ABSTRACT

AYDIN, KAMIL. The Rocking Response of an Unanchored Body Subjected to Simulated Excitation. (Under the direction of Dr. C. C. David Tung and Dr. Vernon Matzen)

The rocking response of rigid bodies with rectangular footprint, freely standing on a horizontal rigid plane is studied analytically. Rigid bodies are subjected to simulated single component of horizontal earthquake time histories. The effect of the baseline correction, applied to simulated excitations, on the rocking response is first examined. The sensitivity of rocking motion to the details of simulated earthquakes and the geometric properties of rigid bodies (i.e., slenderness ratio and size parameter) as well as the coefficient of restitution is investigated. Because of the demonstrated sensitivity of rocking response of rigid bodies to these factors, prediction of rocking stability must be made in the framework of probability theory. Therefore, using a large number of simulated earthquakes, the effects of duration and shape of intensity function of simulated earthquakes on the overturning probability of rigid bodies are next studied. In the case when a rigid body is placed on any floor of a building, the corresponding overturning probability is compared to that of a body placed on the ground. For this purpose, several shear frames with fundamental natural period of vibration ranging from 0.5 sec (stiff) to 2.0 sec (flexible) are employed. Finally, the viability of the energy balance equation which was introduced by Housner in 1963 (The behavior of inverted pendulum structures during earthquakes, Bulletin of Seismological Society of America, 53, 2, 403-417) and widely used by the nuclear power industry to estimate the rocking stability of rigid bodies is evaluated.
THE ROCKING RESPONSE OF AN UNANCHORED BODY SUBJECTED TO SIMULATED EXCITATION

BY

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BIOGRAPHY

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List of Symbols

\(A_{gx}\)  1) Peak horizontal acceleration, 2) Ensemble average of peak horizontal accelerations

\(A_j\)  Amplitude of the \(j^{th}\) component of \(x(t)\) (see Eq. 2.1)

\(a(t)\)  Simulated acceleration time history (see Eq. 2.9)

\(a_x(t)\)  Horizontal base acceleration

\(a_y(t)\)  Vertical base acceleration

\(B\)  Width of a rectangular rigid body

\(e\)  Coefficient of restitution

\(g\)  Acceleration of gravity

\(H\)  Height of a rectangular rigid body

\(h\)  Damping ratio

\(I(t)\)  Intensity function

\(I_O, I_O'\)  Mass moments of inertia about the corner \(O\) and \(O'\) respectively

\(i\)  Dummy index

\(j\)  Dummy index

\(k\)  1) Dummy index, 2) Total stiffness of a floor (see Fig. 7.1)

\(M\)  Mass of a rigid body

\(m\)  1) Total number of discretization points (see Eqs. 2.2 and 2.3), 2) Lumped mass of a floor (see Fig. 7.1)

\(n\)  Total number of components of \(x(t)\)

\(PSD\)  Power spectral density
\( p_j \) Discretized \( j^{th} \) power component (see Eq. 2.2)
\( R \) Distance from center of gravity to the corner \( O \) or \( O' \)
\( R_{k,j}(\omega_k, \omega_j, h) \) Peak acceleration response of a SDOF system with a frequency of \( \omega_k \) and viscous damping ratio of \( h \) (see Eq. 2.4)
\( R_t(\omega_k, h) \) Target acceleration response spectrum (see Eq. 2.3)
\( RS \) Response spectrum
\( RS_t(\cdot) \) Target response spectrum (see Eq. 2.10)
\( S(\cdot) \) Discretized power spectral density function (see Eq. 2.2)
\( S_g(\cdot) \) Power spectral density function (see Eq. 2.1)
\( S_v \) Velocity response spectrum
\( T \) Total duration of an intensity function (or earthquake acceleration time history)
\( T \) Periods of a shear frame (see page 72)
\( T_1 \) Time marking the end of build-up segment of an intensity function
\( T_2 \) Time marking the start of die-off segment of an intensity function
\( T_e \) Equivalent duration of an intensity function (see Eq. 2.5)
\( T_n \) Fundamental natural period of a shear frame
\( t \) Time variable
\( x \) Ratio of \( \omega_j \) to \( \omega_k \)
\( x(t) \) Simulated time history (see Eq. 2.1)
\( u_1, \ldots, u_5 \) Degrees of freedom in a shear frame (see Fig. 7.1)
\( W \) Weight of a rigid body
\( \alpha \) Scale factor
\( \gamma \) Slenderness ratio \( (= \frac{H}{B}) \)
\( \Delta \) Vertical displacement of the center of mass of a body from the position of \( \theta = 0 \) to that of \( \theta = \theta_c \)
\( \Delta \omega_j \) Incremental radian frequency of the \( j^{th} \) component of \( x(t) \)
\( \delta(\cdot) \) Dirac delta function

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\( \theta \)  
Angular rotation of a rigid body

\( \dot{\theta}^-, \dot{\theta}^+ \)  
Angular velocity before and after impact respectively

\( \theta_c \)  
Critical angle of rotation

\( \mu_s \)  
Coefficient of friction

\( \nu_j \)  
Ratio of \( \omega_j \) to \( 2\pi \)

\( \phi_j \)  
Random phase angle of the \( j^{th} \) component of \( x(t) \) (see Eq. 2.1)

\( \omega \)  
Radian frequency

\( \omega_j \)  
\( (= \omega_{j-1} + \Delta \omega_j) \) Radian frequency of the \( j^{th} \) component of \( x(t) \)

\( \omega_k \)  
k\textsuperscript{th} natural frequency of a single-degree-of-freedom (SDOF) system
Chapter 1

Introduction

1.1 Early Studies of Rocking Response Of Rigid Bodies

During earthquakes, some tombstones and monumental columns toppled over but some did not. Researchers tried to correlate these observed behaviors of tombstones and columns to ground shaking intensity. Typical examples are the studies of Japanese researchers Omori (1899), Omori (1900), Ikegami and Kishinouye (1947) and Ikegami and Kishinouye (1950). They were, however, unable to find such correlation. Ishiyama (1982) gives a summary of the other previous studies of response of rigid bodies to earthquake excitations.

To study analytically the rocking response of rigid bodies to ground excitation, it is necessary to first establish the equation of motion of the rigid body and solve it accordingly. This was first done by Housner (1963) following the Chilean earthquakes in 1960. Housner first considered the base motion as consisting of a rectangular or a half-cycle sine-wave pulse. To extend the base motion to more irregular time function,
he modeled the base motion as a white noise random process. He then proposed a heuristic energy balance equation to determine the intensity of base motion that would topple a rocking body with a 50% probability.

Since Housner had already pointed out that the rocking response of a rigid body to base motion is sensitive to the parameters defining the geometry of the body and details of base motion, Yim et. al (1980) decided that the problem should be examined from a probabilistic point of view. Using a limited number of simulated ground motions, Yim et. al solved the equation of a rocking body numerically and produced curves giving the probability of overturning versus base motion intensity for various values of geometric parameters of the body. Aslam et. al, also in 1980, obtained the rocking response of a rigid body to harmonic as well as to simulated earthquake motions both numerically and by laboratory experiment The work of Aslam et. al essentially confirmed the findings of Housner and Yim et. al that it is difficult to predict the response. In 1998, Shao followed the same approach as Yim et. al but used two types of base motions: 50 artificial earthquakes simulated in a way identical to that of Yim et. al and 75 real earthquakes. Shao reached the same conclusion as Yim et. al but showed that real earthquakes give less probability of overturning than simulated earthquakes do.

In recent years the rocking behavior of rigid bodies have been studied using the random vibration approach even though the equation of rocking motion of a rigid body is nonlinear. Dimentberg et. al (1993) employed the method of quasi-conservative averaging to obtain the statistical properties and probability distributions for the time to overturning. Both free-standing and anchored rigid blocks were analyzed under horizontal and vertical base excitations. The excitations were idealized as stationary white noises. Using the same approach, Cai et. al (1995) considered the
base acceleration as a nonstationary process. The approach and computer program of Cai et. al were later utilized by Zhu and Soong (1998) to examine the influence of the size and shape of equipment, properties of the equipment/base interface and vertical as well as horizontal component of seismic excitation. In their study, the earthquake was defined in terms of response spectra as specified by Uniform Building Code (UBC) and National Earthquake Hazard Reduction Program (NEHRP) seismic provisions.

Interest in the problem has extended to the case in which the body is connected to a flexible foundation. This was done by Koh (1986) who idealized the base motion as modulated white noise. Spanos and Koh (1986) also studied the rocking response of a rigid body placed on flexible foundation. Modulated white noise is again used as a model of the horizontal acceleration of the base.

1.2 Objective and Scope of Current Study

Since Shao used only 50 simulated earthquakes, the average response spectrum of which has the same characteristics of that of 4 specified real earthquakes, it is felt that a computer program should be developed that would be able to deal with earthquakes of arbitrary characteristics (response spectrum, for example) and that the number of simulated earthquakes should be greatly increased so that the analysis results of the rocking response would be statistically meaningful.

The process to generate ground motions therefore consists of two stages: 1) conversion of a given earthquake environment (response spectrum) to an equivalent power spectral density function, and 2) simulation of acceleration records (time histories) in the form of sum of a series of sinusoidal functions. The conversion is performed based
on an algorithm developed by Park (1995) The simulation process is iterative. That is, the process is iterated until an acceptable agreement between the given earthquake environment and the calculated response spectra is reached.

The study of Shao also left out a number of issues. They are enumerated as follows.

1) The effect of earthquake duration and the shape of intensity function on the rocking behavior. The duration and the type of intensity function are selected during the synthesis of acceleration records. It is necessary to know how different choices of duration and shape of intensity function would affect the rocking response of rigid bodies under the action of these simulated earthquakes.

2) Rigid bodies of interest may not be always located on the ground (base) of a structural system. Examples are mechanical equipment and electronic devices located on the floors of a building. A rigid body located on a floor is excited by the motion of the floor which is necessarily different from that of the ground.

In 1963, Housner introduced an energy balance equation which enables one to estimate the rocking stability of a body. The equation was given by way of heuristic reasoning and, because of its simple form, has been favored and widely used, with some modifications, by the engineering community.

This study, by using simulated earthquakes and numerical solution of the equation of rocking motion of a rigid body, affords for the first time an opportunity to examine the effectiveness of the energy balance equation.

