ABSTRACT

Qun Wan. Numerical and Theoretical Analysis of Beam Vibration Induced Acoustic Streaming and the Associated Heat Transfer (Under the direction of Dr. Andrey V. Kuznetsov)

The purpose of this research is to numerically and analytically investigate the acoustic streaming and the associated heat transfer, which are induced by a beam vibrating in either standing or traveling waveforms. Analytical results show that the beam vibrating in standing waveforms scatters the acoustic waves into the free space, which have a larger attenuation coefficient and longer propagating traveling wavelength than those of the plane wave. In contrast to a constant Reynolds stress in the plane wave, the Reynolds stress generated by such acoustic wave is expected to drive the free space streaming away from the anti-nodes and towards nodes of the standing wave vibration.

The sonic and ultrasonic streamings within the channel between the vibrating beam and a parallel stationary beam are also investigated. The acoustic streaming is utilized to cool the stationary beam, which has either a heat source attached to it or subjected to a uniform heat flux. The sonic streaming is found to be mainly the boundary layer streaming dominating the whole channel while the ultrasonic streaming is clearly composed of two boundary layer streamings near both beams and a core region streaming, which is driven by the streaming velocity at the edge of the boundary layer near the vibrating beam. The standing wave vibration of the beam induces acoustic streaming in a series of counterclockwise eddies, which is directed away from the anti-nodes and towards the nodes. The magnitude of the sonic streaming is proportional to $\omega^2A^2$ while that of the ultrasonic streaming is proportional to...
$\omega^{3/2}A^2$. Numerical results show that the acoustic streaming induced by the beam vibrating in either standing or traveling waveforms has almost the same cooling efficiency for the heat source and the heat flux cases although the flow and temperature fields within the channel are different.

The hysteresis of the ultrasonic streaming flow patterns associated with the change of the aspect ratio of the channel is numerically investigated. Present research is also extended to a cavity which is driven by a vibrating lid. The ultrasonic streaming induced in the cavity reveals some interesting interactions among the primary eddies, which have never been observed in the classical driven cavity problem.

**Keywords**: acoustic streaming, electronic cooling, CFD, boundary layer, ultrasound, bifurcation, hysteresis, driven cavity.
NUMERICAL AND THEORETICAL ANALYSIS OF
BEAM VIBRATION INDUCED ACOUSTIC STREAMING
AND THE ASSOCIATED HEAT TRANSFER

by

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PAUL D. FRANZON
To my wife Su and first unborn child
BIOGRAPHY

Qun Wan was born in Wuhu, Anhui, People’s Republic of China in 1974. He received his B.E. degree in Thermophysics from University of Science and Technology of China in 1996. After working as manufacturing engineer in Teling (TRANE China) Air-Conditioning Co. Ltd for one year, he went to the National University of Singapore, where he received his M.S. degree in Mechanical and Production Engineering in 2001. Qun had worked as software engineer in Vector Technology Co. Ltd (Singapore) prior to joining North Carolina State University, Raleigh, NC, in August 2000. He is a member of ASME.
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# TABLE OF CONTENTS

**LIST OF TABLES**

| List of Tables | IX |

**LIST OF FIGURES**

| List of Figures | X |

## 1. INTRODUCTION

1.1. BACKGROUND OF THE ACOUSTIC STREAMING .............................................................. 1

1.2. RESEARCH ON COOLING APPLICATION OF THE ACOUSTIC STREAMING COOLING .............. 3

1.3. INTRODUCTION OF PARTS AND CHAPTERS .................................................................. 4

REFERENCES ..................................................................................................................... 8

## PART ONE: FREE SPACE STREAMING

**2. FREE SPACE ACOUSTIC STREAMING FROM VIBRATING SURFACE IN STANDING WAVE FORM** .......................................................................................................................... 13

**ABSTRACT** ................................................................................................................... 13

2.1. INTRODUCTION ........................................................................................................ 13

2.2. THEORY .................................................................................................................. 16

2.2.1. Effect of the Non-Uniform Amplitude of Source Vibration .................................... 18

2.2.2. Standing Wave Source .......................................................................................... 19

2.3. NUMERICAL RESULTS .............................................................................................. 20

2.4. CONCLUSIONS ......................................................................................................... 26

REFERENCES ..................................................................................................................... 27

## PART TWO: SONIC STREAMING

**3. SONIC STREAMING COOLING EFFICIENCY FOR HEAT SOURCE AND HEAT FLUX** ....... 29

**ABSTRACT** ................................................................................................................... 29

**NOMENCLATURE** ............................................................................................................ 30

3.1. INTRODUCTION ........................................................................................................ 32
PART THREE: ULTRASONIC STREAMING

6. ULTRASONIC STREAMING AND ANALYTICAL SOLUTION OF BOUNDARY LAYER STRUCTURES

ABSTRACT

vi
# NOMENCLATURE

## 6.1. BACKGROUND

## 6.2. THEORY

- **6.2.1. Oscillating Boundary**
- **6.2.2. Non-Dimensionalization**
- **6.2.3. Inviscid SFO Solution**
- **6.2.4. Viscous Boundary Layer Near the Stationary Beam (BLS Layer)**
- **6.2.5. Viscous Boundary Layer Near the Oscillating Beam (BLO Layer)**
- **6.2.6. Core Region Streaming and Heat Transfer Across the Channel**

## 6.3. NUMERICAL RESULTS AND ANALYSIS

- **6.3.1. Numerical Scheme**
- **6.3.2. BLS Streaming and BLO Streaming**
- **6.3.3. The Effect of Channel Width**

## 6.4. CONCLUSIONS

## REFERENCES

## APPENDIX

- **6.A.1 Inviscid Solution of Eqs. (6.15)—(6.17)**
- **6.A.2 Solution of Eqs. (6.31)—(6.33) Utilizing the Stream Function Formulation**
- **6.A.3 Solution of Eqs. (6.41)—(6.43) Utilizing the Stream Function Formulation**
- **6.A.4 Nusselt Numbers in Eqs. (6.58) and (6.59)**

## 7. HYSTERESIS IN ACOUSTICALLY DRIVEN CHANNEL FLOW

## ABSTRACT

## NOMENCLATURE

## 7.1. INTRODUCTION

## 7.2. THEORY

## 7.3. NUMERICAL EXPERIMENT AND RESULT

## 7.4. CONCLUSION
REFERENCES .................................................................................................................................................. 169

8. ACOUSTIC STREAMING IN A RECTANGULAR CAVITY INDUCED BY THE VIBRATING LID .................................................................................................................................................. 172

ABSTRACT .................................................................................................................................................. 172

8.1. INTRODUCTION .................................................................................................................................. 172

8.2. THEORY ................................................................................................................................................ 174

8.3. NUMERICAL RESULTS ....................................................................................................................... 177

8.3.1. Effect of the Aspect Ratio \( \Gamma \) on the Primary and Secondary Eddies ........................................ 178

8.3.2. Effect of the Aspect Ratio \( \Gamma \) on the Interactions Between Primary Eddies ...................... 182

8.4. CONCLUSIONS .................................................................................................................................... 185

REFERENCES ............................................................................................................................................. 185

9. CONCLUSIONS ...................................................................................................................................... 187

9.1. Remarks on Free Space Streaming .................................................................................................. 187

9.2. Remarks on Sonic Streaming ......................................................................................................... 188

9.3. Remarks on Ultrasonic Streaming ................................................................................................. 189

9.4. Recommendations for Future Work .............................................................................................. 191

REFERENCES ............................................................................................................................................. 192
LIST OF TABLES

Table 3-1 Parameter values utilized in computations................................................................. 49

Table 3-2 Estimated cooling effect for the constant heat flux and heat source with a constant rate of internal energy generation cases................................................................. 60

Table 4-1 Estimated cooling effect for standing and traveling wave cases......................... 85

Table 5-1 Comparison of cooling effect of the acoustic streaming by experimental and numerical method (Ambient temperature is 20°C)......................................................... 105

Table 6-1 The effect of the channel width on the magnitudes of the horizontal and vertical components of the slip velocity (given by Eqs. (6.37a,b) and (6.47a,b)).
Computations are performed for $\lambda = 25.4\text{mm}$, $Re = 1.356\times10^5$, and $\gamma = 2.474\times10^{-3}$. ............................................................................................................................. 136

Table 6-2 The effect of the channel width on the Nusselt number and the heat transfer coefficient for the constant heat flux and constant temperature cases ($\lambda = 25.4\text{mm}$, $Re = 1.356\times10^5$, and $\gamma = 2.474\times10^{-3}$) ......................................................................... 147
LIST OF FIGURES

Figure 2-1 (a) Non-uniform and (b) Uniform (plane) vibration ............................................. 15

Figure 2-2 Amplitude of acoustic velocity generated by (1) infinite source: (a) amplitude on source surface $x = 0$, (b) amplitude field................................................................. 22

Figure 2-3 Amplitude of acoustic velocity generated by (2) finite source: (a) amplitude on source surface $x = 0$, (b) amplitude field................................................................. 23

Figure 2-4 Traveling wave propagation from the point at $y = 0.0078125m$ on the source surface, solid line: classical plane wave; dotted line: case (1), infinite source; dashed line: case (2), finite source (coincides with the curve for case (1)).......... 24

Figure 2-5 Normalized Reynolds stress distribution along vertical direction, $\delta(y)$............... 25

Figure 3-1 Schematic diagram of the problem for two types of thermal boundary conditions: (a) Constant heat flux at the upper beam, (b) Heat source with a constant rate of energy generation.................................................................................................. 36

Figure 3-2 SIMPLER staggered meshes (grid positions of horizontal, vertical streaming velocity components and streaming pressure are denoted by right arrows, triangles, and circles respectively) and boundary of computation domain (denoted by thick rectangle)..................................................................................................... 47

Figure 3-3 Flow velocity vector field at different moments of time: (a) $t = 0$ period, (b) $t = 1/8$ period, (c) $t = 1/4$ period, (d) $t = 3/8$ period, (e) $t = 1/2$ period, (f) $t = 5/8$ period, (g) $t = 3/4$ period, (h) $t = 7/8$ period .......................................................... 54

Figure 3-4 Isopleths of the vertical and horizontal components of the driving force, $f_b$, in the acoustic streaming equation, Eq. (3.38): (a) horizontal component of the driving force, (b) vertical component of the driving force........................................ 56

Figure 3-5 Isopleths of the mass source term $\dot{m}$ in Eq. (3.37) near vibrating beam........... 56

Figure 3-6 Isobars of the acoustic streaming pressure field $p_{dc}$ ........................................ 58
Figure 3-7 Acoustic streaming velocity field $u_{dc}$: (a) vectors, (b) streamlines .................. 59

Figure 3-8 Isotherms of the air temperature in the gap: (a) constant heat flux at the upper beam, (b) heat source with a constant rate of energy generation ................................. 63

Figure 3-9 Temperature (a) and heat flux (b) distribution across the interface between the upper beam and the heat source and isotherms of the temperature field in the heat source (c) ................................................................. 66

Figure 4-1 Schematic diagram for the heat transfer analysis .................................................. 75

Figure 4-2 Flow velocity at different moments of time during one period (traveling wave is generated on the surface of the lower beam, $\omega = 1000$, air) ........................................... 82

Figure 4-3 Acoustic streaming velocity field $u_{dc}$ ($\omega = 1000$, air) ........................................... 84

Figure 4-4 Normalized temperature fields (increase factor 100, $\bar{\theta} = 160.2K$ for standing wave, $\bar{\theta} = 160.4K$ for traveling wave) (a, c line type: solid $y = 0$, dashed $y = 0.3\text{mm}$, dashdot $y = 0.6\text{mm}$, dotted $y = 0.9\text{mm}$, longdash $y = 1.2\text{mm}$, dashdotdot $y = 1.5\text{mm}$; b, d line type: solid $y = 1.5\text{mm}$, dashed $y = 2.1\text{mm}$, dashdot $y = 2.7\text{mm}$, dotted $y = 3.3\text{mm}$, longdash $y = 3.9\text{mm}$, dashdotdot $y = 4.5\text{mm}$) ........................................... 88

Figure 5-1 Schematic diagram of the cooling device of acoustic streaming .............................. 95

Figure 5-2 Computational meshes for heat source and the gap ............................................. 100

Figure 5-3 Acoustic streaming velocity field $u_{dc}$ in form of vectors (a) and streamlines (b) 101

Figure 5-4 Temperature fields $\theta$ of the gap with bimorph at rest (a) and vibrating (b) ....... 102

Figure 5-5 Temperature fields $\theta$ within the heat source with vibrating bimorph ............... 103

Figure 6-1 (a) Schematic diagram of the infinitely long channel composed by a stationary beam and an oscillating beam; dashed line denotes the mean position of the oscillating beam and the dashed rectangular region denotes a periodic cell. (b) The three-layer structure of the streaming velocity fields within a periodic cell in the channel. .......................................................................................................................... 115
Figure 6-2 Boundary layer streaming near the oscillating beam (BLS streaming), $\delta = 7.76\mu m$. ........................................................................................................................................................................... 133

Figure 6-3 Boundary layer streaming near the stationary beam (BLS streaming), $\delta = 7.76\mu m$. ........................................................................................................................................................................................................................................... 134

Figure 6-4 Stream functions of the core region streaming flow field for various channel widths ............................................................................................................................................................................................................................................................................................................................................................................................................................................................. 141

Figure 6-5 Core region streaming velocity field in vector (a) and streamline (b) forms just before the flow field bifurcation. $\Gamma = 1.299$ ($h = 16.5$mm). ........................................... 142

Figure 6-6 Core region streaming velocity field in vector (a) and streamline (b) forms just after the flow field bifurcation. $\Gamma = 1.307$ ($h = 16.6$mm)........................................... 143

Figure 6-7 Contours of the dimensionless temperature field, $\theta$, caused by core region streaming for the case of constant heat flux (A) and constant temperature (B) just before the flow field bifurcation. $\Gamma = 1.299$ ($h = 16.5$mm). ......................... 144

Figure 6-8 Contours of the dimensionless temperature field, $\theta$, caused by the core region streaming for the case of constant heat flux (A) and constant temperature (B) just after the flow field bifurcation. $\Gamma = 1.307$ ($h = 16.6$mm). ......................... 145

Figure 6-9 The effect of the channel width on the Nusselt number (a) and the heat transfer coefficient (b) for the constant heat flux and the constant temperature cases. ... 146

Figure 7-1 The hysteresis effect on the Nusselt number for constant temperature (a) and constant heat flux (b). (Dashed lines denote the results from Wan and Kuznetsov [1], in which zero initial condition is utilized. The arrows denote the increase or decrease of the aspect ratio) ........................................................................................................................................................................... 165

Figure 7-2 Typical flow patterns and pressure fields in hysteresis region at aspect ratio $\Gamma = 1.1$, (a) flow field streamlines when $\Gamma$ increases, (b) flow field streamlines when $\Gamma$ decreases, (c) flow field vectors when $\Gamma$ increases, (d) flow field vectors when $\Gamma$ decreases, (e) pressure field contours when $\Gamma$ increases, (f) pressure field contours when $\Gamma$ decreases. ........................................................................................................................................................................... 168
Figure 8-1 (a) Schematic diagram of a cavity in which the flow is driven by vibrating the lid and (b) Driving slip velocity at the mean position of the lid .......................... 173

Figure 8-2 Streamlines of the air flow in the driven cavity at Re = 1,000 for the aspect ratio of (a) $\Gamma = 0.125$, (b) $\Gamma = 0.25$, (c) and $\Gamma = 0.375$ .......................................................... 180

Figure 8-3 Velocity vectors of the air flow in the driven cavity at Re = 1,000 for the aspect ratio of (a) $\Gamma = 0.125$, (b) $\Gamma = 0.25$, and (c) $\Gamma = 0.375$ ......................................................... 181

Figure 8-4 Distributions of the horizontal (a) and vertical (b) velocity components along the half-height horizontal cross-section ($y = \pi \Gamma$) for various aspect ratios ........... 183
1. INTRODUCTION

1.1. BACKGROUND OF THE ACOUSTIC STREAMING

Acoustic streaming is the steady, unidirectional fluid flow associated in the vibrating acoustic field. According to Lighthill [1], the acoustic streaming is generally defined as “the mean, or time-independent, motion that is induced in a fluid flow dominated by its fluctuating components.” Acoustic streaming originates from the attenuation of the acoustic field, which is a combined effect of vibrating frequency, viscous dissipation, compressibility, and conductivity of the fluid. (Definition of the attenuation coefficient of fluid can be found in Chapter 2.) The attenuation spatially reduces the vibrating amplitude of the acoustic wave and hence generates Reynolds stress distribution and drive the fluid to form the acoustic streaming.

Generally, the attenuation coefficient of the air is so small that the acoustic wave propagating in a free space filled with air generates very weak acoustic streaming (so-called free space streaming). However, when the acoustic wave propagates near a solid surface, the no-slip boundary conditions of the solid surface reduce the finite vibrating amplitude in the free stream to zero on the solid boundary with a short distance, which is called acoustic boundary layer. The thickness of the acoustic boundary layer is always determined by the viscosity of the fluid and the vibrating frequency of the acoustic wave (Definition of the boundary layer scale can be found in Chapter 6.). The reduction of vibrating amplitude of the acoustic wave is much larger than that in the free space. Hence the associated Reynolds stress and magnitude of the acoustic streaming (so-called boundary layer streaming) is also much larger. Generally, the acoustic streaming near the boundary layer edge does not vanish and instead provide the driving force for the acoustic streaming outside the boundary layer (so-called outer streaming). Although the
outer streaming is not directly generated from the Reynolds stress in the acoustic field, it is
easiest acoustic streaming to detect in the experiment because the free space streaming is too
weak and the acoustic boundary layer is too thin. The free space streaming is covered in Part
One (Chapter 2); the boundary layer streaming is covered in Parts Two and Three (Chapters
3–8); the outer streaming is covered in Part Three (Chapters 6–8).

The acoustic streaming is first discovered in the experiment as “Quartz wind” near the
resonating crystal surface in liquid (Walker and Allen [2]) and the steady air flow motion in the
Kundt tube (Schuster and Matz [3]) in the mid-20th century although Lord Raleigh first
predicted in 19th century the existence of the acoustic streaming in his famous book “Theory of
Sound” [4]. Modern theory of the acoustic streaming is developed by Nyborg [5], Schlichting
[6], and Bradley [7], who utilized the perturbation expansion method to investigate weak
acoustic streaming as the second order effect. The research on sonic streaming in Part Two is
mainly based on theory of Bradley, who emphasizes the acoustic field from a vibrating source
surface. When the acoustic intensity is large enough, e.g. ultrasound, the magnitude of the
acoustic streaming is comparable to that of the acoustic wave. To the author’s knowledge, no
one has ever developed a theory concerning this topic. In Part Three, the author attempts to
propose a new theory of the acoustic streaming near the vibrating surface in ultrasonic
frequency, together with some applications.

Since the free space streaming is very weak (Eckart [8] and Liebermann [9]) and its application
is limited, most research on the acoustic streaming focuses on the interactions of acoustic wave
and the solid surface, which induce much stronger boundary layer streaming and outer
streaming. Such interactions generally fall into two categories: (1) acoustic wave in the free
stream interacting with fixed solid surface (Stuart [10], Lee and Wang [11], and Vainshtein [12]); (2) solid surface vibrating in fluid. Depending on whether the solid surface changes its shape, the Category 2 can be further divided into two subcategories: (2a) solid surface vibrating in a whole (Davidson and Riley [13], Duck and Smith [14], Kim and Troesch [15], Secomb [16]); (2b) solid surface pulsating in standing or traveling waveforms (Vainshtein et al. [17], Hydon and Pedley [18]). In this dissertation, acoustic wave generated from a vibrating beam in Category 2b interacting with a parallel stationary beam in Category 1 is investigated. Detailed introductions of each reference are given in corresponding chapters.

1.2. RESEARCH ON COOLING APPLICATION OF THE ACOUSTIC STREAMING

Cooling

The convective nature of the acoustic streaming enables its heat transfer capacity in Categories 1 (Gopinath and Mills [19, 20], Vainshtein et al. [21], and Mozurkewich [22]), 2a (Richardson [23] and Davidson [24]) and 2b (Ro and Loh [25], and Loh et al. [26]). In Ro and Loh’s experiments, significant temperature drop of the heat source is observed when the acoustic streaming is induced within millimeter scale gap between the heat source and the parallel resonating beam. The beam scatters ultrasound into the gap and large attenuation is due to the small gap size that generates large Reynolds stress and strong boundary acoustic streamings and outer streaming. Inspired by these experimental results, Ms. Tao Wu and the author utilize a similar configuration to investigate the cooling of IC chips by the acoustic streaming under the direction of Drs. Ro and Kuznetsov, respectively. Detailed schematic diagrams are shown in individual chapters. This dissertation is the numerical and theoretical part of the project. The
1.3. INTRODUCTION OF PARTS AND CHAPTERS

This dissertation consists of three parts and nine chapters. Each one of Chapters 6—8 is mainly a published or submitted paper and is relatively independent of each other. Part One, which includes Chapter 2, investigates the free space acoustic streaming due to the vibrating surface in a standing waveform. Part Two, which include Chapters 3—5, investigates the sonic streaming within a channel induced by a vibrating beam in sonic frequency. Due to the weak acoustic intensity of the sonic acoustic field, the acoustic and streaming boundary layers are very large and hence they dominate the whole channel. Such a sonic streaming within a channel is actually a boundary layer streaming which produces a weak second order effect. Cooling efficiencies of the sonic streaming to the heat source and heat flux and the effect of vibrating beam in standing and traveling waveforms are also discussed in Part Two. Part Three, which includes Chapters 6—8, investigates a similar problem as that in Part Two except the vibration frequency is within an ultrasonic range. High acoustic intensity in ultrasonic acoustic field leads to the thin acoustic and streaming boundary layers, the streaming velocity at the edge of which drives outer streaming in the rest of the channel. Such an ultrasonic streaming is as strong as the acoustic vibration. A new theoretical three-layer flow structure with the channel is proposed and the hysteresis effect and driven cavity by the ultrasonic streaming are also discussed.
Chapter 2 (published as ref. [30]) analytically investigates the free space streaming due to the vibrating surface in standing waveform. Analytical results show a longer propagating traveling wavelength and a larger attenuation coefficient than those of the plane wave field. The distribution of the Reynolds stress for the acoustic streaming indicates a small magnitude towards the nodal positions of the vibrating surface and a large magnitude away from the anti-nodal positions of it. The analytical results are all confirmed by the accompanying numerical experiments.

Chapter 3 (published as refs. [31, 32]) numerically investigates the sonic streaming within a channel, which is induced by one of the beams vibrating in sonic frequency. According to Nyborg [5] and Bradley [7], the sonic streaming appears in the second order terms in their perturbation expansion analysis. The attenuation of first order acoustic field generates driving forces for the second order streaming field. In this chapter, the reduced wave equation of the acoustic field is derived and numerically solved using second order Non-Reflective Boundary Conditions (NRBC, Givoli [33]) at the open ends of the channel. The SIMPLER method is adopted in computing the sonic streaming field. The results show four sonic streaming eddies along each standing wavelength. Then the sonic streaming is assumed to be the only convection mechanism to cool the heat source or heat flux that is attached to the stationary beam from the outside. Temperature field from the computation results shows that the temperature drops for the heat source case and the heat flux case are almost the same. Chapter 3 provides theoretical and numerical bases for the rest two chapters of Part Two.

Chapter 4 (published as ref. [34]) continues Chapter 3 to investigate the effect of the vibrating beam in standing and traveling waveforms. The traveling waveform can be achieved in the
experiment by the summation of two standing waveforms with 90 degree phase difference as described in Loh et al. [26]. Numerical boundary conditions at the vibrating beam are set in a similar way. Compared to that in the standing wave case, the flow field in the traveling wave case does not present a symmetric and ordered pattern. Nevertheless, the cooling efficiencies in both cases are almost the same.

Chapter 5 (to be published) continues Chapter 3 to simulate the experiment performed by Ro and Wu [35] and Wu [27]. In their experiment, a bimorph fixed at both ends vibrates in its first mode, where half standing wavelength is generated on the bimorph and the center point has the maximum amplitude. Cooling due to the sonic streaming shows a similar temperature drop. The possible cooling due to the fluid being pushed in and out of the channel due to the acoustic field also shows a significant temperature drop.

Chapter 6 (published as refs. [36, 37]) attempts to find within a channel the mechanism of ultrasonic streaming, in which magnitude is comparable to the acoustic vibrating velocity and cannot be treated as a second order effect. Different from sonic streaming in Part Two, where the perturbation expansion method is used to obtain the second order streaming field, the ultrasonic flow field is assumed to consists of a streaming field, a single frequency oscillating field (SFO), a double frequency oscillating field (DFO) and other multiple frequency oscillating fields. The magnitudes of all of these fields are comparable to each other. The analytical solutions of the streaming and SFO fields can be obtained when the beams are long and multiple standing wavelengths are generated in one of the beams and the flow pattern within the channel can be assumed periodic along the channel. In this case, the inviscid solution of the SFO field is obtained by separation of variables. The matching of the inviscid
solution and the no-slip boundary conditions at the beams reveals the existence of the boundary layers near both beams. The Navier-Stokes equations are thus simplified to boundary layer equations and the solutions to the SFO and streaming fields within the boundary layers are analytically obtained. The solution of streaming field near the stationary beam coincides with that of the boundary layer streaming incurred by a standing wave source in the free stream, which is given by Schlichting and Gersten [38]. However, the solution of streaming field near the oscillating beam is first analytically obtained and is much stronger than that near the stationary beam. This streaming drives the outer streaming in the core region, which is located between two boundary layers. The core region streaming and its cooling effects to heat source and heat flux are therefore numerically obtained. Numerical results show that there are two distinctive flow patterns and the associated cooling efficiencies are quite different. The abrupt change from one flow pattern to the other is further discussed in Chapter 7. Chapter 6 provides theoretical and numerical bases for the rest two chapters of Part Three.

Chapter 7 (published as ref. [39]) continues Chapter 6 to investigate the hysteresis effect associated with changing the channel width. Numerical results show that the abrupt change between two flow patterns depends on whether the channel width increases or decrease. The range of hysteresis is also numerically determined.

Chapter 8 (published as ref. [40]) is motivated by the results in Chapter 6 and investigate the streaming velocity, at the edge of the boundary layer near a vibrating beam in standing waveform, as the driving velocity to the flow within an enclosed cavity. In contrast to the classical driven cavity problem, multiple primary eddies and their interaction with each other are observed and discussed in the numerical experiment.
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PART ONE:

FREE SPACE STREAMING
2. FREE SPACE ACOUSTIC STREAMING FROM VIBRATING SURFACE IN STANDING WAVE FORM

ABSTRACT

The aim of this chapter is to analytically solve the sound field generated by a standing wave induced in a vibrating beam. This case is different from a plane wave which is the traditional way of inducing acoustic streaming. The analytical solution shows that the amplitude non-uniformity can be represented by a non-uniformity coefficient $\gamma$, which characterizes the ratio of the wave number or the attenuation coefficient to their values for the classical plane wave case. The non-uniformity coefficient $\gamma$ is also obtained by resolving the acoustic field utilizing full numerical solution. Numerical and analytical results are in a good agreement. The Reynolds stress generated by a beam vibrating at one of its modes is also calculated. The maximum values of the Reynolds stress are achieved at the anti-node coordinates and small negative minimum values of the Reynolds stress are observed at the node coordinates. An interesting four-vortex-per-wavelength structure is predicted for such sound field.

