ABSTRACT


A noise prediction scheme using time domain noise theory was developed for low speed fans with high inflow distortion. Basic design parameters such as thrust, torque, blade shape and blade thickness are used to predict a baseline noise signature using the Ffowcs Williams and Hawkings equation. Flow disturbances from simple, common shapes in the flow path are approximated and used to determine the noise level produced with installed conditions that ingest such disturbances into the fan. The inputs to the program are restricted to commonly available information that can be determined early in the design cycle.

The process has been verified with an experimental set up in an anechoic chamber. The results of the experiment and the prediction from the program agree within 3 dB over a wide range of angles for the first two blade passing frequencies and a reasonable range of broadband noise.
A Computational Method for Evaluating the Installation Effects of Axial Flow Fan System Noise

by

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DEDICATION

To my sister Karen.
Throughout my life, she has always been there - pushing, challenging, motivating and setting the example. She has been a tremendous role model while always being there to talk. She is my inspiration.
BIOGRAPHY

The author was born in Melbourne, Florida in 1969. Growing up just south of Cape Canaveral opened up many opportunities to follow the space program and the high tech atmosphere that surrounds it. In addition to his love of sciences, the author became an avid sailor. He was active in the local sailboat racing groups where he raced both his sunfish class boat and his parents 27 foot cruiser. He graduated from Melbourne High School in 1987 and started a BS in Mechanical Engineering at Auburn University that summer. While earning his BS, the author worked as an engineering intern at RCA-GE and at Harris Corporation. He was a member of the Auburn University Sailing Team and was instrumental in getting the sailing team recognized by the Dixie Sailing Club out of Montgomery, AL. He earned his BS in December of 1990. The author stayed in Auburn to complete an MS in Mechanical Engineering in December 1992. His thesis title was "On the Computation of Lyapunov-Floquet Transformation Matrices for General Periodic Systems." Soon after graduation he moved to Huntsville, Alabama to join Sverdrup Technology doing rotordynamic analysis of the Space Shuttle Main Engine Turbopumps. In 1994 he moved to Peoria, IL to work on a contract with Caterpillar Inc. He transferred to Clayton, NC in 1995 to work on dynamic analysis for Caterpillar's Building Construction Products Division. In 2001 he moved into a noise control analysis position and began his PhD at NC State.
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NOMENCLATURE

Arc\textsubscript{i}  Swept area of a given element

B  Number of blades

c  Speed of sound in air

\vec{d}\textsubscript{i}  Drag vector on a given element

d\textsubscript{i}  Drag load on a given element

f(\vec{y}, \tau) = 0  Function representing the fan blade surface

F\textsubscript{i}  Force source components

F\textsubscript{0}  General force with no velocity defects

F\textsubscript{gust}  General force including the effects from velocity defects

g  Retarded time

G(\vec{x}, t; \vec{y}, \tau)  Green’s function of the wave equation in unbounded 3d space

H  Boundary layer shape function

H(f)  Heaviside function applied to function f

M  Mach number

m  Harmonic multiple of BPF in Gutin’s formulation

M\textsubscript{r}  Component of the element Mach number in the \hat{r} direction

\dot{M}\textsubscript{r}  Emission time derivative of M\textsubscript{r}

\vec{n}  Unit normal of the data surface

n  Number of blades in Gutin’s formulation

n\textsubscript{i}  Components of the surface area normal of an element

\vec{O}(\vec{x}, t)  Observer position vector relative to the fan face
$O$ Magnitude of the observer position vector

$P$ Total pressure acting on the data surface

$P_{ij}$ Pressure at fluid element $i$ with respect to the fluid element $j$

$P_0$ Atmospheric pressure acting on the data surface

$P_i$ Component of blade loading in the $i$ direction

$p'$ Acoustic pressure

$P_0$ Component of blade loading in the $r$ direction

$Q$ Source strength of a Green’s Function solution

$q$ Thickness source

$Q_{tot}$ Volume flow rate through the fan

$\hat{r}_i$ Unit vector in the $\vec{r}$ direction

$\vec{r}$ Observer position relative to a blade element

$R$ Fan radius

$R_{ele}$ Position of an element along the span of the blade

$Re$ Reynolds number

$t$ Time in the observer space

$Thr_i$ Thrust at a given element

$T$ Total thrust load in Gutin’s formulation

$U_{gust}$ Inflow velocity including velocity defects

$u_i$ Components of the velocity of the fluid

$U_{mean}$ Calculated mean velocity over the swept area of the fan blades

$u_n$ Velocity of the fluid normal to the data surface
\( V_{gust} \) Total apparent velocity including velocity defects

\( V_i \) Outflow velocity at a given element

\( v_{ij} \) Velocity of fluid element i relative to fluid element j

\( V_{mean} \) Total apparent mean velocity with no velocity defects

\( v_n \) Velocity normal of the data surface

\( \hat{x}_i \) Unit vectors of the observer space

\( \vec{x} \) Spatial coordinates in the observer space

\( \vec{y} \) Spatial coordinates in the emission space

\( \Delta P_{static} \) Static pressure across the fan

\( \delta(f) \) Dirac delta applied to a function f

\( \delta^* \) Displacement boundary layer

\( \Gamma_i \) Torque at a given element

\( \Omega \) Fan rotational speed

\( \phi_0 \) Apparent velocity angle

\( \pi \) Pi

\( \rho \) Density of air

\( \tau \) Time in the emission space

\( \tau^* \) Signifies evaluation over the retarded time

\( T_{ij} \) Lighthill’s stress tensor

\( \Theta \) Fan rotation

\( \zeta \) Observer angle from \( \hat{x}_3 \) axis
Chapter 1

Introduction

1.1 Fan Noise in Modern Times

Fans of all shapes and sizes are used in everyday applications. They are in fact one of the largest consumers of generated power in the world. Fan noise has become common in many environments from sources like construction equipment, aircraft overflight, HVAC (air handlers, compressor cooling and evaporators) and electronics [7].

Fan noise has become so ubiquitous, many governmental agencies have begun regulating noise caused primarily by fans. The European Union, for example, has specific product regulations that govern the allowable sound power level of many kinds of machines. A recent provision in those regulations allows for the use of variable speed fans as a means to entice manufacturers to reduce their overall fan noise [9]. Many countries have standards for workplace noise exposure. In construction equipment, factories, and even office settings, fan noise can have a huge impact on the noise exposure level. Fan noise from aircraft drive much of the site location and design of modern airports.

In product design, the desire for low fan noise often is in conflict with other design criteria. In construction equipment, the trade off between low noise from cooling fans and adequate cooling can be a huge challenge. From computers to construction equipment, an increase in machine performance often leads to higher heat loads and therefore more cooling required. In order to keep the products attractive to the consumer, advancements in low noise fans are required, along with advancements in the accuracy and speed of predicting noise levels from practical installations with various installation effects.
1.2 Quiet Fan Design

From this dilemma in product design, a need for tools to aid in quiet fan design exists. In order to effectively design a quiet system one must first have an understanding of both the basic noise mechanisms and the installation effects. The aircraft industry has long recognized this need and responded with leading development for these tools. Propeller noise is generally dominated by its loading scheme [27, 28]. In helicopters quadrupole sources from fluid shear can become significant due to the high tip speeds and high thrust requirements. As such, much of the work in this area has focused on the problems specific to high speed and high thrust propellers.

In a small, low speed fan system, the inflow distortion often dominates [28]. The inflow distortion can often be extreme as cooling fans are packaged close to many flow disturbances like finger guards, cooler cores and grilles. Such inflow distortions can cover the entire fan face, while typical noise testing at a manufacturer is done with more uniform inflow. There exists a need to quantify the effects of the inflow distortions and installation effects on cooling fan noise early in the design phase. This requires a prediction scheme that focuses on the noise sources that are dominant in low speed industrial fans.

1.3 Overview of Analysis Techniques

For many years, engineers have been interested in understanding the noise mechanisms associated with fans. The fundamental mechanisms involved in fan noise are the thickness noise (monopole), noise due to the periodic forces on the air (dipole) and noise due to the viscous shear of the fluid around the fan (quadrupole). The task of aerodynamics is largely in defining the equivalent sources used in the acoustic solution of the problem.

In 1936 Gutin published the defining work on propeller noise [17]. His approach to the periodic noise generated by spinning blades was to examine the power required to spin the fan and the thrust it generates. Gutin set up a velocity potential based on the force transmitted to the fluid by a rotating blade. From this he was able to determine the acoustic pressure created by the blade loading. Later, Lowson developed a helicopter prediction noise code using a series of rotating dipoles to model the forces on the blades [24]. Lowson’s formulation was consistent with Gutin’s result [23]. Neither formulation took into account the thickness noise (monopole source) or volume sources (quadrupoles) such as flow around the tips of the blades and the boundary layer separations.
Meanwhile Ernsthausen in Germany and Deming in the US were investigating the thickness noise [27]. Both benefited from the development of reliable sound measuring devices and used the new technology to help identify the sources of noise in propellers. Ernsthausen made the observation that when a radial obstruction was placed in front of the propeller, the noise at the blade passing frequency (BPF) increased along the axis of rotation. Deming presented results that supported the experimental data showing the importance of thickness noise at high blade tip speeds [11].

Lighthill laid the ground work for modern aeroacoustics with his acoustic analogy which defines source terms for jet noise [29]. In 1951, Lighthill redefined the source of jet noise in terms of a viscous stress tensor. This tensor is directly analogous to the Reynolds stress from the Navier-Stokes equations for low Mach numbers [15]. For sound generation, it turns out that the fine scales of turbulence are irrelevant and the noise is controlled primarily by the large scale turbulent structures. The length scales of this turbulence are largely determined by the basic geometry. Lighthill’s theory is exact for noise generated by solid bodies in a flow stream [15].

In 1969, Lighthill’s theory was extended to flows in the presence of moving surfaces by J. E. Ffowcs Williams and D. L. Hawkings [11]. They represented the sources described in the Lighthill stress tensor as a collection of quadrupoles outside the solid body [15]. The Ffowcs Williams and Hawkings (FW-H) equation has become the basis for most predictive noise codes.

Farassat expanded on the work of Ffowcs Williams and Hawkings using generalized function theory to describe the moving surface in a fluid [12]. He used the generalized derivatives to obtain solutions for the various source terms in the FW-H equation.

The evolution of computational fluid dynamics (CFD) and advanced noise modeling codes based on the FW-H equation have led to new generations of more efficient and quieter fans [26]. As the designers reduce the basic noise of a fan (the noise generated by a fan operating under ideal conditions, also called self-noise), installation effects become increasingly important, and in many cases the dominant source of noise [27]. This is particularly true for low speed, lightly loaded fans, since the basic noise is low and the noise signature is dominated by noise due to installation peculiarities.
1.4 Analysis Applications

With this understanding of the noise mechanisms and the advent of modern computing technology, research into various noise mechanisms has flourished. Aircraft engine manufacturers such as Pratt and Whitney, Sikorsky and Rolls Royce have done extensive work on developing fans with low self-noise [33, 31, 8]. These fans are installed in flow situations that are designed to be as clean as possible. Research into the rotor-stator interaction found that the fundamental blade passing frequency (BPF) is related to not just the number of blades and the fan speed, but also the number of stators. Hanson and others observed that the dipole noise that dominates aircraft fan noise contains discrete tonal noise while the fluctuations in the wakes that cause this noise contribute to the broadband noise [18].

Over the last three decades, we have seen a 10 dB reduction in jet aircraft noise [27, 33]. Farassat and others at NASA’s Langley Research Center have done extensive work in the area of helicopter rotor noise and have been instrumental in developing prediction codes for both subsonic and supersonic fan noise. Farassat has developed several formulations based on the FW-H equation to predict the noise generated by helicopter rotor blades [14]. His Formulations 1 and 1A focus on the monopole and dipole terms in the FW-H equation for subsonic tip speeds. The application was developed for helicopters but the theory is extensible to any kind of axial fan with a subsonic tip speed [12]. For supersonic applications Farassat developed Formulation 3 and Formulation 4. Formulation 3 is a highly complex method and very computationally intensive. Formulation 4 is a simplification of the geometry and calculation of Formulation 3 [14]. Working with J. Casper, Farassat expanded on his work by proposing methods for broadband noise predictions within the same framework [5].

Consumer products industries have also contributed to work in this area. In large fan systems dipole noise dominates in the form of a blade pass frequency tone. Consumer product fans, however, such as the ones found in personal computers, can have major contributions due to inflow disruptions. Unlike helicopters and aircraft engines, consumer product fans tend to have unsteady flow due to their packaging. Fan support brackets, finger guards and complex drive systems create inlet disturbances and nonuniform back pressures which affect the loading on a fan blade. The resulting variation not only changes the noise created by a particular blade, but it can upset the symmetry inherent in the dipole noise signature of a fan. When this happens the phase cancellation of evenly
spaced blades breaks down, resulting in large increases in noise [18]. The high pressure drop of these systems induces higher radial flow along the blades, resulting in spillage across the blade tips. This spillage generates a tip vortex that can produce significant noise as well. Experimentation is currently underway with rotating tip rings, winglets and integrated fan shrouds to try to reduce or eliminate the tip vortex and its associated noise. In most cases, these solutions have little detrimental effect on the power required to spin the fan due largely to elimination of the tip losses [2].

Another area of increasing research is in the noise generated by wind turbines. Though they do not have much disturbance from objects in their airflow path, their sheer size and placement in the open air currents lead to inflow distortions. Here again, tip vortex noise is important. Manufacturers are looking for ways to design the blades to be robust to the inflow distortions and vortex noise associated with these turbines [3].

While much of the background work has been done using axial fans, centrifugal fans are now some of the most widely used fans due to their compactness and relatively low cost. Consequently, centrifugal fans have been studied extensively. Scroll cage fans, a particular subset of centrifugal fans, however, are only just now beginning to be studied. Current work focusing on the regions of flow separation and reattachment in a scroll cage fan offer an opportunity for fan performance improvement and noise reduction [32].

The fan installation is a critical factor for noise in consumer products. Vortices in the airflow are largely responsible for the installation induced noise. They can come in the form of upstream disturbances to the flow, downstream air blowing across various objects, excessive tip vortex noise (from high tip clearances), flow distortion of the inlet air and variations due to back pressure changes. In short there is no substitute for good design.

Fan noise is becoming an issue in a broad range of areas. We are no longer just concerned with loud airplanes. Cooling fans for electronics, cars, construction equipment and buildings create noise that ranges from annoying to damaging, while the challenge of low noise design is becoming more complex. As computational methods are developed and become easier to use, the old empirical design methodologies are being augmented with greater ability to predict noise. Around the world centers for study of fan noise exist. NASA’s Langley Research Center drives much of the research into helicopter and aircraft engine noise. The von Karman Institute for Fluid Dynamics sponsors a lecture series on computational aeroacoustics. IMechE, the qualifying body for mechanical engineering in the United Kingdom, recently sponsored a conference on fans with participation from all
over Europe. While many of the papers dealt with specific noise issues and analyses, the most telling paper presented at the 2004 IMechE conference described how far we have yet to go in terms of systematic innovation in fans [25].

1.5 Cooling System Design Parameters

While most of the research in fan noise has come from the aerospace applications, the needed work is largely applicable to lower speed, lighter load systems. Cooling systems make up a large portion of the fan applications in the modern world. In order to understand the noise mechanisms particular to these kinds of fans, we must understand the basic design strategy for the system. The airflow requirement, of a cooling system, translates directly into the thrust loading on the fan. Consequently the generic scaling rules to estimate the fan noise include parameters to account for increasing airflow. Typically there are scaling rules for fan speed and fan diameter to account for the increased thrust loading as the required airflow is increased. Additionally ducting and items attached to the fan shroud such as coolers and finger guards can change the airflow.

The industry available data can be used to begin the analysis of the fan loading. In order to size the fan correctly, fan manufacturers publish baseline data about the fan. The fan performance curves usually come at several key fan speeds and show the airflow created at varying pressure drops. Often they will include a sound pressure level (SPL) with this kind of data, but this SPL is from the clean inflow case.

The fan performance curves are used to match the airflow with the system restriction or pressure drop. With this set of data the effective fan speed can be determined. Along with this data is a loading curve that shows the motor torque required for a particular fan speed and pressure drop. These two pieces of data form the basis for determining the loading on the fan blades.

What is typically not discussed is how the inflow distortion in either the test rig at the manufacturer or the inflow distortion of the intended system design affects the fan performance. It should be noted that the fan performance curves are representative of a nearly ideal case. The application of these curves to a real world system must include considerations for the installation effects which degrade the fan performance. When the inflow distortions are large, the system designer often uses a system degradation factor to account for the change in airflow. Since this is easily determined by in situ testing,
the decision is made to omit detailed CFD from the design loop. As a result much of the
information required to build a detailed noise model, such as those developed by Farassat
for rotorcraft, is not available. An engineering level tool to determine installed fan
noise levels, with inputs based on available manufacturing data, would provide valuable
information to cooling system designers.

1.6 Using Analysis in Design

The aim of this research is to provide a framework for noise analysis, based on the
Ffowcs Williams and Hawkings equation, that does not require large scale CFD model-
ing for input conditions. The technique described in this paper will provide a method
for computing a sound enhancement factor due to the installation effects, and provides
a framework for a baseline noise analysis that can be used to drive the design of cooling
systems. This technique uses readily available data and simple test parameters to con-
struct a baseline noise model. This model, Simple Lifting Line Acoustic Code (SLLAC),
includes loading on the fan and a simple inflow model to calculate the change from
a baseline noise signature due to particular disturbances. The use of the FW-H equation
allows computation in the time domain so that the blade passing tonals, asymmetric flow
effects and broadband turbulence noise can be modeled.

This research provides guidelines regarding the minimum information required in a
prediction program. It outlines the simplest method to model flow distortions inherent
in the undisturbed installation and from objects in the upstream airflow. Together these
tools can be used to provide a good prediction of the noise from a fan in a commercial
installation.
Chapter 2

Background

2.1 Noise Mechanisms in Fans

There are several different noise mechanisms that are created as a fan blade moves through a medium. The blade thickness creates a pulse, as it passes a given point in space, when it displaces the fluid at the point. The forces acting on the blades create a different pulse as they pass through a given point in space and the viscous stresses in the medium caused by fluid shear in the volume surrounding a fan create another. Each of these noise mechanisms have different characteristics and are treated separately.

2.1.1 Thickness Noise

When a blade moves through air, the displacement of the air to make room for the blade creates a disturbance. This disturbance is characterized as a "mass injection" which classically is described as a monopole source. While the radiation pattern produced is similar to that of a stationary monopole, this is not always the case [12]. As the blade passes a given point, the air displacement resembles a pulsing sphere. This type of noise has a particular radiation pattern as an observer moves in a circle around the fan. Figure 2.1 shows the sound pressure level (SPL) in dB ref. 20\(\mu\)Pa as the observer moves through 360° around the fan with the airflow direction of 0°. In this document, the thrust direction is defined according to the conventions used in the industrial fan industry. The thrust direction is downstream of the fan as it is taken to be the thrust acting on the fluid rather than the loading applied to the fan blade. The observer sitting on the axis of rotation of the fan, the 0° – 180° line, will see no disturbance in the pressure. This
is because each blade is equidistant from the observer and so the only disturbance is a small change in the direction of the pulse origin. Since the fan is symmetric, this small disturbance cancels out with the other blades and is effectively zero.

Figure 2.1: Typical thickness noise SPL, dB ref. 20 $\mu$Pa

The time trace is determined by the thickness distribution of the blade element from the leading to the trailing edge. In practice, this disturbance is not a perfect sine wave, though it is dominated by the first blade pass frequency. In Figure 2.2, the time traces for different observer positions are shown. Each line represents a different observer position around the fan. The two pulses, over one full rotation, are indicative of a two blade fan. The monopole nature of the thickness noise is seen in the time trace since there is no phase change across the rotational plane of the fan.
2.1.2 Loading Noise

The forces acting on the blades of the fan produce a momentary application of a vector load. While the thickness of the blade creates an omnidirectional displacement of the fluid, the force vector creates a displacement of the fluid in the direction of the load. The fluctuating vector creates a dipole disturbance. Since the two forces acting on the blades, thrust and drag, act in different directions, they behave differently.

The thrust force comes from the lift of the fan blades and is characterized by the pressure differential across the fan. The negative pressure upstream of the fan sucks the fluid towards the fan while the positive pressure downstream ejects the fluid away from the fan. For pure axial flow, the thrust force is perpendicular to the plane of rotation. The dipole loading is therefore aligned with the axis of rotation. The noise generated by this thrust is therefore nonexistent on the plane of rotation. For an observer on the axis of rotation, like the monopole, only small changes in the direction of the loading are seen. This sets up a typical dipole radiation pattern having four lobes around the observer’s arc (Figure 2.3). Since the thrust is always in one direction, the pressure side of the fan sees a positive change over static pressure due to this force, while the suction side of the fan sees a negative change over static pressure. Its time trace (Figure 2.4) shows that
the phase changes from positive to negative at the plane of rotation.

![Figure 2.3: Typical thrust noise SPL, dB ref. 20 μPa](image)

The drag loading is always in the plane of rotation. In the undisturbed case, this leaves a zero pressure fluctuation for an observer on the axis of rotation. The dipole rotates with the fan, creating a maximum fluctuation on the plane of the fan face. The radiation pattern, therefore, does not experience phase cancellation at the plane of rotation (Figure 2.5). Similarly the time trace, Figure 2.6, appears like the thickness noise, but with a sharp transition from the maximum peak to the minimum peak.
Figure 2.4: Typical thrust noise time traces, in the far-field, for observer angles between 0° and 360° (see Figure 2.2 for legend)

Figure 2.5: Typical drag noise SPL, dB ref. 20 \( \mu \)Pa
Combining the three steady state noise mechanisms, the phase of the thrust noise plays a part in shaping the overall noise radiation pattern (Figure 2.7 and Figure 2.8).

