GUAN, CHANGHONG. Development of a Closed-loop MEMS Capacitive Force Sensor. (Under the direction of Dr. Yong Zhu).

This thesis describes a closed-loop microelectromechanical system (MEMS) based on lumped-parameter modeling. Analytical models are derived for electrostatic comb drive actuator (CDA) under force-controlled actuation, electrothermal actuator (ETA) under displacement-controlled actuation, capacitive position sensor, including parallel plate capacitive sensor (PPCS) and torsional plate capacitive sensor (TPCS), mechanical equation of motion of a suspended shuttle, viscous air damping, folded flexure. These models are implemented and simulated in finite element analysis softwares (ANSYS and FEMM). System level simulation, implementing PID differential feedback loop, is simulated in a numerical simulation program (MATLAB).

The MEMS die is fabricated by following the standard PolyMUMPs process by MEMSCAP. A series of MEMS packaging process and storage are done in the lab. All peripheral circuitries are self-made.

A commercial capacitive readout IC (MS3110) is first used for open-loop capacitive sensing, which achieves the resolution of 0.05fF, equivalent to 1nm in displacement. Due to the disadvantage of MS3110 in closed-loop, AC bridge capacitance measurement method is then implemented for closed-loop integration. The resolution of AC bridge sensor reaches 0.02fF, equivalent to 0.4nm in displacement. An additional function of AC bridge sensing is accomplished which is simultaneously sensing and actuation of CDA. In the feedback loop, the traditional analog PID controller is designed to transfer the voltage signal of capacitance measurement to the voltage-force transducer which converts feedback voltages to differential feedback force. Since the differential feedback force is limited by clamped voltage, a force-balanced mode is observed under 5V actuation of CDA.
Development of a Closed-loop MEMS Capacitive Force Sensor

by

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DEDICATION

I dedicate this work to my *parents*, without whose caring support it would not have been possible, and to *Jing* for being by my side.
BIOGRAPHY

Changhong Guan was born in Daqing, China on October 15th, 1984, and graduated in 2002 from Daqing Ironman High School, China. He entered Harbin Institute of Technology that autumn in the department of Control Science and Engineering. After his graduation in 2006, he continued his graduate study there for one more year. In August 2007, he was enrolled in the department of Mechanical and Aerospace Engineering at North Carolina State University in Raleigh, US. He focused his research interests on Microelectromechanical System (MEMS) design and fabrication and feedback control system.
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Chapter 1

Introduction

Evolved from integrated circuit industry, microelectromechanical system (MEMS) starts maturing rapidly since 1960s. Besides its success in commercial applications, for instance, pressure sensors[1], ink-jet nozzles [2] and accelerometers [3], etc., MEMS technology also provides a variety of actuators and sensors under classic mechanical and electrical disciplines that could bridge the macroscopic world with the nanoscale science explorations, for example, nanoscale material testing system [4], micromechanical Casimir oscillator [5][6], cell indentation devices [7], etc.

The project described in this thesis is about the development of a closed-loop MEMS device based on capacitive sensing methods. Both actuators and load sensors are described in Chapter 2. Based on different energy transformation mechanisms, electrothermal actuator (ETA) [8][9] and electrostatic comb drive actuator (CDA) [10] are analyzed. Two types of differential capacitive sensors are modeled, parallel-plate and torsional-plate capacitive sensors for lateral and rotational devices, respectively. The capacitive sensing electronics are built upon two approaches, one is commercial capacitive IC chip readout [11] for open-loop sensing and the other is self-made transformer-ratio-arm (TRA) bridge modulation/demodulation capacitive sensing method [12] for closed-loop sensing.

In Chapter 3, a lumped-parameter model [13] of surface micromachined devices is developed by extracting parameters from the layout of our designed surface microsystem, including mass of polysilicon structure, squeeze-film damping coefficient and spring constants. By taking advantage of voltage-force transducer, the closed-loop system is built upon a practical analog Proportional-Integral-Derivative (PID) feedback loop. Structural simulations
and system simulations are performed in ANSYS and MATLAB, respectively. Multi-field solver and coupled-field elements in ANSYS provide a feasible way to solve coupled-field simulation problems in microsystems, thermal-electrostatic-structural coupled-field for instance, although computational cost is still an issue. SIMULINK in MATLAB is a perfect toolbox for control system characterization, in which we optimize PID controller parameters and simulate step responses for both open-loop system and closed-loop system.

Chapter 4 and Chapter 5 describe the detail of experimental setup and results. Comparisons between experimental results and simulations are performed for validation.

In the final chapter, conclusions and discussions are formed about the whole system we built. We then outline the possible future research direction based on the capability of the developed closed-loop system. The appendices contains detailed information about testbed layouts, peripheral electronics, connections, etc..

1.1 Capacitive Sensing Methods

Among a variety of physical sensing methods in MEMS sensor, capacitive sensing is advantageous by comparing with other alternatives. Piezoresistive sensing has inferior performance due to intrinsic resistor thermal noise and strong temperature dependency [14]. Tunneling current sensing, although can achieve low noise floor, requires an extremely small gap (< 10˚A) between tip and electrode and high voltage (> 10V) [15], which makes this kind of devices very expensive to fabricate and difficult to integrate. The obvious characteristics of capacitive sensing are low temperature coefficients, low power dissipation, low noise floor, low fabrication cost and compatibility with VLSI technology scaling, all of which make it commercially implementable in recent years.

Capacitive sensing principles can be allocated to four main categories [16]: resonance, oscillation, charge/discharge and AC bridge methods, all of which are simple in theoretical concept, however, in practice, their implementations are extremely limited due to sensitivity, accuracy, thermal instability and inevitable parasitic contributions.

1.1.1 Resonance Method

The resonance method is capable of measuring both the unknown capacitance and its parallel loss element, which has made the method particularly suitable for measuring
dielectric properties of materials. A typical parasitic capacitance immune measuring circuit is shown in Figure 1.1. The impedance of RLC parallel circuit is

\[
Z_x = \frac{R_x}{j\omega L} + \frac{1}{j\omega C_x} = \frac{j\omega L}{(j\omega)^2 C_x L + j\omega L + j\omega \frac{R_x}{R_x} + 1}
\] (1.1)

in which \(C_x\) is unknown capacitance, \(R_x\) is unknown resistive loss, \(L\) is known inductance and \(\omega = 2\pi f\) is voltage source frequency. The resonance condition is located by the maximum reading of the current detector. The two parasitic capacitances \(C_{s1}\) and \(C_{s2}\) are shunted by voltage source and current detector, respectively. The unknown capacitance \(C_x\) can be determined by

\[
(2\pi f_r)^2 C_x L = 1
\] (1.2)

in which \(2\pi f_r = \omega_r\) is the resonance frequency.

The conductance of the loss element is then given by

\[
R_x = \frac{V_s}{I_r}
\] (1.3)

Figure 1.1: The resonance method for capacitance measurement.

Measurements using the resonance method require several operating steps such as adjusting the resonance frequency, detecting the resonance condition and calculating the unknown capacitance and loss. The first two steps are often done manually, and therefore this method is not suitable for continuously monitoring a physical variable.

### 1.1.2 Oscillation Method

Oscillation method is aimed to build a RC or LC oscillator that depends on the unknown capacitance \(C_x\). The oscillation frequency can be measured, either by using a
digital counter to obtain a digital output, or by using a frequency-to-voltage converter (FVC) to obtain an analog output. The measured frequency is then used to determine the unknown capacitance \( C_x \).

RC oscillator, one of the two oscillation capacitive measurement methods, is the most popular setup used in general purpose capacitance meters. However, it is not suitable for applications requiring a measurement resolution of better than 0.01pF, because its immunity to parasitic capacitance, sensitivity and frequency stability are poor, and more importantly, oscillation frequency is influenced by shunting conductance of \( C_x \).

By comparison, LC oscillator has a better performance in measuring frequency change of oscillation, and a resolution of 0.01fF can be achieved. An improved differential LC oscillator, as shown in Figure 1.2, is capable of measuring differential capacitance with improved transducer drift.

\[
f_x - f_r = \frac{1}{2\pi \sqrt{L}} \left( \frac{1}{\sqrt{f(C_x, C_{s1})}} + \frac{1}{\sqrt{f(C_r, C_{s2})}} \right) \quad (1.4)
\]
in which \( C_x \) and \( C_r \) are unknown capacitor and reference capacitor, respectively, \( C_{s1} \) and \( C_{s2} \) are parasitic capacitors that are included in the measurement as the main disadvantage of this LC oscillation approach.

Figure 1.2: The differential LC oscillator for capacitance measurement.
1.1.3 Charge/discharge Method

One example of charge/discharge method is quad-diode bridge as shown in Figure 1.3. Diodes act like switching components [18], charging capacitors when $D_3$ and $D_4$ are turned ON and discharging capacitors when $D_1$ and $D_2$ are ON. $C_1$ and $C_2$ are used to balance parasitic capacitances introduced by differential capacitors. The output voltage is proportional to $C_{x1} - C_{x2}$ that is measured by differential amplifier.

![Figure 1.3: The diode capacitance bridge for differential capacitance measurement.](image)

This transducer, however, is not a parasitic capacitance immune type and requires four identical diodes to form the bridge. Environmental temperature variation affects transducer sensitivity because of temperature dependency of diode characteristics.

Another category of charge/discharge measurement circuit involves a set of analog FET switches, capacitors and op-amps, as shown in Figure 1.4, which is also known as switched-capacitor method [19][20].

![Figure 1.4: The switched-capacitor circuit (left) and switch timing diagram (right).](image)

The operation of this circuit depends on two non-overlapping clock signals $\phi_1$ and
$\phi_2$ to control switches ON/OFF cycles. An example of phase timing diagram is also shown in Figure 1.4. When $\phi_1$ is ON $\phi_2$ is OFF, capacitors $C_{x1}$ and $C_{x2}$ are charged to values of $Q_1 = C_{x1}V_x$ and $Q_2 = -C_{x2}V_x$, respectively, because common terminal is connected to a virtual ground. The output is zero at this time because the first op-amp behaves like a non-inverting voltage follower with its non-inverting input connecting to ground. When $\phi_2$ is ON and $\phi_1$ is OFF, top and bottom nodes of $C_{x1}$ and $C_{x2}$ are pulled to zero. To keep inverting input at a virtual ground, $C_{x1}$ and $C_{x2}$ are then discharged and differential charge flow to feedback capacitor $C_f$ with a value of $Q_f = V_s(C_{x1} - C_{x2})$ which is also equal to $Q_f = C_fV_o$. The output voltage then swings to a value which is given by

$$V_o = \frac{C_{x1} - C_{x2}}{C_f}V_s$$

(1.5)

Switched-capacitive sensor has been widely analyzed in accelerometer research in recent years. A number of improved modifications have been made to eliminate op-amp input offset voltage and minimize low-frequency noise, for instance, switch chopper stabilization (SCS) [21], correlated double sampling (CDS) [22] and input common-mode feedback (ICMFB) [23][24][25]. The compatibility of IC and MEMS fabrication is another reason that makes this method popular.

1.1.4 AC Bridge Method

AC bridge is recognized as the most accurate and stable method for capacitance measurement [26]. Since the first implementation in micro-displacement transducer [27], AC bridge method has been used extensively in constructing high-precision high-sensitivity capacitive scientific instruments, for instance, capacitance tomography [28], interfacial force microscope (IFM) [12][29], Casimir pendulum [30], Casimir oscillator [5][6].

Two out-of-phase AC sources are required to drive differential capacitors. Two methods are available to generate out-of-phase sources from one single excitation, that are transformer-ratio-arm bridge (TRA) and electronic-ratio-arm bridge (ERA), as shown in Figure 1.5, along with current detector.

