This dissertation includes three essays that consider the role of quality variation within agricultural production when consumers are heterogeneous in their preferences for quality. The first essay, “The Welfare Benefits of USDA Beef Quality Certification Programs,” estimates the consumer welfare benefits from the increase in scope of USDA beef quality certification programs in the 1990’s. Between 1975 and 1999, beef demand fell by 66% (Marsh, 2003) and its share of the overall meat market fell from 48% to 32%. Along with changing consumer preferences and heightened health consciousness, poor quality assurance has been offered as a reason for the decline. Certification programs provide producers a new way to differentiate and brand products as being of higher quality outside the USDA grading program. Between 1994 and 2002, the share of all commercial cattle that are certified rose from 4% to 12% and the share that are both certified and qualify as upper Choice rose from 3% to 8%. An Inverse Generalized Almost Ideal Demand System (IGAIDS) is estimated using weekly data on beef consumption by grade, branded beef, chicken and pork. It is found that the increase in branded beef supplies in 1990’s increased consumer welfare by approximately 2% of beef expenditure.

The second essay, “Grading and Quality Certification in Beef Production,” presents an equilibrium model of demand and production where output qualities are distinguished by quality grade and whether the product is certified. Simulations are used to demonstrate that the entry and exit of producers from the industry eliminates profits
and losses of producers over the long run. Comparative statics and simulations also demonstrate that the size and distribution of total welfare are affected by the number of grades and the placement of grade standards. An overview of the USDA beef quality grading and certification programs is provided and related to the model.

The third essay, “When is Fruit Bundling Fruitless? Sorting, Bundling and Disposal When Quality Information is Asymmetric,” considers the conditions in which sellers allow sorting and discourage it through mechanisms such as bundling. Agricultural goods often vary in quality even when goods are sold at a single price which encourages consumers to sort goods. Consumers can increase their individual gains from trade from sorting but cannot create any new benefits from sorting in aggregate. To eliminate this cost, sellers discourage consumer sorting through marketing mechanisms such as bundling goods. Alternatively, sellers may still allow consumer sorting if quality preferences are such that sorting reduces the price sensitivity of marginal consumers. In this process, consumers with weaker preferences for quality receive higher quality goods in a process introduced as quality discrimination. This essay also shows that sellers may allow sorting to improve expected quality if they intend to dispose of, rather than sell, a portion of their product.
THREE ESSAYS ON QUALITY DIFFERENTIATION

By

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DEDICATION

To my father, William James Ferrier,
whose passion for the social sciences and mathematics sparked my interest in the field of economics.
PERSONAL BIOGRAPHY

Peyton Michael Ferrier was born at Resurrection Hospital in Chicago, Illinois on October 27th, 1975 as the youngest of four children to William and Dolores Ferrier. After graduating from Archbishop John Carroll High School in Radnor, Pennsylvania in 1993, he attended John Hopkins University in Baltimore, Maryland. After graduating with a Bachelor of Arts degree in Economics with a minor in Math Sciences in 1997, he served in the Americorps VISTA program at the Mountain Microenterprise Fund in Asheville, North Carolina for one year. In 1998, he entered the doctoral program in economics at North Carolina State University. In 2001, he married Kathryn M. Stahl at Grace Methodist Church in Aberdeen, Maryland. In 2003, he accepted the position of Visiting Instructor of Economics at Ursinus College in Collegeville, Pennsylvania which he currently holds.
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Chapter 1

Introduction
Agricultural production is uniquely subject to random processes influencing both the quantity and quality of output. While economic models of agricultural markets often control for shocks affecting the quantity demanded or supplied, variation in product quality is often disregarded either because data aggregation prevents the consideration of quality differences across goods or because agricultural grading is presumed to eliminate significant quality variation. In recent years, however, agricultural producers have become increasingly focused on crafting agricultural products for final consumption in retail markets rather than merely producing commodities of a predetermined quality. Often, products are branded to distinguish their unique features and marketed to more specific consumer groups. This process, often referred to as “the industrialization of agriculture,” typically requires improved quality control mechanisms and coordination of the supply chain which may be achieved through contracting and other vertical production relationships.

As more goods with more unique features and more consistent quality become available, consumer heterogeneity plays a larger role in modeling demand and consumer welfare. Improved quality information about existing goods and the creation of new goods can improve allocative efficiency and create substantial consumer benefits but may create additional costs, redistribute the gains from trade, and increase market power. Natural processes often make quality variable and complicate the design of incentive schemes to encourage investment in quality improvement. From the industrial organization literature, discrete choice models of demand including the vertical differentiation model are particularly well suited to considering the role of consumer heterogeneity with regard to product quality. From the contract theory literature,
principal-agent models of production are useful in addressing supply processes characterized by randomness in quality.

The first essay, “The Welfare Benefits of USDA Beef Quality Certification Programs,” estimates the consumer welfare benefits from the increase in scope of USDA beef quality certification programs in the 1990’s. Certification programs provide producers a new way to differentiate the quality of beef products outside the USDA grading program and enable the branding of beef. Certified and branded beef essentially represents a new good with a quality level between that of the Prime and Choice grades. An Inverse Generalized Almost Ideal Demand System (IGAIDS) is estimated using weekly data on beef consumption by grade, branded beef, chicken and pork. The compensating variation from the introduction of branded beef since 1993 is then estimated to have increase consumer welfare by 1.6% of wholesale beef expenditure. The essay also examines whether demand is vertical differentiated by quality grades.

Unlike the USDA grading program, quality standards under certification programs are privately controlled. The second essay, “Grading and Quality Certification in Beef Production,” provides an overview of the production and quality evaluation process in the beef and explains how producers expanded their use of quality certification in the 1990’s in response to consumer concerns regarding poor quality assurance. An equilibrium model of demand and production under certification and grading is presented in which producer entry and exit eliminates profits over the long run. Simulations then demonstrate that the number of grades and the placement of grading standards influence both the size and distribution of the total surplus from trade.
Quality differentiation through grading and certification may not be possible when quality characteristics are not observed by sellers. Goods, such as fruit and produce, may exhibit quality characteristics at the time of purchase that consumers can observe, but are unknown to sellers. In this setting, consumers have an incentive to sort goods for quality. The third essay, “When is Fruit Bundling Fruitless?” considers the conditions in which sellers allow sorting and discourage it through mechanisms such as bundling. Agricultural goods often vary in quality even when goods are sold at a single price which encourages consumers to sort goods. Consumers can increase their individual gains from trade by sorting but cannot create any new benefits by sorting in aggregate. To eliminate this cost, sellers discourage consumer sorting through marketing mechanisms such as bundling goods. Alternatively, sellers may still allow consumer sorting if quality preferences are such that sorting reduces the price sensitivity of marginal consumers. In this process, consumers with weaker preferences for quality receive higher quality goods in a process introduced as quality discrimination. This paper also shows that sellers may allow sorting to improve expected quality if they intend to dispose of, rather than sell, a portion of their product.
Chapter 2

The Welfare Benefits of

USDA Beef Quality Certification Programs
I. Introduction

Between 1975 and 1999, beef demand fell by 66% (Marsh, 2003) and its share of the overall meat market fell from 48% to 32%. Along with changing consumer preferences and heightened health consciousness, poor quality assurance has been offered as a reason for the decline (Lusk et al., 2001; Lamb and Beshear, 1998; Purcell, 1999). In recent years, packers and producers have introduced quality improvement mechanisms including expanded contracting, value-based pricing, and certification to capture potential gains to quality improvement. In doing so, producers branded and differentiated a good that had previously been marketed as a commodity, with the USDA acting only as a third party verifier of quality claims. Between 1994 and 2002, the share of all commercial cattle that were certified rose from 4% to 12% and the share that were both certified and qualify as upper Choice rose from 3% to 8%. While several studies have considered the impact of quality improvement in terms of producer profitability, few have sought to quantify its value to consumers.

The rise of certification and branding has changed the composition of beef available to consumers. Whereas the USDA grading program had been the sole mechanism of identifying beef quality, certification provides an alternative way for consumers to differentiate products and for producers to adjust beef characteristics to consumer preferences. Using weekly data on the consumption of beef by grade, branded beef, chicken and pork, an Inverse Generalized Almost Ideal Demand System (IGAIDS) is estimated. The compensating variation from the increase of branded beef supplies from 3% to 8% of the beef market in 1990’s is found to be approximately 1.6% of wholesale beef expenditure. A conjectured further increase of branded beef supplies of
20% is predicted to have a modest, but not statistically significant, impact on consumer welfare. Additionally, the model is tested for evidence of a vertically differentiated demand structure through tests of the substitution parameters with mixed results.

II. An Overview of the Beef Industry

Historically, beef quality has been distinguished only by the USDA grading program. Developed in the 1930’s, this voluntary federal program separates beef into eight grades based on observable characteristics of the carcass at the time of slaughter. Typically, only three or four of these grades – Prime, Choice, Select, and Standard (in order of decreasing quality) - are available in the consumer markets. Prices of graded beef are well ordered with Prime being the most expensive and lower grades costing less.

Evidence that the USDA grading program poorly measured important quality characteristics and that consumers would pay significant premiums for quality improvement led producer alliances to brand beef using USDA quality certification programs. In these programs, additional observable quality characteristics are measured by USDA graders in addition to those of the regular grading program. For example, under the Certified Angus Beef™ program, cattle must have a hide 51% black in color, show no signs of Brahman genetics, have a yield grade of 3.9 or lower, and quality for the upper Choice grade or better.

As shown in Figure (1), the number of certification programs and participation in these programs grew steadily in the last decade as the proportion of cattle certified rose from approximately 4% to 13% between 1994 and 2002. In that same period, the percentage of commercial US cattle certified that also graded as upper Choice, a requirement for many certification programs, rose from approximately 3% to 8%. In

1 Source: Agricultural Marketing Service (2004)
2002 and 2004, the USDA’s Agricultural Marketing Service (AMS) cutout reports showed that 7.4% of boxed beef was branded.\(^2\)

Under the quality grading system, USDA graders assess both the maturity of the animal and the fat marbling of the meat flesh as younger cattle with more uniform marbling are thought to be superior. In the 1990’s, several meat science studies, including the National Beef Quality Audit (1995) showed that grading does not distinguish key quality characteristics including tenderness. Savell et al. (1987) found that USDA grade standards are ineffective at identifying meat tenderness. Wheeler, Cundiff, and Koch (1999) found that only 5% of the variation in palatability traits is explained by the degree of marbling in beef, the dominant USDA grading criterion. Brooks et al. (2000) corroborates Wheeler’s results in finding that the USDA grade had no effect on tenderness of top loin steaks as measured by Warner Bratzler Shear (WBS) force values.

Evidence also showed that consumers are willing to pay significant premiums for improved quality of their beef purchases. Shackelford et al. (1999) find that 89% of consumers would definitely or probably buy certified “tender select” beef if it were available at their local store. Boleman et al. (1997) found that approximately 95% of consumers are willing to purchase the highest tenderness level offered as certified by the

\(^2\) The definition of branded beef by the AMS is beef that is “produced and marketed under a corporate trademark or under one of USDA’s certified programs where the base of the brand is quality, yield, or breed characteristics of the product which are not unique to any one packer and can be produced by anyone in the industry, regardless of the brand.” On cutout reports, the AMS further explains that branded beef only includes that which also qualifies for the upper Choice category. Slight disparities between the percentage of beef certified and branded are present in the data and may arise because certified cattle are typically smaller than other cattle.

\(^3\) Grading is found to be consistent, however, as the USDA reports a 95% agreement rate across independent evaluations.

\(^4\) Dikeman (1987) and Miller et al. (1995) show that tenderness is the most important palatability attribute of beef.

\(^5\) The WBS tests meat tenderness by measuring the force necessary to cut a cooked steak.
WBS method when offered selection of three products of increasing tenderness and price differentials of $1.10/ kg. Lusk et al. (2001) found that 20% of consumers were willing to pay more than $2.67 more for a steak that was certified as tender using the WBS method and that 51% of consumers were willing to pay a premium for steaks “guaranteed tender” using this method. Several studies\(^6\) have also suggested that the steady erosion of beef demand from 1975 to 1999 is explained, at least in part, by the decline in the quality of beef relative to other meats.

In the 1990’s, producer alliances proliferated to capture these large potential returns\(^7\) by using certification programs to collectively market branded products as shown in Table (1). Grid pricing was developed to create incentives for producers to invest in quality improvement. As several authors have noted (Schroeder et al., 1998; Fuez, 1995), as producers have historically sold cattle on a live- or dressed-weight basis (Schroeder et al., 2002), their compensation is based on average cattle quality across all producers. A moral hazard problem emerges as producers ignore quality improvement in the pursuit of cost reduction by selecting only animals with fast growth rates, above-average feed conversion ratios, and high tolerances to weather and disease stress. Under value-based pricing, quality is explicitly incorporated in compensation as producers receive bonuses or discounts for specific animal characteristics measured at the time of slaughter. For example, under grid-pricing meat packers discount cattle with large humps, a characteristic of exotic (Brahman) cattle, or with too much or too little fat cover.

\(^6\) Purcell (1999); Schroeder, Ward, Minnert and Peel (1998); and Lamb and Breshear (1998)

\(^7\) Still, as late as 2000, Boland and Schroeder write “...at the present time, there is no value-based marketing program that provides economic incentives for producers and processors to market more tender beef primal cuts to consumers. Rapid and accurate measurement of tenderness is needed to implement a value-based marketing program.”
Certification and branding also provides a solution to an asymmetric information problem between buyers and sellers when beef is marketed as a commodity. As shown in Rosen (1976), if quality characteristics are observable at the time of purchase, those characteristics are supplied up to the point where marginal value equals marginal cost. If quality characteristics are not observable, however, the moral hazard problem reduces investment in quality especially if it creates significant opportunity costs, such as lower yields or increased herd mortality. Similarly, Holmstrom and Milgrom (1991) show that if two outputs are negatively correlated – quality and size – while only one output has a market incentive, then the output with the incentive crowds out the output with no incentive. By improving quality assurance, the efficient supply of quality investment is a more likely market outcome.

Certification involves ex post testing of final carcass quality characteristics. In lieu of vertical integration, producers may also increase quality assurance using process verification programs. Under process verification programs, the USDA provides third party verification of specific production practices proscribed ex ante by processors. These programs are not common in US beef production as the small scale and long production lags at the rancher stage inhibit the writing of contracts that cover all possible contingencies, though they are common in poultry and hog production. Purcell and Hudson (2003) also note the specific problem of price volatility in the production chain has made cost-based compensation plans unworkable.

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8 Lawrence (2002) explains that process verification type programs are much more common in Australia where production practices, including a more limited use of grain feeding over grass feeding, make beef quality much more variable.
To improve quality assurance, packers have increasingly used production contracts\(^9\) with producers in which cattle are committed to be delivered to the packer at a specific time and may also be committed to some form of value-based pricing system. While beef has been the slowest to adopt production contracting compared with pork and poultry (Hayenga et al., 2000), evidence from Schroeder et al. (2002) suggest that it is now on the rise. Their survey of cattle producers finds that 52% of sold cattle through some sort of production contract in 2001 and that 65% expected to use production contracts by 2006.

III. Research Questions on Welfare and Demand

Three separate research questions address certification’s effect on the beef industry. The first asks, “What was the value to consumers from the introduction of branded beef?” A counterfactual analysis is performed to estimate the compensating variation that maintains the current utility level (with current prices) while adjusting relative quantities to be equal to the approximate levels of the early 1990’s. Upper Choice certification programs encompassed approximately 3% of U.S. slaughter in 1994 whereas in 2002 they encompassed over 8%. Conservatively, this suggests that branded beef supplies were at least 50% lower in the early 1990’s than today. Unfortunately, because data on branded beef was not collected prior to 2002, this can only be an approximate figure. However, several factors—the large literature on the quality problem, the small number of certification programs, the small volumes of cattle certified, and the small number of alliances—suggest that the portion of branded beef was considerably smaller in the early 1990’s than today. To account for this inexactness, several estimations are performed where the supply of branded beef is reduced by 10%, 20%, and

\(^9\) Hayenga et al., 2000 provide a description of the distinction between marketing and production contracts.
50%. In each estimation, the decrease in branded beef is accompanied by commensurate increases in the supplies of the two nearest quality grades, Choice or Prime. In this manner, the estimation strategy only examines the change in relative qualities while leaving the total supply of beef fixed.

Unfortunately, only aggregate data on branded beef and grades is available. This aggregation necessarily obscures some important quality distinctions as it lumps together “good” certified beef with “bad” certified beef. Two facts, however, mitigate this concern. First, that approximately half of certified beef occurs under the CAB™ program. Second, that the AMS only reports beef as branded if it grades in the upper two-thirds of the Choice grade. A separate problem emerges, however, if the introduction of branded beef lowers the average quality of the remaining beef sold in the Choice grade. The introduction of the intermediate grade of branded beef essentially truncates the distribution of Choice beef and reduces its quality as shown in Chapter 3 of this dissertation. Unfortunately, the lack of disaggregated data on the hedonic characteristics of beef grades prevents the consideration of whether the introduction of an intermediate grade changes the average quality within grades.

A second research question asks, “How much will the expected future expansion of certification programs increase consumer welfare?” In the last decade, the supply of upper Choice certified beef increased an average of 13.4% annually suggesting that a 20% increase in branded beef supplies is plausible over several years. Similarly, Schroeder et al. (2002) survey cattle producers and find that that 62% expect to market

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10 The upper two-thirds of the Choice grade refers to Choice graded beef with moderate marbling scores, as opposed to the remainder of the Choice grade beef with modest and small marbling scores. Greater degrees of marbling are considered desirable as the even dispersion of fat through the meat makes it more tender when cooked.
cattle through a value based pricing system in 2006 versus 45% who did in 2001, though not all cattle marketed under value-based pricing are necessarily branded. To account for differences in conjectures about future supply changes, several scenarios for increases in branded beef supplies –5%, 10% and 20% increases – are simulated with the 20% increase in branded beef scenario argued as being the most plausible. Again, in each estimation, commensurate quality reductions are assumed to occur in either the Choice or Prime grades so that only the effect of shifting quality composition is estimated.

The third question asks, "Is beef demand vertically differentiated?" Vertical differentiation implies that all consumers assess quality in the same manner, but vary in their valuation of it. Beef differentiated by grade seems naturally suited to this framework as grades are directly related to quality measurements and prices are strictly ordered. In this model, the distribution of quality valuations across consumers defines prices that clear the market for any given quantity share. Bresnahan (1987) and Greenstein (1997) have used the vertical differentiation model in a discrete choice framework to consider the introduction of new goods in the auto and computer markets respectively.

The vertical differentiation model assumes that the $j$ goods in the market can be ranked by quality, $q_j$, so that $q_j > q_{j-1}$. For instance, Prime quality is higher quality beef than Choice beef. The variable $\theta_i$ is the $i^{th}$ consumer’s taste for quality so that larger values of $\theta_i$ indicate stronger consumer preferences for quality. The indirect utility to the $i^{th}$ consumer from purchasing the $j^{th}$ good is:

$$ u_i(\theta_i, q_j, p_j) = \begin{cases} \theta_i q_j - p_j & \text{if good } j \text{ is purchased} \\ 0 & \text{if no good is purchased} \end{cases} $$

(1)
Consumers simply pick the good that yields the highest indirect utility or

$$\theta_j q_j - p_j \geq \theta_k q_k - p_k \quad \forall k \neq j$$  \hspace{1cm} (2)

The condition expressed in Equation (2) (along with the condition that consumers only pick a good yielding a positive $\mu_{ij}$) define $\theta_j$, the boundaries of consumer type that are still high enough so that they purchase the $j^{th}$ good, in the equations below.

$$\theta q_{j+1} - p_{j+1} < \theta q_j - p_j \Rightarrow \theta < \frac{p_{j+1} - p_j}{q_{j+1} - q_j}$$ \hspace{1cm} (3)

$$\theta q_j - p_j > \theta q_{j-1} - p_{j-1} \Rightarrow \theta > \frac{p_j - p_{j-1}}{q_j - q_{j-1}}$$ \hspace{1cm} (4)

The distribution of $\theta$ across consumers defines the market share for each good with the probability mass of the range between $(\theta_j, \theta_{j+1})$ defining the market share of the $j^{th}$ good.

If $f(\theta, \Omega)$ is the distribution of consumer preferences dependent on other variables such as income, age, and other demographics, then the quantity shares of each grade are displayed graphically in Figure (2). The market share of each grade multiplied by the number of possible consumers determines the quantity demanded of each grade.

Since the $\theta_j$ determining market shares are only influenced by the prices of adjacent goods in terms of quality, a testable implication of the vertical differentiation model is that the cross price elasticities between non-adjacent goods are zero. In terms of beef demand, a change in the price of branded beef would influence the demand for either Prime or Choice beef but not Select.

VI. Literature Review on Demand Estimation and Welfare Analysis

Because quantities rather than prices of agricultural goods are fixed in the short run researchers have increasingly used inverse demand models rather than ordinary
demand models (Barten and Bettendorf, 1989; Eales and Unnevehr, 1994; Eales, 1996; Eales, Durham, and Wessells, 1997; Beach and Holt, 2001; and Moro and Sckokai, 2002). Agricultural products, including beef, exhibit long lags between initial production and actual harvesting of the good. Moreover, fresh beef is perishable so that supplies cannot be easily stored. Hicks (1956) argues that, in contrast to consumer level demand where prices are likely to be viewed as fixed in the short-run, quantities are fixed in the short run at the market level. Functionally, the difference between imposing price or quantity exogeneity amounts to the difference between whether prices are the right-hand-side independent variables in an ordinary demand system or quantities are the right-hand-side independent variables in an inverse demand system.

