ABSTRACT

GIBSON, LATOSHA MARIE. Skin Friction versus Fire Propagation. (Under the direction of Dr. Kevin M. Lyons.)

In examination of skin friction versus fire propagation, two methods of solution were of interest: (1) the viscous solution of the incompressible stagnation point velocity flow and (2) the Amplification Theory. For stagnation point velocity flow, the velocity is assumed to be zero at the stagnation point for the viscous solution. The Amplification Theory, however, deduces that the velocity is characterized by vortexes at the stagnation point. Therefore it was hypothesized that turbulence intensity through the Amplification Theory would render higher values for skin friction. The accounting of flame stretch was believed to have a small effect on the value of skin friction since the stretched laminar burning velocity is a product of the laminar burning velocity, and the pressure and temperature risen by a small power. Because of the direct correlation between the wall heat flux at the stagnation point and shear stress, the associated analytical heat flux equation utilizing the stagnation velocity gradient as a function of turbulent intensity was believed to be in a closer approximation to empirical values than the heat flux associated with the viscous solution for the incompressible stagnation point flow. Overall, the values from the viscous solution of the stagnation point velocity reported lower values than values of the K-epsilon solution involving premixed combustion. However, factoring stretch decreased the skin friction within the stagnation region. The empirical heat flux formula was shown to be in closer proximity to experimental values than the semi-analytical and theoretical heat flux solution.
SKIN FRICTION VERSUS FIRE PROPAGATION

By

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To Livvy, my niece who has always been livid.
BIOGRAPHY

The author was born in Durham, North Carolina in 1978 and attended Charles E. Jordan High School in 1993. After graduating from High School in 1996, she enrolled at North Carolina Agricultural and Technical State University where she completed her bachelor degree in Mechanical Engineering in May 2000. While in the Aerospace Engineering Graduate Program in North Carolina State University, she had attained the General Electric Teaching Fellowship and Mickey Leland Energy Fellowship. She also participated in an internship program under the direction of Dr. Peter Rozelle of the Clean Energy Systems Division of the office of Fossil Energy at the Department of Energy. She plans to further pursue her interest in turbulent combustion, flame stability, and emissions.
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a  Speed of Light, m/s  Pr  Prandtl Number

a, f, v  Dimensionless Variables  q  Heat Flux, KW/m²

c, k  Wave Number  q_{min}  Disturbance Equation

c1, c2  Constants  R  Radius, m

C_f  Skin Friction  Re  Reynolds Number

cp  Specific heat, J/kg*K  Re_T  Turbulent Reynolds Number

f̅  Mixture Fraction  S_L  Burning Velocity, m/s

G  Equation Representation  S_T  Turbulent Burning Velocity, m/s

g  Acceleration due to Gravity, m/s  T  Temperature, K

g_w  Temperature ratio  Tu  Turbulent Intensity

h  Enthalpy  t  Time

h_f  Convection Constant  U, V  Axial Velocity

k  Conductivity, W/m*K  u, v, w  Velocity components

L  Dimensionless Length \frac{l_j}{d_b}  \tilde{u}, \tilde{v}, \tilde{w}  Velocity components

Markstein length  u_e  Edge Velocity

L_T  Turbulent length Scale  Vm  Mean Velocity

M  Mach Number  x, y, z  Position and Cartesian coordinates

Ma  Markstein Number  \tilde{X}, \tilde{Y}, \tilde{Z}  Coordinates in Figure 3.2

n  Iteration step  Y  Mass Fraction

P  Pressure  Y_c  Yap correction
Greek Symbols

* \( \alpha \) * Disturbance * \( \mu \) * Dynamic Viscosity, kg/m-s

* \( \beta \) * Stagnation Velocity Gradient, s\(^{-1}\) * \( \mu_{\varepsilon} \) * Dissipative Viscosity, kg/m-s

* \( \Gamma, \phi, S_{\phi} \) * Kinetic Energy Representation * \( \nu \) * Kinematic Viscosity, m\(^2\)/s

* \( \Delta x \) * Finite position increment, m * \( \varepsilon \) * Dissipative Rate

* \( \Delta t \) * Time increment, s * * Disturbance Rate

* \( \delta, \delta^* \) * Flame Thickness, Boundary Layer, m * \( \rho \) * Density, kg/m\(^3\)

* \( \zeta \) * Vorticity * \( \sigma_f, \sigma_k, \sigma_{\varepsilon} \)

* \( \eta \) * Dimensionless Parameter, Disturbance * \( \tau_c, \tau_t \) * Chemical and Turbulent

* \( \theta \) * Dimensionless Temperature * * Time Scale, s

* \( \phi \) * Linear Disturbance Parameter * \( \nu \) * Courant Number

* \( \kappa \) * Stretch Rate * \( \psi \) * Stream Line

* \( \lambda \) * Wave Length * \( \omega \) * Reaction Rate,

Superscript

* \( \cdot \) * Perturbation * * Amplified

* \( \sigma \) * Indicator of two dimensional

or Axisymmetric flow

Subscript

* \( b, n, j \) * burner, nozzle, jet * \( I, j \) * index for species, or node

* \( D \) * diameter * \( o \) * at stagnation point

* \( e \) * edge of boundary layer * \( x, y, z \) * Cartesian scalar points

* \( g \) * gas * \( l \) * perturbed component
1 INTRODUCTION

1.1 PURPOSE

Due to the higher demand for power generation and the launching of spacecraft, heat convection is of primary interest. Although skin friction is defined as the force due to drag over the dynamic pressure, the shear stress has a direct effect on the magnitude of heat transfer. Such an experiment investigating skin friction coupled with computational analysis would provide insight into the thermal analysis of components in contact with hot gases while providing an analysis of the worst case scenario. Two primary examples applications of this work are the protective material of space vehicles and the operation of gas turbines.

1.1.1 Aerospace Applications

Heat convection as a measurement of performance of insulating materials for certain space vehicles has been deemed negligible in the past due to the short span in the Earth’s atmosphere [1]. However developing interest for a more viable and commercially available transport to space has shifted the desired mode of takeoff and landing [2]. According to Modlin [2], these hypersonic vehicles are to take-off and land on standard runways and operate within an air breathing corridor of the atmosphere while being powered by air-breathing engines. The air breathing propulsion systems of these airplanes extends the acceleration periods within the denser part of the Earth’s atmosphere prior to reaching cruise and orbital velocities. Thus critical surfaces such as the leading edges of wings, engines, and fuselage would result in severe aerodynamic heating.

According to Modlin, there are two cooling methods to counteract the type of heating described above: Through the use of heat pipes and film cooling. Film cooling is described as
the injection of a coolant into a hot boundary layer that provides a thin protective layer over
the surface to be cooled. This fluid is injected at discrete locations on the surface through
holes or slots. For heat pipes, large heat fluxes are transferred isothermally. The components
of a heat pipe are the outer shell (thin to promote good conduction), the capillary structure,
and working fluid. The heat pipe is said to operate by vaporizing the working fluid at one end
(the evaporator section) and condensing it at the other end of the condenser. The model used
in this research entailed a pipe connected to a heat exchanger while enclosed in a transient
aerodynamic surface. Although it was concluded that the wing leading edge and liquid metal
heat pipe cooling was sufficient for maintaining surface temperature below 1800K in
conjunction with a heat exchanger, the research was not without assumptions and limitations
that could have affected the results.

The focus was on the pipe’s cooling response to aerodynamic heating and the surface
mass transfer cooling effects—not on the detailed boundary layer or flow field response to
coolant mass injection [2]. For this research, 1800K was the maximum allowable temperature
for deployment through the Earth’s atmosphere. However with creep, material deformation
as a function of temperature and time, this temperature threshold may not be acceptable for a
given material. Higher performance through the cooling mechanism or the transient material
may warrant an analysis of the boundary layer flow field in garnering the entire heat flux
through the region of the interface (the joint thin sections of the transient material and outer
pipe material).

For other aerospace vehicles, the effect of aero-convective heating from a material
standpoint is of greater concern. Protective ceramic coating are said to be applied to flexible
insulation for the thermal protection of aerospace vehicles [3]. These coatings are required to
provide the following [3]: (1) Thermal protection to a fabric instantly upon exposure to the high heating environment, (2) Flexibility prior to installation on the vehicle, (3) Freedom from organic materials which may produce volatiles (4) and Adherence to the insulation with minimum weight penalty. For ceramic fabric, requirements of coating are: (1) Protection from oxidation of fibers when exposed to a high heating rate, (2) Increasing the emissivity of the fabric surface, (3) Reductions of the surface temperature by decreasing the catalytic efficiency of the surface.

Overall, the performance of these materials is still limited due to temperature. For example, flexible ceramic insulation, which is said to be easier to fabricate when compared to rigid insulation, is limited in withstanding heating rates at or above 39.5 W/cm² [3]. Meanwhile, the ceramic coating for this insulating material can only provide protection up to 980°C or heating rates at 12.0 W/cm². The emissivity remains un-enhanced since the emittance for the coating remains the same as the fabric [3]. Other coatings crystallize and form cracks upon cooling after heat exposures while some coatings become brittle. Due to the limitations of insulating material, a temperature profile based on a velocity may provide insight into the life of the insulating material and coatings.

1.1.2 Applications for Turbines

In terms of gas and jet turbines, the leading-edge cooling and phase composition change of thermal barrier coatings is of particular concern. The leading edge of airfoils of the first stage stator/nozzles [4] are said to be the most difficult regions to cool. These surfaces encounter the hottest gas temperatures while experiencing high heat transfer rates in their stagnation zones. Very thin boundary layers develop from these stagnation zones where
mainstream hot gases and the cooling jet interact. Acceleration then results due to curvature of the geometry [4].

Furthermore, jet-to-jet interactions due to cooling along the leading edge have been shown to create small separation bubbles (see Figure 6.5) between successive holes due to a pressure gradient upstream [4]. In areas of curvature along the blades and vanes, vortices have developed causing an increase in shear stress and thus temperature gradient in those areas. Therefore, the velocity profile of the boundary layer would have to be analyzed in order to increase the adiabatic efficiency and cooling effectiveness of certain turbine blades.

Aside from the complexity of the flow of hot gases impinging upon turbine blades is the dynamics of the phase change, composition of thermal barrier coatings, and the super alloy material that are coated. The super alloys that are coated are created by extracting slip planes and impurities--enabling the alloys to withstand high temperature. However, the extraction of aluminum results in the loss of corrosion resistance. Since the alloy inner zone changes during higher temperature, the inner zone between the coating and super alloy is of interest. This interaction is not only found to be dependent upon temperature, but the original chemical and phase compositions of the coating and super-alloy.

Such interaction is claimed to be in the in terms of diffusion of elements into the alloy or the diffusion of elements into the coating. Since the elements stored in the coating provides “heat proof oxides” [5], the coating fails as their concentration decreases due to diffusion. Depending upon the composition of the alloy, chromium, molybdenum, tungsten, and titanium also diffuse—but at a rate based on their solubility. Because the stage of each interaction is dependent upon temperature, the higher the temperature is, the greater the rate
of diffusion. However, the temperature gradients of each turbine blade are not uniform during each load.

