ABSTRACT

VALLABH, RAHUL. Modeling Tortuosity in Fibrous Porous Media using Computational Fluid Dynamics. (Under the direction of Dr. Pamela Banks-Lee and Dr. Abdel-Fattah Seyam).

Tortuosity factor is often used to characterize the structure of the pore volume in fibrous porous media. This work involves the determination of tortuosity using computational fluid dynamic (CFD) simulation and particle tracking analysis. A new method has been adopted to generate 3-D geometry for modeling fibrous porous structures using ANSYS® Parametric Design Language (APDL). Computation fluid dynamics has been used to simulate permeability of modeled 3-D fiberweb structures. The simulated permeability results are in good agreement with the models proposed by other authors. The experimental results were found to be slightly higher compared to simulated results and existing models due to the layered configuration of the samples. Permeability is found to be significantly influenced by fiber diameter, and porosity as well as fiberweb thickness. The relationship between air permeability and fiberweb thickness has been used to develop an indirect method for determination of tortuosity factor. Tortuosity factor has also been determined using a more direct method involving CFD simulation and Particle Tracking analysis. Different models established using the direct and indirect methods of determination show that tortuosity is significantly influenced by porosity, fiber diameter and fiberweb thickness, whereas the models available in the literature express tortuosity as a function of porosity only.
Modeling Tortuosity in Fibrous Porous Media using Computational Fluid Dynamics

by
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DEDICATION

To my parents, my sister and friends in grateful recognition of their boundless faith,
encouragement and support.
BIOGRAPHY

Rahul Vallabh was born in Allahabad, Uttar Pradesh Sate, India. He received his elementary and high school education in Bokaro Steel City, India. In 1995, he was admitted to Govt. Sri Krishna Rajendra Silver Jubilee Technological Institute, Bangalore University, Bangalore, India. After four years of study in 1999, he graduated with a Bachelor degree in Textile Technology. In pursuit of further studies, he arrived in United States in August 2003 and began working towards his Masters degree in Textile Technology and Management in North Carolina State University. During this program he worked on thermal barrier properties of needle punched nonwoven materials. He obtained his MS degree in August 2005 and continued to pursue PhD degree in Fiber and Polymer Science.
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Chapter 1 Introduction

In recent decades, fibrous porous media like nonwoven fabrics have emerged as a new class of textiles which are finding new applications and therefore getting increasingly popular. A nonwoven fabric is defined as a sheet or web of individual fibers which are bonded together either by thermal, mechanical or chemical means. Unlike woven and knitted fabrics which are formed of yarns, most nonwovens do not contain yarns and instead are formed of individual fibers (natural or man-made). Yarns, which are the building blocks in woven and knitted fabrics, are arranged in well defined pattern, whereas the arrangement of fibers, which form the building blocks of nonwovens fabrics, is based on orientation distribution function. Therefore, unlike a woven fabric where a geometrically well defined unit is repeated to form the entire fabric, two different areas of a nonwoven fabric may not have identical structures. Therefore internal structure parameters of woven and knitted fabrics can be defined and estimated with high accuracy, whereas it is more difficult to accurately estimate internal structure parameters of nonwoven fabrics. Nonwoven fabrics can be engineered only to some extent, due to limited positive control over structural parameter during formation process.

Nonwovens have found numerous applications in housing, automotive, home furnishings, health care, personal care and hygiene, household, clothing, agricultural and many other industrial and mechanical products.
Nonwovens are being extensively used in products like baby diapers, wipes, thermal and sound insulations, and filters. Transport properties of nonwovens greatly influence the performance of nonwovens for most of these applications. Various structural parameters like the porosity, fiber size, fiber size distribution (especially in natural fibers), pore size and pore size distribution, fiber orientation distribution and pore channel tortuosity influence transport properties of nonwovens.

The internal structure of nonwoven fabrics is a network of interconnected and stand-alone pore channels (Figure 1.1).

![Figure 1.1 Schematic Representation of Interconnected and Stand Alone Pore (Source [1])](image)

Pore channels do not follow a straight line path, but instead follow a tortuous path through the nonwoven structure.
This deviation from the straight line path is quantified by the parameter, pore channel tortuosity, which is defined as the ratio of the actual length of the pore channel to its characteristic length or thickness of fibrous porous media like nonwoven fabric. Pore channel tortuosity describes the geometry of the passage of hydraulic flow, and the channel-network complexity in a porous medium. Tortuosity is an important parameter in nonwoven materials as it influences acoustical and thermal properties. Tortuosity also influences the hydraulic conductivity in porous medium.

While extensive work has been done to determine porosity, pore size distribution and fiber orientation distribution, not much work has been done to determine air channel tortuosity in nonwoven fabrics.

This research aims at developing methods for estimation of air channels tortuosity which is an important parameter influencing the transport properties of nonwoven fabrics. In this work, tortuosity factor will be estimated by means of computer simulation along with an experimental method. The research also aims at studying the relationship between tortuosity and structural parameters of nonwoven fabrics.

References

Chapter 2  Review of Literature

2.1. Introduction

The schematic of tortuous flow paths (or streamtubes) through a porous medium is shown in Figure 2.1.

![Figure 2.1 Schematic of Tortuous Streamtubes through a Porous Medium [1].](image)

Tortuosity is defined by Eq. 2.1

\[ \tau = \frac{L_e}{L} \]

Eq. 2.1
where $L_e$ is the actual length of tortuous streamtubes (or flow path) and $L$ is the characteristic (or straight) length or thickness of a sample along the macroscopic pressure gradient [1].

Tortuosity is related to the typical deviation of the pores in the material from the normal to the material surface [2]. The magnitude of tortuosity can be greater than or equal to unity. Tortuosity has also been defined by some authors [3-5] as the square of the ratio of effective length and characteristic length (thickness) of the samples Eq. 2.2

$$\tau = \left( \frac{L_e}{L} \right)^2$$

Eq. 2.2

2.2. Determination of Tortuosity

The simplest way to model tortuosity is to consider a porous medium having parallel pore channels or streamtubes of fixed diameter, which are inclined such that their axis are at a fixed angle $\theta$ to the normal of the surface of porous media. Tortuosity is then given by Eq. 2.3.

$$\tau = \frac{1}{\cos \theta}$$

Eq. 2.3
However, the angle between the flow lines and normal cannot have a constant value and would vary from point to point along the path, and therefore tortuosity $\tau$ would also vary [6].

### 2.3. Capillary Tube Model

The average velocity, $v$ across the section of a tube can be a useful form of Poisueille Equation as shown in Eq. 2.4 [7].

$$v = -\frac{1}{8\mu} \frac{dp}{dx} a^2$$

Eq. 2.4

In Eq. 2.4, $\mu$ is dynamic viscosity, $a$ is tube radius and $\frac{dp}{dx}$ is the pressure gradient. For tube of any cross-sectional shape, Equation 4 can be generalized to Eq. 2.5.

$$v = \frac{m^2 \Delta P}{k_0 \mu L_e}$$

Eq. 2.5

where $m$ is the hydraulic radius, $L_e$ is the length of the tube, $\Delta P$ is the pressure drop, and $k_0$ is a constant (shape factor). The flow path is random and tortuous in a real porous media, thus the hydraulic radius; $m$ is estimated by dividing the void volume by the wetted surface area of porous medium as given in Eq. 2.6 [7].
where \( V \) is the volume of the porous medium sample, \( \varepsilon \) is the porosity and \( S_0 \) is the specific surface of the porous medium (porous medium surface area per unit volume of porous medium solids) [7].

The average velocity, \( u \) through a porous conduit is described by Darcy’s law [7] which is shown in Eq. 2.7

\[
u = \frac{u}{\varepsilon}
\]

Using Eq. 2.4 - Eq. 2.8, the permeability can be estimated as shown in Eq. 2.9.
\[ k = \frac{\varepsilon^3}{CS_0(1-\varepsilon)^2} \]

Eq. 2.9

where \( S_0 \) is the specific surface area of porous medium, and \( C \) is called the Kozeny constant [7], expressed by Eq. 2.10.

\[ C = \frac{L_e}{L} k_0 \]

Eq. 2.10

2.4. Geometrical Models

Piekaar and Clarenburg [8] theoretically derived tortuosity factor of flow through fibrous porous structures. The derivation was made purely on geometric considerations and involved the following assumptions: (a) all fibers were arranged perpendicular to the direction of flow, with random orientation within the plane, (b) properties of a layer of thickness \( 2\bar{d} \) (\( \bar{d} \) is the mean diameter of fibers) was assumed to represent the whole fibrous structure of thickness \( L >> \bar{d} \), (c) properties of elemental volume of thickness \( 2\bar{d} \) and surface area \( 1^2 \) (\( \bar{d} \) is the mean fiber length), represent the entire volume of the fibrous structure, (d) air molecules take the shortest possible path when passing from one layer to the next, and (e) laminar flow was considered across the thickness of \( 2\bar{d} \), which is too small for a velocity profile to develop.
Assumptions (b) and (c) limit the model to porosities less than or equal to 0.94 which has been mentioned by Piekaar and Clarenburg [8].

In the work by Piekaar and Clarenburg [8], two layers of square array of pores were considered. The position of the first layer was fixed while the second layer was shifted randomly with respect to the first layer. This resulted in two extreme cases; (i) where the pore centers of the two grids are aligned in the flow direction, resulting in tortuosity factor of unity and, (ii) where the center of the pores of the second grid is aligned with the point of intersection of the diagonals connecting the pore centers of the first grid. The transverse distance between the pore centers in the two grids was determined for every position configuration. The transverse distances between the pore centers in various position configurations were averaged and used to determine tortuosity. One of the possible configurations is shown in Figure 2.2. The four pores (in grid two) with center points P are drawn with respect to the pore (of second grid) with center point Q. The position configurations were distinguished into three cases. In the first case, the center point of Q lies in area I shown in, and the entire flow is from Q to P in quadrant (d). In the second case, the center point of pore Q lies in the area II, and so the flow from Q is divided over two pores P. In the third case, the center point of Q lie in area III, and the flow from Q is divided over the four adjacent pores P.
Figure 2.2. Schematic of Two layer Filter Model [8]

The mean transverse distance followed by air molecules, $\overline{U}$, averaged over all possible configuration was calculated. Tortuosity which is the ratio of effective (actual) flow length $L_e$ to depth of the filter $L$, was calculated using Eq. 2.11 [8].

$$\tau = \frac{L_e}{L} = \sqrt{1 + \frac{\overline{U}^2}{4d^2}}$$

Eq. 2.11

Peikaar and Clarenburg [8] calculated the mean transverse distance and found the final expression for tortuosity given in Eq. 2.12 and Eq. 2.13.
\[
\frac{L_e}{L} = \sqrt{1 + \frac{(0.858\sqrt{1.865/\varepsilon} - 1.024)^2 R^2}{4d^2}}
\]

Eq. 2.12

where \( R = 2\overline{m}_{hr} \)

Eq. 2.13

and \( \overline{m}_{hr} \) is the mean hydraulic radius of the pores [8]. Figure 2.3 shows the relation between Tortuosity (determined using Eq. 2.12) and porosity of glass fiber filters with fiber diameter ranging from 0.148 to 2.0 microns. Tortuosity was found to increase with increasing porosity which is contrary to the widely accepted general requirements for tortuosity given by Eq. 2.14 and Eq. 2.15.

\[\tau \geq 1\]

\[\lim_{\varepsilon \to 1} \tau = 1\]

Eq. 2.14

Eq. 2.15
Bo-Ming et al [9] developed a simple geometric model for determining the tortuosity of flow path in a porous media made of particles. They assumed that some of the particles were unrestrictedly overlapped while others were not. They considered two configurations for flow through a porous media of two-dimensional square particles as shown in Figure 2.4.
Figure 2.4. Idealized Configurations of Streamlines for Creeping Flow in Porous Media [9].

The dashed square represents the unit cells having a total volume of $V_t$ (Eq. 2.16).

$$V_t = (b + BF)^2$$ \hspace{1cm} \text{Eq. 2.16}$$

where $b$ is the particle size and $BF$ is the gap size between the particles. $V_{b}$, which is the total pore volume in a unit cell, is given by Eq. 2.17.

$$V_p = (BF)^2 + 2 * BF * b$$ \hspace{1cm} \text{Eq. 2.17}$$
Using the defined volumes above and the definition of porosity, Bo-Ming et al. [9], found the magnitude of gap size $BF$ in terms of porosity, $\varepsilon$, and particle size illustrated by Eq. 2.18.

$$BF = b \left( \frac{1}{\sqrt{1 - \varepsilon}} - 1 \right)$$

Eq. 2.18

The streamline segment $BC$ in the streamline configuration shown in Figure 2.4a was determined by geometric considerations. Owing to the random distribution of particles in the porous media overlapping of particles was considered. Assuming unrestricted overlapping, the tortuosity of the flow is given by Eq. 2.19 (from Figure 2.4a) [9].

$$\tau_a = \frac{BC}{BF}$$

Eq. 2.19

In the case of porous medium with non-overlapping particles, it was assumed that the flow would follow the path $A-B-C-D$ as shown in Figure 2.4b. The tortuosity in that case was given by Eq. 2.20.
\[
\tau_b = \frac{BC + b + BF}{b + BF}
\]

Eq. 2.20

The real tortuosity was then obtained by averaging the two above possible configurations. The model for average tortuosity, \( \tau_{av} \) as shown in Eq. 2.21, was expressed as a function of porosity, \( \varepsilon \) and did not contain any empirical constants.

\[
\tau_{av} = \frac{1}{2} \left[ 1 + \frac{1}{2} \sqrt{1 - \varepsilon} + \sqrt{1 - \varepsilon} \left( \frac{1}{\sqrt{1 - \varepsilon}} - 1 \right)^2 + \frac{1}{4} \right]
\]

Eq. 2.21

Tortuosity predicted by the model as described in Eq. 2.21 was found to be in good agreement with the models proposed by Koponen et al. (Eq. 2.30 and Eq. 2.31) [10-11] and Comiti and Renaud (Eq. 2.33) [12] as shown in Figure 2.5. Unlike the model presented by Bo-Ming and Jian-Hua [9], models proposed by Koponen et al.[10-11] and, Comiti and Renaud [12] have the disadvantage of using empirical constants. However since the model presented by Bo-Ming and Jian-Hua [9] by uses square particle and only considers two dimensional flows, it is not very useful in predicting the tortuosity in fibrous porous material like nonwovens.
Nonwoven fabrics are formed of fibers which are distinct from square particles and pore channels in nonwovens follow a three dimensional path.

In a similar work Mei-Juan et al. [13] developed an analytical model to determine the tortuosity of flow paths in a porous media with spherical particles as opposed to cubic particles used by Bo-Ming et al in their model. The model used four configurations for Newtonian incompressible flow through porous media of two-dimensional spherical particles. In the first three configurations shown in Figure 2.6(a), (b) and (c), the particles were arranged in an equilateral-triangle form.

Figure 2.5 Comparison of model presented by Bo-Ming and Jian-Hua [9] and other models proposed by Koponen et al [10-11] and Comiti and Renaud [12]
The tortuosity of streamlines shown in Figure 2.6(a) is given by Eq. 2.22 [13].

\[ \tau_{1a} = \frac{l_{AB} + l_{BC}}{l_{AO}} \]

\text{Eq. 2.22}

As seen in Figure 2.6(a), straight path length is denoted by \( l_{AO} \), while the actual length path is given by \( l_{AB} + l_{BC} \).

For the streamline configuration shown in Figure 2.6(b), tortuosity is defined by Eq. 2.23 [13].
\[
\tau_{1b} = \frac{l_{AB} + l_{BC}}{l_{AO}}
\]

Eq. 2.23

where \( l_{AB} \) is the length of the streamline tangent to the particle, while \( l_{BC} \) is the arc length of the streamline \( BC \). Mei-Juan et al. [13] also considered the effect of overlapping in the configuration shown in Figure 2.6(b). They suggest that the arc \( BC \) in Figure 2.6(b) might be sheltered from each other due to overlapping, causing the actual length to be \( l_{AB} \) and the corresponding length to be \( l_{AD} \). Therefore, tortuosity, for the configuration shown in Figure 2.6(b) with overlapping is defined by Eq. 2.24 [13].

\[
\tau_{1b'} = \frac{l_{AB}}{l_{AD}}
\]

Eq. 2.24

In a similar situation shown in Figure 2.6(c), Mei-Juan et al. [13] considered that both streamline length and the arc length \( BC \) are overlapped, and therefore the tortuosity was given by Eq. 2.25.

\[
\tau_{1c} = \frac{l_{AB}}{l_{AD}}
\]

Eq. 2.25
where, \( l_{AB} \) is length of the streamline \( AB \) as shown in Figure 2.6(c). The average of tortuosities defined in Eq. 2.22 - Eq. 2.25 is reported as \( \tau \). Mei-Juan et al. [13] considered another configuration in which the particles are arranged in a square form as shown in Figure 2.7. They suggested that the streamline 4, which is close to the particle, is tortuous while streamline 5, which is away from the particle, is almost straight.

Since, the streamline is expected to be tortuous near the particle and almost straight far away from the particle and the fraction of the straight flow path increase while the fraction of tortuous path decrease with increasing porosity, weighted average tortuosity was defined by Eq. 2.26.
\[
\tau_2 = \left(1 - \frac{d}{d+R}\right)\tau_4 + \left(\frac{d}{d+R}\right)\tau_5
\]

Eq. 2.26

where \(d\) is the spacing between the particles and \(R\) is the radius as shown in Figure 2.7. \(\tau_5\), which is the tortuosity of streamline line 5, is equal to 1, while \(\tau_4\) which is the tortuosity of flow path 4, as shown in Figure 2.7 is given by Eq. 2.27.

\[
\tau_4 = \frac{l_{AB} + l_{BC} + l_{CD}}{l_{AB} + 2R + l_{CD}}
\]

Eq. 2.27

The average of tortuosities \(\tau_1\) and \(\tau_4\) gives the approximate tortuosity of flow path through the porous media with spherical particles [13]. The results were found to be in good agreement with the experimental results of Comiti and Renaud [12] (Section 2.6) for porosities higher than 0.4 as shown in Figure 2.8.
Figure 2.8 Comparison between the model proposed by Mei-Juan et al [13] and the empirical correlation proposed by Comiti and Renaud [12]

One major advantage of such geometric model is that they do not involve any empirical factors, however for a fibrous porous media, such a geometric approach will be more complicated as it is more difficult to assume a three dimensional flow in fibrous media.
2.5. Determination of Tortuosity using Computer Simulations

Koponen et al. [10] numerically solved the two-dimensional flow in a random porous medium using lattice-gas model. The two-dimensional porous medium was constructed by random positioning of rectangles of equal size. The porosity of the medium was defined as the ratio of the number of unoccupied sites to the number of all the lattice sites. The number of rectangles was varied between 10 and 68, which corresponded to porosities ranging from 0.9 to 0.5. Figure 2.9 shows an example of flow lines for a configuration of 30 rectangles corresponding to a porosity of 0.74. Tortuosity was calculated using Eq. 2.28 and Eq. 2.29.

\[
\tau = \frac{\sum_{i=1}^{N} \bar{\tau}(r_i)v(r_i)}{\sum_{i=1}^{N} v(r_i)}
\]

Eq. 2.28

\[
\bar{\tau}(r_i) = \frac{L(r_i)}{L_x}
\]

Eq. 2.29

where \(N\) is the number of flow lines, \(\bar{\tau}(r_i)\) is the tortuosity of the \(i\)th flow line starting at point \(r_i\), \(L_x\) is the length of the lattice in the \(x\) (horizontal) direction, and \(v(r_i)\) is the average tangential velocity of the fluid at the starting point \(r_i\).
In the range of porosities tested, Koponen et al. [10] found that tortuosity showed an approximately linear dependence on porosity $\varepsilon$, as shown in Eq. 2.30.

$$\tau = 0.8(1 - \varepsilon) + 1$$

Eq. 2.30

The correlation expressed in Eq. 2.30 would not be able to correctly estimate tortuosity of a three-dimensional flow. The two-dimensionality of the simulation also restricts the use of the correlation to porous structures with high porosity. Tortuosity is plotted as a function of porosity in Figure 2.10 where the solid straight line represents the correlation expressed in Eq. 2.30
In their later work, Koponen et al [11] used lattice-gas simulation considering percolation threshold \( \varepsilon_p \) (0.33) to solve Newtonian incompressible flow through porous medium shown in Figure 2.9. The results of lattice-gas simulation were fitted by the correlation expressed in Eq. 2.31

\[
\tau = 1 + a \frac{(1 - \varepsilon)}{(\varepsilon - \varepsilon_p)^m}
\]

Eq. 2.31

where \( \varepsilon_p \) is the percolation threshold (0.33), \( a = 0.65 \) and \( m = 0.19 \) are constants.
Figure 2.11 shows the simulated values of tortuosity plotted as a function of porosity. Tortuosity was found to monotonously decrease as a function of porosity for $\varepsilon > \varepsilon_p$ and diverge at percolation threshold $\varepsilon_p$.

