Direct-conversion quadrature modulators have been widely used in modern communication systems because of their capability to conserve bandwidth with low cost, low complexity, and small form factor. In this dissertation, three different quadrature modulator applications: 1) generic quadrature modulation system; 2) zero-IF OFDM system; and 3) low-IF OFDM system are reviewed. The different usages and implementation costs of the modulators are compared. Impacts of carrier leakage, gain/phase imbalances, and nonlinear distortion on system performance degradation in terms of spectrum regrowth and waveform quality are analyzed for three applications of the quadrature modulators. The adverse impacts of these physical imperfections demand system-level techniques to accurately characterize them for system design and verification.

A new nonlinear behavioral model is developed for characterization of both correlated/uncorrelated nonlinear distortion and the linear static errors including DC offset and gain/phase imbalances for direct-conversion quadrature modulators. This enables fast and accurate prediction of spectrum regrowth and waveform quality degradation due to these physical imperfections. A low-pass equivalent model structure is developed based on the assumptions that I and Q channels are isolated and the dominant nonlinearities of quadrature modulators are the baseband transconductors. The responses of I and Q inputs are modeled by two independent complex power series. These capture both nonlinear distortion and linear static errors. Amplitude-to-amplitude (AM-AM) and amplitude-to-phase (AM-PM) measurements and 4-point vector network analyzer (VNA) measurements are used to extract the model parameters for characterization of the nonlinear distortion and the linear static errors, respectively. An existing orthogonalization technique for power amplifiers is implemented in the quadrature modulator model to
decompose the correlated and uncorrelated nonlinear distortion. The modeling technique is applied to both passive and active RF quadrature modulators and the models are verified by adjacent channel power ratio (ACPR) and error vector magnitude (EVM) measurements of systems excited by digitally modulated signals.

One fundamental quadrature modulator model assumption, which is validated by the modeling results, is that the baseband transconductors dominate the nonlinear characteristics of integrated quadrature modulators. This motivates design and modeling work for characterization of the nonlinear distortion of a special category of transconductors: bipolar multi-tanh transconductors. Three baseband and bandpass bipolar transconductors: the bipolar differential pair; the multi-tanh doublet; and the multi-tanh triplet; are designed and their nonlinear characteristics are modeled using the same modeling technique as used with the quadrature modulator modeling work: the AM-AM and AM-PM based complex power series model. The bandwidth limitations of the AM-AM and AM-PM based model for characterizing baseband transconductors are studied and measured and an augmented model structure is proposed to overcome the limitations for broadband quadrature modulator applications.
Nonlinear Behavioral Modeling of Quadrature Modulators and Analysis of Impacts on Wireless Communication Systems

by

Minsheng Li

A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the Requirements for the Degree of Doctor of Philosophy

Electrical Engineering

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2007

APPROVED BY:

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Chair of Advisory Committee

__________________________________        __________________________________
Dr. Huaiyu Dai                                            Dr. Mark Johnson
Dedication

To my fiancée, my parents, and my friends…
Biography

Minsheng Li was born on January 13, 1977 in Jilin Province, P. R. China. He received his bachelor’s degree in 2000 from Tsinghua University, Beijing, China.

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He will be joining RF Micro Devices Scotts Valley Design Center in the summer of 2007 as an analog/RF design engineer.
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>ACLR</td>
<td>Adjacent Channel Leakage Ratio</td>
</tr>
<tr>
<td>ACPR</td>
<td>Adjacent Channel Power Ratio</td>
</tr>
<tr>
<td>AM-AM</td>
<td>Amplitude modulation to amplitude modulation</td>
</tr>
<tr>
<td>AM-PM</td>
<td>Amplitude modulation to phase modulation</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
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<tr>
<td>DAC</td>
<td>Digital to Analog Converter</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DSB Modulation</td>
<td>Double Sideband Modulation</td>
</tr>
<tr>
<td>DUT</td>
<td>Device Under Test</td>
</tr>
<tr>
<td>EVM</td>
<td>Error Vector Magnitude</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>HD</td>
<td>Harmonic Distortion</td>
</tr>
<tr>
<td>IM&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Third order intermodulation distortion</td>
</tr>
<tr>
<td>IRR</td>
<td>Image Rejection Ratio</td>
</tr>
<tr>
<td>LSNA</td>
<td>Large Signal Network Analyzer</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>P1dB</td>
<td>1 dB gain compression point</td>
</tr>
<tr>
<td>PA</td>
<td>Power amplifier</td>
</tr>
<tr>
<td>PAR</td>
<td>Peak to Average Ratio</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PSD</td>
<td>Power Spectrum Density</td>
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<tr>
<td>PSS</td>
<td>Periodic Static State</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase-Shift Keying</td>
</tr>
<tr>
<td>RCE</td>
<td>Relative Constellation Error</td>
</tr>
<tr>
<td>RFIC</td>
<td>Radio frequency integrated circuit</td>
</tr>
<tr>
<td>SNDR</td>
<td>Signal to Noise plus Distortion Ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SSB Modulation</td>
<td>Single Sideband Modulation</td>
</tr>
<tr>
<td>VNA</td>
<td>Vector Network Analyzer</td>
</tr>
<tr>
<td>VSA</td>
<td>Vector Signal Analyzer</td>
</tr>
<tr>
<td>WCDMA</td>
<td>Wideband Code Division Multiple Access</td>
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Chapter 1

Introduction

1.1 Motivations

The quadrature modulator is an essential component in modern wireless communication systems because of their high spectrum efficiency. It is used in different manners in different applications including cellular phones, wireless data communication systems, Bluetooth, GPS, and etc. In order to efficiently design and optimize quadrature modulators, one concern is to understand how the quadrature modulators are used in these different applications, and how the physical impairments such as nonlinear distortion, the DC offset, and gain/phase imbalances impact the system performance in different wireless systems. Judicious design decisions can be made only after gaining such knowledge in order to achieve optimized circuit and system designs.

Another concern in quadrature modulator design is finding a way to quickly and accurately characterize the quadrature modulator physical impairments for system design and verification. System-level behavioral modeling techniques are desired because they
can simulate faster with less memory demands than circuit models. Recent literature addressed the modeling of the electrical characteristics of mixers at the circuit level [1–4] but very little work has been done for system-level nonlinear behavioral modeling of quadrature modulators. However, a considerable amount of work has been done for modeling the nonlinear characteristics of power amplifiers [5–14]. Because the nonlinear distortion in a quadrature modulator can cause significant performance degradation in terms of spectral growth and waveform quality, it is desirable to implement the nonlinear behavioral modeling techniques for power amplifiers to quadrature modulators for characterization the nonlinear distortion and other physical imperfections.

1.2 Summary of Research Contributions

Prevailing usage of quadrature modulators are represented by the following three systems: 1) a generic quadrature modulation system; 2) a zero-IF OFDM system; and 3) a low-IF OFDM system. Interestingly, the physical impairments of the quadrature modulators in different applications impact the system performance in different manners as well. Extensive analysis work was done in Chapter 3 to reveal the underlying mechanisms that result in the various impacts of the quadrature modulator physical imperfections in different applications. From these system and circuit designers gain a better understanding about the design tradeoffs, and wisely diagnose the physical imperfections appropriate for different quadrature modulator applications to improve their design and performance. The impact of the nonlinear distortion, of DC offset, and of gain/phase imbalances on quadrature modulator performance degradation in terms of spectral regrowth and waveform quality degradation are summarized in Table 3-4.
In this work, to the best knowledge of the author, for the first time an accurate nonlinear behavioral model is developed for characterization of the correlated/uncorrelated nonlinear distortion and of the linear static errors. These include the DC offset and the gain/phase imbalances of the direct-conversion quadrature modulators. Using this model the modulator performance degradations can be predicted efficiently. The baseband equivalent complex power series model structure is adopted because it is easy to implement without loss of accuracy for quadrature modulator applications. A novel AM-AM and AM-PM measurement technique is developed to extract the nonlinear model parameters. Earlier approach used a swept DC input, but here a swept offset single-tone sinusoidal input in the measurement is used so that the carrier leakage induced dynamic range reduction problem is overcome. For the extraction of the linear error model parameters, an accurate four-point VNA measurement technique is developed that is conveniently performed as only a DC source and a VNA are needed. An orthogonalization technique developed previously [15, 16] for power amplifiers was extended to quadrature modulator applications in this work so that the correlated and uncorrelated nonlinear distortion can be decomposed and thus the waveform quality degradation can be accurately predicted by the model. The details of model development, parameters extraction procedures, and model verifications are presented in Sections 4.1 and 4.2.

In integrated direct-conversion quadrature modulators, usually the baseband transconductors are the dominant nonlinear components, therefore, the nonlinear characteristics of the individual baseband transconductors are investigated to gain more insights regarding nonlinear quadrature modulator modeling. In Chapter 5, the design of a
particular type of linear transconductor, bipolar multi-tanh transconductor, is described and modeled for characterization of its nonlinear responses. The same nonlinear model structure as used in Chapter 4, a complex power series model structure based on the measured AM-AM and AM-PM data, is utilized for characterizing the nonlinear responses of the baseband and bandpass transconductors. The bandwidth limitations due to the input/output parasitics are analyzed and measured for the complex power series model of the baseband transconductors, based on which an augmented two-box model structure is proposed for broadband quadrature modulator applications.

The final contribution is partly related to the nonlinear characterization work which is about the modeling of multisine signals. Multisine signals have gained more and more attentions in behavioral modeling, testing, and characterization of nonlinear devices because they can mimic real communication signals to capture broadband nonlinear characteristics and characterize both in-band and out-of-band distortion. In Section 2.5, a novel FFT based technique for multisine signal generation was presented. Compared to other multisine design techniques, our approach is straightforward and easy to implement.

In summary, these contributions are listed below in the order they appear:

- The development of a novel FFT based technique for generating multisine signal, one particular type of signal for ease of nonlinear characterization (Section 2.5).
- System-level analysis of the impacts of quadrature modulator physical impairments on wireless systems in three different modulator applications: 1) generic quadrature modulations system, 2) zero-IF SSB OFDM system, and 3) low-IF SSB OFDM
system. The impacts of the DC offset, the gain/phase imbalances, and the nonlinear distortion are analyzed in Sections 3.2, 3.3 and 3.4, respectively.

- Behavioral model development and novel parameter extraction procedures for both passive (Section 4.1) and active (Section 4.2) direct-conversion quadrature modulators for characterization of nonlinear distortion and linear static errors.

- Design and modeling of baseband (Sections 5.2 and 5.4.1) and bandpass (Sections 5.2 and 5.4.2) transconductors for characterization of the nonlinear distortion. The bandwidth limitations of complex power series models for characterizing the baseband transconductors were analyzed and measured (Section 5.4.2). The limitations of the balun-based differential probing measurement technique were analyzed for nonlinear differential on-wafer probing characterization (Section 5.3).

1.3 Publications

The above research work leads to the following publications:

Current Publications:


**Future Publications:**


**1.4 Dissertation Organization**

Chapter 2 reviews the three different quadrature modulator applications in wireless communication systems, the background knowledge about the physical impairments of quadrature modulators, and the fundamentals of nonlinear behavioral modeling techniques, all of which serve as the basis for the later chapters. The multisine signal generating technique is documented in Section 2.5. In Chapter 3, the impacts of the quadrature modulator physical impairments on wireless systems in three different modulator applications are discussed. The model development and parameter extraction procedures for passive and active direct-conversion quadrature modulators are elaborated in Chapter 4. The design and modeling of the bipolar transconductors are presented in Chapter 5. Conclusions and future areas of work are presented in Chapter 6. As an extension to the transconductor design and modeling work in Chapter 5, a CMOS RF integrated direct-conversion quadrature modulator is designed and simulation results are presented in Appendix B.
Chapter 2

Nonlinear Modeling of Quadrature Modulator Physical Impairments

2.1 Overview of Quadrature Modulator Applications in Wireless Communication Systems

Since 1901 Guglielmo Marconi successfully transmitted radio signals across the Atlantic Ocean, wireless technology has undergone an incredible revolution, thanks to the inventions of the vacuum tubes [17, 18], the transistors [19], the integrated circuits [20], and the developments of Shannon’s information theory [21] and the cellular system conception. Affordable mobile communications in the areas such as cellular phones, Bluetooth, wireless data communications, global positioning system (GPS) have greatly facilitated human’s daily life today. For example, the Cellular Telecommunications & Internet Association (CTIA) reports that 233,040,781 wireless subscribers exist in US as of the end of 2006 [22]. This huge market in wireless communications drives the industry to
provide competitive services and equipments to raise the market share. A key factor of success is the provision of high performance transceivers with lower cost, lower power consumption, smaller size, higher reliability and more functionality, which pushes the designers to design the transceivers closely approaching the system requirements in order to maximize the yield.

Two types of transceiver architectures are prevailed in modern wireless systems: the super-heterodyne architecture [23–25] and the homodyne, also known as direct conversion architecture [24–26]. In this thesis, the focus is on the modeling of the direct conversion transmitters.

The direct conversion principle was first developed in 1932 as an attempt to surpass the super-heterodyne [26]. It was not widely used for a long time because it has a number of issues including DC offset, I/Q imbalances, and flicker noises. However, it has been resurrected in recent years due to its advantages of adaptability to different standards, small form factor, low cost, reduced bill of materials and low power consumption over super-heterodyne architecture [27]. Also, the improved semiconductor technology and system level optimization of the circuits make it possible to alleviate the above problems. As a good guidance to the designers, in [28], Abidi summarized the key problems of using direct-conversion transceivers and the various solutions to them. As shown in Figure 2-1, a typical direct conversion transmitter consists of a number of concatenated building blocks such as baseband processors, digital-to-analog converters (DAC), variable gain amplifiers (VGA), quadrature modulator, driver amplifier (DA), and surface acoustic wave (SAW) filters.
Among the many components in the wireless transmitters (both super-heterodyne and direct conversion), the quadrature modulator plays an important role by conducting both modulation and frequency conversion functions. The blue dash line enclosed block in Figure 2-1 is the simplified diagram of a direct-conversion quadrature modulator. The local oscillator (LO) input, at the carrier frequency, is split into both inphase and 90° quadrature phase signals which are fed to two balanced mixers. The baseband input signals are subdivided into two independent data streams, $i(t)$ and $q(t)$, and mixed with the in-phase and quadrature carrier signals separately and combined in-phase to generate linear quadrature modulation of the RF carrier. The major advantage of using the quadrature modulators is the increase of the spectrum efficiency because the $i(t)$ and $q(t)$ data streams can be modulated onto the same carriers orthogonally so that different quadrature modulation schemes such as quadrature amplitude modulation, quadrature phase modulation, or theirs combinations can be achieved.

![Figure 2-1 Block diagram of a typical direct conversion transmitter.](image)

One important property of the I/Q quadrature modulator is to perform single-sideband
(SSB) modulation using phasing to suppress the unwanted sideband. If \( i(t) \) and \( q(t) \) are 90 degree out of phase, the resulting modulator output signal is single sideband [29]. This property can be illustrated by a simple example below. Let

\[
i(t) = \cos(\omega_{bb}t); \quad q(t) = \sin(\omega_{bb}t).
\]

(2.1)

where \( \omega_{bb} \) is the baseband frequency. The modulator output is:

\[
w(t) = i(t)\cos(\omega_c t) - q(t)\sin(\omega_c t) = \cos(\omega_{bb}t)\cos(\omega_c t) - \sin(\omega_{bb}t)\sin(\omega_c t).
\]

(2.2)

where \( \omega_c \) is the carrier frequency. By trigonometry,

\[
w(t) = \cos[(\omega_c + \omega_{bb})t].
\]

(2.3)

The quadrature modulator output in Equation (2.3) is a single sideband result. In summary, if the input I and Q signals are 90 degree out of phase, after quadrature mixing with the carrier signals that are 90° out of phase and then summing together, a lower or upper sideband signal results.

Due to the ability of conserving bandwidth, the quadrature modulators have become very popular in modern wireless systems where high data bandwidth is a critical performance metric. The use of quadrature modulators can be categorized into three types of applications depending on the transceiver design and common practice for meeting specifications: 1) generic quadrature modulation systems; 2) zero-IF SSB OFDM systems; and 3) low-IF SSB OFDM systems.

### 2.1.1 Generic Quadrature Modulation Systems

The generic quadrature modulation systems are commonly used for cellular applications such as GSM/EDGE, CDMA, and WCDMA [25]. In these cellular systems,
the baseband information is subdivided and mapped to independent $i(t)$ and $q(t)$ data streams as shown in Figure 2-1 depending on different modulation schemes being used. The I and Q data streams are then upconverted to a single carrier with the quadrature modulators by means of double sideband (DSB) modulation. The carrier signal with amplitude and phase modulation can be represented as:

$$w(t) = \mathfrak{R}\left[\tilde{C}(t)e^{jo\omega t}\right].$$  \hspace{1cm} (2.4)

where $\tilde{C}(t)$ is the carrier information which is referred to as complex envelope.

$$\tilde{C}(t) = A(t)e^{j\theta(t)} = i(t) + jq(t).$$  \hspace{1cm} (2.5)

Plug (2.5) into (2.4), the carrier signal is:

$$w(t) = A(t)\cos[\omega t + \theta(t)] = i(t)\cos(\omega t) - q(t)\sin(\omega t).$$  \hspace{1cm} (2.6)

Viewed in the frequency domain as shown in Figure 2-2, the baseband modulation information of I and Q channel is upconverted to the single carrier $\omega_c$ and overlapped on top of each other orthogonally so that they can be separated and detected at the receiver, which means that using quadrature modulation same amount of RF spectrum can carriers double amount of information (I and Q) and this is the reason why quadrature modulators are able to improve spectrum efficiency.
2.1.2 Zero-IF Single Sideband (SSB) OFDM Systems

The zero-IF OFDM systems are popularly used for wireless data communication applications such as WLAN and WiMAX. The orthogonal frequency division multiplexing (OFDM) is a type of multi-carrier modulation technique, which splits the transmit bandwidth into multiple orthogonal sub-channels that are narrow enough and thus experience a flat fading although the overall radio propagation environment is frequency-selective. The spectra orthogonality is achieved by carefully selecting the carrier spacing so that each sub-carrier is located on all the other sub-carriers’ spectra zero crossing points, as shown in Figure 2-3. Each sub-carrier carries unique narrow-band modulated information in parallel and very high data rate can be obtained with many sub-carriers in a system. The OFDM systems can effectively tackle the inter symbol interference (ISI) problem and allow to achieve high data rate in a frequency-selective radio propagation environment [30, 31]. The OFDM modulation technique is proposed as the air-interface solution for wireless local area networks (WLANs) [32], wireless
metropolitan area networks (WMAN) [33], and possibly for the future fourth-generation mobile cellular wireless systems [34].

**Figure 2-3 Spectrum of a RF modulated signal in a zero-IF SSB OFDM modulation system.**

One implementation of the OFDM transceiver is the zero-IF architecture [35], where all the sub-carriers are located symmetrically around the carrier frequency, as presented in Figure 2-3. The I/Q inputs of a zero-IF OFDM signal to the quadrature modulator are presented in Equation (2.7).

\[
\begin{align*}
i(t) &= \Re\left( \sum_{k=-N}^{N} C_k(t)e^{j\omega_k t} \right) = \sum_{k=-N}^{N} \left[ I_k \cos(\omega_k t) - Q_k \sin(\omega_k t) \right] \\
q(t) &= \Im\left( \sum_{k=-N}^{N} C_k(t)e^{j\omega_k t} \right) = \sum_{k=-N}^{N} \left[ I_k \sin(\omega_k t) + Q_k \cos(\omega_k t) \right]
\end{align*}
\]  

(2.7)

where \( C_k(t) = I_k(t) + jQ_k(t) = A_k(t)e^{j\theta_k(t)} \) represents the location of the symbols within the constellation for the \( k_{th} \) subcarrier at different symbol time, i.e., the modulation information transmitted by the \( k_{th} \) sub-carrier. Notice that \( i(t) \) and \( q(t) \) of a zero-IF OFDM signal are Hilbert transform related so that each sub-carrier is upconverted to the carrier.
frequency by single-sideband modulation. Different from the generic quadrature modulation system, the zero-IF OFDM signals have to employ the quadrature modulators’ single-sideband modulation property to upconvert each sub-carrier. The RF zero-IF OFDM modulated signal is:

$$w(t) = \Re\left\{ \sum_{k=-N}^{N} C_k(t) e^{j\omega_k t} e^{j\omega_{IF} t} \right\} = \sum_{k=-N}^{N} A_k(t) \cos[(\omega_c + \omega_k) t + \theta_k(t)].$$  (2.8)

Equation (2.8) shows that each sub-carrier is modulated by the unique information $A_k(t)$ and $\theta_k(t)$ and upconverted to single-sideband $(\omega_c + \omega_k)$. The spectrum of a zero-IF OFDM signal represented by Equation (2.8) is shown in Figure 2-3. Note that in a real OFDM system, the sub-carrier at the carrier frequency is usually not used to avoid carrier leakage problem, which will be discussed in Section 1.2.1.

### 2.1.3 Low-IF Single Sideband (SSB) OFDM Systems

Another implementation of the OFDM transceiver is the low-IF architecture [36, 37], where all the sub-carriers are located symmetrically around the LO plus an IF frequency $(\omega_{LO} + \omega_{IF})$ instead of the carrier frequency, as shown in Figure 2-4. The I/Q inputs of a low-IF OFDM signal to the quadrature modulator are shown in Equation (2.9). Compared to Equation (2.7), a digital upconversion function $e^{j\omega_{IF} t}$ is added in low-IF architecture.

$$i(t) = \Re\left\{ \sum_{k=-N}^{N} C_k(t) e^{j\omega_k t} e^{j\omega_{IF} t} \right\} = \sum_{k=-N}^{N} \left\{ I_k \cos[(\omega_k + \omega_{IF}) t] - Q_k \sin[(\omega_k + \omega_{IF}) t] \right\}$$

$$q(t) = \Im\left\{ \sum_{k=-N}^{N} C_k(t) e^{j\omega_k t} e^{j\omega_{IF} t} \right\} = \sum_{k=-N}^{N} \left\{ I_k \sin[(\omega_k + \omega_{IF}) t] + Q_k \cos[(\omega_k + \omega_{IF}) t] \right\}.$$  (2.9)

where $C_k(t) = I_k(t) + jQ_k(t) = A_k(t)e^{j\theta_k(t)}$ represents the location of the symbols within the constellation for the $k$th subcarrier at different symbol time, i.e., the modulation.
information transmitted by the \(k\)th sub-carrier. The RF low-IF OFDM signal is:

\[
w(t) = \Re \left( \sum_{k=-N}^{N} C_k(t)e^{j\omega_c t} e^{j\omega_{IF} t} \right) e^{j\omega_{LO} t} = \sum_{k=-N}^{N} A_k(t) \cos[(\omega_c + (\omega_k + \omega_{IF}))t + \theta_k(t)].
\]  

Equation (2.10) shows that each sub-carrier is modulated by the unique information \(A_k(t)\) and \(\theta_k(t)\) and upconverted to single-sideband \([\omega_c + (\omega_k + \omega_{IF})]\). Similar as the zero-IF OFDM signals, the low-IF OFDM signals have to utilize the quadrature modulators’ single-sideband modulation property to upconvert each sub-carrier that carries unique modulation information.

![Figure 2-4 Spectrum of a RF modulated signal in a low-IF SSB modulation system.](image)

Compared to the zero-IF OFDM architecture, the low-IF OFDM architecture has the advantages of reduced in-band distortion and no SNR degradation due to the carrier feedthrough and the quadrature gain and phase imbalances. The drawbacks of the low-IF are the more stringent requirement on the DACs because the sampling rate has to be at least doubled and the increased adjacent channel interferences. The tradeoffs between the
zero-IF OFDM and low-IF OFDM architectures will be discussed in detail in Chapter 4.

2.2 Analog Impairments of I/Q Quadrature Modulators

Imperfections in analog circuits such as device mismatches, nonlinearity and noise will cause degradation of I/Q quadrature modulator performance. Such physical impairments include DC offset, gain/phase imbalances, nonlinear distortion, phase noise, frequency error, spurious, and etc. The analog imperfections produce error products which result in degradation of signal-to-noise ratio (SNR) or equivalently EVM and undesired spectral occupancy of the transmitted signals in the wireless systems. These quadrature modulator impairment impacts are possible to be alleviated. For example, the techniques for estimating and compensating the DC offset and quadrature imbalances were proposed in [38, 39].

In this thesis, the DC offset, gain/phase imbalances, and nonlinear distortion in a direct conversion quadrature modulators were accurately modeled and the effects on the signal quality were analyzed.

2.2.1 DC Offset (Carrier Leakage)

DC offset in a quadrature modulator results from the device mismatches such as threshold voltage mismatch, device size mismatch, resistor mismatch, and etc. It leads to carrier feedthrough, which is a major concern in designing direct-conversion transceivers. In a quadrature modulator, the mismatch can be modeled as a DC offset in the baseband input signals, as shown in Equation (2.11).

\[
\widetilde{C}(t) = [V_{os, i} + i(t)] + j[V_{os, q} + q(t)].
\]
where $V_{OS,I}$ and $V_{OS,Q}$ are the DC offset voltage, $i(t)$ and $q(t)$ are the desired I and Q modulation data.

In a generic quadrature modulation system, the DC offset causes an offset of the signal constellation by the amount of $V_{OS,I}$ and $V_{OS,Q}$ [40] as shown in Figure 2-5 an offset QPSK constellation diagram.

![Figure 2-5 Effect of DC offset on QPSK constellation.](image)

The RF modulated signal with a DC offset is:

$$w(t) = \Re(\tilde{C}(t)e^{j\omega t}) = \left[ V_{OS,I} \cos(\omega_c t) - V_{OS,Q} \sin(\omega_c t) \right] + \left[ i(t) \cos(\omega_c t) - q(t) \sin(\omega_c t) \right]. \quad (2.12)$$

As seen in Equation (2.12), the DC offset produces a carrier leakage term $\left[ V_{OS,I} \cos(\omega_c t) - V_{OS,Q} \sin(\omega_c t) \right]$, which is uncorrelated to the desired signal $\left[ i(t) \cos(\omega_c t) - q(t) \sin(\omega_c t) \right]$ and causes the SNR degradation. In the zero-IF and low-IF OFDM systems, usually there is no sub-carrier at DC so that the carrier leakage will not cause SNR degradation. However, the carrier leakage is still possible to cause violation of
the spectrum mask.

2.2.2 Gain and Phase Imbalances

As shown in Figure 2-1, in the quadrature modulators, the LO output needs to be shifted by 90° to perform quadrature mixing. The analog impairments can cause errors in the 90° phase shift and mismatches of the amplitude of the I and Q channels. These gain and quadrature phase errors result in distortion of the transmitted signal constellation and ultimately the degradation of the bit error rate (BER). The effects of the gain and quadrature phase errors on the transmitted signal constellation can be illustrated by a QPSK example. Suppose the baseband signal \( i(t) \) and \( q(t) \) are:

\[
i(t) = a \\
q(t) = b
\]  \hspace{1cm} (2.13)

where \( a \) and \( b \) are either +1 or -1. With the existence of the gain and phase imbalances, let assume the LO signals fed to the I and Q channels are:

\[
V_{LO,I} = \left( 1 + \frac{\varepsilon}{2} \right) \cos \left( \omega_c t + \frac{\theta}{2} \right) \\
V_{LO,Q} = \left( 1 - \frac{\varepsilon}{2} \right) \sin \left( \omega_c t - \frac{\theta}{2} \right)
\]  \hspace{1cm} (2.14)

where \( \varepsilon \) and \( \theta \) represent the gain and phase errors. The RF modulated output signal is:

\[
w(t) = \left( a \left( 1 + \frac{\varepsilon}{2} \right) \cos \left( \omega_c t + \frac{\theta}{2} \right) - \left( 1 - \frac{\varepsilon}{2} \right) \sin \left( \omega_c t - \frac{\theta}{2} \right) \right).
\]  \hspace{1cm} (2.15)

Simplify (2.15) by trigonometry,
\[ w(t) = \left[ a \left( 1 + \frac{\varepsilon}{2} \right) \cos \left( \frac{\theta}{2} \right) + b \left( 1 - \frac{\varepsilon}{2} \right) \sin \left( \frac{\theta}{2} \right) \right] \cos(\omega t) \]

\[ - \left[ b \left( 1 - \frac{\varepsilon}{2} \right) \cos \left( \frac{\theta}{2} \right) + a \left( 1 + \frac{\varepsilon}{2} \right) \sin \left( \frac{\theta}{2} \right) \right] \sin(\omega t). \]  

(2.16)

By a close examination of (2.16), it can be found the equivalent baseband signal \( i'(t) \) and \( q'(t) \) with the existence of gain and quadrature phase errors are:

\[ i'(t) = a \left( 1 + \frac{\varepsilon}{2} \right) \cos \left( \frac{\theta}{2} \right) + b \left( 1 - \frac{\varepsilon}{2} \right) \sin \left( \frac{\theta}{2} \right) \]  

\[ q'(t) = b \left( 1 - \frac{\varepsilon}{2} \right) \cos \left( \frac{\theta}{2} \right) + a \left( 1 + \frac{\varepsilon}{2} \right) \sin \left( \frac{\theta}{2} \right). \]  

(2.17)

As shown in Equation (2.17), the baseband information was distorted by the I/Q gain and phase errors. Their effects on the transmitted QPSK signal constellation were shown in Figure 2-6: the gain error changes the signal constellation from square to rectangular, and the quadrature phase error causes skew of the constellation.

The I/Q gain and phase imbalances are usually characterized by the single tone SSB
image rejection ratio (IRR) [25]. In such a test, the single tone SSB baseband signals as described in (2.1) were applied to the quadrature modulator with gain and phase errors. The resulting RF spectrum was shown in Figure 2-7. Besides the desired upper sideband ($\omega_c + \omega_{BB}$), there is an image spectrum generated at ($\omega_c - \omega_{BB}$). The IRR is defined as the ratio of the image power spectrum to the desired power spectrum.

$$\text{IRR} = 10 \log \left( \frac{1 - 2\sqrt{\Delta} \cos(\theta) + \Delta}{1 + 2\sqrt{\Delta} \cos(\theta) + \Delta} \right). \quad (2.18)$$

where $\theta$ is the quadrature phase error in degree and $\Delta$ is the square of the ratio of the amplitude of I channel to that of Q channel.

Figure 2-7 Single sideband image rejection.

The quadrature errors in the generic quadrature modulation systems can be seen as the I/Q power leaking onto Q/I. This causes a "distortion floor" that leads to the degradation of the SNR or EVM of the signal. A generic quadrature modulation system is single carrier system. The gain and quadrature phase errors cause the distortion of the signal
constellation as shown in Figure 2-6.

In a zero-IF OFDM modulation system, the results of the I/Q imbalances can be seen as imperfect sideband cancellation. With the existence of the gain/phase imbalances, the \( k^{th} \) sub-carrier generates an image spectrum at the \( -k^{th} \) sub-carrier, as shown in Figure 2-8, which is uncorrelated to the desired signal so that its impacts are very similar to the additive white Gaussian noise (AWGN) degrading the SNR and EVM of the zero-IF OFDM signals. Because the zero-IF OFDM system is a multi-carrier system with each sub-carrier carrying unique information, the I/Q imbalances cause distortion of the constellation of each sub-carrier and the total effect is manifested as a spreading of the constellation in a noise-like fashion [40].

![Image Rejection](image)

**Figure 2-8 Image rejection characteristics for a zero-IF OFDM system.**

The advantage of a low-IF OFDM modulation system is that the I/Q imbalances won’t distort the transmitted signal constellation and cause SNR or EVM degradation. This doesn’t mean that no image spectra are generated from the I/Q imbalances, but the image
spectra are shifted out of the signal bandwidth to the adjacent sideband as shown in Figure 2-9 so that no SNR or EVM degradation occurs. However, it is obvious that the signal in the adjacent channel will suffer because now the image products are the interferences to the adjacent channel.

