ABSTRACT

PANUSITTIKORN, WITOON  Error Compensation Using Inverse Actuator Dynamics
(Under the direction of Thomas A. Dow.)

The use of Non-Rotationally Symmetric (NRS) optical surfaces has increased significantly in recent years. These surfaces can be quickly machined using a diamond turning lathe and a Fast Tool Servo (FTS). However, the dynamic response of the actuator influences the output tool motion resulting in form errors. This dissertation describes an open-loop command modification technique, also know as a deconvolution operation, that can significantly reduce tool motion errors. The technique uses knowledge of the gain and phase response of the dynamic system and the information content of the driving command to modify the amplitude, phase and shape of the input command signal. The modified command signal creates the desired tool motion.

The research made use of a commercial FTS to demonstrate the error compensation technique and implemented a system identification method using Digital Signal Processing (DSP) to determine the closed-loop transfer function of the actuator. This measurements exposed the physical limits of the FTS which constrained the tool speed to 140 mm/s and the nonlinear dynamic behavior of the FTS as a function of the command amplitude. Initial validation of the inverse dynamics technique for a fixed amplitude tool trajectory employed a single transfer function to modify the entire input command signal. The experiments showed that the excursion of the actuator was identical to the desired path after a start up period.
The tool trajectory of an FTS is dependent on the spindle speed and the cross-feed rate which may drift over a long fabrication time and an NRS feature can contain varying amplitude. To address these issues, two adaptive modification schemes using the Short-Time Fourier Transform (STFT) and an equivalent inverse dynamics filter were proposed. These schemes can account for the changing frequency and amplitude content of the driving command using the most recent machining conditions and the appropriate transfer function.

The experimental confirmation of the error compensation scheme involved the fabrications of two off-axis features: a sphere and a cosine groove. The adaptive schemes were used to create an off-line modified input command. Measurements of the machined surfaces using a laser interferometer experimentally validated that the deconvolution operation can extend the usable bandwidth of the FTS to produce the proposed surfaces. The measurements across the machined parts indicated that the form fidelity of the deconvoluted features was improved by 2 orders of magnitude over that of the features produced using the unmodified input command signals.
ERROR COMPENSATION USING INVERSE ACTUATOR DYNAMICS

by

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BIOGRAPHY

Witoon Panusittikorn graduated from Chulalongkorn University, Bangkok, Thailand with his Bachelor of Mechanical Engineering in July, 1996. Then he entered North Carolina State University and received Master of Industrial Engineering and Master of Science in Mechanical Engineering in 1998 and 2001 respectively. His thesis was titled “Modeling and Control of Friction Based Object Transport Using Two Mode Ultrasonic Excitation” in which a magnetic-free material handling system is precisely controlled using ultrasonic vibration. Witoon pursued his Ph.D. in Mechanical Engineering and joined the Precision Engineering Center in August of 2001. In the first year of his tenure, he successfully developed a nonlinear controller for a magnetostrictive actuator to compensate for an intrinsic magnetic hysteresis. Within two years, he implemented a feedforward modification technique that can reduce a motion error caused by the system dynamics and completed his Ph.D. work under the direction of Dr. Thomas A. Dow.
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CHAPTER 1

INTRODUCTION

The use of high precision components with more advance, and more complex geometry has increased significantly in both military and civilian applications, particularly for optical systems [1]. A turning process with a diamond cutting tool can produce machined surfaces with high form fidelity and short production time in which traditional sequential processes of milling, grinding, and polishing hardly accomplish. In lights of the development of fluid-mechanic systems, laser interferometers, temperature and vibration controlled environments, and axes feedback controllers, the improved quality of diamond turned surfaces can be advantageous to all optics industry.

1.1 DIAMOND TURNING AND FAST TOOL SERVO

A turning process with a diamond cutting tool is capable of producing high quality surfaces in a variety of nonferrous materials. Surfaces of revolution are realized by moving the tool and/or workpiece through one-half of a cross-section of the desired shape. The rotation of
the workpiece on a spindle causes a symmetric surface to be machined. Advantages of diamond turning are reduced process time as compared to fly-cutting due to the continuous high cutting speed achieved via rotation of the workpiece, excellent form fidelity and optical quality surface finish. It is desirable to produce optical systems with the diamond turning process that have high spatial frequency features or are non-rotationally symmetric (NRS). The addition of a high-bandwidth actuator is one technique [2-4] that has been used to add low amplitude, sub-revolution features to an asphere or sphere. Examples include torics and off-axis segments of a conic machined on center. Figure 1-1 illustrates a configuration of machining a NRS feature; a DTM is moving its orthogonal axes in x and z directions, and spinning an arm that holds a part off the rotational axis while a FTS is moving in and out in w direction as a function of $\theta$ to generate a high spatial frequency component.

Another demonstration of machined part realized by rotation of a diamond turning machine (DTM) and sinusoidal motion of a fast tool servo (FTS) is a fabrication of inertial confinement fusion targets [13]. These parts were for an investigation of a physics phenomenon at Los Alamos National Labs. Sine wave feature with constant amplitude and frequency was produced on the OD of a brass cylinder as depicted in Figure 1-2.

Challenges of this fabrication were the small size of part, as shown in Figure 1-3, and the small amplitude of the sine wave, less than 5 $\mu$m. While the base machine moved to machine a cylinder, the FTS synchronously superimposed the sinusoid feature into the brass surface.
Figure 1-1. Diamond turning machine (orthogonal stages and air bearing spindle) and a fast tool servo

Figure 1-2. The base machine generates a cylindrical surface while the FTS superimposes a sine wave feature
Figure 1-3. The FTS inscribed a $10\mu m$ P-V sine wave into a brass cylinder

(a) A ring gauge measurement artifact  
(b) A sine wave feature with varying spatial frequencies inscribed on the ID of the gauge

Figure 1-4. Geometric description of the ring gauge
Another demonstration of high precision fabrication is a metrology artifact (or a ring gauge) for a Coordinate Measuring Machine (CMM) [14]. An aluminum ring as shown in Figure 1-4(a) was inscribed with a varying-spatial-frequency sine wave on the ID to determine any anomalies on a surface that contribute to a measurement. The sine wave feature had fixed amplitude of \(5\mu\text{m P-V}\) and continuously increasing wavelength from 0.00125” to 0.250” over the first quadrant of the ring. The other 3 quadrants had mirror images of the first quadrant as shown in Figure 1-4(b).

1.2 NRS Fabrication

The examples of the last section illustrate a range of possible surface shapes feasible using a FTS on a diamond turning machine. Some use the FTS to create the surface on a cylindrical face (fusion targets and ring gauges.) Others require coordinated motion of the FTS and the axes as a function of \(x, w, \text{ and } \theta\) to produce the desired surface shape, such as the off-axis conic machined on axis. In this case, the NRS component is superimposed on the best fit asphere.

Two challenges must be addressed when using the FTS to create NRS surfaces: 1) how to decompose the surface shape into symmetric and non-symmetric component, and 2) how to deal with the different dynamic response of the machine and FTS axes.
1.2.1 Static Decomposition

The motion commands for each axis of the base machine and the FTS must be generated from a 3D geometric description of the desired part. While the fusion targets and the ring gauges are on the cylindrical surface, the features must be located, in some cases, with respect to some reference angle on the part. The part shape must be decomposed into a rotationally symmetric (RS) surface of revolution (i.e., a cross section) for the diamond turning machine (DTM) and a spindle angle dependent motion path of a non-rotationally symmetric (NRS) surface for the FTS. In Figure 1-5, while the DTM axes move to machine an asphere (or RS surface), the FTS superimposes a high frequency feature (or NRS surface).

Figure 1-5. The FTS axis trajectory composes a relatively higher frequency than the base machine axes
The knowledge of a desired trajectory is available in advance by decomposing a 3D geometric description and superimposing a spiral pattern to describe a trajectory of a tool center as shown in Figure 1-6.

![Spiral Path Diagram]

**Figure 1-6.** A spiral path overlaid the part represents a tool excursion associated with a spindle speed and a cross feed.

The principle difficulty is that while the FTS has a high relative bandwidth, it has a limited range of motion when compared to the other axes of the DTM. This surface decomposition process has been solved for a variety of shapes and will not be addressed.

### 1.2.2 Dynamic Convolution

The second complication is the relative dynamic response of the machine axes. Synchronization of axes motion is critical for machining NRS optical systems; therefore, a
small lag time associated with an FTS will result in poor form fidelity for off-axis surfaces and improper placement of NRS features with respect to the rotationally symmetric (RS) base surface and any fiducial. The heterogeneous nature of the axes can result in phase errors in the recomposition of the decomposed command signals. The example in Figure 1-5 shows that the decomposed FTS motion changes direction at least once on each rotation of the spindle whereas the axis motions are almost always in one direction (a waxicon being a notable exception). The phase errors of the base machine axes can usually be ignored as the axis accelerations are moderate and velocity tracking performance of the control system is quite good. However, the FTS usually requires higher acceleration and velocity.

All linear systems including actuators exhibit gain and phase response to varying input command signals. In Figure 1-7, an output response (a solid line) associated with a 250 Hz sinusoidal input signal (a dash line) is a sine wave with its input frequency, but with attenuated amplitude and a phase delay.

These dynamics result in form errors to machined surfaces and make it difficult to utilize the full bandwidth of the axes for precision machining. The influence of the FTS dynamic response can be illustrated by the inscription of the ring gauge artifact. Figure 1-8 shows the amplitude of the machined sine wave observed by an LVDT. This figure shows the measurement indicated degree of attenuation associated with the frequency of the feature. Significant distortion appeared where the high frequencies occurred.
Figure 1-7. An attenuated output excursion with a phase angle of an actuator due to its dynamic characteristics

Figure 1-8. Dynamics of the FTS influences form errors of the machined surface
A first order correction of the FTS phase error is to apply a constant lead time to its motion commands. The magnitude of this phase advance depends only on actuator bandwidth and spindle velocity. However, for motion paths that use a substantial portion of the bandwidth range or contain frequency components close to the bandwidth of the axes, this simple technique yields poor results. The response of a dynamic system to an applied signal is a function of the frequency of that signal. In general, the response results in an attenuated and out-of-phase motion of the output with respect to the input signal. When the system is a fast tool servo and the dynamic input signal is made up of a variety of frequencies, the result is form error in the machined surface.

1.3 REVIEW OF LITERATURE

Many investigators have tried to improve the performance of a dynamic system. In classical controls [5], a pole-zero cancellation scheme modifies the closed-loop dynamic response of the system resulting in significant reduction of steady-state amplitude gain errors. However, this technique cannot produce a zero phase response. As with the dynamic response of an actuator, the dynamics replacement still yields a small phase angle to output excursions. A demonstration of this technique is described in Appendix A. Traditional feedback controller design makes little or no use of the information content of the driving command signal. The time used to acquire a signal and calculate an input command makes the output lag behind the desired signal.
Implementation using stable poles and zeros approximates known resonance, attenuation modes and damping factors of the dynamic system to shape the closed-loop characteristics. As a result, it is sensitive to unknown dynamics, and the unity gain response is limited to the resonant frequencies of the unstable zeros.

Zero Phase Error Tracking Control (ZPETC) [6-10] is a digital feedforward and look-ahead control algorithm that modifies the pole-zero cancellation to reject the dynamic response of stable poles and well-damped zeros. Using knowledge of (entire or partially-known in advance) desired trajectories compensates phase responses of lightly damped.

Feedforward ZPETC has been used in many control schemes such as digital tracking control [6-7] because knowledge of the entire tool excursion is available. A series of feedforward corrective gains expressed by the unstable zeros of the system was attached to exponentially reduce the ZPETC tracking error [8]. A continuous-time control algorithm of the ZPETC [9] was developed with a series of gain compensation for a sinusoidal trajectory. An adaptive control cascading a digital prefilter (DPF) to the ZPETC [10] improves gain and phase response with a low computation cost.

An Adaptive Feedforward Cancellation scheme [11-12] observed tracking errors with respect to periodic trajectories as disturbances with arbitrary harmonic components. The error signal can be illustrated as a summation of sinusoids of known frequencies, but unknown amplitudes. Therefore, a set of harmonic sine wave with negative gains is attached to
resonate an input command signal at harmonic frequencies and eliminate the path error. This control scheme is limited to periodic input command signals.

Another breakthrough of an input command modification technique is an input shaping technique [13] that reduces residual vibration of a flexible mechanical system when following an instantaneous change direction trajectory such as a step. The scheme has been applied to many systems such as a computer disk drive, nanopositioners, etc. Though it seems to be a promising command modification scheme for many applications, the technique does not meet a fundamental requirement of a precision machining; the scheme does not correct errors due to the phase response.

This dissertation describes the development of an inverse dynamics algorithm that enhances both the usable bandwidth and the tracking accuracy of an actuator. If an entire desired input command signal for each axis of a well characterized machine is generated off-line, a new command signal can be derived which will drive the axes to follow the correct motion path. The idea is related to reference feedforward control, but by considering the complete command signal \textit{a priori}, performance of the overall machine tool is greatly enhanced. This technique depends critically on knowledge of the actuator dynamics and does not replace the feedback controller. Rather it improves the closed-loop response of an actuator by compensating for high frequency dynamics that the servo feedback loop cannot correct due to saturation of the drive system. Note that the inverse dynamics scheme, known as deconvolution, has been used only in communication and image processing. The signal processing technique is herein applied to high precision mechatronics.
1.4 OBJECTIVES OF THE RESEARCH

The objectives of the research are as follows:

1. To investigate the dynamic response of a tool axis to an input command signal and construct an algorithm to characterize the inverse of the FTS dynamics.

2. To improve the form fidelity of non-rotationally symmetric (i.e., spindle angle dependent) surfaces machined on a diamond turning lathe by incorporating knowledge of the desired surface and the dynamics of a tool axis into the tool path generation process.

3. To discover the range and quality of features that can be fabricated using the tool axis and then to use that information to select the cutting conditions that optimize the desired surface.

1.5 OUTLINE OF DISSERTATION

In Chapter 2, the principle as well as the fundamental theory behind the inverse dynamics or deconvolution technique will be addressed. This leads to the use of Digital Signal Processing (DSP) to implement the algorithm and circumscribes a practical range of the technique to an actuator. Two validation schemes of the command modification written in Chapter 3 and 6 will be investigated on a Variform FTS.
Chapter 3 introduces a procedure of measuring the dynamic response of an actuator. Then, the FTS will make use of this measurement to perform a preliminary test on the deconvolution. Frequency components of a desired trajectory will be chosen close to the FTS bandwidth, which a output motion is nearly an inverse of its applied input signal. Summary of a significant improvement will, consequentially, be drawn from both simulation and experimental results.

The implementation issues that have came across in the test will be discussed in Chapter 4. Physical limitation plays an important role to describe an operating range of the modified algorithm. Another issue is a hidden dynamic response of an internal position sensor of the Variform FTS. Accordingly, the fault monitored tool motion clouded the true performance of the inverse dynamics algorithm. A use of an laser interferometer will be discussed in detail to identify the actual characteristic of the actuator.

The feedforward and look-ahead scheme can also use a small sliding window of trajectory data to improve tracking performance and to reduce the overshoot of an accelerated axis. Chapter 5 will discuss practical implementation methods of block convolution. Short-Time Fourier Transform and equivalent inverse dynamics filter break a long signal of a desired input command into short pieces before performing the inverse dynamics algorithm.

The second validation is to correct form errors due to gain and phase response on a machined surface. A groove with a sinusoidal cross-section, and a small, concave off-axis sphere machined into a flat surface were selected for this verification. While the first sagittal feature
with constant amplitude demonstrates an intermediate machine test for the modification to a fixed frequency component of a tool excursion, the spherical feature composed of mixed amplitude and frequency contents, which can be varied by changing its dimension, its off-axis distance, and machining parameters such as spindle velocity and cross feed, will be an inclusive investigation. Chapter 6 will describe steps of implementing a deconvolved surface and conclude the performance of the modification technique from the form errors of the machined surfaces.
CHAPTER 2
CONVOLUTION AND DECONVOLUTION

This chapter describes the fundamental mathematics that relates the input command signal, the system dynamics, and the gain and phase response of the output motion. Based on specific assumptions to establish an operating range, the inverse of this input/output operation can be used to create a modified input command that will produce the desired output. Digital Signal Processing (DSP) and Discrete Fourier Transform (DFT) can be used to transfer time-based signal into the frequency domain. Note that the technique can be applicable to any dynamic system; however, for the application in this dissertation (diamond turning of non-rotational symmetric surfaces), the system are electro-mechanical linear and rotary axes. These include a fast tool servo, a long range tool servo, and a rotary tool servo.

2.1 LINEAR, TIME-INVARIANT SYSTEM

To maintain its repeatability and achieve high fidelity, a linear, time-invariant system is a desirable feature for a precision machining axis. The linearity results in homogeneous
characteristics between an output and input signal when the form of the output signal is a copy of the input, but with different phase and amplitude. And it also means that transfer function of the system does not change with time. Figure 1-7 is a good illustration of this property. To its input signal, the response conforms with the same frequency though it is out of phase and has smaller amplitude. Note that the command modification techniques discussed in Section 1.3 also require this assumption.

