

ABSTRACT

ELEY, PETER MADISON. Does using Computer Software make a Difference in Learning Geometric and Probability Concepts? (Under the direction of Lee V. Stiff, PhD).

The purpose of the research project was to see if using computer software in classroom instruction would help students learn geometric and probability concepts better. *Geometer's Sketchpad* and *Probability Explorer* software were used as treatment instructional tools during a two-week summer Geometry and Probability course. Two groups of students were taught using computer software to determine if the software had an effect on student learning. The research method used a pre-test/post-test growth measurement indicator. Both groups of students were given pre-tests about geometry and probability to determine their prior understanding of the subjects. The first group was given a computer software treatment for learning geometry while the second group was taught geometry using traditional classroom methods. After one week, the groups were switched and taught probability. The first group was now taught using traditional classroom methods and the second group used computer software to learn probability. When checking to see if a treatment effect exists, considerations are given for students' prior knowledge of the subject matter and computer skills. After the completion of two weeks of instruction, students were given a post-test to determine negative or positive changes in their achievement. An analysis of the post-test scores using quantitative and qualitative methods indicated that students were better motivated by using computer software. However, results of the data analysis indicated that there was no significant growth in geometric or probabilistic performance by students who received the treatments.

Does Using Computer Software Make a Difference in Learning Geometric and Probability
Concepts?

by
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DEDICATION

I dedicate this Master's thesis to my late grandmother, Mrs. Geneva Evans Alston. My grandmother was a constant source of inspiration for me throughout my academic career. She never missed any of my graduation ceremonies or recognitions of my academic accomplishments. I miss her smile and her home remedies, but most of all, I miss her encouragement.

I love you Grandma, and I hope to see you again after awhile.

With love, from your eldest grandson,

Peter

BIOGRAPHY

Peter Eley currently serves as the North Carolina Mathematics and Science Network Pre-College Program Coordinator in the Center for Mathematics, Science, and Technology Education at Winston-Salem State University and also holds an adjunct instructor position at the university. Mr. Eley earned a Bachelor of Science degree in Pure Mathematics from Elizabeth City State University in the year 2000. Two years later, he completed the Master of Science degree program from North Carolina State University in Applied Mathematics. Peter has taught middle school mathematics in Vance County and high school in Granville County in North Carolina.

He is married to Felicia Lowe Eley, his wife of six years. They reside in Winston-Salem, North Carolina, where he is also a minister at Christ Temple Church.

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CHAPTER I

THE PROBLEM

No Child Left Behind (NCLB, 2001) legislation made educators accountable for students' proficiency in mathematics based on each state's individual standards of performance. As a result of this, innovative ways have been devised to help students learn mathematics and probability concepts better. In many cases, instructional approaches have turned to technology to help in this enormous task. The National Council of Teachers of Mathematics (NCTM, 2000) has endorsed the use of technology for teaching a variety of mathematical content, especially in teaching the concepts of geometry and probability by use of calculators and computers. Using these types of technologies offers the students visual images of mathematical ideas and they facilitate organizing and analyzing data.

Technology in mathematics education aids students in the learning process (NCTM, 2000). NCTM has recognized technology as an important aspect of the teaching/learning process and has identified technology as one of its principles for school mathematics. NCTM says that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p.25). Special technology tools exist to benefit much of what is taught in schools. This is especially true for geometry and probability. Examples of technology that exhibit this are computer software such as *Geometer's Sketchpad*, *Cabri*, *Fathom*, and *Probability Explorer*.

Geometry learning is important because it is a fundamental component of mathematical learning. It “offers ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science” (NCTM, 2000, p. 41). Geometry gives students the opportunity to develop their natural sense of reasoning and justification that are used to build skills across the grades (NCTM, 2000). In its *Principles and Standards for School Mathematics*, NCTM (2000) calls for geometry to be learned using dynamic software as an instructional tool. It is important that a new approach to learning these concepts be found, because the United States (US) performed poorly in geometry and measurement in comparison to other countries according to the Trend in International Mathematics and Science Study (TIMSS) conducted in 2005 by the American Institute for Research.

Learning probability is also important because it allows students to be an informed citizen, employee, and consumer (NCTM, 2002). NCTM endorses the learning of probability and feels that students should be taught a full set of basic skills that include being able to: formulate questions that can be answered with data, analyze data, develop and evaluate inferences, and understand and apply the basic concepts of probability. NCTM further strengthens their position on probability instruction by saying “work in data analysis and probability offers a natural way for students to connect mathematics with other school subjects and with everyday experiences” (p.48). Skills developed in the study of

probability will help students in other realms of study, especially in areas of observation and experimental studies. Data from TIMSS (2005) suggest that the US performed relatively well in statistics and data analysis. Nevertheless, much more work is needed to maintain students' understanding of probability and data analysis.

Many software programs exist that are designed to help students' learning in a variety of subjects such as: *Cabri* (Laborde and Bellemain, 2005), *The Geometer's Sketchpad* (Jackiw, 2001), *Fathom* (Key Press 2001), and *Probability Explorer* (Lee, 2001).

The purpose of this study was to determine the effectiveness of dynamic computer software on the learning of geometry and probability concepts. Moreover, the study analyzed student outcomes on pre and post-tests on geometry transformations and concepts of probability with replacement.

CHAPTER II REVIEW OF THE LITERATURE

DYNAMIC GEOMETRY

Learning geometry can be very difficult for students, because of the abstractness of the concepts. An example of this would be students' understanding of transformations. Students often try to understand transformations by doing them in their heads or with static representations given by pencil and paper. In many cases students fail to make accurate constructions of the figures on which they are trying to perform transformations (Hollebrands 2003). Although students use protractors, rulers, and compasses to create transformations, over time students become weary of the tasks because of the amount of time and steps involved to make accurate figures. Typically, students mimic their instructors and go through exercises without learning and developing needed skills and understandings (Chazan, 1993).

The use of geometry technologies can be incorporated into instruction as a tool for both students and teachers. There have been many studies that incorporate dynamic geometry software into classroom instruction (Chazan, 1993; Edwards, 1997; Hollebrands, 2007; Hoyes & Noss, 1994; Laborde, 1993; Mariotti, 2000; Sanchez, Ernesto, & Sacristan, 2003). Dynamic geometry software gives users the ability to create constructions using equivalent techniques of those associated with a straightedge and compass as well as techniques and methods unique to this dynamic environment. The use of dynamic software

enhances the opportunity for students to learn geometric properties associated with constructions they perform because such constructions may be easily repeated using different figures. The added function of being able to drag computer generated objects also benefits the learning process. Previous research in dynamic geometry software use has produced insight in two areas of importance: “(a) the affects on students’ understandings of geometrical figures and properties, and (b) its influences on students’ deductive reasoning” (Hollebrands & Smith, 2009, in press). In this study, we are concentrating on the affects on students’ understanding of geometrical figures and properties in contrast to using traditional classroom methods to teach these concepts.

Dynamic geometry software empowers students to perform a variety of tasks and encourages them to develop geometric conjectures about observed relationships. This instructional tool is very helpful to students because it strengthens their prior knowledge of a protractor, ruler, compass, and enables them to create and recreate figures quickly. Students are able to do more examples of similar problems with less frustration and more accuracy. Using this tool, students are more likely to understand why deductive proofs are needed, in contrast to empirical proofs, and understand the meaning of geometric properties. For example, students are able to quickly measure angles and length accurately through guided discovery exercises designed to help them develop conjectures about

geometric properties. Students are able to investigate empirically and understand the deductive proof without being able to create the proof from the beginning (Chazan, 1993).

Students learn the difference between constructing figures versus drawing them when using dynamic software. Students who construct figures instead of drawing them can easily test conjectures in this dynamic environment. This allows students to develop their own learning at their own pace through exploring different ways to create a construction where all of the properties of that geometric figure are present.