Figure 2-9 Image rejection characteristics for a low-IF OFDM system.

2.2.3 Nonlinear Distortion

Nonlinear distortion in analog/RF circuit and system design poses a challenging problem to the designers. When an amplitude or phase modulated signal passes through a nonlinear system, new frequency components are usually generated in the output, as shown in Figure 2-10(a). The nonlinearities produce nonlinear distortion both inside and outside the signal band, as shown in Figure 2-10(b). The out-of-band nonlinear distortion leaks into the adjacent channels and causes SNR degradation of the signals in the adjacent bands. Therefore, in wireless system designs, stringent requirements are specified for the allowed maximum out-of-band emissions. Another form of nonlinear distortion is generated by
means of cross modulation that transfers the modulation on the amplitude of the interferer to the amplitude of the desired signal. It is especially an important problem in a multi-channel system [41]. Also shown in Figure 2-10(b), the in-band distortion can be decomposed into two components: one is correlated with the desired signal, which causes gain compression or expansion; the other is uncorrelated with the desired signal and behaves like AWGN to degrade the effective system SNR or equivalently the EVM of the desired transmitted signals, ultimately leads to the degradation of the BER.

Figure 2-10 Illustration of nonlinear distortion: (a) example input and output spectrum in a nonlinear system; (b) zoomed view of the output spectrum.

The effective system SNR is the ratio of the signal power to the total noise plus the uncorrelated in-band distortion power, which is usually referred to as the signal to noise
and distortion power (SNDR) [42, 43]. It is a function of both the nonlinear distortion and the AWGN, as shown in Equation (2.19).

\[
\text{SNDR} = \sqrt{\frac{P_{oc}}{P_{od} + P_n}}.
\]  

(2.19)

where \( P_{od} \) is the uncorrelated distortion power, \( P_n \) is the power of the AWGN, and \( P_{oc} \) is the power of the output components correlated to the input signals.

The EVM is an important figure-of-merit for characterizing the waveform quality for digital modulated signals, which is a measure of the difference between the reference waveform and the actual waveform. It is defined as the square root of the ratio of the mean error vector power to the mean reference power expressed as a %. EVM is directly related to the system SNDR [43]:

\[
\text{EVM(\%)} = \sqrt{\frac{1}{\text{SNDR}}} \times 100\%.
\]  

(2.20)

In OFDM systems, instead of using the term of EVM, Relative Constellation Error (RCE) is used as the figure-of-merit which is exactly equivalent to EVM:

\[
\text{RCE(dB)} = 20\log(\text{EVM}).
\]  

(2.21)

Compare (2.20) and (2.21), it is clear that

\[
\text{RCE(dB)} = -\text{SNDR(dB)}.
\]  

(2.22)

so that RCE is a straightforward measure of SNDR of the OFDM signals.

In the thesis, three classes of digital wireless signals, WCDMA, zero-IF WLAN, and low-IF WLAN signals were extensively used in system analysis and model validation work. The EVM/RCE specifications for these transmit signals were summarized in Table
Modern wireless communications systems such as GSM/EDGE, CDMA, WCDMA, WLAN and WiMAX are multi-channel systems where the RF spectrum is split into multiple channels to accommodate multiple users [46]. Out-of-band emissions from the transmitter leaking into adjacent channels can degrade the SNR of the signals in those channels. Therefore, stringent requirements are specified in various digital wireless standards for maximum allowable out-of-band emissions from the transmitter. ACPR and spectrum mask are two important factors for this use.
As shown in Figure 2-10(b), the spectrum mask specify a set of limiting value on the transmitted signal spectra over frequency and no spectrum can go beyond the mask. ACPR is a key figure-of-merit in quantizing the adjacent channel interference, which is defined as the ratio (in decibel) of the distortion power in a certain bandwidth with a certain frequency offset from the carrier frequency, and the power in the desired channel with a certain information bandwidth. As shown in Figure 2-10(b), the ACPR is calculated as

$$\text{ACPR}_{\text{upper}} = \frac{\int_{f_1}^{f_2} S_{gg}(f) \cdot df}{\int_{f_1}^{f_2} S_{gg}(f) \cdot df}.$$  \hspace{1cm} (2.23)$$

where $f_1$ and $f_2$ are the lower and upper frequency limit of the main channel; $f_3$ and $f_4$ are the lower and upper frequency limit of the adjacent channel. Each wireless standard has its own specification values for the definition of ACPR in order to evaluate the impacts of the out-of-band interferences on the SNR degradation of the user signals in the adjacent channel. Note that WCDMA systems use a different term name, adjacent channel leakage ratio (ACLR), which is exactly the same as ACPR. In WLAN systems, there is no ACPR definition because the EVM is a more stringent system specification.

The ACLR specifications for WCDMA transmit signals were listed in Table 2-1. The spectrum mask for WLAN transmit signals is shown in Figure 2-11.

$$V_{Os,J} \cos(\omega_c t) - V_{Os,Q} \sin(\omega_c t)$$
In [15, 16], the authors successfully developed an orthogonal behavioral model to decompose the correlated and uncorrelated components of the output signal from RF power amplifiers so that the system SNR or EVM degradation can be estimated. In this thesis, this orthogonal behavioral modeling technique for power amplifiers is extended to the direct-conversion quadrature modulators for uncorrelated and correlated in-band distortion decomposition.

2.3 Behavioral Modeling Techniques for Nonlinear Circuits

As a compact representation of a circuit or a system, a behavioral model typically simulates much faster and requires less memory than its circuit-level counterpart, which makes it a powerful tool for system design and verification. Behavioral modeling is a black-box modeling approach to characterize a device since it can be generated without knowing the detailed circuit structure. This property also leads to another advantage of a behavioral model that the vendors can supply the customers only a black-box model
without revealing the detail information of a device to protect the intellectual property [47, 48].

A behavioral model is usually developed based on either simulation or measurement data, which are a set of wisely selected input-output observations. Compared to simulation based model development, a measurement based model development can capture actual nonlinear characteristics of a device since it is not affected by the inaccuracy of the underlying circuit models. Obviously, the accuracy of the behavioral models heavily depends on the adopted model structure and model parameter extraction procedure [47, 49].

In this thesis, a measurement based behavioral model for a direct-conversion quadrature modulator was developed based on the previous modeling techniques for power amplifiers.

2.3.1 Behavioral Modeling for Power Amplifiers

Design of the power amplifier in RF systems is almost the most challenging task due to its nonlinear impairments and high power output/efficiency requirement. A lot of work has been done in the area of behavioral modeling for power amplifiers in order to accurately characterize the nonlinear behavior in a PA [6, 7, 9–14, 50–54]. Most PA models are based on bandpass nonlinearity concepts that only the output signal envelope responses around the carrier frequency were captured known that only the signal envelope carries the useful information and the final PA outputs are bandpass filtered. It differs from the instantaneous PA models that deal with the complete RF signal including all the nonlinear effects and harmonics. The bandpass nonlinearity based PA models greatly ease the
simulation in terms of simulation time and memory requirements by ignoring the carrier effects and the higher order harmonics.

The simplest but very popular PA behavioral model for modeling the nonlinear distortion is the memoryless model. The commonly used baseband equivalent memoryless models are in the form [55]:

\[
\tilde{z}_{RF}(t) = \sum_{k=0}^{K} b_{2k+1} [\tilde{z}(t)]^{k+1} [\tilde{z}^*(t)]^k.
\]  

(2.24)

where \(\tilde{z}_{RF}(t)\) is the baseband equivalent output of the PA and \(\tilde{z}(t)\) is the baseband equivalent input of the PA. \((\cdot)^*\) is the complex conjugate.

The system is called a memoryless model if the coefficients \(b_{2k+1}\) are real, which models the bandpass AM-AM characteristics and can be obtained by single-tone AM-AM power sweep measurement. The models in [50, 51] belong to this genre. It is called a quasi-memoryless model if the coefficients \(b_{2k+1}\) are complex, which models the bandpass AM-AM and AM-PM characteristics and can be developed by polynomial fitting to single-tone AM-AM and AM-PM power sweep measurement data. By incorporating the AM-PM conversion effects in the quasi-memoryless models, the effect of phase shift in the output waveform with different input power levels is accounted for appropriately. The models in [6, 7, 52, 53] are the examples of quasi-memoryless PA models. The memoryless model is simple to develop and the distortion can be directly related to the model parameters. However, the use of memoryless models has restricted conditions. First, the signal bandwidth should be narrow compared to the input and output filters so that they can be viewed as flat filters to the bandpass signals. Second, no odd-order distortion should be
produced from even-order nonlinearities in the active devices [49]. The memoryless models cannot take into account the linear and nonlinear memory effects and the use is limited in wideband and multi-channel systems.

It is well known that PAs present linear memory effects at the input and output of the devices due to the input and output tuned networks. Besides these linear memory effects, there are some dynamic memory effects resulting from the second order interactions, the active-device low-frequency dispersion, electrothermal interactions, and bias circuitry [56–58]. These dynamic memory effects show up only in nonlinear regimes and are called long-term memory effects. Extensions from the memoryless nonlinearities are needed to model the memory effects due to the bandpass system bandwidth limitations. A filter before and after the memoryless block that results in two-box or three-box models are necessary to account for the shifts in AM-AM and AM-PM plots [14, 54]. More complicated model structures are developed to handle the nonlinear memory effects arising from the second order interactions, active-device low-frequency dispersion, electrothermal interactions, and bias circuitry [9–13].

As the advance of the PA behavioral modeling techniques, it seems that the quality of the models is more dominated by the parameter extraction process than the model structure itself. With the models being more sophisticated, the parameter extraction process is getting more complicated and often direct parameter extraction is unable to be achieved [49].

2.3.2 Behavioral Modeling for I/Q Quadrature Modulators

Direct conversion I/Q quadrature modulator plays an important role in RF systems by
performing quadrature modulation and up-converting. It introduces nonlinear distortion mainly from the baseband nonlinearities and linear distortion as well due to carrier leakage, gain and phase imbalances. The impairments in the quadrature modulators can degrade the performance of wireless communication systems. In recent literatures, modeling techniques to capture circuit-level behaviors of RF mixers have been introduced. The authors in [4] develop scattering parameters based models for mixers for use with vector network analyzers to characterize the linear frequency translation behaviors. Reference [1] presents a frequency domain memoryless nonlinear behavioral model for RF mixers. A linearized mixer model as a conversion matrix [2, 3] was developed to characterize both the frequency conversion signals and the main mixing products and the LO leakage. It has limitation due to the intrinsic input to output linearity of the conversion matrix which leads to lack of accuracy in predicting the spectral regrowth. These works focus on the characterization of detail performances of mixer blocks. A behavioral model to describe the system level performance of RF quadrature modulators is however necessary to facilitate RF systems design. In [38], the authors modeled the frequency-dependent gain/phase imbalance and the DC offset by constructing channel models and using least-square estimation to obtain the model parameters in order to pre-compensate the impairments in the digital predistortion linearization system. Huang et al. [39] estimated the gain/phase and DC offset by applying least-square-based technique to the measured instantaneous power at the input and output of the transmitter. In [47], the authors modeled a quadrature modulator nonlinear responses by extracting the model parameters with a pulsed DC signal input. The authors in [59] presents a time-domain baseband equivalent
behavioral model for RF mixers for system-level characterization, however, it only captures the AM-AM nonlinear responses of the mixer and it is desirable to have a frequency domain behavioral model for ease of simulation. To the best knowledge of the author, there is no research work on system-level behavioral modeling of the I/Q quadrature modulators that completely characterizes both linear and nonlinear distortion, which motivates the present work.

2.4 Bandpass Nonlinearity Analysis

The signals in RF systems are narrow-band and band-selected signals and therefore the bandpass nonlinearity concept is widely used in the analysis of nonlinear RF systems. The concept of bandpass nonlinearity was developed in 1950s by information theorists in order to simplify the analysis of the impact of the nonlinear circuits on the degradation in C/N when a modulated RF carrier passes through the nonlinear circuits followed by a bandpass filter centered at the carrier frequency [60]. A complex envelope representation of a modulated carrier signal can be expressed as:

\[
 w(t) = A(t) \cos(\omega_c t + \theta(t)) = \frac{1}{2} \tilde{z}(t)e^{j\omega_c t} + \frac{1}{2} \tilde{z}^*(t)e^{-j\omega_c t}. \quad (2.25)
\]

where \( A(t) \) and \( \theta(t) \) are the amplitude and phase modulation components, \( \omega_c \) is the RF carrier, and \((\ )^*\) is the complex conjugate.

The carrier modulation is contained in the complex envelope, \( \tilde{z}(t) \), which can be represented in either polar or rectangular form

\[
 \tilde{z}(t) = A(t)e^{j\theta(t)} = i(t) + j \cdot q(t). \quad (2.26)
\]
where $i(t)$ and $q(t)$ are the in-phase and quadrature components of the baseband signal. The bandpass nonlinearity can be modeled and analyzed using complex power series [61], complex power series combined with statistical techniques [6, 62], Chebyshev transformations [63], Volterra series analysis [64], and etc.

### 2.4.1 Bandpass Nonlinearity for Power Amplifiers

The bandpass nonlinearity concept simplified the analysis for power amplifiers by eliminating the need to consider other nonlinear terms harmonically related to the carrier frequency since all the distortion components except the ones centered at the carrier frequency are eliminated by the bandpass filter [5, 15], as shown in Figure 2-12 where $w(t)$ is described by Equation (2.25). Note that the application of bandpass nonlinearity requires that the modulation bandwidth is narrow compared to the carrier frequency so that the observed output spectrum is not affected by distortion terms from other harmonics related to the carrier.

![Figure 2-12 Block diagram of the bandpass nonlinearity of a power amplifier.](image)

A simple way to characterize the bandpass nonlinearity for power amplifiers is to use the quasi-memoryless model, i.e.

$$
\tilde{G}[w(t)] = \sum_{k=0}^{K} \tilde{a}_{2k+1} w^{2k+1}(t). 
$$

(2.27)

Note that $\tilde{a}_{2k+1}$ characterizes the instantaneous amplitude and phase responses of the
nonlinearity. By a couple lines of algebra on binomial expansion, the complex envelope of the first zonal filter output \( \tilde{G}_{o_i}[\tilde{z}(t)] \) (the output terms center around the carrier frequency without all the other higher order terms) is [5]:

\[
\tilde{G}_{o_i}[\tilde{z}(t)] = \sum_{k=0}^{K} \left\{ \frac{\tilde{a}_2}{2^{2k}} \left( \frac{2k + 1}{k + 1} \right) \tilde{z}(t)^{k+1} \tilde{z}^*(t)^k \right\}.
\] (2.28)

The typical way to extract the model parameters \( \tilde{a}_2 \) is to do an AM-AM and AM-PM measurement or simulation followed by a least-square curve fitting. Note that the fitted coefficients \( \tilde{b}_{2k+1} \) have already taken into account the carrier effects:

\[
\tilde{b}_{2k+1} = \frac{\tilde{a}_2}{2^{2k}} \left( \frac{2k + 1}{k + 1} \right).
\] (2.29)

### 2.4.2 Bandpass Nonlinearity for Quadrature Modulators

The bandpass nonlinearity of quadrature modulators differs from that of power amplifiers as a result of the fact that the dominant nonlinearities in integrated quadrature modulators are the baseband transconductors. Assuming the I and Q channels are isolated, a bandpass nonlinearity representation for a direct-conversion quadrature modulator is shown in Figure 2-13. Because I and Q baseband transconductors process the baseband information, the outputs of the I and Q channel nonlinearities, \( \tilde{G}_I[i(t)] \) and \( \tilde{G}_Q[q(t)] \) contain the dominant instantaneous amplitude and phase response, which are subsequently up-converted to the RF spectrum terms around the carrier frequency. Different from the power amplifiers, the I and Q channel outputs after the first zonal filter, \( \tilde{G}_{I,o_i}[\tilde{z}_I(t)] \) and \( \tilde{G}_{Q,o_i}[\tilde{z}_Q(t)] \) include the upconverted instantaneous responses out of the I and Q baseband
nonlinearities, and so does the final quadrature modulator output, \( \tilde{G}_{\omega_0} \left[ \tilde{z}_{RF} (t) \right] \).

Figure 2-13 Block diagram of the bandpass nonlinearity of a quadrature modulator.

2.5 Multisine Signal Modeling

Because the development of behavioral models usually relies on input-output observations, the selection of signal excitations used for parameter extraction and validation plays an important role in determining the quality of a behavioral model. Traditionally measured or simulated AM-AM and AM-PM characteristics based on single tone excitation is used to model narrow band nonlinear circuits. A single tone excitation is simple and eases the development of the behavioral models. However, it has its limitations. First, only odd-order nonlinear characteristics can be modeled by a single tone AM-AM and AM-PM data. The even-order nonlinearity can lead to important memory effect which manifest as IM3 asymmetry. Due to the incapability of a single-tone excitation, two-tone excitations were used to capture the even-order nonlinear [9, 57]. Second, a single-tone measurement, or even a two-tone excitation, is insufficient to model a wide-band nonlinear circuit like a multi-channel power amplifier because the characteristics of a wide-band
nonlinear system are different at different frequency, which demands more complicated signal excitations in order to capture the wideband characteristics. Recently much attention has been paid to multisine as the excitation, especially with the aid of large signal network analyzer (LSNA), in order to model the power amplifiers [12, 65–68].

A multisine signal excitation consists of $N$ equally spaced sinewaves, with random or constant amplitudes and phases. Compared to digitally modulated signals, they are easier to generate and simulate by most time/frequency domain simulators. These features render multisine stimuli considerable usefulness not only in behavioral modeling, but also in the areas of characterization of spectrum regrowth both out-of-band and in-band [15, 69–71], device or system testing, and calibration of nonlinear-vector network analyzers [72].

Due to the increasing importance of multisine excitations, in this section an approach for generating multisine to accurately represent digitally modulated signals was presented. Tradeoffs in the number of tones required to accurately represent a particular number of symbols for a given system are discussed. A multisine representation of a CDMA reverse link signal was designed and verified through measurements of ACPR and EVM when the signal is applied to a nonlinear amplifier.

2.5.1 Generation of Bandpass Multisines

Remley [69] examined four types of multisines each with different magnitude/phase relationships to approximate digitally modulated signals in ACPR measurement. Among the four multisines, although the multisine with a closely matched peak-to-average ratio (PAR) to the digital signal had better ACPR results, none of the multisine signals accurately predicted the ACPR of the actual digital modulation. In [73], the authors
designed their bandpass multisines by matching the output power spectral density or higher order statistics of a digital signal in a general nonlinear dynamic system. One limitation of this approach is the expensive computation cost of matching the higher order signal statistics even with small amount of tones. The multisine signals from both efforts were used to primarily model out-of-band spectral regrowth distortion. They are not easily applied towards predicting waveform quality figures-of-merit such as SNR and EVM. In this section, an easy and straightforward approach based on discrete Fourier transform (DFT) transformation was presented to generate a multisine which can accurately represent a digital communication signal.

Construction of multisines from Fourier coefficients begins by defining a bandpass multisine as a finite sum of sinusoids with unique amplitudes and phases:

\[
 x(t) = \sum_{j=-(N-1)/2}^{(N-1)/2} A_j \cos \left[ 2\pi \cdot (f_c + j \cdot \Delta f) \cdot t + \theta_j \right]. \quad (2.30)
\]

where \(N\) is the number of sinusoids, \(A_j\) is the amplitude, \(f_c\) is the carrier frequency, \(\Delta f\) is the frequency spacing, and \(\theta_j\) is the phase. The design goal is to determine the four parameters, \(N, A_j, \Delta_f,\) and \(\theta_j\).

Recall that a discrete signal \(x(n)\) can be represented by its DFT coefficients as:

\[
 x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j2\pi kn/N} \quad n=0,1,2,\ldots,N-1. \quad (2.31)
\]

where \(X(k)/N\) are the normalized DFT coefficients of \(x(n)\). Equation (2.31) can be rewritten in the form as
where $X(k)/N = A_k e^{j\theta_k}$. This means a discrete signal $x(n)$ is equivalent to a sum of
sinewaves whose amplitude and phase are defined by the DFT coefficients of the signal
and frequency spacing is determined by the frequency resolution of the DFT. Thus a
multisine representing sampled signal can be constructed in a straightforward way by using
the DFT coefficients to define the amplitude, phase, and frequency spacing of the tones.

One drawback to use DFT coefficients is that a signal with $N$ sample points will yield
$N$ DFT coefficients that results in a large number of tones required to represent the sampled
signal. However, careful examination of the power spectrum of the RF signals shows that
the majority of the total signal power is contained within a small fraction of the total
bandwidth of the DFT, as shown in Figure 2-14 which is the power spectrum of a typical
IS-95 reverse-link signal. The information content out of this band energy is negligible.
The total number of tones necessary to represent the signal is greatly reduced if the
out-of-band coefficients can be neglected, which give us a possible solution to design a
multisine by truncating the bandwidth of the original signal.
The impact of truncating the bandwidth was quantified by investigating the RMS error introduced by discarding the out-of-band spectral components for different normalized truncation bandwidth. The normalized truncation bandwidth is defined as the ratio of the truncated bandwidth to the modulation bandwidth of the information signal. For instance, the signal modulation bandwidth of a reverse link IS-95 signal is 1.2288 MHz. The truncated bandwidth was decided by comparing the root-mean-squared (RMS) percentage errors between the real IS-95 signal waveform and the equivalent multisine waveform for different truncated bandwidths.

\[
\text{Normalized truncation BW} = \frac{\text{Truncated BW}}{\text{Modulated BW}}. \tag{2.33}
\]

The RMS percentage error was calculated as:
\[ \text{ERR}_{\text{rms}} (%) = 100 \frac{\sqrt{\sum_{k=1}^{N} (w_k - x_k)^2}}{\sum_{k=1}^{N} (w_k)^2}. \] (2.34)

where \( w_k \) is the original signal waveform, \( x_k \) are the time samples of the multisine signal waveform obtained from an inverse DFT (IDFT), and \( N \) is the total number of time points.

For the IS-95 reverse link signal, the RMS percentage errors for different normalized truncated bandwidth are shown in Figure 2-15. The RMS percentage error decreases with an increase of truncated bandwidth and it is acceptable around the normalized truncated bandwidth of 1.2, which results in a 240-tone multisine representing a 163 µs IS-95 reverse-link signal.

![Figure 2-15 RMS percentage error vs. normalized truncation bandwidth for IS-95 signal.](image)

For the IS-95 reverse link signal, the RMS percentage errors for different normalized truncated bandwidth are shown in Figure 2-15. The RMS percentage error decreases with an increase of truncated bandwidth and it is acceptable around the normalized truncated bandwidth of 1.2, which results in a 240-tone multisine representing a 163 µs IS-95 reverse-link signal.

The spectrum plots of the IS-95 reverse link signal and its multisine equivalent constructed with 240 tones are presented in Figure 2-16, which clearly show that the power components out-of-truncation BW have been discarded in the multisine representation of the IS-95 reverse link signal.
Since the frequency, amplitude, and phase are available from the DFT coefficients and sampling frequency, the key design parameter is the number of sinewaves needed to accurately represent the original signal. The number of tones is decided by two factors: the resolution BW that is inversely proportional to the length of the signal, and the truncated
bandwidth. For example, if 1 MHz bandwidth is truncated for reconstruction of signals and the frequency resolution is 10 kHz, the number of tones will be \( \frac{1 \text{ MHz}}{10 \text{ kHz}} = 100 \).

In generating the multisine for the IS-95 reverse-link signal, the signal length was decided by monitoring its statistical properties. The length needs to be long enough so that statistical properties of this realization such as probability density function (PDF), power spectrum density (PSD) should have a good match with a long-duration signal. In this work a signal length of 163us (i.e. frequency resolution is 6.144 kHz) was used to reproduce a multisine for the IS-95 reverse-link signal, corresponding to 800 sampling points with a 4x1.2288 MHz sampling frequency. The signal length was selected to make the frequency resolution be an integer to avoid frequency errors. The frequencies of the bandpass multisine were calculated by a frequency translation from the baseband to carrier frequency [74].

### 2.5.2 Measurement Results for IS-95 Multisine

The accuracy of using the multisine signal representation was assessed by measurements comparing an IS-95 reverse-link signal to its equivalent 240 tone multisine representation, with a normalized truncation bandwidth of 1.2, when applied to a 2GHz nonlinear power amplifier. ACPR and EVM were measured and compared for both excitations over a range of output power levels. The IS-95 standard defines the ACPR as the ratio (in decibel) of the distortion power in 30 kHz, offset by 885 kHz from the carrier frequency, and the power in the desired channel with a bandwidth of 1.2288 MHz [75]. Measurement was conducted using a vector signal analyzer (VSA) to measure the power spectrum and EVM of the two signals. It is worth mentioning that to maintain consistent
measurements between each signal it was necessary to trigger the measurement equipment at the beginning of each signal for the same time duration of approximately 200 data symbols.

With IS-95 reverse-link signal excitation, the 1 dB compression point of the power amplifier occurs at about 13 dBm output power level, as shown in Figure 2-17. The input power sweep is from −22 dBm to 6 dBm, which corresponds to 0 to 17.3 dBm output power range, fully covers the weak and strong nonlinear regions of operation for the device. Notice that the gain compression characteristics of the IS-95 signal are indistinguishable from the multisine signal because the PAR of these two signals that is determined by the spectrum in the main channel are very similar.

![Power Gain Curve for Both IS-95 and Multisine Excitations](image)

**Figure 2-17 Gain compression characteristic of the PA for IS-95 and multisine excitations.**

The IQ data of the IS-95 reverse-link signal were loaded into Agilent E4438C signal generator and carrier modulated to 2 GHz. The multisine signal was loaded into the signal generator using Agilent Enhanced Multitone signal studio that defines a multisine signal
by the power, phase, and frequency of each tone.

ACPR measurement results are presented in Figure 2-18. The ACPR measurements of IS-95 signal and its equivalent multisine match very well when the power amplifier goes into nonlinear region, especially when the PA approaches saturation. In the linear and weak nonlinear region, there are discrepancies between the two excitations at low output power levels, which are due to the truncation of the out-of-band components in the multisine signal (i.e. the out-of-band rejection of the multisine is greater than the original IS-95 signal).

Figure 2-18 Comparison of ACPR for IS-95 and multisine excitations.

The EVM measurements of the two excitations are shown in Figure 2-19. There is excellent agreement between the measured EVM of the IS-95 signal and multisine representation applied to the nonlinear amplifier, which demonstrates that the truncated
multisine signal is an equivalent representation of the original IS-95 signal. Since EVM is related to the in-band distortion, therefore, the multisine can accurately predict the in-band distortion of the original signal. The EVM decreases from the first maximum point because the PA enters the saturation region. The minimum EVM is limited by the measurement noise floor of the signal source and the dynamic range of the VSA equipment.

![Figure 2-19 Comparison of EVM for IS-95 and multisine excitations.](image)

Compared to other approaches such as signal statistical property matching, this DFT based multisine generation method presents an efficient and straightforward way for generating accurate multisine signals for characterization of nonlinear circuits. An IS-95 reverse-link signal was employed to demonstrate this approach. Measured ACPR and EVM of an IS-95 and equivalent multisine signal applied to a nonlinear power amplifier are in excellent agreement. The results showed that the DFT generated multisine can very
accurately predict the ACPR and EVM of the original digital modulated signal.

2.6 Summary

In this Chapter the properties of a quadrature modulator and its three different applications in modern wireless communication systems were reviewed. Three major physical impairments of quadrature modulators, the DC offset, the gain/phase imbalances, and the nonlinear distortion were introduced and their general impacts on system performances were discussed, which motivates the research to develop system-level techniques to accurately and efficiently characterize these quadrature modulator imperfections for system design and verification. Behavioral modeling techniques for this use were reviewed and the underlying bandpass nonlinearity concept developed mainly for power amplifier modeling was extended to quadrature modulator modeling. The differences of the bandpass nonlinearity representation for power amplifiers and quadrature modulators were analyzed. In the last section a particular type of signal, multisine signal, was successfully modeled to accurately represent the CDMA digitally modulated signals.
Chapter 3

Impact Analysis of Physical Impairments of Quadrature Modulators

3.1 SNDR Revisited

Looking in frequency domain, physical impairments of quadrature modulators such as nonlinear distortion, carrier leakage, gain/phase imbalances and noises produce undesired spectrum contents which degrade the signal quality. The undesired signal contents consist of two components. One is correlated to the desired signal, which causes gain compression or expansion. The other component is uncorrelated to the desired signal that results in the degradation of signal EVM [8]. As mentioned in Section 2.2.3, the SNDR is defined as the signal power to the noise and uncorrelated distortion power for characterization of the system performance degradation due to the nonlinear distortion. In the section, the SNDR definition was expanded as the ratio of the desired signal power to the noise and the
uncorrelated interference power including not only nonlinear distortion but also image spectrum and carrier leakage, in order to characterize the degradation of the quadrature modulator performances caused by these physical impairments. With this definition, the SNDR can be calculated by performing cross-correlation between the desired and the actual signal including the impairments as shown in Equation (3.1).

\[
\text{SNDR} = 10 \log_{10} \left( \frac{\left[ y_{\text{act}} \otimes \left( \frac{y_{\text{ideal}}}{y_{\text{ideal}}} \right)^2 \right]}{\left[ y_{\text{act}} - \frac{y_{\text{act}} \otimes y_{\text{ideal}}}{y_{\text{ideal}}^2} \cdot y_{\text{ideal}} \right]^2} \right). \tag{3.1}
\]

where \(y_{\text{act}}\) is the actual signal and \(y_{\text{ideal}}\) is the desired signal. \(\otimes\) denotes cross-correlation.

The numerator \(\left[ y_{\text{act}} \otimes \left( \frac{y_{\text{ideal}}}{y_{\text{ideal}}} \right)^2 \right]\) is the portion of power in the actual signal that is correlated to the desired signal because \(y_{\text{act}} \otimes \left( \frac{y_{\text{ideal}}}{y_{\text{ideal}}} \right)\) is the magnitude of the actual signal projected to the direction of the ideal signal, the square of which gives out the correlated part of power of the actual signal.

The denominator \(\left[ y_{\text{act}} - \frac{y_{\text{act}} \otimes y_{\text{ideal}}}{y_{\text{ideal}}^2} \cdot y_{\text{ideal}} \right]^2\) is the portion of power in the actual signal that is uncorrelated to the desired signal. Rewrite \(\frac{y_{\text{act}} \otimes y_{\text{ideal}}}{y_{\text{ideal}}^2} \cdot y_{\text{ideal}}\) as:

\[
\frac{y_{\text{act}} \otimes y_{\text{ideal}}}{y_{\text{ideal}}^2} \cdot y_{\text{ideal}} = \left[ y_{\text{act}} \otimes \frac{y_{\text{ideal}}}{y_{\text{ideal}}} \right]\left[ \frac{y_{\text{ideal}}}{y_{\text{ideal}}} \right]. \tag{3.2}
\]
It can be seen from Equation (3.2) that \( \frac{y_{\text{act}} \otimes y_{\text{ideal}}}{|y_{\text{ideal}}|^2}. y_{\text{ideal}} \) is the production of the direction of the ideal signal and the magnitude of the actual signal vector projected to the direction of the ideal signal, which is the vector component of the actual signal projected to the direction of the ideal signal. Therefore, \( y_{\text{act}} \frac{y_{\text{act}} \otimes y_{\text{ideal}}}{|y_{\text{ideal}}|^2}. y_{\text{ideal}} \) is the vector component of the actual signal that is perpendicular to the direction of the ideal signal, which is uncorrelated to the desired signal.

\[
y_u = y_{\text{act}} \frac{y_{\text{act}} \otimes y_{\text{ideal}}}{|y_{\text{ideal}}|^2}. y_{\text{ideal}}.
\]

(3.3)

The square (auto-correlation) of (3.3) gives the uncorrelated portion power of the actual signal.

Using the SNDR as defined in Equation (3.1), analysis of the impacts of the physical impairments of quadrature modulators on the signal quality were done in the following sections.

