ABSTRACT

TAYLOR, AMY REBECCA. Students' and Teachers' Conceptions of Surface Area to Volume in Science Contexts: What Factors Influence the Understanding of the Concept of Scale? (Under the direction of Melissa Gail Jones.)

The *National Science Education Standards* emphasize teaching unifying concepts and processes such as basic functions of living organisms, the living environment, and scale (NRC, 1996). Since the relationship of surface area to volume is a pervasive concept that can be found throughout different sciences, it is important for students to not only understand the association of the two, but to also be able to apply it to various situations. The purpose of this study is to investigate the factors that influence the understanding of the concept of scale involving surface area to volume relationships. The first study reported here describes a pilot study with middle school participants in which the correlation between proportional reasoning ability and a student's ability to understand surface area to volume relationships was explored. The results of this study showed there was a statistically significant correlation between proportional reasoning scores and the surface area to volume posttest scores. This correlation was explored further in the second study in which middle school students', high school students', and science teachers' abilities in proportional reasoning, visual-spatial skills, and understanding surface area to volume relationships were assessed. Regression results indicated that all participants' proportional reasoning and visual-spatial scores could be a possible predictor for one's ability to understand surface area to volume relationships. Discussion of the results is followed by implications for teaching scale concepts such as surface area to volume in the science classroom.

Students' and Teachers' Conceptions of Surface Area to Volume in Science Contexts: What Factors Influence the Understanding of the Concept of Scale?

by
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DEDICATION

To my canine cheerleaders, Rustle and Izzy, who slept by me every step of the way.

To my friends, Susan, Gerrie, Jenn, Dave, Luanne, Sue, and Mary, you have been there through it ALL, always filled me with *hope*, and kept me sane.

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To my mom, thank you for believing in me and loving me my whole life through, even when I put rocks in your big red purse.

To my dad, miss you every day.

The sidewalk never ends…

BIOGRAPHY

Amy Rebecca Taylor was born in Kinston, North Carolina in July of 1971. Going to the beach and playing outdoors are the earliest memories. After moving a few times, Rocky Mount, NC became home and the place where once-in-a-lifetime friendships flourished and she was loved unconditionally by wonderful parents. After learning self-discipline as a Marching Gryphon and graduating from Rocky Mount Senior High in 1989, she attended NC Wesleyan College and graduated from East Carolina University with a BS in Science Education. Always wanting to become a teacher, she continued her education at ECU eventually completing a MA in Science Education and getting her first teaching job at the very high school she graduated from. Life eventually took her to Wake Forest Rolesville High School, where she continued to teach biology and environmental science. Upon reflection, she learned so much about herself through the teaching of science for ten years to many wonderful students. Once again, she found herself wanting to grow professionally and after long hours of tortuous decision making (thanks Tina), she decided to become a full-time student one last time! Her doctoral studies have given her many wonderful opportunities and experiences that will fill many scrapbooks (thanks to Gail and the Nano Team). She is very excited about completing her PhD in May of 2008 and to continue collecting memories of the beach by working at University of North Carolina Wilmington, where she is sure to have many visitors.

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INTRODUCTION

This dissertation is organized into two articles, each of which describes a component of the overall dissertation study. The first article describes the initial pilot study determining if proportional reasoning abilities of middle school students are correlated to the ability to understand surface area to volume relationships in a scientific context. The second article discusses the findings of a study investigating students' and teachers' conceptions of surface area to volume in science contexts by determining what factors influence their understanding of the concept of scale. A discussion of proportional reasoning, visual spatial ability, and cognitive development in relationship to implications of learning science concepts is included. In the paragraphs that follow, each article is identified by its title and a brief abstract.

CROSSROADS OF SCIENCE AND MATHEMATICS: THE INTERSECTION OF SCALE AND PROPORTIONAL REASONING

National Science Education Standards emphasize teaching unifying concepts and processes such as basic functions of living organisms, the living environment, and scale (NRC, 1996). Since the relationship of surface area to volume is a pervasive concept that can be found throughout different sciences, it is important for students to not only understand the association of the two, but to also be able to apply it to various situations. This study explored whether or not there is a correlation between proportional reasoning ability and a student's ability to understand surface area to volume relationships. Proportional reasoning scores of middle school students (N = 19) were correlated to pretest and posttest scores of an assessment which measured students' understandings of surface area to volume relationships. The instructional intervention included five consecutive days of investigations of surface area to volume relationships and applications of surface area to volume as a limiting factor in biological and physical systems. There were significant differences in a paired sample t-test from pretest to posttest for the surface area to volume assessment (t = -4.66, p = 0.000). A statistically significant correlation was found for the proportional reasoning scores and the surface area to volume posttest scores (r = 0.73, $p \leq$ 0.000). Relationships between proportional reasoning, estimation skills, visualization abilities and success in solving surface to volume problems are discussed. The implications of the results of this study for learning concepts such as magnitudes of things, limits to size, and properties of systems that change depending on volume and surface are explored.

STUDENTS' AND TEACHERS' CONCEPTIONS OF SURFACE AREA TO VOLUME IN SCIENCE CONTEXTS: WHAT FACTORS INFLUENCE THE UNDERSTANDING OF THE CONCEPT OF SCALE?

This study explored the relationships among proportional reasoning skills and visual-spatial abilities with the understanding of the scale concept of surface area to volume. The *National Science Education Standards* stress teaching unifying concepts and processes that promote connections between and among traditional scientific disciplines (NSES, 1996). Proportional reasoning ability and visual-spatial skills all contribute to learning science but the relationships among these factors has not been fully explored. This study utilizes specific assessments of visual-spatial and proportional reasoning as well as specific application tasks to explore relationships among those abilities with respect to surface area to volume ratios and to answer more specifically the following questions: 1) Is proportional reasoning ability correlated with the ability to understand the relationship between surface area to volume? 2) Is proportional reasoning ability correlated with the ability to apply surface area to volume relationships to applications in science (i.e. limiting factors)? and 3) Is visual-spatial ability correlated with the ability to understand surface area to volume relationships? Correlation and multiple linear regression analyses determined that there is a relationship between one's ability to understand surface area to volume relationships and proportional reasoning, cognitive developmental level, and visual spatial ability. Regression results indicated that all participants' proportional reasoning and visual-spatial scores could be a possible predictor for one's ability to understand surface area to volume relationships. Discussion of the results is followed by implications for teaching scale concepts such as surface area to volume in the science classroom.

CROSSROADS OF SCIENCE AND MATHEMATICS:
THE INTERSECTION OF SCALE AND PROPORTIONAL REASONING

At the heart of science education is scientific literacy, a constantly evolving term that means different things to various people. The most common meaning of scientific literacy is what the general public should know about science and an attainment of an understanding of fundamental scientific concepts (DeBoer, 2000; Reveles, Cordova, & Kelly, 2004). One of the unifying scientific themes within scale is the significant relationship of surface area to volume in biological systems, limits to size, and magnitudes of things (AAAS, 1993). However, the implications of surface area to volume relationships go beyond the standard context to wider influences in nanoscale science, protein folding, and engineering principles. The *National Science Education Standards* attempts to define the essential components of science literacy through an emphasis on teaching unifying concepts and processes which promote connections among traditional scientific disciplines such as basic functions of living organisms, the living environment, and scale (NRC, 1996). In spite of the importance of the theme of scale as a unifying concept that crosses the science domains, it has been largely ignored by teachers and curriculum developers.

Since the relationship of surface area to volume is a pervasive concept that can be found throughout different sciences, it is important for students to not only understand the association of the two, but to also be able to apply it to various situations. The relationship of surface area to volume plays a role in scientific processes such as rate of diffusion, enzymatic activity, rate of chemical reactions, cell growth, and the physics of building structures of different sizes. Ultimately, surface area to volume relationships contribute to limits to size. Organisms can grow only so large before basic biological and chemical processes no longer function, often to the demise of the organism. For example, a bird embryo gets its oxygen by diffusion through pores in the eggshell and so therefore the egg can only be so large. There comes a point when the volume of the egg exceeds the rate that oxygen can diffuse into the egg, leaving the embryo without sufficient gas exchange to survive. The example of the egg's limits to size is just one of many where the concept of scale is applied. Whether the topic is in biology, chemistry, physics, earth or space science, issues of scale are central to understanding science phenomena (Jones, Taylor, Broadwell, & Minogue, in press).

The processes of diffusion and osmosis have been studied more extensively than other processes that are linked to surface area to volume (Odom & Barrow, 2007; Odom & Kelly, 2000). Pinard and Chassé (1977) investigated the pseudoconservation of the volume and surface area of a solid object by utilizing tasks to see whether or not the conservation of one of the two (surface area or volume) of an object would wrongly induce a belief in the conservation of the other. They concluded that dissociating surface area from volume was not possible until the formal level of thinking was obtained and that it was significantly easier in tasks involving the transformation of surface area rather than of volume. Another aspect of volume was studied, (Twidle, 2006), by asking the question of whether the concept of conservation of volume in solids is more difficult than for liquids. Twidle concluded that a hierarchy of development exists within the concepts investigated where liquid volume conservation is the easiest to be mastered, followed by solid volume conservation and, finally, displacement. These conclusions are relevant to the challenges students face when contemplating the relationship of surface area to volume ratios in various scientific situations and may have implications for different instructional methods that are dependent on the cognitive level of students.

Proportional Reasoning

Surface area to volume relationships are often represented as ratios or proportions that can be compared in various situations; for example, comparing the surface area and volume of an ostrich egg to a hummingbird egg. Proportion involves the equivalence of two ratios and the reasoning ability to apply it to various situations (Hiebert & Behr, 1988). Most of studies investigating proportional reasoning have been done in mathematics education research. Jones, Taylor, Broadwell, & Minogue (in press) argued that in order for students to understand concepts of scale in science, proportional reasoning is required. These researchers found proportional reasoning was significantly correlated with students' ability to understand the powers of ten (Jones, et al., in press). Utilizing the *Test of Logical Thinking* (TOLT), Newton and Tobin (1981) suggested that students' difficulty solving proportional reasoning problems has serious implications for science instruction. Historically, proportional reasoning is based on the premise that this ability is correlated with the developmental stage of the student and according to Lamon, (1993) proportional reasoning occurs when a student can demonstrate understanding of the equivalence of appropriate scalar ratios.

There are few studies of proportional reasoning in science contexts even though it plays a significant role in the different domains of science. Hwang (1994) conducted a study focusing on proportional reasoning and self-regulation instruction on students' conceptual change in conceptions of chemical solutions. The results of the Hwang study suggested that the development of proportional reasoning ability can be used as an indicator of students' success in learning chemistry. Along these same lines, Johnson and Lawson (1998) argued that reasoning ability (which includes proportional reasoning) is a possible predictor of biology achievement. Specifically the study indicated that reasoning ability limits the achievement more than prior knowledge no matter what type of biology instruction the students received (expository versus inquiry). Westbrook and Marek (1991, 1992) investigated relationships between reasoning ability and two science concepts and found that there was (a) relationship between reasoning ability and understanding homeostasis, and (b) no relationship of reasoning ability with understanding diffusion concepts.

This present study explored students' understandings of the relationship between surface area to volume and its application in various contexts and investigated the research question: Is there a correlation between proportional reasoning ability and a student's ability to understand surface area to volume relationships?

Methodology

Participants

The participants in this exploratory study were middle school students who attended a five-day science summer camp offered at an informal science center at a large state university located in the southeastern part of the United States. Students (N=19) ranged in age from 11 to 13 years old and included nine females and ten males. Sixteen students were Caucasian, one African American, and two were other ethnicities.

Assessments

Students' proportional reasoning ability was assessed by the *Proportional Reasoning Assessment Instrument* developed by Allain (2000) for middle school students. This assessment was comprised of 10 open-ended items of various difficulty levels. This instrument, in addition to measuring proportional reasoning ability, also revealed common patterns of thinking among the students in the sample. This assessment was examined by a panel of mathematics researchers for validity (criterion and content) and reliability (coefficient-alpha and inter-rater reliability). The reliability coefficient was $\alpha = .71$ and the criterion validity was $r = .69$.

The *Applications of Surface Area to Volume Assessment* (ASAVA) instrument was developed to assess students' understanding of surface area to volume relationships (see Appendix). The instrument consisted of a pretest and posttest with comparable test items and was administered in a group setting. Each test consisted of twelve items including multiple choice, problem solving, explanations, and diagrams. For questions one through six, students were given two wooden cubes with different surface area to volume ratios in which they were asked to indicate which cube had largest surface area versus volume as well as how to calculate the ratio. Assessment items (seven through twelve) assessed students' abilities to apply the relationship of surface area to volume to specific situations dealing with organisms, environments, or experiments. The content validity was established by review from a panel of six experts which included science educators, scientists, and students. After the pretest was administered, the participants completed activities for five consecutive days about the scale of organisms, surface area to volume relationships and its application as a limiting factor (Table 1). On the last day of the science camp, the students were administered the posttest.

Table 1
Descriptions of Surface Area to Volume Investigations

Surface Area to Volume Investigations	Description
Sugar Size	Using raw, granulated, powdered sugar to investigate how surface area affects how it adheres to a surface
Could they really exist?	Debating about why certain creatures could or could not exist. Such as King Kong or giant insects
Surface area to volume of cells	Exploring how surface area to volume as a limiting factor in the size of cells using paper cubes, potato/agar cubes
Insect building competition	Building giant insects with straws, paper clips, etc. in order to show how organisms can only be so big as a result of capability to support mass

Analyses

Student responses to the items on the *Proportional Reasoning Assessment Instrument* were scored for accuracy as well as whether or not an appropriate problem-solving strategy was utilized according to the protocol developed by the authors (Allain, 2000). The *Applications of Surface Area to Volume Assessment* (ASAVA) pretest and posttest were scored for accuracy. Correlations were determined for the proportional reasoning score and the pretest and posttest scores. These scores were analyzed using multiple linear regression. A paired t-test was used to compare pretest and posttest scores. Three assessment items on the ASAVA required students to apply their knowledge to specific contexts (cube surface area to volume as a function of size, surface area to volume in fish gills and rates of diffusion in other biological contexts). Students were asked to explain their reasoning for their responses. The responses were qualitatively analyzed for emergent themes about patterns of reasoning about surface area to volume applications. The inter-rater reliability for the scores on the open-ended items was determined to be 95%.