![Figure 2.11 Simulated Values of Tortuosity as a Function of Porosity](image)

These models however do not correctly simulate fibrous porous medium as it uses square blocks to represent the solid portions. The square blocks cannot represent fibers in a fibrous system. Garza-Lopez et al. [14] developed a model which allowed the systematic analysis of tortuosity of porous medium with well-defined internal structure characterized by an interconnected network of parallel and serial channels.
Garza-Lopez et al [14] numerically calculated the exact values of mean walklengths for a permeant diffusing through the porous system both in the presence and absence of a uniform gradient (bias or external field) acting on the permeant. The ratio of the two walklengths was defined as the tortuosity of the medium. The tortuosity of medium thus defined does not match with tortuosity defined in Section 2.1. The ratio of the walklength for the permeant in the presence of an external field to the thickness of the porous medium would give the tortuosity as defined by in Section 2.1. This model was developed to represent menger sponge (fractal solid) and thus would fail to correctly represent a nonwoven fabric.

2.6. Experimental and Analytical Methods for estimating Tortuosity

Comiti and Renaud [12] employed experimental pressure drop measurements to develop an empirical Equation for tortuosity through a bed of parallelepipedal particles of low thickness-to-side ratio (such as woodchips used in paper industry). Three types of square-based parallelepipedal particles (plates) of height-to-side ratio equal to 0.102, 0.209 and 0.440 were used in the experiments. The pressure drop was measured as a function of the superficial velocity $U_0$ for tightly packed beds. The general Equation of pressure drop in fluid flow through a packed bed is given by Eq. 2.32.
\[ \frac{\Delta P}{H U_0} = M U_0 + N \]

Eq. 2.32

where \( M \) and \( N \) are the slope and intercept respectively of the plot of \( \frac{\Delta P}{H U_0} \) versus \( U_0 \).

The tortuosity was given by Eq. 2.33.

\[ \tau = \left[ \frac{M^2}{N} \frac{2 \gamma \eta \varepsilon^3}{(0.0968 \rho)^2} \right]^{1/4} \]

Eq. 2.33

In Eq. 2.33, \( M \) and \( N \) were determined by linear regression as mentioned earlier, while the shape factor \( \gamma \) was assumed to be 1 for cylindrical pores, \( \eta \) and \( \rho \) are dynamic viscosity and density of the fluid, and \( \varepsilon \) is the porosity of the packed bed. Comiti and Renaud [12] obtained a correlation (Eq. 2.34) by fitting the experimental data with porosity

\[ \tau = 1 + C \ln(1/\varepsilon) \]

Eq. 2.34

where empirical constant \( C = 0.41 \).
This empirical model however is not suitable for nonwoven structures as fibers don’t fall under the category of parallelepipedal particles and also because the shape factor was assumed to be equal to 1 which is not true for a randomly distributed fiber assembly.

Bragg and Pearson [15] developed an analytical model to describe a fibrous filter. The model assumes that helical tubes represent flow passages in a filter, and that the helical tubes have square cross-section. The model predicts pressure drop and particle collection efficiency due to inertial impaction and diffusion. In their work, Bragg and Pearson [8] derived a relationship between tortuosity and porosity. They used the general Equation for laminar flow through a channel (Eq. 2.5) and Davies’ empirical relation (Eq. 2.35) for pressure drop in fibrous filters. The empirical relation developed by Davies [16] is illustrated in Eq. 2.35

\[
\frac{\Delta P A_f d_f^2}{\mu Q L} = 64\alpha^{1.5}(1 + 56\alpha^3)
\]

Eq. 2.35

where \(\Delta P\) is the pressure drop across the filter of depth \(L\), \(d_f\) is the effective fiber diameter, \(\alpha\) is the solidity (1- porosity), and \(Q/A_f\) is the face velocity \(v\) of the fluid. The empirical relation was claimed to be valid for fiber diameters ranging from 1.6 to 80 \(\mu m\) and filter porosities between 0.7 and 0.994.
Bragg and Pearson [15] adopted Kozeny’s argument that the velocity in the filter pores is given by $u_f L_e / \varepsilon L$, where $u_f$ is the face velocity of fluid, $L_e$ is the effective length of pore channels, $L$ is the filter depth and $\varepsilon$ is the porosity of the medium.

Employing this argument along with Hagen-Poiseuille Equation (Equation 4) and Davies empirical Eq. 2.35, Bragg and Pearson [15] expressed the pressure drop across the porous medium as given in Eq. 2.36.

$$\frac{\Delta P}{L} = \frac{u_f L_e^2}{\varepsilon L^2} \frac{k_0 \mu}{m_H} = \frac{4 \alpha^{1.5} (1 + 56 \alpha^3) \mu u_f}{d_f^2}$$

Eq. 2.36

where $d_f$ is the fiber diameter and $m_H$ is the hydraulic radius of the helical tube which were assumed to have square cross-sections ($k_0 = 1.78$) for simplification. Using definitions of hydraulic radius and porosity, Bragg and Pearson [15] showed that permeability can be expressed by Eq. 2.37.

$$k = k_0 \left( \frac{L_e}{L} \right)^2 = \frac{4 \varepsilon^3 (1 + 56 \alpha^3)}{(1 - \varepsilon)^{1/2}}$$

Eq. 2.37
Since $k_0$ is a constant equal to 1.78 (square cross-section), tortuosity $L_c/L$ can be calculated if the porosity of the medium is known. However it would be unrealistic to assume the pore channels to have square cross-sections, thus making this model unsatisfactory for determining tortuosity of actual porous medium like a nonwoven fabric.

2.7. Determining Tortuosity Acoustical Methods

Tortuosity has also been determined using acoustic measurements and theories. Fellah et al. [17] proposed an ultrasonic reflectivity method for measuring porosity and tortuosity of porous materials having a rigid frame. The proposed method was based on measurement of the reflected wave by the first interface of a slab of rigid porous material. The experimental set-up which was used for the ultrasonic measurements in reflected mode is shown in Figure 2.12.
Tortuosity of the porous sample was determined by measuring the reflection coefficient at the first interface for different values of incidence angles. The expression for reflection coefficient at the first interface is given by Eq. 2.38.

\[
\tau(t, \theta) = \frac{\tau_\infty \cos \theta - \varepsilon \sqrt{\tau_\infty - \sin^2 \theta}}{\tau_\infty \cos \theta + \varepsilon \sqrt{\tau_\infty - \sin^2 \theta}} \delta(t)
\]

where \(\tau_\infty\) is the tortuosity, \(\theta\) is the incidence angle, \(\delta(t)\) is the Dirac function. Tortuosity function can be expressed in terms of reflection coefficients \(r_1 = r(t)|_{\theta_1}\) and \(r_2 = r(t)|_{\theta_2}\) corresponding to the incidence angles \(\theta_1\) and \(\theta_2\) respectively as given by Eq. 2.39.
Tortuosity can also be determined using Eq. 2.38, if porosity is known. Fellah et al., [18] proposed another method to determine four parameters namely, porosity \( \varepsilon \), tortuosity \( \tau_\infty \), viscous characteristic length \( \Lambda \), and thermal characteristic length \( \Lambda' \) which characterize the behavior of sound waves in a homogeneous isotropic slab porous material.

The measurements of transmitted and/or reflected signal outside a slab of porous material were used to find the values of the four parameters. An inverse algorithm which was used to determine the values of the parameters is based on a fitting procedure such that the transmitted and reflected signals would best describe the scattering problem in the best possible way (e.g. in the least square sense). The inverse problem is to find the values if parameters \( \varepsilon \), \( \tau_\infty \), \( \Lambda \), and \( \Lambda' \) which would minimize the functions given by Eq. 2.40 and Eq. 2.41

\[
U_1(\varepsilon, \tau_\infty, \Lambda, \Lambda') = \int (r(t) - p'(x,t))^2 \, dt
\]

Eq. 2.40
\[ U_1(\varepsilon, \tau, \Lambda, \Lambda') = \int (r(t) - p^r(x,t))^2 dt \]

Eq. 2.41

Where \( r(t) \) is the experimentally determined reflected signal, \( p^r(x,t) \) is the reflected wave which is predicted, \( s(t) \) is the experimentally determined transmitted signal and \( p^t(x,t) \) is the predicted transmitted wave. However, due to the non-linearity of the Equations, the analytical solution of the inverse problem by conventional method is tedious so, Fellah et al., [18] suggested that \( U_{1,2}(\varepsilon, \tau, \Lambda, \Lambda') \) be expressed in a different form, such that its value can be minimized using the least square method. \( U_{1,2}(\varepsilon, \tau, \Lambda, \Lambda') \) are expressed in Equation Eq. 2.42 and Eq. 2.43.

\[ U_1(\varepsilon, \tau, \Lambda, \Lambda') = \sum_{i=1}^{i=N} (r_i - p^r(x,t))^2 \]

Eq. 2.42

\[ U_2(\varepsilon, \tau, \Lambda, \Lambda') = \sum_{i=1}^{i=N} (s_i - p^t(x,t))^2 \]

Eq. 2.43

where \( r_i \) and \( s_i \) represent the discrete set of values of the reflected experimental signal and \( p^r(x,t) \) is the discrete set of values of the simulated reflected or transmitted signal.
Other similar acoustical methods have also been used to determine tortuosity of rigid porous materials. Umnova et al. [19] used an acoustic method for obtaining tortuosity and porosity of thick rigid porous materials using transmitted and reflected measurements respectively. Tortuosity was determined from the high frequency limit of phase speed which was obtained from transmission data. Tortuosity in porous structures has also been experimentally measured by Johnson et al [20-21] by using the acoustic index of refraction of superfluid $^4$He. Tortuosity of reticulated plastic foam has also been determined as the high frequency limit of squared propagation index determined using narrow-band piezoelectric transducers [22]. In another acoustic method Attenborough et al [23] determined tortuosity of air-filled stereolithographical cancellous bone replicas from measurements using audio frequency pulses in a rectangular waveguide.

2.8. Determining Tortuosity using NMR Measurements

Wang et al. [24] measured permeability, effective porosity and tortuosity of a variety of rock samples using NMR/MRI of thermal and laser-polarized gas. Wang et al [24] defined tortuosity as the square of the ratio of distance actually traveled by a tracer through the pore space to the straight line distance between the two points. The tortuosity in the rock was determined from the inverse of long-time asymptote of $D(t)/D_0$, where $D(t)$ is the $^{129}$Xe (xenon) time-dependent diffusion coefficient, and $D_0$ is the free gas diffusion coefficient.
The experimental procedure has been explained in the work of Wang et al. [24]. Mair et al. [25] used a similar NMR technique to probe porous media structure over the length scales of ~100 – 2000 μm by using $^{129}Xe$ which was imbibed into the pore space. Mair et al. [25] determined the tortuosity of random packed spherical glass beads. Gas diffusion nuclear magnetic resonance (GD-NMR) has also been found to be useful in determination of tortuosity of sandstone and complex carbonate rocks [26]. These technique of determining tortuosity take a long time (typically hours) as pointed out by Davies et al. [27] who proposed a rapid method for determining tortuosity in a bed of 100 μm glass beads. The porosity of the media used in these work were much lower compared to a nonwoven fabric, and therefore it is unclear if the technique would work for a nonwoven fabric. It should also be pointed that these test require highly specialized and expensive instruments.

2.9. Analytical Models Determining Permeability and Tortuosity

Scheidegger [28] and Hillel [29], among others have concluded that a simple correlation does not exists between permeability and volumetric parameters like porosity, as the flow rate is also influenced by area, continuity, shape and tortuosity of pore channels. Thus, porous materials having the same porosity may exhibit different permeability. A medium having numerous small pores will have lower permeability than a medium composed of fewer large pores.
Scheidegger [28] gives a comprehensive review of the capillary models for determining permeability. In the simplest capillary model a porous medium is represented by a bundle of straight, parallel capillaries with uniform diameter $\delta$. Comparing Hagen-Poiseuille Equation and Darcy’s Law, the following relationship was established (Eq. 2.44).

$$k = \varepsilon \bar{d}^2 / 32$$

Eq. 2.44

where $\varepsilon$ is the porosity and $\bar{d}$ is a sort of ‘average’ pore diameter. However, real porous media is far more complicated and so Eq. 2.44 fails to represent the correct relationship between permeability and porosity as it is actually observed. Scheidegger [28] also pointed out that this model would give permeability in one direction only as all the capillaries are parallel and there is no flow orthogonal to the capillaries.

Some modifications were made to the parallel capillary model by considering one-third of capillaries to be in each of the three special directions, thus lowering the permeability by a factor of 3. Scheidegger [28] also suggested the use of pore size distribution $f(\delta)$ instead of pore diameter. However, the parallel model still cannot correctly represent actual porous media.
Scheidegger [28] also reviewed a ‘serial model’ where all the pore spaces are serially lined up, so that fluid would enter one pin hole at one side of a porous medium and travel through very tortuous channels and then emerge at only one pinhole on the other side of porous medium. This model introduces a ratio of the actual length of the flow channels to the length (or thickness) of the porous medium in the relationship between permeability and porosity. This ratio is an excellent definition of tortuosity. However this model was as unrealistic as the parallel capillary model and thus failed to emulate the behavior of real porous media.

Both parallel and serial models neglect the fact that fluid flow path may branch and, later on, join together again. In order to take into account this phenomenon of branching, ‘branching’ type models were introduced in which a capillary can split into one or more branches [28].

Work has also been done to determine tortuosity in unsaturated porous media. The Kozeny-Carmen model is limited to the prediction of permeability of a sample at saturation, therefore Childs and Collis-George [30], Millington and Quirk [31], and Marshall [32] proposed models which expand the concepts to include unsaturated hydraulic conductivities or permeability at lower water contents. In these models tortuosity is defined in terms of effective area available for flow through a sample section, and as affecting the relative hydraulic conductivity.
A model for relative hydraulic conductivity \( (K_r) \) as a function of relative saturation \( (Se) \) was developed by Mualem [33] and is given by Eq. 2.45.

\[
K_r = Se^l \left[ \int_0^{Se} h^{-1} dSe / \left[ \int_0^1 h^{-1} dSe \right] \right]^2 \quad \text{where tortuosity } \tau = Se^l
\]

Eq. 2.45

where \( l \) is an empirical parameter, and \( h \) is the potential at which the relative hydraulic conductivity is calculated. In Eq. 2.45, tortuosity is considered to be less than 1 \( (\tau < 1) \) [34]. This concept of tortuosity is based on the fact that pores are irregular and upon drying, water concentrates in small angles and crevices of the pore system as a water film. Thus with decreasing water content, the path becomes more tortuous. Based on the Mualem-Dagan pore-scale model [34], Kosugi [35] developed a general conductivity model for soils with lognormal pore-size distribution. Two new parameters, \( \alpha \) and \( \beta \), were introduced in the model (Eq. 2.46).
These parameters are related to soil pore tortuosity. In addition to the new parameters, the model also included pore-size distribution.

\[
K_r = Se^\alpha \left[ \frac{\int_0^r r^\beta g(r)dr}{\int_0^\infty g(r)dr} \right]
\]

Eq. 2.46

where \( g(r) \) is the pore-size distribution and \( Se \) is the relative saturation. This approach for tortuosity relates to the magnitude of relative hydraulic conductivity which means that the parameters \( \alpha \) and \( \beta \) represent relative tortuosity [36]. Vervoort [36] calculated tortuosity (Eq. 2.47) as a function of potential by comparing the measured hydraulic conductivities with hydraulic conductivities calculated using a straight capillary model mentioned by Jury et al [37]

\[
\tau = \frac{K_c(h)}{K_m(h)}
\]

Eq. 2.47

where \( K_m(h) \) is the measured hydraulic conductivity and \( K_c(h) \) is the hydraulic conductivity calculated using Eq. 2.48 [37]
\[ K_c(h) = \frac{\phi^2 \Delta \theta}{\mu \rho g} \sum \frac{1}{h^2} \]

where \( \Delta \theta \) is the increments in water content, \( \rho \) is the density of water, \( g \) is the acceleration due to gravity, \( \phi \) is the liquid-solid contact angle, \( \mu \) is the viscosity and \( h \) is the potential.

Vervoort [36] related the hydraulic tortuosity parameters \( \alpha \) and \( \beta \) from the Kosugi [35] model, to mean pore size and found that with increasing mean pore size, tortuosity decreased.

Due to the limitations of the capillary models, more elaborate models would be required to better explain the hydrodynamics of fluid in porous media. A separate set of models were developed based on the assumption that porous medium can be represented by a series of channels which were more elaborate than in the capillary models.
2.10. Probabilistic Models for Permeability and Tortuosity

Childs and Collis-George [30] pointed out that the Kozeny model based on hydraulic radius would fail in the case of a porous medium structure having bundles of capillary tubes with assorted radii.

Childs and Collis-George [30] therefore proposed a probabilistic model expressing the relationship between permeability and pore-size distribution using the case of sand. They represented pore-size distribution function by \( f(r) \), therefore the fraction of total apparent volume which is occupied by pores of radius range \( r \) to \( r + \delta r \) is given by \( f(r) \delta r \). On the cross-section of the sand structure, the fraction of area devoted to the pore group with radius range of \( r \) to \( r + \delta r \) is also given by \( f(r) \delta r \). A fracture at any chosen plane normal to the length will exhibit similar faces with similar pore-size distributions, while the continuous column can be regarded as a random juxtaposition of these two faces. Fractional area devoted to pores having radius in the range of \( \rho \) to \( \rho + \delta \rho \) and \( \sigma \) to \( \sigma + \delta \sigma \) is given by Eq. 2.49 and Eq. 2.50 respectively.

\[
a_\rho = f(\rho)\delta \rho
\]

Eq. 2.49

\[
a_\sigma = f(\sigma)\delta \sigma
\]

Eq. 2.50
Hence on remaking the continuous column, the area associated with the pore sequence \( \rho \rightarrow \sigma \) is given by Eq. 2.51.

\[
a_{\rho \rightarrow \sigma} = f(\rho) \delta \rho. f(\sigma) \delta \sigma
\]

Eq. 2.51

Childs and Collis-George [30] made the following assumptions: a) all the effective resistance to flow is in the sequence is confined to the smaller of the two pores (say \( \sigma \)), b) permeability is entirely contributed by direct sequence and not by any bypass sequences, and c) when two sections are brought in contact, the overall flow across the section depends upon the number of interconnected pairs of pores and their sizes. Since the number of pores of size \( \sigma \) in the area \( a_{\rho \rightarrow \sigma} \) is proportional to \( \sigma^{-2} \), and the rate of flow in each is proportional to \( \sigma^4 \) per unit potential gradient (Hagen-Poiseuille’s Equation), the contribution, \( \delta K \) to the total permeability made by this group of sequences is given by Eq. 2.52, while the total permeability is given by Eq. 2.53.

\[
\delta K = \sigma^2 M f(\rho) \delta \rho. f(\sigma) \delta \sigma
\]

Eq. 2.52
\[ k = M \sum_{\rho=0}^{\rho=R} \sum_{\sigma=0}^{\sigma=R} \sigma^2 f(\rho) \delta \rho \cdot f(\sigma) \delta \sigma \]

Eq. 2.53

The model presented by Childs and Collis-George [30] for permeability has a constant \( M \) which has to be calculated by comparing the theoretical and experimental curves. Millington and Quirk [31], however, found that the permeability at saturation in a porous media of \( \frac{1}{2} \) to 1 mm sand as calculated by Childs and Collis-George [30] did not match with the experimental data. Therefore, Millington and Quirk [31] proposed a new model by pointing that the effective area available for flow in unit area of cross-section is \( \varepsilon^{2/3} \), where \( \varepsilon \) is porosity of porous media. The model for intrinsic permeability proposed by Millington and Quirk [31] is given by Eq. 2.54

\[
K = \varepsilon^{4/3} m^{-2} \left[ r_1^2 + 3r_2^2 + \ldots + (2m-1)r_m^2 \right] / 8
\]

Eq. 2.54

where \( K \) is the intrinsic permeability, which is the property of the medium but independent of nature of fluid, \( \varepsilon \) is porosity of the porous medium, \( m \) is the number of pore classes, and \( r_1, r_2, \ldots, r_m \) are the radii of each of the \( m \) pore classes. The results of the proposed model matched well with the experimental results; however the model offers little scope for computing tortuosity.
Garcia-Bengochea et al. [38] followed the model proposed by Childs and Collis-George [30] to develop a new relationship between pore-size distribution and permeability of ‘silty’ clay. Garcia-Bengochea et al., [38] defined a new parameter called the Pore Size Parameter \( (PSP) \) which is given by Eq. 2.55.

\[
PSP = \sum_{i}^{n} \sum_{j}^{n} \tilde{d}^3 f(d_i) f(d_j)
\]

Eq. 2.55

Where \( f(d) \) is the volume fraction of the pores with diameter between \( d \) and \( d + \delta d \), and \( \tilde{d} \) is the smaller of the two pore diameters \( d_i \) and \( d_j \).

An empirical relation between permeability and pore-size distribution was proposed by Garcia-Bengochea et al. [38] and is given by Eq. 2.56.

\[
k = C_s . PSP^b
\]

Eq. 2.56

where \( C_s \) is a constant called a shape factor, and \( b \) is a regression parameter and was found to be between 1.67 and 4.95 for ‘silty’ clay.
The results are however confined to clay samples only and it is doubtful if the same value of $b$ can be used for fibrous samples.

In order to overcome the empirical nature of the permeability models, Juang and Holtz [39] developed a new probabilistic model relating permeability and pore-size distribution. Juang and Holtz [39] followed an approach which was similar to Childs and Collis-George [30]. They considered a homogeneous soil having interconnected pores which are randomly distributed. The probability of pores with size between $x$ and $x+dx$ (symbolized as $x \rightarrow x = dx$) is $f(x)dx$, where $f(x)$ is the pore size density function of soil. Juang and Holtz [39] considered a soil column of thickness $\Delta y$ and two cross-sections ($i$ and $j$) with identical pore-size density function as shown in Figure 2.13.