### 3.2 DC Offset

Carrier leakage caused by device mismatches in quadrature modulators is a major concern in designing direct-conversion transceivers. In the upconverter, the mismatch can be modeled as a DC offset in the baseband input signals, which are upconverted to the carrier frequency like a spectrum leakage to the carrier and degrade the signal quality.

#### 3.2.1 Generic Quadrature Modulation Systems

In the generic quadrature modulation systems such as WCDMA, the baseband signals
contain useful information at DC. It is impractical to remove the information contents at DC by either using filtering or AC coupling. Therefore, the DC offset in a direct-conversion quadrature modulator can degrade the signal quality. It shifts the constellation of the signal in a generic quadrature modulation system and generates a spectrum term at the carrier frequency. This spectrum is uncorrelated to the input signal and causes interference to the desired signal, which can be proved by Equation (3.3).

Assume a generic quadrature modulated signal is applied to the quadrature modulator. Let

\[ i(t) = [a + b \cos(\omega t)] + c \]
\[ q(t) = [a + b \cos(\omega t)] \]  \hspace{1cm} (3.4)

where \( a \) is the useful information at DC, \( c \) is the DC offset which is uncorrelated to \( a \). Two in-phase sinusoidal signals apply to I and Q channel respectively so that the modulator output has tones at both sidebands. The actual and the ideal output signal are:

\[ y_{act} = i(t) \cos(\omega_c t) - q(t) \sin(\omega_c t) \]
\[ = a \cos(\omega_c t) + a \sin(\omega_c t) + b \cos(\omega t) \cos(\omega_c t) + b \cos(\omega t) \sin(\omega_c t) + c \cos(\omega_c t) \]  \hspace{1cm} (3.5)

\[ y_{ideal} = a \cos(\omega_c t) + a \sin(\omega_c t) + b \cos(\omega t) \cos(\omega_c t) + b \cos(\omega t) \sin(\omega_c t) \]

Using Equation (3.3), the uncorrelated signal vector is \( c \cos(\omega t) \), which is the resulting spectrum of the DC offset.

\[ y_u = y_{act} - y_{ideal} = \frac{a^2}{2} + \frac{a^2}{4} + \frac{b^2}{4} + \frac{0}{4} - \frac{c^2}{4} = c \cos(\omega_c t) \]  \hspace{1cm} (3.6)

Therefore, neglecting the AWGN, the SNDR due to the DC offset is shown in Equation (3.7).
\[ \text{SNDR} = 10 \log \left( \frac{\text{Signal Power}}{\text{Carrier Leakage}} \right) \]  \hspace{1cm} (3.7)

To validate the relationship of (3.7), the internal modulator of the ESG was used and the EVM was measured because SNR and EVM have a direct relationship [16]. A DC offset was deliberately introduced to the modulator. Subsequently, the uplink WCDMA signal was applied to the modulator and the EVM was measured and compared to the EVM predicted by (3.7). As shown in Figure 3-1, the measured and calculated EVM are close within 1%, indicating that carrier leakage is like AWGN in impacting the waveform quality for generic quadrature modulated signals.

![Figure 3-1 Measured and calculated EVM versus carrier leakage for a WCDMA uplink signal.](image)

### 3.2.2 Zero-IF OFDM and Low-IF OFDM Systems

In zero-IF WLAN or zero-IF WiMAX systems, usually the subcarrier at carrier frequency is not used in order to avoid SNR degradation due to the carrier leakage as mentioned in 3.2.1. However, the carrier leakage is possible to cause violation of spectrum mask.
Similar as zero-IF OFDM signal, there is no sub-carrier at carrier frequency because all the sub-carriers are shifted to either upper or lower sideband as described in Section 2.1.3. However, the carrier leakage is now interferences to the adjacent channel and possible to cause violation of spectrum mask.

Similar measurements as described in Section 3.2.1 were done by deliberately introducing DC offsets into zero-IF and low-IF OFDM signals. The carrier leakages were observed for both the zero-IF WLAN and low-IF WLAN cases, as shown in Figure 3-2. As expected, there is no EVM degradation observed with the existence of the carrier leakages.

![Figure 3-2 Measured carrier leakage in zero-IF WLAN and low-IF WLAN systems.](image)

3.3 Gain/Phase Imbalances

For the generic quadrature modulation systems and the zero-IF OFDM modulation systems, the quadrature gain and phase imbalances produce the same amount of image
spectrum terms within the information bandwidth that results in SNR and EVM degradation of the desired signal. The degradation of SNR and EVM due to the quadrature imbalances in the above two systems is proportional to the IRR as presented in Equation (2.18). However, in the zero-IF OFDM system, the receiver equalizer complexity has impacts on the signal quality, which will be discussed in detail in Section 3.3.2.

The impact of the quadrature imbalances on the signals in the low-IF OFDM systems is quite different. Since the center of the modulation bandwidth is shifted from DC to the IF frequency, the image spectrum terms reside in the one of the adjacent channels that is outside the information bandwidth. Therefore, the quadrature gain and phase imbalances have no impact on the SNR and EVM of the signals, but cause an increase in the adjacent channel interference.

### 3.3.1 Generic Quadrature Modulation Systems

In a generic quadrature modulation system, the gain imbalance changes the signal constellation from square to rectangular, and the phase imbalance causes skew of the constellation.

The quadrature gain and phase imbalances produce image spectrum components within the information bandwidth. The uncorrelated image components degrade the SNDR and EVM of the modulator outputs. Using Equation (3.1), the SNDR due to gain and phase imbalance in a generic quadrature modulated system can be calculated.
As described in Section 3.2.1, two in-phase sinusoidal signals can be applied to the modulator to produce a generic quadrature modulated signal output.

\[ i(t) = \cos(\omega t) \]
\[ q(t) = \cos(\omega t) \tag{3.8} \]

As shown in Figure 3-3, the quadrature modulator has gain imbalance of \( \delta \) and phase imbalance of \( \theta \). Gain imbalance \( \delta \) is the ratio of the amplitude \( (A_I) \) of I channel to the amplitude \( (A_Q) \) of Q channel \( (\delta = \frac{A_I}{A_Q}) \).

The actual output signal is:

\[ y_{act} = \cos(\omega t)[(\delta \cos(\omega_c t + \theta))] - \cos(\omega t) \sin(\omega_c t). \tag{3.9} \]

By trigonometry,

\[ y_{act} = \delta \cos(\theta) \cos(\omega_c t) \cos(\omega t) - [\delta \sin(\theta) + 1] \cos(\omega t) \sin(\omega_c t). \tag{3.10} \]

The ideal output signal is:

\[ y_{ideal} = \cos(\omega t) \cos(\omega_c t) - \cos(\omega t) \sin(\omega_c t). \tag{3.11} \]

Plug (3.10) and (3.11) in to Equation (3.1), the numerator is:
Plug (3.10) and (3.11) in to Equation (3.3), the uncorrelated signal vector is:

\[
y_u = y_{\text{act}} - \left[ \frac{\delta \cos \theta + \delta \sin \theta + 1}{4} \right] \frac{1}{\left( \frac{4}{4} + \frac{1}{4} \right)} y_{\text{ideal}}^2 = \left( \frac{\delta \cos \theta - \delta \sin \theta - 1}{2} \right) \cos(\omega t) \cos(\omega t, t) - \left( \frac{\delta \sin \theta + 1 - \delta \cos \theta}{2} \right) \cos(\omega t) \sin(\omega, t). \tag{3.13}
\]

From Equation (3.13),

\[
y_u^2 = (2) \left( \frac{1}{4} \right) \left( \frac{\delta \cos \theta - \delta \sin \theta - 1}{2} \right)^2 = \frac{\left( \delta \cos \theta - \delta \sin \theta - 1 \right)^2}{8}. \tag{3.14}
\]

Therefore, the SNDR with gain and phase imbalance for generic quadrature modulated systems is:

\[
\text{SNDR} = 10 \log_{10} \left( \frac{\left( \delta \cos \theta + \delta \sin \theta + 1 \right)^2}{8} \right) = 10 \log_{10} \left( \frac{\left( \delta \cos \theta + \delta \sin \theta + 1 \right)^2}{8} \right) = \frac{1}{8} \left( \frac{\delta \cos \theta + \delta \sin \theta + 1}{\delta \cos \theta - \delta \sin \theta - 1} \right)^2.
\]

For small \( \delta \), \( \cos \theta \approx 0 \) and \( \sin 2\theta \approx 0 \), therefore,

\[
\text{SNDR} \approx 10 \log_{10} \left( \frac{1 + \delta^2 + 2\delta \cos \theta}{1 + \delta^2 - 2\delta \cos \theta} \right) = \text{IRR}. \tag{3.16}
\]
where IRR is the image rejection ratio as described in Equation (2.18). This equality between SNDR and IRR based on the DSB and SSB sinusoidal calculation indicates that the gain and phase imbalances lead to same amount of SNDR degradation for both DSB and SSB modulation systems. This analytical conclusion was verified further by a MatLab simulation documented in Section 3.3.2.

### 3.3.2 Zero-IF OFDM Signal

The gain and phase imbalances cause the state spreading of the zero-IF OFDM signals constellations (like what phase noise does) instead of distorting the constellations of single-carrier signals because of the total effects of multiple sub-carriers[40]. It is also able to use the IRR Equation (2.18) to predict the SNDR and EVM degradation due to the gain and phase imbalances for this type of signals because the zero-IF OFDM signals can be viewed as a set of multiple SSB modulated subcarriers symmetrically distributed around the carrier frequency. Given the same quadrature gain and phase imbalances, the zero-IF OFDM architectures produces same amount uncorrelated image spectrum which is responsible to the SNR degradation as the generic quadrature modulation systems.

The equality of the IRR, SNDR in a WCDMA system, and SNDR in a zero-IF OFDM system was validated by a MatLab simulation. In the simulation, the behavioral model developed in chapter 4 was used to calculate the modulator output with real WCDMA and zero-IF WLAN signals. Different amount of gain and phase imbalances were deliberately introduced and the carrier leakage and nonlinear distortion were kept at a negligible level. Cross-correlation was done between the input and the model output signal to determine the uncorrelated spectrum due to the gain/phase imbalances. As shown in Table 3-1, the IRR,
SNDR for WCDMA and zero-IF WLAN signals are almost the same, which validates the previous conclusion drawn in Section 3.3.1.

Table 3-1 Simulated IRR and SNDR for WCDMA and zero-IF WLAN signals.

<table>
<thead>
<tr>
<th>Amp_err (dB)</th>
<th>Phi_err (Degree)</th>
<th>IRR_calc (dB)</th>
<th>SNDR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WCDMA</td>
<td>WLAN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>-39.61</td>
<td>-39.61</td>
</tr>
<tr>
<td>0.2</td>
<td>2</td>
<td>-33.59</td>
<td>-33.59</td>
</tr>
<tr>
<td>0.3</td>
<td>3</td>
<td>-30.07</td>
<td>-30.07</td>
</tr>
<tr>
<td>0.4</td>
<td>4</td>
<td>-27.57</td>
<td>-27.57</td>
</tr>
<tr>
<td>0.5</td>
<td>5</td>
<td>-25.63</td>
<td>-25.63</td>
</tr>
<tr>
<td>0.6</td>
<td>6</td>
<td>-24.05</td>
<td>-24.04</td>
</tr>
<tr>
<td>0.7</td>
<td>7</td>
<td>-22.71</td>
<td>-22.69</td>
</tr>
<tr>
<td>0.8</td>
<td>8</td>
<td>-21.54</td>
<td>-21.53</td>
</tr>
<tr>
<td>0.9</td>
<td>9</td>
<td>-20.52</td>
<td>-20.50</td>
</tr>
<tr>
<td>1.0</td>
<td>10</td>
<td>-19.60</td>
<td>-19.58</td>
</tr>
<tr>
<td>1.1</td>
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<td>-18.77</td>
<td>-18.74</td>
</tr>
<tr>
<td>1.2</td>
<td>12</td>
<td>-18.01</td>
<td>-17.98</td>
</tr>
<tr>
<td>1.3</td>
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<td>-17.27</td>
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<td>15</td>
<td>-16.06</td>
<td>-16.01</td>
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<td>1.6</td>
<td>16</td>
<td>-15.50</td>
<td>-15.44</td>
</tr>
<tr>
<td>1.7</td>
<td>17</td>
<td>-14.97</td>
<td>-14.90</td>
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<tr>
<td>1.8</td>
<td>18</td>
<td>-14.47</td>
<td>-14.39</td>
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<tr>
<td>1.9</td>
<td>19</td>
<td>-13.99</td>
<td>-13.91</td>
</tr>
<tr>
<td>2.0</td>
<td>20</td>
<td>-13.54</td>
<td>-13.45</td>
</tr>
</tbody>
</table>

However, in a zero-IF WLAN system, the receiver equalizer complexity has impacts on the signal quality given a certain gain/phase imbalance. The training data for an equalizer in a WLAN system can be: (1) the channel estimation sequence or (2) the entire burst (including both the channel estimation sequence and the data burst) [76]. The option (1) is usually used in a practical WLAN receiver considering the complexity and cost and is compliant to 802.11a standard, although some advanced equalization techniques use the entire burst as the train data [77]. However, if the receiver equalizer is only trained by the channel estimation sequence, the SNR degradation suffers 3dB more degradation than the case when the entire burst was used to train the equalizer. Therefore, for zero-IF OFDM
systems, the receiver equalizer configuration can impact the SNR by 3 dB.

The relationship between the degradation of the SNR and EVM due to the quadrature gain/phase imbalance and the IRR was also validated using the internal modulator of the ESG. The gain and phase imbalances were deliberately introduced to the modulator and then the EVM of the modulator output was measured by supplying a WCDMA downlink signal and a 64 QAM zero-IF WLAN signal. The calculated IRR contour over gain/phase imbalances and the measured EVM were compared in Figure 3-4. Given the same amount of gain/phase imbalances, the measured EVM of WCDMA signals is 3dB better than that of zero-IF WLAN signals (the equalizer is trained by channel estimation sequence only). The measured EVM for both signals has a good match with the calculated IRR, which consolidate the method to use (3.2) and (3.3) to predict the waveform quality for both generic quadrature modulated signals and zero-IF OFDM modulated signals with the existence of gain/phase imbalances.

![Figure 3-4 Measured and Calculated EVM contours versus gain/phase error for WCDMA and zero-IF WLAN signals.](image)
3.3.3 Low-IF OFDM Signal

As discussed in Section 3.2.2, an advantage of the low-IF OFDM architecture is that the carrier leakage is outside of the signal bandwidth and won’t cause SNR or EVM degradation. Another advantage of the low-IF OFDM signals over the zero-IF OFDM signals is that the quadrature gain and phase imbalances induced image interferences have no impacts on the SNR or EVM degradation for the signal itself. Trace (1) in Figure 3-5 shows the measured spectrum of an ideal low-IF WLAN signal without quadrature imbalance, with a carrier frequency of 3.4 GHz. All the subcarriers are within the upper sideband. Trace (2) shows the measured spectrum of a low-IF WLAN signal with deliberately introduced phase error. Since each subcarrier is upconverted to RF frequency by SSB modulation, the image components due to the quadrature imbalances are resided in the lower sideband, as shown in the trace (2) in Figure 3-5. This phenomenon can be explained by the IRR discussions in Section 2.2.2 and low-IF OFDM signal discussions in Section 3.4.1.

Figure 3-5 Measured spectrum plots for a low-IF WLAN signal: (1) without gain/phase imbalance; (2) with phase imbalance.
Obviously, the low-IF architecture has a drawback that the image components now introduce additional adjacent channel interferences, which gain and phase imbalances are now design parameters for meeting the spectrum mask specification. With gain/phase imbalances alone, it is shown that 0.4 dB gain error or 2.5 degree phase error is the maximum in order not to violate the spectrum mask, which puts more stringent requirements on gain/phase errors than the EVM requirement (as shown in Figure 3-4, 0.7 dB gain error or 4.6 degree phase error is the maximum in order not to exceed the worse case RCE: $-25$ dB for 54 Mbps data rate case).

3.4 Nonlinear Distortion

As described in Section 2.2.3, nonlinear systems excited with digital modulated signals generate both in-band and out-of-band nonlinear distortion. The signal being processed in a RF system is band-limited. The nonlinearities in the transmitter system can cause the bandwidth of the transmitted signals to spread out into the adjacent channels due to intermodulation distortion, which is referred to as spectral regrowth [78]. The spectral regrowth phenomenon is explained in the following example where a two-tone signal is applied to a nonlinear system. For simplicity, the nonlinear system is assumed as a memoryless third order nonlinearity:

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t). \quad (3.17)$$

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t. \quad (3.18)$$

Thus,

$$y(t) = a_1 \left(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t\right) + a_2 \left(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t\right)^2 + a_3 \left(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t\right)^3.$$
Expand and apply trigonometry to Equation (3.19), the following terms result:

1) Fundamental terms, where gain compression or expansion has occurred.

\[
\left\{ a_i A_i + \frac{3}{4} a_j A_i^3 + \frac{3}{2} a_j A_i A_j^2 \right\} \cos \omega_i t \quad \text{and} \quad \left\{ a_i A_i + \frac{3}{4} a_j A_i^3 + \frac{3}{2} a_j A_i A_j^2 \right\} \cos \omega_i t.
\] (3.20)

2) 2\textsuperscript{nd} order intermodulation distortion terms:

\[
\left\{ a_i A_i A_j \cos (\omega_i + \omega_j) t \right\} \text{and} \left\{ a_i A_i A_j \cos (\omega_i - \omega_j) t \right\}.
\] (3.21)

3) 3\textsuperscript{rd} order intermodulation distortion terms:

\[
\begin{align*}
\left\{ \frac{3a_i A_i^2 A_j}{4} \cos (2\omega_i + \omega_j) t \right\}, & \left\{ \frac{3a_i A_i^2 A_j}{4} \cos (2\omega_i + \omega_j) t \right\} \\
\left\{ \frac{3a_i A_i^2 A_j}{4} \cos (2\omega_i - \omega_j) t \right\}, & \left\{ \frac{3a_i A_i^2 A_j}{4} \cos (2\omega_j - \omega_i) t \right\}.
\end{align*}
\] (3.22)

4) DC terms and harmonic terms that are not elaborated here because they can usually be filtered out and are not a big concern.

The two 3\textsuperscript{rd} order intermodulation distortion terms \((2\omega_1 - \omega_2)\) and \((2\omega_2 - \omega_1)\) as illustrated in Figure 3-6 are of particular importance because the digital signals are usually narrow-band and the frequency spacing between \(\omega_1\) and \(\omega_2\) is usually small and thus these two terms fall in the vicinity of \(\omega_1\) and \(\omega_2\) and it is impractical to filter out.

![Figure 3-6 Intermodulation in a nonlinear system with two-tone excitation.](image)
Two-tone analysis is a well-understood and straightforward analysis approach and has been popular to circuit or system designers. However, one limitation of this approach is that it cannot provide any useful information about the in-band distortion. Therefore, multisine signal, a class of signal excitation consisting of $N$ equally spaced sinewaves, with random or constant amplitudes and phases, has gain increased attention in nonlinear characterization of both in-band and out-of-band distortion [15, 70, 71], system testing, and nonlinear device modeling [65–68]. In this section, a multitone analysis using a four-tone multisine with equal amplitude and zero phases was done to analyze the impacts of the nonlinear distortion (out-of-band and in-band) in three quadrature modulator applications. Note that the use of equal amplitude and zero-phases multisine signal may overestimate or underestimate the in-band or out-of-band distortion [8, 70], however, the equal amplitude and zero-phases four tone signal suffices in this work for the purpose to qualitatively analyze the different impacts of nonlinear distortion in different quadrature modulator applications.

3.4.1 Nonlinear Distortion in Zero-IF and Low-IF OFDM Systems

As described in Section 2.1, the OFDM signals consist of multiple equally spaced sub-carriers each of which is modulated with unique information. Each baseband sub-carrier is upconverted to RF frequency via single-sideband modulation by the quadrature modulators. To simulate the OFDM signal in the four-tone analysis, an in-phase four-tone sinusoidal signal and a quadrature four-tone sinusoidal signal were applied to I and Q channel respectively to produce a four-tone SSB RF output signal.
\[ i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t) + \cos(\omega_4 t) \]
\[ q(t) = \sin(\omega_1 t) + \sin(\omega_2 t) + \sin(\omega_3 t) + \sin(\omega_4 t) . \]  

(3.23)

The four-tone signal is equally spaced with same amplitude and zero phases. Let \( \Delta \) denotes the frequency spacing of two adjacent tones and \( \omega_{bc} \) denotes the center of the baseband four-tone signal, then the four tones are:

\[
\begin{align*}
\omega_1 &= \omega_{bc} - 3\Delta / 2; \\
\omega_2 &= \omega_{bc} - \Delta / 2; \\
\omega_3 &= \omega_{bc} + \Delta / 2; \\
\omega_4 &= \omega_{bc} + 3\Delta / 2;
\end{align*}
\]

(3.24)

For zero-IF OFDM signals, \( \omega_{bc} = 0 \) so that \( \omega_1, \omega_2, \omega_3, \) and \( \omega_4 \) are symmetrically located around DC, as shown in Figure 3-7 (a). The solid and dash lines are used to illustrate the fact that a baseband real signal has both (+) and (-) spectrum. For example, a real sinusoidal signal \( \cos(\omega_{bb} t) \) can be expressed as:

\[
\cos(\omega_{bb} t) = \frac{1}{2} \left( e^{j\omega_{bb} t} + e^{-j\omega_{bb} t} \right).
\]

(3.25)

By doing a Fourier transform, both (+) and (-) spectrum terms generate for a real signal \( \cos(\omega_{bb} t) \). For low-IF OFDM signals, \( \omega_{bc} = \omega_{IF} \) so that \( \omega_1, \omega_2, \omega_3, \) and \( \omega_4 \) are all located in the same either upper or lower sideband, that is, \( \omega_1, \omega_2, \omega_3, \) and \( \omega_4 \) have the same signs, as shown in Figure 3-7 (b). Similarly, the solid and dash lines are used to illustrated the fact that a baseband real signal has both (+) and (-) spectrum.
Figure 3-7 Four-tone signals for (a) zero-IF OFDM system, (b) low-IF OFDM system.

Assume I and Q channels are isolated so that I and Q channel nonlinearities can be modeled as two independent nonlinearities. Also assume the baseband transconductors dominate the nonlinear characteristics of a quadrature modulator. Then a direct-conversion quadrature modulator can be simplified into the model as shown in Figure 3-8, where \( \tilde{G}_I \) and \( \tilde{G}_Q \) represent the I and Q channel baseband nonlinearities followed by two ideal balanced mixers.

Figure 3-8 Block diagram of the RF quadrature modulator model with I and Q channel nonlinearities at baseband.
Without loss of generality, assume the I and Q channel nonlinearities have the same nonlinear characteristics, which is usually true because of the use of balanced mixers. For simplicity, assume a 3\textsuperscript{rd} order nonlinearity for each channel:

\[ G_I(x) = G_Q(x) = a_1 x + a_3 x^3, \ x = i(t) \text{ or } q(t). \tag{3.26} \]

Plug Equations (3.23) and (3.26) into the model,

\[ w(t) = G_I \left( \cos \omega_1 t + \cos \omega_2 t + \cos \omega_3 t + \cos \omega_4 t \right) \cos(\omega_c t) - \\
G_Q \left( \sin \omega_1 t + \sin \omega_2 t + \sin \omega_3 t + \sin \omega_4 t \right) \sin(\omega_c t). \tag{3.27} \]

Applying trigonometry to Equation (3.27), the following desired spectrum terms result due to the 1\textsuperscript{st} order nonlinearity term \(a_1 x\).

\[ V_{o1}(t) = a_1 \left( i(t) \cos(\omega_c t) - q(t) \sin(\omega_c t) \right) \\
= a_1 \left( \cos(\omega_c t + \omega_1 t) + \cos(\omega_c t + \omega_2 t) + \cos(\omega_c t + \omega_3 t) + \cos(\omega_c t + \omega_4 t) \right). \tag{3.28} \]

As expected, only SSB signal results for zero-IF and low-IF OFDM signals. To save space, Equation (3.29) was used in a form just to show the resulting spectrum terms:

\[ a_1 : (\omega_c + \omega_1), (\omega_c + \omega_2), (\omega_c + \omega_3), (\omega_c + \omega_4). \tag{3.29} \]

The spectrum terms due to the 3\textsuperscript{rd} order nonlinearity for zero-IF OFDM and low-IF OFDM signals are summarized in Table 3-2 and 3-3, respectively.
Table 3-2 Third-order spectrum terms of zero-IF OFDM signals.

<table>
<thead>
<tr>
<th>Description</th>
<th>Scale Factor</th>
<th>Spectrum Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Gain Compression</td>
<td>$\frac{21}{4}a_3$</td>
<td>$(\omega_c + \omega_1), (\omega_c + \omega_2), (\omega_c + \omega_3), (\omega_c + \omega_4)$</td>
</tr>
<tr>
<td>2 In-Band Terms</td>
<td>$\frac{3}{4}a_3$</td>
<td>$[\omega_c + (2\omega_2 - \omega_1)] \rightarrow \omega_3, [\omega_c + (2\omega_2 - \omega_3)] \rightarrow \omega_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c + (2\omega_3 - \omega_2)] \rightarrow \omega_4, [\omega_c + (2\omega_3 - \omega_1)] \rightarrow \omega_2$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}a_3$</td>
<td>$[\omega_c + (\omega_1 + \omega_3 - \omega_2)] \rightarrow \omega_2, [\omega_c + (\omega_1 + \omega_4 - \omega_2)] \rightarrow \omega_3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c + (\omega_1 + \omega_4 - \omega_1)] \rightarrow \omega_3, [\omega_c + (\omega_2 + \omega_3 - \omega_1)] \rightarrow \omega_4$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c + (\omega_2 + \omega_3 - \omega_4)] \rightarrow \omega_1, [\omega_c + (\omega_2 + \omega_4 - \omega_3)] \rightarrow \omega_3$</td>
</tr>
<tr>
<td>3 In-Band Terms$^2$</td>
<td>$\frac{1}{4}a_3$</td>
<td>$(\omega_c - 3\omega_2) \rightarrow \omega_1, (\omega_c - 3\omega_3) \rightarrow \omega_4$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}a_3$</td>
<td>$[\omega_c - (\omega_1 + \omega_2 + \omega_3)] \rightarrow \omega_1, [\omega_c - (\omega_1 + \omega_2 + \omega_4)] \rightarrow \omega_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c - (\omega_1 + \omega_3 + \omega_4)] \rightarrow \omega_3, [\omega_c - (\omega_2 + \omega_3 + \omega_4)] \rightarrow \omega_4$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{4}a_3$</td>
<td>$[\omega_c - (2\omega_3 + \omega_1)] \rightarrow \omega_2, [\omega_c - (2\omega_4 + \omega_1)] \rightarrow \omega_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c - (2\omega_1 + \omega_4)] \rightarrow \omega_3, [\omega_c - (2\omega_2 + \omega_4)] \rightarrow \omega_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c - (2\omega_2 + \omega_2)] \rightarrow \omega_3, [\omega_c - (2\omega_1 + \omega_4)] \rightarrow \omega_4$</td>
</tr>
<tr>
<td>4 Out-of-Band Terms</td>
<td>$\frac{3}{4}a_3$</td>
<td>$[\omega_c + (2\omega_1 - \omega_1)], [\omega_c + (2\omega_3 - \omega_1)], [\omega_c + (2\omega_1 - \omega_3)]$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c + (2\omega_4 - \omega_1)], [\omega_c + (2\omega_4 - \omega_3)], [\omega_c + (2\omega_2 - \omega_3)]$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}a_3$</td>
<td>$[\omega_c + (\omega_1 + \omega_2 - \omega_3)], [\omega_c + (\omega_1 + \omega_2 - \omega_4)]$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c + (\omega_1 + \omega_3 - \omega_4)], [\omega_c + (\omega_2 + \omega_3 - \omega_1)]$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[\omega_c + (\omega_3 + \omega_4 - \omega_1)], [\omega_c + (\omega_3 + \omega_4 - \omega_2)]$</td>
</tr>
<tr>
<td>5 Out-of-Band Terms$^3$</td>
<td>$\frac{1}{4}a_3$</td>
<td>$(\omega_c - 3\omega_1), (\omega_c - 3\omega_4)$</td>
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<td>$\frac{3}{4}a_3$</td>
<td>$[\omega_c - (2\omega_2 + \omega_1)], [\omega_c - (2\omega_1 + \omega_3)], [\omega_c - (2\omega_1 + \omega_4)]$</td>
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<tr>
<td></td>
<td></td>
<td>$[\omega_c - (2\omega_4 + \omega_2)], [\omega_c - (2\omega_1 + \omega_4)], [\omega_c - (2\omega_4 + \omega_3)]$</td>
</tr>
</tbody>
</table>

Note: 1. $[\cdot] \rightarrow \omega_i$ denotes the spectrum term in $[\cdot]$ interferes with the signal spectrum term at $\omega_c + \omega_i$.
2. The spectrum terms are in-band in zero-IF OFDM systems, but shifted out of the signal band to the opposite sideband causing spectrum asymmetry in low-IF OFDM systems.
3. The spectrum terms are located symmetrically around carrier in zero-IF OFDM systems, but all shifted to one sideband causing spectrum asymmetry in low-IF OFDM systems.
Table 3-3 Third-order spectrum terms of low-IF OFDM signals.

<table>
<thead>
<tr>
<th>Description</th>
<th>Scale Factor</th>
<th>Spectrum Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Gain Compression</td>
<td>$\frac{21}{4}a_3$</td>
<td>$(\omega_c + \omega_1), (\omega_c + \omega_2), (\omega_c + \omega_3), (\omega_c + \omega_4)$</td>
</tr>
<tr>
<td>2 In-Band Terms</td>
<td>$\frac{3}{4}a_3$</td>
<td>$[\omega_c + (2\omega_2 - \omega_1)] \rightarrow \omega_1, [\omega_c + (2\omega_2 - \omega_3)] \rightarrow \omega_3$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}a_3$</td>
<td>$[\omega_c + (\omega_1 + \omega_3 - \omega_2)] \rightarrow \omega_2, [\omega_c + (\omega_1 + \omega_4 - \omega_2)] \rightarrow \omega_3$</td>
</tr>
<tr>
<td>Out-of-Band Terms Located Symmetrically around $\omega_{IF}$</td>
<td>$\frac{3}{4}a_3$</td>
<td>$[\omega_c + (2\omega_1 - \omega_2), [\omega_c + (2\omega_1 - \omega_3)]$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}a_3$</td>
<td>$[\omega_c + (\omega_1 + \omega_2 - \omega_3)] [\omega_c + (\omega_1 + \omega_2 - \omega_4)]$</td>
</tr>
<tr>
<td>Out-of-Band Terms Shifted to the Opposite Side of the Desired Signal</td>
<td>$\frac{1}{4}a_3$</td>
<td>$(\omega_c - 3\omega_1), (\omega_c - 3\omega_2), (\omega_c - 3\omega_3), (\omega_c - 3\omega_4)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{4}a_3$</td>
<td>$[\omega_c - (2\omega_2 + \omega_1), [\omega_c - (2\omega_2 + \omega_3)]$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}a_3$</td>
<td>$[\omega_c - (\omega_1 + \omega_2 + \omega_3)] [\omega_c - (\omega_1 + \omega_2 + \omega_4)]$</td>
</tr>
</tbody>
</table>

Note: $[ ] \rightarrow \omega_i$ denotes the spectrum term in [ ] interferes with the signal spectrum term at $\omega_c + \omega_i$.  
67
The 3rd order in-band and out-of-band distortion terms as listed in Tables 3-2 and 3-3 were plotted in Figures 3-9 and 3-10 for the zero-IF OFDM and low-IF OFDM systems, respectively. For simplicity, only the locations of the distortion components and the relative amplitudes of the sum of the components at the same frequency were plotted.

Figure 3-9 Illustration of 3rd order distortion terms from the four-tone analysis for zero-IF OFDM signals.

