ABSTRACT

KRISHNAMURTHY, GAUTHAM. Multifunction Periodic Switching Front-End Circuits for Multi-Antenna Integrated Wireless Receivers. (Under the direction of Dr. Kevin Gard).

In the domain of integrated wireless receivers, multi-antenna techniques are gaining increasing relevance and application as a result of the many advantages they offer. Chief among these advantages are improved throughput by spatial multiplexing in MIMO systems, higher SNR by diversity gains and directional gain / interference cancellation by phased array combining. However, the current state-of-the-art in multi-antenna receivers is to resort to the duplication of front-ends for each separate antenna. This leads to the obvious drawback of higher system area, power and cost.

This research addresses these drawbacks of the traditional approach with a new receiver architecture termed the Switched Front-end Multi-antenna Receiver (SFMR) architecture. The SFMR style employs fast periodic switching within the front-end in order to permit the sharing of as much of the downstream circuitry following the antenna as possible. Two types of periodic switching are primarily investigated: Time-Division Multiplexing (TDM) to exploit spatial multiplexing gains and Coherent Combining (CC) for SNR improvement like in beamforming and diversity systems.

A solution is proposed to address the cross-interference between multiplex channels that affected previous attempts at TDM for multiple receive antennas. The theoretical framework from first principles for the analysis and design of these periodic switching
systems from a radio hardware viewpoint is developed. Closed form relations for noise figure are developed that are useful for quick figure of merit evaluations of the SFMR architecture.

Four front-ends based on the SFMR architecture are implemented as CMOS designs in 0.18 μm technology for 2.4 GHz RF signals to validate the theory. Two designs are single-ended versions of a quad-amplifier array, while one is a differential version of the same. The final design is a dual-antenna differential direct-conversion receiver including all circuit blocks down to the baseband demultiplexing. The differential front-ends are multifunctional in the sense that the circuits may be reconfigured digitally and, with the help of a programmable high-speed digital controller, put in either TDM or CC modes of operation.

All these designs are characterized fully to determine their gain, matching, noise and linearity performance, as also their time-domain operation. Thus, an understanding of the challenges and rewards posed by the SFMR architecture is obtained.
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Multifunction Periodic Switching Front-End Circuits
for Multi-Antenna Integrated Wireless Receivers

by
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DEDICATION

To my parents
BIOGRAPHY

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My parents taught me the value of learning, education and hardwork. They have made extraordinary sacrifices and always been there for me through the years. For these and more they have my deepest respect and appreciation. Special thanks to my brother and sister for making my life more fun and meaningful.
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1 INTRODUCTION

1.1 Motivation

Traditional single antenna wireless communications receivers are hard pressed to provide improvements in system capacity and reliability demanded of present day cellular and broadband wireless systems. These challenges have led to the development and deployment of methods such as diversity, adaptive beamforming and MIMO to boost spectral efficiency of the communication link as well as improve system robustness. One characteristic common to all these methods is the use of multiple antennas at the transmitter and/or receiver, often under the umbrella of smart antenna technology [8], [9].

Digital cellular technologies such as 3.5G and 4G, and wireless networking technologies such as IEEE 802.11n/WLAN and 802.16e/WiMAX have adopted multi-antenna techniques as an integral part of their standards. The other reasons for the use of multiple antennas in wireless transceivers are to support operation across disparate frequency bands and simultaneous multi-standard wireless operation. A sample roadmap that illustrates the inclusion of multi-antenna techniques in future wireless standards is shown in Fig. 1.1.
Fig. 1.1 Wireless roadmap showing profusion of standards using multi-antenna techniques

Implementations of multi-antenna systems have been envisioned over different areas of coverage, varying from short range links upto cellular MAN, as seen in Fig. 1.2.

Fig. 1.2 Multi-antenna wireless system roadmap across different ranges (Credit: [59])

The main multi-antenna techniques are summarized in Fig. 1.3. Diversity techniques at the receiver exploit signal coherence in multiple independent receive paths to improve link
reliability and capacity by either implementing optimal antenna selection or optimal combining of antenna signals [1], [2]. Phased array receivers have a uniformly spaced arrangement of uncorrelated antennas, which produce an electronically steered antenna beam providing high gain in the direction of the beam while rejecting interferers outside the main beam [3]-[5]. Spatial multiplexing MIMO techniques make possible significant gains in channel capacity, spectral efficiency and interference rejection capabilities by transmitting independent data streams over multiple antennas and taking advantage of the rich scattering of transmitted signals – the multipaths – in the environment between the transmitter and receiver [6], [7]. All of the gains may not be simultaneously possible as they tradeoff against each other depending on the signaling scheme and transceiver design. There are tradeoffs between diversity and multiplexing gains available in MIMO [10], thus a radio receiver designed with maximum flexibility by providing for selection between both these would improve throughput [11]. All of these techniques have in common the requirement for multiple antennas at the receiver, the signals from which must be processed and/or combined somewhere in the receive system.
The state-of-the-art in integrated antenna array wireless systems is to do one of the following

1. Replicate the receiver chain by brute force for each antenna element [12]-[15] to leverage the advantages of sophisticated DSP at baseband as shown in Fig. 1.4.
These systems provide maximum flexibility as they can conveniently accommodate diversity, beamforming for interference cancellation and MIMO with spatial multiplexing by software control at baseband. However because of limited reduction of redundancy in transceiver signal paths, these advantages come at the expense of increased die area, power consumption and ultimately system cost. These costs increase proportionally with the number of antenna elements.

2. Combine the signals at RF or IF following phase-shifters implemented either as regular phase shifting networks in the RF [17], IF or LO [18] signal paths. The Cartesian combining technique introduced in [16] uses a combination of variable gain amplifiers at RF and complex downconversion to realize the vector signal combination at IF. The commonly used phased array configurations are shown in Fig. 1.5.

These approaches, although lowering the chip area and power to various degrees depending on the location of signal combining in the receive chain, are cumbersome to implement and control. They also result in the loss of spatial multiplexing (SM) gains.
provided in true MIMO. The loss in SM gains is because individual antenna information is no longer available following signal combining.

Fig. 1.5 Conventional phased array signal combiners
1.2 Previous Work

The twin trends of greater antenna diversity and availability of cheaper digital processing closer to the antenna in the receiver chain has created a need for low power, small silicon footprint hardware in the spirit of low cost CMOS integration. This has spurred the development of receiver circuits which support antenna diversity while keeping duplication of critical high power front-end circuits to the minimum.

Many alternative receiver architectures are possible [22] for accommodating antenna arrays and adaptive beamforming, which are becoming increasingly prevalent because of the numerous advantages they offer [38]-[40]. However, as noted in [37] “From an architectural perspective, the best hope lies with systems where the signal path may be re-used, rather than duplicated, between multiple applications/standards. In the receive signal path, this typically implies keeping the signal wideband at high frequency and performing the filtering at a very low-IF, or baseband, where integrated programmable filter structures may be implemented in either the analog or digital domain.”

Due to the low cost and increasing integration possible with scaled CMOS technologies, one method that is highly attractive is the concept of antenna switching for time division multiplexing (TDM) onto a single receive channel. In this method, which has a long history [24]-[29], a reduction in RF hardware is obtained by switching individual elements of the antenna array onto a single RF channel. The switching is done at a high enough rate for the desired signal attributes at each antenna to be recoverable by digital processing at baseband. An input multiplexing approach for hardware reductions is also exploited in the
design of a HP network/spectrum analyzer [30] where the same receiver hardware is shared among multiple analyzer ports.

The possible use of a rotating antenna switcher to sample antenna signals in a diversity reception system was proposed as early as 1931 by Beverage and Peterson [27] in their work that addresses the fading phenomenon in short wave radiotelegraphy. However, implementation of the switcher was not described in their work. There is however an acknowledgement of likely difficulty in achieving receiver selectivity due to the sidebands generated in such an antenna switching system. Interestingly it is very much the same reasons that has held back or led to abandoned efforts [36] in recent history.

The formal theory of Time Division Multiplexing (TDM) was presented in a treatise by Bennett [24] in 1941. This classic work besides developing the general theory of time division multiplex systems, also discusses its implications on the bandwidth and crosstalk of these systems. Bennett’s treatment also predicted that the restrictions on gain and phase characteristics would be stringent if economy of the bandwidth is desired in the multiplex systems. Some of the origins of what was formalized as the Sampling Theorem by Shannon [23] in 1948 are clearly visible in this early work. A detailed historical account that traces the evolution of the TDM idea to the present day sampling theorem is available in [25]. Another paper on TDM important from an historic context is [26] that analyzes and offers closed form results for multiplex channel crosstalk.

Reducing the number of RF sections in antenna array systems by continuous switching through the array is explained as the “Space Hopping” scheme in [31] for BER improvement in a multipath fading environment. Another similar consideration of the topic is
provided in [32]. Clear system analysis from a TDM perspective is not available in this paper.

The Spatial Multiplexing of Local Elements (SMILE) scheme for digital beamforming [19]-[21] is another recent avatar of the antenna switching idea. The SMILE work demonstrated a low-complexity proof of concept system for processing microwave phased array signals by multiplexing them in time. The prototypes developed showed experimental results for beamforming and direction of arrival estimation. Detailed analysis of the time multiplexing concept as applied to the phased array system was however not presented. The lossy p-i-n diode switches used initially in [19] were replaced with HJFET active switches in [20] for improved noise performance and faster switching. They were both implemented as antenna arrays incorporating discrete switching devices on a circuit board. In the case of passive diode switch, there were problems with noise and speed of the array antenna system. The use of amplifying switching elements alleviated these problems somewhat, however the problems associated with changing antenna loading due to the multiplexing action of the switches still remained [21]. Careful design of the array feed network therefore became necessary. Moreover the developed structures were bulky and not amenable to easy integration. The SMILE architecture as presented by Itoh et al. described the use of multiple channels with multiple ADCs for IF/baseband processing.

More recent work geared towards the sharing of receiver components is presented in [35] in which a low noise amplifier array where a single active amplifier may be selected was demonstrated. This rudimentary design is however primarily intended for selection diversity applications where only one antenna is selected for a long period of time. The circuit
presented in [35] is incapable of high speed operation necessary, as discussed in this dissertation, for multi-antenna techniques other than selection diversity. The degeneration inductor for input matching also cannot be shared as shown, as it no longer is capable of providing the desired real input matching impedance for each of the inputs independently in this position.

In [36], a single pair of ADCs is used for a dual-antenna system by shifting the antenna signals to positive and negative frequencies of a complex spectrum. One major downside to this technique is that it is restricted to systems with two antennas only.

Another more recent work [55], [56] proposes the combining of signals at the RF stage after spreading them with mutually orthogonal code sequences, like in the CDMA system of communications, following which a single receive chain can be utilized. Unlike in regular CDMA where the actual channel data is bandlimited to manageable bandwidths for available DSP to handle in the spreading process, this approach (like the TDM case presented here) runs into the challenge of having to contend with RF input signals that are not necessarily limited to manageable bandwidths. Hence unless the front-end code modulation is done at very high, carefully managed rates, it is expected to suffer difficult noise and interference aliasing problems. A consequence of using high code modulation rates to overcome these problems is the requirement of non-digital de-spreading circuits prior to digitization. Compared to an equivalent TDM system, therefore, the required high switching rates could make the baseband circuitry bigger, more complex and challenging to design. However, the additional baseband hardware could simplify timing synchronization issues compared to a TDM approach. Moreover, successful implementation of this signal
combining scheme could lead to some bandwidth savings compared to techniques such as TDM.

The technique of full range amplitude and phase weighting the antenna signals for microwave beamforming simply by switching them with controlled sampling pulse trains of different time delays and duty cycles is discussed in [31]-[34]. This method is equivalent to the LO shifting method of [18], except that the additional LNA stage of [18] is removed, the analysis is from a sampling perspective and the switching pulses are from a programmable digital controller offering enhanced flexibility.

1.3 Switched Front-end Multi-antenna Receiver (SFMR) Overview

In the Switched Front-end Multi-antenna Receiver (SFMR) introduced in this work the best of both traditional approaches discussed in 1.1 are leveraged by incorporating high speed active (amplifying) switches following the antennas. The system block architecture for the SFMR is shown in Fig. 1.6. The general architecture between the front-end multiplexing and the baseband processing can retain the form of conventional receiver architectures such as superheterodyne, direct conversion, etc. (each will have its own impact on performance though). The most important changes are in the receiver front-end and baseband processing. In the front-end, RF signals from an $N$ element antenna array would be combined into a single RF signal path. At baseband the signals may need to be split into separate antenna data streams by demultiplexing depending on the mode of operation. The front-end switching, baseband deserializing and sampling are all done under synchronous control. With digital processing becoming cheaper and more powerful, this architecture would prove very
attractive for antenna array systems because of the greater flexibility offered with minimal hardware overhead. The digital flexibility would permit the reuse of the same general receiver circuit in different antenna diversity applications such as spatial diversity, adaptive beamforming and MIMO-SM. The hardware – and cost – savings of this multiple antenna receiver architecture are considerable. The number of mixers and ADCs are reduced by a factor of \( N-1 \) for an \( N \) antenna receiver. Also this arrangement could also serve as a generic architecture for multiple antenna RF receivers, the number of antennas being limited only by the bandwidth of the available process technology.

Fig. 1.6 Switched front-end multi-antenna integrated receiver architecture

The two modes of combining operation that this Switched Front-end Multi-antenna Receiver (SFMR) architecture supports are:

1. Time-Division Multiplexing mode: In this mode of operation, only one switch of the array is active at a time. This may be in the form of selection diversity switching or
periodically rotated switching – time division multiplexing – at high rates for maximal-ratio, equal-gain combining or MIMO-Spatial Multiplexing operation. The periodic switching combines the multiple antenna signals sequentially in time and then uses one signal path to downconvert and digitize the single multiplexed signal. The switch rate would usually be much higher than the information symbol rate at the antennas, yet significantly lower than the RF carrier frequency. The multiplexed signal path has to be wideband in order to ensure the multiplexed channels do not interfere with each other, i.e. information from one time slot smear into another time slot, much like the Intersymbol Interference (ISI) encountered in digital communications. This implies that the multiplexed signal cannot be subjected to channel select filtering. This has been a major stumbling block that has prevented the implementation of the TDM scheme in prior work [36] on multi-antenna receivers. The solution proposed in this work is to implement a baseband circuit that demultiplexes the downconverted signal, applies channel select filtering on the individual antenna channels and then remultiplexes them for digitization by a single ADC. This analog baseband circuit is functionally equivalent to a digital decimator, and is indicated as a decimator in Fig. 1.6. Although this may seem counter-intuitive, as the whole purpose of the switched front-end is to prevent unnecessary duplication of circuit blocks, from an overall system perspective there is still a significant cost saving as the number of mixers and ADCs are reduced to one of each. The emergence of discrete-time charge-domain signal processing solutions for alias reject filtering in digital radio architectures [43], [44] allows for the possibility that such a decimator circuit can be devised and practically implemented in the near
future. All sampling and resampling operations right from the front-end onwards (including multiplexing) would have to be under synchronous clock control. The degree of mismatch in synchronization between the multiplex and demultiplex stages due to imperfections in the switch control signal distribution is an important concern in this architecture. These imperfections include skew and jitter, whose effects become more prevalent at higher switching rates. But, the clock derived switch control signals being within a single die are more easily managed compared to wireline or wireless TDM communication links.

2. Coherent Combining mode: In this mode of operation, the signals are complex weighted and combined to perform full range beamforming or diversity combining simply by controlling the time delays and duty cycles of the pulses that control the front-end switches. No special modification of the front-end is required to accommodate this mode of operation, as the same switches can be reused. The only changes are in the waveforms sent to the switches by the digital controller which can be quite sophisticated in the paradigm of the Digital Radio Processor [43]. The decimation block may be bypassed and powered-off in this mode of operation.

Thus, the reconfigurable circuit architecture developed in this work provides for maximum flexibility of the receiver chain replication approach as well as the cost savings of the signal combining approach in a single low complexity integrated system. The main advantages of this SFMR architecture implemented in CMOS are summarized below

1. It allows multiple signals to share a single set of hardware thereby reducing area, power consumption and cost.
2. Excellent high speed switches can be implemented in RF front-end and baseband CMOS circuits.

3. CMOS circuits provide the wide signal path bandwidths inherently required by switched and time division multiplexed signals.

4. Implementation in CMOS processes enables co-integration with the digital modem.

5. The architecture is compatible with existing and developing trends in integrated CMOS receiver design. It can absorb design methods from subsampling receivers as well as traditional direct conversion and Low-IF receivers.

6. A subtle point is that the SFMR is uniquely suited for single chip integration, as discrete modules will not be able to satisfy the switching and performance specs required for its successful implementation.
2 THEORY OF OPERATION

2.1 Time Division Multiplexing (TDM)

2.1.1 Foundations

In the discussion that follows, a distinction needs to be drawn between the channels comprising the RF spectrum and the channels of data from the individual antenna elements. The actual reference if not clear from the context, will be explicitly stated.

The analysis of RF TDM proceeds along the same lines as the theory behind natural sampling that is applied to Pulse Amplitude Modulation in communication theory. Natural sampling is the simplest of sampling processes involving switch gating of analog waveforms as opposed to impulse and flat-top (zero-order hold) sampling. Natural sampling may also be termed rectangular pulse sampling. This type of sampling is therefore well suited for RF signals as other types of sampling place more stringent requirements on the hardware. The only major difference is that conventionally the technique and analysis is applied to bandlimited baseband signals sampled at traditional Nyquist rates and above. In the
multiplexing context, natural sampling will be done at rates significantly lower than the RF carrier frequency, i.e. the RF signal is bandpass sampled.

Fig. 2.1 Spectrum of real-valued bandpass filtered antenna inputs

The basic requirement for recovering data from each of the antenna elements is given by the bandpass sampling theorem \[41\]. Assuming a real-valued bandpass signal bandwidth of \(B\) centered at carrier frequency \(f_c\), as seen in Fig. 2.1, to avoid spectral aliasing each element of the array must be sampled at a rate \(f_s\):

\[
\frac{2f_c + B}{q} \leq f_s \leq \frac{2f_c - B}{q - 1}
\]  
(2-1)

where \(q\) in an integer given by

\[
1 \leq q \leq \left\lfloor \frac{f_c + 0.5}{B} \right\rfloor
\]  
(2-2)

The graphical illustration of equations (2-1) and (2-2) is in Fig. 2.2.
The theoretical minimum sampling rate is a pathological case corresponding to integer band positioning of the bandpass signal, seen in Fig. 2.2 as the tips of the wedges. This minimum sampling rate conditions is simply stated as

\[ f_s \geq 2B \]  \hspace{1cm} (2-3)

If \( u \) as the oversampling factor above this minimum rate, \( f_s \) is rewritten as

\[ f_s = 2uB \]  \hspace{1cm} (2-4)

As depicted in Fig. 2.2 bandpass sampling does not permit \( f_s \) to be arbitrarily chosen as any value satisfying condition (2-3) as this could lead to overlapping of aliases that could
corrupt the complex message signal depending on its band position. Also, the choice of sampling rate must take into consideration system nonidealities and a practical sampling operating point is chosen as shown in Fig. 2.3.

![Fig. 2.3 Practical sampling operating point](image)

The optimum $f_s$ corresponds to the quadrature sampling case and is chosen such that

$$f_s = \frac{4}{2p+1} f_c$$

(2-5)

where $p$ is an integer. This centers the signal spectrum and its aliases in the Nyquist zones, thus simplifying downconversion and any subsequent alias rejection filtering. At this sample rate the phase difference between adjacent samples corresponds to $\pi/2$ at center frequency $f_c$, implying that if the samples are sorted into odd and even sequences they may be processed in quadrature.

In the context of the multiplex system analysis, for clarity sake, sample rate will refer to $f_s$, the pulse rate per antenna channel and multiplex rate to $f_{mux}$, the overall rate for all channels being multiplexed into a single RF stream. For the $N$ element array, the multiplex rate will be
The switch control pulse train for time division multiplexing is shown in Fig. 2.4. The pulse interval is $T$ corresponding to a frequency $f_s = 1/T$, and the pulse “on” duration is $\tau = T/N$. The pulse train corresponding to the $n^{th}$ antenna channel may be expressed as

$$p_n(t) = \sum_{k=\infty}^{\infty} \Pi(t - kT - n\tau) = \sum_{k=\infty}^{\infty} \Pi(t - kT - n)$$

(2-7)

where $\Pi(t)$ is the rectangular function defined as

$$\Pi(t) = \begin{cases} 
1 & \text{if } |t| < \frac{1}{2} \\
\frac{1}{2} & \text{if } |t| = \frac{1}{2} \\
0 & \text{if } |t| > \frac{1}{2}
\end{cases}$$

(2-8)
The Fourier Transform of $p_n(t)$ is given by

$$P_n(f) = \sum_{m=-\infty}^{\infty} P_m e^{-j 2 \pi f n s} \delta(f - m f_s) = \sum_{m=-\infty}^{\infty} P_m e^{-j 2 \pi m n s} N \delta(f - m f_s)$$

(2-9)

where $P_m$ are the coefficients of the rectangular pulse train $p_n(t)$ in its Fourier Series representation and is given by

$$P_m = \frac{\sin(m \pi f_s \tau)}{m \pi} = f_s \tau \text{sinc}(m \pi f_s \tau) = \frac{1}{N} \text{sinc}\left(\frac{m \pi}{N}\right)$$

(2-10)

The unnormalized sinc function is used in equation (2-10). The relationship between the pulse train and its spectrum is depicted in Fig. 2.5.

Fig. 2.5 Fourier Series representation of rectangular pulse train
Since ultimately individual antenna signals are of real importance for any meaningful baseband processing, it is their properties that need to be studied. It is also necessary to ensure that these individual signals are not unduly corrupted by any of the signal processing happening in the time shared receiver. On a per antenna channel basis, the result of the multiplexing operation is analogous to that of natural sampling, and may be expressed in equation form as:

\[ x_{n}(t) = x_{n}(t).p_{n}(t) \]  

(2-11)

where \( x_{n}(t) \) is the time domain RF signal at the \( n^{th} \) antenna. In frequency domain, this may be expressed as the following convolution operation

\[
X_{ns}(f) = X_{n}(f) \ast P_{n}(f) = \sum_{m=-\infty}^{\infty} P_{m} e^{-j2\pi mf_{s}} X_{n}(f - mf_{s}) = \sum_{m=-\infty}^{\infty} P_{m} e^{-j \frac{2\pi m}{N}} X_{n}(f - mf_{s}) \]  

(2-12)

where \( X_{n}(f) \) is the spectrum of the bandpass RF signal at the \( n^{th} \) antenna.

Several important inferences may be drawn from equations (2-10) and (2-12), and Fig. 2.5.

1. The original bandpass spectrum now repeats itself every \( f_{s} \) Hz in the frequency domain.
2. The maximum power in the multiplexed signal spectrum is concentrated at \( f_{c} \) corresponding to the main tone \( (m=0) \) of the pulse spectrum. The contribution of tones decrease as one moves away from the center of the pulse spectrum.
3. As \( \tau \rightarrow 0 \) or equivalently \( N \rightarrow \infty \), pulse train becomes an impulse train and so does the pulse spectrum, spreading energy over all aliases in the multiplexed output equally while simultaneously reducing the signal power at each alias.
4. The zeroth replica \((m=0)\) power per antenna channel drops by \(1/N^2\) as a result of the multiplexing action. This is often expressed as the pulse desensitization factor, the difference in magnitude of the peak pulse power and the main lobe power equal to \(-20 \log (\text{duty cycle})\) or \(20 \log (N)\) in dB scale. At the same time, the total signal power per antenna channel falls by \(1/N\).

