ABSTRACT

EARLY, MORGAN LEIGH. Engineering Professors' Preferences for the Learning of Differential Equations. (Under the direction of Dr. Karen Allen Keene.)

A first semester course in ordinary differential equations is often described as a service course to engineering and hard science majors; how engineering professors envision the course content and tools is important. Additionally, reform efforts in differential equations courses have focused on enhancing students' conceptual understanding of the material. To this end, mathematicians and mathematics educators have developed the notion of relational understanding, which connects procedural and conceptual knowledge. This study reports the results of a nationwide survey of engineering professors who articulate their opinions on the topics in the differential equations curricula, the relational understanding of differential equations students, and how technological advances have impacted the study of differential equations.

Engineering Professors' Preferences for the Learning of Differential Equations

by
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DEDICATION

I dedicate this thesis to my parents. Throughout my life, you have provided the consistency and motivation that I needed to continue to work diligently as unto our Lord Jesus Christ. You two provided excellent examples of a hard work ethic, and constantly supported my academic efforts in every way. Thank you for helping me to remember that this work is not about my accomplishments, but that I am first and foremost a servant to our King. I love you more than you know.

BIOGRAPHY

Morgan Leigh Early was born and raised in Marion, North Carolina by her parents Steve and Miriam Early. She graduated from McDowell High School in the spring of 2004 and began her undergraduate work in both communication and mathematics at North Carolina State in the fall of 2004.

During her time as an undergraduate, Morgan began to work part-time as a teaching assistant for the Mathematics department. She began to feel called to a teaching career during this time. After completing her undergraduate degrees, she continued her educational career as a master's student at North Carolina State, pursuing a degree in Mathematics Education. Upon completion of her Master of Science degree in Mathematics Education, Morgan will begin teaching at Millbrook High School in Raleigh, NC and hopes to one day pursue a PhD.

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My peers, colleagues, and friends: thank you for listening to me talk about differential equations and engineering, even if you were only listening out of support. To my favorite peer, my sister—bouncing ideas off of you is always a pleasure. I would like to thank Dr. Karen Keene for providing wisdom and insight throughout this process. You have taught me how to be a researcher and how to enjoy the process instead of just the final product. Thank you so much for becoming a great mentor and friend.

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Chapter 1
Introduction

Mathematics departments provide a service to many science and engineering fields by teaching prerequisite mathematics courses for these areas. Some of the major participants of mathematics courses are engineering students (Ahmad et. al, 2001, Committee of the MAA, 1967). If we seek to improve the value of engineering students' education, we inevitably must seek to improve their mathematics education. Mathematics professors and engineering professors must work together to meet the needs of engineering students' mathematical understanding, in order to make them more effective learners of engineering.

More specifically, differential equations present themselves very useful for solving different types of systems in various types of engineering. In many cases, a strong understanding of differential equations will also provide students a gateway to avoid rote memorization. One dynamics professor notes that in "most dynamics books all of the equations that are used in dynamics books come from 2 or 3 governing differential equations" (personal communication). He goes on to describe that his students "don't understand the differential equations well enough [and if they try to] memorize all this stuff, they just get balled up…when all they really need to do is understand how to take, for example, F=ma, express it as a differential equation and integrate it twice to get position, but they won't do that" (personal communication). He suggested that his students do not derive solutions because they did not have a strong enough understanding of differential equations.

Since mathematics courses serve a wide variety of science and engineering majors, the demands upon those teaching these courses often change with other fields, and is not limited to the trends in mathematics and mathematics education. With the onslaught of technology in engineering (Boyce, 1994), there have been several shifts in the way one can approach a differential equations problem. Many engineering departments are changing their coursework, as there is "a shift in emphasis from teaching focused on knowledge . . . toward teaching about the process of engineering" (Kent and Noss, 2003, p. 19). In some cases, engineering mathematics is beginning to be taught in engineering departments (Ahmad et. al, 1967).

The process of engineering makes problem solving skills and analysis more critical for today's students than ever before. Thus, since differential equations services many engineering students and the engineering education needs seem to be changing, there is also a need for differential equations curriculum evaluation to see if this course is helping students with the process of engineering. This study incorporates Skemp's (1976) "relational understanding" notion to apply to not only understanding concepts, but also using these concepts in conjunction with other concepts and procedures. Engineers must take knowledge from many areas and combine them to suit their needs in an application setting. There seems to be a strong need for relational understanding among engineers, and this study will seek to explore the opinions of engineering professors about students' relational understanding of differential equations.

We know that engineers need to understand mathematical concepts for modeling purposes so that they may connect those models to their science courses (Blockley, 2002). However, we do not know which topics in ordinary differential equations that are essential to these engineers with the onslaught of new technology. Do engineers need to have relational understanding of science but only procedural or instrumental understanding of ordinary differential equations? This study seeks to identify prevalent opinions of various engineering professors at different universities in order to interpret the most important curriculum topics, the depth of student understanding that engineering students need to grasp in a first year course in differential equations, as well as discuss the importance of technology to engineering students in differential equations.

The content of the first semester course in differential equations must be re-assessed to best meet the needs of our students. To do so, we must assess students' current understandings (see Rasmussen, 2001; Keene, Early, and Gonzalez, in preparation for examples of this), what they need to understand, and how best to teach what they do need to know with the tools at hand in light of the engineering needs. Thus, the following research questions seek to best determine how to meet the future needs of engineering students in an ordinary differential equations course:

- According to engineering educators, what topics in differential equations are most crucial for engineering education and practical engineering?
- What types of relational understanding do engineering educators wish to see in their students' differential equations classes?
- According to engineering educators, how should technology be used in the mathematical education (especially the differential equations education) of engineers?

Once we better understanding the directions that engineering professors desire, we can use their ideas to help see what topics to cover in differential equations and how we should present this information for engineers to be able to utilize the topics in their future courses and careers. We can then design a more effective differential equations curriculum, or if necessary, different curricula for various majors.

Chapter 2
Literature Review

Differential equations courses serve many different types of engineering fields (see Chapter 1 for further detail). In this chapter, I will seek to review the literature discussing the mathematics content which best serves students in engineering, as well as discuss the work that has been done on students' conceptual understanding of mathematics at the undergraduate level. The last several decades have altered the day-to-day operations of an engineer due to the available technologies of our current culture. I will also discuss the known benefits and concerns that have risen from the introduction of new technologies in undergraduate mathematics, specifically the benefits and concerns of technology to the first course in differential equations.

University Mathematics Content for Engineers

Curriculum content should continually be assessed for any mathematics course at the university level to ensure that it is best serving the students who are to benefit from the material. One issue directly related to curriculum development for engineering is that new technologies are being introduced into the engineering workforce. This bears the question of whether or not mathematics curricula are best serving 21st century engineering students. In 1962, the committee on the undergraduate program in mathematics of the Mathematical Association of America met to discuss mathematics curriculum needs of engineering and physics students.

They revised their work, and in 1967, they included the following topics as being important to the first course in differential equations: “linear differential equations with constant coefficients and first order linear systems—linear algebra (including eigenvalue theory) should be used to treat both homogeneous and nonhomogenous problems; first order linear and nonlinear equations with Picard’s method and an introduction to numerical techniques” (p.7). They placed transform methods in a course they entitled “functions of a complex variable,” (p. 32) and also in the partial differential equations course, (p.34) which was separate from the first course in calculus and differential equations. They also included a more extensive study of numerical methods in the partial differential equations course, where students would learn the “replacement of differential equations by difference equations; iterative methods; the method of characteristics [as well as] convergence and error analysis” (p. 35).

Differential equations courses serve many different types of engineering, all of which need to be able to apply these mathematical concepts. In 2003, Kent and Noss were commissioned by the Ove Arup foundation to examine the mathematics being taught to engineers, specifically civil engineers. They noted that while there is a shift in university engineering education, the mathematics courses for these engineers are not necessarily following suit. They note that “engineering mathematics curricula often contain topics that are present for historical reasons, which are no longer used in engineering courses as in the past” (Kent and Noss, p.

36). They suggest more open dialogue between mathematics and engineering faculty in order to address content as well as pedagogical methodology. Thus, the focus for mathematics instructors at the university level should be how to shift our teaching as well from just focusing on knowledge to focusing on collaborating with engineering professors to enhance learning of the problem-solving process.

A working group of the Undergraduate Mathematics Teaching Conference discussed that, as mathematicians, we must note that the goals of the engineering community are different than the goals of the mathematics community (Ahmad et. al, 2001). This working group suggests that if mathematicians teach engineering mathematics, then they "should encourage understanding and thoughtful use, and may communicate…the generality and power of the subject, the range of methods available, and links to other subjects within and outside engineering" (p. 38). However, if engineers teach the mathematics content, the committee suggests that they "allow sufficient time for methods to be understood…ensure pre-requisites are covered, [and should] avoid using [computer] packages as a 'quick fix'" (p. 39). In both cases, they note that engineering professors and mathematics professors must work together to best achieve the usefulness of the mathematics and the understanding of the content being used.

While we often think of engineers as being strong math students who are confident enough in mathematics to go into the engineering field, Kent and Noss (2003) found that many engineers feel uncertain about their mathematics backgrounds, and this may be largely due to their lack of "symbol sense," (Arcavi, 2005) or their conceptual understanding of symbols. Even strong students have to struggle through learning mathematics, and thus, research should continue to explore the need for understanding common pitfalls for these engineering students.

If mathematicians and engineering professors work together to meet curriculum needs, hopefully the mathematics coursework of engineers will align more closely with the mathematics they will eventually use in the workforce. Engineers often proceed into careers that typically use mathematics differently than the way they learned to use mathematics in the classroom. In 2007, the disposition of structural engineers was analyzed at one particular engineering firm. The researcher observed engineers at work and found that they adhere to the rules of the industry, prefer more supporting elements in their buildings/structures. Expertise often meant the ability to make judgment calls, not perform mathematics. Engineering solutions do not have one "right" solution, and many cannot be deductively proven. Sometimes deductively sound arguments were rejected because they were impractical. The author suggests allowing students to reason with elements of a certain context and uncover the mathematics problem (Gainsburg, 2007).

In engineering education at the undergraduate level, there is a growing emphasis on problem-based and project-based learning (Mills and Treagust, 2003), which aids in this preparation of students for a problem-solving career. The main difference between these two approaches is whether or not professors are looking at short-term or long-term goals. Problem-based learning focuses on the acquisition of knowledge and is a short-term problem presented to a class. This may be an activity or problem that takes students a day or two to complete on their own in class.

Project-based learning focuses on students' application of the knowledge they have learned. This type of learning is usually presented as a long term assignment, such as a senior design project, where students have more time to demonstrate knowledge acquired, but must manage their time wisely as they will in their careers. Also, students often have an opportunity to demonstrate knowledge gained in a variety of their different engineering courses when they are assigned project-based learning activities. At least one university is implementing project-based learning in a freshman-level course. An upperclassman works as a project supervisor, but he underclassmen form a team to complete the class. The researchers have found that this course is beneficial to preparing students for later tasks since "70% [of students] thought that the project introduced them to real-world engineering issues [and] 60% of the students taking the course in fall 2007 said they felt more comfortable doing engineering design as a result of the project" (Frank and Mason, 2008, p.10).

Even if students do not spend the whole semester on one project, engineering courses often offer problem-based learning initiatives. Problem-based learning "promotes students active engagement with learning [and its] main goal is to apply knowledge, not just acquire it" (Brodeur, Young, and Blair, 2002, p.2). The problem that students are given is encountered first, learning as they work through the problem. The problems are typically described as lecture substitutes, and may be worked "independently or as part of a group" (Brown and Brown, 2004, p.11).

Engineering educators ask for mathematicians to also incorporate these problem-based and project-based methods into their classrooms. Mills and Treagust (2003) claim that engineers need more teamwork and communication experience, and that engineering professors lack background experience in careers outside of academia. They claim that problem-based learning may be insufficient for students because they do not include a wide range of activities. They instead advocate for teachers to use more project-based learning because they can see a more holistic view of the students' capabilities.