Figure 3-10 Illustration of 3rd order distortion terms from the four-tone analysis for low-IF OFDM signals.
Important information can be observed from Figures 3-9 and 3-10:

1. In zero-IF OFDM systems, each output spectrum term has a mirror spectrum term around the carrier frequency $\omega_c$ because the baseband sub-carriers are symmetric around DC. This is true for all the terms as shown in Figure 3-9, which means that a nonlinear modulator output spectrum of a zero-IF signal is symmetric around the carrier frequency $\omega_c$. The output spectrum symmetry can be seen in Figure 2-4, which shows a measured quadrature modulator output spectrum plot of a zero-IF WLAN signal. Also plotted in Figure 2-4 is the WLAN spectrum mask. The lower and upper sidebands of a zero-IF WLAN transmitted signal will both meet the spectrum mask or both violate it. In Figure 3-11 the simulated correlated and uncorrelated quadrature modulator output spectrum were also shown for illustration of the uncorrelated in-band distortion of the zero-IF WLAN signals.

![Figure 3-11 Spectrum plot of a quadrature modulator output of a zero-IF WLAN signal.](image-url)
However, for low-IF OFDM signals, all the baseband sub-carriers are asymmetrically located in one sideband as shown in Figure 3-7(b). They are upconverted to one sideband of the carrier frequency as well, as seen in Figure 3-10. Different from the zero-IF OFDM case, some of the 3rd order harmonic and intermodulation terms (spectrum terms of group 4 in Table 3-3) shifted to the opposite sideband of the desired signal, as shown in Figure 3-10. Note that the shifted terms include both in-band distortion terms of zero-IF OFDM signals (spectrum terms of group 3 in Table 3-2) and out-of-band distortion terms of zero-IF OFDM signals (spectrum terms of group 5 in Table 3-2). This shift of the 3rd order harmonic and intermodulation terms can be seen in Figure 3-12, which shows a measured quadrature modulator output spectrum plot of a low-IF WLAN signal and the WLAN spectrum mask. It is obvious that one drawback of quadrature modulator application in the low-IF OFDM systems is the increased interferences to the adjacent channels at one side due to the shift of the 3rd order harmonic and intermodulation terms. Together with the amplitude/phase imbalances induced image spectrum (also plotted in Figure 3-12), these increased interferences could potentially cause the violation of the spectrum mask. Also shown in Figure 3-12 are the simulated correlated and uncorrelated quadrature modulator output spectrum for illustration of the uncorrelated in-band distortion of the low-IF WLAN signals.
Figure 3-12 Spectrum plot of a quadrature modulator output of a low-IF WLAN signal.

(2) The quadrature modulators generate more in-band distortion in zero-IF OFDM systems than the low-IF OFDM systems. This is because some of the in-band distortion terms in zero-IF OFDM systems, the distortion products in group 3 as listed in Table 3-2, are shifted out of the signal band and moved into the opposite sideband in low-IF OFDM systems and partly results in the spectrum asymmetry, as shown in Figure 3-10. This renders low-IF OFDM systems another significant advantage that less in-band distortion is generated from the quadrature modulators due to the distortion spectrum terms shift. This property of low-IF OFDM signals will be verified by the measured results in Section 3.4.3. Obviously, the drawback is the increased interferences to the adjacent channels.

(3) In low-IF OFDM systems, there is a design tradeoff in choosing the offset frequency $\omega_{IF}$. As shown in Figure 3-10, the bigger the $\omega_{IF}$, the further the 3$^{rd}$ order
spectrum hump is away from the signal band, and thus it is more likely that these interferences could be filtered out. However, the cost of a larger $\omega_{IF}$ is the increased demands on the DACs. Therefore, there is a design tradeoff in choosing $\omega_{IF}$ between reduced interference and increased DAC costs.

### 3.4.2 Nonlinear Distortion in Generic Quadrature Modulation Systems

Similar four-tone analysis was done to investigate the nonlinear distortion in generic quadrature modulation systems.

For this type of systems, recall in Section 2.1.1 that the baseband I and Q data are independent and they are upconverted to carrier frequency via double-sideband quadrature modulation. The spectrum of the I and Q data are overlapped on top of each other orthogonally in the same RF spectrum. Also, the I and Q channel nonlinearities are assumed to have the same nonlinear characteristics. Therefore, it is a valid simplification to only examine the RF modulated output of one channel, that is, to set one of the channel input to be zero.

\[
i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t) + \cos(\omega_4 t) \\
q(t) = 0
\]  

(3.30)

Similar as zero-IF OFDM signals, $\omega_1, \omega_2, \omega_3,$ and $\omega_4$ are symmetrically located around DC, as shown in Figure 3-7 (a).

Assume the same nonlinear model as in Section 3.4.1 and go through the similar algebra; very similar results were obtained as those presented in Table 3-2 for zero-IF OFDM case. The difference is that in the zero-IF OFDM systems the desired signal and the corresponding nonlinear distortion terms are all single sideband results but in the generic
quadrature modulation systems the desired signal and the corresponding nonlinear distortion terms are all double sideband results. For example:

In the generic quadrature modulation systems, the desired spectrum terms due to the 1\textsuperscript{st} order nonlinearity term $a_1 x$ are double sideband:

\[
\frac{a_1}{2} : (\omega_c + \omega_1), (\omega_c + \omega_2), (\omega_c + \omega_3), (\omega_c + \omega_4),
\]
\[
(\omega_c - \omega_1), (\omega_c - \omega_2), (\omega_c - \omega_3), (\omega_c - \omega_4) \quad \text{(3.31)}
\]

In zero-IF OFDM systems, the desired spectrum terms due to the 1\textsuperscript{st} order nonlinearity term $a_1 x$ are single sideband, as shown in Equation (3.29).

In the generic quadrature modulation systems, the gain compression or expansion terms due to the 3\textsuperscript{rd} order nonlinearity term $a_3 x^3$.

\[
\frac{21}{8} a_3 : (\omega_c + \omega_1), (\omega_c + \omega_2), (\omega_c + \omega_3), (\omega_c + \omega_4),
\]
\[
(\omega_c - \omega_1), (\omega_c - \omega_2), (\omega_c - \omega_3), (\omega_c - \omega_4) \quad \text{(3.32)}
\]

In zero-IF OFDM systems, the gain compression or expansion terms are:

\[
\frac{21}{4} a_3 : (\omega_c + \omega_1), (\omega_c + \omega_2), (\omega_c + \omega_3), (\omega_c + \omega_4). \quad \text{(3.32)}
\]

Similar results were observed for all the other distortion terms. To save space these terms for the generic quadrature modulation case were not listed here. The important observation is that for the generic quadrature modulation case the ratio between the signal and the distortion terms remains the same as for the zero-IF OFDM cases. Therefore, it can be expected that if the zero-IF OFDM signal and the generic quadrature modulated signal have the similar PAR, they should produce similar amount of in-band distortion if no significant memory effects exists in the quadrature modulator. With the memory effects,
the in-band distortion depends not only on the signal PAR, but also other signal characteristics like the locations of the signal peaks. This conclusion was verified by the EVM measurement results as presented in Section 3.4.3.

Similar as the zero-IF OFDM systems, the baseband I and Q data spectra in the generic quadrature modulation systems are symmetric around the DC. The quadrature modulator output spectrum should also be symmetric around the carrier frequency, which is true as seen in Figure 3-13, where a measured quadrature modulator output spectrum and the simulated correlated/uncorrelated distortion of a WCDMA signal are plotted.

![Figure 3-13 Spectrum plot of a quadrature modulator output of a WCDMA signal.](image)

### 3.4.3 EVM Measurement Results

The in-band distortion was characterized by the EVM measurement because the correlated/uncorrelated signal components of a nonlinearity output are related to SNR and
EVM by the Equations (2.19) and (2.20):

Three classes of digital signals: WCDMA signal, zero-IF WLAN signal and, low-IF WLAN signal were fed to a passive direct-conversion quadrature modulator, PolyphaseMicrowave QM3337A [79]. The EVM over an input power range were experimentally measured to characterize the in-band distortion. The three types of signals represent the three applications of quadrature modulators in wireless communications systems: generic quadrature modulation, zero-IF OFDM, and low-IF OFDM. For WCDMA, an uplink pilot signal with a PAR of 3.4 dB and a downlink test model 5 signal with a PAR around 11 dB were used. For zero-IF OFDM and low-IF OFDM, a signal with 16-QAM modulation type and a PAR around 11 dB was used.

The downlink WCDMA signal and the zero-IF and low-IF WLAN signals, which have the similar PAR around 11 dB, have almost the same output 1 dB compression point around 5.5 dBm. This is an indication that comparable amount of gain compression distortion terms were produced for the three different classes of signals, which meets our expectation based on the four-tone analysis in the last two sections. Another observation is that the uplink WCDMA has a much bigger P1dB compression point because of its lower PAR.

The measured EVM of the quadrature modulator output excited with the four signals were plotted in Figure 3-14. As expected, in the nonlinear region, the EVMs excited with the WCDMA downlink signal and the zero-IF WLAN signals are very close, and they are worse than the low-IF OFDM case. This verifies the conclusions drawn in Section 3.4.1 and 3.4.2 that if the generic quadrature modulated signal, the zero-IF OFDM signal, and the low-IF OFDM signal have the similar PAR, the first two signals should generate similar
amount of in-band distortion with the condition of no significant memory effects in the quadrature modulator, while the low-IF OFDM signal should generate less in-band distortion. The good EVM match between the WCDMA downlink signal and the zero-IF WLAN signal leads to another important conclusion: there is no significant memory effect in the QM3337A quadrature modulator. The large discrepancy in the linear region is due to the higher noise floor in the generated WCDMA signals. Another observation is that the WCDMA uplink signal has much better EVM than the other signals due to its smaller PAR.

![Graph](image)

**Figure 3-14** Measured EVM of quadrature modulator output excited with: 1) WCDMA uplink signal; 2) WCDMA downlink signal; 3) zero-IF WLAN 16-QAM signal; 4) low-IF WLAN 16-QAM signal.

### 3.4.4 ACPR Measurement Results

There is no an explicit definition of ACPR for WLAN signals [32] since EVM is the more stringent metric in a WLAN system. For the purpose of validating the conclusions regarding the out-of-band distortion drawn in Section 3.4.1, in this section, the ACPR for
WLAN was defined as the ratio of the total power within the desired 20 MHz bandwidth to the total power in an adjacent 20 MHz channel bandwidth, with a frequency offset of 20 MHz. The measured ACPRs for the QM3337A quadrature modulator excited with the zero-IF WLAN and low-IF WLAN signals, both using 16-QAM modulation scheme, were shown in Figure 3-15.

![Figure 3-15 Measured upper and lower ACPR of quadrature modulator output excited with zero-IF WLAN 16-QAM signal and low-IF WLAN 16-QAM signal.](image)

Figure 3-15 shows that the upper and lower ACPRs of the zero-IF WLAN signal have a very good match, which further proves that no significant memory effects in the quadrature modulator. Second, as expect, due to the shift of the 3\textsuperscript{rd} order harmonic and intermodulation spectrum terms, the upper and lower ACPRs of the low-IF signal have a big difference. The upper ACPR of low-IF signal is larger than the ACPRs of the zero-IF signal, which is then larger than the lower ACPR of the low-IF case. The ACPR results
validate the conclusions drawn in Section 3.4.1.

The ACPR of the WCDMA downlink signal is expected to be equal to that of the zero-IF WLAN signal, but no comparison was done here because the WCDMA and WLAN have different ACPR definitions and it is not possible to compare them.

3.5 Summary

In this chapter, the impacts of the DC offset, gain/phase imbalances and nonlinear distortion of the quadrature modulators were analyzed for three different modulator applications: generic quadrature modulation system, zero-IF OFDM system, and low-IF OFDM system. The important findings were summarized in Table 3-4.

<table>
<thead>
<tr>
<th></th>
<th>I Generic Quad Mod System</th>
<th>II Zero-IF SSB OFDM System</th>
<th>III Low-IF SSB OFDM System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applications</strong></td>
<td>Cellular systems, Voice transmission. (GSM/EDGE, WCDMA,...)</td>
<td>Wireless data transmission. (WLAN, WiMAX, etc)</td>
<td>Wireless data transmission. (WLAN, WiMAX, etc)</td>
</tr>
<tr>
<td><strong>Data Rate</strong></td>
<td>Low</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td><strong>Additional Cost</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>DC offset</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SNR Degradation</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Spectra Emissions</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Gain Phase Errors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SNR Degradation</td>
<td>Yes, SNR=-IRR</td>
<td>Yes, SNR=-IRR, Receiver equalizer, 3 dB more degradation.</td>
<td>No</td>
</tr>
<tr>
<td>Spectra Emissions</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Nonlinear Distortion</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out-of-Band</td>
<td>Same as system II</td>
<td>Same as system I</td>
<td>Sideband asymmetry.</td>
</tr>
<tr>
<td>In-Band</td>
<td>Same as system II</td>
<td>Same as system I</td>
<td>Less than systems I, II.</td>
</tr>
</tbody>
</table>

The DC offset of quadrature modulators produces carrier leakage that is uncorrelated interferences to the desired WCDMA signals and degrades the signal quality. The
degradation can be quantified by the SNDR metric. For zero-IF and low-IF WLAN systems, the carrier leakage causes no SNR or EVM degradation because no sub-carrier is presented at the carrier frequency in the practical systems, however, they may cause violations of spectrum mask.

The gain and phase imbalances produce image spectrum and cause EVM degradation in the WCDMA and zero-IF WLAN systems. The degradation can be predicted by the SNDR, which is proved to be equal to IRR. For zero-IF WLAN systems, the receiver equalizer complexity can cause 3 dB difference in signal EVM with the presence of gain and phase imbalances. The signal EVM in low-IF WLAN systems is not degraded by the gain/phase imbalances because all the image spectrums are outside of signal bandwidth and located in its mirrored sideband. However, this image spectrum will contribute to the adjacent channel interferences and cause violation of spectrum mask.

In the generic quadrature modulation system and the zero-IF OFDM system, the quadrature modulators generate the similar amount of in-band and out-of-band distortion. The nonlinear distortion in the low-IF OFDM systems is quite different. First, the quadrature modulator nonlinear output spectrum is not symmetric because of the shift of part of the 3rd order harmonic and intermodulation terms. Second, less in-band distortion are generated in the low-IF OFDM system due to the fact that some of the in-band distortion terms in the other two systems are shifted out of the signal band and moved into the opposite sideband for the low-IF OFDM case. Both of the above two properties of low-IF OFDM systems lead to the drawback of increased interferences to the adjacent channels. A design tradeoff in choosing the offset frequency \( \omega_d \) can be made for the
low-IF OFDM systems to reduce either the adjacent channel interferences or the DAC costs.
Chapter 4

Behavioral Modeling of Direct-Conversion Quadrature Modulators

The analysis in Chapter 3 shows the adverse impacts of the DC offset, the gain/phase imbalances and the nonlinear distortion of quadrature modulators on the system performances in three different quadrature modulator applications. In this chapter, a baseband equivalent nonlinear behavioral model was developed for accurate characterization of the above physical imperfections in direct-conversion quadrature modulators so that the circuit and system designers can use it to characterize the physical impairments and estimate the modulator performance fast and efficiently. The modeling technique was applied to both a passive and an active quadrature modulator. The model was validated by carrier leakage measurement, image suppression measurement, ACPR measurement, and EVM measurement using practical wireless communication signals.
4.1 Behavioral Modeling for a Passive Direct-Conversion Quadrature Modulator

In this section the details of nonlinear behavioral model development and parameter extraction procedures were presented for a passive direct-conversion quadrature modulator. The nonlinear characteristic of each modulator input was modeled by a complex power series based on AM-AM and AM-PM measurements. The linear static errors of the modulator were modeled by three static terms added into the complex power series based on a four-fixed-point VNA measurement. An existing orthogonalization technique developed previously for power amplifiers was implemented in the quadrature modulator model as well so that the correlated and uncorrelated nonlinear distortion can be decomposed and the modulator output waveform quality (SNR, EVM) can be predicted with this model. WCDMA, zero-IF WLAN signal and low-IF WLAN signal were used to verify the accuracy of the behavioral model.

4.1.1 Model Structure and Assumptions

The quadrature modulator being modeled is manufactured by Polyphase Microwave Inc. with the model number of QM3337A. Table 4-1 lists the important electrical specifications for this quadrature modulator. Its operating frequency range is 3.3 GHz – 3.7 GHz and the baseband I/Q bandwidth is DC-250 MHz, which aims at wideband applications like WiMAX. It has a differential and double balanced structure, which effectively rejects common mode noise and minimizes even-order nonlinear distortion. The block diagram of this modulator is shown in Figure 4-1.
Table 4-1: Key electrical specifications of QM3337A Quadrature Modulator [79].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Range</td>
<td>3300 – 3700 MHz</td>
<td></td>
</tr>
<tr>
<td>LO Power</td>
<td>13 – 16 dBm</td>
<td></td>
</tr>
<tr>
<td>I/Q Baseband Bandwidth</td>
<td>DC – 250 MHz</td>
<td></td>
</tr>
<tr>
<td>Conversion Loss</td>
<td>7 dB</td>
<td></td>
</tr>
<tr>
<td>IIP3</td>
<td>16 dBm</td>
<td></td>
</tr>
<tr>
<td>Output P1dB</td>
<td>−1 dBm</td>
<td></td>
</tr>
<tr>
<td>Carrier Leakage</td>
<td>−44 dBm</td>
<td></td>
</tr>
<tr>
<td>Amplitude Imbalance</td>
<td>±0.05 dB</td>
<td></td>
</tr>
<tr>
<td>Phase Imbalance</td>
<td>±0.05 Degree</td>
<td></td>
</tr>
<tr>
<td>Output Noise Level</td>
<td>−173 dBm/Hz</td>
<td></td>
</tr>
<tr>
<td>LO-RF Isolation</td>
<td>60 dB</td>
<td></td>
</tr>
<tr>
<td>LO-IF Isolation</td>
<td>60 dB</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4-1: Block diagram of QM3337A quadrature modulator, after [79].
Several assumptions were made in this quadrature modulator modeling work.

1) The I and Q channels are isolated so that they can be modeled as two independent bandpass nonlinearities. This is generally true because nowadays most quadrature modulators utilize double-balanced mixer structure that greatly reduces the port-to-port coupling.

2) Compared to other sources of nonlinearity in quadrature modulators, such as mixer cores and parasitic capacitors/resistors, the baseband transconductors are the dominant ones. This is generally true because the baseband transconductors are the V-I converting or gain generation devices.

3) There is no significant memory over the signal bandwidth so that the quasistatic model using a polynomial complex power series can be adopted, which greatly ease the model development. Considering the passive and differential structure of this quadrature modulator, this is a valid assumption since there is no significant memory effects from the second order interactions, the active-device low-frequency dispersion, electrothermal interactions, and bias circuitry impacts [56–58]. In addition, the quadrature modulators are usually not driven to nonlinear regions as strongly as the power amplifiers so that the dynamic memory effects are smaller than the power amplifiers.

Based on the above assumptions, the baseband equivalent model of the quadrature modulator can be constructed as shown in Figure 4-2.
Figure 4-2 Baseband equivalent nonlinear model for QM3337A quadrature modulator.

The model result is the envelope of the modulator RF output, which can be obtained by the quadrature summation of the envelopes of the I/Q channel outputs [80].

\[ \tilde{z}_{RF}(t) = \tilde{z}_I(t) + j \cdot \tilde{z}_Q(t) . \]  

(4.1)

Since

\[ \tilde{z}_{RF}(t)e^{j\omega_0 t} = \tilde{z}_I(t)e^{j\omega_0 t} + j \cdot \tilde{z}_Q(t)e^{j(\omega_0 t + \pi/2)} . \]  

(4.2)

\( \tilde{G}_I \) and \( \tilde{G}_Q \) are complex transfer characteristics of the I and Q channel nonlinearities, respectively. They can be both described as envelope complex power series if only nonlinear characteristics are considered [6, 7, 52].

\[ \tilde{z}_I(t) = \tilde{G}_I[I(t)] = \sum_{n=1, \text{odd}}^{N} \tilde{a}_{I,n} I^n . \]  

(4.3)

\[ \tilde{z}_Q(t) = \tilde{G}_Q[Q(t)] = \sum_{n=1, \text{odd}}^{N} \tilde{a}_{Q,n} Q^n . \]  

(4.4)

where \( \tilde{a}_{I,n} \) and \( \tilde{a}_{Q,n} \) are the complex power-series coefficients that capture the instantaneous amplitude and phase responses of the baseband nonlinearities. A quasistatic
nonlinear device can be accurately characterized by a complex power series model when there is no significant memory within the signal bandwidth, implying flat frequency response and no PM-to-AM conversion. The complex power series coefficients are obtained by fitting a polynomial to the single-tone AM-AM and AM-PM characteristics, as described in [6]. However, only the odd-order envelope coefficients can be extracted from single-tone complex compression measurement (AM-AM and AM-PM), but fortunately, the odd-order terms are the most important since the intermodulation distortion in-band and adjacent to the desired channel are produced by the odd-order terms. In addition, most integrated quadrature modulators utilize differential circuit structures which greatly minimize the even-order nonlinear distortion.

Equations 4.3 and 4.4 were revised to characterize the DC offset, gain imbalance and phase imbalance by adding three more parameters $a_{I,0}, a_{Q,0}, \delta$ and $\theta$.

\[
\tilde{z}_I(t) = \tilde{G}_I[I(t)] = a_{I,0} + \tilde{a}_{I,1}I + \sum_{n=3, \text{odd}}^{N} \tilde{a}_{I,n}I^n. \tag{4.5}
\]

\[
\tilde{z}_Q(t) = \tilde{G}_Q[Q(t)] = a_{Q,0} + \tilde{a}_{Q,1}Q + \sum_{n=3, \text{odd}}^{N} \tilde{a}_{Q,n}Q^n. \tag{4.6}
\]

$a_{I,0}$ and $a_{Q,0}$ are real coefficients characterizing the DC offset of $I$ and $Q$ channel respectively. $\delta$, the ratio of the linear gain between I and Q channel, characterizes the amplitude imbalance. $\theta$, the phase error between the quadrature LO signal to I and Q channel, characterizes the phase imbalance.

To verify the validity of the describing Equations (4.5) and (4.6) for characterizing the gain and phase imbalance, just the first order terms were considered, that is, neglect the
nonlinear distortion terms and the DC offset.

\[
\tilde{z}_I(t) = \tilde{G}_{I,\text{linear}}[I(t)] = \tilde{a}_{I,I} I.
\]  
(4.7)

\[
\tilde{z}_Q(t) = \tilde{G}_{Q,\text{linear}}[Q(t)] = \tilde{a}_{Q,I} \cdot e^{j\phi} Q.
\]  
(4.8)

The RF output is:

\[
v_{\text{out}}(t) = \mathcal{R}[\tilde{z}_{RF}(t)e^{j\Omega t}]
\]

\[
= \mathcal{R}[(\tilde{z}_I(t) + j\tilde{z}_Q(t))e^{j\Omega t}] = \tilde{a}_I (I \cos(\omega_c t) - Q \cdot \delta \cdot \sin(\omega_c t + \theta)).
\]  
(4.9)

Equation (4.9) shows that the baseband equivalent model Equations (4.5) and (4.6) have the correct form for modeling the gain and phase imbalance of the quadrature modulators.

The carrier leakage due to the DC offset is:

\[
v_{cl}(t) = \mathcal{R}[a_{I,0} + ja_{Q,0}e^{j\Omega t}] = a_{I,0} \cos(\omega_c t) - a_{Q,0} \sin(\omega_c t).
\]  
(4.10)

Therefore, the carrier leakage (CL) in a 50 ohm system is:

\[
\text{CL (dBm)} = 10 \log \left[ \frac{a_{I,0}^2 + a_{Q,0}^2}{50} \right] + 30.
\]  
(4.11)

### 4.1.2 Model Parameter Extraction

In this section the techniques for extracting the model parameters were described in detail. These parameter extraction techniques are not specific to the QM3337A quadrature modulator. They are applicable to other types of direct-conversion quadrature modulators.

#### 4.1.2.1 Extraction Techniques for Nonlinear Model Coefficients

The complex power series coefficients which characterize the nonlinear behaviors are obtained by fitting a polynomial to the single-tone AM-AM and AM-PM characteristics.
Recall the discussion in Section 2.4 about the differences of the bandpass nonlinearity representation between power amplifiers and quadrature modulators. Because the major source of nonlinearity in a quadrature modulator is the baseband transconductor, a quadrature modulator output includes the instantaneous amplitude and phase responses of the baseband transconductors that includes not only the first zonal responses but also all the higher order harmonics.

For PA behavioral modeling based on AM-AM and AM-PM parameter extraction technique, the bandpass complex power series coefficients $b_{I,i}$ and $b_{Q,i}$, which relates the complex envelope of the input (a phasor of a single-tone signal) to the complex envelope of the first zonal output responses at the carrier frequency (a phasor of a single-tone signal) need to be extracted by fitting the polynomial to the AM-AM and AM-PM data [6]. For quadrature modulator modeling, if similar bandpass complex power series coefficients $b_{I,i}$ and $b_{Q,i}$ were extracted by fitting a complex polynomial to AM-AM and AM-PM measurements of I and Q channel separately, then a conversion from bandpass coefficients $b_{I,i}$ and $b_{Q,i}$ to low pass model coefficients $\tilde{a}_{I,i}$ and $\tilde{a}_{Q,i}$ which characterize the instantaneous amplitude and phase responses needs to be done by:

$$
\tilde{a}_{I,2n+1} = b_{I,2n+1} \frac{2^n n!(n+1)!}{(2n+1)!}. \tag{4.12}
$$

$$
\tilde{a}_{Q,2n+1} = b_{Q,2n+1} \frac{2^n n!(n+1)!}{(2n+1)!}. \tag{4.13}
$$

An existing approach for AM-AM and AM-PM measurement for quadrature
modulators was presented in [81], where the authors measured the AM-AM and AM-PM characteristics of a RF quadrature modulator by applying a sweeping DC offset to the \( I/Q \) modulator and observing the output responses at the carrier frequency. Note that in this measurement the instantaneous amplitude and phase responses of the modulator nonlinearities (fundamental and higher order harmonics) were overlapped on top of each other at the carrier frequency and being observed. Therefore, the resulting coefficients after the curve fitting were the low pass coefficients \( \tilde{a}_{i,j} \) and \( \tilde{a}_{q,j} \) already and no conversion described by Equations (4.12) and (4.13) was necessary. However, one major limitation of this approach is that the finite carrier suppression performance of the \( I/Q \) modulator greatly limits the measurement dynamic range because the measurement inputs are amplitude swept DC signal that suffers from the DC offset impacts.

In this thesis, to overcome the carrier leakage induced dynamic range limitation, instead of using a sweeping DC offset signal, a baseband 10 kHz sinusoid signal with an exponential amplitude ramp was utilized to measure the AM-AM and AM-PM characteristic of I and Q channel. The measurement setup is shown in Figure 4-3.
Figure 4-3 Measurement setup for AM-AM and AM-PM characterization and ACLR/EVM measurements.

When measuring the AM-AM and AM-PM characteristic of the I channel, an electrical signal generator (ESG) generates a baseband 10 kHz exponential ramp sinusoid signal and fed to the differential I ports of the modulator. The Q channel input of the quadrature modulator is set to zero. The 10 kHz sinusoid is upconverted to RF carrier and the complex envelope of the RF output at 10 kHz offset to the carrier frequency is measured by the vector signal analyzer (VSA). For example, if the carrier frequency is 3.4 GHz, the complex envelope at 3.40001 GHz is measured. The similar approach can be repeated for measuring the AM-AM and AM-PM characteristic of the Q channel nonlinearity. Note that with this measurement approach, the measurement inputs are away from the DC and thus the RF responses are away from the carrier so that the carrier leakage impacts can be avoided. Also, the measured responses are the upconverted first zonal output responses of the baseband nonlinearities. The fitted coefficients are the bandpass coefficients $b_{i,j}$ and $b_{Q,j}$ and a bandpass to low pass coefficients conversion as described in Equations (4.12) and (4.13) needs to be done.
The input power is swept from $-19.7$ dBm to 7.7 dBm. The output 1 dB compression points for the I and Q channel nonlinearities with single tone sinusoidal excitation are both around $(3 - 6 = -3)$ dBm. An 11$^{th}$ order complex power series was fitted to AM-AM and AM-PM data of the I and Q channels of the $I/Q$ quadrature modulator, respectively. The fitted and the measured AM-AM and AM-PM for the I and Q channels match very well as shown in Figure 4-4 and Figure 4-5, respectively, which indicates that the 11$^{th}$ order polynomial fit is sufficient for this input range. Note that the accuracy of the model is guaranteed only up to the input power level being measured.

**Figure 4-4** Measured and predicted AM-AM and AM-PM characteristics of I channel.
Figure 4-5 Measured and predicted AM-AM and AM-PM characteristics of Q channel.

4.1.2.2 Extraction Techniques for Linear Error Model Coefficients

A four-point VNA measurement was done to extract the three model parameters for characterizing the DC offset, gain and phase imbalances. The measurement setup is shown in Figure 4-6.

Figure 4-6 Measurement setup for four-point VNA measurements.

The four input test points are:
1) $I = +B$, $Q = 0$; 2) $I = 0$, $Q = +B$
3) $I = -B$, $Q = 0$; 4) $I = 0$, $Q = -B$

The input voltage setting, $B$, was chosen as 100 mV in order to operate the modulator well within the linear region so that the nonlinear distortion is negligible and only the linear static errors are measured. In linear region, the modulator envelope outputs become:

$$\tilde{G}_I[I(t)] = a_{I,0} + \tilde{a}_{I,1} I = a_{I,0} + (x_{I,1} + jy_{I,1}) I. \quad (4.14)$$

$$\tilde{G}_Q[Q(t)] = a_{Q,0} + \tilde{a}_{Q,1} \cdot e^{j\phi} Q = a_{Q,0} + (x_{Q,1} + jy_{Q,1}) Q. \quad (4.15)$$

Combining Equations (4.1), (4.14), and (4.15):

$$\tilde{z}_{RF}(t) = (a_{I,0} + x_{I,1} I - y_{Q,1} Q) + j(a_{Q,0} + x_{Q,1} I + y_{Q,1} Q). \quad (4.16)$$

The corresponding output complex envelopes were measured using a VNA. Denote the real part of the complex envelopes of the four-point outputs as $\tilde{I}_0, \tilde{I}_1, \tilde{I}_2, \tilde{I}_3$ and the imaginary part as $\tilde{Q}_0, \tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3$. By equating the real and imaginary parts in Equation (4.16) for the four measurement points, the following equations are obtained:

$$a_{I,0} = \frac{1}{4} \sum_{i=0}^{3} \tilde{I}_i \quad \quad \quad a_{Q,0} = \frac{1}{4} \sum_{i=0}^{3} \tilde{Q}_i$$

$$x_{I,1} = (\tilde{I}_0 - \tilde{I}_2) / (2B) \quad \quad \quad y_{I,1} = (\tilde{Q}_0 - \tilde{Q}_2) / (2B). \quad (4.17)$$

$$x_{Q,1} = (\tilde{Q}_1 - \tilde{Q}_3) / (2B) \quad \quad \quad y_{Q,1} = -(\tilde{I}_1 - \tilde{I}_3) / (2B)$$

Knowing $a_{I,0}, a_{Q,0}, x_{I,1}, y_{I,1}, x_{Q,1}, y_{Q,1}$, the gain/phase error terms ($\delta$ and $\theta$) and carrier leakage (CL) can be calculated.
\[
\delta = \left| \frac{x_{Q,1} + jy_{Q,1}}{x_{I,1} + jy_{I,1}} \right|
\]
\[
\theta = \angle (x_{Q,1} + jy_{Q,1}) - \angle (x_{I,1} + jy_{I,1}) . \quad (4.18)
\]

\[
\text{CL (dBm)} = 10 \log \left[ \frac{a_{11,0}^2 + a_{0,0}^2}{50} \right] + 30
\]

By four easy measurements using only a DC voltage source and a VNA, the four-point measurement approach is able to characterize all the modulator static errors.