In addition to the homogeneity, another property of a linear system is the superposition principle; a signal can be decomposed into a number of sub-signals which when applied to the system produce the same result as their sum. These homogeneity and superposition yield an alternative way to determine an output signal by decomposing the input signal into sub-signals, applying the signals individually to the system resulting in sub-output signals, and finally recomposing them to the output.

Mathematically, when a linear system is driven by a periodic function $x(t)$ with an amplitude $A$ and a frequency of $\omega$ radians per second, the steady-state output motion $y(t)$ is attenuated by a gain factor $a$ and is phase shifted by an angle $\phi$.

**Input:** $x(t) = A \sin(\omega t)$

**Output:** $y(t) = a A \sin(\omega t + \phi)$

If the input $x(t)$ contains multiple amplitudes or frequencies, the corresponding output can be determined using superposition; that is, the input signal can be decomposed into single-
frequency components each of which will be attenuated and delayed. The response of the system \( y(t) \) will be the sum of these frequency components.

\[
\text{Input: } x(t) = \sum_i A_i \sin(\omega_i t) \\
\text{Output: } y(t) = \sum_i a_i A_i \sin(\omega_i t + \phi_i)
\]

To machine a desired surface profile with high fidelity, the amplitude attenuation and phase related delay must be eliminated. To achieve this, the input signals are modified prior to applying them to the system such that the attenuation is canceled and the phase is compensated. After the dynamics of the system are identified and the desired motion is decomposed into sinusoids, amplitude \( a_i \) and phase \( \phi_i \) adjustments can be found to manually construct a modified command that moves the actuator in the desired manner.

\[
\text{Modified Input: } x(t) = \sum_i \frac{A_i}{a_i} \sin(\omega_i t - \phi_i) \\
\text{Desired Output: } y(t) = \sum_i a_i \frac{A_i}{a_i} \sin(\omega_i t + \phi_i - \phi_i)
\]

This manual method is slow and cumbersome if the signal contains many frequency components. However, the DSP and DFT can be used to minimize the effort. A technique for system identification and an input signal modification algorithm have been developed and applied to a tool axis to produce the desired output response for high frequency inputs.

Some precision machining tool axes are composed of non-linear mechanisms such as a piezo-driven actuator that exhibit hysteresis. The FTS used in this research (the Variform™

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Fast Tool Servo (FTS)) implements a reference capacitor loop control (discussed in detail in Appendix B) to linearize its closed-loop performance with a broad bandwidth. Despite achieving the linear characteristics, the FTS output shows gain and phase response of a second-order system and is equivalent to a response of a spring-mass-damper system as shown in Figure 2-1. An input command is influenced by the dynamics of the system resulting in an output excursion with an attenuation and a phase delay.

![Figure 2-1. Spring-mass-damper system](image)

The dynamic response of an FTS can be presented in either the time domain (impulse response) or the frequency domain (frequency response). They are both shown in Figure 2-2 and are mathematically equivalent and complete for a linear system. On the left in Figure 2-2(a), the top graph shows the ratio of the amplitude of the output to the input (in dB\(^1\) units) as a function of frequency. At low frequency, the output is equal to the input and the ratio is unity giving a dB value of zero. As the frequency is increased, the amplitude response peaks

\[ \text{dB} = 20 \log_{10} \left( \frac{\text{output}}{\text{input}} \right) \]
at about 200 Hz and then drops rapidly for higher frequencies. The lower graph in Figure 2-2(a) shows the phase angle between the input and the output. At lower frequencies these signals are nominally in phase, but even a small lag of less than a degree represents a significant length of arc if the actuator is used at a large radius from the axis of spindle rotation. As the frequency rises, the output increasingly lags the input. At 100 Hz this lag is about 45° and at 340 Hz it is about 180°.

The impulse response is shown on in Figure 2-2(b) and describes the response of the system with respect to time for an impulse at time zero. The Fourier transform of the impulse response is exactly equal to the frequency response and the inverse Fourier transform of the frequency response is the impulse response.

![Graphs](image)

(a) Dynamic response of a Variform FTS expressed in the frequency domain

(b) expressed in the time domain

**Figure 2-2.** Dynamic response of a FTS can be presented in either the frequency or the time domain
2.2 **Digital Signal Processing (DSP)**

The DSP was originally used in signal processing applications such as noise-rejection in a communication signal. As digital data acquisition systems have become faster and more powerful, the methodology has been fully developed in wide areas such as communications, feedback control, imaging, sensors, and high fidelity music reproduction [1].

2.2.1 **Analog to Digital Conversion**

The motions of electro-mechanical systems are characterized by continuous time-dependent variables. For example, exerting a small movement input \(x(t)\) to a spring-mass-damper system results in a small output motion \(y(t)\). Although both movements change continuously with respect to time, a digital data acquisition system cannot measure this continuity because it contains infinite number of points, and the acquisition system has a limited memory. Rather, the acquisition system produces a finite representation of that motion in the discrete time domain. Accordingly, the continuous time \(t\) becomes,

\[
t = n \ \delta t
\]

where \(n = 0, 1, \ldots, N\) and \(\delta t\) is a sampling time. Thus, a continuous motion \(y(t)\) is represented as a finite series of discrete motion, \(y[n \delta t]\), usually shortened to \(y[n]\) as shown in Figure 2-3.
2.2.2 Sampling Theorem

The basis for digital technology was provided by the Nyquist or sampling theorem and applied to communication theory in 1949 by Shannon and Weaver [2]. It establishes a formal mathematical link between the physical world of continuous signals, both periodic (Fourier series) and aperiodic (Fourier transform), and the reality of modeling physical phenomena based upon discrete and finite observations. The theorem states that a discrete time sequence obtained by sampling a continuous function contains enough information to reproduce the function exactly, provided that the sampling rate is greater than twice the highest frequency contained in the original signal. Furthermore, it provides a means of reconstructing the bandwidth limited signal given the sampled data. Equation (2.1) is an interpolation formula that reconstructs the value of the function $f$ at any time $t \in \mathcal{H}$ from the sequence of samples of $f$ at the discrete time intervals, $k$.

$$f(t) = \sum_{k=-\infty}^{\infty} f[k] \frac{\sin(\pi (k-t))}{\pi (k-t)}$$  \hspace{1cm} (2.1)
The derivation of Equation (2.1) from a Fourier series yields an infinite sum. Its value is zero for all terms outside the time range of the sample, so it can be computed by adding up the values where \( f[k] \) exists. Note however that if \( t \) is outside the time range of the sampled data, then there will still be a value for the reconstructed function. The result is that the sample is repeated both forwards and backwards in time. That is, the sample represents exactly one period of a continuous periodic function.

The sampling process cannot detect any frequency higher than one-half of the sampling frequency (known as the critical frequency), but it obtains amplitude values from the signal nonetheless. Since the reconstruction formula, or any other Fourier series based analysis, can only consider frequencies less than the critical frequency, the total energy of the sampled signal is represented within the frequency band of the sample. If the signal being sampled is not bandwidth limited, then aliasing will occur and the reconstruction will fail to recreate the signal as illustrated in Figure 2-4.

![Figure 2-4](image)

Figure 2-4. An example of aliasing where a 21 Hz sine wave (solid line) is sampled at 20 Hz (dots) and the reconstructed signal (dashed line with dots) is a 1 Hz sine wave.
2.2.3 Superposition

For a linear-time discrete system, the principle of superposition can be used to break a complicated digital signal into a series of individual impulses\(^2\) as shown in Figure 2-5. These impulses can be applied to the dynamic system and the output resulting from each impulse charted according to its magnitude and time of application. Figure 2-5 shows the input signal decomposed into a series of impulses, each sent through and influenced by the dynamic system and finally added together to produce the final output signal. The effect that the system has on each input impulse is called the Impulse Response of the system.

![Diagram of superposition](image)

**Figure 2-5.** Superposition shows how the final output is generated

\(^2\) A signal whose magnitude is zero everywhere except at a single nonzero point.
2.3 **Impulse Response**

The impulse response of a system describes the time response to a normalized impulse applied at $t = 0$. Figure 2-6 depicts the dynamic responses of the system as a result of attenuation and phase shift. Therefore, it is very important to know the exact impulse response to construct a modified signal to cancel those effects.

![Impulse Response Diagram](image)

**Figure 2-6.** Impulse response is the outcome of a normalized impulse to a system.

2.4 **Convolution**

The influence of the dynamic system on an input signal of length $N$ can be determined by applying the convolution operation to the input $x[n]$ and the impulse response $h[n]$ of the system. The convolution operation describes how a linear filter (an actuator) modifies a given input signal (command) to produce an output signal (motion). This process is represented by the * (asterisk) operator and is expressed in Equation (2.2). Each value in the convolved output signal can be calculated with the summation in Equation (2.3), where $M$ is the length of the impulse response. The length of $y[n]$ is $N+M-1$. Convolution is mathematically...
equivalent to polynomial multiplication and can be directly calculated from Equation (2.2) in the time domain.

\[ x[n] \ast h[n] = y[n]. \tag{2.2} \]

\[ y[k] = \sum_{m=0}^{M-1} h[m] x[k-m] \tag{2.3} \]

**Figure 2-7.** Input decomposition to different forms of the components

### 2.5 Discrete Fourier Transform (DFT)

So far, all signals have been described in the time domain; that is, the operations have been applied to a sequence of values collected as a function of time. These operations are sometimes easier to implement when applied in frequency domain; this is especially true for periodic signals. For example, the input signal shown in Figure 2-7 can be decomposed into either 21 time-sequential impulses, or 2 frequency components (which are two sine waves
with different frequency and magnitude.) Performing the convolution operation in the frequency domain can reduce amount of memory required and the computation time. Since only a finite length of signal can be processed, the DFT assumes that an entire signal is made up of the input signal collected over the given time period but repeated again and again. The DFT represents all signals as a sum of sine and cosine components, which facilitates analysis in the frequency domain. The DFT converts $N$ samples in time to $N$ samples in frequency. The more samples that are collected in a fixed time period, the finer the frequency resolution, $\Delta f$, of the resulting frequency analysis. The sampling time $\delta t$ determines the highest frequency that can be represented by the transform, as was discussed in Section 2.2.2.

$$
F_s = \frac{1}{\text{sampling time}} = \frac{1}{\delta t}
$$

$$
\Delta f = \frac{F_s}{N} = \frac{1}{N \delta t}
$$

The DFT decomposes a signal into its constituent sinusoids and typically presents the results in terms of complex sine and cosine components as shown by Equation (2.4). Lowercase letters indicate a signal in time domain (e.g. $h[n]$), while uppercase (e.g. $H[k]$) represents the transformed signal at a specific frequency.

$$
H[k] = DFT(h[n]) = \sum_{n=0}^{N-1} h[n] \times [\cos(2\pi(k \Delta f)n \delta t) - j \sin(2\pi(k \Delta f)n \delta t)]
$$

$$
= \sum_{n=0}^{N-1} h[n] \times \left[ \cos\left(2\pi\left(k - \frac{1}{N}\right)n \delta t\right) - j \sin\left(2\pi\left(k - \frac{1}{N}\right)n \delta t\right) \right] \tag{2.4}
$$

$$
= \sum_{n=0}^{N-1} h[n] \times \left[ \cos\left(\frac{2\pi k n}{N}\right) - j \sin\left(\frac{2\pi k n}{N}\right) \right]
$$

where $1 \leq k \leq N$. 

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Mathematically, the vector of the frequency content resulted from the DFT algorithm in Equation 2.4 is repeatable and symmetric every $N$ samples where $f = 0, F_s, 2F_s$, and so on. The property is also applicable to the negative time (or $n<0$). As a result, the vector repeats itself about $f = -F_s, -2F_s$, and so on. These negative frequencies lead to the exact copies of the vectors from $-F_s/2$ to 0 and $F_s/2$ to $F_s$. When the sampling theorem limits the highest detectable frequency to one-half of $F_s$ over the $N$ input samples, the DFT expresses the frequency content from 0 to $F_s/2$ with a mirror image (or alias) from either $-F_s/2$ to 0 or $F_s/2$ to $F_s$.

**Figure 2-8.** The vector repeats itself at $f = ..., -F_s$ (or $-20$Hz), $0$, $F_s$ (or $20$Hz), $...$
The cosine (real) and sine (imaginary) parts of the DFT of a normalized impulse function repeat themselves for every $N$ samples as illustrated in Figure 2-8. The sampling time in this case is 50 ms to produce an $F_s$ of 20 Hz. In this frequency domain, the data consists of 20 points distributed over the frequency range from the negative half (-$F_s / 2$) to the positive half ($F_s / 2$) with a frequency resolution of 1 Hz. The negative frequencies have no physical meaning but are a mirror image, or *conjugate* pair, of those in the positive frequencies. Their real, even parts (cosine) are the same but the imaginary, odd parts (sine) are inverted to avoid a discontinuity (i.e., $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$.) Since the response is periodic, the distribution in negative frequencies reappears to the right of 10 Hz. As a result, the frequency range can also be thought of as the positive frequencies starting at 0 up to the sampling frequency $F_s$, which in this case is 20 Hz.

If the sampled input and impulse signals are transformed to the frequency domain, convolution becomes a complex multiplication (element by element) of two vectors. The DFT is commonly implemented by the fast Fourier transform (FFT) algorithm which exploits the symmetries in Equation (2.4) to accelerate this calculation. Equation (2.2) can thus be restated in the frequency domain using an FFT operator as shown in Equation (2.5).

$$x[n] * h[n] = y[n] \quad \text{(time domain)}$$

$$\text{FFT}(x[n]) \times \text{FFT}(h[n]) = Y[k] \quad \text{(frequency domain)} \quad (2.5)$$

Therefore, $X[k] \times H[k] = Y[k]$. 

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2.6 Frequency Response

In the time domain, the impulse response of a mechanical system shows how it reacts to a normalized impulse (unity magnitude) applied at time zero. A different picture is created if viewed in the frequency domain. The DFT of the normalized impulse yields an array of frequency coefficients all of which have a magnitude of one as illustrated in Figure 2-9. The frequency content of a swept sine wave is identical to that of a normalized impulse. This implies that the impulse response can be obtained by applying a swept sine wave to the system. Consequently, the frequency response \( H[k] \), or transfer function of the system, can be determined by

\[
H[k] = \frac{Y[k]}{X[k]}
\]

\( X[k] \) is the DFT of a swept sine wave signal used to excite a system, while \( Y[k] \) is the DFT of the time sampled response of the system. \( X[k] \) should be unity for all frequencies, but is usually calculated from \( x[n] \) (the swept sine wave) so that effects of quantization, noise and the finite range of sine frequencies are included in the analysis.

\[\text{Figure 2-9.} \quad \text{Discrete Fourier Transform of a normalized impulse}\]
2.7 Deconvolution

Deconvolution is the inverse of the convolution operation. Knowledge of a desired output \( Y_d[k] \) and a frequency response of the system \( H[k] \) can determine a pre-compensated input command signal \( X[k] \). While the convolution operation is a complex multiplication in frequency domain, the deconvolution is a division, or

\[
X[k] = \frac{Y_d[k]}{H[k]}.
\]

When the output signal \( y_d[n] \) is a desired excursion and \( H[k] \) is the frequency response of an active machine tool axis, the resulting \( X[k] \) (in frequency domain) can be used as an open-loop control signal to the actuator. This \( X[k] \) is, then, efficiently transformed back to the time domain \( x[n] \) using the Inverse Fourier Transform (IFFT command in MATLAB). In short, the input signal \( x[n] \) is obtained from Equation (2.6).

\[
x[n] = \text{IFFT}(X[k]) = \text{IFFT} \left( \frac{\text{FFT}(y_d[n])}{H[k]} \right) \quad (2.6)
\]

As an example, let the desired excursion consist of two 1.0 \( \mu \text{m} \) amplitude sinusoidal profiles with frequencies of 100 and 300 Hz. The digitized signal can be written as

\[
y_d[n \delta t] = 1.0 \times 10^{-6} \sin(2\pi \times 100 n \delta t) + 1.0 \times 10^{-6} \sin(2\pi \times 300 n \delta t).
\]
The transformed excursion $Y_d[k]$ using a sampling frequency of 6000 Hz is illustrated in Figure 2-10. The two peaks in the left half frequency range correspond to the given input frequencies, whereas the others (in the right half range) are their *conjugate* pairs required to perform the inverse transformation.

**Figure 2-10.** Frequency response of the desired excursion

In addition to the desired excursion, the frequency response $H[k]$ or characteristic transfer function of the actuator must be obtained to generate the modified control input $x[n]$ using deconvolution. This frequency response is specific to a particular actuator and must be obtained experimentally.
CHAPTER 3

DYNAMIC SYSTEM IDENTIFICATION AND PRELIMINARY TEST

As a key element of the deconvolution operation, the dynamic response of a system must be precisely identified to create a new input command that yields a desired output. Tomizuka [6-7] addressed issues of unmodeled dynamics that can significantly reduce performance of inverse dynamics schemes. This Chapter will introduce a method that accurately acquire the dynamic characteristics of the system.

