>
<td>♦</td>
</tr>
<tr>
<td>SP12</td>
<td>♦</td>
<td>♦</td>
</tr>
<tr>
<td>SP13</td>
<td>♦</td>
<td>♦</td>
</tr>
</tbody>
</table>
3.2. Reinforcement Details
The steel reinforcing cages for the two prestressed beams, SP13 and SP12, are shown in Figure 3.3 and Figure 3.2 respectively.

![Figure 3.2: Closed stirrup configuration](image)

![Figure 3.3: Open stirrup configuration](image)
Note that the vertical reinforcement on both the inner and outer faces was supplemented with welded wire reinforcement (WWR) when designing with open reinforcement, as shown in Figure 3.3. Also there is larger spacing between the L-bars in the end region in comparison to the specimens designed with closed reinforcement. Note that the outer faces of the spandrels are face down in the forms during casting.

Details of the closed and open reinforcement for the two prestressed beams, SP12 and SP13, are shown in Figure 3.4. The shown vertical reinforcement configuration (stirrups) was also used for the reinforced concrete beams, SP10 and SP11. The primary advantage in using open reinforcement in comparison to conventional closed reinforcement is a substantial savings in labor and material costs. For example spandrel SP10, designed with open reinforcement, has 33% less vertical steel than specimen SP11.

Typical global failure modes found for precast concrete beams include flexure, lateral bending, shear and torsion. Localized failure modes at the connections, punching shear in the ledge, ledge bending and ledge/web detachment have also been observed. It should be noted that all four specimen designs were reinforced with extra flexural reinforcement, approximately 1.5 times the standard requirements, to study the behavior of the end regions which are subjected to heavy combined loading of flexure, shear and torsion. Details of additional extra reinforcement provided follow.
Spandrel SP11
The reinforced concrete control specimen, spandrel SP11, was designed with a closed reinforcing scheme following the Zia-Hsu procedure in the PCI handbook. The specimen was reinforced with Grade 60 entirely closed #4 stirrups, U-shaped #4 bars and longitudinal straight bars of varying diameter.

The web was first reinforced with 99 entirely closed #4 stirrups (see Figure 3.5) at various spacing across the span. After the closed stirrups were put in the casting bed, longitudinal bars of varying diameter and end region #4 U-bars were pulled through and tied in place. The lower half of the web was reinforced with 5 #5 and 4#9 continuous longitudinal bars, 4-30 foot long #9 bars centered at midspan and 2-20 foot long #7 bars centered at midspan, as shown in Figure 3.5. The upper half of the web was reinforced with 6#5 and 2#7 continuous bars. Additional #5 and #6 continuous bars were used to reinforce the ledge. The horizontal
U-bars, spaced between the bottom lateral connection and top of web, were 4.5” wide with 8’ long legs. One additional U-bar, 6” wide with 2’ long legs, was also used to provide confinement around each of the holes in the web used for the column connection.

The ledge to web interface was reinforced with 72 entirely closed #4 stirrups (see Figure 3.5) and special detailing to prevent ledge punching at each of the 9 load points. This ledge detail, used in both SP11 and SP13, consisted of #4 bars welded to 5/16” angle. A sketch of the reinforcing details is shown below in Figure 3.5. Sketches further detailing the reinforcement can be found in Appendix A.

![Figure 3.5: SP11 reinforcing scheme](image)

**Spandrel SP10**
Spandrel SP10 was designed with an open reinforcing scheme following the preliminary design approach for plate-bending, described briefly in Section 2.5. The specimen was
reinforced with Grade 75 W4.0 x W4.0 welded wire reinforcement, Grade 60 C-shaped, U-shaped and L-shaped #4 bars and longitudinal straight bars of varying diameter.

Spandrel SP10 utilized welded wire reinforcement (WWR) style 6 X 6 W4.0 x W4.0 along the outer face starting at 10 feet from each end. The WWR extended the full depth of the web with the first cross wire approximately 1 inch from the bottom of the spandrel. In addition to the outer face mesh, SP10 also had 8 #4 L-shaped bars, at various spacing, spanning the first 9’9” of each end region. Each L-shaped bar was 4’10” high with a 6” leg. After placement of the outer face mesh and L-bars, longitudinal straight bars and end region U-bars were placed in the web. The U-bars, spaced between the bottom lateral connection and top of web, were 4.5” wide with 6’ long legs. Additional U-bars were also used to provide confinement around each of the holes in the web used for the column connection. The longitudinal reinforcement in the lower half of the web and ledge is identical to that found in SP11. 2#7 continuous bars were also used to reinforce the top of the web (reference Figure 3.6).

The inner face reinforcement consisted of 55 additional L-shaped bars at various spacing across the span and weld wire reinforcement style 4 X 4 W4.0 x W4.0 starting at 7 feet from each end. The ledge of SP10 was heavily reinforced to prevent localized failures. The ledge to web interface was reinforced with 54 C-shaped bars and special detailing to prevent ledge punching at each of the 9 load points. This ledge detail, used in SP10 and SP12, consisted of #4 bars welded to 5/16” angle. Sketches further detailing the reinforcement and special ledge details can be found in Appendix A.
Spandrel SP13
The prestressed control specimen, spandrel SP13, was designed with a closed reinforcing scheme following the Zia-Hsu procedure. The specimen was reinforced with 270 ksi prestressing strands, Grade 60 C-shaped, U-shaped and L-shaped #4 bars and longitudinal straight bars of varying diameter. The dimensions of the web and ledge closed stirrups are identical to SP11.

The specimen was prestressed with 13-½” diameter special lo-lax prestressing strands in a 2” grid; 12 in the web and 1 in the ledge. The 11 strands in the lower half of the web and in the ledge were pulled to 31600 lbs (approximately 70% of ultimate) and the 2 strands at the top of the web were pulled to 15800 lbs (approximately 33% of ultimate).

Web reinforcement consisted of 82 entirely closed #4 stirrups at various spacing across the span. After placement of the stirrups in the casting bed, the strands were pulled through and stressed; end region U-bars and longitudinal reinforcement were then tied in place. The
longitudinal reinforcement consisted of 12#4 continuous bars, 2-4 foot long #9 bars centered at midspan and 2-30 foot long #7 bars centered at midspan, as shown in Figure 3.7.

The ledge to web interface was reinforced with 54 entirely closed #4 stirrups and special detailing to prevent ledge punching at each of the 9 load points, as in SP11. Sketches further detailing the reinforcement and special detailing can be found in Appendix A.

![Figure 3.7: SP13 (left) and SP12 (right) Reinforcing Schemes](image)

**Specimen SP12**

Specimen SP12 was designed with an open reinforcing scheme following the preliminary design approach for plate-bending, described briefly in Section 2.5. The specimen was reinforced with 270 ksi prestressing strands, Grade 75 W4.0 x W4.0 welded wire reinforcement, Grade 60 C-shaped, U-shaped and L-shaped #4 bars and longitudinal straight bars of varying diameter.
Welded wire reinforcement style 6 X 6 W4.0 x W4.0 was placed in the casting beds first, stretching across the entire outer face of the specimen. An additional 55 L-shaped bars were placed in the form and the prestressing strands pulled through and stressed. The strand pattern in SP12 is identical to that described for SP13. End region U-bars and longitudinal reinforcement consisting of 2-40 foot long #9 bars, centered at midspan, and 2-30 foot long #7 bars, centered at midspan were tied in place. The ledge was also reinforced with a continuous #6 bar (reference Figure 3.7). The inner face welded wire reinforcement, style 4 X 4 W4.0 x W4.0, was placed on top of the strands starting at 7 feet from each end.

The ledge to web interface was reinforced with 54 C-shaped bars. The special detailing to prevent ledge punching at each of the 9 load points was identical to that in specimen SP10. The dimensions of the ledge C-bar and web L-bar are also identical to SP10. Sketches detailing the reinforcement and special detailing can be found in Appendix A.

**Deck Floor Members**
A set of four double-tees and one single-tee was used to load the specimens along the ledge, at nine discrete load points. The double-tee decks, 10DT26, were 12 feet long and 10 feet wide. The single-tee, one half of a 10DT26, was only 5 feet wide. The deck sections were all reinforced with a combination of welded wire reinforcement, #4 bars and the special connection details needed for the spandrel-deck connections.

**Connection Detailing**
For all four specimens, 8-4x5 ½” Starcon anchors were placed in the inner face of the spandrel to strip them from the forms. 4-8 ton erection anchors were placed along the top of the web to move the specimens into place once inside the testing facility. In the two
specimens designed with an open reinforcing scheme, two sets of back to back C-bars were also placed within 12 inches of each end of the specimens to help confine the end region near the connections. Four inch diameter PVC pipes, two at each end, were cast into the specimens to allow for the lateral support of the spandrel beam to column connection. Five embedded steel plates were cast at the specimen’s inner face for the spandrel-deck welded connection.

3.3. Test Set-up
The general test set-up for the four specimens is shown in Figure 3.8 and Figure 3.10. The overall set-up was designed to match the 3 foot hole spacing in the strong floor at North Carolina State University’s Constructed Facilities Laboratory.
Figure 3.8: Profile View of Test Set-up (a)
Figure 3.9: Profile View of Test Set-up (b)
Figure 3.10: Top View of Test Set-up (a)
Figure 3.11: Top View of Test Set-up (b)
A testing frame consisting of four steel I-beam columns was post-tensioned to the laboratory strong floor. To ensure that the frame remained rigid during testing, the two outside columns were enhanced with stiffener plates. Each pair of columns was then braced together with back to back I-beams to prevent twisting and warping during testing. The spandrels were supported at the web ends by vertical stands and then bolted to the outermost columns with 1 inch diameter B7 rods. The specimens were delivered to the laboratory on a tractor trailer. A 20-ton over-head crane and spreader beam system, as shown Figure 3.12, were used to lift the specimens off of the truck and place on the column/stand supports.

![Figure 3.12: Spreader System for Spandrel Delivery and Removal](image)

Four 12 foot long double-tees (10DT26) and one 12 foot long single-tee spanned the spandrel ledge and a system of reinforced concrete blocks and steel channels post-tensioned to the laboratory strong floor. Two holes were cast into each of the tee-decks, centered along the 10 foot (or 5 foot) width and approximately 2 feet 9 inches from both ends. The holes allowed a
heavy threaded rod to be passed through the double-tees and tied to a system of lower spreader beams and the strong floor below. An upper system of 5.5 foot long spreader beams were centered, widthwise, through the rods and hydraulic jacks placed on top. The load from the jacks was transferred from the upper spreader beams down to the double tee stems through 6 inch by 8 inch steel plates, placed underneath the spreader beams, over the double-tee stems. This set of double-tees and single-tee were used for all four specimens and allowed the specimens to be loaded at nine discreet, evenly spaced points along the ledge. After each use the deck sections were rotated so that the “back” and “front” ends changed positions. Embedded plates at the ends of the double-tees and single-tee and also on the inner-face of the spandrel allowed for welded connection, as shown in Figure 3.13.