$C_1$ and $C_2$ are balance capacitors. The operational amplifier with RC feedback forms current detector. Resistance feedback is necessary to prevent the drift of op-amp output which would eventually saturate the op-amp. The output of current detector is measured by phase-sensitive detector (PSD), also known as lock-in amplifier, to increase
sensitivity and signal-to-noise ratio (SNR) of the transducer. Assume a large value of $R$ (1MΩ) is selected, the output of current detector is given by
\[ V_o = \frac{j\omega R(C_{x1} - C_{x2})}{j\omega RC_f + 1} V_s \approx -\frac{C_{x1} - C_{x2}}{C_f} V_s \quad (1.6) \]
in which $\omega = 2\pi f_s$ is the angular frequency of the AC source and $|j\omega RC_f| \gg 1$.

ERA requires identical amplifiers in two channels which is hard to characterize in practice and wide bandwidth of amplifier has not been an advantage anymore since the appearance of radio-frequency (RF) transformer. Hence, TRA becomes a standard setup for AC bridge method. Negligible input impedance of op-amp provides virtual ground for immunity of parasitic capacitance.

### 1.2 Closed-loop MEMS Sensor

Parallel plate differential capacitors is the most widely used MEMS structure for capacitive sensor. In the schematic shown in Figure 1.6, capacitance pair is represented by
\[ C_1 = \frac{\varepsilon A}{d - x} \quad C_2 = \frac{\varepsilon A}{d + x} \quad (1.7) \]
in which $\varepsilon$ is permittivity of free space, $A$ is the overlapped area, $d$ is the initial gap between electrodes and $x$ is the displacement off central position. Assume $x$ is small compare to $d$, the capacitance difference is expressed by
\[ \Delta C = C_1 - C_2 = \varepsilon A \frac{2x}{d^2 - x^2} \approx \frac{2\varepsilon A}{d^2} x \quad (1.8) \]

This linear relationship is valid when closed-loop feedback system is designed to keep the common electrode around the rest position. Other than that, system accuracy and
Figure 1.6: Differential capacitive sensor schematic and examples: (A) One cell of differential capacitor pair; (B) Transverse comb sensor [4], courtesy of Y. Zhu; (C) Rotational plate with electrodes at bottom; (D) Capacitive strain sensor [31], courtesy of M. Suster.

Figure 1.7: Feedback system with compensator in feedback loop.
frequency response also benefit from feedback loop, because system performance is defined by feedback loop characteristics when system open-loop gain is $|G_o| = |KG_pG_f| \gg 1$, as shown in Figure 1.7.

$$\frac{V_{out}}{F_{in}} = G_c(s) = \frac{G_p(s)}{1 + G_o(s)} = \frac{G_p(s)}{1 + KG_p(s)G_f(s)} \approx \frac{1}{KG_f(s)}$$  \hspace{1cm} (1.9)

in which $G_p(s)$ is the plant transfer function, $K$ is voltage to force transducer gain, $G_f(s)$ is the feedback loop transfer function, $G_o(s)$ and $G_c(s)$ are open-loop and closed-loop transfer functions, respectively. By carefully designed feedback parameters, mechanical resonance can be eliminated which increases sensor bandwidth. Moreover, feedback loop could also reduces Brownian motion noise [32] in sensor, although it does not contribute to improve the signal-to-noise ratio (SNR).

MEMS capacitive sensor is, usually, an underdamped system because of the small damping ratio in squeeze-film and extremely small structural mass. In other word, second order sensor equation of motion has two poles that are located closed to imaginary axis. Thus, a compensator is needed to stabilize the system when external feedback controller is absent, like the cases in integrated on-chip closed-loop accelerometers. The compensator could be placed either in forward path, Figure 1.8 or feedback path, Figure 1.7.

![Figure 1.8: Feedback system with compensator in forward path.](image)

In general, feedback loop is accomplished by differential electrostatic force feedback transducer that converts feedback voltage to force. The principle of electrostatic force will be covered in Chapter 2. And feedback voltage(s) could be generated from either analog or digital feedback loop.

1.2.1 Analog Feedback

Analog feedback scheme is compatible with AC bridge capacitance measurement method and gives continuous analog output, for example, the commercial accelerometer ADXL-50 [3][34], interfacial-force microscopy [12][29] and Casimir pendulum [30][35].
The principle of analog continuous feedback system is the separation of high frequency capacitance measurement AC signal and low frequency feedback signal. A high-pass filter (HPF) [3] or band-pass filter (BPF) [27] could be used to separate capacitive detection signal and force feedback operation electrically. The DC signal results in a charge injection on changing capacitors which affects the output of charge amplifier, as given by [19]

\[
Q_c = C(x)V
\]

\[
i_c = \frac{dQ_c}{dt} = C(x)\frac{dV}{dt} + V\frac{dC(x)}{dx}\frac{dx}{dt}
\]

in which \(V\) could be AC carrier signal or DC feedback signal, either of which could affect current flow across changing capacitor. Multiplier and demodulator are used to extract only the AC sensing signal. Details are described in Chapter 4. In feedback loop, the universal PID controller provides versatility in adjusting system dynamic performance by changing three independent parameters \(K_p\), \(K_i\) and \(K_d\).

1.2.2 Digital Feedback

Digital feedback scheme, also known as Sigma-Delta (Σ-Δ) modulation, has always been combined with switched-capacitor measurement, since the development of first digital accelerometer [36]. The advantage of using a Σ-Δ converter as the feedback method is that the output as well as internal signals is digital. A decimation filter can be applied in order to convert output digital signal to analog signal. A common block diagram is shown in Figure 1.9.

![Figure 1.9: The block diagram of Σ – ∆ closed-loop system.](image)

Not like two-phase clock switch sequence in switched-capacitor method, Σ-Δ feedback system needs more sophisticated clock sequence [23][24]. By incorporating MEMS fabrication and IC manufacturing process, people are able to build the whole system on
a small area, including clock generator, capacitive sensing preamplifier, $\Sigma$-$\Delta$ modulator, compensators and mechanical structures.

Digital feedback voltage is the mean value of modulated voltage in a period of time, but in one cycle, feedback voltage could only be the value of either $+V$ or $0V$. This gives a more stabilized feedback system than analog feedback system (saturation effect).

1.3 Our Approaches

We used a commercial capacitive IC readout, MS3110, to measure the open-loop capacitance change. MS3110 is a compact and integrated AC bridge capacitive sensor, but it does not integrate multiplier inside, which limits its performance in closed-loop as discussed in Chapter 4. Then we switched to another AC bridge setup, a self-made transformer-ratio-arm (TRA) bridge PSD capacitive sensing method. Due to the limitation of frequency mixer, the sensitivity of the self-made PSD could only reach 15fF for equivalent capacitance, which is not sufficient for MEMS sensor with only maximum less than 20fF change. Last, we accomplished the closed-loop system by substituting the self-made PSD with a commercial lock-in amplifier, SR830.
Chapter 2

Actuators and Sensors

Electrothermal actuators (ETA) and electrostatic comb drive actuators (CDA) are two popular actuator categories in MEMS due to their fabrication compatibility with micromachining processes.

Guided by scaling law, electrostatic force has become an attractive candidate for microactuation superior to magnetic force since 1990's [10][37]. Advantages of electrostatic actuation are summarized as simplicity, low power consumption and fast response, while the well recognized disadvantage is high actuation voltage requirement. Recent researches related to electrostatic actuation mainly focused on the optimization of structure shape to maximize electrostatic force [38] and elimination of side effects, out-of-plane levitation for instance, due to high actuation voltage [39][40].

Thermal actuation has been demonstrated as a compact, stable and high-force actuation technique that complements electrostatic actuation [41][42][43][44]. Changes in structural temperature result in mechanical displacement or force output through thermal expansion, contraction or phase change. A variety of applications employed thermal actuators, like linear and rotary microengines [45][46], nanoscale material testing stage [4], optical benches [47]. Considering silicon oriented structural layer, thermal expansion is our main principle of operation.

As described in Chapter 1, load sensors are designed based on differential capacitive sensing due to the sensitivity and linearity. Detailed principles and designs of both actuators and load sensor are described in this chapter.
2.1 Electrostatic Comb Drive Actuator

2.1.1 Electrostatic Force Principle

Electrostatic force between two charged conductors, as shown in Figure 2.1, can be determined from conservation of power of the system, which gives the differential energy balance relation,

\[
\frac{d}{dt} W(Q, x) = V \frac{dQ}{dt} - F_e \frac{dx}{dt}
\]

\[\Rightarrow dW = VdQ - F_e dx\]

(2.1)

(2.2)

where \(W(Q, x)\) is the stored electrical energy, \(V\) is the voltage between the conductors, \(Q = C(x)V\) is the amount of charge on each conductor, \(C(x)\) is the capacitance between conductors, \(x\) is the relative displacement and \(F_e\) is the electrostatic force acting on the conductors.

![Figure 2.1: Schematic of two charged conductors.](image)

Alternatively, energy balance can be expressed in terms of \(V\) and \(x\) as the independent variables, by

\[dW_e = QdV + F_e dx\]

(2.3)

in which the coenergy \(W_e(V, x)\) is defined as \(W_e = QV - W\) and \(dW_e = 2QdV - dW\), in which \(Q = C(x)V\). Integrate Equation 2.3 along an arbitrary path in \(x' - V'\) space, a simple
path is chosen in Figure 2.2,

\[
W_c = \int_0^x F_e(x', V' = 0)dx' + \int_0^V Q(x' = x, V')dV' = \int_0^V C(x)V'dV' = \frac{1}{2}C(x)V^2
\]  \hspace{1cm} (2.4)

Figure 2.2: Path integration in \(x - V\) space to calculate coenergy.

The electrostatic force is then found by taking the partial derivative with respect to \(x\), considering \(V\) is constant,

\[
F_e = \frac{\partial W_c}{\partial x} - Q \frac{\partial V}{\partial x} = \frac{1}{2} \frac{dC(x)}{dx}V^2
\]  \hspace{1cm} (2.5)

Equation 2.5 is the general expression for electrostatic force of any conductor system with given capacitance as a function of position. Usually \(V\) is given by external power source, but \(C - x\) relation needs accurately expressed under the existence of fringing field.

Two major categories of electrostatic actuator geometry are parallel-plate and interdigitated finger, also known as comb drive. The fringing effect is more dominant in comb drive geometry because of small area-gap ratio. A more accurate capacitance approximation is discussed in following sections.

2.1.2 Comb Drive Actuator Design

The first comb drive actuator is demonstrated by Tang [10] in a laterally driven polysilicon resonator, as shown in Figure 2.3 schematically. Two sets of interdigitated
Figure 2.3: Schematic of electrostatic comb drive, top view (top) and side view (bottom).

Comb fingers, movable set (rotor) and stationary set (stator), are patterned from a single polysilicon film. The ground plane is located under the comb fingers and normally is connected to the rotor supporting beam to eliminate electrostatic pull-down force to the substrate. By applying Equation 2.5, a general approximation for electrostatic force in comb drive actuator is given by [10][48],

$$F_e = \frac{1}{2} \frac{dC(x)}{dx} V^2 = \frac{1}{2} \frac{d\left(2N_f \frac{\varepsilon_0 A_h}{g}\right)}{dx} V^2 = \frac{1}{2} \frac{d\left(2N_f \frac{\varepsilon_0 t x}{g}\right)}{dx} V^2 = N_f \frac{\varepsilon_0 t}{g} V^2$$

(2.6)

where $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m is permittivity of air, $N_f$ is the number of rotor fingers, $A$ is overlap area, $t$ is finger thickness and $g$ is separation gap. Assuming no fringing field exists, comb drive electrostatic force is not related to engaged distance. Equation 2.6 is useful to approximate comb drive electrostatic force in a quick manner, however, it is neither accurate nor valid when area-gap ratio is small and ground plane exists.