Results

The results of this study revealed there was a significant correlation between proportional reasoning ability and understanding of surface area to volume relationships. The mean score on the *Proportional Reasoning Assessment* for the 19 participants was 60.84 (SD = 24.48). The mean score on the pretest for the *Applications of Surface Area to Volume Assessment* (ASAVA) was 54.42 (SD = 20.41) whereas the mean score for the posttest increased to 75.89 (SD = 19.71). Results of a paired sample t-test suggested that the significant changes in pretest and posttest for the *Applications of Surface Area to Volume Assessment* (ASAVA) were not due to random chance but instead are probably due to the intervention the students received as a result of completing the surface area to volume application tasks ($t = 4.66$, $p = 0.000$). Proportional reasoning scores were significantly correlated with the surface area to volume posttest scores ($r = 0.73$, $p \leq 0.000$). A high correlation of $r = 0.62$, $p \leq 0.005$ was determined for the surface area to volume pretest and the proportional reasoning score. The results of the multiple linear regression analysis for the three scores (*Proportional Reasoning Assessment* and pretest and posttest of *Applications of Surface Area to Volume Assessment*) indicated that the proportional reasoning score could be a possible predictor for one's ability to understand surface area to volume relationships.

See Table 2 for complete results of the multiple linear regression analysis.

Table 2
Multiple linear regression results
Parameter estimates:

Variable	Estimate	SE	t	p
Intercept	38.28	10.13	3.78	0.00
Proportional Reasoning	0.55	0.17	3.19	0.01
Pre ASAVA	0.07	0.21	0.36	0.73

Dependent Variable: Post ASAVA
Independent Variable(s): Proportional Reasoning Score, Pre ASAVA

Students' reasoning about surface area to volume applications

Students were asked to explain their rationale for which cube (A, B, or C increasing in size) had the greatest surface area to volume ratio on the *Applications of Surface Area to Volume Assessment*. Students who gave the correct answer to this question (in both pre and post assessments) demonstrated an understanding of ratios by giving their answers in ratio format (complete with correct units of measurement) and by discussing the two measurements (surface area and volume) comparatively. For example, one student stated that the ratio for cube C was (1.5:1), cube B was (2:1), and cube A was (6:1), so therefore cube A's ratio is the largest. Some of the students who incorrectly answered the question stated that the largest cube automatically accounted for the larger surface area to volume ratio. Other students indicated that they did not know how to determine the relationship or said that they all had the same surface area to volume ratio. Percentages of correct responses to the increasing cube size item increased 32% from pretest (M = 21) to posttest (M = 53).

Students' ability to apply their knowledge of surface area to volume was assessed when students were asked to predict which type of fish gill (from little branching to highly branched) would absorb oxygen at a greater rate. Correct explanations included that the fish with the highly branched gills would "have more room for oxygen to come in", "has more wrinkles for it to take in oxygen faster", and that it "has the most surface area." Incorrect explanations to this assessment item included that fish A (incorrect) would absorb more oxygen because it is "solid," has more "mass," has more "surface area," or "absorption would occur at same rate." Percentages of correct responses to this item increased 58% from pretest (M = 32) to posttest (M = 90).

Students were also asked to apply their knowledge in four situations involving two different size potato cubes (pretest) or egg slices (posttest) and to predict the absorption rate of liquid. The correct answer given by students was that the largest cube should absorb at the same rate as the small one but it would take longer for the large one to absorb liquid completely. This type of answer is indicative that the students understand that rate of absorption may be the same and that surface area to volume plays a role in how fast the completion of absorption occurs. In summary, proportional reasoning was highly correlated with achievement on the test of knowledge of surface area to volume. Students' knowledge of surface area to volume increased significantly from the pre to post instruction and students were more likely to accurately apply this knowledge to science problems in different contexts.

Discussion

This exploratory study examined students' understandings of the relationship between surface area to volume and its application in various situations. The results of this study indicated that proportional reasoning ability is not only correlated to students' ability to understand surface area to volume relationships but may be a possible predictor for how well a student can comprehend this scaling concept. As a result of this study, three questions emerged as important for further study: (a) What developmental level is necessary for students to fully understand the relationship between surface area and volume? (b) What type of background knowledge is necessary for students to understand surface area to volume relationships and how does it affect the sequence of science instruction? and (c) What other constructs may be involved in understanding of scale applications such as limits to size and surface area to volume relationships?

Proportional reasoning has been linked in other studies to students' higher levels of cognitive thinking (Clark & Kamii, 1996; Karplus, Adi, & Lawson, 1980; Khoury, 2002; Piaget & Inhelder, 1967; Siegler, 1998). Surface area to volume relationships like those assessed in the present study are types of proportions. Previous research suggests that only after students attain multiplicative proportional reasoning ability level, are they then able to understand surface area to volume ratios. Unlike Westbrook and Marek (1991) who found no relationship between reasoning ability and concepts of diffusion, the present study found significant correlation between a related science concept (surface area to volume ratios) and proportional reasoning ability. One possible interpretation of the difference in findings could be that the concept of diffusion demands less abstract reasoning than applying surface area to volume relationships in science contexts.

Researchers now recognize that cognitive developmental stages are not as radical and stage-like as once thought (Siegler, 1998) and levels of proportional reasoning ability are somewhat fluid as a student transitions through phases of how they approach a problem (i.e. use of additive or multiplicative strategies). Different students may approach a proportional reasoning problem with different strategies and perhaps get the same answer, but the thought processes may reflect different levels of reasoning. This variation in strategies was seen in the results of the present study when students were asked to choose the greatest surface area to volume ratio from different size cubes. Some students intuitively chose the largest cube which is actually an incorrect answer. Only students that possessed an understanding of how the two measurements (surface area and volume) related to each other proportionally were able to accurately apply the concept to a specific scientific context. Students at a transitional level of understanding may be able to calculate surface area and/or volume, but may or may not be able to apply and explain the concept to a scientific context such as diffusion of oxygen into eggs or expelling of waste products from a cell. Higher cognitive skills are required for students to be able to take the mathematics concept of surface area to volume ratio and fuse it with a scientific concept such as diffusion to make predictions in specific contexts. In order to fully accomplish this, students must make the transition from concrete understanding of surface area to volume to a more abstract understanding. For example, students were asked to study a picture of a narrow, deep lake and a picture of a shallow, wide lake. Then they were asked to determine which lake would heat up faster. In order to answer this particular question, the students not only had to understand surface area to volume relationships but to apply it to heat transfer. If these results can be replicated and if findings show students must attain a certain level of proportional reasoning ability before they can comprehend surface area to volume relationships, then a resequencing of the science curricula may be necessary.

Historically, mathematics concepts are taught separately from science. The weaving of math and science together may have benefits for both. Biology instruction tends to involve limited mathematics yet many of the surface area to volume relationship applications are linked to biological examples such as surface area of intestines, lungs, or fish gills and the effects they have on the rate of diffusion. For the past century, most American high school students have learned science using a sequence with biology at or near the beginning, and physics at the end (Ewald, Hickman, Hickman, and Myers, 2005). Since proportional reasoning and surface area to volume relationships may require a higher level of cognitive development, could this mean that biological concepts should come later in sequence for science students? Sheppard and Robbins (2002) have noted that the trademark of U.S. science education is the teaching of high school science in the fixed order: biology, then chemistry, and finally physics. The call from some physics educators is that students can and should take physics, a more mathematically-oriented and concrete science, earlier in the course sequence. Ewald et al. (2005) also argued that not only is biology more meaningful to students who have an understanding of chemistry and physics, but that taking physics early may propel them into more challenging science courses in the following years and developing better understandings of concepts like surface area to volume to limits to size.

Other factors like estimation skills or visual-spatial skills could affect an individual's ability to understand and apply surface area to volume ratios. Just like proportional reasoning, estimation is an important skill needed in everyday activities. Estimation ability could affect how a student approaches a surface area to volume problem, especially if that student is not cognizant of formulas that may be used to calculate the ratios. Forrester and

Shire (1995) utilized a three-dimensional "volume" task to investigate how the influence of object size, dimensionality, and prior context affected children's estimation abilities. The results from estimation tasks also drew attention to specific difficulties children seem to have in manipulating more than one spatial domain. It is not fully understood how visual-spatial skills influence a student's ability to conceptualize surface area to volume relationships.

Visual-spatial ability pervades all human experience, from the frontiers of science, to everyday tasks like packing a suitcase (McCormack, 2002). Visual solutions to problems have frequently been recounted by inventors and scientists, but only recently has it been acknowledged by science educators (McCormack, 2002). Thinking with images plays a central role in scientific creativity (graphics, metaphors, and unifying ideas/themes) and the need for visual-spatial cognition in the science classroom is necessary (Mathewson, 1999). As students in the present study completed the *Applications of Surface Area to Volume Assessment* (ASAVA), they had to visualize how the surface area and volume measurements were related to each other in order to answer the questions. For example, students were asked which primate (pygmy monkey, gorilla, lemur, or spider monkey) would have a greater rate of heat loss if placed in a cold environment. Height and weight information for each primate were provided and the students had to understand how size of primates (greater surface area and less volume) contributes to a higher rate of heat loss due to greater surface area and lesser volume. A study by Battista and Clements (1996) explored spatial visualization of three-dimensional cubes and the development of students' meaningful enumeration of cubes in 3-D arrays. The results of the study by Battista and Clement indicated that students' understanding of the measurement of volume is far more complex than previously thought.

Further research is needed to document how surface area and volume are conceptualized as well as applied in science contexts. The role of visual spatial skills in the development of surface area to volume concepts is not known. It is also not clear how different components of cognitive development contribute to a students' understanding of surface area to volume concepts. In conclusion, this study showed that proportional reasoning is positively correlated to students' understanding of surface area to volume relationships and that instructional intervention promotes knowledge of surface area to volume ratios in science contexts. Through fully documenting the trajectory of learning that takes place for surface area to volume concepts, educators can more effectively design instruction that can teach students the unifying concept of scale that underlies the domains of science.

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Appendix

Applications of Surface Area to Volume Assessment
(ASAVA)

Pre-Assessment Questions

For each question below please give what you believe to be the correct response. In some cases you might find the questions to be challenging so try not to get frustrated. Just do your very best and please do not leave any questions blank and give us your best explanation.

Observing the cubes labeled A, B, and C, answer questions 1-6.

1. Which of the following cubes has the largest total surface area?
   a. Cube A
   b. Cube B
   c. Cube C

2. Which of the following cubes has the largest total volume?
   a. Cube A
   b. Cube B
   c. Cube C

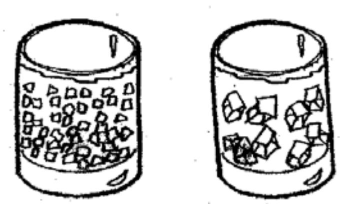
3. Scientists often look at the sizes of organisms and determine the relationship between surface area to volume. Which Cube (A, B, C) has the greatest surface area to volume ratio? (Hint: No calculators are necessary to answer this question.)

4. Explain your reasoning for the answer to question 3.

5. Calculate the total surface area for cube C.

6. Calculate the total volume for cube C.

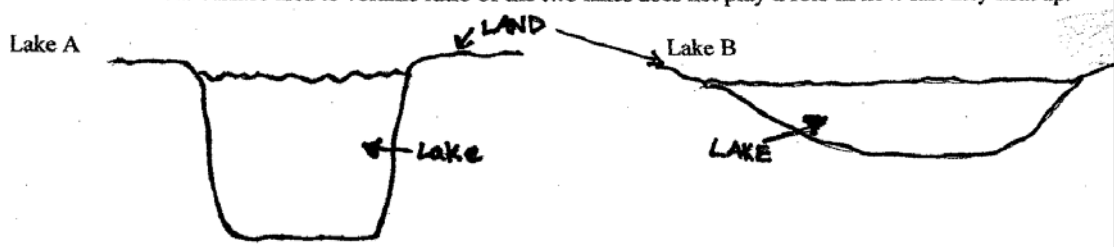
7. On a hot summer day, Sally was pouring glasses of water for her friends after playing volleyball. She made some glasses of water with crushed ice and some with cubed ice. Which glass of water would get colder **faster**?
   a. A glass of water with 5 large ice cubes
   b. A glass of water with crushed ice made from 5 large ice cubes
   c. A glass with no ice at all



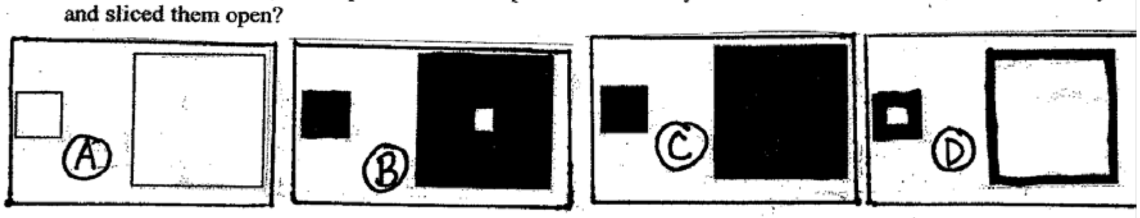
8. Fish need oxygen just like humans and they must obtain the oxygen as water moves through the fish's gills. Fish gills come in a variety of shapes and sizes depending on the type of fish and type of environment it lives in. Which type of gills would absorb oxygen for the fish at a greater rate? **Explain your reasons for your answer.**

9. Organisms on Earth exist in a variety of shapes and sizes. Consider the cells of a 33-foot anaconda snake and a seven-inch worm snake. Which of the following statements is true?
   a. The cells of the anaconda are ten times larger than those of the worm snake.
   b. The worm snake has fewer but much larger cells than the anaconda.
   c. Cells of each snake are about the same size.

