ABSTRACT

LEVEDAHL, BLAINE ALEXANDER. Vehicle Control in Full Unsteady Flow Using Surface Measurements. (Under the direction of committee chair Larry Silverberg).

This dissertation is the first comprehensive attempt to address a new engineering problem: control of a vehicle maneuvering in a full unsteady flow field. The approach to the solution is focused in three main areas: modeling of a vehicle maneuvering in a full unsteady flow field, control of a vehicle maneuvering in a full unsteady flow field, and synthesizing the unsteady fluid loads for use in vehicle control. To model a vehicle maneuvering in a full unsteady flow field, this dissertation develops the Coupled Fluid Vehicle (CFV) model in which the fluid, which is a sum of a finite number of spatially dependent velocity field whose contributions vary with time, is coupled to the vehicle rigid-body equations of motion. To control a vehicle maneuvering in a full unsteady flow field, this dissertation develops the Fluid Compensation Control (FCC) strategy, which gives the control designer an opportunity to include the fluid states, in addition to the vehicle states, in the control law and an opportunity to balance reducing the fluid dynamic load through compensation and reducing the state error through regulation. To synthesize the unsteady fluid loads, this dissertation has attempted to forward current work on the prediction of fluid loads from stagnation and separation point measurements by developing the Kutta principle, which says that the velocity around the vehicle is a smoothly varying function and that it is determined up to a multiplicative constant by its nodes (stagnation, separation, and reattachment points/lines), and by conducting an experiment in an attempt to determine the correlation of the fluid loads from the orientation and separation lines on a 3-dimensional bluff body.
Vehicle Control in Full Unsteady Flow Using Surface Measurements

by
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DEDICATION

This dissertation is dedicated to my wife, Akiko Takamatsu, who despite preparing to bring another into this world found time to support me through this endeavor. And to my grandmother, Soia Mentschikoff, who, without her help, I would not have had any second chances. And I would be remised if not to dedicate it my mother and father, without who I, would not even be here. Finally, この論文は菅原道真のおかげでかんせいできました。 北野天満宮で菅原道真をお参りしてから、がんばることができました。ありがとうございました。
BIOGRAPHY

Blaine Alexander Levedahl was born to Joseph William Levedahl and Alexandria Mentschikoff Levedahl in Manchester, Connecticut on July 2nd, 1976. His passion for all things related to aircraft and reading was obvious from an early age when his mother found that the only things that would keep him from causing mischief were a visit to the local airport to watch the planes land or to the library. After completion of his primary schooling, he began pursuing his undergraduate education in aerospace engineering at North Carolina State University (NCSU). During his undergraduate studies he had the opportunity to work at NASA Langley Research Center (LaRC) on micro aerial vehicles and to study at Waseda University in Japan. After graduation, he began his master’s degree in aerospace engineering at NCSU under a NASA fellowship to work Dr. Dave Cox, at LaRC, and with Dr. Larry Silverberg, at NCSU, on unmanned aerial vehicle formation flight. Upon completion of his master’s degree in aerospace engineering, he transferred to the electrical and computer engineering department at NCSU to gain a better understanding of the hardware that had been implementing the control algorithms he had been designing. During his time in the electrical and computer engineering department, he worked in the Center for Robotics and Intelligent Machines at NCSU on robotic colony behavior and at Northrop Grumman Newport News in Newport News, Virginia with the NNemoI program. After finishing his master’s in electrical engineering, he returned to the mechanical and aerospace engineering department to pursue his doctorate in vehicle control.
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Chapter 1

Introduction

Vehicles are now being developed that can perform rapid maneuvers. In such rapid maneuvers the vehicle very seldom sees a steady flow field. In this situation, the fluid-vehicle system can be said to be maneuvering in a full unsteady flow field. This means the forces and moments experienced by the vehicle are a function of not only the orientation, velocity, and acceleration of the vehicle but also the time history of the flow, that is, the independent degrees of freedom of the fluid.

Because of the lengthened period of time to establish a steady flow field in water compared to air and the larger fluid-to-vehicle mass ratio experience by submersibles, the first encounter with the full unsteady flow regime during an entire maneuver was with an unmanned underwater vehicle (UUV), Northrop Grumman Newport News Experimental Model #1 (NNemo1).\(^1\) NNemo1 was built by Northrop Grumman Newport News (NGNN) in response to the Navy’s request for a new submarine design with a larger payload and the capability to effectively operate in littoral regions. The Navy’s initial opinion of the design was a list of concerns, including porpoising at the surface. To address these concerns, NGNN

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\(^1\) Of course, an aircraft experiences turbulence in flight, which is a full unsteady flow phenomenon; however, this is a very temporary event and not considered a maneuver.
built a scaled free-running model of NNemo1. NNemo1 has an elliptic cross-section and nine external fins: one fairwater, two fixed aft stabilizers, two movable bow planes, and four movable aft control fins called stern planes. NNemo1 is shown in Fig 1-1. Using this free-running model, NGNN successfully addressed the list of concern from the Navy. For example, showing that porpoising did not occur at the surface. In addition, NGNN also was able to demonstrate the rapid maneuvering capability of the design. However, this demonstration led to some previously unseen phenomenon. For example, during a rapid turn, where large side-slip angles were seen, the vehicle would oscillate in roll throughout the turn with excursions of ±40°. In an effort to better predict the turning performance NGNN contracted Vehicle Control Technologies Inc. (VCT) to develop a hydrodynamic model, and based on the hydrodynamic model, a vehicle simulator (1). Although highly informative this simulator failed to predict the rolling in the turn. At this point the author joined the effort with the goal to develop a control system for NNemo1. As the effort progressed, it was clear to the author that a new approach to vehicle modeling and control for vehicles maneuvering in full unsteady flow would be helpful. The goal of this dissertation is to lay the ground work for a new approach to model and control vehicles in full unsteady flow.
This dissertation is organized based on the 3 main questions the author was confronted with while addressing the problem of vehicle control in a full unsteady flow field:

(1) How can a vehicle maneuvering in full unsteady flow be modeled?

(2) Is there a control strategy that can be used to control a vehicle maneuvering in a full unsteady flow field?

(3) What are the sensor requirements to implement the control strategy?

Specifically, Chapter 2 presents a literature survey. Chapter 3 presents the novel coupled fluid vehicle (CFV) model that can be used to simulate a vehicle in full unsteady flow. Chapter 4 presents the novel fluid compensation control (FCC) approach, which uses the fluid forces and moments in a closed-loop control strategy. Chapter 5 discusses the synthesis of the forces and moments from surface measurements on the vehicle. Finally, Chapter 6 gives a summary of the findings.

It should be noted that because the first engineering encounter with the problem of controlling a vehicle maneuvering in a full unsteady flow field was with NNemo1, a UUV; the solution formulation focuses on submarines and UUVs. However, the author is confident that, as the unmanned aerial vehicle (UAV) community continues to develop unmanned combat aerial vehicles (UCAVs), which are limited in their rapid maneuver capability (high g-loading) by the structural limit and not the pilot, they too will encounter the vehicle control in a full unsteady flow field problem. The techniques developed in this dissertation can be extended to UAVs if the future of UCAV development unfolds as the author predicts.
Motivated by the primary operating regimes of current vehicles, research to date has been focused on steady and quasi-steady fluid-vehicle systems. Using steady and quasi-steady flow assumes that the forces and moments experienced by a vehicle during a maneuver can be represented using functions of only the vehicle states. This is typically done through a Taylor Series approximation, as in (2)

\[
M(u, v, w, \dot{u}, \dot{v}, \dot{w}, p, q, r, \delta s, \ldots) = \frac{\partial M}{\partial u} \Delta u + \frac{\partial M}{\partial v} \Delta v + \frac{\partial M}{\partial w} \Delta w + \cdots + \frac{\partial M}{\partial p} \Delta p + \cdots + \frac{\partial M}{\partial \delta s} \Delta \delta s + \cdots
\]  

(1.1)

Where \( u, v, \) and \( w \) are the linear velocities of the vehicle; \( p, q, \) and \( r \) are the rotational velocities of the vehicle; and \( \delta s \) is a control surface deflection. In full unsteady flow, the fluid has characteristics that cannot be accounted for using only the vehicle orientation, velocity, and acceleration as states. The difference between steady, quasi-steady, and full unsteady fluid-vehicle systems is shown in Fig. 2-1.

The problem as formulated in the Introduction spans several different research areas. As such the literature search was quite extensive and many papers and texts were read before
formulating an approach to solve the problem of vehicle control in a full unsteady flow field. For this reason, the following literature survey is broken into 3 subsections pertaining to the 3 main areas this dissertation spans. They are: submarine and UUV research, modal analysis and control, and fluid load prediction from vehicle surface characteristics.

2.1 Submarine and UUV Research

Submarine and Unmanned Underwater Vehicle (UUV) research has been forwarded mostly by the government. The amount of material in the public sector is, therefore, limited. Most of the literature in the area is in conference proceedings. What follows is a review of the
literature, which is available to the public. The focus is divided into 4 areas, fluid dynamics, UUV industry and development issues, biologically inspired aquatics, and UUV dynamics and control. These areas will be discussed in turn and focus will be placed primarily on the most relevant, UUV dynamics and control.

The field of submersible fluid dynamics is long and distinguished and this survey will not even begin to brush the surface. However, some interesting papers under the topic of “hydrodynamic control” will be discussed. Wetzel and Simpson (3), in 1998, demonstrated the need for further investigation into unsteady aerodynamics by showing cases where steady theory falls short. They discussed the differences between steady, quasi-steady, and unsteady flow and compared the steady solution and the experimentally tested solution for a time-dependent side-slip angle of a submarine entering a turning maneuver. The results showed that the steady and unsteady cases had very different flow topologies and that the unsteady separation lags the steady separation. For example, in the steady case 2 areas of separation were observed from the shear stress distribution on the body, however, in the unsteady case separation occurs only once.

This sentiment is echoed by Rhee and Hino’s (4) 2002 study on unsteady turbulent flow around a maneuvering 6:1 prolate spheroid. They apply the 3-dimensional unsteady Reynolds-averaged Navier-Stokes (URANS) method to a prolate spheroid in a pitch-up maneuver. The results, similar to Wetzel and Simpson (3), showed lags in flow field
development and delays in flow field separation when compared to the steady case. However, they do not conduct a comparison to quasi-steady flow.

On an interesting note, Chen and Huang (5), in 2002, used an inverse hydrodynamic problem to find the body geometry from the desired pressure field. The ship hull geometry, which is specified loosely by a B-spline, is iterated with the propeller disk pressure distribution until a resulting pressure distribution is found that minimizes cavitation. The paper neatly demonstrates the inventive direction that fluid modeling is taking.

In response to the increased use of UUVs, the area of UUV industry and development have published several articles that the author has drawn from to establish the current operational trends with UUVs. Notable is Langebrake’s (6) article on the use of UUV sensors in marine research and Griffith’s (7) article on logistics, risks, and procedures concerning UUV operation. The former divides sensors into 2 categories, those for navigation and control and those for research and experimentation. Focusing on the research and experimentation category Langebrake highlights the current UUV sensor technology and implementation of these sensors. The later divides UUV missions into 3 categories: ship-based, shore-based, and completely autonomous. Griffith further divides the missions into 5 segments: launch, outward transit, the work task, inbound transit, and recovery. Within these divisions he discusses different operations and associated tasks.
As more and more UUVs are being developed some engineers and scientists have decided to take their cues from nature. This has resulted in the area of biologically inspired aquatics.

The area was kicked off by Anderson and Kerrebrock’s (8) 1997 paper describing the initial development of the vorticity controlled unmanned undersea vehicle (VCUUV). The VCUUV was designed by Draper Laboratory to mimic the propulsion of the yellow fin tuna \textit{(Thunniform)}. Although the paper does not discuss any construction or testing, it does layout the design of the 4 degree-of-freedom (DOF) linkage system with a rigid caudal fin and the components and placement of the instrumentation package.

Two years later, in 1999, Kato (9) analyzed the pectoral fin of the Black Bass for control of UUVs at Tokai University in Japan. She discussed the use of pectoral fin motion, which is composed of feathering and lead-lag motions, to control a free-swimming model. A fuzzy logic controller was developed to deal with the 7 state variable and 3 control input system of the 2 and 3-DOF prototypes. In-water tests of the prototype swimming from one position to another in a steady current showed good performance. A simulation was also developed using an unsteady vortex lattice method and compared to the experimental results showing good agreement.

Prompted by the development of several free-swimming fish models, Sfakiotakis et al. (10) reviewed the current literature on swimming modes of fish locomotion. This paper begins with the primary division of body and/or caudal fin (BCF) vs. median and/or paired fin (MPF) propulsion. Within these 2 primary types of propulsion are secondary methods:
oscillatory vs. undulatory. They also review the analytical expressions for the forces and 
moments experienced by fish and the fluid modeling approaches to characterize fish 
swimming developed thus far.

The development of free-swimming models has been taken to even higher levels more 
recently. In 2001, motivated by the desire to remove motors from underwater robotics 
because of their constant emission of detectible noises, Penney et al. (11) developed a fish 
model actuated by Nickel-Titanium (Nitinol) shape memory alloys (SMAs). The 
combination of the composite skeleton and Nitinol SMAs was designed to mimic *Thunniform*
swimming, which has been noted as the most efficient swimming form (10). The model can 
perform full tail bends with a maximum deflection of 130mm and S-tail bends.

The area of submarine control, as mentioned in the introduction, is still relatively behind in 
technology. Some pioneering work on submarine control was conducted by Hoffmann and 
Standhagen (12) in 1968. They developed the discrete form of the linearized equations of 
motion in the longitudinal plane and used them to formulate a proportional controller for the 
deepth and the target following problem.

