ABSTRACT

FORD, SHELTON JEROME. The Effect of Graphing Calculators and a Three-Core Representation Curriculum on College Students' Learning of Exponential and Logarithmic Functions. (Under the direction of Lee V. Stiff)

The purpose of this study was to investigate the potential benefits of a multi-representational curriculum on students' understanding of and connections among graphical, tabular, and symbolic representations of algebraic concepts.

The participants of the study were 113 college students enrolled in developmental college algebra at a southern university. This study utilized a quasi-experimental design in which instructors taught the course from a scripted algebraic perspective while the researcher taught the course from a functional approach simultaneously introducing multiple representations.

The effect of a three-core representation curriculum on student success was assessed with a pretests and posttests of nine problems, with three representations; algebraic, graphical, and numerical. Also used were pretests and posttests of ten calculator knowledge questions. The problems were chosen because of their prevalence in most developmental college algebra curricula. The three-core representation curriculum was more successful in increasing student achievement. Students from the three-core representation curriculum scored significantly higher and were significantly more adept in using representational methods other than algebraic to solve the problem. This research showed that a multi-representational curriculum could be effective in expanding students' web of connected knowledge of algebraic and functional concepts.

The Effect of Graphing Calculators and a Three-Core Representation Curriculum on College Students' Learning of Exponential and Logarithmic Functions

by

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DEDICATION

I would like to dedicate this to my mother, Berlene Mason Ford. I sure wish she were here to enjoy this moment with me and see the fruit of her investment. She passed from this world in 2002, and I have really missed her. She was a true lady in every sense of the word. She was a steady, hard-working woman that spoke volumes by example. I am thankful for the encouragement she offered during her stay on earth and for the joy she brought to my life. Her memory is truly the wind the beneath my wings.

BIOGRAPHY

Shelton Jerome Ford was born in Manhattan, New York on November 20, 1971, the son of James and Berlene Ford. At the age of seven, he moved to Chadbourn, NC where he attended and graduated from West Columbus High School in May 1990. He attended North Carolina State University, where he graduated with a Bachelor of Science in Secondary Mathematics Education in May 1995.

Shelton taught mathematics at Garner Senior High School in Garner, NC for four years, 1995 until 1999. During that time, he was also a mathematics and science teacher for the Anderson Academy for Science and Mathematics at North Carolina State University.

Shelton later relocated to Fayetteville, NC in 1999 where he took on the position of Program Coordinator at Fayetteville State University in the Mathematics and Science Education Center. While there he began his graduate work and received his master's of science degree in Mathematics in May 2000.

He began teaching at North Carolina State University as a graduate teacher assistant in August 2002 and entered the Ph.D. program in Mathematics Education under the direction of Lee. V. Stiff, Professor.

Shelton is currently an assistant professor of mathematics education in the Mathematics Department at Winston- Salem State University.

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CHAPTER 1

Introduction

The introduction of graphing calculators in the late 1980s, their widespread availability in the early 1990s, and their wide spread use in mathematics classrooms of today; demonstrate the need for research on learning environments that incorporate the use of graphing calculators. The introduction of graphing calculators involved the need for new visions for mathematics education, many of which called for broader access to deeper mathematics for all students (American Association for the Advancement of Science, 1993, National Council of Teachers of Mathematics, 1989, and National Research Council, 1989). This was especially true of the vision of the 1989 *Curriculum and Evaluation Standards for School Mathematics* produced by the National Council of Teachers of Mathematics (NCTM). The *Standards* asserted that “scientific calculators with graphing capabilities should be available to all students at all times” (p. 124). According to the *Report of the 2000 National Survey of Science and Mathematics Education*, the use of handheld graphing Technology was used by more than 80 percent of the High school mathematics teachers surveyed. Ruthven (1997) presents growing evidence of the graphing calculator prevalence in mathematics education in other countries as well.

Barrett and Goebel (1990) predicted that “the presence of graphing calculators in High school mathematics classrooms would have a significant impact on the teaching and learning of secondary school mathematics in the 1990s” (p. 205). In particular, graphing calculators can improve problem solving (Dick, 1992). According to Dick, the following are some examples of how graphing Technology assists with problem solving:

- Graphing Technology frees up time for instruction by reducing instructional attention given to algebraic manipulation.
- It provides more options for solving problems which is especially useful for students with weaker algebraic skill.
- Students perceive problem solving tasks differently when they are freed from numerical and algebraic computations and able to concentrate on setting up problems and analyzing solutions (Dick, 1992).

In particular, the introduction of graphing calculators was expected to impact the understanding senior secondary students have of what constitutes a “complete graph” of a function, that is, a graph showing “all the relevant behavior” of a given function (Demana & Waits, 1990, p. 216). However, Doerr and Zangor (2000), Penglase and Arnold (1996), Williams (1993), and others suggested there was a lack of detailed documentation about how students make use of graphing calculators in the classroom, especially from the students’ viewpoints. While the types of Technology potentially available in mathematics classrooms have expanded to include computer algebra systems (CAS), as recently as 2002, Zbiek pointed out that “secondary mathematics teachers—like researchers and college instructors—may find the lure of graphing capabilities is more compelling” (p. 132). For this reason, and to add to the knowledge base against which the use of other Technologies can be evaluated, it is imperative that research continues to be conducted into how students use graphing calculators in the learning of functions. No matter how sophisticated other Technologies may be, it is the graphing calculator that students readily have, and as such, it is crucial that the effects of this Technology on the teaching and learning of mathematics are understood by teachers. According to Burrill, Allison, Breaux, Kastberg, Leatham, and Sanchez (2002), “By conducting rigorous studies of important questions and relating the result to classroom practice, we can ensure that handheld Technology contributes in positive ways to improved mathematics education” (p. 56).

Background to the Study

Graphing calculators are common tools used in mathematics classrooms and on assessment tasks of today (Knuth, 2000). Although the term “tool” has various meanings in the literature, here we use Salomon’s (1993) definition. He argues that tools “…need not be a real object.” Tools have a purpose, require skills and knowledge for their use, “serve functions beyond themselves,” and can be distinguished from machines in that “they need to be skillfully operated upon throughout their functioning to achieve their purpose” (pp. 179-80). More specifically, Salomon proposes that educationally useful tools are those that “stimulate Higher order thinking and guide it in a way that makes learners better and more independent thinkers” (p. 181). Hence, graphing calculators are cognitive tools or smart tools (Pea, 1993) because they provide an environment in which students can transcend their cognitive limitations, access more complex thought and activity, and engage in learning which, without the tool, would not be possible. The compulsory use of graphing calculator Technology in the mathematics classroom raises many issues. These include the effects of the Technology on student learning and the extent of changes, if any, in teaching practice as graphing Technology becomes commonplace in the classroom.

Although many studies have been undertaken in the use of graphing calculators, there are still many questions to be asked and answered. Many graphing calculator studies simply "compare the use of the graphing calculator to the use of paper-and-pencil on the same tasks giving only limited insight into how and why students use calculators in the instructional context" (Doerr & Zangor, 2000, p. 144). Penglase and Arnold (1996) in their review of over one hundred papers found that much of the research, in the years between 1990 and 1995, did little to inform and guide practice as it failed to distinguish the role of the tool from that of the instructional process. Burrill et al. in their 2002 review of the research literature note that the core finding is that the type and extent of gains in student learning in the presence of handheld graphing Technology "are a function, not simply of the presence of the graphing Technology, but of how the Technology is used in the teaching of mathematics" (p. i). They concluded that "specific issues regarding the effective use of handheld graphing Technology in the classroom have not yet been adequately addressed" (p. ii). Penglase and Arnold (1996) suggest the need for studies that clearly define the learning environment and address graphing calculator use within the environment.

Graphing Calculators and Functions

Graphing calculators have particular application in the teaching and learning of functions, a key mathematical concept in senior secondary school mathematics (Ferrini-Mundy & Lauten, 1993; Heingraj & Shield, 2002; Knuth, 2000; Leinhardt, Zaslavsky, & Stein, 1990). Ferrini-Mundy and Lauten, for example, see function as a concept "central to modern mathematics" (1993, p. 155). Furthermore, Demana and Waits (1990, p. 222) suggest "deeper understanding and an intuition about functions are important by-products of a technologically rich approach to the teaching and learning of mathematics".

The concept of function contains many interconnected ideas. Generally, a single notation system or representation cannot adequately represent all aspects of a complex idea (Ferrini-Mundy & Lauten, 1993; Kaput, 1989). Hence, Kaput argues that "representation systems, when learned, are used by individuals to structure the creation and elaboration of their own mental representations" (p. 167, 1989). He suggests that multiple connected representations allow learners to combine understanding from different representations in such a way as to build a better understanding of complex ideas and to apply these ideas and concepts more effectively. Kaput (1987) discuss the idea of representation in mathematics education, the translation process, and the continuing difficulties experienced by students in changing from one representation to a second. Zaslavsky, Sela, and Leron (2002) propose that some of these difficulties are a result of confusion "between the algebraic and geometric aspects of slope, scale and angle" (p. 119).

Functions require multiple connected representations for their full understanding. In fact, multiple representations are common and virtually unavoidable in functions (Eisenberg & Dreyfus, 1994). These representation systems include: algebraic, numerical, and graphical (Kaput, 1989, p. 171; Lloyd & Wilson, 1998, p. 252). The graphical representation is sometimes referred to as the visual representation (Acuña, 2002; Eisenberg & Dreyfus, 1994) or geometric (Zaslavsky, Sela, & Leron, 2002). Each of these representations illuminates some, but not all, aspects of the idea of a function. Each "has its intrinsic level of complexity" according to Janvier (1987b, p. 31). The connections between the different representations are key aspects of much mathematics. It is very important that a learner has multiple aspects to their function concept image (Lloyd & Wilson, 1998). Students need to be able to work both within and across representations. The ability to pass from one representation to another when appropriate, the flexibility to use the most appropriate representation in solving a problem, and the ability to “see” one representation when working in another are the most essential components of function sense (Schwarz, cited in Eisenberg & Dreyfus, 1994, p. 46).

For a complete understanding of functions the various representations demand that students move from one to another. Firstly, given the algebraic representation of a function, the students should be able to generate the numerical table and the graph of that function. Secondly, given the graph of a function, the students should be able to generate the numerical table and the algebraic representation of that function. Thirdly, given the numerical table of a function, the students should be able to generate the graph and the algebraic representation of that function. Further, students need to be able to make connections across the three representations.

The use of the graphing calculator should make it much easier for students and teachers to work simultaneously with all three representations. Hence, the depth of students’ understanding of functions would be expected to change with the use of graphing Technology, including increased depth of understanding, and new challenges due to misconceptions becoming apparent with graphing calculator use.

Graphing calculators and their linked representations provide students with immediate feedback as to how changes in one representation affect another. Students then have the opportunity to modify their ideas before inappropriately strengthening misconceptions. However, learning tasks need to be chosen that facilitate this learning process. Teachers need to be careful not to continue past practices that focused mainly on a single representation.

Problem Statement

The particular research questions investigated in this study were motivated by the NCTM *Standards* and the goal of better understanding how the graphing calculator can be used to facilitate understanding of functions. The specific focus of the study was student understanding of algebraic, numerical, and graphical representations of exponential and logarithmic functions, known as "the three core representations." The need to investigate the effect of graphing calculators on students' understanding of the function concept finds further support from Dunham and Osborne (1991) who contend, "There is a critical need for instructional materials that combine visual and algebraic Techniques in a consistent, coordinated approach to functions" (p. 47).

While there is a significant body of research supporting the use of graphing calculators with regard to students' understanding of function and graphing concepts (Durham & Dick, 1994), most of the studies on graphing calculators have compared achievement and/or attitude of graphing calculator and non-graphing calculator users (Wilson and Krapfl, 1994). Missing are studies that vary the way that graphing calculators are used with students involving exponential and logarithmic functions. There is a need to know how using the three core representations in conjunction with a graphing calculator affect the ways that students generate mathematical knowledge, particularly with Low performing students.

Therefore, it was determined to be both reasonable and appropriate to study the impact of the graphing calculator in a multiple representation environment and a traditional curriculum environment among Low performing students. The specific research questions for the study will be stated at the end of Chapter 2.

Defining the Key Terms

Calculator as a computational tool implies that the student's mathematical goal in utilizing the graphing calculator is limited to basic mathematical operations such as addition, subtraction, multiplication, division, conversion from a decimal to a fraction, and finding squares and square roots.

Calculator as a problem-solving tool refers to a student's mathematical goal of developing the concept of function by incorporating the use of the table and graphing features of the calculator.

The *three core representations* refer to Kaput's (1989) definition as graphical, numerical, and algebraic.

Knowledge Pre- and Post-Assessments are assessments given before and after topics are discussed in the research study. There are two (2) types of knowledge assessments: Calculator Knowledge Assessment and Exponential/Logarithmic-Knowledge Assessment.

Student achievement in the study refers to how students perform on the Exponential/Logarithmic Pre- and Post-Assessment. The Exponential/Logarithmic Post-Assessment consists of three (3) Individual Post-tests.

A *Calculator Knowledge Assessment* categorizes students as either: Low Tech Savvy or High Tech Savvy. *Low Tech Savvy* refers to students who scored 10 (ten) points or less on the Calculator Knowledge assessment. *High Tech Savvy* refers to students who scored 11 (eleven) points or more on the Calculator Knowledge assessment.

CHAPTER 2

Review of Literature

Graphing Technology and Its Role in Teaching and Learning Functions

Background

The use of a coordinate system, often attributed to René Descartes, has existed since before the time of Apollonius of Perga (about 300-200 B.C.). This notion was further developed by Oresme into a coordinate system close to our modern version prior to 1361. He provided "an early suggestion of what we now describe as the graphical representation of function" (Boyer, 1991, p. 264) as "he grasped the essential principle that a function of one unknown can be represented as a curve" (p. 265). This graphical representation increased the repertoire of representations available to mathematicians and allowed a greater insight into the concept of functions. However, teacher use of the algebraic representations of functions still dominates the mathematics curriculum (Arnold, 1992, p. 115).

The impact of recent graphing Technology has allowed the production of graphs to change from a time consuming and often difficult activity to a simple and quick occurrence, whether one begins with an algebraic or numerical representation. The easy creation of graphs makes it possible for a large number of graphs to be observed and provides easy access to the graph of various function types. Graphing calculators are not merely providing another way of doing the same old things; they allow us to do new things, including accessing and linking multiple representations almost simultaneously (Dick, 1996; Kaput, 1989, 1992; Pea, 1993).

Without a doubt, functions have an important place in mathematics curriculum (Dubinsky & Harel, 1992; Ferrini-Mundy & Lauten, 1993; Knuth, 2000; Leinhardt, Zaslavsky, & Stein, 1990; Marjanovic, 1999; Yerushlamy & Shternberg, 2001; Zaslavsky, 1997). Marjanovic (1999), although discussing reform in the High school mathematics curriculum during the first half of the twentieth century, proposes "the most important innovation at that period was the introduction of the concept of function into the secondary school, emphasizing the dominant place of that concept in contemporary mathematics" (p. 43).

Context of the Study

The Technology Principle of *Principles and Standards for School Mathematics* (*PSSM*) (2000) signaled the need for Technology to be used by all students. Simply having Technology in the classroom is not enough. There is a corresponding need for teachers to actively seek to change the teaching and learning experiences of students by taking advantage of the Technology. *PSSM* envisions classrooms in which "every student has access to Technology" (p. 25) and is immersed in a learning environment where the Technology "enriches the range and quality of investigations by providing a means of viewing mathematical ideas from multiple perspectives" (p. 25). The *PSSM* Technology Principle (2000) recommends that teachers "enhance their students' learning opportunities by selecting or creating mathematical tasks that take advantage of what Technology can do efficiently and well" and ensure that where students have access to Technology for learning they also have access when their understanding is being assessed. They define appropriate Technology as that which "should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions…used widely and responsibly, with the goal of enriching students' learning of mathematics" (p. 25).

In mathematics education one major interest is the potential of graphing calculators to dramatically change the teaching and learning of functions. The use of graphing Technology has the potential to move the implementation of the mathematics curriculum from the passive transfer of knowledge to students being in the position of taking an active approach to their learning (Connors and Snook, 2001). Mitchelmore and Cavanagh (2000) echo the call of Penglase and Arnold (1996) and others for research to explore changes to learning functions when using graphing calculators. Cavanagh's clinical interview with 25 Year-10 and 25 Year-11 High-achieving students suggested that "many difficulties in using a graphics calculator may be due to an inadequate understanding of some fundamental mathematical ideas including scale, accuracy and approximation, and the link between different representations of functions" (2001, p. 1). Cavanagh suggests that these difficulties are related to shortcomings in the curriculum and exist in both a graphing calculator and a non-graphing calculator environment. Barret and Goebel (1990) suggest access to graphing calculators that allow students to graph both functions and ordered pairs of data will allow students to "investigate and explore mathematical concepts with keystrokes" (p. 205). Furthermore, they state that the graphing calculator will allow a learning environment to exist where students and teachers become partners in developing mathematical understandings and solving problems. Vonder Embse (1992) suggests that the interactive graphing calculator environment allows students to explore and experiment with function relationships, better reconcile numerical and graphical representations, and engage more in mathematical reasoning, connections, and communication thus providing an "ideal environment for teaching and learning mathematics" (p. 65). This study investigates whether students use graphing calculators effectively in their learning of functions, and how the uses of the three core multiple representations affect students' understanding.

The following literature review addresses the relevance of multiple representations — including the importance of the graphical representation particularly in the teaching and learning of functions; differences between graphing calculators and other graphing Technology; the relevance of multiple representations when using graphing Technology; the various ways in which functions can be perceived in a multiple representation environment; and changes to, and implications for, teaching and learning in a graphing Technology learning environment.

MultipleRepresentations

In the following section the three core representations (Kaput, 1989) are detailed, as are their differences and the connections between them. The benefits and concerns of a multiple representation learning environment are also discussed.

