ABSTRACT

PATTERSON, NIKITA COLLINS. A Case Study of an Experienced vs. a Novice Teacher in the Implementation of a New Intermediate Algebra Curriculum. (Under the direction of Karen S. Norwood.)

The mathematics education community, in its call for reform, underscores the importance of mathematics instruction emphasizing the use of multiple representations in the presentation of concepts. One focus of the study was to determine if and how students’ beliefs and attitudes towards technology change as a result of studying quadratic equations using multiple (algebraic, graphical, tabular) representations. Another focus of the study was how teacher beliefs affect their ability to implement a multiple representations curriculum.

The novice instructor’s attitude remained neutral while the experienced instructor’s attitude remained somewhat positive. Of the eight students in the study, six of them showed a more positive attitude towards technology use and multiple representations by the end of the semester. The students increased their calculator use in both classes, particularly in the experienced teacher’s class. The teachers had the greatest impact on the students’ preferences for a particular representation and ease of technology use. Other factors were students’ lack of confidence in technology use, and the structure of the tests. Implications for further research were that teacher training is essential if reform curricula are to be properly implemented.
A CASE STUDY OF AN EXPERIENCED VS. A NOVICE TEACHER IN
THE IMPLEMENTATION OF A NEW INTERMEDIATE ALGEBRA
CURRICULUM

by

NIKITA COLLINS PATTERSON

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Chair of Advisory Committee
DEDICATION

To my husband and best friend, Darvin: Thank you for all of the love and support you have given me. Thank you for all of the sacrifices you made and the joy you have brought to my life. Thank you for believing in me, even when I did not. I am happy that I can share this moment with you. I love you.
BIOGRAPHY

Nikita Danyelle Collins Patterson was born in Cleveland, Ohio on January 3, 1972, the daughter of Arthur and Kathleen Collins. She graduated as the salutatorian from John Marshall High School in 1989. She was in the Dual Degree Engineering Program and received a Bachelor of Science in Chemistry from Spelman College in May, 1994 and a Bachelor of Chemical Engineering from Georgia Institute of Technology in September, 1994. While in the Dual Degree Engineering Program, Nikita was also a scholar in the Women in Science and Engineering (WISE) Program and interned several summers at NASA Lewis Research Center in Cleveland, Ohio. In addition to her academic pursuits, Nikita worked as a choreographer for the dance teams at Morehouse College and Lithonia High School.

Upon graduation, Nikita worked as a chemist and chemical engineer but decided, to enter the education arena in 1995 and began her graduate work at Clark Atlanta University in Atlanta, Georgia. She received her master’s degree in Secondary Mathematics Education in May 1997.

Nikita entered the Ph.D. program in Mathematics Education at North Carolina State University in the fall of 1998. During her time at NCSU, Nikita worked as a Graduate Teaching Assistant with the Math 101 program and went on to become the Coordinator of that program in 2000. While pursuing her doctorate, Nikita also taught mathematics at Community Partners Charter High School in Holly Springs, North Carolina. Her research efforts have focused on multiple representations and the use of technology in the mathematics classroom.
Nikita enjoys spending time with her husband, Darvin, and her two children Kai, age 2, and Aaron, age 1. She also enjoys reading, listening to music, and dancing. Nikita hopes one day to publish a novel or a screen play.
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CHAPTER 1

INTRODUCTION

As educators explore ways to strengthen and energize the teaching of mathematics at the secondary and undergraduate levels, and as technology becomes more common in the classroom, the use of multiple representations is gaining more attention in the curriculum. Research in mathematics education, in its call for reform, underscores the importance of mathematics instruction emphasizing the use of multiple representations enhanced through technology in the presentation of concepts. Multiple representations refer to the presentation of a concept or process from numerical, graphical and symbolic perspectives (i.e., using numerical, graphical, and symbolic representations) (Porzio, 1999).

The National Council of Teachers of Mathematics’ (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) called for a shift in emphasis from a curriculum dominated by memorization…and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understanding, multiple representations and connections, mathematical modeling, and mathematical problem solving (NCTM, 1989). Also, the shift towards a technology-driven society requires that students of mathematics are able to make connections with other disciplines. With the release of the updated *Principles and Standards for School Mathematics* (2000), the NCTM made further recommendations for instructional programs in mathematics that enable all students to:
• create and use representations to organize, record, and communicate mathematical ideas;
• select, apply, and translate among mathematical representations to solve problems;
• use representations to model and interpret physical, social, and mathematical phenomena; (NCTM, 2000, p. 67)

Students need to be able to use a wide range of mathematical representations and make connections among mathematical ideas and with other academic disciplines. As students become mathematically sophisticated, they develop an increasingly large repertoire of mathematical representations and the knowledge of how to use them productively. This knowledge includes choosing specific representations in order to gain particular insights or achieve particular ends (NCTM, 2000).

One theory of learning in mathematics is that multiple representations can be utilized to help students develop deeper, more flexible understandings of concepts and processes (Hiebert & Carpenter, 1992; Kaput, 1989; Kaput, 1992; Porzio, 1999). Dufour-Janvier, Bednarz, and Belanger (1987) identified several motives for using multiple representations in mathematics instruction. They concluded that multiple representations are an inherent part of mathematics, can give multiple concretizations of a concept, can help to decrease certain difficulties students have when trying to complete tasks, and are intended to make mathematics more attractive and interesting.
Functions/Quadratic Equations

This study focused on the concept of solving functions given in the form of quadratic equations. Much work has been done on the concept of function. The mathematics reform movement recognizes that "the concept of function is an important unifying idea in mathematics" (NCTM, 1989, p.189). A function can be viewed in various forms such as a set of ordered pairs, a correspondence, a graph, a dependent variable, a formula, an action, a process, or an object. University students often have considerable difficulty changing from one point of view to another (Selden & Selden, 1992).

There is recent interest in how students come to understand functions, and how well they translate among symbolic, graphical, tabular, and other representations of these functions. Yershalmy and Schwartz (1993) believe that the function is the fundamental object of algebra and that it ought to be present in a variety of representations in algebra teaching and learning from the outset (Yerushalmy & Schwartz, 1993). Because each representation emphasizes and suppresses various aspects of a concept, researchers believe that students gain a more thorough understanding of a function if it is explored using numerical, graphical, and analytical methods (Piez & Voxman, 1997).

To make sense out of multiple representations of a single underlying object - the function under consideration - one must be able to translate readily among the representations so as to understand the common abstraction underlying all of them (Harvey, 1991). To facilitate the transition from the procedural to the more
complex structural conception, a function should be viewed as an abstract mathematical notion, which may be represented graphically, algebraically, or numerically (Mayes, 1995).

According to Embse and Yoder (1998), drawing a static picture representing one state of a functional relationship does not guarantee that students will "see" how changes in the independent variable produce corresponding changes in the dependent variable. An even better situation would be to draw a dynamic picture in which the drawing changes as the independent variable changes. The graphical-and algebraic-solution processes should be used to extend students' understanding of the problem. In addition, these solution processes can build the platform for experimenting with different versions of the original problem to extend and generalize the problem situation (Embse & Yoder, 1998).

**Technology**

One of the main strengths of technology is that it can provide greater and easier access to multiple representations of concepts and processes (Fey, 1989; Goldenberg, 1987; Kaput, 1992; Porzio, 1999). Graphing calculators have extended students' problem-solving repertoire to include visual and numerical techniques (Embse & Yoder, 1998).

Heid (1995) claims that graphing tools will dramatically influence the content of school algebra. She lists six ways in which the content of algebra in a technological world will be affected by graphing tools:
1. Graphing tools allow a ready visualization of relationships.

2. They allow the accurate solution of equations and inequalities not possible through symbolic manipulation alone.

3. They provide numerical and graphical solutions that support solutions found using algebraic manipulation.

4. They promote exploration by students and their understanding of the effect of change in one representation on another representation.

5. They encourage the exploration of relationships and mathematical concepts.

6. They promote "what if" modeling of realistic situations.

   Each of the representations of a mathematical problem may, for an individual student, lend a unique insight; each technique used within a representation becomes part of a student's repertoire of problem-solving skills. Teachers can never be certain which representation is best suited for a particular student or a specific problem. Teachers should make available tools for exploring multiple representations of problems, and when possible, investigate problems with their students using different techniques (Dickey, 1993).

   Some researchers suggest that technology could play an important role in developing a richer understanding and a structural conception of function (Hollar, 1997). The graphing calculator allows for an interconnection among its various modes- data graphing, function graphing, and algebraic - and therefore it helps students to extend certain visualizations during problem situations.
Overview of the Study

In this study, the researcher conducted an experiment with undergraduate mathematics students. Although these students have taken algebra in high school and are typically proficient at most algebraic computations, they still have difficulty understanding how to solve quadratic equations using multiple representations and making connections between the different representations. Before the study began, eight students were interviewed individually and asked to perform several different tasks related to quadratic equations. As they attempted to do the tasks, the interviewer asked questions to determine what each student thought about solving the problem. After the interviews were completed, the instruction took place. The students were interviewed again following the completion of instruction to determine if and how their thinking about quadratic equations and technology use changed.

This study is different from previous studies in several ways. First, there have been numerous studies that have explored the use of technology and multiple representations in calculus instruction. This study employed technology and multiple representations in algebra instruction. Also, much research has been done on the concept of function, but not many studies have looked at quadratic equations as the particular type of functions. Third, various researchers have attempted to determine how well students solve quadratic equations. This study looked at how the students’ use of multiple representations and technology is affected by the instructors’ preferences and comfort in using the technology. Lastly, other research
has been done to determine causes of students’ mathematics anxiety and the effects on students’ achievement in the mathematics classroom (Aiken, 1972; Biller, 1996; Brassell & et al., 1980; Cooper & Robinson, 1991; Fiore, 1999; Frempong, 1998; Gierl & Bisanz, 1995; Goolsby & et al., 1987; Hadfield, Martin, & Wooden, 1992; Hauge, 1991; Hembree, 1990; Ma, 1999; Mitchell & Gilson, 1997). This study attempted to investigate whether the students’ attitudes about the use of technology will affect their anxiety in the mathematics classroom.

**Statement of the Problem**

Research into secondary and university students’, as well as teachers’, conceptions of functions has revealed numerous common misconceptions and difficulties. Techniques for understanding mathematical representations are seldom directly covered in mathematics classes and lack of this understanding underlies many of the misconceptions that impede student progress in algebra. (Brenner et al., 1995)

Examples of the conceptual difficulties identified in educational research literature are…the transition between the tabular, algebraic and graphical representations of a function (Schwarz & et al., 1990). Few students are able to identify equivalence between algebraic and graphical representations of functions, interpret graphs accurately, or develop an intuitive understanding of functions and their representations as graphs (Clement, 1985; Eisenberg & Dreyfus, 1991; Fey, 1984; Goldenberg, 1987; Goldenberg & Kilman, 1990; Harvey, 1991).
In the past, neither teachers nor students have played a role in shaping the traditional language of algebra. Traditionally, the algebraic representation was the only pathway to a solution. Since the traditional representation is not optimal for learning, it makes sense to investigate alternative representations that may be more appropriate. In teaching, we should use a particular representation for the tasks suited to it and go on to others as the pedagogical need arises. (McArthur, 1988).

Multiple representations have frequently been suggested as a way of helping students to abstract the “meaning” from mathematical manipulations. We are interested in exploring representations that facilitate dynamic problem solving, as well as static comprehension. (McArthur, 1988)

Researchers have found that whenever possible, students seem to choose a symbolic framework to process mathematical information rather than a visual one (Eisenberg & Dreyfus, 1991). One of the effects of the growing use of computers in the classroom has been to considerably increase the capacity of visual representation, and beyond, of student action on such visual representations. (Eisenberg & Dreyfus, 1989)

High school students’ algebra experience should enable them to create and use tabular, symbolic, graphical, and verbal representations and to analyze and understand patterns, relations, and functions with more sophistication than in the middle grades (NCTM, 2000).

Teachers of algebra have spent adequate amounts of time teaching two major representations of mathematics – verbal and algebraic, but have spent too
little time on two other critical representations – graphical and numerical (Dickey, 1993). The purpose of the study was to determine if and how students’ understanding of quadratic equations changed as a result of studying the concept using multiple representations. There were two dimensions to this study. The first focus was on if and how teachers’ beliefs about multiple representations and technology use will affect their implementation of the new curriculum. For this section, the following questions were addressed:

1) How will the teachers’ beliefs about technology affect their ability to implement a multiple representations curriculum?
2) How will the teachers’ beliefs about multiple representations affect their ability to implement a multiple representations curriculum?

