SARICA, RABIA ZEYNEP. Wavelet Analyses for Seismic Ground Motion, Simulation, and Stochastic Site Response. (Under the direction of M. S. Rahman.)

The response of soil masses during an earthquake is governed by the characteristics of the ground motions and the soil material properties. The ground motions generated during earthquakes are random and nonstationary with respect to both amplitude and frequency.

Different methods of analysis for time-frequency characteristics of the ground motions have been reviewed and the merits and disadvantages of each are explained. The methods considered here are: Fourier Transform (FT), Short Term Fourier Transform (STFT), Wigner-Ville (WV) Distribution, Hilbert-Huang Transform (HHT), and Wavelet Transform (WT). WT method with a modified version of Littlewood-Paley mother wavelet is found most suitable to analyze the nonstationary characteristics of the seismic ground motion. This method is used to identify some of the nonlinear and nonstationary characteristics of the ground motions recorded during some important earthquakes, namely Northridge-California 1994, Kocaeli-Turkey 1999, and Chi-Chi-Taiwan, 1999. The effect of site distance and site softening on the ground motion characteristics are studied. The information on redistribution of the energy among various frequency ranges as well as its temporal variation is revealed. Responses of Clay and Sand sites with different stiffness-thickness-saturation combinations to different excitations are studied with the use of a computer code for nonlinear analysis of site response (Cyclic1D) and wavelet analysis. Observations for amplification, deamplification, and pore pressure build up are given in both time and frequency domains. A wavelet based formulation to generate earthquake ground motions is also developed. This
method uses a seed acceleration history and a selected design response spectrum to produce many motions that retain the main time-frequency characteristics of the original motion, yet are all different from each other. Finally, a wavelet based method to evaluate non-stationary stochastic response of a soil site, idealized as a uniform layer of linear visco-elastic material, is also developed.
WAVELET ANALYSES FOR
SEISMIC GROUND MOTION, SIMULATION, AND STOCHASTIC
SITE RESPONSE

by

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A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

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________________________ ________________________
Chair of Advisory Committee
Sevgili Annem ve Babam, bu zamana kadar yaptığınız bütün fedakarlıklar, gösterdiğiniz yol ve hersey için sonsuz kere teşekkür. Bu tez ikinize adanmıştır.

For the countless dedications you have made for me, for the courage you gave, for the endless love, and for you are my parents, this dissertation is dedicated to you, my dear mother and father.
BIOGRAPHY

Rabia Zeynep Sarica was born to Sirin and Osman Sarica on February 3, 1976. She graduated with a Bachelor of Science degree in Civil Engineering from Istanbul University in Turkey in 1997. Realizing the importance of knowing a foreign language, she spent almost one year for improving her English. In 1998, she was accepted to the Master of Science program in Civil Engineering Department, Bogazici University, in Turkey. Right after earning her Master of Science degree in 2001, she continued to the doctoral program in North Carolina State University, Department of Civil, Construction, and Environmental Engineering. She has been a Teaching and Research Assistant in both Bogazici and North Carolina State Universities throughout her studies.
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CHAPTER 1
INTRODUCTION

Earthquakes are one of the greatest natural hazards. About 18 earthquakes of magnitude 7.0 or larger occur worldwide every year. Earthquake hazard mitigation requires studies in many areas including geotechnical aspects of earthquake engineering. Ground motion simulation, site response and associated amplification, and soil instability (liquefaction due to pore pressure build up) are among the critical areas. The ground motions generated during earthquakes are nonstationary with respect to both amplitude and frequency. The processes governing the response and instability of soil sites are nonlinear and affected by the nonstationarity of the ground motions. A proper definition of the design ground motion time history is very important for geotechnical and structural engineers.

The commonly used Fourier representation based analyses of these problems are deficient in many ways. Wavelet analysis has emerged as a powerful tool to analyze a signal and recent studies have suggested an effective use of wavelets for the solution to both linear and nonlinear problems. In this study, wavelet analyses are used for the study of ground motion characteristics, generation of random time histories of ground motion, and evaluation of nonstationary stochastic site response.

1.1 Background and Motivation

In the past few decades significant advancement has been made towards the understanding and development of physical and analytical modeling for aforementioned geotechnical
earthquake engineering problems. These may be found in text books (Kramer, 1996), seismic hazard maps (Frankel et. al., 2000), and seismic regulatory guides and codes (Borcherdt, 2002; NEHRP, 2003).

However, there are many issues, which require deeper understanding and better tools for the analysis and simulation of the underlying processes governing ground motion, site response and instability. These issues are related to the following problem areas: (i) ground motion characteristics, (ii) site response and associated amplification/deamplification, (iii) site instability due to liquefaction caused by progressively built up pore water pressure, (iv) ground motion simulation, and (v) stochastic site response.

Currently most of the analyses of ground motion and site response are based on Fourier representation. Frequently used equivalent linear method of site response analysis, commonly used spectral representation of the ground motion, and the derived understanding are all founded on Fourier representation. Fourier’s representation of functions as a superposition of sines and cosines has the ability to analyze a time history of a signal for its frequency content. A disadvantage of Fourier transform (FT) analysis is that frequency information can only be extracted for the complete duration of a signal (Newland, 1993). The basis functions (sines and cosines) used in FT, are non-local and stretch out to infinity. FT provides information about the frequency content of a signal, but it does not give any information about where in time these frequencies or spectral components are located and does a very poor job in approximating sharp spikes in a signal. Therefore, FT works well for stationary data but, cannot deal with the key problems of nonstationarity and nonlinearity. Several methods have been tried to solve this issue, the most popular being Short Time Fourier
Transform (STFT) in which FT analysis is performed over small shifting time windows to better represent local changes. (Strang and Nguyen, 1997). The drawback of this method is that once a particular size for the time window is chosen, that window is the same for all frequencies. A more flexible approach is needed.

Gaupillaud et. al. (1984) initiated the use of wavelet representation to analyze seismic data in geophysical studies for oil exploration. Subsequently, other researchers have contributed to the development of the methodology of wavelet analysis (Mallat, 1989; Daubechies, 1992; Chui, 1992; Meyer, 1992). Newland (1994a, 1994b) used this technique for analyzing vibration signals, and further developed it for engineering applications. The wavelet analysis procedure utilizes basis functions which are scaled and translated versions of a mother wavelet. Long time intervals give more precise low frequency information whereas shorter time intervals give high frequency information. Wavelet analysis has become a popular tool for many scientific and practical applications, especially in the signal processing community. It is used in some areas such as nonlinear filtering or denoising, signal and image compression (even the FBI uses this method to compress its finger-print database), speech coding, medical and biomedical signal and image processing, and communication (Graps, 1995). It has been proposed for use in medical diagnostic tests, such as detecting abnormalities or unusual activities in electro-cardiography and brain signals. In mechanical engineering and in civil engineering it is used as a crack detection tool.

Recently, several applications of wavelet analysis have been made to earthquake engineering problems (Basu and Gupta, 1997, 1998, 1999, 2000; Iyama and Kuwamura,
Wavelet analysis has great potential for investigating the unresolved issues related to the five problems identified above.

Although wavelet analysis was first used in a geophysical problem, there are few studies which utilize this versatile tool in geotechnical engineering compared to other fields which are mentioned above. In this study, the use of wavelet analysis in various aspects of geotechnical earthquake engineering is explored.

The suitability of the wavelet transform (WT) method to obtain the pertinent characteristics of earthquake ground motions as well as other example signals is investigated. A proper mother wavelet to analyze earthquake type of motions has been selected. Using this mother wavelet, effect of site to source distance, and effect of softening on recorded earthquake accelerogram characteristics are studied. To further investigate the nonlinear and nonstationary behavior of different types of soil sites, a numerical investigation is performed using simplified soil columns subjected to both simple harmonic and recorded ground motion. A one-dimensional nonlinear response program is utilized first to get the response of sites to the given excitations. Then the differences between the WT representations of these responses and input motions are observed in both the time and frequency domains.

Responses of structures that are under earthquake loadings are calculated either by the pseudo-acceleration response spectrum method or by use of an acceleration-time history. A smooth design response spectrum takes many possible earthquakes into account in a given zone with a certain probability of occurrence. The design response spectrum method is relatively simple and well-established, and therefore is used frequently in the analysis and aseismic design of conventional buildings. However, structures which are not so common
such as nuclear power plants, very tall buildings, suspension bridges, etc. requires more involved nonlinear time domain analysis of their responses subjected to a random seismic environment. Because of this need, a set of ground motion records should be selected to represent earthquake motions at a site considered as a stochastic process. Although recorded motions are becoming increasingly available nowadays, sufficient historical records are not available and also there are regions at which a sufficient number of earthquake records are not available. Thus, there is a need to produce synthetic motions. Since the sixties, researchers have been trying to develop methods to generate simulated ground motions. The works of Jennings et al (1968), Boore and Joyner (1982), Campbell (1985), Spanos (1983), Lilhanand and Tsen (1988) Iyama and Kuwamura (1999), Mukherjee and Gupta (2002), and Montejo (2004) give the various simulation techniques proposed. In many of the earlier studies, the nonstationarity in the generated motion is provided by multiplying the stationary random process by an envelope function. To account for the nonstationarity directly, a new wavelet based method to produce spectrum compatible acceleration time histories is proposed in this study.

Deterministic prediction of ground motion requires detailed knowledge about the state of the Earth and physical processes which are not usually known. Seismic response of soil masses involves an uncertainty inherent in specifying the ground motion. A rational and economic way to deal with the uncertainty is to work with a stochastic model for earthquake induced ground motion (Rahman and Pal, 1983; Rahman and Hwang, 1994; Yeh and Rahman, 1998; Rahman and Yeh, 1999). These characteristics can be obtained in a stationary sense in frequency domain only using the Fourier transform method. However, an
earthquake-induced ground motion is nonstationary. With the emergence of wavelet analysis recently, it has become possible to account for frequency nonstationarity more conveniently. With this method, statistical characteristics of a nonstationary process can be described by the instantaneous power spectral density function (PSDF) and instantaneous root-mean-square (RMS) values of the process (Basu and Gupta, 1997, 1998, 1999, 2000; Tratskas and Spanos, 2003; Spanos et. al. 2004). In this study, a method to evaluate the nonstationary stochastic response of a soil site is initiated. As a first step, only a uniform layer of soil modeled as a linear visco-elastic material is considered.

1.2 This Study

Throughout this study Wavelet Analysis has been utilized to obtain the ground motion characteristics, generate artificial earthquake ground motions, and obtain the stochastic response of a soil site.

In Chapter 2, different signal analysis methods have been scanned and the merits and disadvantages of each of them are explained. The methods considered here are: Fourier Transform (FT), Short Term Fourier Transform (STFT), Wigner-Ville (WV) Distribution, Hilbert-Huang Transform (HHT), and Wavelet Transform (WT). Example studies show that WT and HHT methods are both suitable for analyzing nonstationary signals. They both can be used to study time-frequency characteristics of these types of signals. However, because WT is easier to use, and provides adequate time-frequency information of a signal, it is found to be a more practical method to study the characteristics of ground motions.

In Chapter 3, the wavelet analysis is used to investigate the characteristics of some of the ground motions recorded during Kobe, Kocaeli and Chi-Chi earthquakes and wavelet
analysis is found useful as it allows the study of ground motion simultaneously in both time and frequency domains. Various characteristics of ground motion are affected in a complex way by the nonlinearity and associated damping of the soils. Some of these characteristics, such as temporal variations of downshift of the predominant frequency at the far sites and liquefied sites are observed with the wavelet analysis.

In Chapter 4, responses of Clay and Sand soils at two different stiffness states, at two different thicknesses, and with two different saturation conditions under sinusoidal and real earthquake motions are studied. First, the responses of these soil-thickness-motion combinations are computed using Cyclic1D (a computer code for nonlinear analysis of site response). Then WT is applied on the responses as well as on the input motions in order to reveal the nonstationary characteristics for different cases. Observations for amplification, deamplification, and pore pressure build up are given in both time and frequency domains.

Chapter 5 presents a wavelet based formulation to generate earthquake ground motions using a seed acceleration history and a selected design response spectrum. Two different time histories are selected, and 20 different artificial motions for each are generated and their time-frequency-amplitude properties examined. This new method generates many motions from a given acceleration time history. Each of these motions retains the main time-frequency characteristics of the original motion, yet they are all different from each other. Therefore, they are suitable for nonlinear response analyses because nonlinear time domain responses will not be the same.

In Chapter 6, a wavelet based method to evaluate nonstationary stochastic response of a soil site is initiated. As a first step, only a uniform layer of soil modeled as a linear visco-
elastic material is considered. A sample formulation is given for the nonstationary stochastic response using wavelet analysis. The site response considered herein is due only to a vertically propagating shear wave. The method presented here can be readily extended for an equivalent linear analysis (of the nonlinear response) of a layered soil site. The stochastic response is evaluated using both the direct method as well as Monte Carlo simulation. The results obtained by both methods are compared to verify the formulation of the direct stochastic method.

The conclusion and limitations of this research, along with some recommendations for future studies are presented in Chapter 7.

1.3 References


Jennings et al P. C., Housner G. W., Tsai N. C. (1968). “Simulated earthquake ground motions”.


CHAPTER 2

TIME-FREQUENCY ANALYSIS OF SIGNALS

2.1. Introduction

In this chapter, various tools available to analyse a nonstationary signal are studied and their relative merits are compared. Earthquake ground motions, by their nature, are transient and non-stationary. A time-varying frequency content is observed in earthquake recordings. The processes governing the response and instability of soil sites are nonlinear and affected by the nonstationarity of the ground motions. Earthquake hazard mitigation requires studies in many areas including geotechnical aspects of earthquake engineering. A proper definition of the design ground motion time history is very important for structural engineers. Signs of soil nonlinearity include decreased spectral ratios of surface to input motion near the dominant frequency of the soil, decreased statistical uncertainty in prediction of peak acceleration, and increased effective period of surface motion (Yu et. al., 1992).

2.2. Analysis of Signals: Methods

In studying time series, several methods have been developed and used by researchers and practitioners. The ones that are used frequently are Fourier Transform (FT), Short-Time Fourier Transform (STFT) and Wigner-Ville Representation. Wavelet Transform (WT) and Hilbert-Huang Transform (HHT) are two recently proposed methods to study the nonstationary and nonlinear characteristics of signals. In this study, use of WT is evaluated further for identifying the characteristics of the earthquake ground motion records. In the
following, first basic information about all of the above methods is provided. Each method has its own advantages and disadvantages depending on where they are used.

2.2.1. Fourier Analysis

The most commonly used method has been the Fourier Analysis. It reveals the frequency content of any signal by decomposing it into sinusoids of different frequency. Fourier Series is used for periodic signals, whereas for the nonperiodic signals there is the Fourier Transform (FT).

2.2.1.1. Fourier Series

In 1822, Fourier published his *Theorie analytique de la Chaleur*, he stated that for any periodic signal \( f(t) \) of period \( T \) for which \( f(t) = f(t + T) \) can be expressed as:

\[
f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(\omega_0 t)
\]  

(2.1)

where \( \omega_0 = \frac{2\pi}{T} \) is the fundamental angular frequency in radians per second and where the coefficients of the cosine and sine terms (Fourier coefficients) are obtained by relating them with \( f(t) \) as follows:
\[ a_0 = \frac{2}{T} \int_{-T/2}^{T/2} f(t) dt \]
\[ a_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(k\omega_0 t) dt \quad (2.2) \]
\[ b_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(k\omega_0 t) dt, \quad k = 0, 1, \ldots, \infty \]

2.2.1.2. Fourier Transform (FT)

If \( f \) is a square integrable function (\( f \in L^2(R) \)), its Fourier transform is defined as

\[ \hat{f}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt, \quad \omega \in R \quad (2.1) \]

where \( i = \sqrt{-1} \), \( \omega \) is the frequency in radians per second, and \( R \) is the set of all real numbers. When \( f \) is real-valued \( \hat{f}(\omega) = \hat{f}(-\omega) \). Since \( f(t) \) does not always exist from minus infinity to plus infinity, practically Fourier transform (FT) of a finite energy signal \( f \) can be written as

\[ \hat{f}(\omega) = \frac{1}{2\pi} \int_{-n}^{n} f(t) e^{-i\omega t} dt, \quad n > 0 \quad (2.2) \]

where \( n \) is a sufficiently large number which usually a power of 2. \( f(t) \) can be regained by the following equation

\[ f(t) = \int_{-n}^{n} \hat{f}(\omega) e^{i\omega t} dt \quad (2.3) \]
Analyzing signals by Fourier transform, called spectral analysis, is a standard technique to obtain information about a periodic signal. Nevertheless, signals to be analysed are often not continuous in time due to sampling procedures that measure an incoming physical signal. To deal with these types of signals, discrete Fourier Transform (DFT) can be used instead of Fourier series (Oonincx, 2000). DFT can be computed in a fast way using an algorithm called butterfly algorithm (Cooley and Tukey, 1965). This algorithm arranges the Fourier coefficients in a way that they are recursively computed. Computing DFT by this way is called the Fast Fourier Transform (FFT). In MATLAB, this algorithm is coded into a built-in function “fft” and the FT results presented in this study are found using this function. The result of FFT is a complex valued array, therefore in order to eliminate the imaginary parts, the square modulus of the FT amplitudes are taken to draw Fourier Amplitude Spectrum (FS).

While FT gives valuable information about frequencies in a seismogram, it is not possible to have any information on temporal location those frequencies. Therefore, it is suitable only for stationary signals. To overcome this problem short-time or short-term Fourier transform was proposed.

2.2.2. Short-Time Fourier Transform (STFT)

The idea in STFT is cutting the original signal into segments of smaller durations and applying FT to obtain the frequency components of each slice. The functions obtained by this crude slicing are not periodic in general and FT will interpret the jumps at the boundaries as discontinuities or abrupt variations of the signal and will introduce higher order harmonics to fit the waveform. To avoid these, the concept of windowing has been introduced. Instead of
localizing by means of rectangular function, a smooth window function, which is close to unity near the origin and decays towards zero at the edges, is used. For this reason STFT is sometimes called windowed Fourier transform. Any square integrable function may be used as a window, but certain criteria should be met. The main property of a good window is its good localization in both time and frequency domains. Some windows are favourable such as Hamming, Hanning, Bartlett, Blackman, Kaiser and Gaussian (Carmona et al., 1998). The reason for the use of these windows is that they have functional forms and their FT is concentrated around $\omega = 0$. The window in the time domain is referred to as the time window, its Fourier transform as the spectral window. Functional forms of some windows are given below (Carmona et al., 1998).

Rectangular: $$g(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Barlett (triangular): $$g(t) = \begin{cases} 2t, & 0 \leq t \leq 1/2 \\ 2(1-t), & 1/2 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Hanning: $$g(t) = \begin{cases} (1 - \cos(2\pi t))/2, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Hamming: $$g(t) = \begin{cases} .54 - .46 \cos(2\pi t), & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Blackman: $$g(t) = \begin{cases} .42 - 5\cos(2\pi t) + .08\cos(4\pi t), & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Gaussian: $$g(t) = \pi^{-1/4} e^{-t^2/2}$$
The signal is multiplied by one of these window functions \( g(t-b) \), where \( g(t) \) is nonzero only in a finite region around time \( b \). Then the Fourier transform of \( f(t)g(t-b) \) is taken, and the window is moved to a different location to perform the same operation. This method is also called Windowed Fourier Transform and its definition is given by

\[
S_f(\omega, b) = \int_{-\infty}^{\infty} f(t)g(t-b)e^{-j\omega t} dt
\]  

(2.4)

The STFT is an energy conserving presentation provided that

\[
\int_{-\infty}^{\infty} |g(t)|^2 dt = 1
\]  

(2.5)

Using Parseval’s equality the following expression may be obtained,

\[
\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |S_f(\omega, t)|^2 d\omega dt
\]  

(2.6)

The signal can be reconstructed from its transform by the following formula,

\[
f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_f(\omega, t)(t-b)e^{j\omega t} d\omega dt
\]  

(2.7)

The fundamental problem with STFT is that it is not possible to have high resolution simultaneously in both time and frequency. A short window choice may cause poor frequency resolution. A long window may reveal the frequency content better at the expense of time resolution. Because, the signal portion in any window is treated as a stationary signal.

2.2.3. Wigner-Ville Representation (WVR)

Wigner-Ville representation of a signal \( f \) is given by (Carmona et al. 1998):

\[
W_f(b, \omega) = \int f(b + \frac{t}{2}) f^*(b - \frac{t}{2}) e^{-j\omega t} dt
\]  

(2.8)
where \( b \) is a time variable, \( \omega \) is a frequency variable, and * denotes complex conjugate.

This process is the correlation of the signal with itself. We may interpret this equation as the computation of a "local" autocorrelation function at each time instant, \( b \), followed by the evaluation of its Fourier transform. This leads to a "local" power spectral density at each time instant, \( t \). In practice, only one realization of the process is available and this forces us to ignore the expectation operator. If expectations of both sides of the above equation are taken:

\[
E\{W_f(b, \omega)\} = \int Cf(b + \frac{t}{2}, b - \frac{t}{2})e^{-i\omega t} dt \tag{2.9}
\]

\( Cf \) is the autocovariance function of the process. In the stationary case, \( Cf \) is not dependent on the time parameter \( b \) and therefore it is constant. Left hand side of this equation is called instantaneous spectral density function and may be denoted as \( S_f(b, \omega) \). In other words, it is an energy density function. In theory, \( S_f(b, \omega) \) is a measure of time-frequency distribution of a random process \( f(t) \) at time \( b \) (Newland, 1993). The Wigner-Ville transform is optimally localized in the time domain for Dirac signals, and in the frequency domain for linear chirps. When \( W_f(b, \omega) \) is integrated over all frequencies the square modulus of the signal is obtained. This is also called the frequency marginal. It is the same as the instantaneous energy, which is obtained by integrating over all frequencies.

\[
\int W_f(b, \omega) d\omega = |f(b)|^2 \tag{2.10}
\]

The total energy is obtained as

\[
\int \int W_f(b, \omega)d\omega db = \int |f(b)|^2 db \tag{2.11}
\]
The power spectrum is obtained when the Wigner-Ville representation is integrated over time. This is sometimes called as the time marginal.

\[ \int W_f(b,\omega)db = |\hat{f}(\omega)|^2 \]  
\[ (2.12) \]

Wigner-Ville distribution is an energy preserving procedure because it satisfies both frequency and time marginal conditions. The difficulties are, \( S_f(b,\omega) \) may be localized at time \( b \), but it covers an infinite range of \( t \), therefore it is very dependent on \( f(t) \) away from the local time. While Fourier spectra are periodic with period equal to the sampling rate, \( W_f(b,\omega) \) is periodic in frequency with period equal to half the sampling rate. This may cause aliasing, which can be removed either by over sampling, or by using the corresponding analytic signal. The definition of an analytical signal is given under the following heading. The distribution is negatively affected by important cross-terms, which limit its practical use. Cross-terms may be adequately reduced by smoothing the distribution over time, but that reduces the time-frequency resolution as well.

2.2.4. Hilbert-Huang Transform (HHT)

Hilbert spectral analysis is a tool for obtaining the instantaneous frequency of a real signal. It may be said that the Hilbert Transform (HT) is the relationship between the real and imaginary parts of the Fourier transform of a one-sided function. HT of an arbitrary real function \( f(t) \) is given as

\[ H(t) = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} f(t') \frac{1}{t'-t} dt' \]  
\[ (2.13) \]
where $PV$ is the Cauchy principle value of the integral. This transform exists for all functions of class $L^p$ (Titchmarsh, 1948). Cauchy principle value of an infinite integral of a function $f$ is defined by

$$PV \int_{-\infty}^{\infty} f(t)dt = \lim_{R \to \infty} \int_{-R}^{R} f(t)dt$$  \hspace{1cm} (2.14)

A signal which has no negative-frequency components is called an *analytic signal*. An analytic signal $Z(t)$ can be generated from a real signal $f(t)$ suppressing the negative frequency components while preserving the positive components as (Reilly, et. al., 1994):

$$Z(t) = f(t) + iH(t) = a(t)e^{i\theta(t)}$$  \hspace{1cm} (2.15)

where

$$a(t) = \sqrt{f^2(t) + H^2(t)}$$  \hspace{1cm} (2.16)

and

$$\theta(t) = \arctan \left( \frac{H(t)}{f(t)} \right)$$  \hspace{1cm} (2.17)

Equation 2.13 defines the HT as the convolution of $f(t)$ with $1/t$; therefore it emphasizes the local properties of $f(t)$ (Huang et. al., 1999).

The instantaneous frequency is defined as:

$$\omega = \frac{d\theta(t)}{dt}$$  \hspace{1cm} (2.18)

HT defines the imaginary part of the analytical signal (Huang et al., 1998, Bendat and Piersol, 1986). $f(t)$ and $H(t)$ are frequently called to be in quadrature because in theory they
are out of phase by $-\pi/2$. In reality, this is true under certain conditions. To obtain meaningful instantaneous frequency, some restrictions should be applied to the data (Boashash 1992a, Bedrosian 1963, Gabor 1946). The signals have to be mono-component, meaning that there should be no riding waves. Therefore, the signals which can be studied by HT are limited to simple free vibrations. The limitation of the data makes this method with a little applicability for actual multi-component vibration problems. In order to solve this problem, Huang et al. (1998) introduced the empirical mode decomposition (EMD) method to make a signal ready for HT analysis. The EMD and HT together are called HHT method.

HHT is a two-step method for analyzing non-stationary and nonlinear data (Huang et al. 1998a, 1999). The first step is EMD and the second step is the HT. EMD is a method for decomposing the signal so that each mode will have only one frequency component at a time (Cohen, 1995). Thus, its HT will provide realistic time-frequency information.

To satisfy these conditions, a class of functions called intrinsic mode functions has been introduced by (Huang et. al. 1998a). An intrinsic mode function satisfies two conditions: First, the number of extrema and the number of zero crossings must be either equal to each other, or they must differ at most by one. Second, the mean value of the envelopes defined by the local maxima and the local minima should be zero at any point, meaning that the functions should be symmetric with respect to the local zero mean. (Huang et al., 1998, 2001).

For complicated data, there may be more than one IMF component; therefore, at a given time, different instantaneous frequencies may be represented. The EMD method is used to
decompose the data into necessary IMF components. Given a signal $f(t)$, the effective algorithm of EMD can be summarized as follows (Flandrin, 2004, Huang et al. 1998, 2003):

As the first step, extrema of $f(t)$ are identified. Secondly, the minima and the maxima are interpolated using cubic spline interpolation. At the end of this second step there should be two envelopes; $f_{\text{min}}(t)$ (for minima) and $f_{\text{max}}(t)$ (for maxima). As the third step, the average is computed as:

$$m_1(t) = \frac{f_{\text{min}}(t) + f_{\text{max}}(t)}{2} \quad (2.19)$$

Then, the detail $h_1(t)$ is extracted as:

$$h_1(t) = f(t) - m_1(t) \quad (2.20)$$

After this fourth step, $h_1(t)$ is treated as the new signal and the above steps are applied until a stopping criterion is met.

$$h_{11}(t) = h_1(t) - m_{11}(t) \quad (2.21)$$

where $m_{11}(t)$ is the average of the maxima and minima envelopes of $h_1(t)$, this may be repeated $k$ times.

When the stopping criterion is met (after $k$ times iteration), the first IMF component is let to be $c_1(t) = h_{1k}(t)$. This is extracted from the signal and residual $r_1(t)$ is obtained as:

$$r_1(t) = f(t) - c_1(t) \quad (2.22)$$

Iteration should be performed on $r_1(t)$ until either no more IMF component can be extracted or the residual meets a predetermined criterion (becomes a constant value).