The objective of this study is to conduct a thorough examination of the rocking response of rigid bodies to base excitation so that the results may be readily and conveniently used with confidence by engineers concerned with rocking stability of rigid bodies.
To achieve the above stated objective, this study will 1) develop computer programs for the generation of artificial earthquakes, for the numerical solution of the equation governing the rocking response of rigid bodies and for the computation of overturning probability, 2) examine the effects of a) the geometric properties of rigid bodies and b) the duration and shape of intensity function of earthquakes, on the rocking response of rigid bodies, 3) compare the rocking response of rigid bodies placed on the ground with that situated on the floors of a building, and 4) evaluate the effectiveness of the energy balance equation for the prediction of the rocking stability of rigid bodies.
Chapter 2

Artificial Generation of Earthquake Excitation

2.1 Introduction

To design a structure for a seismic event, it is necessary to specify the ground motion. Although earthquake ground accelerations that have been recorded in the past could be used for this purpose, they may not have the characteristics of the earthquakes at the locale of the design site. Since the sixties, researchers have been trying to find alternative ways to specify design ground accelerations: simulated earthquakes. The works of Jennings et. al (1968), Hou (1968) and Spanos give brief histories of the various simulation techniques proposed.

The following sections describe the simulation technique used in this study.
2.2 Earthquake Simulation

2.2.1 General

The procedure to generate earthquake time histories is based on the fact that a function can be represented by the sum of a series of sinusoids. That is,

\[ x(t) = \sum_{j=1}^{n} A_j \sin(\omega_j t + \phi_j) \quad 0 < t < \infty \]  

(2.1)

Here \( n \) is the total number of harmonics to be superposed, \( A_j = \sqrt{4S_g(\omega_j)\Delta\omega_j} \) is the amplitude of the \( j^{th} \) component, \( \omega_j = \omega_{j-1} + \Delta\omega_j \) is the frequency of the \( j^{th} \) component, and \( \Delta\omega_j \) is the incremental frequency. The quantity \( \phi_j \) is a random variable (phase angle) uniformly distributed over the interval \([0,2\pi]\). Thus, to obtain \( x(t) \), one generates \( n \) values of \( \phi_j, j = 1, \ldots, n \), using the random number generator computer program. The quantity \( S_g(\cdot) \) is the power spectral density (PSD) function. Many forms of \( S_g(\cdot) \) have been suggested (see, for example, Clough and Penzien (1993)) but we shall not be concerned with which \( S_g(\cdot) \) to use since it is now common practice to first specify the response spectrum from which \( S_g(\cdot) \) is obtained. The manner in which this conversion is carried out will be discussed shortly. Equation 2.1, as it stands, can not be used to represent an earthquake because a real earthquake is of finite duration: it builds up gradually at the initial stage, sustains its intensity for a finite duration and dies off towards the end. To simulate this transient character of an earthquake, Eq. 2.1 is multiplied by a deterministic function, referred to as the intensity function. Various intensity functions have been used in the past for earthquake simulation, four of which are given in Figure 6.1.

In the design of structures against earthquake, seismic loads are often specified in terms of a design response spectrum, which provides an approximate and generally
conservative way to calculate the response quantities in structures of ordinary importance. For such a structure, design response spectrum in conjunction with structural modal analyses have served the design engineer quite well. However, the response spectrum approach can only be used if the structure responds in the linear elastic range. The design of major structures requires an analysis of the response in the inelastic range, in which case a time history analysis of the response of the structure must be carried out. Therefore, it is required that the time history of ground acceleration be specified.

From the above discussion, it is seen that it is customary to generate earthquake ground accelerations that are compatible with a design response spectrum. The specified design response spectrum is, hence, called the target spectrum. A variety of methods have been presented in the last several decades to achieve this purpose. The procedure adopted in this study is to convert a given target response spectrum to an equivalent power spectral density (PSD) function from which compatible ground acceleration time histories may be generated. The following presents the two steps of the simulation process.

2.2.2 Conversion from Response Spectrum (RS) to Power Spectral Density (PSD) Function (Park (1995))

An approach developed by Park (1995) is employed here.

The unknown PSD function of ground acceleration $S_g(\omega)$ is discretized at frequencies $\omega_j$ and considered as the sum of a series of $m$ discretized power components. That is,

$$S_g(\omega) = \sum_{j=1}^{m} S(\omega_j) \Delta \omega_j \delta(\omega - \omega_j) = \sum_{j=1}^{m} p_j \delta(\omega - \omega_j) \quad (2.2)$$
in which \( S(\cdot) \) is the discretized equivalent PSD function (per \( \Delta \omega_j \)), \( \delta(\cdot) \) is the Dirac delta function, \( p_j(\geq 0) \) is the discretized power components and \( \omega_j = \omega_{j-1} + \Delta \omega_j \), which are equally spaced on a logarithmic scale. Approximating the target RS at frequencies \( \omega_k \), the discretized power components \( p_j \) may be obtained by solving a standard least squares problem as

\[
\text{minimize} \quad \sum_{k=1}^{m} \left\{ R_t^2(\omega_k, h) - \sum_{j=1}^{m} p_j R_{k,j}^2(\omega_k, \omega_j, h) \right\}^2 \quad j = 1, m \quad (2.3)
\]

in which \( R_t \) is the target acceleration response spectrum. The term \( R_{k,j} \) in Equation 2.3 is the peak acceleration response of a single-degree-of-freedom (SDOF) system with a vibration frequency of \( \omega_k \) and viscous damping ratio of \( h \). The SDOF system is excited by a sinusoid of unit amplitude and vibration frequency \( \omega_j \). The \( R_{k,j} \) is computed using the peak factor approximation given by Davenport (1964) as

\[
R_{k,j}(\omega_k, \omega_j, h) = \left\{ \sqrt{2 \ln(\nu_j T_e)} + \frac{5.772}{\sqrt{2 \ln(\nu_j T_e)}} \right\} \sqrt{\frac{1 + 4h^2 x^2}{(1 - x^2)^2 + 4h^2 x^2}} \quad (2.4)
\]

in which \( \nu_j = \omega_j/2\pi \), \( x = \omega_j/\omega_k \) and \( T_e \) is the equivalent duration that is related to the intensity function \( I(\cdot) \) as

\[
T_e = \frac{\int_0^T I(\tau) d\tau}{\max\{I(\tau)\}} \quad (2.5)
\]

Here, \( T \) stands for the total duration of the earthquake excitation.

Eq. 2.3 is solved for \( p_j \) using the IMSL Mathematical Libraries and the equivalent PSD function is obtained as

\[
S(\omega_j) = \frac{p_j}{\Delta \omega_j}, \quad j = 1, m \quad (2.6)
\]

An example for obtaining the equivalent PSD function is presented. The target response spectrum (for 5% damping ratio) is chosen to be that given by Regulatory
Guide 1.60 (1973) This response spectrum is used throughout the study. The intensity function chosen here is given by

\[
I(t) = \begin{cases} 
(\frac{t}{3})^2 & , 0 < t < 3 \text{ sec} \\
1.0 & , 3 < t < 16 \text{ sec} \\
e^{-\frac{t-16}{13}} & , 16 < t < 20 \text{ sec} \\
0 & , t > 20 \text{ sec} 
\end{cases} 
\]  

(2.7)

Figure 2.1 shows the equivalent PSD function for the Regulatory Guide 1.60 response spectrum. The response spectrum may be recovered from the PSD function by multiplying the power components \( p_j \) with the peak acceleration response of the SDOF system. Figure 2.2 shows the target and the recovered RS's. It is seen from Fig. 2.2 that the two spectra are closely matched.

![Figure 2.1: Equivalent PSD function for Reg 1.60 Spectrum](image-url)
2.2.3 Generation of Acceleration Records

Upon determining the equivalent PSD function, the earthquake accelerations compatible with the target response spectrum can be simulated by using Eq. 2.1 (and Eq. 2.7).

In the analysis here, \( n \) is set to be equal to \( m \) (see Eq. 2.2) and is selected to be 100. The lower and upper limits of frequency band are 0.15 \( \text{cps} \) and 50 \( \text{cps} \). Therefore, the PSD function and response spectrum (RS) are discretized at 100 frequency points between 0.15 \( \text{cps} \) and 50 \( \text{cps} \). The frequency of the \( j^{th} \) component is

\[
\omega_j = \omega_{j-1} + \Delta \omega_j
\] (2.8)

as mentioned before.
The final simulated acceleration is

\[ a(t) = I(t) \sum_{j=1}^{n} A_j \sin (\omega_j t + \phi_j) \]  

(2.9)

where \( I(t) \) is given by Eq. 2.7. Phase angles, \( \phi_j \), are produced by a random number generator in the IMSL mathematical libraries. Using \( n \) randomly selected value of \( \phi_j \), one obtains one time history \( a(t) \). If one wishes to generate an ensemble of \( N \) \( a(t) \)'s, one randomly selects \( N \times n \) number of \( \phi_j \)'s. The simulated time histories \( a(t) \)'s, thus, will be statistically similar but will be different in details.

The baseline of the time function \( a(t) \) in Eq. 2.9 must be corrected. This is necessary if the behavior of the velocity and the displacement functions given in Figure 2.3 is examined. The figure shows both the uncorrected and baseline corrected accelerations, velocities and displacements, generated by using Regulatory Guide 1.60 as the RS and \( I(t) \) given in Eq. 2.7. It is observed from the uncorrected records that the ground displacement obtained by integrating the uncorrected acceleration time history drifts to a non-zero value at the end of the excitation. Such trend is not realistic and is eliminated by relocating the baseline of the accelerogram in a manner identical to that used for real earthquake records \(^1\). It is noted that the effect of baseline correction is imperceptible on the velocity record and particularly on the acceleration record.

In all subsequent calculations in this study, baselines of the simulated accelerations are always corrected.

To ensure that the acceleration record is compatible with the original target response spectrum, the generated acceleration record is used to produce a response spectrum. The calculated response spectrum is then compared with the target response spectrum \( RS_i \) at a set of specified frequencies \( \omega \). If, at a specified frequency
value, the calculated response spectrum does not match the target response spectrum within the limit of tolerance set, the PSD function is adjusted at that frequency as

\[ S(\omega)_{i+1} = S(\omega)_i \frac{RS_i(\omega)}{RS(\omega)_i} \]  

(2.10)

where the subscript \( i \) refers to the cycle number of iteration. The process is repeated until the match between the target and the calculated response spectra is satisfactory. Equation 2.10 is adopted (with a modification) from the report of Gasparini and Vanmarcke (1976).

Figure 2.4 shows the calculated response spectrum without iteration and Figure 2.5 the calculated response spectrum with iteration, along with the target response spectrum. The response spectrum that is determined after three cycles of iteration closely follows the target response spectrum.

