ABSTRACT

SENGUPTA, BHASWATI Real Options Approach in Migration for two Specific Labor Markets. (Under the direction of Professor John Seater).

This work uses a real options approach to model the migration decision of an individual under very specific labor market conditions where migration is analyzed as a “regime switching” phenomenon. A regime switching model is developed with the possibility of exogenous regime switches, the latter being an innovation of this work.

The first migration decision analyzed is that of an individual considering migration from a rural to an urban labor market that is segmented in nature, consisting of a formal and an informal sector, a common phenomenon observed in many developing countries. A combination of exogenous regime switches are used in the model for an accurate treatment of the “opportunity nature” of finding formal employment once the migrant is in the city. The model also analyzes the value to a migrant of the option to move back and forth between the rural and urban sectors, which is new to the rural-urban migration debate. The exogenous switching formulation developed in this work may be used to model a wide variety of such economic phenomenon where a common dynamic programming problem is augmented to include the possibility of an opportunity arising, that an economic agent may or may not take.

The second problem developed along similar lines concerns the decision of a prospective undocumented Mexican migrant crossing the border to work in the U.S. This model is solved numerically using parameter values obtained from data and qualitative policy prescriptions as suggested by the model are presented. Results suggest that the effectiveness of the INS to modify the probability of apprehension in the interior of the U.S. has a much bigger effect than apprehension at the border in deterring undocumented migration. Also, a decreasing probability of acquiring legal status inside the U.S. does not have a very big effect in deterring migration as compared to increasing border and interior apprehension probabilities or even raising the cost of being an undocumented worker in the U.S.
Real Options Approach in Migration for two Specific Labor Markets

by

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To my late grandmother Indumati Sengupta.
Biography

Bhaswati Sengupta known to everyone as Bonu was born in New Delhi on November 7th, 1973 to Pratima and Arun Kumar Sengupta. She received a Bachelor of Arts in Economics at St. Stephen’s College in Delhi University in 1995. After completing a year of studies at the Delhi School of Economics, she came to the U.S. in 1996 to join the graduate program in Economics at North Carolina State University where she will receive her Ph.D in Economics in August 2003. She is currently working as Assistant Professor of Economics at Grinnell College in Iowa.
Acknowledgements

Firstly, I would like to thank my lucky stars for having the family I do: thank you Ma, Baba, Didi and Steve for being there through this journey and all others.

I would like to thank Dr. Paul Fackler for his immense help at every step of the way in this work. And thank you Dr. Seater for all your advice and patience over the many years this dissertation has taken. Thanks also to Dr. Mitch Renkow for his candid comments and a great sense of humor.

I would like to acknowledge the best friends in the world, for the years of late night/early morning brain-storming and goofing around and everything else. You know who you are, Catherine Skura, Win Leegomanchai, Zulal Denaux and of course, Aaron Hegde, my pillar of strength and best buddy.

Summing up, I would like to thank the Department of Economics for everything I received in exchange for my occasional grumblings about out of state tuition - a lot of human capital, the greatest friends in the world, a chance to meet my future partner in matters of the heart and all other things - Mark Worthington, and a Ph.D in Economics, thanks!
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Chapter 1

Introduction

1.1 A New Approach to Migration

1.1.1 Old vs. New in the Investment Literature

The analogy between the migration decision of an individual and the investment decision of a firm was drawn as far back as Sjaastad (1962). Migration naturally fits into the investment framework as it concerns the decision to incur a present cost in exchange for a stream of future rewards. The “orthodox” theory of investment is based on the Net Present Value Rule, where a firm undertakes an investment if its expected present value exceeds the cost of doing so (assuming a one time cost of investing). Analogously, in the early migration literature, the migration decision was modelled on the simple comparison of the expected present values of incomes between two states, after accounting for the cost of migrating.

One of the fundamental changes that have occurred in modelling investment under uncertainty comes from questioning this simple Net Present Value rule. The work of Dixit and Pindyck (1994) elaborates its deficiencies, namely, this rule implicitly
assumes either of two things:

i) Investment is completely reversible (or there are no sunk costs to making the investment), or,

ii) If investment is irreversible, the opportunity to invest presents itself as a *now or never* proposition.

While some types of investment may meet the above criteria, migration generally does not. Firstly, migration is not completely reversible, even with the possibility of return migration, since the cost of migrating would classify as a sunk cost or one that cannot be recovered. Secondly, while a job offer in the destination region may present itself as a “now or never” proposition, it seems implausible that the opportunity to migrate is lost if not taken in the current period.

Since at least partial irreversibility and the possibility of delay are very real features of most investment opportunities, the foundation of the new work lies on the value of waiting to make the investment and quantifying this value. An analogy is drawn to a financial call option, where the bearer of the option has the right but not the obligation to buy an asset in the future. If a firm decides to exercise the option (invest now), it kills that option (expected value of the “wait and see” alternative). Hence the firm should invest if the discounted sum of expected future rewards exceeds the “full cost” of investing today, the direct cost of investment plus the opportunity cost of exercising the option. Naturally, given irreversibility, more uncertainty in any relevant variable increases the value of waiting.

1.2 The Case of Migration

For the case of migration, irreversibility is only *partial*, if we allow for return migration. Here, in allowing movement across two regions, say, home and abroad (at
the associated costs), we are pricing two options. Upon migration, one receives the stream of rewards and an option to return home. On returning, one gets a stream of rewards and an option to leave again, depending on how labor market conditions pan out. Since in this case the partial irreversibility of moving comes from the sunk costs involved in doing so, and not from the irreversibility of the investment action itself, the questions are a little different, and so is the nature of the solution. The basic intuition behind this is briefly presented below.

The opportunity to return in the future after migrating is an option value. In this case, there are two options, one held in each region (home and abroad). The solution to this problem comprises of the value of being in a particular regime (the “normal returns” augmented by the option value), and associated threshold values that prompt migration away from it, into different regimes. These threshold or “trigger” values associated with the different regimes are of the stochastic state variable whose fluctuations drive migration. One threshold is the wage differential that prompts migration from home to abroad, while the other is the threshold for return migration. The reason for the wedge between the two thresholds is again the partial irreversibility caused by sunk costs of migration. The subtle point is that while any cost to migration forces a wedge between the thresholds, it is the wider implication of irreversibility - a response to the extent of future uncertainty in the wage differential, that affects the size of this wedge. This can be understood by analyzing the behavior of the migrant when the relative wage lies in this wedge between the thresholds. From the way the thresholds have been defined, it is optimal to stay at home or remain abroad if the relative wage indeed falls in this region, so there will be no migration flow either way. This “inactivity” is aggravated by more uncertainty due to the irreversible nature of the decision; a migrant would tend to “wait and see” how conditions turn out before
making the decision to move. This translates into a bigger wedge between thresholds due to the higher indirect cost of more uncertainty. In the absence of any costs to migration, we would have just one threshold value. In this case, if the wage differential is anything but the trigger value, we would expect to see a migration flow, going one way or another.

Modelling migration in this manner is a direct extension of Dixit’s (1989) work on the entry and exit problem of a competitive firm, and was first worked on by Burda (1993) and then by O’Connell (1997). In the entry-exit case, the variable driving the decision process is the price of output, which reflects demand uncertainty. The natural thresholds for entry and exit from “traditional” microeconomics, i.e. the long run average cost and short run minimum average variable cost, respectively, are shown to span a smaller range of prices than those found using the options approach. This implies a bigger range of prices for “inactivity” or staying in a state the firm is currently in, inside or outside of the market. Dixit attributes this difference to the firm’s approach to uncertainty. The former (traditional) approach assumes “static” expectations, where a firm would expect the current price to prevail forever while the latter explicitly takes into account the nature of uncertainty or the stochastic process driving the price of output.

For this work, I will use the same approach as Dixit (1989), with a more general specification of uncertainty, which will be a contribution to existing literature. The basic model will be constructed on the assumption that a migrant may move back and forth between regimes at the associated cost. The innovation of this work would be to add to this kind of regime switching model a possibility of exogenous changes in regime, where one may not choose but is rather, forced to switch regimes. I will also consider a case where the agent may have an opportunity to switch to a different
regime at no cost. The central notion of irreversibility of investment (migration) will be captured in the model by not allowing the migrant to move between certain regimes. The motivation for this kind of specification is that it will allow me to incorporate very specific features of labor markets I am interested in. The general framework could also be used for problems other than migration, that may exhibit similar characteristics.

1.2.1 A Discrete Time Example for Migration

Dixit and Pindyck claim that the “..orthodox theory of investment has not recognized the important qualitative and quantitative implications of the interaction between irreversibility, uncertainty, and the choice of timing.” Along those lines, to illustrate the difference between the orthodox migration rule and this new approach, the following example is presented. This example is of an urban labor market seen often in developing countries. It is typically segmented between the “free entry” informal sector with a lower average wage and a “protected” (through minimum wage regulation) formal sector with a higher average wage. A substantial amount of research has dealt with labor migration from the rural to the segmented urban labor market, but modelling of this migration decision has not seen much change since the 1970’s (specifics are taken up in Chapter 3). The bulk of the documented migration consists of unskilled labor, so human capital plays little role in allocating labor between the informal and formal sectors. According to existing literature “..Migrating workers are essentially participants in a lottery of relatively high paid jobs in the towns” (Stark et al, (1991)). This stochastic nature of the problem is especially interesting, and further emphasizes the need to look more deeply at the effect of uncertainty on the migration decision.
Consider the following: A rural worker makes a wage of 8 per period with cer-

<table>
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<th>PERIOD 1</th>
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<tbody>
<tr>
<td>Rural</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20(F)</td>
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<tr>
<td>Urban</td>
<td>8</td>
<td>15(I)</td>
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<td></td>
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<td>20(F)</td>
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\[ p(f) = 0.1 \]
\[ p(i) = 0.9 \]
\[ p(f) = 0.2 \]
\[ p(i) = 0.8 \]
\[ p(f) = 0.2 \]
\[ p(i) = 0.8 \]

Figure 1.1: Per Period Probabilities and Wages associated with the 3 Sectors

tainty. If he migrates to the city, he is guaranteed informal sector employment where he makes a wage of 8 in the first period. The uncertainty starts in the second period, where the migrant may find formal sector employment with a probability of 0.1 and earn a wage of 20. Otherwise, the migrant stays in the informal sector. Also, the informal sector situation may have become better or worse since period 1, so in period 2, the informal wage could be 1 or 15 with equal probability. If a migrant did make it to the formal sector in the second period, he is assumed to stay there forever, and get a wage of 20 from the second period on. However, if he is forced to stay in the informal sector with a wage of 1 or 15 (depending on how the wage evolved from period 1), he now has a 0.2 chance of graduating to the formal sector and making a wage of 20 forever (the increase in this probability from 0.1 to 0.2 reflects that chances of finding formal work increase with time spent in the city), or stay in the informal
sector with the wage he made in period 2 (1 or 15). All uncertainty is resolved at
the end of the third period, and the migrant keeps the same wage he makes in that
period.

In this example, the uncertainty has two sources. The first comes from the evolu-
tion of the informal sector wage itself while the second comes from a random chance
every period to be selected out of a pool of informal sector workers to move to the for-
mal sector. The wages and associated probabilities are chosen to reflect two features
of this labor market. Firstly, it is always desirable to move to the formal sector if one
gets hired, secondly, the probability of formal sector employment increases with time
spent in the informal sector (attributed to “Network effects”. In the next chapter I
will relax the first assumption and address the second in more detail). For ease of
exposition, the above information is presented in Figure 1.1. \( p(f) \) and \( p(i) \) denote
the probabilities of going to the formal or staying in the informal sector respectively.
The sector corresponding to the wage (in period 2 and 3) is denoted by its initial (R

Given this structure of uncertainty, one can make a comparison between the pre-
dictions of the traditional net present value approach which I will call the Harris-
Todaro approach (one of the earliest works to use that formulation, Todaro(1970))
and the new approach. Under the Harris-Todaro approach,

\[
\text{Expected Net Present Value from the Rural sector} = \frac{8}{1-\rho}
\]

\[
\text{Expected Net Present Value from the Urban sector} = 8 - c + 0.1\left(\frac{20\rho}{1-\rho}\right)
\]

\[+0.9 \left\{ 0.5 \left[ 15\rho + 0.2\left(\frac{20\rho^2}{1-\rho}\right) + 0.8\left(\frac{15\rho^2}{1-\rho}\right) \right] + 0.5 \left[ 1\rho + 0.2\left(\frac{20\rho^2}{1-\rho}\right) + 0.8\left(\frac{1\rho^2}{1-\rho}\right) \right] \right\}
\]

where \( \rho \) is the discount factor that reflects the trade-off for an individual between
consumption in two consecutive periods \(^1\) and \(c\) is the cost of relocating. Using a \(\rho\) of 0.9, the Harris-Todaro approach predicts migration will take place if the net present value from the urban sector exceeds the net present value from the rural sector after accounting for the cost of migration. So migration occurs if \(E(NPV_{URBAN}) \geq E(NPV_{RURAL})\) or \(108.3 - c \geq 80\).

If the migrant has the option to return home after seeing the wage in period 3, the equation will be identical except if wage falls to 1 in that period, the migrant returns home to the rural wage. This kind of a set up was first modelled by Berninghaus and Seifort-Vogt (1991), where the evolution of wages abroad cannot be observed from home. Their contribution is to model migration as an optimal stopping problem, where the worker has the choice of living in a region for the next period, and locate optimally thereafter, or he could receive a terminal payoff by retiring from the decision problem.

The expected net present value from migrating to the urban sector in our case (with the option of returning if conditions turn out unfavorable) then becomes:

\[
\text{Expected Net Present Value from the Urban sector} = 8 - c + 0.1 \left( \frac{20\rho}{1 - \rho} \right) + 0.9 \left\{ 0.5 \left[ 15\rho + 0.2\left( \frac{20\rho^2}{1 - \rho} \right) + 0.8\left( \frac{15\rho^2}{1 - \rho} \right) \right] + 0.5 \left[ 1\rho + 0.2\left( \frac{20\rho^2}{1 - \rho} \right) + 0.8\left( \frac{8\rho^2}{1 - \rho} - c\rho^2 \right) \right] \right\}
\]

So migration occurs if \(E(NPV_{URBAN}) \geq E(NPV_{RURAL})\) or \(128.7 - 1.29c \geq 80\). Notice that the difference in the present values only comes from the last term, which, under the options approach, reflects that a migrant returns home if wage falls to 1 at the associated cost. Since the maximum cost of relocation the migrant is willing to undertake is greater in the second case, it implies he is more willing to migrate.

What drives this result is that in the second case, the migrant also receives an option to return home if future wages fall in the informal sector. The higher the future

\(^1\)For instance, \(\rho\) can be \(1/1 + r\) where \(r\) is the rate of interest.
Table 1.1: Harris-Todaro vs. Option Approach

<table>
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<th>Summarizing, rural to urban migration takes place in the first period under</th>
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<td>i) the Harris-Todaro rule if $c \leq 28.3$</td>
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<tr>
<td>ii) the option approach if $c \leq 37.7$</td>
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uncertainty of informal sector wages, the higher the value of this option. The very real possibility of return is not accounted for in the Harris-Todaro rule, which would predict a lower level of migration.

A third formulation could be a case where a migrant can contemporaneously observe the urban labor market situation from the rural sector. Here a migrant would not only know the informal sector wage at every time but would know if he will get selected by the formal sector upon migration. Obviously, in this case, it might be better to wait in the rural sector and migrate if things look good, or stay home otherwise. This information structure would predict a lower level of migration in period 1, since it is better to migrate after the uncertainty is resolved. This is analogous to the entry decision of a firm modelled by Dixit and Pindyck (1994), where a firm who has all information about the current price of output (where the uncertainty originates) may find that its better to wait and not enter the market even if the current price is higher than average cost. It makes economic sense to use information about the average variation in price to judge whether this is an “outlier” occurrence.

As pointed out by O’Connell (1997), this crucial difference in the information set of prospective migrants has an important bearing on net migration flows. The few works on migration that make use of the options approach assume one or the other about
this information set, i.e., foreign wages are either “locally” (seen only upon migration) or “remotely” observable (seen contemporaneously from home).

The special labor market question posed above has both characteristics. While a migrant may contemporaneously see the evolution of the informal sector wage, the possibility of being “picked” by the formal sector arises only upon migration.
Chapter 2

General Theory

This chapter begins with an introduction/overview of the mathematical techniques used throughout this work. It then proceeds to develop a regime switching model with the inclusion of exogenous regime switches that force the agent to move to a different regime at no cost. This model is laid out in very general terms but is related to migration in the discussion; this is done to make the graduation to its application to specific labor markets more seamless. This general model is a skeleton for chapter 3, where the specifics of an urban labor market in a developing country are used to fill out the discussion and narrow the scope of the model to approximate more closely the features of that labor market. The exogenous regime switching framework is used to capture an “opportunity aspect” where the agent may have an opportunity to move to a different regime at no cost, which he may turn down. The model used in chapter 4 for the case of undocumented Mexican migration is similar to this model (and simpler\footnote{What makes the Mexican migration model simpler is that there is no “opportunity aspect” to capture through the exogenous switches. As will be clearer in Chapter 3, an opportunity open for an amount of time allows the migrant to switch to another regime at no cost. This is an opportunity that the migrant may refuse to take, however. The exogenous switches in the Mexican migration case are ones that “force” a migrant to switch regimes at no cost and provide no choice, for example,}) so all of the theoretical discussion from this chapter carries over to
chapter 4.

2.1 A Brief Description of Mathematical Tools and Applications in Labor Economics

In this section a brief introduction to concepts/tools used in the theory are presented, since these techniques are still new to application in Labor Economics.