By summation, the original signal is recovered as
\[ f(t) = \sum_{p=1}^{n} c_p + r_p \quad (2.23) \]

The above sifting process eliminates riding waves and makes a wave profile more symmetric. However, there are some numerical difficulties with EMD. The sifting process may produce some side effects when it is used more or less than enough number of times (Huang et al. 1998, 2003). New extrema and shift, or exaggeration in the existing ones can be generated by this way. Therefore, stopping criteria must be selected carefully as over-sifting will damage the data and under-sifting will give a false instantaneous frequency result (Huang et al., 2003).

Another possible problem is that the mean of the envelopes can be different from the true local mean for nonlinear data, thus; some asymmetric waveforms can exist even after quite a number of siftings. On the practical side, serious problems may occur because of spline fitting. Near the ends of the signals, cubic spline fitting can have large swings. If they are left untouched, they can propagate inward and corrupt the whole data span, especially in the low-frequency components. Huang et al. (2003) adopted a method of adding characteristic waves at the ends. It is said that this method confines the large swings successfully, but further studies are needed to guarantee that.

HT also has end effects because the numerical method to implement HT is based on FT. The decomposition method cannot separate signals when their frequencies are too close. In this case, they are physically identical. Although EMD will give IMF components, the individual component does not guarantee a well-defined physical meaning. Great caution should be taken in both implementing the algorithm and in interpreting the results.
2.2.5. **Wavelet Transform (WT)**

The wavelet analysis has emerged as a powerful tool to analyze a signal and recent studies suggested an effective use of wavelets for the solution of both linear and nonlinear problems (Grossman and Morlet, 1984). A *wavelet* is a small wave with finite energy, which has its energy concentrated in time or space to serve as a “basis function” for the analysis of transient, nonstationary, or time-varying phenomenon. Its Fourier transform is concentrated around a specific frequency. Therefore, the wavelet still has the oscillating wavelike characteristics, but also has the ability to allow simultaneous time and frequency analysis.

Wavelets are used to analyze signals in a similar way as complex expansions (sine and cosine functions) used in Fourier type analysis. The difference is that the signals are broken down into a series of local basis functions called *wavelets*. Each wavelet is located at a different position on the time axis and is local in the sense that it decays to zero when sufficiently far from its center. The terminology “wavelet” was first introduced, in the context of a mathematical transform, in 1984 by (Grossmann and Morlet, 1984). The Wavelet Transform is a two-parameter expansion of a signal in terms of a particular wavelet basis function or mother wavelet. Temporal analysis is performed with a contracted high frequency version of the prototype wavelet, while frequency analysis is performed with a dilated, low frequency version of the same wavelet. Wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis. Wavelet analysis may be broadly classified as Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT). Information on CWT and DWT is given in the following sections of this chapter.
2.2.5.1. Discrete Wavelet Transform (DWT)

Discrete version of Wavelet analysis involves some signal processing terms such as filtering, downsampling and upsampling. The details of the analysis are described thoroughly in Strang and Nguyen (1995). Multiresolution divides the frequencies into octave bands, from $\omega$ to $2\omega$, instead of uniform bands from $\omega$ to $\omega+\Delta\omega$. Using DWT, a function $a_0(t)$ is resolved to several functions at different levels as:

$$a_j(t) = a_{j+1}(t) + d_{j+1}(t) \quad j \in \mathbb{Z}, \; j = 0..n$$  \hspace{1cm} (2.24)

where, $j$ is the level number representing a particular range of frequency and it is related to the scale $a$ in CWT by the relation:

$$a = 2^j \quad \text{or} \quad j = \log(a)/\log(2)$$  \hspace{1cm} (2.25)

The idea of decomposition and reconstruction (multiresolution) may be illustrated in Fig. 2.1. a’s are local averages at a particular level and d’s are local differences or details. Each detail or level function has a range of particular frequencies out of which the intensity is zero.

The exclusive range of frequency is denoted as $\omega = [\omega_1, \omega_2]$ with

$$\omega_1 = 1/(2^{j+1} \Delta t) \quad \text{and} \quad \omega_2 = 1/(2^j \Delta t)$$  \hspace{1cm} (2.26)

where, $\Delta t$ is the time step of discretized data $a_0(t)$. $a_{j+1}(t)$ is the approximation or the smooth portion of the original signal at level 1 where $d_{j+1}(t)$ is the detail at the same level.

Decomposition is cut at a level where there is no appreciable information remain in the approximation. Mother wavelet should analyze the signal with fewest waveforms. The choice of wavelet decomposition tree can be made according to the above criterion by examining the
entropies of the wavelet coefficients at each decomposition level. In communication theory, entropy is defined as a numerical measure of the uncertainty of an outcome. Therefore, the best basis is the one that produces the least entropy. Thus, if we truncate the series at level $j = n$, the original signal can be reconstructed from the details:

$$a_0(t) = \sum_{j=1}^{n} d_j(t)$$

In the above, the details or level functions $d_j(t)$ are expressed as (Daubechies, 1992):

$$d_j(t) = \sum_{-\infty}^{\infty} C_{j,k} \psi_{j,k}$$

with

$$\psi_{j,k} = 2^{j/2} \psi(2^j t - k)$$

Where, $k$ represents an index on time scale, $\psi_{j,k}$ are the basis wavelet functions and $C_{j,k}$ are corresponding wavelet coefficients. Time starts from zero and goes to $t_e$ ($t = [0 \ t_e]$), thus finally:

$$a_0(t) = \sum_{j=1}^{n} \sum_{k=0}^{t_e} C_{j,k} \psi_{j,k}$$
2.2.5.2. Continuous Wavelet Transform (CWT)

Any signal processing on a computer using real-world data must be performed on a discrete signal. Therefore, continuous wavelet is also a discrete process, but its continuity comes from the flexibility of the set of scales and positions on which it operates. Unlike the discrete wavelet transform, the CWT can operate at every scale. The CWT is also continuous in terms of shifting: during computation, the analyzing wavelet is shifted smoothly over the full domain of the analysed function. If scales and positions are chosen based on powers of two (as in DWT), that is dyadic scales and positions, the analysis will be much more efficient in terms of computational effort.

Continuous analysis is used in this study because it is possible to use a flexible frequency range and it is easier to interpret the results. In contrast, DWT uses frequency only in the octave bands. Although this latter method is computationally less expensive, it does not give a very precise result to interpret, and it is used mostly in signal compression.
Let $\psi(t)$ be a mother wavelet. All other wavelets are obtained by scaling and translating the $\psi(t)$ as follows (Daubechies, 1992; Newland, 1993; Strang and Nguyen, 1995):

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$

(2.31)

Let $f(t)$ is a square integrable function of time, $t$. The CWT of $f(t)$ is defined as:

$$W_{\psi}f_{a,b} = \int_{-\infty}^{+\infty} f(t)\tilde{\psi}_{a,b}(t)dt$$

or

$$W_{\psi}f_{a,b} = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t)\tilde{\psi}_{a,b}\left(\frac{t-b}{a}\right)dt$$

(2.32)

where $a, b \in R, a \neq 0$ and $\sim$ symbol denotes the complex conjugation. In order to keep the energy level the same for different values of $a$ and $b$, the normalizing factor $\frac{1}{\sqrt{a}}$ is used (Zhou and Adeli, 2003).

$$\int_{-\infty}^{+\infty} |\psi_{a,b}(t)|^2 dt = \int_{-\infty}^{+\infty} |\psi(t)|^2 dt$$

(2.33)

$a$ is called scale parameter and $b$ is called translation parameter. When $a$ is increased the wavelet $\psi_{a,b}(t)$ is dilated in time and when $b$ is varied, the signal is translated in time. So, $a$ scales the wavelet in order to capture the local frequency content, and $b$ localizes the wavelet basis function at $t = b$ and its vicinity.

The implementation of CWT is done by first discretizing the signal in time and then sampling it with translation parameters $b$ at scale parameters $a$ to achieve an appropriate
frequency resolution. $b$s are generally chosen to be the same as the sampling points of the original signal $f(t)$. After $a$ and $b$ parameters are selected, basis or mother wavelet are stretched or dilated according to the $a$s and it is translated according to the $b$s in order to produce a family of wavelets $\psi_{a,b}(t)$. The wavelets $\psi_{a,b}(t)$ are multiplied by $f(t)$ at different scales and different translations. The CWT coefficients are obtained by summing the product showing the correlation between the signal and the wavelet functions $\psi_{a,b}(t)$. As a result, a good time resolution is obtained at high frequencies whereas a good frequency resolution is obtained at low frequencies. The original time domain signal can be reconstructed through the inverse wavelet transform (Daubechies, 1992).

$$f(t) = \frac{1}{2\pi c_{\psi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{W_{\psi}f_{a,b}}{a^2} \psi_{a,b}(t) da \, db$$

(2.34)

where, $C_{\psi} = \int_{-\infty}^{+\infty} \left| \hat{\psi}(\omega) \right|^2 d\omega$

(2.35)

and $\hat{\psi}(\omega)$ is FT of $\psi(\omega)$,

$$\hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(t) e^{i\omega t} dt$$

(2.36)

A wavelet analysis is often called a time-scale analysis rather than a time-frequency analysis because the analysis function $\psi(t)$ (or the mother wavelet), is scaled by $a$. Among these wavelets are orthogonal, biorthogonal, and harmonic wavelet systems. Orthogonal wavelets decompose signals into well-behaved orthogonal signal spaces. In 1988, Daubechies introduced a class of compactly supported orthogonal wavelets with growing
smoothness for increasing support. Mallat (1989) and Meyer (1992) presented the theory of multiresolution analysis. The spline family was first studied by Battle (1987), and Chui (1992).

Except for the Haar wavelet, compactly supported orthogonal wavelets are not symmetric about the zero mean, a condition that might be required in some applications such as image processing where symmetry corresponds to linear phase. To obtain symmetry and keep the property of perfect reconstruction, the orthogonal condition was replaced by biorthogonality and the theory of biorthogonal wavelets was established (Chui and Wang, 1992, Cohen et al., 1992, Vetterli and Herley, 1992). Biorthogonal wavelets are more complicated and are defined based on a pair of scaling and wavelet functions.

2.2.5.3. Choice of Wavelet

There is not a unique wavelet representation of a signal as there are different wavelets and not all wavelets work well for all signals. Some may work very well for a specific signal and may not work at all for another signal. So, the problem is, how are we going to find which wavelet is best for our signal?

In most wavelet transform applications, it is required that the original signal be synthesized from the wavelet coefficients. This condition is referred to as perfect reconstruction. In some cases, however, like pattern recognition applications, this requirement can be relaxed. In the case of perfect reconstruction, in order to use the same set of wavelets for both analysis and synthesis, and compactly represent the signal, the wavelets should also satisfy orthogonality condition. By choosing two different sets of wavelets, one for analysis and the other for synthesis, the two sets should satisfy the biorthogonality
condition to achieve perfect reconstruction. In general, the goal of most modern wavelet research is to create a mother wavelet function that will give an informative, efficient, and useful description of the signal of interest. It is not easy to design a uniform procedure for developing the best mother wavelet or wavelet transform for a given class of signals. However, based on several general characteristics of the wavelet functions, it is possible to determine which wavelet is more suitable for a given application.

Transform with orthogonal wavelet bases is energy conserving. This implies that the mean square error (MSE) introduced during the quantization of the WT coefficients is equal to the MSE in the reconstructed signal. For orthogonal filter banks, the synthesis filters are transposes of analysis filters. However, in the case of biorthogonal wavelets, the basis functions are not orthogonal and thus not energy preserving. In this case Parseval’s equality between \( \int |f(t)|^2\,dt \) and \( \sum \sum |W_{\psi f_{a,b}}|^2 \) is not valid. That requires orthogonality.

Parseval’s theorem states that the total power or energy in a signal can be obtained by integrating over all time or all frequencies, and in both domains it is related to the amplitude squared. For a stationary signal with finite power, the frequency spectrum will either contain discrete frequency components (when amplitude of these components are squared, power at each frequency is obtained), or for random signals the squared amplitude spectrum is continuously distributed over frequency and represents “power spectral density” (PSD) which has to be integrated over a finite bandwidth to give finite power. In both cases, the equivalent “power” in the time domain is the mean square value, obtained by integrating the instantaneous squared value (instantaneous power) over a sufficiently long time and dividing
by that time. For transient signals with finite energy (integral of power over time) such as earthquake motions, the squared amplitude of the signals FT represents “energy spectral density” (ESD). When ESD is integrated over frequency, the total energy obtained is the same as the total energy when the instantaneous power of the signal is integrated over the time (Randall, 2000). For a signal with units $U$ (where $U$ represents m, m/sec, m/sec$^2$, g, N, etc.) power has units $U^2$, energy has units $U^2$sec, PSD has units $U^2$/Hz ($U^2$sec), ESD has units $U^2$sec/Hz ($U^2$sec$^2$).

Among other wavelets, a modified version Littlewood-Paley (LP) basis and Harmonic basis by Newland (1993) is found more suitable to analyse earthquake signals when time-frequency behaviour is the main concern, because it gives a better frequency-time resolution than other wavelet bases do (Basu and Gupta 1997a, 1997b, 1999, 2000). The LP basis is characterized by a reasonably fast temporal decay (thus helping to capture the local temporal features) and by very good frequency localization (since the Fourier transform of this basis is defined over a finite interval). The analytical expression for this wavelet is given as:

$$\psi = \frac{1}{\pi(\sqrt{\sigma} - 1)} \frac{\sin(\sigma \pi) - \sin(\pi)}{t}$$

(2.37)

where

$t = (t - b)/a$ ,

$\psi$ is the mother wavelet

$t$ is the duration

$b$ is the time localization parameter (the wavelet is centered around $b$)

$\sigma$ is a constant used in discretizing $a$. When $\sigma=2$ the basis is Littlewood-Paley basis.
Basu and Gupta (1997) used 1.189 whereas in this study, by trial and error 1.5 is found more suitable for both the example signals and recorded earthquake motions. The translated and dilated versions of this basis have the advantage of being mutually orthogonal. The FT of this basis function is defined on finite support and is given as:

\[
\hat{\psi}(\omega_j) = \begin{cases} 
\frac{a_j}{2(\sigma - 1)\pi} & \pi \leq |\omega_j| \leq \sigma\pi \\
0 & \text{otherwise}
\end{cases}
\] (2.38)

This means that dilated versions are characterized by non-overlapping energy bands (Basu and Gupta, 1998). This mother wavelet is shown in Figure 2.2. The translates and dilates of the same wavelet basis is presented in Figure 2.3. The relationship between scale and frequency can be explained by the pseudo-frequency corresponding to each scale. A way to do it is to compute the center frequency \(f_c\) of the wavelet and to use the following relationship:

\[
f_a = \frac{f_c}{a \ dt}
\] (2.39)

where \(a\) is a scale, \(dt\) is the sampling period, and \(f_c\) is the center frequency of a wavelet in Hz. \(f_a\) is the pseudo-frequency corresponding to the scale \(a\), in Hz. The center frequency-based approximation captures the main wavelet oscillations. The idea is to relate the given wavelet with a purely periodic signal of frequency \(f_c\). The center frequency is a convenient and simple characterization of the leading dominant frequency of the wavelet. For a particular scale, the central frequency is found to be \(f_c = \frac{(\sigma + 1)}{2\sigma a \ dt}\).
In the following, several example signals are studied by FT, STFT, HHT and WT methods.

![Modified Littlewood-Paley Basis](image)

Figure 2.2: Modified Littlewood-Paley Basis.

Modified LP Basis ($\psi$) at different time localizations (b) and at different scales (a)

![Waveforms at different times](image)

Figure 2.3: Modified Littlewood-Paley Basis at different scales- $a$ (or pseudo frequencies- $f$) and different locations- $b$ (pf is the pseudo frequency or the central frequency of the wavelet).
2.2.5.4. Energy Calculation Using Wavelet Analysis

It is known that the total energy, $E$, of a signal is the sum of the squares of its values and is written as (Walker 1999):

$$ E = \int_0^t f^2(t) \, dt $$

(2.40)

where $t$ is time. This definition is similar to Arias intensity measure (also termed accelerogram energy). Arias intensity of $f(t)$ is defined as (Kayen and Mitchell 1997):

$$ I = \frac{\pi}{2g} \int_0^t f^2(t) \, dt $$

(2.41)

Where, $g$ is the earth’s gravity. Dimensional unit of Arias intensity integral is given in length/time.

The Wavelet transform decomposes a signal in time and frequency simultaneously. Following the same manner, the energy contents of a signal can be evaluated in time and frequency domain. Here, the energy does not represent the actual physical energy of the system, but, it is proportional to it. The main concern is the variation in components temporal contribution of different frequencies to the total energy. According to the wavelet decomposition equation in Equation 2.32, the unit of $W_{y, \psi f_{a, b}}$ is length/time. $a$ scales the wavelet in order to capture the local frequency content, and $b$ localizes the wavelet basis function at $t = b$ and its vicinity. The total energy of $f(t)$ in terms of different scales (frequencies) is obtained using the Wavelet coefficients:
\[
E = \int_{ab} W_{\psi}^2 f_{a,b} dt
\]  

(2.42)

Although CWT is used, data are handled in discretized form, therefore; the energy level of a given signal is obtained by squaring the modulus of the wavelet coefficients (Farge, 1992), and “instantaneous wavelet energy”, or “wavelet energy density”, \(dE\), is defined as

\[
dE_{a,b,f}(t) = W_{\psi}^2 f_{a,b}
\]  

(2.44)

Cumulative energy contained in \(a^{th}\) scale can be written as:

\[
E_{a,b} = dt \sum_b dE_{a,b,f}(t) = \sum_b W_{\psi}^2 f_{a,b}
\]  

(2.44)

\(dt\) is the sampling period of the accelerogram. Although WT is defined as a time-scale analysis rather than time-frequency analysis, there is an average central frequency (\(f\ in\ Hz\)) associated with each scale the energy of an earthquake motion can be represented in this pseudo frequency-time domain when scales are replaced by the central frequencies. Marginal Wavelet Spectrum (MWS) is obtained when these energies accumulated over time are plotted against the central frequencies.

Cumulative energy contained in \(f^{th}\) frequency up to any time \(t\) can be written as:

\[
E_{f,t} = \Delta t \sum_{t=0}^t dE_{f,t}
\]  

(2.45)

Again, total energy in terms of cumulative energies of different frequencies is:

\[
E = \sum_{f_1}^f E_{f,t}
\]  

(2.46)
If the temporal distributions of the cumulative energies of the different frequencies are normalized with respect to the total energy of the signal, percentage of energy at different time instants for different frequencies may be observed.

\[
\text{percent}E_{f,t} = \frac{E_{f,t}}{E} \times 100
\]  

(2.47)

This equation demonstrates the contribution of an individual frequency component to the total energy at a particular time.

As mentioned in the previous chapter, if the total energies (the final values of the cumulative energy array) contained in all different frequencies are plotted against these frequencies, Marginal Wavelet Amplitude Spectrum (MWS) is obtained. Just as in the Fourier Amplitude Spectrum (FAS), time information is lost in this presentation.

2.3. Example Analyses: Results using FT, STFT, HHT, and WT

2.3.1. Hypothetical Signal 1

A signal \( s = \begin{cases} 
\cos(60\pi t) & 0 \leq t \leq 2.5 \\
\cos(40\pi t) & 2.5 < t \leq 5 \\
\cos(20\pi t) & 5 < t \leq 7.5 \\
\cos(10\pi t) & 7.5 < t \leq 10 
\end{cases} \) is given (Figure 2.4). There are four distinct frequencies (30, 20, 10, and 5 Hz) acting on different time intervals. In this example, this signal is studied using the FT, STFT, HHT and WT. Power Amplitude Spectra for these methods are given together in Figure 2.4. All the methods show where the frequency peaks are. Other than that none of them are exactly the same. Normally, at high frequencies the wavelet is very broad in the frequency space, therefore; the high frequency peaks in the spectrum, unless their energy is significantly high, get smoothed out. At large wavelet scales,
the wavelet is narrower in frequency, therefore the peaks are sharper and have larger amplitude (Torrence and Compo, 1998). If the low frequencies and high frequencies in a signal are equally important, then the squared wavelet coefficients are divided by the square of the scales used in the analysis. By this way the ridges of the coefficients on the time-frequency-energy density plot give the desired amplitude, time and frequency information. But, in situations where the low frequency components are of more interest and the higher frequencies are noises, the original squared wavelet coefficients (before normalizing with the scales) give sharper results in the low frequency range. A Gaussian window with 500 points has been used and the overlap length of 250 points has been selected in this STFT analysis.

Figure 2.4: Time History of “Hypothetical Signal 1”
In Figure 2.6 is shown the three dimensional spectrogram which represents the time-frequency-amplitude relationship. For this figure, all frequencies have the same amplitude as they had in the regular FT analysis. Figure 2.7 shows the two-dimensional STFT spectrogram (time-frequency map) of the signal. Spectrogram is the two dimensional view of the three dimensional time-frequency-amplitude representation, on the time-frequency plane. Frequencies and the time intervals when these frequencies exist may be seen in this figure. STFT method catches the peak frequencies and it draws an envelope to FAS. However, at the frequency boundaries this picture shows spurious frequency values, which is not a desirable effect.

HHT also shows the frequency peaks correctly. Intrinsic mode functions or HHT decomposition modes of the signal is provided in Figure 2.8. It is observed that there are large swings towards the end of the signals in the modes other than the first mode. In order to prevent that, the signal may be made to repeat itself, or it can be zero padded, but each has its own drawbacks. When the signal is not periodic, and not simple as this hypothetical signal, the selection of the portion of the signal to be repeated becomes tricky. With zero padding, the sharp boundary between the signal and the zeros causes problems. It is seen that there are some very high frequency components in the time-frequency distribution of the intrinsic mode functions. However, the amplitude of these frequencies are so low that they do not show up in the three and two-dimensional spectrograms except for some unrealistically high frequencies at 7.5th second. This observation may lead to wrong interpretations.

Time-frequency-amplitude graph with WT and two-dimensional spectrogram corresponding to this are seen in Figure 2.11 and Figure 2.12, respectively. Two-dimensional
wavelet spectrogram is also called a wavelet map. Throughout this thesis, the two terms are sometimes used interchangeably. It is observed that WT gives high resolution at low frequencies. In order to have the same amplitude at the ridges of the different frequency humps, the square of the WT coefficients have been divided by the scales in these figures. If they were not divided by these scales, the volume under each hump would give the same energy amplitude for each frequency. Although WT resolution gets poorer as the frequency increases, its capability to capture even the highest frequency (30Hz) of this signal with excellent time localization is quite acceptable. Among the methods used for this example, WT seems to be the optimal method because it is easy to apply and gives reasonable results. Since we are generally interested in the frequencies less than 20 Hz in a real earthquake, Wavelet transform may be very well suited for determination of time dependent frequency characteristics of earthquakes.
Figure 2.5: Power Spectra of Hypothetical Signal 1 with FT, WT, STFT, and HHT.

Figure 2.6: Three-dimensional STFT spectrogram of Hypothetical Signal 1 by STFT method.
Figure 2.7: STFT Spectrogram of the Hypothetical Signal 1.

Figure 2.8: Intrinsic mode functions or decomposition modes of Hypothetical Signal 1.
Figure 2.9: Three-dimensional spectrogram or time-frequency-amplitude distribution of “Hypothetical Signal 1” by HHT method.

Figure 2.10: Two-dimensional spectrogram of “Hypothetical Signal 1” using HHT.
Figure 2.11: Three-dimensional spectrogram or time-frequency-amplitude distribution of “Hypothetical Signal 1” by WT method.

Figure 2.12: Two-dimensional spectrogram of “Hypothetical Signal 1” using WT.
2.3.2. Hypothetical Signal 2

A slowly varying transient signal \( s = \cos(2\pi t + 0.5\sin(2\pi t))e^{-0.2t} + 0.05\sin(30\pi) \) (Zhang et al., 2004) has been selected as the second example signal to compare the methods (Figure 2.14). If \( 0.5\sin(2\pi t) \) is accepted time dependent phase and \( \cos(2\pi t) \) the carrier frequency, instantaneous frequency can be found by differentiating the time dependent phase with respect to time. Then, \( s \) will have a time-dependent frequency of \( 1 + 0.5\cos(2\pi t) \) Hz and a time independent frequency 15 Hz. However, if we examine this signal by plotting \( \cos(2\pi t + 0.5\sin(2\pi t))e^{-0.2t} \) and \( \cos(2\pi t)e^{-0.2t} \) together, we see that \( 0.5\sin(2\pi t) \) has only a bulging effect on the main signal \( \cos(2\pi t)e^{-0.2t} \). Practically, the signal’s central frequency is 1 Hz.

![Figure 2.13: cos(2\pi t + 0.5 sin(2\pi t))e^{-0.2t} and cos(2\pi t)e^{-0.2t} comparison.](image-url)
The power spectral distributions resulting from FT, STFT, HHT and WT are presented together in Figure 2. 15. The FT and STFT methods give unrealistic high frequencies whereas HHT and WT do not.

The three-dimensional STFT spectrogram (with the specified window size) in Figure 2. 16 does not provide a very correct frequency representation in the lower frequency range. It shows the low frequencies change between 0 Hz and 2.5 Hz whereas the true frequency fluctuation should have taken place around 1 Hz. Inefficiency of the method can be seen further in Figure 2. 17 where the two-dimensional spectrogram is provided. It gives however the amplitude decaying nature of the signal in these last figures. Higher amplitudes are represented by darker colors. In the three dimensional figures, the height of the amplitudes also contribute to the clarity of the picture, naturally, the higher the height the higher the amplitude.

Intrinsic mode functions of this signal from HHT are shown in Figure 2. 18. It is seen that the frequencies in the signal are separated appropriately. Figure 2. 19 shows the three-dimensional time-frequency-amplitude distribution correctly with HHT method. Two-dimensional spectrogram is shown in Figure 2. 20. This figure is basically the projection of Figure 2. 20 over the time-frequency plane. Figure 2. 21 is another two-dimensional representation type for HHT. It shows the change in frequency with time for the first two modes. Nevertheless, it does not provide the amplitude change in this time varying frequency. Although HHT method’s power spectral amplitude plot clearly gives the fluctuation of the lower frequencies, it provides a higher amplitude value for the high frequency (15Hz) component in the signal.
Time-frequency-spectral amplitude relation resulting from WT is shown in Figure 2.22. Even though the frequency resolution is not perfect at 15 Hz it is captured and the actual signal’s frequency is fluctuating around 1 Hz. WT gives a good resolution for low frequencies, 15 Hz is a relatively high frequency in strong ground motions, therefore WT can be used as a tool in estimating the time-frequency-energy content of an earthquake signal.

![Hypothetical Signal 2](image-url)

Figure 2.14: Hypothetical Signal 2.
Figure 2.15: Power Spectra of “Hypothetical Signal 2” with FT, WT, STFT, and HHT.
Figure 2.16: Three-dimensional spectrogram or time-frequency-amplitude distribution for “Hypothetical Signal 2” by STFT method.

Figure 2.17: Two-dimensional spectrogram of “Hypothetical Signal 2” using STFT.
Figure 2. 18: Intrinsic mode functions of “Hypothetical Signal 2” by HHT method.

Figure 2. 19: Three-dimensional spectrogram or time-frequency-amplitude distribution of “Hypothetical Signal 2” by HHT method.
Figure 2.20: Two-dimensional spectrogram by HHT method.

Figure 2.21: Time-frequency distribution of the intrinsic mode functions of “Hypothetical Signal 2” by HHT method.
Figure 2.22: Time-frequency-amplitude distribution of “Hypothetical Signal 2” by WT method.