To consider the contribution of every existing capacitors to electrostatic force, we define partitioning of capacitance per unit length into the stator-to-rotor capacitance, $c_{rs}$, the rotor-to-plane capacitance, $c_{rp}$, and the stator-to-plane capacitance, $c_{sp}$, as shown in

---

**Permittivity of Air**

- $\varepsilon_0$ = 8.854 $\times$ 10$^{-12}$ F/m

**Comb Drive Components**

- **Comb Fingers**: Movable set (rotor) and stationary set (stator)
- **Ground Plane**: Located under the comb fingers and connected to the rotor supporting beam
- **Overlap Area** ($A$)
- **Finger Thickness** ($t$)
- **Separation Gap** ($g$)

---
Figure 2.4, using subscripts $s$, $r$ and $p$ to denote variables relating to the stator fingers, rotor fingers and underlying plane, respectively. Extension of the analysis of Equation 2.3 to three conductors yields,

$$dW_c = Q_s dV_s + Q_r dV_r + f_{e,x} dx$$  \hspace{1cm} (2.7)$$

in which $f_{e,x}$ is the force per unit length in lateral direction. Integrate Equation 2.7 along path shown in Figure 2.5,
\[ W_c = \int_0^x f_{e,x}(x', V_s = 0, V_r = 0) \, dx' + \int_0^V_s Q_s(x, V_s', V_r = 0) \, dV_s' + \int_0^V_r Q_r(x, V_s, V_r') \, dV_r' \]
\[ = \frac{1}{2} c_{sp}(x)V_s^2 + \frac{1}{2} c_{rp}(x)V_r^2 + \frac{1}{2} c_{rs}(x)(V_s - V_r)^2 \]  

(2.8)

in which charges per unit length \( Q_s \) and \( Q_r \) are given by

\[ Q_s = (c_{sp} + c_{rs})V_s - c_{rs}V_r \]  

(2.9)
\[ Q_r = -c_{rs}V_s + (c_{rp} + c_{rs})V_r \]  

(2.10)

Again, apply Equation 2.5 to get electrostatic force per comb finger,

\[ F_{e,x} = \frac{1}{2} \frac{d}{dx} \left( C_{sp} + C_{rs} \right) V_s^2 \]  

(2.11)

in which we switch from per-unit-length capacitance to total capacitance per comb finger. Since the ground plane is electrically connected to the rotor \((V_r = 0)\), Equation 2.11 is simplified as

\[ F_{e,x} = \frac{1}{2} \frac{d}{dx} (C_{sp} + C_{rs})V_s^2 \]  

(2.12)

Neglect global electrical field, the relation between \( C_{rs} \), \( C_{sp} \) and engaged distance is given by

\[ C_{rs} = c_{rs}x \]  

(2.13)
\[ C_{sp} = c_{sp,e}x + c_{sp,u}(L - x) \]  

(2.14)

where \( L \) is the comb finger length, \( x \) is the engaged distance, \( C_{rs} \) is rotor-to-stator capacitance per comb finger, \( C_{sp} \) is rotor-to-stator capacitance per comb finger, \( c_{sp,e} \) and \( c_{sp,u} \) are the capacitance per unit length of the engaged and unengaged fingers, respectively. From Equation 2.12, the lateral electrostatic force of a comb finger is then given by

\[ f_{e,x} = \frac{1}{2} (c_{rs} + c_{sp,e} - c_{sp,u})V_s^2 \]  

(2.15)

Normally, the value of \( c_{sp,e} \) and \( c_{sp,u} \) are different because the engaged rotor finger changes electrical field in the air. Schwarz-Christoffel conformal mapping [49], as a computational efficient method comparing to finite element analysis (FEA), has been applied to express \( c_{rs} \), \( c_{sp,e} \) and \( c_{sp,u} \) explicitly [50][51][52]. Here, I propose an engineering efficient FEA method to calculate the electrical field by utilizing an online free software Finite Element Method Magnetics (FEMM v4.2). The fabricated finger length and width are smaller
than nominal geometry. Two SEM pictures shown in Figure 2.6 are normal comb drive structure and levitation suppression structure, respectively. The comb drive geometry measurement is taken in Revolution v1.6. The actual finger geometry is: total length is 19.1µm; width is 1.8µm; gap is 2.4µm; engaged length is 7.2µm; root radius is 2µm; and tip radius is 0.9µm.

![Figure 2.6](image1.jpg)  
Figure 2.6: A branch of comb drive actuator with ground plane (A) and with levitation suppression suppression (B).

FEMM simulations are shown in Figure 2.7. The value of \( c_{rs} + c_{sp,e} = 4.2412 \times 10^{-11}\text{F/m} \) is calculated in Figure 2.7(B), while \( c_{sp,u} = 2.4592 \times 10^{-11}\text{F/m} \) is calculated in Figure 2.7(C). From Equation 2.15, the electrostatic force per comb finger is given by

\[
f_{e,x} = \frac{1}{2} (c_{rs} + c_{sp,e} - c_{sp,u}) V_s^2 = \frac{1}{2} K_e V_s^2 = \frac{1}{2} \alpha 1.782 \times 10^{-11} V_s^2 \tag{2.16}
\]

where \( K_e \) is defined as electrostatic constant and \( \alpha \) is a form factor, considering the tip fringing field, and defined by the ratio of simulated capacitance over analytical capacitance,
as shown in Figure 2.7(A).

\[
\alpha = \frac{7.3572 \times 10^{-17}}{\varepsilon_0 x_0 t g} = 1.385
\]  

in which engaged length is \(x_0 = 7.2\mu m\), thickness is \(t = 2\mu m\) and gap is \(g = 2.4\mu m\). Thus the simulated electrostatic constant is \(K_e = 2.4681 \times 10^{-11} \text{N/V}^2\). Note that the \(K_e\) given by Equation 2.6 is \(1.4757 \times 10^{-11} \text{N/V}^2\) which is about half of the simulated value.

Figure 2.7: 2-D electrical field simulation in FEMM: (A) top view; (B) engaged fingers cross-section; and (C) unenganged stator fingers cross-section.

Compare the simulated electrostatic constant \(K_e\) with Johnson’s [50] explicit cal-
calculation, which is given by

\[ Q = \frac{\varepsilon_0}{\pi} \left\{ 2 \ln \left( \frac{1}{4} \sqrt{1 + \frac{4h^2}{g^2}} \right) + \ln \left\{ \left[ \left( \frac{c}{g} + 1 \right)^2 - 1 \right] \left( 1 + \frac{2g}{c} \right)^{1+\frac{c}{h}} \right\} + \cdots \right. \]

\[ = 2 \ln \frac{g}{h} + \ln \left( \frac{1}{4} + \frac{h^2}{g^2} \right) + \left\{ \frac{2g}{h} \arctan \left( \frac{2h}{g} \right) + \frac{4h}{g} \arctan \left( \frac{g}{2h} \right) + \pi \left( \frac{2d}{g} + \frac{c}{h} \right) \right\} \]

\[ \bar{Q} = \frac{2\varepsilon_0}{\pi} \left\{ \frac{c\pi}{2h} + 2 \ln \frac{2g}{h} + \ln \left( \frac{1}{4} + \frac{h^2}{(2g+c)^2} \right) + \frac{2g}{h} \arctan \left( \frac{2h}{2g+c} \right) \right\} \]

\[ K_e = Q - \bar{Q} = 2.3187 \times 10^{-11} \text{N/V}^2 \]

in which \( x_0 = 3.6 \mu \text{m} \) is half of engaged length, \( c = 0.9 \mu \text{m} \) is half of finger width, \( g = 1.2 \mu \text{m} \) is half of gap, \( d = 1 \mu \text{m} \) is half of thickness and \( h = 1 \mu \text{m} \) is half of vertical gap. Johnson’s formula deviate from FEMM simulated electrostatic force by 6%, because Johnsons’ formula considers global forces in disengagement direction.

### 2.1.3 Levitation Force

Analogous to Equation 2.11 for lateral force, the comb-drive levitation force is given by

\[ f_{e,z} = \frac{1}{2} \frac{dC_{sp}}{dz} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dz} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dz} (V_s - V_r)^2 \quad (2.18) \]

Still assume \( V_r = 0 \), the vertical force relation reduces to

\[ f_{e,z} = \frac{1}{2} \frac{d}{dz} \left( c_{rs} x + c_{sp,e} x + c_{sp,u} (L - x) \right) V_s^2 \quad (2.19) \]

where \( c_{rs}, c_{sp,e} \) and \( c_{sp,u} \) are independent of lateral position but not vertical. The FEMM simulations are also available to calculate electrostatic force. Since the force is approximately linear with vertical displacement around equilibrium point, in Figure 2.8, it can be modeled with an electrical spring constant, \( k_e \), by

\[ F_{e,z} = k_e (\Delta z_e - \Delta z) \quad (2.20) \]

For the comb drive geometry in Figure 2.6, \( k_e \) corresponds to about 1.55N/m/V\(^2\) per comb finger. If the rotor suspension has spring constant \( k_z \) in vertical direction, the equilibrium displacement is

\[ \Delta z = \frac{k_e}{k_z + k_e} \Delta z_e \quad (2.21) \]
In our design, the vertical stiffness of one folded flexure on comb drive is simulated with the value of $k_z = 4.246\text{N/m}$. For comb drive of 300 fingers, the vertical stiffness is $16.984\text{N/m}$. When the comb drive is actuated under $10\text{V}$, electrical spring constant is $3.348\text{N/m}$, the levitation displacement is $165\text{nm}$. When the comb drive is actuated under $50\text{V}$, electrical spring constant is $8.37\text{N/m}$, the levitation displacement is $330\text{nm}$.

![Plot of Comb Drive Vertical Levitation Force versus Rotor Vertical Displacement](image)

Figure 2.8: Comb drive vertical levitation force versus rotor vertical displacement.

### 2.2 Electrothermal Actuator

Electrothermal actuators (ETA) utilize Joule heating to generate thermal expansion when current passes through the structure. The thermal expansion is then used to produce mechanical deflection out-of-plane or in-plane. For surface micromachined devices, we mainly focus on designing and optimizing in-plane ETA. Two popular in-plane ETA are two-beam ETA \cite{41,42,8} and V-shape, also known as bent-beam, ETA \cite{44,45,46}. Latter shows advantages in one-dimension material testing system so that we duplicated V-shape ETA identically as in \cite{44}. All ETA experimental characterization and multiphysics simulations could refer to \cite{44}. Here we only briefly describe the principles of our V-shape ETA and geometry of designed structures.
2.2.1 Electrothermal-structural Analysis

Figure 2.9: Schematic of a pair of inclined beams subjected to an average temperature increase $\Delta T$. (a) two beams join at the central shuttle; (b) equivalent mechanical representation of one single beam. Courtesy of Y. Zhu.

Electrothermal modeling is necessary to obtain temperature distribution along the inclined beam shown in Figure 2.9 when voltage is applied across anchors. For surface micromachined device with substrate underneath, heat transfer mechanisms are different when the device is operated in air and in vacuum (Two cases are the same when no substrate exist, such as silicon-on-insulator MEMS devices). In air, heat is transferred between the device and substrate through air. The governing equation is given by

$$k_p(T) \frac{d^2T_i}{dx^2} + \frac{V_i^2}{A_i R_i L_i} = \frac{S}{h} \left( \frac{T_i - T_{ref}}{R_T} \right)$$

(2.22)

in which $V_i$ is the voltage across each thermal structure, $R_i = \frac{\rho(T)L_i}{A_i}$ is the resistance of each thermal structure, $\rho(T)$ is the temperature dependent resistivity of polysilicon, $S = \frac{h_i}{w_i} \left( \frac{2h_{air}}{h_n} + 1 \right) + 1$ is the shape factor which accounts on the impact of the shape of the element on heat conduction to the substrate, $A_i$, $L_i$, $w_i$ and $h_i$ are cross-section area, length, width and height of each thermal structure, respectively, $R_T = \frac{h_{air}}{k_{air}} + \frac{h_n}{k_n} + \frac{h_s}{k_s}$ is the thermal resistance between the thermal beam and substrate, $k_p(T)$, $k_{air}$, $k_n$ and $k_s$ are the thermal conductivity of polysilicon, air, silicon nitride and substrate, respectively, $h_{air}$ is the gap between structure and silicon nitride layer, $h_n$ and $h_s$ are the thickness of silicon nitride and substrate, respectively, and $T_{ref}$ is the reference temperature. According to the solution of the differential Equation 2.22 and FEA simulation, the highest temperature change happens in beams and temperature of the shuttle is much lower than thermal beams when device is operated in air.
But in vacuum, heat dissipation by conduction to substrate is only through anchors so that electrothermal model can be simplified by removing the term of heat conduction through the air in Equation 2.22. The governing equation is then given by

\[ k_p(T) \frac{d^2 T_i}{dx^2} + \frac{V_i^2}{A_i R_i L_i} = 0 \]  \hspace{1cm} (2.23)

It could be easily stated that the highest temperature change occurs in the middle of the shuttle. 