The other focus was on if and how students’ beliefs and attitudes towards technology change as a result of studying quadratic equations using multiple (algebraic, graphical, tabular) representations. The questions addressed in this part of the study were:

1) How will the students respond to the multiple representations curriculum?
2) How will the instruction influence the students’ attitudes towards the use of technology in the mathematics classroom?
3) Will the instructor’s preferences and ease of technology usage affect the students’ preferences and attitudes towards technology usage?
4) What representation will students prefer to use to solve quadratic equations?
CHAPTER TWO

LITERATURE REVIEW

Theoretical Framework

The theoretical framework for this study was based on the theory of constructivism as a view of learning. The constructivist theory of knowledge is generally shared by all researchers in the domain of representation (Janvier, 1987). Constructivism is a theory of knowing; students construct their own mathematical knowledge within themselves. Students are not empty vessels into which teachers pour their knowledge. They come to the classroom with many different ideas, whether genuine knowledge or misconceptions, and the teacher, whether aware of it or not, will inevitably build upon these ideas. (Selden & Selden, 1992) The act of cognition is adaptive, in the sense that it tends to organize experience so it “fits” with previously constructed knowledge (Noddings, 1990; Selden & Selden, 1992; von Glasersfeld, 1987). Individual students build their mathematical frameworks from their beliefs, intuitions, and past experiences in trying to understand and make sense of the world. (Schoenfeld, 1987)

The student’s construction of knowledge is a result of psychological and physical activity on the external environment. (Bybee & Sund, 1982). Students come into a classroom with their own experiences and a cognitive structure based on those experiences. These previously formed cognitive structures can be valid, invalid or incomplete. Connections and relationships between old and new ideas must be personally drawn by the student in order for the new idea to become an
integrated part of the student’s memory. Memorized facts or information that has not been connected with the learner's prior experiences will be quickly forgotten. The students’ construction of knowledge can be assisted by using sequences of lessons designed to facilitate the process of development (Bybee & Sund, 1982). In essence, the learner must actively build new information onto his/her existing mental framework for meaningful learning to occur.

The constructivist view of the child acquiring knowledge of the world by operating upon it better fits the goals of those wishing to explain the origin of mathematical knowledge (Connell, 1998). Furthermore, this view leads to insights that are directly applicable in helping children construct their own mathematical abilities.

This study was based on the theories proposed by Piaget, Dubinsky, Hiebert and Carpenter. Piaget’s ideas of assimilation, accommodation, disequilibrium, and schema are used to interpret the students’ cognitive development. Dubinsky’s views on the construction of mathematical knowledge as it relates to Piaget’s reflective abstraction is applied to Piaget’s theory of reflective abstractive to develop a theory on the construction of mathematical knowledge. Hiebert and Carpenter’s theory of learning and understanding is used to help examine students’ abilities to use different mathematical representations and to understand the connections between these representations.

According to Jean Piaget, the cognitive structure changes through adaptation. Adaptation occurs when a child adjusts to his or her environment. A
child’s experience will have an effect on the cognitive structure as the child makes this adaptation. The two components of adaptation are assimilation and accommodation.

Assimilation is an attempt by the child to “interpret” environmental situations in terms of existing cognitive structures. (Bybee & Sund, 1982). Accommodation takes place when the child has to change his existing explanation to fit reality. Core ideas are redefined, reorganized, elaborated, changed and, in short, constructed through the continuous interaction of the individual and the environment. Assimilation and accommodation together lead to cognitive adaptation. Teaching is engaging the learner with materials that require cognitive adaptation. Teachers’ knowledge of students’ thinking is an important guide in planning effective lessons. (Davis & Maher, 1990; Maher & Davis, 1990) Teaching can benefit from the disequilibrium between knowledge taught and the ideas students bring to the classroom with them.

Piaget argued that the general processes of learning mathematics are mainly linked to the idea that the student constructs mathematical structures by abstracting the invariant features of his thinking in problem situations. The student thus develops operative structures in relation to the domain of problems constituting the subject matter of the curriculum (Schwarz & et al., 1990).

Reflective abstraction is also an important theory for this study. Reflective abstraction, as defined by Piaget,
…consists in deriving from a system of actions or operations at a lower level, certain characteristics whose reflection…upon actions or operations of a higher level it guarantees; for it is only to be conscious of the process of an earlier construction through a reconstruction on a new plane (Beth & Piaget, 1966,p. 189).

Piaget argued that the general processes of learning mathematics can be linked to the idea that the student constructs mathematical structures by abstracting the invariant features of his thinking in problem situations (Schwarz & et al., 1990). During this reflection, certain ideas may be abstracted and combined with other existing ideas, later leading to the construction of new ideas. Reflective abstraction occurs when students are constructing new knowledge by solving and interpreting problems (Porzio, 1999). The student thus develops operative structures in relation to the domain of problems constituting the subject matter of the curriculum (Schwarz & et al., 1990).

Dubinsky extended Piaget’s idea of reflective abstraction in conjunction with Piaget’s idea of schema to develop a theory about how mathematical knowledge is constructed. When students are solving problems, reflective abstraction occurs as they construct new knowledge associated with the problem and its solution. If a problem is solved successfully, then a student will somehow assimilate the problem and solution into one or more existing schema. If the problem is not solved successfully, then the student may or may not make
accommodations in existing schema to deal with the unsolved problem. (Dubinsky, 1991)

Hiebert and Carpenter developed a framework for defining students’ understanding. They theorized that knowledge is represented internally or mentally, internal representations can be connected, and internal representations, along with any associated connections, form networks of represented knowledge (Hiebert & Carpenter, 1992). Students can understand a mathematical concept or procedure if the internal representation is part of a system of symbolic knowledge. How many connections are made and the strength of these connections to this internal representation then determine the extent to which the students are able to understand. Students can then examine different representations of the same mathematical idea and construct connections between them. (Hiebert & Carpenter, 1992; Smith, 1997) Under this theory, differences in students’ abilities to use or recognize connections between multiple representations of a concept “can be analyzed in terms of differences in internal networks or represented knowledge likely to be formed by students given the type of instruction they receive.” (Porzio, 1997)

**Multiple Representations**

Representation is useful to many topics in algebra. Graphs convey particular kinds of information visually, whereas symbolic expressions may be easier to manipulate, analyze, and transform. (NCTM, 2000)
This study focused on the impact of graphing calculator use on students’ understanding of different representations (algebraic, graphical, tabular, or numerical) of algebra concepts. This approach provided students with the opportunity to create mental representations of the concepts, which could then help students form more well-connected internal networks of knowledge. Numerous researchers have investigated the effects of instruction using multiple representations of concepts.

Porzio conducted a similar study with college calculus courses in 1997. One class was considered the “traditional” calculus course because little time was spent on anything other than symbolic representations, and graphing calculators were never used. In the graphing calculator course, emphasis was placed on using symbolic and graphic representations when presenting new material and solving problems. The textbook used in the course also presented the concepts algebraically and graphically. All students were required to use graphing calculators in class, on homework, and on exams. The third course was electronic Calculus and Mathematica, in which emphasis was also placed on using symbolic and graphical representations when presenting new material and solving problems. This course emphasized the use of multiple representations generated using Mathematica.

Results from the posttests and interviews showed that the students in the graphing calculator class were able to solve problems using graphic representations but had some difficulty recognizing and making connections with the symbolic representations and lacked conceptual understanding. Several of the students, for
example, knew to take the first derivative of a function but were unable to explain why this operation was used. One important observation made during the study was that although the instructors used multiple representations when presenting concepts, the students were not given enough time for reflective abstraction necessary for them to construct their own knowledge about these representations.

The findings from the study pointed out the need for further research on the effects of instruction emphasizing use of multiple representation in the presentation of concepts. Porzio advises that instructors be sure that multiple representations and technology are not simply tacked onto the existing topics and problems, but are woven into a set of new topics and problems that emphasize multiple representations, connections between representations, and appropriate uses of technology (Porzio, 1997).

Dreyfus and Eisenberg (1982) conducted a study in which they investigated the intuitions of students for certain function concept presented in three settings – graph, table, and diagram. There were 443 pupils in the sample that were taken from grades 6 through 9 at 12 Israeli schools. They found that different groups had different intuitions. Students with high ability preferred the graphical setting for all concepts, while low ability students preferred to use the table setting. The researchers recommend that subconcepts be introduced in a graphical setting for high-level pupils and in a table setting for low-level students. (Dreyfus & Eisenberg, 1982; Keller & Hirsch, 1998)
Harvey et al. (1991) did extensive research, development, and testing of algebra software at the Education Development Center (EDC). This software uses linked multiple representations – where changes in one representation are automatically and immediately reflected in alternative representations (Harvey, 1991). This research involved the help of teachers, mathematicians, and mathematics education specialists. One tool used was Function Analyzer, which allows students to manipulate graphs by translating, reflecting, and stretching. The researchers found that students have many misconceptions about the concept of the function. In testing the software, they concluded that students can acquire significantly deeper understandings of the function concept by working more directly with carefully designed visual representations of functions. (Harvey, 1991).

LaLomia, Coovert and Salas (1988) did a study to determine whether students used tables or graphs to solve tasks. They examined problem solving as a function of display type (table/graph) and numeric function (linear/nonlinear) in four problem solving domains. Twenty-two stimulus problems were developed, and line graph and table displays were constructed for each problem. Half of the displays contained linear numeric functions and the other half contained nonlinear numeric functions. Each display was accompanied by four questions which required: (1) the location of a specific value; (2) trend analysis; (3) data interpolation; and (4) a forecasting decision. Each of the 109 subjects completed six practice and 16 experimental problems. They found that students most often used the table setting when they had to locate a specific number. They also found that
students slightly favored using a graphical setting when asked to do the interpolation and forecasting tasks. (LaLomia, Coover, & Salas, 1988). Overall, subjects performed faster and more accurately for both types of display when solving problems using nonlinear functions.

Keller and Hirsch (1998) conducted a study at Western Michigan University in which they researched students’ preferences for representations of functions. The study involved two sections of first-semester calculus, with the second section requiring a graphing calculator. Data was collected on 79 students. The graphing calculator section was supplemented with graphing calculator-enhanced activities constructed by the instructor and the graphing calculator was frequently an integral part of classroom discussion of concepts and problems. Students were given a Representation Preference Test developed by the investigators as a pre- and post-test. Other information was gathered through informal interviews with five students from the graphing calculator section. They found that students have preferences for various representations of functions and that these preferences differ between concepts presented in a context and those given as purely mathematical situations. (Keller & Hirsch, 1998) They found several factors influencing students’ preferences for representations. These factors included the nature of students’ experiences with each representation, students’ perceptions of the acceptability of using a representation, the level of the task, the context being represented, and the language of the task.
Schwarz, Dreyfus, and Bruckheimer (1990) attempted to ease some of the problems and misconceptions students have with the concept of function by integrating the triple representation model (TRM) – algebraic, graphical, and tabular - curriculum. The TRM curriculum combines open-ended activities with normal classroom activities. The major focus of attention in TRM was on “between” and “within” representation relationships. The students learned by performing tasks in the three representations, then by measuring and comparing the effects of any changes to these tasks in the various representations. The researchers found that the TRM curriculum enabled students to reach high cognitive levels in functional reasoning (Schwarz & et al., 1990).

Technology Uses in the Classroom

Much research has been done to support the use of technology in the mathematics classroom. The NCTM advocates the use of technology in mathematics education. They feel that technology should be an integral part of mathematics education in school. Because mathematics and technology are a part of many aspects of today’s world, students should have access to a full range of technological tools and the guidance of teachers skilled in using theses tools to support the learning of mathematics. Many technological tools have become vital in college and in the workplace. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.
Computer- and calculator-based graphing can change the way we teach mathematics and, more important, the way students learn mathematics. Students are able to handle far more complicated, realistic, noncontrived applications using a technologically enhanced approach. (Waits & Demana, 1988)

Microcomputer and graphing calculator technologies have evolved to the stage where they should be used routinely by mathematics students at all grade levels. However, students and teachers must make sure that a computer-generated graph of a given function is valid (Demana & Waits, 1988). Computer graphing permits students to solve quickly and effectively complex problems whose solutions are not accessible by traditional methods. Computer graphing can serve to motivate and enable students to think about more mathematics. (Demana & Waits, 1988)

A study conducted at The University of North Carolina at Wilmington used a computer algebra system (CAS) that performed the symbolic operations in algebra and calculus and plots functions. Students completed post-class questionnaires and participated in discussions with the researchers. According to the students, the ability to simultaneously see the algebraic and graphic solutions helped them to make connections. The majority of students thought the CAS was very helpful for problem solving, and they appreciated the opportunities for self-assessment provided with the technology.(Smith, 1997)

A study by Mayes (1995) compared a traditional lecture-based college algebra course to an experimental algebra course using the computer as a tool. The study focused specifically on the use of Derive, a symbolic manipulator and
graphing utility. The study involved seven sections of college algebra, four experimental sections and three control sections. Data was collected on 137 subjects: 61 experimental and 76 control. Interviews revealed that many students left the experimental sections because they were unwilling to spend additional time outside of class in the computer lab. The experimental class was exposed to the computer in class, where it was used for symbolic manipulation and graphing, as well as in the lab, where it was used for data generation and analysis. Students in this experimental group attended nine ninety-minute labs outside of class. Mayes found that students in the experimental course scored higher than students in the traditional course on final measures of inductive reasoning, visualization, and problem solving while maintaining equivalent manipulation and computation skills. (Mayes, 1995)

**Students’ Beliefs and Attitudes about Technology Use**

Technology in the classroom can have a positive impact on student learning, in terms of students’ attitudes toward school. Children can actively interact with information and receive feedback on their questions or answers, which helps to decrease their mathematics anxiety and increase their confidence in mathematics.

