Abstract

AKHAVAN TABATABAEI, RAHA. AN INTERACTIVE FRAMEWORK FOR THE PARAMETER DESIGN PROBLEM. (Under direction of Dr. Yahya Fathi.)

We propose an interactive framework for solving the Parameter Design Problem. In this context we consider a response function $Y = h(x_1, ..., x_n)$, where $x_1, ..., x_n$ are random variables with known probability distribution functions, and $h$ is a continuous and differentiable function. Given the distribution functions of $x_1$ through $x_n$ and the function $h$, we use a family of Johnson distributions to approximate the probability density function of $Y$ by using its first four moments as input. This density function can be displayed graphically and compared with the given specification limit of $Y$. Using this approach we develop a computer program that would allow the user to modify the set points of $x_1, ..., x_n$ manually, and immediately observe the impact of this adjustment on the probability density function of $Y$. The user can then interactively search for and determine a satisfactory set of values for these set points. We also present two case studies and solve them by this method. The framework that we propose also provides a platform for employing other techniques such as nonlinear programming or statistical design of experiment in order to assist the user in determining a satisfactory solution for the parameter design problem.
An Interactive Framework for The Parameter Design Problem

By

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Chapter 1

Introduction

In this Chapter we define the Parameter Design Problem and review the literature on this subject. Then we propose an interactive model to solve the Parameter Design Problem.

1.1 Definition of the Problem

Let \( x = (x_1, \ldots, x_n) \) be a vector of design variables (components) for a manufacturing product or a manufacturing process and let \( Y = h(x_1, \ldots, x_n) \) represent an output characteristic of interest. We assume that \( x_1, \ldots, x_n \) are random variables with known probability distributions. Since the output characteristic, \( Y \), is a function of a set of random variables, \( Y \) itself is a random variable whose probability density function and its moments are dependent on probability density functions of \( x_1, \ldots, x_n \) and the form of the response function, \( h \).

Let \( \mu_1, \ldots, \mu_n \) denote the mean values (set points) of \( x_1, \ldots, x_n \). We further assume that \( \mu_1, \ldots, \mu_n \) are controllable parameters (i.e., their values can be determined by the design engineer). In the design phase values of \( \mu_1, \ldots, \mu_n \) are determined such that the nominal value of \( Y \) is equal to its target value, \( \tau \). During the production phase, existence of noise can cause values of \( x_1, \ldots, x_n \) to deviate from their set points \( \mu_1, \ldots, \mu_n \), hence causing \( Y \) to deviate form its target value, \( \tau \).

Parameter Design Problem focuses on determining \( \mu_1, \ldots, \mu_n \) such that not only nominal value of \( Y \) is kept on target, but also the variation or noise in \( Y \) is minimized.

1.2 Literature Review

Initially, the Parameter Design Problem was addressed by G. Taguchi. Since then, many researchers have proposed various methodologies for solving this problem. The most notable work among all is one that is originally proposed by Taguchi and Wu [13] which suggests a model to maximize a performance measure known as ‘signal to noise ratio’. Taguchi and Wu’s method is later discussed in Leon et al. [12] and Box [1].
This methodology is based on using an orthogonal array in experimental design to maximize signal to noise ratio. Fathi [6] states the advantages of this method in being modest in computational requirements and suitable for the situations where the analytical form of the response function between the design variables and the output characteristic $Y$ is not available. The major disadvantage is also stated as terminating at a suboptimal solution.

Box and Fung [2] proposed a constrained nonlinear programming (NLP) model to maximize the signal to noise ratio, and showed that when applicable, this model could result in better solutions. Fathi [7] proposed a constrained nonlinear programming model that minimizes the variance of $Y$. In this method variance of $Y$ is approximated by successive quadratic approximations of the response function. This is only applicable to cases where the response function is known or can be well approximated.

Another approach proposed by Fathi [6] approximates the variance of $Y$ by using a linear approximation. He uses Taylor series expansion to approximate $h$ up to its linear term. From there, mean and variance of $h$ are calculated about the point $(\mu_1, \ldots, \mu_n)$. Then he uses this approximation as the objective function of a NLP model. He constrains the problem by forcing nominal value of $Y$ to be on target, and $\mu_1, \ldots, \mu_n$ satisfy any other technical requirement (constraint). This methodology is referred to as Parameter Design Model with Linear Approximation of Variance or PDM-LAV.

1.3 The Proposed Model

In this thesis we propose a different approach for solving the Parameter Design Problem. This is an interactive approach in which the design engineer can manually adjust the values of the controllable parameters $\mu_1, \ldots, \mu_n$ and see the effect of this adjustment on the probability density function of $Y$, instantaneously.

In this model, we take advantage of a method of approximating the probability density function of a variate by an empirical distribution. In particular we approximate probability distribution of $Y$ by a family of Johnson distributions.

To determine the parameters of the corresponding Johnson distribution, we use the first four moments of the variate $Y$. To this end we approximate these first four moments of
the variate either by using the Taylor series expansion of the function \( h(x_1, \ldots, x_n) \) about the set points \( \mu_1, \ldots, \mu_n \), or (as appropriate) by using a Monte-Carlo simulation model.

Once the first four moments of \( Y \) are evaluated, we identify the Johnson family that it belongs to, and then calculate the distribution parameters of that family. Once the appropriate Johnson family and its parameters are estimated, we form the probability density function (pdf) for \( Y \) and plot its graph. Now a design engineer can visually observe the shape of pdf of \( Y \) at the given set points for \( \mu_1, \ldots, \mu_n \) and compare it with the target value of \( Y \) and its specification limits. He or she can then try different settings for \( \mu_1, \ldots, \mu_n \) as appropriate. For each setting that is tried, the pdf of \( Y \) is graphed and hence the design engineer can see the effect of this new setting on the pdf of \( Y \). This process continues until a desirable set of values for \( \mu_1, \ldots, \mu_n \) is determined.

1.4 Organization of the Thesis

The empirical distributions and methods of approximation are discussed in Chapter 2. The methods for approximating the first four moments of \( Y \) are discussed in Chapter 3. Chapter 4 explains the details of our methodology and the software that is developed for this purpose. In Chapter 5 we present two case studies involving the Parameter Design Model, using this methodology.

The software that we develop can also serve as the basis for a framework in which we may include different approaches in the Parameter Design Problem, i.e. the statistical design of experiments or the nonlinear programming model. In Chapter 6, the features that can be added to the existing model and also other methodologies that can be included in the framework to enhance its capacity and facilitate the search for an optimal combination of set points, are discussed. In this Chapter we also discuss our findings and avenues for further research.
Chapter 2

Approximation by Empirical Distributions

In this Chapter we discuss methodologies of fitting an empirical distribution to a 
variate and in particular we focus on the Johnson distributions.

2.1 Background

In the context of the PDP, for a general form of the response function \( Y = h(x_1, \ldots, x_n) \), 
it is usually difficult to determine the exact form of the pdf for \( Y \), even if the exact form 
of the pdf for \( x_1, \ldots, x_n \) is known. In this case we use an approximation of the pdf of \( Y \) in 
our analysis.

One method for approximating the pdf of \( Y \) is fitting a well-known distribution such as 
normal, Gamma, or lognormal. The most common of these methods is the use of normal 
distribution, since it follows from the central limit theorem that normal distribution 
provides a reasonable representation of many physical phenomena. Gamma and 
lognormal distributions are also used for variables which are bounded at one end, and 
Beta distribution is used for those that are bounded at both ends.

Although these models cover a wide variety of shapes, they do not provide the 
desirable degree of generality required. In Figure 2-1, the regions in the \( (\beta_1, \beta_2) \) plane that 
are covered by the well-known distributions of lognormal, normal, Beta, Gamma and 
student t are shown. In this figure, \( \beta_1 = \left( \frac{\mu_3}{(\sigma^2)^{3/2}} \right)^2 \) represents the square of the 
standardized measure of skewness and \( \beta_2 = \frac{\mu_4}{(\sigma^2)^2} \) represents the standardized measure 
of peakedness. In these expressions \( \mu_3 \) and \( \mu_4 \) are the third and fourth moments of \( Y \) 
about its mean, and \( \sigma^2 \) is its variance.

In this figure, the distributions which do not have any shape parameter, such as normal, 
exponential, and uniform, are represented as single points, whereas others which involve 
different shapes of curves and have one or two shape parameters are represented as lines 
or regions on the \( (\beta_1, \beta_2) \) plane.
While this figure can be used for examining the closeness of a given distribution to a well-known distribution (i.e., uniform, normal, etc.), little can be done about the large area that is not occupied by any of these well-known distributions. This justifies the use of empirical distributions.

A more general technique for approximating the pdf of a random variable would be to use either Johnson or Pearson distributions. In this section we discuss Johnson distribution in some detail and use this distribution throughout the thesis. For discussion on Pearson distribution one can refer to [8] and [10].
2.2 Johnson Distribution

Johnson [11] suggested transforming a standard normal variate as the basis for empirical distributions. One advantage of this transformation is that we can use a table of areas under a standard normal distribution for estimating the percentiles of the fitted distribution.

The general form of this transformation is [8]:

$$z = \gamma + \eta \tau(x; \varepsilon, \lambda), \quad \eta > 0, -\infty < \gamma < \infty, \lambda > 0, -\infty < \varepsilon < \infty,$$

where \(x\) is the variable to be fitted by a Johnson distribution.

In this general form, \(\tau\) is an arbitrary function, \(\gamma, \eta, \varepsilon, \lambda\) are the four parameters of Johnson distribution and \(z\) is a standard normal variate.

Johnson has proposed the following three families for the function \(\tau\):

(a) \(\tau_1(x; \varepsilon, \lambda) = \ln\left(\frac{x - \varepsilon}{\lambda}\right), \quad x \geq \varepsilon,\) \hspace{1cm} (2-2)

(b) \(\tau_2(x; \varepsilon, \lambda) = \ln\left(\frac{x - \varepsilon}{\lambda + \varepsilon - x}\right), \quad \varepsilon \leq x \leq \varepsilon + \lambda\) \hspace{1cm} (2-3)

(c) \(\tau_3(x; \varepsilon, \lambda) = \sinh^{-1}\ln\left(\frac{x - \varepsilon}{\lambda}\right), \quad -\infty < x < \infty\) \hspace{1cm} (2-4)

Hahn and Shapiro [8] find the probability density functions for the three families of Johnson distributions using methods of variable transformation, given that \(z\) is a normal variate.

1-Family that corresponds to \(\tau_1\), also known as Johnson S1 family:

\[f_1(x) = \frac{\eta}{(x - \varepsilon)\sqrt{2\pi}} \exp\left\{-\frac{1}{2} \left[\gamma + \eta \ln\left(\frac{x - \varepsilon}{\lambda}\right)\right]^2\right\},\]

\[x \geq \varepsilon, \eta > 0, -\infty < \gamma < \infty, \lambda > 0, -\infty < \varepsilon < \infty.\]

Setting \(\gamma' = \gamma - \eta \ln \lambda\) gives,

\[f_1(x) = \frac{\eta}{(x - \varepsilon)\sqrt{2\pi}} \exp\left\{-\frac{1}{2} \eta^2 \left[\frac{\gamma'}{\eta} + \ln(x - \varepsilon)\right]^2\right\},\]

\[x \geq \varepsilon, \eta > 0, -\infty < \gamma' < \infty, -\infty < \varepsilon < \infty.\]
This is in fact a generalized form of the lognormal distribution as described in page 99 in [8]. In most of the literature as well as in the model that we develop in this thesis, equation (2-6) is used as the pdf of $S_L$ family.

2- Family that corresponds to $\tau_2$ also known as Johnson $S_B$ family:

$$f_2(x) = \frac{\eta}{\sqrt{2\pi}} \frac{\lambda}{(x-\epsilon)(\lambda-x+\epsilon)} \exp\left\{-\frac{1}{2} [\gamma + \eta \ln\left(\frac{x-\epsilon}{\lambda-x+\epsilon}\right)]^2\right\},$$  \hspace{1cm} (2-7)

$\epsilon \leq x \leq \lambda + \epsilon, \eta > 0, -\infty < \gamma < \infty, \lambda > 0, -\infty < \epsilon < \infty$.

3- Family that corresponds to $\tau_3$ also known as Johnson $S_U$ family:

$$f_3(x) = \frac{\eta}{\sqrt{2\pi}} \frac{1}{\sqrt{(x-\epsilon)^2 + \lambda^2}} \exp\left[-\frac{1}{2} (\gamma + \eta \ln\left(\frac{x-\epsilon}{\lambda}\right) + \left[(\frac{x-\epsilon}{\lambda})^2 + 1\right]^{1/2})^2\right],$$  \hspace{1cm} (2-8)

$-\infty < x < \infty, \eta > 0, -\infty < \gamma < \infty, \lambda > 0, -\infty < \epsilon < \infty$.

Hahn and Shapiro [8] also show the $(\beta_1, \beta_2)$ plane for these Johnson families.

Figure 2-2: $(\beta_1, \beta_2)$ plane for Johnson distributions.
Note that the Johnson $S_L$ and $S_B$ families both have two shape parameters $\gamma$ and $\eta$, one location parameter $\epsilon$, and one scale parameter $\lambda$.

From the defined range of the random variate $x$ in each family, we can see that $S_U$, $S_L$, and $S_B$ distributions are defined for unbounded variates, variates bounded at one end, and variates bounded from both above and below, respectively. However these limitations need not be strictly followed when the distributions are used for approximation. This is similar to approximating a bounded variate with normal distribution which is unbounded by nature. Figures 2-3, 2-4, and 2-5 show some examples of curves from the $S_U$, $S_L$ and $S_B$ families, respectively. It can be observed that these families of distributions give a wide range of shapes; hence they can be used in a variety of situations.