2.1. INTRODUCTION

The linear acoustics theory is nearly analytically complete (Kinsler et al. [1]). The solution for a one dimensional plane wave generated by an infinite plane moving harmonically along its normal direction reveals two traveling waves propagating normal to the plane. Their directions are opposite to each other. When considering attenuation from viscosity, density change, and
heat conduction, the classical theory yields attenuation coefficient of the traveling waves. The attenuation mechanism of the acoustic field ultimately leads to the acoustic streaming phenomenon, which is discussed later on. Acoustic streaming is different from simple acoustic oscillation because it enhances mass and heat transfer. For instance, Wan and Kuznetsov [2] numerically investigated the heat exchange in a gap between two parallel beams, vibration of one of which induces acoustic streaming in the gap.

Piston movement is a good example of an acoustic source that generates a traveling plane wave. However, there are other sources, which can be approximated by a plane surface although each point on the surface may vibrate at different amplitude and even out of phase. For instance, a beam or plate vibrating at a certain mode will generate standing waveform along its surface (Fig. 2-1a). The acoustic field generated by such a source is definitely two-dimensional and not simple. However, when the waveform varies slowly along the source surface, an assumption can be made that the traveling waves generated by each point on the acoustic source surface do not interact with each other. In this case, the shape of the wave at the surface remains unchanged at some distance from the surface. By utilizing this assumption, it is possible to solve such an acoustic field analytically. The solution obtained in this chapter shows that such acoustic field is characterized by a wave number and an attenuation coefficient that differ from those for the classical plane wave of the same frequency. Hence the acoustic streaming associated with the standing wave is different from that of the plane wave case. The non-uniform and plane wave vibrations of the source surface are shown in Figs. 2-1(a) and (b), respectively.
Acoustic streaming is a steady fluid motion found in the acoustic field. So far, three types of acoustic streaming have been discovered. Two of them are generated by the attenuation of the acoustic field. When the sound wave propagates in a free space, the attenuation of the sound wave decreases the vibrating velocity as well as the amplitudes of pressure and density variations and hence induces spatial difference along the propagation direction. Such difference, although small in most cases, generates steady acoustic Reynolds stress, which drives the acoustic streaming. This type of acoustic streaming is always weak due to the small attenuation of the acoustic wave. It is theoretically predicted by Eckart [3] and experimentally confirmed by Liebermann [4] and Zarembo and Shklovskai-Kordi [5]. Other types of acoustic streaming occurring inside the acoustic boundary layer adjacent to a solid surface and outside of it are not considered in this chapter. Good reviews on acoustic streaming can be found in Riley [6], Nyborg [7, 8], and Lighthill [9].

In this chapter, the sinusoid amplitude distribution along the source surface is considered. The wave number $k$ and attenuation coefficient $\alpha$ are obtained. An expression for the Reynolds
stress that drives the acoustic streaming is also obtained. Comparisons with the classical plane wave case are discussed. Numerical solutions are obtained for finite and infinite source surfaces. Analytical solutions are validated by comparison with full numerical solution.

2.2. Theory

According to Rudenko and Soluyan [10], the linear wave equation for a viscous, heat-conducting media is given by

\[
\frac{\partial^2 v}{\partial t^2} - c_0^2 \Delta v - \frac{b}{\rho_0} \frac{\partial}{\partial t} \Delta v = 0
\]  

(2.1)

where \(v\) is the sound velocity field, \(c_0\) is the small disturbance sound speed, and \(\rho_0\) is the media density at rest. The dissipation term is incorporated in the coefficient \(b\), which is a combination of viscosity \(\mu\), bulk viscosity \(\mu_B\), and heat conduction \(k_c\) of the media:

\[
b = \mu_B + \frac{4}{3} \mu + k_c \left( \frac{1}{c_v} - \frac{1}{c_p} \right)
\]  

(2.2)

Equation (2.1) is valid when Reynolds number \(Re\) and Mach number \(M\) are much smaller than unity:

\[
Re = \frac{v_0 \rho_0 c_0}{b \omega} \ll 1 \quad \text{and} \quad M = \frac{v_0}{c_0} \ll 1
\]  

(2.3)

where \(v_0\) is the characteristic source velocity amplitude and \(\omega\) is the vibration frequency.

For the classical plane wave case, \(v_0\) is independent of the position on the surface because all points of the surface vibrate with the same amplitude and in phase. Solution of Eq. (2.1) gives
\[ v(x,t) = u_c \exp\left[i\left(\omega t - k_0 x\right)\right]\exp(-\alpha x) \]  
\[ (2.4) \]
where \( u_c \) is a constant value of \( v_0 \), \( k_0 \) is the wave number, and \( \alpha \) is the attenuation coefficient, which are defined as

\[ k_0 \equiv \frac{\omega}{c_0}, \quad \alpha \equiv \frac{b\omega^2}{2c_0\rho_0} \]  
\[ (2.5a,b) \]
As introduced in the previous section, the acoustic Reynolds stress for the acoustic streaming is non-zero in the attenuated acoustic field. According to Rudenko and Soluyan [10], it can be presented as

\[ F = (0, F_x) \], \quad F_x = -\frac{\partial \overline{\nabla^2 v^2}}{\partial x} - \frac{1}{\rho_0 c_0}\left(\mu_b + \frac{4}{3} \mu\right)\Delta\overline{(v^2)} \]  
\[ (2.6a,b) \]
where the overbars denote time average over a time period. Substituting Eq. (2.4) into Eq. (2.6b), Reynolds stress for the acoustic streaming from the classical plane wave is obtained as

\[ F_x = \left[ a u_c^2 - \frac{1}{\rho_0 c_0}\left(\mu_b + \frac{4}{3} \mu\right)2\alpha^2 u_c^2 \right] \exp(-2\alpha x) \]  
\[ \approx a u_c^2 \exp(-2\alpha x) = av_0^2 \exp(-2\alpha x) \]  
\[ (2.7) \]
The second term in the square bracket in Eq. (2.7) is negligible when \( \left(\frac{M}{\text{Re}}\right)^2 \ll 1 \) according to Rudenko and Soluyan [10]. However, it is retained here to compare with the results from the non-uniform vibration amplitude of the acoustic source.
2.2.1. Effect of the Non-Uniform Amplitude of Source Vibration

Solution (2.4) is valid for the plane surface, all points of which uniformly and harmonically oscillate in phase. Consider a beam vibrating at one of its modes. For such a beam the velocity amplitude varies along the beam’s surface, i.e. \( v_0 = v_0(y) \). Assume that the sound field can be written in the following form

\[
v(x, y, t) = v_0(y) \exp\left[ i(\omega t - k x) \right]
\]  \hspace{1cm} (2.8)

By substituting Eq. (2.8) into Eq. (2.1), the wave number \( k \) for this case is obtained as

\[
k = k_0 \left[ \sqrt{\frac{v_0^*}{v_0 k_0^2} + 1} + \frac{1}{1 + i \frac{b_0}{c_0 \rho_0}} \right] \]  \hspace{1cm} (2.9)

The difference of such wave number from that of the classical plane wave case is a complex coefficient. To simplify this coefficient, assume \( \frac{b_0}{c_0 \rho_0} \ll 1 \), which is valid for frequencies up to a gigahertz if the fluid is air. Equation (2.9) can then be expanded in Taylor series as

\[
k = k_0 \left[ \sqrt{\frac{v_0^*}{v_0 k_0^2} + 1} - \frac{1}{2} \left( \frac{v_0^*}{v_0 k_0^2} + 1 \right) i \frac{b_0}{c_0 \rho_0} + O \left( \left( \frac{b_0}{c_0 \rho_0} \right)^2 \right) \right]
\]  \hspace{1cm} (2.10)

\[
= \gamma k_0 - i \frac{\alpha}{\gamma} = \tilde{k}_0 - i \tilde{\alpha}
\]

where \( \tilde{k}_0 = \gamma k_0 \), \( \tilde{\alpha} = \frac{\alpha}{\gamma} \), \( \gamma = \left( \frac{v_0^*}{v_0 k_0^2} + 1 \right)^{\frac{1}{2}} \) \hspace{1cm} (2.11a,b,c)

From Eq. (2.10) it follows that the source disturbances wave number is \( \tilde{k} \) and the attenuation coefficient is \( \tilde{\alpha} \). Generally, the non-uniformity coefficient \( \gamma = \gamma(y) \) depends on the location on
the source surface. However, when $v_0(y)$ takes the form of a sinusoidal function, the non-uniformity coefficient $\gamma$ remains constant along the whole surface of the source. This happens, for example, when a beam vibrates at any of its modes with both ends fixed, and a standing wave is generated along the beam.

The difference with that of the classical plane wave case is the non-uniformity coefficient $\gamma$ in Eq. (2.11c). The sound velocity field obtained by solving Eq. (2.1) utilizing Eq. (2.8) can be presented as

$$v(x, y, t) = v_0(y)\exp\left[i(\omega t - \vec{k}_0 \cdot \vec{x})\right]\exp(-\alpha x)$$  \hspace{1cm} (2.12)

The Reynolds stress for the acoustic streaming based on Eq. (2.12) is obtained as

$$F_x = \left[\alpha v_0^2 - \frac{1}{\rho_0 c_0} \left(\mu_0 + \frac{4}{3} \mu\right)\left(v_0 v_0^{'} + v_0^{''} + 2\alpha^2 v_0^2\right)\right]\exp(-2\alpha x)$$ \hspace{1cm} (2.13)

Although the second term in the square brackets in Eq. (2.7) is negligible since it is a higher order term, the second term in the square brackets in Eq. (2.13) cannot be neglected because the first term may be zero or of the same scale as the second term when $v_0(y)$ is not constant along the source surface.

2.2.2. Standing Wave Source

Consider a standing wave generated along the beam surface, e.g.

$$v_0(y) = u_z \sin(k_z y)$$  \hspace{1cm} (2.14)
where $k_s$ is the wave number of the standing wave excited in the source surface and $u_c$ is the maximum vibration velocity amplitude on the surface of the source.

In this case, the non-uniformity coefficient $\gamma$ given by Eq. (2.11c) can be presented as

$$\gamma = \left(1 - \frac{k_s^2}{k_0^2}\right)^{\frac{1}{2}} \quad (2.15)$$

The Reynolds stress for the acoustic streaming (2.13) is

$$F_x = \left[\sin^2 k_s x, y - \beta \cos 2k_s x, y\right] \tilde{\alpha} u_t^2 \exp(-2\tilde{\alpha} x)$$
$$= \tilde{\alpha} v_0^2 \exp(-2\tilde{\alpha} x) - \tilde{\alpha} u_t^2 \beta \cos 2k_s x, y \exp(-2\tilde{\alpha} x) \quad (2.16)$$

where the coefficient $\beta$ in the second term is defined as

$$\beta = \frac{1}{\rho_0 c_0} \left(\frac{\mu}{\bar{\rho}} + \frac{4}{3}\mu\right) \frac{k_s^2}{\bar{\alpha}} \quad (2.17)$$

Comparing the Reynolds stress from the plane wave field with that from Eq. (2.16), it is found that an extra term has appeared in Eq. (2.17) in addition to the change of attenuation coefficient. Although it may be small, it is this extra term that can lead to a negative value of the Reynolds stress. This means that the direction of such Reynolds stress at some locations may be opposite to that in other locations.

### 2.3. Numerical Results

A beam of 62.5mm in length with two ends fixed is considered as a sound source to emit acoustic waves into the air. It is assumed that the beam vibrates at the frequency of $f = 21$kHz in its 4th mode. The non-uniformity coefficient in Eq. (2.11c) is then computed to be $\gamma = 0.853$. 
To verify this result, a computer code is written to solve Eq. (2.1) in the reduced form, i.e. by assuming

\[ v = iu(x, y) \exp(iot) \]  

Equation (2.1) is then reduced to

\[ k_0^2 u + \left( 1 + \frac{iob}{c_0^2 \rho_0} \right) \Delta u = 0 \]  

Equation (2.19) is solved numerically in a rectangular region with the sound source velocity specified at the left boundary in accordance with Eq. (2.14)

\[ u(0, y) = v_0(y) = u_c \sin(k_0 y) \]  

where \( u_c \) is set to unity because its value has no effect on linear Eq. (2.19).

Two cases are investigated in these computations: (1) Infinite source, and (2) Finite source. In case (1), the beam is assumed to consist of the infinite number of identical beams that are connected at their ends and oscillate identically. Hence periodic boundary conditions can be utilized at the top and bottom boundaries of the computation domain shown in Fig. 2-2. In case (2), a certain non-vibrating surface is extended beyond the two fixed ends of the vibrating beam and zero gradient boundary conditions are utilized on the top and bottom boundaries of the computation domain shown in Fig. 2-3.

According to Eqs. (2.11a,b), the non-uniformity coefficient \( \gamma \) can be obtained by numerically computing the wave number \( \tilde{k}_0 \) or the attenuation coefficient \( \tilde{\alpha} \) and comparing with their counterparts for the plane wave case. However, the second method is impractical because the acoustic wave attenuation in the air is very small, and a significant attenuation effect occurs
hundreds of meters away from the source. Hence, it is not feasible to compute the attenuation coefficient $\tilde{\alpha}$ to obtain the non-uniformity coefficient $\gamma$. Therefore, the first method is adopted.

In both cases (1) and (2), computations are carried on within one propagation wavelength of the classical plane wave from the surface, i.e. $L = \lambda_0 = 2\pi/k_0$. A non-reflective boundary condition is utilized at the right boundary.

![Diagram](image)

Figure 2-2 Amplitude of acoustic velocity generated by (1) infinite source:
(a) amplitude on source surface $x = 0$, (b) amplitude field
Figure 2-3 Amplitude of acoustic velocity generated by (2) finite source:
(a) amplitude on source surface $x = 0$, (b) amplitude field

Uniform mesh size is $151 \times 401$ for case (1) (infinite source) and $151 \times 601$ for case (2) (finite source), so that the mesh density is the same for both cases. When using proper initial field, case (1) converges much faster than case (2). This may be due to the difference in boundary conditions between these two cases. In case (2), it is expected that the width of the propagation waves is increasing as they leave the source surface. Results are shown in Figs. 2-2 and 2-3.

It is easier to observe the traveling wave propagation from one point on the source surface. Fig. 2-4 shows the horizontal cross-sections of the amplitude fields displayed in Figs. 2-2b and 2-3b at $y = 0.0078125m$. As seen in the graph, there is no noticeable difference between the two
cases. Hence, the finite source can be treated as an infinite source within at least one propagation wavelength. As discussed before, the non-uniformity coefficient $\gamma$ in Eq. (2.11) can be calculated from the data plotted in Fig. 2-4 by comparing half of the propagation wavelength obtained with L/2, i.e.

$$\gamma = \frac{k}{k_0} = \frac{\lambda_0}{\lambda} = \frac{\lambda_0}{\lambda/2} = \frac{0.016333 m}{2} = 0.0094733 m = 0.862$$

(2.21)

The above result is just 1% higher than the theoretical prediction of 0.853 discussed before. Thus, the validity of the theoretical prediction is verified.

Figure 2-4 Traveling wave propagation from the point at $y = 0.0078125 m$ on the source surface, solid line: classical plane wave; dotted line: case (1), infinite source; dashed line: case (2), finite source (coincides with the curve for case (1))
The normalized distribution of the Reynolds stress for the acoustic streaming along a vertical direction can be plotted by using the formula in the square bracket in Eq. (2.16)

$$S(y) = \sin^2 k_x y - \beta \cos 2k_x y$$

Coefficient $\beta$ is calculated to be 0.1684. Equation (2.22) is plotted in Fig. 2-5. As predicted, there are small but noticeable negative values across the propagation direction, which represent the Reynolds stress directed towards the surface. The positive values represent the Reynolds stress directed away from the surface. Comparing with the amplitude variation along
the source in Fig. 2-2, it is found that those negative values correspond to the node locations on the source surface while the maximum positive values correspond to the antinodes. Between each maximum and minimum values, the Reynolds stress distribution acts as a torque to generate one vortex. This means that there is one acoustic streaming circulation between each node and anti-node. Totally, one standing wave length generates four acoustic streaming circulations.

### 2.4. Conclusions

The acoustic field generated by a source with a non-uniform vibration amplitude variation is theoretically investigated in this chapter. It is found that non-uniformity changes the value of the wave number and attenuation coefficient from those of the classical plane wave field by a non-uniformity coefficient $\gamma$. Values of this coefficient from numerical simulations of the acoustic field and theoretical analysis are practically identical. The Reynolds stress for the acoustic streaming is also obtained. By retaining the traditionally neglected term in the Reynolds stress expression for the classical plane wave field, it is found that the acoustic waves generated from the node regions on the source surface produce Reynolds stress directed towards the surface while those from the anti-node regions produce Reynolds stress directed away from the surface. Based on this special property of the Reynolds stress field, a four-vortex-per-wavelength structure of the corresponding acoustic streaming field is predicted.
REFERENCES


PART TWO:

SONIC STREAMING
3. SONIC STREAMING COOLING EFFICIENCY FOR HEAT SOURCE AND HEAT FLUX

ABSTRACT

In this chapter, the efficiency of acoustic streaming for enhancing heat transfer in a channel composed of two parallel beams is studied. A rectangular heat source is attached to the upper beam. The lower beam, kept at a constant and uniform temperature, vibrates and scatters standing acoustic waves into the gap, which induces acoustic streaming in the gap due to the non-zero mean of the acoustic field. By utilizing the perturbation method, the compressible Navier-Stokes equations are decomposed into the first-order acoustic equations and the second-order streaming equations. Only the steady state energy equation associated with the streaming field is of interest because the acoustic field is adiabatic. These governing equations are discretized by the finite-difference method on a uniform mesh and solved numerically. Non-reflective boundary conditions are imposed at the open ends. SIMPLER algorithm is utilized to solve the streaming equation. The cooling effect is investigated by comparing the average temperature of the heated region of the upper beam with and without the acoustic streaming in the gap. Analysis of the streaming flow field reveals a system of steady vortices in the gap that are responsible for heat transfer enhancement. Acoustic streaming generated by vibration of the lower beam with the angular frequency of 1000 rad/s and the amplitude of 100 microns reduces the temperature of the upper beam by 1% for the constant heat flux case and by 0.5% for the case of a heat source with a constant rate of internal heat generation. A more significant cooling effect is expected if the intensity of the acoustic field is increased.
**NOMENCLATURE**

\( a \) \hspace{1cm} \text{thermal diffusivity, m}^2 \text{s}^{-1} \\
\( A_0 \) \hspace{1cm} \text{amplitude of vibration of the lower beam, m} \\
\( c_0 \) \hspace{1cm} \text{small-signal sound speed, m s}^{-1} \\
\( \mathbf{d} \) \hspace{1cm} \text{displacement vector of the vibrating (lower) beam from the mean position, m} \\
\( \mathbf{f}_b \) \hspace{1cm} \text{driving force for acoustic streaming field, N m}^{-3} \\
\( \mathbf{i} \) \hspace{1cm} \text{horizontal unit vector} \\
\( \mathbf{j} \) \hspace{1cm} \text{vertical unit vector} \\
\( L \) \hspace{1cm} \text{length of the channel, m} \\
\( L_b \) \hspace{1cm} \text{length of the standing wave region on the lower beam, m} \\
\( L_t \) \hspace{1cm} \text{length of the heated region on the upper beam, m} \\
\( H \) \hspace{1cm} \text{separation distance between the beams, m} \\
\( \dot{m} \) \hspace{1cm} \text{mass source term for the acoustic streaming equations, kg m}^{-3} \text{ s}^{-1} \\
\( Nu \) \hspace{1cm} \text{Nusselt number} \\
\( \mathbf{r} \) \hspace{1cm} \text{displacement vector of the lower beam (} \mathbf{r} = \mathbf{r}_0 + \mathbf{d} \text{), m} \\
\( \mathbf{r}_0 \) \hspace{1cm} \text{mean position vector of the lower beam (corresponds to} \ y = 0 \text{), m} \\
\( R_s \) \hspace{1cm} \text{streaming Reynolds number}
\( p \) fluid pressure, \( \text{Pa} \)

\( q'' \) heat flux, \( \text{W m}^{-2} \)

\( T \) temperature, \( \text{K} \)

\( T_w \) constant temperature maintained at the lower beam, \( \text{K} \)

\( u \) fluid velocity vector, \( \text{m s}^{-1} \)

\( u \) horizontal component of fluid velocity, \( \text{m s}^{-1} \)

\( u_c \) characteristic acoustic particle velocity, \( \text{m s}^{-1} \)

\( v \) vertical component of fluid velocity, \( \text{m s}^{-1} \)

\( x \) horizontal coordinate, \( \text{m} \)

\( y \) vertical coordinate, \( \text{m} \)

**Greek symbols**

\( \varepsilon \) dimensionless small parameter for the perturbation analysis

\( \mu \) dynamic viscosity, \( \text{kg m}^{-1} \text{s}^{-1} \)

\( \mu_B \) second viscosity, \( \text{kg m}^{-1} \text{s}^{-1} \)

\( \theta \) temperature difference \( (T - T_w) \), \( \text{K} \)

\( \rho \) fluid density, \( \text{kg m}^{-3} \)

\( \omega \) angular vibration frequency, \( \text{s}^{-1} \)

**Subscript**
3.1. Introduction

Acoustic streaming is the formation of a steady secondary vortex in the acoustic field. Acoustic streaming may result from either vibrations of a solid body adjacent to a fluid at rest or from an acoustic standing wave generated in a fluid adjacent to a solid wall (Vainshtein [1]). Lord Rayleigh [2] pioneered the analysis of acoustic streaming in a uniform duct. His investigation was continued by Westerwelt [3], Nyborg [4, 5], and Schlichting [6]. The work by Lighthill [7] revealed that the momentum flux that causes the streaming motion was available due to the attenuation of an acoustic energy flux.

The streaming Reynolds number, $R_s$, was first introduced by Stuart [8]. For a flow between two parallel walls, this parameter can be defined as suggested in ref. [1]:

$$R_s = \frac{3 \nu_0^2 h^2 \omega}{32 \nu c^2} \quad (3.1)$$

where $c$ is the speed of sound, $h$ is the distance between the beams, $\nu_0$ is the amplitude of oscillations, $\nu$ is the kinematic viscosity, and $\omega$ is the frequency of oscillations.
Davidson and Riley [9] performed a visualization of a jet flow from a vibrating cylinder. The flow between two cylinders, where the outer cylinder was at rest and the inner cylinder vibrating, was investigated in Duck and Smith [10]. A similar problem was investigated numerically by Kim and Troesch [11]. Streaming induced by an oscillating sphere was investigated for small streaming Reynolds numbers by Wang [12] and for large streaming Reynolds numbers by Amin and Riley [13]. Lee and Wang [14] investigated the acoustic streaming pattern near a small sphere due to two orthogonal standing waves that had the same frequency but were out of phase. Secondary streaming in a narrow cell caused by a vibrating wall was studied by Vainshtein et al. [15]. Matta et al. [16] investigated the effects of resonant acoustic oscillations on gas phase mixing in a rectangular cavity. Stansell and Greated [17] suggested the utilization of a lattice gas automation fluid modeling technique to study acoustic streaming phenomena arising from the interaction of sound waves with no-slip boundaries in a two-dimensional pipe.

Heat transfer due to forced convection caused by acoustic streaming induced by an oscillating circular cylinder was first studied by Richardson [18] and later by Davidson [19]. Heat transfer due to acoustic streaming from a sphere was studied by Gopinath and Mills [20]. In another paper, Gopinath and Mills [21] investigated heat transfer due to acoustic streaming across the ends of a Kundt tube. The effect of heat transfer enhancement by acoustic streaming between two parallel beams kept at different temperatures was analyzed in Vainshtein et al. [22]. Mozurkewich [23] presented the results of an experimental investigation of heat transfer from a cylinder in an acoustic standing wave generated in a free stream. He established that for a cylinder of fixed diameter and a fixed acoustic frequency, the Nusselt number showed a distinctive variation with acoustic amplitude. At high amplitude, the Nusselt number followed
a steady-flow, forced-convection correlation (time averaged over an acoustic cycle) while at a low amplitude, the Nusselt number had a constant value determined by natural convection.

Another direction of research in acoustic streaming dealt with studies of fluid flow and heat transfer in channels with pulsating walls. Many of these studies arose from considering certain biomechanical problems, for example the attempts to model blood flow in the coronary arteries of large mammals or to improve the understanding of gas transport in the airways of a lung for the purpose of developing a theoretical model of a lung undergoing a high-frequency ventilation. The flow in a parallel-sided channel with pulsating walls was examined by Secomb [24]. The wall motion was assumed to be of a small amplitude and sinusoidal. The oscillatory flow in a tube of slowly varying cross-section was examined by Hall [25]. Hydon and Pedley [26] analyzed fluid flow and solute transport in a long channel whose walls remain parallel but oscillate transversely. Dragon and Grotberg [27] studied oscillatory flow caused by a small-amplitude, long-wavelength, traveling wave generated by an oscillatory pressure gradient. Broday and Kimmel [28] investigated the oscillatory flow in a long elastic tube generated by small periodic radial displacements which formed a standing-wave mode of wall vibrations. Reference [28], contrary to ref. [27], did not resolve the viscoelastic problem to obtain the wall oscillations resulting from the given pressure oscillations, but simply assigned harmonic radial oscillations to the tube wall.

In this chapter, the enhancement of heat transfer between two parallel beams by acoustic steaming is investigated. By vibrating the lower beam, acoustic streaming is generated in the gap. This induces steady convection motion of the fluid in the gap. This convective motion carries the heat away from the upper beam which is heated by a solid heat source with a
constant rate of internal energy generation. This heat source represents a computer chip. The purpose of this investigation is to study the feasibility of utilizing acoustic streaming in a compact system for cooling computer chips in laptop computers.

3.2. DESCRIPTION OF THE PROBLEM

In this chapter fluid flow and heat transfer due to acoustic streaming in the gap between two horizontal beams, the lower of which is vibrating, is investigated. Natural convection effects are neglected. Two types of thermal boundary conditions at the upper beam are investigated: constant heat flux and heating by a heat source with a constant internal energy generation rate (the latter models a computer chip). A standing wave is excited in a lower beam, which can be achieved by coating the lower beam with a PZT material and subjecting it to alternating voltage. The forced convection flow induced by vibration of the lower beam carries the heat from the upper beam to the lower beam, which is then dissipated to the surrounding air. There is also some heat dissipation through the open ends of the channel. The purpose of studying this setup is to investigate the efficiency of acoustic streaming for enhancing heat transfer from the upper beam and discuss its possible applications for the cooling of computer chips. The schematic diagrams of the problem under consideration are displayed in Figs. 3-1a and 3-1b for the constant heat flux and heat source cases, respectively.
Figure 3-1 Schematic diagram of the problem for two types of thermal boundary conditions:
(a) Constant heat flux at the upper beam, (b) Heat source with a constant rate of energy generation
As shown in Fig. 3-1, the length of the channel is \( L \) and the separation distance between the beams is \( H \), the upper beam is subjected to a uniform heat flux distribution \( q'' \) over the length \( L_t \) (or, alternatively, a heat source of length \( L_t \) is attached to the central portion of the upper beam), and a standing wave is excited in the central region of the lower beam of length \( L_b \). The amplitude of the standing wave is \( A_0 \) and the length \( L_b \) may contain more than one wavelength, depending on the wave modes excited on the lower beam. \( L_t \) and \( L_b \) are not necessarily equal to each other, although they are assumed to be equal in this chapter.