Teaching differential equations to engineers.

Engineering educators have been discussing the importance of various topics in differential equations for years. Engineering departments have often disagreed with each other on the importance of certain differential equations topics because of their specific differential equations applications. We have evidence of at least one specific differential equations topic that creates differing view amongst engineering professors—the Laplace transform. Holmberg and Berhnhard (2008) interviewed 22 professors from 5 universities concerning their opinions about the difficulty of the Laplace transform in differential equations. They split teachers into three categories—those who consider the topic difficult for students to learn, teachers that say it may or may not be too difficult for students to learn, and teachers that consider that the transform is not difficult for students to learn. They found teachers in all 3 categories and found that there is no unified view about the student difficulties with the transform or the importance of the Laplace transform to their students' education. They also found that these teachers have different opinions about the links between mathematics, physics, and technology.

While there are differing opinions about the Laplace transform, there seems to be common opinion about approximation methods. For some engineers, nonlinear differential equations have shown to be more common mathematical objects in their engineering work than linear differential equations. A Boston University project sought to embrace the "nonlinear revolution" in engineering—which seeks to "present nonlinear systems on the same footing as linear systems" (Blanchard, 1992, p. 1). Blanchard and associates developed a new textbook, focusing on a wider variety of topics. They sought to "balance between analytic, numeric, and graphical techniques [and] stress qualitative theory" (p. 2). They hope that through their new approach that differential equations will better suit the needs of the engineering community that uses more nonlinear equations than linear ones. These reform efforts in differential equations have started to take hold in many American universities. The reform effort in differential equations is showing to be beneficial to students' conceptual understanding of rate (Rasmussen and King, 2000).

Students' Relational Understanding of Mathematics Content

Differential equations courses traditionally focus on procedures. Students are shown different types of differential equations, and then shown how to solve them (Boyce, 1994). The ways in which we solve these differential equations are referred to as analytic methods. These methods provide exact solutions to differential equations. While these analytic solutions are important, by focusing our attention solely on these many analytical methods without visualization and discussion about function behavior, students are often losing conceptual understanding of many of these topics (Rasmussen and Whitehead, 2003).

As we look to make the differential equations curriculum more applicable to the engineering community, we must also consider if these engineering students correctly understand these new applications. There has been an ongoing discussion over the importance of conceptual versus procedural understanding in the mathematics education community for several years (Thompson and Thompson, 1994; Star, 2005; Pirie and Kieren, 1994). The differential equations course is important to this conversation, as many professors who traditionally teach differential equations only present analytical procedures.

Several researchers have also created new interpretations of student understanding. Donovan (2008) expanded Sfard's (1991, 1992) dichotomy of structural and pseudo-structural understanding. A pseudo-object is an object that can be manipulated, but is not fully connected to a compact whole notion. Donovan used this dichotomy to describe two interviewed students who were given differential equations tasks and asked to work back and forth between function graphs and analytical differential equations. He described three essential elements, according to Sfard, that would describe a student with structural understanding. First, the student with structural understand can recognize the same concept in different situations. Second, he can provide different representations of the same object, and finally, the student with structural understanding will be able to recognize basic ideas or concepts at a glance without a need for solving a procedure. Donovan found that he was able to classify differential equations students with this dichotomy. While both worked back and forth between graphs and equations, the student that he classified as having pseudo-structural understanding was unable to formulate different representations of the concept and had to perform procedures in order to discuss the concept.

Hasenbank and Hodgson (2007) showed in their algebra study that their students needed to connect conceptual ideas to procedural ideas. Instead of promoting conceptual or procedural understanding, they worked under the premise of students developing deep procedural knowledge and developed a framework that assesses a student's deep procedural knowledge. The framework shows that a student with deep procedural knowledge understands the goal/outcome of the procedure and is able to execute procedures that would elicit a solution. The framework also mentions that students with deep procedural understanding will be able to assess the validity of the procedure, compare it to other procedures, and explain where you can use certain procedures and when these procedures are best to utilize.

Similar to deep procedural knowledge, Skemp (1976) also noted that there was an inevitable connection between concepts and procedures in mathematics. Instead of discussing deep procedural understanding, he instead presented a new dichotomy of student understanding of mathematical concepts—instrumental and relational understanding. He notes in his article that loosely using the term "understanding," may imply different meanings in the mathematics community. Instrumental understanding is loosely related to procedural understanding. Typically, instrumental understanding refers to the implementation of a few principles. The student with instrumental understanding has learned to carry out a procedure, but is unsure of where this procedure applies and where it does not apply.

Relational understanding requires students to make connections between different mathematical schemas. This understanding is not fully void of the need for procedures. A student in differential equations will need to learn some procedures, and understand how to do these rightly in order to correctly solve engineering problems. Relational understanding involves rich connections among different concepts, and involves students not only understanding procedures, but also understanding why these procedures are necessary and how they were derived. Students with relational understanding of differential equations will be able to work different types of analytical techniques, but will also be able to compare and contrast these different techniques, explaining why differential equations are classified as they are, and why certain procedures work for certain types of equations. Skemp proposes that if students and teachers are mismatched based on their preferences for instrumental or relational understanding, there will be temporary or long-term disappointment for one or both parties. Thus, if relational understanding is a teaching goal for the differential equations course, we must find if the instructors are also interested in presenting differential equations in a way that would elicit relational understanding.

Others agree that students will understand mathematics better if concepts are part of an internal network of connections. Educational psychologists have found that networks are built gradually and tighten over time. Students have to develop their own understanding and once a student does understand a concept, there is a reduction in the amount that must be remembered. In his review of literature, Hiebert (1992) impressed that students should be taught meaning first before practicing execution. He also suggests that teachers note connections among different representations. Otherwise, misconceptions will exist despite understanding properties because students will separate those incorrect notions into separate islands of knowledge. If teachers make connections clear and help students separate incorrect notions, hopefully students will leave a course with more relational understanding.

Conceptions and Misconceptions in ordinary differential equations.

Any curriculum change is unsuccessful unless it offers a way for students to understand the material presented. Habre (2000) noted that "assessing the success of the new ODE curriculum is incomplete if not complemented by research into student thinking" (p. 456). In order to better assist his students' understanding of the behavior of the solutions to differential equations, Habre conducted a course in which he focused on qualitative approaches to solutions. At the end of the semester, he conducted interviews with several of his students and asked them to work two different problems. He found that even with qualitative tools in their minds, most of them had the tendency to try analytic solutions first. Some of the students noted that the graphs did not seem as sufficient as the analytic solutions. Habre also found that students had a difficult time working fluidly with both types of representations, and did not think about the other representation when working with one.

Rasmussen (2001) interviewed 6 students and developed a framework that shows where students in differential equations need assistance. He found that students need assistance with developing the notion of functions being solutions to differential equations. He also found that students need stronger intuitions and images of differential equations. In his framework, students' understanding of functions as solutions to differential equations includes interpreting solutions as being functions instead of having "mindless graphical manipulation" of symbols. His framework also discusses interpreting equilibrium solutions as being constant solutions that may or may not be found by substituting a "0" into the equation for y and dy/dt. Instead, he says students should focus on equilibrium solutions being seen as quantities. Students' were found to have some intuitions concerning equilibrium solutions, numerical approximations, and stability. However, developing stronger schemas of differential equation solutions and visualization need attention for students to develop stronger conceptualizations of DE solutions.

Keene, Early, and Gonzalez also (in progress) found several student misconceptions. Similar to Rasmussen's findings, they found that some students were unaware that functions were solutions for all differential equations. Some felt that "equation" was a better answer for solutions to ODEs. The students in this study had a poor understanding of equilibrium solutions as well. Many students did not even have a working definition of an equilibrium solution. Beyond having a weak understanding of the algebraic creation of an equilibrium solution, students did not recognize or acknowledge that equilibrium solutions were specific to separable differential equations because of the division action in the method.

The same researchers also investigated students' understandings of the derivative. In separation of variables, as often presented in textbooks, students can simply separate the derivative into two differentials, and then integrate both sides of the equation with respect to two different variables. This method is accurate because of the justification of the chain rule, although in general, integrating with respect to one variable is not an equal operation as integrating with respect to another variable. When examining the separation of variables method, most students had some concern with integrating with respect to 2 different variables, but some of them said that they just accepted it because the teacher taught them to solve the problems this shortcut way. Students accepted what they had been told, even though they always thought it odd or "didn't see clear justification" of the separation of dy/dt.

These researchers also investigated students' understandings of Euler's method. When investigating numerical methods, few students in this study knew whether there would be an over or underestimate depending on the concavity of the actual function. Artigue (1992) and Rasmussen (2001) both found evidence that some students see Euler's method as a "track" of the exact solution curve. Thus, there is a possibility that some students will be unaware of the error created from numerical approximations.

There is limited research concerning misconceptions in differential equations. The common misconceptions that we do know exist may or may not be of importance to engineers. Thus, this study seeks to address which, if any, of these known misconceptions cause major roadblocks in later engineering courses. Once we identify more misconceptions, we also need to focus on how to best address them. Habre (2002) built on research that has shown benefit in writing in calculus classes and examined the effects of writing in an ordinary differential equations course. Students were asked to justify their answers and discuss effects of changing parameters. By the end of the writing-centered course, students widely agreed that writing was essential to the course and that "writing complements the geometrical approach" (p. 7). This survey's aim was to identify the most important misconceptions to the engineering academic community so that we could then later look into efforts other than writing that might be helpful for teaching the relational understanding of differential equations content to engineering students.

Technological Effects on Engineering Students' Mathematical Understanding

Differential equations research has become more popular in the past couple of decades with the onslaught of technologies such as Maple, Mathematica, and MATLAB. Students' understandings of differential equations may be enriched by the use of various technologies at our disposal (Kallaher, 1999). In their literature review on the learning and teaching of ordinary differential equations, Rasmussen and Whitehead (2003) noted that "students' visual understanding of phase portraits, slope fields, and solutions of differential equations is an area where we might consider integrating technology into students' experiences in the classroom" (p. 7).

Beyond visualization, Boyce (1994) noted that technology called for differential equations students to have 3 kinds of experiences: "conceptualization, exploration, and higher-level problem solving" (p. 364). Boyce notes that technology keeps students from thinking about rote action, and instead they have more opportunity to focus on the conceptual understanding, since they don't have to worry as much with manipulative skills. Technology aids in students' explorations as they manipulate to experience the effects of different parameters in systems of differential equations (i.e. determining crucial values). He notes that computer programs can also be used as a reference to discover new mathematical concepts through discovery learning of interpreting computer outputs. Differential equations also offer students a chance to model mathematically. This requires higher-level problem solving and is more directly applicable to engineering than other mathematics topics. Modeling practice increases the higher-order thinking skills of engineers, and preparing them for applying mathematics is the goal of an engineering student's university education. He concluded that technology "will not change the need for men and women who can think clearly, critically, constructively, and creatively about the problems of the day" (p. 371).

While technology seems to have benefits, there may also be drawbacks to the use of technology in the undergraduate classroom. Kent and Noss (2003) note that the increased availability of technology may actually be a detriment to students in engineering. Engineering educators agree that students in their courses graduate "with good knowledge of fundamental engineering science and computer literacy, but they don't know how to apply that in practice" (Mills and Treagust, 2003, p.3).

Puga (2001) investigated the use of several software packages and computer algebra systems in a differential equations course and its effect on modeling. Puga suggests that students could be introduced to modeling differential equations in calculus by using certain software packages. He thinks that this discovery learning could then be continued in a differential equations course. Overall, the students' consensus about using the software was positive. They claimed that it increased their understanding of the course concepts. Yet, there were some new challenges that using the technology introduced. In population modeling, the students continued working despite having negative or non-whole numbers for population.

Technology has shown to be important for the visualization of students in differential equations and is usually well-received by the students. However, we must also be careful how we implement technology into the classroom so that its purpose is aid instead of crutch.

Summary of Literature

Engineering students need to be able to perform procedures in differential equations, but they also need to be able to connect concepts together in order to have a more tightly defined schema of ideas. Skemp (1976) refers to this connection among procedures and concepts as relational understanding.