3.1 VARIFORM FAST TOOL SERVO (FTS)

The Variform is a piezoelectrically driven servo with 400 μm range over a bandwidth of 350 Hz. This system has been used to perform experiments to validate the inverse dynamics algorithm. The FTS is driven by a pair of PZT stacks that expand and contract with the same amplitude but are 180° out of phase. They drive a T-lever that is perpendicularly connected to the tool axis as shown in Figure 3-1. Driving the PZTs in such manner increases the displacement of the axis and needs a differential signal as a command to the high voltage
amplifier. Not only does power the PTZ stacks, the amplifier system of the Variform FTS also implements a reference capacitance to reduce inherent hysteresis, eliminates disturbances by using a feedback controller and shapes the dynamic characteristics of the FTS with analog filters. The tool motion is measured using an LVDT, which completes the loop of the feedback controller.

![Diagram of Variform FTS](image)

**Figure 3-1.** Variform fast tool servo

### 3.2 Frequency Response Measurement

The dynamic response of a system can be presented in either the time domain (impulse response) or the frequency domain (frequency response). The dynamics can be accurately measured by applying a fixed amplitude sine wave of continuously varying frequency as an
input signal to the actuator as shown in an experimental setup in Figure 3-2. A spectrum analyzer (Stanford SRS 780) generates the swept sine wave from which an inverted signal is generated and both are sent to the Variform. The motion of the tool is measured by an internal LVDT and sent back to the analyzer to compare with amplitude and phase of the input signal at each input frequency.

The frequency response can be displayed in different formats but the most widely used form is the amplitude and phase of the output with respect to the input signal. Figure 3-3 illustrates the amplitude and phase form of the frequency response of the Variform to a 20 µm P-V swept sine input signal. The top graph shows the ratio of the amplitude of the output to the input (in dB units) as a function of frequency.

\[ AR_{dB} = 20 \log(AR) \]

Figure 3-2. Diagram of frequency response measurement
At low frequency, the output is equal to the input and the ratio is unity leading to a dB value of zero. As the frequency is increased, the AR peaks at about 200 Hz and then drops rapidly for higher frequencies. The lower graph shows the phase angle between the input and output. At lower frequencies, these signals are nominally in phase but as the frequency rises, the output lags the input. At 100 Hz this lag is about 45°. The gain and phase responses represent the closed loop dynamic characteristics of mechanical and electrical components inside the Variform and the control system.

Figure 3-3. Frequency response of the Variform at 20µm P-V amplitude
3.3 INVERSE DYNAMICS ALGORITHM VALIDATION

3.3.1 ANALYTICAL APPROACH

Figure 3-4 shows the complex number form of the Variform frequency response\(^1\) depicted in Figure 3-3. As with the response shown in Figure 2-10, dynamics of the actuator distribute the gathered frequency response in one half of the sampling frequency range and its conjugate in the other. The frequency response is normalized to cancel out the gain mismatch between the Stanford SRS 780 and the LVDT. If it is transformed back to the time domain, the impulse response is the result as shown in Figure 3-5. The response to the normalized impulse input command signal has the peak amplitude of 0.14 and rapidly decays to zero in less than 8 msec.

A modified input command \(x_{m}[n]\) that will produce a desired motion of the Variform is determined using the deconvolution operator on a desired excursion \(y_{d}[n]\) and the frequency response of the system \(H[k]\) as expressed in Equation 2.6.

\(^1\) The complex number form is an alternative way to present the more familiar frequency response where the amplitude is \(\sqrt{\text{Real}^2 + \text{Imaginary}^2}\) and the phase is \(\tan^{-1}\left(\frac{\text{Imaginary}}{\text{Real}}\right)\) that was in shown in Figure 3-3.
Figure 3-4. The complex number form of the frequency response at 20μm P-V

Figure 3-5. Impulse response of the Variform fast tool servo

Figure 3-6 shows a comparison of two command signals; the top graph shows an unmodified command that is a result from a 160 μm P-V desired trajectory with 100 and 300 Hz frequency components, and the lower graph shows the modified signal. The modified trajectory is advanced to compensate for the phase response of the Variform and the delay through the computer system that was used to command the FTS and acquire the LVDT signal demonstrating its response. All of those adjustments are carried out simultaneously in one deconvolution.

In Figure 3-7, the path difference associated with the modified input $x_m[n]$ (a line with dots) with respect to the desired excursion $y_d[n]$ was reduced to less than 1 μm P-V after about 4 msec. It indicated that after the transient period the motion of the system is essentially close to the desired signal. On the other hand, the path difference associated with the input
command without the deconvolution (a dashed line) is 200 µm P-V which is greater than the amplitude of the desired path.

![Graphs showing desired signal and path error](image)

**Figure 3-6.** Desired signal $y_d[n]$ and modified input signal $x_m[n]$ obtained by deconvolution

**Figure 3-7.** Path error associated with a modified input and unmodified input

### 3.3.2 Initial Experimental Results

Test input signals as shown in Figures 3-8 and 3-10 were sent to the Variform. Both modified and unmodified input commands are padded zeros so that each experiment starts from a stationary state and inverted to account for the negative gain of the LVDT. To correct the phase, the modified signal leaps at the start. However, the Variform filters this discontinuity with a built-in filter and rapidly “catches up” to the desired waveform.

Figure 3-9 illustrates the path error in the Variform response corresponding to a small amplitude input of ±4 volts. Note the close similarity to Figure 3-7. The modified command
signal eliminates actuator path errors due to the attenuation and phase after about 4 ms. A high amplitude input command signal varying between ±10 volts is shown in Figure 3-10. In Figure 3-11, a significant path error appears in the response associated with the modified input command, although it is much smaller than the error produced by an unmodified command signal. The problem is that the system is saturated by the modified command signal and cannot accelerate to the desired velocity. Figure 3-12 shows that the Variform response (a solid line) cannot follow the desired excursion (a dashed dotted line) when the velocity exceeded the maximum. The speed of the actuator response is dictated by hardware limitations. The slew rate of the amplifier and natural frequency of the mechanical system constrain the velocity resulting in a path error in the Variform motion.

**Figure 3-8.** Modified and unmodified inputs at low amplitude (±4 volts)  
**Figure 3-9.** Path error due to modified and unmodified inputs at low amplitude (±80 µm)
Figure 3-10. Modified and unmodified inputs at high amplitude (±10 volts)

Figure 3-11. Path error due to modified and unmodified inputs at high amplitude (±200µm)

Figure 3-12. The slope of the modified command signal response reaches the maximum value due to the physical limits
CHAPTER 4
IMPLEMENTATION ISSUES

4.1 VELOCITY SATURATION AND OPERATING RANGE

Limitations of the deconvolution technique appear with the combination of high frequencies and large amplitudes. In general, an actuator can be considered as a linear spring-mass-damper system whose dynamic response is expressed as a function of frequencies of the applied signals. Though the response results in an attenuated and delayed motion, the output of a linear system contains only the frequencies of the input signals. Since the velocity of the Variform FTS is constrained due to physical limits, the output profile is distorted when a large amplitude and high frequency input signal is applied. The velocity limit as shown in Figure 3-12 reshaped the output response associated with a sinusoidal input to look like a triangle signal as shown in the top graph of Figure 4-1. The bottom graph exhibits the saturated velocity about 140 mm/sec at which the output response crosses zero line. The effects of the distortion on the operating range of the Variform as well as the measured impulse response used in the deconvolution technique can be thoroughly investigated in the frequency domain.
Figure 4-1. FTS response associated with a 200 Hz, 300µm P-V input signal and its velocity

In Figure 4-2(a), the deformed shape (a line with dots) of the FTS response as measured by the internal LVDT sensor is compared with the shape of the desired excursion (a line with diamond marks). The lower plot illustrates the frequency spectrum of the output profile. The distorted response includes the base frequency component of 200 Hz as well as the 3\textsuperscript{rd} (600 Hz) and 5\textsuperscript{th} (1000 Hz) harmonics. Figure 4-2(b) shows the frequency components with respect to the Variform response. While the constituent sinusoid associated with the base frequency is in-phase with the desired signal, the 3\textsuperscript{rd} harmonic is out-of-phase, leading to a triangle-like output response. Note that the amplitude of the constituent sinusoid (a line with circles) is smaller than the amplitude of the distorted output excursion (a line with dots.)
This amplitude difference has a large effect on the accuracy of the impulse response measurement using a spectrum analyzer (Stanford SR780). The analyzer generates a varying frequency input with fixed amplitude (i.e. a swept sine wave) to the FTS, and determines attenuation and phase of the tool motion captured by an internal LVDT at each input frequency. If the output path signal is velocity-saturated, its frequency component associated with the input frequency has, in effect, smaller amplitude than the actual output signal. As a result, the frequency response of the Variform shows significant attenuation. Figure 4-3 depicts the attenuation of the Variform as a function of the frequency and amplitude of the command signal. The frequency response illustrates a large amplitude drop where the input signal contains large amplitudes with high frequencies. This implies that the operating range of the Variform is limited to combinations of amplitude and frequency where the velocity is below the constraint of 140 mm/sec as shown in Figure 4-4.
4.2 Dynamics of the Variform Position Sensor (LVDT)

The Variform FTS uses an LVDT as a position sensor that closes the feedback loop of its tracking controller. To eliminate disturbances in the measurement, the displacement measuring system implements an internal filter that, however, modifies the dynamic response of the monitored axis motion. Therefore, the tool path measured by the LVDT cannot be used for the FTS dynamics measurement technique in Section 3.2. To this end, the actual tool motion was measured with an external capacitance gauge (Lion Precision) placed in front of the tool holder, whose bandwidth is much higher than the LVDT’s. Time-based signals of the Variform axis position measured by the LVDT and the capacitance gauge are displayed on a monitor as shown in Figure 4-5. The actual path was delayed by 21° with respect to a sine wave input command at 100 Hz, while the LVDT displayed a longer delay of 39°. Figure 4-6 illustrated the transfer function between the LVDT and the capacitance
gauge that was determined using the same measurement technique as the actuator dynamic response. Note that the phase response started at $180^\circ$ because the two sensors were measuring in opposite directions. Therefore, the LVDT signal was ahead of the signal of the capacitance gauge by $18^\circ$ at 100 Hz. As with the inverse dynamic response of the FTS, the deconvolution algorithm can also compensate for the feedback sensor's dynamic response.

The transfer function shows that the LVDT implements a second order filter with 500 Hz bandwidth where the amplitude dropped off by 0.707 (of 3 dB) and a filtered output signal was $90^\circ$ out of phase. Another note was that the gain at high frequencies descended about 40 dB over ten increments of the LVDT natural frequency range (500 – 5000 Hz). This implied that the Variform implemented a 2$^{nd}$ order internal filter in its displacement sensor.

**Figure 4-5.** Phase of the tool motion measured by the LVDT and a capacitance gauge (inverted) with respect to a 100 Hz input signal

**Figure 4-6.** Dynamic response measured by the LVDT with respect to a capacitance gauge
LVDT dynamics measurement using a laser interferometer

Despite its high bandwidth, a measurement with a capacitance gauge is limited to a small range (±25µm). It cannot be used to identify the dynamics of the LVDT over the entire range (±200µm) of the Variform. Thus, new instrumentation was needed to characterize the response of the Variform’s position sensor. A laser interferometer makes use of a Doppler effect to create interference between two polarized beams. Since resolution of the measurement is proportional to the wave length \(\lambda\) of He-Ne laser (633 nm), it offers a high accuracy with a long range and a high bandwidth.

4.2.1 Laser operation

A single pass interferometer divides a laser beam which composes of two orthogonal linear-polarization (squares for horizontal and dots for vertical as illustrated in Figure 4-7) into two separate beams. Note that the polarized beams are originally generated with a fixed frequency difference. These polarized beams are then re-combined after directed by a beam-splitter, shifted to circular polarization by ¼ wave plates, reflected by two retroreflectors, and re-shifted to the linear polarization. One polarized beam heads to a stationary, while the other to a moving object, at which Doppler effect modulates the beam frequency and generates interference with the other beam at an external receiver. Afterward, an Axiom 2/20 series interpolates the measurement data to achieve a high resolution of \(\lambda^1/256\) or 2.47 nm per count.

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1 A wavelength of He-Ne laser is 632.32nm

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Figure 4-7. Single-pass laser interferometer configuration

Figure 4-8. Acquisition sequence of the Axiom 2/20 series

* Active low
An output position data (or an accumulated laser count) of the Axiom 2/20 series is refreshed at the register interface with a constant rate of 10 MHz, but can be read at a rate up to 2 MHz. An access of the position data in this constant data rate mode; as illustrated in Figure 4-8 is to send a sample signal to pause the data transfer between a accumulator and a register interface so that the position data can be transmitted out of the 2/20 series, while the counter still keeps updating the measured position of the moving object. Note the accessed data in a 16-bit unsigned format.

4.2.2 Data acquisition system

This research made use of the dSPACE 1104 series to control the hardware systems, such as a control board of the Axiom 2/20 series, an axes controller of a diamond turning machine, and the Variform FTS, from a model in the MATLAB/simulink module. The dSPACE can access in real-time either analog, digital, or encoder signals, and operate on synchronous or asynchronous tasks from various interrupt sources such as timer chips, software, external sources, PWM interrupts. The interface under the constant data rate mode of the laser interferometer system can be implemented at a fixed sample rate (or a synchronous task) for a measurement of FTS dynamic response.

As the most common scheme for a synchronous task in MATLAB/simulink, a timer interrupt service routine can be used to acquire a measured position data from the Axiom register interface. The acquisition made use of a sample rate of 50kHz so each cycle took 20μsec to operate a sequence as shown in Figure 4-9. First, the interrupt sent a sample signal to the
Axiom register interface and also recorded an FTS command signal. Second, since the 2/20 series needs at least 200nsec before the data in the register becomes available, the dSPACE waited for another cycle, then acquired the tool position data. Third, one more cycle delay was needed to release the position held. The Axiom then refreshed new position data from the accumulator to the interface. Since the acquisition sequence took 3 cycles of a 50kHz sample rate, the position data can be updated every 60µsec.

![Diagram](image)

**Figure 4-9.** Acquisition the Axiom 2/20 series using the dSPACE

The components for the dynamics measurement was mounted on an optical table as illustrated in Figure 4-10. A special cable was fabricated to transmit the measured position data (or the laser count with a resolution of 2.47 nm in a 16-bit unsigned binary format) from a parallel port of the Axiom interface to a digital I/O of the dSPACE. Though a measured position can be either a positive or a negative number, a 16-bit unsigned binary represents
only a positive number from 0 to 65535 (or $2^{16}-1$) when converted to a decimal format. The total count covers only a distance of 161.87 µm (65535 counts × 2.47 nm.) The excess displacement is then wrapped around where the count hits the limit (either 0 or 65535) and starts over at the other end. In Figure 4-11, a position data after the format conversion decreases to zero count and continues its profile from the other end. Consequently, an additional process is needed to smooth the wrapped around position profile so that the unwrapped results can be illustrated as Figure 4-12.

Figure 4-10. The measurement setup on an optical table
The result of the measurement agreed with the response observed by the capacitance gauge (in Figure 4-5); the actual tool trajectory had less attenuation and smaller phase lag than what the LVDT displays.

![Figure 4-11. A wrapped around profile of a swept sine wave](image)

![Figure 4-12. An unwrapped profile of the FTS motion with respect to the desired and the signal measured by the LVDT](image)
The format conversion and the unwrap operation can be proceeded in real-time using the dSPACE. The adjusted position data is then transferred to the Stanford spectrum analyzer to determine the dynamic response of the FTS as shown in a schematic in Figure 4-13.

![Figure 4-13. Data flow of the FTS dynamics measurement using a laser interferometer](image)

When the Stanford compares the amplitude and phase of the position data observed by the Axiom to those of the position measured by the LVDT, the dynamic characteristics of the position sensor can be identified. In Figure 4-14, gain and phase response of the LVDT measured by the Axiom laser interferometer was independent of input amplitude and very close to that measured by a capacitance gauge at lower amplitude.
Figure 4-14. Dynamic characteristics of the Variform position sensor
4.3 Nonlinear Characteristics of the Variform FTS

To shape its characteristics to a linear system, the Variform FTS implements a reference capacitor loop to deal with inherent hysteretic behavior of the PZT driving system, constructs a feedback controller to reduce influences of disturbances, and integrates additional filters. Nevertheless, the actuator still exhibits nonlinearly response to varying amplitude input command signals.

![Graph of nonlinearity](image)

**Figure 4-15.** Nonlinearity of the Variform to various amplitudes of input command signals

In Figure 4-15, dynamic responses of the FTS captured by the laser interferometer are dependent on the amplitude of the input command. Though, small gain differences (less than 1%) among the responses are realized at low frequencies, the nonlinearity becomes significant at frequencies close to the bandwidth of the FTS, and even more critical when
combined with large amplitudes. The pre-compensation for the FTS dynamics now becomes more complicated since the response is not only a function of frequency, but also a function of amplitude. While the input amplitude is growing, the peak of the gain response at the natural frequency is decreasing. The peak of the gain response associated with 80\(\mu\)m P-V amplitude is 1.16 at 200 Hz whereas the peak associated with 240\(\mu\)m P-V is only 1.05 at 130 Hz. Note that the established operating range (in Figure 4-4) eases this complication by avoiding conditions when the velocity of a command exceeds the physical limits of the FTS.