5. The composite output of the multiplexer has spectral energy at multiples of \(f_i\) that should not be completely filtered out by any image reject filter following the LNAs. This is because any filtering to remove these spectral bands would lead to “cross-interference” between antenna channels that would make recovery of these channels impossible by simple demultiplexing at baseband. The presence of these aliased bands makes the design of downstream blocks more challenging as it adds to the nonlinear effects in the system. The specs on the noise (image-reject) filter are less stringent in a direct conversion receiver or the Low-IF architecture, hence these appear to be most suitable for the multiplexed receiver.

The strength of these aliased bands would grow as the degree of correlation between identical antenna channels decreases. As an example, consider what happens if identical bandpass signals are given to each of the antenna inputs. In this case all of the undesired replica visible in the individual antenna channel spectra would vanish in the composite signal spectrum. However, this is a special case and cannot be expected in practice due to the random nature of received signals.

Thus, if sections of the RF signal corresponding to each antenna channel are considered individually, replication of the signal spectrum would be evident due to their
sampled nature, the sampling being done with rectangular pulses. The spectral components repeat at intervals of $f_s$ in the frequency domain and their envelope has the characteristic $sinc(x)$ shape. This would show up on demultiplexing the signal at baseband in analog domain into separate antenna channels for further processing. The RF signal spectrum per antenna channel is depicted in Fig. 2.6. The solid line shapes are aliases of the bandpass content lying in the positive frequency region of the original (non-multiplexed) spectrum and the dashed line shapes the negative frequency content that has aliased into the positive region. Quadrature processing would eliminate the negative (or positive) frequencies of the original spectra, thus offering the possibility of reduced $f_s$. This however comes with an area/power penalty.

Fig. 2.6 Spectral components for each antenna channel in multiplexed RF output
The composite output from the multiplexer is simply a combination of the naturally sampled signals from the $N$ antenna channels. Mathematically this may be written in the time domain as

$$x_{\text{mux}}(t) = \sum_{n=0}^{N-1} x_{ns}(t) = \sum_{n=0}^{N-1} x_n(t) \cdot p_n(t) \quad (2-13)$$

where $x_{\text{mux}}(t)$ is the multiplexed output and by using the linearity property of the Fourier Transform, in the frequency domain as

$$X_{\text{mux}}(f) = \sum_{n=0}^{N-1} X_{ns}(f) = \sum_{n=0}^{N-1} X_n(f) * P_n(f) = \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} P_m e^{-j \frac{2 \pi n m}{N}} X_n(f - m f_s) \quad (2-14)$$

where $X_{\text{mux}}(f)$ is the spectrum of the multiplexed output. Since the antenna signals are all bandpass filtered to occupy the same spectral band, chopping up and serializing these pulsed RF signals with the switching waveforms shown in Fig. 1.6 would result in a composite output that has the original frequency band’s spectral energy spread out over the harmonics of the switching function. Spectral bands that would be generated at multiples of the sample rate should not be rejected by any image reject filter following the front-end LNAs, as they are necessary for antenna signal demultiplexing later on. In this respect the output from the LNA stage of the TDM receiver is different compared to that in a conventional single-path receiver. The output spectrum of the TDM front-end would appear as shown in Fig. 2.7.
Assuming a direct conversion scheme, the mixer would translate the entire composite signal spectrum of Fig. 2.6 down to baseband as in the conventional case. The signal spectrum per antenna channel in that case would appear as shown in Fig. 2.8.
The multiplexed signal path cannot be subjected to channel select filtering due to cross-interference considerations as explained later. Hence, the multiplexed signal bandwidth would be too high for a single ADC to be able to handle. Therefore the signals will have to be decimated to lower rates if they are to be digitized by a single ADC. This decimation will require that each of the antenna channels be separated, i.e. demultiplexed in some form, before subjecting them to subband or channel select filtering. The rate change structure will slow the signal rates down to speeds that the ADC can handle.

The decimation, besides low pass filtering the demultiplexed signals, can include characteristics to reject the unwanted components at multiples of $f_s$. The filter cutoff frequency $f_u$ should satisfy
\[
\frac{B_{sub}}{2} < \left| f_a \right| < \frac{f_s}{d} - \frac{B_{sub}}{2}
\]

(2-15)

where \( B_{sub} \) is the bandwidth of the desired wireless subband within the receive band of bandwidth \( B \), and \( d \) is the downsampling factor in the decimation. Also,

\[
B_{sub} \leq \frac{B}{d}
\]

(2-16)

The subband could contain one or more wireless channels, each of bandwidth \( B_{ch} \).

The ADC sample rate may now be written as

\[
\frac{f_{adc}}{d} = \frac{f_{max}}{d} = \frac{Nf_s}{d} = \frac{u}{d} N(2B)
\]

(2-17)

Once digitized, the antenna signals may be subjected to additional channel select filtering to select channels of bandwidth \( B_{ch} \) before the enhanced multi-antenna signal processing steps. The decimation structures’ and the ADC’s own spectral characteristic would be superimposed on the demultiplexed digital data streams. Their spectral response envelope would contain factors of the form \( \text{sinc}(ax) \) where \( a \) is a constant relative to the incoming sample rate. The final spectral shapes may be shaped by these \( \text{sinc} \) envelopes to provide nulls at predetermined frequencies to reject strong interferers.

**2.1.2 Aliasing, Bandwidth and Out-of-Band Interferers**

An important question relevant to the whole discussion is to determine what bandwidth \( B \) should be. It needs to be understood if it is simply the channel bandwidth or the entire RF receive bandwidth. The answer to this stems from frequency translation that is inherent to the multiplexing process. Bandpass sampling considerations indicate that this frequency translation or aliasing action will demand that \( B \) essentially be the entire receive
bandwidth as stipulated by the communications standard. Bandwidth $B$ would ultimately be determined by the band rejection provided by the RF filter immediately after the antenna, as anything outside the passband has potential for moving into the desired signal band due to the multiplexing operation. Therefore, $f_s$ would be much larger than the channel bandwidth $B_{ch}$ which is usually a small fraction of $B$. This in turn implies that antenna switching cannot be done at Nyquist rates corresponding to the symbol rate of encoded digital data.

The problem of aliasing due to multiplexing is analogous to the following issues elucidated in [37] and [35]

a) Reciprocal mixing of Local Oscillator sideband energy (phase noise) with the received RF signal

b) Low alias rejection in low-LO architectures: Low LO $\equiv$ Low sample rate here.

The difference in comparison to the above cases is that the primary purpose of the TDM stage following the antenna is to simply amplify the signal maintaining a low NF, without relocating its position in the spectrum as is the case with mixers or frequency conversion processes. Aliasing of undesired signals due to multiplexing essentially means signals at frequencies of no interest for deciphering the information encoded in the channel become indistinguishable from the desired channel frequency content. This happens simply by virtue of chopping up the continuous RF signal during multiplexing. As they cannot be differentiated any longer from the desired frequencies, they cannot be filtered out. The real danger is that this now happening right at the entrance to the receiver where signal levels could be low and interference levels high. Once the signals are corrupted due to large noise or interference, no amount of diversity combining at baseband can make a difference. In fact,
incorrect multiplexing in the presence of such interferers will only make the problem worse and defeat the purpose of multiplexing in the first place.

![Diagram](image)

**Fig. 2.9** Signal corruption due to in-band aliasing

Just as in subsampling receivers, high frequency spurious content folds into the desired signal band at baseband, a multiplex receiver faces the problem of unfiltered signals (noise or interference) outside the frequency of interest falling into the desired channel at RF frequencies. The graphic in Fig. 2.9 illustrates this clearly. It is for this reason, in a receiver
signal processing is carefully arranged to have large-scale undesired aliasing phenomena occur as late as possible – in the back-end of the receiver – normally at the ADC where sample and quantize takes place. It is also good to note at this stage that aliasing (due to sampling, however ideal that may be) has its own effect on SNR that is independent of SNR degradation due to quantization. To emphasize, signals just owing to their pulsed (sampled) nature can have an appreciably bad SNR without any quantization whatsoever.

Although a challenging technological hurdle, with faster and faster process technologies available, it should be feasible to implement the system for at least a limited number of antennas. Also with the advent of digital RF processing [43], such a system could fit in very well with the rest of the discrete-time receiver architecture. As in those architectures, the sampling rates need to be gradually decreased in a series of filter and decimate steps to achieve the best possible SNR for digital demodulation at baseband. The idea is to keep rates high enough to provide sufficient alias rejection filtering before stepping down the rates.

2.1.3 Channel Select Filtering and Crosstalk

This is the second major concern raised in [36]. The paper concludes that receiver multiplexing is possible only if common channel-select filtering is excluded. Having such a filter would result in “cross-interference” between the signals. However, this is not a new result and is analogous to the well understood Inter Symbol Interference (ISI) topic in digital communications. Simply put, using a narrow band channel select filter on the multiplexed
The minimum theoretical bound for the number \( W \) of switching aliases around the center frequency to be passed in a TDM system to have the possibility of zero interchannel crosstalk is worked out by Bennett [24] to be

\[
W = N
\]  

(2-18)

where \( N \) is the number of multiplexed channels. This bound is specified and valid for multiplex systems that employ impulse sampling. In practice the number of aliases required to minimize crosstalk would be more than \( N \). The actual number of aliases allowed to pass through determines the multiplex signal bandwidth. In this work the number was empirically chosen to be

\[
M = 2N
\]  

(2-19)

In this context, since the noise (image-reject) filtering is eliminated or less stringent in direct conversion architecture or the low-IF architecture, these appear to be most suitable downconversion schemes for the TDM receiver.

The critical bottleneck lies between the downconversion and the baseband ADC stages. As discussed elaborately previously, in order to avoid aliasing related problems, high multiplexing rates will have to be employed at the front-end. However these rates cannot be maintained after downconversion as no ADC will be able to digitize at these rates. Some means of slowing down the signal rate to more manageable levels will have to be developed. The only viable option to address this problem is to demultiplex the antenna channels before subjecting it to channel select filtering and then remultiplex the signals before passing it to a
single ADC. This may be accomplished by either analog switch or discrete time signal processing means and seems like the most tenable solution. Multiple ADCs may be employed following demultiplexing but that would go against the grain of the SFMR concept which is to minimize hardware common to multiple receive paths.

### 2.1.4 Switch Isolation Requirements

In order to prevent leakage and non-coherent combining of signals at the output of the switch combiner that could destroy the desired antenna information content, the designed front-end switches should maintain a certain on-off isolation between the input and output. This minimum requirement may be expressed for each switch channel as

\[
\text{Isolation}_{\text{min}} \text{ (dB)} = \text{DR}_{\text{fade}} \text{ (dB)} + 20 \log (N-1) \tag{2-20}
\]

where \( \text{DR}_{\text{fade}} \) is the dynamic range of a fade, the expected signal power excursion at the antennas in a fading situation. The 20 \( \log (N-1) \) factor arises because of the possibility of coherent combination of signals from the other channels. \( \text{DR}_{\text{fade}} \) is commonly in the 30-40 dB range. Since generally \( \text{DR}_{\text{fade}} >> 20 \log (N-1) \), equation (2-20) may be rewritten as

\[
\text{Isolation}_{\text{min}} \text{ (dB)} \approx \text{DR}_{\text{fade}} \text{ (dB)} \tag{2-21}
\]

In the context of a smart antenna system, though, the availability of powerful digital processing capabilities in current systems suggests that a fair amount of inter-multiplex-channel crosstalk can be factored out by employing strategies similar to RAKE filtering if certain signal characteristics are known apriori.

### 2.1.5 Noise Analysis

The following noise sources are relevant to noise analysis in a TDM system
1. Input thermal noise

2. Device added thermal noise: In the case of a MOS transistor, the drain-referred thermal noise PSD in both ohmic and saturation regions is given by [60] and

\[ S_{ID} = 4kT \Gamma (g_m + g_{mb} + g_{ds}) = 4kT \Gamma g_{ms} \]  

(2-22)

where \( g_m \) is the gate associated transconductance, \( g_{mb} \) the body-effect transconductance, \( g_{ds} \) the drain source conductance, \( g_{ms} \) is the total source conductance, and \( \Gamma \) the thermal noise factor.

3. Phase noise (jitter) of switching clock

In the noise analysis presented heretofore, only the wideband noise sources (1) and (2) that could be aliased inband were being considered. Although it is expected that (1) and (2) noises will dominate, a more complete analysis should also take into consideration the impact of (3).

The analysis of noise and SNR in the TDM case is very much like it is done for mixers, with the LO in the mixer being replaced by the switching clock pulses in this case. Mixers are generally analyzed as Linear Periodic Time Variant (LPTV) systems following the approach laid out in [42]. The sensitivity of the receiver for each of the analyzed receiver configurations may be determined using its relationship with noise figure

\[ P_{\text{min}} \text{ (dBm)} = -174 \text{ dBm} + 10 \log (B_{ch}) + \text{SNR}_{\text{min\_demod}} + NF_{\text{Rx\_config}} \]  

(2-23)

where \( P_{\text{min}} \) = Sensitivity in dBm

\( B_{ch} \) = Receive Channel Bandwidth

\( \text{SNR}_{\text{min\_demod}} \) = Minimum SNR required for Demodulation

\( NF_{\text{Rx\_config}} \) = Noise Figure of the Receiver Configuration being Analyzed
To allow a comparison TDM noise with a simpler case, the noise factor of a single amplifier of gain $G$ or the multiplexing amplifiers in the Selection Diversity case is stated as:

$$F_{amp} = \frac{(S/N)_{in}}{(S/N)_{out}} = \frac{1}{G} \frac{N_{outp}}{N_{inp}} = \frac{1}{G} \left( \frac{G N_{inp} + N_a}{N_{inp}} \right) = 1 + \frac{N_a}{G N_{inp}} = 1 + \frac{N_a}{N_{inp}}$$ \hspace{1cm} (2-24)

where $N_{outp}$ is the available inband output noise power

$N_{inp}$ is the available inband input noise power from the source

$N_a$ is the added noise power from the devices/components

$N_{ai}$ is the input referred added noise power from the device/components

The following general assumptions are made in the analyses that follow:

- The gain $G$ is wideband
- The available input noise $N_{in}(f)$ is white (constant spectral density) and stationary
- The bandpass filter is noiseless; the in-band filter loss is absorbed into the amplifier stage that follows, therefore the in-band gain is 1 and the out-of-band attenuation is $L$.
- The out-of-band input noise is filtered by the anti-aliasing front-end bandpass filter ($N_{ino} = \frac{N_{inp}}{L}$ where $L$ is the attenuation factor). The out-of-band is considered to be at all harmonics outside of the main $m=0$ harmonic of the noise transfer function $H_m(f)$.
- The device added noise is white, stationary and unfiltered ($N_{ao} = N_{ap} = N_a$)
- The switching is ideal, with zero rise and fall times. The switching devices have perfect “on” and “off” states, and are themselves noiseless.

In the case of a time division multiplexer, the noise per multiplexed channel is analyzed first. In order to do this, the multiplexing portion of the circuit is divided into
different stages as shown in Fig. 2.10. The first stage is the filter stage, followed by the amplifier and then the switching stage which is actually responsible for the LPTV properties of the system.

The noise output from the amplifying stage is simply

\[ N_{\text{out amp}}(f) = GN_{\text{out filt}}(f) \] (2-25)

By using the relations developed in [42], the noise output of the LPTV stage may be written as

\[ N_{\text{out}} = \sum_{m=-\infty}^{\infty} |H_m(f_c)|^2 N_{\text{out amp}}(f_c + mf_s) = G \sum_{m=-\infty}^{\infty} |H_m(f_c)|^2 N_{\text{out filt}}(f_c + mf_s) \] (2-26)

where \( H_m(f_c) \) is the noise transfer function at the RF carrier frequency, and is formally defined as

\[ H_m(f_c) = \int_{-\infty}^{\infty} \left[ \frac{1}{T} \int_{0}^{T} h(v+u,u) e^{j2\pi mf_s u} du \right] e^{-j2\pi f_c v} dv \] (2-27)

where \( h(v+u,u) \) is the impulse response of the LPTV system.
In this analysis under the assumptions previously stated, equation (2-27) simplifies to

\[ H_m(f_c) = P_m \]  \hspace{1cm} (2-28)

where \( P_m \) are the Fourier Coefficients of the pulse train, defined in equation (2-10).
In a practical system, all the noise harmonics of equation will not have to be considered and only those within the effective bandwidth $B_{eff}$ around the RF carrier $f_c$ will have to be evaluated. Equation (2-26) may be rewritten as for the practical case

$$N_{out} = \sum_{m=-\frac{B_{eff}}{2f_c}}^{\frac{B_{eff}}{2f_c}} \left| H_m(f_c) \right|^2 N_{out_{trans}}(f_c + mf_s) = G \sum_{m=-\frac{B_{eff}}{2f_c}}^{\frac{B_{eff}}{2f_c}} \left| H_m(f_c) \right|^2 N_{out_{filt}}(f_c + mf_s) \quad (2-29)$$

The attenuation factor $L$ could account for the finite circuit bandwidths and switching signal transitions. In this analysis, however, the worst case scenario of all harmonics through to infinity are considered which allows the use of the following identity

$$\sum_{m=-\infty}^{\infty} \left| H_m(f_c) \right|^2 = \sum_{m=-\infty}^{\infty} P_m = \sum_{m=-\infty}^{\infty} \left( \frac{\sin \left( \frac{m\pi}{N} \right)}{m\pi} \right)^2 = \frac{1}{N} \quad (2-30)$$

Rewriting equation (2-30) after pulling out the $m=0$ term

$$\sum_{m=-\infty}^{\infty} \left| H_m(f_c) \right|^2 |_{m \neq 0} = \sum_{m=-\infty}^{\infty} \left( \frac{\sin \left( \frac{m\pi}{N} \right)}{m\pi} \right)^2 = \frac{1}{N} - \frac{1}{N^2} \quad (2-31)$$

Using equations (2-26) and (2-31), the noise factor of a single antenna channel in the TDM case is given by
\[
F_{\text{TDM_amp}} = \frac{(S/N)_\text{in}}{(S/N)_\text{out}} = \frac{N^2 N_{\text{out}}}{G N_{\text{in}}} = \frac{N^2}{G} \left[ \frac{G}{N^2} N_{\text{in}} \left( 1 + \frac{N}{N_{\text{in}}} \right) + \frac{1}{N} \right] \]

\[
= \left( 1 + \frac{N_{\text{ai}}}{N_{\text{in}}} \right) + (N-1) \left( \frac{G N_{\text{ino}} + N_{a}}{G N_{\text{in}}} \right)
\]

\[
= \left( 1 + \frac{N_{\text{ai}}}{N_{\text{in}}} \right) + (N-1) \left( \frac{1}{L} + \frac{N_{\text{ai}}}{N_{\text{in}}} \right)
\]

\[
F_{\text{TDM_amp}} = \left( 1 + (N-1) \frac{1}{L} \right) + N \frac{N_{\text{ai}}}{N_{\text{in}}} = N \cdot F_{\text{amp}} - (N-1) \left( 1 - \frac{1}{L} \right) \tag{2-32}
\]

The best and worst case noise factors may be obtained as

\[
F_{\text{TDM_amp}} = N \cdot F_{\text{amp}} \quad \text{if } L=1 \text{ (worst case)}
\]

\[
F_{\text{TDM_amp}} = N \cdot F_{\text{amp}} - (N-1) \quad \text{if } L \to \infty \text{ (best case)} \tag{2-33}
\]

Thus from a single channel perspective, without exploiting any of the diversity advantages of the TDM system, the sensitivity of the receiver degrades to

\[
P_{\text{min}} \text{ (dBm)} = -174 \text{ dBm} + 10 \log (B_{\text{ch}}) + \text{SNR}_{\text{min_demod}} + \text{NF}_{\text{system_mux/channel}}
\]

\[
P_{\text{min}} \text{ (dBm)} = -174 \text{ dBm} + 10 \log (B_{\text{ch}}) + \text{SNR}_{\text{min_demod}} + \text{NF}_{\text{systemamp}} + 10 \log (N) \tag{2-34}
\]

There is degradation of \(10 \log (N)\) dB compared to the conventional receiver if only the output from a single multiplex channel is processed at baseband.

Since the noise under consideration is for the front-end amplifier section, it follows from Friis’ noise factor relationship that the system noise figure is impacted by the same factor as the amplifier noise figure. Thus from a single multiplex channel perspective, if no coherent signal combining process such as diversity or beamforming is exploited in the TDM system, the sensitivity of the receiver degrades by a maximum of \(10 \log (N)\) dB. This
degradation compared to the conventional receiver occurs if only the output from a single multiplex channel is processed at baseband. This is of relevance in the MIMO-SM case where individual antenna data streams can be expected to suffer an SNR, hence BER, degradation when compared to an equivalent continuous channel in a conventional multi-antenna receiver. However based on the following capacity equation for MIMO systems

\[ C = B_{ch} \min(M,N) \log_2(1+SNR) \text{ bps} \]  \hspace{1cm} (2-35)

where \( B_{ch} \) is the wireless channel bandwidth, \( M \) the number of transmit antennas and \( N \) the number of receive antennas, the capacity of the MIMO receiver is less dependent on degradation of SNR due to the \( \log \) relationship. The TDM approach still offers \( N \) independent receive channels with which the system capacity has a direct dependence.

The performance of the TDM wireless receiver is also placed in context by comparing its array noise factor with those of the more conventional single antenna receiver and coherent combining receivers. These are considered one at a time below. The available SNR at each antenna is assumed to be

Three receiver types are considered for comparison of output SNRs and array noise factors.

