ABSTRACT

ZHAO, MING. Design, Modeling, and Analysis of User Mobility and Its Impact on Multi-hop Wireless Networks. (Under the direction of Dr. Wenye Wang).

Due to the readily deployable and self-organizing nature of mobile ad hoc networks (MANETs), various demanding military and civilian applications are expected to be widely implemented in MANETs. The fundamental issue in MANETs is that performance degrades dramatically as the increase of path failures due to the complexity of user mobility. Therefore, understanding the impacts and implications of user mobility is essential to the system design, topology control, and routing optimization in MANETs.

In this doctoral study, by observing the limitations of both transient and stationary behaviors in existing random mobility models, we first propose a novel Semi-Markov Smooth (SMS) model. We showed that the proposed SMS model captures the transient user mobility according to the physical law of a smooth movement, and achieves the necessary stationary properties regarding stable average speed and uniform node distribution. Second, based on the SMS model, we investigate the MANET link properties by considering the joint effects of radio channel environments and node mobility. We showed that radio channel characteristics dominate the link performance for slower mobile nodes, while node mobility dominates the link performance for faster mobile nodes. In addition, the link lifetime distribution can be effectively approximated by an exponential distribution with the parameter, characterized by the ratio of average node speed to the transmission range. Third, we studied the impact of human diffusive behaviors on the properties of contact-based metrics, such as inter-meeting time. By investigating the empirical human mobility traces, we demonstrated that both human pause time and trip displacement exhibit a cutoff-power distribution. Furthermore, we showed that the human diffusive rate $r$ effectively characterizes the joint temporal-spatial effect of human mobility regarding pause time and trip displacement on inter-meeting time. Especially, the higher the diffusive rate $r$ is, the longer the power law head is in the distribution of inter-meeting time. Finally, we studied the inherent properties of group mobility in MANETs. We suggested that the group member correlation degree is jointly characterized by the human social correlation degree and the similarity of the users’ relative movements and geographic locations. In addition, we showed that the group evolution behaviors and stability of group structures are based on the group user pair correlation degree. At the end, we propose a novel birth-to-death group mobility (BDGM) model for group mobility based studies in MANETs.
Design, Modeling, and Analysis of User Mobility and Its Impact on Multi-hop Wireless Networks

by

Ming Zhao

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Computer Engineering

Raleigh, North Carolina

2009

APPROVED BY:

Dr. Michael Devetsikiotis

Dr. Khaled Harfoush

Dr. Wenye Wang
Chair of Advisory Committee

Dr. Arne A. Nilsson
DEDICATION

To my parents
Bailing Zhao and Guirong Wang.

To my wife and our son
Lu Qu and Alexander Tianhao Zhao.

To my parents-in-law
Changqi Qu and Chengying Zhu.
BIOGRAPHY

Ming Zhao received the B.S. degree from the Department of Industrial Automation and Instrument, Harbin, Harbin, China, in 1997, and the M.S. degrees from the Department of Electrical and Computer Engineering, China Academy of Telecommunications Technology, Beijing, China, in 2000 and from the Department of Electrical Engineering, State University of New York at Buffalo, Buffalo, USA in 2004, respectively. From 2004 to 2009, he has been pursuing his Ph.D. degree major in the program of computer engineering at the Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, USA. His current research interests include Mobility Modeling; End-to-End Path Analysis; Topology and Mobility Management; Network Connectivity; Routing Design and Performance Analysis in Self-Organizing Mobile Wireless Networks. He has been a member of IEEE since 2004 and a member of Phi Kappa Phi since 2006. He is the recipient of 2006 IEEE GLOBECOM Best Student Paper Award – Communication Networks.
ACKNOWLEDGMENTS

My deepest gratitude goes first and foremost to my advisor Dr. Wenye Wang. I cannot be thankful enough to Dr. Wang for her wisdom, knowledge, support, numerous inspirations and guidance. Without her endless effort and countless support, my PH.D. dream would never become true. Without her guidance and persistent help, this dissertation would not have been possible to be completed. I have benefited immensely from her guidance in academic growth, career development and social behaviors in my life.

I would like to express the cordial appreciation to Dr. Arne A Nilsson, Dr. Michael Devetsikiotis, and Dr. Khaled Harfoush for serving on my committee and providing their invaluable comments and critical suggestions to help me achieve a successful doctoral study.

It was my pleasure and honor to have worked closely with my colleagues during my PH.D. study: Dr. Fei Xing, Dr. Nurcan Tezcan, Yi Xu, Avesh K. Algawal, Shawqi Kharbash, Dr. Xinbing Wang, Dr. Wei Liang, Jung Kee Song, Lei Sun, Entong Shen, Lifang Guo, Chi Yi, and Levi Mason. I have benefited significantly from enlightening discussions, research project study, and more important, life issue and spiritual support with them. They encouraged me often times when I faced the challenging research issues. I would like to thank them for all their help, support, and valuable hints. I wish to thank every my friend for all their help and caring they provided during my PHD study.

I am always indebted my dear parents and sister for raising me up and encouraging me to pursue my dream. They have always given me encouraging phone calls and financial support during my difficult times. A very special appreciation would give to my parents-in-law. They have given me comprehensive support, understanding, and careful caring my lovely baby TianTian and myself during this difficult time. Without their help, I could not concentrate on my study and complete my dissertation. I am so fortunate to have my wonderful wife Lu Qu who stood behind me years and years. Marry her is my biggest success during my life. She takes care of our whole family and gave me the opportunity to focus on my study. Our lovely son TianTian cheers me up everyday and makes my life more enjoyable. This dissertation is dedicated to them.
# TABLE OF CONTENTS

| LIST OF TABLES | viii |
| LIST OF FIGURES | x |

## 1 Introduction
1.1 Motivation ............................................. 1
1.2 Research Objectives ...................................... 4
   1.2.1 Mobility Modeling of MANETs .......................... 5
   1.2.2 Joint Effects of Radio Channels and Node Mobility on Link Dynamics .... 6
   1.2.3 Diffusive Properties of Human Mobility and Its Impact on Contact-based Metrics ........................................... 8
   1.2.4 Inherent Properties of Group Mobility for Mobile Wireless Networks .. 9
1.3 Research Challenges ....................................... 11
1.4 Contributions ........................................... 13

## 2 Design and Study A Unified Mobility Model for Analysis and Simulation of Mobile Wireless Networks ............................................. 17
2.1 Motivation and Related Work .................................. 17
2.2 A Unified Mobility Model: Semi-Markov Smooth (SMS) Model ................. 20
   2.2.1 Model Description ...................................... 20
   2.2.2 Stochastic Process of SMS Model .......................... 24
2.3 Stochastic Properties of SMS Model .................................. 27
   2.3.1 Movement Duration ...................................... 28
   2.3.2 Stochastic Properties of Step Speed .......................... 29
   2.3.3 Trace Length ........................................... 33
2.4 Transient Property: Smooth Movement .................................. 37
2.5 Steady State Analysis ........................................... 38
   2.5.1 Time Stationary Distribution ............................. 39
   2.5.2 Speed Distribution at Steady State ......................... 39
   2.5.3 Average Speed at Steady State ............................. 40
   2.5.4 Spatial Node Distribution .................................. 41
2.6 Simulation Results ........................................... 44
   2.6.1 Assumptions and Parameters ............................. 44
   2.6.2 Average Speed ........................................... 45
   2.6.3 Smooth Movements ........................................... 48
   2.6.4 Uniform Node Distribution .................................. 49
2.7 Impacts and Applications of SMS Model .................................. 50
   2.7.1 Effects of SMS Model ...................................... 51
   2.7.2 Extensions to Group Mobility .................................. 56
2.7.3 Adaption to Geographical Constraints ........................................... 57
2.8 Summary ......................................................................................... 59
2.9 Appendix ......................................................................................... 60
  2.9.1 Derivation of the PMF of movement steps $K$ ......................... 60
  2.9.2 Derivation the CDF of Steady State Speed $v_{ss}$ ................. 62
  2.9.3 Derivation of Expected Steady State Speed $E\{v_{ss}\}$ ......... 64

3 Joint Effects of Radio Channels and Node Mobility on Link Dynamics in Mobile Wireless Networks ................................................................. 67
  3.1 Motivation and Related Work ............................................................ 67
  3.2 Characterization of Radio Links and Mobility ............................... 70
    3.2.1 Effective Transmission Range .............................................. 71
    3.2.2 Smooth Mobility Model ...................................................... 73
    3.2.3 Node-Pair Distance ............................................................. 74
  3.3 Link Lifetime Distribution ............................................................... 76
    3.3.1 Relative Movement: Speed and Distance ............................. 76
    3.3.2 Distance Transition Matrix $P$ ............................................. 79
    3.3.3 Approximation of Link Lifetime Distribution .................... 82
  3.4 Link Stochastic Properties .............................................................. 85
    3.4.1 Average Link Lifetime ....................................................... 85
    3.4.2 Residual Link Lifetime ....................................................... 86
    3.4.3 Link Change Rate and Link Arrival Rate ......................... 88
  3.5 Implications of Link Properties ..................................................... 90
    3.5.1 $k$-hop Path Lifetime ......................................................... 91
    3.5.2 Network Connectivity ....................................................... 94
    3.5.3 Routing Performance ....................................................... 96
  3.6 Summary ......................................................................................... 97

4 Human Diffusive Behaviors: Temporal-Spatial Limitations in Mobile Wireless Networks ........................................................................... 98
  4.1 Introduction .................................................................................... 99
  4.2 Preliminaries and Problem Statement ......................................... 104
    4.2.1 Preliminary of Diffusive Process ....................................... 104
    4.2.2 Definitions ................................................................. 106
    4.2.3 Problem Demonstration .................................................. 107
  4.3 Cutoff power-law: Human Mobility Traces .................................. 110
    4.3.1 Dataset Collection ............................................................ 111
    4.3.2 Data Extraction and Statistics .......................................... 113
    4.3.3 Temporal Domain: Pause Time Property ....................... 115
    4.3.4 Spatial Domain: Trip Displacement Property ................. 116
    4.3.5 Observations ............................................................... 117
  4.4 Scaling Law of Temporal-Spatial Human Diffusive Behaviors: Counter Effects ................................................................. 119
    4.4.1 Continuous-Time Task-Driven Mobility Model ................ 120
    4.4.2 Scaling Law Analysis of Human Diffusive Behaviors ........ 121
# 4.4 Temporal-Spatial Power-law Effects on Inter-meeting Time

- Temporal-Spatial Power-law Effects on Inter-meeting Time: 128
- Human Diffusive Rate Effect on Link lifetime: 130

# 4.5 Coupling Effects of Cutoff power-law Distribution

- Human Social Behaviors Impacts: 133
- Approximation of Cutoff power-law Distribution and Validations: 134
- Cutoff Power-law Effects on Inter-meeting Time: 135

# 4.6 Summary

- Summary: 136

# 5 Understanding the Structure and Dynamics of Mobile Groups in Multi-hop Wireless Networks

- Understanding the Structure and Dynamics of Mobile Groups in Multi-hop Wireless Networks: 139

## 5.1 Introduction

- Introduction: 140

## 5.2 User Pair Correlation

- User Pair Correlation: 144
  - System Model and Definitions: 145
  - Social Correlation between Mobile Users: 147
  - User Pair Correlation Metric: 153

## 5.3 Characteristics of Group Structure

- Characteristics of Group Structure: 157
  - Node Connectness: 157
  - Effect of Node Degree: 164
  - Group Head Selection: 168
  - Effect of Inter-group Edge: 171

## 5.4 Characteristics of Group Evolution

- Characteristics of Group Evolution: 175
  - Quantify Group Evolution Degree: 176
  - Group Stability Measure: 178

## 5.5 Design And Application of Birth-to-Death Group Mobility Model

- Design And Application of Birth-to-Death Group Mobility Model: 180
  - Metrics Applied for BDGM Model: 180
  - Group Initialization Algorithm: 181
  - Group Movement Pattern: 183
  - Routing Performance Evaluation: 184

## 5.6 Summary

- Summary: 186

# 6 Conclusion

- Conclusion: 187

# Bibliography

- Bibliography: 192
LIST OF TABLES

Table 2.1 Effect of Pause Time on Average Speed. .............................................. 47
Table 2.2 Network Connectivity: SMS model vs. RWP model. ............................. 55
Table 2.3 Properties of Different Mobility Models. ............................................. 60
Table 3.1 The ETR with respect to wireless radio environments. ......................... 83
Table 3.2 Comparison: $\overline{T}_L$ and estimated $\hat{T}_L$, for $R_e = 239$ m. .......... 85
Table 3.3 Node density $\sigma_R$ vs. average node degree $E\{d_G(t)\}$. ...................... 95
Table 3.4 Implication of link lifetime. ................................................................. 97
Table 4.1 Human Moving Trace Datasets ....................................................... 114
Table 4.2 Trip statistics of Campus dataset. ..................................................... 115
Table 5.1 Example of User Task Location Profiles ........................................... 151
Table 5.2 Example of Relative Entropy between Mobile Users ........................... 152
Table 5.3 Example of Social Correlation Coefficient between Mobile Users ......... 153
Table 5.4 Example of A Group Formation based on User Pair Correlation .......... 156
Table 5.5 Example of weighted and unweighted node clustering coefficients from Figure 5.4(a). ................................................................. 162
Table 5.6 Example of weighted and unweighted node clustering coefficients from Figure 5.4(b). ................................................................. 162
Table 5.7 Example of weighted and unweighted average node neighbors’ degree from Figure 5.4(a). ................................................................. 167
Table 5.8 Example of weighted and unweighted average node neighbors’ degree from Figure 5.4(b). ................................................................. 167
Table 5.9 Example of Group Head Selection from Figure 5.4(a) ......................... 171
Table 5.10  Example of Group Head Selection from Figure 5.4(b) ................. 171

Table 5.11  Metrics Applied for the Birth-to-Death Group Mobility (BDGM) Model ....... 182
LIST OF FIGURES

Figure 1.1 Inter-dependence of node mobility with related topics and the impact on MANET performance. ................................................................. 5

Figure 2.1 An example of speed and direction transition in one SMS movement. ......... 23
Figure 2.2 Four-state transition process in SMS model. ...................................... 26
Figure 2.3 The PMF and CDF of one SMS movement duration according to different phase duration ranges.......................................................... 29
Figure 2.4 Length vs. time in one SMS movement. ............................................ 34
Figure 2.5 Three smooth SMS movements with different temporal correlation, where $\epsilon_\phi = 0.4\pi$ and $\epsilon_V = 2$ m/sec............................................. 38
Figure 2.6 Average speed vs. simulation time..................................................... 46
Figure 2.7 Average speed vs. phase duration time............................................. 47
Figure 2.8 Distance and trace length............................................................... 48
Figure 2.9 Top-view of node distribution of RWP model and SMS model............... 49
Figure 2.10 Link performance comparison between the RWP and the SMS Model.... 53
Figure 2.11 Connectivity performance comparison between the RWP and the SMS Model... 54
Figure 2.12 Routing performance comparison between the RWP and the SMS Model... 56
Figure 2.13 Group mobility with SMS model.................................................... 58
Figure 2.14 Use SMS model in a Manhattan-like area........................................ 58
Figure 2.15 Different domain intervals of $K$...................................................... 61

Figure 3.1 Probability of link connection between two nodes, where path loss exponent $\xi = 3$, shadow fading $\sigma_s = 5$ dB, and multi-path fading is 3 dB............................................. 71

Figure 3.2 Illustration of ETR vs. link lifetime under different radio channels, where node speed is 2 m/sec.............................................................. 75
| Figure 3.3 | Relative movement trajectory of node-pair \((u, w)\). | 78 |
| Figure 3.4 | Rayleigh distribution approximation of the relative speed. | 78 |
| Figure 3.5 | Approximation of \(P_{ij}\) with respect to \(\varepsilon\). | 81 |
| Figure 3.6 | Link lifetime distribution. | 84 |
| Figure 3.7 | Stochastic properties of link lifetime. | 86 |
| Figure 3.8 | Residual link lifetime: analytical and simulation results. | 87 |
| Figure 3.9 | Derivation of average link arrival rate \(\lambda\). | 89 |
| Figure 3.10 | Node mobility and ETR effects on average link arrival rate. | 90 |
| Figure 3.11 | PMF and PDF of path Lifetime. | 93 |
| Figure 3.12 | Average number of neighbors per node according to node speed \(\bar{V}\) and ETR. | 95 |
| Figure 3.13 | Effective transmission range and node mobility impacts on AODV routing performance. | 96 |

| Figure 4.1 | Separated power-law and exponential behavior of cutoff power-law. | 108 |
| Figure 4.2 | Human moving domain size vs. network domain size. | 108 |
| Figure 4.3 | The mixed power-law and exponential behavior in the cutoff power-law distribution. | 111 |
| Figure 4.4 | Example of an extracted trip. | 113 |
| Figure 4.5 | The CCDF of pause time in Campus dataset. | 116 |
| Figure 4.6 | The CCDF of trip displacement in three dataset. | 117 |
| Figure 4.7 | Continuous time task mobility (CTDM) model. | 120 |
| Figure 4.8 | The CCDF of aggregated task time. | 121 |
| Figure 4.9 | Ambivalent diffusive behavior. | 126 |
| Figure 4.10 | Inter-meeting time upon human diffusive rate \(r\). | 129 |
| Figure 4.11 | Link lifetime upon human diffusive rate \(r\). | 130 |
| Figure 4.12 | Single user weekly trace trajectories in Campus dataset. | 133 |
Figure 4.13  Cutoff power law approximation and validation. .......................... 135
Figure 4.14  Cutoff power-law vs. power-law effects on inter-meeting time. .............. 136
Figure 5.1   Example of human social moving patterns. ........................................ 145
Figure 5.2   Example of 100 task location sites in a network delimited by the Voronoi decomposition. .......................................................... 146
Figure 5.3   Case study of user pair correlation. .................................................. 155
Figure 5.4   Weighted and unweighted clustering coefficient. ................................. 159
Figure 5.5   Example of specifying group head based on physical locations of 11 users. .... 169
Figure 5.6   Weighted edge clustering coefficient comparison. ............................... 174
Figure 5.7   Example of group evolution degree variation with time. ...................... 177
Figure 5.8   Node switch between two groups. .................................................... 180
Figure 5.9   Operation of group initialization algorithm. ...................................... 181
Figure 5.10  Routing performance comparison between the BDGM and the RPGM Model. . . 185
Chapter 1

Introduction

1.1 Motivation

Wireless technologies have enabled freedom of mobility by releasing the constraint of wired connections between correspondent communicating devices. Due to the advance in radio communications and a large number of demanding applications, wireless mobile multi-hop networks have attracted tremendous attentions from both academia and industry in the past decade. A wireless mobile multi-hop network, such as a mobile ad hoc network (MANET), is an infrastructureless wireless network, where all nodes equipped with radio transceivers are capable to move and communicate via radio channels. In MANETs, each node may operate not only as a host but also as a mobile router for discovering and maintaining routes to other mobile nodes. Owing to the limited transmission range, an end-to-end communication in MANETs is carried out dynamically through a number of intermediate relay nodes, which is thus called multi-hop forwarding. The goal of an MANET is to provide a rapidly and easily deployable means of interconnection between mobile nodes, which rely on no fixed infrastructures. Hence, MANET promises to bring the vision of ubiquitous connectivity and pervasive network coverage into reality. Motivated by the readily deployable and self-organizing nature of MANET, imminent applications are expected to be widely used in military (e.g., tactical communication in the battlefield), civilian (e.g., electronic classrooms, convention centers and construction cites), law enforcement (e.g., crowd control, search and rescue), and disaster recovery (e.g., fire, flood and earthquake recovery) [1].

Because of the dynamic node mobility, a link between a node pair breaks when a node moves outside the transmission zone of the other node in MANETs. As the communication paths
rely on a multi-hop fashion, the path failures occur when the links incident to the paths become unavailable. Path failures require immediate response for routing protocols to discover and update new routes [2]. However, frequent routing update messages may incur a high signaling overhead [3, 4] and energy consumption among ad hoc nodes, which seriously deteriorate the utilization of the scarce network resource [5]. Furthermore, due to path failures, excessive transmission delay, packets loss and limited throughput can dramatically degrade the performance in MANETs [6, 2]. These constraints make it much difficult to fulfill the requirement of Quality of Service (QoS)-aware applications such as multimedia transmission and collaborative computing in MANETs [7]. Hence, the performance of the potential applications of MANETs will degrade dramatically as the increase of path failures caused by node mobility [8, 9]. Therefore, compared to traditional wired networks, node mobility is a critical factor to the system performance of MANETs.

Since node mobility plays a significant role on MANET performance, understanding user moving behaviors and associated mobility patterns is essential to routing protocol design, applications planning and system performance evaluation. Ideally, node mobility should be studied through existing trace files of mobile users in MANETs, such as human traces [10, 11, 12] and vehicle traces [13]. However, MANETs have not been implemented and deployed on a large-scale yet, the lack of mobility trace files from real-life applications becomes a main hurdle for characterizing realistic mobility patterns. Consequently, the mobility modeling in MANETs has been an interesting topic because it provides a fundamental supporting tool for simulation and analysis of mobility based research issues [14, 5, 9].

On purpose to evaluate and analyze MANET performance, many mobility models have been proposed to describe moving user behaviors according to diversified application scenarios [15, 14, 5, 9]. Due to straightforward physical interpretation and implementation, random mobility models, such as Random Waypoint (RWP) model [16], in which each mobile node moves without constraint on its velocity, time period and destination, are most widely used [9] in current MANET research. However, because of the complete randomness of user mobility in random mobility models, unrealistic moving behaviors, such as sudden stop, sudden speed change, and sharp turn, occur frequently during the simulations. More important, it has been recently found that random mobility models, especially for RWP model, can lead to biased or even misleading results of analysis and simulations for MANETs [17, 18, 19, 20, 21]. In consequence, the current MANET research based on random mobility models may not correctly indicate the network performance in reality. As an effort to resolve this problem, temporal mobility models which considers the correlation of
node’s moving behavior have been proposed [9]. However, compared with the existing MANET studies upon random mobility models, little work has been done through temporal mobility models. In order to correctly evaluate the MANET performance, people are motivated to revisit analytical and simulation results on MANETs by applying temporal mobility models. In particular, as we discussed above, the link properties such as link lifetime directly manifest MANET performance while they are very sensitive to the user mobility mobility patterns. It is desirable to study the link properties and their correspondent impacts on MANET performance upon temporal models. Furthermore, as humans’ moving behaviors are regulated by their associated societal duties, in a practical MANET, mobile users may either move individually or in a group manner. Because the mobile wireless devices are generally attached to humans, the human mobility directly affects the properties of contact-based metrics, such as contact time and inter-contact time, and there in the routing performance in mobile ad hoc networks (MANETs) [13, 22, 23]. Therefore, a fundamental study of human mobility and its impact on the link level dynamics of mobile devices can greatly benefit the routing design and network performance analysis [14, 13, 22, 23]. In addition, due to the social correlation between humans, mobile users often move in a group way in MANETs. Given a higher correlation degree between mobile users inside a group, the contact time between group members can be much longer than a random node pair, therein the routing performance based on group mobility can be increased. On the other hand, frequent group structure variations according to member leaving and join events in groups can dramatically increase the routing maintenance cost and waste the limit bandwidth resource, therein degrade the network performance. Therefore, understanding the coherence level of group structure and group stability is a key point to characterize the group evolution process, in which the number of group members vary with time and therein, the group mobility impact on the routing performance in MANETs [24, 25].

Upon the above discussions, we notice that node mobility is the most fundamental research problem in mobility related MANET study. Due to the complex and dynamic node mobility, there are many challenging issues in MANET design, planning, and performance evaluation [14, 5, 26, 2, 12, 27, 1]. Therefore, a comprehensive understanding of node mobility and its impact on network performance is necessary before a large-scale of actual deployments of MANET applications. However, current researches on node mobility exhibit many limitations, and the in-depth interpretation of user mobility patterns and their effects upon system results still remains elusive. Motivated by the promising applications and limitation of existing works, we dedicate this Ph.D. study to mobility related research issues in MANETs, that is, “Design, Modeling, and Analysis of
User Mobility and its Impact on Multi-hop Wireless Networks”. Specifically, we categorize our research objectives into four topics which are discussed next regarding the current works and challenges, respectively.

1.2 Research Objectives

In this section, we summarize our research objectives on mobility related issues in MANETs which we aim to achieve during the course of doctoral study. Specifically, Figure 1.1 illustrates the inter-dependence of our four topics of node mobility and the overall impacts on the performance of MANETs. In this doctoral research, we first study mobility modeling, where we design a sound mobility model which not only effectively mimics transient moving behaviors of ad hoc nodes in real environments but also achieves the necessary stationary properties for proper analysis and simulations. As shown in Figure 1.1, node mobility directly impacts the MANET link properties, which further influences the correspondent network connectivity and routing performance. In addition, we find that MANET link performance is influenced by the joint effects of radio channel environments and node mobility. Hence by applying the results from our first work, we want to address the unknown link properties upon these two joint effects in the second work. Further, as almost all the applications in MANETs are tightly coupled with the human daily activities, we move forward to study node mobility in the third work by investigating human mobility according to our collected GPS traces. And then, we analyze the joint temporal-spatial human mobility impact on the property of inter-meeting time. Finally, we notice that practical MANET applications often require a group of users to execute a common task inside the network. Based on our results and understanding of individual node mobility, we target the study of group mobility in our fourth work. Specifically, we study the group correlation degree between mobile users based on human social correlation and similarity of nodes’ movement and their physical distance. And then, we investigate the unknown properties of group structures and group stability for characterizing the group evolution behaviors in MANETs.

In summary, the following four areas are investigated:

- Individual node mobility modeling, steady state analysis, and applications, which is our first Ph.D. topic indexed with (1) in Figure 1.1.

- Stochastic analysis on link dynamics according to node mobility and radio environments,
Figure 1.1: Inter-dependence of node mobility with related topics and the impact on MANET performance.

which is our second doctoral study indexed with (2) in Figure 1.1.

- Empirical study of human mobility patterns in spatial-temporal domains, and Stochastic analysis on their impact on inter-meeting time which is the third Ph.D. topic indexed with (3) in Figure 1.1.

- Group mobility properties analysis and applications, with is the fourth topic of this doctoral study indexed with (4) in Figure 1.1.

1.2.1 Mobility Modeling of MANETs

Owing to the lack of mobility trace files from real-life applications of MANETs, many mobility models have been proposed for both theoretical analysis and simulation studies [15, 14, 5, 9]. In particular, because of the straightforward design and easy implementation, Random Waypoint (RWP) model [16], is most widely used [9] in MANET studies. The stationary moving behaviors of mobility models predominate the performance evaluation, and are critical to analytical study for deriving accurate theoretical expressions [20]. However, two flaws of the RWP model have been found in its stationary behavior. First, Yoon showed that the average node speed of RWP model decreases over time [17]. Second, Bettstetter [18] and Blough et al. [28] respectively observed that RWP
model has non-uniform spatial node distribution at steady state, with the maximum node density in the middle of simulation region given that initial state is uniform distribution. In [29], the authors showed that if a mobility model fails to provide a steady state of average speed, the system evaluation would be misleading and vary dramatically with time. Moreover, the model cannot provide appropriate solution for theoretical analysis. Corresponding to the essence of stable average speed, the virtue of uniform node distribution is two-fold. First, it is especially useful in theoretical studies, in that a system metric of interest, for example network throughput, can be properly calculated and understood without considering otherwise the undesired influence of non-uniform node distribution induced by mobility models [5, 26]. Furthermore, the majority of existing simulation and analytical studies of MANETs assume that mobile users are uniformly distributed in a network. Therefore, the mobility model with steady state uniform node distribution can accurately reflect the analysis and simulation results. In addition, as discussed in previous section, the unrealistic moving behaviors in existing random mobility models including RWP model, can mislead the results of analysis and simulations for MANETs [17, 18, 19, 20, 21].

As an effort to address the problem of unrealistic movements and to provide desirable stationary moving behaviors, mobility models that consider the correlation of node’s moving behavior are desirable. Noticing that existing mobility models do not satisfy these necessary requirements, in our first research work, we are motivated to design a new mobility model, named Semi-Markov Smooth (SMS) model, that can abide by the physical law of moving objects to avoid abrupt moving behaviors, and can provide a microscopic view of mobility such that node mobility is controllable and adaptive to different network environments. Furthermore, the model is expected to be mathematical tractable and bear the desired steady state properties including stable average speed [29] and uniform spatial node distribution [20], so that it can be used to correctly evaluate the performance of MANETs. The detailed design patterns and analysis of our proposed mobility model are elaborated in Chapter 2.

1.2.2 Joint Effects of Radio Channels and Node Mobility on Link Dynamics

As illustrated in Figure 1.1, the MANET link performance is directly influenced by user moving behaviors. Hence, the link lifetime can effectively manifest the network performance, which is also a fundamental metric for planning MANET applications upon node mobility. In particular, excessive transmission delay and packet loss due to the link failure incident to ongoing traffic sessions, severely degrade the performance in MANETs, thus making it difficult to the provisioning
of QoS guaranteed services in MANETs. Therefore, understanding the nature of link dynamics is essential to design mobility-resilient MANETs\cite{30, 9, 26}, maximize routing performance\cite{31, 2}, optimize topology control\cite{32, 33, 34}, and achieve the desired network performance\cite{5, 7}.

Besides the node mobility, the time-varying radio environment is another inherent property of MANETs. Specifically, a received signal of mobile nodes is influenced by three fading effects: large-scale, multipath, and shadowing\cite{35}, so that the radio channel (link) status may vary greatly in different wireless environments, even the node-pair distance is same. In addition, we notice that the link performance degrades dramatically as the increase of the fadings in the radio channel. It turns out that the link performance is very sensitive to both node mobility and radio channel characteristics, which is considered as the major difference from the communication in wired networks. Therefore, because of the inherent nature of MANETs, it is insufficient to analyze the link dynamics from the single effect of node mobility. In other words, it is necessary to study MANET link properties upon the joint effects of node mobility and radio channels.

There has been a large number of studies on the effect of node mobility on link dynamics, such as link lifetime \cite{36, 5, 26, 37, 38}, link change rate \cite{26}, link residual time and link availability \cite{39, 36, 40, 26, 37}. However, little effort has been done to analyze the link dynamics from the joint effects perspective. Specifically, existing studies on link dynamics are mainly focused on random mobility models such as Random Walk (RW) mobility model \cite{41} and a fixed transmission range, which could bring three limitations on the obtained results. First, random models frequently generate abrupt moving behaviors such as sudden stop and sharp turn which are not in comply with smooth motions in real world \cite{42, 41}. Moreover, as mobile nodes do not change velocity within each time epoch, random mobility models cannot describe smooth speed and direction transition in one movement, which however is very common in real moving scenarios. Second, the transmission range of a mobile node in reality varies dynamically due to the time-varying wireless radio environment \cite{32, 33, 43}. In addition, the study in \cite{44} suggested the time scale used to describe node mobility should be less than the time scale for capturing the significant channel variability, which implies smooth mobility models described in small time scale, for instance, the SMS model proposed in our first work, are more preferable for analyzing link properties.

Because of these three limitations, we are motivated to study the link dynamics upon (i) dynamic transmission range regarding the characteristics of mobile wireless environment, and (ii) a smooth mobility model rather than random mobility models. Based on the proposed SMS mobility model in the first work, we aim to accurately study the stochastic link properties including,
link lifetime, link residual time, and link change rate. Moreover, we are interested in finding the weight of impacts on link performance between node mobility and radio environments according to diversified MANET scenarios. The details of this work will be represented in Chapter 3.

1.2.3 Diffusive Properties of Human Mobility and Its Impact on Contact-based Metrics

In fact, majority of mobile nodes in a typical wireless multi-hop network is expected to be either pedestrians carrying wireless enabled devices or vehicles containing humans which support wireless communications. As a result, the human mobility directly affects the link level dynamics between mobile wireless devices, which are characterized by the contact-based metrics, such as contact time and inter-contact time, in mobile ad hoc networks (MANETs) [14, 13, 22, 23]. For instance, the contact time (also called link lifetime) is counted for the time duration when two mobile users have a directly connected link. As a link often disconnects when a mobile user moves outside the transmission range of the other user, the inter-contact time (also called inter-meeting time) is defined as the time period between two consecutive link connections of two mobile users. Therefore, the deep understanding of human mobility pattern and its implications is indispensable to study the properties of contact-based metrics, which in turn, are critical to routing protocol design and network planning.

In reality, the human daily activities are heavily influenced the societal duties and working patterns. This implies that human mobility cannot be fully described by an arbitrary random movement pattern. Therefore, though synthetic mobility models can generate diversified random movement processes of mobile nodes, they can hardly mimic the human moving behaviors in real worlds with societal context. In this regard, it is not sufficient to study human mobility and its impact on MANETs by synthetic mobility models. Compared with synthetic mobility modeling, generalizing mobility models from real human traces is another means to study human mobility, which is clearly more convincing and reliable for MANET research. However, because of the lack of real implemented wireless multi-hop networks and the difficulty of trace collection, seldom human mobility traces are available in the current research community. To solve this problem, several empirical user traces especially for campus networks have been collected recently [12, 13, 22, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55]. However, rather than specifying the movements of individual users, most collected user traces are developed for the prediction of user locations [45, 46, 53, 54], network utilization [47, 49, 51, 52] and network design of WLANs.
The main challenge is that it is not easy to specify the accurate user location and trip movements in these traces. Fortunately, Global Positioning System (GPS) enabled devices have become affordable and have been widely used since recent years. Thus, people start to carefully study the human mobility traces by GPS loggers since 2005. Specifically, the research works focus on human mobility according to spatial effects, such as trip length [56, 57, 22], or temporal effects such as inter-meeting time [12, 55] and pause time [54, 22]. However, as human mobility patterns are influenced by complex social and cultural contexts, it is very difficult to generalize the human mobility patterns upon diversified locations and times. Consequently, there are still many open questions unsolved. For example, what are the inherent properties of human mobility? In other words, without specifying human mobility patterns qualitatively and quantitatively, it is unlikely to accurately analyze the impact of human mobility on MANET performance.

In [56, 57], the authors showed that human mobility patterns and moving capability can be effectively manifested by his/her diffusive capability (order) \( r \). Nevertheless, it is still not clear how to specify the diffusive order \( r \) with respect to different human diffusive behaviors and what is the fundamental human behavior rules with their societal duties in reality. Therefore, the importance and yet limitations of existing research on human mobility strongly motivate us to investigate the human mobility patterns, especially for human diffusive properties, from both spatial and temporal perspectives. To proceed, we collected human GPS traces and record their daily activities for three months (over a thousand hours), upon which we expect to understand, interpret and model the human diffusive behaviors. Furthermore, it has been suggested that human mobility can greatly affect the properties of contact-based metrics in MANETs [12, 55, 54, 22]. Hence, we are motivated to investigate the relation between human diffusive mobility patterns and the property of inter-meeting time in multi-hop wireless networks. Our detailed work is elaborated in Chapter 4.

### 1.2.4 Inherent Properties of Group Mobility for Mobile Wireless Networks

Besides our study on individual user moving behaviors and their impacts on MANETs, we observe that mobile users often exhibit correlated mobility patterns in their movements in practical ad-hoc networks. Such correlated mobility patterns are also referred to as user pair correlation, especially in collaborative networks [58, 59]. Similar to what we observed in individual human mobility, the group user mobility is also regulated by the social duties according to specific MANET application scenarios [58, 60]. Typically, group user activities can be categorized into three types. First, the group mobility can be generally considered as a mission-oriented activity. For example,
to execute a mission which is attacking the enemies’ target, a troop of soldiers move following their command leader in a tactic way. Second, it can often be observed as a location-oriented group mobility in MANETs, in which a number of users move together toward the same destination. For instance, a team of visitors move follow a tour-guider for the same masterpiece of interest in a museum. Third, mobile users could be categorized into groups according to their organizations. For example, on a disaster scene, firefighters, policemen and doctors join three groups for different rescue operations. As practical MANET applications are often involved with group mobility, a number of research works have been elaborated upon the impact of group mobility in MANETs [58, 61, 62, 63, 64, 65]. Recent studies demonstrated that group mobility can benefit the overall network performance, for instance, improving the routing efficiency [24]. Especially, as the size of the network increases, a flat network structure will encounter the scalability problem, which is more difficult to handle in MANETs due to node mobility. If considering each group as a cluster, to enhance scalability, hierarchy routing based on clusters is often applied for a large scale network [2, 64]. In addition, group mobility can improve the efficiency of location management by designing a hierarchical location database architecture upon groups [66] and improve topology control management in MANETs [67]. Compared with the topology changes in a network composed of individual nodes, the network topology is more robust to nodes moving in groups because the correlation of mobile nodes in a group reduces the potential of path failures.

Interestingly, it has been widely observed in mobile social networks that humans with strong social correlations tend to form a social group when they meet at the same community site. Thus, there is a very close resemblance between mobile wireless networks and mobile social networks. Therefore, we are motivated to take the advantage of the resemblance between these two networks to study fundamental characteristics of group mobility. Though integrating human mobility with a variety of social group structures will significantly impact the nature of self-organizing mobile wireless networks, the research of human group mobility in these networks has received less attentions and has not been systematically scrutinized. In addition, the analysis of human social correlation is often neglected in existing group mobility research. Therefore, in this research topic, we focus on the following questions in mobile wireless networks: 1) When and how mobile users organize into groups? 2) How to quantitatively measure the correlation degree between group users? 3) How to evaluate the impact of each node on the dynamic group structures and group stability? By answering these questions, we are able to characterize the individual node moving behaviors and effects within a group.
Furthermore, we observed that in reality, a general group in a network typically experiences the birth-to-death process including group initiation, group evolution and group member disperse in sequence. However, it is still unknown how to capture and predict the detailed group evolution behaviors in mobile wireless networks. As an advanced study, we aim to investigate the group evolution properties based on the analytical results obtained in the first phase of this research. In particular, we want to find out under which conditions the group size and structure can evolve in the network? In addition, we notice that almost all the existing group mobility models have two limitations on describing group moving behaviors. First, existing group mobility models, such as RPGM model, did not capture the group evolution process, there is no node join and node switch event occurs during the entire group movement. Second, the group did not deform during the entire simulation. Thus, there is no birth-to-death group process during the simulation. Hence, existing group mobility models may not be consistent with the group moving behaviors in reality. In order to mimic both birth-to-death group process and group evolution behaviors in MANETs, we are motivated to apply the metrics studied in this work to design a novel birth-to-death group mobility model. The detailed representation of this work is in Chapter 5.

1.3 Research Challenges

During the course of this doctoral research, there are many non-trivial challenges we have to tackle with. In this section, we discuss the major challenges which are categorized into the following groups: radio environment issues, mobility model design issues, human spatial and temporal moving trace collection, diversified human trace dataset requirements, and societal and working environment effects.

- **Mobility model design:** Because of the lack of users traces from real-life mobile wireless networks, many synthetic mobility models have been proposed and widely applied for current MANET research. These models are in general characterized by random movement processes, and they can provide tractable analysis for MANET study. However, it has been found that the complete randomness of mobility pattern in synthetic mobility models can cause unrealistic moving behaviors to occur frequently during the simulations. These unrealistic moving behaviors can lead to biased or even misleading results of analysis and simulations for MANETs. In addition, synthetic mobility models, such as RWP model, may generate unexpected stationary moving behaviors, e.g., the decaying average speed and non-uniform
node distribution, which lead to overoptimistic evaluations on MANET link and routing performance. Furthermore, recent studies show that the finite border of simulation area causes the power-law – exponential decay issues on the CCDF of inter-meeting time and trip displacement. Hence, design an appropriate synthetic mobility model which can not only avoid the unrealistic moving behaviors by complying with the physical law of moving objects in reality but also provide tractable analysis is a challenge issue. More important, the designed model should achieve the desirable stationary moving behaviors while reducing or avoiding the undesired border effect, which makes it more difficult to the design of mobility models. Moreover, people already observed that the network and routing performance is very sensitive to the node mobility patterns. How to design a unified mobility model which can be flexibly and easily controlled to diverse user moving behaviors is not trivial.

- **Small Time Scale of Link Variations in Radio environment:** In general, a received signal of mobile nodes is influenced by three fading effects: large-scale, multipath, and shadowing. Hence, the MANET link status may vary greatly in different wireless environments, even the node-pair distance is same. Due to the limited power consumptions, the transmission range of a mobile node is dynamic according to the time-varying wireless radio environment, which in turn varies the link status between neighboring nodes and the network topology. In consequence, it is really difficult to analyze network topology changes, link dynamics and routing performance in the presence of complex node mobility and the unpredicted radio environments. More specific, the main challenge here is how to tackle the small time scale of radio link variations and random node mobility together on studying the link performance of MANETs.

- **Diversified human trace dataset requirements:** Because of the complicated and diversified human moving behaviors in reality, a single trace dataset which records the human trace in the neighborhood of a certain community site, cannot remarkably exhibit all the inherent properties of human mobility. Thus, the potential defects of limited sample traces, homogeneous social regularities of participants, and small area of movement trajectories in one community site may constrain our observations and results of human mobility. In order to investigate the inherent properties of general human mobility, larger datasets including more number of sample traces, more heterogeneous participants, and city wide movements are preferred for research on human mobility. In addition, we aim to study the human diffusive behaviors in
our doctoral research, which necessarily requires the diversified human traces having different scales of human diffusive (moving) domains. However, most available human trace datasets are campus-wide traces. This imposes a big challenge on collecting different empirical human traces with different scales of human moving domain size.

- **Societal and working environment effects:** Human mobility is closely coupled with the demanding MANET applications. It has been observed that human mobility patterns are influenced by complex social and cultural contexts. The human moving behaviors are affiliated with their duties and working patterns in a certain territory. And the affiliation varies according to different duties in different territories. Particularly for group user moving behaviors, the mission-oriented group mobility is often observed in MANET applications. For example, putting out a fire is the common mission of all firefighters on a disaster scene, where they move closely in a group to execute the task. However, humans are often involved in very complicated societal environments, and the social status of a single person such as a worker, a student, and a shopper, frequently changes upon different spatial-temporal domains. How to characterize the individual and group human mobility patterns is very challenging. Especially for group movements in MANETs, how to specify the group coherence and project the group evolution process under different social and working duties of mobile users is a very challenging issue.

### 1.4 Contributions

In summary, the main contributions of this Ph.D. study is to provide a deep understanding of node mobility behaviors and its impacts to the system design, routing optimization, and performance analysis in mobile wireless networks. To achieve this goal, we studied four mobility related research topics step by step in this doctoral study. As shown below, we list our major contributions obtained by far according to each research topic.

- First, we proposed a novel mobility model, named *Semi-Markov Smooth* (SMS) model, to characterize the smooth movement of mobile users in accordance with the physical law of motion in order to eliminate sharp turns, abrupt speed change and sudden stops exhibited by existing models. We formulated the smooth mobility model by a semi-Markov process to analyze the steady state properties of this model. We rigorously proved that the average node
speed of SMS model is stable with simulation time, hence avoid the well-known speed decay problem of Random Waypoint mobility model. We further showed that the SMS model always maintains the uniform node distribution of mobile nodes with time. Through stochastic analysis, we claimed that this model unifies many good features for analysis and simulations of mobile networks. First, it is smooth and steady because there is no speed decay problem for arbitrary starting speed, while maintaining uniform spatial node distribution regardless of node placement. Second, it can be easily and flexibly applied for simulating node mobility in wireless networks. It can also adapt to different network environments such as group mobility and geographic constraints. To demonstrate the impact of this model, we evaluate the effect of this model on distribution of relative speed, link lifetime between neighboring nodes, and average node degree by ns-2 simulations. This proposed unified mobility model provides a fundamental support for our second research topic.

• Second, we took a new modeling approach that captures the dynamics of radio channels and node movements in small-scale upon microscopic mobility models, such as the SMS model. Specifically, a distance transition probability matrix is designed in order to describe the joint effects of dynamic transmission range due to radio channel fading and relative distance of a node-pair resulting from random movements. We found that the PDF of link lifetime can be approximated by an exponential distribution with parameter characterized by the ratio of average node speed $\bar{V}$ to effective transmission range $R_e$. And the probability distribution function of path lifetime can be effectively approximated by an exponential distribution for any k-hop path, with parameter of $\lambda_k$, which is the summation of exponential parameters of each link along the path. Moreover, we study the PMF of path lifetime which has an immediate impact on end-to-end communications in multi-hop wireless networks in that each path is composed of multiple links. To further understand the implication of link properties, analytical results are used to investigate the upper bound of network connectivity and the associated network performance is evaluated by extensive simulations. We showed that for a large dense network, its network connectivity is bounded by the average node degree, which is equivalent to the multiplication between the average link arrival rate and the average link lifetime.

• Third, by studying the empirical human mobility traces on three different domain scales: campus-wide, city-wide, and county-wide, we demonstrated that both pause time and trip
displacement of human mobility exhibit a cutoff-power distribution. In particular, the cutoff point, defined as characteristic distance $D_c$, in the distribution of trip displacement increases as the human moving domain size increases. Then, we study the joint temporal-spatial effects of pause time and trip displacement on the inter-meeting time of mobile users, when both pause time and trip displacement are characterized by the power-law head in the distribution. We find that the interaction between trip displacement and pause time on human mobility can be manifested by the human diffusive movement patterns [56], which is further characterized by his/her diffusive capability (rate) $r$. Then, by studying the scaling law of human diffusive rate, we show that when the power-law head characterizes both pause time and trip displacement with the power-law coefficients $\alpha$ and $\beta$, respectively, the human diffusive rate $r$ is $r = 2\alpha/\beta$, where $0 < \alpha < 1$ and $0 < \beta < 2$. We further investigate the mixed behaviors of power-law head and exponential tail of a cutoff power-law upon the analysis on empirical human trace datasets with different levels of moving domain sizes. Then, we propose an approximated cutoff power-law distribution, which is featured by a parameter tuple (the power-law coefficient and cutoff point), for instance, $(\alpha, T_c)$ and $(\beta, D_c)$, for pause time and trip displacement, respectively. The approximated cutoff power-law distribution exhibits a close fit with the empirical results from trace files. Finally, with simulation results based on different human diffusive rates, we demonstrate that the human diffusive rate $r$ can effectively characterize the property of contact-based metrics, especially for inter-meeting time, in MANETs. Specifically, we found that superdiffusive rate $r > 1$ leads to the longest power-law head, while the subdiffusive rate $r < 1$ results in the shortest power-law head in the distribution of inter-meeting time. Thus, the higher the diffusive rate is, the longer the power-law head is in the distribution of inter-meeting time.

- Fourth, we studied the inherent properties of group moving behaviors in MANETs. The group moving behaviors and the resulting time-varying group structures are affected by the correlation strength between group members. Hence, we first studied the user pair correlation in MANETs. We suggested that the group member correlation degree is jointly characterized by the human social correlation degree and the similarity of the users’ relative movements and geographic locations. Specifically, we applied information theory to qualitatively measure the human social correlation degree between a user pair in MANETs. Then we investigated the similarity of nodes’ movement and physical distance according to the statistic measure.
We observed that a general group in MANETs typically experiences a birth-to-death process including group initiation, group evolution with member join and leave and group deformation in sequence. Then based on the user-pair correlation, we developed a new metric, called node connectness to characterize the stability level of inter-connections among the node with its group neighbors. And this stability level can reflect the potential node switch and detach event in a group. In addition, we study the routing impact of a node regarding its group structure, which is based on the node neighbors’ degree metric. With this metric, we can improve the routing throughput and bandwidth utilization for multicast traffic inside a group. The overall network routing performance is affected by the group evolution behaviors. Therein, we provided a metric, named group evolution degree, to characterize the group member variation degree due to node detach during the group evolution process. Furthermore, we investigate the condition for group member switching between groups. Finally, we propose a novel birth-to-death group mobility model for both analytical and simulation studies in MANETs. As a case study of routing protocol evaluation, we demonstrate that the widely applied RPGM model stresses AODV much less than the proposed BDGM model, since RPGM model does not take the group birth-to-death evolution behaviors into account, and could bring biased results for group mobility based study.

The rest of the report is organized as follows. Chapter 2 presents our proposed SMS mobility pattern, followed by the steady state analysis and model applications. In Chapter 3, we elaborate the analysis on link properties in the presence of node mobility and radio environments. Chapter 4 presents both empirical and analytical studies of human diffusive behaviors governed by the temporal-spatial limitations of human social behaviors. Chapter 5 first presents the resemblance between mobile social networks and mobile wireless networks. Then we analyze the inherent properties of group mobility and investigate fundamental metrics for characterizing group moving behaviors. Finally, in Chapter 6, we conclude this dissertation, and discuss several future work.
Chapter 2

Design and Study A Unified Mobility Model for Analysis and Simulation of Mobile Wireless Networks

In this chapter, we present the first research topic in this doctoral study, which is to design and analyze a novel Semi-Markov Smooth Mobility (SMS) model for mobile wireless networks. We first motivate this study and review some related works in Chapter 2.1. Next, we describe the detail movement patterns of the proposed SMS mobility model in Chapter 2.2. Then, we analyze the Transient stochastic properties of SMS model regarding the major mobility metrics of movement duration, speed, and trace length in Chapter 2.3. After that we investigate both the transient and the steady state properties of the SMS model in Chapter 2.4 and Chapter 2.5, respectively. In Chapter 2.6, we validate the analysis of SMS steady state properties with simulations. The impacts and applications of SMS model in mobile wireless networks are elaborated in Chapter 2.7. Finally, we summarize our main contributions of the first work in Chapter 2.8.

2.1 Motivation and Related Work

Mobility models are widely used in simulation studies for large-scale wireless networks, especially in the mobility-related research areas, such as routing protocol design [9], link and path duration analysis [5], location and resource management [50, 68, 69], topology control[70, 71, 72], and network connectivity issues [73, 74, 75] in cellular systems, wireless local area networks, and...
mobile ad hoc networks. There are many mobility models which have been well studied in survey papers [15, 14, 9] which are focused on synthetic models, except few experimental studies [10, 11]. The challenge in designing mobility models is the trade-off between model efficiency and accuracy.

Due to easy analysis and simple implementation, random mobility models, in which each mobile node moves without constraint on its velocity, time period and destination, are most widely used [9]. For example, random walk (RW) model was originally proposed to emulate the unpredictable movements of particles in physics, also known as Brownian motion [9]. Compared to the RW model, Random Waypoint (RWP) model [16] is often used to evaluate the performance of mobile ad hoc networks (MANETs) due to its simplicity. However, two flaws have been found in its stationary behavior. First, Yoon showed that the average node speed of RWP model decreases over time [17]. Second, Bettstetter [18] and Blough et al. [28] respectively observed that RWP model has non-uniform spatial node distribution at steady state, with the maximum node density in the middle of simulation region given that initial state is uniform distribution. This implies that analysis and simulations based on RWP model may generate misleading results. To overcome the non-uniform spatial node distribution problem in RWP model, Nain and Towsley recently showed that random direction (RD) model [76] with warp or reflection on the border has uniformly distributed user position and direction in [20]. Recently, Boudec et al. proposed a generic random mobility model, called Random Trip (RT) model, which covers RW, RWP and RD models, as well as their variants [19].

Random mobility models describe the mobility pattern in a macroscopic level, that is, mobile nodes do not change speed or direction within one movement. Thus, random mobility models can fit vehicular and large-scale environments. Therefore, they are insufficient to mimic the minute moving behaviors of mobile users, such as speed acceleration and smooth direction changes within one movement. The movements presented by random models are completely uncorrelated or random, thus demonstrating unrealistic moving behaviors, such as sudden stop, sudden acceleration, and sharp turn frequently occur during the simulation [9]. These abrupt speed and direction change events will influence the network topology change rate, which further significantly affect network performance. In consequence, the simulation results and theoretical derivations based on random mobility models may not correctly indicate the network performance and effects of system parameters.

As an effort to resolve the problem of unrealistic movements and to provide smooth behaviors, mobility models that consider the correlation of node’s moving behavior are proposed,
which are called temporal models [9], such as Smooth Random (SR) model [15] and Gauss-Markov (GM) model [77]. SR model considers the smooth speed and direction transitions for mobile nodes. In the SR model, a node moves at a constant speed along a specific direction until either a speed or direction change event occurs according to independent Poisson process. In the GM model, the velocity of a mobile node at any time slot is a function of its previous velocity with a Gaussian random variable. Although both SR and GM models can be used for either cellular environments or ad hoc networks, they also have application limitations. In the SR model, because of the assumption of Poisson process, the uncertainty of the speed and direction change within each movement make it difficult to evaluate network performance according to a specific node mobility requirement. Moreover, an SR movement may have new speed change during the speed transition and does not stop unless a targeting speed is specified as zero, which is not flexible for movement control. In the GM model, mobile nodes cannot travel along a straight line [14] as long as the temporal correlation (memorial) parameter is not equal to 1, and they do not stop during the simulation. However, in reality, vehicles usually move along a straight line for a period of time and mobile users always move in an intermittent way with a random pause time.

Therefore, we are motivated to design a new mobility model that can abide by the physical law of moving objects to avoid abrupt moving behaviors, and can provide a microscopic view of mobility such that node mobility is controllable and adaptive to different network environments. In summary, this model is expected to unify the desired features as follows:

1. Smooth and sound movements: A mobility model should have temporal features, i.e., a mobile node’s current velocity is dependent on its moving history so that smooth movements can be provided and mobile nodes should move at stable speed without the average speed decay problem [29].

2. Consistency with the physical law of a smooth motion: In order to mimic the kinetic correlation between consecutive velocities in a microscopic level, a mobility model should be consistent with the physical law of a smooth motion in which there exists acceleration to start, stable motion and deceleration to stop for controllable mobility [78, 79].

3. Uniform nodal distribution: As most of analytical studies of MANETs are based on the assumption of uniform nodal distribution, such as network capacity and delay [8], network connectivity, topology control [74] and link change rate [26], a mobility model should be able to generate uniform
spatial node distribution at steady-state. Otherwise, the non-uniform node distribution caused by a mobility model may invoke misleading information and results [20].

4. Adaptation to diverse network application scenarios: In order to properly support rich MANET applications having complex node mobility and network environments, such as group mobility and geographic restriction, a generic mobility model which is adaptive to different mobility patterns is highly desirable.

In addition, compared to the design and simulation studies of mobility modeling in existing literatures, there exists very limited analysis works of mobility models such as steady state analysis of node speed, direction, and node distributions. In order to fill up the gap in the design of microscopic mobility models and the analysis of their steady state properties, we are further motivated to analyze the steady state properties of the proposed SMS model, considered as a unified mobility model, which are essential to understand the system performance and topology dynamics of MANETs.