Based on the applications discussed, there is a strong joint effort between using cooling mechanisms and utilizing resistant thermal properties of selected materials. Although the performance of materials such as protective ceramic coating and thermal barrier coatings are directly tied to temperature and heat flux, the velocity profiles around these materials are quite complex and have a significant effect on the temperature gradients. Therefore, deducing the skin friction at the boundary layers of the flow ill not only serve as key to increasing the cooling effective of cooling mechanisms but will hone in the desired performance of heat resistant materials over a period of time.
1.2 COMPUTATIONAL MODELING OF FLAME IMPINGEMENT

The following represents a set of Cartesian differential equations for the modeling of flames in computational fluid dynamics [6]:

**Cartesian Set**

\[
\frac{\partial (\rho \tilde{u} \phi)}{\partial x} + \frac{\partial (\rho \tilde{v} \phi)}{\partial y} + \frac{\partial (\rho \tilde{w} \phi)}{\partial z} + \left[ \frac{\partial}{\partial x} \left( \Gamma_\phi \frac{\partial (\phi)}{\partial x} \right) - \frac{\partial}{\partial y} \left( \Gamma_\phi \frac{\partial (\phi)}{\partial y} \right) - \frac{\partial}{\partial z} \left( \Gamma_\phi \frac{\partial (\phi)}{\partial z} \right) \right] = S \phi
\]

**Equation**

\[
\begin{align*}
\text{Continuity} & : \phi & & \Gamma_\phi & & S \phi \\
\text{X-momentum} & : \tilde{u} & \mu_\varepsilon & - \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left( \mu_\varepsilon \frac{\partial \tilde{u}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_\varepsilon \frac{\partial \tilde{v}}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu_\varepsilon \frac{\partial \tilde{w}}{\partial z} \right) + \rho g_x = \frac{2}{3} \rho \tilde{\varepsilon} \\
\text{Y-momentum} & : \tilde{v} & \mu_\varepsilon & - \frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left( \mu_\varepsilon \frac{\partial \tilde{u}}{\partial y} \right) + \frac{\partial}{\partial y} \left( \mu_\varepsilon \frac{\partial \tilde{v}}{\partial y} \right) + \frac{\partial}{\partial z} \left( \mu_\varepsilon \frac{\partial \tilde{w}}{\partial z} \right) + \rho g_y = \frac{2}{3} \rho \tilde{\varepsilon} \\
\text{Z-momentum} & : \tilde{w} & \mu_\varepsilon & - \frac{\partial P}{\partial z} + \frac{\partial}{\partial x} \left( \mu_\varepsilon \frac{\partial \tilde{u}}{\partial z} \right) + \frac{\partial}{\partial y} \left( \mu_\varepsilon \frac{\partial \tilde{v}}{\partial z} \right) + \frac{\partial}{\partial z} \left( \mu_\varepsilon \frac{\partial \tilde{w}}{\partial z} \right) + \rho g_z = \frac{2}{3} \rho \tilde{\varepsilon} \\
\text{Mixture fraction} & : \tilde{f} & \frac{\mu_\varepsilon}{\sigma_f} & 0 \\
\text{Turbulent Energy} & : \tilde{k} & \frac{\mu_\varepsilon}{\sigma_k} & G - \rho \tilde{\varepsilon} \\
\text{Dissipation Rate} & : \varepsilon & \frac{\mu_\varepsilon}{\sigma_\varepsilon} & \left( \frac{\varepsilon}{k} \right) (c_1 G - c_2 \rho \tilde{\varepsilon}) \\
\text{Where} & : G = \mu_\varepsilon \left\{ 2 \left[ \left( \frac{\partial \tilde{u}}{\partial x} \right)^2 + \left( \frac{\partial \tilde{v}}{\partial y} \right)^2 + \left( \frac{\partial \tilde{w}}{\partial z} \right)^2 \right] + \left( \frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x} \right)^2 + \left( \frac{\partial \tilde{u}}{\partial z} + \frac{\partial \tilde{w}}{\partial x} \right)^2 + \left( \frac{\partial \tilde{v}}{\partial z} + \frac{\partial \tilde{w}}{\partial y} \right)^2 \right\}
\end{align*}
\]

Due to the nonlinear relation between chemical reaction kinetics and fluid mechanics, modeling flames have proven to be difficult and is therefore limited. Moreover, there is the inclusion of chaos when turbulent mixing and the heterogeneous chemistry of solids such as soot formation is taken into account [6]. Because the fluid flow must be resolved to the
smallest characteristic size to capture those characteristics responsible for momentum transfer, resolution must be no larger than two to three times the Batchelor Scale [6].

In order to perform a computational analysis that determines the skin friction within the propagation of a flame, three primary components of flame impingement must be taken in consideration: boundary conditions at the interface, accuracy of the magnitude and sources of heat flux, and the coupling of the reaction rate with the transport of a turbulent flame. Such conditions not only dictate the accuracy of a numerical solution but the stability and properties of a flame such as flame stretch.

1.2.1 Boundary Conditions at the Interface

First, boundary conditions for partial differential equations such as the Navier Stokes equations are based on the characteristic type or eigenvalue PDE (elliptic, parabolic, and hyperbolic) and will either make a solution well or ill posed. However, in the case of a flame impinging on an object, there is a general concern for the interaction of the species of a burning gas upon the surface of the object and how this accommodates the governing equations for fluid flow. Quenching has also been cited as a condition to be accounted for near the interface [7]. For the mass flux, the following figure represents a diagram of a mass flux at a gas-solid interface with the solid interface fixed [8].

![Figure 1-1 Solid Gas Interface](8)

![Figure 1-2 Interaction at Solid Gas Interface](8)
Where $v_g$ is the velocity of the gas at the interface and ‘n’ is the normal vector.

1.2.1 \[ \rho_{g_+} Y_{i_+} v_{g_+} - \rho_{g_+} D \frac{\partial Y_i}{\partial y_i} \bigg|_{\Gamma} = \rho_{s_+} Y_{i_-} r_{i_-} + \omega_i - \frac{1}{A} \frac{d}{dt} \int \rho_i dS \]

<table>
<thead>
<tr>
<th>Mass flux of Species Convected out</th>
<th>Diffused Out</th>
<th>Supplied to burn</th>
<th>Generated on surface through reaction</th>
</tr>
</thead>
</table>

Here it is assumed that the Ficks Law of Mass Diffusion is valid and there is no subsurface diffusion. If mass accumulation is negligible the equation becomes [8]:

1.2.2 \[ \rho_{g_+} Y_{i_+} v_{g_+} - \rho_{g_+} D \frac{\partial Y_i}{\partial y_i} \bigg|_{\Gamma} = \rho_{s_+} Y_{i_-} r_{i_-} + \omega_i' \]

1.2.3 \[ \rho_{g_+} Y_{i_+} v_{g_+} - \rho_{g_+} D \frac{\partial Y_i}{\partial y_i} \bigg|_{\Gamma} = \omega_i' \]

Here the conservation of species must be satisfied and is influenced by the rate of reaction. Due to the assumption based on Fick’s Law, species in the reaction are assumed to diffuse towards lower concentrations of that particular species. Therefore, each species is believed to move in the same direction of the mixture. In terms of quenching, it was assumed that the Kolmgorov scale was sufficient in gauging the likelihood of a flame to quench [7]. However, this scale was found to overestimate quenching due to its magnitude to flame thickness [7]. In Liakos, a criterion based on turbulence intensity, laminar burning velocity, and the turbulent Reynolds number was used to determine when extinction was likely to occur through the following equations [9]:

1.2.4 \[ \frac{u'}{S_l} > 2 \text{Re}_i^{0.25} \text{ and } \text{Re}_i = \frac{u' L}{v} > 300 \]

When the previous relations are met, the chemical source term is set to zero.
1.2.2 Accuracy and Sources of Heat Flux

For the purpose of this analysis, the accuracy of the heat flux would be of interest since it is a value measured in the vicinity of the stagnation point for a number of experiments in flame impingement. This heat flux which is to be determined computationally would have to account for turbulent flow as well as laminar flow since most applications of flame impingement are turbulent. Because the standard k-ε model has found to over-predict the heat flux, even with the low Reynolds number approximation, the RNG k-ε model is preferred [10]. The RNG k-ε turbulent flow model is a nonlinear model that neglects the energy work terms but employs the yap correction and the parameter fw in the strain tensor to prevent over-prediction of heat transfer [10]. The parameter fw is said to provide better predictions within the stagnation point by diminishing the effect of second order stress tensor terms in region of impingement. The following is the expression of fw:

\[ 5 f_w = 1 - 18w^2 + (72\sqrt{6})w \]

\[ f_w = 1 \quad \text{when} \quad w = 0 \]

\[ f_w = 0 \quad \text{when} \quad w = 1\sqrt{6} \]

In terms of the Yap correction, Yc, this parameter is employed in the diffusivity transport equation, ε. Furthermore, numerical results have been shown to be in better agreement with experimental data when radiation was taken in account [9].

1.2.3 The rate of reaction versus fluid transport

Finally, the reaction rate coupled with fluid transport must be taken in account. The current rate of reaction model, Eddy Break-up (EBU) does not depend on chemistry while
the Eddy Dissipation Concept (EDC) model does not adequately describe the transport of turbulent flame. The EDC model is said to differentiate between the areas of chemical reactions (fine structure regions) and surrounding non-reaction regions [6]. If reactions within the fine structure are perceived as infinitely fast, the mass transfer between the surrounding fluid and fine structure is believed to limit the rate or reaction. The EBU model, however, is based on the calculation of a turbulent, premixed flame with an irreversible one-step reaction.

Liakos has developed a model based on the KPP Theory that hypothesizes a relation between the propagation of velocity and the rate of reaction while assuming a cold edge of a flame. This relation is illustrated in the following for the turbulent velocity:

\[ St = 2 \left( \frac{\mu_f}{\sigma_k} R_f \right)^{1/2} \]

Where  
- The reaction rate  
  \[ R_f = \left[ \tau_c^2 + 0.09 \tau_t^2 \right]^{-1/2} \]
- The turbulent time scale  
  \[ \tau_t = \frac{k}{\varepsilon} \]
- The chemical time scale  
  \[ \tau_c = \frac{\delta}{Sl} \]
- Flame thickness  
  \[ \delta = \frac{\nu}{Sl} \] and Sl is the laminar burning velocity

Here \( R_f \) is a function of the turbulent and chemical time scale. This combustion model is presumed to account for both types of premixed combustion and cover the range of combustion regimes.

In order to determine the skin friction, the velocity profile within the boundary layer must be known. However, the mass flux and the rate of reaction are equally important in determining the boundary layer thickness through the following relation [11]:

\[ \text{(1.2.6)} \]
1.2.7 \[ \dot{M}_w = \frac{d}{dt}[\rho u_c \delta] \]

Through the mass flux at the wall, the viscous region can be discerned from the inviscid region, by differentiating with respect to length, l of the body of interest. Such a value would serve as a value at the interface and can be iterated by a procedure to correct both solutions for the viscous and inviscid region.

1.2.4 Other Computational Considerations: Detecting the Boundary Layer

In terms of the Inverse Method A, the boundary-layer equations are [12]:

1.2.8 Continuity: \[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

With the following boundary conditions:

\[ u(x,0) = v(x,0) = 0 \quad \text{and} \quad \int_0^\infty \left( 1 - \frac{u}{u_c} \right) dy = \delta^*(x) \]

Here the limit of \( u(x,y) \) is \( u_c \) as \( y \) approaches \( \infty \).

In order to distinguish the viscous region from the inviscid region an iterative procedure is used. Such an iterative procedure is the variable secant procedure (Newton-Raphson Method). First, the function \( F(x) \) is expended in the Taylor series about \( x_n \).

1.2.9 Momentum: \[ C[u] \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = u_c \frac{du_c}{dx} + \frac{1}{\rho} \frac{\partial \tau}{\partial y} \]

1.2.10 \[ F(x_n + \Delta x) = F(x_n) + F'(x_n) \Delta x + F''(x_n) \frac{\Delta x^2}{2} \ldots \]
Solving for $\Delta x$ provides the following:

$$1.2.11 \quad x_{n+1} - x_n = \Delta x = - \frac{F(x_n)}{F'(x_n)}$$

Finally, the derivative is replaced by a secant line approximation through two points:

$$1.2.12 \quad F'(x_n) \approx \frac{F(x_n) - F(x_{n-1})}{x_n - x_{n-1}}$$

Combining equation 1.2.11 and 1.2.12 yields the following:

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} \quad x_{n+1} = x_n - \frac{F(x_n)}{F(x_n) - F(x_{n-1})}$$

$$1.2.13 \quad x_{n+1} = x_n - \frac{F(x_n)}{F(x_n) - F(x_{n-1})} x_n - x_{n-1}$$

In using this iterative procedure to determine the boundary layer thickness $x_n$ becomes $u_{e,n}$ and $F = \delta^* - \delta_{BC}^*$. 