From temperature profile, an average temperature change is obtained by integrating over the thermal beam length,

\[ \Delta T = \frac{1}{L_i} \int_0^{L_i} T_i(x) \, dx \]  \hspace{1cm} (2.24)

Thermal expansion along the inclined beams is translated to deflection in \( y \) direction only, because of the symmetry of structure. By solving structural governing equations [44], the vertical displacement and horizontal reaction force can be explicitly expressed by

\[ U = \alpha \Delta T L \frac{\sin(\theta)}{\sin^2(\theta) + \cos^2(\theta) \frac{12I}{AL^2}} \]  \hspace{1cm} (2.25)

\[ P = -\alpha \Delta T E A \frac{\cos(\theta)}{\sin^2(\theta) \frac{AL^2}{12I} + \cos^2(\theta)} \]  \hspace{1cm} (2.26)

where \( A, L, I \) are the beam cross-section area, length and moment of inertia, respectively, \( E \) is Young’s modulus of beam material, \( \alpha \) is the thermal expansion coefficient of beam material.

V-shaped ETA is proved to have huge stiffness comparing to other flexure supported actuators and sensors. The stiffness can also be obtained by solving structural governing equation,

\[ k = \frac{EA}{L} \left( \sin^2(\theta) + \cos^2(\theta) \frac{12I}{AL^2} \right) \]  \hspace{1cm} (2.27)

Quantitively, the stiffness of ETA we designed is 800 times larger than the stiffness of CDA folded flexure so that the V-shape ETA is implemented as position controlled actuator. For more detailed modeling derivation, please refer to [44].

2.2.2 Thermal Actuator Design

From the theoretical modeling of V-shape ETA, a general relationship can be concluded that when beam angle increases \( (\theta > 2^\circ) \), the displacement in active direction
decreases and structure stiffness increases; on the contrary, the displacement increases and stiffness decreases when beam length increases, assuming other geometrical parameters are constant. But the inclined angle cannot be designed sufficiently small to achieve large deformation because axial critical force sets the angle limit when considering beam buckling. The axial critical buckling is written by

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

(2.28)

comparing with axial internal force $P \cos(\theta)$, where $P$ is given by Equation 2.26, we can find the boundary angle.

The beam length is also constrained by fabrication process. It is desired to design a stiff V-shape ETA with a large displacement range to achieve higher resolution and displacement control. It has always been a tradeoff between displacement and stiffness within buckling and length limit. And another way to increase structural stiffness without undermining the displacement range is increasing the number of inclined beam so that we designed the V-shape ETA, as shown in Figure 2.10, with five pairs of 30° inclined thermal beams and six pairs of heat sink beams to reduce the temperature influence on specimen. The device geometrical parameters are: beam length is 300µm, beam width is 7µm, heat sink beam length is 40µm, heat sink beam width is 4µm and thickness is 3.5µm for the whole structure. The calibrated displacement range is around 1µm and stiffness is given by, from Equation 2.27

$$k_{ETA} = 10 \left( \frac{Ebh}{L^2} \sin^2 \theta + \frac{Ebh^3}{L_3 \cos^2 \theta} \right) + \frac{12Eb^3 h}{l^3_{hs}} = 41897\text{N/m}$$

(2.29)

where $E$ is the Young’s Modulus of polysilicon in our application, $L$ and $b$ are the length and width of thermal beams, respectively, $l_{hs}$ and $b_{hs}$ are the length and width of heat sink beams, respectively, and $h$ is the thickness.

ETAs are usually slower than CDAs because thermal time constants are longer than electrical and mechanical time constants. But a relatively high bandwidth, around 1kHz can be expected when considering small thermal mass in our thermal structures [45]. This bandwidth is sufficient in our step actuated material testing system.
2.3 Capacitive Load Sensor

Two categories of capacitive sensing structures are most common used in MEMS devices, one is parallel plate capacitive sensor (PPCS), while the other is torsional plate capacitive sensor (TPCS). With properly designed capacitance measuring circuits, a 0.01fF resolution of capacitance change has been reported constantly [4][5]. A structure of differential capacitors has always been used because of its advantages in parasitic capacitance cancellation, doubled sensitivity and improvement on linearity. Following sections give the modeling in electronic perspective for convenience of system integration.

2.3.1 Parallel-Plate Capacitive Sensor

For a device with CDA and PPCS, the SEM image is shown in Figure 2.11 and corresponding electrical circuits are shown in Figure 2.12. Stators are denoted as $S_1$ and $S_2$ and rotor is denoted as $R$. Three important capacitors are shown, which are the direct differential capacitors $C_1$ and $C_2$ and a mutual capacitor $C_3$. Define $C_0$ is the capacitance value when rotor is at initial position $\Delta d = 0$, $C_1 = C_2 = C_0$. The capacitance are, respectively, given by
Figure 2.11: SEM image of parallel plate capacitive load sensor.

Figure 2.12: Electrical model of parallel capacitive sensor, (a) direct model and (b) equivalent circuit after $\Delta - Y$ transformation.
\[ C_1 = N \varepsilon_0 \left( \frac{A_1}{d_0 + \Delta d} + \frac{A_2}{g} + 0.65 \frac{A_1}{h} \right) \] (2.30)

\[ C_2 = N \varepsilon_0 \left( \frac{A_1}{d_0 - \Delta d} + \frac{A_2}{g} + 0.65 \frac{A_1}{h} \right) \] (2.31)

\[ C_3 = N \varepsilon_0 \left( \frac{A_1}{d_2} + 0.65 \frac{A_1}{h} \right) \] (2.32)

where \( N \) is the number of fingers on rotor, \( \varepsilon_0 \) is the free space permittivity, \( 8.854 \times 10^{-12} \text{F/m} \), \( A_1 \) and \( A_2 \) are the interdigitated area between stators and rotor and between stators and substrate, respectively, \( d_0 \) is the initial gap between stators and rotor, \( \Delta d \) is the displacement of rotor, \( d_2 \) is the gap between stators, \( g \) is the gap between stators and substrate, \( h \) is the beam thickness. The fringing effect is also considered in the capacitance expression.

Equivalent \( \Delta - Y \) transformation is used to give a direct differential capacitance expression. The equivalent capacitances are given by

\[ C_{13} = \frac{C_1 C_2 + C_2 C_3 + C_3 C_1}{C_2} \] (2.33)

\[ C_{23} = \frac{C_1 C_2 + C_2 C_3 + C_3 C_1}{C_1} \] (2.34)

\[ C_{12} = \frac{C_1 C_2 + C_2 C_3 + C_3 C_1}{C_3} \] (2.35)

The capacitance difference we measure in experiments is proportional to displacement.

\[ \Delta C = C_{23} - C_{13} = N \varepsilon_0 A_1 \left( 1 + C_3 \frac{C_1 + C_2}{C_1 C_2} \right) \left( \frac{1}{d_0 - \Delta d} - \frac{1}{d_0 + \Delta d} \right) \]

\[ \approx 2N \varepsilon_0 A_1 \frac{1 + 2C_3/C_0}{d_0^2} \Delta d \propto \Delta d \] (2.36)

While the other equivalent capacitor \( C_{12} \) is approximately a constant value.

\[ C_{12} = C_1 + C_2 + \frac{C_1 C_2}{C_3} \approx 2C_0 + \frac{C_0^2}{C_3} = \text{constant} \] (2.37)

Note that \( C_3 \) is not negligible when \( d_2 \) is comparable to \( d_0 \). It is obvious, from Equation 2.36, that \( \Delta C \) is \( 1 + 2C_3/C_0 \) times larger than \( C_1 - C_2 \), which is 1.71 times to be exact in our design. It is inaccurate if we neglect the value of \( C_3 \).
2.3.2 Torsional-Plate Capacitive Sensor

TPCS equivalent electronics are relatively simple because the mutual capacitance is negligible since it is much smaller than direct capacitance. The only difference between PPCS and TPCS is the translation from linear displacement to rotational angle. The SEM image of TPCS is shown in Figure 1.6C and a cross-sectional view is shown in Figure 2.13.

![Figure 2.13: SEM image of parallel plate capacitive load sensor.](image)

The two bottom electrodes form differential capacitors with torsional plate, the two capacitances can be given when a torsional angle $\theta$ is presented.

$$C_1(\theta) = \varepsilon_0 w \int_0^l \frac{dx}{d_0 - (L - x)\theta} = \varepsilon_0 \frac{w}{\theta} \ln \frac{d_0 - (L - l)\theta}{d_0 - L\theta}$$ (2.38)

$$C_2(\theta) = \varepsilon_0 w \int_0^l \frac{dx}{d_0 + (L - x)\theta} = \varepsilon_0 \frac{w}{\theta} \ln \frac{d_0 + L\theta}{d_0 + (L - l)\theta}$$ (2.39)

where $w$ is the width of bottom electrodes, $l$ and $L$ are length of bottom electrode and half length of top plate, respectively, $d_0$ is the gap between top plate and bottom electrode. Note that we ignore fringing effect because the ratio of gap/width, $d_0/w$, is relatively small.

The differential capacitance is then approximated by

$$\Delta C = C_1 - C_2 = \frac{\varepsilon_0 w}{\theta} \ln \frac{d_0 - (L - l)\theta}{d_0 - L\theta} - \ln \frac{d_0 + L\theta}{d_0 + (L - l)\theta} = \frac{\varepsilon_0 w}{\theta} \ln \frac{d_0^2 - (L - l)^2\theta^2}{d_0^2 - L^2\theta^2}$$

$$\approx \frac{\varepsilon_0 w}{\theta} \ln \frac{d_0^2}{d_0^2 - L^2\theta^2} = \frac{\varepsilon_0 w L^2}{d_0^2} \theta + \frac{\varepsilon_0 w L^4}{2d_0^4} \theta^3 + O(5) \approx \frac{\varepsilon_0 w L^2}{d_0^2} \theta$$ (2.40)

where $L$ is approximately equal to $l$ and first order of Taylor series is used when $\theta$ is small. Thus, in the small signal regime, the capacitance change $\Delta C$ linearly depends on...
the rotation angle $\theta$. Note that the plate surface may not be planar due to residue stress of micro fabrication process. The linear relationship is still valid for curved plate when rotational angle is sufficiently small.

The TPCS is designed with a small stiffness value to increase the force sensitivity.

$$k_{tp} = \frac{2GJ}{l_b}$$  \hspace{1cm} (2.41)

where $G$ is the shear modulus given by $G = \frac{E}{2(1+\nu)}$, $l_b$ is the hinge beam length and $J$ is torsional moment of inertia and $J$. For a slender hinge flexure where the beam width is smaller than thickness, as is the case for most flexure hinges in torsional MEMS applications, the torsional moment of inertia is approximated by [53]

$$J = b^3h \left( \frac{1}{3} - 0.21 \frac{b}{h} + 0.001 \frac{b^4}{h^4} \right)$$  \hspace{1cm} (2.42)

in which $b$ and $h$ are hinge flexure width and height, respectively.
Chapter 3

Closed-loop System Modeling

Two typical micro actuators, ETA and CDA, are implemented in systems. The key component for feedback loop, position sensor, is selected as capacitive sensors, PPCS or TPCS. The most practical control method, PID control, is integrated in the closed-loop system to stabilize the movable mass around its null position in micro structures. The advantages of closed-loop system over open-loop system have been long discussed, for instance improvement on linearity and sensitivity, optimization of frequency response, suppression of noise, etc. But unlike macro mechanical systems where the implementation of the feedback is relatively sophisticated, the control algorithms for MEMS should be kept simple so that they can be directly integrated. Because the most difficult aspect of implementation is related to necessary hardwares for control, rather than control algorithms [54].

The system level modeling and simulations are described in this chapter. The related electronic realizations are followed in the next chapter.