Figure 2-3: Johnson $S_U$ distributions with $\epsilon = 0$, $\lambda = 1$ and various values of the parameters $\eta$ and $\gamma$. 
Figure 2-4: Johnson $S_L$ distributions with $\varepsilon = 0$ and various values of the parameters $\eta$ and $\gamma^*$. 

Figure 2-5: Johnson $S_B$ distributions with $\varepsilon = 0, \lambda = 1$ and various values of the parameters $\eta$ and $\gamma$. 
2.3 Approximation by Johnson Distribution

In order to approximate a variate, \( Y \), using Johnson distributions, we must first determine its corresponding family of Johnson distribution and then determine the parameters of that family. To this end we need to determine the values of \( \beta_1 \) and \( \beta_2 \), and identify the point \((\beta_1, \beta_2)\) in Figure 2-2. If the \((\beta_1, \beta_2)\) point falls close to the log-normal line, then \( S_L \) is chosen. If it falls above or below this line, then \( S_B \) or \( S_U \) should be chosen, respectively. The range of the regions in Figure 2-2 can be extended by use of these parametric equations

\[
\beta_1 = (\omega - 1)(\omega + 2)^2, \quad (2-9)
\]
\[
\beta_2 = \omega^4 + 2\omega^3 + 3\omega^2 - 3. \quad (2-10)
\]

Additional points can be obtained by choosing values for \( \omega \) and solving for the corresponding values of \( \beta_1 \) and \( \beta_2 \), as we discuss later.

To determine parameters of the fitted Johnson distribution (i.e. \( \gamma, \eta, \lambda, \varepsilon \)), two different approaches are used in the literature. One method is based on using a relatively large random sample. This method is discussed in [8] in details. The second approach, which is used in this thesis, is based on using the first four moments of the random variate \( Y \). This method is proposed by Hill, Hill and Holder in [9], where they provide the details of this algorithm and a corresponding computer program. This program takes the mean, the standard deviation, the skewness and the peakedness of a variate as its input. With this input, the algorithm finds the family of Johnson distribution that fits the variate and then calculates the parameters of the fitted distribution. This method is numerical in nature and it is based on the works by Elderton and Johnson [4] and Draper [3]. The complete script of the computer program can be found in Appendix 1. Here, we briefly review the basic ideas of this algorithm.

Let \( \mu, \sigma, \mu_3 \) and \( \mu_4 \) denote the mean, standard deviation, third moment about the mean and fourth moment about the mean of the variate, respectively. Hill et al. [9] further separates the regions in Figure 2-2 by introducing two new families of distributions. One family corresponds to the point \((\beta_1 = 0, \beta_2 = 3)\) which represents the normal pdf. The second family corresponds to the line segment separating the two regions of
“Johnson S_B” and the “Impossible Area”. This line segment is referred to as Johnson S_T. The relationship between $\beta_1$ and $\beta_2$ for this line segment is $\beta_2 = \beta_1 + 1$.

The first step in the algorithm is to determine the family of Johnson distribution that fits the variate. Let $\beta_1^*$ denote the given value for square of skewness and $\beta_2^*$ denote the given value for peakedness of the variate. If $\beta_1^* = 0$ and $\beta_2^* = 3$, then normal pdf is selected. If $\beta_1^* + 1 = \beta_2^*$, then $S_T$ is selected. Otherwise, we solve equation (2-9) with $\beta_1^*$ to determine the corresponding value of $\omega$. We denote this value by $\omega^*$. Then we substitute $\omega^*$ into equation (2-10) and solve for $\beta_2$. Let $\hat{\beta}_2$ denote the corresponding value of $\beta_2$. If $\beta_2^* = \hat{\beta}_2$, then we select $S_L$; if $\beta_2^* > \hat{\beta}_2$, we select $S_B$; and if $\beta_2^* < \hat{\beta}_2$, we select $S_U$.

At this point the algorithm starts to determine the parameters of the selected family of Johnson distribution.

1- $S_L$ Curves:

With $\omega^*$ having been evaluated as above, let

$$\eta = \left(\ln \omega^*\right)^{-1/2},$$

$$\gamma^* = \frac{1}{2} \eta \ln \left\{ \omega^* (\omega^* - 1) / \sigma^2 \right\},$$

$$\varepsilon = \pm \mu - \exp \left\{ (1/2\eta - \gamma^*) / \eta \right\},$$

In equation (2-13) the sign of $\mu$ is determined in each case to be the same as the sign of $\mu_3$. For example, if the third moment of the variate, $\mu_3$, is negative then (2-13) becomes $\varepsilon = -\mu - \exp \left\{ (1/2\eta - \gamma^*) / \eta \right\}$, otherwise, it becomes $\varepsilon = \mu - \exp \left\{ (1/2\eta - \gamma^*) / \eta \right\}$.

2- $S_U$ Curves:

In estimating the parameters of a $S_U$ curve, two cases are identified depending on the value of $\beta_1^*$. If $\beta_1^* = 0$, the corresponding curve is symmetrical, and its associated parameters are as follow.
\[ \omega = \{(2\beta^*_2 - 2)^{\frac{1}{2}} - 1\}^{\frac{1}{2}} \tag{2-14} \]
\[ \eta = (\ln \omega)^{\frac{1}{2}} \tag{2-15} \]
\[ \gamma = 0 \tag{2-16} \]

If \( \beta^*_1 > 0 \) then, the corresponding curve is not symmetrical and the associated values of parameters \( \omega, \eta \) and \( \gamma \) are found by an iterative procedure which is discussed in [4]. This iterative procedure is described in the computer program listing in Appendix 1. In both of these cases, the values of \( \varepsilon \) and \( \lambda \) are then found from the following equations:
\[ \mu_2 = \frac{1}{2} \lambda^2 (\omega - 1)\{\omega \cosh(2\gamma/\eta) + 1\} \tag{2-17} \]
\[ \mu_1 = \varepsilon - \lambda \omega^{\frac{1}{2}} \sinh(\gamma/\eta) \tag{2-18} \]

3- \( S_B \) Curves:
In order to determine the value of \( \eta \) for this family, we consider its boundary values. When the point \((\beta^*_1, \beta^*_2)\) is approaching the \( S_r \) boundary, \( \eta \) approaches zero; when the point is approaching the \( S_l \) boundary, \( \eta \) tends to be the same value as for a \( S_l \) curve [equation (2-11)]. To estimate \( \eta \), it is first set equal to the value from interpolation between these two boundaries. Then Hill et al. [9] use a series of formulas based on work of Draper [3] to evaluate the rest of parameters based on estimated value of \( \eta \), and then improve these values iteratively. These series of formulas are also presented in the computer program listing in Appendix 1.

4-Normal Curves:
Parameter \( \eta \) is set equal to \( 1/\sigma \), and \( \gamma \) is set equal to \( \mu/\sigma \); \( \lambda \) and \( \varepsilon \) are set to 0.

5- \( S_r \) Curve:
For this family let,
\[ \varepsilon = \mu - \frac{\sigma}{\sqrt{1/2 + 1/2}} \left( \sqrt{1 - \frac{4}{\beta_i + 4}} \right), \quad (2-19) \]

\[ \lambda = \varepsilon + \sigma \sqrt{\beta_i + 4}, \quad (2-20) \]

\[ \eta = \sqrt{1/2 + 1/2} \left( \sqrt{1 - \frac{4}{\beta_i + 4}} \right), \quad (2-21) \]

and \( \gamma \) be set to 0.

### 2.4 Examples of Johnson Approximation

In this section we present several numerical examples. In these examples, we compare the probability distribution function of a number of variates that have known probability distributions (e.g., Beta and Gamma, etc.), with their approximations by Johnson distribution, in order to validate this algorithm.

**Example 1**

In this example we fit Johnson distribution to Beta and Gamma distributions with various parameter values. To start the algorithm we need the first four moments of the variate. The first four moments of Beta and Gamma variates are given in [8] with respect to the parameters of each distribution. We take some cases of these variates with different values for their parameters, and use the values of their moments as input to the algorithm. After the algorithm determines the family type and evaluates the parameters of the corresponding family, we insert the parameters in equations (2-6), (2-7) or (2-8) as appropriate. Then we plot the pdf and compare it with the actual pdf of the corresponding distribution as appropriate. Comparing Figures 2-1 and 2-2 shows that both Beta and Gamma curves are in Johnson \( S' \) region. Several examples of comparing the Johnson
distribution and the actual probability density functions of Gamma and Beta are shown here.

Figure 2-6: Fitting Johnson distribution to Gamma with shape parameter =3 and scale parameter =5

Figure 2-7: Fitting Johnson distribution to Gamma with shape parameter =1 and scale parameter =1 (Exponential)
We observe that the fitted Johnson distribution is reasonably close to the actual probability density function of the known distribution in each of these cases.
Example 2

In this example we solve a case that is originally presented in [8]. In this case the corresponding $(\beta_1, \beta_2)$ point is in the $S_U$ region. We find the parameters of Johnson $S_U$ family by the method that we discussed earlier and compare them with the results in [8].

In this example measurements of the coefficient of friction for a metal were obtained on 250 samples. The mean, standard deviation, the skewness and the peakedness are estimated from the sample and are presented in [8] as 0.0345 for mean, 0.0098 for variance, 0.87 for skewness and 4.92 for peakedness, respectively. Plotting the values of $\beta_1 = (0.87)^2 = 0.7569$ and $\beta_2 = 4.92$ in Figure 2-2 suggests the use of a Johnson $S_U$ distribution. Table 2-1 shows the values of the parameters that are found by each method.

<table>
<thead>
<tr>
<th>Parameters of $S_U$</th>
<th>Hahn and Shapiro</th>
<th>Hill, Hill and Hull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>-1.783</td>
<td>-1.785</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2.433</td>
<td>2.446</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0169</td>
<td>0.0169</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.0198</td>
<td>0.0198</td>
</tr>
</tbody>
</table>

Table 2-1: Coefficient of Friction Example

Figure 2-10 shows the fitted Johnson distributions to the parameters of Table 2-1. The two probability density functions closely cover each other.
We consider this approximation method to be reasonably close for the purpose of the interactive model that we propose in Chapter 5.
Chapter 3
Evaluating the First Four Moments of the Response Function

In this Chapter we discuss two methods for determining the moments of $Y$, where $Y = h(x_1, \ldots, x_n)$ as described in Chapter 1. We assume that we know the probability density function for each component variable $x_1, \ldots, x_n$. We further assume that the function $h$ is given in closed form and that it is continuous and differentiable at all points. Both methods that are discussed here are approximation methods, since determining the exact values of the moments of $Y$ can be computationally difficult, if at all possible. One of these two methods is based on Monte-Carlo simulation and the other method is based on the Taylor series expansion of the function $h$.

3.1 Monte-Carlo Simulation

In this method with help of a computer program, capable of generating random numbers, we generate a value for each of the components $x_1, \ldots, x_n$ and determine the corresponding value of $Y$. If we repeat this process $k$ times, we can create a collection of $k$ randomly generated values for $Y$ (i.e., a random sample of $Y$).

Based on this random sample, we can estimate the first four moments of $Y$ by a series of formulas that are adopted from [8].

1- Estimating mean:

$$
\bar{y} = \frac{\sum_{i=1}^{k} y_i}{k} \quad (3-1)
$$

2- Estimating variance:

$$
m_2 = \sigma^2 = \frac{\sum_{i=1}^{k} (y_i - \bar{y})^2}{k} = \frac{k \sum_{i=1}^{k} y_i^2 - (\sum_{i=1}^{k} y_i)^2}{k(k-1)} \quad (3-2)
$$
3-Estimating the third moment:

\[
m_3 = \frac{1}{k} \sum_{i=1}^{k} (y_i - \bar{y})^3 - 3 \frac{1}{k} \left( \sum_{i=1}^{k} y_i^2 \right) \frac{1}{k} \left( \sum_{i=1}^{k} y_i \right) + 2 \left( \frac{\sum_{i=1}^{k} y_i^3}{k} \right)
\]  

(3-3)

4-Estimating the fourth moment:

\[
m_4 = \frac{1}{k} \sum_{i=1}^{k} (y_i - \bar{y})^4 - 4 \frac{1}{k} \left( \sum_{i=1}^{k} y_i^2 \right) \frac{1}{k} \left( \sum_{i=1}^{k} y_i \right) + 6 \left( \frac{\sum_{i=1}^{k} y_i^2}{k} \right) \frac{1}{k} \left( \sum_{i=1}^{k} y_i \right) ^2 - 3 \left( \frac{\sum_{i=1}^{k} y_i^3}{k} \right) \]  

(3-4)

Then the skewness of \(Y\) is estimated as

\[
\sqrt{b_1} = \frac{m_3}{(m_2)^{3/2}},
\]

(3-5)

and the peakedness of \(Y\) is estimated as

\[
b_2 = \frac{m_4}{(m_2)^2}.
\]

(3-6)

Equations (3-1) and (3-2), i.e., \(\bar{y}\) and \(m_2\), can be used as estimates for mean and variance of \(Y\) respectively. Equations (3-5) and (3-6), i.e., \(\sqrt{b_1}\) and \(b_2\), can be used to estimate \(\sqrt{b_1}\) and \(b_2\), respectively.