Figure 1-3 Secant Line Procedure [12]
2 THEORY

In examination of skin friction versus fire propagation, two methods of solution are of interest: (1) the viscous solution of the incompressible stagnation point velocity flow and (2) the Amplification Theory. Velocity is assumed to be zero at the stagnation point for the viscous solution while the Amplification Theory deduces that the velocity is characterized by vortexes at the stagnation point. Therefore it is hypothesized that turbulence intensity through the Amplification Theory will render higher values for skin friction. The accounting of flame stretch is believed to have a small effect on the value of skin friction since the stretched laminar burning velocity is a product of the laminar burning velocity, and the pressure and temperature rose by a small power. Because there is a direct correlation between the wall heat flux at the stagnation point and shear stress, the associated analytical heat flux equation utilizing the stagnation velocity gradient as a function of turbulent intensity will be in a closer approximation to empirical values than the heat flux associated with the viscous solution for the incompressible stagnation point flow.
2.1 THE VISCOUS SOLUTION TO STAGNATION POINT FLOW

For an analytical analysis of flame impingement we consider the viscous solution for stagnation point flow for a blunt body [13].

![Figure 2-1 Stagnation Point Flow](image)

The following set of equations represents the governing equations for Euler Inviscid flow:

\[
\begin{align*}
2.1.1a \quad & \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{w}) \\
2.1.1b \quad & \frac{\partial \vec{w}}{\partial t} = -\nabla \cdot \left( \frac{\vec{w}^2}{2} - (\nabla \times \vec{w}) \times \vec{w} - \frac{1}{\rho} \nabla P + \vec{F}_{b} \right) \\
2.1.1c \quad & \frac{\partial P}{\partial t} = -\rho \nabla \cdot \vec{w} - \vec{w} \cdot \nabla P
\end{align*}
\]

Continuity

Momentum

Energy

With negligible forces due to gravity, and steady irrotational flow, the time derivative terms are drop and the governing equations become the following:

\[
\begin{align*}
2.1.2a \quad & \nabla \cdot (\rho \vec{w}) = 0 \\
2.1.2b \quad & \nabla \cdot \left( \frac{\vec{w}^2}{2} + (\nabla \times \vec{w}) \times \vec{w} \right) = -\frac{1}{\rho} \nabla P \\
2.1.2c \quad & \vec{w} \cdot \nabla h_{o} = 0
\end{align*}
\]

Continuity

Momentum

Energy

Assuming that the gas is perfect, the other properties at the stagnation point are defined as the following:

\[
\begin{align*}
\text{Enthalpy} \quad & \frac{h_{o}}{h} = \frac{T_{o}}{T} = X \quad \text{where} \quad X = 1 + \frac{\gamma - 1}{2} \cdot M^2, \quad M = \frac{w}{a}, \quad \text{and} \quad a^2 = \frac{\gamma P}{\rho} \\
\text{Density} \quad & \frac{\rho_{o}}{\rho} = X^{1/(\gamma - 1)} \\
\text{Pressure} \quad & \frac{P_{o}}{P} = X^{1/(\gamma - 1)}
\end{align*}
\]
Using the properties for perfect gas and rewriting the governing equations in terms of axisymmetric flow yields the following (See Appendix B for the axisymmetric coordinates):

2.1.3a \[ \frac{\partial (\rho w_r)}{\partial x} + \frac{1}{r} \frac{\partial (r \rho w_r)}{\partial r} = 0 \] Continuity

2.1.3b \[ \frac{\partial}{\partial x} \left( \frac{w^2}{2} \right) - w_\theta \frac{\partial w_\theta}{\partial x} - w_r \left( \frac{\partial w_r}{\partial x} - \frac{\partial w_x}{\partial r} \right) + \frac{1}{\rho} \frac{\partial P}{\partial x} = 0 \] x-momentum

2.1.3c \[ \frac{\partial}{\partial r} \left( \frac{w^2}{2} \right) + w_x \left( \frac{\partial w_x}{\partial x} - \frac{\partial w_r}{\partial r} \right) - \frac{w_\theta}{r} \frac{\partial (rw_\theta)}{\partial r} + \frac{1}{\rho} \frac{\partial P}{\partial r} = 0 \] y-momentum

2.1.3d \[ \frac{w_r}{r} \frac{\partial (rw_\theta)}{\partial r} + w_x \frac{\partial (\rho w_x)}{\partial x} = 0 \] z-momentum

2.1.3e \[ \frac{\gamma}{\gamma - 1} \frac{P}{\rho} + \frac{1}{2} w^2 = h_o \] Energy

In a simplified form, equations 3.1.3a-3.1.3b becomes:

2.1.4a \[ \frac{\partial (\rho w_x)}{\partial x} + \frac{\partial (r \rho w_r)}{\partial r} = 0 \] Continuity

2.1.4b \[ w_x \frac{\partial w_x}{\partial x} + w_r \left( \frac{\partial w_r}{\partial x} - \frac{\partial w_x}{\partial r} \right) + \frac{1}{\rho} \frac{\partial P}{\partial x} = 0 \] x-momentum

2.1.4c \[ w_x \left( \frac{\partial w_x}{\partial x} + w_r \left( \frac{\partial w_r}{\partial x} - \frac{\partial w_x}{\partial r} \right) - \frac{w_\theta^2}{r} + \frac{1}{\rho} \frac{\partial P}{\partial r} = 0 \] y-momentum

2.1.4d \[ w_x \frac{\partial (rw_\theta)}{\partial x} + w_r \frac{\partial (rw_\theta)}{\partial r} = 0 \] z-momentum

2.1.4e \[ \frac{\gamma}{\gamma - 1} \frac{P}{\rho} + \frac{1}{2} w^2 = h_o \] Energy

The following is notation used to accommodate two dimensional and axisymmetric flow:

\[ \sigma = \begin{cases} 0 \rightarrow 2d \_ flow & x \rightarrow x_1 \quad | y, r \rightarrow x_2 \quad | z, \theta \rightarrow x_3 \\ 1 \rightarrow axisymmetric \_ flow & h_1 = 1 \quad | h_2 = 1 \quad | h_3 = x_2^\sigma \\ \hat{1}_x \hat{e}_x \rightarrow \hat{1}_x \hat{e}_x \quad \hat{1}_r \hat{e}_r \rightarrow \hat{1}_r \hat{e}_r \quad \hat{1}_\theta \hat{e}_\theta \rightarrow \hat{1}_\theta \hat{e}_\theta \\ u, w_x \rightarrow w_1 \quad | v, w_r \rightarrow w_2 \quad | w, w_z \rightarrow w_3 \end{cases} \]
The governing equations for incompressible two dimensional stagnation flow are:

2.1.5a \[ \frac{\partial(xu)}{\partial x} + \frac{\partial(xv)}{\partial y} = 0 \]

Continuity

2.1.5b \[ u \frac{\partial u}{\partial x} + v \left( \frac{\partial u}{\partial y} \right) = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \nabla^2 u - \frac{u}{x^2} \right) \]

X-Momentum

2.1.5c \[ u \left( \frac{\partial v}{\partial x} \right) + v \left( \frac{\partial v}{\partial y} \right) = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left( \nabla^2 v \right) \]

Y-Momentum

Energy

2.1.5d

\[ \rho cp \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial x} \left( \frac{u}{\rho} + \frac{v}{\nu} \right) - \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} + \frac{\partial}{\partial y} \right) + \kappa \nabla^2 T + 2\mu \left[ \frac{\left( \frac{\partial u}{\partial x} \right)^2 + \frac{\left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial \nu}{\partial \nu} \right)^2 + \left( \frac{\partial \nu}{\partial \nu} \right)^2}{\nu} \right] + \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \]

Where

\[ \nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{1}{x} \frac{\partial}{\partial x} \]

Using the notation for two-dimensional and axisymmetric flow the equations become:

2.1.6a \[ \frac{\partial(x^\sigma u)}{\partial x} + \frac{\partial(x^\sigma v)}{\partial y} = 0 \]

2.1.6b \[ u \frac{\partial u}{\partial x} - \nu \left( \frac{\partial u}{\partial y} \right) = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left[ \nabla^2 u - \sigma \frac{\partial \left( \frac{u}{x} \right)}{\partial x} \right] \]

2.1.6c \[ u \left( \frac{\partial v}{\partial x} \right) + v \left( \frac{\partial v}{\partial y} \right) = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left[ \nabla^2 v - \sigma \frac{\partial v}{x} \right] \]

2.1.6d

\[ \rho cp \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{1}{\rho} \left( \frac{\partial}{\partial x} + \frac{v}{\nu} \right) + \kappa \left( \frac{\nabla^2 T + \left( \frac{\sigma}{x} \right) \frac{\partial T}{\partial x} \right) + 2\nu \left[ \frac{\left( \frac{\partial u}{\partial x} \right)^2 + \frac{\left( \frac{\partial v}{\partial y} \right)^2}{x^2} + \left( \frac{\partial \nu}{\partial \nu} \right)^2}{\nu} \right] + \nu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \]
Where: $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$

Because the density and the dynamic viscosity are considered to be constant, the continuity equation is satisfied by introducing the following steam functions defined as:

$$2.1.7a \quad u = \frac{1}{x} \psi_y$$
$$2.1.7b \quad v = -\frac{1}{x} \psi_x$$

By introducing psi into the momentum equations and eliminating the pressure through cross differentiation the momentum equations the potential inviscid flow can be solved. The equation is in the following form:

$$2.1.8a \quad \left(\psi_y \frac{\partial}{\partial x} - \psi_x \frac{\partial}{\partial y}\right) \nabla^2 \psi = 0 \quad \text{where} \quad \nabla^2 \psi = 0$$

The figure below is a diagram of stagnation point flow normal to the cylinder.

Using the coordinates defined by figure 2-2, the i-component of the stream function is defined by the following equation.

$$2.1.9 \quad \psi_i = U_\infty \left(\frac{\tilde{y} - \frac{R^2 \tilde{y}}{\tilde{x}^2 + \tilde{y}^2}}{\cos \theta} - R\right), \text{ and } x = R\theta = -R \tan^{-1} \left(\frac{\tilde{y}}{\tilde{x}}\right)$$

Using the inverse components the stream function yields:
Because the stagnation region is the only area of interest, the solution is expanded for the large Reynolds number. Therefore the stream function becomes:

\[ \psi = \frac{2U \infty \cdot xy}{R} = a xy \quad \text{where} \quad \psi = \frac{2U \infty}{R} \]

### 2.1.1 Determining the Viscous Solution based on the Inviscid Solution

For flow about a sphere [13]:

\[ \psi = \frac{a}{1 + \sigma} x^{1+\sigma} y = \frac{u}{x^{1+\sigma}} \quad \text{and} \quad \psi = \frac{a}{1 + \sigma} u \]

If the inviscid flow is believed to be approximately correct [13], the streamlines are deflected outward due to viscosity. Thus a function F is set as a function of y.

\[ F = \frac{a}{1 + \sigma} x^{1+\sigma} F'(y) \quad \text{for viscous Flow and} \]

\[ F = \frac{a}{1 + \sigma} x^{1+\sigma} F''(y) \]

Substituting these values into the stream function yields the following:

\[ F'' + \frac{a}{\nu} F'F'' + (\sigma - 1) \frac{a}{\nu} F'F'' = 0 \]

For the velocity components:

\[ u = \frac{1}{x^\sigma} \psi_y = \frac{a}{1 + \sigma} x^{1+\sigma} F' = \frac{a}{1 + \sigma} x F' \]

\[ v = -\frac{1}{x^\sigma} \psi_x = -\frac{1}{x^\sigma} (ax^\sigma F) = -aF \]
On the body where \( y=0 \), the no slip condition applies. Therefore the boundary conditions for function \( F \) is:

\[
2.1.17a \quad F_w = F(0) = 0 \quad \text{ and } \quad 2.1.17b \quad F'_w = F'(0) = 0
\]

And for the wall shear stress:

\[
2.1.18 \quad \tau_w = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)_w = \frac{a\mu}{1+\sigma} x F''
\]

Next the dimensionless variable \( \eta \) and \( f \) is introduced:

\[
\eta = \left( \frac{a}{v} \right)^{1/2} y, \quad f = \left( \frac{a}{v} \right)^{1/2} F, \quad \frac{df}{d\eta} \rightarrow f', \quad \frac{df'}{d\eta} \rightarrow f'', \text{ and } \frac{df''}{d\eta} \rightarrow f'''
\]

