ABSTRACT


This Ph.D. dissertation includes three chapters. In the first chapter I compare absolute to relative performance evaluation under limited liability for the principal. This constraint limits the actual payments the principal will make in low states of nature. The analysis shows that the liquidation value of the firm must be sufficiently large for tournaments to be optimal. In particular, it shows that there exists a critical liquidation value above which tournaments are still optimal even if the limited liability constraint is binding under tournaments. However, below the critical liquidation value piece rate contracts are always optimal. This result is analogous to showing that if the principal were sufficiently risk-averse he would be unable to offer insurance against common shocks by using tournaments and he would resort to piece rate contracts. By contrast, if the principal were less risk-averse than the agent, he would still provide insurance through tournaments. The analysis also shows that the optimality of tournaments over piece rate contracts critically depends on the agent's risk aversion rate, as well as on the variance of common uncertainty. Another important finding is that, under price uncertainty and when the minimum possible price of output is sufficiently high, the principal does not appear to be more concerned about the allocation of profit across states and he tends to use a piece rate.

In the second chapter I compare the same two forms of contracts under liquidity constraints for the agents. A liquidity constraint is an institutional constraint preventing the principal from compensating the agent by an amount smaller than a predetermined level independently of the state of nature. Models used in the standard literature allow the payments to the agents to be negative. A liquidity constraint for the agent may alter the choice the principal makes between tournaments and absolute performance contracts. The analysis shows that in the presence of common uncertainty a principal contracting with risk-averse agents will prefer to offer a tournament even when agents are liquidity constrained. The rationale for this result is that by providing insurance against common shocks through a tournament the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them, while simultaneously providing them with higher-power incentives than under piece rates. A
second claim is that the principal can implement higher-power incentives under a
tournament. Tournaments provide the principal with added flexibility in the determination of
this power when the liquidity constraints are really tight.

In the third chapter, the two compensation forms are compared under an empirical setting.
One example of the principal-agent problem is the instructor-student relationship, which
exhibits many similarities but, also, several differences in comparison to the standard
problem. The analysis is based on an innovative experiment, designed to compare the relative
performance grading methods to the traditional absolute performance method in a real class
environment. Two college classes of the same subject, under the same instructor were
considered. Data on the students’ performance in eight multiple choice homework
assignments, identical for both classes, were collected. The objectives of the experiment are
first, to identify the evaluation method which maximizes the student's effort, second, to
investigate the effects of the evaluation method on social relations and third, to explore the
effect of the relative performance method on answer sharing. The results suggest that,
contrary to the initial expectation, the relative method has a negative impact on student's
effort. Moreover, student's effort is, surprisingly, negatively affected by the power of
incentives. Finally, as expected, the relative method helps in the prevention of answer
sharing.
Tournaments and Piece Rates under Limited Liability, Liquidity Constraints and Peer Effects

by
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Στην μνήμη του Αριστοτέλη και της Χρυσαυγής. Θα ήταν υπερήφανοι σήμερα.

(To the memory of Aristotle and Chrisavgi. They would be proud today.)
BIOGRAPHY

Kosmas Marinakis was born in Athens, Greece in 1978. He was the first child of George Marinakis and Olga Papaioannou. Until the age of three he lived with his grandparents, Aristotle and Chrisavgi Papaioannou in Avarikos, a small village about 200 miles west of Athens. He started and finished his basic education in Athens. In 1996 he was admitted by Athens University of Economics and Business and in 2002 he earned a B.A in Economics. Along with his academic studies, he worked full time at several major advertising and mass media firms of Greece. In 2002 he was named a scholar of the National Scholarship Foundation of Greece and he was admitted by North Carolina State University as a Ph.D. student in economics. He earned a Master’s Degree in 2004 and he is expected to earn a Ph.D. in Economics in 2008.
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I thank my father, who never showed me the way; he just showed me what will happen if I do not find it by myself. I thank my mother, Olga Marinakis, who taught me some tough love.

I am grateful to my teachers because they changed my life. Among them, professor Theodore Lianos inspired me the most. I am proud I was one of his students and I am proud that he believed in me. I express my gratitude to my Ph.D. committee, professors, Theofanis Tsoulouhas, Walter Thurman, Duncan Holthousen and Chuck Knöber for their valuable help and cooperation. I am, also, thankful to Professor David Flath and Professor Douglas Pearce because they helped me more than they were required.

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Chapter 1

Are Tournaments Optimal over Piece Rates under Limited Liability for the Principal?

1.1 Introduction
Following the seminal work of Lazear and Rosen (1981), Holmström (1982), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Shleifer (1985), the literature on relative performance evaluation has focused on two-part piece rate (cardinal) tournaments taking the form \( b + \beta(x_i - \bar{x}) \), where \( x_i \) is agent output and \( \bar{x} \) is average output, and contrasted these schemes with standard linear piece rate contracts of the form \( b + \beta x_i \).\(^1\) The latter are sometimes expressed as "fixed performance standards" when an agent’s performance is evaluated against a fixed standard instead of the average output obtained.\(^2\) Prominent examples of the use of tournaments include contracts for salesmen, contracts for physicians contracting with HMOs, agricultural contracts, promotion tournaments and annual salary raises for faculty.


\(^2\)Rank-order (ordinal) tournaments have received less attention recently partly because they are informationally wasteful by ignoring the agents’ cardinal performance (Holmström (1982)).
Absent limited liability for the principal tournaments constitute a move closer to the First Best. This is because relative performance evaluation partially alleviates the agents’ moral hazard problem by providing information about the value of common shocks. The principal filters away common shocks from the responsibility of agents and charges a premium for this insurance. The move from absolute performance contracts to tournaments is Pareto improving because the principal’s expected profit increases without hurting the agent.

The dominance of tournaments is less clear when the principal is subject to limited liability, and bankruptcy is an issue because the firm’s liquidation value is small or because it is possible for the output state to be quite unfavorable. When switching from absolute performance contracts, such as piece rates, to tournaments the risk premium the principal charges for insurance against common shocks reduces the base payment $b$ the agent receives. Further, the filtering of common uncertainty enables the principal to implement a higher-power incentive scheme by increasing $\beta$. However, because higher effort by the agent reduces his utility, the base payment $b$ will need to adjust to ensure the participation of the agent. Thus, under a tournament the agent will receive better insurance but he will have to exert more effort, and even though the bonus factor $\beta$ under tournament should increase the direction in the adjustment of the base payment $b$ is not clear a priori. Because the total wage bill under tournament is related to the base payment, the direction of the change in the total wage bill under tournament is also ambiguous a priori.\(^3\) When the firm is subject to limited liability, the limited liability constraint limits the actual payments the principal will make in low states of nature. Because it is not clear if total payments go up or down when moving from piece rate contracts to tournaments, tournaments may or may not be better than piece rates under limited liability. Section 2 develops the model we will use to investigate this question.

Our analysis of piece rate contracts in section 3 and of tournaments in section 4

\(^3\)Specifically, the analysis below shows that the total wage bill is the number of agents multiplied by the base payment.
shows that absent limited liability the base payment and, hence, the total wage bill increases under tournament. This is so because the expected bonus payment under tournament is zero, whereas that under piece rate is positive. Therefore, agents expect to be compensated for effort through the base payment in a tournament.\footnote{Under piece rate the bonus also compensates the agents for their effort costs. In fact, the expected bonus exceeds the cost of effort and the base payment is negative.} Thus, in the presence of a limited liability constraint which limits payments in unfavorable states, tournaments may not be dominant over piece rates. The intuition is that contracts with risk neutrality and limited liability for the principal look very much like those that would have been obtained with risk aversion. In other words, if the principal is concerned about the allocation of profit across states, he may no longer offer insurance against common shocks via tournaments and may resort to piece rate contracts or fixed performance standards.\footnote{To the best of our knowledge the only work acknowledging that limited liability for the principal may hinder the use of relative performance evaluation is Tsoulouhas and Vukina (1999). They focus on explaining why tournaments are used in certain industries (e.g., the broiler industry) and not in other industries (e.g., the swine industry).} Section 5 briefly examines how the contractual parameters adjust to changes in the model parameters.

Our comparison of piece rate contracts to tournaments in section 6 shows that, surprisingly, the liquidation value of the firm must be sufficiently small for tournaments to be inferior. In particular, we show that there exists a critical liquidation value above which tournaments are still superior even if the limited liability constraint is binding under tournaments. However, below the critical liquidation value piece rate contracts are always superior. In a sense the limited liability constraint must be really tight for tournaments to be inferior. To the best of our knowledge, this result has never been obtained in the literature. Our finding is analogous to showing that if the principal were sufficiently risk-averse he would be unable to offer insurance against common shocks by using tournaments, and he would resort to piece rate contracts. By contrast, if the principal were less risk-averse than the agent, he would still provide insurance through tournaments.
The analysis also shows in section 6C that the superiority of tournaments over piece rate contracts critically depends on the agent’s risk aversion rate, as well as on the variance of common uncertainty. The more risk-averse the agent is or the higher the magnitude of common uncertainty, the more the agent is willing to pay for insurance or the more the principal can charge for insurance, which raises the principal’s profit. As a result, the range of liquidation values over which tournaments are dominant increases with the risk aversion rate and the magnitude of common uncertainty. The number of agents has a similar effect, in that a large number of agents is necessary to eliminate idiosyncratic noise from the average output obtained by the agents. Hence, more insurance is provided against common shocks when the number of agents is large.

In the main analysis we assume that the price of output is known ex ante. In section 7 we incorporate price uncertainty. With price uncertainty the principal should be even more concerned about the allocation of profit across states, hence, one would expect that tournaments would be less likely to be superior. Our analysis shows that, surprisingly, if the lowest possible price exceeds the bonus factor \( \beta \), the form of the dominant contract is completely unaffected by the presence of price uncertainty. By contrast, if the lowest possible price is smaller than the bonus factor, the increased bankruptcy risk in fact strengthens the need for tournaments by expanding the range of liquidation values over which tournaments are dominant. We trace this surprising result to the potential tension, from the principal’s perspective, between providing insurance to the agent against common shocks and insuring himself against the variability in the total wage bill. In the presence of significant price uncertainty, the principal prefers to offer a tournament in order to eliminate the variability of the total wage bill, even though the limited liability constraint is tighter under significant price uncertainty. To the best of our knowledge, this result as well has never been obtained in the literature.\(^6\)

In all, our analysis shows that the liquidation value of the firm is a far more defini-\(^6\)It is worth noting that in their empirical analysis Tsoulouhas and Vukina (1999) assumed that price volatility should discourage the use of tournaments.
tive factor than the price uncertainty in the determination of the contract the principal should offer. Thus, regardless of price uncertainty, if the liquidation value is really small the principal will prefer to offer a piece rate contract, and if the liquidation value is really large the principal will prefer to offer a tournament. Note, however, that we do not characterize the overall optimal contract. We rather focus on comparing relative performance evaluation schemes, such as tournaments, to absolute performance evaluation schemes, such as piece rates, the way they are used in practice, at least in the case of contracts for salesmen, contracts for physicians contracting with HMOs, agricultural contracts, and annual salary raises for faculty.7

The policy implications of our analysis are, first, that financially constrained firms should use absolute performance standards and refrain from using tournaments because they would increase the firm’s costs in unfavorable states, and second, that firms facing significant price volatility for their products should favor tournaments, unless the firms are in real financial distress.

Section 8 provides our concluding remarks. Appendix A provides the proofs of our comparative statics results which are discussed in section 6C. Appendix B presents an alternative approach for modeling limited liability by using a truncated normal distribution.

1.2 Model

A principal signs a contract with $n$ homogeneous agents.8 Each agent $i$ produces output

---

7 The overall optimal contract may very well be non-linear. The reason for the non-linearity is that contracts adjust to all possible events. Holmström and Milgrom (1987), however, have argued that schemes that adjust compensation to account for rare events may not provide correct incentives in ordinary high probability circumstances.

according to the production function \( x_i = a + e_i + \eta + \varepsilon_i \), where \( a \) is the agent’s known ability, \( e_i \) is the agent’s effort, \( \eta \) is a common shock inflicted on all agents and \( \varepsilon_i \) is an idiosyncratic shock. Both shocks follow independent normal distributions with zero means and finite variances \( \text{var}(\eta) = \sigma^2_\eta \) and \( \text{var}(\varepsilon_i) = \sigma^2_{\varepsilon_i} \), \( \forall \ i \). Each agent’s effort and the subsequent realizations of the production shocks are private information to him, but the output obtained is publicly observed.\(^9\) In the baseline model the price of output is normalized to 1 so that the output produced by the agents is revenue to the principal. The principal compensates agents for their effort based on their outputs by using a piece rate contract or a tournament. Agent preferences are represented by a CARA utility function \( u(w_i, e_i) = \exp \left( -rw_i + \frac{1}{2}r e_i^2 \right) \), where \( r \) is the agent’s coefficient of absolute risk aversion. Note that the cost of effort decreases with agent ability and is measured in monetary units. This utility function has been widely used in the literature (for instance, see Meyer and Vickers (1997)).\(^10\)

One of the advantages of this model is that its results absent limited liability conform with those obtained by Lazear and Rosen (1981) who even though they utilized more general utility functions they relied on first-order Taylor approximations of these functions.\(^11\) The benefit of using a model which provides an analytical solution instead of an approximate solution, at least in the baseline cases, is that the results can be extrapolated in a wide range of parameters without incurring approximation errors.

1.3 The Piece Rate Contract

1.3.A The Piece Rate Contract without Limited Liability

The piece rate contract (R) is the payment scheme in which the compensation to the \( i^{th} \)
agent takes the form \( w_i = b_R + \beta_R x_i \), where \((b_R, \beta_R)\) are the contractual parameters to be determined by the principal. The principal will determine these parameters by backward induction.

First, the principal calculates each agent’s expected utility:

\[
EU_R = -\exp\left\{-r \left[ b_R + \beta_R (a + c_i) - \frac{e_i^2}{2a} - \frac{r \beta_R^2 (\sigma_\eta^2 + \sigma_\epsilon^2)}{2} \right]\right\},
\]

where the expression in square brackets is the certainty equivalent compensation of the agent. Observe that expected utility rises with increases in the expected payment from the principal, reductions in the effort level implemented by the principal and reductions in the variance of the payments. To ensure the compatibility of the contract with agent incentives to perform, the principal calculates the effort level that maximizes (1). First order conditions yield

\[
e_i^* = a \beta_R.
\]

To ensure the compatibility of the contract with agent incentives to participate, the principal selects the value of the base payment, \( b_R \), that satisfies the agent’s individual rationality constraint with equality so that the agent receives no rents but still accepts the contract. The agent receives no rents because the principal is endowed with the bargaining power. For ease of exposition, we normalize the agent’s reservation utility to \(-1\),\(^{12}\) hence, given (1) the agent’s individual rationality constraint implies

\[
EU_R = -1 \iff
\]

\[
\iff b_R = \frac{r (\sigma_\eta^2 + \sigma_\epsilon^2) - a}{2 \beta_R^2} - a \beta_R.
\]

Thus, by choosing the piece rate \( \beta_R \), the principal can precisely determine the agent’s

\(^{12}\)Note that the analysis is directly applicable to any (negative) normalization other than \(-1\).
effort because the agent will optimally set his effort according to (2). In addition, by setting \( b_R \) in accordance with (3) the principal can induce agent participation at least cost. That is, agent incentives to perform are only determined by the bonus factor \( \beta_R \), whereas agent incentives to participate are determined by the base payment \( b_R \).

Given conditions (2) and (3) the principal maximizes his expected total profit

\[
ET\Pi_R = \sum_{i=1}^{n} [Ex_i - Ew_i] = n \left[ a + a\beta_R - \frac{r(\sigma^2_\eta + \sigma^2_\varepsilon)}{2} \beta^2_R \right].
\] (4)

The solution to this problem satisfies

\[
\beta_R = \frac{a}{a + r(\sigma^2_\eta + \sigma^2_\varepsilon)}.
\] (5)

Condition (3) then implies

\[
b_R = -\frac{a^2}{2} \frac{r(\sigma^2_\eta + \sigma^2_\varepsilon) + 3a}{[r(\sigma^2_\eta + \sigma^2_\varepsilon) + a]^2}.
\] (6)

Thus, the principal’s expected profit per agent under the piece rate contract absent limited liability is

\[
E\Pi_R = a + \frac{a^2}{2} \frac{1}{a + r(\sigma^2_\eta + \sigma^2_\varepsilon)}.
\] (7)

1.3.B The Piece Rate Contract with Limited Liability

Suppose now that the principal is subject to limited liability, that is, he cannot be required to pay the agents more than the revenue available to him plus the liquidation value, \( A \), of the firm. This is particularly important if the state turns out to be unfavorable (that is, if production shocks turn out to be unfavorable). The principal could promise payments in these states that are higher than the sum of the revenue available to him and \( A \), in order to reduce payments in high states. But then the principal could renege on his promise by pleading bankruptcy or by threatening to plead bankruptcy,
and reduce the actual payments in low states in accord with the assets available to him. Rational agents, however, cannot be suckered by the prospect of a payment that the principal clearly cannot make. The agents take this constraint into account in deciding whether to participate and which effort to exert (see Kahn and Scheinkman (1985), Innes (1990, 1993a and 1993b) and Tsoulouhas (1996)). They will sign a contract with the principal only if it stipulates that potential losses cannot exceed the firm’s liquidation value. Thus, implicit in the literature above, albeit never clearly stated, is a principle for bankruptcy analogous to the celebrated Revelation Principle.\footnote{According to the Revelation Principle, any equilibrium allocation of any mechanism can be achieved by a truthful direct revelation mechanism.} In particular, outcomes obtained when bankruptcy is a possibility can also be obtained when a limited liability or bankruptcy constraint limits the payments the principal makes in low states so that bankruptcy is prevented. To conclude, in order to provide correct incentives, the principal incorporates a limited liability or bankruptcy constraint (LLP) in determining the contract to offer. The LLP constraint is

\begin{equation}
T \Pi_R(\eta, \varepsilon_1, \ldots, \varepsilon_n) + n \alpha \geq 0, \forall (\eta, \varepsilon_1, \ldots, \varepsilon_n), \tag{8}
\end{equation}

where $\alpha = A/n$ is the liquidation value per agent. For generality, we allow the liquidation value to be negative, that is, we allow the company to be in debt from prior operations, or to have no collateral and be in need to borrow funds exogenously to get started.