10. If there were two lakes of equal volume but different shapes (see sketches A and B). Which statement would be true about which lake would heat up faster?
   a. Lake A would heat up faster because it is deeper (less surface area exposed to the sun's rays).
   b. Lake B would heat up faster because it is shallow (more surface area exposed to the sun's rays).
   c. Lake A and Lake B would heat up at the same rate because they have the same volume.
   d. The surface area to volume ratio of the two lakes does not play a role in how fast they heat up.



11. Anita was conducting an experiment with peeled potatoes and discovered that when she soaked the cubes in dye, they absorbed the dye. If Anita placed two potato cubes of different sizes in the dye for the same amount of time, which picture would represent an unlikely outcome once she took them out of the dye and sliced them open?



12. Explain your answer.

Applications of Surface Area to Volume Assessment
(ASAVA)

Post-Assessment Questions

For each question below please give what you believe to be the correct response. In some cases you might find the questions to be challenging so try not to get frustrated. Just do your very best and please do not leave any questions blank and give us your best explanation.

Observing the cubes labeled A, B, and C, answer questions 1-6.

1. Which of the following cubes has the largest total surface area?
   a. Cube A
   b. Cube B
   c. Cube C

2. Which of the following cubes has the largest total volume?
   a. Cube A
   b. Cube B
   c. Cube C

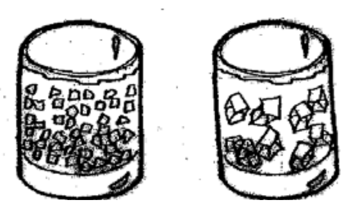
3. Scientists often look at the sizes of organisms and determine the relationship between surface area to volume. Which Cube (A, B, C) has the greatest surface area to volume ratio?

4. Explain your answer to question 3.

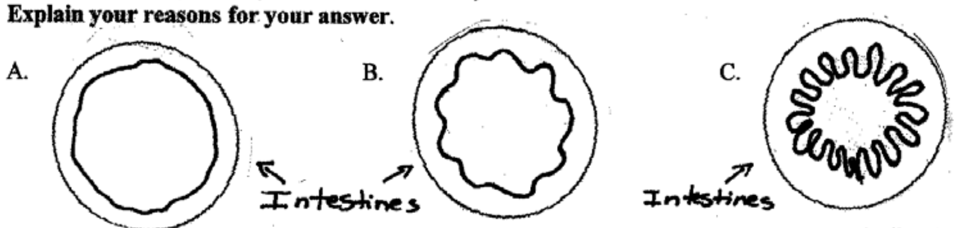
5. Calculate the total surface area for cube C.

6. Calculate the total volume for cube C.

7. On a hot summer day, Sally was pouring glasses of water for her friends after playing volleyball. She made some glasses of water with crushed ice and some with cubed ice. Which glass of water would stay colder **longer**?
   a. A glass of water with 5 large ice cubes
   b. A glass of water with crushed ice made from 5 large ice cubes
   c. A glass with no ice at all



8. Which of these intestines (shown in cross-section) would absorb the most nutrients from food? **Explain your reasons for your answer.**

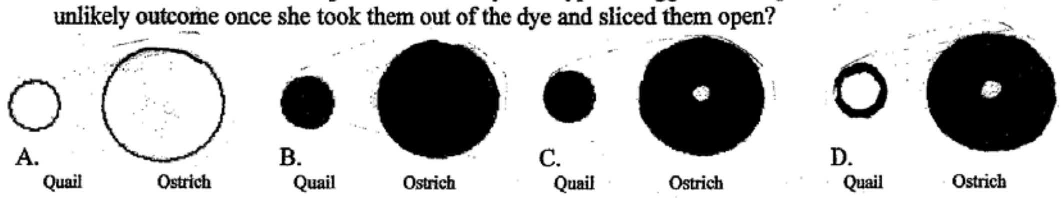


9. Tapeworms are flatworms that do not have circulatory systems and live in the intestines of animals. They must absorb nutrients directly through their surface from their surroundings in order to survive. Which of the following statements is true?
   a. Tapeworms could easily grow to the length and width of a freight train as long as there are some nutrients surrounding it
   b. Tapeworms can grow to a great length but not in width if surrounded by nutrients
   c. Tapeworms are limited to less than 10 inches in length regardless of nutrients.

10. Primates exist in various sizes and weights. Which primate would have a more difficult time staying warm in the winter based on just it's size? **Explain your answer to the right of the chart.**

PRIMATE TYPE	HEIGHT	WEIGHT
 PYGMY MONKEY	15 in. tall	1.25 pounds
 GORILLA	60 inches tall	500 pounds
 LEMUR	40 inches tall	10 pounds
 SPIDER MONKEY	24 inches tall	20 pounds

11. Sarah wanted to make purple hard-boiled eggs for a summer salad. She chose ostrich eggs (150x135 mm) and quail eggs (10x20 mm) to include in the salad. Both types of eggs were boiled, shelled, and placed in purple food coloring for the same amount of time. When she sliced the eggs in half to make the salad she noticed that the food coloring was absorbed by both types of eggs. Which picture would represent an unlikely outcome once she took them out of the dye and sliced them open?



12. Explain your answer to number 11.

STUDENTS' AND TEACHERS' CONCEPTIONS OF SURFACE AREA TO VOLUME IN SCIENCE CONTEXTS: WHAT FACTORS INFLUENCE THE UNDERSTANDING OF THE CONCEPT OF SCALE?

The nascent nanotechnology revolution promises many benefits to humankind and a remarkable convergence of scales looms large on the horizon (Schattenburg & Smith, 2001). At the small scale of nanoscience, advancements in medicine and engineering promise a technology that borders on science fiction. Imagine instead of having a cancer patient's entire body affected by chemotherapy, that nano-sized gold beads or nanoemulsions could deliver treatment directly to the diseased tissue? Whether medicinal nanoemulsions, environmental nanosensors, or engineering of biological fibers, the concept at the core of these discoveries is the relationship between surface area to volume (Hood, 2004; Hooker, 2002; Ma, Kotaki, Inai, & Ramakrishna, 2005; Moraru, Panchapakesan, Huang, Takhistov, Liu, & Kokini, 2003; Shattenburg & Smith, 2001). The present study explored how students conceptualize this critical aspect of scale, surface area to volume relationships. Specifically, the relationships among students' proportional reasoning skills, visual-spatial abilities, and understandings of surface area to volume relationships were examined.

Science impacts the lives of people in diverse ways; therefore, it is critical for students to acquire progressive degrees of scientific literacy in order for them to learn how scientific knowledge is generated, interpreted, and reinterpreted (Reveles, Cordova, & Kelly, 2004). Connected to the discourses of science are the understandings and use of science concepts, habits of mind, critical thinking, and epistemological commitments comprised in conceptions of scientific literacy (AAAS, 1993; DeBoer, 2000). The *National Science Education Standards* stress teaching unifying concepts and processes that promote connections between and among traditional scientific disciplines (NSES, 1996).

Three important themes the National Research Council has identified for science instruction include scale, basic functions of living organisms, and the living environment. As a benchmark of scientific literacy, scale includes such concepts as magnitudes of things, limits to size, and properties of systems that change depending on volume and surface processes; these concepts are included in some fashion at all grade levels (AAAS, 1993). Scale is a pervasive idea throughout many of the unifying concepts in the *National Standards* and is referred to as an understanding that different characteristics, properties, or relationships within a system might change as its dimensions are increased or decreased. In the present study, the study of scale is informed by national (AAAS, 1993; NSES, 1996) and state science standards' (NCDPI, 2004) recommendations as to what middle school and secondary students should know such as cellular structure and functioning which includes how surface area to volume limits size of cell as well as applications to physical sciences.

In a world of technology that has made exploration possible from the tiniest nanoparticle to stars billions of miles away, science students, more than ever, need an understanding of scale and the processes that are affected by changes in scale. Different properties are not affected to the same degree by changes in scale, and large changes in scale typically alter the way that things work in physical, biological, or social systems (AAAS, 1993). Differences in scale are governed by similar physical principles. McGowan (1994) has argued that one of these principles, the relation between length, surface area, and volume, plays an all-pervading role in living things, just as it does in the inanimate world.

The importance of students' understandings of these concepts is paramount to understanding the underlying principles of science. One such example a student might encounter in the life sciences is that of surface area to volume in relation to diffusion and osmosis. Living cells must absorb nutrients and expel waste products through cell transport at a rate that is sufficient for survival and as a consequence the size of the cell is limited. Cell size is limited because as the cell increases in size, its volume increases much faster than its surface area. Without sufficient passage of nutrients into and expulsion of wastes out of the cell, through the cell surface, the cell will die. Since the relationship of surface area to volume is a persistent concept that can be found in various science domains, it is vital for students to not only understand the connection of the two, but to also be able to apply it to scientific contexts. The relationship of surface area to volume plays a role in scientific processes such as cell growth, efficiency of diffusion, enzymatic activity, and the support of physical structures of objects at different sizes. Ultimately, this relationship of surface area to volume plays a role in restricting limits to size.

An organism's size has an integral effect on basic biological and chemical processes. For example, a bird embryo gets its oxygen by diffusion through pores in the eggshell and therefore the egg can only be so large. Compare an ostrich egg (largest known egg on Earth) with a quail egg. Which egg has the largest surface area to volume ratio and how does this affect its rate of oxygen uptake? There is a direct correlation of size and how much time and oxygen it would take for the gas to be dispersed throughout the ostrich egg as compared to the quail egg. These are factors that limit the size of an egg. In this example, students must take into account implications of scale, basic functions of living organisms, and the non-living environment.

How do students understand scale relationships, and what cognitive abilities are necessary for learning these concepts? Are there other factors such as proportional reasoning and visual-spatial abilities that plays a role in an individual's understanding of the concepts found within scale, including surface area to volume ratio?

Understandings of scale

There are few studies that examine scale applications and of those that exist most focus on diffusion and osmosis. Odom & Barrow (2006) used two instruments, *Diffusion and Osmosis Diagnostic Test* (DODT) and the *Certainty of Response* (CRI) scale, to investigate high school biology students' knowledge and certainty about diffusion and osmosis concepts. Results of this study revealed that a significant number of students had misconceptions of the content knowledge. An earlier study by Odom & Kelly (2000) explored the effectiveness of concept mapping, the learning cycle, expository instruction, and a combination of concept mapping/learning cycle in promoting conceptual understanding of diffusion and osmosis. Their findings indicated that the concept mapping groups outperformed the expository group in conceptual understanding of the concept.

Another study exploring diffusion and osmosis focused on how to enhance student learning outcomes by considering the conceptions and prior experience brought to the learning situation (Panizzon & Bond, 2006). This study used the Partial Credit Rasch Model to compare and contrast the conceptions of diffusion and osmosis held by different groups of science students and found that conceptual understanding was independent of the student's level of education. This model was developed for use in school and university learning environments with a focus on the structure of students' responses after a learning experience (Boulton-Lewis, 1998). Results from the analysis indicated that students used similar cognitive processes in the development of conceptual understanding independent to the level of education (Panizzon, 2006).

Thus, the studies suggested that students at a range of educational levels struggle with developing an understanding of diffusion and osmosis, one concept dependent on scale properties. These studies investigated this concept in conjunction with misconceptions, prior experiences, problem solving ability as well as the use of visuals such as concept mapping and concluded that various factors affect students' understanding of diffusion and osmosis.

There are few studies that have examined the conceptual understanding of surface area to volume. One study by Pinard and Chassé (1977) examined the pseudoconservation of the volume and surface area of a solid object. Students' beliefs were tested to determine if they understood that conservation of surface area of an object would impact the conservation of the volume of the same object or visa versa (Pinard & Chassé, 1977). Students from six grade levels completed four different tasks involving models of varying surface areas and volumes. Pinard & Chassé found that dissociating surface area from volume was not possible until students reached the formal level of thinking. In addition, it was significantly easier in tasks involving the transformation of surface area rather than of volume.

More recent Piagetian-based research by Twidle (2006), proposed the question of whether understanding the concept of conservation of volume in solids is more difficult than understanding the conservation of volume in liquids, or is the way we assess conservation giving us an unfair comparison? Twidle noted that traditional methods of assessing liquid volume have involved establishing the quantity of liquid in a container by reference lines or transferring liquid to a new container and that the equivalent process for solids is more complex with calculations and displacement processes. When comparing mastery of concepts of liquid and solid volume conservation using comparable assessments, Twidle concluded that a hierarchy of development exists within the concepts investigated where: (a) liquid volume conservation is the easiest mastered, (b) followed by solid volume conservation and, finally, (c) displacement. This developmental information is important because the challenges students face when contemplating the biological implications of surface area to volume ratios of cells have implications for how teachers should approach this topic. This information can shed light on not only how teachers should teach this topic, but also how to build groundwork for scaffolding students' understandings of surface area to volume.

Students may be unable to understand an abstract concept such as surface area to volume in a cell without the necessary prerequisite skills such as proportional reasoning or visual-spatial skills. There have been few studies investigating how proportional reasoning and visual-spatial ability affect learning surface area to volume, although some studies have looked at the relationships between proportional reasoning, visualization, and other science concepts. For example, Dori & Barak (2001) investigated the effect of the use of virtual and physical models while teaching organic chemistry on students' understanding of new concepts and the spatial structure of molecules. An important value of models in science and science education is their contribution to the visualization of complex ideas, processes, and systems (Dori & Barak, 2001). Just as chemistry students may struggle with the understanding and visualization of molecular formulas and geometric structures, similarly biology students may have difficulty visualizing the proportional ratio of surface area to volume in cells and the limitations that the concept imposes on cell size.