### 2.2 A Unified Mobility Model: Semi-Markov Smooth (SMS) Model

In this section, we present a novel mobility model, Semi-Markov Smooth (SMS) model with detailed mobility patterns. Specifically, within one movement, we elaborate when and how an SMS node changes its speed and direction; how the node accelerates and decelerates the speed; and how strong the temporal correlation is during the speed and direction change. By answering these questions, the moving behaviors of mobile nodes can be deeply understood and well manifested by the proposed model. Further, we describe the stochastic process of the SMS model. We consider it as a *renewal process* with respect to consecutive SMS movements and regard it as a *semi-Markov* process in the study of iterative phases transition.

#### 2.2.1 Model Description

According to the fundamental physical law of a smooth motion, a moving object would experience *speed acceleration*, *stable speed*, and *speed deceleration* during a movement. This implies that a smooth movement should contain multiple moving stages, and a temporal correlation exists during the speed transition. Therefore, based on the physical law of smooth motion, our proposed SMS model consists of four consecutive phases: *Speed Up* phase, *Middle Smooth phase*,
Slow Down phase, and Pause phase. Throughout this dissertation, we define one SMS movement or movement duration as the time period that a node travels from the starting point to its next position that the node stops moving. Each movement is quantized into $K$ equidistant time steps, where $K \in \mathbb{Z}$. The time interval between two consecutive time steps is denoted as $\Delta t$ (sec). The SMS model is an entity model which determines the mobility pattern of each user individually, which is designed with the following assumptions:

- All mobile nodes move independently from others and have the identical stochastic properties.
- For any mobile nodes, every SMS movement has identical stochastic properties.
- Within each SMS movement, a mobile node’s velocity at current time step is dependent on its moving history.

Here, the first two assumptions are similar to the random mobility models for better supporting theoretical studies as well system performance evaluation of MANETs. To be distinguished from random mobility models, the third assumption of the SMS model is especially used for describing microscopic smooth nodal movements. Next, we describe the SMS model based on the single user mobility. The implementation for group mobility and geographic constraints will be explained in Chapter 2.7.

**Speed Up Phase ($\alpha$–Phase)**

For every movement, an object needs to accelerate its speed before reaching a stable speed. In this phase, the movement pattern of an SMS node is exploited from what is defined in SR model [15]. Thus, the first phase of a movement is called Speed up $\alpha$–Phase, which covers the time interval $[t_0, t_\alpha] = [t_0, t_0 + \alpha \Delta t]$. At initial time $t_0$ of a movement, the node randomly selects a target speed $v_\alpha \in [v_{\min}, v_{\max}]$, a target direction $\phi_\alpha \in [0, 2\pi]$, and the total number of time steps $\alpha$ for the speed up phase. $\alpha \in \mathbb{Z}$ and is selected in the range of $[\alpha_{\min}, \alpha_{\max}]$. These three random variables are independently uniformly distributed. Note that $\alpha_{\min}$ and $\alpha_{\max}$ imply the physical speed-up capabilities of the node. For instance, when $\alpha$ is selected close to $\alpha_{\max}$, the transition time is long, but the degree of temporal correlation is strong.

In reality, an object typically accelerates the speed along a straight line. Thus, the direction $\phi_\alpha$ does not change during this phase. To avoid sudden speed change, the node will evenly accelerate its speed along direction $\phi_\alpha$ from starting speed $v(t_0) = 0$, to the target speed $v_\alpha$, which is the
ending speed of \( \alpha \)-phase, i.e., \( v(t_\alpha) = v_\alpha \). Hence, the mobility pattern in speed up \( \alpha \)-phase is represented by a triplet \((\alpha, v_\alpha, \phi_\alpha)\). The acceleration rate at \( \alpha \)-phase, \( a_\alpha \), can be obtained by
\[
a_\alpha = \frac{v_\alpha - v(t_0)}{t_\alpha - t_0} = \frac{v_\alpha}{\alpha \Delta t}.
\]
(2.1)

An example of speed change in \( \alpha \)-phase is shown in Figure 2.1(a), where the node speed increases evenly step by step and reaches the target speed \( v_\alpha \) by the end of this phase.

**Middle Smooth Phase (\( \beta \)-Phase)**

After the initial acceleration, a moving object will have a smooth movement, even though moving speed and direction may change. Accordingly, once the node transits into \( \beta \)-phase at time \( t_\alpha \), the node randomly selects \( \beta \) time steps and moves into \( \beta \)-phase. During time interval \((t_\alpha, t_\beta] = (t_\alpha, t_\alpha + \beta \Delta t)\), where \( \beta \in \mathbb{Z} \) is uniformly distributed over \([\beta_{\min}, \beta_{\max}]\). As a succeeding phase of \( \alpha \)-phase, the initial value of speed \( v_0 \) and direction \( \phi_0 \) in \( \beta \)-phase are \( v_\alpha \) and \( \phi_\alpha \), respectively. The speed and direction during \( \beta \)-phase then change on the basis of \( v_\alpha \) and \( \phi_\alpha \) at a memory level parameter \( \zeta \), which ranges over \([0, 1]\) and is constant for both speed and direction at each time step. Thus, by adjusting parameter \( \zeta \), we can easily control the degree of temporal correlation of velocity between two consecutive steps. For example, let us assume that the standard deviation \( \sigma_v \) and \( \sigma_\phi \) are 1, which implies that the speed or direction difference between two consecutive time steps is less than 1 \( m/s \) or 1 rad within \( \beta \)-phase. Through simulations, we find that this granularity is sufficient to describe user mobility.

Then the step speed (i.e., speed at a time step) and step direction (i.e., direction at a time step) at the \( j^{th} \) time step for an SMS node in \( \beta \)-phase are presented by:
\[
v_j = \zeta v_{j-1} + (1 - \zeta) v_\alpha + \sqrt{1 - \zeta^2} \tilde{V}_{j-1}
= \zeta^j v_0 + (1 - \zeta^j) v_\alpha + \sqrt{1 - \zeta^2} \sum_{m=0}^{j-1} \zeta^{j-m-1} \tilde{V}_m
= v_\alpha + \sqrt{1 - \zeta^2} \sum_{m=0}^{j-1} \zeta^{j-m-1} \tilde{V}_m,
\]
(2.2)
and
\[
\phi_j = \zeta \phi_{j-1} + (1 - \zeta) \phi_\alpha + \sqrt{1 - \zeta^2} \tilde{\phi}_{j-1}
= \phi_\alpha + \sqrt{1 - \zeta^2} \sum_{m=0}^{j-1} \zeta^{j-m-1} \tilde{\phi}_m,
\]
(2.3)
where $\tilde{V}_j$ and $\tilde{\phi}_j$ are two stationary Gaussian random variables with zero mean and unit variance, independent from $V_j$ and $\phi_j$, respectively. By substituting $\beta$ for $j$ into (2.2) and (2.3), the ending step speed $v_\beta$ and ending step direction $\phi_\beta$ of $\beta$-phase are obtained as:

$$v_\beta = v_\alpha + \sqrt{1 - \zeta^2} \sum_{m=0}^{\beta-1} \zeta^{\beta-m-1} \tilde{V}_m,$$

$$\phi_\beta = \phi_\alpha + \sqrt{1 - \zeta^2} \sum_{m=0}^{\beta-1} \zeta^{\beta-m-1} \tilde{\phi}_m.$$ 

Figure 2.1: An example of speed and direction transition in one SMS movement.

Given (2.2) and (2.3), the mobility pattern in $\beta$-phase is represented by a quartet $(\beta, v_\alpha, \phi_\alpha, \zeta)$. As shown in Figure 2.1(a), node speed fluctuates around the speed $v_\alpha$ achieved at the end of $\alpha$-phase in each step of $\beta$-phase. In an SMS movement, $\beta$-phase is the main moving phase, which characterizes the mobility level and direction for a node during the entire movement.

**Slow Down Phase ($\gamma$-Phase)**

According to the physical law of motion, every moving object needs to reduce its speed to zero before a full stop. In order to avoid the sudden stop event happening in the SMS model, we consider that the SMS node experiences a slow down phase to end one movement. In detail, once the node transits into $\gamma$-Phase, i.e., slow down phase, at time $t_\beta$, it randomly selects $\gamma$ time steps and a direction $\phi_\gamma$. Hence, $\gamma$-phase contains the last $\gamma$ time steps of one SMS movement over time.
interval \((t_\beta, t_\gamma] = (t_\beta, t_\beta + \gamma \Delta t]\), where \(\gamma \in \mathbb{Z}\) and is uniformly distributed over \([\gamma_{\min}, \gamma_{\max}]\). The SMS node will evenly decelerate its speed from \(v_\beta\) to \(v_\gamma = 0\). The rightmost part in Figure 2.1(a) shows an example of speed change in \(\gamma\)-phase. In reality, a moving object typically decelerates the speed along a straight line before a full stop. Thus, the direction \(\phi_\gamma\) does not change during the \(\gamma\)-phase. In order to avoid the sharp turn event happening during the phase transition, \(\phi_\gamma\) and \(\phi_\beta\) are correlated. Specifically, \(\phi_\gamma\) is obtained from (2.3), by substituting \(\beta\) for \(j - 1\). At the end of \(\gamma\)-phase, i.e., \(t_\gamma\), a mobile node stops at the destination position of its current movement. Thus, the mobility pattern in \(\gamma\)-Phase is represented by a triplet \((\gamma, v_\beta, \phi_\gamma)\). The deceleration rate \(a_\gamma\) is given by:

\[
a_\gamma = \frac{v_\gamma - v_\beta}{t_\gamma - t_\beta} = -\frac{v_\beta}{\gamma \Delta t}.
\] (2.5)

Figure 2.1(a) shows an example of speed vs. time during one SMS movement, which contains a total number of \(K = 37\) small consecutive steps, where \(\alpha\)-phase contains 12 time steps, \(\beta\)-phase includes 18 time steps, and \(\gamma\)-phase consists of 7 time steps. After each movement, a mobile node may stay for a random pause time \(T_p\). Correspondingly, Figure 2.1(b) illustrates the node’s direction behavior of the same SMS movement. It is clear to observe that the direction is constant in both \(\alpha\) and \(\gamma\)-phase. Specifically, \(\phi_\alpha = 0.8\) rad and \(\phi_\gamma = 0.68\) rad, respectively. And the direction fluctuates around \(\phi_\alpha\) at each step in \(\beta\)-phase based on (2.3). Moreover, the largest direction transition between two consecutive time steps is \(0.23\pi\) between 28th and 29th second, which means only small direction change occurs in each time step. As the difference between \(\phi_\alpha\) and \(\phi_\gamma\) is small in this example, it implies a strong temporal correlation between consecutive velocities for the node. Thus, the target direction \(\phi_\alpha\) effectively predicts the direction of the entire SMS movement.

2.2.2 Stochastic Process of SMS Model

Since the proposed SMS model is based on the physical law of moving objects, the smooth movement is acquired without sharp turns and sudden stop. However, as described in Chapter 2.1 that we also expect a mobility model to have other properties such as steady speed and uniform spacial node distribution. Therefore, we investigate the properties of SMS model based on the analysis of stochastic process.
Renewal Process of SMS Model

For proper nomenclature, random variables used to represent the process in SMS stochastic model are denoted at two levels: all random variables denoted for SMS movements are written in upper case and within each movement, all random variables denoted for trips for each time step are written in normal font. Multi-dimensional variables (e.g., random coordinates in the simulation area) are written in bold face. In the proposed model, all mobile nodes move independently from each other and have the identical stochastic properties, so that it is sufficient to focus on the study of stochastic process of the SMS model with respect to a specific node. Hence, we suppress the node index in the denotations of random variables for simplicity. The discrete-time parameter $i$ indexes the $i^{th}$ movement of an SMS node, where $i \in \mathbb{N}_o$, and $\mathbb{N}_o$ is denoted as a countable non-negative integer space. The discrete-time parameter $j$ indexes the $j^{th}$ time step within one movement of an SMS node. For discrete-time stochastic process in SMS model, all indexes are written in normal font. For example, $v(i,j)$ denotes node speed at $j^{th}$ time step within its $i^{th}$ movement.

For the rest of paper, we use the following denotations for movements and steps within each movement:

- **SMS movement:** Movement Duration $T(i)$ represents the time period of the $i^{th}$ movement from the beginning of $\alpha$–phase to the end of $\gamma$–phase; Movement Position $P(i)$ represents the ending position of the $i^{th}$ movement; and $K(i)$ is total number of time steps of the $i^{th}$ movement, where $K(i) = \alpha(i) + \beta(i) + \gamma(i)$.

- **Steps:** Step Position $d(i,j)$, Step Speed $v(i,j)$, and Step Direction $\phi(i,j)$ denote the ending position, speed, and direction at the $j^{th}$ time step of the $i^{th}$ movement, respectively.

As a new transition happens at the beginning time at each SMS movement, the movement can be regarded as a recurrent event. The movement duration $T(i)$ represents the time between the $(i - 1)^{th}$ and the $i^{th}$ transition of the SMS movement. Since $T(i)$ is an i.i.d. random variable, the movement of SMS model is a renewal process $\{N(t); t \geq 0\}$, where $N(t)$ denotes the number of renewals by time $t$ [80]. Without considering the pause time after each movement, the time instant $S_i$ at which the $i^{th}$ renewal occurs, is given by:

$$S_i = T(1) + T(2) + \ldots + T(i), \quad S_0 = 0, \quad i \in \mathbb{N}.$$  \hspace{1cm} (2.6)

Hence, $N(t)$ is formally defined as $N(t) \triangleq \max\{i : S_i \leq t\}$. If consider that a random pause time $T_p$ comes after each SMS movement, then we denote $\bar{T}(i)$ as the time between the $(i - 1)^{th}$
and the $i^{th}$ transition of the SMS process, where $\bar{T}(i) = T(i) + T_p(i)$. Because $T_p(i)$ is an i.i.d random variable and independent from $T(i)$, $\bar{T}(i)$ is also an i.i.d random variable. Therefore, the consecutive movements of SMS model is a discrete-time renewal process irrespective of pause time.

Semi-Markov Process of SMS Model

Since an SMS movement consists of three consecutive moving phases and a pause $p$-phase, the continuous-time SMS can be represented as an iterative four-state transition process, which is shown in Figure 2.2.

![Four-state transition process in SMS model.](image)

Let $I$ represent the set of phases in SMS process, then $I(t)$ denotes the current phase of SMS process at time $t$, where $I = \{I_\alpha, I_\beta, I_\gamma, I_p\}$. Let $\{Z(t); t \geq 0\}$ denote the process which makes transitions among four phases in $I$ and $\gamma_n$ denote the state of $\{Z(t)\}$ at the epoch of its $n^{th}$ transition. Because the transition time between consecutive moving states in an SMS movement has a discrete uniform distribution, as shown in Figure 2.2, instead of an exponential distribution. By this argument, $\{Z(t)\}$ is a semi-Markov process [81]. This is the very reason that our mobility model is called semi-Markov smooth model because it can be modeled by a semi-Markov process and it complies with the physical law with smooth movement. Accordingly, $\{\gamma_n; n \in \mathbb{N}\}$ is the four-state embedded semi-Markov chain of $\{Z(t)\}$.

We denote $P_{s1s2}$ as the probability that when $\{\gamma_n\}$ enters a state $s1$, the next state will be entered is $s2$. Further, $F_{s1s2}(t)$ denotes the cumulative distribution function (CDF) of the time to make the $s1 \rightarrow s2$ transition when the successive states are $s1$ and $s2$. Let $T_{s1}$ represent the holding-time that the process stays at state $s1$ before making a next transition. Then we use $H_{s1}(t)$ to represent the CDF of holding-time at state $s1$, which is defined as follows [81]:

$$H_{s1}(t) = \sum_{s2 \in I} \text{Prob}\{T_{s1} \leq t \mid \text{next state } s2\} P_{s1s2} = \sum_{s2 \in I} F_{s1s2}(t) P_{s1s2}. \quad (2.7)$$
According to four-state semi-Markov chain \( \{Y_n\} \) shown in Figure 2.2, there is only unidirectional transition between two adjacent states in SMS model, such that the state transition probability \( P_{s_1s_2} = 1 \), if and only if \( s_1s_2 \in \{I_\alpha I_\beta, I_\beta I_\gamma, I_\gamma I_p, I_p I_\alpha\} \). Under this condition, \( H_{s_1}(t) \) is equal to \( F_{s_1s_2}(t) \) in the SMS model. Specifically, \( F_{I_\alpha I_\beta}(t) \), \( F_{I_\beta I_\gamma}(t) \) and \( F_{I_\gamma I_p}(t) \) have discrete uniform distribution, while pause time \( T_p \) can follow an arbitrary distribution, denoted by \( f_{T_p}(t) \). Therefore, from (2.7), the probability density function (pdf) of holding-time \( T_{s_1} \) in SMS model is a discrete uniform distribution when \( s_1 \neq I_p \) and is \( f_{T_p}(t) \) when \( s_1 = I_p \). Therefore, this mobility model follows a semi-Markov process because the pdf of state holding time is not exponential distribution for Markov process.

2.3 Stochastic Properties of SMS Model

Mobility model has a significant impact on both simulation-based and analytical-based study of wireless mobile networks. A deep understanding knowledge of the moving behaviors of this model is critical to properly configure the mobility parameters for a variety of network scenarios; to correctly interpret simulation results, especially for routing protocol design and evaluation; and to provide fundamental theoretical results for other mobility related analytical studies such as link and path lifetime and network connectivity analysis. Therefore, we study the stochastic properties of SMS model in this section.

Although different mobility models lead to different mobility patterns, the stochastic properties of movement duration, speed, and trace length (distance) are of major interests in all mobility models because they are immediate metrics to characterize a node’s movement. In fact, in every mobility model, at least two of these metrics are specified. For example, in RWP model, the mobility pattern of each movement is characterized by the trace length (distance) and speed, whereas RD model combines movement duration, speed and direction together to dominate the mobility pattern.

Based on the model assumption in Chapter 2.2.1, the SMS process is a renewal process, so that it is sufficient to study the stochastic properties of a single movement in SMS model by omitting the index \( i \) and use lower case \( j \) alone to identify the random variable within one SMS movement. For example, \( T \) represents the movement duration, and \( v_j \) denotes the step speed at the \( j^{th} \) time step of this specific SMS movement.
2.3.1 Movement Duration

In each movement, we consider the *movement duration* as the time period from the beginning of $\alpha$–phase to the end of $\gamma$–phase. According to the definition of SMS model in Chapter 2.2.1, $T = K \Delta t$ and $\Delta t$ is a constant unit value, where $K$ is the summation of three independent uniformly distributed random variables $\alpha$, $\beta$, and $\gamma$. Therefore, the duration time of one SMS movement is determined by the number of time steps $K$ and movement duration follows the same probability mass function (pmf) of variable $K$. So, we first derive the distribution of time steps $K$ of SMS model and then, we can find the expected movement duration, $E\{T\}$. For the simplicity of denotation, we set the range of $\alpha$, $\beta$, and $\gamma$ as $[N_{\text{min}}, N_{\text{max}}]$ in the remaining of the paper, i.e., $\alpha_{\text{min}} = \beta_{\text{min}} = \gamma_{\text{min}} = N_{\text{min}}$ and $\alpha_{\text{max}} = \beta_{\text{max}} = \gamma_{\text{max}} = N_{\text{max}}$. The detailed derivation of the pmf of time steps $K$ is shown in APPENDIX 2.9. By integrating the results of three scenarios discussed in APPENDIX 2.9, that is, we combine (2.46), (2.47), and (2.48), the pmf of time steps $K$ is obtained as:

$$
\text{Prob}\{ T = k \Delta t \} = \text{Prob}\{ K = k \} = \begin{cases} 
\frac{k^2 - 6k \cdot N_{\text{min}} + 3k + 9N_{\text{max}}^2 - 9N_{\text{min}}^2 + 2}{2(N_{\text{max}} - N_{\text{min}} + 1)^3}, & \text{Case 1} \\
\frac{6k(N_{\text{min}} + N_{\text{max}}) - 2k^2 - 3(N_{\text{min}}^2 + N_{\text{max}}^2 + 4N_{\text{min}} N_{\text{max}} + N_{\text{min}} - N_{\text{max}} + 2)(N_{\text{max}} - N_{\text{min}} + 1)^3}{2(N_{\text{max}} - N_{\text{min}} + 1)^3}, & \text{Case 2} \\
\frac{k^2 - 6k \cdot N_{\text{max}} - 3k + 9N_{\text{max}}^2 + 9N_{\text{max}}^2 + 2}{2(N_{\text{max}} - N_{\text{min}} + 1)^3}, & \text{Case 3}
\end{cases}
$$

where in Case 1, $3N_{\text{min}} \leq k \leq 2N_{\text{min}} + N_{\text{max}}$; in Case 2, $2N_{\text{min}} + N_{\text{max}} \leq k \leq 2N_{\text{max}} + N_{\text{min}}$; and in Case 3, $2N_{\text{max}} + N_{\text{min}} \leq k \leq 3N_{\text{max}}$. In addition, as the duration time in each moving phase follows uniform distribution and is assumed to be equal over the range $[N_{\text{min}}, N_{\text{max}}]$, the expected value of *Movement Duration* $T$ can be derived as follows:

$$
E\{T\} = (E\{\alpha\} + E\{\beta\} + E\{\gamma\}) \Delta t = \frac{3\Delta t}{2} (N_{\text{min}} + N_{\text{max}}).
$$

Since $\Delta t$ is a constant unit time, in order to simplify the presentation, $\Delta t$ is normalized to unity in the rest of the paper.

The distribution of movement duration can be used to determine the probability of a specific movement period as shown in Figure 2.3 in which the pmf and CDF of one SMS movement duration according to different phase duration range $[6, 30]$ seconds and $[10, 40]$ seconds are demonstrated in Figure 2.3(a) and Figure 2.3(b), respectively.
As each phase duration in the SMS movement has a discrete uniform distribution, from (2.9), the average movement duration is 54 seconds when $[N_{\text{min}}, N_{\text{max}}] = [6, 30]$ seconds, and is 75 seconds when $[N_{\text{min}}, N_{\text{max}}] = [10, 40]$ seconds, respectively. Hence, in Figure 2.3(a), it is clear to see that the peak value of pmf exists at time instant $54^{th}$ and $75^{th}$ seconds, respectively. Because the entire SMS movement duration is within the range $[3N_{\text{min}}, 3N_{\text{max}}]$ seconds, we can see from Figure 2.3(b) that the CDF of SMS movement monotonously increases with time, and becomes 1 when the duration time reaches the maximum movement duration range, i.e., $3N_{\text{max}}$ seconds.

The knowledge of movement duration is useful to understand the dynamics of user mobility, e.g., longer movement duration means a user spends more time in moving. Upon the distribution of the SMS movement duration, we can flexibly configure the SMS model to mimic different types of mobile nodes with specific average movement duration times. Moreover, when applying the SMS model in a cellular network, it is desirable to obtain the distribution of movement duration of mobile users for studying time-based updating schemes.

### 2.3.2 Stochastic Properties of Step Speed

In random models, the study of stochastic properties is based on the movement process. However, being distinguished from random models, one of the main features of the SMS model is that each movement is quantized into many tiny time steps in order to capture the transient behaviors of moving objects from a microscopic view. Therefore, the stochastic properties of the SMS model can be characterized at the macroscopic level (i.e., movement) and microscopic level (time steps).
In order to properly control the SMS model with respect to different mobility parameters for both simulation and analytical study, it is necessary to investigate the properties of SMS model on the microscopic level, which is more challenging than that on the macroscopic level.

Recall the denotations in Chapter 2.2.2, we use step speed to describe the speed within each time step. Therefore, we study the stochastic properties of step speed $v_j$, where $1 \leq j \leq N_{\text{max}}$ for each moving phase. The denotations of random variables applied for studying the stochastic properties of step speed are presented as follows.

\begin{align*}
v_{\alpha}: & \quad \text{the target speed for } \alpha-\text{phase.} \\
f_{v_{\alpha}}(v): & \quad \text{the pdf of target speed } v_{\alpha}. \\
v_{\beta}: & \quad \text{the ending speed in } \beta-\text{phase.} \\
f_{v_{\beta}}(v): & \quad \text{the pdf of step speed } v_{\beta}. \\
v_{\alpha}(j): & \quad \text{the } j^{th} \text{ step speed in } \alpha-\text{phase.} \\
v_{\beta}(j): & \quad \text{the } j^{th} \text{ step speed in } \beta-\text{phase.} \\
f_{v_{\alpha}(j)}(v): & \quad \text{the pdf of } j^{th} \text{ step speed } v_{\alpha}(j) \text{ in } \beta-\text{phase.} \\
v_{\gamma}(j): & \quad \text{the } j^{th} \text{ step speed in } \gamma-\text{phase.} \\
E_{\alpha}\{v\}: & \quad \text{the average speed of a node in entire } \alpha-\text{phase} \\
E_{\beta}\{v\}: & \quad \text{the average speed of a node in entire } \beta-\text{phase} \\
E_{\gamma}\{v\}: & \quad \text{the average speed of a node in entire } \gamma-\text{phase}
\end{align*}

**Step speed in $\alpha$-phase**

From (2.1), the $j^{th}$ step speed in $\alpha$-phase is represented as $v_{\alpha,j} = \frac{v_{\alpha}}{\alpha} j$. We consider two cases of time step index $j$ with the range of $\alpha$-phase duration $[N_{\text{min}}, N_{\text{max}}]$. One is the case when $j \leq N_{\text{min}}$, the other is the case when $N_{\text{min}} < j \leq N_{\text{max}}$. So that the effective $\alpha$-phase duration range according to a specific time step index $j$ is $[\max\{N_{\text{min}}, j\}, N_{\text{max}}]$. Then, the CDF of $j^{th}$ step speed in $\alpha$-phase is derived as follows:

$$
Pr\{v_{\alpha}(j) \leq v\} = Pr\left\{\frac{v_{\alpha}}{\alpha} j \leq v\right\} = Pr\{v_{\alpha} \leq \frac{v \cdot \alpha}{j}\} \\
= E\{1_{\{v_{\alpha} \leq \frac{v}{\alpha}\}}\} = \sum_{m=\max\{N_{\text{min}}, j\}}^{N_{\text{max}}} E\{1_{\{v_{\alpha} \leq \frac{v}{\alpha}\}} \cdot 1_{\{\alpha=m\}}\} \\
= \frac{1}{N_{\text{max}} - \max\{N_{\text{min}}, j\} + 1} \sum_{m=\max\{N_{\text{min}}, j\}}^{N_{\text{max}}} Pr\{v_{\alpha} \leq \frac{v \cdot m}{j}\} \\
= \frac{1}{N_{\text{max}} - \max\{N_{\text{min}}, j\} + 1} \sum_{m=\max\{N_{\text{min}}, j\}}^{N_{\text{max}}} \int_{v_{\text{min}}}^{\frac{v \cdot m}{j}} f_{v_{\alpha}}(v)dv, \quad (2.10)
$$

where $v_{\text{min}} \leq v \leq v_{\text{max}}$. Here, $1_{\{\cdot\}}$ is the indicator function [82]. Thus, if the event that $\{v_{\alpha} \leq \frac{v_{\alpha}}{\alpha}\}$ is true, then $1_{\{v_{\alpha} \leq \frac{v_{\alpha}}{\alpha}\}} = 1$, otherwise $1_{\{v_{\alpha} \leq \frac{v_{\alpha}}{\alpha}\}} = 0$. Note, in the above equation, if $(\frac{v \cdot m}{j}) > \frac{v_{\text{min}}}{\alpha}$,
v_{\text{max}} \& m < N_{\text{max}}), \text{ then } \int_{v_{\text{min}}}^{v_{\text{max}}} f_{v_{\alpha}}(v) dv = \text{Pr}\{v_{\alpha} \leq \frac{v_{\text{min}}}{2}\} = 1. \text{ Moreover, as the time step index } j \text{ increases, the probability } \text{Pr}\{v_{\alpha} \leq \frac{v_{\text{min}}}{2}\} \text{ decreases. Hence, the CDF of step speed in } \alpha\text{-phase is inverse proportion to the time step } j. \text{ In the special case, when } j = N_{\text{max}}, \text{ then the equation (2.10) is equivalent to the expression of CDF of the target speed } v_{\alpha}, \text{ i.e., the ending speed of } \alpha\text{-phase. The average speed of a node } E_{\alpha}\{v\} \text{ in the entire } \alpha\text{-phase is equal to the expected trace length in } \alpha\text{-phase over the average phase duration time. Based on the description of } \alpha\text{-phase, } E_{\alpha}\{v\} \text{ is obtained as:}

$$E_{\alpha}\{v\} = \frac{1}{2} E\{v_{\alpha}\} = \frac{1}{4}(v_{\text{min}} + v_{\text{max}}). \quad (2.11)$$

Hence, for each SMS movement, the average speed of a node in the speed up phase is half of the average target speed } v_{\alpha}, \text{ i.e., half of the expected stable speed, selected for that movement.}

**Step speed in } \beta\text{-phase**}

For simplicity, let us denote } \tilde{V}_{j} \text{ as } \sqrt{1 - \zeta^2} \sum_{m=0}^{j-1} \zeta^{j-m-1} \bar{V}_{m} \text{ in (2.2). Then the speed at the } j^{th} \text{ step of } \beta\text{-phase in (2.2) is equivalent to } v_{\beta}(j) = v_{\alpha} + \tilde{V}_{j}. \text{ Since the summation of } n \text{ independent Gaussian random variables is still a Gaussian random variable [83], } \tilde{V}_{j} \text{ is a Gaussian random variable with zero mean and variance } \sigma_{\tilde{V}_{j}} = 1 - \zeta^{2j}, \text{ i.e., } \tilde{V}_{j} \sim N(0, 1 - \zeta^{2j}). \text{ In this way, } v_{\beta}(j) \text{ is the summation of two independent random variables, } v_{\alpha} \text{ and } \tilde{V}_{j}. \text{ Based on (equation 6-38) in [83], the pdf of } j^{th} \text{ step speed in } \beta\text{-phase, } v_{\beta}(j) \text{ is derived as:}

$$f_{v_{\beta}(j)}(v) = \int_{-\infty}^{\infty} f_{v_{\alpha}}(v-u) f_{\tilde{V}_{j}}(u) I_{\{v_{\text{min}} \leq v-u \leq v_{\text{max}}\}} \, du$

$$= \frac{1}{v_{\text{max}} - v_{\text{min}}} \int_{v_{\text{min}}}^{v_{\text{max}}} \frac{1}{\sqrt{2\pi}\sigma_{u}} e^{-\frac{u^2}{2\sigma_{u}^2}} \, du$

$$= \frac{1}{v_{\text{max}} - v_{\text{min}}} \int_{v_{\text{min}}}^{v_{\text{max}}} \frac{1}{\sqrt{2\pi}\sigma_{u}} e^{-\frac{1}{2\sigma_{u}^2}} \, du$

$$= \frac{G\left(\frac{v-v_{\text{min}}}{\sqrt{1-\zeta^{2j}}}\right) - G\left(\frac{v-v_{\text{max}}}{\sqrt{1-\zeta^{2j}}}\right)}{v_{\text{max}} - v_{\text{min}}}, \quad (2.12)$$

where function } G(\cdot) \text{ is defined as } G(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} \, dy, \text{ and } \sigma_{u} = \sigma_{\tilde{V}_{j}} = 1 - \zeta^{2j}.

Note that there exists a non-zero probability that } \tilde{V}_{j} \text{ has a very large magnitude for the Gaussian random variable } \tilde{V}_{j} \in \mathbb{R}. \text{ However, according to the tail of the Gaussian pdf, the probability that } \tilde{V}_{j} \text{ is selected very far away from its mean value is considerable small. For instance, } P\{\tilde{V}_{j} - E\{\tilde{V}_{j}\} < 3\sigma_{\tilde{V}_{j}}\} = 0.9973 \text{ [83]. Since } E\{\tilde{V}_{j}\} = 0, \tilde{V}_{j} \text{ is mostly selected from the domain}
\[-3\sigma_{V_j}, 3\sigma_{V_j}\] and \(\sigma_{V_j} = \sqrt{1 - \zeta^2_j}\) is less than 1. Thus, one can safely ignore the large values of \(V_j\) during the simulation. To further prevent this potential simulation abnormality, we use a threshold which is the same as \(v_{\text{max}}\) for the step speed selection in \(\beta\)-phase: if the new target speed is higher than \(v_{\text{max}}\) or less than 0 m/sec, the speed will be reselected.

Based on above assumption, with (2.12), the CDF of step speed in \(\beta\)-phase is derived as:

\[
Pr\{v_{\beta}(j) \leq v\} = \int_{v_{\text{min}}}^{v_{\text{max}}} G\left(\frac{v - v_{\text{min}}}{\sqrt{1 - \zeta^2_j}}\right) - G\left(\frac{v - v_{\text{max}}}{\sqrt{1 - \zeta^2_j}}\right) dv \frac{v_{\text{max}} - v_{\text{min}}}{v_{\text{max}} - v_{\text{min}}}. \tag{2.13}
\]

From (2.13), the CDF of step speed in \(\beta\)-phase decreases gradually as the time step \(j\) increases. In turn, the value of \(\sqrt{1 - \zeta^2_j}\) immediately approaches 1, such that the difference of CDFs between consecutive step speeds is imperceptible. This property is distinct from that in \(\alpha\)-phase in which according to (2.10), the CDF of step speed decreases as \(j\) increases. Therefore, the theoretical expressions are consistent with the designed mobility patterns in that the step speed in \(\alpha\)-phase increases gradually and becomes stable in the entire \(\beta\)-phase. Because the step speed in \(\beta\)-phase changes on the basis of target speed \(v_{\alpha}\) with a zero mean Gaussian random variable \(\tilde{V}\), upon (2.19), the average speed of a node in the entire \(\beta\)-phase is:

\[
E_{\beta}\{v\} = E\{v_{\alpha}\} = \frac{1}{2}(v_{\text{min}} + v_{\text{max}}). \tag{2.14}
\]

Hence, the physical meaning of (2.14) indicates that for each SMS movement, the average speed of a node in the middle smooth phase is just the expected stable speed selected for that movement.
Step speed in $\gamma$–phase

From (2.5), the $j^{th}$ step speed in $\gamma$–phase is represented as $v_{\gamma}(j) = v_\beta(1 - \frac{j}{\gamma})$. With the same methodology of derivation in (2.10), the CDF of $j^{th}$ step speed in $\gamma$–phase is given by:

$$Pr\{v_{\gamma}(j) \leq v\} = Pr\{v_\beta(1 - \frac{j}{\gamma}) \leq v\} = E\{1_{\{v_\beta \leq \frac{v}{1 - \frac{j}{\gamma}}\}}\}$$

$$= \sum_{m=\max\{N_{min}, j\}}^{N_{max}} 1_{\{v_\beta \leq \frac{v}{m-j}\}} \cdot 1_{\{\gamma=m\}}$$

$$= \frac{1}{N_{max} - \max\{N_{min}, j\} + 1} \sum_{m=\max\{N_{min}, j\}}^{N_{max}} Pr\{v_\beta \leq \frac{v \cdot m}{m-j}\}$$

$$= \frac{1}{N_{max} - \max\{N_{min}, j\} + 1} \sum_{m=\max\{N_{min}, j\}}^{N_{max}} \int_0^{\frac{v \cdot m}{m-j}} f_{v_\beta}(v)dv,$$  \hspace{1cm} (2.15)

where $v_{min} \leq v \leq v_{max}$. $f_{v_\beta}(v)$ is the pdf of ending step speed in $\beta$–phase, and is derived by substituting $\beta$ for $j$ from (2.12). Note, in (2.15), if $(\frac{v \cdot m}{m-j} > v_{max})$, that is, $m < \frac{v_{max}}{v_{max}-v}$. Thus, if $(\max\{N_{min}, j\} < m < \frac{v_{max}}{v_{max}-v})$, then $\int_{v_{min}}^{\frac{v \cdot m}{m-j}} f_{v_\beta}(v)dv = Pr\{v_\beta \leq v_{max} < \frac{v \cdot m}{m-j}\} = 1$. Compared to the CDF of step speed in $\alpha$–phase, the CDF of step speed in $\gamma$–phase is in proportion to the number of time steps $j$. That is the CDF of step speed in $\gamma$–phase increases as the time step $j$ increases, which implies that step speed decreases gradually in $\gamma$–phase. The average speed of a node in entire $\gamma$–phase is:

$$E_{\gamma}\{v\} = \frac{1}{2} E\{v_\beta\} = \frac{1}{2} E\{v_\alpha\} = \frac{1}{4}(v_{min} + v_{max}).$$  \hspace{1cm} (2.16)

Given the above analysis, it is evident that for each movement, the average speed in $\alpha$–phase is the same as that in $\gamma$–phase, and is half of average speed in $\beta$–phase, which is the expected stable speed selected for that movement. Based on the properties of step speed of a node in each phase, we can further study the trace length, initial average speed and steady state speed distribution of the SMS model in the subsequent sections.

2.3.3 Trace Length

We define the trace length as the length of actual trajectory a node travels and distance as the Euclidean distance between the starting position and ending position of one SMS movement. The correlation between trace length and distance of one SMS movement is dominated by the temporal correlation of velocities of the node during the movement. Especially, if the memory level
parameter $\zeta$ is set 1 for step speed in (2.2) and step direction in (2.3) of $\beta$–phase, then the trajectory of an entire SMS movement is a straight line, which means that the trace length is equal to the distance in one movement. Otherwise, the distance is always smaller than the trace length as the directions in both $\beta$ and $\gamma$–phase are different from the direction in $\alpha$–phase.

Figure 2.4 illustrates the total trace length vs. time according to three phases of one SMS movement. As shown in Figure 2.4, the trace length $L$ includes three parts, such that $L = l_\alpha + l_\beta + l_\gamma$, where $l_\alpha$, $l_\beta$, $l_\gamma$ are the trace length in $\alpha$–$\beta$–, $\gamma$–phase, respectively. The properties of trace length of the mobility models are critical to the research in mobile wireless networks, since the trace length is closely related to the distance in SMS model. When the temporal correlation is strong, mobile nodes associated with longer trace will more easily evoke link and path failures between node pairs due to limited transmission range. Hence, the trace length has impacts on link and path stability, which in turn affects on routing protocol performance [84] and routing design issues [85, 86, 87]. Also, the stochastic properties of trance length and the corresponding distance of one SMS movement can help in location-based design schemes in mobile wireless networks, such as topology control [71, 72] and location management [88]. The stochastic properties of trace length according to three moving phases are derived as follows.

**Trace Length $l_\alpha$ in $\alpha$–Phase** With the movement time and speed, $l_\alpha = \frac{1}{2}a_\alpha \times (\alpha \Delta t)^2$. The CDF of $l_\alpha$ is derived as:
\begin{align}
Pr\{l_\alpha \leq l\} &= \sum_{m=b}^{N_{max}} Pr\{v_\alpha \times \alpha \leq 2l|\alpha = m\} Pr\{\alpha = m\} \\
&= \frac{1}{N_{max} - N_{min} + 1} \sum_{m=b}^{N_{max}} Pr\{v_\alpha \leq \frac{2l}{m}\} \\
&= \frac{1}{N_{max} - N_{min} + 1} \sum_{m=b}^{N_{max}} \int_{v_{\min}}^{\frac{2l}{m}} f_{v_\alpha}(v) \, dv \\
&= \frac{2l}{(N_{max} - N_{min} + 1)} \frac{\psi(N_{max} + 1) - \psi(N_{min})}{v_{\max} - v_{\min}} - \frac{v_{\min}}{v_{\max} - v_{\min}},
\end{align}

(2.17)

where \(\psi(\cdot)\) is the Psi function \(\psi(n) = \frac{\Gamma'(n)}{\Gamma(n)}\) and \(\Gamma(\cdot)\) is the gamma function [89]. From (2.17), the derivative of \(Pr\{l_\alpha \leq l\}\) with respect to \(l\) yields the pdf of \(l_\alpha\) as:

\[
f_{l_\alpha}(l) = \frac{2}{(N_{max} - N_{min} + 1)} \frac{\psi(N_{max} + 1) - \psi(N_{min})}{v_{\max} - v_{\min}}.
\]

(2.18)

Given (2.17) and (2.18), we find that the distribution of \(l_\alpha\) is mainly dependent on distributions of both target speed \(v_\alpha\) and phase duration \([N_{min}, N_{max}]\). Since both target speed and phase duration are uniformly distributed, the distribution of \(l_\alpha\) is solely dominated by the range of target speed \([v_{\min}, v_{\max}]\) and phase duration \([N_{min}, N_{max}]\).

**Trace Length \(l_\beta\) in \(\beta\)-Phase** The travel distance in the entire \(\beta\)-phase can be obtained as:

\[
l_\beta = v_\alpha \beta \Delta t + \sqrt{1 - \zeta^2} \sum_{n=1}^{\beta} \sum_{m=0}^{n-1} \zeta^{n-m-1} \bar{V}_m \Delta t,
\]

(2.19)

which can be simplified as a function of two independent random variables, that is \(l_\beta = l_\beta' + \bar{l}\), where \(l_\beta' = v_\alpha \cdot \beta\), and \(\bar{l} = \sqrt{1 - \zeta^2} \sum_{n=1}^{\beta} \sum_{m=0}^{n-1} \zeta^{n-m-1} \bar{V}_m\). Because random variable \(\alpha\) and \(\beta\) have identical distributions, we have \(l_\beta' = 2l_\alpha\). Hence, the pdf of \(l_\beta'\) is derived as \(f_{l_\beta'}(l) = \frac{1}{2} f_{l_\alpha}(\frac{l}{2})\). On the other hand, we observed that \(\bar{l}\) is a superposed Gaussian random variable, i.e., \(\bar{l} \sim N(0, \sigma_\bar{l}^2)\).

The variance \(\sigma_\bar{l}^2\) of \(\bar{l}\) is derived as:

\[
\sigma_\bar{l}^2 = (1 - \zeta^2) \sum_{m=1}^{\beta} \left(\frac{1 - \zeta^n}{1 - \zeta}\right)^2.
\]

(2.20)

Note that, for a Gaussian random variable \(\bar{l}\), \(P\{\bar{l} - E(\bar{l}) < 3\sigma_\bar{l}\} = 0.9973\) [83]. Based on the same argument on the effective range of \(j^{th}\) step speed \(v_\beta(j)\) in \(\beta\)-phase discussed in Section 2.3.2,
we assume that the effective range of $\tilde{l}$ is $[N_{min} \cdot \max\{v_{min} - 3\sigma_l, 0\}, c \cdot (v_{max} + 3\sigma_l)]$. Through the same methodology for deriving the pdf of step speed in $\beta$–phase in (2.12), the pdf of $l_\beta$ is derived as:

$$
\begin{align*}
  f_{l_\beta}(l) &= \int_{-\infty}^{\infty} f_{l_\beta}(l-u) f_l(u) 1_{\{v_{min} \cdot N_{min} \leq l-u \leq v_{max} \cdot N_{max}\}} \, du \\
  &= \int_{-\infty}^{\infty} \frac{1}{2} f_{l_\beta}(\frac{l-u}{2}) f_l(u) 1_{\{v_{min} \cdot N_{min} \leq l-u \leq v_{max} \cdot N_{max}\}} \, du \\
  &= \frac{1}{(N_{max} - N_{min} + 1)} \psi(N_{max} + 1) - \psi(N_{min}) \int_{\frac{l-v_{min} \cdot N_{min}}{\sigma_l}}^{\frac{l-v_{max} \cdot N_{max}}{\sigma_l}} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \, du \\
  &= \frac{\psi(N_{max} + 1) - \psi(N_{min})}{(N_{max} - N_{min} + 1) \cdot (v_{max} - v_{min})}. \quad (2.21)
\end{align*}
$$

Because the distribution of $l_\beta'$ is solely dominated by the distribution of $l_\alpha$, given (2.21), the distribution of $\beta$–phase trace length $l_\beta$ is dependent on the distributions of both $l_\alpha$, which in turn characterized by the speed range $[v_{min}, v_{max}]$, phase duration $[N_{min}, N_{max}]$, and the memorial parameter $\zeta$.

**Trace Length $l_\gamma$ in $\gamma$–Phase** Similar to $\alpha$-phase, $l_\gamma = -\frac{1}{2} a_{\gamma} \times (\gamma \Delta t)^2$. By using the same methodology for deriving CDF and pdf of $l_\alpha$, we have

$$
\begin{align*}
  \Pr\{L_\gamma \leq l\} &= \frac{1}{N_{max} - N_{min} + 1} \sum_{m=N_{min}}^{N_{max}} \int_{v_{min}}^{\frac{2l}{m}} f_{v_\beta}(v) \, dv, \quad (2.22)
\end{align*}
$$

and

$$
\begin{align*}
  f_{l_\gamma}(l) &= \frac{2}{(N_{max} - N_{min} + 1)} \sum_{m=N_{min}}^{N_{max}} \frac{1}{m} f_{v_\beta}(\frac{2l}{m}), \quad (2.23)
\end{align*}
$$

where $f_{v_\beta}(v)$ is the pdf of ending step speed in $\beta$–phase, which is derived by substituting $\beta$ for $j$ from (2.12). From (2.22) and (2.23), it is clear that the distribution of $l_\gamma$ is mainly dependent on distributions of both ending speed $v_\beta$ in $\beta$–phase and phase duration $[N_{min}, N_{max}]$.

So far we have analyzed the stochastic properties of movement duration, step speed, and trace length of the proposed SMS mobility model. These results can give a deep understanding of the behaviors of this model. Next we study the transient and steady state features of the model in sequence.
2.4 Transient Property: Smooth Movement

In this section, we present the transient smooth moving behaviors of the SMS model based upon its correlation degree characterized by $\zeta$. Throughout this work, we define a smooth nodal movement as:

**Definition 1.** A mobility model generates a smooth movement if the node speed and direction change between any two immediate time steps is less than a small predefined threshold $\epsilon_V$ and $\epsilon_\phi$, respectively.

Here, we demonstrate that SMS model always generates smooth movements. In accordance with the measure of “curvature” for representing the sharpness of a curve, we use the scalar quantity $\Delta \phi$ between two consecutive time steps to indicate the sharpness of the SMS movement. Since an SMS node does not change the direction in $\alpha$– and $\gamma$– phase, the randomness of direction selection in SMS model is characterized in the $\beta$–phase. Based on (2.3), $\Delta \phi_j$ between the $j^{th}$ and $(j-1)^{th}$ time steps in $\beta$–phase is represented as:

\[
\Delta \phi_j = \phi_j - \phi_{j-1} = \sqrt{1 - \zeta^2} \left( (\zeta - 1) \sum_{m=0}^{j-2} \zeta^{j-2-m} \tilde{\phi}_m + \tilde{\phi}_{\zeta-1} \right).
\] (2.24)

From (2.24), $\Delta \phi_j$ is a linear supposition of identical, independent zero-mean Gaussian variables. Hence, $\Delta \phi_j$ is also a Gaussian variable with mean $E\{\Delta \phi_j\} = 0$, and variance

\[
Var\{\Delta \phi_j\} = (1 - \zeta)(2 - \zeta^2 2^{j-2} + \zeta^2 j^{-1}) \sigma_{\phi}^2.
\] (2.25)

We can see from (2.25), $Var\{\Delta \phi_j\}$ monotonously decreases from $2\sigma_{\phi}^2$ to 0, as the temporal parameter $\zeta$ increases from 0 to 1. Therefore, we have

\[
P\{ |\Delta \phi_j - 0| < \epsilon_\phi \} = G\left( \frac{\epsilon_\phi}{\sigma_{\Delta \phi_j}} \right) - G\left( \frac{-\epsilon_\phi}{\sigma_{\Delta \phi_j}} \right) = 1 - \varepsilon,
\] (2.26)

where function $G(\cdot)$ is defined as $G(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy$, and $\varepsilon$ is an infinitesimal number. We observe from (2.25) and (2.26) that $\sigma_{\phi}$ and $\zeta$ are two key factors which determine the sharpness of one SMS movement in that by setting appropriate values of these parameters, (2.26) can be satisfied for any $\varepsilon$. For example, let $\epsilon_\phi = 2\sqrt{2}\sigma_{\phi}$, if $\zeta = 0$, then $\sigma_{\Delta \phi_j} = \sqrt{2}\sigma_{\phi}$ and $\varepsilon = 0.0456$. That means, in the SMS model with the minimum temporal correlation, the probability that the direction change between two immediate time steps is outside the range $(-\epsilon_\phi, \epsilon_\phi)$ is 0.0456. In the same way, we
have $\varepsilon = 0.0048$ for $\zeta = 0.5$, and $\varepsilon = 0$ for $\zeta = 1$. Therefore, we show that SMS model almost surely generates smooth direction transition within each movement when the condition between $\varepsilon_\phi$ and $\varepsilon$ is satisfied in (2.26). Following the similar approach, we can show that SMS model generates smooth speed transition as long as the speed difference is less than $\varepsilon_\nu$, which can be set to $2\sigma_\nu$ in this study$^1$. Figure 2.5 illustrates three SMS moving trajectories given different levels of temporal correlations. It is evident that SMS model can mimic either polyline or straight line movement, regarding the degree of temporal correlation $\zeta$, and more important, SMS model avoids sharp turns all the time.

![Figure 2.5](image.png)

Figure 2.5: Three smooth SMS movements with different temporal correlation, where $\varepsilon_\phi = 0.4\pi$ and $\varepsilon_\nu = 2$ m/sec.

### 2.5 Steady State Analysis

For large-scale wireless networks, the moving behaviors of mobile nodes at steady state predominate the performance evaluation. During the simulation, when the random process of a mobility model reaches the steady state, trace file and the corresponding simulation results are reliable for the performance evaluation. On the other hand, analytical study also requires the steady state distribution of the interested parameter for deriving accurate theoretical expressions. Therefore, we focus on the time stationary distribution of SMS model and steady state analysis of speed in SMS model to demonstrate that there is no speed decay problem. Then, we will demonstrate the nodal distribution at steady-state.

$^1$As $\Delta\phi$ and $\Delta\nu$ are Gaussian variables, there exists a very tiny probability (e.g. less than 0.005 for $\zeta = 0.5$) that direction or speed may have large values in some steps. For simulations, if the new calculated value of $\Delta\phi/\Delta\nu$ is larger than the threshold $\varepsilon_\phi/\varepsilon_\nu$, we reselect the direction/speed.
2.5.1 Time Stationary Distribution

Recall the renewal process described in Chapter 2.2.2. Let $E\{T_{s1}\}$ be the mean value of holding-time that the semi-Markov process spends at state $s1$ before making a next transition, that is

$$E\{T_{s1}\} = \int_0^\infty t \ dH_{s1}(t).$$

(2.27)

Then $E\{T_{s1s1}\}$ denotes the mean recurrence time, i.e., the average first recurrent time after leaving state $s1$ [81]. It is known that for an irreducible semi-Markov process $\{U(t)\}$, if $E\{T_{s1s1}\} < \infty$, then for any state $s1$, the limiting probability $P_{s1}$ exists and is independent of the initial state, for example $U(0) = u$, that is,

$$P_{s1} = \lim_{t \to \infty} \frac{Prob\{U(t) = s1 | U(0) = s2\}}{E\{T_{s1s1}\}} = \frac{E\{T_{s1}\}}{E\{T_{s1s1}\}}. \quad (2.28)$$

According to our SMS model, the embedded semi-Markov chain $\{Y_n\}$ is irreducible since each state is reachable from any other states. Because $\{Y_n\}$ is irreducible, the semi-Markov process $\{Z(t)\}$ is also irreducible. Furthermore, $\{Y_n\}$ is positive recurrent because it is an irreducible semi-Markov chain with finite states. By applying (2.27) into $\{Y_n\}$, we have $E\{T_{s1}\} = E\{\alpha\} = \frac{N_{min} + N_{max}}{2}$, when $s1 \neq I_p$ and $E\{T_{ip}\} = E\{T_p\}$. Therefore, for any state $s1$ of $\{Y_n\}$, $E\{T_{s1s1}\} = E\{\alpha\} + E\{\beta\} + E\{\gamma\} + E\{T_p\} < \infty$, so the the limiting probability of each state exists in the SMS model. In turn, the time stationary distribution $P_v$ for each state $v$ exists in the SMS model according to (2.28). Let $\pi = (\pi_\alpha, \pi_\beta, \pi_\gamma, \pi_p)$ denote the time stationary distribution of SMS stochastic process. By combing (2.27) and (2.28), we derive the time stationary distribution for each phase in the SMS model as following:

$$\pi_m = \lim_{t \to \infty} \frac{Prob\{I(t) = I_m | I \in I\}}{E\{T_m\} + E\{T_p\}}. \quad (2.29)$$

The time stationary properties obtained in the above equation will be used in evaluating the average speed at the initial state so that we can further prove that there is no speed decay in the SMS model. We have studied the stochastic properties of step speed $v_j$, where $1 \leq j \leq N_{max}$ for each phase in Chapter 2.3.2. Now we are interested in the speed distribution at steady state in Chapter 2.5.2 and average speed in Chapter 2.5.3. The nodal distribution will be discussed in Chapter 2.5.4.

2.5.2 Speed Distribution at Steady State

In order to find the average speed at steady state, we derive the speed distribution first. Let $M(t)$ and $M_p(t)$ denote the total number of time steps that a node travels and pauses from...
the beginning till time $t$, respectively. Recall, $N(t)$ represents the number of renewal movements by time $t$. Hence, $M_p(t) = \sum_{i=1}^{N(t)} T_p(i)$. If we consider the last incomplete movement, then $\sum_{i=1}^{N(t)} T(i) \leq M(t) < \sum_{i=1}^{N(t)+1} T(i)$. We denote $v_{ss}$ as the steady-state speed of the SMS model. Then, $F_{v_{ss}}(v)$ of $v_{ss}$ can be derived from the limiting fraction of time when step speeds are less than $v$, as the entire simulation time $t$ approaches to infinity. When $t \to \infty$, $N(t) \to \infty$, such that one can safely omit the impact of fraction time in the last incomplete movement. The formal derivation of $F_{v_{ss}}(v)$ is shown in the APPENDIX 2.9.2. Here, we present the final result of $F_{v_{ss}}(v)$ as:

$$\Pr\{v_{ss} \leq v\} = \frac{1}{E\{T\} + E\{T_p\}} \cdot \left[ E\{T_p\} + \frac{1}{N_{max} - N_{min} + 1} \sum_{m=N_{min}}^{N_{max}} \sum_{j=1}^{m} \left( \int_{v_{min}}^{m} f_{v_{\alpha}}(v) \, dv + \int_{v_{min}}^{v} f_{v_{\beta}}(v) \, dv + \int_{v_{min}}^{m-j} f_{v_{\gamma}}(v) \, dv \right) \right].$$

(2.30)

### 2.5.3 Average Speed at Steady State

A well-known problem of RWP model is that it fails to provide a steady average speed which decreases over time [17]. Thus, we examine whether there exists the speed decay phenomenon in the SMS model for which we need to obtain both the initial average speed $E\{v_{ini}\}$ and the average steady state speed $E\{v_{ss}\}$. More importantly, the average speed at steady state should be independent of initial speed and locations [29].