Finally, it should be pointed out that the response spectrum of a single simulated earthquake acceleration record is not expected to match perfectly the target spectrum. That is, in general the calculated response spectrum as a function will not be a smooth function. To improve the agreement, one must resort to the concept of probability (statistics). Thus, one generates a number of \( a(t) \)'s and calculates the average response spectrum which is then compared with the target response spectrum. As the number of \( a(t) \)'s generated is increased, one expects the agreement between the target and calculated response spectra to improve. This is indeed the case as shown in Figures 2.6, 2.7 and 2.8.

It should be noted that in order to clearly show the effect of iteration scheme, the response spectra given in Figs. 2.4 and 2.5 are in fact the average response spectra obtained using 25 \( a(t) \)'s.
It is customary to use the absolute maximum (peak or trough) of earthquake acceleration time history, denoted by $A_{gx}$, and often referred to as peak acceleration, as a measure of the intensity of an earthquake. The subscript $x$ refers to the fact that the acceleration is in the x-direction. It is also customary to omit the gravitational acceleration $g$ and refer to the intensity of an earthquake as $A_{gx}$. When an ensemble of (real or simulated) earthquakes is used, $A_{gx}$ refers to the ensemble average of the peak acceleration values of the members of the ensemble. The peak acceleration of individual members of the ensemble necessarily differs from the average peak acceleration.

The average $A_{gx}$ value of the ensemble of earthquakes that are simulated from the Regulatory Guide 1.60 is 1.22. That is, the peak acceleration value is $472 \text{ m/sec}^2$. It should be noted that while the average $A_{gx}$ is $472 \text{ m/sec}^2$, the peak acceleration of individual acceleration time history may be greater or smaller than the average value as is the case in Fig. 2.3.
Figure 2.3: Uncorrected and corrected accelerograms, velocities and displacements.
Figure 2.4: Target and simulated response spectra without iteration

Figure 2.5: Target and simulated response spectra with iteration
**Figure 2.6:** Target and simulated response spectra computed from 1 realization

**Figure 2.7:** Target and simulated response spectra computed from 5 realizations
Figure 2.8: Target and simulated response spectra computed from 25 realizations
Chapter 3

Rocking

3.1 Introduction

In recent years, more and more attention has been given to the effect earthquakes on nonstructural elements such as pipes, and mechanical and electrical equipment. Some of these nonstructural elements are freely standing while others are anchored to the supporting floors.

Whether anchored or unanchored, certain vital equipment should be kept operational during a seismic event. Therefore, the stability and movement of such equipment is an important consideration to engineers.

This study examines the stability of freely standing rigid bodies resting on a horizontal rigid base.

An unanchored body subjected to base excitation may assume one of the five modes of response: rest, slide, rock, slide-rock and free flight (Shenton III, H.W. (1995)). This study considers the rocking response. Specifically, it deals with the overturning of rigid bodies due to rocking motion.
3.2 Rocking Motion

A homogeneous rigid body with a rectangular shape in elevation and a rectangular footprint resting on a horizontal rigid plane is shown in Figure 3.1. The supporting base is assumed to move in the plane of the paper so that the motion of the block is two-dimensional.

![Diagram of a rigid body resting on a base with dimensions H and B, c.g., R, \( \theta \), \( \mu_s \), and the supporting base labeled.]

\[ R = \frac{\sqrt{R^2 + B^2}}{2} \] is the size parameter and \( \theta_c = \tan^{-1}(\frac{R}{H}) \) is called the critical angle. The static coefficient of friction between the body and the supporting base is \( \mu_s \).

It is assumed that the coefficient of friction is sufficiently large so that there is no sliding between the body and the base. When the intensity of the ground acceleration is large enough, the body is set into rocking motion. Figure 3.2 shows a rigid body, in a rotated position, subjected to horizontal and vertical base accelerations \( a_x(t) \) and \( a_y(t) \).
The equation of motion of the body in the above orientation is

\[
I_O \ddot{\theta} + W (1 + \frac{a_y(t)}{g}) R \sin(\theta_c - \theta) = -\frac{W}{g} R \cos(\theta_c - \theta) a_x(t)
\]  

(3.1)

where \( \theta \) is the angle of rotation. If the rocking motion is about the corner \( O' \), the equation of motion becomes

\[
I_{O'} \ddot{\theta} - W (1 + \frac{a_y(t)}{g}) R \sin(\theta_c + \theta) = \frac{W}{g} R \cos(\theta_c + \theta) a_x(t)
\]  

(3.2)

In these equations, \( I_O = I_{O'} = \frac{4}{3} MR^2 \) represents the mass moment of inertia of the body about \( O \) or \( O' \), \( g \) the gravitational acceleration, and \( W \) and \( M \) respectively the weight and the mass of the body. The angle \( \theta \) is positive when rocking is around \( O \) and negative when it is around \( O' \).

The above nonlinear differential equations of rocking motion are solved numerically by a procedure developed by Shao (1998). The numerical solution is achieved.
by using a fourth-order Runge-Kutta integration scheme. For the accuracy of the solution procedure, a very short time-step of 1/400 s is used. The rigid body is assumed to be initially at rest.

During the rocking motion of a rigid body, some energy is dissipated at each impact. The extent of energy dissipation depends on the magnitude of coefficient of restitution that is defined as

\[ e = \frac{\dot{\theta}^+}{\dot{\theta}^-} \]  

(3.3)
in which \( \dot{\theta}^- \) is the angular velocity before impact and \( \dot{\theta}^+ \) is the angular velocity after impact. The value of \( e \) depends on the angular velocity before impact and the material properties of the base and the body. Aslam et. al (1980) determined the value of \( e \) by free rocking tests of a concrete body. Their study showed that \( e \) was effectively constant during the rocking motion and could be given a value of 0.925. Therefore, this value will be used in this study (as in Yilm et. al (1980) and Shao (1998)).
Chapter 4

The Sensitivity of Rocking Response of Rigid Bodies

4.1 Introduction

Since ancient times, it has been desired to understand why some free-standing tall and slender objects survived intense ground shakings, while bodies which appear stable were severely damaged. Attempts have been made to explain the phenomenon but they were unsuccessful. This had been the case until several decades ago. Considering the dynamics of the system, Housner was the first to investigate the problem in a systematic manner in 1963. He studied the rocking response of rigid bodies subjected to simple pulse type base excitations and presented an approximate method for the more complex case of earthquake base excitations. Yim et. al (1980) solved the equation governing the rocking motion of rigid bodies numerically. In this way, they were able to demonstrate that the rocking response of rigid bodies is sensitive to system parameters and details of base motion. They pointed out consequently that the rocking response of rigid bodies to base excitations can only be studied in
the framework of probability. In 1998, Shao followed the general approach of Yim et. al but extended the analysis to investigate rigid bodies of more general configurations subjected to more extensive sets of real and simulated earthquakes. Yim et. al used only 24 simulated earthquakes that are characteristics of 4 selected real accelerograms, namely, those of Taft (1952), El Centro (1934 and 1940) and Olympia (1949), while Shao used 75 real earthquakes and 50 simulated earthquakes the latter having the same characteristics of the 4 earthquakes used by Yim et. al.

It is therefore desirable to consider a full set of many (statistically sufficient) simulated earthquakes of arbitrary characteristics. In this way, the conclusions given may be stated with confidence.

4.2 Parametric Study of Response Sensitivity

At present, it is well known that the rocking response of a rigid body is highly sensitive to the details of 1) base acceleration, 2) system parameters and 3) initial conditions of the motion. The system parameters that are considered to have an influence on the response of rocking motion are the size of the body and ratio of the height to the width of the rigid body referred to as the slenderness ratio. To examine the effects of ground accelerations, artificially generated earthquake excitations are used. All the bodies that will be analyzed are assumed to be initially quiescent.

For this purpose, simulated earthquake excitations are generated from the Regulatory Guide 1.60 Spectrum (1973). The intensity function is assumed to be a compound model given in Equation 2.7. An equivalent power spectral density (PSD) function is determined from the target spectrum. Having computed the PSD, acceleration time histories are generated. Twenty five simulated earthquake accelerations
(baseline corrected) are shown in Figures 4.1 and 4.2. Although the duration of earthquakes is assumed to be 20 sec as given by Eq. 2.7, all responses are calculated up to 30 sec.

Before proceeding to the response sensitivity analysis, it is of interest to examine the importance of baseline correction of the simulated ground accelerations because baseline correction may alter the rocking response of rigid bodies. This correction is performed in order to filter out undesired long period components in the earthquake record, in a way identical to that used in processing real earthquakes (See, for more information, Berg, G.V. and Housner, G.W. (1961)).

4.2.1 Baseline Correction and Slenderness Ratio $\gamma$

This section investigates how the method of baseline correction of the simulated base motion affects the response of rigid bodies in rocking motion.

Figures 4.3 and 4.4 are produced from the responses of bodies subjected to earthquake motions which are not baseline corrected. Fig. 4.3 gives the time histories of the angle of rotation of the rigid objects, with $R = 10.0\, ft$ and $e = 0.925$, of slenderness ratios ranging from $\gamma(= H/B) = 3.75$ to $\gamma = 7.0$ exposed to the same simulated ground motion (No.1) with an earthquake intensity of $A_{gx} = 0.6$. It is observed that as the slenderness ratio value increases, the response $\theta$ (angle of rotation) of the body also increases. For values of slenderness ratio larger than 4.5, the response continues to increase significantly and the body topples. It is also of some interest to examine whether the response of a body increases with an increase in the slenderness ratio values when the body is rather stable. To do so, it was necessary to increase the value of $A_{px}$ to 0.8. The results are presented in Fig. 4.4. It is seen that an increase in $\gamma$ causes an increase in the response of the body. This leads to the conclusion that the
rocking response of the body increases with increasing slenderness ratio quantities regardless of the range of values of $\gamma$. This differs from the previous observations made by Shao (1998) and Yim et. al (1980). It is felt that to draw the conclusion that rocking response invariably increases with an increase in $\gamma$, it is necessary to examine whether the application of baseline correction to the simulated earthquake motion would lead to the same conclusion. The responses of the first set of the bodies subjected to baseline corrected simulated earthquake (No. 1) are given in Figure 4.5. Although the body with $\gamma = 6.0$ does not overturn, that with $\gamma = 5.0$ does. Figure 4.6 shows the responses of the second set of bodies with slenderness ratio values smaller than 3.5 subjected to the same baseline corrected earthquake (No. 1) with $A_{ax} = 0.8$ as before. It is seen that these bodies behave in a similar way as the bodies with $\gamma$ values greater than 3.5. That is, response of the body does not necessarily increase uniformly with increasing $\gamma$.