It is easy to show that total profit $T \Pi_R(\eta, \varepsilon_1, \ldots, \varepsilon_n)$ is increasing in the state.\footnote{To be precise, it is increasing in $n \eta + \Sigma \varepsilon_i$.} Therefore, if (8) is satisfied in the lowest possible state (i.e., in the lowest possible realization of production shocks) then it is satisfied in all states. The assumption of normality of the output distribution allowed us to obtain an analytical solution in the baseline case without limited liability. However, with limited liability the distribution must be truncated. In other words, the lowest possible realization of the state must be bounded, otherwise the agent would receive an unbounded negative payment in the lowest states.
However, with a truncated normal distribution no analytical solution can be obtained.\(^{15}\) Given this, either we obtain computational results, or we use normality of the output distribution as a proxy of the truncated normal distribution. Our computations indicate that these results are not qualitatively different than the results that can be obtained when normality of the output distribution is used as a proxy. We assume for simplicity that the lowest possible realization of the state is the one that yields no output, but we also assume that the normal distribution has a sufficiently high mean (in other words, the agent’s known ability \(a\) is sufficiently high) so that the normal distribution is a good proxy of the truncated normal. Then the LLP constraint is satisfied at all states if

\[ b_R \leq \alpha, \] (9)

meaning that the base payment cannot exceed the liquidation value per agent when agents exert effort but obtain no output because of unfavorable shocks. This constraint is binding if

\[ \alpha_R \equiv \frac{-a^2 \, r(\sigma^2 + \sigma^2) + 3a}{2 \left[ r(\sigma^2 + \sigma^2) + a \right]^2} > \alpha, \] (10)

that is, if the solution without the LLP constraint (i.e., condition (6)), which was obtained in the previous section, violates the LLP constraint. In other words, the LLP constraint is binding if the liquidation value of the firm is sufficiently small. When the LLP constraint is binding, the contractual parameters \((\hat{b}_R, \hat{\beta}_R)\) must satisfy the non-linear system consisting of the LLP constraint (9) with equality and the individual rationality constraint (3). Therefore, the piece rate contract with limited liability, when the liquidation value of the firm is sufficiently small satisfies

\[ \hat{b}_R = \alpha, \] (11)

\[ \frac{r(\sigma^2 + \sigma^2) - a \beta^2}{2} \beta_R - a \beta_R - \alpha = 0, \] (12)

\(^{15}\)Appendix B shows how the analysis could be modeled by using a truncated normal distribution.
where (12) derives from (3). Compared to the case without limited liability, (11) and (10) indicate that the base payment $b_R$ is reduced to satisfy the limited liability constraint, that is,

$$\hat{b}_R < b_R. \quad (13)$$

There are two candidate solutions for the piece rate in (12):

$$\hat{\beta}_R = \frac{a \pm \sqrt{a^2 + 2[r(\sigma_n^2 + \sigma_z^2) - a]|\alpha}}{r(\sigma_n^2 + \sigma_z^2) - a}. \quad (14)$$

Given condition (10) and $\beta_R \in [0,1]$, it can easily be shown that the higher root yields a lower profit than the lower root.\footnote{Note that whereas the lower root is inside the interval $[0,1]$ when $r(\sigma_n^2 + \sigma_z^2) \neq a$, the higher root may or may not be inside the interval. However, when both are in the interval, the lower root is optimal as argued above. Obviously, neither root is defined when $r(\sigma_n^2 + \sigma_z^2) = a$.} Therefore,

$$\hat{\beta}_R = \frac{a - \sqrt{a^2 + 2[r(\sigma_n^2 + \sigma_z^2) - a]|\alpha}}{r(\sigma_n^2 + \sigma_z^2) - a}. \quad (15)$$

Observe that the piece rate is not defined if

$$\alpha < \alpha_0 \equiv \frac{a^2}{2[r(\sigma_n^2 + \sigma_z^2) - a]}, \quad (16)$$

because in that case the solution is not a real number. Compared to the case without limited liability (i.e., condition (5)), condition (15) implies

$$\hat{\beta}_R > \beta_R. \quad (17)$$

The rationale behind condition (17) is that because the LLP constraint limits the base payment, the piece rate $\beta_R$ must increase to satisfy the agent’s individual rationality constraint. Given (15) the expected profit per agent under the piece rate contract when
limited liability is binding is

\[ E\hat{\Pi}_R = a \left( \hat{\beta}_R - \hat{\beta}_R^2 \right) - \alpha. \]  \hspace{1cm} (18)

1.4 The Tournament

1.4.A The Tournament without Limited Liability

The (two-part piece rate) tournament or yardstick competition (T) is the payment scheme in which the compensation to each agent is determined by a relative performance evaluation. Specifically the payment scheme is

\[ w_i = b_T + \beta_T (x_i - \bar{x}) = b_T + \beta_T \left( \frac{n-1}{n} x_i - \frac{1}{n} \sum_{j \neq i} x_j \right), \]  \hspace{1cm} (19)

where \( \bar{x} \) is the average output obtained by all agents. Note that under tournament the total wage bill is proportional to the base payment \( b_T \), in particular, \( \sum w_i = nb_T \). Thus, in contrast to the piece rate contract, the principal’s total payment to the agents and, hence, the expected payment per agent are independent of output. This observation is very useful throughout the remaining analysis.

Under a tournament the agent’s expected utility is

\[ EU_T = -\exp \left\{ -r \left( b_T + \beta_T \frac{n-1}{n} (a + e_i) - \beta_T \frac{1}{n} \sum_{j \neq i} (a + e_j) - \frac{e_i^2}{2a} - \frac{n-1}{n} \beta_T^2 \sigma^2 \right) \right\}. \]  \hspace{1cm} (20)

The effort level that maximizes (20) satisfies

\[ e_i^{**} = \frac{n-1}{n} a \beta_T. \]  \hspace{1cm} (21)

Further, the individual rationality constraint implies

\[ EU_T = -1 \iff \]

12
\[
\iffalse b_T = \frac{1}{2} \frac{n - 1}{n} \left( \frac{n - 1}{n} a + r \sigma^2 \right) \beta^2_T. (22) \fi
\]

Then, given conditions (21) and (22), the principal maximizes expected total profit

\[
ET\Pi_T = n \left[ a + \frac{n - 1}{n} a \beta_T - \frac{1}{2} \frac{n - 1}{n} \left( \frac{n - 1}{n} a + r \sigma^2 \right) \beta^2_T \right]. (23)
\]

The solution to the principal’s maximization problem satisfies

\[
\beta_T = \frac{a}{\frac{n - 1}{n} a + r \sigma^2}. (24)
\]

It is straightforward to show that

\[
\beta_T > \beta_R. (25)
\]

where \( \beta_R \) was characterized in condition (5), that is, the principal implements higher-power incentives when common uncertainty is removed from the responsibility of the agent under tournament. Given condition (24), condition (22) can be written as

\[
b_T = \frac{1}{2} \frac{n - 1}{n} a \beta_T,
\]

which implies

\[
b_T = \frac{1}{2} a + \frac{n}{n - 1} r \sigma^2. (26)
\]

Compared to condition (6), condition (26) implies that

\[
b_T > b_R. (27)
\]

As argued above, a priori it was not clear whether the base payment would go up or down under tournament. This is because it would need to be reduced by the risk premium for the insurance against common uncertainty, but at the same time it would need to be
increased to ensure agent participation. It turns out that the second effect is dominant. The principal’s expected profit per agent under tournament and absent limited liability is

$$E\Pi_T = a + \frac{a^2}{2} + \frac{1}{\sqrt{n-1}r\sigma^2}.$$  

(28)

1.4.B The Tournament with Limited Liability

Similar to the case where the principal uses a piece rate and is subject to limited liability, the limited liability constraint under a tournament is satisfied at all states if

$$b_T \leq \alpha,$$  

(29)

and it is binding if the liquidation value of the firm is sufficiently small in the sense that

$$\alpha_T \equiv \frac{1}{2} a + \frac{n}{n-1}r\sigma^2 > \alpha.$$  

(30)

When the LLP constraint is binding, the contractual parameters $\hat{(b_T, \beta_T)}$ must satisfy the non-linear system consisting of (29), with equality, and (22). Therefore, the tournament with limited liability, when the liquidation value of the firm is sufficiently small, satisfies

$$\hat{b_T} = \alpha,$$  

(31)

$$\frac{1}{2} \frac{n-1}{n} \left( \frac{n-1}{n} a + r\sigma^2 \right) \beta_T^2 - \alpha = 0,$$  

(32)

where (32) derives from the agents individual rationality constraint (22). The solution to (32) satisfies

$$\hat{\beta_T} = \sqrt{\frac{2\alpha}{\frac{n-1}{n} \left( \frac{n-1}{n} a + r\sigma^2 \right)}}.$$  

(33)

Observe that in this case the tournament is never defined when the liquidation value per agent, $\alpha$, is negative (because $\hat{\beta_T}$ is not a real number). Conditions (26), (31), (6) and...
(11) imply

\[ b_T > \tilde{b}_T > b_R > \tilde{b}_R, \]  

that is, the base payment increases under tournament, and it is always smaller when the LLP constraint is binding. Conditions (33) and (24) imply

\[ \hat{\beta}_T < \beta_T, \]  

that is, the bonus factor decreases under tournament with limited liability. This is opposite to the result obtained above for piece rates. Under tournament the base payment still needs to be reduced to satisfy the LLP constraint, which reduces the agent’s expected utility, but the bonus factor now needs to be reduced to satisfy the agent’s individual rationality constraint. Providing the agent with lower-power incentives in this case increases his expected utility because his expected wage remains constant but his cost of effort is reduced. This is also consistent with the principal’s objectives because if the principal offered higher-power incentives, given that the total wage bill is independent of output, the LLP constraint would be violated in unfavorable states. Finally, the principal’s expected profit per agent under the tournament when limited liability is binding is

\[ E\hat{\Pi}_T = a + a \sqrt{\frac{2\alpha}{a + \frac{n}{n-1} r\sigma_z^2}} - \alpha. \]  

1.5 The Impact of Changes in \( r, \sigma_y^2 \) and \( \sigma_z^2 \)

Based on the analysis above, we can obtain some useful insights on how the contractual parameters in the two compensation schemes respond to changes in the model parameters. Figure 1 illustrates the impact of changes in \( r, \sigma_y^2 \) and \( \sigma_z^2 \) on the contractual parameters \( b \) and \( \beta \) under both schemes, and on expected profit per agent, by using the values: \( n = 30, a = 10, r = \sigma_y^2 = \sigma_z^2 = 1 \). As the graphs show, when the variance of common
uncertainty or the variance of idiosyncratic uncertainty increase ceteris paribus, then, $b_R$ increases, $\beta_R$ decreases and $E\Pi_R$ decreases. This is so because the agent needs to be compensated more on average in order to accept to sign a contract when there is more total uncertainty. Further, output will depend more on total uncertainty rather than on effort, hence, the principal provides lower powered incentives through a reduced $\beta_R$. Clearly, the same should be true with an increase in the agent’s risk aversion rate, $r$, because an increase in uncertainty given the risk aversion rate, or an increase in the risk aversion rate given uncertainty, should lead to the same results. The graphs indicate that the results are similar indeed.

With respect to tournaments, the variance of common uncertainty does not affect $b_T$, $\beta_T$ or $E\Pi_T$ because tournaments filter away common uncertainty from the responsibility of the agent. By contrast, when idiosyncratic uncertainty increases, ceteris paribus, output variability is less dependent on effort choice. Then there is less need for providing incentives to exert effort and, hence, the agent will be compensated less on average. Thus both $b_T$ and $\beta_T$ decline. Because the agent is expected to exert less effort, expected profit per agent also declines. When the risk aversion rate increases ceteris paribus, then the principal charges more for the insurance against common uncertainty that he provides, hence, $b_T$ declines. The bonus factor $\beta_T$ is also reduced when the agent is more risk-averse because, then, he is more concerned about the idiosyncratic uncertainty against which he is not insured. In this case, the principal benefits by providing lower-power incentives which implement a lower effort for the agent, than by providing higher power incentives at a substantial monetary cost in order to induce high effort by an agent who is not very motivated. The inability to motivate the agent at a reasonable cost also reduces the expected profit per agent.

1.6 The Dominant Contract

The principal’s decision about which payment scheme to offer depends entirely on the expected profit each scheme will yield. As shown below, tournaments are superior provided
Figure 1: The contractual parameters of piece rate contracts and tournaments for different values of $r$, $\sigma^2_{\eta}$, and $\sigma^2_{\varepsilon}$. 
that common uncertainty is sufficient to warrant insurance provision against common shocks via tournaments (which is consistent with the Lazear and Rosen (1981) and Green and Stokey (1983) finding). Interestingly, it is shown that the magnitude of common uncertainty that is required, at least for the CARA case we consider, is only a fraction of idiosyncratic uncertainty. By contrast, piece rate contracts are superior when common uncertainty is significantly small relative to idiosyncratic uncertainty.\footnote{Lazear and Rosen (1981) show that when there is no common uncertainty, workers are risk-averse and have a CARA utility function, then piece rates dominate tournaments.} The implication of this finding is that in practice tournaments should normally be favored over piece rate contracts absent limited liability or when bankruptcy is not an issue. This is because empirical research (for instance, Knoeber and Thurman, (1995)) has found that the magnitude of common uncertainty is approximately equal to that of idiosyncratic uncertainty.

Because tournaments offer better insurance, they lead to lower payments in favorable output states but to higher payments in unfavorable states. When limited liability is introduced, because the LLP constraint limits the payments the principal can make in low states, tournaments may cease to be superior. In other words, when the allocation of profit across states is important in satisfying the limited liability constraint, the principal may not be able to offer insurance against common shocks by using tournaments. Surprisingly, the LLP constraint must be really tight, in the sense that the liquidation value of the firm must be really small for tournaments to be inferior. Otherwise, we show that tournaments will still be superior when the liquidation value of the firm is not really small, but it is sufficient for the LLP constraint to be binding (i.e., when the liquidation value is intermediate). Thus, the allocation of profit across states must be of significant concern in order for piece rate contracts to dominate tournaments.

1.6.A The Dominant Contract without Limited Liability

Absent limited liability recall that expected profit per agent under the piece rate contract
is shown by (7) and under the tournament it is shown by (28). Thus,

$$E\Pi_T \geq E\Pi_R \iff \frac{1}{n-1} \sigma^2 \leq \sigma^2_n. \quad (37)$$

Condition (37) indicates that tournaments are superior provided that the variance of the common shock is larger than only a fraction of the variance of the idiosyncratic shock, where the fraction decreases when the number of agents increases. A large number of agents strengthens the dominance of tournaments over piece rates because idiosyncratic shocks cancel out, which enables the principal to offer better insurance by filtering away common shocks from the responsibility of the agents through the average output obtained by them.\(^{18}\) Therefore, piece rate contracts are superior when the variance of the common shock is sufficiently small or the number of agents is sufficiently small.

1.6.B The Dominant Contract with Limited Liability

In order to contrast the superiority of piece rate contracts against tournaments with limited liability we compare the expected profit per agent under the two schemes. Figure 2 depicts expected profit per agent for various liquidation values per agent, \(\alpha\), under both schemes. The LLP constraint for the piece rate contract is binding for \(\alpha\) values that are smaller than \(\alpha_R\) and the LLP constraint for the tournament is binding for \(\alpha\) values that are smaller than \(\alpha_T\). Recall that \(\alpha_R < 0\) and \(\alpha_T > 0\) were defined in conditions (10) and (30). That is, the LLP constraint under the piece rate contract is binding when the liquidation value per capita is sufficiently negative, while the constraint under the tournament is binding when the liquidation value is positive and sufficiently small.\(^{19}\) Note that there is no discontinuity at \(\alpha_T\) or \(\alpha_R\) because, when \(\alpha_T\) or \(\alpha_R\) is crossed, the base

---

\(^{18}\)Note that under tournament \(\text{Var}(w_i) = \frac{n-1}{n} \beta^2 \sigma^2\) that is, the variance of agent compensation is increasing in \(n\). Thus, from the perspective of the agent the tournament eliminates the common uncertainty from the responsibility of the agent, but on the other hand it introduces idiosyncratic uncertainty from the activities of other agents, which gets larger with more agents.