Piez and Voxman (1997) introduced graphing calculators into their precalculus and calculus classrooms, and also presented concepts using multiple representations. Students were shown several methods for solving quadratic inequalities. The students were then given problems to solve using a method of
their choice and asked to explain why they chose that particular method. Upon closer observation, they noticed that many students preferred a certain type of representation and that it was difficult to encourage these students to try other methods. Some reasons given were that many of the students were using technology for the first time, had limited experience with graphical representations, or had difficulty reading information from a graph (Piez & Voxman, 1997). This leads to the belief that the students' beliefs, attitudes, and proficiency in technology use contribute to their achievement in the mathematics classroom.

Furner conducted a study in which he investigated three types of anxiety: math test anxiety, number anxiety, and math course anxiety. He found a close link between test and math anxiety. Found no correlation between extent of teacher belief in the NCTM Standards and level of anxiety. No significant interaction among grade level, gender, and teacher beliefs with the exception of one significant difference: 8th grade females scored higher than 8th grade males on the anxiety measure. No correlation between teachers with more than five years of experience and those with less than five years of experience. (Furner, 1996)

Results from the student interviews: Students indicated anxiety when called to the board, made to feel bad for wrong answers, timed tests, pen and paper tests, drill and practice, use of large numbers, bored teachers, and past failures. Students liked games, projects, and group work.

While research has shown that graphing calculators may affect student achievement in mathematics, graphing calculators also have the potential to improve
students’ attitudes towards. Wilson and Krapfl’s (Wilson & Krapfl, 1994) review of the research reports that some students using the graphing calculators have indicated that the graphing calculators makes mathematics more enjoyable. Findings also suggest that graphing calculator use can diminish anxiety and increase confidence. (Hollar, 1997; Wilson & Krapfl, 1994)

Schwarz et al. (Schwarz & et al., 1990) found that, in general, the students that studied the function concept in the triple representations model curriculum had a very positive attitude. Although the homework did not require the use of technology, many students would ask for the TRM diskettes in order to use the computer for their assignments. Other students were even quoted as saying that they gained an understanding of mathematics for the first time and they liked “this kind of mathematics” (Schwarz & et al., 1990 p. 257).

Army (Army, 1992) investigated the effects of the use a graphing calculator in a college course in trigonometry. The experimental group completed pre- and post-instruction attitude surveys that were then collected and analyzed. Results showed a positive effect on student attitudes of the experimental group toward the usefulness of mathematics, the usefulness of the graphing calculator as a tool, and the usefulness of real-life applications in solving problems. (The control group did not complete the survey, thus no comparison of the groups was available.)

A study done by Alexander (Alexander, 1993) examined the effects of the use of graphing calculators and computers on students’ attitudes toward mathematics. The experimental group used the TI-81 graphing calculator and the
*Derive* graphing utility, while the control group did not use technology. Results indicated that although there was an increase in the technology group’s understanding of functions and modeling, there was no significant change in the students’ attitudes toward learning mathematics. The students that used technology did, however, comment that the technology did help them to understand the material.

Although early research indicated that the graphing calculator can influence student achievement and attitude positively, some studies are inconclusive or showed negative affects. Coston (1994) found no significant effect gain in mathematics attitude among college algebra classes that used graphing calculators, cooperative learning, or neither. Coston did find, however, that when graphing calculators are combined with cooperative learning in the treatment group, there was a significant negative difference. Coston hypothesized that the students may have been overwhelmed by having to learn the necessary mathematics and use the graphing calculators along with the cooperative learning.

Thomasson (1993) also looked at the effects of using technology on the attitude and achievement of college algebra students. The experimental group used graphing calculators, while the control group did not. Pre- and post-tests were conducted and analyzed to determine changes in the attitude or achievement of the students in each group. The researcher did not find any significant differences in attitude or achievement of the two groups.
Similarly, the previously mentioned study conducted by Mayes (1995) also did not, show an improvement in students’ attitudes towards mathematics or technology, but rather showed a slight decline. The researcher surmised that this drop may have been due to the requirement of students to spend extra time outside class in the computer lab but they did not receive any extra credit towards their grade. Mayes did believe that a change in the structure of the program would correct this problem.
CHAPTER 3

METHODOLOGY

This study follows the case study design. According to Yin (1984), the case study can be used to "describe the real-life context in which an intervention has occurred." (Yin, 1984, p. 25) The intervention in this case is the implementation of a new algebra curriculum which emphasizes the use of multiple representations and technology use.

The purpose of this study was: to investigate students’ understanding of multiple representations (algebraic, graphical, and tabular) as used to solve quadratic equations; to examine the effects of the use of these representations on the students’ attitudes toward mathematics; and to examine the effects on the students’ attitudes toward the use of technology. In particular, this study focused on examining the students’ ability to make connections between the different representations and the students’ ability to use the various forms to solve quadratic equations.

Pilot Study

The researcher, her advisor, and another colleague conducted a pilot study during the 2001 Summer I Session. The participants were students enrolled in one section of MA 101 at NC State University. The class was used to pilot test materials to be used later on with the experimental subjects. This was a way to
informally test research hypotheses about students’ understanding of the concepts when taught using multiple representations.

Three subjects were selected by the instructor to participate in the study. The pretest was administered during the first week of the semester to all students in the class. Questions were typical of those that were asked on the final exam. Four of the questions on the pretest pertained specifically to this study. The students also completed a Technology Attitude Survey and a Mathematics Anxiety Survey to determine their attitudes upon entering the class.

Interviews were conducted with the three students. The students were asked to solve quadratic equations in three different representations. Of the three students, only one was able to solve the quadratic equations using anything other than a paper-and-pencil method. The one student who was able to use the graph and the table was also the student with the highest ability of the three.

The results from the pilot study were not very conclusive but were helpful. One benefit is that the pilot led to a reworking of the pretest instrument and the interview protocol that will be needed for the actual study to take place in the fall.

Course Description

Students were enrolled in two different sections of Intermediate Algebra at a large southeastern university. All students at university were given a Mathematics Placement test before the beginning of their first semester. Students who scored below a certain cutoff were automatically enrolled in Intermediate Algebra. All
students in Intermediate Algebra must have completed Algebra II in high school, although many of them have had mathematics up to, and including, Advanced Placement Calculus. While the course does not satisfy a mathematics requirement, it is needed in order to take any successive courses.

All students enrolled in the Intermediate Algebra course have lectures, classroom discussions, do extensive work on graphing calculators, and do their homework using the Internet tool **WebAssign**. WebAssign (http://webassign.ncsu.edu), is a Web-based assignment delivery system developed by the Physics Department at the university. The system allows instructors to easily build assignments for their students to complete, with automatic delivery and grading of the assignments. The questions can be drawn from the existing database of questions and instructors can also enter new questions into the database. Course instructors are able to view grade reports for completed assignments, and compile reports across all assignments in a given course.

**Participants**

Each semester, the Intermediate Algebra is taught by Graduate Teaching Assistants (TAs) in the Department of Mathematics, Science, and Technology Education. One TA was identified as the novice teacher and the other TA was identified as the experienced teacher.

The novice teacher had limited teaching experience and was beginning a Masters program in Mathematics Education. He was not very confident in using
multiple representations and had a basic proficiency in using the TI-83 graphing calculator. The experienced TA had extensive teaching experience, a Masters degree, and was beginning a Ph.D. program in Mathematics Education. She was also very well trained and confident in using the TI-83 graphing calculator.

Eight students in their first year at the university agreed to participate in this study. Four subjects from both the control group and the experimental group were selected by the instructors to participate in the study. The instructors were asked to exclude students from the study with poor attendance and effort. Nontraditional students were also excluded from the study so that age was not a variable in the study. The participants ranged in age from 17 to 19 and included one European American female, three African American females, and four African American males. They were selected to reflect diversity with respect to race, gender, and academic performance in class. The racial and gender balance was also representative of the racial and gender composition of the two classes.

**Data Collection**

The data collection was guided by the nature of the case study. The collection methods included interviews, classroom observations, audiotape recordings, videotape recordings, and artifact reviews. All classroom observations were audiotaped and transcribed. All interviews were videotaped and transcribed. Field notes taken during the observations focused on any teacher-student interactions that indicated the uses of multiple representations during classroom
instruction. Written artifacts included the pretest, posttest, technology surveys, and observation sheets.

The pretest was administered to the eight subjects prior to instruction of the chapter on quadratic equations. Four questions on the pretest pertained to student preferences for a particular representation--- algebraic, graphical, or tabular. The last two questions required the students to solve quadratic equations using any representation they chose. The questions incorporated the different representations of the concepts taught during the unit. The posttest contained the same six questions exactly like those on the pretest. The posttest was administered to the eight student participants upon completion of the last chapter on quadratic equations.

All Intermediate Algebra students and both instructors completed a Technology Attitudes and Beliefs Survey during the first week of classes. Questions were designed to determine what students think about the use of technology --- graphing calculators in particular. The questions on the survey were based in part on Aiken’s Attitude Survey. The survey was completed using WebAssign, an online homework submission tool. The survey used a Likert scale, with responses ranging from Strongly Disagree to Strongly Agree. The responses were scored using a 1 – 5 rating scale where:

1 = strongly unfavorable to the concept

2 = somewhat unfavorable to the concept

3 = unsure
4 = somewhat favorable to the concept

5 = strongly favorable to the concept

The students completed the Technology Beliefs and Attitude Survey again during the last week of classes to determine if the students’ attitudes towards technology use changed.

Interviews were conducted at the beginning of the unit. The selected students were asked about their attitudes towards mathematics and technology and their level of experience in using the graphing calculator. The interviewees were also asked about multiple representations and solving quadratic equations. Students were interviewed again three weeks after completion of the unit on quadratic equations.

The classroom instruction pertinent to the study was during the unit on polynomial functions, with the emphasis on solving quadratic equations. The unit on quadratic equations was also observed. Both the novice and the experienced teacher were responsible for presenting the material in algebraic, graphical, and tabular representations. The instructors also explained the advantages and disadvantages to using the various representations and guided students in making connections between the forms.

The researcher made daily observations in both classes during the unit on polynomials and the unit on quadratic equations. The purpose of these observations was to document the use of multiple representations by the instructors and the students. The observer tape-recorded the classroom instruction for future analysis.
Data Analysis

Each of the Intermediate Algebra students completed the Technology Attitudes and Beliefs Survey at the beginning of the semester. The questions were coded according to how negative the question was. A student received points for each negative response to a positively worded question or for a positive response to a negatively worded question. If, for example, a student strongly agreed with a question with a negative connotation about technology, that student received 5 points for the question.

Students with high scores would potentially be identified as feeling that technology is not useful in the classroom. Students with low scores would potentially be identified as believing that technology is very appropriate for classroom use.

During the observations, the use of graphical, symbolic, and tabular representations was tabulated by counting how many times each representation was used. The tabulated results were grouped into two different categories:

Teacher-Initiated Use (TIU)--- the teacher initiates the use of a particular representation in the presentation of a concept or solution to a problem

Student-Initiated Use (SIU)--- the student initiates the use of a particular representation in the presentation of a concept or solution to a problem

The researcher also analyzed these results to see if there was any correlation between the instructors' use of representations and the students’ preference for a particular representation.
The goal of the pretest/posttest was to see if the students’ had any preferences for any of the representations and if these preferences changed after instruction. Students were also asked to solve problems that demonstrate quadratic equations in different representations: factoring algebraically, finding intercepts graphically, and finding intercepts on a table.

The interview protocol was modified from the initial sessions to encompass new questions that arose. The interviewer attempted to determine if the students felt better using the graphing calculator after the presentation of the material. The goal was to find out what each student was thinking when asked to solve the equation. The interviewer asked students why they chose a particular method and if they understood the connections. During the interviews, the students were asked to explain their reasoning for using representations while being observed and prompted by the researcher.

