ABSTRACT

ALI, ABDINUR MOHAMED. Comparison and Analysis of Pre-Service Teachers' Knowledge of Measures of Central Tendency and Measures of Dispersion for Both Normal Data and Non-Normal Data Using Multiple Representations. (Under the direction of Dr. Lee Stiff.)

The purpose of this study was to investigate pre-service teachers' knowledge of measures of central tendency and measures of dispersion for both normal and non-normal data using tabular, numerical, and graphical representations. The participants of the study were 93 pre-service teachers at a southeastern university in the United States. Data of the study were collected using statistical questionnaires and analyzed using statistical analysis tools. The researcher found that, if we ignore the representation of the data, there is no significant difference between pre-service teachers' understanding of measures of central tendency and dispersion for either normal or non-normal data. However, the researcher also found that the representation of normal data can make a difference in pre-service teachers' understanding of measures of central tendency and dispersion. It was also found that for tabular and graphical representations of non-normal data, pre-service teachers understood measures of central tendency better than measures of dispersion. The researcher also observed that for both normal and non-normal data, for both measures of central tendency and dispersion, pre-service teachers performed better when using tabular representations than when using the other two representations. With only one exception, for both normal and non-normal data, for both measures of central tendency and dispersion, pre-service teachers performed least well when using graphical representations.

Comparison and Analysis of Pre-Service Teachers' Knowledge of Measures of Central Tendency and Measures of Dispersion for Both Normal Data and Non-Normal Data Using Multiple Representations

by
Abdinur Mohamed Ali

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APPROVED BY:

Dr. Lee V. Stiff Chair of Advisory Committee	Dr. Sarah B. Berenson
Dr. Karen S. Norwood	Dr. Kimberly Weems

DEDICATION

When we meditate on the start of the universe by Big Bang, the evolution of life, and the stars and constellations that we find in the night skies, in humbleness, in awe and respect, and heart full of gratitude, in remembrance to the Source and the Return, I dedicate this work to THOU WHO IS THE GOAL OF MY LIFE with love and joy.

ACKNOWLEDGMENTS

I thank the source of my strength, my Insight, the Source of one universal existence shared by all, and the Return of all after they complete their intrinsic cycles and experience the shift of consciousness which stops the alternating whirlpool waves in the consciousness and removes the phenomenal illusions. I thank my parents, my family members, and my friends for their support. I thank Dr. Lee V. Stiff for his guidance, coaching, mentoring, and counseling. I am very grateful to have an opportunity to study with him because he is truly an enlightened educator. I thank members of my committee—Dr. Sarah Berenson for her mentoring and support, Dr. Karen Norwood for her motivation and support, and Dr. Kimberly Weems for her constant encouragement and support. I thank all of my teachers and all the people who inspired me and transformed my life. I cannot list all of their names because they are too many people. I promise to myself to help my students the same way that you helped me and I promise to myself that I will mentor them, tutor them, guide them, teach them, listen to them, give them a voice, respect their feelings, encourage them, be advocate for them, treat them as unique individuals, and be blessing to them the same way that you have been a blessing to me in my life.

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CHAPTER 1

INTRODUCTION

The purpose of this study is to compare and contrast pre-service teachers' knowledge of measures of central tendency and measures of dispersion for both normal and non-normal data using multiple representations. It is important for pre-service teachers to know how measures of central tendency and measures of dispersion change based on whether the distribution of the data is normal or non-normal. It is also important for pre-service teachers to know how to select appropriate measures of central tendency and measures of dispersion for tabular, numerical, and graphical representations. According to *Principles and Standards for School Mathematics* (NCTM, 2000), there is a great need that teachers of K-12 mathematics should have a good understanding of how to teach statistical concepts and procedures for both normal and non-normal data. However, data analysis and probability standards can only be effectively implemented if teachers are knowledgeable of statistical methods and techniques, and how to teach them.

Pre-service teachers in mathematics education programs, therefore, need to be taught how to teach mathematics and statistics in K-12 schools. In particular, they are required to have a good understanding of statistical concepts such as measures of central tendency and measures of dispersion. Furthermore, pre-service teachers should have sufficient knowledge of how to select the appropriate measures of central tendency and dispersion when the data are either normally or non-normally distributed. For example, if we look at the heights of members of a 5th-grade soccer team, the heights would approximately follow a normal distribution and the measures of central tendency and measures of dispersion would be the mean and variance, respectively. However, if we included the heights of the coach and the assistant coach with the heights of the 5th-grade players, the new data set would be non-normal data. Accordingly, the proper measures of central tendency and measures of dispersion for the new data set would be the median and the range (or inter-quartiles), respectively.

It is very important to distinguish between these two situations because the shapes of the distributions are different. If the proper designation is not made and the mean is used as the measure of central tendency for both situations, then outliers will increase the mean height and the information about the central tendency of the data will be erroneous.

Kader and Perry (2004) advocate that when teaching statistics, middle school teachers need to use multiple representations. They suggested that teachers need to have complete understanding in multiple representations because some students favor one representation over others. Also, some representations are more efficient than others for the teaching and learning of statistics (Moore, 2008). Graphical representations may be used for both qualitative and quantitative data. If the data is qualitative data, then it is appropriate to organize the distribution of the data in bar graphs and pie charts. Bar graphs give the frequencies for each category of the data and pie charts give the percentages for each category of the data. However, if the data is quantitative data, it is appropriate to display the distribution of the data in histograms, in dot plots, or in stem-plots. The general picture of histograms, dot plots, and stem plots can immediately show the extreme values of the data, the skewness, the central tendency, and the dispersion of the data. It will be visually easy to locate the outliers of the data and decide if the overall shape of the data is normally distributed or non-normally distributed. The graphical representation can also be used to determine the type of skewness and how the skewness affects the measures of central tendency and measures of dispersion (Moore, 2008).

Tabular representations or frequency distributions can also be used for both qualitative and quantitative data. If the data is qualitative data such as the ranks of officers in the U.S. Army, they can be arranged in a frequency distribution (Witte & Witte, 2007). For example, the number of captains, colonels, and generals in the United States Army can be represented in tabular form. For ordered or ranked qualitative data, it is more appropriate to use tabular representations than other representations. Also, if the data is quantitative data, the data can be sorted and organized into classes (Witte & Witte, 2007). For example, the frequency distribution of heights of students in a certain course will have two columns. One column will list each student's height and the other column will list the frequency for each student's height. The distribution of the students' heights will give us the overall shape of the data. The shape of the data may either be non-normally or normally distributed. Also, if we have two sets of tabular data, it is easier to compare their relative frequencies (Witte & Witte, 2007). For example, we can compare the frequency distribution of student heights for two different courses.

Numerical representations can be used for both qualitative and quantitative data. If the data is qualitative data, then it is appropriate to organize the data into categories. For example, some surveys use 1 for females and 2 for males. If three females and three males took the a survey then we can list the categories as 1, 1, 1, 2, 2, 2. These numbers are categories and cannot be used for computation. However, if the data is quantitative data then the numbers in the data should be ordered and listed from the smallest to the largest value. Computationally, it is more convenient to use numerical representations than other representations (Witte & Witte, 2007).

Another important reason for using multiple representations is that some students might prefer one representation over another (Gardner, 1987). Gardner created a theory of multiple intelligences and showed how representations of lessons match or mismatch students' intelligences (Armstrong, 1994). Gardner's classification of intelligences are: (a) word smart, (b) picture smart, (c) number smart, (d) music smart, (e) body smart, (f) self smart, (g) people smart, and (h) nature smart. Gardner recommended to teachers that when they are planning a lesson they should consider using one of these eight different intelligences to present that lesson because some representations are more compatible with some intelligence than others.

Specifically, word smart students learn better by talking and listening. Picture smart students learn better with graphs, images, pictures, and charts. Number smart students learn better with number patterns and formulas; music smart students learn better with sound and rhythm. Body smart students learn better with hands on experiences and self smart students learn better with introspections and in solitude. People smart students learn better with cooperative learning and in groups. Nature smart students learn better in field trips and trips to the outdoors.

Gardner (1987) questioned the wisdom of presenting an entire lesson in numerical representations to a student who is picture smart or a visual learner. Likewise, the use of graphs, images, and charts should not be exclusively used for students who are word smart. Thomasenia (2000) suggested that in order to help students learn mathematics, it is important to recognize the various ways students learn mathematics and the use of multiple representations required to meet the needs of students' understanding of mathematics. Hence, mathematics teachers should have the knowledge of how to teach mathematical concepts or statistical concepts using a variety of instructional representations.

Furthermore, mathematics teachers should have some knowledge of how to select the appropriate measures of central tendency and dispersion for both normally distributed data and non-normally distributed data when the data are presented in either a graphical, tabular, or numerical representation (Kader& Perry, 2006). For example, a typical situation in which selecting appropriate measures of central tendency and dispersion might involve is the income distribution of a company. If the owner of the company has the largest income in the company and the rest of the employees combined make less than the income of the owner, it is important to understand that the distribution of salaries in this company is non-normally distributed. Potential future employees would like to know the correct measures of central tendency and measures dispersion for the data presented about the salaries of this company. The reason is that if the mean is used as the measure of central tendency, it will inflate the average income of the company because the mean is influenced by an extreme outlier.

Most of the recent research in statistics education involves the concepts of dispersion, data analysis, statistical inference, sampling distributions, standard deviation, statistical reasoning with software, representation in statistics in middle grades, and the understanding of the concept of average (Canada, 2008, Sharma, 2007, Sorto, 2004, Watson & Moritz, 1999, Watson & Moritz, 2000).

These areas get a lot of attention because the heart of descriptive statistics is measures of central tendency and measures of dispersion. Moreover, sampling distributions play a major role in explaining the concepts of the Central Limit Theorem and statistical inference. The availabilities of many statistical programs in the marketplace have also created new ways to teach statistics using multiple representations.

However, research about mathematics teachers' understanding of measures of central tendency and dispersion for normal and non-normal data is lacking (Leavy, 2004). In fact, such research involving multiple representations is practically non-existent. Hence, research about pre-service mathematics teachers' knowledge of measures of central tendency and measures of dispersion for both normal and non-normal data using multiple representations will bring much needed information to the field.

CHAPTER 2

LITERATURE REVIEW

Pre-service Teachers

The purpose of this study is to investigate pre-service teachers' knowledge of measures of central tendency and measures of dispersion for both normal and non-normal data using tabular, normal, and graphical representations. Although no studies exist that investigate pre-service teachers' knowledge of measures of central tendency and dispersion for both normal and non-normal data with multiple representations, a few studies have investigated the teaching of the individual concepts of measures of central tendency and dispersion, normal data, non-normal data, and statistical reasoning, as well as using multiple representations in a variety of contexts. The following describes that research and creates a pallet against which the focus of this study may be viewed.

The National Council of Teachers of Mathematics (NCTM, 1989) recommended that all K-12 schools include data analysis and probability in their curricula. NCTM recommends that data analysis and probability instructional programs from kindergarten through grade 12 should enable all students to: (1) formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them, (2) select and use appropriate statistical methods to analyze data, (3) develop and evaluate inferences and predictions that are based on data, and (4) understand and apply basic concepts of probability.

In order to implement NCTM standards efficiently, mathematics teachers should know descriptive statistics, inferential statistics, and probability. Descriptive statistics deal with exploration and summarization of the data. Inferential statistics deal with the production of the data and how to generalize from samples to the population (Moore, 2007). Probability deals with the measurement of uncertainty in the data and quantifies the magnitude of the error in the data. Probability helps us to separate the real information from the error in the data (Diamond, 1959).