![Figure 3.13: Typical Welded Connection](image)
A system of reinforced concrete blocks and steel channels was post-tensioned to the laboratory strong floor to support the front end of the double-tee system. The system consisted of three units of three reinforced blocks and one steel channel, each unit supporting 3 tee-stems. As the 5 foot spacing of the double-tee stems did not match up with the 3 foot spacing of the strong floor, a steel channel spanned each unit of reinforced concrete blocks, as shown in Figure 3.14. Bearing pads were placed between the double-tee stems and steel-channels.
The stem-to-ledge slide bearings consisted of very stiff 4 inch by 4 inch cotton duck pad (Capralon) laminated to a layer of Teflon and 10 inch by 5 inch smooth stainless steel plates, roughly 1/8 inch thick. The bearing pads were flush to the front edge of the ledge, centered beneath the stems and adhered to the spandrel ledge with epoxy. The steel plates were adhered to the bottom of the stem and a thin layer of hydraulic oil was placed between the plate and bearing pad to further reduce the friction. Due to the location and size of the bearing pads, the experimental load was applied roughly 2 inches from the front edge of the ledge or 10 inches from the center of the web.
3.4. Instrumentation

**Vertical Reactions**
Two hundred kip load cells were placed at the spandrel ends, as seen in Figure 3.8 and Figure 3.9, under the web, to support the spandrel and measure the main vertical reactions during loading. A Teflon coated bearing pad was placed between each of the load cells and spandrel beam web ends to ensure the most dramatic end reaction as possible by reducing as much friction as possible. The load cells were calibrated at the beginning of each test to output a load of 22.5 kips, approximately half the dead weight of the in-place test set-up. These reactions were monitored during testing to ensure that the specimens were being loaded evenly through the double-tee stems along the ledge. Typical results from the vertical load cells can be found in the appendices (reference Figures B.1, C.1, D.1, and E.1). As the right and left main load cells output indicated an overall balanced system during loading, all reported results are compared using data from the left main load cell.

**Reactions at Columns**
The spandrels were subjected to rotation due to the vertical loading along the ledge. The top of the spandrel rotated towards the inner-face and the bottom of the spandrel rotated towards the outer-face. To capture the lateral forces at the spandrel-column connection, load cells were placed at the top of the inner-face (100 kip load cell) of the specimens and the bottom of the outer-face (50 kip load cell), as shown in Figure 3.8 and Figure 3.9. The load cells were calibrated and zeroed before the specimens were connected to the columns. The load cells and spandrel beams were laterally connected to the column using four B7 rods. Bolts were tightened at both sides of the rods until each of the four lateral reactions read
approximately 4000 pounds. Typical lateral reactions during testing for the spandrel-column connections are given elsewhere (reference Figures B.2, C.2, D.2, and E.2).

**Vertical and Lateral Deflection**

Four-ten inch string potentiometers were used to measure the lateral displacement of the top and bottom of the web outer face at the right quarter-span and midspan locations, as shown in Figure 3.16a. Six string potentiometers were also used to capture vertical displacement of the web and ledge at both quarter-spans and mid-span locations, as shown in Figure 3.16b. The load-deflection curves for each of the four specimens, found in the appendices (reference Figures B.4-8, C.4-8, D.4-8 and E.4-8), plot the vertical and lateral deflections against the total vertical reaction up to failure. The instrumentation was zeroed before each test under the combined dead load of the spandrel and deck sections.

![Figure 3.16: Location of measured deflections on spandrel cross section](image)
**Rotation**

Inclinometers were placed on the outer face at mid-span and left quarter-span, as well as the left profile of the spandrel, to capture beam rotation.

Clockwise rotation, ie the top of the spandrel beam rotating towards the inner face and the bottom of the spandrel beam rotating towards the outer face, was considered positive in this study. To check the accuracy of the recorded data, the inclinometer output was checked against the deflection output from the lateral string potentiometers; it was found that both gave nearly identical readings for spandrel rotation. Linear potentiometers were used to capture the lateral movement at the top and bottom of the inner face web at the specimen ends along the same plane as the spandrel beam-to-column connections. Rotation data can be found in the appendices (reference Figures B.3, C.3, D.3 and E.3).
**Deck-Spandrel Gap**

A linear pot was also used to measure the gap between the bottom of the single-tee stem and the inner face at midspan of the spandrel beam as it increased laterally during loading, see Figure 3.18. The linear pot was calibrated to record the initial gap between the tee stem and inner face at zero applied load.

![Figure 3.18: Deck-Spandrel Gap Instrumentation](image)

In a previous study (Lucier et al 2006), the same test set-up was used to study the behavior of similar slender L-shaped girders. In that study the lateral movement of the deck sections was measured at the block supported end. It was found that the deck sections did not slide away from the spandrel inner face. As the decks were only 12 feet long there was also negligible deformation along the span of the deck due to the imposed loading condition. Therefore the movement between the deck sections and the spandrel is contributed solely to rotation, ie lateral deflection, of the spandrel inner face. Load versus deck-spandrel gap envelopes can be found in the appendices (reference Figures C.17 and E.17)
Concrete Strain
One hundred millimeter PI gages were used along the top and bottom of the web to measure the flexural tensile and compressive strain in the concrete. Six pairs of 200 millimeter PI gages were placed along the top inner face of the beam, three sets in each of the end regions, to capture concrete strain. Two pairs of 300 millimeter PI gages were also placed at the bottom of the inner face of the spandrel, one pair at each end region. The inner-face gages were arranged into rosettes with one gage placed vertically and the other horizontally, aligned along their centerlines, as shown in Figure 3.19. The PI gages were calibrated for displacement and later modified to output millistrain. The naming convention for the inner face PI gages, given in Figure 3.20, will be followed for the remainder of the thesis. Concrete strain data can be found in the appendices (reference Figures B.10-14, C.10-14, D.10-14 and E.10-14).

Data Acquisition
The Micro-Measurements StrainSmart software was used to calibrate and record the various experimental instrument output. During testing, the horizontal and vertical deflections at midspan and the lateral and main vertical reactions were specifically monitored to analyze specimen behavior.
Figure 3.19: Strain Gage Locations

Figure 3.20: Strain Gage Naming Convention
3.5. Load Application

Loading System
A system of 4-120 kip and 1-60 kip hydraulic jacks was used to simultaneously load the spandrel beam through the deck. The jacks were connected with flexible hoses through a series of valves and manifolds to an electric pump. The force from the jacks was transferred to the double-tee stems via spreader beams, as shown in Figure 3.10 and Figure 3.11. The four double-tees, one single-tee deck configuration was chosen to best fit the overall length of the experimental spandrel beams.

Loading Sequence
The load sequence was based off the dead, live and snow load combinations specified by current guidelines. The specified loads for the experimental deck section used, made up of 10DT26 double-tees and single tee are:

- Dead Load=71.6 lbs/ft²
- Live Load=40 lbs/ft²
- Snow Load=30 lbs/ft²

The service loads, per stem, based on a 60 foot long deck are:

- Dead Load=10.74 kips/stem
- Live Load=6.0 kip/stem
- Snow Load=4.5 kips/stem
All four specimens weighed approximately 25.7 kips or 567 lbs/ft. The self weight of a experimental spandrel and half of the 12 foot long deck sections, before external load application, was approximately 45 kips. Therefore the main vertical reaction at “zero load” was actually 22.5 kips. The spandrel beams alone contributed approximately 12.8 kips per main vertical reaction.

The four significant load levels for all specimens are given in Table 3.2. These four values include the initial main vertical reading of 22.5 kips at zero load. The average externally applied load at each of the nine stem to ledge bearing reactions are also given; these values exclude the initial self weight of the test set-up.

Table 3.2: Significant Load Levels in Test

<table>
<thead>
<tr>
<th>Designation</th>
<th>Load</th>
<th>Vertical Spandrel Reaction (kips)</th>
<th>Externally Applied Stem Load (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service</td>
<td>DL+LL</td>
<td>88.1</td>
<td>14.6</td>
</tr>
<tr>
<td>ASCE7 Service with Snow</td>
<td>1.0DL+0.75LL+0.75SL</td>
<td>96.6</td>
<td>16.5</td>
</tr>
<tr>
<td>Service with Snow</td>
<td>1.0DL+1.0LL+1.0SL</td>
<td>108.4</td>
<td>19.1</td>
</tr>
<tr>
<td>Full Factored Design</td>
<td>1.2DL+1.6LL+0.5SL</td>
<td>126.6</td>
<td>23.1</td>
</tr>
</tbody>
</table>

**First loading cycle (Service Load)**

The external loading was applied as shown in Figure 3.21 at approximately 1 kip per second. Due to the constraints of the laboratory environment it was not plausible to test the specimens with a typical 60 foot long deck section. As the deck in the experimental program was only 12 feet, the initial dead load per stem was only 2.15 kips/stem. In order to compensate for the missing 48 feet of dead load for a system with 60 foot decks, an additional 8.59 kips/stem was applied using the hydraulic jacks. Therefore the main vertical reaction for the total equivalent dead load for the system (with 60 foot deck section) is 61.1
kips. Before the load was taken to service load (vertical end reaction of 88.1 kips), the load was paused at the 61 kip load level to check for cracking. The live load was then added to take the specimens to the service load. After reaching service, the load was paused while researchers marked visible cracks and took pictures. The beams were then unloaded and reloaded to the next load level.

Second Loading Cycle (ASCE7 Service with Snow Load)
The second loading cycle was designed to meet the requirements for ASCE7 with snow load \((Vu=96.6 \text{ kips})\). This required the addition of 0.75 live load and 0.75 snow load to the overall dead load of the spandrel beam and a 60 foot deck section. Once again the load was paused to mark cracks and make observations and then the beams were unloaded. This reloading and unloading continued for two more cycles.
Third Loading Cycle (Service with Snow)
The third loading cycle was designed to meet requirements for service with snow ($V_u=108.4$ kips). This required the addition of 1.0 live load and 1.0 snow load to the total equivalent dead load.

Fourth Loading Cycle (Full Factored)
The forth loading cycle was designed to meet requirements for a Full Factored Ultimate Design Load ($V_u=126.6$ kips). The total equivalent dead load was increased by 20%, the live load increased by 60% and the snow load decreased by 50%. This load level was held for 24 hours and then unloaded. The specimens were given one hour to recover to make sure that the requirements of ACI 318 for recovery, clause 20.5, were met.

Cycles to Failure
After the one hour recovery, the specimens were reloaded to a vertical reaction of 140 kips, pausing first at 126.6 kips, and then unloaded. The beams were reloaded to 160 kips and the cycles continued, increasing roughly 20 kips for each new cycle, until failure occurred. The actual cyclic data recorded during the experimental testing for all four specimens can be found in the appendices. Due to the number of data points recorded throughout the testing program, significant data in the body of this thesis will be represented in loading envelopes unless otherwise noted.
4. Experimental Results
This chapter summarizes the properties of materials used in fabricating test specimens, test results for the experimental program and overall specimen behavior based on measured data and observations. The experimental program consisted of four tests to failure of four slender precast L-shaped spandrel beams.