The use of multiple representations has the potential to make learning meaningful and effective. Eisenberg and Dreyfus (1994) suggest that in the study of functions "multiple representations are so ubiquitous … they can hardly ever be avoided" (p. 46). However, in order to realize this potential, the advantages and disadvantages of each representation must be known (Friedlander & Tabach, 2001; Kaput, 1989, 1992). For the numerical representation, also known as tabular representation, the use of numbers allows students to use familiar objects to demonstrate relationships and consider specific cases. However, the lack of generality of this representation may result in overlooking some solutions. The graphical representation provides a visual display, portrays a collection of specific cases, and can always be used when an algebraic solution is beyond the current capabilities of a student, or when no algebraic method exists. Accuracy of solutions may be limited, and scaling may affect interpretations of solutions. As with the numerical representation, only a portion of the domain is visible. Friedlander and Tabach (2001) describe the algebraic (also formula or equation) representation as "concise, general, and effective…sometimes the only method of justifying or proving" (p. 174). However, they did indicate that the exclusive use of the algebraic representation could obstruct mathematical meaning. Thus this obstruction could cause difficulties in some students' interpretation of their results.

Views on mathematical learning suggest that "multiple representations of concepts yield deeper and more flexible understandings" by students (Keller & Hirsch, 1998, p. 1). Making connections among mathematical concepts and representations of those concepts is critical in developing relational understanding because "knowing how they are inter-related enables one to remember them as parts of a connected whole which is easier" (Skemp, p. 159).. Knuth (2000) agrees that the study of multiple representations is important, but suggests that many students leave secondary school "lacking an understanding of the connections between these representations" (p. 500).

The Three Core Representations

The three core representations (Kaput, 1989) used in senior secondary mathematics classrooms and of major interest in this present study are the algebraic representation, the graphical representation, and the numerical representation. Kaput distinguishes between the numerical and graphical representations in the way they sample the domain. The numerical representation "displays discrete, finite samples, whereas coordinate graphs display continuous, infinite samples" (p. 172). He argues that the graphical representation is fuller and simpler, in part due to the condensing of a pair of numbers to a single point. However, he notes graphs do have disadvantages related to misleading views, inferences, and scale. These include the lack of a referential system, conflicts occurring with everyday experiences, and the effects of scaling. However, the graphical representation "leave(s) precise details unclear" according to Goldenberg (1987, p. 197). The shape of a graph is dependent upon the viewing window and this contributes to the confusion between transformations and rescaled views of graphs (Dunham & Osborne, 1991; Goldenberg, 1987). The finite viewing screen of graphing Technology "creates a need for scaling and positioning skills" (Dick, 1992, p. 152) as the graphical behavior of the function under consideration may be outside the current viewing window or distorted by the scale as "zooming in obscures global information and zooming out obscures local information" (p. 153). Advantages of tables include their specific nature and the fact that they are often ordered and evenly spaced. This allows changes in consecutive values to be explicitly determined. Tables provide examples of the relationship but not its exact nature (Goldenberg, 1987), although students need an understanding of the accuracy of numerical values provided by graphing Technology. In contrast, the algebraic representation specifies the exact nature of a relationship but provides neither specific examples nor visual display. In spite of these differences and difficulties, students need all three representations as different features apply differently in different situations (Lloyd & Williams, 1998). Each representation, while conveying some aspects of the function well, leaves other aspects unclear.

Moving from one representation to another is a translation according to Janvier (1987a, 1987b) and Kaput (1987). The graphing calculator has the potential to change teaching and learning with regard to the translation of functions. It allows the Technology to translate actions across representations and provides the learner time to observe and explain the consequences of shifting from one representation to another. “The cognitive linking of representations creates a whole that is more than the sum of its parts.”(Kaput, 1989, p. 179). The graphing calculator helps us “to make fuller use of numerical and graphical Techniques, Techniques typically of greater simplicity and generality” (Kaput, 1989, p. 192) and hence has the potential to improve understanding.

Benefits of Multiple Representations

Several benefits results as an effect of using multiple representations in instruction (Ainsworth, Bibby, and Wood, 1997). First, multiple representations provide support for different ideas and processes, as properties from multiple representations can only enhance the view provided by a single representation. This contrasts with Even’s (1990) view. She suggests it does not follow that understanding a concept in one representation implies understanding the same concept in a second representation. Second, multiple representations clarify concepts and ideas by providing different views of the same idea. Third, multiple representations provide a rich source of views of a concept and create the opportunity to make-connections across the representations. Ainsworth et al. (1997) conclude that if development of better understanding is the desired outcome of the teaching curriculum, then two things need to occur. First, unfamiliar representations should be presented alongside familiar ones, and, second, students need to experiment with constraints available in one representation in order to control outcomes in another. So, although multiple representations can improve student understanding they do not necessarily do so. This concurs with the claims of Even (1990, 1998).

In their substantial review of research on functions and graphs, Leinhardt et al. (1990) concluded that “working simultaneously with at least two linked representations is more manageable” (p. 7) when using graphing Technology. Although each of the representations, and its advantages and disadvantages, should be considered regardless of the medium, it is only with graphing Technology that working simultaneously with representations is truly possible. Demana and Waits (1992) suggest that a global understanding is enhanced as students can see both their algebraic input and the solution or graphical representation simultaneously.

Importance of the graphical representation: visualization. Many authors argue that the real power of graphing calculators is in its graphical representations of functions. The benefits of visualization have been recognized by many researchers, including Brown (1998), Kaput (1992), Smart (1995), Tall (1996) and Underwood (1997). These benefits include the immediate visual feedback provided by the graphing calculator (Selinger & Pratt, 1997; Williams, 1993), its interactive nature, and the opportunity to understand the connections between graphical and other representations (Hector, 1992; Hollar & Norwood, 1999; Kaput, 1992; Wilson & Krapfl, 1994; Connors and Snook, 2001).

Graphical visualization allows students to develop an understanding of the relationships between functions and their graphs. “The global nature of the graphical image means that information may be extracted quickly and easily” (Arnold, 1998, p. 182). Spatial visualization skills are improved through the use of graphing Technology according to Ruthven (1996, p. 448) and Vasquez (1991, as cited in Shoaff-Grubbs, 1994). The quantity of graphs that a student can generate helps them to construct knowledge of a collection of functions and their graphical representation, and provides them with the ability to connect these and make generalizations as a basis for future work. Movshovitz-Hadar (1993) suggests that an increased emphasis on a graphical approach provides students with an understanding of the parameters of a function and the effect these have on the graph. She suggests that the more common algebraic approach does not result in this important understanding (p. 391).

On the other hand, Goldenberg (1987) contends that the graphical representation may cause students to alter correct beliefs and he proposes several reasons for this. It is not because the graph is incorrect but rather the observation skills of students need development. Students often focus on a particular feature of a view of a function and ignore others. Sometimes multiple features are considered but one is treated as dominant. To help overcome these difficulties teachers need to provide students with situations that allow them to focus on all the important aspects of a graph. Students can create illusions, which are false interpretations because of what students only see in the window of the calculator. Students who do not focus on all aspects of a graph can encounter false illusions. Illusions can also be created as a result of several factors including the infinite size of a graph, the position of the viewing window on the function, and the effect of the scale on the view visible.

Not only does the graphical representation allow for an expanded understanding of the concept of function, it also improves spatial skills. However, visual images of functions do not necessarily equate with being able to make connections between algebraic and graphical representations. Again, it is the curriculum and instructional environment that affect both learning outcomes and difficulties experienced.

Is more better? Underwood (1997) questions whether Technology that uses multiple representations "always means better or more efficient (learning) or does the use of multiple representations place new learning demands on the child" (p. 3). She poses the question as to "whether new Technologies offer significantly different ways of representing ideas and knowledge" (p. 5). Wilson and Krapfl (1994) suggest that graphing calculators do just this. The interactive nature of the tool allows students to "experiment and explore, thus fundamentally changing the way they learn important properties of functions" (p. 253). By toggling between "the three most common functional representations … [students are able to] … build conceptual links among these representations" (p. 254). Connors and Snook (2001) supports this view suggesting it as one of the reasons for the fluency attributed to students in a graphical tool environment by Hollar and Norwood (1999). By combining representations students are no longer restricted by the weaknesses of one particular representation. Clearly the graphing calculator learning environment is one where interactions with more than one representation are almost inevitable.

The Multiple Representations of Functions

The easy access to multiple representations has significant implications for the teaching and learning of functions the mathematics classroom. Functions are “multi-faceted” (Lloyd & Wilson, 1998, p. 250) and cannot be fully understood within a single representation environment as each representation illustrates only some of their complexity. The need for multiple representations to express complex ideas, the links between the representations, and the difficulties encountered in relating the local and global views of functions will now be considered.

Complex ideas require multiple representations. According to Coulombe and Berenson a “fluency with multiple representations of mathematical relationships plays a significant role in the successful development of algebraic thinking” (2001, p. 168). This fluency is enhanced for students who learn the algebra of functions in a graphical tool environment according to Hollar and Norwood (1999).

Complex mathematical ideas frequently cannot be expressed with a single representation system according to Asp, Dowsey, and Stacey (1993), Kaput (1989), Markovits, Evelon, and Bruckheimer (1986), Norman (1993), and others. “The idea may require multiple, linked representations for its full expression and these different representations may aid the learner’s understanding of the idea” (Asp, et al., 1993, p. 51). The concept of function is one such complex idea. Ferrini-Mundy and Lauten (1993) suggest students’ interactions with this concept are very complex. Understanding the relationship between different representations of a function is essential for a fully developed concept of function, thus contributing to this complexity.

Being able to make connections between representations is crucial to the underlying concept of functions (Even, 1998; Kaput, 1992; Yerushalmy, 1991). Graphing Technology provides students with the opportunities to make these links. Functions can be represented in a variety of ways including verbal descriptions, ordered pairs, tables, equations, and graphs. Students need to understand and be able to work within each of these representations. They also need to be able to translate freely between them, to understand that "the same function can be represented by each of the above representations" (Markovits et al., 1986, p. 19), and deal with two representations simultaneously. To fully understand the concept of function students must be able to treat the different representations as different systems that by themselves cannot completely describe a function.

The graphing Technology provides the opportunity for students to build a deeper understanding of functions and so explore the complexity of the concept. Understanding the concept of solution and the solution process is also enhanced by graphing Technology. It provides the opportunity to make connections between the algebraic and graphical representation of functions (Duren, 1991). Lessons that take advantage of the power of the graphing Technologies not only generate more data and ideas, but also encourage discussion and negotiation of the meaning of student observations and findings. This enhanced involvement allows students to construct their own understanding of mathematical concepts (p. 24).

Difficulties working in multiple representations. Students experience difficulties working with functions in the algebraic and graphical representations (Billings & Klanderman, 2000; Even, 1993; Kaput, 1989; Piez & Voxman, 1997; Selinger & Pratt, 1997;

Tall, 1996). Many students in Even's study, for example, experienced problems when the scale was changed. Furthermore, they demonstrated a limited concept of function, because they believed that all functions could be represented by an equation and have nice graphs.

Even (1998) collected data via an open-ended questionnaire from 162 college mathematics students who were prospective secondary mathematics teachers. She suggests that not enough attention has been given, by students, teachers, and researchers, to the flexibility to move between representations and other knowledge and understandings. The students had difficulties linking different representations. From this study, Even contends that a distinction between a global approach to the behavior of functions and a point-wise approach is critical.

To deal with functions point-wise means to plot, read or deal with discrete points of a function either because one is interested in some specific points only, or because the function is defined on a discrete set. Reading values from a given graph, or finding the discrete density of a discrete random variable, are examples of a point-wise approach to functions. There are also times when one needs to consider the function in a global way, thus looking at a functions behavior; for example, when one wants to sketch the graph of a function given in algebraic form, or when one wants to find an extremum of a function which is defined on the real numbers. Even suggests that students who could "easily and freely use a global analysis of changes in the graphic representation" (p. 119) had a better, more powerful understanding of the connections between the graphical and algebraic representations of functions, than those students who preferred to use a more local, specific view. However, she warns against concluding that the global view leads to a better understanding of functions and their representations. Further, she notes that a global approach does not equate with understanding. She suggests that a combination of a global and point-wise approach to functions, as well as being critical and flexible is the most powerful. This agrees with the findings of Leinhardt et al. (1990) and Moschkovich, Schoenfeld, and Arcavi (1993). Even's warnings provide a timely reminder that the opportunities opened up by the use of graphing calculators to focus on the global view of functions should not be undertaken at the expensive of local, specific views. This tool allows both of these important aspects of functions to be viewed concurrently.

Access to multiple representations and the connections between them are important for student understanding. While multiple representations allow ideas to be explored more broadly, the additional information they provide can hinder as well as facilitate learning. However, each representation provides only some of the aspects of a concept. Multiple representations need to be used not just because this is easily possible with graphing Technology, but because it is needed to increase student understanding.

The Advantages of Using Graphing Calculators Compared to Other Graphing Technologies

Graphing calculators are not the only Technological tool available to the classroom teacher to facilitate students' development of a rich knowledge base about functions in a multi-representational environment.

Although there is much overlap between the capabilities of a graphing calculator and computer graphing applications, there are some major differences. These differences include ownership, cost, and availability. The last of these includes their portability, access via class sets, and ease of sharing. Also, the fact that the use of the graphing calculator is now mandated in mathematics assessment tasks in some middle, secondary, and college mathematics classes offers another key difference. Although other graphing Technologies may be used in school based assessment tasks, the only electronic graphing Technology that is allowed to be used by students during examinations is the graphing calculator.

In her discussion of the range of Technology now used across the mathematics curriculum, Heid (1997) suggested “perhaps the single most important Technological influence on High school and early college mathematics classrooms has been the graphics calculator” (p. 24). This notion according to Heid (1997) is still prevalent today. Graphing calculators are a personal, user-friendly, and portable Technology (Brown, 1998; Dick, 1996). They are available and inexpensive (Demana & Waits, 1992; Waits & Demana, 2000), able to be accessed at all times, and enable “connections to be made between different representations of mathematical ideas” (Goos, 1998, p. 103). According to Vonder Embse (1992) the graphing calculator provides a unique environment in which to connect different representations. “The graphing calculator bestows a sense of personal ownership on graphs, and that phenomenon alone can make a tremendous difference in the dynamics of the classroom” (Dick, 1996, p. 33). For these reasons, the use of graphing calculators can be considered as distinct from the use of other graphing Technology.

Graphing Calculators: The Available Technology

Computer graphing applications are much less likely to be acquired by schools than graphing calculators for the mathematics classroom (Kissane, 1996; Waits & Demana, 2000). The growing number of computers available in schools has had minimal impact in the teaching of mathematics (Barret & Goebel, 1990; Harskamp, Suhre, & Van Struen, 2000;

Waits & Demana, 2000). Kissane (1996) and Waits and Demana (2000) argue that the graphing calculator is a much more realistic alternative to computer Technology. In fact, given their mandating in assessment tasks, students in senior secondary mathematics classes and college classes are quite likely to own their own calculators, allowing student control over how they use the Technology.

Graphing calculators are much more accessible to students, being available for a very Low cost (Faragher, 1999; Ruthven, 1995; Waits & Demana, 2000). Where students cannot afford to buy their own, schools can provide class sets of graphing calculators for less than the price of a computer (Keiran, 1993; Kissane, 1996; Ruthven, 1992) through favorable purchasing and leasing agreements with graphing calculator companies. Almost every student has access to the tool at school and at home. Although equity questions with regard to cost are frequently raised in the literature (Kissane, 1996), cost does not seem to be a major issue as evidenced by the reported widespread use of graphing calculators in schools.

A Multi-purpose Tool

So after comparing graphing calculators and other graphing Technologies it appears the graphing calculators have many advantages over other Technologies. Graphing calculators have an "influence and impact on mathematics education that has far exceeded that of computers" (Dick, 1996, p. 31). Their use beyond that of a function grapher further suggests the need to consider them as different teaching and learning tools. Their nature as personal Technology, ease of use in the classroom, portability, relative cheapness, relevance to the learning of function, widespread use, and recent mandated usage during assessment in mathematics classrooms all suggest they are different to other graphing Technologies and further suggest the need for research in this area.

Functions and Graphing Technology

The search for an appropriate viewing window teaches students about functional behavior over various domains. Working with a variety of function types and their graphs provides students with the ability to classify graphs according to function type (Hector, 1992). Algebraic methods can be clarified and checked graphically, improving the algebraic understanding of students. All of these situations can be facilitated by the use of electronic graphing Technologies especially graphing calculators.

Leinhardt et al. (1990), in their extensive review of the literature regarding students' understanding of functions and their graphs concur with Hector (1992) that students who are introduced to graphs via the traditional approach have a point to point focus that causes them to overlook the global characteristics of functions. Electronic graphing Technology should overcome the point to point focus since it provides ready access to a large number of graph samples. Also, the type of graphing Technology that encourages students to consider both global and local aspects as being crucial to a well developed understanding of functions takes into account the teaching approach.

Access to a large number of graphs allows students to gain valuable experiences. With guidance, students can develop correct understandings about functions, their graphs, and the connections between the algebraic and graphical representation of functions (Vonder Embse, 1992). The graphing calculator clarifies the algebraic approach; it does not replace it (Hector, 1992).

Wilson and Krapfl (1994) state that because students can easily view families of functions when using a graphing calculator, they are more likely to see the connections between the algebraic representation and the family of graphs. It could, therefore, be argued that any improved learning is the result of increased experience with graphs rather than the use of multiple representations. The use of the Technology does permit students to spend more time on the task of learning and less on the mechanics of producing graphs. Graphing calculators let students "quickly and accurately represent the Cartesian graphs of algebraically defined functions [to] easily adjust the scale of axes [and to] easily link between" representations (Wilson & Krapfl, 1994, p. 253). It is not just regular use of a graphing calculator that is important, according to Ruthven (1990, p. 447), but the way the Technology is used. In particular, both students and teachers need to be encouraged to make greater use of graphical approaches for solving problems.

Graphing calculators allow students access to functions whose complete graphs are not visible in the standard viewing window, scales, and windows other than those presented to them via their textbooks (Connors and Snook, 2001). The scales refer to the increments on the horizontal and vertical axes and the window refer to the maximum and minimum ranges for which the graph is visible to the students on the graphing calculator. Some researchers (e.g., Connors and Snook, 2001; Ruthven, 1990) acknowledge that access to the graphing Technology by itself will not necessarily enable all students to make the connections between the representations. Specific teaching is required, in addition to access to the tool, if all students are to make these connections.

