ABSTRACT

SMELTZER III, STANLEY ST. CLAIR. An inelastic analysis methodology for bonded joints with shear deformable, anisotropic adherends. (Under the direction of Eric C. Klang)

The development of a one-dimensional analysis method for evaluating adhesively bonded joints composed of anisotropic adherends and adhesives that exhibit nonlinear material behavior is presented. The strain and resulting stress fields in a general, bonded joint overlap are determined by using a variable-step, finite-difference solution algorithm to iteratively solve a system of first-order differential equations. Applied loading is given as a system of combined extensional, bending, and shear loads that are applied to the edge of the joint overlap. Adherends are assumed to behave as linear, cylindrically-bent plates using classical laminated plate theory that includes the effects of first-order transverse shear deformation. This provides a capability for modeling differences in the transverse shear modulus between each adherend. Using a total plasticity theory and a modified von-Mises yield criterion, inelastic material behavior is modeled in the adhesive layer. Results for the proposed method are verified using the single-lap joint geometry against previous results from the literature and shown to be in excellent agreement. Convergence of the strain and stress fields determined using the finite-difference solver are described as a function of the number of evaluation points along the length of the joint. Additionally, design studies using the single-lap joint are presented that investigate the effects of changes to the joint overlap, adherend thickness, laminate stacking sequence of the adherend, adherend material properties, and adhesive material properties. Results from the design studies established a nonlinear relationship between changes in the bending and axial stiffness of the adherends due to laminate ply manipulations and a reduction in the inelastic adhesive strain and shear stress responses. Additionally, analyses performed on the bonded joint models that had a difference in the transverse shear stiffness between the upper and lower adherends displayed a minimal effect on the adhesive strain and stress responses.
AN INELASTIC ANALYSIS METHODOLOGY FOR BONDED JOINTS WITH SHEAR DEFORMABLE, ANISOTROPIC ADHERENDS

by

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A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

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Approved by:

[Signatures]

Chairman of Advisory Committee
DEDICATION

This dissertation is dedicated to Melinda, Courtney, Stan, and Claire. Thank you for always being there for me, I love you all.
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LIST OF SYMBOLS

\( A_{ij} \)  extensional stiffness matrix

\( B_{ij} \)  bending-extensional coupling stiffness matrix

\( C_{ij} \)  contracted stiffness matrix

\( C_{ijkl} \)  stiffness matrix

\( D_{ij} \)  bending stiffness matrix

\( E_{11}, E_{22}, E_{33} \)  Young’s moduli of a laminate in the 1, 2, and 3 coordinate directions

\( E_s \)  secant modulus of the adhesive

\( G_{13}, G_{23}, G_{12} \)  shear moduli of a laminate in the 13, 23, and 12 planes

\( h_L \)  thickness of the lower adherend

\( h_U \)  thickness of the upper adherend

\( K_s \)  shear correction factor

\( Q_{ij} \)  plane stress reduced-stiffness matrix

\( \bar{Q}_{ij} \)  plane stress transformed reduced-stiffness matrix

\( Q_x, Q_y \)  transverse force resultants

\( q \)  transverse applied load

\( M_{xx}, M_{yy}, M_{xy} \)  bending and twisting moment resultants

\( N_{xx}, N_{yy}, N_{xy} \)  membrane stress resultants

\( N^t \)  in-plane force

\( S_{ij} \)  compliance matrix

\( t \)  thickness of the adhesive

\( (u, v, w) \)  displacements associated with the coordinate system \((x, y, z)\)
(x, y, z) reference coordinate system
\( \varepsilon_i \) contracted strain tensor
\( \varepsilon_u \) equivalent uniaxial strain
\( \varepsilon_{ij} \) strain tensor
\( \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz} \) normal strain components
\( \phi_x \) rotation about the y-axis
\( \phi_y \) rotation about the x-axis
\( \gamma_{xz}, \gamma_{yz}, \gamma_{xy} \) engineering shear strain components
\( \mu \) elastic Poisson’s ratio of the adhesive
\( \mu_p \) plastic Poisson’s ratio of the adhesive
\( \nu_{13}, \nu_{23}, \nu_{12} \) Poisson’s ratio relating contraction in the \( i \) direction as a result of extension in the \( j \) direction \((i, j) = (1, 2, 3)\)
\( \sigma_i \) contracted stress tensor
\( \sigma_u \) equivalent uniaxial stress
\( \sigma_{ij} \) stress tensor
\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz} \) normal stress components
\( \tau_{xz}, \tau_{yz}, \tau_{xy} \) shear stress components
\(( \cdot )^L \) quantity corresponding to the lower adherend
\(( \cdot )^U \) quantity corresponding to the upper adherend
\( (\cdot)_{,i} \) derivative with respect to \( i \)
CHAPTER 1.0

INTRODUCTION

The information provided in the present chapter is intended to describe the present research problem and motivation for developing a bonded joint analysis method. In this chapter, details are given that provide a general background on joints, a description of a basic, adhesively bonded joint configuration, previous research applicable to the present research program, and the importance of and motivation for developing a bonded joint analysis method. Additionally, an overview of the remaining chapters is provided for the reader.

1.1 Preliminaries

The main job of a structural engineer is to design, build, and maintain structures and structural components. Unless the structure is machined from a solid block of material, the individual components that constitute the overall structure must be joined together. The method of joining the structure varies depending on the applied loads, material, assembly and disassembly requirements, service life, environment, etc. Once the overall requirements of the joint are known, the joining method that meets those requirements is selected. Additionally, the assembly process for the joint itself can have a large effect on the joint's performance in service. There are three major methods, as well as combinations of the three methods, for joining metallic components: bolting, welding, and adhesive bonding. In the case of non-metallic material components, mechanical fastening (bolting), adhesive bonding, or combinations of the two are the predominant joining techniques employed.

A joint can be defined as "connect(ing) two or more parts of a product or system [6]." In the case of a mechanically fastened joint, a bolt joins the respective parts of the system whereas the adhesively bonded joint uses a polymeric material. The choice of whether or not to use a particular type of joining technique is at the discretion of the designer based on
the structural requirements. However, a discussion of the general advantages and disadvantages of each method provides the designer with preliminary selection criteria.

Mechanical fastening is easily the most popular joining technique dating back to the mid to late 19th century as pegs were used to pin wooden structures together. The strongest reason for choosing mechanical fastening over a similar bonded joint is to enable the user to disassemble or accommodate repairs once the structural components are assembled into a final product. Another advantage of bolted assemblies is that the components, i.e. the nuts and bolts, are generally much less susceptible to operating temperatures and environmental degradation than polymer adhesive systems. Mechanically fastened structures are also better suited for complex loading arrangements and asymmetric loading than are adhesive joints since; those situations tend to load the joint perpendicular to the joint surface, which exploits the low peel strength of adhesives. The biggest disadvantage of a mechanically fastened joint compared to an adhesively bonded one is the higher weight and lower efficiency of the joint. In addition, the discrete positioning of bolts along the joint results in a non-uniform distribution of the load through the joint that is manifested as stress concentrations.

A description of the advantages and disadvantages for an adhesively bonded structure, in general, is implied from the contrasting statements for mechanically fastened structures. However, for completeness, a few of the more important characteristics of adhesive joints are described. The two most important advantages of an adhesive joint are closely related. Since the adhesive joint provides a uniform load distribution, a more efficient joint is possible compared to a similar bolted joint. This is always the case for rather thin adherends, which are the pieces of material being joined, and is also possible for rather thick adherends. Another advantage of the adhesive joint is the absence of any protruding shanks or bolt heads. Although this is often a more aesthetic joint requirement, close tolerance situations exist that limit the bolt clearance thus mandating an adhesive joint. Finally, a feature of adhesive joints limited to non-metallic structures is the capability to bond laminates with wide variations in fiber orientations whereas the bolted composite structures are limited to small variations in the laminate configurations in the region of the joint. The reason for this advantage is because composite laminates are
predominantly limited by interlaminar shear strength whereas the predominant failure mode for mechanically fastened composite laminates is in bearing, where ply orientation is a much larger factor than for interlaminar shear strength.

1.2 The Adhesively Bonded Joint

A joint can be as simple as joining two pieces of flat plate metal with a row of bolts or as complicated as joining two cylindrical tubes with a thin layer of adhesive and a few thousandths of clearance between the two pieces. In the case of adhesively bonded joints, a wide variety of joints are available to the designer as discussed by Adams and Wake [1]. A few of the more common joints are the single-lap, double-lap, stepped-lap, single-strap, double-strap, tubular, and skin-stiffener joints. All of these joints with the exception of the skin-stiffener joint are designed to primarily load the adhesive in shear. The skin-stiffener joint usually carries some shear loading, but is primarily designed to carry loads that are normal to the bonded surface.

A general, adhesively bonded joint is comprised of two main components regardless of joint type. First, the structural members of the joint that are to be joined together are referred to as the adherends. Second, these structural members are joined together in a manner that resists separation by a polymeric material called an adhesive. Several additional parameters used to describe different types of joints are the length of the joint overlap, the width of the joint overlap, the thickness of each adherend, and the thickness of the adhesive layer. A more detailed description of these parameters for the present research problem is given in chapters two and three.

1.3 Relevant Literature

The initial characterization of the problems and analysis difficulties associated with an adhesively bonded joint was the classical shear-lag analysis of a single-lap joint by Volkersen [61]. Volkersen identified the incremental deformation of the adherends, but failed to incorporate bending of the adherends that leads to an overall rotation of the joint. This important physical behavior of a single-lap joint was identified in the classical works of both de Bruyne [18] and Goland and Reissner [21]. Specifically, they accounted for the
eccentricity that is present between the applied tensile loading in these joints, identified the resulting bending response, and characterized the multi-axial stress state in the adhesive. Additionally, Goland and Reissner further characterized the single-lap joint behavior by formulating a solution for the normal stress perpendicular to the bonded surface in addition to the previously described shear stress while a joint factor was suggested by de Bruyne that was intended to estimate the failure load of a joint.

The analyses conducted by Goland and Reissner were performed by developing one-dimensional elasticity solutions for two limiting cases: 1) where the adhesive layer is thin and its effect on the flexibility of the joint may be neglected, and 2) where the joint flexibility is mainly due to the adhesive layer. The axial stress was assumed to be zero and the adhesive stresses did not vary through the thickness for both of these cases. The majority of later research studies compared their efforts to the second case of Goland and Reissner as it provided a better approximation of high-strength aerospace joints. One additional important result of their formulation was the development of a bending moment factor that related the in-plane tension load to the bending moment on the edge of the overlap region. A drawback of the method by Goland and Reissner was their failure to properly model the free surface on the edge of the adhesive layer, and provide for zero adhesive shear stress at the edge of the overlap.

A few other notable early investigations related to single-lap joints were produced by Cornell [16], Mylonas [39], and Cherry and Harrison [14]. Important experimental studies that confirmed much of the early analytical work on single-lap joints were performed by Cornell and Mylonas. The research study conducted by Cherry and Harrison suggested an optimum profile for a single-lap joint that involved tapering of the adherends, and was designed to provide a uniform stress state in the adhesive layer. Bensen [4], Sneddon [53], and Kutcha [33] provided thorough reviews that summarized much of the early work on adhesively bonded joints.

Numerous research studies have been conducted since these classical formulations and much of the early work is found in the excellent reviews by Mathews [36] et al. and Roy and Reddy [48,49]. Of these early works, Erdogan and Ratwani [19], Hart-Smith [23-27], Wah [62], Renton and Vinson [46,47], Srinivas [54], and Sawyer [52] made notable
advancements for adhesively bonded joints with composite adherends. Cooper and Sawyer [15], Dattaguru [17] et al., and Pickett and Hollaway [42] performed investigations of bonded joint configurations that explored geometric and material nonlinear effects.

The research performed by Erdogan and Ratwani developed one of the early analytical solutions for a stepped-lap joint configuration, assumed the adherends to be in a state of plane stress, and provided for orthotropic adherend properties. Their solution for the shear stress in an isotropic, elastic adhesive was determined using a strength of materials approach that provided a closed form solution to a system of ordinary differential equations and boundary conditions that were geometry specific. Wah used a similar strength of materials approach for determining the shear and normal stress in a single-lap joint while using classical laminated plate theory to describe the constitutive behavior of the adherends.

Hart-Smith’s voluminous works on the behavior of single, double, stepped-lap, and scarf joints have been widely used by the adhesively bonded joint community. His analytical solutions incorporated the effects of adhesive plasticity as well as thermal mismatch and stiffness imbalance between adherends that resulted in efficient codes for performing parametric studies on a wide array of joint configurations. The single-lap joint analysis method developed by Hart-Smith provided an improvement over the work by Goland and Reissner by using a third free-body diagram and associated boundary conditions to better describe the mechanics of the joint overlap. However, the claim that the different bending moment factor derived by Hart-Smith improved on the earlier one by Goland and Reissner has been disputed [13,57]. Another important aspect of Hart-Smith’s research was the detailed characterization of failure modes in bonded joints with both isotropic and composite adherends.

An important characteristic of adhesively bonded joints with composite adherends is the low transverse stiffness that is often present as a result of the high and ultra-high modulus fibers combined with much lower-modulus polymer resins. Thus, the through-the-thickness properties tend to be dominated by the polymer matrix resulting in a lower transverse stiffness when compared to the in-plane values. Many of the analyses in the early 1970’s had incorporated the multi-directional material properties of composite
laminates, but had neglected the relatively low transverse stiffness possessed by many of the composite material systems when compared to their isotropic counterparts. Renton and Vinson as well as Srinivas accounted for these low transverse stiffness effects by including first-order shear deformation in their formulations. The method developed by Renton and Vinson was an analytical solution of a single-lap joint geometry that included shear deformation for the composite adherends, and determined the linear elastic response for the adherends and adhesive. Srinivas developed a similar method for single-lap, double-lap, and flush joints that included shear deformation as a part of the analytical solution while attempting to approximate the nonlinear geometric effects.

Dattaguru et al. as well as Pickett and Hollaway provided studies that used nonlinear analysis methods to evaluate composite bonded joint configurations. Dattaguru et al. used a geometrically nonlinear finite element analysis method to study the effects of material properties on the strain energy release rates of single-lap joints while Cooper and Sawyer compared linear and nonlinear finite-element results to analyses using the method by Goland and Reissner. Pickett and Hollaway performed analytical and finite-element evaluations to investigate nonlinear material behavior in the form of elastic-perfectly-plastic adhesive modeling to study the performance of single-lap, double-lap, and tubular joints. They used the elastic-plastic analysis method of Hart-Smith to perform their analytical joint evaluations. Additionally, the results from the elastic-plastic joint analyses predicted an increase in joint strength over the linear elastic results while the results from the geometric nonlinear efforts suggested that adhesive properties were a significant factor in the joint responses.

The more recent works by Bigwood and Crocombe [7-9], Yang [65-67] et al., Tsai and Morton [58,59], Mortensen and Thomsen [38], and others [3,10-12,20,29,35] have made significant advances in the analysis of adhesively bonded joints with composite adherends. Bigwood and Crocombe developed a general joint overlap methodology for evaluating isotropic, adhesively bonded joints with inelastic adhesive behavior and subjected to combined loading. By including inelastic adhesive behavior, significant yielding of the adhesive occurred for highly loaded joints. This adhesive yielding transferred additional loading to the interior of the adhesive overlap that resulted in a more fully stressed
adhesive layer. Yang et al. formulated a method using classical laminated plate theory with first-order shear deformation to analyze symmetric and asymmetric single-lap joints subjected to tensile and bending loading. Tsai and Morton evaluated the three-dimensional strain field present in a tension-loaded, single-lap composite joint. Finally, Mortensen and Thomsen developed a numerical, two-dimensional plane stress analysis method for composite joints that modeled the adherends using classical laminated plate theory and the adhesive as a linear elastic material.

A select number of research studies have been conducted on adhesively bonded joints in an effort to identify important parameters and efficient designs [34,37,51,55]. However, only the investigations by Groth and Nordlund [22] and Ojalvo [40] have performed automated optimization of an adhesively bonded joint as opposed to parametric studies. Ojalvo developed a methodology for tailoring an adhesively bonded, double-lap joint by imposing a uniform stress constraint across the adhesive layer. A two-dimensional, plane-strain finite-element analysis method along with an elastic form of Hart-Smith’s closed form analysis were used to adjust the profile of the adherends, and satisfy the stress constraint. Groth and Nordlund performed a similar numerical shape optimization by conducting linear analyses of single-lap, double-lap, and double-strap joints using a two-dimensional, plane-strain finite-element method and evaluating minimum stress and weight functions with a commercially available optimization routine. In both cases, only local tailoring of the adherend thickness profiles were performed using the routines as opposed to gross adjustments of the overall joint geometry. These methods may prove useful for fine-tuning individual joint features; however, the methods appear inefficient and time-consuming for conducting large-scale optimization of generic joint configurations subjected to multiple load states.

1.4 Motivation and Research Objectives

Joining metallic and composite structural components with adhesively bonded joints has become a relatively routine and common practice in the technologically advanced aerospace and automotive sectors [28,31,32,60]. Finite element methods (FEM) or simple special purpose codes (e.g., one- or two-dimensional analytical methods) are primarily
used to obtain the final joint designs in these situations with subsequent verification of the
design through testing. The special purpose codes are generally efficient, user-intensive,
and lend themselves to conducting parametric studies; however, they are limited to one- or
two-dimensional analyses of specific joint configurations. Conversely, FEM are capable
of evaluating joints with complex geometry and loading, but are very inefficient for
conducting design studies and have serious problems with convergence of analysis results
in the regions of interest, i.e. the ends of the joint overlap. The convergence problems are
mainly due to the discontinuity in the normal or peel stress at the ends of the overlap, and
are manifested by decreasing the size of the finite element mesh in those regions. Based
on these difficulties and limitations, the need exists for a rapid-solution methodology
capable of obtaining strain and stress responses for a wide variety of bonded joint
configurations composed of both metallic and non-metallic materials. This methodology
should also include enough mechanics of actual joint behavior so that the resulting
predictive capability of the method provides relatively mature joint designs that only
require minor adjustments. Therefore, the present research program was conducted to
address these concerns by developing an analysis method that provides efficient tailoring
of various joint configurations while incorporating features such as anisotropic adherend
behavior, inelastic adhesive behavior, and first-order shear deformation.

The objective of the present research program is to describe the proposed analysis
method for evaluating general, bonded joint overlaps with anisotropic, shear-deformable
adherends and inelastic adhesive behavior that are subjected to combined tensile, shear,
and bending moment loading. In chapter two, a complete description of the applicable
field equations and development of the equations that govern the behavior of the
anisotropic bonded joint overlap are presented. Chapter three provides a description of the
basic bonded joint model, components of the computer code, and solution methodology.
The different joint models used to verify the present analysis method and investigate
details of joint behavior for single-lap joints are presented in chapter four. Finally, a few
conclusions drawn from the research program and suggestions for future research are
given.
CHAPTER 2.0

ANALYTICAL DEVELOPMENT

The equations presented in the following sections provide the basic framework that govern the motion of a solid body, lead to the development of the equations of equilibrium for a classical and shear-deformable laminated plate, and the equations that govern the behavior of a general, bonded joint overlap. In the first section, the body is referred to as a continuum or continuous medium that does not have the restriction of behaving as an elastic body. The notion of an ideal elastic body, one that will return to its original shape once all external loads are removed, is introduced in section 2.2. In the third section, the basic equations of classical laminated and first-order shear deformable plate theory are derived. Finally, the last section provides a description of the basic mechanics and equations necessary to determine a system of differential equations that can be used to identify the adhesive strain and stress responses in a general, bonded joint overlap.

2.1 Kinematic and Equilibrium Relations

The equations presented in this section describe the motion of a continuum and the related deformation that are independent of the forces causing the motion. As the continuum moves from one configuration (undeformed) to another (deformed), points within the continuum translate and rotate both in a rigid fashion and in relation to one another. The first movement described is referred to as a rigid-body motion while the second type of movement is called strain. The equations used to determine the components of strain are employed to describe the length of a line element between two arbitrary points that lie on a solid body in an undeformed configuration and a deformed configuration [50]. These equations that describe the strain are purely mathematical and do not involve engineering judgment. The components of strain or strain tensor are provided in equations 2.1-2.6 for problems that assume small strains, i.e. linear, where \( \gamma_x \), \( \gamma_z \), and \( \gamma_y \) denote the engineering shear strains and \( u \), \( v \), and \( w \) are
displacements in the x, y, and z directions, respectively.

The strain field given in equations 2.1-2.6 is expressed in terms of displacement gradients. Therefore, each component of strain may be computed once the displacements are known, as long as the displacements are once differentiable (C^1 continuous). However, obtaining a unique solution for the displacements based on a known strain field is not currently possible since there are six independent equations and only three unknowns. An auxiliary set of equations, referred to as the conditions of compatibility, is shown to be a necessary and sufficient condition to assure a continuous, single-valued displacement field. These compatibility equations given by St. Venant are expressed in tensor format by

$$\varepsilon_{ij,nn} + \varepsilon_{mn,ij} - \varepsilon_{im,jn} - \varepsilon_{jn,im} = 0$$

2.7

equation 2.7, which represents six independent equations for the three dimensional case (i,j,m,n=1,2,3). For the interests of plate theory, equation 2.7 is reduced to the two
dimensional case that results in a single compatibility equation of the form in equation 2.8.