4.1. Material Properties
The four tested spandrel beams were fabricated at the same time using the same batch of reinforcement and concrete and produced by the same precast company. The following material characteristics are representative for all tested beams.

Concrete
Four by eight cylinders were made during casting of the experimental beams in order to determine material properties of the concrete and were tested by the fabricator at 28 days. The measured average compressive strength at 28 days was 6500 psi. After testing of the beams, at an age of five months, a two by four in-house core was taken from an uncracked section of spandrel SP12 and the measured compressive strength was found to be 6800 psi.

Reinforcement
Representative samples of #4 bars and WWR were tested to determine their material properties. The samples were tested using a 220-kip capacity MTS universal testing machine with hydraulic wedge grips. The strain was monitored at the center of the samples using a 2” gage length electronic extensometer. The specimens were approximately 2 feet in length. The measured stress-strain data for the #4 bars are shown in Figure 4.1. Samples from the welded wire reinforcement used, containing two cross-wires within their length, were also tested to
determine their characteristics. The length was selected as three times the mesh spacing, therefore containing two cross wires at the center along the length. The measured stress-strain curve for the welded wire reinforcement is shown in Figure 4.2. During testing the loading rate was kept approximately constant at a rate of 0.005 inch/second for all tests.

The measured material properties of three samples for #4 bar reinforcement are given in Table 4.1. The measured results were used to determine the average elastic modulus, yield strength and ultimate strength for the #4 bars. Test results indicate that yielding of the bars occurred at approximately 0.002 as expected for mild steel reinforcement.

Table 4.1:#4 Bar Reinforcement Properties

<table>
<thead>
<tr>
<th></th>
<th>Elastic Modulus (Msi)</th>
<th>Yield stress (Ksi)</th>
<th>0.2% Yield Stress (Ksi)</th>
<th>Ultimate Stress (Ksi)</th>
<th>Ultimate strain (extensometer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>29</td>
<td>61</td>
<td>66</td>
<td>99</td>
<td>.136</td>
</tr>
<tr>
<td>Test 2</td>
<td>30</td>
<td>55</td>
<td>66</td>
<td>99</td>
<td>.220</td>
</tr>
<tr>
<td>Test 3</td>
<td>28</td>
<td>49</td>
<td>65</td>
<td>99</td>
<td>.142</td>
</tr>
<tr>
<td>Average</td>
<td>29</td>
<td>55</td>
<td>65</td>
<td>99</td>
<td>.166</td>
</tr>
</tbody>
</table>
The welded wire reinforcement, used for the open reinforcement configuration, was also tested; however, the results were less uniform. As the WWR was smooth and had a small diameter (0.225 inches for the 4 inch by 4 inch and 0.276 for the 6 inch by 6 inch) it was difficult to keep the extensometer gage in place during testing to measure the elongation.

Since the measured results of the 6 inch by 6 inch tensile tests were more consistent, their material properties were considered to be typical for all the WWR used, as given in Table 4.2. These results were compared to the published values for WWR in the PCI handbook and found to be within the recommended range. None of the WWR samples reached a well defined yielding plateau before ultimate.

Table 4.2: Welded Wire Reinforcement Properties (6 by 6)

<table>
<thead>
<tr>
<th></th>
<th>Elastic Modulus (Msi)</th>
<th>Yeild stress (ksi)</th>
<th>0.2% Yield Stress (ksi)</th>
<th>Ultimate Stress (ksi)</th>
<th>Ultimate strain (extensometer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>31</td>
<td>~</td>
<td>~</td>
<td>77</td>
<td>.0024</td>
</tr>
<tr>
<td>Test 2</td>
<td>32</td>
<td>~</td>
<td>~</td>
<td>74</td>
<td>.0023</td>
</tr>
<tr>
<td>Average</td>
<td>31.5</td>
<td>~</td>
<td>~</td>
<td>76</td>
<td>.00235</td>
</tr>
</tbody>
</table>
All prestressing strands (tendons), used for prestressing two of the specimens were $\frac{1}{2}$” diameter, 7-wire, 270 kip, low-relaxation strands with a nominal area of $0.167 \text{ in}^2$. Material properties of the tendons were based on the published values provided by the PCI handbook. Prestressed tendons were also used as lifting hooks for all tests beams. The typical material properties used for the prestressing strands are summarized in Table 4.3.

<table>
<thead>
<tr>
<th>Elastic Modulus (Msi)</th>
<th>Ultimate Stress (ksi)</th>
<th>Yield Stress (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>270</td>
<td>243</td>
</tr>
</tbody>
</table>
4.2. Overall Structural Behavior of the Test Beams

Crack Pattern
All tested spandrel beams displayed very similar crack patterns at each of the subsequent load limit states. In general, the first visible major cracks were flexural cracks and propagated up from the bottom tension side of the beam towards the compression zone at midspan on both the inner and outer faces. Increasing the applied load caused initiation of shear cracks within the inner face at the end regions. These shear cracks extended up from the end of the ledge, at the ledge-web connection, and lateral bottom reactions, at approximately 45-degree angles. Further increasing the applied loads caused these shear cracks to move towards midspan and flatten out, creating a rainbow cracking pattern across the inner face, as shown in Figure 4.3.

Figure 4.3: Typical Inner Face Cracking Pattern

At higher load levels the inner shear cracks began to propagate above the bottom lateral reactions, moving upward at angles slightly higher than 45 degrees. The shear cracks on the inner face propagated down the web and onto the ledge. It was observed that before the web shear cracks crossed over the top surface of the ledge, they followed the ledge-to-web
connection, up to a few inches, before continuing their skewed path across the top of the ledge and down onto the front of the ledge. Flexural cracks on the outer face crossed the web and ledge bottom near midspan and propagated up the ledge front connecting with existing inner face flexural cracks and initiating more new cracks. The inner face shear cracks propagated up to the top of the web and began to cross over at approximately 45 degree angles. They began to flatten out as they moved towards the outer face, much like midspan inner face shear cracks. It was also observed that while these cracks eventually passed through the outer most lifting hooks at the top of the web (approximately 6’-8” down the span) they never surpassed them.

Three distinct regions were identified by the cracking patterns on the outer face: the disturbed end region, the transition region and the flexure region. It should be noted that as all four specimens were overly reinforced to prevent failures anywhere outside of the “end regions”, the cracking pattern was more exaggerated for these experimental beams than for those designed with typical detailing. The disturbed end region was identified by a diagonal compression zone, recognized by diagonal tension cracks, that extended across the span from the bottom lateral reaction to a length equivalent to one and a half times the height. This zone also contains diagonal cracks, perpendicular to the 45 degree tensile cracks, due to combined torsional and shearing forces. The transition region was identified by a combination of shear and flexural cracking that extended approximately two times the height down the span. The flexure region contained only pure flexural cracks within the center zone of the beam.

Typical cracking patterns for the three zones along the outer face are shown in Figure 4.4. It
should be noted that the cracking pattern was approximately symmetrical about the center span of all tested beams.

The specimens did display minor pre-loading cracks along the top of the web due to initial prestress camber (SP12 and SP13) and transportation and unloading from the truck. There was also some shrinkage cracks observed, particularly in the ledge.

As previously mentioned, after each significant loading level, researchers took time to mark cracks and take pictures before completely releasing the load. Therefore the growth and number of cracks was only noted at these specific load levels, ie shear cracks may have extended up over the spandrel-deck connection on SP10 before a reaction of 66 kips but it was not noted until then.

**Concrete Strain**

PI gages were placed along the inner face to capture cracking within the end regions at the assumed failure plane. Data from the PI gages indicates the inner face end region concrete was in tension, due to plate-bending of the ends. The flexure PI gages on the bottom of the web should also read tensile strain in the concrete due to bending in the bottom of the web.
but in many cases (reference Figures B.12-14, C.12-14, D.12-14 and E.12-14) the data indicated the concrete was in compression. PI gages were bolted onto flat head screws already adhered to the concrete face at the correct dimensions. As multiple cracks formed, on both sides of the nails, the small areas of concrete adhered to the PI gage nail would have been placed in compressed. This recorded behavior was similar for all four tested spandrel beams. Flexure PI gages on the top of the web did indicate that the concrete along the top of the web was in compression.

**Eccentricity**

Measured experimental values have indicated that the eccentricity may decrease as ledge loads are increased. To better understand this concept the following equation was used to determine the values of eccentricity for specimen SP10 as it was loaded up to the full factored design load. The data indicates that eccentricity decreases as the ledge load increases, as shown in Figure 4.5.

\[
e = \frac{(\text{Top Lateral Reaction}) \times (\text{Distance Between Lateral Reactions}) \times \sqrt{2}}{V_I}
\]  

Equation 4-1

![Figure 4.5: Changing of Eccentricity](image-url)
**Reinforced Concrete Specimens**
The first two specimens were constructed using conventional reinforcement. The reinforced control specimen, spandrel SP11, was entirely constructed with closed stirrups following the Zia-Hsu procedure for torsion in the PCI handbook. Spandrel SP10 was constructed with open web reinforcement following the preliminary design approach discussed in Section 2.5. Typical detailing for web and ledge reinforcement were increased by 50% to prevent local or flexural failures outside of the end region. Detailed shop drawings for all tested specimens can be found in Appendix A.

**Cracking Patterns**
The crack pattern of the inner faces, at the end regions of SP10 (bottom) and SP11 (top), under the effect of their respective failure loading is shown in Figure 4.6. The cracks have been enhanced and the deck sections removed for easier viewing. Shear cracks forming a rainbow pattern about midspan and flexural cracks were noted on the inner faces of both specimens. SP10 experienced more densely spaced cracking and the inner face shear and flexural cracks extended further towards the top of the web than for the control specimen, SP11. While SP10 was taken to failure, limitations to the hydraulic jacks and load cells prevented researchers from loading SP11 to its failure, therefore, the resulting crack pattern had fewer visible cracking.
The cracking pattern across the outer faces was also approximately symmetrical about midspan for both beams. Tensile cracks were observed extending down from the top lateral reaction, indicating the formation of a Mode 2 skew-bending compression zone due to combined shear and torsion stresses. This “disturbed end region” cracking pattern spanned approximately 9 feet from both ends for both beams. After the end region, the flexural-diagonal shear crack, due to shear and flexural cracking was observed to extend another 6 feet down the span before transitioning to pure flexural cracks, centered about midspan.
Crack Width

Measured data from the PI gages attached to the concrete face was used to calculate the width of shear cracks. The summation of the shear crack widths, $\sum w$, was determined using the following equation (Shehata 1999):

$$\sum w = (\Delta_H - 0.5l_L \varepsilon_{ct}) \sin \theta + (\Delta_V - 0.5l_L \varepsilon_{ct}) \cos \theta$$

Equation 4-2
where $\theta$ is the measured crack angle to the horizontal axis of the spandrel web, $l_g$ is the PI gauge length, $\varepsilon_{ct}$ is maximum tensile concrete strain of concrete, which was assumed to be 0.0001, and $\Delta_V$ and $\Delta_H$ are the vertical and horizontal displacements measured by the PI gages. The crack angles used in the above equations were determined from photographs taken of crack formation during testing. Determining a single crack angle was difficult as the crack angles not only changed as they moved across the span of the beams but also with increases in applied load. Due to the methods used to determine crack angles and the number of cracks passing through the PI gage stations, confidence in the data is limited to the service load ($V=88.1$ kips).