Proportional Reasoning: A Watershed Concept

For a comprehensive understanding of scale in science, a number of mathematical concepts and skills are required and foremost is the skill of proportional reasoning (Tretter, Jones, & Minogue, 2006). Although research has been done about students' use or understanding of proportional reasoning it has been done from a mathematics perspective. Proportional reasoning plays such a critical role in a student's mathematical development that it has been called a watershed concept, a cornerstone of higher mathematics, and the capstone of elementary concepts (Lesh, Post, & Behr, 1988). Not only is proportional reasoning critical for mathematical understandings, but it is a necessary skill used in a variety of ways in daily life. Teachers often teach proportional reasoning skills by getting students to consider how they think proportionally in daily life (Beckmann, Thompson, & Austin, 2004).

For a mathematician, a proportion is a statement of equality of two ratios, i.e., $a/b = c/d$ (Tourniare & Pulos, 1985). Reasoning about proportions is challenging and can be found in mathematical, scientific, and everyday contexts. Proportional reasoning research has evolved from the early view of proportional reasoning as a global ability, or a manifestation of general cognitive structure, to a more differentiated view of proportional reasoning components and how they are influenced by task and person parameters (Tourniare & Pulos, 1985).

Newton & Tobin (1981) included proportional reasoning in the five reasoning modes having particular relevance to science instruction. Utilizing the Test of Logical Thinking (TOLT) and taking into account different educational levels, various modes of reasoning, and problem context, Newton and Tobin concluded that students' difficulty solving proportional reasoning problems has serious implications for science instruction. The physical and biological sciences contain a myriad of concepts and laws that rely on some form of proportional reasoning for understanding. It isn't clear how cognitive development levels, cognitive abilities, and/or visual spatial skills contribute to students' mastery of these concepts. It appears that the choice of scalar (quantities of same nature) or functional (quantities of different nature) strategy for thinking about proportional reasoning is influenced by the context of the problem (Karplus et al., 1983).

Developmental stages, Cognitive Ability, and Proportional Reasoning

Due to the impact of Piaget's theory on studies that came thereafter it is important to emphasize that although few current studies would be considered "classical Piaget," the field has assimilated his ideas so much that his contributions to current research often are invisible (Flavell, Miller, & Miller, 2002). Piaget postulated that all children progress through four distinct cognitive developmental and stages and abruptly make the transition from one stage to the next sequentially; however, Siegler (1998) suggests that these differences are not radical and stage-like. Piaget viewed the formal operational stage as the culmination of the process of cognitive development where logical and scientific reasoning produce some of the largest changes in thinking (Siegler 1998). Researchers also suggest children's cognitive abilities of dealing with concepts differ across various domains.

Although Piaget's work formed the foundation for the study of children's reasoning, and some researchers today continue within his framework other work has broadened our thinking about cognitive development to examine how children reason about the world as well as the role of knowledge and context (Flavell, Miller, & Miller, 2002). Flavell and his colleagues point out that the use of reasoning skills makes possible new and more abstract knowledge as children detect increasingly abstract relations in the world. However, most of the studies about proportional reasoning have been based on the premise that cognitive ability which employs multiplicative or additive strategies is correlated with the developmental stage of the student. Success rates on proportion problems increase dramatically with age up to adulthood and according to Piagetian theory proportional reasoning emerges as part of the formal stage of cognitive development (Tourniare & Pulos, 1985).

Connecting cognitive development and proportional reasoning

Further evidence that proportional reasoning is developmental was reported by Kwon, Lawson, Chung, and Kim (2000). These researchers tested the hypothesis that maturing prefrontal lobes play a role in the development of proportional reasoning skill because the prefrontal lobes are involved in the inhibition of task-irrelevant information and the representation of task-relevant information. They explored the connection between how growth in adolescence occurs (such as plateaus and spurts) and a student's reasoning ability. Kwon, et al., found a similar age-wise pattern in scientific reasoning ability and in students' ability to learn science concepts. They argued that prefrontal maturity as well as social and physical experiences plays a role in development of scientific reasoning ability and in the acquisition of science concepts.

Lamon (1993) investigated how children learn ratio and proportion with the goal of prescribing how instruction might facilitate the development of proportional reasoning. Utilizing an eight-item written test composed of four different types of semantics, Lamon found that the strategies attached to any semantic type were determined partly by student characteristics and partly by characteristics of the problems. According to Lamon, students demonstrate proportional reasoning when they are able to understand the equivalence of appropriate scalar ratios and the invariance of the function ratio between two measure spaces. The results also suggested that relative thinking is related to higher levels of sophistication in problem solving ability, and that students may benefit from posing questions that may be answered using either absolute or relative thinking in order to engage them in a confrontation with both perspectives. Lamon's study raises the question of whether the perspective students take when solving a problem is linked to the strategies used to "visualize" the problem. Researchers agree that there is a correlation between cognitive development, and proportional reasoning ability, and suggest that instructional methods might facilitate to the development of this type of reasoning (Piaget & Inhelder, 1967; Dean & Frankshouser, 1988; Clark & Kamii, 1996; Kwon, Lawson, Chung, & Kim, 2000). The vast majority of studies of proportional reasoning have been grounded in mathematics education and the research examining proportional reasoning in science contexts is limited.

Proportional Reasoning in Science

Hwang (1994) studied the influence of proportional reasoning and self-regulation instruction on students' conceptual change in conceptions of solutions. The results of that study suggested that the development of proportional reasoning ability may be an indicator of a student's ability to understand chemistry. Another study by Westbrook (1990) examined the relationships of formal reasoning, science process skills, gender, and instructional treatment on promoting conceptual shifts in biology students. The Test of Logical Thinking

(Tobin and Capie, 1980) was utilized to determine students' ability to understand the concepts of diffusion, the cell, circulation, plant food production, and genetics. Results indicated that only instruction was a significant factor influencing shifts in student understanding of a concept (Westbrook, 1990). Reasoning level and prior knowledge did not correlate positively with shifts in student understanding.

Role of visual-spatial ability

Visual-spatial cognition pervades all human experience, from the frontiers of science, to everyday tasks like packing a suitcase (McCormack, 2002). Spatial ability is not a single, undifferentiated construct, but instead is composed of several separate abilities (Hegarty & Waller, 2005). Visual-spatial thinking includes: (a) vision-the process of using the eyes to identify, locate, and think about objects and orient ourselves in the world and (b) imagery-the formation, inspection, transformation, and maintenance of images in the "mind's eye" in the absence of a visual stimulus (Mathewson, 1999).

From 1900 to 1960, visual-spatial research focused on problems of spatial perception and representation. In the 1960's, when the research shifted from behavior to complex internal processes, the amount of attention paid to questions of spatial development, both theoretical and research activity increased dramatically (Eliot, 1987). Spatial ability is important in many different everyday applications and occupations, thus the answers to these questions may have important implications for teaching and learning especially in science. For example, mechanical reasoning and success in scientific domains may be related to facility with spatial tasks, particularly mental rotation (Brownlow, McPheron, & Acks, 2003).

Investigating visual-spatial ability

Allen, Kirasic, Dobson, Long, and Beck (1996) completed a two-part study investigating the relationship between spatial abilities and environmental learning of 100 participants between the ages of 17 and 36 years. The first part consisted of a battery of six psychometric tests selected from the *Kit of Factor-Referenced Cognitive Tasks* (Ekstrom, et al., 1976). The findings offered a heuristic framework for future work aimed at specifying and explaining predictive relationships within the domain of spatial cognition. Even with the use of this framework for predicting relationships within the domain of spatial cognition, one must consider the differences that exist between individuals' spatial strategies. Using a test from the Kit of Factor-Referenced Cognitive Tests as an example, the *Surface Development* test of imagining a paper cube folded or flattened out could be solved using different strategies. A holistic strategic person would actually imagine folding and unfolding the paper cube whereas an analytical person would compute the folding result for each edge. Then they concluded that if we systematically study how people solve spatial tasks, we will gain a better understanding of why some individuals fare better than others in their understanding of various concepts.

Kosslyn and Koenig (1992) argued that many people often think by visualizing objects and events and that visual mental imagery is accompanied by the experience of seeing, even though the object or event is not actually being viewed. For example, which is darker green, a Christmas tree or frozen pea? Kosslyn, et al. further state that because visual imagery shares processing mechanisms with visual perception, we gain insight into what an image representation is and how one must able to interpret the imaged pattern. Image generation is a prerequisite before one can scan an object and determine its spatial properties (Kosslyn, 1992).

A few studies have been conducted that examine how spatial ability, perspectives, and visual imagery influence the way individuals understand or perceive information. Hegarty, Montello, Richardson, Ishikawa, and Lovelace (2006) investigated individual differences in aptitude test performance and spatial layout learning by distinguishing spatial abilities at different scales using paper and pencil tasks in addition to environmental spatial tasks. The results of the study suggested different possible models of the relationship between large and small scale spatial abilities (Hegarty, et al., 2006). One model, the Unitary model, assumes that the two abilities completely overlap as if they both depended on the same cognitive processes; whereas, the Total Dissociation model completely separates the two abilities as if reliant on distinct cognitive processes. A third model, the Partial Dissociation model, proposes that the two abilities rely on some common process while the fourth, Mediation model, would suggest that there is a dissociation with an unknown ability that mediates the relationship. There has been debate over which model best represents the underlying relationship, however, the results from this study indicated that individuals have different spatial abilities (mental rotations, solving mazes, folding and unfolding sheets of paper, hidden figures) and that there are no strong claims that abilities at different scales of space are either completely overlapping (unitary) or completely dissociated (total dissociation) (Hegarty, et al., 2006).

Visual-spatial ability, scale, and the sciences

Visual solutions to problems have frequently been recounted by inventors and scientists, but only recently has the role of visual imagery been acknowledged by science educators (McCormack, 2002). Thinking with images plays a central role in scientific creativity (graphics, metaphors, and unifying ideas/themes) and there is a need for visual-spatial instruction in the science classroom (Mathewson, 1999). Imagine learning chemistry without images of molecules or biology without images of cells. Chavez, Reys, and Jones (2005) documented the difficulties that students have in determining the volume from pictures or drawings of prisms and that spatial visualization, together with logical reasoning, calls on higher-level cognition and processing. Battista and Clements (1996) maintain that students' meaningful enumeration of cubes in 3-D arrays is a fundamental notion in understanding the measurement of volume and they note that the process is far more complex than previously thought.

The use of imagery can serve as a bridge to the formation of abstract concepts. Furthermore, concepts of scale such as magnitude, limits to size, and properties of systems that change depending on volume and surface processes are typically learned through imagery. Mathewson (1999) argued that general scientific concepts such as symmetry, conservation, stability, system, and form and function are learned from a collection of master images of science. For example, one category that is linked to cell surface area to volume ratio is the idea of "boundaries" and could be taught as "interfaces or surfaces" through examples such as cell membranes, weather fronts, etc. Mathewson (1999) also suggested there are categories of visualization in science such as (a) data manipulation, (b) encoding,

(c) Gestalt, and (d) age-appropriate exercises with shadows, mazes, hidden figures, and mental rotations that can prepare the student to use observation and imagery in science. As a result, Mathewson (1999) poses several questions, one of which is pertinent to this study on surface area to volume relationships: What is the relative importance of visual-spatial aptitude compared with other mental abilities for the learning of science?

Gabel and Enochs (1987) examined visual-spatial ability to determine the best sequence to teach metrics (length, area, or volume first). The results indicated that if a student has high visual-spatial ability then teaching length, area and volume in sequence is successful. However, if students had low visual-spatial ability, the volume first approach was more effective. These results contradicted an earlier study by Bilbo and Milkent (1978) which reported that the volume first approach was superior for all students. Gabel and Enoch reason that the volume first approach is beneficial for students with low visual-spatial abilities because students are not required to visualize the area times the height to calculate the volume. These studies imply that visual-spatial ability plays a role in student achievement. A better understanding how visual-spatial ability and proportional reasoning affect the learning of science concepts can inform the instructional methods utilized by teachers and science educators.

Germaine to the question of how students learn surface area to volume relationships is the role that vision plays in encoding and representing the shapes of three-dimensional (3-D) objects (Feldman, 2003). One hypothesis is that object shapes are represented in terms of volumetric components and their spatial configuration and one study examined the basis of volumetric part effects in shape recognition and to elucidate the structure of the shape primitives that mediate 3-D object shape representation (Leek, Reppa, & Arguin, 2005). How the entire surface shape of an object is an important factor for object recognition was examined previously by Leek, et al. (2005). Leek, et al (2005) contrasted volumetric over nonvolumetric configurations and reported that understanding of volumetric configurations may have implications for students' learning topics such as estimation or proportional reasoning.

Results from a study using estimation tasks drew attention to specific difficulties children seem to have in manipulating more than one spatial domain (Forrester and Shire, 1995). Forrester and Shire (1995) used a three-dimensional "volume" task in their study of how the influence of object size, dimensionality, and prior context affected children's estimation abilities. Just like proportional reasoning, estimation is an important skill needed in everyday activities and patterns of results reported did not provide unequivocal support either for a maturational spatial cognition hypothesis regarding estimation abilities or for the overarching importance of context (Forrester, et al., 1995).

Conclusion

Few would argue against the importance of spatial ability in many areas of science inquiry especially those involving scaling, patterning, measuring, interpreting, and visualizing objects as observed from different orientations. Pallrand and Seeber (1984) investigated the nature of the relationship between visual-spatial abilities and achievement in science courses and found that taking a physics course improved visual-spatial abilities and the subsequent success in other science courses. Most of the research within the visual-spatial realm has focused on the physical sciences such as chemistry and physics. The role of visual-spatial ability in learning the biological sciences has not been explored extensively. Visual-spatial thinking in the sciences is an insight that has only recently been acknowledged by science educators even though visual solutions to problems have frequently been recounted by inventors and scientists (McCormack, 2002). For example, Watson and Crick revealed the structure of DNA through use of visual-spatial techniques of thought experiments, sketching, and 3D modeling (Harrison and Treagust, 2000).