It is clear that the baseline correction affects the rocking response of the rigid bodies and the conclusions drawn by Yim et. al and Shao that $\theta$ does not necessarily increase uniformly with an increase in $\gamma$ are in fact correct. It should be kept in mind that baseline correction does not drastically influence the acceleration time history itself (for example, about a maximum of $3\frac{m}{s^2}$ difference existed between the acceleration time history that is corrected and the one that is not corrected). All the acceleration time histories are henceforth determined by using the baseline correction. The baseline corrected acceleration time histories are then input into the program that computes the rocking response.
4.2.2 Size Factor $R$

Considered next is the effect of size parameter $R$ on rocking response $\theta$. Figure 4.7 shows the response for a body of $\gamma = 5.0$ and $e = 0.925$ subjected to the No.1 simulated ground motion with $A_{gx} = 0.5$. $R$ ranges from $6.0\,ft$ to $14.0\,ft$. It is seen that no systematic pattern of the response may be discerned. That is, the maximum angle of rotation is not a monotonic function of $R$. The response of the body with a $\gamma$ value of $2.0$ and $A_{gx} = 0.8$ (everything else being kept the same) is presented in Figure 4.8. It is again noted that no consistent pattern of the effect of size parameter on the response exists.

4.2.3 Coefficient of Restitution $e$

The rigid body with $R = 10.0\,ft$ and $\gamma = 5.0$ is subjected to No.1 simulated earthquake with an intensity of $A_{gx} = 0.5$. The plots in Figures 4.9 and 4.10 are obtained for coefficient of restitution values varying from 0.70 to 0.95. It is expected that the smaller the coefficient of restitution, the smaller the response. This is due to the fact that for small $e$ values, more energy is dissipated at each impact. Although the general trend of response is consistent with the above expectation, close examination shows that there exists slight deviation from the general trend. For example, the maximum angle of rotation $\theta$ for $e = 0.93$ is smaller than that for $e = 0.925$. This is because the amount of energy dissipated depends not only on the value of $e$ but also on the time and value of $\theta$ when impact occurs. It is also observed that as less energy is dissipated for a larger value of $e$, the rocking response generally lasts longer (except in the case with $e = 0.93$).
4.2.4 Earthquake Motions

Figure 4.11 is intended to examine the influence of different members of an ensemble of simulated earthquake motions on the response $\theta$. To do this, simulations No.10, 12, 15, 19 and 25 are employed as forcing functions. The response of the rigid body with $R = 12.0\, ft$ and $\gamma = 5.0$ ($e = 0.925$) under these excitations scaled by a factor of 0.5 is studied. The plots show that the body under No.12, No.15 and No.19 base motions topples, whereas under the rest of the base motions it remains stable. Hence, it may be said that the rocking response is sensitive to the details of ground motion. The same study is repeated for $R = 12.0\, ft$, $\gamma = 2.0$ and $A_{gx} = 0.8$ and the results are in Figure 4.12. The figure similarly shows that rocking response is sensitive to ground motion although the effect is not as pronounced for the body of a smaller slenderness ratio $\gamma = 2.0$.

4.2.5 Scaling Factor $\alpha$

Finally, the effect of scaling factor $\alpha$ is considered. A body with $R = 12.0\, ft$, $\gamma = 5.0$ and $e = 0.925$ is subjected to the No.25 simulated ground excitation. The excitation is scaled by a factor varying from 0.35 to 0.90 by a 0.05 increment. Figures 4.13 and 4.14 show the angle of rotation $\theta$ of the body. It is noticed that the body remains stable when $\alpha = 0.75$ but overturns when $\alpha = 0.60$. 
Figure 4.1: Simulated earthquake motions No.1 to No.13 (baseline corrected)
Figure 4.2: Simulated earthquake motions No.14 to No.25 (baseline corrected)
Figure 4.3: Responses of bodies ($R = 10.0\, ft$, $e = 0.925$) with varying slenderness ratio $\gamma$ under the same simulated base motion (No.1) scaled to $A_{gx} = 0.6$ (baseline uncorrected)
Figure 4.4: Responses of bodies ($R = 10.0 \text{ ft, } e = 0.925$) with varying slenderness ratio $\gamma$ under the same simulated base motion (No.1) scaled to $A_{gr} = 0.8$ (baseline uncorrected)
Figure 4.5: Responses of bodies ($R = 10.0\, ft$, $e = 0.925$) with varying slenderness ratio $\gamma$ under the same simulated base motion (No.1) scaled to $A_{gr} = 0.6$ (baseline corrected)
Figure 4.6: Responses of bodies ($R = 10.0\, ft$, $e = 0.925$) with varying slenderness ratio $\gamma$ under the same simulated base motion (No.1) scaled to $A_{gx} = 0.8$ (baseline corrected)
Figure 4.7: Responses of bodies ($\gamma = 5.0, \epsilon = 0.925$) with varying size parameter $R$ under the same simulated base motion (No.1) scaled to $A_{gz} = 0.5$ (baseline corrected)
Figure 4.8: Responses of bodies ($\gamma = 2.0$, $\epsilon = 0.925$) with varying size parameter $R$ under the same simulated base motion (No.1) scaled to $A_{gz} = 0.8$ (baseline corrected)
Figure 4.9: Responses of bodies ($R = 10.0 ft, \gamma = 5.0$) with varying coefficient of restitution $e$ under the same simulated base motion (No.1) scaled to $A_{g2} = 0.5$ (baseline corrected)
Figure 4.10: Responses of bodies ($R = 10.0ft \gamma = 5.0$) with varying coefficient of restitution $e$ under the same simulated base motion (No.1) scaled to $A_{g2} = 0.5$ (baseline corrected)
Figure 4.11: Responses of a body ($R = 12.0 \text{ ft}, \gamma = 5.0, e = 0.925$) under five simulated base motions, each with peak acceleration = 0.5 (baseline corrected)
Figure 4.12: Responses of a body \((R = 12.0 \text{ ft}, \gamma = 2.0, \epsilon = 0.925)\) under five simulated base motions, each with peak acceleration = 0.8 (baseline corrected)
Figure 4.13: Responses of a body ($R = 12.0\, ft$, $\gamma = 5.0$, $e = 0.925$) under the simulated base motion (No.25) scaled by factors as shown (baseline corrected)
Figure 4.14: Responses of a body \((R = 12.0 ft, \gamma = 5.0, \epsilon = 0.925)\) under the simulated base motion (No. 25) scaled by factors as shown (baseline corrected)
Chapter 5

Statistical Analysis of Overturning

5.1 Introduction

The previous chapter shows that the rocking response of a rigid body is quite erratic in the sense that it is sensitive to the system parameters and details of base motion. Since the excitation of the base on which the body under consideration is located can not be specified with certainty, the only meaningful way to deal with the rocking response of the body is to resort to the theory of probability.

Such probabilistic studies have been carried out in the past. The following reviews the previous works and presents the current work and its results.

5.2 Overturning Probability

5.2.1 Previous Works

Statistical analysis of overturning of a rigid body may be carried out either using the random vibration approach (e.g., Spanos and Koh (1986), Dimentberg et. al (1993), Cai et. al (1995), Zhu and Soong (1998)) or by numerical simulation (e.g.,
Yim et. al (1980), Shao (1998)).

In this study, the approach used by Yim et. al and Shao will be followed. Hence, the literature on rocking response of rigid bodies using the random vibration approach is not reviewed.

Yim et. al (1980) were among the first to conduct probabilistic study of rocking response of rigid bodies using numerical simulation. They used 20 horizontal and 20 vertical ground motions that were generated using a simulation technique. These simulated ground motions have an average response spectrum that resembles that of four real earthquakes, namely, El Centro (1934 and 1940), Taft (1952) and Olympia (1949). Shao (1998) employed 50 simulated earthquake records (for both horizontal and vertical) and 75 real earthquakes. The 50 simulated earthquakes are generated using the same average response spectrum of the 4 earthquakes used by Yim et. al. The 75 real earthquakes are the ones selected from the document provided in CIT-SMARTS. Both Yim et. al and Shao’s findings can be summarized as follows. The overturning probability of a body increases with an increase in slenderness ratio, $\gamma$, for given values of size parameter, $R$, and ground motion intensity. The overturning probability increases with a decrease in size factor for given values of slenderness ratio and ground motion intensity. The overturning probability increases with increasing ground motion intensity for fixed values of size parameter and slenderness ratio.

It is worthwhile to point out that Shao found that the toppling probability of a body is larger using simulated earthquakes than that using real earthquakes.

5.2.2 Current Analysis

The cases covered in the studies of both Yim et. al and Shao are rather limited and the number of simulated earthquakes considered is rather small (20 for Yim et. al...
al and 50 for Shao). Since statistical regularity can not be achieved unless a sufficient number of base excitations are used, it is the interest of this study to carefully select the number of simulated ground excitations so that statistical regularity is ensured.

Simulated earthquakes are obtained using the Regulatory Guide 1.60 (1973) as the response spectrum and the intensity function in Eq. 2.7 (with 20 sec duration). Bodies with two different slenderness ratios, $\gamma = 2$ and 3 and two size parameters, $R = 8.0$ and $20.0 ft$, are considered. The rectangular bodies with these geometric properties are selected because they are comparatively stable against overturning. This would require that the number of earthquakes used be sufficiently large so as to obtain smooth, hence, reliable curves for overturning probabilities. Rigid bodies which are less stable are not analyzed since they would have high likelihood of toppling such that the number of earthquake records needed for the analysis would be small. The coefficient of restitution is fixed at a value of 0.925. The work of Yim et. al showed that the coefficient of restitution has no systematic effect on the response (in a probabilistic study). Therefore, this value is not varied. Average peak ground acceleration is varied from $A_{gz} = 0.3$ to 1.0.

Overturning probabilities are plotted in Figures 5.1, 5.2 and 5.3 against ensemble average peak accelerations. To illustrate how the curves in these figures are obtained, consider the case in which the number of simulated earthquake ground accelerations used is 100. Let the average peak acceleration be $A_{gz} = 0.8$, for example. The rigid body (say $R = 8.0 ft$ and $\gamma = 3$) is subjected to these ground accelerations. The event of toppling is considered to occur if $\theta$ reaches $\pi/2$. The relative frequency (probability) of toppling of the body under these circumstances is then obtained. This produces one point in the first frame of Fig. 5.1.