3.1 Lumped-Parameter Design

3.1.1 Mechanical Equation of Motion

The structures we designed usually has only one degree of freedom, like in-plane position or out-of-plain rotation in single direction. A simplified spring-damper system model can be used to represent dynamic behavior of micro devices. The model is also adequate and sufficient for the purpose of the PID controller design. The dynamical model
of the plant is given by

\[ m\ddot{x} + c\dot{x} + kx = F_{\text{ext}} - F_e(V_{\text{bias}}, x) \]  

(3.1)

where \( x \) is the shuttle position, \( m \) is the effective moving mass, \( c \) is the structure damping, \( k \) is the stiffness of suspension, \( F_{\text{ext}} \) is the external driving force, \( F_e \) is the electrostatic feedback force which is depend on applied bias voltage \( V_{\text{bias}} \) and shuttle position \( x \).

Equation 3.1 is often expressed in the alternative form

\[ \ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = \frac{1}{m} \left( F_{\text{ext}} - F_e(V_{\text{bias}}, x) \right) \]

(3.2)

where \( \omega \), resonant frequency, and \( \zeta \), dimensionless damping ratio, are given by

\[ \omega = \sqrt{\frac{k}{m}} \]  

(3.3)

\[ \zeta = \frac{c}{2\sqrt{km}} \]

(3.4)

### 3.1.2 Viscous Air Damping

The damping level in microstructures determines the stability of devices. For laterally driven structures, viscous drag of the ambient fluid is the dominant dissipative source, like comb drive actuators, while for vertically driven devices, the squeeze film damping is the major source of energy dissipation [55]. The damping factor is also the most difficult parameter to determine analytically, because of the complexity of structures and simplified assumptions. Hence, most of the time, the experimental data are used to estimate the system damping [54]. Here we give an analytical estimation for both one-dimensial laterally driven device and vertically driven rotational device.
For laterally driven devices, the Stokes damping model presents a better damping treatment over Coutette damping model \[55\][56]. An explicit damping coefficient expression is given in \[57\]. From the ambient fluid, the damping coefficient above the shuttle is

\[ c_{s\infty} = \mu \beta A \]  

where the subscript \( s\infty \) denotes that Stokes damper of the fluid-film thickness of infinity, 
\( \beta = \sqrt{\frac{\pi f}{\nu}} \), \( f \) is the oscillation frequency, \( \mu \) and \( \nu \) are the absolute and kinetic viscosity of the fluid, \( \nu = \frac{\mu}{\rho} \), \( A \) is the area of the plate.

The damping coefficient underneath the shuttle is obtained as

\[ c_{sd} = \mu \beta A \frac{\sinh(2\beta d) + \sin(2\beta d)}{\cosh(2\beta d) - \cos(2\beta d)} \]  

If we neglect the effect of etched holes on the shuttle and the damping fluid between fingers, comb fingers and parallel plate capacitive fingers, the system damping coefficient is estimated by

\[ c = c_{s\infty} + c_{sd} \]  

Given an estimated shuttle top and bottom plate surface area \( A = 140000 \mu m^2 \), the damping coefficient of step response laterally driven system is approximately 0.0002N·s/m.

For torsional rectangular plate with many holes on it, the damping coefficient due to squeeze file air flow is given by \[13\]

\[ c_\theta = K_\theta K_z L_x \mu L_y L_x^3 \frac{d^3}{d^3} \]  

where \( K_{\theta\text{eta}} = 0.58 \) and \( K_z = 0.42 \) for square plate are the form factors, \( L_x \) and \( L_y \) are plate length in two directions, \( d \) is the initial gap between plate and substrate.

### 3.1.3 Flexure Design

Four flexures are commonly used in microstructure designs, they are fixed-fixed flexure, crab-leg flexure, folded flexure and serpentine flexure. The fixed-fixed flexure, as shown in Figure 3.2(a), is the most straightforward design of support beams, but it has a very stiff nonlinear spring constant because of extensional axial stress in beams. More importantly, the compressive internal residual stress can cause buckling of the flexure. Later, the crab-leg flexure, as shown in Figure 3.2(b), is designed to minimize the internal stress by
adding the “thigh” section. More than enough, the most frequently used folded flexure, as
shown in Figure 3.2(c), averages the original stress over the entire beam length and each end
of the flexure is free to expand or contract in all directions. A more compliant serpentine
flexure, as shown in Figure 3.2(d), is sometimes used to give a desired stiffness because of
the limitation of beam length. To be more specific, when the structure layer is more than
10µm in SOIMUMPs design, the serpentine flexure is desired to achieve small stiffness. In
order to design a sufficiently small spring constant, the serpentine flexure is more adequate
than extra long folded flexure.

Figure 3.2: Various flexure designs. (a) fixed-fixed flexure. (b) crab-leg flexure. (c) folded
flexure. (d) serpentine flexure.

There is always a limitation for beam length design. One reason is that the fixed
charge in substrate can pull the beam down to touch the substrate based on electrostatic
field. To prevent long beam collapse from residual electrostatic force, the flexure is always
built on the ground plane with zero electrical potential. The other reason is that long beams
are difficult to release after wet etching of the sacrificial layers. A cantilever beam of 1.3mm
long, 2µm wide and 2µm thick has been reported releasing from substrate without sticking.

In our PolyMUMP design, all flexures on load sensors and CDAs are designed
as folded flexure, because, due to 2µm or 3.5µm structural layer thickness and up to 1mm
length variation, a wide range of spring constant can be design by changing beam width
and length. Thus, we conservatively designed folded flexures with maximum beam length 200µm.

Figure 3.3: Free-body diagram of practical folded flexure.

Energy method is used to derive analytical formulas for linear spring constant of folded flexure. We assume small deflection theory, consider only displacement from bending and neglect deformation from shear, beam elongation or shorten. For a given folded flexure schematic and free-body diagram in Figure 3.3, the beam moments on each segment are given by

\[ M_{b1} = M_o - F_x \delta \quad (0 < \delta < L_{b1}) \]  
\[ M_{b2} = M_o - F_x(L_{b2} - \delta) - F_y L_t \quad (0 < \delta < L_{b2}) \]  
\[ M_t = M_o - F_x L_{b1} - F_y \delta \quad (0 < \delta < L_t) \]  

The total energy is given by

\[ U = \int_0^{L_{b1}} \frac{M_{b1}^2}{2EI_{b1}} d\delta + \int_0^{L_{b2}} \frac{M_{b2}^2}{2EI_{b2}} d\delta + \int_0^{L_t} \frac{M_t^2}{2EI_t} d\delta \]  

By considering the symmetrical boundary condition, folded flexure has a guided-end bound-
ary condition, the angle $\theta$ and the motion in vertical direction are constrained to zero.

\[
\begin{align*}
\delta_x &= \frac{\partial U}{\partial F_x} = 0 \quad (3.13) \\
\delta_y &= \frac{\partial U}{\partial F_y} = 0 \quad (3.14) \\
\theta &= \frac{\partial U}{\partial M_o} = 0 \quad (3.15)
\end{align*}
\]

By solving the matrix above, the spring constant in horizontal direction $k_x$ is given by

\[
k_L = \frac{F_x}{\delta_x}
\]

\[
= \frac{3EI_b}{L_{b1}^3} \frac{12\beta L_{b1}^2 \alpha^2 + 4(1 + \beta)L_{b1}L_t + L_t^2}{3(\beta^4 + \beta)L_{b1}^2 \alpha^2 + (\beta^4 + 4\beta^3 - 15\beta^2 + 22\beta + 1)L_{b1}L_t \alpha + (\beta^3 - 3\beta^2 + 3\beta + 1)L_t^2} \approx \frac{6EI_b}{L_b^3} \quad \text{when } \alpha \to \infty \quad \beta = 1 \quad (3.16)
\]

where $I_b = I_{b1} = I_{b2}$ is the beam moment of inertia, $\alpha = \frac{L_t}{L_b}$ is the inertia ratio, $\beta = \frac{L_{b2}}{L_{b1}}$ is the length ratio, $\beta = 1$ when $L_b = L_{b1} = L_{b2}$, for a very stiff truss, $I_t \gg I_b$, $\alpha \to \infty$.

Figure 3.4: Comparison of small deformation linear theory with large deformation theory.
The deflection equations derived above are based on small deformation theory. The exact deformation is simulated in ANSYS considering the beam nonlinearity, as shown in Figure 3.4. Our linear theory gives more than 10% error when deformation is 30% of beam length. The actual device can generate no more than 2\(\mu\)m deformation, thus small deformation theory is acceptable for approximation of the flexure stiffness.

3.2 Force-balanced Feedback Loop

3.2.1 PID Controller

A standard PID controller, also known as three-term controller, in time domain, ideally is given in the parallel form

\[
H_{PID}(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}
\]

where \(K_p\) is the proportional gain, \(K_i = K_p \frac{1}{T_i}\) is the integral gain, \(T_i\) is the integral time constant, \(K_d = K_p T_d\) is the derivative gain, \(T_d\) is the derivative time constant. The functionalities of the three terms are highlighted as follows: \(K_p\), providing an overall control action proportional to the error signal; \(K_i\), reducing steady-state errors through low-frequency compensation; \(K_d\), improving transient response through high frequency compensation.

A Laplace transformation of PID controller, frequently used in control system modeling and design, is given by

\[
H_{PID}(s) = K_p + K_i \frac{1}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}
\]

which could be considered as an extreme form of a phase lead-lag compensator with one pole at the origin and the other at infinity.

In our application, the PID controller is implemented in the feedback loop. The input error signal of controller is the output of capacitance sensor and the output voltage signal is fed to voltage-force transducer to provide force feedback.
3.2.2 Differential Force Feedback

Since the rotor is actuated by force, a force feedback should be applied to the rotor to counterbalance the actuation force. Four possible voltage-force transducer connections have been successfully implemented in feedback loop, as shown in Figure 3.5.

Because of the requirement of zero electrical potential of the rotor, we implement the second voltage-force transducer structure into our feedback loop. The output voltage from PID controller is converted to two out-of-phase voltages with the same amplitude $\pm V_{fb}$. The sensitivity and noise level of capacitance measurement limit the magnification of the feedback loop which converts the displacement of shuttle or capacitance difference to feedback voltage. A bias voltage, $V_{bias}$, is then applied to both stators for linearizing the relationship of the feedback voltage, $V_{fb}$, over small displacements, $\Delta x$, and amplifying the feedback electrostatic force by times. Since the feedback DC voltage should not affect capacitance sensing signal pick-up, a RC network is used to superimpose AC carrier signal,
\[ \pm V_s \sin \omega t, \text{ on DC feedback voltage, } \pm V_{fb} - V_{bias}. \] The combined differential voltages applied to the stators are given by

\[ V_1 = +V_{fb} - V_{bias} \pm V_s \sin \omega t \quad (3.19) \]
\[ V_2 = -V_{fb} - V_{bias} \mp V_s \sin \omega t \quad (3.20) \]

The differential electrostatic feedback force on parallel plate capacitive sensor, neglecting the force components introduced by the AC carrier signal, \( V_s \), is derived from Equation 2.5.

\[
F_{fb} = \frac{1}{2} \frac{dC_1(x)}{dx} V_2^2 - \frac{1}{2} \frac{dC_1(x)}{dx} V_1^2
\]
\[
= \frac{\varepsilon A}{2} \left( \frac{(-V_{fb} - V_{bias})^2}{(d_0 - \Delta x)^2} - \frac{(+V_{fb} - V_{bias})^2}{(d_0 + \Delta x)^2} \right)
\]
\[
= \frac{2\varepsilon A}{d_0^2} V_{bias} V_{fb} \quad \text{when } \Delta x \to 0 \quad (3.21)
\]

where \( \varepsilon \) is still permittivity of free space, \( d_0 \) and \( A \) are the initial gap and interdigitate area between stator and rotor, respectively. When \( \lim \Delta x = 0 \), the feedback force is linearly related to feedback voltage, \( V_{fb} \).