In the case of purely undisturbed forces and blade thickness (which includes the boundary layer), it should be noted that on the axis of rotation there would be no noise. For this position, there is no change in the distance from observer to source with blade rotation. Because of this, and the symmetry of the system, there is no fluctuation in the pressure at the observer. In real applications, this is almost never true. Disturbances in the vicinity of the fan, inflow disturbances and in fact any pressure gradient will break the symmetry of the system, causing the noise to increase, not only on the axis of rotation, but throughout the whole radiation pattern. Even small defects can have a significant impact. It is widely held that the inflow disturbances are the dominant noise mechanisms for multiblade, subsonic axial flow fans [26]. By adding a single velocity defect in the loading of the blade, at a particular location along the span of the blade, the noise level on the axis goes up significantly. In fact, the entire noise profile shows an increase in noise, though not as large as the change on the axis. Figure 2.9 shows an increase of
Figure 2.7: Combined thickness and force noise SPL, dB ref. $20 \, \mu Pa$

Figure 2.8: Combined thickness and force noise time traces, in the far-field, for observer angles between $0^\circ$ and $360^\circ$ (see Figure 2.2 for legend)
40 dB, at an observer location of $\zeta = 180^\circ$, for a 1% force defect when compared to Figure 2.7.

Figure 2.9: Combined thickness and force noise with force defect SPL, dB ref. 20 $\mu$Pa

2.1.3 Broadband Noise

Much of the current work in fan noise is centering around broadband noise sources. Broadband noise can play a major part in the noise signature of a fan system. At moderate Mach numbers it is largely due to the fluctuating loads on the moving blades and stationary surfaces (stators) in the air flow [30]. There are many mechanisms for this and they can be difficult to separate in practice due to their broad spectral coverage.
Table 2.1: Broadband noise mechanisms

<table>
<thead>
<tr>
<th>Mechanism</th>
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</thead>
<tbody>
<tr>
<td>Interaction of blades with inflow turbulence</td>
</tr>
<tr>
<td>Trailing edge noise due to turbulent boundary</td>
</tr>
<tr>
<td>layers or flow separation</td>
</tr>
<tr>
<td>Vortex shedding noise *</td>
</tr>
<tr>
<td>Tip vortices</td>
</tr>
</tbody>
</table>

* either inbound to the blades or radiating off of downstream objects

Since broadband noise is caused by aperiodic fluctuations, it can be treated as a random process. Random fluctuations can be applied to the inflow of the fan, resulting in small random changes to the loading. Outlet effects from vortex shedding are found in the volume source functions.

Guedel proposed that the inflow turbulence produces significant broadband noise when the turbulence intensity is greater than 2% [16]. This is often the case in cooling fan installations and is certainly the case with an axial fan placed behind a cooler core.

Trailing edge noise is caused by the wake from the trailing edge of the blade. This increases as the displacement thickness of the boundary layer increases and becomes significant as the blade approaches stall or the boundary layer separates. In axial flow fan installations these conditions often occur as the pressure drop across the fan face becomes large.

Vortex shedding is also a major contributor to the installation noise. The vortex shedding noise can come in the form of fluctuating flow impinging on the fan blade caused by upstream objects, or as part of the volume source as is the case with objects in the downstream flow. Alić showed the effects of an outlet grill over an axial flow fan. He was able to show that the noise change included effects on the BPF, its multiples and a broadband component related to the vortex shedding from the grille. The quadrupole noise from the vortex shedding was in fact strong enough to mask the 3rd multiple of the BPF in certain situations [1].

Tip vortices are seen in most fan installations where the tips of the blades are open and either no shroud exists or the fit with the shroud allows for air flow to bend around the tip of the blade. They account for some efficiency loss as well as broadband noise since the turbulence from the tip can be recycled to the suction side of the fan.
2.2 History of Fan Noise Analysis

Much of the work on fan noise has been related to the development of aircraft. Early in aviation, there was a desire to understand the noise generated from a propeller. This evolved to extensive work by aircraft engine manufacturers such as Pratt and Whitney and within NASA. Though some of this work eventually found its way into the automotive industry, there was historically little effort placed into low noise cooling system design. The problem of fan noise in automobiles was largely out weighed by the market demand for low cost [34]. As emissions and performance requirements drove larger and larger cooling systems, the need for better performance from the cooling systems grew. Consumer products such as personal computers, appliances and HVAC systems have also experienced the need for quiet fan system design. Current research is focusing on more efficient heat transfer and quieter systems - in effect more airflow per dB.

2.2.1 Gutin

Gutin was the first to propose investigating propeller noise as a set of periodic forces applied to a stationary disc at the face of the propeller. His work is considered foundational in the area of loading noise in fans. The systems that Gutin investigated assumed a uniform loading around the path of the blades and symmetric spacing of the blade pattern [15]. Using the result from Lamb for the acoustic field from a stationary concentrated force, he was able to develop a source array of stationary oscillating forces on the propeller disc. This allowed him to show the relationship between the loading of the propeller blade and the blade passing frequency acoustic pressure levels [10].

In Gutin’s formulation, the forces and resulting solution are represented in the frequency domain. The Gutin model has survived the test of time and pure loading noise is now often referred to as the Gutin noise. Unlike simple propellers, modern fans and compressors consist of large numbers of blades, fixed vanes and significant unsteady loading, Under these conditions, Gutin’s model is not sufficient [15].

2.2.2 Lighthill

Lighthill investigated the relationship of aeroacoustic noise the flow field producing it, using conservation of energy to balance the kinetic energy of shearing motion and acoustic
energy of longitudinal motion. In essence, he used the physics of the fluctuating fluid flow to estimate the generated sound [22].

Lighthill proposed that though pressure fluctuations with the air flow are generally balanced by fluctuations in the fluid acceleration, it is not clear what proportion of their energy is radiated as sound. The whole pressure fluctuation may play its part if provided with a medium which it can excite. Those fluctuations requiring the interaction with solid boundaries are not, however, aerodynamic noise [22].

Lighthill was the first to expand the study of aeroacoustics beyond frequency correlation. His work tackled the general procedure for estimating the intensity of the sound produced in terms of the details of the fluid flow. The method used was to first understand the details of the flow by using aerodynamic principles not concerned with the acoustic propagation, then to use that information to deduce the sound field [22]. This had the limitation of excluding any back reaction of the sound field on the aerodynamics. All the evidence of experiment and theory is that the sound produced by a flow field is so weak relative to the motions producing it that without a resonator present there is not significant back reaction [22].

Lighthill’s theory essentially dealt only with sound radiating from subsonic flows into free space, ignoring not only resonators but all effects of reflection, diffraction, absorption or scattering by solid boundaries. In general, these effects could be sketched in subsequently.

Lighthill showed there were three different ways in which one can cause kinetic energy to be converted into acoustic energy. The first was by forcing the mass in a fixed region to fluctuate, the second was by forcing the momentum in a fixed region to fluctuate and the third was by forcing rates of momentum flux across a fixed volume to vary [22]. The three methods correspond to monopole, dipole and quadrupole sources respectively. Each method is less efficient than the preceding one. This effect is amplified as the wavelength of the sound is increased. Lighthill concluded that the quadrupole sources, given by the Lighthill Stress tensor, \( T_{ij} = \rho v_i v_j + p_{ij} - c_0^2 \rho \delta(ij) \), are the strictly aerodynamic contribution to the sound field. They are also dominated by the direct sound due to the monopole and dipole source distributions.
2.2.3 Tyler and Sofrin

Multi-bladed, low speed fans are essentially an inefficient radiator of sound when they are working in uniform flow [27]. Tyler and Sofrin investigated the effects of stationary blade rows in front of low speed rotors. They were able to show that the BPF of these systems is related to the number of stators as well as the number of blades [33]. Even in subsonic rotors it is possible to generate spinning modes which rotate at \( B/(B - sN) \) times the speed of the rotor, where \( s \) is any integer, \( B \) is the number of blades on the rotor and \( N \) is the number of vanes on the stator. Thus if \( N \) is close to \( B \), the spinning mode can actually rotate supersonically even when the rotor speed is low [33].

While Tyler and Sofrin used this information extensively to show propagation of various modes through ducts, their work has much broader implications. The application of these spinning modes in free-field radiation is similar in that the BPF shifts as does the associated radiation pattern. When the modes spin above the tip critical Mach number, the resulting high speed pattern becomes an efficient radiator [33].

2.2.4 Ffowcs Williams and Hawkings

In 1969, J. E. Ffowcs Williams and D. L Hawkings published what is considered a defining work on the theory of noise produced by sources in motion. They built on the work of Gutin and Lighthill and developed the FW-H equation which details the source terms of an inhomogeneous wave equation. Their work is an extension of Kirchhoff’s method using generalized function theory to describe the moving boundaries of the fan [11].

They observed that not only do the forces and thickness of propellers and fans produce noise, but a volume source related to the Lighthill stress tensor could also be a significant source. Previous to their work, the quadrupole source described in the stress tensor was dismissed as an inefficient radiator of noise. They show that the quadrupole strength can dominate the dipole strength particularly as Mach number increases, number of blades increases and as the harmonic of interest increases. Hence their conclusion was that the quadrupole noise was significant for higher frequency noise [15].

2.2.5 Farassat

Farassat built on the work of Ffowcs Williams and Hawkings as a basis for a comprehensive analysis of rotorcraft noise. Early on his work focused on the noise generated by
One of the common assumptions in acoustic radiation problems is the compactness of source. Farassat removed this restriction in his application of the FW-H equation to helicopter rotor noise [10]. The resulting analysis provides a solution to the acoustic problem that is not limited to the far field and is in fact valid for bodies in arbitrary motion over the entire space. This formulation utilized the first order terms in the FW-H equation representing the blade loading and thickness, neglecting the Lighthill stress tensor term.

Working with K. Brentner, Farassat extended this work to include supersonic tip speeds. The resulting analysis takes a detailed look at the Lighthill stress tensor and the information it contains. He used the Lighthill stress tensor to describe the jump across the shock surfaces among other information that is computed in the CFD codes [11].

Later he worked with J. Casper to describe a generalized turbulence model based on the fluctuating force term in the FW-H equation. Here he does not restrict the loading term to being constant. Instead the far-field surface integral depends on the time derivative and surface gradient of the pressure on the blade [5].

With the pieces in place to describe the noise radiated from aircraft rotors, Farassat has turned to the acoustic scattering problem [21]. Working again with Brentner and a team from Pennsylvania State University, they have developed two analytical models for the acoustic pressure gradient to enable routine acoustic scattering predictions. Both models build on the work of Formulation 1A to provide the acoustic pressure from the rotor. They extend the analysis derived from the gradient of the FW-H equation to develop a similar model for the acoustic pressure gradient [21].

2.2.6 Hanson

While Farassat was focusing on helicopter noise, Don Hanson was conducting similar research into propeller fans using frequency domain techniques [19]. Hanson recognized that for the traditional propeller model with few, long blades, Gutin’s method described the sound field well. In the case of a turbo fan with many, shorter blades, the theory began to break down. In addition, the turbo fan design was such that at high cruise velocities, the fan tip speeds were supersonic [20].

Hanson developed an extension of Gutin’s theory using Goldstein’s version of the acoustic analogy. He described the source terms much like Farassat, but computed the
retarded time problem and the subsequent result in the frequency domain [19]. His compressible lifting surface theory combined the acoustics problem with the aerodynamics and flutter problems in a single theory that was valid in near and far fields with unsteady source terms [20].

2.3 Time Domain vs Frequency Domain

One of the major advantages of the FW-H equation is the ability to evaluate the pressure signature in the time domain. Gutin’s work was completely focused on blade passing frequency. Tyler and Sofrin proposed that much of the broadband noise in ducted fans was related to rotor stator interactions and the resulting sidebanding [33].

Lighthill brought the focus onto the time domain. Farassat built on this work to allow the analysis of an actual time trace model, true broadband analysis and inflow distortion. In addition to simplifying the broadband and inflow distortion inputs, analysis in the time domain removes the far field restriction [13].
Chapter 3

Methodology Development

The analysis method developed here is based on the Ffowcs Williams and Hawkings (FW-H) equation and the methodology of Farassat’s Formulation 1A. This setup allows analysis in both the time domain, and through FFT, the frequency domain. The time domain solutions are useful for understanding both the blade passing frequencies (BPFs) and the broad band noise, while the frequency domain solutions deal primarily with the BPFs.

3.1 Physical System

The first step is understanding the physical system setup, including the relationship between the observer time and space, and the source time and space. The loading on the fan and how that translates into acoustic sources within the FW-H equation is then determined and the representation of the acoustic pressure is developed.

3.1.1 Coordinate Systems

Evaluation of the thickness and loading terms in the FW-H equation requires that the problem be set up in terms of two coordinate systems – one based on the observer space and time and the other based on the emission space and time. The transformation of the equations from the observer time to the source time is non-trivial. It adds a level of complexity to the calculus and is generally not available in a closed form.

Consider an object moving through space, the surface of which can be defined as $f(\vec{y}, \tau) = 0$, and an observer at some location $\vec{O}(\vec{x}, t)$. The coordinate system $(\vec{x}, t)$...
signifies the observer time and space variables while the coordinate system \((\vec{y}, \tau)\) signifies
the source time and space variables. Note that \(\nabla f = \hat{n}\) the unit normal outward from
the surface. The Green’s function of the wave equation in unbounded three dimensional
space is given in Eq. 3.1,

\[
G(\vec{x}, t; \vec{y}, \tau) = \begin{cases} 
0, & \tau > t \\
\delta(\tau - t + r/c)/(4\pi r), & \tau \leq t.
\end{cases} \tag{3.1}
\]

where \(c\) is the speed of sound. Note that the vector from the surface to the observer is
given by \(\vec{r} = |\vec{x} - \vec{y}|\) and the scalar \(r\), the magnitude of \(\vec{r}\), is the distance between the
surface and the observer.

\[\frac{\partial \vec{r}}{\partial x_i} = \hat{r}_i, \tag{3.2}\]

Figure 3.1: Surface normal and surface function \(f(\vec{y}, \tau)\)
Observer

Figure 3.2: Position vectors for observer in the \((\hat{x}_1, \hat{x}_3)\) plane

and

\[
\frac{\partial \vec{r}}{\partial y_i} = -\hat{r}_i,
\]

where \(\hat{r}_i\) are the set of unit vectors in the \(\vec{r}\) direction.

In the case of a stationary object, the relationship between the source time space and the observer time space is known. If \(\vec{r} = |\vec{x} - \vec{y}|\) is the distance from the object to the observer, the source time is \(\tau = t - r/c\). Since \(r\) is not a function of time, the relationship is simple. If the object is moving, then the relationship \(\partial \tau / \partial t\) must be determined.

The first step is to build the model according to the geometry given. Looking at the fan face from the downstream side, a coordinate system is set up to describe the fan position, observer position and forces involved. Figure 3.2 shows the \(\hat{x}_1, \hat{x}_2\) and \(\hat{x}_3\) directions respectively along with the observer position \(\vec{O}(\vec{x}, t)\). To be consistent with the fan described in Chapter 6, the fan position \(\theta\) is taken as zero on the \(\hat{x}_1\) axis and positive in the clockwise direction. The position of the observer relative to the source element on the blade is given by \(\vec{r}\). The \(\vec{r}\) vector can be broken down into its \(\vec{r}_1, \vec{r}_2\) and \(\vec{r}_3\) components, where \(\vec{r}_1, \vec{r}_2\) and \(\vec{r}_3\) are in the \(\hat{x}_1, \hat{x}_2\) and \(\hat{x}_3\) directions respectively. There is a unit vector \(\hat{r}\) in the same direction as \(\vec{r}\) with components \(\hat{r}_1, \hat{r}_2\) and \(\hat{r}_3\). As the fan rotates, the \(\vec{r}\) vector changes as seen in Figure 3.3.
For an observer in the \((\hat{x}_1, \hat{x}_3)\) plane, at a distance \(O\) from the center of the fan and at an angle \(\zeta\) from the \(\hat{x}_3\) axis (\(\zeta\) positive towards the \(\hat{x}_1\) axis), and an element of the fan blade at a distance \(R_{ele}\) along the span of a blade, the \(\vec{r}_1\), \(\vec{r}_2\) and \(\vec{r}_3\) vectors are given by:

\[
\vec{r}_1 = O \sin(\zeta) - R_{ele} \cos(\Theta) \hat{x}_1 \tag{3.4}
\]

\[
\vec{r}_2 = R_{ele} \sin(\Theta) \hat{x}_2 \tag{3.5}
\]

\[
\vec{r}_3 = O \cos(\zeta) \hat{x}_3 \tag{3.6}
\]

### 3.1.2 Forces Acting on the Fluid

The forces acting on the air (equal and opposite to the forces acting on the blade) are shown in Figure 3.4. The thrust load is always acting in the \(\hat{x}_3\) direction while the drag load direction, in the \((\hat{x}_1, \hat{x}_2)\) plane, changes as the blades rotate. The drag load vector acting the air at the blade element is given by Eq. 3.7
\[ \vec{d}_i = d_i[-\sin(\Theta)\hat{x}_1 - \cos(\Theta)\hat{x}_2], \quad (3.7) \]

where \( d_i \) is the magnitude of the drag from the blade element.

### 3.2 Solution Method for Derived Equations

Now with the two coordinate systems consisting of \((\vec{x}, t)\) system in the observer space and time and the \((\vec{y}, \tau)\) system in the emission space and time, one may now examine the various sources to be modeled. The Ffowcs Williams and Hawkings equation describes three general sources and is given in Eq. 3.8,

\[
\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\partial Q}{\partial t} - \frac{\partial F_i}{\partial x_i} + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j},
\]

where \( p' \) is the acoustic pressure. The monopole source strength, \( Q \) represents the mass injection caused by the thickness of the fan blade as it passes a particular location. The
dipole source strength, $F_i$, represents the force injection of the blade loading as the blade passes a particular location. The final term in the equation is a quadrupole term using the Lighthill stress tensor, $T_{ij}$, to represent the shear forces in the fluid. The sources are expressed in terms of a closed surface $f = 0$ that represents the surface of the fan blade. Note that the first two terms are related to the actual surface of the blade and the quadrupole term is a volume source outside of the surface. Also note that the effects of the mean flow on the acoustic pressure have been neglected as the FW-H equation is not based on the convected wave equation. Expanding Eq. 3.8 for the applied sources located on the surface $f = 0$, and using the wave operator notation, $\Box^2$, gives

$$\Box^2 p' = \frac{\partial}{\partial t} \left\{ [\rho u_n - (\rho - \rho_o)v_n] \delta(f) \right\} - \frac{\partial}{\partial x_i} \left\{ [\rho(u_n - v_n)u_i + (P - P_o)n_i] \delta(f) \right\} + \frac{\partial^2}{\partial x_i \partial x_j} [T_{ij}H(f)], \tag{3.9}$$

where $P - P_o$ is the gage pressure acting on $f = 0$ (the data surface), $v_n$ is the normal velocity of the data surface, $u_n$ is the velocity of the fluid normal to the data surface and $T_{ij} = \rho u_i u_j + P_{ij} - (\rho - \rho_o)c^2 \delta_{ij}$. The $\delta(f)$ function is the Dirac delta and physically means that the source is present only on the surface $f = 0$. The $H(f)$ term is the Heaviside function representing the volume term acting on the outside of $f = 0$ ($f$ positive) only. If the data surface is solid, the quantity $(u_n - v_n) \equiv 0$. Assuming an impenetrable surface, Eq. 3.9 becomes

$$\Box^2 p' = \frac{\partial}{\partial t} [\rho_0 v_n \delta(f)] - \nabla \cdot [P\vec{n}\delta(f)] + \frac{\partial^2}{\partial x_i \partial x_j} [T_{ij}H(f)], \tag{3.10}$$

where the first term represents the monopole noise of the blade thickness, the second term represents the dipole noise of the forces on the blades and the third term is the quadrupole noise related to the fluid shear. This work will focus on the thickness and force terms. The information contained in the volume source will be approximated with additions to the loading term related to the turbulence impinging on the fan blade.

**Thickness Noise Solution**

Looking at the first term of the FW-H equation, one can see that $\Box^2 p' = \frac{\partial}{\partial t} [Q \delta(f)]$ has a Green’s Function solution of
\[ 4\pi p'(\vec{x}, t) = \frac{\partial}{\partial t} \int \frac{Q(\vec{y}, \tau)}{r} \delta(g) \delta(f) d\vec{y} d\tau, \tag{3.11} \]

where \( Q \) again is the monopole source strength and \( \delta(g) \) again is the Dirac delta function this time with argument \( g \). The retarded time function \( g \) is defined from Eq. 3.1 as

\[ g = \tau - t + r/c. \tag{3.12} \]

The retarded time function is used as a means of describing the relationship between the observer time and the source time as will be described later.