2.1.1 Brownian Motion

Consider a random walk in discrete time:

\[ x_t = x_{t-1} + \epsilon_t \]  

where \( \epsilon_t \sim \text{i.i.d } N(0, 1) \)

so that the shocks are Standard Normal. If the process starts at \( x_0 = 0 \), it follows that

\[ x_t = \epsilon_1 + \epsilon_2 + \ldots \epsilon_t \quad \text{so} \quad x_t \sim N(0, t). \]

Notice that

\[
E(x_k - x_s) = 0 \quad \text{and} \quad Var(x_k - x_s) = Var(x_k) - Var(x_s) = k - s \quad \text{for} \quad k \geq s.
\]

(2.1)

We would like to construct an analogous process in continuous time. We begin by dividing the time periods into two equal sub-divisions. So the change from \( x_{t-1} \) to \( x_t \) can be seen as the following:

\[
x_t - x_{t-1} = (x_t - x_{t-\frac{1}{2}}) + (x_{t-\frac{1}{2}} - x_{t-1})
\]

(2.2)

being deported back to Mexico from the U.S. This formulation is simpler to handle technically, as we shall see in Chapter 4.

\(^2\text{This exposition combines Hamilton (1994), Dixit (1993) and Trigeorgis (2000)}\)
We can write the above as \( \varepsilon_t = e_{1t} + e_{2t} \), so that the shock \( \varepsilon_t \) can be seen as the sum of two independent Gaussian variables where:

\[
e_{it} \sim i.i.d N(0, \frac{1}{2}) \tag{2.3}
\]

This is by using the properties in 2.1 and associating \( e_{1t} \) as the change between \( x_{t-1} \) and \( x_{t-\frac{1}{2}} \) and \( e_{2t} \) as the change between \( x_t \) and \( x_{t-\frac{1}{2}} \).

By extension if we partition, the interval \( x_t - x_{t-1} \) into \( N \) subperiods, \( x_t - x_{t-1} = e_{1t} + e_{2t} \ldots e_{Nt} \) with \( e_{it} \sim i.i.d N(0, \frac{1}{N}) \), we have a finer grid with the same properties as above. The limit as \( N \to \infty \) is a continuous-time process known as standard Brownian motion.

Non standard Brownian motion has a marginally different form, where given an initial \( x_0 = 0 \), \( x_t \sim N(\mu t, \sigma^2 t) \). \( \mu \) and \( \sigma \) are known as the drift and volatility of the process. We redefine the above random walk as the stochastic process for a variable \( z \), where now for a period of length \( \Delta t \) the change between \( z_t \) and \( z_{t+\Delta t} \) is chosen to be \( \varepsilon_t \sqrt{\Delta t} \) where \( \varepsilon_t \sim i.i.d N(0, 1) \). For \( \Delta t = 1 \), we get back the form of the first equation. For \( \Delta t = \frac{1}{2} \), we would get \( (z_t - z_{t-\frac{1}{2}}) \sim N(0, \frac{1}{2}) \) as in 2.3, etc. If we let the size of the interval go to zero (analogous to \( N \), the number of subperiods \( \to \infty \)), we would write it as the following: As \( \Delta t \to 0 \), \( dz = \varepsilon \sqrt{dt} \), which is the Standardized Brownian motion or Wiener process. Just as we could write a variable \( x \sim N(\mu, \sigma^2) \) as \( x = \mu + \sigma z \) where \( z \) is standard normal, we can write the following:

\[
dx = \mu dt + \sigma dz \tag{2.4}
\]

where \( z \) is a standardized Brownian motion whose increments are mean zero, with variance \( dt \). \( \mu \) is known as the drift term and \( \sigma \) as the volatility term of the process.

The importance of each is better understood if we consider that the mean of \( x_t - x_0 \) is \( \mu t \) while the standard deviation is \( \sigma \sqrt{t} \). For large \( t \), we have \( \sqrt{t} \ll t \), so the trend
or drift term dominates in the long run, while for small t, we have \( t \ll \sqrt{t} \), or the volatility dominates the short run (Dixit, (1993)).

### 2.1.2 Itô’s Lemma

Itô’s Lemma applies to non random functions of variables that follow Brownian motion\(^3\). Suppose \( x \) follows Brownian motion, and \( y = F(x, t) \), and suppose we want to know how changes in \( x \) affect \( y \). Ordinary rules of calculus don’t work here for the following reason: If we take a Taylor’s Series expansion,

\[
dF = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial x^2} (dx)^2 + \frac{1}{6} \frac{\partial^3 F}{\partial x^3} (dx)^3 \ldots ,
\]  

(2.5)

In ordinary calculus we are left with only the first two terms as the higher order terms disappear in the limit. Considering the third term, if we expand \((dx)^2\), we get\(^4\):

\[
(dx)^2 = \mu^2(dt)^2 + 2\mu\sigma(dt)^{\frac{3}{2}} + \sigma^2 dt
\]  

(2.6)

The first two terms involve \((dt)^2\) and \((dt)^{\frac{3}{2}}\), which go to zero faster than \( dt \), but we are still left with the third term in the R.H.S of equation 2.6\(^5\). A quick comparison is made in Table 2.1.

---

\(^3\)It applies more generally to non random functions of variables that are themselves functions of Brownian motion.

\(^4\)This is obtained from the following steps: i) \((dx)^2 = \mu^2(dt)^2 + 2\mu\sigma dt dz + \sigma^2(dz)^2\) which just uses equation 2.4 and, ii) Replacing \( dz \) with \( \sqrt{dt} \). What is most important here is the third term on the R.H.S, \( \sigma^2(dz)^2 \). The justification for replacing \( dz \) with \( \sqrt{dt} \) and not \( \epsilon \sqrt{dt} \) is the following. In the discrete case, we get \((dz)^2 = \epsilon^2 \Delta t\), where \( E(\epsilon) = 0 \) and \( Var(\epsilon) = 1 \). This term is random with mean \( E(\epsilon^2 \Delta t) \) and variance \( E(\epsilon^4 \Delta^2 t) - [E(\epsilon^2 \Delta t)]^2 = (\Delta^2 t)(\epsilon^4 - 1) \). This variance goes to zero faster than \( \Delta t \), so as \( \Delta t \to 0 \), we can replace this term with its mean, \( E(\epsilon^2 dt) = dt \).

\(^5\)The term \((dx)^3\) and higher orders of \( dx \) in the Taylor series (2.5) when expanded only contain \( dt \) terms raised to powers greater than 1. This implies they all go to zero faster than \( dt \) and can be ignored.
2.1.3 The Importance of Itô’s Lemma for Problems with Uncertainty

For simplicity, if we assume i) \( y = F(x) \) (so that \( \frac{\partial F}{\partial t} = 0 \)), and ii) \( \mu = 0 \) or there is no drift term in 2.4, we see that \( E(dx) = \sigma E(dz) = 0 \). Considering \( E(dF) \) however, after substituting 2.4 into equation 1.5, we see that \( E(dF) = \frac{1}{2} \sigma^2 \frac{\partial^2 F}{\partial x^2} dt \neq 0 \).

This comes from Jensen’s Inequality, which implies \( E(dF(x)) \) will exceed \( F(E(dx)) \) if \( F(x) \) is convex and vice-versa if \( F \) is concave. Why is this important? It plays a role in dealing with changes in uncertainty. Even though the expected value of \( x \) may not change, \( E(F(x)) \) could change if we increase the variance of \( x \). Jensen’s inequality plays no role if we use ordinary calculus, since the last term in equation 1.5 is not present.

2.1.4 The Idea behind Value Matching and Smooth Pasting

As introduced in section 1.2, integral to our migration problem, is the concept of threshold or “trigger” values of the state variable. When the stochastic state variable (say the price of output or wage) hits one of these switch points, it prompts the agent to update his choice variable, for example, an active firm shuts down or an individual migrates to a certain region. More technically, at the threshold, an agent exchanges
one value function for another at the associated cost. Most importantly, these thresholds are determined endogenously in the problem. For this reason, questions of this nature are known as free boundary problems. In order to simultaneously “fix” these free boundaries and solve for our value functions, we need two conditions. These are the Value Matching and Smooth Pasting conditions, described intuitively below.

Value matching simply imposes continuity at the threshold. Consider two value functions $V_A(x(t))$ and $V_B(x(t))$ where $x(t)$ is our stochastic state variable. The triggers for moving from A to B and B to A are $x^*_{AB}$ and $x^*_{BA}$ respectively. The costs of doing so are $C_{AB}$ and $C_{BA}$ respectively. The value matching condition would then imply that:

$$V_A(x^*_{AB}) = V_B(x^*_{AB}) - C_{AB} \text{ and } V_B(x^*_{BA}) = V_A(x^*_{BA}) - C_{BA}$$

which shows the indifference between the two (available) alternatives at a boundary.

Smooth Pasting imposes that the value functions meet tangentially at the thresholds, or that $V'_A(x^*) = V'_B(x^*)$. Dixit and Pindyck prove this by contradiction. In a nutshell, if they don’t meet tangentially, they must meet at a kink at the threshold. If there is an upward kink (see right illustration in Figure 2.1.4), then by continuity, say at threshold $x^*_{AB}$, as we move a little to its right, we see that $V_A(x^*_{AB}) > V_B(x^*_{AB}) - C_{AB}$, which violates the definition of this threshold. An analogous argument is made for a downward kink. These conditions are illustrated in the figure below and the next section applies these concepts along with the usual dynamic programming techniques.

---

6See Dixit and Pindyck (1994), Appendix C, Chapter 4.
\[ V_A(x_{AB}) = V_B(x_{AB}) - C_{AB} \]
\[ V_A'(x_{AB}) = V_B'(x_{AB}) \]
\[ V_B(x_{BA}) = V_A(x_{BA}) - C_{BA} \]
\[ V_B'(x_{BA}) = V_A'(x_{BA}) \]

Figure 2.1: Value Matching and Smooth Pasting

2.1.5 Applications in Labor Economics

Applying the real options approach to various problems in labor economics is a relatively new but growing phenomenon. A brief list of citations of these works is presented in chronological order.

The following literature uses the real options approach to model migration. Two works are direct applications of the “the value of waiting” to make an investment modelled by Dixit and Pindyck. Burda (1993) models migration under uncertainty with the wage differential following a stochastic process like the one mentioned above. Assuming that at every given time a prospective migrant knows the available wage in the destination region and drawing a direct analogy to the Dixit and Pindyck result, Burda shows that there might be option value to waiting to migrate even if the Net Present Value rule indicates otherwise. O’Connell (1997) presents a more rigorous treatment of the Burda paper, but includes the possibility of return migration that plays an important role in the migration decision. His model is a direct application of
the entry-exit framework of Dixit (1989). Hanson and Spilimbergo (1996) use a discrete time optimal stopping framework to derive an “apprehensions function” which they empirically estimate, for undocumented immigrants crossing the U.S.-Mexico border.

The following works in managerial labor economics have also drawn on the real options literature. Chen and Zoega (1999) model the hiring and firing decisions of a firm that faces stochastic and exogenous productivity changes and solve for the thresholds for hiring and firing. Chen, Snower Zoega (2001) present a similar model but with uncertainty coming from the demand for the firm’s output. Murlidhar (1992) in his dissertation models the decisions of a multinational firm that takes into account operational flexibility and location choices. Even though the work is from a managerial science perspective, it is one of the first works that uses a stochastic process for wage differentials in two countries to follow Brownian Motion (Chapter 3).

2.2 The Model

This section presents a general formulation of the dynamic programming problem posed by migration to an urban labor market described in section 1.2.1. The questions on that specific labor market are not addressed till the next section. This is done because:

i) it is easier to move from the general case to a particular application.

ii) there may be problems other than the case of migration studied here that fit this formulation and for which this exposition will be more useful.

Without detailing the features of any specific labor market, the general model is still presented with references to migration, in order to maintain the link between the
adopted modelling techniques and basic intuition regarding the migration decision.

The basic dynamic programming technique is to break up the decision sequence into two parts, the immediate period and the continuation beyond that. The idea behind this stems from Bellman’s Principle of Optimality, which says “An optimal policy has the property that, whatever the initial action, the remaining choices constitute an optimal policy with respect to the subproblem starting at the state that results from the initial actions” (Dixit and Pidyck (1994)). The result of this is the Bellman Equation:

\[
V_t(s_t, k_t) = \max_{x=1,2,...,m} \{ f(s_t, k_t) + \delta E \epsilon V_{t+1}(g(s_t, k_t, \epsilon_{t+1})) \} \tag{2.9}
\]

In this discrete time formulation (used initially to illustrate the timing issues), \( V(s_t, k_t) \) is the value function, or the maximum possible sum of current and expected future payoffs. \( f(s_t, k_t) \) is the current reward that depends on the current state of nature, \( s_t \) and the choice variable \( k_t \), that can take on one of \( m \) possible values. \( g(s_t, k_t, \epsilon_{t+1}) = s_{t+1} \) is the state transition equation which shows the state next period as a function of the current state, the current choice of \( k \) and a random error term next period. The expectation is taken with respect to this shock, \( \epsilon_{t+1} \) and \( \delta \) is the per period discount factor\(^7\).

In this completely general setting, migration may be thought of as “Regime Switching”, a problem studied in dynamic optimization, where the agent makes an optimal choice of either staying or switching to a new regime, or every period the agent either stays with the choice of \( k \) from the period before or updates his choice (makes the switch).

\(^7\)This is analogous to \( \rho \) described in the previous chapter.
2.2.1 The Basic Nature of the problem

While the model is set up and solved in continuous time, some of the features of the basic dynamic programming problem are explained in a discrete time framework in this section. This is only to illustrate the timing issues that are not apparent when presented in continuous time.

i) Choice Variable: What we have is a discrete choice dynamic programming problem. An agent, given the state of nature and certain information about its evolution, chooses from a range of discrete regimes to be in. More technically, the action vector $k \in \mathbb{R}$ contains a discrete action variable whose range is an interval on the real line, $\{1, 2, \ldots, m\}$ which show the choice of m regimes. In a migration setting, the $k$ vector spans the range of possible regions the individual can be in. Updating the choice of $k$ implies moving to a new region.

ii) Time Horizon: We work with an infinite horizon, which makes the problem independent of the calendar date $t$, making this recursive equation easier to work with (since this makes the problem identical to all periods).\(^8\)

iii) State Space: The state space is mixed, or the state vector $s \in \mathbb{R}^2$ contains the mixture of one continuous and one discrete state.

a) State variable 1 is the continuous state $S$ that follows a stochastic process as in equation 2.4.

\[ dS = \mu(S)dt + \sigma(S)dz \quad (2.10) \]

where the functional form of $\mu(S)$ and $\sigma(S)$ can be specified based on our economic priors about state variable $S$. In the case of migration, $s$ may be the relative wage between two regions that fluctuates continuously through time and its evolution de-

---

\(^8\)The infinite horizon setting may not do this in the case of a time varying forcing variable, which is not the case here.
termines the choice made between migrating and staying.

b) State variable 2 is the discrete state $j$ which is the regime the agent is currently in, so $j \in \{1, \ldots, m\}$. In the rural-urban migration example, $j \in \{\text{rural sector, urban informal sector, urban formal sector}\}$.

iv) **The Reward Function and Costs of Switching:** The agent gets a flow of payments per unit of time, $f(S_t, k_t)$. In the discrete time framework, this implies that the agent can make the switch at the beginning of the period, otherwise our reward function would be $f(S_t, j_t)$, where $j_t$ is simply where the individual is at the beginning of the period, chosen optimally the period before. In our migration example, the per period reward function can be simply the relative wage earned in a sector. This is dependent on the sector of choice, $k_t$, and the value of the relative wage today, $S_t$.

The switch cost parameters may be arranged as an $m \times m$ matrix $C$ where $C_{ji}$ represents the cost of switching from $j$ to $i$. This need not be symmetric as the cost of moving back and forth between two regimes may be different. Naturally, the diagonal elements will be zero as they represent the cost of not switching.

v) **State Transition Equations:** The State Transition Equations govern the evolution of our state variables.

For a continuous state that fluctuates part deterministically and part randomly, the most general state transition equation in discrete time can be thought of as $S_{t+1} = g(S_t, k_t, \epsilon_{t+1})$ (as in Equation 2.9), so that the state next period is a function of the current state and choice (that are known today), and a random error that makes this stochastic. None of our state variables is this general.

Consider the problem discretized with time period $\Delta$. 
a) The state transition equation for our continuous state $S$ would be:

$$S_{t+\Delta} = g(S_t, \epsilon_{t+\Delta}) = S_t + \mu(S_t)\Delta + \sigma(S_t)\sqrt{\Delta}\epsilon_t$$

(2.11)

where $\epsilon_t$ is i.i.d $\sim N(0,1)^9$. Hence, state $S$ is completely stochastic to the agent as it is not a function of $k$. In the migration case this continuous state variable is simply a relative wage, that evolves randomly for a prospective migrant since one migrant has no affect on the relative wage between two sectors.

b) We define the state transition equation for $j$ as $j_{t+\Delta} = i$ with a Poisson intensity (of a switch to regime $i$) of $\Delta \Lambda_{ji}$, or equivalently, with a Poisson probability of $1 - \exp^{-\Lambda_{ji}\Delta}$. The intensity of the exogenous switch may vary from zero to infinity with the corresponding probability of the exogenous switch varying from zero to one. Analogously, $j_{t+\Delta} = k$ with the Poisson intensity $1 - \Delta \sum_{i=1}^{m} \Lambda_{ji}$. This reflects the possibility that there is no forced switch and the agent sticks with his optimal choice of the regime, $k$.

The parameters given to the agent are

i) $\mu$ and $\sigma$ if assumed to be constants (from Equation 2.11) or the exogenous functional forms of $\mu(S)$ and $\sigma(S)$ need to be specified.

ii) the Cost Parameter matrix $C$

iii) the $m \times m$ matrix $\Lambda$ associated with exogenous switches to other regimes than the current. Its typical element $\Lambda_{ji}$ is the intensity of the exogenous switch between $j$ and $i^{10}$.

iv) a discount rate $\rho$ which gives us the discount factor $\delta$ in equation 2.9$^{11}$.

---

9 This is a discrete time representation of equation 2.10. The discrete time notation is chosen to be consistent with the representation of the other two state transition equations that are easier to interpret in discrete time.

10 Specific applications where such a switch never arises between say regime 1 to 3, would set the corresponding entry in the matrix to zero.

