ABSTRACT

ZHANG, PU. Microstructure Generation of Asphalt Concrete and Lattice Modeling of Its Cracking Behavior under Low Temperature. (Under the Direction of Y. Richard Kim and Murthy N. Guddati)

Fatigue cracking has been pointed out as a major distress in asphalt concrete (AC) pavements. It is well known that cracking performance in AC mainly depends on the mechanical properties of its constituent materials, namely asphalt binder and aggregates. Study of such dependence is the key to effective characterization of the mechanical behavior of AC. Previous studies predicted AC behavior from the mixture properties using extensive physical experiments. As an alternative approach to physical experiments, micromechanical modeling, which is composed of microstructure generation and numerical modeling, is introduced in this study.

Digital imaging processing (DIP) of physical specimens to generate microstructures is first investigated, followed by virtual fabrication, which makes use of the mix properties to virtually fabricate the specimen (or the cross section of specimen for 2D analysis), so that the appearance and mechanical behavior of the actual specimen can be simulated.

The resulting microstructure is then processed to obtain a lattice network that is expected to mimic the mechanical behavior of the AC specimen. Lattice modeling approximates a continuum by using a lattice, with each link representing an intact bond that can be broken at any time to create a microcrack. The cracking process is simulated by successive removal of failed links.
Due to the unrealistic computational cost of direct simulation, the multi-scale approach is adopted to perform microstructural analysis, which considers the effect of different-sized aggregates at different length scales. Such an approach reduces the computational cost significantly, while capturing the mechanical phenomena at various length scales.

The effectiveness of the proposed multi-scale modeling approach is then illustrated by modeling the cracking behavior of the uniaxial tension tests under –10°C.

In the end, the effects of surface energy are studied.
Microstructure Generation of Asphalt Concrete and
Lattice Modeling of its Cracking Behavior
under Low Temperature

by

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1 INTRODUCTION

1.1 Problem

Fatigue cracking, as shown in Figure 1.1, is a common distress in pavement. It occurs when shear and bending forces, mainly due to heavy traffic loading, create stresses that exceed the fracture strength (1).

Figure 1.1 Pavement with Fatigue Cracking Surface

Many researchers have proposed analytical approaches to study the fatigue behavior of asphalt concrete pavement (2~7). At present, the continuum approach is widely applied to asphalt concrete analysis under general types of loads. In particular, the viscoelastic continuum damage model (VCDM), proposed by Kim et al. (4,5), appears to be effective in simulating fatigue damage in asphalt concrete in an effective manner.

VCDM is a phenomenological model that considers the viscoelasticity and damage caused by cracking in an asphalt-aggregate mixture. Through carefully designed
material and damage parameters, VCDM successfully captures the macroscopic fatigue behavior of an asphalt mixture under varying load and environmental conditions. However, these parameters are mixture specific, and it is necessary to perform a new set of experiments to obtain these parameters for each new mix (Figure 1.2). Considering the uncountable number of mixture designs and loading conditions, it is economically inefficient to understand and characterize the material properties solely by experimentation. The obvious question then is whether it is possible to obtain these parameters from component material properties instead of having to resort to expensive experiments for each new mixture design.

1.2 Objective and Methodologies

The objective of this thesis is to develop a procedure, which is able to link the component material properties to the fatigue properties of asphalt concrete mixture without lab experiments.

The methodologies adopted in this thesis are composed of two main parts (Figure 1.2): 1) procedures to obtain microstructure of material being simulated, and 2) numerical modeling to analyze the resulting microstructure.
In this thesis, digital imaging processing (DIP) is first introduced, followed by the development of a virtual fabrication procedure to generate microstructures. DIP techniques have been widely used in civil engineering (8). Macari et al. (9) applied DIP in triaxial tests to measure the volume changes of soil specimens. Terribile (10) applied
DIP to describe soil thin-sections. Dare et al. (11) used DIP to measure the evolution of cracks in concrete structures. Sahu (12) combined Raman spectroscopy with optical microscopy and digital imaging to identify thaumasite in concrete. DIP developed in the study is described in Chapter 2. Noting that DIP requires fabrication of physical specimens that could be expensive, an alternate approach of microstructure generation, namely virtual fabrication, is proposed. Virtual fabrication refers to making use of the mix properties to virtually fabricate the specimen that is expected to mimic the actual specimen in its appearance, and more importantly in its mechanical behavior. The development and evaluation of virtual fabrication is described in Chapter 3.

As for numerical modeling, many researchers have adopted different models to study the fatigue cracking behavior of an asphalt mixture. Birgisson (13) used the boundary element method (BEM) to predict viscoelastic response and crack growth in asphalt mixtures. Buttlar (14) simulated the cracking behavior of IDT test (indirect tensile test) by the discrete element method (DEM), while Sad (15) used the finite element method (FEM) to simulate IDT. In this thesis, lattice modeling, proposed by Feng (16), in which a material is discretized as a lattice consisting of small trusses that can transfer forces, is adopted to study the fatigue cracking of asphalt concrete.

Combining virtual fabrication with lattice modeling through the idea of multi-scale analysis, a seamless procedure, named multi-scale virtual fabrication and lattice modeling (MSVF-LM), is then proposed to study the fatigue cracking behavior of asphalt concrete. The final goal of this thesis is to implement MSVF-LM into a seamless software tool and apply the resulting tool to realistic HMA specimens to illustrate its performance evaluation capability.
1.3 Outline

Seven Chapters are included in this thesis. Chapter 1 describes the significance and objective of this thesis and the adopted methodologies. Chapter 2 gives some basic concepts of digital imaging process, and illustrates the DIP procedure in detail. In Chapter 3, the virtual fabrication procedure is developed by introducing inverse stereology and studying aspect ratio and orientation distributions, and is then evaluated with the help of numerical simulation. Chapter 4 first introduces the background of lattice, followed by the procedure of generating lattice mesh of asphalt concrete. Lattice modeling, which is developed by Feng (16), is also described briefly in this chapter. In Chapter 5, the computational cost of lattice modeling is first studied, followed by introduction of multi-scale approach. Then, the combination of virtual fabrication and lattice modeling through multi-scale approach, namely multi-scale virtual fabrication and lattice modeling (MSVF-LM) procedure is developed. Chapter 6 gives the applications of the MSVF-LM through uniaxial tension tests (UTT) simulation. Chapter 7 studies the effects of surface energy. Chapter 8 concludes the thesis.
2 DIGITAL IMAGING PROCESSING (DIP)

2.1 Introduction

Asphalt concrete is a composite material consisting of interspersed aggregate particles, asphalt binder, and air voids. Their constitutive behavior is defined by the interaction of these constituents. Studying the complex constitutive behavior of asphalt concrete requires accurate representation of its microstructure for subsequent modeling of the complex interaction among its constituents.

Studying the complex constitutive behavior of asphalt concrete requires accurately capturing its microstructure for subsequently modeling the complex interaction among its constituents. Over past few decades, digital imaging techniques have been widely used in the study of asphalt concrete, providing a valuable insight into its microstructure. Eriksen and Wegan (17) conducted microscopic analysis of air voids in asphalt concrete. Yue et al. (18) applied an innovative digital image processing technique to quantify the distribution, orientation, and shape of coarse aggregates. Masad et al. (19,20) used an X-ray tomography technique to analyze images of asphalt mixtures for the purpose of defining aggregate orientation, contacts, and air void distribution.

In this chapter, a digital imaging procedure (DIP) is developed to capture microstructure from cut surfaces of actual specimens.

2.2 Digital Image Definition

A digital image is a two-dimensional array of values representing light intensity. Unlike the traditional coordinate system, in which the origin (0,0) is located at the bottom
left corner, the spatial reference of the pixel (abbreviation of picture element) with the origin (0, 0) is located at the top, left corner of the image. Figure 2.1 shows that an image is a function of the light intensity $f(x, y)$, where $f$ is the brightness of the point $(x, y)$, and $x$ and $y$ represent the spatial coordinates of a pixel. The value of $x$ increases moving from left to right, and the value of $y$ increases from top to bottom.

![Figure 2.1 Spatial Reference of the (0, 0) Pixel](image)

In digital image acquisition, the image of the specimen cut surface is converted into a discrete number of pixels. Each pixel is then assigned a numeric location and a GIV (varying from 0 to 255) that specifies the brightness of the pixel.

### 2.3 DIP Procedure

The DIP procedure for this study essentially involves three steps: (a) digital image acquisition; (b) image processing; (c) data file generation for lattice modeling. (Lattice modeling is introduced in Chapter 4). Figure 2.2 illustrates the DIP procedure very clearly.

First, a cleanly cut surface of the specimen (Figure 2.2a) is scanned by a high-resolution scanner into an 8-bit grayscale digital image (Figure 2.2b), which consists of a single plane of pixels. Each pixel is encoded using a single number representing grayscale values from 0 to 255. Image-analysis software, IMAQ Vision Builder (21), from National Instruments is then used to adjust the brightness and contrast of the
original image, so that most aggregate particles can be isolated from the background of
the binder. After image processing, a black-and-white image (Figure 2.2c) is obtained
and an image file with BMP format is output. This image file cannot be used directly for
the generation of a lattice mesh (Chapter 4). It has to be converted into a data file (Figure
2.2d) with ‘0’ representing a binder pixel and ‘1’ representing an aggregate particle pixel.
The details of these steps are described in the following sections.

![Figure 2.2 DIP Procedure of Actual Asphalt Concrete Specimen: (a) Actual Specimen; (b) Raw Image (gray scale); (c) Processed Image (B/W BMP); (d) Data File.](image)

2.3.1 Digital Image Acquisition

In order to acquire good quality digital images of specimens, an imaging system
with a high resolution is needed. The imaging system should be able to produce images
with high enough quality so that all needed information can be extracted from the images.
The most common imaging systems include digital camera, scanner, or optical camera
with scanner. Five factors contribute to overall image quality, i.e., resolution, contrast,
depth of field, perspective, and distortion (21). The last three factors are mainly related to cameras that are not adopted in this study; therefore, only the first two factors are discussed here. For details about the last three factors, please refer to Appendix I.

Resolution in broad terms is an expression of the amount of information transmitted or received. For a scanner, resolution is the maximum number of pixels per inch. Resolution is measured in dots per inch (dpi). The required resolution of a scanner determined by the smallest feature to be inspected. For example, if the smallest aggregate particles with size of 0.15mm (Sieve size #100), i.e., 0.006in, needs to be recognized, the resolution of the scanner is at least $1/0.006=170dpi$. However, in order to capture the shape of an aggregate particle, more than 2 pixels are needed to represent it. If 3 are chosen, the resolution of the scanner should be $3 \times 170 = 510dpi$. In this study, 600 or larger resolution is chosen for digital image acquisition.

Contrast defines the differences in intensity values between the object under inspection and the background. Contract and resolution are closely related factors contributing to the image quality. Generally, a high-resolution scanner has enough contrast to distinguish objects from the background.

The quality of digital images is not only determined by the imaging system but also determined by the specimen itself. The cut surface of the specimen should be flat and clean so that the digital image can reflect every tiny detail of the specimen cut surface.

### 2.3.2 Image Processing

The main goal of image processing is to recognize aggregate particles, which means to isolate most aggregate particles from the background, binder. This work is so called thresholding in DIP, which can be carried out by image-analysis software. Picture
Publisher (18), Image Pro Plus (22) and IMAQ Vision Builder (21) are popular software used by different researchers.

IMAQ Vision Builder is developed by National Instruments Corporation, which has many functions to process digital images. After isolating aggregate particles from the background, it can analyze the orientation, aspect ratio, size, and other issues of aggregate particles, and then outputs all the information as an Excel file.

To carry out thresholding, i.e., recognize aggregate particles, a threshold of gray intensity value (GIV) should be set so that particles with higher GIV than the threshold can be isolated.

Thresholding consists of segmenting an image into two regions: a particle region and a background region. Particles are characterized by an intensity range. They are composed of pixels with GIV belonging to a given threshold interval. All other pixels are considered to be part of the background. The threshold interval is defined by two parameters, i.e., lower threshold and upper threshold. All pixels that have gray-level values equal to or greater than the lower threshold and equal to or smaller than the upper threshold are selected as pixels belonging to particles in the image. Thresholding works by setting to 1 all pixels that belong to the threshold interval, and setting all other pixels in the image to 0. Thus, a grayscale image, with GIV ranging from 0 to 255 is converted into a binary image (Black and White), with GIV of 0 or 1.

In this study, only one GIV threshold needs to be chosen to distinguish light aggregate particles from dark binder, i.e., GIV smaller than the threshold indicates a binder pixel and is set as 0, while larger GIV indicates an aggregate particle pixel and is set as 1.
It should be mentioned that choosing the appropriate GRV threshold is a longstanding problem in image analysis of engineering materials and especially for asphalt concrete. The difficulty stems from the fact that aggregate particles within the specimen have different colors depending on their mineral composition. Some aggregate particles have dark-gray color close to that of asphalt binder, which makes it difficult to distinguish these aggregate particles from asphalt automatically using the GRV. Currently, this problem still has to be solved manually by painting the dark particles white with the help of image manipulation software (18).

After image processing, an 8 bits uncompressed BMP file can be output by IMAQ. Although this kind of image file has the simplest format (23), it still cannot be used directly to generate lattice mesh until it is converted into a data file.

2.3.3 Data File Generation

(1) Digital Image File Formats

An image file is composed of a header followed by pixel values. Depending on the file format, the header contains image information about the horizontal and vertical resolution, pixel definition, and so on. The common image file formats includes Bitmap (BMP), Tagged image file format (TIFF), Portable network graphics (PNG), Joint Photographic Experts Group format (JPEG), and National Instruments internal image file format (AIPD). Since images typically require a large amount of data to hold them, usually over a megabyte of storage, most image file formats incorporate compression techniques, which make the file formats more complicated.

Since Windows BMP is the simplest image format, which is the native image format in
the Microsoft Windows operating systems and is rare to be in a compressed format, 8 bits uncompressed BMP format images are considered in this research. This kind of image format is illustrated in Figure 2.3:

<table>
<thead>
<tr>
<th>File Header</th>
<th>Bitmap-file header contains information about the type, size, and layout of a device-independent bitmap file.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image Header</td>
<td>Bitmap-image header specifies the dimensions, compression type, and color format for the bitmap.</td>
</tr>
<tr>
<td>Color Table</td>
<td>Color table is defined as an array containing as many elements as there are colors in the bitmap.</td>
</tr>
<tr>
<td>Pixel Data</td>
<td>Pixel Data is an array of bytes that defines the bitmap bits. These are the actual image data, represented by consecutive rows, or &quot;scan lines,&quot; of the bitmap. Each scan line consists of consecutive bytes representing GIV of the pixels in the scan line, in left-to-right order.</td>
</tr>
</tbody>
</table>

Figure 2.3 Bitmap File Structure

(2) Conversion of BMP File

Based on the BMP image file format, a program, namely BMP2DATA (BMP to Data Conversion Processor) is developed in Matlab (24), to get the size of the image (horizontal pixels number (HPN) × vertical pixels number (VPN)), and spatial positions of all the pixels.

The algorithm of BMP2DATA is very straightforward. After opening the BMP file, the width and height of the image, the starting position of the pixel data in the file can be obtained by reading the file header. Then, the file pointer is moved to the starting position, so that the pixel data can be read consecutively. Since the rows of pixel data are stored upside down, which means that the uppermost pixel row stored in the bitmap file appears as the lowest row on the screen, the data should be output in reverse.
The resulting output is a data file with HPN lines by VPN columns elements. The value of all elements is either 0 or 1, which corresponds to the pixels in the image with the color black or white.