![Figure 2.13 Schematic Representation of a Column of Soil (source [39])](image-url)
Juang and Holtz [39] defined the probability, \( P(x_i, x_j) \), of pores of size between \( x_i \) and \( x_i + \delta x_i \) on cross-section \( i \) being connected to pores of sizes between \( x_j \) and \( x_j + \delta x_j \) on cross-section \( j \), for two extreme cases.

Case 1: For \( \Delta y >> x \), the connection between the pores on the two cross-section would be completely random and hence the \( P(x_i, x_j) \) is given by Eq. 2.57

\[
P(x_i, x_j) = f(x_i)f(x_j)dx_i dx_j
\]

Eq. 2.57

Case 2: For \( \Delta y = 0 \), the connections between pores are completely correlated, hence the \( P(x_i, x_j) \) is given by Eq. 2.58

\[
P(x_i, x_j) = f(x_i)dx_i \text{ or } f(x_j)dx_j
\]

Eq. 2.58

Citing Garcia-Bengochea [40], Juang and Holtz [39] put forward Eq. 2.59 and Eq. 2.60 for permeability for Case 1 and 2 respectively.

\[
k = \frac{\rho g \varepsilon^2}{32\mu} \int_0^\infty \int_0^\infty \tilde{x}^2 f(x_i)f(x_j)dx_i dx_j
\]

Eq. 2.59
\[ k = \frac{\rho g \varepsilon^2}{32 \mu} \int_0^\infty x^2 f(x) dx \]

**Eq. 2.60**

where \( k \) is the permeability, \( \rho \) is density of water, \( g \) is the gravitational acceleration, \( \mu \) is the coefficient of absolute viscosity, \( \varepsilon \) is the porosity of soil, and \( \bar{x} \) is the smaller of \( x_i \) and \( x_j \). However, in reality as pointed out by Juang and Holtz [39], the connection between the pores on two cross-sections would be partially correlated. A reasonable assumption would be to consider that there is a greater probability of pores of sizes between \( x_i \) and \( x_i + \delta x_i \) on cross-section \( i \) being connected to pores of same size on cross-section \( j \), than being connected to pores of different sizes. Using the assumption Juang and Holtz [39] introduced a connecting function \( G(y, x_i, x_j) \) and permeability was expressed given by Eq. 2.61.

\[ k = \frac{\rho g \varepsilon^2}{32 \mu} \int_0^\infty \bar{x}^2 G(y, x_i, x_j) f(x_i) f(x_j) dx_i dx_j \]

**Eq. 2.61**

The connecting function \( G(y, x_i, x_j) \) is complex in nature and depends on pore geometry and tortuosity. Juang and Holtz [39] termed the connecting function as ‘governing function’ given by Eq. 2.62.
where \( L \) is an undetermined function of pore-size and \( G \) is the governing function. In Equation 58, if \( L \to \infty \), \( G(x_j) \to 1 \). On the other hand as \( L \to 0 \), \( G(x_j) \) is either 1 or 0 depending upon whether or not \( x_i \) is equal to \( x_j \).

Governing function has also been defined by Mohammadi and Banks-Lee [41] as a product of connecting function, \( g_f \) which is a function of pore geometry and tortuosity function \( \tau_f \) which is a function of thickness. The governing function is given by Eq. 2.63.

\[
G(x_j) = g_f \tau_f = \frac{B}{1 + b(x_j - x_i)^2}.
\]

Eq. 2.63

where \( B \) is the maximum volume of liquid passing through the sample, and \( b \) is a constant that is related to the variance of the pore connection.
For any pores of size \( x_i \rightarrow x_i + dx_i \), the governing function, \( G(x_i) \) is bell-shaped and symmetrical with a mean of \( x_i \rightarrow x_i + dx_i \). Therefore, pores of same size in adjacent cross sections have the greatest probability of being connected to each other.

### 2.11. Measurement of Tortuosity using Image Analysis

Murata [42] estimated tortuosity using several physical experiments and numerical simulations to visualize the streamlines of water through fractures which were artificially prepared by using fractal modeling. According to Murata [42], most of the complex geometries in nature are fractal and that topology of a fracture surface is also fractal. Using variogram method, the fractal model was represented by Eq. 2.64.

\[
\gamma(h) = Vh^{4-2D}
\]

**Eq. 2.64**

where \( \gamma(h) \) is the variogram function and \( h \) is the lag length that is the horizontal distance between the two data point on the profile, \( D (1<D<2) \) is the fractal dimension and \( V \) is a constant called “steepness”. Steepness is defined as the value of variogram at unit lag length. The variogram function was calculated using the semi-variogram function given by Eq. 2.65.
\[ \gamma(h) = \frac{1}{2N} \sum_{i=1}^{N} [z(x_i) - z(x_i - h)]^2 \]

Eq. 2.65

where \( N \) is the number of pairs of data whose lag is \( h \) and \( z(x_i) \) is the profile height at point \( x \) as shown in Figure 2.14. Eq. 2.65 shows that the mean of the square of height difference between two data points is equal to twice the variogram function.

![Diagram](image)

**Figure 2.14. Explanation of Flow Path Length along a Fracture Surface**

The actual flow-path length on the fracture surface, \( L_f \), the projected flow-path length on \( XY\text{-plane}, L \), and the nominal fracture length, \( L_n \), are explained in Figure 2.15.
Tortuosity $\tau$, which is defined as the ratio of actual flow-path length to nominal fracture length is given by Eq. 2.66

$$\tau = \frac{L_t}{L_x} = \alpha (1 + 2\frac{h}{L^{2-D}})^{1/2}$$  \hspace{1cm} \text{Eq. 2.66}$$

where $\alpha$ is called the two-dimensional tortuosity and is given by Eq. 2.67,

$$\alpha = \frac{L}{L_x}$$  \hspace{1cm} \text{Eq. 2.67}$$
While parameters like interlocking size, steepness and fractal dimension can be easily determined by the fractal model, two-dimensional tortuosity cannot be found so easily.

Murata [42] used a method in which the streamlines are visualized and measured directly. The fracture specimen was made by carving modeling wax with a numerical controlled modeling machine according to the numerical surface data generated using Glover’s method. A fracture was prepared by carving two surfaces of a fracture individually and then mating them together. Black water paint was introduced into the fracture as a tracer and then the image of the tracer was taken using a CCD camera.

Image analysis has also been used for determining connectivity and tortuosity [43-45]. Consecutive thin section of soil block was used to obtain pore connectivity as defined by Euler Poincare characteristic, which is a measure of the number of independent loops in a structure. In order to generate serial images of the surface, the surface of the sample was successively ground down by a small distance. Each surface was photographed with a digital camera, and the resulting images were segmented into a binary image of pore space and solid. Finally, the three-dimensional geometry of the pore was reconstructed from the stack of serial image of the surfaces.
Vogel, Moreau and Vogel and Roth [43-45] provide methods to determine connectivity, however no attempts were made to quantify tortuosity. Moreover, the thin section method may not be well suited for nonwoven fabric as it may cause distortion of structure during the sectioning process.

Hou [46] developed a methodology and technique to visualize and quantify nonwoven fabric structure in three-dimension. Using stereology combined with confocal laser scanning microscopy (CLSM) and image processing techniques to analyze three-dimensional void density and fiber orientation. Using optical sectioning, Hou [46] took images of 81 slices and used a volume rendering system (VoxelView) to generate three-dimensional images. The thickness of the nonwoven fabric sample used in this research was only about 400 microns and the penetration depth of the 81st sliced image was approximately 30% of the fabric thickness from the surface. Therefore, this technique of 3D image construction would fail for thicker fabric samples. Huo [46] also did not make any attempt to determine tortuosity.

In another method involving digital image analysis, Scharfenberg and Hesse [47] used digitized image of the pore texture of porous filter material and a skeletonise algorithm to determine tortuosity of macroporous system. The skeletonise algorithm is used to erode gas phase pixels starting at the gas/solid interface resulting in central lines with size of one-pixel.
The resulting skeletonized image was used to determine mean line length of pathways through the macroporous system. Nakashima and Kamiya [48] developed random walk program in Mathematica® to calculate tortuosity of porous rocks using 3-D computer tomography images. The random walk program was used to simulate diffusion of non-sorbing species by performing discrete lattice walk on the largest pore cluster in the region of interest (ROI).

Tortuosity was estimated by the mean-square displacement measurements of non-sorbing walker. An example of the random walk trajectory is shown in Figure 2.16.

![Figure 2.16 Example of Random Lattice-Walk Trajectory in a 2-D Porous Medium [48]](image)
The walker shown in Figure 2.16 migrates on discrete voxels having grey-levels corresponding to that of pore space. After choosing a random point on the image as the starting position at a dimensional integer time \( t' = 0 \), the walker jumps to the nearest pore voxel. For a 3-D image, the mean-square displacement of \( n \) random walkers can be expressed as a function of time \( t' \), Eq. 2.68

\[
    r(t')^2 = \frac{1}{n} \sum_{i=1}^{n} [(x_i(t') - x_i(0))^2 + (y_i(t') - y_i(0))^2 + (z_i(t') - z_i(0))^2]
\]

Eq. 2.68

where \( n \) is the number of random walkers and, \( x_i(t') \), \( y_i(t') \) and \( z_i(t') \) are the 3-D coordinates of walker’s position at time \( t' \).

Tortuosity was determined as the limiting value of the ratio of diffusion coefficient \( D_0 \) in free space to the diffusion coefficient \( D(t) \) in the porous medium as given by Eq. 2.69.

\[
    \tau = \frac{D_0}{d(t)} = \frac{a^2}{d(d(t')^2)} \quad \text{as } t \quad \text{and} \quad t' \to \infty
\]

Eq. 2.69

where \( a \) is dimension of cubic CT voxel, and \( r(t') \) is mean displacement of the walkers expressed as function of dimensionless integer time.
2.12. Estimation of Tortuosity using Diffusivity and Electrical Measurements

Tortuosity has significant influence on hydraulic as well as electrical flow. Electric tortuosity describes the ease or difficulty of electric current passing through a conducting fluid saturating the pore spaces [6]. Two experimental approaches has been followed for direct estimation of tortuosity The first approach involves the measurement of diffusion coefficient of a chosen non-reactive species both in free solution and in a porous media of known porosity. The other approach relates the tortuosity to a measured quantity termed ‘formation factor’ which is obtained from electrical resistivity measurements.

2.12.1. Diffusivity Method

Diffusion of a solute in solution typically is assumed to occur in response to a concentration gradient in accordance to Fick’s Law, which for one dimension is given by Eq. 2.70 [4],

$$ J = -D_0 \frac{dc}{dx} $$

Eq. 2.70

where $J$ is the mass flux, $c$ is the concentration of solute in the liquid phase, $x$ is the direction of transport and $D_0$ is the “free-solution” diffusion coefficient.
Due to the reduced cross-sectional area of flow in a porous media, the concentration of the diffusing species (solute), \( c \), is the concentration in the liquid phase of the pore space. Since fluxes are defined with respect to the total cross-sectional area, Eq. 2.70 is modified to give Eq. 2.71.

\[
J = -D_0 \theta \frac{dc}{dx}
\]

Eq. 2.71

where \( \theta \) is the volumetric water content and is defined by Eq. 2.72.

\[
\theta = \varepsilon S
\]

Eq. 2.72

where \( \varepsilon \) is the porosity of the porous media and \( S \) is the degree of saturation of the porous media, expressed as decimals.

In a porous system, the presence of solid volumes, the diffusion path of species deviates from a straight line as show in Figure 2.17 [4]. Solutes diffuse at slower rates in porous media like soil than in free solution because the pathways for migration are more tortuous in porous media. Consequently the diffusion coefficients of species must be corrected for tortuosity.
The diffusive mass fluxes are also less in soil than in free solution because of the solid volumes in the porous media occupy the same cross-sectional area.

![Figure 2.17 Convoluted Diffusion Path in a Porous Media][4]

Tortuosity of the porous media is accounted for by including a tortuosity factor $\tau$ as in Eq. 2.73

$$J = -D_0 \theta \tau \frac{dc}{dx}$$

Eq. 2.73

Garrouch et al [49] and Shen and Chen [50] listed a number of models for tortuosity based on molecular diffusion data. These models express tortuosity of pore channels in porous material as a function of the ratio of effective bulk diffusion coefficient, $D_e$ in porous media and the diffusion coefficient, $D_0$ in the absence of the porous media.
Shackelford and Daniel [4] and Ullman and Aller [51] defined tortuosity as the ratio of effective diffusivity and diffusivity in the absence of porous media given by Eq. 2.74

$$\tau = \left( \frac{D_e}{D_0} \right)^{1/2}$$

Eq. 2.74

Sherwood et al [5], Dogu and Smith [52] and Moldrup [53] defined tortuosity by introducing volumetric water content, $\theta$ into Eq. 2.74. Therefore, tortuosity was defined as given by Eq. 2.75

$$\tau = \left( \frac{D_e}{D_0} \theta \right)^{1/2}$$

Eq. 2.75

Brakel and Heertjes [54] and Boving and Grathwohl [3] introduced a new dimensional variable constrictivity $\delta$, which accounts for the fact that the cross-section of the pore channel vary over length. Constrictivity becomes important if size of the species becomes comparable to the size of the pores. A new model including constrictivity $\delta$ is given by Eq. 2.76.
\[ \tau = \left( \frac{D_e}{D_0} \theta \theta \right)^{1/2} \]

Eq. 2.76

Empirical relations which express tortuosity as function of porosity have been proposed by Iversen and Jorgensen [55] and Weissberg [56] for marine sediments and randomly overlapping spheres respectively.

These methods involve diffusivity measurement which Shen and Chen [50] and Boudreau [57] noted to be time consuming and specialized. It is also evident that there are disparities between models presented by various authors which make it difficult to choose a model for a particular porous media. Moreover, almost all the above models were developed either for soil, sediments or rocks as porous media, and it is unclear whether the models would be applicable to nonwoven fabrics.
2.12.2. **Electrical Method**

Salem and Chilingarian [6] mathematically derived tortuosity as the square root of the dimensionless formation resistivity factor times porosity as given by Eq. 2.77

\[ \tau = (F\varepsilon)^{1/2} \]

**Eq. 2.77**

where formation resistivity factor, \( F \) is defined as the ratio of bulk resistance [solid matrix and formation liquid] to the liquid resistance. Other models based on formation factor have also been reported by Wyllie and Rose [58], Winsauer et al.[59], Faris et al [60] and Cornell and Katz [61]. However, Salem and Chilingarian [6] rejected these models reasoning that they were developed as simplified models and would not be applicable to complex porous media. Salem and Chilingarian [6] also say that the hydraulic flow and electrical current follow the same path, therefore the hydraulic tortuosity and electrical tortuosity would be identical. This has also been verified by others for porous systems composed of Pyrex glass, porcelain, packed activated alumina, and packed glass beads [62] and sandstone samples [49]. Garrouch et al [49] listed empirical models for tortuosity using electrical conductivity and molecular diffusivity data. Garrouch et al [49] pointed out the difficulties in choosing from the different models for determining tortuosity of a particular porous medium.
Comparing tortuosity obtained from diffusion measurements to the corresponding values obtained from analogous electrical conductivity models, Garrouch et al [49] narrowed it down to models which would best predict the tortuosity values of sandstone core samples.

Equivalence of electrical and hydraulic tortuosity was challenged by Suman and Ruth [63], who proved it to be invalid. It was also pointed out that in the case of real porous media, the equivalency is even more doubtful due to the presence of other factors like the shape of the channels upon which the fluid flow depends which is unlike electric flow that only depends upon the total cross-sectional area of the channels.

2.13. Conclusion

The literature review shows that little work has been done to determine pore channel tortuosity in nonwoven fabrics. The capillary tube models fail to represent the complex pore network in nonwoven fabrics, and the determination of pore channel tortuosity is based on the assumption that the pores have well defined shapes (circular, square etc). Geometric models have represented porous media with spherical or cubic particles, and moreover are limited to determination of 2-D tortuosity. Lattice-gas simulation has also been used to determine tortuosity of porous media which do not adequately represent a fibrous structure of nonwoven fabric.
Experimental technique has been employed to determine pore channel tortuosity using pressure drop measurements across parallelepipedal particle bed, however this techniques assumes cylindrical pores which have shape factor of 1. The pore channels in nonwoven fabrics are not uniform in shape, and therefore determination of shape factor in a nonwoven fabric can be very challenging. Acoustical methods have been used to determine tortuosity, but these methods are only suitable for rigid porous structure. Determination of tortuosity using NMR measurements requires expensive instruments and the method have been only applied to bed of glass beads with porosity much lower than many nonwoven fabric.

Diffusion measurements, hydraulic conductivity and electrical measurements have also been employed to determine tortuosity, however most of these work have been done using porous structures like soil, soil sediments, sandstones, and packed beads. Although a large number of models have been proposed using diffusion, hydraulic conductivity and electrical measurements, no particular model was found to be applicable to all types of porous media. It is also pointed out that hydraulic tortuosity is not equivalent to electrical tortuosity.
Image analysis has also been used to determine pore channel tortuosity in soils using the serial sectioning method. Serial sectioning is not particularly well suited for nonwoven structure as it may cause large structural distortion. Optical sectioning using Confocal Laser Scanning Microscopy has also been used to visualize and generate the internal structure of nonwoven fabrics; however this method is limited to very thin structures in the order of magnitude of milli-inches.

Most of the prior works focused on determining tortuosity of pore channels are limited to porous media like soil, soil sediments and particle beds. Despite the importance of tortuosity in influencing the transport properties in nonwoven fabrics, not enough work has been done to model and determine tortuosity of nonwoven fabrics. The lack of prior work prompted us to focus our research on modeling tortuosity in nonwovens.
References


3-D solid geometries used for determination and analysis of pore channel tortuosity were constructed using ANSYS® Academic Research product. The codes for the operation were written using ANSYS® Parametric Design Language (APDL). A two step approach was used to construct 3-D solid geometry. The first step involved the construction of 3-D solid geometry representing the fiber assembly in nonwoven fabric. The second step involved the extraction of pore volume network from the 3-D fiber assembly.

In the first step fibers are constructed at random locations within a plane to form a layer of fibers assembly. A cell representing the computational region in a single layer fiber assembly is shown in Figure 3.1. All the numbers are in units of length. The shaded area shown in represents the space which contained the starting point of all the generated fibers which belong to the cell.
The operation involving the generation of fibers at random positions within a layer was repeated to generate multi-layered fiber assembly. Empty spaces within the fiber assembly geometry represent the pore volumes. An example of a 4-layer fibrous structure is shown in Figure 3.2.
In the second step the empty spaces were converted into solid volumes so that the pores are represented as solid volumes. This was done by constructing a circular disk volume (Figure 3.3) at each fiber assembly layer with the disk thickness much smaller than the fiber size (diameter), but with radius large enough to include the entire computational area defined in Figure 3.1. The fiber volumes were then successively removed by subtracting them one at a time from the circular disk volume using “Boolean operation” in ANSYS®. The thickness of generated volumes was scaled up to thickness equal to fiber diameter. The resultant volume represents the multi-layer pore volume network containing individual pore volumes at each layer shown in Figure 3.4.
Figure 3.3. Circular Disk Volume Constructed at a Fiber Volume Layer

Figure 3.4. Example of 4-Layer Pore Volume Generated after the Fiber Subtraction Operation
This preliminary 3-D geometry was constructed using the assumptions; a) fibers are non-
continuous and have circular cross-sectional area, b) fibers are without any crimp or
bend, c) in a single layer of fiber assembly, fibers lie in a horizontal plane (parallel to the
face of the nonwoven fabric), d) fibers have random orientation and placement within a
layer, and e) within a layer, the site at which fibers cross has a thickness equal to the fiber
size (diameter). The APDL program codes (See Appendix A) used to generate the
preliminary model allowed direct control over the fiber size (diameter), fiber length and
fabric thickness (number of layers), while the pore volume porosity was indirectly
controlled by changing the number of fibers in a single fiber layer assembly. This
preliminary 3-D geometry, however, had shortcomings in the form “boolean operation
failure” errors. These errors occur during boolean operations when two keypoints are
created closer than the set tolerance limit. “Boolean operation failure” errors frequently
occurred during the fiber volume subtraction operation due to the fact that ANSYS®
requires a larger number of keypoints to define curved surfaces as compared to plane
surfaces. It was also noted that “Boolean operation failure” errors became increasingly
frequent in case of fiber assembly geometries as number of layers increased. Due to the
shortcomings of this preliminary approach, an alternative approach for 3-D geometry
construction was adopted.
The alternative approach assumed the fibers to be in the form of continuous filaments with square cross-sectional area (Figure 3.5). The choice of continuous filaments was made to simulate spunbond nonwoven fabrics or fabrics containing fibers that are much longer than the computational area, while choosing square fiber cross-section area considerably reduced “boolean operation failure” error which frequently occurred while using circular fiber square cross-sectional area. As in the preliminary 3-D geometry, the new approach involves steps involving the fiber assembly generation followed by pore-volume network generation. In the step involving the pore volume generation, fiber volumes are subtracted from circular shaped disk volume.

Figure 3.5. 4-layer Fiberweb Structure with Continuous Fibers having Square Cross-Section
The circular shaped disk in this case had a radius smaller than the computational area in order to model pore structure in a fiberweb with continuous (or long) fibers as shown in Figure 3.6. The APDL codes used for solid geometry construction are included in Appendix A.