11 Where $\delta = 1/(1 + \rho\Delta)$. 
2.2.2 The Dynamic Programming Problem:

The problem is better illustrated by first using a discrete time framework for setting up the Bellman equation. Consider a discretized regime switching model with the possibility of “forced” switches, presented in a very general form using time step $\Delta$:

$$V(S_t, j_t) = \max_{k=1,2,\ldots,m} \left\{ f(S_t, k)\Delta - C_{jk} + \frac{1}{1 + \rho\Delta}E_t[V(S_{t+\Delta}, j_{t+\Delta})] \right\}$$  \hspace{1cm} (2.12)

where the discrete and continuous state are denoted more generally as $j$ and $S$ respectively.

The discrete choice variable is denoted as $k$, where it can take on values $1, 2, \ldots m$, or the agent has the choice to go to any one of $m$ regimes. It is possible that $k = j$ i.e, it is optimal to not switch regimes.

Inside the $\max$ operator:

$f(S_t, k)\Delta = \text{reward received over time interval } \Delta \text{ from choosing regime } k$

$C_{jk} = \text{lump sum cost of moving to regime } k \text{ (zero if } k = j)$

$$\frac{1}{1 + \rho\Delta}E_t[V(S_{t+\Delta}, k)] = \text{continuation value, or the expected value function of time } t + \Delta \text{ discounted over interval } \Delta.$$ 

This value function is evaluated at continuous state $S_{t+\Delta}$ and discrete state $k$ which is optimally chosen at $t$, and by definition, encompasses optimal policies followed in the future.

The uncertainty regarding time period $t + \Delta$ comes from two sources and the agent forms expectations with respect to both:

i) At the beginning of the time interval, we are uncertain about the shock to $S_t$, thereby the value of the continuous state at $t + \Delta$.

ii) There is uncertainty regarding the forced switch to a regime “i” different from the “k” chosen today, giving us the value function $V(S_{t+\Delta}, i)$ instead of $V(S_{t+\Delta}, k)$.
at the end of the interval $\Delta$.

To decompose the expectations operator, we further specify the expectation of ii) as:

$$ E[V(S_{t+\Delta}, k)] = E \left[ \Delta \sum_{i=1}^{m} \lambda_{ki} V(S_{t+\Delta}, i) + \left( 1 - \Delta \sum_{i=1}^{m} \lambda_{ki} \right) V(S_{t+\Delta}, k) \right] $$

where $\Delta \lambda_{ki}$ is the probability over interval $\Delta$ that one is forcibly switched to regime $i$ from regime $k$. The second term in the brackets on the R.H.S is the probability that no such switch occurs, multiplied by the value function of being in the optimally chosen regime $k$. Substituting the above in Equation 2.12 we get the following:

$$ V(S_t, j) = \max_{k=1,2,\ldots,m} \left\{ f(S_t, k) \Delta - C_{jk} + \frac{1}{1 + \rho \Delta} \left[ \Delta \sum_{i=1}^{m} \lambda_{ki} V(S_{t+\Delta}, i) 
+ (1 - \Delta \sum_{i=1}^{m} \lambda_{ki}) V(S_{t+\Delta}, k) \right] \right\} \quad (2.13) $$

To derive the continuous form of the discretized Bellman, it is first simplified by

i) Multiplying both sides of Equation 2.13 with $\frac{1+\rho \Delta}{\Delta}$

ii) Adding and subtracting $\frac{V(S_t, k)}{\Delta}$ and rearranging to get the Bellman equation in the following form:

$$ \rho V(S_t, j) = \max_{k=1,2,\ldots,m} \left\{ f(S_t, k)(1 + \rho \Delta) - \rho C_{jk} \right\} \quad (2.14) $$

$$ + \ E \left[ \sum_{i=1}^{m} \lambda_{ki} V(S_{t+\Delta}, i) - V(S_{t+\Delta}, k) \right] \quad (2.15) $$

$$ + \ E \left[ \frac{V(S_{t+\Delta}, k) - V(S_t, k)}{\Delta} \right] \quad (2.16) $$

$$ + \left( \frac{V(S_t, k) - V(S_t, j) - C_{jk}}{\Delta} \right) \quad (2.17) $$

To get the continuous time limits of each of the parts inside the $\max$ operator of the Bellman equation, we let our time step $\Delta \to 0$ to get the following:
For Part 2.14
\[
\lim_{\Delta \to 0} [f(k, S_t)(1 + \rho \Delta) - \rho C_{jk}] = f(k, S) - \rho C_{jk}
\]

For Part 2.15,
\[
\lim_{\Delta \to 0} E \left[ \sum_{i=1}^{m} \lambda_{ki} [V(S_{t+\Delta}, i) - V(S_{t+\Delta}, k)] \right] = \sum_{i=1}^{m} \lambda_{ki} [V(S, i) - V(S, k)]
\]

For Part 2.16, we make use of Itô’s Lemma:
\[
\lim_{\Delta \to 0} E \left[ \frac{V(S_{t+\Delta}, k) - V(S_t, k)}{\Delta} \right] = \mu V'(S, k) + \frac{1}{2} \sigma^2 V''(S, k) = \frac{dEV}{dt}
\]

For Part 2.17 to not be unbounded above or below, the following relationship must hold with equality for the optimal \( k \):

\[
V(S, j) = \max_{k=1, 2, \ldots, m} \{ V(S, k) - C_{jk} \}
\]

Summarizing, at a given value of \( S \), if it is optimal to switch out of the regime \( j \), then
\[
V(S, j) = \max_{k \neq j} \{ V(S, k) - C_{jk} \}
\]

However, if it is optimal to remain in regime \( j \) then
\[
\rho V(S, j) = f(j, S) + \sum_{i=1}^{m} \lambda_{ji} [V(S, i) - V(S, j)] + \frac{1}{2} \sigma^2 V''(S, j)
\]

This can be easily interpreted as a no-arbitrage condition:

The left hand side of the Equation 2.20 is the return per unit of time of holding an asset using \( \rho \) as the discount rate. This asset is the value of being in regime \( j \), given that optimal policies are followed in the future.

\[\text{Note that this does not imply}\]
\[
\rho V(S, j) = \max_{k=1, 2, \ldots, m} \left\{ f(k, S) - \rho C_{jk} + \sum_{i=1}^{m} \lambda_{ki} [V(S, i) - V(S, k)] + \frac{1}{2} \sigma^2 V''(S, k) \right\}
\]

since this expression may be maximized by a regime that does not satisfy equation 2.19.
On the right hand side of the equation, the first term is the per period reward from being in \( j \). The next two terms taken together are analogous to an expected “Capital Gain” (or loss) from holding the asset, that of being in regime \( j \). The first term represents the expected change in the value received from the exogenous change in regime. The second term represents the value of the option to move to a different regime depending on the evolution of \( S \).

The strategy for solving such models begins by noticing that there is an interval for \( S \) over which, if \( k = j \), it is optimal to remain in regime \( j \). A decision rule consists of choosing the endpoints of this interval and the regimes to switch to if either endpoint is crossed. Suppose that at \( \bar{S} \) the decision is to switch to regime \( k \). It is clearly true that the value just prior to the switch must equal the value just after the switch, i.e., that

\[
V(\bar{S}, j) = V(\bar{S}, k) - C_{jk}.
\]

This value-matching condition holds regardless of whether \( \bar{S} \) is chosen optimal or not.

For the optimal choice \( \bar{S} = S^* \) the smooth-pasting condition also holds.

\[
V'(S^*, j) = V'(S^*, k).
\]

If however, \( C_{jk} = C_{kj} = 0 \) these conditions should be amended. Smooth-pasting will still hold but it is not an optimality condition. The optimality condition is the so-called super-contact condition:

\[
V''(S^*, j) = V''(S^*, k).
\]

One additional issue arises when exogenous switching can occur. Suppose the current regime is \( j \) and that \( \Lambda_{ji} > 0 \). It may be the case that at \( S \), it is optimal to switch out of regime \( i \) immediately (i.e., \( S \) is not in the no-switch interval for regime
It is possible, in fact, that it is optimal to switch back to regime \( j \) immediately, incurring cost \( C_{ij} \) to do so. This means that in order to correctly specify the differential equation for regime \( j \), one needs to determine for each value of \( S \), the regime that would be chosen if a switch from \( j \) to \( i \) occurs. The application to undocumented migration from Mexico to the United States illustrates this point in Chapter 4.

### 2.2.3 Some Specifics of the Solution

In the end, we are looking for a simultaneous solution for our thresholds and the value functions associated with each regime. Moving towards an analytic solution to this problem, we first consider that the Bellman equation satisfies equation 2.2. when it is optimal to be in a particular regime \( j \). This is a second order partial differential equation. However, if we consider it for a fixed \( j \), we have a non-homogenious ordinary differential equation whose solution is the value function for a given regime \( j \). We would then impose the value matching and smooth pasting conditions on the value functions (of a pair of regimes at a time) at the relevant thresholds. Under certain conditions, namely specific distributions of our continuous state \( S \), equation 2.2 yields a second order differential equation with a known form of the solution (the solution will be the value function for the regime). Knowing the functional form of the value functions allows us to simultaneously solve for the thresholds and certain constants in the value functions by imposing value matching and smooth pasting on these functions at the trigger points. The specific distribution of the continuous state variable \( S \) is assumed in subsequent chapters to be geometric Brownian motion which defines the functional forms of \( \mu(S) \) and \( \sigma(S) \) as \( \mu S \) and \( \sigma S \) respectively, so that our equation 2.10 becomes \( dS = \mu S dt + \sigma S dz \). Using this particular distribution for \( S \) gives us a
known functional form for the value function\textsuperscript{13}.

Summarizing (and as we will see in chapter 6 that provides a numerical solution), the solution to this model will comprise of a completely identified value function for each regime and the values of the thresholds of our continuous state that prompt a switch between regimes. The case taken up in subsequent chapters is the movement of undocumented Mexican workers between Mexico and the U.S., the two regimes considered. The solution provides the value functions for Mexico and the U.S., and the trigger values of the relative wage that prompt movement from Mexico to the U.S. and U.S. to Mexico respectively.

\textsuperscript{13}There are other distributions that yield tractable analytical solutions to the dynamic programming problem, for example a mean-reverting process for $S$. The use of geometric Brownian motion over this process (for the case of migration) is justified in Chapter 4.
Chapter 3

The Case of Rural-Urban Migration for a Segmented Urban Labor Market

3.1 Introduction

The Regime switching model in the preceding chapter with the possibility of exogenous shifts was presented in a very general form to preserve the basic intuition behind such models. This section and the next chapter demonstrate how such models can incorporate very specific features of different labor markets. This not only allows for economically interesting questions but also simplifies the problem considerably. The basic setup of the model will of course follow the steps laid out in Chapter 2, but with more structure given to all defining characteristics of the model, for example, our matrix of switch cost parameters. The following literature survey reviews some background information on the labor market considered here, but more impor-
tantly, provides the motivation for choosing specific forms for our state variables, and the basic assumptions that define the model. For this reason, the literature review is divided into subsections, each of which provide justification for modelling choices made in the subsequent section. Each subsection is thus concluded by a few sentences summarizing the information that has direct implications for the model.

3.2 Characteristics of Urban Labor Markets in Developing Countries: A Literature Survey

As was briefly introduced in Chapter 1, the first application of the model concerns the typical urban labor market in many developing countries. The ultimate purpose of this work is to study rural-urban migration in developing countries\(^1\), and more specifically, the migration decision of one prospective migrant given a dichotomous urban labor market with a formal and an informal sector. The fundamental contribution of this work will be to explore more deeply the effect of uncertainty on this migration decision. What then becomes important is to characterize this uncertainty as closely as one can. The nature of the urban labor market an individual faces upon migration plays an important role in this regard.

\(^{1}\)Work in this area goes back to the 1950’s. The informal sector is predominantly a byproduct of rural-urban labor market dynamics which was first approached by Arthur W. Lewis in 1954. He postulated that the formal sector would absorb unskilled labor from the rural areas till the productivity differential between the sectors was eliminated. Though later somewhat disregarded for a number of extreme assumptions like an unlimited supply of unskilled labor from the villages and a zero marginal productivity of this labor, this work is still considered the first attempt to explain the issue within a classical framework.
3.2.1 General Characteristics

What makes this urban labor market particularly interesting is its two distinct segments, the formal and the informal sector. Starting in the late 60’s and deep into the 70’s, numerous articles on the subject were published as the effects of widespread urbanization in South Asia, Africa and South America began to unfold. What was becoming clearly evident was that the “migration to cities outpaced their industrialization” (Moser, 1994). The inability of the formal sector to absorb this labor into productive use (suggesting, among other factors imperfect labor markets due to price controls) gave rise to an informal sector that consisted of a substantial part of the urban labor force, prompting a wide range of works on the issue. It was the International Labour Organization that took the initiative at the time to recognize this urban phenomenon as more chronic than cyclical in nature (Moser, 1994), making available a considerable amount of documentation about the main findings of its research studies, starting from the mid 60’s.

World Bank studies have classified the formal sector in most developing countries as a “protected” sector that arises due to the collective or individual actions of labor unions and the government. One important feature of this formal sector is an institutionally set minimum wage, and most models of rural urban migration attempt to incorporate this reality. Work done by the International Labour Organization starting in the 1970’s supports that the dichotomy is fed by institutional factors. Various forms of “protection” to the formal sector - access to foreign exchange, selective tariff relief on capital and intermediate goods sustain this sector despite high wage rates (Weeks, 1975).

This informal part of the urban labor force generates interest among development

\footnote{For instance, the growth in the labor force outpacing investment in physical capital and technological advances}
economists because of its sheer size alone, accounting for a substantial part of urban employment (20 to 60 percent) in many big cities in the developing world. Some of the numbers quoted in the literature are Bogota - 34.2%, Calcutta - 40%, Brazil - 27.3%, Peru - 40.5%, etc.\(^3\). What generally qualifies to be in the informal sector is that part of the urban labor force that covers small and unorganized family firms, for example - petty traders, domestic help, street peddlers, etc. Its main distinguishing characteristic is a very low capital labor ratio for both human and physical capital, which makes it relatively easy to enter this sector due to the lack of sector specific skills. It sells most of its output to the formal sector with almost all of the value added comprising labor services. The term “informal sector” was coined by British anthropologist Keith Hart (1970)\(^4\), but its most widely used definition comes from ILO studies (1972) that refer to informal economic activity as characterized by “ease of entry, reliance on indigenous resources, family ownership of enterprises, small scale of operation, labor intensive and adapted technology, skills acquired outside the formal school system and unregulated and competitive markets”.

3.2.2 Formal Sector: Implications for the Model

The formal sector is characterized by an institutionally set minimum wage. The literature clearly suggests that this minimum wage is binding and employment is demand determined, so to work in this sector should be considered more of an opportunity (which a migrant may choose to turn down)\(^5\) than a choice. The informal sector is free entry so a migrant may enter the informal sector from the rural sector


\(^4\)Much of his work in this period was based on his extensive study of the sector in Accra, Ghana. An observation he made towards the importance of the informal sector was the “far from peripheral” role of Ghana’s small entrepreneurs.

\(^5\)This issue is taken up in some more detail in section 3.2.5.
or the formal sector depending on labor market conditions.

3.2.3 The Role of the Informal Sector to a Prospective Migrant

Earlier works on rural-urban migration treated the informal sector as nothing more than a waiting ground for formal sector employment. The work of Harris and Todaro (1970) is an important case in point. This well established work was prompted by the condition of the urban labor market in Nairobi, Kenya at the time. The premise of their model is that migration to the cities continues until expected and not actual incomes are equalized between rural and urban areas. Expectations thereby explicitly account for uncertainty of finding employment in the formal sector, which is characterized by an institutionally set minimum wage. This condition is compatible with an equilibrium level of urban unemployment, which was interpreted as being engaged in informal activity, making up the informal sector. This model treats the informal sector as simply a waiting ground for migrants to find better employment and not a viable, thriving sector in itself. This point was later criticized by Cole and Sanders (1985) who argued that rural-urban migration may have more to do with opportunities in the informal sector itself. Moreover, looking at the informal sector in more than the Harris-Todaro dimension goes back to the early seventies. The ILO Kenya report (1972) describes the informal sector as dynamic and capable of generating jobs with the bulk of this sector “economically efficient and profit making”.

6This was the “Tripartite Agreement” instituted by the Kenyan government in 1964 between itself, private firms and unions in the Greater Nairobi area. The agreement was that firms would work to increase employment by 15% in exchange for no further demands for wage increases by the unions. The result was an increase in the urban unemployment in a matter of months due to the resulting migration from rural areas at the prospect of greater ease of finding formal sector employment.
3.2.4 Informal Sector: Implications for the Model

The model should be general enough to account for both views. The “waiting ground” theory would imply that the possibility of being offered formal sector employment adds value to being in the informal sector. Moreover, the model should also account for a possibility that the opportunity to work in the formal sector is worthless (for sufficiently high informal sector wages). This would be similar to the case when there is no formal sector, so rural-urban migration would take place if the value of being in the informal sector exceeds that of being in the rural sector by more than the migration costs.

3.2.5 The Question of Sectoral Choice: Endogenous or Exogenous?

There have been many empirical studies done on employment surveys from big cities in Asia, Africa and especially Latin America. Most of these explore micro labor market issues like the age/sex profiles of workers in the formal versus informal sector, inter-sectoral demands for each other’s goods etc. (House, 1984 i), with the bulk of this work done on city specific survey data.

A substantial part of the debate has centered on the idea of labor market segmentation and the empirical testing of this hypothesis. The two main premises of labor market segmentation are:

1) The returns to workers with the same observed productivity may be different in different sectors of the economy.

2) The primary sector consists of more desirable characteristics like job security, better pay, etc. than the secondary (lower) sector. The market is called segmented not due
to the existence of these sectors but “because of institutional barriers to occupational
mobility between sectors, a worker in the lower sector has less than full access to a job
in the upper sector held by an observationally identical worker.” (Grindling).