2.4 Summary

In this chapter, definition of digital image is first introduced, followed by the description of the DIP procedure, which involves digital image acquisition, image processing by IMAQ, and data file generation. This procedure will be used to get the microstructures of actual fabricated specimen for the verification of the proposed virtual fabrication procedure in Chapter 3.
3 VIRTUAL FABRICATION

3.1 Introduction

The digital imaging techniques require physical fabrication of the specimens. That is, for each mixture to be evaluated, one must fabricate several specimens, digitally image them, and process the images so that they can be used in micro-mechanical simulation. In addition, an uncertainty always exists on whether those specimens are indeed representative of the mixture in question. As a result, a number of specimens need to be evaluated to obtain a statistically valid sample size. Such requirement of physical fabrication could be very expensive and time consuming. In order to reduce, and possibly eliminate the need for physical fabrication, an alternative to digital imaging, namely virtual fabrication, is introduced in this chapter.

Virtual fabrication refers to making use of the mix design information to virtually fabricate the specimen (or the cross-section of specimen for 2D analysis) and is expected to mimic the actual specimen in its appearance and more importantly the mechanical behavior of the specimen. Such an approach starts with inverse stereology (Section 3.2) to convert the volumetric aggregate gradation into 2D apparent gradation. Two important geometrical parameters, i.e., aspect ratio and orientation of aggregate particles, are obtained from statistical analysis of actual specimens and applied to virtual structure generation (Section 3.3). Section 3.4 summarizes the algorithms for virtual fabrication, and the validity of the procedure is then evaluated with the help of DIP and numerical modeling in Section 3.5. For the purpose of eliminating the need of physically fabricating specimens, Section 3.6 studies the particles’ orientation distribution and the relationship
between actual particles and particles’ cut sections aspect ratio distributions.

3.2 Inverse Stereology

3.2.1 Definition of Inverse Stereology

Stereology (25) is the process of predicting the three-dimensional geometrical structure from two-dimensional information, namely geometry of cross-sections on several parallel planes. One of the most commonly used approaches to stereological problems is the so-called statistical-geometrical approach, which measures and classifies a large number of 2D figures or images to build up an actual picture of the average 3D structure. In an analogous manner, inverse stereology, as the name implies, reconstructs the 2D cross-sections based on given 3D information. In the current study, an inverse-stereology based method is developed to convert the 3D volumetric aggregate gradation into 2D apparent area gradation in a cut surface, as illustrated in Figure 3.1.

![Figure 3.1 2D Apparent Area Gradation through Inverse Stereology: (a) 3D Specimen; (b) 2D Cut Surfaces; (c) Volumetric Aggregate Gradation; (d) Apparent Area Gradation.](image-url)
3.2.2 **Inverse Stereology of Ellipsoid**

Here, in order to get the 2D apparent area distribution for ellipsoidal aggregate particles, a specific case of ellipsoid, i.e., sphere, is first studied. The solution for this case can be obtained directly from the analysis of a classical problem, namely the Corpuscle Problem (26). The Corpuscle Problem starts with the derivation of the relationship from the 3D gradation \( f(r) \) to the 2D gradation \( \phi(r) \) of an assembly of perfect spheres.

In the above \( r, r_0, \) and \( R \) are radius, mean radius, and maximum radius of the spherical aggregate particle, respectively. \( f(r) \) is the 3D actual aggregate distribution density, which can be obtained from the following relation.

\[
\Phi(r) = \int_0^r \phi(x) \, dx \quad (3-4)
\]

where \( \Phi(r) \) is the apparent area gradation, which is identical to 2D percent passing (as defined in Figure 3.1d).
Although Equations (3-1) and (3-4) are obtained from the analysis of sphere, they can also be applied to ellipsoid by substituting the radius of sphere, $r$, with geometric mean of the largest and smallest radii of ellipsoid (26).

The above result is used for the typical mixture design shown in Table 3.1 to obtain the 2D apparent area distributions as shown in Figure 3.2.

<table>
<thead>
<tr>
<th>Property</th>
<th>Design</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gradation 19.00mm</td>
<td>¾”</td>
<td>100</td>
</tr>
<tr>
<td>Gradation 12.5mm</td>
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<td>97</td>
</tr>
<tr>
<td>Gradation 9.5mm</td>
<td>3/8”</td>
<td>87</td>
</tr>
<tr>
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</tr>
<tr>
<td>Gradation 2.36mm</td>
<td>#8</td>
<td>35</td>
</tr>
<tr>
<td>Gradation 1.18mm</td>
<td>#16</td>
<td>21</td>
</tr>
<tr>
<td>Gradation 0.600mm</td>
<td>#30</td>
<td>13</td>
</tr>
<tr>
<td>Gradation 0.300mm</td>
<td>#50</td>
<td>9</td>
</tr>
<tr>
<td>Gradation 0.150mm</td>
<td>#100</td>
<td>8</td>
</tr>
<tr>
<td>Gradation 0.075mm</td>
<td>#200</td>
<td>6.1</td>
</tr>
<tr>
<td>Asphalt Content, %</td>
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<td>5.2</td>
</tr>
<tr>
<td>Air Voids, %</td>
<td></td>
<td>4.0</td>
</tr>
<tr>
<td>VMA, %</td>
<td></td>
<td>15.5</td>
</tr>
</tbody>
</table>
Figure 3.2 Conversion of 3D to 2D Aggregate Gradation through Ellipsoid Inverse Stereology: (a) Distribution Density of Volumetric (3D) and Apparent (2D) Aggregate Gradations; (b) Volumetric (3D) and Apparent (2D) Aggregate Gradations.
It can be clearly seen in Figure 3.2, that the percent passing for 2D is larger than that of 3D for the same sieve size. This means there are more fine aggregates in 2D cut sections, which can be explained in a simple way. Imagine a sphere is randomly cut by a horizontal plane, the probability is identical for getting a cut section with a different radius varying from 0 to $R$, which means there are greater chances to get a cut section with size less than $R$. Thus, more fine aggregates are obtained in 2D cut section.

However, this resulting statistical relation has some limitations. The most important one is that an ellipsoid cannot represent the angularity of actual aggregate particles, which causes an error in mechanical properties as large as 35% (See Section 3.5). Thus, a more general aggregate shape of convex polyhedrons is studied to approximate 2D apparent aggregate gradation as follows.

3.2.3 Inverse Stereology of Polyhedron

This study starts with the investigation of polyhedral aggregates with uniform characteristic size $R$ (Figure 3.3 and Figure 3.4), and then extends to the actual aggregate gradation (Figure 3.5). For the sake of simplicity, the effects of aspect ratio and orientation of particles are not considered at this time.
Figure 3.3 Polyhedral Aggregates with Uniform Size: (a) 3D Structure; (b) 2D Cut Surface.

If all the aggregates in Figure 3.3 are assumed uniformly distributed spatially in the specimen, the apparent area distribution in a cut surface (Figure 3.3b) can be obtained through an analysis on a single polyhedron, as shown in Figure 3.4. Imagine the polyhedron is randomly cut by a horizontal plane (Figure 3.4a). The probability is identical for getting a cut section with a different size varying from 0 to \( R \), which means the apparent area distribution density, \( \phi(r) \), is uniform (Figure 3.4b),

\[
\phi(r) = \begin{cases} 
\frac{1}{R} & r \leq R \\
0 & r > R 
\end{cases} \quad (3-5)
\]

The lower case \( r \) indicates the characteristic size of apparent area (i.e., the cross-section area of an aggregate cut surface), and the upper case \( R \) indicates the characteristic size of volumetric aggregate particle. Thus, the apparent area gradation, \( \Phi(r) \), can be obtained by integrating Equation (3-5),
Figure 3.4 A Polyhedral Aggregate Cut by Random Horizontal Plane: (a) A Polyhedron; (b) Distribution Density; (c) Apparent Area Gradation.

\[ \Phi(r) = \int_0^r \phi(x)dx \]  

(3–6)

Although the above equations are only applied to aggregate with uniform characteristic size, they can be easily extended to the more general case of aggregate with varying sizes (Figure 3.5).

Figure 3.5 Polyhedral Particles with Varying Sizes: (a) 3D Structure; (b) 2D Cut Surface.
Assume the aggregate size between any two serial sieve sizes $R_1$ and $R_2$ is uniformly distributed as shown Figure 3.6. Then the volumetric percentage of the aggregate with size between $R-dR/2$ and $R+dR/2$ with respect to the aggregates within this gradation ($R_1$~$R_2$) can be obtained from the following relationship:

$$F(R) = \frac{P_2 - P_1}{R_2 - R_1} dR$$  \hspace{1cm} (3–7)

![Figure 3.6 Volumetric Percentage of the Aggregate with Size of $R$: (a) Aggregate Gradation; (b) Aggregate Size Distribution Density](image)

By letting $dR$ approach zero, one may obtain the apparent area distribution density for aggregate of size $R$ by multiplying Equation (3-7) with Equation (3-5),

$$\phi(r) = \begin{cases} \frac{F(R)}{R} = \frac{P_2 - P_1}{R_2 - R_1} \frac{dR}{R} & r \leq R \\ 0 & r > R \end{cases}$$  \hspace{1cm} (3–8)

The apparent area distribution density for aggregate size varying from $R$ to $R_2$ may be obtained by integrating Equation (3-8),

$$\phi(r) = \int_{R}^{R_2} \frac{P_2 - P_1}{R_2 - R_1} \frac{dR}{R} = \frac{P_2 - P_1}{R_2 - R_1} \ln\left(\frac{R_2}{R}\right)$$  \hspace{1cm} (3–9)

Finally, the 2D apparent area gradation can be obtained by substituting Equation (3-9) into Equation (3-6) for each and every (volumetric) aggregate gradation between two
serial sieve sizes.

Similarly, The result from the analysis of polyhedral aggregates is also used for the typical mixture design shown in Table 3.1 to obtain the 2D apparent area distributions. The distributions are shown in Figure 3.7, which also illustrates that the percent passing for 2D is larger than that of 3D for the same sieve size. By comparing Figure 3.7 with Figure 3.2, one can see that there are more fine aggregates in 2D cut sections when angularity is considered. In Section 3.5, it is demonstrated that 2D apparent area distributions with the consideration of angularity is more reasonable.
3.2.4 Necessity of Inverse Stereology

It is necessary to explain why the inverse stereology has to be adopted. It is obvious that the aggregate gradation should be first taken into account to virtually fabricate a microstructure of HMA. However, how to use the volumetric aggregate gradation correctly to generate 2D HMA microstructures is not clear. Naturally, the first idea is to use it directly. Figure 3.8c gives a virtual HMA microstructure based on volumetric aggregate gradation. The detailed generation procedure is described in Section 3.4. For the sake of comparison, Figure 3.8a gives an image of a cut surface from an actual specimen that has the same volumetric aggregate gradation; and Figure 3.8b gives the microstructure obtained by IMAQ processing. Both b and c only show the aggregate particles larger than 2.36mm (Sieve size #8). One can see clearly that virtual
microstructure has many more large aggregate particles and larger particle density. This implies that volumetric aggregate gradation cannot be used directly to generate 2D virtual microstructures. A conversion of gradation from 3D to 2D, i.e., inverse stereology, has to be adopted.

Figure 3.8 Comparison between Actual & Virtual Microstructures with Same Volumetric Gradation: (a) Cut Surface from Actual Specimen; (b) Imaging Processed Microstructure from Cut Surface; (c) Virtual Microstructure

One should note that the inverse stereology approach only gives apparent area gradation. To generate the 2D aggregate configurations or microstructures, two additional parameters, i.e., aspect ratio and orientation distribution, are needed. However, these two parameters are dependent on several factors, such as particle shape, type of gyratory compactors, and cutting directions (27–29). Such dependence is not only complex but also highly mixture specific, and usually an imaging process can be employed to perform statistical analysis on actual cut surfaces to approximate their values.
In what follows, the definitions of aspect ratio and orientation are first introduced. A statistical curve fitting method is then used to obtain the distributions of these two parameters from the analysis of actual specimen cut surfaces.

3.3 Aspect Ratio and Orientation Distribution

3.3.1 Definitions of Aspect Ratio and Orientation

Aspect ratio is defined as the ratio between maximum intercept and mean perpendicular intercept, as shown in Figure 3.9.

\[
\text{Aspect Ratio} = \frac{\text{Max Intercept}}{\text{Mean Perpendicular Intercept}} \quad (3-11)
\]

\[
\text{Mean Perpendicular Intercept} = \frac{\text{Area of the Convex Hull of the Particle}}{\text{Max Intercept}} \quad (3-10)
\]

![Figure 3.9 Definition of Aspect Ratio](image-url)
*Orientation* is defined as the angle of the longest axis with respect to the horizontal axis, which ranges from 0° to 180° (Figure 3.10).

![Figure 3.10 Definition of Orientation](image)

### 3.3.2 Distributions of Aspect Ratio and Orientation

To approximate the aspect ratio and orientation distribution, a group of HMA specimen cut surfaces is scanned and analyzed using IMAQ. The specimens were cut from 150 mm diameter, 175 mm high gyratory plugs compacted by the Australian Superpave Gyratory Compactor to produce the prismatic geometry. The size of the specimens is 40 mm wide, 60 mm deep, and 150 mm high. The measured data are then processed by Hazen plotting position method (30), a statistical method to obtain the empirical cumulative distribution, which may be fitted by an appropriate distribution function. Equation (3-12) is what Hazen suggested to estimate the cumulative probability of a data point:

$$F_r(x_i) = P_r(X < x_i) = \frac{i - 0.5}{n}, \quad i = 1, 2, \ldots, n, \quad x_1 < x_2 < \ldots < x_n$$

(3–12)

where, Pr(X<xi) denotes the probability that the random value X will have values less than that of the sample xi.

As an example, Figure 3.11a and Figure 3.11b show the data obtained from five
cut surfaces from five HMA specimens made with the same mixture design as shown in Table 3.1. It is observed that aggregates orientation satisfies Beta distribution and aspect ratio satisfies Gamma distribution. The resulting distribution functions are then used to guide the microstructure generation.

Figure 3.11 Distribution of (a) Aggregates Orientation, and (b) Aspect Ratio

It should be pointed out that the method to obtain aspect ratio and orientation distributions in this section still requires actual specimens. In order to eliminate the need
for physical fabrication, Section 3.6 studies the particle orientation distribution and the relationship between actual particles and particle cut section aspect ratio distributions.

### 3.4 Virtual Structure Generation

As seen in the previous section, an inverse-stereology based approach has been successfully developed to convert the volumetric aggregate gradation into the 2D apparent area gradation. In addition, a statistical method is also adopted to obtain the distribution of aggregate orientation and aspect ratio in the cut surface. In this section, these two approaches are combined with a particle generation method (Figure 3.12) to form an automated virtual structure fabrication procedure.

#### 3.4.1 Particle generation algorithm

![Figure 3.12 Particle Generation Procedure: (a) Particle with Arbitrary Shape; (b) Particle with Adjusted Size; (c) Particle with Adjusted Shape; (d) Particle with Adjusted Orientation.](image)

$$X = X \cdot T^S$$

$$X = X \cdot T^O$$

$$X = X \cdot T^{AP}$$

---

29
First, a polygon (octagon) with an arbitrary shape is generated within the area between two serial sieves (shown as the gray area in Figure 3.12a). Coordinates of its vertices are stored in an 8 by 2 matrix $X$. Its size is then adjusted by the following equation:

$$X = X \cdot T^S = X \cdot \begin{bmatrix} I_m^t & 0 \\ I^o_m & I_m^t \\ 0 & I^o_m \end{bmatrix}$$

(3–13)

where $I_m^o$ is the characteristic size of the originally generated polygon, which is defined as the longest axis length of the polygon. $I_m^t$ is the characteristic size of the target polygon, which is a random number between two serial sieve sizes.

