ABSTRACT

SHEHATA, ASHRAF. A New Method for Radioactive Particle Tracking. (Under the direction of Robin Pierce Gardner.)

A system based on the concept of three detectors radioactive particle tracking, to track a particle non-invasively in the three dimensions is presented. It consists of a set of three well collimated detectors mounted on a platform that can be moved to track the radioactive particle vertically through one collimated detector with a horizontal slot opening. The other two collimated detectors with vertical slot opening can be rotated angularly to track the radioactive particle in the planar domain, and deduce the polar coordinates. A complete description of the actual system developed is outlined including the hardware, the automation and control software, and the data acquisition aspects. A critique of the conventional tomographic radioactive particle tracking was established in comparison to the new three detectors system we developed. A number of obvious and valuable advantages of the new method were pointed out. The result presented here are illustrative through a series of benchmark experiments to test and verify the performance of the system. Results of real trajectories of a single radioactive particle moving in air, and in a bed filled with a mass of granular spherical attenuating medium is also presented. Through testing benchmark experiments that include a variety of real time trajectories the success of the tracking system is demonstrated.
A NEW METHOD FOR RADIOACTIVE PARTICLE TRACKING

by

ASHRAF HASSAN SHEHATA

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

NUCLEAR ENGINEERING

Raleigh
2005

APPROVED BY:
DEDICATION

To the Soul of my Father.

I was too late to say goodbye but Father I really did love you, and I miss you so much. I'm doing ok, and it's over now as you always hoped. I hope you are proud, I love you so much. Father I am glad, and I just wish you were here now.
Biography

Ashraf Hassan Shehata was born in Kena, Egypt on November 21, 1966. He obtained his Bachelor of Science in Nuclear Engineering at Alexandria University, Alexandria Egypt in 1990.

In July 1993 he accepted a position with Egypt Atomic Energy Authority as a fellow scientist. In 1998 he obtained his Master of Science in Nuclear Engineering at Alexandria University, Alexandria Egypt. In August 2001 he was admitted as a Doctorate student to the department of Nuclear Engineering at North Carolina State University, Raleigh, North Carolina.

The author has been both a research assistant and a teaching assistant in the Nuclear Engineering department. During his study he was engaged in different research activities conducted at the Center for Engineering Applications of Radioisotopes (CEAR). In addition to his Ph.D. research work, he participated in a variety of research tasks including investigation of various approaches for Radioactive Particle Tracking, and their application to flow mapping and characterization studies, Monte Carlo transport simulations applied to pulsed neutron activation analysis as applied to the Carbon/Oxygen Tools used in the oil well logging industry, Digital Signal Processing for the purposes of digital spectroscopy, Monte Carlo simulation and correction of pulse pileup, Monte Carlo Library Least Squares (MCLLS) and application to bulk analysis, and X-Ray Florescence techniques.
ACKNOWLEDGMENTS

The author would like to express his deepest gratitude to Professor Robin P. Gardner for all the help, guidance and encouragement he gave him throughout the work. Also, the remarks and comments with Dr. Geir Johansen has greatly enriched the work.

Equally, the author appreciates the great assistance of all the staff in the School of Engineering machine shop in designing and fabricating the equipment needed for the present research work. The financial support by the Nuclear Engineering Department (NCSU), and the Center for Engineering Application of Radioisotopes (CEAR-NCSU) greatly enabled the pursuit of this research work and is much appreciated.

Very special thanks to his Mother, Children, and Sisters for their patience, prayers, sacrifices, and encouragements. And last but never the least, his sincerest gratitude to Nourel Hoda Ewida, his Wife, for believing in him.
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1. INTRODUCTION

1.1 General:

Multiphase flow technology plays an important role in the chemical and process industry. Handling systems involving two or more phases is common in areas from the processing of fuels and chemicals to the production of foods, and specialty materials. Despite the wide usage of multiphase systems, the methodology adopted for their design is largely by intuition and rules of thumb rather than on first principles. The main reason for this state of affairs is that the local flow structure is extremely complex and the link between the micro and macro-scale has not been clearly established. Consequently, our understanding of the numerous hydrodynamic problems encountered with multiphase systems remains incomplete. The lack of detailed structural and dynamic information at the micro-scale, and the mathematical difficulties associated with the methods for handling the randomness of the multiphase media are the prime reasons for the inability to treat these flaws purely from a theoretical basis. The successful approach towards the understanding of such complex flows requires reliable data, which in turn depends on the implementation of sophisticated measuring techniques capable of non invasive investigation as well as the ability to provide the required information over the entire flow field. In addition it is desirable that such techniques are amenable for automation to reduce extensive human involvement in the data collection process. Progress in modeling the transport phenomenon in multiphase reactors depends on the availability off such experimental tools which can provide the data for model verification. In this context, very
often, engineers find themselves in need of experiments that will provide valuable information about flow mapping. This information is crucial in order to establish models for Computational Fluid Dynamics (CFD), Residence Time Distribution (RTD), flow characterization etc. This information would be used finally to develop an optimal system design.

Chemical engineers very early realized the merits of nuclear radiation for probing and measuring process characteristics non-invasively. For example, penetrating gamma rays avoid physical and chemical interference with the process and allow large opaque systems to be imaged. The use of radiotracers enables the observations of many structural and dynamic features in fluidized beds. Overall characteristics such as residence and circulation time distributions, and homogenization and mixing of mixtures of fluids and/or solids have also been obtained using Radioactive Particle Tracking (RPT) techniques (Dudukovic, et. al 1997).

An assembly of scintillation detectors placed external to the bed measures the emitted radiation. To infer flow information from a particular constituent in the heterogeneous system, particle(s) marked by radioisotope tracers can be introduced into the system to label the constituent either as a single radioactive particle or as a cloud of radioactive particles. However, use of a single unique radioactive particle, mimicking faithfully a targeted flow constituent, results in information that is more specific and location sensitive, and the usefulness and completeness of measurements with one single tracer particle far outweigh the possibilities offered by the conventional labeled multi-particle injections. Once accurate knowledge of the motion of the tracer is obtained, one can infer a wealth of transient and steady-state information. The instantaneous and local
Lagrangian velocities can be obtained by time-differentiation of the tracer coordinates, and a wealth of other information can be obtained using both Eulerian and Lagrangian frames. This improved understanding of multiphase fluid dynamics offers potential improvements in processing technologies.

### 1.2 Proposed Application of RPT to Pebble Bed Reactors:

In a pebble bed nuclear reactor, fueled pebbles enter at the top of the reactor vessel and pass through the bed and out through the base of the vessel via an extractor. Of special importance are information such as pebble pathway, and relative velocity through the bed. This information is crucial since excessive time spent in parts of the bed could result in severe irradiation and thermal damage to the pebble with possible escape of fission products. The extreme case would be the permanent fixation of a given pebble or group of pebbles in a region of the vessel. The feasibility of the recycling of fuel pebbles in a pebble bed reactor depends on the maintenance of satisfactory pebble flow through the vessel, its outlet, and the pebble extractor. Therefore, such information is important with respect to the basic reactor design calculations, optimization of fuel cycle and burnup calculations, and monitoring of fuel integrity over its lifetime.

The most feasible experimental technique for accurate flow characterization, and studying of the dynamics of pebbles in a pebble bed reactor, is to track pebbles in a simulated scaled pebble bed reactor. In this case the pebbles should be fabricated to accurately model the actual fuel pebbles in terms of materials and physical properties except the simulated pebbles would be dummy (without fuel), and one pebble that need to be studied would be tagged with a radiotracer. Then, the pebble in question would be
tracked by single particle tracking technique(s) as in multiphase flow studies carried out in fluidized beds.
2. **LITERATURE REVIEW**

2.1 **Historical Background:**

Historically, the first application of RPT to determine full-flow-field of particle velocities in multiphase reactors can be traced back to the 1960s, to the pioneering work of Kondukov et al. (1964), Borali et al. (1967), and van Velzen et al. (1974) on gas fluidized and spouted beds. However, due to instrumental and computer limitations, the flow information obtained at that time remained of limited value. Since then, a more elaborate version of RPT was developed by the University of Illinois at Champaign (UI) where the recirculatory patterns of solid particles in bubbling gas fluidized beds were studied (Lin et al. 1985, Moslemian et al. 1989, Sun et al. 1988, Chen et al. 1983). Cooperative investigations allowed two more upgrades of the UI system built at Florida Atlantic University (FAU) and at the Chemical Reaction Laboratory (CREL) at Washington University in Saint-Louis (CREL-WU). The FAU and CREL-WU facilities are currently used for mapping solids and liquid velocity fields in gas-fluidized beds, bubble columns, slurry bubble columns ebullated beds and liquid solid risers (Devanathan et al. 1990, Dudukovic et al. 1991, Dudukovic and Devanathan 1993, Yang et al. 1992, 1993, Moslemian et al. 1992, Kumar et al. 1994). Recently, the work on RPT at Ecole Polytechnique of Montreal was initiated by Chaouki’s Biopro research center to develop a new tracking system devoted to applications to other two and three phase reactor types of interest to industry (Roy et al. 1994, Cassanello et al. 1995, Godfroy et al. 1996, Larachi et al. 1995, Larachi et al. 1996).
In the last decade important progress has been made in the development of advanced non-invasive nuclear particle tracking techniques specifically suited for the characterization of three dimensional flow fields of discrete or continuous phases in dilute or dense and opaque multiphase systems.