The scores on both surveys were compared to see if the students’ or instructors’ technology beliefs changed.
A CASE STUDY OF A NOVICE AND AN EXPERIENCED TEACHER IN THE IMPLEMENTATION OF A MULTIPLE REPRESENTATIONS CURRICULUM

Nikita Patterson
INTRODUCTION

As mathematics educators explore ways to strengthen and energize their pedagogy, and as technology becomes more common in the classroom, the use of multiple representations is gaining more attention in the mathematics curriculum. The term representation, according to the National Council of Teachers of Mathematics (NCTM), “refers to both process and to product- in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself” (NCTM, 2000, p. 67). The phrase multiple representations, as used in this study, refers to the presentation of a concept or process using tables, Cartesian graphs and equations (Confrey, 1990, as cited in Noble, Nemirovsky, Wright, & Tierney, 2001; Porzio, 1999). Dufour-Janvier, Bednarz, and Belanger (1987) concluded that multiple representations are an inherent part of mathematics; can give multiple concretizations of a concept; can help to decrease certain difficulties students have when trying to complete tasks; and are intended to make mathematics more attractive and interesting.

Also, the shift towards a technology-driven society requires that students of mathematics are able to make connections with other disciplines. With the release of the Principles and Standards for School Mathematics (NCTM, 2000), the NCTM made further recommendations for instructional programs in mathematics that enable all students to:
• create and use representations to organize, record, and communicate mathematical ideas;
• select, apply, and translate among mathematical representations to solve problems;
• use representations to model and interpret physical, social, and mathematical phenomena;
• recognize and use connections among mathematical ideas;
• understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
• recognize and apply mathematics in contexts outside of mathematics. (NCTM, 2000)

Techniques for understanding mathematical representations are seldom directly covered in mathematics classes and lack of this understanding underlies many of the misconceptions that impede student progress in algebra. (Brenner et al., 1995) Few students are able to identify equivalence between algebraic and graphical representations of functions, interpret graphs accurately, or develop an intuitive understanding of functions and their representations as graphs (Clement, 1985; Eisenberg & Dreyfus, 1991; Fey, 1984; Goldenberg, 1987; Goldenberg & Kilman, 1990; Harvey, 1991).

In this study, the researcher did a case study of a novice and an experienced teacher in the implementation of a new intermediate algebra curriculum. This reform curriculum stresses the importance of multiple representations in the
presentation of algebra concepts. The purpose of the study was to determine if and how teachers’ beliefs about multiple representations and technology use will affect their implementation of the new curriculum. “Teachers’ educational experiences – both their experiences as students and those as teachers - serve as constraints to their constructions of instructional practices (Millsaps, 2000).” The research was guided by the following questions:

1) How will the teachers’ beliefs about technology affect their ability to implement a multiple representations curriculum?

2) How will the teachers’ beliefs about multiple representations affect their ability to implement a multiple representations curriculum?

Multiple Representations

Numerous researchers have investigated the effects of instruction using multiple representations of concepts (Dreyfus & Eisenberg, 1982; Harvey, 1991; Keller & Hirsch, 1998; LaLomia et al., 1988; Lloyd & Wilson, 1998; Noble et al., 2001; Porzio, 1999; Stein, Baxter, & Leinhardt, 1990). The researchers found that multiple representations enabled students to reach high cognitive levels in functional reasoning. This approach provided students with the opportunity to create mental representations of the concepts, which then helped students to form better-connected internal networks of knowledge. The researchers found that most students were able to solve problems using graphic representations but had some difficulty recognizing and making connections with the symbolic representations and lacked
conceptual understanding. Although the instructors used multiple representations when presenting concepts, the students were not given enough time for reflective abstraction necessary for them to construct their own knowledge about these representations.

**Teachers’ Beliefs**

Many researchers have suggested that a teacher's attitudes towards and beliefs about mathematics will influence their teaching (Fennema & Franke, 1992; Kidder, 1989; Lloyd & Wilson, 1998; Millsaps, 2000; Stein et al., 1990; Thompson, 1992; Wilson, 1994). The conceptions teachers have about mathematical content can affect their ability to implement reform curricula. The word *conceptions*, as used in this study, refers to the teachers’ “general mental structures that encompass knowledge, beliefs, understandings, preferences, and views (Lloyd & Wilson, 1998).

Teachers construct their own knowledge based on the experiences they had as students and the experiences they have once they become teachers (Fennema & Franke, 1992). Kidder (1989) elaborates more on how teachers’ experiences affect their practices:

Like everyone else, teachers learn through experience, but they learn without much guidance. One problem, of course, that experience, especially of that kind that is both repetitious and disappointing, can
easily harden into narrow pedagogical theories…Current research holds that most teachers get set in their ways, both their good and bad one, after about four years of learning by experience. (p.51)

When teachers have the opportunity to reflect about their pedagogy, they become more aware of their instructional practices and any challenges they experience. Teachers may become motivated to make changes in their constructions, either to accommodate or to assimilate the experience (Millsaps, 2000). This change to their instructional practices will have an impact on their students.

**METHODOLOGY**

This study follows the case study design. According to Yin (1984), the case study can be used to "describe the real-life context in which an intervention has occurred."(Yin, 1984, p. 25) The intervention in this case is the implementation of a new algebra curriculum that emphasizes the use of multiple representations and technology use. One focus of this study was to investigate the differences and commonalities between two teachers and how these characteristics affected their implementation of the multiple representations curriculum. In particular, this study attempted to investigate the students’ understanding of multiple representations (algebraic, graphical, and tabular) as used to solve quadratic equations and their
ability to make connections between the different representations as a result of the teachers’ instruction.

Participants

This study took place at a large southeastern university. Each semester, Intermediate Algebra at this university is taught by Graduate Teaching Assistants (TAs) in the Department of Mathematics, Science, and Technology Education. One TA was identified as the novice teacher and the other TA was identified as the experienced teacher based on educational background, teaching experience, and graphing calculator proficiency.

The students used in the study were enrolled in two different sections of Intermediate Algebra at the university. All students enrolled in the Intermediate Algebra course participate in lectures, classroom discussions, use graphing calculators, and submit their homework online. Four students from each class were selected by the novice and experienced teachers, respectively, to participate in the study. The participants ranged in age from 17 to 19 and were selected to reflect diversity with respect to race, gender, and academic performance in their respective classes.

Data Collection

The data collection was guided by the nature of the case study. The collection methods included interviews, classroom observations, audiotape
recordings, videotape recordings, and artifact reviews. All classroom observations were audiotaped and transcribed. All interviews were videotaped and transcribed. Field notes taken during the observations focused on any teacher-student interactions that indicated the uses of multiple representations during classroom instruction. Written artifacts included the pretest, posttest, technology surveys, and observation sheets.

The classroom instruction pertinent to the study was during the unit on polynomial functions, with the emphasis on solving quadratic equations. Both the novice and the experienced teacher were responsible for presenting the material in algebraic, graphical, and tabular representations. The instructors also explained the advantages and disadvantages to using the various representations and guided students in making connections between the forms. The researcher made daily observations in both classes during the unit on polynomials. The purpose of these observations was to document the use of multiple representations by the instructors and the students. The observations were also helpful for the researcher to record the classroom dialogue between the instructors and the students.

Finally, both instructors were interviewed. The interviews focused on each teacher’s educational philosophy, the teachers’ beliefs and attitudes about multiple representations, and graphing calculator use. The interview protocol was slightly modified during the course of the interview to allow further probing.
Data Analysis

The novice teacher was identified by the pseudonym Joe and the experienced teacher was identified by the pseudonym Mary for purposes of confidentiality. During the classroom observations, the researcher tabulated the use of graphical, symbolic, and tabular representations. The researcher also analyzed these results to see if there was any correlation between the instructors' use of representations and the students’ preference for a particular representation.

The goal of the pretest/posttest was to see if the students had any preferences for any of the representations and if these preferences changed after instruction. Students were then asked to solve problems that demonstrated quadratic equations in different representations: factoring algebraically, finding intercepts graphically, and finding intercepts on a table.

Each student was interviewed before and after the instruction on quadratic equations. The goal was to find out what each student was thinking when asked to solve the equation. The interviewer asked students why they chose a particular method and if they understood the connections. During the interviews, the students were asked to explain their reasoning for using representations while being observed and prompted by the researcher.
RESULTS

The Case of Joe

Joe graduated cum laude in mathematics education from a large southeastern university. He is certified in grades 9-12. At the time of the study, Joe was beginning a Masters degree program in mathematics education. Joe entered the Masters program in an attempt to better prepare himself pedagogically. Joe has no teaching experience other than student teaching. He taught two sections of Intermediate Algebra, one of which was used for the study, while he attended class full-time. He has a basic proficiency in using several graphing calculators. He used a calculator for various classes on the high school, undergraduate and graduate level.

Joe feels that his philosophy of education is not yet well defined. He believes that a teacher should be pleasant, interesting, and make mathematics applicable to students by doing “real-world” problems. He likes to create a welcoming environment in his classroom. Joe says that the lecture method is not usually his preferred pedagogical strategy, although he has primarily used it for his Intermediate Algebra classes.

I admit that I have a control issue. I am not ready to give up control of my classes. I know my classes are very teacher-
centered but I do want them to one day be more student-centered.

The researcher observed that Joe’s classes are indeed very teacher-centered. The students take notes while he lectures. He does, however, have a good rapport with the students and several of the students were active participants in the class.

**Joe’s Views on Multiple Representations**

Joe states that to teach with multiple representations is to “present a concept in different media with an object such as a calculator, algebra (doing it on paper) and the Internet.” He considers multiple representations to be necessary although he clearly prefers the algebraic method.

I think the math is clearer that way – algebraically. Besides, I can still see the concept graphically by looking at the algebra.

You can’t teach mental visualizations.

Joe admits that he thinks his students prefer to solve problems algebraically because they have been conditioned that way since high school. He says he realizes that conditioning is the major factor in why he prefers the algebraic method. Because he is aware of this conditioning, he is open to learning other methods and presenting
concepts using multiple representations. He realizes that his classroom instruction could improve drastically if he were to increase his repertoire of teaching strategies.

Joe’s preference for the algebraic method is evident during his classroom instruction. The researcher observed that Joe used the algebraic method approximately 76% of the time when presenting the material and solving problems. Consequently, his students selected the algebraic method 72% of the time when answering questions and solving problems. The researcher also noted that Joe used the calculator only during the last few minutes of each 50-minute class. The calculator was introduced as a way to verify the answers that the students had produced by hand.

**Joe’s Views on Calculator/Technology Use**

The *Technology Attitudes and Beliefs Survey* was completed prior to and following the instruction. The questions were coded according to how negative the question was. Points were assigned for each negative response to a positively worded question or for a strong positive response to a negative question. There were sixteen questions on the survey; therefore, the highest possible score for a negative attitude was 80 points.

**Scale:** 64 – 80 ---- extremely negative attitude  
63 – 48 ---- somewhat negative attitude  
47 – 32 ---- neutral  
31 – 16 ---- somewhat positive attitude  
15 – 0 ---- extremely positive attitude
Table 1. Participants’ Scores on Technology Attitudes and Beliefs Survey (Joe and Students)

<table>
<thead>
<tr>
<th>Subject</th>
<th>Pre-Survey Score</th>
<th>Post-Survey Score</th>
<th>Attitude Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe</td>
<td>40</td>
<td>39</td>
<td>Remained neutral</td>
</tr>
<tr>
<td>Susan</td>
<td>32</td>
<td>30</td>
<td>Became slightly more positive</td>
</tr>
<tr>
<td>Marcus</td>
<td>29</td>
<td>25</td>
<td>Became slightly more positive</td>
</tr>
<tr>
<td>Jennifer</td>
<td>25</td>
<td>25</td>
<td>Remained somewhat positive</td>
</tr>
<tr>
<td>Nick</td>
<td>20</td>
<td>34</td>
<td>Became more neutral</td>
</tr>
</tbody>
</table>

Joe’s scores identify him as having a neutral attitude towards technology use. It is interesting to note that Joe’s attitude was less positive than the students used in the study.

Although his survey score identified him as being neutral towards technology use, Joe is reluctant to use the graphing calculator due to his lack of training. He stated that given a choice, he would not use a calculator in his class until he was more skilled. He feels that in order for him to be successful using the calculator in the presentation of concepts, he needs more support and training. The TAs participated in workshops every other week during the semester. The goal of these workshops was to familiarize the TAs with the multiple representations curriculum and how to use the calculator in the presentation of the concepts. Joe felt that these workshops were helpful but he needed even more preparation.

For successful use of the calculator and multiple representations,

I think I need more training, more structured training, maybe a
handout. I would embrace technology if I were better trained. I need to feel comfortable with the calculator and multiple representations before I can teach it to my class. I need things to be made clear.

Joe also expressed his concerns over how time-consuming it is to use a calculator with his classes. Joe feels that it is tedious to learn how to use the equipment and how to teach the students how to use the equipment. He feels that he has to spend more time teaching students how to use the equipment than teaching them the material. Joe believes that calculators should be allowed on tests and quizzes but not required. He thinks that the Intermediate Algebra class at this university forces calculator use on the students. He thinks students should be allowed to choose how they want to solve a problem. Joe feels that calculator screen shots on tests and quizzes only confuse the students. He thinks that students should not be penalized if they are not skilled at using their calculators, but should be graded based on the algebraic process they use to solve problems.