The major shear crack on the inner face that would ultimately lead to the failure of SP10 was captured by two sets of PI gages on the left side, Station 3 and Station 1 (reference Figure 3.20). Based on the observed failure plane and the strains captured by these PI gage stations it was found that the failure plane was located approximately 40 degrees to the axis of the beam. Measured angles for cracks extending through these two PI gage stations for all of the specimens were approximately 40 to 45 degrees. The SP10 failure plane extended up from the end of the ledge. The width of the shear cracks extending through these PI gages are shown for both the left and right end regions of SP11 and SP10 in Figure 4.9 and Figure 4.10 respectively. SP11 did not experience a skew-bending failure but similar crack patterns to SP10 were observed.
The crack widths of SP10 reached 0.019 inches at the service load which exceeds the allowable value of 0.016 inches set by the Commentary of ACI 318-08, Section 10.6. The crack widths of SP11 reached 0.008 at the service load which is less than the limiting value.
Recovery Criteria

Loading the two specimens to their respective failure loads did not induce any spalling or concrete crushing. After the specimens held the full factored design load (Vu=126.6 kips) for 24 hours, they were unloaded and allowed to recover for an hour to measure the permanent deformation. The following equations were used to check to see if the specimens met the ACI-318 load test recovery criteria.

\[
\Delta_{\text{measured (after 24 hour hold)}} - \Delta_{\text{measured (before 24 hour hold)}} \quad \text{Equation 4-3}
\]

\[
\Delta_{\text{residual}} = \Delta_{\text{(after 1 hour recovery)}} - \Delta_{\text{measured (before 24 hour hold)}} \quad \text{Equation 4-4}
\]

\[
\Delta_{\text{max. allowed}} = 0.25 \times \Delta_{\text{max}} \quad \text{Equation 4-5}
\]

\[
\text{PASS} = \Delta_{\text{max. allowed}} \geq \Delta_{\text{residual}} \quad \text{Equation 4-6}
\]

Test results indicated that both SP10 and SP11 satisfy the recovery criteria for deflection, according to ACI-318-08, during and after the 24 hour load test, as given in Table 4.4 and Table 4.5. All units are given in inches.
Table 4.4: ACI Recovery Criteria for SP11

<table>
<thead>
<tr>
<th></th>
<th>Vertical</th>
<th></th>
<th></th>
<th></th>
<th>Lateral</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Web</td>
<td>Ledge</td>
<td>Top</td>
<td>Bottom</td>
<td></td>
<td>Web</td>
<td>Ledge</td>
<td>Top</td>
</tr>
<tr>
<td>Measured Initial Deflection</td>
<td>0.33</td>
<td>0.22</td>
<td>-0.09</td>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(before 24 hour load)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured Maximum Deflection</td>
<td>1.65</td>
<td>1.18</td>
<td>-0.34</td>
<td>1.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(after 24 hour load)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured Deflection</td>
<td>0.51</td>
<td>0.32</td>
<td>-0.16</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(immediately after release)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured Deflection</td>
<td>0.49</td>
<td>0.31</td>
<td>-0.16</td>
<td>0.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(after 1 hour recovery)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Net Deflection</td>
<td>1.32</td>
<td>0.97</td>
<td>-0.26</td>
<td>0.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual Net Deflection</td>
<td>0.16</td>
<td>0.09</td>
<td>-0.07</td>
<td>0.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(after 1 hour recovery)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Allowed Net Deflection</td>
<td>0.33</td>
<td>0.24</td>
<td>-0.06</td>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(per ACI)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pass/Fail per ACI Recovery</td>
<td>Pass</td>
<td>Pass</td>
<td>Pass</td>
<td>Pass</td>
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<td>Pass</td>
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<td>Criterion</td>
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<td></td>
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</tr>
</tbody>
</table>

Deflection profiles for SP10 and SP11, along the outer face of the bottom of the web, can be
found in the appendices (reference Figures B.15, B.16 and C.15, C.16). The specimen
designed with open reinforcement, SP10, experienced greater overall creep during the load
test than its counterpart, SP11.
**Failure Mode**

Testing of the reinforced concrete control specimen, SP11, was terminated at a load of 250 kips which was the maximum capacity of the testing set-up. At this loading, localized ledge punching, localized concrete crushing, heavy cracking through the web in the end regions and a significant loss of stiffness, both in-plane and out-of-plane, were observed. Major diagonal cracks extended down the front of the ledge on either side of the ledge details, at approximately 45 degrees, as shown in Figure 4.11a. The extensive cracking at the three inner most special ledge details, particularly the one at midspan, led to the belief that this would be a probable failure mode for this beam. At the left end, a large crack also opened up, wrapping around from the back of the specimen, up the side of the web and into the top lateral connection, as shown in Figure 4.11b.

![Possible Failure Modes of SP11](image)
Figure 4.12: SP11 Inner Face Cracking in Right End Region at Ultimate Load
Testing of SP10 was terminated at a measured vertical reaction of 208 kips. At this load level, heavy cracking through the web in the left end region led to a skew bending failure as shown in Figure 4.13. The failure was observed to be more ductile in contrast to the typical failure of skewed failure reported by others. At failure, it was observed that the spandrel bent about a crack which slowly opened up to the final failure. The spandrel beam to the right of the failure plane rolled forward and pulled away from the end of the spandrel beam which was laterally attached to the column.

Figure 4.13: Failure Mode of SP10
4.3. Prestressed Concrete Specimens
The two remaining specimens were prestressed members. The prestressed control specimen, spandrel SP13, was constructed with entirely closed stirrups following the Zia-Hsu method for torsion according to the PCI handbook. Spandrel SP12 was constructed with open web reinforcement following the preliminary plate-bending design procedure, briefly discussed in Section 2.5. Other typical detailing for web and ledge reinforcement was identical for both specimens. Specimen reinforcement details were over designed by 50% to prevent failures outside of the end region. Detailed shop drawings for all the specimens can be found in Appendix A.

It should be noted that during the testing of SP13, the outer most, left spandrel to deck welded connection failed at a main vertical reaction of 88 kips. At this stage the spandrel was unloaded and the connection re-welded. Also, during testing the concrete cover on the left ledge face fell off, due to inadequate concrete cover in this area as shown in Figure 4.14.

Figure 4.14: Loss of Concrete Ledge Cover for SP13
Cracking Patterns

The overall inner face crack pattern (enhanced for easier viewing) for SP13 and SP12 are shown in Figure 4.16 and Figure 4.15 respectively. Similar to SP10 and SP11, specimen SP12 displayed an almost symmetrical rainbow cracking pattern about the midspan. The inner and outer face cracking pattern for SP13 was not symmetrical due to an impending flexural failure to the right of midspan. Flexural cracks along the bottom of the web were not as prevalent on the inner face for either prestressed specimens.

Figure 4.15: SP13 inner-face crack pattern (after failure, decks removed)

Figure 4.16: SP12 inner-face crack pattern (after failure, decks removed)
The cracking pattern on the outer face of SP13 is shown in Figure 4.17 and Figure 4.18. The disturbed end region at the right end of the span extended approximately 9 feet along the span, while it barely spanned the first 7 feet at the left. Diagonal cracks extending out from the flexural failure were indistinguishable from the torsion shear cracks in the transition region at the right end. Pure flexural cracking was not centered at midspan but spanned approximately 5.25 feet between midspan and the left transition region.
The crack pattern across the outer face was symmetrical about the midspan for specimen SP12. Tensile cracks extended approximately 8 feet along the span of SP12 before transitioning to torsion shear and flexural cracking for an additional 10 feet. Pure flexural cracking was observed to be centered across the middle 9.5 feet of span.

Figure 4.19: SP12 Outer-Face Cracking in Right End Region (after failure)
Crack Width

The width of the shear cracks extending through two sets of PI gages, Stations 1 and 3 (reference Figure 3.20) for both the left and right end regions of SP13 and SP12 are shown in Figure 4.20 and Figure 4.21. Neither of the two prestressed specimens exceeded the allowable limit 0.016 inches at the service load with SP12 shear crack widths reaching 0.0016 inches and SP13 reaching 0.0025 inches.

Figure 4.20: Crack Width of Major shear Crack in SP13 Left and Right End Regions

Figure 4.21: Crack Width of Major Shear Crack in SP12 Left and Right End Regions
**Failure Mode**

Testing of the prestressed control specimen, SP13, was terminated when the measured reaction was approximately 240 kips. At this load level heavy cracking occurred within the web, and in particular heavy flexural cracks, at midspan as shown in Figure 4.22. Failure occurred to the right of midspan due to flexure in spite of the fact that the specimen had extra reinforcement to induce failure within the end regions. The failure was sudden and brittle. The tendons at the top of the web buckled in compression at this location as shown in Figure 4.23. Heavy cracking was observed at the ledge at midspan, beneath the single-tee, which suggested that a ledge punching failure was also possible. This heavy cracking beneath the centered ledge load was not observed at any other ledge load locations. Specimen SP13 displayed a large number of cracks, much more than what was observed for SP12.

![Figure 4.22: Failure Mode of SP13 (Right Inner Face, decks removed)](image)
Failure of specimen SP12 occurred at a vertical end-reaction of 185 kips. The brittle failure was due to skew-bending in the left end region as shown in Figure 4.24 and Figure 4.25 for the inner and outer faces respectively. The failure plane on the inner face does not follow a single, unified 45 degree angle. Two skewed diagonal cracks were observed. The first extended from the web-ledge connection and the second down from the top of the web. The two cracks were connected by a small flexural crack within the failure zone. Both the top and bottom skewed cracks were located at approximately 45 degrees to the web centerline, before failure. The skewed failure plane in the outer face compression zone was a more consistent representation of a unified 45 degree crack plane. Several observations were made about the failure of this beam. The bottom of the skew bending failure on the outer face began where the legs of a U-bar ended; this U-bar had three foot long legs. It was noted that a few of the WWR bars along the failure plane were fractured. Failure was more sudden in comparison to SP10.
In SP12’s right end region, a major diagonal crack opened up along the side of the ledge and began to propagate up through the top of the ledge but was not able to extend into the web before failure at the left end.
Recovery Criteria
During testing of specimens SP12 and SP13 to their respective failure, neither showed any signs of spalling or concrete crushing. Measured data while the beams were loaded for 24 hours, unloaded and allowed to recovery for one hour to satisfy the recovery criteria for deflection, per ACI-318-08, is given in Table 4.7 and Table 4.6. All units are given in inches.