1. Single Antenna Receiver

In equation (2-36) and in the analysis that follows, \( N_a \) is assumed to be the overall device added noise in the system and \( G \) the overall system gain.

\[ \text{SNR}_{\text{single}} = \frac{G S_{in}}{G N_{in} + N_a} \]  \hspace{1cm} (2-36)

and
\[ F_{\text{single}} = \frac{\text{SNR}_m}{\text{SNR}_{\text{single}}} = 1 + \frac{N_a / G}{N_{\text{inp}}} = 1 + \frac{N_{ai}}{N_{\text{inp}}} \]  \hspace{1cm} (2-37)

2. Multiantenna Conventional (non-TDM) Diversity / Coherent Combining Receiver

Since the noise is assumed to be uncorrelated, they will add in power, while the signals being coherent will add in amplitude after diversity combining. Therefore

\[
\text{SNR}_{\text{conv.array}} = \left( \frac{N \sqrt{G S_m}}{N (G N_{\text{inp}} + N_a)} \right)^2 = N \left( \frac{GS_m}{GN_{\text{inp}} + N_a} \right) = N \cdot \text{SNR}_{\text{single}}
\]  \hspace{1cm} (2-38)

and

\[
F_{\text{conv.array}} = \frac{\text{SNR}_m}{\text{SNR}_{\text{conv.array}}} = \frac{1}{N} \left( 1 + \frac{N_a / G}{N_{\text{inp}}} \right) = \frac{1}{N} \left( 1 + \frac{N_{ai}}{N_{\text{inp}}} \right) = \frac{1}{N} \cdot F_{\text{single}}
\]  \hspace{1cm} (2-39)

3. Multiantenna TDM Coherent Combining Receiver

Under the previously stated assumptions, with the switching takes place immediately following the amplifiers, the output array SNR is

\[
\text{SNR}_{\text{TDM.array}} = \frac{\left( \frac{1}{N} \sqrt{G S_m} \right)^2}{N \left( \frac{G}{N^2} N_{\text{inp}} + \frac{1}{N^2} N_{\text{ap}} + G \left( \frac{1}{N} - \frac{1}{N^2} \right) N_{\text{naa}} + \left( \frac{1}{N} - \frac{1}{N^2} \right) N_{\text{ap}} \right)}
\]  \hspace{1cm} (2-40)

\[
\text{SNR}_{\text{TDM.array}} = \frac{G S_m}{G \left( \frac{1}{N} + \frac{1}{L} \left( 1 - \frac{1}{N} \right) \right) N_{\text{inp}} + N_a}
\]  \hspace{1cm} (2-41)

\[
\text{SNR}_{\text{TDM.array}} = \frac{\text{SNR}_{\text{conv.array}}}{N} \hspace{1cm} \text{if } L = 1 \text{ (worst case)}
\]  \hspace{1cm} (2-42)

and

\[
F_{\text{TDM.array}} = \frac{\text{SNR}_m}{\text{SNR}_{\text{TDM.array}}} = \left( \frac{1}{N} + \frac{1}{L} \left( 1 - \frac{1}{N} \right) \right) + \frac{N_{ai}}{N_{\text{inp}}}
\]

\[ = F_{\text{single}} - \left( \frac{1}{N} \left( 1 - \frac{1}{L} \right) \right) \]  \hspace{1cm} (2-43)
\[ F_{\text{TDM\_array}} = F_{\text{single}} = N \cdot F_{\text{conv\_array}} \quad \text{if } L=1 \text{ (worst case)} \]

\[ F_{\text{TDM\_array}} = F_{\text{single}} - \left(1 - \frac{1}{N}\right) \quad \text{if } L \to \infty \text{ (best case)} \] (2-44)

It may be summarized

\[ SNR_{\text{single}} \leq SNR_{\text{TDM\_array}} \leq SNR_{\text{conv\_array}} \] (2-45)

or equivalently

\[ NF_{\text{conv\_array}} \leq NF_{\text{TDM\_array}} \leq NF_{\text{single}} \] (2-46)

The conclusion from these results is that in the theoretical limit of infinite signal path bandwidth the difference in noise performance is independent of antenna channel sample rate and is only related to the number of multiplexed channels and the anti-aliasing bandpass filter characteristics. However in the real world, due to the availability of only finite signal path bandwidth, it follows from equation (2-29) and the transfer function coefficients as given by (2-28) and (2-10), that the noise aliasing is minimized with wider pulses and greater sample rates. Wider pulses and higher sampling rates have an inverse relationship, just as the need for wider pulses to minimize noise and narrower pulses to minimize inter-multiplexed-channel crosstalk have an inverse relationship [24]. Thus it must be possible to derive optimum parameters for the sampling pulse train.

### 2.2 Coherent Combining (CC)

In this mode of operation, the front-end amplifying switches no longer operate as multiplexers. Instead their function is to weight and combine the multiple input RF paths into a single receiver stream much like in the traditional Microwave Beamforming methods. This as expected comes about at the expense of the multiplexing diversity advantages of a MIMO
architecture. The weights are conveniently realized as programmable switching functions that are in the form of delay and duty cycle controllable pulse trains [33], [34]. The SFMR of Fig. 1.6 is therefore reconfigured internally to appear as shown in Fig. 2.11 by powering off and bypassing the unnecessary circuits. The hardware changes are primarily in the analog demultiplexing, filtering and remultiplexing section at baseband, as this section need no longer be switched continuously. It may be set to activate only low pass filter path before digitization.

Fig. 2.11 SFMR in Coherent Combining mode

The other significant changes are in the switching and oscillator functions. As opposed to the multiplexing mode of operation, more than one switch may be “on” at a time depending on the parameters of the switching functions for the antenna channels. An important observation for this coherent combining mode is that zeroth harmonic of the switching function cannot be used for implementing the weighting functionality of the signal combiner because it does not experience any phase shift. The best harmonic to choose from
among the remaining is the first harmonic, as this would be generally be responsible for the strongest signal which is important in the amplifying stage of the receiver from an SNR standpoint.

The theoretical treatment for analyzing and synthesizing the switching pulse trains is closely related to the TDM case discussed in section 2.1.1. This is because the bandpass sampling considerations as expressed by equations (2-1) thru (2-5) remain valid. Even the multiplexing results of equations (2-7) thru (2-14) will be seen soon as a special case of the coherent combining mode in the analysis that follows. However, as the signals once combined will not be separated further down the receive chain, the concept of multiplexing at rates given by (2-6) and demultiplexing at baseband is no longer relevant.

The generalized switching pulse train for the \( n^{th} \) antenna channel is as shown in Fig. 2.12, where \([|A_1|, |A_2|] \leq 1\).

![Diagram](image)

**Fig. 2.12 Generalized switching pulse train**

In equation form this would be
\[ p_n(t) = (A_1 - A_2) \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t - kT - t_{0N}}{\tau_n}\right) + A_2 \] (2-47)

with \( \Pi(t) \) being the rectangular function as defined by (2-8). The Fourier Transform of \( p_n(t) \) is given by

\[ P_n(f) = \sum_{m=-\infty}^{\infty} c_{mn} \delta(f - mf_s) \] (2-48)

where \( c_{mn} \) are the coefficients of the rectangular pulse train \( p_n(t) \) in its Fourier Series representation

\[ p_n(t) = \sum_{m=-\infty}^{\infty} c_{mn} e^{jm\pi f_s t} \] (2-49)

and is given by

\[ c_{mn} = (A_1 - A_2) e^{-jm\pi f_s (2t_{0N} + \tau_n)} \frac{\sin(m\pi f_s \tau_n)}{m\pi} = \left(A_1 - A_2\right) f_s \tau_n e^{-jm\pi f_s (2t_{0N} + \tau_n)} \text{sinc}(m\pi f_s \tau_n) \]
\[ c_{0n} = (A_1 - A_2) f_s \tau_n + A_2 \] (2-50)

The unnormalized sinc function is used in equation (2-50). On a per antenna channel basis, the result of the switching operation is analogous to that of natural sampling, and may be expressed in equation form similar to equation (2-11), repeated here for convenience

\[ x_{ns}(t) = x_n(t) \cdot p_n(t) \] (2-51)

where \( x_n(t) \) is the time domain RF signal at the \( n^{th} \) antenna. In frequency domain, this may be expressed as:

\[ X_{ns}(f) = X_n(f) \ast P_n(f) = \sum_{m=-\infty}^{\infty} c_{mn} X_n(f - mf_s) \] (2-52)
where $X_n(f)$ is the spectrum of the bandpass RF signal at the $m^{th}$ antenna. The output of the coherent combining operation is simply a combination of the naturally sampled signals from the $N$ antenna channels. Mathematically this may be written in the time domain as

$$x_{\text{combined}}(t) = \sum_{n=0}^{N} x_n(t) = \sum_{n=0}^{N-1} x_n(t) \cdot p_n(t)$$  \hspace{1cm} (2-53)$$

where $x_{\text{combined}}(t)$ is the output of the combiner and by using the linearity property of the Fourier Transform, in the frequency domain as

$$X_{\text{combined}}(f) = \sum_{n=0}^{N-1} X_n(f) = \sum_{n=0}^{N-1} X_n(f) \cdot P_n(f) = \sum_{n=0}^{N-1} \sum_{m=-\infty}^{\infty} c_{mn} X_n(f - mf_n)$$  \hspace{1cm} (2-54)$$

where $X_{\text{combined}}(f)$ is the spectrum of the combined output.

It may be observed the TDM case specified by equations (2-7) thru (2-14) is actually a special case of equations (2-47) to (2-54), obtained by setting

$$t_{on} = (n - \frac{1}{2})\tau$$

$$\tau_n = \tau = \frac{T}{N}$$

$$A_1 = 1$$

$$A_2 = 0$$

(2-55)

It can also be seen from equation (2-50), that the phase term disappears for $m = 0$. Therefore the zeroth harmonic term of the combined output (2-54) does not experience any phase weighting. The next best alias to select would be the $m = \pm 1$ terms because they would generally be of higher power content. If the downconversion mixer and the filtering is set up so as to select the $m = -1$ alias then the inband output becomes
\[
\hat{x}_{\text{combined}}(t) = \sum_{n=0}^{N-1} c_{-1n} x_n(t) e^{-j2\pi f_n t}
\]

(2-56)

\[
\hat{X}_{\text{combined}}(f) = \sum_{n=0}^{N-1} c_{-1n} X_n(f + f_n)
\]

(2-57)

and the complex weighting function for the \(n^{th}\) antenna is simply

\[
w_n = c_{-1n} = (A_1 - A_2) e^{j\pi f_s (2\tau_n + \tau_n)} \frac{\sin(\pi f_s \tau_n)}{\pi} = \frac{A_1 - A_2}{\pi} \sin(\pi f_s \tau_n) e^{j\pi f_s (2\tau_n + \tau_n)}
\]

(2-58)

The maximum output power for this weighting function is obtained when \(A_1 = -A_2 = 1\). With these pulse parameters, we obtain

\[
|w_n| = \frac{2}{\pi} \sin(\pi f_s \tau_n) \quad \text{and} \quad \angle w_n = \pi f_s (2t_{0n} + \tau_n)
\]

(2-59)

Equation (2-59) makes clear that the RF signal array \(x = [x_0, x_1, \ldots, x_{N-1}]\) can be amplitude and phase weighted simply by controlling the switching pulse train’s duty cycle and time delay respectively. The maximum amplitude is obtained when \(\tau_n = T/2\), i.e. the duty cycle is 50%. By controlling \(\tau_n\) in the interval \([0, T/2]\), the amplitude weighting may be varied as \(0 \leq |w_n| \leq 2/\pi\). With \(\tau_n\) at a fixed value, controlling the pulse delay times \(t_{0n}\) in the interval \([-T/2, T/2]\) allows the phase to be varied as \(\pi f_s \tau_n - \pi \leq \angle w_n \leq \pi f_s \tau_n + \pi\). Accordingly, for a specified amplitude weight and phase shift, the pulse width and initial time may be calculated using

\[
\tau_n = \frac{T}{\pi} \sin^{-1}\left(\frac{\pi}{2} |w_n|\right) \quad \text{and} \quad t_{0n} = \frac{\angle w_n}{2\pi} - \frac{\tau_n}{2}
\]

(2-60)
If the resolution in phase shifts required over $2\pi$ radians is $2\pi/M$ where $M$ is an integer, then this stipulates the time delays to be resolved to fractions of the switching rate given by

$$t_{0_{\text{res}}} = \frac{2\pi / M}{2\pi} T = \frac{T}{M} = \frac{1}{f_c M}$$

(2-61)

The signal transformations in frequency domain are summarized in Fig. 2.13. The spectral plots for the coherent combining mode of operation are similar to the TDM mode shown in Fig. 2.1, Fig. 2.6 - Fig. 2.8 except for one important difference. In this mode, the aliases outside the $m=\pm 1$ harmonic are all spurious signals that need to be tuned out or rejected as much as possible. This is in contrast with the TDM case where as many harmonics as possible need to be retained – the signal needs to be as wideband as possible – in order to be able to demultiplex and extract the individual antenna information. The power levels at each alias in the spectra depend on the pulse parameters $\tau_n, A_1$ and $A_2$ as well as the degree of band and channel select filtering provided. The emphasis at RF frequencies for the power combiner and mixer is the $f_c-f_s$ (or $f_c+f_s$) frequency and not $f_c$ as it is in the case of TDM. It must be noted in the frequency planning and system design that there are certain pulse parameter settings for which the power levels could be higher for harmonics other than $m=\pm 1$. It may be shown however that for most of the settings the $m=\pm 1$ terms will have the greatest signal strength.
2.2.1 Noise Analysis

The analysis of noise and SNR in the Coherent Combining case follows in the same vein as for regular mixers and the TDM case before.

To allow a comparison Coherent Combining noise with a simpler case, the noise factor of a single amplifier of gain $G$ or the multiplexing amplifiers in the Selection Diversity case is as stated in (2-24).

In addition, the following general assumptions are made in the analyses that follow.
• The gain $G$ is wideband
• The available input noise $N_{in}(f)$ is white (constant spectral density) and stationary
• The bandpass filter is noiseless; the in-band filter loss is absorbed into the amplifier stage that follows
• The out-of-band input noise is filtered by the anti-aliasing front-end bandpass filter
  
  \[
  N_{ino} = \frac{N_{inp}}{L}
  \]
  where $L$ is the attenuation factor. The out-of-band is considered to be at all harmonics outside of the desired $m=v$ harmonic of the noise transfer function $H_m(f)$.
• The device added noise is white, stationary and unfiltered ($N_{ao} = N_{ap} = N_a$)

  In the case of a coherent combining amplifier, the noise per input path is analyzed first. In order to do this, the switch and combine portion of the circuit is divided into different stages as shown in Fig. 2.14. The first stage is the filter stage, followed by the amplifier and then the switching stage which is actually responsible for the LPTV properties of the system.

  The relations for noise output from the amplifying stage remain the same as before and are given by equations (2-25) to (2-27).
In this analysis under the assumptions previously stated, equation (2-27) simplifies to

\[ H_m(f_c) = c_m \]  

(2-62)

where \( c_m \) are the Fourier Coefficients of the generalized pulse train as defined in equation (2-50), ignoring the \( n \) dependency.
In a practical system, all the noise harmonics of equation will not have to be considered and only those within the effective bandwidth $B_{\text{eff}}$ around the RF carrier $f_c$ will have to be evaluated. Equation (2-26) may be rewritten as for the practical case

$$N_{\text{conv}} = \sum_{m=-\frac{B_{\text{eff}}}{2f_c}}^{\frac{B_{\text{eff}}}{2f_c}} |H_m(f_c)|^2 N_{\text{out,trans}}(f_c + mf_s) = G \sum_{m=-\frac{B_{\text{eff}}}{2f_c}}^{\frac{B_{\text{eff}}}{2f_c}} |H_m(f_c)|^2 N_{\text{out,filn}}(f_c + mf_s)$$

(2-63)

In this analysis, however, the worst case scenario of all harmonics through to infinity are considered which allows the use of the following identity

$$\sum_{m=-\infty}^{\infty} |H_m(f_c)|^2 = \sum_{m=-\infty}^{\infty} c_m^2 = ((A_1 - A_2)f_s\tau + A_2)^2 + \sum_{m=-\infty}^{\infty} \left( (A_1 - A_2)\frac{\sin(mnf_s\tau)}{m\pi} \right)^2 \quad \text{(2-64)}$$

$$= ((A_1 - A_2)f_s\tau + A_2)^2 + (A_1 - A_2)^2\left(f_s\tau - (f_s\tau)^2\right) = A_1^2f_s\tau + (1 - f_s\tau)A_2^2$$

Note that (2-64) may be equivalently obtained by evaluating the power of the generalized pulse train shown in Fig. 2.12. Rewriting equation (2-64)

$$\sum_{m=-\infty}^{\infty} |H_m(f_c)|^2 = A_1^2f_s\tau + (1 - f_s\tau)A_2^2 - |c_v|^2 \quad \text{(2-65)}$$

where $|c_v|$ is the conversion gain of the desired signal harmonic.

Using equations (2-26) and (2-65), the noise factor of a single antenna channel in the coherent combining case is given by
\[
F_{CC,\text{amp}} = \frac{(S / N)_a}{(S / N)_\text{out}} = \frac{1}{G|c_v|^2} \frac{N_{\text{amp}}}{N_{\text{inp}}}
\]

\[
= \frac{1}{G|c_v|^2} \left[ N_{\text{inp}} + \frac{G(A_1^2 f_s \tau + (1 - f_s \tau) A_2^2 - |c_v|^2) N_{\text{inp}}}{N_{\text{inp}}} \right] + \left( A_1^2 f_s \tau + (1 - f_s \tau) A_2^2 - |c_v|^2 \right) N_{\text{inp}}
\]

\[
= \left(1 + \frac{N_{\text{ai}}}{N_{\text{inp}}} \right) + \left( \frac{A_1^2 f_s \tau + (1 - f_s \tau) A_2^2}{|c_v|^2} \right) - 1 \left( \frac{1}{L} + \frac{N_{\text{ai}}}{N_{\text{inp}}} \right)
\]

\[
F_{CC,\text{amp}} = \left(1 + \left( \frac{A_1^2 f_s \tau + (1 - f_s \tau) A_2^2}{|c_v|^2} \right) - 1 \right) \frac{1}{L} + \left( \frac{A_1^2 f_s \tau + (1 - f_s \tau) A_2^2}{|c_v|^2} \right) \frac{N_{\text{ai}}}{N_{\text{inp}}}
\]

\[
= \frac{A_1^2 f_s \tau + (1 - f_s \tau) A_2^2}{|c_v|^2} F_{\text{amp}} + \left(1 + \left( \frac{A_1^2 f_s \tau + (1 - f_s \tau) A_2^2}{|c_v|^2} \right) - 1 \right) \frac{1}{L} - \frac{A_1^2 f_s \tau + (1 - f_s \tau) A_2^2}{|c_v|^2}
\]

With \( A_1=1, A_2=-1, L=1 \),

\[
F_{CC,\text{amp}} = \frac{1}{|c_v|^2} F_{\text{amp}}
\]

(2-67)

Note for best conversion gain, normally \( v = \pm 1 \) and \( |c_{v,1}| = 2/\pi \) under the previously mentioned conditions.

Thus from a single channel perspective, without exploiting any of the coherent combining advantages of the coherent combining system, the NF of the receiver degrades to

\[
P_{\text{min}} \text{ (dBm)} = -174 \text{ dBm} + 10 \log (B_{ch}) + SNR_{\text{min, demod}} + NF_{\text{system,cc,amp}}
\]

\[
P_{\text{min}} \text{ (dBm)} = -174 \text{ dBm} + 10 \log (B_{ch}) + SNR_{\text{min, demod}} + NF_{\text{system,amp}} - 20 \log |c_v|
\]

(2-68)
There is degradation of $20 \log |c_i| \text{ dB}$ compared to the conventional receiver if only the output from a single antenna channel is processed at baseband.

The SNR ($SNR_{in}$) at each antenna is given by (2-36).

Since the noise under consideration is for the front-end amplifier section, it follows from Friis’ noise factor relationship that the system noise figure is impacted by the same amount. Three receiver types are considered for comparison of output SNRs and array noise factors.

1. Single Antenna Receiver

The SNR ($SNR_{\text{single}}$) and Noise Factor ($F_{\text{single}}$) of a single antenna receiver are given by (2-36) and (2-37) respectively.

2. Multiantenna Conventional Diversity / Coherent Combining Receiver

Since the noise is assumed to be uncorrelated, they will add in power, while the signals being coherent will add in amplitude after diversity combining. Therefore $SNR_{\text{conv_array}}$ and $F_{\text{conv_array}}$ remain the same as in (2-38) and (2-39).

3. Multiantenna Coherent Combining Receiver (this work)

Assuming the switching takes place immediately following the amplifiers

$$SNR_{\text{CC_array}} = \frac{\left( N \cdot |c_i| \sqrt{GS_n} \right)^2}{N(G|c_i|^2 N_{\text{ap}} + |c_i|^2 N_{\text{ap}} + G(A_1^2 f_r \tau + (1 - f_r \tau) A_2^2 - |c_i|^2) N_{\text{ino}}$$

$$+ (A_1^2 f_r \tau + (1 - f_r \tau) A_2^2 - |c_i|^2) N_{\text{ao}})}$$

(2-69)
\[
SNR_{CC\_array} = \frac{N|c_v|^2 GS_{in}}{G\left(|c_v|^2 + \frac{1}{L}\left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2 - |c_v|^2\right)\right)N_{inp}}
\]
\[
+ \left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2\right)N_a
\]
\[
SNR_{CC\_array} = \frac{N|c_v|^2 GS_{in}}{\left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2\right)(GN_{inp} + N_a)}
\]
\[
= \frac{N|c_v|^2}{\left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2\right)}SNR_{single}
\]
\[
= \frac{|c_v|^2}{\left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2\right)}SNR_{conv\_array}
\]
\[
SNR_{CC\_array} = \frac{N|c_v|^2 GS_{in}}{GN_{inp} + N_a}
\]
\[
= N|c_v|^2 SNR_{single}
\]
\[
= |c_v|^2 SNR_{conv\_array}
\]

and
\[
F_{CC\_array} = \frac{SNR_{in}}{SNR_{CC\_array}} = \frac{1}{N|c_v|^2}\left(|c_v|^2 + \frac{1}{L}\left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2 - |c_v|^2\right)\right)
\]
\[
+ \left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2\right)\frac{N_{ai}}{N_{inp}}\right)
\]
\[
F_{CC\_array} = \frac{SNR_{in}}{SNR_{CC\_array}} = \left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2\right)\frac{1 + \frac{N_{ai}}{N_{inp}}}{N|c_v|^2}
\]
\[
= \left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2\right)F_{single}
\]
\[
= \left(A_i^2 f_s \tau + (1-f_s \tau)A_2^2\right)F_{conv\_array}
\]

if \(L=1\) (worst case)
\[ F_{CC_{array}} = \frac{SNR_n}{SNR_{CC_{array}}} = \frac{1}{N|c_v|^2} \left( 1 + \frac{N_{ai}}{N_{inp}} \right) \]

if further \( A_1 = -A_2 = 1 \) with \( L = 1 \) (worst case) \( (2-75) \)

\[ = \frac{1}{N|c_v|^2} F_{single} \]

\[ = \frac{1}{|c_v|^2} F_{conv_{array}} \]

Since \(|c_v| \leq 1\), it may be summarized that if \( N|c_v|^2 \geq 1 \)

\[ SNR_{single} \leq SNR_{CC_{array}} \leq SNR_{conv_{array}} \] \( (2-76) \)

or equivalently

\[ NF_{conv_{array}} \leq NF_{CC_{array}} \leq NF_{single} \] \( (2-77) \)

With \(|c_{\pm 1}| = 2/\pi\), \( N \geq 3 \) for conditions (2-76) and (2-77) to hold. It must be noted that this is without anti-alias filtering.

It is observed that \( SNR_{CC_{array}} \) and \( NF_{CC_{array}} \) with \( \pm 1 \) switching in the limiting cases considered have an \( N \) dependency while there is no such dependency for \( SNR_{TDM_{array}} \) and \( NF_{TDM_{array}} \). The physical reason for this effect is because, with \( \pm 1 \) switching in the coherent combining case, noise power is not altered due to the coherent combining operation if the amplifier gain is not considered whereas in the 0/1 switching for TDM, the noise is actually attenuated as a result of the 0 weighting of the pulse train. This attenuation for the TDM operation is in the same proportion as the number of channels \( N \) being combined, hence that term disappears from the final noise relations. This can be appreciated by observing equations (2-40) and (2-69).
3 Single-Ended Receiver Front-ends

Unlike regular mixers with ideal LOs of 50% duty cycle, the duty cycle of the switching waveforms is dependent on the number of multiplex channels $N$. This affects RF balance and can make a conventional balanced structure unbalanced. Therefore the terms single balanced and double balanced are not sufficient in the context of an RF multiplex system analysis. More useful general terms would be

One Quadrant Switching (1QS) = Unbalanced
Two Quadrant Switching (2QS) $\supseteq$ Single Balanced (with respect to either LO/clock or RF)
Four Quadrant Switching (4QS) $\supseteq$ Double Balanced (with respect to LO/clock and RF)

It must be noted that in a 2QS scheme if RF balance is desired, LO/clock balance is lost. In the design of amplifier circuitry supporting TDM, RF balance is not possible in the 2QS implementation and only possible by 4QS.
3.1 Single-Ended Front-end Designs

Two versions of the single-ended unbalanced (IQS) designs were implemented; both in 0.18\( \mu \) RF-CMOS processes with a 1.8 V supply voltage. The first version did not have thick analog metal while the second had the dual last metal with thick analog metal option.