Then, the polygon’s shape is adjusted by equation (3-14):

$$X = X \cdot T^{ap} = X \cdot \begin{bmatrix} \alpha^t & 0 \\ \alpha^o & 1 \\ 0 & 1 \end{bmatrix}$$

(3–14)

where $\alpha^o$ is the aspect ratio of the original generated polygon. $\alpha^t$ is the aspect ratio of the target polygon, which is a random number according to the aspect ratio distribution function.

Finally, the polygon’s orientation is adjusted by equation (3-15):

$$X = X \cdot T^\theta = X \cdot \begin{bmatrix} \cos(\theta^t - \theta^o) & \sin(\theta^t - \theta^o) \\ -\sin(\theta^t - \theta^o) & \cos(\theta^t - \theta^o) \end{bmatrix}$$

(3–15)

where $\theta^o$ is the orientation angle of the original generated polygon. $\theta^t$ is the one of the target polygon, which is a random number according to the orientation distribution function.

The two distribution functions may be obtained either from the imaging analysis of actual specimen cut surfaces or from an approximate relationship, which is described
in Section 3.6.

3.4.2 **Virtual Structure Generation Algorithm**

The resulting aggregate particle is then randomly placed into the area of the specimen. A check for possible overlaps of the particles is made, and the generated particle is accepted only if there are no overlaps with other previously generated particles inside the specimen. Starting with the largest aggregate, the above procedure recursively proceeds until the apparent area percentage for each and every gradation is reached to form a complete virtual microstructure. This procedure can be illustrated by the following flow chart, Figure 3.13.
3.4.3 Binder Percentage in 2D Cut Sections

A study of quantitative stereology shows that the apparent area fraction determined on cut surfaces can be statistically represented by the actual volume fraction (25). Thus, in the current study, the percentages of aggregate and binder in 2D virtual microstructures are assumed to be the same as those in the volumetric mixture design.
3.4.4 MATLAB Coding

The procedure and algorithm presented above are implemented into a program written in MATLAB, namely ACVF (Asphalt Concrete Virtual Fabrication). Figure 3.14 shows the interface of this program. By inputting the asphalt content, virtual structure size, resolution of the structure, and the 2D apparent gradation obtained from inverse stereology, ACVF can generate the virtual structure graphically and the image data file.

![Figure 3.14 Interface of ACVF](image)

3.5 Application and Verification

The effectiveness of the above virtual fabrication procedure is illustrated in this section with the help of numerical analysis and visual observations. Figure 3.15b shows a virtual microstructure with the 2D apparent gradation given in Figure 3.7b (inverse stereology analysis of polyhedron), while Figure 3.15c shows a virtual microstructure
with the 2D apparent gradation given in Figure 3.2b (inverse stereology analysis of ellipsoid). For the sake of comparison, Figure 3.15a also gives the image processed by IMAQ. In these three images, all the aggregates’ size is larger than 2.36 mm. From Figure 3.15, it can be clearly seen that the micro-structural geometry from the virtually fabricated specimen, which is based on the inverse stereology analysis of the polyhedron, visually matches well with the actual cut surface observed from the specimen. Quantitatively, there are 284 and 281 particles in virtual and actual cut faces, respectively, and the difference of the total aggregate areas between them is about 9%. The other one, however, based on the inverse stereology analysis of ellipsoid, looks denser, in addition to having unrealistic aggregate shapes.

However, the more important criterion to evaluate the quality of virtual fabrication is to verify if the virtual microstructure exhibits realistic mechanistic properties. In the following, uniaxial tensile tests and single edge notch (SEN) tests (Figure 3.16) are used to investigate the mechanical behavior of the virtual microstructures and compared with actual microstructures. Regarding the actual specimen-to-specimen variation and the statistical nature of virtual fabrication procedure, five actual and ten virtual microstructures are tested under the same conditions (material properties, specimen dimension, load conditions, etc.). Aggregates are modeled as rigid inclusions, and binder is modeled as elastic material.
Figure 3.15 Comparison of Actual and Virtual Microstructure: (a) DIP Processed Actual Microstructure; (b) Virtual Microstructure with Polygon; (c) Virtual Microstructure with Ellipse

Figure 3.16 (a) Uniaxial Tensile Test, and (b) Single Edge Notch Test

The results from the uniaxial tensile tests are presented in Table 3.2 in terms of top layer displacements under the static tensile force and the Poison’s ratio of the
microstructures, and also plotted in Figure 3.17 and Figure 3.18 to provide insight into the scatter of data from different realizations.

Table 3.2 Comparison of the Results between Actual and Virtual Microstructures

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Top Layer Displacement</th>
<th>Poison’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from Actual Microstructure</td>
<td>from Virtual Microstructure</td>
</tr>
<tr>
<td>1</td>
<td>1.787</td>
<td>2.161</td>
</tr>
<tr>
<td>2</td>
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<td>2.043</td>
</tr>
<tr>
<td>3</td>
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<td>4</td>
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<td>8</td>
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<tr>
<td>10</td>
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<tr>
<td>Average</td>
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</tr>
<tr>
<td>Difference</td>
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</tbody>
</table>
Figure 3.17 Probability Density Function (PDF) of Top Layer Displacement from Uniaxial Tensile Tests on Actual and Virtual Microstructures

Figure 3.18 Probability Density Function (PDF) of Poison’s Ratio for Actual and Virtual Microstructures
The probability density function (PDF) in Figure 3.17 and Figure 3.18 are the derivative of the probability distribution function. That is, the probability of a displacement between \((\delta, \delta + \Delta\delta)\) can be obtained from the integral of the PDF over \((\delta, \delta + \Delta\delta)\). The displacement corresponding to the peak value of the PDF is the mean value, and the area under the curve is always equal to 1. Generally, a flat curve represents a large deviation from the mean value or large scatter in data distribution. Thus, it can be seen that the mean values of the top layer displacement from both virtual and actual microstructures are close to each other. However, the virtual microstructures show smaller specimen-to-specimen variation. Similar observation can be seen in Poison’s Ratio distribution too.

In what follows, the virtual structures are further evaluated by simulating the fracture tests on SEN specimens. The notch, 7mm long and 3.5mm wide, is located in the middle of left side of the microstructures. All the microstructures are modeled as elastic material with the same mechanical properties (Young’s Modulus, Poison’s Ratio, surface energy, etc.). The peak values of load for the actual and virtual microstructures are listed in Table 3.3. The difference of average peak values of load between actual and virtual microstructures is only 5.0%. For further verification, the envelope of ten virtual microstructures’ load and deflection curves is plotted in Figure 3.19, along with the envelope for five actual microstructures.
Table 3.3 Comparison of the Results between Actual and Virtual Microstructures

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Peak Value of Load</th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>from Actual</td>
<td>from Virtual Microstructure</td>
</tr>
<tr>
<td></td>
<td>Microstructure</td>
<td>Microstructure</td>
</tr>
<tr>
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</tr>
<tr>
<td>Average</td>
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<td>4.10</td>
</tr>
</tbody>
</table>

Difference 5.0%

Figure 3.19 Comparison of Damage Behaviors between Actual and Virtual Microstructures
From Figure 3.19, one can clearly see that the envelope for the virtual specimens (Zone I) falls inside the envelope for actual specimens (Zone II). In summary, the findings from Figure 3.17–Figure 3.19 indicate that, from a statistical point of view, the mechanical behavior of the virtual microstructure is in agreement with that of the actual microstructure. It should be mentioned that the above calculations are carried out by lattice modeling, which will be presented in the following chapters.

From all the evaluations presented above, it can be concluded that the microstructure generated by the proposed virtual fabrication approach can mimic in a realistic manner the mechanical behavior of the material.

3.6 Study of Particle’s Aspect Ratio and Orientation Distribution for IDT Specimens

In the previous sections, the aggregate particle’s aspect ratio distribution and orientation distribution are obtained through digital imaging process of the actual specimen cut sections. The mechanical properties study presented in Section 3.5 shows that, with the proper aspect ratio and orientation distributions of particles, the generated virtual structures can give the satisfying simulation of actual asphalt concrete structure. However, this method still requires physically fabricating some actual specimens. In other words, this method can only reduce the amount of actual specimens, but cannot eliminate the specimen fabrication from the procedure. In order to develop a complete virtual testing procedure this section studies the particles’ orientation distribution and the relationship between actual and cut section particle aspect ratio distributions.

As mentioned in Section 3.3, these two parameters are dependent on several
factors, such as particle shape, type of gyratory compactors, and cutting directions. Since the IDT (Indirect Tensile) test utilizes relatively thin specimens that are more adequate for the two-dimensional lattice model developed in this study, all the studies in this section are based on the IDT specimens. Therefore, for a given gyratory compactor, the particle orientation in a specimen cut surface is mainly determined by the specimen cutting direction in relation to the compaction direction, i.e., the vertical or horizontal cutting directions (31). In this study, 26 different cut surfaces of IDT specimens with different kinds of aggregates and different mix design parameters (Table 3.4) are studied to get the orientation distribution. As for the aspect ratio distribution, it is not only determined by the cutting direction, but also by the particle shape. The relationship between actual particles and particles’ cut sections aspect ratio distributions is studied in this section. This relationship will allow the determination of the particles’ cut sections aspect ratio distribution from the actual particles aspect ratio distribution, which can be obtained by studying the actual particles.

3.6.1 Procedure of IDT Specimen Fabrication

The procedure of fabricating IDT specimens is mainly composed of the following steps:

(1) sieving, which is to separate aggregate particles into different sieve sizes;

(2) batching, which is to put all the aggregate particles together according to a certain aggregate gradation;

(3) mixing, which is to mix the aggregate particles with asphalt according to a certain mix design;

(4) compacting, which is to compact the mixture into a raw specimen. Superpave
Gyratory Compactor (600kPa ram pressure, 1.25° gyration angle and 30 gyr./min gyration speed) and a mold with 150 mm diameter are used to carry on compaction.

(5) sawing, which is to cut the top and bottom of the raw specimen so that a 38 mm thick, 150 mm diameter IDT specimen can be made.

3.6.2 Particle Orientation Distribution for IDT Specimens

In this study, three different kinds of aggregate are adopted. Correspondingly, four different mix designs are used to physically fabricate IDT specimens. Table 3.4 shows the mix design parameters for these three kinds of aggregate.
Table 3.4 Mix Design Parameters

<table>
<thead>
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<th>Aggregate Source</th>
<th>Holly Spring</th>
<th>Haw River</th>
<th>Garner Sand</th>
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</thead>
<tbody>
<tr>
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<td>Coarse</td>
<td>Fine</td>
<td>Coarse</td>
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<td>Mix Design Name</td>
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<td></td>
<td></td>
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<tr>
<td>Gradation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sieve #</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37.5mm</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>25mm</td>
<td>99</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>19mm</td>
<td>89</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>12.5mm</td>
<td>73</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>9.5mm</td>
<td>61</td>
<td>97</td>
<td>97</td>
</tr>
<tr>
<td>4.75mm</td>
<td>39</td>
<td>62</td>
<td>69</td>
</tr>
<tr>
<td>2.36mm</td>
<td>28</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>1.18mm</td>
<td>19</td>
<td>34</td>
<td>31</td>
</tr>
<tr>
<td>0.6mm</td>
<td>14</td>
<td>25</td>
<td>21</td>
</tr>
<tr>
<td>0.3mm</td>
<td>10</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>0.15mm</td>
<td>6</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>0.075mm</td>
<td>3.9</td>
<td>5.6</td>
<td>5.7</td>
</tr>
<tr>
<td>Asphalt Content, %</td>
<td>4.5</td>
<td>5.3</td>
<td>5.9</td>
</tr>
<tr>
<td>Air Voids, %</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>VMA, %</td>
<td>14.1</td>
<td>15.5</td>
<td>16.7</td>
</tr>
</tbody>
</table>

For each of HSC, HSF, and HR mix designs, three IDT specimens are fabricated, and for GS mix design, four IDT specimens are fabricated. Both sides of the specimen cut surfaces are scanned, and the resulting digital images are then processed through IMAQ. The particles’ orientation cumulative distributions (CDF) are then obtained by Hazen Plotting. Figure 3.20 shows the CDFs for these four different cases. From this figure, one can clearly see that the particle orientation distribution in an IDT specimen
cut surface satisfies uniform distribution, i.e., particles tend to orient each possible direction uniformly in a horizontally cutting section.
3.6.3 Particle Aspect Ratio Distribution for IDT Specimens

It can be observed that aggregate particles tend to orient horizontally during gyratory compaction, which has been reported by Kuo (31). Therefore, the aggregate particles in IDT specimens have more chances to be cut along the plane of its longest and
intermediate dimensions, as shown in Figure 3.21b.

Figure 3.21 Three-Dimensional Particle Schematic: (a) Particle Schematic; (b) \(d_l-d_i\) Plane; (c) \(d_l-d_s\) Plane.

Based on this observation, a DIP based procedure shown in Figure 3.22 is adopted here to get the particle aspect ratio distribution. In this procedure, aggregate particles are
first placed on the surface of a scanner. In order to protect the scanner, a transparency is placed on the surface of the scanner. All the particles are arranged so that their longest and intermediate axes are approximately parallel to the scanner surface. The obtained image is then processed by using IMAQ to get each particle’s aspect ratio. After all the particles’ aspect ratios are obtained, their cumulative distribution can be obtained by Hazen Plotting.

Kuo (32) used a similar method to analyze particles, in which a digital camera was used to take images of particles, instead of a scanner. The comparison of the image analysis and manual measurements in his study shows that the DIP based procedure is a good alternative to measure the dimensions of particles. As shown in Figure 3.23, the particles’ longest dimensions, measured by image analysis and manual measurements, are consistent with each other.

![Figure 3.23 Measurements of Longest Dimension d_l](Reprinted from reference 75)

Figure 3.24 shows the cumulative distributions of the three different kinds of aggregates particles, i.e., Holly Spring, Haw River and Garner Sand. For the purpose of comparison, Figure 3.24 also gives the cumulative distributions of the particle aspect
ratio obtained from IDT specimen cut surfaces, which are determined by the DIP procedure. By comparing these CDF curves, one can see that there is a shift of aspect ratio from the scanned particle shapes to the IDT cut sections. In other words, the aspect ratio distributions obtained from the analysis of actual particles cannot be used directly for virtual structure generation. For the purpose of simplification, only particles larger than 4.75 mm are studied.

Figure 3.25 plots all the data points of actual particles and particle cut sections’ aspect ratios with the same value of cumulative distribution for those three kinds of aggregates. Through linear regression, the relationship between these two aspect ratios can be obtained as follows:

$$AP_{2D} = 2.1497 \cdot AP_{3D} - 1.8304$$

(3–16)

where $AP_{2D}$ and $AP_{3D}$ are the aspect ratio from the IDT cut section and the aspect ratio from actual particles, respectively.
In order to verify the validity of this relationship, Figure 3.26 gives the aspect ratio CDFs of each IDT specimen cut surface, CDF of actual particles, and the fitting CDF by (3-15). It is very clear seen that the CDFs of different specimen cut surfaces have an obvious deviation band between each other and the fitting CDF is located inside the band. In other words, the relationship of (3-15) is good enough to approximate the CDF of specimen cut surface.

\[ y = 2.1497x - 1.8304 \]

\[ R^2 = 0.9803 \]
Figure 3.26 Aspect Ratio Cumulative Distributions for: (a) Holly Spring; (b) Haw River Sand; (c) Garner Sand
In order to verify the validity of the relationship further, the same procedure as used in Section 3.5 is adopted here.