3.2 Taylor Series Approximation

Tukey [14] develops a collection of formulas for approximating the first four moments of the random variate \(Y\). These formulas are based on the Taylor series expansion of the function \(h\) about the point \(\mu_1, \ldots, \mu_n\), using its first few terms. The complete forms of these formulas are given in [14], and we employ a simplified version of these formulas as given below. In these formulas the following notations are used for the moments of the components \((x_i, i = 1, 2, \ldots, n)\): mean of \(x_i = \mu_i\), variance of \(x_i = \sigma_i^2\), skewness of \(x_i = \gamma_i\),
and peakedness of $x_i = \Gamma_i$. Also $\mu_Y$ denotes the mean of $Y$, $\sigma_Y^2$ denotes the variance of $Y$, $\phi_Y$ denotes the third central moment of $Y$ about its mean, and $\varphi_Y$ denotes the fourth central moment of $Y$ about its mean.

$$\mu_Y = h(\mu_1, \mu_2, ..., \mu_n).$$  \hfill (3-7)

$$\sigma_Y^2 = \sum h_i^2 \sigma_i^2. \hfill (3-8)$$

$$\phi_Y = \sum h_i^3 \gamma_i \sigma_i^3 \hfill (3-9)$$

$$\varphi_Y \approx \sum h_i^4 (\Gamma_i - 3) \sigma_i^4 + 3 \sigma_Y^4 \hfill (3-10)$$

In these formulas all the summations are form 1 to $n$, and $h_i = \frac{\partial h}{\partial x_i}\bigg|_{\mu_1, \mu_2, ..., \mu_n}$. Formulas for more accurate approximation of these parameters are presented in [14]. These formulas contain higher order terms of the Taylor series approximation.

### 3.3 Numerical Examples

In this section we present two numerical examples and observe the accuracy of these two methods for determining the moments of $Y$. In example 1, we only use the Monte-Carlo simulation method since the corresponding function is linear, and in example 2 we use both methods.

**Example 1**

We consider the following function: $y = x_1 + x_2 + x_3$, and we assume $x_1 \sim N(2, 1^2)$, $x_2 \sim N(3, 1^2)$, and $x_3 \sim N(5, 2^2)$.

Of course in this example it is easy to determine the exact form of the probability distribution function of $y$ and its parameters. It is clear that $y$ has a normal distribution with the following parameters: mean of $y$ is $\mu_y = 2 + 3 + 5 = 10$, variance of $y$ is $\sigma_y^2 = 1^2 + 1^2 + 2^2 = 6$. Skewness and peakedness of $y$ are 0 and 3, respectively since $y$ is a normal random variate.
In order to evaluate these parameters using the Monte-Carlo simulation method, we randomly generate 250,000 values for each component. Then we determine the corresponding values for the function \( y \). We can then estimate mean, variance, skewness and peakedness of \( y \) using equations (3-1) through (3-6). Table 3-1 shows the results from simulating this function versus the values that are calculated analytically.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values from Simulation</th>
<th>Actual Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>10.0090</td>
<td>10</td>
</tr>
<tr>
<td>Variance</td>
<td>6.0034</td>
<td>6</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.0018</td>
<td>0</td>
</tr>
<tr>
<td>Peakedness</td>
<td>2.9953</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3-1: Comparing the results form simulation with actual values

As it is observed, the results are very close to the actual moments. The 95% confidence interval for mean in this example is [9.9993, 10.0186] and for variance is [5.9703, 6.0369].

**Example 2**

We consider the function \( y = x^2 \) and assume that \( x \sim N(\mu, \sigma^2) \). In this example we determine the moments of \( y \) using both approximation methods with different sets of values for \( \mu \) and \( \sigma^2 \). Results are shown in Table 3-2.
Comparison shows that Taylor series approximation method approximates mean and variance of the variate relatively well. However, it is observed that the approximated values for the third and the fourth moments of $y$ are quite inaccurate. In this example we deliberately choose a simple function ($y = x^2$) to show that Taylor series approximation method based on formulas (3-7) through (3-10) gives poor approximation values for the third and the fourth moments, even for such an uncomplicated function. We address this issue further in Chapter 6.
Chapter 4
Methodology

In this Chapter we propose a methodology for developing an interactive model for the Parameter Design Problem, as defined in Chapter 1. We also discuss a computer program that we have developed for this interactive model and present a numerical example.

4.1 The Model

In the context of the Parameter Design Problem as we defined earlier, we need to determine a set of values for the controllable parameters, $\mu_1, \ldots, \mu_n$, so that the corresponding value of the response variable, $Y$, is at or close to its target value $\tau$, with minimum variation. To this end we propose an interactive model and a corresponding computer program, in which the user is able to graphically observe the probability distribution of $Y$ for a given set of values for $\mu_1, \ldots, \mu_n$. We then allow the user to change the values of one or more of the parameters (i.e., $\mu_1, \ldots, \mu_n$) and immediately observe the impact of this change on the graph of the corresponding probability distribution of $Y$. In this model the user can make a subjective assessment of the distribution of $Y$ by comparing it with its target value and its specification limits. Objective measurements such as the corresponding values of the “percent defective” or the “expected loss function” can also be estimated to assist the user in evaluating the quality of the solution. In this manner the user can employ this interactive model to determine an appropriate set of values for $\mu_1, \ldots, \mu_n$.

Obviously, in order to implement this approach, we need to determine the form of the probability distribution function of $Y$ for a given response function $Y = h(x_1, \ldots, x_n)$ and for the given distribution functions of $x_1, \ldots, x_n$. In general, determining the exact form of the distribution function of $Y$ can be difficult, if at all possible. Hence, we propose to approximate this function using an appropriate Johnson distribution function. To this end we take the following two steps.
Step 1-Approximating the First Four Moments of \( Y \)

In Chapter 3 we discuss two methods for approximating the moments of a distribution, i.e., Monte-Carlo simulation and Taylor series approximation. Monte-Carlo simulation is more accurate but more time consuming. Taylor series approximation is less accurate in this case but obviously much faster. In this model we approximate the first four moments of the random variate, \( Y \), with one of these two methods, as appropriate. See Remark 1 below.

Step 2- Fitting Johnson Distribution

After approximating the first four moments of \( Y \), we can use the numerical methods of Hill et al. [9] to find the corresponding Johnson family, as we discuss in Chapter 2. Once the parameters of the Johnson distribution are known, we can form the pdf of the response function, using equations (2-6) through (2-8), and plot it. This pdf can be used to calculate different measures of variation such as estimating the “fraction defective” or the “expected loss function”. Evaluation of these measures can be saved for further comparison with other feasible combinations of set points. We can also provide assistance to the user for determining an appropriate set of values for \( \mu_1, \ldots, \mu_n \). To this end we can employ a non-linear programming model as discusses in [7] or use other techniques.

Remark 1

Ideally we would like to use Taylor series method to approximate the first four moments of \( Y \) for each value of \( \mu_1, \ldots, \mu_n \), since it is much faster than the Monte-Carlo simulation method. But in practice we note that this method is not sufficiently precise for approximating the third and the fourth moments of \( Y \), although it is relatively more accurate for approximating the first two moments of \( Y \) (i.e., mean and variance). Hence, we employ the Monte-Carlo simulation method to approximate the third and the fourth moments. However, we also note that in some situations (depending on the form of the response function), the third and the fourth moments of \( Y \) are less sensitive to changes in \( \mu_1, \ldots, \mu_n \). Thus we employ a mixed strategy in which we initially use Monte-Carlo simulation to approximate all four moments of \( Y \). Then during the interactive phase,
while changing the values of $\mu_1, \ldots, \mu_n$, we assume that the values of the third and the fourth moments of $Y$ remain unchanged; we use Taylor series method to approximate the first two moments of $Y$ during the interactive phase and carry out the analysis accordingly. When the interactive phase is complete, once again we employ a Monte-Carlo simulation model to approximate the third and the fourth moments. If their values at this stage are significantly different from their values at the start of the interactive phase, we update their values and repeat the interactive phase. If, on the other hand, their terminal values are reasonably close to their original values, we terminate the procedure.

**Remark 2**

As a terminating criterion we use the corresponding value of the fraction defective as estimated by using the fitted Johnson distribution. First we evaluate the fraction defective at the original set points, and refer to it as $p_0$. In the interactive phase we try to reduce the value of the fraction defective by changing the values of the set points. Then we evaluate the fraction defective at the end of the interactive phase via the fitted Johnson distribution, using its mean and variance as calculated by Taylor series approximation and its skewness and peakedness as the initial values obtained via Monte-Carlo simulation at the start of the interactive phase. We refer to this value as $p_1$.

Now we employ a Monte-Carlo simulation model to re-evaluate the moments of the response at the current set points. Then we calculate the fraction defective with these updated values of the moments. We refer to this value as $p_2$. If $p_1$ and $p_2$ are sufficiently close to each other we terminate the algorithm. Otherwise we repeat the interactive phase using the updated values of the third and the fourth moments.

In the numerical examples of the next Chapter, if the value of $p_2$ is within 5% of the value of $p_1$, we terminate the procedure, otherwise we repeat the interactive phase. Figure 4-1 shows a flowchart of this procedure.
Figure 4-1: Flowchart of the program
4.2 Computer Program

We have developed a computer program to implement this interactive approach for the Parameter Design Problem. Microsoft Excel with VBA in its background is chosen to develop this model. There are two main reasons for this choice: the high graphical capability of MS Excel to draw plots and charts, and availability of the software on almost every desktop. Since in this method the graphs for the pdf should be plotted every time that the set points are changed, visual features of MS Excel charts make it relatively easy to carry out this approach. Appearance of charts can be edited to the user’s taste and other features can be added as the user wishes. Another reason that makes MS Excel appealing in this context is the fact that it is available on almost any machine in work environments and the users have easy access to the software. Therefore, potential users do not need to install a new software package on their machines.

4.2.1 Modules of the Program

This program consists of three main procedures and some subroutines that are invoked during the implementation of these procedures. The first procedure approximates the moments of the response function; the second procedure fits a Johnson distribution to this function based on the value of its moments; and the third procedure plots the pdf and the specification limits. Figure 4-2 shows a schematic view of this software system. In this figure, sections that are connected to the main program via solid lines are already developed and they are operational. Sections that are connected via dash lines are stipulated for future development.
Procedure A that is designed to approximate the first four moments of the response function, takes the closed form relationship between the response function $Y$ and the components as well as the first four moments of the components as input. This program is designed such that a response function with up to ten arguments can be given as input. Once the input is entered in the designated worksheet (this worksheet is called “Moments of Y”), a macro which is called “FunctionMom” will approximate the first four moments of $Y$ as well as its skewness and peakedness using the Taylor series approximation method. For the Taylor series approximation method we use the formulas of Chapter 3, i.e., (3-7) through (3-10).

To evaluate the partial derivatives in these formulas, we use the symmetric difference quotient formula for numerical approximation of first derivative which is

$$f'(x) = \frac{f(x + \delta) - f(x - \delta)}{2\delta} \quad (4-5).$$

Here, $\delta$ represents incremental changes in variable $x$, and Evans [5] recommends using $\delta = \sigma$ in the context of the above formulas.

From this point, the next part of the methodology, i.e., fitting Johnson distribution, begins and all of the calculations for this part are done by a macro that is called “JOHNSONTYPES”. This macro copies the mean, variance, skewness and peakedness of the response, from the Table in the worksheet “Moments of Y”, to a designated Table in
another worksheet that is called “Johnson”. First the macro checks if there are any flaws in the input to this step. To this end location of \((\beta_1, \beta_2)\) is checked against Figure 2-2 to determine if the point is in the impossible area. If this point is in the impossible area, message “IMPOSSIBLE AREA” will appear as an indication of system failure. Otherwise, the standard deviation is checked for non-negativity. If both of the tests are passed, then a message will appear to show that the input is successfully entered.

In this case, the program proceeds to determine the type of the Johnson family by methods that are discussed in Chapter 2. There will be five possible outputs from this stage, namely \(S_U, S_B, S_L, S_T,\) and Normal. Once the type is determined, the macro begins its procedure to approximate the parameters of the respective type. If the Johnson families \(S_U\) or \(S_B\) are chosen, the program calls the corresponding subroutines for \(S_U\) or \(S_B\) respectively. In the case of \(S_L, Normal,\) and \(S_T\) families, the corresponding procedures are in the main body of the macro “JOHNSONTYPES”. After estimating the parameters of the Johnson distribution as appropriate, the parameters are inserted in a designated table in the worksheet “Johnson”.

At this time the last main macro is called. This macro plots the probability distribution function of the response, \(Y\), as well as the specification limits on a graph.

Now the user can start trying different set points for the components. He or she can see the effect of the change on the distribution of the response for each combination of the set points. The complete script of this program is presented in Appendix 2. Figures (4-3) and (4-4) show a view of worksheets “Moments of Y” and “Johnson” for a numerical example. We include these snapshots of the screen in order to offer a view of the interactive environment, and the numerical values are not meant to correspond to any specific example that we describe here.

In Figure 4-3, the user enters the values for the mean, the standard deviation, the third moment and the fourth moment for each component in rows 2 through 5 of columns B through K. In the specific example of this schematic view, the response, \(Y\), is a function of three components, \(x_1, x_2\) and \(x_3\). In cell N5, the user enters the closed form relationship between the response function and the components. Then the program evaluated the partial derivatives with respect to each component, through the methods described earlier.
Cells A15, A23 and A31 show the values of the partial derivatives with respect to $x_1, x_2$ and $x_3$ for this example, respectively. Then the estimated values of the mean, the standard deviation, the third moment and the fourth moment of $Y$ are calculated in cells N9 through N12, via formulas of Chapter 3. At the end, the skewness and the peakedness of $Y$ are evaluated in cells N13 and N14 from the values of its moments.

Figure 4-3: View of “Moments of Y” worksheet.
4.3 A Numerical Example

Consider an output characteristic, $Y$, which is a function of three components $X_1, X_2$ and $X_3$. The response function is given as $Y = (X_1^2 + 3X_2^2)X_3$. Suppose $X_1, X_2$ and $X_3$ are normally distributed random variables with respective means $\mu_1, \mu_2$ and $\mu_3$, and standard deviations $\sigma_1 = 0.1, \sigma_2 = 0.1$ and $\sigma_3 = 0.01$. Specification limits for $Y$ are, $3,255 \pm 150$. The following are the technical constraints on $\mu_1, \mu_2$ and $\mu_3$:
\[ 2 \leq \mu_1 \leq 10 \\
2 \leq \mu_2 \leq 10 \\
10 \leq \mu_3 \leq 20 \]

We start with the following combination of set points: \( \mu_1 = 5, \mu_2 = 8 \) and \( \mu_3 = 15 \).