Engineering educators have shown a need for more opportunities for their students to explore and create their own mathematical models. They want them to be able to bridge various different concepts from different engineering courses. However, little is known about engineering professors' opinions on how students' bridge mathematical understanding with their engineering knowledge. There is no research to indicate whether or not relational understanding of differential equations content is of importance to these engineering professors. Thus, we need to investigate the importance of relational understanding of mathematical knowledge to the engineering community, especially in differential equations, since it is a course that is so widely used by the engineering community.

Chapter 3
Methodology

This study is structured to address the research questions. In this chapter, I describe the construction of the survey, including the decisions I made on what questions to ask and how I chose participants. I then discuss data collection and data analysis.

Survey Construction

I designed a cross-sectional survey that was a comparative analysis among engineering professors (Creswell, 2009). I formed a stratified sample in order to compare various demographics associated with engineering professors. By using this strategy, I was able to analyze engineering professors' overall responses about curriculum, student understanding, and technology, but I was also able to determine if a specific demographic would tend to lend itself to a particular response about any of these three areas of interest.

I also wanted to elicit the curriculum preferences for the professors, so I decided to blend traditional topics and reform topics in the survey. In order to objectively present this information, I studied both a reform text, *Differential Equations* (Blanchard, Devaney, and Hall, 2002) and a traditional text, *Fundamentals of Differential Equations and Boundary Value Problems* (Nagle, Saff, and Snider, 2004). I chose major topics from both of these books and blended them into the survey with the help of a research advisor. The analytical techniques that I chose to include are based on the types of differential equations that a student might encounter.

Both of these textbooks introduce the separation of variables method and first order linear equations before the other topics. Separation of variables is used for differential equations of the form $\frac{dy}{dt} = p(y) \cdot f(t)$, where p(y) is a function of y and f(t) is a function of t. First Order linear equations must be able to be written in the form of $\frac{dy}{dt} + P(t)y = Q(t)$, where P(t) and Q(t) are functions of t. In general, P(t) and Q(t) are different functions.

The method of undetermined coefficients and variation of parameters are both used for nonhomogeneous differential equations, while auxiliary equations are used for homogeneous equations. Homogeneous equations that can be solved with auxiliary equations are written in the form ay''+by'+cy=0, while nonhomogeneous equations that are solved with the other methods take the form ay'' + by' +cy = f(t). Substitution methods are often used for differential equations that are nonlinear.

Matrix methods are widely used in order to solve large dynamical systems of linear differential equations. Scientists may need several differential equations to depict the model of their physical situation. Matrices handle large amounts of data easily and quickly.

Laplace transforms map functions from one domain into another domain that is simpler algebraically. They are often desired more than other methods for nonhomogenous differential equations because you are not required to find a general and particular solution. Instead, using transforms, one may find the solutions quickly. They may also be used to solve homogeneous differential equations, but usually this method is not preferred over auxiliary equations.

Finally, series solutions are helpful for solving linear differential equations with non-constant coefficients. One may use numerical approximations of the series portions of these solutions to approximate the entire solution to the differential equation.

I decided that to best understand the differential equations curriculum, I should separate the analytical techniques from other types of approaches. I split the rankings into two categories: one where professors rank analytical techniques, and another where professors rank types of approaches to solving and analyzing differential equations.

The second research question aims to determine if research professors' prefer a relational understanding of differential equations, a procedural understanding, or a conceptual understanding. Professors were asked to determine if misconceptions posed problems for them in their engineering curricula. The four misconceptions that were chosen were based on previous research concerning student pitfalls in differential equations (Rasmussen, 2001; Keene, et. al, in progress). I also asked about professors' own preferences concerning student understanding by using the following question:

Which of the following would you prefer?

> 1. A student who can't remember the formula for a numerical technique, but has a general idea of why that technique works. This student has an incorrect solution.
>
> 2. A student who remembers the formula for the technique, but does not know why this method works. This student has a correct solution.

Finally, I wanted to explore the effects of technology to the learning of differential equations for engineering situations. This led to questions designed, with the help of an advisor, to elicit responses about the changes in engineering for the professors and their students due to the technology that is readily available. See Appendix B for the full survey.

Data Collection

I submitted an application to the institutional review board for an exemption to proceed with the research study and was approved for this exemption (See Appendix G). I proceeded to contact professors once I gained this approval.

I first began to contact professors at my own university through personal contacts. Some engineering departments became aware that I was conducting this survey by word of mouth or email from colleagues. The pilot survey included 9 professors from the university. The wording of various questions was modified for clarification purposes, and several items that provided unnecessary results were omitted from the final survey. This pilot survey was closed before I began the actual survey.

Engineering professors from across the United States were contacted to be a part of the data collection online survey. I wanted a wide base of engineering types and engineering schools. I searched for schools with high-ranking engineering programs in both the public and private sectors. High-ranking engineering programs were found through various news sources such as U.S. News and World Report in the last several years. I compiled a list of at least one school from every state that had an engineering program. I used public access emails that were available on the universities' websites in order to contact a wide range of professors from various departments. The specifics of the demographic layouts of these professors may be found in data analysis below.

More than one thousand professors were contacted by email either through the recommendation of other professors or through public access contact information available on university websites. Each professor was emailed individually with a personal address. The email asked them to complete the survey if desired on a volunteer basis. The survey was made available online at the following address using a survey builder created by NC State University's college of agriculture and life sciences:

http://ceres.cals.ncsu.edu/surveybuilder/Form.cfm?TestID=8762

I collected data over a three-month period. Afterwards, I closed the survey. Eighty-three engineering professors completed the survey, yielding a 7.59% return rate. This low return was most likely caused because I only sent one email to each professor chosen. There were no follow-up emails reminding the professors about the survey.

Professors who did complete the survey and gave permission and contact information were contacted for follow up questions about some of the items. I only emailed those professors that indicated in the first survey that they were willing to be contacted with follow-up questions about their responses. I structured the follow-up protocols in such a way that I might gain more insight into the reasons that some of my results had risen. Five professors completed the follow-up survey and 2 of these professors also volunteered to participate in a phone interview.

In the follow-up interviews, I hoped to gain even more information about these same issues. As part of the follow-up survey, I asked for professors who were willing to conduct a phone semi-structured interview to note this on their follow-up survey. Two professors left their information and I conducted 45-minute interviews with both professors at their convenience. Since there had been several months between their original survey and the follow-up survey, I reminded them of some of their pertinent answers in both surveys. I had separate protocols for these follow-up interviews because of their varying responses. These protocols may be found in Appendices E and F.

Information is available concerning the two interviewees—one of these professors is a mechanical engineering professor at a private institution, while the other professor is a civil engineering professor at a public institution. Both participants had many years of teaching experience, and the civil engineering professor also had experience working with industries as a consultant.

Data Analysis

The data was analyzed as a whole and also separated and analyzed based on the following demographics: type of engineering field, public or private university teacher, age, years of experience teaching engineering, and years of experience as a career engineer. These demographics were chosen with the help of an advisor. I hypothesized that these demographics would be significant factors that would cause engineering professors to think differently than their peers about the learning of differential equations. My research advisor also helped me to determine the categories for each of the demographics that follow.

The different types of engineering are as follows: biological/biomedical, chemical, civil (environmental), civil (structural), electrical/computer, mechanical/aerospace, and miscellaneous/other. These groups were determined after the surveys were completed. We discussed key words in the surveys that suggested which professors taught similar concepts and applications of differential equations. There were a large number of civil engineering responses, but the main deciding factor to split this group was this qualitative analysis of the types of concepts taught by these professors. While they will most certainly have some overlap in their coursework, we found the responses of "What courses do you teach?" and "Where do you use differential equations in your coursework?" to be different enough to warrant separating the groups. The numbers of respondents in each category were as follows:

Table 1

Number of Respondents in Each Engineering Field

Biological/Biomedical	5
Chemical	13
Civil (Environmental)	12
Civil (Structural)	10
Electrical/Computer	19
Mechanical/Aerospace	18
Other	6

Twenty of the professors are from private schools, and sixty-three of professors are from public institutions. While I did analyze this data, I had no rationale or consistency amongst the data to draw conclusions. Thus, no information will be reported in the results concerning this demographic.

The age groups of the professors were separated as follows: 20s-30s, 40s, 50s, 60s-70s. The 20s-30s and 60s-70s were grouped together because we typically find few professors in the 20s or 70s age groups in the university. Thus, we decided to have more data in the high end and low end of age range data. Some respondents chose not to enter their age. These respondents were not included in the analysis.

The number of respondents in each category were as follows:

Table 2

Number of Respondents for Each Age Bracket

20s-30s	18
40s	20
50s	30
60s-70s	9

The years of experience teaching engineering were determined in 10-year intervals and the years of experience as a career engineer was separated out in 5-year intervals. I anticipated that more professors would have more experience teaching than working in an engineering field. Thus, I decided to have wider intervals for academic experience than field experience. Specifically, the years of experience teaching engineering were separated out from 0-10 years, 11-20 years, 21-30 years, and greater than 30 years. The years of experience as a career engineer were separated out 0-5 years, 6-10 years, 11-15 years, and greater than 15 years.

The numbers of respondents with the respective years of experience teaching engineering were as follows:

Table 3 *Number of Respondents for Each Category of Teaching Experience*	
0-10	26
11-20	26
21-30	22
>30	5

The numbers of respondents in each of the categories under years of experience in an engineering career were as follows:

Table 4 *Number of Respondents for Each Category of Career Experience*	
0-5	42
6-10	23
11-15	10
>15	5

I decided to first analyze my data by putting all of the survey answers into a master Excel spreadsheet. This quantitative data was later used for both "quantizing" of the frequency of response and "linkage" (Miles and Huberman, 1994, p.42) between the quantitative data from the open-ended questions and interviews. I then compiled all of the data into separate Excel spreadsheets by demographic. For each demographic, the data was separated into the predetermined categories as listed above. After the surveys had been arranged based on demographic category, each of the questions was analyzed. The survey asked for engineering professors to rank curriculum topics and misconceptions' level importance on a scale from 1 to 4, with 1 being very important to their coursework and 4 being of very little to no importance to their coursework. I chose this 4-point scale because I did not wish to have an option that was undecided or neutral. I believed that a 4-point scale would create a clear separation of "yes" and "no" responses of significance. For each of the categories/questions, the results were averaged in order to rank the importance of the topic or misconception to that particular group. Once all of the topics were averaged, the topics were arranged in order from greatest importance (which corresponded to the lowest score) to the least importance (which corresponded to the highest score).

The survey questions about misconceptions being important to engineering curricula were asked as "yes" versus "no" questions. The null hypothesis was that the number of yes responses would equal the number of no responses. Then tests of significance were performed to determine if the number of "yes" responses was significantly lower than the number of "no" responses at the $\alpha = .05$ significance level.

The question that asked professors about a preference for correctness versus student understanding was analyzed in a similar fashion as the misconception questions. Instead of "yes" or "no," they chose whether they preferred a correct answer without understanding or incorrect answer with understanding. I noted in these in my spreadsheet as "correct" and "incorrect," and had each Excel demographic spreadsheet count the number of "correct" and "incorrect" responses in each of my predetermined categories.

Each of the technology questions were open-ended questions. I used open coding for concepts to be "identified and their properties and dimensions to be discovered in the data" (Strauss and Corbin, 1998, p. 101). I searched for key words and phrases among this data, such as "numerical techniques" or "Maple" in all of the questions about technology and then recorded the frequencies of these most popular words. Many people chose not to respond to these questions, or wrote very little about these questions, so there are low frequencies for many of the common phrases. I decided to only analyze this data as a whole and not by different demographics. I did this because I was more concerned in the overall perception in technology, whereas the demographic analysis was intended from the writing of the survey to tell information about engineering professors' opinions of curriculum topics and their students' relational understanding.