Two photon emission-based tracking techniques are currently in application on multiphase systems: the positron emission particle tracking known as PEPT and the gamma ray emission radioactive particle tracking. The second technique is known as CAPTF (Computer-Automated Particle Tracking Facility), CRPT (Computer Automated Radioactive Particle Tracking) or RPT (Radioactive Particle Tracking).

For the sake of conciseness, the last acronym will be employed throughout this dissertation. Both PEPT and RPT use the detection and counting of highly penetrating gamma rays emitted by a single radio-labeled flow follower (tracer) which is dynamically similar to the tracked phase. They use the detected gamma rays to provide the instantaneous coordinates \((x(t), y(t), z(t))\) of the moving tagged particle. The topic of PEPT principles and applications is out of the scope of the present work. Therefore, the present dissertation is exclusively devoted to the radioactive particle tracking technique RPT based on the emission of gamma-rays by which particle location is inferred from the number of gamma-rays detected.
2.2 Radioactive Particle Tracking Principles and Applications:

Combined Computer Automated Radioactive Particle Tracking (CARPT), and Computed Tomography (CT) allow for a non-invasive determination of the flow pattern and for development of models for different types of multiphase reactors which involve complex interactions of various gas-liquid combinations. With CT and CARPT, unique measurements and images of these processes can be made to quantify the flow field in these reactors (Devanathan, Moslemian, and Dudukovic, 1990; Larachi et al., 1996; Larachi, Chaouki, and Kennedy, 1995; and Roy et al., 1995).

In principle, RPT technique is based on using radioactive nuclides that release their energy by the emission of $\gamma$-rays. These highly penetrating $\gamma$ rays can travel substantial distance to a location where a detector may be placed. The counting rate (number of photons registered) depends on the distance between the radioactive particle ($\gamma$-ray emitter) and the detector, the strength of the particle, the detector properties and the material (attenuation properties), the $\gamma$-rays must travel through. This principle is exploited in RPT in which an assembly of scintillation detectors is used to view a single radioactive particle. The number of photons received by a given detector is a measure of the radius of an approximately spherical surface with the detector as its center and the particle located on the surface. The position of the particle can be thought of as the intersection of the surface corresponding to different detectors. In principle, three detectors are sufficient to determine the particle position, but accuracy increases with the number of detectors. Therefore, an array of strategically placed scintillation counters located outside an apparatus monitor the radiation emitted from the isotope or isotopes
being tracked. To infer information from a particular constituent in the heterogeneous system, particles marked by radioisotopes can be introduced to form a cloud of radioactive particles. In turn, the information from these measurements is more specific and location-sensitive.

In practice, the three coordinates of the particle are determined by algorithms based on either phenomenological or empirical approaches that account for the relation between the number of photons detected by each detector and the location of the tracer particle. Besides the distance to the particle, the number of the photons received by the detector depends on the attenuation properties of materials between the particle and a detector as well as the properties of the detector. Accurate tracking requires the detection of a large number of photons due to the statistical fluctuations associated with the random nature of radioactive decay and detection system (Larachi et al., 1996).

However, in order for both fluid and granular flows to be studied with appropriate radioactive flow followers, general radiotracers conditions must be provided (Gardner and Ely). Particularly, radioactive flow followers must match in size, density, shape and buoyancy. Accurate knowledge of the tracer movements in the flow field yields the instantaneous and local Lagrangian velocities by successive time-differentiation of the tracer coordinates.

It should be noted that there are other factors affecting detected counting rates beside the distance of the radioactive particle. The most important of these factors affecting radiation measurements are: the characteristics of the radioactive tracer (\(\gamma\)-ray energy, radio activity level, half life etc.), type of interactions of \(\gamma\)-rays with matter.
(photoelectric absorption, Compton scattering), the solid angle subtended by the irradiated surface of the detector as viewed by the tracer, the detector efficiency, the photo-peak fraction, and the dead time of the acquisition system (Larachi et al., 1996). In addition, another important factor is the pulse pileup, which presents an important source of distortion to acquired energy spectra at high counting rate.

### 2.3 Forward Problem “Relation between counting rate and source location”:

Conventionally, RPT techniques rely on detecting and counting of unscattered gamma rays which follow a straight line from tracer to the detector wherein they contribute to the photo-peak fraction. Theoretically, the number of photo-peak counts $C$ recorded by detector during a sampling time interval $T$ from a point radioactive source of strength $A$ placed at a location $(x, y, z)$ inside a dense medium, as illustrated in Fig.1.1, can be expressed by the following relationship:

$$
C = \frac{T \nu A \phi \varepsilon}{1 + \tau A \phi \varepsilon} \quad (2.1)
$$

where,

$$
\varepsilon = \int \int \int_{\Omega} \frac{p}{r^2} f_1 f_2 \, d \Sigma \quad (2.2)
$$

where $C$ is the counting rate for each detector; $\nu$ is the number of gamma-rays emitted per disintegration; $\varepsilon$ is the total efficiency i.e. the probability that gamma-rays will emerge from the source inside the cylinder without scattering and will interact with the detector; $\tau$ is the dead time per recorded pulse; the $\phi$ is the peak-to-total ratio, $\tau$ is the dead time per recorded pulse, $\Omega$ is the solid angle subtended by the detector surface at the
source and $\mathbf{n}$ is an external unit vector locally perpendicular to $d\Sigma$. $f_1$ is the probability of non-interaction of gamma-rays emitted within $\Omega$ with the material in the cylinder and with the cylinder wall; $f_2$ is the probability of interaction of these gamma-rays along the distance inside the detector, $r$ is the distance between the source and a point $P$ on the outer surface of the detector crystal. Finally, $\mu_R$, $\mu_w$, and $\mu_D$ are the total attenuation coefficients of the reactor inventory, the reactor wall, and the detector material, respectively.

Figure 2.1: Schematic of the NaI detector and tracer in the column under investigation.
From the above definition $f_1$ and $f_2$ can be written as:

\[
 f_1 = \exp(-\mu_r e_r - \mu_w e_w) \quad (2.3)
\]
and

\[
 f_2 = 1 - \exp(-\mu_d d) \quad (2.4)
\]

where, $e_r$, $e_w$, and $d$ are the path length traveled by photon in the reactor medium, reactor wall, and detector, respectively.

The exact analytical model given by equations (1.1) through (1.4) is the governing equation I for the forward problem by which a direct relation is established between different particle locations, and different detectors’ responses. However, it can be seen that it is extremely difficult, if not impossible, to solve the model analytically. In particular, geometric complications are well known in problems involving calculations of solid angles. In addition, geometric complications are further extended to other nonlinear parameters in the above model, such as $e_r$, $e_w$, and $d$ as they are not independent parameters themselves, but rather they are function of the radioactive particle position as well as the direction at which the gamma rays are emitted. Furthermore, in situations of a heterogeneous reactor inventory, the value of $\mu_r$ would be a function of the spatial distribution of the material, i.e. a function of radioactive particle position as well.

All the above mentioned analytical difficulties led to the common practice in RPT community of simplifying the problem by using semi-empirical modeling techniques to obtain linear and non-linear parameters of the above model for a particular system by a series of calibration experiments, and thereafter use the fitted model to obtain particle positions in real tracking measurements. Alternatively, a Monte Carlo simulation is being used in place of experimental calibration to either obtain the needed model parameters, or
to obtain a map of gamma ray counts for a large number of point throughout the system, where each point of the computer generated map gives the 3-D coordinates of the tracer and the corresponding gamma ray counts that would be accumulated by the detector.

2.4 Inverse Problem “Counts mapping and position rendition”:

Three methods have been reported in the literature to determine the location of the particle using the counting rates recorded by the detectors. In the first, the detector is considered as a “virtual point” and the distance to the particle is a polynomial function of the number of counts recorded. For each detector, a polynomial is fitted to calibration data obtained by positioning the tracer at known locations in the column and the position is computed using a weighted least-squares method based on an exact linearization scheme (Lin et al. 1985, Moslemian et al. 1992). The second method uses a rigorous phenomenological model which describes the interactions of radiation with matter for the geometry of the system. A map of detector counts versus location is generated by Monte Carlo simulation and is adjusted to calibration measurements. Subsequently, each location of the tracer is determined by a least-squares search for the grid point on the map which best matches the counts registered in the detectors (Larachi et al. 1994). The third method is an enhanced algorithm where the searches of the second method are replaced by a direct and faster backpropagation neural network model (Godfroy et al. 1996).