The multiplexing front-end was formed by a parallel arrangement of identical LNA unit cells with a single output matching network. High isolation between inputs and output was required for the LNAs to be able to function satisfactorily without affecting each others performance. The cascode LNAs were designed with the switching action intrinsic to their topology. Therefore, another requirement for this architecture was to ensure that the switching speed was not affected due to their parallel connection. The circuit implementation of the multiplexing front-end of Fig. 1.6 was as shown in Fig. 3.1.
3.1.1 RF Switching Amplifiers

The LNA unit cell consisted of the LNA core itself, a CML switch drive buffer and current mirrors for biasing the LNA and CML as shown in Fig. 3.2. Each LNA core received a differential control signal from the switch drive buffer. The LNA core itself was designed around the standard single-ended cascode topology. The cascode amplifier topology provided
high input-output isolation while maintaining a sufficiently low noise figure and adequate gain. The switching was incorporated into the cascode architecture easily by switching the cascode device between cutoff and saturation using a current steering differential pair switch, with one switch being diverted to the supply. The differential switching action also ensured that the each antenna element saw an almost constant input matching regardless of whether the associated channel is ‘on’ or ‘off’. This assisted with maintaining a uniform feed network loading as the ‘on’ switch position rotates through the switch array.

![Fig. 3.2 LNA unit cell](image)
The devices in the cascode LNA were sized first to achieve at least 0 dBm IIP3 linearity for a power spec of 5 mA per LNA. The matching networks were then designed for low noise based on methods exhaustively treated in available literature on the topic [45]-[48]. The inputs and output were matched to 50 \( \Omega \) with all LNA cores in place. The real part of the inductively-degenerated common-source LNA input impedance (3-1) [45] is mostly responsible for the 50 \( \Omega \) input match. The input and output matching were offset from each other for a flatter gain response over a wider bandwidth. The on-chip matching networks were comprised of spiral inductors and Dual MIM capacitors. The inductors were realized over a M1 comb groundplane to obtain higher Q values and greater shielding that would lower influence of substrate noise. Each LNA core had its own current mirror device for biasing.

3.1.2 Switch Drive Buffers

Current Mode Logic (CML) buffers of the type shown in Fig. 3.2 were used to drive the LNA switches in order to reduce signal swings to \( \sim V_{DD}/3 \) at the switch device gates and lower the control signal feedthrough to the RF output. The current consumption per CML buffer was 2 mA. The differential buffers were controlled by the outputs from a digital controller section. These buffers were not included in the first design version.
3.1.3 Digital Controller

The controller was designed for maximum speed to test the limits of the multiplexing capability of the front-end. The controller itself had to cycle through each of the LNA switches in turn at rates governed by relation (2-6). The ring counter topology seen in Fig. 3.3 was used to obtain a one-hot controller output. A high performance, low power, full swing, hybrid logic style [49] was used for the digital logic controller. The Hybrid Latch Flip Flop (HLFF) was used to construct the digital circuits for the superior performance it offers without a large power penalty [50]. Each HLFF included additional logic for the synchronous reset capability as well as output buffering to equalize delays as well as prevent hold time violations.

Fig. 3.3 Digital controller for RF multiplexing

In order to limit the number of control inputs to the circuit, only two digital inputs were provided for: clock and synchronous reset. A synchronous reset was used so that the active LNA core may be inferred with reference to the known clock. A direct-drive Chappell receiver [51] was used as the clock buffer shown in Fig. 3.4 to permit direct interfacing to a 50 Ω pulse generator in the second design version. Simulations with parasitics extracted indicated that the controller was able to operate correctly for rates well over 1 GHz.
3.1.4 Chip Layout

The circuits fabricated in 0.18 μm RF-CMOS processes occupied areas of 1300μm × 830μm and 1300μm × 860μm excluding bondpads for versions 1 and 2 respectively. The circuit die photographs are shown in Fig. 3.5. Minimum recommended ESD protection was provided for on all pads. Extensive use of decoupling capacitors on critical supply and bias lines was made to limit noise. The amplifiers were arranged with respect to the switch control
signals so as to limit large mismatch associated variations when sequentially transitioning from one to the next. A protective guard ring was provided around the digital section to isolate it from the sensitive RF section. The pad arrangement was done to permit chip testing using 3 GSSG probes and 1 GSG probe. The DC inputs were either wirebonded out or probed with DC needles.

### 3.2 Simulation and Experimental Results

Both single-ended front-ends were tested on-wafer by probing. Four probes were needed: 3 GSSG for the RF inputs, clock and reset and 1 GSG for the output. The DC inputs were brought in either through DC probe needles or bond-wires. The probing arrangement for the chips can be seen in Fig. 3.6.

![Fig. 3.6 On-wafer probing of single-ended designs](image)

**Version 1**  
**Version 2**

**3.2.1 Static Amplifier Response**

S-parameter measurements were done to check individual amplifier performance with each LNA biased to consume ~5 mA of current. The results from Version 1 of the design
shown in Fig. 3.7, and summarized in Table 3.1 and Table 3.2 indicated that the on-off switch forward isolation was particularly bad (8 – 9 dB). Although its gain was higher compared to version 2, the poor isolation meant that it could not work effectively in a front-end multiplex system.

With Version 2 of the design, the CML buffer was biased to develop ~600 mV of swing below $V_{DD}$. The measured S-parameters between Port1=RFin1 and Port2=RFout are shown in Fig. 3.8 as the controller cycles through each of the LNAs in turn. The shown response was typical of other channels too. The power gain was 2.6 dB at 2.4 GHz and produced gain over a wide bandwidth of 1.77 GHz around this center frequency. Although the static gain is useful if the LNA is used in its conventional sense, in a multiplexed context a more relevant gain would be the dynamic or conversion gain for the $0^{th}$ harmonic. This power gain would be below the static gain by a factor of $20 \log (N)$ which is 12 dB for the designed circuit. This fact was confirmed by comparing input and output signal spectra. LNA on-off forward isolation was observed to be in the 16-20 dB range. The input matching $S_{11}$ was below -10 dB in the ‘on’ state but rose to -7.6 dB in the ‘off’ state. The poor output matching $S_{22}$ of -3.2 dB contributed to the lower gain of the amplifier.

Values in these ranges could have been predicted by post-layout simulations with the correct set of parasitics extracted. However, this information was not available at the time the chips were sent out for fabrication. The correct parasitic device models to use were understood only after the chips were tested, when the measurement data could be correlated with the simulation results. The disturbance in matching and general performance was identified to be primarily due to unaccounted parasitic resistances in the circuit layout. Once
the correct parasitics were identified and included in the simulations, very good matches between the simulations and measurements was observed, with most values differing by not more than 1-2 dB.

In the absence of the parasitic resistances, with only other parasitic components such as capacitances and diodes extracted, both designs revealed excellent performance with very similar gains of over 13 dB and forward switch isolations of over 37 dB. It follows from this reasoning that one of the reasons for the results from the Version 1 design was worse than Version 2 was because of the lack of thick analog metal and high quality inductors in the process technology used for Version 1. Thus interconnect loss in Version 1 was greater than in Version 2.

However, the lower than desired amplifier performance obtained did not prevent the testing of the RF time multiplexing idea proposed in this work.
Version 1 Measurements

Table 3.1 Active LNA [Ch3] compliance at 2.4 GHz

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Post-layout Simulation</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>S11 (dB)</td>
<td>-12.3</td>
<td>-12.0</td>
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<tr>
<td>S12 (dB)</td>
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<td>-18.7</td>
</tr>
<tr>
<td>S21 (dB)</td>
<td>5.5</td>
<td>4.1</td>
</tr>
<tr>
<td>S22 (dB)</td>
<td>-5.3</td>
<td>-5.0</td>
</tr>
<tr>
<td>NF (dB)</td>
<td>3.9</td>
<td></td>
</tr>
<tr>
<td>IIP3 (dBm)</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Current Consumption (mA)</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Supply (V)</td>
<td>1.8</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 3.2 Inactive LNA [Ch3] compliance at 2.4 GHz [Ch2 On]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Post-layout Simulation</th>
<th>Measured</th>
</tr>
</thead>
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<tr>
<td>S11 (dB)</td>
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<tr>
<td>S12 (dB)</td>
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<tr>
<td>S21 (dB)</td>
<td>-8.2</td>
<td>-5.1</td>
</tr>
<tr>
<td>S22 (dB)</td>
<td>-4.9</td>
<td>-5.3</td>
</tr>
</tbody>
</table>

LNA Switch on-off forward isolation at 2.4 GHz = 7.9 to 9.2 dB
Fig. 3.7 Version 1 design LNA s-parameters for Ch3 (Port1=RFin3 and Port2=RFout)

[— : Ch0_On, – – : Ch1_On, · – · – : Ch2_On, -- : Ch3_On]
## Version 2 Measurements

### Table 3.3 Active LNA [Ch1] compliance at 2.4 GHz

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Post-layout Simulation</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-32.7</td>
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<tr>
<td>S21 (dB)</td>
<td>4.1</td>
<td>2.6</td>
</tr>
<tr>
<td>S22 (dB)</td>
<td>-4.5</td>
<td>-3.3</td>
</tr>
<tr>
<td>NF (dB)</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>IIP3 (dBm)</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Buffer Current (mA)</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>LNA Core Current (mA)</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Supply (V)</td>
<td>1.8</td>
<td>1.8</td>
</tr>
</tbody>
</table>

### Table 3.4 Inactive LNA [Ch1] compliance at 2.4 GHz [Ch2 On]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Post-layout Simulation</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>S11 (dB)</td>
<td>-7.9</td>
<td>-7.6</td>
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<tr>
<td>S12 (dB)</td>
<td>-34.4</td>
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<td>S21 (dB)</td>
<td>-19.0</td>
<td>-18.3</td>
</tr>
<tr>
<td>S22 (dB)</td>
<td>-4.4</td>
<td>-3.2</td>
</tr>
</tbody>
</table>

LNA Switch on-off forward isolation at 2.4 GHz = 16.9 to 21 dB
Fig. 3.8 Version 2 design LNA s-parameters for Ch1 (Port1=RFin1 and Port 2=RFout)

[— : Ch0_On, -- : Ch1_On, · – · – : Ch2_On, – – : Ch3_On]

3.2.2 Multiplex Operation

In order to gauge the usefulness of the circuit as a front-end time division multiplexer for multi-antenna systems leveraging beamforming or diversity advantages, the test setup had to contain methods to introduce known phase/amplitude relationships at the different RF inputs. To that effect, the RF inputs were supplied from a single signal source producing a Gray-encoded QPSK modulated RF signal split into multiple paths for the multiple inputs, with independently controllable phase shifters on each path. The controller was given a free running clock from a pulse generator. The RF output from the circuit went to a vector...
spectrum analyzer whose 70 MHz IF output was then connected to a real time digital oscilloscope sampling at 500 MSamp/s. The VSA IF bandwidth was restricted to 40 MHz, hence a multiplex rate $f_{\text{mux}}$ of 20 MHz was chosen. The per channel pulse rate was therefore 5 MHz. A symbol rate of 1MSymb/s with Root Nyquist filtering ($\alpha =0.35$) was chosen to have RF bandwidth satisfy the Nyquist sampling criterion (2-3). The RF carrier was set to 2.40125 GHz according to (2-5).

The whole setup shown in Fig. 3.9 would therefore appear to have a direct IF architecture, and in relation to Fig. 1.6, the baseband demultiplexing would follow the second mixer (complex digital mixer in this case) of the direct IF arrangement. Unlike regular direct IF implementations, in this case the IF output from the first mixer within the VSA was effectively \textit{oversampled} and digitized by the ADC of the real time digital oscilloscope. This was because the IF signal frequency was within the Nyquist sampling limits of the oscilloscope’s digitization capabilities.

The results presented in this section are from Version 2 of the circuit. The multiplex operation results from the version 1 chip were poor as expected due to the poor (< 10 dB) switch isolation.
Switching pulse feedthrough at multiples of the sample rate $f_s$ (5 MHz) was present in the output of the single-ended designs. The feedthrough spurs as captured by the VSA at the output of the switching front-end can be seen in Fig. 3.10. The output spectrum for Gray-encoded QPSK RF signals at two RF inputs only is shown in Fig. 3.11.
The captured IF signal along with synchronization information in the form of the captured clock and reset signals was transferred to a computer for offline processing. Quadrature digital downconversion to baseband, demultiplexing into multiple channel data streams, matched filtering and decimation with a filter identical to the transmit filter followed next. The demultiplexed signals with inputs on all channels except Ch3 (which was
terminated in 50 Ω) are shown in Fig. 3.12. Some amount of leakage from the other channels is seen in the Ch3 data path. Also the presence additional -4 dB power splitters on the Ch1 and Ch2 signal paths are reflected in their lower amplitude levels compared to Ch0.

The steps from downconversion up to decimation would have to be implemented in analog/mixed-signal hardware if the system were to be realized in practice. The decimated signals being of lower rates can then be remultiplexed again into a signal channel for digitization by a single ADC.

Fig. 3.12 Quadrature path demultiplexed signals
(Pulse train: Demultiplexed downconverted data, Envelope: Filtered baseband Data)
Symbol timing recovery, carrier frequency and phase offset correction was then performed to recover the transmitted data and determine the signal characteristics. A known preamble/training symbol sequence is simulated as having been transmitted to simplify the process and achieve synchronization quickly within a few symbol periods. Timing recovery is done by the squaring timing phase recovery method [53]. The data-aided estimation algorithm for frequency offset correction is discussed in [54]. The phase offset determined based on the known transmit data was used to rotate the complex baseband data on each of the channels. They are shown in Fig. 3.13 with all channels’ carrier phase normalized to Ch0, i.e. rotated with the phase offset factor for Ch0. Phase/amplitude differences between channels may thus be directly inferred from this processed data and compared with the known RF phase/amplitude introduced in the experimental setup. The results of one such comparison are presented in Fig. 3.14 in the form of symbol constellation diagrams. In this case, the phase rotations calculated were -16.4° and -25.2° on Ch1 and Ch2 respectively in relation to Ch0. The computed rotation values in the considered test cases were within the 5° of the actual phase shifter settings. These preliminary experiments confirmed that the system could track phase variations between channels to a fair degree of accuracy, hence highlighting the usefulness of the multiplexing architecture to phased array and other multi-antenna systems. Switch forward isolation greater than the ~18 dB obtained with the implemented Version 2 circuit would certainly improve the performance of the system.
Fig. 3.13 Quadrature path signals with carrier phase normalized to reference Ch0
(O: Expected symbols, ●: Phase rotated, ––: Timing + frequency corrected filtered signals without phase rotation)

Fig. 3.14 Measurement comparison of two sets of per-channel symbol constellations with channel phase offsets normalized to Ch0 showing phase rotation on Ch1 and Ch2
(O: Expected Original Symbol, □: Ch0, ○: Ch1, ◊: Ch2).
4 DIFFERENTIAL FRONT-END I

4.1 Differential Front-end I (DFI) Design

This fully differential design was implemented in a 0.18μm SiGe BiCMOS process that requires a 1.8 V supply voltage. Only the CMOS option was exercised in this work though. This process has all copper interconnects on all metal layers for low loss global interconnects. The design methodology followed for this design was to provide improved performance with respect to the single ended designs as well as provide for maximum flexibility in its working with the use of a larger, improved digital controller. While the single-ended design could only operate in multiplexing mode due to limited functionality of the controller, this design could also operate in the coherent combining mode. Another hallmark of this design was that it eliminated switching clock feedthrough due to its differential topology. Being differential also offered the obvious advantage of rejection of common-mode spurious content.

Just as in the single-ended design, the front-end was comprised of identical differential LNA unit cells connected in parallel to combine RF currents at the output. A
single broadband matching network interfaced with the output. The input and output matching networks were designed for 100 Ω differential impedances that may be converted to single ended 50 Ω impedances using external $\sqrt{2}:1$ balun transformers.

The switching front-end may be set to function as a TDM circuit for RF signals with a low logic level on the ‘mode’ digital input. It actually implemented two quadrant switching (2QS) action in this mode, hence could also work as a coherent combining combiner if the digital controller was so programmed. If maximum gain was desired in the coherent combining combining operation, four quadrant switching (4QS) needed to be applied. This was possible by setting the mode control input to logic high. In this 4QS mode, TDM operation was no longer possible in this implementation as the slightly more involved logic operation involved was not incorporated into the controller design. In both 2QS and 4QS modes, the circuit was balanced with respect to the switching signals – hence the system was at least single balanced with the possibility of being double balanced in the 4 QS mode.
4.1.1 RF Switching Amplifiers

The LNA unit cell seen in Fig. 4.2 consisted of the differential LNA core, a differential CML switch drive buffer and current mirrors for biasing the LNA and CML. The LNA core was driven by a differential control signal from the switch drive buffer. The LNA
core took the form of a differential cascode, with the cascode devices consisting of 3 switching NMOS devices per differential branch. This topology was similar to the classic Gilbert cell mixer structure, with the third switch being used in the multiplexing mode when the signal path had to be disabled. Some digital glue logic was used before the switch drive buffers to activate the correct set of switches depending on the mode control input.

Similar to the single-ended LNA, the devices of the core were first sized to achieve at least 0 dBm IIP3 linearity for a 6 mA current consumption per differential LNA core. The matching networks were then designed for differential 100 Ω matching at the input and output, the output being wideband. The matching networks were comprised of a combination of on-chip spiral inductors and off-chip bond wires, and a single external capacitor.
4.1.2 Switch Drive Buffers

Current Mode Logic buffers as shown in Fig. 4.2 were made differential with 3 outputs for the 3 switches in each differential branch. They developed $\sim V_{DD}/3$ swing below the analog supply. Each differential buffer consumed 4 mA of current from the supply. These buffers were in turn driven by signals from the digital controller.
4.1.3 Digital Controller

The digital controller was constructed of high speed Hybrid Latch Flip-Flops (HLFF), all of which are programmable by means of a ‘prog’ control signal as shown in Fig. 4.3. The controller consisted of a $4 \times 8$ array of flip-flops where each row controlled a single LNA core in the array of front-end amplifiers.
Fig. 4.3 4×8 array digital controller functional view
Waveforms of upto 8-bit length could be stored in each of the register rows and then read out at the differential clock rate to control the switching in the amplifier array. The flexibility of being able to store bits in this fashion allowed the same high speed control circuitry to be used in both modes of front-end operation simply by changing the bit patterns corresponding to each switching LNA. The 8-bit length of the waveform register meant that the period of repetition of the stored bit sequence was $8/f_{clk}$. 

Fig. 4.4 Digital controller for LNA switching consisting of $4 \times 8$ array of flip-flops
In the programming mode (prog=1), serial data may be scanned into the register array with the help of a serial clock. The internal clock signal to all the flip-flops was generated by multiplexing the clocks that arrive from two sources: either the high speed differential clock (clk) or the low speed clock (ser_clk) for serial programming. The high speed differential clock was buffered by the differential clock input buffer [52] shown in Fig. 4.5 that is based on a fully complementary self-bias design. The controller was verified to function correctly up to rates of 1 GHz in post-layout simulations with parasitics extracted.

![Fig. 4.5 Differential clock input buffer](image)

### 4.1.4 Chip Layout

All the NFETs on this chip were chosen to be isolated (triple well) devices to improve the noise immunity of the circuits. The layout for the chip was done so as to permit testing by wirebonding it directly to a 2-layer RF circuit board. Carefully partitioning of the RF and digital portions of the chip was done to minimize the impact of digital switching noise on the...
RF signals. The analog and digital supplies were separated and allotted multiple pads in order to minimize switching noise. Similar precautions were taken for the ground by providing for separate analog and digital grounds spread over multiple pads to reduce the ground bounce.

The amplifiers were also arranged to minimize performance variations while sequentially switching from one to the other due to layout mismatches. All pads had minimum recommended ESD protection. On-chip and off-chip decoupling capacitors were included to contain noise on the supplies and biases. The circuit area (excluding bondpads) was 2460 \( \mu \text{m} \times 1310 \mu \text{m} \).
Fig. 4.7 Chip directly wirebonded on to DFI test PCB

Fig. 4.8 DFI test PCB bottom side
A double-sided FR4 board was made use of as an inexpensive solution to test the IC. Coupled microstrip lines of 100 \( \Omega \) differential impedance are used for the differential RF inputs and outputs. HHM1520 baluns convert the differential RF signals to single-ended. The single-ended RF and high-speed lines are implemented as 50 \( \Omega \) coplanar waveguides. The single-ended clock is made differential with the ADT1-1WT balun. SMA connectors are employed for the clock input, and RF inputs and outputs.

### 4.2 Simulation and Experimental Results

#### 4.2.1 Amplifier Response (TDM mode)

Compared to the single ended case, the overall results are much better in this differential design because of the low loss all-copper interconnects available on metal levels in this process.
Fig. 4.10 Simulated S-parameters for Ch3 (Port1=RFIn3 and Port 2=RFOut)
Fig. 4.11 Simulated noise figure at 25°C for Ch3 (Port1=RFin3 and Port 2=RFout) with Ch3 On / Others Off.
Table 4.1 Active LNA [Ch3] compliance at 2.4 GHz

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Post-layout Simulation</th>
<th>Measured</th>
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<tbody>
<tr>
<td>S11 (dB)</td>
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<td>S12 (dB)</td>
<td>-46.7</td>
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<tr>
<td>S21 (dB)</td>
<td>7.7</td>
<td>7.4</td>
</tr>
<tr>
<td>S22 (dB)</td>
<td>-26.3</td>
<td>-13.7</td>
</tr>
<tr>
<td>NF (dB)</td>
<td>2.1</td>
<td>2.6</td>
</tr>
<tr>
<td>IIP3 (dBm)</td>
<td>&gt; 0</td>
<td>0.3</td>
</tr>
<tr>
<td>Buffer Current (mA)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>LNA Core Current (mA)</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Supply (V)</td>
<td>1.8</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 4.2 Inactive LNA [Ch3] compliance at 2.4 GHz [Ch0 On]

<table>
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<tr>
<th>Parameter</th>
<th>Post-layout Simulation</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-11.4</td>
</tr>
<tr>
<td>S12 (dB)</td>
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<td>S21 (dB)</td>
<td>-60.5</td>
<td>-46.3</td>
</tr>
<tr>
<td>S22 (dB)</td>
<td>-25.6</td>
<td>-14.6</td>
</tr>
</tbody>
</table>

LNA Switch On-Off Isolation at 2.4 GHz = 53.7 dB
The results of the S-parameter measurements done on a single non-switching amplifier channel termed a “static amplifier” are shown in Table 4.1. The matching and gain curves are shown in Fig. 4.12 and Fig. 4.13. It is observed that the switched LNA’s on-off isolation at 2.4 GHz is 53.8 dB. The forward gain S21 peaks at 8.0 dB at 2.23 GHz. The 3 dB bandwidth with reference to this peak gain point is 819.7 MHz.