Then

\[
2.1.18a \quad F' = \frac{dF}{dy} = \frac{dF}{df} \frac{df}{d\eta} \cdot \frac{d\eta}{dy} = \left( \frac{v}{a} \right) \left( \frac{a}{v} \right)^{1/2} f' = f' \rightarrow \frac{F'}{f'} = 1
\]

\[
2.1.18b
\]

\[
F'' = \frac{dF''}{dy'} = \frac{dF''}{df''} \frac{df''}{d\eta} \frac{d\eta}{dy'} = \left( \frac{a}{v} \right)^{1/2} f'' \rightarrow \left( \frac{a}{v} \right)^{1/2} f'' \rightarrow F'' = \left( \frac{a}{v} \right)^{1/2}
\]

\[
2.1.18c
\]

Using dimensional variables for \( \psi \) yields the following relations for viscous flow:

\[
2.1.19 \quad \psi_y = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial f} \left( \frac{\partial f}{\partial \eta} \right) \left( \frac{\partial \eta}{\partial y} \right) \rightarrow \left( \frac{a v}{1+\sigma} \right)^{1/2} f' \left( \frac{a}{v} \right)^{1/2} \rightarrow \frac{a}{1+\sigma} x^{1+\sigma}
\]

\[
2.1.20 \quad \psi_x = \frac{\partial \psi}{\partial x} = \left( \frac{a v}{1+\sigma} \right)^{1/2} (1+\sigma) x^\sigma f \rightarrow \left( a v \right)^{1/2} f x^\sigma
\]

The velocity components become the following:

\[
2.1.21 \quad u = \frac{1}{x^\sigma} \frac{\partial \psi}{\partial y} = \frac{1}{x^\sigma} \left( \frac{a}{1+\sigma} \right) x^{1+\sigma} \rightarrow \frac{a}{1+\sigma} x f''
\]
2.1.22 \[ v = -\frac{1}{x^\sigma} \frac{\partial \psi}{\partial x} = -\frac{1}{x^\sigma} (a \nu)^{1/2} f'(x^\sigma) \rightarrow -(a \nu)^{1/2} f' \]

From the equation 2.1.18, recall the following equation of wall shear stress:

2.1.23 \[ \tau_w = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \frac{a \mu}{1 + \sigma} x f'' \]

For the viscous solution, the wall shear stress is:

2.1.24 \[ \tau_w = \frac{a (a \nu)^{1/2}}{1 + \sigma} \rho x f'' \]

Where: \( f_w = f_w(g_w, \beta) \), \( g_w = \frac{T_w}{T_\infty} \), and \( \beta = \frac{1}{1 + \sigma} \)

The skin friction for the viscous solution for flow around a sphere is:

2.1.25 \[ C_f = \frac{2 \tau_w}{\rho u_i^2} = \frac{2}{\rho} \left( \frac{a}{(1 + \sigma)} \right)^2 \frac{a (a \nu)^{1/2}}{1 + \sigma} \rho x f'' = \frac{1 + \sigma}{(Re_x)^{1/2}} \frac{2R}{x} f'' \]

For purposes of comparison, the heat flux at the wall is calculated: The energy equation for a sphere becomes:

2.1.26

\[ cp \left( \frac{x}{1 + \sigma} f' \frac{\partial T}{\partial x} - f \frac{\partial T}{\partial \eta} \right) = cp \frac{\partial^2 T}{\partial \eta^2} + \left( \frac{ax}{1 + \sigma} \right)^2 \left[ f'' - \frac{1}{(1 + \sigma^2 a)} f' \right] + \frac{\nu}{a} \left( \frac{\partial^2 T}{\partial \eta^2} + \frac{\sigma}{x} \frac{\partial T}{\partial x} \right) + \frac{4 + 2 \sigma}{1 + \sigma} a^2 f'^2 \]

Since temperature \( T \) is only a function of \( \eta \), the other terms are dropped and the temperature profile becomes the following:

2.1.27 \[ \frac{\partial^2 T}{\partial \eta^2} + Pr f \frac{dT}{d\eta} = 0 \]
Next, the temperature profile is dimensionalized and becomes the following:

\[
\frac{d^2 \theta}{d \eta^2} + \text{Pr} \, f(\eta) \frac{d \theta}{d \eta} = 0 \quad \text{where} \quad \theta = \frac{T - T_w}{T_\infty - T_w}
\]

The boundary conditions are: \( \theta_w = 0 \) and \( \theta(\infty) = 1 \)

The wall heat flux then becomes the following:

\[
q_w = -\kappa \left( \frac{\partial T}{\partial Y}_w \right) = -\kappa \left( T_\infty - T_w \right) \left( \frac{a}{v} \right)^{1/2} \left( \frac{d \theta}{d \eta} \right)_w
\]

\[
q_w = 2^{1/2} \text{c} \rho U_\infty \frac{\text{Pr}}{(\text{Re}_\infty)^{1/2}} \left( T_w - T_\infty \right) c_1
\]

\[
q_w = h_f (T_w - T_\infty) \quad \text{where} \quad h_f = 2^{1/2} \text{c} \rho U_\infty \frac{\text{Pr}}{(\text{Re}_\infty)^{1/2}} c_1
\]

Integration is performed to determine the constants \( c_1 \) and \( c_2 \) through the following calculations

\[
\frac{d \left( \frac{d \theta}{d \eta} \right)}{d \theta} = -\text{Pr} \, f d \eta \rightarrow \frac{d \theta}{d \eta} = c_1 \exp \left( -\text{Pr} \int_0^\eta f(\eta')d\eta' \right)
\]

\[
\theta = c_2 + c_1 \int_0^\eta \exp \left( -\text{Pr} \int_0^{\eta'} f d\eta' \right) d\eta^	ext{**}
\]

The following relations are boundary conditions for the dimensionless temperature:

When \( \theta = 0 \quad c_2 = 0 \quad \text{and} \quad \theta(\infty) = 1 \)

Thus the value for constant \( c_1 \) becomes:

\[
1 = c_1 \int_0^\eta \exp \left( -\text{Pr} \int_0^{\eta'} f(\eta')d\eta' \right) d\eta^	ext{**}
\]

\[
c_1 = \left[ \int_0^\eta \exp \left( -\text{Pr} \int_0^{\eta'} f d\eta' \right) d\eta' \right]
\]

21
When the Prandtl number is 1 $c_1$ is equal to the dimensionless parameter $G_w$, which is a function of $\beta$ and $g_w$.

At $Pr = 1 \implies c_1 = G_w' \quad G_w = G_w(\beta, g_w)$
2.2 THE VORTICITY AMPLIFICATION THEORY

The Vorticity Amplification Theory asserts that the sensitivity of the Reynolds Transport of the stagnation point is marked by free stream turbulence [14]. Therefore, the theory proposes that vorticity is amplified due to stretching or vortex filaments (see Figure 2-3). The purpose of this theory was to explain the discrepancies between earlier experimental results and the theory of laminar flow hypothesis [14]. The model of Sutera [15] was aimed to expose the evolution of vorticity as it penetrates the boundary layer and thus justify the amplification of vorticity as input into the boundary layer. The vorticity amplification is said to be a function of the laminar free stream velocity ($V_m$) and a sinusoidal wave as the function of a wavelength lambda. This lambda value, $\lambda$, is said to be a unique and real distinct root.

The velocity with all scalar components (normal to z-axis) is determined to be [16]:

$$2.2.1 \quad V = V_m\left(1 + \sin \frac{2\pi z}{\lambda}\right)$$

where $V_m$ is the average velocity and Turbulence intensity is defined as [17]:

$$2.2.2 \quad Tu = \frac{V_m}{V}$$
The first diagram (above) is a representation of velocity as it approaches the stagnation line on the plane of the cylinder. Vo is the unperturbed velocity that satisfies the Heimenz function while v1 is the perturbed velocity. The variable, k1 is the wave number for the wavelength, λ1. The wave length, λ serves as the indicator of the periodic harmonic disturbance. In figure 2-4, the curvature of the velocity in the form of vortices is acting along the x-axis in the y-z plane. By projecting this plane (see Figure 2-5), the streamlines from a numerical solution can be traced for an arbitrary amplitude [17].

![Figure 2-6 Hiemenz Flow versus Actual Flow [16]](image1)

![Figure 2-7 Potential Normal Velocity Vs Linear Approximation [17]](image2)

Furthermore, the velocity of the Vorticity Amplification Theory is adopted as a more realistic profile than the Heimenz flow represented by the first graph in Figure 2-6. According to Kestin and R T Wood, the potential theory provides a reasonable approximation if the Reynolds number is sufficiently high. However, the Heimenz flow is applicable to the normal velocity that is ten to fifteen boundary layers from the wall [16].

For flat plates, the wavelength λ has been found to damped and thus become negligible depending on the Reynolds number and the length of impingement [18]. However, the wavelength λ for blunt bodies has found to be inherently unstable [16]. Therefore in validating the use of stagnation velocity as a function, the derivation of the neutral
wavelength for the velocity profile will be discussed in terms of satisfying the governing conservative equation. Then this neutral wavelength in terms of the actual wavelength of the perturbed velocity approaching a cylinder will be discussed.

In terms of heat transfer, Smith and Kueth [14] postulated that heat transfer is increased or amplified by eddy generation within the stagnation region. Kestin, Sadeh, S. Bae, and Sung have shown how turbulence within the stagnation region has an effect on heat transfer, while Kueth has discussed the effects of turbulence on laminar skin friction and heat transfer [19][18][17][14]. The relation between heat transfer and the amplification has been determined through the correlation of the Reynolds number, Nusselt Number, and turbulence intensity. The first graph is adopted from Smith and Kueth while the second graph is adopted from van der Meer in terms of the turbulence intensity of isothermal jets.

![Figure 2-8 Nusselt versus Reynolds Number [14]](image)

![Figure 2-9 Turbulent Intensity Versus Reynolds Number [20]](image)

![Figure 2-10 Nusselt Number, Reynolds Number, and Turbulent Intensity [19]](image)
In the first graph above the Nusselt number increases as the Reynolds number increases while the turbulences intensity in the center graph above increase with respect to an increase in the Reynolds number. In the last graph, there seems to be a direct (linear) positive slope between the product of the turbulence intensity and the Reynolds number squared and the product of the nusselt number and the Reynolds number squared. This empirical information supports the notion that heat flux at the wall in the stagnation region is caused by shear stress of the fluid which is driven by an increase in the Reynolds number. For heat transfer, the interest in terms of turbulence intensity would be the point within the stagnation where the source of the shear stress is the highest which is believed to be the highest value of the vorticity [18].

2.2.1 **Derivation of the Neutral Wavelength of Amplified Velocity**

In order to represent the velocity in terms of two velocities (one satisfying the potential function and one represented the perturbed flow) the vorticity transport equation for steady flow is used:

\[ 2.2.3 \quad u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} = \nu \left( \frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} \right) \]

The amplified velocity, \( V^\ast \) can be represented as the following:

\[ 2.2.4 \quad V^\ast(y) = V_o(y) + V_1(y) \cos k_1z \]

Total = Free-stream + Perturbed Velocity

\[ 2.2.5 \quad C^\ast = C_o^\ast + C_1^\ast \]

\( C_o^\ast \) represents the potential flow velocity, while \( C1^\ast \) represents the perturbed velocity.