Figure 3.27 (a) DIP Processed Image, and (b) Virtual Fabricated Structure (Holly Spring Fine)

Figure 3.27 shows a typical DIP processed image of an actual IDT specimen cut surface and a virtual structure from Holly Spring Fine mix design shown in Table 3.4. All the particles less than 4.75 mm are ignored. Since the largest particle size is less than 12.5 mm, a square patch with 75mm×75mm, which can be cut from the processed images or virtual structures, is used to verify the relationship (3-16). In this study, five patches are cut from five DIP processed images and five patches are cut from five virtual structures. Table 3.5 lists the top layer displacements of these patches under uniaxial tensile test and the peak values of load under the single notch test. The difference of the average displacements between actual and virtual cases is 7.8%, and the difference of the average peak load is 11.8%.
Table 3.5 Comparison of the Results between Actual and Virtual Microstructures

<table>
<thead>
<tr>
<th>Patch No.</th>
<th>Top Layer Displacement (Uniaxial Tensile Test)</th>
<th>Peak Load (Single Notch Test)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from Actual Microstructure</td>
<td>from Virtual Microstructure</td>
</tr>
<tr>
<td>1</td>
<td>4.861</td>
<td>4.121</td>
</tr>
<tr>
<td>2</td>
<td>4.737</td>
<td>5.139</td>
</tr>
<tr>
<td>3</td>
<td>4.876</td>
<td>4.224</td>
</tr>
<tr>
<td>4</td>
<td>4.764</td>
<td>4.629</td>
</tr>
<tr>
<td>5</td>
<td>4.896</td>
<td>4.275</td>
</tr>
<tr>
<td>Average</td>
<td>4.827</td>
<td>4.478</td>
</tr>
<tr>
<td>Difference</td>
<td>7.8%</td>
<td>11.8%</td>
</tr>
</tbody>
</table>

3.6.4 Discussion of Aspect Ratio for Vertically Cut Direction Sections

The relationship in Equation (3-15) is only suitable for simulations of horizontally cut specimens’ sections, for example, the IDT simulation. As for simulations of vertically cut specimens’ sections, like the uniaxial tension tests simulation, the aspect ratio of particles should be the ratio between the intermediate and shortest dimensions, since aggregate particles tend to orient horizontally during gyratory compaction. In order to get the shortest dimension of particles, Kuo (32) suggested the following method:

1. Aggregate particles are attached to the adhesive surface in sample trays with two perpendicular Plexiglas walls. Particles are arranged so that their longest axes are approximately perpendicular to the edge of the tray (Figure 3.28).
2. The aggregates (sample trays) are placed under a digital camera, and the projected image can be captured.
(3) The sample trays are then rotated 90 degrees so that the particles are now perpendicular to their original orientation. Double-sided adhesive tape attached to the inside of the initial base of the tray prevents the particles from moving during this process.

Therefore, by substituting a scanner for the digital camera, a similar method could be used to get the aspect ratio of particles. Then, the same procedure as described in Section 3.6.3 can be adopted to get the relationship between actual and cut section particles’ aspect ratio distributions for simulation of vertically cut specimens’ sections.

In the following chapters, due to lack of enough vertical cut specimens with different kinds of aggregate, the generated virtual microstructures for vertical cut specimen tests simulation are still based on the distributions (aspect ratio and orientation) obtained from DIP analysis.
3.7 Conclusion

A virtual fabrication procedure is proposed in this chapter to generate asphalt concrete microstructure directly from the constituents’ information instead of having to physically fabricate specimens. Two important geometrical parameters, i.e., aspect ratio and orientation distributions of aggregate particles for IDT specimen simulation, are also studied in this chapter. The validity of the proposed procedure is then proved with the help of DIP and numerical modeling.

It should be pointed out that the proposed virtual fabrication approach is an inexact approach, since in the inverse stereology analysis, all the particles have the same orientation and are assumed to be cut by a horizontal plan. This is not in agreement with the actual case. Even with the further improvement of introducing the aspect ratio and orientation distributions, this approach is still not an exact method. Although virtual fabrication is an approximate approach in nature, the microstructures generated by this approach are able to provide satisfying results; one can say that this virtual fabrication approach works.
4 LATTICE MODELING

4.1 Introduction

In many materials, such as metal, soil, concrete, as well as asphalt concrete, the cracking phenomenon is driven by the nucleation, propagation, and coalescence of microcracks. This fact indicates that a microlevel simulation of this cracking phenomenon may lead to better qualitative and quantitative understanding of the fatigue performance. Over the past few decades, several micromechanical-based methods have been proposed to study the cracking performance of materials (33~40). In this study, a lattice-based micromechanical model, which is proposed by Feng (16), is adopted to simulate the cracking performance of asphalt concrete.

4.2 Background of Lattice

In the lattice model, a material is discretized as a lattice or mesh consisting of small spring or beam elements that can transfer forces. For example, in a spring (or truss) lattice, elements are hinged together at nodes in the manner, which allows unrestrained relative rotations of individual links. Each link can transmit only axial forces. Microcracking of material is simulated by successively removing elements with stress/strain exceeding the allowable limit.

Lattice-based models have been used over the last 60 years at varying levels of complexity for different engineering applications. In the works of Hrennikoff (41), a regular triangular lattice of truss elements was introduced to solve classical elasticity problems. However, due to the lack of sufficient computational power, lattice-based
models have been restricted to theoretical analysis for a long time. In the late 1980s, lattice-based models were reintroduced by theoretical physicists for simulating cracking processes in heterogeneous materials (42,43). Later, Schlangen and Van Mier applied regular triangular lattice to simulate progressive failure in concrete (44–46). Bolander and Saito proposed another lattice approach to model brittle fracture in homogeneous, isotropic materials using spring networks with random geometry (47). Bhattacharya (48) proposed an equilateral triangular beam lattice to simulate the fracture process in a quasi-brittle material like concrete under direct tension.

All these studies show that the lattice-based model is well suited for the study of fracture processes in cement concrete. In this study, the lattice-based model is adopted and combined with virtual fabrication to characterize the cracking performance of asphalt concrete from its constituent material properties. The lattice-based model is adopted because both cement concrete and asphalt concrete are heterogeneous materials, exhibit similar strain-softening behaviors, and fail progressively by damage-localization and breakdown of their components; and Feng’s work (16) shows that the lattice-based micromechanical model can simulate the cracking performance of asphalt concrete effectively.

### 4.3 Lattice Mesh Generation for Asphalt Concrete

Asphalt concrete exhibits random structure and heterogeneity on the microscopic level. At this level of observation, with the omission of air voids, it is can be regarded as either two-phase (aggregate and asphalt binder) or three-phase (aggregate, asphalt binder and their interface) material. In this thesis, the two-phase system is adopted for the sake
of simplicity.

The asphalt concrete can be represented through two different lattice geometries, regular and random networks. A typical approach to model aggregate composite material with a regular lattice is shown in Figure 4.1.

![Figure 4.1](image)

Figure 4.1 (a) Particle overlay - internal structure, (b) projection on top of a regular triangular lattice, and (c) assigning properties to the beams in each phase. (Reprinted from reference 49)

In Figure 4.1, a regular triangular lattice mesh is projected onto an actual material structure, and properties of each lattice link are assigned according to the material that the link lies over. Because of its simplicity in mesh generation, a regular lattice has been applied to many engineering problems. However, due to its regular geometry, the orientation of the links in the lattice influences the simulated crack patterns, with the cracks tending to follow the mesh lines (Figure 4.2) (50).

![Figure 4.2](image)

Figure 4.2 Simulated crack patterns in (a) square mesh, (b) regular triangular mesh, (c) rotated triangular mesh and (d) random triangular mesh. (Reprinted from reference 51)
To eliminate this disadvantage, the random lattice is adopted in this study to simulate cracking in asphalt concrete. The generation of random lattice is illustrated in Figure 4.3.

![Figure 4.3 Random Lattice Generation](image)

First, a regular square grid with cell-size \( L \) is chosen. In each cell a node is generated randomly. Subsequently, the random lattice is generated by connecting nodes through Delaunay’s triangulation method (52,53), i.e., three points will be connected to form a triangle if their circumscribing circle contains no other points inside.

Feng’s work (16) shows that the distribution function, which controls the node position inside each base cell, affects the mechanical characteristic of the lattice network. Only uniform probability distribution function can generate lattice network with realistic mechanical characteristics.

Figure 4.4 illustrates the random lattice generation procedure based on microstructure of asphalt concrete, i.e., the lattice mesh is directly generated from the configuration of aggregate particles instead of projecting the mesh onto the microstructure.

First, a regular square grid is projected onto the given microstructure (Figure 4.4a) generated by virtual fabrication or DIP. According to the material type of pixels inside, each base cell can be distinguished as “asphalt binder cell”, which contains pixels
representing asphalt binder only (pixel value equals 0), (Figure 4.4b), “aggregate cell”, which contains aggregate pixels only (pixel value equals 1), (Figure 4.4d), or “interface cell”, which contains both mastic and aggregate pixels (pixel value equals 0 or 1), (Figure 4.4c).

Figure 4.4 Lattice Modeling of Asphalt Concrete: (a) Base cell over microstructure surface; (b) Binder base cell and binder node; (c) Interface base cell and Interface nodes; (d) Aggregate base cell and aggregate node; (e) Lattice mesh.
Each asphalt cell of the base mesh contains a single asphalt node whose exact location is determined based on uniform probability distribution function (Figure 4.4b). Since in current modeling, aggregates are looked at as rigid inclusions, it doesn’t matter whether aggregates are discretized as regular mesh or random mesh. For the purpose of visually representing aggregate particles, all aggregates are discretized regularly, i.e., each aggregate cell contains a single aggregate node that is set right on the base cell center to form a regular mesh (Figure 4.4d).

A difference exists with asphalt or aggregate cell; each segment of the aggregate boundary curve in interface cell needs to be represented by a single interface node. Thus an interface cell with several segments of aggregate boundary curves may contain multi interface nodes as shown in Figure 4.4c.

After all nodes are generated, the lattice network is then constructed through Delaunay’s triangulation method. It can be clearly seen that in the resulting lattice mesh (Figure 4.4e), the location and configuration of aggregate particles are precisely represented as dark links and the remaining homogenized mastic is represented as light links in an accurate manner.

To implement the heterogeneity, different material properties including specific surface energy are in turn assigned to binder-to-binder link, aggregate-to-aggregate link and binder-to-aggregate (interface) link. By doing so, the geometry and size of the lattice link and their properties (stiffness, strength, etc) are directly dictated by the geometry of the physical structural of the material.

It should be noted that (i) Figure 4.4 is an illustration of lattice mesh generation. In actual computer coding, the lattice mesh is generated by processing the data file with
element 0 and 1. (ii) The image of microstructure in Figure 3.3a could be obtained from either physically or virtually fabricated specimens using the microstructure generation technique proposed in Chapter 2 and Chapter 3. In the latter case, the material microstructure is generated directly from the aggregate gradation instead of having to process a physically fabricated specimen resulting in a significant reduction in analysis cost and more flexibility in the research process.

4.4 Lattice Modeling of Cracking in Asphalt Concrete

The real value of the lattice model comes from its ability to simulate micro crack-induced damage. Lattice modeling simulates damage in bond-dominated materials by successive removal of failed links representing microcracks. Therefore, a failure criterion is required to trigger the creation of microcracks. This criterion depends on the micromechanical properties of asphalt binder/mastic and the interface between asphalt binder and aggregate particles. Since the breakage of a failed lattice link can be viewed as the creation of a micro crack in the continuum representation, it is reasonable to link the strength of the link to the energy needed to create the associated micro crack. In this section, a surface energy based failure criterion is developed for quasi-brittle material, i.e., asphalt concrete under low temperature.

4.4.1 Failure Criterion for (Quasi-) Brittle Fracture in Lattice Modeling

For quasi-brittle fracture in elastic solids, based on classical fracture mechanics, the required energy to create the crack is a multiple of the crack area and the specific surface energy associated with the broken bond (54,55). In lattice modeling, the lattice link is modeled as linear elastic-brittle, with the failure occurring when the energy stored
in the link exceeds the surface energy of the associated micro crack. Thus, the link is
removed when the linear elastic strain energy ($U_i^{\varepsilon}$) exceeds the multiple of the micro
 crack area (link cross section area, $A_i$) and the specific surface energy of the crack ($\gamma_i$).

\[ U_i^{\varepsilon} \geq \gamma_i \cdot A_i \quad (4-1) \]

Note that the above failure criterion applies only for links in tension, and there is
no compressive failure of the links. Noting that $U_i^{\varepsilon} = E_i A_i \cdot L_i \cdot \varepsilon_i^2 / 2$ ($E_i$, $A_i$, and $\varepsilon_i$ are the Young’s modulus, cross section area, and axial strain of the link), the surface
energy based failure criterion immediately translates into the following strain-based
criterion:

\[ \varepsilon_i < \varepsilon_i' = \frac{2\gamma_i}{E_i L_i} \quad (4-2) \]

Thus, any link subjected to tensile force with strain exceeding its specified failure
strain threshold must be removed. Note that the surface energy, $\gamma_i$, is actually the
interface surface energy. For a binder-to-binder link, $\gamma_i$ is the surface energy of
cohesion, i.e., the energy required to create a unit area crack within the binder, which is
twice the specific surface energy of the asphalt binder,

\[ \gamma_i = 2\Gamma_b, \quad (4-3) \]

where $\Gamma_b$ is the surface energy of the binder. For a binder-to-aggregate link, the surface
energy is:

\[ \gamma_i = \Gamma_a + \Gamma_b - \Gamma_{ab} \quad (4-4) \]
in the above, \( \Gamma_a \) is the aggregate surface energy, and \( \Gamma_{ab} \) derives from the interface effects (56). The implementation of such failure criterion will be illustrated in the following chapters.

4.4.2 Lattice Simulation of Cracking

The above failure criterion is implemented through displacement control to simulate cracking performance of asphalt concrete under low temperature. The loading history, i.e., displacement versus time curve, is divided into many small steps.

At each time step, the analysis is performed in a straightforward manner. Lattice is solved as a linear elastic system under “static load” (instant value of loading history at this time). All the links with the tensile strain larger than the critical failure strain are simultaneously removed from the network, and are not considered in the subsequent analysis. In the above procedure, the time step length, i.e., the displacement increment, has to be small enough to capture the actual material behavior. This is especially important in the case of quasi-brittle failure, which has a very stiff post-peak part in the load-deflection curve. Large time step length will capture neither the peak load value nor the load-deflection curve. On the other hand, reducing the time step length increases the computational cost. Therefore, an optimized time step length has to be determined.

Determining the optimal time step length is a “trial and error” procedure, i.e., an arbitrary time step length is first used to carry out the simulation. Then, a reduced time step is used to carry out the simulation again. If the cracking patterns and reaction-deflection curves of these two trials are closed to each other, this first one can be looked as the optimal time step length. Otherwise, the time step should be reduced again, until the errors between two serial trials are small enough.
4.5 Summary

In this chapter, lattice background is first introduced, followed by the development of lattice mesh generation procedure for asphalt concrete. The random lattice generation procedure based on microstructure of asphalt concrete can capture the location and configuration of aggregate particles precisely. Then, a surface energy based failure criterion is developed for asphalt concrete under low temperature.
5  MULTISCALE VIRTUAL TESTING PROCEDURE
(MSVF-LM)

5.1  Introduction

Since the goal of lattice modeling is to relate the component material properties to
the mixture properties, it is desirable to capture the effect of even the finest aggregates.
However, in order to capture the finest aggregates, the lattice link size should be also fine
enough. This will cause very expensive computational cost. For most situations, the
lattice simulations cannot be carried out on a desktop. This chapter first investigated the
computational cost of lattice modeling, followed by an introduction of multi-scale
procedure. Through integration of virtual fabrication and lattice modeling with multi-
scale procedure, an integrated virtual testing procedure, namely Multi-Scale Virtual
Fabrication and Lattice Modeling (MSVF-LM), is proposed in Section 5.4. An automated
procedure to determine the parameters for MSVF-LM is also introduced in the same
section.