Fig. 4.12  Input and output matching (Port1=RFin3 and Port 2=RFout)

Fig. 4.13  Forward and reverse gain (Port1=RFin3 and Port 2=RFout)

The multiplex operation output for single tone and dual tone RF signals at only one RF input (the others being terminated in 50 Ω) is shown in Fig. 4.14.
4.2.2 Noise Figure Measurements (TDM mode)

The noise figure measurements were done for a sample rate $f_s$ of 40 MHz per channel selected to ensure that device gain is linear over the measurement span set equal to $f_s$. A center frequency of 2.41 GHz was chosen for this sample rate based on equation (2-5). The Agilent PSA E4440A spectrum analyzer along with an Agilent 346A noise source was used for the noise figure measurements. The raw spectral data was analyzed in Matlab using the
modified Y-factor method described in the Appendix to evaluate the correct noise measures for a TDM wireless receiver.

The noise characteristics of the front-end were evaluated for two TDM cases, the multiplexing of $N=2$ and $N=4$ channels. The results presented are indeed the worst case for the designed circuit since they do not involve any additional bandpass anti-alias filtering other than those inherently provided by the circuit’s own matching networks. These results were compared with that of a reference static amplifier. The noisy raw data was averaged over the measurement span in order to obtain the results that are summarized in Table 4.3. The RSS noise figure measurement uncertainty was calculated to be 0.26 dB.

![Plot of Noise Gain vs Frequency for N=2 and N=4 TDM Cases]

**Fig. 4.15** Noise gain per antenna channel (···· Static Amp, — TDM Amp)

In Fig. 4.15, the ratio of output noise attributed to the input noise alone to input noise defined as the noise gain (more details in Appendix) is shown for the two cases considered. We observe that the fall in noise gain in the two cases is slightly more than the theoretical minimum of $10 \log(N)$. One reason for this is the small degree of input noise filtering provided by the matching network at the input. Additional filtering would result in a
lowering in noise gain as determined by the previously presented theory and the relations in the Appendix.

Fig. 4.16 Noise figure per antenna channel [SSB] (⋯ Static Amp, — TDM Amp)

The worst case SSB noise figure per amplifier channel is presented in Fig. 4.16. In both cases they are below the maximum $10 \log(N)$ increase above that of a static amplifier.

Fig. 4.17 Available array noise figure [SSB] (⋯ Static Amp, — TDM Amp)

A better appreciation of the effect of individual channel noise figure results may be obtained by determining the overall array noise figures shown in Fig. 4.17. This involves scaling down the single channel noise figure by a factor of $N$ based on the previously developed theory. Since the complete receiver has not been implemented in this work, only the “available” noise figure based on the amplifier section can be calculated from the
measured data. This is still a relevant measure of performance of the TDM based receiver because of its location in the receiver chain closely following the antenna, hence having a large influence on the overall noise figure.

The noise figure of the TDM amplifiers was observed to improve with higher sample rates $f_s$ as can be expected from equation (2-29). The exact nature of the dependence is yet to be quantified.

A final comparison of the available TDM array noise figure with that of the single amplifier shows that it is better than that of the static amplifier for the cases considered. This is in agreement with the overall theory presented earlier, and inequality (2-46) specifically. It must also be emphasized that the presented results are all worst case since an explicit alias reject filter is not included before the switching amplifiers. With the presence of this filtering the noise figures of the TDM amplifier and array will be appreciably better.
Table 4.3 Summary of noise measurements at 2.41 GHz

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Mean Performance</th>
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</thead>
<tbody>
<tr>
<td>Static Amplifier Available Gain (dB)</td>
<td>7.1</td>
</tr>
<tr>
<td>Static Amplifier Noise Figure (dB)</td>
<td>2.6</td>
</tr>
<tr>
<td>Number of Multiplexed Elements</td>
<td></td>
</tr>
<tr>
<td>N = 2</td>
<td></td>
</tr>
<tr>
<td>N = 4</td>
<td></td>
</tr>
<tr>
<td>TDM Amplifier Noise Gain (dB)</td>
<td>3.9</td>
</tr>
<tr>
<td>Δ1 Noise Gain (dB)</td>
<td>-3.2</td>
</tr>
<tr>
<td>TDM Amplifier Noise Figure (dB)</td>
<td>4.8</td>
</tr>
<tr>
<td>Δ1 Noise Figure (dB)</td>
<td>2.1</td>
</tr>
<tr>
<td>Available TDM Amp Array NF (dB)</td>
<td>1.8</td>
</tr>
<tr>
<td>Δ2 Noise Figure (dB)</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

Δ1: [TDM Amp – Static Amp]

Δ2: [Available TDM Amp Array – Static Amp]

4.2.3 Linearity Measurement (TDM mode)

The same multiplex rate and bias conditions as the noise measurements were retained for the linearity measurements. The inband linearity of the amplifier channels as given by the Third Order Intercept point is unaffected by the TDM operation as is seen in Fig. 4.18. This result is expected as the TDM switching is done at rates that are high enough not to adversely affect either the inband spectrum or the wireless channel spectrum, which is a subset of the inband spectrum.
With the gain flat over the inband frequencies, the effect of the multiplex operation is to attenuate the desired signal component and the IM3 component by the same factor. This causes the amplifiers power output curves to be displaced downwards without affecting the IIP3 of that channel.

![Graph showing linearity measurement plots for a single amplifier channel](image)

**Fig. 4.18** Linearity measurement plots for a single amplifier channel

The combined effect of the increased noise figure and unchanged linearity is a decrease in dynamic range per multiplex channel by the same degree as the noise figure. By the same token, the combined array dynamic range would be better than that of a single antenna receiver but worse than that of a conventional non-TDM array.

### 4.2.4 Performance Comparison

A summarized view of the implemented four-antenna TDM receiver front-end performance vis-à-vis the reference single antenna and conventional multi-antenna receiver front-ends is shown in Table 4.4. In order to appreciate the power utilization implications of
the different approaches, the post-amplifier stages of the receiver are also factored in, although this portion of the receiver is not realized in this work. It is assumed that these stages consume $P_{\text{post-amp}}$ mW in a single antenna system.

Table 4.4 Performance comparison of different receiver types

<table>
<thead>
<tr>
<th>Comparison Metric</th>
<th>Single Antenna</th>
<th>Multi-Antenna TDM</th>
<th>Multi-Antenna Conventional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Available Array NF (dB)</td>
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<td>1.9$^#$</td>
<td>-3.4</td>
</tr>
<tr>
<td>Available Array Signal Gain (dB)</td>
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<td>7.1</td>
<td>19.1</td>
</tr>
<tr>
<td>Net Array Power Estimate (mW)</td>
<td>$10.8 + P_{\text{post-amp}}$</td>
<td>$48.6 + P_{\text{post-amp}}$</td>
<td>$43.2 + 4.P_{\text{post-amp}}$</td>
</tr>
<tr>
<td>Linearity as IIP3/antenna (dBm)</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Array Dynamic Range</td>
<td>Low</td>
<td>Mid</td>
<td>High</td>
</tr>
<tr>
<td>Throughput enhancement possible by Spatial Multiplexing</td>
<td>None (1x)</td>
<td>4x</td>
<td>4x</td>
</tr>
<tr>
<td>Interference Cancellation</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

$^\#$ Without alias reject filtering (worst case)

It can be inferred from this table that the multi-antenna TDM receiver provides a level of performance that is above that of a single antenna system but below that of a full-fledged conventional multi-antenna system. The TDM based multi-antenna receiver offers many of the advantages of the conventional multi-antenna receiver, but at lower power and area costs.
4.2.5 Amplifier Response (CC mode)

The measured typical input and output return loss and the maximum conversion gain per element for the first harmonic is shown in Fig. 4.19. The return loss at the input is 9.1 dB and at the output is 9 dB in the middle of the intended operating frequency range.

The conversion gain was measured for 2.3025 GHz RF carrier frequency switched at a rate $f_s$ of 10 MHz corresponding to an $f_{clk}$ of 80 MHz. This center frequency and switching rate was chosen based on requirements described in 2.1.1. This switching rate and choice of center frequency will support receive signal bandwidths of up to 5 MHz. The maximum conversion gain per amplifier, obtained with control pulse train of 50% duty cycle, around the specified input frequency was measured to be 1.1 dB.

![Fig. 4.19 Input/output return loss and conversion gain for $m=-1$ harmonic for each amplifier](image)

The amplitude weighting per element was verified by comparing the output spectral power for the first harmonic against the prediction of equation (3) for pulse trains of different duty cycles. The output power is normalized with respect to the maximum possible value (obtained with $\tau=0.5$). The results are plotted in Fig. 4.20.
4.2.6 Beamforming Verification (CC mode)

The phase shifting network is constructed for narrowband operation around 2.4 GHz on an FR4 laminate. An assortment of different output phase shifts is obtained from a single RF input by the repeated branching of two-way power dividers ending in delay lines of varying lengths.
Observing the power variation of the first alias for different uniform phase shifts introduced at the array input resulted in the radiation patterns for 0.5\(\lambda\) element spacing and four different scan angles (corresponding to 0\(^\circ\), 45\(^\circ\), -45\(^\circ\) and 90\(^\circ\) phase shifts) as shown in Fig. 4.22. Equal amplitude weighting with \(\tau=0.5\) was applied to all the array inputs. The results indicated that in the current design the beam could be swept over the entire 360\(^\circ\) scan range with a phase shift resolution 45\(^\circ\) corresponding to the 8-bit controller in use.

![Fig. 4.22 Measured and theoretical radiation patterns for 4-element array at (a) 0\(^\circ\) (broadside) (b) -14.5\(^\circ\) (c) 14.5\(^\circ\) (d) -30\(^\circ\) scan angles](image-url)
The beamforming results also confirm a $20 \log(4)=12$ dB increase in directional gain above the conversion gain as a result of coherent signal combining for a 4-antenna system.

**4.2.7 Noise Figure Measurements (CC mode)**

The Agilent PSA E4440A spectrum analyzer and the Agilent 346A noise source were used for these tests. The spectral data was processed in Matlab using the modified Y-factor method described in the Appendix to evaluate the noise figures for the CC based wireless receiver.

The noise performance of the front-end with $N=4$ antennas was evaluated for two cases, with sample rates $f_s$ of 10 and 20 MHz per channel and a center frequency of 2.4 GHz. The results are shown in comparison with that of a reference static amplifier. The noisy data gathered was averaged over the measurement span to produce the results shown in Table 4.5.

![Frequency vs. Noise Gain](image)

**Fig. 4.23** Noise gain per antenna channel (··· Static Amp, — CC Amp)

The noise gain, as defined in the Appendix, and shown in Fig. 4.23 falls by a small factor due to the input noise filtering provided by the input matching network. This fall in noise gain is less than the $10 \log(N)$ dB observed in the TDM case, as a result of the bipolar
switching used in the CC case. Extra filtering would further reduce the noise gain as explained in the prior theory.

![Graph](image)

**Fig. 4.24** Noise figure per antenna channel [SSB] (∧ Static Amp, — CC Amp)

The SSB noise figure per amplifier channel without the alias reject filter is shown in Fig. 4.24. It is greater than that of a static amplifier.

![Graph](image)

**Fig. 4.25** Available array noise figure [SSB] (∧ Static Amp, — CC Amp)

The available CC array noise figure is lesser than that of the static amplifier for the cases considered as seen in Fig. 4.25. All these results match the predictions of the theory presented earlier. It is reiterated that these are all worst case results since additional alias
reject filtering is not present before the switching amplifiers. Adding the alias reject filter would help improve the noise figures.

Table 4.5 Summary of noise measurements at 2.4 GHz

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Mean Performance (Over Span = Switching Rate $f_s$)</th>
<th>$f_s = 10$ MHz</th>
<th>$f_s = 20$ MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplifier Gain (dB)</td>
<td></td>
<td>5.3</td>
<td>5.3</td>
</tr>
<tr>
<td>Amplifier NF (dB)</td>
<td></td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>CC Amp Noise Gain (dB)</td>
<td></td>
<td>5.0</td>
<td>4.6</td>
</tr>
<tr>
<td>$\Delta 1$ Noise Gain (dB)</td>
<td></td>
<td>-0.3</td>
<td>-0.7</td>
</tr>
<tr>
<td>CC Amp NF (dB)</td>
<td></td>
<td>6.2</td>
<td>6.2</td>
</tr>
<tr>
<td>$\Delta 1$ NF (dB)</td>
<td></td>
<td>3.7</td>
<td>3.7</td>
</tr>
<tr>
<td>Available CC Array NF (dB)</td>
<td></td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Delta 2$ NF (dB)</td>
<td></td>
<td>-2.3</td>
<td>-2.3</td>
</tr>
</tbody>
</table>

$\Delta 1$: [CC Amp – Static Amp]

$\Delta 2$: [Available CC Amp Array – Static Amp]
5  DIFFERENTIAL FRONT-END II

5.1  Differential Front-end II (DFII) Design

The DFII CMOS IC was fabricated in a 0.18 μm SiGe BiCMOS process with a 1.8 V supply voltage. Unlike for DFI though, this process does not have all-copper metallization. DFII is an evolution of DFI that includes the rest of the front-end stages of Fig. beyond the switching LNA section. Also, this front-end was designed to support only two antennas, hence can be regarded as a Switched Front-End Dual Antenna system. The additional circuits beyond the switching amplifiers of DFI include a quadrature mixer, a frequency divider, the demux circuit, CML buffers for the demux switches and baseband buffers. The divider generates the quadrature phase LO signals from an external input. The demux splits the TDM signal prior to filtering with the off-chip low pass filters. The baseband buffers drive 50 Ω external loads transformed by the off-chip baluns to 200 Ω. Off-chip low pass filters sit between the balun transformer outputs and the test instrumentation. The output matching following the amplifiers is modified in this front-end version to have parallel spiral inductors and series MIM capacitors in the LC match. Balun transformers are also used at the RF, LO
and clock inputs to convert the single-ended external signals to differential signals required at the chip. Baluns are also used at the baseband outputs for the same purpose. The DFII circuits were designed to provide an overall voltage gain of about 25 dB split between the amplifier and mixer-plus-demux stages.

![Circuit diagram](image)

Fig. 5.1 Circuit implementation of Differential Front-end II

### 5.1.1 RF Switching Amplifiers

This is identical to the circuits used in DFI. The noise matching on the inputs was altered slightly to account for the reduced current draw of 5.6 mA per amplifier compared to DFI.
5.1.2 Switch Drive Buffers

These too were similar to the ones used in DFI. However, in this case extra buffers were added for the demux switches. The CML buffers used to drive the switching devices had an output swing of 600 mV below the analog supply and consumed 1 mA of current per differential branch from the same supply. The buffers’ gate drives were provided by a digital controller. The glue logic was set up so as to operate the correct set of switches depending on the external mode control input and the active multiplex channel select signal.

5.1.3 Direct-Conversion Complex Mixer

The RF output from the output matching network following the switching LNAs is downconverted to DC using the quadrature mixer. The quadrature mixer consists of a pair of Gilbert cell structures operating in parallel to form the I and Q outputs, as shown in Fig. 5.2.
Fig. 5.2 Direct-conversion complex mixer

The commutating differential pairs of the two Gilbert cells share common transconductance devices at the bottom of the stack. The I and Q outputs from the double-balanced structures are available in the form of currents that are folded over using pmos current source devices at the top of the device stack. These pmos current sources are shared with the following stage which is the baseband demux circuitry. The nmos devices at the base and the pmos devices at the top of the structure are carefully biased to remain in saturation as the switching devices of the mixer and demux circuits swing between cutoff and saturation. The quadrature mixer consumes 6mA of current in all from a 1.8 V supply.

5.1.4 LO Divider

A high-speed static 2:1 frequency divider is used to divide down an external 4.8 GHz LO and obtain the I and Q LO signals for the quadrature mixer.

Fig. 5.3 LO divider
The divider core is implemented as a Master-Slave flip-flop in Current Mode Logic. The slave latch output is inverted and fed back to the master latch input as shown in Fig. 5.3. Low capacitive polysilicon resistors serve as CML loads for rapid switching. The frequency divider draws 2 mA of current from the supply and develops an output voltage swing of ~300 mV.

5.1.5 Digital Controller

This too was adapted from DFI. However, since DFII supported only two antennas, only two rows of byte sized circular shift registers were implemented, one for each multiplex channel as shown in Fig. 5.4. In post-layout simulations with parasitics extracted, clock rates of upto 2.2 GHz for the controller were possible.

![Fig. 5.4 Digital controller](image)

5.1.6 Baseband Analog Demux Circuitry

The demux network performs the job of separating the antenna signals prior to channel filtering. The circuit diagram of the demux network is shown in Fig. 5.5. The mixer output in the form of current is demultiplexed by a set of demux switches that are of a folded
cascode nature. These demuxed signals drive a high impedance resistive load to improve gain and minimize current consumption in these switching structures. The resistor outputs are then buffered to drive the external channel select filters and test instruments.

![Baseband analog demux circuit](image)

Fig. 5.5 Baseband analog demux circuit

The baseband circuitry is completely differential too and driven by the same switch control signals used in the amplifier stage. The folded cascode topology, with a reduced device count in the critical RF path, helps minimize the effect of the switching signal skew.
between the mux and demux switches. The demux network consumes a small fraction of the total current from the pmos current source devices that are in common with previous quadrature mixer stage.

### 5.1.7 Baseband Buffers

Open-drain source-coupled differential pairs are used to buffer the demux outputs to the 50 Ω inputs of the test instrumentation. These were designed to provide unity gain, and do not impact the noise figure significantly.

![Baseband Buffer](image)

Fig. 5.6 Baseband buffer
5.1.8 Chip Layout

The circuit area excluding bondpads was 1200 μm × 1500 μm and including them was 2300 μm × 2300 μm. The PCB for DFII adopts much of the same layout techniques and components as the one for DFI. The HHM1596A1 balun is used for differential to single-ended conversion of the LO input. After converting the differential baseband output to single-ended with the ADT4-6T balun, it is filtered by the SCLF-10.7, 10.7 MHz low pass filter.
Fig. 5.8 Chip directly wirebonded on to DFII test PCB

Fig. 5.9 DFII test PCB bottom side
Different views of the wirebonded chip and test board are seen in Fig. 5.8 - Fig. 5.10.

5.2 Simulation and Experimental Results

5.2.1 Static Channel Response (TDM mode)

The results of the measurements done on a single non-switching front-end channel referred to as a “static channel” are first presented.
Fig. 5.11 Input matching for static channel (Port1=RFin1)

![Input Frequency (GHz) vs. Conversion Gain (dB)](chart)

Fig. 5.12 Conversion gain under different static channel conditions

Table 5.1 Static channel front-end performance summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF Center Frequency (GHz)</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>LO Frequency (GHz)</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>S11 (Channel on) (dB)</td>
<td>-16.3</td>
<td>-14.0</td>
</tr>
<tr>
<td>S11 (Channel off) (dB)</td>
<td>-13.0</td>
<td>-15.5</td>
</tr>
<tr>
<td>Gain (Channel on) (dB)</td>
<td>29.4</td>
<td>22.8</td>
</tr>
<tr>
<td>Gain (Channel off) (dB)</td>
<td>-77.1</td>
<td>-47.4</td>
</tr>
<tr>
<td>Gain (Cross channel) (dB)</td>
<td>-28.1</td>
<td>-22.1</td>
</tr>
<tr>
<td>Noise Figure# (dB)</td>
<td>9.7</td>
<td>15.6</td>
</tr>
<tr>
<td>IIP3 (dBm)</td>
<td>-20</td>
<td>-17</td>
</tr>
<tr>
<td>IIP2 (dBm)</td>
<td>42</td>
<td>31</td>
</tr>
</tbody>
</table>

*11 to 12 MHz output frequency range
The input matching and conversion gain plots are seen in Fig. 5.11 and Fig. 5.12. The static channel performance summary is provided in Table 5.1. It is evident that the obtained cross-channel isolation of ~45 dB will have a greater limiting effect on system performance compared to the direct channel isolation result of ~70 dB.

Table 5.2 Front-end circuit power budgeting

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Number of Units</th>
<th>Current (mA) / Unit</th>
<th>Total Current (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LNA Core</td>
<td>2</td>
<td>5.6</td>
<td>11.2</td>
</tr>
<tr>
<td>LNA Drive</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Quadrature Mixer</td>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Demux</td>
<td>2</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>Demux Drive</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>LO Divider</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Current from 1.8 V Supply (mA)</td>
<td></td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>Baseband Buffer</td>
<td>4</td>
<td>14-20</td>
<td>56-80</td>
</tr>
</tbody>
</table>

The power allocation to the different circuit blocks is shown in Table 5.2. The current consumption of the baseband buffers may be excluded from the net current requirement as they have been included for measurement purposes only.

5.2.2 TDM Switching Channel Response (TDM mode)

The “dynamic” performance of the switching channel during time division multiplexing is studied next.
Fig. 5.13  Input matching for switching channel (Port1=RFin1)

Fig. 5.14  Conversion gain for switching channel
Table 5.3 Switching channel front-end performance summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF Center Frequency (GHz)</td>
<td>2.4</td>
</tr>
<tr>
<td>LO Frequency (GHz)</td>
<td>2.4</td>
</tr>
<tr>
<td>Switching Rate / Channel (MHz)</td>
<td>20</td>
</tr>
<tr>
<td>S11 (TDM) (dB)</td>
<td>-15.3</td>
</tr>
<tr>
<td>Gain (TDM) (dB)</td>
<td>17.0</td>
</tr>
</tbody>
</table>

The corresponding matching and gain curves are shown in Fig. 5.13 and Fig. 5.14.

Table 5.3 gives a summarized view of the switching channel performance for a 20 MHz per channel switching rate. As expected the conversion gain drops by \( \sim 20 \log(2) \, \text{dB} = 6 \, \text{dB} \) in relation to the static (non-switching) channel.

### 5.2.3 Noise Figure Measurements (TDM mode)

A sample rate \( f_s \) of 20 MHz per channel was used for the noise figure measurements. The RF spectrum is directly downconverted to DC with a 2.4 GHz LO. The noise figure measurements were done with the Agilent PSA E4445A spectrum analyzer and the Agilent 346B noise source. The raw spectral data was processed in Matlab using the Y-factor method to determine the noise figure per quadrature channel.

The 10 MHz lower frequency limit of the noise source and the presence of output low pass filters restricted the valid output frequency range for accurate noise figure measurements to 10-12 MHz. However the DSB noise figure from 1 MHz to 100 MHz for a static channel...
is obtained by simulations with limited RC parasitic extraction and the results shown in Fig. 5.15.

![Fig. 5.15 Spot noise figure (DSB) at 25°C for Ch1 with Ch1 On / Others Off](image)

The measured noise figures for the static and switching cases in the 11 – 12 MHz span are depicted in Fig. 5.16, Fig. 5.17 and Table 5.4.

![Fig. 5.16 Noise figure per antenna channel [DSB] (× Static Channel, • TDM Channel)](image)

The high noise figures for the receiver are mainly due to the following factors:
1. Additional parasitic resistances in the layout that upset the biasing of the amplifiers leading them to consume more current, and add more noise to the signal, and at the same time reduce the current drawn by the mixer impeding its optimal switching.