The research in the area of using dynamic geometric software to explore geometric transformations brings two important aspects to the fore regarding the understanding of geometry in a dynamic software environment. The distinction between drawing and constructing a figure, and the ways in which computer environments mediate students' understanding of geometry are important notions (Hollebrands, 2007 p.168). Hollebrands analyzes these in her study by giving examples of drawings versus constructions and what makes them different. From this research it was found that it was possible for a teacher and student to interact within a dynamic geometry environment and both of them adopt and use a common language pursuant to mathematical ideas represented by shared visual images. These results are similar to findings discovered by Vygotsky (1978), in which he states that "it's decisively important that speech not only facilitates the child's effective manipulation of objects but also controls the child's own behavior" (p.26).

Verbalizing their thoughts can be quite a challenge for students. But while students are being challenged they develop problem solving, critical thinking, and analytical skills. Engaging students in activities and games involving geometry is a powerful way to help students develop important understandings and skills. When so engaged, students often do not realize that they are learning and putting together geometric properties that they have been studying (Edwards, 1997).

Dynamic geometry software is especially useful when you are working with very abstract ideas such as transformations. Many times students try to learn transformation concepts by a pencil-paper method. It is sometimes difficult for student to see the changes that figures are undergoing and students are often unable to make the connection to what is actually happening. Take for example a reflection transformation. Students in a pencil-paper environment have to construct the: given figure, line of reflection, and reflected figure. If students want to move a figure by stretching it, they may not see how the figure would change. They would have to imagine the figure changing in their heads.

This is not true in a dynamic geometry environment. In the case of a reflection, the student would be able to see that once the figure was moved, the reflection of the figure was in one-to-one correspondence to the original figure. Students are able to see hundreds of examples like this. According to Hollebrands (2007), technology makes the actions of

transformations visible to students in a dynamic environment in such a way that students develop deeper understandings of transformations.

In a study by Edwards (1997) students learned how to use the TGEO microworld to perform geometric transformations. The microworld allowed students to superimpose one shape on another. The activity became a game to the students who could only use geometric transformations to succeed at the task. The “game” helped students develop a working understanding of transformations and provided an environment in which to explore transformations through guided discovery. Although upon completion of the study students were not able to develop their own proofs, they were able to follow and understand proofs provided by the investigator.

Using dynamic geometry software helps empower students in developing concepts about rotations about a given point. The software would allow them to select a point of rotation and the angle amount of rotation they would like to use. They could then observe the rotation and analyze the resulting movement. If the concept of rotation is taught from a more traditional point of view, students would have to either imagine a figure rotating about a given point or use a cut-out of the given figure and model the required rotation as accurately as possible. Students would benefit more from using dynamic geometry software in developing an understanding of this type of transformation.

Geometry software played an important role in the studies of Chazan and Edwards. The studies were able to establish that computers could be used by students to learn key geometric concepts and properties. In both studies students were able to test and explore conjectures made in a more manageable environment. Students were then able to transfer their understanding to traditional situations in their study of geometry.

DYNAMIC PROBABILITY

Learning and teaching probability concepts in middle grades can be a difficult task for many teachers. They may find that when they first begin to teach probability concepts that they are not sure where to start in the development of the basic constructs. Researchers (Borovcnik, Bentz, & Kapadia, 1991; Fischbein, 1975; Hawkins & Kapadia, 1984; Shaughnessy, 1992) suggest that teachers should start with an experimentation approach that relies heavily on objective analysis of repeated experiments. The traditional approach found in most curricula begins with mathematical axioms, rules, and numerical activities that promote memorization rather than understanding. The axioms usually start with simple probability and scaffold to more advanced axioms such as conditional probability. Students often do not grasp the basic concepts from mathematical axioms and rules because students lack the cognitive development needed. It is inferred that students do not ask questions of “how” and “why” because they see probabilistic thinking as a skill and not a process of critical thinking.

There exist many different arguments for how probability should be taught (Borovcnik & Bentz, 1991; Streinbring, 1991; and Shaughnessy, 1992). They vary from traditional rule-driven theoretical approaches to skilled-oriented instruction. Neither of which has been categorized as being effective in teaching students how to effectively use probabilistic reasoning in new situations (Stohl, 2001). Although experimental approaches are often used in school curricula to teach probability concepts, very few employ technological tools.

Dynamic software exists for the sole purpose of helping students understand probability concepts. They include: *Tabletop Jr.*, *Graphers*, *Probability Toolkit*, *Probability Constructor*, and *Probability Explorer*. Although these software programs are commercially available, limited studies (Stohl, 2001) have been conducted to see how effective they are for helping students develop probabilistic reasoning. Konold (1991) developed and used *ProbSim* software with his students and found that students were able to use the simulations that it created to analyze data more effectively. Like Konold, others developed simulation software to meet the need for analyzing empirical data. *ChanceWorld*, developed by Jiang and Potter (1994), was used in a study in which it was found that the use of the computer environment was beneficial to students in helping them overcome common misconceptions about probability. The software program being used in this study is *Probability Explorer (PE)*.

The *Probability Explorer* software was developed and used by Stohl (2001) for her dissertation work on probabilistic reasoning in children using microworlds. “The design of the microworld is based on a constructivist theory of learning, design of mathematical computer microworlds, and research on students’ understanding of probability and rational number concepts”(Drier, 2000). Implications from Stohl’s work suggest that carefully designed microworld tools can facilitate probabilistic reasoning. From her work she was also able to conclude that reflecting on changes in multiple representations during a simulation encouraged students to develop theories-in-action about the Law of Large Numbers (Stohl, 2001).

The probability software programs cited above have a common characteristic they were developed based on research about probabilistic misconceptions and the use of non-technological teaching experiments that enhanced student’s probabilistic reasoning (Stohl, 2001). They each generated data and gave students the opportunity to connect multiple representations to formulate conjectures about the data. *PE* is dynamic software. It allows the student to see experiments in real time. It also gives students the ability to create their own experiments and test their conjectures by exploring. According to Lee (2008) one of the strengths of this software is that it was “purposefully designed as an open-ended learning environment with multiple ways to represent data that engage students in designing, simulating, and analyzing results of probability experiments. The software

operates on a fundamental level in that, data is represented in PE with randomly generated icons that can be sorted, stacked (in a pictograph) or lined up in the sequence in which they occurred. A Pie Graph (relative frequency), Bar Graph (frequency), and Data Table (counts, fractions, decimals, and percents) are also available to display results in both static form as well as dynamically changing during a simulation". Other software such as *ProbSim* and *ChanceWorld* successfully helped students' gain conceptual understanding in probabilistic learning situations. This study will investigate whether the same is true of *Probability Explorer* in a middle grades setting.

CHAPTER III THE METHOD

The purpose of the study was to determine if using dynamic computer software in classroom instruction would help students learn geometric and probability concepts. The instructional variables in the study were traditional classroom instructional methods and instruction involving dynamic computer software. The students were interviewed to obtain background and demographic information. Student learning was measured by their performance on the pre- and post-tests that were given at the beginning and end of the investigation period.

PARTICIPANTS

Nine (9) middle grades students from a public school system in central North Carolina participated in this research study. The students ranged in age from eleven to thirteen years old. Seven (7) male students and two (2) female students took part. The two female students were both African American. Six male students were African American, and one was Asian American. All of the participants were average or above average students (grade C or better) in mathematics. All of the students had basic computer skills in that they knew how to use the mouse and keyboard, they knew how to open computer software programs, and how to follow software program instructions. Student interviews revealed that they had not been exposed to very much geometry or probability instruction in

their formal schooling to date. The socio-economic status of all of the students was determined to be middle class. None of the students had any documented disabilities—physical, emotional, or mental.

The two instructors (Instructors A and B) who taught the geometry and probability classes of the study had very strong backgrounds in mathematics. Both were experienced middle school teachers with a combined 7 years of public school classroom teaching at the middle school level.