A frame \( \eta \) is then attached to the blade such that \( \vec{y} \) is represented in terms of the \( \vec{\eta} \) frame as \( \vec{y} = \vec{y}(\vec{\eta}, \tau) \) and \( d\vec{y} = d\vec{\eta} \). Since the blade surface is defined by \( f(\vec{y}, \tau) = 0 \), in the \( \vec{\eta} \) frame the surface is given by \( f(\vec{y}(\vec{\eta}, \tau), \tau) = 0 \). Expanding Eq. 3.11 to show the dependencies of each of the variables gives

\[ 4\pi p'(\vec{x}, t) = \frac{\partial}{\partial t} \int \frac{Q(\vec{y}(\vec{\eta}, \tau), \tau)}{r(\vec{\eta}, \tau; \vec{x})} \delta(g) \delta(f) d\vec{\eta} d\tau. \tag{3.13} \]

Using the retarded time function, Eq. 3.12, the expression for \( \partial g / \partial \tau \) is given by

\[ \frac{\partial g}{\partial \tau} = 1 + \frac{1}{c} \frac{\partial \vec{r} \cdot \vec{v}_i}{\partial \tau} = 1 - \frac{\hat{r}_i v_i}{c} = 1 - M_r, \tag{3.14} \]

where \( r = |\vec{x} - \vec{y}(\vec{\eta}, \tau)| = r(\vec{\eta}, \tau; \vec{x}) \) and \( M_r \) is the portion of the Mach number, associated with the velocity of the surface, in the direction of the observer. The retarded time equation (Eq. 3.12) shifts from source time to observer time when \( g = 0 \). Evaluating the integral for \( d\tau \) with \( g = 0 \) gives

\[ 4\pi p'(\vec{x}, t) = \frac{\partial}{\partial t} \int \left[ \frac{Q(\vec{y}(\vec{\eta}, \tau), \tau)}{r(1 - M_r)} \right]_{g=0} \delta(f) d\vec{\eta}. \tag{3.15} \]

Evaluating Eq. 3.15 over the space of \( \vec{\eta} \) with the influence of the \( \delta(f) \) reduces the problem to an integral over the surface \( f = 0 \),

\[ 4\pi p'(\vec{x}, t) = \frac{\partial}{\partial t} \int_{f=0} \left[ \frac{Q(\vec{y}(\vec{\eta}, \tau), \tau)}{r(1 - M_r)} \right]_{g=0} dS. \tag{3.16} \]

The source time for a given observer time and blade position is defined as \( \tau^* = \tau|_{g=0} \). Recognizing that for \( M < 1 \), there is only one \( \tau^* \), the \( \partial / \partial t \) can be moved inside the
integral with

\[
\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau^*} \frac{\partial \tau^*}{\partial t}
\]  

(3.17)

such that Eq. 3.16 becomes

\[
4\pi p'(\vec{x}, t) = \int_{f=0} \frac{\partial}{\partial \tau^*} \left[ \frac{Q}{r|1 - M_r|} \right] \frac{\partial \tau^*}{\partial t} dS.
\]  

(3.18)

The retarded time from Eq. 3.12 defines a relationship between \(\tau\) and \(t\) from which \(\partial \tau^* / \partial t\) can be determined. Taking \(\partial / \partial t\) of Eq. 3.12 where \(g = 0\) gives

\[
\frac{\partial \tau^*}{\partial t} - 1 + \frac{\partial}{\partial t} \left[ \frac{r}{c} \right] = 0.
\]  

(3.19)

Applying the chain rule to Eq. 3.19 yields

\[
\frac{\partial}{\partial t} \left[ \frac{r}{c} \right] = -\frac{1}{c} \frac{\partial r}{\partial y_i} \frac{\partial y_i}{\partial \tau}.
\]  

(3.20)

Recalling that \(\vec{r} = |\vec{x} - \vec{y}(\eta, \tau^*)|\),

\[
\frac{\partial r}{\partial y_i} \frac{\partial y_i}{\partial \tau} = -\hat{r} \cdot \vec{v} \equiv -v_r
\]  

(3.21)

where \(v_r\) is the component of the surface velocity in the \(\hat{r}\) direction. It follows that for \(\tau = \tau^*\),

\[
\frac{\partial \tau^*}{\partial t} = \left[ \frac{1}{1 - M_r} \right]_{\tau^*}.
\]  

(3.22)

As \(Q(\vec{y}(\eta, \tau))\) is fixed to the \(\eta\) frame, the derivative of \(Q\) with respect to \(\tau\) is

\[
\frac{\partial}{\partial \tau} Q \equiv \dot{Q},
\]  

(3.23)

where \(\dot{Q}\) is the rate of change of \(Q\) as measured by an observer on \(f = 0\). Rewriting Eq. 3.18,

\[
4\pi p'(\vec{x}, t) = \int_{f=0} \left\{ \frac{\dot{Q}}{r(1 - M_r)} + Q \frac{\partial}{\partial \tau^*} \left[ \frac{1}{r(1 - M_r)} \right] \right\} \frac{\partial \tau^*}{\partial t} dS.
\]  

(3.24)
Let $\xi = r(1 - M_r)$ and Eq. 3.24 becomes

$$4\pi p'(\vec{x}, t) = \int_{f=0} \left\{ \frac{\dot{Q}}{r(1 - M_r)} + Q \frac{\partial}{\partial \xi} \left[ \frac{1}{\xi} \right] \frac{\partial \xi}{\partial \tau^*} \right\} \frac{\partial \tau^*}{\partial t} dS. \quad (3.25)$$

Looking at the partial derivatives,

$$\frac{\partial \xi}{\partial \tau^*} = \frac{\partial r(1 - M_r)}{\partial \tau^*} = -v_r(1 - M_r) - r \frac{\partial M_r}{\partial \tau^*} \quad (3.26)$$

with

$$\frac{\partial M_r}{\partial \tau^*} = \frac{\partial}{\partial \tau^*}[\hat{r}_i M_i] = \frac{1}{c} \left[ \hat{r}_i \frac{\partial v_i}{\partial \tau^*} + \frac{\partial \hat{r}_i}{\partial \tau^*} + v_i \frac{\partial \hat{r}_i}{\partial \tau^*} \right], \quad (3.27)$$

and

$$\frac{\partial \hat{r}_i}{\partial \tau^*} = \frac{\partial}{\partial \tau^*} \frac{r_i v_r}{r} = \frac{1}{r} \left[ r_i \frac{\partial v_r}{\partial \tau^*} - v_i \right]. \quad (3.28)$$

Substituting Eq. 3.28 into Eq. 3.27,

$$\frac{\partial M_r}{\partial \tau^*} = \frac{1}{c} \left[ \hat{r}_i \frac{\partial v_i}{\partial \tau^*} + \frac{\partial \hat{r}_i}{\partial \tau^*} + v_i \frac{\partial \hat{r}_i}{\partial \tau^*} \right] = \hat{r}_i \dot{M}_i + \frac{c(M_r^2 - M^2)}{r}. \quad (3.29)$$

Substituting Eq. 3.29 into Eq. 3.26,

$$\frac{\partial \xi}{\partial \tau^*} = -v_r(1 - M_r) - r \left[ \hat{r}_i \dot{M}_i + \frac{c(M_r^2 - M^2)}{r} \right], \quad (3.30)$$

$$= -r \dot{M}_r - cM_r + cM^2. \quad (3.31)$$

Substituting Eq. 3.22 for $\frac{\partial \tau^*}{\partial \tau}$ and Eq. 3.31 for $\frac{\partial \xi}{\partial \tau^*}$ back into Eq. 3.25 the acoustic pressure radiating due to a thickness source $Q = \rho_0 \nu_n$ is given by

$$4\pi p'(\vec{x}, t) = \int_{f=0} \left[ \frac{\dot{Q}}{r(1 - M_r)^2} \right] dS + \int_{f=0} \left[ \frac{Q(r \dot{M}_r + cM_r - cM^2)}{r^2(1 - M_r)^3} \right] dS. \quad (3.32)$$

**Loading Solution**

Similar to the Thickness Noise Solution, the loading source term given by $\Box^2 p' = -\nabla \cdot [P \vec{n} \delta(f)]$ has a Green’s Function solution of
\[ 4\pi p'(\vec{x}, t) = -\nabla_x \cdot \int \frac{P(\tau)}{r} \delta(g) \delta(f) d\tau, \] (3.33)

where \( \nabla_x \) represents the divergence in the observer time and space, \( f \) is again the surface function and \( g \) is again the retarded time. Taking the divergence gives

\[ 4\pi p'(\vec{x}, t) = -\int \frac{\delta(g)}{r} \nabla \cdot \left[ P(\tau) \right] d\tau - \int \frac{\delta(g)}{r} \nabla \cdot \left[ \frac{\delta(g)}{r} \right] d\tau. \] (3.34)

As \( P(\tau) \) is a function of \( \tau \) alone, the divergence of \( P(\tau) \equiv 0 \). The divergence of the second term of Eq. 3.34 is expanded as,

\[ \nabla \cdot \left( \frac{\delta(g)}{r} \right) = \frac{1}{r} \nabla \delta(g) + \delta(g) \nabla \frac{1}{r}. \] (3.35)

Looking at \( \nabla \delta(g) \),

\[ \nabla \delta(g) = \nabla g \delta'(g) \] (3.36)

where

\[ \nabla g = \frac{\partial g}{\partial x_i} = \frac{\dot{r}_i}{c}, \] (3.37)

and

\[ \delta'(g) = -\frac{\partial}{\partial t} \delta(g). \] (3.38)

Expanding \( \nabla \frac{1}{r} \) gives

\[ \nabla \frac{1}{r} = -\frac{1}{r^2} \nabla r = -\frac{\dot{r}_i}{r^2}. \] (3.39)

Substituting Eq. 3.39 into Eq. 3.35 gives

\[ -\nabla \left( \frac{\delta(g)}{r} \right) = \frac{1}{c} \frac{\partial}{\partial t} \left[ \frac{\dot{r}_i \delta(g)}{r} \right] + \left[ \frac{\dot{r}_i \delta(g)}{r^2} \right]. \] (3.40)

Substituting back into Eq. 3.34 gives

\[ 4\pi p'(\vec{x}, t) = \int \frac{\partial}{\partial t} \left[ \frac{P(\tau)\dot{r}_i \delta(g)}{rc} \right] d\tau + \int \left[ \frac{P(\tau)\dot{r}_i \delta(g)}{r^2} \right] d\tau. \] (3.41)

Again the retarded time equation gives
\[ \partial \tau = \frac{\partial g}{(1 - M_r)}, \quad (3.42) \]

and the integral over \( \tau \) in Eq. 3.41 becomes an integral over the surface such that

\[ 4\pi p'(\vec{x}, t) = \int \frac{\partial}{\partial t} \left[ \frac{P(\tau) \hat{r}_i}{rc(1 - M_r)} \right]_\tau \, dS + \int \left[ \frac{P(\tau) \hat{r}_i}{r^2(1 - M_r)} \right]_\tau \, dS. \quad (3.43) \]

As before, \( \tau^* \) is defined such that \( g = 0 \) and

\[ 4\pi p'(\vec{x}, t) = \int \frac{\partial}{\partial t} \left[ \frac{P(\tau) \hat{r}_i}{rc(1 - M_r)} \right]_{\tau^*} \, dS + \int \left[ \frac{P(\tau) \hat{r}_i}{r^2(1 - M_r)} \right]_{\tau^*} \, dS. \quad (3.44) \]

Recall the following partial derivatives:

\[ \frac{\partial \tau^*}{\partial t} = \frac{1}{(1 - M_r)} \quad (3.45) \]

\[ \frac{\partial P(\tau^*)}{\partial \tau^*} = \dot{P}(\tau^*) \quad (3.46) \]

\[ \frac{\partial \hat{r}_i}{\partial \tau^*} = \hat{r}_iv_i - v_i \quad (3.47) \]

Taking the time derivative in the first term of Eq. 3.44,

\[ \frac{\partial}{\partial t} \left[ \frac{P(\tau) \hat{r}_i}{rc(1 - M_r)} \right]_{\tau^*} = \left[ \frac{\hat{r}_i}{rc(1 - M_r)} \right] \frac{\partial P(\tau^*)}{\partial \tau^*} \frac{\partial \tau^*}{\partial t} + \left[ \frac{P(\tau)}{rc(1 - M_r)} \right] \frac{\partial \hat{r}_i}{\partial \tau^*} \frac{\partial \tau^*}{\partial t} \]

\[ - \left[ \frac{P(\tau) \hat{r}_i}{r^2c(1 - M_r)} \right] \frac{\partial r}{\partial \tau^*} \frac{\partial \tau^*}{\partial t} - \left[ \frac{P(\tau) \hat{r}_i}{rc(1 - M_r)^2} \right] \frac{(1 - M_r)}{\partial \tau^*} \frac{\partial \tau^*}{\partial t}. \quad (3.50) \]

Evaluating Eq. 3.50 term by term,
\[
\frac{\partial}{\partial t} \left[ \frac{P(\tau)\dot{r}_i}{rc(1 - M_r)} \right]_{\tau^*} = \left[ \frac{\dot{P}_r}{rc(1 - M_r)^2} \right]_{\tau^*} + \left[ \frac{P_r M_r}{r^2(1 - M_r)^2} \right]_{\tau^*} - \left[ \frac{P_i M_i}{r^2(1 - M_r)^2} \right]_{\tau^*} + \left[ \frac{P_r M_r}{r^2(1 - M_r)^3} \right]_{\tau^*} + \left[ \frac{P_r (M_r^2 - M^2)}{r^2(1 - M_r)^3} \right]_{\tau^*}. \] (3.51)

Substituting Eq. 3.51 back into Eq. 3.44 gives,

\[
4\pi p'(\vec{x}, t) = \int_{f = 0} \left[ \frac{\dot{P}_r}{rc(1 - M_r)^2} \right]_{\tau^*} dS + \int_{f = 0} \left[ \frac{P_r M_r}{r^2(1 - M_r)^2} \right]_{\tau^*} dS - \int_{f = 0} \left[ \frac{P_i M_i}{r^2(1 - M_r)^2} \right]_{\tau^*} dS + \int_{f = 0} \left[ \frac{P_r M_r}{r^2(1 - M_r)^3} \right]_{\tau^*} dS + \int_{f = 0} \left[ \frac{P_r M_r}{r^2(1 - M_r)^3} \right]_{\tau^*} dS. \] (3.52)

Simplifying Eq. 3.52 yields the acoustic pressure radiating due to a loading source \( P\delta(\vec{x}) \) as

\[
4\pi p'(\vec{x}, t) = \int_{f = 0} \left[ \frac{\dot{P}_r}{rc(1 - M_r)^2} \right]_{\tau^*} dS + \int_{f = 0} \left[ \frac{P_r}{r^2(1 - M_r)^2} \right]_{\tau^*} dS + \int_{f = 0} \left[ \frac{P_r (r\dot{M}_r + cM_r - cM^2)}{r^2c(1 - M_r)^3} \right]_{\tau^*} dS. \] (3.53)

Many investigators would make a far-field simplification by ignoring the higher order terms in these equations. In reality, while they are small for a large observer radius, they cost little in terms of computation to carry along. When in close proximity to the fan face, these terms will make small differences and so are included in this model.

### 3.3 Application of the Solution

The thickness solution in Eq. 3.32 and the loading solution in Eq. 3.53 can now be applied to a physical system. The boundary layer surrounding the blade must be included in the thickness term. The pressure term must be broken into its two components, with
the drag load in the plane of rotation and the thrust load perpendicular to the plane of rotation. The sources are applied in a spanwise distribution along the length of the blade.

### 3.3.1 Lifting Line Loading

Both the thickness solution and the loading solution contain surface integrals over the blade. Looking at a section of the blade at some radius, there exists a single point to which the pressure on that segment can be resolved. This point is on the lifting line of the blade. For this analysis, both the pressure and the resolved thickness at a particular section act on the lifting line. Integrating the thickness source $\rho_0 \nu_n$ over the surface gives the thickness of the blade plus the displacement thickness of the boundary layer. Integrating the pressure over the surface of the blade segment gives the forces acting on that segment in terms of drag and thrust. There is also pressure acting on the end of the blade tip. If the blade tips are small or are of small cross section, this pressure can often be neglected.

### 3.3.2 Phased Sum of Time Domain Acoustic Pressure

The sum of the thickness solution and the loading solution yields the acoustic pressure at a particular point in space in the observer time. The sources acting in the surface integrals reside in the source time however. The transformation from observer time to source time is not a closed form problem in general and must be solved numerically.

#### Table 3.1: Blade position in observer time and source time

<table>
<thead>
<tr>
<th>Blade Position</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observer time</td>
<td>$R_i \Omega t$</td>
</tr>
<tr>
<td>Emission time</td>
<td>$R_i \Omega (t - r/c)$</td>
</tr>
</tbody>
</table>

The $r$ dependency on the position of the blade in observer time is $r = (O - R_i \cos(R_i \Omega(t - r/c)))i + \sin(R_i \Omega(t - r/c))j$. Here, $r$ is a transcendental function for which one must iterate to find $r$ for any given $t$. Looking at the solution of this problem across the span
of a blade, one can see that the position of the fan at the source time is not the same for each element. It in fact creates a swept surface from which the various $r_{\text{vectors}}$ must be calculated. Figure 3.5 shows how for each element of a blade, the source time and therefore fan position is different at a fixed observer time.

![Figure 3.5: Fan rotation for elements along the span of a single blade at a single observer time](image)

Once the relationship between the source time and the observer time is known, the acoustic pressures can be summed in the observer time to obtain a solution for the actual acoustic pressure wave. It is important to maintain the phases of the time domain solutions when summing them to the total noise produced by the fan. It is precisely the phase shift due to the changes in the radius from the source to the observer and the time shift from source time to observer time that creates the noise signature. If these effects were neglected, then the time domain solution would yield a complete cancellation of noise when there is more than one evenly spaced blade.
When the time domain signatures from the various segments on each blade are properly summed, the total fan noise signature in the time domain can be seen. The time domain solution is advantageous as it can be compared directly to measured data. It can also be processed in an FFT method to produce the frequency content in the signal. Since this solution lends itself to clearly showing multiples of the blade passing frequency, tonal noise can be fully understood with a single revolution of the fan. When evaluating broadband noise, the solutions can be averaged over several revolutions to create a broadband level.
Chapter 4

Analysis Inputs

The Ffowcs Williams and Hawkings equation allows one to use basic geometry and forces as inputs, but the formulation of the inputs must be established. As mentioned previously, most of the predictive work in this field has been done with large, high tip speed systems such as rotorcraft, aircraft engines and industrial fans. In these installations, when the tip speed is subsonic, the loading noise tends to dominate. This assumption does not hold for smaller fans and for fans that have a significant effect from the installation. Often the installation effects are far more dominant than the basic loading case. This process is intended to allow the user to model those installation effects as thickness and loading terms in the FW-H equation.

Fan manufacturers typically will provide some analysis of the noise generated under controlled laboratory conditions. While the procedure may vary from manufacturer to manufacturer, the data that they collect can be used as a baseline to understand what is called the Free Inlet - Free Outlet (FIFO) noise.

4.1 Physical Data Input

The first step is to outline the basic operational parameters, shown in Table 4.1. Most of this information is used as direct inputs to the model. The highlighted terms need to be formulated to represent the blade elements that are being evaluated.
Table 4.1: Operational data required for setting up the model

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan Speed</td>
<td>RPM</td>
</tr>
<tr>
<td>Number of Blades</td>
<td>B</td>
</tr>
<tr>
<td>Density of Air</td>
<td>ρ</td>
</tr>
<tr>
<td>Propeller Radius</td>
<td>R</td>
</tr>
<tr>
<td>Observer Position</td>
<td>O</td>
</tr>
<tr>
<td><strong>Thrust Distribution</strong></td>
<td>Thr</td>
</tr>
<tr>
<td><strong>Drag Distribution</strong></td>
<td>Tq</td>
</tr>
<tr>
<td>Blade Thickness</td>
<td>Q</td>
</tr>
<tr>
<td><strong>Inflow Distortion</strong></td>
<td>vdefmat</td>
</tr>
</tbody>
</table>

4.2 Undisturbed Loading Input

The first terms presented are the force terms. Rather than working from a total thrust and a total motor torque as Gutin did, these loads are formulated as a loading distribution along the span of the blade. When more refined CFD data is available, these loads can be applied in a grid across the blade that represents both spanwise and chordwise distribution.

4.2.1 Thrust Load

The thrust load is calculated from the total static pressure and the velocity over the segment. The total static pressure is the total system restriction that the fan sees. It is determined by the whole air flow path. Using fan curves like those shown in Figure 4.1, the operating point can be determined using the fan speed and pressure drop. This operating point gives the total mass flow through the fan.

The total flow rate allows us to compute the mean velocity across the fan face as

$$Q_{tot} = \pi U_{mean}(R_{tip}^2 - R_{hub}^2).$$  \hspace{1cm} (4.1)

Using this mean velocity, a velocity distribution across the fan blades is developed, such
that the mean of the distribution is equal to the calculated mean. The shape of this
velocity profile will depend on the system being analyzed. The effects of a shroud around
the blade tips, chord angle, and sweep can affect the profile.

Once the velocity distribution is established, the thrust load on the blade can be
calculated. The thrust at each element is given by Eq. 4.2 where \( Arc_i \) is the total area
of the arc swept by the segment and \( B \) is the number of blades.

\[
Thr_i = \left( \frac{\rho}{2} V_i^2 + \Delta P_{\text{static}} \right) \frac{Arc_i}{B}.
\]  

(4.2)

### 4.2.2 Torque Load

Similar to the thrust loading, a distribution for the drag load must be determined. The
fan curves give the total motor torque required to spin the fan at the operating point.
The total torque is allocated to each blade by dividing by the number of blades. The
drag profile is assumed to be proportional to the blade element speed squared and is
scaled to match the torque per blade. The drag on a segment is given in Eq. 4.3, where
\( C \) is a scaling constant. The torque per blade \( \Gamma_{\text{blade}} \) is given in Eq. 4.4.
\[ d_i = C(\Omega R_i)^2 \]  \hspace{1cm} (4.3)

\[ \Gamma_{\text{blade}} = \Sigma R_i d_i \]  \hspace{1cm} (4.4)

### 4.2.3 Thickness

In order to calculate the thickness terms, the problem is split into two parts. First the actual blade thickness is determined at the trailing edge of the fan. The trailing edge is used because the maximum boundary layer is assumed to be near the trailing edge of the blade for a curved plate type of blade. If the blade is airfoil shaped, this assumption may not hold.

The more interesting piece of information is the displacement thickness of the boundary layer itself. There are boundary layers on both sides of the fan blade, one in an adverse pressure gradient (holding it closer to the blade) and one in a favorable pressure gradient. Similar to the loading case, we need to look at the particular geometry of the fan to evaluate the boundary layer. Once the displacement thickness of the boundary layer is determined, it needs to be applied in conjunction with the actual blade thickness in the thickness terms of the formulation. Since the blade itself can not vary in shape substantially, during operation, any variation to the thickness term due to inflow velocity changes is applied to the displacement thickness portion only.