In scalar form:

\[ 2.2.6 \quad C_1^\ast = \left\{ u_1^\ast \left( x^\ast, y^\ast \right) \cos k_1^\ast z^\ast; \right. \left. v_1^\ast \left( x^\ast, y^\ast \right) \cos k_1^\ast z^\ast, w_1^\ast \left( x^\ast, y^\ast \right) \sin k_1^\ast z^\ast \right\} \]
Using the previous properties, two independent relations are established. The curl of the perturb velocity equates with the definition of vorticity while the dot product of Co* satisfy the definition of potential flow. The continuity equation is expressed as:

\[
2.2.7 \quad \frac{\partial u_1^*}{\partial x} + \frac{\partial v_1^*}{\partial y} + k_1^* w_1^* = 0
\]

Each component is dimensionalized in order to linearize the vorticity transport equation as demonstrated below:

\[
\frac{x^*}{R}, \frac{y^*}{R}, \frac{z^*}{R} \rightarrow x, y, z \quad \frac{C^*}{V_\infty} = C \\
\frac{\omega^* R}{V_\infty} = \omega \quad k_1^* = k \quad \frac{\lambda^*}{R} = \lambda
\]

Using the dimensionless variables, the linear vorticity transport equation is:

\[
2.2.8 \quad (C_o \cdot \nabla) \omega = (\omega \cdot \nabla) C_o + \frac{2}{Re_D} \nabla^2 \omega
\]

Where the vorticity components are:

\[
2.2.9a \quad \omega_x = \left[ -\frac{1}{k_1} \left( \frac{\partial^2 u_1}{\partial x \partial y} + \frac{\partial^2 v_1}{\partial y^2} \right) + k_1 v_1 \right] \sin k_1 z
\]

\[
2.2.9b \quad \omega_y = \left( k_1 u_1 + \frac{\partial^2 w_1}{\partial y^2} \right) \sin k_1 z
\]

\[
2.2.9c \quad \omega_z = \left( \frac{\partial v_1}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} \right) \cos k_1 z
\]

For this flow the neutral wavelength is determined to be

\[
2.2.10 \quad \lambda_o = 2\pi \left( \frac{\nu}{a} \right)^{1/2} = \frac{2\pi}{(Re_D)^{3/4}}
\]

Where \( \nu \) is the viscosity and \( a \) is the constant for the Heimenz flow.
The neutral wavelength is described as the wavelength where perturbations are non existent and the flow is considered stabled.

2.2.2 Cylinder: The Case for Blunt Body
For the Hiemenz Case for a two dimensional flow across a cylinder the governing equations are[15]:

![Figure 2-11 Hiemenz Flow about a Cylinder [16]](image)

2.2.11 \( \nabla^* \cdot \mathbf{V}^* = 0 \)

2.2.12 \( (\mathbf{V}^* \cdot \nabla)\mathbf{V}^* = \frac{1}{\rho} \nabla P^* + \nu \nabla^2 \mathbf{V}^* \)

Where the following are the velocity components and pressure:

2.2.13 \( u^* = [U_o(y) + u_o^*(y)\cos kz] \)

2.2.14 \( v^* = [V_o(y) + v_o^*(y)\cos kz] \)

2.2.15 \( w^* = \left(\frac{1}{k}\right)v_1^*(y)\sin kz \)

Using the definition of vorticity, the linear disturbance equation is the following in canonical form:

2.2.16 \( \omega^* + \left( \frac{\phi + \frac{\varepsilon}{1 + \varepsilon \eta}}{1 + \varepsilon \eta} \right) \omega' - \left( \alpha^2 - \phi' + \frac{\varepsilon^2}{2 + \varepsilon \eta} \right) \omega = \frac{2\alpha \varepsilon u}{(1 + \varepsilon \eta)^2} \)
Using the boundary conditions below, the linear disturbance equation simplifies to the following:

(1) \( \eta = 0 \)

(2) \( u = v = v' = 0 \)

(3) \( \eta \to \infty; u \to 0, v' \to 0, \phi \to (2\varepsilon)^{-1}, \phi' \to 0 \)

2.2.15 \( \omega^* + (\phi \omega') - \alpha^2 \omega = 0 \)

Setting \( \phi = (2\varepsilon)^{-1} \) and \( \phi' \to 0 \) the solution of vorticity is the following provided that

\( (\eta \to \infty): \)

Based on boundary conditions of \( \lim_{\eta \to 0^+} \omega' = 0 \) and \( \lim_{\eta \to \infty} \omega = 0 \) there are no zeros on the interval as represented by the diagram.

From the Singular Sturm Liouville Problem [16]:

2.2.16 \( \omega(\eta) = f(\eta) \exp\left[-\frac{1}{2} \int_0^\eta \phi(y) dy\right] \) where \( f^* + [u - q(\eta)]f = 0 \)

And \( q(\eta) = \frac{1}{2} \phi^2 - \frac{1}{2} \phi' \)

After analysis of \( q_{\min} = \frac{3}{4} s^2 + \frac{1}{6} s^2 \) and the disturbance, \( \alpha \), it is determined that \( \alpha^2 = -\mu_1 \).

2.2.17 \( \alpha^2 = -q_{\min} = \frac{3}{4} \left( \frac{\phi_O^*}{2} \right)^{2/3} - \frac{1}{6} \left( \frac{2}{\phi_O^*} \right)^{2/3} \)
Thus \( \lambda = \frac{2\pi R}{\alpha \text{Re}^{1/2}} = 1.79\lambda_o \) and the wavelength \( \lambda \) for blunt bodies is inherently unstable[16].

### 2.2.3 Effect of Turbulence on Heat Transfer in the Stagnation Point

According to Bae, Lele, and Sung, the following figure (2-15) illustrates the approximation of velocity and temperature perturbations [18].

Here the temperature and hence heat flux is presumed to be the maximum value where vorticity is the greatest.

The relationship between the turbulent intensity, Reynolds number, and the Nusselt Number is defined as: 2.2.18 \[
\frac{Nu}{\text{Re}^{1/2}} = 0.945 + 3.48 \left( \frac{Tu \text{Re}^{1/2}}{100} \right) - 3.99 \left( \frac{Tu \text{Re}^{1/2}}{100} \right)^2
\]

where Tu is an absolute number.
2.3 FLAME IMPINGEMENT

The following diagram illustrates a schematic of flame impingement on a hemi-nose cylinder.

Semi-analytical convective heat transfer solutions are based on momentum equations for steady flow within the boundary layer [21][22].

\[ v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \beta^2 x + \nu \frac{\partial^2 v_x}{\partial y^2} \]

Based on the potential flow theory, \( x \) is the distance from the stagnation point while \( y \) is normal to the surface [21].

The tangential velocity \( v_x \) is equal to the stagnation velocity gradient, \( \beta \) times position \( x \).

\[ v_x = \beta x \]
Where the velocity gradient at the stagnation point is defined as:

2.3.3 $\beta_s = \left( \frac{\partial v_x}{\partial x} \right)_{x=0, y=\delta}$

Two formulas for the stagnation velocity gradient are the following:

2.3.4 Semi-analytic: $\frac{3v_e}{db}$

Empirical: $\frac{(2.67 + 0.0962Tu) \cdot v_e}{db}$

Where the edge velocity just outside of the boundary layer, $v_e = V_\infty \sin\left( \frac{t_j}{db} \right)$ [23]

Using the stagnation velocity gradient, the heat flux at the stagnation point is:

2.3.5 $q_s = 0.763(\beta_s, \rho_e \mu_e)^{0.5} \Pr_e^{-0.6} \cdot cp_e (t_e - t_w)$ where $0.6 < \Pr_e < 2.0$

The semi-analytical solution is based on the assumption that flow around the body in the boundary layer is laminar, incompressible, and axisymmetric
2.4 FLAME STRETCH

To account for flame stretch, the Markstein and Karlovitz number is used. According to Markstein, the following is the relationship between the stretched laminar flame speed and the laminar unstretched flame speed [8].

\[ S_L = S_{L,0} - L\kappa \]

Where \( S_L \) is the laminar burning velocity

\( S_{L,0} \) is the unstretched laminar burning velocity

\( L \) is the Markstein length and \( \kappa \) is the stretched factor

The Markstein length is said to be proportional to the characteristic flame thickness \( \delta_L \) [8].

Therefore the Markstein number is defined as :

\[ Ma = \frac{L}{\delta_L} \]

The Karlovitz number (Ka) is defined as the non-dimensional factor using the thickness of the unstretched flame and the normal unstretched laminar-flame velocity to form a reference time [8].

\[ ka = \frac{\delta_L}{S_L} \kappa = \frac{\text{residence time for crossing an unstretched flame}}{\text{characteristic time for flame stretching}} \]

The Karlovitz number takes in account stretching and curvature and provides a dimensionless relationship between the laminar unstretched and stretched laminar burning velocity:

\[ \frac{S_L}{S_{L,0}} = 1 + MaKa \]
3 METHODOLOGY

3.1 APPROACH
The heat flux method and surface flame impingement are the basis of the theoretical approach. Both methods are said to be performed under adiabatic as well as stoichiometric conditions.

3.1.1 Heat Flux Method
The purpose of the heat flux method was to determine the adiabatic burning velocity by providing a flat flame. However, by using a burner, heat loss to the flame would have to be accommodated for. Thus the velocity has been determined by extrapolating results to zero heat loss while extrapolation to zero stretch rate--yielding a stretched free burning velocity [24]. Two parameters that separate the heat flux method from other flat burner methods are the burner plate and the cooling circuit configuration. The burner plate is a brass plate with a hexagonal pattern of small holes. This configuration was computationally designed and provides a flat one dimensional flame [24]. Since the flame is flat, the temperature is a sole function of the burner radius. Therefore the stretch is negligibly small (~1s⁻¹) [24]. As for the cooling circuit configuration, the plenum chamber is cooled to the same temperature of the gas mixture entering the chamber (See Figure 3.2). The temperature of the plenum chamber is said to define the temperature of the unburnt gas mixture. To maintain a constant heat flux, the burner plate is heated above the unburnt gas mixture. This is attained through a separate water circuit in the upper part of the burner head [25]. In addition to the water circuit, ceramic material splits the burner into a hot part and a cold part with a cooling jacket built into the lower part. In figure 4.2, heat flux in section one and two are balanced while the influence of
3.1.2 Heat Flux Configuration

![Heat Flux Configuration Diagram]

Figure 3-1 Heat Flux Configuration [25]

![Cooling Configuration Diagram]

Figure 3-2 Cooling Configuration [25]

![Heat Flux Burner Configuration Diagram]

Figure 3-3 Heat Flux Burner Configuration [25]
the heat flux in section three and four are minimized due to the influence of the ceramic material’s insulation.

Other parameters in the heat flux method are the thermostat controllers (maintain parts of burner at constant temperature), the thermocouples (measurement of temperature distribution of burner plate), mixing panel (gas mixture), and mass flow controllers (reduce pressure oscillation in piping for gas flow) [25]. All of the values of laminar burning velocities with the exception of hydrogen gas were taken as empirical values in the simulation for this research.

3.1.3 Flame Impingement: Surface Condition Effects

For the flame impingement research concerning surface condition effects, the four main heat transfer mechanisms of concern were non-luminous radiation heat transfer, equilibrium thermo-chemical heat release (TCHR), catalytic thermo-chemical heat release (TCHR), and mixed thermo-chemical heat release (TCHR) [26]. Non-luminous radiation heat transfer is dependent on the gas temperature, and partial pressure and wavelength of each species in the gas mixture [26] [27]. Thermo-chemical heat release occurs when dissociated species exothermochemically combined when cooling down. Equilibrium TCHR is said to occur through reactions within the boundary layer while catalytic TCHR occurs through chemical diffusion reactions at a surface where species collide [26]. Mixed TCHR is said to be a combination of equilibrium and catalytic TCHR. The purpose of this research was to determine the magnitude of influence of these modes of heat transfer in flame impingement.
The experimental apparatus for this research included a torch with outlet matrix of tiny diffusion flames where jets of fuel were surrounded by an annular wing of oxidizer [26]. The item impinged upon consisted of hasta alloy and was water cooled on the other side (not involving flame impingement) (See Figures 4-4-4-6 ). The total heat flux to each ring was determined using the following equation:

\[ q^* = \frac{\dot{m}_i c_p (T_{out,i} - T_{in})}{A_i} \]

Where \( \dot{m}_i \) is the mass flow rate of the cooling water to each ring

\( T_{out,i} \) is the outlet temperature of the water

\( T_{in} \) is the inlet temperature of the water

\( A_i \) is the surface area of the ring

\( c_p \) is the specific heat constant for the cooling water.

With a nozzle diameter of 38.5 mm, a wall temperature of 323K, and a nozzle velocity of 3.7 m/s, the empirical values of the heat flux from the flame impingement research concerning surface effects was used to compare the heat flux values from the analytical and semi-analytical formula of heat flux.
3.1.4 Flame Impingement: Surface Effects Configuration

Figure 3-4 Configuration of Flame Impingement in the Observance of Surface Effects [26]

Figure 3-5 Top View of Target of Impingement [26]  
Figure 3-6 Side View of Impingement Ring [26]
3.2 **APPARATUS**

The ideal apparatus for this experiment would comprise of a (a) a low Reynolds jet flame nozzle, (b) a laser anemometer, and (c) a steady-state calorimeter.