\(^{19}\)Recall that the tournament is not defined when the liquidation value of the firm is negative because \(\beta_T\) is not a real number in this case. For similar reasons, the piece rate contract is not defined for a sufficiently negative liquidation value (see condition (16)).
payment \( b \) and the bonus factor \( \beta \) adjust in a continuous manner through the agent’s individual rationality constraint which is always binding. Therefore, expected profit \( E\Pi \) also changes continuously.

If the LLP constraint is non-binding under both the piece rate contract and the tournament, that is, if \( \alpha \geq \alpha_T \), then the analysis is identical to that without limited liability. In this case expected profit under tournament is higher independently of the liquidation value per agent, \( \alpha \), provided that condition (37) is satisfied.\(^{20}\) Therefore, the focus is on the case when the LLP constraint is binding under at least one of the contractual forms.

If \( \alpha < \alpha_0 < 0 \), neither the piece rate contract nor the tournament are defined as shown in sections 3B and 4B. The piece rate contract is not defined because \( b_R \) must be reduced to \( b_R = \alpha \) to satisfy the LLP constraint. But this violates the agent’s individual rationality constraint because the bonus factor \( \beta_R \) would need to increase sufficiently in order to satisfy it. But \( \beta_R \) cannot increase sufficiently without making production undertaking unprofitable. In other words, the individual rationality and the LLP constraints

\[^{20}\text{If condition (37) is not satisfied, then, tournaments are dominated by piece rate contracts regardless of whether the LLP constraint is binding or not. That is, } E\Pi_R(\alpha) > E\Pi_T(\alpha), \forall \alpha.\]
cannot be satisfied simultaneously.

If $\alpha_0 \leq \alpha < 0$, then, the LLP constraint for the piece rate contract is binding if $\alpha < \alpha_R$ and non-binding if $\alpha \geq \alpha_R$. In either case, because the tournament is not defined for a negative $\alpha$, the piece rate contract is superior by default. That is a principal who is in debt but makes a contract offer, will propose a piece rate contract. The tournament is not feasible for negative $\alpha$ values because the LLP constraint implies that the base payment $b_T = \alpha < 0$. Under a tournament the wage payment each agent expects to receive, $Ew_i$, is equal to the base payment. Therefore, each agent would expect a negative payment $\alpha$ which would violate his individual rationality constraint. In contrast to the tournament, a piece rate contract is feasible for a negative $\alpha$ because the payment the agent expects to receive is larger than $b_R$, specifically it equals $b_R + a_1 \beta_R + a_2 \beta_R^2$. Therefore the principal can satisfy the agent’s individual rationality constraint by adjusting $\beta_R$ while simultaneously satisfying LLP through a reduced $b_R$.

If $0 \leq \alpha < a_T$, then, the LLP constraint is binding only under tournament. Conditions (7) and (36) imply that there exists a critical value $\alpha^*$ below which the piece rate contact is dominant and above which the tournament is dominant. The critical value, $\alpha^*$ satisfies

$$\alpha^* = a^2 \left[ \frac{1}{a + \frac{n}{n-1} r \sigma^2 \varepsilon} \left( 1 - \sqrt{\frac{r(\sigma^2_{\eta} - \frac{1}{n-1} \sigma^2_{\varepsilon})}{a + r(\sigma^2_{\eta} + \sigma^2_{\varepsilon})}} \right) - \frac{1}{2} \frac{1}{a + r(\sigma^2_{\eta} + \sigma^2_{\varepsilon})} \right] . \quad (38)$$

Thus, when the liquidation value per agent is positive but smaller than the critical value $\alpha^*$, the piece rate contract dominates the tournament. That is, a principal with a small positive liquidation value will find it profitable to refrain from insuring the agent against common uncertainty. The intuition is that when the liquidation value of the firm is sufficiently small, the principal is concerned about the allocation of profit across states because he has to satisfy a tight limited liability constraint. In some sense the

\footnote{To see this note that the expected payment is equal to $b_R + \beta_R (a + e_i)$, where $e_i$ satisfies condition (2).}
suboptimality of offering insurance when the principal is risk-neutral but subject to a tight limited liability constraint is analogous to that had the principal been sufficiently risk-averse without being constrained by limited liability. However, if the liquidation value is larger than the critical value \( \alpha^* \), tournaments will dominate piece rate contracts because the principal will benefit by providing insurance against common uncertainty. Similar to the intuition above, a principal who is not very concerned about the allocation of profit across states will still provide insurance. Therefore, surprisingly, tournaments can still be dominant even if the LLP constraint is binding under tournament.

1.6.C Comparative Statics

In this section we analyze the impact of changes in the coefficient of absolute risk aversion of the agent, \( r \), the variance of the common shock, \( \sigma^2_\eta \), and the number of agent, \( n \), ceteris paribus. As shown in Appendix A, the following relationships hold:

\[
\frac{\partial \alpha^*}{\partial r} < 0, \quad (39)
\]

\[
\frac{\partial \alpha^*}{\partial \sigma^2_\eta} < 0, \quad (40)
\]

\[
\frac{\partial \alpha^*}{\partial n} < 0. \quad (41)
\]

The comparative statics results in relationships (39), (40) and (41) indicate that the critical liquidation value per agent \( \alpha^* \), defined in equation (??) above, decreases when the coefficient of risk aversion, the variance of common shock or the number of agents increase. This means that tournaments dominate piece rate contracts over a wider range of \( \alpha \) values when these parameters increase. The intuition is that the more risk-averse the agent is, the more he is willing to pay for insurance, or the higher the risk premium the principal can charge for insurance. The higher the variability in the common shock the more insurance is provided through the tournament and, as before, the higher the risk premium the principal can charge. Clearly in both cases tournaments become more profitable to
the principal. An increase in the number of agents also makes tournaments dominant over a wider range. A large number of agents is necessary to eliminate idiosyncratic noise from the average output obtained by the agents. Thus, the more agents, the more insurance is provided against common shocks. This leads to a result similar to that obtained above for increases in the variance of common shock.

A numerical example will illustrate the impact of the parameters above. For this example we use the following values: \( n = 30 \), \( a = 10 \), \( r = \sigma_\eta^2 = \sigma_\varepsilon^2 = 1 \). As shown in Figure 3, the impact of \( r \), \( \sigma_\eta^2 \) and \( n \) on \( \alpha^* \) is similar. However, the number of agents has a stronger negative impact initially. Once a sufficient number of agents is reached so that the impact of idiosyncratic noise on relative performance becomes negligible (i.e., once the idiosyncratic noise is virtually eliminated from the variance of \( \bar{x} \)), then additional agents do not have a significant impact on \( \alpha^* \). The impact of all three parameters eventually fades off because \( \alpha^* \) cannot drop below zero.

1.7 Price Uncertainty and Limited Liability

So far we have assumed that the price of output is given and known ex ante. We now extend the analysis to allow for the case when the price is unknown ex ante. It is interesting to investigate whether our results carry over to the case when there is price uncertainty in addition to production uncertainty. In examining this issue we assume that the principal is a price taker. The additional uncertainty the principal bears can, in
principle, have serious consequences on the dominant contractual form. Price uncertainty can make the limited liability constraint tighter. Therefore, price uncertainty coupled with limited liability may limit the principal’s ability to provide insurance against production uncertainty because the principal is more concerned about the allocation of profit across states (in this sense, it is as if the principal were more risk-averse). On the other hand, as demonstrated in the preceding analysis, the total wage bill under tournaments is invariant while under piece rate contracts it is not. This is more important to the principal under price uncertainty because, if total output turns out to be high and price happens to be low, limiting costs is of primary significance. Therefore, under tournaments, there is an interesting trade-off for the principal between providing insurance to the agents and providing insurance to himself against variation in his cost.

In formulating the limited liability constraint under price uncertainty note that there are two candidates for the lowest state. Either output per agent is zero regardless of output price or price is the lowest possible and output is high. For tractability we assume that output per agent is in the interval $[0, x_h]$ with non trivial probabilities, while probability that output is greater than $x_h$ is negligible. Further, output price is normalized to be in the interval $[p_l, p_h]$ with an expected price equal to 1 in order to simplify the comparison to the preceding analysis. Thus, the limited liability constraint under the piece rate contract is:

$$
\begin{align*}
(p_l - \beta_R)x_h - b_R + \alpha \geq 0 \\
(p_h - \beta_R)0 - b_R + \alpha \geq 0
\end{align*}
\implies
\begin{align*}
\alpha \geq b_R - (p_l - \beta_R)x_h \\
\alpha \geq b_R
\end{align*}
$$

Clearly if $p_l \geq \beta_R$, then when the second LLP condition is satisfied the first condition is also satisfied. Hence, if the marginal revenue from an additional unit of output, $p_l$, exceeds the marginal cost, $\beta_R$, the characterization of the piece rate contract is identical to that above in section 3B. By contrast if $p_l < \beta_R$, then when the first condition is binding the second one is non-binding. Thus, when price uncertainty is relatively
large, the LLP constraint is tighter than absent price uncertainty, that is, the constraint becomes binding over a wider range of \( \alpha \) values. In this case, if we denote the contractual parameters by \((\tilde{b}_R, \tilde{\beta}_R)\) and by using a method analogous to that in section 3B, \((\tilde{b}_R, \tilde{\beta}_R)\) must simultaneously satisfy

\[
\begin{align*}
\tilde{b}_R &= \alpha + (p_l - \tilde{\beta}_R)x_h \\
\tilde{\beta}_R &= \frac{r(\sigma^2 + \sigma^2_\varepsilon) - a}{2} \tilde{\beta}_R - a \tilde{\beta}_R
\end{align*}
\]  

(43)

It follows that

\[
\tilde{\beta}_R = \frac{a - x_h + \sqrt{(x_h - a)^2 + 2[r(\sigma^2 + \sigma^2_\varepsilon) - a](\alpha + p_l x_h)}}{r(\sigma^2 + \sigma^2_\varepsilon) - a},
\]

(44)

and

\[
\tilde{b}_R = \alpha - \tilde{\beta}_R x_h.
\]

(45)

Thus,

\[
E\tilde{\Pi}_R = a(\tilde{\beta}_R - \tilde{\beta}_R^2) - \alpha - \tilde{\beta}_R x_h.
\]

(46)

Next we turn to the tournament. Under tournament the LLP constraint is stated as:

\[
\begin{align*}
\left\{ \begin{array}{l}
p_l x_h - b_T + \alpha \geq 0 \\
p_h \cdot 0 - b_T + \alpha \geq 0
\end{array} \right\} \iff \left\{ \begin{array}{l}
\alpha \geq b_T - p_l x_h \\
\alpha \geq b_T
\end{array} \right\}
\]

(47)

Clearly, when the second condition is binding the first one is non-binding. Thus, price uncertainty has no impact on the contractual parameters under tournament. As in section 4B,

\[
\tilde{b}_T = \alpha,
\]

(48)

\[
\tilde{\beta}_T = \sqrt{\frac{2\alpha}{n - 1} \left( \frac{n - 1}{n} a + r \sigma^2_\varepsilon \right)},
\]

(49)
The intuition is that the invariability in the total wage bill under tournament dominates the principal’s concern about providing insurance to the agent when total uncertainty is increased.

Similar to the analysis without price uncertainty, conditions (46) and (50) imply that there exists a critical value $\alpha^\ast$ below which the piece rate is dominant and above which the tournament is dominant. Figure 4 presents three examples depending on the value of the parameters. In the left graph where the parameters are set to $n = 30$, $a = 10$, $r = \sigma_{\eta}^2 = \sigma_{\xi}^2 = 1$, $x_h = 5a$ and $p_l = 0.01$, the piece rate is never superior over the tournament. In the middle graph where the parameters are set to $n = 30$, $a = 10$, $r = \sigma_{\eta}^2 = \sigma_{\xi}^2 = 1$, $x_h = 5a$, and $p_l = 0.4$, the range over which the piece rate is superior is smaller than in the case without price uncertainty. Finally, in the right graph where $n = 30$, $a = 10$, $r = \sigma_{\eta}^2 = \sigma_{\xi}^2 = 1$, $x_h = 5a$, and $p_l = 0.6$, the range over which the piece rate is superior is unaffected.

To summarize our findings, if the lowest possible price $p_l$ exceeds the piece rate $\tilde{R}_p$ the form of the dominant contract for different liquidation values per agent is completely unaffected by the presence of price uncertainty. By contrast if the lowest possible price is

$$E\tilde{\Pi}_T = a + a \sqrt{\frac{2\alpha}{a + \frac{n}{n-1} r \sigma_{\xi}^2}} - \alpha.$$ 

(50)
smaller than the piece rate, for instance, if it is possible that the principal cannot find any buyers for the output produced so that \( p_l = 0 \), then the range of liquidation values over which piece rate contracts are superior is weakly reduced. The intuition why tournaments dominate piece rate contracts over a wider range of liquidation values is, again, that the principal benefits from the removal of the cost variability under tournament. In all, our analysis demonstrates that the liquidation value of the firm is the definitive factor in the determination of the contract the principal should offer. Thus, unless the lowest possible price is extremely small so that the range of liquidation values over which piece rates are dominant is eliminated, if the liquidation value of the firm is sufficiently small, the principal will prefer to offer a piece rate contract.

1.8. Conclusions

Starting with Lazear and Rosen (1981) and Green and Stokey (1983), contract theory has long argued that tournaments dominate piece rate contracts in the presence of common shocks that affect agent performance, when agents are risk averse. Relative performance evaluation via tournaments constitutes a move closer to the First Best because the principal becomes better informed. By removing common uncertainty from the responsibility of the agents, and by charging a premium for this insurance, the principal increases his profit without hurting the agents.

This paper shows that this celebrated result, which, absent limited liability, holds if common uncertainty is larger than only a fraction of the idiosyncratic uncertainty, does not hold at all when the principal is subject to limited liability and the liquidation value of the firm is sufficiently small. Under a limited liability constraint tournaments are dominated by piece rate contracts when the liquidation value of the firm is sufficiently small, because the principal is concerned about the allocation of profit across states. Piece rates allow the principal to decrease the total wage bill if the state of nature is unfavorable and output turns out to be small. By contrast the total wage bill under tournaments is constant and independent of output. Surprisingly, if the liquidation value
is not sufficiently small tournaments dominate piece rates even when the limited liability constraint is binding, which it is for intermediate values. Surprisingly again, when price uncertainty is introduced, if the lowest possible output price is not very small the dominant contract form is completely unaffected by the presence of price uncertainty. By contrast, if the lowest possible price is sufficiently small, specifically if it is smaller than the piece rate, the increased bankruptcy risk strengthens the need for tournaments by expanding the range of liquidation values over which tournaments are superior. The rationale is that from the principal’s perspective there is tension between providing insurance to the agent against common shocks and insuring himself against the variability in the total wage bill. The principal prefers to offer a tournament in order to eliminate the variability of total wages, even though the limited liability constraint is tighter with significant price uncertainty.

To conclude, our analysis shows that the liquidation value of the firm is much more important than the magnitude of price uncertainty in determining the form of the contract the principal should offer. Thus, regardless of price uncertainty, if the liquidation value is really small the principal will prefer to offer a piece rate contract, and if the liquidation value is really large the principal will prefer to offer a tournament. To the best of our knowledge these are novel findings in the theory of tournaments.

Empirical evidence form the broiler, turkey and swine industries in Tsoulouhas and Vukina (1999) suggests that smaller companies do rely more heavily on piece rates rather than on tournaments, as opposed to larger companies. Specifically, the frequency of observing tournaments diminishes as we move from the broiler industry where the firms are largest, to the turkey industry where the firms are medium in size and to the swine industry where the companies are smallest.
2.1 Introduction

Even though linear contracts are only a proxy of the theoretically optimal non-linear contracts, they are popular in several occupations or industries (e.g., sales, physician contracts with HMOs, contracts between processors and farmers, and faculty raises), partly because they are simple to design and easy to implement and enforce.\textsuperscript{22} The most common linear contracts are the piece rate contract and the cardinal tournament. Under the piece rate contract each agent is evaluated according to his absolute performance or according to his performance against a predetermined standard, while under the tournament each agent is evaluated relative to the performance of his peers. In particular, under both schemes each agent receives a base payment and a bonus payment, but the

\textsuperscript{22}The non-linearity of the theoretically optimal contract is partly due to the fact that contracts accommodate all possible events. Holmström and Milgrom (1987), however, have argued that schemes that adjust compensation to account for rare events may not provide correct incentives in ordinary high probability circumstances.
bonus payment is determined by absolute performance in piece rates and by relative performance in tournaments. Following the footsteps of Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and others, the comparison of these two alternative evaluation methods has been the subject of current literature (Tsoulouhas (1999), Wu and Roe (2005 and 2006), Marinakis and Tsoulouhas (2007) and Tsoulouhas and Marinakis (2007)). This comparison is important because it allows us to contrast the efficiency of absolute performance evaluation against relative performance evaluation.