To examine the number of simulated ground accelerations required to produce
smooth overturning probability curves in these figures, the size of ensemble is varied. Fig. 5.1 shows the plots of overturning probabilities of a body with $R = 8.0\, ft$ and $\gamma = 3.0$ under 100, 300, 500, 1000, and 1500 simulated earthquake excitations. As the number of simulated earthquakes increases, the probability curves become smoother. Hence, it may be said that, for a body with $R = 8.0\, ft$ and $\gamma = 3.0$, it is sufficient to use 1000 or 1500 simulated earthquakes to get satisfactory readings for the overturning probability of that body subjected to the earthquake of specified intensity. To see how a smaller slenderness ratio would affect the results, a body with $R = 8.0\, ft$ and $\gamma = 2.0$ is studied. The event of toppling of this body subjected to 300, 500, 1000, 1500 and 2000 simulated motions is plotted in Fig. 5.2. The curves are not as smooth, suggesting that more earthquake records should be used. However, employing more than 2000 records is very time consuming and a rigid body having a slenderness ratio of 2.0 (and $R = 8.0\, ft$) has a small range of overturning probability (from 0.0 to 0.08). This means that although values of probability of overturning are inaccurate, it may be tolerated in practice since the likelihood of overturning is small. A similar observation is made when a body with $R = 20.0\, ft$ and $\gamma = 3.0$ is considered. Fig. 5.3 gives the overturning probabilities of this body using 300, 500, 1000, 1500 and 2000 ground excitations. Again, in this case, using 2000 earthquake motions seems to be enough for satisfactory results although the curves are a bit jagged.

The plots indicate, as noted also by the previous researchers, that given the ground motion intensity, the event of toppling is more likely for bodies of high slenderness ratio than those of low slenderness ratio. It is also observed that the probability of toppling is greater for bodies with small size parameters, i.e., large bodies are less likely to topple under a specific earthquake intensity. Finally, it is noted that the probability that a body topples increases with an increase in ground motion intensity.
5.3 Conclusion

The plots show that overturning probabilities are an increasing function of $A_{ge}$ value.

Statistical regularity can be reached only by using a large number of samples, especially when the body under consideration is relatively stable and the probability of overturning is small. To avoid using excessively large number of samples, our experience shows that the probability curves may be smoothened by using a moderate number of samples together with the method of least squares. The 'optimal' number of samples to be used in the subsequent parts of the study is determined based on this consideration, by trial and error.
Figure 5.1: Overturning probability of bodies with $R = 8.0$ ft and $\gamma = 3.0$ ($e = 0.025$)
Figure 5.2: Overturning probability of bodies with $R = 8.0\, \text{ft}$ and $\gamma = 2.0\, \left(c = 0.025\right)$. 
Figure 5.3: Overturning probability of bodies with $R = 20.0\, ft$ and $\gamma = 3.0$ ($c = 0.025$)
Chapter 6

The Effect of Duration and Shape of Intensity Function of the Simulated Earthquakes on the Rocking Response of Rigid Bodies

6.1 Introduction

The intensity function is needed to obtain the acceleration time histories that reflect the variation of ground shaking with time as happens in real earthquakes. It is therefore important to investigate whether the different intensity functions change the rocking response of rigid bodies.

Furthermore, the duration of earthquakes is also known to affect the response. However, the extent and severity of earthquake duration on rocking response have never been studied.
6.2 The Intensity Functions Employed

A variety of intensity functions have been used in the past. Four of the commonly used ones are given in Figure 6.1. The ones shown in Figs. 6.1a, 6.1b and 6.1d are considered in this study.

For a specific site, the duration of the motion may be suitably estimated by geophysical considerations in conjunction with earthquake simulation or random vibration analysis. However, for the purpose of examining their effect on rocking response, the excitation durations of 10, 15, 20, 25 and 30 sec are selected in this study.

6.3 Results of the Response Analysis

6.3.1 The Effect of Earthquake Duration

The effect of duration of earthquake excitations on the rocking response is studied for rigid bodies with $R$ values of 4.0, 8.0 and 12.0 feet and $\gamma$ values of 3.0 and 4.0. Five sets of 500 earthquake records are generated using the Regulatory Guide 1.60 (1973) response spectrum. These sets have 10, 15, 20, 25 and 30 sec duration respectively.

The boxcar model (rectangular model) of intensity function given in Fig. 6.1a is first considered for simplicity. The rocking response of the rigid bodies is computed for 30 sec of motion for all the durations considered. This means that, if a 20 sec duration is considered, the earthquake excitation lasts 20 sec but the body’s motion is computed for 30 sec.

Figures 6.3-6.8 show the overturning probabilities as functions of the average peak acceleration $A_{sg}$. The durations of 10, 15, 20, 25 and 30 seconds are referred to as Durations 1, 2, 3, 4 and 5 respectively. In general, as the duration of the earthquake
Figure 6.1: Intensity function types a) boxcar or rectangular, b) trapezoidal, c) exponential, d) compound.
time histories increases, the probability of toppling also increases. This holds true for all the 6 bodies analyzed. This result may be due to the fact that as the time passes by, the kinetic energy input to the rocking body by the earthquake motion builds up such that after some level of energy is reached, the body overturns.

The other point that can be seen from the plots is that the toppling probabilities for earthquakes of 25 and 30 sec durations are close to each other. This means that the overturning probabilities would not change much if the duration of excitation is further increased.

Figure 6.9 gives a summary of the effect of duration of excitation on the response of rigid bodies. The figure clearly shows that at all levels of excitation, the body with \( R = 4.0 \text{ ft} \) and \( \gamma = 4.0 \) is the most unstable and that with \( R = 12.0 \text{ ft} \) and \( \gamma = 3.0 \) is the most stable. At \( A_g = 0.5 \) only the body with \( R = 4.0 \text{ ft} \) and \( \gamma = 4.0 \) is significantly excited. At increased levels of excitations \( A_g = 0.6, 0.7, 0.8 \) the more stable bodies are increasingly excited. In most cases, the probability of toppling is not significantly different for excitations of durations 25 sec and 30 sec.

The above conclusions were obtained using the intensity function of the boxcar type as in Fig. 6.1a.

It is of some interest to see what happens if the other intensity functions having the trapezoidal and compound shapes given in Figs. 6.1b and 1d are used. The ratio between \( T \), the total duration of excitation, and \( T_1 \) and \( T_2 \) are specified as shown in Figure 6.2.

The analysis is carried out as previously for \( T = 15, 20, 25 \) and 30 sec and it is found that there are no essential differences between these results and those using the boxcar intensity function.
6.3.2 The Effect of Shape of Intensity Function

The three intensity functions given in Figs. 6.1a, 1b and 1d are considered. For each of the intensity functions, four durations (15, 20, 25 and 30 sec) are used. For each of the intensity function and duration, 500 earthquakes are generated and applied to one of the six rigid bodies analyzed earlier.

Figures 6.10-6.15 show the results. Thus, for each body (say, \( R = 4.0ft \) and \( \gamma = 3.0 \) as in Fig. 6.10) the overturning probability is plotted against \( A_{px} \) for the four excitation durations considered (referred to as Durations 2, 3, 4 and 5). The legends \( I_t1, I_t2, I_t3 \) refer to the intensity functions given in Figs. 6.1a, 1b and 1d respectively. It is seen that the overturning probability values are not significantly affected by which intensity function is used. In subsequent analysis, only the compound type intensity function will be used.
Figure 6.3: Effect of duration on overturning probabilities for the body with $R = 4.0\, ft$ and $\gamma = 3.0$ ($c = 0.925$), using the boxcar intensity function.
Figure 6.4: Effect of duration on overturning probabilities for the body with $R = 8.0$ $ft$ and $\gamma = 3.0$ ($e = 0.925$), using the boxcar intensity function
Figure 6.5: Effect of duration on overturning probabilities for the body with $R = 12.0 \text{ ft}$ and $\gamma = 3.0$ ($e = 0.925$), using the boxcar intensity function
Figure 6.6: Effect of duration on overturning probabilities for the body with $R = 4.0 \, ft$ and $\gamma = 4.0$ ($e = 0.925$), using the boxcar intensity function
Figure 6.7: Effect of duration on overturning probabilities for the body with $R = 8.0\, ft$ and $\gamma = 4.0$ ($e = 0.925$), using the boxcar intensity function.
Figure 6.8: Effect of duration on overturning probabilities for the body with \( R = 12.0 \text{ ft} \) and \( \gamma = 4.0 \) (\( e = 0.925 \)), using the boxcar intensity function
Figure 6.9: Effect of duration on overturning probabilities for all the bodies considered ($e = 0.925$), using the boxcar intensity function
Figure 6.10: Effect of shape of intensity function on overturning probabilities for the body with $R = 4.0\, ft$ and $\gamma = 3.0\, (e = 0.925)$.
Figure 6.11: Effect of shape of intensity function on overturning probabilities for the body with $R = 8.0$ ft and $\gamma = 3.0$ ($e = 0.925$)
Figure 6.12: Effect of shape of intensity function on overturning probabilities for the body with $R = 12.0 \text{ ft}$ and $\gamma = 3.0$ ($e = 0.925$)
Figure 6.13: Effect of shape of intensity function on overturning probabilities for the body with $R = 4.0 \text{ ft}$ and $\gamma = 4.0 \left( e = 0.925 \right)$
Figure 6.14: Effect of shape of intensity function on overturning probabilities for the body with $R = 8.0 \, ft$ and $\gamma = 4.0 \, (e = 0.925)$.
Figure 6.15: Effect of shape of intensity function on overturning probabilities for the body with $R = 12.0 \text{ ft}$ and $\gamma = 4.0 (\epsilon = 0.925)$
Chapter 7

Rocking Response of Rigid Bodies
Subjected to Floor Excitation

7.1 Introduction

In this section, the difference between the response of a rigid body placed on the ground and that of a body located on a floor of a shear frame is examined. The body located on the ground is subjected to ground base excitation of a given average peak acceleration $A_{gx}$. This base excitation is filtered by the shear frame model. The absolute floor accelerations are first determined. The response of the body located on each floor is then determined employing the floor acceleration time history scaled to the same average $A_{gx}$ as that of the ground. The analysis results are presented and some conclusions are drawn at the end.
7.2 Structural model and ground excitations employed

The structure considered is a 5-story uniform shear frame as shown in Figure 7.1.

It was assumed that the structural masses are lumped at each floor level with a value of $0.259 \frac{kips}{sec^2} \text{in}^2$ (or $100 \frac{kips}{G} \text{in}$) and the total story stiffness for each floor is $31.54 \frac{kips}{in}$. The vibrational periods and normal modes are calculated to be, also as given by
Chopra (1995),

\[
T = \begin{bmatrix}
2.000 \\
0.685 \\
0.435 \\
0.338 \\
0.297
\end{bmatrix}
\text{sec.}
\]

\[
[\Phi] = \begin{bmatrix}
0.28463 & 0.76352 & -1.00000 & 0.91899 & 0.54620 \\
0.54620 & 1.00000 & -0.28463 & -0.76352 & -0.91899 \\
0.76352 & 0.54620 & 0.91899 & -0.28463 & 1.00000 \\
0.91899 & -0.28463 & 0.54620 & 1.00000 & -0.76352 \\
1.00000 & -0.91899 & -0.76352 & -0.54620 & 0.28463
\end{bmatrix}
\]

The structure with a fundamental natural period \( T_n = 2 \text{ sec} \) is much more flexible than typical five story frames. It is chosen such that the low frequency content of the ground excitations can be retained and reflected in floor responses. It establishes a margin in the analysis because the rocking response of rigid bodies of interest is sensitive to the low frequency components of input forcing functions. The modal damping ratio is assumed to be 5%.