According to the existing theory (Bradley [29] and Nyborg [30]), a steady acoustic streaming will form between the two beams. This acoustic streaming is induced by harmonic oscillation of the lower beam which has a zero mean displacement. It is this steady acoustic streaming that carries the heat away from the heat source at the upper beam to the lower beam and to the open ends of the channel. The first-order harmonic components of flow velocity form an adiabatic acoustic field and therefore have no cooling capacitance. The maximum acoustic streaming velocity in the gap depends upon the amplitude and frequency of vibration of the lower beam. Exciting a standing wave with an ultrasonic frequency in the lower beam will generate much greater acoustic streaming and hence have a more significant cooling effect. From the results of the experimental investigation and CFX modeling of a similar system reported in Ro and Loh [31] and Loh et al. [32], to achieve effective cooling the size of the gap between the beams must be in the range of 0.1 to 2mm.
3.3. **Theoretical Background**

Relevant theories of acoustic streaming are given in Bradley [29] and Nyborg [30]. Zhao et al. [33] compared two theories and noted that Bradley and Nyborg derived their theories in a similar way. However, the difference is that Nyborg dealt with rectilinear fluid motion while Bradley extended the theory to non-rectilinear cases. Therefore, Bradley’s theory will be used in this investigation.

The conservation equations of mass, momentum, and energy for a compressible fluid can be presented as:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (3.2)
\]

\[
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \left( \frac{\mu_B}{3} \right) \nabla \cdot \mathbf{u} \quad (3.3)
\]

\[
(\mathbf{u} \cdot \nabla) T = a \nabla^2 T \quad (3.4)
\]

No-slip boundary conditions on both fixed and vibrating solid surfaces are utilized. The pressure gradient in the direction normal to solid surfaces is assumed to be zero. Temperature at the lower beam is assumed to be constant and uniform. The central portion of the upper beam of length \( L_c \) is either subjected to a constant heat flux or to a perfect thermal contact with a heat source that has a constant rate of internal energy generation. The peripheral potions of the upper beam are adiabatic. At the open ends, the pressure is set equal to atmospheric pressure; also the horizontal velocity component and temperature are assumed to have zero gradients in the \( x \)-direction. Mathematically, the above boundary condition can be presented as:
\[ \frac{\partial \mathbf{u}(r,t)}{\partial t} |_{r=r_0+d(r_0,t)} = \frac{\partial}{\partial t} \mathbf{d}(r_0,t), \quad \mathbf{u}(r,t) |_{y=H} = 0, \quad \frac{\partial \mathbf{u}}{\partial x} |_{x=0,L} = 0, \quad (3.5a,b,c,d,e) \]

\[ \frac{\partial p}{\partial y} |_{y=0,H} = 0, \quad p |_{x=0,L} = P_{\text{air}} \]

\[ T(x,0) = T_w, \quad -k \frac{\partial T}{\partial y} |_{y=H} = q''(x), \quad \frac{\partial T}{\partial x} |_{x=0,L} = 0 \quad (3.6a,b,c) \]

where \( d \) is the displacement of the lower beam from its mean position (that corresponds to \( y = 0 \) and is denoted by the vector \( r_0 \)), \( r = r_0 + d \) is a vector indicating a point at a displaced position \((x, y)\), and \( T_w \) is the constant temperature at the lower beam.

To obtain equations for the steady component of the flow (the acoustic streaming), the pressure, density, and velocity are written in the form of perturbation expansions utilizing the following dimensionless small parameter:

\[ \varepsilon = \frac{u_c}{c_0} = f(\omega, A_0) \quad (3.7) \]

where \( c_0 \) is the small-signal sound speed and \( u_c \) is the characteristic acoustic particle velocity, which can simply be expressed as the product of the angular frequency and the displacement amplitude, i.e., \( u_c = \omega A_0 \). The perturbation expansions for the velocity \( \mathbf{u} \), pressure \( p \), and density \( \rho \) can be written as, respectively:

\[ \mathbf{u} = \mathbf{u}_0 + \varepsilon \mathbf{u}_1 + \varepsilon^2 (\mathbf{u}_{dc} + \mathbf{u}_2) + O(\varepsilon^3) \]

\[ p = p_0 + \varepsilon p_1 + \varepsilon^2 (p_{dc} + p_2) + O(\varepsilon^3) \]

\[ \rho = \rho_0 + \varepsilon \rho_1 + \varepsilon^2 (\rho_{dc} + \rho_2) + O(\varepsilon^3) \quad (3.8) \]
The variables with subscript ‘dc’ (direct current) are the acoustic streaming variables, which are non-harmonic and time-independent. All other variables are harmonic. Digital subscripts denote the order of the variables. All acoustic steaming variables appear in the second-order terms. The high-order solution is based on the low-order results. Therefore, the first-order equations must be solved utilizing the known zero-order results. In the situation considered in this chapter, the system is initially at rest. Therefore, the zero-order variables representing this initial rest status of the system are constant, i.e., \( u_0 = 0 \), and \( p_0 \) and \( \rho_0 \) are constant. The acoustic streaming equations contain the source terms that are represented through the first-order variables. The acoustic streaming is thus “driven” by the first-order harmonic solution.

The displacement of the vibrating surface can also be expressed in the perturbation approximation form as follows:

\[
\frac{\partial}{\partial t} \mathbf{d}(r_0, t) = u(r, t) \bigg|_{r=r_0+d(r_0,t)} = \varepsilon u_1(r, t) \bigg|_{r=r_0} + \varepsilon^2 \mathbf{d}_1 \cdot \nabla u_1(r, t) \bigg|_{r=r_0} + O(\varepsilon^3) \tag{3.9}
\]

where \( \mathbf{d} = \varepsilon \mathbf{d}_1 \).

This expansion shows that the velocity at the displaced surface can be presented as the summation of terms of different orders, which are calculated at the mean position of the lower beam, \( r_0 \). This makes it possible to utilize a fixed finite-difference grid because the mean position of the lower beam simply coincides with the \( x \)-axis. This greatly simplifies the numerical solution because there is no need to use an adaptive grid to track the vibration of the lower beam. Using the above approximations in the governing equations and boundary
conditions and collecting the terms that have the same powers of \( \varepsilon \), the equations of all orders can be obtained. The resulting equations for the first-order variables are:

\[
\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{u}_1 = 0 \tag{3.10}
\]

\[
\nabla \nabla \cdot \mathbf{u}_1 + \frac{\mu}{\rho_0 c_0^2} \nabla^2 \frac{\partial \mathbf{u}_1}{\partial t} + \frac{\mu}{\rho_0 c_0^2} \nabla \nabla \cdot \frac{\partial \mathbf{u}_1}{\partial t} - \frac{1}{c_0^2} \frac{\partial^2 \mathbf{u}_1}{\partial t^2} = 0 \tag{3.11}
\]

\[
p_1 = c_0^2 \rho_1 \tag{3.12}
\]

These equations must be solved subjected to the following boundary conditions:

\[
\mathbf{u}_1(r, t)|_{r=r_0} = \mathbf{u}(r, t)|_{r=r_0}, \quad \left. \frac{\partial \mathbf{u}_1}{\partial \mathbf{x}} \right|_{x=0, L} = 0 \tag{3.13a,b}
\]

Equation (3.12) represents the dependence of the first-order pressure on the first-order density, and is obtained assuming that the first-order solution exhibits adiabatic behavior like the sound wave, in which the entropy field does not change. This assumption is reasonable since Eq. (3.11) is the sound wave equation with viscous effect and its solution must be adiabatic. The corresponding first-order temperature also exhibits harmonic behavior, which, according to Temkin [34], has a zero mean value. Therefore, velocity in the convection term of the energy equation, Eq. (3.4), can be simply replaced with \( \mathbf{u}_{dc} \). The first-order acoustic equations, Eq. (3.10) — (3.12), are uncoupled, i.e., velocity \( \mathbf{u}_1 \) can be computed independently from Eq. (3.11) and then density \( \rho_1 \) and pressure \( p_1 \) can be computed from Eqs. (3.10) and (3.12) in sequence.

The second-order equations for the steady acoustic streaming and the energy equation for the steady temperature distribution in the gap can be presented as, respectively:
\[ \rho_0 \nabla \cdot \mathbf{u}_{dc} = \dot{m} \quad (3.14) \]

\[-\mu \nabla^2 \mathbf{u}_{dc} - \left( \mu_B + \mu/3 \right) \nabla \nabla \cdot \mathbf{u}_{dc} + \nabla p_{dc} = f_b \quad (3.15)\]

\[ (\mathbf{u}_{dc} \cdot \nabla) \theta = a \nabla^2 \theta \quad (3.16) \]

These equations must be solved subjected to the following boundary conditions:

\[ \mathbf{u}_{dc}(r, t) |_{r = r_0} = \mathbf{(d \cdot \nabla)u}_i (r, t) |_{r = r_0} \left. \frac{\partial \mathbf{u}_d}{\partial x} \right|_{x = 0, L} = 0, \]

\[ \frac{\partial p_{dc}}{\partial y} \bigg|_{y = 0, H} = 0, \quad p_{dc} \bigg|_{x = 0, L} = \text{constant} \quad (3.17a, b, c, d, e) \]

\[ \theta(x, 0) = 0, \quad -k \frac{\partial \theta}{\partial y} \bigg|_{y = H} = q''(x), \quad \frac{\partial \theta}{\partial x} \bigg|_{x = 0, L} = 0 \quad (3.18a, b, c) \]

where the temperature difference, \( \theta \), is defined as:

\[ \theta(x, y) = T(x, y) - T_w \quad (3.19) \]

Two kinds of thermal boundary conditions at the upper beam are considered in this investigation: the central region of the upper beam of the length \( L_t \) is either subjected to a constant heat flux (cf. Fig. 3-1a) or to a perfect thermal contact with a heat source that has a constant rate of internal energy generation (cf. Fig. 3-1b). In the latter case, a conjugate problem of heat conduction in the solid heat source and forced convection in the gap caused by acoustic streaming is solved. The pertinent energy equation that describes conduction in the heat source is:
\[ k_c \nabla^2 \theta + \dot{q} = 0 \]  \hspace{1cm} (3.20)

This equation must be solved subject to the following boundary conditions which postulate that the bottom of the heat source is in perfect thermal contact with the upper beam (which implies that the temperature and heat flux are continuous across the interface) and that three other boundaries of the heat source are adiabatic:

\[-k_c \frac{\partial \theta}{\partial y} \bigg|_{y=H} = q^* (x), \quad \frac{\partial \theta}{\partial y} \bigg|_{y=H+H_c} = 0, \quad \frac{\partial \theta}{\partial x} \bigg|_{x=L \pm L_c} = 0 \]  \hspace{1cm} (3.21a,b,c)

where \( k \) and \( k_c \) are the thermal conductivities of the fluid and the material of the computer chip (typically polysilicon), respectively.

In Eq. (3.17a), the symbol \(< >\) denotes the time-average over one oscillation period. Expressions for the source terms in Eqs. (3.14) and (3.15) can be obtained from the solution of the first-order equations as:

\[ \dot{m} = -\frac{1}{c_0^2} \nabla \cdot \langle I \rangle, \quad I = p_1 u_1 \]  \hspace{1cm} (3.22a,b)

\[ f_b = -\frac{1}{c_0^2} \left( p_1 \frac{\partial u_1}{\partial t} \right) - \rho_0 \langle (u_1 \cdot \nabla) u_1 \rangle \]  \hspace{1cm} (3.23)

In this chapter, the harmonic displacement at the vibrating boundary is assumed to be in the form of the following standing wave:

\[ d(t, x, 0) = j A \sin \omega t \sin kx, \quad \text{when} \ x \in L_b \]  \hspace{1cm} (3.24)

where \( j \) is the vertical unit vector, \( A \) is the standing wave amplitude.

Hence, the velocity at the surface of the lower beam is
and zero outside of the vibrating region. Since the upper beam is not vibrating, the fluid velocity at the surface of the upper beam is zero:

\[ u_i(t, x, H) = 0 \]  \hspace{1cm} (3.26)

The boundary conditions at the left and right open ends of the channel, which are given by Eq. (3.13b), are replaced by the 2\textsuperscript{nd}-order Non-Reflecting Boundary Condition (NRBC) from Givoli [35] to avoid the generation of a numerical reflecting wave at the open ends. Although there is no definite propagating direction of the wave at the open ends, which is somewhat different from the cases considered by Givoli [35], the 2\textsuperscript{nd}-order NRBC is shown to be effective and it considerably improves the convergence of the numerical procedure. The 2\textsuperscript{nd}-order NRBC can be presented as:

\[
\left. \left( \frac{1}{c_0} \frac{\partial^2}{\partial x \partial t} - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} + \frac{1}{2} \frac{\partial^2}{\partial y^2} \right) u_i \right|_{x=0,L} = 0
\]  \hspace{1cm} (3.27)

When time approaches infinity, the solution of the first-order harmonic equations must attain the periodic state. Therefore, the solution of the first-order equations can be presented in a complex number form, as a product of the time independent amplitude and the harmonic multiplier. The harmonic multiplier has the same frequency as the vibration frequency of the lower beam:

\[
u_i(t, x, y) = u_c \text{ Re} \left[ \hat{u}(x, y) \exp(i\omega t) \right]
\]

\[
p_i(t, x, y) = c_0^2 \rho_0 \text{ Re} \left[ \hat{p}(x, y) \exp(i\omega t) \right]
\]  \hspace{1cm} (3.28a,b)
Where \( u_1 = (u_1, v_1), \quad \hat{u} = (\hat{u}, \hat{v}) \) \hspace{1cm} (3.29a,b)

Substituting Eqs. (3.28a,b) and (3.29a,b) into first-order Eqs. (3.10) — (3.13a,b) and NRBC (3.27), the equations for the complex amplitudes \( \hat{u} \), \( \hat{v} \), and \( \hat{p} \) are obtained. These equations have the reduced wave equation form:

\[
\left[ 1 + i\omega (A_\mu + B_\mu) \right] \frac{\partial^2 \hat{u}}{\partial x^2} + i\omega A_\mu \frac{\partial^2 \hat{u}}{\partial y^2} + \left[ 1 + i\omega B_\mu \right] \frac{\partial^2 \hat{u}}{\partial x \partial y} + \frac{\omega^2}{c_0^2} \hat{u} = 0 \hspace{1cm} (3.30)
\]

\[
i\omega A_\mu \frac{\partial^2 \hat{v}}{\partial x^2} + \left[ 1 + i\omega (A_\mu + B_\mu) \right] \frac{\partial^2 \hat{v}}{\partial y^2} + \left[ 1 + i\omega B_\mu \right] \frac{\partial^2 \hat{v}}{\partial x \partial y} + \frac{\omega^2}{c_0^2} \hat{v} = 0 \hspace{1cm} (3.31)
\]

\[
\hat{p} = i \frac{c_0^2 \rho_0}{\omega} \left( \frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} \right) \hspace{1cm} (3.32)
\]

Equations (30)-(32) must be solved subjected to the following boundary conditions:

\[
\hat{u}(x,0) = 0, \hat{u}(x, H) = 0, \hat{v}(x, H) = 0 \hspace{1cm} (3.33a,b)
\]

\[
\hat{v}(x, 0) = \sin kx , \text{ when } x \in L_b \hspace{1cm} (3.33a,b)
\]

\[
\left( - \frac{i}{c_0} \frac{\partial}{\partial x} + \frac{1}{c_0^2} \frac{\partial^2}{\partial y^2} \right) \left. \hat{u} \right|_{x=0,L} = 0 \hspace{1cm} (3.34)
\]

\[
\left( - \frac{i}{c_0} \frac{\partial}{\partial x} + \frac{1}{c_0^2} \frac{\partial^2}{\partial y^2} \right) \left. \hat{v} \right|_{x=0,L} = 0 \hspace{1cm} (3.35)
\]

where
\begin{align}
A_\mu &= \frac{\mu}{\rho_0 c_0^2}, \quad B_\mu = \frac{\mu_0 + \mu/3}{\rho_0 c_0^2} \\
(3.36a,b)
\end{align}

The source terms given by Eqs. (3.22a) and (3.23) can be recast in terms of the amplitudes as follows:

\begin{align}
\dot{m} &= -\frac{1}{2} \rho_0 u_c \text{Re} \left[ \nabla \cdot \left( \overline{p_u} \right) \right] \\
(3.37)
\end{align}

\begin{align}
f_b &= -\frac{1}{2} \rho_0 u_c \omega \text{Re} \left[ i \rho \overline{u} + A \left( \overline{u} \cdot \nabla \right) \overline{u} \right] \\
(3.38)
\end{align}

\begin{align}
\left. \left( (\mathbf{d} \cdot \nabla) \mathbf{u}_i (\mathbf{r}, t) \right) \right|_{t=0} = \frac{1}{2} u_c A \sin kx \text{Re} \left[ \nabla \cdot (i \overline{u}) \right] \\
(3.39)
\end{align}

After \( \mathbf{u}_{dc} \) is computed, the temperature field in the gap can be computed from Eqs. (3.16) and (3.18a,b,c). In the constant heat flux case, \( q''(x) \) is prescribed over the length \( L_r \) of the central portion of the upper beam. In the heat source case, heat conduction equation, Eq. (3.20), is solved to obtain the temperature distribution within the heat source utilizing, as boundary conditions, the continuity of heat flux and temperature across the interface between the heat source and the upper beam.

### 3.4. Numerical Schemes

Equations for the amplitudes of the first-order variables, Eqs. (3.30) — (3.35), steady-state equations for acoustic streaming, Eqs. (3.14) and (3.15), and energy equation, Eq. (3.16), are solved numerically for two different thermal boundary conditions at the upper beam: a constant heat flux and a heat source with a constant internal energy generation. In discretizing
the equations, the second-order central difference scheme is utilized. In solving these
equations, the Gauss-Seidel iteration method is used. A uniform mesh with 501×21 grid points
within 100mm×1.5mm gap is utilized. With this numerical mesh, the region corresponding to
one wavelength (20mm) on the lower beam contains 100 grid points. For computations with
the heat source attached to the upper beam, a uniform mesh with 201×51 grid points within
40mm×5mm cross section of the heat source is used so that the mesh intervals are the same as
the intervals in the air gap. The source terms in the acoustic streaming equations are computed
from the solution for the amplitudes of the first-order variables according to Eqs.
(3.37)–(3.39).

Figure 3-2 SIMPLER staggered meshes (grid positions of horizontal, vertical streaming velocity
components and streaming pressure are denoted by right arrows, triangles, and circles respectively) and
boundary of computation domain (denoted by thick rectangle)
Since the pressure is present in Eq. (3.15), the SIMPLER method on a staggered mesh (Patankar [36]) shown Fig. 3-2 is utilized to overcome the possible false solution caused by the finite-difference discretization of the first-order derivatives. As shown in Fig. 3-2, the meshes of horizontal (denoted by arrows) and vertical (denoted by triangles) streaming velocity components and pressure (denoted by circle) are staggered with each other. Control volumes of each mesh are also shown as various shadow boxes in Fig. 3-2. For those boundaries at which no grid points locate, the boundary condition is represented by the geometrical average of the value on the grid points next to the boundary from both sides. The values on the grid points outside the computation domain are thus updated. For example, the horizontal velocities outside the top boundary are always updated as the reverses of those inside the top boundary due to the no-slip boundary condition.

In the case of a heat source, temperature fields in the gap and in the heat source are solved alternatively to update the heat flux and temperature along the interface between the heat source and the upper beam until they become continuous across the interface. Since the conductivity of the heat source is much larger than that of the air, temperature variation within the heat source is very small. Hence the relaxation number for updating temperatures at mesh points must be very small and the ratio of relaxation numbers for the heat source and the air is chosen as the inverse ratio of their conductivities.
### 3.5. **Numerical Results**

**Table 3-1 Parameter values utilized in computations**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap height</td>
<td>$H = 1.5\text{mm}$</td>
</tr>
<tr>
<td>Lengths of the vibrating and heated regions of the beams</td>
<td>$L_b = L_t = 40\text{mm}$</td>
</tr>
<tr>
<td>Vibration angular frequency</td>
<td>$\omega = 1000\text{rad/s}$</td>
</tr>
<tr>
<td>Wavelength</td>
<td>$\lambda = 20\text{mm}$</td>
</tr>
<tr>
<td>Vibration amplitude</td>
<td>$A_0 = 0.1\text{mm}$</td>
</tr>
</tbody>
</table>

**Constant heat flux case**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat flux</td>
<td>$q'' = 2,500\text{W/m}^2$.</td>
</tr>
</tbody>
</table>

**Heat source case**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat source length</td>
<td>$L_t = 40\text{mm}$</td>
</tr>
<tr>
<td>Heat source height</td>
<td>$H_c = 5\text{mm}$</td>
</tr>
<tr>
<td>Heat source width</td>
<td>$W_c = 40\text{mm}$</td>
</tr>
<tr>
<td>Heat source power</td>
<td>$P = 4\text{W}$</td>
</tr>
</tbody>
</table>

The parameter values utilized in these computations are summarized in Table 3-1. The fluid in the gap is assumed to be air. All computations with acoustic streaming are performed assuming sinusoidal vibration of the lower beam with an angular frequency of 1000 rad/s. To eliminate the effect of the open ends, the total length of the beams is chosen to be $L = 100\text{mm}$ so that there are identical extended regions (30mm each) on both lower and upper beams, as shown in Figs. 3-1(a) and (b). The expended regions of the upper beam are adiabatic and those of the lower beam are non-vibrating. The bulk viscosity $\mu_B$ is also considered and set to $0.6\mu$ according to Temkin [34]. A FORTRAN code has been written to implement a
finite-difference numerical solution of the equations described above. The code takes about 12 CPU hours on the SGI Origin 2400 workstation at the North Carolina Supercomputing Center to obtain the maximum relative error below $10^{-5}$. To check the grid independence of the solution, a finer mesh is also tested and no grid dependence of the solution is observed.

Figs. 3-3(a)—(h) show the flow pattern of the first-order velocity $u_1$ at different moments within one period of vibration $(0, (1/8)\hat{T}, (1/4)\hat{T}, \ldots, (7/8)\hat{T}, \hat{T})$, where $\hat{T}$ is the duration of the period. The velocities at the two beams are given by Eqs. (3.25) and (3.26). The velocity vector field at the end of the period is exactly the same as that in the beginning of the period (Fig. 3-3a) because the flow oscillates with the same frequency as the lower beam. The phase differences between most points within the flow field and the vibrating surface of the lower beam are very small. This can be seen from Figs. 3-3(c) and (g) that show almost a zero velocity vector field when the vibration velocity at the lower beam is zero. This is due to the small viscosity of air and the low vibration frequency studied.
(a) $t = 0$ period

(b) $t = 1/8$ period
(c) $t = 1/4$ period

(d) $t = 3/8$ period
(e) $t = 1/2$ period

(f) $t = 5/8$ period
Figure 3-3 Flow velocity vector field at different moments of time: (a) $t = 0$ period, (b) $t = 1/8$ period, (c) $t = 1/4$ period, (d) $t = 3/8$ period, (e) $t = 1/2$ period, (f) $t = 5/8$ period, (g) $t = 3/4$ period, (h) $t = 7/8$ period
Isopleths of the driving force $f_b$ and the mass source term $\dot{m}$ in the acoustic streaming equations, Eqs. (3.14) and (3.15), are shown in Figs. 3-4(a), (b) and 3-5, respectively. $\dot{m}$ is shown in the vicinity of the vibrating beam in Fig. 3-5 because it takes on non-zero values only in this region. Computations show that the results do not change if $\dot{m}$ is simply set to zero in Eq. (14). The horizontal component of $f_b$, which drives horizontal acoustic streaming velocity, has a much larger value near the lower beam than in other regions of the computational domain. This is the main driving force since the vertical component of $f_b$ has much smaller magnitude.

(a) horizontal component of the driving force
Figure 3-4 Isopleths of the vertical and horizontal components of the driving force, $f$, in the acoustic streaming equation, Eq. (3.38): (a) horizontal component of the driving force, (b) vertical component of the driving force.

Figure 3-5 Isopleths of the mass source term $\dot{m}$ in Eq. (3.37) near vibrating beam.
Isobars of acoustic streaming pressure field and the acoustic streaming velocity field are shown in Figs. 3-6 and 3-7(a), (b), respectively. There is one pressure “valley” in each of the two wavelengths. The acoustic streaming pressure is defined as a pressure relative to the ambient pressure because only the pressure gradient matters. The magnitude of the pressure variation is small compared to the atmospheric pressure of $10^5$ Pa. However, this may be different if the lower beam is vibrating with an ultrasonic frequency. The acoustic streaming velocity forms four circulations within each wavelength, where flow goes up over the nodes and down over the antinodes. The anti-node is the position on the lower beam where the amplitude of the standing wave has its local maximum while the node is the position that does not oscillate (the amplitude equals zero). These results agree with the experimental observations by Ro and Loh [31]. From Fig. 3-7a, it can be seen that the horizontal velocity is much larger than the vertical velocity in most regions of the gap. This is because the height of the gap is much smaller than its length and the wavelength of the vibration of the lower beam. This has an important effect on the cooling as discussed below.
Figure 3-6 Isobars of the acoustic streaming pressure field $p_{dc}$
Cooling of the upper beam is investigated using the acoustic streaming velocity field obtained above. This investigation is performed for both types of heat transfer boundary condition: constant heat flux and perfect thermal contact with a heat source that has a constant rate of internal energy generation. The results are summarized in Table 3-2.

The average heat transfer coefficient is defined as:

\[
\overline{h} = \frac{q}{L_i \overline{\theta}}
\]  

(3.40)

where \(q\) is the total power of the heat source and \(\overline{\theta}\) is the average temperature difference between the two beams. Then the Nusselt number can be expressed as follows:
Without acoustic streaming, the heat transfer is solely controlled by conduction, i.e.

\[ N_{u,\text{cond}} = \frac{\bar{h}_{\text{cond}} H}{k} = \frac{q H}{L_i k \bar{\theta}_{\text{cond}}} = 1 \]  

(3.42)

where \( \bar{\theta}_{\text{cond}} \) is the average temperature difference between the two beams for the case of no acoustic streaming (heat transfer by conduction only).

Hence the Nusselt number can be simplified as the temperature difference ratio:

\[ Nu = \frac{\bar{\theta}_{\text{cond}}}{\theta} \]  

(3.43)

Estimation shows that heat radiation is less than 1% of total heat exchange for the temperature difference between the upper and lower beams shown in Table 3-2; therefore, the effect of radiation is neglected.