The initial state of the SMS model is chosen based on the time stationary distribution $\pi = (\pi_{\alpha}, \pi_{\beta}, \pi_{\gamma}, \pi_p)$ of the SMS process in Chapter 2.5.1. From (2.11), (2.14), (2.16), and (2.29), the initial average speed of the SMS model is derived as:

$$E\{v_{ini}\} = E\{E\{v_{ini} \mid I_m\}\} = E_{\alpha}\{V\} \pi_{\alpha} + E_{I_{\beta}}\{V\} \pi_{\beta} + E_{I_{\gamma}}\{V\} \pi_{\gamma} + 0 \cdot \pi_p = \frac{1}{2}E\{v_{\alpha}\}(E\{\alpha\} + 2E\{\beta\} + E\{\gamma\}) \frac{1}{E\{T\} + E\{T_p\}}.$$  

(2.31)

By differentiating the CDF of steady state speed $v_{ss}$ from (2.30) with respect to $v$, it is observed that the pdf of $v_{ss}$ consists of four distinct components according to each state of $\{Y_n\}$:

$$f_{v_{ss}}(v) = f_{v_{ss}}^{I_{\alpha}}(v) + f_{v_{ss}}^{I_{\beta}}(v) + f_{v_{ss}}^{I_{\gamma}}(v) + f_{v_{ss}}^{I_p}(v).$$

(2.32)
The four components of $f_{vss}(v)$ in (2.32) are given by:

$$f_{vss}(v) = \begin{cases} \frac{1}{N_{max} - N_{min} + 1} \sum_{m=N_{min}}^{N_{max}} \sum_{j=1}^{m} f_{v\alpha}(v) & \text{v of I}_\alpha \\ \frac{1}{N_{max} - N_{min} + 1} \sum_{m=N_{min}}^{N_{max}} \sum_{j=1}^{m} f_{v\beta}(v) & \text{v of I}_\beta \\ \frac{1}{N_{max} - N_{min} + 1} \sum_{m=N_{min}}^{N_{max}} \sum_{j=1}^{m} f_{v\gamma}(v) & \text{v of I}_\gamma \\ \frac{1}{E(T) + E(T_p)} & \text{v of I}_p \end{cases}$$ (2.33)

Corresponding to (2.33), the expectation of steady state speed $E\{v_{ss}\}$ is also composed of four distinct components with regards to each state of embedded semi-Markov chain $\{Y_n\}$, which are derived in APPENDIX 2.9.3. The result is

$$E_u\{v_{ss}\} = \begin{cases} \frac{1}{2}E\{v_u\}(1 + E\{\alpha\}) & u \in I_\alpha \\ \frac{E\{v_u\} E\{\beta\}}{E(T) + E(T_p)} & u \in I_\beta \\ \frac{1}{2}E\{v_u\}(E\{\gamma\} - 1) & u \in I_\gamma \\ \frac{E\{v_u\}}{E(T) + E(T_p)} & u \in I_p \end{cases}$$ (2.34)

Thus, the average speed at steady state, $E\{v_{ss}\}$ of the SMS model can be obtained by combining each item in (2.34):

$$E\{v_{ss}\} = E_\alpha\{v_{ss}\} + E_\beta\{v_{ss}\} + E_\gamma\{v_{ss}\} + E_I\{v_{ss}\}$$

$$= \frac{\frac{1}{2}E\{v_u\}(E\{\alpha\} + 2E\{\beta\} + E\{\gamma\})}{E\{T\} + E\{T_p\}}.$$ (2.35)

By comparing (2.31) with (2.35), we showed that the initial average speed is exactly same as average steady state speed in SMS process, i.e., $E\{v_{ini}\} = E\{v_{ss}\}$. Furthermore, since the step speed is selected independently from the number of time steps in each phase of one movement, our analytical result of average steady state speed in the SMS model is consistent with which proved by Yoon in [29]: there is no speed decay phenomenon in mobility model as long as the speed and time metric for each trip are selected independently. Therefore, we proved that there is no speed decay problem for arbitrary starting speed in SMS model.

2.5.4 Spatial Node Distribution

Another problem with RWP model is that it has non-uniform node distribution, given that the initial distribution is uniform. In this section, we will prove that the SMS model with border wrap has uniform node distribution all the time and validate our results through simulations.
Here, we consider the border effect on the SMS model. When a mobile node reaches a boundary of a two dimensional simulation region, it wraps around and reappears instantaneously at the opposite boundary in the same direction to avoid biased effects on simulation results.

Now we study the spatial node distribution of the SMS model with border wrap by focusing on the successive movements of a single mobile node. Consider a mobile node in an SMS movement, there is a total $K(i)$ time steps in a node’s $i^{th}$ movement. Then the mobility at the $j^{th}$ time step of its $i^{th}$ movement is characterized by a triplet $(d(i,j), v(i,j), \phi(i,j))$. Hence, the complete discrete-time process of the SMS node according to its $i^{th}$ movement is defined as:

$$\{d(i,j), v(i,j), \phi(i,j)\}_{1 \leq j \leq K(i)}.$$ At first, we analyze the node distribution during the first SMS movement, i.e., $i = 1$. For a simple representation, we suppress the movement index $i$ in the denotations of random variables, and normalize the size of the simulation region to $[0,1)^2$. For instance, $d(j)$ denotes the ending position after the $j^{th}$ time step of the first movement, where $d(j) = (X_j, Y_j)$. Then $d(j)$ with border wrap on $[0,1)^2$ is represented by:

$$d(j) = \begin{cases} X_j = X_{j-1} + v_j \cos(\phi_j) - \lfloor X_{j-1} + v_j \cos(\phi_j) \rfloor \\ Y_j = Y_{j-1} + v_j \sin(\phi_j) - \lfloor Y_{j-1} + v_j \sin(\phi_j) \rfloor, \end{cases}$$

(2.36)

where $[\cdot]$ is the floor function. According to the target direction $\phi_\alpha$ of the first movement, the $j^{th}$ step direction $\phi_j = \phi_\alpha + \psi_j$, where $\psi_j$ is the relative direction between $\phi_\alpha$ and direction at the $j^{th}$ time step, which ranges in $[0,2\pi)$. Hence, $\psi_j$ is constant in both $\alpha$–phase and $\gamma$–phase and varies in $\beta$–phase. Based on (2.3), $\psi_j$ is derived as follows:

$$\psi_j = \begin{cases} \frac{0}{1 - \zeta^2 \sum_{m=0}^{\alpha-1} \zeta^{j-\alpha-1-m} \tilde{\phi}_m} & 1 \leq j \leq \alpha \\
\sqrt{\frac{1}{1 - \zeta^2 \sum_{m=0}^{\beta} \zeta^\beta \tilde{\phi}_m}} & \alpha + 1 \leq j \leq \alpha + \beta \\
\sqrt{1 - \zeta^2} \sum_{m=0}^{\beta} \zeta^\beta \tilde{\phi}_m & \alpha + \beta + 1 \leq j \leq K. \end{cases}$$

(2.37)

Consider that $\phi_j$ is also within the range $[0,2\pi)$, then we have

$$\phi_j = \phi_\alpha + \psi_j - 2\pi \lfloor \frac{\phi_\alpha + \psi_j}{2\pi} \rfloor.$$

(2.38)

At time $t = 0$, the node begins to move from the initial position $P(0) = (X_0, Y_0)$, at the speed with $v_1$ along the step direction $\phi_1$, where $\phi_1$ is equal to the target direction $\phi_\alpha$. Based on a specific value $P(0)$ and $\phi_\alpha$, the complete discrete-time process of its first movement is characterized by $\{v_j, \psi_j\}_{1 \leq j \leq K}$. Assume that the movement pattern $\{v_j, \psi_j\}_{1 \leq j \leq K}$ is deterministic, then we have the following Lemma.
Lemma 1. In the SMS model, if the initial position $P(0)$ and the first target direction $\phi_\alpha$ of the mobile node are chosen independently and uniformly distributed on $[0, 1)^2 \times [0, 2\pi)$ at time $t = 0$, then the location and direction of the node remain uniformly distributed all the time.

Proof. Our proof of Lemma 1 is based on the result of Lemma 2.3 and the methodology for proving Lemma 4.1 in [20]. According to Lemma 2.3 of [20], for all $x \in [0, 1)$ and $a \in (-\infty, \infty)$,

$$\int_0^1 1_{\{u + a - \lfloor u + a \rfloor < x\}} du = x.$$  \hspace{1cm} (2.39)

As the node begins to move, with (2.36) and (2.39), the joint probability of ending position and direction of the first step movement is derived as:

$$Pr(X_1 < x_1, Y_1 < y_1, \phi_1 < \theta) = Pr(X_1 < x_1|\phi_1 < \theta) \cdot Pr(Y_1 < y_1|\phi_1 < \theta) \cdot Pr(\phi_1 < \theta)$$

$$= \frac{1}{2\pi} \int_{\phi_1=0}^{\theta} \left( \int_{x_0=0}^{\int_0^1 1_{\{x_0+v_1 \cos(\phi_1) - \lfloor x_0+v_1 \cos(\phi_1) \rfloor < x_1\}} dx_0 \right)$$

$$\cdot \int_{y_0=0}^{\int_0^1 1_{\{y_0+v_1 \sin(\phi_1) - \lfloor y_0+v_1 \sin(\phi_1) \rfloor < y_1\}} dy_0 \right) d\phi_1 = \frac{x_1 y_1 \theta}{2\pi},$$  \hspace{1cm} (2.40)

where $x_1, y_1 \in [0, 1)$ and $\theta \in 2\pi$. The above result shows that $(d(1), \phi_1)$ is uniformly distributed on $[0, 1)^2 \times [0, 2\pi)$. Since $\psi_j = 0$ for each step movement in $\alpha$–phase, by following the same way of derivation in (2.40), we conclude that the ending position and direction for each step movement in $\alpha$–phase are in turn uniformly distributed on $[0, 1)^2 \times [0, 2\pi)$.

For each step movement in $\beta$–phase and $\gamma$–phase, where $\psi_j \neq 0$, based on (2.38) and (2.39) and the result from Eq. (32) of [20], the probability $Pr(\phi_j < \theta)$ is derived as:

$$Pr(\phi_j < \theta) = \frac{1}{2\pi} \int_0^{2\pi} 1_{\{\phi_\alpha+\psi_j-2\pi, \frac{\phi_\alpha+\psi_j}{2\pi}|<\theta\}} d\phi_\alpha = \int_0^{1} 1_{\{u+\psi_j-2\pi, \frac{u+\psi_j}{2\pi}|<\theta\}} du = \frac{\theta}{2\pi},$$  \hspace{1cm} (2.41)

where $u = \frac{\phi_\alpha}{2\pi}$, and $\alpha + 1 \leq j \leq K$. Hence, for the first step movement in $\beta$–phase, i.e., $j = (\alpha + 1)$, we have
\[ Pr \left( X_{\alpha+1} < x_{\alpha+1}, Y_{\alpha+1} < y_{\alpha+1}, \phi_{\alpha+1} < \theta \right) \\
= \frac{1}{2\pi} \int_{\phi_{\alpha}=0}^{\phi_{\alpha}} \int_{\theta}^{\theta} 1 \{ \phi_{\alpha+1} + \psi_{\alpha+1} - 2\pi \lfloor \frac{\phi_{\alpha+1}}{2\pi} \rfloor < \theta \} \cdot \\\n\left( \int_{0}^{1} 1 \{ x_{\alpha} + v_{\alpha+1} \cos(\phi_{\alpha+1}) - [x_{\alpha} + v_{\alpha+1} \cos(\phi_{\alpha+1})] < x_{\alpha+1} \} dx_{\alpha} \cdot \right. \\\n\left. \int_{0}^{1} 1 \{ y_{\alpha} + v_{\alpha+1} \sin(\phi_{\alpha+1}) - [y_{\alpha} + v_{\alpha+1} \sin(\phi_{\alpha+1})] < y_{\alpha+1} \} dy_{\alpha} \right) d\phi_{\alpha} \\
= \frac{x_{\alpha+1} y_{\alpha+1} \theta}{2\pi}. \tag{2.42} \]

Given the result of (2.42), \( (d(\alpha + 1), \phi_{\alpha+1}) \) is also uniformly distributed on \([0, 1)^2 \times [0, 2\pi)\). Through the induction argument, we directly prove that for the successive step movements in \( \beta \)-phase and \( \gamma \)-phase, the uniform distribution always holds. Therefore, we conclude that the position and direction of the node are always uniformly distributed during the first movement. Since pause time does not affect the spatial node distribution in our SMS model, by induction, the uniform node distribution holds for the entire \( i^{th} \) movement. Then, the ending position of the \( i^{th} \) movement \( P(i) \) is uniformly distributed over \([0, 1)^2\). Because SMS process is a renewal process, according to the stochastic property of renewal process, the position in the entire \( (i + 1)^{th} \) movement is also uniformly distributed. Hence, we complete the proof of Lemma 1.

2.6 Simulation Results

In this section, we validate the analytical results in previous section by ns-2 simulations, along with the evaluation of effects of phase duration time and pause time.

2.6.1 Assumptions and Parameters

We integrate our SMS model into the setdest of ns-2 simulator [90], which currently provides both an original and a modified version of RWP model [29]. Each scenario in our simulation contains 1000 nodes moving independently in a square simulation region with size \([0, 1500)m \times [0, 1500)m\) during a time period of 1000 seconds. Then, 30 realizations of each scenario are made through Monte Carlo simulation. Since pause time does not have a significant effect on the built-in RWP model [17], for comparison of average speed and node distribution demonstrated later in Chapter 2.5.4, we set zero pause time for both models. According to the built-in RWP model, the
speed range for all scenarios is \([0, 20]\) m/s. For the SMS model, we set the unit time slot \(\Delta t\) as 1 second and the memory parameter \(\zeta\) as 0.5. The speed range of target speed \(v_\alpha\) in the SMS model is considered in the range of \([0, 20]\) m/s, so that \(E\{v_\alpha\} = 10\) m/s. In real life, it normally takes at least 6 seconds for a car to accelerate its speed from 0 m/s to 20 m/s. Hence, for simulation in the SMS model, when the target speed \(v_\alpha\) is selected close to 20 m/s, we consider at least 6 time steps for speed acceleration. Therefore, the range of phase duration time \([N_{min}, N_{max}]\) is selected either from \([6, 30]\) s or \([10, 40]\) s.

Note, as the speed/direction in each time step of \(\beta\)-phase are affected by a Gaussian random variable, there exists a very small non-zero probability that speed/direction may have a very large value in some time step. To avoid this unwanted event, we set a threshold \(v_{max}\) for the speed and a threshold \(\pi/2\) for the direction change between two consecutive steps, respectively. If the new calculated value of speed/direction is larger than the threshold, we reselect that value.

### 2.6.2 Average Speed

According to the original RWP model in ns-2, all nodes start to move at the initial time, which is named strategy-1 in our evaluation. To be consistent with the simulation strategy for the original RWP model, we let all nodes in the SMS model start to move from the first step of \(\alpha\)-phase. Moreover, each phase duration time is in range of \([6, 30]\) seconds. Given the simulation condition of zero pause time and \(E\{\alpha\} = E\{\beta\} = E\{\gamma\} = 18\) sec, from (2.35), the theoretical result of \(E\{v_{ss}\}\) of SMS model is obtained as: \(E\{v_{ss}\} = \frac{2}{3}E\{v_\alpha\} = 6.7\) m/sec. Both simulation and theoretical results of average speed vs. a time period of 1000 seconds are shown in Figure 2.6, from which we can see that after the first 200 seconds, the average speed of the SMS model with strategy-1 converges to the expected steady state speed 6.7 m/sec and stabilizes at that level, which perfectly matches the theoretical result, whereas the average speed of RWP model continues decreasing with the simulation time.

With the modified RWP model [29], initial locations and speeds of the nodes are chosen from the steady state distributions directly, namely strategy-2 [91, 92, 19]. By this way, the steady state of RWP process is immediately reached at the beginning of the simulation when \(V_{min} > 0\) m/s. Correspondingly, for SMS model, we choose an initial state with a probability according to the time stationary distribution of the SMS process, \(\pi = (\pi_\alpha, \pi_\beta, \pi_\gamma, \pi_p)\) in (2.29). Specifically, if the initial phase of a node is \(\beta\)-phase or \(\gamma\)-phase, the node will choose a random initial step speed, as \(v_{\alpha+1}\) for the first step in \(\beta\)-phase, or \(v_{\beta+1}\) for the first step in \(\gamma\)-phase, which is uniformly distributed over
Figure 2.6: Average speed vs. simulation time.

In Figure 2.6, we can see the average speed of the SMS model according to strategy-2 is stable right from the start of simulation. As we expected, the average speed through strategy-2 has the same expected steady state speed $6.7\text{m/sec}$ as that obtained from strategy-1. Since the average speeds of the SMS model obtained through both strategies do not decay over time regardless of initial speed, the simulation results are consistent with our analytical proof shown in Chapter 2.5.

**Effect of Phase Duration Time** Here we want to observe the effects of the duration time of each phase on steady state speed $E\{v_{ss}\}$ in SMS model. Interestingly, based on our analytical result of $E\{v_{ss}\}$ in (2.35), the average speed at steady state is independent of phase duration time, if the range for each moving phase duration is same, regardless of the interval selection of the range. That is, if $E\{\alpha\} = E\{\beta\} = E\{\gamma\}$ and without pause time, the average speed at steady state in SMS model can always be evaluated as: $E\{v_{ss}\} = \frac{2}{3}E\{v_{a}\}$. To validate our theoretical result, we simulate two scenarios according to phase duration time, [6, 30] seconds and [10, 40] seconds, respectively. Simulation results based on these two scenarios are shown in Figure 2.7.

We can observe from Figure 2.7 that the average speed with strategy-2 converges to the same expected steady state value $6.7\text{m/sec}$ since the beginning of the simulation. By simulation through strategy-1, the average speed based on the range of [6, 30] seconds converges to the steady state faster than that with the range of [10, 40] seconds due to its shorter average movement duration time. In fact, the longer the phase duration time, the longer time is taken for the average speed to converge. We observe that after the first 300 seconds, average speeds of these two scenarios
with both strategies (1 and 2) are asymptotic to the same expected steady state value. Thus, the theoretical proof of $E\{v_{ss}\}$ in (2.35) is validated by simulations.

**Effect of Pause Time** In the proposed SMS model, pause time is the time interval between any two consecutive movements. Let’s look at the impact of pause time on the expected steady state speed. Here, we allow SMS nodes to pause for a random time $t_p$ with a mean value $E\{T_p\}$ after each movement. According to (2.29), the stationary pause phase probability $\pi_p$ is given by $\pi_p = \frac{E\{T_p\}}{E\{T\}+E\{T_p\}}$. Based on (2.35), for the phase duration time selected from $[6, 30]$ seconds, $v \in [0, 20]$ m/sec, the effect of pause time with regards to the stationary pause phase probability $\pi_p$, and steady state speed $E\{v_{ss}\}$ are shown in Table 2.1.

<table>
<thead>
<tr>
<th>$E{T_p}(sec)$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_p$</td>
<td>0</td>
<td>0.16</td>
<td>0.27</td>
<td>0.36</td>
<td>0.43</td>
<td>0.48</td>
<td>0.53</td>
</tr>
<tr>
<td>$E{v_{ss}}(m/s)$</td>
<td>6.67</td>
<td>5.63</td>
<td>4.86</td>
<td>4.29</td>
<td>3.83</td>
<td>3.46</td>
<td>3.16</td>
</tr>
</tbody>
</table>

Table 2.1 demonstrates that as $E\{T_p\}$ increases, $\pi_p$ increases monotonously, whereas $E\{v_{ss}\}$ decreases. Especially, the downtrend of $E\{v_{ss}\}$ is more evident when $E\{T_p\}$ is small. For instance, when $E\{T_p\} = 60$ seconds, and $\pi_p$ is equal to 0.53, i.e., the pause phase becomes a dominant time period in a movement. Correspondingly, $E\{v_{ss}\} = 3.16$ m/s which drops more than half of average speed for $E\{T_p\} = 0$ sec. According to the numerical results shown in Table 2.1,
we find that for SMS model, there always exists a stable average speed at steady state for arbitrary pause time. Therefore, our proposed SMS model eliminates the speed decay problem regardless of pause time.

2.6.3 Smooth Movements

The SMS model is designed by following the physical law of moving objects in order to fulfill the requirement of smooth movements, which is the second desired feature of the SMS model introduced in Chapter 2.1. Thus, there is no unrealistic behaviors such as sharp turns and sudden stops in this model. Here we compare the distance which is the Euclidean distance between the current position and the starting position during one movement, with the trace length which is the length of actual trajectory a node travels during one movement. Different from all macroscopic mobility models, such as RD and RWP model, because direction changes within one SMS movement, the distance is not larger than the trace length. By observing distance evolution during one movement, people can indirectly tell the corresponding moving trajectory. Specifically, if the distance increases monotonously, it implies that the node travels forward with a smooth trace. In contrast, if the distance decreases at some time, it means that the node turns backward during the movement, that is a sharp turn event occurs. In the third case, if the distance is relative stable for some time interval, then we can tell that the node is traveling along a circle trajectory, where the starting position is the center of the circle.

![Figure 2.8: Distance and trace length.](image-url)
The simulation results of both trace length and distance of an SMS movement are shown in Figure 2.8, where \( \alpha = 16 \) seconds, \( \beta = 12 \) seconds, and \( \gamma = 17 \) seconds. We can see that both trace length and distance increase exponentially in both \( \alpha \)-phase and \( \gamma \)-phase, due to the speed acceleration and deceleration, respectively. Furthermore, the trace length \( l_\alpha \) is equivalent to the distance, due to the constant direction \( \phi_\alpha \) in \( \alpha \)-phase. In \( \beta \)-phase, the trace length increases linearly, whereas the uptrend of the distance fluctuates because of the change of direction at every time step. Since the direction \( \phi_\gamma \) does not vary in \( \gamma \)-phase, both the trace length and the distance within \( \gamma \)-phase increase. And the difference between \( \phi_\alpha \) and \( \phi_\gamma \) directly determines the difference between the trace length and distance in \( \gamma \)-phase. Moreover, as the trace length is the upper bound of the distance, the equality holds when the node moves along a straight line during an entire movement. From Figure 2.8, the distance monotonously increases during the entire movement period. Therefore, we conclude that the mobility trace in SMS model is smooth without sharp turns.

### 2.6.4 Uniform Node Distribution

We compare the spatial node distribution of the SMS model with the RWP model to validate our proof through simulation results. For both models, we deploy 1000 nodes uniformly distributed in the simulation region at the initial time. Then, we sample the spatial node position at the 500th and the 1000th second for the SMS model, and at the 1000th second for the RWP model, respectively. A top view of two-dimensional spatial node position of the RWP and the SMS models are shown in Figure 2.9.

![Spatial Nodal Distribution](image)

(a) RWP model at the 1000th sec.  (b) SMS model at the 500th sec.  (c) SMS model at the 1000th sec.

Figure 2.9: Top-view of node distribution of RWP model and SMS model.

According to Figure 2.9(a), it is clear that the node density with RWP model is maximum
at the center of the region, while it is almost zero at the vicinity of the simulation boundary. In contrast, from Figure 2.9(b) and 2.9(c), the node density of the SMS model at the two sampling time instants are similar and mobile nodes are equally located within the simulation region. Since 500th second is the middle stage of the simulation period and 1000th second is the ending time of the simulation, both snapshots of SMS model demonstrate a uniform node distribution. Therefore, we verified our proof in Lemma 1 that the SMS model with border wrap has uniform spatial node distribution.

Since all mobile nodes travel independently and follow the same movement pattern in the SMS model, according to Lemma 1, we conclude that if the positions of all nodes in the first movement are uniformly distributed, they are uniformly distributed on the simulation region all the time.

In summary, we have showed that this SMS model unifies the desired features of mobility models such as smooth movements, stable speed via analysis and simulations, and uniform nodal distribution. To further demonstrate the merits of the proposed model, in the next section, we will discuss how to implement and adapt the SMS model to networking environments. Also, we will illustrate the impacts of the proposed model on link lifetime, and network topology.

2.7 Impacts and Applications of SMS Model

Since the SMS model describes the individual node mobility according to time steps, it can be flexibly controlled to simulate diverse moving behaviors including straight lines or curves; short or long trips; stable or variable speeds; fast or slow speed acceleration and deceleration. Hence, the SMS model can be implemented network simulations requiring flexible control of mobile nodes, for example, the guidance of mobile robots with smooth motion for disaster survivor detection [93] and for navigation across a sensor network [94], or the guidance of mobile agents for better connectivity in MANETs [95]. The implementation of SMS model is not complex even though the analysis of this model involves time steps at microscopic level and movements at macroscopic level. The key to a successful simulation is to specify target speed for α-phase, the time steps for each phase, which determines the granularity of speed change, pause time after a stop, and correlation factor. Next, compared with the RWP model, we will show the the effects of the SMS model on diversified research areas regarding relative speed, link lifetime, node degree, connectivity, and routing performance.
2.7.1 Effects of SMS Model

Almost all mobility models used in current simulation tools, such as RWP model utilized in ns-2, describe completely uncorrelated mobility. The abrupt speed and direction change events, induced by these models, will influence the network topology change rate, which further significantly affects the routing performance of the network. Therefore, the simulation results and theoretical derivations, based on current mobility models in the simulation tools, may not correctly indicate the real-life network performance and effects of system parameters. Since the SMS model provides more realistic moving behaviors than current random mobility models, we apply the model to estimate the system performance and investigate its effect on mobile wireless network studies. Meanwhile, we want to find out whether the evaluation results are much different from random mobility models, for example the RWP model; and how different they are.

Link Performance Effect

To generate different mobility levels for both RWP and SMS models, we respectively set the initial average speed $E\{v_{ini}\}$ as 2, and 15 m/sec. The PDFs of the relative speed of two models according to the different initial average speed $E\{v_{ini}\}$ are shown in Figure 2.10(a). For a pair of neighboring nodes $(u, w)$, the relative speed $V^u$ of node $u$ according to the reference node $w$ consists of two components in terms of X-axis and Y-axis of a Cartesian coordinate system centered at node $w$. Specifically, in the smooth model, the magnitude of $n^{th}$ step relative speed of node $u$ is: $|V^u_n| = \sqrt{(X_n - X_{n-1})^2 + (Y_n - Y_{n-1})^2}$, where $X_{n-1}/X_n$ is the starting/ending coordinate of the $n^{th}$ step relative trip of node $u$ in X-axis, so is $Y_{n-1}/Y_n$ for Y-axis. Since $\Delta t = 1$ sec, $n >> 1$. Based on the central limit theorem (CLT) [83], when $n >> 1$, both i.i.d random variable $X_n - X_{n-1}$ and $Y_n - Y_{n-1}$ can be effectively approximated by an identical Gaussian distribution with zero mean [39]. Furthermore, for any two independent Gaussian RVs, for example $A$ and $B$, with zero mean and and equal variance, the RV $Z = \sqrt{A^2 + B^2}$ has a Rayleigh density. Hence, the relative speed $V^u_n$ in the smooth model has an approximate Rayleigh distribution, which exactly matches the results shown in Figure 2.10(a) of both scenarios with different $E\{v_{ini}\}$. However, because of the speed decay [17] and abrupt velocity change problems [9], the PDF of relative speed in the RWP model varies irregularly during the simulation and tends to have larger proportion in the region of small speed. This phenomenon shows more apparently when $E\{v_{ini}\}$ is large.

As the relative speed has a significant effect on the link lifetime, we further evaluate the
PMF and CDF of link lifetime between these two models. Here, we specify $E\{v_{ini}\}$ as 2 m/sec. To investigate the effect of temporal correlation of node velocity in $\beta$–phase on link lifetime, we respectively set the memorial parameter $\zeta$ as 0, 0.5, and 1 in the SMS model. From Figure 2.10(b), it is observed that the probability mass function (PMF) of link lifetime of SMS model decreases exponentially with time for $\zeta = 0.5$. On the other hand, the PMF of link lifetime resulting from RWP model, as an example of macroscopic mobility models, shows a peak at the 25th second [5, 26]. These outcomes reflect a significant difference in link lifetime distribution between macroscopic and microscopic models. For instance, the probability that link lifetime is 5 sec is about 0.04 in RWP model compared with 0.38 for the SMS model. Such discrepancy may further affect the evaluation of routing and network performance by taking expected link lifetime into account. More elaborated discussions on connectivity and routing performance will be presented later in this section. Let’s look at the simulation results of link lifetime CDF of these two models in Figure 2.10(c). Recall that given (2.3), when $\zeta = 1$, the node velocity has the strongest correlation, and the entire movement is a straight line according to the direction $\phi_\alpha$. In contrast, when $\zeta \neq 1$, the successive direction change in the $\beta$–phase increases the chance of link failures. Thus, from Figure 2.10(c), in the region of short link lifetime, the corresponding CDF for $\zeta = 1$ is evidently less than that for $\zeta = 0$ and $\zeta = 0.5$. Based on (2.31), the SMS model generates stable average speed according to $E\{v_{ini}\}$, regardless of the value of $\zeta$. Given $E\{v_{ini}\} = 2$ m/sec, we find that the expected link lifetime of the SMS model based on different $\zeta$ is almost same and around 100 second. In contrast, for the RWP model, because of the speed decay problem, the relative speed between RWP nodes is generally less than that in SMS model. Hence, the lower mobility level and lower topology change rate increase the link lifetime in the RWP model. From Figure 2.10(c), we can see that the uptrend of CDF of link lifetime for the RWP model is dramatically less than that for the SMS model. Meanwhile, the expected link lifetime of the RWP model is 168 second, which is much longer than that of the SMS model. Therefore, according to Figure 2.10(c), we find that the link lifetime analysis and evaluation in MANETs via macroscopic random mobility models, such as RWP model, may invalidate and even lead to wrong conclusions.

**Connectivity Performance Effect**

Due to different spatial node distributions, mobility models with same initial node density ($\sigma$) would yield different average node degree $\Delta$ during the simulation. Node degree is defined as the number of links a node has with its neighboring nodes at an arbitrary time instant. In our
study, for both RWP and SMS mobility models, we set the node transmission range $R = 250$ m and let 50 nodes move in a square area of size $1401 \times 1401 \text{m}^2$. By this means, each node has initially 4 neighbors in its transmission zone, that is, the node density is $\sigma = 5/(\pi R^2)$. Due to the randomness of mobility, the node degree keep varying over time. Figure 2.11(a) illustrates the percentage of nodes whose node degree is no less than 4 during the ns-2 simulation for both mobility models. We observed that the node degree ($\Delta$) of RWP model is larger than that of SMS model during the simulation. In particular, by comparing their average node degrees $\Delta$, we have $\frac{\Delta_{\text{SMS}}}{\Delta_{\text{RWP}}} = 75\%$. This is because the center-concentrated node distribution increases the average node degree in RWP model [18].

Given the condition of node degree and node mobility in a MANET, there is an often question: whether the network is connected or not? In this dissertation, a connected network is defined as a network in which every node-pair has at least a path. Since mobile nodes move randomly and have a limited transmission range, a MANET cannot be always connected over time. Hence, we are interest in measuring the probability of a connected network, defined as the probability that a network is connected at an arbitrary time instant. Specifically, we let 100 nodes be initially and uniformly located in the network with node density $\sigma = 10/(\pi R^2)$ in both RWP and SMS models, where $R = 250$ m. Then we measure the probability of a connected network by taking a series of snapshots of the network graph at every second during the simulation period of 1000 seconds. In particular, we study this probability according to different mobility levels. The statistical results with respect to different average initial node speed are shown in Figure 2.11(b), Figure 2.11(c) and Table 2.2, respectively. It can be seen in Figure 2.11(b) that in both models, the
probability of a network being connected decreases linearly with the increase of the node speed. Intuitively, a larger average node degree of a network implies a higher probability of the network being connected. Surprisingly, though the average node degree of RWP model is higher than that of SMS model, the probability of a network being connected with SMS model is much better than that with RWP model, regardless of the node speed. For instance, when the average speed is 5 m/sec, the probability of a connected network upon SMS model is 0.93, while it is 0.7 upon RWP model. Figure 2.11(c) illustrates the minimum number of nodes in the largest network component when the network graph is disconnected during the simulation. We find that the number of nodes in the largest network component of RWP model is larger than that of SMS model. Moreover, from Table 2.2, the number of network partitions in RWP model is larger than that in SMS model, especially when nodes move at a faster speed. These observations imply that RWP model tends to generate network partitions with the majority of nodes within one network component, but very few nodes in other components. Hence, we find that although the center-concentrate RWP node distribution increases the node degree of centering nodes, its minimum node density in the edge area is prone to induce the network partition during the simulation. As a result, the network connectivity evaluation based on the RWP model can be underestimated.

![Graphs (a) Node degree, (b) Network connectivity, (c) Network component size.](image)

Figure 2.11: Connectivity performance comparison between the RWP and the SMS Model

**Routing Performance Effect**

Continuously, we compare the routing performance effect between SMS and RWP model by taking the AODV protocol [96] as a case study. Specifically, the studied routing metrics include: *average end-to-end packet delay; average end-to-end network throughput* defined as the percentage...
Table 2.2: Network Connectivity: SMS model vs. RWP model.

<table>
<thead>
<tr>
<th>No. of Network Partitions</th>
<th>Average of Initial Speed</th>
<th>Average of Initial Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v = 2\text{m/sec}$</td>
<td>$v = 5\text{m/sec}$</td>
</tr>
<tr>
<td></td>
<td>RWP</td>
<td>SMS</td>
</tr>
<tr>
<td>1</td>
<td>0.777</td>
<td>0.984</td>
</tr>
<tr>
<td>2</td>
<td>0.223</td>
<td>0.016</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

of packets transmitted by the sources that successfully reach their destinations; and routing overhead defined as the ratio of total size of network control packets to the total size of both network control packets and data packets initiated from the sources during the simulation. Upon the simulation setup, among 50 mobile nodes, the network traffics consist of 20 constant bit rate (CBR) sources and 30 connections. The source-destination pairs are chosen randomly through cbrgen tool of ns-2. Each source sends 1 packet/sec with the packet size 64 bytes. In smooth model, we set the time slot $\Delta t$ of each step as 1 second, the memory parameter $\zeta$ as 0.5, and the range of each moving phase duration time as $[6, 30]$ seconds. Furthermore, we respectively set the initial average speed $E\{v_{ini}\}$ as 2, 5, 10, 15, 20, and 25 m/sec to generate different mobility levels for both models. For better demonstrations, we compare the routing performance metrics according to same average initial speed between these two models.

The simulation results are shown in Figure 2.12. From Figure 2.12, there exists an evident difference of simulation results for all routing metrics between the two models. Specifically, all three measured AODV routing performances of the RWP model outperform the smooth model, regardless of the average initial speed, which is mainly resulted from two reasons. As discussed above, one reason is that the decaying RWP speed keeps reducing the link change rate between two neighboring nodes. Hence, the lower frequency of link and path failures will dramatically reduce the routing overhead and packet delay, while increasing the network throughput. The other reason is that non-uniform RWP node distribution with maximum node density in the center region will increase the connectivity of the majority of nodes, which further “improves” the routing performance. Therefore, the lower mobility level and center concentrated node density of RWP model stresses AODV much less than the smooth model. Therefore, we claim that the routing protocols evaluation based on the RWP model is over optimistic. Thus, the SMS model, which generates stable speeds and maintains
uniform node distribution, is more preferable for network connectivity study and routing protocols evaluation in MANETs.

![Graphs](image)

(a) Average end-to-end packet delay. (b) End-to-end network throughput. (c) Routing overhead ratio.

Figure 2.12: Routing performance comparison between the RWP and the SMS Model.

By far, we comprehensively demonstrate the proposed SMS model effects on network performance evaluation and analysis. As we know, the mobile wireless network environments vary dramatically upon different application scenarios. How to flexibly control a mobility model by simulating diverse user moving behaviors to adapt different application scenarios is a non-trivial question.

Next, for the purpose of case studies, we will discuss how to apply the proposed SMS model in different network scenarios. Briefly, we will show the SMS model adaption of group mobility and vehicle mobility under geographical constraints in sequence.

### 2.7.2 Extensions to Group Mobility

Several group mobility models are described in [9, 14, 58] according to different MANET applications. Among these models, Reference Point Group Mobility (RPGM) model is the most widely used model; however, the similar problems of abrupt moving behaviors and the average speed decay also exist in RPGM model [58], since the leader in RPGM model moves according to the RWP mobility pattern. Moreover, it is difficult to generate mobility scenarios with different levels of spatial dependency between group members and their leader[9].

Here, we study the extension of SMS model to group mobility for eliminating above limitations in RPGM model. In SMS mobility model, we consider each group with one leader and several group members. Initially, the leader lies in the center of the group, and other group members
are uniformly distributed within the geographic scope of the group. The size of the group is dependent on the effective transmission range of the wireless devices and the node density. The group leader dominates the moving behaviors of the entire group, including the node speed, direction and moving duration. Specifically, the velocity of group member $m$ at its $n^{th}$ step is represented as:

$$
\begin{align*}
V^m_n &= V^{Leader}_n + (1 - \rho) \cdot U \cdot \Delta V_{max} \\
\phi^m_n &= \phi^{Leader}_n + (1 - \rho) \cdot U \cdot \Delta \phi_{max},
\end{align*}
$$

where $U$ is a random variable with uniform distribution over $[-1, 1]$. $\Delta V_{max}$ and $\Delta \phi_{max}$ are the maximum speed and direction difference between a group member and the leader in one time step. $\rho \in [0,1]$ is the spatial correlation parameter. When $\rho$ approaches to 1, i.e., the spatial correlation between a group member and the leader becomes stronger, the deviation of the velocity of a group member from that of the leader is getting smaller. Therefore, by adjusting the parameter $\rho$, different SMS group mobility scenarios can be generated.

In SMS group mobility model, the group leader follows the exact mobility patterns defined in the SMS model. The detailed moving behaviors of group members in SMS model are described as follows. At the beginning of an SMS movement, the group leader first selects the target speed $v^{Leader}_\alpha$, target direction $\phi^{Leader}_\alpha$, and phase period $\alpha$ which is the same as all group members. Then, corresponding to $v^{Leader}_\alpha$ and $\phi^{Leader}_\alpha$, the target speed $v^m_\alpha$ and target direction $\phi^m_\alpha$ of group member $m$ are selected from (5.28). For each time step in $\beta$-phase, the speed and direction of a group member are also obtained from (5.28) according to the reference velocity of the leader at that time step. When the leader transits into $\gamma$-phase, similarly, each group member selects its own values of $v^m_\beta$ and $\phi^m_\gamma$ towards a stop. Thus, the entire group will stop after $\gamma$ steps. By this means, every group member can evenly accelerate/decelerate the speed in $\alpha/\gamma$-phase while keeping the similar mobility trajectory to the leader. Figure 2.13 gives an illustration of 5 nodes traveling within one SMS movement period in the proposed SMS group mobility model with correlation parameter $\rho = 0.9$, where $\Delta V_{max} = 5$ m/sec, and $\Delta \phi_{max} = \pi/3$. It is observed that all trajectories of group members are in close proximity to that of the leader, as their spatial correlation are strong. Based on the above demonstration, we conclude that SMS model can be easily adapted to group mobility.

### 2.7.3 Adaption to Geographical Constraints

In real world, the movement of nodes are often under geographical constraints such as streets in a city or pathways of obstacles[14, 9]. To achieve a more realistic movement trajectory,
Jardosh et al. in [97] proposed an obstacle mobility model, under the incorporation of randomly selected obstacles, to restrict both node movement as well as wireless transmissions. Also, Bai et al. discussed a Manhattan mobility model in [9].

We observed that the typical moving behaviors of vehicles match the four-phase mobility pattern in the SMS model very well. Besides, vehicular ad hoc networking (VANET) designed for safety driving and commercial applications is a very important research branch of MANETs. As an example, we discuss how to use SMS model with geographical extension to simulate moving behaviors of vehicles in a Manhattan-like city map. Figure 2.14 illustrates the map applied for our SMS model.

As shown in Figure 2.14, each line segment represents a bi-directional street of the city. The speed limit associated with each type of street is labeled on the right side of the map. In this model, the coordinate of intersection points between streets and the street speed limit are known for all nodes. Initially, mobile nodes are randomly deployed in the streets. Each mobile node
randomly chooses a destination and finds the shortest path using Dijkstra’s algorithm for the next movement. For example, in Figure 2.14, the node located in \((X_1, Y_1)\) will reach the destination \((X_2, Y_2)\) through the intersection point \((X_2, Y_1)\). Because mobile nodes are only allowed to move along the predefined pathways in the map, the adapted SMS model describes a straight line movement without direction change for each trip. Thus, the moving behavior of an SMS node along the street is pseudo-random. For each straight line movement, the moving behaviors of the SMS node comply the following rules:

\[
\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} = \frac{V}{2} + V\beta + \frac{V}{2} \gamma,
\]

where \(V\) is the target speed and will not change during the \(\beta\) phase. The selection of \(V\) is determined by the associated street speed limit \(V_{\text{limit}}\), such that \(V \in \left[V_{\text{limit}} - \epsilon, V_{\text{limit}}\right]\), where \(\epsilon\) is a small positive value. In this way, the SMS movement along the street is a typical movement with even speed acceleration and deceleration without speed decay problem. Moreover, the SMS node can properly stop at the target intersection point, such as \((X_2, Y_1)\) in this example.

### 2.8 Summary

The major contributions of this work include theoretical modeling of smooth movement, steady-state analysis, and simulation studies of the impacts of the proposed mobility model. First, we propose a unified, smooth model, namely Semi-Markov Smooth (SMS) model, which includes four consecutive phases: Speed Up phase, Middle Smooth phase, Slow Down phase, and Pause phase, based on the physical law of a smooth motion. Second, our proposed SMS model is a unified mobility model, which can model the mobility patterns of users at varying levels of granularity, i.e., both the macroscopic level and the microscopic level. As the summary, Table 2.3 illustrates a detail comparison based on properties of current typical mobility models and those of our proposed SMS model, where independent mobility parameters: speed \((V)\), movement duration \((T)\), destination \((D)\) and direction \((\theta)\) with respect to different mobility patterns are also included.

Third, we prove that the proposed SMS model is smooth, which eliminates sharp turns and sudden stops; it is steady because there is no speed decay problem for arbitrary starting speed and it maintains uniform spatial node distribution during the entire simulation period. These nice properties are also verified by ns-2 simulations. Besides, we demonstrate the implementation of the proposed model to serve diverse application scenarios, such as group mobility and geographic
Table 2.3: Properties of Different Mobility Models.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>V, θ</td>
<td>V, D</td>
<td>V, θ, T</td>
<td>V, θ</td>
<td>V, θ, T</td>
</tr>
<tr>
<td>Movement Phases</td>
<td>pause</td>
<td>pause</td>
<td>pause</td>
<td>pause</td>
<td>pause, move phase, pause</td>
</tr>
<tr>
<td>Smoothness</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Speed Decay</td>
<td>May</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Uniform Node</td>
<td>close</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Mobility Scale</td>
<td>macroscopic</td>
<td>macroscopic</td>
<td>macroscopic</td>
<td>microscopic</td>
<td>microscopic</td>
</tr>
<tr>
<td>Unified Model</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controllability</td>
<td>low</td>
<td>Low</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
</tbody>
</table>

restrictions. Finally, we present multiple impacts of the SMS model on mobile wireless networks, such as MANETs: the relative speed between a pair of mobile nodes, which shows a Rayleigh distribution; the CDF of link lifetime and average node degree, which demonstrate that the network connectivity evaluation based on the RWP model can be underestimated, however, the routing protocols evaluation based on the RWP model is over optimistic. Therefore, because the proposed SMS model possesses a variety of nice properties and it satisfies the requirements of a “sound mobility model” introduced in [29], this model can be used as a benchmark mobility model for evaluating the performance of various mobile wireless networks.

2.9 Appendix

2.9.1 Derivation of the PMF of movement steps \( K \)

Let \( K = Z + \gamma \), where \( Z = \alpha + \beta \). Then, we derive the pmf of \( K \) by two steps. First, we derive the pmf of random variable \( Z \), which is the function (summation) of two independent uniform discrete random variables. The similar example for deriving the pdf of summation two independent continuous random variables is shown in section 6-2 in [83]. By substituting \( \alpha_{\min}, \beta_{\min} \) and \( \alpha_{\max}, \beta_{\max} \) with \( N_{\min} \) and \( N_{\max} \), respectively, the pmf of \( Z \) is derived as:

\[
Prob\{Z = Z\} = \begin{cases} 
\frac{Z - 2N_{\min} - 1}{(N_{\max} - N_{\min} + 1)} & 2N_{\min} \leq Z \leq N_{\min} + N_{\max} \\
\frac{2N_{\max} + Z - 1}{(N_{\max} - N_{\min} + 1)} & N_{\min} + N_{\max} \leq Z \leq 2N_{\max} 
\end{cases}
\] (2.45)

Second, we obtain the pmf of \( K \) which is summation of random variable \( Z \) and uniform
random variable $\gamma$. Figure 2.15 illustrates a rectangular region inside the $\gamma - Z$ coordinates, which is the intersection area of the domains between $\gamma$ and $Z$. This rectangular region is split into three zones by two dotted lines. For each zone, the pmf of $K$ is derived with respect to three disjoint intervals of domain $K$.

![Figure 2.15: Different domain intervals of $K$.](image)

For zone 1, where $3N_{\text{min}} \leq k \leq 2N_{\text{min}} + N_{\text{max}}$.

$$
\text{Prob}\{K = k\} = \sum_{r=b}^{k-2N_{\text{min}}} P\{\gamma = r\} P\{Z = k - r\}
= \sum_{r=b}^{k-2N_{\text{min}}} \frac{1}{(N_{\text{max}} - N_{\text{min}} + 1) (N_{\text{max}} - N_{\text{min}} + 1)^2} k - r - 2N_{\text{min}} + 1
= \frac{k^2 - 6k \cdot N_{\text{min}} + 3k + 9N_{\text{min}}^2 - 9N_{\text{min}} + 2}{2(N_{\text{max}} - N_{\text{min}} + 1)^3}.
\tag{2.46}
$$

For zone 2, where $2N_{\text{min}} + N_{\text{max}} \leq k \leq N_{\text{min}} + 2N_{\text{max}}$, by assuming $2N_{\text{min}} + N_{\text{max}} \leq N_{\text{min}} + 2N_{\text{max}}$.
\[
\text{Prob}\{K = k\} = \sum_{r = N_{\min}}^{k - (N_{\min} + N_{\max})} P\{\gamma = r\} P\{Z = k - r\} + \sum_{r = k - (N_{\min} + N_{\max}) + 1}^{N_{\max}} P\{\gamma = r\} P\{Z = k - r\}
\]
\[
= \frac{1}{(N_{\max} - N_{\min} + 1)^3} \left( \sum_{r = N_{\min}}^{k - (N_{\min} + N_{\max})} (2N_{\max} + 1 - k + r) + \sum_{r = k - (N_{\min} + N_{\max}) + 1}^{N_{\max}} (k - r - 2N_{\min} + 1) \right)
\]
\[
= \frac{6k(N_{\min} + N_{\max}) - 2k^2 - 3(N_{\min}^2 + N_{\max}^2) + 4N_{\min} \cdot N_{\max} + N_{\min} - N_{\max}) + 2}{2(N_{\max} - N_{\min} + 1)^3}
\]
\[\text{(2.47)}\]

For zone 3, where \(2N_{\max} + N_{\min} \leq k \leq 3N_{\max}\).

\[
\text{Prob}\{K = k\} = \sum_{r = k - 2N_{\max}}^{N_{\max}} P\{\gamma = r\} P\{Z = k - r\}
\]
\[
= \sum_{r = k - 2N_{\max}}^{N_{\max}} \frac{1}{(N_{\max} - N_{\min} + 1)^2} \frac{2N_{\max} + 1 - k + r}{(N_{\max} - N_{\min} + 1)^2}
\]
\[
= \frac{k^2 - 6k \cdot N_{\max} - 3k + 9N_{\max}^2 + 9N_{\max} + 2}{2(N_{\max} - N_{\min} + 1)^3}
\]
\[\text{(2.48)}\]

By combining Eqs. (2.46 - 2.48), we can obtain the complete pmf of \(K\) shown in (2.8).

**2.9.2 Derivation the CDF of Steady State Speed \(v_{ss}\)**

The cumulative density function (CDF) of steady state speed \(v_{ss}\) can be derived from the limiting fraction of time when step speeds are less than \(v\), as the simulation time \(t\) approaches to infinity. When \(t \to \infty\), \(N(t) \to \infty\), we have
\[
Pr\{v_{ss} \leq v\} = \lim_{t \to \infty} \frac{\sum_{n=1}^{M(t)} 1_{\{v_n \leq v\}} + \sum_{n=1}^{M_p(t)} 1_{\{v_n \leq v\}}}{M(t) + M_p(t)} = \lim_{N(t) \to \infty} \frac{\sum_{i=1}^{N(t)} 1_{\{v(i) \leq v\}} + \sum_{j=1}^{N(t)} 1_{\{v(j) \leq v\}}}{\sum_{i=1}^{N(t)} (T(i) + T_p(i))}
\]

\[
= \lim_{N(t) \to \infty} \frac{1}{N(t)} \sum_{i=1}^{N(t)} \left( \sum_{j=1}^{\alpha(i)} 1_{\{v(i,j) \leq v\}} + \sum_{j=\alpha(i)+1}^{\alpha(i)+\beta(i)} 1_{\{v(i,j) \leq v\}} + \sum_{j=\alpha(i)+\beta(i)+1}^{T(i)} 1_{\{v(i,j) \leq v\}} \right).
\]

(2.49)

Let \(R_i(v)\) denote the total time when step speeds are less than \(v\) in the \(i^{th}\) movement. From (2.49), \(R_i(v)\) represents as:

\[
R_i(v) = \sum_{j=1}^{\alpha(i)} 1_{\{v(i,j) \leq v\}} + \sum_{j=\alpha(i)+1}^{\alpha(i)+\beta(i)} 1_{\{v(i,j) \leq v\}} + \sum_{j=\alpha(i)+\beta(i)+1}^{T(i)} 1_{\{v(i,j) \leq v\}}.
\]

(2.50)

In a discrete-time SMS process, \(\{R(v)\}_i, \{T\}_i\), and \(\{T_p\}_i\) are all i.i.d. random sequences, i.e., \(E\{T(i)\} = E\{T\}, E\{T_p(i)\} = E\{T_p\}\), and \(E\{R(v)_i\} = E\{R(v)\}\). As \(N(t) \to \infty\), by the strong law of large number, from (2.49) and (2.50), \(Pr\{v_{ss} \leq v\}\) is derived as:

\[
Pr\{v_{ss} \leq v\} = \frac{1}{E\{T\} + E\{T_p\}} \left( E\{T_p\} + E\{\sum_{j=1}^{\alpha} 1_{\{v_j \leq v\}} + \sum_{j=\alpha+1}^{\alpha+\beta} 1_{\{v_j \leq v\}} + \sum_{j=\alpha+\beta+1}^{T} 1_{\{v_j \leq v\}}\} \right)
\]

\[
= \frac{E\{R(v)\} + E\{T_p\}}{E\{T\} + E\{T_p\}}.
\]

(2.51)

Based on (2.51), we derived \(E\{R(v)\}\) with respect to \(\alpha\)-phase, \(\beta\)-phase, and \(\gamma\)-phase as follows.

1. The average total time when step speeds in \(\alpha\)-phase of one movement are lower than \(v\) is:
\[
E\{ \sum_{j=1}^{\alpha} 1\{v_j \leq v\} \} = \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{\alpha} E\{ 1\{v_{\alpha}(j) \leq v\} \} E\{ 1\{v=m\} \}
\]
\[
= \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{m} Pr\{\alpha = m\} Pr\{\frac{v_{\alpha}(j)}{m} \leq v\}
\]
\[
= \frac{1}{N_{\max} - N_{\min} + 1} \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{m} \int_{v_{\min}}^{\frac{v_{\alpha}(j)}{m}} f_{v_{\alpha}}(v) \, dv. \quad (2.52)
\]

2. The average total time when step speeds in \(\beta\)-phase of one movement are lower than \(v\) is:
\[
E\{ \sum_{j=\alpha+1}^{\alpha+\beta} 1\{v_j \leq v\} \} = \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{\beta} E\{ 1\{v_{\beta}(j) \leq v\} \} E\{ 1\{v=m\} \}
\]
\[
= \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{m} Pr\{\beta = m\} Pr\{v_{\beta}(j) \leq v\}
\]
\[
= \frac{1}{N_{\max} - N_{\min} + 1} \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{m} \int_{v_{\min}}^{v_{\beta}(j)} f_{v_{\beta}}(v) \, dv. \quad (2.53)
\]

3. The average total time when step speeds in \(\gamma\)-phase of one movement are lower than \(v\) is:
\[
E\{ \sum_{j=\alpha+\beta+1}^{T} 1\{v_j \leq v\} \} = \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{\gamma} E\{ 1\{v_{\gamma}(j) \leq v\} \} E\{ 1\{v=m\} \}
\]
\[
= \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{m} Pr\{\gamma = m\} Pr\{v_{\gamma}(1 - \frac{j}{m}) \leq v\}
\]
\[
= \frac{1}{N_{\max} - N_{\min} + 1} \sum_{m=N_{\min}}^{N_{\max}} \sum_{j=1}^{m} \int_{v_{\min}}^{\frac{v_{\gamma}(j)}{m - \frac{j}{m}}} f_{v_{\gamma}}(v) \, dv. \quad (2.54)
\]

By combining (2.51) to (2.54), we obtain the value of \(E\{R(v)\}\) and show the final CDF result of \(v_{ss}\) in (2.30).