It is assumed that the mass of a rigid body is considerably small compared to that of the floors of a shear frame. This ensures that the mass of a rigid body does not affect the vibrational periods and modal shapes.

The input accelerations to the base of the structures are simulated based on Regulatory Guide 1.60 (1973) response spectrum and using an intensity function \( I(t) \) (see Fig. 6.2d on page 54) with a 20 sec duration.

The absolute floor accelerations are determined. A sample of ground and floor accelerations is given in Figure 7.2. It is seen that the high frequency components of
the ground acceleration are filtered out by the building and the amplitudes of floor accelerations naturally differ from each other and from that of the ground.

The average of the 500 ground accelerations is first computed. The average of the 500 floor accelerations for each floor is adjusted to the same average $A_{gz}$ as that of the ground. The ground and the floor accelerations are then scaled as desired.

### 7.3 Results of the response analysis

The behavior of the rigid bodies located on each floor and on the ground is discussed. The values of $\gamma$ and $R$ of the rectangular rigid bodies analyzed are: $\gamma = 3.0, 4.0$ and $R = 4.0, 8.0, 12.0$ ft. For each floor and the ground, the average peak acceleration, $A_{gz}$, (out of 500 members of the ensemble) ranges from 0.3 to 0.8.

Figures 7.3-7.8 give the overturning probabilities of the bodies versus $A_{gz}$ values. The first figure of the set is for $R = 4.0$ ft and $\gamma = 3.0$. It shows that the maximum likelihood of overturning is obtained for the body placed on the fourth floor. The maximum value of probability of overturning is reduced for the body located on the third, fifth, second and first story in that order. The least probability of toppling is obtained for a body placed on the ground.

This trend also exists for the body with parameters $R = 4.0$ ft and $\gamma = 4.0$ as may be observed from Fig. 7.6. The trend starts to change as the body parameters change such that the bodies become relatively more stable. However, in all the cases the maximum probabilities of toppling are obtained for the fourth and the third floor in that order. The fifth floor accelerations in such cases yield the least likelihood of overturning.

In order to understand the reason for the observed trend of toppling probabilities,
average pseudo-velocity floor response spectra for 5% damping ratio for each story and ground floor are computed. The spectra are determined using an $A_{gx}$ value of 0.6 (assuming that the average peak acceleration for a specific site is 0.6) and the results are given in Figure 7.9. This figure shows the response quantities over a range of vibrational cyclic frequencies. The plots are presented so as to give an idea about the kinetic energy of the earthquake excitations that may be fed into the body. Note, however, that the responses of rigid bodies are not linear and do not have a unique frequency of rotation. It is naturally recognized that response spectrum gives the maximum response of a linear elastic single-degree-of-freedom system. That is, it is not the intention of this study to estimate the $S_v$ value that corresponds to the oscillation of the rigid body of interest.

It is seen from the figure that around the frequency value of 1 $cps$, the $S_v$ values increase in the following order: ground, 1st floor, 2nd floor, 5th floor, 3rd floor and 4th floor. The probabilities of toppling of the body given in Figs. 7.3 and 7.6 also increase in the same order. However, for the bodies that are relatively more stable with the frequency of oscillation in the range below 1 $cps$, such correspondence does not exists. This may be due to the fact that the shear frame currently employed is highly flexible. The flexibility of the shear frame accentuates the contributions of higher modes of vibration of the structure to the total response of the system (or floor accelerations in this case).

For this reason, it was decided to investigate the behavior of bodies situated in a second shear frame. The total story stiffness of the shear frame is increased to 100 $\frac{kips}{in}$ and the floor mass is reduced to 0.051 $\frac{kips \cdot sec^2}{in}$. The fundamental natural period $T_n$ is calculated to be 0.5 sec.

Figure 7.10 shows a sample of the absolute accelerations for each floor and the
Figures 7.11-7.16 give the curves of the probability of toppling for the rigid bodies which have the same parameters as those considered before. It is observed that the ground floor gives the maximum probability of toppling, and the probability of toppling of the body decreases as the floor level on which it is placed increases. This trend exists for all the rigid bodies analyzed here.

The average pseudo-velocity response spectrum for each floor is again computed (with $A_{gx} = 0.6$) and given in Figure 7.17. It is seen that the ground floor produces the maximum values of pseudo-velocity up to a frequency value of 2.6 $cps$. The first, second, third, fourth and fifth floor give the pseudo-velocity values in a decreasing fashion. This trend is identical to the one corresponding to the probability of toppling. This suggests that the pseudo-velocity spectra may be used to give an indication of the relative magnitude of the probability of toppling of a body placed on the floors of a structural system whose fundamental period is around 0.5 $sec$.

Two more shear frames with $T_n = 1$ $sec$ and $T_n = 1.5$ $sec$ are also analyzed. Figures 7.18 and 7.19 (again using $A_{gx} = 0.6$) show the response spectra for the shear frame with $T_n = 1$ $sec$ and $T_n = 1.5$ $sec$. It is seen that there is a gradual transition pattern of the response spectra from a flexible ($T_n = 2$ $sec$) to a stiff structure ($T_n = 0.5$ $sec$). The plots of probability of toppling of the bodies placed on the ground and on the floors of the shear frame versus the ensemble average peak acceleration values are not shown here for brevity.

Comparing the four $S_v$ figures (Figs. 7.9, 7.19, 7.18 and 7.17), it may be observed that as the shear frame gets more flexible, the pseudo-velocity curves in the lower frequency range are more closely-spaced. Therefore, it may be inferred that it is hard to distinguish the severity of the response of a body located on any floor of a flexible
structural system. This is especially true, when the rigid body under consideration is relatively stable against rocking motion.

This study shows that it is practically impossible to find any simple rule connecting the rocking response of a rigid body placed on a floor and on the ground of a frame. This is true for the simple uniform shear frame and is certainly true for a frame of irregular configuration and mass and stiffness distributions.
**Figure 7.2:** Sample of input ground and computed floor accelerations for the shear frame with $T_n = 2 \text{ sec}$
Figure 7.3: Overturning probabilities for a body with $R = 4.0\, ft, \gamma = 3.0\,(e = 0.925)$ placed on the shear frame with $T_n = 2\, sec$
Figure 7.4: Overturning probabilities for a body with $R = 8.0 \text{ ft}$, $\gamma = 3.0$ ($e = 0.925$) placed on the shear frame with $T_n = 2 \text{ sec}$
Figure 7.5: Overturning probabilities for a body with $R = 12.0\, ft$, $\gamma = 3.0$ ($\epsilon = 0.925$) placed on the shear frame with $T_n = 2\, sec$
Figure 7.6: Overtuming probabilities for a body with $R = 4.0\, ft$, $\gamma = 4.0$ ($\epsilon = 0.925$) placed on the shear frame with $T_n = 2\, sec$
Figure 7.7: Overturning probabilities for a body with \( R = 8.0 \text{ ft}, \gamma = 4.0 \) \( (e = 0.925) \) placed on the shear frame with \( T_n = 2 \) sec
Figure 7.8: Overturning probabilities for a body with $R = 12.0 \text{ft}$, $\gamma = 4.0$ ($\epsilon = 0.925$) placed on the shear frame with $T_n = 2 \text{ sec}$
Figure 7.9: Pseudo-velocity response spectrum for the shear frame with $T_n = 2$ sec
Figure 7.10: Sample of input ground and computed floor accelerations for the shear frame with $T_n = 0.5 \ sec$
Figure 7.11: Overturning probabilities for a body with $R = 4.0\, ft$, $\gamma = 3.0$ ($e = 0.925$) placed on the shear frame with $T_n = 0.5\, sec$
Figure 7.12: Overturning probabilities for a body with $R = 8.0$ ft, $\gamma = 3.0$ ($e = 0.925$) placed on the shear frame with $T_n = 0.5$ sec
Figure 7.13: Overturning probabilities for a body with $R = 12.0 \text{ ft}$, $\gamma = 3.0$ ($e = 0.925$) placed on the shear frame with $T_n = 0.5 \text{ sec}$
Figure 7.14: Overturning probabilities for a body with $R = 4.0 \text{ ft}$, $\gamma = 4.0$ ($e = 0.925$) placed on the shear frame with $T_n = 0.5 \text{ sec}$
Figure 7.15: Overturning probabilities for a body with $R = 8.0\, ft$, $\gamma = 4.0$ ($e = 0.925$) placed on the shear frame with $T_n = 0.5\, sec$
Figure 7.16: Overturning probabilities for a body with $R = 12.0 \text{ ft}$, $\gamma = 4.0$ ($\epsilon = 0.925$) placed on the shear frame with $T_n = 0.5 \text{ sec}$
Figure 7.17: Pseudo-velocity response spectrum for the shear frame with $T_n = 0.5$ sec
Figure 7.18: Pseudo-velocity response spectrum for the shear frame with $T_n = 1$ sec
Figure 7.19: Pseudo-velocity response spectrum for the shear frame with $T_n = 1.5$ sec
Chapter 8

Evaluation of Energy Balance Equation for Overturning Potential of Rigid Bodies

8.1 Introduction

This chapter is devoted to the study of the viability of using the energy balance equation first given by Housner (1963) to evaluate the overturning potential of rigid bodies. Such a method may be useful for the industry for a quick and easy analysis of the rocking response of unanchored equipments.