Work on submarines continued on roughly the same path and in 1989 Humphreys (13) 
investigated the effect of nonlinear behavior apparent on UUVs between 21 inch and 5 feet in 
cross-sectional diameter and at cruise speeds up to 20 knots. It is pointed out that in
maneuvering, cross flow is very important and that the characteristic length used in the Reynolds number is the hull diameter. Thus the cross flow Reynolds number is given by

\[ Re_{cf} = \frac{U_o d \sin \alpha}{\mu} \]  

(2.1)

It is also demonstrated that the cross flow drag (force) varies with the cross flow Reynolds number. For laminar flow (Re<10^5), the cross flow drag coefficient is 1.2. For turbulent flow (Re>10^7), the cross flow drag coefficient is 0.4. In the transition region, the cross flow drag coefficient varies. Thus, as shown in the paper, at high speed a UUV operates in the turbulent cross flow region with small variations in cross flow drag coefficient with changes in speed. However, at lower speed the cross flow drag coefficient can vary in the transition region and cause variation in the hydrodynamic moment coefficient, thus affecting the UUVs stability. The paper closes with a stability analysis of a UUV in a turning maneuver to illustrate the above points.

In the same year Cristi and Healey (14) developed a dive controller that uses adaptive pole-placement. The UUV model was linearized around several constant speed operating points. Gain functions were then found for these operating points using pole-placement. These gain functions were applied to dive maneuvers in a closed-loop simulation at various speeds to demonstrate the ability of the adaptive controllers to handle the model outside the linear operating range.
Healey and Cristi (15) also published their study on a rapid UUV depth controller in the same year. In this study they, motivated by the problems surrounding rapid response maneuverability of a UUV as a means of obstacle avoidance, used pole-placement on the longitudinal modes of a UUV. The paper title mentions compensation but, from the authors analysis, the compensation they are referring to is the use of a validated model to reconstruct the full longitudinal states from the depth sensor, which is using an observer not a compensator. The author finds further fault in their choice of gains, which is based on a “eliminate overshoot” design criteria. The design criteria results in minimized rise time which contradicts their goal of rapid maneuverability. The full control algorithm was implemented in open water tests and the control gains were tuned to obtain better performance.

In Japan in 1989, Kato and Kouda (16), motivated by the goal to survey the ocean bed, developed a vehicle navigation and control system. They began from the vehicle design stage in which they used a constrained nonlinear optimization method, the Hooke-Jeeves method, to design the vehicle. The longitudinal and lateral equations of motion with traditional added-mass terms were then derived in state-space form. This multi-input multi-output (MIMO) system was then transformed to a matrix of single-input single-output (SISO) transfer functions. These SISO transfer functions were utilized in the development of the UUV control system with the following goals in mind: to maintain zero vertical speed, pitch angle, roll angle, and yaw angle. The navigational control was implemented in a fuzzy logic controller with 5 fuzzy state variables.
In 1990, Williams et al. (17) motivated by the desire to autonomously navigate through an obstacle field applied position and velocity Kalman filters to handle moving and stationary obstacles. The vehicle and environment were developed and tested entirely in simulation. The sensor model used was a front-facing active sonar with zero-mean, white, Gaussian measurement and ambient noise. The obstacle avoidance logic was based on a 2-dimensional merit function that draws a safety circle around the UUV and determines a new heading based on the minimization of the merit function.

This was followed in 1992 by Healey (18) at the Naval Postgraduate School (NPS) developing a linear-model following control (LMFC) system for a UUV to map uncharted areas. In the design he linearizes the equations of motion around a nominal forward velocity, $U_0$. In accordance with the LFMC strategy, the vehicle system equations are augmented with those in the model. The control gains of this augmented system were then found by minimization of a linear quadratic performance index. A robustness analysis was performed for forward velocities off the nominal velocities to $U=5U_0$.

In 1993, Morrison and Yoerger (19) used system identification to determine the hydrodynamic forces on a vehicle forced to oscillate in the water in 1-dimension. As a force model they used Morrison’s equation, which contains an added-mass term that is proportional to vehicle acceleration and a drag coefficient term. They note that if the vehicle is in oscillatory motion these coefficients are functions of the Keulegan-Carpenter number, a
frequency parameter, and the Reynolds number. Using the results from an experimental setup that consisted of an anchored spring attached to the vehicle, which was given an in-plane disturbance, they attempted a polynomial fit with minimization of the scalar error norm.

In that same year Fossen (20) published his text in which he derives the linear and non-linear 6-DOF equations of motion for submarines and ships. He also presents several techniques, which are in line with modern control theory, to develop guidance and control algorithms for submarines and ships, such as sliding-mode control, optimal control, Kalman filters, among others.

In 1994 Logan (21) conducted a comparison between H-infinity/Mu-synthesis control and sliding-model control on the Draper Laboratory/MIT Sea Grant’s Sea Squirt. The performance of these control systems were analyzed using a simulation of the full-order nonlinear model. The performance of the controllers in the presence of sensor noise was also analyzed.

This was followed in 1995 by Leonard’s (22) formulation to maintain self-controlled motion of a UUV during actuator failure by taking advantage of the nonlinearity of the vehicle dynamics. The dynamics are formulated in a manner that specifies each degree-of-freedom independently as a matrix composed of a rotation and a translation, similar to the Denavit-Hartenberg parameters in robotic systems. The under-actuated vehicle is then modeled by these matrices. Based off these matrices and the property of commutation of matrices, the
ability of the remaining actuators to control motion in the desired translation or rotation of
the failed actuator is determined. Two applications of the theory, yaw maneuver using pitch
and roll actuators and a “parking maneuver,” are presented as examples.

Vehicle Control Technologies Inc. (VCT) was contracted by Florida Atlantic University
(FAU) to develop a hydrodynamic model and dynamic simulation of their Ocean Explorer
UUV (OEX) in 1996. The result was a list of concerns regarding the vehicle, including “flow
separation on the hull and fins due to the steep hull after body design” (23) and sensitivity to
weight and center of gravity changes. Based on this assessment, it was concluded to conduct
in-water testing. Humphreys’ (23) documents the comparison between the simulator and in-
water tested predictions. He further attempts to correlate the two by changing the flap
effectiveness parameter in the simulation model until the responses coincide. The results of
the test indicated that the control surface thickness was too low at 8%. The recommended
minimum to avoid leading edge separation is 12% with typical Navy applications using 15%to 21% airfoils. Also, leading to separation was the high aspect ratio fins of 4.88. Most
control surfaces use an aspect ratio of 1.5 to 3.0. Combining these effects with the low fin
Reynolds number, 110,000 at 2kts. led to the major problems with flow separation.

In the same year, Dreyer et al. (24) weakly coupled the vehicle equations of motion and the
hydrodynamic fluid equations to investigate the maneuvering characteristics of submerged
vehicles. The vehicle dynamics was modeled with the traditional 6-DOF rigid-body
equations of motion. The hydrodynamic forces and moments were calculated using the
unsteady Reynolds-averaged Navier-Stokes (URANS) equations. The equations were combined and evaluated at each time step. This solution was applied to 3 different bodies: a free-falling sphere, a free-falling prolate spheroid, and the SUBOFF model. The results showed the close coupling behavior of the vehicle equations and the fluid equations.

The analysis conducted by Dreyer et al. was continued by Devoudzadeh et al. (25) in which they extended the analysis of the SUBOFF model. Using the same coupled rigid-body dynamic and hydrodynamic equations as in Dreyer et al., Devoudzadeh et al. investigated the unconstrained motion of SUBOFF, roll and yaw moments applied to SUBOFF, and response during crashback – fully reversing thrust such that a vortex ring develops around the propeller. A majority of their focus is on the crashback maneuver and the time history of the vortex ring. In the crashback maneuver, it is seen that the small-amplitude, high-frequency oscillations of the lateral and vertical forces are exhibited, as well as, oscillations in the yaw and pitch rates, which correlate with the large-amplitude, large-wavelength oscillations in the out-of-plane forces.

In 2000, Riedel (26) developed a disturbance compensation controller (DCC), which attempts to extract the physical environment of the UUV through estimation of 1st-order coefficients of a wave dynamic model. This wave dynamics compensator was combined with a sliding-mode control law to achieve improved motion stability. The performance of the DCC was experimentally verified on the NPS Phoenix UUV.
In this same year, Sen (27) investigated the sensitivity of the response of a submarine and an axisymmetric slender body to variations in hydrodynamic coefficients. He defines the sensitivity index to be the relative difference of a maneuvering function over the relative difference of the varying hydrodynamic coefficient. This is in accordance with the standard definition of sensitivity. Sen chooses to use 3 maneuvers for analysis: overshoot in the horizontal and vertical plane and a turning maneuver by constant rudder deflection. The results showed that most of the nonlinear hydrodynamic coefficients have very little influence on the trajectory. Further, the sensitive terms are linear coefficients representing damping forces and moments for the submarine and inertial forces and moments for the axisymmetric body. For both vehicles, the most influential terms were those associated with moments $M_q$ and $N_r$ for the submarine and $M_\dot{q}$ and $N_\dot{r}$ for the axisymmetric body.

On a more application oriented note, Yun, Bachmann, and Suat (28) documented the continued development of the Small AUV Navigation System (SANS) by the NPS in their 2000 paper. The SANS, in its 3rd generation at the time of the papers writing, uses a PC/104 form factor 486 processor running at 133MHz. A water speed indicator, global positioning system (GPS) receiver, compass, inertial measurement unit (IMU), and Ethernet unit all share the PC/104 bus. After major variations in compass reading were noted, use of ground testing (an instrumented golf cart) to determine the source of the errors (vibration, inherent compass error, or noise from the electric motor) was conducted. Also discussed was the use of an asynchronous Kalman filter to improve position measurements in the SANS.
In 2001, Miyamoto et al. (29) discussed a method for developing a proportional-integral-derivative (PID) control system for a UUV. The method begins by decoupling and linearizing the system to obtain longitudinal and lateral equations of motion. The equations of motion are further decomposed into SISO transfer functions. The feedback gain of these SISO transfer functions are derived using partial model matching. Miyamoto et al. claimed without reference that, because the fundamental process characteristics, such as lower-order coefficients of the processor transfer function, are used “the response can be obtained without overshoot or oscillation, allowing lower energy consumption during maneuvering.” The SISO transfer functions are also rewritten to include an anti-windup controller, which prevents the windup of the integrator term since there is no explicit integral and the state after saturation is always bounded. This anti-windup controller is also shown to be capable of being implemented in systems with redundant actuators. The control system design method was implemented on Mitsubishi Heavy Industries UUV, Urashima, and command following tests conducted.

Also from Mitsubishi Heavy Industries, Kobayashi et al. (30) discussed the development of a high-level hardware in-the-loop simulator (HILS) and its use in developing a control algorithm for the Urashima. Four computers were networked and used to represent separate parts of the system. The devices were: the control unit (same as on the actual UUV), the monitor onboard the mother ship (same as on the actual UUV), the UUV simulator (used only for simulation), and the simulator in the mother ship (used only for simulation). A comparison of the simulation results and the actual runs on the Urashima were made.
Kobayashi (31) duplicated the above paper in its entirely in 2002 but renames it “Development and Application of an AUV Maneuvering and Control System Simulator.”

In 2002, Mohamed et al. (32), in an effort to make use of modern techniques in data processing, applied neural networks to predict the hydrodynamic forces and moments on a submarine. A modular neural network with a module for each force and moment output, which consisted of one four node initial layer and one four node hidden layer, was used. The network was trained on experimental data that included circular arc and chirp maneuvers obtained from the Marine Dynamic Test Facility located in Newfoundland. The inputs were displacement and velocity in the y-direction and position and rotation around the z-axis. The outputs were axial forces along the y and z axes and moments around all 3 axes. The results showed that large loads were predicted well, but small loads, which were also high frequency, were not predicted well.

In 2003, Evans and Lane (33) discussed the use of a HILS to integrate simulated and real actuators/sensors on UUVs. The specific tool discussed was CORESIM. Synchronization of the real actuator/sensors and the simulated actuators/sensors was demonstrated. The paper also demonstrates the use of CORESIM for an electromagnetic current meter, a real-time video-based motion control system, a concurrent mapping and localization system in improving pilot navigation and task visualization, and an autonomous docking and deployment vehicle.
In 2004, Smallwood and Whitcomb (34) addressed the trajectory problem for a low-speed maneuvering remotely operated vehicle (ROV) developed at The John Hopkins University. Several candidate controllers were investigated, including a linear proportional-derivative (PD) controller and a family of fixed and adaptive model-based controllers. The results indicate that the model-based controllers outperform the PD controllers over a wide range of operating conditions and that the adaptive controllers outperform the fixed controller.

Also in 2004, as mentioned in Chapter 1, Tureand and Kueny (1), at VCT developed a simulator of the NNemo1 vehicle for NGNN. The effort began with the development of a hydrodynamic model for NNemo1. The hydrodynamic modeling of NNemo1 was conducted by modeling the individual components (hull form and fins) and combining them into a full hydrodynamic model. The hull form was modeled using VCT’s computational fluid dynamic (CFD) code, which uses a combination of potential (steady, inviscid, irrotational) flow and vortex shedding analysis. The fins were modeled using VCT’s HydroV™ code with normal loading functions, stall conditions, and interference effects. The hydrodynamic data was compared to the wind tunnel data from of the Stressor vehicle developed in 1994, which also had an elliptic cross section. The results were in good agreement as was expected because of the vehicle similarity. Two vehicle configurations were analyzed in this manner, with the fairwater fin forward and aft. For both configurations, the hydrodynamic coefficients for the linear and nonlinear terms in the hydrodynamic force and moment equations were generated. The hydrodynamic coefficients were used to develop VCT SimV™, which generates the time-domain response based on the hydrodynamic coefficients.
2.2 Modal Analysis and Controls
Modal analysis and control plays an important part in obtaining the location of nodal lines and the development of the FCC control strategy. This section discusses the development of modal control in chronological order.