Importantly, demand parameters are inconsistent when the exogeneity assumption is misplaced (prices are assumed to be exogenous when they are not or quantities are assumed to exogenous when they are not). As Thurman (1986) emphasizes, a priori assumptions regarding the supply adjustment process have often been imposed haphazardly as the maintained hypothesis without consideration of empirical evidence that might suggest a specific functional form. Thurman advises testing for either quantity or price exogeneity using the Wu-Hausman test. In this technique, demand parameters are first estimated using the instrumental variables (IV) method that is asymptotically unbiased but inefficient. Demand parameters are then re-estimated using reduced form methods assuming that either prices or quantities are exogenous. The IV and ordinary demand estimates are identical if prices are exogenous and this property can be tested using a Wu-Hausman test. Quantity exogeneity is tested in an identical manner. If both quantity and price exogeneity are rejected, Thurman advises that demand estimation
should proceed with IV estimation. Non-rejection of either price or quantity exogeneity, however, may be the result of a low power for the test and does not provide unqualified evidence of price or quantity exogeneity and the choice of appropriate demand model ultimately reverts to the \textit{a priori} beliefs of the econometrician.

The method suggested by Thurman is infrequently implemented due to the difficulty of obtaining appropriate instruments that are correlated with shifts in supply but not demand. Assuming exogenous supplies in the short run seems appropriate given the nature of beef production. Beef production is produced with extremely long lags. Cattle reach the market 24 months after an initial calving decision and cows can produce at most one calf per year.\textsuperscript{11} Poultry and pork have a shorter breeding cycle (4 to 6 months for pigs, 6 to 8 weeks for chickens) and the higher fertility rates so that supply may be more responsive to price in those industries.\textsuperscript{12}

The quantity exogeneity assumption also implies that producers cannot convert low quality cattle into high quality cattle in the short run in response to price changes. This condition is likely to rely significantly on whether high quality cattle are the product of genetic selection (using Angus rather than Brahman genes) or higher quality inputs (extending the feeding times of cattle). If genetics drives quality differentiation, then exogenous quantities of cattle differentiated by grade is plausible. If inputs drive quality differentiation, then exogenous prices are likely to be more common. Little empirical evidence is available on this subject with the exception of Norwood and Lusk (2003, page 14) who find a statistically significant short-term elasticity of technical substitution

\textsuperscript{11} Supply may be somewhat more responsive in that period however as heifers may be slaughtered rather than kept as breeding cows in response to high prices. Similarly, cattle slaughter may be delayed or hastened depending on market prices although constraints on feedlot capacity prevent excessive delay.\textsuperscript{12} Thurman finds that annual price rather than quantity is predetermined in the poultry industry.
of 2.7 for wholesale supplies of Choice and Select beef. At the same time, this magnitude of substitution is likely to be smaller between other adjacent qualities of beef such as branded beef and Choice graded beef due to the more rigorous genetic requirements of the branded beef programs.

As a separate issue, measurements of welfare change only apply to wholesale rather than retail level effects when using disappearance data rather than consumption data as with the estimation used here. Changes in wholesale supply cannot be used to extrapolate to changes in welfare at the retail level. As Brester and Wohlgenant (1993) show, estimates of own-price retail demand elasticities based on disappearance data are inconsistent because they impose the restrictive assumption that wholesale beef and processing inputs are used in fixed proportions in producing the final retail product. If retailers respond to more expensive beef by using beef more efficiently by increasing processing inputs rather than reducing their beef purchases, then some of the retail level welfare losses from restricted supply are dissipated through input substitution. As the ability to substitute processing inputs for beef inputs in production increases (i.e. the marginal rate of technical substitution increases), retail demand for wholesale beef becomes more elastic. The disparity between retail and wholesale demand elasticities prevents the results of this paper from being generalized directly to consumer level effects.

Under the assumption of quantity exogeneity, an Inverse Generalized Almost Ideal Demand System (IGAIDS) model of demand is used to estimate reduced form demand parameters of an inverse utility function. Parameters are estimated using budget shares as the dependent variable and quantities as the independent variables. In
generalized ordinary demand models, consumer expenditure is decomposed into two parts, pre-committed and supernumerary expenditure. Pre-committed expenditures are the fixed proportions of a consumer’s budget devoted to specific goods in the demand system whereas supernumerary expenditure represents the remaining, unallocated budget. In ordinary demand systems, once pre-committed expenditures are subtracted from the consumer’s budget, supernumerary expenditures are then allocated as in ordinary budgeting. In inverse demand systems, the intuition is similar with the exception that expenditures are normalized by income (since only relative marginal valuations are estimated). The distance function is decomposed into pre-committed distance and supernumerary distance. Similarly, once pre-committed shares are subtracted from a consumer’s expenditure, supernumerary shares are allocated from the remaining normalized budget.

In contrast to ordinary demand systems, where differentiation of the expenditure function yields compensated price elasticities, differentiation of the distance function in inverse demand systems yields compensated quantity flexibilities. The distance function gives the scale by which all quantities of all goods must be adjusted to obtain a given level of utility. It is analogous to the cost function, which gives the amount by which income must be adjusted to obtain a given level of utility. For instance, if a consumer is currently consuming \( \{ q_1=3, q_2=4 \} \) to obtain utility 8, the distance function value of 1.1 for the utility level 10 implies that the quantity of all goods must be increased by 10% \( \{ q_1=3.3, q_2=4.4 \} \) to achieve this higher utility level with the same relative quantities.

Price flexibilities give the percentage change in a consumer’s valuation of a good \( i \) in response to a one percent change in quantity \( j \). If the flexibility between two goods is
positive, then the goods are *q-complements* as consumers value good *i* more with increases in good *j* even while holding utility constant. Similarly, if the flexibility is negative, then the goods are *q-substitutes*. Goods are inflexible if the absolute value of their own price flexibility is less than one. In this case, a 1% increase in the consumption leads to decrease in marginal value of that good less than 1%.

Scale flexibilities are the analogue to the income elasticities of ordinary demand systems and represent the change in relative prices that would support a proportionally expanded equilibrium bundle, with income normalized to one. If preferences were homothetic, all scale flexibilities would be equal to -1. In this case, a 1% increase in consumption of all commodities would require no change in relative prices but a 1% increase in total expenditure. Normalized prices, prices divided by total expenditure, must all decrease by 1% to obtain this improved bundle. In reference to homothetic preferences, if the absolute value of scale flexibilities are less than -1 then goods are *necessities* as the marginal valuation of the good is unresponsive to proportional increases in the amount of all goods consumed. If the absolute value of scale flexibilities are greater than -1 then goods are *luxuries* as the marginal valuation is very responsive to the same proportional increases.13

A change in a consumer’s allocation from bundle *A* and *C* can be decomposed into two welfare effects using the distance function in quantity space in the same manner that changes that changes can be decomposed using the expenditure function in price space as shown in Figure (3). The first movement along the existing utility curve from *A* to *B*, the substitution effect, shows the change in relative valuations of goods necessary to reach a bundle of goods yielding the same utility as point *A* but in the same proportion as

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13 See Park and Thurman (1999) for discussion
that of point C. The second movement from B to C, the scale effect, shows the amount that one would need to increase all the utility compensated quantities proportionally (scale quantities) to reach the eventual allocation C.

As Cornes (1994, page 90) shows, the substitution effect and scale effects can obtained from the compensated inverse demand curves. Assume the following identities where \( \zeta_i \) is the uncompensated inverse demand for good \( i \) and \( \psi_i \) is the compensated demand for good \( i \).

\[
p_i = \zeta_i(q, \delta) \Rightarrow p_i = \zeta_i(q, D(q,U)) = \psi_i(q, U) \tag{5}
\]

Noting that the derivative of the distance function is equal to uncompensated inverse demand, Cornes (1994) shows that Equation (5) can be used to derive the following expressions:

\[
\frac{\partial \zeta_i}{\partial q_j} = \frac{\partial \psi_i}{\partial q_j} - \zeta_i \left( \frac{\partial \zeta_i}{\partial \delta} \right) p_i \tag{6}
\]

Multiplying Equation (6) by \( q_j/p_i \) yields Equation (7)

\[
\frac{\partial \zeta_i q_j}{\partial q_j p_i} = \frac{\partial \psi_i q_j}{\partial q_j p_i} - \zeta_i \left( \frac{\partial \zeta_i}{\partial \delta} \right) \frac{\delta p_i}{p_i} \tag{7}
\]

For small changes in quantities, \( p_i, \zeta_i \) and \( \psi_i \) are equal to each other at the optimum and \( \delta \) is equal to 1.

\[
\frac{\partial \zeta_i q_j}{\partial q_j} = \frac{\partial \psi_i q_j}{\partial q_j} - \left( \frac{\partial \zeta_i}{\partial \delta} \right) p_i q_j \tag{8}
\]

The Slutsky equation can be represented in terms of quantities or elasticities. Equations (7) and (8) are the inverse demand analogues to the Slutsky Equation in price and flexibility form.\(^\text{14}\) Equation (7) states that the effect of an increase in the quantity of good

\(^{14}\) Setting \( \delta \) equal to one allows the \( p_i q_j \) term to be interpreted as an expenditure shares. Expenditures across all goods then total one.
\( j \) on a consumer's valuation of good \( i \) is equal to the change that results when the distance function is adjusted (quantities of all goods are changed) so that the consumer achieves the same utility minus the valuation of good \( i \) multiplied by the amount changing the distance function influences the valuation of good \( i \). Equation (8) is slightly more manageable as it implies that the uncompensated flexibility is equal to the compensated flexibility minus the scale flexibility multiplied by consumer expenditure on that good.

The form of the supernumerary distance function is the IAIDS model described by Eales and Unnevehr (1994). This model exhibits many desirable properties that mirror the ordinary AIDS model including (1) derivation from a well defined preference structure (specifically, a logarithmic distance function), (2) flexibility in that the IAIDS preference structure can be interpreted as a local second-order approximation to an unknown preference structure, and (3) ease of imposition of homogeneity and symmetry conditions. The IAIDS model is sometimes further augmented to increase its flexibility and tractability. For instance, a shortcoming of the IAIDS model is its imposition of linearity in the scale curves that Beach and Holt (2001) describe as the analogue to imposing quasi-homotheticity in a direct demand system. In separate work, Beach and Holt (2001) develop the Normalized Quadratic Inverse Demands – Quadratic Scale System to incorporate quadratic terms into scale curves while Moro and Sekokai (2002) derive the Inverse Quadratic Almost Ideal Demand System (IQUAIDS).

Alston, Chalfant and Piggott (2001) show that in many empirical estimation strategies, the IAIDS (and ordinary AIDS) model estimates of real variables (such as market shares, elasticities or flexibilities) are sensitive to the choice of scaling of the exogenous variables when demand shifters are included. They show that the generalized
version of the AIDS and IAIDS models, first developed by Bollino (1987), preserve the desirable theoretic property of being ‘Closed Under Unit Scaling’ (CUUS). As shown by Piggott and Marsh (2004), a generalized expenditure function is defined as follows:

\[ E(p,u) = p \cdot c + \tilde{E}(p,u) \] (9)

The first right-hand-side term represents pre-committed expenditures (prices, \( p \), multiplied by pre-committed quantities, \( c \)) while the second term represents supernumerary expenditure.

Figure (4) shows the effect of an exogenous decrease in the price of \( q_2 \). The variables \( c_1 \) and \( c_2 \) depict pre-committed quantities. The movement from A to B captures the substitution effect. The movement from B to C captures the income effect. The \( q_1 \) axis intercepts multiplied by \( p_1 \) give the expenditure function for a given level of prices and utility and their difference is the compensating variation.

Invoking duality, a generalized distance function is similarly expressed in terms of pre-committed and supernumerary expenditures with the exception that the distance function is depicted in (normalized) price space. The generalized distance function is then written as:

\[ D(q,U) = q \cdot b + \tilde{D}(q,U) \] (10)

Again, the distance is decomposed into pre-committed and supernumerary components. The \( b \) terms represent pre-committed relative marginal valuations. The distance function assumes that relative prices are endogenous and adjust in response to exogenous changes in quantities. Figure (5) depicts an exogenous shift from A to C in price space. Here, the amount of \( q_1 \) increases relative to \( q_2 \). The shift from A to B shows the change in relative valuations of \( q_1 \) and \( q_2 \) while holding utility constant. The shift from B to C decreases all
prices proportionally so that consumers attained the same original utility. The norm (the slope from the ray emanating at the origin) to A is the price ratio \((p_2/p_1)\). The norm to B and the norm to C are equal because they both represent a rescaling of the same relative prices. Moving from A to B indicates that the marginal valuation of \(q_2\) increased relative to \(q_1\). The slope of the tangent to the indirect utility curve is the ratio of quantities consumed at optimum for given levels of utility. Given that prices are normalized, the move from B to C shows the proportional decrease in all prices to reach the higher utility level at C while maintaining the same relative prices as B. Rescaling all prices proportionally is equivalent to rescaling all quantities proportionally. For this reason, a parallel downward shift in the tangent of B so that it intersects C creates a new intersection point to the \(p_1\) axis. This point gives the normalized expenditure (at the original prices) required to obtain utility C. The difference in normalized expenditure from the line intersecting B and the line intersecting C represents the compensating variation in normalized prices. The distance function is equal to the intercepts to the \(p_1\) axis in the same manner that the expenditure function (normalized by \(p_1\)) is equal to the intercepts of the \(q_1\) axis. The distance function is typically characterized as the rescaling of all quantities necessary to obtain the same utility from a price level B as that at price level C. The rescaling is identically depicted as the rescaling of intercepts to the \(p_1\) axis.

As with the quantity dependent AIDS model, pre-committed marginal valuations can be incorporated into the demand model and estimated in empirical settings. The intuitive interpretation of pre-committed marginal valuations is not as transparent as pre-committed quantities in ordinary demand estimation. Pre-committed quantities are sometimes treated as subsistence levels of goods, often by invoking a Stone-Geary utility
function structure. There is no comparable interpretation of pre-committed marginal valuations.

V. Description of the IGAIDS Model

Estimating the IGAIDS model involves simultaneously estimating the pre-committed and supernumerary distance as in Equation (10). The supernumerary component of the distance function is estimated using the Inverse Almost Ideal Demand System as shown by Eales and Unnevehr (1994) which is derived from the logarithmic distance function:

\[
\ln \tilde{D}(U, q) = (1 - U) \ln a(q) + U \ln b(q)
\]

(11)

Components of the supernumerary distance function are then given as:

\[
\ln a(q) = \alpha_0 + \sum \alpha_j \ln q_j + \frac{1}{2} \sum \gamma_i \ln q_i \ln q_j
\]

(12)

\[
\ln b(q) = \beta_0 \prod_j q_j^{-\beta_j} + \ln a(q)
\]

(13)

According to Antonelli’s identity, differentiation of the regular distance function yields compensated inverse demands; Similarly, differentiation of the logged distance function yields compensated share functions, which are budget shares as function of quantities of all goods and utility. Compensated share functions can be converted to ordinary share functions by substituting the utility function into supernumerary distance function.

Defining supernumerary expenditure as \( \tilde{M} = M - \sum_i q_i b_i \), the share equations are:

\[
w_i = \left( \frac{q_i b_i}{M} \right) + \left( \frac{\tilde{M}}{M} \right) \tilde{w}_i(q, \tilde{M})
\]

(14)

The second term in Equation (14) is \( \tilde{M}/M \), the proportion of total expenditure that
is supernumerary, multiplied by \( \tilde{w}_i \), good \( i \)'s share of supernumerary expenditures. Each good’s compensated share of supernumerary expenditure is:

\[
\tilde{w}_i = \alpha_i + \sum \alpha_j \ln q_j + \sum \gamma_{ij} \ln q_j + \beta U_{\theta i} \prod_j q_j^{-\beta_j}
\]  

(15)

Recalling that \( \ln D(U_i, q) \) equals zero at the optimum and isolating for 

\[ U_{\theta i} \prod_j q_j^{-\beta_j} \]  

allows one to solve for the uncompensated share equations of:

\[
\sigma_i = \alpha_i + \sum_j \gamma_{ji} \log q_j + \beta_i \left( \alpha_0 + \sum_k \alpha_k \log q_k + \frac{1}{2} \sum_k \sum_l \gamma_{kl} \log q_k \log q_l \right)
\]  

(16)

Substituting \( \tilde{w}_i \) into Equation (14), the estimated share functions are:

\[
\sigma_i = \frac{q_i b_i}{M} + \frac{\tilde{M}}{M} \left( \alpha_i + \sum_j \gamma_{ji} \log q_j + \beta_i \left( \alpha_0 + \sum_k \alpha_k \log q_k + \frac{1}{2} \sum_k \sum_l \gamma_{kl} \log q_k \log q_l \right) \right)
\]  

(17)

Because \( \beta_0 \) is not identified through the budget shares in Equation (17), it is set to one throughout the estimation\(^{15}\). Pre-committed marginal valuations include a seasonal demand shifter, \( s \), for the summer months so that \( b_i = \tilde{b}_i + d_i \times s \). Alston, Chalfant, and Piggott (2001) show that specifying a demand shift in this manner preserves the CUUS property.

As found with previous studies estimating the AIDS and IAIDS model, the \( \alpha_0 \) parameter is difficult to estimate. Deaton and Muellbauer (1980) explain that the \( \alpha_0 \) parameter can be interpreted as the “outlay required for a minimal standard of living when prices are one.” As the estimated model did not converge when \( \alpha_0 \) was included in the model, this paper followed the practice of previous authors (see Eales and Unnevehr, 1994) by setting \( \alpha_0 \) to zero \emph{a priori}. Convergence is likely to have failed because the

\(^{15}\)Equation (22), later in the paper, shows that the \( \beta_0 \) term is a scalar on utility which can be changed without consequence for real demand parameter estimates.
likelihood function is flat over a substantial range of $\alpha_0$ values. An alternative method of estimating the model without omitting $\alpha_0$ is described by Piggott (1997). A grid search is performed where $\alpha_0$ is set to a range of possible values. In each case, the model is re-estimated treating the $\alpha_0$ term as constant. The value of the likelihood function at the alternative $\alpha_0$ values is used to rank the different model specifications. Finally, the model is re-estimated using the $\alpha_0$ value with the largest likelihood value as the starting point for $\alpha_0$ in estimation. Convergence might occur because the $\alpha_0$ value begins near a local optimum.

Other restrictions drawn from economic theory are imposed including:

$$\sum_k \alpha_k = 1, \sum_k \beta_k = 0,$$ (Adding Up) \hspace{1cm} (18)

$$\sum_k \gamma_{jk} = 0$$ (Homogeneity) \hspace{1cm} (19)

$$\gamma_{ij} = \gamma_{ji} \quad \forall i \neq j$$ (Symmetry) \hspace{1cm} (20)

Adding up restricts supernumerary distance to equal one which implies that supernumerary expenditure shares sum to one. Homogeneity restricts proportional increases in all prices and income to have no influence on the supernumerary expenditure shares. Symmetry restricts the compensated flexibilities of good $i$ and good $j$ to equal the compensated flexibility of good $i$ on good $j$ within supernumerary expenditures although compensated flexibilities are no longer symmetric for all expenditure. When compensated flexibilities are estimated for total expenditures, symmetry no longer holds.

The restrictions resulting from vertical differentiation can also be imposed and tested with the following restrictions:

$$\gamma_{ij} = 0 \quad \forall j \neq i - 1, i, i + 1$$ (21)
Vertical differentiation implies that only goods of adjacent qualities act as substitutes. For instance, Prime beef would not be a substitute for Select, for example. A test of this restriction is whether the $\gamma$ between Prime (where $i = 1$) and Select beef (where $i = 4$) are zero and, in general, whether the $\gamma$’s between non-adjacent quality levels of beef are zero. Unfortunately, this specification only restricts the substitution parameters within the supernumerary expenditures. It is not possible to impose a marginal rate of substitution equal to zero between two goods in the IAIDS model without severely constraining the scale flexibilities.\(^{16}\) The problem arises from the specification of utility in the inverse AIDS model. Noting that the distance function is equal to one at the optimum, utility is represented as:

\[
U = \sum \alpha_j \ln q_j + \frac{1}{2} \sum \gamma_{ij} \ln q_i \ln q_j \beta_0 \prod_j q_j^{-\beta_j} \tag{22}
\]

Notice that this specification implies that an adjustment in $q_j$ will still affect the marginal utility of $q_i$ through the $\beta_j$ term in the denominator even if the $\gamma_{ij}$ between two goods is zero. This specification puts a strong bias against finding zero cross-quantity flexibilities if any substantial scale effects (captured in the $\beta_i$’s) exists.