Study Context

The present study explores the relationships among students' proportional reasoning skills, visual-spatial abilities, and understandings of the scale concept of surface area to volume. As discussed, proportional reasoning ability, cognitive development, and visual-spatial skills all contribute to learning science but the relationships among these factors has not been fully explored. This study utilizes specific assessments of visual-spatial and proportional reasoning as well as specific application tasks to examine relationships among those abilities with respect to surface area to volume ratios and to answer more specifically the following questions:

1. Is proportional reasoning ability correlated with the ability to understand the relationship between surface area to volume?
2. Is proportional reasoning ability correlated with the ability to apply surface area to volume relationships to applications in science (i.e. limiting factors)?
3. Is visual-spatial ability correlated with the ability to understand surface area to volume relationships?

Methodology

Research Design

This exploratory study investigated the relationship between proportional reasoning skills and visual-spatial abilities as possible predictors for students' understanding of surface area to volume. The research questions posed about the correlation between proportional reasoning ability, visual-spatial ability, and one's ability to understand surface area to volume applications were derived from the possible links between cognitive variables and the targeted domains of learning represented in Figure 1. Participants included middle school (seventh and eighth grade students), high school (students that have completed biology), and biology teachers (described further below). All groups completed a series of assessments (described below) including the Test of Logical Thinking Test (TOLT) and two visual-spatial tasks from the *Kit of Factor-Referenced Cognitive Tests,* as well as, the *Applications of Surface Area to Volume Assessment* (ASAVA). The research questions, instruments, and constructs of learning for this study are shown in Table 1.

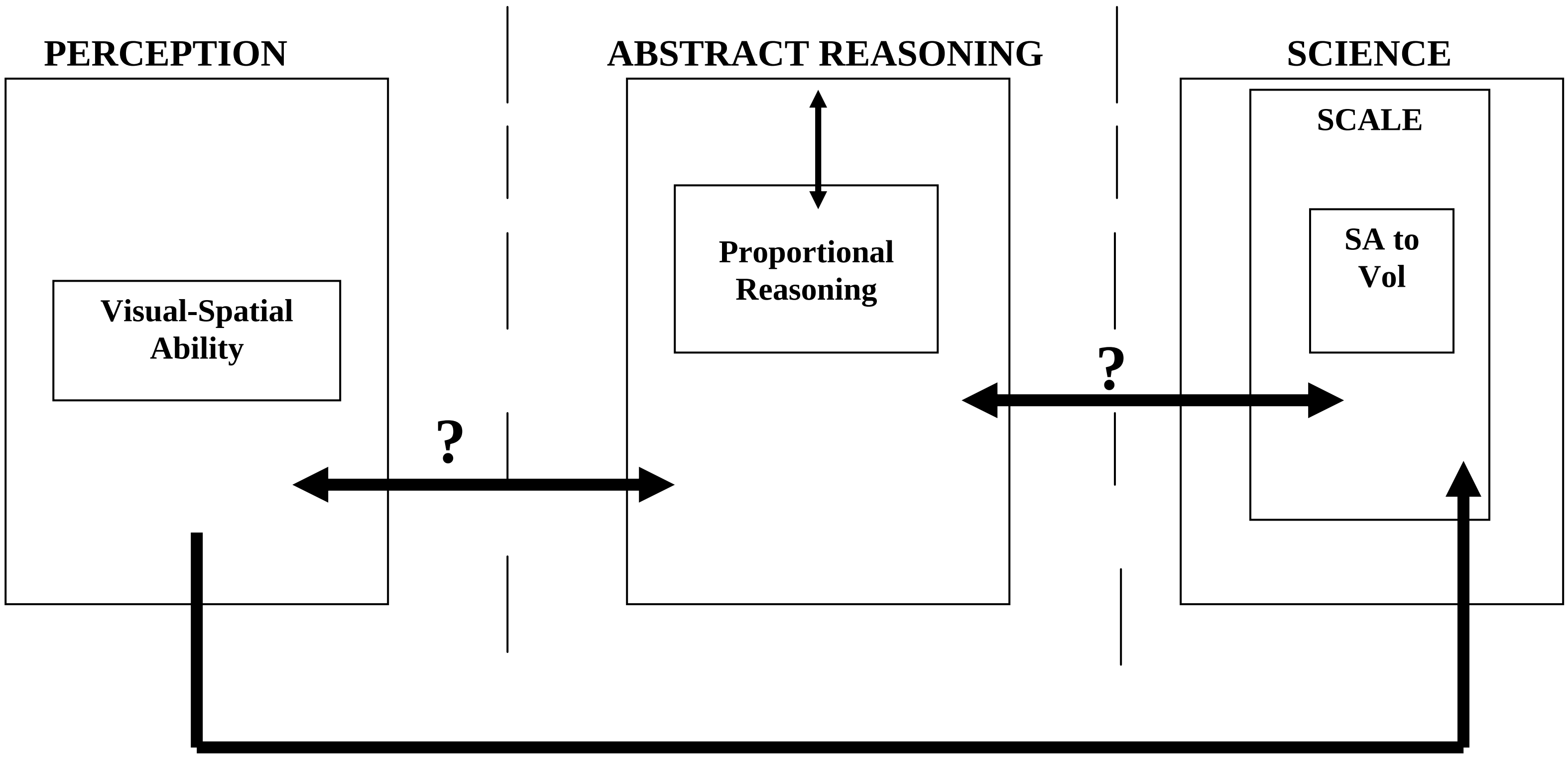


Figure 1. Model of conceptualization of surface area to volume relationships, proportional reasoning ability, and visual spatial ability.

Table 1. *Research questions, Instruments, and Constructs of Learning*

Research Question	Instrument	Constructs of Learning
Is proportional reasoning ability correlated with the ability to understand the relationship between surface area to volume?	TOLT ASAVA	Proportional Reasoning Knowledge of Surface Area to Volume
Is proportional reasoning ability correlated with the ability to apply surface area to volume relationships to applications in science (i.e. limiting factors)?	TOLT ASAVA	Proportional Reasoning Knowledge of Surface Area to Volume
Is visual-spatial ability correlated with a the ability to understand surface area to volume relationships?	Surface Test Storage Test ASAVA	Figural Flexibility Factor Visualization Factor Knowledge of Surface Area to Volume

Study Sites and Participants

Middle School Students.

Middle school participants were volunteers who attended a week-long summer science camp offered in conjunction with a large university at an urban middle school in Eastern US. The middle school participants attended various middle schools from rural, suburban, and urban communities. All but two of the students attending the camp volunteered to participate in the research study. This sample (n=35) was comprised of 17 females and 18 males (seventh and eighth graders) with an ethnic composition of: 9% Asian, 57% Caucasian, 2% Hispanic, 3% East Indian, and 29% African American. The science summer

camp focused on teaching students about levers, scale, magnification, and surface area to volume applications in science. Middle school students were chosen to be assessed as a baseline for documenting the learning trajectory of proportional reasoning skills, visual-spatial skills, and knowledge of surface area to volume relationships. This age was of interest because of the transitions from concrete operational reasoning to formal reasoning that typically begin during the middle school years. Students who have transitioned to the use of formal reasoning are more likely to be better at proportional reasoning (Tobin and Capie, 1980).

High School Students.

High school participants were volunteers from a suburban high school in Eastern US who had completed a biology course prior to this study. This sample (n=45) was comprised of 24 females and 21 males (eleventh and twelfth graders) with an ethnic composition of: 2% Asian, 80% Caucasian, 2% Hispanic, 4% Middle Eastern, and 11% African American. These high school students were chosen as a comparison group since they would likely have reached a developmental level for abstract reasoning, possibly have more advanced visual-spatial abilities, and had completed a biology course in which the surface area to volume applications were taught as part of the curriculum.

Biology Teachers.

Teacher participants were volunteers who attended an *Advanced Placement Program* workshop at a large university in Eastern US. This sample (n=37) of biology teachers was comprised of 28 females and 7 males with an ethnic composition of: 95 % Caucasian and 5 % African American. The mean years of teaching experience of the teacher participants was 10 years.

Instrumentation

The Test of Logical Thinking Test (TOLT).

The Test of Logical Thinking Test (TOLT) was used to assess participants' proportional reasoning ability. This assessment designed by Tobin and Capie (1980), is a group test of formal reasoning ability that required students to solve problems and to justify the solutions obtained. This test is designed to measure five modes of formal reasoning: (a) controlling variables, (b) proportional reasoning, (c) combinatorial reasoning, (d) probabilistic reasoning, and (e) correlational reasoning. The 10 multiple-choice test items require participants to select a correct response, as well as, provide a justification for their response from a number of alternatives and has an established reliability (coefficient $\alpha = .85$). Tobin and Capie (1980) examined criterion-referenced validity of the TOLT for a group 88 students from grades 10 through college and found a correlation of .80 ($p < .0001$), suggesting a strong relationship between the measures of formal reasoning.

Visual Spatial Tasks.

The Storage Test and Surface Development Test, two tasks from the Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Dermen, 1976), were used to measure visual-spatial skills. This kit consists of marker tests for 23 aptitude factors. The *figural flexibility* (XF) factor is the ability to change set in order to generate new and different solutions to figural problems and includes the Storage Test (Ekstrom, et al., 1976). In this test, participants are asked to determine how many objects can be stored in a given space. By looking at two different size cubes, participants try to visualize how many different ways the small cube could be stored in the larger cube. Suggested completion time for this assessment is 3 minutes for each of the two parts of the test which has shown to be reliable ($\alpha = .67$). This particular task was chosen for its emphasis on looking at an object in terms of its "volume."

The *visualization* (VZ) factor uses the Surface Development test which consists of 5 items in each of 6 drawings in which participants try to visualize how a piece of paper can be folded to form some kind of geometric shape. Diagrams are utilized to show how a piece of paper might be cut and folded so as to make a solid form. Dotted lines are used to indicate where the paper could be folded. Suggested time length for the test is 12 minutes. This assessment has been shown to be reliable for both males ($\alpha = .75$) and for females ($\alpha = .77$). This particular test was chosen for its emphasis on looking at an object in terms of its "surface area."

Applications of Surface Area to Volume Assessment (ASAVA)

The Applications of Surface Area to Volume Assessment (ASAVA) was revised from an earlier version utilized in a pilot study and consists of parallel forms for the pretest and posttest (see Appendix). Each form of the ASAVA consists of 20 multiple-choice items that range in level of questioning with the majority of the questions involving application, analysis, and synthesis. The items assessed students' understanding of surface area to volume calculations and applications of the ratio in a science context. Three of the test items have an additional component which asks the students to explain their answer choices. The ASAVA builds on the concept of scale and is informed by national (AAAS, 1993; NSES, 1996) and state science standards' (NCDPI, 2004) recommendations as to what middle school and secondary students should know about cellular structure and functioning which includes how surface area to volume limits size of cells. The scale concept which includes cellular functions is aligned with the content of the ASAVA and features scale and surface area to volume ratios across various science contexts. During the development of ASAVA, national and state goals and objectives were considered for item selection.

A panel of experts consisting of four science educators, one educational psychologist, and a biologist examined the test instrument for the qualities of relevance, balance, and specificity to establish content validity. Two measures of internal consistency were calculated to determine reliability: Kuder-Richardson (K-R 20) and inter-rater reliability. The internal reliability for the ASAVA was determined for both the pretest and posttest. The K-R 20 from the administration for the middle school population was .50 for the pretest and .70 for the posttest. The K-R 20 from the high school population was .60. The K-R 20 for the biology teachers was .70. The inter-rater reliability for the scoring was determined to be 95%. After input from the panel of experts the ASAVA was revised accordingly.

Study Sequence

Part 1: Middle School Students: Baseline for Learning Trajectory.

The proportional reasoning ability and visual-spatial abilities of the middle school participants were assessed using the Test of Logical Thinking, the Storage Test, and the Surface Development Task, respectively. Following these assessments, this group completed the pretest to the Applications of Surface Area to Volume Assessment (ASAVA). The scores from these assessments gave a baseline for the learning trajectory of proportional reasoning, visual-spatial ability, and how they may correlate to the students' understanding of surface area to volume relationships. An intervention consisting of a series of activities focusing on basic calculations of surface area to volume ratios, as well as, application of surface area to volume in various science contexts was implemented (See Table 2 for a description of activities). After completion of the activities, the middle school participants completed the Applications of Surface Area to Volume Assessment (ASAVA) posttest.

Table 2. *Descriptions of Surface Area to Volume Investigations*

Surface Area to Volume Investigations	Description
Sugar Size	Raw, granulated, powdered sugar are used to investigate how surface area affects how sugar adheres to a surface
Could they really exist?	Group exploration activity where students discuss whether or not extreme creatures could or could not exist (such as *King Kong* or giant spiders)
Surface area to volume of cells	Surface area to volume as a limiting factor in the size of cells using different size agar cubes
Insect building competition	Giant insects are built with straws, paper clips, etc. in order to examine limits to size

Part 2: High School Students and Biology Teachers: Comparison Groups.

The high school participants and the biology teachers were also assessed on their proportional reasoning ability, visual-spatial ability, and knowledge of surface area to volume applications by completing the Test of Logical Thinking, Storage Test, Surface Development test, and the Applications of Surface Area to Volume Assessment (ASAVA), respectively. No intervention was employed before or after the ASAVA in Part 2. Scores from the high school students and science teachers were compared to the middle school participants.

Analyses

The Test of Logical Thinking (TOLT) was scored according to the guidelines set by Tobin and Capie (1980). For each correct item the participants received one point for a maximum total of ten points on the TOLT. The two visual-spatial instruments were scored as directed in the Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Dermen, 1976). The Storage test consisted of two different scenarios of participants trying to visually place a small cube with specific dimensions into a larger cube in various ways. One point was awarded to each correct placement of the small cube into the larger cube. If a participant used an alternative placement of the cube in which a "new rule" was employed, two points were given. The Surface Development test which consists of 5 items in each of 12 drawings had a maximum number of 60 points and the scores were determined by calculating the percent correct. The Applications of Surface Area to Volume Assessment (ASAVA) consisted of 20 questions with each item worth one point.

The three explanatory questions from the ASAVA were recorded and analyzed for emergent themes.