2. Parasitic resistors and capacitors that have degraded the matching at the input and the output of LNA stage, pushing it away from the optimal point.

Fig. 5.17 Available array noise figure [DSB] (× Static Channel, • TDM Channel)

Despite the shortcoming in the noise figure magnitudes, they are still consistent with the overall theory discussed earlier. The noise gain per channel falls by $10\log(2) = 3$ dB during TDM, the noise figure per TDM channel is above that of a static channel by less than 3 dB and the TDM array noise figure is below that of the static channel.
### Table 5.4 Summary of noise measurements for 11 to 12 MHz output frequency range

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Mean Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Channel Available Gain (dB)</td>
<td>25.0</td>
</tr>
<tr>
<td>Static Channel Noise Figure (dB)</td>
<td>15.6</td>
</tr>
<tr>
<td>TDM Channel Noise Gain (dB)</td>
<td>22.0</td>
</tr>
<tr>
<td>Δ1 Noise Gain (dB)</td>
<td>-3.0</td>
</tr>
<tr>
<td>TDM Channel Noise Figure (dB)</td>
<td>17.7</td>
</tr>
<tr>
<td>Δ1 Noise Figure (dB)</td>
<td>2.1</td>
</tr>
<tr>
<td>Available TDM Channel Array NF (dB)</td>
<td>14.7</td>
</tr>
<tr>
<td>Δ2 Noise Figure (dB)</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

Δ1: [TDM Channel – Static Channel]  
Δ2: [Available TDM Channel Array – Static Channel]

It is noted that the measured noise figures are worst case values since additional alias reject filtering is not provided for before the LNAs. Including these filters before the switching amplifiers would help improve their noise figures.

### 5.2.4 Time Domain Operation

Data recovery in the time domain for the TDM receiver is verified by feeding the two channels of demultiplexed I-Q baseband signals to four channels of a real time digital oscilloscope sampling at 100 MSamp/s. A gray-encoded QPSK modulated 2.405 GHz carrier RF signal from a RF signal source was split into two paths, with a phase shifter on one path and the other serving as reference, and supplied as antenna signals to the front-end.
The digital controller was setup for a multiplex rate \( f_{\text{mux}} \) of 40 MHz, hence the per channel sample rate was 20 MHz. A symbol rate of 5 MSymb/s with Root Nyquist filtering (\( \alpha = 0.35 \)) was then chosen. The LO was set to 2.405 GHz for zero-IF operation.

![Signal Constellation](image)

**Fig. 5.18** Per-channel symbol constellations showing different phase rotations of Ch1 with respect to Ch0  
(O: Expected Original Symbol, □: Ch0, ○: Ch1)

The captured data is subjected to digital decimation, symbol timing and phase recovery in Matlab. The recovered I-Q data symbol constellations with TDM channel 1 rotated by 30° and 45° with respect to TDM channel 0 is depicted in Fig. 3.14. The effect of the high noise figure and finite inter-TDM-channel cross-interference is reflected in the dispersion of the I-Q scatter plots.

### 5.2.5 Linearity Measurement (TDM mode)

The measured inband linearity in the form of IIP3 and IIP2 may be gauged from the output distortion component power versus input power plots shown in Fig. 5.19. These
measurements are for a static channel, however, as discussed previously they are the same for the TDM case.

![Device Output versus Input Power](image)

**Fig. 5.19** Linearity measurement plots for a single static channel

The low IIP2 is mainly attributed to the higher device mismatch in the differential pair circuit layout. Other reasons for deterioration in both IIP3 and IIP2 linearity performance are parasitics that are unaccounted for in the design process, as well as skewed interstage matching between the amplifier and mixer stages.
6 CONCLUSION

6.1 Research Contributions

This research proposed and implemented the Switched Front-end Multi-antenna Receiver (SFMR) architecture as a possible solution to the hardware duplication problem of conventional multi-antenna receivers. Fast periodic switching in either Time-Division Multiplexing (TDM) or Coherent Combining (CC) schemes made possible the realization of different gains offered by the MIMO mode of communications. This followed a detailed literature survey and review of previous attempts at similar systems and the challenges faced. A solution to the issue of crosstalk between multiplex channels with common channel filtering on the multiplex path in the TDM type of receivers was put forth.

The complete first-order model and theory for understanding the SFMR architecture was developed based on bandpass sampling and periodic switching theory. In particular detailed noise analysis based on LPTV system theory was presented for both TDM and CC modes of operation. This theory was codified into functional simulation models in Matlab that could be simulated to understand and evaluate the system level implications. Simple
closed form noise figure relations in terms of the number of antenna channels and the anti-
alias filter rejection were developed. Noise figure expressions that relate an individual SFMR
amplifier or SFMR amplifier array’s noise figure to that of a regular static amplifier were
also given. These simple relations provide a quick method of determining expected noise
figure of such systems. An important insight gained from the theoretical treatment is that in
general it can be expected that the noise figure (and array output SNR) of a coherent
combining SFMR receiver is better than that of a single antenna receiver but worse than that
of a conventional multi-antenna array. The noise figure relations can easily be generalized to
accommodate the additional filtering and switching non-idealities that would occur in
practice.

The new approach for dealing with multiple input RF channels in integrated multi-
antenna receivers was implemented in the form of four chips that were taped out in 0.18μm
RF-CMOS and SiGe BiCMOS processes that carried novel 2.4 GHz RF front-end circuits
and digital control circuitry for multi-antenna receivers. This approach emphasizes circuit
reuse for area and power savings by switching and combining the RF signal path right from
the antenna front-end onwards.

All these circuits were verified and demonstrated to work successfully by on-wafer
and chip-on-board testing. The measurements involving frequency and time-domain
equipment aided in the characterization of the matching, gain, noise and linearity of the
circuits. Two new strategies for measuring the noise figure of SFMR amplifier arrays that are
modifications of the traditional Y-factor method were devised. The TDM system was shown
to correctly recover signal phase/amplitude information on a per input channel basis. It was
found that maintaining sufficient isolation and containing interference between antenna signal paths was a serious challenge in these circuits. The results from the prototype chips were presented in papers accepted for IEEE publications.

All in all the developed system theory for this architecture was validated. The main conclusions of the work are summarized in Table 6.1 based on the performance gains for MIMO systems specified in [7].

Table 6.1 Performance comparison of different types of receivers

<table>
<thead>
<tr>
<th>Receiver Type</th>
<th>Performance Improvement</th>
<th>Area / Power / Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Array Gain</td>
<td>Diversity Gain</td>
</tr>
<tr>
<td>Single Antenna</td>
<td>No</td>
<td>SNRsingle</td>
</tr>
<tr>
<td>Multi-antenna Conventional</td>
<td>High</td>
<td>SNR_{conv.array} = \frac{N \cdot SNR_{single}}{N}</td>
</tr>
<tr>
<td>Switched Front-end Multi-antenna Receiver (this work)</td>
<td>Low</td>
<td>\geq SNR_{single} \leq SNR_{conv.array}</td>
</tr>
</tbody>
</table>

*SMX = Spatial Multiplexing

Thus, even though the SFMR architecture proposed in this work compromises somewhat on SNR improvement attributed to array gain, it still is capable of offering the other gains (and associated SNR improvements) of a conventional diversity architecture at a much lower cost.
6.2 Future Work

Although only the array gains benefits of the SFMR were demonstrated in this work, its real impact is expected to lie in the spatial multiplexing gain that is possible in the TDM mode. The CC mode would be helpful in a low SNR environment to provide the best output SNR from such a system, while the TDM mode would aid in throughput enhancement according to MIMO principles. A lot more work can be done to analyze these tradeoffs between the two modes. An additional mode that could be compared is the CDM style of operation, as envisaged in [55][56], which too is possible with the same SFMR circuit architecture.

Modifications to the first-order models developed to account for second-order effects such as finite switch signal transition times, finite signal path bandwidths, skew between mux and demux stages and jitter in switching signals are another area of interest. These modifications will then need to be validated with careful circuit measurements.

With regard to actual circuit performance significant improvement is achievable with better design layouts. This would help address the local device level as well as global circuit level mismatch concerns present in the circuits fabricated as part of this work. The three-way switching in the amplifier cascode section that makes the different modes of operation possible need much attention during layout so as to overcome problems such as with clock feedthrough and low IIP2.

Novel circuit ideas to eliminate the switch drive buffers and other such means to reduce the overall power consumption is another area of great interest. Since the switching signal feedthrough is largely rejected by the channel select filters, single-ended circuits
would be preferred in the interest of low power, as long as they maintain sufficient isolation. Removing the on-chip inductors of the amplifier section improves the area advantage of the SFMR.

The switching rates are likely to require boosting for use with many real standards. The circuit performance and validity of a system model to accommodate higher order effects will have to be checked in this situation.

Extensive measurements involving some actual wireless/cellular standards and sophisticated equipment such as BER testers would help address many other questions associated with implementation in real world commercial systems.
REFERENCES


[57] Agilent Application Note 57-2 Noise Figure Measurement Accuracy – The Y-Factor Method, literature number 5952-3706E.


Appendix A: TDM System Model with Channel Impairments in MATLAB

% Demo Version - Diversity System
clear all; close all;

%% System Parameters
fs = 128e6; % 128 MHz pulse rate
fc = 2.4e9; % RF Carrier
nCh = 4; %# Antennas
M = 4; % Alphabet size

%% Simulation Parameters
r2 = 8; %Baseband Samples/Symbol
fsamp = fc*r2; % 19.2 GHz sample freq
tstop = 1e-6; % 1 usec simulation time
t = [0 : 1/fsamp : tstop-1/fsamp]'; %Simulation time
R = fs/4; % Symbol Rate
nSamp = fix(fsamp/R); %Original #Samp/Symbol
r1 = nSamp/r2; %Stage 1 Rate Change
tsymb = downsample(t,nSamp); %Symbol Time
rep_seq = [0; 1; 2; 3]; %Sequence to repeat in original message
nSymb = fix(R*tstop); %Symbols
active_ch = [0 1 2 3];

%% Create Digital Messages
rep_seq = [0; 1; 2; 3]; %Sequence to repeat in original message
m_orig = repmat(rep_seq,fix(nSymb/length(rep_seq)),1); %Original Message
if (rem(nSymb,length(rep_seq)))
    m_orig = [m_orig; m_orig(1:rem(nSymb,length(rep_seq)))];
end
usymb_orig = modulate(modem.pskmod('M',M,'SymbolOrder','gray'),m_orig);
%Original Symbols
m = repmat(m_orig,1,nCh);
usymb_tx = 1/(2^0.5)*modulate(modem.pskmod('M',M,'SymbolOrder','gray'),m);
figure(1)
for ch = 0:nCh-1
    subplot(2,2,ch+1)
    plot(2^0.5*usymb_tx,'db','MarkerSize',6); % Plot the constellation.
    axis([-1.1 1.1 -1.1 1.1])
    xlabel('In-phase')
    ylabel('Quadrature')
    title(['Antenna Channel ',num2str(ch)])
    hold on;
    for jj=1:M
        text(real(usymb_tx(ch+1,jj)),imag(usymb_tx(ch+1,jj)),[' ' num2str(jj-1)]);
    end
    hold off;
end
%% Pulse Shape Baseband Data
u_tx = rcosflt(usymb_tx,R,r2*R,'sqrt',0.35);
delay = 3;
% u = rectpulse(usymb,ceil(fsamp/R));
% delay = 0;
u_tx = u_tx(r2*delay+1:r2*delay+nSymb*r2,:);
t_bb = [t(1) : r1/fsamp : t(end)];
for ch = 0:nCh-1
    figure(2)
    subplot(nCh,1,ch+1)
    stem (tsymb*1e9,real(usymb_tx(:,ch+1)),'k','MarkerEdgeColor','r');
    hold on;
    plot (t_bb*1e9,real(u_tx(:,ch+1))); hold off;
    axis ([0 tstop*1e9 -1.1 1.1])
    if ch==0
        title('In-phase BB Data')
        legend ('Orig. Symbol','Pulse Shaped','Location','NorthWest')
    end
    if ch==nCh-1
        xlabel('Time (nsec)')
    end
    ylabel(['Antenna ',num2str(ch),' Ampl'])
end

figure(3)
subplot(nCh,1,ch+1)
stem (tsymb*1e9,imag(usymb_tx(:,ch+1)),'k','MarkerEdgeColor','r');
hold on;
plot (t_bb*1e9,imag(u_tx(:,ch+1))); hold off;
axis ([0 tstop*1e9 -1.1 1.1])
if ch==0
    title('Quadrature BB Data')
    legend ('Orig. Symbol','Pulse Shaped','Location','NorthWest')
end
if ch==nCh-1
    xlabel('Time (nsec)')
end
ylabel(['Antenna ',num2str(nCh),' Ampl'])
end

f_bb = linspace(-fsamp/(2*r2),fsamp/(2*r2),length(t_bb))./1e9;
figure(4)
subplot(2,1,1)
mag = db(abs(fftshift(fft(u_tx),1)));
plot(f_bb,mag(:,1),f_bb,mag(:,2),f_bb,mag(:,3),f_bb,mag(:,4))
legend('1','2','3','4');
axis([-1 1 -60 50])
xlabel('Frequency (GHz)')
ylabel('Magnitude (dB)')
title('Magnitude Spectrum of Baseband Data')
subplot(2,1,2)
ph = angle(fftshift(fft(u_tx),1))*180/pi;
plot(f_bb,ph(:,1),f_bb,ph(:,2),f_bb,ph(:,3),f_bb,ph(:,4))
140
axis([-1 1 -361 361])
legend('1','2','3','4');
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of Baseband Data')

%% Upconversion
u_up = resample(u_tx,fsamp,r2*R);
foff = 1e5; %Carrier Frequency Offset (Hz)
phi = [0 20 60 45]; %Per channel phase shift (Deg)
figure(5)
for ch = 0:nCh-1
  s(:,ch+1) = real(u_up(:,ch+1).*(2^0.5*exp(j*(2*pi*(fc+foff)*t+phi(ch+1)*pi/180))));
  subplot(nCh,1,ch+1)
  plot(t*1e9,s(:,ch+1)).*(2^0.5*exp(j*(2*pi*(fc+foff)*t+phi(ch+1)*pi/180)))
  axis([400 600 -1 1])
  if ch==0
    title('RF Antenna Signals')
  end
  if ch==nCh-1
    xlabel('Time (nsec)')
  end
  ylabel(['Antenna ',num2str(ch),' Ampl'])
end

%% Generate Switching Pulses
d = [0 : 1/fs : tstop-1/fs]; %Pulse time
figure(6)
for ch = 0:nCh-1
  pt(:,ch+1) = pulstran(t,d+ch/(nCh*fs),'rectpuls',1/(nCh*fs));
  subplot(nCh,1,ch+1)
  plot(t*1e9,pt(:,ch+1))
  axis([0 200 -0.1 1.1])
  if ch==0
    title('Switch Control')
  end
  if ch==nCh-1
    xlabel('Time (nsec)')
  end
  ylabel(['Switch ',num2str(ch),' Ampl'])
end

%% Time-division
x = s.*pt;
figure(7)
for ch = 0:nCh-1
  subplot(nCh,1,ch+1)
  plot(t*1e9,x(:,ch+1))
  axis([400 600 -1.1 1.1])
if ch==0
title('RF Pulse Trains')
end
if ch==nCh-1
xlabel('Time (nsec)')
end
ylabel([['Antenna ',num2str(ch),' Ampl']])
end

f_up = linspace(-fsamp/2,fsamp/2,length(t))./1e9;
figure(8)
subplot(2,1,1)
mag = db(abs(fftshift(fft(x),1)));
plot(f_up,mag(:,1),f_up,mag(:,2),f_up,mag(:,3),f_up,mag(:,4))
legend('1','2','3','4');
axis ([-fsamp/4e9 fsamp/4e9 -40 80])
xlabel('Frequency (GHz)')
ylabel('Magnitude (dB)')
title('Magnitude Spectrum of RF Pulse Trains')
subplot(2,1,2)
ph = angle(fftshift(fft(x),1))*180/pi;
plot(f_up,ph(:,1),f_up,ph(:,2),f_up,ph(:,3),f_up,ph(:,4))
axis ([-fsamp/4e9 fsamp/4e9 -361 361])
legend('1','2','3','4');
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of RF Pulse Trains')

%% Multiplex
xsum = sum(x.').

figure(9)
plot(t*1e9,xsum)
axis ([400 600 -1.4 1.4])
xlabel('Time (nsec)')
ylabel('Ampl')
title('Multilplexed RF')

figure(12)
subplot(3,1,1)
plot(f_up,db(abs(fftshift(fft(xsum)))))
axis ([-fsamp/4e9 fsamp/4e9 -20 90])
xlabel('Frequency (GHz)')
ylabel('Magnitude (dB)')
title('Magnitude Spectrum of Multilplexed RF')
figure(13)
subplot(3,1,1)
plot(f_up,angle(fftshift(fft(xsum)))*180/pi)
axis ([-fsamp/4e9 fsamp/4e9 -361 361])
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of Multilplexed RF')
%% Quadrature Downconvert
rf_down = xsum.*(2^0.5*exp(-j*2*pi*fc*t));
Wp = (fsamp/16)/(fsamp/2); Ws = (fsamp/8)/(fsamp/2);
ord = cheb2ord(Wp, Ws, 1, 90);
[b,a] = cheby2(ord, 90, Ws);
rf_down = filtfilt(b, a, rf_down);

figure(14)
subplot(2,1,1)
plot(t*1e9,real(rf_down))
ylabel('Real Amplitude')
axis ([min(t*1e9) min(t*1e9)+1000 min(real(rf_down))-0.1
max(real(rf_down))+0.1])
title('Complex Baseband Signal')
subplot(2,1,2)
plot(t*1e9,imag(rf_down))
ylabel('Imaginary Amplitude')
axis ([min(t*1e9) min(t*1e9)+1000 min(imag(rf_down))-0.1
max(imag(rf_down))+0.1])
xlabel('Time (nsec)')

figure(12)
subplot(3,1,2)
plot(f_up,db(abs(fftshift(fft(rf_down)))))
axis ([[-fsamp/2e9 fsamp/2e9 -20 80])
xlabel('Frequency (GHz)')
ylabel('Magnitude (dB)')
title('Magnitude Spectrum of Baseband Signal')
figure(13)
subplot(3,1,2)
plot(f_up,angle(fftshift(fft(rf_down)))*180/pi)
xlim ([[-fsamp/2e9 fsamp/2e9])
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of Baseband Signal')

%% Demux
tlimit = [min(t*1e9) max(t*1e9)];
ch_data = repmat(rf_down,1,4).*pt; %Demux
for ch = 0:nCh-1
    figure(15)
    subplot(nCh,1,ch+1)
    plot(t*1e9,real(ch_data(:,ch+1))); 
    axis ([tlimit min(min(real(ch_data)))-0.1 
    max(max(real(ch_data)))+0.1])
    if ch==0
        title('In-phase Path Demultiplexed Signals')
    end
    if ch==3
        xlabel('Time (nsec)')
    end
end
ylabel(['Ant Ch',num2str(ch),'
 Ampl'])
figure(16)
subplot(nCh,1,ch+1)
plot(t*1e9,imag(ch_data(:,ch+1)));
axis ([tlimit min(min(imag(ch_data)))-0.1
max(max(imag(ch_data)))+0.1])
if ch==0
    title('Quadrature-Path Demultiplexed Signals')
end
if ch==3
    xlabel('Time (nsec)')
end
ylabel(['Ant Ch',num2str(ch),'
 Ampl'])
end

%% Decimate
ch_data_dec = 2*resample(ch_data,r2,nSamp);
ch_data_dec_flt = rcosflt(ch_data_dec,R,r2*R,'sqrt/Fs',0.35);
delay = 3; %Delay of rcosflt
bb = ch_data_dec_flt(delay*r2+1:(delay+nSymb)*r2,:); %Adjust for delay
nSamp1 = length(bb); %Stage 1 #Samp/Symbol

for ch = 0:nCh-1
    figure(15)
subplot(nCh,1,ch+1)
hold on; plot (t_bb*1e9,real(bb(:,ch+1)),'--r'); hold off;
if ch==0
    legend('Demuxed','Filtered','Location','NorthWest')
end
figure(16)
subplot(nCh,1,ch+1)
hold on; plot (t_bb*1e9,imag(bb(:,ch+1)),'--r'); hold off;
if ch==0
    legend('Demuxed','Filtered','Location','NorthWest')
end
end

f1 = linspace(-fsamp/2*r2/nSamp,fsamp/2*r2/nSamp,nSamp1)./1e9;
disp_ch = 0; %Display channel
figure(12)
subplot(3,1,3)
mag = db(abs(fftshift(fft(bb(:,disp_ch+1)))));
plot(f1,mag)
axis ([[-fsamp/2e9 fsamp/2e9 min(mag) max(mag)+10])
xlabel('Frequency (GHz)')
ylabel('Magnitude (dB)'
title(['Magnitude Spectrum of Filtered Antenna ' num2str(disp_ch) ' Baseband Signal'])
figure(13)
subplot(3,1,3)
plot(f1,angle(fftshift(fft(bb(:,disp_ch+1))))*180/pi)
axis ([[-fsamp/2e9 fsamp/2e9 -200 200]])
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title(['Phase Spectrum of Filtered Antenna ' num2str(disp_ch) ' Baseband Signal'])

%% Symbol Timing Recovery
for ch = 0:nCh-1
    tphest(ch+1) = -1/(2*pi)*angle(sum((abs(bb(:,ch+1)).^2).*exp(-j*2*pi/r2*[0:nSamp1-1]''))) %Timing Phase Estimate
    if (abs(tphest(ch+1))<1/(2*r2)) %Threshold
        tphestThr(ch+1) = 0;
    else
        tphestThr(ch+1) = tphest(ch+1);
    end
    if (tphestThr(ch+1)<0)
        tphestThr(ch+1) = tphestThr(ch+1)+1;
    end
    disp(sprintf('Timing Phase Estimate for Ch%d = %0.4f
    Samples',ch,r2*tphestThr(ch+1)));
    tsi = r2*[tphestThr+[0:nSymb-1]'']; %Symbol Sampling Instants
    tsi = (1:nSamp1)+r2*tphestThr(ch+1); %In samples
    usymb_tcorr(:,ch+1) = interp1([1:nSamp1+delay*r2]',ch_data_dec_flt(delay*r2+1:2*delay*r2+nSamp1, ch+1),tsi);
end

disp(sprintf(' '))

for ch = 0:nCh-1
    figure(19)
    subplot(nCh,1,ch+1)
    plot(t_bb*1e9,real(bb(:,ch+1)),'or--'); hold on;
    stem(tsi_rt*1e9,real(usymb_tcorr(:,ch+1)),'xb'); hold off;
    axis ([min(t_bb*1e9) max(tsi_rt*1e9) min(min(real(usymb_tcorr)))-0.1 max(max(real(usymb_tcorr)))+0.1])
    if ch==0
        title('Timing Phase Corrected In-phase Baseband Signal')
        legend ('Incoming','Corrected');
    end
    if ch==3
        xlabel('Time (nsec)')
    end
end

ylabel(['Ant Ch',num2str(ch),' Ampl'])
figure(20)
subplot(nCh,1,ch+1)
plot(t_bb*1e9,imag(bb(:,ch+1)),'or--'); hold on;
stem(tsi_rt*1e9,imag(usymb_tcorr(:,ch+1)),'xb'); hold off;
axis ([min(t_bb*1e9) max(tsi_rt*1e9) min(min(imag(usymb_tcorr)))-0.1 max(max(imag(usymb_tcorr)))+0.1])
if ch==0

title('Timing Phase Corrected Quadrature Baseband Signal')
legend ('Incoming','Corrected');

if ch==3
    xlabel('Time (nsec)')
end
ylabel(['Ant Ch',num2str(ch),' Ampl'])
end