Table 4.6: ACI Recovery Criteria for SP13

<table>
<thead>
<tr>
<th>Vertical</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>Ledge</td>
</tr>
<tr>
<td>Measured Initial Deflection (before 24 hour load)</td>
<td>0.14</td>
</tr>
<tr>
<td>Measured Maximum Deflection (after 24 hour load)</td>
<td>1.49</td>
</tr>
<tr>
<td>Measured Deflection (immediately after release)</td>
<td>0.35</td>
</tr>
<tr>
<td>Measured Deflection (after 1 hour recovery)</td>
<td>0.30</td>
</tr>
<tr>
<td>Maximum Net Deflection</td>
<td>1.34</td>
</tr>
<tr>
<td>Residual Net Deflection (after 1 hour recovery)</td>
<td>0.15</td>
</tr>
<tr>
<td>Maximum Allowed Net Deflection (per ACI)</td>
<td>0.33</td>
</tr>
<tr>
<td>Pass/Fail per ACI Recovery Criterion</td>
<td>Pass</td>
</tr>
</tbody>
</table>

Table 4.7: ACI Recovery Criteria for SP12

<table>
<thead>
<tr>
<th>Vertical</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>Ledge</td>
</tr>
<tr>
<td>Measured Initial Deflection (before 24 hour load)</td>
<td>0.16</td>
</tr>
<tr>
<td>Measured Maximum Deflection (after 24 hour load)</td>
<td>1.62</td>
</tr>
<tr>
<td>Measured Deflection (immediately after release)</td>
<td>0.40</td>
</tr>
<tr>
<td>Measured Deflection (after 1 hour recovery)</td>
<td>0.34</td>
</tr>
<tr>
<td>Maximum Net Deflection</td>
<td>1.46</td>
</tr>
<tr>
<td>Residual Net Deflection (after 1 hour recovery)</td>
<td>0.18</td>
</tr>
<tr>
<td>Maximum Allowed Net Deflection (per ACI)</td>
<td>0.37</td>
</tr>
<tr>
<td>Pass/Fail per ACI Recovery Criterion</td>
<td>Pass</td>
</tr>
</tbody>
</table>

The deflection profiles for SP12 and SP13, measured at the outer face along the bottom of the web, can be found in the appendices (reference Figures D.15, D.16 and E.15, E.16). Once again the beam designed with open reinforcement, SP12, did experience slightly larger creep, during the load test.
4.4. Summary of Experimental Results
Both the control reinforced and prestressed spandrel beams, SP11 and SP13, displayed very similar crack patterns and overall behavior to the spandrels designed with the preliminary design approach procedure for open reinforcement, SP10 and SP12. The failure modes of all tested specimens were indicative of skew-bending effects; the crack patterns indicate the specimens were bent about their top lateral reactions causing tension in the inner face concrete and compression in the outer face concrete along an approximate 45 degree failure plane.

Lateral Reactions
The lateral reactions for each of the four specimens are given in Table 4.8 at the full factored design load. At the beginning of each test, the four load cells measuring the lateral reactions were calibrated for an initial reading of 4 kips. It was observed for all four tests that as the load was increased the two bottom lateral reactions were consistently higher than the top lateral reactions, indicating the welded spandrel-deck connections along the inner face were in compression. As these forces were not determined experimentally, an analytical finite element model will be used to predict an approximation of the forces undertaken by the tiebacks.

<table>
<thead>
<tr>
<th></th>
<th>Top Right</th>
<th>Top Left</th>
<th>Bottom Right</th>
<th>Bottom Left</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP10</td>
<td>18</td>
<td>20</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td>SP11</td>
<td>20</td>
<td>20</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>SP12</td>
<td>22</td>
<td>20</td>
<td>27</td>
<td>22</td>
</tr>
<tr>
<td>SP13</td>
<td>22</td>
<td>24</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>21</td>
<td>21</td>
<td>25</td>
<td>23</td>
</tr>
</tbody>
</table>

* all units in kips, data taken after 24 hour load hold
Crack Propagation

The cracking pattern across the inner and outer faces of all four specimens was very similar. A rainbow cracking pattern was observed, symmetrical about midspan on the inner face. The outer face displayed three distinct cracking regions: diagonal tension cracks extending down from the top lateral reaction at the specimen ends and flexure cracks extended into flexure-shear cracking near quarter spans followed by pure flexural cracks into the middle zone of the beams. The measured vertical reaction, in kips, corresponding to the observation of significant crack propagation for each specimen is given in Table 4.9.

<table>
<thead>
<tr>
<th>Table 4.9: Significant Cracking Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Face</td>
</tr>
<tr>
<td>Shear cracks extend over spandrel-deck connection</td>
</tr>
<tr>
<td>Shear cracks cross over the top of the web</td>
</tr>
<tr>
<td>Outer Face</td>
</tr>
<tr>
<td>Shear cracks form in beam region</td>
</tr>
<tr>
<td>Tensile cracks indicate formation of compression zone</td>
</tr>
<tr>
<td>Flexural cracks on outer face reach mid-height of web</td>
</tr>
</tbody>
</table>

As expected, flexural cracks were much more prevalent for the reinforced concrete beams than the prestressed beams. Shear cracks in the outer face also occurred at much lower loads for the reinforced concrete beams than the prestressed beams. In general, the outer web face exhibited vertical cracking extending upwards from the bottom of the beam due to in-plane and out-of-plane flexure. Substantial diagonal cracking was not observed on the outer face due to the opposing directions of the applied shear and torsion stresses on the outer surface.
**Rotation**

As the applied load was increased, the bottom of the spandrel moved down and laterally away from the deck at midspan, while the top moved down and laterally inward towards the deck. Resistance to overturning was provided by the lateral column reactions, creating a warped deflected shape and significant out-of-plane bending deformation along the diagonal cracks in the end regions. Rotations of the four specimens, measured at midspan, are shown in Figure 4.26. The measured rotations for the prestressed specimens were smaller than that measured for the reinforced concrete specimens at the service load. While the measured values indicated that the rotation of the beams with open reinforcement was slightly higher than the control specimens, the difference was negligible, particularly at the service load level.

**Figure 4.26: Spandrel Rotation**
**Midspan Deflections**

Load-deflection envelopes for the reinforced concrete specimens are compared in Figure 4.27 and Figure 4.28. The portions of the graphs that level out at specified load levels are due to pauses in loading during which researchers made observations of the formation of spandrel crack patterns and particularly at the full factored design level when the load was held for 24 hours. At the service load, the specimens experienced very similar vertical and lateral deflections. Up to the full factored load, the difference in midspan lateral deflection between open and closed reinforcement in reinforced concrete beams is negligible. Differences in midspan vertical web and ledge deflections were more noticeable at the full factored load.
Load-deflection envelopes for the prestressed specimens are compared in Figure 4.29 and Figure 4.30. The midspan lateral deflections were nearly identical for the beam designed with open versus closed reinforcement up to the full factored load. The midspan vertical deflections were also nearly identical for the two specimens.
Further load-deflection envelopes comparing the midspan lateral and vertical deflections of reinforced concrete versus prestressed beams can be found in Appendix F.
Behavior mechanisms of the experimental spandrel beams were identified by the three following main regions: disturbed end region, transition region and flexure region. These regions are recognized using the outer face cracking patterns, as the symmetrical inner face rainbow cracking pattern does not easily signify a change in spandrel behavior across the span. This classification is valid for reinforced concrete and prestressed beams. It should be noted that the length and cross section of the spandrel beam as well as loading system will greatly affect the region definitions. The following definitions are for L-shaped spandrel beams with an aspect ratio of 7.5, overall span of 45 feet and 9 point loads along the ledge.

The disturbed end region is classified by the length of the out-of-plane failure zone where the outer faced is exposed to compression stresses and the inner surfaced exposed to tension stresses due to out-of-plane bending effects. Examination of the results for SP10 and SP12 indicates that the disturbed end region for a beam designed with the plate-bending procedure extends 7.5 feet down the span from the spandrel ends, or one and a half times the height of the beam. The transition regions are defined by a combination of flexure cracks extended into flexural shear cracking. The measured transition region for the tested beams was about 10.5 feet which is approximately twice the height of the beam. The flexure region, which is the midspan region, is defined by a zone of pure flexural cracking. The flexure region spans 9.5 feet which is also approximately twice the height. It should be noted that this distinction between the three regions of cracking patterns is not finite. For instance, shear and flexural cracks were observed at the bottom of the outer face web in the disturbed end region, beneath
the compression zone. A sketch of the three regions, as they are defined above, across half of a typical slender L-shaped spandrel beam is shown in Figure 4.31.

![Figure 4.31: Spandrel Region Definition](image)

To augment the experimental results an analytical study was completed to better understand the behavior of slender, precast L-shaped spandrel beams.
5. Analytical Program and Results

This chapter presents the analytical phase of the research program undertaken to predict the behavior of the four large scale spandrel beams tested in this research program and described in Chapter 4. The analytical program will augment the experimental program by allowing for examination of several parameters that may be cost prohibitive and time consuming to be determined experimentally. Data from the experimental program described in Chapters 3 and 4 was used to validate, refine, and calibrate the following analytical models.

The analytical phase of the research consisted of two approaches. The first is to develop and calibrate a three-dimensional nonlinear finite element model (FEM) to study various parameters that have been identified to influence the behavior of slender L-shaped spandrel prestressed and reinforced concrete beams. The second is to use the analytical results from the FEM along with the measured values to develop a rational model based on the principals of equilibrium and compatibility to describe the behavior of precast slender L-spandrel beams. This rational approach should be simple and safe to be used by practitioners to design precast slender L-spandrel beams.

5.1. Basic Concepts of the Finite Element Model

The finite element analysis selected for this study is based on the ANATECH Concrete Analysis Program (ANACAP). The program is capable of analyzing plain, reinforced, or prestressed concrete structures, in either two or three dimensions. The code has extensive nonlinear capabilities and includes advanced concrete material modeling. The built-in material models for concrete and steel require the user to input several key properties including elastic modulus, ultimate strength, and fracture strain.
The FEM includes material modeling that accounts for uniaxial and multiaxial stress/strain states under the framework of isotropic hardening plasticity formulation. The concrete material model is assumed to be linear when the compressive stresses are less than one-half of the compressive strength and follows a strain hardening model until the concrete compressive strength limitations are reached. In order to effectively model the shear performance of concrete, ANACAP reduces the shear modulus of the concrete at cracking and accounts for further reductions as the cracks continue to open and propagate. The concrete model relies on the smeared cracking methodology to predict the propagation of cracks, assuming that all the cracks form perpendicular to the direction of the largest tensile strains. When a crack forms, the normal stresses across the crack reduce, and the forces and stresses around the crack are redistributed to the surrounding concrete and reinforcing elements. While the direction of initiated cracks cannot change, cracks may close to resist compression, and then reopen during cyclic loading. Reinforcing bars are modeled individually as discrete sub-elements within the concrete elements. The stress and stiffness of the reinforcing sub-elements are superimposed on the concrete element in which the reinforcing bar resides.