The follow-up surveys and interviews were conducted so that I might gain more insight into the survey questions. I was alarmed at the low number of professors who were interested in the misconceptions, but were interested in the conceptual understanding of a numerical method. I was also interested why there was a high emphasis on matrix methods. In order to answer these subtopics, I sent these follow-up surveys via email.

For both the follow-up surveys and the interviews, I again used open-coding to find pieces of information that spoke directly to my research questions. These responses were largely used for quotations within my results. Thus, I identified which of the responses addressed each research question. Many responses, or partial responses, were omitted from the results because they did not directly address the research questions at hand. I tried to match the interviewee responses with my other data in order to create a more complete picture of their opinions. Once I organized the interview responses based on research question, I identified commonalities with the two responses and noted common responses in my results.

Chapter 4
Results and Discussion

As previously stated in Chapters 1 and 2, differential equations have traditionally been taught as a series of different analytical techniques. If we are to proceed with reform efforts and incorporate different types of solution strategies, we must first ask which of the analytical techniques must remain in the curriculum, and which techniques we can afford to replace with reform-based curriculum materials. Since the differential equations course serves many engineering departments, asking these professors about differential equations is a good start for determining which topics should be most emphasized in the reform-based curriculum. We must also take a look at the new types of solution strategies that a reform-based curriculum might offer, and analyze which of these are most important as well.

The reform-based curriculum in differential equations seeks to enhance students' conceptual understanding of the material, as well as provide opportunities to connect these ideas with visual representations of the solutions. The reformers speak of addressing the "nonlinear revolution" (Boyce, 1994) in differential equations with these computer-generated visualizations. However, there has been no research on whether or not the engineering professors are concerned about students' understanding of this mathematics content, and very little research has been done concerning how they would like to see technology incorporated into the differential equations classroom.

In the following results, I will discuss my findings concerning engineering professors' opinions of the differential equations curriculum topics. Then I will discuss their opinions of students' understanding of this content. Finally, I will discuss engineering professors opinions about their use and their students' use of technology to solve differential equations.

Differential Equations Curricula for Engineers

In the following section, I will discuss which analytic techniques and which solution strategies are deemed most important by the eighty-three engineering professors that completed my survey. I also include information from follow-up surveys and interviews with several of these professors that offer greater insight into the overall opinions concerning the curriculum content. There will be no results concerning public versus private universities (see Chapter 3).

Analytical techniques.

Engineering professors were asked in the survey to rank eight different analytical methods for solving differential equations on a scale from 1 to 4, with 1 being most important and 4 being least important. For a description of these analytical methods, please see Chapter 3. Table 5 shows the results of these averages. For information on compilation of averages, see Chapter 3.

Table 5 *Comparison of Analytic Technique Preferences with Engineering Field*								
	Biological/ Biomedical	Chemical	Civil (E)	Civil (S)	Electrical/ Computer	Mechanical/ Aerospace	Other	Overall
Separation of Variables	1.60	1.31	1.33	1.60	2.05	1.78	1.50	1.65
First Order Linear/ Integrating Factor	1.60	1.46	1.92	2.70	2.11	2.11	2.00	2.01
Method of Undetermined Coefficients/ Variation of Parameters	1.80	2.00	2.58	2.50	2.53	2.28	1.83	2.30
Auxiliary Equations	2.60	2.08	2.45	2.80	2.26	2.06	2.40	2.31
Change of Variables/ Substitutions	1.80	1.38	1.82	2.44	1.79	2.28	2.00	1.93
Matrix Methods	1.60	2.08	2.25	2.10	1.74	1.78	1.83	1.92
Laplace Transforms	1.80	2.08	2.83	2.70	1.44	1.83	1.83	2.04
Series Solutions	2.60	2.69	2.83	3.00	2.05	2.83	2.50	2.65

After averaging these results, the overall topics of greatest to least importance were:

1. Separation of Variables
2. Matrix methods
3. Substitution methods
4. First order linear method/Integrating Factor
5. Laplace transforms
6. Variation of parameters*
7. Auxiliary equations*
8. Series methods

   *Denotes equal ranking

Trends among different types of engineering became apparent when looking at these analytic solutions. The chemical engineers ranked matrix methods below substitution methods, first order linear, and variation of parameters. Electrical engineers ranked Laplace transforms in first place, and were also the only group to rank series solutions anything other than last place. Their average shows that series solutions were tied with separation of variables in fourth place. I was interested about the importance of transform methods to electrical engineers by asking about these methods with an electrical engineer in his interview:

> I think I wouldn't want other topics to get shorted in the mathematics class because you felt like you had to get to the Laplace transforms because it's what the engineers need. I feel like if a student is well grounded in what a differential equation is doing, then the best place to introduce the transform method is in the context of the engineering class.

Transform methods are often found at the end of a first course in differential equations. Instead of rushing to complete this material, this professor suggests that, while his students do need transforms, he would rather lose their exposure to this in order for them to gain a firmer foundation in the basics of differential equations. Thus, mathematicians who say that they have to cover too much material may be over-generalizing the opinions of the engineering departments at their universities. The mathematics department may be able to move transform methods to an engineering course. Similarly, the same may be done for series solutions since they were ranked lowest of all. Engineering professors with specific requests for mathematicians to cover these topics may find that their students will be better prepared if they learn these particular methods in their courses or in a second course of differential equations.

Matrix methods, unlike transform methods and series solutions, were ranked high on the overall results list. I asked professors in the interview why they thought this high ranking for matrix methods probably occurred in my data. One of my interviewees explained the importance of matrix methods to engineers as follows:

> When they get to applied engineering courses, matrices simply make things easier. Rather than doing all of this stuff longhand, you can do it in matrix format and take all the work out of it. For that reason, I find them very useful… For example, let's say in a structures problem where you have a frame, and you want to analyze that frame and determine the force of all the members. Well, you could do that by writ[ing] the equations in a form of a matrix. Then, you know, you have to invert the set of equations to get the answer.

Matrices not only make differential equations easier for engineers, but they also are used "for a large number of problems," according to one professor. Thus, matrices are used in many instances and can handle large amounts of seemingly complicated data. Instead of lengthening the number of analytical methods into the differential equations course, this finding concerning the importance of matrices suggests that we instead spend more time on more widely used analytical methods. If we eliminate some of the less frequently used analytic methods, instructors will have more time to invest in matrix methods, which are used by a wider variety of engineers.

One professor noted in the follow-up survey that while matrix methods were important, he thought they could be "introduced as needed" in the differential equations course. However, we still must consider that if this topic is used on a more widespread basis, we may be making more efficient use of our time and our students' time by introducing the common topics in depth in the first course in differential equations. If students have a firmer grasp on these more common methods, then they may use technology in a more effective way for other types of differential equations. I address this issue more in Chapter 5.

There is some difference in the ranking of importance of these analytical topics to the electrical and mechanical/aerospace engineering categories, but the overall trend remained fairly constant when analyzed among the other demographics of type of university, age, and levels of experience.

While the ranking may be the same across demographics, age may be a factor for engineering professors' opinions concerning the importance of *all* types of analytical methods for solving differential equations. Older professors tended to rank each of the different techniques with lower scores (more importance) than the younger professors. In Table 6, note that the older professors had lower averages than the younger professors, indicating a desire for emphasis for all of the analytical differential equations topics.

Table 6

Comparison of Analytical Technique Preferences with Age

	20s-30s	40s	50s	60s-70s
Separation of Variables	1.94	1.70	1.47	1.56
First Order Linear/ Integrating Factor	2.39	2.05	1.93	1.67
Method of Undetermined Coefficients/ Variation of Parameters	2.61	2.65	2.07	1.89
Auxiliary Equations	2.53	2.35	2.20	1.89
Change of Variables/ Substitutions	2.35	1.60	1.83	1.63
Matrix Methods	2.22	1.55	2.10	1.78
Laplace Transforms	2.44	2.00	1.90	1.89
Series Solutions	3.17	2.55	2.57	2.00

Similar to the increase of importance to professors with an older age, there is also a trend that the importance of each topic increases with each group according to years of teaching and years of field experience as shown below. With the exception of variation of parameters and undetermined coefficients for the years of career experience, all of the categories show decreased numbers from the least experienced categories to the greatest experience categories. The differences in the values of the less experienced and more experienced is smaller than the difference in the age category, but this may be a result of the small sampling size of the groups of professors with the most experience.

Table 7

Comparison of Analytical Technique Preferences with Years of Teaching Experience

	0-10	11-20	21-30	>30
Separation of Variables	1.77	1.69	1.64	1.25
First Order Linear/ Integrating Factor	2.23	1.88	2.05	1.75
Method of Undetermined Coefficients/ Variation of Parameters	2.58	2.54	1.91	1.63
Auxiliary Equations	2.40	2.42	2.18	2.14
Change of Variables/ Substitutions	2.04	1.88	1.91	1.86
Matrix Methods	1.88	2.08	1.86	1.75
Laplace Transforms	2.19	2.08	1.86	2.00
Series Solutions	2.96	2.54	2.55	2.43

Table 8

Comparison of Analytical Technique Preferences with Years of Career Experience

	0-5	6-10	11-15	>15
Separation of Variables	1.69	1.55	2.00	1.00
First Order Linear/ Integrating Factor	2.00	2.18	1.90	1.40
Method of Undetermined Coefficients/ Variation of Parameters	2.19	2.45	2.10	2.80
Auxiliary Equations	2.35	2.45	1.90	2.20
Change of Variables/ Substitutions	2.05	2.00	1.80	1.20
Matrix Methods	1.98	2.09	1.60	1.60
Laplace Transforms	2.17	2.09	1.50	2.00
Series Solutions	2.63	2.82	2.50	2.40

Thus, those who are older and more experienced seem to think that all analytic techniques in differential equations pertain to their curricula more than the younger and less professors. This finding was consistent with results from the next section, where professors who were older and more experienced ranked each of the solution strategies overall as being more important than their younger and less experienced counterparts.

Solution strategies.

Analytical methods are not the only way that information may be obtained from differential equations. Thus, the differential equations curricula must consider various other approaches to the analysis of differential equations or systems of differential equations. The approaches to differential equations were ranked of greatest (smallest ranking) to least importance (highest ranking) by engineering professors as follows:

1. Modeling scenarios
2. Numerical analysis
3. Analytical methods
4. Systems of differential equations
5. Qualitative analysis
6. Mathematical theory

This ranking was mostly consistent across the different types of engineering departments, which is evident in Table 9, as modeling typically has the lowest average number and mathematical theory typically has the highest average number.

Table 9

Comparison of Solution Strategy Preferences with Engineering Field

	Biological/ Biomedical	Chemical	Civil (E)	Civil (S)	Electrical/ Computer	Mechanical/ Aerospace	Other	Overall
Mathematical Theory	1.80	3.00	2.50	2.80	2.21	2.65	2.50	2.54
Qualitative Analysis	2.00	2.62	2.42	1.90	2.00	2.28	2.00	2.20
Analytical Techniques	1.60	1.54	1.67	2.00	1.68	2.12	1.67	1.78
Numerical Methods	2.20	1.23	1.50	1.60	1.74	1.83	1.83	1.66
Modeling & Applications	1.80	1.23	1.17	1.70	1.47	1.61	1.67	1.48

Table 9 (continued)

	Biological/ Biomedical	Chemical	Civil (E)	Civil (S)	Electrical/ Computer	Mechanical/ Aerospace	Other	Overall
Combination of Methods and Graphs to study Dynamical Systems	1.80	2.08	2.08	2.00	2.26	2.33	2.00	2.14

Bioengineers differed most from the overall list, as is evidenced by the low average bioengineers gave for mathematical theory. They placed analytical approaches first and numerical approaches last. Biological engineers appear to need more focus on mathematical theory and traditional analytic methods. Thus, mathematics majors and biological engineers may benefit from studying differential equations together.

Numerical analysis and qualitative analysis are both seen in reform efforts in mathematics education research. However, differences in importance for engineering educators became obvious in this study. An engineering professor describes talking to a friend of his who is a chemical engineer: "[my friend] said to get a good analytic solution that came out of a problem I was doing, I'd be dead in the water without a computer for getting real results. So he stresses that when he teaches the class, he throws in a few nonlinear equations that you can only get at numerically." From the data, we can see that chemical engineers ranked numerical methods as having more significance than any of the other engineering categories.