4.3.1 Gain scheduling scheme for 3D deconvolution

When a desired machined surface is a non-rotationally symmetric (NRS) feature such as torics, the tool trajectory occasionally displaces in an aperiodic manner, and changes its amplitude at least once a revolution. A tool excursion of a toric feature is the square of a sine wave with twice the frequency of the revolution. Its amplitude is linearly decreased as the tool moves toward the center. To modify such a desired excursion with the inverse dynamics algorithm, the transfer function associated with the appropriate amplitude must be selected. To this end, the nonlinearity of the Variform FTS was investigated with a series of modified input command signals which pre-compensate for the responses of different amplitude. Then, the actual excursion that matches the desired path became the basis for the amplitude selection scheme.

In Figure 4-16, a full-period of a 240\(\mu\)m P-V (or 12 volts P-V) cosine wave with a frequency content of 187 Hz on an otherwise flat trajectory (a line with circles) was generated as a
desired excursion for a nonlinearity test, and modified into different input command signals using the dynamic responses associated with the input amplitudes from 80 to 380\(\mu\)m P-V. The modified input signals associated with small amplitude of 80 and 160\(\mu\)m P-V resemble each other while the remainder had bigger amplitude and more leading phase angles. This means that the selection of the appropriate dynamic response is critical when the desired trajectory has large amplitude (greater than 160\(\mu\)m P-V.)

Since the dynamic response measurement makes use of sinusoids to determine the dynamic characteristics of a system, it is reasonable to base a modified input command for a cosine wave on a value equal to the magnitude. Consequently, the series of the modified input command signals was convoluted with the transfer function associated with 240\(\mu\)m P-V.

**Figure 4-16.** Amplitude dependent modified input signals
The operation was performed in MATLAB resulting in a series of the simulated output trajectories.

Figure 4-17 illustrates the path differences of the modeled output paths with respect to the desired excursion. Since the dynamic response canceled out with its inverse, the path difference of $240\mu\text{m P-V}$ was a flat line (a solid line). The differences of the gain and phase response influenced the degree of the path difference. In particularly for the amplitude of $380\mu\text{m P-V}$, the path difference (a dotted line) was $60\mu\text{m P-V}$ or a quarter of the amplitude of the desired excursion.

![Figure 4-17. Influence of the nonlinearity to the path differences with respect to the desired trajectory](image.png)
The Variform FTS was commanded to displace a tool axis in the air while mounted on an optical table using a series of modified input command signals. As expected, it appeared that the transfer function of the 240µm P-V yielded a close match to the desired trajectory. In Figure 4-18, the actual tool trajectories (x marks) was identical to the desired path (a solid line) except at the transition from a sinusoidal profile to a flat line. The path difference was limited at 2µm P-V for the most part of the cosine wave. At the edges, the difference was less than 10µm. Thus, the gain scheduling scheme will select a transfer function associated with the amplitude of the desired tool path for the inverse actuator dynamics algorithm.

![Graph](image)

**Figure 4-18.** Comparison of the desired and the actual tool trajectory associated with the amplitude of the desired trajectory
CHAPTER 5

BLOCK DECONVOLUTION

The preceding chapters have shown the potential of a deconvolution algorithm that can pre-compensate the dynamic response of an actuator to reduce form errors in a machined part. The amplitude of the tool excursion was scaled and its phase was shifted forward to account for gain and phase response of the actuator [16]. However, as discussed in Section 4.3, the Variform FTS has a different gain response to various amplitudes of input signal. Accordingly, the modification of a whole tool trajectory with one dynamic response may not correctly produce a non-rotationally symmetric (NRS) surface whose the depth-of-cut is changed at particular $r$ and $\theta$.

Another significant issue that must be addressed for the practical implementation of any command signal modification technique is the interdependence of the parametric (i.e., time-based) commands for the multiple axes of a machining system. A desired path of an FTS is described as a parametric function of spindle speed and the axes cross-feed rates of a diamond turning machine (DTM). However, initial machining parameters may drift over the fabrication time. As either velocity changes, the time period of the desired path is also

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altered. Since the operation over the entire command signal is only valid for non-varying machining parameters, a more robust approach would be required to implement the inverse dynamics operation.

Consequently, a long desired excursion must be segmented so that the short pieces are individually modified using the deconvolution technique. Adaptive schemes such as the short-time Fourier transform (STFT) and an equivalent inverse dynamics filter have been developed to use the most recent parameters known during machining. This opens a window for a real-time operation of the modified command technique. The maximum rate at which this adaptation can be performed is same as the impulse response decay rate; that is, the time that it takes for the actuator motion to decay to zero following an impulse input.

5.1 SHORT-TIME FOURIER TRANSFORM (STFT)

The segmentation can be achieved by using a window function applied to a long path signal. Figure 5-1 demonstrates how a rectangular window (Equation (5.1)) and a triangular window function draw a short piece \( x_s[n] \) out of a long sinusoidal signal \( x[n] \). The short signals are the product of the applied signal and the window function as shown in Equation (5.2).

\[
\begin{align*}
  w[n] &= \begin{cases} 
    1 & a \leq n \leq b \\
    0 & \text{otherwise} 
  \end{cases} \\
  x_s[n] &= x[n] \times w[n] 
\end{align*}
\]

1 A window function is a vector of weighting coefficients, which are zero everywhere but within a certain length.
To reverse the process, an overlap-add method is used to reconnect the short pieces. Each individual piece combines with the overlaps of the adjacent pieces as shown in Figure 5-2. Then, the entire combined signal is factored by a gain to correct its amplitude. The applied gain intrinsically depends on the window function and the length of the overlap. In Figure 5-2, the overlap-add using the rectangular window (on the left column) with an overlap length of a half of the window size needs a gain of 0.5 to correct the amplitude of the reconstructed signal whereas the triangular window (on the right column) with the same overlap length can use a gain of 1. Consequently, it is difficult to figure the scale factor in advance. The corrected reconstructed signal is distorted at both ends as a result of the window characteristics, but is a copy of the input signal where the overlap completes.
Figure 5-2. Overlap-add method reconstructs the original signal from small pieces

Figure 5-3 shows a schematic of the STFT deconvolution algorithm that utilizes a moving window function to segment a long desired tool path into short pieces, and transforms the short piece to the frequency domain [17] where the deconvolution operation [18-19] is performed to modify each piece with the dynamic response of the FTS. In the reconstruction process, the modified pieces in the frequency domain are individually transformed back to the time domain, applied again to the window function, rejoined to the adjacent piece with the overlap-add method and factored with a gain to correct its amplitude.
Figure 5-3. Schematic of the short-time Fourier transform

In practice, the length of the window must be short to be sensitive to high frequency components. However, the deconvolution operation needs a lead time to complete the schematic in Figure 5-3 and update all machining conditions; the calculation speed is always slower than the length of the window when implemented in real-time. As a result, the STFT deconvolution cooperates with a look-ahead scheme, generates a modified input signal for a length of, for example, one revolution, and reconnects each of the complete modified input signals. A series of modified signals in Figure 5-4(a) is reconnected to the adjacent pieces at the appropriate time to reconstruct the entire modified signal as shown in Figure 5-4(b).
(a) Series of windowed signals after individual deconvolution

(b) The entire of the modified signal

Figure 5-4. Reconnection process of complete modified input signals
5.1.1 Experimental Setup of the STFT Deconvolution

The STFT command signal modification technique was performed to invert the dynamic response of the Variform FTS. As with the verification in Section 3.3, the actuator was mounted on an optical table and commanded to follow a 200 µm P-V sine wave with combined frequency components of 100 and 300 Hz. In this experiment, dynamics of the LVDT were not compensated so that the desired trajectory can be directly compared with the monitored tool excursion. The time-based tool excursion was windowed, transformed to the frequency domain, modified by the inverse dynamics algorithm [5], then re-synthesized, and finally conveyed to the FTS as a modified command signal. All procedures were performed in real-time with fixed machining parameters. While the FTS was commanded to follow the present trajectory, the input command signal for the future path was produced.

5.1.2 Analytical Approach

In Figure 5-5(a), the modified input signal was calculated using the STFT command modification and the frequency response of the Variform FTS. The initial delay of the modified input command signal occurred about 140 msec to draw the first short piece of the desired trajectory, perform the deconvolution operation, and relay to the FTS. Note that, unlike the modified signal obtained by an entire deconvolution in Figure 3-6, the modified input signal did not jump at the start to correct the phase. The amplitude of the modified input signal was gradually increased due to the weight coefficients of the Hanning window function (expressed in Equation (5.3).) The tool path evolved to the desired excursion after the transient period of 155 msec resulting the steady state path difference less than 1 µm P-V as shown in Figure 5-5(b).
\[ w[k+1] = 0.5 \left( 1 - \cos \left( 2\pi \frac{k}{n-1} \right) \right), \quad k = 0, \ldots, n-1 \] (5.3)

where \( n \) is the length of the window.

(a) Modified input associated with the STFT deconvolution
(b) Path difference with respect to the desired excursion.

**Figure 5-5.** Simulation results of the STFT deconvolution

### 5.1.3 EXPERIMENTAL RESULTS

As with Figure 5-5(a), the modified input command signal started about 140msec to drive the FTS axis along the desired trajectory (a solid line in Figure 5-6(a).) The tool excursion (dashed line in Figure 5-6(a)) was essentially in-phase with the desired trajectory and identical after the transient period of 155msec. However, at the steady state the tool motion slightly mismatched the desired path where the maximum velocity occurred resulting in the path difference about 8\( \mu \)m P-V as depicted in Figure 5-6(b) and larger than that of the model.
The physical limitation of the Variform played a major role that distributed the path error to the output tool trajectory.

The experimental results using the short-time Fourier transform illustrates the potential of the real-time inverse dynamics algorithm. However, it requires careful selection of the window function, the scale factor and the inverse transformation. A simpler implementation of block deconvolution will be introduced in Section 5.2 and be implemented for a machine test.

(a) Desired tool path of 90\(\mu\)m P-V, 90 and 270 Hz and the Variform response

(b) Path difference with respect to the desired excursion

Figure 5-6. Experimental results of the STFT deconvolution in real-time

5.2 EQUIVALENT INVERSE DYNAMICS FILTER

Another approach to the real-time deconvolution development is to build an equivalent inverse dynamics filter which is more efficient than the short-time Fourier transform approach. Equation 2.6 is repeated here to show that a modified input command signal \(x[n]\)
can be determined when a desired tool path \( y[n] \) is transformed to the frequency domain and the deconvolution operation is performed (or a division operation with the frequency response \( H[k] \) of the system)

\[
x[n] = \text{IFFT}
\left( \frac{\text{FFT}(y[n])}{H[k]} \right)
\]

The filter approach is used to eliminate the division operation in the frequency domain. It creates an equivalent inverse dynamics filter as written in Equation (5.4) so that the deconvolution in Equation (2.6) can be re-expressed as a convolution of this inverse filter with the desired motion path as shown by Equation (5.5).

![Figure 5-7. Frequency spectrum of the equivalent inverse dynamic filter](image)

Figure 5-7 depicts the inverse of the FTS’s frequency response at frequencies close to the bandwidth. The gain (the top graph) is amplified and the phase (the bottom) is advanced to compensate for the dynamics of the actuator.

\[
h^{-1}[n] = \text{IFFT}
\left( \frac{1}{H[\omega]} \right)
\]  \hspace{1cm} (5.4)

\[
x[n] = y[n] * h^{-1}[n]
\]  \hspace{1cm} (5.5)
Under machining conditions that vary with time (such as spindle speed), this approach can be implemented using the overlap-add and overlap-save methods. These two algorithms implement block convolution; that is, a segmented convolution of a long parametric desired tool path \( y[n] \) and the inverse dynamic filter \( h^{-l}[n] \). The exact convolution can be created as segmented convolution blocks are reconnected. Note that the length of a dynamic response \( h[n] \) is usually short and so is its inverse response \( h^{-l}[n] \). In practice, these responses are occasionally more concise than the segmented desired signal.

### 5.2.1 **Overlap-Add Convolution Method**

As expressed in Equation (5.6), a linear convolution between a short desired signal \( y[n] \) (with length of \( L \)) and a inverse impulse response \( h^{-l}[n] \) (of \( M+1 \)) results in a modified input signal \( x[n] \) (of \( L+M \)).

\[
x[n] = \sum_{k=-\infty}^{\infty} y[k] h^{-l}[n - k]
\]  

(5.6)

The first \( L \) samples of the modified command represent the correct output of the operation, but the last \( M \) points need to be overlapped and added by the first \( M \) samples of the adjacent modified piece to continue the correct linear convolution. Since the convolution operation takes place between two short signals (the segmented desired signal and the inverse dynamic response), this technique can be implemented functionally with either direct application of Equation (5.6) (in the time domain), or the FFT algorithm (in the frequency domain.)
In the frequency domain, the implementation of the convolution starts by adjusting the length of both the inverse impulse response $h^{-1}[n]$ and the shortened desired signal $y[n]$ to the same length of $L+M$. A vector of zeros with length of $M$ is connected to the end of $y[n]$ and the vector with the length of $L-1$ to $h^{-1}[n]$. This process is known as a “padding zeros” method. The next step is to perform the convolution operation using Equation (5.7) to determine a short piece of the modified signal $x[n]$ with the same length.

$$x[n] = \text{IFFT}(\text{FFT}(y[n]) \times \text{FFT}(h^{-1}[n]))$$

(5.7)

Then, the last $M$ points of the modified signal $x_i[n]$ is overlapped and added with the first $M$ points of the next modified signal $x_{i+1}[n]$ as shown in Figure 5-8.

![Figure 5-8. Block convolution using the overlap-add method](image)

The re-composition process can be illustrated in Figure 5-9(a) through a conjunction of two block modified command signals. In the overlap segment, even though the ripples of the first piece (a line with squares) are out of phase with those of the other piece (a line with
triangles), the sum of the two signals is a smooth modified input command (a solid line) shown by side of the desired trajectory (a dotted line) in Figure 5-9(b).

**Figure 5-9.** Modified input command signal reconstruction using the overlap-add method
### 5.2.2 Overlap-Save Convolution Method

In Equation (5.8), the modification makes use of a circular convolution operation between an inverse impulse response $h'^{-1} [n]$ (with the length of $M+1$) and the fragmented desired path $y[n]$ (of $L$ points) to construct the modified input command signal $x[n]$ (of $L$ points).

$$
x[n] = \sum_{m=0}^{N-1} y[m] h[n - m] \quad 0 \leq n \leq N - 1
$$

Numerically, one difference between the linear and circular convolution is the length of the output product. The linear convolution expands the output signal (to $L+M$ points), whereas the circular convolution constrains it to the length of one input signal (L points.) As a result, the first $M$ points of the circular operation are wrapped around forming an incorrect segment of the output, while the remaining points are correct and identical to those obtained by the linear convolution. The overlap-save convolution algorithm proceeds by dividing the desired path $y[n]$ into small sections of length $L+M$ and overlapping the pieces with the last $M$ points of the preceding section as shown in Figure 5-10. Unlike the overlap-add method, the first $M$ points of the first section are zeros, while those of the other segments similar to the last $M$ points of the preceding ones.

![Figure 5-10. Block convolution using the overlap-save method](image-url)
After performing the circular convolution, the first M points of all segments are disengaged. Then, the remaining outputs are connected together to complete the deconvolution operation.

The two block convolution methods described above have many advantages over the approach using the short-time Fourier transform. First, the division operations, which numerically takes the longest computation time, are only performed during the construction of the inverse dynamics filter instead of every deconvolution operation of each windowed command signal. This makes it much faster calculation resulting in a much efficient implementation. Second, the block convolution is an exact operation, yielding the same answer as a single convolution over a long signal. The STFT algorithm, though, gives different outcomes depending on characteristics of the chosen windowing function. And finally, the reconstruction process of each block convolution method is also much simpler with respect to the STFT.

5.3 IMPLEMENTATION OF INVERSE DYNAMICS FILTER

The signal modification can be implemented as a block convolution in either the time or frequency domain. They are mathematically equivalent to each other and complete the deconvolution operation. Even though the block deconvolution in the frequency domain has an advantage of fast calculation with the fast Fourier transform (FFT) algorithm, since the block convolution algorithm breaks a long signal into short pieces, the implementation using a convolution operation in the time domain is neither a time consuming process. The modification algorithm can be proceeded efficiently in either domains.
An important issue of this modification is that the sample rate of a desired trajectory must match up with that of the inverse filter. In other words, the FFT of the two signals must have an identical frequency range. Otherwise, the modification cannot proceed. However, the sample rate of a desired input signal is rarely identical to the inverse filter’s. The sample rate of the filter is usually less than the input signal.

![Graph](image)

a) Original frequency response of the Variform is increased its frequency range

b) Impulse responses associated with the original and modified frequency responses

**Figure 5-11.** Sample rate adjustment of the FTS frequency response

Consequently, the frequency response of the FTS (a solid line in Figure 5-11(a)) must be expanded. The conjugate of the original response can be translated to either higher (a dotted line) or lower frequencies (a dashed line) than that of the original. When the frequency range is reduced from 1000 to 600 Hz, the new frequency response has a portion of the original response from DC to 300 Hz and creates the mirror image from 300 to 600 Hz. When the range is extended from 1000 to 2000 Hz, however, the exact response inside the gap between the frequency response on the left side and its conjugate is unknown. The straight line binds
the two separated responses with their last value. The change of the frequency range resembles an interpolation of the inverse FFT of the frequency responses (or the impulse response) as shown in Figure 5-11(b). While expanding the range results in an increasing the sample rate of the time-based signal as known as an up-sample method, reducing the range does the otherwise (a down-sample method.) Even though the impulse responses have slight differences with respect to the original, they represent the correct frequency content.