The finite element code is able to capture crack propagation and widespread damage prior to structural failure. The analysis terminates when the displacement convergence at any node in a given solution step cannot be obtained within a selected tolerance. To ensure that analysis termination corresponds with a real structural failure, post-processing allows for examination of specific failure criteria. Selected criteria include concrete compressive strain limits of 0.002 in shear regions and 0.003 in flexural regions. Graphical output such as deformed
shape, cracking patterns, and strain contours can also be used to verify failure, to determine failure mode, and to examine behavior. Additional details describing the ANACAP finite element code can be found elsewhere (Anacap 2003).

5.2. Effect of Relevant Parameters
Several key parameters examined in this study included closed versus open reinforcement, web reinforcement type and ratio and the influence of boundary conditions on the behavior of slender L-shaped spandrel beams.

The influence of deck connections on spandrels is of significant importance when predicting behavior of these beams. The typical welded connections between the inner spandrel face and the deck sections may develop substantial forces which could influence lateral motion and rotation of the spandrel. The analytical model is also being used to evaluate the effect of the friction forces which develop at the bearing reactions between deck stems and spandrel ledge. Initial experimental results indicate that bearing friction at these locations may play a major role in slender spandrel response, in particular the degree of lateral motion. The model will also be used to evaluate the effect of the compressive strength of the concrete.

5.3. FEM Analysis of Slender L-shaped Spandrel Beams
The typical slender spandrel beam considered in this study has a 60 inch deep L-shaped cross-section with an 8 inch web thickness and spans approximately 45.5 feet long. The beam has a continuous ledge which measures 8 inches square and is held back 12 inches from each end to facilitate connection of the spandrel beam to a column.
Considering symmetry, the analysis was based on modeling one-half of a typical slender spandrel beam using 4400 20-node brick elements. The large number of elements was necessary to maintain a sufficiently fine mesh around all of the loading and boundary conditions as well as in the end region where failure is expected to occur. In addition, since out-of-plane behavior was a dominant response for the experimental spandrels, four elements were used through the thickness of the web to capture the out-of-plane skew-bending failure modes. The modeled portion of the beam has 5 ledge loads, 3 tieback connections, 2 lateral end restraints, 1 vertical end reaction, and the symmetry condition at midspan. The finite element mesh and boundary conditions used are depicted in Figure 5.1.

![Figure 5.1: Mesh Configuration for FE Analytical Model](image)

The boundary conditions used in the model were chosen to simulate the connections typically used to support spandrel beams in the field. The spandrel-to-column tiebacks were simulated
by restraining movement in all lateral direction at those two locations in the model. Vertical
movement was restrained at the end to simulate the main vertical reaction. Symmetry
boundary conditions were applied at midspan.

The ultimate concrete compressive strength at the time of testing was specified in the
material models, and was used to determine the elastic modulus, $E_c$, and fracture strain, $\varepsilon_{tu}$,
based on PCI guidelines. The concrete used was a normal weight concrete, with a unit weight
of 150 pcf. Though the designs called for a concrete $f'_c$ of 7600 psi for the reinforced
concrete specimens and 6500 psi for the prestressed specimens it seems likely that all four
beams were cast with the same batch of 6500 psi concrete. Based on measured strength of the
concrete cylinders, it was decided that the compressive strength for all four specimen models
would be 6600 psi.

$$E_c = \left(40000 - \frac{6600}{1000} \times 1.5\right) = 4462096\text{psi}$$ \hspace{1cm} \text{Equation 5-1}

$$\varepsilon_{tu} = \frac{1 \times 6600}{E_c} = 1.48e-4$$ \hspace{1cm} \text{Equation 5-2}

Table 5.1 displays the material properties input into the concrete and reinforcement models
for the appropriate specimen models.

<table>
<thead>
<tr>
<th></th>
<th>Compressive Strength(psi)</th>
<th>Modulus of Elasticity(ksi)</th>
<th>Density (pcf)</th>
<th>Fracture Strain</th>
<th>Yeild Stress (ksi)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>6600</td>
<td>4460</td>
<td>150</td>
<td>1.48e-4</td>
<td>~</td>
<td>0.2</td>
</tr>
<tr>
<td>Deformed Bars*</td>
<td>~</td>
<td>29000</td>
<td>~</td>
<td>~</td>
<td>60</td>
<td>0.3</td>
</tr>
<tr>
<td>WWR**</td>
<td>~</td>
<td>29000</td>
<td>~</td>
<td>~</td>
<td>75</td>
<td>0.3</td>
</tr>
<tr>
<td>Tendons</td>
<td>~</td>
<td>29000</td>
<td>~</td>
<td>~</td>
<td>243</td>
<td>0.3</td>
</tr>
</tbody>
</table>

*C, U and L-shaped vertical reinforcement, Grade 60
**Welded Wire Reinforcement, Grade 75
Each of the 9 applied vertical loads to the spandrel ledge from the deck sections were modeled as a uniform pressure acting on an area equal to the size of the bearing pads used in the experimental program. The load was increased incrementally, at 5 psi per load step, until failure.

Since the behavior of the end-region is of particular interest in this study, care was taken to simulate the transfer length of each prestressing strand in the analytical model. Each strand was split into 10 smaller strands, each strand has one-tenth the area of the original strand and all occupy the same location in the model. The first of the 10 strands stretched the entire length of the spandrel and the tenth strand started at a distance equal to the transfer length away from the end. The remaining eight strands started at equal, incremental distances between the first and tenth strand. A sketch of the transfer length model is shown in Figure 5.2.

![Figure 5.2: Analytical Modeling of Prestressing Strand Transfer Length](image)

The initial prestressing force was a requirement for the tendon material model. The tendons of SP12 and SP13 were pulled to two different levels of initial force, 15.8 kips and 31.6 kips. Using calculated prestress loss levels of 13%, measured for these types of beams, the strands were then identified as having either a stress of 82.31 ksi or 164.62 ksi, respectively.
As with any analytical model, defining appropriate boundary conditions is essential to predicting realistic behavior. Two boundary conditions are particularly challenging in the analysis of slender precast spandrel beam model: the bearing reactions between the deck stems and the ledge, and the welded connections between deck members and the web inner face. As a spandrel beam deforms under load, the ledge tends to rotate outward away from the deck while the top of the web tends to rotate inward towards the deck. This warped shape causes the ledge to slip out from underneath the deck stems as the load increases. Significant friction forces are expected to be developed between each stem of the supported deck sections and the ledge of the spandrel beam which could restrain this motion. In addition, the welded connections between deck sections and inner web face also serve to transfer additional lateral forces to the spandrel. The welded connections are considered to be flexible in the vertical plane since they can rotate easily about thin welds. However, the connections are quite stiff out-of-plane since they are placed in direct tension or compression.

Including the influence of stem-to-ledge bearing friction and deck to spandrel tieback forces is critical in obtaining an accurate analytical model. In both cases, spring elements were used to emulate these connections in the model. For each tieback connection, the stiffness of the spring elements was selected based on the material and cross-sectional properties of the weld plate. Each welded connection was approximately 6 inches by 3 inches and 3/8 inch thick. The overall stiffness, considered an upper-bound limit, was determined using the physical dimensions and material properties of the plates as follows:

\[
K = \frac{EA}{L} = \frac{29e6 \times \left(\frac{3}{8} \times 6\right)}{3} = 21750 \text{ k/in}
\]

Equation 5-3
Five springs were placed at the corner and midsize nodes across the six inch length using a fifth of the overall plate stiffness (4350000 lb/inch) attributed to each spring.

The stiffness of the lateral springs applied at each stem-to-ledge bearing reaction was based on published data for the coefficient of friction for the chosen bearing pads. A thin layer of hydraulic oil was placed between the bearing pads and steel plate causing a “slide bearing” effect in the experimental work, so the range of coefficients of friction used in the analysis was at the lower end. The coefficient of friction for a given bearing pad was assumed to remain constant at all levels of the applied loads. The springs were distributed across each of the 13 nodes that define the stem to ledge bearing areas where the load was applied. During the experimental program, the maximum lateral deflection at the full factored design load, $\Delta_f$, was approximately 1.0 inch for the four tested spandrels and the corresponding maximum full factored per stem load, $N$, was 23.13 kips. The following equation was used to determine the stiffness of the springs emulating a bearing friction of 0.05.

$$F = \mu \times N = 0.05 \times 23.13 = 1.16 \text{ kips} \quad \text{Equation 5-4}$$

Spring Constant $= \frac{F}{\Delta_f} = \frac{1.16}{1.0} = 1.16 \text{ kips/inch} = 1156 \text{ lbs/inch} \quad \text{Equation 5-5}$

Spring elements worked well for modeling friction in this case because friction forces at the bearing reactions increased with increased applied load and lateral deflection.
5.4. Influence of Boundary Conditions
Several analysis cases were performed to determine the behavior of the tested slender
spandrel beams using the described FEM. The analysis indicated that changing the spring
constants for the tieback connections and the friction of the bearing pads had a significant
impact on the predicted behavior. The results, of the SP10 reinforced concrete spandrel beam
reinforced with open stirrups, were compared for the following five cases while all other
boundary conditions and material models remained constant:

<table>
<thead>
<tr>
<th>Case</th>
<th>Tieback Connection Spring Constant (lbs/inch)</th>
<th>Stem-to-Ledge Bearing Friction Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4350000</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>4350000</td>
<td>0.017</td>
</tr>
<tr>
<td>3</td>
<td>4350000</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>435000</td>
<td>0.017</td>
</tr>
</tbody>
</table>

The effect of changing the coefficient of friction from 0.02 to 0.05, Cases 1 and 3, is
compared to the measured values in Figure 5.3. The two coefficients correspond to upper and
lower limits from published data for the selected bearing pads. While the predicted ultimate
carrying capacity did not change significantly between the two cases, the lateral deflection
and rotation of the specimen were greatly influenced by the friction at the deck-to-ledge
bearing. By lowering the coefficient of friction, the top and bottom outer face lateral
predictions were able to match more closely the measured values.
The affect of deck-to-ledge bearing friction on the lateral forces in the tieback connections was also analyzed for Cases 1 and 3, as shown in Figure 5.4. A reduction in the friction coefficient led to a reduction in the lateral forces measured in the springs simulating the tieback connections. Model output indicates that with a coefficient of friction of 0.05, the
forces in the tieback springs were found to be small and within the range of 1 to 1.3 kips.

Changing the coefficient of friction to 0.02, the forces in the tieback springs were found to be within the range of 0.4 to 0.7 kips at an applied load equivalent to full factored design load.

The predicted range seems reasonable and within the same range of the measured values obtained from the experimental data. Measured data also indicated that the welded connections for the tested beams were in compression during testing as the measured bottom lateral reaction were always larger than the top lateral reaction.

Significant reduction of the tieback stiffness by a factor of 10, Cases 2 and 6, did not significantly influence the specimen behavior up to the applied full factored design load. The model predicted a significant change in spandrel rotation and ultimate carrying capacity as the applied load went beyond the full factored design load as shown in Figure 5.5. It should be noted that by lowering the coefficient of friction beyond the published lower limit of 0.02,
predicted lateral deflections and specimen rotations match more closely with measured values.