### 4.1.3 Complete Quadrature Modulator Behavioral Model

For the direct-conversion quadrature modulators, since the majority of the nonlinearity is induced by the baseband transconductors, the nonlinear responses are expected to be independent of the RF frequency. A two-tone sinusoidal signal was applied to the I channel and Q channel separately and the IM\(_3\) product was measured for six RF frequency points: 3 GHz, 3.1 GHz, 3.2 GHz, 3.3 GHz, 3.4 GHz, and 3.5 GHz. A downlink WCDMA signal was also applied to the quadrature modulator and the ACLR at the six frequency points were measured. As shown in Figure 4-7, as expected, the IM\(_3\) products of I and Q channels and the ACLR vary slightly (within 1.5 dBC) for frequency range of 3 – 3.5 GHz.
This property of the quadrature modulator being modeled implies that one set of nonlinear complex power series coefficients extracted at one frequency points (3.4 GHz in this work) can fit for a frequency range of $3 - 3.5$ GHz. This renders the beauty of the final complete model formulation: the nonlinear model coefficients are constant over the frequency range but with different linear error model coefficients, as shown in Table 4-2. This is actually a frequency dependent model where frequency is an input parameter. By using a look-up table to appropriately choose the corresponding linear model coefficients for each frequency point, the modulator responses over frequency can be properly modeled.

Figure 4-7 IM$_3$ and ACLR measurements over RF frequency.
Table 4-2 Behavioral Model Coefficients.

<table>
<thead>
<tr>
<th>Static Linear Errors</th>
<th>DC offset</th>
<th>Gain Err</th>
<th>Phi Err (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{I,0}$</td>
<td>$a_{Q,0}$</td>
<td>$\delta$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>0.0018</td>
<td>-0.0032</td>
<td>0.9960</td>
<td>2.7445</td>
</tr>
<tr>
<td>-0.0049</td>
<td>0.0047</td>
<td>1.0073</td>
<td>2.6921</td>
</tr>
<tr>
<td>-0.0001</td>
<td>0.0028</td>
<td>1.0074</td>
<td>2.0216</td>
</tr>
<tr>
<td>0.0017</td>
<td>0.0027</td>
<td>1.0149</td>
<td>-0.0476</td>
</tr>
<tr>
<td>0.0017</td>
<td>0.0014</td>
<td>1.0045</td>
<td>-0.3313</td>
</tr>
<tr>
<td>0.0033</td>
<td>0.0018</td>
<td>0.9980</td>
<td>-0.5594</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>3</th>
<th>3.1</th>
<th>3.2</th>
<th>3.3</th>
<th>3.4</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{I,0}$</td>
<td>-0.05842 + 0.47786i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{I,1}$</td>
<td>0.21239 + 0.046232i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{I,2}$</td>
<td>-0.20392 - 1.0439i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{I,3}$</td>
<td>0.004129 + 1.5826i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{I,4}$</td>
<td>0.090961 - 1.0033i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{I,5}$</td>
<td>-0.035703 + 0.23708i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{Q,0}$</td>
<td>0.004129 + 1.5826i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$b_{Q,1}$</td>
<td>0.13421 + 0.038073i</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$b_{Q,2}$</td>
<td>0.16119 - 0.9349i</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$b_{Q,3}$</td>
<td>-0.6273 + 1.3725i</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{Q,4}$</td>
<td>0.57698 - 0.84178i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{Q,5}$</td>
<td>-0.17532 + 0.19214i</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

4.1.4 Orthogonalized Model Implementation

In analyzing the impacts of the nonlinear distortion of a nonlinear system on the signal quality degradation, it is desirable to decompose the correlated and uncorrelated distortion components because it is the uncorrelated distortion component that degrades the SNR and EVM. Extensive research work has been done to develop orthogonal polynomial models. In [82, 83], the authors present a technique to identify the uncorrelated nonlinear distortion using orthogonal polynomials, but the bivariate probability density functions of the input signal need to be able to be expressed in the diagonal form. Orthogonal polynomial has also been proposed in [84], but the basis functions are computed online and iteratively so that more computational power is required. In [15, 16], based on the complex power series model structure, the authors developed orthogonal polynomial model for power amplifiers...
using statistical techniques which has no limitations on input signals. In this work, the orthogonal modeling technique in [15, 16] was adopted and implemented for the quadrature modulators model to decompose the correlated and uncorrelated output nonlinear distortion.

One way to calculate the power spectrum of the quadrature modulator model output is to compute the Fourier transform of the output autocorrelation function [85]. The autocorrelation function of the modulator output is obtained using (4.1):

\[ \tilde{R}_{zz}(\tau) = E \left\{ \bar{z}_I(t) + j\bar{z}_Q(t) \right\} \bar{z}_I(t + \tau) + j\bar{z}_Q(t + \tau) \}. \] (4.19)

where \( E(\cdot) \) is the expected value operator, \( (\cdot)^* \) is the complex conjugate, and \( \bar{z}_I(t) \) and \( \bar{z}_Q(t) \) are the output envelopes of the I and Q channel nonlinearities.

Assuming the I and Q channels are isolated and no coupling occurs between these two channels, it follows that,

\[ \tilde{R}_{zz}(\tau) = R_{\bar{z}_I\bar{z}_I}(\tau) + R_{\bar{z}_Q\bar{z}_Q}(\tau) + j[R_{\bar{z}_I\bar{z}_Q}(\tau) - R_{\bar{z}_Q\bar{z}_I}(\tau)] \]. \) (4.20)

The output power spectrum can be solved by taking Fourier transform on the real part of output correlation function:

\[ \tilde{S}_{\bar{z}\bar{z}}(f) = F \left[ \text{Re} \left( \tilde{R}_{zz}(\tau) \right) \right] = \tilde{S}_{\bar{z}_I\bar{z}_I}(f) + \tilde{S}_{\bar{z}_Q\bar{z}_Q}(f) \). \) (4.21)

where \( F \) denotes Fourier transform. Equation (4.21) shows that the output power spectrum of the quadrature modulator equals to the sum of the output spectrums of the I and Q channel, which is consistent with the assumption that the I and Q channel are isolated. Therefore, the orthogonalization model technique developed in [15, 16] for power
amplifiers can be extended to quadrature modulators by orthogonalizing the I and Q channel nonlinearities independently to obtain their orthogonal output spectra and then sum them together to get the orthogonalized quadrature modulator output.

Applying the orthogonalization technique in [15, 16] to I and Q channel nonlinearities respectively, a new set of complex power series coefficients and orthogonal basic functions based on the original coefficients and input signal spectra are obtained. Thus the output of I channel nonlinearity can be decomposed into the correlated component \( \tilde{z}_{IC}(t) \) and the uncorrelated component \( \tilde{z}_{IU}(t) \). Similarly, the output of Q channel nonlinearity can be decomposed into the correlated component \( \tilde{z}_{QC}(t) \) and the uncorrelated component \( \tilde{z}_{QU}(t) \). Then the correlated and uncorrelated output spectrum components of the quadrature modulator output are available by summing the correlated and uncorrelated components of I and Q channel nonlinearities.

\[
\tilde{S}_{zz}(f) = \tilde{S}_{z_{IC}z_{IC}}(f) + \tilde{S}_{z_{IU}z_{IU}}(f)
\]

\[
= \tilde{S}_{z_{IC}z_{IC}}(f) + \tilde{S}_{z_{QC}z_{QC}}(f) + \tilde{S}_{z_{IU}z_{IU}}(f) + \tilde{S}_{z_{RU}z_{RU}}(f)
\]

### 4.1.5 Simulation and Measurement Results

The accuracy of the behavioral model for characterization of nonlinear distortion of the QM3337A quadrature modulator was verified by ACLR and EVM measurements excited by WCDMA signals, zero-IF WLAN signals, and low-IF WLAN signals. The accuracy of the behavioral model for characterization of the linear errors (DC offset, gain/phase imbalance) was verified by the image rejection (IR) measurement and carrier leakage (CL) measurement.
4.1.5.1 Model Validation for Characterization of Out-of-Band Distortion

ACLR is a key parameter in quantifying the spectra regrowth of the nonlinear devices. For WCDMA, in this work, the ACLR at 5 MHz was measured and simulated. There is no an explicit definition of ACPR for WLAN signals [32] since EVM is the more stringent metric in a WLAN system. For the purpose of validation of the out-of-band distortion of the model, the ACPR definition for WLAN signals in Section 3.4.4 was reused here.

Three classes of digital signals: WCDMA signal, zero-IF WLAN signal and, low-IF WLAN signal were fed to a direct-conversion quadrature modulator and the ACPR over an input power range were measured to characterize the out-of-band distortion. The three types of signals represent the three applications of quadrature modulators in wireless communications systems: generic quadrature modulation, zero-IF OFDM, and low-IF OFDM. For WCDMA, an uplink pilot signal with a PAR of 3.4 dB and a downlink test model 5 signal with a PAR around 11 dB were used. For zero-IF OFDM and low-IF OFDM, four signals with different modulation schemes, 64 QAM, 16 QAM, QPSK, and BPSK were used. Their PARs are all close to 11 dB as well. The gain compression characteristics of the quadrature modulator excited by the three classes of signals are summarized in Table 4-3. The output 1 dB compression point for the individual I or Q channel nonlinearity excited by the WCDMA uplink signal is (−2.5−3 = −5.5 dBm), which is as expected about 2.5 dBm lower than the one under single sinusoidal excitation (in Section 4.1.2.1) due to the larger PAR. The output 1 dB compression point excited by the downlink WCDMA and WLAN signals are even lower.
Table 4-3 Summary of gain compression characteristics.

<table>
<thead>
<tr>
<th></th>
<th>1 dB Comp Point (Pout, dBm)</th>
<th>Linear Gain Error* (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WCDMA</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Up_link</td>
<td>−2.5</td>
<td>0.23</td>
</tr>
<tr>
<td>Dn_link</td>
<td>−5.8</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Zero_IF WLAN</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64QAM</td>
<td>−5.3</td>
<td>0.21</td>
</tr>
<tr>
<td>32QAM</td>
<td>−5.2</td>
<td>0.21</td>
</tr>
<tr>
<td>QPSK</td>
<td>−5.1</td>
<td>0.10</td>
</tr>
<tr>
<td>BPSK</td>
<td>−5.1</td>
<td>0.31</td>
</tr>
<tr>
<td><strong>Low_IF WLAN</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64QAM</td>
<td>−5.2</td>
<td>0.25</td>
</tr>
<tr>
<td>32QAM</td>
<td>−5.0</td>
<td>0.16</td>
</tr>
<tr>
<td>QPSK</td>
<td>−5.1</td>
<td>0.10</td>
</tr>
<tr>
<td>BPSK</td>
<td>−5.1</td>
<td>0.10</td>
</tr>
</tbody>
</table>

* The linear gain error between measured and modeled
The linear gain for WCDMA signals is about −6.5 dB
The linear gain for WLAN signals is about −6.2 dB

The ACLR measurement test setup is shown in Figure 4-3. The above digital signals were generated using the ESG. The corresponding baseband I/Q signals were advanced to the I/Q quadrature modulator via the baseband differential output ports of the ESG. The ACPRs were measured by the VSA.

The measured versus the modeled upper sideband and lower sideband ACLR excited with the WCDMA uplink signal are shown in Figure 4-8 and 4-9, from which it is observed that the upper sideband and the lower sideband ACLR are very close for both the measured and modeled results. This ACPR symmetry was observed for all the other WCDMA and zero-IF WLAN signals but not showed here to save space. The ACLR symmetry indicates that there is no significant memory in the quadrature modulator, which validates our assumption that a quasi-memoryless model can be used to model this quadrature modulator.
The measured and the modeled ACLR excited with the two WCDMA signals as listed in Table 4-3 are shown in Figure 4-10. There is an excellent agreement (within 1 dB difference) between the measured and modeled ACLR when the modulator is within the
nonlinear region, indicating that the complex power series model predicts the quadrature modulator out-of-band distortion very accurately for the generic quadrature modulation systems.

![Graph showing ACLR versus output power](image)

**Figure 4-10** Measured and predicted ACLR for (1) WCDMA uplink signal; (2) WCDMA downlink signal.

The four zero-IF WLAN signals as listed in Table 4-3 were used in the ACPR measurement. The measured and modeled ACPR results for the four zero-IF WLAN signals are similar. In Figure 4-11, the ACPR versus output power was only shown for the 64 QAM case, as well as the measured and simulated ACPR difference for four WLAN signals. There is a good agreement (within 1 dB difference) between the measured and the simulated ACLR when the modulator is within the nonlinear region, indicating that the complex power series model predicts the quadrature modulator out-of-band distortion very accurately for the zero-IF OFDM systems.
Figure 4-11 a) Upper plot: ACPR results for a 64 QAM zero-IF WLAN signal; b) Lower plot: Difference of the measured and simulated ACPR for the 4 types of low-IF WLAN signals.

The four low-IF WLAN signals as listed in Table 4-3 were used in the ACPLR measurement. Due to the 3rd order hump as explained in Section 3.4, the lower sideband and the upper sideband ACPR for the low-IF WLAN signal are asymmetric, as shown in Figure 4-12.

Figure 4-12 Upper sideband and lower sideband ACPR for a 64 QAM low-IF WLAN signal.
The differences between the measured versus the modeled ACPR were shown in Figure 4-13, which are very small within 1 dB difference when the modulator is within the nonlinear region, indicating that the complex power series model predicts the quadrature modulator out-of-band distortion very accurately for the low-IF OFDM systems.

![Figure 4-13](image_url)

**Figure 4-13** Difference between the measured and modeled ACPR for low-IF WLAN signals.

4.1.5.2 Model Validation for Characterization of In-Band Distortion

As described in Section 2.2.3, the uncorrelated in-band distortion, SNR, and EVM, are related [43] so that it is equivalent to measure the EVM of the quadrature modulator output signal in order to estimate the uncorrelated in-band distortion. Therefore, the accuracy of the orthogonal model in characterizing the correlated/uncorrelated nonlinear distortion was validated by measuring the EVM of the quadrature modulator output excited by the 3 classes of signals as listed in Table 4-3.

The modeled and measured EVM results for the two WCDMA signal cases are
presented in Figure 4-14. There is a very good match between these two sets of values, indicating that the orthogonal model is able to accurately predict the uncorrelated in-band distortion, or equivalently the quadrature modulator output waveform quality for the generic quadrature modulations systems.

![Figure 4-14 Measured and simulated EVM for 1) an uplink WCDMA signal; 2) a downlink test model 5 WCDMA signal.](image)

The measured and modeled EVM results for the zero-IF and low-IF WLAN signals are presented in Figure 4-15 and 4-16. In these two figures, the upper plot only shows the EVM versus output power for the 64 QAM case; the lower plot shows the differences of the measured and modeled EVM for the four types of WLAN signals, which is small within 1.5 dB. The agreement between the measured and modeled EVM results shows that orthogonal model is able to accurately predict the uncorrelated in-band distortion, or equivalently the quadrature modulator output waveform quality for the zero-IF and low-IF OFDM systems.
Figure 4-15 1) Upper plot: EVM results for a 64 QAM zero-IF WLAN signal; 2) Lower plot: difference between the measured and simulated EVM for the four zero-IF WLAN signals ($RCE = 20\log_{10}(EVM)$).

Figure 4-16 1) Upper plot: EVM results for a 64 QAM low-IF WLAN signal; 2) Lower plot: difference between the measured and simulated EVM for the four low-IF WLAN signals ($RCE = 20\log_{10}(EVM)$).
4.1.5.3 Model Validation for Characterization of Linear Errors

The accuracy of the behavioral model in characterizing the modulator linear errors was verified by the single side band (SSB) image rejection (IR) and carrier leakage (CL) measurements. A SSB sinusoidal signal was applied to the modulator inputs and it is well known that the gain and phase imbalance can be evaluated by the IRR, as presented in Equation (2.18).

The IR and CL were modeled and compared to the measured results at six frequency points: 3.0 GHz, 3.1 GHz, 3.2 GHz, 3.3 GHz, 3.4 GHz, and 3.5 GHz, as shown in Figure 4-17. They are in good agreement with the maxim discrepancy only about 1.3 dBc, which indicates that the linear error model based on four-point VNA measurement is accurate in characterizing the modulator linear static errors.

![Figure 4-17 Measured and modeled image rejection and carrier leakage over frequency.](image-url)
4.1.6 Summary

An orthogonal envelope behavioral model of a passive direct-conversion quadrature modulator was presented in this section. It is an odd order complex power series model with some modifications to incorporate some extra model elements in order to accurately characterize the correlated and uncorrelated nonlinear distortion, carrier leakage, and gain/phase imbalances. The accuracy of this behavioral model was validated by the measurement data.

4.2 Behavioral Modeling for a RF Integrated Direct-Conversion Quadrature Modulator

In this section, the same quadrature modulator modeling technique developed in Section 4.1 was applied to a RF integrated direct conversion modulator, HMC496LP3, which is manufactured by Hittite Microwave Corporation. It is a SiGe wideband direct conversion quadrature modulator aimed for applications such as WLAN, C-band microwave radios, and etc. The operating frequency range of HMC496LP3 is 4 ~ 7 GHz. The baseband bandwidth is from DC to 250 MHz. The top-level diagram of this modulator was shown in Figure 4-18. A behavioral model was developed for HMC496LP3 modulator at 5 GHz for characterization of nonlinear distortion, DC offset, and gain/phase imbalances.
4.2.1 AM-AM and AM-PM Measurement Results

The same AM-AM and AM-PM measurement technique for quadrature modulators using 10 KHz offset single-tone sinusoidal excitation was repeated for HMC496LP3 modulator in order to extract the model parameters for characterizing the nonlinear characteristics. Using the similar measurement setup as shown in Figure 4-3, the output responses at 5.00001 GHz were measured for I channel and Q channel respectively. An 11\textsuperscript{th}-order complex power series was fitted to AM-AM and AM-PM data of the I and Q channel of the I/Q quadrature modulator. The input power is swept from -19.7 dBm to 7.7 dBm. The quality of the fit of the I and Q channels are shown in Figure 4-19 and Figure 4-20 respectively. An 11\textsuperscript{th}-order complex power series was able to fit the AM-AM and AM-PM response curves pretty well.
Figure 4-19 Measured and predicted AM-AM and AM-PM characteristics of I channel.

Figure 4-20 Measured and predicted AM-AM and AM-PM characteristics of Q channel.
4.2.2 4-Point VNA Measurement

The 4-point VNA measurement technique for quadrature modulators was repeated for HMC496LP3 modulator at 5 GHz in order to extract the model parameters for characterizing the static errors including DC offset, gain and phase imbalances. The similar measurement setup as shown in Figure 4-6 was used in this measurement. This accuracy of this extraction technique for this RF integrated quadrature modulator was verified by the SSB image rejection (IR) and carrier leakage (CL) measurement. A SSB sinusoidal signal was applied to the modulator inputs at a RF frequency of 5 GHz. The IR and CL were measured and compared to the modeled results. For the image rejection, the measured and modeled results are 39.4 dBc and 40.9 dBc respectively. For the carrier leakage, the measured and modeled results are 28.7 dBc and 30.2 dBc respectively. They are in good agreement with a discrepancy within 1.5 dBc, which indicates that the 4-point measurement is an accurate way to extract model parameters for the RF integrated modulator static errors as well.

The complete model parameters RF frequency at 5 GHz are presented in Table 4-4, including the complex power series coefficients \( \tilde{b}_{I,i} \) and \( \tilde{b}_{Q,i} \) of order \( N=11 \) for characterizing the I and Q channel nonlinear responses, real coefficients \( a_{I,0} \) and \( a_{Q,0} \) for characterizing the DC offset, \( \delta \) for characterizing gain error and \( \theta \) for characterizing the phase error.
Table 4-4 Model coefficients for HMC496LP3 quadrature modulator.

<table>
<thead>
<tr>
<th>Static Linear Errors</th>
<th>DC offset</th>
<th>Gain Err</th>
<th>Phi Err (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_{1,0}$</td>
<td>$\delta$</td>
<td>$\theta$</td>
</tr>
<tr>
<td></td>
<td>0.0006</td>
<td>1.0010</td>
<td>-1.0400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nonlinear Responses</th>
<th>$b_{1,1}$</th>
<th>$b_{1,3}$</th>
<th>$b_{1,5}$</th>
<th>$b_{1,7}$</th>
<th>$b_{1,9}$</th>
<th>$b_{1,11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.2774 + 0.2390i$</td>
<td>$0.0972 - 0.0980i$</td>
<td>$0.5033 - 0.4216i$</td>
<td>$-1.0122 + 0.8643i$</td>
<td>$0.8593 - 0.6886i$</td>
<td>$-0.2741 + 0.2016i$</td>
</tr>
<tr>
<td></td>
<td>$b_{0,1}$</td>
<td>$b_{0,3}$</td>
<td>$b_{0,5}$</td>
<td>$b_{0,7}$</td>
<td>$b_{0,9}$</td>
<td>$b_{0,11}$</td>
</tr>
<tr>
<td></td>
<td>$-0.2774 + 0.2390i$</td>
<td>$0.1036 - 0.0993i$</td>
<td>$0.4978 - 0.4292i$</td>
<td>$-1.0167 + 0.8868i$</td>
<td>$0.8677 - 0.7138i$</td>
<td>$-0.2773 + 0.2112i$</td>
</tr>
</tbody>
</table>

4.2.3 Simulation and Measurement Results

4.2.3.1 Model Validation for Characterization of Out-of-Band Distortion

A 64-QAM zero-IF WLAN signal was applied to the HMC496LP3 modulator and the ACPR were measured and compared to the modeled results to validate the model characterization of the out-of-band distortion. The gain compression characteristics of the $I/Q$ modulator excited by the 64-QAM zero-IF WLAN signal is shown in Figure 4-21. For the 64-QAM zero-IF WLAN signal, the $I/Q$ modulator goes into gain compression region at an input power of around $-1.5$ dBm.
Figure 4-21 Measured and simulated gain compression characteristic of the I/Q modulator excited by a 64-QAM zero-IF WLAN signal.

The measured versus the simulated ACLR for the upper sideband is shown in Figure 4-22. There is an excellent agreement (within 1 dB difference) between the measured and the simulated ACLR in both linear and strong nonlinear regions, indicating that the memoryless complex power series model predicts the nonlinear distortion characteristics of the quadrature modulator very accurately.
Figure 4-22 Measured and modeled ACPR (upper sideband).

The measured versus the simulated ACLR for the lower sideband is shown in Figure 4-23. The ACPR of the upper sideband and the lower sideband are very close for both the measured data and simulated results. The symmetry indicates that there is no significant memory in the quadrature modulator and thus validate the quasi-memoryless model assumption.

Figure 4-23 Measured and modeled ACPR (lower sideband).
4.2.3.2 Model Validation for Characterization of In-Band Distortion

The accuracy of the orthogonal model was validated by the EVM measurement excited with a 64-QAM zero-IF WLAN signals. The measured and modeled EVM results were shown in Figure 4-24, which shows a good agreement between the modeled and measured EVM. This indicates the orthogonal behavioral model can accurately predict the uncorrelated in-band distortion, or equivalently the quadrature modulator output waveform quality.

![Figure 4-24 Measured and predicted EVM.](image)

4.2.4 Summary

The same modeling technique as described in Section 4.1 was applied to an active RF integrated direct-conversion quadrature modulator to characterize its correlated and uncorrelated nonlinear distortion, carrier leakage, and gain/phase imbalances. The measurement data have a good match between the model results. Combined with the conclusions from Section 4.1, it can be concluded that the orthogonal nonlinear behavioral
model is accurate for both passive and active direct-conversion quadrature modulators for characterization of the correlated/uncorrelated nonlinear distortion, the DC offset, and the gain/phase imbalances.
Chapter 5

Design and Modeling of Bipolar Multi-Tanh Transconductors

One fundamental quadrature modulator model assumption, which was validated by the modeling results, is that the baseband transconductors dominate the nonlinear characteristics of integrated quadrature modulators. This motivated the designing and modeling work in this Chapter aiming at characterizing the nonlinear distortion of a special category of transconductors, bipolar multi-tanh transconductors, in order to gain more insights regarding nonlinear quadrature modulator modeling.

5.1 Basics of Multi-Tanh

Differential pair is probably the simplest but the mostly used baseband transconductor in quadrature modulators. A bipolar differential pair has the well known hyperbolic tangent transfer function [86]:

\[
tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}
\]
\[ I_{\text{out}} = I_E \tanh \frac{V_{\text{in}}}{2V_T}. \]  

(5.1)

where \(I_{\text{out}}\) is the differential output current, \(I_E\) is the emitter bias current, \(V_T\) equals to \(kT/q\).

One step further by doing the derivation, the incremental transconductance of a BJT differential pair is obtained as:

\[ g_m = \frac{d}{d_{V_m}} \frac{I_{\text{out}}}{I_E} = \frac{I_E}{2V_T} \sec h^2 \frac{V_{\text{in}}}{2V_T}. \]  

(5.2)

The Taylor series expansion of (5.1) is:

\[ \frac{I_{\text{out}}}{I_E} = \left( \frac{V_{\text{in}}}{2V_T} \right)^3 + \frac{2}{15} \left( \frac{V_{\text{in}}}{2V_T} \right)^5 + \frac{17}{315} \left( \frac{V_{\text{in}}}{2V_T} \right)^7 \ldots \]  

(5.3)

From (5.3), it is obvious that the nonlinear characteristics of the ideal BJT differential pairs are independent of emitter bias current \(I_E\). Another property of a differential pair transconductor is that no even-order nonlinearities exist, which is due to the cancellation mechanism of the differential structures. The basic BJT differential pairs have limited input range handling capacity before the distortion becomes unacceptable. Therefore, a category of linear transconductors, called “multi-tanh”, was first reported in 1968 [87] as a circuit topology intended for drift compensation in IC amplifiers instead of linearity improvement. Later on it found uses in different kinds of real-word analog/RF systems due to its superior linearity performances. The name of “multi-tanh” was invented and the general multi-tanh principle was shown in [88] in recognition that this circuit structure is a combination of multiple offset tanh functions. A general multi-tanh circuit structure is shown in Figure 5-1.
The basic idea of multi-tanh circuits is to connect multiple differential pairs with different input offsets, i.e. offset tanh functions, in parallel and thus split the individual $g_m$ functions along the input-voltage axis, resulting in an overall $g_m$ function which is flat over a large range of input voltage. Therefore, the input voltage range is increased and the distortion is decreased. The drawback of CMOS multi-tanh technique is the reduced $G_m/I_c$ efficiency, increased noise and complexity [89]. Another limitation of the multi-tanh circuit structure is that the linear input range is limited by the practical emitter area ratio [90].

The dc transfer function of a multi-tanh circuit can be written as:

$$I_{out} = \sum_{j=1}^{N} I_j \tanh \left\{ \frac{V_{in} + V_j}{2V_T} \right\}. \quad (5.4)$$

where $I_j$ is the tail current to the $j_{th}$ stage and $V_j$ is the base offset voltage associated with that stage. Same as the differential pairs, there are no even-order nonlinearities in a multi-tanh system. The total $G_m$ of the $N$-stage multi-tanh is:
\[ G_m = \frac{d I_{out}}{d V_{in}} = \sum_{j=1}^{\infty} \frac{I_j}{2V_T} \sec h^2 \frac{V_{in} + V_j}{2V_T}. \] (5.5)

It worth mentioning that the multi-tanh circuit principle can also be applied to CMOS circuits. In fact, many circuit building blocks in modern wireless systems are utilizing the CMOS version multi-tanh transconductors [35, 91, 92].

### 5.2 Multi-Tanh Transconductors Design

In this section, three different types of baseband and bandpass bipolar transconductors: bipolar differential pair; multi-tanh doublet; and multi-tanh triplet, were designed and fabricated in 0.18 μm IBM BiCMOS technology.

The differential pair transconductor is shown in Figure 5-2. The use of the tail current source \(I_1\) provides two advantages: 1) improved common mode rejection performance; 2) well controlled bias conditions. It was implemented by means of CMOS current mirrors. The designed value of \(I_1\) is 1 mA. The DC transfer function of the BJT differential pair transconductor was presented in Equation (5.1).

![Block diagram of a BJT differential pair transconductor.](image-url)

Figure 5-2 Block diagram of a BJT differential pair transconductor.
The multi-tanh doublet transconductor is shown in Figure 5-3. The two unbalanced BJT differential pairs with an emitter area ratio of $A$ shift the peak of each $g_m$ equally in the opposite direction so as to make the total $G_m$ flat over a broader input range.

$$V_{in} \quad + \quad I_{out} \quad +$$

Figure 5-3 Block diagram of a BJT multi-tanh doublet transconductor.

The offset voltage introduced by an emitter ratio of $A$ is [89]:

$$V_{os} = V_T \log A.$$  \hspace{1cm} (5.6)

because

$$\frac{I_{out}}{I_E} = \frac{I_s e^{\left(\frac{V_{in}}{2V_T}\right)} - AI_s e^{\left(-\frac{V_{in}}{2V_T}\right)}}{I_s e^{\left(\frac{V_{in}}{2V_T}\right)} + AI_s e^{\left(-\frac{V_{in}}{2V_T}\right)}} = \frac{I_s e^{\left(\frac{2\log A}{2V_T}\right)} - e^{\left(-\frac{V_{in}}{2V_T}\right)}}{I_s e^{\left(\frac{2\log A}{2V_T}\right)} + e^{\left(-\frac{V_{in}}{2V_T}\right)}} = \tanh \left(\frac{V_{in} - \log A}{2V_T}\right).$$

It follows that the DC transfer function for a multi-tanh doublet with an emitter area ratio of $A$ is:

$$I_{out} = I_1 \tanh \frac{V_{in} + V_{os}}{2V_T} + I_2 \tanh \frac{V_{in} - V_{os}}{2V_T}.$$  \hspace{1cm} (5.7)
The optimum emitter ratio $A$ for the multi-tanh doublet is 3.73 [89] which was used in this design. The two tail current sources $I_1$ and $I_2$ are equal and the sum of them was set to be 1 mA for comparison with the BJT differential pair transconductor.

The multi-tanh triplet transconductor is shown in Figure 5-4. The two unbalanced BJT differential pairs with an emitter area ratio of $A$ shift the peak of each $g_m$ equally in the opposite direction. The peak of $g_m$ of the middle balanced pair is centered at zero axes. The total effect is to make the total $G_m$ flat over an even broader input range.

![Figure 5-4 Block diagram of a BJT multi-tanh triplet transconductor.](image)

The offset voltage introduced by an emitter ratio of $A$ is presented in Equation (5.6). It follows that the DC transfer function for a multi-tanh triplet with an emitter ratio of $A$ is:

$$I_{out} = I_1 \tanh \left( \frac{V_m + V_{os}}{2V_T} \right) + I_2 \tanh \left( \frac{V_m}{2V_T} \right) + I_3 \tanh \left( \frac{V_m - V_{os}}{2V_T} \right).$$

(5.8)

The optimum emitter ratio $A$ for the multi-tanh triplet is 13, and the ratio of the current sources is $I_2=0.75I_1=0.75I_3$ [89], which was used in this design. The sum of three current sources

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sources was set to be 1 mA for comparison with the other three transconductors.

The baseband transconductors were designed by loading the above three types of transconductors with the same cascode stage but different size resistors to make the voltage gain of the baseband different amplifiers the same. The bandpass transconductors were designed by loading the above three types of transconductors with the same cascode stage but different size tank loads (tuned at 2 GHz) to make the voltage gain of the bandpass different amplifiers the same.

With the above design values, based on the Equations (5.1), (5.7) and (5.8), the linearity performances of the three baseband transconductors are in the following descending order: multi-tanh triplet, multi-tanh doublet, and differential pair, as shown in Figure 5-5, which plots the single-tone gain compression curves for the three baseband transconductors using the Equations (5.1), (5.7) and (5.8). The gain of each transconductor is normalized to zero for comparison purpose.
Figure 5-5 Analytical single-tone gain compression curves of three baseband transconductors.