2.9.3 Derivation of Expected Steady State Speed \(E\{v_{ss}\}\)

Based on (2.33), We derive four components of \(E\{v_{ss}\}\) according to each state of embedded semi-Markov chain \(\{Y_n\}\) as follows.
1. The average steady state speed $E_{\alpha\{v_{ss}\}}$ in $\alpha$–phase is:

$$E_{\alpha\{v_{ss}\}} = \sum_{m=N_{\min}}^{N_{\max}} Pr\{\alpha = m\} \int_{v} v \cdot \frac{m}{f_{\alpha}(v)} dv \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}$$

$$= \sum_{m=N_{\min}}^{N_{\max}} Pr\{\alpha = m\} \sum_{j=1}^{m} \frac{i}{m} \cdot E\{v_{\alpha}\} \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}$$

$$= \frac{1}{2} E\{v_{\alpha}\}(1 + E\{\alpha\}) \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}.$$  \hspace{1cm} (2.55)

2. The average steady state speed $E_{\beta\{v_{ss}\}}$ in $\beta$–phase is:

$$E_{\beta\{v_{ss}\}} = \sum_{m=N_{\min}}^{N_{\max}} Pr\{\beta = m\} E\{v_{\beta}(j)\} \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}$$

$$= \frac{E\{v_{\alpha}\} \sum_{m=N_{\min}}^{N_{\max}} m Pr\{\beta = m\}}{E\{T\} + E\{T_{p}\}} \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}$$

$$= \frac{E\{v_{\alpha}\} E\{\beta\}}{E\{T\} + E\{T_{p}\}}.$$ \hspace{1cm} (2.56)

3. The average steady state speed $E_{\gamma\{v_{ss}\}}$ in $\gamma$–phase is:

$$E_{\gamma\{v_{ss}\}} = \sum_{m=N_{\min}}^{N_{\max}} Pr\{\gamma = m\} \int_{v} v \cdot \frac{m}{f_{\beta}(v)} dv \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}$$

$$= \sum_{m=N_{\min}}^{N_{\max}} Pr\{\gamma = m\} \sum_{j=1}^{m} \frac{j}{m} \cdot E\{v_{\beta}\} \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}$$

$$= \frac{E\{v_{\alpha}\} \sum_{m=N_{\min}}^{N_{\max}} \frac{j}{m} Pr\{\gamma = m\} \sum_{j=0}^{m-1} j}{E\{T\} + E\{T_{p}\}} \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}$$

$$= \frac{1}{2} E\{v_{\alpha}\}(E\{\gamma\} - 1) \frac{E\{T\} + E\{T_{p}\}}{E\{T\} + E\{T_{p}\}}.$$ \hspace{1cm} (2.57)

4. The average steady state speed $E_{T_{p}\{v_{ss}\}}$ in pause phase is:

$$E_{T_{p}\{v_{ss}\}} = \int_{v} v \cdot \frac{E\{T_{p}\}\delta(v)}{E\{T\} + E\{T_{p}\}} dv = 0.$$ \hspace{1cm} (2.58)
By combining (2.55) to (2.58), we obtain the expected steady state speed $E\{v_{ss}\}$, which is shown in (2.35).
Chapter 3

Joint Effects of Radio Channels and Node Mobility on Link Dynamics in Mobile Wireless Networks

In this chapter, we present the second research topic during this doctoral study. Because both node mobility and radio channel characteristics are the inherent properties in mobile wireless networks, such as MANETs, we aim to study their joint effects on link dynamics, which directly manifests the performance of network applications. To proceed in this study, we first motivate the major impacts on link performance in mobile wireless networks and review related works in Chapter 3.1. Next we characterize the joint effect of radio channels and node mobility on link dynamics in Chapter 3.2. Then, we analyze the link lifetime distribution in Chapter 3.3, upon which we further study the stochastic link properties with respect to effective transmission range and different node speeds in Chapter 3.4. Our results do have direct implications on end-to-end path lifetime, network connectivity and routing performance, which are shown in Chapter 3.5. In Chapter 3.6, we summary our main contributions in this work.

3.1 Motivation and Related Work

End-to-end communications are carried out by a set of radio links between node pairs in multihop networks in which each node, except source and destination, behaves as a relay node to forward data packets to its next hop. Therefore, link properties are essential to applications
and services in such networks because they have direct impacts on many performance metrics, such as end-to-end delay, packet losses, and throughput. For example, the link failure incident to ongoing traffic sessions may induce excessive delay and losses, thus making it difficult to the provisioning of quality of service (QoS) in multihop wireless networks. Moreover, link properties, as fundamental characteristics of network dynamics, can also be used to design mobility-resilient multihop networks [30, 9, 26], maximize routing performance [31, 2], optimize topology control [32, 33, 34], and achieve the desired network performance [5, 7].

However, our understanding of link properties is very limited, mainly because they are determined by a set of random factors, such as radio channels, dynamic transmission range, node mobility and node-pair distance in that there exists a link only a pair-node are within the transmission range of each other. Clearly, these random factors are closely dependent on time-varying radio environment and node mobility. Previous works have been focused on examining the effects of node mobility on link dynamics, such as link lifetime [36, 5, 26, 38, 37], link change rate [98, 26], link residual time and link availability [39, 40, 26, 37]. The main results include: i) Markovian model is an effective method to study relative movements and distance of a node-pair [37, 38]. (ii) There exists a peak in the link lifetime distribution based on random mobility models [36, 5, 26]. iii) The PDF of link change inter-arrival time can be approximated by an exponential distribution with fairly high accuracy [26], and iv) For a k-hop path, when k is very large, the path lifetime distribution converges to exponential distribution [99], whereas k ≥ 4 for simulation results [5]. Specifically, the exponential parameter is the sum of the inverses of average link duration times [99].

Existing research on link properties can be categorized as simulation-based and analysis-based study. Among simulation-based studies, Hong et al. and Boleng et al. respectively studied the effects of link change rate [98] and link lifetime [30] on network performance. Later, Gerharz et al. provided the empirical distributions of link lifetime and residual lifetime in [36, 100] obtained from different mobility models including Random Waypoint (RWP) model [16] Gauss-Markov (GM) model [77] and Manhattan (MH) model [41]. Through the statistical analysis of simulation data, Sadagopan et al. measured the distribution of link lifetime with four random mobility models including Reference Point Group Mobility (RPGM), RWP, GM, and MH models in [5]. The results showed that the distribution of the path lifetime can be well approximated by an exponential distribution when the number of hops is larger than 4 for all mobility models. Because the majority of these simulation studies used random mobility models, especially, using the random mobility models with biased steady state properties [41, 18, 29], the obtained results provide limited under-
standing of link dynamics in multihop wireless networks.

Beside the simulation-based study, analytical studies on link properties are mainly based on random mobility models for either link availability prediction, i.e., link residual lifetime or link stability evaluation which is characterized by the distribution of link lifetime. For instance, McDonald et al. first presented a theoretical prediction method to study link availability[39], and later they provided an enhanced analytical framework to extend the random-independent link availability model to capture the effects of correlated movement [101]. A common weakness of these link availability frameworks is that the link which is predicted to be available during a time period may experience link failures within that time interval. To solve this problem, Jiang et al. redefined the availability of a link \( L(T_p) \) as the probability that the link will be continuously available from \( t_0 \) to \( T_p \), while considering possible changes of the relative speed during the time period \( T_p \)[40]. Recently, Qin et al. provided an iterative algorithm for predicting the link availability according to random walk mobility model [7].

On the study of link stability, i.e., the average link lifetime, it is often assumed that the relative speed and direction do not change when a node traverses the transmission zone of its neighbor node[102, 26]. In order to overcome this constraint, Xu et al and Yang et al recently analyzed the link stability by utilizing the information of variation in node relative movement and distance [37], and node mobility in shadowed environments [43], respectively. Wu et al. used a two-state Markovian model where \( S_0 \) represents the link existence otherwise in \( S_1 \) to evaluate link lifetime distribution in [38]. Further, path lifetime is studied using Palm’ theorem[103], Han et al. showed that the distribution of path lifetime can be well approximated by the exponential distribution, when the number of hops is large.

The limitations of existing works on link properties are three-fold. First, existing random mobility models, such as RWP model and its variants, have significant drawbacks toward the steady-state properties of moving speed and nodal distribution, which could lead to defective analysis and simulations on link studies [104]. Second, the time-scale of random mobility models (e.g., moving duration) is generally much larger than the time-scale of radio channels which may change rapidly and distinctly over short distance and time [35, 33]. However, the study in [44] suggested the time scale used to describe node mobility should be much smaller than the time scale for capturing the significant channel variability. Third, it is assumed in previous studies that transmission range of each node is a constant, which is helpful in simplifying the analysis, at the cost of ignoring the effect of radio environments. For instance, Bettstetter provided a wireless channel model and stud-
ied how shadow fading affects the topology and connectivity of multihop wireless networks[32]. Correspondingly, Yang et al. in [43] proposed a link-stability prediction method based on timely user movement information and received signal strength in shadowed environments. In [44], the authors showed that the relative movement of the transmitter-receiver pair can cause significant channel variability, due to the time-varying multi-path propagation, mobility and multiuser interference. Therefore, considering both radio environments and node mobility at similar time-scale is critical to better, or even correctly understanding link and path properties in multihop wireless networks. Therefore, we aim to study the link properties regarding these two independent, yet simultaneously forcible factors (radio channels and smooth node mobility). Our approach is to use a distance transition probability matrix for modeling the node-pair distance after every discrete time step based on a smooth mobility model which captures the small-scale variation in relative distance. By examining the relationship between time-varying transmission range and node-pair distance, we will be able to study link properties.

Somewhat surprisingly, we show that link lifetime distribution can be effectively approximated by an exponential distribution, which is in contrast to previous results that there exists a peak in the distribution function which are mainly obtained from random mobility models [36, 5, 26]. More interestingly, the exponential distribution parameter can be simplified by $\frac{\bar{V}}{R_e}$, where $\bar{V}$ is the average speed and $R_e$ is the effective transmission range (ETR) of a mobile node. Since the path lifetime is determined by the minimum link lifetime en route, we can easily conclude that the PDF of path lifetime also follows an exponential distribution, which greatly relaxes the assumption of large (approach to infinite) hop-count of a path for its distribution converging to exponential [99]. We also find that the impacting factors on both link and residual link lifetime are in the decreasing order of average node speed, ETR, and node-pair distance.

### 3.2 Characterization of Radio Links and Mobility

As the optimal and fixed radio transmission range are rarely achieved in real dynamic wireless channels, we introduce the concept of effective transmission range (ETR) by using radio fading models [33], which is simple, yet can characterize radio propagation, i.e., path loss, shadowing effect, and multi-path fading of wireless links [35]. Hence, the link lifetime between a pair of nodes is determined concurrently by ETR and node-pair distance over-time upon smooth node mobility [104, 105].
3.2.1 Effective Transmission Range

In mobile radio environments, the received signal is generally influenced by three fading effects: large-scale path loss, multi-path fading, and shadowing [35]. For instance, in vehicular movements, mobile nodes usually move at high speeds, so that the large-scale path loss, can be the dominant factor affecting the signal strength with increasing distance. On the other hand, the relative movement of two persons inside a building may be over a short travel distance (order of wavelengths), which is mainly constrained by small-scale fading, also called multi-path fading. Due to the presence of obstacles in the propagation channel, the signal also undergoes shadowing loss. Figure 3.1 shows an example of the probability of link connection between two nodes under different fading conditions upon transmission distance. When the transmission distance is 200 m, it is observed that the probability of link connection is decreased from 1 (with the large-scale path loss only) to 0.5 when shadow fading is included, and it further reduces to 0.28 if all these three propagation mechanisms are in effect. Clearly, the valid node transmission range becomes a random variable featured by channel fading parameters. Moreover, in [106], it is showed that the link and connectivity analysis given the geometric disc abstraction holds for more irregular shapes of a node transmission zone. Therefore, we introduce a novel metric, Effective Transmission Range (ETR), to capture the effect of radio propagation mechanisms.

![Figure 3.1: Probability of link connection between two nodes, where path loss exponent $\xi = 3$, shadow fading $\sigma_s = 5$ dB, and multi-path fading is 3 dB.](image)

**Definition 2.** In a radio channel characterized by the path loss exponent $\xi$, shadowing $X_{\sigma_s}$ and multi-path fading $\chi^2$, Effective Transmission Range (ETR), denoted by $R_{\text{ETR}}$, is the maximum value of the transmission range $R$, which holds the condition $P_{r,dB} \geq P_{0,dB}$ with a very high probability
Let $\tilde{P}_{dB} = P_{t,dB} - L_{0,dB} - 10 \log_{10} E\{\chi^2\}$, where $P_{t,dB}$ is the transmission power, and $L_{0,dB}$ is the average path loss at the reference point that is 1 meter away from the transmitter. $10 \log_{10} E\{\chi^2\}$ is the average multi-path fading in dB [35]. The probability $P$, i.e. $Pr\{P_{r,dB} \geq P_{0,dB}\}$ is represented as:

$$P = \frac{1}{\sqrt{2\pi}\sigma_s} \int_{-\infty}^{\tilde{P}_{dB} - 10 \log_{10} R_e - P_{0,dB}} \exp(-\frac{x^2}{2\sigma_s^2})dx,$$

$$= \frac{1}{2}[1 - erf(\frac{10 \log_{10} R_e + P_{0,dB} - \tilde{P}_{dB}}{\sqrt{2}\sigma_s})], \quad (3.1)$$

where $erf(\cdot)$ is the error function, defined by $erf(z) = \int_0^z \frac{2}{\sqrt{\pi}} e^{-x^2} dx$. From the Definition 2, we have

$$\begin{cases}
\frac{1}{2}[1 - erf(\frac{10 \log_{10} R_e + P_{0,dB} - \tilde{P}_{dB}}{\sqrt{2}\sigma_s})] = P = 0.99, \\
\frac{10 \log_{10} R_e + P_{0,dB} - \tilde{P}_{dB}}{\sqrt{2}\sigma_s} = -1.65.
\end{cases} \quad (3.2)$$

Hence, upon (3.2), we obtain the ETR, denoted by $R_e$, of mobile nodes with specific requirements in a radio environment:

$$\log_{10} R_e = -\frac{2.33\sigma_s + \tilde{P}_{dB} - P_{0,dB}}{10\xi}. \quad (3.3)$$

For simplification, we assume that certain mobile nodes use the same transmission and receiving power threshold, then $P_{t,dB} - L_{0,dB} - P_{0,dB}$ is a constant value denoted by $c$. From (3.3), we find that $R_e$ can written as a function of three fading parameters:

$$R_e = f(\xi, \sigma_s, \chi) = 10^\frac{-2.33\sigma_s - 10 \log_{10} E\{\chi^2\} + c}{10\xi}. \quad (3.4)$$

As an illustration, from (3.4), we find that an increase of 1 dB in either $\sigma_s$ or $E\{\chi^2\}$ only, $R_e$ will be decreased by 16% and 7%, respectively; when path loss exponent $\xi$ increases by 1, e.g. from 3 to 4, $R_e$ will decrease around 30%.

Remark 1. The impacting weight of channel fading on ETR is in the decreasing order of path loss ($\xi$), shadow fading ($X_{\sigma_s}$), and multi-path fading ($\chi^2$).

The introduction of ETR concept has several advantages. First of all, $R_e$ is able to capture the effect of radio channel parameters such as path-loss, shadowing, and multi-path fading. The
effect of any one of these parameters can be found through the close-form of analytical results in terms of \( R_e \). Second, \( R_e \) provides an easy method to characterize radio channels. For instance, Definition 2 and (3.1) can be further extended to include other factors such as coding schemes, power-control, and bit error rates. Finally, by taking \( R_e \) as a given value during the derivation, the analysis can be greatly simplified, while the results, in fact, can be considered as conditioned on \( R_e \). This empowers us to focused on the models and approaches instead of detailed derivation.

### 3.2.2 Smooth Mobility Model

Many literatures have shown that node mobility has significant impact on link properties in multihop wireless networks [98, 100, 5, 9, 26]. Therefore, the mobility model selected for studying link dynamics is critical to the results. It is worth noting that the time-scale of wireless channels is closely dependent on radio propagations. In particular, the path loss is the function of distance, and does not vary with time. As the shadow fading varies with travel distance on the order of tens or hundreds of meters, it generally would not vary within a short time interval. In contrast, the multi-path fading changes within small distance and the fading changing rate is proportional to the receiver velocity. Hence, multi-path fading changes in the order of seconds[35]. Therefore, in order to observe the concurrent influence of radio channels and mobility on link lifetime, we must consider the characteristics of node mobility in the similar time-scale of radio channels. Specifically, the mobility model should satisfy the following requirements:

1. Since the signal strength may change rapidly and distinctly over short travel distance and short time [35, 33], the mobility model need to describe the minute variation of node velocity in small time-scale, by which the up-to-date information of relative movement between two nodes can be easily obtained.

2. To comply with the physical law of a smooth motion [104], the mobility model should capture the temporal correlation of node velocities with smooth speed and direction transition in each movement [77, 42, 41], which results in the frequent variation of relative speed between two nodes during the link connection.

Among existing mobility models [77, 42, 105], the SMS mobility model [105] we proposed in our first work, which is carefully studied in Chapter 2, is chosen as a benchmark because this model allows flexible, small equal-length time steps (\( \Delta t \)) for smooth movement description. The model complies with the physical law of smooth motion: each node accelerates its speed to the
target speed of the movement initially, and decelerates speed before a full stop. Furthermore, the model has nice steady-state properties of uniform nodal distribution for analysis and stable moving speed for simulation verification. 1

This model follows the physical law of a smooth motion, each movement in the smooth model contains three consecutive moving phases: Speed Up ($\alpha$-) phase, Middle Smooth ($\beta$-) phase, and Slow Down ($\gamma$-) phase. Each movement is quantized into random $K$ equal-length time steps, where $K = \alpha + \beta + \gamma$. The time interval between two consecutive time steps is $\Delta t$ (sec). For each movement, a node will select a target direction $\phi_\alpha$ and a target speed $\bar{V}$, which are the average direction and speed of the movement. Specifically, in $\alpha$–phase, it uniformly accelerates its speed to $\bar{V}$ for $\alpha$ time steps along the direction $\phi_\alpha$. For each time step in $\beta$–phase, where the node moves at the stable velocity of the movement, the node speed and direction gently fluctuate around $\bar{V}$ and $\phi_\alpha$, respectively. In $\gamma$–phase, the node uniformly decelerates its speed to 0 for $\gamma$ time steps along a selected direction $\phi_\gamma$. At the end of a movement, the node pauses a random time $T_p$. It is also shown in[104] that the smooth model generates stable node speed and maintains uniform node distribution. These nice properties are crucial to analyze link dynamics.

3.2.3 Node-Pair Distance

The distance between two mobile nodes is denoted by Node-Pair Distance, $\rho$, which is dependent on the relative movements of two nodes. For instance, $\rho_m$ represents the distance between two nodes after $m$ time steps [104]. As an example, Figure 3.2(a) illustrates the relationship between the maximum transmission range $R_{\text{max}}$ and node-pair distance $\rho_m$ under different radio environments. Thus, by comparing the value between the time-varying variable $R_{\text{max}}$ and $\rho_m$ at each time step (normalized to 1 second per time step $\Delta t$), the corresponding link existence can be obtained, which is shown in Figure 3.2(b). It is evident that the link lifetime and breakage rate can vary dramatically under different radio channels. For instance, the frequency of link breakage under channels with additional shadowing and multi-path fading is about 19 times higher than that with path loss only, which is the inverse of average link lifetime.

In this example, both two mobile nodes move at average speed 2 m/sec based on the smooth mobility model [104]. As shown in Figure 3.2(a), if only consider large-scale path loss,
the ETR is considered to be constant of 200 m, hence the link between two nodes is connected as long as their relative distance is less than 200 m. While as consider the other two fading, the ETR transits dramatically within seconds. By comparing the relative distance and the ETR, Figure 3.2(b) illustrates the link lifetime between two mobile nodes with respect to different channel fading. It is interesting to observe that both shadowing and multi-path fading may increase the chance of link connection because of randomness of signal fading. For example, during the time interval [160, 170] sec, the link is connected under both shadowing and multi-path fading while the relative distance is larger than 200m, which is the threshold of transmission range upon the path loss only. However, from the overall impacts, it is evident that link lifetime is much shorter and the link breaks much more frequently when either shadowing or multi-path or both fading effects are considered. Based on this illustration, the frequency of link breakage under all three fading is 19 times larger than that with path loss only and 7 times larger than that with both path loss and shadowing. And their impacts on average link lifetime is based on the reverse ratio of that on the link breakage rate.

As shown in Figure 3.2(a), given a specific radio environment, the maximum transmission range $R_{\text{max}}$ between two nodes varies dramatically with time. Accordingly, the effective transmission range $R_{\text{e}}$, defined in (3.3), can efficiently characterize the valid transmission distance with specific radio fading. In fact, the similar concept of ETR has been already applied in the real industrial world. For example, the Accutech wireless instrumentation products use $1/3$ of the maximum transmission range $R_{\text{max}}$ as the rule-of-thumb for working transmission range [107]. Therefore, the above observations motivate us to analyze the probability distribution of link lifetime by comparing
effective transmission range $R_e$ with node pair distance $\rho_m$.

**Remark 2.** For a pair of nodes $(u, w)$, there exists a link between them if and only if their distance $\rho_m$ is no greater than their symmetric effective transmission range $R_e$.

Thus, the link lifetime $T_L$, in essence, is defined as

$$T_L \triangleq \sup_{m > 0} \{ m \cdot \Delta t : \max \rho_m \leq R_e \}.$$  \hspace{1cm} (3.5)

Note that the node-pair distance $\rho_m$ is dependent on node mobility. Hence, in the next section, we start with the relative movement of a node-pair upon smooth mobility model, and then we derive the link lifetime distribution.

### 3.3 Link Lifetime Distribution

Fundamental link properties can be analyzed by link lifetime, residual link lifetime, and link change rate. The link lifetime is random variable, which is defined as the time from a link appears between a node-pair to the moment this link is broken. The probability distribution function (PDF) of link lifetime demonstrates the basic link property, and can be used to analyze other properties as shown later in Section 3.4. In this section, we propose a new modeling approach for the analysis by several steps: find the relative speed and relative distance first, then define a transition probability matrix to model the distance transition at each time step, followed by the derivation of link lifetime distribution.

#### 3.3.1 Relative Movement: Speed and Distance

For a node-pair $(u, w)$, we use node $u$ as the reference node, which lies in the center of its transmission zone with radius of the effective transmission range $R_e$. As explained in previous section, we use the smooth mobility model [104] in order to match the time-scale variation of radio channels [44] and smooth motion of moving nodes. Thus, the relative distance of a node-pair can be represented by relative positions at each time step. An example of the relative movement trajectory is illustrated in Figure 3.3. We denote $v_m$ as the magnitude of the relative speed vector $\overrightarrow{v}_m$. After the $m^{th}$ time step relative movement, node $w$ lies at the position represented by $(X_m, Y_m)$. Correspondingly, $\rho_m$, the node-pair distance, is the magnitude of the vector $\overrightarrow{\rho}_m$, such that $\rho_m = \sqrt{X_m^2 + Y_m^2}$. We assume that the relative speed $v_m$ and the angle $\psi_m$ of node $w$ are
i.i.d. RVs, then the coordinate $X_m$ and $Y_m$ can be approximated by Gaussian random distribution, when $m >> 1$ [39]. For simplicity, we normalize the time step unit $\Delta t$ to 1 second throughout the rest of the paper, then the $m^{th}$ step relative speed $v_m$ is:

$$v_m = |\vec{v}_m| = \sqrt{(X_m - X_{m-1})^2 + (Y_m - Y_{m-1})^2}, \quad (3.6)$$

where both RV $(X_m - X_{m-1})$ and $(Y_m - Y_{m-1})$ can be effectively approximated by an identical Gaussian distribution with zero mean. Thus, upon the same arguments in [39], when $m >> 1$, $v_m$ can be further effectively approximated by a Rayleigh density represented as:

$$f_Z(z) = \frac{z}{\alpha^2} e^{-\frac{z^2}{2\alpha^2}} U(z) \quad \text{and} \quad E\{z\} = \alpha \sqrt{\frac{\pi}{2}}, \quad (3.7)$$

where $\alpha$ is the parameter of the Rayleigh distribution [83]. To simplify the analysis, we assume that mobile nodes have the same average moving speed $\bar{V}$, though with different mobility pattern. Then the range of relative speed of two nodes is over $[0, 2(\bar{V} + \delta V)]$, depending on either two node moving along the same direction or the opposite direction, where $\delta V$ is the maximum speed deviation of $\bar{V}$ in one movement introduced in the smooth model [104]. Corresponding to (3.7), $\bar{V} = \alpha \sqrt{\pi/2}$, then the PDF of relative speed is:

$$f_V(v) = \frac{v}{(\bar{V} \sqrt{\frac{\pi}{2}})^2} e^{-\frac{v^2}{2(\bar{V} \sqrt{\frac{\pi}{2}})^2}} = \frac{\pi v}{2\bar{V}^2} e^{-\frac{\pi v^2}{4\bar{V}^2}}. \quad (3.8)$$

To validate the expression in (3.8), we obtain the relative speed distribution $f_V(v)$ between two nodes by simulations under different levels of node speed. Figure 3.4 illustrates the PDF of relative speed resulted from both simulation and the theoretical expression from (3.8) versus different values of $\bar{V}$ as $[2, 5, 10, 15, 20]$ m/sec, respectively. It can be observed that the approximated Rayleigh distribution matches very well with the distribution of relative speed obtained by simulations.

**Remark 3.** The relative speed of a node-pair can be approximated by Rayleigh distribution not only for large-scale mobility [39], but also for small-scale smooth mobility. In fact, the smaller the time step of mobility modeling is, the more accurate the approximation yields.

As illustrated in Figure 3.3, $\rho_m$ is a random variable that depends on the current and next positions of node $w$ in relation to node $u$. Specifically, at the $m^{th}$ step, $\vec{\rho}_m = \vec{\rho}_{m-1} + \vec{v}_m$. 


Figure 3.3: Relative movement trajectory of node-pair $(u, w)$.

Figure 3.4: Rayleigh distribution approximation of the relative speed.

Hence, $\rho_m$ can be represented as:

$$\rho_m = |\vec{\rho}_m| = \sqrt{\rho_{m-1}^2 + v_m^2 - 2\rho_{m-1}v_m \cos \psi_m},$$

(3.9)

where $\psi_m$ is uniformly distributed from $[0, \pi)$. From (3.9), $\psi_m$ can be represented as:

$$\psi_m = \arccos \frac{\rho_{m-1}^2 + v_m^2 - \rho_m^2}{2\rho_{m-1}v_m}.$$

(3.10)
We denote \( f_{\rho_m|\rho_{m-1}}(\rho_m \mid \rho_{m-1}) \) as the conditional distribution of relative distance, which is given by

\[
 f_{\rho_m|\rho_{m-1}}(\rho_m \mid \rho_{m-1}) = \int_0^{2(\bar{V} + \delta V)} f_{\rho_m|\rho_{m-1},v_m}(\rho_m \mid \rho_{m-1}, v_m) \cdot f_V(v_m) dv_m = \int_0^{2(\bar{V} + \delta V)} \frac{2\pi \rho_m \cdot f_V(v_m)}{\sqrt{[4\rho_m^2 - (\rho_{m-1}^2 + v_m^2 - \rho_{m}^2)^2]}^{1/2}} dv_m
\]

(3.11)

where \( f_V(v) \) is the PDF of the relative speed. Thus, the conditional probability of node-pair distance can be determined by substituting \( f_V(v) \) obtained from (3.8) into (3.11). The result of (3.11) is useful in understanding the transition between two consecutive steps. However, it is not sufficient to know the node-pair distance at an arbitrary time instant, which is a time-varying variable. In order to examine the node-pair distance at each time step, the effective transmission range \( R_e \) of node \( u \) is quantized into \( n \) equal-length intervals with a width of \( \varepsilon \) meters. Hence, \( R_e = n \cdot \varepsilon \), which indicates that there are \( n \) states within the transmission zone. Each interval \( \varepsilon \) is associated with a state representing the \( u-w \) distance. For example, state \( S_i \) indicates that the \( u-w \) distance interval is over the range \( [(i-1)\varepsilon, i\varepsilon] \), which is shown in the lower half in Figure 3.3. Note, since \( \varepsilon \) is a unit of distance interval, the number of states \( n \) is a variable in proportion to \( R_e \), which is in turn characterized by the wireless environment.

### 3.3.2 Distance Transition Matrix \( P \)

We denote \( P \) as the distance transition probability matrix, to model the distance transition at each time step. Each element \( P_{ij} \) indicates the transition probability that \( u-w \) distance is changed from current state \( S_i \) to next state \( S_j \) after one time step. From Figure 3.3, the link expires after the \( M^{*th} \) time step when the event of \( \{\rho_{M^*} > R_e\} \) first happens. In addition, we use state \( S_{n+1} \) to represent all the \( u-w \) distances that are greater than \( R_e \). Since link connection breaks when node \( w \) reaches state \( S_{n+1} \), we define state \( S_{n+1} \) as the absorbing state of matrix \( P \). This implies that \( P \) is an \( n \) by \( n + 1 \) matrix. The value of \( P_{ij} \) of matrix \( P \) is essential to the analytical study of link dynamics. The details of how to find the link lifetime distribution by using \( P \) will be explained in Section 3.3.3. Next, we derive the approximation of \( P_{ij} \) based on node-pair distance distribution in (3.11).
First, the transition probability $P_{ij}$ can be represented by:

$$P_{ij} = \text{Prob}\{\rho_m \in S_j \mid \rho_{m-1} \in S_i\} = \frac{\text{Prob}\{(j-1)\varepsilon \leq \rho_m \leq j\varepsilon \cap (i-1)\varepsilon \leq \rho_{m-1} \leq i\varepsilon\}}{\text{Prob}\{(i-1)\varepsilon \leq \rho_{m-1} \leq i\varepsilon\}} = \frac{\int_{(j-1)\varepsilon}^{j\varepsilon} \int_{(i-1)\varepsilon}^{i\varepsilon} f_{\rho_m|\rho_{m-1}}(\rho_m|\rho_{m-1})f(\rho_{m-1})d\rho_{m-1}d\rho_m}{\int_{(i-1)\varepsilon}^{i\varepsilon} f(\rho_{m-1})d\rho_{m-1}}. \quad (3.12)$$

It is clear that (3.12) can be obtained by substituting (3.11) into it. However, we find the result of such an expression of (3.12) cannot be simplified to a closed-form representation and is too complicated for computation. Thus, we aim to derive an approximation of $P_{ij}$ for easy analysis.

**Theorem 1.** The transition probability $P_{ij}$ of matrix $P$ can be approximated by

$$
\begin{aligned}
P_{ij} &\approx \frac{0.3\varepsilon}{V}\sqrt{\frac{2j-1}{2i-1}} \left[ \ln \frac{|4(V+\delta V)^2 - \varepsilon^2(j-i)^2|}{\varepsilon^2(j+i-1)^2 - 4(V+\delta V)^2(j-i)^2} \right]^{1/2} \\
&\forall i.
\end{aligned} \quad (3.13)
$$

Recall that $V$ represents average node speed and $\delta V$ is the maximum variation of $V$ according to smooth model [104].

**Proof.** Let $f(x) = e^{\frac{-x^2}{2\varepsilon^2}}$ and $g(x) = \left[ 4\rho_{m-1}^2 - [x - (\rho_m^2 + \rho_m^2)]^2 \right]^{-1/2}$.

With (3.11), we can see that $f(x) > 0$ and $g(x) > 0$, when $x \in [0, 4(\bar{V} + \delta V)^2]$. By using Schwarz inequality [83],

$$\int_a^b |f(x) \cdot g(x)|dx \leq \left[ \int_a^b |f(x)|^2dx \right]^{1/2} \left[ \int_a^b |g(x)|^2dx \right]^{1/2}. \quad (3.14)$$

we have

$$f_{\rho_m|\rho_{m-1}}(\rho_m \mid \rho_{m-1}) \leq \frac{\rho_m}{2V^2} \left[ \int_0^{4(\bar{V}+\delta V)^2} e^{\frac{-x^2}{2\varepsilon^2}}dx \right]^{1/2} \times \left[ \int_0^{4(\bar{V}+\delta V)^2} \frac{dx}{4\rho_{m-1}^2\rho_m^2 - [x - (\rho_m^2 + \rho_m^2)]^2} \right]^{1/2}. \quad (3.15)$$

Then by respectively deriving the integral of $f(x)$ and $g(x)$, plus a bit work on simplification, the approximation of the conditional distribution $f_{\rho_m|\rho_{m-1}}(\rho_m \mid \rho_{m-1})$ can be:
We further apply the Mean-Value theorem to derive the numerical solution of \(P_{ij}\). In particular, according to \(P_{ij}\) defined in (3.12), where \((j - 1)\epsilon \leq \rho_m \leq (i - 1)\epsilon \leq \rho_{m-1} \leq i\epsilon\), if \(\epsilon\) is sufficiently small, we can effectively use the middle point \(i - \frac{\epsilon}{2}\) and \(j - \frac{\epsilon}{2}\) to respectively represent the value of \(\rho_{m-1}\) and \(\rho_m\) [37]. For instance, \(\int_{(i - 1)\epsilon}^{i\epsilon} f(\rho_m) d\rho_m \approx \epsilon \cdot f(i\epsilon - \frac{\epsilon}{2}). \) With this argument and the result from (3.16), \(P_{ij}\) derived in (3.12) can be effectively approximated by \(\tilde{P}_{ij}\):

\[
\tilde{P}_{ij} \approx \epsilon \cdot f_{\rho_m|\rho_{m-1}}[(j - \frac{1}{2}) \cdot \epsilon \mid (i - \frac{1}{2}) \cdot \epsilon] 
\]  

(3.17)

By using the results from (3.16) and (3.17), we can obtain \(\tilde{P}_{ij}\) as shown in Theorem 1. Note, the approximation value of \(\tilde{P}_{ij}\) is normalized along each row of the matrix \(P\) to guarantee the fundamental property of the transition matrix \(P\), i.e., \(\sum_j P_{ij} = 1\), \(\forall i, 1 \leq i \leq n\), as shown in (3.13).

![Figure 3.5: Approximation of \(P_{ij}\) with respect to \(\epsilon\).](image)

To validate the accuracy of this approximate expression, we illustrate the computational error versus \(\epsilon\) between the numerical result of \(P_{ij}\) from (3.12) and (3.13) in Figure 3.5(a). Specifically, we set \(\bar{V} = 15\) m/sec, \(R = 250\) m, the current state \(i = 50\), and let \(\epsilon\) vary from 1 m to 3 m. It
is observed that, as ε decreases, the computational error is reduced, which is up to 0.02 when ε = 1 m. For different speeds, Figure 3.5(b) illustrates the computational error of $P_{ij}$ between (3.13) and (3.12) for $v_0 = 5, 10, 15, 20$ m/sec, respectively. We observed that the average computational error decrease as the growth of the target speed, which is within 0.03 when the target speed $\bar{V}$ is larger than 10 m/sec. This is because the number of possible states in matrix $P$ within each time step is directly proportional to the node target speed, which in turn, is proportional to the accuracy of $P_{ij}$. Therefore, we conclude that $P_{ij}$ approximation can achieve fairly high accuracy (less than 0.03) when ε is sufficient small compared to a specific target speed $\bar{V}$.  

3.3.3 Approximation of Link Lifetime Distribution

Upon Figure 3.3, a communication link between a node-pair forms immediately after the node $w$ crosses the border of node $u$’s transmission zone at time $t_0$. Recall that $T_L$ denotes the link lifetime, which is the time node $w$ continuously lies inside node $u$’s transmission zone. The link expires after the $M^{th}$ time step when the node-pair distance is larger than ETR for the first time since $t_0$. In this example, $T_L = M^* \Delta t$, hence $T_L$ is a random variable from (3.5) and the CDF of link lifetime is $\text{Prob}\{T_L \leq m\}$ for $\Delta t = 1$ s.

Here, we derive the link lifetime distribution based on the distance transition matrix $P$ obtained in previous subsection. We denote by $\pi_i^{(m)}$ the probability that node $w$ lies in state $S_i$ after the $m$th step, and $\pi^{(m)}$ is the row vector whose $i^{th}$ element is $\pi_i^{(m)}$. That is $\pi^{(m)} = \left(\pi_1^{(m)}, \ldots, \pi_i^{(m)}, \ldots, \pi_{n+1}^{(m)}\right)$. And $\pi^{(0)}$ denotes the probability of the initial state that node $w$ lies when the link is initially formed, for instance, according to illustration in Figure 3.3, at time $t_0$, $\pi_i^{(0)} = \text{Prob}\{\rho_0 \in S_i\}$. For simplicity, we denote matrix $P$ as $P = \left[P_1, \ldots, P_j, \ldots, P_{n+1}\right]$ and $P_j$ is the $j^{th}$ column vector of $P$. That means,

$$P_j = [P_{1j}, P_{2j}, \ldots, P_{ij}, \ldots, P_{n+1j}]^T,$$

(3.18)

where $P_{ij}$ is obtained from Theorem 1 in (3.13).

Because $S_{n+1}$ is the absorbing state of the matrix $P$, $[\pi^{(0)}P^{m}]_{(n+1)}$ represents the probability that node $w$ moves outside node $u$’s transmission zone within $m$ time steps. Then the CDF of link lifetime can be obtained by:

$$\text{Prob}\{T_L \leq m\} = \text{Prob}\{\rho_m > R_e | \rho_0 \leq R_e\} = [\pi^{(0)}P^m]_{(n+1)} = \pi_{n+1}^{(m)}.$$  

(3.19)

\footnote{Based on the statistical simulation study, the configuration that $\varepsilon = 1$ m if $\bar{V} \geq 10$ m/sec; otherwise, $\varepsilon = \bar{V}/10$ m can satisfy the high accuracy requirement.}
The probability matrix $P$ is already determined by using Theorem 1. To find the stationary probability $\pi_0$, recall that the range of relative speed of two nodes is over $[0,2(\bar{V} + \delta V)]$. Hence, the maximum distance between a node-pair during each time step is $2(\bar{V} + \delta V)$. This means the maximum number of states $N$ of $P$ can be traveled during one time step is,

$$N = \left\lceil \frac{2(\bar{V} + \delta V)}{\varepsilon} \right\rceil. \quad (3.20)$$

In Figure 3.3, when node $w$ moves across node $u$’s transmission zone, it may be at one of $N$ possible states (from state $S_n$ to $S_{n-N+1}$) at time $t_0$. Here we assume that node $w$ initially lies in these $N$ states with an equal probability as $1/N$ for determining the distribution $\pi_0$. Following (3.19) and (3.20), the PMF of link lifetime distribution is derived as:

$$Prob\{T_L = m\} = Prob\{T_L \leq m\} - Prob\{T_L \leq m - 1\}$$

$$= [\pi_0 P^m]_{(n+1)} - [\pi_0 P^{m-1}]_{(n+1)}. \quad (3.21)$$

In order to have a better understanding the above results, we simulate both the radio environments and smooth node mobility by ns-2. Specifically, the value of ETR can be configured by adjusting an appropriate value of the receiving threshold over the network interface. Because the current ns-2 physical module does not support multi-path (Rayleigh) fading, we set up the receiving threshold according to the Shadowing propagation-model. Specifically, upon (3.4), the value of ETR is chosen from the set $\{94, 149, 200, 239, 286, 342\}$ m, which are obtained by considering typical urban micro-cells ($3 \leq \xi \leq 3.5$) superimposed with shadow fading ($\sigma_s \in [6,9]dB$) [35]. TABLE 3.1 illustrates the relation between ETR with the corresponding radio environments.

<table>
<thead>
<tr>
<th>$ETR$ (m)</th>
<th>94</th>
<th>149</th>
<th>200</th>
<th>239</th>
<th>286</th>
<th>342</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>3.5</td>
<td>3.5</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$\sigma_s$ (dB)</td>
<td>9</td>
<td>6</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

Here, we carried out multiple trials with 50 nodes with $R_e = 239$ m, uniformly distributed in an area of $1401m \times 1401m$ during a time period of 1000 seconds. The smooth user mobility [104] is set to zero pause time, 0.5 for temporal correlation parameter $\zeta$, $[20,40]$ seconds for the moving phase, and $[4,6]$ seconds for acceleration and deceleration phases. Figure 3.6(a) illustrates the
link lifetime distribution with two mobility levels: low level ($V = 2 \text{ m/sec}$) and high level ($V = 20 \text{ m/sec}$). For clear demonstration, we show the results in the log-scale on Y-axis. According to Figure 3.6(a), both theoretical and simulation results demonstrate that link lifetime decreases exponentially with time regardless of the node speed and it decreases much quickly as the node speed is high.

![Figure 3.6: Link lifetime distribution.](image)

Interestingly, by taking a close look, we find that the PMF of link lifetime distribution can be approximated by an exponential distribution with parameter $\frac{V}{R_e}$, which will be discussed in detail in Section 3.4.1, that is,

$$f_{T_L}(t) \approx \frac{V}{R_e} \cdot e^{-\frac{V}{R_e}t} = \frac{V}{f(\xi, \sigma_s, \chi)} \cdot e^{-\frac{V}{f(\xi, \sigma_s, \chi)}t}.$$  \hfill (3.22)

The equation (3.22) in fact represents the PDF of link lifetime with continuous time $t$. It can be seen in Figure 3.6(b) that this approximated exponential distribution characterized by the parameter $\frac{V}{R_e}$, matches very well with the simulation results, especially for high speed. Recall, $R_e = f(\xi, \sigma_s, \chi)$, defined in (3.4), is a function of radio channel parameters: path loss ($\xi$), shadow fading ($X_{\sigma_s}$), and multi-path fading ($\chi^2$). Hence, the parameter $\frac{V}{R_e}$ in (3.22) indicates that the link performance in mobile wireless network can be characterized by joint effects of radio channels and node mobility.

**Remark 4.** The link lifetime distribution can be effectively approximated by an exponential distribution with parameter $\frac{V}{R_e}$, where $V$ is the average speed and $R_e$ is the ETR of a mobile node. This result is in contrast with previous studies that there exists a peak in the distribution function which are mainly obtained from random mobility models [36, 5, 26].
3.4 Link Stochastic Properties

In this section, we discuss link properties such as average link lifetime, residual link lifetime, and link change rate. These link dynamics effectively reveal the changing frequency of network topology [32, 33], efficiency of routing operations [9, 31], and application performance in multihop wireless networks [5, 44].

3.4.1 Average Link Lifetime

From (3.21), the average link lifetime \( T_L \) is given by:

\[
T_L = \sum_{m=1}^{\infty} m (\pi(0) P^m)_{(n+1)} - [\pi(0) P^{m-1}]_{(n+1)}.
\]

(3.23)

Given \( R_e = 239 \) m, both theoretical and simulation results of \( T_L \) with respect to average node speed \( \bar{V} \) are shown in Figure 3.7(a), which match very well. Also, as the node speed increases, \( T_L \) decreases dramatically when \( \bar{V} \) is within the range \([2, 10]\) m/sec, and the downtrend of \( T_L \) slows down when \( \bar{V} > 10 \) m/sec. More interestingly, we find that the \( T_L \) can be estimated by the empirical equation \( \hat{T}_L = R_e / \bar{V} \). Table 3.2 illustrates the results of both theoretical \( T_L \) from (3.23) and estimated \( \hat{T}_L \) with respect to node mobility. The physical meaning of the equation \( \hat{T}_L = R_e / \bar{V} \) is the time a node takes to move across the radius of its neighbor’s transmission zone at its average speed \( \bar{V} \). This result could be used as an engineering approximation of link lifetime in ad hoc networks, especially for low mobility to medium mobility with speed less than 35 km/hour.

Table 3.2: Comparison: \( T_L \) and estimated \( \hat{T}_L \), for \( R_e = 239 \) m.

<table>
<thead>
<tr>
<th>( \bar{V} ) (m/sec)</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_L ) (s)</td>
<td>112.94</td>
<td>50.75</td>
<td>24.89</td>
<td>19.23</td>
<td>15.72</td>
<td>12.68</td>
</tr>
<tr>
<td>( \hat{T}_L ) (s)</td>
<td>119.5</td>
<td>47.8</td>
<td>23.9</td>
<td>15.9</td>
<td>12.0</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Based on the theoretical results on (3.23), we further investigate the ETR effect on average link lifetime \( T_L \) with different node mobility. The results are shown in Figure 3.7(b). We find that the larger \( R_e \) is, the longer the \( T_L \) is obtained, which is consistent with our expectation. However, it can be observed that the ETR has much more significant impact on \( T_L \) for nodes with low mobility than those with high mobility. For example, in the case of low mobility, i.e., the average node speed
is 2 m/sec, $T_L$ varies from 50 sec to around 125 sec when $R_e$ increases from 94 m to 342 m, respectively. In contrast, when the node speed is high, at 25 m/sec, with the same range of $R_e$, the value of $T_L$ varies within 20 sec. We further compare the PMF of link lifetime by simulations between random mobility model without temporal correlation such as RWP model [41] and the temporal mobility model such as Gauss-Markov (GM) model [77] and the smooth mobility model [104]. Given the same initial average node speed and transmission range, from Figure 3.7(c), we observed that the PMF of link lifetime in RWP model is apparently smaller than that of both smooth model and GM model. This is mainly because the temporal mobility models with correlation of node velocity generate more frequent variation of relative velocity between two nodes than the random mobility models. These variations in relative speed, however, are liable to induce link failures. Therefore, compared to smooth mobility models in small time scale, link lifetime resulting from random mobility models in large time scale can be over optimistic[41, 104], which could further cause biased evaluation of routing optimization and network design [5, 9, 26].

![Figure 3.7: Stochastic properties of link lifetime.](image)

(a) Validation of average link lifetime.  
(b) ETR vs. node speed impacts.  
(c) Different mobility models impacts.

**Remark 5.** For multihop wireless networks with lower node mobility, or even without node mobility such as static sensor networks, the average link lifetime $T_L$ is predominated by ETR, i.e., radio channel characteristics. For a network with faster mobile nodes such as vehicular ad hoc networks, $T_L$ is dominated by node speed.

### 3.4.2 Residual Link Lifetime

Residual link lifetime $T_R$ is the remaining link duration after the link is established. It can be interpreted by link availability $L_{ii}^{m,m'}$, which is a probability that a link will be continuously
available at least \( m' \) steps given that the link exists \( m \) time steps with node-pair distance \( \rho_m \) in state \( S_i \).

\[
L(\rho_m^{(i)}, m') = \frac{\text{Prob}\{T_L \geq m' + m\}}{\text{Prob}\{T_L > m \mid \rho_m \in S_i\}}. \tag{3.24}
\]

Therefore, upon the definition of link availability \( L(\rho_m^{(i)}, m') \) in (3.24), the corresponding PMF of residual link lifetime \( T_R \) is represented as

\[
\text{Prob}\{T_R = m'\} = L(\rho_m^{(i)}, m') - L(\rho_m^{(i)}, m' + 1) = [\pi_{i,1}^{(m)} P^{m'+1}]_{(n+1)} - [\pi_{i,1}^{(m)} P^{m'}]_{(n+1)}, \tag{3.25}
\]

where \( \pi_{i,1}^{(m)} \) is a vector in which the \( i \)-th element is equal to 1, while other elements are equal to 0. The physical meaning of \( \pi_{i,1}^{(m)} \) is after \( m \)-th time step, the probability of the node location is \( \text{Prob}\{\rho_m \in S_i\} = 1 \).

---

Figure 3.8: Residual link lifetime: analytical and simulation results.

Figure 3.8(a) illustrates the analytical result of PMF of residual link lifetime \( T_R \) with respect to different node-pair distance when \( \bar{V} = 10 \) m/sec and \( R_e = 239 \) m. We notice that there always exists a peak for the PMF distribution of \( T_R \), which shifts towards right side as the node-pair distance decreases. It implies that larger relative distance is more likely to induce link failures, which is consistent with our snapshot illustration in Figure 3.2, but from a statistical point of view. Given this initial 200 m node-pair distance, Figure 3.8(b) illustrates the average residual link lifetime with respect to node mobility under different ETRs by simulations. It turns out the average residual link
lifetime is much more sensitive to the node mobility than the transmission range. Compared to the results shown in Figure 3.7(b), we also observe that the impacts of transmission range on the average residual link lifetime is very similar to that on the average link lifetime. Furthermore, Figure 3.8(c) illustrates the influence of initial node-pair distance on the average residual link lifetime $T_R$ with respect to different node speeds. In particular, we set the transmission range $R = 239$ m, and the initial node-pair distance is respectively chosen from $\{50, 100, 150, 200\}$ m, when two nodes are connected. It is interesting to see that for a specific average speed, the average residual link lifetime, does not vary significantly with node-pair distance, especially for high speed mobile nodes.

**Remark 6.** Similar to the link lifetime, the impacting factors on residual link lifetime are in the decreasing order of average node speed, ETR, and node-pair distance.

### 3.4.3 Link Change Rate and Link Arrival Rate

Radio links among nodes in multihop wireless networks have an immediate effect on network topology. In this section, we analyze the average link change rate, which is defined as the average number of link changes per second observed by a single node. According to Figure 3.3, the total number of new mobile nodes moving into node $u$’s transmission zone during time interval $[0, t]$ is $N_a(t)$, which denotes the total number of new link arrivals. And the total number of link breakages for the node $u$ during time interval $[0, t]$ is $N_b(t)$. Then, we denote the average link arrival rate as $\lambda = \lim_{t \to \infty} \frac{N_a(t)}{t}$ and the average link breakage rate as $\mu = \lim_{t \to \infty} \frac{N_b(t)}{t}$, respectively. In [26], Samar et al. showed that the average link arrival rate $\lambda$ is equal to the average link breakage rate $\mu$ in multihop wireless networks. Let $\eta_L$ denote the average link change rate. Thus,

$$\eta_L = \lambda + \mu = 2\lambda.$$  

(3.26)

Upon Figure 3.3, the average link arrival rate $\lambda$ is equivalent to the average number of new nodes entering node $u$’s transmission zone at every time step. Thus, we extend the total number of states of matrix $P$ from $n + 1$ to $n + N$, where $N$ is obtained in (3.20). The extended states are shown in Figure 3.9. Hence, a node could enter node $u$’s transmission zone at the next time step, only if it is currently lying in one of the states $\{S_{n+1}, S_{n+2}, ..., S_{n+N}\}$. Let $P_L(n + i)$ denote the probability that a node in state $S_{n+i}$ will move into node $u$’s transmission zone during the next time
Figure 3.9: Derivation of average link arrival rate $\lambda$.

... step movement. Then, $P_L(n + i)$ is given by:

$$P_L(n + i) = \sum_{j=n+i-N}^{n} P_{n+i,j}, \quad 1 \leq i \leq N,$$

(3.27)

where $P_{n+i,j}$ can be obtained from the approximation equation (3.13). Furthermore, for $S_{n+i}$, we denote the region $D_{n+i}$ as the set of all positions that are in distance of $[(n + i - 1)\varepsilon, (n + i)\varepsilon]$ away from the reference node $u$. This set actually covers the region of a circular ring with the outer radius $(n + i)\varepsilon$ and the inner radius $(n + i - 1)\varepsilon$, respectively. Hence, we have the area of $D_{n+i}$, $S(D_{n+i}) = \pi \varepsilon^2[(n + i)^2 - (n + i - 1)^2] = \pi \varepsilon^2(2i + 2n - 1)$. Using the same assumption in [26, 32] that node density $\sigma$ follows the uniform distribution, then $\sigma \cdot S(D_{n+i})$ is the average number of nodes lying within $D_{n+i}$. Therefore, the total number of possible nodes moving into node $u$’s transmission zone at the next time step is the summation of the number of nodes currently lying at all possible regions $D_{n+i}, \quad 1 \leq i \leq N$. Then, the average link arrival rate $\lambda$ is represented as:

$$\lambda = \sum_{i=1}^{N} P_L(n + i) \cdot \sigma \cdot S(D_{n+i}) = \sigma \pi \varepsilon^2 \sum_{i=1}^{N} \sum_{j=n+i-N}^{n} P_{n+i,j} \cdot (2i + 2n - 1).$$

(3.28)

To validate the analytical results of average link change rate $\eta_L$ and average link arrival rate $\lambda$ in (3.26) and (3.28), respectively, we compare the theoretical results with the simulation results according to different node speed in Figure 3.10(a). As can be observed, the analytical results match the simulation results very well. Thus, we validate that the average link change...
rate $\eta_L$ is \textit{two times} as large as the average new link arrival rate $\lambda$. Also, we find that given a fixed transmission change $R_e$, both $\eta_L$ and $\lambda$ grow almost linearly with the increase of the node speed. Based on this relation between $\eta_L$ and the average node speed $\bar{V}$, we can easily estimate the change rate of network topology at different mobility levels. Furthermore, Figure 3.10(b) illustrates the ETR impacts on the link arrival rate $\lambda$ from (3.28). It is clearly to see that the larger an ETR is, the higher $\lambda$ has. As the number of new mobile nodes able to move into the transmission zone increases with the growth of ETR. Moreover, according to Figure 3.10(b), when the average node speed is 2 m/sec, $\lambda$ varies from 0.01 to 0.05 when $R_e$ increases from 94 m to 342 m, respectively. Accordingly, with the same ETR range, $\lambda$ varies from 0.16 to 0.43 when average node speed is 25 m/sec. Thus, it is clear to see that the ETR has much more significant impact on $\lambda$ for nodes with higher mobility than that with lower mobility.

### 3.5 Implications of Link Properties

The knowledge of link stochastic properties under different factors such as radio channel characteristics and node mobility offers deep insights on performance evaluation and improvements in multihop wireless networks. Next, we apply analytical results of link properties to investigate their implications on path lifetime, network connectivity and routing protocol optimization.
3.5.1 k-hop Path Lifetime

To study the path properties, we assume that the stochastic properties of different links incident to a path are identical and links fail independently. In reality, there may exist correlation between two adjacent links which share the same node, so that the adjacent links could break at the same time. Compared to the independent link failure cases along paths in multihop networks, this correlated link failure scenario happens much less. This assumption has been shown to apply well for deriving the path properties [39, 5, 26].

In general, for a $k$-hop path, the link lifetime of each link incident to the path denotes as $T_{L1}, T_{L2}, \ldots, T_{Lk}$, respectively. Then, the path duration time $T_{path}^k$ is equivalent to the minimum duration time among its $k$ incident links, i.e., $T_{path}^k = \min\{T_{Li} \mid 1 \leq i \leq k\}$ [5, 26]. Upon the assumption that links along a $k$-hop path fail independently and have identical stochastic properties, given the CDF of link lifetime $F_L(m)$ derived in (3.19), then the CDF of path lifetime $F_{path}^k(m) = \text{Prob}\{T_{path}^k \leq m\}$ is derived as:

$$F_{path}^k(m) = 1 - \text{Prob}\{\min_{1 \leq i \leq k} T_{Li} > m\} = 1 - [1 - F_L(m)]^k$$

$$= 1 - [1 - \pi^{(0)} P^m(n + 1)]^k. \quad (3.29)$$

Compared (3.19) with (3.29), the CDF of 1-hop path lifetime is exactly the CDF of link lifetime. Here, equation (3.29) uses the initial location $\pi^{(0)}$ associated with each link to derive the path lifetime distribution, which actually implies that all $k$ links of the path are formed at the same time. Especially, upon the description of $\pi^{(0)}$ in Section 3.3.3, (3.29) further indicates that all the incident nodes of the path are located near the border of the transmission zone of their neighbors when the path is being setup, which actually increases the probability of path failure. Hence, (3.29) provides the upper bound CDF of path lifetime.