Figure 2.23: Two-dimensional spectrogram of “Hypothetical Signal 2” by WT method.
Finally, to show that the importance of selection of a proper mother wavelet specific to
the application, the marginal wavelet amplitude spectrum, three and two-dimensional
spectrograms obtained using Haar mother wavelet are shown in Figure 2.24, Figure 2.25, and
Figure 2.26, respectively. The high frequency content is not captured by the Haar wavelet. If
the mother wavelet is chosen properly, there is no reason for not to use Wavelet Analysis
Method in determining the characteristics of nonlinear and nonstationary earthquake signals.

![Figure 2.24: Power Spectra of “Hypothetical Signal 2” with FT, WT(Haar), STFT, and HHT.](image-url)
Figure 2.25: Three-dimensional spectrogram or time-frequency-amplitude distribution of “Hypothetical Signal 2” by WT method using Haar wavelet.

Figure 2.26: Two-dimensional spectrogram by WT method using Haar Wavelet for “Hypothetical Signal 2”.

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2.3.3. Earthquake Ground Motion

In the following example, a real earthquake record (Chi-Chi 1999, TCU129 station NS acceleration component) is studied by the methods above. The time history of the record is given in Figure 2.27. Following the sequence used in the other examples, first the power spectral distributions of the signal according to each method are presented in Figure 2.28. Only HHT gives a different spectrum, it shows a very high peak at 0.15 and 20 Hz frequencies. WT gives several peaks at 0.2Hz, 1.5Hz, 2Hz, 4Hz, and 15Hz. STFT spectrum is rather flat, it gives almost the same amplitudes for the frequencies around 5Hz.

Time-frequency-energy density distribution using STFT is shown in Figure 2.29. Energy density term is used in these figures, because form now on since will deal with earthquake motions we will be talking about the energy distribution at different frequency levels. Figure 2.30 is the two dimensional view of the three dimensional representation on the time-frequency plane. It is seen from these two figures that until 5th second there is only 15Hz, and at 5th second there are frequencies between 7.5 Hz and 13 Hz. From 7th second to 36th second, the frequencies are pretty scattered around 5 Hz. They are very scattered that the fluctuation occurs between 0.005 Hz to 9 Hz.

HHT representation of time-frequency-energy distribution of the same ground motion record is different from STFT representation. It is seen from Figure 2.31 that there is unrealistic high energy content at very low frequencies at the two ends of the time axis (the beginning and the end of the signal). Although the general appearance of the two
dimensional HHT spectrogram (Figure 2.32) is similar to the one with STFT, the above statement about unrealistic high energies at very low frequencies on the two extreme ends of the time axis is noticed easily in the HHT spectrogram. HHT method becomes unstable for some large data sets; indeed it is still a study subject for the people who developed this method. Furthermore it seems that the Matlab code written according to the HHT algorithm, due to the vague criteria on where to stop the sifting process and how to handle the signal end swings, works fine for one signal but does not work for another so well.

Three dimensional time-frequency-energy density distribution with WT is shown in Figure 2.33. Two-dimensional frequency-time spectrogram is given Figure 2.34. These two figures show results similar to the other methods’ results, but the picture is clearer, easier to interpret. There are no additional unrealistic high energies at either very low or very high frequencies, which is a good property of the WT method. At 5th second, energy at 8 Hz seems high, at 7th second 3 Hz and 4 Hz are dominant. From 10th second to 20th second, the central frequency is between 4 Hz and 7 Hz, and from 20th second to 30th second, although slighter in the energy, 5 Hz frequency is apparent.
Figure 2.27: Chi-Chi 1999 Earthquake TCU129 NS acceleration component.

Figure 2.28: Power Spectra of an earthquake record (Chi-Chi 1999, TCU-NS component) with FT, WT, STFT, and HHT.
Figure 2.29: Three-dimensional spectrogram or time-frequency-amplitude distribution for the earthquake record (Chi-Chi 1999, TCU-NS component) by STFT method.

Figure 2.30: Two-dimensional spectrogram of the earthquake record (Chi-Chi 1999, TCU-NS component) using STFT.
Figure 2.31: Three-dimensional spectrogram or time-frequency-amplitude distribution for the earthquake record (Chi-Chi 1999, TCU-NS component) by HHT method.

Figure 2.32: Two-dimensional spectrogram of the earthquake record (Chi-Chi 1999, TCU-NS component) using HHT.
Figure 2.33: Three-dimensional spectrogram or time-frequency-amplitude distribution for the earthquake record (Chi-Chi 1999, TCU-NS component) by WT method.

Figure 2.34: Two-dimensional spectrogram of the earthquake record (Chi-Chi 1999, TCU-NS component) using WT.
As explained in section 2.2.5.4, energy accumulated at different wavelet frequency bands can be calculated according to Equation 2.45. Snapshots of cumulative energy with respect to time at several sample frequency levels for the earthquake motion in this last example are shown in Figure 2.35. This type of figure may be utilized to find the energy accumulation rates of different frequencies (in reality, “accumulation” corresponds to energy release from an earthquake at a particular site at a particular direction). For example, in this figure, it is seen that the largest portion of the energy release at 10 Hz frequency occurs between 5th second and 10th second. Between 10th second and 35th second, although there is still energy release due to 10 Hz, it is seen that 5 Hz central frequency is dominant from 17th second until 40th second.

If we sum up all these separate cumulative energies on frequencies, we can plot the total cumulative energy graph. This graph shows the total energy at an instant due to all the frequency components in the signal. The shape of this graph is the same as the plot of square of the acceleration time history with respect to time.

2.4. Summary and Conclusion

In this chapter, various methods available to analyse nonstationary signals are studied and their relative merits are compared. Particularly, FT, STFT, HHT and WT methods are compared through analysis of three signals. First one is a signal with different stationary frequency contents at different times. Second signal has a slowly varying low frequency and a constant high frequency. The third signal is an earthquake recording from Chi-Chi Taiwan.
1999 Earthquake. A suddenly changing time-varying frequency content is observed in earthquake recordings. This change in frequency cannot be defined as “slowly varying”. Even though HHT is a very promising method for the description of the frequency content of signals with a slowly varying frequency content, Wavelet Transform Method with a modified version of Littlewood-Paley mother wavelet is found most suitable to analyze the nonstationary characteristics of the seismic ground motion. A brief summary providing the main advantages and disadvantages of the methods compared here is provided in Table 2.1.

The processes governing the response and instability of soil sites are nonlinear and affected by the nonstationarity of the ground motions. Earthquake hazard mitigation requires studies in many areas including geotechnical aspects of earthquake engineering. A proper definition of the design ground motion time history is very important for structural engineers. Symptoms of soil nonlinearity include decreased spectral ratios of surface to input motion near the dominant frequency of the soil, decreased statistical uncertainty in prediction of peak acceleration, and increased effective period of surface motion (Yu et. al., 1992). In the following chapter WT analysis will be used to identify some of the nonlinear and nonstationary characteristics of the ground motions recorded during some important earthquakes, namely Northridge-California 1994, Kocaeli-Turkey 1999, and Chi-Chi-Taiwan, 1999.
Figure 2.35: Cumulative energy density snapshots at different frequency levels.

Table 2.1: Comparison of signal analysis methods.

<table>
<thead>
<tr>
<th>Analysis Method</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
</table>
| **FT**          | 1) Good frequency information  
2) Very suitable for stationary signals | 1) Time information is lost  
2) Basis functions are sinusoidal  
3) Spurious high frequencies |
| **STFT**        | Some temporal frequency information is available | 1) Time and frequency resolution depends on the width of the windowing function  
2) The selected window and its length should be |
| **HHT**         | 1) Time-frequency info available  
2) No basis functions needed | 1) Spurious low frequencies depending on the signal  
2) The implementation is very signal dependent and subjective  
4) Can be unstable for long data set |
| **WT**          | 1) Time-frequency info available  
2) No spurious frequencies  
3) Sound analytical description | 1) Mother wavelet selection is important  
2) Low frequency resolution for high frequencies and low time resolution for low frequencies |
2.5. References


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CHAPTER 3

WAVELET ANALYSIS OF GROUND MOTIONS RECORDED DURING EARTHQUAKES

3.1. Introduction

In Chapter 2, the Wavelet method was found to be quite effective for analyzing the nonstationary time series. In this chapter, only the wavelet analysis is used to investigate the characteristics of some of the ground motions recorded during Northridge, Kocaeli and Chi-Chi earthquakes and it is found useful as it allows the study of ground motion simultaneously in both time and frequency domains. Earthquake motion records are nonstationary time series. The nonstationarity exists both as the variation of the intensity of the ground acceleration with time and the variation of the frequency content with time. These nonstationarities may be attributed partly to the different intensity, frequency content and arrival times of the P-wave, S-wave, and surface wave groups, and partly due the finite rupture duration and finite fault area. Temporal frequency-energy distribution, amplitude and frequency content nonstationarities in ground acceleration histories have a significant effect on the dynamic response of structures. Papadimitriou (1990) stated that temporal nonstationarity in the frequency content of the ground motion influences the response of both linear and nonlinear structures.
The wave propagation hazard for a particular site is characterized by the peak ground motion parameters (acceleration and/or velocity) as well as the appropriated propagation velocity (Chen and Scawthorn, 2003). Two different types of waves are propagated from a disturbance source in the Earth; body waves and surface waves. Body waves propagate through the earth, while surface waves travel along the ground surface. Body waves are generated by seismic faulting while surface waves are produced by the reflection and refraction of body waves at the ground surface. Body waves include P-waves (compressional waves) and S-waves (shear waves). The ground motion moves parallel to the direction of propagation in compressional waves. This movement generates alternating compressional and tensile strain. For S-waves the ground moves perpendicular to the direction of propagation. Rayleigh and Love waves are the two main types of surface waves. Because of the nature of the interactions, surface waves are more significant at distances further from the source of the earthquake (Kramer, 1996). The speed at which body waves travel varies with the stiffness of the materials they travel through. Since geologic materials are stiffest in compression, P-waves travel faster than any other seismic waves and therefore they are the first waves which arrive at a site. Fluids, which have no shearing stiffness, cannot bear S-waves. The arrival time and the intensity of each wave type are dependent on the source mechanism, the distance between the source, and the complexity of the earth structure.

High frequency seismograms reflect the random inhomogeneities in the earth. For local earthquakes, in which the motions are recorded usually at distances of less than 100-200 km, “high frequency” means higher than 1 Hz (Sato and Fehler, 1998). Most of the traditional
networks record earthquake signals in the range of 1-30 Hz. Recording of frequencies higher than 30 Hz requires the placement of the seismic sensor in a borehole at depths below the highly attenuating surface layers. In this study, above 1Hz is accepted as high frequency.

The specific objective here is to use the WT for studying the influence of site distance and soil softening on ground motions with respect to the nonstationarity associated with both their amplitude and frequency. The characteristics of the ground motions recorded at varying distances from the fault are studied, and effects of site distance on these characteristics are identified. The differences in the characteristics of the motions at non-liquefied sites and of those at or near the liquefied sites are also studied.

3.2. Earthquake Motions Selected For the Study

In this study ground motions recorded during the earthquakes of Northridge 1994, Kocaeli 1999, and Chi-Chi 1999 are analysed. The Northridge, California, earthquake (M = 6.7; Ms = 6.8) of January 17, 1994 was the most damaging earthquake ever to strike the United States. The Northridge earthquake was recorded by 65 stations of the Los Angeles Strong Motion Array, at epicentral distances between 2 km and 85 km, and at various azimuths from the source (USC). The Kocaeli earthquake with a magnitude of Mw=7.4 hit a very crowded and industrialized city of Turkey, Adapazari, on August 17, 1999. It was originated on the 1400 km long North Anatolian Fault zone. Damage by this earthquake was very huge. The Chi-Chi earthquake (Mw=7.6) of September 21, 1999 in Taiwan was triggered by reactivation of the
pre-existing Che-Lung-Pu fault, and generated a rupture more than 105 km in length. The epicenter is located at 120.810E and 23.850N, near the town of Chi-Chi, Nantou County.

Three ground motion pairs (three stations are located near to the fault rupture, and three are far away from it) at three different site conditions have been selected from each earthquake to investigate the effect of the distance between the site and the fault. Near fault stations are selected to be within 30km from the fault (Rosenblad, 2003). Two motion pairs have been selected from each earthquake to observe any significant changes between the non-softened and softened site ground motion records. The reason for the lesser number of records for non-softened/softened site comparisons is that, there was not enough ground motion records to compare. In some cases, there are more liquefied region motions than non-softened region motions, and in some cases, as is true in Kocaeli Earthquake, even though there is clear evidence of liquefaction in some areas; there is no record on those same areas. USGS classifies the sites based on the average shear wave velocity (Vs) to a depth of 30m. Vs>750 m/sec for site type A, 360<Vs<750m/s (stiff) for site type B, and 180<Vs<360m/s (medium stiff) for site type C (soft). The motions have been collected from the PEER Strong Motion Database programmed and distributed by The National Information Service for Earthquake Engineering and sponsored by the National Science Foundation, USA, and COSMOS Virtual Data Center. They were processed in a uniform manner, with individual components individually filtered. Only one component, which has the largest peak acceleration, has been chosen out of all three components (horizontal NS, horizontal EW, vertical) for each motion. Detailed information about all these motions (site coordinates, site
classification, PGA, PGV, PGD, distance to epicenter, distance to fault rupture, and liquefaction occurrence information) is summarized in Table 3.1 (for distance effects analysis) and Table 3.2 (for softening analysis). In the distance effect section, the sites are chosen from the same USGS class. In the softened-non-softened motions category the effect of USGS site class seems to be uncorrelated whether a site experienced softening or not, meaning that soils at any site class can liquefy under certain circumstances. Therefore, earthquake record selection criteria are not dominated by site class in this case. The motions are selected based on the information on whether any soil softening had been reported at a specific site, and based on the available nearest non-softened stations to sites which had softened during the earthquakes. The reason for this type of analysis is to see the fundamental differences between the characteristics of the motions in non-softened and softened sites. The schematic view of the locations of the selected motions is seen in Figure 3.1.
Bedrock motion

SD (Short Distance)

Softened

Non-softened

Liquefiable region

 Fault

LD (Long Distance)

Rock outcropping motion

Figure 3.1: Schematic view of the locations of the selected motions for the study.
Table 3.1: Information for the earthquake ground motions for distance effects analysis.

<table>
<thead>
<tr>
<th>EARTHQUAKE</th>
<th>MECHANISM</th>
<th>STATION</th>
<th>SHORTEST DISTANCE TO THE RUPTURE (km)</th>
<th>PGA from CORRECTED DATA (g)</th>
<th>LATITUDE</th>
<th>LONGITUDE</th>
<th>SOIL TYPE (USGS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>NS</td>
<td>UD</td>
<td>EW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NORTHRIDGE</td>
<td>Reverse Normal</td>
<td>PUL</td>
<td>11.7</td>
<td>1.585</td>
<td>1.229</td>
<td>1.285</td>
<td>34.334</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CUC</td>
<td>80.6</td>
<td>0.071</td>
<td>0.025</td>
<td>0.051</td>
<td>34.209</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ARL</td>
<td>9.5</td>
<td>0.344</td>
<td>0.552</td>
<td>0.308</td>
<td>34.209</td>
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<tr>
<td></td>
<td></td>
<td>FEA</td>
<td>81.3</td>
<td>0.104</td>
<td>0.024</td>
<td>0.1</td>
<td>34.209</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NWH</td>
<td>10.9</td>
<td>0.583</td>
<td>0.548</td>
<td>0.59</td>
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<tr>
<td></td>
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<td>MJH</td>
<td>88.5</td>
<td>0.037</td>
<td>0.027</td>
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<td>35.07</td>
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<tr>
<td>KOCAELI</td>
<td>Strike Slip</td>
<td>IZT</td>
<td>5</td>
<td>0.152</td>
<td>0.146</td>
<td>0.22</td>
<td>40.790</td>
</tr>
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<td></td>
<td>MSK</td>
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<td>0.03</td>
<td>0.04</td>
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<td>ARC</td>
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<td>0.068</td>
<td>41.065</td>
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<tr>
<td></td>
<td></td>
<td>YPT</td>
<td>4.4</td>
<td>0.268</td>
<td>0.242</td>
<td>0.349</td>
<td>40.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ATK</td>
<td>67.5</td>
<td>0.105</td>
<td>0.064</td>
<td>0.164</td>
<td>40.989</td>
</tr>
<tr>
<td>CHI-CHI</td>
<td>Reverse Normal</td>
<td>TCU046</td>
<td>16.5</td>
<td>0.116</td>
<td>0.104</td>
<td>0.133</td>
<td>24.4683</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KAU069</td>
<td>80.9</td>
<td>0.036</td>
<td>0.019</td>
<td>0.039</td>
<td>22.8873</td>
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<td></td>
<td></td>
<td>TCU128</td>
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<td>0.17</td>
<td>0.097</td>
<td>0.139</td>
<td>24.4162</td>
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<td></td>
<td></td>
<td>TCU008</td>
<td>77.5</td>
<td>0.062</td>
<td>0.025</td>
<td>0.071</td>
<td>25.0092</td>
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<td></td>
<td></td>
<td>TCU049</td>
<td>3.3</td>
<td>0.251</td>
<td>0.171</td>
<td>0.293</td>
<td>24.179</td>
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<td></td>
<td>TCU007</td>
<td>78.8</td>
<td>0.071</td>
<td>0.028</td>
<td>0.06</td>
<td>25.0015</td>
</tr>
</tbody>
</table>

NS: North-south component, EW: East-west component, UD: Up-down component.
<table>
<thead>
<tr>
<th>EARTHQUAKE</th>
<th>MECHANISM</th>
<th>STATION</th>
<th>SHORTEST DISTANCE TO THE RUPTURE (km)</th>
<th>LIQUEFACTION</th>
<th>PGA from CORRECTED DATA (g)</th>
<th>LATITUDE</th>
<th>LONGITUDE</th>
<th>SOIL TYPE (USGS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NS</td>
<td>UD</td>
<td>EW</td>
<td></td>
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<tr>
<td>CHI-CHI</td>
<td>Reverse Normal</td>
<td>TCU070</td>
<td>19.1</td>
<td>NO</td>
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<td>0.085</td>
<td>0.255</td>
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<td>TCU099</td>
<td>17.84</td>
<td>YES</td>
<td>0.172</td>
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<td></td>
<td>TCU129</td>
<td>1.18</td>
<td>NO</td>
<td>0.634</td>
<td>0.341</td>
<td>1.01</td>
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<td></td>
<td>TCU076</td>
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<td>YES</td>
<td>0.416</td>
<td>0.281</td>
<td>0.303</td>
<td>23.91</td>
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<td>Strike Slip</td>
<td>ATK</td>
<td>62.3</td>
<td>NO</td>
<td>0.105</td>
<td>0.064</td>
<td>0.164</td>
<td>40.989</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ATS</td>
<td>78.9</td>
<td>YES</td>
<td>0.249</td>
<td>0.079</td>
<td>0.184</td>
<td>40.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SKR</td>
<td>3.1</td>
<td>NO</td>
<td>0.376</td>
<td>0.259</td>
<td>N/A</td>
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</tr>
<tr>
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<td></td>
<td>YPT</td>
<td>4.4</td>
<td>YES</td>
<td>0.268</td>
<td>0.242</td>
<td>0.349</td>
<td>40.45</td>
</tr>
<tr>
<td>NORTHRIDGE</td>
<td>Reverse Normal</td>
<td>Station 21</td>
<td>26.46</td>
<td>NO</td>
<td>0.42</td>
<td>0.08</td>
<td>0.33</td>
<td>34.082</td>
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<td></td>
<td></td>
<td>Station 17</td>
<td>20.1</td>
<td>YES</td>
<td>0.15</td>
<td>0.09</td>
<td>0.01</td>
<td>34.114</td>
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<td>NWH</td>
<td>4.5</td>
<td>NO</td>
<td>0.583</td>
<td>0.548</td>
<td>0.59</td>
<td>34.387</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SYL</td>
<td>3.6</td>
<td>YES</td>
<td>0.604</td>
<td>0.535</td>
<td>0.843</td>
<td>34.326</td>
</tr>
</tbody>
</table>

NS: North-south component, EW: East-west component, UD: Up-down component.
3.3. Effect of the Distance between Site and the Fault on the Characteristics of the Recorded Motions

The WT analysis is applied to all of the motions in Table 3.1. Short distance and long distance motions are denoted by “SD” and “LD” in the figures, respectively. Northridge, Kocaeli, and Chi-Chi Earthquakes are studied individually, some figures are provided in the main text and all other supporting figures obtained by the analysis are provided in Appendix 3A. These figures are: Time histories, marginal wavelet spectrum time-frequency-energy density distributions of the short and long distance motions.

Worldwide experience repeatedly shows that damages in structures caused by earthquakes are highly dependent on site condition and epicentral distance. Rathje, et. al., (2003) developed event-specific attenuation relationships for the Kocaeli earthquake. These relationships show considerable amplification of peak ground acceleration and spectral acceleration (at a period of 0.3 s) at deep soil sites in the far field, but no amplification in the near-fault region. For spectral accelerations at longer spectral periods (1.0 and 2.0 s), amplification is indicated in both the near field and far field. Amplification factors derived from the Kocaeli earthquake strong motion data are generally larger than those used in current attenuation relationships and building codes. General observation for the effect of site distance is usually a significant energy decrease and shift to lower frequencies at the far sites. However, although the short duration acceleration spikes that have high peak accelerations which cause high frequencies are typical of the near-
fault region, they can occur at sites located at large distances from the epicenter (Stewart et. al., 2001).

3.3.1. Northridge Earthquake

The records at Pacoima Dam (PUL) station as the near fault motion and CUC station as the far fault motion have been selected to represent Class A sites (Figure 3.2). Motion ARL (Short-distance) and FEA (Long-distance) is a pair of motion from type B site. For C type sites, NWH (Short-distance) and MJH (Long-distance) motions are selected for this study.

![Graph](image.png)

Figure 3.2: Acceleration time history of the Northridge earthquake motions at stations PUL and CUC.
For Site A: Figure 3.3 and Figure 3.4 show the energy distribution with respect to time and frequency at the near site station PUL and far site station CUC, respectively. It is seen that total energy drops at the long distance record. The time when the energy is contained at the SD and LD records is very different from each other (Between 0-5 seconds at the SD and between 15-30 seconds at the LD). In Figure 3.3, it is seen that frequencies between 2Hz and 4Hz are effective in the SD record. In Figure 3.4 although 5Hz is more significant than the other frequencies in the time interval between 21st second and 25th second at the LD site, energy around 2.7604 Hz accumulates with time slightly more than 5Hz component does as seen in the marginal wavelet spectrum (Figure 3.5). Figure 3.6 is plotted to show the temporal movement and energy of the dominant frequency for both near site and far site motions. In order to draw this graph, first the dominant central frequency of the SD motion is found from the MWS (3.4 Hz) and the energy density time history which corresponds to that frequency is plotted. Then, at the same frequency, the energy density time history for LD record is plotted. Then the dominant frequency at the LD record is found and the energy density-time history is plotted at that frequency (2.8 Hz) for both SD and LD records. This type of figure may be representative of dominant frequency shifts in time for both SD and LD motions, however; for when comparing near site and far site records the total energy decrease is drastic, therefore the change usually cannot be seen on the same plot. But, with the help of the three dimensional time-frequency-energy graphs, the time interval where the dominant frequency of the LD motion can be seen easily.
Figure 3.3: Time-frequency-energy distribution at PUL.

Figure 3.4: Time-frequency-energy distribution at CUC.
Figure 3.5: Marginal Wavelet Spectra at PUL and CUC stations from Northridge Earthquake.

Figure 3.6: Energy variation at fundamental frequencies at PUL and CUC records.
With the help of Figure 3.3, Figure 3.4, Figure 3.6, Figure 3.7, Figure 3.8 and all other time-frequency-energy distribution figures related to Northridge earthquake records for B and C class sites presented in Appendix 3.A, Table 3.3 is prepared for comparison of observations at different site classes in near site (short distance, SD) and far site (long distance, LD) ground motion records.

**Table 3.3: Northridge earthquake, effects of site class on the SD and LD earthquake records.**

<table>
<thead>
<tr>
<th>Site Class</th>
<th>Record Type</th>
<th>Frequency (Hz)</th>
<th>Time Interval (sec)</th>
<th>Energy Level</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A (stiff)</strong></td>
<td>SD*</td>
<td>3.41</td>
<td>4-7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>3.41</td>
<td>21-24</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>2.76</td>
<td>2.5-12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD*</td>
<td>2.76</td>
<td>15-30</td>
<td>Decreased</td>
</tr>
<tr>
<td><strong>B (medium)</strong></td>
<td>SD*</td>
<td>1.46</td>
<td>2-7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>1.46</td>
<td>10-15</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>2.42</td>
<td>5-12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD*</td>
<td>2.42</td>
<td>14-17</td>
<td>Decreased</td>
</tr>
<tr>
<td><strong>C (soft)</strong></td>
<td>SD*</td>
<td>1.48</td>
<td>3-11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>1.48</td>
<td>15-28</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.07</td>
<td>4-10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD*</td>
<td>3.07</td>
<td>14-16</td>
<td>Decreased</td>
</tr>
</tbody>
</table>

* Frequency is dominant in marked site.

In all of the cases the energy decrease is very obvious. In spite of this energy decrease there are frequency shifts at LD records. For site class A, there is a frequency downshift. The lower frequency is dominant at this site and the duration is prolonged. Downshift is defined as the shift of the peak frequency of the spectrum (Huang et. al., 1999). Lake et al (1977) attributed the
Figure 3.7: Snapshots of energy distribution at fundamental frequencies at ARL and FEA records with respect to time.

Figure 3.8: Snapshots of energy distribution at fundamental frequencies at NWH and MJH records with respect to time.
downshift (to lower frequencies) phenomenon in water waves to the dissipation effects. For site class B and C, the higher frequency is dominant at the LD site (with a very weakened energy). The duration of this higher frequency is short (2-3 sec) in these cases.

3.3.2. Kocaeli Earthquake

IZT and MSK stations’ records constitute a motion pair from Kocaeli Earthquake for class A sites. For soil class B, ARC and MCD motions were selected for comparison of near and far fault motions, respectively. Time histories of fault parallel components are used. For soil class C in Kocaeli Earthquake, YPT (SD) and ATK (LD) motions have been selected.

For Site A: It is noted that the record length of near site motion is one fourth of that of the far fault motion. Previous studies, based on rock recordings in both eastern and western Canada, have revealed a clear and steady increase in this duration with distance, even at distances within a few tens of kilometers of the earthquake source (Atkinson, 1993, 1995). Therefore, it is normal for Kocaeli earthquake long distance motions to have longer durations.

With the help of Figure 3.9, Figure 3.10, Figure 3.11 and the time-frequency-energy distributions in Appendix 3.A, the observations for this earthquake at all site classes are compiled in Table 3.4. In this table, it is seen that for site class A, there is a frequency downshift and energy of the low frequency is prolonged. For site class B, there is a very slight increase in frequency at the LD site, but this frequency is also a very low frequency. It is seen that the duration of this frequency is shortened at LD record. For site class C, a higher frequency is
dominant at LD. Different from the observation at Northridge C class site and Kocaeli B class, the duration of this increased frequency is longer at the LD site.

Figure 3.9: Snapshots of energy distribution at fundamental frequencies at IZT and MSK records with respect to time.

Figure 3.10: Snapshots of energy distribution at fundamental frequencies at ARC and MCD records with respect to time
Figure 3.11: Snapshots of energy distribution at fundamental frequencies at YPT and ATK records with respect to time.

Table 3.4: Kocaeli earthquake, effects of site class on the SD and LD earthquake records.