%% Carrier Frequency Offset Correction (DA)
istop = nSamp1-r2; %Reject last few timing corrected samples
L = 8*r2; %Correlation distance
N = istop-L; %Correlation window
for ch = 0:nCh-1
    usymb_avg_f(ch+1) = mean(usymb_tcorr(L+1:istop,ch+1).*conj(usymb_tcorr(1:N,ch+1)));  
    woff(ch+1) = angle(usymb_avg_f(ch+1))/L;
    disp(sprintf('Carrier Frequency Offset Estimate for Ch%d = %0.4g Hz',ch,woff(ch+1)*(r2*R)/(2*pi)))
end
avg_woff = mean(woff(active_ch+1));
disp(sprintf('Mean Carrier Frequency Offset Estimate for Active Channels = %0.4g Hz/n',avg_woff*(r2*R)/(2*pi)))

for ch = 0:nCh-1
    usymb_fcorr(:,ch+1) = usymb_tcorr(:,ch+1).*exp(-j*avg_woff*[1:nSamp1]).';
    figure(21)
    subplot(nCh,1,ch+1)
    plot(t_bb*1e9,real(usymb_tcorr(:,ch+1)),'or--'); hold on;
    plot(t_bb*1e9,real(usymb_fcorr(:,ch+1)),'.b:'); hold off;
    axis ([min(t_bb*1e9) max(t_bb*1e9) min(min(real(usymb_fcorr)))-0.2 max(max(real(usymb_fcorr)))+0.2])
    if ch==0
        title('Carrier Frequency Offset Corrected In-phase Baseband Signal')
        legend ('Incoming','Corrected');
    end
    if ch==3
        xlabel('Time (nsec)')
    end
    ylabel(['Ant Ch',num2str(ch),' Ampl'])
end
figure(22)
subplot(nCh,1,ch+1)
plot(t_bb*1e9,imag(usymb_tcorr(:,ch+1)),'or--'); hold on;
plot(t_bb*1e9,imag(usymb_fcorr(:,ch+1)),'.b:'); hold off;
axis ([min(t_bb*1e9) max(t_bb*1e9) min(min(imag(usymb_fcorr)))-0.2 max(max(imag(usymb_fcorr)))+0.2])
if ch==0
    title('Carrier Frequency Offset Corrected Quadrature Baseband Signal')
    legend ('Incoming','Corrected');
end

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if ch==3
    xlabel('Time (nsec)')
end
ylabel(['Ant Ch',num2str(ch),' Ampl'])
end

%% Carrier Phase Offset Correction (DA)
usymb_dsmp = downsample(usymb_fcorr,r2);
diff_tol = 5; % 5 degrees
symb_sel_ind = [3 : nSymb]; % Select indices for recovered symbols
m_orig_uniq = length(unique(rep_seq)); % Unique elements in original sequence

diff_usymb = diff(angle(usymb_dsmp))*180/pi; % To determine offset in original sequence
diff_usymb(find(diff_usymb<0)) = diff_usymb(find(diff_usymb<0))+360;
diff_usymb_orig = diff(angle(usymb_orig))*180/pi;
diff_usymb_orig(find(diff_usymb_orig<0)) = diff_usymb_orig(find(diff_usymb_orig<0))+360;

for ch = 0:nCh-1
    if (ch==0)
        for count = 1:m_orig_uniq-1
            if (norm(diff_usymb(2:nSymb-2,1) - diff_usymb_orig(2:nSymb-2))/sqrt(nSymb-3) > diff_tol) % Check if rms of diff_diff is within tol
                m_orig = mod(m_orig + ones(nSymb,1),M); % For gq1b % Rotate original to correct for offset
                % m_orig = circshift(m_orig,1); % For gq4
                usymb_orig = modulate(modem.pskmod('M',M,'SymbolOrder','gray'),m_orig);
            else
                break;
            end
            diff_usymb_orig = diff(angle(usymb_orig))*180/pi;
            diff_usymb_orig(find(diff_usymb_orig<0)) = diff_usymb_orig(find(diff_usymb_orig<0))+360;
        end
    end
    ph_diff(:,ch+1) = angle(usymb_dsmp(:,ch+1)) - angle(usymb_orig);
    cphest(ch+1) = mean(unwrap(ph_diff(symb_sel_ind,ch+1))); % Carrier Phase Estimate
    disp(sprintf('Carrier Phase Offset Estimate for Ch%d = %0.4f Deg.',ch,180/pi*cphest(ch+1)))
    usymb_phcorr(:,ch+1) = usymb_fcorr(:,ch+1)*exp(-j*cphest(ch+1)); % Corrected Symbols
end

disp(' ')

usymb_phase_change = diff_usymb - repmat(diff_usymb_orig,1,4);
figure (29)
subplot(2,1,1)
stem([2:nSymb], usymb_phase_change(:,active_ch+1))
xlabel('Symbol #')

end
ylabel('Phase Change (Deg)')
title('Channel Phase Variation wrt Original Symbols Phase Change')
legend ('Ch 0', 'Ch 1', 'Ch 2');
disp(['Channel Phase Variation Standard Deviation (Deg) = '])
num2str(std(usymb_phase_change(symb_sel_ind(1:end-1),active_ch+1)))
sprintf('
'])

usymb_phcorr_ref0 = usymb_fcorr*exp(-j*cphest(active_ch(1)+1)); %Corrected Symbols Referred to Ch0
for ch = 0:nCh-1
disp(sprintf('Carrier Phase Offset Estimate for Ch%d relative to Ch0 = %0.4f Deg.',ch,180/pi*(cphest(ch+1)-cphest(active_ch(1)+1))))
figure(23)
subplot(nCh,1,ch+1)
plot((tsymb)*1e9,real(usymb_orig)*max(abs(real(usymb_phcorr(:,ch+1))))/max(abs(usymb_orig)),'or:'); hold on;
plot(t_bb*1e9,real(usymb_fcorr(:,ch+1)),'xm--');
stem(t_bb*1e9,real(usymb_phcorr(:,ch+1)),'.b'); hold off;
axis ([tlimit min(min([real(usymb_phcorr) imag(usymb_phcorr)]))-0.2...
max(max([real(usymb_phcorr) imag(usymb_phcorr)]))+0.2])
if ch==0
legend ('Original','Incoming','Corrected','Location','NorthWest');
title('Carrier Phase Offset Corrected In-phase Baseband Signal')
end
if ch==3
xlabel('Time (nsec)')
end
ylabel(['Ant Ch',num2str(ch),' Ampl'])
figure(24)
subplot(nCh,1,ch+1)
plot((tsymb)*1e9,imag(usymb_orig)*max(abs(imag(usymb_phcorr(:,ch+1))))/max(abs(usymb_orig)),'or:'); hold on;
plot(t_bb*1e9,imag(usymb_fcorr(:,ch+1)),'xm--');
stem(t_bb*1e9,imag(usymb_phcorr(:,ch+1)),'.b'); hold off;
axis ([tlimit min(min([real(usymb_phcorr) imag(usymb_phcorr)]))-0.2...
max(max([real(usymb_phcorr) imag(usymb_phcorr)]))+0.2])
if ch==0
legend ('Original','Incoming','Corrected','Location','NorthWest');
title('Carrier Phase Offset Corrected Quadrature Baseband Signal')
end
if ch==3
xlabel('Time (nsec)')
end
ylabel(['Ant Ch',num2str(ch),' Ampl'])
figure(25)
subplot(nCh,1,ch+1); hold on;
stem((tsymb)*1e9,real(usymb_orig)*max(abs(real(usymb_phcorr_ref0(:,ch+1))))/max(abs(usymb_orig)),'g');
plot(t_bb*1e9,real(usymb_fcorr(:,ch+1)),'xm--');
plot(t_bb*1e9,real(usymb_phcorr_ref0(:,ch+1)),'m.'); hold off;
axis ([tlimit min(min([real(usymb_phcorr_ref0)
imag(usymb_phcorr_ref0)]))-0.2 ...
max(max([real(usymb_phcorr_ref0) imag(usymb_phcorr_ref0)]))+0.2])
if ch==0
legend ('Orig Symb','Incoming','Rotated','Location','NorthWest');
title('In-phase Baseband Signal with Carrier Phase Normalized to
Ref. Ch0')
end
if ch==3
xlabel('Time (nsec)')
end
ylabel(['Ant Ch',num2str(ch),' Ampl'])
figure(26)
subplot(nCh,1,ch+1); hold on;
stem((tsymb)*1e9,imag(usymb_orig)*max(abs(imag(usymb_phcorr_ref0(:,ch+1)))
)/max(abs(usymb_orig)),'g');
plot(t_bb*1e9,imag(usymb_fcorr(:,ch+1)),'xm--');
plot(t_bb*1e9,imag(usymb_phcorr_ref0(:,ch+1)),'m.'); hold off;
axis ([tlimit min(min([real(usymb_phcorr_ref0)
imag(usymb_phcorr_ref0)]))-0.2 ...
max(max([real(usymb_phcorr_ref0) imag(usymb_phcorr_ref0)]))+0.2])
if ch==0
legend ('Orig Symb','Incoming','Rotated','Location','NorthWest');
title('Quadrature Baseband Signal with Carrier Phase Normalized to
Ref. Ch0')
end
if ch==3
xlabel('Time (nsec)')
end
ylabel(['Ant Ch',num2str(ch),' Ampl'])
end
disp(sprintf(' '))

%% Recover Symbols
usymb = downsample(usymb_phcorr,r2);
usymb_ref0 = downsample(usymb_phcorr_ref0,r2);
m = demodulate(modem.pskdemod('M',M,'SymbolOrder','gray'),usymb);
%Recovered Message

figure(27)
h_orig = plot(usymb_orig,'or','MarkerSize',12); % Plot the constellation.
hold on;
axis ([-1.1 1.1 -1.1 1.1])
xlabel('In-phase')
ylabel('Quadrature')
title('Signal Constellation with Phase Normalized to Ref. Ch0')
% Include text annotations that number the points.
m_alphabet = [0:M-1];
symb = modulate(modem.pskmod('M',M,'SymbolOrder','gray'),m_alphabet);
for jj=1:M
plot(real(symb(jj)),imag(symb(jj)),'k.');
text(real(symb(jj))+0.06,imag(symb(jj)),num2str(jj-1));
end

present = ismember([0:nCh-1],active_ch);
h_ch = zeros(1,nCh);
ch_label = {'Recovered Ch0'; 'Recovered Ch1'; 'Recovered Ch2'; 'Recovered Ch3'};
for ch = 0:nCh-1
    figure(27)
    if (present(ch+1))
        switch ch
            case 0
                h_ch(1) = plot(usymb_ref0(symb_sel_ind,1)/mean(abs(usymb_ref0(symb_sel_ind,1))),'sb','MarkerFaceColor','c','MarkerSize',6);
            case 1
                h_ch(2) = plot(usymb_ref0(symb_sel_ind,2)/mean(abs(usymb_ref0(symb_sel_ind,2))),'ob','MarkerFaceColor','y','MarkerSize',6);
            case 2
                h_ch(3) = plot(usymb_ref0(symb_sel_ind,3)/mean(abs(usymb_ref0(symb_sel_ind,3))),'dr','MarkerFaceColor','y','MarkerSize',6);
            case 3
                h_ch(4) = plot(usymb_ref0(symb_sel_ind,4)/mean(abs(usymb_ref0(symb_sel_ind,4))),'pk','MarkerFaceColor','g','MarkerSize',6);
        end
    end
end

figure(28)
subplot(nCh,1,ch+1)
plot(m_orig,'or')
hold on;
stem(m(:,ch+1),'xb')
hold off;
if ch==0
    title('Message Data Symbols')
    legend ('Original','Recovered');
end
if ch==3
    xlabel('Symbol #')
end
ylabel(['Ant Ch ',num2str(ch),' Value'])
end

figure(27)
legend([h_orig h_ch(logical(h_ch))],[′Original′; ch_label(logical(h_ch))],′Location′,′NorthWest′)
hold off;
figure (29)
subplot(2,1,2)
stem(abs(usymb_ref0(:,active_ch+1)))
xlabel('Symbol #')
ylabel('Level')
title('Channel Magnitude Variation')
legend ('Ch 0', 'Ch 1', 'Ch 2');
disp(['Channel Magnitude Standard Deviation = '])
num2str(std(abs(usymb_ref0(symb_sel_ind,active_ch+1)))) sprintf('\n'))
Appendix B: Coherent Combining System Model in MATLAB

% Demo Version 1 - Idealized System
clc; clear all; close all;

%% System Parameters
fs = 128e6; % 128 MHz pulse rate
fc = 2.4e9; % RF Carrier
nCh = 4; %# Antennas
M = 4; % Alphabet size
reg_len = 8; %Delay register length

%% Simulation Parameters
r2 = 8; %Baseband Samples/Symbol
fsamp = fc*r2; % 19.2 GHz sample freq
tstop = 1e-6; % 1 usec simulation time
t = [0 : 1/fsamp : tstop-1/fsamp]'; %Simulation time
R = fs/4; % Symbol Rate
nSamp = fix(fsamp/R); %Original #Samp/Symbol
r1 = nSamp/r2; %Stage 1 Rate Change
tsym = downsample(t,nSamp); %Symbol Time
rep_seq = [0; 1; 2; 3]; %Sequence to repeat in original message
nSymb = fix(R*tstop); %#Symbols

%% Create Random Digital Messages
m = randint(round(R*tstop),1,M);
phi = 45; %Per antenna phase shift (Deg)
usymb_orig = 1/(2^0.5)*modulate(modem.pskmod('M',M,'SymbolOrder','gray'),m);
figure(1)
for ch = 0:nCh-1
    usymb_tx(:,ch+1) = usymb_orig .*exp(j*ch*phi*pi/180);
    subplot(2,2,ch+1)
    plot(2^0.5*usymb_tx(:,ch+1),'db','MarkerSize',6); % Plot the constellation.
    axis([-1.1 1.1 -1.1 1.1])
xlabel('In-phase')
ylabel('Quadrature')
title(['Input Channel ',num2str(ch)])
    % Include text annotations that number the points.
    hold on;
    for jj=1:M
        text(real(usymb_tx(ch+1,jj)),imag(usymb_tx(ch+1,jj)),num2str(jj-1));
    end
    hold off;
end

%% Pulse Shape Baseband Data
u_tx = rcosflt(usymb_tx,R,r2*R,'sqrt',0.35);
delay = 3;
u_tx = u_tx(r2*delay+1:r2*delay+nSymb*r2,:);
t_bb = [t(1) : r1/fsamp : t(end)]'
% u = rectpulse(usymb,ceil(fsamp/R));
% delay = 0;
for ch = 0:nCh-1
    figure(2)
    subplot(nCh,1,ch+1)
    stem (tsymb*1e9,real(usymb_tx(:,ch+1)),'k','MarkerEdgeColor','r');
    hold on;
    plot (t_bb*1e9,real(u_tx(:,ch+1))); hold off;
    axis ([0 tstop*1e9 -1.1 1.1])
    if ch==0
        title('In-phase BB Data')
        legend ('Orig. Symbol','Pulse Shaped', 'Location','NorthWest')
    end
    if ch==nCh-1
        xlabel('Time (nsec)')
    end
    ylabel(['Input ',num2str(ch),' Level'])
end
if ch==nCh-1
    xlabel('Time (nsec)')
end

f_bb = linspace(-fsamp/(2*r2),fsamp/(2*r2),length(t_bb))./1e9;
figure(4)
subplot(2,1,1)
mag = db(abs(fftshift(fft(u_tx),1)));
plot(f_bb,mag(:,1),f_bb,mag(:,2),f_bb,mag(:,3),f_bb,mag(:,4))
legend('1','2','3','4');
axis([-1 1 -60 50])
xlabel('Frequency (GHz)')
ylabel('Magnitude (dB)')
title('Magnitude Spectrum of Baseband Data')
subplot(2,1,2)
ph = angle(fftshift(fft(u_tx),1))*180/pi;
plot(f_bb,ph(:,1),f_bb,ph(:,2),f_bb,ph(:,3),f_bb,ph(:,4))
axis([-1 1 -361 361])
legend('1', '2', '3', '4');
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of Baseband Data')
%% Upconversion
u_up = resample(u_tx,fsamp,r2*R);
figure(5)
for ch = 0:nCh-1
    s(:,ch+1) = real(u_up(:,ch+1).*(2^0.5*exp(j*2*pi*fc*t))); %Upconvert
    subplot(nCh,1,ch+1)
    plot(t*1e9,s(:,ch+1))
    axis ([400 600 -1 1])
    if ch==0
        title('RF Antenna Signals')
    end
    if ch==nCh-1
        xlabel('Time (nsec)')
    end
    ylabel(['Antenna ',num2str(ch),' Ampl'])
end

%% Generate Switching Pulses
tau = 2/(reg_len*fs); %Pulse width
t0 = 1/(reg_len*fs); %Pulse delay
d = [0 : 1/fs : tstop-1/fs]'; %Pulse time
figure(6)
for ch = 0:nCh-1
    pt(:,ch+1) = 2*pulstran(t,d+ch*t0,'rectpuls',tau)-1;
    subplot(nCh,1,ch+1)
    plot(t*1e9,pt(:,ch+1))
    axis ([0 200 -1.1 1.1])
    if ch==0
        title('Switch Control')
    end
    if ch==nCh-1
        xlabel('Time (nsec)')
    end
    ylabel(['Switch ',num2str(ch),' Ampl'])
end

%% Pulse-Weight
x = s.*pt;
figure(7)
for ch = 0:nCh-1
    subplot(nCh,1,ch+1)
    plot(t*1e9,x(:,ch+1))
    axis ([400 600 -1.1 1.1])
    if ch==0
        title('RF Pulse Trains')
    end
    if ch==nCh-1
        xlabel('Time (nsec)')
    end
end
ylabel(['Input ',num2str(ch),' Level'])
end

f_up = linspace(-fsamp/2,fsamp/2,length(t))./1e9;
figure(8)
subplot(2,1,1)
mag = db(abs(fftshift(fft(x),1)));
plot(f_up,mag(:,1),f_up,mag(:,2),f_up,mag(:,3),f_up,mag(:,4))
legend('1','2','3','4');
axis([-fsamp/4e9 fsamp/4e9 -40 80])
xlabel('Frequency (GHz)')
ylabel('Level (dB)')
title('Magnitude Spectrum of RF Pulse Trains')
subplot(2,1,2)
ph = angle(fftshift(fft(x),1))*180/pi;
plot(f_up,ph(:,1),f_up,ph(:,2),f_up,ph(:,3),f_up,ph(:,4))
axis([-fsamp/4e9 fsamp/4e9 -361 361])
legend('1','2','3','4');
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of RF Pulse Trains')

%% Combine
xsum = sum(x.').
figure(9)
plot(t*1e9,xsum)
axis ([400 600 -2.0 2.0])
xlabel('Time (nsec)')
ylabel('Level')
title('Combined RF')
figure(10)
subplot(2,1,1)
plot(f_up,db(abs(fftshift(fft(xsum))))))
axis([-fsamp/4e9 fsamp/4e9 -20 90])
xlabel('Frequency (GHz)')
ylabel('Level (dB)')
title('Magnitude Spectrum of Combined RF')
subplot(2,1,2)
plot(f_up,angle(fftshift(fft(xsum)))*180/pi)
axis([-fsamp/4e9 fsamp/4e9 -361 361])
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of Combined RF')

%% Downconvert and Channel Filter
rf_down = xsum.*(2^0.5*exp(-j*2*pi*(fc+fs)*t)); %Downconvert (to ease channel filtering)
Wp = (fsamp/16)/(fsamp/2); Ws = (fsamp/8)/(fsamp/2);
ord = cheb2ord(Wp, Ws, 1, 90);
[b,a] = cheby2(ord, 90, Ws);
rf_down = filtfilt(b, a, rf_down);

figure(11)
subplot(2,1,1)
plot(t*1e9,real(rf_down))
ylabel('Real Amplitude')
axis ([min(t*1e9) min(t*1e9)+1000 min(real(rf_down))-0.1 max(real(rf_down))+0.1])
title('Complex Baseband Signal')
subplot(2,1,2)
plot(t*1e9,imag(rf_down))
ylabel('Imaginary Amplitude')
axis ([min(t*1e9) min(t*1e9)+1000 min(imag(rf_down))-0.1 max(imag(rf_down))+0.1])
xlabel('Time (nsec)')

figure(12)
subplot(2,1,1)
plot(f_up,db(abs(fftshift(fft(rf_down)))))
axis ([-fsamp/2e9 fsamp/2e9 -20 80])
xlabel('Frequency (GHz)')
ylabel('Magnitude (dB)')
title('Magnitude Spectrum of Baseband Signal')
subplot(2,1,2)
plot(f_up,angle(fftshift(fft(rf_down)))*180/pi)
xlim ([-fsamp/2e9 fsamp/2e9])
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of Baseband Signal')

%% Channel Filter
bb_dec = pi/(2*sin(pi*fs*tau))*resample(rf_down,r2,nSamp);
bb_flt = rcosflt(bb_dec,R,r2*R,'sqrt/Fs');
bb = 1/nCh*bb_flt(delay*r2+1:(delay+nSymb)*r2,:); %Adjust for delay
usymb_rx = downsample(bb,r2);

figure(13)
subplot(2,1,1)
stem (tsymb*1e9,real(usymb_orig),'k','MarkerEdgeColor','r'); hold on;
plot (t_bb*1e9,real(bb)); hold on;
plot (tsymb*1e9,real(usymb_rx),'.'); hold off;
axis ([0 tatop*1e9 -2.1 2.1])
title('Filtered In-phase BB Signal')
legend ('Orig. Symbol','Recovered w/f','Rec. Symbol','Location','NorthWest')
xlabel('Time (nsec)')
ylabel(['Input Level'])
subplot(2,1,2)
stem (tsymb*1e9,imag(usymb_orig),'k','MarkerEdgeColor','r'); hold on;
plot (t_bb*1e9,imag(bb)); hold on;
plot (tsymb*1e9,imag(usymb_rx),'.'); hold off;
axis ([0 tatop*1e9 -2.1 2.1])

Filtered Quadrature BB Signal

Orig. Symbol, Recovered w/f, Rec. Symbol, Location, NorthWest

Time (nsec)

Input Level

Magnitude Spectrum of Filtered BB Signal

Frequency (GHz)

Level (dB)

Phase Spectrum of Filtered BB Signal

Frequency (GHz)

Phase (Degrees)

Beamforming Radiation Pattern

d_by_wavelength = 0.5; % Antenna Spacing

fs = t0 = 1/(reg_len*fs); % Pulse delay

beam_dir = -180/pi*asin(fs*t0/d_by_wavelength); % Beam look direction (wrt broadside)