The Cadence SpectreRF Periodic Static State (PSS) simulator [93] was used to estimate the linearity performance of the three baseband transconductors by the two-tone IM$_3$ simulation. The simulated IM$_3$ of the transconductors were shown in Figure 5-6. As expected, the multi-tanh triplet has the best linearity performance, followed by the multi-tanh doublet, and finally the differential pair.
Figure 5-6 Simulated IM\textsubscript{3} over input voltage for the three baseband transconductors.

The improvement of linearity performance of multi-tanh circuits is at the cost of reduced $G_m/I_c$ efficiency. A PSS simulation was run to estimate the $G_m$ of the three transconductors. The simulated $G_m$ with $I_c=1$ mA were shown in Figure 5-7. With the same DC bias current, the BJT differential pair has the largest $G_m$ of 17.97 mA/V, followed by the multi-tanh doublet with a $G_m$ of 12.69 mA/V, and the smallest $G_m$ of 10.68 mA/V for the multi-tanh triplet.
Figure 5-7 Simulated $G_m$ of the differential pair, doublet, and triplet transconductors.
A two-tone IM\textsubscript{3} simulation was run to estimate the linearity performance of the bandpass transconductors. The simulated IM\textsubscript{3} were shown in Figure 5-8.

![IM3 of bandpass Transconductors](image)

**Figure 5-8 Simulated IM\textsubscript{3} over input power for the three bandpass transconductors.**

Similar as the baseband case, for the three bandpass transconductors, the multi-tanh triplet has the widest linear input range, followed by the doublet and finally the differential pair transconductors.
5.3 Measurement Setup for Characterizing Multi-Tanh Transconductors

The multi-tanh transconductors designed in Section 5.2 are not wire-bonded. Therefore, probing measurements are needed to characterize the nonlinear responses of these transconductors. In addition, the transconductors have both inputs and outputs as differential so that differential nonlinear characterization is required. These requirements pose challenges in measurements as described in Sections 5.3.1 and 5.3.2.

5.3.1 Differential Probing with Network Analyzer Only

The first possible measurement setup is shown in Figure 5-9. This method looks straightforward. A four-port VNA and two GSSG (ground-signal, signal-ground) differential probes are required in this measurement setup. Ports 1 and 2 serve as the differential input port and ports 3 and 4 as the differential output port. By calibrating the VNA properly, the reference planes can be set right at the input and output port of the device-under-test (DUT). The differential nonlinear characteristics can be captured by the large-signal S parameters by sweeping the input power from linear to nonlinear region.
Figure 5-9 Measurement setup with network analyzer only.

However, this approach poses great demands on both hardware and software. First, in order to do the nonlinear differential characterization, a four-port, two-source network analyzer must be utilized because when clipping, saturation, or compression effects occur, cross coupling between differential and common modes comes into play and the cross coupling cannot be adequately characterized by measuring responses from just one source at a time. If a two-source, four-port network analyzer is available, a way needs to be found out to differentially control the two sources, and to correctly process the measured four-port single-ended S parameters into mix-mode S parameters to characterize the nonlinear responses of the DUT [94].

Since at the time the multi-tanh transconductors are being characterized there is no a two-source, four-port network analyzer available, this measurement approach was not utilized.
5.3.2 Differential Probing with Network Analyzer and Balun Transformers

The second possible measurement setup is shown in Figure 5-10. In addition to a network analyzer and two GSSG differential probes, two balun transformers are utilized. The two baluns convert the DUT differential input/output into single-ended input/output and then a traditional two-port measurement can be done. The drawback of this method is that the reference planes are now moved to the end of the cables. The baluns and GSSG probes have insertion loss, gain imbalance, and phase imbalances, all of which will impact the measurement results. Such impacts need to be evaluated and possible remedies need to be found out to guarantee the quality of measurement.

![Figure 5-10 Measurement setup with network analyzer and baluns.](image-url)

Figure 5-10 Measurement setup with network analyzer and baluns.

The insertion loss causes inaccuracy in the measured gain but the loss could be able to be calibrated out. The amplitude and phase imbalances can’t be calibrated and analysis of their impacts on DUT nonlinear responses is needed. A simple analysis was done for a baseband BJT differential pair transconductors (a Tanh function).
For a BJT differential pair, its DC transfer function and the Taylor series expansion are shown in Equations (5.1) and (5.3). Assume \( \frac{V_{in}}{2V_T} = \frac{a \cos(\omega t)}{V_T} = b \cos(\omega t) \).

\[
\frac{I_{out}}{I_E} = \left( \frac{V_{in}}{2V_T} \right)^3 + \frac{2}{15} \left( \frac{V_{in}}{2V_T} \right)^5 = \left[ b \cos(\omega t) \right] - \frac{1}{3} \left[ b \cos(\omega t) \right]^3 + \frac{2}{15} \left[ b \cos(\omega t) \right]^5 - ....
\]

(5.9)

By trigonometry, Equation (5.9) can be rewritten as:

\[
\frac{I_{out}}{I_E} = \left( b - \frac{1}{4} b^3 + \frac{1}{12} b^5 \right) \cos(\omega t) + \left( -\frac{1}{12} b^3 + \frac{1}{24} b^5 \right) \cos^3(\omega t) + \left( \frac{1}{120} b^5 \right) \cos^5(\omega t) - ....
\]

(5.10)

The harmonic distortion (HD_x) is used to characterize the nonlinear distortion in this analysis. It is defined as the ratio of the power of the harmonic of interests to the fundamental signal power. For example, HD_3 is the ratio of the 3rd order of harmonic distortion power to the fundamental signal power.

Therefore,

\[
\text{HD}_3 = -\frac{1}{12} b^2 + \frac{1}{24} b^4 - ....
\]

(5.11)

\[
\text{HD}_5 = \frac{1}{120} b^4 - ....
\]

(5.12)

Now suppose there are amplitude imbalances in the differential inputs:

\[
\frac{V_i^+}{\frac{1}{2} V_T} = \frac{a \cos(\omega t)}{V_T} = b \cos(\omega t) .
\]

(5.13)
\[
\frac{V_i}{2 V_T} = \frac{1}{2} \frac{\delta \left[a \cos(\omega t + 180)\right]}{V_T} = -\delta \cdot b \cos(\omega t) . \tag{5.14}
\]

where \( \delta \) is the ratio of the amplitude \((A^+\) of the in-phase input to the amplitude \((A^-)\) of the out-of-phase input \((\delta = \frac{A^+}{A^-})\).

\[
\frac{I_{out}}{I_E} = \frac{e^{\frac{V_i^+}{2V_T}} - e^{\frac{V_i^+}{2V_T}}}{e^{\frac{V_i^+}{2V_T}} + e^{\frac{V_i^-}{2V_T}}} = \frac{e^{b \cos(\omega t)} - e^{-\delta b \cos(\omega t)}}{e^{b \cos(\omega t)} + e^{-\delta b \cos(\omega t)}} = \frac{2}{1 + e^{-(\delta + 1)b \cos(\omega t)}} - 1. \tag{5.15}
\]

Using Taylor series expansion, Equation (5.15) can be rewritten as (let \( x = (\delta + 1) \cdot b \cos(\omega t) \)):

\[
\frac{I_{out}}{I_E} = \frac{2}{1 + e^{-x}} - 1 = \frac{2}{2 - x + x^2/2! - x^3/3! + x^4/4! - x^5/5! + x^6/6! - \ldots} - 1. \tag{5.16}
\]

By polynomial division:

\[
\frac{I_{out}}{I_E} = \left(1 + \frac{x}{2} + \frac{x^3}{24} + \frac{x^5}{240} - \ldots\right) - 1
\]

\[
= \frac{(\delta + 1) \cdot b \cos(\omega t)}{2} - \frac{(\delta + 1) \cdot b \cos(\omega t))^3}{24} + \frac{(\delta + 1) \cdot b \cos(\omega t))^5}{240} - \ldots . \tag{5.17}
\]

Let \( \delta = 1 + \varepsilon \), Equation (5.17) becomes:

\[
\frac{I_{out}}{I_E} = (1 + \frac{\varepsilon}{2}) \cdot b \cos(\omega t) - \frac{1}{3} \left(1 + \frac{\varepsilon}{2}\right) b \cos(\omega t)^3 + \frac{2}{15} \left(1 + \frac{\varepsilon}{2}\right) b \cos(\omega t)^5 - \ldots \tag{5.18}
\]

Compare to Equation (5.9), the net effect of amplitude imbalance is to cause the input signal magnitude variation by \( \frac{\varepsilon}{2} \). Therefore, the HD_3 and HD_5 for the amplitude imbalance case are:
\[ \text{HD}_3 = -\frac{1}{12} \left( 1 + \frac{\varepsilon}{2} \right)^2 b^2 + \frac{1}{24} \left( 1 + \frac{\varepsilon}{2} \right)^4 b^4 - \ldots \]  
(5.19)

\[ \text{HD}_5 = \frac{1}{120} \left( 1 + \frac{\varepsilon}{2} \right)^4 b^4 - \ldots \]  
(5.20)

For small \( \varepsilon \), by neglecting 2\(^\text{nd}\) and higher order terms:

\[ \left( 1 + \frac{\varepsilon}{2} \right)^2 = 1 + \varepsilon + \left( \frac{\varepsilon}{2} \right)^2 \approx 1 + \varepsilon . \]  
(5.21)

\[ \left( 1 + \frac{\varepsilon}{2} \right)^4 \approx (1 + \varepsilon)^2 \approx 1 + 2\varepsilon . \]  
(5.22)

\[ \left( 1 + \frac{\varepsilon}{2} \right)^6 = (1 + \varepsilon)^3 (1 + \frac{\varepsilon}{2}) \approx 1 + 3\varepsilon . \]  
(5.23)

Therefore, the first term of \( \text{HD}_3 \) was varied by a ratio of \( 1 + \varepsilon \); the first term of \( \text{HD}_5 \) was varied by a ratio of \( 1 + 2\varepsilon \); and the first term of \( \text{HD}_7 \) was varied by a ratio of \( 1 + 3\varepsilon \), and so on. It can be roughly concluded that the amplitude imbalances impact the nonlinear responses by the ratio of a weighted \( 1 + \varepsilon \), and the impacts become greater as the order increases.

A Cadence SpectreRF Periodic Static State (PSS) simulation was run to estimate the \( \text{HD}_3 \), \( \text{HD}_5 \), and \( \text{HD}_7 \) for the baseband BJT differential pair with and without amplitude imbalances in the differential input. The simulated \( \text{HD}_3 \), \( \text{HD}_5 \), \( \text{HD}_7 \) over an input swing range with 2 dB and 0 dB amplitude imbalance were plotted in Figure 5-11(a), and the differences between the two cases were shown in Figure 5-11(b). As expected, the
amplitude imbalance has bigger impact on HD\textsubscript{7} than HD\textsubscript{5} and HD\textsubscript{3}. The 2 dB amplitude imbalance cause about 1.5 dB difference in HD\textsubscript{3} in the nonlinear region. Therefore, it is necessary to minimize such imperfection to guarantee measurement accuracy.

Figure 5-11 Simulated HD\textsubscript{3}, HD\textsubscript{5}, HD\textsubscript{7} with 2 dB and without amplitude imbalance.
Now suppose there are phase imbalances in the differential inputs:

\[
\frac{V_i^+}{2 V_T} = \frac{1}{a \cos(\omega t)} = b \cos(\omega t). \tag{5.24}
\]

\[
\frac{V_i^-}{2 V_T} = \frac{1}{a \cos(\omega t + 180 + \theta)} = b \cos(\omega t + 180 + \theta) = - \cos(\omega t) \cos(\theta) + \sin(\omega t) \sin(\theta). \tag{5.25}
\]

where \(\theta\) is phase imbalance between the two half input signals.

\[
\frac{I_{out}}{I_E} = \frac{e^{\frac{V_i^+}{V_T}} - e^{\frac{V_i^-}{V_T}}}{e^{\frac{V_i^+}{2V_T}} + e^{\frac{V_i^-}{2V_T}}} = \frac{e^{b \cos(\omega t)} - e^{-b \cos(\theta) \cos(\omega t)} e^{b \sin(\theta) \sin(\omega t)}}{e^{b \cos(\omega t)} + e^{-b \cos(\theta) \cos(\omega t)} e^{b \sin(\theta) \sin(\omega t)}}. \tag{5.26}
\]

For small \(\theta\), \(e^{b \sin(\theta) \sin(\omega t)} \approx 1\), and \(\theta = \sin(\theta)\), so Equation (5.26) can be rewritten as:

\[
\frac{I_{out}}{I_E} = e^{-b \sin(\omega t)} \frac{2}{1 + e^{-b(\cos(\theta) + 1) \cos(\omega t)}} - 1. \tag{5.27}
\]

Using Taylor series expansion, Equation (5.27) can be rewritten as (let \(x = b(\cos\theta + 1)\cos(\omega t)\) and \(y = b \cdot \theta \sin(\omega t)\)).

\[
\frac{I_{out}}{I_E} = \left(1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \ldots \right) \left(\frac{2}{2 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots}\right) - 1. \tag{5.28}
\]

By polynomial division, Equation (5.28) becomes
\[
\frac{I_{\text{out}}}{I_E} = \left( 1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \ldots \right) \left( 1 + \frac{x}{2} - \frac{x^3}{24} + \frac{x^5}{240} - \ldots \right) - 1
\]

(5.29)

\[
= \left( \frac{x - x^3}{2} + \frac{x^5}{240} - \ldots \right) + \left( - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \ldots \right) \left( 1 + \frac{x}{2} - \frac{x^3}{24} + \frac{x^5}{240} - \ldots \right)
\]

Term (1) – odd order impacts

Term (2) – even order impacts

Equation (5.29) contains two components: the first term (1) produces distortion at all the odd order harmonics due to the same mechanism like the amplitude imbalances, which is can be seen by plugging \( x \) into the term (1):

\[
\frac{I_{\text{out}}}{I_E} = \frac{(\cos \theta + 1) \cdot b \cos(\omega t)}{2} - \frac{[(\cos \theta + 1) \cdot b \cos(\omega t)]^2}{24} + \frac{[(\cos \theta + 1) \cdot b \cos(\omega t)]^3}{240} - \ldots
\]

(5.30)

Compare to Equation (5.17), \( \cos \theta \) is just like the amplitude imbalance term \( \delta \) and causes interferences to all the odd order harmonic terms. However, even for a big \( \theta \) like 10 degree, \( (\cos \theta +1)/2 \) is still very small \((-0.15 \text{ dB})\). Therefore, its impacts on the odd order harmonic products are negligible. Because \( \cos \theta \) is always less than 1, the phase imbalance always lead to gain reduction.

The second term (2) produces distortion at all the even order harmonic products, which can be seen by plugging \( x \) and \( y \) into the term (2):

\[
\text{Term (2) = } \left( - b \cdot \theta \sin(\omega t) + \frac{(b \cdot \theta \sin \omega t)^2}{2!} - \ldots \right) \left( 1 + \frac{(\cos \theta + 1) \cdot b \cos(\omega t)}{2} - \frac{[(\cos \theta + 1) \cdot b \cos(\omega t)]^2}{24} + \ldots \right)
\]

(5.31)

Except the first product \( - b \cdot \theta \sin \omega t \) generate an interfering term at fundamental, all the other products \( \sin 2\omega t, \sin 4\omega t, \sin 6\omega t, \ldots \) produce interferences at even order
harmonics so that their impacts can be neglected because only odd order harmonic components produce distortion within the signal bandwidth.

A Cadence SpectreRF Periodic Static State (PSS) simulation was run to estimate the HD$_3$, HD$_5$, HD$_7$ for the baseband BJT differential pair with and without phase imbalances in the differential input. The simulated HD$_3$, HD$_5$, HD$_7$ over an input swing range with 10 degree and 0 degree phase imbalance were plot in Figure 5-12(a), and the differences between the two cases were shown in Figure 5-12(b). As expected, the phase imbalance causes minor impacts on HD$_3$, HD$_5$, and HD7. The difference in HD$_3$ is close to zero. Therefore, it is safe to say that in this measurement a large phase imbalance can be tolerated.
In summary, the amplitude imbalance in the differential inputs can produce interfering components at odd order harmonics and they can be big enough to degrade the measurement inaccuracy in characterizing the nonlinear responses of the transconductors.

Figure 5-12 Simulated HD$_3$, HD$_5$, HD$_7$ with 10 degree and without phase imbalance.
Therefore, the amplitude imbalances need to be minimized.

The phase imbalance in the differential inputs can produce interfering components at both even and odd order harmonics. However, the impacts are negligible and a big phase imbalances can be tolerated in the measurement.

5.4 Measurement and Simulation Results

The baseband and bandpass transconductors designed in Section 5.2 were measured and behavioral models were developed for characterization of their nonlinear characteristics based on the single-tone AM-AM and AM-PM measurement data. The model results were compared to the measured data, the simulation data, and the analytical data, in order to verify the accuracy of the nonlinear behavioral models for the baseband and bandpass transconductors. The nonlinear performances of these three transconductors were also compared.

5.4.1 Baseband Transconductors

The measurement and modeling results for baseband transconductors were presented in this section. Based on the simulation and measurement data, the bandwidth limitation of the AM-AM and AM-PM based nonlinear behavioral model for broadband quadrature modulator applications was observed and discussed.

5.4.1.1 Modeled and Measured Results

The baseband and bandpass transconductors were measured using the measurement #2 with baluns as shown in Figure 5-10. The small signal gain and the large-signal S-parameters (LSSP) as a function of power and frequency [95, 96] were measured and
collected in Agilent ADS P2D file format [97, 98] for the different baseband transconductors. The measurement data in the P2D file format enable us to easily model the nonlinear characteristics of the transconductors using ADS LSSP based behavioral modeling tools [48, 97, 99].

The measured and Cadence simulated (nominal, fast and slow process conditions) small signal gain and bandwidth results for the differential pair, multi-tanh doublet, and multi-tanh triplet were plotted in Figure 5-13 to Figure 5-15, respectively.

1) For all three transconductors, the measured small signal gain is higher than the simulated one under nominal condition due to the process variations. This makes sense because the three transconductor circuits are from the same wafer lot so they undergone the similar process variations. The measured gain curves are valid because they all fall within the upper and lower corner simulated boundaries.

2) For all three transconductors, the trends of the measured gain curves over frequency match the simulated ones. Also, the measured and simulated 3 dB rolloff frequencies are very close within 3%. This indicates the small signal gain over frequency measurement is accurate and the measured small signal behaviors of the three transconductor circuits are accurate.

3) The 3 dB rolloff frequencies increase in the same order as the $G_m/I_c$ efficiency: multi-tanh triplet, multi-tanh doublet, and the differential pair, which is the expected result because the reverse order of different size resistors were used
to load the three transconductors in order to give the same voltage gain. This leads to the different output RC time constants.

Figure 5-13 Small signal gain-bandwidth plot of baseband differential pair transconductor.
Figure 5-14 Small signal gain-bandwidth plot of baseband doublet transconductor.

Figure 5-15 Small signal gain-bandwidth plot of baseband triplet transconductor.
The gain characteristics with single-tone sinusoidal excitation for the three baseband transconductors were measured and compared to the simulated and analytical results. The results are shown in Figure 5-16 to Figure 5-18. Consistent with the analytical results, the 1 dB compression points vary in an ascending order: differential pair, multi-tanh doublet, and multi-tanh triplet.

The measured, simulated, and analytical gain characteristics of the baseband differential pair transconductor were shown in Figure 5-16. The analytical gain characteristics were calculated by Equation (5.1). The gain curves of all three cases are normalized to 0 dB for comparison purpose, which is valid because the gain is linearly related to the DC bias current for BJT transconductors. The simulated and measured 1 dB gain compression points are as close as 0.6 dB, indicating the accuracy of the measurement. They are up to 1 dB larger than the analytical results possibly due to the higher order effects which are not included in the analytical DC transfer function.
Figure 5-16 Measured, simulated, and analytical gain characteristics of baseband differential pair transconductor.

The normalized gain characteristics of the baseband multi-tanh doublet transconductor were shown in Figure 5-17. The measured, the simulated, and the analytical (by Equation (5.7)) 1 dB compression points are −20.6 dBm, −21.05 dBm, and −21.5 dBm, respectively. The similar conclusion can be drawn for the multi-tanh doublet case as the differential pair case.
Figure 5-17 Measured, simulated, and analytical gain characteristics of baseband multi-tanh doublet transconductor.

The normalized gain characteristics of the baseband multi-tanh triplet transconductor were shown in Figure 5-18. The measured, the simulated, and the analytical (by Equation (5.8)) 1 dB compression points are also close: −18.1 dBm, −19.2 dBm, and −17.1 dBm, respectively. However, for the analytical case, the gain in the lower input range is almost flat, which is different from the measured and simulated cases with a slowly decreased gain curve. The possible reasons for this are the higher order effects of the BJT transistors and the mismatches. These cause $G_m$ fluctuations inside the wide linear input range.
Figure 5-18 Measured, simulated, and analytical gain characteristics of baseband multi-tanh triplet transconductor.

A two tone IM$_3$ test was done to further verify the agreement between the measured data, the simulation data, and the analytical results. For each baseband transconductor, a two tone IM$_3$ simulation using Agilent ADS circuit envelope simulator with the measured P2D file [97, 100] was run. Also, a two tone signal was applied to each transconductor and the IM$_3$ was measured using spectrum analyzer.

Figure 5-19 shows the IM$_3$ of the differential pair transconductor over the input range for the analytical, the PSS simulated, the ADS modeled, and the measured cases. The IM$_3$ trends are the same for the four cases. The measured, the modeled and the simulated IM$_3$ have a very good match with 1.5 dB. The IM$_3$ of the analytical case has a difference about 2.5 dB from the other cases possibly due to the high order effects.
Figure 5-19 Measured, modeled, simulated, and analytical IM$_3$ of baseband differential pair transconductor.

Figure 5-20 shows the IM$_3$ of the multi-tanh doublet transconductor over the input range for the analytical, the PSS simulated, the ADS modeled, and the measured cases. The same conclusion can be drawn for the doublet as the differential pair transconductor case.
Figure 5-20 Measured, modeled, simulated, and analytical IM$_3$ of baseband multi-tanh doublet transconductor.

Figure 5-21 shows the IM$_3$ of the multi-tanh triplet transconductor over the input range for the analytical, the PSS simulated, the ADS modeled, and the measured cases. The measured, the modeled and the simulated IM$_3$ have a difference within 1.5 dB. The analytical case has a much lower IM$_3$ (a IM$_3$ notch) than the other cases in the lower input range due to the $G_m$ fluctuation inside the linear input range for the actual circuits. This is possibly due to the higher order effects of the BJT transistors and the device mismatches which are not included in the analytical DC transfer function. The analytical IM$_3$ curve joins with the other curves as the circuit goes into nonlinear region.
Using the similar nonlinear modeling structure as for the quadrature modulators in Chapter 3, the AM-AM and AM-PM measurement based complex power series models [6, 7, 52, 53] were developed for each baseband transconductor for characterization of the nonlinear responses. ACLR measurements were done to validate the AM-AM and AM-PM based behavioral model for the baseband transconductors. Either I or Q data streams of two WDMA signals were applied to the model and the baseband transconductor circuits. One is the WCDMA uplink pilot signal with a peak-to-average ratio (PAR) of 3.7 dB; the other is a WCDMA downlink test model 5 signal with 30 DPCH and 8 HS-PDSCH, which has a high PAR of 11 dB.

The input 1 dB compression point for each baseband transconductors excited by the two WCDMA signals are summarized in Table 5-1. As expected, the 1 dB input
compression points when excited by the high PAR downlink WCDMA signal are lower than the ones excited by the low PAR uplink WCDMA signal.

<table>
<thead>
<tr>
<th>WCDMA Uplink</th>
<th>Diff_Pair</th>
<th>Doublet</th>
<th>Triplet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>−24.5</td>
<td>−20.0</td>
<td>−17.5</td>
</tr>
<tr>
<td>WCDMA Downlink</td>
<td>−26.2</td>
<td>−21.7</td>
<td>−19.3</td>
</tr>
</tbody>
</table>

There are good agreements between the measured and modeled ACLR for the baseband differential pair, multi-tanh doublet, and multi-tanh triplet transconductors when they are driven into nonlinear regions as shown in Figure 5-22 and Figure 5-23 for differential pair, Figure 5-24 and Figure 5-25 for multi-tanh doublet, and Figure 5-26 and Figure 5-27 for multi-tanh triplet. This indicates that the AM-AM and AM-PM based complex series model can accurately capture the nonlinear responses of the baseband transconductors for narrow band communication signals. Note that there are bigger differences between the measured and modeled ACLR results in very low input power level for all three transconductors. This is not the model problem. It is due to the limited dynamic range of the spectrum analyzer.
Figure 5-22 Measured and modeled ACLR for baseband differential pair transconductor excited by WCDMA uplink pilot signal.

Figure 5-23 Measured and modeled ACLR for baseband differential pair transconductor excited by WCDMA downlink test model 5 signal.
Figure 5-24 Measured and modeled ACLR for baseband multi-tanh doublet transconductor excited by WCDMA uplink pilot signal.

Figure 5-25 Measured and modeled ACLR for baseband multi-tanh doublet transconductor excited by WCDMA downlink test model 5 signal.
Figure 5-26 Measured and modeled ACLR for baseband multi-tanh triplet transconductor excited by WCDMA uplink pilot signal.

Figure 5-27 Measured and modeled ACLR for baseband multi-tanh triplet transconductor excited by WCDMA downlink test model 5 signal.
5.4.1.2 Limitation of the Baseband Memoryless Behavioral Model

The AM-AM and AM-PM based memoryless behavioral model for baseband transconductors has bandwidth limitation for broadband quadrature modulator applications. Low frequency AM-AM and AM-PM characterization data could not be able to accommodate the high frequency responses because as frequency increases, the nonlinearity of the baseband transconductor is not dominated by the transconductor only, the nonlinear parasitic capacitors at both input and output will produce harmonics. The input and output parasitics can cause decrease of the gain, linear phase shift of the AM-PM responses, and nonlinear shifts of the AM-AM and AM-PM responses. In broadband quadrature modulations, these effects can distort the desired signal by ways of frequency dispersion, nonlinear distortion, and etc. These effects can be seen by a careful examination of the AM-AM and AM-PM responses over frequency (up to the 3 dB bandwidth) of the baseband transconductors. Figure 5-28 shows the AM-AM and AM-PM simulation results over frequency and input power for the baseband differential transconductor. As the frequency and input power increases, two things happen. In the linear and weak nonlinear region (about 8 dBm higher than the 1 dB compression point), there is a roughly linear phase shift in the AM-PM responses between the low frequency point and high frequency point due to the parasitics. In the strong nonlinear region, the high frequency AM-PM response phase shift is not linear to the low frequency AM-PM response any more. The phase shift becomes nonlinear as well because of the nonlinear nature of the parasitics.
Figure 5-28 Simulated AM-AM and AM-PM of baseband differential pair transconductor over frequency.

The above conclusions drawn for the AM-AM and AM-PM model bandwidth limitation was verified by comparing the measured and simulated AM-AM and AM-PM responses over frequency for the baseband differential pair, the multi-tanh doublet, and the multi-tanh triplet transconductors, as shown in Figure 5-29 to Figure 5-34. For comparison purpose, the gain and phase are all normalized to zero. For all the three baseband transconductors, the measured and simulated AM-AM and AM-PM responses have a good agreement and the trend of the change of the AM-PM responses over frequency and input power level is similar as the simulated differential pair case. Therefore, it can be concluded the bandwidth limitation exists in the AM-AM and AM-PM based model for broadband quadrature modulator applications.
Figure 5-29 Measured and simulated AM-PM of baseband differential pair transconductor over frequency.

Figure 5-30 Measured and simulated AM-AM of baseband differential pair transconductor over frequency.
Figure 5-31 Measured and simulated AM-PM of baseband multi-tanh doublet transconductor over frequency.

Figure 5-32 Measured and simulated AM-AM of baseband multi-tanh doublet transconductor over frequency.
Figure 5-33 Measured and simulated AM-PM of baseband multi-tanh triplet transconductor over frequency.

Figure 5-34 Measured and simulated AM-AM of baseband multi-tanh triplet transconductor over frequency.
5.4.2 Bandpass Transconductors

Similar as the measurement approach as described in Section 5.4.1, the measurement setup #2 discussed in Section 5.3.2 was used and the LSSP data were collected in P2D file format in order to model and characterize the nonlinear characteristics of the bandpass transconductors.

The bandpass transconductors were designed to work at 2 GHz. The measurement working frequency for the differential pair and the multi-tanh triplet bandpass transconductors is 1.82 GHz, which is different from 2 GHz due to the process variations. However, the multi-tanh doublet doesn’t work properly possibly due to wrong connections with the pads. This bandpass transconductor has a higher measured resonant frequency at 2.37 GHz and doesn’t produce voltage gain therefore it was not characterized.

The normalized gain compression characteristics of the bandpass differential pair transconductor for the analytical, measured, and PSS simulated cases are shown in Figure 5-35. The simulated 1 dB compression point differs from the measured one by about 1 dB. Compare to the baseband differential pair transconductor as shown in Figure 5-16, they are even better than the analytical case, partly due to the same reason as for the baseband transconductor. In a tuned or low frequency amplifier, the output conductance is also a major source of nonlinearity [101]. The use of low Q tank load and parasitic capacitance shunting effect could reduce the nonlinearity of the output transconductance.
Figure 5-35 Measured, simulated, and analytical gain characteristics of the bandpass differential pair transconductor.

The normalized gain compression characteristics of the bandpass multi-tanh triplet transconductor for the analytical, measured, and PSS simulated cases are shown in Figure 5-36. The simulated and measured gain curve has a good match. Similar as the baseband case, in the lower input power range, the measured and simulated gain curves slowly decrease due to the $G_m$ fluctuation, which is different from the analytical case.
Figure 5-36 Measured, simulated, and analytical gain characteristics of the bandpass multi-tanh triplet transconductor.

A two tone IM3 test was done to further verify the agreement between the measured data, the simulation data, and the analytical results. For each bandpass transconductor, a two tone IM3 simulation using Agilent ADS circuit envelope simulator with the measured P2D file [97, 100] was run. Also, a two tone signal was applied to each transconductor and the IM3 was measured using spectrum analyzer.

Figure 5-37 shows the IM3 of the bandpass differential pair transconductor over the input power level for the analytical, the PSS simulated, the ADS modeled, and the measured cases. The IM3 trends are the same for the four cases. The simulated, the modeled, and the measured IM3 are different by about 2 dB. And the simulated and the analytical IM3 are close within 1 dB.
Figure 5-37 Measured, modeled, simulated, and analytical IM₃ of the bandpass differential pair transconductor.

Figure 5-38 shows the IM₃ of the bandpass multi-tanh triplet transconductor over the input power level for the analytical, the PSS simulated, the ADS modeled, and the measured cases. In the nonlinear region, the IM₃ are close for the four cases. The analytical case has a much lower IM₃ than the other cases in the lower input range due to the $G_m$ fluctuation inside the linear input range for the actual circuits. The analytical IM₃ curve joins with the other curves as the circuit goes into nonlinear region.
Figure 5-38 Measured, modeled, simulated, and analytical IM$_3$ of the bandpass multi-tanh triplet transconductor.

The AM-AM and AM-PM measurement based complex power series models [6, 7, 52, 53] were developed for the bandpass differential pair and triplet transconductors for characterization of the nonlinear responses. The same two WCDMA signals as described in Section 5.4.1.1 were used here to verify the behavioral models for the bandpass transconductors.

The input 1 dB compression point for the baseband differential pair and the multi-tanh triplet transconductors excited by the two WCDMA signals are summarized in Table 5-2.