Means and standard deviations of assessments were calculated. Correlations were determined for the proportional reasoning score, visual-spatial scores, and the ASAVA pretest (and posttest scores from the middle school participants) as well as analyzed using multiple linear regression. A paired *t*-test for the pretest and posttest of the Applications of Surface Area to Volume Assessment was calculated for the middle school participants.

Results

Part 1 Middle School Students

The mean score on the TOLT for the 35 middle school participants was 4.11 (SD = 2.40). The mean score on the Surface Development test for the middle school participants was 63.66 (SD=23.67) and their mean score on the Storage test was 11.71 (SD = 3.16). The mean score on the pretest for the ASAVA for the middle school students was 12.14 (SD = 2.38) whereas the mean score for the posttest increased to 12.89 (SD = 3.09). Results of a paired sample *t*-test suggested that the significant changes in pretest and posttest for the Applications of Surface Area to Volume Assessment (ASAVA) were not due to random chance but instead are probably due to the intervention the students received as a result of completing the surface area to volume application tasks $t = 2.102$, $p = 0.043$. See Table 3 for means and standard deviations for assessments.

Table 3. *Mean Scores (Standard Deviations) for Middle School Students*

Instrument	Middle School Students χ	SD
TOLT	4.11	(2.40)
ASAVA		
Pre	12.14	(2.38)
Post	12.89	(3.09)
Surface Development	63.66	(23.68)
Storage	11.71	(3.16)

Part 2 High School Students and Science Teachers

The high school participants who previously completed a biology course in which they learned about surface area to volume ratios in science contexts were compared to the middle school students. The mean score on the TOLT for the 45 high school participants was 7.18 (*SD* = 1.79). The mean score on the Surface Development test for the high school participants was 79.84 (*SD*=19.26) and their mean score on the Storage test was 22.80 (*SD* = 9.17). The mean score on the ASAVA for the high school students was 13.56 (*SD* = 2.66).

The mean score on the TOLT for the 37 biology teacher participants was 6.92 (*SD* = 2.81). The mean score on the Surface Development test for the biology teachers was 79.35 (*SD*=18.10) and their mean score on the Storage test was 14.68 (*SD* = 3.68). The mean score on the ASAVA for the teachers was 16.73 (*SD* = 2.86). See Table 4 for summary of all means and standard deviations of all participants.

Table 4. *Mean Scores (Standard Deviations) for Middle School and High School Students and Biology Teachers*

Instrument	Middle School Students n =35	High School Students n = 45	Biology Teachers n = 37
TOLT	4.11 (2.40)	7.18 (1.79)	6.92 (2.81)
ASAVA			
Pre	12.14 (2.38)	13.56 (2.66)	16.73 (2.86)
Post	12.89 (3.09)		
Surface Development	63.66 (23.68)	79.84 (19.26)	79.35 (18.10)
Storage	11.71 (3.16)	22.80 (9.17)	14.68 (3.68)

Note. The post ASAVA was only given to the middle school participants.

Emergent themes from ASAVA free response

The analysis of the three free response items from the ASAVA showed that the percent correct increased from pre to post. The types of responses the middle students gave for the pretest items only referred to the size of an object with no mention of surface area or volume. After the intervention of activities, the middle school responses on the post items included explanations of surface area to volume and textures of surfaces in question. For the For the item that asked which fish gills would absorb the most oxygen, 66% of the middle schools students' responses changed from typical responses such as "bigger" and "smaller" to explanations that reflected characteristics of surface area such as "more extensions" and "increased surface area." Only two of the high school students included explanations of only size in their answers, with the majority using phrases such as "increased or decreased surface area" or "processes on gills increase surface area." The biology teachers' responses only included references to surface area to volume explanations and none about size. These response patterns suggest that the middle school students began to reason using their knowledge of ratio relationships following instruction whereas the older participants already had an understanding of how surface area to volume ratios could be applied in a specific context. See Table 5 for other representative responses from all participants.

Table 5. *Representative Responses of Participants on Free Response Item #7 on ASAVA*

Middle School		**High School**	**Teachers**
Pre	Post		
The gills are the largest	*They (*the gills*) have the most surface area.*	*The more surface area the more oxygen they can absorb*	*Because the gills with the greatest surface area due to the 'pockets'.*
Gills are largest and can absorb more oxygen	*They have more of an area for the oxygen to be absorbed.*	*Because the sinuses on its gills give it more surface area for the oxygen to move into the gills.*	*Have more invaginations which increase surface area.*
It has the smallest gill, so it will not take in as much.	*The gills with the most surface area and 'arms' to take up the nutrients.*	*They (*the gills*) have a greater surface area for water to be absorbed and thus absorb oxygen faster.*	*Folds in the gills increase the surface area which increases the oxygen uptake.*
It is more open and big	*It's surface area can absorb more*	*The gills with the greatest surface area because the gills would filter more water by the compressed ridges than just a big whole gill.*	*Increased surface area on gills then the more oxygen absorbed.*

Summary of Correlations of Assessments

Middle School Participants.

Statistically significant correlations were identified between the TOLT scores and all other assessments taken by the middle school participants (See Table 6). The ASAVA pretest was highly correlated with the TOLT (r (33) = 0.64, $p \leq 0.01$). The ASAVA posttest and the TOLT had a correlation of r (33) = 0.50, $p \leq 0.01$. The correlated scores on the TOLT and Storage test were r (33) = 0.54, $p \leq 0.01$, and the Surface Development test was highly correlated with r (33) = 0.71, $p \leq 0.01$. The scores of both the pretest and posttest ASAVA were significantly correlated to both visual spatial assessments with the highest correlations determined to be with the Surface Development test (pretest; r (33) = 0.74, $p \leq 0.01$ and posttest; r (33) = 0.51, $p \leq 0.01$).

High School Participants.

Statistically significant correlations were identified between the TOLT scores and all other assessments taken by the high school participants. A correlation of r (43) = 0.49, $p \leq 0.01$ was determined with the ASAVA. The correlation of the TOLT and the Storage test was r (43) = 0.43, $p \leq 0.01$, in addition the correlation between Surface Development test and TOLT was r (43) = 0.32, $p \leq 0.05$. The score on the ASAVA assessment were highly correlated with the Surface Development test (r (43) = 0.37, $p \leq 0.05$). No correlation was found between the ASAVA and the Storage test scores.

Science Teacher Participants.

The biology teachers' correlations followed the same pattern seen in the scores in the middle and high school students'. Scores on the TOLT were significantly correlated with the other assessments. A high correlation of $r\ (35) = 0.57$, $p \leq 0.01$ was found for the TOLT and the ASAVA. The correlation of the TOLT and the Storage test was $r\ (35) = 0.38$, $p \leq 0.05$, whereas the Surface Development test was determined to be $r\ (35) = 0.50$, $p \leq 0.01$. The scores the ASAVA assessment were highly correlated with the Surface Development test $r\ (35) = 0.44$, $p \leq 0.01$). As noted in the high school students' scores, no correlation was found between the ASAVA and the Storage test scores.

Table 6. *Correlations of Assessment Scores for All Participants.*

Assessment	1	2	3	4
		Middle School (n = 35)		
1. TOLT	___	.64 (pre)** .50 (post)**	.71**	.54**
2. ASAVA (pre) ASAVA (post)		___	.74 (pre)** .51 (post)**	.56 (pre)** .45 (post)**
3. Surface Test			___	.56**
4. Storage Test				___
		High School (n = 45)		
1. TOLT	___	.49**	.43**	.32*
2. ASAVA		___	.37*	.09
3. Surface Test			___	.56**
4. Storage Test				___
		Teachers (n = 37)		
1. TOLT	___	.58**	.50**	.38*
2. ASAVA		___	.44**	.26
3. Surface Test			___	.55**
4. Storage Test				___

**p <.05. **p < .01.*

Results of Regression Analysis

Both regression analyses (pre and post ASAVA as the dependent variable) for the scores of TOLT, Storage test, Surface Development test and the Applications of Surface

Area to Volume Assessment indicated that the middle school participants' proportional reasoning and visual-spatial scores could be a possible predictor for the ability to understand surface area to volume relationships. The results of the multiple linear regression analysis for the scores of TOLT, Storage test, Surface Development test and the Applications of Surface Area to Volume pretest indicated that all participants' proportional reasoning and visual-spatial scores could be a possible predictor for one's ability to understand surface area to volume relationships. See Table 7 for complete results of the multiple linear regression analysis for all participants.

Table 7.

Multiple Linear Regression Analysis Results

Parameter estimates:

Variable	Estimate			SE			*t*			*p*		
	Middle	High	Teacher	Middle	High	Teacher	Middle	High	Teacher	Middle	High	Teacher
Intercept	6.54	7.08	10.99	1.11	1.72	1.94	5.91	4.12	5.68	0.00	0.00	0.00
TOLT	0.20	0.63	0.48	0.17	0.22	0.16	1.17	2.91	2.94	0.25	0.01	0.01
Surface	0.05	0.04	0.04	0.02	0.02	0.03	2.88	1.83	1.32	0.01	0.08	0.20
Storage	0.13	0.06	0.04	0.11	0.05	0.13	1.20	1.33	0.31	0.24	0.19	0.76

Dependent Variable: ASAVA

Independent Variable(s): Scores for TOLT (proportional reasoning), Storage Test, and Surface Development Test

Discussion

Past studies have found that the development of proportional reasoning ability can be used as an indicator of the ability to understand various science concepts (Hwang, 1994; Taylor & Jones, 2007; Westbrook, 1990). The present study showed that proportional reasoning is correlated to the ability to understand surface area to volume as well as components of visual-spatial abilities for middle and high school students and science teachers.

Traditionally proportional reasoning ability has been linked to a cognitive developmental level where formal reasoning is evident. The mean scores on the Test of Logical Thinking (TOLT) were significantly different for the middle school students and the high school students indicating a difference in the level of cognitive development. Although one would expect the mean scores on the TOLT to be higher for the science teachers than the high school students, there was instead a slight nonsignificant decrease ($t = 0.481$, $p = 0.634$).

The mean scores on the Applications of Surface Area to Volume Assessment (ASAVA) significantly increased across middle school, high school, and science teachers. Both the pretest and the posttest for the middle school students were significantly lower than the scores of the high school students indicating a less well developed understanding of surface area to volume relationships. The significant results showing the TOLT and ASAVA correlations suggests that understanding surface area to volume is related to the differences in proportional reasoning abilities. The pretest and posttest ASAVA completed by the middle schools students were significantly different. Since the middle schools students participated in the intervention consisting of surface area to volume activities, perhaps their experiential level was increased which led to their trajectory of understanding the relationship between surface area to volume. In addition, the science teachers' mean score on the ASAVA was even higher than that of the high school students. It is likely that teachers' experiences with surface area to volume relationships in science contexts led to the higher mean scores on the ASAVA. As further evidence for the findings of the pilot study that there is a correlation between proportional reasoning and the ability to understand surface area to volume relationships, the present study found once again that proportional reasoning ability is significantly correlated to one's ability to understand surface area and volume in various science contexts. The correlations of the scores on the TOLT and the ASAVA for all participants were found to be significant.

The present study also investigated how proportional reasoning and the ability to understand surface area to volume relationships are linked to visual spatial abilities. The mean scores on the Surface Development task increased significantly from middle school to high school students and were also highly correlated with the scores from the TOLT and ASAVA indicating that there are some underlying relationships between these abilities. The mean score for the science teachers on the Surface Development task was not significantly different from the high school students. Perhaps the factor assessed by the Surface Development task (*visualization*) peaks as students develop formal reasoning indicating that there would not be expected to be a significant difference between high school students and science teachers.

The mean scores for the Storage test increased significantly from middle school to high school students, however, only the mean score on the Storage test for the middle school students was significantly correlated to the TOLT and ASAVA. The high school students' mean score for the TOLT was significantly correlated with the Storage test. Interestingly, the science teachers' mean score on the Storage test fell between the middle and high school students' mean scores. The teachers scored significantly higher than the middle school students, as well as, significantly lower than the high school students. Perhaps the creativity factor in answering the items on the Storage test plays a role in the discrepancy of scores. The scoring of the Storage test according to Ekstrom et al. (1976) allows for one point to be awarded for a correct response and two points awarded for each response that exhibits a "new storage" rule. Hence, the more creative the answers, the more points awarded and the differences in attentiveness to creativity and details were significantly different for the high school students and science teachers. The high school students appeared to have more enthusiasm for the Storage test and therefore could have put more effort into finding creative ways of answering rather than just completing the answers. As was the case for the high school students, only the teachers' mean score for the TOLT was significantly correlated with the Storage test.

The results of the multiple linear regression analysis for the scores of all assessments indicated that the all participants' proportional reasoning and visual-spatial scores could be a possible predictor for one's ability to understand surface area to volume relationships. The correlations confirmed that there are definite beneficial links between the abilities to solve proportionality types of problems, visualize and mentally manipulate objects, and the learning and understanding of surface area to volume relationships. Paivio's (1986) *Dual Coding* theory suggests that there are two classes of phenomena handled cognitively by separate subsystems, one specialized for representation and information processing of nonverbal objects and the other specialized for dealing with language (Paivio, 1986). Furthermore, Paivio (1986) argues that many cognitive tasks require the performer to analyze, evaluate, and manipulate the properties of mental representations in order to respond appropriately. Applying Paivio's theory to the results seen in the present study suggests that for all four assessments participants would not only have to visualize, but also manipulate images, as well as respond verbally. Thus, using both fully incorporates dual coding as suggested by Paivio.

The dual coding processes are implicated in experimental problem solving, concept formation, and reasoning skills and from this perspective the goal of education is to foster the development of verbal and nonverbal systems that can cooperate in beneficial ways in all domains (Paivio, 2007). As noted above in explanation of the scores from the Storage test, creativity goes hand in hand with the abilities to perform on the assessments utilized in the present study. Paivio (2007) argues in great detail about how some of the most pivotal moments in science (i.e. discovery of DNA, Einstein's theory of relativity, and Erlich's side chain theory) were the result of creativity and the use of dual coding. Paivio further explains that these scientists first derived their concept by first modeling it in three dimensional structures relying on imagery juxtaposed to verbal descriptions. Learning science relies heavily on imagery and models in which students must mentally grasp the visual concept and be able to verbally explain what it is that they have learned. The results found in the present study provide evidence that for understanding surface area to volume there are underlying developmental as well as visual components. The degree to which a student applies verbal skills compared to visual skills (dual coding) is not known.