### 4.3 Flow Variations

One of the major effects installations can have are inflow variations. These variations include both inflow turbulence and pressure gradients across a fan. Both of these mechanisms can raise the blade pass tonal level and, in some cases, other multiples of the blade pass frequency. The inflow turbulence can also have a large effect on the broadband noise.

#### 4.3.1 Inflow Distortion

An array of velocity defects is created to model the inflow distortion. In this example, the resulting noise radiation pattern is compared against test data. Adjustments to the
shapes of the inflow distortions can be made to match the levels with the test case, to create a baseline, free inlet - free outlet (FIFO), comparison.

Non-homogeneous inflow can act in exactly the same way that an upstream stator would to induce tonal noise. If the variation in the inflow is axially symmetric we would expect the tones to be shifted from the BPF in the manner described by Tyler and Sofrin[33]. If it is not axially symmetric, then only multiples of the BPF will be seen.

Lift along the fan blade is a function of the outflow velocity squared. Changes in the inflow velocity are assumed to have a direct effect on the outflow velocity and therefore will change this lift force. Since the lift of the blade is perpendicular to the apparent velocity, it has an effect in both the fan thrust and drag loading as seen in Figure 4.2. The thrust load from the fan is therefore changed by $V_{gust}^2 \cos(\phi_0)$ and the drag load changes by $V_{gust}^2 \sin(\phi_0)$.

![Diagram of fan blade and forces](image_url)

Figure 4.2: Thrust and drag forces acting on the air due to lift
4.3.2 Swirl

Another component to the inflow distortion is the swirl of the inflow caused by the momentum change at the rotating blades. In the case where there is no inflow distortion and no velocity defects, the swirl is irrelevant since the whole system remains axially symmetric. When there are changes to this symmetry, either through inflow distortion or specific velocity defects, the swirl changes the location of these defects relative to the emission point at the fan.

4.3.3 Objects in the Flow

Stationary defects such as upstream and downstream objects or stationary ingested turbulence will act as a single stator, producing a pressure defect in a localized area. This raises the tonal noise of the blade pass frequency and its multiples. As multiple defects are placed in the flow, their location as well as the size of the defect influence which BPF multiples are affected. If their placement is axially symmetric, they can change the spinning modes and excite other frequencies[33].

4.3.4 Turbulence

Inflow turbulence is a randomizing factor that increases the broadband noise. The FW-H equation can deal with turbulence explicitly as the fluid shear in the Lighthill Stress Tensor term. In this example, the turbulence is approximated as a randomization of the loading on the blade. While this randomization would also change the boundary layer thickness of the blades, the perturbations are small and the thickness is not a dominant source. Carolus and Stremel analyzed the random pressure fluctuations on a low pressure, low speed fan using small pressure transducers on the fan blades. They found that the turbulence intensity varied from around 1.5% when there is nothing placed in front of the fan to as much as 19% when a thin coarse grid is placed upstream[4]. In the test example presented in Chapter 6, a 1.5% turbulence intensity translates to a total force defect between 0.3% and 0.8% depending on the blade segment.
4.4 Analysis Code

The analysis code was written using Matlab. The code detailed in Appendix A is a comprehensive analytical model including the turbulence model and cataloging of several of the analysis parameters. For a practical application, the code can be streamlined for the type of analysis required. For example if a turbulence model is not required, the computation time can be reduced significantly since the pressure calculation only needs to be done for a single blade and then phased across the whole fan.

The data developed in the previous section is used as the input deck for the model. The analytical model consists of a simple flow model that describes the distortions to the operational parameters of the fan. The simple flow model includes rough approximations for the inflow disturbances, swirl and velocity defects not related to turbulence.

In order to solve for the acoustic pressure, the relationship between the observer time and the source time must be determined. This is referred to as the retarded time problem. This is in general a problem with no closed form solution. A simple iterative scheme is used to determine the \( \tau^* \) value for a given observer position and each element position in the observer time. This computation only needs to be done once and stored for use later in the code.

Whether the turbulence model is being used as a uniform inflow turbulence or applied to specific locations due to measurements or velocity defects, the turbulence model must be applied to each blade segment individually. This increases the complexity of the calculation since, for example, blade element 20 on blade 1 does not see the same loading as blade element 20 on blade 2. The application of the turbulence defects must therefore be done within the main calculation loop over each blade individually. This increases the run time of the code significantly.

With the basic inputs, retarded time calculation and turbulence model complete, the equations derived in Chapter 3 are used to give the acoustic pressure in the observer time, associated with each element of the fan as it rotates through one revolution. This array of pressure data can be constructed into the actual time trace of the acoustic pressure for each element. The time traces are then summed to produce the total acoustic pressure.

Once the acoustic pressure time traces have been calculated they must be transformed into the data required for analysis. It is interesting to note that the sum of each of the elements actually preserves a constant pressure shift which must be removed to look at the acoustic pressure fluctuation. This constant pressure shift is not directly a function...
of the static pressure created by the fan but is merely an artifact of the calculation. Once the mean pressure of the time trace has been set to zero, standard signal processing tools are used to evaluate the pressure trace.
Chapter 5

Classical Examples

5.1 Gutin’s Method Applied to a 2 Blade Propeller

The loading part of the computational model can be compared with Gutin’s theory. Gutin proposed the following equation for the loading noise from a rotating system [17],

\[ p' = \frac{m\Omega}{2\pi cr} \left[ -T\cos(\theta) + \frac{nc\Gamma}{\Omega r^2} \right] J_{mn}(kr\sin(\theta)). \] (5.1)

where \( p' \) is the acoustic pressure level, \( m \) is the harmonic of interest, \( n \) is the number of blades, \( \Omega \) is the fan speed in radian per second, \( T \) is the total thrust, \( \Gamma \) is the motor torque, \( R \) is the fan radius, \( r \) is the radius to the observer, \( \theta \) is the angle of the observer to the axis of rotation, \( k \) is the wave number \( n\omega/c \) and \( J_{mn} \) is the Bessel function of the first kind of order \( m \ast n \).

This equation provides sound pressure levels at the blade passing frequency and its multiples for a given set of blade loads. Since the FW-H equation based model works in the time domain, the FFT of the resultant pressure trace may be used to compare with results from Gutin’s theory. Further, since this form of Gutin’s result does not take into account variations in the loading around the fan face, only the basic loading noise case with no inflow distortion can be compared.

A simple code was developed to calculate the Gutin loading noise for a test case (Appendix B). As a first comparison, inputs similar to Gutin’s are examined. Instead of concentrating the loading at the 75% span position as Gutin did, the loading here is distributed over the outer 55% of the blades.
Applying the condition represented in Table 5.1 to Gutin’s model, the resulting radiation pattern for the first BPF is shown in Figure 5.1. The FW-H based model, again with no inflow distortion, produces similar results (Figure 5.2).

Gutin’s theory makes an assumption regarding the compactness of the fan relative to the observer position. For this reason, when an observer is located in the far field, observer position to fan diameter ratio of 11 as in the example above, the results are in good agreement. When the observer moves closer to the fan face, the results begin to diverge. For an observer radius of 15m and the same loading as in the previous example, the area of low noise near 50 degrees in the radiation pattern persists in the Gutin model case shown in Figure 5.3. This is due to zeros in the Bessel functions. The same area shrinks in the FW-H model shown in Figure 5.4. The near field effects are well handled by the FW-H based model as there is no assumption of compactness.
Figure 5.1: Loading noise SPL, dB ref. 20 $\mu$Pa at 50 m - Gutm model

Figure 5.2: Loading noise SPL, dB ref. 20 $\mu$Pa at 50 m - FW-H model
Figure 5.3: Loading noise SPL, dB ref. 20 $\mu$Pa at 15 m - Gutin model

Figure 5.4: Loading noise SPL, dB ref. 20 $\mu$Pa at 15 m - FW-H model
5.2 Gutin’s Method Applied to the Borg Warner LD7 Test Model

As noted earlier, the inflow distortion becomes the dominant factor in the noise generation for small, low tip speed, lightly loaded fans. In Chapter 6, noise from a Borg Warner LD7 fan will be investigated in detail. The blade loading used in Chapter 6 is presented here using the Gutin and FW-H methods in order to compare the distortion free loading. The basic model inputs, for the LD7 fan, are shown in Table 5.2

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Blades</td>
<td>7</td>
</tr>
<tr>
<td>Speed</td>
<td>2490 RPM</td>
</tr>
<tr>
<td>Propeller Diameter</td>
<td>0.394 m</td>
</tr>
<tr>
<td>Operator Position</td>
<td>2.5 m</td>
</tr>
<tr>
<td>Thrust</td>
<td>29.6 N *</td>
</tr>
<tr>
<td>Drag</td>
<td>4.75 Nm *</td>
</tr>
<tr>
<td>Inflow Distortion</td>
<td>none</td>
</tr>
</tbody>
</table>

* $R^2$ distribution over the full blade

In this example the observer position to fan diameter ratio is approximately 6. It is expected that there are near field effects that the FW-H based model will capture but are not in the Gutin model. This can be seen in Figure 5.5 and Figure 5.6, where the FW-H based model predicts slightly higher sound levels.
Figure 5.5: Loading noise SPL, dB ref. 20 \( \mu \text{Pa} \) at 2.5m for the LD7 at 2490 RPM - Gutin’s model

Figure 5.6: Loading noise SPL, dB ref. 20 \( \mu \text{Pa} \) at 2.5m for the LD7 at 2490 RPM - FW-H model
Chapter 6

Low Speed, High Distortion Case

6.1 Baseline Flow Model Inputs

In the experimental portion of this work, noise measurements on a commercial cooling fan are made in an anechoic chamber. The fan used is a Borg-Warner LD-7 commercial cooling fan that has been trimmed down to 396mm diameter. The fan is mounted to the test chamber with a 3 kW motor through a typical cooling fan knife-edge shroud. In this installation there are flow distortions that need to be accounted for in the FIFO model. Since the shroud is square, flow distortions develop near the corners as the fan rotates. This is caused by the close proximity of the sides of the square shroud at the edges compared to the larger area of the shroud around the corners. The drive motor installation causes significant distortions (Figure 6.1). There is some inflow turbulence within the flow of the test rig which can be seen with a tuft in the inflow air stream. The Reynolds Number across the blade is approximately $4 \times 10^5$ and shows that the flow around the blade is in the turbulent regime. The boundary layer around the blade is therefore expected to contribute significantly to the aerodynamic thickness of the blade. Finally as the shroud does not cover the blade tip completely, the turbulence streaming from the blade tip is significant.

6.1.1 Thrust Load

In the experimental setup, the operating point of the fan is at 2490 RPM with a pressure drop across the fan of 67 Pa. At this operating point, the fan is producing a flow rate of 1.8 cubic meters per second. Equation 4.1 indicates the mean velocity across the fan
is 18.1 meters per second. From this data, an approximation of the velocity distribution across the fan blades is determined. The Borg-Warner LD7 fan is a swept blade, mixed chord angle fan. The tips have some forward sweep to them so that the chord at the tip is larger than the chord at the root. This, combined with the change in chord angle from root to tip, tends to concentrate the outflow velocity towards the tips. For the test case, a relationship proportional to $R_i^2$ has been chosen for the outflow velocity profile. Equation 4.2 is used to calculate the thrust loading. The velocity profile and thrust are shown in Table 6.1. The thrust loads provided have been calculated using the method outlined in Section 4.2.1. In the velocity column the last segment has been scaled back by 50% to account for the tip losses.

### 6.1.2 Drag Load

The total torque load is estimated using the fan curves provided by the manufacturer (Figure 4.1). For the test case, the total torque required to spin the fan at 2490 RPM is 4.75 Nm. The drag profile is assumed to be proportional to $(R_i\Omega)^2$ and is scaled to
Table 6.1: Calculated spanwise outflow velocity and thrust

<table>
<thead>
<tr>
<th>Blade Position</th>
<th>% radius</th>
<th>Outflow Velocity (m/s)</th>
<th>Thrust (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.025</td>
<td>0.00</td>
<td>0.003</td>
</tr>
<tr>
<td>2</td>
<td>0.075</td>
<td>0.00</td>
<td>0.009</td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
<td>0.00</td>
<td>0.015</td>
</tr>
<tr>
<td>4</td>
<td>0.175</td>
<td>0.00</td>
<td>0.020</td>
</tr>
<tr>
<td>5</td>
<td>0.225</td>
<td>0.00</td>
<td>0.026</td>
</tr>
<tr>
<td>6</td>
<td>0.275</td>
<td>0.00</td>
<td>0.032</td>
</tr>
<tr>
<td>7</td>
<td>0.325</td>
<td>0.00</td>
<td>0.038</td>
</tr>
<tr>
<td>8</td>
<td>0.375</td>
<td>0.00</td>
<td>0.044</td>
</tr>
<tr>
<td>9</td>
<td>0.425</td>
<td>0.00</td>
<td>0.050</td>
</tr>
<tr>
<td>10</td>
<td>0.475</td>
<td>7.74</td>
<td>0.084</td>
</tr>
<tr>
<td>11</td>
<td>0.525</td>
<td>9.45</td>
<td>0.106</td>
</tr>
<tr>
<td>12</td>
<td>0.575</td>
<td>11.34</td>
<td>0.137</td>
</tr>
<tr>
<td>13</td>
<td>0.625</td>
<td>13.40</td>
<td>0.180</td>
</tr>
<tr>
<td>14</td>
<td>0.675</td>
<td>15.63</td>
<td>0.237</td>
</tr>
<tr>
<td>15</td>
<td>0.725</td>
<td>18.03</td>
<td>0.314</td>
</tr>
<tr>
<td>16</td>
<td>0.775</td>
<td>20.60</td>
<td>0.414</td>
</tr>
<tr>
<td>17</td>
<td>0.825</td>
<td>23.35</td>
<td>0.543</td>
</tr>
<tr>
<td>18</td>
<td>0.875</td>
<td>26.26</td>
<td>0.706</td>
</tr>
<tr>
<td>19</td>
<td>0.925</td>
<td>29.35</td>
<td>0.911</td>
</tr>
<tr>
<td>20</td>
<td>0.975</td>
<td>16.30</td>
<td>0.352</td>
</tr>
</tbody>
</table>

match the torque per blade. The drag distribution is given in Table 6.2.

6.1.3 Thickness

The effective blade thickness is estimated from the actual blade thickness plus an estimate of the boundary layer displacement thickness. The fan blades have essentially parallel surfaces along the majority of the chord. The blades are therefore assumed to behave similarly to a curved plate in the airflow. Starting with a simple model for a flat plate and then modifying it to compensate for the curvature, the displacement thickness of the boundary layer is determined.

Since the Reynolds number is approximately \(4 \times 10^5\), and there is some modest level of turbulent fluctuation in the inflow, the boundary layer is assumed to be turbulent. The displacement thickness of the boundary layer for a flat plate can be computed using Schlichting’s formula[6],

\[
\delta^* = \frac{1}{1 + n} \frac{0.375x}{Re^{1/5}}.
\]  

(6.1)

where \(n\) is assumed to be 7 for a constant pressure boundary layer. Since the blades are curved, the boundary layer growth will be larger than the flat plate case. Another
Table 6.2: Calculated spanwise drag

<table>
<thead>
<tr>
<th>Blade Position</th>
<th>Element Velocity (m/s)</th>
<th>Radius (m)</th>
<th>Drag (N)</th>
<th>Torque (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.28</td>
<td>0.0049</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>3.85</td>
<td>0.0148</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>6.42</td>
<td>0.0246</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>8.98</td>
<td>0.0344</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>11.55</td>
<td>0.0443</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>14.12</td>
<td>0.0541</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>16.68</td>
<td>0.0640</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>19.25</td>
<td>0.0738</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>21.81</td>
<td>0.0837</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>24.38</td>
<td>0.0935</td>
<td>0.162</td>
<td>0.015</td>
</tr>
<tr>
<td>11</td>
<td>26.95</td>
<td>0.1033</td>
<td>0.198</td>
<td>0.020</td>
</tr>
<tr>
<td>12</td>
<td>29.51</td>
<td>0.1132</td>
<td>0.238</td>
<td>0.027</td>
</tr>
<tr>
<td>13</td>
<td>32.08</td>
<td>0.1230</td>
<td>0.281</td>
<td>0.035</td>
</tr>
<tr>
<td>14</td>
<td>34.65</td>
<td>0.1329</td>
<td>0.328</td>
<td>0.044</td>
</tr>
<tr>
<td>15</td>
<td>37.21</td>
<td>0.1427</td>
<td>0.378</td>
<td>0.054</td>
</tr>
<tr>
<td>16</td>
<td>39.78</td>
<td>0.1526</td>
<td>0.432</td>
<td>0.066</td>
</tr>
<tr>
<td>17</td>
<td>42.35</td>
<td>0.1624</td>
<td>0.490</td>
<td>0.080</td>
</tr>
<tr>
<td>18</td>
<td>44.91</td>
<td>0.1722</td>
<td>0.551</td>
<td>0.095</td>
</tr>
<tr>
<td>19</td>
<td>47.48</td>
<td>0.1821</td>
<td>0.616</td>
<td>0.112</td>
</tr>
<tr>
<td>20</td>
<td>50.05</td>
<td>0.1919</td>
<td>0.684</td>
<td>0.131</td>
</tr>
</tbody>
</table>

The approach is to look at the momentum thickness of the boundary layer $\theta$ using the simple power-law assumptions for the velocity distribution. Cebeci showed that Eq. 6.2 holds for this constant pressure boundary layer [6],

$$\frac{\theta}{x} = 0.036 \frac{1}{Re_x^{1/5}}. \tag{6.2}$$

The boundary layer is assumed to be separated at the end of the blade. Since the shape function $H$ in Eq. 6.3 approaches 1.8 when the boundary layer separates, the momentum thickness at the trailing edge can be related to the displacement thickness there as

$$H = \frac{\delta^*}{\theta} = 1.8. \tag{6.3}$$

From this, $\delta^*$ is calculated as

$$\delta^* = 1.8x \frac{0.036}{Re_x^{1/5}}. \tag{6.4}$$

The assumption of the separated boundary layer and using the power laws gives a 40% increase in the displacement thickness over Schlichting’s flat plate. Since the fan blade experiences flow over both sides, the total displacement thickness is the sum of the
thicknesses on each side plus the thickness of the blade.

The displacement thickness from Eq. 6.4 (shown as $\Delta^*$ in Table 6.3) is used for the boundary layer on each side of the blade. The boundary layer is the only portion of the thickness that is subject to changes with changing air flow. Since the boundary layer calculation is dominated by the blade rotational speed, one can assume that the boundary layer changes in a similar fashion to the drag loading. The total boundary layer thickness that is subject to velocity changes is shown as $Q$ and the fixed blade thickness is assumed to be a constant 2mm.

<table>
<thead>
<tr>
<th>Blade Position</th>
<th>Calc Velocity (m/s)</th>
<th>Width (m)</th>
<th>Re</th>
<th>$\Delta^*$ (m)</th>
<th>$Q$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.28</td>
<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>3.85</td>
<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>6.42</td>
<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>8.98</td>
<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>11.55</td>
<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>6</td>
<td>14.12</td>
<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>7</td>
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<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>8</td>
<td>19.25</td>
<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>9</td>
<td>21.81</td>
<td>0.000</td>
<td>0.00E+00</td>
<td>0.0000</td>
<td>0.0000</td>
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<td>1.40E+05</td>
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</tr>
<tr>
<td>17</td>
<td>48.36</td>
<td>0.111</td>
<td>3.57E+05</td>
<td>0.0006</td>
<td>0.0011</td>
</tr>
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<td>0.124</td>
<td>4.31E+05</td>
<td>0.0006</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

The estimates of the total boundary layer displacement thickness using the separated flow model lead only to approximately 1mm of effective additional thickness to the blade. All of the blade loading and thickness terms are estimated to be acting along the lifting line of the blade. With a higher fidelity (typically CFD) solution, these loads can be distributed chordwise as well as spanwise along the blade.
6.1.4 Inflow Distortion

The installation of the fan in the test rig creates some nonuniform inflow disturbances which must be determined. As stated previously, the mounting struts, drive belt and fan mount position shown in Figure 6.1, all contribute to these inflow disturbances. Areas of nonuniform flow were identified using a 65mm diameter vane anemometer. The velocities measured are an average over the area of the face of the anemometer. The inlet flow data was taken on a 9x9 grid approximately 2 inches upstream from the fan face. The flow was measured only in the axial direction and is used to illustrate the areas of high and low velocity (Table 6.4). The areas labeled in yellow and cross hatched green indicate regions where the flow measurement fluctuated significantly. The green cells (including the cross hatched ones) indicate data taken on the horizontal and vertical axes of the fan face. The inlet data was represented in a contour plot in order to see more clearly the areas of higher and lower flow (Figure 6.2). The circle in Figure 6.2 represents the fan face coverage.