<table>
<thead>
<tr>
<th>Site Class</th>
<th>Record Type</th>
<th>Frequency (Hz)</th>
<th>Time Interval (sec)</th>
<th>Energy Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>SD*</td>
<td>1.98</td>
<td>2-19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>1.98</td>
<td>25-26</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.31</td>
<td>2-18</td>
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<tr>
<td></td>
<td>LD*</td>
<td>0.31</td>
<td>15-35</td>
<td>Decreased</td>
</tr>
<tr>
<td>B</td>
<td>SD*</td>
<td>0.21</td>
<td>3-27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>0.21</td>
<td>20-40</td>
<td>Decreased</td>
</tr>
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<td>30-32</td>
<td>Decreased</td>
</tr>
<tr>
<td>C</td>
<td>SD*</td>
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<td>2-30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>0.31</td>
<td>N/A</td>
<td>Decreased</td>
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<tr>
<td></td>
<td>SD</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>LD*</td>
<td>1.04</td>
<td>25-40</td>
<td>Decreased</td>
</tr>
</tbody>
</table>

* Frequency is dominant in marked site
3.3.3. Chi-Chi Earthquake

The first pair of motions (TCU046, 16.5 km, and KAU069, 80.9 km) is in EW direction. The site class is A according to USGS. For site class B of Chi-Chi Earthquake, motion pair of TCU128 and TCU008 was chosen. Class C site motions TCU049 and TCU007 are selected.

Using Figure 3.12, Figure 3.13, Figure 3.14, and the time-frequency-energy distribution figures in Appendix 3.A, Table 3.5 is constructed to summarize the results observed in Chi-Chi earthquake.

![Figure 3.12: Snapshots of energy distribution at fundamental frequencies at TCU046 and KAU069 records with respect to time.](image)

Figure 3.12: Snapshots of energy distribution at fundamental frequencies at TCU046 and KAU069 records with respect to time.
Figure 3.13: Snapshots of energy distribution at fundamental frequencies at TCU128 and TCU008 records with respect to time.

Figure 3.14: Snapshots of energy distribution at fundamental frequencies at TCU049 and TCU007 records with respect to time.
Table 3.5. Chi-Chi earthquake, effects of site class on the SD and LD earthquake records.

<table>
<thead>
<tr>
<th>Site Class</th>
<th>Record Type</th>
<th>Frequency (Hz)</th>
<th>Time Interval (sec)</th>
<th>Energy Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>SD*</td>
<td>0.104</td>
<td>5-60</td>
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</tr>
<tr>
<td></td>
<td>LD</td>
<td>0.104</td>
<td>N/A</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>1.25</td>
<td>25-40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD*</td>
<td>1.25</td>
<td>40-70</td>
<td>Decreased</td>
</tr>
<tr>
<td>B</td>
<td>SD*</td>
<td>0.21</td>
<td>30-50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>0.21</td>
<td>60-78</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.21</td>
<td>the same</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD*</td>
<td>0.21</td>
<td>the same</td>
<td>Decreased</td>
</tr>
<tr>
<td>C</td>
<td>SD*</td>
<td>0.104</td>
<td>10-55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>0.104</td>
<td>20-70</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.83</td>
<td>30-40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LD*</td>
<td>0.83</td>
<td>28-56</td>
<td>Decreased</td>
</tr>
</tbody>
</table>

* Frequency is dominant in marked site

Contrary to the observations at Northridge and Kocaeli class A sites, there is an increase in frequency at the LD site. The duration of this increased frequency is longer at the LD site than it is at the SD site. For site class B, there is no change in frequency at the LD site, but the duration of the frequency is shortened. For site class C, a higher frequency is dominant at LD. Different from the observation at Northridge C class site and Kocaeli B class, the duration of this increased frequency is a little shorter at the LD site.

To summarize this section; with the wavelet analysis, the differences between the strong motions, recorded at near and far sites can be revealed. The common observation is that the energy of all frequency components of the earthquake motions at long distances from the fault rupture diminishes drastically. In Northridge and Kocaeli earthquakes, at site class A there is
frequency downshift whereas in Chi-Chi earthquake the opposite is observed. When there is frequency downshift at the far sites, the duration of this reduced frequency is almost always prolonged. For site class B and C in all earthquakes, there is either frequency increase or no change in frequency at far sites. Except for the Kocaeli earthquake, duration of these increased frequencies are shorter at the far sites than that of the near sites.

It has been recognized that local site conditions affect the amplitude and frequency content of the seismic ground time histories (Theodulidis and Papazachos, 1992). Generally, at the near sites, the most significant amount of energy of the seismic motions at softer soils is concentrated at the lower frequencies that control the displacements. In this sense, the results of the present study in terms of the local site conditions are in agreement with already known local site effects.

3.4. The Effect of Softening (Due to Pore Pressure Built Up) On the Characteristics of Recorded Motions

3.4.1. Liquefaction and Softening

Liquefaction occurs when saturated, cohesionless materials are transformed from a solid to a near-liquid state. This phenomenon occurs when moderate to severe seismic ground shaking causes pore-water pressure to increase and approach the confinement pressures. It is a highly nonlinear physical phenomenon which is characterized not only by the softening of saturated sands as pore pressure increases, but also by the ejection of water from the soil and associated settlement and lateral movement (Ishihara, 1993; NRC, 1985). Site susceptibility to liquefaction
is a function of the depth, density, and water content of granular sediments, along with the magnitude and frequency of earthquakes in the surrounding region. Softening is the degradation of site stiffness prior to liquefaction. Saturated, unconsolidated silts, sands, and silty sands within 50 feet (15.2 meters) of the ground surface are most susceptible to liquefaction. Liquefaction-related phenomena include lateral spreading, ground oscillation, flow-failures, loss of bearing strength, subsidence, and buoyancy effects.

Nemat-Nasser and Shokooh (1979) introduced the energy concept for the analysis of densification and liquefaction of cohesionless soils. It is based on the idea that during deformation of these soils under dynamic loads, some part of the energy is dissipated into the soil. This dissipated energy is represented by the area of the hysteric shear strain-stress loop and could be determined experimentally. The accumulated dissipated energy per unit volume up to liquefaction considers both the amplitude of shear strain and the number of cycles. Compared with other methods to evaluate liquefaction potential of soils, the energy approach is easy to deal with random loading because the amount of dissipated energy per unit volume for liquefaction is not dependent on the form of loading. Berill et al. (1985), Law et al. (1990), Figueroa (1990), Figueroa et al (1991, 1994) established relationships between pore pressure development and the dissipated energy during the dynamic motion to explore the use of the energy concept in the evaluation of the liquefaction potential of a soil deposit.

Zorappapel and Vucetic (1994) calculated Fourier spectra from ground motions measured at WLA during the 1987 Superstition Hills earthquake (M=6.6) for various time increments. From
these, they noted an increase in the “first characteristic period” (or a decrease in the first characteristic frequency) of the ground motions as a function of time or increased pore water pressure. They also noted reduction in amplitude of short-period (high frequency) ground motions (<0.5 s) as pore pressures increased. In other studies, the effect of soil softening on recorded peak acceleration and damage during the 1994 Northridge earthquake have been investigated (Trifunac and Todorovska, 1996, 1998; Trifunac et. al. 1999). These investigators noted significant reductions of peak acceleration, which is a high frequency characteristic of strong ground motion, on soft sites compared to motions recorded on nearby stiff sites. In the calculated Fourier spectra they also noted shift of peaks toward longer periods as a function of intensity of ground shaking recorded at the same time during the main shock, several after shocks and a few other earthquakes. Todorovska and Trifunac (1999) employed an empirical, energy-based methodology for liquefaction hazard assessment and microzonation mapping. Youd and Carter (2005) analyzed ground motions to provide guidance on the influence of soil softening on ground response and concluded that: 1) soil softening leads to reduced short period (T<1.0 s) spectral accelerations where soil softening occurs early in a strong ground motion sequence, compared to those that would have developed in the absence of softening; 2) soil softening has little influence on short-period (T<1.0 s) spectral response where soil softening occurs late in a ground motion sequence; 3) soil softening and liquefaction may lead to amplification of long period (T>1.0 s) spectral accelerations, particularly if ground oscillation develops.
In this section, site softening effects on strong ground motion characteristics are studied using the motions recorded during Northridge, Kocaeli and, Chi-Chi Earthquakes. It is shown that, in agreement with the previous researchers’ studies, a general energy decrease takes place at the softened sites (with the exception of Kocaeli earthquake). Frequency downshift with a backshift in time is observed in all of the motions. Energy of the lower frequencies is always higher than the energy of higher frequencies in the softened sites.

3.4.2. Northridge Earthquake

The San Fernando Valley is a structural valley in southern California, which is filled with up to 4,500 m of Tertiary sedimentary rock and alluvial sediment (Holzer et al). Holocene age (<10,000 yr) alluvial gravels, sands, and finer sediment with an aggregate thickness ranging from about 8 to 12 m immediately underlie the gently sloping valley floor. This alluvium was deposited by sediment-laden floodwaters that flowed out of the surrounding mountain canyons.

Holzer et. al. (1996) performed a study on the liquefaction and soil failure during 1994 Northridge Earthquake. They conducted detailed subsurface investigations at four sites (Balboa Boulevard, Malden Street, Wynne Avenue, and Potrero Canyon) to determine geotechnical properties of shallow materials and geologic structure. The authors considered several potential mechanisms of ground failure: Liquefaction, Dynamic Failure of Lean Clay, Dynamic Compaction of Dry Material, and Secondary faulting. They found that at three of the sites – Balboa Boulevard, Wynne Avenue, and Potrero Canyon – parts of the Holocene sediment were
susceptible to liquefaction, and should have liquefied and mobilized large shear strains at observed levels of ground shaking, at the fourth site – Malden Street – a low-strength lean clay appeared to have failed under dynamic loading. Despite the lack of direct field evidence of liquefaction, because of the large depth, and small thickness of the liquefied layers and consequent absence of vented material, they concluded that soil failure was primarily caused by soil liquefaction of silty sand. Trifunac and Todorovska (1994) performed a study on the nonlinear soil behavior during 1994 Northridge Earthquake and suggested that the observed reduction of peak accelerations was most probably due to nonlinear soil response.

Two pair of stations used for this earthquake is, USC Stations 21 and 17, and SYL and NWH. The first pair is from Santa Monica Mountains and Hollywood region (Figure 3.15). The second pair is from SYL station at the Joseph Jensen Filtration Plant near Sylmar and at LA Newhall Fire Station. Ground failures at the filtration plant are believed to be related to liquefaction of loose alluvial soils underlying a 12-m-high embankment fill. Liquefaction was observed near stations (17 and SYL). Their time-frequency-energy distribution representations, spectrograms, and cumulative energy graphs are presented in Appendix 3.B. With help of the results for the first pair (Station 21 and Station 17), main observations are discussed in the main body of this chapter.

Figure 3.16 shows the Time-frequency-energy distribution at the non-softened site Station21, it is seen in this figure that the more energy is located around 10sec at frequencies 2.5 Hz and 5 Hz. Figure 3.17 is the same type of figure for the softened site Station17. This figure shows that
overall energy has decreased substantially and seen to be present at lower frequencies (1.5Hz, 2Hz, 3Hz) between 2.5 seconds and 10 seconds. From these two figures, it is concluded that frequency downshift is caused for this pair of motions. The shifting of the dominant frequency is explained by the energy dissipation due to softening of some part of the soil in some sites.

![Graphs of Station 21 and Station 17 acceleration time history](image)

**Figure 3.15**: Acceleration time history of the Northridge earthquake motions at Station 21 and Station 17.

Figure 3.18 is the MWS for the Station 21 and Station 17. As explained in the previous chapter, this figure gives the same information as the FT spectra would give. In Figure 3.19 the above mentioned downshift phenomenon is seen more clearly. The dominant central frequency of the recorded non-softened motion is found from the MWS (2.5 Hz) and the energy density time history which corresponds to that frequency is plotted. Then, at the same frequency, the
energy density time history for softened site is plotted. Figure 3.19 shows that 2.5 Hz is effective around 10th second at the non-softened site, whereas the same frequency is effective at 5th sec at the softened site. It is seen that both non-softened and softened sites have two peaks each at 2.5 Hz. The time intervals between these peaks are very close to each other for both cases, in the non-softened site this interval is 3 seconds, and in the softened site it is 2.5 seconds. It is noted that, there are 5.5 seconds between the peak of 2.5 Hz at the non-softened site and the softened site. This suggests that the softened site is closer to the rupture. Indeed, this is correct, Station17 is 6.46 km closer to the rupture than Station17 is. However this does not effect the observation of softening, because it is obvious that there is a significant energy decrease at this frequency at the softened site. Since this cannot be explained by distance effect, we can attribute this energy decrease to soil softening at the site. After this first step in this figure, the dominant frequency at the softened site is found and the energy density-time history is plotted at that frequency (1.98 Hz). This plot can be called as a snapshot at that frequency. It is seen that 2.5 Hz at is not the dominant frequency at the softened site anymore, instead 1.2 Hz is the dominant frequency. This implies a frequency downshift as suggested by other authors. This downshift is seen in the MWS plot too, but what this new figure reveals is the temporal change in the dominant frequencies at both non-softened and softened sites. Unlike other methods, WT does not require separate FT analyses for different portions of the acceleration time history. At this point, it can be said that due to softening, the fundamental site period increases and this causes the smaller frequencies to be more pronounced at the softened site. Since significant reduction in energy is seen first for 2.5 Hz frequency at 3rd second, the softening might have started at this time point.
Figure 3.16: Time-frequency-energy distribution at Station21.

Figure 3.17: Time-frequency-energy distribution at Station17.
Figure 3.18: Marginal Wavelet Spectra at Stations 21 and Station 17 from Northridge Earthquake.

Figure 3.19: Frequency downshift and energy decrease at fundamental frequencies at Station 21 and Station 17 records with respect to time.
The frequency downshift and energy decrease observations are also valid at the softened site for NWH and SYL stations Figure 3.20. For the non-softened site (NWH) the dominant frequency is 1.5 Hz, and this frequency is located at around 5.5 sec on the time axis. The energy of the same frequency component at the softened site is much less than that of the non-softened site (the reduction is almost 40 times). At the same time the energy of a low frequency component (0.67708 Hz) becomes more visible at the softened site. At the non-softened site this frequency was not significant. This may mean that the softening started around 3rd second at the softened site because this time is where the reduced energy peak at the high frequency occurs for the first time.

Figure 3.20: Frequency downshift and energy decrease at fundamental frequencies at NWH and SYL records with respect to time.
3.4.3. **Kocaeli Earthquake**

Widespread liquefaction-induced ground failures occurred in several localities, including Adapazari, Gölcük, and Sapanca, during the August 17, 1999 earthquake. The combination of the seismic, geologic and geotechnical conditions of these localities controls the occurrence of liquefaction. The thickness of the alluvial deposits reaches around 1100 m depth below the city center of Adapazari. Liquefaction can occur at significant distances from an earthquake source (Ambraseys, 1988). The strong motion network on the North-Anatolian Fault is very sparse (Celebi et al. 2000). Only one record was retrieved from Adapazari (Station SKR), which was on stiff soil in the undamaged part of Adapazari. There were no stations in the urban areas of the Adapazari Basin. The shaking in the basin would have revealed different characteristics such as liquefaction, amplification due to softer media, and basin effects, had there been ground motion records in this region.

The first pair of motion from this earthquake is ATK (non-softened) and ATS (motions). Both of these motions are far away from the source. ATS is a station where significant damage has occurred, due to unusual ground motion amplification. There were also liquefaction observations in the region. At the ATK site there was no damage in the region, and there was no liquefaction. The second pair is SKR (non-softened) and YPT (softened) motions. These motions are from near fault.
Energy time history snapshots at the dominant frequencies of the records ATK and ATS are given in Figure 3.21. The common observation of total energy decrease in the softened site is not seen here due to site amplification. However, energy decrease in high frequencies (frequency downshift) is obvious. It is seen in this figure that the non-softened site’s dominant frequency was 3.6 Hz and softened site’s was 2.1 Hz. Even though ATS was further away from the fault than ATK station was, decreased high frequency energy and increased low frequency energy showed up earlier at the ATS (softened) than ATK. This may be due to the wave propagation path, because the path is not linear as it is assumed to be in general. In large earthquakes such as this one, only one point source assumption is not correct because the rupture process progresses as a series of ruptures along the fault. Waves may emanate from different directions, this is called directivity effect. Constructive interference of these waves may produce large displacements called fling (Benioff, 1955; Ben-Menachem, 1961; Singh, 1985). The 20th second may be the time when the softening started at ATS because it is the point on the time axis where there is a decrease in the high frequency, and an increase in the low frequency.
In Figure 3.22 non-softened site’s (SKR) dominant frequency is 3.6458 Hz where softened site’s (YPT) is 0.7291 Hz. At the non-softened site, 10 sec is the time when 3.6458 Hz is located around. The same frequency first starts to exist (with a 3 times decreased energy) at 7th second until 15th second at the softened site. This 7th second may be the initiation point for the softening at this site. It is noticed that the 0.72917 Hz component covers the time interval between 8th and 12.5th seconds at the softened site and the energy of this component is higher than that of the non-softened site. This observation also, as in the first motion pair, suggests probable site amplification due to the soft thick sediments at the site. Due to softening, part of the earthquake energy is dissipated, but at the same time the fundamental site frequency decreases and this
causes increased energies at reduced frequencies, the amount of increase is dependent on the soil characteristics (depth, relative density etc.) Sedimentary deposits in the form of basins, and in particular deep basins, have a high influence on low frequency ground motions. The strongest effects occur when the earthquake occurs at the basin edge, outside the basin, because entering body waves are trapped by the dipping edge of the basin. Such entrapment of energy results in prolonged duration of low frequency ground motions and in amplification of the amplitudes of velocities and displacements. The location of the earthquake source strongly affects the basin effects at a given site, because such effects are controlled by the geometry of the basin between the site and the earthquake source.
3.4.4. Chi-Chi Earthquake

The liquefaction occurred mainly in the following areas: Nantou, Wufeng, Yuanlin and Taichung. In this segment of the study, we investigate two different sets of motions: the first which are believed to be recorded at sites have suffered softening due to pore water pressure build up, and the other recorded at similar nearby sites but without any softening.

TCU059 is a station near Taichung Harbor in Wufeng, about 55 km northwest of the epicenter. Four piers were out of service immediately after the earthquake due to liquefaction. Major settlement and sand boils were observed. Large surface ground fissures were visible to a
distance of approximately 250 m behind the caisson walls. Water table is roughly estimated at 1.5 m below ground surface (Kayen and Mitchell 1997). The motion TCU070 recorded at a site close to TCU059 has been selected as reference motion. There was no evidence of liquefaction for this station. There is an overall decrease in the energy accumulation at the softened site. One can see the frequency downshift in Figure 3.23 at TCU059 when compared with the TCU070. The dominant frequency at the non-softened site is 1.5 Hz and present mainly between 45\textsuperscript{th} second and 55\textsuperscript{th} second. At the softened site, the energy of this frequency decreases 10 times, and the time at which this frequency is effective shifts back to 35\textsuperscript{th} and 49\textsuperscript{th} seconds. Around 45 and 55 seconds at the softened site, the energy of the low frequency is higher than that of the non-softened site.

Figure 3.23: Frequency downshift and energy decrease at fundamental frequencies at TCU070 and TCU059 records with respect to time.
Evidence of liquefaction in Nantou region during the Chi-Chi earthquake includes sand boils, building subsidence, and lateral spreading. Locations of liquefaction such as sand boils and lateral spreading are mainly along the Mau-Lou River that runs through Nantou and in the Juin-Gong district, which is to the east of downtown Nantou (Stewart et al 2000). The geologic deposit at Nantou generally consists of Holocene alluvial sediments with shallow ground water (2 to 5 m below the ground surface). The area is formed by alluvial sector and deposit plains that are mainly composed of cemented or poorly-cemented clay, silty sand, sand and gravel distributed on valleys and alluvial plateau. TCU076 is the closest ground motion station to the liquefied area. TCU129 is used as the reference motion for TCU076 in this study because there was no evidence of liquefaction around this station and it was close to TCU076.

In Figure 3.24 there is a frequency downshift at TCU076. At TCU129 dominant frequency was 4.2 Hz, whereas it was 0.8 Hz at TCU076. Again, even though these stations are very close to each other (TCU076 being slightly far away from the fault than TCU129), the energy peak (with a reduction) at high frequency at the softened site TCU076 appears earlier than it appears at the non-softened site (35 sec at the non-softened site and 30 sec at the softened site).

3.5. Summary

As a summary, Wavelet analysis was used to study the characteristics of several ground motions recorded during Northridge, Kocaeli and Chi-Chi Earthquakes. Wavelet analysis has potential of providing more insight into ground motion characteristics including the energy content in both frequency and time domain simultaneously. The effect of site distance and site
softening on the ground motion characteristics are studied. The information on redistribution of the energy among various frequency ranges as well as its temporal location was revealed. Following are the conclusions obtained in this chapter.

1) In all of the LD (long distance or far site) cases the energy decrease is obvious. It is seen that the shifting of energy among the frequency bands differs from earthquake to earthquake and within one earthquake from site class to site class. All these observations are presented in tabulated form before in this chapter. Despite the differences, if a general conclusion is to made it is found that: i) there is a frequency downshift with a prolonged and shifted duration for stiff site classes at LD sites (Chi-Chi, Taiwan earthquake case is an exception to this); ii) in all cases for medium stiff and soft sites, there is an energy shift towards a little higher frequency (both SD and LD dominant frequencies may be accepted in the “low” frequency range); iii) The maximum energy at the LD sites are always lower than that of the SD.

2) The effects of softening on the surface site responses are summarized in Table 3.6. According to this table: i) The dominant central frequency is always downshifted in the softened sites, regardless of the name of the earthquake; ii) Duration of the significant energy in the softened sites are shortened and shifted to the early few seconds of the signal; iii) The maximum energy at the LD sites are always lower than that of the SD sites except for one motion pair in Kocaeli, Turkey earthquake).

These observations motivate us to do further investigate the ground motion characteristics using nonlinear ground response analysis which is presented in the next chapter.
Figure 3.24: Frequency downshift and energy decrease at fundamental frequencies at TCU129 and TCU076 records with respect to time.

Table 3.6. Summary of effects of softening on surface site response in different earthquakes.

<table>
<thead>
<tr>
<th>SOFTENING</th>
<th>Site Class</th>
<th>Site</th>
<th>Frequency (Hz)</th>
<th>Time Interval (sec)</th>
<th>Peak Wavelet Energy</th>
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<td></td>
<td></td>
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<td></td>
</tr>
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<td></td>
<td>B</td>
<td>Non-softened</td>
<td>2.5</td>
<td>7-12</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>Softened</td>
<td>1.2</td>
<td>5-10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Non-softened</td>
<td>1.5</td>
<td>4-7.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Softened</td>
<td>0.68</td>
<td>0-12</td>
<td>Decreased</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Non-softened</td>
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<td>30-33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D</td>
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<td>20-25, 28-30</td>
<td>Increased</td>
</tr>
<tr>
<td></td>
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<td>Non-softened</td>
<td>3.65</td>
<td>9-11</td>
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<tr>
<td></td>
<td>C</td>
<td>Softened</td>
<td>0.73</td>
<td>8-12.5</td>
<td>Increased</td>
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<td>Kocaeli</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Non-softened</td>
<td>1.46</td>
<td>45,50, 55</td>
<td>Increased</td>
</tr>
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<td></td>
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<td>Softened</td>
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<td>45, 55</td>
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<tr>
<td></td>
<td>C</td>
<td>Non-softened</td>
<td>4.17</td>
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</tr>
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<td>Chi-Chi</td>
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</tr>
<tr>
<td></td>
<td>C</td>
<td>Softened</td>
<td>0.83</td>
<td>5-7</td>
<td>Decreased</td>
</tr>
</tbody>
</table>
3.6. References


**PEER Report 2001/10.** Pacific Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley.


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CHAPTER 4

GROUND MOTION CHARACTERISTICS ASSOCIATED WITH SITE NONLINEARITY

4.1. Introduction

Nonlinearity of the sites has significant importance on the ground motion characteristics. Local site effects on the nonlinear response have been recognized for many years (Seed and Idriss, 1982). Particularly, the combined effect of the dynamic stiffness and the depth of the soil are very influential (Marek, et. al., 1999). Nonlinearity appears differently in different types of soils. Clays tend to exhibit higher damping than sandy soils (Wiss, 1967). Geology and the types of rocks or sediments under the sites are controlling factors in evaluating the site nonlinearity factor (Yu et. al., 1992; Mayeda et. al., 1992; Castro et. al. 1990; Philips and Aki, 1986). The deep sediments affect surface motions in two opposite ways. Younger and less consolidated sediments may amplify seismic waves several times more than hard rocks do because of different impedance and resonance effects. But, at the same time, because of a reduction in high frequency content due to intrinsic attenuation and wave scattering, this amplification is damped (Boore and Joyner, 1991). Yu et. al. (1992) used a numerical model to calculate the time-domain response of a nonlinear soil-to-base excitation to investigate the characteristics of accelerograms recorded in soil. Their results verified several effects which are believed to be evidences of nonlinearity. According to Cultrera et. al. (1999), nonlinear effects on seismic waves would be an increase in damping and a decrease in propagation velocity, with consequent reduction in high-frequency amplitudes and shifts to lower
frequencies of the spectral resonant peaks of the soil deposit. Nonlinear soil response may be typically defined as the decrease in near-surface amplification of seismic waves as the amplitude of the input wave increases. It is believed that as strain increases, an increasingly hysteretic character of the stress-strain relationship in soils causes this phenomenon. At low strains, that is for the weak ground motion accompanying small earthquakes, the relationship is essentially linear and the amplification due to sediments is well understood in terms of linear elasticity, but for strong ground motions such as large earthquakes there has been always a debate on the associated amplification (Beresnev and Wen, 1996; Ishihara, 1996; Field et. al., 1997).

4.2. Nonlinear Site Response Analysis

There are two main groups of soil models to account for the soil nonlinearity: equivalent linear models, and nonlinear models. A number of studies have been conducted to compare the response of soil deposits using both equivalent linear and direct nonlinear methods and the common observation is that while both methods give similar response spectra, the equivalent linear method underestimates displacements and overestimates accelerations (Constantopoulos et. al., 1973; Finn et. al., 1977, 1978; Yu et. al., 1992).

Nonlinear one-dimensional ground response analysis characterize the stress-strain behaviour of the soil by cyclic stress-strain models such as the hyperbolic model, modified hyperbolic model, Ramberg-Osgood model, Hardin-Drnevich-Cundall-Pyke (HDCP) model, Martin-Davidenkov model, and Iwan-type model. Others have been based on advanced constitutive models such as the nested yield surface model (Kramer, 1996). The Cam-Clay and the modified Cam-Clay models are of this type (Roscoe and Schofield, 1963; Roscoe and
Burland, 1968). In all these models, the nonlinear shear behaviour is commonly described by a shear stress-strain backbone curve.

The models that represent the nonlinear behaviour of soils more accurately are based on advanced constitutive models that use basic principles of mechanics. These models generally require a yield surface that describes the limiting stress conditions for which elastic behaviour is observed, a hardening law that describes changes in the size and shape of the yield surface as plastic deformation occurs, and a flow rule that relates plastic strain increments to stress increments (Kramer, 1996).

In order to implement these models and solve the governing equations in a computer code, finite elements (FE), finite differences, or direct time integration methods can be used. To study the dynamic response of saturated soil systems as an initial boundary value problem, a two-dimensional plane-strain FE code was developed (Parra, 1996; Yang, 2000; Elgamal et al., 2002). Saturated soil is modelled as a two-phase material based on the Biot theory of porous media (Biot, 1962). The formulation is defined by the equation of motion for solid-fluid mixture and the equation of mass conservation for the fluid phase and Darcy’s law. These two governing equations are given in the finite element matrix form as follows:

\[
M\ddot{U} + \int_{\Omega} B^T \sigma' d\Omega + Q_p - f^s = 0
\]

\[
Q^T \ddot{U} + Sp + Hp - f^p = 0
\]

where \( M \) is the mass matrix, \( U \) is the displacement vector, \( B \) is the strain-displacement matrix, \( \sigma' \) the effective stress vector, \( Q \) the discrete gradient operator, \( p \) the pore-pressure vector, \( H \) the permeability matrix, \( S \) the compressibility matrix. A superscript \( T \) denotes matrix transpose and a superposed dot denotes time derivative. The vectors \( f^s \) and \( f^p \)
include the effects of body forces and prescribed boundary conditions for the solid-fluid mixture and the fluid phase respectively.