Figure 3.6 3-D Geometry showing Pore Volume Structure in Fiberweb with Continuous Fibers
3.1. Determination of Pore Channel Tortuosity

Tortuosity is defined as the ratio of actual flow path length average, $L_e$ to the length (thickness), $L$ of the porous medium in the direct of macroscopic flow Eq. 3.1,

$$\tau = \frac{L_e}{L}$$

Eq. 3.1

where $L_e$ is the actual length of tortuous flow path and $L$ is the characteristic (or straight) length or thickness of a sample along the macroscopic pressure gradient. In a preliminary method of determining tortuosity actual path length $L_e$, was calculated using geometric analysis of generated 3-D pore volume geometry. Geometric analysis involved the determination of length of shortest connected path through 3-D geometry. The shortest path was represented by a series of overlap area centroids which were determined using the algorithm shown in Figure 3.7. The algorithm involves selecting a pore volume in the bottom layer which is followed by determination of overlap area between volumes in the adjacent layer, and thereafter forming new volumes with the overlap areas as the base. Each of the area centroids thus found represented individual paths through the 3-D geometry. The corresponding centroids in the subsequent layers were found based on the algorithm described in Figure 3.7.
The shortest path length was calculated by adding the length segments formed by overlap area centroids in adjacent layers. The APDL program codes used for the determination of tortuosity using geometric analysis is given in Appendix B.
Figure 3.7 Algorithm for Determining Centroid of Pore Volumes forming the shortest connected path
The preliminary method however was found to be inaccurate in determining tortuosity as it dramatically overestimated the shortest path length. Therefore the preliminary approach was abandoned for a new approach involving CFD simulation and concepts of particle tracking.

A small change was made in the 3-D geometry allowing its use in CFD simulation and particle tracking analysis. In the pore volume generation procedure described earlier, square shaped disk volumes were used instead of circular disk volumes. This was necessary for the definition of symmetry boundary condition as described in Chapter 4.
An example of 3-D geometry used for CFD simulation is shown in Figure 3.8.

Figure 3.8. 5-Layer 3-D Pore Structure of Fiberweb containing Continuous fibers

APDL program codes for generation of 3-D geometry used for CFD simulation and Particle tracking analysis is given in Appendix C.
Abstract

A method to determine tortuosity in a fibrous porous medium is proposed. A new approach for sample preparation and testing has been followed to establish a relationship between air permeability and fiberweb thickness which formed the basis for the determination of tortuosity in fibrous porous media. An empirical relationship between tortuosity and fiberweb structural properties including porosity, fiber diameter and fiberweb thickness has been proposed unlike the models in the literature which have expressed tortuosity as a function of porosity only. Transverse flow through a fibrous porous media increasingly becomes less tortuous with increasing porosity, with the value of tortuosity approaching 1 at upper limits of porosity. Tortuosity also decreased with increase in fiber diameter whereas increase in fiberweb thickness resulted in the increase in tortuosity within the range of fiberweb thickness tested.

4.1. Introduction

Fibrous porous media like nonwoven material is an assembly of fibers which are bonded to form coherent structures such as webs, sheets and bats. A characteristic feature of a nonwoven material is its high porosity (pore volume fraction). However, in spite of being an open structure, nonwovens have fairly good stability.
It is this property that has made nonwovens a preferred choice of material in many barrier applications such as insulation, filtration and acoustics. Fluid flow properties such as permeability are of immense importance to the performance of nonwoven materials used in these applications. Fluid flow through a porous medium is influenced by the amount and structure of the void (pore) space. While amount of void space is easily quantified by measurement of porosity, characterization of void space structure is more complicated. The path followed by fluid flow through the void space in a porous medium is often quantified by tortuosity. Tortuosity (Eq. 4.1) is defined as the ratio of actual flow path length average, $L_e$ to the length (thickness), $L$ of the porous medium in the direct of macroscopic flow [1-2].

$$\tau = \frac{L_e}{L}$$

Eq. 4.1

The complex nature of void space within nonwoven materials makes it difficult to quantify tortuosity. A simple way to do so would be to assume that a porous medium contains parallel channels of fixed diameter and have their axis inclined at a fixed angle $\theta$ to the normal of the porous medium surface. In this case tortuosity $\tau$, would be equal to $\frac{1}{\cos \theta}$. Such simplified approach however fails to represent the complex nature of the pore channels.
In an early work Piekaar and Clarenburg [2] used geometrical analysis to estimate tortuosity in fibrous porous media. The proposed model predicts higher tortuosity with increasing porosity. The authors however do not explain the reason for such an unusual behavior. Bo-Ming and Jian-Hua [3] and, Mei-Juan et al. [4] used 2D geometrical analysis to estimate tortuosity of flow path in a porous media with square and spherical particles. Experimental methods using pressure drop measurements across bed containing parallelepipedal particles was used by Comiti and Renaud [5] to determine tortuosity with the assumption of shape factor equal to 1. Fellah [6] has proposed method involving ultrasonic reflectivity for measuring porosity and tortuosity of porous materials. Such acoustical methods however are only suitable for porous materials with rigid structures. Tortuosity has also been determined experimentally using Nuclear Magnetic Resonance (NMR) measurements [7-8]. However, works involving determination of tortuosity using NMR method are however so far limited to non-fibrous porous materials. Using methods involving diffusion measurements, various models have been proposed for tortuosity in porous materials like soil, sedimentary rocks, solid sediments, sphere packing [9-12]. Electrical resistance measurements have also been used to determine tortuosity of porous materials like Pyrex glass, porcelain, packed activated alumina, packed glass beads, and sandstones by Garrouch et al [13]. Garrouch et al. [13] also list several other models based on electrical resistance measurements, proposed by other authors.
Validity of models based on electrical resistance measurements has been challenged by Suman and Ruth [14] reasoning that in real porous media, presence of factors like the shape of channels influence the fluid flow unlike electrical flow which only depends upon the total cross-sectional area of channels. Image Analysis techniques involving serial sectioning has also been used by Vogel [15] to find pore connectivity, however no attempt was made to determine tortuosity. Tortuosity has also been determined using lattice-gas and random walk simulation results. Koponen et al [1] used lattice-gas simulation in a 2D media to express tortuosity as a function of porosity. Tomadakis and Sotirchos [16] used random-walk simulation results to determine tortuosity as function of porosity, which was used by Tomadakis and Robertson [17] to derive the expression for viscous tortuosity of fibrous porous media with layered microstructure.

Study of pervious works has thus shown that very little work has been done to determine tortuosity in fibrous nonwoven materials. Some of the methods used for other types of porous materials may not be suitable for fibrous porous materials. A few models which have been porous materials have proposed for fibrous expressed tortuosity as a function of porosity, while ignoring other variables like fiber diameter and porous material thickness. The current work involves development of an experimental method for the determination of tortuosity. Relationship between tortuosity and parameters like porosity, fiber diameter and fiberweb thickness has also been established.
4.2. Determination of Pore Channel Tortuosity

In microscopic terms, Newtonian fluid flow through a porous medium at a low Reynolds number is governed by Darcy’s law (Eq. 4.2) which states that the flux q (discharge rate per unit area, with units of length per time, m/s) of the fluid is proportional to the pressure gradient $\Delta P$ (Pa/m).

$$ q = \frac{-k}{\mu} \Delta P $$

Eq. 4.2

Here, $k$ is the permeability of the porous medium (units of area, $m^2$) and $\mu$ is the dynamic viscosity of the fluid (Pa.s). Using dimensional analysis it has been suggested that permeability of a porous medium can be expressed by Eq. 4.3 [1].

$$ k \frac{f(\varepsilon, \tau)}{S^2} $$

Eq. 4.3

Here $\varepsilon$ is porosity of the porous medium, $\tau$ is the pore channel tortuosity, and $S$ is the specific surface area which is defined as the ratio of total interstitial surface area to the bulk volume of the porous medium. Based on the capillary model which represents a porous material as solid material containing parallel tubes of fixed cross-sectional shape, Carmen-Kozeny equation (Eq. 4.4) expresses permeability as a function of porosity $\varepsilon$ and hydraulic radius $R_h$ as
Here hydraulic radius $R_h$ is equal to $\varepsilon/S$, and $c$ is the Kozeny constant which depends upon the cross section of the tubes. In order to account for the complexity of the actual flow through the porous medium, tortuosity is introduced in the capillary model, in which case permeability is expressed as given by Eq. 4.5[1]

$$k = \frac{\varepsilon R_h^2}{c}$$  

\textbf{Eq. 4.4}

$$k = \frac{\varepsilon^3}{c\tau^2 S^2}$$  

\textbf{Eq. 4.5}

Permeability of a porous medium is influenced by morphology as well as topology of the pore space structure. In case of fibrous porous medium, morphology and topology of the pore space depends upon the fiber diameter, pore volume porosity and fiber orientation distribution. Porosity and fiber diameter being constant, the thickness of fibrous porous medium does not influence the morphology of the pore structure; however thickness does influence the topological characteristics such as tortuosity. Porosity and fiber diameter being constant, decrease in thickness would cause the fluid flow to follow a less tortuous path, thus resulting in reduced tortuosity.
As thickness decreases, the path followed by the fluid flow will become increasingly less tortuous until path becomes almost straight as in the case of nonwoven material with a very small thickness. Using this knowledge of how thickness influences the morphology and topology of pore structure in a porous medium, a new experimental approach has been developed to determine tortuosity of nonwoven porous materials.

Differences in the air permeability of two nonwoven fabrics having the same porosity, fiber diameter and fiber orientation distribution, but having different thickness could be explained by the differences in tortuosity of the fluid flow path in the two porous medium. Tortuosity values $\tau_1$ and $\tau_2$ of two such samples could be related to the respective permeability, $k_1$ and $k_2$ as given by Eq. 4.6.

$$\frac{\tau_1}{\tau_2} = \left(\frac{k_2}{k_1}\right)^{\frac{1}{2}}$$

Eq. 4.6

In the case of nonwoven fabrics with low thickness, where the fluid flow follows almost a straight path, the value of tortuosity could be assumed to be equal to 1. With this assumption, tortuosity $\tau(L)$ of a nonwoven fabric with thickness, $L$ could be determined using Eq. 4.7.
Here \( \tau(L) \) and \( k(L) \) are tortuosity and air permeability respectively of nonwoven fabric with thickness \( L \). \( \tau(L)_{L \to 0} \) and \( k(L)_{L \to 0} \) are the tortuosity and air permeability respectively of nonwoven fabric with very low thickness. Eq. 4.7 is applicable when both nonwoven fabrics have same porosity, fiber diameter and fiber orientation distribution. Tortuosity, \( \tau(L)_{L \to 0} \) of a nonwoven fabric with low thickness could be assumed to be equal to 1, in which case Eq. 4.7 can be written as

\[
\tau(L) = \left( \frac{k(L)_{L \to 0}}{k(L)} \right)^{\frac{1}{2}}
\]

Eq. 4.8

This approach of determining tortuosity would therefore require establishing a relationship between air permeability and nonwoven fabric thickness. Such an analysis would require a sample set where the thickness is varied at different levels of fiber diameter and porosity. However, due to the production process, fiber diameter, porosity and fabric thickness cannot be independently controlled.
In order to overcome this shortcoming, nonwoven fabrics were used in a layered-configuration, which allowed independent control over fabric thickness. The layered-configuration was achieved by layering specimens cut from a sample to form 2-layer, 4-layer and 8-layer configurations. Assuming that each specimen in layered configuration has same porosity, fiber diameter and fiber orientation distribution, pore structure morphology of 2-layer, 4-layer and 8-layer configurations would be the same.

The change in thickness however, would result in different tortuosity values associated with the pore structure of these configurations.

4.3. Sample Description, Methods, and Measurement of Air Permeability

Samples used in this work included spunbond-calendared, spunbond-needled-calendared, and spunbond-hydroentangled nonwoven fabrics. Description of samples is shown in Table 4.1.
Table 4.1 Sample Description

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP45, DP55, DP68, DP90</td>
<td>DuPont Polypropylene (PP) Spunbond-Calendared</td>
</tr>
<tr>
<td>JM1, JM2</td>
<td>Johns Manville Polyester (PET) Spunbond-Needled-Calendared</td>
</tr>
<tr>
<td>SB1, SB2</td>
<td>PP Spunbond-Calendared</td>
</tr>
<tr>
<td>SBH</td>
<td>PP/PET Core-Sheath</td>
</tr>
<tr>
<td></td>
<td>Spunbond-Hydroentangled</td>
</tr>
</tbody>
</table>

Nonwoven fabric samples were cut into 2.5 inch (diameter) circular specimens. Due to the non-uniformity of basis weight, a large variation was observed in the basis weight of the specimens cut from a sample.

Since a layered configuration would require that each specimen in the configuration has same porosity, specimens cut from the samples were grouped together according to their basic weights, such that groups of eight specimens having same or very close basic weights were selected. From these, 2-layer, 4-layer and 8-layer configuration were formed. Each layered configuration was treated as a different sample. A few groups were formed with four specimens instead of eight due to the lack of sufficient specimens with the same basis weights. A total number of 112 samples were formed using this method.
Thickness of nonwoven fabric samples (2-layer, 4-layer and 8-layer configuration) measured at pressure of 100gf/cm², ranged from 0.3 to 4.0 mm, while porosity of the samples ranged from 0.65 to 0.90. Fiber diameter, measured using microscope and image analysis tool ImageJ, ranged between 7.7 to 25.3 microns.

Air resistance (Eq. 4.9) of 2-layer, 4-layer and 8-layer samples was measured using KES-FS Air-Permeability Tester. Air permeability was determined using sample thickness and measured air resistance using Eq. 4.10.

\[ R = \frac{\Delta P}{v} \]  

Eq. 4.9

\[ k = \frac{L\mu}{R} \]  

Eq. 4.10

Here, \( R \) is the measured air resistance \((KPa.s/m)\), \( \Delta P \) is pressure difference \((KPa)\) across the sample thickness \( L \) \((m)\), \( v \) is air flow per unit area of sample \((m^3/m^2.s)\), \( k \) is the air permeability \((m^2)\), and \( \mu \) is the air viscosity \((KPa.s)\).
4.4. Results and Discussion

Air permeability determined using Eq. 4.10, and structural properties of the samples were used to establish an empirical relationship (linear regression model) between air permeability of a sample and its structural properties such as porosity, fiber diameter, and thickness as shown in Eq. 4.11.

\[
k = 6.82e - 06 \left( \frac{d}{1 - \varepsilon} \right) - 2.2e - 11 * \ln(L) + 1.64e - 04 * d * (1 - \varepsilon)^2
\]
\[
-1.71e - 09 * (1 - \varepsilon) - 6.66e - 10
\]

Eq. 4.11

Here \( k \) is the air permeability (m\(^2\)), \( d \) is fiber diameter (m), \( L \) is sample thickness (m) and \( \varepsilon \) is porosity. The empirical equation had an R-Square value of 0.87 and all variables were found to be significant with at least 99% confidence level. The established empirical model was compared with empirical correlations and analytical models proposed by Davies [18], Koponen et. al [19] and, Tomadakis and Robertson [17].

Davies [18] proposed an empirical correlation by fitting transverse permeability data for flow through highly porous layered fibrous structures, Eq. 4.12

\[
k = r^2 \left[ 16(1 - \varepsilon)^{3/2} \left( 1 + 56(1 - \varepsilon)^3 \right) \right]^{-1}
\]

Eq. 4.12
where $r$ is fiber radius and $\varepsilon$ is the porosity of the fibrous structure. Davies’ empirical correlation [18] is valid for porosity, $\varepsilon$ greater than 0.7. Tomadakis and Robertson [17] proposed an analytical model, Eq. 4.13, based on electrical conductivity to predict permeability of fibrous media with layered microstructure.

\[
k = \frac{r^2 \varepsilon}{8 \ln^2 \varepsilon} \times \frac{(\varepsilon - \varepsilon_p)^{\alpha+2}}{(1 - \varepsilon_p)^\alpha [(\alpha + 1)\varepsilon - \varepsilon_p]^2}
\]

Eq. 4.13

where $r$ is the fiber radius, $\varepsilon$ is the porosity, $\varepsilon_p$ is percolation threshold (0.11) and $\alpha$ is a constant (0.785). Like the model proposed by Davies [18], the analytical model proposed by Tomadakis and Robertson [17] is valid for porosity of media greater than 0.7. Koponen et al [19] used lattice-Boltzmann method to solve permeability of random 3D fiber web as a function of porosity. The simulated results were used to obtain a fitted expression given in Eq. 4.14.

\[
k = r^2 A \left[ e^{B(1-\varepsilon)} - 1 \right]^{-1}
\]

Eq. 4.14

where $r$ is the fiber diameter, $\varepsilon$ is porosity, and $A = 5.55$ and $B = 10.1$ are constants.
The effect of porosity on permeability is shown in Figure 4.1, which also illustrates the comparison between our empirical relation (Eq. 4.11) and other empirical and analytical models expressed in Eq. 4.12 - Eq. 4.14.

As can be seen in Figure 4.1, permeability increases with increasing porosity. Comparison between models shows good agreement between the established empirical model (Eq. 4.11) and other models expressed in Eq. 12-14 at lower porosities. At higher porosities however, the established experimental model overestimates permeability as compared to permeability estimated by models proposed by Davies [18] and Koponen et al [19].

![Figure 4.1 Comparison between the Empirical Model (Eq. 4.11) and other Analytical and Empirical Models](image-url)
As seen in Figure 4.1, models proposed by Tomadakis and Robertson [17] also overestimate permeability at higher porosities. All the models however showed similar increasing trend in permeability with increasing porosities. Permeability increases with increasing fiber diameter as can be seen in Figure 4.2. Comparison between different models again indicates that the established empirical model (Eq. 4.11) overestimates permeability. The overestimation of permeability in case of experimental results is due to the use of a layered configuration of the samples.

None of the empirical and analytical models that we found in literature so far investigated the effect of fabric thickness on permeability, however for the range of fabric thickness measured, permeability was found to be significantly influenced by fabric thickness.
Therefore, the established empirical model (Eq. 4.11) includes fabric thickness as one of the variables, and this formed the basis for determination of tortuosity. As can be seen in Figure 4.3, permeability decreased with increasing thickness.

![Figure 4.3 Variation of Permeability with Fiberweb Thickness at $\varepsilon = 0.9$ and $d = 20$ microns](image)

Pore channel tortuosity was determined using Eq. 4.8, where $k(L)_{L \to 0}$ and $k(L)$ were calculated using Eq. 4.11. Permeability $k(L)$ of a sample determined by Eq. 4.11, had a corresponding $k(L)_{L \to 0}$ value which was calculated by setting fiberweb thickness equal to 10 times the fiber diameter, $d$. 
Tortuosity values of samples were then used to develop an empirical relationship (Eq. 4.15) between pore channel tortuosity and variables including porosity, fiber diameter and fiberweb thickness

\[
\ln(\tau) = -7.83 \ln(\varepsilon) - 0.54 \ln(d) + 0.188 \ln(L) + 4.85\varepsilon^2 - 4.49
\]

Eq. 4.15

where \(\tau\) is the pore channel tortuosity, \(\varepsilon\) is the porosity, \(d\) is the fiber diameter and \(L\) is the fiberweb thickness. The empirical model had a high R-square value of 0.84 and all the variables were found to be significant with at least 99% confidence level. Other models for tortuosity found in the literature include those proposed by Koponen et al [1] and, Tomadakis and Sotirchos [16].

Koponen et al [1] used lattice-gas simulation in a 2D media to determine tortuosity. The simulated values of tortuosity \(\tau\), were expressed as function of porosity \(\varepsilon\), given by Eq. 4.16.

\[
\tau = 1 + a \frac{(1 - \varepsilon)}{(\varepsilon - \varepsilon_p)^m}
\]

Eq. 4.16

where \(\varepsilon_p\) is the percolation threshold (0.33), \(a = 0.65\) and \(m = 0.19\) are constants.
Tomadakis and Sotirchos [16] used random-walk simulation results for bulk tortuosity $\tau_b$ to establish the correlation given by Eq. 4.17. Tomadakis and Robertson [17] derived the expression for tortuosity (viscous tortuosity) of fibrous porous media with layered microstructure as a function of porosity $\varepsilon$, and bulk tortuosity $\tau_b$, given by Eq. 4.18.

$$\tau_b = \left( \frac{1 - \varepsilon_p}{\varepsilon - \varepsilon_p} \right)^\alpha$$

Eq. 4.17

$$\tau = \tau_b \left( 1 + \frac{\alpha \varepsilon}{\varepsilon - \varepsilon_p} \right)^2$$

Eq. 4.18

Here $\tau_b$ is the bulk tortuosity, $\tau$ is the tortuosity (viscous tortuosity), $\varepsilon_p$ is the percolation threshold (0.11) and, $\alpha$ is a constant (0.785). Figure 4.4 shows the relationship between tortuosity and porosity as predicted by the current model (Eq. 4.15) and models given by Eq. 4.16 - Eq. 4.18.
As shown in Figure 4.4, pore channel tortuosity decreased with increase in porosity. With respect to predicting the relationship between tortuosity and porosity, a good agreement was found between the current model (Eq. 4.15), model proposed by Koponen et al [1] (Eq. 4.16) and bulk tortuosity (Eq. 4.17). The model proposed by Tomadakis and Robertson (Eq. 4.18) [17] dramatically overestimates tortuosity with its value much higher than 1 even at higher porosity.
All the models found in the literature have expressed tortuosity as a function of porosity, while ignoring the effect of fiber diameter and fiberweb thickness. The current model expressed in Eq. 4.15 includes fiber diameter and fiberweb thickness as variables. Both variables were found to have significant influence on the tortuosity. Figure 4.5 and Figure 4.6 shows the effect of fiber diameter and fiberweb thickness on pore channel tortuosity.