Joe’s bias towards the algebraic method and views about calculator use were evident to his students and affected their approach to solving problems. One student in his class, called “Nick” for this study, elaborated on the subject:

Researcher: If you had a choice, would you just do your whole test without a calculator?
Nick: No, no. I like to have my calculator for convenience and also to see if my answers are actually right. I’m saying I might mess up on the algebra but as far as my answer goes, it should compare with these screens up here, the table and the graph.

R: Could you do a problem without doing it by hand?

N: I could.

R: Like this problem, you could have been done really quickly if you had just looked at the table.

N: I could have, but it’s good to show work. That’s what my instructor always encourages--- showing work, showing how you got it.

And later in the same interview:

Nick: The whole point of the class is based on mathematical concepts and how you could use those. If you can manipulate the calculator, that’s good, but you have to show your work in order to manipulate the calculator. You understand? You have to know the proper procedures in order to use the calculator. You can’t just go in there and say I’m going to use the calculator for all the problems. He says show your work. What are you going to do? Write down, ‘I pressed 2nd calc and this and that’? He’s not going to understand that.
R: But if the problem gives you a quadratic equation and he wants to know the roots…

N: You can do it out by hand.

R: Or you can do it on your calculator. You don’t have to say ‘I pushed 2\textsuperscript{nd} calc’, but you could say ‘I went to the x-axis because that’s where the intercept is’. You could explain it that way because that’s what it means…

N: You could explain it but he’s saying ‘show work’.

R: What if he doesn’t say ‘show work’?

N: But my instructor always says ‘show work’.

R: Because your instructor wants it that way, then that’s the way you have to do it.

N: Yeah. If my instructor didn’t ask me to show work, then I could just easily say what you just said, but if he says show work then you can’t use the calculator for some things. You have to go in and do it out by hand first and check to see if it’s right.
It is apparent that Nick understands that there are different approaches to solving problems but in order to get a good grade he must do things the way his instructors wants.

**Example of Joe’s Classroom Instruction**

Joe’s classroom presentation of solving quadratic equations was somewhat traditional. Although he did not prefer the lecture method, this was the instructional strategy most frequently observed in his classes. Joe’s conceptions about the use of the calculator and multiple representations are reflected in his approach to teaching the lesson on solving quadratic equations. The impact he has on his students’ beliefs and attitudes can be seen during the following exchange:

Joe: *(Writes $4p^2 + p = 3$ on the board)* Let’s try this one. How do I solve this?

Student: Subtract 3 from both sides to get $4p^2 + p - 3 = 0$.

J: Right. *(Factors and uses Principle of Zero Products to get $p = \frac{3}{4}$ or $-1$).* You can also check to see what happens when you graph it.

*(Enters $4p^2 + p = 3$ and into the $Y=$ menu on the calculator.)*

*Displays graph on overhead*

S: But yours is different from mine. (Several other students agree.)

J: Why does mine look different?

S: *(No answers)*
Because you entered the equation the way it is after you subtract 3 from both sides. I actually entered the two sides of the equation into Y1 and Y2 in the Y= menu. Now what do we want to do?

Go to 2nd, calc, zero.

Do we want zero?

What are we trying to do? (Class laughs)

We actually want the intersection. (Goes through method of finding the intersection of the two graphs.)

Do we have to do all of this for the graph?

Yes, we have to find out where these two graphs intersect. We did it algebraically, now we are looking at the graph.

(Speaking to another student) Why do we have to do all of this ways? I already factored it. If one way works, stick with it!

Joe’s use of the graphical representation did not reinforce the lesson on solving quadratic equations. He began with factoring the equation to find the solution and then used the calculator as a means of checking the answer. This illustrates Joe’s lack of comfort and skill in using the graphing calculator. His students were not able to see the benefits of using the calculator. He did not effectively clear up any misconceptions or confusions the students had about solving the problem. The graphical method was seen to be cumbersome and confusing to the students, and
lead to frustration and eventually avoidance of the calculator. As a result, most of Joe’s students preferred to work all problems using paper and pencil.

**The Case of Mary**

Mary has an extensive education and teaching background. Mary has a Bachelor’s and a Master’s degree in mathematics education from a southeastern university. She was a lecturer for two years at the same university where she taught remedial mathematics, college algebra, trigonometry, statistics and Precalculus. She is a National Board Certified teacher who taught on the high school level for several years. At the time of this study, she was beginning a doctoral program in mathematics education.

During the summer of 1994, Mary attended the Technology Tools program. It was a two-week, in-depth program designed to immerse teachers in technology use. The teachers in the program used calculator-based laboratories (CBLs), microcomputer-based laboratories (MBLs), calculators, probes, Internet, MS Word, and spreadsheets. Participants developed classroom lessons as a part of the program. They also attended a follow-up program the summer of 1995.

Mary’s philosophy of education is that “every child can learn”. She believes that every child has the capacity to learn and that it is up to her as a teacher to reach every child. She believes that one method of teaching does not work for every child. She thinks that technology helps to show students the benefits of an education. Mary elaborated on her style of teaching.
I like control. I incorporate technology in order to maintain control. I like hands-on but I pose a problem and guide them to the answer. We work as a whole class.

Mary also stated that she likes to learn about the way she teaches. She feels that she needs time for reflection. She feels the need for time to sit down, reflect on her teaching, and think of ways to improve her instructional strategies. As a result, Mary is continually doing research and reading about mathematics education.

**Mary’s Views on Multiple Representations**

Mary believes that there is not one representation that will be the best for every problem or concept. She feels that the choice of a particular representation has to depend on the context of the problem.

Different problems lend themselves to different representations but, all things being equal, I can do all three. Previously, I would have said the graph was my preference. Now I make an effort to use all representations. I think different students have different preferences. As a group they don’t have a preference. The visual people tend to prefer graphs.
Mary even explained to her students that they could use any method on a test, that the algebraic method was not required for all questions.

On a test I do not expect you to show all of this work. You can look at the table, you can look at the graph; that’s fine. You’ll have problems where you have to just factor and you’re gonna have to show me the work on it. But for solving equations, I have given you several different methods; you can choose any method you want.

Her students, however, still primarily used the algebraic method when taking their tests. It is Mary’s opinion that her students’ preferences for the algebraic representation are due to conditioning. Their high school teachers always restricted calculators on tests and required them to work all problems using the paper-and-pencil method. Mary also believes that the instructions printed on the top of each test contribute to the students’ choices. Because the test instructions say show your work, the students think the only way to show work is to do the whole test algebraically. The students do not think that reporting a calculator answer is equivalent to showing work.

Mary does not see any disadvantages to teaching a curriculum that emphasizes multiple representations. She sees the problem as being a lack of consistency because other mathematics teachers often restrict the use of technology in their classes.
**Mary’s Views on Calculator/Technology Use**

Mary and her students also completed the Technology Attitudes and Beliefs Survey at the beginning and the end of the semester. Mary’s scores were relatively low compared to her students, which is consistent with the views she expressed during her interview and her instructional strategies.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Pre-Survey Score</th>
<th>Post-Survey Score</th>
<th>Attitude Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mary</td>
<td>24</td>
<td>24</td>
<td>Remained somewhat positive</td>
</tr>
<tr>
<td>Mandy</td>
<td>45</td>
<td>22</td>
<td>Became more positive</td>
</tr>
<tr>
<td>Gloria</td>
<td>36</td>
<td>26</td>
<td>Became more positive</td>
</tr>
<tr>
<td>Derrick</td>
<td>32</td>
<td>34</td>
<td>Became slightly less positive</td>
</tr>
<tr>
<td>Justin</td>
<td>30</td>
<td>26</td>
<td>Became slightly more positive</td>
</tr>
</tbody>
</table>

Mary considers the calculator to be useful in the mathematics classroom as long as you do not sacrifice skills when using the calculator. In Mary’s opinion, teachers need to be trained in how to use technology in the teaching of algebra instead of using the calculator as a way to simply check answers done using paper-and-pencil. She believes that calculators allow more connections. She feels that utilizing technology makes students stronger algebraic thinkers.
Connections are necessary for successful use of technology in the classroom. Teachers need a basis in functional connections, need a strong foundation in algebra. Many teachers need to relearn algebra with technology. They could get renewal credit or Masters degree credit.

It is Mary’s view that technology needs to be integrated into the presentation of the concepts and not as an afterthought. Mary says her views on technology use continue to change by connecting, talking, getting ideas, and reflecting together with other educators.

**Example of Mary’s Classroom Instruction**

The lesson was on solving quadratic equations. Prior to this point the class had been working on various polynomials and operations on polynomials such as addition, subtraction, multiplication, and division. Quadratic Equations were introduced as an important type of polynomial that would be used quite frequently. The students were learning different ways to solve quadratic equations. Mary emphasized the advantages and disadvantages of using each representation. She made connections between the three to allow the students to become comfortable and develop their own preferences for which representation they would choose. Mary’s efforts to implement the multiple representations curriculum can be seen in the following class discussion of solving quadratic equations.
Mary: Let’s look at this again. Let’s try this quadratic equation. *(writes on the board)* How do I solve this graphically?

Student: Plug it in.

M: Plug what in?

S: You put in \((x + 2)^2\) and \(9x\) on another line. *(referring to the Y= menu on the calculator.)*

M: Here’s my graph. *(displays graph on the overhead)*. What are you looking for here? I did what he told me, and I put \((x + 2)^2\) into Y1 and \(9x\) into Y2. What am I looking for?

S: Where they are both true.

M: Where they’re both true? What do you mean by that?

S: Where they are the same.

M: Where they *intersect*. Okay I want to pick out where they intersect, the points of intersection for the parabola and the line. How do we see it?

S: Press 2nd, calc, intersect.

M: Okay, somebody said 2nd, calc. You have to be real careful with that. The problem is only one of the intersections is on the screen. The other one is off the screen. So if you go to 2nd, calc and intersect, it’s only gonna find the intersection *on* the screen. In order to find the intersection that is off the screen, we have to set our window differently. Let’s think about that. Where do I want my x’s to start?
What about the y’s? (continues with students on how to correctly adjust the window and find each point of intersection.) So what are my two solutions?

S: 1 or 4.

M: How does the table help? Or does the table help? What am I looking for in the table in this case?

S: Where they’re the same.

M: Where they’re the same. Why am I not looking for where they are equal to zero?

S: Because Y1 is not equal to zero.

M: It’s not equal to zero! It’s equal to 9x. I’m looking for where \((x + 2)^2\) equals 9x. I just want to know where they are the same. I don’t care what they are as long as they’re equal. They’re equal two places, they’re equal right there where both of them equal 9 and right there where both of them are equal to 36. Algebraically, how do we solve that equation? It’s a much more in-depth problem.

S: Set it equal to zero.

M: You have to, first of all, FOIL that out. (continues through steps of doing FOIL to expand the binomial and solve the equation algebraically). You get the same answer \(x = 1\) or \(x = 4\).
Through this classroom exchange between Mary and her students, it is possible to see how she uses the technology during the presentation of the concept of solving quadratic equations. Mary had previously taught her students how to use the calculator to find roots to equations. They were already familiar with what it meant for the left side of a linear equation to equal the right side algebraically, graphically, and tabular. She extended this knowledge to the lesson on solving quadratic equations.

**DISCUSSION**

Research shows that teaching with multiple representations leads to a better understanding of concepts (Hiebert & Carpenter, 1992; Kaput, 1989, 1992; Porzio, 1999). Contrary to the research, however, many educators are reluctant to embrace a multiple representations curriculum. Many educators in the mathematics community believe that higher mathematics must be communicated in a nonvisual framework and this view is deeply rooted (Eisenberg & Dreyfus, 1991).

Eisenberg and Dreyfus (Eisenberg & Dreyfus, 1991) have found that one of the reasons that teachers avoid visual explanations of mathematics is a sociological one; visual is harder to teach. Teaching using multiple representations requires that the teacher rethink the way in which the material is presented. The traditional methods are no longer the most effective when making an effort to reach the diverse learners in the mathematics classroom. The novice instructor, Joe, revealed that he
shared this view. He said in his interview that he was reluctant to teach the visual representations because he was not comfortable teaching them. He realized that his traditional views on how to teach this material would not be effective with this new curriculum. He knew that he needed to rethink and relearn the content in order to teach it to his students. He found it to be a difficult task to communicate to his students about mathematical relationships. He often missed the chance to promote meaningful connections between the concepts and the different representations.

In addition to his discomfort with how to most effectively use multiple representations, Joe hesitated to use the calculator in class because he found it to be difficult to use during instruction. His hesitance was reflected in his teaching and affected his students’ choices for representation use. Joe even admitted to grading students more favorably when they used an algebraic method for solving a problem. Joe further stated that if a student were to use a calculator to solve a problem, that student would have to explain each calculator keystroke. This would be very prohibitive for students when taking tests or quizzes.