Detailed description of the above mentioned methods can be found in the respective references. However, we have employed some of those methods in an attempt to establish a thorough investigation and a subjective critique of the methods. Details of that investigation is to be presented in the following chapter.
3. A DETAILED STUDY OF TRADITIONAL TOMOGRAPHIC RADIOACTIVE PARTICLE TRACKING

The purpose of such a study was to learn and examine the common practice of the conventional tomographic RPT for possible limitations and difficulties when used in different RPT applications. In that study a simplified experimental setup was used, and employed most of the technical procedures described in the literature as reviewed in the previous chapter, including experimental calibration, physical nonlinear modeling, Monte Carlo modeling, and the nonlinear least-squares search. At the end the study reached a number of important conclusions that we used to critique those methods, and identify objectives to overcome the limitations of them.

3.1 Experimental Setup:

In this experiment, a simplified setup was established to carryout the RPT measurements. The reactor, or the bed that was used track the tracer inside was a steel ball mill vessel. The vessel was place on a motor driven rollers that can rotate the vessel consciously at a constant speed. The vessel was surrounded by four sodium iodide scintillation detectors, who will perform the RPT calibration experiments as well as the
actual RPT measurements. Figures 3.1 through 3.3 show a photograph of the system, along with a schematics of the experimental setup.

Figure 3.1: A photograph of the ball mill experimental setup for RPT experiment

Figure 3.2: A Schematic of the side view of the four detectors arrangement
Along with the vessel and detectors arrangement described above, there were the necessary detection electronics. Those included high voltage supplies, linear amplifiers, and single channel analyzers for low level pulse discrimination. In addition, the Sparrow Multi-parameter (Model 1000 Mini-crate) multi-channel analyzer, along with the Kmax software was used to acquire and record the gamma spectra in a multi scalar mode (i.e. in a consecutive dwelled time intervals).

### 3.2 The Dual Energy Approach:

As mentioned earlier with tomographic RPT accuracy and spatial resolution is strongly dependent on the number of detectors used. In a system similar to the we have in hands, the number of detectors needed for RPT would be not less than eight detectors as a minimum. Since the start of the study we had the objective of reducing as much as possible the number of detectors to be used without scarifying much of the RPT accuracy and the spatial resolution of the particle’s positions. Since this method of RPT depends basically on the detection and recording of full energy non-interacting gamma photons,
we had the idea of using a radioactive particle with two “clean” energies. In this case one can record the full energy peaks of both energies independently and simultaneously and then each one of the peaks can be used in system calibration and counts mapping. In this way the number of detectors would be virtually doubled in a way that increases data redundancy for statistical accuracy enhancement purposes. What we mean here by two “clean” energies is that the two energies are separated enough to be sufficiently resolved with the energy resolution of the detectors in hands. Examples of such radioisotopes are $^{60}$Co, $^{46}$Sc, and $^{24}$Na. In the present study a 180 µCi $^{60}$Co source was used. In this case either two SCA’s with a “window” in each or an MCA with two ROI’s can be used for each detector doubling the data available from the same number of detectors.

3.3 Experimental Results and Analysis:

Figure 3.4 shows the raw data collected for one of the energy peaks for each one of the four detectors versus the varying source position, as the cylinder rotates around.

![Figure 3.4: Responses of each of the four detectors vs. radioactive particle position](image-url)
The radioactive particle was attached to the inner surface of the drum which was held rigidly to a fixed coordinates (x, y, z) in each measurement. The calibration data were then taken, where the exact location (x,y,z) of the tracer was readily known while the photo-peaks intensity detected by each detector was acquired by the detection system described in the previous chapter. The calibration continued until all detectors were through.

It was mentioned in section 2.4 that there are three methods to solve the inverse problem of RPT. Two of those methods were employed in this study for the purpose of both demonstration, and investigation. The two used methods are namely, the polynomial fitting to calibration data, and a subsequent weighted least-squares method based on an exact linearization scheme, and the phenomenological Monte Carlo mapping of detector counts versus location, and a subsequent least-squares search for the grid point on the map which best matches the counts registered in the detectors. Each of these two methods is described in more details in the following two sections.

### 3.4 Polynomial Treatment of Calibration and linearization scheme:

In this approach the detector was considered as a virtual point, and the distance between this point and the point where the tracer is located is expressed in terms of a polynomial function of the detector’s response, as follows:

\[
    r_i = f\left(\frac{1}{I}\right) = a_{oi} + \frac{a_{1i}}{I} + \frac{a_{2i}}{I^2} + \frac{a_{3i}}{I^3} \tag{3.1}
\]

where \( r_i \) is the distance between the tracer and the \( ith \) detector, and \( I \) is the photo-peak intensity detected by the \( ith \) detector. The result of such fitting is shown in figure 3.5 for
one of the detectors for the two photo-peaks of interest. Typical fittings would be carried out for each one of the other detectors.

\[ Y = -19.45046 + 1454.5169 X - 18161.5568 X^2 + 106677.18268 X^3 \]

\[ \frac{1}{\text{Intensity (x10^4)}} \]

Data

Polynomial Fit of Det1Peak2_B

Distance (CM)

1/Intensity (x10^4)

\[ Y = 52.53549 - 724.92727 X + 4144.3388 X^2 - 6198.5142 X^3 \]

Figure 3. 5: Polynomial fitting of calibration data acquired by detector.

The availability of eight distance measurements from four detectors (two photo-peaks from each detector) provided data redundancy for the location determination. To take advantage of this redundancy, a weighted least square method based on an exact linearization algorithm was used. The distance between the tracer and the ith detector is governed by the relationship;

\[ r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 \]  \hspace{1cm} (3.2)

where, \((x, y, z)\) denotes the coordinates of the tracer to be solved, \((x_i, y_i, z_i)\) the coordinates of the center of the ith detector, and \(r_i\) the distance to this detector.

Physically, the above equation implies that each detector generates a sphere centered at \((x_i, y_i, z_i)\) with radius \(r_i\). through the tracer \((x, y, z)\). Thus \((x, y, z)\) can always be determined uniquely by four equations, employing one of them to resolve the ambiguity.
of multiple solutions if only three equations have been used. However, the solutions obtained by solving any set of four equations out of \( n \) equations vary, because of the inherent fluctuation of gamma ray count rate and the changing attenuation.

An algorithm to determine the optimum solution is a weighted linear regression method that will be described. Rearranging terms in equation 3.2 yields:

\[
\begin{align*}
  r_i^2 - x_i^2 - y_i^2 - z_i^2 &= (x^2 + y^2 + z^2) - 2xx_i - 2yy_i - 2zz_i \\
  &\quad (3.3)
\end{align*}
\]

Equation 3.3 can be linearized as follows;

Let

\[
\mu = (x^2 + y^2 + z^2)
\]

Thus, equation 3.3 becomes;

\[
\begin{align*}
  r_i^2 - x_i^2 - y_i^2 - z_i^2 &= \mu - 2xx_i - 2yy_i - 2zz_i \\
  &\quad (3.4)
\end{align*}
\]

For, \( i = 1,2,3,\ldots n \) detectors.