The use of Technology not only increases students' experiences with graphs; it provides immediate connections between the graphs and their algebraic representations and allows more time to be spent on the task of learning. This can result in a shift from answering Low level questions to answering conceptual questions involving qualitative reasoning supported by the graphing calculator—a shift from an algebraic and procedural method to a more conceptual approach (van der Kooij, 2001). With graphing calculators as tools "extensive, very specific algebraic training is no longer needed" by students, argues Connors and Snook (2001, p. 101). Instead, students need to focus on the "functional and language-based aspects of algebra" (p. 606) as they explore more complex and real problems. Again, this focus must be made explicit through teaching. It will not automatically happen with the use of graphing calculators in the classroom. Teacher instruction needs to transform to enable students to focus on these visual language aspects of algebra and maximize the opportunities provided by access to graphing calculators.

The use of graphing calculators has the potential to increase student understanding of functions, particularly as they search for an appropriate window; access a large number of graphs; spend less time on Low level tasks; solve freely between representations; and experience a broad range of scales and windows . This potential is just that—potential—teaching must explicitly present and maximize the learning opportunities offered by the Technology.

Steele (1995a) shows that teachers and students working together in a graphing calculator environment developed a very positive attitude toward using graphing calculators. Daily calculator use led to a significant improvement in understanding of general graphing questions, and when teachers emphasized the effects of scaling, students demonstrated a greater ability in finding global views of functions whose key features were not visible in the standard window. Furthermore, the research of Mitchelmore and Cavanagh (2000) Highlights the need for all studies to detail both the previous experiences of the students with the graphing Technology and the type of graphing Technology used.

Changes to Teaching and Learning in a Graphing Calculator Environment

In their discussion of research related to graphing calculators, Dunham and Dick (1994) cite many studies including those by Rich (1991), Ruthven (1990), and Shoaf-Grubbs (1992) which show increased conceptual understanding by students using graphing calculators. In all the studies cited, it was found that the "mere presence of graphing Technology cannot account for the results" (Dunham & Dick, p. 442). Dunham and Dick concur with the words of others in that the combination of Technology use, curriculum, and instruction must all be taken into account when researchers interpret the findings of research studies in this area.

Research findings suggest that experienced users of graphing calculators: (a) are better able to describe a given graph in algebraic terms, (b) are more likely to recognize the important features of a graph (Ruthven, 1990), (c) demonstrate an understanding of the connection between graphical and algebraic representations (Alexander, 1993; Devantier, 1993; Rich, 1991; Ruthven, 1990; Shoaf-Grubbs, 1994), (d) are more likely to relate a graph to its equation (Ruthven, 1990), and (e) are able more likely to be able to find the algebraic representation of a graph (Ruthven, 1990). Although the results of Alexander's study cannot distinguish between the tool and the changed instruction, it is important to note that the pre-and post-test items used were not designed specifically for his study nor any other Technology assisted instruction. Consequently, the findings of Alexander would be expected in other situations where Technology is used to enhance instruction. Many of these findings reinforce those of Goos, Galbraith, Renshaw, and Geiger (2000) in that the mere use of Technology is not enough—the teaching and learning must change if the tool is to be used effectively and exploited to its fullest.

The questions raised by the previous papers concern how teaching and learning change through sustained graphing calculator use. Dick (1996) argues that “graphs generated by the Technology can be used to effectively communicate and discuss the meaning” of the problem context and in doing so “graphing has become a means rather than an end” (p 38). Given that this tool is available, should change in what is currently taught and how we teach be impacted change? How should teaching change in order to maximize the learning opportunities available with the use of the graphing calculator? Clearly, some concepts become accessible to students earlier. But equally the question needs to be asked whether some of the mathematics currently taught is now less relevant, less important, or perhaps even redundant. Also, connections that may be obvious to teachers are not automatically obvious to students. And even where the link between representations is visual, students will not necessarily “see” the links in a meaningful way.

Implications for changes to teaching and learning. “Graphing is a conceptually simple procedure that is very tedious to carry out in practice without Technology” according to Stacey, Kendal, and Pierce (2002, p. 123). Using graphing Technology, the “graph has more functionality than a paper graph: one can zoom in and out to change the picture” (p. 123).

This enables students to view a function and its graph globally rather than as just a collection of points (Dreyfus & Halevi, 1991). The ease with which graphing calculators produce complete graphs for functions provides students with a broad base to compare and contrast functions. The search for an appropriate viewing domain and viewing range to determine a global view of a given function focuses students on the global characteristics of a particular function (Hector, 1992). This is important because when students can easily view families of graphs, they are more likely to see the relationships between the algebraic representation and the family of graphs (Wilson & Krapfl, 1994).

Movshovitz-Hadar (1993) found that students who use graphing Technology are better able to develop their understanding of quadratic functions by building on their existing knowledge and understanding of linear functions. The tool investigates connections between these two types of functions and hence reduces the compartmentalization of mathematics that inhibits learning. Students' knowledge of linear functions, their algebraic and graphical representations, and the connections between these provide the foundations for meaningful construction of the concept of quadratic functions. Movshovitz-Hadar proposes changes to the way quadratic functions are taught. He suggests they be introduced as the product of two linear functions, thus allowing the tool to be effectively exploited, consequently connections are explicitly made for students and learning improved as a result. This advice can be extended to cubic functions. These can be introduced as the product of three linear functions, thus allowing connections between linear, quadratic, and cubic functions to be exploited.

Forster and Taylor (2000) suggest that graphing tools support the learning process by permitting students to check answers or ideas in multiple ways. The use of the graphing calculator makes it possible to learn mathematical concepts in ways completely different from the traditional approach. Moreover, the availability of the graphing calculator and its multiple ways of representing functions “stimulates students’ flexibility in solving problems” (p. 611).

Rich (1991) and Ruthven (1992) found that students made more conjectures and analyzed information more, asked a greater number of Higher-level questions, and saw the relationship between graphs and approximate solutions when using a graphing tool. Rich suggested that teachers using graphing calculators stressed the importance of the graphical representation and the value of approximate solutions in mathematics and actually used examples differently.

Referring to an eleventh-grade class using graphing calculators to prepare for calculus, Lagrange argues “these calculators make the traditional balance of algebraic and graphical-numerical representations redundant, but a new balance is not yet clearly established” (1999, p. 70). Along with Asp et al. (1993) and Piez and Voxman (1997), Lagrange propose that traditional Techniques including work with paper and pencil still need to be used. This continued Technique enhances students’ ability to make sense of the algebraic calculations as well as the graphical and numerical approaches.

Communication. Most studies indicate positive benefits regarding the impact of graphing calculators on classroom communications. Hershkowitz and Schwarz (1999) suggest that graphing calculators appear to provide potential for support of classroom communication. Goos et al.'s (2000) study provided evidence that the use of the graphing calculator "can facilitate social interaction and sharing of knowledge" (p. 318). Lloyd and Wilson (1998) found that an experienced teacher's well-articulated concepts of functions and use of multi-representations in reform materials supported meaningful student discussion.

However, Doerr and Zangor (2000) found that although the use of the graphing calculator inhibited communication in a small group setting, when used as a shared tool by an entire class, it supported mathematical learning. The students in their study, although working as part of a group, obstructed the view on their calculator. This obstruction lead to students progressing in different directions, often resulting in the group breaking apart into individuals.

Farrell (1996) explored changes in student and teacher behavior when graphing calculators were situated in the learning environment. She suggests that students using graphing calculators demonstrate a wider range of roles in the learning process. She found that teachers also took on different roles when using Technology. For example, such teachers spent less time as a task setter and explainer, and more time as a consultant and fellow investigator. There was, however, large variability among teachers in terms of the time spent on the roles of consultant/investigator.

Summary: changes to teaching and learning. The research supports the notion that graphing calculators facilitate less teacher-directed instruction and increased communication, both among students and between teacher and students. Graphing calculator use produces increased student understanding pursuant to more opportunities to explore ideas through a variety of connected representations. Students tend to abandon the view of mathematics learning as rule-following (Boaler, 1997) where students believe "in the need to remember rules" (p. 36) or the demonstration of cue-based behavior where "students (base) their mathematical thinking on what they thought was expected of them, rather than on the mathematics within a question" (p. 37). Although it can be argued that pedagogy necessarily changes with the use of graphing calculators in the classroom, researchers and teachers alike should realize that the greatest benefits of the graphing calculator are achieved through deliberate changes to teaching and learning behaviors (Goos at al., 2000).

Research Questions

This study will compare two different approaches (scripted curriculum vs. three core representations curriculum) to teaching a segment of a college-level developmental mathematics curriculum at the same school to measure student achievement. Students' success rate will be measured in regards to solving problems involving exponential and logarithmic functions. This researcher is interested in the questions:

1. What effect does a three core representation-based curriculum have on college students' understanding of exponential and logarithmic functions?

This question is divided into the following sub-questions that usually involve multiple representations:

a. Are three core representation-based curriculum students able to solve exponential and logarithmic functions algebraically better than students in a traditional college algebra curriculum?

b. Are three core representation-based curriculum students able to solve exponential and logarithmic functions graphically better than students in a traditional college algebra curriculum?

c. Are three core representation-based curriculum students able to solve exponential and logarithmic functions numerically better than students in a traditional college algebra curriculum?

2. What effect does graphing-calculator knowledge have on a college students' understanding of exponential and logarithmic functions?

3. Does the three core representations-based curriculum and graphing calculators interact to effect college algebra students' understanding of exponential and logarithmic functions?

This focus contributes to the ongoing dialogue in mathematics education about the implications of using Technology and a three core representation curriculum in the mathematics classroom.

CHAPTER 3

METHODOLOGY

Overview

This study was designed for the primary purpose of determining the effect graphing calculators have on understanding exponential and logarithmic functions. This chapter describes details of the methodology, the situational context, details of the students involved in the study, how they were selected, and the setting of the study. The research instruments used are presented, as are its administration and the development of these instruments. Where appropriate, links are made to relevant literature. The ways in which the raw data will be analyzed are also presented.

Description of Population and Sample

The College

The study took place at a historically black private college located at a southeastern college in NC. The college is accredited and is a coeducational institution of Higher learning. The college's core curriculum revolves around liberal arts. However, it offers programs in computer science, teacher education, mathematics, natural sciences, and military science, just to name a few. In recent years, the annual enrollment was approximately 1,600 students, representing 36 states, the U.S. Virgin Islands, Jamaica, and 30 foreign countries. A very High proportion of the incoming freshmen are placed into a developmental English course and the *Introduction to College Algebra* course.

The Students in the Study

The research was conducted with freshmen enrolled in *College Algebra*, the second of two sequential developmental courses. This course was offered during the Spring Semester 2007. In the previous Fall Semester, students were enrolled in *Introduction to College Algebra*, the first of two sequential developmental courses. Enrollment in this first course was based on their "*Accuplacer*" test scores. *Accuplacer* test scores become the basis for asserting that students in the study were comparable in ability and/or achievement at the onset.

Instruction and Instructors

College Algebra is taught with a traditional approach, emphasizing algebraic concepts and approaches. Each instructor teaching *College Algebra* is given a written day-by-day script of what to teach for the semester, including the exact examples to illustrate. The textbook used for the course also had a traditional emphasis that is similar to other traditional developmental algebra textbooks (Tobey and Slater, 2001). The syllabus contained details of specific course content to be taught, which also included student assignments for that instructional day.

A graphing calculator was required for all students enrolled in *College Algebra*. All students participating in the study owned their own graphing calculator or borrowed one. The calculators were either a TI-83, a TI-83Plus, or a TI-84. It should be noted that the graphing calculator was mainly used as a computational tool in the scripted course.

Six (6) *College Algebra* classes were involved in the study. Of the five instructors teaching *College Algebra* at the college, only three participated in the study: Instructor 1, Instructor 2, and Instructor 3. Instructors 1 and Instructor 2 taught one Control Group each and did not provide any intervention. Instructor 3, also the researcher, taught two Control Groups and two Experimental Groups. The researcher's teaching load consisted of four (4) classes of *College Algebra*. Those classes were randomly assigned to be either an Experimental Group or a Control Group. Instructors 1 and 2 each taught two (2) sections of *College Algebra*. One of their classes was randomly chosen to be a Control Group. For the study, the students were randomly sorted into the researcher-taught Experimental Group (RTEG), the researcher-taught Control Group (RTCG), and the non-researcher-taught Control Group (NRTCG). Permission to deviate from the scripted instruction was given to the researcher by the department chairperson to conduct the intervention for the topics: exponential functions, logarithmic functions, and problem solving including compound interest.

Quantitative Research Methods

Instrumentation and Data Collection

Several instruments were designed to collect quantitative data. The first instrument, the Calculator Knowledge Assessment (Appendix A), consisted of ten (10) questions to assess participants graphing calculator knowledge. The researcher constructed the specific questions based on the needed skills for basic operations and needed skills in entering functions, such as using the TABLE and the CALCULATE menu on the TI-83, TI-83Plus, or TI-84. The Calculator Knowledge Assessment will be administered to the participants in the control and Experimental Groups before and after the instructional intervention.

The purpose of the two administrations is two-fold. First, scores on the first Calculator Knowledge Assessment is used to characterize participants Tech Savvy Designation as “Low Tech Savvy” students or “High Tech Savvy” students. There will be a total of 16 possible points on this assessment. A student will be characterized as Low Tech Savvy if his/her score was less than or equal to 10 points and High Tech Savvy if the score was at or greater than 11 points. Secondly, Pre- and Post-Calculator Knowledge Assessment scores will be used to determine significant difference in Calculator Knowledge growth.

Quantitative data will be collected to obtain scores on the following three (3) Exponential/Logarithmic Pre- and Post-Assessments (Appendix B and C): Exponential/Logarithmic Pre- and Post-Assessment scores on solving exponential equations, Exponential/Logarithmic Pre- and Post-Assessment scores on solving logarithmic equations, and Exponential/Logarithmic Pre- and Post-Assessment scores on compound interest problems. The Exponential/Logarithmic assessment was designed such that each topic contained only one (1) question for each of the core representations. The assessment items breakdown for each topic on the Exponential/Logarithmic Pre- and Post-Assessment are as follows:

Assessment 1: Three (3) open-ended questions on solving exponential functions using the three (3) representations.

Assessment 2: Three (3) open-ended questions on solving logarithmic functions using the three (3) representations.

Assessment 3: Three (3) open-ended questions on solving compound interest problems using the three (3) representations.

Each participant in the study will take the Exponential/Logarithmic Pre-Assessment for all the topics in one setting at the beginning of the intervention. However, there will be three (3) separate administrations of the Exponential/Logarithmic Post-Assessment. Each of the Exponential/Logarithmic Post-Assessments will be given at the conclusion of instruction for the respective topic. All of the instructors involved in the study will be consistent with this process. At the end of the intervention, the same Calculator Knowledge Assessment will be given to all the participants.

Hypotheses

For this study, the research hypotheses were:

H_1: There is a statistically significant difference between the Knowledge Pre- and Post-Assessment scores of the Control Group and the Experimental Group.

H_2: There is a statistically significant difference between the Knowledge Post-Assessment scores of the Control Group and the Experimental Group.

H_3: There is a statistically significant difference between Exponential/Logarithmic Post-Assessment scores of the Control Group and Experimental Group among the three core representations.

H_4: There is a statistically significant difference between Exponential/Logarithmic Post-Assessment scores between Low Tech Savvy student performance and High Tech Savvy student performance.

H_5: There is a statistically significant difference between Algebraic Representation Post-Assessment scores between Low Tech Savvy student performance and High Tech Savvy student performance.

H_6: There is a statistically significant difference between Graphical Representation Post-Assessment scores between Low Tech Savvy student performance and High Tech Savvy student performance.

H_7: There is a statistically significant difference between Numerical Representation Post-Assessment scores between Low Tech Savvy student performance and High Tech Savvy student performance.

H_8: There is a positive correlation between the Exponential/Logarithmic Post-Assessment scores and the Calculator Knowledge Post-Assessment scores among all students, among groups, and among High and Low Tech Savvy students.

Inter-rater Reliability

Inter-rater reliability is defined as seeking consistency of observations or ratings between two or more raters (Suter, 2006). If the two raters do not agree, or are inconsistent, the researcher will not be certain about the "truth" of the scores (Suter, 2006, p. 242). Therefore, for the purpose of this study, inter-rater reliability will be used to judge how well the two scorers agree in scoring the data on the Calculator Knowledge Pre- and Post-Assessment, Exponential/Logarithmic Pre- and Post-Assessment, and Tech Savvy Designation. The Intra-class Coefficient test will be performed on the data using the statistical software SPSS. The Intra-class Coefficient ranges from 0 to 1, where 1 is 100%

reliability. To attempt to achieve a High inter-rater reliability, two raters, known as Grader 1 and Grader 2, will be trained twice on using the rubrics (Appendix D and E) to score the data. Also raters will be asked to take breaks at designated stopping time.

All questions will be worth 1 point or 2 points. For the questions worth 2 points, students needed to have the correct answer to receive both points. For the questions worth 1 point, students need to have the correct answer to receive a point for that question. A student will receive 1 point on a 2-point question if they either have the correct steps in solving the problem but made an arithmetic mistake, or if they answer half of the question correctly. There will be no partial credit given on any problem worth 1 point. There will be some 2-point problems where no partial credit is given. These particular problems are noted on the instruments.

Data Analysis

Participants' pretest and posttest data will be paired based on ID number. Differences in average Calculator Knowledge Pre-Assessment scores and differences in average Exponential/Logarithmic Pre-Assessment scores for all three groups will be first analyzed using a one-way repeated-measures ANOVA test, to test if the groups are different at the outset. The study will also measure the difference in the average Calculator Knowledge Post-Assessment scores and for the difference in the Exponential/Logarithmic Post-Assessment scores, to see if there is a significant difference between each group at the end of the intervention. A one-way repeated-measures ANOVA test will be used as well for the difference in the posttests between groups. This evaluates the effects of adopting a teaching approach emphasizing simultaneous use of multiple representations, determining if there is a significantly Lower success rate with students in the traditional curriculum. The significance levels will be determined by using the Tukey-Honestly Significant Difference (HSD) test to control for Type I error.

A one-way repeated-measures ANOVA test will be used as well for the difference in the posttests between groups according to sub-topics. The sub-topics are only solving exponential equations, only solving logarithmic equations, and only solving compound interest problems.