<table>
<thead>
<tr>
<th></th>
<th>Diff_Pair</th>
<th>Triplet</th>
</tr>
</thead>
<tbody>
<tr>
<td>WCDMA Uplink</td>
<td>−16.9</td>
<td>−10.5</td>
</tr>
<tr>
<td>WCDMA Downlink</td>
<td>−18.3</td>
<td>−13.1</td>
</tr>
</tbody>
</table>
The simulated and measured ACLR for the both bandpass transconductors excited by the WCDMA uplink pilot signal and the WCDMA downlink test model 5 signal have a good match with a difference within 1 dBc, as shown in Figure 5-39 and Figure 5-40 for the differential pair and Figure 5-41 and Figure 5-42 for the multi-tanh triplet. This indicates that the AM-AM and AM-PM based modeling approach is also an accurate approach in characterizing the nonlinear distortion of the bandpass transconductors. There are some deviations from the measurement data in the lower input power level due to the limited dynamic range of the spectrum analyzer.

![Figure 5-39 Simulated and measured ACLR for the bandpass differential pair transconductor excited by the WCDMA uplink pilot signal.](image-url)
Figure 5-40 Simulated and measured ACLR for the bandpass differential pair transconductor excited by the WCDMA downlink test model 5 signal.

Figure 5-41 Simulated and measured ACLR for the bandpass multi-tanh triplet transconductor excited by the WCDMA uplink pilot signal.
Figure 5-42 Simulated and measured ACLR for the bandpass multi-tanh triplet transconductor excited by the WCDMA downlink test model 5 signal.

5.5 Insights Gained from the Transconductor Modeling Work

The good match between the measured and the modeled results for both baseband and bandpass transconductors as described in Sections 5.4.1 and 5.4.2 indicates that the AM-AM and AM-PM based memoryless model can accurately model the nonlinear characteristics of the transconductors. This result is as expected because there is no significant memory in the transconductors and they are aimed at and tested by narrow-band wireless signals (WCDMA in this case). This builds a solid foundation to use the AM-AM and AM-PM model to do the nonlinear behavioral modeling for modern quadrature modulators for two reasons.

First, as discussed in Chapter 4, the baseband transconductors dominate the nonlinear characteristics of most integrated quadrature modulators. Compared to the baseband $G_m$ stage, the nonlinear contribution of the mixer core, of the nonlinear output impedance, and
of the output parasitics (either negligible or resonant out by LC tank) are small. Therefore, it can be concluded that AM-AM and AM-PM model is able to accurately model the quadrature modulator nonlinear responses for narrow-band signals because it can model the baseband transconductors very accurately.

Second, in modern wireless communication systems, fortunately most applications are narrow-band. For example, the GSM signal bandwidth is 200 kHz [46]; the CDMA signal bandwidth is 1.2288 MHz [46]; the WCDMA signal bandwidth is 3.84 MHz [44]; the WLAN signal bandwidth is 20 MHz [32]; and the WiMAX signal bandwidth is 28 MHz [33]. With these low signal bandwidth, the limitation as discussed in Section 5.4.1.2 does not exist for these applications: all the signal contents inside the signal bandwidth experience almost the same AM-AM and AM-PM transfer functions. There is negligible amount of the linear phase shift due to the input parasitics between the highest and lowest frequency contents inside the signal bandwidth so that almost no frequency dispersion effect occurs that can distort the desired signal.

Therefore, it can be concluded that the multi-tanh transconductor modeling work provides us a solid proof that the nonlinear model based on the single-tone AM-AM and AM-PM measurement data can accurately capture the nonlinear characteristics of the quadrature modulators for narrow-band applications. It is a very useful model to modern wireless communication systems because many of the popular wireless applications are narrow-band.
5.6 Summary

The design and characterization of nonlinear behavior of three types of BJT linear differential transconductors were documented in this chapter. Two possible probe measurement techniques were compared and analyzed. Based on the analysis, the balun based single ended probing technique was adopted due to the limitation of the equipments. The measured nonlinear characteristics for both baseband and bandpass transconductors were compared to the Cadence PSS simulated and the analytical ones. The comparison results showed that the balun based single ended probing measurement technique is able to characterize the devices at both baseband and passband with acceptable accuracy. It also showed that the actual nonlinear behaviors of the baseband transconductor have some deviations from the analytical results possibly due to the higher order effects of the transistors and the mismatches. The bandpass transconductors behaves slightly different from the baseband transconductors due to the tank loading and the parasitics.

The baseband and bandpass transconductors were modeled based on the AM-AM and AM-PM measurement data and curve fitting techniques. A WCDMA uplink signal and a WCDMA downlink signal were applied to the baseband and the bandpass transconductors and the ACLR were measured and compared to the modeled results. There is a good match between the measured and modeled data, which reinforce the conclusion drawn in Chapter 4 that the AM-AM and AM-PM based model can accurately characterize the nonlinear characteristics of the quadrature modulators for narrow-band applications.

The AM-AM and AM-PM based memoryless model has bandwidth limitation for broadband quadrature modulator applications because the impact of the nonlinear
parasitics becomes significant at high frequency and high input power level.
Chapter 6

Conclusions and Future Work

6.1 Summary of Research

The major focus and contribution of this work is the development of a new nonlinear behavioral model for characterization of the correlated/uncorrelated nonlinear distortion, the DC offset, and the gain/phase imbalances of direct-conversion quadrature modulators. With this model, spectral regrowth and waveform quality degradation due to these physical imperfections can be accurately and efficiently predicted before fabrication.

The motivation for the quadrature modulator modeling work is the popular usage of the modulators in modern wireless communication systems, which can be represented by the following three applications: (1) generic quadrature modulator system like WCDMA; (2) zero-IF OFDM system like zero-IF WLAN; and (3) low-IF OFDM system like low-IF WLAN. It is very interesting that these physical imperfections impact the system performance in different way for the three applications. Based on the analysis results in Chapter 3, it is found that the DC offset causes SNR or EVM degradation like the AWGN
in system (1). No SNR or EVM degradation results from the DC offset in the other two systems because there is no subcarrier at DC, but the carrier leakage can possibly cause spectrum mask violation. The gain/phase imbalances result in similar SNR or EVM degradation predictable by the IRR equation in the first two systems, although the receiver equalizer complexity may cause 3 dB more SNR degradation in system (2). The big advantage of system (3) is no SNR or EVM degradation due to the gain/phase imbalances because all the image spectrum products are outsides of the signal bandwidth. The disadvantage is the increased adjacent channel interferences due to the image spectrum. This put more stringent requirements on designers to minimize gain/phase errors to meet the spectrum mask specification. The analysis results regarding the nonlinear distortion are quite interesting. It was found that the systems (1) and (2) produce similar amount of in-band and out-of-band distortion, and their output spectrum are symmetric around carrier. However, in low-IF OFDM systems, because it has all the SSB modulated subcarriers at one sideband, some of the distortion products are shifted out of the signal band and moved to the opposite sideband. The first result is less in-band distortion generated in low-IF OFDM systems. This big advantage also comes with the similar disadvantage: the increased adjacent channel interference. The second result is the asymmetric output spectrum. The above analysis results are very useful to the wireless system and circuit designers in the sense that they can gain a better understanding about the design tradeoffs, and wisely diagnose the physical imperfections appropriate for different quadrature modulator applications to improve their design and performance.

Chapter 4 presents the details of the development and extraction procedure of the
nonlinear behavioral model for direct-conversion quadrature modulators. The AM-AM and AM-PM based low pass equivalent model technique was utilized in this work. This model type is simple and well-understood to system and circuit design engineers so that it can be more efficiently implemented and likely to be adopted by the designers. Besides these nice features, it is also proved in this work that this type of model can capture the nonlinear behaviors of quadrature modulators with excellent accuracy. Novel parameter extraction techniques were developed for extracting both nonlinear model coefficients and linear static model coefficients accurately and efficiently. One important assumption in the modulator modeling work is that the baseband transconductors dominate the nonlinear performance of the whole quadrature modulator. This assumption is validated by the measurement data. With this assumption, a parametric frequency dependent model structure was implemented by using a same set of nonlinear model coefficients and different set of linear error model coefficients over a frequency range. Using this parametric frequency dependent model, the modulator responses (both nonlinear and linear error responses) at different frequency bands can be characterized without repeating the extraction procedure for the nonlinear model coefficients. The nonlinear behavioral model was validated to be able to accurately characterize the correlated/uncorrelated nonlinear distortion, the DC offset, and the gain/phase imbalances so that the spectral regrowth and waveform quality degradation can be accurately predicted. This provides both wireless system and circuit designers a powerful tool to design, verify, and estimate their design performances much more efficiently than using the circuit model counterpart.

As the baseband transconductors usually dominate the nonlinear performance of the
quadrature modulators, in Chapter 5, a particular type of transconductor, multi-tanh transconductors were designed and modeled in order to gain more insights about quadrature modulator nonlinear modeling. The three BJT transconductors being modeled are basic differential pair, multi-tanh doublet, and multi-tanh triplet, each of which has different linearity performances. The nonlinear characteristics of the baseband and bandpass transconductors were modeled based on the measured AM-AM and AM-PM data. The modeled results have a good agreement with the measured data. This indicated that the AM-AM and AM-PM model is able to accurately characterize the nonlinear characteristics of the baseband transconductors, and thus it provides a proof for the modeling work in Chapter 4, knowing the fact that most of modern wireless systems are narrow-band systems. The AM-AM and AM-PM based behavioral model does have bandwidth limitation for broadband quadrature modulator applications because of the existence of the nonlinear parasitics. This limitation is measured and analyzed by examining the AM-AM and AM-PM responses of the transconductors over different frequencies.

6.2 Future Work

As discussed in Section 5.4.1.2, the AM-AM and AM-PM model has bandwidth limitation for broadband quadrature modulator applications. A simple argumentation of the AM-AM and AM-PM model can be used to partly model the input parasitic capacitor by putting a low-pass RC filter before the AM-AM and AM-PM nonlinear modeling block to model the phase shift of the AM-PM responses, as shown in Figure 6-1.
Figure 6-1 A simple augmented AM-AM and AM-PM based two-box model.

This simple augmented two-box model is easy to be implemented and could be able to model the baseband transconductor over a certain bandwidth and input range. As shown in Figure 5-28, for the differential pair case, up to the 3 dB rolloff frequency and an input power level of about 8 dBm higher than the 1 dB compression point, the AM-PM response is roughly in linear proportion to the low frequency AM-PM response, and thus the augmented model as shown in Figure 6-1 could be reasonably accurate to capture the parasitic induced phase distortion. Another supporting argument for this two-box model is that the broadband applications such as ultra wideband (UWB) usually have low output power; therefore, the modulator will be unlikely to work in strong nonlinear region so that the nonlinear phase shift effect is not significant. It deserves more research effort to implement this two-box model for broadband modulator applications.

The quadrature modulators being modeled in this work do not have significant memory and thus the AM-AM and AM-PM memoryless model works pretty well. However, some specific modulators may reveal significant memory depending on the
circuit structure and the working conditions. Therefore, it is desired to implement more sophisticated model like the multi-slice model [9, 57] to quadrature modulators to capture the nonlinear memory effects.

In Chapter 4 a frequency dependent parametric model was implemented to capture the modulator responses over frequency by using one set of nonlinear model coefficients and different set of linear error model coefficients. The linear model coefficients can be chosen appropriately using a look-up table. Although this is a convenient model to incrementally incorporate frequency variation, the frequency points are discrete and thus it is more desirable to develop a continuous frequency dependent model for the quadrature modulators.
Bibliography


[44] 3GPP, "3GPP Technical Specification 25.101, V7.8.0 (2007-06); User Equipment (UE) radio transmission and reception (FDD)."

[45] 3GPP, "3GPP Technical Specification 25.104, V7.7.0 (2007-06); Base station (BS) radio transmission and reception (FDD)."


[79] PolyphaseMicrowave Inc., "QM3337A data sheet."


Appendices
Appendix A MATLAB Code

MatLab Code for Generating the 10 KHz Offset Sinusoidal Signal for Quadrature Modulator AM-AM and AM-PM Measurements (Sections 4.1.2 and 4.2.1)

% This code generate a 10 kHz sinusoidal signal with exponential swept % input amplitude. The signal is uploaded to signal generator and used % to excite the quadrature modulator for offset sinusoidal AM-AM and AM-PM % measurement
clear all; close all;

% setup connection with signal generator
io=agt_newconnection('gpib',0,20);

[status, status_description,query_result] = agt_query(io,'*idn?');
if (status < 0) return; end

f=.01e6; % 10 KHz offset frequency
fs=40*f; % sampling frequency
N=100000; % number of data points
% 1000 dummy points at the beginning to mark out the start of the signal
delay=1000;
% 1000 dummy points at the end to mark out the end point of the signal
tail=1000;

% coefficient in the exponential function to produce appropriate ramp
c1 = 14.3;
idata(1:delay)=0;
idata(delay+1:N)=(exp(c1*[0:N-delay-1]/fs)).*sin(2*pi*f/fs*[delay:N-1]);
idata(N:N+tail)=0;
idata=idata/max(abs(idata)); % normalized the data to range [0 1]
% idata=idata*1e-15; % commented out when characterizing I channel
qdata(1:delay)=0;
qdata(delay+1:N)=(exp(c1*[0:N-delay-1]/fs)).*cos(2*pi*f/fs*[delay:N-1]);
qdata(N:N+tail)=0;
qdata=qdata/max(abs(qdata)); % normalized the data to range [0 1]
qdata=qdata*1e-15; % commented out when characterizing Q channel
IQData=idata+j*qdata; % summarized I and Q channel data

% upload the signal to the signal generator
[status, status_description] = agt_waveformload(io, IQData, 'agtsample', fs, 'play', 'no_normscale');
MatLab Code for Curve Fitting of AM-AM and AM-PM Data (Sections 4.1.2 and 4.2.1)

% This code is to do the polynomial curve fitting on the measured AM-AM and 
% AM-PM data to obtain the power series coefficients

clear all; close all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % A = [];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%
N = 11; % number of the orders

% load the VSA measured modulator output envelope data
load Env_I_14_04_25.mat;
env = double(Y);
t_start = XStart*1000; % convert sec to msec
deltaT = XDelta*1000;
t = t_start + deltaT*[0:(length(env)-1)];

save temp env N t;
clear all;
load temp.mat;

m = 5; % throw away weird points at the end of the offset sinusoidal signal
x1 = find(t==25); % the 1001st data point corresponding to t=25 ms
x2 = find(t==250) - m;
c1 = 0.0143; % exponential function coefficients
% maximum amplitude measured with oscilloscope
Vin_max = 1.15*exp(c1*t(x2))/exp(c1*t(x2+m));
Vin = exp(c1*t(x1:x2));

Vout = env(x1:x2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%
Po = 10*log10(abs(Vout).^2/100*1000*2); % x2 due to two tone
Pin = 10*log10(abs(Vin/2).^2/100*1000*2); % x2 due to two ports (I Ibar)

z = Vin;
y = Vout;

% curve fitting only extracts the odd order coefficients. Duplicate the data
% at the 1st quadrant to generate data at the 3rd quadrant
z_neg = -z;
z_neg = flipud(z_neg);
Z = [z_neg; 0; z];

y_neg = -y;
y_neg = flipud(y_neg);
Y = [y_neg; 0; y];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%
A = [];
for k = 1:length(Z)
    B = [];
    for n = 1:2:N
        B = [B Z(k)^n];
    end
    A = [A; B];
end

rank(A);
cond(A);
svd(A);

% coef=(inv(A'*A))*(A'*Y), the fitted coefficients
coef = pinv(A, 1e-99)*Y

save Icoef_14_04_25 coef
% This code is to calculate the in band signal to uncorrelated distortion ratio and ACLR for the quadrature modulator excited by WCDMA signals

% Set ACPR measurement variables in Hz
acpr_para(1)=5e6; % ACPR offset in ACLR calculation
acpr_para(2)=3.84e6; % adjacent channel bandwidth in ACLR calculation
acpr_para(3)=3.84e6; % main channel bandwidth in ACLR calculation

switch channel
  case 'WCDMA_Up_DPCCH'
    % baseband input power levels controlled by the signal generator
    % attenuation; 7.82 dBm is the measured input signal power when
    % the attenuation is 0 dB for WCDMA uplink pilot signal
    Pin=7.82-18:1:7.82;
  case 'WCDMA_Dn_T5_8HSPDA'
    % baseband input power levels controlled by the signal generator
    % attenuation; 0.17 dBm is the measured input signal power when
    % the attenuation is 0 dB for WCDMA downlink test model 5 signal
    Pin=0.17-13:1:0.17;
end

load Icoef_14.mat
b_I=coef;
load Qcoef_14.mat
b_Q=coef;

for w=1:length(Pin)
  [f,So(:,w),Sc(:,w),Pout(w),Pz_acp_low(w),Pz_acp_hi(w)]=acpr_inband(acpr_para, fs, Pin(w),b_I,b_Q,channel);
end

mainBW = 3.84e6;
offset_main = find((f > -mainBW/2) & (f < mainBW/2));

for j=1:length(Pin)
  Soj=So(:,j);
  Sdj=Sd(:,j);
  Soj=power(10, ((Soj-30)/10));
  Sdj=power(10, ((Sdj-30)/10));
  PWR_in_band=sum(Soj(offset_main));
PWR_in_band_dB(j)=10*log10(PWR_in_band)+30;
Dist_in_band=sum(Sdj(offset_main));
Dist_in_band_dB(j)=10*log10(Dist_in_band)+30;
end

% in band signal to uncorrelated distortion ratio
SUIR= PWR_in_band_dB'-Dist_in_band_dB';

% ACLR lower band
ACLR_Lo=Pout'-Pz_acp_low';

% ACLR upper band
ACLR_Hi=Pout'-Pz_acp_hi';

% This sub-program compute the correlation coefficients based on the input
% signal spectrum terms and the power series model coefficients so that the
% correlated and uncorrelated distortion powers are able to be separated.
% The Orthogonalized coefficients are computed up to 11th order.

function [f,So,Sd,Sc,Pout,Pz_acp_low,Pz_acp_hi]=acpr_inband(acpr_para,
fs,Pin,b_I,b_Q,channel)

% Set ACPR measurement variables in Hz
acpr_offset= acpr_para(1);
acpr_bw= acpr_para(2);
main_bw= acpr_para(3);
for k=1:length(Pin)

ps=sqrt(10^((Pin(k)-3)/10));
[Scc,Sdd,Syy,f]=orthog_spectrum_expand_11(fs,ps,b_I,b_Q,channel);
So(:,k)=10*log10(abs(Syy)/.1); % total power
Sd(:,k)=10*log10(abs(Sdd)/.1); % uncorrelated distortion power
Sc(:,k)=10*log10(abs(Scc)/.1); % correlated distortion power

main_offset=find(f>=-main_bw/2 & f<=+main_bw/2);
Po(k)=sum(abs(Syy(main_offset)));
acp_offset_low=find(f>=(-acpr_offset-acpr_bw/2) &
f<=(-acpr_offset+acpr_bw/2));
acp_offset_hi=find(f>=(acpr_offset-acpr_bw/2) &
f<=(acpr_offset+acpr_bw/2));
% Pzchan is the channel output power
Pz_acp_low(k)=10*log10(sum(abs((Syy(acp_offset_low)))/(100*.001));
Pz_acp_hi(k)=10*log10(sum(abs((Syy(acp_offset_hi)))/(100*.001));
Pout(k)=10*log10(sum(abs(Syy(main_offset)))/(100*.001));
end

function [Scc,Sdd,Syy,f]=orthog_spectrum(fs,ps,b_I,b_Q,channel)
% convert the power series model coefficients to capture the instantaneous
% AM-AM and AM-PM responses
\[ b_{1I} = 2 \cdot b_I(1); \]
\[ b_{3I} = 2 \cdot b_I(2) \cdot \frac{4}{3}; \]
\[ b_{5I} = 2 \cdot b_I(3) \cdot \frac{8}{5}; \]
\[ b_{7I} = 2 \cdot b_I(4) \cdot \frac{64}{35}; \]
\[ b_{9I} = 2 \cdot b_I(5) \cdot \frac{256}{126}; \]
\[ b_{11I} = 2 \cdot b_I(6) \cdot \frac{1024}{462}; \]
\[ b_{1Q} = 2 \cdot b_Q(1); \]
\[ b_{3Q} = 2 \cdot b_Q(2) \cdot \frac{4}{3}; \]
\[ b_{5Q} = 2 \cdot b_Q(3) \cdot \frac{8}{5}; \]
\[ b_{7Q} = 2 \cdot b_Q(4) \cdot \frac{64}{35}; \]
\[ b_{9Q} = 2 \cdot b_Q(5) \cdot \frac{256}{126}; \]
\[ b_{11Q} = 2 \cdot b_Q(6) \cdot \frac{1024}{462}; \]

%%% I correlated with I %%%%%

```matlab
switch channel
    % Load the spectrum terms of I data of the WCDMA uplink pilot signal.  
    % The generation of the spectrum terms is referred to [5].
    case 'WCDMA_Up_DPCCH'
        load mxgen_WCDMA_Up_DPCCH_I
    case 'WCDMA_Dn_T5_8HSPDA'
        % Load the spectrum terms of I data of WCDMA downlink test model 5 signal. 
        % The generation of the spectrum terms is referred to [5].
        load mxgen_WCDMA_Dn_T5_8HSPDA_I
end
```

%%% set the frequency vector%%

```matlab
len=length(Sz1);
m=len/2;
delf=fs/len;
x=-m:(m-1);
f=delf*x;
```

\[ S_{x1x1} = ps^2 \cdot Sz1; \]
\[ S_{x1x3} = ps^4 \cdot Sz13; \]
\[ S_{x1x5} = ps^6 \cdot Sz15; \]
\[ S_{x1x7} = ps^8 \cdot Sz17; \]
\[ S_{x1x9} = ps^{10} \cdot Sz19; \]
\[ S_{x1x11} = ps^{12} \cdot Sz1_11; \]
\[ S_{x3x1} = ps^4 \cdot Sz31; \]
\[ S_{x5x1} = ps^6 \cdot Sz51; \]
\[ S_{x7x1} = ps^8 \cdot Sz71; \]
\[ S_{x9x1} = ps^{10} \cdot Sz91; \]
\[ S_{x11x1} = ps^{12} \cdot Sz11_1; \]
\[ S_{x3x3} = ps^6 \cdot Sz33; \]
\[ S_{x3x5} = ps^8 \cdot Sz35; \]
\[ S_{x3x7} = ps^{10} \cdot Sz37; \]
\[ S_{x3x9} = ps^{12} \cdot Sz39; \]
\[ S_{x3x11} = ps^{14} \cdot Sz311; \]
\[ S_{x5x3} = ps^8 \cdot Sz53; \]
\[ S_{x7x3} = ps^{10} \cdot Sz73; \]
\[ S_{x9x3} = ps^{12} \cdot Sz93; \]
\[ S_{x11x3} = ps^{14} \cdot Sz113; \]
\[ S_{x5x5} = ps^{10} \cdot Sz55; \]
% Compute the correlation coefficients

L31=sum(abs(Sx3x1))./sum(abs(Sx1x1));

L51=sum(abs(Sx5x1))./sum(abs(Sx1x1));
S33=Sx3x3-conj(L31).*Sx3x1-L31.*Sx1x3+(L31.*conj(L31)).*Sx1x1; %
E[u3*conj(u3)]
E[w5*conj(u3)]

S53=Sx5x3-conj(L31).*Sx5x1; % E[w5*conj(u3)]
L53=sum(abs(S53))./sum(abs(S33));

L71=sum(abs(Sx7x1))./sum(abs(Sx1x1));
S73=Sx7x3-conj(L31).*Sx7x1;
L73=sum(abs(S73))./sum(abs(S33));

S55=Sx5x5-L53.*Sx3x5-(L51-L53.*L31).*Sx1x5-(L31.*conj(L53)).*Sx5x3+(abs(L53).^2) .*Sx3x3+(L51-L53.*L31).*conj(L53).*Sx1x3... -conj(L51-L53.*L31).*Sx5x1+L53.*conj(L51-L53.*L31).*Sx5x3+(abs(L51-L53.*L31).^2).*Sx3x3+(L51-L53.*L31).*conj(L53).*Sx1x3...

S75=Sx7x5-conj(L53).*Sx7x3-conj(L51-L53.*L31).*Sx7x1;
L75=sum(abs(S75))./sum(abs(S55));

S77=(Sx7x7+cc5).*Sx7x5+cc3.*Sx7x3+cc1.*Sx7x1+c5.*Sx5x7+(c5.*cc5).*Sx5x5+(c5.*cc3).*Sx5x3+(c5.*cc1).*Sx5x1...
+c3.*Sx3x7+(c3.*cc5).*Sx3x5+(c3.*cc3).*Sx3x3+(c3.*cc1).*Sx3x1+c1.*Sx1x7+(c1.*cc5).*Sx1x5+(c1.*cc3).*Sx1x3+(c1.*cc1).*Sx1x1);
S97=Sx9x7+cc5.*Sx9x5+cc3.*Sx9x3+cc1.*Sx9x1+c5.*Sx5x7+(c5.*cc5).*Sx5x5+(c5.*cc3).*Sx5x3+(c5.*cc1).*Sx5x1;
L97 = \frac{\text{sum}(\text{abs}(S97))}{\text{sum}(\text{abs}(S77))};

L11_1 = \frac{\text{sum}(\text{abs}(Sx11x1))}{\text{sum}(\text{abs}(Sx1x1))};
S11_3 = Sx11x3 - \text{conj}(L31) \cdot Sx11x1;
L11_3 = \frac{\text{sum}(\text{abs}(S11_3))}{\text{sum}(\text{abs}(S33))};
S11_5 = Sx11x5 - \text{conj}(L53) \cdot Sx11x3 - \text{conj}(L51-L53) \cdot L31 \cdot Sx11x1;
L11_5 = \frac{\text{sum}(\text{abs}(S11_5))}{\text{sum}(\text{abs}(S55))};
S11_7 = Sx11x7 + cc5 \cdot Sx11x5 + cc3 \cdot Sx11x3 + cc1 \cdot Sx11x1;
L11_7 = \frac{\text{sum}(\text{abs}(S11_7))}{\text{sum}(\text{abs}(S77))};
c9 = 1; cc9 = \text{conj}(c9);
c7 = -L97; cc7 = \text{conj}(c7);
c5 = (L97 \cdot L75 - L95); cc5 = \text{conj}(c5);
c3 = (L97 \cdot L73 - L97 \cdot L75 \cdot L53 + L95 \cdot L53 - L93); cc3 = \text{conj}(c3);

c1 = (L97 \cdot L71 - L97 \cdot L75 \cdot L51 - L97 \cdot L73 \cdot L31 + L97 \cdot L75 \cdot L53 + L95 \cdot L51 - L95 \cdot L53 \cdot L31 + L93 \cdot L31 - L91);
cc1 = \text{conj}(c1);
S99 = (Sx9x9 + cc7 \cdot Sx9x7 + cc5 \cdot Sx9x5 + cc3 \cdot Sx9x3 + cc1 \cdot Sx9x1 \ldots
+c7 \cdot Sx7x9 + (c7 \cdot cc7) \cdot Sx7x7 + (c7 \cdot cc5) \cdot Sx7x5 + (c7 \cdot cc3) \cdot Sx7x3 + (c7 \cdot cc1) \cdot Sx7x1 \ldots
+c5 \cdot Sx5x9 + (c5 \cdot cc7) \cdot Sx5x7 + (c5 \cdot cc5) \cdot Sx5x5 + (c5 \cdot cc3) \cdot Sx5x3 + (c5 \cdot cc1) \cdot Sx5x1 \ldots
+c3 \cdot Sx3x9 + (c3 \cdot cc7) \cdot Sx3x7 + (c3 \cdot cc5) \cdot Sx3x5 + (c3 \cdot cc3) \cdot Sx3x3 + (c3 \cdot cc1) \cdot Sx3x1 \ldots
+c1 \cdot Sx1x9 + (c1 \cdot cc7) \cdot Sx1x7 + (c1 \cdot cc5) \cdot Sx1x5 + (c1 \cdot cc3) \cdot Sx1x3 + (c1 \cdot cc1) \cdot Sx1x1);
S11_9 = Sx11x9 + cc7 \cdot Sx11x7 + cc5 \cdot Sx11x5 + cc3 \cdot Sx11x3 + cc1 \cdot Sx11x1;
L11_9 = \frac{\text{sum}(\text{abs}(S11_9))}{\text{sum}(\text{abs}(S99))};

aI1 = b1_I + L31 \cdot b3_I + L51 \cdot b5_I + L71 \cdot b7_I + L91 \cdot b9_I + L11_1 \cdot b11_I;
aI3 = b3_I + L53 \cdot b5_I + L73 \cdot b7_I + L93 \cdot b9_I + L11_3 \cdot b11_I;
aI5 = b5_I + L75 \cdot b7_I + L95 \cdot b9_I + L11_5 \cdot b11_I;
aI7 = b7_I + L97 \cdot b9_I + L11_7 \cdot b11_I;
aI9 = b9_I + L11_9 \cdot b11_I;
aI11 = b11_I;

Sd3 = (\text{abs}(aI3)^2) \cdot (S33);
Sd5 = (\text{abs}(aI5)^2) \cdot (S55);
Sd7 = (\text{abs}(aI7)^2) \cdot (S77);
Sd9 = (\text{abs}(aI9)^2) \cdot (S99);

c11 = 1; cc11 = \text{conj}(c11);
c9 = (-L11_9); cc9 = \text{conj}(c9);
c7 = (-L11_7 - L11_9 \cdot L97); cc7 = \text{conj}(c7);
c5 = (-L11_5 - L11_9 \cdot L95 - L11_7 \cdot L75 + L11_9 \cdot L97 \cdot L75); cc5 = \text{conj}(c5);

c3 = -(L11_3 - L11_5 \cdot L53 - L11_7 \cdot L73 - L11_9 \cdot L93 + L11_9 \cdot L97 \cdot L73 + L11_9 \cdot L95 \cdot L53 + L11_7 \cdot L75 \cdot L53 - L11_9 \cdot L97 \cdot L75); cc3 = \text{conj}(c3);

c1 = (L11_9 \cdot L91 - L11_9 \cdot L97 \cdot L71 - L11_9 \cdot L95 \cdot L51 - L11_9 \cdot L93 \cdot L31 + L11_9 \cdot L97 \cdot L75 \cdot L51 + L11_9 \cdot L97 \cdot L73 \cdot L31 \ldots
cc1=conj(c1);
Sd11=(abs(aI11).^2).*(Sx11x11+cc9.*Sx11x9+cc7.*Sx11x7+cc5.*Sx11x5+cc3.*Sx11x3+cc1.*Sx11x1...+c9.*Sx9x11+(c9.*cc9).*Sx9x9+(c9.*cc7).*Sx9x7+(c9.*cc5).*Sx9x5+(c9.*cc3).*Sx9x3+(c9.*cc1).*Sx9x1...+c7.*Sx7x11+(c7.*cc9).*Sx7x9+(c7.*cc7).*Sx7x7+(c7.*cc5).*Sx7x5+(c7.*cc3).*Sx7x3+(c7.*cc1).*Sx7x1...+c5.*Sx5x11+(c5.*cc9).*Sx5x9+(c5.*cc7).*Sx5x7+(c5.*cc5).*Sx5x5+(c5.*cc3).*Sx5x3+(c5.*cc1).*Sx5x1...+c3.*Sx3x11+(c3.*cc9).*Sx3x9+(c3.*cc7).*Sx3x7+(c3.*cc5).*Sx3x5+(c3.*cc3).*Sx3x3+(c3.*cc1).*Sx3x1...+c1.*Sx1x11+(c1.*cc9).*Sx1x9+(c1.*cc7).*Sx1x7+(c1.*cc5).*Sx1x5+(c1.*cc3).*Sx1x3+(c1.*cc1).*Sx1x1);
SddI=Sd3+Sd5+Sd7+Sd9+Sd11; % total output power of I channel
SccI=(abs(aI11).^2).*Sx1x1; % correlated distortion power of I channel
SyyI=SccI+SddI; % uncorrelated distortion power of I channel