It should be noted that the strain compatibility equations are satisfied by default when

\[ \varepsilon_{11,12} + \varepsilon_{22,11} - 2\varepsilon_{12,12} = \frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} - 2\frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} = 0 \]  

2.8

the strains are computed from a displacement field. Thus, the compatibility equations require verification only when the strains are computed from a stress field that is in equilibrium.

The equations presented next are used to describe the static equilibrium of forces acting on a continuum, which are classified as internal and external. Internal forces are those that resist the tendency of one portion of the continuum to be separated from another. External forces act on the outside of the continuum and are classified in two categories: 1) body forces and 2) surface forces. Body forces act on the elements of volume or mass within the continuum, e.g. gravitational or magnetic forces. Surface forces are contact forces acting on an element of area on the boundary surface of the continuum, e.g. surface tractions or applied pressure. Based on this general description of the different types of forces that may be applied to a continuum, a relationship between the applied forces (loads) and internal stresses is required to allow the solution of problems in continuum and solid mechanics.

The well-known relationships from elementary plate theory that relate the in-plane force resultants and moments to the applied loads are the Euler-Lagrange equations of equilibrium given by equations 2.9-2.11 where \( \mathbf{N}^i \) represents an in-plane force term and \( \mathbf{q} \)

\[ \frac{\partial \mathbf{N}_x}{\partial x} + \frac{\partial \mathbf{N}_y}{\partial y} = 0 \]  

2.9

\[ \frac{\partial \mathbf{N}_{xy}}{\partial x} + \frac{\partial \mathbf{N}_{xy}}{\partial y} = 0 \]  

2.10
is a transverse applied load. Additionally, the variables $N_{ij}$ and $M_{ij}$ are the familiar stress and moment resultants, respectively, which will be defined later. These equations may be determined by summing the forces and moments on a differential plate element or by applying energy principles and the calculus of variations to the basic plate equations [63].

2.2 Constitutive Relations

The equations developed in this section describe the basic material information for a continuum and provide a relationship between the kinematic and equilibrium equations. As a body is deformed isothermally, a linear relationship is determined between the state of stress and state of strain. This relationship governs the body’s response to load as it is deformed and as it recovers to its original or unloaded state.

2.2.1 Generalized Hooke’s law

The generalized Hooke’s law that relates the nine components of the stress tensor to the strain tensor is the most general form of the constitutive equation and is expressed as

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl}$$  \hspace{1cm} 2.12

where $C_{ijkl}$ is referred to as the stiffness matrix. Using the symmetric property of the stress and strain tensors, $\sigma_{ij} = \sigma_{ji}$ and $\varepsilon_{kl} = \varepsilon_{lk}$, equation 2.12 is rewritten using contracted notation as

$$\sigma_i = C_{ij} \varepsilon_j$$  \hspace{1cm} 2.13
\[
\begin{align*}
\sigma_1 &= \sigma_{11}, \quad \sigma_2 = \sigma_{22}, \quad \sigma_3 = \sigma_{33}, \quad \sigma_4 = \sigma_{23}, \quad \sigma_5 = \sigma_{13}, \quad \sigma_6 = \sigma_{12}, \\
\varepsilon_1 &= \varepsilon_{11}, \quad \varepsilon_2 = \varepsilon_{22}, \quad \varepsilon_3 = \varepsilon_{33}, \quad \varepsilon_4 = \varepsilon_{23}, \quad \varepsilon_5 = \varepsilon_{13}, \quad \varepsilon_6 = \varepsilon_{12}.
\end{align*}
\]

Equation 2.13 is written explicitly to display the 21 independent material constants

\[
\begin{bmatrix}
\sigma_1 \\ \\
\sigma_2 \\ \\
\sigma_3 \\ \\
\sigma_4 \\ \\
\sigma_5 \\ \\
\sigma_6
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{33} & C_{34} & C_{35} & C_{36} \\
C_{44} & C_{45} & C_{46} \\
(sym.) & C_{55} & C_{56} \\
(sym.) & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\ \\
\varepsilon_2 \\ \\
\varepsilon_3 \\ \\
\varepsilon_4 \\ \\
\varepsilon_5 \\ \\
\varepsilon_6
\end{bmatrix}
\]

that define an anisotropic material. An anisotropic material is identified as a material without any planes of symmetry. A material that possesses one plane of material symmetry is termed monoclinic and has 13 independent material constants. If a material possesses two planes of symmetry, then a third plane of symmetry is present by default. A material with three planes of symmetry is known as an orthotropic material and has 9 independent material constants that are given by

\[
\begin{bmatrix}
\sigma_1 \\ \\
\sigma_2 \\ \\
\sigma_3 \\ \\
\sigma_4 \\ \\
\sigma_5 \\ \\
\sigma_6
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{22} & C_{23} & 0 & 0 & 0 \\
C_{33} & 0 & 0 & 0 \\
C_{44} & 0 & 0 \\
(sym.) & C_{55} & 0 \\
(sym.) & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\ \\
\varepsilon_2 \\ \\
\varepsilon_3 \\ \\
\varepsilon_4 \\ \\
\varepsilon_5 \\ \\
\varepsilon_6
\end{bmatrix}
\]

An alternative format for displaying the material constants for an orthotropic material is given by the strain-stress constitutive formulation of
where $S_y$ is referred to as the compliance matrix. The strain-stress form of the constitutive equation is used to conveniently display basic material testing information. Since most material characterization tests are conducted by applying a load or stress and measuring the resulting deformation, the constants of the compliance matrix are obtained in a more direct fashion than are those of the stiffness matrix. In that regard, the individual components of the compliance matrix can be written in terms of the engineering constants as
2.2.2 Plane stress constitutive relations

A simplification from the generalized Hooke’s law is obtained for problems that can assume certain stresses are negligible. For the case of plane stress, the stresses that act transverse to a plane may be assumed zero thus eliminating some of the terms in the generalized constitutive equation. This assumption is reasonable for most laminated plates, since the thickness dimension is much smaller than the length or width. Therefore, for a thin plate the relationship in equation 2.17 is reduced to

$$
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_6
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & 0 \\
S_{12} & S_{22} & 0 \\
0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix}
$$

where

$$\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2.$$  

The strain-stress relation in equation 2.18 can be inverted to obtain the stress-strain constitutive equation in plane stress; such that,

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_6
\end{bmatrix}
$$

where the $Q_{ij}$ values are called the plane stress-reduced stiffness terms. They can be defined in terms of the compliance components as well as the engineering constants as follows

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}$$
An additional set of equations is produced when the transverse normal stress is neglected while allowing nonzero transverse shear stresses. Using equation 2.17, the following relationship is included with equation 2.20 for shear deformable plates

\[
Q_{12} = \frac{S_{12}}{S_{11}S_{22} - S_{12}^2} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}
\]

\[
Q_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}
\]

\[
Q_{66} = \frac{1}{S_{66}} = G_{12}
\]

The relationships in equations 2.20 and 2.25 can be extended to plates that have plane anisotropy by using the coordinate transformation relations for a second order tensor. The transformation relations are necessary to describe the material properties for rotated orthotropic laminae within a given laminate. Using the transformation relations, a stress-strain relationship similar to those in equations 2.20 and 2.25 are obtained with the exception that the matrices are now fully populated. These modified properties are referred to as transformed reduced-stiffness terms, and are labeled with a bar in contrast to the non-rotated terms. Thus, the transformed stress-strain relationship that is used to describe the material behavior for an anisotropic structure can be written with \((x,y,z) = (1,2,3)\) as
\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix} = 
\begin{bmatrix}
\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\
\overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\
\overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\sigma_{yz} \\
\sigma_{xz}
\end{bmatrix} = 
\begin{bmatrix}
\overline{Q}_{44} & \overline{Q}_{45} \\
\overline{Q}_{45} & \overline{Q}_{55}
\end{bmatrix}
\begin{bmatrix}
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix}
\]

where

\[
\overline{Q}_{11} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{22} \sin^4 \theta
\]

\[
\overline{Q}_{12} = Q_{12} (\cos^4 \theta + \sin^4 \theta) + (Q_{11} + Q_{22} - 4Q_{66}) \cos^2 \theta \sin^2 \theta
\]

\[
\overline{Q}_{22} = Q_{22} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \cos^2 \theta \sin^2 \theta + Q_{11} \sin^4 \theta
\]

\[
\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66}) \cos^3 \theta \sin \theta + (Q_{12} - Q_{22} + 2Q_{66}) \cos \theta \sin^3 \theta
\]

\[
\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66}) \cos \theta \sin^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \cos^3 \theta \sin \theta
\]

\[
\overline{Q}_{66} = Q_{66} (\cos^4 \theta + \sin^4 \theta) + (Q_{11} + Q_{12} - 2Q_{12} - 2Q_{66}) \cos^2 \theta \sin^2 \theta
\]

\[
\overline{Q}_{44} = Q_{44} \cos^2 \theta + Q_{55} \sin^2 \theta
\]

\[
\overline{Q}_{45} = (Q_{55} - Q_{44}) \cos \theta \sin \theta
\]

\[
\overline{Q}_{55} = Q_{55} \cos^2 \theta + Q_{44} \sin^2 \theta
\]

2.3 Laminated Plate Theory

A description of the field equations has been given in the previous sections for a general body with arbitrary loading. In this section, basic assumptions and restrictions are used to develop the field equations for an anisotropic, laminated plate; commonly referred to as classical laminated plate theory (CLPT). In section 2.3.3, certain restrictions in CLPT are relaxed leading to laminate equations that include transverse shear effects and
are referred to as first-order shear deformable plate theory (FSDPT). The basic assumptions and restrictions for CLPT are:

1. The Kirchoff-Love hypothesis holds true; that is, straight lines perpendicular to the mid-plane before deformation remain
   a) straight,
   b) inextensible, and
   c) normal
   to the midplane after deformation. The first two assumptions provide that the transverse displacement is not a function of the thickness (transverse coordinate) and the transverse normal strain ($\varepsilon_{33}$) is zero. The third assumption leads to transverse shear strains ($\varepsilon_{13}$ and $\varepsilon_{23}$) that are zero.

2. The plate is constructed of an arbitrary number of layers that are perfectly bonded together.

3. The material of each layer is linear elastic and has at least two planes of symmetry; that is, orthotropic.

4. The displacements, $u$, $v$, $w$, and strains are small compared to the plate thickness.

5. Certain nonlinear terms in the EOM involving products of stresses and displacement gradients are retained to include in-plane forces.

6. Each layer has a uniform thickness.

7. Rotary inertia terms are negligible.

8. There are no body forces.
9. The plate is thin, i.e. the plate thickness is much smaller than the other physical dimensions.

10. Transverse shear stresses (\(\sigma_{13}\) and \(\sigma_{23}\)) on the top and bottom surfaces (\(z = \pm h/2\)) of the laminate are zero.

2.3.1 Displacements and strains

A plate with thickness \(h\) and composed of an arbitrary number of layers is referred to as a laminated plate [44]. The individual layers of a laminated plate are referred to as laminae or plies. Each ply within the laminate has a principal set of material coordinates \((\chi^i_k)\) that provide the orientation of the fiber at an angle \(\theta_k\) to the laminate coordinates, \(X^i\), where \(k\) refers to the lamina or ply number. Kirchoff-Love assumptions result in deformation being due entirely to bending and inplane stretching. As a result of the Kirchoff-Love hypothesis, the displacements take the following form

\[
\begin{align*}
    u(x, y, z) &= u_o(x, y) - z \frac{\partial w_o}{\partial x} \quad 2.37 \\
    v(x, y, z) &= v_o(x, y) - z \frac{\partial w_o}{\partial y} \quad 2.38 \\
    w(x, y, z) &= w_o(x, y) \quad 2.39
\end{align*}
\]

where \(u_o\), \(v_o\), and \(w_o\) are the midplane displacements in the \(x\), \(y\), and \(z\) coordinate directions, respectively. The assumed displacement field given in equations 2.37-2.39 is used in conjunction with the infinitesimal strain tensor, equations 2.1-2.6, to provide the kinematic relations for CLPT as

\[
\begin{align*}
    \varepsilon_{xx} &= \frac{\partial u_o}{\partial x} - z \frac{\partial^2 w_o}{\partial x^2} = u_{o,xx} - z \cdot w_{o,xx} \quad 2.40
\end{align*}
\]
\[
\varepsilon_{yy} = \frac{\partial v_o}{\partial y} - z \frac{\partial^2 w_o}{\partial y^2} = v_{o,yy} - z \cdot w_{o,yy} \quad 2.41
\]

\[
\gamma_{xy} = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} - 2z \frac{\partial^2 w_o}{\partial x \partial y} = u_{o,xy} + v_{o,xx} - 2z \cdot w_{o,xy} \quad 2.42
\]

\[
\gamma_{xz} = \frac{1}{2} \left( -\frac{\partial w_o}{\partial x} + \frac{\partial w_o}{\partial y} \right) = 0 \quad 2.43
\]

\[
\gamma_{yz} = \frac{1}{2} \left( -\frac{\partial w_o}{\partial y} + \frac{\partial w_o}{\partial x} \right) = 0 \quad 2.44
\]

\[
\varepsilon_{zz} = 0 \quad 2.45
\]

These strain-displacement equations can be written in a more convenient format as

\[
\varepsilon_{xx} = \varepsilon_{xx}^o + z \cdot \kappa_x \quad 2.46
\]

\[
\varepsilon_{yy} = \varepsilon_{yy}^o + z \cdot \kappa_y \quad 2.47
\]

\[
\gamma_{xy} = \gamma_{xy}^o + z \cdot \kappa_{xy} \quad 2.48
\]

where \( \varepsilon_{xx}^o, \varepsilon_{yy}^o, \gamma_{xy}^o \) are the midplane strains and \( \kappa_x, \kappa_y, \) and \( \kappa_{xy} \) are the bending strains or curvatures and are defined as

\[
\varepsilon_{xx}^o = \frac{\partial u_o}{\partial x} = u_{o,x} \quad 2.49
\]

\[
\varepsilon_{yy}^o = \frac{\partial v_o}{\partial y} = v_{o,y} \quad 2.50
\]

\[
\gamma_{xy}^o = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} = u_{o,xy} + v_{o,xx} \quad 2.51
\]
\( \mathbf{K}_x = -\frac{\partial^2 w_o}{\partial x^2} = -w_{o,xx} \)  

2.52

\( \mathbf{K}_y = -\frac{\partial^2 w_o}{\partial y^2} = -w_{o,yy} \)  

2.53

\( \mathbf{K}_{xy} = -2\frac{\partial^2 w_o}{\partial x \partial y} = -2w_{o,xy} \)  

2.54

### 2.3.2 Lamina and laminate constitutive relations

The values of \((N_{xx}, N_{yy}, N_{xy})\) and \((M_{xx}, M_{yy}, M_{xy})\) as previously discussed are the common force and moment resultants from classical plate theory and can be defined in matrix form for a laminated plate as

\[
\begin{align*}
\begin{bmatrix}
N_{xx} \\
N_{yy} \\
N_{xy}
\end{bmatrix}
&= \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{bmatrix} \, dz \\
&= \begin{bmatrix}
M_{xx} \\
M_{yy} \\
M_{xy}
\end{bmatrix}
\end{align*}
\]

where \(z_k\) refers to the distance from the neutral axis for each ply. Substituting the constitutive relationship given by equation 2.26 and the kinematic relations from equations 2.46-2.48 into equations 2.55 and 2.56 gives

\[
\begin{align*}
\begin{bmatrix}
N_{xx} \\
N_{yy} \\
N_{xy}
\end{bmatrix}
&= \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{bmatrix}
\tilde{\varphi}_{11} & \tilde{\varphi}_{12} & \tilde{\varphi}_{16} \\
\tilde{\varphi}_{12} & \tilde{\varphi}_{22} & \tilde{\varphi}_{26} \\
\tilde{\varphi}_{16} & \tilde{\varphi}_{26} & \tilde{\varphi}_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx}^o + \mathbf{K}_x \\
\varepsilon_{yy}^o + \mathbf{K}_y \\
\gamma_{xy}^o + \mathbf{K}_{xy}
\end{bmatrix} \, dz
\end{align*}
\]

2.57

21
which may be reduced further by performing the through-the-thickness integration. After integrating equations 2.57 and 2.58, the results form the well-known constitutive relationship

\[
\begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx}^o + \kappa_x \\ \varepsilon_{yy}^o + \kappa_y \\ \gamma_{xy} + \kappa_{xy} \end{bmatrix} \cdot z \, dz \quad 2.58
\]

where \( A_{ij} \) are the extensional stiffnesses, \( B_{ij} \) are the bending-extensional coupling stiffnesses, and \( D_{ij} \) are the bending stiffnesses. These equations may be written in a more compact format using matrix notation as

\[
\begin{bmatrix} (N) \\ (M) \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \cdot \begin{bmatrix} (\varepsilon^o) \\ (\kappa) \end{bmatrix} \quad 2.61
\]

where

\[
(A_{ij}, B_{ij}, D_{ij}) = \int_{-\frac{t}{2}}^{\frac{t}{2}} \bar{Q}_{ij}(1, z, z^3) \cdot dz \quad 2.62
\]
Equation 2.61 can be expressed in terms of displacements using the kinematic relationships for strains and curvatures from equations 2.49-2.54 as

\[
\begin{bmatrix}
N_{xx} \\
N_{yy} \\
N_{xy} \\
M_{xx} \\
M_{yy} \\
M_{xy}
\end{bmatrix} = \begin{bmatrix}
[A] \\
[B]
\end{bmatrix}
\begin{bmatrix}
\frac{u_o \cdot u_x}{x} \\
\frac{v_o \cdot v_y}{y} \\
\frac{u_o \cdot v_x}{x} + \frac{v_o \cdot u_x}{x} \\
-w_o \cdot u_{xx} \\
-w_o \cdot u_{yy} \\
-2w_o \cdot u_{xy}
\end{bmatrix}
\]

2.63

2.3.3 First-order shear deformable plate theory

The displacement field for a laminate using first-order shear deformable plate theory (FSDPT) is

\[
u(x, y, z) = u_o(x, y) + z\phi_x(x, y) \quad 2.64
\]

\[
v(x, y, z) = v_o(x, y) + z\phi_y(x, y) \quad 2.65
\]

\[
w(x, y, z) = w_o(x, y) \quad 2.66
\]

where \(\phi_x\) and \(\phi_y\) correspond to the rotation of a transverse normal about the y and x axes, respectively. The displacement field in equations 2.64-2.66 can be used to generate kinematic relations for FSDPT from small strain theory using equations 2.1-2.6 which provides

\[
\varepsilon_{xx} = \frac{\partial u}{\partial x} + z \frac{\partial \phi_x}{\partial x} = \frac{u_o}{x} + z \cdot \phi_x \cdot x \quad 2.67
\]

\[
\varepsilon_{yy} = \frac{\partial v}{\partial y} + z \frac{\partial \phi_y}{\partial y} = \frac{v_o}{y} + z \cdot \phi_y \cdot y \quad 2.68
\]
\[\varepsilon_{zz} = 0\]  
\[\gamma_{xy} = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} + z \left( \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) = u_{o,xy} + v_{o,xy} + z \left( \phi_{x,xy} + \phi_{y,xy} \right)\]  
\[\gamma_{xz} = \frac{\partial w_o}{\partial x} + \phi_x = w_{o,xy} + \phi_x\]  
\[\gamma_{yz} = \frac{\partial w_o}{\partial y} + \phi_y = w_{o,xy} + \phi_y\]  

and can be substituted into the constitutive relationship in equation 2.61 to obtain

\[
\begin{bmatrix}
N_{xx} \\
N_{yy} \\
N_{xy} \\
M_{xx} \\
M_{yy} \\
M_{xy}
\end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{bmatrix}
u_{o,xy} \\
v_{o,xy} + v_{o,xy} \\
\phi_{x,xy} \\
\phi_{y,xy} \\
\phi_{x,xy} + \phi_{y,xy}
\end{bmatrix}.
\]

The strain relations in equations 2.67-2.72 can be written in a format similar to those for CLPT as

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{yz} \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix} \varepsilon'_{xx} \\
\varepsilon'_{yy} \\
\gamma'_{yz} \\
\gamma'_{xz} \\
\gamma'_{xy}
\end{bmatrix} + z \begin{bmatrix} \kappa_x \\
\kappa_y \\
\kappa_{yz} \\
\kappa_{xz} \\
\kappa_{xy}
\end{bmatrix}.
\]

where the midplane strains and curvatures are as defined previously and can be shown in the expanded format as displacements which provides the following results:
Unlike classical plate theory, the transverse shear strains are nonzero, so the relationships for the transverse shear stresses are generated from the displacement field for a shear deformable plate. A relationship from equilibrium that relates the transverse force resultants and the transverse shear stresses is written as

\[
\begin{align*}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{yx} \\
\gamma_{xy}
\end{bmatrix} &=
\begin{bmatrix}
u_{o,x} \\
v_{o,y} \\
w_{o,y} + \phi_{y} \\
u_{o,y} + \nu_{o,x}
\end{bmatrix} + z
\begin{bmatrix}
\phi_{x,x} \\
\phi_{y,y} \\
0 \\
0
\end{bmatrix}.
\end{align*}
\]

2.75

and can be written in a form that accounts for integrating through the thickness of a laminated plate as

\[
\begin{align*}
\begin{bmatrix}
Q_y \\
Q_x
\end{bmatrix} &=
\frac{h}{2}
\int_{-\frac{h}{2}}^{\frac{h}{2}}
\begin{bmatrix}
\sigma_{xz} \\
\sigma_{yz}
\end{bmatrix}
\cdot dz
\end{align*}
\]

2.76

where \( K_s \) is a shear correction factor. Substituting the constitutive relationship from equation 2.27 into 2.77 and performing the integration yields

\[
\begin{align*}
\begin{bmatrix}
Q_y \\
Q_x
\end{bmatrix} &=
K_s
\begin{bmatrix}
A_{44} & A_{45} \\
A_{45} & A_{55}
\end{bmatrix}
\begin{bmatrix}
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix}.
\end{align*}
\]

2.77

2.78
which can be written in terms of the displacements using the kinematic relations in equations 2.71 and 2.72 to show

\[
\begin{bmatrix}
Q_y \\
Q_x
\end{bmatrix} = K_s \begin{bmatrix}
A_{44} & A_{45} \\
A_{45} & A_{55}
\end{bmatrix} \begin{bmatrix}
w_{o,y} + \phi_y \\
w_{o,x} + \phi_x
\end{bmatrix}.
\]

2.79

Although the application of a shear correction factor to the right hand side of equation 2.79 is traditional, a brief explanation for the use of this factor in the present study is in order.