Mathematics teachers should know how to explore data using multiple representations (Thomasenia, 2000). They should know how to explore distributions of coin tosses and dice flips or any other types of samples using multiple representations. For example, we can simulate a coin toss by using Excel software and use different charts and graphs available in Excel to explore the overall shape of the data. Random number generators can be used to simulate coin tosses by using zeroes and ones. The zeroes are counted as tails and ones are counted as heads. Then, the results of the simulation can be represented graphically, numerically, or in tabular form. Also, the results from the coin tosses can be generalized to represent the proportion of heads or tails in the population of coin tosses. Furthermore, mathematics teachers should know probability because it measures the reliability and accuracy of the generalization from samples to the population. For example, the proportion of heads and tails are not 50-50 immediately. It takes large number of tosses possibly 100, 000 times before the proportion converges to 50 percent heads and 50 percent tails. The sequence of proportions can be plotted against number of tosses and the convergence of the proportion of heads to 50 percent can be observed graphically. Therefore, sample proportions can be generalized to population proportions (Moore, 2007) and graphical representations can be used to show this convergence of sample proportions to the population proportion.

In order to facilitate the NCTM standards, teachers should know how to test the normality of data. The only way to know when to use the appropriate statistical test is to do normality tests. For example, if the incomes of eight employees from the National Football League (NFL) are compared to the incomes of eight employees from Burger King, we cannot assume normality for the two different samples and use a t-test for the two independent samples. When the sample sizes are small, violations of normality can nullify the accuracy and reliability of t-tests (Witte & Witte, 2007).

It is also necessary for mathematics teachers to know how to use appropriate measures of central tendency and measures of dispersion when data is either normally or not normally distributed. Unfortunately, there are no research findings that adequately address concerns associated with teaching measures of central tendency and dispersion for either normal or non-normal data. Moreover, the role played by the representation of that data (tabular, numerical, or graphical) has not been explored (Leavy, 2004).

The American Statistical Association (GAISE, 2005) developed guidelines for teaching and learning statistics in 2005. The foundation of these guidelines is built on NCTM's *Principles and Standards for School Mathematics* (2000). The main goal of these guidelines is the development of statistically literate citizens (Kader & Perry, 2006). The statistically literate citizen should be able to interpret and evaluate the statistical claims made in newspapers, in radio and television advertisements for new products, and in public opinion polls.

The American Statistical Association guidelines also emphasize data collection design, exploration of data, and the interpretation of results. As an example, in the data collection stage, students in a particular class can take a survey, say, to determine each student's favorite music. In the exploration stage, students can tally the frequency count of each choice and find the median, mean, and mode of the data. The data can then be represented in bar graph, pie graph, or in tabular form (frequency count form). Finally, students can compare one individual's music preference to another individual's music preference, and one individual's music preference to the group's music preference. In the interpretation stage, students can generalize the results of the survey to their class but students should not generalize the results to another class (GAISE, 2005).

As illustrated above, high school graduates should be able to collect data, analyze data, interpret results, and understand the role of variability in the use of data (Kader & Perry, 2006). Hence, the need for statistical literacy demands that K-12 teachers have strong backgrounds in, and a good understanding of, statistics and probability. Consequently, it is important that K-12 teachers of mathematics have a strong understanding of statistics. This is especially true for middle grades and high school math teachers. Therefore, in this study we will examine pre-service middle grades and high school teachers and how well they understand the statistics that they will teach in their grades 6-12 classrooms. The focus of this study is on pre-service teachers' knowledge of measures of central tendency and measures of variability in a variety of contexts. According to Garfield (2002), how to reason about statistical measures of central tendency and variability are two fundamental concepts pre-service teachers need to grasp.

Since there are so few studies that investigate the questions about measures of central tendency and variability that we have raised, the following literature review will look at research studies that addressed statistical measures of central tendency and variability at the elementary, middle, and high school levels in order to gain an understanding of what others have done.

<u>Elementary and middle school pre-service teachers</u>

A study done by Groth and Bergner (2006) indicated that pre-service teachers' procedural and conceptual understandings of measures of central tendency are different. In their study, Groth and Bergner investigated pre-service elementary teachers' conceptual and procedural knowledge (rules and algorithms) of mean, median, and mode. Participants were 46 pre-service elementary teachers. Students were given written questionnaires and they were asked to compare and contrast mean, median, and mode. That is, the pre-service teachers were to show how the concepts are different and show how they are similar. Pre-service teachers were also expected to define mean, median, and mode. This study used four levels to describe the procedural and conceptual knowledge of pre-service teachers. They are the uni-structural, multi-structural, relational, and extended abstract levels of understanding. The uni-structural level is when pre-service teachers show knowledge in how to calculate each measure. The multi-structural level is when pre-service teachers show knowledge in how each measure can be used as a data analysis tool. The relational level is when pre-service teachers can explain that all three measures can be used as the measures of center for the data. The extended abstract level occurs when pre-service teachers show knowledge of how to select the right measure when one of the three measures is a better measure of the center of the data. Groth and Bergner found that a majority of pre-service teachers showed uni-structural and multi-structural levels of understanding of measures of central tendency.

The overall results of the study indicated that the majority of the pre-service teachers only had procedural knowledge (rules and algorithms) of mean, median, and mode.

A study conducted by Leavy and O'loughlin (2006) identified pre-service teachers' understanding of the role and function of the mean as the statistical measure of central tendency. Participants were 263 pre-service elementary teachers in their first year of a Bachelor of Education degree at a college of education in the southwest of Ireland. Participants were given five tasks to perform in order to gain insight into pre-service teachers' understanding of "mean." The overall findings of this study were: 57% of the participants used the mean correctly to compare two data sets, 21% of the participants gave accurate answers for a weighted means problem, and 88% of the participants constructed a data set to match a known mean value. The results also indicated that 25% of the students had some conceptual understanding of the mean, 75% of the students showed limited computational-based understanding of the mean, and 25% of the students confused the mean with the mode. The result of the study supported Groth and Bergner's results (2006). That is, the majority of the pre-service teachers have limited computational understanding of the concept of mean.

Dispersion is one of the most important statistical measures that is taught in schools (Moore, 1990; GAISE, 2005). A study carried out by Sharma (2007) investigated pre-service teachers' understanding of dispersion. Participants were 24 pre-service teacher education students who were given written questionnaires in sampling and distribution contexts in order to explore their understanding of statistical dispersion. For the sampling question, there were two hospitals–a large hospital and a small hospital–for which the chance of having a boy or a girl baby was the same. Pre-service teachers were asked to decide if on a particular day a hospital was more likely to record 80 percent or more female births? For the distribution question, students were asked to imagine throwing a die 60 times and then predict how many times each number (1, 2, 3, 4, 5, or 6) might show up. The overall findings of the study showed that 29% of the pre-service teachers gave the appropriate responses to the sampling question and 8% of the pre-service teachers gave the appropriate responses to the distribution question. Hence, it was found that pre-service teachers performed better on the sampling question than on the distribution question.

A study conducted by Canada (2008) discussed pre-service teachers' understanding of dispersion. Participants were 30 pre-service teachers at a university in the Northwest of the United States. Participants were given written questionnaires in order to examine their understanding of variability in sampling contexts. In the first week of the class before the pre-service teachers had any instruction on probability and statistics, they were first surveyed and then asked to draw, with replacement, six samples of size 10 from a jar containing 60 red candies and 40 yellow candies and indicate how many red candies might be in each of the six samples? Next, the pre-service teachers were given sampling experiments involving computer programs and were given instructions on using probability simulations and graphs. Following the instruction, pre-service teachers were given a second survey and were asked to draw, with replacement, 6 samples of size 100 from a jar containing 600 red candies and 400 yellow candies and indicate how many red candies might be in each of these six samples? The researchers found that on the first survey, 13.3% of the students gave explanations explicitly involving reasoning about measures of central tendency and dispersion. This result showed that a large number of pre-service teachers had difficulties with measures of central tendency and measures of dispersion. However, on the second survey after instructional interventions were used, pre-service teachers showed improvement in their knowledge of measures of central tendency and dispersion. Specifically, 30% of the pre-service teachers gave explanations explicitly involving reasoning about measures of central tendency and dispersion. It was also determined that on the first survey, 26.7% of the pre-service teachers gave explanations using either measures of central tendency or dispersion but not both, while on the second survey, 40% of pre-service teachers' gave such explanations. On the first survey, 30% of pre-service teachers made mistakes involving explanations using additive thinking or informal notions of chance; on the second survey, only 10% of pre-service teachers made such mistakes. Finally, the results indicated that pre-service teachers' understanding of dispersion improved from the first survey to the second.

Canada (2006) examined pre-service teachers' concepts of dispersion in a probability context. The participants of the study were 30 pre-service elementary teachers at a northwestern university in the United States. The participants were given two surveys. The first survey was administered before the instructional interventions of the study and the second survey was given after the instructional interventions. In the first survey, pre-service teachers were asked to imagine someone tossing a fair coin 50 times and predict how many times the outcome would be heads. In the instructional interventions, pre-service teachers were given hands-on activities and conducted computer simulations similar to the coin-tossing experiment. In the second survey, pre-service teachers were asked to imagine someone spinning a half-black and half-white spinner 50 times and predict how many times the spinner would land on black. The first survey and the second survey questions are similar. The study found that pre-service teachers' concepts of dispersion improved following the instructional interventions.

A study conducted by Groth and Bergner (2005) examined pre-service elementary school teachers' metaphors for the concept of statistical sampling. Participants of the study were 54 pre-service elementary teachers at a northeastern university in the United States. In this study, each of the pre-service teachers was asked to write a metaphor for statistical sampling. Then, the metaphors were categorized in order to see if they covered the important characteristics of statistical sampling. The result of the study showed that the majority of the pre-service teachers mentioned sampling as being a part of a whole but did not construct metaphors for the randomness and dispersion aspects of sampling. The study found that less than 20% of the pre-service teachers could write metaphors which covered the important aspects of a statistical sample.

A study conducted by Canada (2004) examined elementary teachers' conceptions of dispersion. Participants of the study were 30 pre-service elementary teachers at a northwestern university in the United States. Participants were given a survey about measures of dispersion followed by an instructional intervention and then a survey about measures of dispersion to determine the effects of the intervention. The instructional interventions consisted of class activities in which spinners, dice, and coin tosses were used to focus on the concepts of dispersions. The overall finding of the study suggested that pre-service teachers had difficulties with the concepts of dispersion.

A study conducted by Heaton and Mickelson (2002) investigated the learning and teaching of statistical investigations to pre-service teachers. Participants of the study were

44 pre-service elementary teachers at a mid-western university in the United States. In this study, in one of the assignments, pre-service teachers were asked to pose questions, carry out data collections and data analysis, and report the findings. The results of the study showed that pre-service teachers had difficulties with posing good research questions. In another assignment, the classroom practices of the pre-service teachers conducting statistical investigations and using technology were observed. The overall finding of the study suggested that pre-service teachers did not develop depth in statistical knowledge and method in teaching statistical investigations.

Godino, Cañizares, and Díaz (2003) examined teaching probability to pre-service primary school teachers through computer simulations. Participants of the study were 132 pre-service elementary teachers. Pre-service teachers were asked to complete questionnaires about probability and statistical reasoning and conduct simulation experiments. The questionnaires had 5 problems. In the first problem, a six-sided die was tossed and pre-service teachers were asked to predict the likelihood of certain outcomes. In the second problem, pre-service teachers were asked to predict which hospital is more likely to record 80% or more female births if half of all new-born babies are girls and half are boys, and Hospital A records an average of 50 births a day while Hospital B records an average of 10 births a day. In the third problem, two dice are thrown simultaneously and pre-service teachers were asked to predict the chances of obtaining either a 5 and a 6 on a roll, or a 5 and a 5 on a roll. In the fourth problem, three dice are thrown simultaneously and pre-service teachers were asked to predict which outcomes are most likely? In the fifth problem, three dice are thrown simultaneously and pre-service teachers were asked to predict which outcomes are least likely? After the written questionnaires were completed, pre-service teachers were asked to simulate the five problems cited above using computers and compare their findings with their classmates, and then compare their findings to their written responses to the questionnaire. The researchers found that pre-service teachers had misconceptions about randomness and dispersion of probability events. The result of this study agrees with the results of Sharma (2007) which indicated that the majority of pre-service teachers had problems with the Law of Small Numbers or statistical sampling.