%%%%%%% Q correlated with Q %%%%%%%%%%
switch channel
  case 'WCDMA_Up_DPCCH'
    % Load the spectrum terms of Q data of WCDMA uplink pilot signal.
    % The generation of the spectrum terms is referred to [5].
    load mxgen_WCDMA_Up_DPCCH_Q
  case 'WCDMA_Dn_T5_8HSPDA'
    % Load the spectrum terms of Q data of WCDMA downlink test model 5 signal.
    % The generation of the spectrum terms is referred to [5].
    load mxgen_WCDMA_Dn_T5_8HSPDA_Q
end

Sx1x1=ps^2*IQscale^2*Sz1;
Sx1x3=ps^4*IQscale^4*Sz13;
Sx1x5=ps^6*IQscale^6*Sz15;
Sx1x7=ps^8*IQscale^8*Sz17;
Sx1x9=ps^10*IQscale^10*Sz19;
Sx1xl1=ps^12*IQscale^12*Sz11_1l;
Sx3x1=ps^4*IQscale^4*Sz31;
Sx3x3=ps^6*IQscale^6*Sz33;
Sx3x5=ps^8*IQscale^8*Sz35;
\[ S_{x3x7} = p_s^{10} \cdot \text{IQscale}^{10} \cdot S_{z37}; \]
\[ S_{x3x9} = p_s^{12} \cdot \text{IQscale}^{12} \cdot S_{z39}; \]
\[ S_{x3x11} = p_s^{14} \cdot \text{IQscale}^{14} \cdot S_{z311}; \]
\[ S_{x5x3} = p_s^{8} \cdot \text{IQscale}^{8} \cdot S_{z53}; \]
\[ S_{x7x3} = p_s^{10} \cdot \text{IQscale}^{10} \cdot S_{z73}; \]
\[ S_{x9x3} = p_s^{12} \cdot \text{IQscale}^{12} \cdot S_{z93}; \]
\[ S_{x11x3} = p_s^{14} \cdot \text{IQscale}^{14} \cdot S_{z113}; \]
\[ S_{x5x5} = p_s^{10} \cdot \text{IQscale}^{10} \cdot S_{z55}; \]
\[ S_{x5x7} = p_s^{12} \cdot \text{IQscale}^{12} \cdot S_{z57}; \]
\[ S_{x5x9} = p_s^{14} \cdot \text{IQscale}^{14} \cdot S_{z59}; \]
\[ S_{x5x11} = p_s^{16} \cdot \text{IQscale}^{16} \cdot S_{z511}; \]
\[ S_{x7x5} = p_s^{12} \cdot \text{IQscale}^{12} \cdot S_{z75}; \]
\[ S_{x9x5} = p_s^{14} \cdot \text{IQscale}^{14} \cdot S_{z95}; \]
\[ S_{x11x5} = p_s^{16} \cdot \text{IQscale}^{16} \cdot S_{z115}; \]
\[ S_{x7x7} = p_s^{14} \cdot \text{IQscale}^{14} \cdot S_{z77}; \]
\[ S_{x7x9} = p_s^{16} \cdot \text{IQscale}^{16} \cdot S_{z79}; \]
\[ S_{x7x11} = p_s^{18} \cdot \text{IQscale}^{18} \cdot S_{z711}; \]
\[ S_{x9x7} = p_s^{16} \cdot \text{IQscale}^{16} \cdot S_{z97}; \]
\[ S_{x11x7} = p_s^{18} \cdot \text{IQscale}^{18} \cdot S_{z117}; \]
\[ S_{x9x9} = p_s^{18} \cdot \text{IQscale}^{18} \cdot S_{z99}; \]
\[ S_{x9x11} = p_s^{20} \cdot \text{IQscale}^{20} \cdot S_{z911}; \]
\[ S_{x11x9} = p_s^{20} \cdot \text{IQscale}^{20} \cdot S_{z119}; \]
\[ S_{x11x11} = p_s^{22} \cdot \text{IQscale}^{22} \cdot S_{z1111}; \]

% Compute the correlation coefficients

L_{31} = \frac{\sum \text{abs}(S_{x3x1})}{\sum \text{abs}(S_{x1x1})};

L_{51} = \frac{\sum \text{abs}(S_{x5x1})}{\sum \text{abs}(S_{x1x1})};
S_{33} = S_{x3x3} - \text{conj}(L_{31}) \cdot S_{x3x1} - L_{31} \cdot S_{x1x3} + (L_{31} \cdot \text{conj}(L_{31})) \cdot S_{x1x1};

E[u3*conj(u3)]

S_{53} = S_{x5x3} - \text{conj}(L_{31}) \cdot S_{x5x1};

L_{53} = \frac{\sum \text{abs}(S_{53})}{\sum \text{abs}(S_{33})};

L_{71} = \frac{\sum \text{abs}(S_{x7x1})}{\sum \text{abs}(S_{x1x1})};
S_{73} = S_{x7x3} - \text{conj}(L_{31}) \cdot S_{x7x1};
L_{73} = \frac{\sum \text{abs}(S_{73})}{\sum \text{abs}(S_{33})};

S_{55} = S_{x5x5} - L_{53} \cdot S_{x5x3} - (L_{51} - L_{53} \cdot L_{31}) \cdot S_{x1x5} + (\text{abs}(L_{53})^2 \cdot S_{x3x3} - (L_{51} - L_{53} \cdot L_{31}) \cdot \text{conj}(L_{53}) \cdot S_{x1x3};

S_{75} = S_{x7x5} - \text{conj}(L_{53}) \cdot S_{x7x3} - (L_{51} - L_{53} \cdot L_{31}) \cdot S_{x1x5} + (\text{abs}(L_{53})^2 \cdot S_{x3x3} - (L_{51} - L_{53} \cdot L_{31}) \cdot \text{conj}(L_{53}) \cdot S_{x1x3};

L_{91} = \frac{\sum \text{abs}(S_{x9x1})}{\sum \text{abs}(S_{x1x1})};
S_{93} = S_{x9x3} - \text{conj}(L_{31}) \cdot S_{x9x1};
L_{93} = \frac{\sum \text{abs}(S_{93})}{\sum \text{abs}(S_{33})};
S_{95} = S_{x9x5} - \text{conj}(L_{53}) \cdot S_{x9x3} - (L_{51} - L_{53} \cdot L_{31}) \cdot S_{x1x5} + (\text{abs}(L_{53})^2 \cdot S_{x3x3} - (L_{51} - L_{53} \cdot L_{31}) \cdot \text{conj}(L_{53}) \cdot S_{x1x3};
L_{95} = \frac{\sum \text{abs}(S_{95})}{\sum \text{abs}(S_{55})};
c7 = 1; cc7 = \text{conj}(c7);
c5 = -L_{75}; cc5 = \text{conj}(c5);
c3 = -(L_{73} - L_{75} \cdot L_{53}); cc3 = \text{conj}(c3);
c1=-(L71-L75*L51-L73*L31+L75*L53*L31); cc1=conj(c1);
S77=(Sx7x7+cc5.*Sx7x5+cc3.*Sx7x3+cc1.*Sx7x1+c5.*Sx5x7+(c5.*cc5).*Sx5x5+(c5.*cc3).*Sx5x3+(c5.*cc1).*Sx5x1);
+3.*Sx3x7+(c3.*cc5).*Sx3x5+(c3.*cc3).*Sx3x3+(c3.*cc1).*Sx3x1+c1.*Sx1x7+(c1.*cc5).*Sx1x5+(c1.*cc3).*Sx1x3+(c1.*cc1).*Sx1x1);
S97=Sx9x7+cc5.*Sx9x5+cc3.*Sx9x3+cc1.*Sx9x1;
L97=sum(abs(S97))./sum(abs(S77));
L11_1=sum(abs(Sx11x1))./sum(abs(S1x1x1));
S113=Sx11x3-conj(L31).*Sx11x1;
L113=sum(abs(S113))./sum(abs(S33));
S115=Sx11x5-conj(L53).*Sx11x3-conj(L51-L53.*L31).*Sx11x1;
L115=sum(abs(S115))./sum(abs(S55));
S117=Sx11x7+cc5.*Sx11x5+cc3.*Sx11x3+cc1.*Sx11x1;
L117=sum(abs(S117))./sum(abs(S77));
c9=1; cc9=conj(c9);
c7=-L97; cc7=conj(c7);
c5=(L97*L75-L95); cc5=conj(c5);
c3=(L97*L73-L97*L75*L53+L95*L53-L93); cc3=conj(c3);
c1=(L97*L71-L97*L75*L51-L97*L73*L31+L97*L75*L53*L31+L95*L51-L95*L53*L31+L93*L31-L91); cc1=conj(c1);
S99=(Sx9x9+cc7.*Sx9x7+cc5.*Sx9x5+cc3.*Sx9x3+cc1.*Sx9x1);
+7.*Sx7x9+(c7.*cc7).*Sx7x7+(c7.*cc5).*Sx7x5+(c7.*cc3).*Sx7x3+(c7.*cc1).*Sx7x1;
+5.*Sx5x9+(c5.*cc7).*Sx5x7+(c5.*cc5).*Sx5x5+(c5.*cc3).*Sx5x3+(c5.*cc1).*Sx5x1;
+3.*Sx3x9+(c3.*cc7).*Sx3x7+(c3.*cc5).*Sx3x5+(c3.*cc3).*Sx3x3+(c3.*cc1).*Sx3x1;
+1.*Sx1x9+(c1.*cc7).*Sx1x7+(c1.*cc5).*Sx1x5+(c1.*cc3).*Sx1x3+(c1.*cc1).*Sx1x1;
S119=Sx11x9+cc7.*Sx11x7+cc5.*Sx11x5+cc3.*Sx11x3+cc1.*Sx11x1;
L119=sum(abs(S119))./sum(abs(S99));
aQ1=b1_Q+L31*b3_Q+L51*b5_Q+L71*b7_Q+L91*b9_Q+L11_1*b11_Q;
aQ3=b3_Q+L53*b5_Q+L73*b7_Q+L93*b9_Q+L113*b11_Q;
aQ5=b5_Q+L75*b7_Q+L95*b9_Q+L115*b11_Q;
aQ7=b7_Q+L97*b9_Q+L117*b11_Q;
aQ9=b9_Q+L119*b11_Q;
aQ11=b11_Q;
Sd3=(abs(aQ3).^2).*S33;
Sd5=(abs(aQ5).^2).*S55;
Sd7=(abs(aQ7).^2).*S77;
Sd9=(abs(aQ9).^2).*S99;
c11=1; cc11=conj(c11);
c9=(-L119); cc9=conj(c9);
\[ c7 = -(L_{117} - L_{119} \cdot L_{97}); \quad cc7 = \text{conj}(c7); \]
\[ c5 = -(L_{115} - L_{117} \cdot L_{95} + L_{119} \cdot L_{97} \cdot L_{75}); \quad cc5 = \text{conj}(c5); \]
\[ c3 = -(L_{113} - L_{115} \cdot L_{53} - L_{117} \cdot L_{93} + L_{119} \cdot L_{97} \cdot L_{73} + L_{119} \cdot L_{97} \cdot L_{95} \cdot L_{73} - L_{119} \cdot L_{97} \cdot L_{95} \cdot L_{53}) \]
\[ cc3 = \text{conj}(c3); \]
\[ c1 = (L_{119} \cdot L_{91} - L_{119} \cdot L_{97} \cdot L_{95} \cdot L_{51} + L_{119} \cdot L_{93} \cdot L_{31} - L_{119} \cdot L_{97} \cdot L_{75} \cdot L_{51} - L_{119} \cdot L_{95} \cdot L_{53} \cdot L_{31} - L_{119} \cdot L_{97} \cdot L_{75} \cdot L_{53} - L_{119} \cdot L_{97} \cdot L_{73} \cdot L_{31}) \]
\[ cc1 = \text{conj}(c1); \]
\[ S_{d11} = (\text{abs}(a_{Q11})^2) \cdot (S_{x11x11} + \text{cc}_9 \cdot S_{x11x9} + \text{cc}_7 \cdot S_{x11x7} + \text{cc}_5 \cdot S_{x11x5} + \text{cc}_3 \cdot S_{x11x3} + \text{cc}_1 \cdot S_{x11x1} \ldots \]
\[ + c9 \cdot S_{x9x11} + c9 \cdot \text{cc}_9 \cdot S_{x9x9} + (c9 \cdot \text{cc}_7 \cdot S_{x9x7}) + (c9 \cdot \text{cc}_5 \cdot S_{x9x5}) + (c9 \cdot \text{cc}_3 \cdot S_{x9x3}) + (c9 \cdot \text{cc}_1 \cdot S_{x9x1}) \ldots \]
\[ + c7 \cdot S_{x7x11} + c7 \cdot \text{cc}_7 \cdot S_{x7x9} + (c7 \cdot \text{cc}_7 \cdot S_{x7x7}) + (c7 \cdot \text{cc}_5 \cdot S_{x7x5}) + (c7 \cdot \text{cc}_3 \cdot S_{x7x3}) + (c7 \cdot \text{cc}_1 \cdot S_{x7x1}) \ldots \]
\[ + c5 \cdot S_{x5x11} + c5 \cdot \text{cc}_5 \cdot S_{x5x9} + (c5 \cdot \text{cc}_5 \cdot S_{x5x7}) + (c5 \cdot \text{cc}_5 \cdot S_{x5x5}) + (c5 \cdot \text{cc}_5 \cdot S_{x5x3}) + (c5 \cdot \text{cc}_5 \cdot S_{x5x1}) \ldots \]
\[ + c3 \cdot S_{x3x11} + c3 \cdot \text{cc}_3 \cdot S_{x3x9} + (c3 \cdot \text{cc}_5 \cdot S_{x3x7}) + (c3 \cdot \text{cc}_5 \cdot S_{x3x5}) + (c3 \cdot \text{cc}_3 \cdot S_{x3x3}) + (c3 \cdot \text{cc}_3 \cdot S_{x3x1}) \ldots \]
\[ + c1 \cdot S_{x1x11} + c1 \cdot \text{cc}_9 \cdot S_{x1x9} + (c1 \cdot \text{cc}_7 \cdot S_{x1x7}) + (c1 \cdot \text{cc}_5 \cdot S_{x1x5}) + (c1 \cdot \text{cc}_3 \cdot S_{x1x3}) + (c1 \cdot \text{cc}_1 \cdot S_{x1x1}) \]
\[ S_{ddQ} = S_{d3} + S_{d5} + S_{d7} + S_{d9} + S_{d11}; \quad \% \text{total output power of Q channel} \]
\[ S_{ccQ} = (\text{abs}(a_{Q1})^2) \cdot S_{x1x1}; \quad \% \text{correlated distortion power of Q channel} \]
\[ S_{yyQ} = S_{ccQ} + S_{ddQ}; \quad \% \text{uncorrelated distortion power of Q channel} \]
\[ S_{yy} = S_{yyI} + S_{yyQ}; \quad \% \text{total modulator output power} \]
\[ S_{cc} = S_{ccI} + S_{ccQ}; \quad \% \text{correlated modulator output distortion power} \]
\[ S_{dd} = S_{ddI} + S_{ddQ}; \quad \% \text{uncorrelated modulator output distortion power} \]
Appendix B Design of a RF Integrated Direct-Conversion Quadrature Modulator

A 2.4GHz direct-conversion quadrature modulator was designed and taped out in 0.18 \( \mu \text{m} \) IBM RFCMOS process. It targeted at WLAN application where OFDM modulation schemes were used. The first design purpose is to analyze the design tradeoffs of using two different baseband transconductors: a CMOS differential pair and a CMOS multi-tanh triplet. Auxiliary circuitry was added to measure the contribution of the baseband transconductor to the total nonlinear distortion, which serves as the second design purpose to verify that the baseband transconductors are the dominant nonlinear components in the direct-conversion quadrature modulators.

B.1 Architectures of Quadrature Modulator

Direct-conversion quadrature modulators are widely used in modern wireless transceivers such as WCDMA, WLAN, WiMAX systems. They perform quadrature modulation on the baseband I and Q data streams, which are upconverted to carrier frequency by two mixers whose LO signals are in quadrature. The outputs of the two mixers are summed together, usually in current domain, to produce the RF modulated signal.

The core circuit block of a quadrature modulator is the up-conversion mixer. The design of a mixer involves many tradeoffs such as conversion gain, LO drive requirement, noise, linearity, voltage supply, port-to-port isolation, power consumptions, and etc. In
designing a CMOS mixer, usually the first design decision needed to make is whether to use a passive or an active mixer. Passive mixers operate the FET transistors in linear region as switches and demonstrate excellent linearity performance which leads to the increased dynamic range. The drawbacks are the conversion loss and the requirement for large LO drive. The active mixers usually consists of tail current sources, baseband transconductors, mixer core, and resistive or tank load. Due to the baseband transconductors, active mixers provide conversion gain. They also require smaller LO drive compared to the passive counterpart, but they produce more nonlinear distortion [102]. The reduced LO drive required by an active mixer is a big advantage over the active mixers in contemporary CMOS wireless transceiver design because the continuing shrink of the supply voltage with the progress of technology nodes. The smaller LO drive provides better LO-RF/LO-IF isolation as well. In addition, the conversion gain of active mixers reduces the noise generated in the subsequent cascaded stages [25]. Therefore, CMOS wireless transceiver designs favor the active mixers.

Another important decision in designing a mixer is to choose between single-balanced and double-balanced structures. An unbalanced mixer has both baseband and LO signals as single-ended; a single-balanced mixer features a single-ended baseband signal and a differential LO signal; while a double-balanced mixer has both baseband and LO signals as differential. Compared to unbalanced mixer, the single-balanced mixers provide rejection of baseband signal leaking into the RF output, but the LO feedthrough to RF port can not be rejected. The major advantage of the double-balanced mixer over the single-balanced mixer is the rejection of both baseband signal leakage and LO feedthrough at the cost of
extra current consumption and larger input-referred noise [25]. Another advantage of double-balanced structure is the reduced even-order distortion due to the differential configuration at both baseband and LO ports. The extent of the port-to-port isolation and even-order distortion reductions highly depends on the symmetry in the circuits which is degraded due to device mismatches and imperfections. The LO feedthrough rejection is critical in direct-conversion upconverter design because the LO leakage sits right on top of the upconverted DC. In time-domain modulation systems like WCDMA, it is impractical to perform low-pass filtering or AC coupling so that there is useful information contained at DC. The LO feedthrough will corrupt such information contents and degrade the signal quality. Therefore, it is desirable to adopt a double-balanced mixer structure. Double-balanced Gilbert cell mixer is the most commonly used one among the various types of active mixers.

The in-phase and quadrature LO signals can be generated either by a 90 degree phase shifter with a VCO frequency equal to the carrier frequency or by a divide-by-2 frequency divider with a VCO frequency twice the carrier frequency. In a direct-conversion upconverter, it is desirable to use the divide-by-2 LO signal generation scheme in order to avoid the deleterious impacts resulting from the power amplifier coupling which is referred to as injection pulling or injection locking [103]. A buffer stage may be necessary between the divide-by-2 circuit block and the mixer core in order to drive the large, or possibly variable capacitive load.

Usually a tank load is needed to resonate off the parasitic capacitances of the mixer core. However, it may be possible to use a resistive load if the operating frequency is not
too high and the transistors in the mixer core are sized reasonably small. This is more likely to be true for the advanced technology nodes like 90 nm CMOS process, 65 nm CMOS process, and beyond.

Usually the next stage after the quadrature modulator is the driver amplifier, which is likely to be single-ended. Therefore, most time a differential to single end converter is needed [104]. In this work, since the research focus is on the modulator only, a differential to single end converter was not implemented. Instead, an output buffer was designed to buffer the tank load impedance to $50 \, \Omega$ for the testing purpose.

**B.2 Linearization Techniques for Quadrature Modulators**

Direct conversion quadrature modulators are widely used in modern wireless systems which utilize sophisticated digital modulation schemes like QPSK, 8-/16-/32-/64-QAM in order to increase the spectrum efficiency, ultimately the system throughput. Since most currently used modulation schemes feature non-constant envelope and large PAR, the quadrature modulators, driver amplifiers, and power amplifiers are required to be sufficiently linear in order to keep the integrity of the useful information and meet the stringent spectrum emission specifications.

In CMOS up-converters, the baseband transconductors, mixer switches, and the tank or resistive load contribute to the total nonlinear distortion. The baseband transconductors convert the voltage inputs into current outputs. They are the major sources of nonlinearity in the up-converter due to the inherent nonlinear $V_{gs} - I_d$ relationship of MOSFET. Mixer switches contribute to the mixer nonlinear distortion due to non-ideal switch actions [104].
With appropriate LO drives, the mixer switches’ nonlinearity contributions can be small compared to the baseband transconductors[91]. The resistive/tank load can also introduce nonlinear distortion in an up-converter due to the nonlinear resistances. However, this is a negligible source of nonlinearity compared to the other two because the resistive load or the effective tank resistance is small and linear.

Because of the big impact of baseband transconductors on the nonlinear distortion, various techniques have been developed to make the baseband transconductors more linear. In this category, the most popular and straightforward technique is common source with source degeneration by utilizing the local negative feedback principle at the cost of reduced $G_m/I_c$ efficiency. Another widely used baseband linearization technique is the CMOS multi-tanh, where multiple CMOS differential pairs with different effective bias are connected in parallel to achieve the overall constant $G_m$ over a broader input range [35, 91, 92]. The drawback of CMOS multi-tanh technique is the reduced $G_m/I_c$ efficiency, increased noise and complexity. Multiple gated transistor (referred to as derivative superposition in HEMT community) is another baseband transconductor technique without reducing the $G_m/I_c$ efficiency based on third order harmonic cancellation using a secondary transistor [105]. However, the extent of cancellation is sensitive to PVT variations [104].

Instead of linearization on the circuit-level, system-level linearization techniques are also extensively explored by researchers, such as feed-forward technique [106], pre-distortion approach [107], and signal injection method [108].
B.3 Quadrature Modulator Design

A double balanced Gilbert cell type mixer operating at 2.4 GHz was designed in 0.18\(\mu m\) IBM RFCMOS process. The top-level diagram of this modulator was shown in Figure B-1. Two baseband signal streams \(i(t)\) and \(q(t)\) were up-converted to RF frequency by two up-converters with LO signals in quadrature. The outputs of the two up-converters were summed together in current domain as the output signal of the quadrature modulator. The two 90 degree out-of-phase LO signals were generated by two static divide-by-two fully differential frequency dividers, which are realized as two high speed current-mode-logic (CML) latches in a negative feedback loop [25]. The CML latches consist of a differential pair and a regenerative pair. They were designed and optimized for best performance based on [109, 110]. The divide-by-two divider can provide two half-frequency signals which are in quadrature. However, the quadrature property could be degraded if the inputs are not ideally differential, or the inputs do not have 50% duty cycle, or device mismatches exist [25]. The first divide-by-two divider was used solely for testing purpose in order to present a good differential input to the second divide-by-two circuit. However, for safety reason, it can be bypassed by a control bit (VcLO) if it turns out to be malfunctioned.
Figure B-1 Top-level diagram of the quadrature modulator.
Looking into the up-converter block, there are two baseband transconductors, a common source differential pair and a CMOS triplet, which share the same mixer core and the tank load. The two transconductors were designed to have the same $G_m$ for comparison purpose. Selection between the two options can be made manually through a control bit (Vc1). With this configuration, the tradeoffs in terms of using the two transconductors can be analyzed. The outputs of the two baseband transconductors were also routed to a cascade stage (common gate configuration) with resistive load. Since a low frequency cascade stage is pretty linear, an approximation can be made that observing the cascade stage output is a measure of the nonlinearity characteristics of the baseband transconductors. The routes of the baseband transconductor outputs can be selected between the mixer core and the cascade stage manually through a control bit (Vc2). In this way, one can tell if the baseband transconductor is the major source of nonlinearity in an up-converter by comparing the modulator output and the cascade stage output. The coding of the three control bits was summarized in Table B-1.

### Table B-1 Description of the three control bits.

<table>
<thead>
<tr>
<th>Control Bit</th>
<th>Value</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vc1</td>
<td>0</td>
<td>Differential Pair</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>CMOS Triplet</td>
</tr>
<tr>
<td>Vc2</td>
<td>0</td>
<td>Mixer Core</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Cascade Stage</td>
</tr>
<tr>
<td>VcLO</td>
<td>0</td>
<td>Not Bypass 1st Divider</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Bypass 1st Divider</td>
</tr>
</tbody>
</table>
The bandgap reference circuit block and the current mirror circuit blocks provide the
bias current to the major modulator building blocks. Output buffers were designed to buffer
the outputs of both the quadrature modulator and the cascade stage to 50 Ω for testing
purpose.

B.4 Simulation Results

The quadrature modulator (mixer stage and cascade stage) was simulated using
Cadence SpectreRF simulator. Periodic Steady State (PSS) analysis was used to analyze
the nonlinear performance, conversion gain, and image suppression. Periodic Noise
(Pnoise) analysis along with PSS analysis was used to simulate the noise characteristics of
the modulator. The tradeoffs between the differential pair and the CMOS triplet baseband
transconductors were investigated based on the simulation results. The performance
comparisons were summarized in Table B-2.

Table B-2 Summary of the simulation results of the quadrature modulator.

<table>
<thead>
<tr>
<th></th>
<th>Differential Pair</th>
<th>Triplet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Supply (V)</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>Current Drain* (mA)</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Input Voltage (peak, mV)</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Output Voltage (rms, mV)</td>
<td>305</td>
<td>245</td>
</tr>
<tr>
<td>Output Impedance (ohm)</td>
<td>545</td>
<td>495</td>
</tr>
<tr>
<td>Conversion Gain* (dB)</td>
<td>9.8</td>
<td>8</td>
</tr>
<tr>
<td>IM3 (modulator, dBC)</td>
<td>−39.5</td>
<td>−53</td>
</tr>
<tr>
<td>IM3 (cascade stage, dBC)</td>
<td>−40</td>
<td>−55</td>
</tr>
<tr>
<td>Noise (dBc/Hz @ 190 MHz)</td>
<td>−156</td>
<td>−154</td>
</tr>
<tr>
<td>Image Suppression (dBc)</td>
<td>−40.5</td>
<td>−40.9</td>
</tr>
</tbody>
</table>

* Current drain and conversion gain are for one mixer only
B.4.1 Transconductance $G_m$

A DC sweep simulation was done to simulate the effective $G_m$ for the two different transconductors. The plot shown in Figure B-2 (a) shows the $G_m$ plot for the CMOS triplet baseband transconductor (Gm_triplet_total), which is composed of three individual $g_m$ pairs: gm_right_pair, gm_center_pair, and gm_left_pair so that the total effective Gm is flat over a much broader input range than a differential pair transconductor, as shown in Figure B-2 (b). The DC based $G_m$ simulation provides a straightforward explanation for the improvement of linearity performance by using a CMOS triplet over a differential pair, however, one limitation of such simulation is that it cannot take the high frequency phenomenon into consideration. The effective $G_m$ of the two transconductors were designed to be equal for comparison purpose, which was verified by PSS simulation.
Figure B-2 $G_m$ plot: (a) for CMOS triplet transconductor (b) for CMOS triplet transconductor and differential pair transconductor.

B.4.2 Conversion Gain and IM3

As described in Section 2.1, WLAN systems utilize OFDM modulation schemes
containing multiple sub-carriers. Each individual sub-carrier is single-sideband modulated to carrier frequency which is different from the double-sideband modulated systems such as WCDMA systems. Therefore, to measure the IM₃, instead of using an in-phase single-tone on both I and Q channels which is suitable for WCDMA systems [104], a two-tone signal was applied to I channel and a two-tone signal with the same amplitude but quadrature phase compared to its I channel counterpart was applied to Q channel to measure the IM₃ for the quadrature modulator used for WLAN systems. Therefore, the two quadrature two-tone signal will be upconverted to a two-tone signal at carrier frequency. A PSS simulation was used to simulate the modulator output frequency spectra and IM₃ could be measured from the fundamental and intermodulation terms.
Figure B-3 Simulated IM₃ for (a) quadrature modulator w/ differential pair transconductor and (b) quadrature modulator w/ CMOS triplet transconductor.
The IM$_3$ were measured at the maximum output voltage level as specified in Table B-2. As shown in Figure B-3, the CMOS triplet version quadrature modulator (53 dBc) has about 13 dBc IM$_3$ improvement compared to the differential pair counterpart (39.5 dBc).

The same input excitations were applied to the baseband transconductors with cascade stage. The IM$_3$ of the cascade stage outputs were measured as shown in Figure B-4.
Figure B-4 Simulated IM₃ for: (a) cascade stage w/ differential pair transconductor and (b) cascade stage w/ CMOS triplet transconductor.
The IM$_3$ of the cascade stage is 39.6 dBc when connecting with a differential pair transconductor and is 55 dBc for a CMOS triplet transconductor. They are a little better compared to the IM$_3$ of the quadrature modulator outputs due to the high frequency nonlinear effects of the parasitic capacitors, but very close within 2 dBc. Therefore, it can be concluded that the major source of nonlinearity in the active RF integrated direct-conversion quadrature modulator is the baseband transconductor.

The conversion gain is defined for the I/Q mixer only. It was able to be measured with the same simulation setup for IM$_3$. Since the input two-tone signal at I and Q channel are in quadrature, the output of the I and Q mixers are summed together in-phase. Therefore, the conversion gain can be calculated by looking at either one of the two tones:

\[
\text{Conversion Gain} = \left(\frac{V_{RF,\text{rms}}}{V_{BR,\text{rms}}}\right)_{\text{onetone, I/Q}}
\]  \hspace{1cm} (B.1)

The simulated conversion gain for the CMOS triplet version mixer is about 1.5 dB smaller than the differential pair counterpart. This is because the CMOS triplet (6 mA) consumes 2 mA more DC current than the differential pair (4 mA) in order to provide the same $G_m$. This leads to the smaller output impedance of the CMOS triplet version mixer than the other. Also, another outcome due to the larger DC current is that the switches in the CMOS triplet version mixer are simultaneously on for a longer time than its differential pair counterpart given the same LO drive, which wastes more AC current generated by the baseband transconductor as a common-mode signal and leads to conversion gain reduction.
B.4.3 Noise

PNoise followed by a PSS simulation with the maxim output voltage was run to get the noise performance of the two versions of quadrature modulators. The spot noises were plotted in Figure B-5. The CMOS triplet version quadrature modulator generates about 2 dBC more noise than its differential pair counterpart partly due to the more switching noise from the mixer core (longer simultaneously on time). Another source of the larger noise is from the triplet transconductor due to more device noise sources.
Figure B-5 Spot noise for (a) quadrature modulator w/ differential pair transconductor and (b) quadrature modulator w/ CMOS triplet transconductor.
C.4.4 Image Suppression

A single-tone signal in quadrature was applied to I and Q channel respectively for testing the image suppression of the modulator. The output is a single-sideband single tone signal and the gain/phase imbalances are reflected by the ratio of the desire signal power to the image power. The simulated results are shown in Figure B-6. The simulation results did no show noticeable image suppression difference between the two versions of quadrature modulators; both have about 41 dBc image suppressions.
Figure B-6 Image suppression: (a) quadrature modulator w/ differential pair transconductor and (b) quadrature modulator w/ CMOS triplet transconductor.
B.5 Summary

The design of this 2.4 GHz quadrature modulator serves two research purposes:

1) Investigate the tradeoffs of using two different baseband transconductors: differential pair transconductor and CMOS triplet transconductor. The CMOS triplet transconductor has worse $G_m / I_d$ efficiency. To provide the same $G_m$, the CMOS triplet needs to use more devices and burn 2 mA more DC current than its differential pair counterpart, which leads to smaller conversion gain and worse noise performances. The gain of these costs is the increased linearity performance and better dynamic range. With about 13 dBc improvement of IM$_3$ and 2 dBc more noise, the dynamic range increases by about 2 dB.

2) The IM$_3$ performances of the modulator and the cascade stage are very close, which indicates that the major source of nonlinearity in a direct-conversion quadrature modulator is the baseband transconductor. This finding consolidates the assumption made in the previous modeling work.