Differences in pretest and posttest scores will be measured for each group using a one sided (greater than 0) paired t-test for a mean difference (posttest-pretest). This test examines student success, thus, the effectiveness teacher instruction had on students' graphing calculator knowledge and the effectiveness teacher instruction and use of graphing calculators has on student achievement with the three core multiple representations. The Spearman Rank correlation coefficient test is used to examine the correlations between the second administration of the Calculator Knowledge Post-Assessment scores and Exponential/Logarithmic Post-Assessment scores for each group. This evaluates to what extent teacher instruction emphasizing multiple representations and the use of graphing calculators has on student achievement.

Participants were characterized as either Low Tech Savvy or High Tech Savvy in all groups, based on their Calculator Knowledge Pre-Assessment scores. The Spearman rank correlation coefficient test will be conducted to measure how the Exponential/Logarithmic Post-Assessment scores compared between Low Tech Savvy students in the Experimental Group and all Low Tech Savvy students in the Control Groups, between High Tech Savvy students in the Experimental Group and all High Tech Savvy students in the Control Groups, and between Low Tech Savvy students in the Experimental Group and all High Tech Savvy students in the Control Groups. Also, the Spearman rank correlation coefficient test will be used to compare Exponential/Logarithmic Post-Assessment scores of Low Tech Savvy and High Tech Savvy students in the Experimental Group. These analyses describe the effectiveness of instruction and use of Technology on student achievement regardless of their calculator skills.

Confidence intervals and effect sizes were also reported for the study. In mathematics education, it has been suggested that reporting effect sizes can add additional support for the conclusions gathered from the significance testing as well as for the practical importance of the study (Capraro, 2004). Confidence intervals and effect sizes also contribute to the field by providing a benchmark for future studies with the same parameters, allowing for results to be supported.

Multiple Roles of the Researcher

As a teacher/researcher, at times my role was unclear. How much should I be the teacher? How much should I be the researcher? Stake (1995) argues that the roles overlap, in that the role of the researcher is "to inform, to sophisticate, to assist the increase of competence and maturity, to socialize, and to liberate … (and these) are responsibilities of the teacher" (pp. 91-2). As the teacher of the participants some researchers (Gay, 1987, White, 1998) may suggest that I create interference in the experimental scene merely by being in the room. Alternatively, one can accept the view that in the classroom environment, I am a constant factor so I have the advantage of blending in. The view of the latter concurs with Eisner (1997), that as a teacher researcher in my own classroom, I bring a richness of experience and expertise to the experimental setting. I bring "insider knowledge" (Eisner, p. 265) to the research process. I have "theoretical sensitivity" (Strauss & Corbin, 1998), merely by having been in the classroom and also as a reader of academic papers in my studies. "Stake has argued that the best understandings of educational phenomena are likely to be held by those closest to the educational process" (MacDonald, 1982, p. 26). Thus, my role as a teacher/researcher offers a unique and focused perspective in the analysis process of the data. It also permits students to immediately feel comfortable and connected to the research process.

CHAPTER 4

RESULTS

Students in all of the *College Algebra* sections in the study agreed to participate in the study. Pretest data was collected on 51 students in Researcher-Taught Experimental Group (RTEG) classes, 52 students in the Researcher-Taught Control Group (RTCG), and 57 students in the Non-Researcher-Taught Control Group (NRTCG). Due to attrition and absenteeism, posttest data was collected on 40 students in the RTEG, 38 students in RTCG, and 35 students in the NRTCG. Since the study compared pretest and posttest data, students who were in attendance on one collection day but not the other days could not be used in the study.

Table 1: Summary Results for Student Participation by Group

Group	Beginning Number of Participants	Ending Number of Participants
RTEG	51	40
RTCG	52	38
NRTCG	57	35

Inter-Rater Reliability Results

Due to the open-ended nature of the instruments used in this project, it was necessary to test the Inter-rater reliability of the scores from the graders of the instruments. The Inter-rater reliability, as measured by the Intra-class correlation coefficient (ICC), was calculated for 5 situations between Grader 1 and Grader 2. Inter-rater reliability was identified between

Grader 1 and Grader 2 for the Calculator Knowledge Pre-Assessment, the Calculator Knowledge Post-Assessment, the Exponential/Logarithmic Pre-Assessment, the Exponential/Logarithmic Post-Assessment, and the Tech-Savvy designation (Low Tech or High Tech). Tech Savvy Designation is defined as whether a student was classified as High Tech Savvy or Low Tech Savvy based on the grader's scoring. The inter-rater reliability was calculated using the two-way mixed effect model under the absolute definition (see http://www.nyu.edu/its/statistics/Docs/intracls.html). The inter-rater reliability was less than acceptable for all tests except for the Calculator Knowledge Pre-Assessment with a single measure Intraclass Correlation of .9613 and an average measure Intraclass Correlation of .9803. Further ICC results are presented below with a 95% confidence interval.

Table 2: Inter-Rater Reliability Results Between Grader 1 and Grader 2

Test	Single Measure ICC	ICC 95% Confidence Interval	Average Measure ICC	ICC 95% Confidence Interval
Calculator Knowledge Pre-Assessment	.9613	(.9444, .9732)	.9803	(.9714, .9864)
Calculator Knowledge Post-Assessment	.6712	(.5556, .76913)	.8033	(.7144, .8645)
Exponential/Logarithmic Pre-Assessment	.4765	(.3219, .6067)	.6455	(.4867, .7553)
Exponential/Logarithmic Post-Assessment	.6807	(.5682, .7682)	.8100	(.726, .8690)
Tech Savvy Designation (Low or High)	.8820	(.8334, .9171)	.9373	(.9091, .9568)

Based on the Inter-rater reliability scores, the consistency was not High enough to justify randomly selecting one grader or rater results to test the hypotheses and answer the research questions. Therefore, results are presented separately for both Graders 1 and Grader 2 and compared for consistency.

Development and Explanation of Cases

The College Board developed the *Accuplacer* to help provide to colleges the level of skill for a student and to determine the appropriate mathematics course that a student should take. Students in the study were placed into the course *College Algebra* based on their score. This placement is sufficient to assume that the students were the same in Calculator Knowledge and Exponential/Logarithmic at the outset. Since the Inter-rater reliability scores were not High enough between graders, two (2) cases were developed to analyze the data and to answer the research questions presented within this study. The two cases will now be described.

Case A is defined as follows: assume that all students start with a similar Exponential/Logarithmic and Calculator Knowledge background based on the *Accuplacer* test. Thus, only Exponential/Logarithmic Post-Assessment results were used to answer the questions/hypotheses. Therefore, the code "A1" means that the hypotheses are answered under Case A assumptions using only Grader 1 results and the code "A2" means that the hypotheses are answered under Case A assumptions using only Grader 2 results.

The maximum points on the Calculator Knowledge Pre-Assessment or the Calculator Knowledge Post-Assessment are 16 points. The maximum points allowed on the Exponential/Logarithmic Pre-Assessment or the Exponential/Logarithmic Post-Assessment are 18 points.

Case A1: Assume that all students start with a similar Exponential/Logarithmic and Calculator Knowledge background based on the *Accuplacer* test. Thus, only Exponential/Logarithmic Post-Assessment results were used for Case A1. The analysis is completed using only Grader 1 results where the Experimental Group (EG) is compared to the combined Control Group (CG) which consists of RTCG and NRTCG. Mean scores for each assessment by group according to Grader 1 are shown in Table 3.

Table 3: Case A Mean Scores for Grader 1 by Assessment for the Experimental Group and Control Group

GROUP (N)	TEST	MEAN(STD)
EG (40)	Calculator Knowledge Pre-Assessment	6.35(2.815)
	Calculator Knowledge Post-Assessment	10.23(3.416)
	Exponential/Logarithmic Pre-Assessment	1.83(1.752)
	Exponential/Logarithmic Post-Assessment	13.23(5.545)
CG (73)	Calculator Knowledge Pre-Assessment	6.3(2.815)
	Calculator Knowledge Post-Assessment	7.66(3.481)
	Exponential/Logarithmic Pre-Assessment	1.40(1.622)
	Exponential/Logarithmic Post-Assessment	6.08(3.796)

Case A2: Assume that all students start with a similar Exponential/Logarithmic and Calculator Knowledge background based on the *Accuplacer* test. Only Exponential/Logarithmic Post-Assessment results were used for Case 2. The analysis is completed using only Grader A2 results where the Experimental Group (EG) is compared to the combined Control Group (CG) consisting of RTCG and NRTCG. Mean scores for each assessment by group according to Grader 2 are shown in the Table 4.

Table 4: Case A Mean Scores for Grader 2 by assessment for the Experimental Group and Control Group

GROUP (N)	TEST	MEAN(STD)
EG (40)	Calculator Knowledge Pre-Assessment	5.83(2.591)
	Calculator Knowledge Post-Assessment	10.13(3.314)
	Exponential/Logarithmic Pre-Assessment	2.20(1.977)
	Exponential/Logarithmic Post-Assessment	5.85(3.499)
CG (73)	Calculator Knowledge Pre-Assessment	4.85(3.418)
	Calculator Knowledge Post-Assessment	7.81(3.121)
	Exponential/Logarithmic Pre-Assessment	1.63(1.822)
	Exponential/Logarithmic Post-Assessment	5.85(3.499)

If the Knowledge Pre-Assessments on Exponential/Logarithmic functions and Calculator Knowledge had not been administered, then I would have rightly assumed that all the groups were the same at the outset. However, I wanted to assess the students in addition to the *Accuplacer* test. So I administered my own calculator and exponential/logarithmic tests, known as Calculator Knowledge Pre-Assessment and Exponential/Logarithmic Pre-Assessment. The graders did not agree that all three groups (RTEG, RTCG, and NRTCG) were identical in Calculator Knowledge and Exponential/Logarithmic at the outset. However, the graders did agree that groups RTEG and RTCG were the same. So Case B reports only about groups RTEG and RTCG, since both graders agreed they were the same, based on the Tukey HSD multiple comparison test.

Case B is defined as follows: Since both graders agreed that the RTEG and the RTCG were the same at the onset based on the Knowledge Pre-Assessments, then both Knowledge Pre- and Post-Assessment results will be used to answer the questions/hypotheses between these two groups only. Therefore, the code “B1” means that the hypotheses are answered under Case B assumptions using only Grader 1 results and the code “B2” means that the hypotheses are answered under Case B assumptions using only Grader 2 results.

The maximum points on the Calculator Knowledge Pre-Assessment or the Calculator Knowledge Post-Assessment are 16 points. The maximum points allowed on the Exponential/Logarithmic Pre-Assessment or the Exponential/Logarithmic Post-Assessment are 18 points.

Case B1: Do not assume that all students start with a similar Exponential/Logarithmic and Calculator Knowledge as measured by the Exponential/Logarithmic Pre-Assessment and Calculator Knowledge Pre-Assessment, respectively. The analysis is completed using only Grader 1 results where the RTEG is compared to the RTCG. Based on Knowledge Pre-Assessment results, both Exponential/Logarithmic Pre- and Post-Assessment results were used for Case B1. Means for each assessment by group according to Grader 1 are shown in Table 5.

Table 5: Case B Mean Scores for Grader 1 by Assessment for the Researcher-Taught Experimental Group and Researcher-Taught Control Group

GROUP (N)	TEST	MEAN(STD)
RTEG(40)	Calculator Knowledge Pre-Assessment	6.35(2.815)
	Calculator Knowledge Post-Assessment	10.22(3.416)
	Exponential/Logarithmic Pre-Assessment	1.83(1.752)
	Exponential/Logarithmic Post-Assessment	13.23(5.545)
RTCG(38)	Calculator Knowledge Pre-Assessment	4.79(3.465)
	Calculator Knowledge Post-Assessment	8.89(3.562)
	Exponential/Logarithmic Pre-Assessment	1.74(1.735)
	Exponential/Logarithmic Post-Assessment	8.24(3.292)

Case B2: Do not assume that all students start with a similar content and Calculator Knowledge as measured by the Exponential/Logarithmic Pre-Assessment and Calculator Knowledge Pre-Assessment, respectively. The analysis is completed using only Grader 2 results where the RTEG is compared to the RTCG. Based on Knowledge Pre-Assessment results, both Exponential/Logarithmic Pre- and Post-Assessment results were used for Case B2. Means for each assessment by group according to Grader 2 are shown in Table 6.

Table 6: Case B Mean Scores for Grader 2 by Assessment for the Researcher-Taught Experimental Group and Researcher-Taught Control Group

GROUP (N)	TEST	MEAN(STD)
RTEG(40)	Calculator Knowledge Pre-Assessment	5.83(2.591)
	Calculator Knowledge Post-Assessment	10.12(3.314)
	Exponential/Logarithmic Pre-Assessment	2.20(1.977)
	Exponential/Logarithmic Post-Assessment	12.05(5.053)
RTCG(38)	Calculator Knowledge Pre-Assessment	5.13(3.581)
	Calculator Knowledge Post-Assessment	9.16(2.766)
	Exponential/Logarithmic Pre-Assessment	2.18(1.984)
	Exponential/Logarithmic Post-Assessment	7.79(2.915)

For Case B, a One-way ANOVA analysis was used to compare the mean scores of each group of students on the Exponential/Logarithmic Pre-Assessment as well as the Calculator Knowledge Pre-Assessment. Results are presented for Case B in Table 7. The dependent variable is the Knowledge Pre-Assessment scores and the factor is Teaching Group: RTEG, RTCG, or NRTCG. Tukey's HSD is used to control Type I error without being overly conservative (see http://www.tufts.edu/~gdallal/mc.htm). There is a significant difference between groups for CASE B1 Calculator Knowledge Pre-Assessment ($F(2,110) = 3.686$, $p=.028$, $p<.05$) and CASE B2 Exponential/Logarithmic Pre-Assessment ($F(2,110) =4.903$, $p=.009, p<0.01$). Post hoc Tukey-Honestly Significant Difference (HSD) tests indicated that for B1, RTEG scored Higher on the Calculator Knowledge Pre-Assessment than the NRTCG ($p<.05$) and for B2 RTEG scored Higher on the Exponential/Logarithmic Pre-Assessment than both the RTCG ($p<.05$) and NRTCG ($p<.05$).

Table 7: Case B ANOVA Results for Comparison of Means on Pre- and Post-Assessments

Case	Assessment	Groups	Mean Difference	STD. Error	P-Value
B1	Exponential/Logarithmic Pre-Assessment	RTEG-RTCG	.09	.374	.970
	Exponential/Logarithmic Pre-Assessment	RTEG-NRTCG	.80	.382	.098
	Exponential/Logarithmic Pre-Assessment	RTCG-NRTCG	.71	.387	.165
	Calculator Knowledge Pre-Assessment	RTEG-RTCG	1.56	.710	.076
	Calculator Knowledge Pre-Assessment	RTEG-NRTCG	1.78	.726	.042*
	Calculator Knowledge Pre-Assessment	RTCG-NRTCG	.22	.735	.953
B2	Exponential/Logarithmic Pre-Assessment	RTEG-RTCG	.02	.414	.999
	Exponential/Logarithmic Pre-Assessment	RTEG-NRTCG	.117	.423	.018*
	Exponential/Logarithmic Pre-Assessment	RTCG-NRTCG	1.16	.428	.022*
	Calculator Knowledge Pre-Assessment	RTEG-RTCG	.69	.715	.598
	Calculator Knowledge Pre-Assessment	RTEG-NRTCG	1.28	.731	.190
	Calculator Knowledge Pre-Assessment	RTCG-NRTCG	.59	.740	.706

Here, there is a clear advantage in both calculator knowledge and knowledge of exponential/logarithmic functions as indicated by Graders 1 and 2, respectively. Due to these differences NRTCG will not be used in Case B results. NRTCG will only be used when Pre and Post Assessments are used in combination for growth measures. In Case B, the differences in pre-test outcomes are taken into account within the calculations. NRTCG will be used in Case A results, since the NRTCG is incorporated with the RTCG into a Control Group (CG).

From this point on in the analysis of the results, the only acronyms used to designate the Experimental Group and the Control Group will be EG and CG, respectively. Case A and Case B consist of the same students who got the treatment taught by the researcher. Therefore, EG will be used to represent the Experimental Group for both cases. Also, both Case A and Case B will use CG to represent the Control Group. However, the Control Group (CG) for Case A consists of all students who did not get the treatment in the study. And the Control Group (CG) for Case B consists of all students taught by the researcher who did not get the treatment.

Analysis of Knowledge Pre- and Post-Assessment Results

The analysis of the first hypothesis is to determine the knowledge growth for each group on the Calculator Knowledge Pre- and Post-Assessment and on the Exponential/Logarithmic Pre- and Post-Assessment. The mean differences between each group are calculated. This hypothesis only applies to Case B.

H_1: There is a statistically significant difference between the Knowledge Pre- and Post-Assessment scores of the Control Group and the Experimental Group.

This hypothesis will be explained by looking at two (a-b) sub-hypotheses.

H_{1a}: There is a statistically significant difference between the Pre- and Post-Calculator Knowledge Assessment scores of the Control Group and the Experimental Group.

I used the repeated measures ANOVA to examine the growth of Calculator Knowledge as measured by the Calculator Knowledge Pre- and Post-Assessment between two groups: EG (N=40) and CG (N=38). There was a significant effect of groups on growth as shown in Table 8.

Table 8: Case B Repeated Measures ANOVA Results for Calculator Knowledge Growth

CASE	TEST	Source	df	F-ratio	P-value
B1	Calculator Knowledge Assessment	Intercept	1	783.400	p<0.001*
		GroupCode	2	11.194	p<0.001*
B2	Calculator Knowledge Assessment	Intercept	1	876.390	p<0.001*
		GroupCode	2	10.167	P<0.001*

Post-Hoc Tukey-Honestly Significant Difference (HSD) tests indicated that for both graders, the EG experienced significantly greater growth than CG.

Table 9: Case B Tukey-Honestly Significant Difference results for Mean Difference for Calculator Knowledge Growth

Case	Assessment	Groups (I-J)	Mean Difference (I-J) (Std.Error)	95% Confidence Interval	P-Value
B1	Calculator Knowledge Assessment	EG-CG	1.45(.589)	(.05,2.84)	P<0.05*
B2	Calculator Knowledge Assessment	EG-CG	1.57(.561)	(.07,2.31)	P<0.05*

H_{1b}: There is a statistically significant difference between the Exponential/Logarithmic Pre- and Post-Assessment scores of the Control Group and the Experimental Group.