![Image](image.png)

(a) A sinc function symmetrically propagates to the past and the future

(b) The frequency-selective property yields an ideal lowpass filter

**Figure 5-12.** Comparison of a sinc function and a sine wave

A drawback of the frequency expansion with this method is that the inverse FFT of the new frequency response becomes a non-causal zero phase filter. An example of this kind of filter is an ideal lowpass filter as known as a sinc filter (as expressed in Equation (5.9)) which requires an infinitely long signal from the past to the future. Figure 5-12(a) illustrates the
sinc function in comparison with an in-phase, 0.5 Hz sine wave. While the sinusoid propagates with fixed amplitude, the sinc filter symmetrically decays over positive and negative time.

\[ y(t) = \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \]  

(5.9)

In Figure 5-12(b), the Fourier transform of the sine wave returns an impulse at 0.5 Hz (a dashed line). The sinc filter (a solid line), on the other hand, results in a rectangular window whose the frequency band of non-zero gains is sharply cut off at 0.5 Hz. This results in a frequency-selective property of the sinc filter which was used in the frequency expansion method in Figure 5-11(a) to limit the gain response at 0.5 in frequencies between 500 and 1500 Hz.

As a result, the inverse Fourier transform of the frequency response or the impulse response became a long signal distributed into the past and the future as illustrated in Figure 5-13. The past kernel (non-zero components) of the response was wrapped around and appeared at the end of the long signal. If these two separated kernels are rejoined, then the rest of the signal which contains dispensable values can be discarded, and the computation between the desired signal and the inverse filter can be accelerated.
Adding a delay with $\zeta$ samples to the impulse response (or mathematically equivalent to shifting the phase response with a constant lag of $e^{-i2\pi \zeta}$ as written in Equation (5.10)) can avoid the non-causal problem.

$$h_{\text{shifted}}[n] = \text{IFFT}(H(f)e^{-i2\pi \zeta})$$  \hspace{1cm} (5.10)

where

$\zeta$  \hspace{1cm} Delay (samples)

The kernel that is wrapped around to the end of the signal is then regrouped with the rest of it to complete the impulse response. The number of the delay samples must be greater than the largest group delay$^2$ of the response. Figure 5-14 indicates that the adjustment would be at

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$^2$ The average delay of the filter as a function of frequency
least 4 samples. As the separated kernels rejoin, the long impulse response in Figure 5-13 can now be shorted so that it has only the concise kernel as displayed in Figure 5-15.

![Graph showing Group delay vs Frequency](image)

**Figure 5-14.** Group delays of all FTS responses are less than 4 samples

As with the impulse response, the inverse Fourier transform of an inverse dynamics filter also encounters the non-causal problem. Application of a constant phase shift can eliminate the wrap around and complete the filter as shown in Figure 5-16. Note that the inverse filter (Figure 5-16) looks similar to a mirror image of the impulse response (Figure 5-15.)

Now the input command modification or the convolution of a desired input command and the impulse response of the inverse filter can be operated in the time domain to pre-compensate gain and phase response of an actuator. Because of the linear property of the convolution operation, when a desired trajectory is modified by the lagged inverse dynamics filter, the result is a modified input command signal with the same amount of phase delay. Thus, after the command modification, the modified signal must be shifted forward to restore the correct
phase. The analytical approach and the experimental results of this method will be discussed thoroughly in Chapter 6 as implemented to the diamond turning process of a NRS feature into a flat surface.

Figure 5-15. Shifted impulse response of the FTS

Figure 5-16. Shifted inverse dynamics filter
CHAPTER 6

TEST MACHINED SURFACE

The inverse dynamics algorithm pre-accounts for gain and phase response to a time-based signal. In a turning process, a time based signal is based on a spiral pattern described as a function of radius $r$ and rotating angle $\theta$. After applying the deconvolution algorithm, the modified input command of the FTS axis (or the w-axis) along the spiral path is known. It would have been best to directly use this modified command signal to machine the desired surface if the base machine axes can create the identical spiral excursion over the part during continuous machining while the FTS axis follows the modified tool excursion. However, when a work piece is rotated at high speed and the tool axis is mounted on a slow moving x-slide of a DTM, the initiation of the spiral pattern is not an easy task. An alternative implementation is to form a look-up table dependent on $r$ and $\theta$, or a deconvolved surface from the spiral pattern, and then determine the modified input signal for the w-axis at run time. This section will describe the procedures of this transformation.
6.1 Off-Axis Sphere

A spherical optical surface has been chosen to demonstrate the inverse dynamics error compensation technique. An concave, off-axis sphere is machined into a flat surface because the command trajectory is not periodic and its frequency content can be modified by changing the off-axis distance and/or spindle speed. Most important though is that form errors of the machined surface can be evaluated using a laser interferometer if an appropriate \textit{f-number} (the ratio of focal length to entrance pupil diameter) is selected.

The sagittal component of the tool path to machine this surface is a set of nearly circular trajectories with different widths and depths depending on the radial position of the tool. An example of one pass of the tool is shown in Figure 6-1. This picture shows the tool path beginning at a particular height above the surface, then plunging into the part, and finally returning to that height. The deviation from a circular trajectory is a result of the shape of the intersection of the sphere with the spiral radial tool position and the correction needed for the nose radius of the tool. Because the depth of the sphere is too large to create in a single pass across the part, the surface will be created by a series of passes following the same trajectory but removing 10 \( \mu \text{m} \) per pass.

\textbf{Figure 6-1.} Tool cutting motion along spiral path to create a concave sphere
An off-axis sphere with a radius $R$ can be mathematically described as in Equation (6.1) where motions of the base machine axes ($x$ and $y$) are parametric functions of the off-axis distance $r$ and spindle rotation $\theta$. While the base axes and spindle generate a spiral pattern overlaid on the machined surface, the FTS axis ($w$) plunges in and out as expressed in Equation (6.2).

\[
(x - a)^2 + (y - b)^2 + (w - c)^2 = R^2
\]  

(6.1)

where

\[
x = r \cos \theta
\]

\[
y = r \sin \theta
\]

\[
w = c + \sqrt{R^2 + 2r a \cos \theta - a^2 + 2r b \sin \theta - b^2 - r^2}.
\]  

(6.2)

Time-variation of $r$ and $\theta$ can be accounted by re-writing them as functions of axis cross-feed and spindle speed, respectively. Then, the FTS command signal required to generate the entire surface can be generated as a time-based vector at any desired resolution.

### 6.1.1 DECONVOLVED SURFACE

A NRS feature to be added by the FTS can be either expressed as a parametric function of $r$ and $\theta$, or portrayed as a cloud of data points. When its geometric description and the machining conditions such as a spindle speed, a cross feed rate, etc. are determined, a deconvolved surface (or a look-up table) can be constructed using the following procedure:
1. **Create a desired surface.** A series of concentric rings and sections described by even spaced $r$ and $\theta$ as shown in Figure 6-2(a) is created. An off-rotational-axis sphere (a circle) is surrounded by an array of dots to map the depth of the desired surface. The amplitude of the sphere can be depicted as a dependent of $r$ and $\theta$ in Figure 6-2(b). Note the deformation is a result of transfiguration from a polar plot (a wedge-shaped table) to a plane figure.

(a) The geometry of a desired surface is described in the polar coordinate system  
(b) A map represents the amplitude of the surface as a function of $r$ and $\theta$

**Figure 6-2.** Create an amplitude map of the desired surface

2. **Compensate for a tool nose radius.** With respect to the orientation of desired surface described in Figure 6.3(a), the spindle revolves counter-clockwise, the $x$-slide of the base machine translates toward the center, and a cutting tool stays at $y=0$ and moves only in the $w$-direction. Along the solid line across the center of the sphere in Figure 6-3(a), the cutting points of the tool on the desired surface can be illustrated as Figure 6-3(b). Due to the tool radius, the center of tool is offset in the $x$ and $w$ direction such that the circumference of the
tool tangentially touches the surface. The mapping array of the desired surface in the previous step must be compensated so that the array represents the position of the tool center at which the FTS axis displaces it in and out of a machined part. Accordingly, a trajectory of the tool center (a dash line in Figure 6-4) is squeezed toward the center of the sphere (a solid line). The compensation algorithm is mathematically described in detail in Appendix E.

(a) The FTS axis needs a tool radius compensation in the transverse direction of the tool path  
(b) The tool center tangentially offsets from the desired surface

**Figure 6-3.** The tool radius compensation for an off-axis sphere
3. **Generate a spiral pattern overlaid on the part.** Associated with machining parameters, a helical profile (a red line in Figure 6-5(a)) mimics the tool path over the desired surface. Its amplitude represents the displacement of the w-axis (as shown in Figure 6-5(b)) that can be determined from the mapping array of the tool position in step 2.

![Figure 6-5. Generate a spiral pattern over the machined part](image)

(a) A revolving path represents a tool path over the part  
(b) The amplitude of the w-axis is interpolated from the tool position map
4. **Smooth the desired tool path.** Some desired surfaces such as a concave sphere on a flat consist of sharp corners at which the FTS must produce an infinite acceleration to machine this discontinuity. However, the Variform has an acceleration limit as well as a velocity constraint (as discussed on Chapter 4.1). In Figure 6-6, a sharp corner occurs at the conjunction between the arch and the flat (a dashed line) is, in turn, smoothed out (a solid line.) In addition, the smoothing scheme can also monitor the velocity profile of the desired path. If it exceeds the physical limit of an actuator (140 mm/s for the Variform), the given spindle speed must be reduced, or the geometry be changed to reduce the speed of the FTS.

![Figure 6-6. Acceleration reduction at sharp corners](image)

5. **Apply the deconvolution operation.** A modification input command signal can be formulated using either the inverse dynamics algorithm for the entire path, or a block deconvolution in which the modification can be proceeded in the time, or the frequency domain. If a Variform FTS were a linear, time-invariant system, each inverse scheme would
create the same modified tool trajectory. However, the characteristics of the FTS is intrinsically dependent on the amplitude of an input signal. Consequently, the block deconvolution with a gain scheduling scheme (discussed in Section 4.3.1) is the best way to generate a modified path that accounts for the nonlinearity of the servo. And finally, the modified spiral trajectory re-constructs a deconvoluted surface so that the modified input command of the FTS can be determined while machining the surface.

6.1.2 **ON-AXIS SPHERE: CONTROLLED EXPERIMENT**

In Figure 6-7, on-axis concave spheres with 35.89 mm radius and 350 µm depth of cut were diamond-turned on 1”-radius copper plated blocks by the base axes to investigate the best surface finish and form fidelity achievable by an ASG-2500 DTM. The form error associated with a 1.512-mm-nose-radius tool was 12.5 nm RMS as shown in Figure 6-8 and the surface roughness was 3.5 nm RMS as depicted in Figure 6-9.

![Figure 6-7. On-axis sphere diamond turned on copper-plated blocks](image)

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6.1.3 ANALYTICAL APPROACH OF FABRICATION AN OFF-AXIS SPHERE

A small concave sphere with a sag of 120 µm P-V and a radius of 12.33 mm was selected as a desired test surface. The description resulted in a 3.4 mm-wide feature on a flat plane. When the center of the sphere was at 20 mm off the rotational axis, the FTS machined the part only once per revolution. For most of the machining time, the FTS stayed at a stationary level and plunged into the part with frequency of 153 Hz when turned at 500 rpm. A time-based input command signal, accordingly, looked similar to a series of impulses as illustrated in Figure 6-10(a). As with a solid line displayed in Figure 6-3, the profile of the impulse evolved from a DC, followed a circular path, and gradually blended back to the adjacent flat line. Since the impulse series cannot be represented by a single sine wave, the frequency spectrum (as shown in the top graph of Figure 6-10(b)) distributed over a wide range of the
The main frequency content, though, is in the bandwidth of the Variform. Accordingly, this off-axis sphere can be fabricated by the FTS.

A command following test was performed with the Variform mounted on an optical table and the motion monitored by a laser interferometer. In Figure 6-11(a), the desired trajectory (a solid line) was modified by the overlap-add block deconvolution as discussed in Section 5.2.1. A long time-based signal was segmented into short pieces, deconvoluted individually in the frequency domain, and re-transformed back to the time domain. The modification resulted in a twisted shape of the input command signal (a solid line) that rose up a steep slope, made a small dip to change the axis velocity prior to the peak of the sphere, descended continuously, and created another small wave to decelerate the tool and gradually stop it at the DC level. This modified shape of the input command yielded an output trajectory (a dashed line) close to the desired excursion. Performance of the deconvolution technique was
compared to a simple gain and phase adjustment at which the input command signal had same profile as the desired excursion, but the phase was shifted and the amplitude scaled (as illustrated in 6-11(b).) As shown in Figure 6-12, the error (or path difference with respect to the desired tool path) associated with the simple correction technique was twice that of the modified input command, 12 \( \mu m \) P-V versus 6 \( \mu m \) P-V. The investigation indicated that not only did the amplitude and phase, but also the shape of an input command improved the tool trajectory better than that produced by the simple correction.

(a) Twisted shape of the modified input command yielded a circular path

(b) The unmodified command made use of a simple phase and gain adjustment

**Figure 6-11.** Comparison of the deconvolution and the gain and phase correction
Figure 6-12. Path differences associated with modified and unmodified input signals

Figure 6-13. An off-axis sphere was fabricated into a copper blank with a 1.012 mm radius diamond tool on the ASG 2500 DTM
6.1.4 Experimental Results

Figure 6-13 shows a turning setup of an 1"-radius test copper blank, similar to the ones used in the fabrication of the on-axis spheres, and a Variform FTS with a 1.012 mm-radius diamond tool on the ASG 2500 DTM. While machining, the servo stayed 20 µm above the flat surface and created a 3 mm-wide machined feature that was 100 µm deep.

![Figure 6-13](image)

(a) Deconvoluted input command resulted in a path difference of 6 µm P-V

(b) The difference associated with the unmodified command was three times of that of the deconvoluted input command

**Figure 6-14.** Comparison of the profiles of the machined surfaces in the tool trajectory direction

It appeared that the machined spherical surfaces cannot be characterized by the laser interferometer because the figure errors exceeded the tolerance of the instrument. The local slope of the machined surfaces was so steep that the interference patterns (or fringes) cannot be identified by the detector. As a result, a Talysurf profilometer was used to measure the
form fidelity of the machined surfaces associated with the modified (shown in Figure 6-14(a)) and unmodified input command (in Figure 6-14(b).) The traces in the tool path direction across the center of the part (solid lines in the top graphs) represented the tool trajectories that had been machined into the copper blank. Their deviation from the desired circular profile (dashed lines) illustrated the form errors (in the bottom graph) of the diamond turned features. Both the magnitude and shape of the path differences from the machining experiments matched the results in Figure 6-12. The modified input command reduced the profile error by one-third (±2 \( \mu m \)) when compared to the unmodified command (±6 \( \mu m \)).

Figure 6-15. With a lower spidle speed, the modification yielded even better form fidelity.
To investigate the improvement in form accuracy, another off-axis sphere was made. The geometric and machining parameters of the new feature as well as the deconvolution operation were similar to those in the previous cut, but the spindle speed was decreased from 500 to 250 rpm. This change decreased the frequency content of the desired surface and, as expected, resulted in reduced form error. As shown in Figure 6-15, the maximum error was within ±1 µm and most of the error was located close to the edges, where the smoothing operation altered the slope of the sphere. This improvement meant that proper machining parameters and correct transfer function of the actuator can improve the fidelity of the machined surface.

Figure 6-16. The shape of the reflection at the center of the spherical features clearly show the form error of the machined surfaces
In Figure 6-16, the reflection across the center of the diamond turned features illustrates the profiles of the machined surfaces. The reflection inside the unmodified sphere (at the left) captures a large amplitude, but low frequency wave similar to the path difference shown in Figure 6-14(b). The modified surface machined at 500 rpm (center) exhibits low amplitude ripples with a high frequency component. And lastly, the feature turned at a low speed of 250 rpm (right) reflects the best sphere. If a perfect sphere is machined into a flat surface, edge of the sphere becomes an exact circle. The shadow of the unmodified surface, however, illustrates a twisted circle due to the phase response of the Variform FTS. The shape of the modified spheres, on the other hand, displays circle features.