For a more general case, let the $u$-$w$ link shown in Figure 3.3 be the $i$-th hop of a $k$-hop path, where $i \leq k$. That is, once node $w$ moves into node $u$’s transmission zone, the $u$-$w$ link is formed, so does the path. In this case, during the path setup process, except the $u$-$w$ link, all other $k - 1$ links are already active. Hence, the residual link lifetime distribution upon the knowledge of the relative distance between two nodes of the other $k - 1$ links should be applied for analyzing a more accurate path lifetime distribution. In particular, once node $w$ moves into transmission range of node $u$, the $k$-hop path starts to form. In particular, we respectively denote the relative distance between neighboring nodes as $\rho^1, \rho^2, \ldots, \rho^{k-1}$ according to each link during the path setup. Hence,
by combing (3.19), (3.24) and (3.29), the CDF and the PMF of path lifetime distribution is given by:

\[
F_{\text{path}}^k(m) = 1 - [1 - F_L(m)] \cdot \prod_{h=1}^{k-1} L(\rho^h, m)
\]

\[
= 1 - [1 - \pi^{(0)} P^m(n + 1)] \cdot \prod_{h=1}^{k-1} L(\rho^h, m). \tag{3.30}
\]

Correspondingly, from (3.30), the PMF of path lifetime is

\[
\text{Prob}\{T_{\text{path}}^k = m\} = F_{\text{path}}^k(m) - F_{\text{path}}^k(m - 1). \tag{3.31}
\]

Figure 3.11(a) presents an example of a PMF of 4-hop path lifetime, with the comparison of simulation results, theoretical values of the upper bound (3.29) and more accurate calculation from (3.31). In this example, the link sequence of the 4-hop path is represented as \(w \leftrightarrow u \leftrightarrow x \leftrightarrow y \leftrightarrow z\). We let the relative distance of \(u-x\) link, \(x-y\) link and \(y-z\) link be the 200 m, 150 m, and 100 m, respectively. The node target speed \(\bar{V}\) is 10 m/sec and has \(R_e = 239\) m. As we average, the simulation result has a better match with theoretical calculation from (3.30) than the upper bound analytical result from (3.29). Especially, the resulting PMF of the upper bound path lifetime from (3.29) is much higher than the other two PMFs when the path lifetime is over \([0, 7]\) sec. Thus, Figure 3.11(a) validates our analysis on (3.29) and (3.30). In addition, upon Figure 3.11(a), we find that the PMF of path lifetime decreases exponentially with time for both theoretical and simulation results. In particular, the PMF value decreases dramatically within the first 20 sec. According to (3.30), the PMF value drops from 0.16 at 1st sec to 0.02 at the 20-th sec. And both the theoretical and analytical PMF values almost approach to 0 after the first 50 sec.

Continue to this example, Figure 3.11(b) presents the theoretical CDF results of path lifetime from (3.30) with different hops, where the average node speed of each node is 10 m/sec, and \(R_e = 239\) m. In particular, the CDF of 1-hop path lifetime is exactly the CDF of link lifetime. It is evident that given a fixed time instant, the CDF value of path lifetime increases with the rise of the number of hops along the path. While based on this illustration, the CDF of 3-hop path lifetime is almost overlapped with that of 4-hop path lifetime, it implies that there is a very slight difference when the number of hops incident to a path is larger than 3. As shown in Figure 3.11(b), the path lifetime is less than 15 seconds with probability 0.6 regardless the number of hops. According to
Given (3.30) and (3.31), the average path lifetime with $k$ links, $T_{path}^k$, is represented as:

$$T_{path}^k = E\{T_{path}^k\} = \sum_{m=1}^{\infty} m \cdot \text{Prob}\{T_{path}^k = m\}. \quad (3.32)$$

Compare to the analysis of average link change rate $\eta_l$ in Section 3.4.3, we are discussing the property of the average path change rate $\eta_P^k$ in the following. If we assume that the paths generated between two nodes $w$ and $z$ always contain $k$ links, then, let $N_P^k(t)$ represent the total number of $k$-hop paths generated between two nodes during time interval $[0, t]$. And $T_P^k(i)$ denotes as the $i^{th}$ path lifetime between the two nodes. In real-life, a pair of nodes $w$ and $z$ cannot always find a new path immediately after the path failure, due to the limited available link resources incident to the path. Hence, we have $\sum_{i=1}^{N_P^k(t)} T_P^k(i) \leq t$. Based on this argument, we obtain the upper bound of the average path change rate $\eta_P^k$ as:

$$\eta_P^k = \lim_{t \to \infty} N_P^k(t) / t \leq \lim_{t \to \infty} N_P^k(t) / \sum_{i=1}^{N_P^k(t)} T_P^k(i) = 1/T_{path}^k. \quad (3.33)$$

The equality of (3.33) holds under the condition that a new $w,z$-path is always being setup immediately after the previous path failure. From (3.33), we showed that the upper bound of the average path change rate $\eta_P^k$ is the inverse of the average path lifetime $T_{path}^k$. 

Figure 3.11: PMF and PDF of path Lifetime.

a high probability 0.9, the lifetime is less than 60 seconds for the 3-hop and 4-hop paths, and the lifetime is less than 100 seconds and 160 seconds for the 2-hop and 1-hop path, respectively.

Given (3.30) and (3.31), the average path lifetime with $k$ links, $T_{path}^k$, is represented as:

$$T_{path}^k = E\{T_{path}^k\} = \sum_{m=1}^{\infty} m \cdot \text{Prob}\{T_{path}^k = m\}. \quad (3.32)$$

Compare to the analysis of average link change rate $\eta_l$ in Section 3.4.3, we are discussing the property of the average path change rate $\eta_P^k$ in the following. If we assume that the paths generated between two nodes $w$ and $z$ always contain $k$ links, then, let $N_P^k(t)$ represent the total number of $k$-hop paths generated between two nodes during time interval $[0, t]$. And $T_P^k(i)$ denotes as the $i^{th}$ path lifetime between the two nodes. In real-life, a pair of nodes $w$ and $z$ cannot always find a new path immediately after the path failure, due to the limited available link resources incident to the path. Hence, we have $\sum_{i=1}^{N_P^k(t)} T_P^k(i) \leq t$. Based on this argument, we obtain the upper bound of the average path change rate $\eta_P^k$ as:

$$\eta_P^k = \lim_{t \to \infty} N_P^k(t) / t \leq \lim_{t \to \infty} N_P^k(t) / \sum_{i=1}^{N_P^k(t)} T_P^k(i) = 1/T_{path}^k. \quad (3.33)$$

The equality of (3.33) holds under the condition that a new $w,z$-path is always being setup immediately after the previous path failure. From (3.33), we showed that the upper bound of the average path change rate $\eta_P^k$ is the inverse of the average path lifetime $T_{path}^k$. 

Figure 3.11: PMF and PDF of path Lifetime.

a high probability 0.9, the lifetime is less than 60 seconds for the 3-hop and 4-hop paths, and the lifetime is less than 100 seconds and 160 seconds for the 2-hop and 1-hop path, respectively.
Note that the derivation of CDF and PMF path lifetime in (3.30) and (3.31) depends on the knowledge of relative distance between each node pair. Recall that in (3.22), we showed that the PMF of link lifetime distribution can be approximated by an exponential distribution with parameter $\lambda_{R_{el}}$, which does not rely on the geographic information of a node pair. Since the path lifetime is determined by the minimum link lifetime en route, and assuming link fails independently, we can easily conclude that the approximation of path lifetime PDF also follows exponential distribution with the parameter $\lambda_{P}^k$ [83]. This analysis greatly relaxes the assumption of large (approach to infinite) hop-count of a path for its distribution converging to exponential [99]. In particular, the parameter $\lambda_{P}^k = \sum_{l}^{k} \lambda_{R_{el}}$, where $R_{el}$ and $V_{l}$ are the associated ETR and average node speed for the $l^{th}$ link along the $k$-hop path. Following this argument, upon (3.33), we can easily see that the average path change rate $\eta_{P}^k$ is upper bounded by $\lambda_{P}^k$, that is, $\eta_{P}^k \leq \lambda_{P}^k$.

Remark 7. The probability distribution function of path lifetime can be effectively approximated by an exponential distribution for any $k$-hop path, with parameter of $\lambda_{P}^k$, which is the summation of exponential parameters of each link along the path, and also the upper bound of the average path change rate $\lambda_{P}^k$.

3.5.2 Network Connectivity

Now we apply the average link lifetime $T_{link}$ and average link change rate $\eta_{L}$ to investigate their impacts on average node degree, and further network connectivity. Let $\kappa(G(t))$ and $E\{d_{G(t)}\}$ be the network connectivity and average degree of a multihop wireless network $G(t)$, respectively. Then we have $\kappa(G(t)) \leq E\{d_{G(t)}\}$. Thus, $E\{d_{G(t)}\}$ is the upper bound of the connectivity of $G(t)$. Let each node in $G(t)$ be associated with a queuing system. For instance, in the node $u$’s system, an arrival event means the event that a node moves inside node $u$’s transmission zone, and a departure event represents a node moves outside its transmission zone. Then, according to the Little’s law of a queuing system: the average number of customers in the system, i.e., $E\{d_{G(t)}\}$, is equal to the average arrival rate of customer to the system, i.e., $\lambda$, multiplied by the average system time per customer, i.e., $T_{link}$. Therefore, we can apply $T_{link}$ in (3.23) and $\eta_{L}$ in (3.26) and (3.28) to estimate the upper bound connectivity of a multihop wireless network,

$$\kappa(G(t)) \leq E\{d_{G(t)}\} = \lambda \cdot T_{link} = \frac{1}{2} \eta_{L} \cdot T_{link}.$$  

(3.34)
In fact, $E\{d_{G(t)}\}$ implies the maximum number of disjoint end-to-end paths available for a large dense multihop network. Therefore, the knowledge of $\kappa(G(t))$ and $E\{d_{G(t)}\}$ from (3.34) can benefit path selection and routing design especially for a large dense multihop network. Here, by combining analytical results from (3.23), (3.28), and (3.34), we demonstrate the relationship among average node degree $E\{d_{G(t)}\}$, node mobility $\bar{V}$, and the effective transmission range $ETR$ upon the smooth mobility model including 50 mobile nodes. The simulation results are illustrated in Figure 3.12, which reveal that the average node degree increases linearly with the growth of the transmission range. It is consistent with our expectation that the larger transmission range will potentially increase the network connectivity. Also, within a fixed transmission range, the decreased $T_{\text{link}}$ caused by high node mobility is compensated by the increase of $\eta_L$. Therefore, as shown in Figure 3.12, the average node degree is almost the same with different node speed under a fixed transmission range. Furthermore, since 50 nodes are uniformly located in the network size 1401m $\times$ 1401m, the node density $\sigma_R$ per transmission zone varies with transmission range $R$. As an example, Table 3.3 illustrates the impacts of transmission range on the node density and average node degree.

<table>
<thead>
<tr>
<th>$R_e$ (m)</th>
<th>94</th>
<th>149</th>
<th>200</th>
<th>239</th>
<th>286</th>
<th>342</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_R = \frac{No.}{\pi R_e^2}$</td>
<td>0.7</td>
<td>1.8</td>
<td>3.2</td>
<td>4.6</td>
<td>6.5</td>
<td>9.3</td>
</tr>
<tr>
<td>$E{d_{G(t)}}$</td>
<td>0.52</td>
<td>1.62</td>
<td>2.82</td>
<td>4.35</td>
<td>6.05</td>
<td>7.9</td>
</tr>
</tbody>
</table>

Figure 3.12: Average number of neighbors per node according to node speed $\bar{V}$ and ETR.
3.5.3 Routing Performance

The effective design of routing protocols should take into account the crucial factors such as wireless channel characteristics, node density and node mobility. As explained earlier in this paper, the complex interactions of these factors directly determine the link properties in a multihop network. Thus, the link properties such as average link lifetime can be utilized as the effective indicators of network performance and the metrics for designing mobility adaptive routing protocols [30].

In this section, we investigate the impacts of link dynamics on routing performance by taking AODV as a case study in ns-2. The network traffic is composed of 20 constant bit rate (CBR) sources and 30 connections among total 50 nodes. And each source sends 1 packet/sec with the packet size 64 bytes. The value of ETR is chosen from the set \{94, 149, 200, 239, 286, 342\} m, which are obtained by considering radio channels shown in TABLE 3.1. From Figure 3.13(a) and 3.13(b), it can be seen that the performance of average end-to-end packet delay and throughput increases substantially as the rise of ETR \(R_e\). However, in Figure 3.13(b) when \(R_e \leq 239\)m, the routing performance is not acceptable for practical applications, because of network dis-connectivity due to lack of neighboring nodes.

![Figure 3.13](image-url)

(a) Average Packet Delay. (b) Network Throughput. (c) Routing Overhead Ratio.

Figure 3.13: Effective transmission range and node mobility impacts on AODV routing performance.

Interestingly, from Figure 3.13(c), we find that the routing overhead increases when \(R_e\) rises from \([94, 286]\)m, and it starts to reduce regardless of node speed when \(R_e > 286\)m. This is because the increasing number of neighboring nodes is large enough to almost always form a connected network, while dramatically reducing the number of path updates. Thus, we find that routing
protocols should only be evaluated and studied under certain range of node density, where the statistics of link properties can be well applied to improve the routing efficiency as well as the network performance. Based on the analytical results in Figure 3.7(b) and simulation results in Figure 3.13, Table 3.4 illustrates an example of the mapping between average link lifetime and network performance under different network circumstances, where the node speed is 15 m/sec, which can be regarded as a typical vehicular ad hoc network in downtown area under both shadowing and small-scale fading. Hence, in order to improve the overall network performance, we can either increase the average node degree or adjust the transmission range to extend average link lifetime. Therefore, we have the following observation regarding the effect of node density on routing performance.

<table>
<thead>
<tr>
<th>End-to-End Delay (sec)</th>
<th>0.43</th>
<th>0.22</th>
<th>0.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-to-End Throughput</td>
<td>69%</td>
<td>86%</td>
<td>91%</td>
</tr>
<tr>
<td>Average Node degree $E{d_{G(t)}}$</td>
<td>4.45</td>
<td>5.65</td>
<td>7.39</td>
</tr>
<tr>
<td>Average link lifetime $T_L$ (sec)</td>
<td>21.23</td>
<td>23.40</td>
<td>27.08</td>
</tr>
</tbody>
</table>

**Remark 8.** The routing performance varies sensitively and can be effectively improved by applying the knowledge of link dynamics when the number of nodes per transmission zone, $\sigma_R$, changes from 3 to 10.

### 3.6 Summary

In this research work, we presented a modeling approach to study the joint effects of radio channels and node mobility on link properties by using a transition probability matrix. We have found that i) radio channel characteristics predominate the link performance for slower mobile nodes, while node mobility dominates the link performance for faster mobile nodes; ii) link lifetime can be effectively approximated by exponential distribution with parameter $\overline{V}/\overline{E}$; iii) the impacting factors on both link and residual link lifetime are in the decreasing order of average node speed $\overline{V}$, ETR, and node-pair distance; and iv) $k$-hop path lifetime can also be characterized by an exponential distribution for any arbitrary hop-count $k$. By demonstrating the implications of link properties on path lifetime, node degree, and routing performance, the proposed approach and results provided several new findings that can be readily applied to system design such as topology control and routing optimization.
Chapter 4

Human Diffusive Behaviors: Temporal-Spatial Limitations in Mobile Wireless Networks

By far, we have studied the individual node mobility according to the proposed SMS model in our first research topic. By considering the intrinsic properties of node mobility and radio channel characteristics of MANETs, we analyzed their joint effects on link dynamics upon SMS model in our second research topic. In summary, our first two topics of node mobility are based on the synthetic mobility models. In fact, in a typical wireless multi-hop network, mobile devices are generally by humans. As a result, the human mobility directly affects the link level dynamics between mobile wireless devices, which are characterized by the contact-based metrics, such as inter-meeting time, in MANETs. By far, the inherent properties of human mobility, such as human pause time, and human trip displacement, still remain elusive, which therein brings a main hurdle for studying human mobility impact on contact-based metrics in MANETs. Therefore, we move forward to study human mobility in the third work by investigating human diffusive behaviors upon three diversified human trace datasets. As human diffusive movement patterns are jointly affected by human pause time and human trip displacement, we further explore human diffusive movement pattern to study human mobility impact on inter-meeting time in MANETs.

In this chapter, we first motivate this study and review related works of human mobility and its impact on contact-based metrics in Chapter 4.1. Next, we introduce the preliminaries of
diffusive process and demonstrate the problem studied in this work in Chapter 4.2. By studying three diversified human trace datasets with different human moving domain sizes, we suggested that the cutoff power law distribution is an invariant property of human mobility regarding pause time and trip displacement in Chapter 4.3. In Chapter 4.4, we study the joint spatial and temporal effects on the scaling law of human diffusive behaviors, when either the distribution of pause time, or trip displacement, or both are characterized by the power-law head. And we discuss the implication of human diffusive mobility patterns on the property of inter-meeting time. Furthermore, in Chapter 4.6, we study the properties of cutoff power-law distribution of human mobility metrics, by taking both power-law head and the exponential tail, i.e., the whole shape of the distribution, into account. Finally, we summary this study in Chapter 4.7.

4.1 Introduction

Human mobility directly affects the properties of contact-based metrics, such as contact time and inter-contact time, in mobile ad hoc networks (MANETs). These contact-based metrics further characterize the link level dynamics between wireless devices attached to humans. For instance, the contact time (also called link lifetime) is counted for the time duration when two mobile users have a directly connected link. As a link often disconnects when a mobile user moves outside the transmission range of the other user, the inter-contact time (also called inter-meeting time) is defined as the time period between two consecutive link connections of two mobile users. Therefore, it has been found that human mobility patterns have a large effect on link performance, and therein, routing performance in MANETs [14, 13, 22, 23].

Human mobility patterns are reflected in both temporal and spatial domain. Specifically, human pause time in temporal domain and human trip displacement in spatial domain are the two fundamental mobility metrics, as each of them can significantly affect the stochastic properties of contact-based metrics. In spatial domain, it has been found that border-effect directly affects inter-contact time for existing random mobility models [108]. Basically, the finite border size indicates the maximum trip displacement a mobile user can take in a simulation area [108, 23], the authors showed that a finite border size, i.e., a finite trip displacement, always generates a mixture of power-law and exponential behavior in the inter-meeting time distribution. In other words, the finite trip displacement leads to a cutoff power-law distribution of the inter-meeting time, which has a power-law head and exponential tail. In addition, it has been found in [23] that the scale of trip dis-
placement determines the power-law head effect on the distribution of inter-meeting time. Thus, for humans with infinite trip displacement, it leads to an exclusive power-law distributed inter-meeting time.

Accordingly, for pause time effect in temporal domain, we take a closer look at two extreme cases: zero pause time and infinite pause time as an example. In the former case, a mobile node does not stop, even though direction or speed may change, then the property of inter-meeting time is mainly determined by the property of trip displacement [109, 108]. In the latter case, an infinite pause time means that a node does not move, and it is equivalent to a static node. It has been recently shown that the contact time can be approximated by the exponential distribution [110]. These new findings collectively show that both trip displacement and pause time have a large impact on the properties of contact-based metrics.

However, compared to the recently increasing attention on the properties of contact-based metrics, such as inter-meeting time [111, 108, 55, 22, 23], less work has been done on investigating the inherent properties of individual human mobility patterns regarding pause time and trip displacement. And in fact, it has been demonstrated that the joint temporal-spatial effect of human pause time and trip displacement can directly influence the behaviors of contact-based metrics [22] according to the simulation results, which however, is still unclear by far. Motivated by the above observations, in this work, we study the stochastic properties of human trip displacement with the size scale of human moving domain and human pause time. Based on this first-hand observation of human trip displacement and the pause time, we then investigate their joint effects on inter-meeting time in MANETs.

The study on mobility patterns of moving objects including humans in nature is not a new problem. Interestingly, people have respectively found in biology [112] and physics [113] that the long trip displacements of animals’ foraging habits and the long sojourn time of moving particles are ubiquitous. Then, does trip displacement and pause time of human mobility exhibit power-law property as what have been found in animal behaviors in biology and particle movement in physics? As a partial answer to this question, several works have presented some results based on human moving trace files, yet different observations. In [22], by studying human walking traces in different areas, the authors found that both pause time $T_p$ and trip displacement $L$ follow a truncated (cutoff) power-law distribution, similar to what has been observed in the distribution of inter-meeting time [108, 55]. In addition, in [114], the authors showed the collected human mobile phone call traces exhibit a cutoff power-law distribution of trip displacement. However, another
recent human mobility study based on the spreading traces of bank notes hold by humans suggested that both human pause time and trip displacement can exhibit an exclusive power-law distribution [56]. By taking a close look on these studies, we found that the traces collected from [22, 114] are from individual human moving traces. And individual human trajectories inevitably have a finite moving domain size $D_m$, which is constrained by the temporal and spatial regularities of human social moving behaviors [114]. However, the bank notes can be exchanged between humans during their travel [56], so that the bank note trajectories extend to an “infinite” moving domain size, which results in exclusive power-law behaviors on both trip displacement and pause time.

To further investigate this outright discrepancy in different human mobility patterns, especially the scaling order effect, we studied the empirical human mobility traces on three different domain $D_m$ scales: campus-wide, city-wide, and county-wide. By analyzing the collected traces, we find that the trip displacement exhibits the cutoff power-law behavior for all three trace datasets, which have finite moving domain size with different scales. Furthermore, the location of cutoff point in the CCDF of trip displacement increases as the human moving domain size $D_m$ increases. In addition, we also observed the cutoff power-law distribution of pause time from the campus-wide traces which record the complete temporal information of students’ daily activities. Therefore, compared our results with existing ones [22, 114, 56], there is a strong evidence that the finite human moving domain size $D_m$ and the temporal limit of human social activities (which results in a finite human pause time) lead to a mixture of power-law and exponential behavior of human mobility regarding trip displacement and pause time, while leading to exclusive power-law distribution with the extremes of infinite domain.

By investigating the empirical human traces, we already knew that the cutoff power-law distribution of both trip displacement and pause time contains two parts: power-law head and exponential tail, separated by a cutoff point. In particular, the value of cutoff point directly affects the entire shape of the cutoff power-law distribution. When the value of cutoff point is very small, the power-law head is very short, so that almost the entire shape of the cutoff law distribution is characterized by the exponential tail. In contrast, when the value of cutoff point is considerable large, the shape of the cutoff power-law distribution is characterized by the power-law head. As a result, the distribution of trip displacement and pause time can exhibit either power-law like of exponential like behavior. Therefore, in this work, we first study the separated power-law and exponential effects of the cutoff power-law distribution on temporal and spatial human mobility metrics, i.e., pause time and trip displacement, and in turn, the resulting influence of human mobility on
inter-meeting time. Clearly, according to the value of cutoff point, there are four combinations
of temporal-spatial human mobility patterns: \((\text{exponential-like pause time, exponential-like trip}
\text{displacement})\); \((\text{exponential-like pause time, power-law-like trip}
\text{displacement})\); \((\text{power-law-like pause time, exponential-like trip}
\text{displacement})\); and \((\text{power-law-like pause time, power-law-like trip}
\text{displacement})\). In the first case, when both trip displacement and pause time are considered
as exponentially distributed, it is the case simulated by almost all existing mobility models [14]
which mimic user moving behaviors with short step-length and pause time in a network. Under
this condition, the authors in [23] recently showed that this class of mobility models will lead to
the same level of average inter-meeting time, therein, resulting a similar network performance. As
the research on inter-meeting time underlying random mobility models has been widely studied, we
are more interested in finding the answers of human mobility effect on inter-meeting time with the
other three cases.

In particular, in this work, we focus on the case when power-law characterizes the distrib-
ution of trip displacement and pause time at the same time. The reason is explained as follows.
Recent study of human mobility in spatial domain [22] suggested that the exclusive power-law ef-
fect of trip displacement may lead an overly optimistic routing performance, as the high occurrences
of long trips intensify the chance of meeting destinations in DTNs. In contrast, for the studies of
human mobility in temporal domain [111, 55], they respectively suggested the exclusive power-law
effect of pause time results in an overly pessimistic routing performance. Hence, the power-law
of trip displacement typically brings contradictory effect on network performance compared to the
power-law of human pause time. Such a conflict actually undermines our understanding of net-
work performance due to human mobility. Then, the following question needs to be studied: What
are the joint temporal-spatial effects of power-law pause time and power-law trip displacement
on the inter-meeting time of mobile users? To answer this question, we find that the ambivalent
interaction between trip displacement and pause time can be manifested by the human diffusive
movement patterns [56], which is further characterized by his/her diffusive capability (rate) \(r\). That
is, \(MSD(t) \propto t^r\), where \(MSD(t)\) denotes the mean square human travel displacement during the
diffusive (traveling) process time \(t\) [112, 56]. Then, a succeeding question comes out: how is the
diffusive rate \(r\) determined with respect to temporal and spatial stochastic properties of human mov-
ing patterns? Hence, in this work, we are motivated to study the scaling law of human diffusive rate
\(r\) to address this question. Upon this answer, we can further characterize the joint temporal-spatial
effects of human mobility on inter-meeting time with a specific human diffusive rate. Interestingly,
we find and prove that the human diffusive rate $r$ also can characterize the human mobility patterns with other two cases: (exponential-like pause time, power-law-like trip displacement) and (power-law-like pause time, exponential-like trip displacement). Therefore, we suggest that the human diffusive rate is a good solution to study human mobility pattern impact on contact-based metrics in MANETs.

Furthermore, we notice that when the cutoff point resides in the middle area of the distribution, both power-law head behavior and exponential tail behavior collectively characterize the cutoff power-law distribution. In this case, instead of studying the separated power-law and exponential effect on human mobility, we need to investigate the mixture behaviors of power-law head and exponential tail in the distribution of trip displacement and pause time, in order that their joint impacts on the properties of inter-meeting time in MANETs can be correctly evaluated. Otherwise, either the exclusive power-law or exponential effect alone on temporal-spatial human mobility metrics can lead to a biased or even misleading result of inter-meeting time [111, 55]. Ideally, the mixed power-law and exponential behavior should be analyzed upon the whole shape of the cutoff power-law distribution. However, a close form of a cutoff power-law distribution of human mobility metrics is still unknown. Therefore, in the second part of this work, we aim to mathematically model the entire shape of the cutoff power-law distribution of human trip displacement and pause time by investigating the collected empirical human trace datasets. Based on this first-hand observation, we are able to evaluate the joint temporal-spatial effect of human mobility, characterized by the mixed power-law and exponential behaviors, on inter-meeting time.

Our contributions in this work can be summarized as the following:

- By studying the empirical human mobility traces on three different domain scales: campus-wide, city-wide, and county-wide, we validate that both pause time and trip displacement of human mobility exhibit a cutoff-power distribution, as those have observed in inter-contact time (ICT) [55, 108]. In other words, the moving patterns of humans are affected by their temporal limits of social activities in a finite moving domain; thus it has similar properties as contact-based metrics. In addition, due to different social behaviors, the human moving domain size $D_m$ varies, which further induces the variation of power-law coefficient and cutoff point in the distribution. In particular, the cutoff point, defined as characteristic distance $D_c$, in the distribution of trip displacement increases as the human moving domain size $D_m$ increases. As a result, the distinct human social behaviors may lead to different human mobility patterns, which in turn, affects the performance of contact-based metrics in MANETs.
When the power-law head characterizes the distribution of human mobility, we define the power-law coefficients $\alpha$ and $\beta$ for the pause time and trip displacement, respectively. Accordingly, the human diffusive capability (rate) $r$ is, $r = 2\alpha/\beta$, where $0 < \alpha < 1$ and $0 < \beta < 2$. This result is obtained by using a continuous time task-driven mobility (CTDM) model to effectively demonstrate human diffusive moving behaviors regulated by their associated societal duties.

We investigate the mixed behaviors of power-law head and exponential tail of a cutoff power-law upon the analysis on empirical human trace datasets with different levels of moving domain sizes. Then, we propose an approximated cutoff power-law distribution, which is featured by a parameter tuple. For either temporal or spatial domain, a parameter tuple has two elements: the power-law coefficient and cutoff point, $(\alpha, T_c)$ and $(\beta, D_c)$, respectively. We further fit the approximation technique to the results from trace files and find that the parameter tuple can capture the CCDF of each dataset, but with different values that are determined by their social behaviors.

4.2 Preliminaries and Problem Statement

In this section, we first introduce the preliminaries of diffusive process theory to be used to study temporal-spatial effects of human mobility. Then, we give definition of cutoff points in the distribution of pause time and trip displacement, which will be used throughout this chapter. Finally, we demonstrate examples of human moving behaviors within different scale of moving domain size. By comparing the human moving domain size with the studied network size, we demonstrate that either the power-law head, or the exponential tail, or the entire shape of the cutoff power-law distribution can characterize the trip displacement and pause time of mobile users in the studied network. As a result, the human mobility will affect the contact-based mobility metrics of the network in very different ways.

4.2.1 Preliminary of Diffusive Process

For the easy understanding of human diffusive movement patterns, we introduce the definitions of a mobile node (human) diffusive process as follows.
**Definition 4.2.1.** Let $M(t)$ denote a node position at time $t$. Then, $S(t)$ is the displacement of the node movement process by time $t$, i.e., $S(t) = |M(t) - M(0)|$.

**Definition 4.2.2.** The mean square displacement $MSD(t)$ is defined as $MSD(t) = E\{S(t)^2\} = E\{|M(t) - M(0)|^2\}$.

**Diffusive Movement Patterns**

In this work, one of major goals is to study the effect of human diffusive mobility patterns on inter-meeting time in mobile wireless networks. The diffusive process is originated from physics, which can effectively characterize the power-law behavior of step length and pause time of a particle movement process [115, 113, 112, 116]. Specifically, in physics, the node diffusive process is manifested by its diffusive rate $r$ [115, 113].

**Definition 4.2.3.** The stochastic process of the moving trace $\{M(t)\}$ is diffusive at the rate $r \in (0, 2]$, if $MSD(t) \propto t^r$.

According to [113], there are two categories of diffusive movement patterns, Normal diffusive and Anomalous diffusive.

**Definition 4.2.4.** The normal diffusive is defined as a movement pattern in which $MSD(t) \propto t^r$, where $r = 1$. That is the mean square displacement $MSD(t)$ grows linearly with time $t$.

**Definition 4.2.5.** The anomalous diffusive is defined as a movement pattern in which $MSD(t) \propto t^r$, where $r \neq 1$. Apparently, the nodes can have two anomalous diffusive movement patterns: Superdiffusive ($r > 1$) and Subdiffusive ($0 < r < 1$) [113]. Thus, Superdiffusive/Subdiffusive movement pattern is characterized by faster/slower-than-linear growth of the $MSD(t)$.

Recent study in [22] demonstrated that the human diffusive movement pattern plays an essential role in contact-based metrics and routing performance evaluation in mobile wireless networks. Specifically, they showed that the inter-meeting distribution via various mobility models is closely related to mobile nodes diffusive mobility patterns. The more diffusive the mobility is, the shorter tail its inter-meeting distribution becomes. Furthermore, they demonstrated that the heavier tail of trip displacement, the shorter tail distribution of routing delay is. In contrast, the heavier tail of pause time, the heavier tail distribution of routing delay is. However, there is no clear solution yet to characterize the human diffusive mobility pattern in [22], which motivates our study in this work.
Scaling Order of A Diffusive Process

Compared to the node diffusive rate $r$, the scaling order $\rho$ is another essential metric for characterizing the node diffusive process. In physics, both diffusive rate $r$ and the scaling order $\rho$ of a particle movement process have been thoroughly studied by a one-dimensional continuous time random walk (CTRW) model [113]. Specifically, in CTRW model, $X_i$ denotes the independent identically distributed (i.i.d) displacement vector of a particle at the $i^{th}$ step, and $N(t)$ denotes the total number of steps occurred in the time interval $[0, t]$. Also, let $\Delta t$ be the time increment (pause time) between two successive steps, that is $\Delta t = t/N$. Then $S_N(t)$, the position of the particle by time $t$, is represented by $S_N(t) = \sum_{n=1}^{N(t)} X_n$. As the diffusive process time experiences very long, i.e., $t \to \infty$, $N(t) \to \infty$. Thus, $S_N(t)$ may have infinite variance, which makes the MSD($t$) not mathematically tractable. In this case, regarding the scaling law of node diffusive process [113], the node diffusive process can be characterized by the scaling order $\rho$ of $S_N(t)$.

**Definition 4.2.6.** Let $Y_N(t) = \frac{S_N(t)}{t^\rho}$, if the limiting pdf $\lim_{t \to \infty} f_{Y_N(t)}(y) = f_Y(y)$ is independent of time $t$, then $S_N(t)$ has a scaling order $\rho$, represented as $S_N(t) \sim t^\rho$.

Let $f_{S(t)}(x,t)$ denote the pdf that a particle (node) at the location $x$ from the origin at time $t$. In one-dimensional CTRW model $S(t) = S_N(t)$, then upon the scaling order $\rho$ in Definition 4.2.6, the relation of pdf $f_{S(t)}(x,t)$ and the limiting pdf $f_Y(y)$, for a large $t$, is given by [56],

$$f_{S(t)}(x,t) \sim \frac{1}{t^\rho} f_Y\left(\frac{x}{t^\rho}\right). \tag{4.1}$$

### 4.2.2 Definitions

In this section, we first present definitions of cutoff point in the distribution of human mobility metrics in temporal and spatial domain, that is, *characteristic time* $T_c$ and *characteristic distance* $D_c$, respectively.

Due to the temporal and spatial limitation of human social moving behaviors [114], i.e., a finite pause time and a finite moving domain size, we suggest that there always exists a cutoff point in the CCDF of human mobility metrics in both temporal and spatial domain. In particular, the cutoff point indicates the decay transition from “power-law” to “exponential” in a CCDF distribution. Thus, we define this cutoff point as *characteristic time*, denoted by $T_c$, in the cutoff power-law distribution of human mobility metrics in temporal domain, such as pause time. Accordingly, we
define \textit{characteristic distance} $D_c$ as the cutoff point in a cutoff power-law distribution of human spatial mobility metrics, such as trip displacement.

For easily understanding the problem demonstration described in next section, we describe useful terms regarding human mobility first.

- **Trip Displacement $L$**: The Euclidean distance between source and destination of a trip.
- **Pause Time $T_p$**: The time duration between two consecutive trips.
- **Human Moving Domain Size $D_m$**: The side length of a square-shaped human moving domain, which covers all the areas an individual mobile user ever visits during the trace collection period.
- **Network Size $D$**: The side length of the studied network with a square shape.
- **Observation Time $T$**: The time duration for performance evaluation of a network under study, which is equivalent to the simulation time period of mobility models.

### 4.2.3 Problem Demonstration

In order to help understand the problems studied in this work, we now take a close look on different shapes of cutoff power-law distributions according to the location (value) of the cutoff point. The cutoff point delimits the whole cutoff power-law distribution into two parts: power-law head and exponential tail. However, as illustrated in Figure 4.1, due to the different value of cutoff point in the distribution, a cutoff power-law can exhibit distinguished mixture behavior of power-law and exponential behavior. In particular, for case 1, as the value of cutoff point is very small, the power-law head is very short, so that the cutoff power-law distribution will be characterized by the exponential behavior. In contrast, when the value of the cutoff point is considerable large, the distribution will featured by the power-law behavior. As we will demonstrated later in Section 4.3, by studying real human trace files, both trip displacement and pause time are characterized by cutoff power-law. Then, regarding the specific value of cutoff point ($T_c$ and $D_c$), there are four combinations of temporal-spatial human mobility patterns: (exponential-like pause time, exponential-like trip displacement); (exponential-like pause time, power-law-like trip displacement); (power-law-like pause time, exponential-like trip displacement); and (power-law-like pause time, power-law-like trip displacement). Thus, the complexity of joint temporal-spatial effect of human mobility
bring a huge hurdle to understand how human mobility impacts contact-based metrics in mobile wireless networks.

![Cutoff Point](ccdf-power-law-exponential.png)

(a) Case 1: Very small cutoff point  
(b) Case 2: Very large cutoff point

Figure 4.1: Separated power-law and exponential behavior of cutoff power-law.

To be more specific on human moving behaviors in a network, we illustrate three distinct examples of human mobility patterns in a network in Figure 4.2. In example 1 (case 1) the trip displacement and pause time are mainly characterized by the exponential tail behavior and in contrast characterized by the power-law head behavior in example 2 (case 2). As shown in Figure 4.2, a MANET with network size $D$ is illustrated in each case. In particular, each network is divided into multiple community sites. According to studies in social networks [117], human daily activities are strongly regulated by their associated societal duties and working patterns in networks. Thus, due to different human societal behaviors, the moving domain size and the pause time range of mobile users can vary dramatically.

![Network Diagram](manet-community-site.png)

(a) Case 1: $D_m << D$.  
(b) Case 2: $D_m >> D$.

Figure 4.2: Human moving domain size vs. network domain size.

In Figure 4.2(a), the moving domain size $D_m$ of a mobile user is within a community site inside the network. In this case, $D_m$ is much less than the network size $D$, and the trip displacement is typically very short. If the pause time is also short, such that both $D_c$ and $T_c$ are very small, then
the power-head effect in the distribution of trip displacement and pause time can be “neglected”. Since existing mobility models can mimic both trip displacement and pause time of mobile nodes with exponential distribution [14], the mobility impacts on contact based metrics have been widely studied [23]. Therefore, the investigation of human mobility in this case is out of scope of this work.

On the other hand, for Case 2, the mobile user moving domain size $D_m$ is much larger than the network size $D$ of interest. As shown in this example, the community sites that a mobile user visits are located within four different network areas. To this mobile user, it does not make sense that he/she will bounce back during the travel when he/she reaches the invisible boundary of the network. Hence, the mobile user can often move across the entire domain of the studied network during a single trip. This is the case for the distribution of trip displacement where $D_c > D$. Hence, the power-law head effect will dominate human trip displacement behavior within the network area $D$. In addition, previous studies demonstrated that the pause time of human can be in the order of half of day [118] or even days [56], which can be larger than the observation time (simulation time) $T$ for the studied network, that is, the characteristic time $T_c$ of pause time can be larger than a preselected observation time $T$. In consequence, the power-law head in the distribution of both trip displacement and pause time characterize the human moving behaviors reflected in the network.

However, the following issue still remains unsolved: what are the joint temporal-spatial power-law effects of pause time and trip displacement on the inter-meeting time of mobile users? In this work, we aim to use human diffusive rate $r$ to characterize this joint effect, and further investigate the relationship between $r$ and the properties of inter-meeting time. In particular, we evaluate the impact of different diffusive rates according to superdiffusive ($r > 1$), subdiffusive ($r < 1$), and normal diffusive ($r = 1$) on the property of inter-meeting time.

Clearly, according to Figure 4.1, there are two remaining cases of combined temporal-spatial human mobility patterns which are not demonstrated here. One is the combination of (exponential-like pause time, power-law-like trip displacement), the other is (power-law-like pause time, exponential-like trip displacement). Interestingly, by studying the scaling law of human diffusive behavior in Section 4.4, we show that the former one leads to a subdiffusive mobility pattern ($r < 1$), which indicates that the power-law of pause time has more influence than that of exponential distributed trip displacement on the joint human mobility. In contrast, for the latter case, as the power-law behavior of trip displacement outperforms the exponentially distributed pause time, it results in a superdiffusive mobility pattern ($r > 1$). Accordingly, the questions of temporal-spatial human mobility impact on inter-meeting time in these two cases can still be addressed by studying
the relationship of diffusive rate $r$ and the distribution of inter-meeting time.

By far, we demonstrated the temporal and spatial human mobility metrics characterized by the separated power-law and exponential behaviors of a cutoff power-law. This is under the conditions that either cutoff point is very large or very small in the cutoff power-law distribution. However, as shown in Figure 4.3(a), when the cutoff point resides in the middle area of the distribution (Case 3), clearly, power-law head behavior and exponential tail behavior collectively characterize the cutoff power-law distribution. The corresponding example of mobile user behavior is shown in Figure 4.3(b). In this example, a mobile user can frequently visits several community sites inside the network, such that the user moving domain size $D_m$ is almost the same order as the network size $D$. In addition, the mobile user often takes both longer inter-site trips and shorter intra-site trips in the network. As the characteristic distance $D_c$ of trip displacement is less than $D_m$, hence in this case, $D_c$ is less than or in the same order as the network size $D$. Because the inter-site trip displacements are in the same order of the network size $D$, it implies a non-negligible value of $D_c$ for the mobile user. Hence, both the power-law head and exponential tail could collectively characterize the distribution of the trip displacement. Similarly, when the regular pause time of the mobile user is less than or in the same order as the network observation (simulation) time, the mixed power-law and exponential effect on pause time distribution is necessarily to be considered to study the human mobility metrics, and in turn, their effects on inter-meeting time. Intuitively, the mixed power-law and exponential behavior need to be analyzed from the whole shape of the cutoff power-law distribution. However, a mathematical close form of a cutoff power-law distribution of trip displacement and pause time is still unknown. Therefore, in this work, we aim to mathematically model the whole shape of the cutoff power-law distribution of human mobility metrics by a parameter tuple: (power-law coefficient and cutoff point). Upon the proposed cutoff power-law distribution for both trip displacement and pause time, the impact of human mobility with the mixture of power-law and exponential behaviors on inter-meeting time can be further investigated.

Based on this problem demonstration, the above interesting yet challenging issues mentioned in Case 2 and Case 3 collectively motivate our entire study in this work.

### 4.3 Cutoff power-law: Human Mobility Traces

Recall that our first question in this study is whether human trip displacement in a finite moving domain size always exhibits a cutoff power-law distribution. If so, does the temporal limi-
tation of human mobility also result in a cutoff power-law distribution of human temporal mobility metrics, such as pause time? To answer this question, we studied the empirical human mobility traces on three different domain scales: campus-wide, city-wide, and county-wide, respectively. By analyzing these collected human traces, we are additionally interested in the following questions: 1) Does human moving domain size $D_m$ affects the location of characteristic distance $D_c$ in the cutoff power-law distribution of trip displacement? 2) What is the relation between the average pause time and the characteristic time $T_c$, and respectively, the average trip displacement and the characteristic distance $D_c$ from the collected traces?

Furthermore, our main objective of this work is to study the effect of human diffusive mobility pattern, which is collectively determined by human trip displacement and pause time, on inter-meeting time in mobile wireless networks. However, we found that there is no existing available human trace dataset which can provide the complete information of human successive daily travel activities regrading both trip displacement and pause time. This brings a challenge for comparing and validating our analytical results in this study. Hence, to overcome this issue, we collect student daily travel traces over our campus area during three months (over a thousand hours). Next, we introduce the detailed information of the collected campus-wide traces, and the other city-wide and county-wide trace dataset as well.

### 4.3.1 Dataset Collection

To conduct our research, we let a small group volunteer students carry GPS loggers during each travel, either by walk or vehicles. Each GPS logger takes measurements every 10 seconds, and
records the time-stamped dataset, including current time, latitude, longitude, and speed. By this means, the trace dataset contains both human spatial and temporal information necessary for this study. In this work, we named this trace dataset as Campus dataset.

However, we notice that our collected trace files are limited to students only, such that their majority movements are in the neighborhood of a 2 km wide campus, which is the similar to a small scale of human moving domain size, i.e., $D_m \leq 3$ km defined in [114]. Thus, our collected trace dataset is a typical campus-centric human moving trace. Consequently, the potential defects in the Campus dataset of limited sample traces, homogeneous social regularities of participants (students), and small area of movement trajectories may constrain our observations and results of human mobility. In order to investigate the inherent properties of general human mobility, the larger dataset including more number of sample traces, more heterogeneous participants, and larger scale of human moving domains are still preferred for this study. More important, to find out the inherent cutoff power-law property of human mobility, human traces with different scales of moving domain size are necessary for this study.

Fortunately, we obtained two more trace datasets from the public, which are Lexington [119] and Seattle [120], respectively. For convenience, we name each dataset according to the location where the traces are collected. The majority traces in Lexington dataset are car traces and within a city-wide moving domain scale. Briefly, the Federal Highway Administration collected the personal travel data at Lexington area in central Kentucky [119] in 1996. The sample collection comprised 100 households including 216 drivers in total. It turns out that the daily moving domain size of Lexington residences is typically in the orders of tens km, which is similar to the middle scale of human moving domain size defined in [114].

Accordingly, the Seattle dataset collected traces of the actual movement of buses in the Seattle, Washington area King County Metro bus system, which is covering a 5100-square kilometer area [120]. This is the case for a middle-to-large scale of human moving domain size defined in [114]. Here, we consider each bus movement from the start to the immediate stop between any two bus stations as a proxy of human movement when people take the bus. Although the Seattle dataset cannot be exclusively treated as human moving traces, as mentioned in [120], it can fairly reflect a county-wide human mobility upon bus vehicles.

In short, compared to Campus dataset, these two available datasets provide the large-scale and long-time human moving traces. However, the pause time records associated with a single person for consecutive trips is missing in these two datasets, which brings the limitation for studying
properties of pause time upon different trace datasets.

4.3.2 Data Extraction and Statistics

Chapter4-Data Extraction

In this work, we consider a human movement as a trip movement if the source and destined location sites are different. Specifically, we define a trip as a human travel from a starting location point to a destined location point. As the moving direction can change frequently during a trip, similar to [22], we define the line segment associated with each direction as a leg in one trip. Thus, a trip is composed of multiple legs. If we consider a leg as a vector, then the displacement of a trip, denoted by $S(t)$, is the magnitude of the vector summation of the trip. Clearly, the trip displacement is always equal or less than the trip length. To determine trip destinations from each subject’s raw dataset, we utilize the same trace extracting strategies introduced in [118]. Briefly, we separate trips if there are more than 3 minutes gap between two consecutive waypoints in the trace. By this means, we are able to collect leg length, starting and ending trip/leg positions, direction changes, velocity, and pause time within each trip. In addition, we use the direction model introduced in [22] to differentiate legs in one trip, where the relative angle between two successive legs is larger than a direction threshold $\theta_{th}$. An example of the trip extracted by the direction model is shown in Figure 4.4. In the figure, the dotted line represents the GPS trace, marked by the logger waypoints every 10 seconds. After running the direction model, the trip is abstracted and decomposed into three legs. Specifically, $\theta_i$ and $l_i$ denotes the direction and the length of the $i^{th}$ leg, respectively. And the entire trip direction $\theta$ is measured by the starting position and ending position of the trip. Correspondingly, $L$ is denoted by the displacement of the trip, i.e., the Euclidean distance between source and destination of the trip. The time duration between two consecutive trips is the pause time $T_p$.

![Figure 4.4: Example of an extracted trip.](image)
Data Statistics

After data extraction, Table 4.1 illustrates the summary information of each dataset. We see that while the majority of human traces studied in this work are made by cars, the type of trips in Campus dataset is more diversified than the others. Specifically, 78.4% of total traces are made by cars. In contrast, 94.9% of total traces in Lexington dataset are city-wide driving traces, while the county-wide Seattle dataset contains only bus trips from random selected 20 buses. Therefore, the majority of collected human traces studied in this work are made by cars.

<table>
<thead>
<tr>
<th>Name</th>
<th>No. of Users</th>
<th>No. of Trips</th>
<th>Type of Trips</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Car</td>
</tr>
<tr>
<td>Campus</td>
<td>7</td>
<td>306</td>
<td>78.4%</td>
</tr>
<tr>
<td>Lexington</td>
<td>216</td>
<td>1885</td>
<td>94.9%</td>
</tr>
<tr>
<td>Seattle</td>
<td>20</td>
<td>9582</td>
<td>0%</td>
</tr>
</tbody>
</table>

As the Campus dataset contains the most complete human travel logs during successive days, also due to the limited work space, we mainly describe the trace statistics of Campus dataset in this work. The statistics of Campus dataset are illustrated in Table 4.2. We find that on average the students have only few trips and visiting location sites per day. In detail, according to all daily traces, the percentage that students take less than 5 trips per day is 72.3%, and visit less than 6 different locations each day is 84%. These are common phenomena in human mobility also exhibited in Lexington dataset, where people on average take 5.2 trips and visit less than 6 different locations per day. Moreover, for Campus dataset, certain sites such as homes and labs are revisited multiples times during one day. This implies that humans, not only students, limit their activities to a few key sites in their daily routine. Most of them are occasionally mobile during a day and spend a considerable amount time at certain places, such as home and offices. These findings are consistent with the pioneering works on human mobility [48]. It implies that human mobility has strong degree in both temporal and spatial regularities [114]. Interestingly, we observe that the trips resulting from different students, illustrated in Table 4.2, share the common properties. Specifically, we find that the average trip displacement is at the order of 1000 meters. The average travel time per trip is typically less than 30 minutes, while the average pause time is 10 times longer than the average
travel time of each user.

<table>
<thead>
<tr>
<th>Student ID</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg trip displacement (m)</td>
<td>2333</td>
<td>2131</td>
<td>1456</td>
<td>1594</td>
<td>7920</td>
</tr>
<tr>
<td>Avg travel time (min)</td>
<td>24.2</td>
<td>20.1</td>
<td>8</td>
<td>8.8</td>
<td>33.87</td>
</tr>
<tr>
<td>Avg pause time (min)</td>
<td>291.5</td>
<td>285</td>
<td>423</td>
<td>194</td>
<td>368.5</td>
</tr>
<tr>
<td>Avg sites visited daily</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Different sites visited weekly</td>
<td>15</td>
<td>14</td>
<td>8</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

As introduced in Section 4.2.1, the human mobility is subject to spatial-temporal effects. Specifically, trip displacement is a factor in spatial domain and pause time is a temporal factor. Given the extracted trace statistics, next we investigate the inherent properties of pause time and trip displacement in sequence. Unless otherwise specified, our findings on these human mobility metrics hold for all trace datasets.

### 4.3.3 Temporal Domain: Pause Time Property

Because Campus dataset records complete information of consecutive trips per user per week, we can obtain the CCDF pause time of one single student and the aggregated CCDF of all students in log-log scale, which are shown in Figure 4.5. We observe that these two CCDF values decay in a very similar way. Specifically, both CCDF values decrease linearly within the range $[10, 360]$ minute, beyond which they drops exponentially. Thus, we observe a “power-law – exponential” transition in the CCDF of pause time. That is, the pause time in human mobility also follows a cutoff power-law distribution. Recall that in Section 4.2.2, we define this cutoff point as the characteristic time ($T_c$) in pause time distribution. By comparing Figure 4.5(a) with Figure 4.5(b), we can see that the order of characteristic time $T_c$ in both distributions is almost same, which is at the order 360 minute (6 hours), regardless of the number of users. This indicates that humans with same social duties in a network domain, e.g., students on campus, are very likely to share the same properties on the pause time. Recall that Table 4.2 illustrates the average pause time for each student in Campus dataset. For aggregated student traces, the average pause time is 304 minutes, which is interestingly very close to the characteristic time $T_c$ observed in Figure 4.5.
4.3.4 Spatial Domain: Trip Displacement Property

Here, we are interesting to find the property of human trip displacement. Figure 4.6 shows the trip displacement CCDF of a single user/bus and the aggregated CCDF of all users/buses in three datasets in log-log scale. Similarly to what is observed in CCDF of inter-contact time [55], from Figure 4.6(a), we can see that regardless of the datasets, the CCDF value of a single user always decreases linearly up to a certain point, i.e., cutoff point, thus suggesting an initial power-law decay of the distribution. Accordingly, this cutoff point is defined as the characteristic distance ($D_c$) of the trip displacement. In addition, after the characteristic distance $D_c$, we find that the CCDF value drops exponentially. Thus the exponential decay in the tail part of the distribution is also observed. Therefore, we validate our conjecture that human trip displacement in a finite moving domain size always exhibits a cutoff power-law distribution.

Though it is very difficult to pinpoint the exact $D_c$ value in each CCDF distribution of trip displacement shown in Figure 4.6, we can clearly identify the order of $D_c$ value, which is the transition area from power-law decay to exponential decay in each distribution. Interestingly, we notice that the order of $D_c$ is different from users in different datasets. Specifically, for Campus dataset, as students mainly move in the proximity of 2km-wide campus, the $D_c$ of the student is at the order of 1000 m. Recall that Table 4.2 illustrates the average trip displacement for each student. We can see that the value of $D_c$ is less than while in the same order as the average trip displacement of a student. For instance, the average trip displacement of student (ID 1) is 2333 m and the average trip displacement of student (ID 3) is 1456 m. Meanwhile, for the city-wide Lexington dataset, a person living in Lexington area has a larger range of daily moving trips, whose
Figure 4.6: The CCDF of trip displacement in three datasets.

$D_c$ is at the order of 3000 m, due to his/her more diversified social activities than students. In contrast, in the county-wide Seattle dataset, each bus generally moves around a major circle of the county, which represents a middle-to-large human moving domain size. Thus, the bus’ daily traces can cover more location areas than those covered by individuals. Therefore, the bus trace generates the largest $D_c$, at the order of 5000 m, of trip displacement among three datasets. Based on this observation, we hypothesize that the value of $D_c$ in the distribution of trip displacement varies according to different human social duties reflected in the spatial domain. Basically, the larger average trip displacement an individual has, the larger $D_c$ for the distribution. In other words, the value of $D_c$ is proportional to the scale of human moving domain $D_m$.

Moreover, we calculate the aggregated CCDF trip displacement of all users in each dataset shown in Figure 4.6(b). By comparing Figure 4.6(a) with Figure 4.6(b), we observe that both CCDF values decay in a very similar way. It further implies that the cutoff power-law distribution of trip displacement is a universal property of human mobility. Moreover, we find that the $D_c$ value of aggregated trip displacement is almost the same as that of a single user in each dataset. It implies that humans with the same type of social duties or work patterns are very likely to have similar travel activities.