In matrix form, equation 3.4 can be written as follows:

\[
\begin{bmatrix}
  1 & -2x_1 & -2y_1 & -2z_1 \\
  1 & -2x_2 & -2y_2 & -2z_2 \\
  \vdots & \vdots & \vdots & \vdots \\
  1 & -2x_n & -2y_n & -2z_n
\end{bmatrix}
\begin{bmatrix}
  X \\
  \beta
\end{bmatrix} =
\begin{bmatrix}
  \mu \\
  \beta
\end{bmatrix}
\]

Let;

\[
|\beta| =
\begin{bmatrix}
  \mu \\
  X \\
  Y \\
  Z
\end{bmatrix}_{4x1}
\]

and;

\[
|Z| =
\begin{bmatrix}
  r_1^2 - x_1^2 - y_1^2 - z_1^2 \\
  r_2^2 - x_2^2 - y_2^2 - z_2^2 \\
  \vdots & \vdots & \vdots \\
  r_n^2 - x_n^2 - y_n^2 - z_n^2
\end{bmatrix}_{n\times4}
\]
Thus;

\[ |X| |\beta| = |Z| \]  \hspace{1cm} (3.5)

Note that \( |X| \) is a constant depending on the measurement arrangement and consists of the coordinates of the centers of \( n \) detectors, \( |\beta| \) consists of the coordinates of the tracer, and \( |Z| \) consists of the observed distances from each detector. It is noted that equation 4.5 is valid only when the measurements of \( r_1, r_2, \ldots, r_n \) are of zero variance. Therefore a more realistic form is;

\[ |Z| = |X| |\beta| + |E| \]  \hspace{1cm} (3.6)

where,

\[ |E| = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \]

is the error vector of the estimation for \( |\beta| \). According to the statistical nature of gamma ray, it can be assumed that \( E_i \) is a random variable with variance \( \sigma_i^2 \). The least square estimate of \( |\beta| \) can be obtained by differentiating error sum of squares \( |E|^T x |E| \) with respect to each element in \( |\beta| \). This results in the normal equations of regression (Draper N. R., and Smith H. 1966).

\[ \left( |X|^T |X| \right) |\beta| = |X|^T |Z| \]  \hspace{1cm} (3.7)

where, \( |\beta| \) is the estimated value of \( |\beta| \) to minimize \( |E|^T x |E| \). Therefore,

\[ |\beta| = \begin{bmatrix} \mu_e \\ x_e \\ y_e \\ z_e \end{bmatrix} = \left( |X|^T |X| \right)^{-1} |X|^T |Z| \]  \hspace{1cm} (3.8)

Moreover, according to (Draper N. R., and Smith H. 1966), it can be shown that the variance and covariance of the estimates can be given by;
\[|V(b)| = \left(\|X' X\|^1\right) \sigma^2 \]  

(3.9)

Where, the diagonal terms are the variance and the off-diagonal terms are the covariance.

It should be noted that equation 3.8 implies that at least 4 detectors are required to provide non-singular determinant of \(\|X' X\|\). Also equation 3.9 shows that the resolution of location determination can be improved by increasing the value of the determinant \(\|X' X\|\) as well as by decreasing the fluctuation of signals. Therefore, to reduce the uncertainty of location determination it is necessary to both increase the number of detectors, and in the same time uniformly distribute detectors around the cylinder. These steps would increase the values of:

\[
\sum_{i=1}^{n} (x_i - \bar{x}_b)^2 ,
\]

\[
\sum_{i=1}^{n} (y_i - \bar{y}_b)^2 ,
\]

and

\[
\sum_{i=1}^{n} (z_i - \bar{z}_b)^2
\]

where

\[
\bar{x}_b = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

\[
\bar{y}_b = \frac{1}{n} \sum_{i=1}^{n} y_i
\]

\[
\bar{z}_b = \frac{1}{n} \sum_{i=1}^{n} z_i
\]

and therefore will increase the determinant of \(\|X' X\|\).

It should be noted that distance measurements by the detectors are not equally accurate. Thus, in order to be able to obtain an optimal solution for equation 3.8, an
appropriate weighting function must be introduced. As a result, equation 3.8 is modified such that the diagonal elements of the variance of $|E|$ are equal. Since $E_i$ and $E_j$ are uncorrelated for $i \neq j$, the variance of $|E|$ becomes,

$$
|\mathcal{V}(E)| = |W|\sigma^2 = \begin{bmatrix}
\sigma_i^2 & 0 \\
0 & \sigma_j^2 \\
0 & \sigma_k^2 \\
\end{bmatrix}
$$

(3.10)

So far, linear regression theory is being used and equation 3.5 can be written as

$$
|X|^T |W|^{-1} |X| |b| = |X|^T |W|^{-1} |Z|
$$

(3.11)

where, $|W|$ is the weighting function defined by;

$$
|W|^{-1} = \begin{bmatrix}
\frac{1}{\sigma_i^2} & 0 \\
0 & \frac{1}{\sigma_j^2} \\
0 & \frac{1}{\sigma_k^2} \\
\end{bmatrix}
$$

(3.12)

An empirical relation for $\sigma_i$ was used (Lin, J.S., M.M. Chen, and B.T. Chao, 1985), suggesting that;

$$
\sigma_i = r_i^3
$$

(3.13)

Therefore, the tracer position is given by solving for the vector $|b|$, which can be obtained from equation 3.11 as follows;

$$
|b| = (|X|^T |W|^{-1} |X|)^{-1} |X|^T |W|^{-1} |Z|
$$

(3.14)
and its variance is given by;

$$V(b) = \left( |X|^T |W^{-1}| X \right)^{-1} \sigma^2 \tag{3.15}$$

Figure 3.6 shows the results of the above algorithm.

Figure 3.6: Tracer position as obtained by weighted linear regression method vs tracer position.

3.5 **Non-Linear and Monte Carlo Modeling:**

It was mentioned in section 2.3 when describing the forward problem of RPT, that there are a number of serious analytical difficulties in applying an exact model.
correlating detector’s counting rate to particle’s position. The practical solution is simplifying the problem by using semi-empirical modeling techniques to obtain linear and non-linear parameters of the exact model (equation 2.1) for a particular system by a series of calibration experiments, and thereafter use the fitted model to obtain particle positions in real tracking measurements. We have adapted this approach and applied it to the dual energy data we have in hands. The following nonlinear model was fitted to the data we have:

\[
C_i = A \times \left\{ \frac{\left( e^{-\mu_R S_R} \times (1 - e^{-\mu_D S_D}) \right)}{r_i^2} \right\} + B
\]  \hspace{1cm} (3.16)

where, \(C_i\) is total photo-peak counts of \(i_{th}\) detector, \(r_i\) is distance from tracer to \(i_{th}\) detector, \(\mu_R\) is the attenuation coefficient of mill wall and charge, \(S_R\) is the distance traveled in wall and charge, \(\mu_D\) is the attenuation in detector, \(S_D\) is the distance traveled in detector, \(B\) is background, and \(A\) is a constant proportional to the source intensity. The results of fitting this model to the obtained empirical data are shown in figures 3.7 and 3.8.
\[ C = (K/r^2)e^{-\mu \omega w}(1-e^{-\mu d}) + B \]

- \( K = 9993.39 \)
- \( \mu_w = 7.11260 \)
- \( \mu_d = -22.6997 \)
- \( B = 766.287 \)

**Figure 3.7:** Fitting results of nonlinear model vs. detector response at different positions

**Figure 3.8:** The same fitting results in terms of the distance \( r \) between center of detector and source position
It can be shown that the dual energy approach provided a reliable redundancy of data, that eventually resulted in an obvious good model fitting results as can be seen from the figures above. Alternatively, a Monte Carlo simulation can be used in place of experimental calibration to either obtain the needed model parameters, or to obtain a map of gamma ray counts for a large number of point throughout the system, where each point of the computer generated map gives the 3-D coordinates of the tracer and the corresponding gamma ray counts that would be accumulated by the detector. Such a map was generated in the present study, and it is presented in figure 3.9:

Figure 3.9: Monte Carlo generated map of detector’s response vs. different particle’s positions
Figure 3.10 shows a comparison between a sample Monte Carlo calculated detector counting rate and the corresponding measured one.

It can be seen clearly the discrepancies between simulation and experimental data. A first examination of the above figure indicates that the discrepancy is consistently increasing as the counting rate increases. It is also observed that the measured counting rates are always less than the corresponding simulated counting rates. Those two observations show that the discrepancy is associated with high counting rates and in the form of measured counting rates that are less than they should be. This clearly indicates that the counting rate losses are due to detector dead time and pulse pileup, which are the
typical counting rate losses associated with high counting rate measurements. However, the experimental data shown in figure 3.10 were corrected for detector’s dead time using the non-paralizable model (G. Knoll 1990). Therefore, the pulse pileup is a major reason for high counting rate losses. In any case, those losses will be one of the basis upon which we will build our critique of the current RPT methods.

3.6 Critique on Present RPT Techniques:

The RPT techniques described above have some serious limitations and disadvantages. Three very well identified problems will be mentioned and described here. First of all, in order to accurately track a radioactive particle it was shown the one would need a large number of detectors for tracking, and their gain and zero would have to be known at all times. As mentioned previously, it is sufficient in principle to use three detectors, and the particle position would be the intersection of the three spheres corresponding to the three detectors. However, this is not practical since the process involves a least-squares search for position rendition, whether using calibration data, a physical model, or Monte Carlo count maps. Position reconstruction in this way (least-squares search) inherently contains a certain amount of uncertainty, and in order to overcome these errors, it is necessary to increase the number of detectors to have more accurate particle positions and/or better spatial resolution.