Note that once the DC feedback voltages are applied to stators, because of the electrical potential between stators and substrate, the out-of-plane deflection of fixed beam is expected. When the DC feedback voltage exceeds the cantilever pull-in voltage, the beam will stick to the substrate. Hence, a voltage clamp is necessary to limit the feedback within cantilever operational safety range. Also, the out-of-plane deflection cause the change of feedback electrostatic force, here we ignore the electrostatic field change with respect to deflections of stators.

The relationship between differential electrostatic feedback force and feedback voltage for shuttle displacement of \(-1 \mu m, -0.5 \mu m, 0 \mu m, 0.5 \mu m, 1 \mu m\) is shown in Figure 3.6 when bias voltage is set as 5V. The feedback force is linear to feedback voltage when \( \Delta x = 0 \mu m \) as predicted in Equation 3.21. Compare with actuation force generated from 300CDA of 10V actuation voltage and 1200CDA of 5V actuation voltage, the feedback force is sufficient to prevent shuttle motion if the clamp voltages are set \( \pm 9 \pm 0.7V \).

Because the characteristic of electrostatic force is attractive, a polarity change related to bias voltage is shown in Figure 3.7. When the bias voltage is small, the change of polarity will result in positive feedback which can cause latch-up. Either increase the bias
voltage or limit the displacement by decreasing the actuation voltage, could eliminate the latch-up condition.

### 3.3 System Simulation and Analysis

We simulated both open-loop and closed-loop system response in Simulink of Matlab R2007a. Here we take device D3 as an example, which is a MEMS device actuated by CDA of 300 fingers and sensed by differential parallel plate capacitive sensor of 64 pairs. The open-loop transfer function is given by

$$G_o(s) = \frac{1}{ms^2 + cs + k}$$

(3.22)

Since the system input is voltage, an electrostatic constant, $K_e$, is used to convert input voltage to force.

The block diagram of open-loop simulation is shown in Figure 3.8. The electrostatic constant is $K_e = 4.08 \times 10^{-9}$ N/V$^2$. The mass is geometrical dependent parameter which is approximated by $m = 8.15 \times 10^{-10}$ kg. Both damping coefficient $c = 2 \times 10^{-4}$ N·s/m and spring constant $k = 20.467$ N/m are from analytical models discussed above.
Figure 3.7: Polarity of differential electrostatic feedback force under shock condition.

Figure 3.8: The block diagram of open-loop simulation in Simulink.
The open-loop step response is shown in Figure 3.9. Under the step force of 1\,\mu N, the movable shuttle gives a 50nm displacement in 45\,\mu s.

Figure 3.9: Open-loop step response of 1\,\mu N.

Simulation of the closed-loop system response, as shown in Figure 3.10, is to interrogate the efficiency of PID controller and optimize the three-term gain in PID. Two more constants are: \( K_{cap} = 8.22 \times 10^5 \) V/m, is the factor that converts shuttle displacement into output voltage; \( K_f = 1.36 \times 10^{-6} \, \text{N/V}^2 \), is also the electrostatic constant that evaluates the electrostatic force provided from stators.

The closed-loop transfer function is given by

\[
G_c(s) = \frac{G_o(s)}{1 + K_{cap} K_f G_o(s) H_{PID}(s)} = \frac{s}{ms^3 + (c + K_{cap} K_f K_d)s^2 + (k + K_{cap} K_f K_p)s + K_{cap} K_f K_i} \tag{3.23}
\]

When the PID gains are chosen as practical values, \( K_p = 100 \), \( K_i = 10^7 \) which is equivalent to \( R = 100\Omega \) and \( C = 0.001\mu F \), \( K_d = 0.0003 \) which is equivalent to \( R = 1k\Omega \) and \( C = 0.3\mu F \), the system is working in force-balanced mode. For comparison, a step response of closed-loop system is shown in Figure 3.11. The shuttle gives a peak displacement of 6.22nm under 1\,\mu N step force which is already 10 times smaller than open-loop response. Due to
the function of integration term, shuttle is stabilized around the initial position within $45\mu s$ which is comparable to open-loop step response settling time.

To clearly present the closed-loop system working under force-balanced mode, a continuous random step actuation is simulated, as shown in Figure 3.12. The bigger the actuation voltage step is, the large overshoot displacement the system can generate. After the constant settling time, approximately $45\mu s$, the initialized shuttle position...
should be expected. Compare the bode diagrams between open-loop system and closed-loop system, as shown in Figure 3.13, PID controller in the feedback loop decreases the loop gain by 20dB and both low-frequency and high-frequency actuation are suppressed in closed-loop system.

![Figure 3.12: Continuous step actuation for closed-loop system.](image)

A more realistic system simulation is performed considering the time constant of capacitance measurement (Lock-in amplifier SR830), as shown in Figure 3.14. When time constant on the front panel is selected as 300ms, Slope/Oct is selected as 18dB, the measuring electronics are modeled by three low-pass filters in series, \( \frac{1}{T s + 1} \), in which \( T = 0.3 \). The bias voltage is separated from voltage-to-force transducer constant \( K_f \) and set as 5V fixed. The voltage clamps limit the two channel feedback voltage from \( \pm 9 + 0.7 \). Since the model of capacitance measurement provides a dominant pole in forward path, which overwhelmed the dynamics of micro structures, the parameters of PID controller are adjusted to \( K_p = 5, K_i = 15 \) and \( K_d = 2 \) for optimal closed-loop response. The step response of the modified closed-loop system is shown in Figure 3.15. The shuttle moves to a certain position which is defined by structural open-loop transfer function in the beginning. Because of the sensing time constant, the shuttle is gradually balanced in 1.2s which is much longer than settling time 45\( \mu \)s shown in Figure 3.15.
Figure 3.13: Bode diagram of open-loop system (A) and closed-loop system (B).

Figure 3.14: The block diagram of a more realistic closed-loop system.
Figure 3.15: Closed-loop step response considering time constant of capacitance measurement.

Figure 3.16: Continuous step actuation for closed-loop system considering time constant of capacitance measurement.
Another continuous step response, related to modified closed-loop system, is shown in Figure 3.16. The output voltage of PID controller is given as green line, the value of which is limited by the clamp voltage. The bias voltage can amplify the electrostatic feedback force, generated by this small error voltage, by several times.

Several assumptions have been made to simplified the simulated model: 1. No time constant for CDA to generate applied load; 2. The damping coefficient is a constant value proportional to velocity, not relating to position; 3. Stators are rigid, no deformation is modeled under feedback electrostatic force; 4. Gains adjustment of PID controller are ideal, for instance, practical integral term is $G_i(s) = \frac{1}{K_i s^2 + p_i}$, $p_i$ is the integral phase constant, derivative term is $G_d(s) = \frac{K_d s}{\tau_d s + 1}$, $\tau_d$ is the derivative time constant; 5. Small deformation modeling guarantees the linear behavior of spring constant and electrostatic feedback force; 6. No electrical and mechanical noise has been considered in modeling.
Chapter 4

Experimental Testbed

With properly designed microstructures, we utilized a commercial multi-user MEMS process for microfabrication, which is also known as MUMPs. Specifically, we chose one of three standard processes, PolyMUMPs, for three-layer polysilicon surface micromachining fabrication. Process description and mandatory design rules are available in [60], which is highly recommended to review prior to design submission. After completing the 2-D layout drawing in a designated commercial software L-edit v12.6 (Tanner Research, Monrovia, CA), a recommended GDSII format file was submitted online to a commercial foundry (MEMSCAP, Research Triangle Park, NC). A typical run time from MEMSCAP is about three months. For 1 standard die site service, we received 15 dies per run with 10.1mm×10.1mm die size. Note that the die size will be updated due to new subdicing technique.

The closed-loop capacitive sensing structures are only one part of this multi-project chip shown in Figure A-1. Other micromechanical research projects also share the 10.1mm×10.1mm die area, including comb drive and thermal actuators tension, compression and bending testbeds for one-dimensional nanomaterial, nanowire resonators, Casimir oscillators and nanowire cantilever adhesion testing devices. In order to make the electrical connection off-chip, on-chip devices are routed to 100 peripheral gold pads.

In this chapter, a description of PolyMUMPs fabrication process will be presented followed by detailed electrical connections in system integration.
4.1 PolyMUMPs Process

PolyMUMPs process is a three-layer polysilicon surface micromachining process derived from the work performed at the Berkeley Sensors and Actuators Center (BSAC) at the University of California in the late 80s and early 90s. Several modifications and enhancements have been made to increase the flexibility and versatility of the process for the multi-user environment. The basic process includes, 7 physical levels: Nitride (0.6µm), Poly0 (0.5µm), 1st oxide (2.0µm), Poly1 (2.0µm), 2nd oxide (0.75µm), Poly2 (1.5µm), Metal (0.5µm), and 8 lithography layers: POLY0, ANCHOR1, DIMPLE, POLY1, POLY1_POLY2_VIA, ANCHOR2, POLY2, METAL.

A fabrication flow chart of key steps is shown in Figure 4.1 for illustration. The device to be fabricated is a torsional plate suspended by sides thin torsional rods which are anchored to the substrate by support post. Overall view of the device is shown in Figure 1.6(C). Starting from heavily doped N+ silicon wafer covered by low stress nitride and Poly0 layers. A standard photolithography process transfers the POLY0 pattern onto the wafer, Figure 4.1(A). The 1st oxide layer (2.0µm) is deposited on the wafer by low temperature CVD and patterned by ANCHOR1 and then processed through RIE to remove the oxide from the anchor area. DIMPLE mask is not shown in this example, but it should be note that DIMPLE is critical for any suspended long features, like cantilever beam, big plate, etc.. It may greatly reduce the risk of stiction when devices are actuated. ANCHOR1 defined where Poly1 will be attached to the substrate, and the thickness of the 1st oxide defines how far above the substrate Poly1 will sit after release. Figure 4.1(B) shows Poly1 is patterned by POLY1. The 2nd oxide layer (0.75 µm) is deposited, conformally coating the topography on the wafer and defining the separation of Poly1 from Poly2. The mask POLY1_POLY2_VIA defines the contact regions between Poly1 and Poly2, Figure 4.1(C). The wafer is then coated with Poly2 and patterned by POLY2, ANCHOR2 is not shown in the design. After the Poly2 layer is etched and the wafer is stripped, the basic mechanical structure is complete, Figure 4.1(D). Metal layer, sometimes, is deposited to increase conductivity, adhering to Poly2. A lift-off template is used to deposit the metal layer on the wafer and patterned by METAL mask, Figure 4.1(E). Followed by a 2 minute soaking in concentrated HF, all the sacrificial oxide layers are removed and the moveable mechanical parts are released. The plate is now free to rotate about side supporting beams.
Figure 4.1: PolyMUMPs fabrication highlighted processing features.
4.2 Electrical Connections

The 100 peripheral gold pads on die were designed to match with 100-pin ceramic pin grid array (PGA) package (Spectrum Semiconductor Materials Inc., San Jose, CA). First, when 15 dies arrived in a tape ring package (MUMPs 82), we used tweezer to remove each die from the adhesive tape to Gelbox (Gel-Pak, Hayward, CA). Second, we spread the PGA cavity with electrically conductive adhesive paste (Ablebond 8700E, Emerson&Cuming, Canton, MA) evenly and attached one of the chips to the cavity on top of paste. Third, in order to cure the paste fast, we put filled PGA package on to a hot plate for two hours with a set temperature 175K. Last, when the package was cool down,
the 100 gold pads on die were wire bonded to 100 leads around the cavity. Wire bonding was performed by a commercial packaging institution (RTI, Research Triangle Park, NC, Wire Bonder Type: West Bond 454647E). After accomplishing wire bonding, a glass lid (Spectrum Semiconducter Materials Inc., San Jose, CA) was covered on top to prevent dust or moisture in the air from undermining devices on die. Figure 4.2 shows the packaging process. A 13×13 169 pins Zero Insertion Force (ZIF) PGA socket (Digi-key, Thie River Falls, MN) was then used for electrical connection between PGA package and PCB on which all off-chip electronics are designed. All PGA packages should be stored in a dry environment, like nitrogen desiccator or vacuum oven, when they are not being tested.