A plasticity-based constitutive model with emphasis on simulating the cyclic mobility response mechanism and associated pattern of shear strain accumulation is used in this code. They incorporated this constitutive model into a two-phase (solid-fluid), fully coupled finite element code and implemented this model in a one-dimensional computer program called Cyclic1D (Yang et. al., 2004). Calibration of the model has been done based on a unique set of laboratory monotonic and cyclic triaxial tests and dynamic centrifuge experiments (Lai et. al., 2004). The calibration focused on reproducing the prominent characteristics of dynamic soil response as dictated by the cyclic mobility mechanism. The program is still being improved, and it has been chosen in this study because of its uncomplicated use and because it incorporated many physical properties of soils.

This model’s constitutive model is based on the framework of multi-surface plasticity (Prevost, 1985; Parra, 1996). According to classical convention of plasticity, it is assumed that nonlinearity and anisotropy result from plasticity and the material elasticity is linear and isotropic (Hill, 1950). The yield function is a conical surface in principal stress space (Figure 4.1). In this figure, the hardening zone is defined by a number of similar yield surfaces with a common apex at \(-p_0\) along the hydrostatic axis. The outermost surface is designated as the failure surface.
Responses of Clay and Sand sites at two different stiffness states, at two different thicknesses, and with two different saturation conditions under sinusoidal and real earthquake motions are studied. First, the nonlinear response of these soil-thickness-motion combinations is computed using Cyclic1D. Then WT analysis is used to study the characteristics of the response as well as on the input motions in order to reveal their nonstationarity. The soil type, motion type, and soil thickness combinations for this part of the study are presented in Table 4.1. Soil column profile and the excitation conditions are shown in Figure 4.2. Shear wave velocity of cohesionless soils varies approximately in proportion to \((p_m)^{1/4}\) where \(p_m\) is effective mean confinement. Shear wave velocities for cohesionless soils are based on the empirical formula of Seed and Idriss (1970). Friction angles for cohesionless soils are based on Table 7.4 (p.425) of Das, B.M. (1983). Mass density is based on Table 1.4 (p.10) of Das, B.M. (1995). Undrained shear strength for cohesive soils are based on Table 7.5 (p.442) of Das, B.M. (1983).

In Cyclic1D, Rayleigh damping is assumed, i.e., \(C = A_mM + A_kK\)
where $M$ is the mass matrix, $C$ is the viscous damping matrix, $K$ is the initial stiffness matrix. $A_m$ and $A_k$ are two constants. The damping ratio curve $\zeta(f)$ is calculated based on

$$\zeta = \frac{A_m}{4\pi f} + A_k \frac{\pi f}{\omega}.$$ Where $f$ is frequency. $C$ is selected to be 5% over 1-10 Hz range in this study.

4.3. Characteristics of Site Response: Simple Excitations

4.3.1. Clay Sites

In this part of the study, the surface responses of shallow and deep, and stiff and soft clay layers to “sn1 and sn3”, and “sn2 and sn4” excitations are compared separately since the frequency content is different for these two pairs. sn1 and sn3 are low frequency motions, their frequency is 0.5Hz, where sn2 and sn4 are high frequency motions with 10Hz. Figure 4.3, and Figure 4.5 show the wavelet energy distribution of the base motions sn1-sn3, sn2-sn4, and Lake Piru dam site Northridge earthquake motion with respect to time and frequency. The duration is 15 sec for all the signals. In the time-frequency-energy distribution figures of sn1 and sn3 however, the real duration and the duration obtained by the analysis are not the same, the apparent duration the analysis gives, although the energy of it is too small, is a little longer (i.e. the duration in the graph ends at 17th sec). In other words, wavelet energy spectrum has a tail in time which lasts 2 sec (at this frequency). This means that there is some energy leakage or smearing effect, but, as it is said earlier the energy of this tail is quite low compared to the 15 sec portion of the signal. Nevertheless, the effect of this smearing can be eliminated from the response 3D wavelet energy spectrum by subtracting the 2 sec from whatever the end value of the time duration. For the sn2 and sn4, smearing in time is not an issue due to the nature of Wavelet analysis; that is the time
resolution for higher frequencies is always better than the time resolution for low frequencies.

Table 4. 1: The soil-motion-type-thickness combinations for response analysis.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Soil State</th>
<th>Soil Thickness</th>
<th>Fundamental Frequency $f_0$</th>
<th>Saturation</th>
<th>Motion Type</th>
<th>Motion Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLAY</td>
<td>Stiff</td>
<td>5m</td>
<td>20 Hz</td>
<td>Saturated</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100m</td>
<td>1 Hz</td>
<td>Saturated</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td>Soft</td>
<td>5m</td>
<td>5 Hz</td>
<td>Saturated</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100m</td>
<td>0.25 Hz</td>
<td>Saturated</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5m</td>
<td>12.75 Hz</td>
<td>Dry</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100m</td>
<td>0.6375 Hz</td>
<td>Dry</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td>Dense</td>
<td>5m</td>
<td>12.75 Hz</td>
<td>Saturated</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100m</td>
<td>0.6375 Hz</td>
<td>Saturated</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td>Loose</td>
<td>5m</td>
<td>9.25 Hz</td>
<td>Dry</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100m</td>
<td>0.4625 Hz</td>
<td>Dry</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td>SAND</td>
<td></td>
<td>5m</td>
<td>9.25 Hz</td>
<td>Saturated</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100m</td>
<td>0.4625 Hz</td>
<td>Saturated</td>
<td>Sinusoidal</td>
<td>sn1; sn2; sn3; sn4</td>
</tr>
</tbody>
</table>

Earthquake Lake Piru
1) Stiff Clay (SFC):
Mass Density: 1900 kg/m³
Initial Shear Strength: 75 kPa
Shear Wave Velocity Vs at 10m depth: 400 m/s

2) Soft Clay (SC):
Mass Density: 1300 kg/m³
Initial Shear Strength: 18 kPa
Shear Wave Velocity Vs at 10m depth: 100 m/s

3) Dense Sand (DS):
Mass Density: 2100 kg/m³
Shear Wave Velocity Vs at 10m depth: 255 m/s
Friction Angle: 40 deg
Permeability: 6.6x10⁻⁶ m/s
Rayleigh damping coefficients:
Am=0, Ak=2x10⁻³

4) Loose Sand (LS):
Mass Density: 1700 kg/m³
Friction Angle: 29 deg
Shear Wave Velocity Vs: 180 m/s
Permeability: 6.6x10⁻⁶ m/s
Rayleigh damping coefficients:
Am=0, Ak=2x10⁻³

Base Rock Motions:

1) \( s_{n1} = (9.81)(0.1) \sin (1\pi t) \text{ m/s}^2, 0 \leq t \leq 15 \)
   \( s_{n1} = 0, \text{ otherwise} \)

2) \( s_{n2} = (9.81)(0.1) \sin (20\pi t) \text{ m/s}^2, 0 \leq t \leq 15 \)
   \( s_{n2} = 0, \text{ otherwise} \)

3) \( s_{n3} = (9.81)(0.5) \sin (1\pi t) \text{ m/s}^2, 0 \leq t \leq 15 \)
   \( s_{n3} = 0, \text{ otherwise} \)

4) \( s_{n4} = (9.81)(0.5) \sin (20\pi t) \text{ m/s}^2, 0 \leq t \leq 15 \)
   \( s_{n4} = 0, \text{ otherwise} \)

5) Northridge Earthquake Lake Piru Dam Rock Motion

Figure 4.2: Soil profiles and excitation combinations.
Figure 4.3: Time-frequency-energy distribution of the sinusoidal base motions a) “sn1”, b)”sn3”

Figure 4.4: Time-frequency-energy distribution of the sinusoidal base motions a)”sn2”, b)”sn4”

Figure 4.5: Time-frequency-energy distribution of the Lake Piru Dam input accelerogram.
4.3.1.1. Shallow Stiff Clay

For low frequency inputs, the acceleration-time histories of the surface and input motions are the same (Figure 4.6 and Figure 4.7.) Both MWS and FS for base and surface motions show the correct frequency of the signals at 0.5Hz (Figure 4.8 and Figure 4.9). When surface to base MWS ratio and FS ratio are studied (Figure 4.10 and Figure 4.11), it is seen that MWS surface to base ratio gives unity for all frequencies meaning that there is no shift in the frequency content of the signal at all, this also means that there is no amplitude amplification. However, FS ratio gives amplifications at some high frequencies which are spurious, especially at 9Hz. No frequency shifts are seen in time-frequency-wavelet energy graphs either in Figure 4.12 and Figure 4.13. Since this one figure is capable of showing the shifts of energy among different frequencies, the former type of figures (FS and MWS) will not be presented here from now on. From all these figures, it can be said that for shallow stiff clays at low frequencies: the surface response is not amplified, there is no shift in frequency, and the amplitude of input does not affect the response characteristics. All these can be said by only looking at the three dimensional time-frequency-energy-density plots.

For high frequencies, the surface acceleration is amplified and the amplitude of the input does not affect the surface motion. In terms of maximum accelerations in the signals, for both cases the surface acceleration amplitude is approximately 1.6 times greater than the base acceleration (Figure 4.14 and Figure 4.15). If one looks at the time-frequency-energy distributions of the input and output motions (Figure 4.18 and Figure 4.19), the square root of the ratio is approximately 1.6 (the wavelet energy of the signal is calculated by squaring the
signal’s wavelet coefficients). It is seen that there is no particular frequency that is amplified in the response.

The characteristic or fundamental site period \( T_s \) \((= \frac{2\pi}{\omega_0} = \frac{4H}{V_s})\) of this stiff shallow clay is 0.05 sec. The period of vibration corresponding to the fundamental frequency is called the fundamental site period. The characteristic site period is a useful indication of the period or frequency of vibration at which the most significant amplification may occur. Characteristic site frequency is 20Hz for this case, but the maximum input frequency is 10Hz. Therefore, there is some amplification for high frequency and no amplification for low frequency input.

Essential observation for this case is that there is no amplification or deamplification at low frequencies, and there is some amplification at high frequencies. There is no frequency shift. The amplitude of input does not affect the response characteristics.

4.3.1.2. Deep Stiff Clay

There is deamplification in the surface response energy for both low and high frequencies. The higher the frequency the higher the deamplification is. For all input frequencies, the higher the amplitude, the higher the deamplification. Related figures are presented from Figure 4.20 to Figure 4.27.

In the case of high frequency inputs, there is a significant energy shift towards lower frequencies. This is expected because the characteristic site frequency is low (1Hz) compared to the high frequency input motion.

The low frequency component in surface response has two peaks in time, one is at the beginning of the signal and the other is at the end of the input signal (15th second). The first peak ceases at 15th sec, where the second peaks rises, and the energy of this second gradually
decreases as time passes. It is visible in time-frequency-energy plots that there is still energy at 30th second. There is some energy shift through a higher frequency (around 1 Hz). This is expected too because the characteristic frequency (fo) is higher than the frequency of low frequency signal. This 1 Hz high frequency stops at 15th sec. whereas the original low frequency (0.5 Hz) clearly exists until 25th sec.
Figure 4.6: Time histories of base excitation sn1 and surface response for 5m thick saturated stiff clay.

Figure 4.7: Time histories of base excitation sn3 and surface response for 5m thick saturated stiff clay.

Figure 4.8: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated stiff clay to excitation sn1.

Figure 4.9: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated stiff clay to excitation sn3.
Surface to Base Spectral Ratios for 5m Saturated SFC for sn1: \(a_{\text{max}} = 0.1\text{g}\) and \(f=0.5\text{Hz}\)

**Figure 4.10:** Marginal Wavelet and Fourier surface to base spectral amplitude ratios for 5m thick saturated stiff clay to excitation sn1.

Surface to Base Spectral Ratios for 5m Saturated SFC for sn3: \(a_{\text{max}} = 0.5\text{g}\) and \(f=0.5\text{Hz}\)

**Figure 4.11:** Marginal Wavelet and Fourier surface to base spectral amplitude ratios for 5m thick saturated stiff clay to excitation sn3.

Wavelet Energy at Soil Surface for 5m Saturated SFC for sn1: \(a_{\text{max}} = 0.1\text{g}\) and \(f=0.9\text{Hz}\)

**Figure 4.12:** Time-frequency-energy distribution of the response motion of 5m thick saturated stiff clay due to excitation sn1.

Wavelet Energy at Soil Surface for 5m Saturated SFC for sn3: \(a_{\text{max}} = 0.5\text{g}\) and \(f=0.9\text{Hz}\)

**Figure 4.13:** Time-frequency-energy distribution of the response motion of 5m thick saturated stiff clay due to excitation sn3.
Time Histories of Base and Surface Motions for 5m Saturated SFC for sn2: \(a_{\text{max}} = 0.1\text{g}\) and \(f=10\text{Hz}\)

Figure 4.14: Time histories of base excitation sn2 and surface response for 5m thick saturated stiff clay.

Time Histories of Base and Surface Motions for 5m Saturated SFC for sn4: \(a_{\text{max}} = 0.5\text{g}\) and \(f=10\text{Hz}\)

Figure 4.15: Time histories of base excitation sn4 and surface response for 5m thick saturated stiff clay.

MWS and FS for 5m Saturated SFC for sn2: \(a_{\text{max}} = 0.1\text{g}\) and \(f=10\text{Hz}\)

Figure 4.16: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated stiff clay to excitation sn2.

MWS and FS for 5m Saturated SFC for sn4: \(a_{\text{max}} = 0.5\text{g}\) and \(f=10\text{Hz}\)

Figure 4.17: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated stiff clay to excitation sn4.
Figure 4.18: Time-frequency-energy distribution of the response motion of 5m thick saturated stiff clay due to excitation sn2.

Figure 4.19: Time-frequency-energy distribution of the response motion of 5m thick saturated stiff clay due to excitation sn4.

Figure 4.20: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated stiff clay to excitation sn1.

Figure 4.21: Time-frequency-energy distribution of the response motion of 100m thick saturated stiff clay due to excitation sn1.
Figure 4.22: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated stiff clay to excitation sn3.

Figure 4.23: Time-frequency-energy distribution of the response motion of 100m thick saturated stiff clay due to excitation sn3.

Figure 4.24: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated stiff clay to excitation sn2.

Figure 4.25: Time-frequency-energy distribution of the response motion of 100m thick saturated stiff clay due to excitation sn2.
4.3.1.3. Shallow Soft Clay

In practice it can be said that there is very little or no frequency shift for the case of shallow soft clay soil subjected to the example sinusoidal motions. There is no change in the surface motions due to low frequency excitation. There is deamplification due to high frequency input and in this case the higher the input amplification the higher the deamplification. Figures for this case are presented from Figure 4.28 to Figure 4.35.
There is some lower frequency (5Hz) energy component in the surface motion in response to the high frequency inputs. This frequency occurs at the beginning and at the end of the response motion. The 5 Hz corresponds to the characteristic frequency of this site.

Compared to the shallow stiff clay, the difference is in the response to the high frequency motion. High frequencies are amplified in the stiff clay case, while they are de-amplified in the soft clay. The amplitude of the input acceleration was irrelevant for the surface response in the former case, where for this latter case the deamplification seems to be related to the maximum acceleration in the base input acceleration. The larger the amplitude, the larger the deamplification is. The reduction and/or shift of the peaks during strong motion are indications of the nonlinearity. They are due to the nonlinear response of the soil that causes change in the stiffness of the medium dependent on the waveform amplitudes (Huang et. al., 2001).

4.3.1.4. Deep Soft Clay

For all input motions there is a very significant deamplification in the surface response energy. Figures related to this case are Figure 4.36 through Figure 4.43. The higher the frequency, the higher is the deamplification. The higher the amplitude of the input, the higher is the deamplification in the surface motion.

For the high frequency inputs, there is a dramatic shift towards a very low frequency that the high frequency in the response vanishes. The characteristic site frequency is 0.25Hz. The shifting of frequencies to this low frequency is apparent even in the already low frequency input case (0.5Hz). In the case of high frequency case, wavelet energy of the signal, although reduced substantially, is redistributed around 0.25Hz and the duration of this shifted low frequency component in surface response is prolonged.
For low frequency input, there is also some energy shift through a higher frequency. The duration of this high frequency is not as long as the duration of the low frequency. This is similar to the stiff deep clay case.

Compared to the shallow soft clay, the difference is most in the responses to low frequencies. For the shallow layer, there was almost no change in the amplitude of the response where there is deamplification for the deep layer. Also, the duration of the surface responses is always longer for the deep soft clay case, especially for the high frequency input. When compared to the stiff deep clay, although a little bit less in the stiff soil, soft and stiff clays behave similarly in terms of deamplification and frequency shift in the response. The difference is the lower frequencies which the higher frequency is shifted to, and the level of deamplification. In the deep stiff clay case, the original high frequency remains in the surface motion, although reduced in energy.
Figure 4.28: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated soft clay to excitation sn1.

Figure 4.29: Time-frequency-energy distribution of the response motion of 5m thick saturated soft clay due to excitation sn1.

Figure 4.30: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated soft clay to excitation sn3.

Figure 4.31: Time-frequency-energy distribution of the response motion of 5m thick saturated soft clay due to excitation sn3.
Figure 4.32: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated soft clay to excitation sn2.

Figure 4.34: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated soft clay to excitation sn4.

Figure 4.33: Time-frequency-energy distribution of the response motion of 5m thick saturated soft clay due to excitation sn2.

Figure 4.35: Time-frequency-energy distribution of the response motion of 5m thick saturated soft clay due to excitation sn4.
Figure 4.36: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated soft clay to excitation sn1.

Figure 4.37: Time-frequency-energy distribution of the response motion of 100m thick saturated soft clay due to excitation sn1.

Figure 4.38: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated soft clay to excitation sn3.

Figure 4.39: Time-frequency-energy distribution of the response motion of 100m thick saturated soft clay due to excitation sn3.
Figure 4.40: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated soft clay to excitation sn2.

Figure 4.41: Time-frequency-energy distribution of the response motion of 100m thick saturated soft clay due to excitation sn2.

Figure 4.42: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated soft clay to excitation sn4.

Figure 4.43: Time-frequency-energy distribution of the response motion of 100m thick saturated soft clay due to excitation sn4.
4.3.2. Sand Sites

In saturated sandy soils pore pressure rises progressively due to the energy dissipation during cyclic loading. Nemat-Nasser and Shokooh (1979) have developed a model for the relationship between dissipated energy and pore pressure build up. Figueroa and Saada (1994) presented an energy-per-unit-volume approach which can incorporate non-uniform loading, in defining the liquefaction of a soil deposit. They considered the influence of relative density, shear strain, and confining pressure in the development of liquefaction. There are other factors that affect the liquefaction potential of a soil deposit such as, grain size distribution, type of applied loading, drainage characteristics of the deposit, soil structure and cementation of the sand forming deposit, intensity and duration of ground shaking, strain history of deposit, amount of entrapped air in the deposit, and thickness of the deposit (Prakash, 1981). Ni et.al., (1997) studied the effect of depth on the response. By using a stress-dependent model with impulse base excitation, they investigated the nonlinear behavior of various soil deposits under a variety of conditions. Their results show that (1) the saturated soil deposit has a smaller surface amplitude and significantly lower resonant frequency than the unsaturated soil deposit of the same thickness; (2) for the saturated soil conditions, the larger the base excitation, the lower the surface amplification and the resonant frequency; (3) the deep soil deposits show lower surface amplification and resonant frequency compared to the response of shallow deposits; (4) when shallow and deep deposits are compared, the shallow deposits develop much higher residual pore-water pressure; and (5) the amplification and residual pore-water-pressure response of deposits deeper than 100 m or so are very similar.
In this section, softening observations are related to the temporal amplitude and the frequency content of the excitation and response signals depending on the density and the depth of the sand layer. In the following, some figures are presented after the context, all others are presented in Appendix 4B.

4.3.2.1. Shallow Dry Dense Sandy Sites

For low frequency input, there is no distinction between the input and response motions regardless of the input amplitude (Figure 4.44 to Figure 4.47). In the case of high frequency excitations, amplification of acceleration occurs at the surface. Therefore signal energy gets amplified. The higher the amplitude, the lesser the amplification is. There is no frequency shift or prolonged or shortened energy duration in any case.
Time Histories of Base and Surface Motions for 5m Dry DS for sn1: $a_{\text{max}}=0.1g$ and $f=0.5\text{Hz}$

Figure 4.44: Acceleration histories of base and surface motions for shallow dry dense sand for sn1 input motion.

Wavelet Energy at Soil Surface for 5m Dry DS for sn1: $a_{\text{max}}=0.1g$ and $f=0.5\text{Hz}$

Figure 4.45: Time-frequency-energy distribution of base and surface motions for shallow dry dense sand for sn1 input motion.

Time Histories of Base and Surface Motions for 5m Dry DS for sn3: $a_{\text{max}}=0.5g$ and $f=0.5\text{Hz}$

Figure 4.46: Acceleration histories of base and surface motions for shallow dry dense sand for sn3 input motion.

Wavelet Energy at Soil Surface for 5m Dry DS for sn3: $a_{\text{max}}=0.5g$ and $f=0.5\text{Hz}$

Figure 4.47: Time-frequency-energy distribution of base and surface motions for shallow dry dense sand for sn3 input motion.
Figure 4.48: Acceleration histories of base and surface motions for shallow dry dense sand for sn2 input motion.

Figure 4.49: Time-frequency-energy distribution of base and surface motions for shallow dry dense sand for sn2 input motion.

Figure 4.50: Acceleration histories of base and surface motions for shallow dry dense sand for sn4 input motion.

Figure 4.51: Time-frequency-energy distribution of base and surface motions for shallow dry dense sand for sn4 input motion.
4.3.2.2. Shallow Saturated Dense Sandy Sites

(Figure 4.52 to Figure 4.59) For low frequencies, regardless of its amplitude, there is no amplification or de-amplification in the surface response and there is no frequency shift and temporal changes in the energy content. There is no shift to upper or lower frequencies. From this observation and the observation from the dry dense sand case, it is understood that for this thickness, saturation does not play a role in amplification and energy re-distribution among different frequencies.

For high frequency inputs, there is amplification in the case of low amplitude, and there is de-amplification in the case of high amplitude input excitation. Deamplification gradually increases towards the end. When we look at the profile of effective confinement, we see that liquefaction occurs at every depth (Figure 4.60). If we compare the amplification amount in this case and in the case of dry sand, it is seen that saturation reduced the degree of amplification in the low amplitude case. There is no frequency shift in either input amplitudes. Only for the high amplitude case, there is a little elongation of the response duration.
Figure 4.52: Acceleration-time histories of base excitation sn1 and surface response for 5m deep saturated dense sand.

Figure 4.53: Time-frequency-energy distribution of the response motion of 5m thick saturated dense sand due to excitation sn1.

Figure 4.54: Acceleration-time histories of base excitation sn3 and surface response for 5m thick saturated dense sand.

Figure 4.55: Time-frequency-energy distribution of the response motion of 5m thick saturated dense sand due to excitation sn3.
Figure 4.56: Acceleration-time histories of base excitation sn2 and surface response for 5m thick saturated dense sand.

Figure 4.58: Acceleration-time histories of base excitation sn4 and surface response for 5m thick saturated dense sand.

Figure 4.57: Time-frequency-energy distribution of the response motion of 5m thick saturated dense sand due to excitation sn2.

Figure 4.59: Time-frequency-energy distribution of the response motion of 5m thick saturated dense sand due to excitation sn4.
4.3.2.3. Shallow Dry Loose Sandy Sites

(Figure 4.61 to Figure 4.68) For low frequencies, No amplification or de-amplification occurs in the surface response. The amplitude of the input at this frequency does not affect this result.

For high frequencies, amplification of energy is observed for both input amplitudes, but not as much as it was in the dense sand case. There is no frequency shift. For high amplitude, the amplification gradually increases until 15th second, and there is a small elongation in the response energy (2 seconds more than the original energy).

4.3.2.4. Shallow Saturated Loose Sandy Sites

(Figure 4.69 to Figure 4.78) For low frequencies, energy deamplification takes place after the first few seconds. No frequency shift occurs, but the energy prevails at the surface a little bit longer than it does at the base. Amplitude change does not change the situation. Effective
confinement profile suggests softening at every depth. The time at which this occurs first is not known, this will be studied in detail with a real earthquake motion in one of the following sections. For now, it is only observed that the softening phenomenon has happened in the first few seconds, because of the energy drop corresponding to this time.

For high frequencies, there is significant energy de-amplification for both amplitudes, but de-amplification is greater when the amplitude of the input is greater. There are low frequencies until around 2 seconds in the low amplitude case. Other than this, there is no frequency shift. When these observations are compared with the dry case, it is seen that saturation caused the deamplification of the surface energy. Liquefaction occurrence is certain as it is shown by the effective confinement vs. depth graph.
Figure 4.61: Acceleration histories of base and surface motions for shallow dry loose sand due to excitation sn1.

Figure 4.62: Time-frequency-energy distribution of the response motion of 5m thick dry loose sand due to excitation sn1.

Figure 4.63: Acceleration histories of base and surface motions for shallow dry loose sand due to excitation sn3.

Figure 4.64: Time-frequency-energy distribution of the response motion of 5m thick dry loose sand due to excitation sn3.
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Spectral Amplitude (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FS-Base</td>
</tr>
</tbody>
</table>

**Figure 4.65:** Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry loose sand to excitation sn2.

**Figure 4.66:** Time-frequency-energy distribution of the response motion of 5m thick dry loose sand due to excitation sn2.

**Figure 4.67:** Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry loose sand to excitation sn4.

**Figure 4.68:** Time-frequency-energy distribution of the response motion of 5m thick dry loose sand due to excitation sn4.
Figure 4.69: Acceleration histories of base and surface motions for shallow saturated loose sand due to sn1.

Figure 4.70: Time-frequency-energy distribution of the response motion of 5m thick saturated loose sand due to excitation sn1.

Figure 4.71: Acceleration histories of base and surface motions for shallow saturated loose sand due to sn3.

Figure 4.72: Time-frequency-energy distribution of the response motion of 5m thick saturated loose sand due to excitation sn3.
Figure 4.73: Acceleration histories of base and surface motions for shallow saturated loose sand due to sn2.

Figure 4.75: Acceleration histories of base and surface motions for shallow saturated loose sand due to sn4.

Figure 4.74: Time-frequency-energy distribution of the response motion of 5m thick saturated loose sand due to excitation sn2.

Figure 4.76: Time-frequency-energy distribution of the response motion of 5m thick saturated loose sand due to excitation sn4.
Figure 4.77: Shear strain profile for the 5m thick saturated loose sand due to excitation sn1.

Figure 4.78: Excess pore water pressure development with time at different depths for 100m thick saturated loose sand due to the earthquake excitation.
4.3.2.5. Deep Dry Dense Sandy Sites

For low frequencies, for both amplitudes, there is amplification and additional higher frequency (1.5Hz) after 15\textsuperscript{th} second (Figure 4.79 to Figure 4.82). This suggests that the energy amplification is associated with the high frequency existence at the surface motion. If we compare the observation here with the observation at the shallow dry dense sand, we see that depth of the sand lead to the amplification in energy and some frequency shift to a higher frequency. In both cases (shallow and deep), the amplitude of the input did not significantly influence the response.