5.2  Study of Computational Cost

Asphalt concrete is a kind of heterogeneous, continuous material. The lattice
method is to discretize the continuous material into many truss elements connected at
nodes, which means each lattice link represents a small area (or volume) of the material.
Obviously, this is a kind of approximate method, the more links (the smaller link size),
the more accurate the simulation will be. However, reducing the link size will cause an
unacceptable increment in computation cost. This is the biggest disadvantage of the
lattice method. Therefore, the size of the lattice link that should be used is discussed in this section so that computation cost and simulation accuracy are both acceptable.

Two main factors affect the computation time of lattice modeling. One is computer speed, which is related to CPU speed, RAM capacity, hard drive speed, etc. The other is lattice link size, i.e., the characteristic length of the lattice link, which is defined as the ratio between the rectangular base cell size (Figure 4.3) and the specimen size.

The former factor is a kind of hardware issue, which is not discussed here. The latter one is studied by using different link sizes to analyze a homogeneous elastic patch with 1×1 unit size under uniaxial tension. The results are shown in Table 5.1.

<table>
<thead>
<tr>
<th>Lattice Link Characteristic Length</th>
<th>D/50</th>
<th>D/100</th>
<th>D/180</th>
<th>D/200</th>
<th>D/300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computation Time (second)</td>
<td>8</td>
<td>23</td>
<td>59</td>
<td>93</td>
<td>3583</td>
</tr>
</tbody>
</table>

These computations were carried out on a PC with P4, 2.2GHz CPU, 1.0GB RAM, 7200RPM hard drive. As shown in Table 5.1, for a simple linear elastic problem, D/300 or even more less is still acceptable. However, modeling cracking behavior of asphalt concrete requires many iterations, and each iteration can be looked at as a simple linear elastic problem. For example, if D/300 is used to analyze a damage problem with 500 iterations, the time will be approximately $3,600 \times 500 = 1,800,000 \text{sec} = 500 \text{hrs} = 21 \text{days}$. This is unacceptable. Therefore, based on the analysis above, lattice size less than D/200 is reasonable to use according to the current computer speed level.
Since asphalt concretes are composite materials, in order to catch the mechanic properties of the mixture, the lattice link size should be small enough to capture the shape of particles. The question then is how small the lattice link size should be to get satisfying results. To answer this question, ten \(3\times3\) in\(^2\) (150mm\(\times\)150mm) microstructures generated by virtual fabrication procedure, under uniaxial tensile test, are studied with different lattice link sizes. The smallest aggregates’ size for this study is 2.36mm. Table 5.2 lists the top layer displacement of the microstructures, and Figure 5.1 gives the distributions of the displacement. For the FEM calculation, a very fine mesh is used (element size is less than 0.2 mm), and the results from FEM are assumed as accurate ones. All the structures have the same boundary conditions and load level. From Figure 5.1, one can see that lattice link size of 0.5mm can match the FEM results very well. Therefore, lattice link size should be at least \(1/5\) (2.36mm/0.5mm=5) of the smallest particle’s size.

Table 5.2 Top average vertical displacements under uniaxial tensile load

<table>
<thead>
<tr>
<th>Structure</th>
<th>FEM</th>
<th>Lattice Link Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.5mm</td>
</tr>
<tr>
<td>1</td>
<td>0.14714</td>
<td>0.14575</td>
</tr>
<tr>
<td>2</td>
<td>0.15279</td>
<td>0.15046</td>
</tr>
<tr>
<td>3</td>
<td>0.15061</td>
<td>0.15176</td>
</tr>
<tr>
<td>4</td>
<td>0.14895</td>
<td>0.14736</td>
</tr>
<tr>
<td>5</td>
<td>0.14761</td>
<td>0.14615</td>
</tr>
<tr>
<td>6</td>
<td>0.15768</td>
<td>0.1547</td>
</tr>
<tr>
<td>7</td>
<td>0.15389</td>
<td>0.15291</td>
</tr>
<tr>
<td>8</td>
<td>0.15225</td>
<td>0.1501</td>
</tr>
<tr>
<td>9</td>
<td>0.14481</td>
<td>0.14435</td>
</tr>
<tr>
<td>10</td>
<td>0.14644</td>
<td>0.14618</td>
</tr>
<tr>
<td>Average</td>
<td>0.15022</td>
<td>0.14897</td>
</tr>
</tbody>
</table>
Typically, the smallest sieve size of aggregate gradation is 0.075mm, and the specimen’s diameter for IDT test is usually 150mm. As discussed above, the reasonable lattice size is 1/5 of the aggregate’s size, which is 0.0015mm, about 1/10000 of the specimen size, more than the current desktop’s computational capacity. Therefore, a multi-scale procedure has to be adopted.

5.3 Multi-scale Procedure

Essentially, a multi-scale model considers the effect of different-sized aggregates at different length scales. Such an approach reduces the computational cost significantly, while capturing the mechanical phenomenon at various length scales. The basic idea of
multi-scale modeling is to model the effects of different sized aggregates at different length scales (Figure 5.2).

Figure 5.2 Multiscale Modeling Approach

At the largest scale (scale 1), the coarse aggregates are explicitly modeled, whereas the finer aggregates and the binder are combined to obtain an effective material, referred to as scale 1 mastic. It is then necessary to obtain the mechanical properties of scale 1 mastic. Noting that these properties are dependent on the finer aggregate gradations as well as the binder properties, lattice modeling is again used to obtain these properties through a serial of numerical tests on representative volume elements (RVEs). Such an analysis is performed at a smaller length scale (scale 2) where some of the finer aggregates are explicitly modeled. Again, for the reasons of computational cost it may be necessary to approximate the effect of the remaining (even finer) aggregates in the form
of scale 2 mastic. This procedure is recursively applied until all the fine aggregates of significance are considered. An important point to note is that the scale $n$ mastic properties are obtained by taking the average of multiple simulations at scale $n+1$.

The application of multi-scale modeling and its effectiveness will be further discussed in the following section.

5.4 Integrated Virtual Testing Methodology - MSVF-LM

Through multi-scale modeling, the microstructure generation procedures and the microstructural lattice methodology can be integrated to form an enhanced virtual testing procedure, namely multi-scale virtual fabrication and lattice modeling (MSVF-LM) as illustrated in Figure 5.3. Given the aggregate gradation of the mixture (in terms of aggregate volume), inverse-stereology methods are applied to obtain the aggregate gradation in terms of the aggregate area percentage -- this transformation is important because the lattice modeling methodology currently simulate only two-dimensional specimens. The resulting 2D apparent aggregate gradation in a cut surface then may be divided into a series of sub-regions based on different scale length of observation.

Analysis starts with the virtual fabrication of a group of representative volume elements (RVEs) at the last (smallest) scale, scale $n$. Since heterogeneity at scale $i$ is ignored at the next scale i-1, scale $n$ material patches can be regarded as part of a homogeneous mastic in the material patches generated at larger scale, i.e., mastic at scale $n-1$. These material RVEs are then discretized into random truss lattice networks. The link properties of the binder are chosen such that the global structural behavior of the lattice network matches that of the material simulated. The lattice link size is primarily
determined by the minimum aggregate size in the microstructure and the computational cost, as described in the previous section. The lattice analysis takes the surface energies and modulus of the binder and aggregates and obtains the effective stiffness and surface energies of material RVEs at scale \( n \). This is accomplished by the use of virtual uniaxial tensile and single notch tests on RVEs. The resulting stiffness and surface energies from all the RVEs are statistically averaged to eliminate the element-to-element variation due to the random structural generation nature. The processed mastic properties are in turn used on the larger-scale (scale \( n-1 \)) lattice modeling. This procedure is recursively applied until all the sub-regions of the gradation are considered. Then, in particular, the microstructure of the specimen, i.e., at scale 1 level, is simulated so that its mechanical response and damage performance can be predicted.
Figure 5.3 Integrated Virtual Testing Procedure for Multiscale Analysis
5.5 **Determination of Parameters in MSVF-LM**

Since lattice modeling represents a continuum with a discrete random truss network, the resolution of the representation (i.e., lattice characterize length) and the link properties are critical to lattice analysis. In this section, an automated procedure built on “trial and error approach” is introduced to determine the lattice size and major parameters for analysis including the number of analysis scales, RVE size for mastic analysis, and resolution for virtual fabrication at each analysis scale.

As shown in Figure 5.4, such procedure starts with the determination of the RVE size and maximum aggregate (sieve) size to be modeled at current scale level. In general, the size of RVE should be 4~5 times the largest aggregate size, so that the mechanical properties of the mastic can be represented. Since for the first (largest) scale analysis, there is no need to determine the maximum aggregate and RVE (specimen) size, the procedure can directly begin with the initial trial of lattice size assumed based on experience. If the resulting lattice system size (total number of nodes and degree-of-freedoms approximated from lattice mesh generation) exceeds the capacity of current PC power, increase the lattice size until the upper limit of current PC capacity is reached. Once the lattice size is determined, the minimum aggregate size is approximated by another explicitly developed criterion, i.e., to capture the effect of an aggregate particle on the mixture mechanical behavior, the lattice size should be less than 1/5 of the aggregate size. Next, an appropriate resolution for digital image of material microstructure is determined so that the microstructure can be easily discretized into desired lattice mesh. This procedure is recursively applied to the analysis at a different scale until all the aggregates of significance are modeled.
Figure 5.4 Parameters Determination for Multiscale Analysis

Initial trial of Lattice Size

Calculate the total number of DOF for the system based on the size of specimen being simulated

If system size is less than but close to the upper limit of PC capacity?

RVE size should be at least 4-5 times the largest aggregate (sieve) size

Determine the RVE size

Determine the maximum aggregate (sieve) size for analysis at current scale level (from the upper level scale analysis)

Increase lattice size

Lattice link size should be at least 5 times smaller than the aggregate sieve size.

Determine the minimum aggregate (sieve) size that can be effectively modeled in lattice

All the aggregates of interest have been modeled?

Yes

END

No

Yes
5.6 Summary

In this chapter, the computational cost of lattice modeling is first investigated, followed by introduction of multi-scale procedure. Through integration of virtual fabrication and lattice modeling with multi-scale procedure, an integrated virtual testing procedure, namely Multi-Scale Virtual Fabrication and Lattice Modeling (MSVF-LM), is then proposed. An automated procedure to determine the parameters for MSVF-LM is introduced in the end.
6 UNIAXIAL TENSION TEST SIMULATION

6.1 Introduction

In this chapter, the validity of the proposed multi-scale virtual fabrication and lattice modeling (MSVF-LM) methodology in Chapter 5 is verified through simulating uniaxial tension tests (UTT) under low temperature and fast loading rates.

6.2 Test Setup

The uniaxial tension tests are carried out in a UTM-25 (with a 5,620 lb load cell) test machine, which is equipped with a temperature control chamber. This closed-loop servo-hydraulic testing system is capable of delivering a wide range of loading frequencies (0.1-25 Hz) and loading rates at various temperatures (-10 to 40°C). A fully computerized system allows the user to easily control the machine and to accurately program the tests (57). Figure 6.1 shows the general uniaxial tests setup.

Figure 6.1 General Uniaxial Tests Setup
The specimen is fabricated with a 12.5 mm Maryland state highway administration Superpave mix design (Table 3.1), 75 mm x 150 mm with LVDTs (Linear Variable Differential Transducers) installed in the middle (Figure 6.2) (58). The tests are performed under -10°C and constant crosshead rates of 0.0135, 0.0005, and 0.00019 strain/second, respectively. The function of LVDT is to get the actual specimen’s loading rate because of the machine compliance (58).

Figure 6.2 Uniaxial Tension Test Specimen

6.3 Asphalt Mechanical Properties

6.3.1 Linear Viscoelasticity and Generalized Maxwell Model

Asphalt can be regarded as a linear viscoelastic material, which satisfies the following two conditions (61):
(1) The homogeneity (proportionality) condition is obeyed: an increase in the input (stress or strain) by an arbitrary factor must increase the response (strain or stress) by the same factor;

(2) The linear superposition principle is valid, i.e., the strain response due to the combination of two arbitrary but different stress inputs must equal the sum of the strain responses resulting from the two stress inputs each acting separately. This condition can be also expressed in terms of strain inputs and stress responses.

There are many mechanical analogs, such as Maxwell model, Voigt model, etc., to represent the linear viscoelasticity of the binder. In this study, the Generalized Maxwell model is chosen, in which the relaxation modulus is presented by the Prony series expansion. The Generalized Maxwell consists of a spring and $m$ Maxwell elements connected in parallel, as shown in Figure 6.3.

\[
E(t) = E_\infty + \sum_{i=1}^{m} E_i \rho_i^{-t},
\]

where $E_\infty$ (the equilibrium modulus), $E_i$ (relaxation strengths), and $\rho_i$ (relaxation
times, $\rho_i = \eta_i/E_i$ ) are all positive constants. In the following sections, these constants are obtained by fitting this expression to the available experimental data.

6.3.2 Experimental Data

The asphalt binder used in this study was an unmodified PG 64-22 obtained from the Paulsboro, New Jersey terminal of the Citgo Asphalt Refining Company. An extensive testing program was performed to characterize the rheological properties of the binder over a wide range of temperatures using both conventional and Superpave tests. The details of the testing program and results can be found in (59). Here, only dynamic shear rheometer (DSR) data and bending beam rheometer (BBR) data for Rolling Thin Film Oven Test (RTFOT) aged conditions are listed in Table 6.1 and Table 6.2 for the derivation of the relaxation modulus master curve.
Table 6.1 Dynamic Shear Rheometer Data for RTFOT Conditions.