Figure 4.5 Variation of tortuosity with fiber diameter at $\varepsilon = 0.9$, and $L = 5\text{mm}$
Increase in fiber diameter with other structural parameter remaining constant, resulted in a decrease in tortuosity, while increase in the fiberweb thickness results in the increase in tortuosity.

4.5. Conclusions

The current empirical model for permeability was found to be in good agreement with other empirical, analytical and simulation models found in the literature which has expressed permeability as a function of porosity and fiber diameter. Permeability was found to increase with increase in pore volume porosity.

Figure 4.6 Variation of Tortuosity with Fiberweb Thickness at $\varepsilon = 0.9$, $d = 15$ microns
Permeability also increased with increase in fiber size. While other models have ignored the effect of fiberweb thickness on permeability, in the range of fiberweb thickness measured, permeability was found to be significantly influenced by fiberweb thickness. This formed the basis for the determination of pore channel tortuosity. With respect to the predicting the relationship between tortuosity and porosity, the empirical model developed for tortuosity was in good agreement with other models in the literature which have expressed tortuosity as a function of porosity while ignoring other structural parameters like fiber diameter and fabric thickness. In the current empirical model, structural parameters like porosity, fiber diameter and fabric thickness were found to significantly influence tortuosity. As porosity approached the value 1, tortuosity reduced to near the value of 1 indicating that the pore channels have minimal tortuosity at porosities close to 1. The current empirical model also showed that tortuosity decreased with increase in fiber diameter, where as it increased with increase in thickness of porous media.
References


Chapter 5  Prediction of Tortuosity in Fibrous Porous Media using CFD Simulation and Particle Tracking Method

Abstract

Two computational models to predict tortuosity of fibrous porous media were developed using computational fluid dynamics (CFD) simulation and Particle Tracking method. The first computation model involved an intermediate step in which air permeability of fibrous porous media was simulated using CFD. 3-D geometries used in these models were generated in ANSYS® Academic Research product to simulate fibrous porous microstructure. In terms of predicting the relationship between tortuosity and porosity of fibrous media, the proposed computational models were found to be in good agreement with the models available in the literature. The proposed models showed that transverse flow through a fibrous porous media increasingly becomes less tortuous with increasing porosity, with the value of tortuosity approaching 1 at upper limits of porosity. While other models found in the literature have expressed tortuosity as a function of porosity only, the proposed computational models show that apart from porosity other structural parameters such as fiber diameter and fiberweb thickness also have a significant influence on tortuosity. Tortuosity also decreased with increase in fiber diameter whereas increase in fiberweb thickness resulted in the increase in tortuosity.
The proposed computational models for tortuosity were also found to be in good agreement with the empirical model presented in Chapter 4. Unlike the models in the literature which have expressed permeability as function of porosity and fiber diameter, the current computational model for permeability also included fiberweb thickness.

5.1. Introduction

A characteristic feature of a nonwoven material is its high porosity (pore volume fraction), which has found many applications such as insulation, filtration and acoustics. Fluid flow property such as permeability is an important influencing factor which determines the performance of nonwoven materials used in these applications. Characterization of the void space within fibrous porous material therefore is of immense importance. While the amount of void space is easily determined in terms of porosity, characterization of the structure of the void space is complicated. One aspect of void structure characterization is the path followed by fluid flow through a porous medium which is often quantified by tortuosity or tortuosity factor. Tortuosity factor is defined in Eq. 5.1 as the ratio of actual flow path length average, $L_c$ to the length (thickness), $L$ of the porous medium in the direction of macroscopic flow [1]

$$\tau = \frac{L_c}{L}$$

Eq. 5.1
The complex structure of the pore space results in the transported material following a
tortuous path through the porous media. Transport parameters like hydraulic permeability
and electrical conductivity are therefore directly influenced by tortuosity. Higher values
of tortuosity would indicate that the transported material has to follow longer and more
complicated path resulting in higher resistance to the flow. Due to its significant
influence on the transport property of porous media, tortuosity has direct impact on
filtration, absorbance, heat transfer and acoustic applications.

Determination of tortuosity is essential to the understanding of the mechanism of
hydraulic flow and therefore it has been typically attempted to establish a correlation
between hydraulic permeability and tortuosity. Tortuosity factor was first associated with
the Carmen-Kozeny capillary model for predicting permeability. There it was used to
account for the tortuous path followed by the fluid through a porous medium. In an early
work Piekaar and Clarenburg [2] used geometrical analysis to estimate tortuosity in
fibrous porous media. Their model predicts higher tortuosity with increasing porosity.
They, however did not explain the reason for such an unusual behavior. Bo-Ming and
Jian-Hua [3] and, Mei-Juan et al. [4] used 2D geometrical analysis to estimate tortuosity
of the flow path in a porous media with square and spherical particles. Tortuosity has also
been determined using lattice-gas and random walk simulation results. Koponen et al [5]
used lattice-gas simulation in a 2D media to express tortuosity as a function of porosity.
Tomadakis and Sotirchos [6] used random-walk simulation results to determine tortuosity as function of porosity, which was then used by Tomadakis and Robertson [7] to derive the expression for viscous tortuosity of fibrous porous media with layered microstructure. Monte Carlo random walk simulation has also been used by Rigby and Gladden [8]. An algorithm to determine tortuosity of void space porous media containing particles and fibers with low aspect ratio has been proposed by Donohue and Wensrich [9]. The algorithm involves determination of continuous chains of voids connecting the bottom and top layers of fibers in the assembly. Their results showed that tortuosity increased with increasing packing efficiency.

It is thus clear that very little work has been done to model tortuosity in fibrous nonwoven materials. A few models which have been proposed for fibrous porous materials have expressed tortuosity as a function of porosity, while ignoring other variables like fiber diameter and material thickness. This work involves modeling tortuosity factor in fibrous porous media using Computation Fluid Dynamics (CFD) and Particle Tracking Method. The modeling allowed establishing the relationships between tortuosity and structural parameters like porosity, fiber diameter and fabric thickness.
5.2. Determination of Tortuosity

In the capillary model where porous media is represented as solid material containing parallel tubes of fixed cross-sectional shape, permeability is expressed as a function of porosity $\varepsilon$ and specific surface area $S$, as shown in Carmen-Kozeny equation (Eq. 5.2) [5, 10],

$$k = \frac{\varepsilon^3}{cS^2}$$

Eq. 5.2

where $\varepsilon$ is porosity of the medium, and $c$ is the Kozeny constant which depends upon the cross section of the tubes, and $S$ is the specific surface area which is defined as the ratio of total interstitial surface area to the bulk volume of the porous medium.

In order to account for the complexity of the actual flow through the porous medium, tortuosity factor $\tau$, is introduced in the capillary model (Eq. 2), and permeability is expressed as given by Eq. 5.3[5, 10]

$$k = \frac{\varepsilon^3}{c \tau^2 S^2}$$

Eq. 5.3
Permeability of a porous medium is therefore influenced by morphology as well as topology of the pore space structure. In case of fibrous porous medium, morphology and topology of the pore space depends upon the fiber diameter, pore volume porosity, fiber orientation distribution and thickness of fibrous porous media. Porosity and fiber diameter being constant, the thickness of fibrous porous medium does not influence the morphology of the pore structure. However, thickness does influence the topological characteristics such as tortuosity. Porosity and fiber diameter being constant, decrease in thickness would cause the fluid flow to follow a less tortuous path, thus resulting in reduced tortuosity. Using this knowledge of how thickness influences the morphology and topology of pore structure in a porous medium, a new experimental approach was developed to determine tortuosity of nonwoven porous materials (see previous chapter). Difference in the air permeability of two nonwoven fabrics having the same porosity, fiber diameter and fiber orientation distribution, but having different thickness could be explained by the difference in tortuosity of the fluid flow path in the two porous medium.

Tortuosity values $\tau_1$ and $\tau_2$ of two such samples could be related to the respective permeability, $k_1$ and $k_2$ as given by Eq. 5.4.

$$\frac{\tau_1}{\tau_2} = \left(\frac{k_2}{k_1}\right)^{\frac{1}{2}}$$

Eq. 5.4
In case of nonwoven fabrics with low thickness, where the fluid flow follows almost a straight path, the value of tortuosity could be assumed to be equal to 1. If \( \tau(L) \) and \( k(L) \) are tortuosity and air permeability of nonwoven fabric with thickness \( L \). \( \tau(L)_{L \to 0} \) and \( k(L)_{L \to 0} \) are the tortuosity and air permeability of nonwoven fabric with very low thickness, tortuosity \( \tau(L) \) of a nonwoven fabric with thickness, \( L \) could be determined using Eq. 5.5

\[
\frac{\tau(L)}{\tau(L)_{L \to 0}} = \left( \frac{k(L)_{L \to 0}}{k(L)} \right)^{1/2}
\]

Eq. 5.5

where tortuosity \( \tau(L)_{L \to 0} \) can be assumed to be equal to 1. Therefore Eq. 5.5 can be written as

\[
\tau(L) = \left( \frac{k(L)_{L \to 0}}{k(L)} \right)^{1/2}
\]

Eq. 5.6

This approach of determining tortuosity is similar to the one used in the experimental method, and therefore requires establishment of a relationship between air permeability and nonwoven fiberweb thickness.
In order to simulate permeability, 3-D fiberweb geometries were generated with varying construction parameters including the thickness of fiberweb which could independently be controlled unlike in the case of actual nonwoven materials.

5.3. Generation of Solid Geometry

3-D geometry was developed to model layered fibrous microstructure of a nonwoven fiberweb. 3-D geometries were generated in ANSYS® Academic Research product using program codes written in ANSYS® Parametric Design Language (APDL). A two step approach was used to generate 3-D geometry to simulate nonwoven permeability and particle tracking. Step 1 involved the construction of 3-D geometry representing fibrous microstructure of nonwoven fiberweb. In order to generate a layered microstructure, fibers were arranged such that their axes lay on parallel planes. Within each plane the fibers were arranged with random orientation distribution. Step two involved the conversion of pore space within the microstructure into solid volume by using volume subtraction function in ANSYS®. Volume subtraction was performed in each plane (layer) separately, thus resulting in a layered structure representing the pore volume network.
In addition to the assumption of layered microstructure, the 3-D geometry also assumed the fiber to be in the form of continuous filaments with square cross-sections. The choice of square cross-section over circular cross-section was made to overcome the shortcoming experienced while performing volume subtraction when circular cross-section was chosen. A larger number of “keypoints” (coordinate points) are required to define curved surfaces, which frequently caused the “keypoints” to lie closer than the set tolerance limit which led to “boolean operation errors” during the volume subtraction operation.

Lesser number of “keypoints” required to define plane surfaces in case of square fiber cross-section considerably reduced the “boolean operation errors”. The model also assumed the fibers to be straight and that the thickness at the site of fiber crossing is equal to the fiber diameter. The program code written in APDL allowed fiberweb properties such as fiber size (diameter), fiberweb thickness, and porosity to be controlled directly or indirectly. While fiber size and fiberweb thickness were controlled directly, porosity of fiberweb was indirectly controlled by varying the number of fibers in the geometry. An example of resulting 3-D geometry after the volume subtraction step is shown in Figure 5.1.
Porosity of the fiberweb was determined using Eq. 5.7

\[ \varepsilon = \frac{V_{\text{pores}}}{V_{\text{total}}} \]

Eq. 5.7

Where total pore volume, \( V_{\text{pores}} \) is equal to the volume of the generated 3-D geometry, while \( V_{\text{total}} \) is calculated using Eq. 5.8.

\[ V_{\text{total}} = D^2 \cdot N \cdot d \]

Eq. 5.8
where $D$ is the side length of computational area (shown in Figure 1.), $N$ is the number of fiber layers in the 3-D geometry, and $d$ is the fiber size (diameter). After the fiber subtraction operation (step two) the resulting pore volumes layers were added to form a single pore volume network. Fiberweb geometries were generated with fiber diameter $d$, ranging from 10 – 40 microns, number of fiber layers $N$, between 2-100 and porosity $\epsilon$, between 0.77 – 0.94.

Determination of permeability using CFD requires calculation of pressure drop. To ensure that pressure drop calculations are carried out in regions far away from any velocity gradient, fluid volumes were added at the two ends of the 3-D geometry (Figure 5.3) thus creating fluid entry and exit regions.

The resulting 3-D geometries were meshed using ANSYS ICEM-CFD which is an advanced meshing tool within ANSYS WORKBENCH. Tetra Meshing using Octree method was employed to generate mesh geometry containing tetrahedral meshes. Some examples of 3-D fiberweb geometries are shown in Figure 5.2.
(a) $d = 35$ microns, $l_y = 5$, $\varepsilon = 0.81$
(b) $d = 15$ microns, $l_y = 25$, $\varepsilon = 0.87$
(c) $d = 10$ microns, $l_y = 50$, $\varepsilon = 0.94$
(d) $d = 25$ microns, $l_y = 100$, $\varepsilon = 0.91$

Figure 5.2 Examples of 3-D fiberweb Geometry used for CFD simulation and Particle Tracking Analysis.
5.4 Air Permeability Simulation

In macroscopic terms, a Newtonian fluid flow through a porous medium at a low Reynolds number is governed by Darcy’s law (Eq. 5.9) which states that the flux $q$ (discharge rate per unit area, with units of length per time, m/s), of the fluid is proportional to the pressure gradient $\nabla P$ (Pa/m).

$$q = -\frac{k}{\mu} \nabla P$$

Eq. 5.9

Here, $k$ is the permeability of the porous medium (units of area, m$^2$) and $\mu$ is the dynamic viscosity of the fluid (Pa.s). Permeability of modeled 3-D fiberweb geometry was determined using a commercial Computational Fluid Dynamics (CFD) software package, ANSYS-CFX by assuming a steady-state laminar flow through fiberweb media. Finite volume method in ANSYS-CFX was used to solve continuity and conservation of momentum equations expressed in Eq. 5.10 and Eq. 5.11 respectively.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

Eq. 5.10

$$\rho \frac{Dv}{Dt} = -\nabla p + \mu \nabla^2 v$$

Eq. 5.11
where $p$ is pressure, $v$ is fluid velocity, $\rho$ is fluid density, and $\mu$ is fluid viscosity. A single phase fluid domain was created with air at 25°C as the fluid with laminar flow. Symmetry boundary conditions were applied to the four sides of the 3-D geometry, whereas wall boundary condition with no slip flow was defined at fiber surfaces. Figure 5.3 shows 3-D fiberweb geometry containing 50 layers with fiber diameter $d$, of 10 microns and porosity $\varepsilon$ equal to 0.94. Outlet and inlet boundary conditions were applied to fluid inlet and outlet zones as shown in Figure 5.3.
Inlet fluid velocity was set at 0.05 m/s such that a low Reynolds number close to 0.1 was achieved for all fiberweb geometries, thus ensuring laminar flow. Figure 5.4 shows pressure profile of simulated fluid flow through the fiberweb geometry shown in Figure 5.3.

Figure 5.4 Pressure Profile of a Simulated Fluid flow through a 3-D Fiberweb Geometry containing 10 micron fibers and having a porosity of 0.94
Pressure drop across the sample thickness and superficial fluid velocity, determined from the CFD solution were used to determine permeability using Darcy’s law. Simulations were run using the modeled 3-D fiberweb geometries with varying structural parameters. Table 5.1 shows structural parameter description of 3-D fiberweb geometries used in the simulations. The results of CFD simulations are included in the Appendix E and G.

Table 5.1 Structural parameter description of 3-D fiberweb geometries used in fluid flow simulations

<table>
<thead>
<tr>
<th>Number of Simulations</th>
<th>Fiber Diameter, microns</th>
<th>Porosity</th>
<th>Number of Fiber Layers</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>40</td>
<td>0.815 – 0.848</td>
<td>2, 5, 10, 25, 50</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>0.782, 0.772</td>
<td>10, 20</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
<td>0.892 – 0.905</td>
<td>2, 5, 10, 25, 50</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>0.839 – 0.853</td>
<td>2, 5, 10, 50</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
<td>0.804 – 0.813</td>
<td>2, 5, 10, 25, 50</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>0.910 – 0.917</td>
<td>2, 5, 10, 25, 50, 75</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>0.835 – 0.859</td>
<td>2, 10, 20, 50</td>
</tr>
<tr>
<td>7</td>
<td>25</td>
<td>0.918 – 0.934</td>
<td>2, 5, 10, 25, 50, 75, 100</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>0.907 – 0.921</td>
<td>2, 5, 10, 25, 50</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>0.862 – 0.886</td>
<td>2, 5, 25</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.943, 0.965</td>
<td>50, 50</td>
</tr>
</tbody>
</table>
5.5. Particle Tracking Simulation

Particle tracking method uses a more direct approach towards determining tortuosity in fibrous porous media. Particle tracking method uses a more direct approach towards determining tortuosity in fibrous porous media. Particle flow through porous fibrous media was modeled in ANSYS-CFX using a Lagrangian particle tracking multiphase model. Particles were treated as a separate dispersed phase which is discretely distributed in a continuous phase, water. Lagrangian tracking model was used to characterize the flow behavior of a dispersed phase by tracking Individual particles from their injection point until they escape the domain. One-way coupling was set between the dispersed phase and the continuum phase so that the presence of the dispersed phase (particles) does not influence the continuous phase flow field. The particle trajectories however are influenced by the fluid flow, therefore making the particles act merely as tracers. Particles were tracked by calculating particle displacement using forward Euler integration of particle velocity, $v_i^o$ over time-step, $\delta t$ as shown in Eq. 5.12.

$$x_i^n = x_i^o + v_i^o \delta t,$$

\textbf{Eq. 5.12}

where $x_i$ is the particle position, and the superscripts $n$ and $o$ refer to new and old values respectively.
Particle velocity at the beginning of a time-step was assumed to prevail over the entire
time-step. A new particle velocity was calculated at the end of each time-step by solving
the momentum equation for a discrete particle (Eq. 5.13).

\[ m_p \frac{dv_p}{dt} = F, \]

Eq. 5.13

where \( m_p \) is the particle mass, \( v_p \) is the particle velocity and \( F \) represents the forces acting
on the particle including the drag force and buoyancy force due to gravity. Buoyancy
force was neglected due to the low volume fraction of the dispersed particle phase. Drag
force acting on the particle was computed using the Schiller Naumann model for sparsely
distributed particle. The drag force, \( F_D \) on a particle is given by Eq. 5.14.

\[ F_D = \frac{1}{2} C_D \rho_F A |v_F - v_p| |v_F - v_p| \]

Eq. 5.14

where \( \rho_F \) is the density of fluid, \( A \) is the effective cross-section area of the particle, \( v_F \)
and \( v_p \) are the velocity of fluid and particle respectively, and \( C_D \) is the drag coefficient
which is computed using the Schiller Naumann correlation (Eq. 5.15).
\[ C_D = \frac{24}{Re} \left(1 + 0.15 Re^{-0.687}\right) \]

Eq. 5.15

where \( C_D \) is the drag coefficient and \( Re \) is the Particle Reynolds Number. The same mesh geometries used in simulating permeability were used for particle tracking simulation as well. Single-phase fluid domain with laminar flow was created using water as the fluid. The simulation used solid particles with 0.1 micron diameter and a shape factor of 1. A one-way coupling between the particles and continuous fluid phase was used to predict the particle path based on the flow-field. Use of one-way coupling ensured that particles did not influence the continuous phase flow field. “Cone” injection method was used to inject particles through a circular plane of radius 500 microns, lying in the inlet fluid zone just below bottom layer of fibers. “Cone angle” was set to zero, so that the particles were injected in the direction perpendicular to the injection plane. Particles were injected at 5000 seeding points randomly distributed on the injection plane. Symmetry, wall, inlet and outlet boundary condition were applied in a similar way as shown in Figure 5.3, however inlet fluid (water in this case) was set at 0.005 m/s to ensure a low Reynolds number. At the wall boundary condition coefficient of restitution of 1 was set in order to ensure that the particles bounce back when they hit the fiber surface, therefore resulting in maximum number of through-tracks (tracks running all through the thickness and exiting at the outlet, shown in Figure 5.3.
Tortuosity factor was determined using the classical definition of tortuosity (Eq. 5.1) where; actual length average \( L_e \) is calculated using Eq. 5.16

\[
L_e = \frac{L_{\text{total}}}{N}
\]

**Eq. 5.16**

where \( L_{\text{total}} \) is total length of all the through tracks and \( N \) is the total number of through tracks. Particle tracking simulations were run using 3-D fiberweb geometries with varying structural parameters. Table 5.2 shows structural parameter description of 3-D fiberweb geometries used in the simulations. The results of particle tracking simulations are included in Appendix H.
Table 5.2 Structural parameter description of 3-D fiberweb geometries used in particle tracking simulations

<table>
<thead>
<tr>
<th>Number of Simulations</th>
<th>Fiber Diameter, microns</th>
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<th>Number of Fiber Layers</th>
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</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0.959 – 0.965</td>
<td>10, 25, 50</td>
</tr>
</tbody>
</table>
3-D geometry generation, mesh generation, permeability simulation and particle tracking simulations were performed on Linux Clusters having dual Xeon compute nodes with Intel Xeon processors and 2 Gigabytes memory per core. The time required for running the geometry generation and meshing operation varied from 1 to 4 hours depending upon the size and complexity of the 3-D geometry. On several occasions the geometry generation and the mesh generation operation failed due to complex nature and size of 3-D geometry, therefore complex geometries often required several repeat runs to successfully complete the geometry generation and meshing operations. Some repeat runs of in case of geometry generations to achieve desired porosity values.
Both air permeability and particle tracking simulations were also run on the Linux Cluster. Each of these simulations took up to 6 hours to finish depending upon the number of elements generated in the meshing operation.