In addition to his reluctance to use the technology, the manner in which he used the technology also influenced his students’ use of the technology. Joe stated, and the researcher observed, that he only used the calculator in the last few minutes of the class. With the visual representations being used as an afterthought, the students began to see them as only a way to “check” their answers after having done the problems algebraically.
In contrast, Mary incorporated the graphing calculator into her instruction throughout the class period. She used each of the three representations almost equally (algebraic – 25.9%, graphical – 29.0%, tabular – 16.1%, and a combination of the representations 29%). Similarly, her students also opted to use the various representations to solve problems and did not totally rely on the algebraic method. It is interesting to note, however, that although her students expressed preferences for different representations during their interviews, they all primarily used the algebraic method when completing tests or quizzes. Because the instructions said to “Show your work for full or partial credit”, the students thought this meant they were required to solve the problems algebraically unless told otherwise. Joe’s students expressed the same views. They also believed that they were supposed to solve all problems algebraically.

Mary’s positive attitude, strong calculator proficiency, extensive teaching experience and beliefs played a crucial role in her implementation of the reform curriculum. Mary’s conceptions contributed to a classroom environment in which she encouraged students to employ a variety of representations and make connections among them.

CONCLUSION

It is not reasonable to expect Joe and Mary to approach the multiple representations curriculum in the same manner based on their dissimilar levels of experience and teaching styles. It is, however, reasonable to expect that Joe can
effectively implement this reform curriculum with the proper guidance and preparation. This training must include how to make connections between the various representations, and how to integrate technology into the classroom.

**Implications for Teaching**

Since the algebraic representation is not always optimal for learning, it makes sense to investigate alternative representations that may be more appropriate. These other representations, as stated earlier, are graphical and tabular. Teachers should use the representation that is best suited for certain tasks and go on to others as the instructional need arises (McArthur, 1988). Teachers may use available technological tools to assist them in the visual representations of the various concepts.

Using technology which allows for a visual display of mathematical statements does not, however, alone guarantee that students will build the mental constructs necessary to understand the mathematical statements (Cuoco, 1994). When implementing a new curriculum, teachers need to use more verbal communication. It is important that teachers learn to ask different questions in the classroom. Teacher questioning plays an important part in helping students make connections and to balance their use of verbal, tabular, graphical, and symbolic representations of functions (Driscoll, 1999).

It is also necessary for instructors to better assist their students in making the connections between the different representations. Multiple representations and
technology use must not be an afterthought when presenting topics and problems, but need to be integrated into the material at the onset. There is a need for a set of new topics and problems that emphasize multiple representations, connections between representations, and appropriate uses of technology (Porzio, 1997).
A CASE STUDY OF THE EFFECTS OF A TEACHER’S BELIEFS ON THE STUDENTS’ BELIEFS ABOUT MULTIPLE REPRESENTATIONS

Nikita Patterson
INTRODUCTION

The mathematics education community, in its call for reform, underscores the importance of mathematics instruction emphasizing the use of multiple representations in the presentation of concepts. The term *representation*, according to the National Council of Teachers of Mathematics (NCTM), “refers to both process and to product- in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself” (NCTM, 2000, p. 67). Kaput (1985) offers a preliminary definition of representation:

“In its broadest sense a representation is something that stands for something else, and so must inherently involve some kind of relationship between symbol and referent, although each may itself be a complex entity (p. 383).

The phrase *multiple representations*, as used in this study, refers to the presentation of a concept or process using numerical, graphical and symbolic viewpoints, i.e. tables, Cartesian graphs, and equations (Confrey, 1990, as cited in Noble et al., 2001; Porzio, 1999). Researchers have found that multiple representations can lead to students gaining a richer understanding of mathematical concepts and relationships (Hiebert & Carpenter, 1992; Kaput, 1989; Kaput, 1992; Porzio, 1999). Dufour-Janvier, Bednarz, and Belanger (1987) concluded that multiple representations are an inherent part of mathematics; can give multiple concretizations of a concept; can help to decrease certain difficulties students have when trying to complete tasks; and are intended to make mathematics more attractive and interesting.
With the integration of technology into the mathematics classroom, students now have more opportunities to access multiple representations (Fey, 1989; Goldenberg, 1987; Kaput, 1992; Porzio, 1999). “New forms of representation associated with electronic technology create a need for even greater instructional attention to representation.” (NCTM, 2000, p. 67). Graphing calculators, for example, have broadened students' problem-solving repertoire to include visual and numerical techniques (Embse & Yoder, 1998). One of the effects of the growing use of computers in the classroom has been to considerably increase the capacity of visual representation, and beyond, of student action on such visual representations. (Eisenberg & Dreyfus, 1989)

Although the benefits of visualizing mathematical concepts are often advocated, many students are reluctant to accept them; they prefer algorithmic over visual thinking (Eisenberg & Dreyfus, 1991). The purpose of the study is to determine if and how students’ beliefs and attitudes towards technology change as a result of studying quadratic equations using multiple (algebraic, graphical, tabular) representations. The research was guided by the following questions:

1) How will the students respond to the multiple representations curriculum?

2) How will the instruction influence the students’ attitudes towards the use of technology in the mathematics classroom?

3) Will the instructor’s preferences and ease of technology usage affect the students’ preferences and attitudes towards technology usage?
4) What representation will students prefer to use to solve quadratic equations?

**Multiple Representations**

Representation is useful to many topics in algebra. Traditionally, the algebraic representation was the only way to solve a problem. Since the traditional representation is not always the most favorable for learning, it makes sense to investigate alternative representations that may be more effective. “Different representations often illuminate different aspects of a complex concept or relationship” (NCTM, 2000, p. 69).

Numerous researchers have investigated the effects of instruction using multiple representations of concepts (Dreyfus & Eisenberg, 1982; Harvey, 1991; Keller & Hirsch, 1998; LaLomia et al., 1988; Lloyd & Wilson, 1998; Noble et al., 2001; Porzio, 1999; Stein et al., 1990). These studies have shown that students do, in fact, have preferences for which representation is best to use and that these choices normally depend on the context of the problem situation.

**Quadratic Equations and Functions**

There is recent interest in how students come to understand functions, and how well they translate among symbolic, graphical, and tabular representations of these functions. Yerushalmy and Schwartz (1993) believe that because functions are fundamental to algebra and should be taught using multiple representations.
“Because of the complex nature and manifold uses of the function concept, functional situations lend themselves to a variety of representations, including equations, graphs, tables, and verbal descriptions.” (Lloyd & Wilson, 1998, p. 251) Researchers believe that investigating a function using numerical, graphical, and analytical methods can help students gain a more thorough understanding of that function (Piez & Voxman, 1997). Research into secondary and university students—as well as teachers’—conceptions of functions has revealed numerous common misconceptions and difficulties. Techniques for understanding mathematical representations are seldom directly covered in mathematics classes and lack of this understanding underlies many of the misconceptions that impede student progress in algebra. (Brenner et al., 1995)

Examples of the conceptual difficulties identified in educational research literature are…the transition between the tabular, algebraic and graphical representations of a function (Schwarz & et al., 1990). Few students are able to identify equivalence between algebraic and graphical representations of functions, interpret graphs accurately, or develop an intuitive understanding of functions and their representations as graphs (Clement, 1985; Eisenberg & Dreyfus, 1991; Fey, 1984; Goldenberg, 1987; Goldenberg & Kilman, 1990; Harvey, 1991).

Effects of Technology Use on Student Achievement and Attitude

Much research has been done to support the use of technology in the mathematics classroom (Aiken, 1972; Demana & Waits, 1988; Mayes, 1995; Smith,
1997; Williams, 1994). The NCTM advocates the use of technology in mathematics education. They feel that technology should be an integral part of mathematics education in school. “Computers and calculators change what students can do with conventional representations and expand the set of representations with which they can work.” Because mathematics and technology are a part of many aspects of today’s world, students should have access to a full range of technological tools and the guidance of teachers skilled in using these tools to support the learning of mathematics. Many technological tools have become vital in college and in the workplace. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.

Technology in the classroom can have a positive impact on student learning, in terms of students’ attitudes toward school. Children can actively interact with information and receive feedback on their questions or answers, which helps to decrease their mathematics anxiety and increase their confidence in mathematics.

Many studies have been conducted to investigate how the technology use in the mathematics classroom affects students’ beliefs and attitudes (Alexander, 1993; Army, 1992; Coston, 1994; Furner, 1996; Hollar, 1997; Mayes, 1995; Piez & Voxman, 1997; Thomasson, 1993; Wilson & Krapfl, 1994).

**METHODOLOGY**

This study follows the case study design. According to Yin (1984), the case study can be used to "describe the real-life context in which an intervention has
occurred." (Yin, 1984, p. 25) The intervention in this case was the implementation of a new algebra curriculum that emphasizes the use of multiple representations and technology use.

The purpose of this study was: to investigate students’ understanding of multiple representations (algebraic, graphical, and tabular) as used to solve quadratic equations; to examine the effects of the use of these representations on the students’ attitudes toward mathematics; and to examine the effects on the students’ attitudes toward the use of technology. In particular, this study is focused on examining the students’ ability to make connections between the different representations and their ability to use the various forms to solve quadratic equations.

Course Description

Students were enrolled in two different sections of Intermediate Algebra at a large southeastern university. All students at this university were given a Mathematics Placement test before the beginning of their first semester. Students who scored below a certain criteria were automatically enrolled in Intermediate Algebra. All students in Intermediate Algebra must have completed Algebra II in high school, although many of them have completed mathematics up to Advanced Placement Calculus. While the course does not satisfy a mathematics requirement, it is needed in order to take any successive courses.
All students enrolled in the Intermediate Algebra course participate in lectures, classroom discussions, do extensive work on graphing calculators, and do their homework using the Web-based assignment delivery system, *WebAssign* (http://webassign.ncsu.edu).

**Participants**

Each semester, Intermediate Algebra at this university is taught by Graduate Teaching Assistants (TAs) in the Department of Mathematics, Science, and Technology Education. One TA was identified as the novice teacher and the other TA was identified as the experienced teacher. The novice teacher had no teaching experience other than student teaching, was beginning a Masters program in mathematics education, was not very confident in using multiple representations and had a basic proficiency in using the TI-83 graphing calculator. The experienced teacher had extensive teaching experience, National Board Certification, a Masters degree, was beginning a doctoral program in mathematics education and was also very well trained and confident in various forms of instructional technology.

The participants in the study were eight first-year students in their first year at a large southeastern university. Four subjects in each class were selected by their respective instructors to participate in the study. The instructors were asked to exclude students from the study with poor attendance and lack of class participation. Nontraditional students were also excluded from the study so that age was not a variable in the study. The participants ranged in age from 17 to 19 and included one
European American female, three African American females, and four African American males. They were selected to reflect diversity with respect to race, gender, and academic performance in class. The racial and gender balance was representative of the racial and gender composition of the two classes, but was not representative of the course in general.

**Data Collection**

The data collection was guided by the nature of the case study. The collection methods included interviews, classroom observations, audiotape recordings, videotape recordings, and artifact reviews. All classroom observations were audiotaped and transcribed. All interviews were videotaped and transcribed. Field notes taken during the observations focused on any teacher-student interactions that indicated the uses of multiple representations during classroom instruction. Written artifacts included the pretest, posttest, technology surveys, and observation sheets.

The pretest was administered to the eight subjects prior to instruction of the chapter on quadratic equations. Four questions on the pretest pertained to student preferences for a particular representation---algebraic, graphical, or tabular. The last two questions asked the students to solve quadratic equations using any representation they preferred. The questions incorporated the different representations of the concepts taught during the unit. The posttest contained the same six questions identical to those on the pretest. The posttest was administered
to the eight student participants upon completion of the last chapter on quadratic equations.

During the first week of classes, Intermediate Algebra students and both instructors completed a Technology Attitudes and Beliefs Survey. Questions were designed to determine what students think about the use of technology --- graphing calculators in particular. The questions on the survey were based in part on Aiken’s Attitude Survey (Aiken, 1972). The survey used a Likert scale, with responses ranging from Strongly Disagree to Strongly Agree. The students completed the Technology Beliefs and Attitude Survey again during the last week of classes to determine if the students’ attitudes towards technology use changed.

Prior to instruction of the chapter involving quadratic equations, interviews were conducted. The selected students were asked about their attitudes towards mathematics and technology and their level of experience in using the graphing calculator. The interviewees were also asked about multiple representations and solving quadratic equations. Students were interviewed again three weeks after completion of the unit on quadratic equations.

The classroom instruction pertinent to the study was presented during the unit on polynomial functions, with the emphasis on solving quadratic equations. Both the novice and the experienced teacher were responsible for presenting the material in algebraic, graphical, and tabular representations. The instructors also explained the advantages and disadvantages to using the various representations and guided students in making connections between the forms. The researcher made
daily observations in both classes during the unit on polynomials. The purpose of these observations was to document the use of multiple representations by the instructors and the students.