Absent liquidity considerations, when agents are risk averse and production is subject to sufficiently large common shocks the tournament is a superior incentive scheme to the piece rate. This is because the tournament uses the information generated by the performance of the group of participating agents as a whole, while the piece rate does not. Specifically, if the disturbance in the output of each agent is correlated with the disturbances in the output of the other agents, the information contained in the average production can be very useful to the principal in creating a contract which is a step closer to the First Best. Moreover, under the tournament, if the principal is risk-neutral or is less risk-averse than the agent, an insurer–insured relationship can be developed between principal and agent allowing for a Pareto improvement of the contract. That is, the principal will offer insurance to the risk averse agent by filtering away the common shock from his responsibility. Insurance will make the agent more tolerant to a higher-power incentive scheme and, therefore, the agent is expected to increase his effort level.

One might conjecture that the superiority of tournaments over piece rates may not survive under liquidity constraints. Marinakis and Tsoulouhas (2007) have shown that the optimality of tournaments over piece rates breaks down when the risk-neutral principal is subject to a limited liability (bankruptcy) constraint, which limits the payments a principal can make, provided that the liquidation value of the principal’s enterprise

\[\text{The base payment ensures agent participation and the bonus provides incentives to perform. Under tournament an agent receives a bonus if his performance is above that of his peers, and a penalty otherwise.}\]
is sufficiently small. This is so because tournaments increase payments in unfavorable states, but these are the states in which the limited liability constraint comes into play. The intuition is that contracts with risk neutrality and limited liability for the principal look very much like those that would have been obtained with risk aversion. In other words, if the principal is concerned about the allocation of profit across states, he will no longer offer insurance against common shocks via tournaments and will resort to piece rate contracts or fixed performance standards. This paper investigates the optimality of tournaments over piece rates when the agent, instead, is subject to a liquidity constraint which introduces ex post limitations on the minimum payment the agent can accept or the maximum penalty that can be imposed on him (Innes 1990, 1993a and 1993b). The liquidity constraint prevents the principal from compensating the agent by an amount smaller than a predetermined level in all states of nature.

The models used by Lazear and Rosen (1981), Green and Stokey (1983), Meyer and Vickers (1997) and others, allow the payments to the agents to be negative. In particular, under both the piece rate and the tournament payment schemes, if the agents produced a sufficiently low output they would usually have to pay the principal. Thus, according to the standard literature, if the production of an agent is sufficiently low the principal will penalize the agent by imposing a negative compensation and acquire whatever output the agent produced. This is certainly inconsistent with what we observe in reality.

The liquidity constraint is partly an institutional constraint on contracts. It is imposed by law for several industries in numerous countries. Such legislation aims at removing the burden of excessive penalties imposed on agents for negative outcomes beyond their control, rather than at maximizing social welfare. However, a liquidity constraint for the agent may alter the choice the principal makes between tournaments and absolute performance contracts. This can be due to a number of reasons. Some of these reasons are in favor of tournaments and some are in favor of piece rates. First, by increasing payments to the agents in unfavorable states, tournaments are more likely
to satisfy tight liquidity constraints for the agents. Second, by providing insurance,
tournaments may satisfy the liquidity constraints for the agents without paying rents
to them. This is so because tournaments increase the compensation to the agents in
unfavorable states but they reduce the payments in favorable states. By contrast piece
rates may pay the agents ex ante rents when the liquidity constraints are tight (i.e., when
the minimum required payment to the agents is high), which reduces the principal’s profit.
If piece rates pay ex ante rents to the agents, they could be dominant over tournaments
from the principal’s perspective only if implemented effort under piece rates were higher.
But, in general, tournaments allow the principal to implement higher-power incentives
than piece rates, which enhances the dominance of tournaments. Third, agents may be
unable to pay for insurance especially in low states of nature if the liquidity constraints
are tight, which works against tournaments. Fourth, the attitude of the principal and
the agents toward risk may change. Liquidity constraints may make the agents more
tolerant to risk, in the sense that if the agents know that their liability is limited, they
may become indifferent among the range of states over which the liquidity constraint is
binding. This is certainly in accord with Laffont and Martimort (2002) who state (see
p.121):

“A limited liability constraint on transfers implies higher-powered incentives
for the agent. It is almost the same as what we would obtain by assuming
that the agent is a risk lover. The limited liability constraint on transfers
somewhat convexifies the agent’s utility function.”

On the other hand, the liquidity constraints for the agents are expected to make the
principal care about the allocation of payments and, hence, profit across states to satisfy
the liquidity constraints and ensure agent participation. When the principal becomes less
tolerant to risk, while agents simultaneously become more tolerant to risk and, therefore,
they are not willing to pay enough for insurance, the principal may find it suboptimal to
offer insurance to the agent through a tournament and may resort to piece rates again.
Thus, in all, it is not a priori clear if tournaments, which are normally superior over piece rates when production is subject to common shocks, maintain their superiority under liquidity constraints for the agents.

Our analysis shows that, surprisingly, in the presence of common uncertainty a principal contracting with risk averse agents will prefer to offer a tournament even when agents are liquidity constrained. This finding is diametrically opposite to the result for the case when the principal, instead, is subject to limited liability. The rationale for this result follows directly from the discussion above. It turns out that by providing insurance against common shocks through a tournament, so that payments to the agents in unfavorable states increase and payments in favorable states decrease, the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them while simultaneously providing them with higher-power incentives than under piece rates. The individual rationality constraints for the agents are always binding under tournaments, whereas under piece rates they are non-binding (that is, the agents receive ex ante rents) when the liquidity constraints for the agents are really tight (that is, when the minimum payment required to satisfy the liquidity constraints is high). This finding establishes our claim that the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them under tournament. Our second claim, that the principal can implement higher-power incentives under tournament, follows from the fact that the piece rate cannot be defined for a piece rate larger than one (in the sense that the principal would not make an offer such that marginal cost exceeded marginal revenue) whereas the tournament is defined for a larger bonus factor. The larger the minimum payment satisfying an agent’s liquidity constraint, the higher the power of incentives the principal provides. In other words, the principal counterbalances the increase in the base payment, which is required to satisfy the liquidity constraint, with higher-power incentives in order to curb agent rents and in order to reduce the likelihood that output is low. Tournaments provide the principal with added flexibility in the determination of
this power when the liquidity constraints are really tight. On the other hand, regardless of whether the principal offers a piece rate or a tournament, the liquidity constraints for the agents are non-binding (that is, in some sense, agents receive ex post rents) when the minimum payment required to satisfy the liquidity constraints is low. In that case, the analysis is similar to the benchmark case in Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983), and tournaments are optimal under sufficient common uncertainty.

The empirical application that stems from our analysis is that firms should adopt relative performance evaluation via tournaments over absolute performance evaluation via piece rates regardless of whether the agents are liquidity (wealth) constrained or not. This finding enhances the generality of the results obtained in Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983). For instance, in the case of processor companies contracting with farmers who most often are liquidity constrained, processors need not fear that the farmers’ liquidity issues detract from the superiority of tournaments.\textsuperscript{24}

Even though the issue we analyze has been largely overlooked by the current tournament literature, the introduction of liquidity constraints on the agent side is not novel. Bhattacharya and Guasch (1988) examine the efficiency of tournaments with heterogeneous agents. They argue that tournaments that are based on comparisons across ability levels are more efficient than tournaments that are based on comparisons within cohorts of similar ability agents. However, this result is reversed when agents are subject to limited liability (liquidity) constraints, because tournaments with comparisons across cohorts are more likely to lead to negative payments. Kim (1997) analyzes a setting with a risk neutral principal and a risk neutral agent when the agent’s liability is limited. He shows that the optimal contract is a bonus contract in which the principal and the agent share the output, and the agent receives an additional fixed bonus only when output is

\textsuperscript{24}Wealth constraints can certainly be a concern in contracts for salesmen as well.
greater than some predetermined level. Demougin and Garvie (1991) examine two forms of constraints for risk neutral agents: non-negativity constraints for the transfers to the agents and ex post individual rationality constraints for the agents. They show that the principal cannot implement the First Best and agents earn informational rents. Courty and Marschke (2002) analyze a framework with liquidity constraints and budget balancing. They show that when the difference in agent budgets is large enough, the liquidity and budget balancing constraints bind, thereby reducing the effectiveness of incentives. Demougin and Fluet (2003) focus on examining the cost of providing incentives through rank-order tournaments when agents care about the fairness of their payoffs relative to that of others, and agents are subject to limited liability which makes rents possible. They show that the presence of more envious contestants reduces the principal’s cost of providing incentives, when rents must be paid, because the agents will motivate themselves to perform even with lower rents from the principal. Kräkel (2007) analyzes a model with risk-neutral agents who are subject to limited liability, but face no common uncertainty inflicted on their productive activities, to make the point that the Lazear and Rosen (1981) finding of equal incentive efficiency for piece rates and for rank-order tournaments does not necessarily carry over when limited liability is introduced. In particular, piece rates dominate tournaments if idiosyncratic risk is high. This is an intuitive result because, even absent limited liability, a tournament would be suboptimal by introducing idiosyncratic noise from the activity of other agents onto the payment to any given agent. Therefore, the introduction of limited liability should not change that, but it should change the specification of the piece rate. Namely, given risk-neutrality, limited liability should entail a move from the "selling the enterprise to the agent" solution (i.e., a piece rate of 1) to a piece rate of less than one, because the liquidity constraint prevents the sale of the enterprise to the agent. We differ from Kräkel in a number of important respects. We assume the existence of common uncertainty which provides scope for tournaments. We also assume that agents are risk-averse to incorporate the insurance aspect
of tournaments, and we show that tournaments in our setting are dominant over piece rates with or without limited liability. Last but not least, note that similar to Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Malcomson (1984) we are not looking for the optimal contract, instead, we contrast the efficiency properties of absolute to relative performance evaluation.

Section 2 presents our model, section 3 presents the benchmark case without liquidity constraint and section 4 presents our results when the agents are liquidity constraint. Section 5 determines the dominant compensation scheme and section 6 concludes.

2.2 Model
A principal signs a contract with \( n \) homogeneous agents. Each agent \( i \) produces output according to the production function \( x_i = a + e_i + \eta + \varepsilon_i \), where \( e_i \) is the agent’s effort, \( \eta \) is a common shock, \( \varepsilon_i \) is an idiosyncratic shock and \( a \) is a positive constant. The idiosyncratic shocks, \( \varepsilon_i \), and the common shock follow independent distributions. Each agent’s effort and the subsequent realizations of the shocks are private information to him, but the output obtained is publicly observed. The principal compensates agents for their effort based on their outputs by using a piece rate contract or a tournament. Agent preferences are represented by a CARA utility function \( u(w_i, e_i) = \exp(w_i + \frac{1}{2 \alpha} e_i^2) \), where the agent’s coefficient of absolute risk aversion is set equal to 1 for simplicity. The cost of effort is measured in monetary units. Each agent has a reservation utility \( -\exp(-\bar{u}) \).

2.3 Piece Rates and Tournaments without Liquidity Constraints
We start by deriving the optimal contractual variables for the piece rate and the tournament without liquidity constraints for the agents. We assume that the total production disturbance, \( \varepsilon_i + \eta \), follows a normal distribution with zero mean and variance equal to \( c/\sqrt{2\pi} \), and the idiosyncratic shock, \( \varepsilon_i \), follows a normal distribution with zero mean and variance equal to \( d/\sqrt{2\pi} \).

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25 Tsoulouhas and Marinakis (2007) examine a model with ex post heterogeneous agents to make the point that agent heterogeneity compromises the insurance function of tournaments.

26 As will become obvious in the remaining analysis, this assumption on the variance simplifies the
The piece rate contract \((R)\) is the payment scheme in which the compensation to the \(i^{th}\) agent is \(w_i = b_R \beta_R x_i\), where \((b_R, \beta_R)\) are the contractual variables to be determined by the principal. The principal determines these parameters by backward induction. Thus, the principal calculates each agent’s expected utility

\[
EU_R = - \exp \left( -b_R - \beta_R (a + e_i) + \frac{e_i^2}{2a} + \frac{\beta_R^2 c}{2\sqrt{2\pi}} \right). \tag{51}
\]

To ensure the compatibility of the contract with agent incentives to perform, the principal calculates the effort level that maximizes (51). First order conditions yield

\[
e_i = a \beta_R. \tag{52}
\]

To ensure the compatibility of the contract with agent incentives to participate, the principal selects the value of the base payment, \(b_R\), that satisfies the agent’s individual rationality constraint with equality so that the agent receives no rents but still accepts the contract. The agents individual rationality constraint satisfies \(EU_R = -\exp(-\pi)\), where \(EU_R\) is determined by (51) and (52). Solving for \(b_R\) implies

\[
b_R = \bar{u} + \frac{\sqrt{2\pi}}{2} \beta_R^2 - a \beta_R. \tag{53}
\]

Thus, by choosing the piece rate \(\beta_R\), the principal can precisely determine the agent’s effort because the agent will optimally set his effort according to (52). In addition, by setting \(b_R\) in accordance with (53) the principal can induce agent participation at least cost. That is, agent incentives to perform are only determined by the bonus factor \(\beta_R\), whereas agent incentives to participate are determined by the base payment \(b_R\).

\[\text{exposition.}\]

\[\text{27Note that the concavity of the utility function implies that first order conditions are sufficient.}\]
Given conditions (52) and (53) the principal maximizes his expected total profit

\[ \text{ET} \Pi_R = \sum_{i=1}^{n} [Ex_i - Ew_i] = n \left[ a + a\beta_R - \frac{c}{\sqrt{2\pi}} + \frac{a}{2}\beta_R^2 - \bar{u} \right]. \] (54)

The solution to this problem satisfies

\[ \beta_R = \frac{a}{a + \frac{c}{\sqrt{2\pi}}}. \] (55)

Condition (53) then implies

\[ b_R = \bar{u} - \frac{a^2}{2} \frac{c}{\sqrt{2\pi}} + 3a \left[ \frac{c}{\sqrt{2\pi}} + a \right]^2. \] (56)

Given conditions (55) and (54) expected profit per agent is

\[ E\Pi_R = a + \frac{1}{2} \frac{a^2}{a + \frac{c}{\sqrt{2\pi}}} - \bar{u}. \] (57)

The tournament (T) is the payment scheme in which the compensation to each agent is determined by a relative performance evaluation. Specifically, \( w_i = b_T + \beta_T(x_i - \bar{x}) \), where \( \bar{x} \) is the average output obtained by all agents and \( (b_T, \beta_T) \) are the contractual variables to be determined by the principal. Under a tournament the agent’s expected utility is

\[ EU_T = -\exp \left( -b_T - \beta_T \frac{n-1}{n} (a + e_i) + \beta_T \frac{1}{n} \sum_{j \neq i} (a + e_j) + \frac{e_i^2}{2a} + \frac{1}{n} - \frac{1}{n} \frac{\beta_T^2 d}{\sqrt{2\pi}} \right). \] (58)

The effort level that maximizes (58) satisfies \( e_i = \frac{n-1}{n} a\beta_T \). (59)
Further, the individual rationality constraint $EU_T = -\exp(-\bar{u})$ implies

$$b_T = \bar{u} + \frac{1}{2} \frac{n-1}{n} \left( \frac{n-1}{n} a + \frac{d}{\sqrt{2\pi}} \right) \beta_T^2.$$  (60)

Then, given conditions (59) and (60), the principal maximizes expected total profit

$$ET\Pi_T = n \left[ a + \frac{n-1}{n} a \beta_T - \frac{1}{2} \frac{n-1}{n} \left( \frac{n-1}{n} a + \frac{d}{\sqrt{2\pi}} \right) \beta_T^2 - \bar{u} \right].$$  (61)

The solution to the principal’s maximization problem satisfies

$$\beta_T = \frac{a}{\frac{n-1}{n} a + \frac{d}{\sqrt{2\pi}}},$$  (62)

therefore,

$$b_T = \bar{u} + \frac{1}{2} a + \frac{a^2}{n-1 \sqrt{2\pi}}.$$  (63)

Given (62) and (61) expected profit per agent is

$$E\Pi_T = a + \frac{1}{2} a + \frac{a^2}{n-1 \sqrt{2\pi}} - \bar{u}. $$  (64)

By comparing (54) to (64) it is easy to show that

$$E\Pi_T > E\Pi_R \Leftrightarrow \frac{n}{n-1} d < c,$$  (65)

that is, tournaments are superior when total uncertainty is large relative to the idiosyncratic uncertainty (equivalently, when common uncertainty is relatively large) and when the number of agents is large. This so because tournaments eliminate common uncertainty but they add the average individual noise of others.
It is also straightforward to show that

\[ \beta_T > \beta_R \]  

and

\[ b_T > b_R. \]  

The rationale behind (67) is that the expected bonus payment under tournament is zero, whereas that under piece rate is positive. Therefore, agents expect to be compensated for effort through the base payment in a tournament. The intuition behind (66) is that the principal implements higher-power incentives when common uncertainty is removed from the responsibility of the agent under tournament.

2.4 Piece Rates and Tournaments with Liquidity Constraints

Next we turn to the case with liquidity constraints for the agents. The liquidity constraint is

\[ w_i \geq w, \]  

where \( w \) is the minimum permissible payment. The liquidity constraints for the agents necessitate a support for the production shocks which is bounded below and above. The support must be bounded below so that in the worst possible output state the liquidity constraints are still satisfied (obviously they cannot be satisfied with an output space which is unbounded below). For a similar reason, the support must be bounded above to eliminate the case when the payment under tournament is below the minimum required to satisfy the liquidity constraint when average output is unbounded above.\(^{28}\) With bounded support for the production shocks one might expect that the First Best is always implementable by punishing the agent severely for outcomes outside the support (see p.