Paparistodemou, Potari, and Pitta (2006) investigated how prospective teachers teach probability to children. Participants of the study were 23 pre-service elementary teachers in Cyprus. Pre-service teachers were observed while they were teaching probability lessons to children. The overall finding of the study suggested that pre-service teachers had difficulties in using open-ended questions to explain concepts such as randomness, certainty, the probable, and the impossible in their teaching of probability.

A study done by Sorto (2004) discussed pre-service teachers' understanding of data analysis. Participants were 42 pre-service teachers. Participants were given eight tasks. For each task, participants were asked a number of questions to measure their statistical knowledge in general, and their statistical knowledge for teaching. The overall findings of this study indicated that pre-service teachers did better with questions related to statistical knowledge than questions related to statistical knowledge for teaching.

Carter (2005) investigated knowledge and understanding of probability and statistics topics by pre-service pre-K-8 teachers. Participants were 342 pre-service teachers at southern university in the United States. Participants were asked to complete multiple choice questions about measures of central tendency, measures of dispersion, and probability. The study found that many pre-service teachers had conceptual errors about probability and statistics. For example, only 11% of the pre-service teachers understood how to use the Law of Large Numbers. Nearly half of the pre-service teachers, 47%, had only one representation of measures of central tendency.

Leavy (2004) examined how pre-service teachers described the appropriate representative measures of central tendency and dispersion for five different distributions of data. Participants were 283 Irish pre-service elementary teachers in their 2nd year of a 3-year undergraduate program. Participants were given five tasks. For each task, participants were asked to identify the appropriate summary statistics (e.g., measures of central tendency or measures of variability). The five tasks represented: (a) positively skewed numerical data (Task 1), (b) negatively skewed numerical data (Task 2), (c) positively skewed graphical data (Task 3), negatively skewed graphical data (Task 4), and bimodal symmetrical graphical data (Task 5). Leavy found that: 76% of pre-service teachers used a measure of central tendency and 2 % of pre-service teachers used a measure of dispersion in Task 1; 85% of pre-service teachers used a measure of central tendency and 11% of pre-service teachers used a measure of dispersion for Task 2; 60% of pre-service teachers used a measure of central tendency and 26% of pre-service teachers used a measure of dispersion in Task 3; 64% of pre-service teachers used a measure of central tendency and 27% of pre-service teachers used a measure of dispersion for Task 4; and 66% of pre-service teachers used a measure of central tendency and 16% of pre-service teachers used a measure of dispersion Task 5.

For all five tasks, pre-service teachers were expected to use both measures of central tendency and measures of dispersion; the use of one measure was not acceptable.

Leavy found that the type of representation had an influence on pre-service teachers' choice of an appropriate summary statistics. Measures of central tendency were used more often for non-graphical data and measures of dispersion were used more often for graphical data. The overall findings of the study suggested that 41% of the pre-service teachers used measures of central tendency as summary statistics for the distribution of any data. Thus, measures of dispersions are not widely used by pre-service teachers to describe the distribution of data in summary statistics.

Another study performed by Leavy (2006) observed the development of pre-service teachers' understanding of the concept of distribution. Participants were 23 pre-service teachers in a one-year master's degree program at a university in the United States. Participants were asked to perform two experiments, namely, the Lentil Growth Experiment and the Popcorn Experiment. For each experiment, pre-service teachers were asked to compare the distributions of the experimental data. The participants of the study were divided into eight groups and the experiments were done over a 15-week semester. In the first stage, lentil growth experiments were performed. In the Lentil Growth Experiment, pre-service teachers were asked to record the heights of lentil sprouts. Pre-service teachers were asked to collect the data under four different conditions: lentil grown with water and lighted conditions, lentil grown with water and non-lighted conditions, lentil grown with lemon and lighted conditions, and lentil grown with lemon and non-lighted conditions. Then, each group was asked to compare the data from all other groups including their own data. Also, each group was asked to describe the appropriate measures of central tendency and measures of variability for each data distribution. In the second stage of the investigation, instructional interventions were used. Students had group discussions about data modeling and data analysis. Also, advantages and disadvantages of graphical representations and how to locate the measures of central tendency and measures of variability were explained. In third stage, the Popcorn Experiment was performed and same eight groups worked with this experiment and were asked to measure the distance each popped kernel landed from the popper and select the appropriate measures of central tendency and measures of dispersion for data distribution. For the Lentil Growth Experiment, out of the 8 groups, 4 groups used bar graphs and means as the measures of central tendency, and one group used box-and-whisker plots with both measures of central tendency and measures of variability. For the Popcorn Experiment, out of the 8 groups, 5 groups used stem-and-leaf graphs and one group used box-and-whisker plots. The overall findings of the study suggested that for both the Lentil Growth Experiment and the Popcorn Experiment, most pre-service teachers used measures of central tendency as summary statistics to compare distributions. The results of this study support the findings of another study done by Leavy (2004) which reported that pre-service teachers focused more often on measures of central tendency as a summary statistics, and that the majority of the pre-service teachers had difficulties with the construction of graphical representations of data.

High school pre-service teachers

Gfeller, Niess, and Lederman (1999) investigated how high school pre-service teachers used multiple representations in solving arithmetic mean problems. Participants were 13 pre-service science teachers and 6 pre-service math teachers in a one-year Master of Arts in Teaching program. All of the participants had Bachelor of Science degrees. These pre-service teachers were asked to solve problems involving means found in different contexts–problems may have required students to find means or missing values in data sets whose means were known for data given in both numerical and graphical formats. An added requirement was that each solution had to have two different explanations.

The study investigated two research questions: (a) Do pre-service science teachers view the mean differently than pre-service math teachers with respect to their use of the computational algorithm? and (b) Do pre-service science and math teachers view the mean differently with respect to their use of fair share or balancing deviations? (Fair share occurs when data points are distributed into equal piles).

The findings of the study indicated that no significant difference was found between the pre-service science and math teachers' use of the computational algorithm for both numerical and graphical data. Moreover, the computational algorithm was the most widely used method for solving the problems for both groups of teachers. However, a difference was found between the two groups in their use of the concept of fair share or balancing deviations. Specifically, the pre-service math teachers used fair share more than the science pre-service teachers.

Bowen and Roth (2005) investigated pre-service science teachers' interpretation of data and graphs. Although this study is about high school science teachers, the researchers examined pre-service teachers' understanding of data collection and data analysis which are relevant to statistics education. The participants of this study were 25 pre-service science teachers who were enrolled in a teacher preparation program in a Canadian university. The Canadian teacher preparation program only accepted applicants who had already completed a degree in some content area. The participants of the study were given two tasks. In the first task, pre-service secondary teachers were asked to design their investigations by collecting data, transforming the data, and interpreting the transformed data. In the second task, participants were asked to interpret a set of raw data in order to investigate their data and graph-analysis skills. The study clearly demonstrated that these pre-service science teachers were not ready to teach data collection and data analysis.

Watson (2000) examined whether pre-service mathematics teachers' understanding of sampling is intuitively based or mathematically based. Intuitive understanding is reaching correct conclusions without mathematical arguments, and mathematical understanding is the use of ratios, fractions, or use of binomial and normal distributions for reaching correct conclusions. Participants of the study were 33 pre-service secondary mathematics teachers in Australia. Pre-service teachers were given a sampling problem about a large hospital and a small hospital. In one year, each hospital recorded when 60% or more of the babies born were boys. Then, pre-service teachers were asked to decide which hospital had fewer days with more than 60% male births or more days with more than 60% male births. Watson found that 15 pre-service teachers used intuition, 12 pre-service teachers used mathematics and 6 pre-service teachers used both mathematics and intuition.

A study done by Gonzales and Pinto (2008) examined pre-service high school mathematics teachers' concepts of graphical representations. The participants of the study were four pre-service secondary mathematics teachers in Spain. The participants were asked to classify 20 graphical representation problems and translate between different graphical representational systems. The graphical representations used in the study were histograms, bar graphs, stem-and-leaf graphs, and frequency polygon graphs. The researchers found that pre-service teachers had limited knowledge of graphical representations.

In another study, Groth (2005) examined pre-service high school teachers' understanding of stochastics. Groth (2005) used the same sampling problem as Watson (2000). Participants of this study were 14 pre-service high school teachers at a mid-Atlantic university in the United States. Pre-service teachers were given a sampling problem about a large hospital and a small hospital. In one year, each hospital recorded when 60% or more of the babies born were boys. Then, pre-service teachers were asked to decide which hospital had fewer days with more than 60% boy births or more days with more than 60% boy births. The course used an online discussion board and each pre-service teacher was asked to make at least four postings to the online discussion board about the sampling problem. Then, the responses of the pre-service teachers were categorized into five levels. Level 1 is pre-structural and is the response that does address the task at hand. Level 2 is uni-structural. This is where one relevant aspect is shown in the response. Level 3 is multi-structural. Here, several relevant aspects are shown in the response but not integrated. Level 4 is relational and is defined by having several relevant aspects shown in the response that are integrated. Level 5 is extended abstract. In this case, the response goes beyond the immediate task and introduces new levels of abstraction.

The study found that 10 responses showed Level 2 understanding, one response showed Level 3 understanding, two responses showed Level 4 understanding, and one response showed Level 5 understanding of the sampling problem. Thus, the pre-service teachers in this study were not functioning at a very high level of understanding. Indeed, the results of this study agree with the findings of Sharma (2007), Watson (2000), and Godino, Cañizares, and Díaz (2003) that reported that the majority of pre-service teachers had difficulties with the Law of Small Numbers or statistical sampling.

Gonzales and Pinto (2008) examined pre-service high school mathematics teachers' concepts of graphical representations. The participants of the study were four pre-service secondary mathematics teachers in Spain. The participants were asked to classify 20 graphical representation problems and translate between different graphical representational systems. The graphical representations used in the study were histograms, bar graphs, stem-and-leaf graphs, and frequency polygon graphs. The study found that pre-service teachers had limited knowledge of graphical representations.

The next series of studies investigated in-service teachers' knowledge of measures of central tendency and measures of dispersion.

In-service Teachers

Elementary and middle school in-service teachers

Jacobbe (2008) examined elementary school teachers' understanding of the mean and median. The participants of the study were three elementary teachers in the United States. Over a course of 18 months the three teachers were interviewed and completed assessments and questionnaires. The interviewer visited each teacher's classroom at least 12 times over the 18 months. On a written questionnaire, the three teachers were given eight questions about the mean and the median. Two teachers got 63% of the eight questions correct and the third teacher got 75% of the eight questions correct. The results of the study showed that two of the three teachers had procedural knowledge of the measures of center and one of three teachers had conceptual understanding of the measures of center. However, the teacher who demonstrated conceptual knowledge of measures of center did not have associated procedural knowledge. This result was surprising. When pre-service teachers were asked to describe what does the mean represent and what is the difference between the mean and the median, pre-service teachers with procedural knowledge only described correctly how to calculate the mean and the median but could not describe correctly the conceptual difference between the two. On the contrary, the teacher with conceptual knowledge described correctly the difference between the mean and the median but could not calculate correctly the measures of center. The study suggests that teachers may lack the connection between procedural and conceptual aspects of measures of center. It also showed that teachers may not understand when one measure is more appropriate or more useful than the other.