### 4.3.5 Observations

By studying the human mobility traces with different scales of moving domain size, we summary our observations as follows.
• The cutoff power-law behaviors of human trip displacement are observed in all three datasets, which is consistent with the studies of [22, 114]. Similarly, the cutoff power-law property of pause time is also seen in Campus dataset. Interestingly, the human mobility demonstrates the same cutoff power-law behavior as what has been observed in inter-contact time (ICT) [55, 108]. Thus, the contact-based metrics are affected by human moving behaviors in a network, which are further influenced by the temporal and spatial limits of social activities in the network. For an intuitive explanation, due to temporal and spatial limitations of human social moving behaviors, there is a finite size of pause time and human moving domain size. As a result, the cutoff points $T_c$ and $D_c$ exist inevitably in the distribution of pause time and trip displacement, respectively. Therefore, we validate that human trip displacement in a finite moving domain size always exhibits a cutoff power-law distribution.

• For the spreading traces of bank notes hold by humans in [56], the bank note exchange between humans is not constrained by the temporal and spatial limitations of human social moving behaviors. Hence, this nation-wide traces have an “infinite” moving domain size. Therein, it results in an exclusive power-law distribution of human trip displacement, as power-law has the “scale-free” property [113, 112, 116].

• With different scales of moving domain size, we find that the characteristic distance $D_c$ varies with different human trace dataset. In particular, the value of $D_c$ is proportion to the scale of human moving domain $D_m$. If comparing our results of pause time in Campus dataset with the existing results of pause time in human walker traces in [22], we can also observe that characteristic time $T_c$ varies in different traces, and the larger average pause time is, the larger $T_c$ has for that trace.

• We find that the values of characteristic distance $D_c$ and characteristic time $T_c$ are less than, while in the same order, as the value of corresponding average trip displacement and pause time for each trace. In addition, humans with the same type of social duties or work patterns are very likely to have the same order of $D_c$ and $T_c$ in the distribution of trip displacement and pause time.

By far, we have validated that cutoff power-law characterizes human mobility in temporal and spatial domains. But it is still not clear how to characterize the human moving behaviors upon the joint effect of human pause time and trip displacement, especially when these two metrics are
characterized by the power-law head in the distribution. To answer this question, we study the scaling law of human diffusive behaviors in the next section.

4.4 Scaling Law of Temporal-Spatial Human Diffusive Behaviors: Counter Effects

In previous section, we have observed that cutoff power-law characterizes human trip displacement and pause time. Recall that, as demonstrated Case 2 in Section 4.2.3, the trip displacement and pause time of mobile users can be characterized by power-law in a network. The power-law trip displacement and pause time indicate that human mobility are characterized by very long trips and very long pause time. Recent studies respectively suggested that the exclusive power-law effect of trip displacement may lead an overly optimistic routing performance [22], while the exclusive power-law effect of pause time results in an overly pessimistic routing performance [111, 55]. Hence, the power-law of trip displacement typically brings contradictory effect on network performance compared to the power-law of human pause time. Then we have the following question: under which condition, the temporal factor of pause time, or the spatial factor of trip displacement, has a more significant impact on the inter-meeting time? In recent studies [56, 22], it suggests that the ambivalent interaction between trip displacement and pause time can be manifested by the human diffusive movement patterns [56], which is further characterized by his/her diffusive capability (rate) $r$. Motivated by this observation, we aim to study the scaling law of human diffusive rate $r$ when both trip displacement and pause time are characterized by power-law. Then, we investigate how to explore the human diffusive rate $r$ to specify the joint temporal-spatial power-law effect of human mobility on inter-meeting time.

To achieve this goal, it is necessary to measure the human diffusive behaviors according to his/her diffusive capacity (diffusive rate $r$). Hence, in this section, we first develop a continuous time task-driven mobility (CTDM) model for studying the properties of human diffusive behaviors. Upon this model, we then study the scaling law of human diffusive behaviors featured by diffusive rate $r$. By applying this analytical result, we further study the direct influence of different diffusive mobility patterns, which characterizes the contradictory power-law effect between trip displacement and pause time, on the properties of inter-meeting time.
4.4.1 Continuous-Time Task-Driven Mobility Model

In this section, we elaborate the mobility pattern of the proposed continuous-time task-driven mobility (CTDM) model, which is used to describe human diffusive moving behaviors. In fact, our designed CTDM model is a variant of the continuous time random walk (CTRW) model, which is widely used to describe particle movements in physics [113, 56]. Briefly, in a CTRW model, a walker (particle) pauses a random time between two jumps and then randomly jumps from one location to another location. The major limitation of the CTRW model is that it does not consider the travel time a walk spends between two jumps, which hence cannot directly apply for studying human moving behaviors. Corresponding to the CTRW model, our proposed CTDM model interprets the human diffusive moving behaviors upon their social duties in the network. Specifically, humans only stay at task locations, travels randomly from one task location to another one. And they do not execute any task during the travel. Figure 4.7(a) demonstrates an example of two consecutive trips in the CTDM model. As we can see, the trip displacement is defined as the displacement between two consecutive task locations. And task time is defined as the time interval from the instant a user arrives at the current task location to the instant that the user reaches the next task location. In other words, the task time is the summation of the current task execution time and the travel time to the next task location.

![Diagram showing trip displacement and task time in CTDM model.](a)

(b) One-dimensional CTDM model.

Figure 4.7: Continuous time task mobility (CTDM) model.

Note that, given the extracted Campus trace statistics of pause time and travel time in Table 4.2, it is clear to see when the travel time is short, the pause (task execution) time predominates the value of task time. However, the distribution property of the task time is still unknown so far. Hence, we calculate the CCDF of the aggregated task time from the Campus dataset which is shown in Figure 4.8. Corresponding to the definition of aggregated pause time shown in Figure 4.5(b), the
aggregated task time accounts for the collected task times of all volunteer students extracted from the Campus dataset. Interestingly, we notice that the CCDF of the aggregated task time in Figure 4.8 matches very well with the CCDF of aggregated pause time in Figure 4.5(b). Clearly, the cutoff power-law also characterizes the task time of the CTDM model. Especially, the characteristic time of the aggregated task time, which is at the order of 360 minutes, is the same as that in pause time. In addition, the empirical CCDF power-law slope values (0.15) are also the same. Therefore, in the following analysis, it is feasible to substitute the power-law coefficient $\alpha$ of pause time for the power-law coefficient of task time.

![Figure 4.8: The CCDF of aggregated task time.](image)

For simplicity, in what follows, we study the human diffusive behaviors in one dimensional CTDM model, as shown in Figure 4.7(b). Specifically, each mobile node in the CTDM model moves back and forth along the X-axis with time. Without the boundary limitation, the CTDM model exhibits the scaling diffusive behaviors in the long run. Intuitively, the power-law of trip displacement implies that long distance trips may frequently occur in the human trajectories, which leads to a superdiffusive ($r > 1$) movement pattern. In contrast, the power-law of pause time indicates that long pause times may often occur between consecutive trips, which results in a subdiffusive ($r < 1$) movement pattern. However, it is still unknown how to characterize the human diffusive movement pattern according to the contradictory effect induced by the power-law behaviors of trip displacement and pause time, respectively. To address this issue, we study the scaling law of human diffusive behaviors next.

### 4.4.2 Scaling Law Analysis of Human Diffusive Behaviors

Recall that in this study, $T_p$ denotes the pause time, and $L$ denotes the trip displacement of a mobile user. To start the analysis of the scaling law of human diffusive behaviors, we first describe the power-law distribution of human mobility regarding the pause time $T_p$ and trip displacement $L$.
as follows:

\[ f_{T_p}(t) \sim \frac{1}{t^{1+\alpha}} \quad \alpha > 0, \quad (4.2) \]

and

\[ f_L(l) \sim \frac{1}{l^{1+\beta}} \quad \beta > 0, \quad (4.3) \]

where \( \alpha \) and \( \beta \) are defined as the power-law coefficient of pause time and trip displacement, respectively. With the pdf definition of pause time in (4.2), its CCDF exhibits the linear relationship between \( \log f_{T_p}(t) \) and \( \log t \) with slope \(-\alpha\) in the log-log coordinate. This is the exact case shown in Figure 4.5, where \( \alpha = 0.15 \) for student pause time. The same rule applies for CCDF of trip displacement in Figure 4.6, where \( \beta \) is close to 0.2 for all three datasets. In reality, \( \alpha \) and \( \beta \) typically lie in the range of \((0, 2)\) for human mobility [56, 114]. Therefore, the proposed CTDM model is able to characterize the scaling diffusive behaviors of mobile users. Next, we analyze the ambivalent temporal-spatial power-law effects on human diffusive behaviors in terms of the diffusive rate \( r \).

To proceed, let \( \varphi(L, \tau) \) denote the pdf of the joint probability density that a trip displacement (task displacement) from the current task location to the new task location is \( L \) and the current task time requires a total time \( \tau \). As discussed in Section 4.4.1, in the CTDM model, we use the pause time in the current task location to approximate the corresponding task time \( \tau \). Accordingly, the pdf of trip (task) displacement is \( f(L) = \int_0^\infty \varphi(L, \tau) d\tau \), and the task time pdf, i.e., the pause time pdf, is \( w(\tau) = \int_0^\infty \varphi(L, \tau) dL \). As illustrated in Figure 4.7(b) of the one-dimensional CTDM model, \( \eta(x, t) \) denotes the pdf that a human is arriving at the task location \( x \) at time \( t \). Given the example, \( X' \) and \( X \) are the locations of task \( i \) and task \( j \), respectively. Similar to the relationship between \( \varphi(L, \tau) \) and \( \eta(x, t) \) of particle movement process in CTRW model [113], in our CTDM model,

\[ \eta(x, t) = \int_{-\infty}^\infty \int_0^\infty \eta(x', t') \varphi(x - x', t - t') dt' dx' + \delta(x)\delta(t), \quad (4.4) \]

where \( \delta(x) \) and \( \delta(t) \) indicate that the movement process starts at the origin point at time \( t = 0 \). The RHS of equation (4.4) indicates that the joint probability that the mobile user arrive at the previous task location \( x' \) at time \( t' \) and the task time at the task location \( x' \) lasts \( t - t' \). As a result, the mobile user arrives at the current task location \( x \) at time \( t \). Therefore, the pdf \( f(x, t) \) that a person lies in a task position \( x \) at time \( t \), i.e., the pdf of trip displacement \( x(t) \), is

\[ f(x, t) = \int_0^t \eta(x, t') \psi(t - t') dt', \quad (4.5) \]
where \( \psi(t) = 1 - \int_0^t w(\tau) d\tau \), and \( w(\tau) \) is the corresponding pause time in task location \( x \). Hence, \( \psi(t) \) is the surviving probability of task time \( w(\tau) \), which means the probability that the pause time is longer than \( t \). Recall that \( MSD(t) = E\{x(t)^2\} \) in Definition 4.2.2, then the value of \( MSD(t) \) can be easily derived for a given \( f(x, t) \). Accordingly, the diffusive rate \( r \) can be further obtained.

However, the derivation of joint density \( f(x, t) \) directly from (4.5) is quite challenging. In [121], the authors showed that the (Spatial, Temporal) space of \( (x, t) \) can be reverted to (Fourier, Laplace) space \( (k, s) \), i.e., \( (x, t) \rightarrow (k, s) \), where the property of \( E\{x^2(t)\} \sim t^r \) of the CTRW process can be mapped to a much simpler condition of \( f(k, s) \) when both \( k \rightarrow 0 \), and \( s \rightarrow 0 \). Specifically, in the Fourier-Laplace space, following the same methodology of deriving particle diffusive movement in CTRW model [121], by combining (4.4) and (4.5), \( f(x, t) \) can be reverted to

\[
f(k, s) = \frac{1 - w(s)}{s} \cdot \frac{1}{1 - \varphi(k, s)}.
\] (4.6)

From (4.6), the second moment of \( f(x, t) \), i.e., the \( MSD(t) \) is obtained as [113],

\[
E\{x^2(t)\} = \int x^2 f(x, t) dx = -\frac{\partial^2}{\partial k^2} f(k, t) \bigg|_{k=0} = -\frac{\partial^2}{\partial k^2} \mathcal{L}^{-1} \left[ \frac{1 - w(s)}{s} \cdot \frac{1}{1 - \varphi(k, s)} \right]_{k=0} ,
\] (4.7)

where \( \mathcal{L}^{-1} \) is the inverse Laplace transform. Given (4.7), it is necessary to find \( \varphi(k, s) \) in deriving \( E\{x^2(t)\} \). As shown in Figure 4.7, the trip displacement \( L \) and task time \( t \) in the CTDM model are independent. Hence, we have \( \varphi(L, \tau) = f(L) \cdot w(\tau) \). Accordingly, \( f(k) \) and \( w(s) \) are independent in the Fourier-Laplace space \( (k, s) \), i.e., \( \varphi(k, s) = f(k) \cdot w(s) \). Before we continue to analyze the human diffusive process in the CTDM model, we will use the following major results of particle movements in CTRW model proved in [121, 113].

**Lemma 2.** Suppose that the spatial step increment \( f(L) \) and temporal increment \( w(\tau) \) of a user movement process \( x(t) \) are characterized by Fourier transform \( f(k) \) and Laplace transform \( w(s) \). For small values of \( k, s \),

\[
\begin{align*}
\lim_{k \to 0} f(k) &= 1 - C_1 k^a + O(k^{2a}) \sim 1 - C_1 k^a, \\
\lim_{s \to 0} w(s) &= 1 - C_2 s^b + O(s^{2b}) \sim 1 - C_2 s^b.
\end{align*}
\] (4.8)
In Lemma 2, the coefficient values of $C_1$, $C_2$, $a$ and $b$ vary with the distributions of $f(L)$ and $w(\tau)$, respectively. Specifically, a distribution can be either a light tailed or a heavy tailed (e.g. power-law) distribution. In [122, 113], it has been shown that the generalized Fourier and Laplace transforms in (4.8) can be further represented with respect to the tail property of distributions.

**Lemma 3.** Following the condition of Lemma 2, the Fourier and Laplace transforms of $f(L)$ and $w(\tau)$ are represented as

\[
\begin{align*}
\lim_{k \to 0} f(k) &\sim 1 - C_3 k^2 & \text{if } f(l) \sim \text{light tail distribution} \\
\lim_{k \to 0} f(k) &\sim 1 - C_\beta |k|^\beta & \text{if } f(l) \sim l^{-(1+\beta)} \quad 0 < \beta < 2,
\end{align*}
\]

And for the Laplace transform of $w(\tau)$,

\[
\begin{align*}
\lim_{s \to 0} w(s) &\sim 1 - C_4 s^1 & \text{if } w(t) \sim \text{light tail distribution} \\
\lim_{s \to 0} w(s) &\sim 1 - C_\alpha s^\alpha & \text{if } w(t) \sim t^{-(1+\alpha)} \quad 0 < \alpha < 1,
\end{align*}
\]

where $C_3$, $C_4$, $C_\beta$, and $C_\alpha$ are constants.

Recall that the Definition 4.2.6 introduced in Section 4.2.1, that is, a node diffusive process can be characterized by its scaling order $\rho$ of $S_N(t)$, i.e., $S_N(t) \sim t^\rho$. As shown in [113], if the trip displacement $f(L)$ and pause time $w(t)$ of a particle movement process $x(t)$ are characterized by Fourier transform $f(k)$ and Laplace transform $w(s)$ upon (4.8), then the following Theorem is obtained.

**Theorem 2.** Under the condition of a node diffusive process in Lemma 2, the user’s one-dimensional position $x(t)$ always has a scaling relation with $t$ as $x(t) \sim t^{b/a}$. That is, the node diffusive process $x(t)$ is characterized by its scaling order $\rho$, where $\rho = b/a$.

Next, given the existing results of node diffusive process of Lemma 2 and 3 and Theorem 2 in physics, we obtain our main result of scaling law of human diffusive behaviors here.

**Theorem 3.** Following the same conditions as represented in Theorem 2, if there exists a finite value of $E\{Y^2\}$ in (4.12), then the human diffusive rate is double of the scaling order, that is $r = 2\rho = 2b/a$ and $MSD(t) \sim t^{2b/a}$.

**Proof.** Recall that in Definition 4.2.6, if a walker’s position $x(t)$ has a relation with $t$ at the scaling order $\rho$, then the limiting pdf $f_Y(y)$ of the scaling position $Y = \lim_{t \to \infty} x(t)/t^\rho$ is independent of
This implies that for a large time $t$, \[ f(x,t) \sim \frac{1}{t^\rho} f_Y\left(\frac{x}{t^\rho}\right). \] (4.11)

Hence, if a movement process has the scaling ratio $x(t) \sim t^\rho$ and $Y = x(t)/t^\rho$, upon (4.11), the MSD$(t)$ is given by
\[
E\{x(t)^2\} = \int_{x \in \mathbb{R}} x(t)^2 f(x,t) dx \propto \int_{x \in \mathbb{R}} x(t)^2 \frac{1}{t^\rho} f_Y\left(\frac{x}{t^\rho}\right) dx \propto t^{2\rho} \int_{y \in \mathbb{R}} y^2 f_Y(y) dy \propto t^{2\rho} E\{Y^2\}. \] (4.12)

Because the existence of $D_c$ in the cutoff power-law of trip displacement $x(t)$, no matter how large the value of $D_c$ is, the variance of trip displacement $E\{x(t)^2\} = C_5$ is finite. Thus, $E\{Y^2\} = \lim_{t \to \infty} E\{x(t)^2/t^{2\rho}\} = C_6$ is also finite. Therefore, given (4.12), we have
\[
MSD(t) = E\{x(t)^2\} \propto t^{2\rho} E\{Y^2\} \propto t^{2\rho} \propto t^r. \] (4.13)

Therefore, by plugging $\rho = b/a$ into (4.13), we have $r = 2b/a$, that is, $MSD(t) \sim t^{2b/a}$.

**Theorem 3** tells that the scaling law of human diffusive movement pattern is characterized by the ratio of distribution coefficient pair $(a, b)$. That is the scaling law of human diffusive movement patterns is dominated by the interaction between human trip displacement and pause time, which are critical human spatial and temporal mobility metrics. Next, we analyze the human diffusive rate $r$ according to different distributions (light tail and power law) of human pause time and trip displacement.

**Ambivalent Temporal-Spatial Interactions on Human Diffusive Behaviors**

Here, we assume that the pdf of both trip displacement $f(L)$ and task (pause) time $w(t)$ follows a power-law distribution, characterized by the coefficient $0 < \beta < 2$ and $0 < \alpha < 1$, respectively. Hence, the corresponding Fourier and Laplace transforms are $f(k) \sim 1 - C_\beta |k|^\beta$ and $w(s) \sim 1 - C_\alpha s^\alpha$. Fortunately with the Theorem 2 and Theorem 3, we are able to specify the ambivalent interaction between trip displacement and pause time on human diffusive mobility patterns. By applying these two Theorems, the *ambivalent diffusive* process [56] is characterized by the scaling relation $x(t) \sim t^{\alpha/\beta}$, and the diffusive rate $r = 2\alpha/\beta$. Figure 4.9 illustrates the
joint temporal-spatial effect on the ambivalent diffusive pattern. It is clear to see that the ratio of $2\alpha/\beta$ directly manifests the diffusive feature of human mobility. Therefore, we have the following Theorem.

**Theorem 4.** The interaction between the power-law effect of task time ($0 < \alpha < 1$) and trip displacement ($0 < \beta < 2$) leads to an ambivalent diffusive pattern with the diffusive rate $r = 2\alpha/\beta$. Specifically, the human diffusive behavior is a superdiffusive behavior when $\beta < 2\alpha$, otherwise a subdiffusive behavior when $\beta > 2\alpha$. Particularly, it becomes a normal diffusive behavior when $\beta = 2\alpha$.

Figure 4.9 illustrates the ambivalent human diffusive mobility patterns according to competing influences between task time and long trip displacement. We can see that upper triangle area represents the supperdiffusive movement pattern when $\beta < 2\alpha$. In contrast, the lower triangle area represents the subdiffusive movement pattern when $\beta > 2\alpha$. The diagonal of the entire rectangle represents the normal diffusive movement pattern when $\beta = 2\alpha$.

![Diagram showing ambivalent diffusive behavior](image)

Figure 4.9: Ambivalent diffusive behavior.

By far we have shown that the human diffusive rate $r$ can effectively characterize the human mobility pattern when both trip displacement and pause time follow a power-law distribution. Recall that as discussed in Section 4.2.3, the temporal-spatial human mobility patterns can be featured by distributions, such as the combination of (exponential-like pause time, power-law-like trip displacement) and (power-law-like pause time, exponential-like trip displacement). Hence, in next section, we further show that the diffusive rate $r$ can also effectively characterize the human mobility pattern in these two cases.
**Superdiffusive Behavior: Exponential-like pause time and power-law-like trip displacement**

Now we consider the human mobility scenarios characterized by very long trip displacements but overall short pause times. That is, the pause time $w(t)$ follows a light tailed distribution, such as exponential distribution, meanwhile the trip displacements $f(L)$ has a power-law distribution. Specifically, based on Lemma 3, we have $w(s) \sim 1 - C_2 s^1$ and $f(k) \sim 1 - C_3 |k|^\beta$. Following the same methodology, we show that the scaling law of this type of human movement process is $x(t) \sim t^{1/\beta}$. Notice that, when $0 < \beta < 2$ and $x(t) \sim t^{1/\beta}$, the variance of trip displacement could be infinite. In this case, from (4.12), $E\{Y^2\} \to \infty$. Because $E\{Y^2\}$ is infinite, Theorem 3 cannot be directly applied for this case. However, according to the CTDM model, the longest trip displacement is just equal to the longest distance $L_{\text{max}}$ among all the task locations. Because all the task locations are always inside a finite human moving domain $D_m$, hence $L_{\text{max}}$ is finite in reality. And the finite $L_{\text{max}}$ results in a finite value of $E\{Y^2\}$. Therefore, by applying Theorem 3, we have

$$E\{x(t)^2\}_{L_{\text{max}}} \propto \int_{L_{\text{min}}}^{L_{\text{max}}} x(t)^2 f(x,t)dx \propto t^{2/\beta}, 0 < \beta < 2,$$

where $E\{x(t)^2\}_{L_{\text{max}}}$ denotes the MSD of the movement process bounded by the human moving domain boundary $L_{\text{max}}$. Clearly, equation (4.14) indicates the scaling order $r = 2/\beta > 1$, when $0 < \beta < 2$, that is, the joint spatial-temporal effects of power-law trip displacements and lighted tailed pause times lead to a superdiffusive behavior featured by the diffusive rate $r = 2/\beta$.

**Subdiffusive Behavior: Power-law-like pause time and exponential-like trip displacement**

In this section, we consider the human mobility scenarios characterized by very long pause times but overall short trip lengths. In other words, we study the human moving scenarios where the pause time $w(t)$ has a power-law distribution, but the trip displacements $f(L)$ has a light tailed distribution, such as exponential distribution. In particular, upon Lemma 3, we have $w(s) \sim 1 - C_\alpha s^\alpha$ and $f(k) \sim 1 - C_3 k^2$. By applying Theorem 3, we can directly obtain the scaling law of the movement process $x(t) \sim t^{\alpha/2}$, i.e., $MSD(t) \propto t^\alpha$. Clearly, when $\alpha < 1$, it leads to a subdiffusive human behavior pattern. In fact, by substituting the value of $w(s)$ and $f(k)$ into (4.6), after a tedious calculus from (4.7), we can obtain the $MSD(t)$ as

$$E\{x(t)^2\} = \frac{2C_3}{C_\alpha \Gamma(1+\alpha)} \cdot t^\alpha \propto t^\alpha, \ 0 < \alpha < 1.$$
Thus, the result in (4.15) validates Theorem 3. Therefore, we conclude that the joint spatial-temporal effects of short trip displacements and long (fractal) task times lead to a subdiffusive behavior featured by the diffusive rate $r = \alpha$, where $\alpha$ is the power-law coefficient of the task time.

4.4.3 Temporal-Spatial Power-law Effects on Inter-meeting Time

In previous section, Theorem 4 provides a guideline for identifying the scaling law of human diffusive behaviors. This theorem can be well applied for human mobility scenarios when power-law characterizes the distributions of human trip displacement and pause time at the same time. To this end, we are able to address the question raised in Section 4.1: what are the joint temporal-spatial power-law effect of pause time and trip displacement on the inter-meeting time of mobile users? Therefore, in this section, by applying Theorem 4, we study how human diffusive rate affect the contact-based metrics, regarding the link lifetime and the inter-meeting time.

As described in Section 4.2.1, the human diffusive mobility patterns can be categorized into three types: subdiffusiv behavior ($r < 1$), normal diffusive behavior ($r = 1$), and superdiffusive behavior ($r > 1$). Based on the definitions of these diffusive mobility patterns, human subdiffusive behavior exhibits a type of human moving behavior in which humans tend to stay at certain location areas for a long time. Since $r = 2\alpha/\beta < 1$, it implies that the power-law effect of pause time (featured by the coefficient $\alpha$) has more influence than trip displacement (featured by the coefficient $\beta$) on inter-meeting time. In contrast, for superdiffusive behavior, i.e., $r = 2\alpha/\beta > 1$, humans tend to move forward to the locations outside the current network.

To simulate different human diffusive behaviors, a mobility model which can control the trip displacement and the pause time is preferred. Note that it has been proved that the impact of pause time and trip displacement on contact-based metrics, such as inter-meeting time, exhibits an invariant property on the first movement statistics over finite domain for random mobility models [23]. Among existing mobility models, RWP model setups the trip displacement and pause time for each trip. Therefore, in this study, we simulate the standard random waypoint (RWP) mobility model [123] with two variations regarding the distribution of trip displacement and pause time, such that different human diffusive behaviors in terms of diffusive rate can be obtained. Note that, we can also modify the RD or SMS model to achieve this simulation. Just for each trip, we can random select a direction with a random trip time. The longer the trip time, the longer the trip displacement becomes.

Specifically, in this simulation, we configured three diffusive rates: Subdiffusiv rate $r =$
0.2, normal diffusive rate \( r = 1 \), and superdiffusive rate \( r = 1.8 \), respectively. The diffusive rate is obtained by applying Theorem 4 to configure the power-law coefficient of \( \alpha \) and \( \beta \) of pause time and trip displacement of the RWP model with 50 mobile nodes during the simulation.

![Figure 4.10: Inter-meeting time upon human diffusive rate \( r \).](image)

The simulation results of the inter-meeting time distribution upon diffusive rate \( r \) are shown in Figure 4.10. We find that the power-law head of inter-meeting time varies dramatically according to the diffusive rate. Specifically, for the case of superdiffusive rate \( r = 1.8 \), the power-law head is up to 2000 seconds. In contrast, for the normal diffusive rate \( r = 1 \), the power-law head of inter-meeting time ends at the order of 500 seconds, while it ends at the order of 100 seconds, when diffusive rate \( r = 0.2 \). Exhibited by superdiffusive rates, mobile nodes tend to move outside the current network area, and the long trip displacement decreases the meeting chance of mobile nodes. Therefore, the power-law head of inter-meeting time will become longer, while the exponential tail will decay faster in the CCDF distribution. As the diffusive rate decreases, the impact of pause time on node mobility increases, therein, the power-law head of inter-meeting time will become shorter. For the normal diffusive behavior \( (r = 2\alpha/\beta = 1) \), the trip displacement plays almost an equal role with the pause time on inter-meeting time. In this case, as shown in Figure 4.10, the resulting power-law head in the inter-meeting time will be larger than that from subdiffusive pattern \( (r = 0.2) \), while shorter than that from superdiffusive pattern \( (r = 1.8) \).

**Remark 9.** Human diffusive rate can effectively characterize the joint temporal-spatial power-law effect of pause time and trip displacement on the inter-meeting time of mobile users. In addition, as the power-law-like pause time and exponential-like trip displacement leads to a subdiffusive rate \( r = \alpha \), while the exponential-like pause time and power-law-like trip displacement leads to a superdiffusive rate \( r = 2/\beta \). Hence, we suggest that the effects according to superdiffusive rate and...
subdiffusive rate on inter-meeting time can also be applied for these two cases. Among these three human diffusive behaviors, we find that superdiffusive rate leads to the longest power-law head, while subdiffusive rate results in the shortest power-law head in the distribution of inter-meeting time. Thus, the higher the diffusive rate is, the longer the power-law head is, while the shorter the exponential tail becomes in the distribution of inter-meeting time.

**Remark 10.** As the exclusive power-law distributions of pause time and trip displacement can either under-estimate or over-estimate the actual system performance in DTNs [12, 22]. We suggest that human mobility with superdiffusive rate contributes the network performance improvements in DTNs, while the network performance degrades with human mobility characterized by subdiffusive rate. This is another explanation that why human mobility can either assist [124, 22] or harm [12, 55] the network performance.

### 4.4.4 Human Diffusive Rate Effect on Link lifetime

With the same simulation setup as described in the previous section, we study the human diffusive rate impact on link lifetime between mobile users. Compare to inter-meeting time, the link lifetime is the contact time period when two mobile users can directly communicate, i.e., become the neighbor between each other. The detailed description has been introduced in Chapter 2. The simulation results of PMF link lifetime according to different diffusive rate $r$ are shown in Figure 4.11.

![Figure 4.11: Link lifetime upon human diffusive rate $r$.](image)

Here, the simulation results of link lifetime are based on the RWP model. Recall that it has been observed that macroscopic mobility models such as RWP model, typically generate a peak
in the shape of PMF link lifetime [5, 26]. Figure 4.11 illustrates exactly the case, where there is a peak in the time at the order of 40 seconds. Interestingly, from the figure, we found that the PMF of link lifetime is insensitive to the diffusive rate. The PMF of link lifetime is almost the same with different diffusive rate (movement patterns). This is because human diffusive rate demonstrates the human diffusive capability regarding his/her travel displacement with time, as time goes by. In contrast, as shown in Figure 4.11, the typical link lifetime between two mobile users is less than 250 seconds, where the average moving speed is 10 m/sec. Therefore, when a mobile node (user) meets and crosses the transmission zone of another mobile node during the relative very short time period, their relative moving behaviors are almost the same and generally are not affected by their diffusive movement patterns. Another explanation is that the link lifetime between two neighboring users is generally much less than their respective trip duration time, especially when two users move in the reverse direction. In contrast, the human diffusive rate is characterized upon the time scale several orders larger than a user’s general trip duration time. Therefore, the human diffusive rate cannot directly reflect the property of link lifetime metric.

**Remark 11.** By comparing Figure 4.10 and Figure 4.11, we suggest that human diffusive rate can effectively reflect the power-law head of inter-meeting time but not the link lifetime in MANETs. In other words, the human diffusive rate indicates the potential short of long inter-meeting time between mobile users as time goes by. In contrast, because the order of link lifetime is generally much less than the order of time for characterizing the human diffusive rate, human diffusive rate cannot effectively reflect the link lifetime in multi-hop wireless networks.

### 4.5 Coupling Effects of Cutoff power-law Distribution

In previous section, we have studied the human mobility effect on inter-meeting time when either trip displacement or pause time or both are characterized by power-law. However, we notice that this is not the only case of human mobility pattern reflected in MANETs. As demonstrated Case 3 in Section 4.2.3, when \( D_c < D \), and \( T_c < T \), the power-law head and exponential tail collectively characterize the distribution of the trip displacement and pause time. Therefore, the mixed power-law and exponential effect on the distribution of pause time and trip displacement need to be collectively considered to further correctly analyze their impacts on inter-meeting time. For instance, human trip displacement could lead to either overly opportunistic or pessimistic routing performance when it follows a exclusive power-law or exponential distribution, respectively [22].
Otherwise, either the exclusive power-law or exponential effect alone on temporal-spatial human mobility metrics can lead to a biased or even misleading result of inter-meeting time, therein, a misleading performance evaluation for the studied network. To tackle this issue, in this section, we first study the property of the entire cutoff power-law distribution for both trip displacement and pause time. Then, we develop two approximated cutoff power-law distributions for both temporal and spatial mobility metrics upon empirical human trace datasets. We demonstrate that the proposed cutoff power-law distributions have a close fit with the results from the three human trace datasets. Finally, we apply these two proposed distributions to study the impact of human mobility with the mixture of power-law and exponential behaviors on inter-meeting time.

4.5.1 A Close Look at Cutoff Power-Law

In this section, we intend to find out which parameters are critical to identifying a cutoff power distribution of human mobility metrics. Specifically, a cutoff power-law distribution manifests power-law property before the cutoff point \( D_c \) or \( T_c \), while decaying exponentially after the cutoff point. Clearly, the cutoff point is the most important parameter, which directly delimits power-law head domain and exponential tail domain in the distribution. In other words, the cutoff point determines the impact of power-law head and exponential tail on the human mobility metrics, which in turn, the impact on the properties of contact-based metrics in MANETs. In addition, as shown in Figure 4.5 and Figure 4.6, the power-law coefficient \( \alpha \) or \( \beta \) of human mobility metrics characterizes the shape of power-law head and the power-law decaying rate in the cutoff distribution. As elaborated in Section 4.4, we find that the power-law coefficient significantly affects the human diffusive patterns, which further results in different stochastic properties of contact-based metrics. Thus, the power-law coefficient is also essential to specify a cutoff power-law distribution. Therefore, in this study, we suggest that the cutoff power-law distribution of corresponding temporal and spatial mobility metrics is always featured by a parameter tuple (temporal or spatial power-law coefficient, characteristic time or distance), e.g., \( (\alpha, T_c) \) and \( (\beta, D_c) \). However, upon the results shown in Figure 4.5 and Figure 4.6, we observed that the cutoff power-law coefficient and cutoff point vary with different human trace files. Then, what is the reason for such a discrepancy in different human trace files? The recent study [114] suggested that human moving trajectories are highly dependent on the temporal-spatial limitations due to their social roles and behavior regularities. Hence, we conjecture that human social behavior congruity significantly affects the values of cutoff point and power-law coefficient in the cutoff power-law distributions of human mobility.
As our collected Campus dataset contains the complete temporal-spatial information of students’ consecutive daily activities, next, we investigate how human social behaviors influence the human mobility upon the Campus dataset.

4.5.2 Human Social Behaviors Impacts

Here, as a case study in Campus dataset, we illustrate a single user’s all visited locations during 10 days in Figure 4.12(a). Among these locations, the lab (located in the (0, 0) coordinate) and home are the two most preferred locations. The corresponding sequential trace trajectories are shown in Figure 4.12(b), among which 72% of total 64 trips are traveled in campus vicinities. It implies that most of human daily trips are govern by his/her major social duties. Also there are multiple repetitive trips between two sites. For instance, in Figure 4.12(b), we notice that the joint line between lab and home site is darker than others because of the overlapping of repetitive trajectories during 10 days. These travel phenomena demonstrate that human moving trajectories are highly dependent on the temporal-spatial limitations due to their social roles and behavior regularities.

Figure 4.12: Single user weekly trace trajectories in Campus dataset.

Furthermore, recent empirical studies showed that human regular activities often follow diurnal cycle patterns [48, 55, 114]. In addition, our proposed CTTM model also consents with this observation as humans execute tasks periodically, especially based on a diurnal cycle. This explains why the value of $T_c$ is at the order of hours in Campus trace dataset shown in Figure 4.5. Hence, upon the analysis of the collected human trace dataset in Figure 4.12, we suggest that the routine social behaviors have limited the range of both human pause time and travel distance. That
is why both $T_c$ and $D_c$ invariably exist in distributions of human temporal and spatial metrics from all collected datasets. In addition, due to different social roles such as students and salesmen, the typical task execution time (pause time), task displacement (trip displacement), and the period of return time of major task locations are different for humans. In consequence, power-law coefficients $\alpha$ and $\beta$ and cutoff points $T_c$ and $D_c$ vary with different types of human social roles.

4.5.3 Approximation of Cutoff power-law Distribution and Validations

By far, we have discussed that regular social behaviors cause the inherent cutoff power-law properties in both temporal and spatial human mobility metrics. Consequently, the distribution of corresponding temporal and spatial mobility metrics is always featured by a parameter tuple (temporal or spatial power-law coefficient, characteristic time or distance), e.g., $(\alpha, T_c)$ and $(\beta, D_c)$. However, a close form of a cutoff power-law distribution which characterizes this parameter tuple of human mobility metrics is unknown in existing studies. Then an interesting question is: how to mathematically describe a cutoff power-law distribution of a mobility metric according to its associated parameter tuple? The answer to the question is necessary to further investigate the cutoff power-law of human mobility on contact-based metrics and routing performance in mobile wireless networks. To address this issue, in what follows, we provide a feasible solution by modeling the pdf of human trip displacement and pause time.

By carefully observing the empirical CCDF distributions in Figure 4.5 and Figure 4.6, evidently, the cutoff power-law distribution is dominated by the power-law coefficient $\alpha$ ($\beta$) before the cutoff point $T_c$ ($D_c$). Rather than a sharp transition, the distribution smoothly transits from a power-law decay to an exponential decay in the proximity of the cutoff point. When the value of pause time (trip displacement) is much larger than the cutoff point, the distribution exhibits a fully exponential tail. By complying these observations, we provide the approximated cutoff power-law pdfs for both trip displacement and pause time as follows,

\[
\begin{align*}
    f_{T_p}(t) &\sim t^{-(\alpha+1)} \cdot e^{-\lambda \frac{t}{T_c}}, \\
    f_L(l) &\sim l^{-(\beta+1)} \cdot e^{-k \frac{l}{D_c}},
\end{align*}
\]

(4.16)

where $\lambda$ and $k$ are constants. Further, $\frac{\lambda}{T_c}$ and $\frac{k}{D_c}$ indicate the exponential decay rate against with the corresponding power-law decay rate $\alpha$ and $\beta$ in the pdf of pause time and trip displacement, respectively. Given (4.16), this coupling effect on the approximated pdf is over the entire domain. We compare our approximated trip displacement CCDF with the empirical CCDF of Campus dataset in
Figure 4.13(a). We can see that the decreasing rate is closely matched over the entire domain between these two distributions. To further validate the effectiveness of the proposed equation (4.16), we further compare the CCDF approximation with the corresponding empirical distribution of trip displacement in Seattle and Lexington datasets, and pause time in Campus dataset. The results are shown in Figure 4.13. It is clear that the approximated distributions are fully supported by the empirical results, which in together validate our equation (4.16) of cutoff power-law distributions of human mobility. Especially, the significance of influence on the distribution made by these dual effects are differentiated by the cutoff point.

![Figure 4.13: Cutoff power law approximation and validation.](image)

One remarkable merit of our proposed approximated cutoff power-law distribution in (4.16) is that this pdf distribution is featured by the parameter tuple (temporal or spatial power-law coefficient, characteristic time or distance), e.g., $(\alpha, T_c)$ and $(\beta, D_c)$. This parameter tuple can effectively reflect the power-law and exponential effect on the distribution, which in turn are governed by the human social behaviors. However, to determine the value of this tuple parameters for both temporal and spatial human mobility metrics is a very challenging issue. As shown in Figure 4.6 and Figure 4.5, the value of $D_c$ and $T_c$ is selected based on the order of 1000 meters and 100 minutes, respectively. To investigate and correctly specify the tuple $(\beta, D_c)$ and $(\alpha, T_c)$ values upon different human social behaviors and corresponding trace datasets is our on-going research work.

### 4.5.4 Cutoff Power-law Effects on Inter-meeting Time

In this section, we investigate the cutoff power-law effects of human mobility on inter-meeting time. Here, we use the approximated cutoff power-law pdfs for both trip displacement and
pause time from (4.16). And the corresponding tuple \((\beta, D_c)\) and \((\alpha, T_c)\) parameters are referred by the the Campus dataset, which are shown in Figure 4.13(a) and Figure 4.13(c), respectively.

![Figure 4.14: Cutcoff power-law vs. power-law effects on inter-meeting time.](image)

Similar to the simulation setup in Section 4.4.3, we let 50 mobile nodes move according to RWP model in the simulation zone \((5000m \times 5000m)\). The simulation results of the inter-meeting time distribution upon diffusive rate \(r\) are shown in Figure 4.10. For a better presentation, we compared the inter-meeting time distribution obtained by mobile nodes upon the cutoff power-law (\(\alpha = 0.2, \beta = 0.2\)), with those obtained by mobile nodes with superdiffusive rate (\(r = 1.8\)) and normal diffusive rate (\(r = 1\)), respectively. The simulation results are shown in Figure 4.14. Interestingly, we observed that the cutoff power-law of human mobility on inter-meeting time has a very similar effect made by mobile users with normal diffusive rate (\(r = 1\)). This is because the cutoff power-law leads to negligible effect of both very large trip displacement and very long pause time for mobile nodes. As a result, in the case of cutoff power-law on human mobility, the human trip displacement plays almost an equal role with pause time on inter-meeting time, which is very similar to the effect indicated by human mobility with normal diffusive rate.

### 4.6 Summary

In this work, we investigated the human diffusive behaviors from an empirical study upon three diversified trace datasets according to different size scales of human moving domains. Interestingly, upon these trace dataset study, we suggested that the cutoff power-law distribution is an “invariant” property of human mobility, because human mobility is affected by the temporal and spatial limits of human social behaviors. Thus, it has similar cutoff power-law properties as
contact-based metrics, such as inter-meeting time.

Given the trace observation results, we find that the distinct human social behaviors may lead to different human mobility patterns. The temporal-spatial limitations of human social behaviors result in the cutoff power-law distribution of mobility metrics, such as pause time and trip displacement. Because the cutoff point could be small or large in these two distributions, respectively, the distribution of pause time and trip displacement can be characterized by either power-law-like or exponential-like distributions.

It has been found that the contact-based metrics, such as inter-meeting time, are jointly affected by the human pause time in temporal domain and human trip displacement in spatial domain. This joint temporal-spatial effect of human mobility can be further characterized by the human diffusive patterns (diffusive rate). Therefore, by studying the scaling law of human diffusive behaviors, we showed that when power-law characterizes both pause time and trip displacement with the power-law coefficients $\alpha$ and $\beta$, respectively, the human diffusive rate $r$ is, $r = 2\alpha/\beta$, where $0 < \alpha < 1$ and $0 < \beta < 2$. Thus, based on different combinations of $(\alpha, \beta)$, the human diffusive rate $r$ can be superdiffusive ($r > 1$), subdiffusive ($r < 1$), and normal diffusive ($r = 1$), respectively. Furthermore, we demonstrate that the human diffusive rate can effectively characterize the power-head shape of inter-meeting time. Specifically, the superdiffusive rate ($r > 1$) leads to the longest power-law head, while the subdiffusive rate ($r < 1$) results in the shortest power-law head in the distribution of inter-meeting time. Thus, the higher the diffusive rate is, the longer the power-law head becomes in the distribution of inter-meeting time.

Furthermore, we notice that when the cutoff point resides in the middle area of the distribution, both power-law head behavior and exponential tail behavior collectively characterize the cutoff power-law distribution. In this case, the mixed power-law and exponential behavior should be analyzed upon the whole shape of the cutoff power-law distribution. Accordingly, we investigated this mixed power-law head and exponential tail behavior based on the empirical human trace datasets. As another contribution, we proposed an approximated cutoff power-law distribution, which is featured by a parameter tuple (the power-law coefficient and cutoff point), for instance, $(\alpha, T_c)$ and $(\beta, D_c)$, for pause time and trip displacement, respectively. In this case, the probability of either long trip displacement or long pause time is negligible in human movements due to the limited power-law head effect. As a result, the interplay between power-law head and exponential tail is almost equal. From the simulation, we demonstrate that cutoff power-law of human mobility has a very similar effect on inter-meeting time as what human mobility with normal diffusive rate
does.
Chapter 5

Understanding the Structure and Dynamics of Mobile Groups in Multi-hop Wireless Networks

Current research community has put main efforts on the study of node mobility in terms of individual user behaviors. In order to enhance the existing works on individual node mobility, we carefully studied individual user moving behaviors, fundamental human mobility and its impacts on MANETs in our first three doctoral research topics. However, in reality, mobile users are often involved in team activities in which they execute a common task in a MANET. As a result, there are many demanding applications of MANETs where mobile nodes are organized into groups to coordinate their movements. Examples include military tasks, disaster recovery operations. Thus, to achieve the desired application performance upon group moving behaviors in MANETs, it is critical to understand the fundamental properties of group mobility, or in more general scopes, group moving behaviors and group structures. However, less work has been down in this research field, and there are limitations of existing metrics to characterize the group properties. Therefore, we are motivated to analyze the inherent properties of group mobility in our last doctoral work.

In this chapter, we first motivate this work and review the existing solutions and corresponding limitations on group mobility study in Chapter 5.1. Then, we study a fundamental group mobility characteristic metric, named user pair correlation, which characterizes the correlation degree of between a user pair in Chapter 5.2. By exploring the user pair correlation metric, we study
the characteristics of group structure in in Chapter 5.3. Then, we further investigate the properties of
group evolution behaviors and measure the corresponding group stability in Chapter 5.4. In Chap-
ter 5.5, we apply the metrics studied in this work to develop a novel birth-to-death group mobility
model. Finally, we summary this study in Chapter 5.6.

5.1 Introduction

Existing studies demonstrated that group mobility can improve the routing efficiency and
network performance [58, 14, 24]. For instance, in [14], it showed that the reference point group
mobility (RPGM) model [58] with both intergroup and intragroup communicates has the lowest
average hop count compared with random mobility models such as random waypoint (RWP) model,
which in turn, leads to a high data packet delivery ratio. As mobile users have strong correlation
with moving speed inside a group, the study in [125] showed that both link and path duration last
longer in RPGM model than RWP model. As a result, RPGM models achieves a higher throughput
and less routing cost than RWP model. In addition, recently study showed that group mobility can
also improve the packet delivery ratio than random node mobility in DTNs [25].

Group mobility is in fact a result of many real applications in mobile wireless networks.
For instance, a group of soldiers execute a military mission in a battlefield and a group of firefighters
cooperate disaster recovery operations in a MANET [58, 61, 59, 65]. Thus, to achieve the desired
application performance upon group moving behaviors in MANETs, it is critical to understand the
fundamental properties of group mobility. The study of group mobility or in more general scopes,
group moving behaviors and group structures, is not a brand new topic. First, to characterize group
moving behaviors, existing studies generally use the correlation degree between group members to
characterize their relative node movements. Specifically, the correlation degree is either quantified
by the similarity of node moving speed [59] or the physical distance [58]. Both of node speed
similarity and distance similarity partially reflect the correlation degree of mobile users, since in re-
ality group mobile users move more closely and at a similar speed and direction than mobile nodes
outside the group. However, the common limitation of these two metrics is that given the instan-
taneous physical locations or moving speed information of mobile nodes, it is difficult to discern
the future node’s group movement patterns. In addition, the authors suggested that group mobility
based on the distance similarity, such as RPGM model is insufficient to predict network partition-
ing because of independent group movements [61]. This implies that group mobility characterized
by different user correlation metrics may exhibit different moving behaviors in the network, which can further lead to different routing performance. Second, to characterize group structures, existing studies have proposed several centrality metrics. In [62], the authors use the reference point of the geographical center of the group for selecting group head, which maintains the intragroup routing and guides the entire group node movement. Clearly, due to the random node mobility of group members, the geographical center of the group can vary frequently, which cannot effectively characterize the group structure. Another more widely used centrality metric in group structure study is called betweenness [126, 25]. Betweenness centrality measures the number of times a node falls on the shortest path between two other nodes, it can represent the importance of node as a potential traffic relay for other nodes in the system [25]. The first limitation of betweenness metric is that it requires the global routing information for calculating the shortest paths between node pairs in the network. Second, the betweenness did not consider the correlation degree on each link between group members. As different routing protocols may select different shortest paths of a node pair according to the link stability level instead of number of hops [127, 36], the betweenness value may provide biased information for measuring the routing importance of a node in a group. Therefore, though existing studies provided contributing solutions for characterizing the group structure and group moving behaviors, the inherent properties of group mobility in MANETs still remain elusive, which motivate our study in this work.

In fact, the distinct group moving behaviors from random individual mobile nodes, it is the strong correlation degree between mobile users. If the correlation degree among mobile users is high, the link stability regarding the contact time among mobile users is also high, which results in a stable group structure. Then, what are the main factors affecting the correlation degree of a user pair? Previous studies have demonstrated that both similarities of mobile speed and distance can partially reflect the correlation degree between group members. We suggest that the human social correlation is another and even more important factor, as the human social relationship may vary much more slowly than either moving speed and physical distance between mobile users [128, 129]. In addition, the human social networks can map to MANETs as mobile devices are generally attached to humans in MANETs. Thus, each group in MANETs can be considered as a local human social community. And the group moving behaviors are in fact characterized by the human social moving behaviors. For instance, a group students walk together to a same classroom, due to the same social duty in a campus. Accordingly, the social based group mobility study has attracted lots of attention during recent years [130, 126, 25]. However, there is lack of a mechanism to quantify
the social correlation of mobile users in MANETs. Interestingly, it has been recently found that in mobile social networks, humans inherently come together at a popular location site and form social groups [131, 129]. Enlightened by this observation, as the first contribution in this work, we applied information theory to qualitatively measure the social correlation strength between user pairs upon their location preference in MANETs. Then, we investigated the user pair correlation between mobile nodes by jointly taking social correlation strength, similarity of nodes’ movement and physical distance into consideration.

By applying the analytical result of user pair correlation, we further investigate the properties of group structures. In social networks, there are two key metrics to characterize the complex human social structures in the network. One is node clustering coefficient, the other is betweenness [132, 133]. Specifically, the metric of node clustering coefficient indicates the density of link connections among a node’s neighbors, which is also called the local group cohesiveness in social networks [134]. An interesting phenomenon has been found in social networks is that the node clustering coefficient appears to be far greater than in equal size random networks based on stochastic models [135, 126]. The authors suggest that this is strictly related to the fact that humans usually organize themselves into communities. Correspondingly, the betweenness indicates that who is the most influential people in the network are, as he/her control the flow of information between most others. Thus, the people with highest betweenness in social networks also result in the largest increase in typical distance between other people. Since there is a close resemblance between human social communities in social networks and human based groups in MANETs, then, can we directly apply these two social network oriented metrics to study the properties of group structure. Unfortunately, we found both metrics have limitations. First, both metrics are topological metrics which do not consider the effect of correlation degree on each link within a group. Intuitively, the clustering properties of a node with strong correlated neighbors contribute more on maintaining a stable group structure than that with weak correlated neighbors. The weak correlation between group users can imply a weak social correlation degree; highly different mobile speed; or large physical distance. In either case, the chance that a disconnection of a weak correlated link is much higher than that of a strong correlated link. Hence, in this study we develop a new metric, named node connectness, which combines the topological information with the correlation degree of links to characterize the clustering property of a group. As a result, we demonstrated that node clustering coefficient metric indeed overestimates the stability of inter-connections within a group.

Regarding the betweenness metric, as we mentioned earlier, it requires the global routing
information to obtain the number of shortest paths. In addition, the frequent changes of network topology may frequently affect the shortest paths selection in MANETs. Even for counting the shortest paths inside a group, without considering the correlation degree, the path selection could be totally different according different routing metrics such as minimum hop count and link stability level [127, 36]. Then, instead of applying betweenness metric, what metrics can be used to characterize the impact of each node on the group structure and routing paths of a group? In this work, we demonstrate that the combination of node degree and average node neighbors’ degree can effectively evaluate the routing function role of each node inside a group. It has been observed in social networks that humans try to attach to people who are already well connected with other people to expand their social connections [129]. As a result, some node in the social network topology has a very high node degree. In addition, it has been also found in many social networks, well connected nodes often connect between each other, which dramatically reduces the average shortest path length between two nodes in a social network. This is the fundamental reason of “small world” phenomenon in social networks [117, 133]. Thus, we suggest that a node with both high node degree and high average neighbors’ node degree in a group will potentially reduce the overall shortest path between nodes inside a group. And this node will have more impact on traffic flow and routing maintenance than other nodes of a group. If this node detaches from the group, both the group structure and routing paths can be dramatically changed. The recent study in [25] provide a validation on our suggestion, they show that if mobile nodes forward the packets to the their neighbors with the highest node degree each hop, it has the similar packet delivery ratio as forwarding to the node with highest betweenness at each hop. However, the betweenness of a node in the study is off-line calculated, which increases the difficulty for routing design and implementation in real applications.

Furthermore, we observed that in reality, different groups may gather together for cooperating certain goals [24], such that the inter-group connection is important for routing and data exchange between two neighboring groups. If all the inter-group links are disconnected, the group is isolated from other groups in the network, and the application services based on intergroup communications are disrupted [61, 63, 136]. Therefore, understanding the property of inter-group links is also important for group mobility based applications in MANETs. In this work we focus on the following question: how to specify the correlation strength of inter-group links? In social networks, it has been suggested that the social strength of a tie between two nodes increases with the overlap of their friendship circles [137]. Motivated by the study in social networks, we developed a new metric
called *inter-group edge clustering coefficient* to measure the correlation strength of an inter-group link, which is further characterized by the correlation strength between these two gateway nodes with their common neighbors in each group. The intuition behind this metric is that the more common neighboring nodes of two gateway nodes reside in the overlapped area between two groups, the more potential connections between two groups, which overall increase the correlation strength between two groups.

Another interesting phenomena of human group movements in a MANET is that the group size (the total number of group members) can vary often times. This process is called group evolution process [129, 132]. For instance, students can switch between different groups for taking different classes in campus, doctors can frequently switch between different rescue teams for treat wounded people during disaster rescue operations. The frequent node switch events inside a group may dramatically increase routing cost inside each group. For example, when the group structure is greatly changed due to the node leaving and join event, it may trigger the group (clustering) reconstruct event to maintain routing information among mobile nodes inside the group, while it has been demonstrated that the frequent re-clustering computational overhead can outweigh the clustering-based routing benefit for routing efficiency in MANETs [61, 24]. Therefore, understanding and characterizing the group evolution behaviors is critical for routing design and performance evaluation in MANETs. However, far less work has been done in this field. Thus, in the later part of this study, we are motivated to investigate the group evolution behaviors in MANETs. In particular, we investigate the condition for node switching between neighboring groups. We suggest that a node can switch to the other group, when it has stronger correlation with its neighbors in the new group. Finally, by applying the studied metrics for characterizing group moving behaviors and group structure dynamics, we propose a novel birth-to-death group mobility for both analytical and simulation studies in MANETs.