A flow trending matrix was developed using the inlet velocities measured along the horizontal and vertical axes of the fan (shown in green in Table 6.4). This matrix was

<table>
<thead>
<tr>
<th>Grid Location</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
<th>8</th>
<th>9</th>
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</thead>
<tbody>
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<td>4.2</td>
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</tr>
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<td>4.2</td>
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<td>6.2</td>
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<td>5.9</td>
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</tr>
<tr>
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<td>6.1</td>
<td>7</td>
<td>7</td>
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<td>7.1</td>
<td>5.7</td>
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</tr>
<tr>
<td>4</td>
<td>5.9</td>
<td>7.2</td>
<td>7.3</td>
<td>6.4</td>
<td>5.6</td>
<td>6.1</td>
<td>6.8</td>
<td>6.6</td>
<td>5.15</td>
</tr>
<tr>
<td>5</td>
<td>6.2</td>
<td>7.3</td>
<td>7.5</td>
<td>5.8</td>
<td>4.6</td>
<td>5.7</td>
<td>6.7</td>
<td>6.2</td>
<td>6.25</td>
</tr>
<tr>
<td>6</td>
<td>5.3</td>
<td>7.1</td>
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<td>6.8</td>
<td>6.1</td>
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<td>7.6</td>
<td>7.75</td>
<td>7.3</td>
<td>5.1</td>
<td>4.1</td>
</tr>
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<td>8</td>
<td>3.2</td>
<td>4</td>
<td>5.45</td>
<td>6.8</td>
<td>5.45</td>
<td>6.55</td>
<td>5.7</td>
<td>3.8</td>
<td>2.7</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>2.7</td>
<td>3.3</td>
<td>4</td>
<td>4.2</td>
<td>3.7</td>
<td>2.75</td>
<td>2.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>
then used as the starting point for describing the whole velocity defect scheme. Figure 6.3 shows the trending matrix applied over 20 spanwise segments, turning through 360°. The x-axis is the blade position and the y-axis is a defect factor that represents a change from the mean velocity \((U_{gust} - U_{mean})/U_{mean}\). Similar plot formats will be used throughout to describe the inflow velocity distortions. In addition to the trend from axis to axis, there are 4 smaller individual peak distortions. Individual velocity defects were added, in these locations, to create the whole velocity defect matrix. The addition of these defects is shown in Figure 6.4. Finally the velocity defect matrix has to be scaled back to keep the total inlet velocity consistent with the measured total fan flow. This is done by adjusting the mean velocity defect, over one full rotation, on each spanwise element back to zero (Figure 6.5). The details of how the distortion profile was chosen are provided in section 6.3.
Figure 6.3: Base inlet velocity defect model for various blade elements

Figure 6.4: Inlet velocity defect model with smaller distortions for various blade elements (see Figure 6.3 for legend)
The velocity defects translate mainly into changes in the thrust loading. The change in inflow velocity will change the lift generated by the blade. Since the actual thrust load acts perpendicular to the apparent inflow velocity, a small component of the drag will change as well. The effects are estimated as $dL \sin \phi_0$ and $dL \cos \phi_0$, for the drag load and thrust load respectively, where $\phi_0$ is the angle of the total apparent inflow velocity (Figure 4.2). The change in load acts proportionally to $V_{inlet}^2$, where $V_{inlet}$ is the total apparent velocity impinging on the blade. It includes both the velocity due to blade rotation and the velocity due to mass flow. To calculate the change in loading, the velocity defects were accounted for in the total apparent velocity at each element and squared. This total was divided by the square of the mean inlet velocity to give a normalized change in load,

$$F_{gust} = F_0 \frac{V_{gust}^2}{V_{mean}^2}. \tag{6.5}$$

From this information, the loading defect scheme is created. This change in loading, that stays relatively stationary throughout the test, creates the fixed disturbances in loading that generate the BPF and its multiples.
6.1.5 Turbulence

Two different types of turbulence are examined for use in the velocity defect model. The first is a generalized homogeneous inflow turbulence. The second is a turbulence model where specific areas of the fan face are identified as having a stationary perturbation.

A small amount of homogeneous inflow turbulence is applied across the whole blade. The last element of the blade is assigned a higher turbulence intensity level to account for the tip turbulence and associated losses. This uniform inflow turbulence can be described directly by a random component to the forces on the blade. The locations where increased velocity fluctuation occurred, (Table 6.4), represent a 10% turbulence intensity. This fluctuation in flow can be modeled as a random change in the inflow component and results in a change in loading similar to the inflow disturbances described earlier, but with a random variation of the magnitude. In both cases the turbulence is represented by a normalized random function, that has been scaled by the turbulence intensity, to create the change in loading. This loading defect is then applied to each blade element.

6.2 Test Measurements

The test rig, used for measuring the baseline SPL, consisted of placing the fan in the wall of an anechoic settling chamber with a long anechoic duct. The fan was mounted to a 3kW motor and positioned in a knife edge shroud in the wall of the settling chamber. The chamber was constructed of wood with 50mm, open cell, urethane foam mounted on both sides of the fan wall and the remaining inside walls. The anechoic duct was mounted in the opposite wall of the chamber and was lined with 50mm foam. The setup was designed to keep the noise from the upstream side of the fan out of the test cell. Details of the test cell are described in Appendix F.

An array of 17 measurement points were established, in an anechoic chamber, on the downstream side of the fan. The microphones, shown in Figure 6.6, were placed at 10 degree intervals on either side of the axis of rotation at the height of the center of the fan. In the positive $\zeta$ direction, the microphones were placed 2.5 meters from the fan face. For the measurements in the negative $\zeta$ direction, the size of the chamber limited the observer radius to 2 meters from the fan face. A schematic of the layout is shown in Figure 6.7. Due to reflections from the edge of the settling chamber, the measurements at $\zeta > 50^\circ$ and $\zeta < -50^\circ$ are somewhat higher than expected.
Figure 6.6: Microphones used for measurements

Figure 6.7: Location of microphones relative to the text fixture
The noise at each position was measured on a Bruel and Kjaer Pulse system, taking 8 Hz bandwidth noise levels. The data was processed to pull out the BPFs, the overall sound pressure level and a broadband index consisting of the noise between 1250 Hz and 2900 Hz. The noise levels at 2m were scaled to match the data from 2.5 meters to put together a complete radiation pattern, from $-80^\circ$ to $80^\circ$, on the downstream side of the fan.

With the exception of the data presented on broadband noise, the radiation patterns plotted are the SPL of the first BPF only. The frequency plots confirm that the noise level is dominated by the first BPF and it is therefore used for the comparisons of the noise model to the measured data. Figure 6.8 shows a typical spectrum where the primary BPF is shown at 290 Hz. The multiples of the BPF are significantly lower than the primary. The broadband noise in this spectrum remains below the multiples of the BPF through 2900 Hz. In section 6.6, full spectrum plots from 100 to 2900 Hz are presented at two observer locations. These plots are used to examine the higher orders of the BPF and the broadband noise.

![Figure 6.8: Sample frequency plot showing dominance of the first BPF](image-url)
6.3 Noise Model Comparison - Free Inlet - Free Outlet (FIFO) Case

In Chapter 5, the Gutin noise example for the LD7 fan shows the noise expected due purely to the loading on the fan in the absence of inflow distortions (Figure 5.6). The noise from the undisturbed blade loading case is clearly much lower than the measured baseline. When the thickness terms are included, the overall noise increases a nominal amount, but still does not represent the measured noise levels. Figure 6.9 shows the pressure level of the primary BPF for the LD7 analytical model with thickness terms and no inflow distortion. Figure 6.10 shows the same data plotted with the measured data for the primary BPF. It should be noted that in this undisturbed case, the analytical model predicts only a single BPF with no multiples.

Table 6.5: Model inputs for basic loading case

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency, Basic Loading</th>
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</thead>
<tbody>
<tr>
<td>Thrust Load</td>
<td>$R^2$†</td>
</tr>
<tr>
<td>Drag Load</td>
<td>$R^2$</td>
</tr>
<tr>
<td>Thickness</td>
<td>$2\delta^*$</td>
</tr>
<tr>
<td>Inflow Distortion</td>
<td>none</td>
</tr>
</tbody>
</table>

† Last blade element 50% $R^2$

6.3.1 Inflow Distortion Models

To bring the analytical model in agreement with the measured data, the inflow velocity distortions must be considered. The velocity measurements, plotted in Figure 6.2 show a basic gradient from axis to axis with 4 areas of higher flow between the axes. When measured with the anemometer, the first 3 are roughly equivalent to a 20% peak increase in flow. Approximating the distortions as ramp functions, a 20% peak flow increase over the width of these distortions is applied. The last distortion shows a peak velocity increase of a slightly lower level, but over a wider arc. To account for this a 15% peak flow distortion over the larger arc is applied. This inflow distortion is thus modeled as a saw-tooth type pattern, shown in Figure 6.11. When this inlet distortion is used directly
Figure 6.9: SPL at primary BPF, dB ref. 20 \( \mu \)Pa

Figure 6.10: SPL at primary BPF, dB ref. 20 \( \mu \)Pa with measured data
in the model, the total sound pressure levels do not yet agree with the measured data (Figure 6.12).

Table 6.6: Inputs for the measured velocity defect model

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
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<tbody>
<tr>
<td>Loading</td>
<td>Basic Loading</td>
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<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
</tr>
<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
</tr>
<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>20% Peak Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>15% Peak Ramp **</td>
</tr>
</tbody>
</table>

* 41 degree span
** 82 degree span

Figure 6.11: Inlet distortion pattern for the measured velocity defect model for various blade elements (see Figure 6.3 for legend)
Figure 6.12: SPL at primary BPF, dB ref. 20 $\mu$Pa, with the measured velocity defect model

To improve the comparison with the measured data, the effective width of the distortions can be changed while scaling the magnitude to keep the velocity defect constant. To achieve this, the widths of the distortion peaks have been reduced by 38% and the peak levels scaled accordingly. This keeps the average velocity change in agreement with the measurements. Figure 6.13 and Figure 6.14 show the improved velocity defect model inlet distortion and the corresponding acoustic radiation pattern.
Table 6.7: Inputs for the improved velocity defect model

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
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<tbody>
<tr>
<td>Loading</td>
<td>Basic Loading</td>
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<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
</tr>
<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
</tr>
<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Peak Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Peak Ramp **</td>
</tr>
</tbody>
</table>

* 26 degree span
** 52 degree span

Figure 6.13: Inlet distortion pattern for the improved velocity defect model for various blade elements (see Figure 6.3 for legend)

These results show that the noise levels are very much dominated by the inflow distortion patterns. Appendix D shows more details on how the changes in the shape of
the distortions affect the overall levels and various multiples of the blade pass frequency.

Another factor involved in this comparison relates to the swirl of the inlet velocity. A flow distortion measured on the vertical axis for example is actually felt some arc length in the direction of the fan rotation away. So even though a distortion could be measured along the vertical axis for example, it is actually generated somewhat further back in the fan rotation. Though in reality this distortion would no longer be even radial, due to the close proximity of the measurements and objects placed upstream, they are approximated as radial in nature. The amount of rotation of this distortion will depend on the angle of inflow to the fan face. A tuft was used to approximate the inlet swirl angle at 25°. The resulting inlet distortion pattern and SPL are shown in Figure 6.16 and Figure 6.17 respectively.
Figure 6.15: Tuft used to estimate the inflow swirl angle

Table 6.8: Inputs for the improved velocity defect model with 25° swirl

<table>
<thead>
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<th>Input Parameters</th>
<th>Input Dependency</th>
</tr>
</thead>
<tbody>
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<td>Basic Loading</td>
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<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
</tr>
<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
</tr>
<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Peak Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Peak Ramp **</td>
</tr>
<tr>
<td>Swirl</td>
<td>25°</td>
</tr>
</tbody>
</table>

* 26 degree span
** 52 degree span
Figure 6.16: Inlet distortion pattern for the improved velocity defect model with \(25^\circ\) swirl for various blade elements (see Figure 6.3 for legend)

Figure 6.17: SPL at primary BPF, dB ref. \(20 \mu\)Pa, with the improved velocity defect model and \(25^\circ\) swirl
6.3.2 Free Inlet - Free Outlet (FIFO) Results

As illustrated above, when changes to the inflow distortions are taken into account, the analytical model results can vary dramatically. Not only does the magnitude of the radiation pattern change, its shape may change as well. For the LD7 analytical model, the apparent velocity angle $\phi_0$ was computed from the mean inflow velocity and the blade element speed. The apparent velocity angle varied from 36° at the hub to 20° near the tip. The swirl was measured at 25°. The velocity defect model was taken as a linear gradient between the axial positions with 4 individual defects superimposed. The 4 individual defects were modeled as ramp functions, three of which spanning 26° and the fourth spanning 52°. The magnitudes of the ramps were scaled to match the average measured velocities.

The measured SPL and the analytical model results for the first BPF are shown in Figure 6.17. The analytical model results compare well with the measured data over the majority of the radiation pattern. The measured data for $\zeta > 50^\circ$ and $\zeta < -50^\circ$ show some discrepancy with the model data due to reflections of noise inherent in the test setup geometry. As the microphones move closer to the plane of rotation of the fan, the measured data shows higher levels than expected, diverging from the analytical model. In the range of $-50^\circ \leq \zeta \leq 50^\circ$, the first BPF shows an average deviation from the measured data -1.1 dB. The average error over the center 70° is -0.2 dB. The comparison on the second BPF shows an average drop of 5.1 dB over the primary BPF. The analytical model showed an average error over $-50^\circ \leq \zeta \leq 50^\circ$ of -2.6 dB.

The main factor in the velocity defect model is the position and magnitude of the individual defects. Small effects are seen with changes in the swirl angle and the apparent velocity angle. Further discussion of the effects of the apparent velocity angle on the thrust and drag can be found in Appendix C, further discussion on the inlet distortion shape can be found in Appendix D and further discussion about the effect of the inlet swirl can be found in Appendix E.

6.4 Effect of a Flat Plate Obstruction

The baseline inflow distortion is an installation effect by itself. It is the result of the essential parts of the system that make the fan run. A second kind of installation effect is one that is induced by placing an object in the flow upstream of the fan. Such flow
disturbances need to be carefully investigated in a manner similar to the baseline inflow
distortion discussion in the previous section. In the following sections, three different
approaches to modeling the velocity defect are presented.

6.4.1 Simple Plate Velocity Defect Model

In order to evaluate the effect of an object upstream, a representation of the resulting
change in the flow across the fan blade must be developed. The test rig was set up with
a 30mm wide plate placed 25mm in front of the fan face, and extending from the center
of the fan hub radially outward past the blade tip. Figure 6.18 shows a photo of the
upstream side of the fan face with the blade superimposed for this fan along the \( -\hat{x}_2 \)
axis.

![Fan face with blade superimposed at -\( \hat{x}_2 \) axis](image)

The noise signature is driven mainly from the loading near the tips, due both to the
higher speed and the increased loading. This can be advantageous when analyzing the
velocity defects from objects placed upstream. For this test case, a very simple model
for the velocity defects was created.

If the contraction towards the hub can be ignored, the simplest model applies the defects along the entire radial position that is covered by the blade tip. For the 30mm plate, this is a shadow between blade position 31 and blade position 39. Applying the simple plate velocity defect model in Table 6.9 to this area gives a nominal increase to the sound pressure level over the FIFO baseline. The resulting inlet distortion pattern and SPL are shown in Figure 6.19 and Figure 6.20. It is clear that this model does not predict the total change in sound level.

Table 6.9: Inputs for the simple plate velocity defect model

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
</tr>
</thead>
<tbody>
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<td>Loading</td>
<td>Basic Loading</td>
</tr>
<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
</tr>
<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
</tr>
<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Ramp **</td>
</tr>
<tr>
<td>Swirl</td>
<td>25°</td>
</tr>
<tr>
<td>Plate Model</td>
<td>Parabolic over 23 deg arc</td>
</tr>
<tr>
<td>Peak Plate Velocity Defect</td>
<td>70% FIFO velocity</td>
</tr>
<tr>
<td>Edge Effect</td>
<td>none</td>
</tr>
</tbody>
</table>

* 26 degree span
** 52 degree span

6.4.2 Simple Plate Velocity Defect Model with Edge Effects

When an object is placed upstream of the fan, there is a reduction in the total flow through the fan due to the increased resistance of the flow path. The majority of the flow however goes around the object and is ultimately moved through the fan. In the second plate velocity defect model, the flow is diverted around the object. The aim is to balance the flow defect with the additional flow around the edges. Table 6.10 gives the inlet velocity distortion model inputs, with the distortion and analytical results presented
Figure 6.19: Inlet distortion pattern for the simple plate velocity defect model for various blade elements (see Figure 6.3 for legend)

Figure 6.20: SPL at primary BPF, dB ref. 20 $\mu$Pa, with the simple plate velocity defect model
in Figure 6.21 and Figure 6.22. The addition of the edge effects bring the analytical model in close agreement with the measured SPL.

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading</td>
<td>Basic Loading</td>
</tr>
<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
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<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
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<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Ramp **</td>
</tr>
<tr>
<td>Swirl</td>
<td>25°</td>
</tr>
<tr>
<td>Plate Model</td>
<td>Parabolic over 23 deg arc</td>
</tr>
<tr>
<td>Peak Plate Velocity Defect</td>
<td>70% FIFO velocity</td>
</tr>
<tr>
<td>Edge Effect</td>
<td>50% of defect flow</td>
</tr>
</tbody>
</table>

* 26 degree span
** 52 degree span

### 6.4.3 Contracting Flow Model

Both of the previous models were chosen for their simplicity. The fact is that the flow into the fan face is somewhat contracted radially (in addition to the swirl mentioned in previous sections). There is a stagnation point near the center of the fan face, where there is zero velocity. Moving radially outward from this point, the fluid velocity has a strong radial component. The fluid near the stagnation point moves past the root of the fan blades as it crosses to the fan face. Similarly, fluid from outside the fan face is pulled radially inward. This contraction near the fan face changes the effective shape of the wakes encountered by the fan blades.

If one investigates the shape of the wake of this plate (before applying the swirl) it can be seen that, towards the tip, the wake covers between blade locations 31 and 39, but closer towards the center of the fan, the wake expands. If the plate was very thin, this contraction would create little distortion to the wake. The spreading of the wake is
Figure 6.21: Inlet distortion pattern for simple plate defect model with edge effects for various blade elements (see Figure 6.3 for legend)

Figure 6.22: SPL at primary BPF, dB ref. 20 μPa, with the plate velocity defect model with edge effects
significant, however, for a 30mm wide plate. Figure 6.23 shows the shadow created by the plate with this radial flow contraction taken into account.

![Figure 6.23: Inlet distortion pattern for 30mm plate with full contracting flow model](image)

Using this more complex model for the shadow of the plate with the same 70% velocity defect, the velocity defects shown in Figure 6.24 get more complex than those from the simple plate velocity defect model. Figure 6.25, however, indicates little change in the analytical model results.
Table 6.11: Inputs for the contracting flow model with edge effects

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
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<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
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<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Ramp **</td>
</tr>
<tr>
<td>Swirl</td>
<td>25°</td>
</tr>
<tr>
<td>Plate Model</td>
<td>Contracting flow model</td>
</tr>
<tr>
<td>Peak Plate Velocity Defect</td>
<td>70% FIFO velocity</td>
</tr>
<tr>
<td>Edge Effect</td>
<td>50% of defect flow</td>
</tr>
</tbody>
</table>

* 26 degree span
** 52 degree span

6.4.4 Plate Model Results

The velocity defect distortions have been kept simple in this example because the objective is to develop a noise prediction scheme using only simple flow models. Though the actual shape of these defects can be examined closer with CFD, the main result for the primary BPF is in good agreement with the simple models. The main issue is to ensure that the defect flows are balanced with increased flow around the edges of the defect.

The resulting level for the first BPF, shown in Figure 6.25, again agrees with the measured data over the majority of the radiation pattern. In the range of $-50^\circ \leq \zeta \leq 50^\circ$ the first BPF shows an average deviation from the measured data of -1.1 dB. The average error over the center 70$^\circ$ is -0.6 dB. The second BPF did not compare as well. The analytical model shows an average error over $-50^\circ \leq \zeta \leq 50^\circ$ deg. of -6.8 dB. Since the second BPF is an average of 5 dB below the primary BPF, the effect of this error on the overall sound pressure level (OASPL) is a reduction of approximately 1 dB. Further refinements in the shape of the distortion caused by the plate velocity defect would provide better comparison.
Figure 6.24: Inlet distortion pattern for the contracting flow model with edge effects for various blade elements (see Figure 6.3 for legend)

Figure 6.25: SPL at primary BPF, dB ref. 20 μPa, with the contracting flow model with edge effects
6.5 Turbulence Models

Up to this point, the velocity defect models have been used to predict the primary BPF. Random fluctuations in the blade loading, due to ingested turbulence, tip turbulence and trailing edge turbulence, create broadband noise. The velocity defect models, in the previous sections, have had no random flow to generate the broadband noise. As mentioned in Chapter 2, the turbulence terms are mainly found in the Lighthill Stress Tensor for the FW-H equation. Since the information necessary to construct this tensor is not available without detailed CFD analysis, a generic random process model is presented. A broadband factor was constructed from the sum of the acoustic pressure levels between the 4th and 10th BPF (1250 and 2900 Hz). This broadband factor can be used to evaluate the broadband noise predictions in a way similar to the primary BPF in previous sections. A radiation pattern for the broadband factor has been established from the measured data and is compared to the analytical model results.

6.5.1 Uniform Inflow Turbulence

The velocity defect models from the previous cases are used to investigate a random fluctuation in the inflow velocity. Because of the random elements in the analytical model, each blade element load must be calculated separately, greatly increasing the calculation time. The uniform inflow turbulence model uses the improved velocity defect model from section 6.3 with a 0.7% randomized blade loading (corresponding to a 1.5% turbulence intensity) to produce an increase in the broadband noise floor (Table 6.12). The results from the analytical model in Figure 6.26 represent an average of the calculated broadband noise over 10 revolutions of the fan. The analytical model results and the measured broadband noise levels differ by approximately 3 dB. Increasing the random fluctuation increases the broadband noise level.
Table 6.12: Inputs for the FIFO model with uniform inflow turbulence

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading</td>
<td>Basic Loading</td>
</tr>
<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
</tr>
<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
</tr>
<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Ramp **</td>
</tr>
<tr>
<td>Swirl</td>
<td>25°</td>
</tr>
<tr>
<td>Defect</td>
<td>none</td>
</tr>
<tr>
<td>Edge Effect</td>
<td>none</td>
</tr>
<tr>
<td>Turbulence Model</td>
<td>0.7% random force defect</td>
</tr>
</tbody>
</table>

* 26 degree span
** 52 degree span

Figure 6.26: SPL of the broadband factor, dB ref. 20 μPa, for the FIFO model with uniform inflow turbulence
The measured broadband factor for the test with the 30mm plate test increased slightly over the measured broadband factor for the FIFO case. Using the same turbulence model with the contracting flow model from section 6.4.3, Table 6.13, produces a prediction for the broadband factor that has a similar error as the previous case.