For the IGAIDS model, equations for the compensated, uncompensated and scale flexibilities are:

Uncompensated Own Quantity Flexibility ($f_{ij}$)

\[
f_{ij} = -1 + \left( \frac{1}{Mw_i} \right) \left[ q_i b_i (1 - w_i) + \tilde{M} \left( \gamma_{ij} - \beta_i \left( \frac{q_i b_i}{M} + \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln q_j \right) \right) \right] \tag{23}
\]

Uncompensated Cross Quantity Flexibility ($f_{ij}$)

\(^{16}\)Similarly, a parameter restriction cannot be defined which imposes the scale flexibility of a good to be zero in the IAIDS model as an income elasticity cannot be imposed to be zero in an AIDS model.
\[ f_{ij} = \left( \frac{1}{Mw_i} \right) \left[ q_i b_i (1-w_i) + \tilde{M} \left( \gamma_{ij} - \beta_i \left\{ \frac{q_i b_i}{M} + \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln q_j \right\} \right) \right] \tag{24} \]

Scale Flexibility \((f_i)\)

\[ f_j = 1 + \left( \frac{1}{M} \right) \left[ -q_j b_j + (M - \tilde{M}) \left( \gamma_{ij} - \beta_j \left\{ \frac{q_j b_j}{M} + \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln q_j \right\} \right) \right] \tag{25} \]

Compensated Flexibility \((f_{ij}^h)\)

\[ f_{ij}^h = f_{ij} - w_j f_j \tag{26} \]

The compensating variation\(^{17}\) is a well-established measure of consumer welfare effect resulting from changes to price or quantity (See Chapter 7 of Deaton and Muellbauer, 1998 for discussion). With ordinary demand systems, the compensating variation is typically calculated as the change in the expenditure function, the cost of a consumer reaching a utility level at a given set of prices. With inverse demand systems, however, the compensating variation is calculated the change in distance function from a quantity shift multiplied by income. As discussed in Kim (1997) and Beach and Holt (2001), the compensating variation is as follows:

\[ CV = M^0(D(u^0, q^0) - D(u^0, q^0)) \tag{27} \]

As Figures (5) and (6) show, the distance function is analogous to the expenditure function of ordinary demand systems. Once re-scaled by income, \(M^0\), the distance function equals the cost of reaching a given utility level with different proportions of goods. Holding utility constant, the difference in distance at different quantities multiplied by income gives the compensating variation of a quantity change.

\(^{17}\) The difference between the two measures is whether utility is held constant at its initial level or its final level. When income effects are zero, these two measures are equivalent. Deaton and Meullbauer (1998) show that the compensating variation will provide systematically lower estimates of welfare changes of normal goods than the equivalent variation.
VI. Description of the Data

The described demand system is estimated with 108 weekly observations of prices and quantities of four grades of beef (Prime, Choice, Select, and Ungraded, in order of quality), branded beef, pork, and chicken between January 1st, 2002 and January 30th, 2004. Summary statistics of the data are provided in Table (2). Beef prices are the total carcass values of different grades of beef; Beef quantities are computed from total wholesale loads (40,000 pound shipments) from packing plants. Both values are publicly available from the Agricultural Marketing Service Branch of the USDA weekly price reports made available after the passage of the Mandatory Price Reporting Act of 1999. Pork and chicken data are obtained from the Livestock Marketing Information Center. Pork prices are based on total carcass cutout values and quantities are based on dressed weights. Chicken prices are the 12-city broiler price and quantities are the total live-weights of chickens. Chicken weights are adjusted to reflect dressed weight quantities by multiplying by 0.71, the approximate dressed weight to live weight ratio of chickens reported by the USDA National Agricultural Statistics Service (2004). Quantities are in millions of pounds and prices are in cents per pound.

Notably, Prime graded beef is a small proportion of total beef expenditures and exhibits little variation in quantity. Chicken and pork represent roughly 40% and 25% of consumer meat expenditures. Collectively, beef represents the remaining 35% of consumer expenditure with the Choice, Select and Ungraded categories representing the largest portions of beef. The price difference between Select and Ungraded beef is small.

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18 There are certain exemptions for reporting to small packing plants which represent a small portion of the market.
19 The actual ratio is obtained by dividing pounds of broilers certified as ready to cook by live weights. In the Tennessee data listed in the references (Keenersen, Farm Facts, 2004), this ratio is 811,567/1,117,809 or 0.726. Earlier reports generate a ratio of 0.71.
and the prices are very closely correlated. For this reason, Ungraded beef is excluded from the estimation of the remaining demand system.

The data appears consistent with outside sources. Using the 2000 U.S. Bureau of Census estimate\(^{20}\) of a US population of 281 million, the average total expenditure on wholesale chicken, pork and beef in the US is approximately $150 per year. The Consumer Expenditure Survey of the Bureau of Labor Statistics\(^{21}\) estimates that average consumer expenditure of chicken, pork and beef to be $542 dollars but this figure should be discounted by approximately 40% to account for the wholesale-retail price spread\(^{22}\) and then further discounted to account for the role of imports.

\textbf{VII. Estimation Results}

The demand system was estimated by fitting Equation (17) for the three beef grades, branded beef, and pork and chicken as a seemingly unrelated regression using the PROC MODEL function in SAS, which jointly estimates the shared parameters of the demand system while weighting the errors of the share equations by the inverse of the covariance matrix. The dummy variable for the summer month is defined as those weeks of the summer between the last week of May (Memorial Day) and the first week of September (Labor Day). Summary statistics for the seemingly unrelated regression are provided in Table (3). Estimates of the parameters estimated in Equation (17) are given in Tables (4), (5) and (6) with chicken being the excluded good from estimation.

The demand model predicts well with nearly 84\% of the variation in beef expenditure shares and 44\% of the variation in pork expenditure shares being explained.

\(^{20}\)Source: U.S. Census Bureau, \url{http://eire.census.gov/popest/estimates.php}


\(^{22}\)The Economic Research Service of the USDA estimated that the average per pound sale price of beef is $3.749 at the retail level and $2.229 at the wholesale level or approximately 60\% of the retail price. Source: USDA, ERS, \url{http://ers.usda.gov/Briefing/foodpricespreads/meatpricespreads/beef.htm}
by the quantity variables. Gamma parameters are significant with notable exception of those between Prime and Select and those between Branded and Select. Seasonal variables positively impact demand but are not significant with the Branded, Choice and Select grades. None of the $\beta$ parameters are significant. Evaluating the flexibility formulas at all data points and taking the average of these estimates gives the estimates of compensated and uncompensated flexibilities provided in Tables (7) and (8).

The scale flexibilities imply that pork and all grades of beef are luxuries while chicken is not.\textsuperscript{23} The small (in absolute terms) own quantity flexibilities imply that the demand elasticities are large.\textsuperscript{24} The large asymmetry between cross quantity flexibilities between chicken and pork to beef may be due to the large difference in budget shares between those goods upon which the flexibility formulas depend. Since the cross quantity flexibilities between each grade of beef are negative, they indicate that all beef grades are \textit{q-substitutes} for all other beef grades. So, for example, an increase in holdings of Choice beef decreases the marginal value of Select beef.

\textbf{VII.A. Vertical Differentiation}

The vertical differentiation restrictions were tested by using a small-sample likelihood ratio test\textsuperscript{25} to consider whether the gamma parameters between non-adjacent grades – Prime-Select, branded-Select, and Prime-Choice – were zero. In the first two cases, the gamma values were not found to be statistically different from zero which partially supports the vertical differentiation hypothesis. However, because the gamma parameters only capture the substitution effects in the demand system, both the

\textsuperscript{23} Park and Thurman (2001) show that the weak connection between the scale flexibility and the income elasticity prevents stronger statements from being made.

\textsuperscript{24} Beach and Holt (2003) similarly argue that small quantity flexibilities imply large elasticities due to the reciprocal relationship between the two variables.

\textsuperscript{25} See Piggott and Marsh (2004) for a description of the small sample adjustment to the likelihood ratio test.
compensated and uncompensated cross quantity flexibilities between these goods will not equal zero as Equations (24) and (26) indicate. Because evidence on vertical differentiation of the demand system is mixed and to maintain flexibility in the estimation process, the vertical differentiation restrictions were not imposed in the remaining estimation sections.

**VII.B. Potential Future Consumer Welfare Gains**

To project the compensating variation from expected future increases in branded beef supplies, several alternative assumptions are made regarding supply changes in the simulations. Table (9) presents conjectured future increases to branded beef supplies and simulations of their associated compensating variations. Welfare estimates were obtained through simulations in three steps. First, 1000 simulated draws on the parameters were made given the distribution of parameters in Tables (4), (5), and (6). Second, the compensating variation for each period was calculated using equation (27) and averaged over periods. Finally, the standard deviation of the compensating variation for each average compensating variation is used to construct confidence intervals for the compensating variation.

To isolate the effect of improved quality rather than increases in the total volume of output, each conjectured increase in branded beef is associated with a commensurate decrease in the volume of another grade. Between 2002 and 2004, branded beef represented approximately 7.5% of total beef expenditure and 6.8% of total beef volume. Therefore, a 5% increase in branded beef supplies would only increase its volume share to 7.1% while a 20% increase would only raise it to 8.2%. Between 1995 and 2002, the

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26 Larger simulations using 2000, 3000, 4000 and 10000 draws found similar results suggesting that convergence is approached.
percentage increase in the number of cattle certified average 14.6% while the percentage increase in those certified and qualifying as upper Choice averaged 2.7% of cattle. A 20% increase in branded beef supplies over 5-10 year period seems the most plausible scenario for future supply increases. As shown in Table (2), the quantity of Prime graded beef exhibited relatively little variation compared to other grades of beef as measured by its small standard deviation. Moreover, because branded beef volume share is so much larger than Prime beef, equal volume changes for each grade produce a much larger change in the share for Prime beef. For these reason, it seems unlikely that substantial portions of increases in branded beef supplies would be drawn from Prime beef supplies.

As Table (9) shows, a 20% increase to the supply of branded beef\textsuperscript{27} that is drawn only from Choice is predicted to increase consumer welfare by 0.0345% of total wholesale beef expenditure or $12.807 million on an annual basis this is asserted to be the most plausible scenario for the near future. If the change, however, is smaller and a substantial portion is drawn from Prime beef rather than Choice beef, increased branded beef supplies actually generate a welfare loss. In general, the potential gains from increases in branded beef supplies are modest and not statistically significant.

\textbf{VII.C. Consumer Welfare Increases in the 1990’s}

Measurements of the compensating variation from the introduction of branded beef in the 1990’s are obtained in a similar manner. Mean estimates and confidence intervals are simulated with 1000 different random parameter draws from the distribution of estimated demand system parameters. Again, because exact data on the aggregate

\textsuperscript{27} This change is equal to an average shift of 1.54 million pounds of production from Choice to branded on a weekly basis.
volume of Branded beef is not available prior to 2002, several probable cases for changes in Branded beef supply are presented.

As Table (10) shows, a 10% decrease in branded beef supply associated with a commensurate increase in Choice beef supplies is estimated to generate a $26.1 million annual loss or a 0.07% decrease in consumer welfare. The welfare effect of decreased branded beef supplies is highly sensitive to the supply response across grades as even modest increases in Prime supplies mitigates most of the gains of decreases in branded beef’s share. Branded beef supplies are likely to have been 50% less in the early 1990’s as today primarily through improvements in the quality of cattle previously only qualifying as Choice. In this case, the increase in branded beef supplies is associated with a welfare gain of $230 million annually or approximately 1.6% of total consumer expenditure on wholesale beef equaling $14,302 million, result is significant at the 5% level. Distributed uniformly across 280 million U.S. residents, this effect increases consumer welfare by about $0.80 per resident. In magnitude, this result is less than other studies on new good introduction, such as those of the Boskin Commission (Boskin et al., 1996).\textsuperscript{28} They find that the CPI exhibits an upwards bias of approximately 0.6% for produce and 0.3% for other food at home as a result of quality change and new good introduction. On an annualized bias over the 10-year period between 1994 and 2004, this papers estimate is 0.16%.

\textbf{VIII. Conclusion}

Agricultural producers have increasingly attempted to add value to products by increasing processing, improving quality, and branding. USDA certification programs

\textsuperscript{28} See discussion in Gordon (2002).
represent one way market integrators can organize diffuse production to tailor products to consumer demands. Quality improvement and increased product variety improves consumer welfare by better meeting consumer needs and preferences. In the 1990’s, the welfare improvements gains from the introduction of branded beef were substantial and amount to approximately 1.6% of consumer expenditure on beef annually. Future gains from increased supplies of branded beef are modest and not statistically significant.

USDA certification programs play a vital role in coordinating the quality improvements generated by branded beef. As quality certification in the meat and agricultural industries expands and allows for a larger number of differentiated products, substantial consumer benefits may arise from increased quality and variety. Though discussions of agricultural marketing programs, such as beef certification programs, often focus narrowly on producer benefits, consumer benefits are often substantial as well.
IX. Bibliography

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<th>Year</th>
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<tr>
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<td>0.0819</td>
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<td>0.2595</td>
<td>0.4068</td>
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<td>St. Dev.</td>
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<td>0.0035</td>
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<td>0.1154</td>
<td>0.1369</td>
<td>0.3068</td>
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Prices (in US Cents unadjusted for inflation)

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<th>St. Dev.</th>
<th>Min.</th>
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<td>118.81</td>
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Quantities (in Millions of Pounds)

<table>
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<th>Min.</th>
<th>Max.</th>
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<td>69.721</td>
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<td>44.200</td>
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<tr>
<td>Max.</td>
<td>56.761</td>
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Total Expenditure

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<tr>
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<th>Weekly in Millions</th>
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<td>Mean</td>
<td>$ 814.3166</td>
<td>$42,458.47</td>
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<td>$ 86.0442</td>
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<td>$ 986.513</td>
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Table 2-3 - Summary Statistics on the Seemingly Unrelated Regression

<table>
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<tr>
<th>Pred. Share</th>
<th>SSE</th>
<th>MSE</th>
<th>Root MSE</th>
<th>R-Square</th>
<th>Adj. R-Squ.</th>
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<td>$3.38\times10^{-8}$</td>
<td>$0.000184$</td>
<td>$0.9754$</td>
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<td>Branded</td>
<td>$0.000172$</td>
<td>$1.71\times10^{-6}$</td>
<td>$0.00131$</td>
<td>$0.9057$</td>
<td>$0.8997$</td>
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<tr>
<td>Choice</td>
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<td>$0.000029$</td>
<td>$0.00537$</td>
<td>$0.8519$</td>
<td>$0.8424$</td>
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<td>$0.00359$</td>
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<td>Hogs</td>
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Table 2-4 - Estimation Results for the Gamma Parameters

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<th>Choice</th>
<th>Select</th>
<th>Pork</th>
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<tr>
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<td>&lt;.0001</td>
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<td>&lt;.0001</td>
<td>0.3607</td>
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<td>p-value</td>
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### Table 2-5 - Estimation Results for the Alpha and Beta Parameters

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<th>P-value</th>
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<tr>
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<td>0.014482</td>
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<td>Pork</td>
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<td>-0.14703</td>
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Table 2-6 - Estimation Results For Pre-committed Marginal Valuations

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<th>P-value</th>
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Table 2-7 - Estimated Compensated Cross Quantity Flexibilities

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<td>Branded Quantity</td>
<td>Other Quantities</td>
<td>Distance (x 1000)</td>
<td>Comp. Var. (Millions)</td>
<td>Stan. Dev.</td>
<td>95% Confid. Interval</td>
<td></td>
</tr>
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<td>------------------</td>
<td>-----------------</td>
<td>-------------------</td>
<td>-----------------------</td>
<td>------------</td>
<td>------------------</td>
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<tr>
<td>5% increase</td>
<td>Reduction in Choice</td>
<td>1000.2275</td>
<td>8.16</td>
<td>6.480</td>
<td>{4.54,20.86}</td>
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<tr>
<td>10% increase</td>
<td>Reduction in Choice</td>
<td>1000.3555</td>
<td>13.01</td>
<td>12.66</td>
<td>{-11.79,37.82}</td>
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</tr>
<tr>
<td>10% increase</td>
<td>¾ Reduction in Choice, ¼ Reduction in Prime</td>
<td>999.6945</td>
<td>-11.23</td>
<td>16.80</td>
<td>{-44.15,21.70}</td>
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<tr>
<td>10% increase</td>
<td>½ Reduction in Choice, ½ Reduction in Prime</td>
<td>999.8053</td>
<td>-6.35</td>
<td>35.93</td>
<td>{-76.77,64.0}</td>
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<tr>
<td>20% increase</td>
<td>Reduction in Choice</td>
<td>1000.3782</td>
<td>12.81</td>
<td>25.45</td>
<td>{-37.06,62.68}</td>
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</table>
Table 2-10 - The Effect of the Introduction of Branded Beef in the 1990’s

<table>
<thead>
<tr>
<th>Branded Quantity</th>
<th>Other Quantities</th>
<th>New Distance (x 1000)</th>
<th>Comp. Variation (Millions)</th>
<th>Stan.Dev.</th>
<th>95% Conf. Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% decrease</td>
<td>Increase in Choice</td>
<td>999.6824</td>
<td>-11.442</td>
<td>6.504</td>
<td>{-24.19, 1.31}</td>
</tr>
<tr>
<td>10% decrease</td>
<td>Increase in Choice</td>
<td>999.2926</td>
<td>-26.120</td>
<td>14.108</td>
<td>{-53.77, 1.53}</td>
</tr>
<tr>
<td>10% decrease</td>
<td>¾ increase in Choice,</td>
<td>999.9667</td>
<td>-0.488</td>
<td>17.866</td>
<td>{-18.35, 17.38}</td>
</tr>
<tr>
<td></td>
<td>¼ increase in Prime</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% decrease</td>
<td>½ increase in Choice,</td>
<td>1000.5729</td>
<td>19.829</td>
<td>24.870</td>
<td>{-28.92, 68.57}</td>
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<td></td>
<td>½ increase in Prime</td>
<td></td>
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<tr>
<td>20% decrease</td>
<td>Increase in Choice</td>
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<td>-64.793</td>
<td>30.666</td>
<td>{-124.90, -4.69}</td>
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<td>50% decrease</td>
<td>Increase in Choice</td>
<td>993.6447</td>
<td>-230.875</td>
<td>136.471</td>
<td>{-498.35, 36.6}</td>
</tr>
</tbody>
</table>
Figure 2-1 - Percentage of the Commercial Slaughter Certified
Figure 2-2 - Market Shares of Goods Vertically Differentiated by Quality
Figure 2-3 - The Distance Function in Quantity Space
Figure 2-4 - The Expenditure Function and Compensating Variation
Figure 2-5 - The Distance Function and Compensating Variation
Chapter 3

Grading and Quality Certification

in Beef Production
I. Introduction

Between 1976 and 1999, beef demand fell by 66% (Marsh, 2003) and beef’s share of the overall meat market fell from 48% to 32%. Beef inventories fell from 45 million head to below 33 million head. This decline has been linked to a decline in beef quality relative to other protein sources, such as chicken or pork, in that period. To improve quality assurance, private producer organizations expanded their use of certification programs as part of a concerted effort to develop branded beef. Under certification programs, USDA graders evaluate cattle carcasses on both the USDA quality grade standards and a separate set of standards determined by a private producer organization. The producer organization can then market the beef under a branded label. Branded beef now accounts for approximately 8% of all U.S. boxed beef.

This paper provides an overview of the beef industry in the United States and the process by which beef quality is evaluated and graded including a discussion of the rise of beef quality certification programs and value-based pricing. An equilibrium model of production under grading then specifies the supply and demand structure for different quality grades of beef and shows how the supply of different grade proportions may become unresponsive to changes in producer payments. Simulations are used to demonstrate that the entry and exit of producers from the industry eliminates profits and losses of producers over the long run. Comparative statics are applied to the model showing the effects of both changing a grade standard and increasing the number of possible grades. Finally, simulations demonstrate that the size and distribution of the total surplus to trade depends on the placement and number of grades despite the fact that consumers are risk neutral and grading is costless.
II. An Overview of Beef Quality Evaluation

The earliest efforts to standardize the quality of consumer beef arose with enactment of the Pure Food and Drug Act and the Meat Inspection Act following the publication of Upton Sinclair’s *The Jungle* in 1906, which exposed unsanitary practices in Chicago’s meat packing industry. At the behest of large retailers and hotel chains, the USDA Federal Meat Grading Service was developed in 1926 to harmonize the state grading systems with national standards for interstate sales (Food Safety and Inspection Service, USDA, 2002). After World War II, rising beef demand and falling grain and transportation prices allowed producers to expand the use of feedlots to improve yields prior to slaughter. Developments in animal husbandry and genetic manipulation further increased cattle yields, and boxing, where beef is processed into smaller, more manageable cuts for retail use, also helped lower costs.

In the late 1970’s, the trend of rising beef demand abruptly reversed. Between 1976 and 1999, the beef demand is estimated to have fallen nearly 66% (Marsh, 2003) as the poultry industry grew steadily. This fall in demand has been attributed to several factors, including reductions in the relative prices of chicken and other meat products, changes in consumer tastes, and health concerns. Poor quality assurance has also been cited by several authors as a reason for the decline\(^1\). In 1987, the American Angus Association began expanding the Certified Angus Beef Program, which was initially developed in 1978 in response to concerns that changes in the Prime and Choice standards in USDA grading program made them too inclusive of lower quality beef (Certified Angus Beef Program, 1999). Administered by the USDA but developed and controlled by producer groups, USDA certification programs grew steadily in number.

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\(^1\) Purcell (1999); Schroeder, Ward, Minnert and Peel (1998); and Lamb and Breshear (1998)
and size during the 1990’s such that approximately 13% of all cattle are currently certified. Once certified, beef can be marketed under a branded label to retail outlets.