Labor market segmentation has been empirically supported in many developing
and developed countries, but is attributed to different factors. It is tested by a
simple procedure. The labor market is divided into segments along lines suggested by
theory. Earning equations are then estimated for each segment and pairwise Chow
tests are conducted to see if the estimated coefficients (of say, level of skills) vary
across segments (Cohen and House (1996)).

For a developing country, the formal-informal sector dichotomy is said to cause
this segmentation and one way to test it is to see whether the returns to schooling are
different depending on which sector one works in. Other factors like sex of the worker,
nature of the enterprize (public/private) and scale of the firm are also considered to
see along which line is segmentation most severe.\(^7\)

There is one interesting difference between premise 1) and 2) listed above. Dis-
 crimination studies (whether wages for workers of equal productivity differ with sex
or race) can test premise 1) of labor market segmentation and yield unbiased esti-
mates since the characteristic on which segmentation is based (race, sex, or another
independent variable) is truly exogenous to the worker (Cohen and House (1996))
and his earning stream. When testing segmentation along formal/informal lines, we
appeal more to premise 2) due to the existence of a primary and secondary sector with
the noted characteristics. However, as Fields (1980) pointed out, this is not enough
\(^7\)For developed economies, segmentation studies consider the internal labor market as the “pri-
mary segment” of the market, with the most desirable jobs being filled by internal candidates through
promotions, etc. Desirable characteristics of this sector include long promotion ladders, more job
security, etc. The “secondary segment” is lower in the hierarchy with various points of entry and
has features that discourage labor force attachment (House (1984) ii).
to show segmentation. The researcher should be able to show that workers in the secondary sector are not able to access the higher wage function, so which sector an individual works in is not a choice variable, but beyond the worker’s discretion. This issue is much larger than an empirical concern, and two views are well represented in the literature.

I. The first view is that we definitely see a primary/secondary sector type pattern in the urban labor markets. World Bank studies have classified the formal sector as “protected”, so wage levels and working conditions are unavailable to job seekers unless they somehow cross the protective barriers that arise due to collective or individual actions of the unions and the government (Mazumdar, 1981). This difficulty of access to upper sector jobs addresses the exogeneity of the sector to the worker. This is supported by work from both sociologists and economists who report a strong networking system that aids in finding formal sector employment for new migrants. The urban social network is heavily based on kinship, area of origin, etc, which are again exogenous to the worker. A few works that find support for labor market segmentation are House (1984, for Cyprus), Marcouiller et al. (1997, for El Salvador and Peru), Banerjee (1983, for India), etc.

II. The second view takes a more critical look at the lower/upper sector demarcation that assumes the informal sector is a residual sector with workers who wish to be working in the upper sector. An increasing number of works now describe the redundancy of this dualistic framework (Kannappan (1985). They declare that the informal sector is very heterogeneous in nature, with sections that acquire skills with the intent of using it for a permanent livelihood (House, 1984, Cole and Sanders, 1985) and who choose to stay in the sector. This challenges the older view of lack of

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8See Cohen and House (1996) for a detailed list of references.
sectoral choice on the part of the workers. No clear consensus on the issue has yet emerged.

3.2.6 Sectoral Choice: Implications for the Model

The question of exogenous vs. endogenous sector of work is very important in our regime switching model as the regimes are defined as the different sectors - rural, urban-informal and urban-formal. The model assumes that the opportunity to work in the formal sector is completely exogenous to the migrant. However, given the opportunity, he may either move or stay in the informal sector.

This opportunity aspect is captured by defining our informal sector as consisting of two sub-sectors. One in which there is no possibility of moving to the formal sector, the other where it is costless to do so. An exogenous regime change may move the migrant between these sub-sectors. Using a discrete time exposition, there is a small probability every period that the migrant is moved from the “without opportunity” to the “with opportunity” sub-sector so that an opportunity to work in the formal sector opens up, which a migrant may refuse to take. If such a migrant is currently working in the “with opportunity” sub-sector, then there exists a probability every period that he is moved to the “without opportunity” sub-sector. A big probability of this switch implies that the opportunity to work in the formal sector does not last very long.

Hence, an exogenous regime shift approach can be used to model the “opportunity” aspect of certain phenomena. An opportunity to undertake an action may arise exogenously to the agent, which he may or may not make use of. This is done by defining a regime which is identical to the current regime except here, the optimizer can undertake a certain action at no cost, an option not available in the current
regime. The individual being switched to this regime is synonymous to the opportunity arising. Being switched back to the current regime if the action is not undertaken is synonymous to the opportunity closing up.

3.3 The Model

3.3.1 Setup

As in chapter 2, the model is set up and solved in continuous time, but the evolution of certain variables is described in discrete time for a clearer exposition of timing issues so as to preserve the basic economic intuition behind such models.

i) **Choice Variable:** The agent’s choice variable \( x \) is a set of four regimes:
   a) The Rural Sector,
   b) The Urban Informal sub-sector with no possibility of moving to the formal sector,
   c) The Urban Informal sub-sector from which it is costless to move to the formal sector,
   d) The Urban Formal Sector.

   The choices available to the migrant at a point in time are themselves state dependent; they depend on the regime a migrant currently works in. For instance, for a migrant working in the informal sub-sector b), the formal sector is not a part of the choice set of regimes.

   ii) **Time Horizon:** The agent faces an infinite time horizon.

   iii) **State Space:** The mixed state space consists of 1 continuous and 1 discrete state.

   a) State variable 1 is the continuous state variable \( W_I \) which is the wage made by a migrant in the informal sector (in either of its sub-sectors). It follows the following
stochastic process:
\[ dW_I = \sigma W_I dw \]

or a geometric Brownian motion without a drift term\(^9\).

b) State variable 2, \( j \) is the sector of work an individual is currently in, so \( j \in \{ \text{rural}, \text{informal}(O), \text{informal}, \text{formal} \} \).

where \( \text{informal} (O) \) is the informal sub-sector that presents the opportunity to work in the formal sector, and \( \text{informal} \) is the sub-sector without any such opportunity.

iv) **Reward Function and Cost of Switching regimes:** The reward function is defined as \( f(j) = W_j \) where \( W_j \) is the per period return from being in sector \( j \) defined as the wage of that sector. \( W_F \) or the formal sector wage is assumed to be fixed at the minimum wage, which is higher than \( W_R \), the rural wage\(^{10}\), also assumed to be fixed. So the uncertainty comes only from \( W_I \). Since expected present values of income *differences* drive migration, and since these differences are stochastic, we preserve the basic nature of questions about uncertainty and the migration decision. Keeping the dimensionality of problem small\(^{11}\) is a modelling choice made to avoid the “curse of dimensionality” and guarantee a solution.\(^{12}\)

The matrix of switch costs is constructed to reflect the following: In our regime switching framework, switches not considered feasible in the model are guaranteed by imposing the associated switch-costs to be \( \infty \). Switch costs that are irrelevant because it is never optimal to switch (even at zero cost) are defined with a bar. For

\(^9\)We could instead assume a positive rate of drift for the informal sector wage and assume that the formal sector minimum wage and the rural wage also drift up at the same rate. This would not make any qualitative difference, since it is relative wages that ultimately drive migration.

\(^{10}\)The formal wage exceeding the rural is not a strong assumption and is in accordance with the literature.

\(^{11}\)Instead of having all 3 sector specific wages being stochastic state variables

\(^{12}\)The problem arises from the fact that the value functions need to satisfy *partial* differential equations, which are often difficult or impossible to solve using simple techniques. Special cases exist where we can reduce the dimensionality of the problem without affecting the predictions of the model, but these require very specific conditions. See Dixit and Pindyck (pages 207-211).
example, an individual would never move from the formal to the rural sector since the income differential is positive and invariant with respect to time.

If we write down a cost matrix $C$, with rows indicating the sector one is currently in and columns indicating the sector to switch to, we have the representation in matrix form in table 3.1. A summary of the implications of the matrix of cost parameters follows.

a) An individual cannot directly move from the rural to the urban formal or the informal (O) sector (this is consistent with the general observation of network effects, so that a migrant may find formal sector employment only after he is in the city).

b) Once in the informal sector without opportunity, the individual cannot choose to move to the the informal (O) sector (this is only possible as an exogenous switch) and of course neither to the formal sector. These assumptions imply the “opportunity” factor in finding formal sector employment, that is further elaborated below.

c) In the informal (O) sector, the individual may move at no cost to the formal sector, so being in this sector represents the opportunity of working in the formal sector. It is never optimal to move from this sector to the informal sector without opportunity

Table 3.1: Cost of Switching between Regimes

<table>
<thead>
<tr>
<th>SWITCH COSTS</th>
<th>Rural</th>
<th>Informal</th>
<th>Informal(O)</th>
<th>Formal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td>0</td>
<td>$C_{RI}$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Informal</td>
<td>$C_{IR}$</td>
<td>0</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Informal(O)</td>
<td>$\bar{C}_{IR}$</td>
<td>$\bar{0}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Formal</td>
<td>$\bar{C}_{IR}$</td>
<td>$\bar{0}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
since it pays the same wage but provides no opportunity, making the corresponding cost irrelevant. It is also never optimal to move from this sector to the rural sector, since the migrant may move to the formal sector at no cost, and the formal sector wage is always above the rural wage.

d) It is never optimal to move from the formal sector to the informal sector without opportunity since it is costless to move to the sub-sector with opportunity.

The diagonal elements are all zero as they reflect the costs of not switching. The only relevant costs are those associated with switching back and forth between the rural and the informal sector. All other costs are either zero, infinity or not relevant since the corresponding switches are never optimal.

v) State Transition Equations:

a) The evolution of state 1, $W_I$ is governed by equation 3.1.

b) As in the general model from chapter 2, $j_{t+1}$ is simply $x_t$ or the sector of work at the beginning of the next period is that which is optimally chosen this period.

The matrix Λ shows us the probabilities of being exogenously shifted to a different regime. This has the typical element $\Lambda_{ji}$ that is the probability of being switched between regime $j$ and $i$ at zero cost. Again, if rows indicate the current sector of work while the columns index a sector to switch to, our Λ is represented in table 3.2. The only exogenous switches in the model occur between the two sub-sectors of the informal sector, so the probabilities of interest to our model are $\lambda_{IO}$ and $\lambda_I$. $\lambda_{IO}$ is effectively the probability of finding a formal work at the minimum wage after entering the informal labor force and $\lambda_I$ effectively is the probability of this opportunity closing, so that a high $\lambda_I$ means that the migrant will have a small span of time to decide whether or not to join the formal labor force before the choice no
Table 3.2: Probability of Exogenous Regime Switches

<table>
<thead>
<tr>
<th>PROBABILITIES OF EXOGENOUS REGIME SWITCHES</th>
<th>Rural</th>
<th>Informal</th>
<th>Informal(O)</th>
<th>Formal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td>–</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Informal</td>
<td>0</td>
<td>–</td>
<td>$\lambda_{IO}$</td>
<td>0</td>
</tr>
<tr>
<td>Informal(O)</td>
<td>0</td>
<td>$\lambda_{I}$</td>
<td>–</td>
<td>0</td>
</tr>
<tr>
<td>Formal</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>–</td>
</tr>
</tbody>
</table>

longer available. Both of these probabilities are assumed as constant.\textsuperscript{13}

Exogenous to the agent are i) Cost Matrix $C$, ii) Probability Matrix $\Lambda$ iii) $\sigma$ from equation 3.1 iv) Discount rate $\rho$. Endogenous to the agent is his sector of work, given the alternatives.

### 3.3.2 Value Functions for the three sectors

The Bellman equation satisfied by a value function in a given regime (when optimal to stay in the regime) was derived in Chapter 2. For each discrete state (sector), we can define a value function of being in that sector. The general form of the value function defined in Chapter 2 was $V(s, j)$. For this case, the continuous state $s = W_I$, the informal sector wage. $j$ is the sector of work. The representation of the value functions will use a simpler notation, given below:

Value of being in the rural sector:

$V(W_I, Rural) \equiv V_R(W_I)$

\textsuperscript{13}If networking increases with the amount of time spent in the city and is important in finding formal sector employment, it would be more realistic to have $\lambda_{IO}$ increase with time. However, this makes our problem time dependent, making it considerable hard to solve.
Value of being in the informal sector without opportunity:
\[ V(W_I, Informal) \equiv V_I(W_I) \]

Value of being in the informal sector with opportunity:
\[ V(W_I, Informal(O)) \equiv V_{IO}(W_I) \]

Value of being in the formal sector:
\[ V(W_I, Formal) \equiv V_F(W_I) \]

The state variable \( j \) for each value function is denoted in the subscript as \( V_j \).

We begin with the Bellman equations that are satisfied by \( V_F(W_I) \) and \( V_R(W_I) \) since they are simpler and symmetric. For the rural and formal sectors, there is no exogenous switch of a regime for the migrant originating in either of these two sectors. So the value of being in the formal sector \( V_F(W_I) \) would satisfy the Bellman equation (or that our optimal \( x \) is the formal sector itself):

\[ \rho V_F(W_I) = W_F + \frac{\sigma^2}{2} W_I^2 V''_F(W_I) \]  \hspace{1cm} (3.2)

or

\[ W_F + \frac{\sigma^2}{2} W_I^2 V''_F(W_I) - \rho V_F(W_I) = 0 \] \hspace{1cm} (3.3)

Equation 3.2 can be interpreted as the per period\(^{14}\) return from being in the formal sector as the sum of the formal sector wage and the option to migrate to the informal sector if the informal sector wage rises enough\(^{15}\). This ordinary non-homogenous differential equation has a known form of the solution so we use the guess and verify method to try a solution of the form : \( V_F(W_I) = W_I^\beta \). We substitute this solution and its second derivative \( V''_F(W_I) = (\beta - 1)\beta W_I^{\beta - 2} \) into equation 3.3 to get the solution

\(^{14}\)Using discrete time exposition.
\(^{15}\)The migrant would only migrate to the informal sub-sector with opportunity from the formal sector.
for the homogenous part:

\[
\frac{\sigma^2}{2} W_i^2 (\beta - 1) \beta W_i^{\beta - 2} - \rho W_i^\beta = 0
\]

This equation would hold if (rearranging):

\[
W_i^\beta \left[ \frac{\sigma^2}{2} (\beta - 1) \beta - \rho \right] = 0
\]

or that \(\beta\) solves the following quadratic:

\[
\theta(\beta) = \left[ \frac{\sigma^2}{2} (\beta - 1) \beta - \rho \right] = 0
\]

This quadratic has one negative and one positive root. This is motivated by the fact that this parabola opens upward and the value of the quadratic at \(\beta = 0\) and \(\beta = 1\) are both equal to \(-\rho\) (See Figure 3.1). The solution for the homogenous part is:

![Figure 3.1: The Fundamental Quadratic](image)

Since the general solution can be written as a linear combination of any two independent solutions, where \(\beta_1\) and \(\beta_2\) are the positive and negative roots of the quadratic \(\theta(\beta)\) respectively. This part of the value function measures the value of the option to
migrate to the informal sector (with opportunity) if conditions make it the optimal strategy\textsuperscript{16}. Since this option would be worthless if the wage of the individual in the informal sector goes to zero, we constrain the coefficient of the negative root ($A_2$) to be zero (otherwise the option value will go to infinity with $W_I \to 0$). Adding a particular solution to the homogenous solution, the solution to our differential equation is:

$$V_F(W_I) = A_F W_I^{\beta_1} + \frac{W_F}{\rho}$$

(3.4)

So the first part of our value function for the formal sector consists of the option to migrate to the informal sector, while the second part is simply the present value of the wage stream earned in the formal sector, as

$$\frac{W_F}{\rho} = \int_0^\infty W_F e^{\rho t} dt.$$  

The rural sector is completely analogous to the formal sector, as it has the same homogenous part of the differential equation as the formal sector, and thereby the same quadratic and its roots $\beta_1$ and $\beta_2$ apply to this problem. The value function for the rural sector is:

$$V_R(W_I) = A_R W_I^{\beta_1} + \frac{W_R}{\rho}$$

(3.5)

Again, the solution comprises of two parts. The first part is the value of the option to migrate to the informal sector without opportunity (the same real wage is paid in both the sub-sectors of the informal economy). We use the same positive root as for the formal sector $\beta_1$ for the same reason that as the informal sector wage goes to zero, the option to migrate to it should be worthless. The second part is the present value of the wage stream in the rural sector.

We have a different value function for each sub-sector of the informal labor market.

\textsuperscript{16}The reason for leaving the formal sector is simply that the informal wage a migrant earns is high enough for this move to be optimal. This is a very real possibility, and adheres to the views of economists/anthropologists who see pockets of the informal sector as viable and productive enterprises that may for some time offer a wage higher than the formal sector.
For the value function of the informal sector without opportunity, we get the following:

$$\rho V_I(W_I) = W_I + \frac{\sigma^2}{2} W_I^2 V''_I(W_I) + \lambda_{IO}[V_{IO}(W_I) - V_I(W_I)]$$  \hspace{1cm} (3.6)$$

Using a discrete time exposition, this equation is interpreted as the per period return to being in this sector as the sum of the informal wage, the option to migrate to the rural sector and the possibility of an opportunity to work in the formal sector. This value function holds for a range of wages for which it is not optimal to switch from the informal(O) to the formal sector. For a range of wages for which this move would be optimal, we can use the following representation:

$$\rho V_I(W_I) = W_I + \frac{\sigma^2}{2} W_I^2 V''_I(W_I) + \lambda_{IO}[V_F(W_I) - V_I(W_I)]$$  \hspace{1cm} (3.7)$$

In this region, the migrant would move to the formal sector if given the opportunity (or that $V_{IO}(W_I)$ would be exchanged for $V_F(W_I)$ at no cost).