Modal control was developed in 1982 when Meirovitch and Baruh (35) showed how one can control a system by controlling its modes. The method was called ‘independent modal-space control’ and enabled designers to regulate the motion of a dynamic system on a mode-by-mode basis. In 1983, Meirovitch and Silverberg (36) showed that independent modal-space control is globally optimal. In 1985, Baruh and Silverberg (37) examined the robustness of independent modal-space control and showed why it’s insensitive to errors in the parameters of the model. In 1986, Silverberg (38) showed that independent modal-space control becomes decentralized and the modes become uniformly damped when the interest lies in regulating settling time. In 1992, Weaver and Silverberg (39) applied sensors and actuators at nodal locations along a beam for active sensing and control of vibration. In 1995, Rossetti and Sun (40) applied uniform damping to the problem of controlling ring vibration. In 1997, Washington and Silverberg (41) developed a uniform damping/stiffening control method to regulate a systems peak overshoot and it’s settling time. Most recently, in 2005, Silverberg and Levedahl (42) applied uniform damping/stiffening to control formations of autonomous aircraft.
2.3 Fluid Load Prediction from Vehicle Surface Characteristics
This section discusses two closely related areas of research that were leveraged by the author in formulating the approach of controlling a vehicle maneuvering in a full unsteady flow field outlined in this dissertation. Specifically, the prediction of the forces and moments on a body from the stagnation, separation, and reattachment points on the body surface and techniques to predict the stagnation, separation, and reattachment points on the surface of a body.

In 1994, Goman and Khrabrov (43) showed that the unsteady forces and moments on a 2-dimensional airfoil through high angles of attack can be expressed uniquely by the instantaneous measurements of the separation point, \( x \), and the angle of attack, \( \alpha \). For example, the lift coefficient could be calculated from

\[
C_L(\alpha, x) = \frac{\pi}{2} (1 + \sqrt{x})^2 \alpha
\]

The other forces and moments are expressed similarly. In the process, they developed a function for the instantaneous separation point location \( x \) as an internal state of the system

\[
\tau_1 \frac{dx}{dt} + x = x_0 (\alpha - \tau_2 \dot{\alpha})
\]

Where \( x_0 \) is the steady-state location of separation, \( \alpha \) and \( \dot{\alpha} \) are the angle-of-attack and angle-of-attack rate, \( \tau_1 \) is the relaxation time constant that defines the transient fluid dynamic effects, and \( \tau_2 \) is the total time delay of the separation point due to quasi-steady effects such as circulation and boundary layer convective lags. The process they proposed to calculate the unsteady forces and moments was:
1. Find the characteristic times, $\tau_1$ and $\tau_2$, which relate the instantaneous separation point location $x$ to the steady-state separation point location, through Eq. (2-3) by curve fitting experimental data.

2. Use Eq. (2-3) with the characteristic times to calculate the instantaneous separation point location $x$ from the steady-state separation point location $x$, angle-of-attack $\alpha$, and the angle-of-attack rate $\dot{\alpha}$.

3. Use Eq. (2-2) to find the lift coefficient from the angle-of-attack $\alpha$ and the instantaneous separation point location $x$.

In 1994 and 1996, Goman, Khrabrov, and Usoltsev (44) and Fan and Lutz (45), respectively, validated this method in several experiments.

Researchers soon noticed that if the instantaneous separation point can be measured directly from sensors on the body surface, then Eq. (2-2) by itself will give lift coefficients in real-time and Eq. (2-3) doesn’t need to be used at all. The result was a new 3-part process for obtaining the unsteady forces and moments:

1. Through steady and unsteady testing determine the forces and moments on the body and the instantaneous separation point locations for different angles of attack.

2. Correlate the forces and moments to instantaneous separation point location and angle-of-attack.

3. Use angle-of-attack and instantaneous separation point location to obtain the lift coefficient.
The process for the other forces and moments follow similarly. As an added benefit, because in most cases the separation point move around in specific areas, the sensors need only be placed in areas where the separation occurs. It was also noted by Goman and Khrabrov (43) that the stagnation point is a function of angle-of-attack and could be used in place of angle-of-attack measurements.

Despite having developed the elegant theory of the correlation between forces and moments and stagnation and separation points on 2-dimensional airfoils; and demonstrated it through experimentation, the problem of how to effectively determine the stagnation and separation points still existed. To address this problem, hot-film shear stress sensors with a thermal anemometer were employed.

A hot-film sensor consists of a 0.25μm-thick nickel sensing element and 2mm-thick copper leads. They are vacuum deposited on a flexible insulated substrate that can be affixed to curved surfaces and can be found as single sensors or as sensor arrays on sheets. Hot-film works by measuring the amount of heat transferred to the fluid from the sensor, which is a function of the state of the boundary layer, fluid properties, and the relative speed of the vehicle through the fluid. A change in the heat transfer is seen as a change in the resistance of the hot-film element.

To convert the change in resistance to a voltage which can be read by the data acquisition system a thermal anemometer is used. Three types of thermal anemometers are available:
1. Constant Current Anemometry (CCA) – holds the current of the sensor constant.
2. Constant Temperature Anemometry (CTA) – holds the resistance of the sensor constant.
3. Constant Voltage Anemometry (CVA) – holds the voltage of the sensor constant.

The most successfully of these being CVA, which has been demonstrated in several wind tunnel, ocean, and flight tests by Sarma, Comote-Bellot, Lankes, Faure, and Mangalam over the period from 1998 to 2004 (46), (47), (48), (49), (50), and (51). CVA has several characteristics that have made it so successful (51):

1. CVA does not require critical adjustments to account for changes in flow conditions.
2. CVA has a high signal-to-noise ratio (SNR).
3. CVA has a negligible temperature drift.
4. CVA is practically immune to electro-magnetic interference (EMI), radio frequency interference (RFI), and cable-capacitance effects.

The most relevant of the above papers is Mangalam’s (51), in which he uses the formulation developed by Goman and Khrabrov and given in Eqs. (2-2) and (2-3) to find the lift coefficient on a 2-dimensional airfoil in steady and unsteady conditions in water. To measure the separation location on the airfoil Mangalam uses a hot-film/CVA system. In this work he also presents the fact that, since the location of flow bifurcation points (stagnation, separation, and reattachment points) are found from comparing the sensors in the sensor array, the hot-film/CVA system does not need to be calibrated to determine the flow bifurcation points. That is, the sensors forward and behind experience a phase reversal in the
time history plots and the sensor at the flow bifurcation point experiences a double frequency.
Chapter 3

The Coupled Fluid-Vehicle Model

The coupled fluid-vehicle (CFV) model is formulated below; first equations describing the motion of the fluid are developed, then equations describing the motion of the vehicle are developed, and finally the equations describing the fluid are coupled with the equations describing the motion of the vehicle. The approach differs from the traditional formulation which assumes steady potential flow as developed in Newman (52). The approach here can be used in full unsteady analysis, as explained in more detail later. The coordinates of the inertial frame are \(x\) (longitudinal), \(y\) (lateral), and \(z\) (depth). The coordinates of the vehicle frame are \(X\), \(Y\), and \(Z\) and the origin is located at the vehicle’s center of mass, \(C\). These are shown in Fig. 3-1.

Figure 3 - 1: Inertial and Vehicle Coordinate Systems
3.1 The Fluid
A fluid element’s general stress state is represented by the stress matrix, \( \sigma \). The pressure is

\[
p = -\frac{1}{3} \text{trace}(\sigma)
\]  

(3-1)  

The pressure can be removed from \( \sigma \), as shown in Fig 3-2, leaving the complementary stress state

\[
\sigma_0 = \sigma + pI
\]  

(3-2)

![Figure 3 - 2: Differential Fluid Element](image)

In which \( I \) is the 3x3 identity matrix. The force vector acting on the element is

\[
d\vec{F} = \nabla \cdot \sigma dV = (\nabla \cdot \sigma_0 - \nabla p)dV
\]  

(3-3)  

In which \( dV \) is the volume. From Newton’s Second Law, the force vector is

\[
d\vec{F} = \rho \ddot{a}_f dV
\]  

(3-4)  

In which \( \rho \) is the mass density and \( \ddot{a}_f \) is acceleration. Thus, the fluid motion is described as

\[
\ddot{a}_f = \frac{1}{\rho} (\nabla \cdot \sigma_0 - \nabla p)
\]  

(3-5)
Referring to Fig. 3-1, the fluid elements position vector, velocity vector, and acceleration vector are

\[ \mathbf{r}_f = \mathbf{r}_c + \mathbf{r} \quad (3-6) \]
\[ \mathbf{v}_f = \mathbf{v}_c + \omega \times \mathbf{r} + \mathbf{V} \quad (3-7) \]
\[ \mathbf{a}_f = \mathbf{a}_c + \omega \times \mathbf{r} + \omega \times (\omega \times \mathbf{r}) + 2\omega \times \mathbf{V} + \mathbf{A} \quad (3-8) \]

In which lower case quantities are measured in the inertial frame, upper case quantities are measured in the vehicle frame, and the subscript \( f \) refers to the fluid element. The vehicle’s velocity vector, acceleration vector, angular velocity vector, and angular acceleration vector are respectively, \( \mathbf{v}_c, \mathbf{a}_c, \omega, \) and \( \dot{\omega} \). Substituting Eq. (3-8) into Eq. (3-5) yields the equations that govern the motion of the fluid

\[ \mathbf{a}_c + \omega \times \mathbf{r} + \omega \times (\omega \times \mathbf{r}) + 2\omega \times \mathbf{V} + \mathbf{A} = \frac{1}{\rho} (\nabla \cdot \mathbf{\sigma}_0 - \nabla p) \quad (3-9) \]

Equation (3-9) is a set of three partial differential equations. An approximate solution is found by expressing the flow field \( \mathbf{V} \) as a linear combination of fundamental flow fields \( \mathbf{V}_n \) as

\[ \mathbf{V}(\mathbf{r}, t) = \sum_{n=1}^{N} \mathbf{V}_n(\mathbf{r})q_n(t) \quad (3-10) \]

In which \( q_n(t) \) is a generalized coordinate that expresses the level of participation of \( \mathbf{V}_n \) in \( \mathbf{V} \).

Equation (3-9) is an Eulerian description of the flow measured in the vehicle frame. The fundamental flow fields \( \mathbf{V}_n \) are admissible functions that satisfy the geometric boundary conditions on the vehicle surface. They are not required to satisfy any conditions other than the geometric boundary conditions although it is often convenient to select them from the
well-established set of admissible functions associated with potential flow. By time
differentiation, the acceleration of the flow in the vehicle frame is

\[ \vec{A} = \nabla \vec{V} \cdot \vec{V} + \frac{\partial \vec{V}}{\partial t} \]  

(3-11)

Substituting Eq. (3-10) and it’s derivative with respect to time into Eq. (3-11) yields

\[ \vec{A} = \sum_{n=1}^{N} \sum_{m=1}^{N} \nabla \vec{V}_n (\vec{r}) \cdot \vec{V}_m (\vec{r}) q_n (t) q_m (t) + \sum_{n=1}^{N} \vec{V}_n (\vec{r}) \dot{q}_n (t) \]  

(3-12)

The flow field \( \vec{v}_f \) in Eq. (3-7) is assumed to be stationary in the vehicle far field, which is expressed mathematically by the conditions

\[ \vec{v}_f (\infty) = 0 \]  

(3-13)

and

\[ \vec{a}_f (\infty) = 0 \]  

(3-14)

These conditions are used to relate the generalized coordinates and their time derivatives to the vehicle’s velocity vector, \( \vec{v}_C \), and acceleration vector, \( \vec{a}_C \). Substituting Eqs. (3-10) and (3-13) into Eq. (3-7) and substituting Eqs. (3-12) and (3-14) into Eq. (3-8) yields the conditions

\[ 0 = \vec{v}_C (t) + \omega (t) \times \vec{r} (\infty) + \sum_{n=1}^{N} \vec{v}_n (\infty) q_n (t) \]  

(3-15)

and

29
\[ 0 = \ddot{a}_C + \dot{\omega}(t) \times \dot{r}(\infty) + \omega(t) \times (\omega(t) \times \dot{r}(\infty)) + 2\omega(t) \times \sum_{n=1}^{N} \tilde{V}_n(\infty)q_n(t) \]
\[ + \sum_{n=1}^{N} \sum_{m=1}^{N} \nabla \tilde{V}_n(\infty) \cdot \tilde{V}_m(\infty)q_n(t)q_m(t) + \sum_{n=1}^{N} \tilde{V}_n(\infty)\dot{q}_n(t) \]

(3-16)

### 3.2 The Vehicle

The vehicle is modeled as a rigid body acted on by a fluidic force vector \( \vec{f}_F \), a control force vector \( \vec{f}_C \), a gravity force vector \( \vec{f}_G \), a fluidic moment vector \( \vec{m}_F \), and a control moment vector \( \vec{m}_C \). The equations that govern the vehicle’s translations and rotations are

\[ m\ddot{a}_C = \vec{f}_F + \vec{f}_C + \vec{f}_G \]

(3-17)

and

\[ J\dot{\omega} + \dot{\omega} \times J\omega = \vec{m}_F + \vec{m}_C \]

(3-18)

In which \( m \) denotes the vehicle’s mass and \( J \) is the inertia matrix.