The second problem would be particularly significant when tracking a radioactive particle in heterogeneous stochastically distributed granular media in beds or columns, such as pebble beds, fluidized resin ion exchangers, mills and powders mixers, etc. The fact that stochastic granular media are randomly packed within the bed adds difficulties
and uncertainties to the modeling of radiation transport through the bed. This can be seen on the following simulation results. To investigate the role of different packing on the radiation transport modeling, we have carried out a series of Monte Carlo simulations with pebbles packed differently inside a vessel and calculate the responses of an array of detectors located outside the vessel. This is a situation very similar to situation of using tomographic radioactive particle tracking. The Monte Carlo models used in this study are shown in the figures 3.11 and 3.12 below.

![Figure 3.11: Cubic packing of pebbles inside a vessel with a porosity of 0.476.](image1)

![Figure 3.12: Orthorhombic (Clear Passage) packing of pebbles inside a vessel with a porosity of 0.3954.](image2)
The results of these simulations are shown in figures 3.13 and 3.14 below. Figure 3.13 shows a comparison between the total responses of the detectors corresponding to the packings described above, while figure 3.14 shows the same comparison for the full energy responses of the detectors.

Figure 3.13: Total responses of detectors for two different packings.

Figure 3.14: Full energy responses of detectors for two different packings.
It can be seen that different packings have resulted in changes of detectors’ responses. These changes are observed in value as well as the symmetry of the responses. In conclusion, radiation transport modeling in a heterogeneous random packing can result in uncertainties in the simulated responses that can prevent proper application to Monte Carlo mapping of the radioactive particle positions. Therefore, computational techniques, based on normal radiation transport modeling, may not be effective for application to particle tracking in such beds.

Finally, losses (e.g. dead time and pulse pileup) associated with high counting-rate measurements typical for particle tracking measurements, when a wide range of distances from the particle to detectors is involved, most likely will require further corrections to the measurements, which in turn adds more difficulties and makes the techniques particularly inappropriate for online tracking measurements.

The above mentioned limitations represented a justification for searching and investigating alternative that might overcome all or some of those limitations. Such alternative will be presented in the following chapter.
4. AN ALTERNATIVE THREE-DETECTOR

METHOD FOR RADIOACTIVE

PARTICLE TRACKING

In search of alternative approaches to overcome the limitations of the tomographic RPT just described, an interesting experimental study was found that was carried out by a research group in Australia (Gatt et. al 1973) and (Geeson et. al. 1973) for pebble bed nuclear reactors. An experimental investigation was made of the transit of individual pebbles in random packing of equal size spherical pebbles. Pebbles are used to form beds in a vessel with a concave base and single axial outlet. The pebbles flow downwards under the influence of gravity through a rotating extractor in the base. A radioactively tagged pebble is inserted at the top of the bed and as it descends with the pebble packing its position is detected and recorded at intervals using a tracking device. In this way tracks and velocities of pebbles seeded in the bed are determined, and were found to be generally of ‘streamline’ form. Using the tracks of tagged pebbles, patterns and effects of flow zones could be deduced, which is very similar to flow characterization studies for fluidized beds.
4.1 **Review of Previous Work on the Approach:**

In 1973 the Australian atomic energy committee carried out an experimental investigation on the flow behavior of pebbles flowing in a pebble bed reactor. In this experiment, which investigates pebble paths and velocities in the bed, the pebbles used simulate pebble bed nuclear reactor fuel elements in size, shape, surface and density. An outline of the pebble bed is given in figure 4.1. The vessel is filled to the required height with pebbles, thus forming the bed. A radioactively tagged pebble is inserted at the top of the bed and extraction and recycling of the bed is commenced using the extractor and the conveyor. As the tagged pebble descends with the bed, its position is detected and recorded at intervals using the tracking device. When the tagged pebble emerges it is separated from the main stream and the experiment is stopped.

![Figure 4.1: An outline of a Pebble Bed](image-url)
Throughout each trial a tagged pebble of the same sphericity, diameter, specific gravity and surface finish as those constituting the pebble bed was used. A typical tagged pebble is shown in figure 4.2.

![Figure 4.2: Tagged Pebble](image)

The tagged pebble was designed so that it had a small hole drilled to allow insertion of a radioactive $^{60}$Co source. After insertion of the source a close fitting zirconite plug is fitted and glued flush with the pebble surface. The plug has a convex end so as to continue the outer spherical surface of the pebble. To achieve satisfactory tracking of the tagged pebble, the activity of the sources used must account for the attenuation by the bed during their movement through the vessel. Maximum activity used was approximately 0.5 curie for the pebble moving through the center of the vessel.

Movement of the tagged pebble, seeded at the different radial positions, is followed by a tracking device illustrated in figure 4.3.
The tracking system uses a group of three collimated radiation detectors placed outside the vessel to obtain the position of the tagged pebble. The three detectors are mounted on a platform whose height can be varied. First, the center detector (with Horizontal slits), is used to establish the vertical position of the tagged pebble, and then the outer pairs (with vertical slits) rotate until both are aligned with it. Its location is then defined by the height of the platform and the angular position of the outer detectors. The equipment was designed to locate the tagged pebble in successive positions automatically (e.g. at given time intervals), display the results and record them for computer processing.
4.2 Preliminary Feasibility Study:

In this section we will describe preliminary experiments that were conducted to investigate the tracking technique described above. The objectives of this experiment were to investigate the potential of the technique and investigate the limitations through some sensitivity and error analysis. Such a study could give us, as will be shown in the discussions, an idea about the feasibility of the technique as well as the requirements needed for an accurate, reliable, and informative tracking system.

For the purposes of the present experiments, a collimated detector was used to manually track a radioactive source (\(^{137}\text{Cs}\)) in air. The experiment was carried out in a two-dimensional domain by fixing the height of the source. A coordinates transformation algorithm was established to deduce the cylindrical 2-D coordinates \(r\) and \(\theta\). Figure 4.4 shows a schematic of the layout of the pebble tracking system described above. This schematic represents the top view of the layout.

![Figure 4.4: A schematic Layout of the Measurement Setup](image-url)
From the geometry shown, one can write;

\[ KP = RK \tan \alpha_1 \quad (4.1) \]

and

\[ KP = KQ \tan \alpha_2 \quad (4.2) \]

Thus,

\[ \frac{KQ}{RK} = \frac{\tan \alpha_1}{\tan \alpha_2} \quad (4.3) \]

Thus, combining equations (4.3) and (4.4) yields;

\[ RK + KQ = L \quad (4.4) \]

But since,

Thus combining equations (4.3) and (4.4) yields;

\[ \frac{\tan \alpha_1}{\tan \alpha_2} = \frac{L - RK}{RK} = \frac{L}{RK} - 1 \]

\[ \therefore \frac{L}{RK} = \frac{\tan \alpha_1 + 1}{\tan \alpha_2} \]

& \[ RK = \frac{L}{\tan \alpha_1 + 1} \]

\[ \therefore RK = \frac{L \tan \alpha_2}{\tan \alpha_1 + \tan \alpha_2} \quad (4.5) \]

From the geometry in figure 4.4;

\[ PT = \frac{L}{2} - RK \]
Substituting for \( RK \) from equation 4.5 results in;

\[
PT = L \left[ \frac{1}{2} - \frac{\tan \alpha_2}{\tan \alpha_1 + \tan \alpha_2} \right] \tag{4.6}
\]

Similarly, from equation 4.1 \( KP \) is given by,

\[
KP = RK \tan \alpha_1
\]

and by substituting for \( RK \) from equation 4.5 we have;

\[
KP = \left[ \frac{L \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 + \tan \alpha_2} \right] \tag{4.7}
\]

and thus \( OT \) is given by;

\[
OT = S - \frac{L \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 + \tan \alpha_2} \tag{4.8}
\]

Now, \( r \) and \( \theta \) can be calculated from,

\[
r = \sqrt{(OT)^2 + (PT)^2} \tag{4.9}
\]

\[
\theta = \tan^{-1} \frac{PT}{OT} \tag{4.10}
\]
The above derivation shows that, the deduced coordinates $r$ and $\theta$ are functions of $\alpha_1$, $\alpha_2$, $L$, and $S$. The dimensions of the experimental setup determine the values of $L$ and $S$, and they are fixed. However, $\alpha_1$, $\alpha_2$ are the measurable independent variables that determine the coordinates $r$ and $\theta$ at any given instant. Therefore, it is important to find out how precise $\alpha_1$, $\alpha_2$ measurements need to be in order to achieve a specific level of accuracy of $r$ and $\theta$. This can be seen by calculating the rate of change of $r$ and $\theta$ over some range of change of $\alpha_1$, $\alpha_2$. Figure 4.5 shows qualitatively the results of such sensitivity analysis.