We first consider the solution to equation 3.7. For the solution to the homogenous part, we follow the same kind of steps as for the formal and rural sectors. The solution $W_I^\beta$ holds if

$$W_I^\beta \left[ \frac{\sigma^2}{2} (\beta - 1)\beta - (\rho + \lambda_{IO}) \right] = 0$$

or that $\beta$ solves the following quadratic:

$$\theta_I(\beta) = \left[ \frac{\sigma^2}{2} (\beta - 1)\beta - (\rho + \lambda_{IO}) \right] = 0$$

which is slightly different than the quadratic for the formal and rural sectors so has different roots. We express the solution in the form:

$$V_I(W_I) = A_{I2} W_I^{\beta_2} + A_{I3} W_I^{\beta_3}$$

The complete solution to our differential equation 3.7, after adding the particular solution is:

$$A_{I2} W_I^{\beta_2} + A_{I3} W_I^{\beta_3} + \lambda_{IO} A_F W_I^{\beta_1} + \frac{W_I}{\rho + \lambda_{IO}} + \frac{\lambda_{IO} W_F}{\rho (\rho + \lambda_{IO})}$$  \hspace{1cm} (3.8)$$
Note that this solution is only for the case where a migrant would take the opportunity of formal work (or the opportunity has value) if so offered. The first term represents the option to migrate away from the informal sector. The second term reflects the probability of being offered formal employment multiplied by the option to return back to the informal sector (if it becomes optimal in the future). The third term is the present value of the wage stream in the informal sector, discounted by an augmented rate, which includes the probability of finding formal employment (as this would prompt movement away from the sector). The last term is more difficult to interpret, and can be seen as the discounted expected wage stream from holding the opportunity to move to the formal sector.

For the value function of the informal sector with opportunity, $V_{IO}(W_I)$ is the solution to the following Bellman Equation:

$$\rho V_{IO}(W_I) = W_I + \frac{\sigma^2}{2} W_I^2 V''_{IO}(W_I) + \lambda_I [V_I(W_I) - V_{IO}(W_I)] \quad (3.9)$$

The first term on the right hand side is again the informal sector wage. The second term is the option to move to the formal sector if optimal. The third term reflects the expected change in the value function if the opportunity of formal sector employment goes away (or that the individual is switched from regime informal(O) to regime informal).

Taking the two value functions of the Informal and the Informal(O) sector when the informal wage lies in the region where it is not optimal to switch to the formal sector given the opportunity, we have a system of second order linear differential equations that must be solved simultaneously. The system consists of the two equations:

$$\rho V_{IO}(W_I) = W_I + \frac{\sigma^2}{2} W_I^2 V''_{IO}(W_I) + \lambda_I [V_I(W_I) - V_{IO}(W_I)] \quad (3.10)$$

$$\rho V_I(W_I) = W_I + \frac{\sigma^2}{2} W_I^2 V''_I(W_I) + \lambda_{IO} [V_{IO}(W_I) - V_I(W_I)] \quad (3.11)$$
Following the methodology of solving simultaneous differential equations through substitution, we use equation 3.10 to write $V_I(W_I)$ in terms of $V_{IO}(W_I)$ so that:

$$V_I(W_I) = \frac{1}{\lambda_I} \{W_I + \frac{\sigma^2}{2}W_I^2V''_{I0}(W_I) - (\rho + \lambda_I)V_{IO}\} \quad (3.12)$$

We substitute $V_I(W_I)$ and its second derivative that we get by differentiating equation 3.12 in equation 3.11 to get a fourth order differential equation in $V_{IO}(W_I)$:

$$V'''_{I0}(W_I)^4 + \frac{\sigma^4}{4\lambda_I} + V''_{I0}(W_I)^3\frac{\sigma^4}{\lambda_I}$$

$$+ V''_{I0}(W_I)^2\left\{\frac{\sigma^2}{2\lambda_I}(\sigma^2 - (2\rho + \lambda_I + \lambda_{IO}))\right\}$$

$$+ V_{I0}(W_I)\left\{(\rho + \lambda_I)(\rho + \lambda_{IO})\right\}$$

$$+ W_I\left\{1 - \frac{\rho + \lambda_{IO}}{\lambda_I}\right\} = 0 \quad (3.13)$$

Using the guess and verify method and using a form of the value function as in the formal and rural sectors, we substitute the solution of the form: $V_{IO}(W_I) = (W_I)^{\beta}$ and its derivatives into the homogenous part of the differential equation. It can be shown that:

$$V_I(W) = \frac{W}{\rho} + A_{I4}W^{\beta_4} + A_{I5}W^{\beta_5}$$

and

$$V_{IO}(W) = \frac{W}{\rho} + A_{O4}W^{\beta_4} + A_{O5}W^{\beta_5}$$

where $A_{I4}$, $A_{I5}$, $A_{O4}$ and $A_{O5}$ are constants to be determined and $\beta_4$ and $\beta_5$ are the two negative roots of the quartic equation

$$\left(\rho + \Lambda_{IO} - \frac{1}{2}\sigma^2\beta(\beta - 1)\right)\left(\rho + \Lambda_I - \frac{1}{2}\sigma^2\beta(\beta - 1)\right) - \Lambda_{IO}\Lambda_I = 0$$

It is easy to verify the quartic equation is satisfied by defining $\beta_4$ to be the negative root of the quadratic $\rho - \frac{1}{2}\sigma^2\beta(\beta - 1) = 0$ and $\beta_5$ to be the negative root of the quadratic $\rho + \Lambda_{IO} + \Lambda_I - \frac{1}{2}\sigma^2\beta(\beta - 1) = 0$. The constants must, furthermore, satisfy $A_{I4} = A_{O4}$ and $\Lambda_{IO}A_{I5} = -\Lambda_I A_{O5}$.
3.3.3 Nature of the Solution

The nature of the solution can be best described with the graphical illustration in Figure 3.2. The behavior of our continuous state variable $W_I$ prompts the agent to switch between the regimes defined. We can define these points along a real line spanned by $W_I$ and the solution will comprise of these trigger wages. There is a wedge between the rural to urban and the urban to rural threshold, which is the range of inactivity, or if the informal wage does indeed fall in this range, there would be no migration flow either way.

However, the informal(O) to formal trigger is the same as the formal to informal trigger, because the migrant can move back and forth between the two regimes at no cost. If there are no costs to moving between two regimes, migration between those two sectors is completely reversible, since the irreversibility associated with the sunk costs disappear.

To solve for our three trigger points and the 8 coefficients for our value functions, we would need 11 equations. These are given by

i) $V_R(W_{RI}) = V_I(W_{RI}) - C_{RI}$
ii) \( V_I(W_{IR}) = V_R(W_{IR}) - C_{IR} \)

iii) \( V_F(W_{FO}) = V_{IO}(W_{FO}) \)

iv) \( V'_F(W_{FO}) = V'_{IO}(W_{FO}) \)

v) \( V_I(W_{FO}^+) = V_I(W_{FO}^-) \)

vi) \( V'_I(W_{FO}^+) = V'_I(W_{FO}^+) \)

vii) \( A_{I4} = A_{O4} \)

viii) \( \Lambda_I A_{I5} = -\Lambda_{IO} A_{O5} \)

ix) \( V'_R(W_{RI}) = V'_I(W_{RI}) \)

x) \( V'_I(W_{IR}) = V'_R(W_{IR}) \)

xi) \( V''_F(W_{FO}) = V''_{IO}(W_{FO}) \)

The fifth and sixth conditions indicate that \( V_I \) should be continuous and differentiable at \( W_{FO} \).

Due to the lack of data on many of the parameter values, this model is not solved numerically. However, the theoretical contribution of this work is to add to the rural urban migration literature, a new methodology in dealing with uncertainty.

In this model, the prospective rural-urban migrant considers three factors in his decision to migrate - the returns from working in the informal sector, the chances of being picked by the formal sector (an opportunity he may turn down depending on the informal sector wage) and an option to return to the rural sector if conditions in the urban sector turn out to be unfavorable. An important theoretical implication of this setup is that a migrant working in the informal sector derives value from all of these factors and even from the ability to come back to the informal sector from the formal sector if it is optimal in the future to do so. The issue of future flexibility in the choice of sector of work (and its value to prospective migrants) has not received any attention in the rural urban migration debate, and this work attempts to fill that
void.

In studying the wage differences that ultimately prompt rural to urban migration: one may look at the average wages prevailing in the urban sector (compared to the rural sector) that are affected by numerous factors like labor legislation in the formal sector, improvements in productivity in the informal sector, or rural wages affected by returns to labor in agriculture and a host of other factors. One can also consider the informal sector as having a whole distribution of wages, based on the level of skill of the migrants so that the wage earned is affected by migrant characteristics. This model does not attempt to explain these underlying characteristics of the relative wage structure (between the three sectors), but instead deals with the migrant’s reaction to the variance of these relative wages and the uncertainty associated with work in the urban informal sector.
Chapter 4

Application: The Case of

Undocumented Migration from

Mexico to the United States

4.1 Introduction

The role of uncertainty in the migration decision can be further illustrated for another special case, that of undocumented Mexican migration to the United States. As with the last case, a migrant may also be exogenously switched to a different regime at no cost to him. In the last chapter, exogenous regime switching was used to model the opportunity aspect of certain phenomena, like finding formal sector work. In this chapter, the use for this kind of specification allows us to study factors like deportation, a shift in regime outside the control of the migrant. Hence, a slightly simpler specification of the model is used to accommodate features of the market for unskilled undocumented labor from Mexico.
Illegal Mexican migration contributes to about 54 percent of the total undocumented population of the U.S., with another 10 percent coming from Central America, mainly from El Salvador, Guatemala, Honduras and Nicaragua (INS, 1996). The majority of these migrants cross the Mexico-U.S. border without documents, while a much smaller fraction arrive legally on tourist visas and overstay the allowed duration.\(^1\) The result is an undocumented Mexican immigrant population residing in the U.S. last estimated at 3.1 million in 1997. The estimated number of illegal immigrants from Mexico who establish residence in the U.S. has averaged around 230,000 annually between the years 1990-1996. Accounting for emigration, death and legalization, the INS estimates that this undocumented population grew by 150,000 annually over the period 1988-1996\(^2\). This number is \textit{not}, however, the estimated number of illegal \textit{entries} during a year which is estimated to be more than tenfold of this value over the same period.

\subsection*{4.1.1 Historical Trends}

Historically, the first notable increase in undocumented migration was caused by the end of the Bracero Program in 1964. This was a bi-national agreement that instituted a guest-worker program in 1942, partly in response to labor shortages caused by the 2nd World War. This let millions of Mexican workers (largely agricultural) enter the U.S. on temporary legal visas. This program was extended through the 1950’s and was unilaterally terminated by the U.S. after successful lobbying efforts of organized labor in 1964(Orrenius (2001)). As a result, many “Braceros” simply started to migrate illegally, because of the abrupt change in their status. This phenomenon was

\footnote{Only 16\% of the undocumented Mexican workers are nonimmigrant overstays compared to 91\% of all other (non Central American) countries.}

\footnote{Most official estimates of this population are based on the difference between the alien population in the Census and the size of the \textit{legal} alien population as measured by the INS.}
facilitated by an important residual of the Bracero program - the “network effects”. The program had established vital links between employers in the U.S., recruiters on both sides of the border and the migrant workers that lowered the risks of migrating to the U.S. even after the termination of the program. Studies based on survey data clearly establish the importance of these networks in increasing the propensity to migrate. (Massey et al. (1987)).

Since the actual number of undocumented entries into the U.S. cannot be directly measured, these are estimated based on the number of apprehensions at the border by the INS, after controlling for factors like increased apprehension activity due to additional funding allocated to the Border Patrol, etc. National surveys in Mexico support the estimates of Massey and Singer (1995), who report the following general trend: The Gross inflow of undocumented migrants grew on average by 20% from 1965 to 1978, when it reached nearly 1.5 million, levelling off till Mexico’s economic crisis in the early eighties that prompted another period of rapid expansion, peaking at 3.8 million in 1986. This number fell into the 2.5 to 3 million range till 1990. Their estimate gives a total of 36.5 million entries by undocumented Mexicans over the period 1965-1990, which they say is broadly consistent with a recent national survey in Mexico, that reveals that one-third of all Mexicans have been to the U.S. at some point in their lives³.
Figure 4.1: Estimated Gross and Net undocumented in-migration from Mexico
Source: Massey and Singer (1995)

4.2 The Question of Permanent Versus Temporary Migration

The issue of permanent vs. temporary migration is of obvious importance to a regime switching model. If we are only dealing with permanent migration, a simpler setup which models migration as an optimal stopping problem may be used, where upon migration, the worker retires from the decision problem. This setup is used by Hanson and Spilimbergo (1996) to motivate the specification of an apprehensions function, which they estimate to establish the relative importance of different variables on total apprehensions at the border. Important data and estimation issues from their

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\(^3\)Current Population of Mexico: 99,600,000.
work are discussed in more detail in Chapter 4. The optimal stopping setup in the investment literature is used to model irreversible investment and for the migration case, this simply precludes return migration.

The motivation for a regime switching framework is that the possibility of returning home is an important factor in the individual’s decision to migrate abroad. This setup allows the migrant to move back and forth between regimes, thus accounting for temporary migration. Hence, choosing regime switching over an optimal stopping framework is justified by the widely observed phenomenon of temporary migration of undocumented workers from Mexico. The empirical literature on this subject clearly supports that, contrary to popular belief, the significant part of illegal migration is temporary. We just have to compare INS estimates of additions to the illegal Mexican population (comprising of illegal immigrants who establish residence here) and compare them to the Massey and Singer (1995) estimates of total illegal entries from Mexico to see the difference.

Figure 4.1 shows estimated gross and net undocumented migration that illustrates the importance of return migration. Gross migration steadily increases from the early eighties following the Mexican Crisis and falls substantially following the passage of the Immigration Reform and Control Act in 1986\(^4\).

Many relevant empirical works in this area draw their conclusions from survey data gathered from interviews of Mexican communities from the “core” sending regions, or those areas identified as having well developed migratory traditions. Kassoudji (1992) presents a detailed list of references that conclude that “a significant portion of the migrating population migrates temporarily and repetitively...One study of a long term core sending region in Mexico and its associated communities in the U.S.

\(^4\)This decrease is attributed more to the fact that many undocumented workers were legalized and now crossed the border freely than a more effective Border Patrol through increased funding.
classified about 50% of the migrants as temporary and the rest as U.S. oriented or permanent” (Mines and de Janvry (1982)). In another study of one such community, the probability of a young man in Mexico migrating illegally before his fortieth birthday was above 0.8 throughout the 1980’s, the probability of making the second trip was near 1 through the same period and the probability of making a third trip never fell below 0.75 (Donato et al. (1992)).

In more aggregate terms, Massey and Singer (1995) predict that over the period of 1965 to 1990, 86% of all illegal entries into the U.S. were offset by departures, which would imply a net inflow of only 5.2 million workers over this 25 year period\(^5\). A recent work by Durand, Massey and Zenteno(2001) contends that Mexican migration to the U.S. has become even more cyclical or short-term in the 90’s.

### 4.2.1 Permanent Vs. Temporary Migration:

**Implications for the Model**

A regime switching setup is adopted for modelling purposes to account for movement between the regimes - Mexico and U.S. This framework allows us to impute a value to the option of returning home in the future.

### 4.3 A Brief Summary of Policy Responses

The case of rural urban migration in developing countries and its relation to growing informal urban labor markets (discussed in chapter 3) has not typically given rise to any policy implemented at the federal government level to deter this migration flow, but has resulted in a few attempts at dealing with the resulting informal sector

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\(^5\)The INS had put this figure at 3.1 million in 1996.
population in the cities, mostly at the local government level. The case of undocumented migration to the U.S. in that sense is more interesting since it has seen a history of policy reactions at the U.S. federal government level precipitating a big percentage of funds of the INS allotted to the Border Patrol to deal with this issue on a daily basis.

While a sizeable literature deals with the passage of IRCA (Immigration Reform and Control Act) in 1986 and its effect on illegal immigrants from Mexico, Orrenius (2001) presents a more detailed overview of U.S. policy on the issue. The following information is taken from this work, with some additions made to to the IRCA discussion.

After the end of the Bracero program, it was the Carter administration that approved increased INS funding to fight the influx of undocumented migrants in the late 1970’s. All time high border apprehension probabilities in the period are attributed to the effectiveness of the policy.

The next big step was taken by the Reagan administration through the implementation of IRCA in 1986 that outlined significant changes to deter undocumented immigration which had peaked after the Mexico crisis in the early 1980’s. The three most important implications of IRCA were:

i) Employer sanctions against those who knowingly hired undocumented migrants (as opposed to before its passage, when it was illegal to work without documents but not illegal to hire these workers).

ii) More funds allocated to the INS and border enforcement, with a stated goal of at least 50% more border personnel in 1987 and 1988 than in 1986\textsuperscript{6}.

\textsuperscript{6}However, not all apprehensions are made at the border which account for about 50-70% of the apprehensions, the rest are made by non-linewatch patrols that perform farm and ranch checks, traffic checks at road blocks, etc.
iii) An IRCA amnesty that gave legal status to agricultural workers and long term U.S. residents.\footnote{As a result 1.8 million of regular and 1.3 million of special agricultural workers (SAWS) were legalized, two-thirds of the total came from Mexico.}

Ten years after IRCA was passed, another significant policy response came from the Clinton administration with the passage of IIRIRA (The Illegal Immigration and Reform and Immigrant Responsibility Act). This act further increased penalties on illegal entrants and mandated a doubling of the Border Patrol by 2001. Even more significantly, welfare reform legislation passed at this time denied illegal immigrants access to most public benefits.