Table 4-1 shows the approximated values of the mean, variance, skewness and peakedness of \( Y \) at these set points by Monte Carlo simulation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>3,255.6</td>
</tr>
<tr>
<td><strong>Std Dev</strong></td>
<td>73.479</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>0.036404</td>
</tr>
<tr>
<td><strong>Peakedness</strong></td>
<td>2.9941</td>
</tr>
</tbody>
</table>

Table 4-1: Results of MC simulation at the initial set points

The program approximates the distribution of this function to be Johnson \( S_L \). The fraction defective at this combination is approximately \( p_0 = 3.45\% \). Figure 4-5 shows the graph of probability density function of \( Y \).

![PDF AND LIMITS](image)

Figure 4-5: Pdf of \( Y \) with starting set points
We now start the interactive phase of the program and after trying a number of combinations of set points for $\mu_1, \mu_2,$ and $\mu_3,$ we find the combination $\mu_1 = 7.34,$ $\mu_2 = 8.0935$ and $\mu_3 = 13$ to be a better combination than the first one. For this combination Table 4-2 shows the approximated values of the mean, variance, skewness and peakedness of $Y$ at the new set points. Note that the mean and variance are calculated by Taylor series method and skewness and peakedness are from the Monte-Carlo simulation at the starting combination (i.e., at the start of the interactive phase).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$3,255.0677$</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$65.9983$</td>
</tr>
<tr>
<td>Skewness</td>
<td>$0.036404$</td>
</tr>
<tr>
<td>Peakedness</td>
<td>$2.9941$</td>
</tr>
</tbody>
</table>

Table 4-2: Results of MC simulation at the new set points

The program approximates the distribution of this function as Johnson $S_L$ too. The corresponding value of the fraction defective for this combination is $p_i = 2.32\%$. Figure 4-6 shows the graph of probability density function of $Y$.

![PDF AND LIMITS](image_url)
Now we simulate the response function with this combination of set points by Monte-Carlo simulation, and compare its first four moments with the values of Table 4-2. Table 4-3 shows the results from simulation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>3255.03</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>65.921</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>0.032912</td>
</tr>
<tr>
<td><strong>Peakedness</strong></td>
<td>2.9932</td>
</tr>
</tbody>
</table>

Table 4-3: Results of MC simulation at the final stage

The fraction defective here is \( p_2 = 2.32\% \). Figure 4-7 shows the probability density function of this combination with moments estimated by Monte-Carlo simulation, which of course closely resembles Figure 4-6.

As it is observed, the values for the fraction defective for this new set point from both methods are equal. So, we accept the combination \( \mu_1 = 7.34, \mu_2 = 8.0935 \) and \( \mu_3 = 13 \) as
the final answer. This combination keeps the mean (nominal) value of the response function almost on target and reduces its variance and the fraction defective.
Chapter 5
Case Studies

In this Chapter we present two case studies to illustrate the interactive model and the corresponding software, for the Parameter Design Problem. Throughout this Chapter all the assumptions of Chapter 1 for Parameter Design Problem hold. In each case we start with the original set points of the components and then find other set points that decrease the variation of the response function while keeping its nominal value on target. The final results are then verified with Monte-Carlo Simulation.

5.1 Deflection of a Coil Spring

5.1.1 Statement of the Problem

We have adopted this example from [6] and applied some changes to it. A detailed description of the problem can be found in [6]. Let \( \delta \) denote the deflection along the axis of the spring. As discussed in [6] deflection, \( \delta \), is a function of three components, wire diameter, \( d \), coil diameter, \( D \), and number of active coils, \( N \). The relationship is given as:

\[
\delta = \frac{D^3 N}{143750d^4}
\]

We assume that each component is a normally distributed random variable with given values of its mean and standard deviation as below:

\[
d \sim N(0.0517, 0.00174^2),
\]

\[
D \sim N(0.357, 0.0197^2),
\]

\[
N \sim (11.29, 185^2).
\]

The technical requirements on the set points of the components are:

\[
0.05 \leq \mu_d \leq 0.20
\]

\[
0.25 \leq \mu_D \leq 1.30
\]

\[
2 \leq \mu_N \leq 15
\]

We further assume that the set points of each component can be modified (i.e., it is a controllable parameter). The target value for \( \delta \) is 0.50 in. and the upper and lower specification limits are given as 0.60 in. and 0.40 in. respectively.
5.1.2 Distribution of $\delta$ at the Original Set Points

We use the given values for the mean and the standard deviation of the components, and values 0 and 3 for their skewness and peakedness (the components are all normal variates). A Monte-Carlo Simulation of this function with the original set points gives the following results in Table 5-1.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.51037</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.10995</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.5929</td>
</tr>
<tr>
<td>Peakedness</td>
<td>3.631</td>
</tr>
</tbody>
</table>

Table 5-1: Results from Monte-Carlo Simulation at the initial set points

When these values are fed to the procedure for Johnson fitting, the result is an $S_z$ distribution. The corresponding value of the fraction defective is $p_0 = 34.64\%$.

Figure 5-1 shows the plot for this pdf. In this plot the target value of $\delta$ (0.5 in) and its specification limits (LSL=0.4 and USL=0.6) are clearly marked.

Figure 5-1: Spring Deflection case with original set points.
From Figure 5-1 we observe that the original set points for the components make the mean of the response function slightly off target (i.e., not exactly equal to 0.5 in). We also observe that there is a relatively high probability for deflection of the spring to violate its specification limits, as reflected in the corresponding value of the fraction defective. Histogram of the data set from Monte-Carlo simulation is shown in Figure 5-2 for comparison.

5.1.3 Interactive Stage

At this stage we modify the values of each controllable parameter, \( \mu_d \), \( \mu_D \), and \( \mu_N \), and observe the impact of this modification on the pdf of \( \delta \). To choose new values for these set points such that the nominal value of \( \delta \) stays on target (0.5 in), we can use the partial derivatives of the function with respect to each component (which are evaluated at the current set points), as a guide for increasing or decreasing the set points of each
component. If the partial derivative with respect to a specific component is positive, it implies that increasing the value of that component results in an increase in the mean value of $Y$ and if the partial derivative is negative, increasing the value of that component results in a decrease in the mean value of $Y$.

Table 5-3 shows some of the different combinations of set points that we tried, in decreasing order of their respective values of fraction defective. In this table we only present those combinations of the set points that satisfy the technical constraints and result in $\mu_\delta \approx 0.5$. We tried many other combinations of values for $\mu_d$, $\mu_D$, and $\mu_N$ in the context of designing the parameters interactively.

<table>
<thead>
<tr>
<th>Set Points</th>
<th>Response</th>
<th>Johnson Type</th>
<th>Fraction Defective</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_d$</td>
<td>$\mu_D$</td>
<td>$\mu_N$</td>
<td>$\mu_\delta$</td>
</tr>
<tr>
<td>0.05640</td>
<td>0.4826</td>
<td>6.460</td>
<td>0.49919</td>
</tr>
<tr>
<td>0.05642</td>
<td>0.4826</td>
<td>6.467</td>
<td>0.49903</td>
</tr>
<tr>
<td>0.06081</td>
<td>0.6200</td>
<td>4.110</td>
<td>0.49832</td>
</tr>
<tr>
<td>0.06500</td>
<td>0.7474</td>
<td>3.067</td>
<td>0.49901</td>
</tr>
<tr>
<td>0.07598</td>
<td>0.8796</td>
<td>3.514</td>
<td>0.49917</td>
</tr>
<tr>
<td>0.13660</td>
<td>1.3000</td>
<td>11.360</td>
<td>0.49865</td>
</tr>
</tbody>
</table>

Table 5-2: Improvement of fraction defective for spring deflection

Some of the combinations in this table are suggested in Table 2 of [6]. Note that for all of the combinations in Table 5-2, we use the same skewness and peakedness that we found for the original combination, via Monte-Carlo simulation presented in Table 5-1 (i.e., skewness = 0.5929 and peakedness = 3.631). Figure 5-3 shows the probability density function of deflection at the best combination of set points, i.e., $\mu_d = 0.1366$, $\mu_D = 1.3$, and $\mu_N = 11.36$, with the skewness and peakedness form Monte-Carlo
simulation of the original combination. The value of the fraction defective at this combination of set points is \( p_1 = 0.80\% \).

\[
\text{PDF AND LIMITS FOR SPRING DEFLECTION}
\]

![PDF and Limits for Spring Deflection](image)

Figure 5-3: Best case with skewness and peakedness of the starting point

Now as it is explained in Chapter 4, we calculate the moments of the response function at this best combination, by Monte Carlo simulation. Table 5-3 shows the values from Monte-Carlo simulation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.49986</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.035071</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.21865</td>
</tr>
<tr>
<td>Peakedness</td>
<td>3.0732</td>
</tr>
</tbody>
</table>

Table 5-3: Results from MC simulation for the best case in Table 5-2

The program fits a Johnson \( S_B \) distribution using the moments in Table 5-3. The fraction defective for this distribution is \( p_2 = 0.48\% \). Figure 5-4 shows the pdf of this distribution.
The fraction defective $p_2 = 0.48\%$, calculated with moments from Monte-Carlo simulation, is not sufficiently close to $p_1 = 0.80\%$ (more than 5\% which is our limiting value for termination as described earlier). Comparison between Figures 5-3 and 5-4 shows a difference in the shape of these two probability density functions as well. This discrepancy comes from the difference in skewness and peakedness used in each case. Therefore, we take this best case combination as a new starting point and repeat the interactive phase. We keep the values for skewness and peakedness that are found by Monte-Carlo simulation, i.e., skewness = 0.21865 and peakedness = 3.0732, for all of the combinations that we try, and we calculate their mean and variance by Taylor series approximation. Table 5-4 shows some of these trial combinations.
As it is shown in this table, the last combination results in the least fraction defective compared to other combinations that we have tried. The corresponding value of fraction defective at this combination is \( p_1 = 0.43\% \). Figure 5-5 shows the probability density function for this combination.

![PDF AND LIMITS FOR SPRING DEFLECTION](image)

Figure 5-5: Best case with skewness and peakedness of the starting point
Monte-Carlo simulation for this combination gives the following results:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.50146</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.034773</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.21883</td>
</tr>
<tr>
<td>Peakedness</td>
<td>3.0719</td>
</tr>
</tbody>
</table>

It is observed that the skewness and peakedness from the Monte-Carlo simulation are relatively close to the skewness and peakedness of the new starting point. The corresponding value of the fraction defective is $p_2 = 0.45\%$. Since this value is within 5% of the value of $p_1$, we can stop here and keep this combination as an acceptable solution.

5.2 An Output Transformerless Pull-Push Circuit

5.2.1 Statement of the Problem

This example deals with an Output Transformerless (OTL) Pull-Push Circuit. A graphical representation of this circuit is given in Figure 5-6. We have adopted this example form [6] with slight modifications.

![Figure 5-6: Graphical representation of the OTL pull-push circuit](image-url)
The performance characteristic of interest is $V_m$ and the response function is given by

$$V_m = (V_{b1} + V_{be1}) \frac{\beta R_s}{\beta R_s + R_f} + (E_c - V_{be3}) \frac{R_f}{\beta R_c + R_f} + \frac{V_{be2} R_f \beta R_s}{(\beta R_c + R_f) R_{c1}}$$

where

$$V_{b1} = \frac{E_c R_{b2}}{R_{b1} + R_{b2}} \quad \text{and} \quad R_o = R_{c2} + R_L$$

with $R_L = 90\text{Ohms}$, $V_{be1} = V_{be3} = 0.65\ \text{Volts}$, $V_{be2} = 0.74\ \text{Volts}$, and $E_c = 12\ \text{Volts}$.

$R_{b2}, R_{b1}, R_f, R_{c2},$ and $R_{c1}$ are resistances, and $\beta$ is the current gain of the transistor. The means of these components are denoted by $\mu_1$ through $\mu_6$ and standard deviations are denoted by $\sigma_1$ through $\sigma_6$, respectively. Parameters $\mu_1$ through $\mu_6$ are controllable. All the components are assumed to be normally distributed. The technical requirements of $\mu_1$ through $\mu_6$ are as below:

$$25000 \leq \mu_1 \leq 70000$$
$$50000 \leq \mu_2 \leq 150000$$
$$649.4 \leq \mu_3 \leq 2053.5$$
$$237.1 \leq \mu_4 \leq 749.9$$
$$1271.1 \leq \mu_5 \leq 2260.3$$
$$73 \leq \mu_6 \leq 280$$

Target value for $V_m$ is $6\ \text{Volts}$. We further assume that the specification limits for $V_m$ are $6 \pm 0.2$.

The original set points and the standard deviations of the components are given as below.

$$\mu_1 = 48930, \mu_2 = 117100, \mu_3 = 2054, \mu_4 = 237.1, \mu_5 = 1271, \text{ and } \mu_6 = 73$$

$$\sigma_i = 0.02 \mu_i \quad \text{for} \quad i = 1, \ldots, 5, \text{ and } \sigma_6 = 0.15 \mu_6.$$

5.2.2 Variation with the Original Set Points

With the original set points (mean values) and standard deviations as above and values 0 and 3 for the skewness and peakedness of the components (resulting from their
normality), Monte-Carlo simulation gives the values for the mean, standard deviation, skewness and peakedness of \( V_m \) as in Table 5-5.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>6.0073</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.11652</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.4208</td>
</tr>
<tr>
<td>Peakedness</td>
<td>3.6557</td>
</tr>
</tbody>
</table>

Table 5-5: Results from Monte-Carlo Simulation for the original set points

When the distribution of \( V_m \) is approximated with a Johnson distribution through our model, the resulting pdf is Johnson \( S_U \). Figure 5-7 shows this pdf.