TREATMENT

Students in the study were divided into two groups: Group 1 and Group 2. Instructor A taught both geometry and probability using the appropriate dynamic software programs (*Geometer's Sketchpad* and *Probability Explorer*). Instructor B taught both geometry and probability in a traditional manner without using any dynamic software programs.

The study was conducted over a two-week period. Each group of students received two-hour-long lessons each day of the study. See Table 3.1 for the study's research design. See Appendix A for information about the instruction.

Table 3.1 Research Design

| Research Design | | |
|-------------------------|---|--|
| | Group 1 | Group 2 |
| Geometry (Week 1) | Sketchpad-based Instruction (Instructor A) | Traditional Instruction (Instructor B “control”) |
| Probability (Week 2) | Traditional Instruction (Instructor B “control”) | Probability Explorer- based instruction (Instructor A) |

A group of nine (9) sixth-grade students were randomly divided into two smaller groups. The first group (Group 1) had five students and the second group (Group 2) had four students. Each participant was interviewed by the researcher who asked seven questions about their experiences with technology and their likes and dislikes regarding the use of technology to learn mathematics. The interviews helped to gain more information about students’ feelings toward the math content. (See Appendix B.) These data were recorded and saved for analysis and comparison to post-interview data.

A pre-test and a post-test were created for the geometry and probability treatments, so that students’ scores could be compared at the end of the study. The geometry and probability pre-post tests were created from the teaching materials that the instructors used in the class. Problems from each day of instruction were selected from in-class materials. The problems covered all of the instruction that took place over the period of the study. At

the beginning of the study, the two groups of students were given two pre-tests to measure their understanding of geometry and probability. At the conclusion of the first week, a post-test in geometry was administered to Groups 1 and 2. At the conclusion of the second week, a post-test in probability was administered to Groups 1 and 2.

STATISTICS

A two-sample t-test was used to detect differences between the groups' performance on the pre- and post-tests. The t-test allows us to observe differences between two samples with the results depending on the spread of the observations as well as on the two means (Moore, 2004). It was assumed that the variances were unequal in all of the cases.

MATERIALS

The classroom in which the traditional instructional approach was implemented was a typical mathematics classroom. The classroom in which the instructional treatment was implemented was equipped with a presentation device and a projection screen on which the teacher could project software-driven instruction. Students receiving the instructional treatments used handouts provided by the instructor. (See Appendices C and D.) Each student in the treatment groups had access to a Dell Latitude laptop computer with wireless internet capability and a Texas Instruments 83-Plus calculator. The computers were equipped with *Geometer's Sketchpad* and *Probability Explorer* software packages. Students also used a Dell 256mb flash drive to save their work from the laptops.

INSTRUCTION

Instruction for the *Geometer's Sketchpad* treatment was taught using supplemental handouts developed by Karen Hollebrands. (See Appendix C.) Hollebrands created these materials, which were modified in the study for middle school students, to teach high school students geometry by incorporating dynamic geometry software instruction into their geometry curriculum. The handouts are very detailed and give step-by-step instruction on transformations. (See Appendix C.) The materials consist of activities that allow students to explore properties of transformations in a dynamic environment. Instructor A was a facilitator for his students, providing help and support as needed. Each day's lesson included a discussion about the lesson objectives designed to help students better understand the content. *Geometry: Applying, Reasoning, Measuring* (Larson, Boswell, & Stiff, 2000), pages 394-412, was used as a reference for definitions and study problems. The traditional geometry class used the same textbook as its source of instruction. Standard tools such as protractors, pattern blocks, wax paper, and the chalkboard were used in the traditional classroom to help student to learn lesson objectives.

Instruction for the *Probability Explorer* treatment was taught using supplemental handouts and computer generated files developed by Hollylynne S. Lee. Lee created these materials to help in-service teachers incorporate dynamic probability software instruction into their probability curriculum. The handouts give detailed instruction on probability

concepts, including pre-loaded computer files. (See Appendix D.) No textbook was used in this class, students were taught probability using the handouts provide. (See Appendix D.) The traditional probability class used number cubes, spinners, playing cards, and the chalkboard to help student to learn lesson objectives.

On the first days of geometry and probability instruction, both Groups 1 and 2 were given a pre-test to determine their existing knowledge and skills for the subject matter. The participants were taught geometry during the first week. Group 1 was randomly determined to receive the treatment instruction using the *Geometer's Sketchpad* software. During the first week, Group 2 received traditional geometry instruction. Consequently, Group 2 would receive the *Probability Explorer* treatment for the probability instruction given the second week. Accordingly, Group 1 would be taught probability using traditional teaching methods.

GEOMETRY TREATMENT

During Week 1 of the geometry lessons, both groups studied transformations: reflections, rotations, and translations. On Day 1, the students took a pre-test on transformations and were introduced to transformations. The treatment group, Group 1, received its introduction to the software *Geometer's Sketchpad* and received handouts on the names of transformations: “*Explorations I-V*” in Appendix C (Hollebrands, 2006). Instructor A used *Geometer's Sketchpad* to complete the handouts. The control group,

Group 2, used traditional teaching methods to meet the same lesson objectives. On Day 2, a review of the previous day's lesson was provided to both Groups 1 and 2. Then, the new lesson objectives were introduced. The treatment group, Group 1 was taught translations using *Geometer's Sketchpad*. The students were given handouts to complete the tasks in translations (See Appendix C.). The control, Group 2, was taught using materials such as protractors to demonstrate translations. This pattern of instruction for the treatment and control group follows throughout the rest of this research project. On Day 3 of the project a review of translations was given to both groups. Then, appropriate lessons on rotations were introduced to each group. On Day 4 a review of rotations was provided and new lessons on reflections were taught. On Day 5, a review of all of the transformations was provided followed by a post-test.

PROBABILITY TREATMENT

The lesson objective taught in Week 2 of the project was about how to compute simple probability with replacement. On Day 1, students took a pre-test on probability and were then introduced to computing simple probability with replacement. Instructor A taught Group 2 students how to use *Probability Explorer*. They were then shown how to compute simple probability with replacement using handouts that were provided (See Appendix D.). The *Probability Explorer* software helped students in computing simple probabilities by demonstrating the connections between theory and empirical situations.

Probability Explorer did not compute the probabilities for students but became a visual aid in helping them see what was going on and keeping track of data. Group 1, now the control group, learned simple probability from using manipulatives, number cubes, and handouts. This pattern of instruction for the treatment and control groups was followed throughout the rest of the week. That is, prepared activities that were used to teach the probability concepts were used with both groups, but Group 2 used *Probability Explorer* to explore relationships while Group 1 used hand-held manipulatives. On Day 2, a lesson on simple probability with replacement was continued. The “*Guess My Marbles I, Guess My Marbles II* and *Guess My Marbles III*” activities (See Appendix D.) were designed to teach simple probability with replacement (Lee, 2006). These activities allowed Group 2 students to create their own experiments using *Probability Explorer* as an instructional tool. The control group, Group 1, used manipulatives to achieve a similar experience. On Day 3 of the project, simple probability with replacement was continued using “*The Fish Study*” (Lee, 2006). The fourth and final day of the teaching experiment, students reviewed the lesson objective that was covered during the four day period. After the review, both groups of students were then given a post-test on the concepts.

Pre- and post-test scores were analyzed. The research hypotheses were as follows:

H1: No significant differences exist between the pre- and post-test geometry scores of Group 1 & Group 2. (P-value ≤ 0.05)

H2: No significant differences exist between the pre- and post-test probability scores of Group 1 & Group 2 (P-value ≤ 0.05)

CHAPTER IV RESULTS AND DISCUSSION

PRE-INTERVIEWS

The students who participated in the study were interviewed prior to their instruction to acquire information about their backgrounds and academic status. On the pre-interview, 5 out of 9 students said they enjoyed learning mathematics. All of the students said they enjoyed using technology. Each child said they had a computer available to them at home. Although the students in the study indicated that they had been exposed to using technology in an educational setting, only 2 out of 9 of them had used computer software to do either geometric or probability assignments. All of the students did indicate that they felt that using computer software would help them to learn geometric and probability concepts in a meaningful way.