<table>
<thead>
<tr>
<th>Freq, rad/sec</th>
<th>1</th>
<th>1.59</th>
<th>2.51</th>
<th>3.98</th>
<th>6.31</th>
<th>10</th>
<th>15.9</th>
<th>25.1</th>
<th>39.8</th>
<th>63.1</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td>2363000</td>
<td>3239000</td>
<td>4358000</td>
<td>5875000</td>
<td>7800000</td>
<td>10290000</td>
<td>13990000</td>
<td>18120000</td>
<td>23160000</td>
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<td>37110000</td>
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<td>35</td>
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<td>605400</td>
<td>875400</td>
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<td>5392000</td>
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<td>16080</td>
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<td>1121000</td>
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<td>888.2</td>
<td>1386</td>
<td>2142</td>
<td>3293</td>
<td>5041</td>
<td>7698</td>
<td>11620</td>
<td>17560</td>
<td>26340</td>
<td>39220</td>
</tr>
<tr>
<td>70</td>
<td>145.7</td>
<td>228.6</td>
<td>358.8</td>
<td>564.1</td>
<td>880.4</td>
<td>1368</td>
<td>2108</td>
<td>3254</td>
<td>4999</td>
<td>7624</td>
<td>11530</td>
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<td>43.94</td>
<td>70.42</td>
<td>110.2</td>
<td>174.7</td>
<td>275.6</td>
<td>433.6</td>
<td>679.7</td>
<td>1059</td>
<td>1646</td>
<td>2545</td>
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<tr>
<td>95</td>
<td>9.216</td>
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<td>56.35</td>
<td>89.02</td>
<td>141.2</td>
<td>222.5</td>
<td>350.3</td>
<td>551.5</td>
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<tr>
<td>105</td>
<td>3.912</td>
<td>6.365</td>
<td>9.89</td>
<td>15.80</td>
<td>25.04</td>
<td>39.53</td>
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<td>97.89</td>
<td>155.5</td>
<td>246.2</td>
<td>386.4</td>
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<td>115</td>
<td>1.970</td>
<td>3.256</td>
<td>5.049</td>
<td>7.838</td>
<td>12.43</td>
<td>19.64</td>
<td>31.18</td>
<td>49.07</td>
<td>77.91</td>
<td>123.2</td>
<td>193.4</td>
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<table>
<thead>
<tr>
<th>Phase Angle, Degrees</th>
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<tbody>
<tr>
<td>Freq, rad/sec</td>
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<tr>
<td>Temp, C</td>
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</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>45</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>70</td>
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<td>80</td>
</tr>
<tr>
<td>95</td>
</tr>
<tr>
<td>105</td>
</tr>
<tr>
<td>115</td>
</tr>
</tbody>
</table>
Table 6.2 Bending Beam Rheometer Data for RTFOT Conditions.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>-12 C</th>
<th>-18 C</th>
<th>-24 C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time, s</td>
<td>S, MPa</td>
<td>m</td>
<td>S, MPa</td>
</tr>
<tr>
<td>8</td>
<td>261</td>
<td>0.330</td>
<td>512</td>
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<tr>
<td>15</td>
<td>210</td>
<td>0.363</td>
<td>446</td>
</tr>
<tr>
<td>30</td>
<td>161</td>
<td>0.399</td>
<td>375</td>
</tr>
<tr>
<td>60</td>
<td>121</td>
<td>0.435</td>
<td>308</td>
</tr>
<tr>
<td>120</td>
<td>88</td>
<td>0.471</td>
<td>248</td>
</tr>
<tr>
<td>240</td>
<td>63</td>
<td>0.508</td>
<td>194</td>
</tr>
</tbody>
</table>

6.3.3 Master Curves of $|G^*|$ and Phase Angle at -10°C from DSR

Since the temperature of UTT tests is -10°C in this study, the master curves at -10°C should be determined. However, the lowest temperature for the DSR test is 15°C, as listed in Table 6.1. Therefore, the master curve at 15°C should be derived first, and then it can be shifted to -10°C. Figure 6.4 gives the curves of $|G^*|$ versus frequency in logarithmic scale.

![Figure 6.4 $|G^*|$ versus Frequency Curves at Different Temperatures](image-url)
In order to get the master curve at 15°C, shift factors for all the higher temperatures should be determined first. The shift factor is defined as:

\[ a_T = \frac{t_T}{t_{T_R}} = \frac{f_{T_R}}{f_T} \]  

(6–2)

where \( t_T(f_T) \), and \( t_{T_R}(f_{T_R}) \) are time (frequencies) for temperature \( T \) and reference temperature (15°C ), when the \(|G^*|s\) are equal to each other. Table 6.3 lists the shift factors of all the higher temperatures to 15°C.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>( a_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>0.049459</td>
</tr>
<tr>
<td>35</td>
<td>0.004957</td>
</tr>
<tr>
<td>45</td>
<td>0.000645</td>
</tr>
<tr>
<td>60</td>
<td>4.4E-05</td>
</tr>
<tr>
<td>70</td>
<td>1.09E-05</td>
</tr>
<tr>
<td>80</td>
<td>3.27E-06</td>
</tr>
<tr>
<td>95</td>
<td>6.65E-07</td>
</tr>
<tr>
<td>105</td>
<td>2.85E-07</td>
</tr>
<tr>
<td>115</td>
<td>1.48E-07</td>
</tr>
</tbody>
</table>

In order to get the shift factor of -10°C, a semi-empirical relationship, namely Williams-Lander-Ferry (WLF) equation (62), as shown in (6-3), is adopted in this study:

\[ \log a_T = \log \frac{t_T}{t_{T_R}} = \frac{C_1(T-T_R)}{C_2 + T-T_R} \]  

(6–3)

where \( C_1 \) and \( C_2 \) are coefficients.

Equation (6-3) can be easily converted into the following linear equation:

\[ \log a_T = \frac{-\log a_T}{C_2 + C_1} \cdot C_2 + C_1 \]  

(6–4)

where \( \frac{-\log a_T}{C_2 + C_1} \), and \( \log a_T \) can be obtained directly from the data in Table 6.3. Through linear regression, \( C_1 \) and \( C_2 \) can then be obtained. As shown in Figure 6.5, for this study, \( C_1 \) and \( C_2 \) are -13.32 and 93.506, respectively.
Therefore,
\[ \log a_{-10C@15C} = \frac{-13.32 \cdot (-10 - 15)}{93.506 - 10 - 15} = 4.86 \]  \hspace{1cm} (6-5) 
and
\[ a_{-10C@15C} = 10^{4.86} = 72544 \]  \hspace{1cm} (6-6) 
and
\[ a_{15C@-10C} = \frac{1}{a_{-10C@15C}} = \frac{1}{72544} \]  \hspace{1cm} (6-7)

Once the shift factor of 15°C to -10°C is obtained, the master curves of 15°C can be shifted to -10°C, as shown in Figure 6.6.
The same procedure is adopted to get the master curves of phase angle, as illustrated in Figure 6.7.
Through equations (6-8) –(6-11), \( \text{E}’ \) and \( \text{E}” \) can be obtained for the derivation of Prony series.

\[
|G^*| = \frac{|E^*|}{2(1+\nu)} \quad (6–8)
\]

\[
E' = E' + E''i \quad (6–9)
\]

\[
E' = |E^*| \cdot \cos(\Phi) \quad (6–10)
\]

\[
E'' = |E^*| \cdot \sin(\Phi) \quad (6–11)
\]

where \(|E^*|\) is the magnitude of dynamic modulus, \(\nu\) is Poison’s Ratio, which is assumed as 0.5 (61). \(E’\), \(E”\), and \(\Phi\) are storage modulus, loss modulus, and phase angle, respectively. Figure 6.8 gives the master curve of \(|E^*|\) at -10°C.

![Figure 6.8 Master Curve of \(|E^*|\) at -10°C from DSR](image)

It should be pointed out that the above procedure can only obtain half part of the \(|E^*|\) master curve, i.e., the lower frequency part. The other half can be obtained through analysis of the BBR test data.
6.3.4 Master Curve of $|E^*|$ from BBR

Table 6.2 lists the creep stiffness $S$ at different low temperatures. Based on the relationship of (6-12) (63), the creep compliance can be easily obtained.

\[
D(t) = \frac{1}{S(t)} \quad (6-12)
\]

Figure 6.9 shows the curves of creep compliance at different temperatures. Based on the shift factors listed in Table 6.4, the master curve of creep compliance at -12°C can be obtained by shifting the curves at lower temperatures.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>$a_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12</td>
<td>1</td>
</tr>
<tr>
<td>-18</td>
<td>13.075</td>
</tr>
<tr>
<td>-24</td>
<td>392.25</td>
</tr>
</tbody>
</table>

Since -10°C and -12°C are close to each other, a simple approximate method is adopted in this section to get the shift factors for -10°C. As shown in Figure 6.10, $\log(a_T)$
and temperature are assumed to have a linear relationship. Through linear regression, the relationship is obtained as follows:

$$\log(a_T) = -0.2161 \cdot T - 2.6537$$  \hspace{1cm} (6–13)

Therefore,

$$a_{-10C@-12C} = 10^{-0.493} = 0.322$$  \hspace{1cm} (6–14)

and

$$a_{-12C@-10C} = \frac{1}{a_{-10C@-12C}} = 3.11$$  \hspace{1cm} (6–15)

Once the shift factor of -12°C to -10°C is obtained, the master curves of -12°C can be shifted to -10°C, as shown in Figure 6.11.
By using linear viscoelastic theory, dynamic modulus $E^*$ can be predicted from creep compliance $D$ (64). This method is summarized below.

Through linear viscoelastic theory, it can be found that

$$E^* \times D^* = 1 \quad (6–16)$$

where the complex compliance is defined as

$$D^* = D' - iD'' \quad (6–17)$$

where $D'$ and $D''$ are storage and loss compliance, respectively.

Therefore, the storage and loss modulus $E'$ and $E''$ can be obtained as:

$$E' = \frac{D'}{|D'|^2} \quad (6–18)$$
\[ E'' = \frac{D''}{|D'|^2} \]  
(6–19)

where
\[
|D'| = \sqrt{D'^2 + D''^2} \]  
(6–20)

Thus, as long as D’ and D’’ are known, dynamic modulus E* can be predicted. The creep compliance is usually assumed to be expressed in the generalized power law (GPL) form, i.e.,
\[ D(t) = D_0 + D_1 t^m \]  
(6–21)

where D_0, D_1, and m are regression constants. Once regression is done, through Laplace Transform, D’ and D’’ can be obtained as follows:
\[
D' = D_0 + D_1 \Gamma(m + 1)(2\pi f)^{-m} \cos\left(\frac{m\pi}{2}\right) \]  
(6–22)

\[
D'' = D_1 \Gamma(m + 1)(2\pi f)^{-m} \sin\left(\frac{m\pi}{2}\right) \]  
(6–23)

In order to get the regression constants, different values of m are assumed from 0.1 to 1 to get different groups of D_0 and D_1. Figure 6.12 shows the R-squared values for different m. The R-squared value is an indicator that ranges in value from 0 to 1 and reveals how closely the estimated values for the trendline correspond to the actual data. A trendline is most reliable when its R-squared value is at or near 1.
From Figure 6.12, one can see that $m=0.48$ gives the largest R-squared value, 0.9988. Table 6.5 lists the regression constants for fitting $D(t)$ at -10°C. Figure 6.13 gives the actual data and fitting $D(t)$ master curve at -10°C, which illustrates that the fitting curve matches the actual data very well.

### Table 6.5 Regression Constants

<p>| | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>$m$</td>
<td>0.48</td>
</tr>
<tr>
<td>$D_0$</td>
<td>0.0009</td>
</tr>
<tr>
<td>$D_1$</td>
<td>0.0018</td>
</tr>
<tr>
<td>R-squared value</td>
<td>0.9988</td>
</tr>
</tbody>
</table>
By putting the constants in Table 6.5 into the equations (6-16) ~ (6-23), the magnitude of dynamic modulus $|E^*|$ can be obtained. Figure 6.14 gives the master curve of $|E^*|$ at -10°C. This curve is another half part of the whole master curve.
By combining Figure 6.8 and Figure 6.14, the whole master curve of $|E^*|$ can be obtained, as shown in Figure 6.15. From Figure 6.15, one can see that there is a gap between the DSR curve and the BBR curve. This is mainly because of the difference between these two tests, such as different physical hardening effect, different cooling media, and different specimen thickness (61). Since the specimen in DSR is 1mm or 2mm thick, while the specimen in BBR is 6.2mm thick, considering of the effect of thin film, which will be discussed in the following section, the BBR data is shifted horizontally, so that the two curves can be overlapped, as shown in Figure 6.16.
6.3.5 Prony Series Coefficients Determination

The constants in equation (6-1) can be obtained by fitting this expression to $E'$ and $E''$, which are obtained from the available experimental data, i.e., $E^*$, based on equations (6-9) ~ (6-11). $E'$ and $E''$ can be expressed in terms of $E_i$, $\rho_i$, and angular frequency (65).

\[
E'(\omega) = E_\infty + \sum_{i=1}^{m} \frac{\omega^2 \rho_i^2 E_i}{\omega^2 \rho_i^2 + 1} \tag{6–24}
\]

\[
E''(\omega) = \sum_{i=1}^{m} \frac{\omega \rho_i E_i}{\omega^2 \rho_i^2 + 1} \tag{6–25}
\]

Various methods of fitting have been proposed; e.g., the collocation method was proposed by Schapery (66) and the multidata method by Cost and Becker (67). In this
study, the collocation method is adopted to get the constants of Prony series, as listed in Table 6.6.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\rho_i$</th>
<th>$E_i$ (MPa)</th>
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</thead>
<tbody>
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<td>1</td>
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<td>213.73</td>
</tr>
<tr>
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<td>0.0001</td>
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<tr>
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<td>0.1</td>
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</tr>
<tr>
<td>9</td>
<td>100</td>
<td>94.61</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
<td>31.89</td>
</tr>
<tr>
<td>11</td>
<td>10000</td>
<td>7.13</td>
</tr>
<tr>
<td>12</td>
<td>100000</td>
<td>1.01</td>
</tr>
<tr>
<td>13</td>
<td>1000000</td>
<td>0.12</td>
</tr>
<tr>
<td>$E_\infty$</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Therefore, the relaxation modulus $E(t)$ can then be obtained from equation (6-1). Figure 6.17 gives the curve of $E(t)$.
6.3.6 Thin Film Effect

Asphalt concrete consists mostly of aggregate particles coated by thin films of binder. The average binder thickness typically ranges between 0.01 and 0.013 mm (10–13 micron) (68,69). There is considerable experimental evidence (70–73) that shows that when a liquid is confined in a narrow gap (thin films), new dynamic behavior emerges. The principal effects are a considerably increased viscosity and great elastic strength of the liquid near the surface of a solid compared to the same material in bulk form. This effect is highly pronounced with asphalts (74–78). Huang (79) studied the effect of film thickness on the viscosity of asphalts in contact with aggregate surface by using the sliding plate microviscometer with machined aggregate plates. Figure 6.18 shows the experimental data curves of the viscosity versus film thickness, and Table 6.7 lists the data for the curve of the limestone plate.

![Graph showing viscosity versus film thickness for Asphalt AAD-1 with both limestone plates and glass plates at 25 °C (Reprinted from Ref. 79)]
Table 6.7 Viscosity at shear rate of 0.05 /sec, Pa·s

<table>
<thead>
<tr>
<th>Film thickness</th>
<th>10 µm</th>
<th>20 µm</th>
<th>35 µm</th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity</td>
<td>113282</td>
<td>83326</td>
<td>80628</td>
<td>57055</td>
<td>65864</td>
<td>59373</td>
</tr>
</tbody>
</table>

From Figure 6.18, one can see that the viscosity is almost a constant after the binder’s thickness is over 150 microns. The factor between thin film (10 microns) and thick film (over 150 microns) is equal to 1.91 (113282/59373), which means that the viscosity of the thin film is almost 1.91 times the viscosity of the thick film.

Based on the Cox-Merz rule (80), the binder stiffness can be estimated by the following equation (81):

\[
\eta = \frac{G_b^*}{\omega} \left(\frac{1}{\sin \delta}\right)^{3.639+0.1314\omega-0.000901\omega^2} \quad (6–26)
\]

where \(\eta\), \(\delta\), and \(G_b^*\) are binder viscosity, phase angle, and dynamic shear modulus, respectively.