![Figure 4.5: Results of Sensitivity Analysis](image)

It can be seen from the figure, that the surface of the values of $r$ and $\theta$ as a function of $\alpha_1$, $\alpha_2$ is quite steep, and thus small changes of $\alpha_1$, $\alpha_2$ result in appreciable changes of $r$ and $\theta$, implying large sensitivity. This result is very important if one is to increase the precision of the coordinates of the tracked pebble. In this case a greater precession of the measured angles $\alpha_1$, $\alpha_2$ is crucial for accurate and precise measurements.
The accuracy of the values of $\alpha_1$, $\alpha_2$ need to be enhanced in two ways. The first, enhancing the resolution of the measuring device responsible for reading out the values of $\alpha_1$, $\alpha_2$. This can be accomplished by considering the use of highly accurate sophisticated displacement sensors, such as optical encoders, employing laser technology. Some of these sensors have a resolution up to 0.00025 degrees. The second required enhancement is related to condition upon which $\alpha_1$, $\alpha_2$ are measured. Recall that, whenever $\alpha_1$, $\alpha_2$ are to be measured; this should be corresponding to a detected maximum of the output signal of the detectors. Therefore, the way that maximum is isolated and specified along with the corresponding $\alpha_1$, $\alpha_2$ will affect the final results of $r$ and $\theta$. An illustration of an effective method for this purpose will be shown in the next section.

The response of a collimated rotating detector as a function of the rotation angle of the detector is shown in figure 4.6.

![Figure 4.6: Detector Response as a function of the angular position of detector](image)
As mentioned before, the key point in measuring precisely any given angular position of the detector is the identification of the angle corresponding to the maximum counting rate. In order to investigate an appropriate mathematical approach to extract those maxima, a Monte Carlo simulation study was carried out similar to the one described in section 3.6. In this case the responses of a collimated detector with a slot opening were simulated corresponding to different source positions along the vertical direction as shown on figures 4.7 below showing the Monte Carlo models constructed for this simulation.

Figure 4. 7: Monte Carlo Model used to simulate collimated detector’s responses.
Figure 4.8: Monte Carlo simulated response of a collimated detector.

Figure 4.8 shows the result of the Monte Carlo simulation of the collimated detector corresponding to various source positions. Figures 4.9 and 4.10 show a comparison between two fitting schemes of the points around the maximum to conclude the most appropriate approach to identify the maximum detector response and the corresponding source position.

Figure 4.9: Five points maximum identification scheme.
The source is presumably located exactly at position 0.0 mm. Using the five points maximum identification scheme shown above resulted in a maximum corresponding to a position of 0.0343 mm. Carrying the same procedure with a three points scheme as shown in the figure below; resulted in a maximum that is corresponding to source position exactly located at 0.0 mm. It is obvious that the three points scheme is far more accurate, and therefore, the mathematical approach used in the present study for identifying such a maximum will be base on using two points surrounding the maximum along with the peak point and fit them to a parabola as illustrated in figure 4.11 (Bevington 1969).
As shown in figure 4.11, the parabola fit very well to the given three points, and since such parabola is given by;

\[
y = ax^2 + bx + c
\]  
(4.11)

where \( x \) is representing the angular position of the detector (\( \alpha_1 \) or \( \alpha_2 \)), and \( a, b, \) and \( c \) are the fitting coefficients. Therefore, the maximum can be identified in usual way of differentiating and equaling to zero;

\[
\frac{\partial Y}{\partial x} = 0 = 2ax + b
\]  
(4.12)

and thus, the angle corresponding to the maximum is given by,

\[
\therefore \alpha = \frac{-b}{2a}
\]  
(4.13)
4.3 Preliminary Experimental Results and Error Analysis:

Following the procedure described above, and applying the algorithms discussed earlier, a few preset points were selected for positioning the radioactive source, and the particle coordinates \((r \text{ and } \theta)\) were deduced in the way described above. The results of such measurements are presented in figure 4.12.

![Figure 4.12: Preliminary tracking results compared to actual source positions.](image-url)
As can be noticed from the Figure, some errors are associated with the measurements in the range of few percents of the preset actual positions. These errors will be discussed in the following. The errors mentioned above in the results of determining the coordinates of the radioactive particle can be classified into two major categories. The first is the error associated with the uncertainties of specifying the maximum counting rates and the corresponding angular positions of the detector through the parabolic fitting described above, since there is always an inherent error with the fitting process. The second category is the systematic error resulting from the precision and accuracy limitations of the measurement system itself (where the term systematic implies) (Gardner, and Ely).

In order to quantify those errors, the standard error propagation techniques are used to indicate the level of confidence in the obtained measurements. The standard error propagation calculation is based on obtaining the error of the estimated quantity as a function of the errors of the individual independent measured quantities in terms of their standard deviation. The standard deviations $\sigma_r$ and $\sigma_\theta$ of $r$ and $\theta$ can be obtained from;

$$\sigma_r^2 = \left( \frac{\partial r}{\partial \alpha_1} \right)^2 \sigma_{\alpha_1}^2 + \left( \frac{\partial r}{\partial \alpha_2} \right)^2 \sigma_{\alpha_2}^2 + \left( \frac{\partial r}{\partial L} \right)^2 \sigma_L^2 + \left( \frac{\partial r}{\partial S} \right)^2 \sigma_S^2$$ (4.14)

$$\sigma_\theta^2 = \left( \frac{\partial \theta}{\partial \alpha_1} \right)^2 \sigma_{\alpha_1}^2 + \left( \frac{\partial \theta}{\partial \alpha_2} \right)^2 \sigma_{\alpha_2}^2 + \left( \frac{\partial \theta}{\partial L} \right)^2 \sigma_L^2 + \left( \frac{\partial \theta}{\partial S} \right)^2 \sigma_S^2$$ (4.15)

Where $\sigma_{\alpha_1}$, $\sigma_{\alpha_2}$, $\sigma_L$, and $\sigma_S$, are the standard deviations of $\alpha_1$, $\alpha_2$, $L$, and $S$ (see figure 4.4), respectively. Carrying out the derivatives in equations 4.14, and 4.15;
\( \left( \frac{\partial r}{\partial \alpha} \right)^2 = \frac{2L}{2 - \tan \alpha + \tan \alpha_2} \left[ \frac{\tan \alpha_2}{(\tan \alpha_1 + \tan \alpha_2)^2} \left( \tan \alpha + (\tan \alpha_1)^2 \right) \right] + \frac{2L}{2 - \tan \alpha + \tan \alpha_2} \cdot \left[ S - L \tan \alpha \tan \alpha_2 \left( \tan \alpha + (\tan \alpha_1)^2 \right) \right] \left( \frac{L \tan \alpha, tan \alpha_2}{(\tan \alpha_1 + \tan \alpha_2)^2} \right) \left( \tan \alpha + (\tan \alpha_1)^2 \right) \right] \end{align*}
The results of such error analysis are shown in figure 9 for another set of points in the form of an associating error bars with the estimated particle positions.
In this analysis the values of $\sigma_{\alpha_1}$, $\sigma_{\alpha_2}$, are assumed to be equal to $\pm 0.5$ degree which corresponds to the minimum readable angle in the present setup, and $\sigma_L$ and $\sigma_S$ were assumed to be 1 mm.

It can be seen that in spite of the many limitations of the present measurement system, the results of the estimated particle’s coordinates are within the anticipated errors of such measurements, indicating a good feasibility for the technique, and promising a much more reliable result after the implementation of automation and control technology. Therefore, this encouraged us to implement the approach, and build an integrated tracking system, and fully automate the system with the implementation of high tech electromechanical systems, that greatly enhanced the accuracy and ability.
5. IMPLEMENTATION OF THE THREE-DETECTOR RPT AND SYSTEM BENCHMARKING

In this chapter we will describe the implementation and benchmarking of a tracking system based on the three detectors dynamic tracking approach. The system was designed and built at the Center for Engineering Applications of Radioisotope, North Carolina State University. The many parts of the system was designed and constructed from scratch like collimators, platform, motion mechanics …etc. Other parts of the system were adapted from commercially available components like stepper motors, motion controllers and motor drives …etc. We will first start with some design aspects of some crucial parts of the system. In particular, we will describe design optimization of the collimators in order to obtain the best possible results with regard to spatial resolution and accuracy.

5.1 Collimator Design Optimization:

As was mentioned previously the tracking concept is based on using well collimated detectors with a slotted openings, so that the detectors’ responses will be maxima when the collimated detectors are oriented in a way that the slot is directly facing (on line of sight) the radioactive particle. However, the observed maxima are required to
be very well resolved and isolated in order to establish a high resolution positioning. A series of analytical and experimental studies were conducted to investigate the effect of different design parameters of the collimator on the observed detectors’ maxima. As will be seen two main design parameters were found to be of greatest effect, namely, the slot width and the slot depth. Figure 5.1 shows a schematic of the front part of the collimator with the slot opening, and defines the above mentioned two design parameters.

Figure 5.1: A schematic of the slot opening portion of the collimator.