We observe that the mean of \( V_m \) is very close to its target value (i.e., \( \mu_{V_m} \approx 6 \)) but the probability that \( V_m \) violates its specification limits is considerably high. The corresponding value of the fraction defective is \( p_0 = 8.27\% \). The resulting histogram is shown in Figure 5-8.
5.2.3 Interactive Stage

At this stage we try several combinations of set points. Some of these combinations are taken from Table 4 in [6]. Table 5-6 shows those combinations that result in reasonably lower values of fraction defective, compared to the original case. These combinations are shown in decreasing order of their respective fraction defective values. Note that we have used the skewness and peakedness from the Monte-Carlo simulation of the starting combination.
Table 5-6: Improvement in fraction defective

In this table, the best combination of set points appears in the last row, i.e., $\mu_1 = 58750$, $\mu_2 = 100000$, $\mu_3 = 1539.9$, $\mu_4 = 237.1$, $\mu_5 = 1467.8$, and $\mu_6 = 280$. The corresponding value of the fraction defective is $p_1 = 1.76\%$.

Now we conduct a Monte-Carlo simulation with the set points of the best case combination. Table 5-7 shows the results and Figure 5-9 shows the fitted Johnson $S_B$ distribution.

Table 5-7: Results from Monte-Carlo Simulation for the best case set points

<table>
<thead>
<tr>
<th>Set Points</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>$\mu_2$</td>
</tr>
<tr>
<td>56500</td>
<td>87000</td>
</tr>
<tr>
<td>55000</td>
<td>85000</td>
</tr>
<tr>
<td>55835</td>
<td>75000</td>
</tr>
<tr>
<td>56590</td>
<td>87010</td>
</tr>
<tr>
<td>58750</td>
<td>100000</td>
</tr>
</tbody>
</table>

Table 5-7: Results from Monte-Carlo Simulation for the best case set points

| Mean      | 6.011 |
| Standard Deviation | 0.08267 |
| Skewness  | 0.02016 |
| Peakedness| 2.9879 |

Table 5-7: Results from Monte-Carlo Simulation for the best case set points
Figure 5.9: Fitted Johnson distribution of the best case set points.

The corresponding values of the fraction defective for this distribution is $p_2 = 1.65\%$.
Since the value $p_2$ is not within 5% of $p_1$, we resume the interactive stage with updated values of the skewness and peakedness as 0.02016 and 2.9879, respectively. We use this combination as a new starting point and resume trying different combinations. Table 5-8 shows the results of this trial.

<table>
<thead>
<tr>
<th>Set Points</th>
<th>Response</th>
<th>Johnson Types</th>
<th>Fraction Defective</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>$\mu_2$</td>
<td>$\mu_3$</td>
<td>$\mu_4$</td>
</tr>
<tr>
<td>55500</td>
<td>85500</td>
<td>1044</td>
<td>500</td>
</tr>
<tr>
<td>55000</td>
<td>85500</td>
<td>1044</td>
<td>500</td>
</tr>
<tr>
<td>55500</td>
<td>85000</td>
<td>1044</td>
<td>600</td>
</tr>
<tr>
<td>40000</td>
<td>62000</td>
<td>1044</td>
<td>600</td>
</tr>
</tbody>
</table>

Table 5-8: Improving variation with a new starting combination
In Table 5-8, the combinations are sorted in decreasing order of their fraction defective. As before, the skewness and peakedness from the Monte-Carlo simulation of the new starting combination (i.e., from Table 5-7) are used as the skewness and the peakedness at all combinations. The combination with the least fraction defective is the combination at the last row of Table 5-8, with value of $P_1 = 1.51\%$

Now we conduct a Monte-Carlo simulation with the last combination in this table. The results are shown in Table 5-9.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.994</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.08237</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.01889</td>
</tr>
<tr>
<td>Peakedness</td>
<td>3.003</td>
</tr>
</tbody>
</table>

Table 5-9: Results from Monte-Carlo simulation of the best case

We can observe that the values of skewness and peakedness by Monte-Carlo simulation are reasonably close to the values in Table 5-7 for the best case combination. The corresponding value of the fraction defective is $P_2 = 1.55\%$. Since this value is very close to the value of $P_1 = 1.51\%$ (within 5% of $P_1$), we accept the corresponding combination as the combination that results in the least fraction defective. The set points for this combination are $\mu_1 = 40000, \mu_2 = 62000, \mu_3 = 1044, \mu_4 = 600, \mu_5 = 1270, \mu_6 = 280$. 

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Chapter 6
Summary of Findings and Avenues for Further Research

In this Chapter we discuss our findings from solving Parameter Design Problems with this interactive framework. We also discuss avenues for further research.

6.1. Summary of the Proposed Model

In the Parameter Design Problem, as we define in Chapter 1, we want to determine a set of values for $\mu_1, \ldots, \mu_n$, such that the mean of $Y$ remains at (or close to) its target value $\tau$ and its variation is minimized. To solve this problem we propose a method by which the user can manually change the values of $\mu_1, \ldots, \mu_n$ and observe the effect of this change on the variation of $Y$. To build this model, we approximate the probability distribution of $Y$ by a Johnson distribution as we discuss in Chapter 2. To find the required inputs (the first four moments of $Y$) for fitting a Johnson distribution, we use Taylor series approximation method or Monte-Carlo simulation as appropriate and as we explain in Chapter 3. We have developed a computer program that we can use to solve the Parameter Design Problems by this method. This program is discussed in Chapter 4. In Chapter 5 we solve two case studies using this model. In this Chapter we discuss our findings, and offer several suggestions that can help improving this model.

6.2 Findings

As it is shown in Chapter 5 the results form this model conform to the results of other models for solving Parameter Design Problems e.g., non-linear programming. However, with this model the user can visually observe the probability distribution function of $Y$ at each combination of set points for $\mu_1, \ldots, \mu_n$ and compare different combinations with each other. We no longer have to assume that $Y$ has a normal probability distribution function. We can also calculate the value of the fraction defective more accurately.

In solving different cases with this model, we find that it will be very useful if we have a guide for changing our set points. For example, it is useful to know that increasing which set points lead to increase in mean of $Y$. In our case studies in Chapter 5, we used
the partial derivatives of the function $h$ with respect to each variable, $x_1, \ldots, x_n$, for this purpose.

We also find that by adding some new features to this model, we can have an interactive framework for solving Parameter Design Problems. In the next section we offer our suggestions for these new features.

6.3 Avenues for Further Research

This model is the building block of an interactive framework that a designer can use to implement different methods of solving the Parameter Design Problem. Other features may be easily added to this framework in order to facilitate and enhance its capacity as described below.

Calculating Measures of Variation

We can add a module to this model that calculates measures of variation such as “expected loss”. Performance measures such as $C_p$ or $C_{PK}$ can also be estimated for each combination. Estimating these measures will help the designer in comparing different combinations from several points of views.

A Systemic Method for Changing the Set Points

As we discussed earlier, it is very helpful if we have a guide for changing the set points and finding new feasible combinations. We can add a module to the existing model that suggests new combinations to the user. This feature can be designed such that set points are changed systematically. Then the user can compare the different combinations that are recommended by the program, by observing the corresponding probability distribution functions on the screen.

A Nonlinear Programming Module

A useful feature to be added to this framework is a module that employs an appropriate optimization technique for finding an optimal set of values for $\mu_1, \ldots, \mu_n$. The objective function of this model can be the loss function, the fraction defective, or any other
measure that is appropriate for the user. If the respective model is a standard nonlinear programming model, then we can potentially use Excel Solver to analyze the model.

**Statistical Models**

A module for solving the Parameter Design Problem using statistical methods can also be added to this framework. In this part we can use the techniques of design of experiments to find an optimal set of values for the design variables of a Parameter Design Problem.

**Finding a More Accurate Way of Approximating the Moments**

One avenue for further research can be on the subject of approximating the first four moments of the variate \( Y \), by a more accurate method. Such a method helps to enhance the overall performance of the interactive algorithm.

As a first step towards this goal, for several examples we tried using the extended versions of the formulas of Chapter 3 for estimating the moments of \( Y \) [i.e., formulas (3-7) through (3-10)] as given in [5]. However, the approximated values that we obtained for the third and the fourth moments in this manner were still not sufficiently accurate for the purpose of this algorithm.
References


Appendix 1

Script of the Computer Program
Developed by Hill et al. [9]
APPLIED STATISTICS

ACKNOWLEDGEMENTS

We are indebted to J. Draper for permission to make use of his M.Sc. thesis, and to the referee for many useful comments.

REFERENCES


SUBROUTINE JNORM(XBAR, SD, B21, B22, ITYPE, GAMMA, DELTA, XMAL, XI, *
* IFAULT)
C
C ALGORITHM AS (9) APPL. STATIST. (1970) VOL.2, NO.3
C
C TYP2: TYPE AND PARAMETERS OF A JOHNSON CURVE
C
C WITH GIVEN FIRST FOUR MOMENTS
C
C REAL XBAR, SD, B21, B22, GAMMA, DELTA, XMAL, XI, TOL, B1, B2, Y,
* X, U, W, ZERO, ONE, TWO, THREE, FOUR, HALF, QUART
C
C LOGICAL FAULT
C
C DATA TOL, .01/
C DATA ZERO, ONE, TWO, THREE, FOUR, HALF, QUART
* DATA .0, 1.0, 2.0, 3.0, 4.0, 0.5, 0.25/
* IFAULT = 1
* IF (SD, LT, ZERO) RETURN
* IFAULT = 0
* XI = ZERO
* XMAL = ZERO
* GAMMA = ZERO
* DELTA = ZERO
* IF (SD, GT, TOL) GOTO 10
* LTYPE = 1
* XI = XBAR
* RETURN
10 B1 = B21 * B22
B2 = B21
* IFAULT = .FALSE.
C
C TEST WHETHER LOGNORMAL (OR NORMAL) REQUIRED
C
C IF (B2, LT, ZERO) GOTO 30
20 IF (ABS(B1), LT, TOL) GOTO 70
* GOTO 80
C
C TEST FOR POSITION RELATIVE TO BOUNDARY LINE
C
30 IF (B1, LT, B1 + TOL) GOTO 60
IF (B2, LE, B2 + TOL) GOTO 50
C
C SIGN DISTRIBUTION
C
40 LTYPE = 5
Y = HALF + HALF + SIGN((B1 + FOUR / (B1 + FOUR))
IF (B1, GT, ZERO) Y = ONE + Y
X = SD + SIGN(Y * (ONE + Y))
XI = XBAR - Y * X
XMAL = XI + X
* DELTA = Y
RETURN
50 IFAULT = 2
RETURN
STATISTICAL ALGORITHMS

Q0 IF (ABS(BETA) .GT. TOL, CM, OBS(2) = THREE) .GT. TOL) GOTO 50
C
C NORMAL DISTRIBUTION
C
70 ITYPE = 4
DELTA = ONE / SD
GAMMA = -SQRT(2) / SD
RETURN
C
C TEST FOR POSITION RELATIVE TO LOGNORMAL LINE
C
80 U = ONE / THREE
X = HALF + R1 + ONE
Y = R51 * SQRT(QUART * R1 + ONE)
W = (X + Y) ** U + (X - Y) ** U + ONE
V = W + W + (THREE + W - (W + W)) - THREE
IF (W .LT. ZEROMIN, ZEROMAX, FAULT) B2 = 0
X = U - B2
IF (ABS(X) .GT. TOL) GOTO 80
C
C LOGNORMAL (S2) DISTRIBUTION
C
ITYPE = 1
XLAM = SQRT(WAR, R1)
U = XLAM + YR0B
X = ONE / SQRT(1 + Y)
DELTA = X
Y = WAPE + X * ALOG(W + (W + ONE) / (SD + SD))
GAMMA = Y
XI = 0 - EXP((HALF / X - T) / T)
RETURN
C
C S2 OR S1 DISTRIBUTION
C
90 IF (X .LT. ZEROMIN) GOTO 100
ITYPE = 2
CALL SUFFIXWAR, SD, R1, R2, GAMMA, DELTA, XLAM, XI
RETURN
100 ITYPE = 3
CALL SUFFIXWAR, SD, R1, R2, GAMMA, DELTA, XLAM, XI, FAULT
IF (FAULT) RETURN
C
C FAILURE - TRY TO FIT APPROXIMATE RESULT
C
IF (FAULT) GOTO 90
IF (R2 .GT. T1 + TWO) GOTO 90
GOTO 40
END
C
C SUBROUTINE SUFFIXWAR, SD, R1, R2, GAMMA, DELTA, XLAM, XI
C ALGORITHM AS 99.1 APPL. STATIST. (1965) VOL.15, NO.2
C FINDS PARAMETERS OF JOHNSON SS CURVE WITH
C GIVEN FIRST FOUR MOMENTS
C
REAL WAR, SD, R1, R2, GAMMA, DELTA, XLAM, XI, TOL, R1, R2, W, Y,
X, R1, Z, V, A, B, N, ZERO, INT, TWO, THREE, EVEN, SIX,
* EIGHT, NINE, TEN, NINE, ONE, TWO, THREE, FOUR, SIX, SEVEN, EIGHT, NINE, TEN,
* HALF, THREE, TWP0 / 0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 0.0,
* 10.0, 15.0, 20.0, 30.0, 40.0, 50.0, 60.0, 70.0, 80.0
DATA T1, T2, T3, T4 / 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
3 = R1 - T3
D = (W - R2) / (WAR ** (W - R2) / R2) - (WAR ** (-R2))
D = SQRT(1 + D ** 2) - 1
IF (ABS(D) .GT. TOL) GOTO 50