An important fact from elementary beam theory that also applies to plates is that the transverse shear stresses are parabolic in the thickness direction. However, looking back at the earlier described strain field in equation 2.75, it is evident that the transverse shear strains are represented as constant values through the thickness of the plate. This characterization of the transverse shear strains is a result of assumptions made for the displacement field. Since the transverse shear strains are constant through the thickness of the plate, the resulting shear stresses are also constant. Although there are no provisions within the kinematics of first-order shear-deformable plate theory to remedy the inconsistency between the actual parabolic variation of transverse shear stress and the assumed constant value, the values of the shear forces may be corrected by applying a shear correction factor to the stiffness matrix. This has the result of modifying the transverse shear stiffness of the plate. Additionally, the values for an appropriate shear correction factor can vary with extensional stiffness, Poisson’s ratio, transverse shear stiffness, and geometry for a given plate [5,45,64]. Thus, to perform the analyses in the present study in a uniform manner, a value of one was chosen for the shear correction factor.

2.4 Anisotropic Bonded Joint Formulation

A description of the basic geometry and a differential element for a joint overlap that is contained within a general adhesively bonded joint is given in Figure 2.1. The adhesively bonded joint is composed of two laminated composite plates, referred to as upper and lower adherends, and an adhesive layer. The adherends are assumed to behave
as linear elastic, cylindrically bent plates under the condition of plane strain while the adhesive layer is modeled as a nonlinear, isotropic material. Specifically, components of shear and normal stress within the adhesive layer are nonlinear functions of the adhesive strains. In addition, the effects of transverse shear deformation in the adherends are included using first-order shear deformable plate theory. As a result of the assumption of cylindrical bending, only a cross-section of the entire joint is modeled; therefore, the loading in the figure is given in terms of a unit width joint. The terminology for the basic joint geometry and loading that is given in the figure will be used to develop the adhesively bonded joint analysis methodology in the remainder of this section. Superscript letters, $U$ for the upper adherend and $L$ for the lower adherend, are used to identify variables for each adherend, while the stresses for the adhesive layer do not have any special notation.

### 2.4.1 Bonded joint equilibrium

To begin, expressions that relate the unit width joint loading and the adhesive stresses are desirable to formulate the system of equations for the general adhesively bonded joint. These equations are formulated using the horizontal and vertical force and moment equilibrium on each free-body diagram in Figure 2.1b. The overlap section is divided at the mid-plane of the adhesive layer, which provides half of the total adhesive thickness along the bottom surface of the upper adherend and half along the upper surface of the lower adherend. Summing forces in the direction of the positive $x$ and $z$-axes and moments about the $y$-axis through the mid-plane of each adherend, the state of equilibrium for the upper adherend is determined to be

\[
\sum F_x : \quad N_{xx}^U + dN_{xx}^U - N_{xx}^U - \tau_{xz} dx = 0
\]

\[
\sum F_x : \quad Q_{x}^U + dQ_{x}^U - Q_{x}^U - \sigma_{zz} dx = 0
\]
\[
\sum M_y = M_{xx}^U - (M_{xx}^U + dM_{xx}^U) + Q_x^U \left( \frac{dx}{2} \right) + (Q_x^U + dQ_x^U) \quad 2.82
\]

\[
-\tau_{xz} dx \left( \frac{h_U + t}{2} \right) = 0
\]

where \( h_U \) is the upper adherend thickness and \( t \) is the adhesive thickness. The force and moment equilibrium equations are manipulated to find the following ordinary differential equations for the upper adherend

\[
\frac{dN_{xx}^U}{dx} = \tau_{xz} \quad 2.83
\]

\[
\frac{dQ_x^U}{dx} = \sigma_{zz} \quad 2.84
\]

\[
\frac{dM_{xx}^U}{dx} = Q_x^U - \tau_{xz} \left( \frac{h_U + t}{2} \right) \quad 2.85
\]

A similar set of equations are obtained for the lower adherend and are expressed as

\[
\frac{dN_{xx}^L}{dx} = \tau_{xz} \quad 2.86
\]

\[
\frac{dQ_x^L}{dx} = -\sigma_{zz} \quad 2.87
\]

\[
\frac{dM_{xx}^L}{dx} = Q_x^L - \tau_{xz} \left( \frac{h_L + t}{2} \right) \quad 2.88
\]

where \( h_L \) is the lower adherend thickness.

### 2.4.2 Classical laminated plate formulation

Assuming simple adherend bending, a relationship relating the curvature of the overlap and the moment loading may be determined for adherends that behave as
cylindrically bent plates. Using the relationship from elementary plate theory for the
deflection of an elemental strip [56], the displacements of the joint overlap in cylindrical
bending for each adherend are given by

\[
\frac{d^2 w_o^U}{dx^2} = - \left( \frac{A}{D} \right)^U \mathbf{M}_{xx}^U \\
\frac{d^2 w_o^L}{dx^2} = - \left( \frac{A}{D} \right)^L \mathbf{M}_{xx}^L
\]

where \( A = A_{11} A_{66} - A_{16}^2 \), \( B = B_{11} A_{66} - B_{16} A_{16} \), \( C = A_{11} B_{16} - A_{16} B_{11} \)
and \( D = AD_{11} - BB_{11} - CB_{16} \).

Since the joint overlap is assumed to behave in cylindrical bending, all terms that contain
derivatives with respect to \( z \) may be neglected. Applying the assumption of cylindrical
bending to the kinematic equations 2.49-2.54 provides

\[
\varepsilon_{xx}^o = \frac{\partial u_o}{\partial x} = \frac{du_o}{dx} = u_o^x
\]

\[
\varepsilon_{yy}^o = \frac{\partial v_o}{\partial y} = 0
\]

\[
\gamma_{xy}^o = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} = \frac{dv_o}{dx} = v_o^x
\]

\[
\kappa_x = - \frac{\partial^2 w_o}{\partial x^2} = - \frac{d^2 w_o}{dx^2} = -w_o^{xx}
\]

\[
\kappa_y = - \frac{\partial^2 w_o}{\partial y^2} = 0
\]

\[
\kappa_{xy} = 2 \frac{\partial^3 w_o}{\partial x \partial y} = 0.
\]
Thus, the constitutive equation for a classical laminated plate can be reduced using by substituting equations 2.91-2.96 into equation 2.61 to obtain

\[
\begin{bmatrix}
N_{xx} \\
N_{xy} \\
N_{yy} \\
M_{xx} \\
M_{xy} \\
M_{yy}
\end{bmatrix} = \begin{bmatrix} [A] & [B] \end{bmatrix} \begin{bmatrix}
u_{o,x} \\
v_{o,y} \\
-w_{o,xx} \\
0 \\
0 \\
0
\end{bmatrix}.
\]

Inverting equation 2.97 provides the strain-stress constitutive relationship

\[
\begin{bmatrix}
u_{o,x} \\
v_{o,y} \\
-w_{o,xx} \\
0 \\
0 \\
0
\end{bmatrix} = \begin{bmatrix} [A] & [B] \end{bmatrix}^{-1} \begin{bmatrix}
N_{xx} \\
N_{xy} \\
N_{yy} \\
M_{xx} \\
M_{xy} \\
M_{yy}
\end{bmatrix}.
\]

which can be used to determine the longitudinal and bending normal strain in each adherend. Since the adherends are assumed to behave in cylindrical bending, the loading perpendicular to the joint cross-section is neglected, that is \(N_{yy} = N_{xy} = M_{yy} = M_{xy} = 0\). Therefore, the terms for longitudinal and bending normal strain can be determined from equation 2.98 by performing the necessary matrix multiplications with the remaining terms to find

\[
u_{o,x} = (A_{11})^{-1} N_{xx} + (B_{11})^{-1} M_{xx}
\]

\[-w_{o,xx} = (B_{11})^{-1} N_{xx} + (D_{11})^{-1} M_{xx}\]
where $\varepsilon_{xx}^L = u_y^L_x$ and $K_x = -w_o^L_{xx}$. Assuming laminate symmetry, the longitudinal and bending normal strain can be combined to form an expression for the total normal strain along the x-axis at the adherend-adhesive interface for each adherend. Recalling equation 2.46, expressions for the total normal strain in the upper and lower adherends can be written as

\[
\varepsilon_{xx}^U = \frac{du^U}{dx} = (A_{11}^U)^{-1} N_{xx}^U + z(D_{11}^U)^{-1} M_{xx}^U \tag{2.101}
\]

\[
\varepsilon_{xx}^L = \frac{du^L}{dx} = (A_{11}^L)^{-1} N_{xx}^L + z(D_{11}^L)^{-1} M_{xx}^L \tag{2.102}
\]

where the value of $z$ is determined at any point through the thickness of the adherend. These equations can be further reduced since it is necessary to determine the strain at the adhesive-adherend interface, thus the value of $z$ in equations 2.101 and 2.102 will be half the thickness of the respective adherend. Substituting a negative value of the adherend thickness for the upper adherend and a positive value of the adherend thickness in the lower adherend to correspond with the location of the respective adhesive-adherend interfaces results in

\[
\varepsilon_{xx}^U = \frac{du^U}{dx} = (A_{11}^U)^{-1} N_{xx}^U - \frac{h_u}{2} (D_{11}^U)^{-1} M_{xx}^U \tag{2.103}
\]

\[
\varepsilon_{xx}^L = \frac{du^L}{dx} = (A_{11}^L)^{-1} N_{xx}^L + \frac{h_L}{2} (D_{11}^L)^{-1} M_{xx}^L. \tag{2.104}
\]

Relationships for the shear and transverse normal strain in the adhesive layer can be determined in terms of the longitudinal and transverse displacements of the upper and lower adherends; such that,

\[
\gamma_{xz} = \frac{1}{t} (u^U - u^L) \tag{2.105}
\]
\[ \varepsilon_{zz} = \frac{1}{i \left( w^u - w^L \right)} \]  

which results in a constant value for the shear and normal strain through the thickness of the adhesive.

The adhesive is assumed to behave as an isotropic, elastic-plastic material. The inelastic material behavior is modeled using a total plasticity theory [30] which provides for quasi-static loading that is applied in increasing proportions, otherwise referred to as proportional loading. There are no provisions within the total plasticity theory to account for unloading or cyclical loading. Since the adhesive is loaded in a manner that primarily results in shear and transverse normal stresses in the adhesive, the axial normal stress in the adhesive, \( \sigma_{xx} \), is neglected. Additionally, the transverse shear strains, \( \gamma_{xy} \) and \( \gamma_{yz} \), are assumed to be negligible since the adhesive is assumed to be in a state of plane strain. Using these assumptions, the following constitutive relationship is reduced from the three-dimensional stress state

\[
\begin{bmatrix}
\varepsilon_{zz} \\
\varepsilon_{xy} \\
\gamma_{xz}
\end{bmatrix} = \frac{1}{E_s} \begin{bmatrix}
1 & -\mu_p & 0 \\
-\mu_p & 1 & 0 \\
0 & 0 & 2(1+\mu_p)
\end{bmatrix} \begin{bmatrix}
\sigma_{zz} \\
\sigma_{xy} \\
\tau_{xz}
\end{bmatrix}
\]

where \( E_s \) is the secant modulus and \( \mu_p \) is referred to as the plastic Poisson’s ratio and defined as

\[ \mu_p = \frac{1}{2} \left[ 1 - \frac{E_s}{E} (1-2\mu) \right]. \]

The plastic Poisson’s ratio ranges from an elastic value, \( \mu \), to the asymptotic value of 0.5 for the fully plastic case. The secant modulus, \( E_s \), is defined by the slope of a line on the
uniaxial stress-strain curve taken from the origin to a point along the curve denoted by \( \sigma_u \) and \( \varepsilon_u \); i.e.,

\[
E_s = \frac{\sigma_u}{\varepsilon_u}.
\]

Inverting equation 2.107, setting \( \varepsilon_{yy} = 0 \) using the assumption of plane strain in the adhesive, and then performing the necessary matrix operations to expand the equations allows the relationships that model the adhesive stress-strain behavior to be obtained. As a result, the adhesive stress-strain relations are given as

\[
\sigma_{zz} = \frac{E_s \varepsilon_{zz}}{1 - \mu_p^2},
\]

\[
\sigma_{xy} = \frac{E_s \mu_p \varepsilon_{zz}}{1 - \mu_p^2},
\]

\[
\tau_{xz} = \frac{E_s \gamma_{xz}}{2(1 + \mu_p)}.
\]

A description of the adhesive yielding behavior is provided by

\[
\varepsilon_u = \frac{1}{2s(1 - \mu_p^2)} \left[ \left( 1 + \mu_p \right)(s - 1)\varepsilon_{zz} + \left[ \left( (s - 1)^2 \left( 1 + \mu_p \right)^2 \right. \right. \\
\varepsilon_{zz}^2 + 4s \left( 1 - \mu_p^2 + \mu_p^2 \right) \gamma_{xz}^2 \right]^{0.5} \right]
\]

where, \( \varepsilon_u \), is referred to as the equivalent uniaxial strain term and \( s \) is a material parameter. This yield model is a modified version of the von Mises yield criterion that accounts for both the normal and shear components of strain as well as the effects of
hydrostatic loading [43]. The value of s used in this study was 1.3, which Raghava et al. determined to be applicable for polymer-type materials. A numerical model used to estimate the uniaxial stress-strain response of a particular adhesive is also required. A model that provides a good approximation of the yielding behavior for polymer materials is given by the hyperbolic-tangent approximation attributed to Prager

$$\sigma = A \tanh \left( \frac{\epsilon E_s}{A} \right)$$

where A is referred to as the asymptotic stress value that corresponds to the uniaxial failure stress for a particular adhesive. The level of yielding is determined along the joint overlap by determining an equivalent uniaxial strain from the computed strain field and comparing that to an adhesive yield stress. Therefore, by using equations 2.110-2.114 the complete inelastic strain field in the adhesive layer is determined in an iterative manner for a given joint loading and geometry configuration.

The field equations that govern the behavior of the bonded joint overlap are now determined using the previous equations developed in this section. Substituting equation 2.112 into 2.83 to find,

$$N^U_{xx,xx} = \left[ \frac{E_s}{2(1+\mu_p)} \right] \cdot \gamma_{zz}$$

and equation 2.110 into 2.84 to find,

$$Q^U_{xx,zz} = \left[ \frac{E_s}{(1-\mu_p^2)} \right] \cdot \varepsilon_{zz}$$

and equation 2.112 into 2.85 to find,
provides three field equations that relate the applied loads on the upper adherend to the adhesive strains. Using the kinematic relationship of equation 2.106 and taking two derivatives with respect to \( x \) the following equation is determined

\[
\varepsilon_{zz,xx} = \frac{1}{t} \left( w_{U,xx} - w_{L,xx} \right)
\]

Substituting the moment-curvature relationships for cylindrical bending of a laminated plate in equations 2.89 and 2.90, it follows that

\[
\varepsilon_{zz,xx} = \frac{1}{t} \left( \frac{A^L}{D^L} M^L_{xx} - \frac{A^U}{D^U} M^U_{xx} \right).
\]

In order to reduce the complexity of the system of equations governing the behavior of the general, bonded joint overlap, it is desirable to develop the differential equations for the bonded joint overlap as functions of only the upper adherend unknowns. So, an expression for the moment loading on the lower adherend in terms of upper adherend values is needed to make equation 2.119 a function of only the upper adherend unknowns. A description of the loading on a general section of the bonded joint overlap is shown in Figure 2.2. Using this figure, moment equilibrium is calculated about a point on the right-hand side of the lower adherend that lies on the centerline of the adherend to show

\[
\sum M_y = M_{xx}^U + M_{xx}^L - M_{xx}^U - M_{xx}^L + x \left( Q_{xx}^U + Q_{xx}^L \right) - h^* \left( N_{xx}^U - N_{xx}^L \right) = 0
\]

where

\[h^* = t + \left( \frac{h_U + h_L}{2} \right).\]

Solving for the lower adherend moment
\[ M'_{xx} = M^L_{xx} + M^L_{x} - M'_x + x \left( Q^L_{o} + Q^L_{a} \right) - h^* \left( N^U_{xx} - N^L_{o} \right) \]  \hspace{1cm} 2.121

which is substituted into the expression in equation 2.119 to find

\[ \varepsilon_{zz,xx} = \frac{1}{t} \left\{ \frac{A^L}{D^L} \left[ M^L_{o} + M^L_{x} - M'_x + x \left( Q^L_{o} + Q^L_{a} \right) - h^* \left( N^U_{xx} - N^L_{o} \right) \right] - \frac{A^U}{D^U} M^U_{xx} \right\}. \]  \hspace{1cm} 2.122

Thus, a second-order differential equation for the normal adhesive strain has been obtained in terms of upper adherend unknowns and the known applied loading. Additionally, it is desirable to express equation 2.122 as a combination of two first-order differential equations as the other field equations for the general, bonded joint overlap are all expressed as first-order differential equations. Separating equation 2.122 into two first-order differential equations provides

\[ \frac{dF}{dx} = \frac{1}{t} \left\{ \frac{A^L}{D^L} \left[ M^L_{o} + M^L_{x} - M'_x + x \left( Q^L_{o} + Q^L_{a} \right) - h^* \left( N^U_{xx} - N^L_{o} \right) \right] - \frac{A^U}{D^U} M^U_{xx} \right\} \]  \hspace{1cm} 2.123

and

\[ \frac{d\varepsilon_{zz}}{dx} = F \]  \hspace{1cm} 2.124

where \( F \) is a general function.