Where $W$ is the collimator’s slot width, and $D$ is the collimator’s slot depth. In the above figure assume that the source is located at a distance $S$ from the outer side of the collimator. From the geometry in the figure it can be seen that the collimator’s opening has a Range Of View “$ROV$” that can be easily seen to be given by:

$$
Range of View (ROV) = (R - \frac{W}{2}) + (R - \frac{W}{2})
= 2R - W
$$

(5.1)
Also from the geometry shown in figure 5.1 it can be shown that triangles $abc$ and $ade$ are similar. Thus;

$$\frac{W}{D} = \frac{R}{(S + D)}$$

$$\therefore R = \left(\frac{W}{D}\right) \times (S + D) \quad (5.2)$$

Now we carry on with the algebra;

$$\therefore ROV = 2R - W$$

$$= \frac{2W}{D} (S + D) - W$$

$$= W \left[ \frac{2(S + D)}{D} - 1 \right]$$

$$= W \left[ \frac{2S}{D} + 2 - 1 \right]$$

Thus,

$$\therefore ROV = W \left( \frac{2S}{D} + 1 \right) \quad (5.3)$$

Therefore, if we are to maximize the resolution of the position of the source we have to minimize $ROV$. Thus from equation 5.3 minimum $ROV$ would require a minimum $W$, as well as a maximum $D$. To verify the above analysis regarding collimator’s resolution, an experimental study was carried out with two collimators of two different sets of design parameters. The first collimator used in that study had a slot width $W$ of 3mm and a slot depth $D$ of 25.4 mm. According to equation 5.3, the $ROV$ of the collimator is 26.6 mm for a source located at a distance $S$ of 100 mm. The second collimator used has a value of $W$ of 1 mm and a value of $D$ of 50.8 mm. Thus in this case, for a source located a same distance $S$ of 100 mm, $ROV$ is equal to 4.94 mm. That is more than five times reduction in the value of ROV, i.e. five times enhancement in the collimator resolution. Figure 5.2
shows a comparison between the responses between two detectors each collimated with one of the above described collimators.

![Graph showing comparison between two differently collimated detectors.](image)

**Figure 5.2: A comparison between responses of two differently collimated detectors.**

It is clear from the figure above the degree of enhancement of the resolution of the observed maxima. However, the figure also shows an almost two orders of magnitude reduction in the counting rate. Therefore, a tradeoff compromise must be considered that would also include the source activity to obtain a good resolution as well as a reasonable signal strength to avoid worsening the statistics of the counting rate.

As well, a series of Monte Carlo simulations of the responses of the collimated detectors were carried out to investigate the reliability of collimator design, specifically to investigate the effect of attenuating media present between the radioactive particle and
the collimated detector. Figures 5.3 and 5.4 show the Monte Carlo model investigated and a sample result of such simulation.

Figure 5.3: Monte Carlo model used to simulate the response of collimated detector in the presence of an attenuating medium.

Figure 5.4: Sample result of the Monte Carlo simulation of the response of the collimated detector.
5.2 Design and Construction of the RPT Platform:

Figure 5. 5: A photograph of the RPT system Constructed.
Figure 5.5 above shows a photograph of the tracking system constructed at CEAR based on the three detectors tracking concept. The system is composed of an aluminum platform that accommodated the three collimated detectors. The two side detectors are placed on a rolling bearings that are free to rotate, and are coupled to the motors through a timing pulleys driven by timing belts. The timing belts are found to be optimum for accurate positioning and to prevent slippage. The platform is moved vertically a whole by two vertical side screws, which are driven by a separate motor and through timing pulleys and a timing belt, as well. The screws are coupled to the platform through two large brass nuts attached to the platform. The system can accommodate vessels of sizes up to 60 cm diameter by 80 cm height.

Underneath the platform, there are the electronics used by the tracking system. These include high voltage power supplies supplying high voltage to the detectors, linear amplifiers to magnify signals produced from detectors, and single channel analyzers used to discriminate against low level pulses. At the top of the system (a close up photo is shown at right bottom), there is a source positioning mechanism specially designed to hold the radioactive particle at known coordinates, for the purposes of testing, verifying, and benchmarking the tracking system.

5.3 Development of Motion Control:

The tracking system is required to be fully automated and computer controlled with minimal intrusion during the course of a tracking experiment. Therefore, and for this purpose it was required that a computer to perform control of the motion and running the motors to accurately locate each detector to the appropriate position. In this context it was
decided to use stepper motors as the deriving motors. This has two advantages to the automation and control system. First, stepper motors are very easily controlled by digital computers. In fact the control and operation of stepper motors is based on the logical pulse train of zeros and ones produced by a digital microcontroller or a computer. Second, stepper motors are moved step by step, and each step is identical in the amount of displacement in a particular axis of motion the step produces. Thus, the number of steps the computer will order the stepper motor to move will be corresponding to a specific position the axis of motion will be at. So, keeping track of the detectors’ position is as simple as keeping track of the number steps the stepper motor has moved relative to an initial reference point.

In general, a digital motion control system is composed of the main components shown in figure 5.6 below.

![Figure 5.6: Main components of a stepper motor motion control system.](image)

The first and main component to the left of the above figure is the motion control software hosted by a computer interfacing with the second component, which is the motion controller. The function of the motion controller is to convert the motion logic produce by the motion software into a logic pulse train that convey the targeted position in terms of a number of steps and direction of motion to the third component. The third component is the motor power drive, which receive the logic signal and direction pulses
from the motion controller and amplify that signal into a power signal with a current that can drive the stepper motor the required number of steps in the required direction. Details of the design and operation of steppers motors are out of the scope of the present dissertation, but the point is that stepper motors are a good and simple choice to be employed in such a fully automated tracking system.

In implementation of the above scheme for motion control of the tracking system, we have employed the necessary components. A personal computer with a Pentium IV processor was used to host the motion controller as well as the motion and automation software, which will be described in more details in the following section. The motion controller use was a National Instruments four axis stepper and servo PCI controller. The motion controller is connected to a the motor power drive. The motor drive is also a National Instruments four axis external stepper drive. The motor drive is capable of driving four stepper motors simultaneously. Three of these will be responsible of moving and positioning the detectors and the platform, and the fourth will be used to continuously move the source in a predetermined trajectory to test and verify the tracking accuracy of the system. Finally, a National Instruments data acquisition PCI board was used as well to acquire amplified signals produced by the detectors.

5.4 Development of the Automation and Control Program:

As mentioned earlier, the tracking system is required to be fully automated and computer controlled. This necessitate to have on top of the tracking system an automation and control computer program, that will carry out the motion logic of the detectors on the platform, keep track of the instantaneous positions of the detectors, acquire and record
the response signals from each detector, and perform the coordinates algorithms and present the tracking results in an interactive graphical user interface. Several computer programming languages are capable of carrying out those functions such as Visual Basic or C++. However, for the purpose of compatibility and investigation of a programming language that is relatively uncommon in nuclear measurements, we have decide to employ *National Instruments LABVIEW* as the programming language to develop the automation and control program needed for the tracking system.

*LABVIEW* stands for (Laboratory Virtual Instrument Engineering Workbench). The main features of the language are the development environment based on graphical programming concept. *LABVIEW* uses terminology, icons, and relies on graphical symbols rather than textual language to describe programming actions. *LABVIEW* is integrated fully for communication with hardware and plug-in data acquisition boards. Virtually, *LABVIEW* turns the computer into a cluster of instruments beside the normal computational functions of the computer. Thus, by using computer-based instruments one can design measurement and automation systems that implements low-cost, flexible personal computer technology. In addition, *LABVIEW’s* graphical programming language provides an easy means to quickly design and implement complex test and measurement and automation applications. Another important feature of *LABVIEW* language is that it is based on the data flow programming concept. It is known that traditional text-based programming relies on top-down design, where programmer must write lines of code that execute line by line. However, *LABVIEW* is based on graphical programming as can be seen on figure 5.7 below, which shows a sample programming statement (if you will). Here, whenever the data necessary for the execution of a function
is available, the function will be executed regardless of its sequence and/or its position in the diagram. This is what is called operations on the concept of data flow. Details of LABVIEW programming language can be found in the related references at the end of this dissertation.

Figure 5. 7: Part of a LABVIEW block diagram containing some arithmetic functions.

In addition to the special features of LABVIEW as a programming language, LABVIEW has all of the normal features and functionality one can find in any conventional programming language. Therefore, we have adapted LABVIEW to carry out motion control of the stepper motors, data acquisition of detectors’ signals, perform a rate meter function to produce a counting rate proportional parameter, carry out the counting rates maxima detection algorithms, and finally present the tracking results both graphically, and in data format.