The surface profile and error measurements in Figure 6-14 and 6-15 were measured in the direction of the tool motion across the sphere or circumferential direction. If these spheres are measured in radial direction (cross feed direction), a different shape emerges as shown in Figure 6-17. Here the error remains small ($\pm 2 \mu m$) but the shape is much different. One explanation may lie in the tool nose radius correction needed to fabricate the spherical shape. For this experiment, the size of the tool radius (1.012 mm) is close to the size of the spherical feature. Calculating the tool path to correct for the nose radius is a difficult task for this application due to the circular tool path, the desired spherical shape, and any variation in the tool radius over the angle of contact. For these reasons, an experiment that is easier to test the deconvolution algorithm was devised and is presented in the next Section.
6.2 COSINE GROOVE

A cylindrical cavity with a fixed amplitude sinusoidal cross-section profile as illustrated in Figure 6-18 was proposed as a new test part shape. The feature can be described as a full period of a cosine wave whose slope smoothly changes into a flat. As a result, no sharp corners are in the tool trajectory. And since the cavity has a constant depth, the same tool excursion can be replicated for every revolution. Therefore, the turning process does not need tool radius compensation. The modification technique can now be clearly validated without any influences of other corrective schemes.
6.2.1 Analytical Approach of Fabrication a Cosine Groove

When machined at 561 rpm, an 18-degree wide cosine groove with a 240 µm sag contains a frequency component of 187 Hz close to the peak of the gain response of the Variform FTS as shown in Figure 6-19. At this frequency, the output trajectory will be amplified and out of phase by 52° resulting in the path difference with respect to the desired cosine wave similar to a sine wave. The designated tool path was modified by the inverse dynamics filter using the block deconvolution with the overlap-add method. The operation proceeded in time domain where a desired excursion was segmented and individually convoluted with the equivalent inverse dynamics filter as discussed in Section 5.3.
Figure 6-19. The cosine profile of the test part has a frequency component close to the peak of the gain response and near the physical limits of the Variform.

The simulated output trajectory (points in Figure 6-20) associated with a modified input command (circles) was close to the desired path (squares). The estimated error (a dash line) was less than 200 nm P-V.

Figure 6-20. A simulated output tool path associated with a modified input command signal yielded the path difference of 200 nm P-V.
The modified input signal was then experimentally verified with the FTS on an optical table with the displacement measured by a laser interferometer. Figure 4-18 is repeated here in Figure 6-21(a) to illustrate the agreement of the FTS motion (x marks) observed by an interferometer and that of the simulation (in Figure 6-20.) The output trajectory was in-phase and the path difference was ±1 \( \mu \text{m} \) near the peak and 8\( \mu \text{m} \) at the edges. The mismatch between the gain response used in the deconvolution operation and the actual response of the FTS may cause the path difference of the actual tool trajectory larger than that of the model.

On the contrary, the tool trajectory (x marks in Figure 6-21(b)) associated with an unmodified signal was, as expected, 10 \( \mu \text{m} \) bigger, and 52° out of phase with respect to the desired path (a solid line). The dynamic response of the Variform caused a sinusoidal profile of the path difference (a solid line) with significant magnitude of 225 \( \mu \text{m} \) P-V.

Figure 6-21. Comparison of the path differences associated with the modified and unmodified input commands
Since the Variform was performed in vicinity of the FTS physical limits where the highest speed of the tool motion is restricted, the velocity was charted to determine whether the operation was out of the linear boundary of the Variform FTS. If a tool excursion is a cosine wave, its velocity (or derivative) is a sine wave. In Figure 6-22(a), a full sine wave of the velocity profile shows that the FTS was performed within its limits. The unmodified excursion, on the other hand, exhibited a more distorted sine wave on the tool speed (in Figure 6-22(b)). This meant that the input command modification technique can extend the usable bandwidth of the Variform.

![Velocity Profiles](image)

(a) A twisted sinusoidal velocity of the modified tool path  
(b) A more distorted velocity profile of the unmodified path

**Figure 6-22.** The velocity profiles of the modified and unmodified tool path
6.2.2 Experimental Results

The cylinder cavity with a cosine cross section profile with the dimension described in the previous Section was fabricated into a copper blank in which a flat surface was produced on the side as a reference feature. The location of the desired machined surface was designated at $0^\circ$ with respect with the reference flat. In Figure 6-23, the cosine groove close to the circumference of the copper blank was machined using the modified input command whereas the other surface close to the center was associated with the unmodified input command.

![Modified and Unmodified Surfaces](image)

Figure 6-23. The machined surface associated with the modified and unmodified input commands
A comparison of the form errors between modified and unmodified surfaces was made using the Talysurf profilometer and the phase angles of the machined surfaces were measured by using a microscope and the slide stage of the Zygo New-View interferometer to locate the surfaces in the Cartesian coordinate before transforming to the polar coordinate for $r$ and $\theta$. The profile of the machined surface associated with the deconvoluted input command (measured along the trace (A) in Figure 6-24) is depicted as a solid line in the top graph of Figure 6-25(a), and the profile of the unmodified surface (measured along the trace (B)) is illustrated in Figure 6-25(b). The path differences with respect to the desired surface profile shown in the bottom graphs of both Figure 6-25(a) and (b) show that the unmodified surface was $50^\circ$ out of phase resulting in a path difference of $122 \ \mu m$ P-V, and the deconvoluted surface was in-phase yielding a significant reduced form errors by two orders of magnitude ($4 \ \mu m$ P-V) as shown in the bottom graph of Figure 6-25(c). However, the average of the path difference is about $10 \ \mu m$. This means that the amplitude of the modified surface was short by $10 \ \mu m$ with respect to the desired surface.

**Figure 6-24.** Direction of the profile measurement with respect to the reference flat.
The amplitude and phase of the machined surfaces mismatched those estimated in Section 6.4.1. Since the desired operation is to command the Variform at 187 Hz with 240 μm P-V, the FTS produces the maximum amplitude and 52° phase angle. The attenuated amplitude and the reduced phase angle of the machined surface indicated that the FTS was commanded at frequencies lower than 187 Hz. Figure 6-26(a) and (b) illustrate the monitored signals of the spindle and the desired input command during the machining. Since the spindle axis of the ASG 2500 DTM is an open-loop system, when the spindle is commanded at 561 rpm (9.35 Hz), the axis actually spins at 532 rpm (8.87 Hz) as shown in Figure 6-26(a). The wrong spindle speed resulted in a reduced frequency content of the cosine feature from 187 Hz to 179 Hz.

(a) Measured surface profile associated with the modified input command
(b) Measured surface profile associated with the unmodified input command

(Figure continued)
(c) The deconvoluted surface was offset by 10 μm with respect to the desired surface

Figure 6-25. The surface profile measurements of the modified and unmodified surfaces

(a) The spindle speed was slower than the desired condition

(b) The FTS was operated at a lower frequency than the desired condition

Figure 6-26. The form errors in the machined surface is a result of the wrong spindle speed
6.3 SUMMARY

The deconvolution technique results in higher form accuracy of machined surfaces than that of the unmodified input command even though the machining parameters are different from the designated operation. The machining experiments corroborate the simulated results and the tool path measurements using the laser interferometer as the Variform is mounted on the air table. The figure errors can be decreased by two orders of magnitudes due to the significantly reduced phase response. However, the simple phase and gain adjustment of the desired input command cannot eliminate the dynamic response of the FTS. The shape of the input command signal is modified to yield the output tool path close to the desired excursion.
CHAPTER 7

CONCLUSIONS

An open-loop control scheme using Digital Signal Processing (DSP) was proposed to correct form errors caused by the dynamics of the Variform fast tool servo. Fundamental concepts of DSP were introduced to explain the causes of attenuated and delayed excursion, to measure the dynamic characteristics of the FTS, and to formulate a corrective algorithm. A desired tool path is modified using the deconvolution or the inverse dynamics algorithm to create a new input command that counteracts the 2\textsuperscript{nd} order dynamics of the actuator yielding a significant reduction in steady-state form errors.

Initial experimental validation

The magnitude of the error was reduced by almost 3 orders of magnitude and the analytical simulation predicted the experimental behavior very closely. The response showed practically no delay from the desired excursion after a startup interval lasting approximately 4 ms. In practice, this startup period occurs before machining begins and thus does not effect the fidelity of a surface. Since the entire command signal needed to machine a surface can be generated \textit{a priori}, the deconvolution is performed once for a given set of machining
parameters (e.g., spindle speed, cross-feed). The technique critically depends on knowledge of the actuator impulse response.

**Operating range of the Variform FTS**

The limitations of this technique appear with the combination of high frequencies and large amplitudes. Actuator motion is constrained by the maximum velocity (140 mm/sec) of the electro-mechanical system resulting in an uncorrectable error. The optimal machining condition is assured when the Variform FTS functions within its operating range in which the velocity is less than its constraint. Out of this range, the Variform FTS loses its driving acceleration and distorts a sinusoid command signal to a triangle-like output motion. These restrictions is incorporated into a procedure for automatically decomposing an arbitrary motion path into its constituent sinusoids and producing a modified command signal that drives an actuator to follow the desired profile. Inputs to this procedure also include the actuator impulse response and the machining parameters.

**LVDT dynamic response**

The LVDT implements a 2nd order internal filter on the position measurement resulting in a lag behind the actual tool motion. The measurements using a laser interferometer corroborated that of a capacitance gauge in which the bandwidth of the position sensor is about 500 Hz and the dynamic characteristics are independent to an input amplitude. The dynamic response can be eliminated using the similar method as the inverse FTS dynamics algorithm.
Nonlinear dynamic characteristics of the Variform FTS

The measurements using a laser interferometer shows that the FTS response is a function of both frequency and amplitude of an input signal. A basis of a gain schedule that resulted in the best command following performance was determined. The dynamic response associated with the peak-to-valley of the input command signal yields a path closest to the desired tool motion. The gain scheduling scheme was used in the machined surface test.

Block Deconvolution

Inverse dynamics algorithms in real-time were developed to adapt to varying amplitude and machining conditions using the appropriate dynamic response and the most recent parameters. The short-time Fourier transform applied to a sliding window function over a desired excursion and deconvolved with the dynamic response in the frequency domain. Simulation of the corrective scheme under a fixed machining condition indicated that the steady-state form errors associated with a modified input command were significantly reduced (to 1 \( \mu \text{m} \) P-V) after a startup interval. However, the experiments resulted in a bigger path difference (8 \( \mu \text{m} \) P-V) than that of the model because the implementation of the STFT requires careful selection of the window function, the scale factor and the inverse transformation to result in a correct command modification.

When an equivalent inverse dynamics filter is used, complication of the deconvolution operation is now implemented as a convolution operation (or multiplication in the frequency domain). Block convolution using overlap-add and overlap-save methods was introduced to
develop the inverse dynamics algorithm in *real-time*. This approach yields a simpler as well as faster implementation than the STFT method.

**Machined Surfaces**

An off-axis sphere was selected as a surface-machining test of both deconvolution schemes. The profile of the machined surface measured by a Talysurf profilometer showed that the figure errors in the modified surface was less by 1/3 when compared to that in the unmodified surface. Though the modified trajectory of the FTS was experimentally validated that its profile was close to the desired trajectory, the size of the tool close to the size of the sphere made it difficult to compensate for a tool nose radius resulting in the form errors of the machined sphere.

A cosine groove was proposed as an alternative shape of the machined test part. When the FTS was commanded to fabricate the features at its natural frequency, the unmodified tool excursion was $90^\circ$ out of phase resulting in the path difference over $200\mu m$ P-V, and was distorted due to the physical limits of the actuator. On the other hand, the modified tool path was identical to the desired trajectory. It is experimentally verified that the input command modification technique can extend the usable bandwidth of the Variform. Even though the features were fabricated at an unintended spindle speed, the off-line modified input command yielded a better form accuracy in which the figure errors caused by the phase response was corrected to the machined cosine grooves than the unmodified surface.
Since the ASG 2500 DTM does not have a closed-loop controller for the spindle axis, it is likely to create a speed error resulting in a mismatch gain and phase response included in the modified input command signal. When the FTS is commanded by an off-line modified input signal, it cannot be changed if the spindle speed drift over during continuous fabrication. A better way to deal with this uncertainty of the spindle speed is to generate the modified command on-line during machining.

**Future work**

Though the experiments showed that the command following of the FTS was significantly enhanced when using a modified input command signal, there was a room for improving the figure errors of the machined surfaces. The command of the FTS must incorporate with the multiple axes of the base machine which may be unintentionally changed during the fabrication time. The initial implementation discussed in Chapter 5 will be proceeded. Here the *real-time* algorithm must integrate the gain scheduling scheme to select the appropriate dynamic response to create a modified input command signal. A challenge of this operation is the conjunction between two commands modified by different dynamic response. The connection may not smooth due to different gain and phase included in the command signals. A smoothing filter will be implemented to reduce this discontinuity. The computation time is also critical to operate the *real-time* deconvolution algorithm incorporating with the gain scheduling scheme. Many implementation issues such as a *real-time* tool nose radius compensation should be further investigated.
REFERENCES


APPENDIX A

POLE-ZERO CANCELLATION ALGORITHM

Assumptions:
1. Impulse response of a dynamic system can be represented as a transfer function model with known zeros and poles.
2. Disturbances to the system are minimized.

Pole-zero cancellation algorithm pre-filters an input command signal to compensate for a dynamic response (poles) of a system by implementing its zeros to be exactly the same as poles of the dynamic system. Equation (A.1) expresses dynamics of a second-order system (or transfer function of an output to an input) in a continuous frequency domain as

\[
\text{Frequency response of the system } = \frac{\text{Output}(s)}{\text{Input}(s)} = \frac{s + z}{s^2 + 2 \zeta \omega_n s + \omega_n^2}
\]  
\hspace{3cm} (A.1)

where

- \(s\) Laplace operator
- \(\omega_n\) Natural frequency
- \(\zeta\) Damping ratio
- \(z\) zero of the system

Consequently, the inverse dynamics filter can be implemented as written in Equation (A.2).
Inverse dynamics filter = \[
\frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s + z}
\] (A.2)

However, filters whose order of their zeros is higher than the poles’ cannot be implemented (in time domain). Additional poles with the frequency responses much higher than the Nyquist frequency of the system bandwidth are implemented in the filter.

Figure A-1 illustrates a diagram of the pole-zero cancellation to a second-order dynamic system with a bandwidth of 350 Hz and a damping ratio of 0.6. As a desired trajectory, a sine wave of 100 and 300 Hz was modified by the inverse filter to compensate for phase and gain dynamic response before applied to the dynamic system. The poles of the filter were selected at 2000 Hz with a critical damped system characteristic. Equation 3 shows the system dynamics and the inverse dynamics filter.

\[
\text{Output}(s) = G(s) = \frac{(2\pi \times 350)^2}{s^2 + 2 \times 0.6 \times 350s + (2\pi \times 350)^2}
\]

\[
\text{Input}(s) = G^{-1}(s) = \frac{s^2 + 2 \times 0.6 \times 350s + (2\pi \times 350)^2}{(s + 2\pi \times 2000)^2}
\] (A.3)
Figure A-1. Schematic of input command modification using the pole-zero cancellation

The inverse filter resulted in a modified input trajectory (solid line) such that its phase was advanced with respect to the desired trajectory (dash line) and the amplitude gain was compensated as shown in Figure A-2. The modified trajectory started with a big step to correct the phase dynamic response. However, this open loop cancellation is not a zero phase error corrective scheme. Rather, it replaces low-bandwidth dynamics of a system with a second-order dynamic response with a higher bandwidth. Still, this replacing dynamics has a small phase response at low frequencies. As a result, this open loop scheme cannot totally eliminate the phase error with respect to the desired trajectory as illustrated in Figure A-3. As a result, a PID controller was added to reduce the phase response.
In Figure A-4, a schematic shows a feedback loop with the PID controller. Gains of the controller were

\[
\begin{align*}
K_p &= 9.5 \\
K_i &= 100000 \\
K_d &= 0.00005.
\end{align*}
\]

Let \( H(s) \) be a transfer function of the PID controller so that the transfer function of Figure A-4 can be rewritten as shown in Equation A.4.

\[
\frac{Output(s)}{Input(s)} = \frac{H(s) \cdot G^{-1}(s) \cdot G(s)}{1 + H(s) \cdot G^{-1}(s) \cdot G(s)} \tag{A.4}
\]
Basically, the feedback loop tuned the system using a set of gains to yield new dynamic response. However, it cannot achieve a zero phase and unity gain response. In fact, the closed loop control only reduced the steady state path error as estimated by a model shown in Figure A-5. In addition, input command signals generated by any feedback loop are a function of errors. Therefore, the corrective command always lags behind the error.

In contrast to the feedback loop, the deconvolution scheme estimates amount of dynamic errors and calculate a new input command ahead of time to achieve a desired tool path. As a result, the algorithm yields zero phase and unity gain for a wide portion of the frequency range.

Conclusively, the pole-zero cancellation replaces the system dynamics with a board bandwidth frequency response to reduce the path error. The scheme can correct gain response; however, phase error cannot be totally eliminated. Additional PID control improves the tracking performance, but it always lags behind the error. A feedforward with a look-ahead scheme using inverse dynamics algorithm can result in zero phase and unity gain response to the dynamic system.

Figure A-5. Steady state path difference as applying the closed loop pole-zero cancellation (model)
APPENDIX B

REFERENCE CAPACITOR LOOP CONTROL

Variform fast tool servo (FTS) transforms electrical power to mechanical motion using piezoelectric property. However, the displacement of the PZT stacks are inherently nonlinear with respect to an applied voltage exhibiting a hysteresis curve. Ge and Jouaneh [27] described the nonlinear response in macroscopic manner using an empirical parametric model of Preisach operation. The characteristic appears as an asymmetric loop between ascending and descending direction. Zhoung, Smith, and Seelecke [26] proposed an energy based model of a piezoelectric actuator for nano-positioning. Their quadrature method reflects the kinetics of each crystal element in a piezoceramic. However, the method carries a lot of computation load. The simplified model using a parametric implementation on only one direction deflection improves the computation speed by 3 orders of the magnitude with respect to the quadrature method.