For high frequency inputs, as opposed to the low frequency cases, there is deamplification for both amplitudes. The larger the amplitude of the input, the larger the deamplification is. Frequency downshift to a lower frequency (1Hz) is obvious in both cases. For both amplitudes this low frequency component lasts longer that the original frequency of the input motion (10Hz). Also, the energy of the 1Hz is larger at the beginning and at the end of the original signal. Prolonged duration suggests release of energy after the actual motion has not stopped. When these results are compared with the ones which were obtained from shallow (5m) dry dense sand case, it is clear that the depth increase caused the deamplification of energy and frequency shift. The amplitude of the input is important for the deep sites.

4.3.2.6. Deep Saturated Dense Sandy Sites

For low frequencies, there is amplification in both low and high amplitude input cases (figures are presented in Appendix 4). For low amplitude, there is a higher frequency (1Hz) after 15\textsuperscript{th} second, in the response time-frequency-energy density plot. Amplification increases toward the end of the original signal time (15sec). For the high amplitude case,
again there is an additional higher frequency component, but it is 1.5Hz and it exists together with the original low frequency (0.5Hz). The duration for energy at 0.5Hz is prolonged a little in both cases.

For high frequencies, deamplification is observed for both amplitudes. Interestingly, deamplification in the case of high amplitude is less than the deamplification in the case of low amplitude (Figure 4.83 to Figure 4.86). Although the minimum effective confinement is zero at all depths, the excess pore pressure-time plots show some negative pore pressure developments. This is observed only for this case. Those negative pore pressures might have been caused by the dilation of the dense sand. Many experimental researches suggest a possible strong influence of soil-skeleton dilation at large cyclic shear strain excursions during liquefaction. Such phases of dilation can result in significant regain in shear stiffness and strength, and are deemed to control observed liquefaction induced deformation in many actual situations (Holzer et al. 1989, Zeghal and Elgamal 1994). But still, the existence of liquefaction is undeniable. For both amplitudes, there is frequency shift to 1Hz, and this frequency prevails a long time. For high amplitude, existence of the 1Hz is more visible, and although the duration is prolonged, its energy is concentrated at two temporal locations; the beginning and the end of the original signal. Compared to the dry case, there is more deamplification in this case. This can be explained by the existence of softening.
Figure 4.79: Marginal Wavelet spectra and Fourier spectra for base and surface motions for deep dry dense sand for sn1 motion.

Figure 4.80: Time-frequency-energy distribution of the response motion deep dry dense sand for sn1 motion.

Figure 4.81: Marginal Wavelet spectra and Fourier spectra for base and surface motions for deep dry dense sand for sn3 motion.

Figure 4.82: Time-frequency-energy distribution of the response motion deep dry dense sand for sn3 motion.
Figure 4.83: Marginal Wavelet spectra and Fourier spectra for base and surface motions for deep saturated dense sand for sn2 motion.

Figure 4.84: Time-frequency-energy distribution of the response motion deep saturated dense sand for sn2 motion.

Figure 4.85: Marginal Wavelet spectra and Fourier spectra for base and surface motions for deep saturated dense sand for sn4 motion.

Figure 4.86: Time-frequency-energy distribution of the response motion deep saturated dense sand for sn4 motion.
4.3.2.7. Deep Dry Loose Sandy Sites

For low frequencies, there is less amplification in the higher amplitude case than there is in the low amplitude case. Amplification for the lower amplitude is more than the amplification in the dry and saturated dense sand sites (Figure 4.87 and Figure 4.90). This observation tells that the relatively high density is the reason for the increased amplification for this input motion. Higher frequencies are introduced in the responses (1Hz for low amplitude, 1.5Hz for higher amplitude). 1.5Hz exists together with the original low frequency 0.5Hz, whereas 1Hz starts after 15\textsuperscript{th} second. From this and previous observations, it is seen that the input amplitude has an effect on the frequency shift. Because, for both dense and loose sands; frequency is shifted to 1Hz for low amplitude inputs, and to 1.5Hz for high amplitudes. The duration of the 0.5Hz is prolonged a little in both cases.

For high frequencies, on the contrary to the shallow dry loose soil for the same motion, there is deamplification of energy for both amplitudes. In the former case, there was amplification for the low input case. There is frequency downshift to 1Hz. This frequency prevails longer than the high frequency.

4.3.2.8. Deep Saturated Loose Sandy Sites

For low amplitudes, a serious deamplification is observed in both amplitude cases. The interesting observation here is that the peak acceleration is increased. But the duration of this peak acceleration in the acceleration time history is very limited, and the frequency of it is reduced very much. The already low frequency is reduced further (to around 0.25 Hz) and response duration is shortened. This peak acceleration increase and liquefaction at the same time phenomenon occurred in reality in Kocaeli 1999, Turkey earthquake. More interestingly, it happened at a far site distance. There were both softening and significant
Figure 4.87: Marginal Wavelet spectra and Fourier spectra for base and surface motions for deep dry loose sand for sn1 motion.

Figure 4.88: Time-frequency-energy distribution of the response motion deep dry loose sand for sn1 motion.

Figure 4.89: Marginal Wavelet spectra and Fourier spectra for base and surface motions for deep dry loose sand for sn3 motion.

Figure 4.90: Time-frequency-energy distribution of the response motion deep dry loose sand for sn3 motion.
shaking evidences at this site. Effective confinement and pore pressure ratios show that for both amplitudes there is liquefaction at every depth for the deep saturated loose sand.

For high frequencies, very significant energy deamplification for both input amplitudes (Example figures are presented in Figure 4.91, Figure 4.92, and Figure 4.93). For low amplitude, frequency downshift is clear at 1Hz, whereas for higher amplitude the downshift occurs at different frequencies (0.5Hz, 1Hz, and 2Hz.) In the low amplitude case, the 1Hz exists at all times but with an energy emphasis at the first and 15th seconds. As differently, in high amplitude case, the different low frequency are located in different time intervals (in more detail, 0.5Hz exists during the first five seconds, 1Hz during the first 3 seconds, and 2Hz until only 1 second.) As opposed to the low amplitude case, the high frequency component’s duration is shortened very much (from 15 seconds down to five seconds.) For low amplitude, similar to the dry loose sand, there is no prolonged energy at 10Hz, but there is no shortening of the duration either.

![Figure 4.91: Acceleration histories of base and surface motions for deep saturated loose sand for sn2 motion.](image)
Figure 4.92: Time-frequency-energy distribution of the response motion of 100m thick saturated loose sand due to excitation sn2.

Figure 4.93: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated loose sand to excitation sn2.

4.4. Characteristics of Site Response: Earthquake Motion

The response of the same soil sites which were studied in earlier sections, now subjected to a real earthquake motion (Northridge earthquake 1994 - Lake Piru Dam motion) is investigated. Additionally, wherever possible, the depth when sand liquefaction first occurred is found by the help of the effective confinement stress and strain profiles, and then the frequencies present at that time are extracted from the time-frequency-wavelet energy graph.
These frequencies and energy they carry are compared with the results from the dry sand analysis for the same depth and the same time in order to reveal the effect of pore pressure to the response motion nonlinearity and nonstationarity. The time-frequency-energy distribution of the Lake Piru station earthquake record is presented in Figure 4.5. It is seen from this figure that the energy of this record is mainly concentrated around 2Hz, 3Hz, and 4Hz until around 5th second.

### 4.4.1. Shallow Stiff Clay Site

Compared to the input motion, the peak wavelet energy is slightly amplified in the surface response. There is almost no shift in the frequency content (Figure 4.95).

### 4.4.2. Shallow Soft Clay Site

There is amplification in the energy and energy shift to higher frequencies. Two peaks occur in the surface motion for the higher frequency region. Higher frequencies prevail longer than that of the shallow stiff clay site (Figure 4.96 and Figure 4.97).

### 4.4.3. Deep Stiff Clay Site

There are deamplification and frequency downshift of the energy (Figure 4.98 and Figure 4.99). One peak with a temporal translation of 1sec occurs in the response at the 2Hz at the beginning of the signal. There are two consecutive small peaks at 1.7Hz between 12 and 17 seconds, and finally there are 3 peaks until 30th sec in the 1Hz frequency component. The energy at 1 Hz prevails for a long time (until 40th second at least).

### 4.4.4. Deep Soft Clay Site

Deamplification in energy is greater than that of the stiff deep site. Frequency downshift is very clear. One big peak occurs at 3Hz, and the significance of 2Hz, as well as 1Hz is increased. 2 Hz prevails longer in this case, and 1Hz also prolongs.
Figure 4.94: Marginal Wavelet spectra and Fourier spectra of base and surface motions for 5m thick saturated stiff clay due to earthquake excitation.

Figure 4.95: Time-frequency-energy distribution of the response motion of 5m thick saturated stiff clay due to earthquake excitation.

Figure 4.96: Marginal Wavelet spectra and Fourier spectra of base and surface motions for 5m thick saturated soft clay due to earthquake excitation.

Figure 4.97: Time-frequency-energy distribution of the response motion of 5m thick saturated soft clay due to earthquake excitation.
Figure 4.98: Marginal Wavelet spectra and Fourier spectra of base and surface motions for 100m thick saturated stiff clay due to earthquake excitation.

Figure 4.99: Time-frequency-energy distribution of the response motion of 100m thick saturated stiff clay due to earthquake excitation.

Figure 4.100: Marginal Wavelet spectra and Fourier spectra of base and surface motions for 100m thick saturated soft clay due to earthquake excitation.

Figure 4.101: Time-frequency-energy distribution of the response motion of 100m thick saturated soft clay due to earthquake excitation.
4.4.5. Shallow Dry Dense Sandy Site

Amplification in energy occurs. There is frequency downshift from 2Hz, 3Hz, and 4Hz to 0.8Hz, 1Hz, and 2Hz. Time location is very similar to the original input (Figure 4.102).

4.4.6. Shallow Saturated Dense Sandy Site

Saturation has no effect on the response characteristics for this case. The observations are the same as the shallow dry dense sand case (Figure 4.103).

4.4.7. Shallow Dry Loose Sandy Site

The observations of the WT analysis for this case are not much different from shallow dry dense sand and shallow saturated dense sand. Only 2Hz frequency is more pronounced in this case (Figure 4.104).

4.4.8. Shallow Saturated Loose Sandy Site

There is significant deamplification in energy and frequency downshift. Prevailed energy accumulation at 0.2Hz and at 1Hz is observed. Frequencies higher than 1Hz practically do not exist after three seconds in the response. Therefore, we may say that the time-frequency-energy graph suggests that there is liquefaction at this site. Indeed, effective confinement vs. depth figure tells the same story. It may be assumed that this liquefaction took place in the first 3 seconds (Figure 4.105).

4.4.9. Deep Dry Dense Sandy Site

Significant amplification in energy is observed. Frequency downshift to 1Hz and 2Hz is present (Figure 4.106).

4.4.10. Deep Saturated Dense Sandy Site

Energy amplification although not as much as the dry sand is observed. Some of the energy of the input shifts to 1Hz at the surface. Energy of the 1Hz is concentrated between 10th and
15\textsuperscript{th} seconds. There is also prolonged energy at this frequency until 30\textsuperscript{th} second. No softening is observed, because the effective confinement is never zero at any depth (Figure 4.107).

4.4.11. Deep Dry Loose Sandy Site

Similar observations as the deep dry and saturated dense sands are obtained. The energy shifts to 0.5Hz, instead of 1Hz (Figure 4.108).

4.4.12. Deep Saturated Loose Sandy Site

Deamplification of energy and frequency downshift to 1Hz are observed again (Figure 4.109). This time duration of the energy at this frequency is limited to between zero and five seconds. There is also another very low frequency component which exists between two and fifteen seconds. Although the energy of this component is not very significant, existence of it strongly suggests presence of liquefaction. This observation is a result of the investigations of the responses of the same soil columns to simple excitations and a result of the observations made in the previous chapter of this study. So, the existence of extremely low frequencies in an earthquake acceleration may be accepted as an indication for soil softening below the site where the acceleration history is recorded.

It is seen from effective confining stress (Figure 4.110) that the liquefaction took place at every depth above 85m. However, from this figure we cannot deduce which portion of the soil liquefied first. If we look at excess pore pressure-time histories (Figure 4.111), we can find when pore pressure reaches its maximum at a particular depth. Still, at this time (when the pore pressure is maximum at any depth) soil may or may not soften. It depends on the confining stress at that instant at that depth.

When there is liquefaction at a certain depth, the acceleration responses above that depth seem to be almost unchanged (Elgamal, et. al., 2002). Liquefaction, in a way, isolates the
upper layers from experiencing large strains. Therefore, we may say that the depth that observes the maximum strain undergoes liquefaction first.

In order to observe the change in time-frequency-energy with depth for this particular case, WT is applied at several depths. It is indeed seen that the main characteristics of the time-frequency-energy graphs do not change much at the levels above where the maximum strain occurs (9m). The strain-depth relation for this case is shown in Figure 4.112. The only change after this point is that high frequencies (what is remained of them) continue to shift towards the lower frequency (1Hz). This causes the energy of the low frequency to increase.
Figure 4.102: Time-frequency-energy distribution of the response motion of 5m thick dry dense sand due to the earthquake excitation.

Figure 4.103: Time-frequency-energy distribution of the response motion of 5m thick saturated dense sand due to the earthquake excitation.

Figure 4.104: Time-frequency-energy distribution of the response motion of 5m thick dry loose sand due to the earthquake excitation.

Figure 4.105: Time-frequency-energy distribution of the response motion of 5m thick saturated loose sand due to the earthquake excitation.
Figure 4.106: Time-frequency-energy distribution of the response motion of 100m thick dry dense sand due to the earthquake excitation.

Figure 4.107: Time-frequency-energy distribution of the response motion of 100m thick saturated dense sand due to the earthquake excitation.

Figure 4.108: Time-frequency-energy distribution of the response motion of 100m thick dry loose sand due to the earthquake excitation.

Figure 4.109: Time-frequency-energy distribution of the surface motion of 100m thick saturated loose sand due to the earthquake excitation.
Figure 4.110: Effective confinement vs. depth for thick saturated loose sand under the earthquake loading.

Figure 4.111: Excess pore water pressure development with time at different depths for 100m thick saturated loose sand due to the earthquake excitation.

Figure 4.112: Shear strain vs. depth for thick saturated loose sand under the earthquake loading.
Following figures (Figure 4.113 through Figure 4.118) shows the time-frequency-energy variations at different depths. It should be noted that the maximum energy in the surface response decreases sharply in five seconds, and this five seconds is the time when the pore pressure at 9m (maximum strain) reached its peak. With this observation, it may be said that, from surface time-frequency-energy distributions, the time when soil softening first occurs can be deduced approximately. Meanwhile, the frequency component which is present and dominant at that time frequency is known also. If some main characteristics of the site such as shear wave velocity are known, the depth at which liquefaction occurred may be back calculated.

Wavelet analysis therefore may be a useful tool in identifying whether softening took place at a certain site. Often times sand boilings, soil subsidence or other liquefaction related phenomena are not observed at the surface. This new interpretation may be used also to find the liquefaction potential of a free field on which there are no man made structures. In this study, we know the exact soil conditions and the input motion, then we use a computer program to simulate the site response, but in reality, the soil conditions are never known completely and it is very difficult to measure pore pressures. If seismographs are placed at those regions, and earthquake recordings are taken and examined by WT, most possible locations for softening for different earthquake motions may be determined and may be used in remediation works. This is out of the scope of this dissertation, but may be a future research topic.
Figure 4.113: Time-frequency-energy distribution of the surface motion of 100m thick saturated loose sand due to the earthquake excitation at 85m.

Figure 4.114: Time-frequency-energy distribution of the surface motion of 100m thick saturated loose sand due to the earthquake excitation at 60m.

Figure 4.115: Time-frequency-energy distribution of the surface motion of 100m thick saturated loose sand due to the earthquake excitation at 15m.

Figure 4.116: Time-frequency-energy distribution of the surface motion of 100m thick saturated loose sand due to the earthquake excitation at 12m.
Figure 4.117: Time-frequency-energy distribution of the surface motion of 100m thick saturated loose sand due to the earthquake excitation at 11m.

Figure 4.118: Time-frequency-energy distribution of the surface motion of 100m thick saturated loose sand due to the earthquake excitation at 9m.
4.5. Summary and Conclusions

Four types of sinusoidal motions and one earthquake motion have been used to observe the time and frequency characteristics of the motions at the surface of clay and sand sites with wavelet analysis. Parameters studied in the artificial sinusoidal motions were the frequency and amplitude. The soil parameters changing were the depth, the stiffness, and the saturation conditions.

At very low frequencies, stiff and soft shallow clays show the same behavior. There is no amplification or deamplification in the response regardless of the amplitude of the input excitation. At high frequencies, amplification in energy is observed at the stiff shallow clay site surface response for both input amplitudes. As opposed to stiff shallow clay, soft shallow clay sites exhibits deamplification under high frequency shaking conditions. For higher amplitude case, the deamplification is a little more. There is very small localized frequency downshift in soft shallow clay. In practice it can be said that there is no frequency shift for the case of shallow clay sites subjected to the example sinusoidal motions.

The obvious and expected result for deep clays is deamplification. The characteristic site frequency is 1 second. This explains the deamplification and the frequency shift to 1Hz from 10Hz frequency input signal. The higher the frequency and the amplitude of the input, the larger the deamplification is. This information is completed by the time-frequency-wavelet energy figures. The duration of the surface responses is always longer for the deep soft clay case, especially for the high frequency input. When deep soft clay is compared with the stiff deep clay, soft and stiff clays behave similarly in terms of deamplification and frequency shift in the response. The difference is at the lower frequencies which the higher frequency is shifted to, and at the level of deamplification. In the deep stiff clay case, the original high
frequency remains in the surface motion, although reduced in energy, but for the deep soft clay site there is even no 10Hz frequency content remaining in the response. These results are summarized in Table 4. 2.

For the excitation during earthquake, for shallow clay sites, the peak wavelet energy is slightly de-amplified. There is some shift in the energy towards higher frequencies. Higher frequencies prevail longer in soft clay than that of the stiff clay site. There is deamplification and frequency downshift of the energy in both stiff and soft deep sites. The energy at 1 Hz prevails for a long time (until 40\textsuperscript{th} second at least) in both cases. Deamplification in energy is greater in the soft deep clay site than that of the stiff deep site. 2 Hz prevails longer in the soft clay case. A table is provided for quick review of the observations (Table 4.3)

Table 4. 2: Summary of observations for clay sites under the sinusoidal excitations with different frequency and amplitude contents.

<table>
<thead>
<tr>
<th>CLAY (Simple Excitations)</th>
<th>Amplification/Deamplification of energy</th>
<th>Frequency shift</th>
<th>Temporal changes in frequency and energy content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow Stiff (fo = 20Hz)</td>
<td>Amplification at high frequencies</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Shallow Soft (fo = 5Hz)</td>
<td>De-amplification with high frequencies. The larger the amplitude the larger the de-amplification.</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Deep Stiff (fo = 1Hz)</td>
<td>Deamplification: the higher the frequency and amplitude the higher the de-amplification.</td>
<td>Downshift and shift towards characteristic site frequency.</td>
<td>Prolonged low frequency response durations</td>
</tr>
<tr>
<td>Deep Soft (fo = 0.25Hz)</td>
<td>More Deamplification: the higher the frequency and amplitude the higher the de-amplification.</td>
<td>More Downshift and shift towards characteristic site frequency.</td>
<td>Prolonged low frequency response durations</td>
</tr>
</tbody>
</table>
Table 4.3: Summary of observations for clay sites under the earthquake excitation.

<table>
<thead>
<tr>
<th>Clay (Earthquake Excitation)</th>
<th>Amplification or Deamplification of energy</th>
<th>Frequency shift</th>
<th>Temporal changes in frequency and energy content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow Stiff (fo = 20Hz)</td>
<td>NO</td>
<td>To higher frequencies (very little)</td>
<td>No</td>
</tr>
<tr>
<td>Shallow Soft (fo = 5Hz)</td>
<td>Slight deamplification</td>
<td>To higher frequencies</td>
<td>Prolonged high frequencies.</td>
</tr>
<tr>
<td>Deep Stiff (fo = 1Hz)</td>
<td>Deamplification</td>
<td>To lower frequencies</td>
<td>Prolonged low frequencies</td>
</tr>
<tr>
<td>Deep Soft (fo = 0.25Hz)</td>
<td>More deamplification</td>
<td>To lower frequencies</td>
<td>Prolonged low and high frequencies</td>
</tr>
</tbody>
</table>

Table 4.4 summarizes the observations for sand sites for simple sinusoidal excitations. It is seen that most of the time shallow sand sites do not amplify or de-amplify the base accelerations and there is no energy shift to upper or lower frequencies. However, saturation and relative density changes this behavior significantly. For deep sites, the amplification behavior is rather input frequency dependent. For low frequencies, response accelerations are amplified, whereas for high frequencies, they are de-amplified. Frequency downshift behavior is similar to this, depending on the characteristic site period, there is an energy shift to higher frequencies for low input frequencies, and there is an energy shift to lower frequencies for high input frequencies. Table 4.5 shows the general observations for the earthquake record. In this case, there is always frequency downshift for the shallow sand sites, and except for saturated loose sand case, there is always energy amplification. For dense sites the energy de-amplification and frequency downshift occurs in all cases. Liquefaction or soil softening is characterized by concentrated energy spikes at the beginning of the response time-frequency-energy plot, and a prolonged or localized significantly low frequency content.
Table 4.4: Summary of observations for sand sites under the sinusoidal excitations with different frequency and amplitude contents.

<table>
<thead>
<tr>
<th>SAND (For Simple Excitations)</th>
<th>Softening</th>
<th>Amplification/Deamplification of energy</th>
<th>Frequency shift</th>
<th>Temporal changes in frequency and energy content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow Dry Dense (fo = 12.75Hz)</td>
<td>NO</td>
<td>Amplification for high frequency input</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Shallow Saturated Dense (fo = 12.75Hz)</td>
<td>only for high frequency high amplitude input</td>
<td>- Some amplification for high frequency low amplitude input - Deamplification for high frequency high amplitude input</td>
<td>NO</td>
<td>Energy drop after first few seconds</td>
</tr>
<tr>
<td>Shallow Dry Loose (fo = 9.25Hz)</td>
<td>NO</td>
<td>- Some amplification for high frequency low amplitude input - Deamplification for high frequency high amplitude input</td>
<td>NO</td>
<td>Little elongation in signal duration</td>
</tr>
<tr>
<td>Shallow Saturated Loose (fo = 9.25Hz)</td>
<td>Always</td>
<td>Deamplification</td>
<td>- Minimal downshift for low frequency low amplitude input - Downshift for high frequencies</td>
<td>- Prolonged energy for low frequency low amplitude input - Low frequencies at the first few seconds for high frequency inputs</td>
</tr>
<tr>
<td>Deep Dry Dense (fo = 0.6375Hz)</td>
<td>NO</td>
<td>- Amplification for low frequency inputs - Deamplification for high frequency inputs</td>
<td>- To a higher frequency for low frequency inputs - To lower frequencies for high frequency inputs</td>
<td>- Prolonged duration for low frequency inputs - Prolonged low frequencies for high frequency inputs</td>
</tr>
<tr>
<td>Deep Saturated Dense (fo = 0.6375Hz)</td>
<td>Always</td>
<td>- Amplification for low frequency inputs - Deamplification for high frequency inputs</td>
<td>- To a higher frequency for low frequency inputs - To lower frequencies for high frequency inputs</td>
<td>- Prolonged duration for low frequency inputs - Prolonged low frequencies for high frequency inputs - Location of energy concentration of low frequency changes with the amplitude for high frequency inputs</td>
</tr>
<tr>
<td>Deep Dry Loose (fo = 0.4625Hz)</td>
<td>NO</td>
<td>- Amplification for low frequency inputs - Deamplification for high frequency inputs</td>
<td>- To a higher frequency for low frequency inputs - To lower frequencies for high frequency inputs</td>
<td>- Prolonged duration for low frequency inputs - Prolonged low frequencies for high frequency inputs</td>
</tr>
<tr>
<td>Deep Saturated Loose (fo = 0.4625Hz)</td>
<td>Always</td>
<td>- Deamplification</td>
<td>- Amplification of the peak acceleration for low frequency low amplitude input</td>
<td>Shortened duration for low frequency low amplitude input - Prolonged low frequency energy for low frequency high amplitude input - Prolonged low frequency and shortened high frequency energy for high frequency high amplitude inputs</td>
</tr>
</tbody>
</table>
### Table 4.5: Summary of observations for sand sites under the earthquake excitation.

<table>
<thead>
<tr>
<th>SAND (For Earthquake)</th>
<th>Softening</th>
<th>Amplification or Deamplification of energy</th>
<th>Frequency shift</th>
<th>Temporal changes in frequency and energy content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow Dry Dense (fo = 12.75Hz)</td>
<td>NO</td>
<td>Amplification</td>
<td>Downshift</td>
<td>NO</td>
</tr>
<tr>
<td>Shallow Saturated Dense (fo = 12.75Hz)</td>
<td>NO</td>
<td>Amplification</td>
<td>Downshift</td>
<td>NO</td>
</tr>
<tr>
<td>Shallow Dry Loose (fo = 9.25Hz)</td>
<td>NO</td>
<td>Amplification</td>
<td>Downshift</td>
<td>NO</td>
</tr>
<tr>
<td>Shallow Saturated Loose (fo = 9.25Hz)</td>
<td>YES</td>
<td>De-amplification</td>
<td>Downshift</td>
<td>- Concentration of high frequency energy at the very beginning - Prolonged low frequencies</td>
</tr>
<tr>
<td>Deep Dry Dense (fo = 0.6375Hz)</td>
<td>NO</td>
<td>De-amplification</td>
<td>Downshift</td>
<td>NO</td>
</tr>
<tr>
<td>Deep Saturated Dense (fo = 0.6375Hz)</td>
<td>NO</td>
<td>De-amplification</td>
<td>Downshift</td>
<td>- Concentration between 10 and 15 sec - Prolonged low frequencies</td>
</tr>
<tr>
<td>Deep Dry Loose (fo = 0.4625Hz)</td>
<td>NO</td>
<td>De-amplification</td>
<td>Downshift (even to a lower frequency)</td>
<td>- Concentration between 10 and 15 sec - Prolonged low frequencies</td>
</tr>
<tr>
<td>Deep Saturated Loose (fo = 0.4625Hz)</td>
<td>YES</td>
<td>Significant de-amplification</td>
<td>Significant downshift (even to a lower frequency)</td>
<td>- Limited duration only at the beginning for the dominant downshifted frequency - Prevalence of very low frequency in a certain time interval</td>
</tr>
</tbody>
</table>

In summary, WTs of artificial and real input motions and the responses of selected idealized soil columns to these motions help observing the following characteristics,

1) Except for the saturated shallow loose sand site, amplification of the base acceleration occurs in all shallow sand cases. In the saturated loose site, de-amplification takes places instead.

2) Saturated loose sand sites whether they are shallow or dense, always experience softening.

3) Existence of softening in a site can be sensed by the time-frequency-wavelet energy density distributions at the surface. There is huge energy deamplification, a significant frequency downshift with an accompanying energy concentration during the first few seconds of the shaking, and prolonged very low frequencies. If the site is sandy and
saturated, it is very likely that softening or liquefaction has occurred. An approximate method to estimate the depth of the first softening occurrence is suggested here.

4) For deep dense or loose sands, the amplification and frequency shift depend on the input frequency. If the input has low frequency, the energy is amplified and the frequency is shifted up; and if the input has high frequency, the energy is de-amplified and the frequency is down shifted.

5) Similar to deep sand sites, deep clay sites de-amplify energy when the input frequency is high. Amplification did not occur for the low frequencies in the particular cases we investigated.