As mentioned above, the 15 dies from MEMSCAP (MUMPs 82) are multi-project chips which are mostly designed for SEM experiments. Figure A-1 is the schematic layout view with identification labels for different types of devices. The function for each device is described in Table 4.1. Two major types of actuators included in design are electrostatic comb drive actuator (CDA) and electrothermal actuator (ETA). For CDA, 300 and 1200 comb finger setups are used to provide different electrostatic forces. For ETA, 5 pair of 30 degree V-shape thermal beams are designed to generate useful displacement range with large stiffness. In addition, 6 pair of heat sink beams are designed to prevent specimen

<table>
<thead>
<tr>
<th>ID</th>
<th>Device Function Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1, T2, T3</td>
<td>NW\textsuperscript{a} tension devices. VETA\textsuperscript{b}, PPCS\textsuperscript{c}.</td>
</tr>
<tr>
<td>T4, T5</td>
<td>NW tension devices. CDA\textsuperscript{d}, PPCS.</td>
</tr>
<tr>
<td>C1, C2</td>
<td>NW compression devices. VETA and CDA, PPCS.</td>
</tr>
<tr>
<td>B1, B2</td>
<td>NW bending devices. CDA, PPCS.</td>
</tr>
<tr>
<td>P1, P2</td>
<td>Casimir torsional plate. Bottom electrodes capacitive sensor.</td>
</tr>
<tr>
<td>D1, D2, D3</td>
<td>Closed-loop system testbeds. CDA, PPCS.</td>
</tr>
<tr>
<td>O1, O2, O3</td>
<td>NW Oscillators. Bottom electrode actuation.</td>
</tr>
<tr>
<td>S1, S2</td>
<td>NW adhesion devices. Horizontal and vertical actuation.</td>
</tr>
<tr>
<td>Z1</td>
<td>Cell tension device. CDA, PPCS.</td>
</tr>
<tr>
<td>Z2, Z3</td>
<td>CDA levitation suppression trial.</td>
</tr>
<tr>
<td>Z4</td>
<td>2nd Oxide layer insulation trial.</td>
</tr>
</tbody>
</table>

\textsuperscript{a} NW is one-dimensional nano-material, like CNT and nanowires.

\textsuperscript{b} VETA is short for V-shape electrothermal actuator (AKA bent-beam TA).

\textsuperscript{c} PPCS is short for parallel-pate capacitive sensor.

\textsuperscript{d} CDA is short for electrostatic comb drive actuator.
heating. Load sensors with different stiffness are designed to test different types of nano-materials. Table 4.2 lists geometrical parameter details and stiffness of each device. Besides nano-material testing devices including tension, compression and bending, other devices are designed to provide the stage to explore adhesion properties, Casimir force, etc.. Several trial devices that placed in the middle area and connect to internal pads can be actuated on probe station.
Table 4.2: Parameters of devices on chip (MUMPs 82)

<table>
<thead>
<tr>
<th>ID</th>
<th>Pins</th>
<th>Actuator (Units are $\mu$m and $\mu$N/$\mu$m)</th>
<th>Sensor (Units are $\mu$m and $\mu$N/$\mu$m)</th>
<th>Unit is $\mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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$^a$ ETP is short for electrostatic torsional plate.

$^b$ Stiffness unit of torsional plate is Nm/rad.

$^c$ TPCS is short for torsional plate capacitive sensor.
4.3 System Integration

Once the PGA package is sitting on the ZIF PGA socket, the devices on die is ready to connect with external circuitry which is design in an online-free, user-friendly PCB software PCB123 v2 and manufactured by a commercial PCB service (Sunstone Circuit, Mulino, OR). We designed two different circuits on board according to open-loop and closed-loop capacitive sensing applications. For open-loop capacitive sensing circuit, we adopted the similar experimental setup as in [4] by using a commercially available IC chip, MS3110, while for closed-loop capacitive sensing, the traditional AC-bridge sensing method is used for feedback loop [12]. All circuit details are described in this section, including advantages and disadvantages of both sensing methods.

4.3.1 MS3110 Open-loop Sensing

MS3110 (Irvine Sensors Corp., Costa Mesa, CA) provides an ultra-low noise, high resolution, internal parameters programmable capacitive readout interface for either differential capacitor pair or a single capacitor. It has been successfully used in many reported capacitive microsensors [4][61][62] because of its resolution of more than 0.1fF in practical measurement and programmable low-pass filter (LPF) bandwidth from 0.5 to 8kHz. An evaluation/programming board MS3110BDPC is designed to adjustable all the internal parameters, such as reference voltage, carrier signal frequency, feedback/parallel capacitor, output gain, etc.. Once the setting for a certain application is decided, a +16V power is supplied to write all the information into an internal EEPROM.

A system schematic diagram is shown in Figure 4.3. Four external filter capacitors are necessary for MS3110 stabilization. Differential MEMS capacitors represent parallel plate sensor or torsional plate on PolyMUMPs die we designed. Pin 4 and Pin 6 supply AC carrier signal for capacitance bridge which is 100kHz square wave signal with a peak-to-peak amplitude of 2.25V. Pin 5 is kept at a 2.25V electrical potential and connected to a common electrode that is usually the movable part of the MEMS structures. Other functional pins are used for internal registers programming. We utilized digital multimeter (Agilent 34401A) to measure the output of MS3110 (Pin 14) and acquired the output data on computer by connecting multimeter and PC with RS-232 cable.

The advantages of MS3110 are high resolution, stability, low drift and more impor-
tantly adjustable internal balancing capacitors. All these features are good for open-loop capacitive sensing, but it is limited when it is integrated with feedback loop circuit. First, feedback voltage should not be fed to common electrode because once the electrical potential of common electrode is alternated from reference voltage, MS3110 would output only 0V or 5V which depends on the potential on common electrode (larger or smaller than reference voltage). Second, an alternative feedback method is adding the differential feedback voltages to carrier signals. But no matter what circuit is used (operational amplifier adder or RC network) for superimposing DC signal on AC carrier signal, the carrier signal will be slightly changed due to nonidentical op-amps or block capacitors. Thus, theoretical transfer function of the chip is no longer accurate. Last, even the carrier signal could be kept as original, DC feedback voltage also introduces linear offset in output sensing signal. Because when parallel capacitor \( C(x) \) is charged by sensing carrier signal \( V_s \), the capacitor charge is written by

![Figure 4.3: Open-loop capacitive sensing using MS3110.](image)
\[ Q = C(x)V_s \] (4.1)

The current in capacitor is the time derivative of charge
\[ i_C = C(x) \frac{dV_s}{dt} + V_s \frac{\partial C}{\partial x} \frac{dx}{dt} \] (4.2)

When the carrier signal \( V_s \) is AC signal whose frequency is higher than mechanical resonance frequency, the parallel capacitor does not respond to high frequency electrostatic actuation, hence the term \( \frac{\partial C}{\partial x} \) can be ignored. Integrated by feedback capacitor \( C_f \), the output of op-amp is given by
\[ V_O = -\frac{C(x)}{C_f} V_{AC} \] (4.3)

But when \( V_s \) is DC signal, because of \( \frac{dV_s}{dt} = 0 \), the output of op-amp is turned to be
\[ V_O = -\frac{1}{C_f} \frac{\partial C}{\partial x} V_{DC} \] (4.4)

Thus, when feedback DC voltage is superimposed on high frequency AC carrier signal, the output voltage will not only include capacitance information but also capacitance change, \( \frac{\partial C}{\partial x} \) term, which cannot be eliminated by post low-pass filter (LPF).
\[ V_O = -\frac{C(x)}{C_f} V_{AC} - \frac{1}{C_f} \frac{\partial C}{\partial x} V_{DC} \] (4.5)

4.3.2 AC Bridge Closed-loop Sensing

In order to integrate feedback loop, another AC bridge capacitive sensing method is designed by implementing phase-sensitive detector (PSD, also known as lock-in amplifier) to eliminate DC feedback voltage introduced offset in capacitance measurement. Briefly introduce PSD principle by assuming AC carrier signal is \( V_{AC} \sin(\omega t) \), the output of pre-amp in Equation 4.6 is given by
\[ V_O(t) = -\frac{C(x)}{C_f} V_{AC} \sin \omega t - \frac{1}{C_f} \frac{\partial C}{\partial x} V_{DC} \] (4.6)

as shown in Figure 4.4. \( V_O(t) \) is then multiplied by the same AC carrier signal in the same phase, for example, the output of frequency mixer is given by
\[ V_{mix} = V_O(t)V_{AC} \sin(\omega t) \]
\[ = -\frac{C(x)}{C_f} V_{AC}^2 \sin^2 \omega t - \frac{1}{C_f} \frac{\partial C}{\partial x} V_{DC} V_{AC} \sin(\omega t) \]
\[ = -\frac{1}{2} \frac{C(x)}{C_f} V_{AC}^2 + \frac{1}{2} \frac{C(x)}{C_f} V_{AC}^2 \cos(2\omega t) - \frac{1}{C_f} \frac{\partial C}{\partial x} V_{DC} V_{AC} \sin(\omega t) \] (4.7)
in which high frequency terms, $2\omega$ and $\omega$, will be eliminated by following LPF. Hence, the output of LPF is only capacitance related signal,

$$V_{\text{output}} = -\frac{1}{2} \frac{C(x)}{C_f} V_{AC}^2$$  \hspace{1cm} (4.8)

A self-made PSD is built by referring to previous work [12][63]. A high-frequency RF transformer T1-6T-X65 (Mini-Circuits, Brooklyn, NY) is used for transferring 100kHz AC carrier signal from function generator (Agilent 33250A) to AC-bridge. A low-noise op-amp, OP37, is selected as pre-amplifier. A frequency mixer, TFM-3 (Level 7), and a RF amplifier, MAN-2, consist the major part of PSD. The output of frequency mixer is then fed to a cascaded LPF network with a cut-off frequency around 34mHz. The system resolution is calibrated as 15fF which is insufficient for parallel-plate capacitive sensor of maximum 20fF change. The poor performance is generally because of the quality of frequency mixer. For level 7 frequency mixer, the RF input permits only less than 600mVpp signal which limits the gain of band-pass filter (BPF), and by connecting IF output to op-amp load, a 400mVpp mixed output signal is expected, thus the LPF output range of $\pm150$mV (rms of 400mVpp) for 10pF difference sets the resolution.

A much better resolution of at least 0.1fF can be achieved by replacing self-made PSD with commercial lock-in amplifier SR830 (SRS, Stanford Research System, Sunnyvale, CA). A comprehensive and straightforward analog feedback loop is built upon general purpose op-amps, UA741. A protective diode voltage clamp sets the differential feedback voltage in the range of $\pm9+0.7$V (Values may vary based on battery status). Five adjustable parameters in feedback loop are: 1. Voltage offset, to obtain error voltage; 2. Proportional gain, $K_p$; 3. Integral gain, $K_i$; 4. Derivative gain, $K_d$; and 5. Voltage bias, to adjust electrostatic feedback force in proportion. Refer to Section 3.2 for the detailed discussion about force-balanced feedback loop.

Keep in mind that any circuitry shown above must be verified by the user in their particular application. No endorsement of any specific manufacturer’s product is intended. The experimental results of both sensing methods are shown in the following chapter. More experimental testbed pictures are shown in Appendix.
Figure 4.4: AC-bridge capacitive sensing circuit with PID feedback controller.
Chapter 5

Testbed Experimental Results

In this section, we present our experimental results of the MEMS testbed. Followed the work done in [4], we performed the capacitance measurement based on the IC readout, MS3110, for open-loop system. A self-made AC bridge capacitance sensing circuit was then tested to interrogate closed-loop force feedback system. Both open-loop and closed-loop results are given for verification.