Alternatively, the Variform FTS implements an amplifier with a loop charge feedback circuit. Newcomb and Flinn [21] showed that the nonlinear effect is reduced when driving the PZT as a function of electric charge. Main, Garcia, and Newton [2] illustrated the physical model of a PZT stack and proposed a control equation of the PZT deflection $\Delta l$ associated with the total free charge $Q_t$. The relation can be simplified as

$$Q_t = a \Delta l.$$  

where $a$ is a constant. The derived equation will be shown in Appendix D.
Comstock [23] presented a charge-feedback circuit concept for the PZT position controller. A reference capacitor \( C \) is connected in series with the stack as shown in Figure B-1.

![Diagram of a capacitor](image)

**Figure B-1.** Diagram of a reference capacitor loop

The capacitance of the reference is a constant \( C \). Since an operational-amplifier input draws no current, the amount of current supplied to the PZT goes to the reference capacitor \( I_C = I_t \). This means that the series connection constrains the charge on the PZT stack and the reference capacitor to be the same \( (Q_c = Q_e) \). In addition, the op-amp generates the output \( V_{out} \) so that the voltage difference between inputs becomes zero. Therefore, voltage at point A is equal to input voltage \( (V_A = V_{in}) \). The voltage across the reference capacitor \( V_A \) is governed by charge \( Q_c \) and capacitance \( C \).

\[
Q_c = C V_A
\]

Therefore, the input voltage is proportional to the charge across the piezoelectric stack.

\[
Q_t = C V_{in}
\]

Note that the input voltage \( V_{in} \) supplied to the PZT stack is amplified \( a_{amp} \) times greater than the control voltage \( V_{control} \).
Though this simple diagram delivery the charge control with respect to the input voltage, the circuit has some drawbacks. Other references [24, 25] have developed expanded investigations of this elementary circuit analysis. With the charge-loop amplifier concept, the input voltage $V_{in}$ is literally proportional to the deflection $\Delta l$ of the PZT stack.

The amount of charge stored on series capacitors is less than that on either one of them alone with the same voltage. The additional circuit reduces the sensitivity of the deflection to the input voltage. Indirectly, it helps the actuator to reduce an effect of the PZT physical limits in which the stack motion reaches its velocity constraint.

A position loop is constructed around the charge loop with feedback position signal derived from a linear variable differential transformer (LVDT). The operating diagram of the Variform under the inverse dynamic technique is illustrated in Figure B-2.

The modified desired tool path is the reference signal to the position feedback loop. The nonlinear piezoelectric drive including dynamic of the tool fixture becomes a linear system in the operating bandwidth when incorporating with the charge-loop amplifier. Conclusively, the model of the Variform FTS is derived as
Piezoelectric property:

\[ Q_t = a \Delta l \]

Charge-loop amplifier:

\[ Q_t = C V_{in} \]

Therefore, the PZT deflection can be written as a function of the input voltage as

\[ \Delta l = \frac{C}{a} V_{in} \]

It is proportional to the control voltage \( V_{control} \)

\[ \Delta l = \frac{C a_{amp}}{a} V_{control} \]

Let a PZT force be represented as a spring force model.

\[ F_t = k_t \Delta l \]

where \( k_t \) is the stiffness of the PZT.
The T-lever amplifies the magnitude and changes the orientation of the driving force. The moment about point \( O \) is zero as shown in Figure B-3. Note that the Variform drives two piezoelectric stacks in the opposite direction of each other.

\[ F_x h = 2F_i b \]

The end of the T-mechanism connects to the straight linear motion arrangement. The H flexures provide high lateral, but low axial stiffness. The dynamic response of this tool fixture and driving force is

\[ m\ddot{x} + c\dot{x} + k_x x = F_x \]

where

- \( m \) is an effective mass of the tool fixture
- \( c \) is a damping coefficient
- \( k_x \) is a stiffness
- \( x \) is a tool motion

The governing equation of the tool motion as a function of the control voltage in the Variform FTS is formulated as

\[ m\ddot{x} + c\dot{x} + k_x x = \frac{2bC k_x a_{amp}}{a h} V_{control} \quad \text{or} \quad \ddot{x} + c'\dot{x} + k'_x x = K' V_{control} \]

where

\[ c' = \frac{c}{m}, \quad k'_x = \frac{k_x}{m}, \quad K' = \frac{1}{m} \frac{2bC k_x a_{amp}}{a h} \]

The Laplace transform of this open loop system is
Therefore, the closed-loop position control of the FTS system is

\[ TF(s) = \frac{G(s)}{1 + G(s)} = \frac{K'}{s^2 + c's + k' + K'} \]

Since the fraction of the stiffness of the H flexure to the tool fixture mass is so small with respect to the coefficient \( K' \) obtained from the PZT stiffness, the coefficient \( k' \) is negligible.

As a result, the Variform closed loop model is simply represented as

\[ TF'(s) = \frac{K'}{s^2 + c's + K'} \]

The characteristic equation of this system is illustrated as

\[ \frac{X(s)}{V_{\text{control}}(s)} = \frac{\omega_n^2}{s^2 + (2ζ\omega_n)s + \omega_n^2} \]

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

where

\( \zeta \) is a damping ratio

\( \omega_n \) is an undamped natural frequency

\( \omega_d \) is a damped natural frequency
Table B-1. Mechanical properties of the Variform FTS components

* obtained from manufacturer technical specification

^ obtained from matwave.com

The coefficient $a$ as a proportion of the total charge $Q_t$ to the stack deflection $\Delta l$ is

$$\frac{1}{a} = \frac{t d_{33}}{\varepsilon_{33}^T A_{PZT}} = 0.08 \text{ Coulomb/m}.$$ 

Then, the coefficient $K'$ as a ratio of force and control voltage $V_{control}$ is
As a result, the natural frequency of this closed-loop system is

\[ \omega_n = \sqrt{K'} = 363 \text{ Hz}. \]

The damping ratio \( \zeta \) is

\[ 2\zeta \omega_n = \frac{c}{m} \]

\[ \zeta = \frac{c}{2m \omega_n} = 0.83 \]

The damped natural frequency is

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} = 202 \text{ Hz}. \]

The estimated parameter yields resemble characteristics of the Variform FTS such as

Closed loop –3 dB bandwidth (\( \omega_b \)): 350 Hz

Damped natural frequency (\( \omega_d \)): 180 Hz.
APPENDIX C

INVERTING AMPLIFIER

When operating the Variform from a single command signal, a second signal must be generated that is the inverse of the first. Some commercial controllers (e.g. the Delta Tau PMAC) provide this option for the output. However, to find the frequency response of the Variform using a Spectrum Analyzer's single channel output, the inverted signal must be generated. Figure C-1 shows a way to accomplish this using an operational amplifier and two resistors. The gain of this circuit can be written as,

\[
\frac{V_{out}}{V_{in}} = - \frac{R_f}{R_i}
\]

If the two resistors are selected with the same magnitude, a unity gain inverter will be the result. This simple circuit can be built using a single operational amplifier 741 chip, a 100K-ohm resistor, and a potentiometer.

![Schematic of an inverse amplifier](image)

**Figure C-1.** Schematic of an inverse amplifier
APPENDIX D

PIEZOELECTRIC PROPERTY

Main, Garcia, and Newton [25] demonstrated a model of a piezoelectric stack in the third polarization direction, which can be written as a function of strain $S$, stress $T$, and electric charge on an individual layer of positive electrode $Q_f$.

$$S_3 = \left( \frac{1}{Y_{33}} - \frac{d_{33}^2}{\varepsilon_{33}^T} \right) T_3 + \frac{d_{33} Q_f}{\varepsilon_{33}^T A}$$

where

$A$ Electrode area, m$^2$

$d_{33}$ Piezoelectric constant

$\varepsilon_{33}^T$ Permittivity at constant stress, F/m

$Y_{33}$ Young’s modulus, Pa

For an $n$-layer PZT stack, the deflection $\Delta l$ and total free charge $Q_t$ can be expressed as

$$\Delta l = S_3 n t$$

$$Q_t = Q_f n$$

where $t$ is thickness of a PZT layer. Then, the deflection of the stack is

$$\Delta l = n t \left( \frac{1}{Y_{33}} - \frac{d_{33}^2}{\varepsilon_{33}^T} \right) T_3 + \frac{t d_{33} Q_f}{\varepsilon_{33}^T A}.$$
APPENDIX E

TOOL RADIUS COMPENSATION

Radius of a tool nose offsets to the machined surface in which the center of the tool is on the normal direction of the surface. For an on-axis sphere whose the center is on the spindle axis and the tool symmetrically wraps around the surface, the compensation is simply a constant offset of the tool radius. However, the simple technique cannot be applied to an off-axis sphere. The center of the sphere is far off from the spindle of rotation and the tool path is no longer symmetric about the center of the surface.

Figure 6-3(b) is repeated here. The center of the tool is shifted from the contact point in both x and w direction. Since the dotted line (a) is normal to the dashed line (b), the slope of line (a) is equal to the negative inverse of the slope of line (b) \[ \frac{dw}{dx_{(b)}} = -\frac{dx}{dw_{(a)}}. \]

The angle \( \theta \) can be expressed as

\[
\theta = \tan^{-1}\left(\frac{dw}{dx}\right) + \frac{\pi}{2}.
\]

Then the location of the tool center \((x_c, w_c)\) is written as a function of the location of the contact points \((x_n, w_n)\) and the tool radius \(R_T\) as
\[ x_c = x_n - R_T \cos \theta \]
\[ w_c = w_n - R_T \sin \theta \]

In Figure E-1, the trajectory of the tool center (a dash line) is offset from the desired surface (a solid line) with a constant distance. If the circle represents a tool, its center moves on the dashed line while the circumference touches on the desired surface.

**Figure E-1.** Tool center is shifted in x and w direction to compensate for the nose radius
APPENDIX F

MATLAB CODE FOR A DECONVOLUTED SURFACE

MATLAB file: Drive_FTS_command.m is a main program calling all sub programs

```matlab
clc, close all;

open parameters;
open Cosine_Lookup; % Generate a desired surface for a look-up table
open DeconOLAPadd_LookupX;

Cosine_Lookup;
sim('simDrive_FTS',total_time-delta_t);
[x,y] = pol2cart(theta,R);
figure, plot3(x,y,zvolt_out*20e-3,’.’),grid;
save zVOLT_out.mat zvolt_out;
DeconOLAPadd_LookupX;
```

MATLAB file: parameter.m contains all parameters of the machining conditions

```matlab
% Parameters of a slot with a sinusoid cross section
sag = 0.240; % mm
% stay in the air at 20um from flat surface
% total depth of cut is sag-0.02 = 0.100 mm
r_tool = 0.4801; % mm
Ro = 21; % mm
Rl = 18; % mm
width = pi/20; % rad
spin = 561.0; % rpm
feed = 4.0; % mm/min
encoder = 5000.0; % line/rev
R_step = 0.010; % mm Radial resolution of the table
% location of the sphere %
Tcenter = pi/2;
Tmin = Tcenter-width;
Tmax = Tcenter+width;
buffer = pi/288*0;
R_index = [ Ro : -R_step : Ri ]; %
theta_index = [ Tmin-buffer : 2*pi/encoder : Tmax+buffer ]; %
n_steps = floor((Ro-Ri)/feed*spin*encoder);
total_time = (Ro-Ri)*60/feed;
Fs = 2^11;
NEWencoder = ceil(encoder/(n_steps/total_time)*Fs)*5;
delta_t = 1/Fs;
volt_offset = -sag*1e3/40; % volt
```

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MATLAB File: Cosine_Lookup.m creates an array map of the desired surface

warning off MATLAB:divideByZero;
clear; close all;
parameters;
% Create a data grid of even spaced R and theta
[Rtab,Ttab] = meshgrid(R_index,theta_index);

% Constant Z in each rev, then Z is a function of theta only
Zrev = sag/2.*(-cos([0:2*pi/length(theta_index):2*pi-2*pi/length(theta_index)]') + 1);
delta_t = 1/Fs;
t = [0 : delta_t : total_time-delta_t]';
R = (Ro-Ri)-t*feed/60 + Ri;
theta = 2*pi*((t*spin/60)-floor(t*spin/60));
Z = sag/2.*(-cos(2*pi/(pi/10)*theta) + 1);

% Replicate Z to every rev
[dum, Ztab] = meshgrid(R_index, Zrev);
% Look-up table accepts only increasing vector
% Flip R and Z around
R_index_inc = fliplr(R_index);
T_index_inc = theta_index;
Z_tab_inc = fliplr(Ztab);
% Contour plot in Cartesian coordinate
[Xtab,Ytab] = pol2cart(Ttab,Rtab);
% convert amplitude to volt
Z_tab_inc = Z_tab_inc/20e-3;
save zSMOOTH.mat R_index_inc T_index_inc Z_tab_inc;
clear, load zSMOOTH.mat;
parameters;
delta_t = 1/Fs;
t = [0 : delta_t : total_time-delta_t]';
R = (Ro-Ri)-t*feed/60 + Ri;
theta = 2*pi*((t*spin/60)-floor(t*spin/60));

MATLAB file: DeconOLAPadd_LookupX.m modifies an input command with the dynamic
response of the FTS.

warning off MATLAB:divideByZero;
clear;
load zVOLT_out.mat; % Spiral path of the input command
parameters;
% Shift the desired tool path (in volt) so that it starts from zero
% Frequency Response of the entire path
[f, mag, phase, fq] = freqKEN(zvolt_out,Fs);

% Deconvolution with a fixed amplitude
% Use a TF data from LVDT so that the code can be verified with an %experiment
zvolt_ml = deconvX(zvolt_out,Fs);
% Deconvolution with amplitude selective scheme
zvolt_m3D = deconvX3D_v3(zvolt_out,Fs,NEWencoder);
zvolt_m3D = zvolt_m3D(:,1) + volt_offset;

% Create the spiral path
t = [0 : delta_t : total_time-delta_t]';
R = (Ro-Ri)-t*feed/60 + Ri;
theta = 2*pi*((t*spin/60)-floor(t*spin/60));

% Generate a look-up table
% Geometric parameters for R and Theta
R_step = 0.01; % mm Radial resolution of the look-up table
encoder = 5000; % line/rev Angular resolution of the look-up table
R_index = [ Ro : -R_step : Ri ];
theta_index = [ Tmin-15*pi/180 : 2*pi/encoder : Tmax+10*pi/180 ];

[i] = find( theta >= min(theta_index) & theta <= max(theta_index) );
zDec = zvolt_m3D(i);
TDec = theta(i);
RDec = R(i);

clear R theta i
[R_tab, T_tab] = meshgrid(R_index,theta_index);
zDecTab = griddata(TDec,RDec,zDec, T_tab,R_tab,'cubic');

i=isnan(zDecTab);
w=find(i==1);
zDecTab(i) = volt_offset;

Rdec_index_inc = fliplr(R_index);
Zdectab_inc = fliplr(zDecTab);
Tdec_index_inc = theta_index;

Rdec_tab_inc = fliplr(R_tab);
Tdec_tab_inc = T_tab;

save zVOLT_DEC.mat Rdec_index_inc Tdec_index_inc Zdectab_inc;
save RT_tab.mat Rdec_tab_inc Tdec_tab_inc;
save zVOLT_DEC_index.mat Rdec_index_inc Tdec_index_inc

% Offset the desired surface
load zSMOOTH.mat;
parameters;
Z_tab_inc = Z_tab_inc + volt_offset;
save zSMOOTH.mat R_index_inc T_index_inc Z_tab_inc;

MATLAB File: deconvX.m modifies an input command with the dynamics measured by the LVDT

function [zvolt_m] = deconvX(zvolt,Fs)
%DECONVX() Deconvolve a signal in the time domain with TF measured by LVDT
% Input
%  zvolt signal vector
% Fs sample frequency of the signal in Hz
% Output
% zvolt_m modified signal

% Load TF of the FTS measured by LVDT
load HnewTool.mat; % File contains [freq, H]
F = 2000; % sample rate of H in Hz
% Transfer function
% Interpolate the TF to new sample rate, Fs
Freq = freq;
overlap_len = 100;
deconv_len = Fs-overlap_len;
f_step = Fs/(deconv_len+overlap_len)/2;

% Generate conjugate and expand frequency response from DC to Fs
[freq1, H1]=freqrespX(freq, Re1, Im1, f_step, Fs);

% Regularization part
% Here is 0<\theta<1 is the floor of your equalizer.
% The higher \theta, the smoother your signal.
\theta = 0.8
H2 = H1;
I = [abs(H2)<\theta];
H2(I) = \theta*exp(\sqrt{-1}*angle(H2(I)));

% Impulse response of the FTS
h = ifft(H2);
heq = ifft(1./H2.*exp(-sqrt(-1)*[0:length(H2)-1]'/(length(H2))*2*pi*delay));