Beam steering angle (scan angle) = %0.1f Degrees

Array input inter-element phase shift = %0.1f Degrees, 180*2*fs*t0);

Phase Difference vs. Input Beam Angle for a 4-element, d_by_wavelength spaced uniform array

Input Beam Angle vs. Phase Difference for a 4-element, d_by_wavelength spaced uniform array
array_input = exp(j*ph_diff1*[0:nCh-1]);
weights = 2/pi*sin(pi*Fs*tau)*exp(j*pi*2*Fs*t0*[0:nCh-1]');
array_factor = array_input * weights;
figure(16)
mag = abs(array_factor)/max(abs(array_factor));
plot(theta1,db(mag+eps))
axis([-180 180 -50 1])
xlabel('Angle relative to broadside (degrees)')
ylabel('Normalized Received Power (dB)')
title(['Radiation pattern for a 4-element ', num2str(d_by_wavelength), '\lambda spaced uniform array'])
figure(17)
polardb(theta1*pi/180,db(mag+eps),-50)
title(['Radiation pattern for a 4-element ', num2str(d_by_wavelength), '\lambda spaced uniform array in dB scale'])
figure(18)
polarabs(theta1*pi/180,mag)
title(['Radiation pattern for a 4-element ', num2str(d_by_wavelength), '\lambda spaced uniform array in linear scale'])
% Noise Analysis - Idealized System
clc; clear all; close all;

%% System Parameters
fs = 128e6; % 128 MHz pulse rate
B = 128e6; % Bandwidth
fc = 2.4e9; % RF Carrier
N = 4; %# Antennas
reg_len = 8; %Delay register length
phi = -45; %Per antenna phase shift (Deg)
pin = -30; %Signal power dBm
noise_in_per_hz = -174; %Noise power dBm/Hz
noise_added_per_hz = -166.33; %Added noise power dBm/Hz
pwr_gain = 10; % dB
n = -1; %Desired signal harmonic

%% Simulation Parameters
fsamp = fc*8; % 19.2 GHz sample freq
tstop = ceil(1e-6*fs)/fs; % 1 usec simulation time
t = [0 : 1/fsamp : tstop-1/fsamp]'; %Simulation time

%% RF Signal Creation
figure(1)
for ch = 0:N-1
    s(:,ch+1) = 10^(pin/20)*sqrt(2*1e-3)*cos(2*pi*fc*t+ch*phi*pi/180);
%Phase Shifted RF Signals
subplot(N,1,ch+1)
    plot(t*1e9,s(:,ch+1))
    axis ([0 20 min(min(s))+0.1*min(min(s)) max(max(s))+0.1*max(max(s))])
if ch==0
    title('RF Antenna Signals')
end
if ch==N-1
    xlabel('Time (nsec)')
end
ylabel(['Antenna ',num2str(ch),' Ampl'])
end

f_up = linspace(-fsamp/2,fsamp/2,length(t))./1e9;
figure(2)
subplot(2,1,1)
    mag = abs(fftshift(fft(s)/length(t),1));
    plot(f_up,db(mag(:,1)))+30,f_up,db(mag(:,2)))+30,f_up,db(mag(:,3)))+30,f_up,db(mag(:,4))+30)
    legend('0','1','2','3');
    axis ([[-fsamp/4e9 fsamp/4e9 -140 0])
    xlabel('Frequency (GHz)')
    ylabel('Level (dBm)')
    title('Magnitude Spectrum of RF Antenna Signals')
subplot(2,1,2)
ph = angle(fftshift(fft(s)/length(t),1))*180/pi;
plot(f_up,ph(:,1),f_up,ph(:,2),f_up,ph(:,3),f_up,ph(:,4))
% axis ([-fsamp/4e9 fsamp/4e9 -361 361])
legend('0','1','2','3');
xlabel('Frequency (GHz)')
ylabel('Phase (Degrees)')
title('Phase Spectrum of RF Antenna Signals')
sig_pwr_in_t = db(mean(s.^2)/1e-3,'power'); %from time domain
disp(sprintf('Net signal power (measured) = %0.0f, %0.0f, %0.0f, %0.0f
  dBm', sig_pwr_in_t));
sig_pwr_in = db(max(mag(ceil(length(t)/2):end,:)))+30 %from frequency
domain
sig_in_ind =
  ceil(length(t)/2)+find(mag(ceil(length(t)/2):end,1)==max(mag(ceil(length(t)
  /2):end,1)))-1;

% Add Noise
noise_pwr_in_net = noise_in_per_hz + 10*log10(fsamp) %(dBm)
danl = noise_pwr_in_net - 10*log10(length(t)) %displayed average noise
  level (dBm)
noise_in = wgn(length(t),N,noise_pwr_in_net,'dBm');
s_noisy = s+noise_in;

f_up = linspace(-fsamp/2,fsamp/2,length(t))./1e9;
figure(3)
s subplot(3,1,1)
mag = abs(fftshift(fft(s)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,d
  b(mag(:,4))+30)
legend('0','1','2','3');
axis ([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of RF Antenna Signals')
subplot(3,1,2)
mag = abs(fftshift(fft(noise_in)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,d
  b(mag(:,4))+30)
legend('0','1','2','3');
axis ([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of AWGN')
subplot(3,1,3)
mag = abs(fftshift(fft(s_noisy)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,d
  b(mag(:,4))+30)
legend('0','1','2','3');
axis ([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Noisy RF Antenna Signals')

ant_noise_in_net = db(mean((s_noisy-s).^2)/1e-3,'power');
disp(sprintf('Net input noise power per antenna = %0.2f, %0.2f, %0.2f,
%0.2f dBm', ant_noise_in_net));

pts_per_B = ceil(B/(fsamp/length(t)));
mag = abs(fftshift(fft(s_noisy-s)/length(t),1));
ant_noise_in_B = db(sum(mag(sig_in_ind-fix((pts_per_B-1)/2):sig_in_ind+fix((pts_per_B-1)/2),:).^2,1),'power')+30;
disp(sprintf('Input noise power per antenna over channel BW = %0.2f,
%0.2f, %0.2f, %0.2f dBm', ant_noise_in_B));

snr_in_per_ant = sig_pwr_in_t-10*log10(2)-ant_noise_in_B;
disp(sprintf('Input SNR/antenna = %0.2f, %0.2f, %0.2f, %0.2f dB',
snr_in_per_ant));

%% Filter+Amplify RF Signal
s_amped = s*10^(pwr_gain/20);
s_noisy_amped = s_noisy*10^(pwr_gain/20);

%% Device Added Noise
noise_pwr_added_net = noise_added_per_hz + 10*log10(fsamp) %(dBm)
danl1 = 10*log10(10^(danl/10*10^(pwr_gain/10))+10^((noise_pwr_added_net-
10*log10(length(t)))/10)) %displayed average noise level (dBm)
noise_added = wgn(length(t),N,noise_pwr_added_net,'dBm');
s_noisy_out = s_noisy_amped + noise_added;

figure(4)
subplot(3,1,1)
mag = abs(fftshift(fft(s_amped)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,db(mag(:,4))+30)
legend('0','1','2','3');
axis([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Amplified RF Antenna Signals')

subplot(3,1,2)
mag = abs(fftshift(fft(s_noisy_out-s_amped)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,db(mag(:,4))+30)
legend('0','1','2','3');
axis([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Input+Added AWGN')

subplot(3,1,3)
mag = abs(fftshift(fft(s_noisy_out)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,db(mag(:,4))+30)
legend('0','1','2','3');
axis ([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Amplified Noisy RF Antenna Signals')

amp_noise_out_net = db(mean((s_noisy_out-s_amped).^2)/1e-3,'power');
disp(sprintf('Net output noise power/antenna following amplifier = %0.2f, %0.2f, %0.2f dBm', amp_noise_out_net));

mag = abs(fftshift(fft(s_noisy_out-s_amped)/length(t),1));
amp_noise_out_B = db(sum(mag(sig_in_ind-fix((pts_per_B-1)/2):sig_in_ind+fix((pts_per_B-1)/2),:).^2,1),'power')+30;
disp(sprintf('Output noise power/antenna following amplifier over channel BW = %0.2f, %0.2f, %0.2f, %0.2f dBm', amp_noise_out_B));

% mag = abs(fftshift(fft(s_amped)/length(t),1));
% snr_out_per_amp = db(mag(sig_in_ind,:))+30-amp_noise_out_B
snr_out_per_amp = sig_pwr_in_t-10*log10(2)+pwr_gain-amp_noise_out_B;
disp(sprintf('Output SNR/antenna following amplifier = %0.2f, %0.2f, %0.2f dB', snr_out_per_amp));
disp(sprintf('NF/antenna following amplifier = %0.2f, %0.2f, %0.2f, %0.2f dB', snr_in_per_ant-snr_out_per_amp));

%% Generate Switching Pulses
tau = 4/(reg_len*fs); %Pulse width
t0 = 1/(reg_len*fs); %Pulse delay
A = [1 -1]; %Pulse Amplitudes
d = [0 : 1/fs : tstop]'; %Pulse time
pulse_train = pulstran(t,d,'rectpuls',tau);
figure(5)
for ch = 0:N-1
pt(:,ch+1) = (A(1)-A(2))*circshift(pulse_train,round(ch*t0*fsamp))+A(2);
subplot(N,1,ch+1)
plot(t*1e9,pt(:,ch+1))
% ylim([-1.1 1.1])
axis ([980 1000 -1.1 1.1])
if ch==0
    title('Switch Control')
end
if ch==N-1
    xlabel('Time (nsec)')
end
ylabel(['Switch ',num2str(ch),' Ampl'])
end

%% Pulse-Weight
x = s_amped.*pt;
x_noisy = s_noisy_out.*pt;
figure(6)
for ch = 0:N-1
    subplot(N,1,ch+1)
    plot (t*1e9,x(:,ch+1))
    axis ([0 20 min(min(x))+0.1*min(min(x)) max(max(x))+0.1*max(max(x))])
    if ch==0
        title('RF Pulse Trains')
    end
    if ch==N-1
        xlabel('Time (nsec)')
    end
    ylabel(['Input ',num2str(ch),' Level'])
end
figure(7)
subplot(3,1,1)
mag = abs(fftshift(fft(x)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,db(mag(:,4))+30)
legend('0','1','2','3');
axis ([[-fsamp/4e9 fsamp/4e9 -140 0]])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Pulse Weighted RF Antenna Signals')
subplot(3,1,2)
mag = abs(fftshift(fft(x_noisy-x)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,db(mag(:,4))+30)
legend('0','1','2','3');
axis ([[-fsamp/4e9 fsamp/4e9 -140 0]])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Pulse Weighted AWGN')
subplot(3,1,3)
mag = abs(fftshift(fft(x_noisy)/length(t),1));
plot(f_up,db(mag(:,1))+30,f_up,db(mag(:,2))+30,f_up,db(mag(:,3))+30,f_up,db(mag(:,4))+30)
legend('0','1','2','3');
axis ([[-fsamp/4e9 fsamp/4e9 -140 0]])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Pulse Weighted Noisy RF Antenna Signals')

mag = abs(fftshift(fft(x)/length(t),1));
srch_ind_tol  = 1;  %tolerance in indices for search
harm_srch_ind = sig_in_ind+n*round(fs*length(t)/fsamp)-srch_ind_tol:
sig_out_ind = sig_in_ind+n*round(fs*length(t)/fsamp)-srch_ind_tol+(find(mag(harm_srch_ind,1)==max(mag(harm_srch_ind,1)))-1);
%signal out index
sig_pwr_out = db(mag(sig_out_ind,:))+30  %from frequency domain
conv_gain = sig_pwr_out - sig_pwr_in \ dB

sw_noise_out_net = db(mean((x_noisy-x).^2)/1e-3,'power');
disp(sprintf('Net output noise power/antenna following switching = %0.2f, %0.2f, %0.2f dBm', sw_noise_out_net));

mag = abs(fftshift(fft(x_noisy-x)/length(t),1));
sw_noise_out_B = db(sum(mag(sig_out_ind-fix((pts_per_B-1)/2):sig_out_ind+fix((pts_per_B-1)/2),:).^2,1),''power'')+30;
disp(sprintf('Output noise power/antenna following switching over channel BW = %0.2f, %0.2f, %0.2f, %0.2f dBm', sw_noise_out_B));

snr_out_per_sw = sig_pwr_out-sw_noise_out_B;
disp(sprintf('Output SNR/antenna following switching = %0.2f, %0.2f, %0.2f dB', snr_out_per_sw));
disp(sprintf('NF/antenna following switching = %0.2f, %0.2f, %0.2f dB', snr_in_per_ant-snr_out_per_sw));

%% Combine
x_sum = sum(x.');
x_noisy_sum = sum(x_noisy.');

figure(8)
plot(t*1e9,x_noisy_sum)
axis ([0 20 min(min(x_noisy_sum))+0.1*min(min(x_noisy_sum)) max(max(x_noisy_sum))+0.1*max(max(x_noisy_sum))])
xlabel('Time (nsec)')
ylabel('Level')
title('Combined RF')

figure(9)
subplot(3,1,1)
mag = abs(fftshift(fft(x_sum)/length(t)));
plot(f_up,db(mag)+30)
axis([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Combined RF Antenna Signals')
subplot(3,1,2)
mag = abs(fftshift(fft(x_noisy_sum-x_sum)/length(t)));
plot(f_up,db(mag)+30)
axis([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Combined AWGN')
subplot(3,1,3)
mag = abs(fftshift(fft(x_noisy_sum)/length(t)));
plot(f_up,db(mag)+30)
axis([-fsamp/4e9 fsamp/4e9 -140 0])
xlabel('Frequency (GHz)')
ylabel('Level (dBm)')
title('Magnitude Spectrum of Combined Noisy RF Antenna Signals')
mag = abs(fftshift(fft(x_sum)/length(t)));  
comb_sig_pwr_out = db(mag(sig_out_ind))+30  
comb_conv_gain = comb_sig_pwr_out - (pin-10*log10(2)) %dB

comb_noise_out_net = db(mean((x_noisy_sum-x_sum).^2)/1e-3,'power');  
disp(sprintf('Net combined output noise power = %0.2f dBm',  
comb_noise_out_net));

mag = abs(fftshift(fft(x_noisy_sum-x_sum)/length(t)));  
comb_noise_out_B = db(sum(mag(sig_out_ind-fix((pts_per_B-  
1)/2):sig_out_ind+fix((pts_per_B-1)/2)).^2,1),'power')+30;  
disp(sprintf('Combined output noise power over channel BW = %0.2f',  
comb_noise_out_B));

snr_out_comb = comb_sig_pwr_out-comb_noise_out_B;  
disp(sprintf('Combined output SNR = %0.2f dB', snr_out_comb));  
disp(sprintf('Effective Array NF = %0.2f dB',mean(snr_in_per_ant)-  
snr_out_comb));
Appendix D: Relative Measurement Technique for Noise Figure Measurement of TDM Front-End

The Y-factor method is the most common noise figure measurement technique used in practice [57]. Since the TDM front-end has frequency conversion effects and multiple side bands are inherent in its working, a modified Y-Factor method has to be adopted to correctly characterize its noise figure.

The calibrated excess noise ratio (ENR) specified for the noise source is related to the hot noise temperature \( T_h \) and standard noise temperature \( T_0 \) as

\[
ENR_{cal} = \frac{T_h}{T_0} - 1 \quad (D1)
\]

Correcting for the actual physical cold noise temperature \( T_c \), the corrected ENR is written as

\[
ENR_{corr} = \frac{T_h - T_c}{T_0} = ENR_{cal} + 1 - \frac{T_c}{T_0} \quad (D2)
\]

The overall noise added by the switching (frequency translating) device is a combination of frequency translated source noise and the noise due to the device alone

\[
N_{overall}(T_c, T_s) = (G_{ASB} - G_s) kT_s B + \hat{N}_a(T_c) \quad (D3)
\]

where \( \hat{N}_a(T_c) \) is the noise due to the device alone at physical temperature \( T_c \), \( T_s \) is the source noise temperature, \( G_s \) is the desired signal conversion gain (of the zeroth harmonic in the TDM case) and \( G_{ASB} \) is the noise gain defined as the ratio of the net output noise due to the input noise alone to the input noise at the same frequency. By definition, \( G_{ASB} \) is an all side band measure. In non-frequency translating devices such as regular amplifiers \( G_{ASB} = G_s \).
The measured Y-factor is actually the all side band (ASB) Y-factor $Y_{ASB}$ expressed as

$$Y_{meas} = Y_{ASB} = \frac{N_h}{N_c} = \frac{G_s k T_0 B + N_{overall}(T_c, T_s)}{G_s k T_0 B + N_{overall}(T_c, T_s)}$$

$$= \frac{G_{ASB} k T_0 B + \hat{N}_a(T_c)}{G_{ASB} k T_0 B + \hat{N}_a(T_c)}$$

from which

$$\hat{N}_a(T_c) = G_{ASB} k B \left( \frac{T_h - Y_{meas} T_c}{Y_{meas} - 1} \right)$$

and the generally calculated noise factor is actually the all side band (ASB) noise factor $F_{ASB}$ expressed as

$$F_{ASB} = \frac{N_{out}}{G_{ASB} k T_0 B} = \frac{G_{ASB} k T_0 B + \hat{N}_a(T_c)}{G_{ASB} k T_0 B}$$

$$= 1 + \frac{1}{T_0} \left( \frac{T_h - Y_{meas} T_c}{Y_{meas} - 1} \right)$$

The relevant measure of noise performance in the TDM context is the Single Sideband (SSB) Noise Factor $F_{SSB}$ as defined by IEEE [58] and given by

$$F_{SSB} = \frac{N_{out}}{G_s k T_0 B} = 1 + \frac{N_{overall}(T_c, T_s)}{G_s k T_0 B} = \frac{G_{ASB} k T_0 B + \hat{N}_a(T_c)}{G_s k T_0 B}$$

Substituting for $\hat{N}_a(T_c)$ in (D7) from (D5) above and simplifying we have

$$F_{SSB} = \frac{G_{ASB}}{G_s} F_{ASB}$$

$G_{ASB}$ is determined as part of the standard Y-factor noise measurement technique. In addition, $G_s$ will have to be measured too to determine $F_{SSB}$ of the TDM wireless receiver.

In relation to the previous analysis of a single TDM amplifier channel in Section 2.1.5, it is readily found that
\[ G_{\text{asb}} = \frac{G}{N} \left( \frac{1}{N} + \frac{1}{L} \left( 1 - \frac{1}{N} \right) \right) \]

\[ G_s = \frac{G}{N^2} \]

\[ \hat{N}_a(T_c) = \frac{N_s}{N} \]

As it is not possible to accurately determine the added noise \( \hat{N}_{a,TDM\_amp}(T_0) \), hence the \( F_{ASB} \), of a single TDM amplifier channel directly, an indirect relative measurement technique is adopted to obtain this value. This approach involves the measurement of net output noise during TDM operation after terminating all the inputs to the TDM array with the same cold noise terminations. Assuming that the \( N \) TDM channels are all identical, we may then determine the ratio \( R \) of the output noise from the TDM array to the output noise from a single static amplifier as

\[
R = \frac{N_{out\_TDM\_array}}{N_{out\_amp}} = \frac{N(G_{asb}kT_cB + \hat{N}_{a,TDM\_amp}(T_c))}{GkT_cB + \hat{N}_{a\_amp}(T_c)}
\]

\[
= \frac{N(G_{asb}kT_cB + (F_{ASB\_TDM\_amp} - 1)G_{asb}kT_0B)}{GkT_cB + (F_{amp} - 1)GkT_cB}
\]

\[
= \frac{NG_{asb}(T_c + (F_{ASB\_TDM\_amp} - 1)T_0)}{G(T_c + (F_{amp} - 1)T_0)}
\]

Simplifying (D10) further we obtain

\[
F_{ASB\_TDM\_amp} = \frac{RG}{NG_{asb}} \left( F_{amp} - 1 \right) + \left( T_c \frac{T_0}{T_c} \right) \left( 1 - \frac{T_c}{T_0} \right)
\]

(D11)

All the factors on the right side of (D11) being either measured or known beforehand, we can determine \( F_{ASB} \) of the TDM amplifier. The single sideband noise figure \( F_{SSB} \) of this TDM amplifier may then computed using (D8).
Appendix E: Measurement Technique for Noise Figure Measurement of CC Front-End

In the coherent combining mode of operation, the output noise includes the contribution from all the input channels, although only one of them is subject to the cold to hot noise variation that is part of the normal Y-factor measurement procedure. Assuming equal contributions from the $N$ identical channels to the output noise, the measured Y-factor which is actually the all side band (ASB) Y-factor $Y_{ASB}$ may therefore be expressed as

$$Y_{\text{meas}} = Y_{ASB} = \frac{N_h}{N_c} = \frac{(G_{kT_eB} + N_{\text{overall}}(T_e, T_h)) + (N - 1)(G_{kT_eB} + N_{\text{overall}}(T_e, T_h))}{N(G_{kT_eB} + N_{\text{overall}}(T_e, T_h))}$$

$$= \frac{G_{ASB}kT_eB + (N - 1)G_{ASB}kT_eB + N \cdot \hat{N}_a(T_e)}{N(G_{ASB}kT_eB + \hat{N}_a(T_e))} \quad \text{(E1)}$$

from which

$$\hat{N}_a(T_e) = G_{ASB}kB \left( \frac{1}{N} \left( T_h + (N - 1)T_e \right) - Y_{\text{meas}}T_e \right) \frac{Y_{\text{meas}} - 1}{Y_{\text{meas}} - 1} \quad \text{(E2)}$$

and the generally calculated noise factor is actually the all side band (ASB) noise factor $F_{ASB}$ expressed as

$$F_{ASB} = \frac{N_{\text{out}}}{G_{ASB}kT_0B} = \frac{G_{ASB}kT_eB + \hat{N}_a(T_e)}{G_{ASB}kT_0B}$$

$$= 1 + \frac{1}{T_0} \left( \frac{1}{N} \left( T_h + (N - 1)T_e \right) - Y_{\text{meas}}T_e \right) \frac{Y_{\text{meas}} - 1}{Y_{\text{meas}} - 1} \quad \text{(E3)}$$

The relevant measure of noise performance in the coherent combining context is the Single Sideband (SSB) Noise Factor $F_{SSB}$ as defined by IEEE [58] and given by
\[ F_{\text{SSB}} = \frac{N_{\text{out}}}{G_s k T_0 B} = 1 + \frac{N_{\text{aver}}(T_c, T_0)}{G_s k T_0 B} \]
\[ = \frac{G_{\text{ASB}} k T_0 B + \hat{N}_a(T_c)}{G_s k T_0 B} = \frac{G_{\text{ASB}}}{G_s} + \frac{\hat{N}_a(T_c)}{G_s k T_0 B} \]

(E4)

Substituting for \( \hat{N}_a(T_c) \) in (E4) from (E2) above and simplifying we have

\[ F_{\text{SSB}} = \frac{G_{\text{ASB}}}{G_s} \left( \frac{1}{N} \left( T_b + (N-1)T_c - Y_{\text{meas}} T_c \right) \right) = \frac{G_{\text{ASB}}}{G_s} F_{\text{ASB}} \]

(E5)

\( G_{\text{ASB}} \) and \( F_{\text{ASB}} \) are determined as part of the standard Y-factor noise measurement technique. In addition, \( G_s \) will have to be measured too to determine \( F_{\text{SSB}} \) of the coherent combining wireless receiver.