From the engineering perspective, the numerical methods are important for solving many differential equations that are unsolvable without approximation methods. A civil engineer stated that in his field, “there are very few useful analytical solutions.”

Numerical techniques and qualitative analysis were placed in separate categories because numerical techniques actually help to find a solution to a differential equation, while qualitative analysis can just tell us about certain aspects of the differential equation or how systems of differential equations interact. While engineering professors agree that we must continue to focus on emphasizing numerical techniques in differential equations courses, the emphasis on qualitative analysis may not be as important to the engineering community, which was ranked fifth out of the six options for solution strategies. Other than qualitative analysis, we will also see that mathematical theory is of little importance directly to the engineering curriculum. Thus, engineering professors tend to want differential equations to emphasize numerical techniques more than qualitative analysis and mathematical theory. In the interviews, one professor noted that theory that is necessary for applications often comes up in engineering graduate school and that undergraduate mathematics needs to focus on other concepts.

The interviewed engineering professors also commented that they would like for their students to be more comfortable with seeing problems of varying types before coming to engineering courses. Thus, they would like for their students to be exposed to approaching novel problems in mathematics courses as well as in engineering courses. There exists a great disconnect between the differential equations curriculum's typical emphasis and the emphasis engineers place on modeling and applications. Students are presented in differential equations courses with several engineering applications problems (the brine problem, for example). However, project-based learning is finding itself more in engineering education curricula and not in mathematics education curricula. In order to aid the engineering community, project-based learning during differential equations may be helpful preparation for modeling they will do in a higher-level course such as partial differential equations.

Yet again, professors who were older typically ranked all categories as being more important than the younger professors, as evidenced by the lower scores for older groups below:

Table 10

Comparison of Solution Strategy Preferences with Age

	20s-30s	40s	50s	60s-70s
Mathematical Theory	2.94	2.16	2.60	2.44
Qualitative Analysis	2.39	1.95	2.33	2.22
Analytical Techniques	2.28	1.84	1.60	1.78
Numerical Methods	1.89	1.55	1.70	1.56

Table 10 (continued)

	20s-30s	40s	50s	60s-70s
Modeling & Applications	2.00	1.45	1.40	1.11
Combination of Methods and Graphs to study Dynamical Systems	2.50	2.20	1.93	1.89

The importance in difference among older and younger professors concerning the importance of analytical techniques has already been discussed in the first section of the results. Overall, for the ordering of all topics, there are similarities among each age group, with all groups finding that modeling, numerical methods, and analytical techniques as being the three most important solution strategies.

For teaching and career experience of engineers, the differences among older and younger professors are not as clear. Among the oldest and youngest groups, there is a lower score for all categories except for mathematical theory. For those with the most teaching experience and for those with the most career experience, we see higher values for teaching mathematical theory than among the younger and less experienced professors. Since the older and more experienced professors have lower scores for the other topics, we may assume that these professors feel strongly that mathematical theory is the least important way for engineering students to obtain the most applicable information for success in their other engineering courses.

Table 11 *Comparison of Solution Strategy Preferences with Years of Teaching Experience*				
	0-10	11-20	21-30	>30
Mathematical Theory	2.6	2.5	2.45	2.75
Qualitative Analysis	2.27	2.23	2.05	2.38
Analytical Techniques	1.92	1.85	1.55	1.88
Numerical Methods	1.65	1.73	1.77	1.13
Modeling & Applications	1.69	1.5	1.36	1.13
Combination of Methods and Graphs to study Dynamical Systems	2.23	2.19	2.14	1.88

This low importance for mathematical theory is even more prevalent among those who have more career experience. These professors show a large gap between mathematical theory and the other strategies. Those with more field experience also greatly show that they care more about dynamical systems than their less experienced colleagues.

Table 12 *Comparison of Solution Strategy Preferences with Years of Career Experience*				
	0-5	6-10	11-15	>15
Mathematical Theory	2.44	2.73	2.10	3.00
Qualitative Analysis	2.24	2.27	2.20	1.80
Analytical Techniques	1.88	1.64	1.70	1.60
Numerical Methods	1.76	1.55	1.80	1.60
Modeling & Applications	1.60	1.36	1.20	1.40
Combination of Methods and Graphs to study Dynamical Systems	2.43	2.00	1.90	1.40

The topics other than mathematical theory also show that career experience causes professors to rank strategies more important, but the margin of difference is smaller for the other categories. Dynamical systems are less important to those with very little career experience. This same difference was less in the teaching experience scenario than the career experience scenario. Thus, we may conclude that dynamical systems may be more important for those in engineering careers than for engineering researchers. More research needs to be done to confirm this hypothesis.

Engineers' Relational Understanding of Differential Equations

We have information from the previous section about specific content that needs to have greater emphasis for engineering students in differential equations. In this section I discuss the results of the participant's responses concerning the understanding students have of specific content within some of the important analytical techniques.

Several different common differential equations misconceptions made by students were listed. Professors then decided if these misconceptions were important to their specific curricula or not. If they checked "yes," then this indicated that the misconception was a concern for their curricula and if they checked "no," this indicated the misconception was not important to address for their specific curricula. I decided to analyze the number of "yes" responses and "no" responses by using hypothesis testing. My null hypothesis for my analysis was that there would be an equal proportion of yes and no responses from the professors as to whether or not these misconceptions were significant. The alternative hypothesis was that the proportion of "yes" responses was lower than the proportion of "no" responses.

$H_0 : p_1 = p_2$
$H_a : p_1 < p_2$

I chose this alternative hypothesis because I assumed that there would be more "no" responses from professors than "yes" responses if there was any difference. Since there is limited research on this issue, I wanted to assume that relational understanding is of little importance to the professors. Thus, having the hypothesis test constructed this way would tell me which categories had more "no" responses than other categories, even if they both had more "no" responses than "yes" responses. I believed this would help cause topics that were least helpful to engineers to surface more clearly.

From this point on, I will refer to the misconceptions by the following labels. The order of the number was determined by the order in which I presented the misconceptions in the survey.

> Misconception 1: Students sometimes show that they do not realize that the solution of a differential equation is the function in the original differential equation.

Misconception 2: In separation of variables, students sometimes think that they are permitted to separate dy and dt because it is a fraction.

Misconception 3: Students sometimes cannot make the distinction that equilibrium solutions exist in separable differential equations but not linear differential equations.

Misconception 4: Students sometimes show that they do not understand that approximation methods do not follow the exact solution curve exactly, and based on the error of the approximation, the approximation may deviate from the actual function solution at an increasingly large rate as t tends toward infinity.

In the analysis of every one of these misconceptions, the number of "yes" (significant to their curricula) responses was lower than the number of "no" (not significant to their curricula) responses for the overall category. All four of the misconceptions had a significant *p-value* at the $\alpha = .05$ significance level, indicating that the number of "yes" responses for the whole population will likely be smaller than the number of "no" responses. However, the analysis of specific categories is more telling of which misconceptions are more important to certain types of engineering professors.

In the follow-up survey, one professor noted that Misconception 1 and Misconception 4 alarmed him the most, and found it surprising that these two were not ranked higher by engineering professors. These results are especially surprising since separation of variables and numerical methods were both ranked high in the curriculum portion of this survey. In the misconception data, one may notice that certain types of engineering professors may have skewed the overall ranking of important curriculum topics in the earlier portion of the survey. Also, more research needs to be done to investigate the important subtopics within the curriculum topics that do need to be covered, and in what context these misconceptions should be addressed.

Misconception 1: Students' lack of understanding of function as solution.

This misconception seems to be the most basic of the four presented because it involves the goal of solving any type of differential equation. However, the importance of this misconception to engineering professors is not clear.

The number of "no" and "yes" responses, and the approximate *p-values* for the hypothesis tests for significant difference in number of responses were as follows:

Table 13

Comparison of Misconception 1 Importance by Engineering Field

	Biological	Chemical	Civil (E)	Civil (S)	Electrical	Mechanical	Other	Overall
No	2	8	5	6	11	11	4	47
Yes	2	3	6	4	8	7	2	32
p-values	.5000	.0655	.6179	.2643	.2451	.1131	.0516	.0455

There were no significant differences in "yes" and "no" responses for any of the different engineering categories. While there were typically more "no" responses, these were not significant. However, civil engineering professors and electrical engineers showed that they had higher *p-values*, which indicates less difference in the number of "yes" and "no" responses. Environmental civil engineers actually had more "yes" responses than "no" responses, indicating that they may have more concern for the function as solution problem than other types of engineers. The chemical engineering professors and miscellaneous group had the lowest *p-values*, suggesting that they may care less than the other professors about this particular misconception. There is no evidence from my qualitative data as to why this difference occurs for this misconception.

Overall, the results were statistically significant that the number of "yes" responses would be lower than the number of "no" responses. The hypothesis test for the proportion of "yes" responses being less than half has an overall *p-value* of .0455, indicating that the number of "yes" responses is significantly lower for this misconception, even though there was no significant difference in each category.

This particular misconception seems to be more important to those in their 40s and those in their 60s and 70s (see Table 14). There is no significant difference between "yes" and "no" responses for these two age groups. There are a significantly lower number of "yes" responses than "no" responses for the 20s-30s age group. Since there is this up and down shift, the years of experience seems to be more telling of how one may distinguish importance as one progresses as a teacher.

Table 14 *Comparison of Misconception 1 Importance by Age*				
	20s-30s	40s	50s	60s-70s
No	12	9	18	4
Yes	5	11	10	4
p-values	.0446	.6736	.0655	.5

Teaching experience shows a similar trend to age, but it is more distinctive. The least experienced teachers and younger teachers having the greatest percentage of "no" responses. However, responses from professors with varying years of teaching experience also shows that those with little and those with the greatest number of experience have significantly fewer "no" responses than the other groups. The middle groups are very closely split on this misconception.

Table 15 *Comparison of Misconception 1 Importance by Teaching Experience*				
	0-10	11-20	21-30	>30
No	18	13	11	4
Yes	7	12	11	2
p-values	.0139	.4207	.5000	.0516

Teachers seem to go through a shift as to whether or not this misconception is important. As they start and end their careers, they are less likely to be concerned with students explicitly stating that a solution is a function. Mid-career, however, half of these professors found that this distinction was important. As stated above in the transcript, this split may have occurred because of the interpretation of the wording of the misconception.

Career experience, on the other hand, seems to be the biggest indicator of concern for the function as solution misconception. With more career experience, we see an increased percentage in the number of "yes" percentages, with the least career experience exhibiting a 44% rate for the number of "yes" answers to an 80% rate for the group with the most field experience. There is also a major distinction among the *p-values* for this group as evidenced below, showing that the youngest group has a significant *p-value*, indicating the number of "yes" responses is lower than the number of "no" responses. The high *p-value* of the most experienced group actually indicates that the results for this group would show significance if the alternative hypothesis was that number of "yes" responses would be greater than the number of "no" responses. Thus, there is strong evidence to indicate that those with more career experience care more about the function as solution misconception.

Table 16

Comparison of Misconception 1 Importance by Career Experience

	0-5	6-10	11-15	>15
No	27	12	5	1
Yes	12	9	5	4
p-values	.0082	.2578	.5000	.9099

More than any other misconception, this one seems to be where the professors have the greatest split in number of "yes" the concept is important responses and "no" the concept is not important responses, which I will discuss further with Table 13. This distinction was clarified by the interviews with the two professors who responded with different answers.