### 3.3 Coupling the Fluid and the Vehicle

The fluid and the vehicle are coupled by the fluidic force vector \( \vec{f}_F \) and the fluidic moment vector \( \vec{m}_F \). Each is expressed in terms of the stresses acting on the vehicle’s surface \( S \) as

\[ \vec{f}_F = \oint_S [\sigma_0 - pI]\vec{n}dS \]

(3-20)

and
\[ \vec{m}_F = \oint_S \vec{r} \times [\sigma_0 - p\mathbf{I}]\vec{n}dS \]

(3-21)

In which \( \vec{n} \) is a unit vector normal to the surface element \( dS \) and \( \vec{r} \) is the position vector from C to the surface element. The pressure on the vehicle surface can be written as

\[ p = p_0 + \int_{r_0}^{r} \nabla p \cdot d\vec{r} = p_0 + \int_{r_0}^{r} \left[ \nabla \cdot \sigma_0 - \rho \vec{a}_f \right] dS \]

(3-22)

where \( p_0 \) is the pressure at a reference point on the vehicle’s surface. Substituting Eq. (3-22) into Eqs. (3-20) and (3-21) yields

\[ \vec{f}_F = \oint_S \sigma_0 \vec{n}dS - \oint_S \left[ p_0 + \int_{r_0}^{r} \left[ \nabla \cdot \sigma_0 - \rho \vec{a}_f \right] \cdot d\vec{r} \right] \vec{n}dS \]

(3-23)

and

\[ \vec{m}_F = \oint_S \vec{r} \times \sigma_0 \vec{n}dS - \oint_S \vec{r} \times \left[ p_0 + \int_{r_0}^{r} \left[ \nabla \cdot \sigma_0 - \rho \vec{a}_f \right] \cdot d\vec{r} \right] \vec{n}dS \]

(3-24)

Since \( \oint_S \vec{n}dS = 0 \) and \( \oint_S \vec{r} \times \vec{n}dS = 0 \), simplification of Eqs. (3-23) and (3-24) yields

\[ \vec{f}_F = \oint_S \left[ \sigma_0 - \int_{r_0}^{r} \nabla \cdot \sigma_0 \cdot d\vec{r} \right] \vec{n}dS - \oint_S p_r \vec{n}dS \]

(3-25)

and
\[
\mathbf{m}_F = \oint_S \mathbf{r} \times \left[ \mathbf{\sigma}_0 - \int_{r_0}^r \nabla \cdot \mathbf{\sigma}_0 \cdot d\mathbf{r} \right] \mathbf{n} dS - \oint_S p_r \mathbf{r} \times \mathbf{n} dS
\]

(3-26)

In which \( p_r \) denotes the relative pressure and is

\[
p_r = -\int_{r_0}^r \rho \mathbf{\bar{a}}_f \cdot d\mathbf{\bar{r}}
\]

(3-27)

A CFV model whose number of independent degrees of freedom is equal to the number of degrees of freedom is called a quasi-steady model. Quasi-steady models undergo simplifications by employing steady-flow analysis tools, notably stability derivatives to approximate fluidic forces and moments. Predictions with quasi-steady CFV models will disagree significantly with the actual physical behavior under full unsteady conditions because of the absence of the fluid’s independent degrees of freedom. But, prediction with full unsteady CFV models will disagree with the physical behavior observed in an operational setting, too. The first problem that arises with full unsteady models is associated with the admissible functions. To capture full unsteady behavior the number of admissible functions must be extremely large. The second problem, which is even more severe, is that it is unclear how to set the initial conditions in a CFV model. Indeed, the flow’s history is not known in an operational setting making real-time predictions beyond the current state-of-the-art.
### 3.4 CFV Model Example

Below the CFV model approach is applied to a Swimmer Delivery Vehicle (SDV) (20) which is modeled as an elliptically-shaped vehicle maneuvering in the x-y plane. The semi-major and semi-minor lengths of the elliptical body are \(a = 1.83\text{m (6ft)}\) and \(b = 0.305\text{m (1ft)}\), respectively. The vehicle is controlled through variable thrust actuation. Further, to simplify the analysis the vehicle will be assumed to be neutrally buoyant so that the weight force balances with the buoyancy force. The shear stress will also be neglected.

The fundamental flow fields \(\vec{V}_n (n = 1, 2, 3, 4)\) will be constructed from a Joukowsk transformation of fundamental flow fields around a cylinder of radius \(R\) (53). Accordingly, the two coordinates \(\zeta\) and \(\eta\) (and corresponding unit vectors \(\vec{e}_\zeta\) and \(\vec{e}_\eta\)) are mapped to \(X\) and \(Y\) (and corresponding unit vectors \(\vec{I}\) and \(\vec{J}\)) by

\[
X = \zeta \left[ 1 + \left( \frac{A}{r} \right)^2 \right]
\]

and

\[
Y = \eta \left[ 1 - \left( \frac{A}{r} \right)^2 \right]
\]

In which \(r^2 = \zeta^2 + \eta^2\). Notice at infinity that \(X = \zeta\) and \(Y = \eta\). The cylinder is mapped to an ellipse with semi-major axis \(a\) and semi-minor axis \(b\) by letting \(R = \frac{a+b}{2}\) and \(A = \sqrt{\left( \frac{a+b}{2} \right) \left( \frac{a-b}{2} \right)}\). The four fundamental flow fields around a cylinder are

\[
\vec{V}_{1,Cyl} = \left[ 1 + \left( \frac{R^2}{r^2} \right) - 2 \left( \frac{\zeta^2 R^2}{r^4} \right) \right] \vec{e}_\zeta - 2 \left[ \frac{\zeta \eta R^2}{r^4} \right] \vec{e}_\eta
\]

\[
\vec{V}_{2,Cyl} = -2 \left[ \frac{\zeta \eta R^2}{r^4} \right] \vec{e}_\zeta + \left[ 1 + \left( \frac{R^2}{r^2} \right) - 2 \left( \frac{\eta^2 R^2}{r^4} \right) \right] \vec{e}_\eta
\]
\[ \vec{V}_{3,\text{cyl}} = \vec{k} \times (\zeta \hat{\zeta} + \eta \hat{\eta}) \]  
(3-32)

\[ \vec{V}_{4,\text{cyl}} = \vec{k} \times \left( \frac{\vec{\rho}}{r} \right) \left( \frac{\zeta}{r} \hat{\zeta} + \frac{\eta}{r} \hat{\eta} \right) \]  
(3-33)

The components of the fundamental flow fields (as well as the components of any other vectors) transform like the unit vectors, which transform according to

\[ \vec{I} = \frac{\partial X}{\partial \zeta} \hat{\zeta} + \frac{\partial X}{\partial \eta} \hat{\eta} \]  
(3-34)

and

\[ \vec{J} = \frac{\partial Y}{\partial \zeta} \hat{\zeta} + \frac{\partial Y}{\partial \eta} \hat{\eta} \]  
(3-35)

In which

\[ \frac{\partial X}{\partial \zeta} = 1 + \left( \frac{A}{r} \right)^2 - 2 \left( \frac{A}{r} \right)^2 \left( \frac{\zeta}{r} \right)^2 \]  
(3-36)

\[ \frac{\partial X}{\partial \eta} = -2 \left( \frac{A}{r} \right)^2 \left( \frac{\zeta}{r} \right) \left( \frac{\eta}{r} \right) \]  
(3-37)

\[ \frac{\partial Y}{\partial \zeta} = 2 \left( \frac{A}{r} \right)^2 \left( \frac{\zeta}{r} \right) \left( \frac{\eta}{r} \right) \]  
(3-38)

\[ \frac{\partial Y}{\partial \eta} = 1 + \left( \frac{A}{r} \right)^2 - 2 \left( \frac{A}{r} \right)^2 \left( \frac{\eta}{r} \right)^2 \]  
(3-39)

Thus, the fundamental flow fields of the ellipse which are shown in Fig. 3-3 are given by

\[ \vec{V}_r = \left( \frac{\partial X}{\partial \zeta} V_{r,\text{cyl},\zeta} + \frac{\partial X}{\partial \eta} V_{r,\text{cyl},\eta} \right) \vec{I} + \left( \frac{\partial Y}{\partial \zeta} V_{r,\text{cyl},\zeta} + \frac{\partial Y}{\partial \eta} V_{r,\text{cyl},\eta} \right) \vec{J} \quad r = 1, 2, 3, 4 \]  
(3-40)

The first 2 fundamental flow fields and the fourth are potential flows; they satisfy incompressibility (\( \nabla \cdot \vec{V} = 0 \)) and they’re irrotational (\( \nabla \times \vec{V} = 0 \)). The third flow field satisfies only incompressibility. Notice that \( \vec{V}_1(\infty) = \vec{I}, \vec{V}_2(\infty) = \vec{J}, \vec{V}_3(\infty) = \vec{K} \times \vec{r}(\infty), \) and \( \vec{V}_4(\infty) = 0. \) Substituting Eq. (3-41) into Eqs. (3-15) and (3-16) gives
\[ q_1(t) = -v_{cx} \]  
\[ q_2(t) = -v_{cy} \]  
\[ q_3(t) = -\omega \]  
\[ \dot{q}_1(t) = -a_{cx} - \omega v_{cy} \]  
\[ \dot{q}_2(t) = -a_{cy} - \omega v_{cx} \]  
\[ \dot{q}_3(t) = -\dot{\omega} \]  

In which \( \dot{\nu}_C = v_{cx}\dot{I} + v_{cy}\dot{J} \), \( \dot{a}_C = a_{cx}\dot{I} + a_{cy}\dot{J} \), and \( \ddot{\omega} = \omega \ddot{K} \). Equations (3-41) through (3-46) express the first three generalized coordinates of the flow and their time derivatives in terms of the vehicle’s three planar degrees of freedom \( v_{cx}, v_{cy}, \) and \( \omega \).

Figure 3 - 3: Fundamental Flow Fields (a) \( V_1 \), (b) \( V_2 \), (c) \( V_3 \), and (d) \( V_4 \)
When taking more than three fundamental flow fields, Eqs. (3-41) through (3-46) are still satisfied provided the additional flow fields are stationary at infinity. The additional fundamental flow fields, when not constrained, produce independent degrees of freedom of the fluid. In this example, the fourth fundamental flow field above will be a dependent degree of freedom and will be applied to the coupled fluid-vehicle equations below. It will constrain the flow to zero at a particular point on the body, that is, enforce the Kutta condition.

Substituting Eqs. (3-10) and (3-12) into Eq. (3-8) and the result into Eq. (3-27) yields the relative pressure

\[ p_r = -\int_{r_0}^{r} \rho \left[ \ddot{a}_c + \omega \times \ddot{r} - \omega^2 \dot{r} + 2\omega \times \sum_{n=1}^{4} \vec{V}_n(\vec{r}) q_n(t) \right. \]

\[ + \sum_{n=1}^{4} \sum_{m=1}^{4} \nabla \vec{V}_n(\vec{r}) \cdot \vec{V}_m(\vec{r}) q_n(t) q_m(t) + \left. \sum_{n=1}^{4} \vec{V}_n(\vec{r}) \dot{q}_n(t) \right] \cdot d\vec{r} \]

(3-47)

Equation (3-47) can be simplified to

\[ p_r = \sum_{m=1}^{4} \Lambda_m q_m + \sum_{m=1}^{4} \sum_{n=1}^{4} \Omega_{mn} q_m q_n \]

(3-48)

The coefficients in Eq. (3-48) are

\[ \Lambda_1 = \rho \int_{X_0}^{X} \left( -1 + V_{1x} \right) dX + \rho \int_{Y_0}^{Y} V_{1y} dY \]

\[ \Lambda_2 = \rho \int_{X_0}^{X} V_{2x} dX + \rho \int_{Y_0}^{Y} \left( -1 + V_{2y} \right) dY \]
\[ \Lambda_3 = \rho \int_{X_0}^{X} (Y + V_{3x}) \, dX + \rho \int_{Y_0}^{Y} (-X + V_{3y}) \, dY \]

\[ \Lambda_1 = \rho \int_{X_0}^{X} V_{4x} \, dX + \rho \int_{Y_0}^{Y} V_{4y} \, dY \]

and

\[ \Omega_{11} = \omega_{11} \]

\[ \Omega_{22} = \omega_{22} \]

\[ \Omega_{44} = \omega_{44} \]

\[ \Omega_{12} = \Omega_{21} = \omega_{12} \]

\[ \Omega_{14} = \Omega_{41} = \omega_{14} \]

\[ \Omega_{24} = \Omega_{42} = \omega_{24} \]

\[ \Omega_{13} = \Omega_{31} = \omega_{13} + \rho \int_{X_0}^{X} \left( \frac{1}{2} - V_{1x} \right) \, dX + \rho \int_{Y_0}^{Y} V_{1y} \, dY \]

\[ \Omega_{23} = \Omega_{32} = \omega_{23} + \rho \int_{X_0}^{X} \left( -\frac{1}{2} + V_{2x} \right) \, dX - \rho \int_{Y_0}^{Y} V_{2y} \, dY \]

\[ \Omega_{34} = \Omega_{43} = \omega_{34} + \rho \int_{X_0}^{X} V_{4x} \, dX - \rho \int_{Y_0}^{Y} V_{4y} \, dY \]

\[ \Omega_{33} = \omega_{33} - \frac{\rho}{2} [(X^2 + Y^2) - (X_0^2 - Y_0^2)] + 2\rho \int_{X_0}^{X} V_{3y} \, dX - 2\rho \int_{Y_0}^{Y} V_{3x} \, dY \]

in which

\[ \omega_{mn} = \frac{\rho}{2} \int_{X_0}^{X} \left[ \frac{\partial V_{mx}}{\partial X} \frac{\partial V_{nx}}{\partial Y} + \frac{\partial V_{nx}}{\partial X} \frac{\partial V_{mx}}{\partial Y} \right] \, dX \]

\[ + \frac{\rho}{2} \int_{Y_0}^{Y} \left[ \frac{\partial V_{my}}{\partial X} \frac{\partial V_{ny}}{\partial Y} + \frac{\partial V_{ny}}{\partial X} \frac{\partial V_{my}}{\partial Y} \right] \, dY \]
If the forces and moments components due to the shear effects are neglected, as was mentioned in the problem statement, then the fluidic forces and moments on the vehicle are given by Eqs. (3-25) and (3-26) with $\sigma_0 = 0$ or

$$\bar{f}_F = -\int_S p_r \bar{n} dS$$

(3-49)

and

$$\bar{m}_F = -\int_S p_r \bar{r} \times \bar{n} dS$$

(3-50)

If the problem is planar (or the vehicle is assumed to rotate but not spin); then the $\omega \times J \omega$ term can be neglected. Further, assuming that the vehicle is neutrally buoyant, Eqs. (3-17) and (3-18) can be combined into

$$\mathbf{M}_0 \ddot{\mathbf{v}} = \bar{\mathbf{F}}_F + \bar{\mathbf{F}}_C$$

(3-51)

Where

$$\mathbf{M}_0 = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J \end{bmatrix} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_4 \end{bmatrix} \quad \mathbf{\dot{v}} = \begin{pmatrix} v_{C_x} \\ v_{C_y} \\ \omega \end{pmatrix} \quad \bar{\mathbf{F}}_F = \begin{pmatrix} f_{F_x} \\ f_{F_y} \\ m_F \end{pmatrix} \quad \bar{\mathbf{F}}_C = \begin{pmatrix} f_{C_x} \\ f_{C_y} \\ m_C \end{pmatrix}$$