Finally, in 2001 Congress passed LIFEA (The Legal Immigration and Family Equity Act) which allowed approximately 650,000 illegal immigrants to apply for legal residency.\footnote{This is not regarded as a “total” amnesty as workers pay $1,000 for applying for legal residency.}

\section{4.4 Border Patrol Apprehensions}

The U.S. Border Patrol polices the nation’s international boundaries, seeking to apprehend any individual attempting to enter the U.S. illegally, with most of its activity concentrated at the Mexico border where it is the most common. The importance of Border Patrol for Mexican migration becomes important if we consider the following: there are two ways of working illegally in the U.S.:

i) By entering without inspection (EWI) or

ii) overstaying the duration of a legal visa (Nonimmigrant Overstays). 84\% of undocumented Mexican workers are EWI’s compared to just 9\% of undocumented workers from the rest of the world. In response, the number of Border Patrol hours policing the Mexican border has quadrupled over the last 25 years.
While INS data shows the number of border apprehensions growing from 200,000 in 1970 to 1.68 million in 1998, these numbers are modest when compared to the volume of attempted illegal crossings, with the probability of border apprehension actually peaking in the late 1970’s. Estimated probabilities of apprehension at the border have ranged from 0.2 to 0.4 from the mid sixties to the early nineties, averaging at 0.33 for the period (Massey and Singer (1995)). Figure 4.2 shows the calculated probabilities along with the actual recorded apprehensions by the INS. 

Figure 4.2: Border Apprehension Probability and Total Apprehensions  
Source: Mexican Migration Project, INS

There are two issues concerning border patrol activities:

i) How effective are Border Patrol activities? We see that the percentage of migrants apprehended at the border is only about one third, but more importantly from a

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9 Probabilities are estimated as the ratio of the number of repeat migrants to the total number of apprehensions.

10 The passage of IRCA resulted in a drop in apprehension rates, but this is not attributed as
modelling perspective, ii) how does the possibility of being apprehended affect the migration decision?

Since in the regime switching framework, the choice variable is the regime an agent wishes to be in, it becomes crucial to address the issue of endogeneity of migrating to the U.S. illegally, for a prospective migrant. If border enforcement can prevent the migrant from ever entering the U.S.\(^{11}\), then working in the U.S. must be seen as an “opportunity”, that is completely exogenous to the migrant, and analogous to finding work in the formal sector in the last chapter. However, there is no evidence to support this view. Drawing on data from both sides of the border, the emerging consensus in the literature is that apprehension does not prevent future attempts to cross the border. Once apprehended, most migrants accept an offer of voluntary departure and are merely returned by bus to the nearest border town in Mexico. These apprehended migrants stay in the border towns and repeatedly try to re-enter the U.S. until they succeed (Passel, Bean, and Edmonston (1990); Kossoudji (1992))\(^{12}\). An analysis based on surveys of seven Mexican communities by Donato, Durand and Massey 1992 find that “…the number of attempts at crossing the border was always one greater than the number of apprehensions, that is, all migrants simply tried until they succeeded in entering the U.S. Apprehended or not, every migrant who attempted to enter the U.S. eventually got in.”\(^{13}\) Concluding, the issue of migrating without documents much to lower illegal migration, but the amnesty clause in IRCA, that gave legal papers to about 2 million workers, who could now cross the border without the fear of apprehension (Espenshade (1994)). Morover, apprehension rates picked up again two or three years after IRCA was passed.

\(^{11}\)From a modelling perspective, this would mean working in the U.S. illegally would not be in the choice set of a prospective migrant.

\(^{12}\)The penalty for the first apprehension is the same as any succeeding apprehension, so the direct penalty does not rise with the number of attempts made.

\(^{13}\)The survey data were chosen to represent a broad range of economic bases in Mexico, including several rural agrarian villages, a mining town, a small commercial city, a mid-sized industrial city, a neighborhood in a large metropolitan area, etc. The survey was administered from December to January for 1987-1988 and 1989-1990, where the chosen months reflect times when most temporary and recurrent migrants are at home with their families (there is a huge outflow of Mexican migrants
to the U.S. will be considered as endogenous to a prospective migrant. However, apprehension at the border must figure in some other way in the migration decision, and it is interpreted as increasing the direct and indirect costs of migration. This is taken up in a following section.

4.4.1 Border Patrol:

Implications for the Model

An apprehension at the border does not restrict our choice set since it doesn’t prevent a migrant from entering the U.S. However, border apprehension is costly to a prospective migrant (he spends time in a border town, forgoes wages and may pay a “Coyote” to be smuggled across the border). In the model, a higher probability of border apprehension affects the migration decision by increasing the expected cost of migrating without documents.

4.5 Cost of Migrating without Documents

A migrant may choose to cross the Mexico-U.S. border with or without the help of a “coyote” or a “people smuggler”. The following information on coyotes is from Kassoudji (1992): A coyote can be hired in a migrant’s hometown or along the border and he typically accompanies the migrant to his ultimate destination using rafts, drainage pipes and experience of the desert. Many of the Coyotes are modern day descendants of railroad recruiters with strong networks on both sides of the border, supplying a specific number of migrants to a specific employer. The reported costs of hiring coyotes range from $400 to $700 depending on the final destination (Or-

from the U.S. at these times due to Christmas, etc.).
Though there has been an ever increasing use of coyotes as a percentage of crossings since the 1970’s, there is only a small statistically significant effect of stricter border enforcement on coyote use (Massey et al. (1992)). However, some works contend that using a coyote on a subsequent trip is a direct cost of being apprehended previously (Espenshade (1994), Orrenius (2001)). “There is general agreement that being apprehended at the border raises both the direct and the opportunity cost of undocumented migration. A border apprehension extends the duration of an undocumented trip and increases the expenses associated with reaching \textit{el Norte}, including possibly the expense of hiring a coyote after repeated failures to enter the United States.” (Espenshade (1994))

4.5.1 Border Apprehension and Coyote Costs: Implications for the Model

For modelling purposes, two simplifying assumptions are made:

i) If a migrant is apprehended at the border, he hires a coyote at the associated cost and successfully crosses over. Apprehension increases the normal costs of moving by the amount paid to the coyote.

ii) The cost of moving from U.S. to Mexico is the same as moving from Mexico to the U.S. \textit{if} one is not apprehended at the border. Hence,

\begin{equation}
\text{Expected cost of moving to the U.S.} = P_A(\text{Coyote Cost} + C_{MU}) + (1 - P_A)(C_{MU})
\end{equation}

where $P_A$ is the probability of being apprehended at the border and $C_{MU}$ is the cost of moving to the U.S. if not apprehended (same as that of moving from the U.S. to Mexico).

\footnote{Some reported coyote costs are: Houston $500, San Antonio $450, Los Angeles $700.}
4.6 Working without Documents inside the U.S.

Once a migrant has successfully crossed the border with or without the use of a coyote, there is a small probability per unit of time that he is apprehended in the interior and deported to Mexico and another small probability that he instead acquires legal status. Either of these can be considered a forced switch to a new regime, with the regimes being Mexico and working legally in the U.S., respectively.\footnote{While the former regime is in the choice set of the undocumented migrant (he can return to Mexico whenever it is optimal), the second regime is outside his choice set and may become available due to factors outside his control.}

4.6.1 Being Apprehended inside the U.S.:

Implications for the Model

Most of the INS effort is concentrated at the border to deter illegal entry. However, once an illegal migrant is inside the U.S., apprehensions by INS authorities are much smaller as compared to border apprehensions. For instance, between 1986-1998, the number of interior apprehensions were estimated as being between 3-8 percent of the total undocumented workforce. Poor enforcement of employer sanctions mandated by IRCA is also acknowledged by the INS, with inadequate funding for work-site raids so that enforcement in the interior of the U.S. is much more lax than at the border. This policy is often described as “once you’re in, you’re in” (Orrenius (2001)).

Even though the chances of being caught inside the U.S. are small, it is included in the model and is interpreted as the probability that a migrant is forced to switch regimes by being sent back to Mexico. In the light of recent events, if this probability increases in the future with more INS funds allotted to apprehending undocumented aliens in the interior, policy questions may be addressed using this model.
4.6.2 Acquiring Legal Status inside the U.S.:

Implications for the Model

Figure 4.3: Percentage of Undocumented Workers Legalized Annually
Source: Mexican Migration Project

Once a migrant makes it across the border and is working in the U.S. illegally, there is a chance per unit of time that he acquires legal status. The model assumes that this probability is exogenous to the migrant and completely policy determined. For example, when the Govt. grants Amnesty to large sections of the undocumented labor force as under the IRCA, this probability increases\(^\text{16}\). Figure 4.3 shows the percentage of undocumented workers legalized annually, which is taken as the proxy

\(^{16}\)In reality, this probability is dependant on migrant specific characteristics like having a son or daughter in the U.S., etc, but is ignored in the aggregate framework of analysis. Another option is to make this probability time dependant, say as going up with time spent in the U.S. Unfortunately, this completely changes the nature of the problem as this probability would now have to be treated as a state variable, and would no longer be a simple parameter. The resulting partial differential equations from the Bellman equation would no longer be tractable with a known analytical solution.
for this probability. Since the model focuses on the decision of an undocumented migrant, we assume (as a prior) that entering the U.S. legally was not an option, so was outside the choice set of an optimizing migrant. However, once across the border, working in the U.S. legally is described as another regime with a small chance every period that the migrant is able to switch to this regime. Once in this regime, a migrant is able to move back and forth between Mexico and the U.S. legally. This phenomenon may be seen as the movement of Mexican migrants with Green Cards between the U.S. and Mexico.

### 4.7 Wage Differences Between Mexico and the U.S.

Since relative returns of being in the U.S. versus Mexico should ultimately drive migration, and there is a possibility that an undocumented migrant acquires legal status in the U.S., a comparison of wages between legal and illegal immigrants becomes important when modelling the decision of an illegal worker.

Estimates of wages earned by undocumented workers have come mainly from survey data, the earliest estimates (North and Houstoun (1976)) showed that undocumented workers earned wages 37% less than legal workers. An identical wage gap was estimated by Massey (1987) in a 1987 survey. Rivera-Batiz (1999) finds that the wage earned by legal workers to be 42% more than illegal immigrants. Whether this gap is due to differences in characteristics between legal and illegal immigrants or suggests discrimination against undocumented workers is still not resolved in the literature.\(^\text{17}\).

Similarly, in the case of legal workers, the differences between the legal U.S. wage

\(^{17}\)Rivera-Batiz suggests the presence of the latter as he finds only 48% of the wage gap explained by measured characteristics like the level of human capital, proficiency in English, etc.
and the Mexican wage would affect the decision to migrate between the two countries.

One must also recognize that it is more than simply wage differences that affect
the migration decision, it seems natural that there is some implicit value to being at
home (due to the obvious familial/cultural ties, etc.) that is simply not captured by
a purely economic model like the one developed in this work.

4.7.1 Wage Differences: Implications for the Model

For this model, I will assume that a prospective undocumented migrant responds
to the relative return between being in the U.S. illegally and being in Mexico. The
wage of the former is assumed as being the same as the legal wage, but with the mi-
grant incurring a cost of being undocumented, creating the estimated wedge between
the return to legal and illegal work. This could be interpreted as the cost of avoid-
ing detection. “Because of their need to be sheltered from detection, undocumented
workers are often employed in marginal jobs or in declining industries that offer com-
paratively low pay” (Rivera-Batiz(1999)). The possibility of more informal contracts
in such marginal industries with payments made in cash are greater, a factor that
helps avoiding detection as these contracts may not leave a paper trail. Hence,

\[
\text{Effective Relative Wage of an Undocumented Worker} = (1 - C_U)W
\]

where \( W \) is the relative U.S. to Mexico wage and \( C_U \) is the per period cost of being
illegal in the U.S. The per period return to working in Mexico is \( W_M = 1 \) as labor in
Mexico is the numeraire.

For the case of legal Mexican workers, we may ignore such detection avoidance
costs so that the

\[
\text{Effective Relative Wage of a documented Worker} = W
\]
4.8 Setup of the Model

i) **Time Horizon:**
An infinite time horizon framework is adopted which makes the problem considerably simpler than a finite time horizon. Without a “terminal” date, the problem for the optimizer becomes identical every period, making the optimization problem independent of calendar date $t$.

ii) **State Space:**
The state space contains variables whose values define the state of nature for the optimizer in any given period. The state space defined for the migration problem is mixed, consisting of one continuous and one discrete state.

The continuous state for this problem $W$, is the relative U.S. to Mexico wage which fluctuates continuously through time.

The discrete state $j$ is the regime the migrant is currently in, which takes on four possible discrete values. Hence, $j = \{MI, UI, ML, UL\}$, where the first and second letter denote location and status respectively. Hence our four regimes are

$MI = $ Mexico without documents to work in the U.S.,
$UI = $ U.S. without documents,
$ML = $ Mexico with documents to work in the U.S. and
$UL = $ U.S. with documents.

iii) **Choice Variable:**
The Choice variable $k$ is also discrete, where $k = \{MI, UI, ML, UL\}$. The Choice set is defined in the most general form, when in fact this choice set may be different for migrants starting out in different regimes. For illustration, in any given period $t$, say the state of nature is $W_t$ (the current relative wage) and $j = MI$ (or that the migrant is currently in Mexico without U.S. work authorization), he may choose optimally from $\{MI, UI\}$, i.e, he either decides to stay in Mexico or migrate to the U.S. to work illegally.
In the model, the choice set is technically restricted for migrants starting out in different regimes by defining the costs of moving to some other regimes as infinity. This effectively makes some moves impossible by assumption.

Hence the discrete state is the regime a migrant is currently in, while the optimal value of our discrete choice variable is the regime a migrant moves to. This difference becomes clearer in a discrete time exposition:
The current discrete state is the migrant’s location at the beginning of this period. The optimal discrete choice made this period is where the migrant will be at the beginning of next period, which then becomes the discrete state for that period.

iv) State Transition Equations:
The State transition equations govern the behavior of the state variables over time.

The continuous state $W$ is governed by the stochastic process:

$$dW = \sigma W d\omega$$

(4.1)

which is a geometric Brownian motion without drift. Justification for this assumption is made in section 4.9.

Describing the state transition for $j$ in discrete time is simply that in the next period, its value is $k$, or the regime at the beginning of the next period is that which is optimally chosen today.

v) The Reward Function:
The reward function gives us the per-period return to being in a particular regime. For our case, the reward function associated with a regime is simply the relative wage earned in that regime.

Being in the U.S. without documents earns the migrant a per period effective return:

$$W_{\text{undocumented}} = W_{UI} = (1 - C_U)W$$
where $C_U$ is the per period cost to being undocumented.

Working in the U.S. legally earns the migrant the following return:

$$W_{\text{documented}} = W_{UL} = W$$

The reward function in Mexico (whether one has U.S. work authorization or not) is

$$W_{\text{Mexico}} = W_{MI} = W_{ML} = 1$$

since it is the Numeraire.

So while $W$ is stochastic, by definition $W_M$ will be fixed. Since it is finally income differentials/relative wages that should drive the migration decision, we are allowing this differential to be stochastic and change over time. We can write our reward function more generally as $f(k, W) = W_k$, so that $f(MI, W) = 1$, $f(ML, W) = 1$, $f(UI, W) = (1 - C_U)W$ and $f(UL, W) = W$.

v) The Cost of Regime Switches:

For a legal worker, the cost associated with moving to U.S. from Mexico $C_{UM}$ is the same as $C_{MU}$, the cost of moving from Mexico to U.S. and is known with certainty.

For an illegal immigrant, the expected cost of moving from Mexico to the U.S. depends on the probability of apprehension at the border, $P_A$:

Expected cost of moving to the U.S. = $P_A(C_{\text{Coyote Cost}} + C_{MU}) + (1 - P_A)(C_{MU})$

This formulation assumes that if apprehended at the border, the migrant hires a Coyote to successfully cross over, so that the normal costs ($C_{MU}$) are augmented by what he pays the Coyote. If not apprehended then he crosses the border incurring only $C_{MU}$. This is discussed in more detail in Section 4.5.1. Since the Coyote costs are always greater than $C_{MU}$, we may write the illegal moving cost as $C_{MU} + C_I$ so that the undocumented move in expected value is always greater than the normal costs of moving, where $C_I = P_A(C_{\text{Coyote Cost}})$. 
The switch cost assumptions between the four regimes is summarized in the following Matrix, where a typical element $C_{ij}$ is the cost of switching from regime $i$ to regime $j$.

$$C = \begin{bmatrix}
MI & UI & ML & UL \\
MI & 0 & C_{MU} + C_I & \infty & \infty \\
UI & C_{UM} & 0 & \infty & \infty \\
ML & 0 & \bar{C}_{MU} & 0 & C_{MU} \\
UL & \bar{C}_{UM} & 0 & C_{UM} & 0
\end{bmatrix}$$

The diagonal elements are of course zero since they represent the cost of not switching. The costs represented by $\infty$ show the switches that are not possible unless an exogenous event reduces these costs to zero. For instance, one cannot move from the UI to the UL regime by choice unless an exogenous event allows it. The Costs represented with bars above them are costs that are irrelevant since those switches are never optimal, even at zero cost, for instance, one does not become illegal once one acquires legal documents.

viii) **Exogenous switches between regimes:**

Once a migrant is working in the U.S. without documents, there is a small probability that he will acquire legal status, this probability is denoted as $P_{UL}$. If this event occurs, then from the UI regime, the migrant is switched to the UL regime at zero cost. There is also a small probability that he is apprehended by the INS and deported back to Mexico, denoted by $P_{UM}$. In this case, from the UI regime, the migrant is forced to move to the MI regime.
The exogenous switching probability matrix may be summarized as follows:

\[
\Lambda = \begin{bmatrix}
MI & UI & ML & UL \\
MI & 0 & 0 & 0 & 0 \\
UI & P_{UM} & 0 & 0 & P_{UL} \\
ML & 0 & 0 & 0 & 0 \\
UL & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

ix) **Other Parameters:**
The other parameters in the model are \( \rho \) and \( \sigma \) which are assumed to be fixed across time and invariant with respect to both the continuous and the discrete state.