One professor noted why he thought the misconception was important: "It's only when they get to classes like the ones I and other faculty members of us that teach in engineering where it becomes important to know that the solution to a differential equation is another equation, and the reason it is if you don't know that, you don't know whether you have to solve a differential equation or not." The following professor explains why he chose that understanding the solution to be a function was not imperative:

> So, if a student says I solved the differential equation and here's the curve that represents the position of the map over time, and I said oh, now is that a function? And they said, oh, well, I guess so, I guess I wouldn't worry too much about that because they solved the problem, they understand the position of the map of the function of time, they read the graph and interpret
>
> the result. Whether or not they're sort of consciously coming out with the result, oh this is a function x(t) and pairing along with everything we know about functions, you know continuous, single value, or whatever, I guess I

> wouldn't worry too much about that because I kind of think that all the mathematics of what is a function has kind of receded into the background because now what they're doing is focusing on what they are doing to the result.

He then went on to describe the main distinction he had in his mind about the major misconception concerning solutions of differential equations:

> On the other hand, if a student says they solved the differential equation and they got a number, they didn't get x(t), they got a number, probably that the solution of the differential equation was a constant, with no time dependence whatever, I would say let's talk about what differential equations are supposed to be telling us here and that I think would be a real problem.

While the second professor didn't mark that "yes" this misconception is important on his survey, he still has inclinations that students need to understand time dependence, which is the nature of the function solution. The same reasoning came from the other interviewee who marked "yes": "What I do try to point out and the misconception that I'm talking about is very often if I say what is a solution if I write a differential equation on the board and say what is the solution and students will give me answers as though it was a number." Thus, the wording of the question may have resulted in different answers if I had written in the survey that students think that the result of a differential equation is a number and not a function. Thus, the emphasis on a solution not being a number was the same response for both professors, despite their differing answers on the survey.

Misconception 2: Thinking of dy/dt only as a finite entity.

The second misconception arises from just the separation of variables technique, and is not a problem for all differential equations like the first misconception. In separation of variables solutions with a derivative dy/dt, we offer to our students a shortcut that demonstrates that we can place all of the y values on one side of the equation, and all of the t values on the other side of the equation. However, splitting dy/dt is not a mathematical generalization. The limit has already been taken when we are looking at the definition of dy/dt, and is not the same as Δy divided by Δt. The number of "no" and "yes" responses, and the approximate *p-values* for the hypothesis tests for significant difference in number of responses were as follows:

Table 17

Comparison of Misconception 2 Importance by Engineering Field

	Biological	Chemical	Civil (E)	Civil (S)	Electrical	Mechanical	Other	Overall
No	2	8	7	8	10	13	4	52
Yes	2	5	5	2	9	5	2	30
p-values	.5000	.2033	.2810	.0287	.4090	.0294	.2061	.0075

There were significantly fewer "yes" responses than "no" responses for the structural civil engineers and the mechanical engineers. There is no evidence to show why this distinction has occurred in these two groups. Biological engineers had the highest *p-values* of .5, which indicates an even split in "yes" and "no" responses. This group ranked analytical techniques higher on their solution strategies list than some of the other strategies. Thus, doing a by-hand calculation of the separation of variables method probably occurs more often with biological engineers than some of the other groups.

In the other demographic categories, there were typically more "no" responses in all categories and there was very little variability among the different categories. Thus, the only group that I found that indicates that the fraction misconception is of significant concern is for biological engineers.

During one of the phone interviews, I explained that while dy/dt splitting in separation of variables is a legitimate shortcut to a correct solution, students sometimes take away the notion that dy is the same as the difference Δy . While this nuance does not hinder understanding of the separation of variables method, there may be other situations where this distinction is necessary to make.

Some engineering professors may inadvertently encourage this mathematically incorrect notion:

> On that method of modeling, what we're doing here is say it's this part delta x and as delta x gets small low and behold we have the partial derivative, and we do that carefully. With experience, instead of

> calling that thickness delta x, you start calling it dx because you kind of know that that's where you're going in the end because you've done it enough times you sort of know… The problem can be that when you're learning that, if you conflate the steps too soon and the student reduces it to an algorithm before they really catch on, then I think they can get into the business of mistaking delta x for dx.

Students may be attaining this incorrect notion of equality between $\frac{\Delta y}{\Delta t}$ and $\frac{dy}{dt}$ because of engineering professors' examples. In many cases, the engineers begin to think of these mathematical representations as being the same thing, because in their situations, doing so yields a correct answer, even if the notion is slightly incorrect. One professor notes on his follow-up survey: "The genius of the Leibnitz's notation $\frac{dy}{dt}$ is that it CAN be treated as a fraction in all practical situations I can think of. And even in calculus we ENCOURAGE students to treat it as such, for it is very helpful in manipulation."

More work needs to be done concerning this misconception to see if the misconception causes problems in other topics in engineering other than for separation of variables. Upon talking with one of the engineering professors, I found that some of the respondents were thinking about whether or not the misconception causes problems in the ability to do a separation of variables problem. We know that the fraction concept does not result in a wrong answer here. However, more work needs to be done to determine if there are problems in other topics with students incorrectly splitting a $\frac{dy}{dt}$ expression into finite elements.

Misconception 3: Students' lack of understanding of the production of equilibrium solutions.

Another misconception, or lack of conception, that was found with the separation of variables technique specifically was that students were not able to distinguish that equilibrium solutions were only created if a differential equation was separable. This misconception seemed to be noticed more by biological, chemical, and civil engineers more than electrical/computer and mechanical/aerospace engineers.

The number of "no" and "yes" responses, and the approximate *p-values* for the hypothesis tests for significant difference in number of responses were as follows:

Table 18

Comparison of Misconception 3 Importance by Engineering Field

	Biological	Chemical	Civil (E)	Civil (S)	Electrical	Mechanical	Other	Overall
No	2	5	5	5	12	12	4	45
Yes	2	6	5	5	6	5	1	30
p-values	.5000	.6179	.5000	.5000	.0793	.0446	.0901	.0418

Thus, there is significant evidence to show that mechanical engineers, and perhaps electrical engineers, care less about equilibrium solutions than the other types of engineers. After conducting my interview with the electrical engineer, I realized that this is due to the differential equations that arise in electrical and mechanical engineering problems, and the lack of times that separation of variables is needed to solve their application problems.

I was discussing with an electrical engineer in the following transcript about the necessity (or lack of necessity) of distinguishing that equilibrium solutions can only be found if a differential equation is separable.

> Interviewer: So I was curious with you guys if it kind of mattered if students understand that that division right there is what's creating that equilibrium solution?
>
> Professor: Yeah, truthfully, it's not a problem I've encountered a lot in my teaching because in the classes I teach we don't get equations like this [separation of variables].

He had previously mentioned that in electrical engineering, many times "a lot of differential equations that [they] solve have constant coefficients." He described what they encounter as having "df/dt plus constant times f equals right hand side, not df/dt plus g(t)f(t) equals right hand side." Having constant coefficients was, in his opinion, beneficial because he says, "a lot of nasty business disappears, and that was where I was thinking, well in my context, I see a lot of equilibrium solutions."

While he sees equilibrium solutions, these apparently come from differential equations that are both linear and separable. In general, however, he does not have equations that are just separation of variables problems. He admitted that he himself did not have a strong distinction between these 2 methods and equilibrium solutions in his mind, and would have to think more if this notion would really be helpful for his own students' understanding. Thus, he had never found this equilibrium solution creation misconception to be a problem in his courses.

Increased age may also be an indicator of the importance of misconceptions to engineering professors. The first three age groups had similar percentages of "yes" responses (.4, .37, and .41, respectively). However, the final age group had a higher percentage rate of "yes" responses, yet again, of .625, which means they had more "yes" responses than "no" responses, unlike the other categories. This indicates that those of the highest age may care more than the younger professors about this distinction for equilibrium solutions.

Table 19

Comparison of Misconception 3 Importance by Age

	20s-30s	40s	50s	60s-70s
No	9	12	16	3
Yes	6	7	11	5
p-values	.2206	.1335	.1685	.7611

As professors get older and gain more experience, they may see a greater number of different types of differential equations in their field. Thus, this particular misconception may not happen often, but once professors get older, they may begin to note that this could be a helpful distinction for students to make in a differential equations course. This is confirmed by the analysis on teaching experience in the table below:

Table 20

Comparison of Misconception 3 Importance by Teaching Experience

	0-10	11-20	21-30	>30
No	28	16	13	3
Yes	11	7	9	3
p-values	.0033	.0301	.4129	.5000

The two groups with less teaching experience have significantly more "no" responses than "yes" responses. Thus, those with more teaching experience show more interest in making a distinction about the creation of equilibrium solutions.

More teaching experience and more field experience result in a greater number of professors indicating the importance of this distinction about equilibrium solutions. However, as with other misconceptions, there is more support that field experience is more telling of the importance of this misconception than teaching experience.

Table 21

Comparison of Misconception 3 Importance by Career Experience

	0-5	6-10	11-15	>15
No	26	11	4	2
Yes	11	9	5	3
p-values	.0068	.3264	.6293	.6736

Thus, knowledge of equilibrium solutions may be more important in an engineering career than an engineering course. More research needs to be done to investigate the opinions of career engineers and how they may use equilibrium solutions in their work.

Misconception 4: Students' lack of recognition in the flaws of numerical methods.

In both the surveys and in the interviews, the engineering professors continually emphasized the importance of numerical methods to their coursework. However, as noted earlier in this section on misconceptions, the data shows that numerical methods are more important based on the types of application that an engineer encounters. The number of "no" and "yes" responses, and the approximate *p-values* for the hypothesis tests for significant difference in number of responses were as follows:

Table 22

Comparison of Misconception 4 Importance by Engineering Field

	Biological	Chemical	Civil (E)	Civil (S)	Electrical	Mechanical	Other	Overall
No	2	5	6	7	14	11	5	50
Yes	2	7	5	3	5	6	1	29
p-values	.5000	.7190	.3821	.1038	.0197	.1131	.0516	.0091

Chemical engineers actually showed more "yes" responses than "no" responses. In interviewing, I found that chemical engineers use numerical solutions for many of the nonlinear differential equations that come up in their applications. Biological engineers and civil engineers had a more even split among their responses, while electrical, mechanical, and miscellaneous engineering professors find this misconception less important. These responses seem to correlate directly with the amount that each of these types of engineering use approximation methods for nonlinear differential equations.

Those who were older yet again showed more interest in this misconception, with the younger two groups having significantly more "no" responses than "yes" responses.

Table 23

Comparison of Misconception 4 Importance by Age

	20s-30s	40s	50s	60s-70s
No	12	14	15	4
Yes	5	6	13	4
p-values	.0446	.0367	.3520	.5000

Again, those with more field experience have more "yes" responses than "no" responses for this misconception. This finding was even more distinctive than that of the comparison between teaching experience categories. The interviewed professors noted that they were wary of students of the future not recognizing the errors that may result from using technology. Thus, they wanted to create very analytically astute students who used technology with extreme caution. The data below may support that those with career experience also find that this caution of approximation error is necessary in the workforce:

Table 24 *Comparison of Misconception 4 Importance by Years of Career Experience*				
	0-5	6-10	11-15	>15
No	26	15	6	2
Yes	13	6	4	3
p-values	.0188	.0250	.2643	.6736

The importance of misconceptions to distinctive groups of engineering professors.

In all of the misconceptions data, the overall results showed that the number of "yes" responses were significantly smaller than the number of "no" responses. Overall, this leads me to believe that these particular misconceptions are not considered serious problems for engineering students. However, this does not cause me to conclude that the engineering professors do not care about these misconceptions at all.

Those who were older, had more teaching experience, and especially those who had more career experience, showed that these misconceptions are more important than the overall data would suggest. The specific misconceptions also showed different results among different types of engineering. The second and third misconception both focused on the separation of variables technique, and those who do not use this technique obviously did not find it as important as the first and fourth more general misconceptions. While numerical methods were seemingly important overall, I became aware in the misconceptions study that some engineers use these more than others depending on the types of differential equations that arise in application problems. Thus, the large number of "no" responses cannot allow us to conclude that engineering professors do not care about the misconceptions. Instead, researchers might realize that misconceptions will be important based on the content of the misconception to certain types of engineers. Also, the younger and less experienced engineering professors need to be educated on the research about misconceptions in differential equations so that they may reach these conclusions about students' understanding more quickly.

Engineering professors' preference for relational understanding of approximation methods.