5.6. Results and Discussion

Using the simulated results of permeability (included in Appendix E) a linear regression model was developed in which permeability was expressed as a function of construction parameters of 3-D fiberweb geometry. The model expressed in Eq. 5.17 had an R-square value of 0.97, and all the structural parameters were found to have significant affect on tortuosity.

\[
k = 4.44e - 06(d / (1 - \varepsilon)) - 9.10e - 11ln(L) + 1.79e - 04d(1 - \varepsilon)^2 5.85e
- 09(1 - \varepsilon) - 9.82e - 11
\]

Eq. 5.17

Here \(k\) is the air permeability (m\(^2\)), \(d\) is fiber diameter (m), \(L\) is fiberweb thickness (m) and \(\varepsilon\) is porosity. The model established from simulated air permeability results was compared with other empirical and analytical models proposed by Davies [11], Koponen et al. [12], and Tomadakis and Robertson [7].
Davies proposed an empirical correlation by fitting transverse permeability data for flow through highly porous layered fibrous structures, Eq. 5.18

\[ k = r^2[16(1-\varepsilon)^{3/2}(1 + 56(1-\varepsilon)^3)]^{-1} \]

Eq. 5.18

where \( r \) is fiber radius and \( \varepsilon \) is the porosity of the fibrous structure. Davies’ empirical correlation [11] is valid for porosity, \( \varepsilon \) greater than 0.7. Tomadakis and Robertson [7] proposed an analytical model, Eq. 5.19, based on electrical conductivity to predict permeability of fibrous media with layered microstructures

\[ k = \frac{r^2\varepsilon}{8\ln^2\varepsilon} \times \frac{(\varepsilon - \varepsilon_p)^{\alpha+2}}{(1-\varepsilon_p)^\alpha[(\alpha+1)\varepsilon - \varepsilon_p]^2} \]

Eq. 5.19

where \( r \) is the fiber radius, \( \varepsilon \) is the porosity, \( \varepsilon_p \) is percolation threshold (0.11) and \( \alpha \) is a constant (0.785). Like the model proposed by Davies [11], the analytical model proposed by Tomadakis and Robertson [7] is valid for porosity of media greater than 0.7.
Koponen et al. [12] used lattice-Boltzmann method to solve permeability of random 3D fiber web as a function of porosity. The simulated results were used to obtain a fitted expression given in Eq. 5.20

\[ k = r^2 A [e^{B(1-\varepsilon)} - 1]^{-1} \]

Eq. 5.20

where \( r \) is the fiber diameter, \( \varepsilon \) is porosity, and constants \( A = 5.55 \) and \( B = 10.1 \) are constants. The model given in Eq. 5.17 was also compared with an empirical relation established from the experimental results of air permeability (Chapter 4), Eq. 5.21 [13]

\[
k = 6.82e^{-06} \left( \frac{d}{1-\varepsilon} \right)^{-2.2} - 11 \ast \ln(L) + 1.64e^{-04} \ast d(1-\varepsilon)^2 - 1.71e^{-10} - 09 \ast (1-\varepsilon) - 6.66e^{-10}
\]

Eq. 5.21

Here \( k \) is the air permeability (in units of m\(^2\)), \( d \) is fiber diameter (m), \( L \) is fiberweb thickness (m) and \( \varepsilon \) is porosity. The comparison between established model (Eq. 5.17) and the other empirical and analytical models (Eq. 5.18 - Eq. 5.21)and the effect of pore volume porosity on permeability is shown in Figure 5.6. All the models show that permeability increases with higher porosity. At lower porosity, the empirical model (Eq. 5.17) is found to be in good agreement with models proposed by Davies [11],
Koponen et al [12] and Tomadakis and Robertson [7]. At higher porosities, the results were in more agreement with the results of Tomadakis and Robertson [7], as compared to the results of Davies [11] and Koponen et al [12]. The model established based on the CFD simulation results also showed good agreement with the empirical model proposed in Chapter 4.

![Figure 5.6 Comparison between the Empirical Model (Eq. 5.17) and other analytical, empirical and simulation models](image)

The over estimation of permeability by the proposed model (Eq. 5.17) was found to be due to the coarser mesh size used in the meshing operation. Six 10 layer 3-D geometries with fiber size ranging from 15- 40 microns and porosities between 0.81- 0.93 were re-meshed using finer mesh size, in order to find the effect of meshing on simulated results of air permeability.
The simulated results of permeability were found to be lower than those obtained with coarser mesh size, and were also in good agreement with the results obtained from models proposed by Davies [11], Koponen et al [12], and Tomadakis and Robertson [7]. Figure 5.7 shows permeability values of six 3-D geometries determined using fine and coarse mesh size along with the values calculated using other empirical and analytical models.

![Figure 5.7 Comparison Between Simulated Results Using Coarsely and Finely Meshed 3-D Geometries.](image)

Although better results were obtained by using fine meshed 3-D geometry, meshing with small mesh size often failed due to the complexity of the 3-D geometry.
The complex nature of the 3-D geometry also made it very difficult to locate the problems within the 3-D geometry thus making it extremely difficult to repair the inappropriately meshed regions. The simulated results of permeability in case of coarsely meshed geometries followed the same trend as the results obtained in case of finely meshed geometries evident from Figure 5.7. Similarly, the model (Eq. 5.17) obtained from simulated permeability results using coarsely meshed geometry also followed a similar trend as the models proposed by Davies [11], Koponen et al [12] and Tomadakis and Robertson [7].

Permeability increased with increasing fiber diameter as can be seen in Figure 5.8. Comparison between different models again indicated good agreement with other empirical and analytical models, except for the empirical model (Eq. 5.21) based on the experimental results of permeability, which overestimated permeability due to the layered structure of the samples used [13].
The simulated results of permeability were found to be significantly influenced by fiberweb thickness, for the range of fiberweb thickness used. Therefore, both the models based on CFD simulation results (Eq. 5.17) and experimental results (Eq. 5.21) include fiberweb thickness as one of the variables, and this formed the basis for determination of tortuosity. Empirical, analytical and numerical models found in literature so far have not investigated the effect of fabric thickness on permeability. As can be seen in Figure 5.9, permeability decreased with increasing thickness.
Tortuosity factor was determined using Eq. 5.6, where $k(L)_{L \rightarrow 0}$ and $k(L)$ were calculated using the established model based on simulated results (Eq. 5.17). For the permeability $k(L)$ of a fiberweb, a corresponding value $k(L)_{L \rightarrow 0}$ was calculated by setting fabric thickness equal to 4 times the fiber diameter $d$, while keeping the porosity and fiber diameter same. Values of tortuosity of fiberweb geometries were then used to establish a model (Eq. 5.22) which expressed tortuosity as a function of structural parameters of fiberweb geometry such as porosity, fiber diameter and fiberweb thickness.
Here, \( \tau \) is tortuosity factor, \( d \) is fiber size (diameter), \( L \) is fiberweb thickness and, \( \varepsilon \) is porosity.

The proposed model for tortuosity has an R-square value of 0.83 and all the structural parameters were found to have significant influence on tortuosity. Length of the particle tracks through the fiberweb, determined from solution of particle tracking simulations, was used to determine the tortuosity using the classical definition of tortuosity given by Eq. 5.1. The tortuosity results (see Appendix E) were used to establish a model (Eq. 5.23) which established a relationship between tortuosity and fiberweb structural parameters.

\[
\ln(\tau^2) = -0.24 \ln(d) + 0.11 \ln(L) - 1.2 \varepsilon^2 + 1.18
\]

\textbf{Eq. 5.22}

\[
\tau = 0.049d - 0.84 \sqrt{d} + 0.076 \ln(l_y) - 0.048 \sqrt{l_y} - 0.4 \ln(\varepsilon)
+ 0.87 \ln(d) + 0.0019l_y + 1.17
\]

\textbf{Eq. 5.23}

Here, \( \tau \) is tortuosity factor, \( d \) is fiber size (diameter), \( l_y \) is number of fiber layers in 3-D fiberweb geometry and, \( \varepsilon \) is porosity. The model expressed in Eq. 5.23 has an R-square value of 0.91.
The proposed models expressed in Eq. 5.22 and Eq. 5.23 were compared with the empirical model developed using experimental results of permeability (Chapter 4), and other models of tortuosity factor proposed by Koponen et al [5] and Tomadakis and Sotirchos [6]. Koponen et al [5] used lattice-gas simulation in a 2D media to determine tortuosity. The simulated values of tortuosity $\tau$, were expressed as function of porosity $\varepsilon$, given by Eq. 5.24

$$\tau = 1 + a \frac{(1 - \varepsilon_p)}{(\varepsilon - \varepsilon_p)^m}$$

**Eq. 5.24**

where $\varepsilon_p$ the percolation threshold (0.33) and, $a = 0.65$ and $m = 0.19$ are constants. Tomadakis and Sotirchos [6] used random-walk simulation results for bulk tortuosity $\tau_b$, to establish a correlation given by Eq. 5.25. Tomadakis and Robertson [7] derived the expression for tortuosity (viscous tortuosity) of fibrous porous media with layered microstructure as a function of porosity $\varepsilon$, and bulk tortuosity $\tau_b$, given by Eq. 5.26.

$$\tau_b = \left( \frac{1 - \varepsilon_p}{\varepsilon - \varepsilon_p} \right)^a$$

**Eq. 5.25**
\[ \tau = \tau_b \left( 1 + \frac{\alpha \varepsilon}{\varepsilon - \varepsilon_p} \right)^2 \]

Eq. 5.26

Here, \( \tau_b \) is the bulk tortuosity, \( \tau \) is the tortuosity (viscous tortuosity), \( \varepsilon_p \) is the percolation threshold (0.11) and, \( \alpha \) is a constant (0.785). Comparisons were also made with the empirical model based on the experimental results of permeability, Eq. 5.27 (Chapter 4). 

\[ \ln(\tau) = -7.83 \ln(\varepsilon) - 0.54 \ln(d) + 0.188 \ln(L) + 4.85 \varepsilon^2 - 4.49 \]

Eq. 5.27

As shown in Figure 5.10, pore channel tortuosity decreased with increase in porosity. Comparison between the models showed a good agreement between the empirical model (Eq. 5.22), particle Tracking method (Eq. 5.23), Koponen et al [5] (Eq. 5.24) and bulk tortuosity (Eq. 5.25). The model based on the experimental results of permeability (Eq. 23) was also found to be in good agreement with the established models. The model proposed by Tomadakis and Robertson (Eq. 5.26) [7] overestimated tortuosity with its value much higher than 1 even at higher porosity.
Figure 5.10 Variation of Tortuosity with Porosity for Transverse Flow through Fibrous Porous Media

All the models found in the literature have expressed tortuosity as a function of porosity, while ignoring the effect of fiber diameter and fiberweb thickness. The effect of porosity on tortuosity is also illustrated in Figure 5.11 which shows the comparison between two fiberweb geometries having the same thickness and fiber diameter while differing only in porosity. The 3-D geometry having a lower porosity was found to have higher tortuosity value.
The empirical models based on CFD simulation and particle tracking method (Eq. 5.22 and Eq. 5.23 respectively) included fiber diameter and fiberweb thickness as variables. Both variables were found to have significant influence on the tortuosity. Figure 5.12 and Figure 5.13 shows the effect of fiber size (diameter) and fiberweb thickness on tortuosity factor.
Figure 5.12 Variation of tortuosity with fiber diameter at $\varepsilon = 0.9$, and $L = 5\text{ mm}$

Figure 5.13 Variation of Tortuosity with Fabric Thickness at $\varepsilon = 0.9$, $d = 15\text{ microns}$
Increase in fiber diameter with other fabric parameter remaining constant, resulted in a decrease in tortuosity, while increase in the fabric thickness resulted in the increase in tortuosity factor. Comparison between the models based on simulated permeability, experimental permeability and particle tracking results showed a good agreement among the established computational and empirical models for tortuosity.

Using the simulated results of tortuosity obtained tortuosity is plotted as a function number of layers, $l_y$ and solid volume fraction, $1-\varepsilon$ at different values of fiber diameter, $d$ shown in Figure 5.14 - Figure 5.18.

![Figure 5.14 Tortuosity as a function of Number of Layers and Solid Volume Fraction at Fiber Diameter $d = 10$ microns](image)
Figure 5.15 Tortuosity as a function of Number of Layers and Solid Volume Fraction at Fiber Diameter $d = 15$ microns

Figure 5.16 Tortuosity as a function of Number of Layers and Solid Volume Fraction at Fiber Diameter $d = 25$ microns
Figure 5.17 Tortuosity as a function of Number of Layers and Solid Volume Fraction at Fiber Diameter $d = 35$ microns

Figure 5.18 Tortuosity as a function of Number of Layers and Solid Volume Fraction at Fiber Diameter $d = 35$ microns
The plots in Figure 5.14 - Figure 5.18 show that increase in fiber volume fraction results in particles traveling a more tortuous path through fiberweb structure. Tortuosity also increases with increase in the number of layers. Comparison between these plot shows that increase in the fiber diameter resulted in the increase in tortuosity.

5.7. Conclusions

The complexity of the pore space which can be characterized by tortuosity has significant influence on the transport property of fibrous porous media. Modeling tortuosity of fibrous porous media therefore gives a good understanding of filtration, absorbance, heat transfer and acoustic applications. In order to achieve the objective of modeling tortuosity in fibrous porous media, a new model for permeability was proposed based on results of CFD simulations. Unlike the models found in the literature which have expressed permeability as a function of porosity and fiber diameter, the proposed model shows that permeability of fibrous structure is significantly influenced by porosity, fiber diameter as well as fiberweb thickness. The proposed model for permeability was found to be in good agreement with other empirical, analytical and simulation models found in the literature as well as with the proposed empirical model based on experimental results of permeability. Permeability was found to increase with increase in pore volume porosity as well as fiber diameter.
While other models have ignored the effect of fabric thickness on permeability, in the range of fabric thickness measured, permeability was found to be significantly influenced by fabric thickness. This formed the basis for the determination of tortuosity factor. Two computational models for tortuosity were proposed based on CFD simulation and Particle Tracking method.

Unlike the models available in the literature which have expressed tortuosity as a function of porosity only, the new models proposed in this work included other structural parameters as well like fiber diameter and fiberweb thickness. With respect to predicting the relationship between tortuosity and porosity, the proposed computation model was found to be in good agreement with other models available in the literature. Tortuosity was found to decrease with increase in porosity, with the value of tortuosity increasingly becoming closer to 1 as porosity approaches 1. This indicated that the pore channels have minimal tortuosity at porosities close to 1.

The computational models also showed that tortuosity decreased with increase in fiber diameter, whereas it increased with increase in fiberweb thickness. The computational models were also found to be in good agreement with the proposed empirical model based on the experimental results of permeability.
References


Chapter 6  Conclusions

3-D geometry was developed to model fibrous porous material using ANSYS Parametric Design Language (APDL). The generated 3-D geometries were used to simulate permeability of fibrous porous media. A relationship was established between simulated air permeability and fiberweb structural parameters like fiber diameter, porosity and fiberweb thickness. Similarly an empirical relationship was also determined using experimental results of air permeability. The simulated results of permeability were found to be in good agreement with other models in the literature for predicting permeability. The experimental results were found to be slightly overestimating permeability due to the layered configuration of the samples. Both simulated and experimental results of permeability were found to be significantly influenced by fiber diameter, porosity as well as fiberweb thickness. Within the range of sample tested permeability was found to decrease with increasing fiberweb thickness. This significant influence of fiberweb thickness on permeability formed the basis for an indirect method of determination of tortuosity factor. Tortuosity factor was also determined using a more direct method involving particle tracking analysis. Unlike the models available in the literature which have expressed tortuosity as a function of prosody only, the new models proposed in this work included other structural parameters as well like fiber diameter and fiberweb thickness. Tortuosity was found to increase with increase in fiberweb thickness whereas increase in fiber diameter resulted in lower tortuosity.
With respect to predicting the relationship between tortuosity and porosity, the proposed computation model was found to be in good agreement with other models available in the literature. A close agreement was also seen between the results obtained from empirical model and computational models.
Chapter 7 Future Work

In this work, tortuosity of fibrous porous material has been determined using particle tracking analysis as well as analysis of experimental and simulated permeability results. As future work, a diffusion based analysis could also be used to determine tortuosity. It would also be interesting to look at the comparison between diffusive and hydraulic tortuosity. The preliminary algorithm involving geometric analysis of 3-D pore structure in fibrous porous material overestimates tortuosity. The algorithm could be modified to generate more accurate results. Image analysis techniques in conjunction with Digital Volumetric Imaging could be another potential method for determination of tortuosity in fibrous porous materials.

3-D geometry used for CFD simulation and particle tracking analysis assumed random fiber orientation. Therefore, study of effect of fiber orientation distribution on tortuosity could not be made. 3-D fiberweb geometry having varying fiber orientation distribution can be used to study the effect of fiber orientation distribution.
Appendix A: Preliminary APDL Codes for Modeling 3-D Fiberweb Geometry

APDL program codes for 3-D fiberweb generation using non-continuous fiber with circular cross section

/prep7
num=5 !number of layers
LLL=100 !length of fiber
R=0.5 !radius of fiber cross-section
yy=0
LL=0 !line counter initiation
aa=0 !area counter initiation
fnum=25 !Number of fibers in one layer
vnn=1001 !starting volume number after subtraction
*do,jj,0,num-1,1 !change number of layers here
   *do,ii,0,(fnum-1)*11,11
      kk=ii+fnum*11*jj
      xx=rand(0,100)
      zz=rand(0,100)
      theta=rand(0,360)
      *do,nn,1,3,1
         *if,nn,eq,1,then
            xx1=xx
            yy1=yy
            zz1=zz
            theta1=theta
            pp1=kk+nn
            k,pp1,xx1,yy1,zz1
         *elseif,nn,eq,2
            xx2=xx1+(LLL*cos(theta1))
            yy2=yy
            zz2=zz1+(LLL*sin(theta1))
            pp2=kk+nn
            k,pp2,xx2,yy2,zz2
         *else
            xx3=xx1+(LLL*cos(theta1))
            yy3=yy
            zz3=zz1+(LLL*sin(theta1))
            pp3=kk+nn
            k,pp3,xx3,yy3,zz3
         *endif
   *endif
*endif
xx3=xx1
yy3=yy1+.1
zz3=zz1
pp3=kk+nn
k,pp3,xx3,yy3,zz3
*endif
*enddo
l,pp1,pp2
circle,pp1,R,pp2,pp3
LL1=LL+1
LL2=LL+2
LL3=LL+3
LL4=LL+4
LL5=LL+5
al,LL2,LL3,LL4,LL5
aa1=aa+1
LL=LL+13
aa=aa+6
vdrag,aa1,,,,LL1
*enddo

yy=yy+1
*enddo
APDL Program Codes for Generating 3-D Geometry of Pore Structure

/prep7

num=5 !number of layers
LLL=100 !length of fiber
R=0.5 !radius of fiber cross-section
yy=0
LL=0 !line counter initiation
aa=0 !area counter initiation
fnum=25 !Number of fibers in one layer
vnn=1001 !starting volume number after subtraction
*do,jjj,0,num-1,1 !jjj is the layer number
  !making solid disk for every layer of fibers
  *get,kpcount,kp,,count !kpcount is the keypoint count
  k,kpcount+1,50,-0.05+jjj,50
  k,kpcount+2,50,0.05+jjj,50
  k,kpcount+3,50,-0.05+jjj,51
  l,kpcount+1,kpcount+2
  *get,lcount,line,,count !lcount is the line count
circle,kpcount+1,175,kpcount+2,kpcount+3
  al,lcount+1,lcount+2,lcount+3,lcount+4
  *get,acount,area,,count !acount is the area count
  numstr,volu,vnn
  vdrag,acount,,,,lcount
  *get,vcounta,volu,,count

!Subtracting fibers from the disk
   /uis,msgpop,3
   boptn,nwarn,1
   vsbv,vnn,1+fnum*jjj,,delete,delete
   *do,ff,2,fnum,1
     *get,vcountb,volu,,count
     vnew=vcountb-vcounta+2
     *dim,vnum,array,1,vnew
     *if,vnew,eq,1,then
       vnum(1)=vlnext(vnn-1)
     *else
*do,i,2,vnew,1
vnum(1)=vlnext(vnn-1)
vpree=vnum(1,i-1)
vnum(1,i)=vlnext(vpre)
*enddo

*endif
*do,i,1,vnew,1
vv=vnum(1,i)
vsbv,vv,ff+fnum*jjj,delete,keep
*enddo
vdele,ff+fnum*jjj,ff+fnum*jjj,1,1
*del,vnum
*enddo
!vnn=vnn+100
!numstr,volu,vnn
!*do,jjn,1,50,1
!cyl4,jjn,-10-jj,0.5,,,,1
!*enddo
!vdele,1+(fnum*jjj),fnum*(1+jjj),1,1
!
!vdele,l+(fnum*jjj),fnum*(1+jjj),1,1
vnn=vnn+200
numcmp,kp
numcmp,line
numcmp,area
*enddo
!delete all the fibers
!*do,iii,0,num-1,1  !change number of layers here
!vdele,l+(fnum*iii),fnum*(1+iii),1,1
!*enddo
!delete largest volume (disk) in each layer
vsum,lab
*do,jj,0,num-1,1
vol1=vlnext(1000+200*jj)
vol2=vlnext(vol1)
*do,i,1000+200*jj,1100+200*jj,1
*if,vlnext(i),lt,1100+200*jj,then
*get,vlarge1,volu,vol1,volu
*get,vlarge2,volu,vol2,volu
*if,vlarge1,gt,vlarge2,then
vol2=vlnext(vol2)
*else
else
vol1=vol2
vol2=vlnext(vol1)
*endif
*else
*endif
*enddo
vdele, vol1, vol1, 1, 1
*enddo

!delete extra volumes
!*do, jj, 0, num-1, 1
!vdele, 1101+(200*jj), 1101+(200*jj)+50, 1, 1
!*enddo

!moving pore volumes
*do, jj, 0, num-1, 1
mm1=1001+200*jj
mm2=1001+200*jj+99
vgen,, mm1, mm2, 1, 0, -0.9*jj, 0,, 1
*enddo

!scale Volume in Y-direction
*do, jj, 0, num-1, 1
mm1=1001+(200*jj)
mm2=1001+(200*jj)+99
vlscale, mm1, mm2, 1, 1, 10, 1,, 1
*enddo

APDL Program Codes for 3-D Geometry using Continuous Fibers with Square Cross-Section.