The last field equation is obtained by taking a derivative of the kinematic relationship in equation 2.105 to obtain

\[ \gamma_{zz,xx} = \frac{1}{t} \left( \frac{du^U}{dx} - \frac{du^L}{dx} \right). \]  \hspace{1cm} 2.125

Substituting the constitutive relations of 2.103 and 2.104 into the above equation yields

\[ \gamma_{zz,xx} = \frac{1}{t} \left\{ \left( A^U_{11} \right)^{-1} N^U_{xx} - \frac{h^L}{2} \left( D^U_{11} \right)^{-1} M^U_{xx} \right\} - \left\{ \left( A^L_{11} \right)^{-1} N^L_{xx} + \frac{h^U}{2} \left( D^L_{11} \right)^{-1} M^L_{xx} \right\} \} \]  \hspace{1cm} 2.126
which again needs to be modified to remove the unknowns from the lower adherend. Using the free-body diagram in Figure 2.2, a relationship for the lower adherend force resultant, \( N_{xx}^L \), is obtained in terms of upper adherend values by summing force equilibrium in the horizontal direction to find

\[ N_{xx}^L = N_{xx}^U + N_o^L - N_{xx}^L. \tag{2.127} \]

The value of the moment resultant was previously determined in equation 2.121 and can be used along with equation 2.127 to rewrite the equation for the shear strain gradient as

\[
\gamma_{xx} = -\frac{1}{l}\left\{ \left( A_{11}^U \right)^{-1} N_{xx}^U - \frac{h_L}{2} \left( D_{11}^L \right)^{-1} M_{xx}^L - \left( A_{11}^L \right)^{-1} \left( N_o^U + N_o^L - N_{xx}^L \right) \right\} - \frac{h_L}{2} \left( D_{11}^L \right)^{-1} \left[ M_{xx}^L + M_o^L - M_{xx}^L + x \left( Q_o^U + Q_o^L \right) - h^i \left( N_{xx}^L - N_o^L \right) \right] \]

which is now in a more preferable format that contains only upper adherend unknowns and known values of geometry, material properties, and applied loading. Rearranging, equation 2.128 is expressed in the following final format

\[
\gamma_{xx} = -\frac{1}{l}\left\{ N_{xx}^L \left[ \left( A_{11}^U \right)^{-1} + \left( A_{11}^L \right)^{-1} + \frac{h_L}{2} \left( D_{11}^L \right)^{-1} \right] \right\} - \left( A_{11}^L \right)^{-1} \left( N_o^U + N_o^L \right) \]

\[
-\frac{h_L}{2} \left( D_{11}^L \right)^{-1} \left[ M_{xx}^L - M_o^L + h^i N_{xx}^L \right] - x \frac{h_L}{2} \left( D_{11}^L \right)^{-1} \left( Q_o^U + Q_o^L \right) \]

\[ \tag{2.129} \]

Therefore, equations 2.115-2.117, 2.123, 2.124, and 2.129 constitute a system of first-order differential equations developed for an anisotropic, bonded joint overlap. These field equations are then solved to determine the force and moment resultants along the overlap and the strains in the adhesive as a function of the joint length.
2.4.3 First-order shear deformable plate formulation

As in the case of a classical laminated plate, the assumption of cylindrical bending is used to formulate an expression for the moment-curvature relationship. Thus, the expressions in equations 2.89 and 2.90 will be used again in this section. Reducing the kinematic relationships in equations 2.67-2.72 for a shear deformable plate based on the assumption of cylindrical bending; such that, the terms that contain derivatives with respect to y are neglected to find

\[ \varepsilon_{xx} = \frac{\partial u_o}{\partial x} + z \frac{\partial \phi_x}{\partial x} = \frac{du_o}{dx} + z \frac{d\phi_x}{dx} = u_{o,x} + z\phi_{x,x} \]  \hspace{1cm} 2.130

\[ \varepsilon_{yy} = \frac{\partial v_o}{\partial y} = 0 \]  \hspace{1cm} 2.131

\[ \gamma_{xy} = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} + z \left( \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) = \frac{dv_o}{dx} + z \frac{d\phi_y}{dx} = v_{o,y} + z\phi_{y,x} \]  \hspace{1cm} 2.132

\[ \gamma_{xz} = \frac{\partial w_o}{\partial x} + \phi_x = w_{o,x} + \phi_x \]  \hspace{1cm} 2.133

\[ \gamma_{yz} = \frac{\partial w_o}{\partial y} + \phi_y = \phi_y \]  \hspace{1cm} 2.134

Using these results, the constitutive equation for a laminated plate with first-order shear deformation can be reduced from the form in equation 2.73 to obtain

\[
\begin{bmatrix}
N_{xx} \\
N_{yy} \\
N_{sy} \\
M_{xx} \\
M_{yy} \\
M_{sy}
\end{bmatrix} = \begin{bmatrix} [A] & [B] \end{bmatrix} \begin{bmatrix}
u_{o,x} \\
0 \\
v_{o,y} \\
\phi_x \\
\phi_y \\
0
\end{bmatrix}
\]

2.135

and equation 2.79 to obtain the transverse shear resultant as
\[ Q_x = K_s \left[ A_{55} \right] \left\{ w_{o,x} + \phi_x \right\}. \] \hspace{1cm} 2.136

Inverting equation 2.135, eliminating terms that have been neglected due to the assumption of cylindrical bending, and rewriting provides the following constitutive equation

\[
\begin{pmatrix}
  u_{o,x} \\
  0 \\
  v_{o,x} \\
  \phi_{x,x} \\
  0
\end{pmatrix} = \begin{bmatrix}
  A & B \\
  [B] & [D]
\end{bmatrix}^{-1} \begin{pmatrix}
  N_{xx} \\
  0 \\
  0 \\
  0 \\
  M_{xx}
\end{pmatrix} \hspace{1cm} 2.137
\]

Expanding equation 2.137 to determine the normal strain in the axial direction and the rotation of the cross-section about the y-axis as

\[
u_{o,x} = (A_{11})^{-1} N_{xx} + (B_{11})^{-1} M_{xx} \hspace{1cm} 2.138
\]

\[
\phi_{x,x} = (B_{11})^{-1} N_{xx} + (D_{11})^{-1} M_{xx} \hspace{1cm} 2.139
\]

where \( \varepsilon_{xx} = u_{o,x} \) and \( \kappa_x = \phi_{x,x} \). Equation 2.138 is identical to equation 2.99 from the earlier formulation for CLPT while equation 2.139 is similar to the bending strain from CLPT, but contains the additional rotation experienced by the joint cross-section due to transverse shear strain. Using equation 2.136, the total rotation can be written as a combination of the bending strain and shear force as

\[
\frac{Q_x}{K_s A_{55}} = w_{o,x} = B_{11}^{-1} N_{xx} + D_{11}^{-1} M_{xx} \hspace{1cm} 2.140
\]

or in terms of the bending strain
Equations 2.138 and 2.141 represent the longitudinal and bending normal strain including shear deformation, and are used to determine an expression for the total normal strain along the x-axis at the adherend-adhesive interface for each adherend. Recalling equation 2.46 and using these equations for the longitudinal and bending normal strain, expressions for the total normal strain in the upper and lower adherends are derived assuming laminate symmetry to find

\[
\varepsilon_{xx}^U = \frac{du^U}{dx} = (A_{11}^U)^{-1}N_{xx}^U + z \left[ (D_{11}^U)^{-1}M_{xx}^U - \frac{Q_{x,x}^U}{K_s A_{55}^U} \right] \tag{2.142}
\]

\[
\varepsilon_{xx}^L = \frac{du^L}{dx} = (A_{11}^L)^{-1}N_{xx}^L + z \left[ (D_{11}^L)^{-1}M_{xx}^L - \frac{Q_{x,x}^L}{K_s A_{55}^L} \right] \tag{2.143}
\]

Proceeding with the development in a similar manner as the CLPT formulation, the value for \( z \) is determined as the distance from the laminate midplane to the adhesive-adherend interface for each respective adherend, which results in

\[
\varepsilon_{xx}^U = \frac{du^U}{dx} = (A_{11}^U)^{-1}N_{xx}^U + \frac{h_U}{2} \left[ (D_{11}^U)^{-1}M_{xx}^U - \frac{Q_{x,x}^U}{K_s A_{55}^U} \right] \tag{2.144}
\]

\[
\varepsilon_{xx}^L = \frac{du^L}{dx} = (A_{11}^L)^{-1}N_{xx}^L + \frac{h_L}{2} \left[ (D_{11}^L)^{-1}M_{xx}^L - \frac{Q_{x,x}^L}{K_s A_{55}^L} \right] \tag{2.145}
\]

Finally, these values for the total normal strain in each adherend are used in equation 2.125 to formulate the following equation that provides an expression for the gradient of the shear strain,
\[ \gamma_{xx,x} = \frac{1}{t} \left\{ \left( A_{11}^U \right)^{-1} N_{xx}^U - \frac{h_U}{2} \left[ \left( D_{11}^U \right)^{-1} M_{xx}^U - \frac{Q_{x,x}^U}{K_b A_{55}^L} \right] \right\} \]

\[ - \left[ \left( A_{11}^L \right)^{-1} N_{xx}^L + \frac{h_L}{2} \left[ \left( D_{11}^L \right)^{-1} M_{xx}^L - \frac{Q_{x,x}^L}{K_b A_{55}^L} \right] \right] \]

which may be further reduced by substituting for the values of the lower adherend unknowns. Substituting the values of equations 2.121 and 2.127 into 2.146 leads to

\[ \gamma_{xx,x} = \frac{1}{t} \left\{ \left( A_{11}^U \right)^{-1} N_{xx}^U - \frac{h_U}{2} \left[ \left( D_{11}^U \right)^{-1} M_{xx}^U - \frac{Q_{x,x}^U}{K_b A_{55}^L} \right] \right\} \]

\[ - \left[ \left( A_{11}^L \right)^{-1} \left( N_o^U - N_o^U \right) + \frac{h_L}{2} \left[ \left( D_{11}^L \right)^{-1} \left( M_o^U + M_o^L \right) \right] \right] \]

\[ - M_{xx}^U + x \left( Q_o^U + Q_o^L \right) - h' \left( N_{xx}^U - N_o^U \right) \]

Collecting related terms and reducing yields equation 2.148. Finally, expressions for the

\[ \gamma_{xx,x} = \frac{1}{t} \left\{ N_{xx}^U \left[ \left( A_{11}^U \right)^{-1} + \left( A_{11}^L \right)^{-1} \right] - \left( A_{11}^L \right)^{-1} \left( N_o^U + N_o^U \right) \right\} \]

\[ - M_{xx}^U \left[ \frac{h_U}{2} \left( D_{11}^U \right)^{-1} - \frac{h_L}{2} \left( D_{11}^L \right)^{-1} \right] + Q_{x,x}^U \left( \frac{h_U}{2K_b^L A_{55}^L} \right) + Q_{x,x}^L \left( \frac{h_L}{2K_b L A_{55}^L} \right) \]

\[ - \frac{h_L}{2} \left( D_{11}^L \right)^{-1} \left[ x \left( Q_o^U + Q_o^L \right) + M_o^U + M_o^L + h' N_o^U \right] \}

gradient of the shear force from equation 2.116 and a similar expression for the lower adherend are substituted into equation 2.148. The resulting expression is given by...
\[ \gamma_{xx} = \frac{1}{t} \left\{ N_{xx}^U \left[ (A_{ii}^U)^{-1} + (A_{ii}^L)^{-1} + \frac{h^L}{2} (D_{ii}^L)^{-1} \right] - (A_{ii}^L)^{-1} (N_{xx}^U + N_{xx}^L) \right. \\
- M_{xx}^U \left[ \frac{h_U}{2} (D_{ii}^U)^{-1} - \frac{h_L}{2} (D_{ii}^L)^{-1} \right] \\
+ \varepsilon_y \left[ \frac{E_y}{(1-\mu_y^2)} \right] \left[ \left( \frac{h_U}{2K_s A_{ys}} \right) - \left( \frac{h_L}{2K_s A_{ys}} \right) \right] \right\} \\
- \frac{h_L}{2} (D_{ii}^L)^{-1} \left[ x (Q_{o}^U + Q_{o}^L) + M_{o}^U + M_{o}^L + h^L N_{o}^U \right] \right\} \tag{2.149} \\
\]

which shows the relationship between the gradient of the adhesive shear strain and the applied loading, upper adherend unknowns, and normal adhesive strain. Thus, equations 2.115-2.117, 2.123, 2.124, and 2.149 constitute the system of differential equations for the anisotropic, bonded joint overlap that takes into account shear deformation.
CHAPTER 3.0

BONDED JOINT MODEL AND FORTRAN CODE

The present chapter provides a description of the basic joint model and Fortran code used to perform the inelastic, adhesively bonded joint analyses based on the method developed in the previous chapter. First, details on the joint geometry and loading are provided. Second, a description of the basic elements of the Fortran 95 code, the basic program flow, and the methodology for conducting an inelastic analysis are presented.

3.1 The Bonded Joint Model

The basic geometry and edge-joint loading for a general bonded joint overlap was given in Figures 2.1 and 2.2. Although the geometry is representative of a single-lap joint, changes to the differential element loading provide the capability for evaluating several different joint types. However, all the analyses performed during the current research program evaluated a single-lap joint geometry. A more detailed description of the single-lap joint geometry and nomenclature is shown in Figure 3.1 while a depiction of a single-lap joint subjected to a uniform axial tension is provided in Figure 3.2. The upper adherend thickness is denoted by \( h_u \), the lower adherend thickness by \( h_l \), the adhesive thickness by \( t \), and the total joint overlap length as \( C \). Additionally, for all the figures contained in this work, the geometry of the adherend and adhesive components was chosen merely for illustration purposes and were not drawn to scale.

3.2 Basic Components of the Fortran Code

A Fortran 95 code was written to solve the system of first-order differential equations given in section 2.4.3 using a variable-step, finite-difference solution algorithm developed by the IMSL. The basic components of the Fortran code, hereafter referred to as ABJO (anisotropic bonded joint optimizer), are a main program with input/output, composite laminated plate analysis, inelastic material behavior, boundary value solver, and adhesive stress subprograms. All of these major components are briefly described in the following
paragraph while a detailed explanation of their interrelations and overall program flow are provided in the next section.

The main program initializes parameters and makes subprogram calls to either perform a linear elastic or inelastic analysis of a general, bonded joint overlap. Joint geometry, edge-joint loading, material properties, choice of analysis type, and English or SI unit designation are provided to the code through an input file. An example of an ABJO input file is given in Appendix C. Information provided during the input phase is reproduced in the output file along with the solution vectors to the system of differential equations and the post-processed stress vectors. An example of an ABJO output file is given in Appendix D. The composite laminated plate routine determines the plane-stress constitutive matrices for the upper and lower adherends while the inelastic material behavior subprogram updates the adhesive yielding. Explicit calculation of the system of differential equations, partial derivative of the differential equations with respect to each dependent variable (Jacobian matrix), and boundary conditions are determined by the boundary value solver subprograms. A detailed description of the boundary value solver is provided in the next paragraph. Finally, the adhesive stress subprogram utilizes the normal and shear strain solution vectors determined from the boundary value solver and the vectors of elastic or inelastic adhesive material properties to obtain the resulting normal and shear stress vectors. In the case of an inelastic analysis, the adhesive material properties are iteratively updated until yielding reaches an equilibrium state.

The boundary value solution algorithm, BVPFD, is an ISML math library that is capable of solving a parameterized system of ordinary differential equations with boundary conditions at two points using a variable order, variable step size finite difference method. BVPFD was developed using the subprogram PASVA3 developed by Pereyra [41]. The discretization of the adhesive region is determined by applying the trapezoidal rule along a nonuniform mesh. The local truncation error at each grid point is kept uniform, thus providing an adaptively chosen set of grid point locations. A description of the grid points and their location within the bonded joint model is shown in Figure 3.3. As discussed in the previous chapter, the grid points are located at the centerline of the adhesive layer. Deferred correction of the initial mesh is employed to reduce costly computations of the
Jacobian matrix. The solution of the system of differential equations is obtained by employing Newton’s method with step control. The addition of the step control correction procedure aids convergence for problems that are not sufficiently smooth. Global discretization error estimates are calculated to control the computation of the overall solution. The parameterization option is available to aid in the solution of nonlinear systems of differential equations. Since the system of differential equations derived in the previous chapter are piecewise linear, the parameterization option was not used in this study.

Basic information pertaining to each bonded joint problem is passed to BVPFD through the individual subprograms. Data provided to BVPFD through the subprograms includes the form of each differential equation, the form of the Jacobian matrix, the total number of differential equations, the initial conditions at the boundaries, the number and type of boundary conditions, initial and maximum number of grid points, and an error control parameter. The output received from BVPFD is in the form of solution vectors that contain the number of each grid point, the final x-coordinate of each grid point, final values of the dependent variables at each grid point, and an estimate of the global discretization error for each dependent variable.

## 3.3 Solution Method

An example of the overall program flow using ABJO is shown in Figure 3.4. Program execution begins with the initialization of all the variables, matrices, and control parameters. Next, the details of the bonded joint problem to be solved are read from an input file. The details of the joint model are then passed to the boundary solver algorithm. Using the joint information from the original input file, the boundary value solver calculates values for the system of differential equations, the Jacobian matrix, and the boundary condition in order to determine a solution matrix. Since the information for this initial analysis was based on the input data, the original linear elastic material properties are used to obtain a linear elastic solution matrix. A decision to conduct further analyses or write the information from the linear elastic solution to an output file is made according to a logical variable that was read from the input file. If the logical variable is false, then
the program writes the information for the linear elastic solution to an output file and program execution stopped. If the logical variable is true, then a routine is called that uses the current adhesive strain field to determine the amount of yielding in the adhesive layer based on the equivalent strain term, equation 2.113. Specifically, the equivalent strain is substituted into the adhesive stress-strain representation of equation 2.114 to calculate an equivalent stress value, which is used along with the equivalent strain to update the secant modulus and plastic Poisson’s ratio. Once the adhesive material properties are updated, the boundary value solution algorithm is called to perform another analysis based on the updated adhesive material properties. Convergence is determined by using the solution matrix from the previous analysis to compare with the solution matrix from the current analysis. If the difference between the two solutions is within the limits of the convergence criteria, then the data from the current analysis is written to an output file and program execution stopped. If the difference between the previous and current solutions is too large, the program loops back to the yield criteria subroutine in order to update the adhesive material properties. These newly modified properties are used to perform yet another analysis. This loop continued until a converged solution is obtained or a maximum number of iterations exceeded.

The number of joint models evaluated during the current research program was in excess of fifty. Although some models required additional grid points to identify a solution, a valid solution was obtained for all the models evaluated. Additionally, using the author-developed Fortran code and IMSL routine, all of the joint models were analyzed in less than five seconds using a laptop personal computer with an 800 Mhz. micro-processor with 128 megabytes of RAM.
CHAPTER 4.0

RESULTS AND DISCUSSION

Analytically predicted results of the single-lap, adhesively bonded joints considered in this study are presented in this chapter. These results were obtained using the Fortran 95 program, ABJO, which was thoroughly described in the preceding chapter. First, a section describing the validity of the code is provided; where, the current methodology is compared with single-lap joint analyses from previous research studies. Next, the results from several design studies are used to illustrate the effects of changes in adherend and adhesive material properties, adherend thickness, joint overlap length, and laminate stacking sequence. Additionally, the convergence of the boundary value solution algorithm is demonstrated using a case from the design study.

4.1 Code Validation

The analyses described in this section were performed to validate the analytical and numerical capabilities of the ABJO code against previously cited analysis methods in the literature. A total of three comparisons were made between ABJO and other bonded joint analysis methods. In the first case, ABJO results obtained from a linear elastic analysis of a single-lap joint model with quasi-isotropic adherends are compared with results obtained using the classical analysis by Goland and Reissner for a joint with the same geometry and material properties. The second case was performed to compare ABJO results obtained from an inelastic analysis of the same single-lap joint model with results obtained using the elastic-plastic analysis method by Bigwood and Crocombe. The joint geometry and material properties for the models used in the first two comparison cases are shown in Tables 4.1 and 4.2, respectively. The loading for each of the first two comparison cases was a uniform tensile load of 400 Newtons and is depicted in Figure 3.2. The last comparison was performed to compare ABJO results obtained from a linear elastic analysis of a single-lap joint with anisotropic adherends to results from the linear elastic analysis method by Yang of the same joint. Additionally, the loading applied to the single-
lap joint in the third case is a pure bending moment of 2 N-mm/mm, opposed to the uniform tensile loading from the two previous cases. The joint geometry and material properties for the last case are given in Tables 4.3 and 4.4, respectively.

The material properties used in the first comparison case between ABJO and the analysis method by Goland and Reissner differed slightly. In order to compare the two methods, models that had identical geometry and equivalent material properties were needed to perform the analyses. Furthermore, since the analysis method by Goland and Reissner assumes linear elastic adhesive behavior, the corresponding ABJO analysis was performed without considering adhesive yielding. The material properties used in the ABJO analysis are given under the column denoted by the quasi-isotropic model heading in Table 4.2, while the properties for the Goland and Reissner analysis are listed in the same table under the isotropic model heading. Using the lamina material properties and laminate stacking sequence given in Table 4.2 for the quasi-isotropic model, the in-plane average laminate properties were determined to be $E_{11} = E_{22} = 69.5 \text{E}03 \text{ N/mm}^2$ and $\nu = 0.29$. These values for the quasi-isotropic model are close, but slightly lower than the corresponding properties in the isotropic model. However, variations between the material properties of the adherends in each model were minor enough that a reasonable comparison between the different analysis methods could be performed using the two models.

The results for the first comparison case between a linear elastic ABJO analysis and the linear elastic analysis method by Goland and Reissner are shown in Figure 4.1. The shear and normal stress response for these analyses are plotted in Figures 4.1a and 4.1b, respectively, as a function of the overlap length for the joint. A comparison between the strains for each method was not given since the analysis method by Goland and Reissner only determines adhesive stresses. The results from the ABJO analysis are displayed using a solid line and stress terms with a superscript A while the results for the classical analysis by Goland and Reissner are displayed with a dashed line and stress terms using a superscript GR. Excellent agreement between the present analysis method and the classical method by Goland and Reissner is seen for almost the entire elastic shear and normal stress fields in each figure. The largest deviation between the two methods is
found at the location of the peak shear stress on the ends of the joint overlap. An approximately 15% lower peak shear stress is predicted by the present analysis method compared to that of Goland and Reissner. This difference is most likely due to the tendency of the analysis method developed by Goland and Reissner to be conservative for most cases as a result of approximations made during the formation of their solution. These approximations are well documented [2,15] and are a result of neglecting higher-order terms within the differential equations of equilibrium as well as simplifications made to certain mathematical expressions. One other possibility for the difference in the peak shear stress is the slight discrepancy between the in-plane stiffness of the adherends in the quasi-isotropic and isotropic joint models, which was discussed in the preceding paragraph. Although the plate/laminate properties for the two models are nearly identical, the lamina properties and stacking sequence chosen for the quasi-isotropic model provided a slightly more compliant joint than the isotropic model. As a result, the peak shearing stress determined from the analysis of the quasi-isotropic model would tend to be lower than that for the isotropic model.