One may not conclude that relational understanding is insignificant to the engineering community, as previously discussed. This is further evidenced below in my specific question addressing the relational understanding of numerical methods. By a wide margin, the engineering professors chose that they preferred a student who can't remember the formula for a numerical technique, but has a general idea of why that technique works over a student who remembers the formula for the technique, but does not know why this method works. The latter was also said to have a correct answer, whereas the former had an incorrect solution. Thus, most preferred an incorrect solution with understanding more than a correct solution without understanding.

One of the interviewed professors made this comment after discussing the discomfort some engineers had felt about the calculus reform movement: "I know I want them to understand what they're doing, but I also want them to be able to solve problems. I don't want them to just be algorithmic about it so that they just do silly things—apply a method when it isn't legitimate." This comment encourages the notion that engineers are not simply focused on procedural; they have a necessity for correct answers, but also understand the need for interpreting that correct answer. Thus, procedural understanding seems to be a better descriptor of the desire of these engineering professors. Their students do need to have some understanding of procedures to understand the correctness of their solutions. Yet, they also need to be analytical with those solutions.

Table 25

Number of Respondents Preferring Understanding to Correctness

	Incorrect Response, but Understanding	Correct Response, Little or No Understanding
All Respondents	70	10

I made my null hypothesis that the preferences would be equal, and my alternative hypothesis was that the number of correct responses would be less than the number of incorrect responses. The *p-value* for this situation is approximately 0. Thus, there is very strong evidence that more professors prefer an incorrect response with understanding than a correct response with little or no understanding.

The only professors who chose that they would prefer a correct solution from a student with little or no understanding of a numerical method to a student with an incorrect solution who had strong understanding were all age 50 or above. This initially seemed alarming since more of the older professors indicated that they were concerned with the four misconceptions.

Table 26

Comparison of Preference for Understanding/Correctness with Age

	20s-30s	40s	50s	60s-70s
Correct Response, Little or No Understanding	0	0	8	2
Incorrect Response, but Understanding	18	18	22	7

During interviews, I became aware of the weightiness of a correct solution upon older professors. Ideally, the professors would have students who both understand and have a correct solution. However, for some professors the correct solution has more importance because of the dangers of an incorrect solution. Understanding the physics of a structure may be more important than understanding the differential equation's numerical method that models the structural system. Many professors noted that they need students to be wary of their solutions. Some address this by saying they want them to be wary of the limits of a numerical method. Others say they want them to be wary of the actual outputs of their differential equation solution and be able to interpret the correctness of the model to the physical situation.

The misconceptions data indicated that increased career experience was an indicator that a professor would be more concerned about students' relational understanding of differential equations. Increased career experience seems to be an even stronger indicator of a preference for an incorrect response with understanding. The greatest number of correct responses occurred with those who had little or no career experience, and those with the most career experience had no one respond that they would prefer a correct response.

Table 27

Comparison of Preference for Understanding/Correctness with Career Experience

	0-5	6-10	11-15	>15
Correct Response, Little or No Understanding	5	2	3	0
Incorrect Response, but Understanding	36	19	6	5

Technological Impact on Differential Equations in Engineering Education

Career engineers, engineering professors, and engineering students have seen a change in engineering in the past 20 or 30 years due to the growing popularity of computer programs such as Mathematica, MatLab, Maple, and MathCad, to name a few. Once engineers had personal computers at their fingertips, the software followed suit, and the practice of engineering became visualization of mathematical and physical models became more quickly and precisely generated.

The majority of the surveys showed that technology was seen as having an impact on how both professors now teach their engineering courses and also how they themselves approach engineering problems. The following software programs were cited by the participants as being used by themselves and/or their students, in decreasing order of use.

Table 28

Number of Respondents Noting Software

MatLab	22
Mathematica	5
Maple	5
MathCad	2

Twenty-nine professors noted in their open-ended responses spoke of numerical solutions or techniques being easily accessed due to these technologies. Thus, most of the time when I dialogued with these professors, they spoke of using these types of technology for numerical solutions as opposed to using technology for differential equations in general. They use computer software more for numerical techniques than any other type of solution method. Twenty-three professors noted that students have the ability to do more involved problems and provide more detailed answers due to technological approximations of differential equations. One professor notes, “It’s now possible for an undergraduate to develop a sophisticated numerical simulation of a complex structure or flow using commercial software.” Four professors noted that technology also makes differential equations easier for their students to solve.

Similar to the research cited in the literature review, technology has had a similar impact on these engineering professors as it has had for mathematics professors. The same professor goes on to say, “It’s dangerous as well because sophisticated analyses can often appear authoritative but be wrong in ways that are trickier to sort out than simple ones.” Students have a greater capacity to store and manipulate large amounts of data. They can easily find a solution to a differential equation or system of equations. However, their ability to interpret this data may be hindered by technology.

Engineering professors noted that not only were students less likely to be familiar with even simple analytic methods such as separation of variables, but they also failed to note that they had erratic results that were more obviously noted by students of the past. Four professors wrote in their surveys about a harmful reliance on technology, specifically calculators, and nine professors stated that their students either could not interpret solutions and/or they don’t understand the importance of what the software is computing for them. In the phone interviews one professor noted, “My concern is you’ve got a bunch of engineers out there using software that will do this stuff for you and generate solutions, but they have no clue and don’t know how to determine if that solution is correct. Now, there’s a problem.”

Students may not note these incorrect results for various reasons. Some students may notice and correct their program, but others may turn in their incorrect result. I consulted with an engineering professor on this matter. He classified students into 3 different categories:

> There’s one group that will kill themselves to get the right answer. They will fight to get that right answer, whatever it is. There’s the other group that doesn’t even know enough to know how to even make sense of the answer that they get. There’s 2 groups and it’s hard to distinguish between the 2 groups. One is the students are so weak that they don’t even realize what they answer ought to look like…The other one is more devious. They know the answer is wrong, but they’re too lazy to figure it out so they’re going to turn it anyway hoping that you won’t see it or that there’s some kind of partial credit…A lot of that is laziness, quite frankly. They know the answer’s

wrong, but they're not going to spend the time to make the answer right.

Mathematics technology research has shown us that the first two groups exist. As discussed in the literature review, researchers have been aware for several years that technology can possibly hinder one from analyzing a result. However, there has not been evidence to suggest that the third case may be a possibility when considering teaching mathematics with technology. In a school environment, students may be unconcerned with a correct answer as long as they know they will receive partial credit for their work. Thus, professors need to emphasize in their grade reflection the necessity of using technology to properly assist us in giving precise mathematical solutions.

Both interviewees noted that they use computer-generated demonstrations when they are able to do so and both noted the importance of students' visualization, as noted in the research (Rasmussen and Whitehead, 2003). Professors often noted things such as "We visualize more and more." In showing visualization, qualitative analysis will become a natural outflow in classroom discussions.

While the types of solution strategies showed that engineering professors ranked qualitative methods low on the list, these approaches are still found to be important for engineers in some areas, even if they are not the most important strategies overall. In one of the phone interviews, another professor notes:

> It's still valuable for a student to be able to look at a function and get a qualitative idea of what it does. The good old sinc function—sinx over x, we call it sinc, I don't know what mathematicians call it—sin of pi x over pi x, it shows up a lot in electronics, and you know a student ought to be able to look at thing and sketch it. They ought to be able to look at it and say, these are the 0 crossings because I think it conveys intuition also about what's going on with the underlying physics, and it's that intuition that ultimately we want the mathematics to reveal something about the physics and it's that connection, that fire in the equation, that we want the students to have.

These qualitative methods are inherently tied in with the visualization aspect of differential equations, and should not be eliminated from the course, but simply added to the discussion of graphs of differential equations. The amount of time spent discussing qualitative methods may not need to be emphasized as much, but there is value in their presence in the curricula.

While technology offers very important visual information for engineers, as noted in the literature, students need to be wary of the outputs created by technology and be able to interpret the correctness of a result (Rasmussen and Whitehead, 2003). Due to this need for alertness to a correct solution, one professor describes his hesitance to move away from emphasizing analytic methods: “I think we [engineering professors] have this uneasy feeling that there are these automated methods, but we’re worried that students are losing something in the process that they will start trusting what the computer tells them without being properly critical of the result. “

Technology should be incorporated for various different curriculum topics that were shown to be important in the first section of the results. Once educators establish which topics are important, and begin to incorporate technology into the differential equations classroom they must remember to only do so if this is beneficial to students’ relational understanding. We know from previous research (see Chapter 2) that technology aids in students’ visualization of differential equations situations. We now also know that most engineering professors would like for technology to be implemented with the study of numerical methods. We should remember, however, to treat technology as a dangerous friend—an aid that must be used with careful supervision. Otherwise, results will go unchecked, and students will see technology as a crutch instead of as a tool that requires analytic interpretation.

Chapter 5
Discussion and Conclusion

Engineering professors, like mathematicians, want students to make rich connections among techniques to solve differential equations. A significant number of responses showed that engineering professors prefer strong mathematical understanding to a correct answer. They also want students to have a critical eye as they analyze graphical and approximate solutions. Unfortunately, many professors have noted that their students do not come away from differential equations with the ability to make these connections and critically analyze solutions. One of the professors noted the following about his students' understanding of undergraduate-level mathematics currently: "They [the students] will argue that the way it's done in math, they get very little out of. They simply don't develop an appreciation for this stuff, or are not developing an appreciation for it until they get to my class. In some cases, by the time they figure out that it's important, it's too late."

This same professor said he did not think that this was the fault of the mathematics department, but we as educators must consider that we could always try to teach deeper cognitive thinking in our courses. Students should leave a differential equations course confident in the mathematics that they have learned and comfortable with applying this knowledge in various scenarios. The ability to apply this knowledge and interpret various situations shows that students can not only do differential equations, but can interpret differential equations, which also means they have relational understanding of the differential equations concepts.

Differential equations material will continue to appear in engineering students' courses. Thus, the responsibility of the first semester differential equations teacher should not be taken lightly due to the amount of applications for this material. In this study, I sought to see how the mathematics community can better serve engineering students by asking the following research questions:

- According to engineering educators, what topics in differential equations are most crucial for engineering education and practical engineering?
- What types of relational understanding do engineering educators wish to see in their students' differential equations classes?
- According to engineering educators, how should technology be used in the mathematical education (especially the differential equations education) of engineers?

Summary of Results

I addressed these research questions through surveys, follow-up surveys, and phone interviews. In the following summary, I will discuss the major findings from each of the three research questions.

Differential equations course curriculum.

The engineering professors ranked the following analytical techniques as being very important to their curricula or not very important at all to their curricula. The averages of their rankings indicate the following ranking from greatest to least importance:

1. Separation of Variables
2. Matrix methods
3. Substitution methods
4. First order linear method/Integrating Factor
5. Laplace transforms
6. Variation of parameters*
7. Auxiliary equations*
8. Series methods

*Denotes equal ranking

I learned that matrix methods should be emphasized more in the differential equations course and that transform methods and series methods may be misplaced if taught in the first semester of differential equations. Many different engineering departments use matrix methods, and often, engineering applications will result in systems of differential equations that are easily modeled by matrix methods. Transform methods and series methods are only emphasized by a few engineering departments. These departments may wish to take on these topics and incorporate them in engineering courses. If not, then they could also be placed in a second course of differential equations.

The analytical methods are not the only concepts covered in differential equations courses. Engineering professors ranked modeling scenarios and numerical analysis as being more important solution strategies than analytical techniques. The notion that engineering professors just want students to be able to use mathematics to get an answer in the midst of engineering is short-sided. Engineering professors noted that their students also need intuition about the correctness of a mathematical output.

Engineering students' understanding of differential equations.

The misconceptions that I studied all showed overall that there were significantly less "yes" this misconception is important to my curriculum responses than "no" this is not important to my curriculum responses. However, there was more interest shown from older and more experienced professors and the overall results showed lower *p-values,* which indicated that engineering professors are concerned more about misconceptions as a collective whole. Experience was examined as teaching experience and experience in an engineering career. Typically, both showed that more experience led to an increased interest in the misconception, but the career experience showed this even more than the teaching experience. Some of the misconceptions were more important to certain types of engineers, but this data needs to be investigated further as to why certain types of engineers care more about certain misconceptions.