POST-INTERVIEWS

All of the participants felt that the technologies that they were exposed to helped them learn the geometric and probability concepts they were taught in the study. The students expressed how much they enjoyed using the geometry and probability software and completing the activities during instruction. When asked if they now enjoyed mathematics after being exposed to the math concepts in a new way, 8 out of 9 students stated that they now enjoyed mathematics after using the computer software to learn mathematical ideas. This was an increase of 3 students, which now enjoyed math because

of the technology. This could indicate that technology can be used to motivate students in learning new mathematical concepts.

DATA ANALYSIS

Table 4.1 shows the scores and the differences between scores on the pre- and post-tests given in geometry and probability.

Table 4.1 Pre-Post Test Scores

| | | Geometry | | |
|-----------------------|---|--------------------|------------------|-------------------|
| Group (1 or 2) | Treatment Type & Instructor (A or B) | Pre-test | Post-test | Difference |
| 1 | Sketchpad(A) | 35 | 35 | 0 |
| 1 | Sketchpad(A) | 35 | 65 | 30 |
| 1 | Sketchpad(A) | 25 | 60 | 35 |
| 1 | Sketchpad(A) | 10 | 5 | -5 |
| *1 | Sketchpad(A) | 50 | 45 | -5 |
| | average | 31 | 42 | 11 |
| | | | | |
| 2 | Traditional(B) | 25 | 40 | 15 |
| 2 | Traditional(B) | 55 | 85 | 30 |
| 2 | Traditional(B) | 30 | 85 | 55 |
| 2 | Traditional(B) | 65 | 55 | -10 |
| | average | 43.75 | 66.25 | 22.5 |
| | | Probability | | |
| Group (1 or 2) | Treatment Type & Instructor (A or B) | Pre-test | Post-test | Difference |
| 1 | Traditional(B) | 29 | 33 | 4 |
| 1 | Traditional(B) | 14 | 29 | 15 |
| 1 | Traditional(B) | 14 | 33 | 19 |
| 1 | Traditional(B) | 29 | 29 | 0 |
| | average | 21.5 | 31 | 9.5 |
| | | | | |
| 2 | Probability Explorer(A) | 38 | 33 | -5 |
| 2 | Probability Explorer(A) | 29 | 38 | 9 |
| 2 | Probability Explorer(A) | 24 | 29 | 5 |
| 2 | Probability Explorer(A) | 24 | 33 | 9 |
| | average | 28.75 | 33.25 | 4.5 |

*Note “one traditional student” dropped out of second study on Probability

Tables 4.2 and 4.3 show mean difference scores and standard deviations for the pre-test/post-test differences for geometry and probability, respectively. It was found that the control group (Group 2) in geometry had a higher mean difference score than the treatment group (Group 1). Difference scores were analyzed using Student's t-test for $\alpha = .05$. The results of this analysis are found in Table 4.2. As seen in the table, there was no significant difference between the mean difference scores in geometry for Groups 1 and 2.

Table 4.2 Descriptive Statistics for Geometry

| t-Test: Two-Sample Assuming Equal Variances Pre-Test / Post-Test Difference for Geometry | | |
|--|-------------------------------|----------------------------------|
| | <i>Treatment Geometry</i> | <i>Traditional (control)</i> |
| Mean | 11 | 22.5 |
| Variance | 392.5 | 741.6666667 |
| Observations | 5 | 4 |
| Hypothesized Mean Difference | 0 | |
| df | 5 | |
| t Stat | -0.707887807 | |
| P(T<=t) one-tail | 0.255319356 | |
| t Critical one-tail | 2.015048372 | |
| P(T<=t) two-tail | 0.510638711 | |
| t Critical two-tail | 2.570581835 | |
| Standard Deviation (Treatment Geometry) | 19.81161276 | |
| Standard Deviation (Traditional Geometry) | 27.23355773 | |

Similarly, it was found that the control group (Group 1) in probability had a higher mean difference score than the treatment group (Group 2). Again, difference scores were

analyzed using Student's t-test for $\alpha = .05$. The results of this analysis are found in Table 4.3. As seen in the table, there was no significant difference between the mean difference scores in probability for Groups 1 and 2.

Table 4.3 Descriptive Statistics for Probability

| t-Test: Two-Sample Assuming Equal Variances Pre-test / Post-test Difference for Probability | | |
|---|------------------------------|------------------------------|
| | <i>Treatment Probability</i> | <i>Traditional (control)</i> |
| Mean | 4.5 | 9.5 |
| Variance | 43.66666667 | 80.33333333 |
| Observations | 4 | 4 |
| Hypothesized Mean Difference | 0 | |
| Df | 6 | |
| t Stat | -0.89802651 | |
| P(T<=t) one-tail | 0.201882952 | |
| t Critical one-tail | 1.943180274 | |
| P(T<=t) two-tail | 0.403765903 | |
| t Critical two-tail | 2.446911846 | |
| Standard Deviation (Treatment Probability Explorer) | 6.608075867 | |
| Standard Deviation (Traditional Probability) | 8.96288644 | |

DISCUSSION

According to Table 4.2, students did not benefit from using *Geometer's Sketchpad* to learn concepts about geometric transformations. There is an 11.5 point difference in the averages suggesting that traditional classroom methods used to teach transformations work

better than dynamic geometry software. However, if you look back at Table 4.1 you can see that one student's improvement of 55 points could easily skew the overall mean.

In geometry, the post-test scores of students who had Instructor B were, on average, 24 points higher than Instructor A. In probability, the post-test scores of students who had Instructor A were approximately, 2 points higher than Instruction B. (See Table 4.1.) From the data we can say that Instructor B was better at teaching the probability and geometry lesson objectives as evident by the mean difference scores in Tables 4.2 and 4.3.

On the other hand, we note that the students in Instructor A's classes had to learn more in the same time period according to Table 4.4. The students had to learn how to use the computer software, thus giving them less time to concentrate on the learning objectives. Undoubtedly, this represents a limitation of the study.

Table 4.4 Topics of Instruction

| Topics covered | |
|---|---|
| <i>Instructor A</i> | <i>Instructor B</i> |
| Learn software basics | Teach specific concepts |
| Teach specific concepts | Ensure students are using traditional manipulates correctly |
| Teach software basics for that specific concept (repeated for each new concept) | Guided Practice Time for Students with Instructor |
| Ensure students using software correctly | |
| Guided Practice Time for Students with Instructor | |

The t-test was used to analyze the data for the geometry pre-test/post-test difference scores.

The hypothesis was that the geometry software treatment scores would be the same as traditional treatment scores with p-value equal to .05. The t-test yielded a one-tailed p-value of 0.25. Therefore, we conclude that no significant difference exist between the geometry software treatment scores and traditional treatment scores.

The t-test was also used to analyze the data for the probability difference scores. The t-test yielded a one-tailed, p-value of 0.20. Therefore, we conclude that no significant difference exist between the probability software treatment scores and traditional treatment scores.

CONCLUSION

The purpose of this study was to find evidence that suggest that using computer software would make a difference in students learning geometry and probability concepts. The research study was conducted by having students separated into two groups with one group as the traditional (control) and the other as the treatment group. The students took pre and post-tests on the content of the study. The scores from these tests were analyzed using quantitative methods.

The statistics from the geometry part of the study suggested that students in the traditional classroom environment outperformed those in the computer software oriented classroom. Although the use of software did not result in students performing better than their counterparts, it is believed, based on post-interviews that the computer-oriented classrooms did have an impact on how the students felt about the subject matter. The students were more motivated to complete task because they were interested in using the computer to do so. Still, it can be noted that there was no significant difference between the software treatment group and the traditional group.