\(^{28}\) An alternative approach would be to consider a modification of the payment schemes such that the agent still receives the minimum payment required to satisfy his liquidity constraint, \( w \), specifically, consider \( \max\{w, w_i\} \) where \( w_i \) is determined by the scheme. However, the analysis in this case is intractable.
Note, however, that the liquidity constraints of the agents prevent severe punishment of them.

The requirement of bounded support eliminates unbounded distributions such as the normal we used in section 3 (which is typically used in the literature for the setting without liquidity constraints). The normal distribution is necessary to obtain a closed form solution for the case without liquidity constraints. Further, a truncated normal distribution provides neither a closed form solution nor a numerical one. However, we were able to obtain significant insight through a numerical analysis by assuming that the idiosyncratic and the common shocks follow independent uniform distributions, in which case the sum of these shocks follows a triangular distribution. Specifically, the idiosyncratic shocks, \( \varepsilon_i \), follow independent uniform distributions with support \([-d, d]\) and, therefore, the total production shock, \( v_i \equiv \varepsilon_i + \eta \), follows a triangular distribution with density \( f(\cdot) \), the support of which is assumed to be \([-c, c]\) with zero mean. The following lemmata apply to piece rates and tournaments with liquidity constraints.

**Lemma 1** Under piece rates at least one of the individual rationality and the liquidity constraints for each agent binds depending on the values of the model parameters.

**Proof.** The proof is straightforward by noting that if both constraints were non-binding, then, the principal would reduce the payments to the agent until one of the two constraints became binding (that is, until the agent received no rents in an ex ante or in an ex post sense). As shown in section 3, solving without the liquidity constraint for each agent (in which case the individual rationality constraint is obviously binding) implies that the contractual variables \( (b_R, \beta_R) \) satisfy conditions (56) and (55) and therefore the payment \( w_i \) may or may not satisfy the liquidity constraint in all states depending on the values of parameters \( \underline{w}, \overline{w}, a \) and \( c \). Therefore, when the individual rationality constraint is binding, the liquidity constraint is binding or non-binding (the latter when \( \underline{w} \) is relatively low). Solving without the individual rationality constraint (in which case the liquidity constraint is obviously binding in the lowest possible state) implies that the payments
to the agent may or may not satisfy the individual rationality constraint depending on
the values of parameters \(w, \overline{u}, a\) and \(c\) again. Therefore, when the liquidity constraint is
binding the individual rationality constraint is binding or non binding (the latter when \(w\) is relatively large).

**Lemma 2** Under tournaments the individual rationality constraint for each agent is al-
ways binding and the liquidity constraint for each agent is binding or non-binding de-
pending on the values of the model parameters.

**Proof.** First, similar to Lemma 1, the two constraints cannot simultaneously be non-
binding. Solving without the individual rationality constraint (in which case the liquidity
constraint is obviously binding in the lowest possible state) implies that \(b_T = w + \beta_T d\).
This is so because \(w_i = b_T + \beta_T (x_i - \overline{x}) = w\) and, given that \(\varepsilon_i \in [-d, d]\), if the number
of agents is sufficiently large \(x_i - \overline{x} \overset{D}{\rightarrow} uniform[-d, d]\). Then, since the principal’s profit
per agent is \(\Pi_i = x_i - b_T = a - w + \left(\frac{n-1}{n} a - d\right) \beta_T + \varepsilon_i + \eta\), it follows that expected profit
per agent is \(E \Pi_T = a - w + \left(\frac{n-1}{n} a - d\right) \beta_T\). To maximize this expected profit the principal
chooses the maximum \(\beta_T\) that satisfies the individual rationality constraint with equality
so that the agent accepts the contract. Therefore, the individual rationality constraint
is always binding. As shown in section 3, solving without the liquidity constraint (in
which case the individual rationality constraint is obviously binding) implies that the
contractual variables \((b_T, \beta_T)\) satisfy (63) and (62) and therefore the payment \(w_i\) may
or may not satisfy the liquidity constraint in all states depending on the values of parameters
\(w, \overline{u}, a\) and \(d\). Therefore, when the individual rationality constraint is binding, the
liquidity constraint is binding or non binding (the latter when \(w\) is relatively low).

We start by analyzing the piece rate case. The piece rate scheme can be written as
\(w_i = b_R + \beta_R (a + e_i + v_i)\). As Lemma 1 indicates, the individual rationality constraint
can be binding or not. Because of this, the procedure for determining the contractual
variables is somewhat different than the one we followed above for the case without
liquidity constraints (without liquidity constraints the individual rationality constraints are always binding). With liquidity constraints, we determine the base payment \( b_R \) through these constraints, and the piece rate \( \beta_R \) from the profit maximizing condition. Then we check whether this solution satisfies the individual rationality constraints.

Clearly, if the payment satisfies the liquidity constraint (68) in the lowest possible state, then, it satisfies the constraint in all states because the payment scheme is increasing in the state. Therefore, if the constraint is binding in the lowest state, then it is non-binding in all states. From the agent’s perspective, given that the principal controls incentives through the payment scheme, the worst state is the one in which the principal provides him no incentives to perform and the production state turns out to be the worst, that is, \( e_i = 0 \) and \( v_i = -c \). In the remaining analysis we focus on the case when the liquidity constraint is binding in the lowest possible state. Therefore, the principal will set

\[
b_R = w - \beta_R (a - c).
\]  

(69)

The expected utility for the agent is

\[
EU_i = - \int_{-c}^{c} \exp (-\beta_R v_i) f(v_i) dv_i \exp \left( \frac{-w - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a}}{2a} \right).
\]  

(70)

To provide correct incentives to the agent, the principal calculates the effort level \( e_i \) that maximizes (70). First order conditions yield

\[
- \int_{-c}^{c} \exp (-\beta_R v_i) f(v_i) dv_i \exp \left( \frac{-w - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a}}{2a} \right) \left( -\beta_R + \frac{e_i}{a} \right) = 0
\]  

(71)

and, because \( \int_{-c}^{c} \exp (-\beta_R v_i) f(v_i) dv_i \) and \( \exp \left( \frac{-w - \beta_R c - \beta_R e_i + \frac{e_i^2}{2a}}{2a} \right) \) cannot be equal to zero, it follows that

\[
e_i = a\beta_R.
\]  

(72)

The principal’s profit per agent is

\[
\Pi_i = (1 - \beta_R) x_i - b_R = a + a\beta_R - a\beta_R^2 - w - \beta_R c +
\]
Then the expected profit per agent is

$$E\Pi_R = a + a\beta_R - a\beta_R^2 - w - \beta Rc. \quad (73)$$

Maximizing the expected profit with respect to $\beta_R$ yields

$$\beta_R = \frac{a - c}{2a}. \quad (74)$$

Hence, given the contractual variables and the optimal effort level for the agent, the expected profit per agent is

$$E\Pi_R = \frac{5}{4}a + \frac{1}{4}c^2 - \frac{1}{2}c - \frac{w}{4}. \quad (75)$$

Note that condition (75) indicates that the principal will make an offer only if $w$ is relatively low, otherwise production is unprofitable.

Given conditions (69) and (74), the individual rationality constraint requires

$$- \int_{-c}^{c} \exp \left( -\frac{a - c}{2a} v_i \right) f(v_i) dv_i \exp \left( -\frac{w}{4} - \frac{3}{8}c + \frac{5}{8}c^2 + \frac{1}{8}a \right) \geq - \exp(-\bar{u}). \quad (76)$$

Clearly, (76) may or may not hold, depending on the values of parameters $w$, $\bar{u}$, $a$ and $c$. If it holds, then the contractual variables to be offered by the principal satisfy (69) and (74). If (76) does not hold, that is, if $\beta_R$ in (74) violates the individual rationality constraint, then the individual rationality constraint is binding. In this case, $\beta_R$ must be determined through the individual rationality constraint with equality. Given (70), (72) and the density function for $v_i$, the individual rationality constraint is written as

$$\int_{-\bar{c}}^{\bar{c}} \exp (-\beta_R v_i) \frac{c - |v_i|}{c^2} dv_i \exp \left( \frac{1}{2} a\beta_R^2 - \beta Rc \right) \exp(-w) = \exp(-\bar{u}). \quad (77)$$
Given that \( c > 0 \), (77) is equivalent to

\[
\frac{1 + \exp(2\beta_Rc) - 2\exp(\beta_Rc)}{\beta_R^2e^2} \exp\left(\frac{1}{2}a\beta^2_R - \beta_Rc\right) \exp(\bar{u} - \bar{w}) - 1 = 0. \tag{78}
\]

A closed form solution for \( \beta_R \) is impossible to obtain from (78). As a result we have to rely on computational methods in order to determine the piece rate values \( \beta_R \) which are individually rational. Our computations proceed as follows: We derive the contractual variables from equations (69) and (74) assuming that the liquidity constraint is binding in the lowest state and ignoring the individual rationality constraint. Then we check if the individual rationality constraint (76) is satisfied by the solution (in which case it is non-binding) or if it is violated (in which case it is binding). If (76) is found to be binding, then the piece rate \( \beta_R \) is derived by the solution of (78) using a Newton algorithm and \( b_R \) is still determined by (69). In this case, when we have multiple solutions for \( \beta_R \), we keep the one maximizing the principal’s profit. If (76) is found to be non-binding we keep the solutions from equations (69) and (74).

Next, we turn to the tournament case. Recall that under the tournament the compensation to each agent is \( w_i = b_T + \beta_T(x_i - \overline{x}) \), which can be written as \( w_i = b_T + \beta_T(e_i - \overline{e}) + \beta_T\vartheta_i \), where \( \vartheta_i \equiv e_i - \overline{e} \), with \( \overline{e} \) denoting the average effort and \( \overline{e} \) denoting the average idiosyncratic shock. Given that the agents are homogeneous, the contract is uniform for all agents and the optimal effort level is equal in equilibrium for all agents. Thus, the compensation to each agent can be expressed as \( w_i = b_T + \beta_T\vartheta_i \).

As shown in the proof of Lemma 2,

\[
b_T = \overline{w} + \beta_Td. \tag{79}
\]

Similar to piece rates, if the liquidity constraint is binding in the lowest state, then it is non-binding in all states, because the payment under tournament is also increasing in
the state. The agent’s expected utility is

\[ EU_i = - \int_{-d}^{d} \exp(-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i \exp \left( -w - \beta_T d - \beta_T \frac{n-1}{n} e_i + \beta_T \frac{1}{n} \sum_{j \neq i} e_j + \frac{e_i^2}{2a} \right). \]

(80)

The effort level that maximizes the agent’s expected utility satisfies

\[ - \int_{-d}^{d} \exp(-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i \exp \left( -w - \beta_T d - \beta_T \frac{n-1}{n} e_i + \beta_T \frac{1}{n} \sum_{j \neq i} e_j + \frac{e_i^2}{2a} \right) \cdot \left( -\beta_T \frac{n-1}{n} + \frac{e_i}{a} \right) = 0. \]

(81)

Because the product of the first two terms in the equation cannot be equal to zero, it follows that

\[ e = \frac{n-1}{n} a \beta_T. \]

(82)

Given Lemma 2, which states that the individual rationality constraint is always binding under tournaments with liquidity constraints, the principal chooses the value of the bonus factor \( \beta_R \) that satisfies the agent’s individual rationality constraint with equality. Thus, (79) and (82) imply

\[ EU_i = - \int_{-d}^{d} \exp(-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i \exp \left( -w - \beta_T d + \frac{1}{2} \left( \frac{n-1}{n} \right)^2 a \beta_T^2 \right) = - \exp(-\pi). \]

(83)

Note that in equilibrium \( x_i - \bar{x} \sim D \rightarrow \text{uniform}[-d, d] \), when the number of agents is sufficiently large. Hence, \( \int_{-d}^{d} \exp(-\beta_T \vartheta_i) f(\vartheta_i) d\vartheta_i \) converges to

\[ \int_{-d}^{d} \exp(-\beta_T \vartheta_i) \frac{1}{2d} d\vartheta_i = \frac{\exp(\beta_T d) - \exp(-\beta_T d)}{2\beta_T d} \]

(84)
Then, (83) becomes

\[-\exp(\beta_T d) - \exp(-\beta_T d) \exp \left( -w - \beta_T d + \frac{1}{2} \left( \frac{n - 1}{n} \right)^2 a \beta_T^2 \right) = -\exp(-\overline{u}) \Leftrightarrow \]

\[\Leftrightarrow \exp \left( \frac{1}{2} \left( \frac{n - 1}{n} \right)^2 a \beta_T^2 - \beta_T d \right) \exp(\overline{u} - w) = \frac{2\beta_T d}{\exp(\beta_T d) - \exp(-\beta_T d)}. \] (85)

Clearly, similar to the piece rate case, equation (85) has no closed form solution. A solution can only be obtained by computational methods (recall that we use a Newton algorithm). The principal’s profit per agent is \( \Pi_i = x_i - b_T = a - w + \left( \frac{n - 1}{n} a - d \right) \beta_T + \varepsilon_i + \eta. \)

Hence, given the optimal base payment and the optimal effort level for the agent, the expected profit per agent is

\[ E\Pi_T = a - w + \left( \frac{n - 1}{n} a - d \right) \beta_T. \] (86)

where \( \beta_T \) can only be determined numerically by solving (85).

### 2.5 The Dominant Contract Under Liquidity Constraints

The principal’s decision about which compensation scheme to offer depends entirely on expected profits. Clearly, under both schemes, expected profits decline when a liquidity constraint is introduced in addition to the other constraints. Our analysis indicates that these profits decline faster under piece rates as the liquidity constraint becomes tighter. The intuition behind our result is that the liquidity constraint distorts the agent’s incentives to perform because it reduces the penalty the principal can impose for unfavorable outcomes. Therefore, the principal needs to provide higher-power incentives. By filtering common shocks from the responsibility of the agent, tournaments make the agent more tolerant to higher-power incentives, hence, it is easier for the principal to implement higher-power incentives under tournament than under piece rates. Moreover, the piece rate \( \beta_R \) cannot exceed 1 (i.e., because marginal cost cannot exceed marginal revenue).
By contrast, the bonus factor $\beta_T$ can exceed 1 which enables the implementation of higher-power incentives.

Figure 1 illustrates that tournaments are dominant over piece rates when liquidity constraints are introduced. In particular, panel (a) shows that expected profit is always strictly larger under tournament regardless of the value of $w$, that is, regardless of how tight the liquidity constraint is. For the case without liquidity constraints expected profits per agent are calculated by using conditions (57) and (64). Note that in our numerical analysis we assume that condition (65) holds, that is, we assume that common uncertainty is sufficiently large relative to the idiosyncratic uncertainty. For the case with binding liquidity constraints expected profits per agent are calculated by using condition (73) where $\beta_R$ is determined either by (74) or by the numerical solution of (78), and condition (86) where, again, $\beta_T$ is determined numerically by (85). Obviously, for the range over which the liquidity constraint is non-binding, expected profit is flat and independent of $w$. We confirmed this result for all possible values of common uncertainty, provided that there is some common uncertainty. With no common uncertainty, expected profits are trivially equal when the liquidity constraint is binding under piece rates. Nevertheless, even in this case, tournaments are expected to yield strictly higher profits for a sufficiently large $w$. Further, an increase in the minimum permissible wage $w$ decreases the expected profit under both schemes, but it does so much faster under piece rates. In fact, piece rates cannot be defined at all after a critical value of $w$ is passed (see point B in panel (a)), because the principal needs to offer a piece rate larger than 1 to provide correct incentives to the agent. However, given that the piece rate $\beta_R$ cannot exceed 1, piece rates cannot be defined.\footnote{In other words, the principal does not find it profitable to make an offer that the agent will accept.} In interpreting the results depicted in Figure 1, note that for $w$ in the range up to A the individual rationality constraint under piece rates is binding and the liquidity constraint is non-binding. For $w$ in the AB range the individual rationality constraint under piece rates is binding or non-binding and the liquidity constraint is
Figure 5: The expected profit per agent and the contractual variables for the piece rate contract and the tournament.
binding. Under tournaments, the individual rationality constraint is always binding (see Lemma 2). Lastly, for $w$ in the range up to $C$ the limited liability constraint under tournament is non-binding.

Panel (b) indicates that the base payment is always larger under tournament, but it increases with $w$, that is, when the minimum acceptable payment increases the base payment must also increase to provide correct incentives to the agent to participate. Further, panel (c) indicates that both the piece rate $\beta_R$ and the bonus factor $\beta_T$ increase when $w$ increases. There are two reasons for this: First, because the base payment increases when $w$ increases, the principal must provide the agents with higher-power incentives in order to exert more effort and make up in lost profit due to the increase in the base payment. Second, when $w$ increases, the principal provides the agents with higher-power incentives in order to minimize the likelihood that output is low and the principal is forced by the liquidity constraint to pay the minimum acceptable wage to the agent when, absent the constraint, it would have been optimal to pay less or impose a penalty. Again, note that piece rates are not defined for a piece rate above 1, whereas under tournaments the principal can continue to provide incentives to the agents through a bonus factor larger than 1, which explains the increased dominance of tournaments over piece rates for large values of the minimum acceptable payment $w$.