6) In general (especially for deep sites) peak energy of the frequencies other than fundamental frequency tends to shift to, either a lower or a higher frequency depending on which side of the fundamental frequency they are initially at.

7) Even though there is an obvious decrease in the total energy in the time-frequency-energy density plots, energy of a particular frequency at a particular time may get amplified.

4.6. References


APPENDIX 4.A

Surface Response Motions to Simple Excitations and Earthquake Motions for Clay Sites.
Figure 4A.1: Time histories of base excitation sn1 and surface response for 5m thick saturated soft clay.

Figure 4A.2: Time histories of base excitation sn3 and surface response for 5m thick saturated soft clay.

Figure 4A.3: Time histories of base excitation sn2 and surface response for 5m thick saturated soft clay.

Figure 4A.4: Time histories of base excitation sn4 and surface response for 5m thick saturated soft clay.
Time Histories of Base and Surface Motions for 100m Saturated SFC for sn1: $a_{\text{max}}=0.1\text{g}$ and $f=0.5\text{g}$

Figure 4A.5: Time histories of base excitation sn1 and surface response for 100m thick saturated stiff clay.

Time Histories of Base and Surface Motions for 100m Saturated SFC for sn3: $a_{\text{max}}=0.5\text{g}$ and $f=0.5\text{Hz}$

Figure 4A.6: Time histories of base excitation sn3 and surface response for 100m thick saturated stiff clay.

Time Histories of Base and Surface Motions for 100m Saturated SFC for sn2: $a_{\text{max}}=0.1\text{g}$ and $f=10\text{Hz}$

Figure 4A.7: Time histories of base excitation sn2 and surface response for 100m thick saturated stiff clay.

Time Histories of Base and Surface Motions for 100m Saturated SFC for sn4: $a_{\text{max}}=0.5\text{g}$ and $f=10\text{Hz}$

Figure 4A.8: Time histories of base excitation sn4 and surface response for 100m thick saturated stiff clay.
Time Histories of Base and Surface Motions for 100m Saturated SC for sn1: $a_{\text{max}}=0.1g$ and $f=0.5Hz$

Figure 4A.9: Time histories of base excitation sn1 and surface response for 100m thick saturated soft clay.

Time Histories of Base and Surface Motions for 100m Saturated SC for sn2: $a_{\text{max}}=0.1g$ and $f=10Hz$

Figure 4A.11: Time histories of base excitation sn2 and surface response for 100m thick saturated soft clay.

Time Histories of Base and Surface Motions for 100m Saturated SC for sn3: $a_{\text{max}}=0.5g$ and $f=0.5g$

Figure 4A.10: Time histories of base excitation sn3 and surface response for 100m thick saturated soft clay.

Time Histories of Base and Surface Motions for 100m Saturated SC for sn4: $a_{\text{max}}=0.5g$ and $f=10Hz$

Figure 4A.12: Time histories of base excitation sn4 and surface response for 100m thick saturated soft clay.
Figure 4A.13: Time histories of base excitation LakePiru1 and surface response for 5m thick saturated stiff clay.

Figure 4A.14: Time histories of base excitation LakePiru1 and surface response for 5m thick saturated soft clay.

Figure 4A.15: Time histories of base excitation LakePiru1 and surface response for 100m thick saturated stiff clay.

Figure 4A.16: Time histories of base excitation LakePiru1 and surface response for 100m thick saturated soft clay.
APPENDIX 4B

Acceleration Time Histories, MWS and FS plots, and Time-Frequency-Energy Density Figures (figures which are not presented in the main body of the study) for Sand Sites Due to Simple and Earthquake Excitations
Figure 4.B. 1: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry dense sand due to excitation sn1.

Figure 4.B. 2: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated dense sand due to excitation sn1.

Figure 4.B. 3: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry loose sand due to excitation sn1.

Figure 4.B. 4: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated loose sand due to excitation sn1.
Figure 4.B. 5: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry dense sand due to excitation sn3.

Figure 4.B. 6: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated dense sand due to excitation sn3.

Figure 4.B. 7: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry loose sand due to excitation sn3.

Figure 4.B. 8: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated loose sand due to excitation sn3.
Figure 4.B. 9: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry dense sand due to excitation sn2.

Figure 4.B. 10: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated dense sand due to excitation sn2.

Figure 4.B. 11: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry loose sand due to excitation sn2.

Figure 4.B. 12: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated loose sand due to excitation sn2.
Figure 4.B. 13: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry dense sand due to excitation sn4.

Figure 4.B. 15: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry loose sand due to excitation sn4.

Figure 4.B. 14: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated dense sand due to excitation sn4.

Figure 4.B. 16: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated loose sand due to excitation sn4.
Time Histories of Base and Surface Motions for
100m Dry DS for sn1: $a_{max}=0.1g$ and $f=0.5\text{Hz}$

Figure 4.B. 17: Input and surface response acceleration time histories for 100m thick dry dense sand due to excitation sn1.

Time Histories of Base and Surface Motions for
100m Dry LS for sn1: $a_{max}=0.1g$ and $f=0.5\text{Hz}$

Figure 4.B. 19: Input and surface response acceleration time histories for 100m thick dry loose sand due to excitation sn1.

Time Histories of Base and Surface Motions for
100m Saturated DS for sn1: $a_{max}=0.1g$ and $f=0.5\text{Hz}$

Figure 4.B. 18: Input and surface response acceleration time histories for 100m thick saturated dense sand due to excitation sn1.

Time Histories of Base and Surface Motions for
100m Saturated LS for sn1: $a_{max}=0.1g$ and $f=0.5\text{Hz}$

Figure 4.B. 20: Input and surface response acceleration time histories for 100m thick saturated loose sand due to excitation sn1.
Time Histories of Base and Surface Motions for 100m Dry DS for sn3: $a_{\text{max}}=0.5g$ and $f=0.5\text{Hz}$

Figure 4.B. 21: Input and surface response acceleration time histories for 100m thick dry dense sand due to excitation sn3.

Time Histories of Base and Surface Motions for 100m Dry LS for sn3: $a_{\text{max}}=0.5g$ and $f=0.5\text{Hz}$

Figure 4.B. 23: Input and surface response acceleration time histories for 100m thick dry loose sand due to excitation sn3.

Time Histories of Base and Surface Motions for 100m Saturated DS for sn3: $a_{\text{max}}=0.5g$ and $f=0.5\text{Hz}$

Figure 4.B. 22: Input and surface response acceleration time histories for 100m thick saturated dense sand due to excitation sn3.

Time Histories of Base and Surface Motions for 100m Saturated LS for sn3: $a_{\text{max}}=0.5g$ and $f=0.5\text{Hz}$

Figure 4.B. 24: Input and surface response acceleration time histories for 100m thick saturated loose sand due to excitation sn3.
Time Histories of Base and Surface Motions for 100m Dry DS for sn2: $a_{max}=0.1g$ and $f=10$Hz

Base Motion
Surface Motion

Figure 4.B. 25: Input and surface response acceleration time histories for 100m thick dry dense sand due to excitation sn2.

Time Histories of Base and Surface Motions for 100m Dry LS for sn2: $a_{max}=0.1g$ and $f=10$Hz

Base Motion
Surface Motion

Figure 4.B. 27: Input and surface response acceleration time histories for 100m thick dry loose sand due to excitation sn2.

Time Histories of Base and Surface Motions for 100m Saturated DS for sn2: $a_{max}=0.1g$ and $f=10$Hz

Base Motion
Surface Motion

Figure 4.B. 26: Input and surface response acceleration time histories for 100m thick saturated dense sand due to excitation sn2.

Time Histories of Base and Surface Motions for 100m Saturated LS for sn2: $a_{max}=0.1g$ and $f=10$Hz

Base Motion
Surface Motion

Figure 4.B. 28: Input and surface response acceleration time histories for 100m thick saturated loose sand due to excitation sn2.
Figure 4.B. 29: Input and surface response acceleration time histories for 100m thick dry dense sand due to excitation sn4.

Figure 4.B. 31: Input and surface response acceleration time histories for 100m thick dry loose sand due to excitation sn4.

Figure 4.B. 30: Input and surface response acceleration time histories for 100m thick saturated dense sand due to excitation sn4.

Figure 4.B. 32: Input and surface response acceleration time histories for 100m thick saturated loose sand due to excitation sn4.
Figure 4.B. 33: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick dry dense sand due to excitation sn2.

Figure 4.B. 34: Time-frequency-energy distribution of the response motion of 100m thick dry dense sand due to excitation sn2.

Figure 4.B. 35: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick dry dense sand due to excitation sn4.

Figure 4.B. 36: Time-frequency-energy distribution of the response motion of 100m thick dry dense sand due to excitation sn4.
Figure 4.B. 37: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated dense sand due to excitation sn1.

Figure 4.B. 38: Time-frequency-energy distribution of the response motion of 100m thick saturated dense sand due to excitation sn1.

Figure 4.B. 39: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated dense sand due to excitation sn3.

Figure 4.B. 40: Time-frequency-energy distribution of the response motion of 100m thick saturated dense sand due to excitation sn3.
Figure 4.B. 41: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick dry loose sand due to excitation sn2.

Figure 4.B. 43: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick dry loose sand due to excitation sn4.

Figure 4.B. 42: Time-frequency-energy distribution of the response motion of 100m thick dry loose sand due to excitation sn2.

Figure 4.B. 44: Time-frequency-energy distribution of the response motion of 100m thick dry loose sand due to excitation sn4.
Figure 4.B. 45: Input and surface response acceleration time histories for 5m thick dry dense sand due to earthquake excitation.

Figure 4.B. 46: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry dense sand due to earthquake excitation.

Figure 4.B. 47: Input and surface response acceleration time histories for 5m thick dry dense sand due to earthquake excitation.

Figure 4.B. 48: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated dense sand due to earthquake excitation.
**Figure 4.B. 49:** Input and surface response acceleration time histories for 5m thick dry loose sand due to earthquake excitation.

**Figure 4.B. 50:** Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick dry loose sand due to earthquake excitation.

**Figure 4.B. 51:** Input and surface response acceleration time histories for 5m thick saturated loose sand due to earthquake excitation.

**Figure 4.B. 52:** Marginal Wavelet spectra and Fourier spectra for base and surface motions for 5m thick saturated loose sand due to earthquake excitation.
Figure 4.B. 53: Input and surface response acceleration time histories for 100m thick dry dense sand due to earthquake excitation.

Figure 4.B. 55: Input and surface response acceleration time histories for 100m thick saturated dense sand due to earthquake excitation.

Figure 4.B. 54: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick dry dense sand due to earthquake excitation.

Figure 4.B. 56: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated dense sand due to earthquake excitation.
Figure 4.B. 57: Input and surface response acceleration time histories for 100m thick dry loose sand due to earthquake excitation.

Figure 4.B. 58: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick dry loose sand due to earthquake excitation.

Figure 4.B. 59: Input and surface response acceleration time histories for 100m thick saturated loose sand due to earthquake excitation.

Figure 4.B. 60: Marginal Wavelet spectra and Fourier spectra for base and surface motions for 100m thick saturated loose sand due to earthquake excitation.
CHAPTER 5

GENERATION OF GROUND MOTION TIME HISTORIES USING
WAVELETS

5.1 Introduction

Responses of structures that are under earthquake loadings are calculated either by pseudo-acceleration response spectrum method or by use of an acceleration-time history. A smooth design spectrum takes many possible earthquakes into account in a given zone with a certain probability of occurrence. Design spectrum method is relatively simple and well-established, and therefore is used frequently in the analysis and aseismic design of the conventional buildings. However, structures which are not so common such as nuclear power plants, very tall buildings, suspension bridges etc. requires more involved nonlinear time domain analysis of their response subjected to a random seismic environment. Because of this need, a set of ground motion records should be selected to represent earthquake motions at a site considered as stochastic process. Although recorded motions are becoming increasingly available nowadays, sufficient historical records are not available and also there are regions at which the number of earthquake records are not sufficient. Thus, there is a need to produce synthetic motions. Another advantage of simulated time histories is that they can be made much more site-specific than recorded ground motions, which are from a different earthquake and represent different wave propagation and site conditions than those that pertain to the site.
Various methods of creating artificial motions have been developed to date. The works of Jennings et al (1968), Boore and Joyner (1982), Campbell (1985) and Spanos (1983) give brief histories of various simulation techniques proposed. They can be grouped as: 1) Synthetic accelerograms generated from models of the seismic fault rupture; 2) Artificial accelerograms generated to match a target response spectrum.

The goal of this chapter is to develop a wavelet procedure to generate artificial ground motions by modifying a recorded accelerogram so that the generated accelerograms’ response spectra match a specified design spectrum.

5.2 Generation of Spectrum Compatible Artificial Ground Motions

Procedures for generating time histories from a specific response spectrum have been proposed by several researchers in the past. There are several ways of generating artificial motions by modifying existing records such as: shifting the frequency content of the spectrum by changing the time step to match the target spectrum; adding two or more records to produce a spectrum with the frequency content and amplitude of the target spectrum; and modifying frequency content by frequency domain methods.

Scanlan and Sachs (1974) assume that the motion is composed by a series of harmonic waves with amplitudes equal to the ordinates of the undamped velocity response spectrum. Phase shifts are considered as random variables uniformly distributed over \([0, 2\pi]\) range. Tsai (1972) uses recorded motions and a filtering technique. The amplitudes of the motion are suppressed or amplified at different frequencies until the design response spectrum is satisfactorily matched. In another procedure, random vibration theory is used to compute the mean square response of a simple damped oscillator. By solving the so called first-passage
problem, a relation between the design velocity response spectrum and the power spectral density is arrived (Vanmarcke, 1969; Corotis et al, 1972; Gasparini and Vanmarcke, 1976). Once the power spectral density, $S(\omega_n)$, is obtained, a trigonometric series expansion is used to generate the time history of motion:

$$f(t) = I(T) \sum_n \left[2S(\omega_n)\Delta\omega_n \frac{1}{2} \sin(\omega_n T + \theta_n) \right]$$  \hspace{1cm} (5.1)$$

where $I(T)$ is a deterministic intensity function used to simulate the transient character of real earthquakes, $\Delta\omega_n$ is the frequency step, $\theta_n$ is a random phase angle having a uniform probability density function within the $[0, 2\pi]$ interval.

Romo-Organista (1976) employed a method similar to the one explained above. The main difference of his method was that he included a compatibility coefficient in the formula to ensure that the energy contained in the generated motion is equal to the energy content of the design spectrum. Iyengar and Rao (1979) and Preumont (1980, 1984) proposed similar methods.

These methods are based on the iterative evaluation of the Fourier amplitudes so they match the response spectrum of the artificial time history with the target spectrum within a specified tolerance, but they did not account for the temporal variation of the frequency characteristics of the ground motion, and simply relied on a white noise signal with a time-dependent modulating function to simulate the amplitude non-stationarity. However, a non-linear system would respond differently to two accelerograms with different non-stationary characteristics, even though those are compatible with the same response spectrum (and thus, are responded to by the linear systems in the same way) and have same durations.
The widely used SIMQKE methodology may produce accelerograms with deficient characteristics such as unrealistically high duration and numbers of cycles of motion, especially for inelastic analysis (Naeim and Lew, 1995). Lilhanand and Tsen (1988) adjusted actual earthquake time histories to match a design spectrum while minimizing the perturbations of their original characteristics. This is a time domain procedure. The time at which the spectral response of a time history occurs (ti), is not perturbed by making a small adjustment on the time history. Iyama and Kuwamura (1999) developed a technique to simulate earthquake accelerations by wavelet inverse transform on the condition that target time-frequency characteristics are specified. Mukherjee and Gupta (2002) has also proposed a wavelet based procedure to produce artificial motions.

Recently, Montejo (2004) addressed this concern by developing a wavelet based procedure to modify a recorded accelerogram so that the response spectrum of the revised record matches a specified design spectrum. He stated in his work that the selection of the proper wavelet for synthetic motion generation is very important and only modified Littlewood-Paley basis function and the new basis function proposed by him (Impulse Response Wavelet) give good results. In the numerical calculations he used the new impulse response wavelet. The mother wavelet for this new basis is:

$$\psi(t) = e^{-\zeta \Omega t} | \sin \Omega t$$

where $\zeta$ and $\Omega$ are the parameters which govern the decaying and time variation properties. They are identified with the damping ratio $\zeta$ and the natural frequency $\Omega$ of a single degree of freedom oscillator. The FT of $\psi(t)$ is defined by:
\[
\psi(\omega) = \left| \int_{-\infty}^{\infty} e^{-\zeta \omega t} \sin(\Omega t) e^{-i\omega t} \, dt \right|
\]

\[
\psi(\omega) = \frac{4i \zeta \omega \Omega^2}{\omega^4 + 2(\zeta^2 - 1) \omega^2 \Omega^2 + (\zeta^2 + 1) \Omega^4}
\]

All previously mentioned wavelet decomposition and reconstruction formulae are valid for this wavelet too. The admissibility condition constant to reconstruct the signal from decomposed signal can be obtained using Equation 2.35.

\[
C_{\psi} = \frac{-4\zeta \left(\zeta^2 - 1\right) + \pi \left(\zeta^2 + 1\right)^2 + 2 \left(\zeta^2 + 1\right)^2 \tan^{-1}\left(\frac{\zeta}{2} - \frac{\zeta^2}{2}\right)}{4\zeta \left(\zeta^2 + 1\right)^2 \Omega^2}
\]

The value of \( \zeta \) controls the time domain localization whereas the value of \( \Omega \) controls the dominant frequency of wavelet. It is found convenient that \( \zeta = 0.05 \) and \( \Omega = \pi \).

An important concept in this method is the “detail functions”. The detail functions \( d_j(t) \) are obtained by fixing the scale \( a \), and sum on the position \( b \). The formulation is given as follows:

\[
d_j(t) = \frac{1}{a^2} \int_{-\infty}^{\infty} W_{\Psi} f_{a,b} \psi \left( \frac{t - b}{a} \right) \, db
\]

Using these detail functions the original signal is reconstructed by:

\[
f(t) = \frac{1}{C_{\psi}} \int_{0}^{\infty} d(a,t) \, da
\]
Wen and Gu (2004) proposed a method to simulate nonstationary random processes based on Hilbert spectra. In this method, using empirical mode decomposition method (EMD) a random process is decomposed into intrinsic mode functions (IMFs) first. The Hilbert transform of the IMFs yield the instantaneous amplitude and frequency, therefore the Hilbert spectrum can be obtained as a function of time and frequency. This method is described earlier in detail in Chapter 2. The average of the Hilbert spectra over the samples is then defined as the Hilbert spectrum of the process and this same spectrum from one realization is used as the target in the simulation of the processes. Although the implementation becomes easy and no further assumption is required, assumption of only one sample’s spectrum as the target can lead to limited variation in frequency and time content of the other realizations generated by the method. After the EMD and Hilbert transform the time series $f(t)$ can be expressed as follows:

$$f(t) = \text{Re} \left\{ \sum_{j=1}^{n} a_j(t) e^{i\theta_j(t)} \right\} + r_n(t)$$

(5.8)

Where $\text{Re}$ denotes the real part, $a_j(t)$ controls the amplitude variation, and $r_n(t)$ represents the residue that can be the mean trend or a constant. Instantaneous frequency of the IMF is given by

$$\omega_j(t) = \frac{d\theta_j(t)}{dt}$$

(5.9)

It is suggested that the underlying random process can be represented by introducing a random element to the Equation 5.8 (Wen and Gu, 2004) as in the following.
\[ f(t) = \text{Re} \left\{ \sum_{j=1}^{n} a_j(t) e^{i \theta_j(t) + \phi_j} \right\} + r_n(t) \]  \hspace{1cm} (5.10)

In which \( \phi_j \) is an independent random phase angle uniformly distributed between 0 and \( 2\pi \).

In this study a similar approach is applied in obtaining random processes using wavelet decomposition and reconstruction. A uniformly distributed phase angle between 0 and \( 2\pi \) is introduced in the wavelet reconstruction formulation.

**5.3 A Simplified Wavelet Based Method for Simulation of Ground Motion**

In this procedure, a recorded accelerogram is decomposed into a finite number of time histories by wavelet decomposition with a dominant frequency and those histories are scaled iteratively such that the resultant time history is compatible with the specified design spectrum. The Matlab code developed by Montejo (2004) to produce wavelet based spectrum compatible artificial ground motions has been modified to generate a set of random time series from only one record. The randomness is introduced to the mother wavelet at the level of reconstruction by providing uniformly distributed random phase over the range \([0, 2\pi]\).

The procedure is outlined in the following steps:

1) A spectral acceleration design spectrum to match is chosen. A typical acceleration time history \( f(t) \) is also selected and sampled at \( Nt \) discrete points.

2) Scale, a, and translation, b, parameters are defined.

3) CWT of the acceleration history is calculated using the discrete version of Equation 2.32.
\[ W_{\psi} f_{a_j, b_j} = \frac{1}{\sqrt{a_j}} \sum_{k=1}^{N_t} f(t_k) \psi \left( \frac{t_k - b_j}{a_j} \right) dt \] (5.11)

4) Detail functions are calculated using the discrete version of Equation 5.6.

\[ d_j(t) = \frac{dt}{a_j^2} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} W_{\psi} f_{a, b} \psi \left( \frac{t - b_n}{a_j} \right) \] (5.12)

At this step instead of Equation 5.2, the following equation is used.

\[ \psi_j(t) = e^{-\zeta \Omega |t| + \phi_j} \sin(\Omega t + \phi_j) \] (5.13)

where \( \phi_j \) is a random number between 0 and 2\( \pi \). By introducing this term, randomness in both time and frequency is provided. At each level, a slightly different mother wavelet is used by this way. This alteration of the mother wavelet makes the resulting constructed signal to be different from the original one, which is a desirable property in simulation.

5) The new accelerogram is reconstructed using discrete version of Equation 5.7

\[ f(t) = \sum_{j=1}^{N-1} d_j(t) \Delta a \] (5.14)

where \( \Delta a \) is the frequency spacing

6) The ground response spectrum of the reconstructed signal at the values of the periods defined by the discrete values of \( a_j \) is calculated. Then the ratios between the target and the calculated spectra are calculated:

\[ \gamma_j = \frac{\left[ Sa(T_j) \right]_{\text{target}}}{\left[ Sa(T_j) \right]_{\text{reconstructed}}} \] (5.15)
7) The detail functions are calculated by these ratios and a new accelerogram is obtained using Equation 5.14.

8) This process continues until the ratios become sufficiently close to 1 or a pre-determined maximum number of iteration is reached.

9) Verification of the convergence of the iterative process is done by the use of the Root-Mean-Square error measure. The error may be calculated in each iteration using:

\[
e(\%) = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \left( \frac{PSA(T_j)_{\text{target}} - PSA(T_j)_{\text{reconstructed}}}{PSA(T_j)_{\text{target}}} \right)^2} \times 100 \quad (5.16)
\]

5.4 Example Analyses

Artificial motions are produced with a design response spectrum and two acceleration records. The records are taken from rock sites at Northridge 1994 earthquake. One is Pacoima Dam record (PUL) at a distance of 8 km (closest distance) to the fault rupture, and the other one is Rancho Cucamonga record (CUC) at a distance of 80 km to the fault rupture. The seismic provisions of Unified Building Code (UBC) and National Earthquake Hazards Reduction Program (NEHRP) are the most common codes in defining the design ground motions in terms of smooth response spectra. Here, pseudo acceleration design response spectrum (for 10% probability of exceedance in 50 years) is constructed using NEHRP provisions for Northridge region USGS Class A sites (Figure 5.1). Class A corresponds to rock sites. Original acceleration records and corresponding response spectra are shown in Figure 5.2.
Figure 5.1: Design response spectrum for Pacoima and Rancho Cucamonga according to NEHRP provisions.

Figure 5.3 shows several artificial motions generated upon using the procedure described above using the first record (Pacoima Dam – PUL). The response spectrum of the record when it is first modified before any iterations and the response spectrum of the generated motion at the end of iterations together with the target spectrum of the generated motion PUL5 is shown in Figure 5.4. Figure 5.5 shows the RMS error with the iteration numbers for the same generated motion, it is seen that the process converges quickly (at 10th iteration error is 7% for this case). As it is seen from Figure 5.3, all the motions are different from each other yet the shape and the peak ground acceleration values are similar. The total durations of the generated motions are elongated (it is seen in the Husid plot presented in Figure 5.6 that the effective duration of the generated motion is longer than the original motion by 52.9%. This observation is valid for the rest of the generated motions too and can be seen in Figure 5.7. From the same figure, it is understood that the energy accumulation pattern is very similar to each other for the generated motions, which is a good thing.
Figure 5.2: Pacoima Dam and Littlerock Original Records and Their Response Spectra.
Figure 5.3: Example Generated Motions Based on Pacoima Dam Record.
Figure 5.4: Response spectrum of the generated motion(PUL5) when it is first modified before any iterations and at the end of iterations together with the target spectrum.

Figure 5.5: RMS Error After Each Iteration During the Generation of PUL5 Synthetic Motion.
Figure 5.6: Husid’s plot for the original motion PUL and for one of the generated motions.

Figure 5.7: Husid’s plot for the original motion PUL and for many other generated motions.
Figure 5.8 and Figure 5.9 show the Fourier power spectrum and marginal wavelet spectrum for the original motion PUL and for one of the generated motions (PUL5). It is seen from these figures that the frequency content is different in the generated motion but is not altered too much. Time-frequency variation in the shape of wavelet spectrogram for the original Pacoima Dam record and for some of the generated acceleration histories are presented in Figure 5.10. In this figure, it is seen that the dominant frequency for all the generated motions is between 3Hz to 5Hz, and there are also some higher and lower frequencies depending on the random numbers generated. Therefore it can be said that the produced motions retain the main characteristics of the seed motion, but at the same time each of them are different from each other in terms of both frequency content and temporal variation of these frequencies. Therefore, these motions should be appropriate for nonlinear response analysis.

When the procedure is applied to LIT (Littlerock) record and all the figures constructed for PUL record are produced for this motion, it is seen that this time the frequency content is modified significantly. This may be because of the fact that the initial response spectrum of the motion was relatively different from the target spectrum. Thus, in order to match the response spectrum of the seed record to the target spectrum significant high frequency content is introduced into the generated motions (Figure 5.16, Figure 5.17, Figure 5.18). In Figure 5.17, it is observed that the generated motion still carry three distinct frequencies, but the values of these frequencies are shifted to higher values. Some example generated motions, their initial and final response spectrum together with the target spectrum, and the RMS error for convergence are presented in Figure 5.11, Figure 5.12, and Figure 5.13.
respectively. Duration of the generated motions again is elongated (Figure 5.14 and Figure 5.15).

5.5 Summary and Conclusion

For common structures the design response spectrum application in assessing the responses of these structures may be adequate. But, for some other more sensitive structures such as nuclear power plants, very tall buildings and suspension bridges where nonlinearity becomes significant, there is a need to perform rigorous time domain nonlinear analysis. Since nonlinear analyses are very time dependent, in order to have reliable results there should be more than one earthquake acceleration histories at hand. Unfortunately, these “should be” earthquakes are most of the time absent. Thus, there is a need to produce many ground motions artificially. In this chapter some available ground motion simulation methods are reviewed. There are two main groups of methods, the first groups uses the source and path mechanisms to produce earthquake motions while the second group uses the existing records and a target spectrum to reproduce different earthquakes. Because it is not easy to single out the many physical parameters related to an earthquake motion, and because of the more convenient implementation procedure, focus is given to the spectrum compatible generation of acceleration histories. Within this group there are different methods also (Montejo, 2004; Wen and Gu, 2004). The disadvantage of Montejo’s method was it was producing only one motion from a given seed acceleration history, while the drawback of Wen and Gu’s HHT based random earthquake motion generation method was the use of only one record’s response spectrum as the target spectrum. The proposed method provides an improvement to the drawbacks of these methods in order to be able to obtain more than one
artificial motion at the end of the motion generation process. According to this method, first an acceleration time history is decomposed into its wavelet coefficients at different scales (corresponding to different central frequencies); then the detail functions are obtained (detail functions are the reconstructed versions of the coefficients at specific central frequencies). While obtaining these detail functions, a wavelet basis, which is slightly different from the original basis, is used so that the reconstructed signal will not be exactly the same as the original one. The difference between the original wavelet basis and the reconstruction wavelet basis comes from the introduced random phase angle. Phase angles are selected to be uniformly distributed random numbers between \([0 \ 2\pi]\). At each frequency level, a different number is used. Each of these detail functions are multiplied by the corresponding ratios between the design response spectrum and the signal’s response spectrum at the corresponding periods (or frequencies). Finally, these modified detail functions added back to get a different acceleration history.