The second part of the research study looked at probability concepts. The post-test scores do not have evidence that suggest students scored better when they were able to use the probability software. The statistical analysis suggests that there is not a significant difference in the post-test in either case. It cannot be concluded that students who learned

probability using the software as a whole did better than students who did not. Students who used the software did not do better we feel because they had to learn how to use the software in a small window of time. The software allowed the students to build tables of compiled data, which seemed to enhance student understanding of related probability concepts but the data did not reflect that. By constructing examples the students were able to explore learning probability in different ways that could be more meaningful to them. It was also noted that a teacher effect may exist, however it was beyond the scope of this study.

The data from this study suggest that a teacher effect may be present. Instructor B had better overall results than Instructor A. Any future study must account for instructional differences pursuant to the teachers participating.

Lastly, it appears that using computer software to help students learn geometric and probability concepts is effective as a motivational tool as conclude from the post-interviews. We found that students enjoy using computer programs during instruction and there is evidence that students are actively engaged as a result of using computer software to complete classroom tasks. Unexpectedly, we found no evidence to infer that software use in geometry and probability helps students to learn geometry and probability concepts better or faster than by traditional methods.

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APPENDIX A

Research Plan

Week 1 (June 19- June 23) (Classroom Instruction)

Day 1: Pre-test and Introduction to the basic tools, menus, and commands for using the software program The Geometer's Sketchpad. Constructing basic geometrical figures (squares, rectangles, parallelograms, triangles (isosceles, right)).

Introduction to vectors and **translations**

Solve problems such as: Find the image or pre-image of a figure under a translation. Given the pre-image and image determine the translation vector that was used.

Day 2: Investigating the compositions of two or **translations**. Introduction to tessellations.

Introduction to rotations - Center and angle of **rotation** and properties that are preserved by rotations. Solve problems such as:

Day 3: Investigating the composition of two or more **rotations**.

Introduction to **reflections**

Solve problems such as: Find the image or pre-image of a figure under a particular reflection. Determine the location of the mirror line given a pre-image and image.

Day 4: Investigating compositions of reflections. Use of **reflections** to solve “real world” problems (e.g., miniature golf GSP game)

Identifying unknown transformations using the GSP matching task (translations, reflections, rotations). Creating a Kaleidoscope

Day 5: Review Post test
Translations, rotations, reflections

Week 2 (June 26-29) (Classroom Instruction)

Day 1: Probability: Pre-Test, Introduction to *Probability Explorer*

Day 2: Probability: Guess My Marbles I, II & III with replacement activity

Day 3: Probability: The Fish activity with replacement activity

Day 4: Probability: Review and Post -Test

APPENDIX B

Interview Questions for Does technology make a difference project.

Pre-Interview

1. Do you like mathematics?
2. Do you like technology?
3. Do you have a computer at home?
4. Have you ever used a technology do school work?
5. Have you ever used technology to geometry of statistics?
6. Did at least one of your parents graduate from college?
7. Do you feel that the computer will help you to learn math?

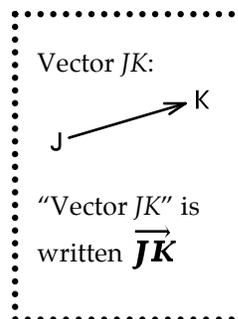
Post-Interview

1. Did you feel that the technology helped you to learn math?
2. Do you feel like you learn the math material?
3. Did you enjoy the activities that were given?
4. Do you like geometry and statistics after using the technology?

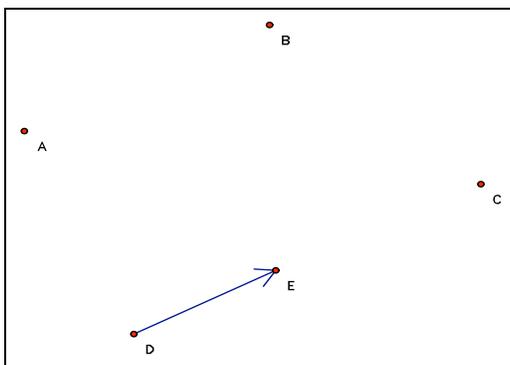
APPENDIX C

EXPLORATION I: Translations

A *translation* is a function that maps objects a specified distance in a specified direction. A way that mathematicians communicate a specified distance and a specified direction is through a *vector*. In drawings, a vector looks like an arrow in the intended direction and whose distance indicates the specified distance.

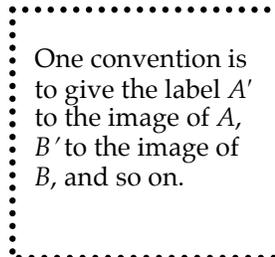


1. Open the GSP sketch, “Translate.” Create three points, and label them A, B, and C. Now, click the tool button , use the pull down menu to select vector and then click and drag to create a vector. Label it \overrightarrow{DE} . Your finished sketch might look something like this.



2. Select points D and E. Choose “Mark Vector” from the Transform menu. Select points A, B, and C and choose Translate from the Transform menu. The result should be a set of three new points configured just like the original ones, but in a new location.

Each new point is called the *image* of the



corresponding original point. Each original point is called the *pre-image* of the corresponding new point.

If your image points are not already labeled, use the Label tool to label them.

- 3a. Predict what you think will happen if you move the “head” (in some dynamic geometry tools, the “head” will appear as an arrow endpoint) of the vector and record your prediction below:

b. Try it out. What happens? Explain why.

- 4a. Predict what you think will happen if you move the “tail” of the vector and record your prediction below.

b. Try it out. What happens? Explain why.

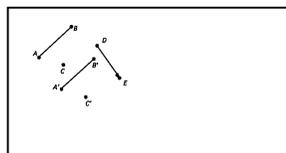
5. Using the selector tool, move the entire vector without changing its length or direction.

a. What happens? Explain why.

b. Move the entire vector so that its “tail” is at point C. Where is its head? Explain why.

- c. If the tail is at point A or point B, predict where the head of the vector will be in each of these cases.

6. Connect points A and B with a segment, and connect points A' and B' with a segment. Your sketch should look something like the one to the right. Now, move the entire vector so its tail, D , sits at a point somewhere along \overline{AB} . No matter which point along \overline{AB} you choose, its head then sits at a point on $\overline{A'B'}$. Why?



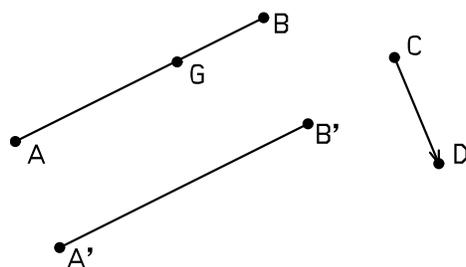
7. Drag point A and describe what happens. Why?

1

- b. What happens to $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ as you move the head of the vector? Explain why.

11. Using the same vector of translation, translate the *new* points (A' , B' , and C'). Label their images A'' , B'' , and C'' . Now move the head of the vector.
- Explain what happens to these newest points.
 - How far is A'' from A' ?
 - How far is A'' from A ?

12. This following picture shows the segment \overline{AB} and its image after a translation by \overrightarrow{CD} . Point G was marked afterwards. If you were to translate G by \overrightarrow{CD} , where do you think its image would land? Explain.



Translations¹Name _____
Date _____

1. Open a new sketch. Construct \overline{AB} . Using this segment and translations, construct a parallelogram. Use properties of translations to explain why your construction works.

2. Consider each of the following questions. Explain your answer.
 - a. Does a translation preserve lengths of segments?

 - b. Does a translation preserve areas of polygons?

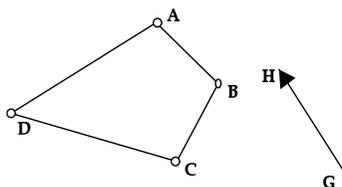
 - c. Does a translation preserve measures of angles?

 - d. Under any single translation, are any points mapped to themselves (that is, are there any fixed points)?