The laser anemometer would measure the velocity and the turbulent intensity of the gas by measuring the time of tracer particles flight through two separated focused laser beams [28]. Laser anemometers are said to have three components (1) a transmitting device by which laser beams are directed to a certain point in the flow path where probe volume is fared, (2) a receiving device which is aligned with the probe volume in order to decipher the scattering light from particles passing through and (3) an electronic device which processes photomultiplier signals and delivers velocity and turbulence information [29]. For calibration, careful selection of size of particles for seeding would have to be performed. Particles must be large enough to scatter sufficient light to produce a signal but small enough to follow the flow of gas [29].

For the measurement of heat flux, a steady-state calorimeter as designed by Hargrave and Kilham would be used. This calorimeter was composed of a hemi-nosed shaped body (22mm in diameter) a glycol cooling circuit, and a 3.2mm diameter copper rod exposed to flame gases (see Figure 3-9) [30]. Four thermocouples were attached along the length of the rod, one at the stagnant point, one at the cold end, and two along the length of the rod [30]. This configuration is said to have provided a differentiation between the stagnation temperature difference and the copper rod’s thermal properties.
3.2.1 Apparatus: Components

Figure 3-7 Laser Doppler Anemometer [31]

Figure 3-8 Process of Laser Doppler Anemometer [32]

Figure 3-9 Steady-State Calorimeter [30]
The low Reynolds jet flame of interest was calibrated from a small industrial rapid heating burner to attain premixed combustion [20]. The combustion taking place in the burner chamber is said to have been completed prior to flow reaching the exit of the burner [20]. A small disk was used as a flame holder.

![Image of Low-Reynolds Jet Flame](image)

**Figure 3-10 Low-Reynolds Jet Flame [20]**

The Empirical Nusselt Number for this burner is defined as [20]:

\[
3.2.1 \quad Nu = 0.57 \text{Re}^{0.525}
\]
3.3 MODEL

The computational model entails a solution based on the Turbulent Kinetic Energy Model discretized over a finite volume domain. The upwind (first order for the initial case) scheme is used to determine the velocity for each node while pressure is accounted for through the SIMPLE Pressure-Correction Approach. The following provides an overview of the computational model for the scope of this research.

3.3.1 Pressure-Velocity Coupling

The SIMPLE Method is based on a cyclic series of guess and correct operations to solve the governing equations [12]. The velocity components are initially calculated based on the momentum equations and guessed pressure field. The pressure and velocity of each node is corrected to satisfy the continuity equation until the solution converges. For the procedure, pressure and the velocity components are denoted as:

3.3.1 \[ P = P_o + P' \]  \hspace{1cm} 3.3.2 \[ u = u_o + u' \]  \hspace{1cm} 3.3.3 \[ v = v_o + v' \]

where the subscript \( O \) represents intermediate values and the superscript “'” represents correction components. Because the velocity corrections are assumed to be zero at previous iteration steps, the velocity components are related to the momentum equations such that:

3.3.4 \[ u' = -A \frac{\partial P'}{\partial x} \]  \hspace{1cm} \[ v' = -A \frac{\partial P'}{\partial y} \] where \( A \) is a time increment divided by density.

Once the continuity equation is satisfied, the pressure correction is zero at each point.

3.3.5 \[ P = P_o + \alpha_p P' \]
3.3.2 Turbulent Kinetic Energy

For turbulent flow, the kinetic energy and dissipation rate are related through the following equations with the specified model constants:

**Turbulent Kinetic Energy**

\[
\frac{D\bar{k}}{Dt} = -\frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{Pr_k}\right)\frac{\partial \bar{k}}{\partial x_j}\right] + \left(2\mu_T S_{ij} - \frac{2}{3} \rho \bar{k} \frac{\partial \bar{\epsilon}}{\partial x_j}\right) \frac{\partial u_i}{\partial x_j} - \rho \bar{\epsilon}
\]

**Dissipation Rate**

\[
\frac{D\bar{\epsilon}}{Dt} = -\frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{Pr_\epsilon}\right)\frac{\partial \bar{\epsilon}}{\partial x_j}\right] + C_{\epsilon 1} \frac{\bar{\epsilon}}{k} \left(2\mu_T S_{ij} - \frac{2}{3} \rho \bar{k} \frac{\partial \bar{\epsilon}}{\partial x_j}\right) \frac{\partial u_i}{\partial x_j}
\]

**K-Epsilon Model Constants**

<table>
<thead>
<tr>
<th>$C_\mu$</th>
<th>$C_{\epsilon 1}$</th>
<th>$C_{\epsilon 2}$</th>
<th>$Pr_k$</th>
<th>$Pr_\epsilon$</th>
<th>$Pr_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>1.44</td>
<td>1.92</td>
<td>1.0</td>
<td>1.3</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The kinetic energy, dissipation rate, and dissipation time is related to the turbulent length scale such that [12]:

\[
l_\tau = C_D \frac{\bar{\epsilon}^{\gamma/2}}{\bar{k}^{\gamma/2}} = C_D \frac{\bar{k}^{\gamma/2}}{\bar{\omega}} = C_D \bar{k}^{\gamma/2}
\]

However, turbulent combustion is dictated by the Karlovitz Number, Damkohler Number, and Turbulent Reynolds Number which determines the turbulent length scale. Provided that the Schmidt number (ratio of kinematic viscosity to the mass diffusivity) is unity, the Karlovitz, Damkohler, and Turbulent Reynolds respectively are:

\[
Ka = \left(\frac{u'}{S_L}\right)^{\gamma/2} \left(\frac{L_\tau}{\delta_L}\right)^{-\gamma/2} \quad Da = \left(\frac{L_\tau}{\delta_L}\left(\frac{u'}{S_L}\right)^{-1}\right) \quad \text{and} \quad Re_\tau = \frac{u'L_\tau}{v}
\]
The equation for the progress variable is

$$\frac{\partial (\rho c)}{\partial t} + \frac{\partial (\rho u_i c)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\mu_t}{S c_t} \frac{\partial c}{\partial x_i} \right) + \rho \dot{\omega} \quad \text{where} \quad c = \frac{\sum_{i=1}^{n} Y_i'}{\sum_{i=1}^{n} Y_i', ad}$$

To account for flame stretch the following equations are employed [32]. The expression for the critical rate of strain is:

$$3.3.13 \quad g_{cr} = \frac{BS_t}{\alpha^2} \quad \text{where} \quad B \quad \text{is a constant based on the burner.}$$

The turbulent dissipation rate at the critical rate is:

$$3.3.14 \quad \varepsilon_{cr} = 15\nu g_{cr}^2$$

The Stretch Factor $G''$ is obtained by integrating the log-normal distribution of the turbulence dissipation rate, $\varepsilon$:

$$3.3.15 \quad G = \frac{1}{2} \text{erfc} - \frac{1}{2\sigma} \left[ \ln \left( \frac{\varepsilon_{cr}}{\varepsilon} \right) + \frac{\sigma}{2} \right] \quad \text{and} \quad \sigma = \mu_{\varepsilon} \ln \left( \frac{L}{\eta} \right)$$

Where erfc is an error function and $\sigma$ is the standard deviation of

### 3.3.3 Finite Volume

Fine volume is a discretization methodology where dependent variables are evaluated at the center of volume (cells) by solving the integral forms of governing equations.

$$3.3.15 \quad \frac{\partial}{\partial t} \int_{abcd} u dx dy + \oint (Edy - Fdx) = 0$$

![Figure 3-11 Two-Dimensional Finite Volume Mesh [12]](image)
Where E and F represents the moment fluxes for the two-dimensional Navier Stokes Equation [11]

3.3.4 Finite Difference Scheme: First and Second Order Upwind Difference

The Upwind (First Order and Second Order) is an explicit scheme used for the Navier Stokes equation. The scheme is stable provided that the wave parameter c is positive and the Courant number $\nu$ is between 0 and unity $0 \leq \nu \leq 1$. The upwind first order scheme is [11]:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + c \frac{u_j^n - u_{j-1}^n}{\Delta x} = 0 \quad \text{where} \quad \nu = \frac{c\Delta t}{\Delta x}$$

The second-order upwind scheme employs a corrector and predictor and algorithm which can be combined in one step [11]:

3.3.17

**Predictor:**

$$u_j^{n+1} = u_j^n - \frac{c\Delta t}{\Delta x} (u_j^n - u_j^{n})$$

**Corrector:**

$$u_j^{n+1} = \frac{1}{2} \left[ u_j^n + u_j^{n+1} - c \frac{\Delta t}{\Delta x} (u_j^{n+1} - u_{j+1}^{n+1}) - c \frac{\Delta t}{\Delta x} (u_j^n - 2u_{j-1}^n + u_{j-2}^n) \right]$$

**One Step:**

$$u_j^{n+1} = u_j^n - \nu (u_j^n - u_{j-1}^n) + \frac{1}{2} \nu (\nu - 1) (u_j^n - 2u_{j-1}^n + u_{j-2}^n)$$
3.4 PROCEDURE

Three cases involving the k-epsilon equation, premixed without flame stretch, and premixed with stretch was performed for Methane, Propane, and Hydrogen. Prior to performing calculations was the calibration of the grid. The mesh chosen for the theoretical and computational analysis was a two-dimensional cylinder mesh (cyl.msh). This cylinder was centered at origin (0,0) and originally had a diameter of one meter. The diameter was later scaled to one centimeter. Because the stagnation region was the surface of interest, five equidistant points, approximately 22.5 degrees apart were selected within the second quadrant of the cross-section of the circle. Therefore, the product of the angle from the horizontal within the second quadrant and the radius was taken to be the surface integral and the x-coordinate ($s=r\theta$).

As a part of the model, a Reynolds number for the diameter of the isothermal jet (as described in the apparatus) was assumed. With the Reynolds number constant, the axial velocity of the free stream was determined by multiplying the kinematic viscosity of the reactants and then dividing this product by the diameter of the nozzle which was 13.8mm. With the axial velocity determined for a particular fuel, the theoretical skin friction was calculated using equations 2.1.11a, 2.1.12b, and 2.1.25. The parameter $G_w$ was determined through the interpolation of values provided by ref [13] (See Appendix B).

This same axial velocity for each fuel was used as a boundary condition for Fluent. Other parameters included in the Fluent program were the turbulent intensity, turbulent length scale, and the critical strain rate. Using the empirical formula provided by ref [20], the Nusselt number was calculated using the assumed Reynolds number. The turbulent intensity
was determined by using the formula (equation 2.2.18) by Kestin and R T. Wood while the perturbed velocity was determined based on the definition of turbulent intensity. The chemical mixing time was assumed to be equal to the turbulent mixing time. Thus, the Damköhler number and Schmidt number was set to unity in order to calculate the turbulent length scale through equation 3.3.10. The pressure and stoichiometric ratio was assumed to be the same conditions of the laminar burning velocity of various fuels for the Heat Flux Method. The value of hydrogen burning velocity, however, was taken from ref [33]. The critical strain rate was calculated using equation 3.3.12. All other parameters such as the viscosity and the specific heat were computed through the program Gas Equilibrium. These parameters were entered in Fluent as the initial and boundary conditions where appropriate (see Appendix C) prior to running the case for each fuel. Once boundary and initial conditions were set, a solution was iterated for the following three cases:

1. K-Epsilon Equation without Premixed Combustion
2. Premixed Combustion Without Stretch
3. Premixed Combustion with Stretch

Once the solution has been iterated, the tangent points, as discussed earlier, were set in Fluent for comparison of theoretical values of the stagnation point velocity to computational values.