Finally, both instructors were interviewed. The interviews focused on the teachers’ beliefs and attitudes about multiple representations, graphing calculator use, and each teacher’s educational philosophy. The interview protocol was slightly modified during the course of the interview to allow further probing.

Data Analysis

All Intermediate Algebra students and both instructors completed the Technology Attitudes and Beliefs Survey at the beginning of the semester. The questions were scored according to how negative the question was. Points were given for each negative response to a positively worded question or for a positive response to a negatively worded question. If, for example, a person strongly agreed with a question with a negative connotation about technology, that person received 5 points for the question. Students or instructors with high scores would potentially be identified as feeling that technology is not useful in the classroom. Students or instructors with low scores would potentially be identified as believing that technology is very appropriate for classroom use. The survey was completed again at the end of the semester. The scores on both surveys were compared to see if the students’ or instructors' technology beliefs changed.
During the observations, the researcher tabulated the use of graphical, symbolic, and tabular representations. The tabulated results were grouped into two different categories:

- Teacher-Initiated Use (TIU)--- the teacher demonstrated the use of a particular representation in the presentation of a concept or solution to a problem.

- Student-Initiated Use (SIU)--- the student suggested the use of a particular representation during a classroom discussion or as a solution to a problem.

The researcher also analyzed these results to see if there was any correlation between the instructors' use of representations and the students’ preference for a particular representation.

The goal of the pretest/posttest was to see if the students had any preferences for any of the representations and if these preferences changed after instruction. Students were also asked to solve problems that demonstrated quadratic equations in different representations: factoring algebraically, finding intercepts graphically, and finding intercepts on a table.

The interview protocol was modified from the pilot study and the initial interview sessions to encompass new questions that arose. The interviewer attempted to determine if the students felt better using the graphing calculator after the presentation of the material. The goal was to find out what each student was thinking when asked to solve the equation. The interviewer asked students why
they chose a particular method and if they understood the connections. During the interviews, the students were asked to explain their reasoning for using representations while being observed and prompted by the researcher.

RESULTS

Technology Attitudes and Beliefs Survey

Each of the students in MA 101 completed the Technology Attitudes and Beliefs Survey at the beginning of the semester. The questions were coded according to how negative the question was. A student received points for each negative response to a positively worded question or for a strong positive response to a negative question. If, for example, a student strongly agreed with a question with a negative connotation about technology, that student received 5 points for the question. There were sixteen questions on the survey; therefore, the highest possible score for a negative attitude was 80 points.

Scale: 64 – 80 ---- extremely negative attitude  
63 – 48 ---- somewhat negative attitude  
47 – 32 ---- neutral  
31 – 16 ---- somewhat positive attitude  
15 – 0 ---- extremely positive attitude
Table 3. Participants’ Scores on Technology Attitudes and Beliefs Survey (Novice Teacher and Students)

<table>
<thead>
<tr>
<th>Subject</th>
<th>Pre-Survey Score</th>
<th>Post-Survey Score</th>
<th>Attitude Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe--- Novice Teacher</td>
<td>40</td>
<td>39</td>
<td>Remained neutral</td>
</tr>
<tr>
<td>Susan</td>
<td>32</td>
<td>30</td>
<td>Became slightly more positive</td>
</tr>
<tr>
<td>Marcus</td>
<td>29</td>
<td>25</td>
<td>Became slightly more positive</td>
</tr>
<tr>
<td>Jennifer</td>
<td>25</td>
<td>25</td>
<td>Remained somewhat positive</td>
</tr>
<tr>
<td>Nick</td>
<td>20</td>
<td>34</td>
<td>Became more neutral</td>
</tr>
</tbody>
</table>

Table 4. Participants’ Scores on Technology Attitudes and Beliefs Survey (Experienced Teacher and Students)

<table>
<thead>
<tr>
<th>Subject</th>
<th>Pre-Survey Score</th>
<th>Post-Survey Score</th>
<th>Attitude Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mary—Experienced Teacher</td>
<td>24</td>
<td>24</td>
<td>Remained somewhat positive</td>
</tr>
<tr>
<td>Mandy</td>
<td>45</td>
<td>22</td>
<td>Became more positive</td>
</tr>
<tr>
<td>Gloria</td>
<td>36</td>
<td>26</td>
<td>Became more positive</td>
</tr>
<tr>
<td>Derrick</td>
<td>32</td>
<td>34</td>
<td>Became slightly less positive</td>
</tr>
<tr>
<td>Justin</td>
<td>30</td>
<td>26</td>
<td>Became slightly more positive</td>
</tr>
</tbody>
</table>

The results of the survey show that none of the students have a negative attitude towards the use of technology. The results show that most of the students are very open to using graphing calculators in the mathematics classroom. The scores for the instructors are very consistent. The novice instructor has a significantly higher score than the experienced instructor. That means the experienced instructor has a more positive attitude than the novice instructor about
the use of technology in the mathematics classroom. It is also noteworthy that the novice instructor's score is in fact higher than his students' scores. The experienced instructor's score is much lower than her students' scores. There is no relationship between the students' scores on the attitude pre-survey and the teachers’ scores because the pre-survey was done before class instruction began.

**Pretest/Posttest**

The students took a pretest prior to instruction on the concepts covered and a posttest upon completion of the material. The first portion of the test featured representation preferences. Students were asked to select which representation of the problem they would prefer to use to solve the problem. The posttest was identical to the pretest. The focus was on whether the students’ preferences changed as a result of instruction.

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Graph Pretest</th>
<th>Graph Posttest</th>
<th>Table Pretest</th>
<th>Table Posttest</th>
<th>Equation Pretest</th>
<th>Equation Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe --Novice</td>
<td>1</td>
<td>2</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Mary --Experienced</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>10</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

The majority of students chose the tabular representation, then the algebraic method. Many acknowledged during their interviews that they felt the tabular method was “more precise”. Students' preferences were consistent with scores on the technology beliefs and attitudes survey. Students with lower scores on the
survey tended to use technology more. Students with more negative attitude scores chose the algebraic method more frequently than the table or the graph. The lower the student's score, the more likely that student was to select the table or the graph.

In comparing the students of the novice teacher to the students of the experienced teacher, both teachers' students seemed to prefer the tabular or algebraic method. One student from each class chose to use a graph to solve a problem on the pretest. The experienced teacher's students all had higher scores on the attitude survey and therefore selected the algebraic method more frequently than the novice teacher’s students.

The number of times students selected the graphical representation was much greater on the posttest. The number of times students chose the tabular representation remained almost the same. The difference was in the individual student choices. There was a significant decrease in the number of times students chose the algebraic method. Two students in the experienced teacher’s class showed a dramatic change in their approach to solving the problems. The student identified as ES2 changed from a mixture of algebraic and tabular methods to tabular only. The student identified as ES3 changed from a mixture of all three representations to graphical only.

<table>
<thead>
<tr>
<th>Table 6. Number of Times Certain Students Chose a Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Gloria</td>
</tr>
<tr>
<td>Derrick</td>
</tr>
</tbody>
</table>
**Instructor and Student Representation Use**

The researcher observed the instruction of both the novice teacher and the experienced teacher. Audiotapes of the classes were made. A tally was recorded each time the instructor or student initiated the use of an algebraic, graphical, or tabular representation in the classroom.

<table>
<thead>
<tr>
<th></th>
<th>Algebraic</th>
<th>Graphical</th>
<th>Tabular</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe</td>
<td>75.6</td>
<td>12.2</td>
<td>4.9</td>
<td>7.3</td>
</tr>
<tr>
<td>Joe’s Students</td>
<td>72.2</td>
<td>16.7</td>
<td>11.1</td>
<td>0</td>
</tr>
<tr>
<td>Mary</td>
<td>25.9</td>
<td>29.0</td>
<td>16.1</td>
<td>29.0</td>
</tr>
<tr>
<td>Mary’s Students</td>
<td>34.2</td>
<td>26.3</td>
<td>21.1</td>
<td>18.4</td>
</tr>
</tbody>
</table>

The results show a relationship between the uses of representation that the instructor chose and those that the students chose. Nick, a student in the novice instructor’s class, directly mentioned the novice instructor's calculator use in the interview:

**Researcher:** So your last preference is to use the graph. Do you ever use the graph?

**Nick:** The only time I use the graph is like when I have to try to find the y-intercept it will be easier to just look right here on the graph. Like I can find the y-intercept for these equations: -3
and down here it looks like positive 10. That's the only time I
would use the graph.

R: Does your instructor ever use the graph?
N: Yeah, he uses it for maximums, mins, local max, local mins,
stuff like that. Most of the time we just do it by hand or we
just use the table.

This suggests that the novice instructor prefers to use the algebraic or tabular
method for solving problems. The instructor does, however, use the graphical
method for finding maximum and minimum points on graphs.

DISCUSSION

Research shows that teaching with multiple representations leads to better
understanding of concepts. This study attempted to verify these claims. The
students appeared to increase their calculator use in both classes, especially in the
experienced teacher’s class. The students showed a marked preference for using the
available technology to solve problems on the posttest. Upon interviewing the
students, however, it was discovered that they tended to favor the algebraic
representation when taking tests or quizzes in class. This was due to several factors:
a) teacher influence, b) the students’ lack of confidence in using the technology, c)
actual wording of the test questions.
**Teacher Influence**

The most significant influence on the students’ comfort level with technology usage was the actual instructor. Teacher influence is the largest factor in how the students view using the technology. Teacher conceptions regarding multiple representations are important. The novice teacher acknowledged that he is grounded in the lecture method. He believes that students must know how to do all of the algebra by hand and use the graphing calculator as a way of checking their answers. He acknowledged that he taught everything the algebraic way and introduced the calculator at the end of the lesson, almost as an afterthought. He also stated that he stressed that his students show all of the algebraic steps when answering questions on a test or quiz.

The novice instructor’s bias towards the algebraic method was evident to his students and affected their decision to use a particular method when solving a problem. One student, Nick, in his class elaborated on the subject:

Researcher: If you had a choice, would you just do your whole test without a calculator?

Nick: No, no. I like to have my calculator for convenience and also to see if my answers are actually right. I’m saying I might mess up on the algebra but as far as my answer goes, it should compare with these screens up here, the table and the graph.

R: Could you do a problem without doing it by hand?
N: I could.

R: Like this problem, you could have been done really quickly if you had just looked at the table.

N: I could have, but it’s good to show work. That’s what my instructor always encourages--- showing work, showing how you got it.

And later in the same interview:

N: The whole point of the class is based on mathematical concepts and how you could use those. If you can manipulate the calculator, that’s good, but you have to show your work in order to manipulate the calculator. You understand? You have to know the proper procedures in order to use the calculator. You can’t just go in there and say I’m going to use the calculator for all the problems. He says show your work. What are you going to do? Write down, ‘I pressed 2nd calc and this and that’? He’s not going to understand that.

R: But if the problem gives you a quadratic equation and he wants to know the roots…

N: You can do it out by hand.

R: Or you can do it on your calculator. You don’t have to say ‘I pushed 2nd calc’, but you could say ‘I went to the x-axis
because that’s where the intercept is’. You could explain it that way because that’s what it means…

N: You could explain it but he’s saying ‘show work’.

R: What if he doesn’t say ‘show work’?

N: But my instructor *always* says ‘show work’.

R: Because your instructor wants it that way, then that’s the way you have to do it.

N: Yeah. If my instructor didn’t ask me to show work, then I could just easily say what you just said, but if he says show work then you can’t use the calculator for some things. You have to go in and do it out by hand first and check to see if it’s right.

**Students’ Technology Beliefs and Attitudes**

The material was presented to the students using multiple representations, yet many students still preferred to solve their problems using the paper-and-pencil algebraic method. This was due to the students’ mistrust of the technology. Mandy, a student in the experienced teacher’s class, elaborated on this mistrust. Although she had a very positive attitude towards the use of the graphing calculator, she had very little confidence in her own ability to use it correctly:
Researcher: If you saw this problem on a test or quiz and you weren’t given any screens but you had your calculator, how would you solve it?

Mandy: I would probably add or I’d probably subtract 16 and factor that out and then go ahead and solve it.

R: So you would do it algebraically?

M: Mm-hmm, because I don’t know how to put that into my calculator. I think that would be y equals negative x squared, like I could put that in my y= screen. But honestly if that was on a test I would just go ahead and do the equation and then probably plug it in my calculator and see if I got the right answer.

R: If you are given the screens you know how to use them, but if you have to do it yourself and put it into the calculator, then you would rather just do it by hand.

M: Yeah. If I was given the table I’d know what to do with it, but I think if I was doing a test…like, I get nervous when I take tests. I’d look at that and I’d panic. I wouldn’t know how to make sense of it.

R: If you were given the equation and just the graph, you wouldn’t know how to use the graph?

M: Yeah. Because like the tests and quizzes that we’ve taken and we’ve only had the graph, I panicked. I’d equate it out and look at my table.

R: You just draw a blank?

M: Yeah, basically. I panic when I see the graphs.