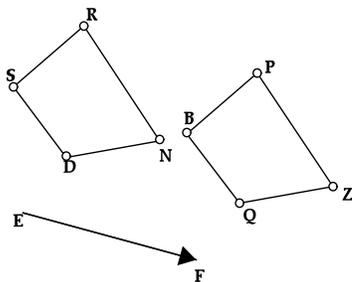
 - e. Does a translation map parallel lines to parallel lines?

¹ These activities are adapted from, *Technology-Intensive Secondary School Mathematics Curriculum*, produced by Heid, M.K., & Zbiek, R.M. (2004).

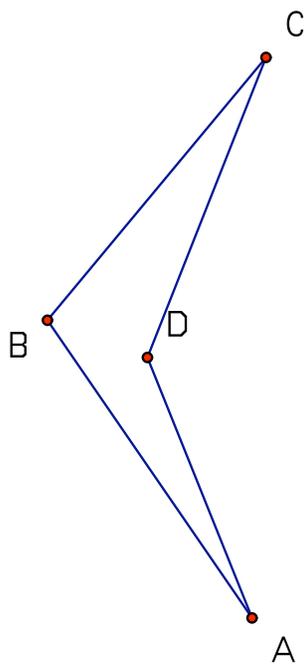
- f. Does a translation map perpendicular lines to perpendicular lines?
3. How is a vector similar to and different from a ray?
4. Sketch the pre-image of polygon ABCD where polygon ABCD is the image under translation by the vector GH.



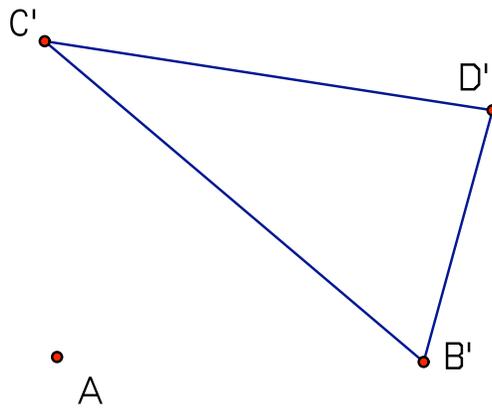
5. Using proper notation, describe the mapping that is shown below.



- f. Does a rotation map perpendicular lines to perpendicular lines?
- g. Does a rotation map collinear points to collinear points?
3. Sketch the image of polygon ABCD when it is rotated by 30 degrees about point A.

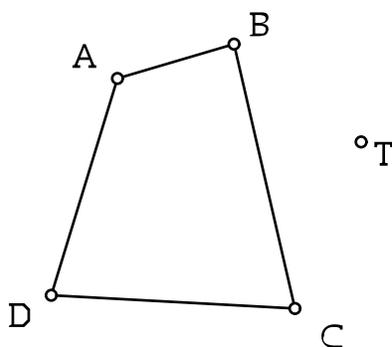


4. Sketch the pre-image of polygon $B'C'D'$ where polygon $B'C'D'$ is the image under a rotation about point A through an angle of 90° .

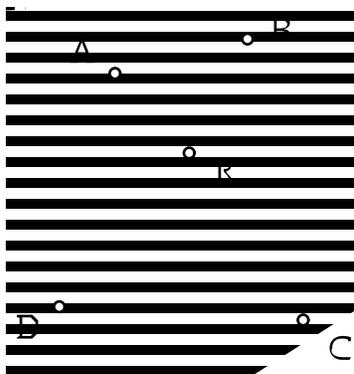


Rotations

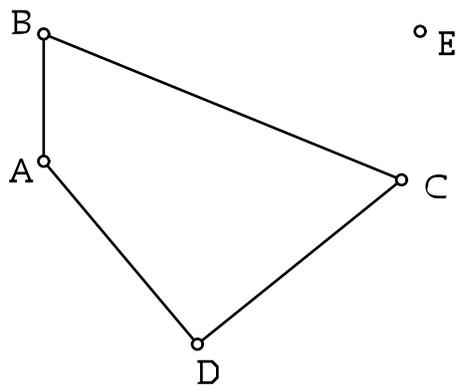
2. _____ S
sketch the image of polygon ABCD when it is rotated 130° about point T.



2. Sketch the image of polygon ABCD when it is rotated by 130° about point R.

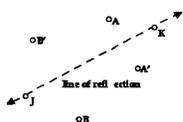


3. Sketch the pre-image of polygon ABCD where polygon ABCD is the image under a rotation about point E by 65° .



EXPLORATION V: Reflections²

A third type of transformation is a *reflection*. Reflecting is like producing a mirror image with a line of reflection as the mirror. A reflection, M_l , maps all points in the plane over the line l .



The letter M is a convenient reminder that this function is a reflection. Reflecting a figure is like producing a mirror image.

Identifying a reflection requires that you specify:

- The line of reflection.

In the figure above, M_{JK} has been applied to all of the points in the plane. Of course, it would be impossible to view what happens to every point so two selected points are labeled A and B . The images of points A and B under the reflection M_{JK} are labeled A' and B' , respectively.

We name this mapping the function M_{JK} .

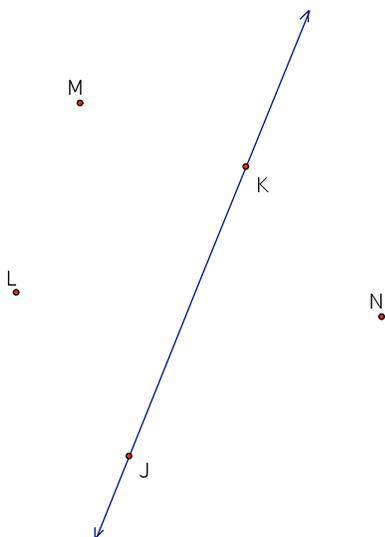
To say that M_{JK} maps point B to point B' , use the following shorthand notation:

$$M_{JK}(B) = B'$$

Read this equation as "The image of point B under the reflection over line JK is point B' ."

² These activities are adapted from, *Technology-Intensive Secondary School Mathematics Curriculum*, produced by Heid, M.K., & Zbiek, R.M. (2004).

1. Open a new GSP sketch. Click on the line tool and draw \overleftrightarrow{JK} . Click on the point tool and create three points, L , M , and N , not on the line. Your sketch should look similar to the picture that follows.



- a. Choose the selector arrow. Click on line JK and choose “Mark Mirror” from the Transform menu. Choose the selector arrow and click on points L, M, and N. Choose “Reflect” from the Transform menu. Observe what happens when you drag \overleftrightarrow{JK} and when you drag points L, M, and N. Describe how a point and its image under reflection are related.

- b. Choose the segment tool. Connect L and L' with a line segment. Do the same thing with points M and M' and with points N and N'. Mark and

label the points where the segments intersect \overline{JK} . Measure the distances from L to the intersection of segment LL' and line JK. Then measure the distance from L' to the intersection. Repeat these two steps for points M and M' , and for points N and N' . What do you notice about the distances?

2. Can the pre-image point and image point ever be on the same side of the line of reflection? Explain.
3. Each line segment joining a (*pre-image, image*) pair intersects the line of reflection. What can you say about the angles formed as these segments intersect the line of reflection? Why does this make sense?
4. Describe the relationship between \overline{JK} and $\overline{NN'}$.
5. Move a pre-image point so that it coincides with its image point. Where does this occur? At how many different locations does this occur? Explain.
6. Delete segments LL' , MM' , and NN' without deleting their endpoints. Create segments LM and $L'M'$. What do you notice about these two segments?
7. Create Triangles LMN and $L'M'N'$. What is true about these two triangles?

APPENDIX D**GUESS MY MARBLES I**

There is a Bag containing 10 Marbles. The marbles could be any of six colors: white, black, red, green, blue, and yellow.

We are able to take a sample from this bag by choosing one marble at a time with replacement—meaning each time we pull a marble out and record its color, we put it back in the bag.

You should open the file called *10marbles.pbe* in Probability Explorer. Use the run button to simulate choosing a marble from the bag.

Goal: Predict how many marbles of each color are in the bag based on results from sampling.