Groth (2009) analyzed characteristics of teachers' conversations about teaching mean, median, and mode. Participants were nine elementary and middle school teachers. Teachers were asked to read an article about students' difficulties in learning mean, median, and mode. Then, teachers were asked to discuss how the teaching of measures of central tendency can be improved in schools. The discussion of the article took place in online discussion board. The conversation lasted one week and each teacher was asked to make at least four posts to the online discussion board. Four teachers suggested that the meanings of the measures of center should not be taught to young students. Two teachers suggested that the meanings of the measures of center should be taught to young students, and two teachers suggested it should be taught with box-and-whisker plots. Another teacher suggested that technology will be needed in order to teach the meanings for measures of center.

At the end the study, the claims and the supporting data of the teachers were categorized and analyzed. The study reported that many teachers in the study believed that young students should not be taught conceptual knowledge about measures of central tendency. Instead, they should be taught procedural knowledge. The findings of this study suggest that if the students have difficulties with understanding measures of central tendency, teachers' beliefs about not teaching the topics conceptually may contribute to student difficulties.

Burgess (2007) investigated teacher knowledge needed and used in teaching statistics at elementary schools. The participants were four elementary school teachers in their second year of teaching in New Zealand. The participants were videotaped while teaching statistics in an elementary school. For each teacher, four statistical lessons were videotaped and analyzed for the study. Then, there were follow-up interviews after each lesson. The findings of the study suggested that teachers need specialized knowledge of the content of statistics in order not to create misconceptions in students' learning of statistics at early age.

Meletiou-Mavrotheris, Paparistodemou, and Stylianou (2009) investigated how the integration of technology in professional development for elementary teachers enhanced their understanding of statistics. The participants of the study were 12 in-service elementary school teachers in Cyprus. In the initial stage of the professional development seminar, teachers were given written questionnaires. In the written questionnaires, they were given data about areas of countries in Europe, population numbers, population density, percentage of urban population, per capita income, average life expectancies of women and men in Europe, patients per doctor, students per teacher, percentage of TV sets per household, and literacy in Europe. Then, teachers were asked to find the mean life expectancies of women and men, and per capita income of Europe and compare them to those of Cyprus. At this stage, all the teachers focused on calculating the measures of central tendency and did not use measures of dispersion. All of the teachers used only numerical calculations and did not use graphs. In the second stage, the teachers were asked to work in groups and explore the same questions with Tinkerplots software. After using Tinkerplots, all of the teachers started to focus on both the measures of central tendency and the measures of dispersion for the analysis of data. Teachers also explored numerous relations between the two measures using graphs. The group activities were videotaped and the teachers were interviewed. The results of the study suggest that in the initial stages of the study, in-service teachers used measures of central tendency exclusively and did not consider measures of dispersion. However, in the final stages of the study, results suggested in-service teachers' understandings of dispersion had improved.

Sanchez and Garcia (2008) examined middle school teachers' concepts of statistical dispersion. The participants of the study were six middle school teachers in Mexico. In the first stage, the participants were given a written questionnaire. The participants were asked to imagine that a die is thrown 60 times and predict the number of times each of the numbered faces will show up. At another time, they were asked to imagine that a die is thrown 1000 times and predict the number of times each of the numbered faces will show up. In the second stage, the teachers carried out a guided activity to answer the same problems from a written questionnaire using computer simulations. In the third stage, the results of the two previous stages were compared and contrasted. In the final stage, the participants were interviewed and videotaped. The results of the study indicated that before the guided activity, teachers' responses were almost deterministic and they predicted that for the 60 throws, each numbered face of the die will appear 10 times. After the guided activity, the teachers realized because of dispersion ideas that each number will appear in a range of values. Each number might appear more than 10 times or less than 10 times or exactly 10 times. The overall finding of the study indicated that the in-service teachers had difficulties in understanding the concept of dispersion.

High school in-service teachers

Peters (2009) examined AP statistics teachers' perceptions about dispersion. Participants of the study were 16 high school statistics teachers. Teachers were given questionnaires to complete. The results of the study indicated that the concepts of statistical dispersion can be classified into three types, namely, expected but explainable and controllable (EEC), noise in signal and noise (NSN), and expected with deviations from expectation, (EDE) (Peters, 2009, p. 329). Teachers with expected but explainable and controllable concepts focused on the design of the data and saw variation as something that needs to be controlled. Teachers with noise in signal and noise concepts focused on the data analysis and they saw variation as something that needs to be explored. However, the teachers with expected with deviations from expectation concepts focused on inference. They saw variation as something that needs to be modeled and expected. The study found that only five out of 16 high school teachers showed connected reasoning across the three types of statistical dispersions.

Silva and Coutinho (2008) explored how secondary mathematics teachers reason about dispersion of a univariate distribution. The participants of the study were nine Brazilian in-service teachers who were taking teacher training course in statistics. The nine teachers were given a frequency distribution with seven age groups and the number of people in each age group. Then, a histogram for the frequency distribution was created and the teachers were asked to analyze the variation of the data. The study revealed that the teachers could calculate mean, median, and standard deviations but did not know how these concepts were related and that none of the teachers showed a complete understanding of dispersion.

In an earlier study, Silva and Coutinho (2006) investigated how secondary mathematics teachers reason about dispersion. The participants of the study were 10 Brazilian in-service teachers. The teachers created questionnaires and collected data from 110 teachers. Then, participants were asked to find the characteristics of the ages for the 110 teachers who took part in the survey and the length of tenure in the profession. The characteristics of the data are measures of central tendency and measures of dispersion. When the teachers were asked to analyze the dispersion in the data, some of them said that if the standard deviation is high, then the dispersion of the data is large. The results of the study indicated that the teachers focused on high or low standard deviations without connecting the standard deviation values to the scattered values around the mean. Accordingly, the study demonstrated that these in-service teachers knew how to compute measures of dispersion but they did not know how to relate measures of dispersion to the measures of central tendency.

Hammerman and Rubin (2006) investigated how the shape of data affects in-service teachers' analysis of two distributions in a software exploration environment. The participants were nine in-service teachers. Six were high school teachers and three were middle school teachers in the United States. The participants used two representational tools–Tinkerplots and Fathom. Six teachers used Tinker plots and three teachers used Fathom. Then teachers were asked to analyze two sets of data. One set was a symmetric distribution about the height and gender of a group of Australian teenagers. The second set of the data was a skewed distribution about the gender of a group of Australian teenagers and the money earned by these teenagers from part-time jobs. The results indicated that teachers used only measures of central tendency to describe the normal data or symmetrically distributed data, but for the skewed distribution or non-normal data, the teachers mostly used a cut point or a single value to compare how much data which was below or above the cut point. The study found that most of the teachers used only measures of central tendency for both symmetric and skewed distributed data when using either Tinkerplots or Fathom.

According to a study conducted by Eichler (2007) in Germany, in-service teachers have multiple belief systems about how to plan for statistics and probability instruction. Participants in the study were 13 high school teachers who were observed while teaching statistics and probability classes. Based on the explanations and the teaching approaches used by the teachers, they were classified into four types of approaches: the traditionalist approach, the application approach, the everyday-life approach, and the structural approach. Traditionalist teachers teach statistics and probability as a mathematics course using algorithms and abstractions. Application-oriented teachers combine theory with applications. Everyday-life oriented teachers use real-life social statistics, and everyday statistics reported the news in order to teach how to interpret and critique real-world situations. Structural-oriented teachers promote the teaching of statistics and probability as a demonstration of logical connections or as the study of general logical systems.

Summary

As described above, many research studies have examined pre-service and in-service teachers' understanding of measures of central tendency and dispersion. Surprisingly, there is no research about pre-service or in-service teachers' knowledge of measures of central tendency and dispersion for either normal and non-normal data using tabular, numerical, and graphical representations. This research study will break new ground by examining pre-service teachers' understanding of measures of central tendency and dispersion in these representational contexts.

Importance of the Problem

An important goal of teaching measures of central tendency and dispersion to K-12 students is for students to become proficient in identifying an appropriate summary statistics for given data. Summary statistics reduces many data points into two measures. These measures give the overall picture of the data. It is crucial that these two measures should be appropriate to the given data. The appropriateness of the selection of summary statistics is linked to the data type and the representation type. If the selections of the two measures are incorrect, then the overall picture of the data will be incorrect as well.

Unfortunately, pre-service and in-service teachers' knowledge of measures of central tendency and dispersion in the context of whether the data is normal data or non-normal data, or whether the data representation is tabular, numerical, or graphical representations is missing from the research literature.

This study investigates pre-service teachers' knowledge of measures of central tendency and measures of dispersion for both normal and non-normal data using tabular, numerical, and graphical representations. This research study will also benefit teacher education programs as a tool to gauge prospective math teachers' knowledge of these statistical measures.

Research Questions

This study examines pre-service teachers' knowledge of measures of central tendency versus measures of dispersion for normal and non-normal data. Each research question may be posed for each of three representations: tabular, numerical, and graphical. Similarly, each research question may be posed for each of four student classifications: sophomores, juniors, seniors, and licensure-only students. Hence, the statements of the research questions have been abbreviated with this in mind. Thus, the study was conducted to answer the following research questions.

1. Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data for each student classification?

2. Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data using a given representation?

3. Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data using a given representation for each student classification?

4. Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data for each student classification?

5. Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data using a given representation?

6. Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data using a given representation for each student classification?

7. For each student classification, is there a correlation between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data for a given representation?

8. For each student classification, is there a correlation between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data for a given representation?

9. For each student classification, is there a correlation between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for all normal data regardless of the type of representations?

10. For each student classification, is there a correlation between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for all non-normal data regardless of the type of representations?

11. For each student classification, is pre-service teachers' knowledge of measures of central tendency for normal data across representations statistically significant?

12. For each student classification, is pre-service teachers' knowledge of measures of dispersion for normal data across representations statistically significant?

13. For each student classification, is pre-service teachers' knowledge of measures of central tendency for non-normal data across representations statistically significant?

14. For each student classification, is pre-service teachers' knowledge of measures of dispersion for non-normal data across representations statistically significant?

CHAPTER 3

METHODOLOGY

Quantifying the Constructs of the Problem

A questionnaire that measures pre-service teachers' knowledge of measures of central tendency and measures of dispersion for both normal and non-normal data was created using test items from Assessment Resource Tools for Improving Statistical Thinking (ARTIST, 2007), a website providing a variety of assessment resources for teaching first courses in Statistics. A given score on the questionnaire will be the operational definition of student knowledge. Student knowledge will be determined for each classification—sophomores, juniors, seniors, and licensure-only students.

Participants of the Study

The target population for this research study was pre-service teachers who were math education majors at a land-grant research university in the Southeast. Participants were selected from the sophomore, junior, and senior classes; and the group known as licensure-only students were also included in the study. All of the pre-service teachers in the math education courses were invited to participate in the study. There were 28 sophomores, 10 juniors, 40 seniors, and 15 licensure-only pre-service teachers who agreed to participate. Subjects were given code numbers prior to the statistical analysis of the data in order to protect their identities.

Constructing the Statistical Questionnaires

A statistical questionnaire was used as the assessment instrument for the study; it may be found in Appendix A. The statistical questionnaire contained 26 questions designed to cover measures of central tendency and measures of dispersion for both normal and non-normal data with tabular, numerical, and graphical representations.

Each representation had eight questions—four questions about measures of central tendency and measures of dispersion for normal distribution and another four questions about measures of central tendency and measures of dispersion for non-normal or skewed distributions.

The items of the questionnaire were randomly arranged, resulting in six different questionnaires. Each of the six questionnaires was either coded as 1 (tabular), or 2 (numerical), or 3 (graphical) per the sequence of items found on the questionnaire. Hence, a questionnaire in which the tabular representations were followed by the graphical representation, followed by the numerical representation was labeled as Q132. Accordingly, the resulting six statistical questionnaires were identified by their permutations as: Q123, Q231, Q132, Q312, Q132, and Q213.