For the second part, heat flux values were determined for each fuel using the adiabatic temperature and an assumed wall temperature of 400 K. Heat flux values for each fuel were calculated using the theoretical, semi-analytical, and empirical formula (equations 3.1.31, 3.3.4, and 3.3.5). \( Gw' \) was determined through the interpolation of values provided by ref [13] (See Appendix B.). The turbulent intensity calculated previously was used for the empirical stagnation velocity gradient. The Prandtl Number at the edge of the boundary
layer, however, was taken from ref. [23]. The average of the specific heat of the reactants and the specific heat at the sink towards the wall of the surface was calculated. This value was used in the semi-analytical and empirical equation. Heat flux values were also calculated using empirical data from the research regarding the effects of surface conditions from ref [26]. To attain the turbulent intensity for the empirical formula, the velocity (3.7m/s), diameter (38.5mm) and viscosity was used to determine the Reynolds Number. The Nusselt number was calculated using Equation 3.2.1. Likewise, the turbulent intensity was determined using the formula by Kestin and R.T. Wood.
4 RESULTS

4.1 DISCUSSION OF RESULTS

In terms of the skin friction, values based on the viscous solution for Stagnation Point Flow is initially higher than the computational results—but approaches zero as the fluid propagates. With the exception of methane in the first case, the computational skin friction values are initially lower than theoretical values but increase midway through the second quadrant. The skin friction values, however, remain well above zero. Even with stretch accounted for, the computational skin friction values remain higher than the theoretical values of the stagnation point flow values midway through the second quadrant. The behavior of the theoretical values implies that the no-slip condition assumes a linear deceleration of the velocity—thus a decrease in the shear stress (in the absence of flow separation).

For flame stretch, it was believed that skin friction would increase. However, when stretch was factored in, the skin friction decreased by a factor of approximately 500% for each of the fuels within the first quadrant. This decrease is probably due to the stretch due to curvature and strain which probably distorts the axial velocity. However, the skin friction XY Plot over the entire circle implies that the velocity may be increased further downstream from the stagnation region while being decreased within the stagnation region. This suggestion is further supported by the velocity, pressure, and progress contour plots that show a distortion of the values (pressure, velocity, and progress variable) after flame stretch has been factored in.
The heat flux values, for the most part, display consistent behavior for each of the solutions. The theoretical heat flux decreases as the length of the jet from the target increases. The heat flux for the semi-analytical and empirical formula gradually increased and then decreased as the length of the jet increased. Although the semi-analytical formula reported higher heat flux values than the empirical formula, the empirical formula was in closer proximity than the semi-analytical formula. The theoretical formula from the stagnation point velocity underpredicted the heat flux data.

Overall, the cases for the hydrogen reported higher skin friction values than methane and propane. The case for propane reported the highest peak of skin friction in comparison to methane. Methane, however, had a greater range of higher skin friction values when compared to propane velocities. Because propane and methane have comparable burning velocities as well as axial velocities, the turbulent length scale may be partly responsible for the range and magnitude of the skin friction values. Hydrogen had a significantly higher axial as well as burning velocity—but a significantly lower turbulent length scale. This explains the overall higher skin friction values.

4.2 CONCLUSION

The values from the viscous solution of the stagnation point velocity reported lower values than values of the K-epsilon solution involving premixed combustion. However, factoring stretch decreases the skin friction within the stagnation region. The empirical heat flux formula is closer to experimental values than the semi-analytical and theoretical heat flux solution.
### 4.3 FUEL PROPERTIES AND BOUNDARY CONDITIONS

Table 1 Fuel Properties Used For Cases and Heat Flux Calculations

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Methane</th>
<th>Propane</th>
<th>Hydrogen</th>
</tr>
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<tbody>
<tr>
<td>Schmidt Number</td>
<td>Dimensionless</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Pressure</td>
<td>Atmosphere K</td>
<td>2226</td>
<td>2267</td>
<td>2389.8</td>
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<tr>
<td>Adiabatic Temperature</td>
<td>No Dimension m/s</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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<td>Stoichiometric Ratio</td>
<td>No Dimension</td>
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<td>0.395</td>
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<td>Laminar Burning Velocity</td>
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<td>10600</td>
<td>10600</td>
</tr>
<tr>
<td>Reynolds Number</td>
<td>Dimensionless m/s</td>
<td>12.28</td>
<td>11.62</td>
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<td>Free-Stream Velocity</td>
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<td>1.197</td>
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<td>Unburnt Density</td>
<td>Kinematic kg/m/s</td>
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<td>1.78E-05</td>
<td>1.83E-05</td>
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<tr>
<td>Reactant Viscosity</td>
<td>Kinematic m^2/s</td>
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<td>1.48E-05</td>
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<td>Prandtle Number at edge</td>
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<td>.6560</td>
<td>.6510</td>
<td>.7251</td>
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<td>Specific Heat</td>
<td>J/kg/K</td>
<td>1075.13</td>
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<td>Specific Heat at Sink *</td>
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<td>Dimensionless m</td>
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<td>0.069</td>
<td>0.069</td>
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<td>Turbulent Length Scale</td>
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<td>7.59E-05</td>
<td>4.42E-06</td>
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<td>Critical Strain rate</td>
<td>Unstretched Dimensionless</td>
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<td>1.80E+08</td>
<td>1.80E+08</td>
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<tr>
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<td>Stretched</td>
<td>3313.674</td>
<td>4409</td>
<td>69377</td>
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</table>
4.4 **SKIN FRICTION VALUES FOR METHANE, PROPANE, AND HYDROGEN**

**Table 2 Skin Friction Values for Methane**

<table>
<thead>
<tr>
<th>x-value (m)</th>
<th>Velocity u (m/s)</th>
<th>Reynolds Number</th>
<th>Theoretical Skin Friction</th>
<th>Computational Skin Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>K-Epsilon Solution</td>
<td>w/o stretch</td>
</tr>
<tr>
<td>0.000</td>
<td>0.214</td>
<td>0.040</td>
<td>69.683</td>
<td>4.275</td>
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<td>0.002</td>
<td>4.822</td>
<td>20.061</td>
<td>0.653</td>
<td>15.559</td>
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<td>0.004</td>
<td>9.645</td>
<td>80.243</td>
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<td>14.467</td>
<td>180.546</td>
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<td>31.829</td>
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<td>0.008</td>
<td>19.289</td>
<td>320.971</td>
<td>0.082</td>
<td>23.681</td>
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**Table 3 Skin Friction Values for Propane**

<table>
<thead>
<tr>
<th>x-value (m)</th>
<th>Velocity u (m/s)</th>
<th>Reynolds Number</th>
<th>Theoretical Skin Friction</th>
<th>Computational Skin Friction</th>
</tr>
</thead>
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<td></td>
<td></td>
<td></td>
<td>K-Epsilon Solution</td>
<td>w/o stretch</td>
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<td>0.000</td>
<td>0.203</td>
<td>0.037</td>
<td>71.683</td>
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<td>0.008</td>
<td>18.253</td>
<td>301.802</td>
<td>0.084</td>
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**Table 4 Skin Friction Values for Hydrogen**

<table>
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<th>x-value (m)</th>
<th>Velocity u (m/s)</th>
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<th>Theoretical Skin Friction</th>
<th>Computational Skin Friction</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td>K-Epsilon Solution</td>
<td>w/o stretch</td>
</tr>
<tr>
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<td>0.042</td>
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<td>3.149</td>
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<tr>
<td>0.002</td>
<td>6.515</td>
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<td>0.632</td>
<td>18.964</td>
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<tr>
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<td>13.030</td>
<td>84.435</td>
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<tr>
<td>0.006</td>
<td>19.545</td>
<td>189.979</td>
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<tr>
<td>0.008</td>
<td>26.060</td>
<td>337.741</td>
<td>0.079</td>
<td>24.585</td>
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4.4.1 Skin Friction Values of the Theoretical Solution versus the Computational Solution without Stretch

Figure 4-1 Theoretical and Computational Skin Values without Flame Stretch for Methane

Figure 4-2 Theoretical and Computational Skin Friction Values without Flame Stretch for Propane

Figure 4-3 Theoretical and Computational Skin Friction Values without Flame Stretch for Hydrogen
4.4.2 Skin Friction Values of the Theoretical Solution versus the Computational Solution with Stretch

Figure 4-4 Theoretical and Computational Skin Friction Values with Flame Stretch for Methane

Figure 4-5 Theoretical and Computational Skin Friction Values with Flame Stretch for Propane

Figure 4-6 Theoretical and Computational Skin Friction Values with Flame Stretch for Hydrogen
4.4.3 Skin Friction Values of the Computational Solution without Stretch versus the Computational Solution with Stretch

Figure 4-7 Skin Friction Values for Methane with Stretched versus Unstretched

Figure 4-8 Skin Friction Values for Propane with Stretched versus Unstretched

Figure 4-9 Skin Friction Values for Hydrogen Stretched versus Unstretched
4.4.4 The Skin Friction Values of the K-Epsilon Solution versus the Computational Solution with Stretch

Figure 4-10 Skin Friction Values for Methane with the K-Epsilon Solution and Premixed with Stretch

Figure 4-11 Skin Friction Values for Propane with the K-Epsilon Solution and Premixed with Stretch

Figure 4-12 Skin Friction Values for Hydrogen with the K-Epsilon Solution and Premixed with Stretch
4.4.5 Skin Friction over the Profile for Fluent

Figure 4-13 Methane without Stretch

Figure 4-14 Methane with Stretch

Figure 4-15 Propane without Stretch

Figure 4-16 Propane with Stretch

Figure 4-17 Hydrogen without Stretch

Figure 4-18 Hydrogen with Stretch
### 4.5 HEAT FLUX VALUES

#### Table 5 Heat Flux Values for Each Fuel

<table>
<thead>
<tr>
<th>Fuel Type</th>
<th>Value of L (-----)</th>
<th>Edge Velocity (m/s)</th>
<th>Theoretical H-f (KW/m²*K)</th>
<th>Turbulent Intensity (-----)</th>
<th>Stagnation Velocity Gradient (s^-1)</th>
<th>Heat Flux Values (KW/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Semi-Analytical</td>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Theoretical</td>
<td>Semi-Analytical</td>
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<td></td>
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<td></td>
<td></td>
<td>Empirical</td>
<td>Empirical</td>
</tr>
<tr>
<td>Methane</td>
<td>8.33</td>
<td>16.344</td>
<td>0.223</td>
<td>0.069</td>
<td>363.199</td>
<td>227.365</td>
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<tr>
<td>Propane</td>
<td>8.33</td>
<td>15.466</td>
<td>0.234</td>
<td>0.069</td>
<td>343.679</td>
<td>728.708</td>
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<tr>
<td>Hydrogen</td>
<td>8.33</td>
<td>31.277</td>
<td>0.220</td>
<td>0.069</td>
<td>695.047</td>
<td>1045.378</td>
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#### Table 6 Comparison of Heat Flux Values Using Empirical Data

<table>
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<tr>
<th>Value of L (-----)</th>
<th>Edge Velocity (m/s)</th>
<th>Theoretical H-f (KW/m²*K)</th>
<th>Turbulent Intensity (-----)</th>
<th>Stagnation Velocity Gradient (s^-1)</th>
<th>Heat Fluxes</th>
</tr>
</thead>
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<td></td>
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<td>Semi-Analytical</td>
<td>Theoretical</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>Theoretical</td>
<td>Semi-Analytical</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Empirical</td>
<td>Empirical</td>
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<tr>
<td>0.5</td>
<td>2.661</td>
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<td>4</td>
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<td>0.070</td>
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<tr>
<td>6</td>
<td>-1.551</td>
<td>0.108</td>
<td>0.070</td>
<td>-34.461</td>
<td>201.071</td>
</tr>
</tbody>
</table>

Experimental (KW/m²)
Heat Flux at the Stagnation Point

Figure 4-19 Heat Flux Values for Various Solutions
5 REFERENCES

1. Demitrius A Koutides, Huy K. Trans and S. Amanda Chill Composite Flexible Insulation for Thermal Protection of Space Vehicles


31. Measurement Principles of LDA 03/22/06
http://www.dantecdynamics.com//LDA/Princip/index.html

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http://www.hw.ac.uk/mecWWW/cbb/page8.html

33. Turbulent Flame Speed 2/23/06

APPENDIX A

A.1 ILLUSTRATIONS FOR INTRODUCTION: AEROSPACE APPLICATIONS

A.2 ILLUSTRATIONS FOR INTRODUCTION: TURBINE APPLICATIONS
6 APPENDIX A

A. 1 ILLUSTRATIONS FOR INTRODUCTION: AEROSPACE APPLICATIONS

Figure 6-1 Figure A.1 Pipe for Aero-convective Cooling [2]