Production of beef involves three distinct links in the supply chain: ranching, feeding and packing. Ranchers maintain and feed the herd of reproductive animals and select its genetic makeup. Feedlots then purchase the cattle at around the age of 18 months from ranchers at auction and prepare them for slaughter by altering the animals’ diet to add weight. The feeder ultimately decides when and to which meatpackers the cattle are sold. The meatpacker then slaughters the animal, separates it into cuts and either sells it as boxed beef to supermarkets or restaurants or processes it further before retail sale.

At the wholesale and retail levels, beef has historically been differentiated only by cut and quality, although in recent years producers have also begun marketing branded beef, such as Certified Angus Beef. In quality evaluation, USDA graders evaluate a carcass’s characteristics at the packing plant after slaughter and assign it both a yield and quality grade. When cattle are certified, the grader may also perform a cursory examination of the live cattle in addition to the regular grading process. Quality grades are based on two main features of the carcass: maturity, determined by examining the bones, cartilage and flesh of the carcass; and marbling, determined by examining the fat dispersion through the meat. Graders then assign the carcass to one of eight quality grades (Prime, Choice, Select, Standard, Commercial, Utility, Cutter, and Canner), but typically only the Choice and Select grades are marketed to consumers. Standard and Commercial grades are often marketed as ungraded\(^2\) or “store brand” beef and may be

\(^2\)At the wholesale level, carcasses marketed without a grade do not have a quality identification attached with a roller and are, therefore, called “no roll”.
processed into products such as ground meat before sale. Prime beef, typically representing less than 2% of graded beef, is sold primarily to restaurant and hotel chains (Food Safety and Inspection Service, USDA, 2002). Table (1) depicts summary statistics on weekly wholesale boxed beef sales between January 2002 and February 2004 disaggregated by grade.

Because quality improvement of beef is determined by actions taken at the rancher and feeder levels, USDA grading acts as an *ex post* measurement of quality. Schroeder et al. (1998) explain that marketing methods for live cattle have often offered poor incentives to improve quality. When cattle are sold from feedlots to meat packers, three payment schemes have been used: live-weight, dressed-weight, or value-based pricing. Under live- and dressed-weight pricing, payments are made on either the weight of the live animal or the weight of the carcass after the animal is dressed but, importantly, the payment does not depend on a quality assessment of the animal. Alternatively, under value-based pricing, carcasses are assessed for quality traits such as marbling, percentage kidney-pelvic-heart fat, the fat thickness over the 12 rib, and the size of the rib-eye, and those quality scores are multiplied by a grid of premiums and discounts\(^3\) to adjust a base price paid for each cattle. Feedlots and ranchers may also divide these premiums as well. Unlike live-weight and dressed-weight pricing which base payments on the average quality of cattle and introduce a free rider problem with regard to investments in quality improvement, value-based pricing strengthens the incentive for producer investment in quality improvement.

Value-based pricing programs are often used in conjunction with USDA quality certification programs. In these programs, USDA graders evaluate animals for visible

\(^3\) For this reason, value based pricing is often referred to as “grid pricing.”
carcass characteristics defined by the certification program in addition to those under the USDA grading program. The oldest and most visible certification program is the Certified Angus Beef (CAB) program, which was created in 1978 and has grown steadily since 1987. In this program, live cattle must have a hide that is 51% black, have genetics traceable to registered Angus cattle, and must not have large humps to ensure that it has not been cross bred with Brahman. After slaughter, the cattle are also evaluated for carcass weight, rib eye size, and marbling.

While value-based pricing is the mechanism by which cattle producers are compensated, certification programs are the mechanism by which beef is marketed at the retail level. Ownership of the brand name belongs to the developer of the certification program and is licensed to packers, fabricators, distributors, restaurants and retail stores by the American Angus Association. Other producer organizations, however, can produce similar certification programs, such as Washington Beef’s Quality Plus Angus Beef and Creekstone Farm’s Black Angus Beef, or they can focus on different breeds, such as Certified Hereford Beef or Tyson’s Classic (Red) Angus Beef. Currently, standards exist for 47 certification programs (Agricultural Marketing Service, 2004), most of which have been developed recently, as shown in Table (2). Figure (1) shows the percentage of total commercial cattle slaughter that was certified since 1993 and the percentage of the slaughter that was certified and qualified as upper Choice.

The increase in certification and value-based pricing is the result of evidence showing that consumers are willing to pay significant premiums for beef verified to be of

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4 The actual criterion for certification for the Certified Angus Beef Program was developed in 1978 by the American Angus Association in response to perceived weakening of the USDA grading standards, but the program was not used on any large scale until 1987, when yearly sales grew to 50 millions pounds annually before leveling off. Growth then accelerated rapidly after 1993.
higher quality. Lusk et al. (2001) found that 20% of consumers were willing to pay more than $2.67 over the cost of a comparable steak if it was certified as tender using the Warner Bratzler method. Similarly, Fuez and Umberger (2001) found that 29% of consumers were willing to pay $1.30 more for a USDA Choice steak over a USDA Select steak. Shackelford et al. (1999) find that 89% of consumers would definitely or probably buy certified “tender select” beef if it were available at their local store. Such premiums exist because grading is thought to poorly measure important quality characteristics of beef. Wheeler, Cundiff and Koch (1994) find that only 5% of the variation in palatability traits are explained by the degree of marbling in beef, the prime USDA grading criterion. Savell et al. (1987) also find that USDA grade standards are ineffective at identifying meat tenderness.

As an alternative to certification, quality assurance may be improved by increased control over inputs of the production process, an *ex ante* method of quality verification. Under process verification programs, USDA agents audit the process by which beef is produced to ensure that it has followed a predetermined manual of operations developed by a producer group. Few process verification programs currently exist in the U.S including PM Beef Group’s program named “Ranch to Retail™”. In Australia, process verification is more common as producers have faced difficulty gaining access to export markets following BSE outbreaks and ensuring cattle quality to foreign buyers owing to the higher percentages of grass-fed cattle (Lawrence, 2002).

To ensure steady supplies of cattle of a given quality, marketing contracts have been used increasingly in cattle sales (Schroeder et al., 2002). Marketing contracts include forward contracts, which commit the cattle to specific packers for sale at a
predetermined time, and contracts which make producer payments subject to a formula such as those determined by value-based pricing (Hayenga et al., 2000). Contracting arrangements have come under sharp criticism for limiting the number of marketing outlets for non-contracted producers, which could lead to market power abuses. Under many marketing contracts, the base price for the contract is tied to the spot market price for non-contracted cattle. As increasing supplies of cattle are committed to specific packers under contract ("captive supplies"), large packers can more easily manipulate the spot market price on the small number of remaining uncommitted cattle. In 2002, the narrowly defeated Johnson amendment to the Packers and Stockyards Act of 1921 would have made it unlawful for meat packers to "own, feed, or control livestock intended for slaughter" for more than 14 days prior to slaughter, except when the meat packers are cooperatively owned by the livestock producers or the meat packer "slaughters less than 2 percent of the head of that type of livestock slaughtered in the United States." Similar state laws are already in place in Iowa and Nebraska.

In the 2004 case *Pickett v. IBP*, a jury found that IBP’s (now Tyson’s) use of captive supplies had an anti-competitive effect on the cash market for fed cattle and that its use of captive supplies lowered the cash market price. The jury awarded the 35,000 plaintiffs in the class action lawsuit $1.28 billion for violations of the Packers and Stockyards Act which proscribes market power abuses in the grain and livestock industries. The verdict was later thrown out in April of 2004 by U.S. District Court Judge Lyle Strom and is currently pending appeal.
III. Moral Hazard in Quality Production

Information asymmetries regarding product quality can create vastly different market outcomes. Akerlof (1970) shows that markets become highly inefficient when quality information is asymmetric due to adverse selection problems, while Rosen (1974) shows that markets are perfectly efficient when quality information is complete and symmetric. While Akerlof’s argument relies on a fixed distribution of product qualities, his main conclusion is that high quality products become unavailable when consumers cannot verify a product’s quality. This outcome is easily generalized to cases in which producers can only improve quality at a cost (Ligon, 2002). In this case, a moral hazard problem arises as producers reduce their investment in quality improvements while free-riding on the quality improvements of other producers. Schroeder et al. (1995) criticize live- and dressed-weight pricing for making cattle producer payments dependent on the average quality of all cattle rather than on that of the individual producer.

Within the cattle industry, the opportunity cost of improving beef quality may well be increases in cattle size and heartiness. As improvements in breeding progressed through the 1960’s and 1970’s, producers may have become more willing to trade off size for quality. Holmstrom and Milgrom (1991) argue that when two desirable traits are negatively correlated but only one trait has a market incentive, the incentivized trait crowds out the other trait. In the beef industry, incentives for weight gain may have crowded out quality improvement as new hybrid breeds of cattle were developed during the 1960’s and 1970’s with Bos Indicus, or exotic\(^5\), genetics. While these hybrid breeds

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\(^5\) Exotic is industry jargon for Indian breeds including Brahmans and their crossbreds. English breeds, such as Hertford, Angus, and Shatham, were the first cattle to arrive in the U.S. Continental breeds, which arose on the European continent and include Charolais, Chianina, and Maine-Anjou, are less prevalent because of their late arrival in the U.S. Exotic breeds grow larger and are able to withstand harsher climate conditions but produce a less palatable meat.
performed adequately on the USDA grading system, evidence shows that these breeds may be less tender than the European breeds (Wheeler et al., 1994). Certification, which often examines the animals for breed characteristics in addition to those of the grading system, sought to address this problem.

This quality problem is exacerbated by the diffuse nature of the supply chain. Severe diseconomies of scale at the rancher level necessitate production to be separated across several links of the supply chain and inhibit vertical integration. Long production lags, variable prices for feed inputs, and supply shocks due to climate change prevent quality information from being easily transferred from producers to processors. Hennessey (1996) argues that vertical integration can mitigate the moral hazard problem when quality information is asymmetric. In this model, producers under-invest in quality improvement because processors cannot differentiate between good and bad quality products with certainty. Processors must then test products to determine whether they are high or low quality, while producers who have invested in quality improvement have a higher chance of being of high quality. Processors test the product without knowledge of whether investment has occurred. When quality testing is not accurate, the return to investing on quality improvement diminishes and fewer firms invest. In a vertically integrated producer-processor arrangement, testing is not required to determine the proportion of high quality. Instead, the true change in proportion of high quality products is used to construct the incentive to invest, rather than the proportion that tests as high quality.

Without integration, production contracts can still be devised to induce quality investment by making producer payments contingent on grading and certification
As risk-averse producers bear increasing burdens of the risk of stochastic quality production, the payment necessary to induce any level of product quality increases (Grossman and Hart, 1986). Similarly, Laffont and Martimort (2002, page 162) show that the agency cost of inducing producers to invest in quality improvement increases as the observed quality output becomes a poorer measure of whether the producer has invested in quality improvement.

IV. An Equilibrium Model of Grading

As producer payments become increasingly tied to indicators of quality performance and producer groups increasingly develop and expand certification programs to supplement regular grading, it becomes increasingly important to understand the welfare consequences of changing the standards by which beef is graded. Beef grading is coordinated on the national level and, while subject to periodic revision, its quality standards are inflexible from the standpoint of individual producers. This section provides an equilibrium model of grading when quality production is stochastic.

Welfare consequences from changing an existing grade standard and introducing a new grade are then considered. Grade standards may not maximize the potential gains from trade which may cause producer or consumer groups to organize to establish independent quality standards. The Certified Angus Beef program, for example, cites the 1976 change in the USDA grading standards which increased the permissiveness of the choice grade in the formation of its certification standard in 1978 (Certified Angus Beef Program, 1999).

This model assumes that quality standards for each grade are initially determined exogenously by the government. The standards for each grade and the distribution of consumer preferences determine market shares, prices, and average quality of each grade.
and can be used to determine consumer and producer surpluses. Entry and exit by outside producers over the long run drives the producer surplus to zero. Comparative statics are then presented for the effects of changing a grading standard and of introducing a new grade. Simulations show both the consumer and producer welfare effects. Simulations also demonstrate cases in which the consumer surplus either increases or decreases depending on the changes to the grading standards and the distribution of qualities.

IV.A. Producers and Supply

The market for graded products is modeled as three separate links in the supply chain: the producer, the sorter and the consumer. Each of the risk-neutral producers creates a single output with quality, $q$. Increased input of producer effort, $e$, improves product quality subject to a random error, and this quality is observable to both the producer and sorter after production occurs. The sorter’s only role in production is to verify whether the producer’s quality, $q$, has surpassed the minimum quality standard for a given grade, $x_i$. The minimum quality standard for the lowest grade, $x_1$, and the minimum quality standard for the highest grade is $x_{\tilde{i}}$, where $\tilde{i}$ is the best grade. The sorter pays the producer a payment, $G_i$, if the product’s quality surpasses $x_i$. Risk-neutral consumers are unable to verify the actual quality of the good at the time of purchase and therefore base their purchase on the expected quality of each grade, denoted $\mu_i$. Sorters sell each graded product to consumers for $P_i$. Consumer preferences for quality, $\theta$, are distributed across all consumers based on a strictly positive parametric distribution. Over the long run, producers and sorters make zero profits, and the payment
to producers for making a grade, \( G_i \), is equal to the price charged to consumers for that grade, \( P_i \).

Producers are modeled as \( N \) identical agents who produce a good of quality \( q \) subject to an additive error. For simplicity, quality production is assumed to be linear, as follows:

\[
q = e + \lambda \quad \text{where} \quad \lambda \sim F(0, \sigma) \tag{1}
\]

After production, quality is observable to both the sorter and producer. The market share of grade \( i \) is denoted \( \pi_i(e) \), which is equal to the probability that \( q \) is between \( x_i \) and \( x_{i+1} \) for a given level of effort. For each grade, the market share is:

\[
\begin{align*}
\pi_i(e) &= P(x_i < q < x_i \mid e) = F(x_i + e) - F(x_i) \\
\pi_{i+1}(e) &= P(x_i < q < x_{i+1} \mid e) = F(x_{i+1} - e) - F(x_i - e) \\
\pi_i(e) &= P(x_i < q) = 1 - F(x_i - e) \tag{2}
\end{align*}
\]

Table (4) presents an index of notations for grades and producer payments using the current USDA beef grading system as an example. Weekly Agricultural Marketing Service (AMS) reports\(^6\) include Prime as the best quality grade and Ungraded as the lowest. Though not listed in the table, the AMS reports also include Branded beef as a quality category. This quality category as defined by the AMS includes beef that is:

“…produced and marketed under a corporate trademark or under one of USDA’s certified programs where the base of the brand is quality, yield, or breed characteristics of the product which are not unique to any one packer and can be produced by anyone in the industry, regardless of the brand.”

On cutout reports, the AMS further explains that Branded beef only includes that in the upper Choice category. This quality category essentially represents a new intermediate

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\(^6\) Available at [http://www.ams.usda.gov/lsmnpubs/mbeefw.htm](http://www.ams.usda.gov/lsmnpubs/mbeefw.htm)
quality grade with standards between Choice and Prime that has been introduced through a certification program.

Producers are assumed to be risk-neutral with a cost of exerting effort of \( h(e) \) that is increasing and convex. The producer surplus is then:

\[
PS(e) = N\left( \sum_i G_i \pi_i(e) - h(e) \right)
\]

Substituting in probabilities, Equation (3) can be alternatively written as

\[
PS(e) = N\left( G_i + \sum_{i=1}^{i-1} \left( (G_i - G_{i+1}) F(x_{i+1} - e) \right) - G_i F(x_i - e) - h(e) \right)
\]

Optimal producer effort, \( e' \), is the effort level that maximizes the producer’s surplus. Noting that \( \partial F(x_i - e)/\partial e \) is equal to \(-f(x_i - e)\), a local first order condition for an optimum is:

\[
\frac{\partial PS(e)}{\partial e} = N\left( G_i f(x_i - e') + \sum_{i=1}^{i-1} \left( (G_i - G_{i+1}) f(x_{i+1} - e') \right) - \frac{\partial h(e')}{\partial e} \right) = 0
\]

The first two terms in Equation (5) represent the marginal benefit of increasing effort in terms of changes in probabilities of making a certain grade. The third term represents the marginal cost of effort. As is typical, optimization implies that producers increase effort until the marginal cost of effort is equal to the marginal benefit. The possible non-monotonicity of the marginal benefit curve implies that the first-order condition yields multiple solutions, a problem that is noted by Laffont and Martimort (2002) among others. Interestingly, only increases in \( G_i \), the payment to the uppermost grade, are certain to increase producer effort for the intuitive reason that higher payments to lower quality grades may discourage the production of higher quality.
An example of the production process is graphically depicted in Figure (2) where the error in production, $\lambda$, is uniform with parameters zero and one. Assuming that each grade has a positive market share, then $\pi_i(e')$ is $x_2 - e' + \frac{1}{2}$, $\pi_i(e')$ is $x_{i+1} - x_i$ and $\pi_i(e')$ is $e' + \frac{1}{2} - x_i$. From Equation (3), the producer surplus is equal to:

$$PS(e') = N\left(G_1(x_1 - e' + \frac{1}{2}) + \sum_{i=2}^{7-1} G_i(x_{i+1} - x_i) + G_7(e' + \frac{1}{2} - x_7) - h(e')\right) \quad (6)$$

Notice that effort, $e$, only affects the surplus through the probabilities of the highest and the lowest categories, respectively. For intermediate grades, increases in effort equally change the probability of moving both into and out of the grade. Only changes to probabilities of qualifying for the highest and lowest categories are not offset. As is shown in Figure (2), the density of grades 2 and 3 remain the same regardless of whether effort is $e_1$ or $e_2$.

The same phenomenon can arise for marginal changes in effort under alternative distributions. At the optimum effort level, the first order condition in equation (5) is satisfied at the global optimum effort level, $e'$. The comparative static solution in equation (7) below represents the effect of an increase in the payment, $G_i$, on the market share of grade $i$, an intermediate grade\(^7\). Recall that the distribution $f(\lambda)$ is the distribution of the error in production, $\lambda$, where $q = e' + \lambda$. The marginal effect of effort on the probability that quality surpasses a certain grade, $P(q > x_i)$ or $P(\lambda > x_i - e')$, is the probability density function evaluated at $f(x_i - e')$. The effect of an increase in the payment for a grade is shown in Equation (7). The intermediate steps for this result are shown in Appendix (A).

\(^7\) An intermediate grade is neither the highest nor lowest possible quality grade.
\[
\frac{\partial \pi_i}{\partial G_i} = \frac{\partial \pi_i}{\partial e^*} \frac{\partial e^*}{\partial G_i} = -\frac{[f(x_{i+1} - e') - f(x_i - e')]^2}{G_i \frac{df(x_i - e')}{de} - \frac{\partial^2 h(e')}{\partial e^2} + \sum_{i=2}^{t-1} (G_i - G_{i+1}) \frac{df_{i+1}(e')}{de}}
\] (7)

Equation (7) indicates that the market share of good \(i\) is invariant to an increase in \(G_i\) if \(f(x_{i+1} - e')\) and \(f(x_i - e')\) are equal. The intuition for this result stems from an inspection of Figure (2). Once the densest part of the quality distribution is captured within the boundaries of grade 3, no change in producer effort can change the proportion of goods qualifying for the grade. The supply of a grade \(i\) may still increase in response to an increase in \(G_i\), but only by inducing more producers to enter the market (increasing \(N\)) and not by causing producers to adjust effort levels. Once \(f(x_{i+1} - e)\) and \(f(x_i - e)\) are equal, it is likely that production of graded goods behaves as if grades are produced in fixed proportions. Therefore, if consumer preferences for beef were to shift so as to increase the price of Choice graded beef, perhaps due to a shift in diets or the invention of a complementary food processing good, producers will initially readjust their effort levels to increase the share of Choice they produce. The ability to increase the share of Choice produced is limited, however, once the producers have centered their effort levels within the upper and lower bounds of the Choice grade. If the price of Choice continues to rise, there is no further adjustment to effort that increases the production of Choice and only entry of outside producers or an adjustment of grade standards can increase Choice output.

Figure (3) shows the distribution of quality when the error in quality production is normal. Between the upper and lower portions of Figure (3), effort increases in response to an increase in the payment to a producer for making grade 3. While the market share
of grade 3 has now increased, changes in effort can no longer increase the market share of grade 3, as $f(x_3)$ and $f(x_4)$ are equal. Now, only changes in the number of producers can influence the supply of grade 3. Essentially, the fattest part of the quality distribution is now captured in grade 3. Because the error in quality production determines the market share, the supply of a specific grade may not increase in response to an increase in producer payment.

**IV.B. Consumers and Demand**

Consumer demand follows the vertical differentiation model of demand used in Mussa and Rosen (1978). Consumer tastes for quality, $\theta$, are distributed according to the cumulative distribution $S(\theta)$ with a strictly positive support across $M$ possible risk-neutral consumers. Consumers with larger $\theta$ values have stronger preferences for quality.