Data Collection

The statistical questionnaires were administered to sophomores, juniors, seniors, and licensure-only students. Students were randomly assigned to a questionnaire. A time limit for the questionnaire was established at 45 minutes.

Data Analysis

In order to properly analyze the questionnaire data, it was important to determine if the responses to each questionnaire item were normal or non-normal. Possible ways to determine the normality of the responses is to use the Kolmogorov-Smirnov Test or the Shapiro-Wilk Test.

The Kolmogorov-Smirnov Test is similar to the chi-square test. However, instead of comparing the expected and observed frequencies, the Kolmogorov-Smirnov Test compares the cumulative relative frequencies of the sample and the cumulative Gaussian relative frequencies (Ott, Mendenhall, & Larson, 1978), followed by taking the differences of the respective cumulative relative frequencies. The Kolmogorov-Smirnov Test uses the maximum deviation of the differences while ignoring the sign of the differences of the relative frequencies. If the maximum absolute differences have a p-value less than 0.05, then the sample did not come from a normally distributed population.

The Shapiro-Wilk Test (Shapiro & Wilk, 1965) is also used to check whether data are normally distributed or not. In the Shapiro-Wilk Test the sample scores are ordered from the smallest value to the largest value. Then, the sample values are correlated with normal distribution values. If the p-value is less than 0.05, then the test indicates that the sample is not drawn from a normally distributed population.

It was decided to use both the Kolmogorov-Smirnov Test and the Shapiro-Wilk Test because they work in different ways. Hence, using them together would check the robustness of the normality of the responses. If the results of Kolmogorov-Smirnov Test and the Shapiro-Wilk Test did not agree, we selected the results of the Shapiro-Wilk Test because it is more robust test of the two.

When we compared the scores from two groups, if the response data were not normal then we used the Wilcoxon Test. In the Wilcoxon Test (Hollander & Wolfe, 1999), the assumption of the null hypothesis is that there are no differences in medians of the two groups. If the p-value is less than 0.05, then the medians are different and the null hypothesis is rejected. Furthermore, when we compared the scores from three groups or four groups, if the normality tests indicated that the response data were not normal, and the scores were from independent samples, then we used the Kruskal-Wallis Test. The Kruskal-Wallis Test is used to find the differences in medians. If the scores were from related samples, then the Friedman Test was used to detect differences in medians for the different groups instead of using the ANOVA test to find differences among means. The Friedman Test uses a chi-square distribution to detect differences among medians (Hollander & Wolfe, 1999). In the Friedman Test the scores are placed in a table with rows and columns. The rows are called blocks and each row represents a specific student; the columns are called treatments and each column represents the specific student's score for tabular, numerical, or graphical representations. The scores of each row are ranked from the smallest value to the largest value and then the sum of the ranks of each column is determined. The chi-square test is used to determine whether the total ranks from the columns are different. If the total ranks from each column are equal then the null hypothesis cannot be rejected. However, if the total ranks from each column vary greatly from one column to another column, then null hypothesis is rejected and alterative hypothesis was accepted.

As indicated, in order to properly analyze the questionnaire data, the responses to each questionnaire item were examined. When we tested the responses to each item using the Kolmogorov-Smirnov and Shapiro-Wilk Tests, we found all of the responses to questionnaire items to be non-normal. The detailed results of these findings are given in Appendix B. Hence, the research questions were analyzed with the Wilcoxon Test, Kruskal-Wallis Test, and the Friedman Test (Hollander & Wolfe, 1999).

For the correlation research questions, Spearman rank correlations were used. Spearman rank correlations compare the ranks of two variables and are used to detect the existence of an association between two variables. If the two variables are unrelated then the Spearman correlation coefficient is zero. The closer the correlation coefficient is to 1 or -1, the stronger the association is between the two variables (Siegel, 1956).

It should be noted that throughout the paper, if no significant difference was found between pre-service teachers' understanding of measures of central tendency and measures of dispersion then it is said that "the pre-service teachers understood measures of central tendency and dispersion equally well." In particular, since there was no significant difference, the intent here is to say that the pre-service teachers understood the concepts "equally well" or "equally poorly."

CHAPTER 4

RESULTS

Given the number and verbal complexity of the research questions, each research question will be examined in turn within given sub-headings.

Comparing Measures of Central Tendency and Dispersion for Normal Data

Research Question 1: Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data for each student classification?

We compared pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data separately. Sophomore pre-service teachers did better in measures of dispersion than measures of central tendency for normal data. However, juniors, seniors, and licensure-only pre-service teachers understood measures of central tendency and measures of dispersion for normal data equally well. A summary of the results for each student classification is displayed in Table 1.

Table 1 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Each Classification

Classification	*N*	*z-score*	*p-value*
Sophomores	28	-3.034	0.002**
Juniors	10	-0.712	0.476
Seniors	40	-1.510	0.131
Licensure-only	15	-1.425	0.154

* $p < 0.05$, ** $p < 0.01$

Pursuant to Research Question 1, we compared sophomores, juniors, seniors, and licensure-only pre-service teachers' statistical knowledge of descriptive statistics (i.e., measures of central tendency and dispersion combined) for normal data for combined representations. The mean rank scores of pre-service teacher's knowledge of descriptive statistics for normal data for each classification are displayed in Table 2.

Table 2 – Mean Rank Scores of Pre-service Teachers' Knowledge of Descriptive Statistics for Normal Data for Each Classification

Classification	*N*	*Mean Rank Score*
Sophomores	28	37.25
Juniors	10	39.40
Seniors	40	51.83
Licensure-only	15	57.40

An analysis of the results indicated that sophomores, juniors, seniors, and licensure-only pre-service teachers' knowledge of descriptive statistics for normal data were not equal. The results are displayed in Table 3.

Table 3 – Kruskal-Wallis Test of Pre-Service Teachers' Knowledge of Descriptive Statistics for Normal Data

	DF	χ^2 *-value*	*p-value*
Classification	3	8.157	0.043*

* $p < 0.05$, ** $p < 0.01$

A post hoc analysis indicated that seniors had a better knowledge of descriptive statistics for normal data than sophomores. The results also showed that licensure-only pre-service teachers had a better knowledge of descriptive statistics for normal data than sophomores. The results are in Table 4.

Table 4 – Post Hoc Analysis of Pre-Service Teachers' Knowledge of Descriptive Statistics for Normal Data

	z-score	*p-value*
Sr. vs. Soph.	-2.698	0.007**
LO vs. Soph.	-1.985	0.047*

Note: Soph. = Sophomores, Sr. = Seniors, LO = Licensure-only

* $p < 0.05$, ** $p < 0.01$

Research Question 2: Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data using a given representation?

The focus of this question is on the representation. We compared pre-service teachers' knowledge of measures of central tendency and measures of dispersion for each representation. The overall findings indicated that pre-service teachers did better in measures of dispersion than measures of central tendency for normal data for numerical representations. The results suggest that pre-service teachers understood measures of central tendency and measures of dispersion for normal data using tabular and graphical representations equally well. The results are displayed in Table 5.

Table 5 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Each Representation

Representation	*N*	*z-score*	*p-value*
Tabular	40	-0.480	0.631
Numerical	40	-4.743	0.000**
Graphical	40	-1.605	0.109

* $p < 0.05$, ** $p < 0.01$

Research Question 3: Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data using a given representation for each student classification?

For normal data described by tabular representations, there were no significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for sophomores, juniors, seniors, or licensure-only pre-service teachers. The results are found in Table 6.

Table 6 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Tabular Representations

Classification	*N*	*z-score*	*p-value*
Sophomores	28	-0.302	0.763
Juniors	10	0.000	1.000
Seniors	40	-0.619	0.536
Licensure-only	15	-1.342	0.180

* $p < 0.05$, ** $p < 0.01$

For normal data described by numerical representations, there were significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for both sophomores and seniors. However, there were no significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for juniors and licensure-only students. The results are shown in Table 7.

Table 7 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Numerical Representations

Classification	*N*	*z-score*	*p-value*
Sophomores	28	-3.987	0.000**
Juniors	10	-1.508	0.132
Seniors	40	-4.442	0.000**
Licensure-only	15	-0.276	0.783

* $p < 0.05$, ** $p < 0.01$

For normal data described by graphical representations, there were significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for seniors. However, there were no significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for either of the sophomores, juniors, or licensure-only students. Table 8 shows the results.

Table 8 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Graphical Representations

Classification	*N*	*z-score*	*p-value*
Sophomores	28	-0.474	0.635
Juniors	10	0.000	1.000
Seniors	40	-2.019	0.044*
Licensure-only	15	-0.905	0.366

* $p < 0.05$, ** $p < 0.01$

Comparing Measures of Central Tendency and Dispersion for Non-normal Data

The next set of research questions investigates pre-service teachers' knowledge of non-normal data.

Research Question 4: Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data for each student classification?

The results indicated that juniors, seniors, and licensure-only pre-service teachers understood measures of central tendency and measures of dispersion for non-normal data equally well. However, a difference between sophomore pre-service teachers' knowledge of measures of central tendency and measures of dispersion was statistically significant. Sophomores did better with measures of central tendency than with measures of dispersion for non-normal data. The results are displayed in Table 9.

Table 9 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Each Classification

Classification	*N*	*z-score*	*p-value*
Sophomores	28	-3.933	0.000**
Juniors	10	-0.728	0.467
Seniors	40	-1.697	0.090
Licensure-only	15	-1.348	0.178

* $p < 0.05$, ** $p < 0.01$

Next, we compared sophomores, juniors, seniors, and licensure-only pre-service teachers' knowledge of descriptive statistics (i.e., measures of central tendency and dispersion combined) for non-normal data for combined representations. The mean rank scores of pre-service teacher's knowledge of descriptive statistics for non-normal data for each classification are given in Table 10.

Table 10 – Mean Rank Scores of Pre-Service Teachers' Knowledge of Descriptive Statistics for Non-normal Data for Each Classification

Classification	*N*	*Mean Rank Score*
Sophomores	28	46.54
Juniors	10	43.50
Seniors	40	51.18
Licensure-only	15	39.07

The results indicated that sophomores, juniors, seniors, and licensure-only pre-service teachers' knowledge of non-normal data were not statistically different. The results suggest that all of the pre-service teachers understood descriptive statistics for non-normal data equally well. The results are displayed in Table 11.

Table 11 – Kruskal-Wallis Test of Pre-Service Teachers' Knowledge of Descriptive Statistics for Non-normal Data

	DF	χ^2 *-value*	*p-value*
Classification	3	2.510	0.474

* $p < 0.05$, ** $p < 0.01$

Research Question 5: Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data using a given representation?

The focus of this question is on multiple representations for non-normal data. We compared pre-service teachers' knowledge of measures of central tendency and measures of dispersion for a given representation for non-normal data. The results indicated that pre-service teachers did better in measures of central tendency than measures of dispersion for non-normal data with both tabular and graphical representations. The results suggest that pre-service teachers understood measures of central tendency and measures of dispersion for non-normal data with numerical representations equally well. Table 12 displays the results.

Table 12 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Each Representation

Representation	*N*	*z-score*	*p-value*
Tabular	40	-2.798	0.005**
Numerical	40	-0.175	0.861
Graphical	40	-3.878	0.000**

* $p < 0.05$, ** $p < 0.01$

Research Question 6: Is there a significant difference between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data using a given representation for each student classification?

For non-normal data described by tabular representations, there were significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for sophomores. However, there were no significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for juniors, seniors, or licensure-only students. Table 13 shows the results.