Equation (3-48) can then be substituted into the planar form of Eqs. (3-49) and (3-50) and the result substituted into Eq. (3-51) to yield the vehicle equations of motion

$$\sum_{m=1}^{4} M_{lm} q_m = -\sum_{m=1}^{4} \sum_{n=1}^{4} D_{lmm} q_m q_n + f_{C_l} \quad l = 1,2,3$$

(3-52)
\[ M_{1m} = \int_t \Lambda_m n_x \, dl - m \delta_{1m} \]
\[ M_{2m} = \int_t \Lambda_m n_y \, dl - m \delta_{2m} \]
\[ M_{3m} = \int_t \Lambda_m (Xn_y - Yn_x) \, dl - J \delta_{3m} \]
\[ D_{1mn} = \int_t \Omega_{mn} n_x \, dl + m \delta_{2m} \delta_{3m} \]
\[ D_{2mn} = \int_t \Omega_{mn} n_y \, dl - m \delta_{1m} \delta_{3m} \]
\[ D_{3mn} = \int_t \Omega_{mn} (Xn_y - Yn_x) \, dl \]

Notice the (3-52) represents three equations in terms of four generalized coordinates. The three equations describe the motion of the body and the four coordinates describe the motion of the body and the fluid. The motion of the fluid is constrained by the Kutta condition as shown below. The tangential component of the fluid in the body frame is constrained to zero at the trailing edge \( \vec{r}_0 \), written as

\[ 0 = \vec{V}(\vec{r}_0, t) \cdot \vec{t}(\vec{r}_0) = \sum_{n=1}^{N} \vec{V}_n(\vec{r}_0) q_n(t) \cdot \vec{t}(\vec{r}_0) \]  

(3-53)

In which \( \vec{t}(\vec{r}_0) \) is a vector in the tangential direction at the trailing edge. Thus, the Kutta condition can be written as the constraint.

\[ \vec{q}_D = \begin{bpmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bpmatrix} = \begin{bpmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ k_1 & k_2 & k_3 \end{bpmatrix} \begin{bpmatrix} q_1 \\ q_2 \\ q_3 \end{bpmatrix} = K \vec{q} \]  

(3-54)

In which
\[ k_n = -\frac{\vec{V}_n(\vec{r}_0) \cdot \vec{\dot{r}}(\vec{r}_0)}{\vec{V}_n(\vec{r}_0) \cdot \vec{\dot{r}}(\vec{r}_0)} \]

The equations of motion, in terms of the independent degrees of freedom, are

\[ \sum_{m=1}^{3} B_{lm} \dot{q}_m = -\sum_{m=1}^{3} \sum_{n=1}^{3} C_{lmn} q_m q_n + f_c \quad l = 1,2,3 \quad (3-55) \]

In which \( B = MK \) and \( C_l = K^T D_l K \).
Chapter 4

Fluid Compensation Control Strategy

Clearly, the states of a vehicle and the surrounding fluid are coupled. In the absence of direct measurements of fluid states, the system is unobservable; the fluid states cannot be deduced from the vehicle’s states. Reduced-order models, such as the CFV model developed in the previous Chapter, is a way to account for the motion of the fluid. This could lead one to the conclusion that the fluid states are observable. Whether the CFV model employs $N=3$ admissible functions or more, it would be incorrect to conclude that the system is observable on the basis of an observable model. In practice, the fluid contains a large number of degrees of freedom. To date, no model has been developed that captures all of the possible fluid-vehicle interactions. Furthermore, it is to be seen whether “safe” models can be constructed for the design of closed-loop observers because the performance of closed-loop observers, whether they capture the fluid-vehicle interactions, are sensitive to parameter uncertainty. Indeed, CFV models of vehicles that account for the independent degrees of freedom of the fluid, in the absence of direct measurements of fluid states, possess a high level of parameter uncertainty.
An alternative approach, which will be examined in the next Chapter, is to synthesize the fluidic loads from direct fluid measurements and utilize them in as part of a control strategy. This Chapter develops the FCC strategy for vehicle control. The strategy will then be applied by example to a simple one degree of freedom system and the SDV model maneuvering in the x-y plane from the previous Chapter.

4.1 General Fluid Compensation Control Strategy

The Fluid Compensation Control (FFC) strategy is broken into three parts. The first part is called the tracker. The tracker prescribes open-loop forces and moments (like the computed torque method) that theoretically causes the vehicle to follow the desired path in the absence of fluid forces and other disturbances that would otherwise cause the vehicle to go off-course. This part of the algorithm can be implemented autonomously (automatically) or through human commands (manually). The second part is called the regulator. The regulator prescribes feedback forces and moments that oppose the error between the vehicle’s actual path and its desired path. The third part of the FCC strategy is called the fluid compensator. The fluid compensator prescribes a force and moment that is aimed at canceling all or part of the fluidic load acting on the vehicle that interfere with the regulation. The second and third parts of the FCC strategy are implemented automatically. Figure 4-1 shows the participation of the parts in the FCC strategy.

The particular control algorithm that one uses depends on the actuators available and their type, that is, the realization. For demonstration purpose, and because the focus of this
development has been to underwater vehicles, the examples below assumed that the actuation is being realized by a gimbaled propeller or vectored thrust. Active control surfaces are not being used. Nevertheless, it should be understood that the general methodology applies to other actuator realizations. The general form of the control algorithm is

$$\tilde{F}_C = \tilde{F}_T + \tilde{F}_R - (1 - \beta)\tilde{F}_{FC}$$  \hspace{1cm} (4-1)

In which $T$ refers to the tracker, $R$ refers to the regulator, and $FC$ refers to the fluid compensator. The parameter $\beta$ is a relaxation gain that dictates the level of relaxation of the compensation from no relaxation when $\beta = 0$ to full relaxation when $\beta = 1$. The compensation may be unnecessary in slow maneuvers and more necessary in rapid maneuvers.
4.2 FCC Example: Simple 1-D System
The 3 mass-damper model used to initially investigate the FCC strategy applied to a simple system with vehicle and fluid states is shown in Fig. 4-2. As mentioned in the introduction, in aircraft, the behavior of the surrounding fluid is generally not a problem because the time constant is small and the mass of the fluid moving around the vehicle is small. However, as the mass of the surrounding fluid increases with respect to the vehicle mass, the behavior of the surrounding fluid becomes ever more important. This will be shown in the analysis that follows. This model differs from the standard added-mass model because it models the dynamics of the surrounding fluid by adding the extra states due to the surrounding fluid to the system equations of motion. In the development that follows the fluid is considered to be water, however, this is not strictly necessary. The normalized equations of motion for this system will now be derived.

Figure 4 - 2: 3 Mass-Damper Model

The free-body diagram for the vehicle mass and the surrounding water masses is shown in Fig. 4-3. These free-body diagrams yield the 3 equations of motion
\[ m\ddot{x} = c(\dot{x}_2 - \dot{x}) - c(\dot{x} - \dot{x}_1) + F \]  
\[ m_w \ddot{x}_1 = c(\dot{x} - \dot{x}_1) - c\dot{x}_1 \]  
\[ m_w \ddot{x}_2 = c(\dot{x} - \dot{x}_2) - c\dot{x}_2 \]

In the above system equations, Eq. (4-2) represents the vehicle dynamics and Eqs. (4-3) and (4-4) represent the dynamics of the surrounding water. Note that the vehicle equation is coupled with the surrounding water equations.

To normalize the above system equations, they are first divided by \(m\) and then the substitution \(\frac{c}{m} = \gamma\) is made to yield

\[ \ddot{x} = \gamma(\dot{x}_2 - \dot{x}) - \gamma(\dot{x} - \dot{x}_1) + \frac{F}{m} \]  
\[ \left(\frac{m_w}{m}\right) \ddot{x}_1 = \gamma(\dot{x} - \dot{x}_1) - \gamma\dot{x}_1 \]  
\[ \left(\frac{m_w}{m}\right) \ddot{x}_2 = \gamma(\dot{x} - \dot{x}_2) - \gamma\dot{x}_2 \]

Defining the normalized time to be \(\tilde{t} = \gamma t\), where the time has been nondimensionalized by multiplying by the systems natural damping in \(\frac{\text{rad}}{s}\), gives the first and second derivatives with respect to normalized time to be
\[
\frac{d}{dt} (\cdot) = \frac{d}{d(\frac{t}{\gamma})} (\cdot) = \gamma \frac{d}{d\tilde{t}} (\cdot) = \gamma \frac{\ddot{\cdot}}{} \\
\frac{d^2}{d^2 t} (\cdot) = \frac{d^2}{d^2(\frac{t}{\gamma})} (\cdot) = \gamma^2 \frac{d^2}{d^2\tilde{t}} (\cdot) = \gamma^2 \frac{\dddot{\cdot}}{}
\]

Where \(\frac{\cdot}{}}\) and \(\frac{\cdot}{}}\) denote the derivatives of the variables in \((\cdot)\) with respect to normalized time, \(\tilde{t}\).

Converting the derivatives with respect to dimensional time, \(t\), to derivatives with respect to non-dimensional time, \(\tilde{t}\), and then dividing through by \(\gamma^2\) in Eq. (4-5) through (4-7) gives

\[
\ddot{x} = \dot{x}_2 - 2\dot{x} + \dot{x}_1 + \frac{F}{m\gamma^2} \\
\left(m_w\right) \ddot{x}_1 = \ddot{x} - 2\ddot{x}_1 \\
\left(m_w\right) \ddot{x}_2 = \ddot{x} - 2\ddot{x}_2
\]

These are three coupled 2\textsuperscript{nd} order non-dimensional ordinary differential equations with respect to non-dimensional time. To obtain non-dimensionality, they have been normalized with respect to the systems damping rate, \(\gamma\). To make them more readable, they can be put into state-space form to yield

\[
\begin{bmatrix}
\frac{m_w}{m} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \frac{m_w}{m}
\end{bmatrix}
\begin{bmatrix}
\frac{\ddot{x}_1}{x_1} \\
\frac{\ddot{x}}{x} \\
\frac{\ddot{x}_2}{x_2}
\end{bmatrix}
+ \begin{bmatrix}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
\frac{\dot{x}_1}{x_1} \\
\frac{\dot{x}}{x} \\
\frac{\dot{x}_2}{x_2}
\end{bmatrix}
= \begin{bmatrix}
\frac{F}{m\gamma^2} \\
0
\end{bmatrix}
\]

In this example, to control the system through the vehicle force, \(F\), two different techniques will be compared: with full fluid compensation (\(\beta = 0\)) and without fluid compensation (\(\beta = 1\)).
Control without Compensation

The control force without compensation is

\[ F = -g(x - x_d) - h(x - \dot{x}_d) + m\ddot{x}_d + 2c\dot{x}_d \]  \hspace{1cm} (4-14)

Converting the control force to derivatives with respect to non-dimensional time and non-dimensionalizing the force by dividing it by \( m\gamma^2 \) gives

\[ \frac{F}{m\gamma^2} = -\bar{g}(x - x_d) - \bar{h}(\dot{x} - \dot{x}_d) + \ddot{x}_d + 2\dot{x}_d \]  \hspace{1cm} (4-15)

Where \( \bar{g} \) and \( \bar{h} \) are the normalized position and velocity feedback gains and are given by

\[ \bar{g} = \frac{g}{m\gamma^2} \quad \text{and} \quad \bar{h} = \frac{h}{m\gamma} \]  \hspace{1cm} (4-16)

Substituting this normalized force into the normalized system equations, Eq. (4-13), and redefining the states to be \( x - x_d \), \( x_1 \), and \( x_2 \) the uncompensated closed-loop system equations in state-space form are

\[
\begin{bmatrix}
\frac{m_w}{m} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \frac{m_w}{m}
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x} - \ddot{x}_d \\
\ddot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
2 & -1 & 0 \\
-1 & (2 + \bar{h}) & -1 \\
0 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x} - \dot{x}_d \\
\dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & \bar{h} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x - x_d \\
x_2
\end{bmatrix} = 0
\]  \hspace{1cm} (4-17)

To investigate the system behavior for variations in the effective of the water mass \( \frac{m_w}{m} \), the desired tracking force is set equal to zero – the goal is to maintain position. The states become \( x, x_1, \) and \( x_2 \) and the system with only regulation becomes

\[
\begin{bmatrix}
\frac{m_w}{m} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \frac{m_w}{m}
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x} \\
\ddot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
2 & -1 & 0 \\
-1 & (2 + \bar{h}) & -1 \\
0 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x} \\
\dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & \bar{h} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x \\
x_2
\end{bmatrix} = 0
\]  \hspace{1cm} (4-18)

This system is shown graphically in Fig. 4-4.
To examine the effect of the mass ratio on the system, Matlab™ was used to calculate the maximum, minimum, and average damping rates from the characteristic equation for $\bar{g} = 2\pi$, $\bar{h} = 0 \rightarrow 6$, and mass ratios $\frac{m}{m_w} = 0.125 \rightarrow 1.0$ in increments of 0.125. The results for the maximum and average damping rates are shown in Figs. 4-5 and 4-6, respectively. The minimum damping of the system was always zero. This is due to a zero root that is associated with the surrounding water that is unaffected by the control gains, no matter what they are.
Figure 4-5 shows that the maximum damping rate increases as we increase the velocity gain, \(\bar{h}\), for all mass ratios which is expected. Also seen is that as the surrounding fluid gets relatively heavier \(m/m_w \downarrow\), the maximum damping rate for a prescribed gain goes down. In other words, a relatively heavier surrounding fluid requires a higher velocity gain and, thus, more power consumption, to achieve a desired decay envelope. Finally, as the velocity gain becomes larger \(\bar{h} > 7\) the effect of added mass becomes less. This makes physical sense because so much damping is being added that it has made the inertial effects small.