5.1 MS3110 Capacitance Measurement

Aforementioned in Chapter 4, MS3110 has advantages in open-loop capacitance measurement. The reported resolution [4] could reach up to 0.05fF in capacitance change, which is equivalent to 1nm in displacement. The output voltage of MS3110 was converted to capacitance value by the calibrated formula.

\[
V_{out} = \text{Gain} \times V^{2P25} \times 1.14 \times \frac{C_2 - C_1}{C_f} + V_{ref} \quad (5.1)
\]

\[
\Delta C = \frac{(V_{out} - V_{ref}) \times C_f}{\text{Gain} \times V^{2P25} \times 1.14} \quad (5.2)
\]

where \(V_{out}\) is the output voltage signal we measured from digital multimeter, \(\text{Gain}, V^{2P25}\) and \(C_f\) are all internal programmable parameters.

Figure 5.1 shows a typical capacitance measurement curve of ETA-PPCS open-loop system. The voltage is converted to capacitance by Equation 5.1 and the displacement
from image analysis is converted to capacitance change by

\[ \Delta C = N\varepsilon_0 L t \frac{2x}{g^2} \]  

(5.3)

where \( N = 64 \) is the finger number of PPCS, \( \varepsilon_0 \) is the permittivity of free space, \( L = 118\mu m \), \( t = 2\mu m \) and \( g = 2\mu m \) are finger length, thickness and gap, respectively, \( x \) is the displacement obtained from image analysis.

The actuation voltage could reach up to 8V, the ETA could generate 500nm in displacement with 30fF capacitance change. The measured capacitance change agrees well with image analysis considering optical error.

Figure 5.1: Capacitance measurement of ETA-PPCS open-loop system.

The CDA-PPCS open-loop system was also tested, as shown in Figure 5.2. The actuator was a CDA with 300 comb fingers, the electrostatic force is calculated by using the electrostatic constant simulated in Chapter 2. The capacitance measurement still agrees well with the image analysis. The deviation between the analytical solution and experimental results is because that the calculated stiffness of folded flexure is larger than the real stiffness due to the over etching during the fabrication. The actuation voltage could reach up to 50V for CDA(300). Because CDA is the force control actuator, the displacement depends on the stiffness of supporting flexure. The experiment was performed on a device with stiffness of
20.5N/m, hence the CDA(300) could generate 1µm in displacement with 70fF capacitance change.

\[ \Delta C = \left( \frac{V_{out}}{Expand \times 10V} + Offset \right) \times Sensitivity \times \frac{\sqrt{2}C_f}{V_s} \]  

(5.4)

where \( Expand \) is the control parameter on the front panel which could be 1,10 or 100, \( Offset \) is a control function which could change the output by percentage range in \( \pm 105\% \), \( Sensitivity \) sets the resolution of the measurement range from 1nV to 1V, though 20mV was normally used, \( C_f \) is the feedback capacitor on our choice and \( V_s \) is the amplitude of the input carrier signal. Because the lock-in amplifier gives outputs the root-mean-square (rms) value of an AC signal, we need to take \( \sqrt{2} \) into consideration during the conversion.

The system under test was CDA(1200)-PPCS with stiffness 43.3N/m. Before we test the closed-loop system, we need to confirm that the capacitance measurement is

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**Figure 5.2:** Capacitance measurement of CDA(300)-PPCS open-loop system.

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5.2 AC Bridge Capacitance Measurement

5.2.1 Commercial PSD: SR830

The output voltage is converted to capacitance change by

\[ \Delta C = \left( \frac{V_{out}}{Expand \times 10V} + Offset \right) \times Sensitivity \times \frac{\sqrt{2}C_f}{V_s} \]  

(5.4)
accurate and trustable. An open-loop measurement experiment is shown in Figure 5.3. The measurement agrees well with the analytical value and confirmed by image analysis.

![Capacitance measurement of CDA(1200)-PPCS open-loop system.](image)

Figure 5.3: Capacitance measurement of CDA(1200)-PPCS open-loop system.

The step response time is related to the time constant we chose, ranging from $10\mu s$ to 30s. The larger the time constant is, the longer it will take for lock-in amplifier to output a more stable value. But the tradeoff is the rising time which is important for a closed-loop system. By choosing a normal time constant value of 300ms, 2 seconds is required to output a stable value as shown in Figure 5.4. Since the simulated step response is comparable to the experimental response, the simulated parameters of PID controller can be taken as a reference for selecting the electronic components, like resistors and capacitors.

### 5.2.2 Simultaneous Sensing and Actuating of CDA

Aforementioned in Chapter 4, PSD is able to extract AC carrier signal out of DC actuation signal. Since the mechanical structure is an attenuator to high frequency signal, as shown in Figure 3.13, the movable shuttle only responds to DC actuation voltage when a 100kHz carrier signal is superimposed on DC signal. The simplest way to combine the AC and DC signals together is by taking advantages of passive electronic components, inductors and capacitors. Because inductor blocks AC signal, while capacitors blocks DC signal, a
common RLC network, as shown in Figure 5.5, is apparently able to supply a combined power source. Though the AC coupled capacitor is in series with CDA capacitor, the input capacitance can be estimated by CDA capacitance only when AC coupled capacitance value far exceeds CDA capacitance value.

\[ C_{input} = \frac{C_{AC}C_{CDA}}{C_{AC} + C_{CDA}} = \frac{C_{AC}}{1 + \frac{C_{AC}}{C_{CDA}}} \approx C_{AC} \]  

(5.5)

Figure 5.5: Electrical diagram of simultaneous sensing and actuating for CDA.

The result of simultaneous sensing and actuation of CDA is shown in Figure 5.6. The structure under test was an independent CDA with 1200 comb fingers for bending application. The capacitance change is given by \( \Delta C = \alpha \frac{\sin t}{\theta} \Delta x \), where \( t \) is thickness and
$g$ is gap. Considering the fringing effect, a constant from FEMM simulation, $\alpha = 2.237$, was applied to the conversion from displacement to capacitance change. The experimental capacitance measurement does not agree well with the image analysis because the fringing constant may not be accurate when ground plane is present and tip fringing is not considered in this case. Also, the result does not quite follow the parabolic fitting curve due to the above reasons. A similar experiment will be performed on a SOI MEMS structure we recently developed, as shown in Figure A-2. Because the SOIMUMPs fabrication process provides thicker structural layer, 10$\mu$m or 25$\mu$m, and not conductive substrate, insulation layer, a better parabolic relationship between capacitance change and actuation voltage is expected.

![Figure 5.6: Capacitance measurement on CDA while actuation.](image)

### 5.2.3 Closed-loop Results

We used the same RLC network, shown in Figure 5.5, to superimpose differential feedback DC voltage on 100kHz AC carrier signal. A comparison between open-loop and closed-loop capacitance measurement output is shown in Figure 5.7. When 1200CDA was actuated under 5V, the output difference was apparent between open-loop and closed-loop. The bias voltage was set as 5V and the feedback voltages were not saturated. When increase
the actuation voltage, the feedback voltages will saturate at clamped voltage and counter-balanced force cannot initialize the movable shuttle. The experiment will be performed in SEM to verify the quality of the closed-loop by image analysis.

Figure 5.7: Closed-loop capacitance measurement on 1200CDA while actuated by 5V.
Chapter 6

Conclusions

In this work, we have demonstrated that a complex surface microelectromechanical system (MEMS) can be modeled with lumped-parameter components that are incorporated into component-level simulation tools, such as ANSYS and FEMM, and system-level simulation tools, such as Simulink in MATLAB. We have developed and adopt analytical models for several components, electrostatic comb drive actuator (CDA), electrothermal actuator (ETA), parallel plate capacitive sensor (PPCS), torsional plate capacitive sensor (TPCS), folded flexure and viscous air damping, all of which are frequently used in a broad class of microsystems.

The modeling of CDA has been fully discussed for a long time since the first invention [10]. The most accurate modeling of CDA is 3-D FEA of the whole structure in ANSYS, but not only a couple of fingers, to be specific, because of the global electrostatic force. This kind of FEA simulation is difficult to construct and time consuming. Our FEMM approach provide a fast and relatively accurate way to obtain the electrostatic constant by measuring the real geometry of comb fingers. The modeling of ETA is basically the duplication of the structure reported in [44]. All FEA simulation results and experimental data agree well with previous work.

Besides the classic PPCS, the modeling of TPCS is also derived in Chapter 2. Given a linear relation between differential capacitance and rotational angle, TPCS is similar as PPCS. Note that the calculation for torsional stiffness of hinge is not trivial. An accurate expression of torsional moment of inertia is given for system modeling. The verification of ANSYS coupled field simulation is not shown in this thesis.
A straight forward system modeling is described in Chapter 3. Because of the advantage of residual stress relief, the folded flexures are designed for all the movable structures on die, stiffness of which is modeled structurally and verified by FEA simulation. The estimation of viscous air damping, however, is hard to verify by simulation or experiments and may result in 10% to 20% discrepancy as reported [56]. The PID feedback control is proved to be useful in stabilizing the movable shuttle by system-level simulation in Simulink. A differential voltage-force feedback transducer is built to close the loop. The parameters used in the PID controller and simulated signals in the loop provide a reference and guidance for empirical system.

Two approaches have been used for capacitance measurement with high resolution. One is commercial capacitance readout IC which provide resolution of 0.05fF equivalent to 1nm in displacement. The other is AC bridge with phase sensitive detector (PSD) module. The self-made PSD gives the resolution of 15fF equivalent to 300nm in displacement which is even worse than optical microscope. A commercial lock-in amplifier, SR830, substitutes the self-made PSD and improves the resolution up to 0.02fF equivalent to 0.4nm in displacement. The good agreements are shown in the results of open-loop capacitance measurement.

Closed-loop performance is limited by noise level and PPCS structure. In order to eliminate noise fluctuation, a large time constant of lock-in amplifier is required to output a stable value which defines the system dynamic response. Due to the geometry of designed PPCS, the counter-balanced feedback force, considering clamped voltage, constrains the actuation voltage under 10V. An open-loop step response of 5V actuation voltage is compared with closed-loop step response under the same circumstances. The force-balanced mode is apparent.

6.1 Future Work

- The obstacle of the dynamic system is the noise level due to multiple sources, like interconnect noise, electronic noise and Brownian noise. A better experimental approach is placing the electronics in the vacuum or shielding the circuits in the Farad cage to isolate environmental noise source. Other noise sources need to be fully characterized.

- The analytical representation of CDA is highly related to thickness/gap ratio. A complicated calculation is needed to estimated the electrostatic force from CDA when the
structural thickness is small, conductive ground plane is present and vertical stiffness is comparable to horizontal stiffness. A SOIMUMPs design has been submitted for a more analytical reliable CDA structure with $10\mu m$ structural thickness, as shown in Figure A-2. Calibrations will be required for these devices.

- Applying feedback voltage to PPCS has disadvantages: one is, amplitude of feedback voltage is constrained by fixed beam pull-in voltage because of the ground plane; the other is, length and number of sensor finger is limited by fabrication process. Hence, the balanced force is relatively small compare with actuation force, which suppressed the balanced range. To better apply the force-balanced function, a comb drive sensor is designed to generate an equivalent feedback force with respect to comb drive actuation, as shown in Figure A-2.

- One application of closed-loop capacitive sensor is integration of nanomaterial testing system. Once the load sensor is initialized, the load force is proportionally related to output voltage. Combined with the elongation of specimen, the mechanical properties of the nanomaterials can be investigated.

- The other application is closed-loop surface force measurement system on TPCS, like interfacial force microscopy (IFM). When a surface force is applied to TPCS, the force-balanced loop overcomes the “snap-in” phenomenon and makes the interrogation of surface force in less than 100nm possible.
Bibliography


Appendices
APPENDIX A

Figure A-1: The layout drawing of PolyMUMPs 10mm×10mm chip. The labels represent different types of devices.
Figure A-2: The layout drawing of SOIMUMPs 10mm×10mm chip. The labels represent different types of devices.
APPENDIX B

Figure B-1: Photograph of external electronics when using MS3110 to measure capacitance, evaluation board and control board are connected through 8 by 2 pin array, control board and test board are connected through 15-ribbon cable.
Figure B-2: Photograph of external electronics when using self-made AC bridge to measure capacitance, all connections are through 15-ribbon cable, cable directions of inside and outside of flange are different.

Figure B-3: Photograph of all experimental apparatus for power source, data acquisition and image analysis.