Let $\mu_i$ be the expected quality of goods that qualify for grade $i$ so that:

$$\mu_i = \int_{x_i}^{x_{i+1}} \frac{f(\lambda)}{\pi_i(e)}(e + \lambda) d\lambda$$  \hspace{1cm} (8)

An individual consumer who chooses a good in grade $i$ receives the following indirect utility as a function of expected quality, $\mu_i$, and price, $p_i$:

$$u(\mu_i, p_i) = \theta \mu_i - p_i$$  \hspace{1cm} (9)

Consumers select goods that fulfill both the individual rationality (IR) constraint and the incentive compatibility (IC) constraint. The IR constraint implies that the consumer must receive a surplus at least as large as their reservation utility, $U$, so that:

$$u(\mu_i, p_i) = \theta \mu_i - p_i \geq U$$  \hspace{1cm} (IR)  \hspace{1cm} (10)

---

8 In similar work by Ligon (2001), consumers are assumed to maximize utility by choosing between quality and a numeraire good. In his model, heterogeneity in consumer’s income rather than in quality preferences drives the selection of different grades.
The IC constraint implies that the consumer receives an indirect utility at least as large as that from selecting a different good.

\[ \theta u_i - p_i \geq \theta u_j - p_j \quad \text{for all } i \neq j \]  

(IC)  

The IR constraint is only binding for consumers at the lowest quality grade as consumers of higher qualities receive a positive surplus when the IC constraint is satisfied.

As the number of producers, \( N \), is smaller than \( M \), the number of potential buyers, producers set prices so that only the \( \frac{N}{M} \) proportion of possible consumers with the largest \( \theta \) purchase goods as shown in Figure (4). In general, sellers seek to maximize the producer surplus by pricing to segment the market by preference type. Their pricing will ensure that only consumers with \( \theta > \theta_i \) purchase grade \( i \) and only consumers with \( \theta_{i+1} > \theta > \theta_i \) purchase grade \( i \). This allocation is efficient in that the consumers with the strongest preferences for quality receive the highest quality goods.

For the highest quality grade, the quantity supplied is \( N\pi_i \), which is the proportion of output qualifying for grade \( \bar{i} \), multiplied by the number of producers. The quantity demanded is \( M \left( 1 - S(\theta_i) \right) \). The boundary for preferences for which consumers purchase the top graded good, \( \theta_i \), is:

\[ \theta_i = S^{-1}(1 - \frac{N}{M}\pi_i) \]  

(12)

The boundary for preferences on the second highest graded good is:

\[ \theta_{i+1} = S^{-1}(1 - \frac{N}{M}(\pi_i + \pi_{i-1})) \]  

(23)

Subsequent boundaries for lower grades are obtained similarly. The smallest value of \( \theta \) for which a consumer still purchases a good is \( S^{-1}(1 - \frac{N}{M}\sum \pi_i) \) or \( S^{-1}(1 - \frac{N}{M}) \). Figure (4) shows a graphical depiction and Table (4) presents a hypothetical case with four
grades. Prices are set to maximize the producer’s surplus while ensure that both the IR and IC constraints are satisfied for each grade. Of the \( \frac{N}{M} \) proportion of consumers that purchase goods, the top \( \pi_i \) proportion of consumers purchase the highest quality goods with average quality \( \mu_i \). The next \( \pi_{i-1} \) proportion of consumers purchases the grade \( i-1 \) with average quality \( \mu_{i-1} \). The portion of the distribution greater than \( \theta_i \) represents the portion of all possible consumers to which goods are sold. Of that truncated distribution, the probability mass between \( \theta_i \) boundaries represents the portion of active consumers purchasing each grade.

With the \( \theta_i \) values depicted in Figure (4), prices can be determined by solving first for the price of the lowest graded good. Prices of the lower quality goods can then be used to determine the prices of the next highest graded good. Since higher graded goods are of higher quality, consumers will always prefer the higher grade to the lower grade good if their prices are equal. Similarly, consumers with higher values of \( \theta \) have a greater willingness to pay for quality than other consumers. For higher quality goods, sellers will raise the price until consumers with low \( \theta \)'s drop out of the market and consumers with high \( \theta \)'s are indifferent between buying that good and the lower quality good. Consumers with low \( \theta \)'s essentially have only one consumption choice, the lowest quality good. Profit maximizing sorters charge consumers of the lowest graded good the price that sets the indirect utility of the marginal consumer (with preferences of \( \theta_i \)) equal to \( \bar{U} \), which is hereafter assumed to be zero. At that price, the participation constraint binds for the marginal consumer, or:

\[
P_i = \theta_i \mu_i
\]
While the marginal consumer of the second grade receives no surplus to trade, intermediate consumers with $\theta$ between $\theta_1$ and $\theta_2$ receive a positive consumer surplus owing to their stronger preferences for quality.

For higher graded goods, the sorter must set prices to ensure that the consumer does not switch to a lower quality good. In other words, the IC constraint of Equation (18) must be satisfied in addition to the participation constraint. The price which solves both these constraints is:

$$P_{i+1} = \theta_i \left( \mu_{i+1} - \mu_i \right) + P_i$$  \hspace{1cm} (15)

For the second quality grade, the marginal consumer now receives a surplus of $\theta_i \left( \mu_{i+1} - \mu_i \right)$. Consumers of the second grade would purchase the second graded good if the price were equal to $\theta_2 \mu_2$ and no other grade were available. At this price, the consumer’s surplus from purchasing the good exceeds his reservation utility. The availability of lower grades, however, offers the consumer a surplus. The presence of lower grades obligates the seller to reduce the price of second graded good in the manner analogous to the way an increase in a consumer’s reservation utility forces the seller to lower the price of the grade 1 goods. The value of this price reduction is known as the *information rent* within contract theory as it reflects the value of the consumer’s private knowledge of their preferences for quality. While consumers with the stronger preferences pay higher prices for higher graded good, they also receive increasingly larger information rents.

The iterative nature of prices under the vertical differentiation structure links prices across all goods as shown in Table (4). For example, a decrease in the price of chicken that increases the reservation utility of the typical consumer, $\bar{U}$, decreases the
price of the lowest quality good, which then causes the prices of all higher graded goods to decrease as well. Alternatively, a decrease in the number of producers, \( N \), increases the price of all grades through its initial effect on the lowest graded good.

**IV.C. Supply and Demand in Equilibrium**

In a fashion similar to that of the monopolistic competition of demand, the producer and sorter surplus converges to zero over the long run equilibrium\(^9\). Sorters earn a zero surplus as payments from sorters to producers, \( G_i \), equal the payments from consumer to sorters, \( P_i \), in a competitive market for sorters. Producer entry and exit eliminates the producer surplus over the long run as prices, weighted by the shares produced of each quality grades, equal the cost of effort.

When producers earn a positive surplus (profits), outside producers enter the market and existing producers expand operations. As \( N \) increases, the prices of the lowest graded good, \( P_1 = S^{-1}(1 - \frac{N}{M})\mu_1 \), and all higher graded goods decrease. The expanded supply drives prices down until the expected producer surplus is equal to zero. Conversely, producer losses encourage exit from the industry. As firms leaves the industry, \( N \) decreases and prices increase which eliminates producer losses over the long run. Other things equal, the distribution of qualities is invariant to the number of producers. Because firm entry and exit generates price effects, however, the optimal effort level chosen by producers is likely to change. In the simulation section, the producer, consumer, and total surpluses are simulated for a given set of grades in a

\(^9\)Ligon (2001) approaches grading from a continuous grading framework and models consumers in a slightly different utility framework. Ligon focuses on the pricing structures that emerge under different information structures and does not address discrete grading.
market, assuming that effort is constant, and shows that the producer surplus is decreasing in the number of firms.

IV.D. The Effect of Changing a Grading Standard

Grade standards are often outside the immediate control of either producer or consumer groups, and are not changed frequently. Comparative statics show that changes in grade standards may significantly influence the size of the total surplus to trade in the short run, and the distribution of that surplus between producers and consumers.

The consumer surplus from trade is:

\[
CS = M \sum_i \left( \theta_i \int_{\beta_i} (\theta, P_i) s(\theta) d\theta \right) = M \sum_i \left( \mu_i \int_{\beta_i} (\theta) s(\theta) d\theta - \theta_i \right) - N \sum_i \pi_i P_i
\]

(23)

The first term represents the benefits to the proportion of active consumers who purchase the good, while the second represents payments to producers. Notice that the second element of Equation (23) is simply the revenue from the producer surplus defined in Equation (4). In considering the effect of a grade standard change on the consumer surplus, it is useful to assume that effort, \( e \), is constant. Essentially, this specification assumes that the distribution of quality is fixed, a plausible assumption if the cost of effort function, \( h(e) \), is vertical or if production decisions regarding effort are fixed in the short run. With the quality distribution fixed, grading only serves to reallocate qualities across consumers of different preference types. In this case, the total surplus is the sum of the producer surplus, Equation (4), and the consumer surplus, Equation (24), or:

\[
TS = PS + CS = M \left( \mu_1 \int_{\beta_1} s(\theta) d\theta + \mu_2 \int_{\beta_2} s(\theta) d\theta + \mu_3 \int_{\beta_3} s(\theta) d\theta + \mu_4 \int_{\beta_4} s(\theta) d\theta \right) - Nh(e)
\]

(24)
A change in the standard for grade 4 only affects the portions of the producer surplus derived from changes in revenue from goods making grade 3 and grade 4, represented as $PS_3$ and $PS_4$. The change in producer surplus is then:

$$\frac{\partial PS}{\partial x_4} = \frac{\partial PS_4}{\partial x_4} + \frac{\partial PS_3}{\partial x_4} = N\left(\frac{\partial (\pi_4 P_3)}{\partial x_4} + \frac{\partial (\pi_4 P_4)}{\partial x_4}\right)$$

(25)

The change in $PS_3$ from a change in $x_4$ is:

$$\frac{\partial PS_3}{\partial x_4} = N\left(f(x_4 - e)\theta_3 (\mu_3 - \mu_2) + \ldots + (F(x_4 - e) - F(x_3 - e)) \frac{\partial \mu_3}{\partial x_4}\right) > 0$$

(26)

Raising $x_4$ increases the number of goods in grade 3, making the first term of Equation (26) positive, and increases the quality and price of grade 3, making the second term positive as well. The change in $PS_4$ from a change in $x_4$ is:

$$\frac{\partial PS_4}{\partial x_4} = N\left(- f(x_4 - e)\theta_4 (\mu_4 - \mu_3) + \theta_3 (\mu_3 - \mu_2) + \ldots \right)$$

$$+ N\left(1 - F(x_4 - e)\right)\left(\frac{\partial \mu_4}{\partial x_4} - \frac{\partial \mu_3}{\partial x_4}\right) + \theta_3 \left(\frac{\partial \mu_3}{\partial x_4}\right)$$

(27)

Raising $x_4$ decreases the number of goods in grade 3, making the first term negative. Raising $x_4$ also increases the quality of grade 4, but may reduce the price of grade 4 if $\frac{\partial \mu_3}{\partial x_4}$ is larger than $\frac{\partial \mu_4}{\partial x_4}$. For this reason, the net effect on the producer surplus from a change in $x_4$ is ambiguously signed.

The derivatives $\partial PS/\partial x_4$, $\partial CS/\partial x_4$, and $\partial TS/\partial x_4$ show the effect that changes in the grade standard for the highest grade have on the producer surplus, the consumer surplus, and the total surplus respectively.
Let \( CS_i \) equal \( \left( \frac{\mu_i}{\rho} \theta_s(\theta) d\theta - N\pi_i P_i \right) \), the consumer surplus that accrues only to consumers of good \( i \). Changing \( x_4 \), the grade standard for grade 4, only affects consumers of grades 3 and 4 so that:

\[
\frac{\partial CS}{\partial x_4} = \frac{\partial CS_4}{\partial x_4} + \frac{\partial CS_3}{\partial x_4} \quad (28)
\]

Consumers of the third graded good receive a total surplus of:

\[
CS_3 = M\mu_3 \int_{\theta_i}^{\theta_4} \theta(\theta) d\theta - N\pi_3 P_3
\]

\[
= M\mu_3 \int_{\theta_i}^{\theta_4} \theta(\theta) d\theta - N\pi_3 (\theta_3 (\mu_3 - \mu_2) + \ldots) \quad (29)
\]

A change in \( x_4 \) changes the consumer surplus as follows:

\[
\frac{\partial CS_3}{\partial x_4} = M \left( \frac{\partial\mu_3}{\partial x_4} \int_{\theta_i}^{\theta_4} \theta(\theta) d\theta + \mu_3 \frac{\partial \int_{\theta_i}^{\theta_4} \theta(\theta) d\theta}{\partial x_4} \right) - N \left( f(x_4 - e)P_3 - \pi_3 \theta_3 \left( \frac{\partial\mu_3}{\partial x_4} \right) \right) \quad (30)
\]

This change in Equation (30) is ambiguously signed. The terms \( \frac{\partial\mu_3}{\partial x_4} \) and \( \frac{\partial \int_{\theta_i}^{\theta_4} \theta(\theta) d\theta}{\partial x_4} \) are both positive, ensuring that the first term is positive. Increasing \( x_4 \) raises the standard of grade 4 but also expands the upper bound for grade 3. As higher quality goods are now included in grade 3, its quality and its market share increases. This second term is ambiguously signed. Increasing the upper boundary for grade 3 increases the number of consumers paying \( P_3 \), decreasing their surplus but also increases the quality those consumers receive.

The change in \( CS_4 \) is similar:
The effect of the same grading change on consumers of the highest graded good is:

\[
\frac{\partial CS_4}{\partial x_4} = M \left( \int_{\theta_4}^{\infty} \frac{\partial \mu_4}{\partial x_4} (\theta) d\theta + \mu_4 \int_{\theta_4}^{\infty} (\theta) s(\theta) d\theta \right) \\
- N \left( - f(x_4 - e)(\theta_4 (\mu_4 - \mu_3) + \theta_3 (\mu_3 - \mu_2) + \ldots) \right) \\
- N(1 - F(x_4 - e)) \left( \frac{\partial \theta_4}{\partial x_4} (\mu_4 - \mu_3) + \theta_4 \left( \frac{\partial \mu_4}{\partial x_4} - \frac{\partial \mu_3}{\partial x_4} \right) + \theta_3 \frac{\partial \mu_3}{\partial x_4} \right)
\]

Equation (32) is also ambiguously signed. The sign of the first term is ambiguous as \( \frac{\partial \mu_4}{\partial x_4} \) is positive, while \( \int_{\theta_4}^{\infty} (\theta) s(\theta) d\theta / \partial x_4 \) is negative because raising the standard for grade 4 increases its average quality while decreasing its market share. The second term is negative, as fewer consumers pay \( P_4 \) after the grade change. The third term, representing the price effect, is more difficult to interpret. Increasing the quality standard of grade 4 simultaneously increases the quality of both grades 3 and 4\(^{10} \), while also raising \( \theta_4 \), since \( \frac{\partial \theta_4}{\partial x_4} \) is positive. As the average quality of grade 3 increases, its price also increases. Since prices are linked across goods, the price of grade 4 also increases. Again, the net effect of the third term is ambiguous. Equation (33) compiles the two effects below.

---

\(^{10} \) (\( \frac{\partial \mu_4}{\partial x_4} , \frac{\partial \mu_4}{\partial x_4} > 0 \) )
\[
\frac{\partial CS}{\partial x_4} = M \mu_4 \left( \frac{\partial}{\partial x_4} \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta + \frac{\partial}{\partial x_4} \int_{\theta_5}^{\theta_6} (\theta) s(\theta) d\theta \right) + Nf(x_4 - e)(\theta_4 (\mu_4 - \mu_3))
\]
\[+ \left(-N\pi_3 \theta_3 + N\pi_4 (\theta_3 - \theta_4) + M \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta \right) \frac{\partial \mu_3}{\partial x_4}
\]
\[+ \left(M \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta + \theta_4 \pi_4 \right) \frac{\partial \mu_4}{\partial x_4} + (\mu_4 \pi_4 - \mu_3 \pi_3) N \frac{\partial \theta_4}{\partial x_4} - N \pi_3 \theta_3 \mu_2
\]
\[= M \left( \mu_4 \frac{\partial}{\partial x_4} \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta + \frac{\partial}{\partial x_4} \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta \right) + \frac{\partial \mu_4}{\partial x_4} \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta \right) - \frac{\partial PS}{\partial x_4}
\]

Combining Equation (37) and the effect described in Equations (26) and (27), the net change in the total surplus is given below.

\[
\frac{\partial TS}{\partial x_4} = M \left( \frac{\partial \mu_3}{\partial x_4} \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta + \mu_3 \frac{\partial}{\partial x_4} \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta \right) + \frac{\partial \mu_4}{\partial x_4} \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta \right) + \frac{\partial}{\partial x_4} \int_{\theta_4}^{\theta_5} (\theta) s(\theta) d\theta \right)
\]

Equation (34) is also neither necessarily positive nor necessarily negative.

Altering \(x_4\) influences the total surplus both through its effect on average qualities and its effect on the \(\theta_i\) terms which define the boundaries of consumer preferences for each grade. The simulation section shows that lowering the level of \(x_4\) may either increase or decrease the total surplus from trade.

**IV.E. The Effect of Introducing a New Grade**

To consider the effect of introducing a new grade between the third and fourth, let \(x_{3.5}\) represent a new quality standard between \(x_3\) and \(x_4\). This new grade might represent branded beef that arose in the 1990’s in addition to the USDA grading program. Assume that this new grade is marketed in an identical manner to those on the ordinary
grading system. Prices, quality standards, \( \theta \) preferences, and market shares are denoted under the new standards with an apostrophe so that average qualities are \( \mu'_1, \mu'_3, \) and \( \mu'_4 \), prices are \( P'_3, P'_{3.5} \) and \( P'_4 \), and \( \theta \) preferences are \( \theta'_3, \theta'_{3.5} \) and \( \theta'_4 \). The intermediate grade only influences the qualities of the third grade so that \( \mu'_1 < \mu'_3 < \mu'_{3.5} \) and \( \mu'_4 = \mu_4 \). As the market share of grade 4 is unaffected so that \( \pi'_4 = \pi'_4 \), its theta boundary is unaffected so that \( \theta_4 = \theta'_4 \). The market shares of the new third grade, \( \pi'_{3.5} \), and the intermediate grade, \( \pi'_3 \), sum to the share of the original third grade, \( \pi_3 \), which implies that \( \theta_3 = \theta'_3 < \theta'_{3.5} \). The price of grade 3 falls with the introduction of the new grade due to the reduction in quality so that \( P'_3 < P_3 \) while the price of the intermediate good is larger than that of grade 3 before the introduction so that \( P'_{3.5} > P_3 \). Because the price of the grade 4 good is tied to the price of lower-graded goods through information rents, its price may increase or decrease.

The producer surplus change under the old grading system is

\[
PS^{Old} = \sum_i \pi_i P_i - h(e) = N \left( \ldots + \pi_3 P_3 + \pi_4 P_4 - h(e) \right)
\]  

while the producer surplus under the new grading system is:

\[
PS^{New} = \sum_i \pi'_i P_i - h(e) = N \left( \ldots + \pi'_3 P'_3 + \pi'_{3.5} P'_{3.5} + \pi'_4 P'_4 - h(e) \right)
\]

For simplicity, it is again assumed that effort, \( e \), is unchanged. The net change in the producer surplus is:

\[
\Delta PS = N \left( \left( P_3 - P'_3 \right) \pi'_3 + \left( P_3 - P'_{3.5} \right) \pi'_{3.5} + \left( P_4 - P'_4 \right) \pi'_4 \right)
\]

or, alternatively, after simplification:

\[
\Delta PS = \theta'_1 (\mu'_1 - \mu'_3) (\pi'_3 + \pi'_{3.5} + \pi'_4) - \theta'_{3.5} (\mu'_{3.5} - \mu'_3) (\pi'_{3.5} + \pi'_4) + \theta'_4 (\mu'_4 - \mu'_3) \pi'_4
\]
Notice that if the introduction of the new grade is trivial so that \( x_{3.5} = x_4 \), then \( \mu_3 = \mu_3' \), \( \pi_{3.5}' = 0 \), and \( \theta_{3.5}' = \theta_4' \), then there is no change in the producer surplus.

The change in the consumer surplus can also be isolated into its effect on grade 3 and grade 4. Where the original consumer surplus is:

\[
CS^{OLD} = \ldots + M\mu_3 \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta - N\pi_3 P_3 + M\mu_4 \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta - N\pi_4 P_4
\]  

(39)

The new consumer surplus is:

\[
CS^{NEW} = M\mu_3' \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta - N\pi_3' P_3' + M\mu_4' \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta - N\pi_4' P_4'
\]  

(40)

The change in consumer surplus can then be simplified to:

\[
\Delta CS = CS^{OLD} - CS^{NEW}
\]  

(41)

\[
\Delta CS = M\left( (\mu_3 - \mu_3') \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta + (\mu_3 - \mu_{3.5}') \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta + (\mu_4 - \mu_4') \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta \right)
\]  

(42)

\[
\Delta CS = M\int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta
\]

The change in total surplus is the first term in Equation (42) or:

\[
\Delta TS = M\left( (\mu_3 - \mu_3') \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta + (\mu_3 - \mu_{3.5}') \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta + (\mu_4 - \mu_4') \int_{\theta_i}^{\theta_i'} \theta s(\theta) d\theta \right)
\]  

(43)

Equation (43) shows that the total surplus may either increase or decrease even after accounting for the redistribution of the surplus between consumers and producers. This result stems from consumers selecting goods based on average quality and being risk-
neutral. Introducing a new good can sufficiently change average qualities to increase or
decrease the total surplus from trade. Moreover, introducing a new grade improves
allocative efficiency by increasing the segmentation of the market by quality type. The
simulations also demonstrate that Equation (43) is ambiguously signed.