8.2 Analysis

In 1963, Housner provided an equation for estimating the rocking stability of a rigid body. Roughly speaking, he equates the energy required to overturn a body with the energy input from the base to the body that is computed from the velocity response spectrum, $S_v$ (Housner, G.W. (1963)).
Specifically, the equation takes the form

$$\frac{1}{2} W \frac{MR^2}{I_o} S_v^2 \cos^2 \theta_c = W \Delta$$

(8.1)

where, as defined in Chapter 3, $W$ and $M$ are respectively the weight and the mass of the body, $I_o = \frac{1}{6} MR^2$ is the mass moment of inertia of the homogeneous body with a rectangular shape in elevation about the point $O$ or $O'$. The quantity $\Delta = R(1-\cos \theta_c)$ is the vertical displacement of the center of mass of the body from the position $\theta = 0$ to that when $\theta = \theta_c$. From Equation 8.1, the required value of $S_v$ for overturning is

$$S_v = \frac{1}{\cos \theta_c \sqrt{\frac{8}{3} g R(1 - \cos \theta_c)}}$$

(8.2)

where $\theta_c = \tan^{-1} \frac{1}{\gamma}$. It is seen that the mass of the body is not a factor in Equation 8.2 (nor in Eq. 8.1). Housner noted that when $S_v$ satisfies Eq. 8.2, there is a 50 percent probability that the ground motion will overturn the body. He also indicated that Eq. 8.1 is arrived at assuming that the ground acceleration is such that ‘the average velocity response spectrum (undamped) is a constant’.

Since in this study we have developed computer programs to generate as many earthquakes as necessary and can compute the response of rigid bodies and obtain the probability of overturning, it would be worthwhile to utilize the tools available to verify the energy balance equation.

For this purpose, 200 artificial earthquakes are produced and several rigid bodies are subjected to these ground excitations. The earthquakes are specified by the response spectrum of Regulatory Guide 1.60 with a 5% damping ratio (see Fig. 2.2) and the intensity function $I(t)$ is that given by Equation 2.7.

For a given body, a plot of probability of overturning versus average ground peak acceleration $A_{gr}$ is produced as was done in the previous chapters. From this plot,
the $A_{gx}$ value that corresponds to 50 percent probability of overturning is determined. The ordinates of the 200 acceleration time histories are adjusted and the 200 pseudo-velocity response spectra with a damping ratio of 5% are produced from which the average pseudo-velocity response spectrum is obtained.

For a rigid body with $R = 4.0\, ft$ and $\gamma = 2.0$, the average $A_{gx}$ value corresponding to 50% overturning probability is 1.02. Figure 8.1 shows this spectrum. Since the pseudo-velocity response spectrum is not constant but varies (widely) with frequency, it is not possible to specify which value of $S_v$ is to be used to compare with the required $S_v$ value given by Eq. 8.2. If we choose the maximum value of $S_v = 80.75\frac{in}{sec}$ in Fig.

![Figure 8.1](image)

**Figure 8.1:** The average $S_v$ spectrum corresponding to 50% overturning probability of a body with $R = 4.0\, ft$ and $\gamma = 2.0$ using 200 simulated ground motions ($A_{gx} = 1.02$)
8.1 and compare it with the required $S_v = 80.75 \text{ in/sec}$ from Eq. 8.2, we see that these two values are exactly the same.

Encouraged by this result, the analysis is extended to rigid bodies with nine slenderness ratio values, $\gamma = 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5$, and 6.0 and eight size parameter values, $R = 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0$, and 11.0 $ft$. That is, altogether, 72 different bodies are analyzed. The following first two tables give the maxima of the average velocity response spectra generated and the required $S_v$ values in Eq. 8.2. Table 8.3 shows the ratios of generated response velocities to the required $S_v$ values.

<table>
<thead>
<tr>
<th>$R \setminus \gamma$</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
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<tr>
<td>4.0 $ft$</td>
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<td>45.92</td>
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<td>45.92</td>
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<td>49.08</td>
<td>45.12</td>
<td>41.96</td>
</tr>
<tr>
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<td>88.66</td>
<td>75.21</td>
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</tr>
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</table>

**Table 8.1:** The generated $S_v$ values in $\text{ in/sec}$

The agreement between the required $S_v$’s and maximum values of the average velocity response spectra is extremely good. In other words, the energy balance equation does what Housner claimed that it would do. That is, if the equation is satisfied, the probability of overturning is approximately 50%.

It should be noticed that the generated $S_v$ (see, for example, Fig. 8.1) is not constant and in order to compare the required $S_v$ with the generated $S_v$, one must decide which value of generated $S_v$ to use. In the current case, the maximum value of generated $S_v$ is used (as indicated previously) and the result is found to be quite
<table>
<thead>
<tr>
<th>$R \setminus \gamma$</th>
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<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
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<td>65.01</td>
<td>55.54</td>
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Table 8.2: The required $S_v$ values (Eq. 8.2) in $\frac{m}{sec}$

<table>
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<tr>
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<th>3.5</th>
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<td>1.08</td>
<td>1.08</td>
<td>1.08</td>
<td>1.09</td>
</tr>
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Table 8.3: The ratios of generated $S_v$’s to the required $S_v$ values

satisfactory. Since, for a given body, the determination of $A_{sx}$ value corresponding to 50% probability of overturning involves no judgement, it would be of some interest to plot the required $S_v$ values against the $A_{sx}$ values obtained for 50% overturning probability for the 72 bodies considered. Figure 8.2 presents such a plot. The solid line is obtained by curve fitting \(^1\). The relationship between the required $S_v$ and $A_{sx}$ is linear and the 72 points all lie extremely close to the solid line, indicating that the

\(^1\)A nonlinear curve fitting routine in MATLAB has been employed. The resulting linear function is given by $A_{sx}(g) = -0.056 + 0.015S_v$
energy balance equation predicts accurately the energy level required to overturn a body (with a 50 percent probability).

![Diagram showing the relationship between $S_v$ and $A_{gx}$](image)

**Figure 8.2:** The $S_v$ and $A_{gx}$ values required to topple 72 different bodies placed on the ground

We now wish to examine if the energy balance equation may be used for the determination of overturning potential of a rigid body located on the floor of a building. The shear frame model that is used in the previous chapter (given in Fig. 7.1) is reproduced in Figure 8.3a and is first employed. The fundamental natural period of the shear frame is $T_n = 2$ sec.

The 200 simulated earthquake motions are filtered through the building to get the absolute floor accelerations. The average $A_{gx}$ value of each floor is adjusted to be the same as that of the ground. The 72 different bodies are placed on each floor of
Figure 8.3: a) Shear frame with $T_n = 2.0$ sec, b) Shear frame with $T_n = 3.0$ sec

the 5 story structure. Once the floor $A_{gE}$ values corresponding to 50% probability of
overturning are determined, they are plotted against the required $S_v$ values (in Eq.
8.2).

It is important to note that the structural model used has a fundamental natural
period of 2.0 sec. This period is in general on the high side for a 5-story structure,
implying that the building is quite flexible. Therefore, it is expected that the low fre-
quency content of the earthquake ground motions will be amplified. Since the rocking
response of the rigid bodies usually has a small oscillation frequency (Shunmugavel,
(1980)), this model may be considered a suitable one to use to examine the viability
of employing the energy balance equation for the stability analysis of rigid bodies
placed on floors.

Figures 8.4-8.8 show the plots of the required $S_v$ versus $A_{gE}$ values for each floor.
The points representing the 72 bodies are somewhat more scattered than those given
in Fig. 8.2 when the bodies are located on the ground. This simply means that
the energy balance equation as applied to the rigid bodies placed on the floors is not as good as when it is applied to the bodies placed on the ground. The scatter, however, is so small that for all practical purposes, the energy balance equation is still considered quite acceptable.

As mentioned earlier, to verify the validity of the energy balance equation, it would be desirable to compare the required $S_v$ with the generated $S_v$ values, when it is applied to a body placed on a floor. The average floor velocity response spectra, however, have several spikes (except that of floor 4) at certain frequencies and using the maximum or any other measure of the spectrum may lead to erroneous or ambiguous conclusions. This is illustrated in Figure 8.10. These spectra are obtained based on the floor excitations that are scaled by the $A_{gr}$ values which correspond to 50% probability of overturning. As can be seen from the plots of the individual story pseudo-velocity response spectra, no $S_v$ value can be identified which can be used to match the required $S_v$ value. If, however, one realizes that the frequency of 'oscillation' of a rocking body tends to be rather 'small' and if one uses the maximum values of $S_v$ in Fig. 8.10 in the frequency range between 0.0 and 0.6 cps, say, it may be expected that a fairly good agreement between the required $S_v$ and generated $S_v$ may be achieved.

Note once again that the above structure analyzed is a rather flexible shear frame. If the structure becomes stiffer, it is expected that the scatter in the plots of required $S_v$ versus $A_{gr}$ will be less. To show this, the uniform shear frame with $T_n = 0.5$ sec that was analyzed in Chapter 7 is again considered here. Figures 8.11-8.13 show the required $S_v$ plotted against $A_{gr}$ values corresponding to 50% probability of overturning. Only the response of rigid bodies located on three floors are studied because of the large amount of computer time needed to run the analysis. From the figures, it is
seen that the scatter of the calculated data from the $S_v-A_{g_x}$ line is greatly reduced as compared with the case of $T_n = 2.0 \text{ sec}$, indicating that the energy balance equation yields even more reliable results for the determination of the energy level required to overturn a body in this case.

It is now desired to examine a new structure which is even more flexible than the shear frame with $T_n = 2 \text{ sec}$. Such a structure will amplify the low frequency content of the ground excitations. This results in the floor accelerations that may have considerable low frequency energy contents. These floor accelerations, then, may increase the rocking response of the rigid bodies. Therefore, the shear frame shown in Figure 8.3b with a fundamental period of 3 sec is considered. According to Uniform Building Code (1994), such a structure is highly physically irregular in the vertical configuration (due to top floor mass). It also falls within the displacement controlled region of the response spectrum given on page 179 of UBC while most of the building-type structures are generally expected to be in the acceleration or velocity controlled regions. The structural model in Fig. 8.3b is selected so that an extreme case could be examined.

Figures 8.15-8.17 show the $A_{g_x}$ values obtained for the present shear frame versus the required $S_v$ values. Only three floors are studied again here due to the large amount of computation time needed. The fact that, in each figure, the points are all clustered around the straight line shows that the energy balance equation works equally well when a body is placed on the floors of an extremely flexible structure.
8.3 Application of the Energy Balance Equation

Having shown the robustness of the energy balance equation, it remains to show how it can be applied. At a site locale where a body is placed, the earthquake environment, usually given in terms of a design response spectrum, can be assumed to be known.