2.6. Conclusion

A familiar result in the principal-agent literature is that when agents are risk averse and production is subject to relatively large common shocks the tournament is a superior compensation scheme to the piece rate. The superiority of tournaments over piece rates may not survive under liquidity constraints. Prior research (for instance, Marinakis and Tsoulouhas (2007) for limited liability on the principal) would lead someone to expect the same result even when limited liability is imposed on the agent instead of the principal. In addition, one might also expect that limited liability would make the agents more tolerant to risk (in the sense that liquidity constraints convexify the agent’s utility.
function) and the principal less tolerant to risk (in the sense that the principal cares about the allocation of payments across states in order to satisfy the liquidity constraints). The reduced interest of agents in getting insurance, as well as the reduced ability of the principal to provide it, might also diminish the scope for tournaments. However, there is a fundamental difference between limited liability on the principal side and limited liability on the agent side. Under limited liability for the principal, the agents cannot be suckered by the prospect of payments the principal cannot make, therefore, the principal introduces a constraint to provide correct incentives to the agents. The constraint puts a maximum on the payment to the agents in low states and, hence, the solution looks like that if the principal were risk averse. Under liquidity constraints for the agents, instead, the agents will not sign a contract with the principal unless it satisfies these constraints and the principal incorporates the constraints to make sure that the agents participate. The constraints put a minimum on the payments to the agents in low states. Our analysis builds on this fact to show that in the presence of common uncertainty a principal contracting with risk averse agents will prefer to offer a tournament even when agents are liquidity constrained.

The rationale for our result is that by providing insurance against common shocks through a tournament, so that payments to the agents in unfavorable states increase and payments in favorable states decrease, the principal can satisfy tight liquidity constraints for the agents without paying any ex ante rents to them while simultaneously providing them with higher-power incentives than under piece rates. The larger the minimum payment satisfying an agent’s liquidity constraint, the higher the power of incentives the principal provides. In other words, the principal counterbalances the increase in the base payment, which is required to satisfy the liquidity constraint, with higher-power incentives in order to curb agent rents and in order to reduce the likelihood that output is low. Tournaments provide the principal with added flexibility in the determination of this power.
Chapter 3

Incentives, Effort and Peer Effects: A Field Experiment

3.1 Introduction

A well-known result in the principal-agent literature is that the principal is able to influence the agents’ choice of effort by providing the appropriate incentives. Numerous contract theory articles (for example, Holmström (1982) or more recently, Marinakis and Tsoulouhas (2007)) have shown that under simple linear contracts, such as piece rates or cardinal tournaments, the agent’s effort can be set directly by the principal at the level he considers optimal. Undoubtedly, the instructor-student grading relationship is an instance of the principal-agent model, in which agents choose the optimal level of effort in order to keep the cost of effort low but also, to maximize the utility gained from monetary compensation by the principal, who can observe nothing but final production. Similarly, in the instructor-student regime, students choose the optimal level of study in order to keep the disutility from study low but also, to maximize the utility gained from grades assigned by the instructor, who can only observe their performance in tests, exams and homework assignments. Given these similarities, a natural question is raised: can the instructor make students study harder by adjusting the grading method?
This is an interesting and meaningful question for the theory of incentives because, apart from the many similarities, the instructor - student regime exhibits a number of important qualitative differences compared to the standard principal - agent model. The key difference is that, under the standard principal - agent model, agents are considered to care only about their personal interest while, in the instructor - student regime, students often tend to involve altruism or envy in their decisions. This is because the classroom is a highly socialized environment in which students form long term social relationships and thus, apart from self interest, popularity is an important concern for students’ choices. In this sense, if the instructor is not able to affect the students’ performance by adjusting the grading method, then incentives work differently under the two settings and this would indicate that peer effects are of crucial importance for the theory and practice of incentives. Apart from the classroom, peer effects might play an important role in agents’ decisions in many other competitive environments with social characteristics. An example would be the situation in which the existing employees of a firm compete for a promotion. If the candidates for the position have developed strong social relationships, they may not prefer to take actions that maximize the likelihood of getting the promotion if these actions isolate them from their peers. Additional examples include associate attorneys who compete to become partners in a law firm and faculty annual raises based on performance relative to peer performance.

The paper adopts an experimental approach to investigate the effect of incentives in the instructor - student grading relationship. Two alternative grading methods were used in the grading of homework assignments in two separate sections of a college course. Both sections were taught by the author of this paper during the same semester. The first section is evaluated according to a relative method, while the second section is evaluated according to the traditional absolute performance method. Towards the middle of the semester the grading methods were reversed. A fixed effects model was used for analysis of the panel data.
The contrast between absolute and relative performance evaluation has been the subject of numerous articles in the contract theory literature. Upon investigation of the effect of incentives on performance and effort, the contrast of these two compensation methods can yield valuable insights. Under the traditional grading system, the instructor calculates each student’s score according to the student’s absolute performance. In a multiple choice assignment, for instance, if the student answers correctly 20 out of the 25 questions, his score will be 80/100 independently of the performance of the rest of the class. Under absolute grading, the power of incentives cannot be adjusted by changing the amount of points per correct answer. If, for example, the instructor gave 5 points per correct answer instead of 4 the student’s score would be 100/125 which is equivalent to 80/100.\footnote{Students are not considered to exhibit any form of 'grade illusion'. In other words, they do not care for nominal changes in their grades.} Under a relative grading system each student is evaluated according to the difference of individual performance from the average performance of the class. A usual form of the relative method is when each student’s score is equal to $b + \beta (x_i - \bar{x})$ where $b$ and $\beta$ are positive parameters defined by the instructor and $x_i - \bar{x}$ is the difference of individual performance from the average performance. Under the relative evaluation method the power of the incentives can be adjusted by changing $\beta$. In a sense, the relative method allows the provision of incentives by transferring points from below the average performance students to above the average performance students or vice versa.

The classroom turns out to be the ideal environment for experimenting with incentives. The subjects are not studied in the laboratory but in their natural environment doing exactly what they are used to doing in their entire school life. Subjects have to think seriously about their effort choices under each grading method because their decisions affect their popularity in class, and more importantly, their GPA. Moreover, the duration of the experiment, a whole semester, is long enough for every subject to understand the details of the experiment and how the subject is going to be affected by it. Participants anticipated that the experiment would impact real aspects of their life.
and, therefore, their response to the incentive structure is of great interest. The experimental approach in addressing economic questions is not novel. McCabe, Rigdon and Smith (2002) used a laboratory experiment to test reciprocity in simple two-person trust games. Holt and Laury (2002) conducted a laboratory experiment offering a menu of paired lottery choices to infer the degree of risk aversion of subjects. Wu and Roe (2007) used an experimental setting to investigate how different enforcement methods affect the efficiency of contracts.

The experimental results were surprising. First, the data show that relative performance evaluation is less effective than absolute performance evaluation with respect to students’ performance. This is in contrast with many prominent theoretical articles like Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983) which indicate that relative evaluation is superior to absolute evaluation when it is costless to observe and compare agents’ performance, agents are risk averse and performance includes a sufficient level of common noise. The superiority of the relative evaluation is because it provides a sort of insurance by filtering out the common noise from agent’s compensation. Even when agents are risk neutral and they are not interested in the provision of insurance by the principal, Lazear and Rosen show that both evaluation methods are equally effective. However, the paper’s result is perfectly in line with a recent empirical investigation of incentives. Bandiera et al. (2005) found that, when the principal switched payment methods from a relative performance tournament to an absolute performance piece rate, productivity significantly increased. Similarly, Wu and Roe (2005) used a laboratory experiment to test the difference of effort levels under a tournament and under a fixed performance standard contract. They concluded that exerted effort was significantly higher under the fixed performance standard, which is an absolute evaluation method.

The second experimental result is even more unanticipated and has not been observed in the literature before. The experiment indicates that the power of incentives has a
negative impact on students’ performance. That is, when the class is graded according
to a relative method, an increase in the power of incentives leads students to decrease
effort in an attempt to lower the social cost, which arises from the likelihood that the
student will become the "curve breaker". The importance of peer effects was evident in
the classroom. Students participating in the experiment stated that, even if they did
not care if the class as a whole performed better or worse than they did, they did care
to minimize their score’s deviation from the scores of their group of friends. In other
words, social blocks of students, formed before or during the semester, tend to connect
their utility with the deviation of their individual performance from the average when
grading is according to the relative method. The participants in such blocks experience a
decrease in their utility when they score high and this becomes the reason for their friends
to receive a lower score. To some extent, this is similar to the notion of inequity aversion
proposed by Fehr and Schmidt (1999) according to which agents can be either altruistic
or envious towards their peers and they do not strictly pursue their self interests.

The paper also investigates whether the use of a relative grading method prevents
answer sharing. Under absolute grading, answer sharing is a common phenomenon,
especially when the assignments consist of multiple choice questions. In fact, under
absolute grading, answer sharing may be optimal because it increases the popularity
of a student at zero cost. In this case, homework answers become a public good and
even students of low ability may appear to perform well. Conversely, when a relative
evaluation method is in effect, a cost is inflicted on those who give their answers away
because, by doing so, the class average increases and the expected score of those who
gave their work away decreases. This causes a strong disincentive for answer sharing.
The data confirm that the relative method prevents answer sharing.

The data have been weighed according to the item discrimination index, a correlation
measurement between student responses to a particular question and total score on all
other questions on each assignment. Questions with a high discrimination index tend
to require more work from the student than questions with a low discrimination index. By weighing the performance of students on each question according to the question’s discrimination index, the data provide a more clear description of effort choices. This is due to two reasons: First, the discrimination index assigns a weight to each question with respect to the question’s discriminating ability and secondly, the number and the degree of non-discriminating questions vary from assignment to assignment and, thus, the discrimination index is valuable in normalizing the performance observations in every assignment.

Section 2 provides a theoretical framework for the results. The model suggests that students’ preferences are affected by social factors in addition to self interest. That is, students are assumed to gain utility from two sources: first, from their individual performance reduced by the disutility of effort and, second, from reducing the deviation from the average score of the social block the student belongs to. In section 3 the experimental design is described while section 4 portrays the treatment of the data. Section 5 presents the results and finally, section 6 discusses the implications and concludes the paper.

3.2 Theoretical Background

Consider a risk neutral principal and $N$ risk neutral agents of heterogeneous ability. Agents must individually undertake a task which involves production of output. Each agent $i$ produces output according to the production function $x_i = a_i + e_i + \varepsilon_i + \eta$, where $a_i > 0$ is the agent’s ability known to him ex ante, $e_i$ is the agent’s effort, $\varepsilon_i$ is an idiosyncratic shock and $\eta$ is a random shock common to all agents. Both shocks follow independent normal distributions with means of zero and finite variances $\text{var}(\eta) = \sigma^2_\eta$ and $\text{var}(\varepsilon_i) = \sigma^2_{\varepsilon_i}, \forall i$. Moreover the idiosyncratic shocks for each agent are independent. Upon completion of the task, the principal compensates the agents according to a predetermined compensation scheme. If the principal offers a piece rate contract (absolute
method) the compensation to the $i$th agent will be

$$w_i = \gamma x_i$$  \hfill (87)$$

where $\gamma$ is the piece rate coefficient. If the principal offers a cardinal tournament with coefficients $b$ and $\beta$ (relative method) the compensation to the $i$th agent will be

$$w_i = b + \beta(x_i - \bar{x}_i)$$  \hfill (88)$$

where $\bar{x}_i = \frac{1}{N-1} \sum_{j=1}^{N} x_j$. Agents are exogenously assumed to have accepted the terms of the agreement and, thus, there is no question about agent participation. Even if an agent performs the task exerting zero effort, his expected production will still be positive. Therefore, his expected compensation will be higher than if he does not perform the task at all. The principal is not assumed to be the residual claimant of the difference between total production and compensation to the agents. However, the principal’s satisfaction is increasing in $\sum_{i=1}^{N} x_i$. During the production process the $N$ agents form $M$ groups in which the participants develop social relationships. These groups are referred to as "social blocks" throughout the paper. The blocks are not necessarily of equal size but an agent can belong only to one block. The payoff for each agent, $U_i$ is determined by

$$U_i = w_i - \frac{c_i^2}{2a_i} - \lambda \delta_i (w_i - \bar{w}_{-i,m})^2$$  \hfill (89)$$

where $m$ indexes the block in which $i$ is participating, $\bar{w}_{-i,m}$ is the average compensation of the agents participating in block $m$ excluding agent $i$;\footnote{The exclusion of agent $i$ from the calculation of the mean serves for simplification purposes and it does not alter qualitatively any of the model’s results. If $i$ was included in the block mean, the deviation from the mean would be equal to $\frac{M_i-1}{M_i} w_i - \frac{1}{M_i} \sum_{j=1}^{M_i} w_j$ where $M_i$ is the size of the block to which agent $i$ belongs. However, in the experiment, the standard definition of mean was used.} $\delta_i$ is a parameter indicating the impact of the deviation from the block average on agent’s payoff and $\lambda$ is a binary variable.
which is zero when compensation is absolute and one when compensation is relative. The first two terms of the payoff function (89) denote the self interest of the agent from his personal compensation. The third term denotes the decrease in agent’s payoff when his personal payoff deviates from that of the other agents in the same block. Due to the presence of $\lambda$ the last term will matter only under a tournament. Consequently, only under a relative performance evaluation method can an agent be considered responsible for this deviation.

**The Choice of Optimal Effort under the Piece Rate**

Agents choose effort in order to maximize expected payoff. Under the piece rate, the agent’s goal is to maximize his expected payoff. That is,

$$\max_{e_i} EU_i = \max_{e_i} \left[ \gamma Ex_i - \frac{e_i^2}{2a_i} \right] = \max_{e_i} \left[ \gamma (a_i + e_i) - \frac{e_i^2}{2a_i} \right].$$

The solution satisfies

$$e^n_{PR} = a_i \gamma. \quad (90)$$

That is, the optimal effort depends increasingly on ability and the piece rate, $\gamma$. Note that, because the agent’s choice on effort does not affect other agents, $\delta_i$ does not enter the optimal effort derivation.

**The Choice of Optimal Effort under the Tournament**

Under the tournament, when the agent takes the effort choices of other agents as given, the agent’s problem can be stated as

$$\max_{e_i} EU_i = \max_{e_i} \left\{ Ew_i - \frac{e_i^2}{2a_i} - \delta_i \beta^2 E(w_i - \bar{w}_{-i,m})^2 \right\} =$$

$$= \max_{e_i} \left\{ b + \beta E \left[ (a_i - \bar{a}) + (e_i - \bar{e}) + (\varepsilon_i - \bar{\varepsilon}) \right] - \frac{e_i^2}{2a_i} - \delta_i \beta^2 E \left[ a_i^2 + e_i^2 + \varepsilon_i^2 + a_i^2 + \bar{e}^2 + \bar{\varepsilon}^2 + ... + 2(-a_i \bar{a} - e_i \bar{e} - \varepsilon_i \bar{\varepsilon} + a_i e_i + \bar{a} \bar{e} - a_i \bar{e} - a_i e_i + (e_i - \bar{e})(\varepsilon_i - \bar{\varepsilon}) + (a_i - \bar{a})(\varepsilon_i - \bar{\varepsilon})) \right] \right\} =$$

59
\[
= \max_{e_i} \{ b + \beta (a_i - \bar{a}) + \beta (e_i - \bar{e}) - \frac{e_i^2}{2a_i} - \ldots \\
- \delta_i \beta^2 [a_i^2 + e_i^2 + \bar{a}^2 + \bar{e}^2 + E \varepsilon_i \bar{e} + a_i e_i + \bar{a} \bar{e} - a_i \bar{e} - \bar{a} e_i] \}.
\]

where "\( \bar{\cdot} \)" over a variable without subscript denotes block averages excluding the \( i \)th agent. The solution satisfies

\[
e_i = a_i \beta - 2a_i \delta_i (a_i - \bar{a} + e_i - \bar{e}) \beta^2.
\] (91)

That is, under the tournament, the optimal effort depends on the power of incentives, \( \beta \) the altruism index \( \delta_i \), the ability deviation, \( a_i - \bar{a} \) and the effort deviation, \( e_i - \bar{e} \).

Assuming that blocks comprise agents of similar ability and that agent \( i \) has no reason to assess a non-zero value to the ability deviation, \( a_i - \bar{a} \), (91) can be written as

\[
e_{i}^* = \frac{a_i \beta}{1 + 2a_i \delta_i \beta^2} + \frac{2a_i \delta_i \beta^2}{1 + 2a_i \delta_i \beta^2} \bar{e}.
\] (92)

where \( \bar{e} \) is taken as given by agent \( i \). Notice that for sufficiently high values of \( \beta \) the power of incentives will negatively affect the amount of optimal effort. This is because, when the strength of incentives is high, the social cost from increasing effort dominates the self interest of higher compensation.

Comparison of the Piece Rate and the Tournament

It would be interesting to contrast the optimal effort for the two schemes under the same power of incentives (that is, when \( \gamma = \beta \)). From (90) and (92) it can easily be shown that

\[
e_{i}^* = \frac{e_{PR}^*}{1 + 2a_i \delta_i \beta^2} + \frac{2a_i \delta_i \beta^2}{1 + 2a_i \delta_i \beta^2} \bar{e}.
\] (93)

Relationship (93) shows that for sufficiently high values of \( \delta \) the effort under the piece rate will be higher than that under the tournament. The rationale behind this result is that the higher the importance of altruism to the agent, the higher the social cost.
of the agent’s effort under the tournament and, thus, the agent is expected to reduce
effort when a relative scheme is in effect. In this environment, the effect of incentives on
performance becomes an empirical question.