V. Simulation Results

Three simulation experiments demonstrate the entry and exit process of reaching
long-run equilibrium and the ambiguous effects on welfare of both changing a grade
standard and introducing a new quality grade. The first simulation demonstrates that the
producer surplus is decreasing in the number of producers in the industry. The long run
equilibrium number of producers is then found for a given consumer preference
distribution and set of quality standards. The second simulation demonstrates that the
producer, consumer, and total surplus changes as the grade standard for the highest
quality grade ranges over a continuum of levels. The grade standards that maximize each
of those values are different, implying that control over grade standards may have value
to producers and consumers. The third simulation demonstrates that the producer,
consumer and total surplus also changes with the introduction of a new grade between the
existing grade 3 and grade 4 standards. Surprisingly, the introduction a new grade can
reduce the total surplus depending on where it is placed.

Simulations were performed assuming that there are 20,000 potential consumers
with preferences drawn from the beta distribution. An $N$ number of producers make a
single good with a random quality determined by Equation (2). Effort is set to the level

\footnote{Estimates not reported here were also performed with $\theta$ being drawn from the other distributions with
similar results. Distributions that do not have closed supports, such as the normal and gamma distributions
can show large swings from the presence of outliers. These distributions showed more variation between
simulations especially with regard to welfare measures of the highest quality grades.}
of 200 and the error in quality production is normal with a mean of 60 and a standard deviation of 25. In each of the simulations, the effort level is fixed at 200 with a cost of 8. This specification implies that effort is trivially inexpensive until the level of 200 and then prohibitively expensive afterwards. More appropriately, however, this specification can alternatively be interpreted as producers not being able to re-optimize effort levels in response to a change in grading standards. In the first simulation experiment, values are calculated for \( N \) between 8000 and 12000; In the other simulations, \( N \) is fixed at 10,000.

Given a set of quality and preference distributions, grade shares are determined using the grading standards in Equation (3) and expected qualities are determined as in Equation (15). Shares define \( \theta \) boundaries as in Equation (19) and (20) and these values are used to determine prices as in Equations (21) and (22). Finally, the integrals that define consumer surplus in Equation (23) are numerically estimated by calculating each simulated consumer’s individual surplus, then dividing by the total number of active consumers, \( N \).\(^{12}\) The producer surplus stated in Equation (4) is also estimated with prices and market shares.

The first simulation considers the number of producers in the market over the long run for grades fixed at the following levels: grade 1, 200; grade 2, 235; grade 3, 260; and grade 4, 290. Figure (5) relates the total producer surplus to the number of firms in the industry. The jaggedness of the line is caused by the re-simulation of qualities with each calculated producer surplus. The trend shows that the producer surplus is decreasing in \( N \). In accordance with the earlier discussion, a positive producer surplus induces entry, which drives down prices and dissipates producer revenues. Table

\(^{12}\) To account for error using numerical estimation with random distributions, the simulation was performed with 1,000,000 and 500,000 simulated consumers and producers. The results were essentially identical.
(5) presents welfare measures and prices for several $N$ values in the simulation. Figure (5) and Table (5) show that the producer surplus is decreasing in the number of firms and that the producer surplus is closest to zero and the market is in long run equilibrium when there are 10,348 firms in the market.

The second simulation considers the effect of changing the standard for grade 4, $x_4$, by varying it over 1000 evenly spaced points between 261 and 350\textsuperscript{13}. Each simulation occurs with 10,000 producers and 20,000 potential consumers. Preference parameters are drawn from the beta distribution with alpha 0.2 and beta 0.8, and the quality error distribution is again normal with mean 60 and standard deviation 25. As with each of the simulations, producer effort is fixed at the level of 200 with cost 8 throughout the simulation. The standard for the first three grades are again assumed to be 200, 235, and 260, respectively. Figures (6), (7) and (8) show the total surplus, total producer surplus, and total consumer surplus as $x_4$ varies. The increasing jaggedness of the line as $x_4$ increases is the result of the small number of goods with high qualities. Table (6) shows that adjusting grading standards affects the size of the total surplus and that the total surplus is maximized when $x_4$ is approximately 278.89. The distribution of the total surplus is also dependent on the grading standard, as the consumer surplus is maximized when $x_4$ is 267.71, while the producer surplus is maximized when $x_4$ is 292.75. Moreover, the grading standard that optimizes the total surplus is not that which optimizes the producer or consumer surplus.

Control over the standards of grading has value to producers and consumers. Grades improve allocative efficiency, as they divide the quality distribution more finely.

\textsuperscript{13} A quality boundary could not be set equal to 260 as this would set the market share of grade 3 to zero which causes computational problems.
and therefore allow for a better matching of qualities with consumer preferences. The setting of grades also influences average qualities of individual grades, which alters the size and distribution of the total surplus to trade. Periodically, producer groups, such as the American Angus Association after 1976, express discontent with grading standards. Alterations to standards for grades are typically subject to public comment preceding the introduction of a new market rule by the AMS. Harris, Cross and Savell (1988) note that the National Cattlemen’s Beef Association is very active in the development of grade standards. While this paper does not attempt to evaluate claims of producer and retail groups regarding grade standards, it does show that control over the institutional mechanism controlling grading is valuable to interested parties.

The third set of simulations illustrates the effect of introducing a new quality grade by simulating 200 new grade standards between grade 3 and grade 4 in the interval between 261 and 289. Figure (9), (10), and (11) show the sizes of the total surplus, producer surplus, and consumer surplus as they vary over the grade standard with the remaining grades. The right and left endpoints on the graph show the approximate values that occur when no new grade is added. Table (7) shows welfare estimates and prices for several values of $x_{3.5}$. In general, the grade standard which maximizes the total surplus, 274.65, differs from the standard that maximizes the producer surplus, 276.9, although it is close to the standard which maximizes the consumer surplus, 273.24.

The maximum total surplus, consumer surplus and producer surplus are higher both when grades are adjusted and when new grades are added. Figures (9), (10), and (11) suggest that the introduction of a new grade strictly increases the total surplus as well as the consumer surplus and the producer surplus. Figure (12), however, shows that
this result is sensitive to the choice of quality distribution. In this simulation, effort remains at 200, but now the distribution of the error in quality is now the beta distribution with parameters of 3 and 4$^{14}$. In this figure, the total surplus is shown to be decreasing for a distinct range of grade values indicating that a new grade can decrease the total surplus in certain situations.

Introducing a new grade benefits consumers by allowing for a better distribution of qualities while increasing the power of sellers to price discriminate as more grades are offered. At the same time, more consumption options on the market force producers to offer larger information rents to consumers. A larger number of grades need not benefit both consumers and producers, as indicated by Equation (43). In this case, the change in the average quality of grades from reassigning goods between grades is sufficiently large as to reduce the total surplus, as shown in Figure (12).

The number of grades again has important impacts on total welfare and its distribution between consumers and producers. These effects are purely informational and occur in the absence of both consumer and producer risk aversion and supply effects. As producer groups use certification to create finer divisions of quality in the beef industry, significant welfare improvements may arise even in the absence of incentive effects that change the level of producer investment in quality improvement.

VI. Summary and Conclusions

The 1990’s witnessed a substantial expansion of quality certification programs in the beef industry. As certification programs potentially changed the control over the

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$^{14}$ The error distribution is widened and re-centered so that it ranges between 0 to 112. All grades are thus assured of having positive market shares.
standards for and the number of quality grades, new concerns arose regarding their
effects on consumer and producer welfare. This paper presents an equilibrium model of
quality grading where grading is costless and economic agents are risk neutral.
Comparative static analysis and simulations show that placement and number of quality
standards have significant effects on the size and distribution of the total benefits to trade.
Moreover, finer division of the quality distribution through the introduction of a new
grade may not improve consumer welfare or total welfare.

The expansion of beef branding and certification is often lauded for improving
incentives for quality improvement. This paper shows that a significant redistribution of
welfare may be associated with short run changes in grading standards. As certification
and branding become more prominent, finer distinctions in quality become available.
While this potentially improves allocative efficiency, it also allows producers to segment
the market by consumer type. While this paper ignores market power, branding
introduces the possibility of price-setting or quantity-restricting behavior which may be
an important topic for future research.
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Wheeler, T. L., Shackelford, S. D. and Koohmaraie, M.,
### VIII. Tables and Figures

**Table 3-1 - Summary Statistics on Weekly Output of Beef Grades**

<table>
<thead>
<tr>
<th>Mean</th>
<th>Prime</th>
<th>Brand</th>
<th>Choice</th>
<th>Select</th>
<th>Ungraded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shares</td>
<td>0.77%</td>
<td>6.80%</td>
<td>31.08%</td>
<td>25.30%</td>
<td>36.06%</td>
</tr>
<tr>
<td>Price (Dollars/cwt)</td>
<td>144.43</td>
<td>133.43</td>
<td>127.58</td>
<td>118.81</td>
<td>117.51</td>
</tr>
<tr>
<td>Quantities (in Millions)</td>
<td>1.730</td>
<td>15.246</td>
<td>69.721</td>
<td>56.761</td>
<td>80.894</td>
</tr>
</tbody>
</table>
### Table 3-2 - Certification Program Formation

<table>
<thead>
<tr>
<th>Year</th>
<th>Active Certification Programs:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>3</td>
</tr>
<tr>
<td>1994</td>
<td>3</td>
</tr>
<tr>
<td>1995</td>
<td>5</td>
</tr>
<tr>
<td>1996</td>
<td>7</td>
</tr>
<tr>
<td>1997</td>
<td>8</td>
</tr>
<tr>
<td>1998</td>
<td>12</td>
</tr>
<tr>
<td>1999</td>
<td>20</td>
</tr>
<tr>
<td>2000</td>
<td>26</td>
</tr>
<tr>
<td>2001</td>
<td>34</td>
</tr>
<tr>
<td>2002</td>
<td>36</td>
</tr>
<tr>
<td>2003</td>
<td>38</td>
</tr>
<tr>
<td>2004</td>
<td>47</td>
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Table 3-3 - Index of Notations for Supply

<table>
<thead>
<tr>
<th>Grade</th>
<th>Index</th>
<th>Payment</th>
<th>Market Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime</td>
<td>( i = 4 )</td>
<td>( G_i )</td>
<td>( \pi_4 = 1 - F(x_4 - e) )</td>
</tr>
<tr>
<td>Choice</td>
<td>( i = 3 )</td>
<td>( G_3 )</td>
<td>( \pi_3 = F(x_4 - e) - F(x_3 - e) )</td>
</tr>
<tr>
<td>Select</td>
<td>( i = 2 )</td>
<td>( G_2 )</td>
<td>( \pi_2 = F(x_2 - e) - F(x_1 - e) )</td>
</tr>
<tr>
<td>Ungraded</td>
<td>( i = 1 )</td>
<td>( G_1 )</td>
<td>( \pi_1 = F(x_2 - e) - F(x_1 - e) )</td>
</tr>
</tbody>
</table>
Table 3-4 - Index of Notations for Demand

<table>
<thead>
<tr>
<th>Grade</th>
<th>Index</th>
<th>Price, Market Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime</td>
<td>$\tilde{i} = 4$</td>
<td>$P_4 = \theta_4 (\mu_4 - \mu_3) + \theta_3 (\mu_3 - \mu_2) + \theta_2 (\mu_2 - \mu_1) + \theta_1 \mu_1 - \bar{U}$ [\theta_4 = S^{-1} \left(1 - \frac{N}{M} \pi_4 \right)]</td>
</tr>
<tr>
<td>Choice</td>
<td>$i = 3$</td>
<td>$P_3 = \theta_3 (\mu_3 - \mu_2) + \theta_2 (\mu_2 - \mu_1) + \theta_1 \mu_1 - \bar{U}$ [\theta_3 = S^{-1} \left(1 - \frac{N}{M} \left(\pi_4 + \pi_3 \right) \right)]</td>
</tr>
<tr>
<td>Select</td>
<td>$i = 2$</td>
<td>$P_2 = \theta_2 (\mu_2 - \mu_1) + \theta_1 \mu_1 - \bar{U}$ [\theta_2 = S^{-1} \left(1 - \frac{N}{M} \left(\pi_4 + \pi_3 + \pi_2 \right) \right)]</td>
</tr>
<tr>
<td>Ungraded</td>
<td>$i = 1$</td>
<td>$P_1 = \theta_1 \mu_1 - \bar{U}$ [\theta_1 = S^{-1} \left(1 - \frac{N}{M} \left(\pi_4 + \pi_3 + \pi_2 + \pi_1 \right) \right)]</td>
</tr>
<tr>
<td></td>
<td>TS</td>
<td>PS</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>--------</td>
</tr>
<tr>
<td>$N = 8,000$</td>
<td>982,510</td>
<td>68,922</td>
</tr>
<tr>
<td>$N = 9,000$</td>
<td>998,980</td>
<td>36,664</td>
</tr>
<tr>
<td>$N = 10,000$</td>
<td>1,009,500</td>
<td>8,410</td>
</tr>
<tr>
<td>$N = 10,348$</td>
<td>1,012,500</td>
<td>19</td>
</tr>
<tr>
<td>$N = 11,000$</td>
<td>1,014,400</td>
<td>-16,116</td>
</tr>
<tr>
<td>$N = 12,000$</td>
<td>1,017,800</td>
<td>-36,256</td>
</tr>
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</table>
Table 3-6 - Welfare Measures and the Grade 4 Standard

<table>
<thead>
<tr>
<th>$X_4$</th>
<th>TS</th>
<th>PS</th>
<th>CS</th>
<th>Prices $P_1, P_2, P_3, P_4$</th>
<th>$\theta_i$ $\theta_1, \theta_2, \theta_3, \theta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>278.89</td>
<td>1,002,200</td>
<td>4,229</td>
<td>998,000</td>
<td>3.89, 4.77, 7.69, 18.33</td>
<td>0.0174, 0.0353, 0.1434, 0.4366</td>
</tr>
<tr>
<td>292.75</td>
<td>996,700</td>
<td>8,976</td>
<td>987,730</td>
<td>3.89, 4.77, 8.39, 33.93</td>
<td>0.0174, 0.0353, 0.1434, 0.8566</td>
</tr>
<tr>
<td>267.71</td>
<td>997,100</td>
<td>-4.818</td>
<td>1,001,900</td>
<td>3.89, 4.77, 6.95, 11.79</td>
<td>0.0174, 0.0353, 0.1434, 0.2292</td>
</tr>
</tbody>
</table>
Table 3-7 - Welfare Measures and the Grade 4 Standard

<table>
<thead>
<tr>
<th></th>
<th>TS</th>
<th>PS</th>
<th>CS</th>
<th>Prices (P₁,P₂,P₃,P₄,P₅)</th>
<th>θ₁</th>
<th>θ₂</th>
<th>θ₃</th>
<th>θ₄</th>
<th>θ₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₄ = 274.65</td>
<td>1.023,100</td>
<td>11,056</td>
<td>1,012,000</td>
<td>3.88,4.76,7.40, 12.40,27.81</td>
<td>0.0173,0.0353,0.1426, 0.3425,0.7474</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X₄ = 277.04</td>
<td>1.022,900</td>
<td>11,128</td>
<td>1,011,800</td>
<td>3.88,4.76,7.55, 13.32,27.76</td>
<td>0.0173,0.0353,0.1426, 0.3882,0.7474</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>X₄ = 273.24</td>
<td>1.023,000</td>
<td>10,965</td>
<td>1,012,100</td>
<td>3.87,4.76,7.32, 11.88,27.89</td>
<td>0.0173,0.0353,0.1426, 0.3170,0.7474</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 3-1 - Percentage of the Total Commercial Slaughter Certified
Figure 3-2 - Market Shares when the Error Distribution is Uniform
Figure 3-3 - Production of Grades when the Production Error is Normal
Figure 3-4 - Market Shares and Consumer Preferences
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Figure 3-6 - Total Surplus and the Grade 4 Standard
Figure 3-7 - Producer Surplus and the Grade 4 Standard
Figure 3-8 - Consumer Surplus and the Grade 4 Standard
Figure 3-9 - Total Surplus and the Introduction of a New Grade
Figure 3-10 - Producer Surplus and the Introduction of a New Grade
Figure 3-11 - Consumer Surplus and the Introduction of a New Grade
Figure 3-12 - Total Surplus and the Introduction of a New Grade
IX. Appendix

Appendix A - Equations (A1) through (A6) show the intermediate steps to deriving Equation (7).

\[
\frac{\partial \pi_i}{\partial e_i} = \frac{\partial [F(x_{i+1} - e') - F(x_i - e')]}{\partial e_i^*} = -f(x_{i+1} - e') + f(x_i - e') \quad (A1)
\]

\[
PS(e) = N \left( -G_i F(x_i - e') + \sum_{i}^{i-1} ((G_i - G_{i+1}) F(x_{i+1} - e')) + G_i - h(e') \right) \quad (A2)
\]

\[
\frac{\partial PS(e')}{\partial e'} = N \left( G_i f_i(x_i - e') + \sum_{i}^{i-1} ((-G_i + G_{i+1}) f(x_{i+1} - e')) - \frac{\partial h(e')}{\partial e'} \right) \quad (A3)
\]

\[
\frac{\partial^2 PS(e')}{\partial e'^2} = N \left( -f(x_{i+1} - e') + f(x_i - e') \right) \quad (A4)
\]

\[
\frac{\partial^2 PS(e')}{(\partial e')^2} = N \left( G_i \frac{-\partial f(x_i - e')}{\partial e'} + \sum_{i}^{i-1} \left( (-G_i + G_{i+1}) \frac{-\partial f(x_{i+1} - e')}{\partial e'} \right) - \frac{\partial^2 h(e')}{(\partial e')^2} \right) \quad (A5)
\]

\[
\frac{\partial^2 PS(e')}{(\partial e')^2} \frac{\partial e'i}{\partial G_i} = \frac{f(x_{i+1} - e') - f(x_i - e')}{G_i \frac{-\partial f(x_i - e')}{\partial e'} + \sum_{i}^{i-1} \left( (-G_i + G_{i+1}) \frac{-\partial f(x_{i+1} - e')}{\partial e'} \right) - \frac{\partial^2 h(e')}{(\partial e')^2}} \quad (A6)
\]
Chapter 4

When is Fruit Bundling Fruitless?

Sorting, Bundling and Disposal When

Quality Information is Asymmetric
I. Introduction

Goods often vary in quality even when sold at a single price. In this setting, consumers expend significant time and effort searching for higher quality goods. Although individual consumers can increase their gains to trade by identifying goods in the upper portion of quality distribution, homogeneous consumers in aggregate cannot create any new benefits through sorting, as it only redistributes the existing distribution of qualities. Sorting does lower prices, however, by the cost of sorting goods.

Barzel (1977, 1982) argues that sellers design market mechanisms in these circumstances to prevent or discourage sorting. For example, sellers will bundle goods or have attendants disperse them. Medical offices may have receptionists assign doctors to patients; mechanics may similarly be assigned to cars. In both cases the consumer’s value of trade may be significantly altered by the worker assigned to them. Sellers have a strong incentive to incorporate the value of quality differences into prices. This may be impossible, however, if sellers cannot observe quality as with produce which perishes on display or second hand clothing which has a large fashion component or if there are significant barriers to charging multiple prices, as with doctor’s services.

When consumers are homogeneous, the allocation of qualities to different consumers has no impact on the total gains from trade. When consumers are heterogeneous, the total gains from trade are improved when high quality goods are reallocated from consumers with weak preferences to those with strong preferences. When information is asymmetric, meaning that consumers but not sellers can identify the quality of individual goods, sellers are unable to appropriate the benefits of reallocating qualities across consumers through the pricing mechanism. Sorting, however, lowers
prices. When consumers sort goods, they remove the highest quality goods from the distribution and leave the remaining truncated distribution of goods with a lower expected quality. When expected quality falls, sellers are forced to lower prices for consumers who do not sort. For this reason, sellers may prohibit or discourage sorting even though it may improve allocative efficiency.

Despite these concerns, prohibitions on consumer sorting are rare. This paper presents two possible reasons why sellers may still allow sorting even when products vary in quality. First, sorting may allow sellers to allocate higher quality goods to consumers with weak preferences for quality. While this allocation of qualities is inefficient, it reduces the price sensitivity of marginal consumers which allows sellers to raise the single price charged to all consumers. This strategy, in which higher quality goods are assigned to consumers with weak preferences for quality, is introduced in this paper as quality discrimination. Sorting accomplishes quality discrimination if weak valuations of quality are correlated with low sorting costs across consumers. Here, sellers commit to discarding a portion of their goods. Knowing that all consumers sort, sellers increase their expected quality by throwing out the leftover portion of supply after the higher quality goods are removed. Disposal is shown to increase profits when consumer preferences are heterogeneous, sorting costs are low, and wholesale prices are low relative to resale prices.

II. Literature Review
Agricultural goods sold in retail outlets often vary in quality but sell at a single price. In this setting, individual consumers benefit from sorting goods to find higher quality goods in the available distribution. In aggregate, however, sorting goods cannot
benefit consumers who value quality equally as it merely rearranges the consumer surplus across existing consumers but does not enlarge it. At the same time, sorting may create significant opportunity costs to individual consumers in terms of time and effort spent identifying product quality. When sorting is prohibited, the market clearing price under sorting prohibitions is higher by the amount of the sorting costs.