I used the repeated measures ANOVA to examine the growth of Calculator Knowledge as measured by the Exponential/Logarithmic Pre- and Post-Assessment among two groups: EG (N=40) and CG (N=38). There was a significant effect of groups on growth as shown in Table 10.

Table 10: Case B Repeated Measures ANOVA Results for Exponential/Logarithmic Growth

CASE	TEST	Source	df	F-ratio	P-value
B1	Exponential/Logarithmic	Intercept	1	479.167	p<0.001*
		GroupCode	2	42.561	p<0.001*
B2	Exponential/Logarithmic	Intercept	1	452.080	p<0.001*
		GroupCode	2	36.034	p<0.001*

Post hoc Tukey-Honestly Significant Difference (HSD) tests indicated that for ALL Cases EG experienced significantly greater growth than CG as shown in Table 11.

Table 11: Case B Tukey-Honestly Significant Difference Results for Mean Difference for Exponential/Logarithmic Growth

Case	Assessment	Groups (I-J)	Mean Difference (I-J) (Std.Error)	95% Confidence Interval	P-Value
B1	Exponential/Logarithmic	EG-CG	2.54(.545)	(1.24,3.83)	P<0.001*
B2	Exponential/Logarithmic	EG-CG	2.14(.546)	(.84,3.44)	P<0.001*

Analysis of Knowledge Post-Assessment Results

The analysis of the second hypothesis is to determine if the scores on the Calculator Knowledge Post-Assessment for each group are significantly between each other. The same analysis is done for the Exponential/Logarithmic Post-Assessment. The mean differences between each group are calculated.

H_2: There is a statistically significant difference between the Knowledge Post-Assessment scores of the Control Group and the Experimental Group.

This hypothesis will be explained by looking at five (a-e) sub-hypotheses.

H_{2a} There is a statistically significant difference between the Calculator Knowledge Post-Assessment scores of the Control Group and the Experimental Group.

I used an independent-samples t-test to examine Calculator Knowledge as measured by Calculator Knowledge Post-Assessments between two teaching groups: EG and CG for Case A and EG and CG for Case B. Levene's Test for Equality of Variances was not significant ($p>0.05$) for all cases, therefore equal variances were assumed. There was a significant difference among Case B ($p<0.05$) and there was a significant difference among Case A ($p<.001$) as presented in Table 12. The mean differences are presented in Table 13.

Table 12: Independent-samples t-test Results for the Calculator Knowledge Post-Assessment Between Groups

CASE	df	T-statistic	p-value
A1 EG-CG	111	3.774	P<0.001*
A2 EG-CG	111	3.692	P<0.001*
B1 EG-CG	76	2.684	P<0.05*
B2 EG-CG	76	2.395	P<0.05*

Table 13: Mean Difference Results for the Calculator Knowledge Post-Assessment Between Groups

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG-CG	2.57(.680)	(1.219,3.915)	P<0.001*
A2 EG-CG	2.32(.628)	(1.073,3.560)	P<0.001*
B1 EG-CG	1.33(.790)	(-.243,2.904)	P<0.05*
B2 EG-CG	1.97(.693)	(-.413,2.347)	P<0.05*

H_{2b} There is a statistically significant difference between the Exponential/Logarithmic Post-Assessment scores of the Control Group and the Experimental Group.

I used an independent-samples t-test to examine math content knowledge as measured by the Exponential/Logarithmic Post-Assessment among two teaching groups: EG and CG for Case A and EG and CG for Case B. Levene's Test for Equality of Variances was not significant (p>0.05) for all cases, therefore equal variances were assumed. There was a significant difference between each of the groups in all cases as presented below in Table 14. The mean differences are presented in Table 15.

Table 14: Independent-samples t-test Results for the Exponential/Logarithmic Post-Assessment Between Groups

CASE	df	T-statistic	p-value
A1 EG-CG	59.480	7.268	P<0.001*
A2 EG-CG	59.938	6.906	P<0.001*
B1 EG-CG	64.016	4.859	P<0.001*
B2 EG-CG	62.959	4.589	P<0.001*

Table 15: Mean difference results for the Exponential/Logarithmic Post-Assessment between groups

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG-CG	7.14(.983)	(5.176,9.109)	P<0.001*
A2 EG-CG	6.20(.898)	(4.405,7.997)	P<0.001*
B1 EG-CG	4.99(1.026)	(2.938,7.039)	P<0.001*
B2 EG-CG	4.26(.928)	(2.405,6.116)	P<0.001*

H_{2c} There is a statistically significant difference between the Exponential Equations Assessment scores of the Control Group and the Experimental Group.

I used an independent-samples t-test to examine exponential equations math content knowledge as measured by Exponential/Logarithmic Post-Assessment among two teaching groups: EG and CG for Case A and EG and CG for Case B. Levene’s Test for Equality of

Variances was not significant ($p>0.05$) therefore equal variances were assumed. There was a significant difference between each of the groups in all cases as presented in Table 16. The mean differences are presented in Table 17.

Table 16: Independent-samples t-test results for Exponential Equations on the Exponential/Logarithmic Post-Assessment Between Groups

CASE	df	T-statistic	p-value
A1 EG-CG	111	11.338	P<0.001*
A2 EG-CG	111	10.807	P<0.001*
B1 EG-CG	76	7.788	P<0.001*
B2 EG-CG	76	7.430	P<0.001*

Table 17: Mean difference Results for Exponential Equations on the Exponential/Logarithmic Post-Assessment Between Groups

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG-CG	2.73(.241)	(2.254,3.208)	P<0.001*
A2 EG-CG	2.46(.227)	(2.007,2.909)	P<0.001*
B1 EG-CG	2.19(.281)	(1.628,2.746)	P<0.001*
B2 EG-CG	1.94(.258)	(1.424,2.455)	P<0.001*

H_{2d} There is a statistically significant difference between the Logarithmic Equations Assessment scores of the Control Group and the Experimental Group.

I used an independent-samples t-test to examine logarithmic equations math content knowledge as measured by the Exponential/Logarithmic Post-Assessment among two teaching groups: EG and CG for Case A and EG and CG for Case B. Levene's Test for Equality of Variances was not significant ($p>0.05$) for all cases, therefore equal variances were assumed. There was a significant difference between each of the groups in all cases as presented in Table 18. The mean differences are presented in Table 19.

Table 18: Independent-samples t-test Results for Logarithmic Equations on the Exponential/Logarithmic Post-Assessment Between Groups

CASE	Df	T-statistic	p-value
A1 EG-CG	55.305	3.782	P<0.001*
A2 EG-CG	52.224	3.142	P<0.01*
B1 EG-CG	66.968	2.090	P<0.05*
B2 EG-CG	59.437	1.662	p>0.05

Table 19 Mean Difference Results for Logarithmic Equations on the Post-Content-Knowledge Assessment Between Groups

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG-CG	1.45(.335)	(.788,2.117)	P<0.001*
A2 EG-CG	1.10(.350)	(.397,1.800)	P<0.001*
B1 EG-CG	.89(.419)	(.039,1.713)	P<0.001*
B2 EG-CG	.91(.367)	(-.124,1.346)	P<0.001*

H_{2e} There is a statistically significant difference between the Solving Compound Interest Problems Assessment scores of the Control Group and the Experimental Group.

I used an independent-samples t-test to examine solving compound interest problems math content knowledge as measured by Exponential/Logarithmic Post-Assessment among two teaching groups: EG and CG for Case A and EG and CG for Case B. Levene's Test for Equality of Variances was not significant ($p>0.05$) for all cases therefore equal variances were assumed. There was a significant difference between each of the groups in all cases as presented in Table 20. The mean differences are presented in Table 21.

Table 20: Independent-samples t-test Results for Compound Interest Word Problem on the Exponential/Logarithmic Post-Assessment Between Groups

CASE	Df	T-statistic	p-value
A1 EG-CG	55.251	6.944	P<0.001*
A2 EG-CG	51.199	6.262	P<0.001*
B1 EG-CG	67.860	5.199	P<0.001*
B2 EG-CG	62.906	5.000	P<0.001*

Table 21: Mean Difference Results for Compound Interest Word Problem on the Exponential/Logarithmic Post-Assessment Between Groups

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG-CG	2.42(.348)	(1.721,3.118)	P<0.001*
A2 EG-CG	2.229(.366)	(1.558,3.028)	P<0.001*
B1 EG-CG	1.99(.383)	(1.228,2.759)	P<0.001*
B2 EG-CG	1.98(.396)	(1.189,2.772)	P<0.001*

Analysis of the Exponential/Logarithmic Post-Assessment Results

The analysis of the third hypothesis is to determine if the scores on the algebraic representation assessment, the graphical representation assessment, and the numerical representation assessment for each group are significantly different between each other. Tech Savvy designation for each student is not considered in this hypothesis. The mean differences between each group are calculated.

H_3: There is a statistically significant difference between Exponential/Logarithmic Post-Assessment scores of the Control Group and Experimental Group among the three core representations.

This hypothesis will be explained by looking at 3 (a-c) sub-hypotheses.

H_{3a}: There is a statistically significant difference between the Algebraic Representation Post-Assessment scores of the Control Group and the Experimental Group.

I used an independent-samples t-test to examine algebraic representation knowledge as measured by Exponential/Logarithmic Post-Assessment Algebraic questions among two teaching groups: EG and CG for Case A, and EG and CG for Case B. The means for each group are presented in Table 22. Levene's Test for Equality of Variances was not significant ($p>0.05$) for all cases, therefore equal variances were assumed. There was no significant difference among any cases ($p>0.05$). The mean differences are presented in Table 23.

Table 22: Mean Scores for the Algebraic Representation Assessment Between Groups

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Algebraic Representation Assessment(40)	3.93(1.670)
	CG Algebraic Representation Assessment(73)	3.22(1.931)
A2 EG-CG	EG Algebraic Representation Assessment(40)	3.80(1.636)
	CG Algebraic Representation Assessment(73)	3.21(1.922)
B1 EG-CG	EG Algebraic Representation Assessment(40)	3.93(1.670)
	CG Algebraic Representation Assessment(38)	4.29(1.523)
B2 EG-CG	EG Algebraic Representation Assessment(40)	3.80(1.636)
	CG Algebraic Representation Assessment(38)	4.26(1.519)

Table 23: Mean Difference Results for Algebraic Representation Assessment Between Groups

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG-CG	.71(.363)	(-.013,1.424)	P>.05
A2 EG-CG	.59(.359)	(-.117,1.307)	P>.05
B1 EG-CG	-.36(.362)	(-1.086,.357)	P>.05
B2 EG-CG	-.46(.358)	(-1.176,.250)	P>.05

H_{3b}: There is a statistically significant difference between the Graphical Representation Post-Assessment scores of the Control Group and the Experimental Group.

I used an independent-samples t-test to examine graphical representation knowledge as measured Exponential/Logarithmic Post-Assessment Graphical questions between two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 24. Levene's Test for Equality of Variances was not significant (p>0.05) for all cases, therefore equal variances were not assumed. There is a significant difference among all cases (p<0.001). The mean differences are presented in Table 25.

Table 24: Mean Scores for the Graphical Representation Assessment Between Groups

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Graphical Representation Assessment(40)	3.90(1.892)
	CG Graphical Representation Assessment(73)	.36(.856)
A2 EG-CG	EG Graphical Representation Assessment(40)	3.45(1.739)
	CG Graphical Representation Assessment(73)	.16(.472)
B1 EG-CG	EG Graphical Representation Assessment(40)	3.90(1.892)
	CG Graphical Representation Assessment(38)	.63(1.076)
B2 EG-CG	EG Graphical Representation Assessment(40)	3.45(1.739)
	CG Graphical Representation Assessment(38)	.26(.554)

Table 25: Mean Difference Results for Graphical Representation Assessment Between Groups

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG-CG	3.54(.315)	(2.910,4.178)	P<0.001*
A2 EG-CG	3.29(.280)	(2.720,3.851)	P<0.001*
B1 EG-CG	3.27(.346)	(2.576,3.961)	P<0.001*
B2 EG-CG	3.19(.289)	(2.605,3.769)	P<0.001*

H_{3c}: There is a statistically significant difference between the Numerical Representation Post-Assessment scores of the Control Group and the Experimental Group.

I used an independent-samples t-test to examine numerical representation knowledge as measured by Exponential/Logarithmic Post-Assessment Numerical questions between two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 26. Levene's Test for Equality of Variances was not significant ($p>0.05$) for all cases, therefore equal variances were not assumed. There is a significant difference among all cases ($p<0.001$). The mean differences are presented in Table 27.

Table 26: Mean Scores for the Numerical Representation Assessment Between Groups

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Numerical Representation Assessment(40)	2.60(2.110)
	CG Numerical Representation Assessment(73)	.25(.722)
A2 EG-CG	EG Numerical Representation Assessment(40)	2.17(1.866)
	CG Numerical Representation Assessment(73)	.21(.576)
B1 EG-CG	EG Numerical Representation Assessment(40)	2.60(2.110)
	CG Numerical Representation Assessment(38)	.45(.950)
B2 EG-CG	EG Numerical Representation Assessment(40)	2.17(1.866)
	CG Numerical Representation Assessment(38)	.37(.751)

Table 27: Mean Difference Results for Numerical Representation Assessment Between Groups

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG-CG	3.54(.344)	(1.660,3.047)	P<0.001*
A2 EG-CG	1.97(.303)	(1.359,2.580)	P<0.001*
B1 EG-CG	2.15(.367)	(1.416,2.889)	P<0.001*
B2 EG-CG	1.81(.319)	(1.166,2.447)	P<0.001*

Analysis of Exponential/Logarithmic Post-Assessment Results between Low Tech Savvy and High Tech Savvy Students

The analysis of the fourth hypothesis is to determine if the scores for Low Tech Savvy students and High Tech Savvy students differ significantly between each other on the

Exponential/Logarithmic Post-Assessment. The mean differences between each group are calculated.

H_4: There is a statistically significant difference between Exponential/Logarithmic Post-Assessment scores between Low Tech Savvy student performance and High Tech Savvy student performance.

This hypothesis will be explained by looking at 5 (a-e) sub-hypotheses.

H_{4a}: There is a statistically significant difference between the Exponential/Logarithmic Post-Assessment scores of Low Tech Savvy students found in the Control Group and their counterparts found in the Experimental Group.

I used Between-subjects one-way ANOVA to examine math content knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 28. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant among all cases ($p<0.01$). The mean differences are presented in Table 29.

Table 28: Means for the Exponential/Logarithmic Post-Assessment Between Low Tech Savvy Students in Each Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Low Tech (27)	11.41(5.493)
	CG Low Tech (49)	6.51(3.507)
A2 EG-CG	EG Low Tech (28)	11.00(5.200)
	CG Low Tech (46)	5.80(2.979)
B1 EG-CG	EG Low Tech (27)	11.41(5.493)
	CG Low Tech (26)	8.50(2.804)
B2 EG-CG	EG Low Tech (28)	11.00(5.200)
	CG Low Tech (23)	7.48(2.233)

Table 29: Mean Difference Results for the Exponential/Logarithmic Post-Assessment Between Low Tech Savvy Students

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG Low Tech-CG Low Tech	4.90(1.010)	(2.26,7.53)	P<0.001*
A2 EG Low Tech-CG Low Tech	5.20(.967)	(2.67,7.72)	P<0.001*
B1 EG Low Tech-CG Low Tech	3.32(1.053)	(.47,5.96)	P<0.05*
B2 EG Low Tech-CG Low Tech	3.52(1.040)	(.50,6.54)	P<.05*

H_{4b}: There is a statistically significant difference between the Exponential/Logarithmic Post-Assessment scores of High Tech Savvy students found in the Control Group and their counterparts in the Experimental Group.

I used between subjects one-way ANOVA to examine math content knowledge as measured by Exponential/Logarithmic Post-Assessment among High Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 30. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.01$). The mean differences are presented in Table 31.

Table 30: Means for the Exponential/Logarithmic Post-Assessment Between High Tech Savvy Students Found in the Control Group and Their Counterparts in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG High Tech (13)	17.00(3.416)
	CG High Tech (24)	5.21(4.273)
A2 EG-CG	EG High Tech (12)	14.50(3.849)
	CG High Tech (27)	5.93(4.305)
B1 EG-CG	EG High Tech (13)	17.00(3.416)
	CG High Tech (12)	7.67(4.250)
B2 EG-CG	EG High Tech (12)	14.50(3.849)
	CG High Tech (15)	8.27(3.770)

Table 31: Mean difference Results for the Exponential/Logarithmic Post-Assessment Between High Tech Savvy Students Found in the Control Group and Their Counterparts in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG High Tech-CG High Tech	11.79(1.451)	(8.01,15.58)	P<0.001*
A2 EG High Tech-CG High Tech	8.57(1.400)	(4.92,12.23)	P<0.001*
B1 EG High Tech –CG High Tech	9.33(1.534)	(4.88,13.78)	P<0.001*
B2 EG High Tech-CG High Tech	6.23(1.431)	(2.08,1039)	P<0.001*

H_{4c}: There is a statistically significant difference between the Exponential/Logarithmic Post-Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Experimental Group.

I used Between-subjects one-way ANOVA to examine math content knowledge as measured by Exponential/Logarithmic Post-Assessment between Low Tech Savvy students and High Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 32. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.01$) for Case A1 and B1 and ($p>.05$) for Case A2 and Case B2. The mean differences are presented in Table 33.

Table 32: Means for the Exponential/Logarithmic Post-Assessment Between Low Tech Savvy students and High Tech Savvy Students Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1/B1	EG High Tech (13)	17.00(3.416)
	EG Low Tech (27)	11.41(5.493)
A2/B2	EG High Tech (12)	14.50(3.849)
	EG Low Tech (28)	11.00(5.200)

Table 33: Mean difference Results for the Exponential/Logarithmic Post-Assessment Between Low Tech Savvy Students and High Tech Savvy students Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1/B1 EG High Tech - EG Low Tech	5.59(1.422)	(1.88,9.30)	P<0.01*
A2/B2 EG High Tech - EG Low Tech	3.50(1.392)	(-.13,7.13)	P<0.05*

H_{4d}: There is a statistically significant difference between the Exponential/Logarithmic Post-Assessment scores of Low Tech Savvy students in the Experimental Group and High Tech Savvy students found in the Control Group.