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STATISTICAL ALGORITHMS

IF (D.LT.0.004) GOTO 10
F = .52415 / (D * (D - 2.162) + 11.345)
GOTO 20
10 D = 1.223 * D
20 F = E + F + 1.3E
IF (F.LT.1.1) GOTO 25
D = (0.0635 + F - 0.495) * (THREE - F) ** (-0.479)
GOTO 30
25 D = 0.8 *(F - ONE)
C
GET G AS FIRST ESTIMATE OF GAMMA
C
30 G = ZERO
IF (D.LT.VT) GOTO 70
IF (D.GT.ZERO) GOTO 40
G = (0.2466 + D ** 1.727) + 0.5385 * D ** 0.495
GOTO 70
40 IF (D.LT.2.5) GOTO 50
U = 0.0614
T = 0.5384
GOTO 60
50 U = 0.0614
T = 0.5384
60 G = B1 ** (U + D + Y) - (0.0614 + D * (1.0614 + D = 0.7077))
70 W = 0
C
MAIN ITERATION STARTS HERE
C
80 M = M + 1
FAULT = M.LT. LIMIT
IF (FAULT) RETURN
C
GET FIRST SIX MOMENTS FOR LATEST G AND D VALUES
C
CALL MOM(G, D, MM, FAULT)
IF (FAULT) RETURN
S = MM(1) + MM(4)
H1 = MM(1) - S
FAULT = MM(1A).ZERO
IF (FAULT) RETURN
T = SQRT(W)
G1A = T + H1
H1A = MM(4) - MM(1) + (ONE 1 + T * ONE)
T1 = MM(3) / H1A
B1 = MM(4) - MM(1) + (ONE / MM(3)) - MM(1)
* (SIX + MM(3) - THREE + S))
MOMA = MM / H1A
W = G + D
U = D + D
C
GET DERIVATIVES
C
DO 100 J = 1, S
DO 110 K = 1, A
T = K
IF (J.EQ.1) GOTO 90
S = (W - T) * MM(K + 1) + (T + ONE) * 
* (B1 + T1) / T1 + MM(K + 1)) / M
GOTO 100
90 S = MM(K) - MM(K)
100 CONTINUE
T = MM(1) + MM(4)
S = MM(1) + MM(4)
Y = T0(T)
DERIV(2) = (DEL) - THREE * (S + MM(2)) + D1(1) - T * MM(1)
* ONE + M ** T / H1 / H1A
DERIV(3) = (DEL) + (DEL) + MM(1) + D1(1) * MM(1)
* (SIX + MM(3) - THREE + S) + D1(1) ** (S - T + MM(1))
* - T0(T) + T / H1A / H1A

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120 CONTINUE

T = ONE / (DERIV(1) * DERIV(4) - DERIV(4) * DERIV(1))
U = (DERIV(4) + (MCT - R21) * DERIV(1) * (R202 - R2)) * T
V = (DERIV(1) + (R20 - R21) * (R211 - R2)) * T

STATISTICAL ALGORITHMS

H = 0.5 * H
40 T = W
U = Y
V = B * H
X = T * W
A(I) = U * Q / E
DO 50 T = 2, 6
50 A(I) = A(I - 1) / X
V = Y
P = X * H
W = 0
C
C START OF INNER LOOP
C
C TC EVALUATE INFINITE SERIES
C
60 W = M + 1
IF (W.GT. LIMIT) GOTO 140
DO 70 I = 1, 6
70 B(I) = A(I)
U = 0 - P
Z = (P)
IF (U.GT. -EXP(X)) Z = EXP(U) + 2
T = T + P
L = Y * U
IF (.NOT.(L)) S = EXP(T) + (L)
F = EXP(-Y)
Q = P
DO 90 T = 1, 6
AA = A(I)
P = P / Z
AB = AA
AA = AA + P
IF (AA.GT. 0.0) GOTO 100
IF (1) GOTO 80
G = Q / Z
AB = AA
AA = AA + Q
L = AB * 0.0
AB
80 A(I) = AA
C
C CONTINUE
C
100 W = W + X
V = W / R
DO 110 I = 1, 6
110 A(I) = V * A(I)
DO 120 I = 1, 6
IF (A(I).GT. EPS) GOTO 140
IF (A(I).LT. EPS) GOTO 140
IF (ABS(A(I)) .GT. (1.0) / (A(I)) .GT. W) GOTO 60
120 CONTINUE
C
C END OF INNER LOOP
C
C
V = N 
DO 130 I = 1, 6
130 A(I) = V * A(I)
DO 140 I = 1, 6
IF (ABS(A(I)) .GT. (1.0) / (A(I)) .GT. W) GOTO 20
140 CONTINUE
C
C END OF OUTER LOOP
C
RETURN
140 PRINT *, 'THUS,'
RETURN
END
Appendix 2

Script of the Computer Codes
In Microsoft Excel VBA
Subroutine “FunctionMOM”

Sub FunctionMOM()

' This macro gets the first four moments of up to ten variables and the functional
relationship between these variables and the variate Y as input and calculates the first
four moments of Y based on formulas of Tukey [14] up to their first terms

Sheets("Moments of Y").Select
' Calculate skewness and Curtosis for variables
Range("B6").Select
ActiveCell.FormulaR1C1 = "=R[-2]C/R[-3]C^3"
Range("B7").Select
ActiveCell.FormulaR1C1 = "=R[-2]C/R[-4]C^4"
Range("B6:B7").Select
Selection.AutoFill Destination:=Range("B6:K7"), Type:=xlFillDefault
Range("B6:K7").Select

' Copy means of variables into 10 subtables for parital derivatives
Range("B2:K2").Select
Selection.Copy
Range("B9:K11").Select
ActiveSheet.Paste
Range("B17:K19").Select
ActiveSheet.Paste
Range("B25:K27").Select
ActiveSheet.Paste
Range("B33:K35").Select
ActiveSheet.Paste
Range("B41:K43").Select
ActiveSheet.Paste
Range("B49:K51").Select
ActiveSheet.Paste
Range("B57:K59").Select
ActiveSheet.Paste
Range("B65:K67").Select
ActiveSheet.Paste
Range("B73:K75").Select
ActiveSheet.Paste
Range("B81:K83").Select
ActiveSheet.Paste

'Copy formula for relationship of Y and design variables into "Mean of Y" cells
Range("N5").Select
Selection.Copy
Range("B12").Select
Selection.PasteSpecial Paste:=xlPasteFormulas, Operation:=xlNone, _
    SkipBlanks:=False, Transpose:=False
Range("B20").Select
Selection.PasteSpecial Paste:=xlPasteFormulas, Operation:=xlNone, _
    SkipBlanks:=False, Transpose:=False
Range("B28").Select
Selection.PasteSpecial Paste:=xlPasteFormulas, Operation:=xlNone, _
    SkipBlanks:=False, Transpose:=False
Range("B36").Select
Selection.PasteSpecial Paste:=xlPasteFormulas, Operation:=xlNone, _
    SkipBlanks:=False, Transpose:=False
Range("B44").Select
Selection.PasteSpecial Paste:=xlPasteFormulas, Operation:=xlNone, _
    SkipBlanks:=False, Transpose:=False
Range("B52").Select
Selection.PasteSpecial Paste:=xlPasteFormulas, Operation:=xlNone, _
'Make Mean X+/‐Sigma for all the variables
Range("B10").Select
ActiveCell.FormulaR1C1 = "+R[‐1]C+R[‐7]C"
Range("B11").Select
ActiveCell.FormulaR1C1 = "+R[‐2]C‐R[‐8]C"
Range("C18").Select
ActiveCell.FormulaR1C1 = "+R[‐1]C+R[‐15]C"
Range("C19").Select
ActiveCell.FormulaR1C1 = "+R[‐2]C‐R[‐16]C"
Range("D26").Select
ActiveCell.FormulaR1C1 = "+R[‐1]C+R[‐23]C"
Range("D27").Select
ActiveCell.FormulaR1C1 = "=R[‐2]C‐R[‐24]C"
Range("E34").Select
ActiveCell.FormulaR1C1 = "=R[‐1]C+R[‐31]C"
Range("E35").Select
ActiveCell.FormulaR1C1 = "=R[‐2]C‐R[‐32]C"
Range("F42").Select
ActiveCell.FormulaR1C1 = "+R[-1]C+R[-39]C"
Range("F43").Select
ActiveCell.FormulaR1C1 = "+R[-2]C-R[-40]C"
Range("G50").Select
ActiveCell.FormulaR1C1 = "+R[-1]C+R[-47]C"
Range("G51").Select
ActiveCell.FormulaR1C1 = "+R[-2]C-R[-48]C"
Range("H58").Select
ActiveCell.FormulaR1C1 = "+R[-1]C+R[-55]C"
Range("H59").Select
ActiveCell.FormulaR1C1 = "+R[-2]C-R[-56]C"
Range("I66").Select
ActiveCell.FormulaR1C1 = "+R[-1]C+R[-63]C"
Range("I67").Select
ActiveCell.FormulaR1C1 = "+R[-2]C-R[-64]C"
Range("J74").Select
ActiveCell.FormulaR1C1 = "+R[-1]C+R[-71]C"
Range("J75").Select
ActiveCell.FormulaR1C1 = "+R[-2]C-R[-72]C"
Range("K82").Select
ActiveCell.FormulaR1C1 = "+R[-1]C+R[-79]C"
Range("K83").Select
ActiveCell.FormulaR1C1 = "+R[-2]C-R[-81]C"

'Make F(X+-Sigma) for all the variables
Range("B12").Select
Selection.AutoFill Destination:=Range("B12:B14"), Type:=xlFillDefault
Range("B12:B14").Select
Range("B20").Select
Selection.AutoFill Destination:=Range("B20:B22"), Type:=xlFillDefault
Range("B20:B22").Select
Range("B28").Select
Selection.AutoFill Destination:=Range("B28:B30"), Type:=xlFillDefault
Range("B28:B30").Select
Range("B36").Select
Selection.AutoFill Destination:=Range("B36:B38"), Type:=xlFillDefault
Range("B36:B38").Select
Range("B44").Select
Selection.AutoFill Destination:=Range("B44:B46"), Type:=xlFillDefault
Range("B44:B46").Select
Range("B52").Select
Selection.AutoFill Destination:=Range("B52:B54"), Type:=xlFillDefault
Range("B52:B54").Select
Range("B60").Select
Selection.AutoFill Destination:=Range("B60:B62"), Type:=xlFillDefault
Range("B60:B62").Select
Range("B68").Select
Selection.AutoFill Destination:=Range("B68:B70"), Type:=xlFillDefault
Range("B68:B70").Select
Range("B76").Select
Selection.AutoFill Destination:=Range("B76:B78"), Type:=xlFillDefault
Range("B76:B78").Select
Range("B84").Select
Selection.AutoFill Destination:=Range("B84:B86"), Type:=xlFillDefault
Range("B84:B86").Select

'Calculate partial derivative with respect to each variable
Range("B15").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-12]C)"
Range("B23").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-20]C[1])"
Range("B31").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-28]C[2])"
Range("B39").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-36]C[3])"
Range("B47").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-44]C[4])"
Range("B55").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-52]C[5])"
Range("B63").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-60]C[6])"
Range("B71").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-68]C[7])"
Range("B79").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-76]C[8])"
Range("B87").Select
ActiveCell.FormulaR1C1 = "=(R[-2]C-R[-1]C)/(2*R[-84]C[9])"

'Copy the results into "Johnson worksheet"
Range("N9:N10").Select
Selection.Copy
Sheets("Johnson").Select
Range("B2:B3").Select
'Sheets("Moments of Y").Select
'Range("N13:N14").Select
'Application.CutCopyMode = False
'Selection.Copy
'Sheets("Johnson").Select
'Range("B4:B5").Select
'Run macros for determining the family and parameters of Johnson and graph the pdf
JOHNSONTYPES
GRAPHS

End Sub
Subroutine “JOHNSONTYPES”

'MAIN PART OF THIS MACRO IS ORIGINALLY PROPOSED BY HILL, HILL, AND HOLDER "FITTING JOHNSON CURVES BY MOMENTS" APPLIED STATISTICS, 25 (1976) 180-189

Sub JOHNSONTYPES()
'This macro determines the family of Johnson distribution that fits a variate given its first four moments, it also determines the corresponding parameters of the distribution

Const TOL = 0.01
'DEFINE VARIABLES

Dim X As Double
Dim Y As Double
Dim U As Double
Dim W As Double
Dim XBAR As Double
Dim SD As Double
Dim RB1 As Double
Dim B1 As Double
Dim B2 As Double
Dim BB2 As Double
Dim LAMBDA As Double
Dim DELTA As Double
Dim GAMMA As Double
Dim XI As Double
Dim XLAM As Double
Dim B3 As Double

XBAR = Range("B2")
SD = Range("B3")
RB1 = Range("B4")
B2 = Range("B5")
B1 = RB1 ^ 2

Worksheets("Johnson").Range("D2:F2").ClearContents
Worksheets("Johnson").Range("I1:I4").ClearContents
Worksheets("Johnson").Range("A11:A19").ClearContents

'CHECK FAILURES
Range("D2").Select
If B2 < B1 + 1 Then
    ActiveCell.FormulaR1C1 = "IMPOSSIBLE AREA"
ElseIf Range("B3") < 0 Then
    ActiveCell.FormulaR1C1 = "NEGATIVE STANDARD DEVIATION"
Else: ActiveCell.FormulaR1C1 = "SUCCESSFUL COMPLETION"
End If