For the motion control part, there is a certain logic followed to establish targeted positions of the stepper motors. That logic is based mainly on the following argument:
Suppose that at one instant the tracker had located the radioactive particle. Then assume that the particle will move in a presumably unknown direction (which is supposed to be the actual case). Now the dynamic electromechanical tracker we have should start moving accordingly to follow the radioactive particle. But since there is no information about the direction the source will move to, the system have no way to know whether to move up or down (to follow the vertical motion), nor to move left or right (to follow radial motion). So if the detector in question happens to move in the wrong direction, it could completely lose track of the particle. So there have to be a way to resolve this issue in order to have the detectors keep track of the radioactive particle. The logic we have used to resolve the problem was based on what we called a **Scanning Technique**. In this technique, the detector in question would move in an oscillating fashion around the point where the particle is located. In other words it scans a range of angular or vertical positions, and the acquired detector response would have a distribution over that range. The maximum of that distribution is then corresponding to the position where the particle is located. The range that the detector will oscillate around the source is meant to be dynamic not a static one. In other words, that range will be based on the previously established source position, and each established position will be used to update the oscillation range for the next cycle of oscillation.

In addition to the motion control part of the program, there is the data acquisition part which acquires amplified pulses from the linear amplifier connected to detectors. This part will perform in addition a rate meter function by which the program will produce a mean value that is directly proportional to the number of pulses acquired per unit time. A snapshot of the user interface of that part is shown on figure 5.8.
Figure 5.8: A snapshot from the interface of the rate meter module of the automation program.

It is clear on the figure above the individual acquired pulses. The program was designed to produce a root mean square voltage that is directly proportional to the number of pulses acquired in a preset time interval (usually 10 milliseconds in this application). This way, we have counting rates in arbitrary units and we can thereafter perform the maxima detection algorithm.

5.5 Benchmark Experiments and Their Results:

In this section a series of benchmark experiments was carried out with the tracking system designed and constructed. The experiments were designed in a way to make the radioactive particle move in a preset trajectory by independently move the source with a stepper motor with an independent controlling program. This way it is
assured that the time trajectory of the particle is pre-known, that is it is known in advance where the particle will be located at what time. Therefore, a comparison can be established between the deduced particle’s positions by the tracker and those pre-known positions. The motion of the source is a combination of vertical and angular motion in experiments conducted in air and in water and is only a vertical motion in a heterogeneous randomly packed attenuating medium (marbles in our experiment).

Figure 5.9 shows a photograph of the first of these experiments, in which the source was moved in air (i.e. with negligible attenuation).

![Figure 5.9: A photograph of the benchmark experiment carried out in air.](image)

A snapshot of the graphical user interface part of the computer automation and control program that interactively and dynamically shows the radioactive particle's position in real time is shown in figure 5.10. The snapshot was taken at the end of the trajectory of the radioactive particle tracked. The *LABVIEW* program was developed in a way to present the deduced positions of the radioactive particle in a real time 3-D plot as a visual
representation of the particle’s track, as well as to export these information in a tabulated data format.

Figure 5.10: A snapshot of the radioactive particle track as presented by the developed Labview Program.

A similar experiment was carried out for a source pulled up vertically in a vessel filled with glass marbles randomly packed in the vessel. The purpose of this experiment was to test the tracking system in situations similar to pebble bed reactors packing and find whether the randomly distributed granular attenuating media will affect the performance and accuracy of the tracking system. Again the source motion was independently controlled by a stepper motor resulting in a predetermined trajectory of the source which was only vertical in this case. Figure 5.11 shows a photograph of the experiment including the accommodated vessel, and figure 5.12 shows a snapshot of the particle track as presented by the developed LABVIEW program.
Figure 5.11: A photograph of the benchmark experiment carried out in marbles.

Figure 5.12: A snapshot of the radioactive particle track at the end of the particle’s motion.
A comparison between the predetermined particle’s positions and the particle’s positions deduced by the tracking system is shown in figure 5.13. The comparison shows a great matching between the two sets of coordinates. The deviations between the deduced coordinates are also observed to be much less than the estimated errors calculated by equations 4.14 and 4.15 and shown as error bars in the plot below.

Figure 5.13: A comparison between actual and measured radioactive particle trajectories.
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Table 5.1: Numerical comparison between actual and tracked X coordinates.
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Table 5.2: Numerical comparison between actual and tracked Y coordinates.
6. DISCUSSION AND CONCLUSIONS

In principle, the three detectors technique shows a reasonable feasibility in employment for investigating flow paths of pebbles inside pebble beds. In practice, the constructed tracking system has potential advantages over the traditional tomographic RPT. The most obvious advantage is the tremendous reduction in the number of detectors required to perform the tracking. Only three detectors will do the job. In addition, the system avoids many of the complicated spectroscopy, and the complications associated with high counting rates radiation measurements as dead time and pulse pile up. Also, the system showed a reliable performance when tracking radioactive particle in heterogeneous stochastic distribution of granular attenuating medium regardless of the packing scheme of the granular medium and without the need for complicated modeling. Finally, the built system provides a very convenient fully automated tracking method. The developed controlling program provides a real time online tracking with a 3-D interactive dynamic visualization of the particle’s track during the experiment running.

However, the system does have a limitation that worth to point out. Particularly, the system has a maximum limiting tracking speed. The tracking speed is limited by the fact that the mechanical loads in the system are considerably large, and requires high moving power and special mechanical design to overcome the large inertias that can produce slippage when moved at high speeds. The slowest axis of motion in the system constructed is the vertical motion, as it is the largest mechanical load of all axis. The maximum stabilized tracking speed that could be achieved with our system was about 1 cm per minuet. While such a tracking speed is reasonable for slowly moving flows, such
as flowing pebbles in a pebble bed reactor, it might not be applicable in faster fluidized beds. However, design modifications and incorporation of higher power equipment can provide much higher tracking speeds, increasing the various possible application of the system.

For example, and as mentioned before, the detector in question moves in an oscillating fashion around the point where the particle is located. In other words it scans a range of angular or vertical positions, and the acquired detector response would have a distribution over that range. The maximum of that distribution is then corresponding to the position where the particle is located. This is a rather simple technique that needs only three detectors. However, this technique has the disadvantage of limited tracking speed. Suppose that the particle will make a rapid movement in one direction, while the detector is scanning positions in the other direction. The system in this case is somewhat “wasting” time scanning positions in the wrong direction, and it will take a longer time to move back to the right direction. Even if the detector was scanning in the same direction of the particle movement, if the particle was moving with a higher speed than the scanning speed, the detector will not end up with a response distribution that has a maximum. Then the detector will eventually loose track of the particle for high speed particle’s motion. Therefore this technique is somewhat limited to low speed particle tracking, like in pebble bed reactors.

This can be overcome, and therefore enhance the tracking speed of the tracker, as can be seen in the following argument. As seen in the schematic shown in figure 6.1 below, two identical collimated detectors paired so that their response distribution would intersect at the particle’s location. Thus assuming identical responses distributions, as
shown below, an error signal between the two detectors of the pair equals to zero (± a preset value accounting for statistical and/or electronics fluctuation) would correspond to the orientation of the midpoint of the pair toward the location of the particle.

Figure 6. 1: A schematic of paired collimated detectors.

Now if the particle starts a movement, the absolute value of the continuously measured error signal from the pair will change from zero (± the preset value), which will instantly initiate a movement of the pair to return to the zero error signal position, which in turn corresponds to the particle location. The sign of the changing error signal will simultaneously predetermine the direction to which the detector pair should be moved. For example, suppose the error signal equals the RED signal minus the BLUE signal in the figure above. Thus, a change in the error signal that will make it negative means the particle have moved to the right (toward the BLUE detector). Similarly, a larger positive value of the error means the particle have moved to the left (toward the RED detector).
Therefore, an instantaneous anticipation of the direction of motion of the detectors is established, without wasting any time in scanning a range of positions where the particle is not located in. In addition, tracking in this way would require the detectors to be moved small incremental movements, resulting in faster tracking. In this case the only limitations on the tracking speed would be pure mechanical, depending on the mechanical loads and the power of the motors moving those loads. However, such technique might posses other technical difficulties such as drifting. Therefore, further investigation is recommended if the proposed technique is to be implemented.

The present research project was initiated basically for the purpose of implementation to flow mapping and characterization of flowing pebbles in a pebble bed reactor. In such a study we propose building a prototype scaled vessel and monitor graphite pebbles flowing in the vessel under realistic reactor operating conditions of temperatures and coolant gas pressure. Such a study would require a tremendous research and technical efforts, but we believe that the outcome is worth the effort to establish verified flow models, and to optimize the fuel cycle of such reactors. We hope that one day we can continue on the proposed study, and use the tracking system constructed to perform the tracking of pebbles in such scaled pebble bed reactor.
7. LIST OF REFERENCES