Figure 5.5: Effect of Spandrel-Deck Connection Stiffness

Complete removal of the deck-to-spandrel tieback connection, Cases 1 and 4, had a greater influence on the predicted behavior of the specimen as shown in Figure 5.6. The model indicated that the welded connection between the inner web face and deck sections significantly increased the predicted ultimate carrying capacity for the slender spandrel as well as prevented large lateral movement at the top of the spandrel.
To better visualize the effect of the tieback connection, rotations of the spandrel beam at midspan for beams with tiebacks and without tiebacks, Cases 1 and 4, are shown in Figure 5.7. The centerlines of the inner face of the web are marked to depict the overall movement of the specimen at midspan from zero load to ultimate. Note that the model shows that for the beams with tieback connection, the specimen rotates about the connection and deflects in the

Figure 5.6: Effect Modeling of Tieback Connection
same vertical plane. Without the tieback connection the beam deflects vertically and also has
greater overall lateral movement and rotation.

![Figure 5.7: Rotation at Ultimate Load](image)

It should be highlighted that the modeled spandrel-deck tieback connections have been
greatly simplified by modeling the forces with a spring constant in the lateral direction.
Experimentally, the 12 foot long decks did not slide at the end supports, and had limited deck
deflection and end rotation during loading. In field conditions, loading of a typical 60 foot
deck could have more vertical deflection and rotation in comparison to the one used in the
test. These longer deck sections may affect the magnitude and direction of the forces in the
tieback. However, the analytical model undertaken in this research was intended to simulate
the test configuration conducted at the Constructed Facility Laboratory at NC State.

A final comparison assuming no tieback connection or deck-to-ledge friction, Cases 5 and 1,
is shown in Figure 5.8. The comparison indicates that without tiebacks or bearing friction,
both the top and bottom of the specimen will initially move laterally in the same positive direction before the specimen begins to rotate.

Figure 5.8: Effect of Modeling Slide Bearing and Tieback Connections

(a)

(b)
The analysis indicated that the tieback connection and bearing friction coefficients are important parameters in modeling boundary conditions and could significantly affect the forces in and deformation of the specimen. Based on the comparisons several conclusions were made as follows:

1. Reducing the friction of the bearing pads increases lateral movement.

2. Using a friction coefficient in the range of 0.02 and 0.05 does not affect the predicted ultimate load carrying capacity.

3. Changing the friction coefficient and the stiffness of the tiebacks does not significantly affect vertical deflections.

4. Reducing the stiffness of the tiebacks reduces the predicted ultimate carrying and bottom lateral movement and increases top lateral movement.

5. Further refinement of the model will require analysis beyond just changing the friction coefficient and tieback stiffness boundary conditions.

**5.5. Influence of Concrete Strength**

Analysis was completed to determine the effect of the concrete compressive strength on predicted behavior. Results of the analysis indicate that there is no significant change in predicted deflection or rotation up the full factored design load, as shown in Figure 5.9. An increase in the ultimate carrying capacity from 190 kips to 221 kips was observed for an increase in concrete compressive strength from 6.6 ksi to 7.6 ksi.
5.6. Influence of Creep

It is important to note that the four experimental specimens were all tested with frictionless slide bearings at both the main vertical reactions and deck to ledge bearings. Also all tested spandrel beams were loaded for 24-hours at their factored load levels and then unloaded to
determine the degree of permanent deformation. Each beam was then reloaded in load increments of 20 kip up to failure. The creep measured by the experimental phase, due to pauses in the cyclic loading and sustained loading during the 24 hour hold, was not modeled in the analytical program. Measured values shown in Figure 5.9b are re-plotted in Figure 5.10 without the “creep effects” for all cyclic loading levels up to failure. The comparisons are much closer to the experimental specimen stiffness, up to 1.3 times the full factored design load, for these beams.

![Figure 5.10: Removal of Creep Effects from Experimental Results](image)
5.7. Closed Stirrups versus Open Reinforcement
The other parameter considered in this analysis was the use of closed stirrups versus open reinforcing configurations. The following parameters, as shown in Table 5.3, were kept constant for the analysis of the four tested beams:

Table 5.3: FEM Properties for Parameter Analysis

<table>
<thead>
<tr>
<th>Tie-back Connection</th>
<th>Stem-to-Ledge Bearing</th>
<th>Concrete Compressive Strength (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring Constant (lb/in)</td>
<td>Friction Constant</td>
<td>435000</td>
</tr>
</tbody>
</table>

Note that the control specimens, SP11 and SP13, were designed using current design methods therefore, they were reinforced by closed stirrups. The reinforcing ratio of the closed stirrups was 33% higher than the vertical open reinforcement used for specimens SP10 and SP12.

Results from the analysis show a noticeable difference in the ultimate load carrying capacity for prestressed beams, shown in Figure 5.11. The results indicate that specimens designed with closed reinforcement had a 13% increase in ultimate carrying capacity. However, the use of open reinforcement did not affect the vertical and bottom lateral deflections as they were nearly identical up to the full factored design load. The lateral movement and rotation for the beams designed with open reinforcement were slightly higher in comparison to the beams designed with closed stirrups with increasing load levels, significantly higher after the full factored design load.
Reinforced concrete slender spandrels with open and closed reinforcement had similar ultimate carrying capacity and nearly identical deformation and rotation up to the full factored design load, as shown in Figure 5.12. Beams designed with open stirrups
experienced slightly higher deflection and rotation at failure in comparison to the open reinforcement configuration.

![Diagram of FEM Comparison for Reinforced Concrete Specimens](image)

**Figure 5.12:** FEM Comparison for Reinforced Concrete Specimens

### 5.8. Cracking Pattern and Failure Mode

The typical cracking pattern observed at the inner face cracking pattern for the reinforced concrete, whether reinforced with closed stirrups or open reinforcement, is shown Figure
5.13a. In this figure, the deck members were removed and the cracking pattern was enhanced for easier viewing. Recall that the failure mode in the end region for a reinforced concrete specimen designed with the open reinforcement, spandrel SP10, was a skew-bending failure as shown in Figure 5.13b.

![Figure 5.13: Reinforced Concrete Inner Face Cracking Pattern and Failure Mode](image)

Predicted crack patterns from the FEM were comparable to that observed experimentally for the reinforced concrete beams, as shown in Figure 5.14a. The crack pattern predicted from the analysis was initially flexural cracks along the bottom of the web within midspan range first and then spread out towards the ends of the spandrels to form flexural-diagonal shear cracks and finally skew-bending cracks formed leading to failure.

The predicted failure mode was skew-bending in the end-region along a primary crack angle of approximately 45-degrees, as shown in Figure 5.14b. Along this skewed-diagonal crack, the shear strains at the inner face indicated the concrete was in tension perpendicular to the failure plane and in the range of 0.002.
Figure 5.14: Analytical Predictions for Reinforced Concrete Specimen

(a) FEM Crack Pattern
(b) FEM Shear Strain Contours

Figure 5.14: Analytical Predictions for Reinforced Concrete Specimen
The predicted behavior of the prestressed specimens near failure is represented by Figure 5.15; similar shear strains and cracking pattern were predicted for both SP12 and SP13. The model indicated initiation of minor flexural cracks near midspan first followed by the initiation of flexural cracks within the midspan region, extending to flexural-diagonal shear cracks towards the end regions. In spite of the flexural failure of spandrel SP13, the model indicated a skew-bending failure in the end regions of both prestressed specimens.
5.9. Proposed Rational Design Method
Analysis of the experimental data and the results from finite element model has led to the development of a rational design method for the reinforcement in the end region of slender, precast L-shaped spandrel beams. The proposed method is based on equilibrium of forces at the failure surface for the design of the reinforcement required for the inner and outer faces. The proposed rational method will be evaluated by comparing the reinforcements provided for specimen SP10 in the experimental program. It should be noted that this method is proposed only for slender, precast spandrel beams, with an aspect ratio of 7.5, and for the use of open reinforcement instead of closed stirrups.

A sketch of the three regions for a typical slender L-shaped spandrel beam is shown again in Figure 5.16. The shear, moment and torsion diagrams are also shown in the same figure for the applied stem loads only, up to the full factored design load. Note how closely the beam region definitions are aligned to drops in shear, moment and torque across the span for a beam loaded at nine evenly spaced load points.
The method, briefly summarized in the following steps, provides reinforcement for the disturbed end region only. Reinforcement in the transition and flexure regions is to be designed using current code equations for shear and flexure. Details regarding calculation of ledge hanger capacity and other potential secondary-failure modes in precast slender members may be found elsewhere (PCI 2004).
1. Assume a constant eccentricity, \( e \), equal to the distance between the applied ledge load and web centerline of the beam, as shown in Figure 5.17.

![Figure 5.17: Eccentricity of Applied Load](image)

2. Design the required total shear reinforcement \((2*{A_v})\) neglecting eccentricity, where \( A_{v1} \) is the area of a single bar or wire required on the web inner face:

\[
A_{v1} = \left( V_a - V_c \right) \left( \frac{s}{d} \right)
\]

Equation 5-6

Where:
- \( d \) = Effective depth of web from web top to centroid of bottom tensile steel
- \( f_y \) = Yield strength of steel reinforcement
- \( s \) = Spacing between vertical bars
- \( V_c = 2\sqrt{f_c b_w d} \)
- \( V_a \) = Vertical reaction due to applied stem loads at full factored loading

3. Assuming a 45 degree failure plane, the plate-bending component of the applied torsion, \( T_{ub} \), can be determined from the applied torsion, \( T_u \), Figure 5.18a, as follows:

\[
T_{ub} = \frac{V_a e}{\sqrt{2}}
\]

where \( T_u = V_a e \)

The plate-bending torsion, \( T_{ub} \), can be analyzed into two components, tension, \( T \), and compression, \( C \), with a lever arm, \( d_w \), along the assumed failure plane, as shown in Figure
5.18b. The lever arm, \( d_w \), that separates these two equal and opposite forces, can be determined using the following equation:

\[
d_w = 0.95 \times \text{effective depth of web thickness}
\]

The tensile force, which is in equilibrium with the compressive force, \( C \), can be determined as follows:

\[
T = \frac{T_{ub}}{d_w} = C
\]

This compressive force, \( C \), is resisted by the compressive zone of the concrete on the outer face, as shown in Figure 5.18b.

The total amount of reinforcement, needed in the vertical, \( A_{v1} \), and lateral, \( A_{l1} \), directions, along the inner face, are determined as follows:
Therefore the area of one bar or wire required on the inner face to resist the torsion can be determined as follows:

\[
A_{i1} = \frac{T_{ub}}{d_w} \left( \frac{1}{f_y} \right) \frac{1}{\sqrt{2}} \left( \frac{1}{2} \right)
\]

\[
= A_{i1}
\]

Equation 5-7

4. Shear flow and torsion are additive on the inner face and subtractive on the outer face of the slender spandrel beams, as shown in reference Figure 2.2. Therefore, the total vertical reinforcement on the inner face is determined to be \(A_{i1} + A_{i2}\).

This total inner face vertical reinforcement should be at least equal to or more than that required for hanger reinforcement.