Table 13 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Tabular Representations

Classification	*N*	*z-score*	*p-value*
Sophomores	28	-2.225	0.026*
Juniors	10	-0.921	0.357
Seniors	40	-1.756	0.079
Licensure-only	15	-1.941	0.052

* $p < 0.05$, ** $p < 0.01$

For non-normal data described by numerical representations, there were no significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for sophomores, juniors, seniors, or licensure-only students. These results are displayed in Table 14.

Table 14 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Numerical Representations

Classification	*N*	*z-score*	*p-value*
Sophomores	28	0.000	1.000
Juniors	10	-1.000	0.317
Seniors	40	-0.500	0.617
Licensure-only	15	-0.816	0.414

* $p < 0.05$, ** $p < 0.01$

For non-normal data described by graphical representations, there were significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for sophomores. However, there were no significant differences between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for either of the juniors, seniors, or licensure-only students. The results are in Table 15.

Table 15 – z-Scores for the Difference between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Graphical Representations

Classification	*N*	*z-score*	*p-value*
Sophomores	28	-4.490	0.000**
Juniors	10	-0.447	0.655
Seniors	40	-0.617	0.537
Licensure-only	15	-1.058	0.290

* $p < 0.05$, ** $p < 0.01$

Correlations of Student Knowledge for Normal or Non-normal Data

The following set of research questions examines correlations between pre-service teachers' knowledge of measures of central tendency and dispersion for either normal or non-normal data.

Research Question 7: For each student classification, is there a correlation between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for normal data for a given representation?

For normal data described by tabular representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for any student classification. Spearman rank correlations are displayed in Table 16.

Table 16 – Spearman Rank Correlations between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Tabular Representations

Classification	*N*	*Spearman's ρ-Coefficient*	*p-value*
Sophomores	28	-0.161	0.412
Juniors	10	0.163	0.653
Seniors	40	-0.174	0.284
Licensure-only	15	0.139	0.622

* $p < 0.05$, ** $p < 0.01$

For normal data described by numerical representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for either of the sophomores, juniors, seniors, or licensure-only students. Table 17 gives the results.

Table 17 – Spearman Rank Correlations between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Numerical Representations

Classification	*N*	*Spearman's ρ-Coefficient*	*p-value*
Sophomores	28	0.259	0.184
Juniors	10	-0.577	0.08
Seniors	40	0.285	0.075
Licensure-only	15	0.311	0.259

* $p < 0.05$, ** $p < 0.01$

For normal data described by graphical representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for any of the student classifications. The results of these correlations are found in Table 18.

Table 18 – Spearman Rank Correlations between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Graphical Representations

Classification	*N*	*Spearman's ρ-Coefficient*	*p-value*
Sophomores	28	0.262	0.178
Juniors	10	0.567	0.087
Seniors	40	-0.286	0.074
Licensure-only	15	0.414	0.125

* $p < 0.05$, ** $p < 0.01$

Research Question 8: For each student classification, is there a correlation between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data for a given representation?

For non-normal data described by tabular representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for either of the sophomores, juniors, seniors, or licensure-only students. The results are shown in Table 19.

Table 19 – Spearman Rank Correlations between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Tabular Representations

Classification	*N*	*Spearman's ρ-Coefficient*	*p-value*
Sophomores	28	-0.014	0.945
Juniors	10	-0.452	0.190
Seniors	40	-0.033	0.842
Licensure-only	15	0.156	0.578

* $p < 0.05$, ** $p < 0.01$

For non-normal data described by numerical representations, there were significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for sophomores, juniors, and seniors. However, there was no significant correlation between licensure-only pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data using numerical representations. The results are displayed in Table 20.

Table 20 – Spearman Rank Correlations between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Numerical Representations

Classification	*N*	*Spearman's ρ-Coefficient*	*p-value*
Sophomores	28	0.441	0.019*
Juniors	10	.816	0.004**
Seniors	40	0.372	0.018*
Licensure-only	15	0.440	0.100

* $p < 0.05$, ** $p < 0.01$

For non-normal data described by graphical representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for any of the student classifications. Table 21 shows these results.

Table 21 – Spearman Rank Correlations between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Graphical Representations

Classification	*N*	*Spearman's ρ-Coefficient*	*p-value*
Sophomores	28	0.170	0.387
Juniors	10	0.623	0.054
Seniors	40	-0.188	0.246
Licensure-only	15	-0.416	0.123

* $p < 0.05$, ** $p < 0.01$

Research Question 9: For each student classification, is there a correlation between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for all normal data regardless of the type of representations?

For all normal data regardless of the type of representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for either of the sophomores, juniors, and seniors. However, there was significant correlation between licensure-only pre-service teachers' knowledge of measures of central tendency and measures of dispersion for all normal data regardless of the type of representations. Table 22 gives these results.

Table 22 – Spearman Rank Correlations between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Normal Data for Each Classification

Classification	*N*	*Spearman's ρ-Coefficient*	*p-value*
Sophomores	28	0.355	0.064
Juniors	10	-0.225	0.532
Seniors	40	0.073	0.653
Licensure-only	15	0.650	0.009**

* $p < 0.05$, ** $p < 0.01$

Research Question 10: For each student classification, is there a correlation between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for all non-normal data regardless of the type of representations?

For all non-normal data regardless of the type of representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for any of the student classifications. Table 23 shows these outcomes.

Table 23 – Spearman Rank Correlations between Pre-Service Teachers' Knowledge of Measures of Central Tendency and Dispersion for Non-normal Data for Each Classification

Classification	*N*	*Spearman's ρ-Coefficient*	*p-value*
Sophomores	28	0.281	0.147
Juniors	10	0.326	0.359
Seniors	40	-0.025	0.880
Licensure-only	15	0.138	0.625

* $p < 0.05$, ** $p < 0.01$

Pre-service Teachers' Overall Knowledge of Measures of Central Tendency or Measures of Dispersion for Normal and Non-normal Data

The next set of research questions addresses pre-service teachers' understanding of measures of central tendency or measures of dispersion across representations.

Research Question 11: For each student classification, is pre-service teachers' knowledge of measures of central tendency for normal data across representations statistically significant?

The Friedman Test was used to examine pre-service teachers' knowledge of measures of central tendency for normal data across representations. The results indicated that there were significant differences across representations of pre-service teachers' knowledge of measures of central tendency for normal data. The results of the test are displayed in Table 24.

Table 24 – Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Central Tendency for Normal Data across Representations

Classification	*N*	*DF*	χ^2 *-value*	*p-value*
Sophomores	28	2	36.738	0.000**
Juniors	10	2	8.857	0.012*
Seniors	40	2	20.117	0.000**
Licensure-only	15	2	5.200	0.074

* $p < 0.05$, ** $p < 0.01$

Since there were significant differences across representations, a post hoc analysis was required. The Wilcoxon Test was used to perform the post hoc analysis. The results of the post hoc analysis indicated that sophomore pre-service teachers did better in the measures of central tendency with tabular representations than measures of central tendency in numerical representations, and they also did better in measures of central tendency with tabular representations than measures of central tendency in graphical representations. In addition, sophomores did better in measures of central tendency with numerical representations than measures of central tendency in graphical representations. Junior pre-service teachers did better in measures of central tendency for both tabular and numerical representations than measures of central tendency in graphical representations. Senior pre-service teachers did better in measures of central tendency in tabular representations than measures of central tendency for both numerical and graphical representations. Also, senior pre-service teachers did better in measures of central tendency for graphical representations than measures of central tendency in numerical representations. The results of the post hoc analysis are given in Tables 25, 26, and 27.

Table 25 – Post Hoc Analysis for Sophomores (*N* = 28) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Central Tendency for Normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular > Numerical	-3.852	0.000**
Tabular > Graphical	-4.512	0.000**
Numerical > Graphical	-2.399	0.016*

* $p < 0.05$, ** $p < 0.01$

Table 26 – Post Hoc Analysis for Juniors (*N* = 10) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Central Tendency for Normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular = Numerical	-1.633	0.102
Tabular > Graphical	-2.226	0.026*
Numerical > Graphical	-2.449	0.014*

* $p < 0.05$, ** $p < 0.01$

Table 27 – Post Hoc Analysis for Seniors (*N* = 40) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Central Tendency for Normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular > Numerical	-4.260	0.000**
Tabular > Graphical	-1.966	0.049*
Numerical < Graphical	-2.676	0.007**

* $p < 0.05$, ** $p < 0.01$

Research Question 12: For each student classification, is pre-service teachers' knowledge of measures of dispersion for normal data across representations statistically significant?

The responses to Research Question 12 were analyzed using the Friedman Test. The results indicated that there were significant differences between pre-service teachers' knowledge of measures of dispersion for normal data across representations. The results of the Friedman Test are presented in Table 28.

Table 28 – Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Dispersion for Normal Data across Representations

Classification	*N*	*DF*	χ^2 *-value*	*p-value*
Sophomores	28	2	38.583	0.000**
Juniors	10	2	9.556	0.008**
Seniors	40	2	27.457	0.000**
Licensure-only	15	2	6.500	0.039*

* $p < 0.05$, ** $p < 0.01$

A post hoc analysis used the Wilcoxon Test since there were significant differences across representations. Sophomore, junior, and senior pre-service teachers did better in measures of dispersion for both tabular and numerical representations than measures of dispersion in graphical representations. However, licensure-only pre-service teachers did better in measures of dispersion in tabular representations than measures of dispersion in graphical representations. The results of the post hoc analysis are given in Tables 29, 30, 31, and 32.

Table 29 – Post Hoc Analysis for Sophomores ($N = 28$) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Dispersion for Normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular = Numerical	-0.378	0.705
Tabular > Graphical	-4.144	0.000**
Numerical > Graphical	-4.235	0.000**

* $p < 0.05$, ** $p < 0.01$

Table 30 – Post Hoc Analysis for Juniors ($N = 10$) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Dispersion for Normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular = Numerical	-1.000	0.317
Tabular > Graphical	-2.226	0.026*
Numerical > Graphical	-2.373	0.018*

* $p < 0.05$, ** $p < 0.01$

Table 31 – Post Hoc Analysis for Seniors (*N* = 40) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Dispersion for Normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular = Numerical	0.000	1.000
Tabular > Graphical	-3.500	0.000**
Numerical > Graphical	-3.810	0.000**

* $p < 0.05$, ** $p < 0.01$

Table 32 – Post Hoc Analysis for Licensure-only (*N* = 15) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Dispersion for Normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular = Numerical	-1.414	0.157
Tabular > Graphical	-2.126	0.033*
Numerical = Graphical	-1.633	0.102

* $p < 0.05$, ** $p < 0.01$

Research Question 13: For each student classification, is pre-service teachers' knowledge of measures of central tendency for non-normal data across representations statistically significant?

The responses to Research Question 13 were analyzed using the Friedman Test. The results of the responses indicated that the medians for measures of central tendency for non-normal data across representations were not equal. The results of the Friedman Test are presented in Table 33.

Table 33 – Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Central Tendency for Non-normal Data across Representations

Classification	*N*	*DF*	χ^2 *-value*	*p-value*
Sophomores	28	2	10.667	0.005**
Juniors	10	2	0.222	0.895
Seniors	40	2	3.389	0.184
Licensure-only	15	2	2.977	0.226

* $p < 0.05$, ** $p < 0.01$

A post hoc analysis of the non-normal data used the Wilcoxon Test. The results showed that sophomore pre-service teachers did better in measures of central tendency with numerical representations than measures of central tendency with graphical representations. The results of the post hoc analysis are given in Table 34.