The strain and resulting stress response for the second bonded joint comparison between ABJO and the isotropic, inelastic analysis method by Bigwood and Crocombe are presented in Figures 4.2a and 4.2b, respectively. The solid lines in each figure show the results for the present method while the isotropic analysis results are displayed as dashed lines. Additionally, stresses and strains from the anisotropic analysis are denoted by terms with a superscript $A$ while the isotropic analysis is denoted by terms with a superscript $I$. The joint geometry and material properties for the models used to compare ABJO with the analysis method by Bigwood and are identical to the previous case and are located in Tables 4.1 and 4.2, respectively. Again, as noted in the previous case, a slight variation is seen between the isotropic adherend properties and the quasi-isotropic adherend properties. In Figure 4.2a, a very good agreement is provided between the anisotropic and isotropic solutions with a reduction in shear and normal strain from the isotropic to the anisotropic solutions seen as the solution moves from the center of the overlap towards the joint edges. In general, the solution for the strains at the joint edges is more sensitive to changes in joint parameters due to the peak value of the strains occurring at those locations.
Therefore, the more compliant behavior of the adherends in the quasi-isotropic model result in a reduction of the shear and normal stress similar to the previous case for Goland and Reissner, but to a greater degree due to the addition of inelastic adhesive behavior. One additional point to note in Figure 4.2b is the small drop in the shear stress at the four and eight millimeter points along the joint overlap. This represents the interface between the elastic and the plastic zones in the adhesive layer. Although the strain response is continuous over the entire overlap as evident by the curves in Figure 4.2a, calculations for the shear stress response result in a discontinuity at the elastic-plastic zone interface. Essentially, the discontinuity at the interface is present because of a mismatch between the equations used to determine the adhesive properties in the elastic and plastic regions. The adhesive properties are constant in the elastic region according to the linear elastic constitutive equation. At the initiation of yielding, the adhesive properties are determined using the hyperbolic-tangent approximation, which provides slightly different properties from the adjacent elastic values. A detailed discussion of this effect is provided in the following section describing the design study cases.

A very significant reduction in peak shear and normal stress is obtained between the inelastic solution comparing the analysis of Bigwood and Crocombe and the elastic solution for Goland and Reissner. The joint configuration for the two analyses is identical yet the peak shear stress for the elastic solution in Figure 4.1a was approximately 120 N/mm$^2$ while the peak shear stress for the inelastic solution in Figure 4.2b was approximately 44 N/mm$^2$, which corresponds to a 67% reduction. Similarly for the normal stress, a reduction of 67% exists between the 150 N/mm$^2$ elastic value and the 50 N/mm$^2$ inelastic value. Although the inelastic solution is a more realistic response to the loading, this significant reduction in adhesive stresses is not obtained without a cost as only a few millimeters of the joint overlap are now stressed below 20 N/mm$^2$ compared to almost eight millimeters in the elastic joint. Although a much more efficient joint is obtained by allowing a certain portion of the joint to yield, an important note of caution is prudent for joints that are intended for long-term use, multiple cycles, or hazardous environments. Joints that are intended for these uses may still be designed by taking into account the inelastic adhesive behavior; however, it is crucial that a section at the center of
the joint overlap remain well below the yield stress level to serve as an elastic reserve. The need to provide a reasonably long, lightly loaded elastic trough at the center of fatigue-loaded joints was first discussed in detail by Hart-Smith [23-28].

The results of the last comparison between the anisotropic analysis using ABJO and the anisotropic, elastic analysis method by Yang are shown in Figures 4.3a and 4.3b. In these figures, the adhesive shear and normal stress fields are shown as a function of the nondimensionalized joint overlap. The ABJO results presented in these figures were determined using just the linear elastic adhesive properties since the analysis method by Yang does not include the effect of adhesive yielding. Additionally, a comparison between the strain fields for each method was not presented due to the omission of these results by Yang. As in the first two cases, the ABJO results are displayed using a solid line and stress terms with a superscript $A$; while the results for the analysis method by Yang are displayed using a dashed line and a superscript $Y$. The correlation between the two analysis methods is excellent with less than a 1% difference in the peak shear and normal stress values. Shear stress solutions are almost identical while a small difference in the normal stress response for the two methods is evident at the points on the joint overlap just before the ends of the joint. This difference appears to be a result of localized, nonlinear effects that make the solution sensitive near the joint ends due to large gradients in the normal stress field in those regions. The large gradients are a result of the bending deformation that occurs in the joint overlap adjacent to the ends of the joint. Specifically, the bending occurs as the applied load is gradually increased thus removing the eccentricity between the two force vectors. However, the difference is not an important factor in the stress response of the joint since the normal stress at that location is not critical compared to the global stress field.

A few final remarks concerning the ABJO analyses in this section are now given. The ABJO analysis code was compared to three different analysis methods that provided independent validations of the linear elastic, inelastic, and anisotropic modeling and analysis capabilities of the code. Additionally, the models were subjected to two different applied loadings: uniform tension and pure bending, which revealed some of the general-use capabilities that are integral to the method. Finally, all of the analyses conducted to
validate the code had models with between thirty and fifty grid points. Convergence was not the reason for the level of discretization for the validation models, but was chosen mostly to ease the display of the final results. In fact, there was virtually no change in the solution for analyses conducted of models with even half the number of original grid points. A detailed description of the convergence using different levels of discretization with the boundary value solver is given in section 4.2.2.

4.2 Design Studies

The design studies presented in this section were conducted to determine the sensitivity of the adhesive layer’s strain/stress response to changes in different joint parameters using the ABJO inelastic analysis methodology. The approach for performing the design studies was to conduct an initial preliminary study followed by a more comprehensive parametric study. The preliminary study was performed as several mini-parametric studies used to evaluate selected features of the single-lap joint geometry, and the effect changes in these features had on the resulting strain/stress response of the adhesive layer. The topics investigated during the preliminary study were the length of the joint overlap, thickness of the adherends, and convergence of the ABJO code. Additionally, the preliminary study was used to identify a preferred joint configuration that would be used during the subsequent parametric study. The parametric study investigation was conducted to evaluate several key features of a single-lap type joint using the preferred joint configuration selected during the preliminary study. Furthermore, by selecting a preferred joint configuration for the parametric study, the number of joint features to be evaluated was reduced to a more manageable quantity.

A primary function of the preliminary study was to identify a suitable single-lap joint configuration that would provide a significant amount of adhesive yielding without causing excessive strains. However, this could have resulted in an endless quest, as quite a few of the joint parameters were candidates for adjustment. For example, simultaneous manipulation of two of the joint parameters could have resulted in counter-balancing effects for a third parameter. Thus, a choice was made to fix the overlap length and thickness of the adherends while investigating changes to the material properties in the
joint. This seemed to be a prudent decision since a designer is often constrained by the size and length of the structure that may be used in a particular joint region, e.g. increasing the thickness of a wing skin from 0.1 inch to one-fourth of an inch is not normally feasible. Although this is not always the case, this situation is often present due to limitations on the structure and the plethora of materials available with wide ranges of application. In summary, the preliminary design study was performed to identify a preferred overlap length and thickness for the upper and lower adherends.

A description of the materials and parameters chosen for use in the parametric design study is given in Table 4.5. The table consists of three different laminate stacking sequences or layups, two different carbon/epoxy materials, two different adhesives, and one factor that evaluated the effect of shear deformation between the upper and lower adherends. The material properties for the FM300 and Araldite 2015 adhesives and the AS4/3501-6 and P75/954-3 carbon/epoxy composites are given in Tables 4.6 and 4.7. These material properties were obtained from a mix of manufacturer’s data and data generated from NASA test programs and, while intended to be representative of the actual materials, should not be used as design values.

The reason for choosing the adhesives and carbon/epoxy materials in Table 4.5 for the parametric design studies was to illustrate the differences in the strain/stress response for each joint configuration as a result of using materials with contrasting properties. The AS4/3501-6 material is a high-strength, standard-modulus carbon/epoxy composite material used in aircraft and launch vehicle applications that are strength driven while the P75/954-3 is a high-modulus carbon/cyanate ester composite material used in spacecraft and lightweight structural applications that are stiffness driven. Additionally, the cyanate ester resin provides low-outgassing of volatiles and moisture uptake when compared to conventional epoxy resins that is essential for many optical spacecraft requirements. As for the adhesives, the FM300 is an elevated cure, modified epoxy film adhesive that is primarily used in aircraft and launch vehicle applications to bond metal and composite structures while Araldite 2015 is a room temperature cure, two-part epoxy paste adhesive that is used in automotive applications for general bonding of sheet molding compound and carbon/epoxy structures. In terms of material behavior, the FM300 adhesive is
essentially a high modulus, low elongation material while Araldite 2015 is a low-modulus, high elongation adhesive that is much more compliant than the more brittle FM300. Additionally, the FM300 possesses nearly twice the failure shear strength of the Araldite 2015, which makes it capable of sustaining relatively high loads in comparison.

4.2.1 Joint overlap study

The first design study was conducted to determine the effect of different overlap lengths on the inelastic adhesive response and amount of yielding that could be obtained using ABJO. The joints evaluated consisted of a quasi-isotropic, $[\pm 60,0]_{4s}$, layup of AS4/3501-6 for the upper and lower adherends and Araldite 2015 adhesive. This arrangement of materials and layup were also used for the investigation on adherend thickness that will be discussed in the next section. The combination of a quasi-isotropic layup of AS4/3501-6 for the adherends and Araldite 2015 for the adhesive was chosen for the preliminary design study to provide the most compliant joint based on the joint parameters given in Table 4.5. It was estimated that the most compliant joint would result in the greatest adhesive strains. Therefore, determining a design load that resulted in a converged solution from the boundary solver routine using a very compliant joint should allow the remaining, stiffer joints to be solved. Based on this rationale and several initial joint analyses, a uniform tensile loading of 750 lbf. was selected as the design load to be applied to all the models in the joint overlap study.

The only joint parameter evaluated in the analyses for the joint overlap study was the length of the joint overlap. Three different overlap lengths were used to determine the resulting strain/stress response from ABJO: one-half-inch, one-inch, and two-inches. Based on experience, this seemed to be a suitable range for the study. The thickness of the upper and lower adherends and adhesive were held constant at 0.12- and 0.005-inches thick, respectively, for all the analyses in the joint overlap study. Additionally, no effects of changes in the transverse shear stiffness were evaluated during the joint overlap study.

The results of the ABJO analyses conducted for the joint overlap study are shown in Figures 4.4-7, and show the adhesive strains and stresses for each overlap length as a function of the nondimensionalized overlap. In each figure, the response for the one-half-
inch overlap length are represented by the solid line and circular symbol, the one-inch overlap length by the dashed line and square symbol, and the two-inch overlap length by the dashed-dot line and diamond symbol. Additionally, only a selected number of data points are displayed along each line. In Figures 4.4 and 4.5, the shear and normal strain responses for the three overlap lengths revealed the expected result that the shortest overlap resulted in the highest magnitude peak strains as well as the highest average strain field, while the longest overlap resulted in the lowest magnitude peak strains as well as the lowest average strain field. Although the shear strain in the case of the one-half-inch overlap was almost eight thousand micro-strain, this value was below the ultimate uniaxial tensile strain for the adhesive and most likely would not have resulted in a joint failure.

A similar trend was revealed for the shear and normal stress responses in Figures 4.6 and 4.7, respectively, with the shortest overlap length resulting in the highest magnitude average shear and normal stress across the joint overlap while the longest overlap length had the lowest magnitude average stresses. However, the most important feature in these figures was provided by the inelastic shear stress responses in Figure 4.6. A significant amount of yielding was displayed by the adhesive in the case of the one-half-inch overlap while a much smaller amount of yielding was determined for the one-inch overlap, and virtually no yielding in the case of the two-inch overlap. Based on these results, a one-half-inch overlap was chosen to perform the remaining parametric design studies. Additionally, a sharp jump in the shear stress was seen in the responses for the one-half-inch and one-inch overlaps. These occurred approximately at the 0.05 and 0.95 locations along the nondimensionalized overlap for the one-inch overlap length and the 0.2 and 0.8 locations for the one-half inch overlap length. This phenomenon was briefly described in section 4.1 and only appears at the transition between the elastic and plastic zones due to the method used to calculate the adhesive stress fields. A detailed discussion of this phenomenon is provided in section 4.2.2 along with a discussion of the convergence of the solution using ABJO.

One final interesting result concerning the shear stress responses was noted in relation to the design of adhesively bonded joints. The stress response for the two-inch overlap resulted in a peak shear stress of approximately 1,900 psi. with essentially no adhesive
yielding. However, the peak shear stress for the one-half-inch overlap was only 2,180 psi., less than 10% higher, due to a significant amount of yielding in the adhesive layer. A linear elastic analysis that had not accounted for the effects of yielding would have erroneously predicted a much greater peak shear stress for either the one-half-inch or one-inch configurations that could have severely reduced the efficiency of the joint design. However, this significant reduction in adhesive peak shear stress is not obtained without a cost as only 20% of the one-half-inch overlap length joint was stressed below 1,000 psi. compared to almost 90% for the longer overlap length joint. Although a much more efficient joint may be obtained by allowing a certain portion of the joint to yield, a designer should be very careful when choosing to allow inelastic adhesive behavior to occur. The rationale for limiting the stress in a portion of certain bonded joints was previously cited in the works of Hart-Smith.

4.2.2 Convergence study

The results presented in this section are intended to investigate the convergence of the adhesive strains and stresses using the boundary value solution algorithm, BVPFD, as well as substantiate the number of grid points used during the validation and design studies. As discussed in section 3.3, the number of grid points is determined according to the boundary value solver based on upper and lower limits provided by the user. The number of grid points used to conduct the ABJO analyses for the majority of the design study cases was thirty, but was as high as fifty in some cases. The total number of grid points was similar for the validation cases in section 4.1, where the total number of grid points varied between thirty and fifty as discussed near the end of that section. In the validation cases, the primary reason for setting a lower limit on the number of grid points was to ease the plotting and comparison of results for the different analyses methods. However, the results for the design study cases produced regions of interest that required finer grids to accurately display the solution for the stress field in those areas.

The regions of interest that required additional attention were not areas identified from the original solution for the strain responses, but were discovered as a result of post-processing the strain data to show the shear and normal stress responses. Although the
strain responses were smooth and continuous, discontinuities were present in the stress field due to differences in the methods used to calculate the adhesive properties in the elastic and plastic zones. This disconnect between the two responses made it difficult to modify the mesh used by the boundary value solver, since the solver had obtained a converged solution based only on the strain field. An additional difficulty surrounding the use of the boundary value solver was the adaptive meshing algorithm that made it virtually impossible for the user to place grid points at certain locations of interest along the joint overlap. Therefore, the modification of a particular grid for a joint overlap to investigate certain regions of interest could only be addressed by increasing the minimum number of grid points.

The geometry, loading, and material properties of the single-lap joint configuration used to conduct the analyses for the convergence study were identical to the one-half-inch joint overlap configuration evaluated during the joint overlap study. During the convergence study, this joint configuration was used to perform analyses of models with twenty, thirty, and fifty grid points to assess the convergence rate of the strain and resulting stress solutions. Results for the shear and normal strain are given in Figure 4.8 for the models using twenty and thirty grid points while the results for the thirty and fifty grid point models are given in Figure 4.9. The correlation between the three models was excellent with less than 1% difference between the solutions for the models with twenty, thirty, and fifty grid points. Similar correlation was obtained for the shear and normal stress fields shown in Figures 4.10 and 4.11 for the twenty, thirty, and fifty grid point models; however, differences of 1-5% were identified between the twenty grid point model and the thirty and fifty grid point models at locations along the shear stress response in the yielded regions of the joint overlap.

A closer evaluation of the strain and stress responses was conducted to investigate the overall convergence and the convergence of the solutions in the yielded region of the joint overlap. The results in Figures 4.12 and 4.13 provide a detailed look at the shear strain response from Figures 4.8 and 4.9, respectively, in the region surrounding the first grid point on the edge of the joint overlap. Since this is normally the region in the strain response where correlation between the solutions is the worst, it was chosen to investigate
the convergence of the solution in more detail. Although it was not distinguishable in the global strain response, the small 1\% difference that existed between the solutions for the model with twenty grid points and the other two models was made evident in Figure 4.12. In Figure 4.13, it is clear that the solution for the strain has completely converged to a final solution with less than 0.1\% difference between the two solutions. Additionally, the solution for the strain is shown to be converging from below as the model with fifty grid points has a slightly greater strain response than the model with thirty grid points.

The results of the stress response in the yielded region near the edge of the joint overlap are shown in Figures 4.14 and 4.15 for the models with twenty, thirty, and fifty grid points. Looking at the results for the shear stress in Figure 4.14, a small difference was determined between the solutions using the models with twenty and thirty grid points. The twenty grid point model provided a rather course definition of the shear stress response in the plastic region while the thirty grid point model identified a smoother curve with a higher peak shear stress. Comparing the results in Figure 4.15 between the thirty and fifty grid point models, the solutions appear to have converged as little difference was determined between the solutions with one notable exception. This exception in the convergence of the shear stress results was identified for all three models at approximately the 0.15 nondimensionalized overlap location. At that location, sharp discontinuities were found between the solutions regardless of the number of grid points used in the model. The reason for the sharp discontinuity at this location was due to the method used to calculate the shear stress. The adhesive material properties used to calculate the shear stress in the elastic region of the joint were the linear elastic input values while the material properties used to calculate the shear stress in the yielded region were determined by degrading those values according to the level of yielding. As a result, a difference between the adhesive material properties in the elastic and plastic regions was always present, but varied for each joint configuration depending on the stiffness of the joint, loading level, and adhesive behavior. This phenomenon while visually alarming did not occur at a critical location in the stress field and was essentially a localized effect. Furthermore, this discontinuity was clearly a function of post-processing the original strain solution as the
results displayed in Figures 4.8 and 4.9 did not contain any discontinuity in the strain fields.

The results of the analyses provided in this section have demonstrated the convergence of the solution using the boundary value solution algorithm for models using between twenty and fifty grid points. Additionally, anomalies in the solution for the stress response and their effect on the joint behavior were addressed. Since all the models evaluated in the validation cases and the design studies used between thirty and fifty grid points, the information presented in this section provides strong evidence that all the solutions presented in the present work had reached convergence.

4.2.3 Adherend thickness study

The results of the ABJO analyses conducted to determine the effects of adherend thickness on the strain and resulting stress response of the adhesive layer are given in Figures 4.16 through 4.19. Two values of the adherend thickness were used to investigate the effect on the strain/stress response for both the FM300 and Araldite 2015 adhesives. The remaining parameters used to create the joint model were an overlap length of one-half-inch, adherends with a quasi-isotropic, $[\pm 60,0]_{\text{ns}}$, layup of AS4/3501-6, and a uniform axial tension load of 750 lbf. The thinner adherend had a thickness of 0.06-inches, which corresponded to a twelve-ply quasi-isotropic laminate, while the thicker adherend was twenty-four plies with a total thickness of 0.12-inches. Additionally, all the models analyzed contained upper and lower adherends of equal thickness.

The shear and normal strain results given in Figures 4.16 and 4.17, respectively, show a large difference in the adhesive responses due to changes in adherend thickness. The Araldite 2015 results are represented by a solid line and circle symbol for the thinner adherend while the thicker adherend results are displayed using a long dashed line and square symbol. The FM 300 results are represented by a dash-dot line and diamond symbol for the thinner adherend with the thicker adherend results being displayed using a short dashed line and triangle symbol. Looking at the overall shear and normal stress fields, only minor differences were determined in the center portion of the joints related to changes in adherend thickness. The more compliant adhesive, Araldite 2015, exhibited
greater peak strains compared to the FM300 joints as well as a more highly strained field across the entire length of the joint overlap as expected. The major differences in the responses occurred at the ends of the overlap regions as the joint models with thinner adherends provided the highest peak shear and normal strain values. At the ends of the joint overlaps, the thinner adherends exhibited strains that were approximately 50% higher than their thicker counterparts. Based on these results, the value of the peak strains for the joints with thinner adherends were excessive and in the range of the uniaxial failure strains for each adhesive. These failure strains are displayed as the endpoints of the stress-strain responses in Figure 4.20, which was determined for each adhesive using the hyperbolic tangent approximation model of equation 2.114. Although this joint configuration was likely the most compliant of all the models to be analyzed in the parametric design study and none of the other models would experience peak strains in this range, the peak strains associated with the thinner adherends were deemed too high. Therefore, the adherend thickness of 0.12-inches was selected as the nominal value for the models used in the parametric design studies.

A few points concerning the shear and normal stress responses in Figures 4.18 and 4.19, respectively, were noted. Joint configurations using both adhesive types exhibited a certain degree of yielding regardless of adherend thickness. As expected, the lower modulus Araldite 2015 adhesive provided a wider yield zone at the ends of the overlap; however, a much smaller difference in the peak shearing strain was demonstrated between the Araldite 2015 joints when compared to the joints using the higher modulus FM 300. Additionally, the discontinuity between the elastic and plastic regions does not appear to be as pronounced for the joint configurations in the adherend thickness study compared to the prior results from the joint overlap. A final interesting observation was noted based on the normal stress response from the joints evaluated in the adherend thickness study and the basic mechanics of a single-lap joint.