Table 3-2 Estimated cooling effect for the constant heat flux and heat source with a constant rate of internal energy generation cases

<table>
<thead>
<tr>
<th></th>
<th>Average temperature difference the heated region of the upper beam to the lower beam (°C)</th>
<th>Nusselt number</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant heat flux case</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Air conduction only</td>
<td>137.61</td>
<td>1.000</td>
</tr>
<tr>
<td>Acoustic streaming is on</td>
<td>136.26</td>
<td>1.010</td>
</tr>
<tr>
<td><strong>Heat source case</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Air conduction only</td>
<td>135.94</td>
<td>1.000</td>
</tr>
<tr>
<td>Acoustic streaming is on</td>
<td>135.27</td>
<td>1.005</td>
</tr>
</tbody>
</table>
Computational results predict a large temperature difference (136.26°C for the constant flux case and 135.27°C for the heat source case) between the upper and lower beams. The Nusselt number is larger than unity by only 0.5 to 1%. This means that the heat transfer enhancement by acoustic streaming is not very large in this case because the acoustic streaming velocity is very small (40 and 7 mm/s are the maximum horizontal and vertical velocities, respectively) and the heat flux (2,500W/m³) is large. However, the perturbation expansions given by Eq. (3.8) suggest that \( u_{dc} \) is of the order of \( O\left(\varepsilon^2 = \omega^2 A_n^2 / c_n^2\right) \). If the frequency and/or the amplitude of vibration are increased (which means the acoustic intensity is increased), the acoustic streaming velocity will increase proportional to the square of the product of the vibration frequency and the amplitude of vibration. This means that the heat transfer efficiency of the suggested device will be much better for the ultrasonic vibration of the lower beam. However, this is just an estimation because when the acoustic intensity increases, the non-linear effect will be enhanced and the acoustic streaming velocity field may be completely different from what is observed for the low frequency case investigated in this chapter.

Isotherms of the temperature field in the gap for the constant heat flux case are shown in Fig. 3-8a; the temperature distribution along the surface of the upper beam for this case is shown by solid and dotted lines in Fig. 3-9a. It can be observed in Fig. 3-9a that there are a number of peaks in the temperature distribution, which means that there are hot spots on the surface of the upper beam. These spots occur exactly above the nodes of the standing wave on the lower beam. This is due to heat transfer by the air flow when the flow goes horizontally along the heated surface of the upper beam. In the beginning of this “journey,” the flow has just come up
after giving up the heat to the cooled surface of the lower beam and hence has a larger cooling capacity. At the end of this “journey,” the flow has been heated by the surface of the upper beam for quite a while and hence its cooling capacity drops to its minimum, just like in a heat exchanger. In reality, such a “steep” temperature distribution will not occur because internal conduction inside the IC chip will smooth out the peaks of the temperature distribution. Hence the heat flux distribution along the interface between the IC chip and the upper beam will adjust accordingly. The temperature of the air in the gap will also adjust.

(a) constant heat flux at the upper beam
The above prediction is confirmed by computational analysis of the problem in which a heat source with a constant rate of energy generation is attached to the upper beam. The thickness of the heat source (5mm in this case) and the intensity of the heat generation are chosen to realistically represent a computer chip. The chip is assumed to be adiabatic at every side except its bottom. The total heat rate that goes into the gap from the chip is exactly the same as in the previous computations where constant heat flux was assumed at the upper beam. The thermal conductivity of the heat source is assumed to be that of polysilicon \( k = 157 \text{ W/mK} \).
Temperature fields in the heat source and in the gap are shown in Figs. 3-8b and 3-9c, respectively. The temperature distribution along the surface of the upper beam for this case is shown by the dashed and dash dotted lines in Fig. 3-9a. A number of interesting observations can be made. First, all the “peaks” and “valleys” observed in the uniform heat flux case disappear; the temperature distribution along the interface between the heat source and the upper beam is almost uniform. This is because the thermal conductivity of polysilicon is thousands of times greater than that of the air. In other words, a slight temperature difference within the heat source causes significant heat flux that smoothes out any temperature variations within the chip. In fact, the temperature difference within the chip is less than 0.1%, which means that the heat source case almost exactly corresponds to the isothermal upper beam situation. Another important observation is that the mean temperature of the heated region of the upper beam is by 1K lower than that for the constant heat flux case (cf. Table 3-2). As it follows from Table 3-2, if there is no acoustic streaming, the average temperature of the heated region of the upper beam for the constant heat flux case is 1.67K higher than that for the heat source case. If acoustic streaming is turned on, the average temperature of the heated region of the upper beam drops by 1.35°C (from 137.61°C to 136.26°C) for the constant heat flux case and by 0.65°C (from 135.94°C to 135.27°C) for the heat source case. This means that turning acoustic streaming on gives a larger temperature decrease in the constant heat flux case. Therefore, the value of the Nusselt number is larger in the constant heat flux case (1.010) than in the heat source case (1.005). This is consistent with the classical internal forced convection result, in which the Nusselt number for a circular tube with an isoflux wall (4.36) is larger than that for a circular tube with an isothermal wall (3.66) (Bejan [37]). However, because the average temperature with no acoustic streaming for the constant heat flux case is much higher
than that for the heat source case, the predicted average temperature of the heated region of the upper beam for the heat source case (135.27°C) is still lower than that for the constant heat flux case (136.26°C). From Fig. 3-9b, it can be observed that in the heat source case the heat flux distribution is very non-uniform and that the heat flux increases near the edges of the heated region (which correspond to $x = 0.03$ and $0.07$ m, respectively).

(a) temperature distribution across the interface between the upper beam and the heat source
Figure 3-9 Temperature (a) and heat flux (b) distribution across the interface between the upper beam and the heat source and isotherms of the temperature field in the heat source (c).

(b) heat flux distribution across the interface between the upper beam and the heat source

(c) isotherms of the temperature field in the heat source
3.6. **CONCLUSIONS**

In this chapter, a perturbation method is utilized to analyze the acoustic streaming in the gap between two beams, one of which is vibrating and scattering sound waves into the gap. The first-order harmonic velocity field is computed numerically. This field works as the driving force for the second-order steady state acoustic streaming. Cooling effect due to acoustic streaming is estimated and a small but detectable temperature drop is predicted. The results obtained in this chapter suggest that if a higher (ultrasonic) vibration frequency or a higher vibration amplitude is utilized, the proposed device may be used instead or in conjunction with a heat sink for cooling of computer chips. This prediction needs to be verified by further numeric simulations for higher acoustic intensity cases.

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4. SONIC STREAMING FROM VIBRATING BEAM IN STANDING AND TRAVELING WAVEFORMS

**ABSTRACT**

The acoustic streaming flow and heat exchange mechanisms are investigated between a chip as a heat source and a PZT-covered beam scattering acoustic sound waves into the gap. Convective-like acoustic streaming forms in the gap and heat exchange between the two beams is enhanced because of that. The governing equations are asymptotically decomposed into the first order hyperbolic wave equations and second order time-independent acoustic streaming equations by a small parameter, which is defined as the ratio of the characteristic vibration velocity and the local sound speed. The first order harmonic equations are numerically solved in the reduced wave equation form. The steady acoustic streaming equations are then driven by the first order harmonic field and are solved by the SIMPLER method. Two types of acoustic waveforms from the acoustic source at the lower beam are compared in the flow pattern and heat exchange effect. The results show that although the flow patterns differ very much for different waveforms, there are only slight differences in the effect of the cooling effect.

4.1. **INTRODUCTION**

In the acoustic field where nonlinear effect is prominent, the steady fluid circulations called acoustic streaming will occur. When nonlinear effects, such as viscosity, induce enough phase difference between velocities and pressure, the interaction, which has a non-zero mean,
provides driving forces for the secondary acoustic streaming. Acoustic streaming becomes significant near the fluid-solid interface, where the solid boundary effect increases the Reynolds stresses and hence enhances the acoustic streaming. Some typical examples are a vibrating solid body adjacent to a fluid at rest and a free stream acoustic standing wave adjacent to a solid wall (Vainshtein [1]). Lord Rayleigh [2] attempted to analyze the acoustic streaming in a uniform duct. Further research of acoustic streaming was carried on by Westerwelt [3], Nyborg [4, 5], and Schlichting [6]. Lighthill [7] pointed out that the momentum flux that drives the streaming motion is caused by the attenuation of an acoustic energy flux. Nyborg [8] continued the early work of his own and placed emphasis on the effect of ultrasonic frequency on the acoustic streaming. Bradley [9] investigated the acoustic streaming in a non-rectilinear domain. Other recent reviews can be found in Riley [10, 11].

Two types of acoustic waves, standing and traveling waves, are typically utilized to induce the acoustic streaming. Davidson and Riley [12] performed a flow visualization of a jet flow from a vibrating cylinder. The flow between two cylinders with the outer cylinder at rest and the inner cylinder vibrating was investigated by Duck and Smith [13]. Kim and Troesch [14] numerically solved a similar problem. Acoustic streaming in a narrow cell caused by a vibrating wall was studied by Vainshtein et al. [15]. For the standing wave excitation, Lee and Wang [16] investigated the acoustic streaming field near a small sphere at rest and induced by two orthogonal standing waves with the same frequency but out of phase, which were excited in the fluid.

Acoustic streaming can affect heat transfer by forced convection. Richardson [17] and Davidson [18] later studied this mechanism for an oscillating circular cylinder. Later,
Gopinath and Mills [19] investigated heat transfer by acoustic streaming across the ends of a Kundt tube. The effect of heat transfer enhancement by acoustic streaming between two parallel beams kept at different temperatures was analyzed in Vainshtein et al. [20]. Mozurkewich [21] showed the results of an experimental study of heat transfer from a cylinder in a free stream acoustic standing wave. He concluded that the Nusselt number varies with acoustic amplitude if the diameter of a cylinder and the acoustic frequency are fixed. At high amplitudes, the Nusselt number coincides with that given by a steady-flow, forced-convection correlation (one period mean). At low amplitudes it keeps a constant value, which is determined by natural convection analysis.

Recent research in acoustic streaming focuses on pulsating walls as an acoustic source. The flow in a channel bounded by two parallel pulsating walls was examined by Secomb [22]. Hydon and Pedley [23] analyzed fluid flow and solute transport in a long channel whose walls remain parallel but oscillate transversely. Broday and Kimmel [24] investigated the oscillatory flow in a long elastic tube generated by small periodic radial displacements, which formed a standing-wave mode of wall vibrations.

Ro and Loh [25] experimentally studied the cooling effect of acoustic streaming excited by a vibrating beam positioned near the heat source. Significant temperature drop of the heat source surface was observed. Either a standing or a traveling wave was generated on the surface of the beam. They found that the cooling effect for both types of the acoustic waves is almost the same. In Chapter 3, Wan and Kuznetsov [26] numerically investigated a case similar to that experimentally investigated in Ro and Loh [25]. They first studied the sonic standing wave
effect on the properties of the acoustic streaming and the heat exchange within the gap. Then they further estimated the ultrasonic situation and predicted much better cooling effects.

The heat dissipation of an integrated circuit (IC) board becomes a main concern when the density and power of the IC components on the board increases rapidly. In this chapter, the cooling effect induced by acoustic streaming is studied similarly to problem investigated in Chapter 3 by Wan and Kuznetsov [26] and but considering both standing and traveling waves. Acoustic streaming is expected to enhance the cooling efficiency. The problem is modeled in a 2D rectangular domain. The flow patterns and temperature distributions are obtained by solving numerically conservation equations with appropriate boundary conditions. The effect of different types of acoustic waves (standing versus traveling waves) that are generated on the surface of the beam, on the velocity field in the gap and heat transfer enhancement, is the main interest of this chapter. The results are compared with the experimental findings of Ro and Loh [25].

4.2. STATEMENT OF THE PROBLEM

In this chapter the heat transfer enhancement by the acoustic streaming between two parallel beams is studied. Natural convection is neglected. The acoustic streaming is induced by the sound wave scattered from the lower beam, on whose surface either standing or traveling wave is generated. It is assumed that the temperature of the lower beam is constant and uniform. A rectangular solid heat source with spatially uniform heat generation is placed on the upper beam. The induced steady acoustic streaming circulation is expected to decrease the temperature difference between the beams. By coating the lower beam with PZT, its vibration
frequency can be made very high (in the ultrasonic range), which can induce significant acoustic streaming velocities. The main purpose of this chapter is to compare the heat transfer efficiency for the cases of acoustic streaming induced by a standing or a traveling wave generated on the surface of the lower beam.

The schematic diagram of the problem is displayed in Fig. 4-1.

Two beams with the same length $L$ are placed parallel to each other. The separation distance between the beams is $H$. Solid heat source of length $L_t$ and width $H_c$ producing constant volumetric heat generation $\dot{q}$ is symmetrically placed on the top of the upper beam. All the other three sides of the heat source are insulated except the interface with the upper beam. The
vibration region on the lower beam is $L_b$ long and is also symmetrically placed right beneath the heat source. The vibration amplitude is $A_0$. In this chapter, the lengths of the vibration region and the heat source are set to be equal, $L_t = L_b$.

Existing theories suggest that steady acoustic streaming is expected to occur between two beams because of the harmonic vibration of the lower beam. This steady acoustic streaming enhances heat transfer between the beams. The magnitude of the acoustic streaming depends on the system configuration, gap size, vibration parameters, etc. Ultrasonic vibration is expected to generate much greater acoustic streaming and hence produce remarkable cooling effect. Ro and Loh [25] considered a similar system experimentally. Either a standing or a traveling wave was generated on the surface of the lower beam. There is almost no difference in heat transfer efficiency for these two cases as was observed in their experiments. This chapter attempts to explain theoretically this surprising experimental outcome.

### 4.3. Theoretical Background

#### 4.3.1. Standing Wave Case

For the theory of standing wave case, one can refer to Sec. 3.3.

#### 4.3.2. Traveling Wave Case

The harmonic displacement of the vibrating boundary, if a sine traveling wave is generated on the surface of the lower beam, can be written as:
\[ \mathbf{d}(t, x, 0) = jA \sin(\omega t - kx), \text{ when } x \in L_b \]  

(4.1)

Hence the velocity at the boundary is

\[ \mathbf{u}_1(t, x, 0) = j\omega A \cos(\omega t - kx), \text{ when } x \in L_b \]  

(4.2)

\[ \mathbf{u}_1(t, x, H) = 0 \]  

(4.3)

Equation (4.2) shows that the first order velocity field for the traveling wave boundary condition can be composed of the velocity fields computed for two different types of standing wave boundary conditions. The velocity amplitude vector considered in the previous section, \( \mathbf{u} \), is redefined as \( \mathbf{u}_{sn} \) to represent the velocity amplitude vector of the sine-type standing wave boundary condition. Further, a similar vector \( \mathbf{u}_{cs} \) is defined for the cosine-type standing wave boundary condition. Then the first order velocity field for the traveling wave boundary condition can be written as:

\[ \mathbf{u}_1(t, x, y) = u_c \text{ Re} \left\{ \mathbf{u}_{cs}(x, y) - i\mathbf{u}_{sn}(x, y) \right\} \exp(i\omega t) \]  

(4.4)

### 4.4. Numerical Results

The first order amplitude equations (3.30) — (3.35), the acoustic streaming equations and energy equations (3.14) — (3.23), coupled with heat source conduction equations (3.20) and (3.21a,b,c), can be solved numerically. A second order central difference scheme is used and the Gauss-Seidel iteration is utilized. Uniform meshes with 501\times21 grid points for 100mm\times1.5mm gap and 201\times41 grid points within 40mm\times3mm heat source are utilized. In
this mesh scheme, one wavelength (20mm) of the lower beam vibration contains 100 grid points. To solve the acoustic streaming equations, the driving source terms are computed from the first order amplitude results obtained by numerically solving Eqs. (3.37)—(3.39). Since the pressure is also present in Eq. (3.15), the staggered grid method SIMPLER (Patankar [27]) is utilized to overcome the possible false solution caused by the finite difference discretization of the first order derivatives.

The parameter values utilized in computations are: listed in Table 3-1 and volumetric heat generation (\( \dot{q} \)) is 3.3MW/m\(^3\). The fluid is assumed to be air. The thermal conductivity of the heat source is assumed to be the same as of polysilicon (\( k = 157W/m^\circ K \)) to simulate a computer chip. To eliminate the open end effect, a larger total length of the beam, \( L = 100\)mm, is used so that there are non-vibrating regions (30mm on each side of the beam), as shown in Fig. 4-1. The bulk viscosity \( \mu_B \) is also considered and set to 0.6\( \mu \) according to Temkin [28]. A FORTRAN code was written to solve the discretized equations with these parameters. It was run on the SGI Origin 2400 workstation at North Carolina Supercomputing Center for about 24 hours until the maximum relative error became smaller than 10\(^{-5}\). A Finer mesh was also tested and the same results were obtained. In the following discussion, the results for standing and traveling wave cases are compared.
(a) $t = 0$ period

(b) $t = 1/8$ period
(c) $t = 1/4$ period

(d) $t = 3/8$ period
(e) $t = 1/2$ period

(f) $t = 5/8$ period
Figure 4-2 Flow velocity at different moments of time during one period
(traveling wave is generated on the surface of the lower beam, $\omega = 1000$, air)
Figs. 3-3 (a)—(h) and 4-2 (a)—(h) show the typical flow patterns of the first order velocity, $u_1$, at different moments of time during one time period (0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, and 7/8). The boundary velocities at two beams conform to Eqs. (3.25) or (4.1), and (3.26). The phase difference of most points within the flow field and the points on the vibrating boundary is very small. This conclusion is justified by Figs. 3-3c, which shows that an almost zero velocity field in the gap is observed when the displacement of the surface of the lower beam is maximum (the velocity of the beam surface at that moment of time is zero). Although there is no zero velocity distribution at the lower beam for the traveling wave case, the fluid in the gap responds to the lower beam vibration quite synchronously, as shown in Figs. 4-2 (a)—(d). This may be due to the small viscosity of air and the low frequency studied. In the ultrasonic region, the phase difference is expected to be large and hence have significant effect on acoustic streaming.

The horizontal component of the driving force $f_b$, which drives the horizontal acoustic streaming velocity component, has a much larger value near the lower beam. Actually this is the main driving force since the vertical component of $f_b$ has much smaller magnitude. It should be noted that $\tilde{m}$ is the compensation for the replacement of $\rho_1$ with $\rho_0$ in the first order mass conservation equation (3.10). It can be observed that $\tilde{m}$ is much smaller compared to the driving force $f_b$ and only accumulates in the vicinity of the vibrating beam for both traveling and standing wave cases. A noticeable difference is that the “mass source” term $\dot{m}$ in the traveling wave case is about 100 times larger than in the standing wave case. The locations where the driving force takes on its maximum and minimum values are also not the same for the traveling and standing wave cases.
Figure 4-3 Acoustic streaming velocity field $u_{ac} (\omega = 1000, \text{air})$
The magnitudes of pressure variation for both cases are also small compared to the atmospheric pressure of 10^5 Pa. However, this may change if an ultrasonic frequency is used to excite the lower beam. The acoustic streaming velocity fields are shown in Figs. 3-7 (a)—(b) and 4-3 (a)—(b). The acoustic streaming velocity patterns exhibit more differences for the two cases. In the standing wave case, there are four symmetric circulations in each wavelength, where flow goes up over the vibration nodes and goes down over the antinodes. (Anti-node is where the standing wave has its local maximum while node never oscillates.) These results agree with the experimental observations by Ro and Loh [25]. In the traveling wave case, more pseudo-symmetric circulations and much larger velocities are observed in the vicinity of the vibrating lower beam. This may be due to a large “mass source” term, \( \dot{m} \), and the fact that the traveling wave has a dominant propagation direction (to the right in this case). There are also some similarities between the two cases. From Figs. 3-7a and 4-3a, one can see that the horizontal velocity components are much larger than the vertical components in most regions. This is because the height of the gap is much smaller than both the length of the beam and the wavelength of oscillation. This has some effect on the cooling results as discussed below.

<table>
<thead>
<tr>
<th>Table 4-1 Estimated cooling effect for standing and traveling wave cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase factor (times)</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>Standing wave case</td>
</tr>
<tr>
<td>Standing wave case</td>
</tr>
<tr>
<td>Traveling wave case</td>
</tr>
<tr>
<td>Without convection</td>
</tr>
</tbody>
</table>

85
Cooling effect is investigated using the acoustic streaming velocity field obtained above. However, the acoustic streaming velocity is very small (in the standing wave case the magnitudes of the horizontal and vertical velocities are 40 and 7\,mm/s, respectively) because of the low vibration frequency. From the perturbation approximation in Eq. (3.8), $u_{dc}$ is of the order of $O\left(\varepsilon^2 = \frac{\omega^2 A_0^2}{c_0^2}\right)$. Hence the high frequency effect can be estimated by multiplying $u_{dc}$ by an appropriate increase factor. For example, for the ultrasonic frequency of 20kHz, the appropriate increase factor is 15775. In the following discussion, the increase factor is set to be 100 to make sure that the cooling effect for high frequency case is not overestimated. The cooling effect estimation for both cases is shown in Table 4-1, which shows that the effect of increasing frequency to the ultrasonic range (estimated by increase factor), is dramatic. In the ultrasonic range, the temperature drop is expected to be much smaller than for the best existing heat sinks. It is also interesting that the difference in the cooling effect for the standing and traveling wave cases is negligible. Although the generation of a traveling wave in a lower beam instead of a standing wave results in a deformation of the circulation pattern of the air in the gap, the overall structure of the circulation pattern remains unchanged. Indeed, comparing Figs. 3-7b and 4-3b, one can see that in both cases the number of main circulations is the same (four). The magnitudes of convection velocities are also very close (cf. Figs. 3-7a and 4-3a). Therefore, the overall convection effect is the same, and the temperature difference between the upper and lower beams is also the same in these two cases.
(a) air (standing wave)

(b) chip (standing wave)
Figure 4-4 Normalized temperature fields (increase factor 100, $\bar{\theta} = 160.2K$ for standing wave, $\bar{\theta} = 160.4K$ for traveling wave) (a, c line type: solid $y = 0$, dashed $y = 0.3\text{mm}$, dashdot $y = 0.6\text{mm}$, dotted $y = 0.9\text{mm}$, longdash $y = 1.2\text{mm}$, dashdotdot $y = 1.5$; b, d line type: solid $y = 1.5\text{mm}$, dashed $y = 2.1\text{mm}$, dashdot $y = 2.7\text{mm}$, dotted $y = 3.3\text{mm}$, longdash $y = 3.9\text{mm}$, dashdotdot $y = 4.5$)
Temperature fields in the heat source and in the gap are shown in Figs. 4-4 (a, b) for the standing wave case. Both fields have been normalized by the mean interface temperature difference with the lower beam, $\bar{\theta} = 160.2K$. Normalized temperature, $\theta^*$, is defined as the ratio of the temperature ($\theta$) and its mean value along the horizontal direction ($\bar{\theta}$):

$$
\theta^* = \frac{\theta}{\bar{\theta}}
$$

(4.5)

A number of interesting observations can be made. The temperature variation along the bottom of the chip is almost negligible. This is because the thermal conductivity of polysilicon is thousands of times greater than that of the air. In other words, slight temperature difference in the chip can cause significant variation in heat flux. This is also true for the temperature field inside of the heat source shown in Fig. 4-4b, where the maximum temperature is only 0.1% larger than the mean temperature of the interface. Results for the traveling wave case computed for the same parameter values are shown in Figs. 4-4(c) and (d). The mean temperature difference between the two beams, $\bar{\theta}$, is 160.4K. Although the cooling effect is almost the same, the regions of lower temperature occupy more space in the gap for the traveling wave case. The temperature field inside the heat source is also more uniform for the traveling wave case. The temperature fields in the gap and in the chip for the traveling wave case are not symmetric because the traveling wave has a definite direction of propagation.

4.5. **Conclusions and Remarks**

In this chapter, the perturbation approximation method is utilized to analyze the acoustic streaming in a gap between two beams. Either a standing or a traveling wave is generated on
the surface of the lower beam. The first order harmonic velocity field in the gap is computed numerically. This first order velocity field works as a driving force for the second order steady state acoustic streaming. The velocity field for the low frequency excitation of the lower beam is computed and the high frequency (ultrasonic) cooling effect is estimated based on the low frequency results. Considerable temperature drop for the ultrasonic case is predicted. This prediction needs to be verified by further numerical simulations of the real ultrasonic frequency case. The case of a traveling wave generated on the surface of the lower beam is compared with the case of a standing wave generated. This comparison reveals that although the generation of a traveling wave in the lower beam results in a deformation of a circulation pattern of the air in the gap, this does not significantly affect the temperature difference between the plates. This result is supported by the experimental observations of Ro and Loh [25]. Because it takes much larger energy input to generate a traveling wave, the conclusion is that the utilization of a standing wave will result in a better overall performance of this novel cooling device.

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5. SONIC STREAMING FROM A BEAM VIBRATING IN HALF STANDING WAVELENGTH AND COMPARISON WITH EXPERIMENT

5.1. Theory

The acoustic field in a fluid with attenuation is always accompanied by the unidirectional flow called acoustic streaming. Bradley [1] and Nyborg [2] analyzed this phenomenon by assuming that the acoustic streaming does not affect the first order acoustic field and appears in the second order terms. Utilizing the perturbation method and the compressible Navier-Stokes equations, the first order acoustic equations and second order acoustic streaming equations are obtained. The acoustic field with attenuation is governed by the following first order acoustic equations:

\[
\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{u}_1 = 0 \tag{5.1}
\]

\[
\nabla \nabla \cdot \mathbf{u}_1 + \frac{\mu}{\rho_0 c_0^2} \nabla^2 \frac{\partial \mathbf{u}_1}{\partial t} + \frac{\mu + \mu_B}{\rho_0 c_0^2} \nabla \nabla \cdot \mathbf{u}_1 - \frac{1}{c_0^2} \frac{\partial^2 \mathbf{u}_1}{\partial t^2} = 0 \tag{5.2}
\]

\[
p_1 = c_0^2 \rho_1 \tag{5.3}
\]

where \( \mathbf{u}_1 = (u_1, v_1) \) is the acoustic velocity vector; \( \rho_1 \) and \( p_1 \) are the acoustic density and acoustic pressure, respectively; \( \rho_0 \) is the density of the fluid at rest; \( c_0 \) is the speed of sound; \( t \) is time; \( \nabla \) is the differential operator in coordinates \((x, y)\); \( \mu \) and \( \mu_B \) are the fluid dynamic viscosity and the bulk viscosity, respectively.
The streaming field is governed by the second order acoustic streaming equations as

\[ \rho_0 \nabla \cdot \mathbf{u}_{dc} = \dot{m} \]  \hspace{1cm} (5.4)

\[ - \mu \nabla^2 \mathbf{u}_{dc} - (\mu \beta + \mu/3) \nabla \nabla \cdot \mathbf{u}_{dc} + \nabla p_{dc} = \mathbf{f}_b \]  \hspace{1cm} (5.5)

where \( \mathbf{u}_{dc} = (u_{dc}, v_{dc}) \) is the streaming velocity vector and \( p_{dc} \) is the streaming pressure. The acoustic field drives the streaming field through the acoustic intensity \( I = p_1 \mathbf{u}_1 \) (\( \dot{m} = -\frac{1}{c_0^2} \nabla \cdot \langle \mathbf{I} \rangle \)) and the driving force \( \mathbf{f}_b = -\frac{1}{c_0^2} \left\langle \frac{\partial \mathbf{u}_1}{\partial t} \right\rangle - \rho_0 \langle (\mathbf{u}_1 \cdot \nabla)\mathbf{u}_1 \rangle \), where < > denotes time average of the variable.

\[ \text{Figure 5-1  Schematic diagram of the cooling device of acoustic streaming} \]

In this chapter, the cooling efficiency of the acoustic streaming within a channel is investigated. As shown in the schematic diagram in Fig. 5-1, the channel is composed of a bimorph beam at the bottom, which vibrates in its first mode at a certain electric field intensity, and a stationary beam on the top, which is attached to a rectangular solid heat source outside of the channel.
The heat source is characterized by a constant internal heat generation rate and is insulated on all sides except the side attached to the stationary beam. Under these circumstances, the heat generated within the heat source can only be dissipated through the channel. The bimorph beam is kept at a constant temperature and the thickness of both beams is negligible. The coordinate setup for this problem is also shown in Fig. 5-1. With the regions near both ends fixed, the bimorph beam vibrates in its first standing wave mode in the middle of the beam (denoted as \( L_b \)) with the displacement \( d(t, x) \) described as

\[
d(t, x) = jA \sin \omega t \sin kx \quad \text{(when} \ x \in L_b),
\]

(5.6)

where \( A, \omega \) and \( k \) are the displacement amplitude, angular frequency and the wave number of the acoustic field, respectively, and \( j \) is the unit vector along the \( y \) direction. According to Wan and Kuznetsov [3], derivation of the beam displacement Eq. (5.6) gives the beam vibration velocity, which by Taylor expansion is the sum of the acoustic velocity at the mean position of the vibrating beam

\[
\mathbf{u}_1(t, x, 0) = j \omega A \cos \omega t \sin kx \quad \text{(when} \ x \in L_b),
\]

(5.7)

and the acoustic streaming velocity at the mean position of the vibrating beam

\[
\mathbf{u}_{ac}(x, 0) = \left( \mathbf{d} \cdot \nabla \right) \mathbf{u}_1(t, x, 0) \quad \text{(when} \ x \in L_b),
\]

(5.8)

and higher order negligible components.