**Control with Compensation**

The control force with full fluid compensation is

\[
F = -g(x - x_d) - h(\dot{x} - \dot{x}_d) - \frac{c\ddot{x}_1 - c\ddot{x}_2}{2c\ddot{x}_d} + m\ddot{x}_d + 2c\ddot{x}_d \tag{4-19}
\]

Where the compensation forces \(c\ddot{x}_1\) and \(c\ddot{x}_2\), can be measure using a synthesis of the fluidic forces and moments as discussed in the next Chapter. Converting the control force to
derivatives with respect to non-dimensional time and non-dimensionalizing the force by dividing it by \( m\gamma^2 \) gives

\[
\frac{F}{m\gamma^2} = -\bar{g}(x - x_d) - \bar{h}(\dot{x} - \dot{x}_d) - \ddot{x}_1 - \ddot{x}_2 + 2\ddot{x}_d
\]

Where \( \bar{g} \) and \( \bar{h} \) are the normalized position and velocity feedback gains and are given by Eq. (4-16). Substituting this normalized force into the normalized system equations, Eq. (4-13), and redefining the states to be \((x - x_d), x_1, \) and \( x_2 \) the compensated closed-loop system equations in state-space form are

\[
\begin{bmatrix}
mw/m & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & mw/m
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
2 & -1 & 0 \\
0 & (2 + \bar{h}) & 0 \\
0 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_d
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & \bar{h} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= 0
\]

(4-21)

Here the external forces of the water have been compensated for by adding a sensor system to measure the forces transmitted to the vehicle. This measurement is used to subtract out the uncontrollable states of the water from the vehicle equation. Thus, effectively reducing the problem to a single differential equation with one unknown and eliminating the added complication of the water from the problem. This new simplified system is shown in Fig. 4-7.
To obtain a solution to the closed-loop, normalized, equations of motion, where the effect of the states due to the surrounding fluid have been removed by applying a compensator, the vehicle equation of motion is written as

\[ \ddot{x} + (2 + \bar{h})\dot{x} + \bar{g}x = 0 \quad (4-22) \]

Notice that the dependence on the water states, \( x_1 \) and \( x_2 \), no longer exists, as shown by the system model in Fig. 4-7. Separating the damping and the stiffening components in a typical 2\(^{nd}\) order differential equation gives

\[ \ddot{x} + 2\alpha \dot{x} + (\alpha^2 + \beta^2)x = 0 \quad (4-23) \]

Comparing Eqs. (4-22) and (4-23) gives

\[ \bar{h} = 2(\alpha - 1) \quad \text{and} \quad \bar{g} = \alpha^2 + \beta^2 \quad (4-24) \]

Here \( \bar{h} \) can be found exactly from the combination of the above and

\[ \alpha = \frac{\ln(10)}{T_s} \quad (4-25) \]

Where \( T_s \) is the desired settling time to attenuate 90\% of the disturbance. By combining \( \alpha \) and a desired closed-loop frequency of oscillation, \( \beta, \bar{g} \) can be specified.

It should be noted that the form of Eq. (4-23) was chosen because the quadratic equation gives

\[ \lambda_{1,2} = \frac{-2\alpha \pm \sqrt{(2\alpha)^2 - 4(\alpha^2 + \beta^2)}}{2} \quad (4-26) \]

Which has roots

\[ \lambda_{1,2} = -\alpha \pm i\beta \quad (4-27) \]
Thus, the settling time and peak overshoot for the closed-loop system response can be specified independently. The compensator has effectively eliminated the fluid from the control system model which has allowed the response of the vehicle to specified exactly. As will be seen in the second example of this Chapter, it is not always beneficial to remove the water completely from the system. For example, if it is helping the vehicle through a maneuver, it would be wise for the controller to use the fluidic force or moment. Hence the motivation for the relaxation parameter, $\beta$, in the FCC formulation.

It should also be noted that the system naturally has no frequency of oscillation. To keep the system from being over-damped, suffering from a slow rise time, all the stiffness must be artificially added by the designer in the form of $\beta$.

4.3 FCC Example: SDV Maneuvering in the $x$-$y$ Plane

Substituting Eq. (4-1) into the CFV equations of motion developed for the SDV maneuvering in a plane (Eq. (3-51) gives

$$M_0 \ddot{\hat{v}} = \tilde{F}_F + \tilde{F}_T + \tilde{F}_R - (1 - \beta)\tilde{F}_{FC} \quad (4-28)$$

This section first discusses the design of the tracking, the regulator, and the compensator for a general vehicle maneuvering in a plane. Second, some general stability and robustness characteristics of the FCC strategy applied to a general vehicle maneuvering in a plane are discussed. Finally, the FCC strategy is applied to two specific maneuvers using the SDV CFV model developed in the previous Chapter.
Tracker Design
The design of the tracker, whether autonomous or governed by human commands, begins with the selection of a desired linear velocity \( \dot{v}_D \) and angular velocity \( \dot{\omega}_D \). Substituting \( \dot{v}_D \) and \( \dot{\omega}_D \) and their time derivatives into Eq. (4-28) and letting \( \ddot{F}_R = 0 \), \( \beta = 0 \), and \( \ddot{F}_F = \ddot{F}_{FC} \) yields the tracking force

\[
\ddot{F}_T = M_0 \dot{v}_D
\]  
(4-29)

Regulator Design
The design of the regulator begins by considering Eq. (4-28) letting \( \beta = 0 \) and \( \ddot{F}_F = \ddot{F}_{FC} \). Also, let \( \ddot{E} = \dot{v}_C - \dot{v}_D \) represent the velocity error vector and substitute Eq. (4-29) into Eq. (4-28) to get

\[
M_0 \ddot{E} = \ddot{F}_R
\]  
(4-30)

The purpose of the regulator is to reduce the velocity error vector \( \ddot{E} \) to zero. Among the family of linear regulators, the interest lies in controlling settling time. It follows that the different components of the velocity error vector need to be damped uniformly, which requires a regulator of the form

\[
\ddot{F}_R = -2\alpha M_0 \ddot{E}
\]  
(4-31)

In which \( \alpha \) is the uniform damping rate of the vehicle’s motion. Equation (4-31) can be realized using one or more gimbaled propellers or thrust vectoring nozzles. An alternative is to employ a propeller or thrust vector system that applies a constant level of thrust, which requires a regulator of the form

\[
\ddot{F}_R = -F_R \ddot{e}_V
\]  
(4-32)
In which $F_R$ denotes the magnitude of the thrust vector and $\vec{e}_V = \frac{\vec{E}}{|\vec{E}|}$ denotes the unit vector in the direction of $\vec{E}$. This regulator applies a constant level of thrust that varies in direction. Again, the particular form of the regulator depends on the control realization.

**Fluid Compensator Design**

The fluid compensator requires an estimate of the fluidic load obtained by synthesizing $\vec{F}_F$. Neglecting synthesis errors, the estimate is used to produce the fluid compensator $(1 - \beta)\vec{F}_F$. The relaxation gain $\beta$ prevents the control load from being higher than is realizable or degrades the control load when the fluid force or moment vector is in the desired direction to assist the maneuver. The fluidic residue $\beta\vec{F}_F$ acts as a disturbance on the vehicle. As the residue increases in size, the possibility increases that the vehicle can become unstable. Therefore, the interest generally lies in minimizing the residue. The availability of the synthesized $\vec{F}_F$ makes it possible to adapt the tracker to reduce the residue. For example, the desired path of the vehicle can be changed to head directly into or away from the flow.

**Stability and Robustness**

The real-time estimate of the fluidic load makes it possible to guarantee the vehicle’s stability. Substituting Eqs. (4-29) and (4-32) into (4-28) gives

$$M_0 \dot{\vec{v}} = \vec{F}_F + M_0 \dot{\vec{v}}_D - F_R \vec{e}_V - (1 - \beta)\vec{F}_F$$

Equation (4-33) can be rearranged to yield

$$M_0 \dot{\vec{E}} = \beta\vec{F}_F - F_R \vec{e}_V$$

Premultiplying Eq. (4-34) by $\vec{E}^T$ gives
\[
\frac{dT}{dt} = \frac{d}{dt}\left(\frac{1}{2} \tilde{E}^T M_0 \tilde{E}\right) = \tilde{E}^T M_0 \dot{\tilde{E}} = \beta \tilde{E}^T \tilde{F}_F - F_R \tilde{E}^T \tilde{e}_V \quad (4-35)
\]

Where \( T \) is the pseudo-kinetic energy and the right side of the equation if the pseudo-power.

Since \( M_0 \) is a positive definite matrix, the velocity error vector approaches zero when the pseudo-kinetic energy approaches zero. It follows from Eq. (4-35) that the velocity error vector \( \tilde{E} \) approaches zero when

\[
\frac{dT}{dt} = \beta \tilde{E}^T \tilde{F}_F - F_R \tilde{E}^T \tilde{e}_V < 0 \quad (4-36)
\]

Dividing by \( F_F E \) yields

\[
\beta \cos(\delta) - f_R < 0 \quad (4-37)
\]

In which \( \cos(\delta) = \tilde{e}_F \cdot \tilde{e}_V, \tilde{e}_F = \frac{\tilde{F}_F}{F_F} \) and \( f_R = \frac{F_R}{F_F} \). The angle between the fluidic load vector \( \tilde{F}_F \) and the velocity error vector \( \tilde{E} \), denoted by \( \delta \), is called the control angle. The vehicle’s stability is guaranteed when Eq. (4-37) is satisfied. Equation (4-37) is a basic condition that needs to be satisfied for any control algorithm that is used with the realization described by Eq. (4-32). It also governs the trade-off that can be made between regulation and compensation. First, notice that the condition is independent of the vehicle’s inertia properties so stability is being guaranteed independent of the vehicle’s inertia properties.

When \( \beta = 0 \) the fluid compensator cancels the fluidic force, in which case stability is guaranteed with theoretically no regulation \( (f_R = 0) \). When \( \beta > 0 \) the fluid compensator cancels only a portion of the fluidic load, in which case stability is guaranteed when \( f_R \) is sufficiently large. As was noted previously, however, it is not necessarily desirable to cancel the fluidic load as will be shown below. Equation (4-37) shows that the need for fluid compensation depends on the control angle.
From Eq. (4-37), the fluidic load helps reduce the vehicle’s pseudo-kinetic energy when $\cos(\delta) < 0$. Therefore, when $\delta > 90^\circ$ or $\delta < -90^\circ$ the relaxation gain can be set to 1. This can be seen in a pictorial manner in Fig. 4-8. Next, let $-90^\circ < \delta < 90^\circ$ (i.e. $\cos(\delta) > 0$). It can be desirable to hold the magnitude of the thrust level constant through a maneuver.

Denoting the magnitude of $\vec{F}_C$ by $F_C$ and letting $\dot{\vec{v}}_D = 0$, it follows from Eq. (4-33) that

$$\left(\frac{F_C}{F_F}\right)^2 = [-f_R \hat{e}_V - (1 - \beta) \hat{e}_F][ - f_R \hat{e}_V - (1 - \beta) \hat{e}_F]$$

(4-38)

Thus, $f_R$ satisfies the quadratic equation $0 = f_R^2 + 2(1 - \beta) \cos(\delta) f_R + [(1 - \beta)^2 - f_C^2]$ in which $f_C = \frac{F_C}{F_F}$. Its solution is

$$f_R = -(1 - \beta) \cos(\delta) + \sqrt{\Delta}$$

(4-39)

In which $\Delta = f_C^2 - (1 - \beta)^2 \sin^2(\delta)$. We find that $\beta = 1$ in all of these cases.

Figure 4–8: Vehicle Stability Vectors
**SDV CFV Simulation with FCC Strategy**

In this subsection, we apply the FCC strategy to the SDV model developed in the previous Chapter using the CFV modeling approach. Two maneuvers are considered: a turn in calm water and a straight run in the presence of a lateral water disturbance. The turn is a $180^\circ$ turn with a non-dimensional turning radius of $r' = \frac{\text{length of vehicle}}{\text{radius of turn}} = 0.6$. The straight translation is in a 6m wide canyon in an oscillatory wave motion. In each situation either yaw and forward velocity regulation or full compensation feedback was used. The regulation feedback gains were 50 for yaw and forward velocity and were not optimally found. However, they are sufficient to illustrate a general comparison.

The tracking force was generated by specifying the desired x-position and the yaw angle with 5th order polynomials in time and applying the boundary conditions: $x(0) = \dot{x}(0) = \ddot{x}(T) = \psi(0) = \dot{\psi}(0) = \ddot{\psi}(T) = 0$, $\dot{x}(0) = \ddot{x}(T) = v$, $\psi(T) = 180^\circ$, and $x(T) = x_f$ for the turning maneuver and $x(0) = \dot{x}(0) = \ddot{x}(T) = \psi(0) = \dot{\psi}(0) = \ddot{\psi}(0) = \psi(T) = 0$, $\dot{x}(0) = \ddot{x}(T) = v$, and $x(T) = x_f$ for the straight run in a canyon. The resulting tracking position and orientation are

$$x(t) = x(T) \left[ 10 \left( \frac{t}{T} \right)^3 - 15 \left( \frac{t}{T} \right)^4 + 6 \left( \frac{t}{T} \right)^5 \right]$$  \hspace{1cm} (4-40)

$$\psi(t) = \psi(T) \left[ 10 \left( \frac{t}{T} \right)^3 - 15 \left( \frac{t}{T} \right)^4 + 6 \left( \frac{t}{T} \right)^5 \right]$$  \hspace{1cm} (4-41)

Where the maneuver time is $T$. The tracking forces and moments are obtained by differentiating twice with respect to time to yield the desired accelerations and multiplying by the vehicle mass and moment of inertia. Figures 4-9(a) and (b) show the tracking force.
and moment for the turning maneuver and the straight run in a canyon, respectively. It should be mentioned that this is not the only way to generate tracking forces. The best tracking forces and moments are a matter of optimization. However, the focus of this example is to illustrate the usefulness of the FCC strategy and the above tracking force will suit this purpose.