## 4.9 Geometric Brownian Motion and Relative Wage

In assuming a geometric Brownian motion for our state variable, the relative U.S. to Mexico wage, we are assuming that this variable, \( W \) follows a random walk, or that there is complete persistence of shocks in the level of relative wages. While geometric Brownian motion makes the analysis considerably simpler, a justification for this specification has to be made over other specifications that also lead to analytically tractable solutions for the value functions, for example a mean reverting (stationary) process where the relative wage would tend to revert to some *normal* level. Economic theory dictates that over the long run, the relative wage should regress to some proportion of the relative marginal products of unskilled labor in the two countries (given less than complete integration of labor markets). This ratio, in turn is affected by numerous factors like the capital labor ratios, rates of technological advance, etc, variables that may or may not show persistence in their time series properties. Hence, it is very difficult to predict the properties of the distribution of relative wages.
A simple plot of the data points towards a difference stationary random walk representation. A plot of the first differences in the relative wage (see Figure 4.4) appears to be stationary. Empirically, both the Augmented Dickey Fuller’s test and the Phillips Perron test fail to reject the presence of a unit root even at the 15% significance level. This result is robust to many different specifications (without an intercept term, with an intercept term, with an intercept term and a time trend, with each specification tested for AR orders of up to 10). Hence, twenty years of data (with one observation every two months) of the relative wage do not support stationarity. One possible reason may be that even if the relative wage is mean reverting, the rate of mean reversion is so small that 20 years of data do not capture this reversion. For the time period under consideration, the assumption of geometric Brownian motion over mean reversion is borne out by the time series properties of relative wages.
4.9.1 Regions of the State Space:

Having discussed the general approach to solving such models in Chapter 2 and working through the specifics for the Rural-Urban migration case, the solution approach is not presented in great detail in this chapter. Instead, a complete discussion on the regions of the state elaborating the basic nature of the problem is presented below,

\( W \), our continuous state variable representing the U.S. to Mexico relative wage can take on the range of values in its state space. The nature of the problem and the solution can be illustrated by breaking up this state space into different regions, each one associated with a different optimal reaction to the current state of nature - the value of \( W \) and \( j \) (whether the migrant is currently in Mexico or the U.S. and under
The regions of the state space are separated by the trigger values of $W$. If the wage is higher than the lower trigger, the migrant stays in the U.S., but if it hits the lower trigger, the migrant returns home. The triggers are different for undocumented than undocumented workers. Clearly, the U.S. to Mexico relative wage would have to be higher for an undocumented than a documented worker to cross the border given that the former would possibly incur a higher cost to do so and accept a lower effective wage (due to the cost of working undetected by the INS). In general, if the wage is lower than the upper trigger, a prospective migrant stays in Mexico and once it hits the upper trigger, he migrates to work in the U.S. Referring to Figure 4.5, the regime of the migrant is given on the left hand side, while the bold vertical lines mark a regime switch when the wage reaches that point so that the trigger wages are illustrated by $W_{12}, W_{21}, W_{34}$ and $W_{43}$.

Consider a migrant starting out in the first regime MI. In the region MI 1 for relative wage in the range $[0, W_{12}]$ the migrant stays in Mexico so that his value function satisfies:

$$\rho V_1(W) = 1 + \frac{1}{2} \sigma^2 W^2 V_1''(W)$$

In the region MI 2 for relative wage in the range $[W_{12}, \infty]$, the value function must satisfy

$$V_1(W) = V_2(W) - C_{12}$$

where $C_{12}$ is the cost of moving without documents to the U.S., discussed in detail earlier.

Now consider the case where the migrant starts out in the UI regime. If the wage falls below $W_{21}$ into the UI 1 regime, the migrant returns to Mexico so that his value
function satisfies:

\[ V_2(W) = V_1(W) - C_{21} \]

In the UI 2 and UI 3 regions, a migrant stays in the U.S. However, the reason we break up the the range of wages from \([W_{21}, \infty]\) into UI 2 and UI 3 is that in the region UI 2 (wage range \([W_{21}, W_{12}]\)), if a migrant is caught working without documents and is deported, he stays in Mexico. The value function for being in the U.S. without documents then satisfies:

\[
(\rho + \lambda_{21} + \lambda_{24})V_2(W) = (1 - C_u)W + \frac{1}{2}\sigma^2W^2V''_2(W) + \lambda_{21}V_1(W) + \lambda_{24}V_4(W)
\]

However, in region UI 3 for wages in the range \([W_{12}, \infty]\), if a migrant is deported to Mexico, he returns to the U.S. at the associated cost.

The ML and UL cases are analogous to MI, so that in the region ML 1 (on \([0, W_{34}]\)), the value function satisfies:

\[
\rho V_3(W) = 1 + \frac{1}{2}\sigma^2W^2V''_3(W)
\]

while in the region ML 2 (on \([W_{34}, \infty]\)), the value function satisfies:

\[ V_3(W) = V_4(W) - C_{34} \]

In the region UL 1 (on \([0, W_{43}]\)), the value function satisfies:

\[ V_4(W) = V_3(W) - C_{43} \]

and in the region UL 2 (on \([W_{43}, \infty]\)), the value function satisfies:

\[
\rho V_4(W) = W + \frac{1}{2}\sigma^2W^2V''_4(W)
\]
Note that we may have a possible paradoxical result where if a migrant in the UI regime acquires legal status, he would immediately return to Mexico. This would happen if $W_{43}$ were above $W_{21}$ and the wage lay in this region at the time the migrant acquired legal papers. However, it is unlikely $W_{43}$ would exceed $W_{21}$, or that a legal worker would return home before an undocumented worker in response to a wage decrease. This is because the illegal worker makes a fraction of the legal wage in the first place and the costs of returning for undocumented workers are not significantly greater than for documented workers. As we will see in our estimation, this prediction is borne out by the parameters of the data and intuitive ordering of the trigger wages given in the figure above hold.

4.10 The Solution:

The solution will comprise:

a) The four trigger wages $W_{12}^*, W_{21}^*, W_{34}^*$ and $W_{43}^*$.

b) The five value functions

i) $V_1(W)$ on $[0, W_{12}]$

ii) $V_2(W)$ on $[W_{21}, W_{12}]$

iii) $V_3(W)$ on $[W_{12}, \infty]$

iv) $V_4(W)$ on $[0, W_{34}]$

v) $V_4(W)$ on $[W_{43}, \infty]$

Following the methodology detailed in Chapter 3 for solving second order differential equations in a variable that follows Geometric Brownian Motion, we get the following:

$$V_1(W) = \frac{1}{\rho} + A_m W^{\beta m} \text{ on } [0, W_{12}]$$
The first part of the value function is the discounted stream of relative wages earned in Mexico while the second reflects the value of the option to Migrate to the U.S. in the future if conditions make migration optimal.

\[
V_2(W) = \frac{(1 - C_u)\rho + \lambda_{21}}{\rho(\rho + \lambda_{21} + \lambda_{24})} W + \frac{\lambda_{21}}{\rho(\rho + \lambda_{21} + \lambda_{24})} + \frac{\lambda_{21}}{\rho(\rho + \lambda_{21} + \lambda_{24})} A_m W^{\beta_m} \\
+ \frac{\lambda_{24}}{\lambda_{21} + \lambda_{24}} A_U W^{\beta_u} + A_1 W^{\beta_1} + A_2 W^{\beta_2} \text{ on } [W_{21}, W_{12}]
\]

On the right hand side of the Equation, the first term expanded as

\[
\frac{W(1 - C_u)}{\rho + \lambda_{21} + \lambda_{24}} + \frac{\lambda_{24}W}{\rho(\rho + \lambda_{21} + \lambda_{24})}
\]

reflects the a) the present value of the undocumented wage stream, where we now have an augmented discount rate, where the usual discount rate \(\rho\) is augmented by the Poisson intensities that reflect the possible movement away from the current regime and b) The probability of becoming legal multiplied by the discounted legal wage stream one would receive when legalization occurs, which is then discounted to the current period using the augmented discount rate.

The second term on the right hand side is completely analogous to the one just described above, but in case of deportation to Mexico (where the discounted wage stream is \(1/\rho\)). Taking them together would reflect the expected value received from being exogenously switched, discounted to the current period (but not including the option value of moving away from those regimes).

The third and fourth terms reflect the conditional probabilities of being switched to each of the alternative regimes multiplied by the option value of moving out of those regimes in the future, if optimal.

The fifth and sixth term together reflect the option value of moving out of the UI regime if again, it becomes optimal to do so.

Similarly,
\[ V_2(W) = \frac{(1 - C_u)\rho + \lambda_{24}}{\rho(\rho + \lambda_{24})} W - \frac{\lambda_{21}C_{12}}{\rho + \lambda_{24}} + A_U W^{\beta_u} + A_3 W^{\beta_3} \] on \([W_{12}, \infty)\)

\[ V_3(W) = \frac{1}{\rho} + A_M W^{\beta_m} \] on \([0, W_{34})\)

\[ V_4(W) = \frac{1}{\rho} W + A_U W^{\beta_u} \] on \([W_{43}, \infty)\)

where (following the exact methodology of the last chapter) \(\beta_m\) and \(\beta_u\) are the positive and negative roots of

\[ 0 = \rho - \frac{1}{2}\sigma^2\beta(\beta - 1) \]

\(\beta_1\) and \(\beta_2\) are the two roots of

\[ 0 = \rho + \lambda_{24} + \lambda_{21} - \frac{1}{2}\sigma^2\beta(\beta - 1) \]

and \(\beta_3\) is the negative root of

\[ 0 = \rho + \lambda_{24} - \frac{1}{2}\sigma^2\beta(\beta - 1) \]

The four value functions defined (in different regions of the state space) above have six undetermined coefficients, \(A_1, A_2, A_3, A_m, A_M, A_U\). We impose six side conditions to determine these:

i) \(V_2(W_{21}) = V_1(W_{21}) - C_{21}\)

ii) \(V_1(W_{12}) = V_2(W_{12}) - C_{12}\)

iii) \(V_2(W_{12}^-) = V_2(W_{12}^+); iv) V_2'(W_{12}^-) = V_2'(W_{12}^+);\)

v) \(V_4(W_{43}) = V_3(W_{43}) - C_{43}\)

vi) \(V_3(W_{34}) = V_4(W_{34}) - C_{34}.\)

The first, second, fifth and sixth are the usual value matching conditions while the third and the fourth impose continuity and differentiability of \(V_2\) at \(W_{12}\) that separates regions UI 2 and UI 3.
To determine the locations of the switch points, we impose the four smooth-pasting conditions:

i) \( V'_1(W_{21}) = V'_2(W_{21}) \)

ii) \( V'_1(W_{12}) = V'_2(W_{12}) \)

iii) \( V'_3(W_{43}) = V'_4(W_{43}) \)

iv) \( V'_3(W_{34}) = V'_4(W_{34}) \)

Unfortunately, since all of these 10 equations are highly non-linear functions of \( W \), we do not have an analytic solution. We can however, pass this system of 10 non-linear equations through a root finding algorithm such as Broyden, to solve for our 10 unknowns. Together these unknowns completely determine the four value functions (over all the regions of the state space) and the four trigger points in our model. This is discussed in more detail in the next chapter.
Chapter 5

Description of the Data and Numerical Methods

The first part of this chapter describes the data sets and sources used to assign parameter values in the numerical estimation of the model. These parameter values are used in the preliminary estimation and then varied one at a time to see the implications of the model on the migration decision, by analyzing their numerical effect on the trigger wages. The data set used spans the period 1980 to 1995.

To solve the model in terms of the five value functions (in six unknowns) and the four trigger wages, we need to solve the set of ten non linear equations implied by the conditions outlined in Section 4.10. The second part of the chapter briefly describes the Broyden root finding algorithm that may be used to solve such a set of non linear equations.
5.1 Parameter Values and Data Sources

5.1.1 $\sigma$

Wage and price data for Mexico are from the IMF International Financial Statistics (IFS). This wage series is a monthly index of the nominal average hourly wage of production labor in manufacturing. This is deflated by the Mexican CPI to get real wages. Following Hanson and Spilimbergo’s (1996) work on relative wages and illegal immigration from Mexico, the choice of manufacturing wage as the representative wage for prospective migrants is justified by the similarity of educational levels of Mexican migrants in the U.S. to manufacturing workers in Mexico.

The U.S. wage series used is one constructed by Hanson and Spilimbergo and was kindly provided by the authors. This wage series is constructed to represent the wage a prospective migrant expects to earn if he or she successfully crosses the border. They construct an expected hourly wage based on the labor-force participation of Mexican-born individuals in the United States. For the raw wage series, they use the U.S. Bureau of Labor Statistics data on monthly average hourly wages for production labor in seven nonagricultural U.S. industries: construction, manufacturing, transportation, wholesale trade, retail trade, finance/insurance/real estate, and services. They then calculate the expected U.S. wage as the weighted-average hourly wage in these industries, where they use the industry share of non-agricultural Mexican-born workers as weights. The weights used are calculated using employment data on Mexican-born individuals from the Public Use Microsample of the 1980 and 1990 U.S. Census of Population. The weight is calculated as the average industry share of Mexican-born employment in the two sample years. This is then deflated by the U.S. CPI.
The assumed distribution for the relative U.S. to Mexico wage $W_U$ is geometric Brownian motion which implies that the change in the logarithm of $W_U$ is normally distributed with variance $\sigma^2T$ where $T$ is the total number of time periods in consideration. To find an estimate of $\sigma^2$, a new series measuring the change in $\log(W_U)$ is constructed. This series spans 1980-1995 and is measured every two months. The annual variance is then estimated by multiplying the variance of this series by 6. The estimate of the standard deviation $\sigma$ is then estimated to be 0.06.

### 5.1.2 $P_A$

Massey and Singer (1995) estimate the probabilities of apprehension at the border using data from the Mexican Migration Project (MMP). The MMP is a collaborative research project based at the Population Studies center at the University of Pennsylvania and the University of Guadalajara. It is the result of an ongoing multidisciplinary study of Mexican Migration to the United States and contains data gathered since 1982 in surveys administered every year in Mexico and the United States. The MMP Database is currently one of the most concise and vast data set of its kind in existence. It comprises of 71 communities with more than 11,000 households surveyed in Mexico and more than 700 households surveyed in the United States.

As described in the literature review, the common observation is that a prospective migrant repeatedly tries to enter the United States until he or she succeeds. Using this assumption, they calculate the probability of apprehension as $P_A = R/A$ where $R$ is the number of repeat migrants (those who have been apprehended two or more times), and $A$ is the total number of apprehensions. This estimate is constructed using the repeated trials model proposed by Espenshade (1994), illustrated by table 5.1.2. If $F$ is the size of a cohort making the first attempt to cross the border in a given period,
Table 5.1: Repeated Trials Model for Calculating Probability of Apprehension, Espenshade (1994)

<table>
<thead>
<tr>
<th>Attempt Number</th>
<th>Size of Group Making the Attempt</th>
<th>Number Apprehended on This Attempt</th>
<th>Number Entering U.S. Successfully on This Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$F$</td>
<td>$F \cdot p$</td>
<td>$F \cdot (1 - p)$</td>
</tr>
<tr>
<td>2</td>
<td>$F \cdot p$</td>
<td>$F \cdot p^2$</td>
<td>$F \cdot p(1 - p)$</td>
</tr>
<tr>
<td>3</td>
<td>$F \cdot p^2$</td>
<td>$F \cdot p^3$</td>
<td>$F \cdot p^2(1 - p)$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$i$</td>
<td>$F \cdot p^{i-1}$</td>
<td>$F \cdot p^i$</td>
<td>$F \cdot p^{i-1}(1 - p)$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>Column sums</td>
<td>$F/(1 - p)$</td>
<td>$Fp/(1 - p) = A$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

A fraction $p$ of this cohort is apprehended and $F(1 - p)$ evade detection to successfully cross the border, leaving $Fp$ to make a second attempt. On the second attempt, $Fp^2$ are apprehended and the remaining $Fp(1 - p)$ cross successfully. The model assumes that all migrants making the attempts eventually cross over successfully. The INS has data on the total number of apprehensions - the sum of column 3, and the number of repeat offenders (apprehended two or more times) - the sum of column 3 starting from row 2. This gives us

$$R/A = \left\{ \frac{Fp^2/(1 - p)}{Fp/(1 - p)} \right\} = P_A$$

Taking an average over our time period yields an estimate of 0.33 for $P_A$.

### 5.1.3 $C_U$

The cost to being undocumented in the U.S. is assumed to be reflected as a wage gap between legal and illegal Mexican migrants. Three major studies done over a
period of three decades find very similar estimates for this wage gap. The first of these is a survey of 793 illegal immigrants apprehended by the INS undertaken by North and Houston (1976) in 1975 which found that illegal migrants earned 37% less than the average industry wage. An identical wage gap is estimated by Douglas Massey in a 1987 survey of 232 immigrants in their region of origin in Mexico. Finally, Rivera-Batiz (1999) finds this gap as 41.8% using information provided by a national sample of illegal immigrants surveyed by the U.S. Department of Labor. This data set is the Legalized Population Survey (LPS of 1989-1992) consisting of 6,193 illegal immigrants residing in the U.S. in 1987/1988, who sought permanent residence through the IRCA (Immigration Reform and Control Act) in 1986. Based on the above mentioned works, an estimate of 0.63 for $C_U$ is used for the numerical analysis. Rivera-Batiz finds that human capital, occupational and demographic characteristics explain about 48% of this wage gap, so the more conservative estimate of the wage gap was chosen for the analysis.

5.1.4 $\lambda_{UL}$

The proxy used for the probability of acquiring legal status $P_{UL}$ is the number of Mexican born persons who become legal residents as a percentage of the total Mexican born population in the Mexican Migration project. From this probability, the Poisson intensity was estimated and used in the numerical analysis for $\lambda_{UL}$. Note that if the Poisson intensity of a legalization is $\lambda_{UL}$, then the probability that a legalization occurs is $1 - \exp^{-\lambda_{UL}}$. This is using the properties of the Poisson distribution, where the latter is simply 1 - the probability that no legalization occurs. As the intensity varies from 0 to infinity, the corresponding probability varies from 0 to 1. The MMP data set constructs this variable through the following ratio. The numerator is composed
of both non-IRCA and IRCA related legalizations. The denominator used is based on the U.S. Census Bureau for the three censal years in the project (1970, 1980 and 1990), with a second degree polynomial fitted to these data points to determine the population in the inter-censal years. An average for our time span yields an estimate of 0.027 for this probability $P_{UL}$ in the probability matrix. The corresponding $\lambda_{UL}$ is about 0.0274.