Figure 6-2 Comparison of RCG and PCC Coated Material after Exposure to 35 W/cm² for 120 seconds [3]
A.2 ILLUSTRATIONS FOR INTRODUCTION: TURBINE APPLICATIONS

Figure 6-3 Passage About a Turbine Blade [4]

Figure 6-4 Computational Flow about holes of impingement for Turbine Blades [4]
Figure 6-5 Computational Flow about holes of impingement for Turbine Blades [4]

Figure 6-6 Computational Flow about holes of impingement for Turbine Blades [4]
APPENDIX B

B. 1 COORDINATES FOR AXISYMMETRIC FLOW

B.2 DIMENSIONLESS PARAMETERS FOR THE THEORETICAL SOLUTION FOR STAGNATION POINT VELOCITY
7 APPENDIX B

B.1 COORDINATES FOR AXISYMMETRIC FLOW

For axisymmetric flow $\hat{e}_x, \hat{e}_r$, and $\hat{e}_\theta$ form the orthonormal basis. By letting $h_x=1$, $h_r=1$, and $h_\theta=r$. The operators in axisymmetric flow are:

$$\nabla = \hat{e}_x \frac{\partial}{\partial x} + \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{\partial}{\partial \theta}$$

$$\vec{A} = A_x \hat{e}_x + A_r \hat{e}_r + A_\theta \hat{e}_\theta$$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x} A_x + \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta}$$

$$\nabla \times \vec{A} = \frac{1}{h_x h_r h_\theta} \begin{vmatrix} h x e_x & h r e_r & h \theta e_\theta \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} \\ h x A_x & h r A_r & h \theta A_\theta \end{vmatrix} = \begin{vmatrix} e_x & e_r & e_\theta \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} \\ A_x & A_r & A_\theta \end{vmatrix}$$

$$\nabla \times \vec{A} = \frac{1}{r} \left( \frac{\partial (r A_\theta)}{\partial r} + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} \right) \hat{e}_x + \left( \frac{1}{r} \frac{\partial (A_x)}{\partial \theta} + \frac{\partial A_x}{\partial x} \right) \hat{e}_r + \left( \frac{\partial A_r}{\partial x} + \frac{\partial A_x}{\partial r} \right) \hat{e}_\theta$$

Figure 7-1 Axisymmetric Coordinates
### B.2 DIMENSIONLESS PARAMETERS FOR THE THEORETICAL SOLUTION FOR STAGNATION POINT VELOCITY

#### Table 7 $f_s''(\beta, g_w)$ [13]

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<th>$\beta$</th>
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#### Table 8 $G_w'(\beta, g_w)$ [13]

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<td>6.5</td>
<td>0.5752</td>
</tr>
<tr>
<td>7.0</td>
<td>0.5611</td>
</tr>
<tr>
<td>7.5</td>
<td>0.5660</td>
</tr>
<tr>
<td>8.0</td>
<td>0.5691</td>
</tr>
<tr>
<td>8.5</td>
<td>0.5712</td>
</tr>
<tr>
<td>9.0</td>
<td>0.5768</td>
</tr>
</tbody>
</table>

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APPENDIX C

C.1 PROCEDURE FOR USING FLUENT
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C.1 PROCEDURE FOR USING FLUENT

STEP I: GRID
After opening Fluent, You are prompted for the number of dimensions.

Select 2D and Click on RUN

After the console window appears Click FILE $\rightarrow$ READ $\rightarrow$ CASE
Select cyl.msh in the working directory

Check and scale the dimensions for the mesh file by selecting GRID $\rightarrow$ CHECK and then GRID $\rightarrow$ SCALE
With centimeters units chosen for unit conversion, the grid was scaled by clicking on SCALE.

To display the Grid, select **DISPLAY ➔ GRID ➔ GRID**

**Figure 8-4 Cyl.msh Grid**

**STEP II: MODEL**

Next the model must be defined. Since the flow is steady select **DEFINE ➔ MODELS ➔ SOLVER** and retain default settings for steady flow and a segregated solver.

**Figure 8-5 Solver Panel**
In order to enable the premixed combustion option, a viscous model must be defined. Select **DEFINE → MODELS → VISCOUS**.

![Viscous Model Panel](image1.png)

**Figure 8-6 Viscous Model Panel**

Select the k-epsilon model and retain the default settings. Finally define the species models through the following commands:

**DEFINE → MODELS → SPECIES → TRANSPORT AND REACTION** ...

Select **premixed-Combustion**.

![Species Model Panel](image2.png)

**Figure 8-7 Species Model Panel**

Retain adiabatic conditions, and enter 1 for the Schmidt number.

The main console later prompts the user for information needed regarding the adiabatic temperature, and properties for the reactants.
Therefore, for the materials, select **DEFINE → MATERIALS**, type pre-mixed mixture in the materials name, and enter the properties. (For values used for each fuel see Table I)
STEP III: BOUNDARY CONDITIONS

Select **DEFINE → BOUNDARY CONDITIONS. . .**

For the fluid zone, click SET, and select premixed-mixture.
For the inlet zone, select velocity inlet (click on SET) and enter values for the turbulent intensity, turbulent length scale, and the velocity (See Table I for values for each fuel).
For the outlet zone, select pressure (click on SET) and enter values for the turbulent intensity and turbulent scale. Retain the default settings for pressure and the progress variable.

STEP IV: SOLUTION

To begin the solution process select **SOLVE → CONTROLS → SOLUTION. . .**

To check for convergence, only the Flow and Turbulence Equation is Enabled. Select First Order Upwind for Discretization (For the second and third case, select Second Order Upwind for each fuel) and retain the default under-relaxation factors.
Next, set the tolerance for each residual. (See Appendix D for the residual plot for each case)

![Figure 8-12 Residual Monitor Panel](image1)

Next the solution must be initialized. Select **INITIALIZE → INITIALIZE**. . .

![Figure 8-13 Initialization Panel](image2)

Compute from the inlet zone and click on INIT.
For accuracy in the discretization, the grid is adapted. Select ADAPT→REGION...

Set X-Min and X-Max to .01 and .2 respectively. Then, set Y-Min and Y-Max to 0 and .125. Click Mark. Then change Y-Min and Y-Max to -.125 and 0 respectively and click Mark again.

Before iterating, save the case and data. Select: WRITE→CASE&DATA. . . Enter a file name and retain the binary files option for accessibility for Windows and UNIX.

Next select SOLVE→ITERATE. The following window appears:
The solution will either converge (based on the selected tolerance) as indicated by a message in the console window or will either proceed through the set number of iterations. Two hundred and fifty iterations were entered for the number of iterations for the first case.

The following window appears during the iteration sequence:

![Residual Plot](image)

Figure 8-16 Residual Plot

Again, save the case and data and overwrite the current file. Now enable the premixed combustion: **SOLVE ➔ CONTROLS ➔ SOLUTION**. Iterate 200 more times and save this case and data as a different file. To include stretch, change the value for the critical stretch factor gcrit (See Table 1). Initialize the solution and iterate 250 times for the third case.

**STEP V POST-PROCESSING.**
To calculate the skin friction at desired points along the second quadrant of the circle.

![Point-Surface Console](image)

Figure 8-17 Point-Surface Console

Select **SURFACE ➔ POINT**
With the grid displayed and the SELECT POINT with MOUSE button activated, five approximately equidistant points (22.5º apart) are set and are labeled as point-a, point-b, point-c, point-d, and point-e. Each point must intersect two lines of the grid in addition to the surface of the cylinder for accuracy.

The following is a display of the cylinder and selected points

![Figure 8-18 Grid with Circular Profile and Selected Points](image)
Once the points are created the skin friction can be found. Select **REPORT → SURFACE INTEGRALS**.

Then select **Vertex Avg** under **Report Type**. Under **Field Variable** select **Wall Fluxes**. . . **Skin Friction**. Select the desired point and click **COMPUTE**.

![Figure 8-19 Surface Integral Properties Report Console](image)

To view the skin friction for the entire surface, select **PLOT → XY PLOT** and retain the default settings for the coordinates. Select wall fluxes and then skin friction. (See Appendix D for all vector and contour Plots).

For the Contour plots, select **DISPLAY → CONTOUR**
Contour plots are available for the pressure, velocity, and the progress variable.

To see the velocity, select **DISPLAY → VECTOR**. Set the scale to 1.
APPENDIX D

D.1 PRESSURE, VELOCITY (VECTOR), PROGRESS VARIABLE CONTOUR PLOTS, AND RESIDUAL PLOTS
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D.1 PRESSURE, VELOCITY (VECTOR), PROGRESS VARIABLE CONTOUR PLOTS, AND RESIDUAL PLOTS

Methane Fuel with the K-Epsilon Solution

Figure 9-1 Pressure Contour Plot of Methane using the K-Epsilon Solution

Figure 9-2 Velocity Contour Plot of Methane using the K-Epsilon Solution

Figure 9-3 Velocity Vector Plot of Methane using the K-Epsilon Solution

Figure 9-4 Iteration Plot of Methane using the K-Epsilon Solution
Methane Fuel Case without Flame Stretch

Figure 9-5 Methane Pressure Contour Premixed Solution (w/o Stretch)

Figure 9-6 Methane Progress variable Plot Premixed (w/o Stretch)

Figure 9-7 Methane Vector Contour Premixed Solution (w/o Stretch)

Figure 9-8 Methane Iteration Plow Premixed Solution (w/o Stretch)
Methane Fuel Case with Flame Stretch

Figure 9-9 Methane Pressure Contour Premixed (w Stretch)

Figure 9-10 Methane Progress Variable Contour Premixed (w Stretch)

Figure 9-11 Methane Vector Contour Premixed (w Stretch)

Figure 9-12 Methane Iteration Plot Premixed (w Stretch)
Propane Fuel Case with the K-Epsilon Solution

Figure 9-13 Pressure Contour Plot of Propane using K-Epsilon Solution

Figure 9-14 Velocity Profile Plot of Propane using K-Epsilon Solution

Figure 9-15 Velocity Vector Plot of Propane using K-Epsilon Solution

Figure 9-16 Iteration Plot for Propane using K-Epsilon Solution
Propane Fuel Case without Flame Stretch

Figure 9-17 Propane Pressure Contour Premixed (w/o Stretch)

Figure 9-18 Propane Progress Variable Contour Premixed (w/o Stretch)

Figure 9-19 Propane Vector Contour Premixed (w/o Stretch)

Figure 9-20 Propane Iteration Plot Premixed (w/o Stretch)
Propane Fuel Case with Flame Stretch

Figure 9-21 Propane Pressure Contour Premixed (w Stretch)

Figure 9-22 Propane Progress Variable Contour Premixed (w Stretch)

Figure 9-23 Propane Velocity Contour Premixed (w Stretch)

Figure 9-24 Propane Iteration Plot Premixed (w Stretch)
Hydrogen Fuel Case with K-Epsilon Solution

Figure 9-25 Pressure Contour for Hydrogen using K-Epsilon Solution

Figure 9-26 Velocity Vector Plot for Hydrogen using K-Epsilon Solution

Figure 9-27 Velocity Contour Plot for Hydrogen using K-Epsilon Solution

Figure 9-28 Iteration Plot of Hydrogen using K-Epsilon Solution
Hydrogen Fuel Case without Flame Stretch

Figure 9-29 Hydrogen Pressure Contour Premixed (w/o Stretch)

Figure 9-30 Hydrogen Progress Variable Contour Premixed (w/o Stretch)

Figure 9-31 Hydrogen Velocity Contour Premixed (w/o Stretch)

Figure 9-32 Hydrogen Iteration Plot Premixed (w/o Stretch)
Hydrogen Flame Stretch with Flame Stretch

Figure 9-33 Hydrogen Pressure Contour Premixed (w Stretch)

Figure 9-34 Hydrogen Progress Variable Contour Premixed (w Stretch)

Figure 9-35 Hydrogen Velocity Contour Premixed (w Stretch)

Figure 9-36 Hydrogen Iteration Plot Premixed (w Stretch)