Table 6.13: Model inputs for contracting flow model with uniform inflow turbulence

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading</td>
<td>Basic Loading</td>
</tr>
<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
</tr>
<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
</tr>
<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Ramp **</td>
</tr>
<tr>
<td>Swirl</td>
<td>25°</td>
</tr>
<tr>
<td>Defect</td>
<td>Contracting flow model</td>
</tr>
<tr>
<td>Edge Effect</td>
<td>50% of defect flow</td>
</tr>
<tr>
<td>Turbulence Model</td>
<td>0.7% random force defect</td>
</tr>
</tbody>
</table>

* 26 degree span
** 52 degree span
Figure 6.27: SPL of the broadband factor, dB ref. 20 $\mu$Pa, for the contracting flow model with uniform inflow turbulence

### 6.5.2 Stationary Turbulence Defects

To refine the turbulence model, additional stationary turbulence was placed in the areas where the velocity fluctuation was high. Figure 6.4 shows these areas in yellow. For these areas the velocity fluctuation measured was approximately +/- 10%. The analytical model inputs for the FIFO and contracting flow models, with stationary turbulence, are shown in Table 6.14 and Table 6.15. The analytical model results for the FIFO and contracting flow models, with stationary turbulence defects, are shown in Figure 6.28 and Figure 6.29. In both cases the analytical model results for the broadband factor compare well with the measured data.
Table 6.14: Inputs for the FIFO model with stationary turbulence velocity defects

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading</td>
<td>Basic Loading</td>
</tr>
<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
</tr>
<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
</tr>
<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Ramp **</td>
</tr>
<tr>
<td>Swirl</td>
<td>25°</td>
</tr>
<tr>
<td>Defect</td>
<td>none</td>
</tr>
<tr>
<td>Edge Effect</td>
<td>none</td>
</tr>
<tr>
<td>Turbulence Model</td>
<td>uniform inflow with 10% flow turb</td>
</tr>
</tbody>
</table>

* 26 degree span  
** 52 degree span

![Figure 6.28: SPL of the broadband factor, dB ref. 20 μPa, for the FIFO model with stationary turbulence velocity defects](image)

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Table 6.15: Inputs for the contracting flow model with stationary turbulence velocity defects

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Input Dependency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading</td>
<td>Basic Loading</td>
</tr>
<tr>
<td>Apparent Velocity Angle</td>
<td>Calculated</td>
</tr>
<tr>
<td>Base Inflow Distortion</td>
<td>Base Measured</td>
</tr>
<tr>
<td>Inflow Distortion Peaks 1-3</td>
<td>28% Ramp *</td>
</tr>
<tr>
<td>Inflow Distortion Peak 4</td>
<td>22% Ramp **</td>
</tr>
<tr>
<td>Swirl Defect</td>
<td>25°</td>
</tr>
<tr>
<td>Edge Effect</td>
<td>Contracting flow model</td>
</tr>
<tr>
<td>Turbulence Model</td>
<td>uniform inflow with 10% flow turb</td>
</tr>
</tbody>
</table>

* 26 degree span
** 52 degree span

Figure 6.29: SPL of the broadband factor, dB ref. 20 μPa, for the contracting flow model with stationary turbulence velocity defects
6.5.3 Turbulence Model Results

Similar to the FIFO and plate models, a radiation pattern for the broadband factor has been established from the measured data. A combination of a 0.7% random force defect over the entire surface of the fan and a 10% random velocity fluctuation, applied to specific areas, was used to create a stationary turbulence velocity defect model. The resulting level for the FIFO broadband factor, shown in Figure 6.28, matches well with the measured data. In the range of $-50^\circ \leq \zeta \leq 50^\circ$, the broadband factor shows an average deviation from the measured data of 0.5 dB. The results shown in Figure 6.29, for the same stationary turbulence velocity defect model applied to the contracting wake model, gives an average deviation from the measured data of -1.4 dB.

6.6 Disc Defect

Changes in the location of the object were investigated using a 50mm diameter disc, placed near the tips of the fan blades. Four locations are modeled, $\theta = 90, 120, 150$ and $180^\circ$, with the outer edge of the disc aligned with the swept arc of the blade tips. The test setups for the 4 locations are shown in Figure 6.30. Measured SPL data was taken with the microphones at $\zeta = 0^\circ$ and $\zeta = 30^\circ$. 
Figure 6.30: Test setup for 4 disc locations
Since the disc is small relative to the fan blades, the velocity defects generated by this model, shown in Figure 6.31, are only affecting the outer 4 blade elements. As the disc velocity defect is moved around the 4 locations, note that the velocity distortions from the disc and the velocity distortions due to the FIFO test setup behave differently.

Figure 6.31: Inlet distortion patterns for the disc velocity defect model at 4 locations for various blade elements (see Figure 6.3 for legend)
Figure 6.32 shows the SPL spectrum for the analytical model results for the 4 disc locations compared to the measured data at observer locations $\zeta = 0^\circ$ and $\zeta = 30^\circ$.

Figure 6.32: SPL spectra, dB ref. 20 $\mu$Pa, for the disc velocity defect model at 4 locations

The analytical model results show good agreement, with the measured SPL, for the disc at $\theta = 90^\circ$ (Figure 6.32(a)) and at $\theta = 150^\circ$ (Figure 6.32(c)). With the disc at $\theta = 90^\circ$, the analytical model results for the BPF differed from the measured data by 1.5 dB for the observer at $\zeta = 0^\circ$ and by 3 dB for the observer at $\zeta = 30^\circ$. With the disc at $\theta = 150^\circ$, the comparison for the BPF was within 2 dB for both observer locations.

The comparisons for the BPF with the disc at $\theta = 120^\circ$ and $\theta = 180^\circ$ were not as good. With the disc at $\theta = 120^\circ$, the disc velocity defect completely covers one of the
inflow distortion peaks (Figure 6.31(b)). As a result, the SPL for the BPF predicted by the model is 14 dB below the measured result. With the disc at $\theta = 180^\circ$, the disc velocity defect again covers a significant portion of the inflow distortion (Figure 6.31(d)), resulting in a 5.5 dB difference between the predicted and measured results. It is clear that where multiple flow distortions overlap, more refinement of the inflow distortion model is required than the simple tools give.
Chapter 7

Conclusion and Discussion

7.1 Analytical Model Results

The Ffowcs Williams and Hawkings equation has been used as the state of the art for evaluating the noise created by rotorcraft for many years. These systems generally have tip speeds either near Mach 1 or greater. They also have inflow distortions that are large scale either due to forward flight rotor tilt or phenomenon like rotor-stator interaction. The blade loading itself, including effects from the inflow distortion, is the major player in the tonal noise created by these systems and the broadband noise is generated from the shear layers in the turbulence at the trailing edge and tips of the blades.

In smaller fan systems, the tip speeds are often much lower and the blade loading noise is dominated by a set of inflow distortions. When designing these systems, CFD modeling is rarely used, so detailed information about the blade loading and inflow distortions is not readily available. The methodology described in this paper provides a mechanism for evaluating the blade loading and inflow distortions in order to create a baseline noise model for a generalized installation.

Evaluation of the inflow distortion is critical to developing an acoustic model that represents the actual system. This study provides a framework for building a simple inflow distortion model that can be used with the Ffowcs Williams and Hawkings equation to represent the noise generated by a real fan system. Though the model inputs are taken from crude measurements, the comparison to the actual system was favorable. Finer scale measuring tools such as a hot wire anemometer or detailed CFD can provide better inputs for the inflow distortion model.
The velocity defects models presented in these examples show the necessary flow specification. The objects chosen were sufficiently large to be illustrated with the rotational grid size used and to yield significant changes in the measured SPL of the fan system. In order to model smaller velocity defects, the grid size should be refined to match.

Since this model does not use information about the shear layers in the fan outflow, the broadband model is quite limited. While some of the drivers of turbulence noise can be simply approximated, to further refine this model requires more information than is readily available. The effects of the turbulent shear contained in the Lighthill stress tensor can contribute significantly to the high frequency broadband noise. Broadband modeling is an active area of research both at NASA and in the broader fan noise community.

7.2 Adaptation to CFD

Refining the model inputs will yield better agreement with the baseline measured data. The use of CFD to create the model inputs provides the flexibility to either refine the data applied to this lifting line type of model or expand further into a chordwise grid. Expanding the model in this way does come with its costs, primarily computational time. The grid size for the examples presented above have been chosen to balance computational time with refining the results. With the inflow distortion measurements taken, a rotational grid of 140 fan positions around the circumference was more than sufficient to describe the flow. The test objects used for the velocity defect models were also large enough to use with this size grid. With finer scale measurements or with CFD inputs, the grid size can be refined to capture smaller scale distortions. Where significant features of the FIFO inflow distortion and upstream obstructions overlap, more detailed inputs and a finer scale grid in the model are needed to yield good results.

7.3 Future Work

This work provides the framework for using the FW-H equation in modeling low speed, lightly loaded systems. The cases presented in this work are simple in nature. The extension of this work will involve modeling a full cooling system including multiple upstream obstructions. The simple inflow velocity distortion models are also applicable to large wind turbines where the sheer size of the system guarantees unsteady inflow.
Ultimately this methodology can be integrated into a design tool that allows a designer to assess changes to improve the noise level of a given fan system.
REFERENCES


Appendix A

Matlab Code for SLLAC Model with Velocity Defects and Turbulence

```matlab
1 clear all;
2
3 %set the basic fan parameters
4 c=334;
5 rho=1.28;
6 RPM=2490;
7 rr=0.197;
8 B=7;
9 bseg=20;
10 gridfac=1;
11 aspect=gridfac*20;
12 segment=B*aspect;
13 circseg=36;
14 oprad=2.50;
15 bladefact=1;
16 aattack=99;
17 swirl=10;
18 savefile='StationaryTurbA';
19
20 Omega=RPM*2*pi/60;
21 halfseg=rr/bseg/2;
```
dr=rr/bseg;

for looper=0:bseg-1
    rrvec(looper+1)=(halfseg+looper*dr);
    vfan(looper+1)=rrvec(looper+1)*Omega
end;

tface=[0 0 0 0 0 18.05 18.05 18.05 18.05 18.05 18.05 18.05 ... 18.05 18.05 18.05 18.05];
BladeAngle=[0 0 0 0 0 0 45 43.5 42 40.5 39 37.5 36 34.5 33 31.5 30];

vomega=sqrt(vface.*vface+vfan.*vfan);
%AngAttack=aattack*[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1];
AngAttack=360/2/pi*atan(vface./vfan);

BPF=RPM/60*B;
DragFactor=sin(2*3.14159*AngAttack/360)*1
ThrustFactor=cos(2*3.14159*AngAttack/360)*1
fsamp = Omega*segment/2/pi;

%Set up the Loading on the blades
%Set the magnitude for Total Torque, Total Thrust and Thickness
Tqmag=1.;
Thrmag=1;
Qmag=rho*1;
Tipmag=1*rho/2*(Omega*rr)^2;
Omegas=Omega;
defscale=1*8/5;
%defscale=0;
BaseFlowOn=1;
%basescale=-1;
basescale=-1.06;
bpct=0.00;
% Set the shape for the torque, thrust and thickness vectors

% Proportional to r^2 with last element clipped for tip losses
ThrShape=[0.002912977 0.008738932 0.014564887 0.020390842 0.026216797 ... 
0.032042751 0.037868706 0.043694661 0.049520616 0.084146635 ... 
0.106161968 0.137106995 0.179774946 0.237488899 0.314147858 ... 
0.414272822 0.543052862 0.706391194 0.910951255 0.352292758];
TqShape=[0 0 0 0 0 0 0 0 0 0.162373241 0.19835623 ... 
0.237937519 0.281117107 0.327894993 0.378271179 0.432245663 ... 
0.489818447 0.550989529 0.61575891 0.684126591];
QShape=[0 0 0 0 0 0 0 0 0 0.001006776 0.001024191 ... 
0.001041248 0.001057681 0.001073298 0.001087976 0.001101667 ... 
0.001114394 0.001137332 0.001159845 0.001181969];
Tipshape=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.003*0.123];
QBase=[0 0 0 0 0 0 0 0 0 0.002 0.002 0.002 ... 
0.002 0.002 0.002 0.002 0.002 0.002 0.002];

% Initialize matrices as zeros
vdefmat=zeros(segment,bseg);
sdefmat=zeros(140,4,bseg);
Thetamat=zeros(circseg,segment,B,bseg);
Tdot=zeros(circseg,segment,B,bseg);

r1mat=zeros(circseg,segment,B,bseg);
r2mat=zeros(circseg,segment,B,bseg);
r3mat=zeros(circseg,segment,B,bseg);
rmat=zeros(circseg,segment,B,bseg);

% Set the parameters for velocity defects
% defectdev sets the randomization for random turbulence, spacedefon
% sets whether there is spacial turbulence on
% statdefon equals 0 for no stationary defects, 1 for allow stationary defects
\begin{verbatim}
statdefon=1;
spacedefon=1;
PlateDef=statdefon*-0.70;
Platedefon=0;
LossFact=0.59;
defectdev=1;
dsmag=0.7;
InTurb1=1;

% dmag is the delta magnitude that indicates the fan is at a
% stationary defect location

dmag=0.1;

Atqdefsize=0.1*InTurb1;
Athrdefsize=0.1*InTurb1;
Btqdefsize=0.1*InTurb1;
Bthrdefsize=0.1*InTurb1;

%dsz is the Defect Size for Random Blade Turbulence
dszn=dsmag*[0 0 0 0 0 0 0.01 0.01 0.01 0.01 0.01 0.01 ... 
0.01 0.01 0.01 0.01 0.01 0.05];
ds7z=dszn;

% Input a matrix for the stationary defects
% 140x3 matrix for each blade segment
% First column is the angle for the defect, second is the thrust
defect, third is the drag defect

% enter data here
for defang=1:segment;
    sdefmat(defang,1,1:bseg)=2*pi*defang/segment;
end;

%Set FFT Index
\end{verbatim}
for idx=1:segment/2;
freqvector(idx)=(idx-1)*RPM/60;
end;

% Perform the Retarded Time Calculation and save the Theta and Thetadot matrices
% Loop over the observer angle zeta
  for zloop=1:circseg;
    zloop
    zeta=pi*zloop*2/circseg;
    zt(zloop)=zeta;
  end;

% Set up the observer position
  op1=oprad*sin(zeta);
  op2=0;
  op3=oprad*cos(zeta);

% Set up acoustic pressure at each observer time by solving for
% a rotational position of blade 1 in each observer time
% Since at each observer time plus bladepass time another blade is at
% the same position, the pressure matrix for each blade can be filled in

  for step=0:segment-1;
    for bladeseg=1:bseg;
      r0(bladeseg)=oprad-(rr*(bladeseg/bseg)-halfseg);
      Mach(bladeseg)=Omega*(rr*bladeseg/bseg-halfseg)/c;
    end;
  end;

% Loop over fan rotation angle. Sets which observer time steps will
% correlate to a given blade position. bp(1) and bp(2) should differ
% by aspect such that aspect time steps later, blade 2 is in the same
% position as blade 1 was (same theta1)

  for loop=1:B;
bp(loop)=step+1+(loop-1)*aspect;
while bp(loop)>segment
    bp(loop)=step+1+(loop-1)*aspect-segment;
end;
end;

% Determine observed R vector by a shooting technique over the retarded
% time. First initialize the observer time.

time=step/segment/Omega*2*pi;
obstime(step+1)=time;

% Using the initial guess for R (as r0) iterate until the new r vector
% is essentially unchanged from the starting r0 vector

for bladeseg=1:bseg;
    for loop=1:10;

    % Estimate a blade angle theta0 based on the estimated observer distance r0
    % This assumes that we are starting the time clock when the observer sees
    % the fan at the starting point. The blade angle is then reversed by an
    % amount relating to the speed of the blade and the time it takes the sound
    % to get to the observer
    theta0=(time-(r0(bladeseg))/c)*Omega;

    % Set up R vector from source to observer using the estimated theta0
    R1=(op1-(rr*bladeseg/bseg-halfseg)*cos(theta0));
    R2=op2+(rr*bladeseg/bseg-halfseg)*sin(theta0);
    R3=op3;

    % Use the length of the R vector to set up Unit Vectors in R direction
    % and determine a new estimate for blade position, theta1
    r=sqrt(R1^2+R2^2+R3^2);
    r1=R1/r;
    r2=R2/r;
    r3=R3/r;
\[ \theta_1 = (t \cdot r/c) \cdot \Omega; \]

% Reinitialize the r0 length to the length of the R vector
\[ r0(b) = r; \]

% Set the theta1 value from the iteration in the thetafan(timestep) vector
% This step may be eliminated if there is no need for a record of the
% actual theta values

% loop over each blade - Calculates the load with spatial randomization
% for each blade at a position of theta1 and places the appropriate
% acoustic pressure in the PR matrix.
\[
\begin{align*}
\text{for bladeloop}=1:B; \\
\quad \text{if step+1+segment/B*(bladeloop-1)} <= \text{segment;} \\
\qquad \text{index1} = \text{segment/B*(bladeloop-1);} \\
\quad \text{else;} \\
\qquad \text{index1} = \text{segment/B*(bladeloop-1)} - \text{segment;} \\
\quad \text{end;} \\
\quad \text{shift} = \text{step+1+index1;} \\
\quad \text{Thetamat(zloop,shift,bladeloop,b)} = \theta_1; \\
\quad \text{end;} \\
\end{align*}
\]

% Save permanent copies of the Theta matrix and Tdot matrix.
% Note that the Tdot matrix is equivalent to 1/|1-Mr|
\[
\begin{align*}
\text{for zloop}=1:circseg; \\
\quad \text{for bladeseg}=1:bseg; \\
\quad \text{for bladeloop}=1:B; \\
\quad \text{Tdot(zloop,1,bladeloop,b)} = \ldots \\
\quad \quad \text{Thetamat(zloop,1,bladeloop,b)} = \ldots \\
\quad \quad \quad \text{-Thetamat(zloop,segment,bladeloop,b)} = \ldots \\
\end{align*}
\]
/(2*3.14159/segment);
if bladeloop==1;
Tdot(zloop,1,bladeloop,bladeseg)=...
(Thetamat(zloop,1,bladeloop,bladeseg)... 
-Thetamat(zloop,segment,bladeloop,bladeseg)... 
+2*3.14159)/(2*3.14159/segment);
end;
for shift=2:segment;
    
    ThetaNew=Thetamat(zloop,shift,bladeloop,bladeseg);
    ThetaOld=Thetamat(zloop,shift-1,bladeloop,bladeseg);
    if Thetamat(zloop,shift,bladeloop,bladeseg)... 
        -Thetamat(zloop,shift-1,bladeloop,bladeseg)<-1
    ThetaOld=Thetamat(zloop,shift-1,bladeloop,bladeseg)-2*pi;
    end;
    Tdot(zloop,shift,bladeloop,bladeseg)=...
    (ThetaNew-ThetaOld)/(2*pi/segment);
end;
end;
end;

% Set up testrun and testrun2 variables to allow for looping over 
% various inlet swirl conditions or turbulence runs

testrun=swirl
for testrun2=1:10;

% Set up the inflow velocity field

vdefmat=zeros(segment,bseg);

Level1Defect=defscale*0.2*statdefon;
Factor1=1;
Factor2=1;
Factor3=1;
Factor4=1;
gridfac=5;

%Load the base flow matrix baseflowmat
load baseflowmat
for defang=1:segment-swirl;
    for looper=1:bseg;
        vdefmat(defang+testrun,looper)=BaseFlowOn*statdefon*basescale...
            +BaseFlowOn*statdefon*baseflowmat(defang,looper);
    end;
end;
for defang=1:swirl;
    for looper=1:bseg;
        vdefmat(defang,looper)=BaseFlowOn*statdefon*basescale+BaseFlowOn...
            *statdefon*baseflowmat(segment-testrun+defang,looper);
    end;
end;

% Set up the 4 peak defects
Level1Defect=defscale*0.2*statdefon;
CenterP=10+testrun;
LowE=CenterP-gridfac;
HiE=CenterP+(gridfac-1);
for defang=LowE:CenterP-1;
    Factor1=1/(CenterP-LowE)*(defang-(LowE-1));
    for looper=1:20;
        vdefmat(defang,looper)=vdefmat(defang,looper)+Factor1*Level1Defect;
    end;
end;
for defang=CenterP:HiE;
    Factor2=1/(CenterP-LowE)*(HiE+1-defang);
    for looper=1:20;
        vdefmat(defang,looper)=vdefmat(defang,looper)+Factor2*Level1Defect;
Level2Defect=defscale*0.2*statdefon;

CenterP=27+testrun;
LowE=CenterP-gridfac;
HiE=CenterP+gridfac-1;

for defang=LowE:CenterP-1;
   Factor1=1/(CenterP-LowE)*(defang-(LowE-1));
   for looper=1:20;
      vdefmat(defang,looper)=vdefmat(defang,looper)+Factor1*Level2Defect;
      end;
   end;
for defang=CenterP:HiE;
   Factor2=1/(CenterP-LowE)*(HiE+1-defang);
   for looper=1:20;
      vdefmat(defang,looper)=vdefmat(defang,looper)+Factor2*Level2Defect;
      end;
end;

Level3Defect=defscale*0.2*statdefon;