Figures 4-10 through 4-12 show the control action and the vehicle path for the 180° turn. Figure 4-10 considers control in which there is no regulation or fluid compensation, only tracking. Notice that the tracking force is zero but the tracking moment is non-zero. There is no disturbance acting on the vehicle during the maneuver. The solid line is the desired path and the dashed line is the actual path. It is seen that the desired vehicle does not follow the desired path. Figure 4-11 considers the case in which there is no fluid compensation, just regulation and tracking. The regulation force reacts to correct for the tracking error. The regulation moment corrects for errors in turning. Notice that a steady-state error builds up because the regulation uses only velocity feedback. The steady-state error consists of a steady motion that is offset by a constant amount. Figure 4-12 considers control in which there is fluid compensation and tracking, but no regulation. In the simulation there is no need for regulation because the fluid compensation is full ($\beta = 0$); it perfectly cancels the fluidic force and moment. The vehicle follows the path perfectly.

Figures 4-13 through 4-15 show the control action and vehicle path for the straight run. Notice that the disturbance is acting while the vehicle is still accelerating. Figure 4-13
considers control in which there is no regulation or fluid compensation, just tracking. The solid line is the desired path and the dashed line is the actual path. The double arrow reminds the reader that the vehicle is acted on by an up-and-down wave. Shortly after the maneuver begins at about 1.5m the vehicle goes unstable and enters a spin. Figure 4-12 considers control consisting of tracking and regulation and no fluid compensation. The regulation force reacts to the drag that acts on the vehicle; its shape corresponds to the vehicle’s acceleration profile. Notice that the regulation moment follows the moment disturbance due to the wave. Since the regulation only feeds back velocities, a steady-state error arises. The steady-state error consists of a steady motion of the vehicle that is offset by a constant amount. Figure 4-15 considers control consisting of tracking and fluid compensation, but no regulation. In the simulation there is no need for regulation because the fluid compensation is full ($\beta = 0$); it perfectly cancels out the fluidic force and moment. The vehicle follows the desired path perfectly.
Figure 4 - 9: Tracking Forces and Moments for (a) 180° Turn and (b) Straight Run
Figure 4 - 180° Turning Maneuver with Tracking
Figure 4 - 11: 180° Turning Maneuver with Regulation
Figure 4 - 12: 180° Turning Maneuver with Fluid Compensation
Figure 4 - 13: Straight Run through a Canyon with Tracking
Figure 4 - 14: Straight Run through a Canyon with Regulation
Figure 4-15: Straight Run through a Canyon with Fluid Compensation
Chapter 5

Synthesis of Forces and Moments

The fluid dynamic load is produced by distributions of pressure and shear stress over the vehicle surface. Pressures are commonly measured using pressure taps and shear stresses are less frequently measured, but there are shear stress sensors for this purpose (50) (49). For the purposes of control, the question arises how to practically synthesize the fluid dynamic loads from sensor measurements. The fluid dynamic load is related to the distributions of pressure and shear stress by Eqs. (3-25) and (3-26) or by Eqs. (3-49) and (3-50) when shear stresses are being neglected.

This Chapter examines the synthesis of fluid dynamic loads, from measurements on the vehicle surface. Two different concepts will be discussed: (1) the use of surface pressure measurements and (2) the use of stagnation and separation points/lines. The later was investigated using both theoretical and experimental development and both approaches will be discussed.
5.1 Synthesis from Pressure Measurements

For the purposes of this discussion assume that the shear effects are being neglected.

Equations (3-49) and (3-50) are essentially spatial filters. The fluid dynamic load can be synthesized by performing a quadrature, according to which Eqs. (3-49) and (3-50) are approximated by

\[
\tilde{f}_F = \sum_{s=1}^{M} \tilde{a}_s p_s
\]

(5-1)

\[
\tilde{m}_F = \sum_{s=1}^{M} \tilde{b}_s p_s
\]

(5-2)

In which \(\tilde{a}_s\) and \(\tilde{b}_s\) represent shape functions, \(p_s\) is the relative pressure located at the \(s^{th}\) node, and \(M\) is the number of nodes. The accuracy of the synthesized fluid dynamic load is higher than the accuracy of the pressure measurements. As a rough estimate, a standard deviation \(\sigma_p\) of pressure measurements produces a standard deviation \(\sigma_F = \frac{\sigma_p}{\sqrt{M}}\) of the fluid dynamic load. Furthermore, the components of the pressure distribution that are orthogonal to the resultant load, e.g. small zero-mean wavelength variations in pressure, do not produce an appreciable resultant load and hence do not appreciably degrade the synthesis. Moreover, for the purposes of autonomous control, the accuracy of the synthesized load does not need to be very high provided a regulator is used in conjunction with the fluid compensator. It follows that real-time synthesis of fluid dynamic loads, although not being performed in existing vehicles, is viable.
5.2 Synthesis from Fluid Stagnation and Separation: Theoretical

As discussed in the introduction, modeling full unsteady flow around a vehicle is fundamentally different from modeling steady flow around a vehicle. This same situation exists for characterizing full unsteady flow from experimental testing or real-time measurements used in vehicle control. Steady flow is characterized through experimentally obtained fluid dynamic derivatives. Using these derivatives, fluid dynamic loads are expressed in terms of vehicle states, for example, a freestream velocity $U_\infty$, an angle $\alpha$, an angular rate $\dot{\alpha}$, and an angular acceleration $\ddot{\alpha}$. The corresponding full unsteady problem, because of the independent degrees of freedom of the fluid, cannot employ fluid dynamic derivatives. Even worse, initial conditions are required at every point in the flow to predict how a full unsteady flow evolves. In physical practice, this is beyond the state of the art and will likely be beyond the state of the art for years to come. Therefore, the question arises whether there is another way to characterize full unsteady flow and the corresponding fluid dynamic loads.

As mentioned in the introduction, it is well known that steady and unsteady fluid dynamic loads on airfoils can be predicted from the locations of stagnation, separation, and reattachment points (43) (51). Indeed, as was seen in Chapter 3 and is commonly used in airfoil design, the Kutta condition, which prescribes where the flow separates on a body, helps predict the fluid dynamic load by constraining a fluid degree of freedom to rigid body degree of freedom. These two applications of using surface characteristics to predict the influence of the fluid on the body, seem to suggest that the mathematically nature of the
flow’s velocity distribution around the body can predict the fluid dynamic loads. That is, the velocity distribution around a body is a smoothly varying function that is determined to a multiplicative constant by its nodes (stagnation, separation, and reattachment points or lines). This property, referred to here as the Kutta principle, would hold when the flow varies continuously, whether steady or unsteady. This section develops a mathematical argument in favor of the Kutta principle and suggests its use for characterizing fluid dynamic loads in unsteady flow, whether for experimental testing or for vehicle control.

This section begins with the development of a nodal theorem for estimating fluid dynamic loads. The nodal theorem provides a set of idealized conditions under which the fluid dynamic load is uniquely determined by the nodal points. In practice, these conditions are not met exactly, but can be approximated. The remainder of the section develops approximation methods. Note that the extrapolation of critical information from the nodes of a distribution, although not practiced in the fluid dynamics community, has been successfully exploited in other communities (e.g., (39)).

**Velocity Distribution over a Body**
The nodes of the velocity distribution over a body surface, \( v(P) \), are points or lines depending on the formulation or the physics. In planar problems in which only cross sections of a flow are being considered and the fluid dynamic loads are being examined on a per-length basis, the nodes are points. The nodes are also points in some three-dimensional
attached flows, such as the case of attached flow around a sphere. Otherwise, the nodes are lines and the curvature of the line must be considered.

**Nodal Points**
The locations \( P_1, P_2, \ldots \) on the body for which \( v(P) = 0 \) are called nodal points. The following point theorem for the Kutta principle uniquely determines the flow from the nodal points.

*Theorem 1*: Let the speed \( v \) on the surface of the body be a linear combination of the basis flows \( \{v_n(P)\}_{n=1}^N \). Also, take a reference measurement of speed \( v_0 = v(P_0) \), where \( v_0 \neq 0 \). The nodal points are \( P_1, P_2, \ldots, P_{N-1} \). The flow is uniquely determined by \( v_0 \) and \( P_1, P_2, \ldots, P_{N-1} \).

*Proof*: Write \( v \) as

\[
v(P, t) = \sum_{n=1}^{N} v_n(P) q_n(t)
\]

(5-3)

Where \( q_1(t), q_2(t), \ldots, q_n(t) \) are coefficients. Evaluating Eq. (5-3) at \( P_0, P_1, P_2, \ldots, P_{N-1} \) gives

\[
v_0 = \sum_{n=1}^{N} v_n(P_0) q_n(t), \quad 0 = \sum_{n=1}^{N} v_n(P_1) q_n(t), \quad \ldots \quad 0 = \sum_{n=1}^{N} v_n(P_{N-1}) q_n(t)
\]

(5-4)

Written in matrix-vector form

\[
v_0 1 = Aq
\]

(5-5)

where
\[ \mathbf{1} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{A} = \begin{bmatrix} v_1(P_0) & v_2(P_0) & \cdots & v_N(P_0) \\ v_1(P_1) & v_2(P_1) & \cdots & v_N(P_1) \\ \vdots & \vdots & \ddots & \vdots \\ v_1(P_{N-1}) & v_2(P_{N-1}) & \cdots & v_N(P_{N-1}) \end{bmatrix} \]

Assuming that \( \mathbf{A} \) is full rank,

\[ \mathbf{q} = \mathbf{A}^{-1} \mathbf{1} v_0 \quad (5-6) \]

The coefficients are uniquely determined by the locations of the nodal points up to the multiplicative constant \( v_0 \). It follows that \( v(P) \) is uniquely determined by the nodal points up to a multiplicative constant.

Experimental tests could be used to characterize flow around a body on the basis of Theorem 1. Tests would be run for different sets of nodal points and, in a given run, one reference speed \( v_0 \) would be measured. In other flows that have the same nodal points, the fluid dynamic loads scale as a function of \( v_0 \). In steady flow, fluid dynamic loads are proportional to \( v_0^2 \), in which case they scale by \( v_0^2 \) (52).

**Nodal Point Approximations**

Theorem 1 is an idealization, because a flow is not an exact linear combination of basis flows. Attached steady flow around a cylinder (54) is considered next. This example is illustrative of steady or unsteady flow. The speed is

\[ v = \sqrt{v_x^2 + v_y^2} \quad (5-7) \]

where

\[ v_x = 1 + \frac{R^2}{r^2} - 2\frac{R^2 x^2}{r^4} - k \frac{R y}{r^2} \]
\[ v_y = -\frac{2R^2xy}{r^4} + \frac{Rx}{r^2} \]

And \( k \) prescribes circulation. On the surface,

\[ v_x = \frac{1}{r^2} [2R^2 - kyr - 2x^2] \quad \text{and} \quad v_y = \frac{1}{r^2} x(kR - 2y) \quad (5-8) \]

In terms of \( \theta \), \( v_x = \sin \theta [2 \sin \theta - k] \), and \( v_y = -\cos \theta (2 \sin \theta - k) \). The tangential component of the velocity is

\[ v_t = v_x \sin \theta - v_y \cos \theta = 2 \sin \theta - k \]

And the speed is \( v = |2 \sin \theta - k| \). Theorem 1 can be applied to the tangential components of velocity (replacing \( v \) everywhere with \( v_t \)). The functions \( \{1, \sin \theta\} \) form a basis from which \( v_t \) is uniquely and exactly determined by the nodal points \( P_1 \) and \( P_2 \).

In unattached flow, additional nodal points appear. The vortices around the cylinder alternate in direction, and so the tangential component of velocity on the surface is an oscillating function of \( \theta \). Assume \( N-1 \) nodal points and consider the basis \( \{1, \theta, \theta^2, ..., \theta^{N-1}\} \) of \( N-1 \)th order polynomials. The tangential component of velocity is written in factored form as

\[ v_t = A \prod_{r=1}^{N-1} (\theta - \theta_r) \]

\((5-9)\)

where

\[ A = \frac{v_t(\theta)}{\prod_{r=1}^{N-1} (\theta_0 - \theta_r)} \]
where $\theta_0$ is the angle of the reference measurement. It is obvious that Theorem 1 holds in the case of the polynomial set by simply inspecting Eq. (5-9). Equation (5-9) also suggests a more general method of approximating $v$ from nodal points and nodal lines.

The factors in Eq. (5-9) can be viewed as the constraints $c_r(\theta) = \theta - \theta_r = 0 \; (r = 1, 2, ..., N - 1)$ placed on the coordinate $\theta$. In terms of $x$ and $y$, the constraints are $c_r(x, y)$ and a second approximation of the speed is expressed in the general form

$$v(x, y) = A \prod_{r=1}^{N-1} c_r(x, y)$$

(5-10)

The constraints $c_r(x, y)$ can represent straight lines that intersect the circle at the nodal points. Denote the nodal points on the circle by $(x_r, y_r)$ $(r = 1, 2, ..., N - 1)$. Then for straight-line constraints,

$$v(x, y) = A \prod_{r=1}^{N-1} (R^2 - x_r x - y_r y)$$

(5-11)

**Nodal Lines**

In three-dimensional flow, the speed is expressed, like Eq. (5-10), in the general form

$$v(x, y, z) = A \prod_{r=1}^{N-1} c_r(x, y, z)$$

(5-12)
The constraints $c_r$ are now surfaces. The surfaces intersect the body at nodal points or along the body surface on nodal lines. The speed on the body, when expressed in this form, is uniquely determined by its nodes up to a multiplicative constant, such as in the case of Theorem 1.