The pseudo-velocity response spectra for the ground and for the floors (scaled to have the same $A_{gr}$ value as that of the ground) of a building are first obtained. Such response spectra were produced in Chapter 7 for the uniform 5-story shear frame with $T_n = 2$ sec in Fig. 7.9 ($T_n = 1.5$ sec in Fig. 7.19, $T_n = 1$ sec in Fig. 7.18 and $T_n = 0.5$ sec in Fig. 7.17), all having a modal damping ratio of 5% and an $A_{gr}$ value of 0.6. It should be noted here that the required $S_v$ in Eq. 8.2 refers to undamped pseudo-velocity response spectrum. This suggests that undamped pseudo-velocity response values should be used for comparison with the required $S_v$ values. Such a computation is accordingly carried out. The spectral values are found to be highly amplified to such an extent that these values can not be used for comparison. Therefore, it is decided that the required $S_v$ values obtained from Eq. 8.2 is to be compared with those values determined from the response spectra with 5% damping ratio. As an example, consider now a body with $R = 4.0 ft$ and $\gamma = 2.0$ placed on the first floor of the shear frame with $T_n = 2$ sec. Fig. 7.9 shows that in the low frequency range (frequency < 0.6 cps), the maximum $S_v$ value is approximately $45.00 \frac{in}{sec}$. Eq. 8.2 or Table 8.2 shows that the required $S_v$ is $80.75 \frac{in}{sec}$ (> $45.00 \frac{in}{sec}$). The body is ‘safe’ against overturning.

If the body under consideration is less stable; say, it has $R = 4.0 ft$ and $\gamma = 4.0$, the required $S_v$ is $39.59 \frac{in}{sec}$ which is less than $45.00 \frac{in}{sec}$. The body, placed on the first
floor of the shear frame, will therefore overturn with a probability more than 50%.

If the same body is placed on the first floor of a stiffer 5-story uniform shear frame with $T_n = 0.5 \text{ sec}$, Fig. 7.17 shows that in the low frequency range (frequency $< 3 \text{ cps}$), the $S_v$ value is approximately $34.00 \text{ in/sec}$ which is less than $39.59 \text{ in/sec}$. The body is therefore considered safe against overturning.

Before closing this section, it is well to revisit the information generated so far with the hope that the task of determining the stability of an unanchored body placed on the floors of a building using the energy balance equation may be made even easier.

For this purpose, reference is made for Figs. 7.9, 7.17, 7.18 and 7.19. These figures show that in low frequency range, the pseudo-velocity response spectral value of a floor is always smaller than that of the ground. This is more evident for a stiff ($T_n = 0.5 \text{ sec}$) than a flexible ($T_n = 2 \text{ sec}$) structure. This means that a body which is safe against overturning on the ground ought to be safe when it is placed on the floors of a building and one may make such claim with more certainty for a stiff than a flexible structure. Such evidence may also be found in Fig. 8.9 which is constructed by combining the lines in Figs. 8.2 and 8.4 to 8.8 relating the required $S_v$ with $A_{gr}$ values for the 5-story uniform shear frame with $T_n = 2 \text{ sec}$. From Fig. 8.9, it is seen that, for a body with given $R$ and $\gamma$ values whose required $S_v$ for overturning (with a 50% probability) is, say, $100 \text{ in/sec}$, the $A_{gr}$ value is 1.4 if it is placed on the ground but it requires a stronger shaking to overturn the body with $A_{gr} = 1.6$ if it is placed on the 5th floor. This figure also shows that if the body is placed on the 4th floor, the $A_{gr}$ value is 1.3. That is, a somewhat less energetic shaking would overturn the body placed on the 4th floor than if it is placed on the ground. A similar figure is constructed for stiffer 5-story shear frame with $T_n = 0.5 \text{ sec}$ and given in Figure 8.14. This figure indicates that, as long as a body placed on the ground is safe against
overturning, it will be safe if it is placed on any floor of the building.

Fig. 8.18 is the combined $A_{px}$ versus the required $S_u$ diagram for a nonuniform 5-story shear frame of Fig. 8.3b with $T_n = 3 \text{ sec}$. It is clear that, for such a flexible structure, the statement made earlier no longer holds true.
**Figure 8.4:** The $S_v$ and $A_{gx}$ values for Floor 1 of the shear frame in Fig. 8.3a ($T_n = 2.0$ sec)
Figure 8.5: The $S_v$ and $A_{gx}$ values for Floor 2 of the shear frame in Fig. 8.3a ($T_n = 2.0 \, sec$)
Figure 8.6: The $S_v$ and $A_{gx}$ values for Floor 3 of the shear frame in Fig. 8.3a ($T_n = 2.0$ sec)
Figure 8.7: The $S_v$ and $A_{gx}$ values for Floor 4 of the shear frame in Fig. 8.3a ($T_n = 2.0$ sec)
Figure 8.8: The $S_v$ and $A_{gx}$ values for Floor 5 of the shear frame in Fig. 8.3a ($T_n = 2.0$ sec)
Figure 8.9: The $S_v$ and $A_{gx}$ values for all the floors of the shear frame in Fig. 8.3a ($T_n = 2.0$ sec)
Figure 8.10: The average floor velocity response spectra of the shear frame in Figure 8.3a.
Figure 8.11: The $S_v$ and $A_{gx}$ values for Floor 1 of the shear frame with $T_n = 0.5$ sec
Figure 8.12: The $S_v$ and $A_{gx}$ values for Floor 4 of the shear frame with $T_n = 0.5$ sec
Figure 8.13: The $S_v$ and $A_{gx}$ values for Floor 5 of the shear frame with $T_n = 0.5$ sec
Figure 8.14: The $S_v$ and $A_{gx}$ values for 3 floors of the shear frame with $T_n = 0.5$ sec
Figure 8.15: The $S_v$ and $A_{gx}$ values for Floor 1 of the shear frame in Fig. 8.3b ($T_n = 3.0$ sec)
Figure 8.16: The $S_v$ and $A_{px}$ values for Floor 4 of the shear frame in Fig. 8.3b ($T_n = 3.0$ sec)
Figure 8.17: The $S_v$ and $A_{px}$ values for Floor 5 of the shear frame in Fig. 8.3b ($T_n = 3.0$ sec)
Figure 8.18: The $S_v$ and $A_{px}$ values for 3 floors of the shear frame in Fig. 8.3b ($T_n = 3.0$ sec)
Chapter 9

Summary and Conclusions

9.1 Introduction

This chapter briefly summarizes the research done and outlines the main conclusions. It also gives recommendations for future work.

9.2 Summary

A rigid body that freely stands on a supporting plane subjected to an earthquake excitation may assume a wide range of responses. These responses are described as rest, slide, rock, slide-rock and free flight. This study deals with the rocking motion.

The problem of rocking motion is highly nonlinear. The nonlinearity involved cannot be linearized by the common approaches for weakly nonlinear systems. The problem, therefore, is less understood. The relevant research towards the understanding the rocking response of rigid bodies has been done only in the last forty years. The problem of rocking has many applications. Some of the examples may be given as: computers or electronic equipment in a nuclear power plant, cabinets, scaffoldings
and art objects in a museum.

Rocking motion of a rigid body is sensitive to 1) the details of base excitation, 2) geometric properties of the body and 3) initial conditions of motion. In this study, the first two factors are studied and, assuming all the bodies analyzed are initially at rest, the third factor is not studied. For this reason, the rocking behavior of a rigid body is best studied using statistical means. Statistical analysis of rocking behavior of rigid bodies can be carried out by three methods: 1) Simulation (Monte-Carlo Simulation, for example), 2) Random Vibration Theory and 3) Energy Balance Equation, which was derived by Housner (1963). In this study, simulation technique is employed.

Rocking motion is studied for rigid bodies subjected to simulated earthquake excitations. Simulation procedure consists of two steps: 1) conversion of a given response spectrum to an energy spectrum (power spectral density function) and 2) generation of earthquake acceleration time histories in the form of a series of harmonic functions modulated by an intensity function to capture the transient nature of a real earthquake motion.

The research herein examines the effect of earthquake duration as well as that of shape of intensity function.

In the case of a rigid body located on the floors of a building system, the body is excited by a set of accelerations that are necessarily different from the ground accelerations. Hence, it is desired to investigate if there exists any difference between the response of a rigid body placed on the ground and that of a body placed on a floor of a structural system.

The last subject examined in this study is the viability and applicability of the energy balance equation derived by Housner in 1963. This equation basically estimates the rocking stability of a rigid body. By using the tools available in this study,
this equation is investigated for the rigid bodies located on the ground and on the floors of a shear frame building.

9.3 Conclusions

The results obtained in this research can be enumerated as follows:

a) Rocking response of a rigid body is highly sensitive to 1) slenderness ratio \( \gamma \) and 2) size parameter \( R \). In other words, no consistent pattern of the effect of slenderness ratio and size parameter exists. The same also holds true for 3) coefficient of restitution but the sensitivity is not as severe. It is next observed that a rigid body responds differently to each member of ensemble of earthquake accelerations, pointing to the response sensitivity to 4) the details of earthquake excitations. In this connection, it should be mentioned that the simulated earthquakes must be baseline corrected. Finally, the effect of 5) scaling factor \( \alpha \) is investigated. Again there is no systematic pattern of the effect of scaling factor on the rocking motion.

b) When the theory of probability is used to predict the rocking response of a rigid body, the above sensitiveness starts to disappear. This shows that the theory of probability is the only meaningful way to deal with the rocking response of rigid bodies. It should be mentioned that the theory of probability yields statistically regular results only if a large ensemble (of excitation) is used. This is more evident if the rigid body of interest is relatively stable. It is found that 1) overturning probabilities are an increasing function of peak base acceleration value, 2) as the slenderness ratio \( \gamma \) increases, the overturning probability increases and 3) as the size parameter \( R \) increases, the overturning probability decreases.

C) 5 duration values (10, 15, 20, 25 and 30 sec) and 3 intensity functions are
used to examine their effect on the rocking response of rigid bodies. It is found
that 1) as the duration of earthquake increases, the probability of overturning also
increases. The overturning probabilities for earthquakes of 25 and 30 sec duration
do not significantly differ, suggesting that, 2) for earthquakes with durations longer
than 30 sec, the overturning probabilities would not change much. 3) The overturning
probability values are not notably affected by which type of intensity function is used.

d) From the response of a rigid body placed on the ground, it is highly unlikely
to predict the response of a body placed on a floor of shear frame. This conclusion
is drawn for a uniform shear frame model. It is expected, therefore, the difference
between any two floor levels would greatly differ if the structural frame is of irregular
configuration and mass and stiffness distribution.

e) The energy balance equation accurately estimates the energy level required to
overturn a body with a 50 percent probability equally well for a body placed on the
ground and on the floor of a building.

9.4 Future Work

2) earthquake simulation procedure. 

This study has considered the rocking response of a rigid body subjected to hori-
izontal component of simulated excitation. A study to consider both horizontal and
vertical simulated earthquakes can be conducted. Also a 3-D analysis of rocking mo-
tion and a rocking behavior that couples with the other response modes (sliding and
bouncing, for example) may as well be studied.

Several regular and one irregular shear frame models have been employed herein
to examine the effect of floor accelerations. The structural model may be made more
complex.
Bibliography


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