/prep7
*GET,DIM,ACTIVE,0,TIME,WALL
DIM=DIM*3600
*DIM,DUMMY,ARRAY,DIM
*VFILL,DUMMY(1),RAND
*DEL,DIM
*DEL,DUMMY
/uis,msgpop,3
boptn,nwarn,1
wpstyl,defa
num=10 !number of layers
LL=650 !length of fiber
dd=3 !fiber diameter
yy=0
aa=0 !area counter initiation
fnum=30 !Number of fibers in one layer
*do,jjj,1,num,1 !change number of layers here
wpstyl,defa
*do,jj,1,fnum,1
  !wpstyl,defa
  *get,kpcount,kp,,count
  pp=rand(300,325) !smaller ranger produces more uniform distribution of pores
  pptheta=rand(0,360)
  xx=pp*cos(pptheta)
  zz=pp*sin(pptheta)
  theta=rand(0,360)
  k,kpcount+1,xx,yy,zz
  k,kpcount+2,xx+dd*cos(theta),yy,zz+dd*sin(theta)
  k,kpcount+3,xx,yy+dd,zz
  kwplan,,kpcount+1,kpcount+2,kpcount+3
  seg=rand(-5,5)
  *if,seg,le,0,then
    blc4,0,0,dd,dd,-LL
  *else,
    blc4,0,0,dd,dd,LL
  *endif
*enddo
yy=yy+dd+1
*enddo

wpstyl,defa
/prep7
wpstyl,defa

num=4  !number of layers
LLL=100  !length of fiber
s=1  !stack number
fnum=30  !Number of fibers in one layer
vnn=1001+num*200*(s-1)  !starting volume number after subtraction
jj=0
*do,jjj,(s-1)*num,(s*num)-1,1 !jjj is the layer number
!making solid disk for every layer of fibers
  wpstyl,defa
  wprota,0,-90,0
  wpoffs,,,4*jjj
  numstr,volu,vnn
  !blc5,0,0,450,450,3  !square disk
  cyl4,0,0,250,,,,3  !circular disk
!Subtracting fibers from the disk
/uis,msgpop,3
boptn,nwarn,1
vsbv,vnn,1+fnum*jj,,delete,delete
*do,ff,2,fnum,1
  vsel,s,volu,,vnn,vnn+199,1,0
*get,vcountb,volu,,count
  vnew=vcountb
*dim,vnum,array,1,vnew
  *if,vnew,eq,1,then
    vnum(1,1)=vlnext(vnn-1)
  *else
    *do,i,2,vnew,1
      vnum(1,1)=vlnext(vnn-1)
      vpre=vnum(1,i-1)
      vnum(1,i)=vlnext(vpre)
  *endif
*endif
vsel, all
*do, ii, 1, vnew, 1
    vsbv, vnum(1, ii), ff+fnum*jj, delete, keep
*enddo
vdele, ff+fnum*jj, ff+fnum*jj, 1, 1
*del, vnum
*enddo

vnn=vnn+200
numcmp, kp
numcmp, line
numcmp, area
jj=jj+1
*enddo
vdele, 1, fnum*num, 1, 1

! moving pore volumes
*do, jj, 1, num, 1
    mm1=1001+200*jj
    mm2=1000+200*(jj+1)
vgen, mm1, mm2, 1, 0, -1*jj, 0, ,, 1
*enddo
Appendix B: Preliminary Approach for the Determination of Tortuosity using Geometric Analysis

APDL Codes for Determination of Tortuosity using Geometric Analysis in case of 3-D Geometry with Non-continuous Fiber with Circular Cross-Section

/prep7
!generate pore channels (solids)
!step one (Layer1 and Layers2 volume intersection)
  n=5          !number of layers
  bb=0         !column counter
  vn=1         !starting volume number

*dim,centpath,array,100,100,10  !stores keypoints along the shortest path
*dim,volpath,array,100,100,10   !stores volumes associated with shortest path
*dim,overlapvol,array,100,100,10 !stores overlap volumes
*dim,leff,array,1,100           !stores effective length of shortest paths corresponding to volumes in first layer

boptn,keep,yes
numcmp,area
numcmp,line
numcmp,kp
*do,ii,1000,1098,1               !selects pores volumes in first layer
  bb=bb+1
  vsel,all
  ksel,all
  *if,vlnext(ii),gt,1099,exit,
    *del,areakeep
    *del,centkp
  cc=1
*do,jj,1200,1300,1
  *get,acount,area,,count
  *if,vlnext(jj),gt,1400,exit
    vinp,vlnext(ii),vlnext(jj)
    *get,acountnew,area,,count
    *if,acountnew,gt,acount,then
      numstr,volu,vn
      *do,i,acount+1,acountnew,1
        asel,s,area,,i,i,1,0
        asum,lab
*get,centx,area,,cent,x  
*get,centy,area,,cent,y  
*get,centz,area,,cent,z  
asel,all  
ksel,all  
*get,kpcount,kp,,count  
centkp0=kpcount+1  
k,centkp0,centx,centy,centz  
centpath(1,bb,cc)=centkp0  
vext,i,i,1,0,1,0  
overlapvol(1,bb,cc)=vn+cc  
volpath(1,bb,cc)=vlnext(jj)  
cc=cc+1  
*enddo  
*endif  
*endo  
*do,kk,1,cc-1,1 !for number of overlap vols created  
*do,LL,3,n,1 !for number of layers  
*dim,centkp,array,1,100  
!temporarily stores centroids of overlap areas in a layer  
*dim,areakeep,array,1,100  
!temporarily stores overlap areas  
*dim,volkeep,array,1,100  
!temporarily stores volumes  
aaa=1  
*do,mm,1200+200*(LL-2),1300+200*(LL-2),1 !for volumes in a layer  
*get,acount,area,,count  
vinp,overlapvol(LL-2,bb,kk),vlnext(mm)  
*get,acountnew,area,,count  
*if,acountnew.gt,acount,then  
  *do,i,acount+1,acountnew,1  
    asel,s,area,,i,i,1,0  
    asum,lab  
    *get,centx,area,,cent,x  
    *get,centy,area,,cent,y  
    *get,centz,area,,cent,z  
    asel,all  
    ksel,all  
  *endo  
*endif
*get,kpcount,kp,count
centkp0=kpcount+1
k,centkp0,centx,centy,centz
centkp(1,aaa)=centkp0
areakeep(1,aaa)=i
volkeep(1,aaa)=vlnext(mm)
aaa=aaa+1
*enddo
*endif
*enddo
*if,aaa-1,gt,1,then,
ksel,s,kp,centpath(1,bb,kk),centpath(1,bb,kk),1
ksel,s,kp,centkp(1,1),centkp(1,aaa-1),1
centpath(LL-1,bb,kk)=kpnear(centpath(1,bb,kk)
*do,j,1,aaa-1,1
  *if,centpath(LL-1,bb,kk),eq,centkp(1,j),then
  vext,areakeep(1,j),areakeep(1,j),1,0,1,0
  overlapvol(LL-1,bb,kk)=cc+LL-3
  volpath(LL-1,bb,kk)=volkeep(1,j)
  *endif
  *enddo
*enddo
*del,areakeep
*del,volkeep
*del,centkp
ksel,all
asel,all
vsel,all
*enddo

vn=vn+100
*enddo
Determination of Tortuosity using Geometric Analysis in case of 3-D Geometry with Continuous Fiber with Circular Cross-Section

/prep7
!generate pore channels (solids)
!step one (Layer1 and Layers2 volume intersection)
wpstyl,defa
n=10 !number of layers
bb=0 !column counter
d d = 3 !fiber diameter
numstr,volu,1
*dim,centpath,array,100,100,20 !stores keypoints along the shortest path
*dim,volpath,array,100,100,20 !stores volumes associated with shortest path
*dim,overlapvol,array,100,100,20 !stores overlap volumes
*dim,leff,array,1,100,100 !stores effective length of shortest paths corresponding to volumes in first layer
*dim,contrl,array,100,100,20
boptn,keep,yes
numcmp,area
numcmp,line
numcmp,kp
*do,ii,1000,1050,1 !selects pores volumes in first layer
bb=bb+1
vsel,all
ksel,all
asel,all
*if,vlnext(ii),gt,1099,exit
*del,areakeep
*del,centkp
*del,kopleep
!volii=vlnext(ii)
!*status,volii
cc=1
volpath(1,bb,cc)=vlnext(ii)
overlapvol(1,bb,cc)=vlnext(ii)
*do,jj,1200,1300,1  !volumes in second layer
*get,acount,area,,count
*if,vlnext(jj),gt,1400,exit
  vinp,vlnext(ii),vlnext(jj)
  *get,acountnew,area,,count
  !areanum=acountnew
 !*status,acountnew
  *if,acountnew,gt,acount,then
   *do,i,acount+1,acountnew,1
     asel,s,area,,i,i,1,0
     asum,lab
     *get,centx,area,,cent,x
     *get,centy,area,,cent,y
     *get,centz,area,,cent,z
     asel,all
     ksel,all
     vsel,all
     *get,kpcount,kp,,count
     centkp0=kpcount+1
     k,centkp0,centx,centy,centz
    !*status,cc
     voljj=vlnext(jj)
    !*status,voljj
     centpath(2,bb,cc)=centkp0
     ctrl(2,bb,cc)=1
     !k,centkp0+1,centx,centy-1,centz
     !centpath(1,bb,cc)=centkp0+1
     vext,i,i,1,0,dd,0
     *get,vlow,volu,1000,nxtl
     !next lowest volume number below 1000
     overlapvol(2,bb,cc)=vlow
     volpath(2,bb,cc)=vlnext(jj)
     volpath(1,bb,cc)=vlnext(ii)
     aa=aa+1
     cc=cc+1
  *enddo
*endif
*enddo
!*status,cc
*if,cc,gt,1,then
*do,kk,1,cc-1,1 !for number of overlap vols created
  ksel,all
  asel,all
  vsel,all
*do,LL,3,n,1 !third layer onwards
  *del,areakeep
  *del,volkeep
  *del,centkp
  ksel,all
  asel,all
  vsel,all
  *dim,centkp,array,1,100
  !temporarily stores centroids of overlap areas in a layer
  *dim,areakeep,array,1,100
  !temporarily stores overlap areas
  *dim,volkeep,array,1,100
  !temporarily stores volumes
  aaa=1
  *do,mm,1200+200*(LL-2),1300+200*(LL-2),1
  !volumes in the current layer
  *if,vlnext(mm),gt,1300+200*(LL-2),or,vlnext(mm),eq,0,exit
    *get,acount,area,,count
    aa1=overlapvol(LL-1,bb,kk)
    aa2=vlnext(mm)
    !*status,aa1
    !*status,aa2
    !*status,kk
    !*status,bb
    vinp,overlapvol(LL-1,bb,kk),vlnext(mm)
    *get,acountnew,area,,count
    *if,acountnew,gt,acount,then
      *do,i,acount+1,acountnew,1
        asel,s,area,,i,i,1,0
        assum,lab
        *get,centx,area,,cent,x
        *get,centy,area,,cent,y
        *get,centz,area,,cent,z
        asel,all
        ksel,all
*get,kpcount,kp,count
  centkp0=kpcount+1
k,centkp0,centx,centy,centz
centkp(1,aaa)=centkp0
areakeep(1,aaa)=i
volkeep(1,aaa)=vlnext(mm)
aaa=aaa+1
*enddo
*enddo
!*status,aaa
*if,aaa,gt,1,then
!ksel,s,kp,,centpath(LL-1,bb,kk),centpath(LL-1,bb,kk),1
ksel,s,kp,,centkp(1,1),centkp(1,1),1
*do,j,1,aaa-1,1
  ksel,a,kp,,centkp(1,j),centkp(1,j),1
*endo
!*status,centkp,1,1,1,5
centpath(LL,bb,kk)=knear(centpath(LL-1,bb,kk))
contrl(LL,bb,kk)=1
!cp=centpath(LL,bb,kk)
!*status,cp
*do,j,1,aaa-1,1
  cpp=centkp(1,j)
!*status,cpp
  *if,centpath(LL,bb,kk),eq,centkp(1,j),then
    vext,areakeep(1,j),areakeep(1,j),1,0,dd,0
    vsel,all
    *get,vlow,volu,1000,nxtl
    overlapvol(LL,bb,kk)=vlow
    !olap=vlow
   !*status,olap
    volpath(LL,bb,kk)=volkeep(1,j)
  *endif
*endo
*elseif,aaa,eq,1,then
  bbb=aaa
*do,mm,1200+200*(LL-2),1300+200*(LL-2),1
!for volumes in a layer
*if,vlnext(mm),gt,1300+200*(LL-2),or,vlnext(mm),eq,0,exit
*get,acount,area,,count
!aa3=volpath(LL-1,bb,kk)
!aa4=vlnext(mm)
vinp,volpath(LL-1,bb,kk),vlnext(mm)
*get,acountnew,area,,count
*if,acountnew,gt,acount,then
  *do,i,acount+1,acountnew,1
    asel,s,area,,i,i,1,0
    asum,lab
    *get,centx,area,,cent,x
    *get,centy,area,,cent,y
    *get,centz,area,,cent,z
    asel,all
    ksel,all
  *get,kpcount,kp,,count
  centkp0=kpcount+1
  k,centkp0,centx,centy,centz
  centkp(1,bbb)=centkp0
  areakeep(1,bbb)=i
  volkeep(1,bbb)=vlnext(mm)
  bbb=bbb+1
  *enddo !loop i
*endif
*enddo !loop mm

!status,bbb
!if,bbb,gt,1,then,
  !ksel,all
!ksel,s,kp,.,centpath(1,bb,kk),centpath(1,bb,kk),1
  ksel,s,kp,.,centkp(1,1),centkp(1,1),1
  *do,j,1,bbb-1,1
    ksel,a,kp,.,centkp(1,j),centkp(1,j),1
  *enddo
  centpath(LL,bb,kk)=knear(centpath(LL-1,bb,kk))
  contrl(LL,bb,kk)=0
  *do,j,1,bbb-1,1
  *if,centpath(LL,bb,kk),eq,centkp(1,j),then
    vext,areakeep(1,j),areakeep(1,j),1,0,dd,0
    vsel,all
*get,vlow,volu,1000,nxtl
overlapvol(LL,bb,kk)=vlow
!lap=vlow
!*status,lap
    volpath(LL,bb,kk)=volkeep(1,j)
    *endif
    *endif
!else
    *exit
*endif
*endif
*enddo
*enddo
*endif
*endif
*endif
!loop LL
*enddo
!loop kk
*endif
!if statement cc
*do,rr,1,cc,1
    len=dd
    *do,qq,2,n,1
        *if,centpath(qq,bb,rr),eq,0,exit
        *if,contrl(qq,bb,rr),eq,1,then
            len=len+dd
        *else,
            lenseg=distkp(centpath(qq-1,bb,rr),centpath(qq,bb,rr))
            len=lenseg+len
        *endif
        *endif
    leff(1,bb,rr)=len
*enddo
!*status,bb
*enddo
!loop ii
!*status,volpath,1,num,1,50,1,20
!*status,leff,1,1,50,1,20
*status,bb
*status,cc
save,out30F10Lcont450sq_650_300.db
Appendix C: APDL Codes for 3-D Geometry Construction

APDL program Codes for 3-D Geometry Construction for CFD and Particle Tracking Analysis

/prep7
*GET,DIM,ACTIVE,0,TIME,WALL
DIM=DIM*3600
*DIM,DUMMY,ARRAY,DIM
*VFILL,DUMMY(1),RAND
*DEL,DIM
*DEL,DUMMY
/uis,msgpop,4
boptn,nwarn,1
nn=1                !number of stacks
yy=0
*do,ss,1,nn,1        !stack number
wpstyl,defa

numstr,volu,1
num=5                !number of layers per stack
LL=160               !length of fiber
dd=2.5               !fiber diameter
sep=5                !layer separation

aa=0                 !area counter initiation
fnum=45              !Number of fibers in one layer

*do,jjj,1+(ss-1)*num,num*ss,1     !change number of layers here
wpstyl,defa
*do,jj,1,fnum,1
   *get,kpcount,kp,,count
     pp=80
     pptheta=rand(0,360)
     xx=pp*cos(pptheta)
     zz=pp*sin(pptheta)
     theta=rand(0,360)
k,kpcount+1,xx,yy,zz
k,kpcount+2,xx+dd*cos(theta),yy,zz+dd*sin(theta)
kwplan,,kpcount+1,kpcount+2,kpcount+3
seg=rand(-5,5)
*if,seg,le,0,then
   blc4,0,0,dd,dd,-LL
*else,
   blc4,0,0,dd,dd,LL
*endif

*enddo
yy=yy+dd+sep
*enddo
*enddo
*do,s,1,nn,1
wpstyl,defa
vnn=10001+num*200*(s-1) !starting volume number after subtraction
jj=(s-1)*num
*do, jjj,(s-1)*(dd+sep)*num,(s*num-1)*(dd+sep),dd+sep !jjj is the layer number
   !making solid disk for every layer of fibers
   wpstyl,defa
   wpnota,0,-90,0
wpoffs,,,jjj
   numstr,volu,vnn
   ! cyl4,0,0,75,,,,dd !circular disk
   blc5,0,0,100,100,dd !square disk
   !Subtracting fibers from the disk
   /uis,msgpop,3
boptn,nwarn,1
vsbv,vnn,1+fnum*jj,delete,delete
*do,ff,2,fnum,1
vsel,s,volu,,vnn,vnn+199,1,0
*get,vcountb,volu,,count
vnew=vcountb
*dim,vnum,array,1,vnew
*if,vnew(eq,1,then
   vnum(1,1)=vlnext(vnn-1)
*else
   *do,i,2,vnew,1
      vnum(1,1)=vlnext(vnn-1)

vpre=vnum(1,i-1)

vnum(1,i)=vlnext(vpre)
*enddo
*endif
vsel,all
*do,ii,1,vnew,1
  vsvb,vnum(1,ii),ff+fnum*jj,delete,keep
*enddo
vdele,ff+fnum*jj,ff+fnum*jj,1,1
*del,vnum
*enddo
vnn=vnn+200
numcmp,kp
numcmp,line
numcmp,area
jj=jj+1
*enddo
vdele,1+fnum*(s-1)*num,fnum*num*s,1,1
!moving pore volumes
xx=(s-1)*num
*do,jj,(s-1)*num,s*num-1,1
  mm1=10001+200*jj
  mm2=10000+200*(jj+1)
  vgen,,mm1,mm2,1,0,-1*sep*xx,0,,,1
  xx=xx+1
*enddo
vsel,all
jj1=10000+200*(s-1)*num
jj2=10000+200*s*num
vsel,s,volu,,jj1,jj2,1,1
vadd,all
wpstyl,defa
ksel,all
lsel,all
asel,all
vsel,all
*enddo
vsum
save,5ly25d45fsqc.db
Appendix D: Fiber Diameter Measurement

Sample sb1 at 10X magnification  
Sample sb2 at 10X magnification

Sample jm1 at 40X magnification  
Sample jm2 at 40X magnification

Figure 8.1 Sample images under an optical microscope
Figure 8.2 Sample images under an optical microscope
Sample sbh at 40X magnification

Figure 8.3 Sample image under an optical microscope
Table 8.1 Fiber Diameter Measurement Results

<table>
<thead>
<tr>
<th>Fiber Diameter (mm)</th>
<th>sb1</th>
<th>sb2</th>
<th>jm1</th>
<th>jm2</th>
<th>sbh</th>
<th>dp45</th>
<th>dp55</th>
<th>dp68</th>
<th>dp90</th>
<th>Avg (mm)</th>
<th>Std Dev.</th>
<th>CV</th>
<th>New Avg (mm)</th>
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<tr>
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<td>0.0097</td>
<td>0.0069</td>
<td>0.0232</td>
<td>0.0249</td>
<td>0.0333</td>
<td>0.0269</td>
<td>0.0143</td>
<td>0.0009</td>
<td>0.0630</td>
<td>0.0143</td>
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<td>sb2</td>
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<td>0.0111</td>
<td>0.0092</td>
<td>0.0078</td>
<td>0.0257</td>
<td>0.0248</td>
<td>0.0266</td>
<td>0.0286</td>
<td>0.0158</td>
<td>0.0011</td>
<td>0.0696</td>
<td>0.0158</td>
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<td>jm1</td>
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<td>0.0163</td>
<td>0.0085</td>
<td>0.0097</td>
<td>0.0069</td>
<td>0.0237</td>
<td>0.0249</td>
<td>0.0225</td>
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<td>0.0281</td>
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<td>0.0099</td>
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<td>0.0231</td>
<td>0.0222</td>
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<td>0.0261</td>
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<td>New Avg (mm)</td>
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</table>
The t-test showed the lack of significant difference between average fiber diameters of samples dp55, dp68, and dp90, therefore the samples were considered to have a common average fiber diameter value 25.3 microns. Similar t-test showed no significant difference between fibers diameters of samples jm1 and jm2, therefore a common diameter average of 9.5 microns was used for samples jm1 and jm2.
Appendix E: Experimental and Simulated Results of Permeability, and Simulated Results of Tortuosity

Table 8.2 Experimental Results of Air Permeability

<table>
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<tr>
<th>Sample ID</th>
<th>Number of Layers</th>
<th>Fiber Size (m)</th>
<th>Thickness (m)</th>
<th>Porosity</th>
<th>Air Perm (m^2)</th>
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<td>Sample ID</td>
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<td>Fiber Size (m)</td>
<td>Thickness (m)</td>
<td>Air Perm (m^2)</td>
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Table 8.4 Experimental Results of Air Permeability (Continued)

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<th>Number of Layers</th>
<th>Porosity</th>
<th>Fiber Size (m)</th>
<th>Thickness (m)</th>
<th>Air Perm (m^2)</th>
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Table 8.5 Experimental Results of Air Permeability (Continued)

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Table 8.6 Simulated Results of Air Permeability

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<th>Pressure Difference</th>
<th>Flow out (m/s)</th>
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Table 8.7 Simulated Results of Air Permeability

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<th>Pressure Difference</th>
<th>Flow out (m/s)</th>
<th>Thickness (m)</th>
<th>Permeability (m²)</th>
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Table 8.9 Results of Tortuosity obtained using Particle Tracking Simulations

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Appendix F: Analysis of Variance

Analysis of Variance performed using experimental results of air permeability.