Equation (6-26) shows that binder stiffness is proportional to its viscosity. In another word, for the binder used in Figure 6.18, its thin film stiffness is 1.91 times the thick film stiffness. As for the uniaxial tension tests, the binder properties are mainly obtained by DSR test, in which the bind thickness is 1mm or 2mm, much larger than the typical average binder thickness in asphalt concrete. Therefore, for the lattice simulation, the binder properties should be multiplied by a factor, which is determined by many reasons, such as aggregate type, binder type, and binder thickness. Due to lack of the exact experimental data of viscosity versus film thickness for the Georgia granite aggregate and the unmodified PG 64-22 binder, which are adopted to make the actual specimens, the factor 1.91, obtained from Figure 6.18 and Table 6.7, is adopted approximately for the lattice simulation.
6.4 Mastic properties determination through MSVF-LM

To illustrate the multi-scale lattice modeling procedure, a four-scale approach is used to model the UTT tests. The given (volumetric) design aggregate structure is first converted into the aggregate gradation in terms of the aggregate area percentage using the inverse-stereology proposed in Chapter 3. In this application, the resulting aggregate area gradation is divided into four regions: scale-1 for coarse aggregates between 2.36~19 mm, scale-2 for fine aggregates between 0.6 ~ 2.36 mm, scale-3 for aggregates between 0.15~0.6 mm, and scale-4 for aggregates below 0.15mm (Figure 6.19a). The segment of aggregate area gradation for scale-1 is used to create the UTT specimen where the coarse aggregates are explicitly represented and mastic (or “scale-1 mastic”) is considered as a homogeneous medium. The segment of aggregate area gradation for scale-2 or scale-3, however, has to be extended to become a new independent aggregate area gradation. The new aggregate area gradations may be used to generate material RVEs (Figure 6.19b) for the mastic analysis at scale 2, scale 3 and scale 4, respectively. The dimension and lattice link size of these structures, as shown in Figure 6.19c, are explicitly determined through the procedure proposed in Section 5.5.

Due to the linear viscoelastic property of asphalt, as discussed in Section 6.3.1, the mastic mechanical properties at each scale can be obtained through elastic analysis. Table 6.8 lists the elastic material properties used in lattice modeling (56,59).

<table>
<thead>
<tr>
<th></th>
<th>Aggregate (Georgia Granite)</th>
<th>Binder (AAK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus</td>
<td>50 GPa</td>
<td>209 MPa</td>
</tr>
<tr>
<td>Surface Energy</td>
<td>206 ergs/cm²</td>
<td>27.4 ergs/cm²</td>
</tr>
</tbody>
</table>
Once the virtual fabrication of the microstructures in Figure 6.19c is completed, the lattice modeling is then applied for the scale 4 mastic RVEs first. Each lattice realization takes the modulus of the binder (209 Mpa) and aggregates (50 Gpa for Granite) and obtains the effective stiffness of scale 4 mastic. This is accomplished by the use of uniaxial tensile tests. The resulting (statistically averaged) mastic stiffness is then used in the lattice modeling of the material RVEs at scale 3 to obtain the stiffness of scale 2 mastic. The same procedure is adopted on scale 2 RVEs to obtain the stiffness of scale 1 mastic for the lattice analysis of UTT at scale 1 level.

Similarly, the surface energy for each scale is obtained by the use of single edge notch tests. The resulting (statistically averaged) mastic surface energy for a scale is then used in the lattice modeling of the material RVEs at the upper scale, until the mastic surface energy of scale 1 mastic is obtained. Table 6.9 lists the mechanical properties of each scale.

<table>
<thead>
<tr>
<th>Table 6.9 Mastic Mechanical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Elastic Modulus (Mpa)</td>
</tr>
<tr>
<td>Surface Energy of Mastic (ergs/cm²)</td>
</tr>
<tr>
<td>Surface Energy of Aggregate (ergs/cm²)</td>
</tr>
<tr>
<td>Interfacial Surface Energy between Mastic-Aggregate (ergs/cm²)</td>
</tr>
<tr>
<td>Link Surface Energy</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

The interfacial surface energy between mastic and aggregate is assumed to be the same as the interfacial surface energy between binder and aggregate (16), which is determined by the Lifshitz–van der Waals component and the Acid-Base component of aggregates’ and binder’s surface energy (56). Chapter 7 will explain this in detail.
Figure 6.19 Multiscale Lattice Elastic Modeling of UTT

(a) Volumetric Aggregate Gradation

(b) For RVE generation at scale 4

(c) Lattice Size = 0.004mm
Specimen size = .75x.75mm²

(d) Moduli of RVEs at scale

(b) For RVE generation at scale 3

(c) Lattice Size = 0.02mm
Specimen size = 4x4 mm²

(d) Moduli of RVEs at scale

(b) For RVE generation at scale 2

(c) Lattice Size = 0.064 mm
Specimen Size = 13x13mm²

(d) Moduli of RVEs at scale

For specimen scale generation

Lattice Size =0.3mm
Specimen Size = 40x150mm²

Scale 1
Scale 2
Scale 3
Scale 4
The elastic mastic analysis in Figure 6.19 shows that the mastic stiffness at scale 2 (1516 MPa) is about 7.25 times the binder’s stiffness (209 MPa). Therefore, the mastic relaxation modulus should be 7.25 times the binder’s relaxation modulus, because of the linear viscoelastic property of asphalt. Considering the thin film affect, the binder’s relaxation modulus, as shown in Figure 6.17, should multiply a factor 13.85 (7.25×1.91) as the input for the UTT viscoelastic simulation.

6.5 Lattice Simulation Results and Analysis of UTT

The UTT under three different constant crosshead rates, i.e., 0.0135, 0.0005, and 0.00019 strain/second, are simulated in this section. The corresponding LVDT loading strain rates are shown in Figure 6.20.
Figure 6.20 LVDT Strain Rates: (a) 8.16e-4 strain/second, (b) 4.38e-5 strain/second, and (c) 2.48e-6 strain/second.

Figure 6.21 shows the lattice simulation result for the 8.16e-4 strain/second LVDT strain rate, in which the stiffness of lattice simulation matches the experimental stiffness very well. However, the failure load of lattice simulation is higher than the experimental data. Since the surface energy determines the failure load, which will be discussed in Chapter 7, the comparison between these two failure loads means that the mastic surface
energy is enlarged through multi scale analysis. Therefore, it is necessary to study how much the mastic surface energy is enlarged. Figure 6.22 gives the results of surface energy factor study. It shows that the 1/3 factor can give a good failure load close to the experimental data.
The factor $1/3$ of surface energy is then used to carry out all other simulations. Figure 6.23 shows lattice simulations of $8.16e-4$ strain/second LVDT strain rate for four different virtual structures. All the simulations match the experimental data very well. Furthermore, the corresponding cracking patterns for these 4 virtual structures are given in Figure 6.24. The observed cracking patterns are in agreement with the typical cracking patterns observed in UTT tests of asphalt concrete.

![Figure 6.23 Lattice Simulations of 8.16e-4 strain/sec. for 4 Different Virtual Structures](image)

However, for the same input of mastic properties, i.e., relaxation modulus and surface energy, the lattice simulations cannot give reasonable results for the lower LVDT strain rates, $4.38e-5$ and $2.48e-6$ strain/second, respectively. The simulation results are shown in Figure 6.25. From this figure, one can see that the simulations are much softer than the experimental data for lower strain rates. This means that in reality, asphalt behaves more elastic when it is mixed with aggregates in asphalt concrete. The following section will study this issue.
Figure 6.24 Cracking Patterns for: (a) vf07, (b) vf08, (c) vf09, and (d) vf10.

Figure 6.25 Lattice Simulations for Different LVDT Strain Rates
6.6 Lattice Simulation Based on Backcalculated Binder Relaxation Modulus

As shown in Figure 6.25, the binder’s properties obtained from BBR and DSR, can only give good simulation results for very fast loading rates. They give softer predictions for lower loading rates. This indicates that asphalt becomes more elastic when it is mixed with aggregates in asphalt concrete. Therefore, the binder’s properties need to be modified to make it more elastic.

In order to get the proper modification of binder’s properties, the actual mixture of asphalt concrete needs to be known. The relaxation modulus of the actual mixture is obtained from (58), as shown in Figure 6.26. The constants of Prony Series are listed in Table 6.6.
Table 6.10 Constants of Prony Series for Mixture

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\rho_i$</th>
<th>$E_i$ (MPa)</th>
<th>$i$</th>
<th>$\rho_i$</th>
<th>$E_i$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>633.7416</td>
<td>13</td>
<td>10000</td>
<td>1309.578</td>
</tr>
<tr>
<td>2</td>
<td>0.031623</td>
<td>911.9228</td>
<td>14</td>
<td>31622.78</td>
<td>841.2066</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>1137.733</td>
<td>15</td>
<td>100000</td>
<td>506.6924</td>
</tr>
<tr>
<td>4</td>
<td>0.316228</td>
<td>1474.597</td>
<td>16</td>
<td>316227.8</td>
<td>294.8273</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1828.38</td>
<td>17</td>
<td>1000000</td>
<td>168.3299</td>
</tr>
<tr>
<td>6</td>
<td>3.162278</td>
<td>2227.544</td>
<td>18</td>
<td>3162278</td>
<td>98.70693</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>2596.036</td>
<td>19</td>
<td>10000000</td>
<td>55.91963</td>
</tr>
<tr>
<td>8</td>
<td>31.62278</td>
<td>2876.862</td>
<td>20</td>
<td>31622776</td>
<td>38.07956</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>2970.132</td>
<td>21</td>
<td>1E+08</td>
<td>16.28208</td>
</tr>
<tr>
<td>10</td>
<td>316.2278</td>
<td>2816.594</td>
<td>22</td>
<td>3.16E+08</td>
<td>25.90254</td>
</tr>
<tr>
<td>11</td>
<td>1000</td>
<td>2418.642</td>
<td>23</td>
<td>1E+09</td>
<td>-15.0368</td>
</tr>
<tr>
<td>12</td>
<td>3162.278</td>
<td>1871.682</td>
<td>24</td>
<td>3.16E+09</td>
<td>36.97246</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$E_\infty$</td>
<td>40</td>
</tr>
</tbody>
</table>

The mixture relaxation modulus cannot be used as the input material properties directly. The inputs for UTT simulation at scale 1 are the material properties of scale 2 mastic. In order to get the scale 2 mastic properties, the scale 1 structures’ stiffness is first investigated. By adopting the same procedure, as discussed in Section 6.4, through virtual uniaxial tensile tests, the scale 1 structure elastic modulus can be obtained by inputting the scale 2 mastic modulus (1516MPa). As listed in Table 6.11, the scale 1 structure elastic modulus is 3468MPa. Therefore, the relaxation modulus of scale 2 mastic is equal to the mixture relaxation modulus (Figure 6.26) times a factor 0.44 (1516/3468).

Table 6.11 Scale 1 Structure Modulus

<table>
<thead>
<tr>
<th>Case</th>
<th>Modulus(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>vf07</td>
<td>3347</td>
</tr>
<tr>
<td>vf08</td>
<td>3632</td>
</tr>
<tr>
<td>vf09</td>
<td>3453</td>
</tr>
<tr>
<td>vf10</td>
<td>3439</td>
</tr>
<tr>
<td>mean</td>
<td><strong>3468</strong></td>
</tr>
</tbody>
</table>
However, by using the factor 0.44, the lattice simulation still cannot give satisfying predictions. Through several trials, it is found that the factor 0.5 can produce pretty good predictions.

Figure 6.27 Lattice Simulation by Backcalculated Binder Relaxation Modulus

Figure 6.28 Lattice Simulation by Backcalculated Binder Relaxation Modulus and Adjusted Surface Energy
Figure 6.27 shows the lattice simulations, in which the backcalculated binder relaxation modulus is adopted, but the surface energy isn’t adjusted. One can see that with the backcalculated binder relaxation modulus, the lattice simulations can predict good stiffness for all the three different loading rates. When the factor 1/7 is adopted to adjust surface energy also, both stiffness and failure load can be predicted properly for all the loading rates, as shown in Figure 6.28.

The reason that the backcalculated binder properties can give good predictions can be explained by putting the moduli of original binder relaxation modulus (Figure 6.17) and the backcalculated binder, which is obtained by multiplying the actual mixture relaxation modulus a factor (0.5*1/7.25), together, as shown in Figure 6.29. It can be clearly seen in this figure that the modified binder is more elastic and its viscoelasticity is also changed. The reason for this phenomenon is that the binder in asphalt concrete is thin film, which is more elastic, as discussed in Section 6.3.6.
6.7 Summary

The proposed multi-scale virtual fabrication and lattice modeling (MSVF-LM) methodology is verified through simulating uniaxial tension tests (UTT) under low temperature and fast loading rates. The comparison between numerical simulation and physical experimental data shows that given proper material properties, MSVF-LM is able to precisely predict the crack patterns and the mechanical response, and shows excellent promise for micromechanical modeling of asphalt concrete.
7 EFFECTS OF SURFACE ENERGY

7.1 Introduction

Surface energy is one of the most important material parameters for lattice modeling of cracking. In this chapter, this parameter is studied through indirect tensile tests (IDT) and uniaxial tension tests (UTT) simulations.

7.2 Surface Energy

Surface energy is the energy needed to create a unit area of a new surface (crack) within mastic or between the mastic and aggregate. For asphalt concrete, surface energy is mainly composed of a non-polar component and an acid-base component (82,83), i.e.,

\[ \Gamma = \Gamma^{LW} + \Gamma^{AB} \]  (7–1)

where \( \Gamma \), \( \Gamma^{LW} \) and \( \Gamma^{AB} \) are surface energy of asphalt or aggregate, Lifshitz–van der Waals component of surface energy and Acid-Base component of the surface energy, respectively.

\( \Gamma^{LW} \) can be obtained from lab testing. However, \( \Gamma^{AB} \) can be divided into a Lewis acidic surface parameter and a Lewis basic surface parameter (82), which can also be obtained from lab testing. The relation among the \( \Gamma^{AB} \) and its components is:

\[ \Gamma^{AB} = 2\sqrt{\Gamma^+\Gamma^-} \]  (7–2)

where \( \Gamma^+ \) and \( \Gamma^- \) are Lewis acid component and Lewis base component of surface...
interaction, respectively.

### 7.3 Adhesion and Cohesion

In asphalt concrete, cracks occur either at the asphalt-aggregate interface or within the asphalt mastic. The interfacial strength between asphalt binder and aggregate is called adhesion. The strength within the mastic is called cohesion. From a thermodynamics point of view, for cohesion, surface energy is the energy required to create a unit area crack within the binder or mastic under a vacuum. For adhesion, surface energy is the energy required to create a unit crack area at the interface between two dissimilar bodies (mastic-aggregate) in a vacuum. The relationship between cohesion and surface energy is 

\[ \Delta G_i^C = 2\Gamma_i \]  

(7–3)

Similarly, cohesion has two components, the Lifshitz-van der Waals part, \( \Delta G_i^{clW} \), and the acid-base part, \( \Delta G_i^{cAB} \), i.e.,

\[ \Delta G_i^C = \Delta G_i^{clW} + \Delta G_i^{cAB} \]  

(7–4)

The relationship between adhesion and surface energy is:

\[ \Delta G_{ij}^a = \Gamma_i + \Gamma_j - \Gamma_{ij} \]  

(7–5)

where \( \Gamma_{ij} \) is the interfacial surface energy between i and j. There are two components for interfacial surface energy as described in the following:
\[ \Gamma_{ij} = \Gamma_{ij}^{LW} + \Gamma_{ij}^{AB} \quad (7-6) \]

The Berthelot geometric mean is used to calculate the Lifshitz-van der Waals component of surface energy as follows (82):

\[ \Gamma_{ij}^{LW} = (\sqrt{\Gamma_{i}^{LW}} - \sqrt{\Gamma_{j}^{LW}})^2 \quad (7-7) \]

The acid-base component of surface energy due to the complementarity of the acid-base interaction is defined as follows:

\[ \Gamma_{ij}^{AB} = 2(\sqrt{\Gamma_{i}^{-}} - \sqrt{\Gamma_{j}^{-}})(\sqrt{\Gamma_{i}^{+}} - \sqrt{\Gamma_{j}^{+}}) \quad (7-8) \]

Therefore, adhesion can also be described by the Lifshitz-van der Waals part, \( \Delta G_{i}^{aLW} \), and the acid-base part, \( \Delta G_{i}^{aAB} \), i.e.,

\[ \Delta G_{ij}^a = \Delta G_{ij}^{aLW} + \Delta G_{ij}^{aAB} \quad (7-9) \]

\[ \Delta G_{ij}^{aLW} = 2\sqrt{\Gamma_{i}^{LW} \Gamma_{j}^{LW}} = \sqrt{\Delta G_{i}^{cLW} \Delta G_{j}^{cLW}} \quad (7-10) \]

\[ \Delta G_{ij}^{aAB} = 2\sqrt{\Gamma_{i}^{+} \Gamma_{j}^{-}} + 2\sqrt{\Gamma_{i}^{-} \Gamma_{j}^{+}} \quad (7-11) \]

### 7.4 Experimental Data of Surface Energy

In this section, three different aggregates and two different binders, i.e., six different combinations are studied to see how surface energies affect cracking behavior of asphalt concrete. Table 7.1 lists the surface energy of different kinds of binder and
aggregate, which are measured by lab experiments. Those data are from (56). Table 7.2 and Table 7.3 list the cohesion and adhesion strengths of those different combinations, which are obtained through the analysis in Section 7.3. These data are input information for lattice modeling of IDT test simulation.