Instead of solving the time-dependent wave equations (3.10)—(3.12), this chapter utilizes the reduced wave equations given by Eqs. (3.10)—(3.12), which are derived by Wan and Kuznetsov [3] in Chapter 3. These equations can be presented in the steady state and complex number forms, as
\[
\begin{align*}
\left[1 + i\omega(A_\mu + B_\mu)\right]\frac{\partial^2 \hat{u}}{\partial x^2} + i\omega A_\mu \frac{\partial^2 \hat{u}}{\partial y^2} + \left[1 + i\omega B_\mu \right] \frac{\partial^2 \hat{v}}{\partial x \partial y} + \frac{\omega^2 c_0^2}{c_0^2} \hat{u} = 0 & \quad (5.9) \\
\end{align*}
\]

\[
\begin{align*}
\omega A_\mu \frac{\partial^2 \hat{v}}{\partial x^2} + \left[1 + i\omega(A_\mu + B_\mu)\right] \frac{\partial^2 \hat{v}}{\partial y^2} + \left[1 + i\omega B_\mu \right] \frac{\partial^2 \hat{u}}{\partial x \partial y} + \frac{\omega^2 c_0^2}{c_0^2} \hat{v} = 0 & \quad (5.10) \\
\end{align*}
\]

\[
\hat{p} = \frac{c_0^2 \rho_0}{\omega} \left( \frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} \right) & \quad (5.11)
\]

where \( \hat{u} = (\hat{u}, \hat{v}) \) is the amplitude of the vibrating acoustic velocity; \( \hat{p} \) is the amplitude of the vibrating pressure; and \( i \) is the imaginary unit. The attenuation related parameters are

\[
A_\mu = \frac{\mu}{\rho_0 c_0^2} \quad \text{and} \quad B_\mu = \frac{\mu_B + \mu/3}{\rho_0 c_0^2}. \]

According to Wan and Kuznetsov [3] in Chapter 3, the driving terms in the Eqs. (3.14) and (3.15) can be presented through the acoustic amplitudes, as

\[
\begin{align*}
\hat{m} = -\frac{1}{2c_0^2} \text{Re} \left[ \nabla \cdot (\hat{\rho} \hat{u}) \right] \quad \text{and} \quad \hat{f}_b = -\frac{1}{2c_0^2} \text{Re} \left[ i\hat{\rho} \hat{u} + \rho_0 (\hat{\rho} \cdot \nabla) \hat{u} \right] & \quad (5.12a,b)
\end{align*}
\]

where \( \text{Re}(a) \) represents the real part of the variable \( a \) and the overbar denotes the conjugate of the complex variable.

Similarly, the amplitude of the beam vibrating acoustic velocity at mean position can be derived from Eq. (3.25) as

\[
\hat{u}(x,0) = 0, \quad \hat{v}(x,0) = \omega A \sin kx \quad \text{(when} \ x \in L_b) \quad \text{(5.13)}
\]

The acoustic streaming velocity at the mean position of the vibrating beam, Eq. (5.8), can be rewritten as
Since the vibrating acoustic field is generally adiabatic, the accompanying unidirectional streaming flow contributes to the convective heat transfer and can carry away the heat from the hot surface. According to Wan and Kuznetsov [3] in Chapter 3, the steady state energy equation for the streaming flow is

\[(\mathbf{u}_{dc} \cdot \nabla)\theta = a \nabla^2 \theta \quad (5.15)\]

where \(\theta(x, y) = T(x, y) - T_w\) is the difference between temperature \(T(x, y)\) and the reference temperature \(T_w\); and \(a\) is the thermal diffusivity of the fluid.

If there is a solid heat source behind the hot surface, the energy equation for the heat source at steady state is

\[k_c \nabla^2 \theta + \dot{q} = 0 \quad (5.16)\]

where \(k_c\) is the thermal conductivity of the solid and \(\dot{q}\) is the constant heat generation rate within the solid.

### 5.2. Numerical Schemes and Results

To simulate the experimental results of the cooling efficiency of acoustic streaming, which is performed by Ro and Wu [4] and Wu [5], one can first obtain numerically the acoustic field within the gap, which is governed by Eqs. (3.30)—(5.11) and driven by the acoustic source (Eq. (5.13)) on the vibrating beam. Thus, the driving terms in Eqs. (5.12a,b) can be calculated and
the streaming field governed by Eqs. (3.14) and (3.15) together with the boundary condition Eq. (3.39) can also be numerically obtained. Consequently, the convection temperature field of the acoustic streaming, which is governed by Eq. (3.16) and coupled with the heat source conduction field governed by Eq. (3.20) at the stationary beam, can be obtained numerically. The cooling efficiency is investigated by comparing the mean temperatures of the heat source surface when the acoustic streaming is on and off. The temperature field with acoustic streaming off can be obtained by simply setting the acoustic streaming velocity $u_{ac}$ to zero, then heat conduction is the only cooling mechanism.

The boundary conditions at the open ends of the channel for all the governing equations are zero-gradient boundary conditions except for the first order acoustic Eqs. (3.30)—(5.11), where the second order non-reflective boundary conditions are applied (Givoli [6], Wan and Kuznetsov [3], Chapter 3). On the surface of the stationary beam and at the fixed part of the vibrating beam, no-slip boundary conditions and zero pressure gradients at normal directions are utilized. Adiabatic boundaries conditions are utilized on all sides of the heat source except at the interface with the stationary beam, where the temperature and heat flux of the heat source side couples with those of the fluid at the other side in the channel.

All of the governing equations are discretized by the finite volume method and numerically solved by the Gauss-Seidel iteration method. In addition, the SIMPLER method is utilized in solving the acoustic streaming equations (3.14) and (3.15). A uniform $501 \times 21$ mesh is generated in the channel domain and a uniform $94 \times 51$ mesh is generated in the heat source domain. Both meshes are shown in Fig. 5-2. The parameters from the experiments are: 280Hz vibrating frequency, 130µm vibrating amplitude, 1$^{st}$ mode resonance, 2.5” length of vibrating
region, 1.4”×1” ×0.4” silicon heat source with 3.9W power, 650µm gap filled with air. In these computations, the fixed regions near both ends of the bimorph are set to be 2.5”. Since the numerical model is two-dimensional, the heat generation rate is calculated based on the power and the dimensions of the heat source.

The acoustic streaming from computational results is shown in vector and streamlines forms in Figs. 5-3(a) and (b). The maximum streaming velocity is found to be 0.821m/s at \((x, y) = (0.08001m, 0.00013m)\) and \((0.11049m, 0.00013m)\) in present coordinates. These two locations are horizontally 0.01524m away from the anti-node and 0.00013m above it.

The streaming flow goes from the anti-node and down towards two nodes and moves from two nodes to the anti-node along the vibrating beam. This phenomenon agrees with the experimental report from Loh [7] and numerical results from Wan and Kuznetsov [3] in Chapter 3.
Figure 5-3 Acoustic streaming velocity field $u_{ac}$ in form of vectors (a) and streamlines (b)
Figure 5-4 Temperature fields $\theta$ of the gap with bimorph at rest (a) and vibrating (b)
Temperature fields with and without acoustic streaming are displayed in Figs. 5-4(a) and (b), respectively. Initially without acoustic streaming (cf. Fig 5-4a), the heat transfer within the channel is dominated by conduction and the averaged temperature difference $\theta$ along the heat source surface is 102.91°C. When there is acoustic streaming existing in the channel (cf. Fig 5-4b), the $\theta$ drops to 96.7656°C and more heat is dissipated towards the open end, which is shown by the isothermal lines. The cooling effect is 6.1444°C, or 5.97% of the initial value. Because of the much larger heat conductivity of the silicon heat source than that of the air in the channel, there is negligible temperature variation within the heat source, as shown in Fig. 5-5 and along its surface. This is similar to that reported in Wan and Kuznetsov [3] and Chapter 3.

![Figure 5-5 Temperature fields $\theta$ within the heat source with vibrating bimorph](image-url)
Since there are two open ends of the channel, the vibrating acoustic field pushes the fluid in and out of the channel. Because the temperature within the channel is higher than the ambient temperature, it is natural to ask whether the acoustic field has any cooling effect, although theoretically it is adiabatic. A rough estimation is made by using the first order acoustic velocity field in the unsteady energy equation and specifying the temperature at two open ends as the ambient temperature. Computations are carried on the existing meshes and the steady state temperature along the heat source is found to be 93.84°C and the temperature drop is 9.07°C or 8.82% of the initial value. These values are listed in Table 5-1. The higher initial temperature difference in the computational results may be due to the 2-D numerical model, which does not account for cooling effect in the spanwise dimension in the experiment. The total cooling effect in the computations has a larger percentage of the initial temperature difference. This may be due to the fact that the ambient temperature is set to the bimorph temperature in the computations. In the experiment, the bimorph is hotter than the ambient temperature so as to dissipate of heat from the channel. The comparison of the experimental and numerical results in Table 5-1 also reveals that the acoustic streaming is not the only cooling mechanism for the present configuration. The acoustic field may also contribute to cooling.

5.3. **CONCLUSIONS**

A numerical model for the cooling efficiency of the bimorph is established. The comparisons of the computation results with the experimental result show that the acoustic streaming has certain cooling effects and the acoustic field may also have some cooling capacity.
Table 5-1 Comparison of cooling effect of the acoustic streaming by experimental and numerical method

(Ambient temperature is 20°C)

<table>
<thead>
<tr>
<th>Temperature difference from the ambient (bimorph), $\theta$ (°C)</th>
<th>Initial</th>
<th>Steady State</th>
<th>Drops</th>
<th>Percentage of initial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment (refs. [4, 5])</td>
<td>70.2</td>
<td>63.2</td>
<td>7.0</td>
<td>9.97%</td>
</tr>
<tr>
<td>computation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>acoustic streaming cooling</td>
<td>102.91</td>
<td>96.7656</td>
<td>6.1444</td>
<td>5.97%</td>
</tr>
<tr>
<td>acoustic cooling</td>
<td>102.91</td>
<td>93.84</td>
<td>9.07</td>
<td>8.82%</td>
</tr>
<tr>
<td>total cooling effect</td>
<td>102.91</td>
<td>87.6956</td>
<td>15.2144</td>
<td>14.79%</td>
</tr>
</tbody>
</table>

REFERENCES


4 Ro, P.I. and Wu, T., private communication.


PART THREE:
ULTRASONIC STREAMING
6. ULTRASONIC STREAMING AND ANALYTICAL SOLUTION OF BOUNDARY LAYER STRUCTURES

Abstract

In this chapter the oscillating and streaming flow fields in a channel composed of two long parallel beams, one of which is stationary and the other one oscillates with an ultrasonic frequency in a standing wave form are investigated. The perturbation technique is utilized under the assumption that the oscillation amplitude is much smaller than the channel width and that the Reynolds number, which is defined by the oscillating frequency and the standing wave number, is much greater than unity. A three-layer structure of both the oscillating and streaming flow fields, which is composed of two very thin boundary layers near the beams and the core region between the boundary layers, is found in the channel. The oscillating velocity fields in all three layers are obtained analytically. The streaming fields within both boundary layers are also obtained analytically based on the oscillating fields. It is found that the streaming velocities approach constant values at the edges of the boundary layers and thus provide slip velocities for the streaming field in the core region. The core region streaming velocity field is then obtained numerically by solving the Navier-Stokes equations in the stream function–vorticity formulation. Based on the core region streaming field, which dominates most part of the channel, the temperature field is computed for two cases: both beams are kept at constant but different temperatures (case A) and the oscillating beam is kept at a constant temperature while the stationary beam is subjected to a uniform constant heat flux (case B). Cases of different channel widths are computed and a critical width is found. When
the channel width is smaller than the critical one, for each half standing wavelength distance along the beams, two symmetric eddies are observed, which occupy almost the whole width of the channel. In this case, the Nusselt number increases with the increase of the channel width. After the critical width, two layers of asymmetric eddies are observed near the oscillating beam and the Nusselt number decreases and approaches unity with further increase of the channel width. The abrupt change of the streaming field and the Nusselt number as the channel width goes through its critical value may be due to a bifurcation caused by instability of the vortex structure in the fluid layer.

**NOMENCLATURE**

\( a^* \) displacement of the oscillating beam from its mean position in the \( y^* \) direction, [m]

\( a^* \) displacement vector of the oscillating beam, [m]

\( A \) amplitude of the standing wave generated in the oscillating beam, [m]

\( c_p \) specific heat at constant pressure, [J / kg K]

\( f \) frequency of the standing wave generated in the oscillating beam, [Hz]

\( h \) channel width, [m]

\( H \) convective heat transfer coefficient, [W / m² K]

\( j \) unit vector in the \( y^* \) direction

\( k \) wave number of the standing wave generated in the oscillating beam, [m⁻¹]

\( K \) thermal conductivity, [W / m K]
$Nu$  Nusselt number

$p^*$  pressure, [Pa]

$q$  heat rate through the stationary beam, [W]

$q^{''}$  heat flux through the stationary beam, [W / m$^2$]

$Q$  dimensionless channel width, $kh$

$R$  function of the dimensionless channel width defined in Eq. (6.A17), $\sqrt{2Re \tanh Q}$

$Re$  Reynolds number, $\omega / \nu k^2$

$t$  dimensionless time, $\omega t^*$

$t^*$  time, [s]

$T^*$  temperature, [K]

$T_{\text{cond}}^*$  temperature of the stationary (lower) beam for the case when the upper beam does not oscillate, [K]

$T_h^*$  temperature of the oscillating beam, [K]

$u$  dimensionless horizontal velocity component, $u^* / u_c$

$u^*$  horizontal velocity component, [m / s]

$u_1$  dimensionless horizontal velocity component of the single frequency oscillating (SFO) velocity

$u_2$  dimensionless horizontal velocity component of the double frequency oscillating (DFO) velocity
\( u_c \) characteristic velocity, \( \omega A \), [m / s]

\( \tilde{u}_Q \) dimensionless SFO slip velocity distribution at the oscillating beam

\( \tilde{u}_z \) dimensionless SFO slip velocity distribution at the stationary beam

\( U \) dimensionless horizontal velocity component of the streaming field

\( v \) dimensionless vertical velocity component, \( v^*/u_c \)

\( v_1 \) dimensionless vertical velocity component of the SFO velocity

\( v_2 \) dimensionless vertical velocity component of the DFO velocity

\( v^* \) vertical velocity component, [m / s]

\( \mathbf{v}^* \) velocity vector, [m / s]

\( \mathbf{v}_1^* \) SFO velocity vector, [m / s]

\( \mathbf{v}_2^* \) DFO velocity vector, [m / s]

\( \tilde{v}_Q \) dimensionless SFO vertical velocity distribution at the oscillating beam

\( \hat{v}_Q \) transformed dimensionless vertical velocity near the oscillating beam,

\[ \sqrt{2Re} \left[ \tilde{v}_Q(x) \cos t - v(x, y, t) \right] \]

\( \hat{v}_z \) transformed dimensionless vertical velocity near the stationary beam, \( \sqrt{2Re} v \)

\( V \) dimensionless vertical velocity component of the streaming field

\( \mathbf{V}^* \) streaming velocity vector, [m / s]

\( \hat{V}_Q \) transformed dimensionless vertical streaming velocity near the oscillating beam
\( \hat{V}_z \)  transformed dimensionless vertical streaming velocity near the stationary beam

\( x \)  dimensionless horizontal coordinate, \( kx^* \)

\( x^* \)  horizontal coordinate, [m]

\( y \)  dimensionless vertical coordinate, \( ky^* \)

\( y^* \)  vertical coordinate, [m]

**Greek symbols**

\( \alpha \)  thermal diffusivity, \([m^2/s]\)

\( \delta \)  viscous boundary layer scale, \( \sqrt{\nu/2\omega} \), [m]

\( \varepsilon \)  ratio of the standing wave amplitude to the channel width, \( A/h \)

\( \phi \)  dimensionless specific pressure, \( k\phi^*/\omega u_c \)

\( \phi^* \)  specific pressure, \( p^*/\rho \), [m\(^2\)/s\(^2\)]

\( \phi_1^* \)  SFO specific pressure, [m\(^2\)/s\(^2\)]

\( \Phi^* \)  specific pressure of the streaming field, [m\(^2\)/s\(^2\)]

\( \gamma \)  dimensionless amplitude of the standing wave generated in the oscillating beam, \( kA \)

\( \Gamma \)  ratio of the channel width to the period of streaming along the channel, \( h/(\lambda/2) \)

\( \eta_Q \)  boundary layer coordinate near the stationary beam, \( \sqrt{2Re} \ y \)

\( \eta_z \)  boundary layer coordinate near the oscillating beam, \( \sqrt{2Re}(Q - y) \)
λ wavelength of the standing wave generated in the oscillating beam, [m]

ν fluid kinematic viscosity, [m² / s]

θ dimensionless temperature, \((T^* - T_0^*)/(T_{\text{cond}}^* - T_0^*)\)

ρ fluid density, [kg / m³]

ω angular frequency of the standing wave generated in the oscillating beam, [rad / s]

Ω dimensionless vorticity of the streaming field

ψ dimensionless stream function

Ψ dimensionless stream function of the streaming field

Ψ_z dimensionless stream function of the streaming field in the boundary layer near the stationary beam

Ψ_Q dimensionless stream function of the streaming field in the boundary layer near the oscillating beam

6.1. **BACKGROUND**

The acoustic streaming phenomenon, which is the steady fluid motion generated in an acoustic field, has been extensively investigated. “Quartz wind” (Walker and Allen [1]) and the steady flow motion in the Kundt tube (Schuster and Matz [2]) are among the first discovered acoustic streamings. Acoustic streaming is mainly generated due to the attenuation of the acoustic field, which usually occurs when the acoustic wave propagates in a free or large space (Starritt et al.
The acoustic streaming is always very weak in the former case in the air due to the small attenuation coefficient of the air (Wan and Kuznetsov [6]). In the latter case, it is much stronger since the no-slip boundary condition reduces the finite acoustic wave amplitude to zero at the solid surface through a thin boundary layer and hence causes large attenuation in the boundary layer. The outer streaming occurs when the streaming velocity at the edge of the acoustic boundary layer near the solid surface does not vanish and provides the driving force for the outer streaming. Because of their larger magnitude, the streaming near the solid surface and the outer streaming associated with it are more interesting than the free space streaming and gain more attention. Good reviews on acoustic streaming are found in Riley [7], Nyborg [8, 9], and Lighthill [10, 11].

Acoustic streaming in a channel that consists of two parallel beams can be considered to be streaming near a solid surface because it occurs in a bounded space. The acoustic wave is generated either in the free stream (Vainshtein [5]) or by oscillation of one or both of the beams. Secomb [12] has analytically obtained the streaming field in a uniformly pulsating channel or a pipe.

However, a more practical case is when a standing wave is induced in one of the beams. This can be easily achieved by coating the beam with a piezoelectric material and applying appropriate voltage through the coating. Since the acoustic streaming occurs within the channel, heat transfer enhancement across the channel is expected. Acoustic enhancement of heat transfer between two parallel plates was first investigated by Vainshtein et al. [13]. Wan and Kuznetsov [14, 15, 16] investigated numerically the streaming field and the resulting cooling efficiency in a channel, which is bounded by two parallel beams. Either a standing or a
traveling wave was induced in one of the beams. This chapter considers a similar configuration for the ultrasonic oscillating frequency, as shown in Fig. 6-1a. The analysis shows a three-layer structure of oscillating and steaming fields, which is composed of two very thin acoustic boundary layers near the beams (the BLO layer near the oscillating beam and the BLS layer near the stationary beam) and a core region (CR) in the most part of the channel. Fig. 6-1b shows the three-layer structure of the streaming velocity flow fields between two adjacent nodes on the oscillating beam in one period. The BLO streaming carries the fluid that comes from the core region from the node positions to the anti-node positions and then returns it to the core region. The BLO steaming thus drives the fluid circulation in the core region. The CR streaming is composed of a series of counter-rotational eddies driven by the acoustic boundary layer near the oscillating beam. The BLS streaming also consists of counter-rotational eddies.
6.2. **Theory**

6.2.1. **Oscillating Boundary**

As shown in Fig. 6-1a, two-dimensional incompressible flow in a long channel, which is bounded by a stationary beam at $y^* = 0$ and an oscillating beam whose mean position is at $y^* = h$, is considered. Standing wave is induced in the oscillating beam. The displacement $a(x^*, t^*)$ of the oscillating beam from its mean position is assumed to be much smaller than the width of the channel. It is also assumed that $a(x^*, t^*)$ is strictly vertical (the horizontal component is neglected). Hence, the displacement can be described in $(x^*, y^*)$ coordinates as

$$a^*(x^*, t^*) = j a^* = j \varepsilon h \sin \omega t^* \sin kx^*$$  \hspace{1cm} (6.1)
where dimensional variables are denoted by an asterisk, \( t^* \) is the time, and \( k \) and \( \omega \) are the wave number and the angular frequency of the standing wave, respectively. The ratio \( \varepsilon \equiv A/h \) of the standing wave amplitude \( A \) and the channel width \( h \) is much smaller than unity according to the assumption.

According to Bradley [17], the velocity at the oscillating surface can be decomposed into the oscillating and streaming velocities at the mean position \( y^* = h \) of the oscillating beam utilizing a Taylor series expansion

\[
\mathbf{v}^*\mathbf{|}_{y^* = h, a^*} = \frac{\partial \mathbf{a}^*}{\partial t}\mathbf{|}_{y^* = h, a^*} + \left( \mathbf{a}^* \cdot \nabla \right) \frac{\partial \mathbf{a}^*}{\partial t}\mathbf{|}_{y^* = h} + O \left( \mathbf{a}^*^2 \right) \tag{6.2}
\]

The first term on the right-hand side of Eq. (6.2) is the single frequency oscillating (SFO) velocity at the mean position while the second term contains the double frequency oscillating (DFO) velocity and the streaming velocity at the mean position. The streaming velocity can be easily obtained by time-averaging (denoted by \( < > \) the second term on the right-hand side of Eq. (6.2).

Substituting Eq. (6.1) in Eq. (6.2), the SFO velocity \( \mathbf{v}_1^* \), the streaming velocity \( \mathbf{V}^* \), and the DFO velocity \( \mathbf{v}_2^* \) at the mean position can be expressed as

\[
\mathbf{v}_1^* (x^*, h, t^*) \equiv \frac{\partial \mathbf{a}^*}{\partial t}\mathbf{|}_{y^* = h} = j u_\varepsilon \cos \omega t^* \sin \kappa x^* \tag{6.3}
\]

\[
\mathbf{V}^* (x^*, h) \equiv \left( \mathbf{a}^* \cdot \nabla \right) \mathbf{v}_1^*\mathbf{|}_{y^* = h} \tag{6.4}
\]

\[
\mathbf{v}_2^* (x^*, h, t^*) \equiv \left( \mathbf{a}^* \cdot \nabla \right) \mathbf{v}_1^*\mathbf{|}_{y^* = h} - \mathbf{V}^*\mathbf{|}_{y^* = h} \tag{6.5}
\]

where the characteristic velocity is defined as
As seen in Eq. (6.4), streaming velocity only depends on the SFO velocity \( v_1^* \) at the mean position of the oscillating beam. As shown later, the SFO velocity provides the driving force for the streaming velocity inside the channel. Since the DFO velocity does not have any effect on the streaming velocity or heat transfer, it will not be considered further. Similar to the decomposition given by Eq. (6.2), velocity and pressure fields inside the channel are also decomposed into the SFO fields and the streaming fields, as:

\[
v^*(x^*, y^*, t^*) = v_1^*(x^*, y^*, t^*) + V^*(x^*, y^*, t^*)
\]

\[
\phi^*(x^*, y^*, t^*) = \phi_1^*(x^*, y^*, t^*) + \Phi^*(x^*, y^*, t^*)
\]

where the specific pressure \( \phi^* = p^* / \rho \) in Eq. (6.8) is defined as the ratio of the pressure \( p^* \) to the fluid density \( \rho \).