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: perm

Number of Observations Read 112
Number of Observations Used 112
Backward Elimination: Step 0

All Variables Entered: R-Square = 0.8656 and C(p) = 5.0000

Analysis of Variance

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Bounds on condition number: 10.451, 90.866

Variable selection terminated as the selected model is a perfect fit.
Analysis of Variance using Simulated Permeability Results

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: perm

Number of Observations Read 48
Number of Observations Used 48

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.9654 and C(p) = 5.0000

Analysis of Variance

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Bounds on condition number: 16.089, 130.07

Variable selection terminated as the selected model is a perfect fit.
Analysis of Variance using Experimental Results of Tortuosity

The SAS System
The REG Procedure
Model: MODEL1
Dependent Variable: logtau

Number of Observations Read 112
Number of Observations Used 112

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.8442 and C(p) = 5.0000

Analysis of Variance

<table>
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Variable Parameter Estimate Standard Error Type II SS F Value Pr > F
Intercept        -4.48754  1.04964  0.19168   18.28  <.0001
logpor           -7.83359  1.47952  0.29398   28.03  <.0001
logdia           -0.54493  0.03431  2.64587   252.31 <.0001
logthick         0.18790  0.01569  1.50331   143.35 <.0001
pord              4.85562  1.17720  0.17841   17.01  <.0001

Bounds on condition number: 128.39, 1019.8

All variables left in the model are significant at the 0.1000 level.
### Analysis of Variance using Simulated Results of Tortuosity

The SAS System
The REG Procedure
Model: MODEL1
Dependent Variable: logtau

Number of Observations Read 51
Number of Observations Used 51

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.8311 and C(p) = 4.0000

#### Analysis of Variance

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Bounds on condition number: 2.9255, 20.168

All variables left in the model are significant at the 0.1000 level.
Analysis of Variance using Tortuosity Results obtained using Particle Tracking Analysis

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: tortuosity

Number of Observations Read 49
Number of Observations Used 49

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.9059 and C(p) = 8.0000

Analysis of Variance

<table>
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<th>Mean Square</th>
<th>F Value</th>
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Parameter Estimates

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Bounds on condition number: 37670, 403300

All variables left in the model are significant at the 0.1000 level.
Appendix G: Results of CFD Simulation

3-D geometry: d= 25 microns, ly = 100, \( \varepsilon = 0.918 \)

Pressure Profile
\( \nabla P = 1.66 \text{ Pa}, k = 1.40 \times 10^{-9} \text{ m}^2 \)

3-D geometry: d= 25 microns, ly = 75, \( \varepsilon = 0.926 \)

Pressure Profile
\( \nabla P = 1.06 \text{ Pa}, k = 1.64 \times 10^{-9} \text{ m}^2 \)

Figure 8.4 3-D geometry and pressure profile of simulated fiberwebs with d = 25 microns
3-D geometry: d = 25 microns, ly = 50, $\varepsilon = 0.921$

Pressure Profile
$\nabla P = 8.54\times 10^{-1}$ Pa, $k = 1.36\times 10^{-9}$ m$^2$

3-D geometry: d = 25 microns, ly = 25, $\varepsilon = 0.934$

Pressure Profile
$\nabla P = 2.90\times 10^{-1}$ Pa, $k = 2.00\times 10^{-9}$ m$^2$

Figure 8.5 3-D geometry and pressure profile of simulated fiberwebs with d = 25 microns
3-D geometry: \(d= 25\) microns, \(l_y = 10\), \(\varepsilon = 0.925\)

Pressure Profile
\(\nabla P = 1.44e-01\) Pa, \(k = 1.58e-09\) m\(^2\)

3-D geometry: \(d= 25\) microns, \(l_y = 5\), \(\varepsilon = 0.934\)

Pressure Profile
\(\nabla P = 4.90e-02\) Pa, \(k = 2.37e-09\) m\(^2\)

Figure 8.6 3-D geometry and pressure profile of simulated fiberwebs with \(d = 25\) microns
3-D geometry: \(d = 25 \text{ microns}, \ ly = 75, \ \varepsilon = 0.910\)

Pressure Profile
\[\nabla P = 1.42 \text{ Pa}, \ k = 1.22 \times 10^{-9} \text{ m}^2\]

3-D geometry: \(d = 25 \text{ microns}, \ ly = 50, \ \varepsilon = 0.914\)

Pressure Profile
\[\nabla P = 8.6 \times 10^{-1} \text{ Pa}, \ k = 1.35 \times 10^{-9} \text{ m}^2\]

Figure 8.7 3-D geometry and pressure profile of simulated fiberwebs with \(d = 25 \text{ microns}\)
3-D geometry: d= 25 microns, ly = 25, \( \varepsilon = 0.917 \)

Pressure Profile
\( \nabla P = 4.05 \times 10^{-1} \text{ Pa}, k = 1.44 \times 10^{-9} \text{ m}^2 \)

3-D geometry: d= 25 microns, ly = 10, \( \varepsilon = 0.914 \)

Pressure Profile
\( \nabla P = 1.62 \times 10^{-1} \text{ Pa}, k = 1.43 \times 10^{-9} \text{ m}^2 \)

Figure 8.8 3-D geometry and pressure profile of simulated fiberwebs with d = 25 microns
3-D geometry: d = 25 microns, \( l_y = 5 \), \( \varepsilon = 0.914 \)

Pressure Profile
\[ \nabla P = 7.66 \times 10^{-2} \text{ Pa}, \quad k = 1.52 \times 10^{-9} \text{ m}^2 \]

3-D geometry: d = 25 microns, \( l_y = 50 \), \( \varepsilon = 0.859 \)

Pressure Profile
\[ \nabla P = 1.69 \text{ Pa}, \quad k = 6.72 \times 10^{-10} \text{ m}^2 \]

Figure 8.9 3-D geometry and pressure profile of simulated fiberwebs with d = 25 microns
3-D geometry: $d = 25$ microns, $l_y = 20$, $\varepsilon = 0.844$

Pressure Profile
$\nabla P = 7.94 \times 10^{-1} \text{ Pa}$, $k = 5.74 \times 10^{-10} \text{ m}^2$

3-D geometry: $d = 25$ microns, $l_y = 10$, $\varepsilon = 0.841$

Pressure Profile
$\nabla P = 4.09 \times 10^{-1} \text{ Pa}$, $k = 5.57 \times 10^{-10} \text{ m}^2$

Figure 8.10 3-D geometry and pressure profile of simulated fiberwebs with $d = 25$ microns
3-D geometry: \( d = 25 \) microns, \( ly = 2, \varepsilon = 0.8355 \)

Pressure Profile
\[ \nabla P = 6.48 \times 10^{-2} \text{ Pa}, k = 7.17 \times 10^{-10} \text{ m}^2 \]

3-D geometry: \( d = 15 \) microns, \( ly = 50, \varepsilon = 0.913 \)

Pressure Profile
\[ \nabla P = 8.21 \times 10^{-1} \text{ Pa}, k = 8.49 \times 10^{-10} \text{ m}^2 \]

Figure 8.11 3-D geometry and pressure profile of simulated fiberwebs with \( d = 25 \) and 15 microns
3-D geometry: $d = 15$ microns, $l_y = 25$, $\varepsilon = 0.919$

Pressure Profile

$\nabla P = 3.61 \times 10^{-1} \text{ Pa}$, $k = 9.66 \times 10^{-10} \text{ m}^2$

3-D geometry: $d = 15$ microns, $l_y = 10$, $\varepsilon = 0.907$

Pressure Profile

$\nabla P = 1.71 \times 10^{-1} \text{ Pa}$, $k = 8.15 \times 10^{-10} \text{ m}^2$

Figure 8.12 3-D geometry and pressure profile of simulated fiberwebs with $d = 15$ microns
Figure 8.13 3-D geometry and pressure profile of simulated fiberwebs with \( d = 15 \) microns.
3-D geometry: \( d = 15 \) microns, \( l_y = 25 \), \( \varepsilon = 0.871 \)

Pressure Profile
\( \nabla P = 6.82 \times 10^{-1} \) Pa, \( k = 5.11 \times 10^{-10} \) m\(^2\)

3-D geometry: \( d = 15 \) microns, \( l_y = 5 \), \( \varepsilon = 0.886 \)

Pressure Profile
\( \nabla P = 1.09 \times 10^{-1} \) Pa, \( k = 6.40 \times 10^{-10} \) m\(^2\)

Figure 8.14 3-D geometry and pressure profile of simulated fiberwebs with \( d = 15 \) microns
3-D geometry: \( d = 15 \) microns, \( l_y = 2 \), \( \varepsilon = 0.862 \)

Pressure Profile
\[ \nabla P = 4.38 \times 10^{-2} \text{ Pa}, \quad k = 6.36 \times 10^{-10} \text{ m}^2 \]

3-D geometry: \( d = 35 \) microns, \( l_y = 50 \), \( \varepsilon = 0.893 \)

Pressure Profile
\[ \nabla P = 1.24 \text{ Pa}, \quad k = 1.32 \times 10^{-9} \text{ m}^2 \]

Figure 8.15 3-D geometry and pressure profile of simulated fiberwebs with \( d = 15 \) and 35 microns
3-D geometry: \( d = 35 \) microns, \( l_y = 25, \varepsilon = 0.899 \)

Pressure Profile
\( \nabla P = 5.59e-01 \) Pa, \( k = 1.46e-09 \) m\(^2\)

3-D geometry: \( d = 35 \) microns, \( l_y = 10, \varepsilon = 0.894 \)

Pressure Profile
\( \nabla P = 2.29e-01 \) Pa, \( k = 1.42e-09 \) m\(^2\)

Figure 8.16 3-D geometry and pressure profile of simulated fiberwebs with \( d = 35 \) microns
Figure 8.17 3-D geometry and pressure profile of simulated fiberwebs with $d = 35$ microns

3-D geometry: $d = 35$ microns, $l_y = 5$, $\varepsilon = 0.892$

Pressure Profile
$\nabla P = 1.05 \times 10^{-01}$ Pa, $k = 1.55 \times 10^{-09}$ m$^2$

3-D geometry: $d = 35$ microns, $l_y = 2$, $\varepsilon = 0.906$

Pressure Profile
$\nabla P = 3.46 \times 10^{-02}$ Pa, $k = 1.88 \times 10^{-09}$ m$^2$
Figure 8.18 3-D geometry and pressure profile of simulated fiberwebs with $d = 35$ microns
Figure 8.19 3-D geometry and pressure profile of simulated fiberwebs with \( d = 35 \) microns

\[
\n\n\begin{align*}
\text{Pressure Profile} \\
\n\n\n\end{align*}
\]

\[
\n\n\begin{align*}
\n\n\n\end{align*}
\]

\[
\n\n\begin{align*}
\n\n\n\end{align*}
\]

\[
\n\n\begin{align*}
\n\n\n\end{align*}
\]
Figure 8.20 3-D geometry and pressure profile of simulated fiberwebs with d = 35 microns

3-D geometry: d= 35 microns, ly = 50, \( \varepsilon = 0.804 \)

Pressure Profile

\( \nabla P = 3.38 \text{ Pa, } k = 4.81 \times 10^{-10} \text{ m}^2 \)

3-D geometry: d= 35 microns, ly = 25, \( \varepsilon = 0.811 \)

Pressure Profile

\( \nabla P = 1.43 \text{ Pa, } k = 5.68 \times 10^{-10} \text{ m}^2 \)
Figure 8.21 3-D geometry and pressure profile of simulated fiberwebs with $d = 55$ microns

3-D geometry: $d = 35$ microns, $l_y = 10$, $\varepsilon = 0.814$

Pressure Profile

$\nabla P = 5.72e-01$ Pa, $k = 5.69e-10$ m$^2$

3-D geometry: $d = 35$ microns, $l_y = 5$, $\varepsilon = 0.809$

Pressure Profile

$\nabla P = 2.77e-01$ Pa, $k = 5.87e-10$ m$^2$
3-D geometry: d = 35 microns, ly = 2, $\varepsilon = 0.808$

Pressure Profile
$\nabla P = 8.45 \times 10^{-2}$ Pa, $k = 7.71 \times 10^{-10}$ m$^2$

3-D geometry: d = 40 microns, ly = 50, $\varepsilon = 0.829$

Pressure Profile
$\nabla P = 2.65$ Pa, $k = 7.01 \times 10^{-10}$ m$^2$

Figure 8.22 3-D geometry and pressure profile of simulated fiberwebs with d = 35 and 40 microns
Figure 8.23 3-D geometry and pressure profile of simulated fiberwebs with d = 40 microns
3-D geometry: $d = 40$ microns, $l_y = 5$, $\varepsilon = 0.815$

Pressure Profile
$\nabla P = 2.92 \times 10^{-1}$ Pa, $k = 6.37 \times 10^{-10}$ m$^2$

3-D geometry: $d = 40$ microns, $l_y = 2$, $\varepsilon = 0.848$

Pressure Profile
$\nabla P = 6.10 \times 10^{-2}$ Pa, $k = 1.22 \times 10^{-9}$ m$^2$

Figure 8.24 3-D geometry and pressure profile of simulated fiberwebs with $d = 40$ microns
3-D geometry: d = 40 microns, ly = 20, $\varepsilon = 0.772$

Pressure Profile
$\nabla P = 1.79 \text{ Pa}, k = 4.14 \times 10^{-10} \text{ m}^2$

3-D geometry: d = 40 microns, ly = 10, $\varepsilon = 0.782$

Pressure Profile
$\nabla P = 7.11 \times 10^{-1} \text{ Pa}, k = 5.22 \times 10^{-10} \text{ m}^2$

Figure 8.25 3-D geometry and pressure profile of simulated fiberwebs with d = 40 microns
Figure 8.26 3-D geometry and pressure profile of simulated fiberwebs with d = 10 microns
Appendix H: Results of Particle Tracking Simulation

Figure 8.27 Particle Tracking Simulation Result: $\lambda = 100$, $d = 25$ microns, $\varepsilon = 0.917$, $\tau = 1.065$
Figure 8.28 Particle Tracking Simulation Result: \( l_y = 75 \), \( d = 25 \) microns, \( \varepsilon = 0.910 \), \( \tau = 1.073 \)
Figure 8.29 Particle Tracking Simulation Result: \( l_y = 75 \), \( d = 25 \) microns, \( \varepsilon = 0.926 \), \( \tau = 1.059 \)
Figure 8.30 Particle Tracking Simulation Result: $l_y = 50$, $d = 35$ microns, $\varepsilon = 0.804$, $\tau = 1.125$
Figure 8.31 Particle Tracking Simulation Result: $l_y = 50$, $d = 25$ microns, $\varepsilon = 0.859$, $\tau = 1.072$
Figure 8.32 Particle Tracking Simulation Result: $l = 50$, $d = 25$ microns, $\varepsilon = 0.921$, $\tau = 1.066$
Figure 8.33 Particle Tracking Simulation Result: $l_y = 50$, $d = 25$ microns, $\varepsilon = 0.914$, $\tau = 1.061$
Figure 8.34 Particle Tracking Simulation Result: ly = 50, d = 10 microns, ε = 0.943, τ = 1.088
Figure 8.35 Particle Tracking Simulation Result: \( l_y = 50, d = 10 \) microns, \( \varepsilon = 0.965, \tau = 1.053 \)
Figure 8.36 Particle Tracking Simulation Result: $l_y = 25$, $d = 35$ microns, $\varepsilon = 0.899$, $\tau = 1.076$
Figure 8.37 Particle Tracking Simulation Result: $l_y = 25$, $d = 35$ microns, $\varepsilon = 0.854$, $\tau = 1.080$
Figure 8.38 Particle Tracking Simulation Result: $l_y = 25$, $d = 25$ microns, $\varepsilon = 0.919$, $\tau = 1.064$
Figure 8.39 Particle Tracking Simulation Result: $l_y = 25$, $d = 25$ microns, $\varepsilon = 0.917$, $\tau = 1.059$
Figure 8.40 Particle Tracking Simulation Result: $y = 25$, $d = 15$ microns, $\varepsilon = 0.920$, $\tau = 1.074$
Figure 8.41 Particle Tracking Simulation Result: ly = 25, d = 15 microns, ε = 0.871, τ = 1.115
Figure 8.42 Particle Tracking Simulation Result: $l_y = 25$, $d = 10$ microns, $\varepsilon = 0.947$, $\tau = 1.080$
Figure 8.43 Particle Tracking Simulation Result: $l_y = 25$, $d = 10$ microns, $\varepsilon = 0.962$, $\tau = 1.053$
Figure 8.44 Particle Tracking Simulation Result: $l_y = 20$, $d = 40$ microns, $\varepsilon = 0.772$, $\tau = 1.154$
Figure 8.45 Particle Tracking Simulation Result: \( y = 20 \), \( d = 25 \) microns, \( \varepsilon = 0.844 \), \( \tau = 1.102 \)
Figure 8.46 Particle Tracking Simulation Result: $l_y = 10$, $d = 40$ microns, $\varepsilon = 0.782$, $\tau = 1.133$
Figure 8.47 Particle Tracking Simulation Result: \( l_y = 10 \), \( d = 40 \) microns, \( \varepsilon = 0.838 \), \( \tau = 1.108 \)
Figure 8.48 Particle Tracking Simulation Result: \( l_y = 10 \text{, } d = 35 \text{ microns, } \varepsilon = 0.814 \text{, } \tau = 1.111 \)
Figure 8.49 Particle Tracking Simulation Result: \( \ell_y = 10, d = 25 \) microns, \( \varepsilon = 0.841, \tau = 1.093 \)
Figure 8.50 Particle Tracking Simulation Result: \( l_y = 10 \), \( d = 25 \) microns, \( \varepsilon = 0.925 \), \( \tau = 1.045 \)
Figure 8.51 Particle Tracking Simulation Result: $l_y = 10$, $d = 25$ microns, $\varepsilon = 0.914$, $\tau = 1.050$
Figure 8.52 Particle Tracking Simulation Result: $l_y = 10$, $d = 15$ microns, $\varepsilon = 0.907$, $\tau = 1.079$
Figure 8.53 Particle Tracking Simulation Result: \( l_y = 10, d = 10 \) microns, \( \varepsilon = 0.945, \tau = 1.077 \)
Figure 8.54 Particle Tracking Simulation Result: $l_y = 10$, $d = 10$ microns, $\varepsilon = 0.959$, $\tau = 1.032$
Figure 8. 55 Particle Tracking Simulation Result: $l_y = 5$, $d = 35$ microns, $\varepsilon = 0.809$, $\tau = 1.090$
Figure 8. 56 Particle Tracking Simulation Result: $\lambda y = 5$, $d = 35$ microns, $\varepsilon = 0.839$, $\tau = 1.082$
Figure 8.57 Particle Tracking Simulation Result: \( ly = 5, d = 15 \) microns, \( \varepsilon = 0.886, \tau = 1.066 \)
Figure 8.58 Particle Tracking Simulation Result: \( l_y = 5 \), \( d = 15 \) microns, \( \varepsilon = 0.916 \), \( \tau = 1.53 \)