Table 34 – Post Hoc Analysis for Sophomores (*N* = 28) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Central Tendency for Non-normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular = Numerical	-1.496	0.135
Tabular = Graphical	-0.812	0.417
Numerical > Graphical	-3.051	0.002**

* $p < 0.05$, ** $p < 0.01$

Research Question 14: For each student classification, is pre-service teachers' knowledge of measures of dispersion for non-normal data across representations statistically significant?

The responses to Research Question 14 were analyzed using the Friedman Test. The null hypothesis of the Friedman Test is that the medians of the scores are all equal. The results indicate that junior, senior, and licensure-only pre-service teachers' knowledge of measures of dispersion across representations is same. However, the results show that the medians of the responses of the measures of dispersion for non-normal data across representations are not equal for sophomore pre-service teachers. The results are presented in Table 35.

Table 35 – Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Dispersion for Non-normal Data across Representations

Classification	*N*	*DF*	χ^2 *-value*	*p-value*
Sophomores	28	2	17.133	0.000**
Juniors	10	2	0.923	0.630
Seniors	40	2	2.420	0.298
Licensure-only	15	2	2.450	0.294

* $p < 0.05$, ** $p < 0.01$

A post hoc analysis of the results used the Wilcoxon Test. The results of the Wilcoxon Test indicated that sophomore pre-service teachers did better in measures of dispersion for tabular representations than measures of dispersion for graphical representations, and they also did better in measures of dispersion for numerical representations than measures of dispersion for graphical representations. The results of the post hoc analysis are presented in Table 36.

Table 36 – Post Hoc Analysis for Sophomores (N = 28) of the Friedman Test Results for Pre-Service Teachers' Knowledge of Measures of Dispersion for Non-normal Data across Representations

Representation Comparisons	*z-score*	*p-value*
Tabular = Numerical	-0.959	0.337
Tabular > Graphical	-3.260	0.001**
Numerical > Graphical	-3.522	0.000**

* $p < 0.05$, ** $p < 0.01$

CHAPTER 5

DISCUSSION AND CONCLUSION

The purpose of this study was to investigate pre-service teachers' knowledge of measures of central tendency and measures of dispersion for both normal and non-normal data using tabular, numerical, and graphical representations. The participants of the study were 93 pre-service teachers at a southeastern university in the United States. This chapter is divided into four sections—the first section provides the findings of the study, the second section identifies teaching implications from the study, the third section provides the limitations of the study, and the fourth section offers recommendations for future research.

Findings

Comparing measures of central tendency and dispersion for normal data

Sophomore pre-service teachers did better in measures of dispersion than measures of central tendency for normal data. On the other hand, juniors, seniors, and licensure-only pre-service teachers understood measures of central tendency and measures of dispersion for normal data equally well (see Table 1). We compared sophomores, juniors, seniors, and licensure-only pre-service teachers' knowledge of descriptive statistics (i.e., measures of central tendency and dispersion combined) for normal data for combined representations. Seniors and licensure-only pre-service teachers had a better knowledge of descriptive statistics for normal data than sophomores (see Tables 2, 3, and 4).

When we focused on multiple representations, the results (Table 5) indicated that pre-service teachers did better in measures of dispersion than measures of central tendency for normal data for numerical representations. The results also suggested that all the pre-service teachers understood measures of central tendency and measures of dispersion for normal data using tabular and graphical representations equally well.

For normal data described by tabular representations (Table 6), all the pre-service teachers understood measures of central tendency and measures of dispersion equally well. For normal data described by numerical representations (Table 7), sophomores and seniors did better with measures of dispersion than with measures of central tendency. Juniors and licensure-only pre-service teachers understood measures of central tendency and measures of dispersion equally well. For normal data described by graphical representations (Table 8), seniors did better with measures of central tendency than with measures of dispersion. However, sophomores, juniors, and licensure-only pre-service teachers understood measures of central tendency and measures of dispersion equally well.

Comparing measures of central tendency and dispersion for non-normal data

The results indicated that juniors, seniors, and licensure-only pre-service teachers understood measures of central tendency and measures of dispersion for non-normal data equally well. However, sophomores did better with measures of central tendency than with measures of dispersion for non-normal data (see Table 9).

We compared sophomores, juniors, seniors, and licensure-only pre-service teachers' knowledge of descriptive statistics (i.e., measures of central tendency and dispersion combined) for non-normal data for combined representations (see Tables 10 and 11). It was determined that sophomores, juniors, seniors, and licensure-only pre-service teachers understood measures of central tendency and measures of dispersion equally well.

When we focused on multiple representations for non-normal data, the results (Table 12) indicated that pre-service teachers did better with measures of central tendency than with measures of dispersion for non-normal data with both tabular and graphical representations. The results also suggested that pre-service teachers understood measures of central tendency and measures of dispersion for non-normal data with numerical representations equally well. For non-normal data described by tabular representations, the results (Table 13) indicated that sophomores did better with measures of central tendency than with measures of dispersion. Juniors, seniors, or licensure-only pre-service teachers understood measures of central tendency and measures of dispersion equally well. For non-normal data described by numerical representations, the results indicated that all the pre-service teachers understood measures of central tendency and measures of dispersion equally well (see Table 14). For non-normal data described by graphical representations, sophomores did better with measures of central tendency than with measures of dispersion. On the other hand, the results indicated that juniors, seniors, and licensure-only pre-service teachers understood measures of central tendency and measures of dispersion equally well (see Table 15).

Student knowledge correlations for normal or non-normal data

For normal data described by tabular, numerical, and graphical representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for each classification (see Tables 16, 17, and 18).

For non-normal data described by tabular and graphical representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for any of the pre-service teachers (see Tables 19 and 21). For non-normal data described by numerical representations, there were positive correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for sophomores, juniors, and seniors. However, there was no significant correlation between licensure-only pre-service teachers' knowledge of measures of central tendency and measures of dispersion for non-normal data with numerical representations (see Table 20).

For all normal data regardless of the type of representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for either the sophomores, juniors, or seniors. However, there was a positive correlation between licensure-only pre-service teachers' knowledge of measures of central tendency and measures of dispersion for all normal data regardless of the type of representations (see Table 22).

For all non-normal data regardless of the type of representations, there were no significant correlations between pre-service teachers' knowledge of measures of central tendency and measures of dispersion for each classification (see Table 23).

Pre-service teachers' overall knowledge of measures of central tendency or measures of dispersion for normal and non-normal data

For normal data across representations, the results (Table 24) indicated that there were significant differences among pre-service teachers' knowledge of measures of central tendency. A post hoc analysis indicated that sophomore pre-service teachers did better in tabular representations than numerical representations and better in tabular representations than graphical representations. Sophomores also did better in numerical representations than graphical representations.

Junior pre-service teachers did better in tabular and numerical representations than graphical representations. Senior pre-service teachers did better in tabular representations than numerical and graphical representations, and better in graphical representations than numerical representations (see Tables 25, 26, and 27).

For normal data across representations, the results (Table 28) indicated that there were significant differences among pre-service teachers' knowledge of measures of dispersion. A post hoc analysis indicated that sophomore, junior, and senior pre-service teachers did better in tabular and numerical representations than graphical representations. However, licensure-only pre-service teachers did better in tabular representations than graphical representations (see Tables 29, 30, 31, and 32).

For non-normal data across representations, the results (Table 33) indicated that there was a significant difference for sophomore pre-service teachers' knowledge of measures of central tendency. A post hoc analysis showed that sophomore pre-service teachers did better in numerical representations than graphical representations (see Table 34).

For non-normal data across representations, the results (Table 35) indicated that there was a significant difference for sophomore pre-service teachers' knowledge of measures of dispersion. A post hoc analysis showed that sophomore pre-service teachers did better in tabular and numerical representations than graphical representations (see Table 36).

Discussion of the Findings

The findings may be summarized into five observations. This first observation is that, generally, there is no significant difference between pre-service teachers' understanding of measures of central tendency and dispersion for either normal or non-normal data. It is worth nothing that for both normal and non-normal data, there were significant differences between these descriptive statistics for sophomores. It was also found that as pre-service teachers progressed through student classifications, they understood these descriptive statistics better for normal data but not for non-normal data.

The second observation is that for normal data, pre-service teachers understood measures of dispersion better than measures of central tendency for numerical representations. Stated differently, pre-service teachers understood measures of central tendency and dispersion equally well for tabular and graphical representations. For non-normal data, pre-service teachers understood measures of central tendency better than measures of dispersion for tabular and graphical representations. Similarly, pre-service teachers understood measures of central tendency and dispersion equally well for numerical representations.

The third observation is that the representation of normal data can make a difference in pre-service teachers' understanding of measures of central tendency and dispersion. For non-normal data, this seems less likely to be the case.

The fourth observation is that there is no correlation between pre-service teachers' understanding of measures of central tendency and dispersion for normal data for any representation. However, there is a positive correlation between pre-service teachers' understanding of measures of central tendency and dispersion for non-normal data for numerical representations only. If we ignore the representation of the data, then, generally, there is no significant correlation between measures of central tendency and dispersion for either normal or non-normal data.

The fifth and final observation is quite surprising. We found for both normal and non-normal data, for both measures of central tendency and dispersion, that pre-service teachers performed better using tabular representations than the other two representations. With only one exception, for both normal and non-normal data, for both measures of central tendency and dispersion, pre-service teachers performed least well when using graphical representations.

Teaching Recommendations

Based on the findings of this study, several instructional approaches for teaching pre-service teachers about measures of central tendency and dispersion for normal and non-normal data are recommended. It should be noted that each descriptive statistic should be routinely presented in each of the three representations (i.e., tabular, numerical, and graphical) for each data type (normal and non-normal).

Recommendation 1. Measures of central tendency and dispersion should receive equal attention in instruction for both normal and non-normal data. These descriptive statistics should be taught together.

Recommendation 2. For normal data, more attention should be given to instruction about measures of central tendency for numerical representations. For non-normal data, more attention should be given to instruction about measures of dispersion for tabular and graphical representations.

Recommendation 3. Although the data suggest that for normal data, the selection of a given representation matters more than it does for non-normal data, teachers must still pay close attention to the selection of representations for both normal and non-normal data in order increase the likelihood that pre-service teachers will better understand both measures of central tendency and dispersion.

Recommendation 4. Although statistical instruction should involve all three representations, more attention should be given to the use of graphical representations for teaching measures of central tendency and dispersion for both normal and non-normal data.

Limitations of the Study

One limitation of the study is that we cannot make any “cause and effect” statements regarding the results. The correlations used in the study can only give us the association, direction, and strength between the variables of the study.

A second limitation of the study is the amount of information we collected about the participants. For example, if we had looked at students’ transcripts we might find that some students took more statistics courses than others, or may have taken such courses closer to the time of the study which may have impacted their performance on the assessment. Similarly, knowing students’ course-taking history, we could have grouped participants for analysis based on the number and type of statistics courses studied.

Recommendations for Further Study

Several recommendations for further study follow.

Recommendations 5. This study should be repeated using a larger sample size in several different mathematics education programs.

Recommendation 6. Two separate questionnaires—one questionnaire for normal data and a second questionnaire for non-normal data—should be used.

Recommendation 7. A separate interview component should be included in the study to determine pre-service teachers' reasoning for their responses to the questionnaire.

Recommendation 8. The study should be repeated each year for given cohorts of pre-service teachers (Freshmen, Sophomores, Juniors, and Seniors) as they progress through their mathematics education program. This would permit the researcher to observe trends in students' growth of measures of central tendency and dispersion.

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APPENDICES

APPENDIX A

Statistical Questionnaires

Statistical Understanding Questionnaire

Name:____

Circle the appropriate response for each question below.