I used between subjects one-way ANOVA to examine math content knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students in teaching group EG and High Tech Savvy students in teaching group CG, and between EG and CG. The means for each group are presented below. There is a significant difference between each of the groups for Case A ($p<.001$). There was a significant difference between each of the groups in Case B ($p<.05$) as presented in Table 34 below. Tukey's HSD is used to control for Type I Error. The results were not significant ($p>0.05$). The mean differences are presented in Table 35.

Table 34: Means for the Exponential/Logarithmic Post-Assessment between Low Tech Savvy Students in the Experimental Group and High Tech Savvy Students Found in the Control Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Low Tech (27)	11.41 (5.493)
	CG High Tech 24()	5.21 (4.273)
A2 EG-CG	EG Low Tech (28)	11.00(5.20)
	CG High Tech (27)	5.93(4.305)
B1 EG-CG	EG Low Tech (27)	11.41(5.493)
	CG High Tech (12)	7.67(4.250)
B2 EG-CG	EG Low Tech (28)	11.00(5.200)
	CG High Tech (15)	8.27(3.770)

Table 35: Mean Difference results for the Exponential/Logarithmic Post-Assessment Between Low Tech Savvy Students in the Experimental Group and High Tech Savvy Students Found in the Control Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG Low Tech-CG High Tech	6.20(1.182)	(3.12,9.28)	P<0.001*
A2 EG Low Tech-CG High Tech	5.07(1.088)	(2.23,7.91)	P<0.001*
B1 EG Low Tech-CG High Tech	4.74(1.329)	(2.31,-7.60)	P<.05*
B2 EG Low Tech-CG High Tech	4.73(1.182)	(3.14,8.29)	P<.05*

Analysis Algebraic Representation Post-Assessment scores between Low Tech Savvy and High Tech Savvy Students

The analysis of the fifth hypothesis is to determine if the scores for Low Tech Savvy students and High Tech Savvy students differ significantly between each other on the Algebraic Representation Assessment. The mean differences between each group are calculated.

H_5: There is a statistically significant difference between Algebraic Representation Post-Assessment scores between Low Tech Savvy student performance and High Tech Savvy student performance.

This hypothesis will be explained by looking at 5 (a-e) sub-hypotheses.

H_{5a}: There is a statistically significant difference between the Algebraic Representation Post-Assessment scores of Low Tech Savvy students found in the Control Group and their counterparts found in the Experimental Group.

I used between subjects one-way ANOVA to examine algebraic representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students in two teaching groups: EG and CG for Case A, and EG and CG for Case B. The means for each group are presented in Table 36. There was no significant difference between each of the groups in all cases. Tukey's HSD is used to control for Type I Error. The results were not significant ($p>0.05$). The mean differences are presented in Table 37.

Table 36: Means for the Algebraic Representation Assessment Between Low Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1	EG Low Tech (27)	3.44(1.739)
	CG Low Tech (49)	3.47(1.827)
A2	EG Low Tech (28)	3.57(1.752)
	CG Low Tech (46)	3.33(1.777)
B1	EG Low Tech (27)	3.44(1.739)
	CG Low Tech (26)	4.58(1.301)
B2	EG Low Tech (28)	3.57(1.752)
	CG Low Tech (23)	4.48(1.238)

Table 37: Mean Difference Results for the Algebraic Representation Assessment Between Low Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG Low Tech-CG Low Tech	-.02(.429)	(-1.14,1.09)	P>0.05
A2 EG Low Tech-CG Low Tech	.25(.438)	(-.90,1.99)	P>0.05
B1 EG Low Tech –CG Low Tech	-1.13(.427)	(-2.37,.11)	P>0.05
B2 EG Low Tech-CG Low Tech	-.91(.450)	(-2.21,.40)	P>0.05

H_{5b}: There is a statistically significant difference between the Algebraic Representation Post-Assessment scores of High Tech Savvy students found in the Control Group and their counterparts found in the Experimental Group.

I used between subjects one-way ANOVA to examine algebraic representation knowledge as measured by Exponential/Logarithmic Post-Assessment among High Tech Savvy students in two teaching groups: EG and CG for Case A, and EG and CG for Case B. The means for each group are presented in Table 38. There was no significant difference between each of the groups in Case B. Tukey's HSD is used to control for Type I Error. The results were not significant ($p>0.05$) for Case A. The mean differences are presented in Table 38.

Table 38: Means for the Algebraic Representation Assessment Algebraic Representation Assessment Scores Between High Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG High Tech (13)	4.92(.954)
	CG High Tech (24)	2.71(2.074)
A2 EG-CG	EG High Tech (12)	4.33(1.231)
	CG High Tech (27)	3.00(2.166)
B1 EG-CG	EG High Tech (13)	4.92(.954)
	CG High Tech (12)	3.67(1.826)
B2 EG-CG	EG High Tech (12)	4.33(1.231)
	CG High Tech (15)	3.93(1.870)

Table 39: Mean difference results for the Algebraic Representation Assessment Algebraic Representation Assessment Scores Between High Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG High Tech-CG High Tech	1.27(.616)	(.61,3.82)	P>0.05
A2 EG High Tech-CG High Tech	1.33(.634)	(-.32,2.99)	P>0.05
B1 EG High Tech –CG High Tech	1.26(.623)	(-.55,3.06)	P>0.05
B2 EG High Tech-CG High Tech	.40(.619)	(-1.40,2.20)	P>0.05

H_{5c}: There is a statistically significant difference between the Algebraic Representation Post-Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Control Group.

I used between subjects one-way ANOVA to examine algebraic representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students and High Tech Savvy students in the two teaching groups: EG and CG for Case A, and EG and CG for Case B. The means for each group are presented in Table 40. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.05$). The mean differences are presented in Table 41.

Table 40: Means for the Algebraic Representation Assessment Scores of Low Tech Savvy Students and High Tech Savvy Students Found in the Control Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	CG Low Tech (49)	3.47(1.827)
	CG High Tech (24)	2.71(2.074)
A2 EG-CG	CG Low Tech (46)	3.33(1.777)
	CG High Tech (27)	3.00(2.166)
B1 EG-CG	CG Low Tech (26)	4.58(1.301)
	CG High Tech (12)	3.67(1.826)
B2 EG-CG	CG Low Tech (23)	4.48(1.238)
	CG High Tech (15)	3.93(1.870)

Table 41: Mean Difference Results for the Algebraic Representation Assessment scores of Low Tech Savvy students and High Tech Savvy Students Found in the Control Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 CG High Tech-CG Low Tech	-.76(.446)	(-1.92,.40)	P<0.05*
A2 CG High Tech-CG Low Tech	-.33(.443)	(-1.48,.83)	P<0.05*
B1 CG High Tech –CG Low Tech	-.91(.543)	(-2.49,.66)	P<0.05*
B2 CG High Tech-CG Low Tech	-.54(.530)	(-2.08,.99)	P<0.05*

H_{5d}: There is a statistically significant difference between the Algebraic Representation Post-Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Experimental Group.

I used between subjects one-way ANOVA to examine algebraic representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students and High Tech Savvy students in the two teaching groups: EG for Case A and EG for Case B. The means for each group are presented in Table 42. There was no significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were not significant ($p>0.05$). The mean differences are presented in Table 43.

Table 42: Means for the Algebraic Representation Assessment Scores of Low Tech Savvy students and High Tech Savvy Students Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1/B1	EG Low Tech (27)	3.44(1.739)
	EG High Tech (13)	4.92(.954)
A2/B2	EG Low Tech (28)	3.57(1.752)
	EG High Tech (12)	4.33(1.231)

Table 43: Mean Difference Results for the Algebraic Representation Assessment Scores of Low Tech Savvy Students and High Tech Savvy Students Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1/B1 EG High Tech-EG Low Tech	1.48(.604)	(-.10,3.05)	P>0.05
A2/B2 EG High Tech-EG Low Tech	.76(.630)	(-.88,2.41)	P>0.05

H_{5e}: There is a statistically significant difference between the Algebraic Representation Post-Assessment scores of Low Tech Savvy students in the Experimental Group and High Tech Savvy students found in the Control Group.

I used between subjects one-way ANOVA to examine algebraic representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students in teaching group EG and High Tech Savvy students in teaching group CG and between CG and CG. The means for each group are presented in Table 44. There was no significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were not significant ($p>0.05$). The mean differences are presented in Table 45.

Table 44: Means for the Algebraic Representation Assessment Scores of Low Tech Savvy Students in the Experimental Group and High Tech Savvy Students Found in the Control Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Low Tech (27)	3.44(1.739)
	CG High Tech (24)	2.71(2.074)
A2 EG-CG	EG Low Tech (28)	3.57(1.752)
	CG High Tech (27)	3.00(2.166)
B1 EG-CG	EG Low Tech (27)	3.44(1.739)
	CG High Tech (12)	3.67(1.826)
B2 EG-CG	EG Low Tech (28)	3.57(1.752)
	CG High Tech (15)	3.93(1.870)

Table 45: Mean Difference results for the Algebraic Representation Assessment scores of Low Tech Savvy students in the Experimental Group and High Tech Savvy students found in the Control Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG Low Tech-CG High Tech	.74(.502)	(-.57,2.05)	P>0.05
A2 EG Low Tech-CG High Tech	.57(.493)	(-.71,1.86)	P>0.05
B1 EG Low Tech-CG High Tech	-.22(.540)	(-1.79,1.34)	P>0.05
B2 EG Low Tech-CG High Tech	-.36(.511)	(-1.85,1.12)	P>0.05

Analysis Graphical Representation Post-Assessment scores between Low Tech Savvy and High Tech Savvy Students

The analysis of the sixth hypothesis is to determine if the scores for Low Tech Savvy students and High Tech Savvy students differ significantly between each other on the Graphical Representation Assessment. The mean differences between each group are calculated.

H_6: There is a statistically significant difference between Graphical Representation Post-Assessment scores between Low Tech Savvy student performance and High Tech Savvy student performance.

This hypothesis will be explained by looking at 5 (a-e) sub-hypotheses.

H_{6a}: There is a statistically significant difference between the Graphical Representation Post-Assessment scores of Low Tech Savvy students found in the Control Group and their counterparts found in the Experimental Group.

I used between subjects one-way ANOVA to examine graphical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 46. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.001$). The mean differences are presented in Table 47.

Table 46: Means for the Graphical Representation Assessment Scores of Low Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Low Tech (27)	3.56(2.044)
	CG Low Tech (49)	.35(.879)
A2 EG-CG	EG Low Tech (28)	3.36(1.890)
	CG Low Tech (46)	.13(.453)
B1 EG-CG	EG Low Tech (27)	3.56(2.044)
	CG Low Tech (26)	.58(1.102)
B2 EG-CG	EG Low Tech (28)	3.36(1.890)
	CG Low Tech (23)	.17(.491)

Table 47: Mean difference Results for the Graphical Representation Assessment Scores of Low Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG Low Tech-CG Low Tech	3.21(.310)	(2.40,4.02)	P<0.001*
A2 EG Low Tech-CG Low Tech	3.23(.265)	(2.54,3.92)	P<0.001*
B1 EG Low Tech –CG Low Tech	2.98(.353)	(1.96,4.00)	P<0.001*
B2 EG Low Tech-CG Low Tech	3.18(.312)	(2.28,4.09)	P<0.001*

H_{6b}: There is a statistically significant difference between the Graphical Representation Post-Assessment scores of High Tech Savvy students found in the Control Group and their counterparts found in the Experimental Group.

I used between subjects one-way ANOVA to examine graphical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among High Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 48. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.001$). The mean differences are presented in Table 49.

Table 48: Means for the Graphical Representation Assessment Scores of High Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG High Tech (13)	4.62(1.325)
	CG High Tech (24)	.38(.824)
A2 EG-CG	EG High Tech (12)	3.67(1.371)
	CG High Tech (27)	.22(.506)
B1 EG-CG	EG High Tech (13)	4.62(1.325)
	CG High Tech (12)	.75(1.055)
B2 EG-CG	EG High Tech (12)	3.67(1.371)
	CG High Tech (15)	3.93(1.870)

Table 49: Mean Difference Results for the Graphical Representation Assessment Scores of High Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG High Tech-CG High Tech	4.24(.446)	(3.08,5.40)	P<0.001*
A2 EG High Tech-CG High Tech	3.44(.383)	(2.44,4.44)	P<0.001*
B1 EG High Tech –CG High Tech	3.87(.514)	(2.37,5.36)	P<0.001*
B2 EG High Tech-CG High Tech	3.27(.430)	(2.02,4.51)	P<0.001*

H_{6c}: There is a statistically significant difference between the Graphical Representation Post-Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Control Group.

I used between subjects one-way ANOVA to examine graphical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students and High Tech Savvy students in the two teaching groups: EG and CG for Case A, and EG and CG for Case B. The means for each group are presented in Table 50. There was not a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were not significant ($p>0.05$). The mean differences are presented in Table 51.

Table 50: Means for the Graphical Representation Assessment scores of Low Tech Savvy Students and High Tech Savvy Students Found in the Control Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	CG Low Tech (49)	.35(.879)
	CG High Tech (24)	.38(.824)
A2 EG-CG	CG Low Tech (46)	.13(.453)
	CG High Tech (27)	.22(.506)
B1 EG-CG	CG Low Tech (26)	.58(1.102)
	CG High Tech (12)	.75(1.055)
B2 EG-CG	CG Low Tech (23)	.17(.491)
	CG High Tech (15)	.40(.632)

Table 51: Mean Difference Results for the Graphical Representation Assessment Scores of Low Tech Savvy Students and High Tech Savvy Students Found in the Control Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 CG High Tech-CG Low Tech	.03(.322)	(-.81,.87)	P>0.05
A2 CG High Tech-CG Low Tech	.09(.268)	(-.61,.79)	P>0.05
B1 CG High Tech –CG Low Tech	.14(.448)	(-1.13,1.47)	P>0.05
B2 CG High Tech-CG Low Tech	.23(.368)	(-.84,1.30)	P>0.05

H_{6d}: There is a statistically significant difference between the Graphical Representation Post-Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Experimental Group.

I used between subjects one-way ANOVA to examine graphical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students and High Tech Savvy students in the two teaching groups: EG for Case A and EG for Case B. The means for each group are presented in Table 52. There was a significant difference between each of the groups. Tukey's HSD is used to control for Type I Error. The mean differences are presented in Table 53.

Table 52: Means for the Graphical Representation Assessment Scores of Low Tech Savvy Students and High Tech Savvy Students Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1/B1	EG Low Tech (27)	3.56(2.044)
	EG High Tech (13)	4.62(1.325)
A2/B2	EG Low Tech (28)	3.36(1.890)
	EG High Tech (12)	3.67(1.371)

Table 53: Mean Difference results for the Graphical Representation Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1/B1 EG High Tech-EG Low Tech	.41(.337)	(-.59,1.38)	$P<0.001$*
A2/B2 EG High Tech-EG Low Tech	.31(.381)	(-.68,1.30)	$P<0.001$*

H_{6e}: There is a statistically significant difference between the Graphical Representation Post-Assessment scores of Low Tech Savvy students in the Experimental Group and High Tech Savvy students found in the Control Group.

I used between subjects one-way ANOVA to examine graphical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students in teaching group EG and High Tech Savvy students in teaching group CG. The means for each group are presented in Table 54. There was a significant difference between each of the groups in all cases as presented below. Tukey’s HSD is used to control for Type I Error. The results were significant ($p<0.001$). The mean differences are presented in Table 55.

Table 54: Graphical Representation Assessment Scores of Low Tech Savvy Students in the Experimental Group and High Tech Savvy Students Found in the Control Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Low Tech (27)	3.56(2.044)
	CG High Tech (24)	.38(.824)
A2 EG-CG	EG Low Tech (28)	3.36(1.890)
	CG High Tech (27)	.22(.506)
B1 EG-CG	EG Low Tech (27)	3.56(2.044
	CG High Tech (12)	.75(1.055)
B2 EG-CG	EG Low Tech (28)	3.36(1.890)
	CG High Tech (15)	.40(.632)

Table 55: Mean Difference Results for the Graphical Representation Assessment Scores of Low Tech Savvy Students in the Experimental Group and High Tech Savvy Students Found in the Control Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG Low Tech-CG High Tech	3.18(.363)	(2.29,4.13)	P<0.001*
A2 EG Low Tech-CG High Tech	3.13(.298)	(2.36,3.91)	P<0.001*
B1 EG Low Tech-CG High Tech	2.81(.445)	(1.51,4.10)	P<0.001*
B2 EG Low Tech-CG High Tech	2.96(.355)	(1.93,3.99)	P<0.001*

Analysis Numerical Representation Post-Assessment scores between Low Tech Savvy and High Tech Savvy Students

The analysis of the seventh hypothesis is to determine if the scores for Low Tech Savvy students and High Tech Savvy students differ significantly between each other on the Numerical Representation Assessment. The mean differences between each group are calculated.

H_7: There is a statistically significant difference between Numerical Representation Post-Assessment scores between Low Tech Savvy student performance and High Tech Savvy student performance.

This hypothesis will be explained by looking at 5 (a-e) sub-hypotheses.

H_{7a}: There is a statistically significant difference between the Numerical Representation Post-Assessment scores of Low Tech Savvy students found in the Control Group and their counterparts found in the Experimental Group.

I used Between-subjects one-way ANOVA to examine numerical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 56. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.001$). The mean differences are presented in Table 57.

Table 56: Means for the Numerical Representation Assessment Scores of Low Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Low Tech (27)	1.89(1.948)
	CG Low Tech (49)	.20(.577)
A2 EG-CG	EG Low Tech (28)	1.64(1.830)
	CG Low Tech (46)	.09(2.85)
B1 EG-CG	EG Low Tech (27)	1.89(1.948)
	CG Low Tech (26)	.35(.745)
B2 EG-CG	EG Low Tech (28)	1.64(1.830)
	CG Low Tech (23)	.13(.344)

Table 57: Mean Difference Results for the Numerical Representation Assessment Scores of Low Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG Low Tech-CG Low Tech	1.68(.298)	(.91,2.46)	P<0.001*
A2 EG Low Tech-CG Low Tech	1.56(.263)	(.87,2.24)	P<0.001*
B1 EG Low Tech –CG Low Tech	1.54(.341)	(1.96,4.00)	P<0.001*
B2 EG Low Tech-CG Low Tech	1.51(.308)	(.62,2.41)	P<0.001*

H_{7b}: There is a statistically significant difference between the Numerical Representation Post-Assessment scores of High Tech Savvy students found in the Control Group and their counterparts found in the Experimental Group.