Columns("D:D").EntireColumn.AutoFit

If Range("D2") = "SUCCESSFUL COMPLETION" Then

'DETERMINE WHICH TYPE

'TEST WHETHER LOGNORMAL OR NORMAL REQUESTED
If B2 >= 0 Then
    GoTo 30
End If
20    If Abs(RB1) <= TOL Then
    GoTo 70
Else: GoTo 80
End If

'TEST FOR POSITION RELATIVE TO BOUNDARY LINE

30 If B2 > B1 + TOL + 1 Then
    GoTo 60
End If

'CALCULATE PARAMETERS OF ST DISTRIBUTION

40 Range("F2").Select
    ActiveCell.FormulaR1C1 = "ST"
    Y = 0.5 + 0.5 * Sqr(1 - 4 / (B1 + 4))
    If RB1 > 0 Then
        Y = 1 - Y
    End If
    X = SD / Sqr(Y * (1 - Y))
    XI = XBAR - Y * X
    XLAM = XI + X
    DELTA = Y

'INSERT PARAMETERS INTO CELLS I1:I4

    Range("I1").Select
    ActiveCell.FormulaR1C1 = 0
    Range("I2").Select
    ActiveCell.FormulaR1C1 = DELTA
    Range("I3").Select
    ActiveCell.FormulaR1C1 = XLAM
    Range("I4").Select
    ActiveCell.FormulaR1C1 = XI

Exit Sub
60   If Abs(RB1) > TOL Or Abs(B2 - 3) > TOL Then
       GoTo 80
   End If

   'CALCULATE PARAMETERS OF NORMAL DISTRIBUTION AND INSERT
   INTO CELLS
70     Range("F2").Select
       ActiveCell.FormulaR1C1 = "NORMAL"
       DELTA = 1 / SD
       GAMMA = -XBAR / SD
       Range("I1").Select
       ActiveCell.FormulaR1C1 = GAMMA
       Range("I2").Select
       ActiveCell.FormulaR1C1 = DELTA

   Exit Sub

   'TEST FOR POSITION RELATIVE TO LOGNORMAL LINE
80     U = 1 / 3
       X = 0.5 * B1 + 1
       Y = RB1 * Sqr(0.25 * B1 + 1)
       W = (X + Y) ^ U + (X - Y) ^ U - 1
       U = W ^ 2 * (3 + W * (2 + W)) - 3
       If B2 < 0 Then
           B2 = U
       End If
       X = U - B2

       If Abs(X) > TOL Then
           GoTo 90
'COMPUTE THE PARAMETERS OF THE DISTRIBUTIONS OTHER THAN ST AND NORMAL

'SL
Range("F2").Select
ActiveCell.FormulaR1C1 = "SL"

XLAM = Sgn(RB1)
LAMBDA = XLAM
U = XLAM * XBAR
X = 1 / Sqr(Log(W))
DELTA = X
Y = 0.5 * X * Log(W * (W - 1) / (SD ^ 2))
GAMMA = Y
XI = U - Exp((0.5 / X - Y) / X)
Range("I1").Select
ActiveCell.FormulaR1C1 = GAMMA
Range("I2").Select
ActiveCell.FormulaR1C1 = DELTA
Range("I3").Select
ActiveCell.FormulaR1C1 = LAMBDA
Range("I4").Select
ActiveCell.FormulaR1C1 = XI

Exit Sub

'SB OR SU DISTRIBUTION

90 If X > 0 Then
GoTo 100

Else
  Range("F2").Select
  ActiveCell.FormulaR1C1 = "SU"
  SUFIT
  Exit Sub
End If

100  Range("F2").Select
  ActiveCell.FormulaR1C1 = "SB"
  SBFIT

End If

' GRAPHS

End Sub
Subroutine “SUFIT”

'MAIN PART OF THIS MACRO IS ORIGINALLY PROPOSED BY HILL, HILL, AND HOLDER "FITTING JOHNSON CURVES BY MOMENTS" APPLIED STATISTICS, 25 (1976) 180-189
Sub SUFIT()

'EXTRA
Const TOL = 0.01
'DEFINE VARIABLES

XBAR = Range("B2")
SD = Range("B3")
RB1 = Range("B4")
B2 = Range("B5")
B1 = RB1 ^ 2

If Range("F2") = "SU" Then

    Dim W1, WM1 As Double
    Dim z As Double
    Dim V As Double
    Dim A As Double
    Dim B As Double
    Dim XV As Double

    B3 = B2 - 3
    'W IS FIrST ESTIMATE OF EXP(DELTA^(-2))
    W = Sqr(Sqr(2 * B2 - 2.8 * B1 - 2) - 1)
If Abs(RB1) > TOL Then
   GoTo 10
Else
   Y = 0
   GoTo 20
End If

Do
10      W1 = W + 1
   WM1 = W - 1
   z = W1 * B3
   V = W * (6 + W * (3 + W))
   A = 8 * (WM1 * (3 + W * (7 + V)) - z)
   B = 16 * (WM1 * (6 + V) - B3)
   'ADD EXTRA VARIABLES TO REDUCE COMPLEXITY
   XV = (3 + W * (9 + W * (10 + V)))
   Y = (Sqr(A * A - 2 * B * (WM1 * XV - 2 * W1 * z)) - A) / B
   XV = (4 * (W + 2) * Y + 3 * W1 ^ 2) ^ 2
   z = Y * WM1 * XV / (2 * (2 * Y + W1) ^ 3)
   V = W ^ 2
   XV = (B1 * (B2 - 1.5 - V * (1 + 0.5 * V)))
   W = Sqr(Sqr(1 - 2 * (1.5 - B2 + XV / z)) - 1)
Loop Until Abs(B1 - z) <= TOL

'END OF ITERATION

Y = Y / W
Y = Log(Sqr(Y) + Sqr(Y + 1))
If RB1 > 0 Then
   Y = -Y
20     X = Sqr(1 / Log(W))
      DELTA = X
      GAMMA = Y * X
      Y = Exp(Y)
      z = Y ^ 2
      XV = (0.5 * W * (z + 1 / z) + 1)
      X = SD / Sqr(0.5 * (W - 1) * XV)
      XLAM = X
      LAMBDA = XLAM
      XI = (0.5 * Sqr(W) * (Y - 1 / Y)) * X + XBAR

      Range("I1").Select
      ActiveCell.FormulaR1C1 = GAMMA
      Range("I2").Select
      ActiveCell.FormulaR1C1 = DELTA
      Range("I3").Select
      ActiveCell.FormulaR1C1 = LAMBDA
      Range("I4").Select
      ActiveCell.FormulaR1C1 = XI

      End If
      End Sub
Subroutine “SBFIT”

'MAIN PART OF THIS MACRO IS ORIGINALLY PROPOSED BY HILL, HILL, AND HOLDER "FITTING JOHNSON CURVES BY MOMENTS" APPLIED STATISTICS, 25 (1976) 180-189

Sub SBFIT()

If Range("F2") = "SB" Then
'EXTRA
Const TOL = 0.01
'DEFINE VARIABLES

XBAR = Range("B2")
SD = Range("B3")
RB1 = Range("B4")
B2 = Range("B5")
B1 = RB1 ^ 2

Const TT = 1# * 10 ^ (-4)
Const LIMIT = 50

Dim RTB1 As Double
Dim E, F As Double
Dim D As Variant
Dim G As Variant
Dim M As Double
Dim J, K As Long
Dim NEG As Long
Dim FAULT As Integer
RTB1 = RB1
RB1 = Abs(RTB1)

If RTB1 < 0 Then
    NEG = 1
Else
    NEG = 0
End If

'GET D AS FIRST ESTIMATE OF DELTA

E = B1 + 1
U = 1 / 3
X = 0.5 * B1 + 1
Y = RB1 * Sqr(0.25 * B1 + 1)
W = (X + Y) ^ U + (X - Y) ^ U - 1
F = W * W * (3 + W * (2 + W)) - 3
E = (B2 - E) / (F - E)

If Abs(RB1) > TOL Then
    GoTo 5
End If
F = 2
GoTo 20

5    D = 1 / Sqr(Log(W))
If D < 0.64 Then
    GoTo 10
End If
F = 2 - 8.5254 / (D * (D * (D - 2.163) + 11.346))
GoTo 20
10   F = 1.25 * D
20   F = E * F + 1

If F < 1.8 Then
   GoTo 25
End If
D = (0.625 * F - 0.408) * (3 - F) ^ (-0.479)
GoTo 30
25   D = 0.8 * (F - 1)

'GET G AS FIRST ESTIMATE OF GAMMA
30     G = 0
       If B1 < TT Then
         GoTo 70
       End If

       If D > 1 Then
         GoTo 40
       End If
G = (0.7466 * D ^ 1.7973 + 0.5955) * B1 ^ 0.485
GoTo 70
40 If D <= 2.5 Then
   GoTo 50
End If
U = 0.0124
Y = 0.5291
GoTo 60
50   U = 0.0623
     Y = 0.4043
G = B1 ^ (U * D + Y) * (0.9281 + D * (1.0614 * D - 0.7077))

M = 0

'MAIN ITERATION STARTS HERE

M = M + 1
If M > LIMIT Then
    FAULT = 1
Else: FAULT = 0
End If

'GET FIRST SIX MOMENTS FOR LATEST G AND D VALUES
Range("A11").Select
ActiveCell.FormulaR1C1 = G
Range("A12").Select
ActiveCell.FormulaR1C1 = D

MOMSUB

HMU(1) = Range("A13")
HMU(2) = Range("A14")
HMU(3) = Range("A15")
HMU(4) = Range("A16")
HMU(5) = Range("A17")
HMU(6) = Range("A18")
FAULT = Range("A19")

If FAULT = 1 Then Exit Sub

S = HMU(1) ^ 2
H2 = HMU(2) - S
If \( H2 \leq 0 \) Then

\[
FAULT = 1
\]

Else: \( FAULT = 0 \)

End If

If \( FAULT = 1 \) Then Exit Sub

\[
T = \sqrt{H2}
\]

\[
H2A = T \times H2
\]

\[
H2B = H2^2
\]

\[
H3 = HMU(3) - HMU(1) \times (3 \times HMU(2) - 2 \times S)
\]

\[
RBET = \frac{H3}{H2A}
\]

\[
H4 = HMU(4) - HMU(1) \times (4 \times HMU(3) - HMU(1) \times (6 \times HMU(2) - 3 \times S))
\]

\[
BET2 = \frac{H4}{H2B}
\]

\[
W = G \times D
\]

\[
U = D \times D
\]

'GET DERIVATIVES

For \( J = 1 \) To 2

For \( K = 1 \) To 4

\[
T = K
\]

If \( J = 1 \) Then GoTo 90

\[
S = \frac{((W - T) \times (HMU(K) - HMU(K + 1)) + (T + 1) \times (HMU(K + 1) - HMU(K + 2)))}{U}
\]

GoTo 100

90 \( \quad S = HMU(K + 1) - HMU(K) \)

100 \( \quad DD(K) = T \times S / D \)

110 \( \quad Next \ K \)

\[
T = 2 \times HMU(1) \times DD(1)
\]
S = HMU(1) * DD(2)  
Y = DD(2) - T  
DERIV(J) = (DD(3) - 3 * (S + HMU(2) * DD(1) - T * HMU(1)) - 1.5 * H3 * Y / H2) / H2A  
DERIV(J + 2) = (DD(4) - 4 * (DD(3) * HMU(1) + DD(1) * HMU(3)) + 6 * (HMU(2) * T + HMU(1) * (S - T * HMU(1))) - 2 * H4 * Y / H2) / H2B  
120 Next J

T = ONE / (DERIV(1) * DERIV(4) - DERIV(2) * DERIV(3))  
U = (DERIV(4) * (RBET - RB1) - DERIV(2) * (BET2 - B2)) * T  
Y = (DERIV(1) * (BET2 - B2) - DERIV(3) * (RBET - RB1)) * T

'FORM NEW ESTIMATES OF G AND D

G = G - U  
If B1 = 0 Or G < 0 Then  
   G = 0  
End If  
D = D - Y  
If Abs(U) > TT Or Abs(Y) > TT Then  
   GoTo 80  
End If

'END OF ITERATION

DELTA = D  
XLAM = SD / Sqr(H2)  
LAMBDA = XLAM  
If NEG = 1 Then  
   GoTo 130  
End If  
GAMMA = G
GoTo 140

130  GAMMA = -G
     HMU(1) = 1 - HMU(1)
140  XI = XBAR - XLAM * HMU(1)

    Range("I1").Select
    ActiveCell.FormulaR1C1 = GAMMA
    Range("I2").Select
    ActiveCell.FormulaR1C1 = DELTA
    Range("I3").Select
    ActiveCell.FormulaR1C1 = LAMBDA
    Range("I4").Select
    ActiveCell.FormulaR1C1 = XI

    End If

    End Sub
Subroutine “MOMSUB”

'THIS MACRO IS ORIGINALLY PROPOSED BY HILL, HILL, AND HOLDER
"FITTING JOHNSON CURVES BY MOMENTS" APPLIED STATISTICS, 25 (1976)
180-189
Sub MOMSUB()

    Const zz = 1# * 10 ^ -5
    Const VV = 1# * 10 ^ -8
    Const LIMIT = 500
    Const RTTWO = 1.414213562 'SQRT(2)
    Const RRPI = 0.5641895835 'RECIPIRAL OF SQRT(PI)
    Const EXPA = 80# 'A VALUE SUCH THAT EXP(EXPA) DOES NOT QUIT
    CAUSE OVERFLOW
    Const EXPB = 23.7 'A VALUE SUCH THAT 1.0+EXP(-EXPB)MAYBE
    TAKEN TO BE 1.0


Dim A(6) As Double
Dim B(6) As Double
Dim C(6) As Double
Dim i, L As Long
Dim FAULT As Integer