How many statistics courses did you have in high school?

a. 1 b. 2 c. 3 d. more than 3 e. 0

What is your current GPA?

a. below 2.0 b. 2.0 - 2.49 c. 2.5 – 2.99 d. 3.0 – 3.49 e. 3.5-4.0

What is your gender?

a. Male b. Female

What is your classification?

a. freshman b. sophomore c. junior d. senior e. other

For what teaching level are you preparing?

a. middle grades math b. middle grades math/science c. high school math

If your teaching level is high school, what is your specialization?

a. math b. computer science c. statistics

Statistical ideas and relationships are presented in numerical, tabular, and graphical representations. For each set of questions below, circle the letter that identifies the correct/appropriate response.

Numerical representations

Q1: For a perfectly normal distribution of data, the mean and median are

a. the same
b. always different
c. possibly the same, possibly different
d. insufficient information

Q2: A perfectly normal distribution of 30 scores has a median score of 21. If the highest score in the list increases by 3 points, what will the median be?

a. 21
b. 21.5
c. 24
d. Cannot be determined without additional information.
e. none of these

Q3: If you are told that a set of data has a mean of 25 and a variance of 0, which of the following must be true?

a. Someone has made a mistake.
b. There is only one element in the population.
c. There are no elements in the population.
d. All the elements in the population are 25.
e. None of the above.

Q4: The sample variance of the following sample S={3,3,3,3,3}is:

a. 0
b. 3
c. 9
d. 11.25
e. 45

Q5: If the mean, median, and mode of a data set are 5, 6, and 7 respectively, then the shape of the distribution of data is:

a. skewed to the left
b. not skewed
c. skewed to the right
d. normal
e. bimodal.

Q6: Which of the following measures of central tendency tends to be most influenced by an extreme score?

a. median
b. mode
c. mean

Q7: The scores in a data set that have the greatest impact on the variance of the data set are scores that are:

a. above the mean.
b. below the mean.
c. nearest the mean.
d. farthest from the mean.

Q8: The mean of the following data set D = {22, 23, 63, 74, 78, 78, 89, 91, 91, 94} is 70.3, and the modes are 78 and 91. The nature of the skewness of the data set is:

a. negative (to the left)
b. zero
c. positive (to the right)
d. not determined
e. both positive and negative

Tabular representations

Q1: Find the sample mean of the following data set given in the table:

X	Frequency of X
2	1
3	2
4	1

a. 3.9
b. 4
c. 36
d. 12
e. 36/10
f. 3

Q2: Find the sample mode of the following data set given in the table:

X	Frequency of X
2	1
3	2
4	1

a. 2 d. 4
b. 3
c. 1 e. 4.5

Q3: The animals in two pet-shops, A and B are described below in tables.

Pet-shop A	Frequency of animals
Rabbits	1
Cats	21
Dogs	1

Pet-shop B	Frequency of animals
Cats	21

Given the information in the tables, which statement below is true:

a. Data set of pet-shop A is homogenous
b. Data set of Pet-shop B is heterogeneous
c. Pet-shop A has less dispersion than pet-shop B
d. Pet-shop B has less dispersion than pet-shop A

Q4: Two workers on the same job show the following results over a long period of time.

	Worker A	Worker B
Mean time for completing a job (in minutes):	25	25
Standard deviation (in minutes):	2	4
Median (in minutes):	25	25
Inter-quartile (in minutes):	2.7	5.4

True or False: Worker B is faster than Worker A.

a. True
b. False

Q5: Find the sample mean of the following data set given in the table:

X	Frequency of X
2	1
3	2
4	3

a. 3
b. 2
c. 20
d. 20/9 = 2.22
e. 20/6 = 3.33

Q6: Find the sample mean of the following data set given in the table:

X	Frequency of X
0	4
1	3
2	1
3	0
4	1

a. 9/5
b. 9
c. 9/4
d. 1
e. none of the above

Q7: Each sample was randomly obtained from the production of the hotdog manufacturer listed.

Company	Hot-dog Length (inches)
A	5, 5, 5, 5, 5
B	7, 7, 7, 7, 7
C	9, 9, 5, 1, 1

Which of the following statements is false?

a. Company A has the same dispersion as Company C.
b. Company B has the same dispersion as Company A.
c. Company B has a different dispersion than Company C.
d. Company A has a different dispersion than Company C.

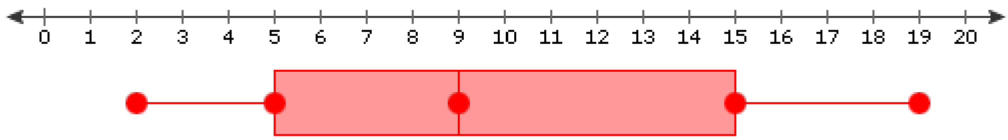
Q8: Two workers on the same job show the following results over a long period of time.

	Worker A	Worker B
Mean time for completing a job (in minutes):	20	26
Standard deviation (in minutes):	15	6
Median (in minutes):	10	20
Inter-quartile (or middle of the numbers) (in minutes):	20.25	8.1

True or False: Worker B is faster than Worker A.

a. True
b. False

Q1. Which of the following data sets matches the box-and-whisker plot shown below?



a. 2, 5, 9, 15, 19

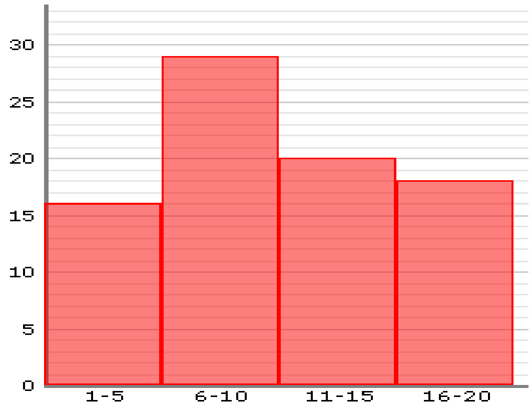
b. 2, 3, 5, 9, 15, 17, 19

c. 2, 5, 6, 9, 14, 15, 19

d. 1, 2, 3, 5, 9, 15, 17, 19, 21

Q2. You conduct a survey asking students how many CDs they own. The results are shown in the graph below.

Y-axis: Students



X-axis: CDs

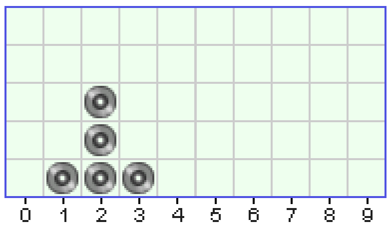
What is the mode of this grouped data?

a. 6 b. 8 c. 10 d. 29 e. 6-10

Q3. In the line plot shown below, which of the following will not be equal to 8?

a. mean   b. median   c. mode   d. all three measures equal 8

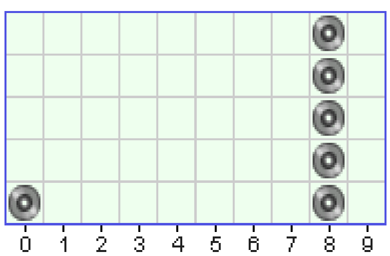
Q4. If a new data item with a value of 9 was added to the line plot below, which of the following would change?



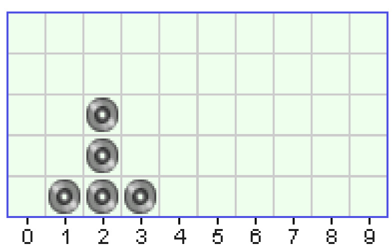
a. mean   b. median   c. mode   d. all three measures will change

Q5. In the following two line plots, which line plot has more dispersion? Circle the correct answer.

Line Plot A



Line Plot B



a. Line Plot A   b. Line Plot B

Questions from graphical representations are adopted from explore learning with modifications.

Q6. Draw the graph of a distribution whose median, mean, and mode are the same. Sketch your drawing below.

Q7. Draw the graph of two normal distributions whose means are the same and whose standard deviations are different. Sketch your drawing below.

Q8. Draw the graph of a distribution whose standard deviation is zero. Sketch your drawing below.

APPENDIX B

Normality Tests for Research Equations Using

Kolmogorov-Smirnov (KS) and Shapiro-Wilk (SW) Tests

Normal data	Sophomore		Junior		Senior		Licensure-only	
Tabular representations	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion
	N=28 KS P<0.01** SW P<0.01**	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Numerical representations	N=28 KS P<0.01** SW P<0.01**	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.05* SW P<0.05*	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Graphical representations	N=28 KS P<0.01** SW P<0.01**	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.05* SW P<0.01**	N=10 KS P<0.05* SW P<0.05*	N=40 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.05* SW P<0.01**	N=15 KS P<0.01** SW P<0.01**

Normality Test for the Responses from Research Question #6

Non-normal data	Sophomore		Junior		Senior		Licensure-only	
Tabular representations	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion
	N=28 KS P<0.01** SW P<0.01**	N=28 KS P<0.01** SW P<0.01**	N=10 KS P>0.01** SW P<0.01**	N=10 KS P>0.05 SW P<0.01*	N=40 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Numerical representations	N=28 KS P<0.01** SW P<0.01**	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Graphical representations	N=28 KS P<0.01** SW P<0.01**	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.05*	N=40 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**

Normality Test for the Responses from Research Question #1

Sophomore		Junior		Senior		Licensure-only	
Normal data		Normal data		Normal data		Normal data	
Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion
N=28 KS P<0.05* SW P<0.05*	N=28 KS P<0.05* SW P<0.05*	N=10 KS P<0.05* SW P<0.05*	N=10 KS P<0.05* SW P<0.05*	N=40 KS P<0.05* SW P<0.05*	N=40 KS P<0.05* SW P<0.05*	N=15 KS P<0.05* SW P<0.05*	N=15 KS P<0.05* SW P<0.05*

Normality Test for the Responses from Research Question #4

Sophomore		Junior		Senior		Licensure-only	
Non-normal data		Non-normal data		Non-normal data		Non-normal data	
Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion	Measures of Central tendency	Measures of Dispersion
N=28 KS P<0.05* SW P<0.05*	N=28 KS P<0.05* SW P<0.05*	N=10 KS P<0.05* SW P<0.05*	N=10 KS P<0.05* SW P<0.05*	N=40 KS P<0.05* SW P<0.05*	N=40 KS P<0.05* SW P<0.05*	N=15 KS P<0.05* SW P<0.05*	N=15 KS P<0.05* SW P<0.05*

Normality Test for the Responses from Research Question #11

Normal data	Sophomore	Junior	Senior	Licensure-only
Tabular representations	Measures of Central tendency	Measures of Central tendency	Measures of Central tendency	Measures of Central tendency
	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Numerical representations	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.05* SW P<0.05*	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Graphical representations	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.05* SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.05* SW P<0.01**

Normality Test for the Responses from Research Question #12

Normal data	Sophomore	Junior	Senior	Licensure-only
Tabular representations	Measures of Dispersion	Measures of Dispersion	Measures of Dispersion	Measures of Dispersion
	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Numerical representations	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Graphical representations	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.05* SW P<0.05*	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**

Normality Test for the Responses from Research Question #13

Non-normal data	Sophomore	Junior	Senior	Licensure-only
Tabular representations	Measures of Central tendency	Measures of Central tendency	Measures of Central tendency	Measures of Central tendency
	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Numerical representations	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Graphical representations	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**

Normality Test for the Responses from Research Question #14

Non-normal data	Sophomore	Junior	Senior	Licensure-only
Tabular representations	Measures of Dispersion	Measures of Dispersion	Measures of Dispersion	Measures of Dispersion
	N=28 KS P<0.01** SW P<0.01**	N=10 KS P>0.05 SW P<0.05*	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Numerical representations	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.01**	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**
Graphical representations	N=28 KS P<0.01** SW P<0.01**	N=10 KS P<0.01** SW P<0.05*	N=40 KS P<0.01** SW P<0.01**	N=15 KS P<0.01** SW P<0.01**