### 6.2.2. Non-Dimensionalization

The behavior of the fluid in the channel is governed by two-dimensional incompressible mass and momentum conservation equations:

\[
\nabla^* \cdot v^* = 0
\]

\[
v^*_{,x} + (v^* \cdot \nabla^*)v^* = -\phi^*_{,x} + \nu \Delta^* v^*
\]

These equations must be solved subject to no-slip boundary conditions at both beams (stationary and oscillating):
\begin{equation}
\mathbf{v}^*(x^*,0,t^*) = 0 \tag{6.11}
\end{equation}

\begin{equation}
\mathbf{v}^*(x^*,h,t^*) = (\mathbf{v}_1^* + \mathbf{V}^*) \bigg|_{y = h} \tag{6.12}
\end{equation}

Since the channel is long and the standing wave generated in the oscillating beam is periodic, a periodic flow field is assumed in the channel. Therefore, periodic boundary conditions are utilized for the \(x\) coordinate along the channel, i.e.

\begin{align}
\mathbf{v}^* \left( x^* + \frac{2\pi}{k}, y^*, t^* \right) &= \mathbf{v}^* \left( x^*, y^*, t^* \right) \\
\phi^* \left( x^* + \frac{2\pi}{k}, y^*, t^* \right) &= \phi^* \left( x^*, y^*, t^* \right) \tag{6.13a,b}
\end{align}

The following dimensionless variables are utilized:

\begin{align}
&x \equiv kx^* , \quad y \equiv ky^* , \quad t \equiv \omega t^* , \\
&u \equiv \frac{u^*}{u_c} , \quad v \equiv \frac{v^*}{u_c} , \quad \phi \equiv \frac{k \phi^*}{\omega u_c} \tag{6.14}
\end{align}

Governing Equations (6.9) and (6.10) and corresponding boundary conditions (6.11), (6.12), and (6.13a,b) are recast into the following dimensionless scalar form:

\begin{align}
u_x + v_y &= 0 \tag{6.15} \\
u_x + \gamma (u u_x + v u_y) &= -\phi_x + \frac{1}{\text{Re}} \left( u_{xx} + u_{yy} \right) \tag{6.16} \\
v_x + \gamma (u v_x + v v_y) &= -\phi_y + \frac{1}{\text{Re}} \left( v_{xx} + v_{yy} \right) \tag{6.17}
\end{align}

The dimensionless form of boundary conditions is:
\[ u(x,0,t) = 0, \quad v(x,0,t) = 0 \] \hspace{1cm} (6.18a,b)

\[ u(x,Q,t) = (u_1 + U)|_{y=Q}, \quad v(x,Q,t) = (v_1 + V)|_{y=Q} \] \hspace{1cm} (6.19a,b)

\[ u(x+2\pi,y,t) = u(x,y,t), \quad v(x+2\pi,y,t) = v(x,y,t), \] \hspace{1cm} (6.20a,b,c)

\[ \phi(x+2\pi,y,t) = \phi(x,y,t) \]

and

\[ u_1(x,Q,t) = 0, \quad v_1(x,Q,t) = \cos t \sin x \] \hspace{1cm} (6.21a,b)

\[ U(x,Q) = \gamma \sin x \left[ \sin t \frac{\partial u_1}{\partial y} \right]_{y=Q}, \] \hspace{1cm} (6.22a,b)

\[ V(x,Q) = \gamma \sin x \left[ \sin t \frac{\partial v_1}{\partial y} \right]_{y=Q} \]

where \( Q \) is the dimensionless channel width, \( \gamma \) is the dimensionless oscillation amplitude, \( \text{Re} \) is the Reynolds number, and \( \delta \) is the dimensional viscous boundary layer scale. These parameters are defined as:

\[ Q = kh, \quad \gamma = \varepsilon Q = kA \] \hspace{1cm} (6.23a,b)

\[ \text{Re} = \frac{\omega}{\nu k^2} = \frac{1}{2(\delta k)^2}, \quad \delta = \sqrt{\frac{\nu}{2\omega}} \] \hspace{1cm} (6.24a,b)

6.2.3. Inviscid SFO Solution

The SFO solution of Eqs. (6.15)—(6.17) for the inviscid flow in the core region of the channel (the core region is separated from both beams by viscous boundary layers whose thickness
scale is \( \delta \) when Reynolds number is much greater than unity can be obtained by dropping off the viscous term in momentum Eqs. (6.16)—(6.17) and utilizing only boundary conditions given by Eqs. (6.18b) and (6.21b). The no-slip boundary conditions given by Eqs. (6.18a) and (6.22a) are ignored. From Appendix 6.A.1, the SFO solution is

\[
\begin{align*}
 u_1(x,y,t) &= \frac{1}{\sinh Q} \cos t \cos x \cosh y \\
 v_1(x,y,t) &= \frac{1}{\sinh Q} \cos t \sin x \sinh y \\
 \phi_1(x,y,t) &= \frac{1}{\sinh Q} \sin t \sin x \cosh y + \frac{\gamma}{2 \sinh^2 Q} \cos^2 t \left( \sin^2 x - \sinh^2 y \right)
\end{align*}
\]  

(6.25)

(6.26)

(6.27)

From Eq. (6.25) it is obtained that the SFO slip velocity distribution at the oscillating beam \((y = 0)\) is

\[
\tilde{u}_z(x) = u_1(x,0,t) = \frac{\cos x}{\sinh Q}
\]

(6.28)

and the SFO slip velocity distribution at the standing wave beam \((y = Q)\) is

\[
\tilde{u}_Q(x) = u_1(x,Q,t) = \frac{\cos x}{\tanh Q}
\]

(6.29)

This reveals the existence of viscous boundary layers near both beams, where \(u_1\) changes from zero at the beams to the SFO slip velocities at the boundary layer edge. When the dimensionless channel width \(Q\) is large enough, \(\tilde{u}_z\) and \(\tilde{u}_Q\) approach zero. When this happens, the boundary layer near the oscillating beam (BLO layer) still exists and its thickness is independent of the channel width.
6.2.4. Viscous Boundary Layer Near the Stationary Beam (BLS Layer)

To investigate the BLS layer structure, a dimensionless BLS layer coordinate near the stationary beam at \( y = 0 \) is defined based on the dimensionless boundary layer thickness scale \( \delta k \). The vertical velocity is transformed accordingly to simplify the resulting equations:

\[
\eta_z = \frac{y}{\delta k} = \sqrt{2 \text{Re} \cdot y}, \quad \hat{v}_z = \sqrt{2 \text{Re} \cdot v} \quad (6.30a,b)
\]

Governing equations (6.15)—(6.17) are then simplified as

\[
\frac{\partial u}{\partial x} + \frac{\partial \hat{v}_z}{\partial \eta_z} = 0 \quad (6.31)
\]

\[
\frac{\partial u}{\partial t} + \gamma \left( u \frac{\partial u}{\partial x} + \hat{v}_z \frac{\partial u}{\partial \eta_z} \right) = -\frac{\partial \phi}{\partial x} + \frac{1}{2} \frac{\partial^2 u}{\partial \eta_z^2} \quad (6.32)
\]

\[
\frac{\partial \phi}{\partial \eta_z} = 0 \quad (6.33)
\]

These equations must be solved subject to the following boundary conditions:

\[
u(x,\eta_z,t)\bigg|_{\eta_z=0} = 0
\]

\[
u(x,\eta_z,t)\bigg|_{\eta_z=\infty} = u_i(x,y,t)\bigg|_{y=0} = \tilde{u}_z(x)\cos t \quad (6.34a,b,c)
\]

\[\hat{v}_z(x,\eta_z,t)\bigg|_{\eta_z=0} = \sqrt{2 \text{Re} \cdot v_i(x,y,t)}\bigg|_{y=0} = 0
\]

where terms of the order of \( O(\text{Re}^{-1}) \) and higher are neglected.
According to Eq. (6.33), pressure is constant across the BLS layer. Therefore, the pressure throughout the BLS layer is equal to the inviscid SFO pressure at the BLS layer edge, which is obtained from Eq. (6.27), i.e.

\[
\phi(x, \eta_z, t) = \phi(x, y, t)]_{y=0} = \frac{1}{\sinh Q} \sin t \sin x + \frac{\gamma}{2\sinh^2 Q} \cos^2 t \sin^2 x
\]  

(6.35)

Streaming velocity can be obtained from Eqs. (6.A6) and (6.A7a,b) (details are given in Appendix 6.A.2).

\[
U(x, \eta_z) = \gamma \tilde{u}_z \frac{d\tilde{u}_z}{dx} \Psi_{\eta_z}(\eta_z)
\]  

(6.36a)

\[
V(x, \eta_z) = \frac{1}{\sqrt{2\operatorname{Re}}} \tilde{V}_z(x, \eta_z) = -\frac{\gamma}{\sqrt{2\operatorname{Re}}} \frac{d}{dx} \left( \tilde{u}_z \frac{d\tilde{u}_z}{dx} \right) \Psi_{\eta_z}(\eta_z)
\]  

(6.36b)

where \( \Psi_{\eta_z}(\eta_z) \) is given by Eq. (6.A9) and the prime in Eq. (6.36a) denotes the derivative with respect to \( \eta_z \).

The periodicity of the streaming flow along the channel can be proved by explicitly calculating the \( x \)-dependency of Eqs. (6.36a,b), i.e., \( \tilde{u}_z \frac{d\tilde{u}_z}{dx} \) and \( \frac{d}{dx} \left( \tilde{u}_z \frac{d\tilde{u}_z}{dx} \right) \) utilizing Eq. (6.28). The results can be expressed in multiples of \( \sin 2x \) or \( \cos 2x \). This shows that the streaming flow has a period of \( \pi \) along the channel. In contrast, the SFO flow has a period of \( 2\pi \) (cf. Eqs. (6.25), (6.26), and (6.A6)).

The \( x \) and \( y \) components of the streaming velocity at the BLS layer edge (i.e., the slip streaming velocity components at the stationary beam) are obtained as
\[
U(x, y)_{y=0} = U(x, \eta_{Z})_{\eta_{Z} \to \infty} = -\frac{3}{4} \gamma \ddot{u}_{x} \frac{d\dot{u}_{y}}{dx} = \frac{3}{8} \frac{\gamma \sin 2x}{Q^2}
\]

\[
V(x, y)_{y=0} = V(x, \eta_{Z})_{\eta_{Z} \to \infty} = -\gamma \frac{d}{dx} \left( \frac{u_{x} d\ddot{u}_{y}}{dx} \right) \left[ \frac{13}{8} \frac{1}{\sqrt{2} \text{Re}} \right] \left( \frac{1}{4} \right)
\]

\[
= \frac{13}{8} \frac{\gamma \cos 2x}{\sinh^{2} Q \sqrt{2 \text{Re}}} \quad (6.37a, b)
\]

The ratio of the slip streaming velocity components at the stationary beam

\[
\frac{U(x, y)_{y=0}}{V(x, y)_{y=0}} = -\frac{13}{3} \frac{1}{\sqrt{2} \text{Re}} \cot 2x \quad (6.38)
\]

reveals that the vertical component is much smaller than the horizontal component in most \( x \) locations except for the node and anti-node positions of the standing wave on the oscillating beam where the horizontal component is zero.

6.2.5. **Viscous Boundary Layer Near the Oscillating Beam (BLO Layer)**

The BLO layer coordinate \( \eta \) near the oscillating beam is defined based on the BLO layer thickness scale similar to that utilized near the stationary beam. The BLO layer coordinate \( \eta \) is directed downward and its origination point coincides with the mean position of the oscillating beam. Different from Eq. (6.30b), a difference between vertical velocity anywhere in the BLO layer and its value at the BLO layer edge is considered. This vertical velocity difference vanishes at the mean position of the oscillating beam.
where the vertical velocity distribution at the BLO layer edge is found to be

\[
\tilde{v}_q(x) = \frac{v_l(x, y, t)|_{y=\eta}}{\cos \phi} = \sin \phi
\] (6.40)

After terms of the order of $O(\text{Re}^{-1})$ and higher are neglected, governing Eqs. (6.15)—(6.17) are simplified as

\[
\frac{\partial \hat{u}}{\partial t} + \gamma u \frac{\partial \hat{u}}{\partial x} + \left( \tilde{\hat{v}}_q - \sqrt{2 \text{Re}}\tilde{\tilde{v}}_q \cos \phi \right) \frac{\partial \hat{u}}{\partial \eta_q} = -\frac{\partial \phi}{\partial \eta_q} + \frac{1}{2} \frac{\partial^2 \hat{u}}{\partial \eta_q^2} (6.42)
\]

\[
\frac{\partial \phi}{\partial \eta_q} = 0 (6.43)
\]

These equations must be solved subject to the following boundary conditions

\[
u(x, \eta_q, t)|_{\eta_q=0} = U(x, Q) = -\sqrt{2 \text{Re}} \sin \phi \left( \sin \phi \frac{\partial \hat{u}}{\partial \eta_q} \right)_{\eta_q=0}
\]

\[
u(x, \eta_q, t)|_{\eta_q=\infty} = u(x, y, t)|_{y=0} = \tilde{u}_0(x) \cos \phi
\] (6.44a, b, c)
Similar to Eq. (6.35), the pressure is obtained as the inviscid SFO pressure distribution at the BLO layer edge from Eq. (6.27), i.e.

$$\phi(x, \eta_0, t) = \left. \phi(x, y, t) \right|_{y=Q} = \frac{1}{\tanh Q} \sin t \sin x + \frac{\gamma}{2 \sinh^2 Q} \cos^2 t \sin^2 x - \frac{\gamma}{2} \cos^2 t$$

(6.45)

Streaming velocity can be obtained from Eqs. (6.A10) and (6.A11a,b) (details are given in 6.A.3).

$$U(x, \eta_0) = \gamma \tilde{u}_0 \frac{d\tilde{u}_0}{dx} \Psi'(\eta_0)$$

$$V(x, \eta_0) = \frac{1}{\sqrt{2 \text{Re}}} \tilde{v}_0(x, \eta_0) = -\frac{\gamma}{\sqrt{2 \text{Re}}} \frac{d}{dx} \left( \tilde{u}_0 \frac{d\tilde{u}_0}{dx} \right) \Psi'(\eta_0)$$

(6.46a,b)

where $\Psi(\eta)$ is given by Eq. (6.A18).

Similar to the BLS streaming discussed in Section 6.2.4, the BLO streaming also has a period of $\pi$ along the channel.

The $x$ and $y$ components of the streaming velocity at the BLO layer edge (i.e. the slip streaming velocity components at the oscillating beam) are obtained as

$$U(x, y)\big|_{y=Q} = U(x, \eta_0)\big|_{\eta_0 \to \infty}$$

$$= \gamma \tilde{u}_0 \frac{d\tilde{u}_0}{dx} \left[ -\frac{3}{4} - R(Q) + O((Q-y)^2) \right] \bigg|_{y=0}$$

(6.47a,b)

$$= -\gamma \tilde{u}_0 \frac{d\tilde{u}_0}{dx} R(Q) = \frac{\gamma \sqrt{2 \text{Re}}}{2 \tanh Q} \sin 2x$$
\[ V(x,y)|_{y=0} = -V(x,\eta Q)|_{y=0} \]
\[ = \gamma \frac{d}{dx} \left( \tilde{u}_Q \frac{d\tilde{u}_Q}{dx} \right) \left[ \frac{13}{8} \frac{1}{\sqrt{2} Re} + \frac{1}{2} \tanh Q + O(Q - y) \right] |_{y=0} \]
\[ = \gamma \frac{d}{dx} \left( \tilde{u}_Q \frac{d\tilde{u}_Q}{dx} \right) \tanh Q = -\frac{\gamma}{2 \tanh Q} \cos 2x \]

The ratio of the slip streaming velocity components at the oscillating beam, given by

\[ \frac{V(x,y)}{U(x,y)}|_{y=0} = -\frac{1}{\sqrt{2} Re} \cot 2x \]  

(6.48)

reveals that the vertical component is much smaller than the horizontal component in most \( x \) locations except the node and anti-node positions of the standing wave on the oscillating beam where the horizontal component is zero, which is similar to that near the stationary beam.

The analysis of the flow structure predicts steady counter-rotational flow circulations in the core region, which are driven by the slip velocity at the oscillating beam and slowed down by the slip velocity at the stationary beam, as confirmed by numerical results later on. Further analysis shows that the slip velocity components at the stationary beam are negligible compared to those at the oscillating beam.

6.2.6. Core Region Streaming and Heat Transfer Across the Channel

According to Eq. (24b), the boundary layer thickness is proportional to the inverse of the square root of frequency. Therefore, if the channel width is sufficiently large or if the vibration frequency is sufficiently high (ultrasonic), the boundary layers at both beams do not overlap in the center of the channel, they are separated by a core region. The slip streaming velocity
components at the oscillating beam (cf. Eqs. (6.47a,b)) provide a driving force for the streaming flow motion in the core region (CR streaming). This streaming flow motion is governed by full steady-state incompressible Navier-Stokes equations

\begin{align}
U_x + V_y &= 0 \quad (6.49) \\
\gamma U_x V_y &= -\phi_x + \frac{1}{Re} \left( U_{xx} + U_{yy} \right) \quad (6.50) \\
\gamma (U V_y + V V_y) &= -\phi_y + \frac{1}{Re} \left( V_{xx} + V_{yy} \right) \quad (6.51)
\end{align}

that must be solved subject to the slip boundary conditions given by Eqs. (6.37a,b) and (6.47a,b).

The period of the CR streaming flow along the channel, \( \pi \), is determined by that of the slip velocities at both beams, which are discussed in Sections 6.2.4 and 6.2.5. Numerical solution for the CR streaming flow computed for one period along the channel will be discussed later on.

For cooling applications, it is important to investigate the cross-channel heat transfer efficiency of the flow fields obtained above. Two types of flow (the SFO flow and the streaming flow) exist in three regions (the BLO and BLS layers and the core region). The SFO flow is similar to an acoustic wave (both are harmonic oscillations). Therefore, the SFO flow can be treated as adiabatic and its contribution to heat transfer is negligible. When the Reynolds number is large, the viscous boundary layer thickness is negligible compared to the channel width. Vortices within these boundary layers provide an efficient heat transfer mechanism; therefore, thermal resistance created by these boundary layers is negligibly small.
Hence, only the CR streaming flow must be considered in computing the heat transfer across the channel.

The heat transfer efficiency of the CR streaming can be obtained by solving the following energy equation (which is written neglecting the viscous dissipation and the pressure work):

$$U^* \frac{\partial T^*}{\partial x^*} + V^* \frac{\partial T^*}{\partial y^*} = \alpha \left( \frac{\partial^2 T^*}{\partial x^*^2} + \frac{\partial^2 T^*}{\partial y^*^2} \right)$$

(6.52)

Two types of boundary conditions are investigated: constant heat flux (A) and constant beam temperature (B). In the case (A), the oscillating beam is kept at a constant temperature $T_h^*$ while constant heat flux $q''$ is imposed at the stationary beam. In the case (B), both stationary and oscillating beams are kept at different constant temperatures. Radiative heat flux between the beams is neglected because parameters for cases (A) and (B) are chosen so that heat transfer by radiation is much smaller than heat transfer by convection.

The same non-dimensionalization scheme as that given by Eq. (6.14) is utilized for coordinates and velocities and the dimensionless temperature $\theta$ is defined as:

$$\theta \equiv \frac{T^* - T_h^*}{T_{0\text{cond}}^* - T_h^*}$$

(6.53)

where $T_{0\text{cond}}^*$ is the temperature of the stationary (lower) beam for the case when the upper beam does not oscillate (in this case heat transfer between the beams is assumed to be by conduction only). In the case (A), $T_{0\text{cond}}^*$ is equal to $T_h^* + \frac{h}{\alpha \rho c_p} q''$ and in the case (B), $T_{0\text{cond}}^*$ is equal to a constant temperature at the stationary beam no matter whether the other beam oscillates or not.
The resulting dimensionless equation is

\[
U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \frac{1}{\gamma \text{Re Pr}} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \tag{6.54}
\]

This equation must be solved subject to the following boundary conditions

Case (A): \( \theta(x,0) = 0, \quad \left. \frac{\partial \theta}{\partial y} \right|_{y=0} = -\frac{1}{Q} \tag{6.55a,b} \)

Case (B): \( \theta(x,Q) = 0, \quad \theta(x,0) = 1 \tag{6.56a,b} \)

A periodic boundary condition similar to that given by Eqs. (6.13a,b) also applies for both cases (A) and (B):

\[ \theta(x + \pi, y) = \theta(x, y) \quad \tag{6.57} \]

As shown in Appendix 6.A.4, the Nusselt numbers can be expressed as

Case (A): \( Nu = \frac{\pi}{2x} \int_0^x \theta(x,0) dx \tag{6.58} \)

Case (B): \( Nu = -\frac{Q}{\pi} \int_0^x \frac{\partial \theta}{\partial y} \bigg|_{y=0} dx \tag{6.59} \)
6.3. **Numerical Results and Analysis**

6.3.1. **Numerical Scheme**

The CR streaming is obtained numerically by solving Eqs. (6.49)—(6.51) subject to the boundary conditions given by Eqs. (6.37a,b) and (6.47a,b) utilizing stream function – vorticity ($\Psi$-\(\Omega\)) formulation:

\[
\nabla^2 \Psi = -\Omega \tag{6.60}
\]

\[
\nabla^2 \Omega = \gamma \operatorname{Re} \left( \frac{\partial \Psi}{\partial y} \frac{\partial \Omega}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial \Omega}{\partial y} \right) \tag{6.61}
\]

Equations (6.60) and (6.61) must be solved subject to boundary conditions at the two beams which are obtained by using the definition of stream function, 

\[
\frac{\partial \Psi}{\partial y} = U, \quad -\frac{\partial \Psi}{\partial x} = V,
\]

and the periodic condition along the channel

\[
\frac{\partial \Psi}{\partial x} \bigg|_{y=0,Q} = -V(x,y) \bigg|_{y=0,Q}, \quad \frac{\partial \Psi}{\partial y} \bigg|_{y=0,Q} = U(x,y) \bigg|_{y=0,Q} \tag{6.62}
\]

\[
\Psi(x+\pi,y) = \Psi(x,y) \tag{6.63}
\]

The temperature field and the Nusselt number associated with the CR streaming are thus obtained by numerically solving Eq. (6.54) for both cases (A) and (B).

Equations (6.60), (6.61), and (6.54) are discretized on a rectangular 100×200 uniform mesh by the finite difference method and solved by the Gauss-Seidel iteration method. Grid independence of the results is tested by testing the code on finer meshes.
The standing wave generated in the oscillating beam is assumed to have wavelength \( \lambda = 25.4\text{mm} \), amplitude \( A = 10\mu\text{m} \), and frequency \( f = 21\text{kHz} \). The fluid in the channel is air. Hence the Reynolds number \( \text{Re} \) equals to \( 1.356 \times 10^5 \). Because the Reynolds number is large enough, this means that the boundary layer theory described above is valid for these computations.

6.3.2. BLS Streaming and BLO Streaming

As explained in the previous section, when the channel width is sufficiently large or the oscillation frequency is sufficiently high (so that the momentum boundary layers are thin enough), the BLS and the BLO layers are separated from each other by a core region. In addition, the BLS layer thickness is independent of the channel width. This conclusion is obtained by examining the stream function for the BLS layer, \( \Psi_z \) in Eq. (6.A9), which is only a function of the BLS layer coordinate \( \eta_z \) and hence does not change with the channel width. Although \( \Psi_\phi \) in Eq. (6.A18) shows a dependence on the dimensionless channel width \( Q \), this dependence becomes negligible for large \( Q \) when \( \tanh Q \approx 1 \). Hence, for a large channel width, the BLO layer thickness does not change with the channel width either. From Eqs. (6.36a,b) and (6.46a,b) it is found that when the channel width increases, the magnitudes of the \( x \) and \( y \) components of boundary layer streaming velocity approach zero and a constant value, respectively, because the SFO slip velocity distributions at the beams, \( \tilde{u}_x \) and \( \tilde{u}_y \), defined in Eqs. (6.28) and (6.29) approaches zero and \( \cos x \), respectively, as discussed in Sec. 6.2.3.

Utilizing the parameters specified in the previous section and a typical channel width of \( h = 12.7\text{mm} \) (half of the standing wavelength), the streaming velocity fields in both BLS and BLO
layers, which are given by Eqs. (6.37a,b) and (6.47a,b), are shown in Figs. 6-2 and 6-3. Near the anti-node positions the BLO streaming is “pushed” into the core region while near the node position the BLO streaming is “pulled” from the core region. Between the adjacent anti-node and node regions, the BLO streaming goes from the node position to the anti-node position. This mass exchange between the BLO layer and core region makes it hard to determine the exact thickness of the BLO layer. In fact, the BLO streaming and the core region streaming form a complete circulation loop within the gap, as shown in Figs. 6-5 and 6-6. The fluid in the core region is driven by the BLO streaming and slowed down by the BLS streaming. The magnitude of the BLS streaming is very small and for practical purposes it can be treated as a stagnant no-slip layer. The thickness of the BLS layer is found to be about $1.9 \times \delta = 14.78 \mu m$ in Fig. 6-3b. According to Fig. 6-1b, there must be an additional small vortex between the BLS layer and the core region; however, the fluid velocity in the BLS layer is so small that it is hardly of any practical significance. With the current resolution of the numerical mesh, it is not possible to resolve the exact structure of the flow between the BLS layer and the core region, which should be done in the future. However, because of a very small magnitude of velocity in this region this future investigation is only of theoretical interest.
Figure 6-2 Boundary layer streaming near the oscillating beam (BLS streaming), $\delta = 7.76 \mu m$. 

(a) vectors

(b) streamlines
Figure 6-3 Boundary layer streaming near the stationary beam (BLS streaming), $\delta = 7.76 \mu$m.
6.3.3. **The Effect of Channel Width**

Various channel widths, ranging from 0.5mm to 30mm, are tested to investigate the effect of the channel width. All channel widths investigated are much larger than the standing wave amplitude so that the ratio of the amplitude to the channel width is much smaller than unity. In the following discussion, the ratio \( \Gamma = h/(\lambda/2) \) of the channel width to the period of streaming along the channel is used to characterize the channel width.

As discussed in Sec. 6.2.6, slip streaming velocity components at the oscillating and stationary beams, which are given by Eqs. (6.37a,b) and (6.47a,b), provide the driving and slowing down mechanisms, respectively, for the CR streaming governed by Eqs. (6.49)—(6.51). The magnitudes of the slip streaming velocity for the specified values of parameters and various widths are listed in Table 6-1. As discussed in Sec. 6.2.5, the horizontal component of the slip velocity at the oscillating beam is the driving force for the core region streaming. Its dimensionless value ranges from 5.2354 (at \( h = 0.5\text{mm} \)) to 0.64426 (as \( h \to \infty \)). Since the velocity components are all non-dimensionalized by characteristic velocity \( u_c \) and \( u_c = 1.3195\text{m/s} \) for the specified parameter values, the dimensional horizontal component of the slip velocity at the oscillating beam is calculated to range from 6.9080\text{m/s} (at \( h = 0.5\text{mm} \)) to 0.8501\text{m/s} (as \( h \to \infty \)).
Table 6-1 The effect of the channel width on the magnitudes of the horizontal and vertical components of the slip velocity (given by Eqs. (6.37a,b) and (6.47a,b)). Computations are performed for \( \lambda = 25.4\text{mm} \), \( Re = 1.356 \times 10^5 \), and \( \gamma = 2.474 \times 10^{-3} \).

(A) Dimensional channel width
(B) Ratio of the channel width to the period of streaming velocity field along the channel
(C) Dimensionless channel width
(D) Magnitude of the horizontal component of the slip velocity at the stationary beam
(E) Magnitude of the vertical component of the slip velocity at the stationary beam
(F) Magnitude of the horizontal component of the slip velocity at the oscillating beam
(G) Magnitude of the vertical component of the slip velocity at the oscillating beam

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</table>
Contours of the stream functions of the CR streaming fields for various channel widths and for one period along the channel are shown in Fig. 6-4. It is found that the flow field changes dramatically when $\Gamma$ changes from 1.299 (which corresponds to $h = 16.5$mm) to 1.307 (which corresponds to $h = 16.6$mm). Detailed CR streaming fields just before and right after this bifurcation are displayed in Figs. 6-5 and 6-6 as a velocity vector field and in the streamline form, respectively. Before the jump the flow field exhibits combinations of symmetric and counter-rotational eddies, which occupy the whole channel width. The flow leaves the anti-nodes of the oscillating beam, goes towards the stationary beam and then goes back to the nodes of the oscillating beam. After the jump the flow field consists of combinations of asymmetric counter-rotational eddies, which exist near the oscillating beam. Since the boundary condition given by the slip velocity at the oscillating beam is symmetric about the anti-node (or node) position, the flow field after the jump can physically choose either of the two patterns displayed in Figs. 6-4(d) and (e). This is because the symmetry after the jump is broken by any tiny fluctuation on either side of the symmetry, which can result in the flow choosing either of two possible configurations. Computational errors can efficiently model real physical fluctuations. This bifurcation of the flow field is somewhat similar to the bifurcation that occurs in a driven enclosure when the depth of the enclosure is increased (Pan and Acrivos [18]).
(a) $\Gamma = 0.315 \ (h = 4 \text{mm})$

(b) $\Gamma = 0.787 \ (h = 10 \text{mm})$
(c) $\Gamma = 1.299 \ (h = 16.5\text{mm})$
(d) $\Gamma = 1.307 \ (h = 16.6 \text{mm})$
Figure 6-4 Stream functions of the core region streaming flow field for various channel widths

\( \Gamma = 2.362 \) \((h = 30\text{mm})\)
Figure 6-5 Core region streaming velocity field in vector (a) and streamline (b) forms just before the flow field bifurcation. $\Gamma = 1.299 \ (h = 16.5\text{mm})$. 
Figure 6-6 Core region streaming velocity field in vector (a) and streamline (b) forms just after the flow field bifurcation. $\Gamma = 1.307 \ (h = 16.6\text{mm})$.