5.1.5 $\lambda_{UM}$

For the probability of interior apprehension, the proxy used is the number of arrests of illegal immigrants in the U.S. interior as a percentage of the total estimated undocumented population. Espenshade (1994) puts this average annual probability ($P_{UL}$ in the probability matrix) at 1-2%, and the estimate of the Poisson intensity $\lambda_{UM}$ used is 0.02.

5.1.6 $CoyoteCost, C_{MU}(C_{UM})$

All the migration cost variables are specific to the geographic location of the destination region, so a rough average is used to aggregate across regions. The average Coyote cost used for the numerical analysis is $500 (based on an average of rates mentioned in Kassoudji, 1992), and $C_{MU}(C_{UM})$ is assumed $100$. This is a very rough average\footnote{Given that travel costs vary significantly depending on mode of travel, distance crossed, etc.} taken from an informal survey of bus fares (obtained from various web sites) from central Mexico (where a large part of the migrating population originates) to some of the major border towns, and then an average of travel costs to approximately 300 miles north of the border town on the U.S. side of the border. This average was based on Greyhound bus fares in the U.S. The total cost of the this trip was found
to be roughly $100.

Both of these costs are divided by the average annual Mexican wage earnings. This is to be consistent with the specification of the problem in terms of the Mexican wage, so these costs may be interpreted as opportunity costs, in terms of lost hours of work in Mexico over the time period in consideration.

5.2 Broyden’s Root Finding Method

The exposition of Broyden’s Method in this section is taken from Fackler and Miranda (2002).

In solving a set of nonlinear equations, the problem can be viewed as a nonlinear root finding problem, where we have a given function $f$ that maps $\mathbb{R}^n$ to $\mathbb{R}^n$, and we need to find the $n$-vector $x$ which is the root of $f$ such that:

$$f(x) = 0$$

Analogously, our set of 10 nonlinear equations conditions laid out at the end of chapter 4 can be put more generally in the following form:

i) $V_2(W_{21}) - (V_1(W_{21}) - C_{21}) = 0$

ii) $V_1(W_{12}) - (V_2(W_{12}) - C_{12}) = 0$

iii) $V_2(W_{12}^-) - V_2(W_{12}^+) = 0$

iv) $V_2'(W_{12}^-) - V_2'(W_{12}^+) = 0$

v) $V_4(W_{43}) - (V_3(W_{43}) - C_{43}) = 0$

vi) $V_3(W_{34}) - (V_4(W_{34}) - C_{34}) = 0$

vii) $V_1'(W_{21}) - V_2'(W_{21}) = 0$

viii) $V_1'(W_{12}) - V_2'(W_{12}) = 0$

ix) $V_3'(W_{43}) - V_4'(W_{43}) = 0$
x) $V'_3(W_{34}) - V'_4(W_{34}) = 0$

where we must find the 10x1 vector $x = [A_1, A_2, A_3, A_m, A_M, A_U, W'^*_{12}, W'^*_{21}, W'^*_{34}, W'^*_{43}]^T$ that satisfies the above.

Newton’s method replaces the nonlinear problem with a series of simpler linear problems whose solutions converge to that of the nonlinear one and most rootfinding methods are variations of it. Newton’s method takes a starting value provided by the user for the root of function $f$. Subsequent iterates are computed by solving a linear rootfinding problem by replacing the nonlinear function $f$ with its linear approximation obtained by its first order Taylor approximation about the previous iterate. Hence, for a univariate rootfinding problem, given a guess for the root $x^{(k)}$, $f$ is replaced by its linear approximation about $x^{(k)}$,

$$f(x) \approx f(x^{(k)}) + f'(x^{(k)})(x - x^{(k)}) = 0$$

giving us the iteration rule:

$$x^{(k+1)} = x^{(k)} - \frac{f(x^{(k)})}{f'(x^{(k)})}$$

One drawback of the Newton method is that both the function and its derivative must be evaluated at every iteration, and in many cases the derivative may be inconvenient or expensive to evaluate. The secant method is a variant of the Newton method that replaces $f'(x^{(k)})$ with its finite-difference approximation from the function values at the previous two iterates, yielding the iteration rule:

$$x^{(k+1)} = x^{(k)} - \frac{x^{(k)} - x^{(k-1)}}{f(x^{(k)}) - f(x^{(k-1)})} f(x^{(k)})$$

Consider the graphical illustration of the secant method in Figure 5.1: Firstly, two distinct guesses of the root - $x(0)$ and $x(1)$ are provided to start the algorithm. The
function is then approximated by the secant line passing through \( x(0) \) and \( x(1) \), whose root - \( x(2) \) is the new estimate of the root of \( f \). The next step gives us \( x(3) \) as the root of the secant passing through \( x(1) \) and \( x(2) \), thereby the new estimate of the root of \( f \). Hence the secant method approximates the function \( f \) by the secant line through the previous two iterates.

Broyden’s method is a multivariate generalization of the secant method, and the algorithm generates at every iteration a vector that approximates the root of \( f \) and a matrix that approximates the Jacobian of this function at this root. \( x^{(0)} \), an initial guess of the root and \( A^{(0)} \), an initial guess of the Jacobian are needed to start the algorithm\(^2\). Again, a linear rootfinding problem is solved by replacing \( f \) with its first

\(^2\)Often, \( A^{(0)} \) is set equal to the numerical Jacobian of \( f \) at \( x^{(0)} \)
order Taylor approximation about the last iterate for the root, say $x^{(k)}$:

$$f(x) \approx f(x^{(k)}) + A^{(k)}(x - x^{(k)}) = 0$$

The root approximation rule for Broyden is:

$$x^{(k+1)} \leftarrow x^{(k)} - (A^{(k)})^{-1} f(x^{(k)})$$

The Broyden method then updates the Jacobian approximant by the following iteration rule:

$$A^{(k+1)} \leftarrow A^{(k)} + \frac{(f(x^{(k+1)}) - f(x^{(k)}))}{(x^{(k+1)} - x^{(k)})^T} \left( x^{(k+1)} - x^{(k)} \right)$$

The algorithm “Broyden”\(^3\), used for the numerical estimation uses an “inverse update” rule. This generates a sequence of root vectors $x^{(k)}$ and matrices $B^{(k)}$ that approximate the inverse of the Jacobian, rather than the Jacobian itself. This method is computationally more efficient than the Jacobian update and most implementations of Broyden’s methods use this strategy.

\(^3\)From the Compecon Toolbox accompanying *Applied Computational Economics and Finance* (2002)
Chapter 6

Numerical Results and Policy Implications

This first part of this chapter briefly presents the numerical results for the trigger wages and the value function associated with each regime/status combination, using the parameter values discussed in Chapter 5.

The second part of this chapter presents the results obtained by changing these parameter values one at at time, so as to study the sensitivity of the trigger wages to the parameters of the model. The behavior of the trigger wages helps us analyze the implications of the regime switching model on the migration decision of illegal migrants on both sides of the border and provides some general policy implications.

6.1 Numerical Estimation

The first step is to impose the value matching and smooth pasting conditions discussed in chapter 5, using parameter values from the data. The estimated four trigger values are given in the following figure, first introduced in the discussion on
the regions of the state space. The discussion is not repeated here but the figure is reproduced with the exact trigger values to keep in perspective the original problem posed.

<table>
<thead>
<tr>
<th>MI</th>
<th>MI 1</th>
<th>W_{12}^* = 1.9825</th>
</tr>
</thead>
<tbody>
<tr>
<td>UI</td>
<td>UI 1</td>
<td>W_{21}^* = 1.0101</td>
</tr>
<tr>
<td></td>
<td>MI</td>
<td>UI</td>
</tr>
<tr>
<td></td>
<td>UI 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ML</td>
<td>W_{34}^* = 1.1213</td>
</tr>
<tr>
<td></td>
<td>ML 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UL</td>
<td>W_{43}^* = 0.8946</td>
</tr>
<tr>
<td></td>
<td>UL 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UL</td>
<td></td>
</tr>
</tbody>
</table>

**RELATIVE US-MEXICO WAGE**

Figure 6.1: Estimated Trigger Wages

The values of the relative trigger wages tell us that the U.S. wage should rise to more than 1.9 times that of the manufacturing wage in Mexico to prompt undocumented migration. The U.S. wage falling below the Mexican wage will prompt return migration of undocumented workers.

For documented workers, these corresponding values of the relative wage are 1.1 and 0.89.

Secondly, the graphical representation in figure 6.2 makes the comparison of the value functions for each region/status combination for a whole range of relative wages.
6.2 Policy Implications

The third and final step in the numerical analysis, is the variation of parameters to study the sensitivity of the trigger wages in order to derive policy implications from the model. Note that none of these parameters have any bearing on the triggers facing documented migrants, but the discussion of this type of migration is important since undocumented workers face a possibility of having their status changed which makes it another aspect that affects their migration decision.
6.2.1 Probability of Border Apprehension

Varying the probability of apprehension at the border through tighter enforcement increases the Mexico to U.S. trigger wage, thereby deterring undocumented migration. For every 0.1 increase in the probability of border apprehension, approximately an increase of 0.02 in the relative wage is required for the migrant to attempt to cross the border. However, achieving this 10% increase in the current apprehension rate implies approximately 170,000 more arrests by the Border Patrol per year, given the same volume of undocumented attempts.

The total number of line-watch hours of the Border Patrol has increased by around 4% for the sample period. A simple calculation of apprehensions per line-watch hour gives us an approximate average of one apprehension every two line-watch hours,
which implies that line-watch hours would have to rise by around 10% (6% more than the sample average) to achieve the end of increasing apprehensions also by 10%, given the same number of attempts.

The probability of border apprehension also has a deterrent effect on return migration of undocumented workers, since the relative wage must fall further before they return home. This is intuitively obvious since it may be better to stay in the U.S. for longer periods if the expected costs of returning go up.

6.2.2 Apprehension of Undocumented Workers in the Interior of the U.S.

![Effect of the Probability of Interior Apprehension on Trigger Wages](image)

Figure 6.4: Effect of Interior Apprehension

The effect of a stronger INS presence in the interior that increases the probability
of apprehension\textsuperscript{1} inside the U.S. is presented in Figure 6.4. As discussed in the literature review, the INS has been notoriously lax in its implementation of interior policy, mostly due to inadequate funding. This policy variable, however, may take on importance with the increased INS funding proposed in the light of the 9/11 events, as interior INS vigilance casts a wide net when apprehending individuals affiliated to terrorist organizations.

The probability of interior apprehension affects both the trigger wages. It increases the Mexico to U.S. trigger, so it deters migration to the U.S. by decreasing the \textit{expected} returns from migrating. The paradoxical result from the estimation is that it deters return migration as it lowers the U.S. to Mexico trigger (but to a smaller extent). The trigger wages are not well behaved beyond a probability of 0.8 and thus the analysis is presented for probabilities up to the point, this still covers a huge range given that the current probability is around 0.02. The same is true for the probability of acquiring legal status. This is because in this range the U.S to Mexico Trigger for undocumented workers actually falls below the 0.89 range, which is the trigger for legal Mexican workers in the U.S returning home! This will give us the paradoxical result that undocumented workers on achieving legal status immediately return to Mexico! Since this contingency is highly unlikely, it has not been modelled, or the value functions in different regions of the state space have not been defined to include this possibility. Only the possibility that the legal U.S. to Mexico trigger would lie below the corresponding trigger for undocumented workers is included in the analysis. While our estimates of the trigger wages bear out this intuitive reasoning, this contingency may arise in other cases.

\textsuperscript{1}The probability of deportation was allowed to vary from zero to close to one and the corresponding Poisson intensities were used in the analysis. Very high probabilities imply intensities approaching infinity, so the feasible values are used. The values on the X-axis, however are the probabilities, to be consistent with the discussion.
6.2.3 Probability of Border Apprehension vs. Probability of Deportation from the Interior of the U.S.

Figure 6.5: Border Apprehension vs. Deportation from the Interior

To compare the effects of deportation versus apprehension at the border, we can look at the sensitivity of the Mexico to U.S. Trigger to these two variables. Figure 6.5 illustrates their respective effects.

Clearly, increasing the probability of deportation from the interior has a much stronger effect than increasing the probability of border apprehension. Interior apprehension being a bigger deterrent to migration is intuitively obvious, since the costs associated are much higher for an undocumented worker.

For instance, if 8 out of every ten workers in the U.S. was deported from the interior, the Mexico to U.S. trigger would have to be around 2.6 to prompt in-migration.
While the same probability of apprehension at the border merely results in a trigger wage 1.63.

It is also clear, however, that it is much harder to police the interior than the U.S. Mexico border, and a 0.1 increase in the interior apprehension probability may come with such high associated costs than it may still be more efficient to concentrate INS effort at the border. Though not explicitly stated by the INS, this may be the reason that some analysts describe INS policy as “once you’re in, you’re in”.

6.2.4 Cost to Working in the U.S. without Documents

![Graph showing the effect of the cost of working illegally on trigger wages](image)

Figure 6.6: Cost to Working in the U.S. without Documents

While for the numerical analysis, the cost to working without documents $C_U$ is taken as the wage gap between legal and illegal workers, this variable may be defined
to include many indirect costs, like denial of many public benefits to undocumented workers through welfare reform legislation that was passed in 1996.

The effects of this cost variable are illustrated in Figure 6.6. Increasing the cost to being undocumented increases the Mexico to U.S. trigger significantly (as compared to a higher rate of border apprehension) so it has a strong deterrent effect on migration. It also has a strong positive effect on the Mexico to U.S. trigger so results suggest that these costs induce significant return migration of undocumented workers.

For instance, approximately doubling the cost to working illegally ($C_U$) to 0.75 doubles the U.S. to Mexico relative trigger to about 2.25, from the current value of around 1.1.

### 6.2.5 Probability of Acquiring Legal Status in the U.S.

For modelling purposes, the probability of becoming legal was taken as an exogenous policy variable. This was done to avoid the problem from becoming time dependent, so as to get an analytical solution to the differential equation implied by the Bellman. It is of course more realistic to assume that the probability of acquiring legal status is dependent on migrant characteristics. But at least a part of this probability can be considered policy determined, say the granting of amnesty to undocumented migrants under certain regimes, etc.

From Figure 6.7, we see that it decreases the Mexico to U.S. trigger, or encourages undocumented migration (most drastically till the probability hits around 20%, which is approximately 18% more than its current value), but only to a point. The intuition behind this is that the higher the wage gap between illegal and legal workers, the more effective this probability will be. This probability also has a strong negative (but diminishing, for the same reasons) effect on the U.S. to Mexico trigger, or that
it discourages return migration, also an intuitively obvious result.

Consider the probability of acquiring legal status rising to 0.8. The Mexico to U.S. trigger would fall a little below 1.2, so it would encourage in-migration by lowering the trigger from its current value at 2. However, this effect is not much different than if this probability were to be only 0.4, as the trigger wage would then only be around 1.23. The gains from becoming legal maybe underestimated in the model, simply because we may be underestimating its flip side, the costs to being illegal that might be in part unmeasurable.

Summarizing, the policy implications of this model are that interior deportation has the biggest deterrent effect on undocumented migration, when compared to other deterrent measures as increasing the rate of border apprehension, or making it harder
for illegals to get legal status inside the U.S. On that same note, increasing the probability of acquiring legal status (say, through more frequent amnesty grants) is not predicted to have a very big effect on in-migration.
Chapter 7

Conclusion

This work draws on the developments made over the last fifteen years in the treatment of uncertainty in modelling investment decisions which have some degree of irreversibility associated with them. The interaction of irreversibility and uncertainty brings out some interesting implications for optimal behavior in any sort of investment decision, including migration. A few works have applied some of these developments to the case of migration.

In this work, the phenomenon of migration is modelled as “regime switching”, where movements in an underlying state variable may cause an agent to switch to a different regime. The innovation of this work is to add certain features to a standard regime switching model that allows us to closely approximate the features of two labor markets: a typical segmented urban labor market observed in many developing countries, and the market for unskilled Mexican workers in the U.S. These observed features of the labor markets are used to characterize the nature of uncertainty facing migrants in each case. These features are approximated by using a regime switching model with the possibility of exogenous changes in regime.

The first part of this work models the migration decision of rural-urban migrants
in developing countries. The model used for this case is developed in chapter 2, which adds an “opportunity” aspect to a regime switching model that allows the agent to move to another regime at no cost. For this particular case, the prospective rural-urban migrant considers three factors in his decision to migrate - the returns from working in the informal sector, the chances of being picked by the formal sector (an opportunity he may turn down depending on the informal sector wage) and an option to return to the rural sector if conditions in the urban sector turn out to be unfavorable. An important theoretical implication of this setup is that a migrant working in the informal sector derives value from all of these factors and even from the ability to come back to the informal sector from the formal sector if optimal in the future. The issue of future flexibility in the choice of sector of work (and its value to prospective migrants) has not received any attention in the rural urban migration debate, and this work attempts to fill that void, contending that flexibility must feature in the migration decision. This model is not solved numerically due to the paucity of data on various parameters in the model but is more a contribution to the existing rural urban migration literature in the treatment of uncertainty in the migration decision.

The second part of this work also considers the case of exogenous regime changes, where an agent may not choose, but is rather forced to switch regimes (at no cost). The model developed for this case in chapter 4 is a slight variation of that developed in chapter 2. This specification is adopted as it better approximates the second case of interest, undocumented Mexican migration to the United States. Undocumented workers travel back and forth between the U.S. and Mexico, but may be deported back to Mexico from the U.S., an example of an “exogenous regime change”. The model includes most of the relevant factors in this migration phenomenon, for instance the
probability of apprehension at the border, the probability of acquiring legal status inside the U.S., coyote costs incurred for illegal crossings, etc. This model is solved numerically by using parameter values from data, providing some interesting policy implications. As a deterrent to undocumented migration, the model predicts that increasing the chances of deportation from the interior of the is far more effective than apprehension at the border, increasing the costs to being illegal or decreasing the chances of getting legal status.
Bibliography