3.3 The Experiment
The experiment took place at the North Carolina State University during the Spring
semester of 2007. Two separate sections of a "Principles of Macroeconomics" course
were taught by the author of this paper. The first section consisted of 42 students and
the second had 40 students. Both sections were in session two times a week, back to
back. The style and the content of the lectures were identical for both sections. Students
were informed from the class syllabus, which became available the first day of classes,
that by enrolling in one of these two sections they would be evaluated according to an
experimental method. No student dropped or added the class from the first day of classes
to the end of the semester\textsuperscript{32}.

The goal of the experiment was to investigate the effect of incentives on students’
effort. Incentives were determined by the grading method in each assignment and effort
was measured indirectly by performance. Performance in multiple choice questions is
jointly affected by effort and ability and we are not able to identify the individual effects
of effort and ability on performance. However, each student’s ability is not expected to
change significantly during the semester and, thus, changes in performance will be most
likely due to changes in the student’s effort. Moreover, the midterm and the final exam
performance for each subject can provide an estimate for each student’s ability because
they took place in class and they were graded according to the absolute grading method
only. Even though the observed performance in such exams is also affected by both effort
and ability, the homework performance, relative to exam performance, is a good measure
of pure effort. Nevertheless, The term pure effort is used in the paper to describe the

\textsuperscript{32}Principles of Macroeconomics are classes of high demand by students at NC State. Most of these
sections are full early and students rarely drop the class after the first day of classes.
actions a student can take in order to increase performance in homework but, however, cannot take during an in class exam. Examples of such actions can be studying notes or textbooks, online research, attending office hours or increasing attention during the lecture.

Subjects were required to turn in eight homework assignments during the semester. Homework was announced to count for 40% towards the final score in the course while the two midterms combined with the final examination had a total weight of 60%. The homework weight was set higher than the usual 20 - 30% in order to increase the importance of homework in the determination of the final grade and, thus, add to the robustness of the results. Each assignment consisted of 25 multiple choice questions. Every multiple choice question offered four alternative responses, out of which only one was correct. The assignments and the deadlines were common for both sections but the grading methods were always different. Students had to log in to the class web site using their personal university id and password in order to download the homework questions. They were given seven to ten days to work on the homework and then they had to submit their answers. Each student was allowed only one submission and late submissions were not accepted. From the 25 questions in each assignment, 23 questions were related to course material and two questions were about the grading method. The latter aimed to familiarize the students with the grading system. Every homework question was written by the instructor and had never been used before in order to prevent leakages. The grading method for each section was announced on the document containing the assignment’s questions. This ensured that before exerting effort every student was informed about the grading method in his or her section and also about the grading method used in the other section. The grading methods varied as follows. For the first four assignments, section 1 was evaluated according to a progression of tournaments (see Table 1) while section 2 was evaluated according to the traditional piece rate (4 points per correct answer). Section 2 served as the control section. For the last four assignments the experimental setting
was reversed. Section 1 became the control section and was evaluated according to the piece rate and section 2 was evaluated according to the same progression of tournaments used earlier for section 1. In the progression of tournaments the strength of incentives increased from assignment to assignment. Table 1 shows the grading methods for each section.

Table 1: The grading methods used in the two sections. \( x_i \) stands for the number of correct answers for the \( i \)th student and \( \overline{x} \) stands for the section average

<table>
<thead>
<tr>
<th></th>
<th>Section 1</th>
<th>Section 2</th>
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<tbody>
<tr>
<td>hw #1</td>
<td>( 75 + 2(x_i - \overline{x}) )</td>
<td>( 4x_i )</td>
</tr>
<tr>
<td>hw #2</td>
<td>( 75 + 4(x_i - \overline{x}) )</td>
<td>( 4x_i )</td>
</tr>
<tr>
<td>hw #3</td>
<td>( 75 + 6(x_i - \overline{x}) )</td>
<td>( 4x_i )</td>
</tr>
<tr>
<td>hw #4</td>
<td>( 75 + 8(x_i - \overline{x}) )</td>
<td>( 4x_i )</td>
</tr>
<tr>
<td>hw #5</td>
<td>( 4x_i )</td>
<td>( 75 + 2(x_i - \overline{x}) )</td>
</tr>
<tr>
<td>hw #6</td>
<td>( 4x_i )</td>
<td>( 75 + 4(x_i - \overline{x}) )</td>
</tr>
<tr>
<td>hw #7</td>
<td>( 4x_i )</td>
<td>( 75 + 6(x_i - \overline{x}) )</td>
</tr>
<tr>
<td>hw #8</td>
<td>( 4x_i )</td>
<td>( 75 + 8(x_i - \overline{x}) )</td>
</tr>
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The selection of values for the experimental parameters were based upon previous teaching experience. The base compensation for the tournament was set to 75 and remained unchanged throughout the duration of the experiment. This value was selected because the average score in similar assignments for a large number of past sections of the same course was 75/100 with a notably small variance. The piece rate was set to 4 points per question in order to normalize the maximum possible score to 100. This was useful for comparison purposes between the piece rate and the tournament scores. The number of homework assignments was set to eight because it was necessary to have an even number of assignments and there was not enough time for ten assignments.
The variable of interest for the experiment was the coefficient of the deviation between individual scores and average score in the tournament ($\beta$). The values 2, 4, 6 and 8 were selected to provide a reasonable range of incentive power under the tournament. When the tournament coefficient was 2 (indicated by 'beta05' in the Tables) the tournament incentives were 50% lower in power compared to the piece rate (with coefficient equal to 4). When the coefficient of the tournament was set to 4 (indicated by 'beta10') the power of incentives was identical for both schemes and when it was set to 6 ('beta15') and 8 ('beta20') the tournament imposed 50% and 100% higher power incentives than the piece rate.

The midterm and final exams served as an indirect measurement for students’ ability. The tests were identical for both sections, with closed book and closed notes. In addition, the tests were administered in the classroom by the author and students were not given the option of a makeup test. The papers from both sections were put together, shuffled and marked blindly by the author. The grading system for the tests was the traditional piece rate. All tests were 50% multiple choice and 50% essay questions. The selection of this format was an a priori safe choice, in order to identify potential problematic situations. For example, it is not rare that a hard working student underperforms in multiple choice assignments. However, this precaution turned out to be unnecessary because the performance in multiple choice questions was not significantly different than performance in essay questions. This can be seen in Figure 1.

3.4 Data Treatment

The data collected from the experiment consist of a panel of observations for the total number of correct answers for each student and for each of the eight assignments. When the raw data was used to test various statistical hypotheses, student performance turned out to be unaffected by the incentive structure of the grading method. This result is in contrast with basic economic intuition. Even though the fixed effects model controls for individual and assignment effects, the raw dataset evidently contained a significant
Figure 6: The performance of each student in the essay question portion versus the multiple choice portion for each exam.
amount of noise because the observations were not weighted. Regression Table 2 illustrates the impact of incentives on effort according to the raw data. The binary variables \( hw2 - hw8 \) control for assignment effects relative to \( hw1 \), which is the omitted variable. A similar structure of 81 binary variables was used to control for individual effects. The regression coefficients for these variables are not shown in Regression Table 2 in order to economize on space. The binary variables \( beta05, beta10, beta15 \) and \( beta20 \) take the value 1 if the tournament coefficient was 2, 4, 6 or 8 and take the value 0 otherwise. The regression coefficients of these variables measure the effect of each tournament on performance relative to the effect of the piece rate, which is the omitted variable. From this regression it is unclear how incentives affect performance because, only when the tournament coefficient is 6, does there seem to be a significant relationship between incentives and performance. In this case, performance decreases by 7.3 points out of the relative to the performance under the traditional piece rate. Table 2 suggests that raw performance is not a representative index of effort choices.
In the experiment the quality of multiple choice questions varied from question to question and from assignment to assignment. Multiple choice questions are considered of good quality, when only able students who know the material well or challenged students who work hard, can come up with the correct response. On the contrary, low quality questions allow students who do not prepare, to get the answer right. Furthermore, questions that just require high observance skill or ability to use logic in generic situations may add noise to the dataset because success in such questions is not necessarily correlated with high effort. An unknown student aptly pointed this issue out in the semester’s evaluation:

“[…] Some multiple choice questions are confusing and/or misleading and
they do not simply test the knowledge of the material but they are made to
test how skilled we are spotting the tricky part of the question. [...]

The difference in the ability of a question to discriminate high from low effort stu-
dents can be illustrated in the following example which considers two questions used in
the first homework assignment of the experiment.

1. Doug is the head of the NCSU department of economics and Carol is his
secretary. Doug has high managerial ability and he can also type 100 words
per minute. Carol has lower managerial ability than Doug and she can type
80 words per minute. However, Doug is the department’s manager and Carol
is the typist. Which of the following is correct?

A. Carol has the absolute advantage in typing.
B. Doug has the comparative advantage in typing.
C. Carol has the comparative advantage in management.
D. Doug has the absolute advantage in typing.

2. If household income increases and, ceteris paribus, the supply of a good
increases then the good is:

A. A luxury.
B. An inferior good.
C. A normal good.
D. None of the above is correct.

The first question tests whether the student has mastered the definitions of absolute
and comparative advantage in a real life situation. On the contrary, the second question
can easily trick someone. Household income and the supply of a good are not related in
principle, but in the way this question is written, the typical student tends to think that
the question examines the definitions of normal and inferior goods rather than whether income and supply are in fact related. In this question, the majority of students answered C because they only paid attention to the direction of the change. This distracted them from noticing that the question referred to a supply increase rather than to a demand increase. In such questions, success depends more on observance skill rather than on effort.

Performance will be a better index of effort if we weigh the questions according to an index of discriminating ability. For this purpose the item discrimination index is used to determine the weight of each question.\textsuperscript{33} The item discrimination index measures the correlation between the item response and the overall performance on the assignment. As a correlation measurement, the discrimination index appears in many variations in popular grading software packages. The experimental data in this paper was transformed according to one of the most common and simple discrimination indexes. That is,

\[
d_q = \frac{\overline{X}_{c,q} - \overline{X}}{S_X} \sqrt{\frac{P_{c,q}}{1 - P_{c,q}}}
\]

where \(d_q\) is the discrimination index for question \(q\), \(\overline{X}_{c,q}\) is the mean score of those who answered question \(q\) correctly, \(\overline{X}\) is the mean score on the assignment, \(S_X\) is the standard deviation of scores on the assignment and \(P_{c,q}\) is the proportion of those who answered \(q\) correctly. Obviously, the index is constructed to take values between zero and one. After this index was used for weighting each question\textsuperscript{34} in the dataset, the transformed dataset clearly showed statistically significant patterns for the effects of incentives on effort. These patterns are discussed in the next section. The discrimination index evidently reduced the noise in the dataset. For instance, for the questions in the previous example, not surprisingly, the discrimination index assigned 25\% less weight to the second question than the first one. Notably, when the second question of the previous example was reused

\textsuperscript{33}See Pyrczak (1973) for an extensive investigation of the item discrimination index.

\textsuperscript{34}The weight for question \(q\) is \(d_q\). Then, the weighted performance is \(d_q\) if the student’s answer was correct and 0 otherwise.
in another section after the experiment, with answer D rephrased as: "Income and supply are not related", the majority of students picked D and the discrimination index of the rephrased question was not significantly different than that of the first question of the example above. Table 3 illustrates the descriptive statistics for the raw and weighted performance in each section for each assignment.

Table 3: Descriptive statistics for raw and weighted performance. Data per section and per assignment.

<table>
<thead>
<tr>
<th></th>
<th>Hw 1</th>
<th>Hw 2</th>
<th>Hw 3</th>
<th>Hw 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Section:</td>
<td>Absolute</td>
<td>Absolute</td>
<td>Absolute</td>
<td>Absolute</td>
</tr>
<tr>
<td>Std.:</td>
<td>1.88</td>
<td>0.88</td>
<td>2.69</td>
<td>1.25</td>
</tr>
<tr>
<td>Max Possible:</td>
<td>25</td>
<td>15.02</td>
<td>25</td>
<td>13.27</td>
</tr>
<tr>
<td>Max Observed:</td>
<td>25</td>
<td>15.02</td>
<td>25</td>
<td>13.27</td>
</tr>
<tr>
<td>Min Observed:</td>
<td>17</td>
<td>11.26</td>
<td>15</td>
<td>8.54</td>
</tr>
<tr>
<td></td>
<td>Hw 5</td>
<td>Hw 6</td>
<td>Hw 7</td>
<td>Hw 8</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Section:</td>
<td>Relative (beta = 2)</td>
<td>Relative (beta = 4)</td>
<td>Relative (beta = 6)</td>
<td>Relative (beta = 8)</td>
</tr>
<tr>
<td>Avg.:</td>
<td>21.82</td>
<td>13.61</td>
<td>19.95</td>
<td>11.10</td>
</tr>
<tr>
<td>Std.:</td>
<td>2.10</td>
<td>1.03</td>
<td>2.47</td>
<td>1.24</td>
</tr>
<tr>
<td>Max Possible:</td>
<td>25</td>
<td>15.02</td>
<td>25</td>
<td>13.27</td>
</tr>
<tr>
<td>Max Observed:</td>
<td>25</td>
<td>15.02</td>
<td>25</td>
<td>13.27</td>
</tr>
<tr>
<td>Min Observed:</td>
<td>16</td>
<td>10.52</td>
<td>13</td>
<td>6.82</td>
</tr>
<tr>
<td></td>
<td>Hw 9</td>
<td>Hw 10</td>
<td>Hw 11</td>
<td>Hw 12</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Section:</td>
<td>Relative (beta = 2)</td>
<td>Relative (beta = 4)</td>
<td>Relative (beta = 6)</td>
<td>Relative (beta = 8)</td>
</tr>
<tr>
<td>Avg.:</td>
<td>20.46</td>
<td>14.38</td>
<td>17.78</td>
<td>11.43</td>
</tr>
<tr>
<td>Std.:</td>
<td>2.42</td>
<td>1.45</td>
<td>3.81</td>
<td>2.14</td>
</tr>
<tr>
<td>Min Observed:</td>
<td>12</td>
<td>8.70</td>
<td>9</td>
<td>6.17</td>
</tr>
</tbody>
</table>

Another issue concerning the data was the decision on how to handle missing observations. From the 656 possible submissions a total of 38 submissions were missing.
A missing observation could be either because a student had a personal reason not allowing submission in time or because the student decided not to exert any effort for the assignment. The two reasons are qualitatively different. The first reason does not indicate intention for exerting zero effort while the second reason does so. Fortunately, the way missing observations were handled did not materially affect the results. Therefore, it was considered reasonable to handle the missing observations as follows: All missing observations counted as zero performance unless a student submitted less than 6 out of the 8 assignments. In the latter case, all the observations of these students (zero or non-zero) were dropped. According to the selected method, only 16 observations were dropped (2 students, both from the second section). The rationale behind this choice was that students who failed to submit more than two assignments were considered to be uninterested in the homework and the grading method, so they should be left out of the experiment.

3.5 Results

The fixed effects model was used for capturing the effects of the grading method and the power of incentives on performance. Sets of binary variables were used to control for various effects. Specifically, the set of seven binary variables hw_dums was used to control for assignment difficulty effects. Each binary variable, hw_h, in this set takes the value 1 for the hth assignment and the value 0 otherwise. The regression coefficients of the hw variables measure the difficulty of each assignment relative to the difficulty of the first assignment because the omitted variable was hw_1. The set of 81 binary variables ’id_dums’ controlled for idiosyncratic effects among subjects. Each id_i variable of this set indicates if the assignment was submitted by the ith student. The omitted binary variable was id_1. The variable id_1 represents the student with the highest performance in tests (the homework scores were not used for ranking the subjects), so the regression coefficients on the id_i will always appear to be negative. The set of binary variables ’rel_dums’ is used to capture the effects of the coefficients of the tournament on performance. As before,
the notation of the binary variables is explained in the following table.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Grading Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>'beta05’</td>
<td>$75 + 2(x_i - \bar{x})$</td>
</tr>
<tr>
<td>'beta10’</td>
<td>$75 + 4(x_i - \bar{x})$</td>
</tr>
<tr>
<td>'beta15’</td>
<td>$75 + 6(x_i - \bar{x})$</td>
</tr>
<tr>
<td>'beta20’</td>
<td>$75 + 8(x_i - \bar{x})$</td>
</tr>
</tbody>
</table>

The regression coefficients of the variables in this set indicate the effects of the power of incentives on performance relative to the effect of the four-point piece rate.\(^{35}\) The binary variable 'tourbot' takes the value 1 when a tournament is used and when the student is ranked in the bottom 20 students of his or her section according to performance in tests, and 0 otherwise. The binary variable 'tourtop' takes the value 1 when a tournament is used and when the student is ranked in the top 20 of students of his or her section according to performance in tests, and 0 otherwise. The variables tourtop and tourbot are useful to define $tt = tourtop + tourbot$. The latter is used to test the hypothesis of equality of the regression coefficients of tourtop and tourbot.