The heat transfer efficiency through the channel is numerically investigated by computing the dimensionless temperature field within the channel for the case (A) of constant heat flux and the case (B) of constant temperature. Dimensionless temperature fields for these two cases are displayed in Figs. 6-7 and 6-8, respectively.
Figure 6-7 Contours of the dimensionless temperature field, $\theta$, caused by core region streaming for the case of constant heat flux (A) and constant temperature (B) just before the flow field bifurcation.

$\Gamma = 1.299 \ (h = 16.5\text{mm})$. 
Figure 6-8 Contours of the dimensionless temperature field, $\theta$, caused by the core region streaming for the case of constant heat flux (A) and constant temperature (B) just after the flow field bifurcation.

$\theta = \Delta T / \Delta T_{cond}$

$\Gamma = 1.307 \ (h = 16.6\text{mm})$.

Nusselt number and heat transfer coefficient are computed based on the numerically obtained dimensionless temperature field. The results are shown in Figs. 6-9 (a) and (b) and summarized in Table 6-2. The Nusselt number reaches its maximum value before the jump, decreases down to nearly unity after the jump, and continuously approaches unity as the channel width increases. Although the heat transfer coefficient also decreases after the jump, it takes on greater values for smaller channel widths. This can be explained by larger slip velocities for smaller channel widths, as listed in Table 6-1.
Figure 6-9 The effect of the channel width on the Nusselt number (a) and the heat transfer coefficient (b) for the constant heat flux and the constant temperature cases.
Table 6-2 The effect of the channel width on the Nusselt number and the heat transfer coefficient for the constant heat flux and constant temperature cases ($\lambda = 25.4$mm, $Re = 1.356 \times 10^5$, and $\gamma = 2.474 \times 10^{-3}$)

<table>
<thead>
<tr>
<th>$\Gamma = \frac{h}{\lambda/2}$ (mm)</th>
<th>$Q \equiv kh$</th>
<th>Nusselt Number $Nu$</th>
<th>Heat Transfer Coefficient (W/m$^2$K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Const Flux</td>
<td>Const Temp</td>
</tr>
<tr>
<td>0.039</td>
<td>0.5</td>
<td>1.5443</td>
<td>1.4458</td>
</tr>
<tr>
<td>0.079</td>
<td>1</td>
<td>1.8828</td>
<td>1.8777</td>
</tr>
<tr>
<td>0.157</td>
<td>2</td>
<td>2.0935</td>
<td>2.4238</td>
</tr>
<tr>
<td>0.315</td>
<td>4</td>
<td>3.2566</td>
<td>3.8416</td>
</tr>
<tr>
<td>0.472</td>
<td>6</td>
<td>5.3086</td>
<td>5.4219</td>
</tr>
<tr>
<td>0.630</td>
<td>8</td>
<td>6.6900</td>
<td>6.6383</td>
</tr>
<tr>
<td>0.787</td>
<td>10</td>
<td>7.8092</td>
<td>7.6625</td>
</tr>
<tr>
<td>0.945</td>
<td>12</td>
<td>8.7602</td>
<td>8.5310</td>
</tr>
<tr>
<td>1.102</td>
<td>14</td>
<td>9.5579</td>
<td>9.2514</td>
</tr>
<tr>
<td>1.260</td>
<td>16</td>
<td>10.2101</td>
<td>9.8331</td>
</tr>
<tr>
<td>1.307</td>
<td>16.6</td>
<td>1.2013</td>
<td>1.2013</td>
</tr>
<tr>
<td>1.575</td>
<td>20</td>
<td>1.1217</td>
<td>1.1218</td>
</tr>
<tr>
<td>2.362</td>
<td>30</td>
<td>1.0484</td>
<td>1.0486</td>
</tr>
</tbody>
</table>

6.4. **Conclusions**

The flow field in a channel composed by two parallel beams, one of which is stationary and the other oscillating in a standing waveform, is decomposed into a series of flow fields of different frequencies when the Reynolds number is large. The single frequency oscillating (SFO) flow field is analytically obtained both in the core region and the boundary layers near both beams (BLS and BLO layers). The streaming fields in both BLS and BLO layers are also analytically obtained based on the SFO field. It is found that the streaming velocity at the edge of the
boundary layers does not vanish. The core region (CR) streaming is then numerically obtained based on the slip velocities at the two boundary layer edges. When the channel width is increased above a critical value, the CR streaming field changes abruptly from a symmetric counter-rotational eddy structure, which occupy the whole channel, to an asymmetric counter-rotational eddy structure, which is concentrated near the oscillating beam. This bifurcation may be due to the instability in the system. The heat transfer efficiency is investigated for both constant heat flux and constant beam temperature cases by numerically solving the energy equation based on the CR streaming field. The results reveal a jump in the Nusselt number and heat transfer coefficient when the bifurcation occurs. Nusselt number increases before the jump and it decreases after it. The heat transfer coefficient decreases as the channel width increases.

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**APPENDIX**

6.1.1 Inviscid Solution of Eqs. (6.15)—(6.17)

It is assumed that the vertical component of the SFO velocity is separable into multiples of three single-variable functions

\[ v_i(x,y,t) = F(y) v_i(x,h,t) = F(y) \cos t \sin x \]  

(6.A1)

In this case from Eqs. (6.18b) and (6.22a) follows that

\[ F(0) = 0, \quad F(Q) = 1 \]  

(6.A2)

The velocity component \( u_1 \) can be obtained from continuity Eq. (6.15) by integration

\[ u_1 = \int u_1 \, dx = -\int v_1 \, dx = F'(y) \cos t \cos x + c_1(y,t) \]  

(6.A3)

where \( c_1(y,t) \) is the velocity profile of the flow in the channel when the upper beam does not oscillate. In our case, \( c_1(y,t) = 0 \) because there is no flow motion without the oscillation of the upper beam. Utilizing Eq. (6.A1) in Eqs. (6.15) and (6.16) and neglecting viscous terms in these equations, one obtains

\[ F'(y) = F(y) \]  

(6.A4)

The solution of Eq. (6.A4) subject to boundary condition (6.A2) is

\[ F(y) = \frac{\sinh y}{\sinh Q} \]  

(6.A5)

The inviscid solutions given by Eqs. (6.25)—(6.27) are thus obtained.
6.A.2 Solution of Eqs. (6.31)—(6.33) Utilizing the Stream Function Formulation

According to Schlichting and Gersten [19], the solution of Eqs. (6.31)—(6.33) in the stream function form is

\[
\psi(x, \eta_z, t) = \tilde{u}_z \left[ \psi_{\eta_1}(\eta_z) \cos t + \psi_{\eta_i}(\eta_z) \sin t \right] \\
+ \gamma \tilde{u}_z \frac{du_x}{dx} \left[ \psi_z(\eta_z) + \psi_{\eta_2}(\eta_z) \cos 2t + \psi_{\eta_i}(\eta_z) \sin 2t \right] \tag{6.A6}
\]

\[
u(x, \eta_z, t) = \frac{\partial \psi}{\partial \eta_z}, \quad v(x, \eta_z, t) = -\frac{\partial \psi}{\partial x} \tag{6.A7a,b}
\]

where

\[
\psi_{\eta_1}(\eta_z) = \eta_z + \frac{1}{2} e^{-\eta_z} \left( \cos \eta_z - \sin \eta_z \right) - \frac{1}{2} \tag{6.A8a,b}
\]

\[
\psi_{\eta_2}(\eta_z) = \frac{1}{2} e^{-\eta_z} \left( \cos \eta_z + \sin \eta_z \right) - \frac{1}{2}
\]

\[
\psi_z(\eta_z) = \frac{13}{8} \eta_z^2 - \frac{3}{4} e^{-\eta_z} - \frac{1}{8} e^{-\eta_z} \sin \eta_z - \frac{3}{2} e^{-\eta_z} \cos \eta_z - e^{-\eta_z} \sin \eta_z \tag{6.A9}
\]

6.A.3 Solution of Eqs. (6.41)—(6.43) Utilizing the Stream Function Formulation

It is assumed that the stream function has the following form

\[
\psi(x, \eta_0, t) = \tilde{u}_0 \left[ \psi_{\eta_0}(\eta_0) \cos t + \psi_{\eta_1}(\eta_0) \sin t \right] \\
+ \gamma \tilde{u}_0 \frac{du_x}{dx} \left[ \psi_{\eta_0}(\eta_0) \cos 2t + \psi_{\eta_1}(\eta_0) \sin 2t \right] \tag{6.A10}
\]

\[
u(x, \eta_0, t) = \frac{\partial \psi}{\partial \eta_0}, \quad v(x, \eta_0, t) = -\frac{\partial \psi}{\partial x} \tag{6.A11a,b}
\]
Substituting Eqs. (6.A10) and (6.A11a,b) into governing Eqs. (6.41)—(6.43) and collecting the terms containing $\sin t$, $\cos t$, as well as the terms independent of time $t$, the equations for the SFO flow stream function $\psi_{cl}$, $\psi_{sl}$, and the streaming flow stream function $\Psi$ are obtained as

$$\psi_{cl}'' - 2\psi_{cl}' = 0, \quad \psi_{sl}'' + 2\psi_{cl}' = 2 \quad (6.A12a,b)$$

$$\Psi'' = \psi_{cl}''^2 + \psi_{sl}''^2 - \psi_{cl}''\psi_{cl}' - \psi_{sl}''\psi_{sl}' - \frac{1}{\cosh^2 Q} + R(Q)\psi_{sl}'' \quad (6.A13)$$

These equations must be solved subject to the following boundary conditions that follow from Eqs. (6.44a,b,c)

$$\psi_{cl}(0) = \psi_{cl}'(0) = 0, \quad \psi_{cl}'(\infty) = 1 \quad (6.A14a,b)$$

$$\psi_{sl}(0) = \psi_{sl}'(0) = 0, \quad \psi_{sl}'(\infty) = 0 \quad (6.A15a,b)$$

$$\Psi_{cl}(0) = \frac{1}{2} R(Q)\psi_{cl}'(0) \quad \Psi_{sl}'(0) = \frac{1}{2} R(Q)\psi_{sl}''(0) \quad (6.A16a,b)$$

where the parameter $R(Q)$, which is a function of the dimensionless channel width, $Q$, is

$$R(Q) = \sqrt{2Re \tanh Q} \quad (6.A17)$$

Solving Eqs. (6.A12a,b) subject to boundary conditions given by Eqs. (6.A14a,b) and (6.A15a,b) results in the exactly same solution that is given by Eqs. (6.A8a,b). Utilizing Eqs. (6.A8a,b) and boundary conditions given by Eqs. (6.A16a,b), the solution of Eq. (6.A13) can be obtained by integration as

$$\Psi_{cl}(\eta) = \Psi_{sl}(\eta) + \left[ \frac{1}{2} R(Q) - R(Q)\eta \right] \frac{1}{6} \eta^3 \tanh^2 Q - R(Q)e^{-\eta} \cos \eta \quad (6.A18)$$

where $\Psi_{sl}(\eta)$ is given by Eq. (6.A9).
6.4 Nusselt Numbers in Eqs. (6.58) and (6.59)

In the case (A), constant heat flux goes through the channel no matter whether the beam oscillates or not. Without beam oscillation, heat conduction is the only heat transfer mechanism (since radiation and natural convection are neglected) and the heat flux can be represented as 

\[ q^* = -K \frac{T_b^* - T_{0,\text{cond}}^*}{h} \]

When the beam oscillates, convection becomes important and 

\[ q^* = H(T_0^* - T_h^*) \]

Hence, utilizing Eq. (6.53), the Nusselt number for the constant heat flux case can be expressed as

\[
Nu \equiv \frac{Hh}{K} = \frac{T_{0,\text{cond}}^* - T_b^*}{T_0^* - T_h^*} = \frac{1}{\frac{\partial}{\partial y}} \int_0^\pi \theta(x,0) dx
\]

(6.19)

In the case (B), constant beam temperature is prescribed at both beams. The total heat rate coming through the oscillating beam

\[
q = \int_0^\pi q^*(dx \cdot 1) = -K \int_0^\pi \frac{\partial T^*}{\partial y} \bigg|_{y=0}^\pi \left. \frac{\partial T^*}{\partial y} \right|_{y=0} dx
\]

is carried to the stationary beam by convection, i.e., 

\[ q = H \cdot (\pi \cdot 1)(T_{0,\text{cond}}^* - T_h^*) \]

Hence, utilizing Eq. (6.53), the Nusselt number for the constant oscillating beam temperature case can be expressed as

\[
Nu \equiv \frac{Hh}{K} = -O \int_0^\pi \frac{\partial \theta}{\partial y} \bigg|_{y=0}^\pi dx
\]

(6.20)
7. HYSTERESIS IN ACOUSTICALLY DRIVEN CHANNEL FLOW

Abstract

The purpose of this chapter is to investigate the hysteresis phenomenon in the acoustic streaming flow in a channel. The channel is bounded by one stationary beam and one beam vibrating in a standing wave form. According to Wan and Kuznetsov [1] in Chapter 6, the acoustic streaming within the channel is driven by the slip velocity at the edge of the acoustic boundary layer near the vibrating beam. The streaming flow exhibits two different types of flow pattern at small and large channel widths. In this chapter, a transition between these two flow patterns and the corresponding jump in the Nusselt number as the channel width changes is investigated. As a result of extensive numerical investigations, a hysteretic region is discovered. When the channel width falls into the hysteresis region, the flow pattern and the heat transfer efficiency depends on whether the channel width is increasing or decreasing. Outside of the hysteresis region, such dependence does not exist.

Nomenclature

\( a^* \) displacement of the oscillating beam from its mean position in the \( y^* \) direction, [m]

\( a^* \) displacement vector of the oscillating beam, [m]

\( A \) amplitude of the standing wave generated in the oscillating beam, [m]
frequency of the standing wave generated in the oscillating beam, [Hz]

channel width, [m]

unit vector in the \( y^* \) direction

wave number of the standing wave generated in the oscillating beam, [m\(^{-1}\)]

Nusselt number

dimensionless channel width, \( kh \)

function of the dimensionless channel width defined as \( \sqrt{2} \text{Re} \tanh Q \)

Reynolds number, \( \omega / \nu k^2 \)

dimensionless time, \( \omega t^* \)

time, [s]

temperature, [K]

temperature of the stationary (lower) beam for the case when the upper beam does not oscillate, [K]

temperature of the oscillating beam, [K]

dimensionless horizontal velocity component, \( u^* / u_c \)

horizontal velocity component, [m / s]

characteristic velocity, \( \omega A \), [m / s]

dimensionless vibrating velocity at the boundary layer edge

dimensionless horizontal velocity component of the streaming field
\( v \)  
dimensionless vertical velocity component, \( v^* / u_c \)

\( v^* \)  
vertical velocity component, \([\text{m / s}]\)

\( x \)  
dimensionless horizontal coordinate, \( kx^* \)

\( x^* \)  
horizontal coordinate, \([\text{m}]\)

\( y \)  
dimensionless vertical coordinate, \( ky^* \)

\( y^* \)  
vertical coordinate, \([\text{m}]\)

Greek symbols

\( \phi \)  
dimensionless specific pressure, \( k\phi^* / \omega u_c \)

\( \phi^* \)  
specific pressure, \([\text{m}^2 / \text{s}^2 ]\)

\( \gamma \)  
dimensionless amplitude of the standing wave generated in the oscillating beam, \( kA \)

\( \Gamma \)  
ratio of the channel width to the period of streaming along the channel, \( h / (\lambda / 2) \)

\( \eta \)  
boundary layer coordinate near the oscillating beam, \( \sqrt{2 \text{Re}}(Q - y) \)

\( \lambda \)  
wavelength of the standing wave generated in the oscillating beam, \([\text{m}]\)

\( \nu \)  
fluid kinematic viscosity, \([\text{m}^2 / \text{s}]\)

\( \theta \)  
dimensionless temperature, \( (T^* - T_h^*) / (T_{0,cond}^* - T_h^*) \)

\( \omega \)  
angular frequency of the standing wave generated in the oscillating beam, \([\text{rad} / \text{s}]\)

\( \Omega \)  
dimensionless vorticity of the streaming field
\( \psi \)  
\[ \text{dimensionless stream function} \]

\( \Psi \)  
\[ \text{dimensionless stream function of the streaming field near the vibrating beam} \]

\( \psi_c \)  
\[ \text{dimensionless stream function of the streaming field in the core region} \]

### 7.1. *Introduction*

Hysteresis is not a new topic in fluid mechanics. The presence of solid boundaries may cause hysteresis in a flow. Le Gal et al. [3] investigated the hysteretic effect of the forced Stuart-Landau equation and predicted a wake region behind an oscillating cylinder. Defina and Susin [4] analyzed hysteresis in a supercritical flow approaching a solid obstacle. Lou et al. [5] numerically and experimentally studied the hysteresis phenomena occurring in a flow over a fixed square cylinder at various angles of attack. Beliaev and Hassanizadeh [6] developed models of capillary hysteresis with dynamic effects to explain the capillary pressure—saturation relationship that has dynamic memory effect on two-phase flow in porous media. Yang et al. [7] studied the flow motion of monodisperse polybutadienes melt in a barrel, and found that the hysteresis of the melt flow occurs when a stick-slip transition of an extremely large magnitude is present. Perlin and Schultz [8] reviewed the study of the capillary effect on the surface waves, including the hysteresis due to the pressure—saturation relationship. The hysteretic phenomena are also reported in the study of biofluids due to the nonlinear stress-strain relationship. Sun and De Kee [9], for example, modeled the rheological behavior of the blood and penicillin suspension.
In the previous research in Chapter 6, Wan and Kuznetsov [1, 2] studied the acoustic streaming flow in a channel comprised of two beams, one of which being stationary and the other one vibrating in the standing wave form. Their investigation of beam vibration at sonic frequency [10] in Chapter 3 shows that the generated acoustic streaming is so weak that the cooling effect is not significant due to the small acoustic intensity. In the later paper [1] and Chapter 6, they investigated the ultrasonic vibration of high acoustic intensity and reported a sudden change of flow pattern when the channel width comes through a critical value. The heat transfer efficiency and the Nusselt number exhibit a similar behavior, and the latter reaches its maximum value at the critical channel width. Loh et al. [11] and Ro and Loh [12] performed experiments for a similar arrangement. They also found that there is a maximum temperature drop for a certain channel width.

In this chapter, the hysteresis in the transformation between two distinct flow patterns, which was initially reported in Wan and Kuznetsov [1] and in Chapter 6, is numerically investigated. The width of the hysteretic region is numerically determined.

7.2. Theory

It is well known that the acoustic field generates acoustic streaming, which is a unidirectional fluid flow, with the presence of acoustic attenuation. The acoustic streaming due to the attenuation of the acoustic wave propagating in a free space in the air is always very weak [12]. Strong acoustic streaming occurs near the solid boundary [1, 2, 11, 12, 10] where the no-slip boundary condition exists. Good reviews on acoustic streaming are given by Riley [7], Nyborg [8, 9], and Lighthill [10, 11].
In Chapter 6, Wan and Kuznetsov [1] investigated the flow and heat transfer efficiency in a two-dimensional channel bounded by two long parallel beams. As shown in Fig. 6-1a, one beam is kept stationary and the other one vibrates in a standing wave form with the amplitude $A$, the wave number $k$ (which corresponds to the wavelength $\lambda$), and the angular frequency $\omega$. The displacement of the surface of the vibrating beam, $a^*$, in the direction normal to mean position of the beam, $j$, is

$$a^*(x^*, t^*) = ja^* = JA \sin \omega t^* \sin kx^* \quad (7.1)$$

where the asterisks denote dimensional variables, $t^*$ is the time, and $x^*$ is the coordinate along the mean position of the vibrating beam.

When the vibration amplitude $A$ is much smaller than the channel width $h$ and the Reynolds number defined as $Re = \frac{\omega}{\nu k^2}$ is sufficiently large, they analytically found a three-layer structure within the channel as shown in Fig. 6-1b, utilizing a perturbation method. The three-layer structure found is composed of an inviscid core region and two thin acoustic boundary layers near both beams due to the attenuation of vibration flow through the boundary layers. The non-zero acoustic streaming velocities at the edges of the two viscous boundary layers are the driving velocities for the core region streaming, although the magnitude of the streaming velocity near the stationary beam can be neglected when it is compared with that near the vibrating beam. Near the vibrating beam, the vertical streaming component can also be neglected because it is negligibly small compared to the horizontal one. Hence, the horizontal component of the acoustic streaming velocity at the edge of the acoustic boundary layer near the vibrating beam is the main driving velocity for the core region streaming. The
streaming velocity at the boundary layer edge can be considered as a slip driving velocity at the mean position of the beam since the boundary layer thickness is small compared to the channel width. This slip driving velocity moves core region streaming flow in the channel.

The dimensionless governing equations for the acoustic boundary layer near the vibrating beam, which are obtained by simplifying the incompressible Navier—Stokes equations, are found in Wan and Kuznetsov [1] and in Chapter 6 to be

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial \eta} = 0
\]  

(7.2)

\[
\frac{\partial u}{\partial t} + \gamma \left[ u \frac{\partial u}{\partial x} + (v - \sqrt{2\text{Re}_u} \sin x \cos t) \frac{\partial u}{\partial \eta} \right] = -\frac{\partial \phi}{\partial x} + \frac{1}{2} \frac{\partial^2 u}{\partial \eta^2}
\]  

(7.3)

\[
\frac{\partial \phi}{\partial \eta} = 0
\]  

(7.4)

where the coordinates \((x^*, y^*)\) are non-dimensionalized by \(1/k\); time \(t^*\) by \(1/\omega\); velocity components \((u^*, v^*)\) by the magnitude of the beam vibration speed, \(u_c = \omega A\); and the specific pressure \(\phi^*\) by \(\omega u_c/k\). \(\gamma = kA\) and \(Q = kh\) are the dimensionless vibration amplitude and channel width, respectively. \(\eta = \sqrt{2\text{Re}}(Q - y)\) is the boundary layer coordinate directed from the vibrating beam to the center of the channel and \(v(x, \eta, t)\) is the difference of the vertical velocity component in the boundary layer from that at the surface of the vibrating beam.

In Chapter 6, Wan and Kuznetsov [1] found that the analytical solution of Eqs. (6.41)—(6.43) presented in the stream function form is
\[
\psi(x, \eta, t) = \tilde{u} [\psi_{s1}(\eta) \cos t + \psi_{s1}(\eta) \sin t] \\
+ \gamma u \frac{d\tilde{u}}{dx} \left[ \Psi(\eta) + \psi_{s2}(\eta) \cos 2t + \psi_{s2}(\eta) \sin 2t \right]
\]  
(7.5)

where \( \tilde{u}(x) = \frac{\cos x}{\tanh Q} \) is the vibrating velocity at the boundary layer edge.

The steam function for the streaming flow in Eq. (7.5) is found to be

\[
\Psi(\eta) = \left[ \frac{13}{8} - \frac{3}{4} \eta - \frac{1}{8} e^{-2\eta} - \frac{1}{2} \eta e^{-\eta} \sin \eta - \frac{3}{2} e^{-\eta} \cos \eta - e^{-\eta} \sin \eta \right] \\
+ \left[ \frac{1}{2} R(Q) - R(Q) \eta + \frac{1}{6} \eta^3 \tanh^2 Q - R(Q) e^{-\eta} \cos \eta \right]
\]  
(7.6)

where \( R(Q) = \sqrt{2 \text{Re} \tanh Q} \).

From Eq. (7.6), one can find that the slip driving velocity is

\[
U(x, y) \bigg|_{\gamma=\eta} = \frac{\gamma \sqrt{2 \text{Re}}}{2 \tanh Q} \sin 2x
\]  
(7.7)

Equation (6.47a,b) reveals that the spatial period of the slip driving velocity along the channel is half of that of the standing wave vibration of the beam. Therefore, the resulting boundary streaming and the core region streaming repeat their flow pattern. The period of the streaming flow equals to the distance between two adjacent nodes of the standing wave (half the wavelength of the standing wave) along the channel. As shown in Eq. (6.47a,b), the magnitude of the slip velocity is a function of beam vibration parameters and the channel width. The nature of the hyperbolic tangent function leads to the result that the slip velocity is independent of the channel width for large gap or small standing wavelength, when \( Q \) is large enough so that \( \tanh Q \to 1 \).
When the Reynolds number is large enough, the thickness of the acoustic boundary layers near both beams is negligible and the core region streaming dominates the entire channel. According to Wan and Kuznetsov [1] in Chapter 6, the governing equations for the core region streaming flow and the temperature field are the steady incompressible Navier—Stokes equations in stream function—vorticity ($\Psi—\Omega$) formulation and the steady energy equation, as given below

\[ \nabla^2 \Psi = -\Omega \]  
\[ \nabla^2 \Omega = \gamma \, \text{Re} \left( \frac{\partial \Psi}{\partial y} \frac{\partial \Omega}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial \Omega}{\partial y} \right) \]  
\[ U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \frac{1}{\gamma \, \text{Re} \, \text{Pr}} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \]

where $\text{Pr}$ is the Prandtl number of the air and $\theta = \frac{T^* - T_{h^*}}{T_{0\,\text{cond}^*} - T_{h^*}}$ is the dimensionless temperature, which is defined as the ratio of the temperature differences from that of the stationary beam $T_{h^*}$, which is always kept constant. $T^*$ is the air temperature in the channel and $T_{0\,\text{cond}^*}$ is the temperature of the stationary beam for the case when the other beam does not vibrate. Two types of boundary conditions are imposed on the vibrating beam: constant heat flux (A) and constant temperature (B). The Nusselt number for each case can be calculated when the temperature field is numerically obtained: $Nu = \frac{\pi}{\int_0^{\pi} \theta(x,0) \, dx}$ for case (A) and $Nu = -\frac{Q}{\pi} \left[ \frac{\partial \theta}{\partial y} \right]_{y=0}^{\pi} \, dx$ for case (B).
7.3. **Numerical Experiment and Result**

In Wan and Kuznetsov [1] and in Chapter 6, the core region streaming and temperature field in the channel for various aspect ratios \( \Gamma = h/(\lambda/2) \) are numerically obtained by using zero initial conditions and iterating Eqs. (6.60)—(6.54) until convergence criteria are satisfied. Physically, this numerical scheme represents the start-up of the beam vibration from the stagnant air until the core streaming reaches a steady state. The reported critical value of \( \Gamma = 1.3 \), at which the bifurcation occurs, shows that if starting from the stagnant air, the steady core region streaming and temperature field choose one or another flow pattern when the aspect ratio falls below or above 1.3.

In the present chapter, the aim is to investigate the hysteresis effect of the core region streaming and temperature fields. The hysteresis effect is expected to be observed when the channel width changes quasi-steadily. The infinitely slow change of the channel width is essential to ensure that the core region streaming and the temperature fields remain quasi-steady. Numerically, this can be realized by using results of the smaller aspect ratio as the initial conditions for the larger aspect ratio computation when the ratio increases and vice versa.