5. The applied torsion acts in the opposite direction of the shear on the outer face, therefore, the need for shear vertical reinforcement, \(A_{v2}\), may be reduced.

\[
A_{v2} = \frac{(V_a - V_e)}{2f_y} \left( \frac{s}{d} \right) - \frac{V_e e}{4d_w f_y} \left( \frac{s}{d} \right)
\]

\[
= \frac{V_a \left( 1 - \frac{V_e}{V_a} - \frac{e}{2d_w} \right)}{2f_y} \left( \frac{s}{d} \right)
\]

Equation 5-8

\[
> 0.75 \sqrt{f_c \left( b_w \ast s / f_y \right)}
\]

Equation 5-9

Note that when \(e/2d_w\) is greater than \((1-V_c/V_u)\), shear reinforcement is not required on the outer face, therefore, the minimum shear requirements, per ACI, must be met.
However, as the amount of torque in the member is greatly affected by boundary conditions, it is left to the designer to decide if one-half vertical shear is more appropriate for outer face reinforcement.

6. There are two components of torsion acting action at the 45-degree failure surface: plate bending and twist. The plate bending component is reflected in the previous design equations in Steps 3-5. The influence of the twist component, \( \tau_T \), observed in the experimental testing, results in a slightly warped failure plane. Two possible models to determine the resistance to the twist, along the assumed failure plane, are presented below (Klein 2008).

   a. Out-of-plane shear stress, increasing in proportion to the distance from the center of twist as shown in Figure 5.19a, is determined as follows:

   \[
   \tau_{nw-p} = \frac{2.12T_u}{d_w d^2}
   \]

   b. Out-of-plane shear stress, uniform above and below the center of twist as shown in Figure 5.19b, is determined as follows:

   \[
   \tau_{nw-u} = \frac{1.41T_u}{d_w d^2}
   \]
The twist component of torsion, $T_{\text{ut}}$, taken about the failure plane, is dependent on out-of-plane shear strength but it is not practical to provide shear reinforcement through the width of the web. It is assumed in this analysis that the torsional strength, for linear out-of-plane shear stress, is dependent on the concrete contribution to shear or $2\sqrt{f_c b_w d}$. As resistance to vertical shear is also provided by the same concrete, the resistance to torsion may be less than expected. It is then assumed that the concrete contribution to torsional resistance, for uniform out-of-plane shear stress, is then a component of the total shear or $\sqrt{2}\sqrt{f_c b_w d}$.

The concrete resistance to torsion for a linear distribution of out-of-plane shear stress is given by:

$$T_u = 0.94 \cdot \sqrt{f_c b_w d^2}$$  \hspace{1cm} \text{Equation 5-10}
A similar resistance for a uniform distribution of out-of-lane shear stress is given by:

\[ T_u = \sqrt{f'_{c} d_w d^2} \quad \text{Equation 5-11} \]

These equations indicate that an unreinforced specimen, with identical cross section and concrete compressive strength to that of the experimental specimens, have sufficient capacity to resist this twist component of torsion. Using Equation 5-10 as an upper bound limit, twist resistance was approximately 1400 kip-inches for the tested specimens; the factored design torsion was only 1266 kip-inches.

**Evaluation of Proposed Rational Design Methodology**
Analysis of specimen SP10 was completed to determine the amount of steel reinforcement in the end region, as designed using the preliminary design approach discussed in Section 2.5. Calculations to determine the amount of required hanger steel indicated that in the end region, hanger reinforcement would not control in the design of SP10. Therefore, end region inner face steel for SP10 was based on requirements needed for shear and torsion.

The vertical and horizontal reinforcement crossing the assumed failure plane on the inner face was transformed 45 degrees to determine the total area of reinforcement normal to the assumed failure plane. Note that the welded wire reinforcement does not cross the assumed failure plane on either face and is therefore not accounted for in end region reinforcement.

On the inner face, across the assumed failure plane, there is one C-bar and seven L-bars in the vertical direction and legs from seven U-bars in the horizontal direction. Transforming this reinforcement by 45 degrees, a total area of 2.23 in\(^2\) crosses the failure plane on the inner face. Outer face reinforcement consists of one C-bar and four L-bars in the vertical direction.
and legs from seven U-bars in horizontal direction, for a total of 1.78 in\(^2\) across the assumed failure plane on the outer face.

To determine the total amount of orthogonal reinforcement required to cross the assumed failure plane on the inner face, the following equations were derived from the proposed rational design method.

\[
A_{v1}\left(\frac{d}{s}\right) = \frac{(V_a - V_c)}{2f_y} \sqrt{2} \tag{Equation 5-12}
\]

\[
(A_{t1} + A_{t2})\left(\frac{d}{s}\right) = \frac{\sqrt{2}T_w}{2f_yd_w} \tag{Equation 5-13}
\]

The total amount of orthogonal reinforcement on the inner face is then \((A_{v1} + A_{t1} + A_{t2})\frac{d}{s_i}\).

The equations for the outer face reinforcement were also updated to find the total amount of steel required orthogonal to the web centerline. As the value for \(e/2d_w\) is greater than \((1-V_c/V_a)\), the minimum shear requirements, per ACI, must be met or:

\[
A_{v2} = 0.75 \star \sqrt{\frac{f_c}{f_y}} \frac{b_w s}{f_y} \star \sqrt{2} \tag{Equation 5-14}
\]

\[
(A_{t2})\left(\frac{d}{s}\right) = \frac{\sqrt{2}T_w}{4f_yd_w} \tag{Equation 5-15}
\]

The total amount of orthogonal reinforcement on the outer face is then \((A_{v2} + A_{t2})\frac{d}{s_i}\). To determine the minimum value of outer face vertical steel, an 8 inch spacing was assumed.

**Conclusions**

While the preliminary approach designed SP10 with slightly higher outer face end region reinforcement, the proposed rational method required higher end region inner face reinforcement, as shown in Table 5.4.
When comparing the differences between the design methods, it was noted that the proposed rational method had a lower phi factor, higher eccentricity and smaller lever arm than the preliminary approach, also shown in Table 5.4. Two other key differences should also be noted:

- Outer face shear requirements only require the minimum amount of steel in the rational method while the preliminary approach assumes half vertical shear on both sides. Note that one-half vertical shear reinforcement is conservative and may be well suited for practice.

- Shear and torsion demand dramatically reduces at each of the ledge loads heading towards midspan; the first stem load crosses the hypothetical failure plane approximately 2.75 feet into the disturbed end region. Currently the proposed rational method neglects this reduction in force, resulting in overly conservative shear design, while the preliminary design approach incorporates the reduction into vertical shear steel requirements.

<table>
<thead>
<tr>
<th></th>
<th>Preliminary Design Approach</th>
<th>Proposed Rational Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Face Reinforcement (in²)</td>
<td>2.23</td>
<td>2.6</td>
</tr>
<tr>
<td>Outer Face Reinforcement (in²)</td>
<td>1.78</td>
<td>1.14</td>
</tr>
<tr>
<td>Phi Factor for Shear and Torsion</td>
<td>0.9</td>
<td>0.75</td>
</tr>
<tr>
<td>Eccentricity, e (inches)</td>
<td>9.06</td>
<td>10</td>
</tr>
<tr>
<td>Lever Arm, d₀ (inches)</td>
<td>6.75</td>
<td>6.4</td>
</tr>
</tbody>
</table>
6. Summary and Conclusions

6.1. Overview
The main objective of the research program, presented in this thesis, is to study the behavior of slender precast L-shaped spandrel beams, reinforced with an open stirrup configuration, under the effects of typical loading conditions. Current codes and guidelines typically require heavily congested, closed stirrup for specimens subjected to combined shear and torsion. The objective of the research includes also an introduction of a design guideline based on equilibrium and compatibility of the failure plane. The concept considered in this thesis is based on out-of-plane bending to design the required open web reinforcement and accounting for flexure, shear, and torsion due to the applied load. The potential benefits of the simplified detailing for precast slender spandrel beams are significant. Open reinforcement greatly enhances constructability of these types of beams in comparison to the traditional closed stirrups, therefore, providing considerable savings of construction time and cost.

The research program consisted of an experimental program including testing of four full scale slender precast concrete L-spandrel beams at the Constructed Facilities Laboratory, North Carolina State University. Boundary conditions used in the test are similar to field conditions, including the loading conditions by using typical double-tee prestressed members representing flooring or roofing of parking structures. The loading configuration creates combined torsion, shear and bending loading conditions similar to field conditions. The main parameters considered in the study were the use of open versus closed stirrups and reinforced concrete versus prestressed concrete slender L-spandrels.
Data from the experimental phased was used to evaluate the effectiveness of the proposed open stirrup configuration on slender L-spandrel behavior versus the typical closed stirrups. Test results were used to validate and refine a non-linear finite element model which was used to study various parameters believed to affect the behavior of L-spandrels. Based on the experimental and analytical studies, a rational design methodology is proposed for the design of slender precast L-spandrels using open reinforcement.

6.2. Conclusions
Based on the experimental and analytical modeling studies, the following conclusions were made:

1. L-shaped spandrel beams with continuous ledges do not exhibit any signs of concrete spalling or spiral cracking, typically observed for beams subject to torsion reported in the literature review.

2. At service load levels, there is virtually no difference in behavior when using open stirrups in comparison to beams reinforced with closed stirrups.

3. Test results indicate that out-of-plane affects dominate end region behavior of slender L-shaped spandrel beams and failure modes for beams designed with open reinforcement as bottom lateral deflections exceed the vertical deflections at failure.

4. The spandrel-deck connections significantly effect the lateral displacement induced by bending about the weak principal axis.

5. Shear cracks in the end regions extended up from the bottom of the beams at approximately 45 degrees before flattening out towards midspan.
6. The crack pattern is used to define three distinct regions along the span of a typical slender L-shaped spandrel beam: disturbed end region, transition region, flexure region.

7. Using open reinforcement to combat plate-bending effects provides sufficient reinforcement in the end region and reduces overall cost and fabrication time for slender L-shaped spandrel beams.

**6.3. Future Work**

The challenge in creating such a design guideline is in evaluating the large number of relevant factors, both experimentally and analytically, and defining appropriate limitations for the new approach. It is believed that once completed the research effort presented here will allow the industry to fully embrace the concept of using open reinforcement for slender precast spandrel beam construction. Based on the results and conclusions of the research program and analytical study, the following recommendations are made for future work:

- Test spandrel beams with different aspect ratios to examine the validity of the proposed design method for the transition from slender to shallow members.

- Test spandrel beams with discrete haunches, instead of continuous ledge.

- Study the influence of friction at the deck to ledge bearing and the effect of changing the welded connection to deck members.
References


[17] Zia, P. and Hsu, T. 2004. Design for Torsion and Shear in Prestressed Concrete Flexural Members. *PCI Journal* 49(3): 34-42. May-June. (This paper was previously presented at the American Society of Civil Engineers Convention, October 16-20, 1978, Chicago, IL, reprint #3424.)


Appendices
Appendix A. Reinforcement Details
M1b

PLATES AND STRUCTURAL SHAPES—ASTM A36
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