CenterP=47+testrun;
LowE=CenterP-gridfac;
HiE=CenterP+gridfac-1;

for defang=LowE:CenterP-1;
   Factor1=1/(CenterP-LowE)*(defang-(LowE-1));
   for looper=1:20;
      vdefmat(defang,looper)=vdefmat(defang,looper)+Factor1*Level3Defect;
      end;
end;
for defang=CenterP:HiE;
   Factor2=1/(CenterP-LowE)*(HiE+1-defang);
   for looper=1:20;
      vdefmat(defang,looper)=vdefmat(defang,looper)+Factor2*Level3Defect;
      end;
end;
for looper=1:20;
  vdefmat(defang,looper)=vdefmat(defang,looper)+Factor2*Level3Defect;
end;
end;
Level4Defect=defscale*0.15*statdefon;
CenterP=87+testrun;
LowE=CenterP-gridfac*2;
HiE=CenterP+gridfac*2-1;
for defang=LowE:CenterP-1;
  Factor1=1/(CenterP-LowE)*(defang-(LowE-1));
  for looper=1:20;
    vdefmat(defang,looper)=vdefmat(defang,looper)+Factor1*Level4Defect;
  end;
end;
for defang=CenterP:HiE;
  Factor2=1/(CenterP-LowE)*(HiE+1-defang);
  for looper=1:20;
    vdefmat(defang,looper)=vdefmat(defang,looper)+Factor2*Level4Defect;
  end;
end;

%Set the object velocity defects
%Contracting flow plate model
%EdgeR=[0 0 0 0 0 0 0 0 0 1 12 21 26 28 29 30 30 30 30 30];
%EdgeL=[0 0 0 0 0 0 0 0 0 70 58 49 45 43 41 40 40 40 40 40];

%Simple plate model
EdgeR=[0 0 0 0 0 0 0 0 0 31 31 31 31 31 31 31 31 31 31 31];
EdgeL=[0 0 0 0 0 0 0 0 0 39 39 39 39 39 39 39 39 39 39 39];

% set up the velocity jets on either side of the defect
jetwidth=8;
for spanloop=10:bseg;
    Aedge=EdgeR(spanloop)+testrun;
    Bedge=EdgeL(spanloop)+testrun;
    for defang=Aedge:Bedge;
        defangg=defang;
        if defang<1;
            defangg=defang+segment;
        end;
        vdefmat(defangg,spanloop)=vdefmat(defangg,spanloop)+PlateDef*...((Bedge-Aedge)/2)^2-((defang-Aedge)-(Bedge-Aedge)/2)^2).../(Bedge-Aedge/2)^2)*Platedefon;
    end;
    Aedge=EdgeR(spanloop)+testrun-jetwidth;
    Bedge=EdgeR(spanloop)+testrun;
    for defang=Aedge:Bedge;
        defangg=defang;
        if defang<1;
            defangg=defang+segment;
        end;
        vdefmat(defangg,spanloop)=vdefmat(defangg,spanloop)-PlateDef*...*LossFact*(((Bedge-Aedge)/2)^2-((defang-Aedge)-(Bedge-Aedge)/2)^2)...(Bedge-Aedge/2)^2)/((Bedge-Aedge/2)^2)*Platedefon;
    end;
    Aedge=EdgeL(spanloop)+testrun;
    Bedge=EdgeL(spanloop)+jetwidth+testrun+1;
    for defang=Aedge:Bedge;
        defangg=defang;
        if defang<1;
            defangg=defang+segment;
        end;
        vdefmat(defangg,spanloop)=vdefmat(defangg,spanloop)-PlateDef...
*LossFact*(((\(\text{Bedge-Aedge}/2\)^2-((\text{defang-Aedge})-... (\(\text{Bedge-Aedge}/2\)^2)/(\(\text{Bedge-Aedge}/2\)^2)*Platedefon;
end;
end;

for defang=1:segment;
    for looper=1:bseg;
        vpct=vdefmat(defang,looper)+1;
        if vomega(looper)>0;
            sdefmat(defang,2,looper)=ThrustFactor(looper)...*((vomega(looper)^2+vface(looper)^2*vpct^2).../(vomega(looper)^2+vface(looper)^2)-1);
            sdefmat(defang,3,looper)=DragFactor(looper)...*(vomega(looper)^2+vface(looper)^2*vpct^2).../(vomega(looper)^2+vface(looper)^2)-1);
            sdefmat(defang,4,looper)=0*(vpct^2-1);
        end;
    end;
end;

figure(5)
plot(vdefmat)
axis([0 140 -1 1])
set(gca,'XTick',0:35:140)
set(gca,'XTickLabel',\{'0'\}',90\}',180\}',270\}',360\}','FontSize',14)
set(gca,'YTick',-1:0.2:1)
set(gca,'YTickLabel',-1:0.2:1,'FontSize',14)
xlabel('Fan Rotation, deg.\}','FontSize',14)
ylabel('Defect Factor\}','FontSize',14)

%Perform the Pressure Calculations

testnum=num2str(testrun2);
dottxt='.txt';
rectxt='vecs.txt';
deftxt='def.txt';
% Basic Dipole Formulation Engine;
% The following variables must be set;
% c, RPM, rr, Tqmag, Thrmag, TqShape, devectdev, ThrShape, B, bseg, dmag,
% aspect, circseg, oprad, dsz;
% The following values are set
% Omega = RPM*2*pi/60, fsamp = Omega*segment/2/pi, BPF = RPM/60*B,
% halfseg = rr/bseg/2, segment = B*aspect, sdefmat = zeros(segment, 3, bseg);

sdefth = zeros(circseg, segment, B, bseg);
sdefdr = zeros(circseg, segment, B, bseg);
sdefthick = zeros(circseg, segment, B, bseg);
Frmat = zeros(circseg, segment, B, bseg);
Qmat = zeros(circseg, segment, B, bseg);
Frdotmat = zeros(circseg, segment, B, bseg);
Frdot1mat = zeros(circseg, segment, B, bseg);
Frdot2mat = zeros(circseg, segment, B, bseg);
Mrmat = zeros(circseg, segment, B, bseg);
Mrdotmat = zeros(circseg, segment, B, bseg);
FiMimat = zeros(circseg, segment, B, bseg);
Thrmat = zeros(circseg, segment, B, bseg);
Tqmat = zeros(circseg, segment, B, bseg);
Tqdotmat = zeros(circseg, segment, B, bseg);
Tipmat = zeros(circseg, segment, B, bseg);
Tipdotmat = zeros(circseg, segment, B, bseg);
Thrdotmat = zeros(circseg, segment, B, bseg);
Tipz = zeros(bseg);
turbdefmat = zeros(segment, bseg);
Pr = zeros(circseg, segment, B, bseg);
sdThr = zeros(circseg, segment, B, bseg);
sdDr = zeros(circseg, segment, B, bseg);
stdefmat = zeros(segment, 4, bseg);
defect1 = zeros(segment, bseg);
defect2 = zeros(segment, bseg);
defect3 = zeros(segment, bseg);
defect4 = zeros(segment, bseg);
defect5 = zeros(segment, bseg);
defect6=zeros(segment,bseg);
defect7=zeros(segment,bseg);
checkmat=zeros(circseg,segment,bseg);
bch=normrnd(0,1,B);
for l2=1:B;
    bladechange(l2)=bch(l2)*bpct;
end;

%Loop over the observer angle zeta
for zloop=1:circseg;
    zloop

%Define defect randomization size
for bladeseg=1:bseg;

    % Set up a vector for torque and thrust defects
    def=normrnd(0,defectdev,segment,B)*spacedefon;
    for l1=1:segment;
        for l2=1:B;
            defect(l1,l2,bladeseg)=def(l1,l2);
        end;
    end;

    Tqz(bladeseg)=Tqmag*TqShape(bladeseg);
    Thrz(bladeseg)=Thrmag*ThrShape(bladeseg);
    Qz(bladeseg)=Qmag*QShape(bladeseg);
    Tipz(bladeseg)=Tipmag*Tipshape(bladeseg);
end;

zeta=pi*zloop*2/circseg;
zt(zloop)=zeta;

%Set up the observer position
op1=oprad*sin(zeta);
% Set up acoustic pressure at each observer time by solving for
% a rotational position of blade 1 in each observer time
% Since at each observer time plus bladepass time another blade is at
% the same position, the pressure matrix for each blade can be filled in
% 
  for step=0:segment-1;
    for bladeseg=1:bseg;
      r0(bladeseg)=oprad-(rr*(bladeseg/bseg)-halfseg);
      Mach(bladeseg)=Omega*(rr*bladeseg/bseg-halfseg)/c;
    end;
    TP(step+1,zloop)=0;
  end;

% Loop over fan rotation angle. Sets which observer time steps will
% correlate to a given blade position. bp(1) and bp(2) should differ
% by aspect such that aspect time steps later, blade 2 is in the same
% position as blade 1 was (same theta1)

  for loop=1:B;
    bp(loop)=step+1+(loop-1)*aspect;
    while bp(loop)>segment
      bp(loop)=step+1+(loop-1)*aspect-segment;
    end;
  end;

% Determine observed R vector by a shooting technique over the retarded
% time. First initialize the observer time.

time=step/segment/Omega*2*pi;
obstime(step+1)=time;

% Using the initial guess for R (as r0) iterate until the new r vector
% is essentially unchanged from the starting r0 vector

  for bladeseg=1:bseg;
% Estimate a blade angle theta0 based on the estimated observer distance r0
% This assumes that we are starting the time clock when the observer sees
% the fan at the starting point. The blade angle is then reversed by an
% amount relating to the speed of the blade and the time it takes the sound
% to get to the observer

theta0=Thetamat(zloop,step+1,1,bladeseg);
theta0=Thetamat(zloop,step+1,1,bladeseg);

% Set up R vector from source to observer using the estimated theta0
R1=(op1-(rr*bladeseg/bseg-halfseg)*cos(theta0));
R2=op2+(rr*bladeseg/bseg-halfseg)*sin(theta0);
R3=op3;

% Use the length of the R vector to set up Unit Vectors in R direction
% and determine a new estimate for blade position, theta1
r=sqrt(R1^2+R2^2+R3^2);
r1=R1/r;
r2=R2/r;
r3=R3/r;
theta1=(time-r/c)*Omega;

% Reinitialize the r0 length to the length of the R vector
r0(bladeseg)=r;

% Set the theta1 value from the iteration in the thetawave(timestep) vector
% This step may be eliminated if there is no need for a record of the
% actual theta values
thetawave(step+1,bladeseg)=theta1;

% loop over each blade - Calculates the load with spatial randomization
% for each blade at a position of theta1 and places the appropriate
% acoustic pressure in the PR matrix.
Comment the for bladeloop line and uncomment the bladeloop=1 line
% and the for bladeloop =1:B line after the pressure calculation
% to speed calculation when not using turbulent defects.

for bladeloop=1:B;

%bladeloop=1;

% Inflow Turbulence flow defect - Normal Random Pressure
InflowATq=Atqdefsize*statdefon;
InflowATH=Athrdefsize*statdefon;
InflowBTq=Btqdefsize*statdefon;
InflowBTh=Bthrdefsize*statdefon;

turb1=normrnd(0,1,segment,B);
turb2=normrnd(0,1,segment,B);
turb3=normrnd(0,1,segment,B);
turb4=normrnd(0,1,segment,B);
turb5=normrnd(0,1,segment,B);
turb6=normrnd(0,1,segment,B);
%turb1=zeros(10,7);
%turb2=zeros(10,7);
%turb3=zeros(10,7);
%turb4=zeros(10,7);

for defang=40:48;
    turbdefmat(defang,15:20)=turb1(defang,B)*InflowATH;
end;

for defang=68:105;
    turbdefmat(defang,15:20)=turb2(defang,B)*InflowBTh;
end;

for defang=1:segment;
    for looper=1:bseg;
        vpct=turbdefmat(defang,looper)+1;
if vomega(looper)>0;
   stdefmat(defang,2,looper)=ThrustFactor(looper)*((vomega(looper)^2 ... 
       +vface(looper)^2*vpct^2)/(vomega(looper)^2+vface(looper)^2)-1);
   stdefmat(defang,3,looper)=DragFactor(looper)*((vomega(looper)^2 ... 
       +vface(looper)^2*vpct^2)/(vomega(looper)^2+vface(looper)^2)-1);
   stdefmat(defang,4,looper)=0*(vpct^2-1);
end;
end;
end;

%Set the force defect

for sdloop=1:segment;
   obj1=theta1;
   while obj1<0;
      obj1=obj1+2*pi;
   end
   if abs(sdefmat(sdloop,1,bladeseg)-obj1)<dmag;
      sdefth(zloop,bp(bladeloop),bladeloop,bladeseg)=... 
         0+sdefmat(sdloop,2,bladeseg)+stdefmat(sdloop,2,bladeseg);
      sdefdr(zloop,bp(bladeloop),bladeloop,bladeseg)=... 
         0+sdefmat(sdloop,3,bladeseg)+stdefmat(sdloop,3,bladeseg);
      sdefthick(zloop,bp(bladeloop),bladeloop,bladeseg)=... 
         0+sdefmat(sdloop,4,bladeseg)+stdefmat(sdloop,4,bladeseg);
   end;
end;

ds=dszn;

   if bladeloop==7;
      ds=ds7;
   end;

Thr=Thrz(bladeseg)*(1+bladechange(bladeloop))... 
   +(defect(step+1,bladeloop,bladeseg)*dsz(bladeseg)...
   +sdefth(zloop,bp(bladeloop),bladeloop,bladeseg))... 
   *Thrz(bladeseg)*(1+bladechange(bladeloop));

Tq=Tqz(bladeseg)+(defect(step+1,bladeloop,bladeseg)*dsz(bladeseg)...

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\( + s_{\text{defdr}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) \cdot T_{q}(b_{\text{loop}}) \);

\[ Q = Q_{\text{Base}}(b_{\text{loop}}) + Q_{z}(b_{\text{loop}}) + (\text{defect}(\text{step}+1, b_{\text{loop}}, b_{\text{loop}}) \cdot d_{\text{sz}}(b_{\text{loop}}) + s_{\text{defthick}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) \cdot Q_{z}(b_{\text{loop}}) ; \]

\[ \text{Tip} = \text{Tip}_{z}(b_{\text{loop}}) + (\text{defect}(\text{step}+1, b_{\text{loop}}, b_{\text{loop}}) \cdot d_{\text{sz}}(b_{\text{loop}}) + s_{\text{defth}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) \cdot \text{Tip}_{z}(b_{\text{loop}}) ; \]

\% Uncomment this line for faster calculation when not using turbulence.
\%
\% for bladeloop=1:B;

\[ T_{q\text{mat}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = T_{q} ; \]
\[ T_{\text{tipmat}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = T_{\text{ip}} ; \]
\[ T_{\text{rmat}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = T_{\text{rs}} ; \]
\[ Q_{\text{mat}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = Q_{z} ; \]
\[ r_{1\text{mat}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = r_{1} ; \]
\[ r_{2\text{mat}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = r_{2} ; \]
\[ r_{3\text{mat}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = r_{3} ; \]
\[ r_{\text{mat}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = r ; \]

\[ s_{\text{dThr}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = 1 + \text{defect}(\text{step}+1, b_{\text{loop}}, b_{\text{loop}}) \cdot d_{\text{sz}}(b_{\text{loop}}) \cdot s_{\text{dThr}}(z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) \];

\[ s_{\text{dDr}}(z_{\text{loop}}, b_{\text{loop}}, z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) = 1 + \text{defect}(\text{step}+1, b_{\text{loop}}, b_{\text{loop}}) \cdot d_{\text{sz}}(b_{\text{loop}}) \cdot s_{\text{dDr}}(z_{\text{loop}}, b_{\text{loop}}, b_{\text{loop}}) ; \]

\end{for}

\text{for} z_{\text{loop}}=1:z_{\text{circseg}} ;
\text{zloop}
for step=1:segment;
    for bladeloop=1:B;
        for bladeseg=1:bseg;

            theta1=Thetamat(zloop,step,bladeloop,bladeseg);
            thetadot=Tdot(zloop,step,bladeloop,bladeseg);
            Thr=Thrmat(zloop,step,bladeloop,bladeseg);
            Tq=Tqmat(zloop,step,bladeloop,bladeseg);
            Tip=Tipmat(zloop,step,bladeloop,bladeseg);
            Q=Qmat(zloop,step,bladeloop,bladeseg);
            r=rmat(zloop,step,bladeloop,bladeseg);
            r1=r1mat(zloop,step,bladeloop,bladeseg);
            r2=r2mat(zloop,step,bladeloop,bladeseg);
            r3=r3mat(zloop,step,bladeloop,bladeseg);

            if step>1;
                Throld=Thrmat(zloop,step-1,bladeloop,bladeseg);
                Tqold=Tqmat(zloop,step-1,bladeloop,bladeseg);
                Qold=Qmat(zloop,step-1,bladeloop,bladeseg);
                Tipold=Tipmat(zloop,step-1,bladeloop,bladeseg);
                r1old=r1mat(zloop,step-1,bladeloop,bladeseg);
                r2old=r2mat(zloop,step-1,bladeloop,bladeseg);
                r3old=r3mat(zloop,step-1,bladeloop,bladeseg);
            end;

            if step==1;
                Throld=Thrmat(zloop,step-1+segment,bladeloop,bladeseg);
                Tqold=Tqmat(zloop,step-1+segment,bladeloop,bladeseg);
                Qold=Qmat(zloop,step-1+segment,bladeloop,bladeseg);
                Tipold=Tipmat(zloop,step-1+segment,bladeloop,bladeseg);
                r1old=r1mat(zloop,step-1+segment,bladeloop,bladeseg);
                r2old=r2mat(zloop,step-1+segment,bladeloop,bladeseg);
                r3old=r3mat(zloop,step-1+segment,bladeloop,bladeseg);
            end;

        if bladeloop==0;
            Thr=bladefact*Thr;
        end;

    end;
end;
Tq = bladefact * Tq;
Q = bladefact * Q;
end;
if bladeloop == 0;
    Thr = 1 / bladefact * Thr;
    Tq = 1 / bladefact * Tq;
    Q = 1 / bladefact * Q;
end;
if bladeloop == 7;
    r1 = r1 * bladefact;
end;

% Set up Rotational Mach Number and M dot in the R direction;
Mr = r1 * (-Mach(bladeseg)) * sin(theta1) + r2 * (-Mach(bladeseg) * cos(theta1));
Mrdot = r1 * (-Mach(bladeseg) * thetadot * Omegas * cos(theta1)) + r2 * (+Mach(bladeseg) * thetadot * Omegas * sin(theta1));
M2 = Mach(bladeseg) * Mach(bladeseg);

% Set up Force vectors in the R direction;
Fr = (r1 * (-Tq * sin(theta1)) + r2 * (-Tq * cos(theta1)) + r3 * Thr + r2 * -Tip * sin(theta1) + r1 * Tip * cos(theta1));

Thrdot = (Thr - Throld) / (2 * pi / segment) * Omegas;
Tqdot = (Tq - Tqold) / (2 * pi / segment) * Omegas;
Tipdot = (Tip - Tipold) / (2 * pi / segment) * Omegas;
r1dot = (r1 - r1old) / (2 * pi / segment) * Omegas;
r2dot = (r2 - r2old) / (2 * pi / segment) * Omegas;
r3dot = (r3 - r3old) / (2 * pi / segment) * Omegas;

Frdot1 = (r1 * (-Tq * thetadot * Omegas * cos(theta1)) + r2 * (+Tq * thetadot * Omegas * sin(theta1)) + (-r1 * Tip * thetadot * Omegas * sin(theta1)) + r2 * -Tip * thetadot * Omegas * cos(theta1));
Frdot2 = (r1 * (-Tqdot * sin(theta1)) + r2 * (-Tqdot * cos(theta1)) + r3 * Thrdot...
\[ + r2 \cdot \text{Tipdot} \cdot \sin(\theta_1) + r1 \cdot \text{Tipdot} \cdot \cos(\theta_1) \]
defect5(l1,l2)=sdThr(zloop,l1,5,l2);
defect6(l1,l2)=sdThr(zloop,l1,6,l2);
defect7(l1,l2)=sdThr(zloop,l1,7,l2);
end;
end;

% Generate total pressure curve
for loop=1:B;
    for n=1:segment;
        for bladeseg=1:bseg;
            TP(n,zloop)=TP(n,zloop)+Pr(zloop,n,loop,bladeseg);
        end;
    end;
end;

%Remove Mean Pressure from Total Pressure
meann = mean(TP);
for n=1:circseg;
    for loop=1:segment;
        pprime(loop,n)=TP(loop,n)-meann(n);
    end;
end;

%Do FFT on pprime(observer_time,zeta)
xf = fsamp*(0:segment/2-1)/segment;
xf = xf’;
yf = xf;
freqdata = xf;
%frequency = 0*(1:circseg);
xflen = length(xf);
freqmat = zeros(xflen-1,circseg);
%frequency1 = zeros(xflen-1,circseg);
for n=1:circseg;
    for loop=1:segment;
        freqdata(loop)=pprime(loop,n);
    end;
end;
w_sig = hann(segment).*freqdata;
851\[ yf = \text{abs}(\text{fft}(w_{sig}))*2/\text{segment}; \]
852\[ yf = 1.5*yf(1:\text{segment}/2); \]
853\[ \text{for loop}=1:\text{xflen}; \]
854\[ \quad \text{freqmat}(\text{loop},n)=yf(\text{loop}); \]
855\[ \text{end}; \]
856\[ \text{end}; \]
857\[ \text{RMS} = \text{std}(p_{prime}); \]

\%Set the level in dB
860\[ \text{Prms}=20*\text{log10}(\text{RMS}/20e-6); \]
861\[ \text{FreqdB}=20*\text{log10}(\text{freqmat}/20e-6); \]
862\[ \text{for pos}=1:\text{circseg}; \]
863\[ \quad \text{for freq}=1:\text{xflen}; \]
864\[ \quad \quad \text{if FreqdB(freq,pos)<0}; \]
865\[ \quad \quad \quad \text{FreqdB(freq,pos) = 0}; \]
866\[ \quad \quad \text{end}; \]
867\[ \quad \text{end}; \]
868\[ \quad \text{if Prms(pos)<0}; \]
869\[ \quad \quad \text{Prms(pos)=0}; \]
870\[ \quad \text{end}; \]
871\[ \text{end}; \]