What strategy did you use? Why?

What is your best prediction for what is in the bag? Why is this your best prediction?

GUESS MY MARBLES II

There is a Bag containing 100 Marbles. The marbles could be any of six colors: white, black, red, green, blue, and yellow.

We are able to take a sample from this bag by choosing one marble at a time with replacement—meaning each time we pull a marble out and record its color, we put it back in the bag.

You should open the file called *100marbles.pbe* in Probability Explorer. Use the run button to simulate choosing a marble from the bag.

Goal: Predict how many marbles of each color are in the bag based on results from sampling.

What strategy did you use? Why?

What is your best prediction for what is in the bag? Why is this your best prediction?

How was your data collection and reasoning in making a prediction similar and different when you knew the bag had 10 marbles or 100 marbles?

Guess My Marbles III

DIRECTIONS:

Design a bag of marbles with 12 marbles, with either 2, 3, or 4 colors of your choice. Do not show your bag of marbles to any other student. HIDE the Weight Tool and the Bag of Marbles (under the OPTIONS menu).

SWITCH COMPUTERS with a partner. Now you must use the tools in Probability Explorer to try to predict what is in your partner's bag of marbles.

2. Based on data collected what is your BEST prediction for what is in the bag?

2. Why is this your best prediction? Explain your reasoning for your prediction, including what evidence you have from data collected in Probability Explorer.

The Fish Study

Scientists are studying how fish population in particular lakes change over time. To conduct their study, they populated each of the four lakes with the same number and types of fish. They are interested in knowing about the growth of fish in the lakes. Specifically, they asked us to investigate whether there is evidence that the distribution of fish in the four lakes is still the same now that some time has passed.

Your job is to study and describe the distribution of fish in the four lakes and provide a convincing argument for your claim.

Then we need to determine how the lakes are alike or different and support our claim.

The Lake files needed for your study are:

LakeAlta.pbe

LakeBeta.pbe

LakeDiamond.pbe

LakeGamma.pbe

Open each of these files in Probability Explorer and use the Run button to simulate catching fish from the lake. Remember that once a fish is caught and recorded, it is thrown back in the lake so that it can survive.

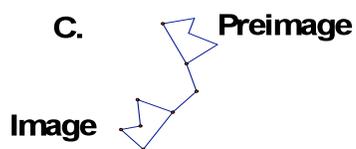
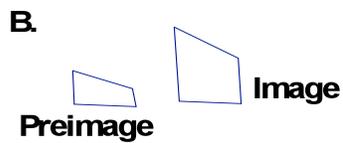
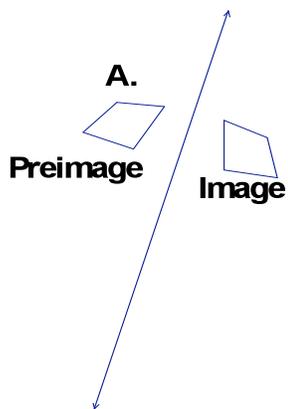
APPENDIX E**Pre- Test**

Transformation Concepts

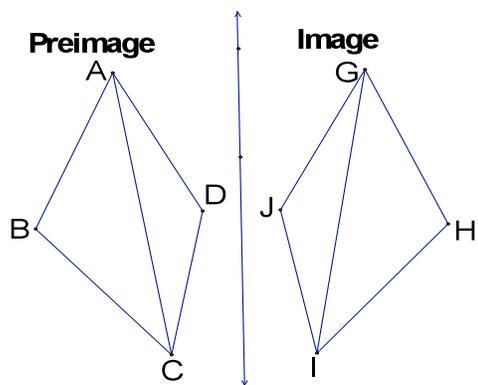
Name: _____

Teacher: _____

1. Which of the following are true about a transformation that has **isometry**?
 - a. it preserves length
 - b. it preserves angle measurements
 - c. it preserves parallel lines
 - d. it preserves distance between points
 - e. all of the above
2. Which of the following figures appear to be an isometry? (circle each one)



Answer questions 3-5 using the figure below.

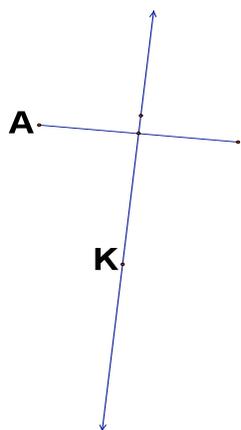


3. Name the preimage of GH _____
4. Name the image of DC _____
5. Name triangle that appears to be congruent to ADC _____

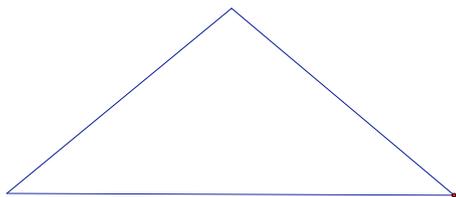
Name: _____

Teacher: _____

6. Circle the **line of reflection**?



7. How many lines of symmetry does this triangle have?

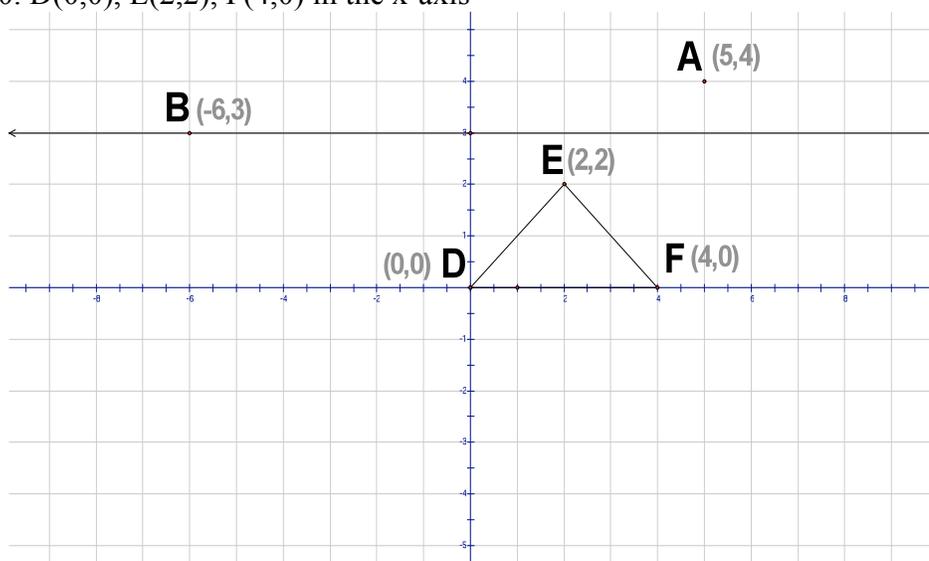


Name: _____

Teacher: _____

Using the graph paper provided, graph the reflection of each

8. $A(5,4)$ in the x-axis
9. $B(-2,3)$ in the line $y=3$
10. $D(0,0)$, $E(2,2)$, $F(4,0)$ in the x-axis



11. Rays drawn from the center of rotation to a point and its image form an angle called?

- a. rotation b. angle of rotation c. center of rotation d. none of these

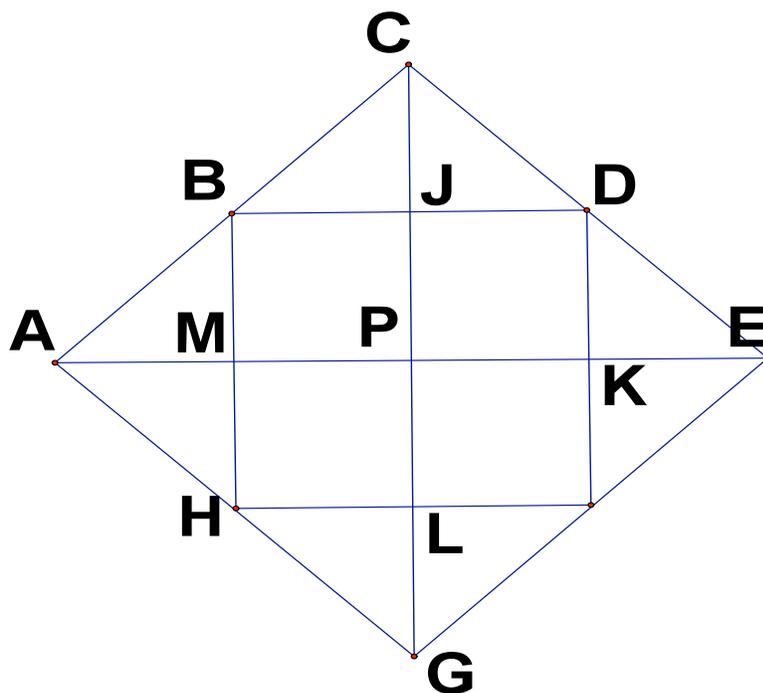
12. Which figure does not have rotational symmetry?