The main time-frequency characteristics of the seed motion are retained in the generated motion depending on the initial match between the target spectrum and the seed’s response spectrum. The selected motion’s response spectrum to begin with should be similar to the target spectrum as much as possible where ever applicable. Because if they are too different from each other, the frequency content of the generated motions will be quite different from the original record. In order to overcome this problem, multi-band spectrum matching methods are proposed in the literature. These methods use multiple earthquake recordings for the different regions of the target spectrum to produce more desirable artificial earthquakes. But to keep it simple, it is out of the scope of this chapter.
Figure 5.8: Fourier and wavelet power spectra for original motion PUL.

Figure 5.9: Fourier and wavelet power spectra for generated motion PUL5.
Figure 5.10: Comparison of the Time-Frequency Distributions of the Some of the Generated Motions Using Pacoima Dam (PUL) Record.
Figure 5.11: Example Generated Motions Based on Littlerock Record
Figure 5.12: Response spectrum of the generated motion (LIT5) when it is first modified before any iterations and at the end of iterations together with the target spectrum.

Figure 5.13: RMS Error After Each Iteration During the Generation of LIT5 Synthetic Motion.
Figure 5.14: Husid’s plot for the original motion LIT and for one of the generated motions.

Figure 5.15: Husid’s plot for the original motion LIT and for many other generated motions.
Figure 5.16: Fourier and wavelet power spectra for original motion LIT.

Figure 5.17: Fourier and wavelet power spectra for generated motion LIT5.
Figure 5.18: Comparison of the Time-Frequency Distributions of the Some of the Generated Motions Using Littlerock (LIT) Record.

5.6 References


CHAPTER 6

NONSTATIONARY STOCHASTIC RESPONSE OF SOILS USING WAVELETS

6.1 Introduction

Deterministic prediction of ground motion requires detailed knowledge about the state of the Earth and physical processes which are not usually known. Seismic response of soil masses involves an uncertainty inherent in specifying the ground motion. A rational and economic way to deal with the uncertainty is to work with a stochastic model for earthquake induced ground motion (Rahman and Pal, 1983; Rahman and Hwang, 1994; Yeh and Rahman, 1998; Rahman and Yeh, 1999). A stochastic process which gives a probabilistic description of the ground motion may be employed to account for this uncertainty in such knowledge (Papadimitriou, 1990). Statistical characteristics of a nonstationary process can be described by the generalized power spectral density function (PSDF) and the root-mean-square (RMS) values of the process (Basu and Gupta, 1998; 2000a).

These characteristics can be obtained in a stationary sense in frequency domain only by Fourier transform method. However, an earthquake-induced ground motion is nonstationary. Priestley proposed the evolutionary spectral density function to model the earthquake accelerations. The evolutionary spectral density preserves the concept of the
power-frequency relationship used in stationary processes (Soong and Grigoriu, 1993). A stationary process, \( X(t) \), can be expressed in the form

\[
X(t) = \int_{-\infty}^{\infty} e^{i\omega t} dZ(\omega)
\]  
(6.1)

where \( Z(\omega) \) is a zero-mean, complex valued stochastic process with orthogonal increments. When \( X(t) \) is nonstationary, this equation is not valid. An alternative representation maybe presented as:

\[
X(t) = \int_{-\infty}^{\infty} a(t, \omega)e^{i\omega t} dZ(\omega)
\]  
(6.2)

where \( Z(\omega) \) is again an orthogonal process and \( a(t, \omega)e^{i\omega t} \) is a deterministic oscillatory function. If \( a(t, \omega) \) varies slowly with \( t \), \( a(t, \omega)e^{i\omega t} \) keeps the physical significance as an amplitude-modulated harmonic function. The evolutionary power spectral density associated with \( X(t) \) then is found as:

\[
S(t) = |a(t, \omega)|^2 S(\omega)
\]  
(6.3)

where \( S(\omega) \) is the power spectral density function of the associated stationary process.
With the emergence of wavelet analysis recently, it has become possible to account for frequency nonstationarity more conveniently. Basu and Gupta (1997; 1998; 1999; 2000a; 2000b) developed the analytical formulations for stochastic responses of linear and non-linear single-degree-of-freedom (SDOF) and multi degree of freedom (MDOF) systems based on the wavelet-based characterization of earthquake ground motions. Tratskas and Spanos (2003) and Spanos et. al. (2004) addressed the estimating the power spectrum of non-stationary stochastic processes using dyadic, generalized, and filtered harmonic wavelets. They have shown that the popular concept of a separable evolutionary spectrum involving a deterministic modulating envelope is subject to interpretation.

For stochastic study, the input motions may be obtained using these approaches: 1) Real accelerograms form past earthquakes; 2) Synthetic accelerograms generated from models of the seismic fault rupture; 3) Artificial accelerograms generated to match a target response spectrum. In this study, the third approach is used with the generation of artificial accelerograms according to the method described in Chapter 4.

In this study, a method to evaluate non-stationary stochastic response of soil site is initiated. As a first step, only a uniform layer of soil modeled as a linear visco-elastic material is considered. A sample formulation is given for the nonstationary stochastic response using wavelet analysis. The site response considered herein is due only to a vertically propagating shear wave. The method developed here can be readily extended for an equivalent linear analysis (of nonlinear response) of a layered soil site. The stochastic response is evaluated using both the direct method as well as Monte Carlo simulation. They
are presented in separate sections in the following. The results obtained by both methods are compared to verify the formulation of direct stochastic method.

6.2 Transfer Function and Monte Carlo Simulation

One-dimensional ground response analyses are based on the assumption that all the boundaries are horizontal and the response of a soil layer is caused mostly by the vertically propagating SH-waves from the underlying bedrock. The soil and the bedrock surface are assumed to extend infinitely in the horizontal direction. Ground motion predictions with this assumption have shown in reasonable agreement with measured responses in many cases.

Wave equation is given as:

\[ \rho \frac{\partial^2 x}{\partial t^2} = G \frac{\partial^2 x}{\partial z^2} + \eta \frac{\partial^3 x}{\partial z^2 \partial t} \]  \hspace{1cm} (6.4)

Figure 6.1: Comparison of the power spectral density functions from the stochastic wavelet solution and the Monte Carlo simulation solution for the case of Pacoima dam site 1994 Northridge Motion.
The solution to this wave equation is in the form:

\[ x(z,t) = U(z)Te^{i\omega t} \]  \hspace{1cm} (6.5)

\[ \frac{\partial^2 x}{\partial t^2} = U(z)T(-\omega^2)e^{i\omega t} \]  \hspace{1cm} (6.6)

\[ \frac{\partial^2 x}{\partial z^2} = \frac{dU^2}{dz^2}Te^{i\omega t} \]  \hspace{1cm} (6.7)

\[ \frac{\partial^3 x}{\partial z^2 \partial t} = \frac{dU^2}{dz^2}i\omega\eta Te^{i\omega t} \]  \hspace{1cm} (6.8)

\( U(z) \) can be given as:

\[ U(z) = Ee^{ik^*z} + Fe^{-ik^*z} \]  \hspace{1cm} (6.9)

where \( k^* \) is the complex wave number. It is given as:

\[ k^* = \frac{\omega}{V_s(1 + i\xi)} \]  \hspace{1cm} (6.10)

\[ V_s = \frac{G}{\sqrt{\rho}} \]  \hspace{1cm} (6.11)

Then,
\[ x(z, t) = T(Ee^{ikz} + Fe^{-ikz})e^{i\omega t} \]  \hspace{1cm} (6.12)

When the above equations are substituted back in the first equation and the stress at the surface is zero boundary condition \( \frac{\partial x}{\partial z} = 0 \) at \( z = 0 \) is applied, it is found that \( E = F \) and

\[ x(z, t) = 2TE \cos(k^*z)e^{i\omega t} \]
\[ x(H, t) = 2TE \cos(k^*H)e^{i\omega t} \]  \hspace{1cm} (6.13)

Therefore, the transfer function between the base and the surface displacement becomes:

\[ H(\omega) = \frac{\cos(k^*z)}{\cos(k^*H)} \]  \hspace{1cm} (6.14)

Assuming a deterministic solution method is available, the Monte Carlo simulation technique consists of establishing a computer program for a process and exercising this model repeatedly to generate sets of observations for all the random variables or functions in the calculation, and appropriate responses and performance measures. Conceptually, the results obtained from Monte Carlo simulation are similar to the experimentation results. Therefore, these results are treated statistically. Monte Carlo technique is often used to validate approximate analytical solution models, but generally is a last resort to solve complex engineering problems involving random variation, when analytical solution methods are not available.

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A time history of recorded motion can be thought as one sample from the entire ensemble representing the random process. The two important characteristics to give attention in the sample time history are the frequency content and nonstationarity. For our case, the soil response (in terms of acceleration) is found in the frequency domain by the transfer function method for each of the simulated motions, and the PSDF (power spectral density function) and RMS (root mean square values) are computed by averaging the surface responses.

Power spectral density function (PSDF), \( S(\omega) \), is frequently used in describing the frequency content of a ground motion. It can be used to estimate the statistical properties of a ground motion and to compute stochastic response using random vibration techniques (Clough and Penzien, 1975; Vanmarcke, 1976; Yang, 1986).

Where \( T_d \) is the duration of the motion, the root-mean-square (RMS) or pseudo root-mean-square acceleration (PRMSA), or the standard deviation, \( \sigma_o \), of the motion is given as (Kramer, 1996; Basu and Gupta, 1998):

\[
\sigma_o^2 = \frac{1}{T_d} \int_0^{T_d} (\ddot{x}_g)^2 dt
\]  

(6.15)

Utilizing the Parseval’s theorem, RMS can be expressed in frequency domain.

\[
\sigma_o^2 = \frac{1}{\pi T_d} \int_0^{\omega_n} (Cn)^2 d\omega
\]  

(6.16)
where \( \omega_n \) is the highest frequency in the Fourier series (Nyquist frequency).

Another method of obtaining the PSDF of ground acceleration is taking the Fourier transform of its auto-correlation function.

\[
S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega\tau} R(\tau) d\tau
\]  \hspace{1cm} (6.17)

where \( R(\tau) \) is the auto-correlation function.

In practice, the power spectral density can be estimated by taking the mean square of the Fourier transform of all realizations as follows:

\[
S(\omega) = \frac{1}{T_d} \sum_{n=1}^{N} |C_n|^2
\]  \hspace{1cm} (6.18)

6.3 Wavelet Based Stochastic Response Analysis (Direct Method)

As every function may be expressed in terms of wavelet coefficients an input acceleration time history, \( \ddot{x}(H,t) \), at the base of a uniform soil column may be expressed as follows:

\[
\ddot{x}(H,t) = f(t) = \sum_{i} \sum_{j} \frac{K\Delta b}{a_j} W_{\psi} f(a_j,b_i) \psi_{a_j,b_i}(t)
\]  \hspace{1cm} (6.19)

where, \( H \) is the thickness of the soil column and
\[ K = \frac{1}{4\pi C_{\psi}} (\sigma - \frac{1}{\sigma}) \]

with the admissibility condition

\[ C_{\psi} = \int_{-\infty}^{\infty} \frac{\hat{\psi}^2(\omega)}{|\omega|} d\omega < \infty \] (6.20)

Applying the separation of variables rule, the displacement at any depth and time, \( x(z,t) \) can be described using wavelet basis function and coefficients as follows:

\[ x(z,t) = U(z)x(t) \] (6.21)

\( x(t) \) can be written in terms of wavelet coefficients as in the following,

\[ x(t) = \sum_{i} \sum_{j} \frac{K\Delta b}{a_j} W_{\psi}(a_j, b_i) \psi_{a_j, b_i}(t) \] (6.22)

combining Equations 6.19, 6.20, 6.21, and 6.22:

\[
\rho U \left[ \sum_{i} \sum_{j} \frac{K\Delta b}{a_j} W_{\psi}(a_j, b_i) \psi_{a_j, b_i}(t) \right] = \\
G \frac{d^2 U}{dz^2} \left[ \sum_{i} \sum_{j} \frac{K\Delta b}{a_j} W_{\psi}(a_j, b_i) \psi_{a_j, b_i}(t) \right] + \eta \frac{d^2 U}{dz^2} \left[ \sum_{i} \sum_{j} \frac{K\Delta b}{a_j} W_{\psi}(a_j, b_i) \psi_{a_j, b_i}(t) \right]
\] (6.23)

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Fourier transform can be applied to both sides the FT of the wavelet function and its derivatives can be written as follows (Basu and Gupta, 2000a):

\[
\text{FT}(\psi) = \hat{\psi}; \quad \text{FT}(\psi') = i\omega \hat{\psi}; \quad \text{FT}(\psi'') = -\omega^2 \hat{\psi}
\]  

(6.24)

Where \( \omega \equiv \omega^* \) (central frequency)

\[
\sum_i \sum_j \frac{1}{d_j} W_{ij}(a_j, b_j) (-\omega^2 \rho U) \hat{\psi}_{a_j, b_j}(\omega) =
\sum_i \sum_j \frac{1}{d_j} W_{ij}(a_j, b_j) G \frac{d^2 U}{dz^2} \hat{\psi}_{a_j, b_j}(\omega) + \sum_i \sum_j \frac{1}{d_j} W_{ij}(a_j, b_j)(i\omega \eta) \frac{d^2 U}{dz^2} \hat{\psi}_{a_j, b_j}(\omega)
\]

(6.25)

When the above formulation is simplified the following equation is obtained,

\[
-\omega^2 \rho U = (G + i\omega^* \eta) \frac{d^2 U}{dz^2}
\]

(6.26)

where the complex shear modulus is defined as \( G^* = (G + i\omega^* \eta) \), and the square of the FT of the wavelet basis is found as:

\[
|\hat{\psi}(\omega)|^2 = \psi_{a_j, b_l}(\omega) \psi^*_{a_k, b_l}(\omega) = \frac{\sqrt{a_j a_k}}{2(\sigma - 1)\pi} \chi[\pi / a_j, \sigma / a_j](\omega) \chi[\pi / a_k, \sigma / a_k](\omega) e^{i(b_l - b_l)\omega}
\]

(6.27)

\( \chi[.] \) is the indicator function which is unity on the interval [.] and zero anywhere else.
\[
\frac{d^2 U}{dz^2} = -\frac{\omega_j^*}{V^*^2}
\]

where \( V^*^2 = \frac{G^*}{\rho} \) and \( k_j^* = \frac{\omega_j^*}{V} \)

\[
\frac{d^2 U}{dz^2} = (ik_j^*)^2 U
\]

\[U(z)\text{ can again be expressed as, } (Ee^{ik_j^*z} + Fe^{-ik_j^*z})\]

\[x(z,t) = \sum_i \sum_j \frac{K\Delta b}{a_j} W_{\psi} x(a_j,b_i)\psi_{a_j,b_i}(t)(Ee^{ik_j^*z} + Fe^{-ik_j^*z})\]

As a boundary condition, the stress at the surface should be zero;

\[\frac{\partial x}{\partial z}(0,t) = 0 \text{ therefore, } E = F\]

\[u(z,t) = \sum_i \sum_j \frac{K\Delta b}{a_j} W_{\psi} x(a_j,b_i)\psi_{a_j,b_i}(t) 2E \cos(k_j^*z)\]

At the base level,
\[ \ddot{u}(H,t) = C \sum_i \sum_j \frac{K\Delta b}{a_j} W_{ij} x(a_j, b_i) \psi_{a_j, b_i}(t) \cos(k_j^*H) = \]

\[ C \sum_i \sum_j \frac{K\Delta b}{a_j} W_{ij} f(a_j, b_i) \psi_{a_j, b_i}(t) \cos(k_j^*H) \]  

(6.32)

where \( C = 2E \)

\[ CW_{ij} x(a_j, b_i) \cos(k_j^*H) = W_{ij} f(a_j, b_i) \]  

(6.33)

Then

\[ CW_{ij} x(a_j, b_i) = \frac{1}{\cos(k_j^*H)} W_{ij} f(a_j, b_i) \]  

(6.34)

Therefore;

\[ \ddot{u}(z,t) = \sum_i \sum_j \frac{K\Delta b}{a_j} W_{ij} f(a_j, b_i) \psi_{a_j, b_i}(t) H(\omega_j) \]  

(6.35)

where

\[ H(\omega_j) = \frac{\cos(k_j^*z)}{\cos(k_j^*H)} \]  

(6.36)

The PSDF given in Section 6.2 by itself is a stationary random process, therefore its statistical parameters do not change with time. Actual ground motion accelerograms on the
other hand are non-stationary random processes. This nonstationarity is frequently modelled by multiplying a stationary time history with a deterministic intensity function (Hou, 1968; Shinozuka, 1973; Saragoni and Hart, 1974). Changes in frequency content during the motion have been described using an evolutionary power spectrum approach (Priestley, 1965, 1967; Liu, 1970). Instantaneous PSDF of the same process may also be obtained by using wavelet transform. The formulation is given as (Basu and Gupta, 1998):

\[
S_i(\omega)|_{t=b_i} = \sum_j K E_{W_i^2 f(a_j,b_i)} H(w) |^2 \psi_{a_j,b_i}(w) \]

To obtain more information and characteristics of the response process \( x(0,t) \), the statistical moments of the PSDF can be evaluated. In terms of wavelet coefficients, \( n^{th} \) moment of the instantaneous output PSDF may be expressed as:

\[
m_n|_{t=b_i} = \int_0^\infty K E_{W_i^2 f(a_j,b_i)} \omega^n X[\omega_j,\omega_{j+1}](\omega)H(\omega) d\omega
\]

The closed form equations can be given for the first two moments when the integral is evaluated.

\[
m_0|_{t=b_i} = \sum_j K E_{W_i^2 f(a_j,b_i)} f_{o,j}
\]

where
\[ I_{\alpha,j} = \int_{\omega_j}^{\omega_{j+1}} |H(\omega)|^2 d\omega = \int_{\omega_j}^{\omega_{j+1}} \left[ \frac{1}{\cos(k^*H)} \right]^2 d\omega = \frac{\omega V^*}{H} \tan(\omega A) \] (6.40)

where \( V^* \) is complex shear wave velocity.

\[ m_0|_{t=b_t} = \sigma_i^2 \] is the instantaneous mean-square (MSA) value. Therefore, RMSA is obtained when the square root of the zeroth moment is taken. The different moments of the instantaneous PSDF may be used to obtain several statistical parameters of the process \( x(0,t) \).

### 6.4 Example Analysis

A 20m-thick uniform layer of dry dense sand is used in this part of the study. All other parameters of this soil are the same as the dense sand used in Chapter 4. Viscous damping is taken as 5\%. A real rock motion from Pacoima dam site during 1994 Northridge earthquake was used to generate 20 artificial earthquakes in Chapter 5. After applying baseline correction for not to obtain non-zero velocities at the end of the motions, the responses at the surface of this soil site is obtained for each motion using the wave equation and the frequency domain transfer function method. This is well-known Monte Carlo simulation. Stochastic wavelet response is then found at the surface, by first obtaining the wavelet coefficients at the base and using the same transfer function. These results are compared with the results obtained from the Monte Carlo simulation results.
The instantaneous PSDFA and RMSA responses of this soil site have been obtained. The ensemble is obtained for the Pacoima dam site during the 1994 Northridge earthquake. For the stochastic characterization of the ground motion process, the wavelet coefficients have been calculated for the whole ensemble, for \( j = 1 \) to 600, and \( i = 1 \) to 2000, with \( \Delta b = \Delta t = 0.02 \text{sec} \). The squares of these coefficients have been averaged over the ensemble to give the expected values, \( E\left[ W_{ij}^2 \psi \left( a_j, b_t \right) \right] \). Then the instantaneous PSDF of the acceleration is found by the Equation 6.40. The RMSA response is obtained by the Equations 6.42 and 6.43.

For the simulation part, the PSDF and RMSA are obtained from the Fourier coefficients of the surface response motions by the use of Equations 6.38 and 6.39. When the instantaneous PSDF and RMSA are both integrated over time individually, similar results as in the stationary case are obtained. Therefore these parameters from both methods can be compared. The comparison of the PSDFs obtained from the wavelet analysis and Monte Carlo analysis is given in Figure 6.2. It is seen that both graphs are in general agreement. Comparison of the RMSAs is given in Figure 6.3. This figure also shows a general agreement between the two methods. Figure 6.4 shows the instantaneous RMSA obtained from the stochastic wavelet formulation. It shows the temporal variation in root mean square acceleration value at different frequencies. A similar figure can be obtained for the PSDF also. These types of figures cannot be obtained from the classical stationary process.
Figure 6.2: Comparison of the power spectral density functions from the stochastic wavelet solution and the Monte Carlo simulation solution for the case of Pacoima dam site 1994 Northridge Motion.

Figure 6.3: Comparison of pseudo root-mean-square acceleration responses from wavelet and simulation formulation for the case of Pacoima dam site 1994 Northridge Motion.
6.5 Summary and Conclusion

In this chapter, a wavelet-transform based approach has been applied to obtain the stochastic response of a single layer dense sand soil to earthquake excitation. This method provides the temporal frequency-amplitude information. The temporal variation of energy in a ground motion process is different for the different bands of frequencies. Instantaneous (time-varying) PSDF and RMSA of the response are obtained in terms of the wavelet coefficients of the base excitations. Monte Carlo simulation is performed to calculate the surface response for each base excitation separately. Then, the PSDF and RMSA are obtained using the
surface response information in this case. These parameters, from the wavelet and from the simulation, are then compared (for the wavelet method, the marginal case of the instantaneous values are used). The results are in general agreement.

This study investigated only the linear response of a single layer soil site. However, this approach should be extended to equivalent linear analysis of multi-layered nonlinear site response for the results to be physically meaningful.

6.6 References


CHAPTER 7

CLOSURE

7.1 Summary

Ground motions recorded during earthquakes are nonstationary both with respect to amplitude and frequency. The processes governing the response and instability of soil sites are nonlinear and affected by the nonstationarity of the ground motions. A proper definition of the design ground motion time history is very important for geotechnical and structural engineers. Potential for using wavelet analysis as a tool to obtain time-frequency characteristics of the ground motions is explored in this study.

The suitability of the wavelet transform (WT) method to obtain the pertinent characteristics of earthquake ground motions as well as other example signals is investigated. A proper mother wavelet (a modified version of Littlewood-Paley wavelet), to analyze earthquake type of motions, has been selected. Using this mother wavelet, effect of site to source distance, and effect of softening on recorded earthquake accelerogram characteristics are studied. To further investigate the nonlinear and nonstationary behavior of different types of soil sites, a numerical investigation is performed using simplified soil columns subjected to both simple harmonic and recorded ground motion. A wavelet based method of generating an ensemble of ground motion time histories is proposed. These generated motions are used in obtaining the stochastic site response directly in the framework of wavelet analysis. The results obtained from this direct method are compared with the results obtained by Monte
Carlo simulation. Monte Carlo simulation method involves solving the deterministic equation for response for each of the randomly generated motions and then analyzing these deterministic responses by a statistical method.

7.2 Conclusions

(1) In Chapter 2, various methods available to analyse nonstationary signals are studied and their relative merits are compared. Wavelet Transform Method with a modified version of Littlewood-Paley mother wavelet is found suitable to analyze the nonstationary characteristics of the seismic ground motion.

(2) In Chapter 3, wavelet analysis is used to identify some of the nonlinear and nonstationary characteristics of the ground motions recorded during some important earthquakes, namely Northridge-California 1994, Kocaeli-Turkey 1999, and Chi-Chi-Taiwan, 1999. It is found useful as it allows the study of ground motion simultaneously in both time and frequency domains. Various characteristics of ground motion are affected in a complex way by the nonlinearity and associated damping of the soils. The effect of site distance and site softening on the ground motion characteristics are studied. Temporal changes in the frequency content, especially in the form of frequency downshift, and energy decrease in softened sites has been demonstrated. The wavelet analysis of ground responses at far sites depicts the decrease in the energy content of the original motion clearly.

(3) In Chapter 4, responses of Clay and Sand sites at two different stiffness states, at two different thicknesses, and with two different saturation conditions under sinusoidal and real earthquake motions are studied. First, the responses at these soil-thickness-motion conditions are computed using Cyclic1D. Then wavelet analysis is applied on the responses as well as...
on the input motions in order to reveal the nonstationary characteristics for different cases. Many times, a frequency shift in the surface response towards the characteristic frequencies of the sites is observed. Based on the observations for amplification, deamplification, and pore pressure build up in both time and frequency domain, it is concluded that softening is characterized by a WT energy distribution with a distinct pattern characterized by a dramatic frequency downshift and localized energy content during the early few seconds of the response. When there is no surface evidence of liquefaction and no field measurements, WT may be a neat tool to identify the existence of softening at that site after the earthquake.

(4) In Chapter 5, a wavelet analysis approach in generating earthquake ground motions is found useful as it allows producing motions which have the same response spectrum. The generated motions have similar frequency and non-stationary characteristics for a given seed acceleration-time history, but random in nature, therefore they can be used in nonlinear site response studies because nonlinear site response is quite sensitive to even very small changes in the input.

(5) In Chapter 6, a wavelet based method to evaluate non-stationary stochastic response of a soil site is initiated. The stochastic response is evaluated using both the direct method as well as Monte Carlo simulation. The results obtained by both methods are compared to verify the formulation of the direct stochastic method.

7.3 Suggestions for Future Studies

This study represents one of the very few attempts to use wavelet analysis for geotechnical earthquake problems. Most of the time was spent on the development of analytical tools.
Many simplifying assumptions have been made in the models and formulations used. The following recommendations are made for further studies:

(1) Additional wavelet functions should be examined or devised for better resolution of ground motion.

(2) For the study of nonstationary time series, Hilbert Huang Transform is the closest rival to Wavelet Transform. The development of some hybrid method may be attempted to provide a better frequency resolution in high frequency region and a better time resolution for the low frequency region.

(3) Further attempts should be made to categorize various observations made in relation to time-frequency characteristics of surface ground motions vis a vis site conditions. These may further be utilized to develop site specific nonstationary spectra for use in design.

(4) The result of wavelet based method of generating ground motion time histories should be compared with those from other existing methods. This method may be enhanced further.

(5) The formulation developed for the nonstationary stochastic site response is limited to a uniform layer of linear visco-elastic material. This should be generalized to handle nonlinearity of the material behavior through equivalent linearization. Also, in order to model a layered site, a finite element model be developed for discretization and associated numerical scheme of solving the resulting equations.

(6) In this study, it has been shown that energy in ground motion process is significantly different for different bands of frequency. Also wavelet analysis allows for expressing the response in terms of non-overlapping energy bands of different frequencies. Therefore, this
opens the door for using frequency dependent soil parameters in the evaluation of site response including dissipated energy, which may lead to a better energy based method for evaluating the pore pressure response and the liquefaction potential. Frequency dependent soil parameters may be used also in stochastic site response calculations.