The general behavior exhibited by a single-lap joint in uniaxial tension is to allow rotation of the joint overlap section in order to align the load path. As this rotation occurs, normal stress values begin to increase at the edges of the overlap regions due to the bending of the joint. The bending increases with higher loads until the joint fails by
tearing the adherends apart at the adhesive interface beginning at the joint ends due to the excessive normal stress experienced by the adhesive. As a result of this tearing behavior, the normal stress is often referred to as the adhesive peel stress. A characteristic response of joints subjected to high axial tension loads is to develop sharp peaks in the compressive normal stress just inboard of the joint edges due to the majority of the bending occurring at the ends of the joint overlap. This response is present in Figure 4.19 for both joints using the lower modulus Araldite 2015 adhesive and the thin adherend/FM300 combination while the joint using the higher modulus FM300 with the thicker adherends had a smooth transition. Of the four joints evaluated, the joint with the FM300 adhesive and thicker adherends would be expected to possess the highest joint bending stiffness and least amount of deformation due to rotation. In summary, these observations using the normal stress response can provide insight into the level of loading imposed on a particular joint as well as the resulting bending behavior.

4.2.4 Parametric study

The ABJO analyses conducted to perform the parametric study were based on the twenty-four models constructed from the parameters listed in Table 4.5. Four parameter types were evaluated: laminate type, adherend material type, adhesive material type, and the effect of shear deformation. For example, the components of the first joint model were upper and lower laminates of AS4/3501-6 with a \([±60°, \, 0°]_{4s}\) layup that were bonded together using FM 300, and the upper and lower laminates have equal values of the transverse shear modulus. A subsequent model had all the same parameters except the adherend material type would be changed to P75/954-3. Continuing this rationale to determine all twenty-four models resulted in a full-factorial design, which provided the data to investigate the effect that changing each parameter had on the resulting joint response. In the remainder of this section, results from the parametric study are presented by organizing the figures into two main sets. The first set of figures provide results from the parametric study by plotting the stress and strain responses for combinations of adherend and adhesive material types as a function of laminate stacking sequence that do not include the effects of changes to the transverse shear modulus. The second set of
figures display the results that investigated the effect of changes in the transverse shear modulus between the upper and lower adherends.

The individual joint parameters were selected to investigate the effect of specific characteristics on the adhesive response. At the beginning of section 4.2, a brief discussion was given on the parameters used as adherend and adhesive material types and the rationale for their inclusion in the parametric study. The parameter for increasing the transverse shear modulus by 30\% in the lower adherend while leaving the upper adherend unchanged was utilized to investigate the effects of shear deformation on the adhesive response. The final parameter was employed to evaluate the adhesive response based on changes in the axial and bending stiffness of the adherends using different laminate stacking sequences or layups. The composition of each laminate layup was carefully selected to provide material properties for each adherend that would improve insight into characteristics of the adhesive responses.

The three different laminate stacking sequences used to describe the adherends were selected to provide a baseline laminate, a laminate with increased bending stiffness, and a laminate with increased axial and bending stiffness. The \([\pm 60^\circ, 0^\circ]_{4s}\) quasi-isotropic layup was used as the baseline laminate with which the analyses of the other laminate types could be compared. The \([0^\circ, \pm 45^\circ, 90^\circ]_{3s}\) quasi-isotropic layup was chosen to provide a laminate with the same axial stiffness and an almost 50\% increase in the bending stiffness when compared to the \([\pm 60^\circ, 0^\circ]_{4s}\) layup. Qualitative comparisons of the adhesive strain and stress responses using these two laminates were useful for making observations about the effects of changes to the joint bending stiffness that were independent of any other joint changes. The \([\pm 30^\circ, 0^\circ]_{4s}\) layup was a directional configuration chosen to provide a substantial increase in both axial and bending stiffness over the other two layup configurations. The axial stiffness for the \([\pm 30^\circ, 0^\circ]_{4s}\) layup was increased by approximately 70\% over the other two layups while the bending stiffness was increased over the \([\pm 60^\circ, 0^\circ]_{4s}\) and \([0^\circ, \pm 45^\circ, 90^\circ]_{3s}\) layups by approximately 100\% and 50\%, respectively. These three laminate stacking sequences, when used in the adherend construction for the joint models of the parametric study, provided useful joint characteristics for identifying basic trends in the adhesive strain and stress responses. In

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summary, using the $[\pm 60^\circ, 0^\circ]_{4s}$, $[0^\circ, \pm 45^\circ, 90^\circ]_{3s}$, and $[\pm 30^\circ, 0^\circ]_{4s}$ layups for the adherends of a joint provided a basic model that was the most compliant of the three, a model with increased bending stiffness over the basic model, and a model with significant axial and bending stiffness compared to the other two models, respectively.

The results for the first group of joint models are shown in Figures 4.21 through 4.24, and are plotted as a function of the nondimensionalized joint length. These models used the low modulus Araldite 2015 adhesive and the standard modulus AS4/3501-6 composite material as the constituent materials in the joints. This combination of joint materials provided the most compliant joints of all the models in the parametric study. Additionally, differences in the transverse shear moduli between the upper and lower adherends were not included in these analyses. The results for each of the laminate stacking sequences are denoted in the figures by using a solid line and circle symbol for the $[\pm 60^\circ, 0^\circ]_{4s}$ layup, a long dashed line and square symbol for the $[0^\circ, \pm 45^\circ, 90^\circ]_{3s}$ layup, and a dashed-dot line and diamond symbol for the $[\pm 60^\circ, 0^\circ]_{4s}$ layup. One last note concerning the descriptions of the remaining figures in this section is needed. Since the remaining figures described in what follows all pertain to inelastic analyses using ABJO, the term inelastic applies to all the results and will not be included with each individual description for the sake of clarity.

The results for the adhesive shear and normal strain in Figures 4.21 and 4.22 demonstrated responses that are classical representations of a single-lap joint subjected to uniaxial tension. The adhesive shear strain responses in Figure 4.21 have a parabolic shape with a lightly loaded region in the center of the joint and peak strains occurring at the ends of the overlap. The adhesive normal strain responses in Figure 4.22 have a similar shape except the data through the center section of the joint is in compression and the response in that region has a flattened shape. A comparison of the adhesive strain responses for each layup type revealed that they are very similar in the center region of the joint with larger differences occurring near the overlap ends. The largest difference in the strain fields occurred between the $[\pm 60^\circ, 0^\circ]_{4s}$ layup model and the other two layup models. In particular, the $[0^\circ, \pm 45^\circ, 90^\circ]_{3s}$ layup and $[\pm 30^\circ, 0^\circ]_{4s}$ layup models appeared to have almost identical responses except near the location of the maximum strains at the ends of the overlap. As expected, the results for the $[\pm 60^\circ, 0^\circ]_{4s}$ layup model provided the
highest maximum shear and normal strains of the three layup types since it was the most compliant joint. Although failure was not predicted using ABJO, the shear and normal strains predicted using the $[\pm 60^\circ, 0^\circ]_{4s}$ layup model were in the neighborhood of the uniaxial failure strain of the adhesive material alone, which suggests that the joint would be very near failure if not already failed. The maximum shear and normal strain values for the other two layup models showed reductions of similar magnitude compared to the maximum strains of the $[\pm 60^\circ, 0^\circ]_{4s}$ layup model. This relatively equal reduction in magnitude was seen despite the differences in the axial and bending stiffness between the $[0^\circ, \pm 45^\circ, 90^\circ]_{3s}$ and $[\pm 30^\circ, 0^\circ]_{4s}$ layup models. In the case of the shear responses, the difference in the maximum shear strains between the $[\pm 60^\circ, 0^\circ]_{4s}$ layup model and the other two layup models was approximately 25-30\% while the difference between the $[0^\circ, \pm 45^\circ, 90^\circ]_{3s}$ and $[\pm 30^\circ, 0^\circ]_{4s}$ layup models was only about 8\%. Therefore, the increase in both bending and axial stiffness for the $[\pm 30^\circ, 0^\circ]_{4s}$ layup model did not appear to provide a significant reduction in shear and normal strain compared to the $[0^\circ, \pm 45^\circ, 90^\circ]_{3s}$ layup model. This phenomenon suggests a nonlinear relationship exists between an increase in the bending stiffness of the adherends and the resulting inelastic adhesive strain response for a low strength, low modulus adhesive. Additionally, increasing the axial stiffness of the adherends appeared to have a very small, if any, effect on the adhesive strain response.

The adhesive shear stress response given in Figure 4.23 displayed significant yielding while it was more difficult to discern the level of yielding from the normal stress plot in Figure 4.24. As discussed in the previous studies, a sharp jump between the elastic and plastic regions was present in all the shear stress responses, but does not indicate a true increase in shear stress. The shape of the adhesive shear stress response is characterized by fully stressed regions near both ends of the joint that traverse the first and last two-tenths of the nondimensionalized joint overlap with the stress dropping rapidly towards a minimum point at the center of the joint. The fully stressed regions are the areas of the joint overlap where yielding of the adhesive material has occurred. This was the typical adhesive shear stress response for all the results evaluated using ABJO with slight variations occurring in the length of the yield region. Furthermore, this shear stress response resembles classic elastic-plastic responses noted by other researchers [24].

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A comparison of the shear and normal stress responses for the three laminate types in Figures 4.23 and 4.24 provided a somewhat different trend than the previously discussed strain responses. Correlation of the shear stress responses for each layup type across the entire nondimensionalized overlap length was again close with the \([0^\circ, \pm 45^\circ, 90^\circ]_3s\) and \([\pm 30^\circ, 0^\circ]_4s\) layup models providing almost identical responses. Looking at the peak shear stress regions, the \([\pm 60^\circ, 0^\circ]_4s\) layup displayed a higher maximum shear stress than either the \([0^\circ, \pm 45^\circ, 90^\circ]_3s\) or \([\pm 30^\circ, 0^\circ]_4s\) layup models which was similar to the trend seen in the shear strain results. However, the difference in the maximum shear stress between the \([\pm 60^\circ, 0^\circ]_4s\) layup and the other two layup models was much smaller for the shear stress results, about 8-10\%, compared to the previously discussed 25-30\% for the strain results.

In the case of the normal stress response in Figure 4.24, only slight differences between the responses were apparent across the entire joint overlap. While the normal strain results provided a large difference in the responses at the ends of the joint overlap, the normal stress provided very close correlation at the ends with a larger difference in the solutions occurring through the center region of the joint. As a result, the normal stress responses made it easier to qualitatively discern the differences in bending behavior for each joint. Results for the normal stress response of the \([\pm 60^\circ, 0^\circ]_4s\) layup model were more of a W-shape compared with the U-shaped responses for the other two layup models, which indicates a larger amount of rotation and bending at the ends of the joint overlap for the former joint. Looking at the maximum normal stress values for each layup type revealed almost no benefit to changes in bending or axial stiffness between the three responses. One possible explanation for this behavior is the significant yielding that takes place in the region of the joint overlap near the ends. Since the yielding reduces the stiffness of the adhesive in that region, large increases in the normal strain only translate as small, almost insignificant, increases in normal stress.

The results for the next group of joints modeled are provided in Figures 4.25 through 4.28. These models used the P75/954-3 high-modulus, composite material for the adherends along with the Araldite 2015 adhesive from the previous models. Differences in the transverse shear moduli between the upper and lower adherends were not included in these analyses either. Thus, these models only differed from the previous models by
significantly increasing the stiffness of the adherends. The shape of the normal and shear strain responses in Figures 4.25 and 4.26 are identical to those predicted using the AS4/3501-6 adherends and Araldite 2015 adhesive. The \([\pm 60^\circ, 0^\circ]_4s\) layup model again provided a strain response that differed from the other two layup models across the entire joint overlap while the difference in shear and normal strain responses between the \([0^\circ, \pm 45^\circ, 90^\circ]_3s\) and \([\pm 30^\circ, 0^\circ]_4s\) layup models was even closer than in the previous comparison. An important difference between the models with the P75/954-3 adherends and the previous comparison using AS4/3501-6 adherends was the value of the maximum strains. Approximately 30\% lower shear and normal strains were determined with the present models compared to the models using AS4/3501-6 adherends. The reduced value of the shear and normal strains with the present models also suggest it is further from failure based on the stress-strain response in Figure 4.20. The maximum shear strain values for the \([0^\circ, \pm 45^\circ, 90^\circ]_3s\) and \([\pm 30^\circ, 0^\circ]_4s\) layup models were identical while the maximum normal strain values were even closer for the present models than the previous models with AS4/3501-6 adherends. A difference between the maximum strains for the \([\pm 60^\circ, 0^\circ]_4s\) layup model and the \([0^\circ, \pm 45^\circ, 90^\circ]_3s\) and \([\pm 30^\circ, 0^\circ]_4s\) layup models was again found with the P75/954-3 adherend models, but to a lesser degree than then the models using AS4/3501-6 adherends.

Results for the shear and normal stress responses of the P75/954-3 adherend with Araldite 2015 adhesive models are given in Figures 4.27 and 4.28. The basic shape and characteristics are, as in the strain results, similar to the stress responses from the previous models using AS4/3501-6 adherends. However, two important distinctions were noted. The first notable difference occurred for the shear stress responses displayed in the yielded regions of Figure 4.27. Due to the increase in adherend stiffness from the P75/954-3 material, the length of the plastic zone was reduced by approximately 50\%. A much closer correlation between the three plotted responses was also determined for the analyses of the P75/954-3 adherend models. Additionally, a much sharper discontinuity between the elastic and plastic regions was provided for the present analyses, which suggests that additional refinement of the model in the area of the interface may be needed. The second difference involved the shape of the normal stress response for the \([\pm 60^\circ, 0^\circ]_4s\) layup
model. The $[\pm 60^\circ, 0^\circ]_{4s}$ layup model displayed a W-shaped normal stress response in the previous comparison using the AS4/3501-6 adherends; however, in Figure 4.28 the shape is more U-shaped with no peaking at the center of the joint overlap. This type of a response suggests a more uniform bending of the joint overlap section as opposed to large bending rotations at only the ends of the joint overlap. This behavior is indicative of joints possessing high-stiffness adherends.

Observations from the strain and stress results for the models with P75/954-3 adherends and Araldite 2015 adhesive are provided to reinforce trends from the previous model while suggesting an additional relationship. First, an insignificant difference between strain responses for the $[0^\circ, \pm 45^\circ, 90^\circ]_{3s}$ and $[\pm 30^\circ, 0^\circ]_{4s}$ layup models was demonstrated. This result reinforced the idea that significantly increasing both the bending and axial stiffness of the adherends by changing layups had a minimal effect on the resulting strain and stress responses. The P75/954-3 adherend models further demonstrated the insensitivity of the normal stress response to changes in the axial and bending stiffness of the adherends due to different laminate stacking sequences. Finally, one additional observation made based on results from the models with P75/954-3 adherends and an Araldite 2015 adhesive was that a large increase in the adherend stiffness due to a material change reduced the sensitivity of the adhesive strain and stress responses to changes in the laminate stacking sequences for the adherends.

The results shown in Figures 4.29 through 4.32 provide the adhesive shear and strain responses plotted versus the nondimensionalized joint length for models using the high modulus FM 300 adhesive and the standard modulus AS4/3501-6 composite material as the constituent materials for the joint. Additionally, differences in the transverse shear moduli between the upper and lower adherends were not included in these analyses. The basic shape and characteristics for the strain and stress responses were the same as the two previous groups of models discussed. The shape of the shear strain responses in Figure 4.29 are more U-shaped than the two previous groups of models, which suggests the center section of the joint overlap was lightly-loaded. Additionally, the normal strain responses in Figure 4.30 possess a notable W-shape indicating significant bending localized near the ends of the joint overlap. The $[\pm 60^\circ, 0^\circ]_{4s}$ layup model again provided a strain response
that differed from the other two layup models except the difference was confined to the regions near the ends of the joints. Almost no difference in the shear and normal strain responses between the $[0^\circ, \pm 45^\circ, 90^\circ]_{3s}$ and $[\pm 30^\circ, 0^\circ]_{4s}$ layup models was demonstrated, similar to the models using the P75/954-3 adherends and Araldite 2015 adhesive. A reduction in the maximum strain values for the present models was also obtained over the two previous groups of models. However, the previous groups of models had joints bonded with the Araldite 2015 adhesive while the adherends in the present models used the FM300 adhesive. Thus, the increased stiffness of the FM300 adhesive limited the maximum strains in both the normal and shear responses. Although the maximum and overall strain responses were reduced using the FM300 adhesive, the maximum strains in the adhesive were still dangerously close to the failure strains of the uniaxial tension tests plotted in Figure 4.20 for this adhesive. Therefore, it was surmised that the adhesive in the joint overlap using the AS4/3501-6 adherends and FM300 adhesive combination was highly loaded similar to the models containing AS4/3501-6 adherends and an Araldite 2015 adhesive layer.

Results for the adhesive shear and normal stress responses of the models using the AS4/3501-6 adherends and FM300 adhesive combination are provided in Figures 4.31 and 4.32. The basic shape and characteristics differed from the stress responses from the previous models using the Araldite 2015 adhesive. The maximum shear and normal stress values were much higher for the responses of the AS4/3501-6 adherends and FM300 adhesive models than the previous models using the Araldite 2015 adhesive. Additionally, a short yield region was present in the shear stress responses even though the maximum strain responses previously discussed indicated a highly-loaded joint. Differences between stress responses for any of the three joint layup types was very small for the present joint models with the $[\pm 60^\circ, 0^\circ]_{4s}$ layup model providing a slightly higher maximum shear and normal stress than the other two layup types. The adhesive normal stress results in Figure 4.32 resulted in the most pronounced W-shaped response of all the previous results, and indicated that these joint models probably exhibited the most joint rotation near the ends of the overlap.
A brief discussion of the trends determined from the strain and stress responses in Figures 4.29 through 4.32 served to again reinforce most of the previous developments. A moderate increase in the bending stiffness of the adherends due to changes in the laminate stacking sequence provided moderate reductions in the strains with small reductions in the stress values. Increasing the stiffness of the joint, in this case by using a high-modulus adhesive, tended to reduce the effectiveness of making changes to the bending and axial stiffness of the adherends in order to reduce the maximum strains and stresses. Finally, although small differences in the normal stress responses were demonstrated between the three different layup types in Figure 4.32, the normal stress responses still appeared to be relatively insensitive to changes in the adherend axial and bending stiffness as a result of modifying the laminate stacking sequence.

Figures 4.33 through 4.36 provide the results for the joint analyses that used the high modulus FM 300 adhesive and the high modulus P75/954-3 composite material as the constituent materials for the joint. The differences in the transverse shear moduli between the upper and lower adherends were not included in these analyses. The basic shape and characteristics of the strain and stress responses obtained from the present models using the P75/954-3 adherends and FM300 adhesive parameters were very similar to the previous results from the models with AS4/3501-6 adherends and an FM300 adhesive layer. Due to the obvious similarity between the adhesive responses for the present models and the adhesive responses for the models using the AS4/3501-6 adherends with an FM300 adhesive layer, only a few main points are presented. The maximum strain values shown in Figures 4.33 and 4.34 were the lowest of all the previous models evaluated, which suggests that this adherend and adhesive combination resulted in the stiffest overall joint. The \([±60°, 0°]_{4s}\) layup model again provided a strain response that differed from the other two layup models. However, the magnitude of the differences between the strain responses for each layup type in Figures 4.33 and 4.34 were some of the smallest overall. This observation appears to strengthen the trend that an increase in the overall joint stiffness reduces the effectiveness of changes in the axial and bending stiffness of the adherends due to changes in the laminate layup. One last observation from Figure 4.36 was that the disparity in the adhesive normal stress responses for each layup was greater.
for the analyses of the present models than any of the other previously models analyzed. This concludes the description of results from ABJO analyses without the effect of changing the transverse shear stiffness between the upper and lower adherends.

The results presented in the remainder of this section were obtained from ABJO analyses of models with different transverse shear moduli in the upper and lower adherends. Specifically, a transverse shear modulus value from Table 4.6 was used for the upper adherend, depending on the type of adhesive in the joint model, while the value of the transverse shear modulus used in the lower adherend was 30% greater than the value of the upper adherend. The decision to increase the transverse shear stiffness by 30% was made by selecting a value that was close to a technologically attainable upper bound. Twelve analyses of joint models using the parameters from Table 4.5 were performed using the following basic joint geometry: 0.12-inch thick adherends, a 0.005-inch thick adhesive layer, and a one-half inch joint overlap. The shear and normal stress responses from four of the analyses are given in Figures 4.37 and 4.38, respectively. The results given by the solid line and circle symbol refer to a model using the Araldite 2015 adhesive with no transverse shear moduli difference between the upper and lower adherends while the results denoted by the long-dashed line and square symbol refer to a model using the same adhesive and a 30% difference between the transverse shear moduli. Similarly, the results given by the dot-dashed line and diamond symbol refer to a model using the FM300 adhesive with no transverse shear moduli difference between the upper and lower adherends while the results denoted by the short-dashed line and triangle symbol refer to a model using the same adhesive and a 30% difference between the transverse shear moduli.