Consider attached steady flow around a sphere. The speed is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

(5-13)

where

$$v_x = 2 + \frac{R^3}{r^3} - \frac{3R^3x^2}{r^5} - k_x \frac{Ry}{r^2} + k_y \frac{Rz}{r^2}$$

$$v_y = -\frac{3R^3xy}{r^5} + k_z \frac{Rx}{r^2} - k_x \frac{Rx}{r^2}$$

$$v_z = -\frac{3R^3xz}{r^5} + k_x \frac{Ry}{r^2} - k_y \frac{Rx}{r^2}$$

and $k_x$, $k_y$, and $k_z$ prescribe circulation about the $x$, $y$, and $z$ axes, respectively. On the surface

$$v_x = \frac{1}{R^2} \left[3R^2 - 3x^2 + R(-k_x y + k_y z)\right]$$

$$v_y = \frac{1}{R^2} \left[-3xy + R(k_x x - k_x z)\right]$$

$$v_z = \frac{1}{R^2} \left[-3xz + R(k_x y + k_y x)\right]$$

The $\theta$ and $\phi$ components of velocity are

$$v_\theta = -v_x \sin \theta + v_y \cos \theta = -3 \sin \theta - k_x \cos \phi \cos \theta - k_y \cos \phi \sin \theta + k_z \sin \phi$$

$$v_\phi = v_x \cos \phi \cos \theta + v_y \cos \phi \sin \phi - v_z \sin \phi = 3 \cos \phi \cos \theta - k_x \sin \theta + k_y \cos \theta$$
The speed is \( v = \sqrt{v_\theta^2 + v_\phi^2} \). The constraints \( c_r(x, y, z) \) can represent flat planes that intersect the sphere at the nodal points \( x_r, y_r, \) and \( z_r \) \( (r = 1, 2, ..., N - 1) \). For the flat-plane constraints that intersect at nodal points

\[
v(x, y, z) = A \prod_{r=1}^{N-1} (R^2 - x_r x - y_r y - z_r z)
\]

(5-14)

**Illustrative Examples**

Consider first the cylinder and then the sphere. Figure 5-1 shows the flow around a cylinder for \( k = -1.75, -1.00, 0, 1.00, \) and \( 1.75 \). The flow was measured on top of the cylinder \( (\theta_0 = 90^\circ) \). The middle column compares the exact \( v_t(\theta) \) with the polynomial approximation of Eq. (5-9). For improved accuracy, an 8\textsuperscript{th}-order polynomial was used that has zeros at \( \theta_1 + j360^\circ \) and \( \theta_2 - j360^\circ \) \( (j = -1, 0, 1, \) and \( 2) \). The right column compares the exact \( v(\theta) \) with the constrained method of Eq. (5-11).

Figure 5-2 shows the flow around a sphere for \( k_z = -1.75, -1.00, 0, 1.00, \) and \( 1.75 \). The flow was measured at the top of the sphere. The right column compares the exact \( v(\theta, \phi) \) with the constrained approach of Eq. (5-14).

Referring to Fig. 5-1, the 8\textsuperscript{th}-order approximation shown in the middle column is more accurate than the constrained method shown in the right column. The errors in both approximations are larger when the nodal points in the flow are close to the measurement
point. It appears to be best to measure the flow away from the nodal points. Figure 5-2 shows similar results regarding the nodal point’s location and the measurement.
Flow Field

\( v_\theta(\theta) \) verse \( \theta \) 8th-order polynomial \( v(\theta) \) verse \( \theta \) constrained method

Figure 5 - 1: Approximating \( v(\theta) \) Over a Cylinder (solid = exact, dashed = approximate)
Streamlines $v(\theta)$ verse $\theta$ constrained method

Figure 5 - 2: Approximating $v(\theta)$ Over a Sphere (solid = exact, dashed = approximate)
5.3 Synthesis from Fluid Stagnation and Separation: Experimental

As discussed in Chapter 2, Goman and Khrabrov (43) developed a theory that correlates the unsteady forces and moments to the stagnation and separation points on a 2-dimensional airfoil and Mangalam (51) applied the theory using a hot-film/CVA system to predict the forces and moments from the stagnation and separation points on an airfoil in water. In this dissertation an ambitious attempt to predict the unsteady forces and moments on a NNemo1 style body, a bluff 3-dimensional body, from the yaw angle and the separation lines was attempted. Specifically, an unsteady test rig was designed and constructed, a 2/10\textsuperscript{th} scale NNemo1 style test model was built and tested to determine hot-film placement locations, the hot-film were mounted on the test model, and tested in the NCSU subsonic wind tunnel\textsuperscript{2}. The unsteady test rig and the test model mounted in the NCSU wind tunnel are shown in Fig. 5-3. The effort was broken into 2 phases: the design and construction phase and the testing and analysis phase. These will be discussed in turn.

\textsuperscript{2} Testing was to be conducted at the Virginia Institute of Marine Science (VIMS) flume facility, but before testing could take place a nor’ester hit the facility, destroying the flume motor. VIMS decided not to repair the motor since plans were already underway to close the facility at the end of 2009. This resulted in the testing taking place at the NCSU subsonic wind tunnel.
**Design and Construction Phase**
The unsteady test rig was an original design to mount on top of the flume or wind tunnel and conduct dynamic rotations. The rig consists of an H-frame base, a rotary stage, a control box, and an adjustable sting, which can be adjusted to the desired tunnel depth and to keep the test model center of rotation in the desired location. All components except the rotary stage were designed by the author and constructed in-house. The test rig was also designed to allow the mounting of linear rails and a linear actuator in the future to drive the rotary stage dynamically in a lateral direction, thus allowing the simulation of a turn of any diameter.

The test model was constructed from cross-sections of a NNemo1 style body using plug and mold construction techniques. This process, which was conducted completely by the author in-house, is shown in Fig. 5-4. The resulting test model was made from carbon fiber skins with 2 internal bulkheads made from carbon fiber with a Nomex® honeycomb core. The aft bulkhead, which was mounted at the desired $x_{cg}$ location, contained an aluminum hard point embedded in the honeycomb for mounting the load cell. The aluminum hard point was drilled to be able to mount the load cell borrowed from the Naval Surface Warfare Center – Carderock (NSWC-CR), which is water-proof and can handle larger loads, and the load cell available at the NCSU wind tunnel.

**Testing and Analysis Phase**
Since turns were where the instabilities were observed with NNemo1 and NNemo1 is known to experience extremely large angles of sideslip, sideslip was the focus during the testing phase. Testing in the NCSU wind tunnel applied several constraints to the testing, which
were: a maximum wind speed of 90ft/s was attainable before the fan blades began to stall and the NCSU sting was limited to 22° in angle-of-attack and 15° in sideslip. The second constraint on sideslip was alleviated by rotating the model 90° on the mount as shown above in Fig. 5-3(a).

(a) Cross-section templates mounted on Fermica

(b) Templates sandwiched between foam.

(c) Rough sanded plug.

(d) Sanding epoxy coated plug.

(e) Molds pulled from plug and skins pulled from mold.

(f) Closing skins and fitting internals

Figure 5-4: Test Model Construction
The testing phase began by tufting the test body and mounting it in the NCSU wind tunnel using the NCSU load cell and sting to investigate the flow around the test model in static conditions. This was done to determine the location and movement of separation for the placement of the hot-film sensors. Sideslip angles of 16°, 19°, and 22° were investigated by observing the tuft movement and the movement of a string probe place upstream that traced the streamlines over the model as shown in Fig. 5-4. After conducting these tests it appeared that separation was occurring well before the locations observed during the smoke tests of NNemo1 at the Langley Full-Scale Tunnel (LFST) conducted by NGNN. It was decided to install a trip strip on the nose of the test body (similar to that on the free-running NNemo1) to force the transition of the boundary layer from laminar to turbulent and delay separation until

(a) Aft running tufts
(b) String probe
(c) Starboard running tufts

Figure 5 - 5: Initial Investigation of Separation Location
a more realistic location. Theses test were then run again and it appeared that the location of separation was in a similar location to that of the LFST tests.

Three arrays of 8 hot-film sensors were then mounted at the locations were separation was believed to occur. The mounting of the hot-film sensors is shown in Fig. 5-6 and the resulting hot-film sensor nomenclature is given in Fig. 5-7. Utilizing the 8 channel CVA unit on loan from NGNN (see Fig. 5-8), the 24 hot-film sensors were annealed using an overheat voltage of $V_w = 0.5V$ for over an hour each. After annealing the sensors, each array was tested to ensure that it was responding to an increase in flow velocity by blowing over the sensor. The results of this test for each array, both unfiltered and filtered with a 2nd-order Butterworth filter with a cut-off frequency of 20Hz, are given in Figs. 5-9 through 5-11. Having confirmed that all the sensors in the 3 arrays were performing properly, a test matrix of velocities of 50, 70, and 90ft/s and sideslip angles of 16°, 19°, and 22° was run. At each test point, data from sensors arrays 1, 2, and 3 were taken at 1 kHz for 20 seconds. The data was then analyzed and no indication of separation was observed. The problem was believed to be caused by the overheat voltage of $V_w = 0.5V$. The idea was that there was not enough heat being supplied to the hot-film to replace the heat being convected away by the flow. So, the overheat voltage was increased to a value of $V_w = 1.0V$ and data from the 3 sensor arrays sampled at 1 kHz for 50 seconds at sideslips angles of 16°, 19°, and 22° at a tunnel velocity of 90ft/s were taken. Again, analysis of this data did not show any indications of separation.

In an attempt to find a location of separation on the test model several different tufted locations were investigated. These different locations, shown in Fig. 5-12 did not give
evidence of any separation. Because of the difficulty of determining separation locations with the hot-film sensors during steady conditions, use of the unsteady test rig did not seem a viable testing approach and was not conducted. Further, since the data collected from the hot-film sensors did not help identify separation, it is not given in this dissertation. As will be discussed in the conclusion, it is the author’s belief that the experiment was too ambitious and a more incremental testing regimen would yield better results.

Figure 5 - 6: Hot-Film Sensor Installation

(a) Application of MacTac® adhesive. (b) Removal of hot-film from rigid backing
(c) Smooting placed sensor onto MacTac®. (d) Routing wires.
(e) Soldering leads on hot-films.
Figure 5 - 7: Hot-Film Sensor Array Nomenclature

Figure 5 - 8: Testing Control Desk

Figure 5 - 9: Unfiltered and Filter Response to Functional Test [Array #1]
Figure 5 - 10: Unfiltered and Filtered Response to Functional Test [Array #2]

Figure 5 - 11: Unfiltered and Filtered Response to Functional Test [Array #3]

(a) Tufts along top of test model.  (b) Tufts along side of test model.

Figure 5 - 12: Second Investigation of Separation Location
Chapter 6

Conclusion

This dissertation is the first comprehensive attempt to address a new engineering problem: the control of a vehicle maneuvering in a full unsteady flow. The approach to the solution is focused in three main areas: modeling of a vehicle maneuvering in a full unsteady flow, control of a vehicle maneuvering in a full unsteady flow, and synthesizing the fluid loads for use in control of a vehicle maneuvering in a full unsteady flow. In this dissertation a chapter was dedicated to each of these areas.

As discussed, this problem was first encounter by a UUV conducting rapid maneuvers. As such most of the formulation was focused on UUV applications. But, as mentioned, it is the author’s opinion that UAVs, as their maneuverability is increased will also encounter this problem and it is the author’s hope that this dissertation will provide insight.

Chapter 3 addressed modeling of a vehicle maneuvering in a full unsteady flow. It outlined a new modeling approach called the Coupled Fluid Vehicle model (CFV). The CFV model represents the full unsteady flow around the vehicle as the sum of a finite number of spatially dependent velocity fields whose contributions vary with time. These time dependent
variables are found by coupling the sum of flow fields to the vehicle’s rigid-body equations of motion and applying the boundary conditions in the far-field. For flows in which shear stress is being neglected the time dependence can be exclusively a function of the vehicle’s rigid-body states; this is the quasi-steady flow assumption. The steady flow assumption can be obtained if the fluidic inertia terms are further neglected. The number of unconstrained admissible functions for quasi-steady flow is equal to the number of independent degrees-of-freedom of the fluid. As additional admissible functions are taken, the viscous effects are more thoroughly taken into account. This was demonstrated through the inclusion of the Kutta condition, which is the separation of the fluid at the trailing edge due to the flows inability to produce an infinite pressure gradient and reverse directions, via an example. The relationship between the contributions of the additional admissible functions (their time dependence) and the locations of the separation lines on the vehicle is a subject of further study. In principle, the number of additional admissible functions may be minimized through these relationships. The CFV formulation gives tremendous insight into the divisions of steady, quasi-stead, and full unsteady flow and served as a platform on which to develop potential control laws.

Chapter 4 addressed control of a vehicle in full unsteady flow. It outlined a new control strategy called Fluid Compensation Control (FCC). In the FCC strategy the fluid forces and moments on the vehicle are found in real-time and, based on a relaxation parameter, compensated for partially or completely. The FCC strategy overcomes the problem of model uncertainty by having a direct measure of the fluid states in the control law. It also enables a
control methodology where vehicle stability can be guaranteed, thereby widening the maneuverability and safe operating range in the presence of disturbances. In general, the FCC strategy gives the control designer an opportunity to include the fluid states, in addition to the vehicle states, in the control law and an opportunity to balance reducing the fluid dynamic load through compensation and reducing the state error through regulation. The application of the FCC strategy was also demonstrated by example through a CFV model.

Chapter 5 addressed synthesizing the fluidic load for use in vehicle modeling and control. The approach of synthesizing the fluidic loads included only those methods with sensors on the vehicle surface and was divided into those that measure pressure and those that measure stagnation and separation points/lines. When pressure measurements are made the synthesis of the forces and moments can be made by performing a quadrature. When stagnation and separation point/lines are measured, a correlation to the fluidic forces and moments need to be found. From a theoretical perspective this dissertation attempted to characterize fluid dynamic loads in full unsteady flow by developed the Kutta principle, which says that the velocity around the vehicle is a smoothly varying function and that it is determined up to a multiplicative constant by its nodes (stagnation, separation, and reattachment points/lines). The principle was proved through a theorem and demonstrated by application to steady attached flow around a rotating cylinder and a rotating sphere using an 8th-order polynomial and a constrained method fit. From an experimental perspective, an ambitious effort was undertaken to correlate the fluid forces and moments and the angle and separation location on a NNemo1-like body. Even though the ground work had been laid by demonstrating this
capability on a 2-dimensional airfoil, as discussed in Chapter 2, going directly to a 3-dimensional bluff body proved too difficult. It is the author’s opinion that a three step testing effort in which the theory is demonstrated first on a 3-dimensional streamlined body, then on a 2-dimensional bluff body, and finally tackling the 3-dimensional bluff body.
Bibliography