I used between subjects one-way ANOVA to examine numerical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among High Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. The means for each group are presented in Table 58. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.001$). The mean differences are presented in Table 59.

Table 58: Means for the Numerical Representation Assessment Scores of High Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG High Tech (13)	4.08(1.656)
	CG High Tech (24)	.33(.963)
A2 EG-CG	EG High Tech (12)	3.42(1.311)
	CG High Tech (27)	.41(.844)
B1 EG-CG	EG High Tech (13)	4.08(1.656)
	CG High Tech (12)	.67(1.303)
B2 EG-CG	EG High Tech (12)	3.42(1.311)
	CG High Tech (15)	.73(1.033)

Table 59: Mean Difference Results for the Numerical Representation Assessment Scores of High Tech Savvy Students Found in the Control Group and Their Counterparts Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG High Tech-CG High Tech	3.74(.429)	(2.63,4.86)	P<0.001*
A2 EG High Tech-CG High Tech	3.01(.381)	(2.01,4.00)	P<0.001*
B1 EG High Tech –CG High Tech	3.41(.497)	(1.97,4.85)	P<0.001*
B2 EG High Tech-CG High Tech	2.68(.423)	(1.45,3.91)	P<0.001*

H_{7c}: There is a statistically significant difference between the Numerical Representation Post-Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Control Group.

I used between subjects one-way ANOVA to examine numerical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students and High Tech Savvy students in the two teaching groups: EG and CG for Case A, and EG and CG for Case B. The means for each group are presented in Table 60. There was not a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were not significant ($p>0.05$). The mean differences are presented in Table 61.

Table 60: Means for the Numerical Representation Assessment Scores of Low Tech Savvy Students and High Tech Savvy Students Found in the Control Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	CG Low Tech (49)	.20(.577)
	CG High Tech (24)	.33(.963)
A2 EG-CG	CG Low Tech (46)	.09(.285)
	CG High Tech (27)	.41(.844)
B1 EG-CG	CG Low Tech (26)	.35(.745)
	CG High Tech (12)	.67(1.303)
B2 EG-CG	CG Low Tech (23)	.13(.344)
	CG High Tech (15)	.73(1.033)

Table 61: Mean difference results for the Numerical Representation Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Control Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 CG High Tech-CG Low Tech	.13(.310)	(-.68,.94)	P>0.05
A2 CG High Tech-CG Low Tech	.32(.266)	(-.37,1.02)	P>0.05
B1 CG High Tech –CG Low Tech	.32(.434)	(-.94,1.58)	P>0.05
B2 CG High Tech-CG Low Tech	.60(.363)	(-.45,1.66)	P>0.05

H_{7d}: There is a statistically significant difference between the Numerical Representation Post-Assessment scores of Low Tech Savvy students and High Tech Savvy students found in the Experimental Group.

I used between subjects one-way ANOVA to examine numerical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students and High Tech Savvy students in the two teaching groups: EG for Case A and EG for Case B. The means for each group are presented in Table 62. There was a significant difference between each of the groups in all cases as presented below. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.001$). The mean differences are presented in Table 63.

Table 62: Means for the Numerical Representation Assessment Scores of Low Tech Savvy Students and High Tech Savvy Students Found in the Experimental Group

Case	GROUP (N)	MEAN(STD)
A1/B1	EG Low Tech (27)	1.89(1.948)
	EG High Tech (13)	4.08(1.656)
A2/B2	EG Low Tech (28)	1.64(1.830)
	EG High Tech (12)	3.42(1.311)

Table 63: Mean Difference for the Numerical Representation Assessment Scores of Low Tech Savvy Students and High Tech Savvy Students Found in the Experimental Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1/B1 EG High Tech-EG Low Tech	2.19(.420)	(1.09,3.28)	P<0.001*
A2/B2 EG High Tech-EG Low Tech	1.77(.379)	(.78,2.76)	P<0.001*

H_{7e}: There is a statistically significant difference between the Numerical Representation Post-Assessment scores of Low Tech Savvy students in the Experimental Group and High Tech Savvy students found in the Control Group.

I used between subjects one-way ANOVA to examine numerical representation knowledge as measured by Exponential/Logarithmic Post-Assessment among Low Tech Savvy students in teaching group EG and High Tech Savvy students in teaching group CG. The means for each group are presented in Table 64. There was a significant difference between each of the groups in all cases. Tukey's HSD is used to control for Type I Error. The results were significant ($p<0.001$). The mean differences are presented in Table 65.

Table 64: Means for the Numerical Representation Assessment Scores of Low Tech Savvy Students in the Experimental Group and High Tech Savvy Students Found in the Control Group

Case	GROUP (N)	MEAN(STD)
A1 EG-CG	EG Low Tech (27)	1.89(1.948)
	CG High Tech (24)	.33(.963)
A2 EG-CG	EG Low Tech (28)	1.64(1.830)
	CG High Tech (27)	.41(.844)
B1 EG-CG	EG Low Tech (27)	1.89(1.948)
	CG High Tech (12)	.67(1.303)
B2 EG-CG	EG Low Tech (28)	1.64(1.830)
	CG High Tech (15)	.73(1.033)

Table 65: Mean Differences for the Numerical Representation Assessment Scores of Low Tech Savvy Students in the Experimental Group and High Tech Savvy Students Found in the Control Group

CASE	Mean Difference (Std. Error)	95% Confidence Interval	p-value
A1 EG Low Tech-CG High Tech	1.56(.349)	(.64,2.47)	P<0.001*
A2 EG Low Tech-CG High Tech	1.24(.296)	(.46,2.01)	P<0.001*
B1 EG Low Tech-CG High Tech	1.22(.431)	(.37,2.07)	P<0.001*
B2 EG Low Tech-CG High Tech	1.33(.350)	(.42, 1.92.)	P<0.001*

Analysis of Correlation between Post-Calculator Knowledge and Post-Exponential/Logarithmic

The analysis of the eighth hypothesis is to determine if there is a significant relationship between student scores on the Exponential/Logarithmic Post-Assessment and student scores on the Calculator Knowledge Post-Assessment. These results will help to inform as to whether or not the graphing calculator has an effect on student achievement when learning the three core representations (algebraic, graphical, and numerical.)

H_8: There is a positive correlation between the Exponential/Logarithmic Post-Assessment scores and the Calculator Knowledge Post-Assessment scores among all students, among groups, and among High and Low Tech Savvy students.

This hypothesis will be explained by looking at 3 (a-c) sub-hypotheses.

> H_{8a}: There is a positive correlation between the Exponential/Logarithmic Post-Assessment scores and the Calculator Knowledge Post-Assessment scores among all students.

I used the Spearman rank correlation coefficient test to determine if a positive correlation exists between Exponential/Logarithmic Post-Assessment and Calculator Knowledge Post-Assessment among all students for each grader in each case. There was a positive correlation among all students (N=113) for both graders in each case as presented in Table 66.

> H_{8b}: There is a positive correlation between the Exponential/Logarithmic Post-Assessment scores and the Calculator Knowledge Post-Assessment scores among students in the experimental and control groups.

I used the Spearman rank correlation coefficient test to determine if a positive correlation exists between Exponential/Logarithmic Post-Assessment and Calculator Knowledge Post-Assessment among students in two teaching groups: EG and CG for Case A and EG and CG for Case B. There was a positive correlation for each of the groups in all cases as presented in Table 66.

> H_{8c}: There is a positive correlation between the Exponential/Logarithmic Post-Assessment scores and the Calculator Knowledge Post-Assessment scores among High and Low Tech Savvy students.

I used the Spearman rank correlation coefficient test to determine if a positive correlation exists between Exponential/Logarithmic Post-Assessment and Calculator Knowledge Post-Assessment among Low Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. There was positive correlation between each of the groups in all cases as presented in Table 66.

I also used the Spearman rank correlation coefficient test to determine if a positive correlation exists between Exponential/Logarithmic Post-Assessment and Calculator Knowledge Post-Assessment among High Tech Savvy students in two teaching groups: EG and CG for Case A and EG and CG for Case B. There was positive correlation between each of the groups in all cases as presented in Table 66.

Table 66: Spearman Rank Correlation Results of the Exponential/Logarithmic Post-Assessment and Calculator Knowledge Post-Assessment Between Groups

CASE	GROUP	Spearman's Rho P-Value	N
A1/B1	All	.527 P<0.001*	113
A2/B2	All	.512 P<0.001*	113
A1	EG	.430 P<.01*	40
	CG	.213 P>.05	73
A2	EG	.355 P<.001*	40
	CG	.227 P>.05	73
B1	EG	.430 P<.01*	40
	CG	.325 P<.05*	38
	NCG	.168 P>.05	35
B2	EG	.355 P<.05*	40
	CG	.272 P<.05*	38
	NCG	.177 p>.05	35
A1/B1	Low Tech Savvy	.559 P<.001*	30
A1/B1	High Tech Savvy	.517 P<.01*	13
A2/B2	Low Tech Savvy	.607 P<.001*	31
A2/B2	High Tech Savvy	.400 P<.01*	12

CHAPTER 5

Conclusions, Discussion, and Recommendations

The purpose of this research was to investigate the potential benefits of teaching College Algebra from three core representations functions: algebraic, graphical, and numerical. This chapter is organized into three parts in order to discuss the findings of the research. First, a brief summary of the study is followed by conclusions, based on quantitative results reported in Chapter 4, which are presented in relation to the research questions. The next section of the chapter is a discussion of limitations and recommendations of the study on the use of multiple representations in teaching College Algebra. Recommendations for teaching the concept of functions and for future research makes up the last part of this chapter.

Conclusions

The participants of the study were 113 college students enrolled in *College Algebra* at a historically black private college located in North Carolina. The study utilized a quasi-experimental design in which one set of instructors taught the course using scripted materials and the researcher taught the course using a functional approach in which content was taught simultaneously using the three core representations: algebraic, graphical, and numerical. The content of the study was exponential equations, logarithmic equations, and compound interest problems.

All of the students involved in the study were taught the same topics over the same period of time. The scripted curriculum emphasized solving problems only from the algebraic representation perspective. Students in this curriculum (n=73) were taught algebraic manipulation skills. Students in the intervention curriculum (n=40) were also taught algebraic manipulation skills along with the graphical and numerical representations. Connections among the three core representations were highlighted during instruction and students were assessed throughout the intervention using the three core representations. Students in both curricula were required to have a graphing calculator.

At the beginning of the course, both groups were given pretests to measure their Exponential/Logarithmic Knowledge and Calculator Knowledge. Two graders were used to score the responses to these measures using the Inter-rater Reliability Test to test for consistency. The consistency was not high enough to justify randomly selecting one grader or rater results to test the hypotheses and answer the research questions. Therefore, the summary and conclusions will be explained using two cases: Case A and Case B. Each case considers either the results of Grader 1 or Grader 2 separately.

Case A assumes that all students start with a similar content and Calculator Knowledge based on the *Accuplacer* test. For Case A, all students in the traditional curriculum are the Control Group (CG) and all students in the intervention are the Experimental Group (EG). Case B only uses scores from students in the Researcher Taught Control Group (CG) and the Researcher Taught Experimental Group (EG) to test the hypotheses and answer the research questions.

At the end of the semester, a posttest was given to all students participating in the study. The following section addresses the findings of the study in relation to the research questions per case.

Conclusions Pertaining to Case A for Grader 1 and Grader 2

Research Question 1

> What effect does a three core representation-based curriculum have on college students' understanding of exponential and logarithmic functions?

Both Graders 1 and 2 concluded that there was a statistically significant difference between students in the Experimental Group (EG) and the Control Group (CG) on the Post-Calculator Knowledge test scores. All of the students are assumed to have had the same Calculator Knowledge before the intervention, due to their placement in *College Algebra*. This result indicates that students in the Experimental Group gained in knowledge on the ability to apply graphing calculator procedures helpful for the concept of functions.

Each grader also agreed that the Experimental Group outperformed the Control Group on the Exponential/Logarithmic Post-Assessments involving only exponential equations, involving only logarithmic equations, and involving only solving compound interest problems. This result is to be expected since the traditional curriculum does not emphasize the graphical and numerical representation.

The intent of the study was to also examine how students who were designated as High Tech Savvy or Low Tech Savvy would perform compared to their counterparts. When Low Tech Savvy students in the Experimental and Control Group are compared to each other, the one-way ANOVA test shows both graders agree there was a significant difference between the students on the Exponential/Logarithmic Post-Assessment. When students who have been designated as High Tech Savvy in the Experimental are compared to students who have been designated as High Tech Savvy in the Control Group are compared to each other, the one-way ANOVA test shows both graders agree there was a significant difference between the students on the Exponential/Logarithmic Post-Assessment. When Low Tech Savvy students in the Experimental Group and High Tech Savvy students in the Experimental Group are compared to each other, the one-way ANOVA test shows both graders agree the High Tech Savvy students had a Higher gain on the Exponential/Logarithmic Post-Assessment. This result shows that the intervention did not create a ceiling for learning with the High Tech Savvy students. Even though the Low Tech Savvy students did well, the High Tech Savvy students did even better.

Lastly, when Low Tech Savvy students in the Experimental Group and High Tech Savvy students in the Control Group are compared to each other, the One-way ANOVA test shows both graders agree there was a significant difference between the students on the Exponential/Logarithmic Post-Assessment; the High Tech Savvy students had a Higher gain on the Exponential/Logarithmic Post-Assessment. This result suggests ~~could indicate~~ that students who are not a part of the intervention and already operating at a High level of knowledge in using the graphing calculator can create for themselves ways to be successful.

To what extent a three core representation-based curriculum have on college students' understanding of exponential and logarithmic functions is divided into the following sub-questions that usually involve multiple representations. Again, these three core representations are taught simultaneously.

a) Are three core representation-based curriculum students able to solve exponential and logarithmic functions algebraically better than students in a traditional college algebra curriculum?

Both graders found no statistically significant difference on the Algebraic Representation Post-Assessment scores between the Control Group and Experimental Group. All instructors taught algebraic manipulation skills to the students participating. There was also agreement among the graders that there was no statistically significant difference on the Algebraic Representation Assessment between Low Tech Savvy students in the Experimental and Control Group, between High Tech Savvy students in the Experimental and Control Group, between Low Tech Savvy and High Tech Savvy students in the Experimental Group, and between Low Tech Savvy students in the Experimental Group and High Tech Savvy students in the Control Group.

However, the data from the two graders indicates that the High Tech Savvy students in the Control Group outperformed the Low Tech Savvy students in the Control Group. This result is not surprising, considering it would be expected that students operating at a Higher graphing Calculator Knowledge skill level would perform better.

b) Are three core representation-based curriculum students able to solve exponential and logarithmic functions graphically better than students in a scripted college algebra curriculum?

Both graders found a statistically significant difference on the Graphical Representation Assessment scores between the Control Group and the Experimental Group. Only the Experimental Group was taught solving exponential and logarithmic functions from a graphical perspective. The graders also agreed that the Low Tech Savvy students in the Experimental Group outperformed the Low Tech Savvy students in the Control Group. This result importantly indicates that even if calculator knowledge is challenging at first, students have the capacity to think critically from a graphical perspective and learn to apply new calculator knowledge to functions.

The High Tech Savvy students in the Experimental Group outperformed the Low Tech Savvy students in the Experimental Group on Graphical Representation Assessment. There was a statistically significant difference between the Low Tech Savvy students in the Experimental Group and the High Tech Savvy students in the Control Group. The graders found no significant difference on the Graphical Representation Assessment scores between the Low Tech Savvy students and High Tech Savvy students in the Control Group. Clearly the mere presence of the graphing calculator and the ability to know where certain features are does not constitute an increase of mathematical knowledge. Graphical representations require the ability to not only understand the local view of the graph, but also the global view of functions and those functions applied within a context.

c) Are three core representation-based curriculum students able to solve exponential and logarithmic functions numerically better than students in a traditional college algebra curriculum?

Both graders found a statistically significant difference on the Numerical Representation Assessment scores between the Control Group and the Experimental Group. Only the Researcher/Instructor taught solving exponential and logarithmic functions from a graphical perspective. The graders also agreed that the Low Tech Savvy students in the

Experimental Group outperformed the Low Tech Savvy students in the Control Group. This result importantly indicates that even if calculator knowledge is challenging at first, students have the capacity to think critically from a numerical perspective and learn to apply new calculator knowledge to functions.

The High Tech Savvy students in the Experimental Group outperformed the Low Tech Savvy students in the Experimental Group on Numerical Representation Assessment. There was a statistically significant difference between the Low Tech Savvy students in the Experimental Group and the High Tech Savvy students in the Control Group. The graders found no significant difference on the Numerical Representation Assessment scores between the Low Tech Savvy students and High Tech Savvy students in the Control Group. Again, the mere presence of the graphing calculator and the ability to know where certain features are does not constitute an increase of mathematical knowledge.

Research Question 2

> What effect does graphing calculator knowledge have on a college students' understanding of exponential and logarithmic functions?

Case A does not use the Calculator Knowledge Pre-Assessment data, since it is assumed that the students were the same at the onset. Thus, determining growth was not possible. However, the Post-Calculator Knowledge data was used to determine statistical significance between the groups. The two graders agreed that there is a statistically significant difference on the Calculator Knowledge Post-Assessment between the Control

Group and the Experimental Group. This result is consistent with the results regarding the graphical and numerical representations in research question one.

Student participants were asked to solve the same exponential equation algebraically, graphically, and numerically. When the scores of these three problems are combined for each student and compared within their respective group, the graders agreed that the Experimental Group significantly outperformed the Control Group. This result is true for the logarithmic equation problem and the solving compound interest problem. It can be concluded that the intervention of graphical representation and numerical representation was effective, since there was no significant difference between groups using only the algebraic representation as noted in Table 22.

Research Question 3

> Does the three core representations-based curriculum and graphing calculators interact to effect college algebra students' understanding of exponential and logarithmic functions?