Implications

There is increasing evidence that imagery plays an important role in learning and doing science (Paivio, 2007 & Mathewson, 1999). Knowing how proportional reasoning and visual spatial ability may affect a student's ability to understand a concept such as surface area to volume has implications on how concepts may be introduced, taught, and experienced. The results of the pilot study concluded that proportional reasoning was positively correlated to students' understanding of surface area to volume relationships and that instructional intervention promotes knowledge of surface area to volume ratios in science contexts (Taylor & Jones, 2007). Furthermore, one of the questions that arose from that pilot study was: What other constructs may be involved in understanding of scale applications such as limits to size and surface area to volume relationships? The present study established that not only proportional reasoning plays a role in how someone understands surface area to volume relationships and their applications, but also the significance of visual spatial ability in this process.

If imagery and visual spatial cognition indeed play a role in how well someone understands science concepts including surface area to volume, then educators need to be aware of what types of visual spatial abilities are most beneficial for science class. These results suggest that if students lack cognitive development, proportional reasoning, or visual spatial skills that may be unsuccessful in mastering surface area to volume instruction. Further research is needed to examine how surface area to volume concepts are learned in different science domains (biology, physics, and chemistry) and whether or not the domain makes a difference in one's understanding of the concept. Research that documents one's visual spatial ability affects the learning of science concepts is needed. Do different science domains require different types of visual spatial ability? Would pre-teaching proportional reasoning to different age groups affect of learning surface area to volume concepts more effectively?

In summary, this study showed that learning and applying scale specifically understanding surface area to volume, is dependent on students' abilities to apply proportional reasoning and visual-spatial skills. With the major advancements in science at the extremes of scale (cosmic to nano), the challenges of teaching these new areas are significant. For science educators, it may be more critical than ever to unpack the complexities that underlie cognition of science concepts and phenomena.

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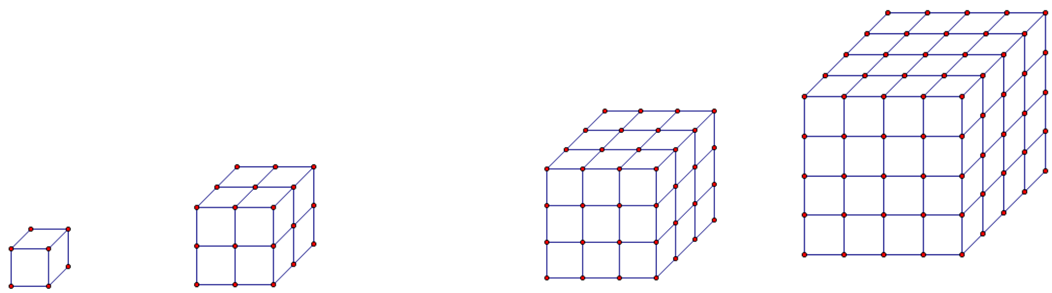
Appendix

Applications of Surface Area to Volume Assessment
PRE ASAVA

For each question below please give what you believe to be the correct response. In some cases you might find the questions to be challenging so try not to get frustrated. Just do your very best and please do not leave any questions blank .

Place all answers on bubble sheet. Write explanations when prompted.

Study the cubes labeled A, B, C, and D and answer questions 1-5.



A B C D

1. Which of the following cubes has the largest total surface area?
    a. Cube A
    b. Cube B
    c. Cube C
    d. Cube D

2. Which of the following cubes has the largest total volume?
    a. Cube A
    b. Cube B
    c. Cube C
    d. Cube D

3. Scientists often look at the sizes of organisms and determine the relationship between surface area to volume. Which Cube (A, B, C, or D) has the greatest surface area to volume ratio? (Hint: No calculators are necessary to answer this question.)

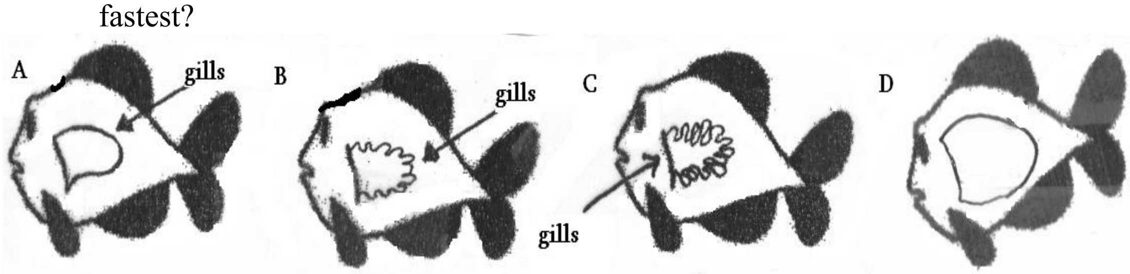
    a. A
    b. B
    c. C
    d. D

4. What is the total surface area for cube C?
    a. 3 square units
    b. 12 square units
    c. 54 square units
    d. 96 square units

5. What is the total volume for cube C?
    a. 3 cubed units
    b. 18 cubed units
    c. 27 cubed units
    d. 54 cubed units

6. On a hot summer day, Sally was pouring glasses of water for her friends after playing volleyball. She put crushed ice in some of the glasses and cubed ice in the others. Which glass of water would get colder **faster**?
    a. A glass of water with 5 large ice cubes
    b. A glass of water with crushed ice made from 5 large ice cubes
    c. A glass with no ice at all
    d. Neither, both at same rate

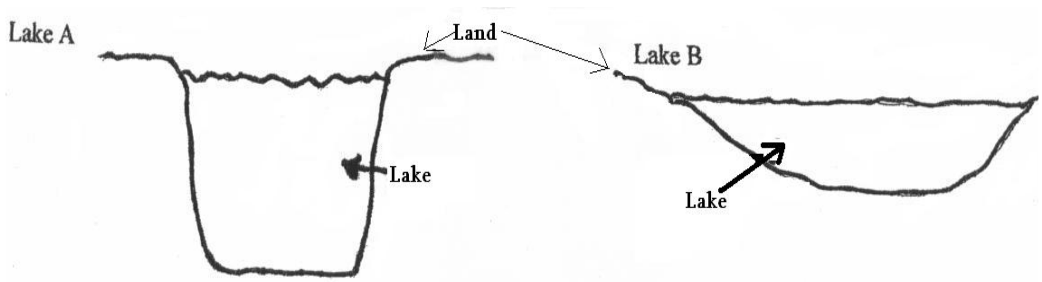
7. Fish need oxygen just like humans. They use the oxygen as water moves through their gills. Fish gills come in a variety of shapes and sizes depending on the type of fish and type of environment it lives in. Which type of gills would absorb oxygen the fastest?



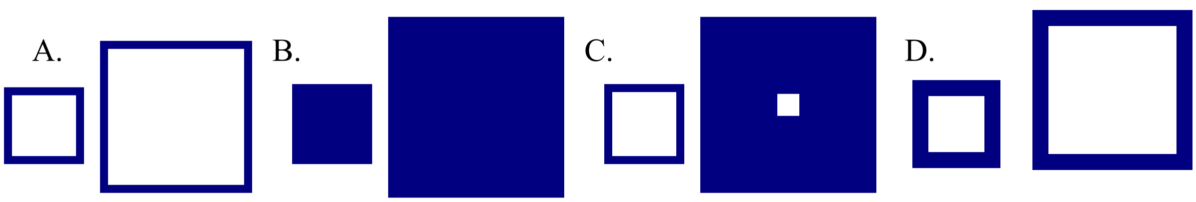
EXPLAIN YOUR ANSWER:

8. Organisms on Earth exist in a variety of shapes and sizes. Consider the cells of a 33-foot anaconda snake and a seven-inch worm snake. Which of the following statements is true?
    a. The cells of the anaconda are ten times larger than those of the worm snake.
    b. The worm snake has fewer but much larger cells than the anaconda.
    c. The cells of each snake are the same size, but the anaconda has many more cells than the worm snake.
    d. The cells of each snake are the same size, but the worm snake has many more cells than the anaconda.

9. Sketches A and B show two lakes of equal volume but different shapes. Which statement would be true about which lake would heat up faster?
   a. Lake A would heat up faster because it is deeper (less surface area in contact with air).
   b. Lake B would heat up faster because it is shallow (more surface area in contact with air)
   c. Lake A and Lake B would heat up at the same rate because they have the same volume.
   d. The size and shape of the two lakes does not play a role in how fast they heat up.



10. Anita was conducting an experiment using peeled potatoes. She discovered that when she soaked them in dye, they absorbed the dye. If Anita placed two potato cubes of different sizes in the dye for the same amount of time and then sliced them open which picture might represent an **unlikely** outcome of the experiment?



EXPLAIN YOUR REPONSE:

11. A farmer wanted to protect his tomatoes from insects. He has three full baskets of tomatoes which have the same mass. One basket contains huge tomatoes, one basket has medium tomatoes, and one basket has cherry tomatoes. He sprayed a chemical on the surfaces of all types of tomatoes to protect them. The cherry tomatoes are small in circumference (1.5 inch), the medium tomatoes are 4 inches in circumference, and the huge tomatoes are 10 inches in circumference. Which statement below is **not** true?

    a. In order to protect the entire surface of each tomato, the farmer will have to spray more chemical on the basket of huge tomatoes than the basket of cherry tomatoes.
    b. In order to protect the entire surface of each tomato, the farmer will have to spray more chemical on the basket of cherry tomatoes than the basket of medium tomatoes.
    c. In order to protect the entire surface of each tomato, the farmer will have to spray less chemical on the basket of medium tomatoes than the basket of huge tomatoes.
    d. In order to protect the entire surface of each tomato, the farmer will have to spray less chemical on the basket of cherry tomatoes than the basket of huge tomatoes.

12. If you wanted to have a dessert at your birthday party that has the smallest surface area to volume ratio, which of the following would you choose?

a. **Cupcake**	**b. Cake**	**c. Donut Hole**	**d. Jelly Donut**
(4 x 3 inches)	**(12 x 24 inches)**	**(1 x 1 inches)**	**(5 x 1 inches)**

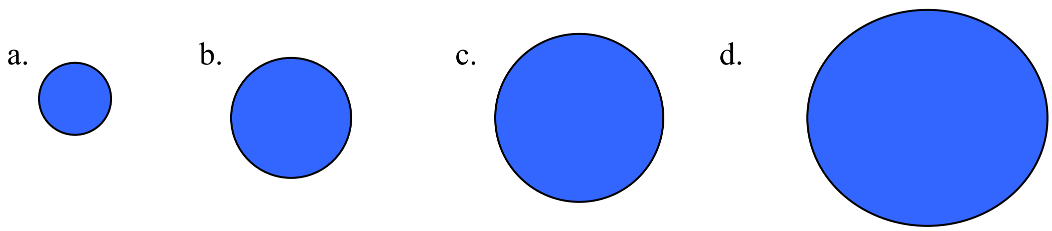
13. One hundred (100) people wearing red shirts held hands and made a circle. They squeezed as many people wearing blue shirts into their circle as possible. Then, ten (10) people wearing green shirts held hands and made another circle. This group squeezed as many people wearing orange shirts into their circle as possible. If a message needed to get to the centermost person in each circle from the outside, which group could get the message whispered the fastest?

    a. The red shirt circle could get message to their centermost person faster because there are more people passing message along.
    b. The red shirt circle could not get message to their centermost person faster because the large volume of people would slow the message down.
    c. The green shirt circle could get message to their centermost person faster because there are fewer people passing the message along.
    d. The green shirt circle could not get message to their centermost person faster because small volume of people would slow the message down.

14. Eggs come in various sizes depending on the kind of bird. The baby chicks inside the eggs must absorb oxygen through pores in the eggshell. Ostrich eggs are the largest eggs on Earth with a volume of 1,567 milliliters. A chicken egg is smaller than an ostrich egg with a volume of 59 milliliters. A bluebird egg (30 milliliters) is smaller than a chicken egg and a hummingbird egg (0.60 milliliters) is the smallest of all. Based on the volume of the following four eggs, which egg would have the best rate of oxygen absorption?

    a. Ostrich egg (1,567 ml)
    b. Chicken egg (59 ml)
    c. Bluebird egg (30 ml)
    d. Hummingbird egg (0.60 ml)

15. Which sphere has the largest surface area to volume ratio?



16. Which plastic coffee mug will keep coffee warmer longer?

    a. A short mug with wide opening at top
    b. A short mug with a narrow opening at top
    c. A tall mug with wide opening at top
    d. A tall mug with a narrow opening
    e.

17. Which of the following metric units is correct when labeling surface area of a one cm cube?

    a. cm
    b. cm^2
    c. cm^3
    d. cm^4

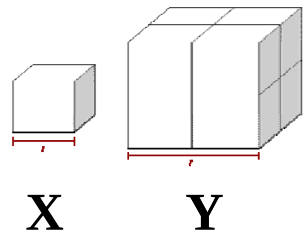
18. Which of the following metric units is correct when labeling the volume of a one cm cube?

    a. cm
    b. cm^2
    c. cm^3
    d. cm^4

19. Bird feathers are critical for flight and insulation. A bird spends a good deal of time cleaning and grooming its feathers by applying oil or bathing in water. Birds spread oil with their beak over their feathers for waterproofing. If birds need to spread the oil evenly over the entire surface of their feathers, describe the shape of a feather that would take the most oil?