\% File Section
873\[ \%Ftotal=10*\text{log10}(\text{freqtotal}); \]
874\[ \text{fname=\text{strcat}(\text{savefile},\text{testnum},\text{dottxt})} ;\]
875\[ \text{save(\text{strcat}(\text{savefile},\text{testnum},\text{dottxt})...} \]
876\[ \quad \quad \text{,'Prms','c','RMS','c','FreqdB','c','defect1','c','Tqz','c', ...} \]
877\[ \quad \quad \quad \text{'Thrz','-ASCII')} \]
878\[ \quad \text{end}; \]
879\[ \text{for zeep}=1:\text{xflen}; \]
880\[ \quad \text{FreqAxis(zeep)=(zeep-1)*RPM/60}; \]
881\[ \text{end}; \]
882\[ \text{Freq60=FreqdB(:,3)}; \]
Freq0 = FreqdB(:, circseg);
figure(1);
polar(zt, Prms);
figure(2);
plot(freqmat);
figure(3);
plot(FreqdB);

fname = strcat(savefile, testnum, vectxt)
save(strcat(savefile, testnum, vectxt), 'Freq60', 'c', 'Freq0', '-ASCII')
fname = strcat(savefile, testnum, deftxt)
save(strcat(savefile, testnum, deftxt), 'Freq60', 'c', 'Freq0', '-ASCII')
figure(4);
plot(FreqAxis, Freq60, 'r', FreqAxis, Freq0, 'b');

disttxt = 'Distortion';
polartxt = 'Polar';
timetxt = 'Time';
print(figure(1), '-dpdf', strcat(savefile, polartxt));
print(figure(4), '-dpdf', strcat(savefile, timetxt));
print(figure(5), '-dpdf', strcat(savefile, disttxt));
end;
% Enter the Radius of the fan R and the Hub radius Ro
R=0.197
Ro=0.05

% s is the number of steps from Ro to R
s=20

% Enter the Lift force vector P and the Drag force vector M
% from the hub element to the tip element
P=-1*[0.002912977 0.008738932 0.014564887 0.020390842 0.026216797 ... 
0.032042751 0.037868706 0.043694661 0.049520616 0.084146635 ... 
0.106161968 0.137106995 0.179774946 0.237488899 0.314147858 ... 
0.414272822 0.543052862 0.706391194 0.910951255 0.352292758];
M=[0 0 0 0 0 0 0 0 0.162373241 0.19835623 ... 
0.237937519 0.281117107 0.327894993 0.378271179 0.432245663 ... 
0.489818447 0.550989529 0.61575891 0.684126591];

% Enter the fan speed in RPM
RPM=2490

% Enter the number of blades B
B=7

% Enter the order of the harmonic of interest m
m=1

Omega=2*pi*RPM/60;
rstep=(R-Ro)/(s-1);
RR=Ro:rstep:R;
OR=1./RR;
k=B*Omega/334;
for z=1:1000
    alpha=z*pi/1000;
    a(z)=alpha;
    L=2.5;
    PP=k*B/(2*pi*L)*(-P*cos(alpha)+(m*B/(k)*M.*OR));
    J=besselj(m*B,k*RR*sin(alpha));
    Pr(z)=abs(PP*J');
    Pressure(z)=20*log10(abs(Pr(z))/20e-6);
    if Pressure(z)<0;
        Pressure(z)=0;
    end;
end
figure(2);
polar(a,Pr)
figure(3);
polar(a,Pressure)
print(figure(3),'-dpdf','GutinLD7Result');
Appendix C

Effects of Apparent Velocity Angle

The test cases presented in Appendix C come from the inflow distortion models used in Chapter 6. The first section presents the effect of changing the apparent velocity angle $\phi_0$ on the noise prediction, using the FIFO inlet distortion model from section 6.3.1. The second section uses the simple plate velocity defect model with edge effects from section 6.4.2 to further illustrate the effect of changing the apparent velocity angle. The aim of this section is to show how using the apparent velocity angle in the loading variation calculation changes the radiation pattern.

The apparent velocity angle is used to calculate the change in lift and drag due to a change in the inflow velocity. Figure 4.2 shows the relationship between the apparent velocity and the thrust and drag forces due to the blade lift. The cases presented look at a constant apparent velocity angle along the entire span of the fan blade for $\phi_0 = 0^\circ, 10^\circ, 20^\circ, 30^\circ$. The final case in each section is using an apparent velocity angle calculated from the velocity profile model for each blade element.

C.1 FIFO

Figure C.1 shows the predicted SPL of the primary BPF, for the baseline case of $\phi_0 = 0^\circ$, plotted with the measured SPL. Here the variations in flow contribute solely to changes in the thrust loading. Figure C.2 shows the analytical prediction for $\phi_0 = 10^\circ$, along with the prediction for $\phi_0 = 0^\circ$ and the measured SPL. This case shows a slight shift of the radiation pattern anti-clockwise. Figure C.3 shows the the analytical result for $\phi_0 = 20^\circ$. Again, the curve rotates slightly anti-clockwise. Similar results are seen with
$\phi_0 = 30^\circ$ in Figure C.4. Finally the analytical result for $\phi_0$ calculated from the total apparent velocity is shown in Figure C.5.

Changes in the calculated apparent velocity angle create only a small change to the predicted noise levels. Observer locations near the plane of the fan face see the greatest influence. Since the calculated apparent velocity angle is between $20^\circ$ and $36^\circ$ across the span of the blade, it is reasonable that the varying apparent velocity angle case in Figure C.5 is not really an improvement over the constant 30 degree case. Simplifying the calculation of this angle to a fixed value is thus justified.

Figure C.1: SPL at primary BPF, dB ref. 20 $\mu$Pa, inflow at $\phi_0 = 0^\circ$
Figure C.2: SPL at primary BPF, dB ref. 20 μPa, inflow at $\phi_0 = 10^\circ$.

Figure C.3: SPL at primary BPF, dB ref. 20 μPa, inflow at $\phi_0 = 20^\circ$. 
Figure C.4: SPL at primary BPF, dB ref. 20 $\mu$Pa, inflow at $\phi_0 = 30^\circ$

Figure C.5: SPL at primary BPF, dB ref. 20 $\mu$Pa, calculated inflow apparent velocity angle
C.2 Simple Blade Model

The simple blade model also shows an anti-clockwise shift as the apparent velocity angle is increased. Figure C.6 shows the analytical result for the SPL of the primary BPF, for the baseline case of the simple blade inflow defect model with edge effects and $\phi_0 = 0^\circ$, compared with the measured SPL. Again in this case, the variations in flow contribute solely to changes in the thrust loading. Figure C.7 shows the analytical result for $\phi_0 = 10^\circ$, compared with the model prediction for $\phi_0 = 0^\circ$ and the measured SPL. As in the FIFO model, there is a shift of the radiation pattern anti-clockwise. Figure C.3 shows the analytical result for $\phi_0 = 20^\circ$, and Figure C.4 shows the analytical result for $\phi_0 = 30^\circ$. Again, similar results to the FIFO case are observed as the curves rotate slightly anti-clockwise. The results for the calculated apparent velocity angle, shown in Figure C.10 again show little difference relative to the 30 degree constant case.

![Diagram showing SPL at primary BPF, dB ref. 20 $\mu$Pa, blade model, inflow at $\phi_0 = 0^\circ$](image)

Figure C.6: SPL at primary BPF, dB ref. 20 $\mu$Pa, blade model, inflow at $\phi_0 = 0^\circ$
Figure C.7: SPL at primary BPF, dB ref. 20 µPa, blade model, inflow at $\phi_0 = 10^\circ$

Figure C.8: SPL at primary BPF, dB ref. 20 µPa, blade model, inflow at $\phi_0 = 20^\circ$
Figure C.9: SPL at primary BPF, dB ref. 20 \( \mu \text{Pa} \), blade model, inflow at \( \phi_0 = 30^\circ \)

Figure C.10: SPL at primary BPF, dB ref. 20 \( \mu \text{Pa} \), simple blade model, calculated inflow apparent velocity angle
C.3 Conclusion

The apparent velocity angle is used in the model to relate the change in lift across the fan blade with the change in thrust in drag loading due to inflow velocity distortions. In practice, this angle changes with these inflow distortions. The change to the noise levels when this angle is modeled within a reasonable bound is small, and is within the expected precision of the model. For this reason, the calculation of the apparent velocity angle does not require using a complicated model. Using a fixed average of the apparent velocity angle across the blade yields acceptable results.
Appendix D

Effects of Inlet Distortion Shape

Choosing the shape of the inlet distortions is key to obtaining realistic results from the SLLAC model. In Chapter 6, the inflow distortion was known only in large scale. It was shown in those examples that changes to the inflow distortion shape have significant effects on the noise generated by the system. Using the 4 individual peak velocity distortions detailed in Chapter 6, this section presents different methods for creating the inflow velocity defect model. These distortions are centered around the fan face at $\theta = 22^\circ$, $\theta = 68^\circ$, $\theta = 120^\circ$ and $\theta = 222^\circ$. For simplicity, each distortion has a peak velocity of 20% over the mean velocity, rising over 21° of rotation followed by an equal decrease to the base level. Figure D.1 shows graphically how these distortions are placed on the fan face. Comparisons of directivity patterns for the first BPF are shown along with selected observer locations for looking at higher order BPF peaks.
Figure D.1: Base inflow velocity model definition for all blade elements

D.1 Primary BPF Radiation Pattern

The undisturbed loading result from section 6.3.1 shows that the predicted SPL of the primary BPF is much lower than the measured SPL (Figure D.2). When the measured distortions are applied as ramp functions over the regions, as indicated in Figure D.1, a significant increase in the SPL is seen (Figure D.3).

Several different shapes can be used to describe the inflow velocity defects, by keeping the mean velocity over the defect constant. For example, a step function of the same width and half the peak velocity change will provide the same mean velocity distortion (Figure D.4). Using this step function in the velocity defect model produces a significant decrease in noise. Figure D.5 shows the SPL of the primary BPF for the step velocity distortion model. The decrease in noise is due to the fact that the $\dot{P}$ term in Eq. 3.53 dominates the acoustic pressure level when the forces fluctuate. In the step function case the $\dot{P}$ term is high only near its edges, while the ramp function maintains a significant contribution from $\dot{P}$ over its entire width.
Figure D.2: SPL at primary BPF, dB ref. 20 $\mu$Pa, with no inflow distortion

Figure D.3: SPL at primary BPF, dB ref. 20 $\mu$Pa, with base inflow velocity distortion
Figure D.4: Half peak, step velocity distortion for all blade elements

Figure D.5: SPL at primary BPF, dB ref. 20 µPa, with half peak, step distortion
Doubling the width of the defects at half of the velocity again achieves the same mean velocity distortion. Figure D.6 shows the velocity distortion for the ramp and step models applied over a wider arc. Here the individual distortions begin to overlap creating a smoother change and correspondingly lower $\dot{P}$. The predicted SPL for the primary BPF shows a significant reduction in the SPL as a result (Figure D.7 and Figure D.8.

![Ramp Defects](image1)

**(a) Ramp Defects**

![Step Defects](image2)

**(b) Step Defects**

Figure D.6: Inflow velocity distortions, double the width of base distortions for all blade elements
Figure D.7: SPL at primary BPF, dB ref. 20 $\mu$Pa, with 2x width ramp inflow distortion

Figure D.8: SPL at primary BPF, dB ref. 20 $\mu$Pa, with 2x width step distortion
Shrinking the width of the inflow distortions and increasing the velocity, again maintaining the same mean velocity distortion, increases the levels. Figure D.9 shows that the inflow distortions remain distinct while the magnitude of $\dot{P}$ increases. The predicted SPL in Figure D.10 and Figure D.11 show an increase over the base ramp and step inflow model results.

![Graph showing inflow velocity distortions](image)

(a) Ramp Defects

![Graph showing inflow velocity distortions](image)

(b) Step Defects

Figure D.9: Inflow velocity distortions, half the width of base distortions for all blade elements
Figure D.10: SPL at primary BPF, dB ref. 20 $\mu$Pa, with half width ramp inflow distortion

Figure D.11: SPL at primary BPF, dB ref. 20 $\mu$Pa, with half width step distortion
Halving the width again (Figure D.12) produces similar results as seen in Figure D.13 and Figure D.14.

![Graph](image)

(a) Ramp Defects

![Graph](image)

(b) Step Defects

Figure D.12: Inflow velocity distortions, quarter the width of base distortions all blade elements
Figure D.13: SPL at primary BPF, dB ref. 20 $\mu$Pa, with quarter width ramp inflow distortion

Figure D.14: SPL at primary BPF, dB ref. 20 $\mu$Pa, with quarter width step distortion
It is clear that the sound level of the primary BPF can be influenced simply by the peak magnitude of the velocity distortion applied. Care must be taken though to ensure that the velocity distortions remain distinct. The shapes presented are simple models to implement and appear sufficient for representing the distortions required to model the primary BPF. Thus, for any upstream flow distortion, one must only obtain a measure of the velocity defect at any downstream location. Conservation of momentum will then determine the peak magnitude. The evolution of the shape of the velocity defect is unimportant and the defect can be included using simple ramp functions with the appropriate magnitude. If the upstream disturbances are many or are close together, the defects may overlap and the method becomes invalid.

D.2 Multiples of the BPF

In general the primary BPF combined with the broadband noise level of the fan control the overall sound pressure level. Depending on the shape of the inflow distortion, the higher multiples of the BPF can become important. In this section, the size and shape of the velocity distortions are used to change the prediction for multiples of the BPF. Rather than construct a radiation pattern, the SPL, in 8 Hz bandwidth, is presented up to 9 times the BPF, measured at two observer locations $\zeta = 0^\circ$ and $30^\circ$.

Figure D.15 and Figure D.16 show the narrow band prediction for each of the 4 distortion widths presented above. As the widths of the individual distortions decrease, the multiples of the BPF increase.

Choosing the distortion shape has a pronounced effect on the calculated noise levels. The basic shape of the radiation pattern for the primary BPF stays the same whether step functions or ramp functions are used. While the simple shapes presented can produce good comparisons with the measured data for the lower BPFs, the agreement decreases with the higher multiples. Since the primary BPF dominates the higher orders, using the simple shapes to predict these levels is sufficient.
Figure D.15: SPL, dB ref. 20 $\mu$Pa, at $\zeta = 0^\circ$ and $30^\circ$ for ramp distortions
Figure D.16: SPL, dB ref. 20 μPa, at \( \zeta = 0^\circ \) and 30° for step distortions
Appendix E

Effects of Inlet Swirl

The shape of the SPL radiation pattern can also be affected by changing the amount of swirl in the inflow. Axial fans create a residual swirl of the inflow due to the momentum change as the blade accelerates the airflow. This swirl can be visualized using a tuft to indicate the direction of the inflow velocity. The effect of this swirl is that a fluid particle passing across the vertical axis, some distance away from the fan face actually meets the fan face at some displaced angle from the axis. The faster the fan spins, the more inlet swirl is created. Another parameter affecting the inlet swirl is the pressure drop across the fan. In cases where the inlet distortion is negligible, the inlet swirl is not important due to the axial symmetry. When the inlet distortions are applied, the phase of their effect on the blade relative to the observer position is critical. In effect, a 3 dimensional radiation pattern could be constructed by plotting the results with varying degrees of inlet swirl. In this Appendix, the effects of varying degrees of inlet swirl, on the overall shape and magnitude of the calculated SPL for the primary BPF, are presented using the FIFO velocity defect model from section 6.3.1 and the blade with contracting flow model from section 6.4.3.

E.1 FIFO Velocity Defect Model

The FIFO velocity defect model from section 6.3.1 is shown in Figure E.1 for swirl angles of $\theta = 0^\circ, 20^\circ, 40^\circ, 60^\circ, 80^\circ$ and $90^\circ$. The inflow distortion pattern is simply shifted around the rotation of the fan for each case, as clearly demonstrated in the following figures.
Figure E.1: Inflow distortions for varying swirl angles (see Figure 6.3 for legend)
The 0 degree swirl case, corresponding to purely axial flow, is shown in Figure E.2. As the swirl in the inflow model is increased, the shape of the radiation pattern begins to shift slightly. Figures E.3 through E.7 show how the first BPF radiation pattern changes as the swirl goes from 20 to 90 degrees. The change in shape is relatively small.

Figure E.2: SPL at primary BPF, dB ref. 20 μPa, FIFO model, swirl angle = 0°
Figure E.3: SPL at primary BPF, dB ref. 20 $\mu$Pa, FIFO model, swirl angle = 20°

Figure E.4: SPL at primary BPF, dB ref. 20 $\mu$Pa, FIFO model, swirl angle = 40°
Figure E.5: SPL at primary BPF, dB ref. 20 \( \mu \)Pa, FIFO model, swirl angle = 60°

Figure E.6: SPL at primary BPF, dB ref. 20 \( \mu \)Pa, FIFO model, swirl angle = 80°
Figure E.7: SPL at primary BPF, dB ref. 20 $\mu$Pa, FIFO model, swirl angle = 90°

### E.2 Blade Velocity Defect Model

The blade with contracting flow velocity defect model from section 6.4.3 is shown in Figure E.8 for swirl angles of $\theta = 0°, 20°, 40°, 60°, 80°$ and 90°. As in the previous section, the inflow distortion pattern, including both the FIFO velocity defect and the velocity defect from the blade, is shifted around the rotation of the fan for each case.
Figure E.8: Inflow distortions for varying swirl angles (see Figure 6.3 for legend)
The 0 degree swirl case, again corresponding to purely axial flow, is shown in Figure E.9. As in the FIFO model, as the inlet swirl is adjusted upward, the shape of the radiation pattern begins to shift slightly. Figures E.10 through E.14 show the changes to the primary BPF as the swirl goes from 20 to 90 degrees. Again, the change in shape is relatively small.

Figure E.9: SPL at primary BPF, dB ref. 20 \( \mu \)Pa, blade model, swirl angle = 0°
Figure E.10: SPL at primary BPF, dB ref. 20 $\mu$Pa, blade model, swirl angle = 20°

Figure E.11: SPL at primary BPF, dB ref. 20 $\mu$Pa, blade model, swirl angle = 40°
Figure E.12: SPL at primary BPF, dB ref. 20 $\mu$Pa, blade model, swirl angle = 60°

Figure E.13: SPL at primary BPF, dB ref. 20 $\mu$Pa, blade model, swirl angle = 80°
For the simple inflow distortion cases the changes to the overall SPL radiation pattern are small. The results show that for the observer location $\zeta = 0^\circ$ there is no change, with varying swirl angles, as expected. As the swirl angle increases, observer locations near $\zeta = 90^\circ$ and $\zeta = 270^\circ$ show the greatest change in SPL. Even in the presence of large inflow distortions, the shape change is still small.

The net effect of the swirl angle is equivalent to shifting the plane of the observer by the negative of the swirl angle. This allows for a simple mechanism to develop a 3 dimensional radiation pattern, even for complex velocity distortions.
Appendix F

Test Chamber and Fan Setup

The experimental validation of the SLLAC model was done in the anechoic chamber in Broughton Hall at NC State University. To get measurements from the downstream side of the fan, an apparatus for mounting the fan to a drive motor and an anechoic duct and settling chamber had to be built.

F.1 Fan Mounting

The fan used in this validation is a Borg Warner LD7 fan that has been trimmed to 396mm diameter. The fan was mounted to a Perkins fan mount bearing through an aluminum spacer and drive pulley, shown in Figure F.1. This apparatus was attached to a frame with a 3kW motor to drive the fan. The frame was attached to the side of the settling chamber with the fan sitting centered in a knife edge shroud, as shown in Figure F.2
Figure F.1: Downstream side of the fan test rig showing the fan mounting system

Figure F.2: Downstream side of the fan test rig showing the mounting frame with motor
F.2 Settling Chamber and Duct

One of the major adaptations to the NC State anechoic chamber that was required was the installation of a duct system to mask the upstream fan noise and bring undisturbed air to the fan face. The 2m x 2m x 2.4m settling chamber was constructed of wood and attached to an anechoic termination. The termination consisted of a 0.5m square tube, 2m in length with a 1.2m square muffler on the end (Figure F.3). The inside of the muffler, duct and settling chamber were all lined with 50mm foam to minimize the noise transmitted through the opening (Figure F.4 and Figure F.5).

Figure F.3: Inlet muffler for bringing air into the settling chamber
Figure F.4: Inside of the square tube for air intake

Figure F.5: Tube outlet into the settling chamber
The muffler brought in air from the outside of the anechoic chamber through openings in its side. The air traveled down the tube into the settling chamber where it passed through 2 layers of screen (Figure F.6). The screens were used to provide a slight pressure drop and distribute the flow from the tube, outward across the whole settling chamber, before entering the fan face (Figure F.7). An inclined manometer (Figure F.8) was used to measure the pressure drop through the muffler, tube, chamber and screens. The low pressure side was connected to a tube that ran into the muffler, down the tube and to the downstream side of the screens. The high pressure side was connected to a tube that ran into the anechoic chamber on the downstream side of the fan thus providing the total static pressure drop. At an airflow rate of 108 m$^3$/s, the average velocity at the inlet to the muffler was 7.8 m/s. Inside the duct, the average velocity was 7.4 m/s. At the settling screens the average velocity was slowed to 0.4 m/s before entering the fan face at an average velocity of 18 m/s.

Figure F.6: Screens inside the settling chamber
Figure F.7: Upstream face of the fan, mounted in the wall of the settling chamber

Figure F.8: Manometer mounted to the side of the inlet muffler
F.3 Disturbance Mount

To test the upstream velocity defects, a system for mounting the test articles had to be mounted to the settling chamber. Brackets were mounted to the wall of the chamber and a transverse rod attached to allow the placement of the test articles. The articles were mounted to the rod using aluminum blocks (Figure F.9). Care was taken to keep the blocks as far from the inlet flow as possible, minimizing their effect on the inflow distortions.

Figure F.9: Mounting brackets used to attach test articles for velocity defects