The results in Figure 4.37 and 4.38 were typical of each of the twelve analyses conducted. The results for both the adhesive shear and normal stress responses from the analyses with no effects due to changes in the transverse shear modulus between the adherends provided very little difference from the responses for the analyses that included the 30% difference between the transverse shear moduli. The largest difference between the responses with and without the effects of a 30% change in the transverse shear moduli
were obtained for the joint configuration shown in Figure 4.39. A close evaluation of the responses in Figure 4.39 provided an almost indistinguishable difference between the two solutions for each adhesive; with the largest difference of approximately 1% occurring between the responses determined for the model using the FM300 adhesive. The slight differences between the two solutions for the FM300 adhesive occurred in regions located to either side of the joint overlap’s center as well as in the yielded regions at the ends of the joint overlap. There were no distinguishable differences between the two responses for the Araldite 2015 adhesive in Figure 4.39. Finally, the results from the analyses of the other models used to evaluate the effects of a 30% change in the transverse shear stiffness were not presented here based on the previously discussed findings and the similarity of the remaining analyses to the responses shown in Figures 4.37 through 4.39.

The results of the models investigating the effects of shear deformation, thus far, have demonstrated a minimal benefit on the adhesive strain/stress responses by including this additional adherend behavior. Since more of an effect due to shear deformation in these analyses was expected, a few additional models were evaluated. The first model evaluated used the same configuration as in Figure 4.39 with the FM300 adhesive except the 30% difference between the transverse shear moduli was increased to 60%. Although possibly beyond a technologically attainable value, this was certainly felt to be an upper limit for the transverse shear stiffness. The shear stress results of the two ABJO analyses performed on the models with and without the 60% difference in the transverse shear moduli are presented in Figure 4.40. Once again, the shear stress results for these models demonstrated a minimal effect when using adherends with different transverse shear moduli. Even with the difference between the transverse shear stiffness doubled over the previous efforts, the largest difference demonstrated between the responses was approximately 3%. However, a more noticeable shift of the shear stress response was evident for the model that was affected by a change in transverse shear modulus. In summary, the results from the analyses provided in Figure 4.40 suggest that a 60% difference in the transverse shear stiffness between the upper and lower adherends did not provide a significant difference in the shear stress responses.
The results for the next set of analyses were intended to evaluate a joint model that had undergone extensive yielding and used relatively compliant adherends. The purpose of these evaluations was to investigate the effect that load level along with changes to the transverse shear stiffness of the upper and lower adherends had on the adhesive stress responses. Results from analyses performed using the joint parameters from Table 4.1 and 4.2 for the joint configuration by Bigwood and Crocombe were obtained for a joint model that had 0.12-inch thick adherends, a 0.005-inch thick adhesive layer, and a one-half inch joint overlap. The shear stress responses are given in Figure 4.41 with the results from the baseline model displayed using a solid line and circle symbol and the results from the modified adherend model displayed using a dashed line and square symbol. The baseline model had upper and lower adherends with identical transverse shear moduli while the modified model had a lower adherend with a transverse shear modulus that was 30% higher than the upper adherend. A noticeable difference between the shear strain responses is readily apparent in Figure 4.41. As the load transfers from the upper adherend to the lower adherend on the left end of the joint overlap, the stiffer, lower adherend forces additional adhesive shearing near the end, but decreases more rapidly compared to the baseline joint as the solution moves inboard. A reverse of this trend is seen on the right end of the joint overlap as the maximum shear stress is reduced for the modified joint. The maximum shear stress on the right end of the joint overlap was approximately 45 N/mm² and occurred at about the 0.8 joint station while the maximum shear stress on the left end of the joint was approximately 41 N/mm² and occurred near the 0.2 joint station. This was approximately a 9% difference in the maximum shear stress values for the modified model from one end of the joint overlap to the other. Additionally, a 7% decrease in the maximum shear stress was demonstrated by the modified model compared to the baseline model on the left end of the joint overlap while a 2.5% increase in the maximum shear stress was determined for the modified model compared to the baseline model on the right end of the joint overlap. Thus, by modifying only the transverse shear stiffness between the upper and lower adherends a drop in the shear stress was obtained at one end of the joint overlap while incurring a slight increase at the opposite end. Therefore, the results from the ABJO analyses of the Bigwood and Crocombe models suggest a change of 30%
between the transverse shear moduli of the upper and lower adherends produced a moderate effect on the shear stress response. Although the normal stress responses for the Bigwood and Crocombe model were not shown, an indistinguishable difference was obtained between the two responses.

The results for the final model evaluated are shown in Figure 4.42. This model was the same joint configuration as an earlier model used to obtain results presented in Figure 4.37 for the FM300 adhesive. As previously discussed, the model had 0.12-inch thick adherends, a 0.005-inch thick adhesive layer, and a one-half inch joint overlap in addition to using the FM300 adhesive and a layup. These analyses were conducted to further determine the effect of load level on the adhesive response using one of the joint configurations from the parametric study.

The results given by the solid line and circle symbol refer to a 750 lbf uniaxial tension load with no transverse shear moduli difference between the upper and lower adherends while the results denoted by the long-dashed line and square symbol refer to a model using the same loading and a 30% difference between the transverse shear moduli. Similarly, the results given by the dot-dashed line and diamond symbol refer to a model using a 950 lbf uniaxial tension load with no transverse shear moduli difference between the upper and lower adherends while the results denoted by the short-dashed line and triangle symbol refer to a model using the same loading and a 30% difference between the transverse shear moduli. The shear stress responses in Figure 4.42 demonstrated very little difference between the solutions with and without changes in the transverse shear stiffness between adherends. Therefore, these results support the previously discussed results in Figures 4.37 through 4.40 that changes in transverse shear stiffness appear to have little effect on the adhesive shear and normal stress response.
CHAPTER 5.0

CONCLUDING REMARKS

5.1 Conclusions

A new method for evaluating general bonded joint overlaps with anisotropic, shear-deformable adherends and inelastic adhesive behavior that are subjected to combined tensile, shear, and bending moment loading was presented. Rapid solution of a wide variety of joint configurations is possible with the Fortran code ABJO, which was developed from the proposed theory using a finite-difference solution algorithm. Therefore, an analysis tool has been developed that is capable of tailoring the stiffness of composite adherends in an adhesively bonded joint. Additionally, efficient joint designs were obtained by allowing adhesive yielding that can lead to significantly decreased maximum shear and normal stresses in the adhesive.

A number of joint analyses were conducted using the anisotropic bonded joint optimizer to verify and demonstrate the joint tailoring capabilities of the code. The results of the proposed method were verified using three single-lap joint configurations from the literature, two with isotropic and one with orthotropic adherends. Excellent correlation was obtained for the three verification cases, which were selected to demonstrate both the inelastic and anisotropic capabilities of the method. Further analyses were conducted to demonstrate the joint tailoring capabilities of the method using a single-lap joint geometry. Design studies were performed to evaluate details of the joint geometry and the effects of certain material parameters on the adhesive strain and stress responses. Specifically, an additional case was presented that showed the effect of varying the transverse shear stiffness between the upper and lower adherends for quasi-isotropic and orthotropic laminate stacking sequences.

The results of the design studies that were conducted using the anisotropic bonded joint algorithm produced several interesting observations. First, there appeared to be little if any effect of shear deformation on all but one of the joint configurations evaluated
during the design studies. Second, a nonlinear relationship between changes in the bending and axial stiffness of the adherends through modifications to the laminate layup and a reduction in the inelastic adhesive strain and shear stress fields was determined. Specifically, a moderate increase in the adherend bending stiffness of approximately 45% through modifications to the laminate layup provided moderate reductions of the inelastic strain responses with a smaller reduction to the inelastic shear stress responses. However, further combined increases of 50% to the bending and 100% to the axial stiffness of the adherends through a change in laminate stacking sequence provided minimal to no additional reduction of the resulting inelastic strain and stress responses. Additionally, any amount of increase in the axial and bending stiffness of the adherends through a change in laminate stacking sequence produced minimal to no effect on the normal stress responses for all the models evaluated. Therefore, in summary, modeling the inelastic behavior of adhesives and anisotropic adherend stiffness changes in structural joints made of non-metallic materials is important for designing efficient joints since important joint tailoring effects were captured using this newly developed method.

5.2 Recommendations for Future Research

A list of recommended areas that may provide additional features or refinements to the anisotropic bonded joint analysis method is provided below. Further development of the present analysis method is encouraged in the following areas:

1. Extension of the inelastic method to two dimensions for the purpose of: a) Investigating the effects of anisotropic laminate behavior on the adhesive response via a plane stress modeling formulation or b) Investigating the details of the adhesive response through the thickness of the joint overlap via a plane strain modeling formulation

2. Incorporate a higher-order adhesive model into the present method that would provide a more detailed adhesive response

3. Replace the current yield model with one that includes incremental plastic strain to study the effects of cyclic loading and strain hardening on the adhesive layer
4. Extend the kinematic equations to include the effects of geometric nonlinear joint behavior

5. Combine the present analysis method with an optimization technique to automate the joint design process
REFERENCES


APPENDIX A

Table 4.1  Geometry of the single-lap joint models used to compare ABJO with the analysis methods by Goland and Reissner and Bigwood and Crocombe.

<table>
<thead>
<tr>
<th></th>
<th>Upper adherend thickness, $h_U$ (mm)</th>
<th>Lower adherend thickness, $h_L$ (mm)</th>
<th>Adhesive thickness, $t$ (mm)</th>
<th>Overlap length, $C$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.0</td>
<td>2.0</td>
<td>0.05</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Table 4.2  Material properties for the components of the single-lap joint models used to compare ABJO with the analysis methods by Goland and Reissner and Bigwood and Crocombe.

<table>
<thead>
<tr>
<th>Lamina Property&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Quasi-isotropic Model</th>
<th></th>
<th>Isotropic Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper Adherend</td>
<td>Lower Adherend</td>
<td>Adhesive</td>
</tr>
<tr>
<td>$E_{11}$, (N/mm&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>162,000</td>
<td>162,000</td>
<td>1,875</td>
</tr>
<tr>
<td>$E_{22}$, (N/mm&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>19,300</td>
<td>19,300</td>
<td>1,875</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.31</td>
<td>0.31</td>
<td>0.4</td>
</tr>
<tr>
<td>$G_{12}$, (N/mm&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>11,000</td>
<td>11,000</td>
<td>-</td>
</tr>
<tr>
<td>Laminate stacking sequence (degrees)</td>
<td>[0,45,-45,90]&lt;sub&gt;s&lt;/sub&gt;</td>
<td>[0,45,-45,90]&lt;sub&gt;s&lt;/sub&gt;</td>
<td>-</td>
</tr>
<tr>
<td>Yield stress, (N/mm&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>-</td>
<td>-</td>
<td>40.0</td>
</tr>
</tbody>
</table>

<sup>a</sup>Subscripts 1 and 2 denote the longitudinal (fiber) and transverse (matrix) directions of an anisotropic lamina, respectively.
Table 4.3  Geometry of the single-lap joint model used to compare ABJO with the analysis method by Yang.

<table>
<thead>
<tr>
<th></th>
<th>Upper adherend thickness, $h_u$ (mm)</th>
<th>Lower adherend thickness, $h_l$ (mm)</th>
<th>Adhesive thickness, $t$ (mm)</th>
<th>Overlap length, $C$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper adherend thickness</td>
<td>3.0</td>
<td>3.0</td>
<td>0.1</td>
<td>50.0</td>
</tr>
</tbody>
</table>

Table 4.4  Material properties for the components of the single-lap joint model used to compare ABJO with the analysis method by Yang.

<table>
<thead>
<tr>
<th>Lamina Property$^a$</th>
<th>Upper Adherend</th>
<th>Lower Adherend</th>
<th>Adhesive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$, (N/mm²)</td>
<td>181,000</td>
<td>181,000</td>
<td>960</td>
</tr>
<tr>
<td>$E_{22}$, (N/mm²)</td>
<td>10,300</td>
<td>10,300</td>
<td>960</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.28</td>
<td>0.28</td>
<td>0.4</td>
</tr>
<tr>
<td>$G_{12}$, (N/mm²)</td>
<td>7,170</td>
<td>7,170</td>
<td>-</td>
</tr>
<tr>
<td>Laminate stacking sequence (degrees)</td>
<td>$[0,90_2,0,90,0]_s$</td>
<td>$[0,90_2,0,90,0]_s$</td>
<td>-</td>
</tr>
</tbody>
</table>

$^a$Subscripts 1 and 2 denote the longitudinal (fiber) and transverse (matrix) directions of an anisotropic lamina, respectively.
Table 4.5  Adhesively bonded joint parameters used in the ABJO analyses for the parametric design study.

<table>
<thead>
<tr>
<th>Laminate Stacking Sequence</th>
<th>Adherend Material Type</th>
<th>Adhesive Material Type</th>
<th>Transverse Shear Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>[±60,0]₄ₛ</td>
<td>AS4/3501-6</td>
<td>FM 300</td>
<td>No change between adherends</td>
</tr>
<tr>
<td>[0,±45,90]₃ₛ</td>
<td>P75/954-3</td>
<td>Araldite 2015</td>
<td>Lower adherend increased by 30%</td>
</tr>
<tr>
<td>[±30,0]₄ₛ</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Total number of design study models was 24.

Table 4.6  Lamina material properties for the adherends used in the ABJO analyses for the parametric design study.

<table>
<thead>
<tr>
<th>Lamina Propertyᵃ</th>
<th>AS4/3501-6 (GPa)</th>
<th>P75/954-3 (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$, Msi</td>
<td>20.5 (141)</td>
<td>44.0 (303)</td>
</tr>
<tr>
<td>$E_{22}$, Msi</td>
<td>1.4 (9.6)</td>
<td>0.85 (5.9)</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>.29</td>
<td>0.3</td>
</tr>
<tr>
<td>$G_{12}$, Msi</td>
<td>0.71 (4.9)</td>
<td>0.62 (4.3)</td>
</tr>
<tr>
<td>$G_{13}$, Msi</td>
<td>0.502 (3.5)</td>
<td>0.31 (2.1)</td>
</tr>
</tbody>
</table>

ᵃSubscripts 1 and 2 denote the longitudinal (fiber) and transverse (matrix) directions of an anisotropic lamina, respectively. Subscript 3 is perpendicular to the plane of the lamina.
Table 4.7  Material properties for the adhesives used in the ABJO analyses for the parametric design study.

<table>
<thead>
<tr>
<th></th>
<th>FM 300</th>
<th>Araldite 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$, Msi (GPa)</td>
<td>0.420 (2.9)</td>
<td>0.181 (1.25)</td>
</tr>
<tr>
<td>$v$</td>
<td>0.39</td>
<td>0.41</td>
</tr>
<tr>
<td>$\sigma_{\text{yield}}$, Ksi (MPa)</td>
<td>6.0 (41)</td>
<td>3.0 (21)</td>
</tr>
<tr>
<td>$\sigma_{\text{asym}}$, Ksi (MPa)</td>
<td>9.0 (62)</td>
<td>4.8 (33)</td>
</tr>
</tbody>
</table>
APPENDIX B

a) A general bonded joint with a joint overlap

b) Differential element of the joint overlap

Figure 2.1  Bonded joint configuration and element loading.
Figure 2.2 Loading equilibrium for a general section of the bonded joint overlap.

Figure 3.1 Basic geometry and nomenclature for the bonded single-lap joint models.

Figure 3.2 Uniform axial tension loading of a single-lap joint.
Figure 3.3  Description of grid point locations on a generic bonded joint model.

Figure 3.4  Flowchart describing the execution of the anisotropic bonded joint optimizer (ABJO).
Figure 4.1 Correlation of the stress distributions from a linear elastic ABJO analysis of a single-lap joint subjected to axial tension to results using the analysis method by Goland and Reissner [21].
Figure 4.2 Correlation of the strain and stress distributions from an inelastic ABJO analysis of a single-lap joint subjected to axial tension to results by Bigwood and Crocombe [9].
Figure 4.3 Correlation of the shear and normal stress distributions from an elastic ABJO analysis of a single-lap joint in pure bending to results by Yang [65].
Figure 4.4 Shear strain distribution as a function of joint overlap for Araldite 2015 in a single-lap joint with \([±60°, 0°]_{as}\) adherends of AS4/3501-6 subjected to a 750 lbf. axial tension.

Figure 4.5 Normal strain distribution as a function of joint overlap for Araldite 2015 in a single-lap joint with \([±60°, 0°]_{as}\) adherends of AS4/3501-6 subjected to a 750 lbf. axial tension.
Figure 4.6 Shear stress distribution as a function of joint overlap for Araldite 2015 in a single-lap joint with $[\pm 60^\circ, 0^\circ]_6$ adherends of AS4/3501-6 subjected to a 750 lbf. axial tension.

Figure 4.7 Normal stress distribution as a function of joint overlap for Araldite 2015 in a single-lap joint with $[\pm 60^\circ, 0^\circ]_6$ adherends of AS4/3501-6 subjected to a 750 lbf. axial tension.
Figure 4.8 Convergence of the Araldite 2015 strain in a single lap joint as a function of the nondimensionalized overlap length, analysis cases using twenty and thirty grid points.

Figure 4.9 Convergence of the Araldite 2015 strain in a single lap joint as a function of the nondimensionalized overlap length, analysis cases using thirty and fifty grid points.
Figure 4.10 Convergence of the Araldite 2015 stress in a single lap joint as a function of the nondimensionlized overlap length, analysis cases using twenty and thirty grid points.

Figure 4.11 Convergence of the Araldite 2015 stress in a single lap joint as a function of the nondimensionlized overlap length, analysis cases using thirty and fifty grid points.
Figure 4.12 Convergence of the Araldite 2015 peak shear strain in a single lap joint as a function of the nondimensionalized overlap length, analysis cases using twenty and thirty grid points.

Figure 4.13 Convergence of the Araldite 2015 peak shear strain in a single lap joint as a function of the nondimensionalized overlap length, analysis cases using thirty and fifty grid points.
Figure 4.14 Convergence of the Araldite 2015 peak shear stress in a single lap joint as a function of the nondimensionalized overlap length, analysis cases using twenty and thirty grid points.

Figure 4.15 Convergence of the Araldite 2015 peak shear stress in a single lap joint as a function of the nondimensionalized overlap length, analysis cases using thirty and fifty grid points.
Figure 4.16 Shear strain distribution as a function of adherend thickness and adhesive type for a single-lap joint with [±60°, 0°]_4s adherends of P75/954-3 subjected to a 750 lbf. axial tension.

Figure 4.17 Normal strain distribution as a function of adherend thickness and adhesive type for a single-lap joint with [±60°, 0°]_4s adherends of P75/954-3 subjected to a 750 lbf. axial tension.
Figure 4.18 Shear stress distribution as a function of adherend thickness and adhesive type for a single-lap joint with $[\pm 60^\circ, 0^\circ]_4$ adherends of P75/954-3 subjected to a 750 lbf. axial tension.

Figure 4.19 Normal stress distribution as a function of adherend thickness and adhesive type for a single-lap joint with $[\pm 60^\circ, 0^\circ]_4$ adherends of P75/954-3 subjected to a 750 lbf. axial tension.
Figure 4.20 Stress-strain responses for the adhesives used in the parametric design study based on the hyperbolic-tangent material model.

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Figure 4.22 Araldite 2015 normal strain distribution as a function of laminate stacking sequence for a single-lap joint with AS4/3501-6 adherends subjected to a 750 lbf. axial tension.

Figure 4.23 Araldite 2015 shear stress distribution as a function of laminate stacking sequence for a single-lap joint with AS4/3501-6 adherends subjected to a 750 lbf. axial tension.
Figure 4.24 Araldite 2015 normal stress distribution as a function of laminate stacking sequence for a single-lap joint with AS4/3501-6 adherends subjected to a 750 lbf. axial tension.

Figure 4.25 Araldite 2015 shear strain distribution as a function of laminate stacking sequence for a single-lap joint with P75/954-3 adherends subjected to a 750 lbf. axial tension.
Figure 4.26 Araldite 2015 normal strain distribution as a function of laminate stacking sequence for a single-lap joint with P75/954-3 adherends subjected to a 750 lbf. axial tension.

Figure 4.27 Araldite 2015 shear stress distribution as a function of laminate stacking sequence for a single-lap joint with P75/954-3 adherends subjected to a 750 lbf. axial tension.
Figure 4.28 Araldite 2015 normal stress distribution as a function of laminate stacking sequence for a single-lap joint with P75/954-3 adherends subjected to a 750 lbf. axial tension.

Figure 4.29 FM 300 shear strain distribution as a function of laminate stacking sequence for a single-lap joint with AS4/3501-6 adherends subjected to a 750 lbf. axial tension.
Figure 4.30 FM 300 normal strain distribution as a function of laminate stacking sequence for a single-lap joint with AS4/3501-6 adherends subjected to a 750 lbf. axial tension.

Figure 4.31 FM 300 shear stress distribution as a function of laminate stacking sequence for a single-lap joint with AS4/3501-6 adherends subjected to a 750 lbf. axial tension.
Figure 4.32 FM300 normal stress distribution as a function of laminate stacking sequence for a single-lap joint with AS4/3501-6 adherends subjected to a 750 lbf. axial tension.

Figure 4.33 FM300 shear strain distribution as a function of laminate stacking sequence for a single-lap joint with P75/954-3 adherends subjected to a 750 lbf. axial tension.
Figure 4.34 FM300 normal strain distribution as a function of laminate stacking sequence for a single-lap joint with P75/954-3 adherends subjected to a 750 lbf. axial tension.

Figure 4.35 FM300 shear stress distribution as a function of laminate stacking sequence for a single-lap joint with P75/954-3 adherends subjected to a 750 lbf. axial tension.
Figure 4.36 FM300 normal stress distribution as a function of laminate stacking sequence for a single-lap joint with P75/954-3 adherends subjected to a 750 lbf. axial tension.