**Effort and Performance under the Two Methods**

Table 5 illustrates the effect of incentives under the two compensation methods when we control for idiosyncratic effects. The dependent variable in the regression model for Table 5 is 'weighted performance' and the regressors are id_dums and rel_dums. When we use the discrimination index to weigh the data there is no need to use the hw_dums in the regression to control for assignment effects. This is because the weighing of the dataset according to the discrimination index has already normalized the dataset for assignment score to the usual maximum of 100. If one considers performance to be the total of correct answers, the maximum performance would be 25 and the piece rate is the unitary piece rate. The same logic applied to the tournament will yield power of incentives to be 0.5, 1, 1.5 and 2 which explains the names of the rel_dums binary variables, beta05, beta10, beta15 and beta20.

\(^{35}\)The 4 point piece rate was used in the experiment to achieve normalization of the 25 question assignment score to the usual maximum of 100. If one considers performance to be the total of correct answers, the maximum performance would be 25 and the piece rate is the unitary piece rate. The same logic applied to the tournament will yield power of incentives to be 0.5, 1, 1.5 and 2 which explains the names of the rel_dums binary variables, beta05, beta10, beta15 and beta20.
assignment difficulty effects. Recall that the omitted variable from rel_dums is the binary variable which is 1 when the absolute method is used. Therefore, the coefficients of the id_dums represent the total weighted score of the four relative methods relative to the absolute method. As one can see in Table 2, the maximum total weighted score for each assignment varies from 11.57 (hw 7) to 16.54 (hw 5). The average maximum total weighted score is 14.41 and one can use this average to provide some quantitative interpretation of the results. The regression results in Table 5 verify the claim made with relationship (93) in section 2. That is, when $\gamma = \beta = 4$ (unitary incentive power for both schemes), performance, and thus effort, seem to be lower under the tournament than under the piece rate. This means that when the tournament was used performance dropped by 0.86 units out of the 14.41 (5.96 % decrease). The difference appears to be quite significant with a p-value equal to 0.0271. Once again, the intuition behind this result is that under the tournament, the choice of effort inflicts a social cost on the student because, as effort becomes higher, it decreases the compensation to other students for whom the student might care. This observation is in line with recent empirical papers (see Bandiera et al. (2004)).

---

36 When the set hw_dums was included in this regression the coefficients of these variables were not significant.
Table 5: Fixed effects regression: Incentives on weighted performance

Model: $wperf = id\_dums \ rel\_dums$

<table>
<thead>
<tr>
<th>OLS</th>
<th>Coef.</th>
<th>t-stat</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>intrcpt.</td>
<td>13.9484</td>
<td>12.6311</td>
<td>0.0000</td>
</tr>
<tr>
<td>beta05</td>
<td>1.9013</td>
<td>4.9002</td>
<td>0.0000</td>
</tr>
<tr>
<td>beta10</td>
<td>-0.8599</td>
<td>-2.2163</td>
<td>0.0271</td>
</tr>
<tr>
<td>beta15</td>
<td>-0.9358</td>
<td>-2.4118</td>
<td>0.0162</td>
</tr>
<tr>
<td>beta20</td>
<td>-1.0845</td>
<td>-2.7951</td>
<td>0.0054</td>
</tr>
</tbody>
</table>

The Effect of the Incentive Power on Effort under Tournament

According to Regression Table 5, again, performance seems to be negatively affected by the coefficient of the tournament, $\beta$. That is, when the incentive power is low ($beta05$) performance is even higher than the performance under the piece rate (13.18% higher). Notice that, when $\beta$ and $\gamma$ deviate, (93) is no longer a valid relationship to consider for comparing the two methods. For $\beta$ equal to or higher than the piece rate ($beta10$, $beta15$ and $beta20$) the tournament causes progressively lower effort (reductions of 5.96%, 6.52% and 7.53% respectively) According to the p-values for the t-statistics for the regression coefficients these results are highly significant. Moreover the null hypothesis that the coefficients for $beta05$, $beta10$, $beta15$ and $beta20$ are all equal is rejected with an F-statistic equal to 17.07, for which the p-value for 3 and 556 degrees of freedom is practically zero. This striking result has never been observed in the literature and it indicates that in highly socialized environments incentive effects may be reversed. One can state that
students seem to care a lot about the opinion of others which is not a novel observation in sociology.

Answer Sharing

Tables 6 and 7 examine the difference in performance for each section’s top 20 students and bottom 20 students.

Table 6: How are top and bottom students affected by the introduction of a tournament

Model: \( w_{perf} = tourbot \times tourtop \)

<table>
<thead>
<tr>
<th>OLS</th>
<th>Coef.</th>
<th>t-stat</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>intrcpt.</td>
<td>11.5700</td>
<td>61.2357</td>
<td>0.0000</td>
</tr>
<tr>
<td>tourbot</td>
<td>-0.8360</td>
<td>-2.5962</td>
<td>0.0096</td>
</tr>
<tr>
<td>tourtop</td>
<td>0.4088</td>
<td>1.2278</td>
<td>0.2200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>obs.</th>
<th>d.o.f.</th>
<th>R_sq</th>
<th>var</th>
<th>F-stat</th>
<th>Sig. F</th>
</tr>
</thead>
<tbody>
<tr>
<td>640</td>
<td>637</td>
<td>0.0180</td>
<td>11.4236</td>
<td>5.8316</td>
<td>0.0031</td>
</tr>
</tbody>
</table>

Table 7: Test: When a tournament is in effect, do bottom students drop more than top students?

Model: \( w_{perf} = tt \times tourtop \)

<table>
<thead>
<tr>
<th>OLS</th>
<th>Coef.</th>
<th>t-stat</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>intrcpt.</td>
<td>11.5700</td>
<td>61.2357</td>
<td>0.0000</td>
</tr>
<tr>
<td>tt</td>
<td>-0.8360</td>
<td>-2.5962</td>
<td>0.0096</td>
</tr>
<tr>
<td>tourtop</td>
<td>1.2448</td>
<td>3.2900</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>obs.</th>
<th>d.o.f.</th>
<th>R_sq</th>
<th>var</th>
<th>F-stat</th>
<th>Sig. F</th>
</tr>
</thead>
<tbody>
<tr>
<td>640</td>
<td>637</td>
<td>0.0180</td>
<td>11.4236</td>
<td>5.8316</td>
<td>0.0031</td>
</tr>
</tbody>
</table>

Students are ranked according to their midterm and final exam performance only. The analysis assumes that students ranked high, according to tests, are less likely to turn in
answers copied from others. The model in Table 6 contrasts the performance response of top and bottom students when the grading method was switched from a piece rate to a tournament. Top students seem to have a slight (non significant) positive performance response (approximately 5.8%) to the switch, while bottom students’ performance drops significantly by almost 8.67%.\textsuperscript{37} A plausible explanation for this result is that some answer sharing took place while a piece rate was used and it was discontinued after the introduction of the tournament. Social relationships developed within the social blocks may render answer sharing optimal under an absolute grading method. In this case, high ability students have no reason to avoid sharing their work with others because there is no cost in doing so. Moreover, students who share gain valuable popularity. Under answer sharing, homework answers become a public good and even low ability students may appear to perform well. Conversely, when a relative evaluation method is in effect, a cost is imposed on those who share, because answer sharing increases the class average and decreases the expected score of those who shared. Regression Table 7 tests the significance of the difference in performance of top and bottom students through the p-value of the coefficient of the variable $tourtop$. The difference is also highly significant with an extremely low p-value of 0.0011.

### 3.6 Discussion and Conclusions

The principal - agent problem exhibits many similarities with the instructor - student problem. Given these similarities, one might assume that the instructor can lead students to study harder just by adjusting the grading method used in the class assignments. However, a major difference between the principal - agent and the instructor - student relationship, is that student behavior is more likely to involve altruism. Under a relative evaluation method, altruistic behavior clearly decreases the student’s individual payoff with respect to grades but, on the other hand, it increases the student’s popularity among

\textsuperscript{37}Notice that the regression coefficients in Panel 3 contrast the piece rate with the four tournaments used in the experiment combined together.
his fellow students. The latter turns out to be highly desirable in the student community up to the point that it may appear to offset the loss in individual payoff.

When a relative incentive structure is utilized within a socialized environment, agents have to weigh two opposite factors. The first refers to pure self interest. When the agent increases his effort, his compensation is increased at the expense of his peers. This is due to the function of relative incentive compensation schemes to reward those who perform above the average and equally penalize those who perform below the average. The second factor refers to the social interest of the agent. When the agent increases his effort, he experiences an increase in his social cost because he causes his peers to be penalized. The interaction of these two factors suggests that, when agent behavior is sufficiently affected by altruism, a relative evaluation method will discourage effort compared to an absolute method because of the excess social costs it creates. In other words, under a relative evaluation method, effort may be negatively affected by the power of incentives.

The paper uses an experimental approach to investigate how altruism impacts the function of incentives. The experiment considers two different grading methods for eight homework assignments in two separate sections of the same college course. One section is evaluated according to a relative method while the other is evaluated according to the traditional absolute performance method. Then, by using a fixed effects model the results are compared. The dataset consists of an 82 student by 8 assignment panel of observations for homework performance. In addition data on performance of students in three in-class tests are used to provide information for the generic ability of students. The paper proposes an innovative transformation for the panel data collected from the experiment. To distinguish between questions requiring high amounts of effort and questions requiring other skills, output is weighed according to the item discrimination index. This index is used by many internet platforms for class management (Aplia, Blackboard etc.) or multiple choice grading software packages which accompany scantron machines.

The experimental results were quite surprising. First, relative performance evalu-
ation, through a tournament, seems to have a significant negative impact on students' performance compared to absolute performance evaluation, through the piece rate. This is not what one would expect having in mind the theoretical articles of Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983) who conclude that, if it is costless to compare agents’ performance, relative evaluation cannot lead agents to exert effort lower than the effort they would exert under an absolute method. Of course, these authors consider the principal-agent problem within a standard, perfectly competitive environment, in which agents strictly pursue their self interests. Furthermore, empirical work, that has verified the theoretical findings of the above articles, like Knoeber and Thurman (1994 and 1995) and Tsoulouhas and Vukina (2001), contrast the two methods within competitive environments, in which agents may not even know who they are competing against. When altruism is involved in the agents’ decisions, relative evaluation inflicts a social cost on those who try hard. Altruism is without doubt a characteristic of in-class competition. Classmates develop strong, social relationships, which place relative compensation methods at a disadvantage. It is apparent that, during the experiment, several participating students showed their discomfort towards the relative method by characterizing it as a "cut throat method". From the experiment it is evident that students seriously consider the social cost imposed on them under the relative method and significantly reduce their effort. This result is in line with other empirical investigations, like Bandiera et al. (2005), who include altruism in their analysis.

The second result is also striking and has not appeared in the literature before. The experiment clearly shows that, as incentives increase in power, students respond by decreasing their effort in an attempt to lower their social cost. This behavior shows that students dislike the transfer of points from those who performed lower than the average to those who performed better than the average. Some participating students stated that, even if they do not care about the performance of the class as a whole relative to theirs, they feel uncomfortable experiencing situations in which their performance was an outlier.
relative to the performance of their group of friends. This indicates that social blocks are formed amongst students. The participants of such blocks connect their utility with the deviation of their individual performance from the block’s average performance. When a relative evaluation method is used and the power of incentives increases, the social cost inflicted on those who outperform the block becomes higher and their effort is expected to drop. On the other hand, under an absolute grading method, the formation of social blocks is not expected to alter the effort choices for students because individual results have no impact on the other participants of the block and high effort is not accompanied by social cost.

A third issue examined in the paper is the occurrence of answer sharing. Answer sharing can be verified by conducting a simple "means test" for homework and test scores for the class. Typically, it is expected that students who benefit from answer sharing will exhibit significant differences between their homework and test performance. For this purpose students were split into two groups according to their average performance in tests only. The means test showed that answer sharing occurred less under the relative method than under the absolute method.

All the experimental results conform to the peer effects hypothesis. Social behavior is an important factor that should be taken into consideration when one examines the principal - agent problem. The implication of the paper is that higher power incentives will not be effective in a social environment characterized by strong peer effects.
References


Appendix
Appendix

Proof of condition (40):

\[
\frac{\partial \alpha^*}{\partial \sigma^2} < 0 \iff
\]

\[
\iff \frac{ra^2}{2(a + r\sigma^2_\varepsilon + r\sigma^2_\eta)^2} \left[ 1 - \frac{1}{\sqrt{\frac{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon}{a + r\sigma^2_\varepsilon + r\sigma^2_\eta}}} \right] < 0 \iff
\]

\[
\iff \frac{1}{\sqrt{\frac{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon}{a + r\sigma^2_\varepsilon + r\sigma^2_\eta}}} > 1 \iff
\]

\[
\iff \frac{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon}{a + r\sigma^2_\varepsilon + r\sigma^2_\eta} < 1 \iff
\]

\[
\iff -\frac{1}{n-1}r\sigma^2_\varepsilon - r\sigma^2_\varepsilon < a. \quad (A1)
\]

Condition (A1) is satisfied because the LHS is negative and the RHS is positive. Q.E.D.

Proof of condition (41):

\[
\frac{\partial \alpha^*}{\partial n} < 0 \iff
\]

\[
\iff \frac{\sqrt{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon} + \frac{n(a + \frac{n}{n-1}r\sigma^2_\varepsilon)}{2\sqrt{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon}}}{\sqrt{a + r\sigma^2_\varepsilon + r\sigma^2_\eta}} > 1 \iff
\]

\[
\iff \sqrt{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon} + \frac{n(a + \frac{n}{n-1}r\sigma^2_\varepsilon)}{2\sqrt{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon}} > \sqrt{a + r\sigma^2_\varepsilon + r\sigma^2_\eta} \iff
\]

\[
\iff r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon + \frac{n^2(a + \frac{n}{n-1}r\sigma^2_\varepsilon)^2}{2(r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon)} + \ldots
\]

\[
\ldots + 2\sqrt{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon} \frac{n(a + \frac{n}{n-1}r\sigma^2_\varepsilon)}{2\sqrt{r\sigma^2_\eta - \frac{1}{n-1}r\sigma^2_\varepsilon}} > a + r\sigma^2_\varepsilon + r\sigma^2_\eta \iff
\]
\[
\Leftrightarrow n^2 \left(a + \frac{n}{n-1}r\sigma_\varepsilon^2\right)^2 + (n-1) \left(a + \frac{n}{n-1}r\sigma_\varepsilon^2\right) > 0. \tag{A2}
\]

Given condition (37), condition (A2) holds. Q.E.D.

Regarding condition (39), one can show that:

\[
\frac{\partial \alpha^*}{\partial n} = a^2 \left[ \frac{\alpha^2 + \sigma_\varepsilon^2 - \frac{a(\sigma_\eta^2 - \frac{1}{n-1}\sigma_\varepsilon^2)}{r\left(\frac{1}{n-1}\sigma_\varepsilon^2\right)}(a + \frac{n}{n-1}r\sigma_\varepsilon^2)}{2 \left(a + r\sigma_\varepsilon^2 + r\sigma_\eta^2\right)^2} - \left(1 - \sqrt{\frac{r(\sigma_\eta^2 - \frac{1}{n-1}\sigma_\varepsilon^2)}{a + r(\sigma_\eta^2 + \sigma_\varepsilon^2)}}\right) \frac{n}{n-1}\sigma_\varepsilon^2 \right]. \tag{A3}
\]

This expression cannot be simplified in any meaningful way that would enable us to sign the derivative, however, the statement was easily verified by computational techniques.

An alternative approach for modeling limited liability would be to assume that \(x_i \in [\bar{x} - \delta, \bar{x} + \delta]\) follows a doubly truncated normal distribution with the same mean and variance as in the model presented. The benefit of such an approach would be the ease in which one could consider a lower \((\bar{x} - \delta)\) and a higher \((\bar{x} + \delta)\) state of nature. However, the drawback of this technique would be that

\[
E \left[-\exp \{r\beta_R(\eta + \varepsilon_i)\}\right] = \exp \left\{-\frac{r^2\beta_R^2(\sigma_\eta^2 + \sigma_\varepsilon^2)}{2}\right\} \cdot B(\beta_R, \sigma_\eta^2, \sigma_\varepsilon^2, r), \tag{B1}
\]

where \(B(\cdot)\) is a form of multiplicative truncation bias satisfying

\[
B(\beta_R, \sigma_\eta^2, \sigma_\varepsilon^2, r) = \int_{-\gamma}^{\gamma} \frac{1}{2\pi} \exp(-\frac{(\eta + \varepsilon_i)^2}{2})d(\eta + \varepsilon_i), \tag{B2}
\]

with

\[
\gamma = \frac{\delta}{\sqrt{\sigma_\eta^2 + \sigma_\varepsilon^2}} - r^2\beta_R^2\sqrt{\sigma_\eta^2 + \sigma_\varepsilon^2}. \tag{B3}
\]

Clearly, from conditions (B1) - (B3) if output follows a doubly truncated normal dis-
tribution, a closed form solution for $\beta_R$ (and similarly for $\beta_T$) is impossible to obtain analytically. Even though, a computational solution for the contractual parameters can be obtained, it leads to numerical solutions for specific parameters without providing more general results. Nevertheless, the numerical results obtained conform with the predictions obtained in the analysis above. For more information see Aitchison and Brown (1963) or Cohen (1959).