The following computation parameters are utilized: vibration standing wavelength \( \lambda = 25.4\text{mm} \), frequency \( f = 21\text{kHz} \), and amplitude \( A = 10\mu\text{m} \). The fluid is air. A 101×201 uniform rectangular mesh is adopted and the finite difference method and Gauss-Seidel iteration are utilized. Grid independence of the results is checked by running the code on finer meshes.
Figure 7-1 The hysteresis effect on the Nusselt number for constant temperature (a) and constant heat flux (b). (Dashed lines denote the results from Wan and Kuznetsov [1] and Chapter 6, in which zero initial condition is utilized. The arrows denote the increase or decrease of the aspect ratio)
Figs. 7-1(a) and (b) show the hysteresis effect on the Nusselt number for constant heat flux at the vibrating beam (A) and constant temperature of the vibrating beam (B) cases. As the aspect ratio $\Gamma$ increases up to 1.33, the Nusselt number increases towards its maximum 9.93 (for case A) or 10.33 (for case B) and jumps abruptly to 1.20 for both cases at $\Gamma = 1.33$. After $\Gamma = 1.33$, the Nusselt number decreases and approaches unity as the aspect ratio increases. When the aspect ratio $\Gamma$ decreases from a large value down to 1.02, the Nusselt number increases and jumps from 1.53 for both cases to 8.86 (for case A) or 9.12 (for case B) at $\Gamma = 1.02$. After $\Gamma = 1.02$, the Nusselt number decreases and approaches unity as the aspect ratio increases. The overlap region between $\Gamma = 1.02$ and $\Gamma = 1.33$ is the hysteresis region, where the core region streaming and temperature fields depend on whether the aspect ratio increases or decreases. This hysteresis effect may be due to the influences of the two solid beams on the fluid, the vortexes in which occupy the entire channel or dominate the vicinity of the vibrating beam depending on the change of the aspect ratio. Figs. 7-2(a) and (c) show two possible core region streaming fields at $\Gamma = 1.10$ in the hysteresis region. The two symmetric vortexes dominating the whole channel shown in Figs. 7-2(a) and (c) represent a typical pattern when the aspect ratio increases up to $\Gamma = 1.33$. As shown in Figs. 7-1(a) and (b), the corresponding Nusselt number is much larger than unity and, therefore, denotes larger heat transfer efficiency. In contrast, the two asymmetric vortexes shown in Figs. 7-2(b) and (d) stand for a typical pattern when the aspect ratio decreases down to $\Gamma = 1.02$. The corresponding Nusselt number is only a little larger than unity, which means very poor convection. This can be explained by the flow velocity vectors in Fig. 7-2d, where the strong vortexes exist only in the vicinity of the vibrating beam, leaving most part of the channel filled with a nearly stagnant fluid. The
respective pressure fields are shown Figs. 7-2(e) and (f), where large pressure gradients are observed near the anti-node of the vibrating beam. In Fig. 7-2e, the larger symmetric pressure gradient near the anti-nodes enable stronger fluid flow from the vibrating beam to the stationary beam, as shown in Fig. 7-2a. In contrast, the weaker and asymmetric pressure gradient near the anti-nodes can only drive the fluid near the vibrating beam, as shown in Fig. 7-2c.
Figure 7-2 Typical flow patterns and pressure fields in hysteresis region at aspect ratio $\Gamma = 1.1$, (a) flow field streamlines when $\Gamma$ increases, (b) flow field streamlines when $\Gamma$ decreases, (c) flow field vectors when $\Gamma$ increases, (d) flow field vectors when $\Gamma$ decreases, (e) pressure field contours when $\Gamma$ increases, (f) pressure field contours when $\Gamma$ decreases.
7.4. **Conclusion**

The acoustic streaming flow in a channel, which is bounded by one beam at rest and one beam vibrating in a standing wave form, is driven by the slip velocity at the edge of the acoustic boundary layer near the vibrating beam. As the aspect ratio increases or decreases, the flow changes between a symmetric type of pattern and an asymmetric one. Numerical results show a hysteresis region, where the flow pattern depends on whether the aspect ratio increased or decreased. As the aspect ratio increases, the Nusselt number increases and reaches a maximum value at the right boundary of hysteresis region. Further increase of the aspect ratio leads to a jump of the Nusselt number down to 1.20, after that, the Nusselt number decreases towards unity. As the aspect ratio decreases, the Nusselt number increases slowly above unity and jumps to another maximum value, which is much larger than unity, but smaller than the maximum value when the aspect ratio increases. The Nusselt number decreases from this maximum value towards unity when the aspect ratio continues to decrease.

**References**


13 Wan, Q. and Kuznetsov, A.V., “Effect of non-uniformity of source vibration amplitude on the sound field wave number, attenuation coefficient and Reynolds stress for the


8. ACOUSTIC STREAMING IN A RECTANGULAR CAVITY INDUCED BY THE VIBRATING LID

ABSTRACT

This chapter investigates fluid flow in a rectangular cavity whose lid (the upper boundary) vibrates in one standing wavelength. The fluid in the cavity is driven by the sinusoidal streaming velocity at the edge of the viscous boundary layer near the lid. When the vibration Reynolds number is sufficiently large, the boundary layer thickness is negligible. The properties of the primary and secondary eddies and the interaction between the primary eddies are discussed in detail for the cavities of three different aspect ratios.

8.1. INTRODUCTION

The classical driven cavity problem with uniform driving velocity at the lid (the upper boundary) has been extensively studied. Because of its simple configuration and exhibition of many important phenomena in incompressible flows, this problem is a basic study in fluid mechanics. It is also relevant to some industrial applications such as coating, mixing, etc. A very good review of the driven cavity problem is given in Shankar and Deshpande [1].

In the classical driven cavity problem, the fluid velocity at the lid is assumed uniform. Recent analysis (Wan and Kuznetsov [2, 3] and Chapter 6) shows that if a beam is vibrated in a viscous fluid, there is a viscous boundary layer attached to the surface of the beam. If the
thickness of this boundary layer is small, the streaming velocity at the edge of this boundary layer can be considered as a slip velocity at the surface of the beam which drives the rest of the fluid. In Chapter 6, Wan and Kuznetsov [2, 3] analytically and numerically investigated vibrating and streaming fields within and outside the viscous boundary layer.

Figure 8-1 (a) Schematic diagram of a cavity in which the flow is driven by vibrating the lid and (b) Driving slip velocity at the mean position of the lid
In this chapter, the flow in a cavity with its lid vibrating in one wavelength of a standing wave is investigated, as shown in Fig. 8-1a. When the vibration Reynolds number is large, the boundary layer thickness at the vibrating lid is small and can be neglected. In this case, the streaming velocity at the edge of the boundary layer acts as a slip velocity which drives the flow in the whole cavity. Different from the classical driven cavity problem, the slip velocity that results from vibrating the lid is not uniform, but has a sinusoidal distribution along $x$ as shown in Fig. 8-1b. One of the advantages of this configuration is that the volume of the fluid within the cavity remains the same at any time during the vibration, which makes it possible to consider a cavity filled with an incompressible fluid. Another advantage is that multiple primary eddies are expected to occur and interact with each other in such cavity.

### 8.2. Theory

In Chapter 6, Wan and Kuznetsov [2, 3] considered a two-dimensional channel bounded by two long parallel beams. One beam is stationary and the other one is vibrating in a standing wave form with the amplitude $A$, the wave number $k$, and the angular frequency $\omega$. The displacement of the surface of the beam, $a^*$, in the direction normal to the beam, $j$, is

$$a^*(x^*,t^*) = jA^* \sin \omega^* t^* \sin kx^*$$

where the asterisks denote dimensional variables, $x^*$ is the coordinate along the mean position of the vibrating beam, and $t^*$ is the time. When the channel width $h$ is much larger than the vibration amplitude $A$ and the vibration Reynolds number $Re_\omega = \frac{\omega}{\nu k^2}$ is large enough, they utilized a perturbation method and analytically found within the channel a three-layer structure,
which is composed of two thin viscous boundary layers near both beams and an inviscid core region. They further found that non-zero unidirectional streaming velocities exist at the edges of both viscous boundary layers due to the attenuation of vibration flow through the boundary layers. The magnitude of the streaming velocity near the stationary beam is much less than that near the vibrating beam and can be neglected. Near the vibrating beam, the vertical streaming component is so small compared to the horizontal one that it can be neglected. According to Wan and Kuznetsov [2, 3] in Chapter 6, the horizontal streaming velocity component at the edge of the boundary layer near the vibrating beam is found to be

\[
U^* (x^*, y^*)\big|_{y^*-h} = \frac{k_\omega A^2 \sqrt{2} Re}{2 \tanh (kh)} \sin (2kx^*)
\]

Equation (6.47a,b) shows that the magnitude of the streaming velocity at the boundary layer edge is a function of the beam vibration parameters and the channel width. For large gap or small standing wavelength, \( kh \) is large enough so that \( \tanh (kh) \to 1 \) and \( U^* \) is independent of the channel width. Since the boundary layer thickness is small compared to the channel width, the streaming velocity at the boundary layer edge can be treated as a driving slip velocity at the mean position of the beam (cf. Fig. 8-1b). This driving velocity moves the streaming flow in the entire channel.

The problem of computing flow in a cavity displayed in Fig. 8-1a is thus reduced to computing flow in a cavity with fixed walls (the upper boundary now coincides with the mean position of the vibrating lid). Vibration of the lid is accounted for by imposing a horizontal velocity distribution (given by Eq. (6.47a,b)) at the mean position of the lid, as shown in Fig. 8-1b. Equation (6.47a,b) can be recast as
where the amplitude $U_c$ can be computed from Eq. (6.47a,b).

Two-dimensional steady incompressible Navier—Stoke equations are considered. Different from Wan and Kuznetsov [2, 3] in Chapter 6, the streaming driving velocity amplitude $U_c$ is chosen as the velocity scale, instead of the vibrating velocity amplitude $u_c$. The dimensionless variables are defined as

$$x \equiv kx^*, \quad y \equiv ky^*, \quad U \equiv \frac{U^*}{U_c}, \quad V \equiv \frac{V^*}{U_c}$$

(8.4)

Governing equations in the stream function—vorticity form can be presented as

$$\nabla^2 \Psi = -\Omega$$

(8.5)

$$\nabla^2 \Omega = \text{Re} \left( \frac{\partial \Psi}{\partial y} \frac{\partial \Omega}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial \Omega}{\partial y} \right)$$

(8.6)

where the Reynolds number is defined as

$$\text{Re} \equiv \frac{U_c}{\nu k}$$

(8.7)

Equations (6.60) and (6.61) must be solved subject to the following boundary conditions:

$$\left. \frac{\partial \Psi}{\partial y} \right|_{y=2\pi} = \sin 2x \quad \text{and} \quad \Psi(x, 2\pi) = 0 \quad \text{at the mean position of the lid},$$

(8.8)

$$\Psi = \frac{\partial \Psi}{\partial n} = 0 \quad \text{at all other fixed walls},$$

where $n$ is the coordinate normal to the corresponding wall and
\[ \Gamma \equiv \frac{h}{\lambda} = \frac{kh}{2\pi} \quad (8.9) \]

is the aspect ratio of the height of the cavity to its length. In Eq. (8.9), \( \frac{2\pi}{k} \) is the wavelength of the standing wave excited in the lid. The dimensionless cavity height is \( kh = 2\pi \Gamma \).

### 8.3. Numerical Results

The driven cavity problem described above is investigated numerically in a dimensionless rectangular domain, which is \( 2\pi \) in length and \( 2\pi \Gamma \) in height. A uniform rectangular mesh of 201×201 nodal points is utilized in this domain to capture possible eddies of small size. Equations (6.60) and (6.61) with boundary conditions (8.8) and vorticity boundary condition (Pozrikidis [4]) are first discretized by finite difference method and then solved by the Gauss-Seidel iteration method. Convergence of iterations is declared when the relative maximum error becomes smaller than \( 10^{-8} \).

To ensure that convection dominates the cavity, the value of the Reynolds number is chosen to be 1,000 for all computations. As shown in Fig. 8-1b, one wavelength of vibration of the lid is composed of four regions which are separated by either nodes or anti-nodes. Within each region, the driving slip velocity starts from zero at the node, reaches its maximum at the point located halfway between the adjacent node and anti-node, and decreases to zero at the anti-node. Therefore, it is expected that there are four primary eddies lined up horizontally within the cavity corresponding to four regions of driving slip velocity. It is obvious that
primary eddies are those driven directly by the slip velocity at the lid. Usually primary eddies also drive secondary eddies.

8.3.1. **Effect of the Aspect Ratio $\Gamma$ on the Primary and Secondary Eddies**

Figures 8-2(a)–(c) show the numerically computed streamlines of the air flow in this driven cavity at $Re = 1,000$ for various aspect ratios, $\Gamma = 0.125, 0.25, \text{ and } 0.375$. As expected, there are four primary eddies in each cavity. For $\Gamma = 0.25$, these eddies are of roughly square shape. Besides, it is observed that in all cavities there are also several secondary eddies, which appear near the both fixed wall corners and the vertical center line at the bottom of the cavity. Secondary eddies in the bottom corners appear to decrease in size with the increase of the aspect ratio $\Gamma$, as seen by comparing Figs. 8-2(a)–(c). According to Moffatt [5], these eddies are driven by the primary eddies and there are infinite sequences of high order eddies of diminishing size and intensity as the corner is approached. It is still unclear however whether similar infinite sequences of high order eddies also exist near the vertical center-line at the bottom of the cavity. This is because in this position there is no corner composed of two fixed walls, which is a necessary condition for the infinite sequences.

In Fig. 8-2a, although the aspect ratio $\Gamma$ is decreased by 50% from that in Fig. 8-2b, four primary eddies do not stretch horizontally. They instead keep roughly a square shape and the rest of the space in the cavity is occupied by four secondary eddies, whose size is comparable with that of the primary ones but have much weaker intensity as shown in the vector plot in Fig. 8-3a. This is different from the flow pattern in a classical driven cavity of similar aspect ratio,
which is driven by a uniform lid velocity and exhibits a horizontally stretched primary eddy (Pan and Acrivos [6]). The reason may be mainly due to the distribution of the driving velocity. This chapter deals with the sinusoid driving velocity distribution which is large only near the points (midpoints) that are located halfway between the adjacent nodes and anti-nodes. The primary eddies are mainly driven by the streaming velocity near these midpoints instead of the entire lid. Therefore, the primary eddies are not stretched and tend to keep the square shape. This is particularly true for the cavities of small aspect ratios as shown in Fig. 8-2a, where the primary eddies accumulate near two anti-nodes because the driving slip velocity is directed towards the anti-nodes.

In addition, there are two more secondary eddies near both top corners under the driving lid, as shown in Figs. 8-2(b) and (c). The size of these top corner secondary eddies appear to increase with the increase of the aspect ratio $\Gamma$, as seen by comparing Figs. 8-2(b) and (c). These top corner secondary eddies, however, are not observed in Fig. 8-2a.

Figures 8-3(a)—(c) show the velocity vectors of the air flow in the driven cavity at Re = 1,000 for various aspect ratios. These vectors denote relative magnitudes of the flow velocities within the cavity. As expected, the secondary eddies are much weaker than the primary eddies.
Figure 8.2: Streamlines of the air flow in the driven cavity at Re = 1,000 for the aspect ratio of
(a) $\Gamma = 0.125$, (b) $\Gamma = 0.25$, (c) and $\Gamma = 0.375$
Figure 8-3 Velocity vectors of the air flow in the driven cavity at $Re = 1,000$ for the aspect ratio of
(a) $\Gamma = 0.125$, (b) $\Gamma = 0.25$, and (c) $\Gamma = 0.375$
8.3.2. Effect of the Aspect Ratio \( \Gamma \) on the Interactions Between Primary Eddies

When there are multiple primary eddies, these primary eddies may interact. If two primary eddies are separated by secondary or even higher order eddies, their influence on each other is indirect and hence cannot be called a true interaction. The interaction only occurs when two primary eddies are adjacent to each other and share a common interface with a non-zero velocity along it. When two eddies are in interaction, their shape and characteristics will not be the same as those for each individual eddy alone because of the existence of a non-zero velocity at the interface between them.
Figure 8-4 Distributions of the horizontal (a) and vertical (b) velocity components along the half-height horizontal cross-section \((y = \pi \Gamma)\) for various aspect ratios.

The interactions never occur in the classical driven cavity in which the uniform lid velocity drives only one primary eddy. In the present research, the steady state flow is considered. Since sinusoid driving slip velocity occurs along the horizontal lid, the primary eddies formed in the cavity are also lined up horizontally. As a result, the interaction interfaces are the vertical lines normal to the lid. Possible locations of the interaction interfaces are near the node and anti-node positions where the driving slip velocity diminishes to zero. The \(x\) coordinates of possible locations are \(\frac{\pi}{2}\), \(\pi\), and \(\frac{3\pi}{2}\) (0 and \(2\pi\) are the coordinates of the fixed walls and are omitted).
Figures 8-4(a) and (b) show the distribution of the horizontal and vertical velocity components along the half-height horizontal cross-section ($y = \pi \Gamma$) for various aspect ratios. From Fig. 8-4b, it is found that there are interaction interfaces near two anti-node positions at $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ for all aspect ratios. This is because the driving slip velocity is directed towards the anti-nodes, pushing the fluid away from the anti-nodes vertically down to the bottom and thus forming an interaction interface between the primary eddies. The vertical velocity near the node position at $x = \pi$, however, exhibits a different behavior for different aspect ratios. For the aspect ratios of $\Gamma = 0.25$ and 0.375, the vertical velocity takes on a positive value, which means the flow is directed upwards and rotates in the same direction as the flow on the opposite sides of the primary eddies. (Along the interaction interface, the flow is directed upwards and at the opposite sides of the interacting primary eddies the flow is directed downwards.) These interactions are shown more clearly in Figs. 8-2(b), (c) and 8-3(b), (c). The vertical velocity at $x = \pi$ tends to increase as the aspect ratio increases although it reaches a local minimum. Two local maximums near the interface come from two interacting primary eddies, the distance between these maximums gets smaller as the aspect ratio increases and the two eddies interact more intensively with each other. For the aspect ratio of $\Gamma = 0.125$, however, the opposite happens. The vertical velocity at $x = \pi$ is negative and there is no interaction between the primary eddies, which is also seen in Figs. 8-2a and 8-3a.

As shown in Fig. 8-4a, the horizontal velocity along the half-height horizontal cross-section is much smaller than the vertical one because the primary eddy centers are located near the
half-height horizontal cross-section and the fluid in the eddy tends to flow normally to the radial direction of the eddy.

8.4. **CONCLUSIONS**

The lid of a cavity, which vibrates in a standing wave form, generates a sinusoidal acoustic streaming driving velocity at the edge of the viscous boundary layer near the lid. If the boundary layer thickness is small, this streaming velocity at the edge of the boundary layer acts as a driving slip velocity for the flow in the cavity. The flow in the cavity is composed of multiple primary eddies, each of which is driven by the streaming velocity in the corresponding region bounded by one node and one adjacent anti-node on the lid. The sizes of the secondary eddies vary for different aspect ratios of the cavity. The primary eddies keep a square shape in the cavity of aspect ratios smaller than 0.25, generating secondary eddies of considerable sizes. Interactions between primary eddies occur near the anti-node positions because the driving slip velocity is directed towards the anti-nodes. In the cavities of large aspect ratios, the primary eddies at both sides of the node positions also tend to interact and the intensity of this interaction increases as the aspect ratio increases.

**REFERENCES**


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9. CONCLUSIONS

The acoustic streaming, which is induced by a beam vibrating in standing or traveling waveform, and its associated cooling efficiency are numerically and analytically investigated. Due to the very small attenuation coefficient of the air, free space streaming in the air is so weak that the associated heat transfer is negligible. When a parallel stationary beam is placed near the vibrating beam, the acoustic attenuation within the gap is much larger than that in the free space. The acoustic streaming within the gap and cooling effect of the heat source or heat flux attached to the stationary beam are investigated both in sonic and ultrasonic range. The numerical results of the sonic streaming agree well with the experiments. The analytical and numerical results of the ultrasonic streaming show the structures of the boundary layer streaming and the outer streaming in the core region.

9.1. REMARKS ON FREE SPACE STREAMING

Free space streaming is generated when the vibrating beam scatters propagating traveling acoustic waves into the free space. Different from the traditional plane wave, such an acoustic wave has a larger wavelength and attenuation coefficient when the beam vibrates in the standing waveform. The difference is a factor, which is analytically obtained as the coefficient of non-uniformity, \( \gamma \). The explicit expression of \( \gamma \) and the Reynolds stress that drives the free space streaming for any arbitrary vibrating waveform are derived in Chapter 2. Comparison of the Reynolds stresses for the plane wave and standing wave shows that both stresses decay exponentially along the propagation direction. The decay for the standing wave case is larger
due to the larger attenuation coefficient. Normal to the propagation direction, the stress of the plane wave remains constant while that of the standing wave reaches its maximums along the anti-nodal positions and smaller minimums along the nodal positions. Such Reynolds stress distribution will drive the free space streaming that is directed away from the anti-nodal positions and towards the nodal positions. Hence, four eddies of free space streaming are expected along one wavelength of the standing wave vibration of the beam.

9.2. Remarks on Sonic Streaming

Sonic streaming is investigated in the gap between the vibrating beam and a stationary parallel beam in Part Two (Chapters 3—5). In Chapter 3, the acoustic streaming, which is induced by the standing wave vibration, and the associated cooling effect of the heat source and heat flux attached to the stationary beam is numerically investigated and compared. Due to the low intensity of the sonic acoustic waves, the boundary layer thickness of the sonic streaming exceeds the gap width. Therefore, the whole gap is occupied by the boundary layer streaming. The computational results clearly show four eddies of the sonic streaming along each standing wavelength. The streaming velocity is directed down to the nodes, from nodes to anti-nodes, and away from the anti-nodes. It is also found that the cooling efficiencies for both cases are almost the same. In the heat source case, there is little temperature variance along the heat source surface provided that the conductivity of the heat source (e.g. of the silicon chip) is much larger than that of the air.

In Chapter 4, the comparison of standing and traveling wave vibrations of the beam shows no difference in the cooling efficiency to the heat source although the sonic streaming for the
traveling wave case does not exhibit a symmetric flow pattern. However, the sonic streaming is steady and not carried away while the waves are traveling along the vibrating beam. Explanation for the experimental results of Wu and Ro [1] is given in the computation in Chapter 5, where the only half standing wave form is resonated on the vibrating beam. Large vibration amplitude (130µm) compensates the low sonic frequency (280Hz) and resulting acoustic field induces the sonic streaming that has significant cooling capacity. The oscillating acoustic field within the gap also acts as an oscillating piston that pushes the air in and out of the gap through two open ends. It is probably the combined effect of these two mechanisms that contribute to the cooling effect.

9.3. Remarks on Ultrasonic Streaming

The acoustic streaming induced in high intensity acoustic field is supposed to have magnitude that is comparable to the acoustic vibration and hence high cooling efficiency. In Part Three, ultrasonic vibration of the beam induced the acoustic streaming that is strong enough to invalidate the perturbation expansion method utilized in Part Two. Nevertheless, the total flow field can still be separated as streaming, Single Frequency Oscillating (SFO), Double Frequency Oscillating (DFO), and multiple frequency fields.

The analytical solution of the inviscid SFO field shows the existence of the boundary layers near both beams and also provides free stream condition for them. Further analytical solutions of the streaming and SFO field within two boundary layers reveals non-trivial streaming velocities at the edge of the boundary layers, which in turn drives the flow in the core region.
between the two boundary layers. As shown in Eq. (8.2), the driving velocity at the vibrating beam is a function of frequency, amplitude, wave number, viscosity of the fluid, and the gap size. Most importantly, it is proportion to \( \omega^2 A^2 \) instead of \( \omega^2 A^2 \) in sonic streaming. In other words, the increase of the amplitude induces larger ultrasonic streaming than the increase of frequency. The boundary layer thickness is proportional to the inverse of the square root of frequency, as shown in Eq. (6.24b).

The core region streaming driven by the streaming velocities at the edge of the boundary layers is thus obtained numerically. The boundary layer streaming near the oscillating beam (BLO streaming) and the core region streaming forms a series of complete eddies, which is similar to those in Part Two. The boundary streaming near the stationary beam (BLS streaming) forms another series of eddies, which are too small and too weak to post significant impact on the cooling efficiency. However, when the aspect ratio of the gap increases over a critical value, the core region streaming shows an asymmetric flow pattern, where the strong eddies accumulate near the oscillating beam leaving the rest of the gap with almost no fluid motion. This phenomenon and the hysteresis effect are further discussed in Chapter 7, where the length of hysteresis region is numerically determined. The cooling capacities of the core region streaming for both heat source and heat flux cases are also obtained in the form of the Nusselt number. As the aspect ratio increases, the Nusselt number increases to approximately 10 before the critical value and jumps to nearly unity when the critical value is exceeded. This means the core region streaming contribute to the increasing cooling efficiency before the critical value and has almost no effect after the critical value is exceeded. The corresponding experiments by Wu and Ro [1] do not achieve any cooling effect because the ultrasonically
resonating bimorph (beam) generates a lot of heat into the gap and increase the heat load too much for the ultrasonic streaming. Without the ultrasonic streaming, the gap with continuous heat dissipation inside cannot be maintained at a constant temperature. This an indirect evidence of the cooling effect of the ultrasonic streaming.

The cavity driven by the ultrasonic streaming, discussed in Chapter 8, is a new area of research. Similar to the core region streaming, the fluid in a rectangular cavity can be driven by the non-zero streaming at the edge of the boundary layer edge near a lid that is vibrating in ultrasonic frequency. In contrast to the classical driven cavity problem, where only one primary eddy exists, there are multiple primary eddies in the cavity, and the number of primary eddies depends on the number of standing waves generated in the lid. Some primary eddies always interact directly while other primary eddies interact directly or indirectly through secondary eddies, depending on the aspect ratio of the cavity.

9.4. **Recommendations for Future Work**

The free space streaming field, although very weak in magnitude, is very interesting to investigate. Analytical solution may be obtained for the free space streaming induced by an infinitely long beam vibrating in a standing wave form. The large free space and its boundary conditions may be challenging for numerical computations. It is also interesting to investigate the free stream acoustic wave interacting with passive compliant wall or liquid surface, which responds to the acoustic pressure and shear stress according to its own elasticity and surface tension. The responding behavior of the compliant wave or liquid surface may have some strengthening or reduction of the free space streaming.
When the bimorph (beam) vibrates in its first mode, only half standing wave is generated on its surface. In this case the total volume between the gap and the parallel beam cannot be constant. The heat exchange mechanism through two open ends due to the air flow being pushed in and out of the gap needs more realistic boundary conditions, which may be very complicated.

The ultrasonic streaming within the gap display two layers of eddies: BLO and core region streamings; BLS streaming. Although BLS streaming is relatively much weaker than BLO and core region streaming, its effect of the core region streaming needs further investigation. It might have some frictional effect on the core region streaming due to the same eddy rotation direction as that of the core region streaming.

The vibrating beam scatter acoustic waves into the gap, which is investigated in this dissertation in detail, and out into the free space on the other side. Possible weak free space streaming might occur and have some capacity of cooling. This can be investigated by removing constant temperature boundary condition at the vibrating beam and setting the temperature boundary condition somewhere in the free space, which may also be very complicated.

It is also recommended to investigate a 3-D acoustic streaming since the acoustic sources are usually 3-D. Direct Numerical Simulation (DNS) may be involved; further theoretical research has same level of importance.

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