Barzel (1977, 1982) argues that sellers use a variety of subtle and innocuous marketing mechanisms to discourage consumers from sorting goods. By bundling goods such as apples or potatoes sold in bags, sellers reduce the variation in quality of the final product which removes the incentive for consumers to sort. Kenney and Klein (1983) argue that diamonds are bundled in a similar manner to discourage diamond buyers from the costly practice of presale quality examination. Typically, diamonds are sold in unopened packages containing many diamonds and buyers are severely discouraged from inspecting packages with threat of blacklisting from future transactions. By randomly bundling diamonds, sellers lower the variance in quality any package relative to that of individual sales so that buyers are less likely to make any quality examination.

Kenney and Klein argue that the bundling of first run movies through the process of block booking also prevents expensive rescheduling. After movies are exhibited, their quality becomes common knowledge and theater owners attempt to reject or shorten the performance runs of lower quality movies. Under block booking, theater owners commit to displaying multiple movies from a given distributor prior to their release date. Block booking lowers the variance in quality of any given bundle of movies relative to that of a single movie. Gains from unexpectedly good movies offset losses from unexpectedly bad ones.
When quality is variable and observable, sellers price goods to incorporate quality differences and dissuade consumers from sorting. In considering contracts for the right to remove timber between timber land owners and loggers, Leffler and Rucker (1991) find that contracts for rights to harvest timber are more likely to be on a per unit basis rather than lump sum when quality is heterogeneous or pre-sale examination of quality is expensive. When harvest contracts are lump sum fees, loggers pay a fixed amount regardless of the volume of timber removed from a tract and will harvest timber until the marginal cost of harvesting equals the market price of timber. In this case, loggers have an incentive to expend resources examining the tract for species composition, tree quality, and density as more valuable tracts improve logger returns even though this expense lowers their willingness to pay for harvest rights. When harvest contracts are per unit, logger fees are a function of the number of trees removed. Because this type of contract raises the marginal cost of tree harvesting, fewer trees are removed and the tract is harvested less intensively than when lump sum fees are charged. At the same time, loggers have little incentive to examine the quality of individual tracts of land. In choosing which contract type to offer, land owners trade off between the inefficient under-harvesting with per unit contracts and costly pre-sale quality measurement with lump sum contracts. The authors find that where pre-sale measurement costs are low or tract quality is homogenous, the lump sum contracts are more likely to be offered.

Leffler, Malishka, and Rucker (2001) also consider the alternative cases where fruit and vegetables are priced per unit (by the each) or per pound. Pricing fruit per pound eliminates the incentive for consumers to sort through goods to find the larger items but forces cashiers to weigh the produce, which is more time consuming than
simply counting it. The authors find that produce is more likely to be sold per pound, when it has a larger variance in size or a larger per pound value. Alternatively, goods are more likely to be sold per unit when the consumers have more heterogeneous valuations of goods.

Beside bundling and explicitly pricing quality traits, sellers may attempt to restrict sorting in other ways. For instance, sellers may use attendants to disperse goods, such as produce at a supermarket, or secretaries to assign tasks to individual workers, such as mechanics assigned to cars or doctors assigned to patients. Supermarket and retail stores may reserve a portion of a good’s inventory from the store shelf to decrease the benefit from sorting and thereby discourage the practice. Sellers may deliver goods to buyers rather than allowing consumers to select goods. For example, lumber sold in retail outlets is occasionally delivered directly to a car to discourage customers from sorting the lumber for defects.

Prohibitions or other impediments to sorting, however, are far from universal and sorting is often encouraged, even though it potentially reduces the value of the remaining distribution of quality. For example, premium grocery stores and specialty markets, such as pick-your-own strawberry patches, encourage sorting but may also charge consumers a premium for the opportunity to select a higher quality good. Sorting and quality inspection of used goods, such as second hand clothing, occurs regularly, even though the goods may be sold at a single price. In these instances, sellers may be unable to inexpensively identify quality in their products or credibly commit to given quality level, so that charging price premiums for higher quality goods is impossible. For example, produce may decay while on the shelf only after it has been displayed. Sellers may be
able to improve expected product quality by committing to discard a fixed portion of goods once consumers have sorted out the best items.

Kantor et al. (1997) estimate that 707 million pounds of fresh fruit and 999 million pounds of fresh vegetables or approximately 2% of the total produce went unsold at the retail level in 1995. Some goods have much higher rates of disposal. For instance, disposal rates for potatoes and cabbages, which are very durable, are lower than for bananas, tomatoes and melons. A typical supermarket disposes of approximately 6-7% of very perishable food items outside of the primary market and that this figure may be twice as high at premium supermarkets. Unsold produce may be discounted and sold on a “reduced” rack or donated to a food bank, but often it is simply hauled off as refuse. Often produce is damaged by the act of sorting which, ironically, may create the quality differences that causes sorting to occur. Quality disparity for this reason, considered as well by Leffler, Maliska and Rucker (2001), may either obligate sellers to throw out a portion of their produce or, alternatively, ban sorting altogether.

By committing to discard a certain percentage of goods after they are sorted, sellers raise the goods’ expected value of any individual good purchased. A store that discards a larger percentage of goods will have a higher expected quality. For instance, a store that discards and restocks its shelves when the bottom 20% of the goods remain will have a higher quality than a store that replenishes its shelves only after it worst 5% remain.

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1 Personal communication with Ed Estes, a former produce buyer for a large supermarket chain and a professor in the department of Agricultural and Resource Economics at North Carolina State University
III. **The Vertical Differentiation Model of Demand**

The vertical differentiation model of demand, described by Mussa and Rosen (1977) and Laffont and Martimort (2002) among others, has been used extensively to consider the purchasing behavior of consumers, who vary in their willingness to pay for that quality. In this model, the $i^{th}$ consumer purchasing a single good of quality $q$ and price $p$ receives the following indirect utility:

$$U(\theta_i, q) = \theta_i q - p$$ (1)

The variable $\theta_i$ measures the $i^{th}$ consumer’s preference for quality. Larger $\theta$ values indicate a higher willingness to pay for quality. Across all consumers, the $\theta_i$’s are distributed according to the probability distribution function $f(\theta)$ over the support $(0, m)$. It is assumed that each consumer purchases a single unit of a good, but consumers purchasing multiple units are simply represented as multiple points on the distribution of $\theta$.

The quality of goods is distributed according to the probability distribution function $g(q)$ with mean $\mu$ and a positive support. While sellers (retailers) can adjust wholesale volume at a constant unit wholesale cost of $c$, the quality of their goods is stochastic and independent of the sales volume. For example, a supermarket can order more apples from a supplier, but that does not affect the quality of its apples. The utility specification imposes risk neutrality and allows the purchasing decision to be generalized to be a function of price and expected quality.

III.A. **Bundling to Inhibit Sorting**

When consumers do not sort goods, either due to prohibitions or other discouragements against sorting, they base their purchases on the expected quality of all
goods in the distribution, \( \mu \). Bundling, for instance, reduces the variance of quality across purchased units and consumers may expect to receive the average quality from any given bundle. Other strategies, such as seller prohibitions on sorting, essentially distribute goods randomly which also leads consumers to anticipate receiving an average quality good.

Let \( M \) represent the total number of possible consumers who may purchase a single good and \( N \) represent the total volume of wholesale goods marketed through the retail outlet. For simplicity, assume that wholesale goods are supplied perfectly elastically at cost \( c \). A consumer will purchase a bundled good with expected quality \( \mu \), if the good is priced such that the consumer receives a utility greater than their reservation utility. Assuming that the consumer’s reservation utility is zero, a consumer purchases a good if:

\[
\theta_i \geq \frac{p}{\mu}
\]

The portion of consumers with positive utility is the portion whose \( \theta \) values exceed \( \frac{p}{\mu} \) or \( 1 - F\left(\frac{p}{\mu}\right) \) where \( F(\theta) \) is the cumulative distribution function of \( \theta \). The quantity of the good demanded at price \( p \) is the market size multiplied by the proportion of consumers who purchase, or:

\[
D(p) = M \times \left(1 - F\left(\frac{p}{\mu}\right)\right)
\]

Market clearing implies that the retail quantity demanded, \( D(p) \), is equal to wholesale supply, \( N \). The market clearing price is:

\[
p = \mu F^{-1}(1 - \frac{N}{M})
\]

and seller profits are:
\[ \Pi = \left( \mu F^{-1}(1 - \frac{N}{M}) - c \right) N \]  

(5)

If \( f(\theta) \) is the uniform distribution over \((0, m)\), then the profits are

\[ \Pi = \left( \mu m(1 - \frac{N}{M}) - c \right) N \]  

(6)

Solving the first-order conditions shows that the optimal price, quantity, and profit in equilibrium are:

\[ p' = \frac{M}{2} \]  

(7)

\[ N' = \left( \mu m - c \right) \frac{M}{2\mu m} \]  

(8)

\[ \Pi' = (p' - c)N' = \left( \mu m - c \right) \frac{M^2}{4\mu m} - c \left( \mu m - c \right) \frac{M}{2\mu m} \]  

(9)

Profits are increasing in market size, \( M \), and average quality, \( \mu \), and decreasing in cost, \( c \).

**III.B. Quality Discrimination**

In some cases, it is possible for sellers to identify quality differences in their own product but not to charge separate prices for goods of different qualities due to legal restrictions or high transaction costs. If sellers have specific knowledge of each consumer’s preference for quality, \( \theta_i \), but can charge only a single price, then they can increase profits by strategically assigning qualities across consumers to reduce the price sensitivity of the marginal consumer. Doing so raises the price that sellers can charge all consumers while still clearing the market.

An example would be that of a pastry chef who knows the quality of all her pastries and has intimate knowledge of all her customers. If the chef knows that certain customers are marginal in their decision to purchase – he does not like pastries or he has a small income – she may instead offer that customer the best pastries rather than just randomly selecting them off the shelf. Other customers – who purchase pastries regularly
and can be relied upon to buy the good with less regard for quality—will receive the remaining pastries that are necessarily of a lower quality. The pastry chef might alternatively have offered all customers the option of purchasing low price, low quality or high price, high quality pastries. In this case, the outcome is reversed. Customers with strong quality preferences purchase the higher quality goods, while consumers with weaker preferences for quality purchase the lower quality goods.

In this example, the pastry chef assigns only higher quality goods to consumers who value quality less occurs because she cannot charge customers different prices. The process whereby sellers assign qualities to different consumers to reduce the price sensitivity of marginal consumers is introduced as quality discrimination. Unlike perfect price discrimination, which is efficient in both production and allocation, quality discrimination is extremely inefficient in terms of allocation. Under quality discrimination, the highest quality goods are distributed to consumers who value them least, the consumers with the lowest valuation of quality. At the same time, the lowest quality goods are distributed to consumers who value quality most. This distribution of quality lowers the total gains from trade by reducing allocative efficiency but increases seller profits.

Assume supply is fixed and equal to $N'$, the quantity from equation (8). If those same goods were distributed according to the decision rule, $q(\theta_i)$, which assigns individual consumers goods of quality $q$ based on their $\theta_i$, then the maximization problem for sellers is as follows:

$$\max_{P, q(\theta)} (P - c) * N' \text{ subject to } \theta_i q(\theta_i) - P \geq 0 \quad (9)$$
subject to \[ N' = M \int_{\psi(\theta)}^{\mu} f(\theta)d\theta \]

The first constraint ensures that each consumer receives at least a non-negative utility. The second constraint ensures that the demand for goods equals the supply. From this specification, it is clear that quality is assigned according to the following rule:

\[ q(\theta_i) \geq P / \theta_i \] \hspace{1cm} (10)

This distribution rule implies that higher quality goods should be assigned to consumers who value quality less. The actual price that sellers can charge in equilibrium is bound by the distributions of quality and consumer preferences, but the pattern of assignment remains the same: consumers with stronger preferences for quality receive lower quality products.

A further example of how quality discrimination works involves two types of baseball fans, diehard and fair weather. Diehard fans love seeing baseball games and have high preferences for quality. They will pay $65 for a front row seat close to the field and $35 for an upper deck seat far from the field. Fair weather fans are largely indifferent to seeing baseball games and do not appreciate improvements in quality as diehard fans do. Fair weather fans will pay $40 for a front row seat and $20 for an upper deck seat. If allowed to charge separate prices, sellers make $85 by charging $65 for front row seats and $20 for upper deck seats. If sellers cannot charge separate prices for different seats and they cannot identify fans by type, they will sell all seats at a price of $65, and a portion of the seats will go unsold. Sellers make only $65. If sellers still cannot charge separate seat prices, but they can identify fans by type, they will sell all tickets at a price of $35 dollars, but now fair weather fans will sit in the front row and
diehards will sit in the upper deck. Sellers will now make $70 from ticket sales. Note that the only way to induce fair weather fans to pay the higher price of tickets is to offer them a higher quality. This redistribution of quality, however, assigns the best seats to the fan type that appreciates them least. It is easy to construct opportunities for market improvement such as fair weather fans offering to trade seats with diehards for a price between $30 and $15. But if legal restrictions, information problems (tickets do not state where the seats are in the stadium), or high transaction costs prevent trade in secondary markets, the inefficient distribution remains.

III.C. Market Clearing Under Asymmetric Information with Sorting

Information is asymmetric when consumers can identify product quality but sellers cannot. For instance, in the case of produce, quality differences may not emerge until after goods are displayed as with produce that naturally ripens or decays on display, or produce that degrades in quality as it is handled. Alternatively, information asymmetries may emerge if quality testing is expensive for sellers but not for buyers. Consumers may also be better able than sellers to identify product quality when the goods have a large fashion element, such as used clothing at thrift stores.

Identifying quality is costly to consumers due to the time and effort required to sort goods. When the $i^{th}$ consumer sorts goods they incur the cost $\lambda_i$. For simplicity, $\lambda_i$ is assumed to be bounded to fall in the range of 0 to $l$ and fixed, which eliminates complicating factors such as intensity or duration of search. Consumers are defined by their preference for quality, $\theta$, and their sorting costs, $\lambda$, which are distributed across all consumers according to the joint distribution of $f(\theta, \lambda)$. 
Consumers who sort incur the cost of sorting to receive a higher quality good. It is assumed here that consumers who sort goods base their purchase only on an unbiased expectation of quality. For example, if 10% percent of consumers sort goods, those consumers would receive a good with the expected quality of the top 10% and the 90% of consumers who did not sort would receive the expected quality of the bottom 90%. Let $q_s^e$ be the expected quality of sorted goods, $q_n^e$ be the expected quality of goods that are not sorted, and $\pi$ be the proportion of buyers who sort. The expected quality of the upper $\pi$ portion of the quality distribution is $q_s^e$. The expected quality of the lower $1-\pi$ portion of the quality distribution is $q_n^e$. Let the term $\Delta q^e$ denote $(q_s^e - q_n^e)$, the difference in expected quality between sorted and non-sorted goods. For example, if quality is distributed uniformly between $\mu-n$ and $\mu+n$, then the expected quality of sorted goods, $q_s^e$, is equal to $\mu+(1-\pi)n$; the expected quality of non-sorted goods, $q_n^e$, is equal to $\mu-\pi n$; and the difference in expected quality between the two, $\Delta q^e$, is equal to $n$. This example is depicted in Figure (1).

Consumers who are permitted to sort goods face three options: buying a good and sorting for quality, buying a good and not sorting for quality and not purchasing a good. For a consumer to purchase a good, he must expect to receive a utility that exceeds their reservation utility, a property referred to as the individual rationality (IR) constraint within adverse selection models (Laffont and Martimort, 2002). Given that a consumer makes a purchase, he will sort goods only if he expects to receive a utility from sorting greater that from not sorting, a property referred to as the incentive compatibility (IC) constraint. Consumers who buy and sort goods always have the option of buying and not
sorting. If they were to buy the lower quality non-sorted good, they would receive a positive consumer surplus (that is, their indirect utility would exceed always exceed their reservation utility.) For this reason, the IR constraint is not binding for consumers choosing the high-quality sorted good. The surplus that these consumers receive is known as their information rent. It is binding for marginal consumers buying the low-quality non-sorted good because sellers price these goods to capture the entire consumer surplus. The binding IR constraint is:

\[ \theta_i \geq \frac{P}{q_n^e} \]  

(IR) \hspace{2cm} (13)

The incentive compatibility constraint for consumers buying high quality sorted goods is:

\[ \theta_i q_s^e - P - \lambda_i \geq \theta_i q_n^e - P \]  

(IC) \hspace{2cm} (14)

\[ \theta_i \geq \frac{\lambda_i}{\Delta q^e} \]  

(IC) \hspace{2cm} (15)

The incentive compatibility constraint shows that only the \( \pi \) portion of buyers whose \( \theta_i \) exceed \( \lambda_i / \Delta q^e \) will buy sorted goods. The remaining (1- \( \pi \)) portion of consumers have \( \theta_i \) values greater than \( P / q_n^e \) to ensure that they receive a positive surplus, but less than \( \lambda_i / \Delta q^e \) since they do not sort.

**III.D. Comparing the Sorting and Bundling Equilibriums**

In considering whether to allow or discourage consumer sorting, sellers compare the price associated with the level of quantity demanded under each system. Other things equal, if the quantity demanded under sorting is higher than that when sorting is prevented, then sellers allow consumers to sort.
Figure (2) divides the joint distribution of $\theta$ and $\lambda$ into several regions implied by Equations (13) and (15). If sorting is permitted, buyers with $\theta$ values greater than $\lambda/\Delta q^e$ purchase and sort goods, while buyers with $\theta$ values between $\lambda/\Delta q^e$ and $p/q_n$ still purchase goods, but do not sort. If goods are bundled (so that expected quality is $\mu$), only buyers with $\theta$ values greater than $p/\mu$ purchase the product. When sorting is allowed, consumers in regions II, III, IV, and V purchase the good. Alternatively, when sorting is prohibited through bundling, buyers in regions I, II, III, and V purchase the product. If quality is distributed uniformly between $(\mu-n)$ and $(\mu+n)$ as was previously assumed, then $\Delta q^e$ is $n$ and $q_n^*$ is $\mu-n$. Figure (2) can then be represented in Figure (3).

The proportion of consumers sorting goods, $\pi$, has thus far been treated as exogenous. In fact, this variable can be specified as the ratio of consumers in regions I, II, and III, the sorters, to those in regions I, II, III, and V. So, the proportion of sorters can be specified as:

$$\pi = \frac{\int_0^{\lambda/\Delta q^e} \int_0^{(\mu-n)} f(\theta, \lambda) d\lambda d\theta}{\int_0^{\lambda/\Delta q^e} \int_0^{(\mu-n)} f(\theta, \lambda) d\lambda d\theta + \int_0^{(\mu+n)} f(\theta, \lambda) d\lambda d\theta} = \frac{\text{Regions I, II, and III}}{\text{Regions I, II, III and V}}$$

Even under fairly simple parameterizations of $f(\theta, \lambda)$ and $g(q)$, obtaining an explicit analytic specification of $\pi$ is confounded by the fact that expected quality levels are themselves functions of $\pi$. For example, in the uniform distribution case stated above,

---

2 Figure (2) deals with the case where $l$ is less than $\Delta q^e$. A graph depicting the opposite case is in Appendix A.
the solution to π is the root of a cubic equation which, although analytically tractable, is difficult to interpret\(^3\).

Even without an explicit solution for π, it is still possible to consider whether allowing or discouraging sorting increases demand in a partial equilibrium setting. Assume that the price of goods is set equal to \( p' \) the profit maximizing price under bundling, which is described in Equation (4). At this price the quantity demanded, which is the area of regions II, III, IV and V multiplied by the market size, \( M \), is equal to the wholesale supply, \( N \). If the probability mass of regions I and IV are equal, then the quantity demand is equal when sorting is both allowed and discouraged. If the probability mass of region I exceeds region IV, then more customers are gained by allowing sorting than lost. This excess demand implies that the market will clear at a higher price when sorting is allowed than when it is prohibited. Alternatively, if region I is smaller than region IV, then larger numbers of customers are lost than gained when sorting is allowed. In this case, the market will clear at a lower price when sorting is prohibited or discouraged, such as when fruit is bundled.

When quality is distributed uniformly, an increase in \( n \) indicates that quality is more variable. In figure 3, as \( n \) increases, the slope of \( \frac{\Delta}{n} \) flattens and the horizontal line \( \frac{p}{\mu - m} \) shifts upward. Both of these effects make region V smaller relative to region I implying that a larger percentage of consumers will sort when allowed to do so. The intuition for this result is that sorting is a fixed cost, but the returns to sorting increase as quality becomes more variable.

\(^3\) See Appendix B for discussion of specifics and an overview of the difficulties using the cubic root method.
The sorting strategy acts as a quality discrimination mechanism because it assigns higher quality sorted goods to consumers who value quality little (the region I consumers) while assigning lower quality goods to consumers who value them more (the region V consumers). The seller can only accomplish price discrimination by exploiting the fact that buyers in region I have a low cost to sorting. By sorting goods, these consumers receive a higher expected quality and are now willing to purchase a good that they would not if it were merely average quality. By sorting goods, however, they truncate the quality distribution offered to buyers not sorting. In the absence of a price change, consumers in regions IV and V are worse off, consumers in region I are better off, and consumers in regions II and III face ambiguous changes in welfare when sorting occurs. Buyers in regions IV and V who have a strong preference for quality and high cost of sorting prefer sellers to prohibit sorting. Alternatively, buyers in region I who have weak preferences for quality and low cost of sorting prefer to be allowed to sort goods.