EXPLAIN YOUR RESPONSE:

20. How many of cube X could fit inside of cube Y?

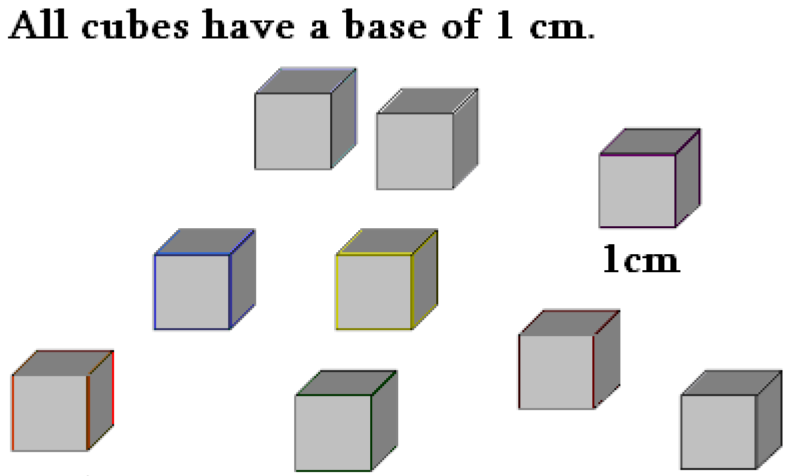


a. Two
b. Four
c. Eight
d. Sixteen

21. If a 15 foot woman and a one inch man existed, which one would have a faster rate of body heat loss?

a. The 15 ft. woman would have a faster rate of heat loss because she has more volume compared to her surface area.
b. The 15 ft. woman would not lose any body heat because she has more volume compared to her surface area.
c. The one inch man would have a faster rate of heat loss because he has less volume compared to his surface area.
d. The one inch man would not lose any body heat because he has such a small volume.

22. Find the total surface area of all the cubes below:



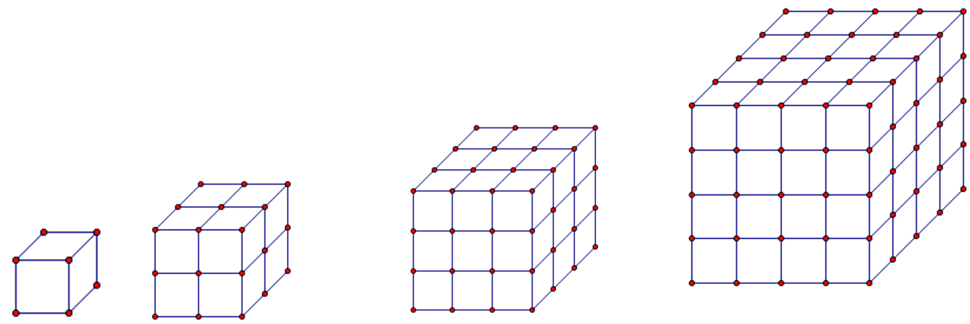
a. 9 cm^2
b. 36 cm^2
c. 15 cm^2
d. 54 cm^2

Applications of Surface Area to Volume Assessment
POST ASAVA

For each question below please give what you believe to be the correct response. In some cases you might find the questions to be challenging so try not to get frustrated. Just do your very best and please do not leave any questions blank.

Place all answers on bubble sheet. Write explanations when prompted.

Study the cubes labeled A, B, C, and D and answer questions 1-5.



A B C D

1. Which of the following cubes has the smallest total surface area?
   e. Cube A
   f. Cube B
   g. Cube C
   h. Cube D

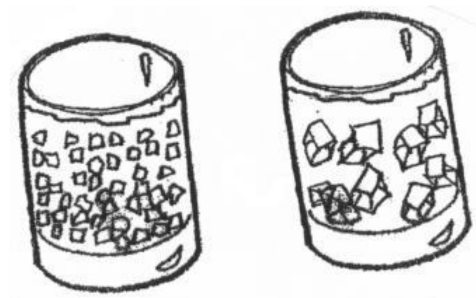
2. Which of the following cubes has the smallest total volume?
   a. Cube A
   b. Cube B
   c. Cube C
   d. Cube D

3. Scientists often look at the sizes of organisms and determine the relationship between surface area to volume. Which Cube (A, B, C, or D) has the smallest surface area to volume ratio? (Hint: No calculators are necessary to answer this question.)

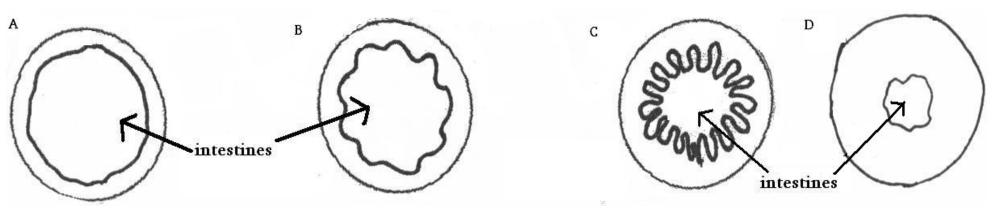
4. What is the total surface area for cube D?
   a. 4 square units
   b. 12 square units
   c. 64 square units
   d. 96 square units

5. What is the total volume for cube D?
    a. 4 cubed units
    b. 16 cubed units
    c. 64 cubed units
    d. 96 cubed units

6. On a hot summer day, Sally was pouring glasses of water for her friends after playing volleyball. She made some glasses of water with crushed ice and some with cubed ice. Which glass of water would stay colder **longer**?
    a. A glass of water with 5 large ice cubes
    b. A glass of water with crushed ice made from 5 large ice cubes
    c. A glass with no ice at all
    d. Neither, both at same rate



7. Which of these intestines (shown in cross-section) would absorb the most nutrients from food?



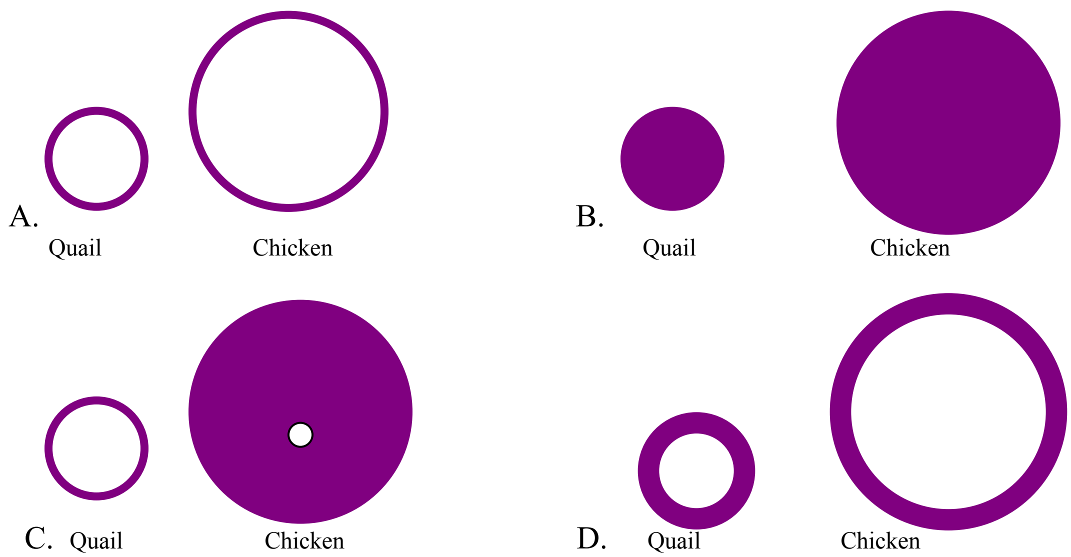
EXPLAIN YOUR ANSWER:

8. Tapeworms are segmented worms that live in the intestines of animals and must absorb nutrients from their surroundings in order to survive. Which of the following statements is true?
    a. Tapeworms could easily grow to the length and width of a freight train as long as there are some nutrients surrounding it
    b. Tapeworms can grow to a great lengths but not in width if surrounded by nutrients
    c. Tapeworms are limited to less than 10 inches in length regardless of nutrients.
    d. None of the statements are true

9. Primates exist in various sizes and weights. Which primate would have a more difficult time staying warm in the winter based on just its size? **Explain your answer to the right of the chart.**

PRIMATE TYPE	HEIGHT	WEIGHT
PYGMY MONKEY	15 in. tall	1.25 pounds
GORILLA	60 inches tall	500 pounds
LEMUR	40 inches tall	10 pounds
SPIDER MONKEY	24 inches tall	20 pounds

10. Sarah wanted to make purple hard-boiled eggs for an egg salad. She chose chicken eggs (4x6 cm) and quail eggs (1x2 cm) to include in the salad. Both types of eggs were boiled, shelled, and placed in purple food coloring for the same amount of time. When she sliced the eggs in half to make the salad she noticed that the food coloring was absorbed by both types of eggs. Which picture would represent an **unlikely** outcome once she took the eggs out of the dye and sliced them open?



EXPLAIN YOUR ANSWER:

9. The rate of cooling for a sphere made of gold is greatest when…
    a. the volume is large
    b. the surface area is large
    c. the ratio of surface area to volume is large
    d. the ratio of surface area to volume is small

10. If you wanted to have a dessert at your birthday party that has the largest surface area to volume ratio, which of the following would you choose?

a. **Cupcake** **(4 x 3 inches)** **b. Cake** **(12 x 24 inches)** **c. Donut Hole** **(1 x 1 inches)** **d. Jelly Donut** **(5 x 1 inches)**

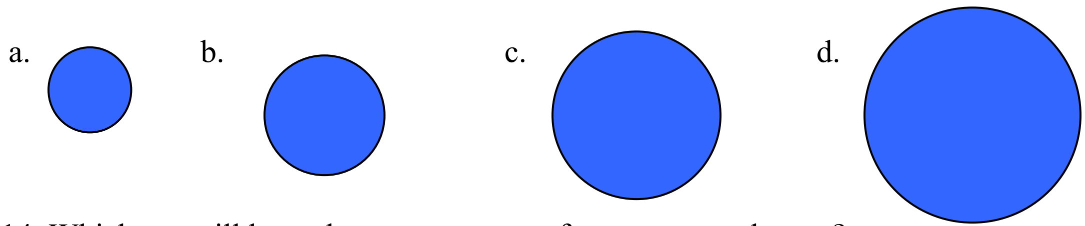
11. One hundred people wearing red shirts held hands and made a circle. They squeezed as many people wearing blue shirts into their circle as possible. Then, ten people wearing green shirts held hands and made another circle. This group squeezed as many people wearing orange shirts into their circle as possible. If a message needed to get to the centermost person in each circle from the outside, which group would take longer to whisper the message to the center?

    a. The red shirt circle could get message to their centermost person faster because there are more people passing message along.
    b. The red shirt circle could not get message to their centermost person faster because the large volume of people would slow the message down.
    c. The green shirt circle could get message to their centermost person faster because there are fewer people passing the message along.
    d. The green shirt circle could not get message to their centermost person faster because small volume of people would slow the message down.

12. Eggs come in various sizes depending on the kind of bird. The baby chicks inside the eggs must absorb oxygen through pores in the eggshell. Ostrich eggs are the largest eggs on Earth with a volume of 1,567 milliliters. A chicken egg is smaller than an ostrich egg with a volume of 59 milliliters. A bluebird egg (30 milliliters) is smaller than a chicken egg and a hummingbird egg (0.60 milliliters) is the smallest of all. Based on the volume of the following four eggs, which egg would have the best rate of oxygen absorption?

    a. Ostrich egg (1,567 ml)
    b. Chicken egg (59 ml)
    c. Bluebird egg (30 ml)
    d. Hummingbird egg (0.60 ml)

13. Which sphere has the smallest surface area to volume ratio?



14. Which pot will keep the same amount of soup warmer longer?

a. A short pot with wide opening at top
b. A short pot with a narrow opening at top
c. A tall pot with wide opening at top
d. A tall pot with a narrow opening

15. Which of the following metric units is correct when labeling surface area of a one mm cube?

a. mm
b. mm^2
c. mm^3
d. mm^4

16. Which of the following metric units is correct when labeling the volume of a one mm cube?

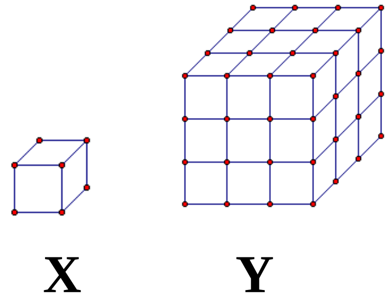
a. mm
b. mm^2
c. mm^3
d. mm^4

17. Jared was going hiking and he wanted make dried apples to go into his trail mix to take with him. Which statement is correct in describing which apple would lose water faster?

a. The peeled apple would lose water faster than sliced apple because it has the peel has been completely removed.
b. The peeled apple would lose water faster because it has greater surface area than the sliced apples.
c. The sliced apple would lose water faster than peeled apple because a portion of the peel remains on the slices.
d. The sliced apple would lose water faster because it has greater surface area than the peeled apple.

EXPLAIN YOUR ANSWER:

20. How many of cube X could fit inside of cube Y?



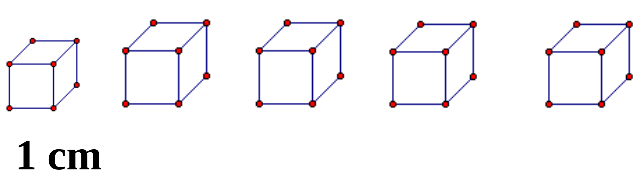
a. Three
b. Nine
c. Eighteen
d. Twenty-seven

21. If a 15 foot woman and a one inch man existed, which one would have a slower rate of body heat loss?

a. The 15 ft. woman would have a faster rate of heat loss because she has more volume compared to her surface area.
b. The 15 ft. woman would not lose any body heat because she has more volume compared to her surface area.
c. The one inch man would have a faster rate of heat loss because he has less volume compared to his surface area.
d. The one inch man would not lose any body heat because he has such a small volume.

22. Find the total surface area of all the cubes below:

All cubes have a base of 1 cm.



a. 5 cm^2
b. 10 cm^2
c. 25 cm^2
d. 30 cm^2