### 5.2 User Pair Correlation

In this section, we study a fundamental group mobility characteristic metric, named *user pair correlation*, which characterizes the correlation degree of between a user pair. The correlation degree indicates the link stability regarding the contact time among mobile users. The correlation degree directly characterizes the similarity of user movements inside the group. Thus, it is an essential metric applied for investigating the inherent properties of group moving behaviors and
group structures. However, there is still lack of a mechanism to qualitatively and quantitatively specify the correlation degree of a user pair in MANETs. We notice that a user pair correlation can be affected by three factors regarding the similarity of the member nodes’ movements [59, 63], the proximity in physical displacement between mobile node [58], and more important, the social correlation between mobile users[126]. Then, we first qualitatively measure the social correlation of mobile users in a network by applying relative entropy upon the user location profiles in the network. Then, we study the similarity of user movements based on the information of physical locations and relative speed between mobile users. Finally, we integrate the social correlation factor with the geographical location factor and moving speed factor of mobile users to quantify their correlation by the developed user pair correlation metric.

5.2.1 System Model and Definitions

It has been observed that the human social behaviors heavily influence the human mobility in a MANET [126, 114]. Thus, a mobile user moves to different task locations according to his/her social duties. For instance, a student goes to a library in campus first, and then goes to a shopping mall in sequence. Figure 5.1 illustrates an example of human moving pattern driven by his/her time-varying social duties in a network.

Based on the human moving pattern demonstrated in Figure 5.1, when a mobile user executes a task, he/she moves only within the corresponding task location site. Specifically, let us consider a MANET region is composed of many task locations. We use the Voronoi decomposition algorithm [97] to delimit the task location regions in a network with two-dimensional plane. We assume that there is a set of task locations: $S = \{S_1, S_2, S_3, ..., S_S\}$, which are randomly located in the network. Then by applying the Voronoi decomposition algorithm, the network topology is split by $S$ convex polygon Voronoi cells. As a result, each Voronoi cell represents a task location area. In particular, the Voronoi cell associated with each task location site $s$ covers all points closer to $s$.
than to any other site. Figure 5.2 illustrates an example of total 100 task locations (Voronoi cells) in a square network with side length of 5000 m. Upon the study in social networks, humans generally execute the same type of task when they are present at the same task location. As demonstrated in Figure 5.2, a mobile user executes tasks in location $L_i$, $L_T$ and $L_k$ in sequence. After finishing the social task at location $L_i$, the mobile user moves to task location $L_T$ along a straight line. Though the task location is abstracted into a point inside the Voronoi cell as shown in the Figure 5.2, each Voronoi cell is in fact a location

Figure 5.2: Example of 100 task location sites in a network delimited by the Voronoi decomposition.

**Task Location Preference Set**

In existing studies, it is commonly assumed that mobile users are allowed to move inside a network all the time, and each mobile user can visit any task locations in the network [58, 14].

However, in real life, as observed in [24, 138], the time that mobile users spend on each task location area vary dramatically. On reason is that the social interest and the social task regularities on each task location are different for mobile users. Specifically, for a network containing total $M$ task locations, we let $\mathcal{M}$ denote as the corresponding task location set, i.e., $\mathcal{M} = \{l_1, l_2, \ldots, l_M\}$. Because the social task duties and regularities vary remarkably, the sojourn time at each task location for humans is also different. For example, a person may visit and be present at office location site for 8 hours per day, and visit shopping mall once per week. Thus, let $p(l_i)$ denote the steady state probability that a mobile user stays at location $l_i$ in the network. $p(l_i)$ is proportion to the user sojourn time fraction of task location $l_i$ in the location set $\mathcal{M}$. Under this condition, we have $p(l_i) = \frac{E(T_i)}{\sum_{j=1}^{M} \frac{E(T_j)}}$, where $E(T_i)$ is the expected sojourn time for the node in location $l_i$. In general, the value of $p(l_i)$ is affected by the task execution time and the frequency of
task operations in the network. For simple representation, let the set \( \{ p_1, p_2, \ldots, p_M \} \) denote the location preference probability set, in which the elements are permuted in a non-decreasing order. Thus, \( p_M \) is the probability of user presence in his/her most preferred task location, and \( p_1 \) is the probability of his/her least preferred task location.

**Definition 3.** Given a network with \( M \) locations, \( LP_x \) is defined as the location preference set of user \( x \). The element in set \( LP_x \) is the task location index, which is always listed in a monotonically increasing order. For instance, let \( LP_x = \{ x_1, x_2, \ldots, x_M \} \), then \( p(x_i) \leq p(x_j) \), for \( i < j \).

In [126], the authors suggested that people with strong social links are likely to be geographically colocated often or from time to time. Then given the location preference profile of each mobile user inside a MANET, next, we specify the social correlation between mobile users.

### 5.2.2 Social Correlation between Mobile Users

It has been widely observed that mobile users within the same group typically have strong social tie among each other [117, 24, 126, 25]. For example, a group students taking the same class are very likely in the same department and therein exhibit a strong social correlation between each other. The soldiers in a group, moving at a battle field, are very likely in the same company. Hence, human social correlation plays essential role on characterizing the group correlation between mobile users. In [126], the authors provides a community based mobility model, which collects mobile nodes into groups according to the social relationship among individuals. However, the work is lack of a specific mechanism to specify the social correlation of mobile users. Interestingly, in [117], the authors observed that people tend to build up correlation when they play similar social roles at the same task location. Moreover, the results of social networks [139, 132] respectively show that collaboration events between the same mobile user pairs can be repeated often times, and a higher frequency of collaboration acts usually indicates closer social relationship. Especially, in MANETs, due to the limited transmission range, the social correlation between a pair of mobile users is affected by their geographical locations. Basically, when the distance between two mobile users are less than the transmission range, they have a link connection, so that their direct social interaction occurs. Therefore, we consider the task location profile corresponds to the location preference set of a mobile user. According to \( LP_x \) in Definition 3, the location preference probability set of user \( x \) is \( \{ p(x_1), p(x_2), \ldots, p(x_M) \} = \{ p_1, p_2, \ldots, p_M \} \). Thus, the location \( x_1 \) is user \( x \)'s lowest location preference, and \( x_M \) is the highest location preference in the network. Similarly, for the task
location preference set \( LP_y \) of user \( y \), his/her task location preference probability set is denoted as \( \{q_1, q_2, \cdots, q_M\} \), where \( q_i \leq q_j \), for \( i < j \). Note that, the element in set \( LP_x \) and \( LP_y \) are same as the element in set \( \mathbb{M} \), but the permutation of location index is different, which is based on per user task location preference. The intuition here is that as human mobility in a network heavily depends on his/her social duties, mobile users have similar location preference can indicate a similar social behaviors, which further indicate a strong social correlation.

From information theory, we know that relative entropy measures the “distance” (difference) between a pair of probability mass function [140]. A smaller difference means a high similarity of these two functions. Hence, we apply the relative entropy theory to analyze the difference of the user location preference set between mobile users in a MANET. Specifically, the relative entropy \( D(p||q) \) between two probability mass functions \( p \) and \( q \) is defined as [140],

\[
D(p||q) = \sum_{x_i \in \mathcal{X}} p(x_i) \log \frac{p(x_i)}{q(x_i)}.
\] (5.1)

From (5.1), it is evident that the relative entropy is always non-negative.

**Definition 4.** Given (5.1), we define the relative entropy of location presence probability between users \( x \) and \( y \) as \( R(x, y) \). That is, \( R(x, y) = D(p(LP_x)||q(LP_y)) \).

Regarding Definitions 3 and 4, the following property of \( R(x, y) \) of location presence probability holds.

**Lemma 4.** Given a user pair \( (x, y) \) in the network containing \( \mathcal{M} \) task locations, and \( \{p(x_1), p(x_2), \cdots, p(x_M)\} = \{p_1, p_2, \cdots, p_M\} \), then \( R(x, y) \) has the minimum value \( R_{\text{min}} \), when \( \{q(x_1), q(x_2), \cdots, q(x_M)\} = \{q_1, q_2, \cdots, q_M\} \), i.e., \( LP_x = LP_y \). Especially, \( \min R(x, y) = 0 \), when \( p(x_i) = q(x_i) \).

In contrast, \( R(x, y) \) has the maximum value \( R_{\text{max}} \), when \( \{q(x_M), q(x_{M-1}), \cdots, q(x_2), q(x_1)\} = \{q_1, q_2, \cdots, q_M\} \). That is, the location index in the set \( LP_x \) is exactly listed in the reverse order in the set \( LP_y \).

**Proof.** Since the network consists of total \( \mathcal{M} \) locations, there are \( \mathcal{M}! \) possible permutations in the location preference set \( LP \) for each mobile user, but a unique location preference probability set. Thus, for user \( y \), among these \( \mathcal{M}! \) permutations, \( LP_y \) could be either \( LP_y = \{x_1, x_2, \cdots, x_M\} \) or \( LP_y = \{x_M, x_{M-1}, \cdots, x_2, x_1\} \). Accordingly, user \( y \)'s location preference probability set could be represented as \( \overline{q} = \{q(x_1), q(x_2), \cdots, q(x_M)\} \), or \( \overline{q} = \{q(x_M), \cdots, q(x_2), q(x_1)\} \), respectively. Note that, regardless of the permutations of location sequence in \( LP \), for user \( y \), his/her location
preference probability set is unique, i.e., \( \overline{\mathbf{q}} = \{q_1, q_2, \cdots, q_M\} = \mathbf{q} \), and \( q_i \leq q_j \) when \( i < j \). In particular, in set \( \overline{\mathbf{q}} \), the value of \( q(x_i) \) monotonically increases as \( i \) increases. In contrast, in set \( \mathbf{q} \), the value of \( q(x_i) \) monotonically decreases as \( i \) increases. With the above preliminary explanation, we first show that \( R(x, y) = R_{\min} \), when \( LP_y = \overline{LP_y} \).

Assume by way of contradiction that there is a permutation \( LP'_y \) in the location preference set, such that \( LP_y' \neq \overline{LP_y} \), and \( R(x, y) = D(p(LP_x)||q(LP'_y)) = R_{\min} \). Then there exists at least a pair of locations \((x_i, x_j)\) in \( LP'_y \), where \( i < j \) and \( q_i < q_j \), such that \( LP'_y = \{x_1, \cdots, x_j, \cdots, x_i, \cdots, x_M\} \). In this case, we can obtain another location preference set \( LP''_y \) by just switching the position of \( x_i \) and \( x_j \) in \( LP'_y \), that is, \( LP''_y = \{x_1, \cdots, x_i, \cdots, x_j, \cdots, x_M\} \), and we have \( R(x, y) = D(p(LP_x)||q(LP''_y)) \). Upon (5.1),

\[
D(p(LP_x)||q(LP'_y)) - D(p(LP_x)||q(LP''_y)) = (p(x_j) - p(x_i)) \log \frac{q_j}{q_i} = (p_j - p_i) \log \frac{q_j}{q_i} > 0. \tag{5.2}
\]

Note that, in (5.2), since \( i < j \), then \( p(x_j) > p(x_i) \) for user \( x \), meanwhile \( q_j > q_i \) for user \( y \). Hence, we have \( D(p(LP_x)||q(LP'_y)) \) is less than \( D(p(LP_x)||q(LP''_y)) \), which is impossible, since \( D(p(LP_x)||q(LP'_y)) = R_{\min} \). Therefore, we show that \( R(x, y) \) has the minimum value \( R_{\min} \) when \( LP_y = \overline{LP_y} \) and \( \overline{\mathbf{q}} = \{q(x_1), q(x_2), \cdots, q(x_M)\} \). Based on this conclusion, upon (5.1), it is straightforward to see that when \( p(x_i) = q(x_i) \), \( R_{\min} = D(p(LP_x)||q(LP''_y)) = 0 \).

Following the same methodology, we assume that there is a location preference set for user \( y \), \( \overline{LP}_y \neq \overline{LP}_y \), and \( R(x, y) = D(p(LP_x)||q(LP_y)) = R_{\max} \). Since \( LP_y \neq \overline{LP}_y \), there exists at least a pair of locations \((x_i, x_j)\) in \( LP_y \), where \( i < j \), such that \( \overline{LP}_y = \{x_M, \cdots, x_i, \cdots, x_j, \cdots, x_1\} \) and \( \{q(x_M), \cdots, q(x_i), \cdots, q(x_j), \cdots, q(x_1)\} = \{q_1, q_2, \cdots, q_M\} \).

Then, we can always obtain another location preference set \( LP''_y \) by just switching the position of \( x_i \) and \( x_j \) in \( LP_y \), that is, \( LP''_y = \{x_M, \cdots, x_j, \cdots, x_i, \cdots, x_1\} \). Similar to the derivation of (5.2), we have

\[
D(p(LP_x)||q(LP''_y)) - D(p(LP_x)||q(LP_y)) = (p(x_j) - p(x_i)) \log \frac{q_j}{q_i} = (p_j - p_i) \log \frac{q_j}{q_i} \geq 0. \tag{5.3}
\]
The result from (5.3) is contrary to our assumption that \( D(p(LP_x)||q(LP_y)) = R_{\text{max}} \).
As \( LP_y \) is selected arbitrarily from \( \mathcal{M}! \) permutations in the location preference set \( LP \) for user \( y \) except \( LP_y \), we showed that \( R_{\text{max}} = D(p(LP_x)||q(LP_y)) \).

**Remark 12.** LEMMA 4 indicates that the better match the location preference profiles between two mobile users, i.e., the smaller \( R(x,y) \), hence the stronger the social correlation they may have. In other words, the relative entropy measure \( R(x,y) \) introduced in Definition 4 is an effective metric to evaluate the human social correlations in mobile wireless networks.

**Case Study**

In order to validate our proof and results, we demonstrate a case study of calculating \( R(x,y) \) among four mobile users in a network. Specifically, Table 5.1 illustrates four user task location profiles in a given network which includes total 8 locations. The corresponding task location set is \( \mathcal{M} = \{A,B,C,D,E,F,G,H\} \). We see that node \( x \) and node \( u \) have a very similar location preference in the network. Specifically, the preferred first four locations of node \( x \) and node \( u \) are \( \{H,G,F,E\} \) and \( \{H,G,E,F\} \), respectively. In addition, the last preferred two locations \( \{B,A\} \) of node \( x \) and node \( u \) are also same. It means that the locations where node \( x \) stays with most of time and least of time in the network is almost same as node \( u \). It indicates that node pair \((x,u)\) has a similar social behaviors in the network. Also, since they can frequently meet each other in a same location area, the chance they can collaborate for a social task in that location is also high. In either way, node \( x \) and node \( u \) can have a strong social correlation. For node pair \((u,z)\), they also exhibit a close similarity on their preferred first three locations \( \{H,G,E\} \), which suggests that node pair \((u,z)\) can also have a strong social correlation. But they have a distinct difference on the preference of location \( A \). Because node pair \((u,z)\) has a more similar location preference than node pair \((u,z)\), we expect that node pair \((x,u)\) has a stronger social correlation than node pair \((u,z)\). In contrast, we notice that node \( y \) has an exactly reverse order of location preference with node \( x \). That is, the locations where node \( x \) stays with most of time is the locations where node \( y \) stays with least of time and verse visa. Thus, node \( x \) and node \( y \) exhibit distinct social behaviors in the network, therein, we expect they may have a very weak social correlation or even no social correlation between either other.

Upon empirical human moving traces [13, 114], recent studies have found that humans
Table 5.1: Example of User Task Location Profiles

<table>
<thead>
<tr>
<th>Location Preference ↑</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sojourn Time Weight $w_i$</td>
<td>$2^1$</td>
<td>$2^2$</td>
<td>$2^4$</td>
<td>$2^4$</td>
<td>$2^6$</td>
<td>$2^6$</td>
<td>$2^8$</td>
<td>$2^8$</td>
</tr>
<tr>
<td>$LP_x$ of user $x$</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
</tr>
<tr>
<td>$LP_y$ of user $y$</td>
<td>H</td>
<td>G</td>
<td>F</td>
<td>E</td>
<td>D</td>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>$LP_z$ of user $z$</td>
<td>B</td>
<td>D</td>
<td>C</td>
<td>F</td>
<td>A</td>
<td>E</td>
<td>H</td>
<td>G</td>
</tr>
<tr>
<td>$LP_u$ of user $u$</td>
<td>A</td>
<td>B</td>
<td>D</td>
<td>C</td>
<td>F</td>
<td>E</td>
<td>G</td>
<td>H</td>
</tr>
</tbody>
</table>

are expected to return certain locations, such as home and office, upon their diurnal cycle patterns. Because of the inherent regularities of human social behaviors, mobile users visit and stay at different locations with different regularities. Recall that $p(l_i)$ denotes the steady state probability that a mobile user stays at location $l_i$ in the network, and $p(l_i)$ is proportion to the user sojourn time fraction of task location $l_i$ in the location set $M$. Thus, $p(l_i) = \frac{E(T_i)}{\sum_{j=1}^{M} E(T_j)}$. As an example, we set $w_i = f(i) = 2^i$ as the sojourn time weight coefficient of the $i$-th preferred location in the location preference set. As shown in Table 5.1, $w_i$ is a monotonically increasing function of the preference order $i$ in the location preference set. Under this condition, the probability $p_i$ that a mobile user staying at his/her $i$-th location is derived as

$$p_i = \frac{E(T_i)}{\sum_{j=1}^{M} E(T_j)} = \frac{w_i}{\sum_{m=1}^{M} w_m} = \frac{2^i}{\sum_{j=1}^{M} 2^i} = \frac{2^i}{2^{M+1} - 2^i}.$$ \hspace{1cm} (5.4)

Given (5.4), we can directly obtain the location preference probability set for each user.

For instance, according to user $x$, his/her location preference probability set is $\{p(A), p(B), \cdots, p(H)\}$ $= \{p_1, p_2, \cdots, p_8\} = \{0.0039, 0.0078, 0.0157, 0.0314, 0.0627, 0.1255, 0.2510, 0.5020\}$. Corresponding to user $x$, $LP_y = \overline{LP_y} = \{H, G, F, E, D, C, B, A\}$. By combining (5.1) and (5.4), from Lemma 12, we have $R_{\text{max}} = D(p(LP_x)||q(\overline{LP_y})) = 5.0627$. Through the same means, we can obtain the relative entropy measure between each user pair, respectively. Table 5.2 illustrates the result of relative entropy between these four pairs of mobile users. For instance, $R(x, z) = D(p(LP_x)||q(LP_z)) = 1.6353$, $R(z, x) = D(p(LP_z)||q(LP_x)) = 0.9922$, $R(x, u) = D(p(LP_x)||q(LP_u)) = 0.0824$.

In this example, mobile users have the same location preference probability set $\{p_i, 1 \leq i \leq M\}$, but distinguished location preference set $LP$. Under this condition, by applying Lemma 4, the maximum relative entropy $R_{\text{max}}$ of location presence probability functions between any user pair is a unique value. According to Lemma 12, in this example, $R_{\text{max}} = R(x, y) =$
Table 5.2: Example of Relative Entropy between Mobile Users

<table>
<thead>
<tr>
<th>Relative Entropy</th>
<th>user x</th>
<th>user y</th>
<th>user z</th>
<th>user u</th>
</tr>
</thead>
<tbody>
<tr>
<td>user x</td>
<td>0</td>
<td>5.0627</td>
<td>1.6353</td>
<td>0.0824</td>
</tr>
<tr>
<td>user y</td>
<td>5.0627</td>
<td>0</td>
<td>4.5686</td>
<td>4.7333</td>
</tr>
<tr>
<td>user z</td>
<td>0.9922</td>
<td>4.0706</td>
<td>0</td>
<td>0.8157</td>
</tr>
<tr>
<td>user u</td>
<td>0.0824</td>
<td>4.984</td>
<td>1.5373</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ D(p(LP_x)||q(LP_y)) = 5.0627. \]

**Remark 13.** In the example, because LP_u is the closest match to LP_x, while LP_y is the most different one, we have \( R(x, u) \ll R(x, z) \ll R(x, y) = R_{\text{max}} \). The results shown in Table 5.2 validate our expectation discussed in the beginning of this section, the node pair \((x, u)\) exhibits the strongest social correlation strength among all user pairs. Node pair \((u, z)\) also exhibits a strong social correlation strength as a close match between LP_u and LP_z. As \( R_{\text{max}} = R(x, y) \), node pair \((x, y)\) may have a very weak social correlation or even no social correlation.

**Remark 14.** Note that, from Table 5.2, we see that \( R(x, z) \neq R(z, x) \), which manifests the non-symmetric property of relative entropy measure [140]. In addition, the value of location preference probability \( p_i \) will decreases with the increase of total number of locations in the network, which in turn, increases the range of relative measure \( R(x, y) \). As a result, the more locations a network contains, the larger range of \( R(x, y) \) can have.

In contrast, as specified in human social networks [117], the range of human social correlation coefficient is over \([0, 1]\), which does not vary with the community location and network size. And the typical social tie (strength) between a pair of humans is symmetric [117], since the human mutual interactions can be viewed as being symmetric [126]. To solve this two issues, we propose a human social correlation metric \( SC(x, y) \), which is a function of \( R(x, y) \), as follows

\[
SC(x, y) = SC(y, x) = 1 - \frac{R(x, y) + R(y, x)}{2R_{\text{max}}}. \tag{5.5}
\]

Given (5.5), it is clear to see that the human social correlation metric \( SC(x, y) \) is symmetric to any user pair, and the range of \( SC(x, y) \) for any user pair in a network is mapped into \([0, 1]\). These two properties comply with the specifications the social correlation metric defined in [117].
In detail, \( SC(x, y) = 1 \) represents the strongest social correlation of a user pair could be in a given network when \( R(x, y) = R(y, x) = 0 \). This is the case that two mobile users have an exactly same location preference file. This further implies that they may play same social roles and take very similar social duties in the network. By applying (5.5) in the previous example, Table 5.3 illustrates the estimated social correlation degree between each user pair. Compared to Table 5.2, we have the same result that node pair \((x, u)\) has the strongest social correlation, and node pair \((u, z)\) also has strong social correlation, while node pair \((x, y)\) has no social correlation.

<table>
<thead>
<tr>
<th>Social Correlation</th>
<th>user x</th>
<th>user y</th>
<th>user z</th>
<th>user u</th>
</tr>
</thead>
<tbody>
<tr>
<td>user x</td>
<td>1</td>
<td>0</td>
<td>0.7405</td>
<td>0.9837</td>
</tr>
<tr>
<td>user y</td>
<td>0</td>
<td>1</td>
<td>0.1468</td>
<td>0.0407</td>
</tr>
<tr>
<td>user z</td>
<td>0.7405</td>
<td>0.1468</td>
<td>1</td>
<td>0.7676</td>
</tr>
<tr>
<td>user u</td>
<td>0.9837</td>
<td>0.0407</td>
<td>0.7676</td>
<td>1</td>
</tr>
</tbody>
</table>

By far, we have developed the social correlation metric for quantifying the social correlation degree between a user pair in a MANET. We consider each group in a MANET as a local human social community, and the group moving behaviors can be characterized by the human social moving behaviors. For instance, a group students walk together to a same classroom, due to the same social duty in a campus. As a result, the social correlation can be used to specify the correlation degree between a pair of mobile users. Next, we investigated the user pair correlation between mobile nodes by jointly taking social correlation strength, similarity of nodes’ movement and physical distance into consideration.

### 5.2.3 User Pair Correlation Metric

Compared to social correlation metric, existing studies of group mobility modeling characterize the correlation degree of mobile users regarding their relative movements. The relative movements between a node-pair can provide information in two aspects: the temporal information according to relative speed, and the spatial information according to distance between two mobile users. It has been suggested in [61] that two mobile users with similar speed, i.e., small relative speed, tends to have strong temporal correlation. This temporal correlation is also simulated by mobility models such as Gauss-Markov mobility model [141]. Accordingly, the authors suggest...
that in reality group users usually move more closely than others, hence they use distance between mobile users to characterize the correlation degree between a node pair [58]. However, there is no specific metric to quantify the similarity coefficient of a node pair by jointly considering the temporal-spatial factors regarding their relative speed and distance. Fortunately, with the similarity measure defined in statistical analysis [142], we are able to quantify the similarity coefficient of two mobile users with their relative movement information. In [142], the statistical distance between two 2-dimensional observations for a user pair \((x, y)\) is

\[
d(x, y) = \sqrt{w_1 \cdot (x_1 - y_1)^2 + w_2 \cdot (x_2 - y_2)^2}, \tag{5.6}
\]

where each user has two attributes, for instance, \(x' = [x_1, x_2]\) and \(y' = [y_1, y_2]\). \(w_1\) and \(w_2\) are the weight coefficients for each attribute of the user, by default, \(w_1 = w_2 = 1\). Therefore, by simply replacing the two attributes of users \(x\) and \(y\) with relative speed \(\Delta v\) and distance \(\Delta d\), then we have

\[
d(x, y) = \sqrt{w_1 \cdot (\Delta v)^2 + w_2 \cdot (\Delta d)^2}. \tag{5.7}
\]

From (5.7), \(d(x, y)\) indicates the closeness (similarity) between two mobile users regarding their relative speed and distance. \(d(x, y)\) increases when either the relative speed \(\Delta v\) or distance \(\Delta d\) increases. Clearly, \(d(x, y)\) approaches to 0, when \(\Delta v\) and \(\Delta d\) decrease to 0. This is the case that two very adjacent mobile users move at the same speed, which indicates a very high similarity between them. Furthermore, in [142], the authors showed that it is always possible to construct similarities of a user pair given their statistical distance. Specifically, we define the similarity metric \(\tilde{S}_{x,y}\) of a user pair in MAENTs as,

\[
\tilde{S}_{x,y} = \frac{1}{1 + d(x, y)}, \tag{5.8}
\]

where \(0 \leq \tilde{S}_{x,y} \leq 1\) is the similarity coefficient between mobile user \(x\) and \(y\) according to the statistical distance metric \(d(x, y)\). Recall that the user is characterized by both their social correlation degree \(S(x, y)\) and similarity \(\tilde{S}_{x,y}\). Therefore, by combining (5.5) and (5.8), we provide a new metric called user pair correlation \(\rho_{x,y}\) to capture the correlation degree between two mobile users.

**Definition 5.** The user pair correlation \(\rho_{x,y}\) between a user pair \((x, y)\) is given by

\[
\rho_{x,y} = (1 - \lambda)S(x, y) + \lambda\tilde{S}_{x,y}, \tag{5.9}
\]
where $\lambda$ is the weight coefficient of the similarity metric $\tilde{S}_{x,y}$.

From (5.9), Figure 5.3 illustrates an example of user pair correlation according to the weight coefficient $\lambda$. In this example, the social correlation $SC(x, y)$ of the user pair is 0.95, and it would not change in the studied network. In contrast, the similarity metric $\tilde{S}_{x,y}$ varies remarkably with time according to the relative movement between the user pair. As shown in the figure, when $\lambda = 0.1$, from (5.9), the social correlation $SC(x, y)$ dominates the user pair correlation, and therein leads to a comparably stable value with simulation time. In the second case, when $\lambda = 0.9$, the user pair correlation is mainly characterized by their similarity metric. Since the relative distance and speed between the user pair can vary dramatically with time, as shown in the figure, the user pair correlation also varies remarkably with time. And the value is much less than their social correlation by taking the similarity coefficient into account.

Remark 15. Given (5.9), the weight coefficient $\lambda$ characterizes the importance order between social correlation and similarity for quantifying the correlation degree between a user pair. Clearly, if $\lambda = 1$, then only the similarity of user pair regarding their relative speed and distance is considered for measuring their correlation degree. In contrast, if $\lambda = 0$, then the social correlation degree is equivalent to the correlation degree of a user pair.

The major difference between group mobility and individual node mobility in MANETs is that mobile users inside the group have a much higher correlation degree than mobiles nodes outside the group. Thus, for group mobility modeling, it generally suggests a threshold $\rho_{th}$ as the required correlation threshold for mobile nodes to join a group [39]. Thus, a node pair $(x, y)$ can form a group, if $\rho_{x,y} > \rho_{th}$. According to this way, we demonstrate an example that how a group...
can be formed according to user pair correlation among mobile users. For convenience, we continue
to follow the example for estimating the social correlation between four mobile users in Table 5.3,
by adding the information of similarity metric $\tilde{S}_{x,y}$ of a user pair. In this example, the value of $\tilde{S}_{x,y}$
is randomly assigned for each pair, and the weight coefficient $\lambda = 0.5$ from (5.9). Accordingly,
Table 5.4 illustrates the relationship of each user pair according to user pair correlation.

<table>
<thead>
<tr>
<th>User Pair</th>
<th>$SC(x, y)$</th>
<th>$S_{x,y}$</th>
<th>$\lambda$</th>
<th>$\rho_{x,y}$</th>
<th>$(&lt;, =, &gt;)$</th>
<th>$\rho_{th}$</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x, y)$</td>
<td>0</td>
<td>0.8</td>
<td>0.5</td>
<td>0.4</td>
<td>$&lt;$</td>
<td>0.5</td>
<td>non-group members</td>
</tr>
<tr>
<td>$(x, z)$</td>
<td>0.7405</td>
<td>0.33</td>
<td>0.5</td>
<td>0.5353</td>
<td>$&gt;$</td>
<td>0.5</td>
<td>group members</td>
</tr>
<tr>
<td>$(x, u)$</td>
<td>0.9837</td>
<td>0.601</td>
<td>0.5</td>
<td>0.7968</td>
<td>$&gt;$</td>
<td>0.5</td>
<td>group members</td>
</tr>
<tr>
<td>$(y, z)$</td>
<td>0.1468</td>
<td>0.5</td>
<td>0.5</td>
<td>0.3234</td>
<td>$&lt;$</td>
<td>0.5</td>
<td>non-group members</td>
</tr>
<tr>
<td>$(y, u)$</td>
<td>0.0407</td>
<td>0.6</td>
<td>0.5</td>
<td>0.3204</td>
<td>$&lt;$</td>
<td>0.5</td>
<td>non-group members</td>
</tr>
<tr>
<td>$(z, u)$</td>
<td>0.7676</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6338</td>
<td>$&gt;$</td>
<td>0.5</td>
<td>group members</td>
</tr>
</tbody>
</table>

From Table 5.4, we can see that the user pairs $(x, z)$, $(x, u)$ and $(z, u)$ have high user pair
correlation which is larger than a predefined correlation threshold, i.e., $\rho_{th} = 0.5$, for forming a
group. Though, user $y$ exhibits a high similarity with other three mobile users regarding $\tilde{S}_{x,y}$, since
user $y$ has a very weak social correlation with these three neighbors. It implies that user $y$ may
move away from other three users in a short time, as user $y$ has different social activity intentions
in locations of the network from the other three users. As a result shown in Table 5.4, user pairs
$(x, u, z)$ form a group. For instance, user $x$ considers user $u$ and user $z$ as his/her group members, as
they have strong social correlation between each other meanwhile a high similarity of their relative
movements.

By far, we have developed a user pair correlation metric, which integrates three factors:
the social correlation, the similarity of mobile speed, and the proximity in physical displacement
between a user pair. From the above example, we demonstrate that how mobile users form a group
according to their user pair correlations. Once a group is formed, the user pair correlation
indicates the correlation strength between each member pair in the group, which in turn, reflects the
stability and reliability of intra-connections inside the group. Therefore, by exploring this user pair
correlation metric, we study the characteristics of group structure in the next section.
5.3 Characteristics of Group Structure

A large number of existing studies of group mobility has demonstrated the importance of group structure [58, 39, 62, 24, 126, 25] for routing design and network performance in MANETs. For instance, in [58, 39], the studies demonstrated that a stable group structure can extend the link connection duration between mobile users, and improve the routing performance. In delay tolerant networks (DTNs), [126, 25] respectively suggested that the stable group structure leads to a better packet delivery ratio, especially when the mobile speed is high. In [62], the authors demonstrated that well structured mobility models increase the performance of reactive routing protocols than the unstructured mobility models in MANETs. In addition, the authors in [24] suggested that unstable group structures can significantly induce extra routing cost for maintaining communication within and between groups and therein degrade the network performance. These studies showed that understanding the inherent group structures is essential to optimizing service planning and efficient routing in mobile wireless networks [61, 65]. Therefore, we study the characteristics of group structure in this section.

5.3.1 Node Connectness

The property of group structure is based on the status of inter-connections among mobile nodes inside the group. To have a clear view on a group structure, the general way is to investigate link connections of the group from its corresponding graph topology. Thus, we first introduce the graph theory terms [143] used to study the properties of group structures in this work.

Definition 6. A group $G$ consists of group member set $V(G)$, which has the associated link connection set $E(G)$. Let $h_{u,w}(G)$ or simply $h_{u,w}$ denote the least hop length (number of hops) of a $u, w -$path. Thus, $h_{u,w} = 1$ if node $u$ is a neighbor of node $w$. Let $N_i$ denote the node $i$’s neighbor set and $\Delta_i$ be the degree of node $i$. Hence, we have $N_i = \{v_j|e_{i,j} \in E\}$ and $\Delta_i = |N_i|$, where $e_{i,j} \in E$ is equivalent to $h_{i,j} = 1$.

In social networks, graph theory [143] has been widely applied for characterizing the structures of a human social society. Especially, the metric node clustering coefficient is a key metric to characterize the density of link connections among a node’s neighbors [135, 128, 133], which is also called the local group cohesiveness in social networks [134]. An interesting phenomenon has been found in social networks is that the node clustering coefficient appears to be far greater than
in equal size random networks based on stochastic models \cite{135, 126}. The authors suggest that this is strictly related to the fact that humans usually organize themselves into communities. Since each group in MANETs can be considered as a local human community, then, it seems the metric of node clustering coefficient can be also applied to characterize the property of group structure in MANETs. Hence, in this section, we focus on the following questions: 1) Whether node clustering coefficient can effectively characterize the group structure in MANETs? 2) If not, what is a good metric for group structure study?

To tackle these two questions, we first have a close look on node clustering coefficient metric. The node (vertex) clustering coefficient is originate from graph theory to measure the local link density between the node and its neighbors. Specifically, in \cite{143}, the clustering coefficient $C_i$ of a vertex $i$ ($v_i$) in a graph quantifies how close the vertex $i$ and its neighbors are to form a clique, the highest link density of a graph. The clustering coefficient of node $i$ ($v_i$) is defined as

$$C_i = \frac{2|e_{j,k}|}{\Delta_i(\Delta_i - 1)}, \forall j, \forall k \in N_i, e_{j,k} \in E.$$

From (5.10), $0 \leq C_i \leq 1$. And a high clustering coefficient of a vertex $i$ implies a well interconnected neighborhood of the vertex $i$. Specifically, in (5.10), as node $j$ ($v_j$) and node $k$ ($v_k$) are neighbors of node $i$, the numerator $2|e_{j,k}|$ is determined by the number of edges between neighbors of node $i$. Thus, when the edge $e_{j,k}$ is present, there exists a corresponding triangle edges $(i, j, k)$ in the graph, which contributes the local inter-connections at node $i$’s neighborhood. Therefore, the more number of triangles incident to node $i$ exists, the higher the clustering coefficient is, and therein, a higher chance that node $i$ and its neighbors to form a clique.

As a example shown in Figure 5.4(a), node $x$ has 5 neighbors \{u, b, c, d, z\}. And there are total 6 edges between neighbors of node $i$: \{e_{b,c}, e_{b,d}, e_{b,z}, e_{c,d}, e_{c,z}, e_{d,z}\}. In this case, from (5.10) the node clustering coefficient of node $x$ is $C_x = \frac{2 \times 6}{5(5-1)} = 0.6$. By the same way, we can easily derived that $C_b = C_c = C_d = C_z = 1$ and $C_u = 0$. Thus, from node $b$’s point of view, all its neighbors mutually connected each other, which results in a complete inter-connections (clique) in node $b$’s neighborhood. In contrast, since node $x$ is the only neighbor of node $u$, its node clustering coefficient is 0. As shown in this example, the majority of nodes (b, c, d, z) of the group have a complete inter-connections among their neighbors, which suggest the group members are closely connected each other. As a result, the entire group has a very high group cohesiveness. Group cohesiveness is a term defined in social network to describe the strength bringing group members closer together \cite{117}. 

\[C_i = \frac{2|e_{j,k}|}{\Delta_i(\Delta_i - 1)}, \forall j, \forall k \in N_i, e_{j,k} \in E.\]
However, there is a limitation for the metric of node clustering coefficient, as it is a pure topological metric which does not consider the effect of correlation degree on each link within a group. Intuitively, the clustering properties of a node with strong correlated neighbors contribute more on maintaining a stable group structure than that with weak correlated neighbors. The weak correlation between group users can imply a weak social correlation degree; highly different mobile speed; or large physical distance. In either case, the chance that a disconnection of a weak correlated link is much higher than that of a strong correlated link. In [134], the authors studied the scientific collaboration network and the world-wide air-transportation network, by assigning a weight of each edge of the network graph proportional to the intensity or capacity of the connections among the various elements of the network. Interestingly, they found that the effect of weighted graph on the underlying topological structure of the network shows a distinct difference than that evaluated by a unweighted graph. In particular, they found that the clustering has a minor effect in the organization of the air-transportation network when the largest part of interactions (air traffic) in occurring on edges not belonging to the inter-connected triples. For example, when routing and data traffic go through more often along edges $e_{u,x}$, $e_{x,z}$, $e_{x,d}$ and $e_{z,d}$ in Figure 5.4(a), respectively, as these links have more strong correlation (link stability) than other links inside the group. Thus, we observed that the user pair correlation on each link within the group in MANETs is also different, which in fact leads to a weighted cluster. Therefore, to correctly evaluate the property of a group structure, the user pair correlation on each link is necessary to be considered.
According to the study [134] in social networks, the authors found that the triangle user pairs with strong social interactions have more impact on determining the structure of social organizations in a weighted complex network. Similarly, a mobile wireless network is also a weighted complex network when the user pair correlation is assigned into each edge of a group (cluster) in the network. Therefore, the weight of user pair correlation between different neighboring pairs must be considered to accurately estimate the clustering property. It worth noting that in graph theory [143] the clustering coefficient of an undirected graph \( G \) in (5.10) can also be obtained from the ratio of the number of triangles to the number of triples embed in subgraphs of \( G \). In particular, each triangle represents a three-interconnected vertexes and each trip represents a subgraph of 2 edges within 3 vertexes. Hence, by taking the contribution of user pair correlation of each link into the consideration, we define a metric called node connectness to measure the weighted node clustering coefficient of node \( i \), which is represented as

\[
C^w_i = \frac{2 \cdot \sum_{j,k \in N_i} \frac{\rho_{i,j} + \rho_{i,k} + \rho_{j,k}}{3} \cdot 1\{h_{j,k}=1\}}{\Delta_i(\Delta_i - 1)}, \tag{5.11}
\]

where \( 1\{\cdot\} \) is the indicator function. Distinguished from (5.10), the weight contribution for each node \( i \)'s neighbor in the triangle including node \( i \) is specifically calculated in (5.11). Consistently, when the weights of all edges are the same, which is equivalent to a non-weighted network, we have \( C^w_i = C_i \), where \( 0 \leq C^w_i \leq 1 \).

To explain this discrepancy, we demonstrate two cases for calculating a weighted and unweighted node clustering coefficient in Figure 5.4. Specifically from Definition 5, let the required correlation threshold of group mobility \( \rho_{th} = 0.1 \). And mobile nodes apply (5.9) to calculate the user pair correlation with each neighbor. By comparing the relation between user pair correlation of each link with \( \rho_{th} = 0.1 \), a group is formed. As shown in Figure 5.4, there are total six nodes in a group, and there are three levels of user pair correlation \( \rho = \{0.2, 0.5, 0.8\} \) between group members.

In Case 1, as we have already demonstrated, the unweighted clustering coefficient of node \( x \) is \( C_x = 0.6 \), which shows a comparably well interconnected neighborhood of node \( x \). It suggests that most neighbors of node \( x \) have a link connection between each other. However, we see that the weight (user pair correlation) of edges forming the interconnected triangle vertexes \( (x, b, c) \) are smaller than those forming the interconnected triangle vertexes \( (x, d, z) \). Specifically, the user pair correlation of each link in triangle \( (x, b, c) \) is \( \rho_{x,b} = \rho_{b,c} = \rho_{x,c} = 0.2 \), while the user pair
correlation of each link in triangle \((x, d, z)\) is \(\rho_{x,d} = \rho_{d,z} = \rho_{x,z} = 0.5\). Thus, node \(x\) has a higher correlation strength with its neighboring pair \((x, d)\) and \((x, z)\) than the neighboring pairs \((x, b)\), and \((x, c)\). In other words, the stability of link connections in triangle \((x, d, z)\) is higher than that in triangle \((x, b, c)\). As a result, the triangle \((x, d, z)\) induces more influence in determining the clustering property of node \(x\) than the triangle \((a, b, c)\). Even more, we see that among all neighbors of node \(x\), though node \(u\) has the strongest correlation with node \(x\), it has no direct connections with other nodes, which therein has less contribution on clustering property of node \(x\). Hence, by applying (5.11), the weighted on node \(x\), we have

\[
C^w_x = \frac{2}{3\Delta_x(\Delta_x - 1)} \cdot \left[ (\rho_{x,b} + \rho_{b,c} + \rho_{x,c}) + (\rho_{x,b} + \rho_{b,d} + \rho_{x,d}) + (\rho_{x,b} + \rho_{b,z} + \rho_{x,z}) \\
+ (\rho_{x,c} + \rho_{c,z} + \rho_{x,z}) + (\rho_{x,c} + \rho_{c,d} + \rho_{x,d}) + (\rho_{x,d} + \rho_{d,z} + \rho_{x,z}) \right] = 0.19. \tag{5.12}
\]

From (5.12), as we expected, \(C^w_x = 0.19 < C_x = 0.6\). This is because, there are several low weighted inter-connected edges in the neighborhood of node \(x\). Hence, it leads to a weak clustering coefficient, which shows the conflict observation if without considering the user user pair correlation effect, where \(C_x = 0.6\).

Furthermore, by comparing unweighted clustering coefficient of node \(x\) between Figure 5.4(a) and Figure 5.4(b), \(C_x = 0.5\) in Figure 5.4(b), which is less than \(C_x = 0.6\) in Figure 5.4(a). This is because, the number of inter-connections between node \(x\)’s neighbor in Figure 5.4(a) is higher than that in Figure 5.4(b). However, by applying (5.11) to calculate the node connectness metric of node \(x\), in Figure 5.4(b), we have \(C^w_x = 0.4\), which is larger than \(C^w_x = 0.19\) in Figure 5.4(a).

**Remark 16.** The node clustering coefficient only focuses on the density of inter-connections in the neighborhood of the node. Without considering the correlation strength of user pairs inside the group, the metric of node clustering coefficient will overestimate the stability of inter-connections of the group structure. Therefore, to correctly evaluate the property of a group structure, the user pair correlation on each link is necessary to be considered, which is based on the node connectness metric. By comparing the value of node connectness of node \(x\) in both Figure 5.4(a) and Figure 5.4(b), we see that user pair correlation can outweigh the number of inter-connections on determining the node clustering property inside the group. This is true in reality, since mobile users with high correlation may have more stable link connection between each other, which contribute the stability of the group structure. In contrast, even mobile users have more inter-connections but with
low correlation strengths, these links are more easily broken, which contributes the dynamics of the group structure variation, i.e., a reverse effect on stability of the group structure.

For a more detailed demonstration, Table 5.5 and Table 5.6 illustrate the weighted and unweighted node clustering coefficients for all nodes based on the group structure topology in Figure 5.4(a) and Figure 5.4(b), respectively. As we can see, without considering the user pair correlation effect on the edge in the group topology, the value of node connectness (unweighted clustering coefficient) of each node in Figure 5.4(b) is no larger than the corresponding one in Figure 5.4(a). As the number of inter-connections in Figure 5.4(a) is larger than Figure 5.4(b). However, we also notice that in the later case, the user pair correlation is much stronger in edges, which indicates more stable link connections between members in the group, and in turn, a stable local structures in the neighborhood of each node. Hence, the value of $C_i$ obtained from (5.10) can provide misleading information for evaluating the local group coherence, and the corresponding property of group structure. In contrast, as we expected, the weighted node clustering coefficient $C_i^w$ from (5.11) can effectively indicates the stability level of inter-connections of each group member with his/her neighbors in the group.

Table 5.5: Example of weighted and unweighted node clustering coefficients from Figure 5.4(a)

<table>
<thead>
<tr>
<th>Mobile Nodes:</th>
<th>$x$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$z$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$ from (5.10)</td>
<td>0.6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$C_i^w$ from (5.11)</td>
<td>0.19</td>
<td>0.25</td>
<td>0.25</td>
<td>0.3167</td>
<td>0.3167</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.6: Example of weighted and unweighted node clustering coefficients from Figure 5.4(b)

<table>
<thead>
<tr>
<th>Mobile Nodes:</th>
<th>$x$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$z$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$ from (5.10)</td>
<td>0.5</td>
<td>1</td>
<td>0.833</td>
<td>1</td>
<td>0.833</td>
<td>0</td>
</tr>
<tr>
<td>$C_i^w$ from (5.11)</td>
<td>0.4</td>
<td>0.8</td>
<td>0.667</td>
<td>0.8</td>
<td>0.667</td>
<td>0</td>
</tr>
</tbody>
</table>

Remark 17. By comparing (5.10) with (5.11), $C_i^w$ is no larger than $C_i$. Upon Table 5.6, in Case 2, the value of clustering coefficient $C_b = 1$ and node connectness $C_b^w = 0.8$, respectively. In contrast, from Table 5.5, in Case 1, the value of clustering coefficient $C_b = 1$ and node connectness $C_b^w = 0.25$, respectively. Compared between Figure 5.4(a) and Figure 5.4(b), if the value of $C_i$ and
\( C_{iw} \) is close, it indicates that inter-connections between node \( i \) and its neighbors are more likely formed by the edges with stronger user pair correlation, otherwise, they are formed by the edges with weaker user pair correlation. The larger difference between \( C_i \) and \( C_{iw} \), the weaker the user pair correlation in these edges, which leads to a potential unstable local group structures. For instance, in Case 1, \( C_b = 1 \) and \( C_{wb} = 0.2 \) as most of inter-connections among node \( b \)’s neighbors have a weak correlation, where \( \rho = 0.2 \).

**Remark 18.** In Figure 5.4(a), node \( u \) has only one neighbor, as a result, both \( C_u \) and \( C_{uw} \) is 0. Hence, node \( u \) may easily be detached from the group than other mobile nodes, especially when the correlation between node \( u \) and node \( x \) is weak, as shown in Case 2. The disconnection between node pair \((u, x)\) will result in both group structure and group size variation. In contrast, if the node clustering \( C_i \) is high, such as \( C_b \) in Figure 5.4(b), the higher \( C_{iw} \) is, more stable local group structure of node \( i \) becomes, which contribute the stability of the group structure.

To this end, we demonstrate that the developed metric node connectness, which is considered as a weighted clustering coefficient, can effectively characterize the stability level of inter-connections of each node with all its neighbors. With the knowledge of the weighted clustering coefficient for each node in a group, we can easily use the average weighted cluster coefficient to express the statistical coherence level of entire group, which is defined as group coherence \( C_g \). The group coherence metric characterizes the average stability level of inter-connections among all nodes in the group, which is represented as

\[
C_g = \frac{1}{V(G)} \sum_{i \in V(G)} C_{iw}. \tag{5.13}
\]

According to the results in Table 5.5 and Table 5.6, we can easily obtain the group coherence of two groups shown in Figure 5.4(a) and Figure 5.4(b), respectively.

\[
C_{g1} = \frac{1}{6} \times (0.19 + 0.25 + 0.25 + 0.3167 + 0.3167 + 0) = 0.2206, \tag{5.14}
\]

and

\[
C_{g2} = \frac{1}{6} \times (0.4 + 0.8 + 0.667 + 0.8 + 0.667 + 0) = 0.5556. \tag{5.15}
\]

Similar to the property of \( C_{iw} \), the high value of \( C_g \) implies two aspects of the inherent group structure in MANETs. One is the stability level of link connections regarding the user pair correlation between mobile nodes, another is the wellness of inter-connections among all group
members inside the group. By comparing (5.14) with (5.15), it is clear to see that the group coherence $C_{g_2}$ in Figure 5.4(b) is much higher than $C_{g_1}$ in Figure 5.4(a). Though the total number of inter-connections inside group of Case 2 is less than that in Case 1, because the overall user pair correlation of mobile users in Figure 5.4(b) is much larger than that in Figure 5.4(b), we have a much higher group coherence in Case 2 compared with Case 1. Hence, it again indicates that for characterizing the group structures, the user pair correlation can outweigh the number of inter-connections on determining the structure property of the group.

Interestingly, we observed that $C_u^w = 0$ for both cases in Figure 5.4, as node $u$ only has one neighbor. As a result, the single-connected group member reduces the group coherence dramatically. Especially, for the example in Figure 5.4(b), the user pair correlation $\rho_{x,u} = 0.2$, this weak correlation can increase the chance of detach of node $u$ from the group. Hence, the node degree of mobile nodes also affect the property of a group structure. Upon (5.13), we expect that a well balanced node degree of mobile nodes in a group can increase the group coherence. However, one limitation of node connectness metric and the resulting group coherence metric is that they cannot directly indicate the node degree impact on a group structure. Therefore, we are motivated to investigate a new metric, which can characterize the node degree effect on group structure in the next section.

5.3.2 Effect of Node Degree

In social networks, besides the metric of clustering coefficient, the metric of betweenness is widely applied to study the social network structure [135, 128, 133]. Specifically, the betweenness metric indicates that who is the most influential people in the network are, as he/her control the flow of information between most others. Thus, the people with highest betweenness in social networks also result in the largest increase in typical distance between other people. However, there are limitations for directly applying the betweenness metric to study the structure of human based groups in MANETs. First, the betweenness metric requires the global routing information to obtain the number of shortest paths, which may not available for mobile nodes inside a group. In addition, different from social network topology, because of node mobility, the topology of MANETs can frequently change, which may lead to frequent variations of shortest paths of mobile nodes in MANETs. Even for counting the shortest paths inside a group, without considering the correlation degree, the path selection could be totally different according different routing metrics such as minimum hop count and link stability level [127, 36]. Then, instead of applying betweenness metric,
what metrics can be used to characterize the impact of each node on the group structure and routing paths of a group? To address this question, in what follows, we elaborate that the combination of node degree and average node neighbors’ degree can effectively evaluate the impact of each node on the group structure and routing selection.

Given observations in social networks, for quickly expanding social connections, humans try to attach to people who are already well connected with other people [129]. Similar to the social networks, the node degree distribution of mobile nodes in a group is also important in MANETs. In particular, according to the applications of packet replication and forwarding in DTNs, it has been found the node degree of group head and group gateway of each group significantly impacts on the packet delivery ratio [126, 25]. In addition, for fast information dissemination, aggregation and exchange in a network, the node degree of all neighbors of a node is also an important factor, especially when the total number of group members is large. Another phenomenon has been observed in social networks is that a well connected person tends to connect other well-connected persons in the network for quickly increasing the social connections, which leads to a power law distribution of node degree in the social networks [139]. This is the fundamental reason of “small world” phenomenon in social networks. Hence, the nodes’ degree associate with each mobile node is also an important metric, which can reflect the information of a group structure.

For studies on social network structure, the metric of average node neighbors’ degree has been recently applied for estimating the property of network structure [134, 139]. Specifically, for a vertex $i$ with degree $\Delta_i$ and neighbor set $N_i$, the average node neighbors’ degree is

$$\Delta_{nn,i} = \frac{1}{\Delta_i} \sum_{j \in N_i} \Delta_j.$$ (5.16)

For instance, as shown in Figure 5.4(a), the node $x$ has total 5 neighbors. Its neighbors’ degrees are: $\Delta_u = 1, \Delta_b = \Delta_c = \Delta_d = \Delta_z = 4$. Thus, from (5.16), the average node neighbors’ degree of node $x$ is $\Delta_{nn,x} = \frac{1}{5}(1 + 4 + 4 + 4 + 4) = 3.4$. By comparing the degree correlation between $\Delta_i$ and $\Delta_{nn,i}$, people can tell whether a high degree vertex tends to be connected to other high degree vertexes or vice verse [139]. For instance, in Figure 5.4(a), $\Delta_b = 4$ and $\Delta_{nn,b} = 4.25$. This result implies that one of more neighbors of user b’s has a larger node degree than user b, which is the case for user $x$, where $\Delta_x = 5$. In this example, we see that $\Delta_b \sim \Delta_{nn,b}$. Clearly, there are another two cases can be observed in a group structure: 1) $\Delta_i \gg \Delta_{nn,i}$ and $\Delta_i < \Delta_{nn,i}$. For the first case, when $\Delta_i \gg \Delta_{nn,i}$, it indicates that most of node $i$’s neighbors have a small number
of connections with other group members. Hence, these node $i$’s neighbors may locate on the edge area of a group. Because these nodes do not have many directly connections with other group members, the impact of node $i$ on its neighbors regarding the link connection and data exchange is significant. For the second case, $\Delta_i \ll \Delta_{nn,i}$, it implies that that some of node $i$’s neighbors have a larger number of connections with other group members, and these neighbors tends to locate the center area of the group. A recent study in [25] demonstrated when each node forwarding the packets to its neighbors with the maximum node degree, it will increase the packet delivery ratio compared to forwarding packets to a random neighbor. Thus, it suggests the node with high node degree can potential have more impact on routing exchange and packet forwarding inside the group. An intuitive explanation is that mobile nodes with high node degree have more chance to fall into shortest paths than nodes with less node degree.

Furthermore, in social networks, two properties regarding node degree and the impact on network structures have been studied in [128], which are assortative mixing behavior and disassortative mixing behavior. Specifically, for node $j$, if it has a large node degree, meanwhile $\Delta_{nn,j}$ is also very large, then it indicates that high degree nodes tend to be connected with other high degree nodes inside the network, which is called assortative mixing behavior between nodes inside the network. In contrast, if $\Delta_{nn,j}$ is small, this contrary trend shows a disassortative mixing behavior between nodes in the network. A directly result of assortative mixing behavior in the network is that the average of shortest path will be reduced. While in disassortative mixing behavior, the node $j$ has the most impact on its neighbors for expanding social (routing) connections. Therefore, we see that when the node degree increases, its impact on group structure also increase. According to its average node neighbors’ degree, it may either exhibit an assortative mixing behavior or disassortative mixing behavior.