- a. trapezoid b. parallelogram c. regular octagon d. none of these

Name: _____

Teacher: _____

USE THE FIGURE TO ANSWER QUESTIONS 13-15

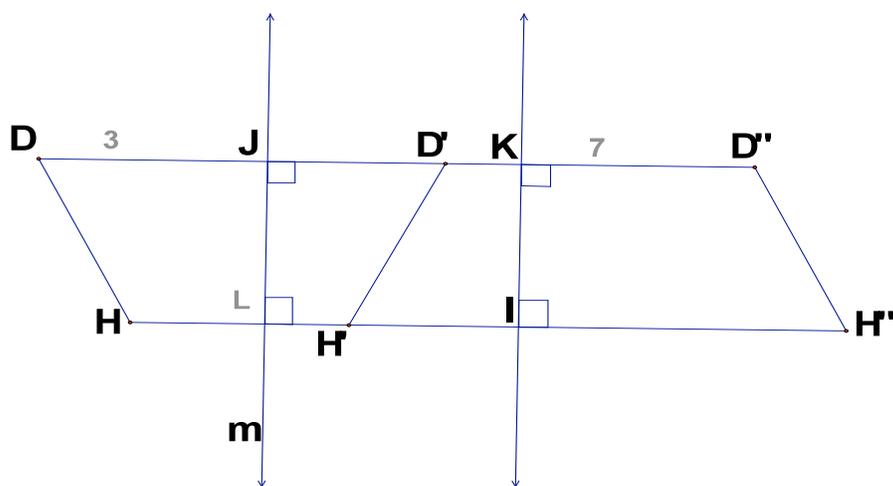


13. 90 degrees clockwise rotation of AB about P _____
14. 90 degrees clockwise rotation of AB about P _____
15. 180 degrees rotation of KEF about P _____
16. True or false: A translation is an isometry? _____

Name: _____

Teacher: _____

USE FIGURE TO ANSWER QUESTIONS 17-18



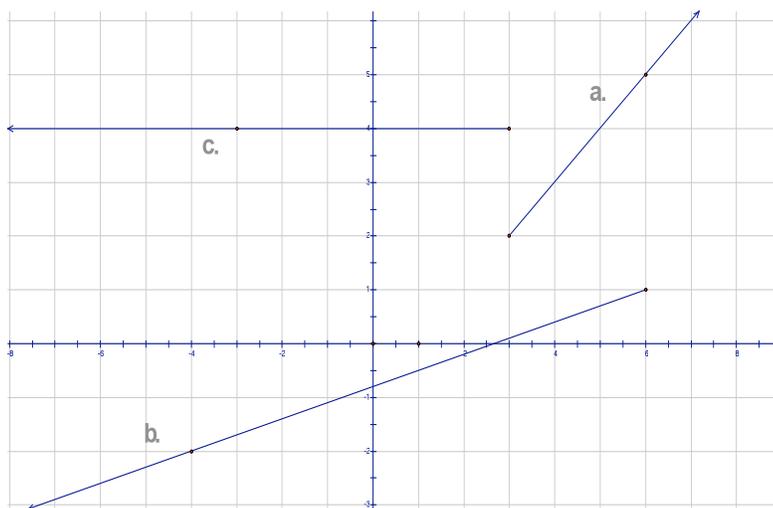
17. Name some congruent segments _____

18. Does $LI = JK$? Explain

Name: _____

Teacher: _____

USE THE GRAPH TO ANSWER QUESTIONS 19-20



Match the vector components (Approximate)

18. Find vector $\langle -11, 0 \rangle$ a. b. c.19. Find vector $\langle 4, 4 \rangle$ a. b. c.20. Find vector $\langle -14, -4 \rangle$ a. b. c.

Appendix F

Probability Pre/Post Test

Use the narrative for problems 1-4

You have a jar that has 18 marbles in it. Five of the marbles are blue, four are orange, and nine are purple. *Find the probabilities of picking each color if you assume that each marble has an equal chance of being picked from the jar.*

1. P (purple) a. $\frac{2}{9}$ b. $\frac{1}{2}$ c. 0 d. $\frac{5}{18}$
2. P (orange) a. 0 b. $\frac{2}{9}$ c. $\frac{5}{18}$ d. $\frac{1}{2}$
3. P (blue) a. $\frac{5}{18}$ b. 0 c. $\frac{1}{2}$ d. $\frac{2}{9}$
4. P (red) a. $\frac{1}{2}$ b. $\frac{5}{18}$ c. $\frac{2}{9}$ d. 0

Use the narrative for problems 4-14

Suppose that you toss a coin and roll a six-sided die for the following problems.

5. What is the probability of obtaining tails and a five? a. $\frac{1}{12}$ b. $\frac{7}{12}$ c. $\frac{1}{2}$
6. What is the probability of obtaining tails or five? a. $\frac{1}{12}$ b. $\frac{7}{12}$ c. $\frac{1}{2}$
7. What is the probability of obtaining heads and a two? a. $\frac{1}{12}$ b. $\frac{7}{12}$ c. $\frac{1}{2}$
8. What is the probability of obtaining tails? a. $\frac{2}{3}$ b. $\frac{1}{2}$ c. $\frac{7}{12}$
9. What is the probability of obtaining one, two, three, or four? a. $\frac{2}{3}$ b. $\frac{1}{2}$ c. $\frac{7}{12}$
10. What is the probability of obtaining heads or a two? a. $\frac{2}{3}$ b. $\frac{1}{2}$ c. $\frac{7}{12}$
11. What is the probability of obtaining heads and an odd number? a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$

12. What is the probability of obtaining heads or an odd number? a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$

13. What are the probability of obtaining heads and a five? a. $\frac{7}{12}$ b. $\frac{1}{4}$ c. $\frac{1}{12}$

14. What is the probability of obtaining heads or a five? a. $\frac{7}{12}$ b. $\frac{1}{4}$ c. $\frac{1}{12}$

Use the narrative for problems 14-20

A single card is selected from an ordinary deck of cards. Find the probabilities for the following.

15. P (five of hearts) a. $\frac{1}{13}$ b. $\frac{5}{13}$ c. $\frac{1}{52}$ d. $\frac{1}{4}$

16. P (Five) a. $\frac{1}{13}$ b. $\frac{5}{13}$ c. $\frac{1}{52}$ d. $\frac{1}{4}$

17. P (even number) a. $\frac{1}{13}$ b. $\frac{5}{13}$ c. $\frac{1}{52}$ d. $\frac{1}{4}$

18. P (heart and a jack) a. $\frac{4}{13}$ b. $\frac{5}{13}$ c. $\frac{1}{52}$ d. $\frac{1}{4}$

19. P (diamond) a. $\frac{4}{13}$ b. $\frac{5}{13}$ c. $\frac{1}{52}$ d. $\frac{1}{4}$

20. P (heart or jack) a. $\frac{4}{13}$ b. $\frac{5}{13}$ c. $\frac{1}{52}$ d. $\frac{1}{4}$