Even though seller revenue may increase under sorting, it is likely to reduce the consumer surplus for three reasons. First, sorting diverts goods to consumers in region I, who have weak values of quality, from consumers in region IV, who have strong values of quality. Second, sorting diverts higher quality goods to consumers in region I from consumers in region V, who value the added quality more. Third, sorting in itself is costly and the total benefits to consumers are necessarily reduced by the sum of the sorting costs. The only scenario in which quality discrimination increases welfare is if it induces a large enough increase in the number of consumers to counteract the other effects.
An analogy can be drawn between allowing sorting as a strategy to quality discriminate and issuing coupons as a strategy to price discriminate. In both cases, an improvement in the consumer’s indirect utility is offered if the consumer undertakes an added cost – the time it takes to clip coupons or the time it takes to sort goods. If all consumers used coupons, the value of trade to consumers would fall by the opportunity cost of clipping the coupons. Issuing coupons is profitable only if consumers’ cost of clipping coupons is negatively correlated with their price elasticity of demand. Then, consumers who receive the lower price are those who would not have purchased under the higher listed selling price that regular consumers receive. Allowing sorting is similar to issuing coupons in that it gives consumers the option of sorting goods to receive a higher quality. Allowing sorting only accomplishes price discrimination if consumers’ cost to sorting goods is negatively correlated with their responsiveness in demand to quality changes\(^4\). It is worth noting, however, that coupon use has no effect on the value of trade to other consumers while sorting by one consumer necessarily reduces the value of trade to other consumers.

**IV. Disposal as Quality Improvement**

The quality discrimination motive for allowing consumer sorting is to raise the market clearing price by increasing the willingness to pay of marginal consumers. Often, however, sellers may opt not clear and instead set prices such that a portion of output goes unsold. If the distribution of quality is extremely skewed towards high quality – so that most goods are high quality and a small percentage of goods are extremely bad – then eliminating a small number of low quality goods may increase average quality substantially. If sellers could identify quality, they would discard these goods to improve

\(^4\) I thank Charles Knoeber for this helpful analogy.
the average quality of the remaining goods rather than attempting to sell their entire supply. However, if sellers cannot identify quality at a low cost, they may allow consumers to sort goods to identify the best quality. Sellers then discard the remaining portion of goods that are necessarily of the lowest quality. By committing to a certain level of disposal, sellers raise expected product quality, even if sellers themselves cannot identify quality.

This section identifies conditions in which it is more profitable for sellers to dispose of a portion of their goods once sorting has occurred rather than attempt to sell their entire stock. Disposal is profitable when wholesale costs are lower, when a marginal change in disposal has a large effect on expected quality, and when consumers are more heterogeneous in their preferences for quality. Although computational difficulties prevent a direct comparison of profits in the disposal equilibrium of this section with those of the previous section, conditions can be derived to show that disposal is profitable in certain situations.

In order for sellers to use disposal to increase expected product quality, all consumers must sort goods. Because consumers only have a single consumption option – to buy the goods which they have sorted or to not buy goods at all – only the individual rationality (IR) constraint binds. Letting $q^*_d$ represent the expected quality of goods under disposal, consumers purchase goods, if they receive a positive utility in purchasing so that:

\[ \theta q^*_d - \lambda - p \geq 0 \]  
\[ \theta \geq (\lambda + p)/q^*_d \]

where $\theta$ represents the disposal decision, $\lambda$ represents the wholesale cost, and $p$ represents the purchase price. These inequalities ensure that disposal is profitable in the context of the model.
Figure (4) shows the regions of the \( f(\theta, \lambda) \) distribution that correspond to whether consumers purchase goods under free disposal. Let \( \delta \) be the percentage of goods that are sold and \( (1 - \delta) \) be the percentage of goods thrown out. As disposal increases (so that \( \delta \) decreases), the expected quality of goods sold increases (\( q^e_\delta \) increases.) Because disposal raises the expected quality, \( q^e_\delta \) is greater than the average quality if all goods are sold. For example, if quality is uniformly distributed in the range \((\mu + n, \mu - n)\), the expected quality of goods under disposal, \( q^e_\delta \), is \( \mu + (1 - \delta)n \).

Assuming that quality preferences and sorting costs are distributed joint uniformly as in the previous section, the demand for goods under free disposal is simply the area of the upper left hand portion of the distribution in Figure (4) multiplied by \( M \), the market size, or \( \frac{Mq^e_\delta}{2ml} \left( m - \frac{P}{q^e_\delta} \right)^2 \).

Again, letting \( c \) represent wholesale costs, sellers maximize profits which are:

\[
\Pi = (\delta P - c) \times N 
\]  

(19)

Equation (20) represents the supply and demand equality where the right hand side is demand and the left hand side is supply, which is the wholesale volume, \( N \), multiplied by the portion of wholesale volume that is actually sold, \( \delta \).

\[
\delta N = \frac{Mq^e_\delta}{2ml} \left( m - \frac{P}{q^e_\delta} \right)^2 
\]  

(20)

Solving for price in equation (20) yields:

\[
P = \frac{q^e_\delta m}{\sqrt{\frac{2ml\delta Nq^e_\delta}{M}}} 
\]  

(21)

Substituting the price in equation (21) back into equation (19) enables profit to be expressed solely as a function of wholesale quantity, \( N \), and expected quality, \( q_\delta \). Sellers
optimize profits in a partial equilibrium by solving the first order condition with respect to $N$ assuming a fixed level of $\delta$. Note that $q_d'$ is constant, since increasing the amount sold, $N$, does not influence expected quality:

$$\frac{\partial \Pi(N, \delta)}{\partial N} = \delta P - c + N\delta \frac{\partial P}{\partial N} = 0 \quad (22)$$

$$\frac{\partial \Pi(N, \delta)}{\partial N} = \delta \left( q_d' m - \sqrt{\frac{2ml\delta q_d'}{M}} \right) - c + \delta N \left( - \sqrt{\frac{\delta mlq_d'}{2NM}} \right) = 0 \quad (23)$$

$$N^D = \frac{2M}{9\delta q_d'm} \left( q_d' m - \frac{c}{\delta} \right)^2 \quad (24)$$

Prices under disposal are obtained by substituting $N^D$ back into the equation (21):

$$P^D = q_d'm - \sqrt{\frac{2ml\delta q_d'}{M}} \sqrt{\frac{2M}{9ml\delta q_d'} \left( q_d'm - \frac{c}{\delta} \right)^2} \quad (25)$$

$$P^D = q_d'm - \frac{2}{3} q_d'm + \frac{2}{3} \frac{c}{\delta} = \frac{1}{3} q_d'm + \frac{2}{3} \frac{c}{\delta} \quad (26)$$

Finally, substituting $P^D$ and $N^D$ into equation (19) yields profits as a function of the still unknown variables $q_d$ and $\delta$:

$$\Pi^D = \left( \delta \left( \frac{1}{3} q_d'm + \frac{2}{3} \frac{c}{\delta} \right) - c \right) \frac{2M}{9ml\delta q_d'} \left( q_d'm - \frac{c}{\delta} \right)^2 \quad (27)$$

$$\Pi^D = \frac{2M}{27ml\delta q_d'} \left( q_d'm - \frac{c}{\delta} \right)^3 \quad (28)$$

Because the known distribution of quality is invariant to wholesale volume, $N$, the expected quality under free disposal, $q_d$, is exactly defined if $\delta$ is known. Profits in equation (28) are solely a function of $\delta$. The optimal level of disposal, $\delta^D$, is obtained by
solving the first-order conditions of the profit function. The solution is shown in equation (30):

\[
\frac{\partial \Pi^D}{\partial \delta} = \frac{2M}{27ml} \left( q_d m - \frac{c}{\delta} \right)^2 \left( -\frac{q_d'}{q_d^2} \left( q_d m - \frac{c}{\delta} \right) + \frac{1}{q_d} \left( q_d' m + \frac{c}{\delta^2} \right) \right) = 0 \tag{29}
\]

\[
\delta^D = \frac{c}{2q_d m} \pm \frac{1}{q'} \sqrt{\frac{c^2 q_d'^2}{4q_d^2} \frac{m^2}{m} - \frac{6q_d' c}{m}} \tag{30}
\]

The variable \( q_d' \) is the marginal effect of a change in disposal on expected quality or \( \frac{\partial q_d}{\partial \delta} \).

Both \( q_d \) and \( q_d' \) are functions of \( \delta^D \), making this an imperfect solution for analytic purposes. Again, assuming that \( q \) is distributed uniformly between \( \mu-n \) and \( \mu+n \), an explicit solution for \( \delta \) is the solution to equation (32).

\[
\frac{\partial \Pi}{\partial \delta} = -cn \delta - 2nm \delta^2 (\mu + (1-\delta)n) + 3c(\mu + (1-\delta)n) = 0 \tag{31}
\]

\[
2nm \delta^3 - (2n^2 m + 2nm\mu) \delta^2 - 4c \delta n + 3c \mu + 3cn = 0 \tag{32}
\]

Unfortunately, like the sorting equilibrium, the optimal level of disposal is the root to a cubic equation, and, therefore, difficult to interpret analytically.\(^5\)

While an exact analytical solution for the profit maximizing level of disposal is intractable, it is possible to find conditions in which some disposal is profitable. As long as the marginal effect of \( \delta \) on profits is negative when all goods are sold, disposal will increase profits. In terms of equation (29), disposal increases profits if \( \frac{\partial \Pi^D}{\partial \delta} < 0 \) when no disposal occurs (\( \delta = 1 \)). In this case, the marginal effect of a change in disposal is:

\[
\frac{\partial \Pi^D}{\partial \delta} = \frac{c \cdot q_d'}{q_d} + 2q_d' m + 3c \leq 0 \tag{33}
\]

\(^5\) See Appendix C for the exact derivation of equation (31)
If the derivative in equation (32) is negative, then some disposal is profitable. In relation to wholesale costs, \( c \), some disposal is profitable so long as:

\[
c \leq 2q'_d q_d m \left( q'_d + 3q_d \right)^{-1}
\]  

(34)

Unsurprisingly, disposal is more likely to be profitable when wholesale costs, \( c \), are lower or when the marginal effect of disposal on quality, \( q'_d \), increases. Additionally, as \( m \) increases, disposal is more likely. Since \( m \) can be interpreted as the range of consumer preferences for quality, this suggests that disposal is more profitable as consumer preferences become more heterogeneous.\(^6\)

The optimal level of disposal expressed in equation (29) can alternatively be expressed in terms of the elasticity of the equilibrium quantity under disposal with respect to disposal rates, \( \varepsilon_{N,\delta} \), and elasticity of price with respect to disposal rates, \( \varepsilon_{P,\delta} \).\(^7\)

\[
\frac{\partial \Pi}{\partial \delta} = (\delta P - c) \frac{\partial N}{\partial \delta} \frac{\delta}{N} + \left( 1 + \frac{\delta}{P} \frac{\partial P}{\partial \delta} \right) \delta P = 0
\]

(35)

\[
\delta^D = \frac{c}{P} \left( \frac{\varepsilon_{N,\delta}}{\varepsilon_{N,\delta} + \varepsilon_{P,\delta} + 1} \right)
\]

(36)

The terms \( \varepsilon_{N,\delta} \) and \( \varepsilon_{P,\delta} \) can both be expected to be negative. The quantity elasticity, \( \varepsilon_{N,\delta} \), is negative because when sellers discard fewer goods (\( \delta \) increases), they require less in wholesale supply (\( N \) decreases). The price elasticity, \( \varepsilon_{P,\delta} \), is negative because as sellers discard fewer goods (\( \delta \) increases), the expected product quality drops and the price falls (\( p \) decreases). Notice that as the price-cost ratio increases (\( c/p \) approaches zero), disposal increases (\( \delta^D \) approaches zero). Disposal increases the price

\(^6\) While increasing \( m \) increases the range of consumer tastes, it also increases the average consumer preference for quality. This result must be qualified by noting the difficulty in distinguishing which effect is driving the result.

\(^7\) See Appendix D for the exact derivation of equation (35)
firms can charge by raising quality at the cost of unsold wholesale supply. Disposal is cheaper when wholesale goods are less expensive. When price is very responsive to changes in disposal rates \((\varepsilon_{p,\delta} \text{ is large in absolute value})\), the level of disposal also increases \((\delta^D \text{ approaches zero})\).

V. Conclusion

Barzel argues that sellers will bundle goods to prevent consumers from sorting goods because sorting only dissipates the gains from trade in aggregate. This paper demonstrates that when information is asymmetric and consumer’s cost of sorting goods is correlated with their preferences for quality then sorting may allow sellers to quality discriminate. Like price discrimination, quality discrimination reduces the consumer surplus. When sellers are unable to charge multiple prices for their goods, they quality discriminate by exploiting the fact that consumers with weak preferences for quality have low sorting costs. Consumers who sort goods receive higher qualities, even though they have weaker quality preferences.

Testing the quality discrimination explanation of consumer sorting may prove difficult empirically, as the information asymmetries affecting sellers are also likely to influence researchers. One potential empirical test may leverage the fact that vertical differentiation models often assumed that incomes and consumer preferences for quality are closely correlated and high income consumers are similarly likely to have higher sorting costs. For example, older shoppers with limited incomes and low sorting costs will receive better goods, since they are more likely to sort goods. Young shoppers with strong preferences for quality and high sorting costs will receive lower quality goods, because they do not sort goods.
This paper also predicts several conditions in which disposal can be used by producers to improve the expected quality of goods. First, retailers are more likely to use disposal to improve the quality of goods with low wholesale costs than of those with high wholesale costs. Second, retailers are more likely to use disposal when a small increase in disposal greatly increases the expected quality of a good. In this setting, disposal is more profitable when the quality distribution is skewed towards higher quality goods. Third, disposal is more likely to be used when there is a larger variance in consumer types. Again, strength of quality preferences is often assumed to be positively correlated with income level. In such a case, this paper then predicts that as the range of consumer incomes rises, sellers are more likely to use disposal to increase quality. This prediction is consistent with the casual observation that stores with wealthier shoppers are more likely to allow consumer sorting and to discard larger portions of their wholesale purchases. Furthermore, prices are higher in grocery stores with wealthier shoppers, which is consistent with this paper’s finding that disposal is more common when prices are higher. While disposal might initially be considered waste from the standpoint of goods going unused, this paper shows that disposal may be viewed as a quality improvement mechanism and that disposal might be planned by retailers.
VI. Bibliography


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Appendix A. - The case where $l$ is less than $\Delta q^e$

If $l$ is less than $\frac{m}{n\Delta q^e}$, Figure (1) is now represented by Figure (A):

**Figure (A) – Division of Consumers Under Sorting**

Appendix (B) - Cubic Roots and Solving for $\pi$ and $\delta$

This discussion is drawn from Weisstein (1999). The solutions to $\pi$ and $\delta$ in equations (16) and (32) both require solving a cubic equation. These equations are respectively represented in equations (B1) and (B2) below.

\[
\left\{\pi^3\left(mn^2(1+Pn) - \pi^3\left(n(2\mu - Pm - m^2n^2)\right) + \pi\left(m^2\mu + 2n\mu + P(n^2 - m\mu - n^2m\mu) - m^2\mu^2\right)\right) = 0 \right. \quad \text{(B1)}
\]

\[
2mn\delta^3 - (2n^2m + 2nm\mu)\delta^2 - 4c\delta^3 + 3c\mu + 3cn = 0 \quad \text{(B2)}
\]

Unfortunately, this method yields analytic results that are very difficult to interpret. A summary of the method of finding cubic roots is shown for the following equation:

\[
z^3 + a_2z^2 + a_1z + a_0 = 0 \quad \text{(B3)}
\]
Substituting $z = x - \frac{1}{3}a_2$, convert equation (B1) to:

$$x^3 + \left( a_2 + \frac{1}{3}a_1^2 \right)x^1 - \left( \frac{1}{3}a_1a_2 - \frac{2}{37}(a_2)^3 - a_0 \right) = 0$$  \hspace{1cm} (B4)

Setting $p = a_2 + \frac{1}{3}a_1^2$ and $q = \frac{1}{3}a_1a_2 - \frac{2}{37}(a_2)^3 - a_0$ yields:

$$x^3 + px^1 = q$$  \hspace{1cm} (B5)

Substituting $x = w - \frac{\delta}{3w}$ converts equation (B3) into:

$$\left( w^3 \right)^2 - q\left( w^3 \right) + \frac{1}{37}p^3 = 0$$  \hspace{1cm} (B6)

Using the quadratic equation $w^3$ can be solved for as:

$$w^3 = \frac{1}{2}q \pm \sqrt{\left(\frac{1}{2}q \right)^2 - \frac{4}{27}p^3}$$  \hspace{1cm} (B7)

The solution to the cubic roots for $\delta$ can be recovered by substituting back into the equation below.

$$w = \frac{1}{2}\sqrt{\left(\frac{1}{2}q \pm \sqrt{\left(\frac{1}{2}q \right)^2 - \frac{4}{27}p^3} \right)}$$  \hspace{1cm} (B8)

Notice that the solution to the cubic root equation is the cube root of a square root. Given specific values of $n$, $m$, $\mu$, $P$, and $c$, interior solutions to the profit maximizing level of $\delta$ and $\pi$ can be obtained as solutions for equations (16) and (31). However, these solutions are not easily interpreted when $f(\theta, \lambda)$ is joint uniform, unless very restrictive assumptions are made regarding the parameters of the model.

**Appendix (C) - Derivation of Equation (32)**

$$\frac{\partial \Pi}{\partial \delta} = -cn\delta - 2nm\delta^2(\mu + (1 - \delta)n) + 3c(\mu + (1 - \delta)n) = 0$$  \hspace{1cm} (C1)

$$-cn\delta + 2nm\delta^3 - 2n^2m\delta^2 - 2nm\mu\delta^2 + 3c\mu + 3cn - 3c\delta n = 0$$  \hspace{1cm} (C2)

$$2nm\delta^3 - (2n^2m + 2nm\mu)\delta^2 - 4c\delta n + 3c\mu + 3cn = 0$$  \hspace{1cm} (C3)
Appendix (D) – Derivative of Equation (35)

\[
\frac{\partial \Pi}{\partial \delta} = (\delta P - c) \frac{\partial N}{\partial \delta} + \left( P + \delta \frac{\partial P}{\partial \delta} \right) N = 0 \quad (D4)
\]

\[
\frac{\partial \Pi}{\partial \delta} = (\delta P - c) \frac{\partial N}{\partial \delta} \delta N + \left( 1 + \frac{\delta}{P} \frac{\partial P}{\partial \delta} \right) \delta P = 0 \quad (D5)
\]

\[
\frac{\partial \Pi}{\partial \delta} = (\delta P - c) \epsilon_{N,\delta} + (1 + \epsilon_{p,\delta}) \delta P = 0 \quad (D6)
\]

\[
\frac{\partial \Pi}{\partial \delta} = \delta P \left( 1 + \epsilon_{N,\delta} + \epsilon_{p,\delta} \right) = c \epsilon_{N,\delta} \quad (D7)
\]

\[
\delta = \frac{c \epsilon_{N,\delta}}{P \left( 1 + \epsilon_{N,\delta} + \epsilon_{p,\delta} \right)} \quad (D8)
\]
Figure 4-1 - Expected Qualities under Sorting
Figure 4-2 - The Division of Consumers under Sorting
Figure 4-3 - The Division of Consumers under Sorting when Quality is Uniformly Distributed
Figure 4-4 - Division of Consumers under Disposal
Chapter 5

Conclusion
This dissertation adds to the growing body of literature addressing the differentiation of products when consumers are heterogeneous in their quality preferences. Improved quality differentiation and new good introduction has substantial impacts on consumer and producer welfare. The rise of quality certification programs in the 1990s increased consumer welfare by approximately 1.6% of beef expenditure. While consumer welfare increased in this case, the introduction of a new quality grade or the change in a grade standard may reduce consumer welfare in other cases, depending on the placement of grade standards and the distribution of quality. These changes also redistribute the total surplus from trade between producers and consumers. Alternatively, when quality information is asymmetric between sellers and consumers, grading may be impossible. Depending on the distribution of quality preferences and sorting costs, sellers may allow sorting in order to quality discriminate and, thereby, appropriate larger portions of the consumer surplus. Alternatively, in some situations, sellers may allow sorting as a way to improve expected product quality.

Additional empirical verification of the models presented in this dissertation is important work for the future. In the models considered in chapter two, estimating the cost of improving cattle quality would provide important insight into the prospects of future increases in the proportion of quality certified beef and allow for consideration of the producer welfare benefits of certification as well. In chapter three, more complete data sets on the production inputs of individual producers and consumer choice of quality differentiated goods on a household level would allow for the estimation of explicit supply and demand functions. These functions might then be used to conjecture regarding the optimal grading system. In chapter four, observable population statistics
may be adequate instruments for estimating the distributions of consumer preferences for quality and sorting costs in different markets. These estimates might then inform the theoretical predictions of how sellers use bundling and disposal to maximize profits.

As certification and branding decentralize control over quality standards, the paradigm of production of separate commodity goods distinguished only by grade becomes tenuous. Heterogeneity in the quality of agricultural goods often highlights the role of quality information in creating incentives for quality improvement. As agricultural production becomes more oriented toward the creation of final consumer outputs, market coordinators are more closely managing quality in supply chains through contracting and output testing. A result of this increased management, a larger number of more differentiated goods are sold with smaller market shares to more specific consumer groups. Many empirical studies of the introduction of new goods have found that it creates large, often unmeasured, consumer welfare gains. This work corroborates those findings but emphasizes that in many cases, quality differentiation may be associated with a redistribution of the total gains to trade as well.