Figure 4.37 Effect of a 30% difference in the transverse shear modulus between the upper and lower adherends, $[\pm 60^\circ, 0^\circ]_4$, laminates of AS4/3501-6, on the shear stress distribution of a single-lap joint subjected to a 750 lbf. axial tension.
Figure 4.38 Effect of a 30% difference in the transverse shear modulus between the upper and lower adherends, $[\pm 60^\circ, 0^\circ]_s$ laminates of AS4/3501-6, on the normal stress distribution of a single-lap joint subjected to a 750 lbf. axial tension.

Figure 4.39 Effect of a 30% difference in the transverse shear modulus between the upper and lower adherends, $[\pm 30^\circ, 0^\circ]_s$ laminates of P75/954-3, on the shear stress distribution of a single-lap joint subjected to a 750 lbf. axial tension.
Figure 4.40 Effect of a 60% difference in the transverse shear modulus between the upper and lower adherends, [±30°, 0°], laminates of P75/954-3, on the normal stress distribution of a single-lap joint subjected to a 750 lbf. axial tension.

Figure 4.41 Effect of a 30% difference in the transverse shear modulus between the upper and lower adherends on the shear stress distribution of the single-lap joint from the results by Bigwood & Crocombe [9].
Figure 4.42 Effect of a 30% difference in the transverse shear modulus between the upper and lower adherends, $[\pm 60^\circ, 0^\circ]_4$, laminates of AS4/3501-6, on the shear stress distribution of FM 300 as a function of load level.
APPENDIX C

Design Study Case: 3HMAR0, Single-Lap Joint in Tension

* Inelastic Material Property Flag: T for inelastic adhesive behavior, F for elastic adhesive behavior

* T

* Number of plies for upper adherend

* 24

| 44.0e06,0.85e06,0.3,0.62e06,0.005, 30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, -30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 0.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, -30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 0.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, -30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 0.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, -30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 0.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, -30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 0.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, -30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 0.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, -30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 0.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, -30.0 |
| 44.0e06,0.85e06,0.3,0.62e06,0.005, 0.0 |
* Upper adherend transverse shear modulus
0.31e06
*

* Number of plies for lower adherend
*
24

<table>
<thead>
<tr>
<th>Density</th>
<th>Modulus</th>
<th>Angle</th>
</tr>
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<tbody>
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<td>0.85e06</td>
<td>0.3, 0.62e06, 0.005, 30.0</td>
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<tr>
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<td>0.85e06</td>
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<td>0.3, 0.62e06, 0.005, 0.0</td>
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<td>0.3, 0.62e06, 0.005, -30.0</td>
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<td>44.0e06</td>
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<td>0.3, 0.62e06, 0.005, 0.0</td>
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<td>44.0e06</td>
<td>0.85e06</td>
<td>0.3, 0.62e06, 0.005, 30.0</td>
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</tr>
<tr>
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<td>0.85e06</td>
<td>0.3, 0.62e06, 0.005, -30.0</td>
</tr>
</tbody>
</table>

* Lower adherend transverse shear modulus
0.31e06
*

* Additional Joint Material Properties:
*
* Upper Adherend Density
0.058
  *  Upper Adherend Tensile Stress Allowable
60.0E03
  *  Lower Adherend Density
0.058
  *  Lower Adherend Tensile Stress Allowable
60.0E03
  *  Adhesive Secant Modulus of Elasticity
0.181E06
  *  Adhesive Plastic Poisson's Ratio
0.410000
  *  Adhesive Density
0.065
  *  Adhesive Shear Allowable
3000.0
  *  Adhesive Normal Stress Allowable
1200.0
  *
  *  Joint Geometry:
  *
  *    Adhesive Thickness
0.0050000
  *  Joint Length
0.5000
  *
  *  Joint Loading:
  *
  *    Moment Resultant on left side of upper adherend (MOU)
-43.38
  *    Moment Resultant on left side of lower adherend (MOL)
0.0
  *    Shear Resultant on left side of upper adherend (VOU)
-13.99
  *    Shear Resultant on left side of lower adherend (VOL)
0.0
  *    Stress Resulant on left side of upper adherend (NOU)
* Stress Resultant on left side of lower adherend (NOL)

* Select units for input/output (must be a real):
  * 1. - MKS
  * 2. - English

* Choose printer setting for information on the boundary value routine
  * 1. - No convergence information
  * 2. - Print convergence information

1.
APPENDIX D

Design Study Case: 3HMAR0, Single-Lap Joint in Tension

UPPER ADHEREND MATERIAL PROPERTIES

Number of plies: N = 24

Lamina properties for ply 1:
- $E_1 = 440.0 \times 10^5$ psi
- $E_2 = 85.0 \times 10^4$ psi
- $G_{12} = 62.0 \times 10^4$ psi
- $v_{12} = 0.30$
- $t = 0.0050$ in
- Ply angle = 30.0 degrees

Lamina properties for ply 2:
- $E_1 = 440.0 \times 10^5$ psi
- $E_2 = 85.0 \times 10^4$ psi
- $G_{12} = 62.0 \times 10^4$ psi
- $v_{12} = 0.30$
- $t = 0.0050$ in
- Ply angle = -30.0 degrees

Lamina properties for ply 3:
- $E_1 = 440.0 \times 10^5$ psi
- $E_2 = 85.0 \times 10^4$ psi
- $G_{12} = 62.0 \times 10^4$ psi
- $v_{12} = 0.30$
- $t = 0.0050$ in
- Ply angle = 0.0 degrees

Lamina properties for ply 4:
- $E_1 = 440.0 \times 10^5$ psi
- $E_2 = 85.0 \times 10^4$ psi
- $G_{12} = 62.0 \times 10^4$ psi
- $v_{12} = 0.30$
- $t = 0.0050$ in
- Ply angle = 30.0 degrees

Lamina properties for ply 5:
- $E_1 = 440.0 \times 10^5$ psi
- $E_2 = 85.0 \times 10^4$ psi
- $G_{12} = 62.0 \times 10^4$ psi
- $v_{12} = 0.30$
- $t = 0.0050$ in
- Ply angle = -30.0 degrees
Lamina properties for ply 6:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
   t = 0.0050 in         Ply angle =  0.00E+00 degrees

Lamina properties for ply 7:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
   t = 0.0050 in         Ply angle =  30.0E+00 degrees

Lamina properties for ply 8:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
   t = 0.0050 in         Ply angle = -30.0E+00 degrees

Lamina properties for ply 9:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
   t = 0.0050 in         Ply angle =  0.00E+00 degrees

Lamina properties for ply 10:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
   t = 0.0050 in         Ply angle =  30.0E+00 degrees

Lamina properties for ply 11:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
   t = 0.0050 in         Ply angle = -30.0E+00 degrees

Lamina properties for ply 12:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
   t = 0.0050 in         Ply angle =  0.00E+00 degrees
Lamina properties for ply 13:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
  t = 0.0050 in       Ply angle = 0.00E+00 degrees

Lamina properties for ply 14:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
  t = 0.0050 in       Ply angle = -30.0E+00 degrees

Lamina properties for ply 15:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
  t = 0.0050 in       Ply angle = 30.0E+00 degrees

Lamina properties for ply 16:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
  t = 0.0050 in       Ply angle = 0.00E+00 degrees

Lamina properties for ply 17:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
  t = 0.0050 in       Ply angle = -30.0E+00 degrees

Lamina properties for ply 18:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
  t = 0.0050 in       Ply angle = 30.0E+00 degrees

Lamina properties for ply 19:
E1 = 440.0E+05 psi  E2 = 85.0E+04 psi
G12 = 62.0E+04 psi  v12 = 0.30
  t = 0.0050 in       Ply angle = 0.00E+00 degrees
Lamina properties for ply 20:

\[
\begin{align*}
E_1 &= 440.0 \times 10^5 \text{ psi} & E_2 &= 85.0 \times 10^4 \text{ psi} \\
G_{12} &= 62.0 \times 10^4 \text{ psi} & v_{12} &= 0.30 \\
t &= 0.0050 \text{ in} & \text{Ply angle} &= -30.0^\circ 
\end{align*}
\]

Lamina properties for ply 21:

\[
\begin{align*}
E_1 &= 440.0 \times 10^5 \text{ psi} & E_2 &= 85.0 \times 10^4 \text{ psi} \\
G_{12} &= 62.0 \times 10^4 \text{ psi} & v_{12} &= 0.30 \\
t &= 0.0050 \text{ in} & \text{Ply angle} &= 30.0^\circ 
\end{align*}
\]

Lamina properties for ply 22:

\[
\begin{align*}
E_1 &= 440.0 \times 10^5 \text{ psi} & E_2 &= 85.0 \times 10^4 \text{ psi} \\
G_{12} &= 62.0 \times 10^4 \text{ psi} & v_{12} &= 0.30 \\
t &= 0.0050 \text{ in} & \text{Ply angle} &= 0.0^\circ 
\end{align*}
\]

Lamina properties for ply 23:

\[
\begin{align*}
E_1 &= 440.0 \times 10^5 \text{ psi} & E_2 &= 85.0 \times 10^4 \text{ psi} \\
G_{12} &= 62.0 \times 10^4 \text{ psi} & v_{12} &= 0.30 \\
t &= 0.0050 \text{ in} & \text{Ply angle} &= -30.0^\circ 
\end{align*}
\]

Lamina properties for ply 24:

\[
\begin{align*}
E_1 &= 440.0 \times 10^5 \text{ psi} & E_2 &= 85.0 \times 10^4 \text{ psi} \\
G_{12} &= 62.0 \times 10^4 \text{ psi} & v_{12} &= 0.30 \\
t &= 0.0050 \text{ in} & \text{Ply angle} &= 30.0^\circ 
\end{align*}
\]

Transverse Shear Modulus, \( G_{13} = 3.10 \times 10^5 \text{ psi} \)

Stacking sequence of plies:

\[
\begin{align*}
30.00 \\
-30.00 \\
0.00 \\
30.00 \\
-30.00 \\
0.00 \\
30.00
\end{align*}
\]
Laminate Stiffness Components:

A Matrix:

\[
\begin{bmatrix}
3.80E+06 & 6.60E+05 & -6.16E-04 \\
6.60E+05 & 3.38E+05 & -1.48E-04 \\
-6.16E-04 & -1.48E-04 & 7.03E+05
\end{bmatrix}
\]

B Matrix:

\[
\begin{bmatrix}
-7.05E-02 & -1.37E-02 & -9.01E-04 \\
-1.37E-02 & -5.71E-03 & -3.24E-04 \\
-9.01E-04 & -3.24E-04 & -1.32E-02
\end{bmatrix}
\]

D Matrix:

\[
\begin{bmatrix}
4.34E+03 & 8.83E+02 & 1.81E+02 \\
8.83E+02 & 4.39E+02 & 6.26E+01 \\
1.81E+02 & 6.26E+01 & 9.36E+02
\end{bmatrix}
\]

Density = \(5.800E-02 \text{ lbm/in}^3\)
Tensile Stress Allowable = 60.00E+03 psi

**LOWER ADHEREND MATERIAL PROPERTIES**

Number of plies: \( N = 24 \)

Lamina properties for ply 1:
- \( E_1 = 440.0 \times 10^5 \) psi
- \( E_2 = 85.0 \times 10^4 \) psi
- \( G_{12} = 62.0 \times 10^4 \) psi
- \( v_{12} = 0.30 \)
- \( t = 0.0050 \) in
- Ply angle = 30.0E+00 degrees

Lamina properties for ply 2:
- \( E_1 = 440.0 \times 10^5 \) psi
- \( E_2 = 85.0 \times 10^4 \) psi
- \( G_{12} = 62.0 \times 10^4 \) psi
- \( v_{12} = 0.30 \)
- \( t = 0.0050 \) in
- Ply angle = -30.0E+00 degrees

Lamina properties for ply 3:
- \( E_1 = 440.0 \times 10^5 \) psi
- \( E_2 = 85.0 \times 10^4 \) psi
- \( G_{12} = 62.0 \times 10^4 \) psi
- \( v_{12} = 0.30 \)
- \( t = 0.0050 \) in
- Ply angle = 0.00E+00 degrees

Lamina properties for ply 4:
- \( E_1 = 440.0 \times 10^5 \) psi
- \( E_2 = 85.0 \times 10^4 \) psi
- \( G_{12} = 62.0 \times 10^4 \) psi
- \( v_{12} = 0.30 \)
- \( t = 0.0050 \) in
- Ply angle = 30.0E+00 degrees

Lamina properties for ply 5:
- \( E_1 = 440.0 \times 10^5 \) psi
- \( E_2 = 85.0 \times 10^4 \) psi
- \( G_{12} = 62.0 \times 10^4 \) psi
- \( v_{12} = 0.30 \)
- \( t = 0.0050 \) in
- Ply angle = -30.0E+00 degrees

Lamina properties for ply 6:
- \( E_1 = 440.0 \times 10^5 \) psi
- \( E_2 = 85.0 \times 10^4 \) psi
- \( G_{12} = 62.0 \times 10^4 \) psi
- \( v_{12} = 0.30 \)
t = 0.0050 in       Ply angle = 0.00E+00 degrees

Lamina properties for ply 7:
    E1 = 440.E+05 psi       E2 = 85.0E+04 psi
    G12 = 62.0E+04 psi     v12 = 0.30
    t = 0.0050 in       Ply angle = 30.0E+00 degrees

Lamina properties for ply 8:
    E1 = 440.E+05 psi       E2 = 85.0E+04 psi
    G12 = 62.0E+04 psi     v12 = 0.30
    t = 0.0050 in       Ply angle = -30.0E+00 degrees

Lamina properties for ply 9:
    E1 = 440.E+05 psi       E2 = 85.0E+04 psi
    G12 = 62.0E+04 psi     v12 = 0.30
    t = 0.0050 in       Ply angle = 0.00E+00 degrees

Lamina properties for ply 10:
    E1 = 440.E+05 psi       E2 = 85.0E+04 psi
    G12 = 62.0E+04 psi     v12 = 0.30
    t = 0.0050 in       Ply angle = 30.0E+00 degrees

Lamina properties for ply 11:
    E1 = 440.E+05 psi       E2 = 85.0E+04 psi
    G12 = 62.0E+04 psi     v12 = 0.30
    t = 0.0050 in       Ply angle = -30.0E+00 degrees

Lamina properties for ply 12:
    E1 = 440.E+05 psi       E2 = 85.0E+04 psi
    G12 = 62.0E+04 psi     v12 = 0.30
    t = 0.0050 in       Ply angle = 0.00E+00 degrees

Lamina properties for ply 13:
    E1 = 440.E+05 psi       E2 = 85.0E+04 psi
    G12 = 62.0E+04 psi     v12 = 0.30
    t = 0.0050 in       Ply angle = 0.00E+00 degrees
Lamina properties for ply 14:
  $E_1 = 440.0 \times 10^5 \text{ psi}$  
  $E_2 = 85.0 \times 10^4 \text{ psi}$  
  $G_{12} = 62.0 \times 10^4 \text{ psi}$  
  $\nu_{12} = 0.30$  
  $t = 0.0050 \text{ in}$  
  Ply angle = $-30.0 \times 10^0 \text{ degrees}$

Lamina properties for ply 15:
  $E_1 = 440.0 \times 10^5 \text{ psi}$  
  $E_2 = 85.0 \times 10^4 \text{ psi}$  
  $G_{12} = 62.0 \times 10^4 \text{ psi}$  
  $\nu_{12} = 0.30$  
  $t = 0.0050 \text{ in}$  
  Ply angle = $30.0 \times 10^0 \text{ degrees}$

Lamina properties for ply 16:
  $E_1 = 440.0 \times 10^5 \text{ psi}$  
  $E_2 = 85.0 \times 10^4 \text{ psi}$  
  $G_{12} = 62.0 \times 10^4 \text{ psi}$  
  $\nu_{12} = 0.30$  
  $t = 0.0050 \text{ in}$  
  Ply angle = $0.00 \times 10^0 \text{ degrees}$

Lamina properties for ply 17:
  $E_1 = 440.0 \times 10^5 \text{ psi}$  
  $E_2 = 85.0 \times 10^4 \text{ psi}$  
  $G_{12} = 62.0 \times 10^4 \text{ psi}$  
  $\nu_{12} = 0.30$  
  $t = 0.0050 \text{ in}$  
  Ply angle = $-30.0 \times 10^0 \text{ degrees}$

Lamina properties for ply 18:
  $E_1 = 440.0 \times 10^5 \text{ psi}$  
  $E_2 = 85.0 \times 10^4 \text{ psi}$  
  $G_{12} = 62.0 \times 10^4 \text{ psi}$  
  $\nu_{12} = 0.30$  
  $t = 0.0050 \text{ in}$  
  Ply angle = $30.0 \times 10^0 \text{ degrees}$

Lamina properties for ply 19:
  $E_1 = 440.0 \times 10^5 \text{ psi}$  
  $E_2 = 85.0 \times 10^4 \text{ psi}$  
  $G_{12} = 62.0 \times 10^4 \text{ psi}$  
  $\nu_{12} = 0.30$  
  $t = 0.0050 \text{ in}$  
  Ply angle = $0.00 \times 10^0 \text{ degrees}$

Lamina properties for ply 20:
  $E_1 = 440.0 \times 10^5 \text{ psi}$  
  $E_2 = 85.0 \times 10^4 \text{ psi}$  
  $G_{12} = 62.0 \times 10^4 \text{ psi}$  
  $\nu_{12} = 0.30$  
  $t = 0.0050 \text{ in}$  
  Ply angle = $-30.0 \times 10^0 \text{ degrees}$

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Lamina properties for ply 21:
   $E_1 = 440.0 \times 10^5$ psi  $E_2 = 85.0 \times 10^4$ psi
   $G_{12} = 62.0 \times 10^4$ psi  $\nu_{12} = 0.30$
   $t = 0.0050$ in  Ply angle = 30.0E+00 degrees

Lamina properties for ply 22:
   $E_1 = 440.0 \times 10^5$ psi  $E_2 = 85.0 \times 10^4$ psi
   $G_{12} = 62.0 \times 10^4$ psi  $\nu_{12} = 0.30$
   $t = 0.0050$ in  Ply angle = 0.00E+00 degrees

Lamina properties for ply 23:
   $E_1 = 440.0 \times 10^5$ psi  $E_2 = 85.0 \times 10^4$ psi
   $G_{12} = 62.0 \times 10^4$ psi  $\nu_{12} = 0.30$
   $t = 0.0050$ in  Ply angle = -30.0E+00 degrees

Lamina properties for ply 24:
   $E_1 = 440.0 \times 10^5$ psi  $E_2 = 85.0 \times 10^4$ psi
   $G_{12} = 62.0 \times 10^4$ psi  $\nu_{12} = 0.30$
   $t = 0.0050$ in  Ply angle = 30.0E+00 degrees

Transverse Shear Modulus, $G_{13} = 3.10 \times 10^5$ psi

Stacking sequence of plies:

    30.00
    -30.00
     0.00
    30.00
    -30.00
     0.00
    30.00
    -30.00
     0.00
    30.00
    -30.00
     0.00
Laminate Stiffness Components:

A Matrix:

\[
\begin{array}{ccc}
3.80E+06 & 6.60E+05 & -6.16E-04 \\
6.60E+05 & 3.38E+05 & -1.48E-04 \\
-6.16E-04 & -1.48E-04 & 7.03E+05 \\
\end{array}
\]

B Matrix:

\[
\begin{array}{ccc}
-7.05E-02 & -1.37E-02 & -9.01E-04 \\
-1.37E-02 & -5.71E-03 & -3.24E-04 \\
-9.01E-04 & -3.24E-04 & -1.32E-02 \\
\end{array}
\]

D Matrix:

\[
\begin{array}{ccc}
4.34E+03 & 8.83E+02 & 1.81E+02 \\
8.83E+02 & 4.39E+02 & 6.26E+01 \\
1.81E+02 & 6.26E+01 & 9.36E+02 \\
\end{array}
\]

Density = 5.800E-02 lbm/in^3

Tensile Stress Allowable = 60.00E+03 psi
ADHESIVE MATERIAL PROPERTIES

Secant Modulus = 181.0E+03 psi
Plastic Poisson's Ratio = 0.410
Density = 6.500000E-02 lbm/in^3
Shear Stress Allowable = 30.00E+02 psi
Normal (Peel) Stress Allowable = 12.00E+02 psi

JOINT GEOMETRY

Upper Adherend Thickness = 0.120 in
Lower Adherend Thickness = 0.120 in
Adhesive Thickness = 0.005 in
Hstar = 0.125 in
Length of joint = 0.50 in

APPLIED LOADS:

MOU = -43.38 N-mm/mm      MOL = 0.00 N-mm/mm
QOU = -13.99 N/mm         QOL = 0.00 N/mm
NOU = 750.00 N/mm        NOL = 0.00 N/mm

---------> NO JOINT OPTIMIZATION PERFORMED

--------->ADHESIVE MODELED AS AN INELASTIC MATERIAL

FINAL RESULTS OF THE ANISOTROPIC ADHESIVELY BONDED JOINT ANALYSIS

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<th>NxxU</th>
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