G = Range("A11")
D = Range("A12")
For i = 1 To 6
    HMU(i) = 0
Next i

For i = 1 To 6
    C(i) = 0
Next i

W = G / D

'TRIAL VALUE OF H

If W > EXPA Then
    GoTo 140
End If

E = Exp(W) + 1
R = RTTWO / D
H = 0.75
    If D < 3 Then H = 0.25 * D

K = 1
GoTo 40

'START OF OUTER LOOP

20   K = K + 1
    If K > LIMIT Then GoTo 140
For i = 1 To 6
30     C(i) = A(i)
Next i

'NO CONVERGENCE YET- TRY SMALLER H

H = 0.5 * H

40     T = W
        U = T
        Y = H^2
        X = 2 * Y
        A(1) = 1 / E
        For i = 2 To 6
50             A(i) = A(i - 1) / E
        Next i
        V = Y
        F = R * H
        M = 0

'START OF INNER LOOP TO EVALUATE INFINITE SERIES

60     M = M + 1

    If M > LIMIT Then GoTo 140

    For i = 1 To 6
70         B(i) = A(i)
        Next i
        U = U - F
        z = 1
If $U > -\text{EXPB}$ Then
\[ z = \exp(U) + z \]
End If

$T = T + F$

If $T > \text{EXPB}$ Then
\[ L = 1 \]
Else: $L = 0$
End If

If $L = 0$ Then $S = \exp(T) + 1$

$P = \exp(-V)$
$Q = P$

For $i = 1$ To 6
\[ AA = A(i) \]
$P = P / z$
$AB = AA$
$AA = AA + P$
If $AA = AB$ Then GoTo 100

If $L = 1$ Then GoTo 80

$Q = Q / S$
$AB = AA$
$AA = AA + Q$
If $AA = AB$ Then $L = 1$

80 $A(i) = AA$
90     Next i

100    Y = Y + X
       V = V + Y

For i = 1 To 6
    If A(i) = 0 Then GoTo 140
    If Abs(A(i) - B(i)) / A(i) > VV Then GoTo 60
110     Next i

'END OF INNER LOOP
V = RRTP * H

For i = 1 To 6
120      A(i) = V * A(i)
     Next i

For i = 1 To 6
    If A(i) = 0 Then GoTo 140
    If Abs((A(i) - C(i)) / A(i)) > zz Then GoTo 20
     Next i

'END OF OUTER LOOP

For i = 1 To 6
    HMU(i) = A(i)
     Next i
Range("A13").Select
ActiveCell.FormulaR1C1 = HMU(1)
Range("A14").Select
ActiveCell.FormulaR1C1 = HMU(2)
Range("A15").Select
ActiveCell.FormulaR1C1 = HMU(3)
Range("A16").Select
ActiveCell.FormulaR1C1 = HMU(4)
Range("A17").Select
ActiveCell.FormulaR1C1 = HMU(5)
Range("A18").Select
ActiveCell.FormulaR1C1 = HMU(6)
Range("A19").Select
ActiveCell.FormulaR1C1 = FAULT

Exit Sub

140 FAULT = 1
'WRTIE THE OUTPUT TO EXCEL
End Sub
Subroutine “GRAPHS”

Sub GRAPHS()
' This macro draws the graph of the density function of the fitted Johnson distribution

Dim GAMMA, DELTA, LAMBDA, XI, X, LOLIM, UPLIM, XBAR, SD, STP,
    LOLIMSTP, USL, LSL, MAXY As Variant
Dim TEMPRANGEX, TEMPRANGEY, USLRANGE, LSLRANGE, SLYRANGE As Range
Dim i As Double

' Enter the parameters of Johnson distribution, mean, variance and spec limits from
    worksheet "Johnson" as input
GAMMA = Range("I1")
DELTA = Range("I2")
LAMBDA = Range("I3")
XI = Range("I4")
XBAR = Range("B2")
SD = Range("B3")
USL = Range("E6")
LSL = Range("E7")

Application.ScreenUpdating = False
' If the variate is in the impossible area terminate
If Range("D2") = "IMPOSSIBLE AREA" Then
    GoTo 10
End If
' Generate data for graph of SL function
If Range("F2") = "SL" Then
' find the startind point of the graph by comparing XI and zero
    Range("G12").Select
ActiveCell.FormulaR1C1 = "=MAX(R[-8]C[2],0)"

'Find the upper and lower limits for x axis
LOLIM = XBAR - 5 * SD
UPLIM = XBAR + 10 * SD

'Insert the density function in cell B10 and copy to cells B10 through B110
Range("B10").Select
ActiveCell.FormulaR1C1 = "=(R2C9 / (Sqrt(2 * Pi()) * (RC[-1] - R4C9))) * Exp(-0.5
   * R2C9 ^ 2 * (R1C9 / R2C9 + LN(RC[-1] - R4C9)) ^ 2)"
Range("B10").Select
Selection.AutoFill Destination:=Range("B10:B110") ' & Trim(Str(10 + (UPLIM -
   LOLIM) * 10))), Type:=xlFillDefault

'Insert values of the x axis in column A
STP = (UPLIM - LOLIM) / 100
Range("A10").Select
LOLIMSTP = LOLIM
For i = 0 To 100
   ActiveCell.Offset(i, 0).Value = LOLIMSTP + STP
   LOLIMSTP = LOLIMSTP + STP
Next i

End If

'Generate data for graph of normal function
If Range("F2") = "NORMAL" Then

'Find the upper and lower limits for x axis
LOLIM = XBAR - 4 * SD
UPLIM = XBAR + 4 * SD

'Insert formula for the density function of normal in cells B10 through B110
Range("B10").Select
ActiveCell.FormulaR1C1 = "=(1/SQRT(2*PI()))*EXP(-(RC[-1]-R2C)^2/(2*R3C^2))"
Range("B10").Select
Selection.AutoFill Destination:=Range("B10:B110") ' & Trim(Str((UPLIM - LOLIM) * 100 + 10)), Type:=xlFillDefault
'Insert values for the x axis in column A
STP = 8 * SD / 100
Range("A10").Select
LOLIMSTP = LOLIM
For i = 0 To 100
    ActiveCell.Offset(i, 0).Value = LOLIMSTP + STP
    LOLIMSTP = LOLIMSTP + STP
Next i
End If

'Generate data for graph of SB function
If Range("F2") = "SB" Or Range("F2") = "ST" Then

'Find the upper and the lower bound for x
LOLIM = XI
UPLIM = XI + LAMBDA

'Insert density function in cells B10 through B110
Range("B10").Select
ActiveCell.FormulaR1C1 = "=(R2C9/SQRT(2*PI()))*(R3C9/((RC[-1]-R4C9)*(R3C9-RC[-1]+R4C9)))*EXP(-.5*(R1C9+R2C9*LN((RC[-1]-R4C9)/(R3C9-RC[-1]+R4C9)))^2)"

Range("B10").Select
Selection.AutoFill Destination:=Range("B10:B110") ' & Trim(Str(10 + (UPLIM - LOLIM) * 10))), Type:=xlFillDefault
'Insert values of x in column A
STP = LAMBDA / 100
Range("A10").Select
LOLIMSTP = LOLIM

For i = 1 To 100
    Range("A10") = LOLIM
    ActiveCell.Offset(i, 0).Value = LOLIMSTP + STP
    LOLIMSTP = LOLIMSTP + STP
Next i

End If

'Generate data for graph of SU function
If Range("F2") = "SU" Then

'Find upper and lower bounds for x
LOLIM = XI - 4 * LAMBDA
UPLIM = XI + 4 * LAMBDA

'Insert density function in cells B10 through B110
Range("B10").Select
ActiveCell.FormulaR1C1 = "=(R2C9/SQRT(2*PI()))*(1/SQRT((RC[-1]-R4C9)^2+R3C9^2))*EXP(-.5*(R1C9+R2C9*LN((RC[-1]-R4C9)/R3C9+(((RC[-1]-R4C9)/R3C9)^2+1)^.5))^2)"

Range("B10").Select
Selection.AutoFill Destination:=Range("B10:B110") ' & Trim(Str(10 + (UPLIM - LOLIM) * 10)), Type:=xlFillDefault
'Insert values of x in column A
STP = 8 * LAMBDA / 100
Range("A10").Select
LOLIMSTP = LOLIM

For i = 1 To 100
    Range("A10") = LOLIM
    ActiveCell.Offset(i, 0).Value = LOLIMSTP + STP
    LOLIMSTP = LOLIMSTP + STP
Next i
End If

'Generate data for marking the specification limits
Range("G11").Select
ActiveCell.FormulaR1C1 = "=MAX(RC[-5]:R[98]C[-5])"
MAXY = Range("G11")

Range("E10").Select
Range("E10").FormulaR1C1 = 0
STP = MAXY / 100
For i = 1 To 100
    ActiveCell.Offset(i, 0).Value = STP * i
Next i

For i = 0 To 100
    ActiveCell.Offset(i, -2).Value = USL
    ActiveCell.Offset(i, -1).Value = LSL
Next i
Application.ScreenUpdating = True
'DRAW CHART BASED ON 100 DATA POINTS
'SET THE RANGES FOR THE DATA SOURCES OF X, F(X), USL AND LSL
Set TEMPRANGEX = Range("A10:A110") ’& Trim(Str((UPLIM - LOLIM) * 10 + 10)))
Set TEMPRANGEY = Range("B11:B110") ’& Trim(Str((UPLIM - LOLIM) * 10 + 10)))
Set USLRANGE = Range("C10:C110")
Set LSLRANGE = Range("D10:D110")
Set SLYRANGE = Range("E10:E110")

'ADD DATA TO CHART
Range("A10").Select
Charts.Add
ActiveChart.ChartType = xlXYScatterSmoothNoMarkers
ActiveChart.SetSourceData Source:=Sheets("Johnson").Range("E2")
ActiveChart.SeriesCollection.NewSeries
ActiveChart.SeriesCollection.NewSeries
ActiveChart.SeriesCollection.NewSeries
ActiveChart.SeriesCollection.NewSeries(1).XValues = TEMPRANGEX
ActiveChart.SeriesCollection.NewSeries(1).Values = TEMPRANGEY
'NAME THE DATA SERIES
ActiveChart.SeriesCollection.NewSeries(1).Name = "=""PDF"
ActiveChart.SeriesCollection.NewSeries(2).XValues = USLRANGE
ActiveChart.SeriesCollection.NewSeries(2).Values = SLYRANGE
ActiveChart.SeriesCollection.NewSeries(2).Name = "=""LSL"
ActiveChart.SeriesCollection.NewSeries(3).XValues = LSLRANGE
ActiveChart.SeriesCollection.NewSeries(3).Values = SLYRANGE
ActiveChart.SeriesCollection.NewSeries(3).Name = "=""USL"

'DEFINE NAME AND LOCATION OF CHART AND NAME THE AXES
ActiveChart.Location Where:=xlLocationAsObject, Name:="Johnson"
With ActiveChart
  .HasTitle = True
  .ChartTitle.Characters.Text = "PDF AND LIMITS"
  .Axes(xlCategory, xlPrimary).HasTitle = True
  .Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "X"
  .Axes(xlValue, xlPrimary).HasTitle = True
  .Axes(xlValue, xlPrimary).AxisTitle.Characters.Text = "F(X)"
End With

'CHANGE COLOR AND STYLE OF CHART
ActiveChart.PlotArea.Select
  With Selection.Border
    .ColorIndex = 16
    .Weight = xlThin
    .LineStyle = xlContinuous
  End With
With Selection.Interior
  .ColorIndex = 2
  .PatternColorIndex = 1
  .Pattern = xlSolid
End With
ActiveChart.ChartArea.Select
ActiveChart.SeriesCollection(2).Select
  With Selection.Border
    .ColorIndex = 57
    .Weight = xlMedium
    .LineStyle = xlContinuous
  End With
  With Selection
    .MarkerBackgroundColorIndex = xlNone
    .MarkerForegroundColorIndex = xlNone
.MarkerStyle = xlNone
.SMOOTH = True
.MarkerSize = 3
.Shadow = False

End With
ActiveChart.PlotArea.Select
ActiveChart.SeriesCollection(3).Select
With Selection.Border
 .ColorIndex = 57
 .Weight = xlMedium
 .LineStyle = xlContinuous
End With
With Selection
 .MarkerBackgroundColorIndex = xlNone
 .MarkerForegroundColorIndex = xlNone
 .MarkerStyle = xlNone
 .Smooth = True
 .MarkerSize = 3
 .Shadow = False
End With
ActiveChart.SeriesCollection(1).Select
With Selection.Border
 .ColorIndex = 57
 .Weight = xlThick
 .LineStyle = xlContinuous
End With
With Selection
 .MarkerBackgroundColorIndex = xlNone
 .MarkerForegroundColorIndex = xlNone
 .MarkerStyle = xlNone
 .Smooth = True
.MarkerSize = 3
.Shadow = False

End With
ActiveChart.ChartArea.Select
ActiveChart.SeriesCollection(3).Select
With Selection.Border
  .ColorIndex = 3
  .Weight = xlMedium
  .LineStyle = xlContinuous
End With
With Selection
  .MarkerBackgroundColorIndex = xlNone
  .MarkerForegroundColorIndex = xlNone
  .MarkerStyle = xlNone
  .Smooth = True
  .MarkerSize = 3
  .Shadow = False
End With
ActiveChart.ChartArea.Select
ActiveChart.SeriesCollection(2).Select
With Selection.Border
  .ColorIndex = 50
  .Weight = xlMedium
  .LineStyle = xlContinuous
End With
With Selection
  .MarkerBackgroundColorIndex = xlNone
  .MarkerForegroundColorIndex = xlNone
  .MarkerStyle = xlNone
  .Smooth = True
  .MarkerSize = 3
  .Shadow = False
End With
.Shadow = False
End With
ActiveChart.ChartArea.Select

10 End Sub