And later in the same interview:

R: In class, your instructor shows you how to do it algebraically, and using the graph, and using the table. Do you think she has a preference? Can you tell which one she uses the most?
M: She prefers to use the calculator just because she says it’s easier.

R: Which thing on the calculator do you think she prefers, the graph or the table?

M: I think that she kinda prefers to use the graph but she has taught us how to use the table. I don’t know. I kinda say the graph.

R: But when she’s talking about how to use the graph are you tuning her out?

M: (Laughs) Yeah, basically because I just haven’t gotten it. I don’t understand it. And if I understand the table and the table works for me, I’m gonna go with the table.

R: You just wait until she gets to the table and that’s when you tune back in?

M: (Laughs) Yeah. Like I try to understand it but I just get so confused, and to me there’s so many steps to remember, and I don’t trust myself. I just know I’m gonna get it wrong. And I’m afraid that if I remember all this information then something else I know down pat I’m going to become a little hazy on. So I don’t really waste the effort on remembering those when I know the table works.

R: Do you like using the calculator?

M: Now that I understand it, yeah. Like these past maybe, like, four weeks. Before I was like, “why did I spend all this money for this calculator?” But now that’s she’s taught us how to use everything, I’m comfortable using it. At the beginning of that class I hated it and wanted like a little, simple, nothing calculator. But now I prefer this calculator, because she gave us all the shortcuts. Like, we had a WebAssign due and I did the whole thing on the calculator and I had never done that. I was like, “alright, cool!” Like, I finally understand it.

R: Do you think you will use it in other classes?

M: Yeah. I won’t throw it out.
R:  Do you think calculators ought to be allowed in math classes?

M:  Yeah, definitely. Like the real one, the TI-83 or whatever. I know if I didn’t have it I’d be in trouble.

This lack of self-confidence in the ability to use the calculator and get accurate results was echoed in the novice instructor’s classroom. Nick also discussed how he felt calculator answers could be misleading:

Researcher:  So you think that if you didn’t know how to do it by hand, you wouldn’t be able to put it in your calculator. You wouldn’t be able to understand what you got.

Nick:  Yeah, I wouldn’t be able to understand what I got. I can put it in my calculator because our teacher shows us how to put it in the calculator but if I didn’t know how to do it out by hand then I would be confused about what answers I would get. I’m saying the calculator is right not all of the time. I’m saying the calculator is right most of the time, but not all of the time.

R:  The calculator’s not right all of the time?

N:  Nope. For instance, if you put in –2 and square it, it’s going to give you –4 because you didn’t put the parentheses up there. Now if my teacher hadn’t shown me how to put the parentheses up there, I wouldn’t know what to do and I would write down –4. It’s just misconceptions. People would get confused.
Structure of the Tests

The results were slightly misleading with regard to technology use due to the structure of the tests. When asked about a preference for solving problems, the students often indicated that they would choose to use the graph or table. Upon further probing, however, it was determined that they only opted to use the table when it was given to them on a test or quiz. If the table screen were not provided, they would not solve the problem using their own calculators. The students stated that they would only do the problem algebraically.

The test instructions also had an effect on the students’ use of the graphing calculator. All tests or quizzes have instructions that state, “Show all work for partial or full credit”. All students interviewed said they understood this to mean that they were required to show the work algebraically in order to receive credit. One student (ES3) explained that this often prevented him from using his calculator. He explained that if he were to use his calculator to solve a problem, he would then have to work the problem out on his test algebraically in order to receive points. In essence, he would have to do the problem twice. He stated that he usually didn’t have time to do twice the work so in the interest of time, he would only do it algebraically.

It is also important to note that one of the novice teacher’s students, Marcus, stated that he often chose to use the table but does not actually know how to use a calculator himself. As a result of classroom instruction, he knows how to find the answers if a screenshot of the table is given on a quiz or test, but does not know how
to produce the table on a calculator himself. He says he avoids graphs as much as possible, even if they are provided. It is also interesting to note that he scored relatively low on the attitude survey, indicating that he had a positive attitude towards technology use. The student is not averse to technology use; he simply does not own a calculator.

CONCLUSION

Teachers of algebra have spent adequate amounts of time teaching two major representations of mathematics – verbal and algebraic, but have spent too little time on two other critical representations – graphical and numerical (Dickey, 1993). In this study, the novice instructor acknowledged that he is more likely to stress the algebraic method when presenting a concept to his students but that he is more than willing to change his teaching style if he is provided with more training in how to teach using multiple representations. He explained that he is not comfortable teaching his students how to view a concept from the different perspectives because he does not know how to do it himself. He strongly believes that students have different learning styles and need to be taught using numerous pedagogical strategies.

This implies that teacher training is essential if reform curricula are to be properly implemented. Teachers can never be certain which representation is best suited for a particular student or a specific problem. Teachers should make available tools for exploring multiple representations of problems, and when
possible, investigate problems with their students using different techniques (Dickey, 1993). For this to be done successfully, the teachers must be well equipped with the means to assist their students in these explorations. The TA’s in this study are required to participate in sessions in which they learn more strategies for presenting the material using multiple representations. It may also be beneficial for these instructors to observe other teachers who model effective instruction using multiple representations.

The reluctance students have about visualizing mathematics may be a result of the way mathematics is presented and communicated by most teachers (Eisenberg & Dreyfus, 1989). According to this study, teachers had the greatest impact on the students’ preferences for a particular representation and ease of technology use. Once the teachers become more comfortable with the technology and the various representations, their students will also become more comfortable. It was apparent that the students in the experienced teachers’ class were more comfortable with the use of technology because she was comfortable with the technology herself. Their attitudes about technology use became more positive as a result of her positive attitude.
REFERENCES


Furner, J. M. (1996). *Mathematics Teachers' Beliefs about Using the National Council of Teachers of Mathematics Standards and the Relationship of These Beliefs to Students' Anxiety toward Mathematics* (ED406427). Tuscaloosa, AL.


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APPENDIX A: Instructor and Student Representation Use Observation Sheet

<table>
<thead>
<tr>
<th></th>
<th>Algebraic</th>
<th>Graphical</th>
<th>Tabular</th>
<th>Algebraic and Graphical</th>
<th>Algebraic and Tabular</th>
<th>Graphical and Tabular</th>
<th>Algebraic, Graphical and Tabular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor Use</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Use</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX B: Technology Beliefs and Attitude Survey

Directions: Each question on the survey expresses a feeling that a particular person has towards the use of technology in an algebra classroom. Your are to express, on a 5-point scale, the extent of agreement between the feeling expresses in each statement and your own personal feeling. The 5 points are: Strongly Agree (SA), Agree (A), Unsure (U), Disagree (D), and Strongly Disagree (SD). You are to circle the letter which best indicates how closely you agree or disagree with the feeling expressed in each statement as it concerns you.
Please give a candid response to each survey question. There is no incorrect response (blank responses do not count).

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Students should be taught how to use graphing calculators in math class.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>2. Graphing Calculators should be available for all students to use in every math class.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>3. Students should use a graphing calculator only for difficult calculations in application problems.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>4. Students should be allowed to use graphing calculators for homework and tests.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>5. Graphing calculators should only be used for checking answers.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>6. Graphing calculators should be used for graphing and analyzing functions.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>7. Students who use a graphing calculator are more likely to have difficulty learning basic algebra.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>8. Given a choice, I prefer to solve an algebra problem using paper-and-pencil.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>9. Given a choice, I prefer to solve an algebra problem using the graph feature of a calculator.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>10. Given a choice, I prefer to solve an algebra problem using the table feature of a calculator.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
<tr>
<td>11. Given a choice, I prefer to solve an algebra problem numerically using the calculator.</td>
<td>SA</td>
<td>A</td>
<td>U</td>
<td>D</td>
</tr>
</tbody>
</table>
12. Using a graphing calculator to solve problems is like “cheating”.  

13. If the instructor allows, I intend to continue using a graphing calculator in future math classes.


15. I find the use of a graphing calculator to be a distraction in learning math concepts.

16. I would recommend using a graphing calculator to my friends taking math classes.
APPENDIX C: Teacher Interview Protocol

Materials Needed: tape recorder or video camera, videotapes or audiotapes, Interview Protocol sheet.

Task 1: BACKGROUND
- Undergraduate Education
- Graduate Education
- Work/Teaching Experience
- Certifications, if any

Task 2: PHILOSOPHY OF EDUCATION
- Teaching/Education Goals
- Learning Style
- Teaching Style
- How Students Learn
- Career Goals

Task 3: TECHNOLOGY BELIEFS AND ATTITUDE
- Amount of graphing calculator experience
- Should calculators be allowed on tests/quizzes? Required?
- Level of confidence in using graphing calculator?
- Do students appear to
- Calculator use in other classes?
- Encourage students to use calculator?
- Would you use the calculator if it were not required for MA 101?
- Further probes: mental math, forcing the technology

Task 4: MULTIPLE REPRESENTATIONS
- Do you think it is necessary to teach using multiple representations?
- What does it mean to you to "teach using multiple representations"?
- What do you see as the advantages/disadvantages?
- Do students benefit from this method of instruction?
- Do you have a preference? If so, what is it?
- Does the type of problem matter?
- What do you see as your students' preference? Why?

Task 5: FURTHER PROBES
APPENDIX D: Student Interview Protocol

Materials Needed: tape recorder or video camera, graphing calculator, worksheets containing the six quadratic equation problems. Allow students 15-20 minutes to complete the worksheet.

Task 1: Point to first question pertaining to representation preference.
Say: Why did you choose this particular representation to solve this problem? Describe how you would actually use this representation to find the solution.

Possible Probe: Is this how you normally solve these types of problems on a quiz or test? Does your instructor normally use this representation for this type of problem?

Task 2: Point to the second representation preference question.
Say: Why did you choose this particular representation to solve this problem? Describe how you would actually use this representation to find the solution.
Possible Probe: Is this how you normally solve these types of problems on a quiz or test? Does your instructor normally use this representation for this type of problem?

Task 3: Point to the third representation preference question.
Say: Why did you choose this particular representation to solve this problem? Describe how you would actually use this representation to find the solution.
Possible Probe: Is this how you normally solve these types of problems on a quiz or test? Does your instructor normally use this representation for this type of problem? Does the graph mean anything to you in terms of the falling object and time? What other method could you have used to solve this problem?

Task 4: Point to the fourth representation preference question.
Say: Why did you choose this particular representation to solve this problem? Describe how you would actually use this representation to find the solution.
Possible Probe: Is this how you normally solve these types of problems on a quiz or test? Does your instructor normally use this representation for this type of problem? Does the graph mean anything to you in terms of the falling object and time? What other method could you have used to solve this problem?
Task 5: Point to question five, solving a quadratic equation. Say: Please describe what representation you used to solve this problem.

Follow-up: Describe how you solved the problem and what the solution means.

Possible probes: Why did you set up your equation that way?
Why did you choose to solve the problem this way?
Can you think of another way to solve the problem?

Task 6: Point to question five, solving a quadratic equation. Say: Please describe what representation you used to solve this problem.

Follow-up: Describe how you solved the problem and what the solution means.

Possible probes: Why did you set up your problem that way?
Why did you choose to solve the problem this way?
Can you think of another way to solve the problem?

Task 7: Probing questions:
Do you see connections between the different problems?
Do you see any connections between the different representations?
Do you think you can use all three representations for any quadratic equation?
When do you think it's best to use the table? The graph? Do it algebraically?
Why do you dislike using a particular representation?
Do you use your calculator on quizzes or tests?
Does your instructor often use the calculator?
Have you gotten more confident in using the calculator?
If a screen is given to you, do you know how to use it? If the screen is not given, which representation do you prefer?
Which representation do you think your instructor prefers?
Did you use a graphing calculator before this class?
If previously not allowed, do you agree?
$F$ is a function described by the graph, table, and equation below.

Window: $(-10,10,1,-20,20,2)$

\[ F(x) = 2x^2 - 7x - 4 \]

1. To find the value of $F(-2)$ I would:
   a) use the graph  
   b) use the table of values  
   c) use the equation

2. To find the values of $x$ when $F(x) = 0$ I would:
   a) use the graph  
   b) use the table of values  
   c) use the equation
An object is dropped off the top of a building that is 250 feet above the ground. The distance, in feet, above the ground at $x$ seconds is represented by the following equation, graph and table:

$$P(x) = -16x^2 + 250$$

3. To find when the object hits the ground I would:
   a) use the graph  
   b) use the table of values  
   c) use the equation

4. To find the time when the object will pass a window that is 150 ft above ground level I would:
   a) use the graph  
   b) use the table of values  
   c) use the equation
5. Solve using the graph, tables or equation:

\[ x^2 + x - 30 = 0 \]

Window: (-10,10,1,-50,50,10)

6. Solve using the graph, table or equation:

\[ -x^2 + 6x + 16 = 0 \]

Window: (-6,20,2,-10,30,2)