However, one limitation of the metric $\Delta_{nn,i}$ is that it does not consider the effect of correlation degree between node $i$ with its neighbors. As we discussed in the previous section, if the user pair correlation $\rho_{i,j}$ between a node pair $(i, j)$ is small, then the link $e_{i,j}$ may be easily disconnected often times. As a result, node $i$ may also lose the connection with node $j$’s neighbors. In this case, we expect that the influence of node $i$ on node $j$’s neighbors is small. By applying the similar rule of (5.11) here, we define the average weighted neighbors’ degree $\Delta_{nn,i}^w$ as

$$\Delta_{nn,i}^w = \frac{1}{\Delta_i} \sum_{j \in N_i} \rho_{i,j} \cdot \Delta_j. \quad (5.17)$$

In (5.17), $\rho_{i,j}$ is the edge weight (user pair correlation) between node $i$ and its neighbor $j$, ...
which indicates the influence level of node $i$ on node $j$’s neighbors. Consistently, when each edge weight is 1, which is a non-weighted group, we have $\Delta_{nn,i} = \Delta^w_{nn,i}$. As a case study, Table 5.7 and Table 5.8 illustrate the weighted and unweighted average node neighbors’ degree for all nodes based on the group structure topology in Figure 5.4(a) and Figure 5.4(b), respectively.

Table 5.7: Example of weighted and unweighted average node neighbors’ degree from Figure 5.4(a)

<table>
<thead>
<tr>
<th>Mobile Nodes:</th>
<th>$x$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$z$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{nn,i}$ from (5.16)</td>
<td>3.4</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>4.25</td>
<td>5</td>
</tr>
<tr>
<td>$\Delta^w_{nn,i}$ from (5.17)</td>
<td>1.28</td>
<td>0.85</td>
<td>0.85</td>
<td>1.525</td>
<td>1.525</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5.8: Example of weighted and unweighted average node neighbors’ degree from Figure 5.4(b)

<table>
<thead>
<tr>
<th>Mobile Nodes:</th>
<th>$x$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$z$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{nn,i}$ from (5.16)</td>
<td>3</td>
<td>4.33</td>
<td>3.75</td>
<td>4.33</td>
<td>3.75</td>
<td>5</td>
</tr>
<tr>
<td>$\Delta^w_{nn,i}$ from (5.17)</td>
<td>2.4</td>
<td>3.47</td>
<td>3</td>
<td>3.47</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

As we can see from Table 5.7 and Table 5.8, $\Delta_{nn,x} = 3.4$ in Figure 5.4(a), which is larger than $\Delta_{nn,x} = 3$ in Figure 5.4(b). However, by looking the metric of $\Delta^w_{nn,i}$, the conflict result is obtained, where $\Delta^w_{nn,x} = 1.28$ in Figure 5.4(a), while $\Delta^w_{nn,x} = 2.4$ in Figure 5.4(b). This due to the different contribution of user pair correlation between node $x$ with its neighbors. By this means, the proposed metric average weighted neighbors’ degree $\Delta^w_{nn,i}$ can characterize the effect of correlation on each link. The reason that $\Delta^w_{nn,x} = 2.4$ in Figure 5.4(b) is larger than $\Delta^w_{nn,x} = 1.28$ in Figure 5.4(a), is because in Case 2, node $x$ has more strong correlation, therein, potential longer link connection time, with its neighbors, such that the impact of node $x$ on its neighbors will be larger than the first case. This user pair correlation effect can be observed evidently by comparing $\Delta^w_{nn,b}$ between Case 1 and Case 2. As node $b$ has a weak correlation with all its neighbors in Case 1, the impact of node $b$ on its neighbors will be dramatically increased.

**Remark 19.** By comparing (5.16) with (5.17), $\Delta^w_{nn,i}$ is no larger than $\Delta_{nn,i}$. If the value of $\Delta_{nn,i}$ and $\Delta^w_{nn,i}$ is close, it indicates that inter-connections between node $i$ and its neighbors are more likely formed by the edges with stronger user pair correlation. This is the case for mobile nodes in Figure 5.4(b) and Table 5.8. In particular, when the node degree $i$ is large and has strong correlation with its neighbors, then if $\Delta^w_{nn,i}$ is also high, node $i$ exhibits an assortative mixing behavior with its neighbors; otherwise if $\Delta^w_{nn,i}$ is small, node $i$ exhibits an disassortative mixing behavior with its neighbors;
In this section, we demonstrate that the metric $\Delta_{\text{nn},i}^{w}$ can reflect the impact of the neighbors’s node degree associate with each node in a group. In particular, by using the proposed metric $\Delta_{\text{nn},i}^{w}$, we can evaluate the relationship regarding associative mixing or disassociate mixing behavior between mobile users [128] in the group. Therefore, we suggest that the metric of average weighted neighbors’ degree can effectively provide the complementary information to the node connectness metric for specifying the group structure.

5.3.3 Group Head Selection

In previous section, we analyzed that mobile nodes can have different impacts on group structures, which may further impact on routing and data exchange among group members inside a group [63, 24, 25]. In particular, within each group, it often selects a node as a group head regarding its significant impact on routing and group moving behaviors. Basically, there are two main functions of a group head. First, the group head maintains the internal routing information of the group and exchanges hierarchical routing information between neighboring groups, which is the key point for the routing scalability of cluster-based MANETs. In addition, group head and group gateway of each group normally form a virtual backbone for inter-group routing in MANETs [24]. With the help of group head, the routing cost can be dramatically reduced. Second, for existing group mobility modelings in MANETs, it often requires a leader per group for collaborating group member moving behaviors [58, 62, 65, 144]. These phenomena are also often observed in real group mobility scenarios. For instance, a group of visitors follows a guider’s moving track in a museum; or a group of soldiers follows a commander’s order to execute a military mission in a battlefield. As have been shown in [58, 62, 25], the group head can significantly affect the routing performance in MANETs. It has been found that if the group head frequently changes inside a group, the routing performance regarding throughput and packet delay and extra routing cost can dramatically degraded [24]. Hence, properly selecting the group head among all nodes inside a group is an important issue in group mobility based study. Therefore, in this section, we aim to investigate the fundamental requirements for a group head selection.

In existing group mobility models, there are three general ways for group head selection. The first way, which is the simplest way, is to predefine a group head or randomly select a node during the initial stage of simulation [58, 63]. As the group head is completely randomly selected, there is no convinced support for this strategy. The second solution is to select the node lying the geographical center of the group as the group head [62], which suggests that the location of
the group head can effectively capture the spatial and geographical features of the group structure. However, with the following example, we demonstrate the limitations of the second solution.

![Figure 5.5: Example of specifying group head based on physical locations of 11 users.](image)

Figure 5.5 illustrates a simple example of physical group structure. In this example, node C becomes the centering node, i.e., group head, upon its centrality geographical location of the group. As shown in this example, being the group head, node C reach all the group nodes within 2 hops. However, distinguished from the fixed cluster structure in a static graph, the group structure may change remarkably with time due to the dynamic node mobility in MANETs. Specifically, in this example, by considering the node connectness metric, node B may have stronger correlation with its neighbors than node C. Especially, if the edging node D or node E leaves the group due to a weak user pair correlation with node C, node B could have more significant contribution on group coherence and stability than node C. In this case, node B can outperform node C as the group head. Thus, the geographical information alone cannot accurately identify the group head.

The third solution in existing mobility models is to use the centrality metric to measure the importance of a node in a network, for instance, the betweenness centrality measures the number of times a node falls on the shortest path between two other nodes. Thus, the group head is considered as the node with the maximum betweenness inside a group [126, 25]. This strategy has the merit as the selected node has the most importance on the routing maintenance in the group, which is main function expected for the group head. However, it also has limitations as we mentioned before. First, as the network topology in MANETs frequently changes, which leads to frequent changes of end-to-end paths. As a result, the number of betweenness centrality for each node may vary dramatically with time. Second, and more important, the betweenness centrality metric does not consider the user pair correlation effect on links among group users. As we have demonstrated in this section, for
the metrics of $C_i^w$ and $\Delta_{nn,i}^w$, without considering the user pair correlation influence between group members, the counter-intuitive findings can happen. Then, what are the requirements for the group head selection?

To answer this question, we suggested three requirements upon the observations and analytical results we obtained in this work.

- First, the node degree of the group head should be high in a group. In a human-based community (group), it has been found that people tends to attach a well-connected person [117], which makes the well-connected person as the communication center to other people. Accordingly, we expected a group head should also be well-connected in the group for maintaining routing information and data exchange.

- Second, the average weighted neighbors’ degree $\Delta_{nn,i}^w$ should be high for the group head, for the purpose of efficient packet forwarding [126, 25], data replications [65] and routing and topology control [24, 67]. This requirement can be reflected by the metric of average weighted neighbors’ degree.

- Third, the node connectness of the group head should be high. Clearly, in order to effectively maintain a stable group structure, it is expected that the stability level of inter-connections in the neighborhood of the group head should be high. And this requirement can be reflected by the metric of node connectness.

In summary, to achieve these three requirements, compared with other group members, the group head is expected to have the combinations: high node degree, high node clustering coefficient, and high average node neighbors’ degree. Based on the above analysis, we integrate the analytical results derived in (5.11) and (5.17) for selecting the group head $G_{\text{head}}$.

\[
G_{\text{head}} = \{ i | \Delta_j \cdot C_i^w \cdot \Delta_{nn,j}^w \geq \Delta_j \cdot C_j^w \cdot \Delta_{nn,j}^w, \forall j \in V(G) \}. \tag{5.18}
\]

Following the same examples shown in Figure 5.4(a) and Figure 5.4(b), Table 5.9 and Table 5.10 illustrate the results based on the group head selection metric in (5.18) for each mobile node. We see that in the first example, node $d$ and node $z$ have the maximum value according to (5.18). Correspondingly, node $b$ and node $d$ have the maximum value in the second example. Hence, these two nodes are the candidates of the group head in each group. Note that, for simple demonstration purpose, the user pair correlation values are same for many links in Figure 5.4, and
these value will be different when estimated in real cases. Hence, we expected the group head selection rule according to (5.18) will select a unique group head for each group.

Table 5.9: Example of Group Head Selection from Figure 5.4(a)

<table>
<thead>
<tr>
<th>Mobile Nodes:</th>
<th>$x$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$z$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_i$</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$C_i^w$ from (5.11)</td>
<td>0.19</td>
<td>0.25</td>
<td>0.25</td>
<td>0.3167</td>
<td>0.3167</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta_{nn,i}^w$ from (5.17)</td>
<td>1.28</td>
<td>0.85</td>
<td>0.85</td>
<td>1.525</td>
<td>1.525</td>
<td>4</td>
</tr>
<tr>
<td>$G_H$ Candidate from (5.18)</td>
<td>1.216</td>
<td>0.85</td>
<td>0.85</td>
<td>1.9319</td>
<td>1.9319</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.10: Example of Group Head Selection from Figure 5.4(b)

<table>
<thead>
<tr>
<th>Mobile Nodes:</th>
<th>$x$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$z$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_i$</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$C_i^w$ from (5.11)</td>
<td>0.4</td>
<td>0.8</td>
<td>0.667</td>
<td>0.8</td>
<td>0.667</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta_{nn,i}^w$ from (5.17)</td>
<td>2.4</td>
<td>3.47</td>
<td>3</td>
<td>3.47</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$G_{head}$ Candidate from (5.18)</td>
<td>4.8</td>
<td>8.328</td>
<td>8.004</td>
<td>8.328</td>
<td>8.004</td>
<td>0</td>
</tr>
</tbody>
</table>

5.3.4 Effect of Inter-group Edge

In previous section, we discussed three requirements for group head selection. In reality, many demanding mobile wireless applications require a collaboration between groups. Thus, besides the group head, the group gateway is also an important group member in terms of its functional roles for exchanging routing and data information with a neighboring group. Being a group gateway, the node has at least one inter-group link, which can access the neighboring group. Though the node pair attached to an inter-group link belongs to two different groups, it is possible that they also have a correlation between each other. For example, in a disaster rescue operation, large number of people can be associated with multiple teams such as fire fighters team, medial rescue team and policemen team. Hence, these people can cooperate among teams for collaborated rescue operations. In this case, there are close collaborations between neighboring groups, which communicate between each other via the inter-group links. However, if these inter-group links are unstable and often disconnected, then the network (group) partition events will frequently occur, which will dramatically degrade the network performance [61, 136]. Hence, the link stability of inter-group links...
is important for routing and application performance in a MANET, which performs a hierarchical routing among groups [24]. Therefore, in this study, we estimate the link stability according to the user pair correlation. Then, an interesting question is: how to specify the correlation strength of an inter-group link between two groups? In what follows, we tackle this issue.

Recently, the authors in [137] studied the network structure of mobile communication networks based on the communication traces of millions of mobile phone users. They found two interesting results given the traces. First, the studied mobile communication network consists of small local clusters. In particular, the majority of the strong ties are found within the clusters, indicating that users spend most of their on-air time talking to members of their immediate circle of friends. In contrast, most links connecting different communities are visibly weaker than the links within the communities. These weaker correlated links are in fact similar to the inter-group links we studied in MANETs. Second, the authors state that the strength of a tie between two nodes increases with the overlap of their friendship circles. Specifically, the authors measured the relative topological overlap of the neighborhood of a user pair \((i, j)\),

\[
O_{ij} = \frac{n_{ij}}{(\Delta_i - 1) + (\Delta_j - 1) - n_{ij}},
\]

(5.19)

where \(O_{ij}\) represents the proportion of their common friends, and \(n_{ij}\) is the number of common neighbors between node pair \((i, j)\). From (5.19), when \(n_{ij}\) approaches to 0, \(O_{ij}\) approaches to 0 accordingly. This is the case that the link between node \(i\) and node \(j\) bridges between two different communities, such that node \(i\)’s friends and node \(j\)’s friends are in different communities. According to empirical phone call traces, the authors found that the strength tie of the link between node pair \((i, j)\) increases as their common neighbors \(n_{ij}\) increases.

Motivated by this study [137], we aim to evaluate the correlation strength of an inter-group link by jointly considering two effects: 1) the number of common neighbors with the node pair associated with the inter-group link and 2) the user pair correlation between these neighbors with the node pair. As two examples shown in Figure 5.6, \(e_{ij}\) is the inter-group edge between groups \(G_1\) and \(G_2\). Thus, node \(i\) is the group gateway of \(G_1\) and node \(j\) is the group gateway of \(G_2\). For simplicity, we let the common neighbors of node pair \((i, j)\) either lie in group \(G_1\) or in group \(G_2\). In other words, their common neighbors are either being a group member in \(G_1\) or in \(G_2\). From the topological perspective, the existing of a common neighbor \(k\) implies that there is a triangle connection among the node tuple \((i, j, k)\), with the edge set \(\{e_{i,j}, e_{i,k}, e_{k,j}\}\). According to the observations of real mobile communication networks in [137], the strength of a tie between
two nodes increases with the overlap of their friendship circles. By following this claim, upon the examples in Figure 5.6, the number of node pair \((i, j)\)'s common neighbors increases, indicating that the number of triangles (friendship circles) which cross the edge \(e_{ij}\) also increases. As a result, the stronger correlation between node pair \((i, j)\) becomes.

Note that for each triangle made between node \(i\) and node \(j\) and their common neighbor, node \(i\) and node \(j\) will contribute one degree (edge) from \(\Delta_i\) and \(\Delta_j\), respectively. With a simple calculation, given the node degree \(\Delta_i\) and \(\Delta_j\), the maximum number of possible triangles (friendship circles) which include the edge \(e_{ij}\) can be formed is \(\left\lfloor \frac{(\Delta_i - 1) + (\Delta_j - 1)}{2} \right\rfloor\). Here, \(\lfloor x \rfloor\) is the floor function, means the largest integer which is less than or equal to \(x\). Similar to the analysis in (5.19), we define a new metric of edge clustering coefficient \(C_{ij}\) as the ratio of the number of triangles the edge \(e_{ij}\) is currently attached to the maximum number of triangles that could be potentially included

\[
C_{ij} = \frac{2|Tr_{ij}|}{(\Delta_i - 1) + (\Delta_j - 1)},
\]

where \(|Tr_{ij}|\) denotes the number of triangles formed between node pair \((i, j)\). From (5.20), we see that the range of \(C_{ij}\) is over \([0, 1]\). If there is no common neighbors between node \(i\) and node \(j\), \(C_{ij} = 0\), which leads to the weakest strength of edge \(e_{ij}\). On the other hand, when \(|Tr_{ij}| = \frac{(\Delta_i - 1) + (\Delta_j - 1)}{2}\), \(C_{ij} = 1\), which leads to the strongest strength of edge \(e_{ij}\), as all neighbors of node \(i\) are also neighbors of node \(j\). This may happen when the topology of \(G_1\) and \(G_2\) is highly overlapped. Thus the equation (5.20) is consistent with what have been observed in mobile communication networks [137]. Hence, it can properly reflect the link strength between two nodes in a unweighted graph. However, we have presented in this work that the group structure need to be correctly studied in a weighted graph, by taking the edge weight of user pair correlation into the consideration. In analogy with the analysis of the node connectnesd (i.e., weighted node clustering coefficient) in Section 5.3.1, next, we study the weighted inter-group edge clustering coefficient \(C_{ij}^w\). According to the analysis in (5.20), \(C_{ij}^w\) is represented as

\[
C_{ij}^w = \frac{\sum_{k \in N_i \cup N_j, k \neq i, j} (\rho_{i,k} + \rho_{j,k}) \cdot 1_{\{h_{i,k}=h_{j,k}=1\}}}{\sum_{k \in N_i, k \neq j} \mathcal{E}_i + \sum_{k \in N_j, k \neq i} \mathcal{E}_j},
\]

where \(\mathcal{E}_i = \rho_{i,k} \cdot 1_{\{h_{j,k}=1\}} + 1_{\{h_{j,k} \neq 1\}}\), and \(\mathcal{E}_j = \rho_{j,k} \cdot 1_{\{h_{i,k}=1\}} + 1_{\{h_{i,k} \neq 1\}}\). In detail, \(\mathcal{E}_i\) represents the total possible weighted edge contribution for forming the triangles of node \(i\)'s neighbor set \(N_i\). If the edge \(e_{j,k}\) exists, then the weight is \(\rho_{i,k}\), otherwise is 1. The same rule applies for node \(j\)'s neighbor set \(N_j\), which results in \(\mathcal{E}_j\). By comparing (5.21) with (5.20), it is easy to prove that
In addition, if without considering the edge weight contribution in (5.21), these two equations (5.20) and (5.21) are equivalent.

Figure 5.6: Weighted edge clustering coefficient comparison.

The intuition behind the metric weighted inter-group edge clustering coefficient $C_{ij}^w$ in (5.21) is that the more common neighboring nodes of two gateway nodes reside in the overlapped area between two groups, the more potential connections between two groups, which overall increase the correlation strength between two groups. As a case study, Figure 5.6 illustrates the two examples of the correlation strength of groups $G_1$ and $G_2$ according to the weighted edge clustering coefficient. Specifically, for the example in Figure 5.6(a), there is small number of common neighbors between group gateways $i$ and $j$. While in Figure 5.6(b), there are large common neighbors and stronger correlation between group gateways $i$ and $j$. As a result, $C_{ij}^w = 0.89$ in Figure 5.6(b), which is much larger than $C_{ij}^w = 0.39$ obtained in Figure 5.6(a). Thus, the result of matches our expectation, that the correlation strength between two groups increases when more common neighboring nodes of two gateway nodes reside in the overlapped area between two groups, which is Case 2 shown in Figure 5.6(b). The correlation strength between two groups will also increase when the correlation between two gateway nodes with their common neighbor is high. Thus, we demonstrate that the metric of weighted inter-group edge clustering coefficient can effectively characterize the correlation strength between two neighboring groups upon inter-group links.
5.4 Characteristics of Group Evolution

In previous section, we elaborated several metrics for characterizing the group structure regarding different node status in the group: group member, group head, and group gateway. Given the large number of observations [117, 144, 132], an interesting phenomena of human group movements in a MANET is the group evolution. Group evolution represents the variation of group size (the total number of group members) with time. In MANETs, people tend to join a group when executing the same task at a certain location. Once the task is finished, they may detach from the group and be ready to form a new group or directly join another group. Thus, a general group movement typically experiences a birth-to-death process. Furthermore, when different groups meet in the same task location area, there is possibility that existing group members can switch to other groups upon their own judgments. In consequence, the group size and group structure can vary often times in MANETs. Based on these observations, the group evolution process is a very complicated process, as its behavior can be affected by many factors in real application scenarios. In addition, when the group structure is greatly changed due to the node leaving and join event, it may trigger the group (clustering) reconstruct process to maintain routing information among existing mobile nodes inside the group. However, it has been demonstrated that the frequent re-clustering computational overhead can outweigh the clustering-based routing benefit for routing efficiency in MANETs [61, 24]. Therefore, understanding and characterizing the birth-to-death process of group evolution behaviors can greatly benefit the routing design and performance evaluation in MANETs.

In parallel, there have been several studies in social networks to investigate how human community grows [129, 132]. In particular, the authors in [132] first provided an solution for quantifying social group evolution based on the k-clique percolation algorithm. However, little has been done to investigate the properties of group evolution and measure the corresponding group stability in mobile wireless networks. Therefore, we tackle this interesting yet very challenging issue in this section. More specific, we focus on the following questions: 1) How to quantify the group evolution degree regarding the variation of group membership during the group evolution process? 2) What is the group stability level during the group evolution process? 3) When do node switch events occur during the group evolution process?
5.4.1 Quantify Group Evolution Degree

When a group arrives at a new task location, an existing group member may meet new neighbors which are already present at the location. These new neighbors can be an individual member who is not in any group or a member of other groups. Upon the social interactions with existing group members, the new detected individual neighbors may decide to join the group, if they tend to execute the same task with the group. In addition, mobile users may switch between two groups upon their own judgments when two groups cross each other. In either case, the group evolution event occurs. In this section, we aim to quantify the group evolution degree regarding the variation of group membership during the group evolution process.

Typically, we take the entire group evolution process as a group birth-to-death process. By this way, we find that there are two important metrics which can characterize the birth-to-death group process: the group size $S_G$ and the group lifetime $T_G$. Specifically, we assume at time $t_0$, a group $G(t_0)$ is formed. After time $t$, the group status is represented by $G(t_0 + t)$. Similar to the study for quantifying group evolution in social network [132], here, we let $|G(t_0) \cap G(t_0 + t)|$ be the number of common members in $G(t_0)$ and $G(t_0 + t)$. Accordingly $|G(t_0) \cup G(t_0 + t)|$ denotes the number of members in the union of $G(t_0)$ and $G(t_0 + t)$. Then, we can use the auto-correlation function $A(G, t)$ to quantify the overlap between two states $G(t_0)$ and $G(t_0 + t)$ of the group,

$$A(G, t) = \frac{|G(t_0) \cap G(t_0 + t)|}{|G(t_0) \cup G(t_0 + t)|}, \quad t \leq T_G. \tag{5.22}$$

Since time $t_0$, the group $G(t_0 + t)$ experienced $t$ time steps. Then, we discuss two cases here. First, there is no member change during this $t$ time steps, that is, $|G(t_0) \cap G(t_0 + t)| = |G(t_0) \cup G(t_0 + t)|$. From (5.22), we have $A(G, t) = 1$. It means the auto-correlation between the initial status $G(t_0)$ and the current status at $G(t_0 + t)$ is 1. And there is no group evolution event occurs. For the second case, there is total $n$ new members who join the group $G$ and total $m$ number of members leave the group $G$. If let $S_G(t_0)$ represent the number of group members at the initial stage, from (5.22), we have

$$A(G, t) = \frac{S_G(t_0) - m}{S_G(t_0) + n}, \quad t \leq T_G. \tag{5.23}$$

Given (5.23), we can see that the value of $A(G, t)$ decreases with the growth of either $m$ or $n$. And $A(G, t)$ decreases faster when both $m$ and $n$ increase. In other words, when either new members join or old members leave the group, the value of $A(G, t)$ will decrease. In the extreme
case, when \( S_G(t_0) = m \), \( A(G, t) \) approaches to 0, which indicates that all the nodes initially forming the group \( G \) have all left the group.

As a case study, Figure 5.7 illustrates the group evolution degree during 100 time steps. In this simulation, there are total 50 mobile nodes in a network. The group initially contains 30 group members, with a group correlation threshold \( \rho_{th} = 0.5 \). Thus, at each time step, a new member will join the group if it has a user pair correlation with an existing group member larger than \( \rho_{th} \). And the old member will detach the group when it has all user pair correlations less than \( \rho_{th} \). Recall that in Section 5.2.3, the user pair correlation is characterized more by their social correlation, and more stable when \( \lambda \) is less.

As we expected, when \( \lambda = 0.1 \), the user pair correlations among group members are stable and therein less node join/detach event occurs. However, \( \lambda = 0.9 \), the user pair correlation can be less than \( \rho_{th} = 0.5 \), when their relative movement vary dramatically. Thus, the node join/detach event can frequently occur with time. From (5.22) and (5.23), we observed that the trend of \( A(G, t) \) is decreasing with time and approaches to 0. Note that the fluctuating of \( A(G, t) \) indicates that the previous detached old group member can rejoin the group, which increases the value of \( A(G, t) \).

![Figure 5.7: Example of group evolution degree variation with time.](image)

From (5.22), it is clear that the range of \( A(G, t) \) is over \([0, 1]\). As shown in Figure 5.7, the smaller \( A(G, t) \) is, the larger degree of group evolution degree is. In contrast, the larger \( A(G, t) \) is, the smaller degree of group membership variation becomes. Note that, if \( A(G, t) \) approaches to 0 in a short time, it means the group member join and detach happen frequently, which results in an unstable group structure and invokes routing cost and performane degradation. This happens also when the group correlation \( \rho_{th} \) is small, so that it is very easy for new members joining into the group or old member detaching the group. In contrast, when the group correlation \( \rho_{th} \) is high,
group members have a strong user pair correlations between each other. Hence, the group structure is stable, and the group evolution degree is small, i.e., \( A(G, t) \) is high, which is the case when \( \lambda = 0.1 \) shown in Figure 5.7. Therefore, we demonstrate that auto-correlation function \( A(G, t) \) is an effective metric for quantifying the group evolution degree during its birth-to-death process.

### 5.4.2 Group Stability Measure

In previous section, we use the auto-correlation function \( A(G, t) \) to quantify the group evolution degree. When \( A(G, t) = 1 \), there is no node join and switch event happening for the group \( G \) during \( t \) time steps. Accordingly, we consider this group \( G \) is stable in terms of its group size. It has been found that a stable group can benefit the routing efficiency and service sustaining in mobile wireless networks [136, 64, 65, 66]. In this section, we aim to characterize the group stability level during the group evolution process.

Upon the real communication traces of mobile phone users, the authors showed that in the local structures of a mobile communication network, the internal connections between nodes within a community are denser and closer than the external ones [137]. This observation is consistent with what have been found in the network structures of human based communities [117]. Accordingly, we apply this result for characterizing human based groups in MANETs. Specifically, for a node \( i \) in a group \( G \), the node \( i \)'s degree \( \Delta_i \) includes two parts, that is, \( \Delta_i = \Delta_i^{in}(G) + \Delta_i^{out}(G) \). \( \Delta_i^{in}(G) \) and \( \Delta_i^{out}(G) \) represent the number of link connections of node \( i \) with its neighbors within and without the group \( G \), respectively. Following this denotation, we define the group stability level at time \( t \) as follows.

**Definition 7.** A group \( G(t) \) consisting of group member set \( V(G(t)) \) at time \( t \) is **stable in a strong level** if \( \forall i \in V(G(t)) \)

\[
\sum_{j \in N_i^{in}} \rho_{i,j} > \sum_{k \in N_i^{out}} \rho_{i,k} \quad \& \quad \Delta_i^{in}(G(t)) > \Delta_i^{out}(G(t)). \tag{5.24}
\]

The group \( G \) is **stable in a weak level** if \( \forall i \in V(G(t)) \)

\[
\sum_{j \in N_i^{in}} \rho_{i,j} > \sum_{k \in N_i^{out}} \rho_{i,k} \quad \& \quad \sum_{i \in V(G(t))} \Delta_i^{in}(G(t)) > \sum_{i \in V(G(t))} \Delta_i^{out}(G(t)). \tag{5.25}
\]

According to Definition 7, the group stability level represents the resistance degree of variation of group structure and group members with time. From (5.24), for a stable group in a
strong level, each group member has more link connections within the group than those outside the group. Accordingly, from (5.25), for a stable group in a weak level, the sum of all connections within the group is larger than the total connections toward the nodes outside the group. It is clear that the condition of (5.24) satisfies (5.25), but the reverse case is not true. More important, in Definition 7, a stable group in both strong and weak levels must satisfy the condition that for each group member, the sum of user pair correlation with its neighbors within the group is greater than that with neighbors outside the group. If this condition can not be satisfied, we consider the group is unstable, as its member can have overall stronger social correlations with its neighbors outside the current group. That is, \( \sum_{j \in N_{i}^{in}} \rho_{i,j} < \sum_{k \in N_{i}^{out}} \rho_{i,k} \). For instance, as the example shown in Figure 5.6, when two groups geometrically overlapped, if the group gateway \( i \) find that it has more neighbors with strong correlations in \( G_{2} \), and it has less neighbors with weak correlations in \( G_{1} \), the node \( i \) can then switch to group \( G_{2} \).

**Node Switch Condition**

As described in previous section, the node switch event occurs when two neighboring groups cross each other, and the mobile user resides the geometrically overlapped area between two groups. From Definition 7, a group is stable regarding the group size variation when the condition of (5.24) or (5.25) is satisfied. Therefore, the condition that node \( i \) can switch between groups should meet the reverse conditions of (5.24) and (5.25), which is

\[
\sum_{j \in N_{i}^{in}} \rho_{i,j} < \sum_{k \in N_{i}^{out}} \rho_{i,k}.
\] (5.26)

The switch condition in (5.26) suggests that a node can switch to the other group, when it has stronger correlation with its neighbors in the new group.

As an example shown in Figure 5.8(a), groups \( G_{1} \) and \( G_{2} \) communicate via their gateways, where node \( i \) is the gateway of group \( G_{1} \). In this example, node \( i \) has three neighbors (blank circles) in \( G_{1} \) and 5 neighbors (solid circles) belonging to group \( G_{2} \). Then, we calculate the values of left hand side and right hand side of the inequality equation (5.26) as follows,

\[
\sum_{j \in N_{i}^{G_{1}}} \rho_{i,j} = 0.8 + 0.8 + 0.5 = 2.1 < \sum_{k \in N_{i}^{G_{2}}} \rho_{i,k} = (0.8 + 0.8 + 0.8 + 0.5 + 0.5) = 3.4 \] (5.27)
Hence, given (5.27), the node $i$ has stronger correlation with its neighbors in group $G_2$ than $G_1$, thus it satisfies the node switch condition, then node $i$ can switch from $G_1$ to $G_2$, which is the case shown in Figure 5.8(b).

To this end, we have studied the metrics for characterizing the group structure and group stability. And we suggest that the group evolution process is a common process of group moving behaviors in MANETs. However, we notice that almost all the existing group mobility models have two limitations on describing group moving behaviors. First, existing group mobility models, such as the reference point group mobility (RPGM) model [58], did not capture the group evolution process, there is no node join and node switch event occurs during the entire group movement. Second, the group did not deform during the entire simulation. Thus, there is no group birth-to-death process during the simulation. Hence, existing group mobility models may not be consistent with the group moving behaviors in reality. In order to mimic both birth-to-death group process and group evolution behaviors in MANETs, in next section, we apply the metrics studied in this work to design a novel birth-to-death group mobility model.

5.5 Design And Application of Birth-to-Death Group Mobility Model

5.5.1 Metrics Applied for BDGM Model

In this section, we propose a novel birth-to-death group mobility (BDGM) model. In the proposed BDGM model, a general group typically experiences the birth-to-death process including group initiation, group evolution and group deformation in sequence. Before describing the move-
ment pattern of the BDGM model, we first demonstrate how to apply the metrics studied in this work for designing a group mobility model. Specifically, Table 5.11 illustrates the metrics applied for the proposed BDGM model.

At the initial stage of the simulation, we first use the user pair correlation $\rho_{x,y}$ metric to quantify the correlation between each mobile pair in the simulation area. Thus, if the user pair correlation value of a link is larger than the predefined group correlation threshold $\rho_{th}$, then two nodes will form a new group or join into an existing group which is in the group forming process. Once a group is formed, we can apply the metrics of node degree $\Delta_i$, node connectness $C_{w}^i$, and average neighbors’ degree $\Delta_{wn,i}$ for group head selection. We assume the group life time, which is the task execution time $T_{G}$ is randomly selected by the group head. During the group movements, when two groups meet each other, by applying the node switching condition upon the tuple pair $(\sum_{j \in N^i} \rho_{i,j}, \sum_{k \in N^{out}_{i}} \rho_{i,k})$ for each node inside the overlapped area of two groups, the node can decide whether switch to the other group or not. Once the group lifetime reaches $T_{G}$, i.e., the expected task is completed, the group will deform. Therefore, the birth-to-death process of a group $G$ in the simulation area is performed. After that, the mobile nodes of the deformed group will either join other existing groups in the current location area or reform a new group with his/her neighbors.

### 5.5.2 Group Initialization Algorithm

In previous section, we summarized the metrics applied for the BDGM model in Table 5.11. In this section, we focus on the group initialization algorithm for forming a group among mobile nodes.

![Figure 5.9: Operation of group initialization algorithm.](image)

Figure 5.9 illustrates a group initialization operation in the proposed BDGM model. At the beginning, the group initiator node $s$ in location $I_s$ sends a group forming query message to all its neighbors including the required user pair correlation threshold $\rho_{th}$, and the targeting task location
Table 5.11: Metrics Applied for the Birth-to-Death Group Mobility (BDGM) Model

<table>
<thead>
<tr>
<th>Metric</th>
<th>Metric Description</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>user pair correlation $\rho_{x,y}$</td>
<td>The user pair correlation $\rho_{x,y}$ between neighboring users $(x,y)$ depends on their social correlation $SC(x,y)$ and similarity metric $\tilde{S}<em>{x,y}$. If $\rho</em>{x,y} &gt; \rho_{th}$, then two users join the same group.</td>
<td>Group initialization</td>
</tr>
<tr>
<td>$\Delta_i, C^w_i, \Delta^w_{nn,i}$</td>
<td>The node with the highest value from (5.18) given the node degree $\Delta_i$, weighted clustering coefficient $C^w_i$, and average weighted neighbors degree $\Delta^w_{nn,i}$ within a group is selected as the group head.</td>
<td>Group head selection</td>
</tr>
<tr>
<td>Task location $(l_I, l_T), (v_{head}, \theta_{head})$</td>
<td>Once the group is formed, the entire group moves from current task location $l_I$ to the destined task location $l_T$. The group nodes follow the movement pattern $(v_{head}, \theta_{head})$ of the group head.</td>
<td>Group moving behaviors</td>
</tr>
<tr>
<td>$(\sum_{j \in N^\text{in}<em>{i}} \rho</em>{i,j}, \sum_{k \in N^\text{out}<em>{i}} \rho</em>{i,k})$</td>
<td>When two groups meet in the same task location, each node can switch from existing group $G_i$ to new group $G_j$, when the switch condition $\sum_{j \in N^\text{in}<em>{i}} \rho</em>{i,j} &lt; \sum_{k \in N^\text{out}<em>{i}} \rho</em>{i,k}$ satisfies.</td>
<td>Group evolution</td>
</tr>
<tr>
<td>Task execution time $T_G$</td>
<td>After the expected task execution time $T_G$, i.e., the expected group lifetime, all the group nodes detach from the group. They move in their own way and are ready to join into a new group.</td>
<td>Group member detach</td>
</tr>
</tbody>
</table>

$L_T$. By comparing the user pair correlation value with $\rho_{th}$, each neighbor decides whether it will join the group or not. Upon the decision result, each neighbor will put its node ID either into the group member list $Q_G$ or non-group member list $Q_{G^c}$. As shown in Figure 5.9, after the first run of query, $Q_G = \{s, z, x, y\}$ and $Q_{G^c} = \{v, u\}$. Within each run, the node indexes in $Q_G$ are permuted according to the decreasing order of user pair correlation value, for instance, $\rho_{z,s} > \rho_{x,s}$. Then, the new joined neighbors in $Q_G$ continue to send the group forming query message to their neighbors with a new query message sequence number, respectively. Similar to the breadth-first-search (BFS) algorithm, after each run, the new added group member in $Q_G$ is one more hop away the group initiator $s$. When either all individual nodes in location $l_I$ or all individual nodes which can be reached by node $s$ in location $l_I$ have been queried, the group initialization operation stops. As the result, all the group members are listed in $Q_G$. In this example, there are total 7 nodes joining the group.
5.5.3 Group Movement Pattern

In this section, we describe the detailed movement pattern of the proposed BDGM model. According to Table 5.11, once a group is formed, the group moving behaviors continue the following procedures.

1. **Group Head Selection**: Given the member list in $Q_G$, the group head is selected according to the group head selection rule (5.18). Then the head will choose the expected group task time $T_G$, and starts to move toward the targeting task location, for instance, $L_T$.

2. **Group Moving Behaviors**: Similar to existing group mobility models, we let the group members follow the mobility pattern of the group head. Specifically, during a group movement, each group member moves according to the velocity (include the speed $v$ and direction $\theta$) of the group head. The correlation degree of velocities is heavily dependent on the corresponding user pair correlation between group member and the group head. By this means, the velocity of a group member $j$ can be easily controlled as follows,

$$
\begin{align*}
    v_j &= v_{\text{head}} + (1 - \rho_{j,\text{head}}) \cdot U \cdot \Delta v_{\text{max}} \\
    \theta_j &= \theta_{\text{head}} + (1 - \rho_{j,\text{head}}) \cdot U \cdot \Delta \theta_{\text{max}},
\end{align*}
$$

(5.28)

where $U$ is a random variable with uniform distribution over $[-1, 1]$. $\Delta v_{\text{max}}$ and $\Delta \theta_{\text{max}}$ are the maximum speed and direction difference between a group member and the group head in one movement epoch. Thus, when the user pair correlation $\rho_{j,h}$ approaches to 1, the deviation of the velocity of a group member from that of the leader is getting smaller. Note that, for members who are not the immediate neighbors of the group head, their node velocities during the group movement can be obtained by just replacing the group head index with one of their neighbors which has already updated his/her velocity from (5.28). Therefore, all members can follow the group head’s movement in location $L_T$.

3. **Group Evolution**: During the group movement interval $T_G$, group members can switch to neighboring groups according to the switching policy (5.26). Also a new individual node already in location $L_T$ can join the group when its user pair correlation with an existing group member is larger than $\rho_{th}$ required for that group.

4. **Group Member Detach**: When the group timer $T_G$ is expired, members automatically detach from the group. Then, the previous group members will prepare for joining a new group when a new group forming query message is received. In addition, they can join other existing groups upon the corresponding user pair correlation threshold $\rho_{th}$.
5.5.4 Routing Performance Evaluation

As a direct application, the proposed BDGM model can be applied for routing design and performance evaluation in MANETs. In this section, we compare the routing performance effect between BDGM and RPGM model by taking the AODV protocol [96] as a case study. Specifically, the studied routing metrics include: average end-to-end packet delay; average end-to-end network throughput defined as the percentage of packets transmitted by the sources that successfully reach their destinations; and routing overhead defined as the ratio of total size of network control packets to the total size of both network control packets and data packets initiated from the sources during the simulation. Upon the simulation setup, among 50 mobile nodes, the network traffics consist of 20 constant bit rate (CBR) sources and 30 connections. The source-destination pairs are chosen randomly through cbrgen tool of ns-2. Each source sends 1 packet/sec with the packet size 64 bytes. At the initial stage, for both BDGM and RPGM models, every 10 nodes form a group, that is, there are 5 groups are assigned at the beginning of the simulation. Furthermore, we respectively set the initial average speed $E\{v_{ini}\}$ as 2, 5, 10, 15, 20, and 25 m/sec to generate different mobility levels for both models. In both models, we let mobile nodes follow the movement of the group leader. In the RPGM model, the movement randomness between group members is based on the distance of reference point at each time step. Accordingly, in the BDGM model, the movement randomness between group members are based on their social correlation and similarity coefficients. The total simulation time is 1000 seconds. In BDGM model, the weight coefficient is set as $\lambda = 0.5$. Thus, the social correlation and movement similarity metric play an equal role on estimating the user pair correlation. Different from RPGM model, mobile nodes can detach the current group and move upon its own “random walk” in the simulation area. In addition, the entire groups in the BDGM model will deform and reform with time. In this simulation, we let all groups in BDGM model deform 3 times during the entire simulation period of 1000 seconds. For easy control, all groups in the BDGM model deforms at the same time. Then, after a random time, all the mobile nodes reform the group according to the group initialization algorithm. Thus, the group birth-to-death evolution process happens 3 times for all groups in the BDGM model. However, in the RGPM model, group members follow the movement of the pre-selected group leader during the entire simulation, thus, the groups in RPGM model never deform during the simulation. For better demonstrations, we compare the routing performance metrics according to same average initial speed between these two models.
The simulation results are shown in Figure 5.10. From this figure, there exists an evident difference of simulation results for all routing metrics between these two models. It is clear to see that the routing performance for both models degrades as the increase of the node speed. Compared to the BDGM model, the performance degradation is much less in RPGM model, especially when node speed is high. Furthermore, all three measured AODV routing performances of the RPGM model outperform the BDGM model, regardless of the average initial speed.

As discussed above, the major reason for the results is that RPGM model is much more stable than BDGM model during the simulation. The group deform and reform process in the BDGM model invokes more routing cost than RPGM model. Because group members can death the existing group and move upon its own way during the simulation, the average hop count of end-to-end path in BDGM model is around 2.25, which is larger than 1.5 in RPGM model, therein results in a longer end-to-end packet delay. As the routing overhead and end-to-end packet delay in BDGM model is higher than RPGM model, BDGM model has less average throughput than RPGM model. Therefore, since there is no node detach and group deformation in RPGM model, it stresses AODV much less than the BDGM model.

In addition, the group mobility can be also applied in vehicular ad-hoc networks (VANETs), where vehicles in geographical proximity from groups. And in reality, the birth-to-death group evolution process frequent happens in both MANETs and VANETs, provided that the group formation is regulated by human social behaviors in the network. Thus, we suggested that the routing protocols evaluation based on the widely applied RPGM model is over optimistic. Therefore, the proposed BDGM model, which describes the typical group birth-to-death evolution process, is more prefer-
able for routing protocol design and network performance evaluation in MANETs. To apply the BDGM model for researches in VANETs is our further study.

5.6 Summary

In this work, we studied the inherent properties of group moving behaviors in MANETs. We notice that the group moving behaviors and the resulting time-varying group structures are directly affected by the correlation strength between group members. Hence, we first studied the correlation degree between mobile users in MANETs. Specifically, we applied information theory to qualitatively measure the social correlation strength between a user pair in MANETs. Then we investigated the correlation degree $\rho_{i,j}$ between a node pair $(i, j)$ by taking social correlation strength, similarity of nodes’ movement and physical distance into consideration. To characterize the inherent properties of group structures, we demonstrated that unweighted node clustering coefficient overestimates the clustering property of group structure in MANETs, then we developed a new metric of node connectness $C^w_i$ to evaluating the stability level of inter-connections of each group member with his/her neighbors inside the group. We found that the user pair correlation can outweigh the number of inter-connections on determining the structure property of the group. Then we study the node impact on the group structure based on the average weighted node neighbors’ degree $\Delta^w_{nn,i}$, which can be applied for group head selection. We suggest that when the node degree $i$ is large and has strong correlation with its neighbors, then if $\Delta^w_{nn,i}$ is also high, node $i$ exhibits an assortative mixing behavior with its neighbors; otherwise if $\Delta^w_{nn,i}$ is small, node $i$ exhibits an disassortative mixing behavior with its neighbors. We further investigate the correlation strength between two neighboring groups based on the property of the inter-group link, which characterizes the stability of link connection between two neighboring groups. By characterizing the group evolution features, we studied the group evolution degree regarding the variation of group membership during the group evolution process. Furthermore, we investigated the condition for group member switching between groups. Finally, to apply the metrics we studied in this work, we propose a novel birth-to-death group (BDGM) mobility for both analytical and simulation studies in MANETs. As a case study of routing protocol evaluation, we demonstrate that the widely applied RPGM model stresses AODV much less than the proposed BDGM model, and could lead to an over-optimistic result without taking group birth-to-death evolution behaviors into account. How to properly apply the BDGM model for diversified application scenarios in wireless networks is our further study.
Chapter 6

Conclusion

During the course of this Ph.D. study, we have been focusing on 1) mobility modeling design; 2) radio link and path properties analysis; 3) human trace collections and analysis of human diffusive moving behaviors and their impacts on inter-meeting time; and 4) Group mobility properties analysis and modeling design. By combining all the results of these inter-dependent and equally important four pieces of research works, finally we achieved all of our research objectives of this doctoral study. To this end, we obtained an in-depth understanding of user mobility and its impact on multi-hop wireless networks. In this Chapter, we conclude this dissertation.

At the beginning of this Ph.D. study, we noticed that the node mobility is a dominating factor to the system performance of MANETs. However, the knowledge and understanding of people on node mobility and its impacts on mobile wireless networks are still very limited. Therefore, we are motivated to tackle with the research issues of node mobility in mobile wireless networks as our overall PH.D. research topic.

The first research issue we immediately faced is that because of the complete randomness of user mobility in random mobility models, unrealistic moving behaviors could occur frequently during the simulations. More important, people have found that random mobility models, especially for RWP model, can lead to biased or even misleading results of analysis and simulations for MANETs. In order to correctly reflect the network performance while providing flexible controls of node mobility to adapt diversified network scenarios, in Chapter 2, we proposed a novel Semi-Markov mobility model. Each SMS movement includes three consecutive phases: *Speed Up* phase, *Middle Smooth* phase, and *Slow Down* phase. Thus, the entire motion in the SMS model is smooth and consistent with the moving behaviors in real environment. Through steady state analysis, we
demonstrate that SMS model has no average speed decay problem and always maintains a uniform spatial node distribution. The analytical results are validated by extensive simulation experiments. In addition, we showed that the SMS model can be easily and flexibly applied for simulating node mobility in wireless networks. It can also adapt to different network environments such as group mobility and geographic constraints. To demonstrate the impact of this model, we evaluate the effect of this model on distribution of relative speed, link lifetime between neighboring nodes, and average node degree by ns-2 simulations.

Next, due to the limited transmission range, the end-to-end communication in mobile wireless networks has to be via a multi-hop way. Therefore, link properties are essential to applications and services in such networks, however, are still unclear to research community. We found that the time-scale of existing random mobility models generally cannot capture the time-scale of radio channels, hence is insufficient to capturing the significant channel variability. Furthermore, previous studies on link properties assumed that the transmission range of each node is a constant value regardless the variation of wireless radio environments, which however is not true in reality. Therefore, by applying the results from our first work, we want to address the important yet unknown link dynamics in mobile wireless networks. As our second research topic, we elaborated our stochastic analysis of link properties by jointly considering the radio channels and node mobility effects in Chapter 3. We first provided the concept of effective transmission change of a mobile node, which guarantees the effective communication of the node with a very high probability, meanwhile varying according to the variation of radio environments. Then, we showed that link lifetime distribution can be effectively approximated by an exponential distribution, which is in contrast to previous results that there exists a peak in the distribution function which are mainly obtained from random mobility models. More interestingly, the exponential distribution parameter can be simplified by $\frac{\bar{V}}{R_e}$, where $\bar{V}$ is the average speed and $R_e$ is the effective transmission range (ETR) of a mobile node. Since the path lifetime is determined by the minimum link lifetime en route, we concluded that the PDF of path lifetime also follows an exponential distribution, which greatly relaxes the assumption of large (approach to infinite) hop-count of a path for its distribution converging to exponential. We also found that the impacting factors on both link and residual link lifetime are in the decreasing order of average node speed, ETR, and node-pair distance. In addition, we showed that for a large dense network, its network connectivity is bounded by the average node degree, which is equivalent to the multiplication between the average link arrival rate and the average link lifetime.

After this study, we noticed that in reality, the wireless devices are basically carried by
humans in a typical wireless multi-hop network. As a result, the human mobility directly affects the link level dynamics between mobile wireless devices, which are characterized by the contact-based metrics, such as inter-meeting time, in MANETs. By far, the inherent properties of human mobility, such as human pause time, and human trip displacement, still remain elusive, which therein brings a main hurdle for studying human mobility impact on contact-based metrics in MANETs. Therefore, we move forward to study human mobility in the third work in Chapter 4 by investigating human diffusive behaviors and their impact on contact-based metrics, such as inter-meeting time. To study this work, it is necessary to have a complete trace log information of human daily travel activities in both temporal and spatial domains for investigating human diffusive movement patterns. However, the trace dataset which satisfies this requirement is unavailable. Thus, to conduct our research, we collected student daily travel traces over NCSU campus for three months. Also we acquired another two trace datasets which contain the human travel information in city-wide, and county-wide, respectively. These three trace datasets are characterized by different size scales of human moving domains. Interestingly, upon these trace dataset study, we suggested that the cutoff power-law distribution is an “invariant” property of human mobility, because human mobility is affected by the temporal and spatial limits of human social behaviors. Thus, it has similar cutoff power-law properties as contact-based metrics, such as inter-meeting time. Given the trace observation results, we analyzed the reasons of the existence of characteristic time $T_c$ and characteristic distance $D_c$ in the cutoff power-law distribution of mobility metrics, and found that the parameter tuple (temporal or spatial power-law coefficient, characteristic time or distance) is dominated by temporal-spatial limitations of human social behaviors. As a result, the distinct human social behaviors can lead to different human mobility patterns, which in turn, affect the performance of contact-based metrics in MANETs. Then, by studying the scaling law of human diffusive behaviors, we showed that the human diffusive rate $r$ can characterize the joint temporal-spatial effect on human mobility pattern, when either pause time or trip displacement or both are characterized by power-law. Specifically, we showed that when the power-law head characterizes both pause time and trip displacement with the power-law coefficients $\alpha$ and $\beta$, respectively, the human diffusive rate $r$ is, $r = 2\alpha/\beta$, where $0 < \alpha < 1$ and $0 < \beta < 2$. Thus, based on different combinations of $(\alpha, \beta)$, the human diffusive rate $r$ can be superdiffusive ($r > 1$), subdiffusive ($r < 1$), and normal diffusive ($r = 1$), respectively. Then, we studied the human diffusive rate effects on the inter-meeting time and link lifetime. We found that the diffusive rate can effectively indicate the property of inter-meeting time, while the link lifetime distribution is insensitive to the diffusive rate. Regarding the inter-meeting time, we
found that superdiffusive rate $r > 1$ leads to a longest power-law head, while the subdiffusive rate $r < 1$ results in the shortest power-law head in the distribution of inter-meeting time. Thus, the higher the diffusive rate is, the longer the power-law head is, while the shorter the exponential tail becomes in the distribution of inter-meeting time. Finally, we investigated the mixed behaviors of power-law head and exponential tail of a cutoff power-law upon the analysis on empirical human trace datasets with different levels of moving domain sizes. Then, we proposed an approximated cutoff power-law distribution, which is featured by a parameter tuple (the power-law coefficient and cutoff point), for instance, $(\alpha, T_c)$ and $(\beta, D_c)$, for pause time and trip displacement, respectively. The approximated cutoff power-law distribution exhibits a close fit with the empirical results from trace files.

Besides our study on individual user moving behaviors and their impacts on MANETs, we observe that mobile users often exhibit correlated mobility patterns in their movements in practical ad-hoc networks. Understanding the group moving behaviors is important for routing design and performance analysis in MANETs. Interestingly, it has been widely observed in mobile social networks that humans with strong social correlations tend to form a social group when they meet at the same community site. Thus, there is a very close resemblance between mobile wireless networks and mobile social networks. Therefore, we are motivated to take the advantage of the resemblance between these two networks to study fundamental characteristics of group mobility. Thus, as the last integral piece of this doctoral research, in Chapter 5, we studied the inherent properties of group moving behaviors in MANETs. We notice that the group moving behaviors and the resulting time-varying group structures are directly affected by the correlation strength between group members. Hence, we first studied the correlation degree between mobile users in MANETs. Specifically, we applied information theory to qualitatively measure the social correlation strength between a user pair in MANETs. Then we investigated the correlation degree $\rho_{i,j}$ between a node pair $(i, j)$ by taking social correlation strength, similarity of nodes’ movement and physical distance into consideration. To characterize the inherent properties of group structures, we demonstrated that unweighted node clustering coefficient overestimates the clustering property of group structure in MANETs, then we developed a new metric of node connectness $C_i^w$ to evaluating the stability level of inter-connections of each group member with his/her neighbors inside the group. We found that the user pair correlation can outweigh the number of inter-connections on determining the structure property of the group. Then we study the node impact on the group structure based on the average weighted node neighbors’ degree $\Delta_{nn,i}^w$, which can be applied for group head selection. We suggest
that when the node degree $i$ is large and has strong correlation with its neighbors, then if $\Delta_{nn,i}^w$ is also high, node $i$ exhibits an assortative mixing behavior with its neighbors; otherwise if $\Delta_{nn,i}^w$ is small, node $i$ exhibits a disassortative mixing behavior with its neighbors. We further investigate the correlation strength between two neighboring groups based on the property of the inter-group link, which characterizes the stability of link connection between two neighboring groups. By characterizing the group evolution features, we studied the group evolution degree regarding the variation of group membership during the group evolution process. Furthermore, we investigated the condition for group member switching between groups. Finally, to apply the metrics we studied in this work, we propose a novel birth-to-death group mobility for both analytical and simulation studies in MANETs. As a case study of routing protocol evaluation, we demonstrate that the widely applied RPGM model stresses AODV much less than the proposed BDGM model, and could lead to an over-optimistic result without taking group birth-to-death evolution behaviors into account. How to properly apply the BDGM model for diversified application scenarios, such as multicast group routing, in wireless networks is our further study.
Bibliography


[9] Fan Bai, Narayanan Sadagopan, Bhaskar Krishnamachari, and Ahmed Helmy. IMPOR-
TANT: A Framework to Systematically Analyze the Impact of Mobility on Performance of 

[10] Libo Song, David Kotz, Ravi Jain, and Xiaoning He. Evaluating Location Predictors with 

[11] Francisco Chinchilla, Mark Lindsey, and Maria Papadopouli. Analysis of Wireless Infor-
mation Locality and Association Patterns in a Campus. In Proceedings of IEEE INFOCOM, 
2004.

[12] Augustin Chaintreauz, Pan Hui, Jon Crowcroft, Christophe Diotz, Richard Gassy, and James 

[13] Xiaolan Zhang, Jim Kurose, Brian Levine, Don Towsley, and Honggang Zhang. Study of 
2007.

Networks Research. Wireless Communication and Mobile Computing (WCMC): Speci-


Waypoint Mobility Model for Wireless Ad Hoc Networks. IEEE Transactions on Mobile 


[107] Accutech: Factors Affecting Transmission Distance.


[119] GPS for Personal Travel Lexington Area Travel Data Collection Test, 1997.