Table 7.1 List of Component Surface Energy (ergs/cm²)

<table>
<thead>
<tr>
<th>Component</th>
<th>( \Gamma )</th>
<th>( \Gamma_{LW} )</th>
<th>( \Gamma_{AB} )</th>
<th>( \Gamma^+ )</th>
<th>( \Gamma^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Georgia Granite</td>
<td>206.5</td>
<td>133.2</td>
<td>73.3</td>
<td>24.1</td>
<td>96.0</td>
</tr>
<tr>
<td>Colorado Limestone</td>
<td>87.3</td>
<td>79.9</td>
<td>7.3</td>
<td>0.1</td>
<td>206.5</td>
</tr>
<tr>
<td>Rhine River Moraine Gravel</td>
<td>278.1</td>
<td>137.3</td>
<td>140.7</td>
<td>8.3</td>
<td>598.0</td>
</tr>
<tr>
<td>AAD</td>
<td>27.14</td>
<td>8.59</td>
<td>18.55</td>
<td>5.96</td>
<td>14.48</td>
</tr>
<tr>
<td>AAM</td>
<td>48.35</td>
<td>9.33</td>
<td>39.02</td>
<td>17.79</td>
<td>21.40</td>
</tr>
</tbody>
</table>

Table 7.2 Cohesion Strength (ergs/cm²)

<table>
<thead>
<tr>
<th>Component</th>
<th>( \Delta G_{i \Lambda}^{c_{LW}} )</th>
<th>( \Delta G_{i \Lambda}^{c_{AB}} )</th>
<th>( \Delta G_{i \Lambda}^{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Georgia Granite</td>
<td>266.4</td>
<td>146.6</td>
<td>413</td>
</tr>
<tr>
<td>Colorado Limestone</td>
<td>159.8</td>
<td>14.6</td>
<td>174.4</td>
</tr>
<tr>
<td>Rhine River Moraine Gravel</td>
<td>274.6</td>
<td>281.4</td>
<td>556</td>
</tr>
<tr>
<td>AAD</td>
<td>17.18</td>
<td>37.1</td>
<td>54.28</td>
</tr>
<tr>
<td>AAM</td>
<td>18.66</td>
<td>78.04</td>
<td>96.7</td>
</tr>
</tbody>
</table>

Table 7.3 Adhesion Strength (ergs/cm²)

<table>
<thead>
<tr>
<th>Component</th>
<th>Georgia Granite</th>
<th>Colorado Limestone</th>
<th>Rhine River Moraine Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAD</td>
<td>130</td>
<td>123</td>
<td>210</td>
</tr>
<tr>
<td>AAM</td>
<td>176</td>
<td>177</td>
<td>304</td>
</tr>
</tbody>
</table>

7.5 IDT Simulation by Elastic Models

An indirect tensile test on an asphalt concrete specimen under low temperature is simulated through the MSVF-LM approach in this section. As shown in Figure 7.10,
diameter and thickness of the specimen are 150 mm and 38 mm, respectively. The same mix design as for UTT tests is used to virtually fabricate the IDT specimen (Table 3.1). Since the purpose of this section is to study the effects of surface energy, the modulus of binder is assumed as 152 MPa, the moduli of all the three kinds of aggregate are assumed same as 50 GPa, and the surface energy data are listed from Table 7.2 and Table 7.3.

![Indirect Tensile Test](image)

**Figure 7.1 Indirect Tensile Test**

The procedure and results of numerical simulation are given in Figure 7.2~ Figure 7.8. Figure 7.2 illustrates the procedure to determine the moduli of each scale. Same as for the simulation of UTT, the volumetric design aggregate gradation is first converted into the apparent area gradation through inverse stereology (Figure 7.2a). The resulting area gradation is then divided into four regions, 4.75~19 mm for scale 1, 1.18~4.75 mm for scale 2, 0.3~1.18 mm for scale 3, 0~0.3 mm for scale 4. These four area gradation segments (Figure 7.2b) are used to generate IDT virtual specimens and mastic RVEs (Figure 7.2c) for each scale, respectively. For scales 2~4, by running uniaxial virtual
tensile tests, the modulus of mastic of each scale can be obtained. The statistical average values (Figure 7.2d) are then used as the input material properties for the upper scale. Once the modulus of the mastic for scale 2 is obtained, by combining with the surface energy obtained through virtual single notch testing of scale 2, the IDT virtual testing can be carried out (Figure 7.2e).

Figure 7.3~ Figure 7.8 illustrate the procedure of determining cohesion strength of mastic and adhesion strength of mastic and aggregate. For scales 2~4, by running single notch virtual tests (Figure 7.3a ~ Figure 7.8a), the cohesion strength of mastic, which is defined as the total energy lost divided by the macro cracking length, is obtained (Figure 7.3b ~ Figure 7.8b). Then the adhesion strength can be obtained according to equation (7-5) (Figure 7.3c ~ Figure 7.8c). By inputting the modulus, the cohesion and adhesion strengths of scale 2, the IDT cracking pattern and load-deflection curve of each combination can be obtained (Figure 7.3d ~ Figure 7.8d).

By comparing Figure 7.3a ~ Figure 7.8a, one can see that different combinations have different cracking patterns for mastic. However, Figure 7.3d ~ Figure 7.8d show that the cracking patterns of IDT specimens for different combinations are almost the same. This means that surface energy affects the micro crack pattern but has little effect on the macro crack pattern. Furthermore, by comparing the load-deflection curves, one can say that surface energy does affect the failure load. In order to make this clearer, Figure 7.9 shows all the load-deflection curves of the six different combinations. From this figure, one can see that surface energy does not affect the stiffness but does affect the failure strength of the specimen, i.e., the larger the surface energy, the larger the failure strength the specimen has.
Figure 7.2 Mastic Modulus Determination for IDT Using MSVF-LM
Figure 7.3 Mastic Cohesion and Adhesion Strengths Determination for IDT Using MSVF-LM (Colorado Limestone and AAD (CD))
Figure 7.4 Mastic Cohesion and Adhesion Strengths Determination for IDT Using MSVF-LM (Colorado Limestone and AAM (CM))
Figure 7.5 Mastic Cohesion and Adhesion Strengths Determination for IDT Using MSVF-LM (Georgia Granite and AAD (GD))
Figure 7.6 Mastic Cohesion and Adhesion Strengths Determination for IDT Using MSVF-LM (Georgia Granite and AAM (GM))
Figure 7.7 Mastic Cohesion and Adhesion Strengths Determination for IDT Using MSVF-LM (Rhine River Moraine Gravel and AAD (RD))
Figure 7.8 Mastic Cohesion and Adhesion Strengths Determination for IDT Using MSVF-LM (Rhine River Moraine Gravel and AAM (RM))
7.6 UTT Simulation Results

In the previous section, the effects of surface energy are studied by using elastic models to simulate IDT tests. In this section, the effects of surface energy will be studied by using viscoelastic models to simulate uniaxial tension tests, i.e., uniaxial tension tests on an asphalt concrete specimen with different binder properties, under $-10^\circ C$ and $8.16 \times 10^{-4}$ strain/second LVDT strain rate, are simulated. Using the same procedure as described in Section 6.3, the relaxation moduli of AAD and AAM can be derived from the dynamic shear modulus $G^*$ data in reference 85. Table 7.4 lists the Prony Series constants for AAD and AAM, and the corresponding relaxation modulus curves are plotted in Figure 7.10.
Table 7.4 Constants of Prony Series for AAD and AAM

<table>
<thead>
<tr>
<th>i</th>
<th>$\rho_i$</th>
<th>$E_i$ (MPa)</th>
<th>$\rho_i$</th>
<th>$E_i$ (MPa)</th>
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<tr>
<td>1</td>
<td>1E-09</td>
<td>2290918.908</td>
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<td>2334848.991</td>
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<tr>
<td>2</td>
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<td>1E-08</td>
<td>2635558.36</td>
</tr>
<tr>
<td>3</td>
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<td>0.000001</td>
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<tr>
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<td>2852534.616</td>
<td>0.00001</td>
<td>2524527.892</td>
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<tr>
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<tr>
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<tr>
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<td>10</td>
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<td>12301.23722</td>
</tr>
<tr>
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</table>

Figure 7.10 Relaxation Moduli of AAD and AAM
By multiplying the relaxation moduli the thin film factor 1.91 and the scale factor 7.25, the obtained mastic relaxation moduli are then used as the inputs for lattice simulation.

The surface energy values of mastic are obtained through MSVF-LM. Figure 7.11 and Figure 7.12 illustrate the procedure of determining cohesion strength of mastic and adhesion strength of mastic and aggregate. For scales 2~4, by running single notch virtual tests (Figure 7.11a and Figure 7.12a), the cohesion strength of mastic, which is defined as the total energy lost divided by the macro cracking length, is obtained (Figure 7.11b and Figure 7.12b). Then the adhesion strength can be obtained according to equation (7-5) (Figure 7.11c and Figure 7.12c). By inputting the modulus, the cohesion and adhesion strengths of scale 2, the stress-strain curves for AAD and AAM can be obtained, as shown in Figure 7.13.

From Figure 7.13, one can get the similar observation as the study in the previous section, i.e., surface energy does affect the failure strength of the specimen, the larger the surface energy, the larger the failure strength the specimen has.
<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Image] SEN Virtual Testing of RVEs at scale 4</td>
<td><img src="image.png" alt="Graph" /> Mastic Cohesion of RVEs</td>
<td><img src="image.png" alt="Graph" /> Mastic and Aggregate Adhesion of RVEs</td>
</tr>
<tr>
<td>[Image] SEN Virtual Testing of RVEs at scale 3</td>
<td><img src="image.png" alt="Graph" /> Mastic Cohesion of RVEs</td>
<td><img src="image.png" alt="Graph" /> Mastic and Aggregate Adhesion of RVEs</td>
</tr>
<tr>
<td>[Image] SEN Virtual Testing of RVEs at scale 2</td>
<td><img src="image.png" alt="Graph" /> Mastic Cohesion of RVEs</td>
<td><img src="image.png" alt="Graph" /> Mastic and Aggregate Adhesion of RVEs</td>
</tr>
</tbody>
</table>

Figure 7.11 Mastic Cohesion and Adhesion Strengths Determination for AAD
Figure 7.12 Mastic Cohesion and Adhesion Strengths Determination for AAM
7.7 Summary

Surface energy is studied through IDT and UTT tests simulation in this chapter. The results show that surface energy affects cracking patterns of mastic. Furthermore, surface energy affects the failure strength of the specimen, i.e., the larger the surface energy, the larger the failure strength the specimen has.
8 SUMMARY AND FUTURE WORK

In this thesis, micromechanical modeling, which is composed of microstructure generation and lattice modeling, is introduced to study the cracking behavior of asphalt concrete under low temperature.

The digital imaging processing (DIP) of physical specimens to generate microstructures essentially involves digital image acquisition, image processing by IMAQ, and data file generation. Although DIP can accurately capture the microstructure of asphalt concrete, it requires fabrication of physical specimens that could be expensive. As an alternate approach of microstructure generation, virtual fabrication is proposed and tested, which is able to generate asphalt concrete microstructure directly from the constituents’ information instead of having to physically fabricate specimens. Two important geometrical parameters, i.e., aspect ratio and orientation distributions of aggregate particles for IDT specimen simulation, are also studied.

Lattice modeling approximates a continuum by using a lattice, so that the cracking process can be simulated by successive removal of failed links. A random lattice mesh generation procedure, which can precisely capture the location and configuration of aggregate particles, is developed. The resulting lattice mesh can then be analyzed by numerical solvers.

The computational cost of lattice modeling is investigated, which shows that a multi-scale procedure is necessary for simulating laboratory experiments. Through integration of virtual fabrication and lattice modeling with the multi-scale procedure, the Multi-Scale Virtual Fabrication and Lattice Modeling (MSVF-LM), is then proposed.

The effectiveness of MSVF-LM is investigated with the help of the quantitative
prediction of the uniaxial tension tests. The comparison between numerical simulation and physical experimental data shows that MSVF-LM is able to precisely predict the crack patterns and the mechanical response, and shows excellent promise for micromechanical modeling of asphalt concrete.

The results of surface energy effects study show that surface energy doesn’t affect the stiffness but does affect the failure strength of the specimen. The larger the surface energy, the larger the failure strength the specimen has.

However, the generated virtual microstructures for UTT simulation are still based on the distributions (aspect ratio and orientation) obtained from DIP analysis of physical fabricated specimen. Therefore, the vertical particle cut sections’ aspect ratio and orientation distributions for different specimens with different kinds of aggregates still need to be investigated further, so that a sufficient database and some empirical relationships can be established.

Also, the effect of thin film needs to be studied more quantitatively, so that the proper binder properties can be used as the input for the lattice simulation.

Furthermore, a viscoelastic fracture law needs to be developed and evaluated, so that the cracking behavior of asphalt concrete at high temperature and low loading rate could be simulated effectively.
REFERENCE


17. Eriksen, K., and Wegan, V., Optical Methods for the Evaluation of Asphalt Concrete and Polymer-modified Bituminous Binders, *Report Note 244*, Danish Road Institute, 1993


APPENDIX I - IMAGE QUALITY CONTROL FACTORS*

For an imaging system, there are five factors that overall image quality, i.e. resolution, contrast, depth of field, perspective, and distortion.

Resolution in broad terms is an expression of the amount of information transmitted or received. For a scanner, resolution is the maximum number of pixels per inch. Resolution is measured in dots per inch (dpi).

Contrast defines the differences in intensity values between the object under inspection and the background. Contrast and resolution are closely related factors contributing to the image quality.

Depth of Field of a lens is its ability to keep objects of varying heights in focus. If objects have to be inspected with various heights, the lens should be able to maintain the image quality as the objects move closer to and further from it.

Perspective errors often occur when the camera axis is not perpendicular to the object being inspected. Figure A.1a shows an ideal camera position. Figure A.1b shows a camera imaging an object from an angle. Figure A.2a shows a grid of dots. Figure A.2b illustrates perspective errors caused by a camera imaging the grid from an angle. Therefore, it is very important to position the camera perpendicular to the object to reduce perspective errors.
Distortion is a geometric aberration caused by optical errors in the camera lens. A typical camera lens introduces radial distortion. This causes points that are away from the lens’s optical center to appear further away from the center than they really are. Figure A.2c illustrates the effect of distortion on a grid of dots. When distortion occurs, information in the image is misplaced relative to the center of the field of view.

* All this information is from IMAQ Vision Concepts Manual by National Instruments.