% Finish setting up parameters
% Overlap-add Deconvolution
% Modify input command length (deconv_len+overlap_len) at a time
n_steps = length(zvolt);
rev = ceil(n_steps/deconv_len);
zvolt_m(rev*deconv_len+overlap_len) = 0;
zvolt_m = zvolt_m(:);
zvolt (rev*deconv_len+overlap_len) = 0;
num = conv([ zvolt([1:deconv_len]); zeros(overlap_len,1) ], heq);
zvolt_m(1:deconv_len + overlap_len) = num(1:deconv_len + overlap_len);
for k = 1 : rev-1
  dum = zvolt_m( [1:overlap_len] + k*deconv_len );
  num = conv( [zvolt([1:deconv_len]+k*deconv_len); zeros(overlap_len,1) ], heq);
  zvolt_m([1:deconv_len+overlap_len]+k*deconv_len)
    = num([1:deconv_len + overlap_len] + k*deconv_len);
  zvolt_m([1:deconv_len+overlap_len]+k*deconv_len) = zvolt_m([1:overlap_len]+k*deconv_len) + dum;
end
zvolt_m = zvolt_m(delay+1:end); % shift backward to correct phase
zvolt_m = zvolt_m(1:n_steps); % deconvoluted of the tool center path
zvolt = zvolt(1:n_steps);
MATLAB file: deconvX3D_v3.m modify an input command with a 3D frequency response of the Variform

```matlab
function [zvolt_m] = deconvX3D_v3(zvolt,Fs,deconv_len)

% DECONVX3D( ) Deconvolve a signal in the time domain with TF measured by laser interferometer
% [zvolt_m] = deconvX3D(zvolt,Fs,deconv_len)
% Input
% zvolt signal vector
% Fs sample frequency of the signal in Hz
% deconv_len length of block deconvolution
% Output
% zvolt_m modified signal
% Length of convolution a deconv_len (length L) with
% an inverse filter (length M) is L+M-1. So the length
% of overlap between 2 pieces is M-1.
overlap_len = 49;

% Frequency resolution
f_step = Fs/(deconv_len+overlap_len);
% Generate conjugate and expand frequency response from DC to Fs
% of the frequency response of the FTS
[freq1, H1]=freqrespX3D_sn0113(f_step, Fs);

% Regularization part
% Here is 0<th<1 is the floor of your equalizer.
% The higher th, the smoother your signal.
th = 0.8
H2 = -H1;
I = [abs(H2)<th];
H2(I) = th*exp(sqrt(-1)*angle(H2(I)));

h2 = real(ifft(H2));
figure, hold
for i=1:10
    grpdelay(h2(1:overlap_len+1,i),1,Fs,Fs);
end
title('Group Delay of h1');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Equivalent inverse dynamics filter
% Because of the regularization, the impluse response of
% the inverse filter spreads out into positive and negative time.
% Add a constant phase or delay of N points to correct the effect
% where N > max(group delay)
delay = 15;
[m,n] = size(H2);
[dum,H_len] = meshgrid([1:n],[0:length(H2)-1]/(length(H2)));
Heq = 1./H2 .*exp(-sqrt(-1)*H_len*2*pi*delay);

% Equivalent inverse dynamic filter
heq = ifft(Heq);
heql = real(heq(1:overlap_len+1,:)); % Keep only non-zero values
figure, plot(real(heql(:,5))), grid
ylabel('Inverse Dynamics Filter');

% Impulse response of the FTS
```

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\[ H_3 = H_2 \cdot \exp(-\sqrt{-1} \cdot H_{\text{len}} \cdot 2 \cdot \pi \cdot \text{delay}); \]
\[ h = \text{ifft}(H_3); \]
\[ h_3 = \text{real}(h(1:\text{overlap}_{\text{len}}+1,:)); \quad \% \text{Keep only non-zero values} \]
\[ \text{figure, plot(real(h_3(:,3))), grid} \]
\[ \text{ylabel('Impulse response of FTS');} \]
\[ \% \text{Estimate the path error due to the nonlinearity} \]
\[ \text{num} = \text{zeros}(\text{deconv}_{\text{len}}+\text{overlap}_{\text{len}},10); \]
\[ \text{for } i = 1:10 \]
\[ \text{num(:,i) = conv( zvolt([1:deconv_{\text{len}}]), h_3(:,i));} \]
\[ \text{end} \]
\[ \text{figure, plot(zvolt(1:deconv_{\text{len}}),'--'), hold, ...} \]
\[ \text{plot(num(deley+1:end,[1,3,5,7,10])), grid} \]
\[ \text{xlabel('Sample'), ylabel('Unmodified Output Signal'), ...} \]
\[ \text{legend('Desired', '40\text{\mu m P-V}', '120\text{\mu m P-V}', '200\text{\mu m P-V}', ...} \]
\[ \text{'280\text{\mu m P-V}', '380\text{\mu m P-V}');} \]
\[ \text{title('conv( Desired, h )');} \]
\[ \text{err1 = zvolt(1:deconv_{\text{len}}) - num(delay+1:delay+deconv_{\text{len}},1);} \]
\[ \text{err3 = zvolt(1:deconv_{\text{len}}) - num(delay+1:delay+deconv_{\text{len}},3);} \]
\[ \text{err5 = zvolt(1:deconv_{\text{len}}) - num(delay+1:delay+deconv_{\text{len}},5);} \]
\[ \text{err7 = zvolt(1:deconv_{\text{len}}) - num(delay+1:delay+deconv_{\text{len}},7);} \]
\[ \text{err10 = zvolt(1:deconv_{\text{len}}) - num(delay+1:delay+deconv_{\text{len}},10);} \]
\[ \text{figure,plot([1:deconv_{\text{len}}]/Fs,[err1*20 err3*20 err5*20 err7*20 err10*20 ]),grid} \]
\[ \text{xlabel('Sample'), ylabel('Path Difference due to nonlinearity (\text{\mu m})'), ...} \]
\[ \text{legend('40\text{\mu m P-V}', '120\text{\mu m P-V}', '200\text{\mu m P-V}', '280\text{\mu m P-V}', ...} \]
\[ \text{'380\text{\mu m P-V}');} \]
\[ \text{title('Desired - Unmod_{TF_i}');} \]

%%% Finish setting up parameters

% Overlap-add Deconvolution
% Modify input command length (deconv_{\text{len}}+overlap_{\text{len}}) at a time
\[ \text{n_{steps} = length(zvolt);} \]
\[ \text{rev = ceil(n_{steps}/deconv_{\text{len}});} \]
\[ \text{zvolt_{m}(rev*deconv_{\text{len}}+overlap_{\text{len}}) = 0;} \]
\[ \text{zvolt_{m} = zvolt_{m}(:);} \]
\[ \text{zvolt (rev*deconv_{\text{len}}+overlap_{\text{len}}) = 0;} \]

% Convolution
\[ \text{zout(rev*deconv_{\text{len}}+overlap_{\text{len}}) = 0;} \]
\[ \text{zout = zout(:);} \]

% Modify the first piece of the input command
\[ \text{amp = round( abs((max(zvolt(1:deconv_{\text{len}}))-min(zvolt(1:deconv_{\text{len}}))))/2 )} \]
\[ \text{if amp <= 0, amp = 1; end;} \]
\[ \text{if amp > 10, amp =10; end;} \]
\[ \text{heq_{num} = heq1(:,amp);} \]
\[ \text{num = conv( zvolt([1:deconv_{\text{len}}]), heq_{num});} \]
\[ \text{zvolt_{m}(1:deconv_{\text{len}} + overlap_{\text{len}}) = num([1:deconv_{\text{len}}+overlap_{\text{len}}]);} \]
\[ \text{dum2 = num([1:deconv_{\text{len}}+overlap_{\text{len}}]);} \]

%%% Estimate the output by convoluting the modified with
% the impulse of the FTS
\[ \text{h_{num} = h_3(:,amp);} \]
\[ \text{num = conv( zvolt_{m}(1:deconv_{\text{len}}), h_{num});} \]
\[ \text{zout(1:deconv_{\text{len}} + overlap_{\text{len}}) = num([1:deconv_{\text{len}}+overlap_{\text{len}}]);} \]

% Estimate the error if choosing a wrong TF

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num = zeros(deconv_len+overlap_len,10);
for i = 1:10
    num(:,i) = conv( zvolt_m([1:deconv_len]), h3(:,i) );
end

figure, plot(zvolt(1:deconv_len),'--'), hold, ...
    plot(num(delay*2+1:end,[1,3,5,7,10])), grid
xlabel('Sample'), ylabel('Unmodified Output Signal'),
legend('Desired','40\mum P-V','120\mum P-V','200\mum P-V', ...
    '280\mum P-V','380\mum P-V');
title('conv( Modified_{TF_3}, h )');

for k = 1 : rev-1
    dum = zvolt_m([1:overlap_len] +k*deconv_len);
    amp = round(abs(max(zvolt([1:deconv_len] +k*deconv_len)) - ...
        min(zvolt([1:deconv_len] +k*deconv_len))))/2
    if amp <= 0, amp = 1, end;
    if amp > 10, amp =10, end;
    heq_num = heq1(:,amp);
    num = conv( zvolt([1:deconv_len]+k*deconv_len), heq_num );
    zvolt_m([1:deconv_len+overlap_len]+k*deconv_len)=
        num([1:deconv_len+overlap_len]);
    zvolt_m((1:overlap_len)+k*deconv_len) =
        zvolt_m([1:overlap_len]+k*deconv_len) + dum;
end

% Estimate the output of the modified command
    dum = zout([1:overlap_len] +k*deconv_len);
    h_num= h3(:,amp);
    num = conv( zvolt_m([1:deconv_len]+k*deconv_len), h_num );
    zout([1:deconv_len+overlap_len]+k*deconv_len) =
        num([1:deconv_len+overlap_len]);
    zout([1:overlap_len]+k*deconv_len) =
        zout([1:overlap_len]+k*deconv_len) + dum;
end

% Entire modified command signal
    zvolt_m = zvolt_m(delay+1:end); % shift backward to correct phase
    zvolt_m = zvolt_m(1:n_steps); % deconvoluted tool center path
    zvolt_m = zvolt_m(:);

% Entire estimated output of the modified input
    zout = zout(delay*2+1:end); % shift backward to correct phase
    zout = zout(1:n_steps); % deconvoluted the tool center path
    zout = zout(:);
    err = (zvolt - zout)*20; % micron
figure, plot([1:length(zvolt)],zvolt, [1:length(zvolt_m)],zvolt_m,'g', ...
    [1:length(err)],err,'r'), grid
xlabel('Sample'), ylabel('Signal'),
legend('Desired (volt)', 'Mod Input (volt)', 'Output_{laser} (volt)', 'Error (\mum)');

MATLAB file: freqrespX3D_sn0113.m generate the frequency response of the Variform

#0113 to the desired frequency range
function [freq2, H2] = freqrespX3D_sn0113(f_step, Fs)
% FREQRESPX3D() Interpolate TF to desired sample rate and resolution
% [freq2, H2] = freqrespX3D(f_step, Fs)
% Input
% f_step frequency resolution of new TF
% Fs sample rate of new TF
% Output
% freq2 frequency vector of new TF
% H2 new TF
% TF of the Variform #0113 measured by laser interferometer
% contain [fq, H(:,1:10)]
load sn0113.mat;
% construct frequency vector for new TF
norm_freq = fq;
delF = norm_freq(2)-norm_freq(1);
f_pad_left = [0 : delF : norm_freq(1)-delF]';
f_pad_right = [ ];
if Fs > norm_freq(end)
    f_pad_right = [norm_freq(end)+delF : delF : Fs/2]';
end
% Pad real and imaginary part with their last value
npad = [f_pad_left; norm_freq; f_pad_right];
REpad = interp1(norm_freq, real(H), npad, 'nearest', 'extrap');
IMpad = interp1(norm_freq, imag(H), npad, 'nearest', 'extrap');
freq = npad;
Re = REpad;
Im = IMpad;
% Increase number of data points to desired resolution
freq_new = [0:f_step:Fs/2-f_step]';
Re = interp1(freq, Re, freq_new, 'cubic', 'extrap');
Im = interp1(freq, Im, freq_new, 'cubic', 'extrap');
freq = freq_new;
% Mirror image of TF to the other half
H = Re + sqrt(-1)*Im;
dummy = length(freq);
freq2 = [freq; freq(dummy)+freq(1:dummy)+f_step];
H2 = [-H; conj(-H(dummy:-1:2:1))];
figure, plot(freq2, abs(H2(:,1)), freq2, abs(H2(:,2)),
            freq2, abs(H2(:,3)), freq2, abs(H2(:,4)), ...
            freq2, abs(H2(:,5)), freq2, abs(H2(:,6)), ...
            freq2, abs(H2(:,7)), freq2, abs(H2(:,8)), ...
            freq2, abs(H2(:,9)), freq2, abs(H2(:,10))), grid
figure, plot(freq2, abs(H2(:,2)), freq2, abs(H2(:,4)), ...
            freq2, abs(H2(:,6)), freq2, abs(H2(:,8)), ...
            freq2, abs(H2(:,10))), grid
xlabel('Frequency (Hz)'), ylabel('Gain FTS/input'),
legend('80\mum P-V', '160\mum P-V', '240\mum P-V', ...
       '320\mum P-V', '380\mum P-V');
title('Frequency Response of the FTS'),
MATLAB/simulink: simDrive_FTS.mdl creates a tool path along the spiral pattern \((r, \theta)\).
The map array of the desired surface is in the Look-Up Table (2-D) block.

MATLAB/simulink: drive_fts_dec_groove_v3.mdl interpolates an input command signal at run time from the map array of the deconvoluted surface. Then the command is generated as a differential input signal for the Variform FTS.

The inside of “Command Selection” subsystem is a condition to choose the modified or unmodified map array.
The map array block is similar to “simDriveFTS.mdl”
MATLAB/simulink: Axiom_laser.mdl generates an input command signal for the Variform FTS, controls the data access from Axiom laser 2/20 and transforms a 16-bit position data of the tool motion into a decimal number.

The format conversion is proceeded in “16-bit I/O” Subsystem.
APPENDIX G

GAIN SCHEDULING SCHEME TEST

Description: A Variform tool axis is to follow a variable amplitude trajectory when the desired path is a 200 Hz sine wave whose amplitude is increased two folds from 120 to 240 μm P-V (a dashed line in Figure G-1.) Since the slope of the trajectory is abruptly changed, discontinuity happens at the transition. The entire desired signal is modified using a transfer function associated with a single input amplitude resulting in the smooth modified input command signals associated with 120 and 240 μm P-V shown as solid and dotted lines in Figure G-1. Note that the biggest amplitude of the modified input command signals occurs at the transition of the desired amplitude due to the discontinuity.

![Desired and modified input command signals](image)

**Figure G-1.** Desired and modified input command signals
Both modified input command signals are in phase to each other, but their amplitude is different due to the nonlinear gain response of the Variform FTS. In Figure G-2, the difference at 120 \( \mu \text{m} \) P-V is \( \pm 3 \ \mu \text{m} \), while at 240 \( \mu \text{m} \) P-V is \( \pm 7 \ \mu \text{m} \).

**Figure G-2.** Difference of the modified input command signals

**Figure G-3.** Nonlinear gain response results in the input command difference
The nonlinearity of the dynamic gain response to different amplitude influences the degree of the command difference. In Figure G-3, while the phase angles associated with 120 and 240 \( \mu \text{m P-V} \) at 200 Hz (dotted and dashed lines in Figure G-3(b)) are close to 60 degree, the gain responses of both amplitude as shown in Figure G-3(a) are different by 5 %.

**Figure G-4.** Modified input command signal using a block deconvolution and gain scheduling scheme

When the command modification was implemented using a block deconvolution and the gain scheduling scheme, the modified input command signal no longer had a smooth transition of the changing amplitude for 120 to 240 \( \mu \text{m P-V} \). In Figure G-4, the amplitude of the modified input command signal was abruptly changed at the transition. This amplitude change may result in a velocity saturation of the output tool trajectory. Figure G-4 shows that at the transition the velocity (a dotted line with square marks) of the modified input command (a solid line with dots) is about 300 mm/sec exceeding the physical limits of the FTS (140
The estimation will be corroborated by the experimental results where the output tool motion of the Variform is measured by a laser interferometer.

![Graph showing velocity of modified input signal exceeding physical limits of Variform](image)

**Figure G-5.** Velocity of the modified input signal exceeds the physical limits of the Variform

### EXPERIMENTAL COROBORATION OF GAIN SCHEDULING SCHEME

The tool excursion associated with the modified input command signals as shown in Figure G-1 was measured using a laser interferometer while the Variform was mounted on an optical table. Figure G-6 (a) and (b) illustrate the results of the modification for entire input commands using the transfer functions associated with 120 and 240 µm P-V (solid and dotted lines in Figure G-1) individually. The effect of the Variform’s nonlinear
characteristics is shown here. The steady state path difference was minimized where the amplitude of the desired tool path associated with the applied transfer function.

**Figure G-6.** Tool trajectories associated with transfer functions of 120 and 240 µm P-V
On the other hand, the modified input command associated with the block deconvolution resulted in better command following than the axis motion associated with the modification for the entire path. In Figure G-7, the error profile (a dotted line) was minimized for both 120 and 240 $\mu$m P-V desired trajectory. However, at the conjunction where the desired amplitude changed from 120 to 240 $\mu$m P-V, the tool velocity was constrained resulting in a path difference of 20 $\mu$m. The modified input command using the block deconvolution needs an additional scheme to smooth the profile of the input signal reducing the path error in the transient period.

![Image of Figure G-7](image)

**Figure G-7.** Tool motion associated with block deconvolution