This question looked to find a positive correlation between the Exponential/Logarithmic Post-Assessment scores and the Calculator Knowledge Post-Assessment scores between groups. The graders agreed that there is not a positive correlation between students in the Control Group. This result is not surprising. Students in the Control Group did not receive any type of intervention. It is possible that some of the Control Group students were familiar with finding solutions graphically and numerically, but were uncertain on how to use the graphing calculator to aid in finding those solutions. The data from both graders did find that a relationship exists between Calculator Knowledge and Exponential/Logarithmic assessments for students in the Experimental Group. This includes students who were High Tech Savvy in the Experimental Group students who were Low Tech Savvy in the Experimental Group.

Conclusions Pertaining to Case B for Grader 1 and Grader 2

Research Question 1

> What effect does a three core representation-based curriculum have on college students' understanding of exponential and logarithmic functions?

Both Graders 1 and 2 concluded that there was a statistically significant difference between students in the Researcher Taught Experimental Group (EG) and the Researcher Taught Control Group (CG) on the Post-Calculator Knowledge test scores. All of the students are assumed to have had the same calculator knowledge before the intervention, due to their placement in the *College Algebra* course. This result indicates that students in the EG gained in knowledge on the ability to apply graphing calculator procedures helpful for the concept of functions.

Each grader also agreed that the EG outperformed the CG on the Exponential/Logarithmic Post-Assessments involving only exponential equations, involving only logarithmic equations, and involving only solving compound interest problems. This result is to be expected since the traditional curriculum does not emphasize the graphical and numerical representation.

The intent of the study was to also examine how students who were designated as High Tech Savvy or Low Tech Savvy would perform among their counterparts. When Low Tech Savvy students in the EG and CG are compared to each other, the one-way ANOVA test shows both graders agree there was a significant difference between the students on the Exponential/Logarithmic Post-Assessment. When High Tech Savvy students in the EG and CG are compared between each other, the one-way ANOVA test shows both graders agree there was a significant difference between the students on the Exponential/Logarithmic Post-Assessment. When Low Tech Savvy students in the EG and High Tech Savvy students in the EG are compared between each other, the one-way ANOVA test shows both graders agree the High Tech Savvy students had a Higher gain on the Exponential/Logarithmic Post-Assessment. This result shows that the intervention did not create a ceiling for learning with the High Tech Savvy students. Even though the Low Tech Savvy students did well, the High Tech Savvy students did even better.

Lastly, when Low Tech Savvy students in the EG and High Tech Savvy students in the CG are compared between each other, the one-way ANOVA test shows both graders agree there was a significant difference between the students on the Exponential/Logarithmic Post-Assessment; the High Tech Savvy students had a Higher gain on the Exponential/Logarithmic Post-Assessment. This result suggests that students who are not a part of the intervention and already operating at a High level of knowledge in using the graphing calculator can create for themselves ways to be successful.

To what extent a three core representation-based curriculum have on college students' understanding of exponential and logarithmic functions is divided into the following sub-questions that usually involve multiple representations. Again, these three core representations are taught simultaneously.

a) Are three core representation-based curriculum students able to solve exponential and logarithmic functions better algebraically than students in a traditional college algebra curriculum?

Both graders found no statistically significant difference on the Algebraic Representation Post-Assessment scores between the CG and EG. All instructors taught algebraic manipulation skills to the students participating. There was also agreement among the graders that there was no statistically significant difference on the Algebraic Representation Assessment between Low Tech Savvy students in the EG and CG, between High Tech Savvy students in the EG and CG, between Low Tech Savvy and High Tech Savvy students in the EG, and between Low Tech Savvy students in EG and High Tech Savvy students in the CG.

However, the data from the two graders indicates that the High Tech Savvy students in the CG outperformed the Low Tech Savvy students in the CG. This result is not surprising, considering it would be expected that students operating at a Higher graphing Calculator Knowledge skill level would perform better.

b) Are three core representation-based curriculum students able to solve exponential and logarithmic functions better graphically than students in a traditional college algebra curriculum?

Both graders found a statistically significant difference on the Post Graphical Representation Assessment scores between the CG and the EG. Only the

Researcher/Instructor taught solving exponential and logarithmic functions from a graphical perspective. The graders also agreed that the Low Tech Savvy students in the EG outperformed the Low Tech Savvy students in the CG. This result importantly indicates that even if calculator knowledge is challenging at first, students have the capacity to think critically from a graphical perspective and learn to apply new calculator knowledge to functions.

The High Tech Savvy students in the EG outperformed the Low Tech Savvy students in the EG on the Graphical Representation Assessment. There was a statistically significant difference between the Low Tech Savvy students in the EG and the High Tech Savvy students in the CG. The graders found no significant difference on the Graphical Representation Assessment scores between the Low Tech Savvy students and High Tech Savvy students in the CG. Clearly the mere presence of the graphing calculator and the ability to know where certain features are does not constitute an increase of mathematical knowledge. Graphical representations require the ability to not only understand the local view of the graph, but also the global view of functions and those functions applied within a context.

c) Are three core representation-based curriculum students able to solve exponential and logarithmic functions better numerically than students in a traditional college algebra curriculum?

Both graders found a statistically significant difference on the Numerical Representation Assessment scores between the CG and the EG. Only the Researcher/Instructor taught solving exponential and logarithmic functions from a graphical perspective. The graders also agreed that the Low Tech Savvy students in the Experimental Group outperformed the Low Tech Savvy students in the CG. This result importantly indicates that even if calculator knowledge is challenging at first, students have the capacity to think critically from a numerical perspective and learn to apply new calculator knowledge to functions.

The High Tech Savvy students in the EG outperformed the Low Tech Savvy students in the EG on the Numerical Representation Assessment. There was a statistically significant difference between the Low Tech Savvy students in the CG and the High Tech Savvy students in the CG. The graders found no significant difference on the Numerical Representation Assessment scores between the Low Tech Savvy students and High Tech Savvy students in the CG. Again, the mere presence of the graphing calculator and the ability to know where certain features are does not constitute an increase of mathematical knowledge.

Research Question 2

> What effect does graphing calculator knowledge have on a college students' understanding of exponential and logarithmic functions?

Case B uses the Pre- and Post-Calculator Knowledge Assessment scores and the Exponential/Logarithmic Pre- and Post-Assessment scores to determine a statistically significant difference between the between the EG and the CG. Both graders agreed that the EG obtained a significantly Higher growth on both assessments than the CG.

Also, the Post-Calculator Knowledge data was used to determine statistical significance. The two graders agreed that there is a statistically significant difference between the Post-Calculator Knowledge between the CG and the EG. This result is consistent with the results regarding the graphical and numerical representations in research question one.

Student participants were asked to solve the same exponential equation algebraically, graphically, and numerically. When the scores of these three problems are combined for each student and compared within their respective group, the graders agreed that the Experimental Group significantly outperformed the Control Group. This result is true for both the logarithmic equation problem and the solving compound interest problem. It can be concluded that the intervention of graphical representation and numerical representation was effective, since there was no significant difference between groups using only the algebraic representation as noted in Table 22.

Research Question 3

> Does the three core representations-based curriculum and graphing calculators interact to effect college algebra students' understanding of exponential and logarithmic functions?

This question looked to find a positive correlation between the Exponential/Logarithmic Post-Assessment scores and the Calculator Knowledge Post-Assessment scores between the Control Group and the Experimental Group. The graders agreed that there is not a positive correlation between students in the Control Group. This result is not surprising. Students in the Control Group did not receive any type of intervention. It is possible that some of the Control Group students were familiar with finding solutions graphically and numerically, but were uncertain on how to use the graphing calculator to aid in finding those solutions. The data from both graders did find that a relationship exists between Calculator Knowledge and Exponential/Logarithmic assessments for students in the Experimental Group. This includes students who were High Tech Savvy in the EG students who were Low Tech Savvy in the EG.

Even though the Inter-Rater Reliability Test indicated that the consistency between the raters was not high enough to justify randomly selecting one grader or rater, it is important to realize that the statistical results were exactly the same for both Cases.

Discussion

One of the factors providing motivation for the present study was the need for expanded research on the effect of graphing calculators on college students' learning of exponential equations, logarithmic equations, and solving compound interest word problems from a three core representation perspective. The concept of function is an important unifying idea in mathematics. In the United States, functions traditionally are introduced in algebra I, and play a central role in algebra II, and permeate all the pre-calculus and calculus. In fact, some scholars (Chazan, 1993; Schwartz, 1992) have suggested that functions become central to a first course in algebra by combining the study of algebraic, graphical, and numerical representations.

Graphing calculators allow investigation of functions through tables, graphs, and equations in ways that were not possible before their increased use. Dick (1992) and Wilson and Krapfl (1994) suggested that the use of multiple representations, interpretation from one representation to another, and analysis requiring the connection between the three core representations are key to understanding functions. Each of these representations is readily available in a graphing calculator environment. Further, graphing calculators allow the focus to be on understanding and setting up and interpreting results (Dick, 1992; Hopkins, 1992).

In a Technology-rich classroom environment, Technology impacts not only what is taught and how it is taught but what students learn and how they learn it. Technology also impacts how that learning is assessed. Indeed, if Technology is an integral part of instruction, then the *Principals and Standards of School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1995) recommended, Technology should be an important feature of assessment so that assessment and instruction are aligned.

Using multiple representations does not guarantee that a student will be able to make the connections back to the original problem (Dufour-Javier, Bednarz, & Belanger, 1987; Nobel et al., 2001). Multiple representations enhance the algebraic thinking that is currently present when viewing a problem situation. If the algebraic thinking is in separate isolated ideas, the representations in turn will be separate isolated ideas that have no connection and offer no help in solving the problem. Driscoll describes "…a part of student sense-making in their use of multiple representations is to hold onto an awareness of the original problem context" (Driscoll, 1999, p. 151). This awareness is cultivated when the student has experience to draw from about the problem situation.

A major idea in research on functions deals with the importance of students moving comfortably between representations (Breener et al., 1997; Keller & Hirsch, 1998; NCTM, 2000). “The skill in choosing or building representations, together with interpretive skills will likely soon outstrip computational skills in importance by a wide margin” (Kaput, 1987, p. 21). Even though the support is strong for students’ use of multiple representations in learning mathematical and algebraic concepts (Breener et al., 1997; Janiver, 1987; Noble et al., 20010) there are legitimate concerns about using too many representations and causing more confusion than understanding (Dufour-Janiver et al., 1987; Noble et al., 2001).

Recommendations

Implications for Teaching

The goal of instruction is to teach students how to learn, rather than about mathematics. Traditional *College Algebra* courses tend to focus strongly on students learning rules and following procedures, rather than thinking about concepts or about thinking. A three core representation curriculum allows students to think more deeply about the context and gives students multiple ways to check results.

Students taught by an instructor with an algebraic manipulation orientation to teaching do not build conceptual understanding. Students only gain limited procedural understanding. The focus of instruction needs to be on helping students to understand more than how to get answers. Given the complexity and depth of the topics discussed in any mathematics course, it is important for the instructor to be continuously aware of the needs of the students and the ways in which the students can be aided in learning.

Students need to know that the best methods for solving a problem arise from understanding it deeply, and that understandings are personal, not prescribed. They should understand that a solution to a problem is not unique to the particular method used to determine it. In this way, students are led to see that there is meaning beyond the procedures and algorithms used to solve problems. They will also have the opportunity to be confronted with and examine the fact that mathematics is not separate from their personal world of sense and understanding, as with the compound interest problem. The students in the traditional curriculum did not have these opportunities, and made few connections with graphical and numerical representations. Students need to know that they have both the freedom and the responsibility to approach a problem in any what that makes sense to them, for which they can build justifications for the sense that it makes (algebraic, graphical, numerical).

Algebraic manipulations cannot help students to gain understanding of functions if the students are not taught to analyze and interpret all aspects of the mathematics involved. Students in the Experimental Group were able to see connections between representations and the mathematics being used to present those representations, since the intervention did focus on these connections. Without this analysis and interpretation, mathematics will become a separate, disconnected topic which students approach algebraically or procedurally.

Graphing calculators provide opportunities for students to connect graphical images, Algebraic expressions, and sets of related numerical values. Students need to learn to represent functions numerically as tables of input-output pairs, algebraically as algebraic representations, and graphically as plots of input-output points. Students need to translate across these representations and connect these representations to physical contexts.

This study produced support for the use of the graphing calculator as a justifiable and a promising tool for the teaching and learning of mathematics from a multiple representation perspective. The graphing calculator holds promise as a key for genuine reform in mathematics education. Graphing calculator Technology requires mathematics teachers to rethink how and what they teach. The teacher's role must shift from dispensing information to facilitating learning. Less emphasis must be placed on procedures and manipulative skills and more emphasis on concepts and higher order thinking skills.

Future Research Considerations

The results of this study compel additional studies for determining the most effective methods for integrating graphing calculator Technology into mathematics education. Although a limited amount of research involving exponential and logarithmic functions has been done, much more needs to be done in this area of mathematics involving multiple representations. The research needs to involve shifts and transformations, as well as additional applications involving exponential and logarithmic functions.

Additional research needs to also involve Low achievers, and how their ability to move through representations impacts their mathematical knowledge. Even though the study suggests that the intervention was successful for students who had an initial Low knowledge of graphing calculators, more research can assist in validating the results obtained.

Research on which of the three core representations the student participants preferred is of interest. Student participants in the study were required to legibly describe their thinking process on solving the problems graphically and numerically. Analyzing how many students who understood the process but had difficulty executing the process is of interest. The results from this inquiry can inform the mathematics education community about conceptual understanding in regards to knowing what to do and how to do it.

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APPENDICES

Appendix A – Calculator Knowledge Assessment Instrument

College Algebra Calculator Knowledge Assessment
Student graphing Calculator Knowledge

1. Do you know how to use a graphing calculator? Yes____ No_____ Somewhat_____

As best as you can, explain how to use the graphing calculator in the following situations below. For example, write the steps you use to complete the task, including which calculator buttons are used.

2. Write how you would enter $\frac{4}{8+6}$ into your graphing calculator and then give the answer. If the answer is a decimal, write it to 4 decimal places without rounding.

3. How do you make a number negative on your graphing calculator?

4. How do you square 7.178 on your graphing calculator? Give the answer.

5. How do you find the square root of 58 on your graphing calculator? Give the answer.

6. Use your calculator to write π to 8 decimal places.

7. Suppose you want your graphing calculator display to show only four decimal places. How would you make that happen for $\frac{45}{7898}$. Give the answer without rounding.

8. How do you retrieve the last computation performed by the calculator?

9. How do you change the size of the window on your graphing calculator?

10. a. Enter $y_1 = x^2 - 5x + 9$ into your graphing calculator.
    b. Generate the table of values for $y_1 = x^2 - 5x + 9$ on the graphing calculator screen.
    c. Complete the entries in the given table shown below.

X	Y_1
	59
	135
23	

11. Use your graphing calculator to find the intersection point of $y_1 = 2x - 5$ and $y_2 = -4x - 1$.

Appendix B – Exponential/Logarithmic Pre-Assessment Instruments

College Algebra Graphing Calculator Exponential/Logarithmic Pre-Assessment
You are allowed to use your calculator.

1. Solve $3^{2x-5} = 9$.

2. Solve $3^{2x-5} = 9$ graphically. Explain your process.

3. Solve $3^{2x-5} = 9$ using a table or Numericaly. Explain your process.

4. Solve $\log_2(5x-7) = 3$.

5. Solve $\log_2(5x-7) = 3$ graphically. Explain your process.

6. Solve $\log_2(5x-7) = 3$ using a table or Numericaly. Explain your proces

7. Solve $\log_2(5x-7)=3$ using a table or Numericaly. Explain your process.

8. \$100 is invested at 6% compounded annually. How much money is in the account after a period of 3 years?

A. Solve the problem Algebraically.

B. Solve the problem graphically. Explain your process.

C. Solve the problem using a table or Numericaly. Explain your process.

Appendix C – Exponential/Logarithmic Post-Assessment Instruments

College Algebra Post-Exponential/Logarithmic 1
Solving Exponential Equations
You are allowed to use your calculator.

1. Algebraically solve for x: $3^{2x-5} = 9$

2. Solve $3^{2x-5} = 9$ graphically. Explain your process.

3. Solve $3^{2x-5} = 9$ using a table or Numericaly. Explain your process.

College Algebra Post-Exponential/Logarithmic 2
Solving Logarithmic Equations
You are allowed to use your calculator.

1. Algebraically solve for x: $\log_2(5x-7)=3$

2. Solve $\log_2(5x-7)=3$ graphically. Explain your process.

3. Solve $\log_2(5x-7)=3$ using a table or Numericaly. Explain your process.

College Algebra Post-Exponential/Logarithmic 3
Solving Compound Interest Problems
You are allowed to use your calculator.

$100 is invested at 6% compounded annually. How much money is in the account after a period of 3 years?

1. Solve the problem Algebraically.

2. Solve the problem graphically. Explain your process.

3. Solve the problem using a table or Numericaly. Explain your process.

Appendix D – Rubric for Calculator Knowledge Assessment

TI 83/83 Plus/84 Calculator Knowledge Assessment Rubric		
Question	1-point	2-Points
2	Correctly explained how to enter the problem into the calculator.	Correctly explained how to enter the problem into the calculator AND calculated answer correctly.
3	Correctly explained how to make a number negative.	
4	Correctly explained how to square a number.	Correctly explained how to square a number AND calculated the answer correctly.
5	Correctly explained how to find the square root of a number.	Correctly explained how to find the square root of a number AND calculated the answer correctly.
6	Correctly found pi to 8 decimal places.	
7	Correctly explained how to display the given problem to four decimal places.	Correctly explained how to display the given problem to four decimal places AND calculated the answer correctly
8	Correctly explained how to retrieve the last computation performed on the calculator.	
9	Correctly explained how to change the size of the window.	
10		Correctly entered the given function and used the generated table of values to complete the entries in the given table.
11		Correctly used the intersect feature to find the intersection point of the two given functions.
*Some problems were only 1-Point or 2-point only.		

Appendix E – Rubric for Exponential/Logarithmic

Each problem on the Exponential/Logarithmic is worth two (2) points total. One (1) point is awarded if the process to solve the problem using a graphing calculator was written correctly. One (1) point is awarded if the solution to the problem was derived correctly by the requested core representation.