From the survey data and from the qualitative data of this study, one may see that engineering professors, like mathematics educators, desire for students to have relational understanding of mathematics material. The most significant finding, statistically speaking, was the finding that professors preferred having a student who had an incorrect numerical solution, but understood the basic idea of the numerical approximation. This was in comparison with a student who had a correct numerical solution, but did not understand the mathematical process.

Mathematicians and engineers must continue their discussions with one another in the university setting. They must keep re-assessing the important topics in mathematics, and which topics should be eliminated or placed in a different type of course to best meet the needs of all students in that course. The goals of the mathematics and engineering educator should be united in students' relational understanding of the mathematics material. This is especially important in the differential equations course because of the amount of applications of differential equations to a vast variety of engineering problems.

Teaching differential equations with technology.

Many engineering professors said they used technology to solve numerical approximations of differential equations. Thus, their views of using technology for engineering widely focused on using technology to approximate differential equations with numerical techniques. When approximating differential equations numerically with software packages, engineering professors discussed that they wanted their students to be cautious of the results of the software packages. Engineering students need to recognize when they have made an input error, based on the mathematical outputs. Thus, engineering professors said they are wary of using too much technology because they want students to recognize correct mathematics before they have a computer calculate a result. This way, if an answer is illogical, they will be able to recognize the error and correct it. The professors said this allowed undergraduate students to solve more difficult problems now that only graduate-level students would have attempted in the past. Yet, more than ever, students need a critical eye once they have computer-generated solutions.

The professors also noted that technology was important for the visualization of the differential equations at hand and that having a qualitative look at differential equations was also helpful for building intuition. So while qualitative methods were ranked lower in the curriculum portion of important solution strategies, engineering professors still see value in qualitative methods. Thus, these methods may not need to be a large portion of the curriculum, but they are an important part of the curriculum.

Limitations of the Results

There were only 83 total responses to the differential equations survey. For analyzing the whole group, this was a large enough data set in order to be able to assume a normal distribution. However, for many of the smaller categories, an assumption that the categories followed a normal distribution may be inaccurate. More survey data would have been helpful, but due to limited time and taking volunteers, I decided that I wanted at least 50 responses in order to draw more precise conclusions so that at least the overall data would have been more likely to represent the whole population of engineering professors. This study also operated on a volunteer basis. Thus, there is a possibility that the population of engineering professors interviewed here may skew the data because they are more interested in student understanding of differential equations and took the survey because of their interest.

Further investigation still needs to be done to investigate which types of mathematical misconceptions in differential equations matter the most to engineering professors. There may be more distinctions than were made evident in this study between types of engineering. The contents of this survey may have also been too narrow for the true opinions of the professors to come out. Professors often responded that the misconceptions were not pertinent to their courses, but that by the end of the senior year, their students should have conquered these misconceptions.

Also, some of the survey data may be skewed if the professors did not understand the survey questions. Some of the professors noted in the follow-up surveys and in the interviews that the misconception concerning equilibrium solutions was somewhat confusing. They were considering a differential equation that was both separable and linear and thinking that a differential equation did not have to be separable to have an equilibrium solution because they were classifying these cases that were both separable and linear as linear. Mathematically, my intention was to discuss separable equations in general, but engineering professors were not reading it as being in general, but saying that there were some linear differential equations that did have equilibrium solutions. So, while I had changed many problems with the pilot survey, there still were some problems with the wording of the actual survey that could have altered some responses.

The nature of the survey itself was also narrow. Professors had to choose "yes" the misconception is important/relevant or "no" it is not. "It depends" was not an option in order to help rank the level of importance of these misconceptions, if possible. Yet, in the interviews, "it depends" became a very legitimate answer. Some professors noted that the misconceptions are important but that "these…misunderstandings are not common among the students [they] teach," so some may have chosen "no" because the misconceptions do not occur often enough for them to be overly concerned. This is not to demean the importance of the misconceptions, but more may have chosen "no" because of reasons such as this one. While professors did have a chance to expand upon their choices, they did not always choose to do so, and thus, further insight needs to be gained about the effects of misconceptions in ordinary differential equations.

Suggestions for the First Course in Differential Equations

The survey did show that there are some different needs for differential equations among different engineering departments. Thus, educators need to assess how to best serve all students at once in the differential course, or alter the types of differential equations courses we offer to our students. As previously discussed, transform methods and series solutions should be left to specific engineering departments to teach or they should be placed in another differential equations course.

If a university decides to only offer one first course in differential equations, mathematics and electrical engineering departments at some universities should consider allowing the electrical engineering departments to teach transform methods in the engineering coursework as needed. This would allow for more time to focus on matrix methods and systems of differential equations in further detail in the first semester differential equations course.

However, universities may consider forming two or more differential equations courses, depending on a students' major. Mathematicians and engineering faculty should continue to work together to discuss the best topics to cover in a first semester differential equations for engineering students. Pure mathematics majors and bioengineering may require a course with a more analytical approach, electrical and mechanical engineering may wish to have a course that includes transforms if this will not be taught in their departments, and/or the other engineering departments may want to focus more on numerical methods for solving differential equations.

Mathematicians need to focus more on the understanding of engineering students in differential equations. While they may seem to be strong mathematics students, a teacher should not assume that these students will automatically understand differential equations content. Mathematicians need to continue to work with engineering professors to determine which misconceptions are most important for engineering students.

Mathematics professors should also give students visualization opportunities when possible with technology that is available. They should also give students opportunities to explore more in-depth problems that require modeling skills. This will be helpful to those in applied fields in their later coursework and career paths. They should familiarize themselves with software packages that can calculate and/or approximate differential equations.

Similar types of research need to be conducted to determine the needs of other majors such as mathematics, physics, and chemistry majors, which are also typically required to take a first course in differential equations. This would help gain a better picture of how the first course in differential equations can best serve the whole university community.

Each university will need to decide how best to offer differential equations content based on its population of students. Due to the nature of differential equations and its use by so many different types of fields, discussions about the curricula and students' understanding of the content will need to continue to take precedence in course directors' discussions in the future.

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APPENDICES

Appendix A: Pilot Differential Equations Survey

Title: Differential Equations in Engineering

Actual survey was originally made available online:
http://ceres.cals.ncsu.edu/surveybuilder/Form.cfm?TestID=8451

We appreciate your thoughtful responses to the following questions.

Thank you for taking the time to complete this survey. By taking this survey, you are consenting to allowing your responses to be used in research, which may result in publication. Your responses will help in our understanding of how best to revise the teaching of differential equations to best suit the needs of those in engineering fields. Please note that the first several questions are demographic questions for analysis purposes only and any connection with you or your university specifically will not be included in research findings.

If you would like to comment on any of the problems, there is a designated space at the end of the survey. Please include the question number with your comments.

Demographics

University:

Particular Engineering Field:

Age:

Years of Experience Teaching Engineering:

Years of Engineering Experience in a Non-academic setting:

Specific Courses you are currently teaching:

General Questions

1. Explain to a mathematician how and when you use ordinary differential equations in the engineering courses that you teach.

2. Rank the 5 techniques that are most important to *your curricula*, with 1 being what you consider *the most important method for students to learn for applications in your field*. For those past 5, please select "None."

Separation of Variables

Solving Linear ODEs with an Integrating Factor

Method of Undetermined Coefficients and/or Variation of Parameters for solving nonhomogenous linear equations of higher order

Using Auxiliary Equations for solving homogeneous first and second order linear

Change of Variables and Substitutions

Matrix Methods from Linear Algebra to solve systems/Eigenvalues and Eigenvectors

Laplace Transforms

Series Solutions (Power Series, Cauchy-Euler, Frobenius, etc.)

Other

3. Differential equations is changing as a course in many universities. There is increasing use of technology and approximation techniques for gathering general information about differential equation solutions. We would like to know whether you find these types of strategies more useful than straight computation for solutions. Rank the following with 1 being most important *to you in your field*.

Mathematical Theory that provides insight for possible ODE solutions (Existence/Uniqueness Theorem)

Qualitative Analysis (slope fields, stability, equilibria, bifurcations, long term behavior)

Analytical Techniques (methods as described in Question number 2)

Numerical Methods (approximations of differential equation solutions either by hand or with technology; examples include Euler, Taylor, and Runge-Kutta Methods)

Modeling and Applications (simple engineering problems that use differential equations in order to derive a final solution)

Combining Methods and Graphs to analyze Dynamical Systems

Other

4. Do you find teaching applications and modeling in differential equations courses useful to your particular area of interest? Explain if desired.

5. Are differential equations modeling problems too limited and simple? Are they too “cookie cutter”? Explain if desired.

6. Please provide suggestions for helpful modeling questions for differential equations.

7. Do the following misconceptions cause problems in the engineering coursework that you teach?

a. not knowing that the solution of a differential equation is the function in the original differential equation
yes/no
b. in separation of variables, thinking that it is okay to separate dy and dt because they think it is like a fraction
yes/no
c. not knowing that equilibrium solutions exist in separable differential equations but not linear differential equations
yes/no
d. not understanding that approximation methods do not follow the exact solution curve
yes/no

8. Which of the following would you prefer?

1. A student who can't remember the formula for a numerical technique, but has a general idea of why that technique works. This student has an incorrect solution.

2. A student who remembers the formula for the technique, but does not know why this method works. This student has a correct solution.

9. Has technology altered the math curricula that your engineers need or the content that you teach? If so, how?

10. Has technology affected the way you approach engineering problems? If so, how?

Extra explanations or comments about any of the survey items:

Would you be willing to be contacted for clarification on any of these survey items? If so, please include your email address and/or phone number.

Appendix B: Updated Differential Equations Survey

Title: Engineering Professors' Differential Equations Course Preferences

Actual survey was originally made available online:
http://ceres.cals.ncsu.edu/surveybuilder/Form.cfm?TestID=8762

Thank you for taking the time to complete this survey. By taking this survey, you are consenting to allowing your responses to be used in research, which may result in publication. Your responses will help in our understanding of how best to revise the teaching of differential equations to best suit the needs of those in engineering fields. Please note that the first several questions are demographic questions for analysis purposes only and any connection with you or your university specifically will not be included in research findings.

If you would like to comment on any of the problems, there is a designated space at the end of the survey. Please include the question number with your comments.

Demographics

University:

Particular Engineering Field:

Age:

Years of Experience Teaching Engineering:

Years of Engineering Experience in a Non-academic setting:

Specific Courses you are currently teaching:

General Questions

1. Explain to a mathematician how and when you use ordinary differential equations in the engineering courses that you teach.

2. Rank the following techniques, with 1 being what you consider a very important method for students to learn for applications in your field and 4 being a technique that is not important at all to your field.

Separation of Variables

Solving Linear ODEs with an Integrating Factor

Method of Undetermined Coefficients and/or Variation of Parameters for solving nonhomogenous linear equations of higher order

Using Auxiliary Equations for solving homogeneous first and second order linear

Change of Variables and Substitutions

Matrix Methods from Linear Algebra to solve systems/Eigenvalues and Eigenvectors

Laplace Transforms

Series Solutions (Power Series, Cauchy-Euler, Frobenius, etc.)

3. Please note any other analytic techniques that you feel are important to your curricula.

4. Differential equations is changing as a course in many universities. There is increasing use of technology and approximation techniques for gathering general information about differential equation solutions. We would like to know whether you find these types of strategies more useful than straight computation for solutions. Rank each of the following with 1 being very important for coursework in your field and 4 being irrelevant to coursework in your field.

Mathematical Theory that provides insight for possible ODE solutions (Existence/Uniqueness Theorem)

Qualitative Analysis (slope fields, stability, equilibria, bifurcations, long term behavior)

Analytical Techniques (methods as described in Question number 2)

Numerical Methods (approximations of differential equation solutions either by hand or with technology; examples include Euler, Taylor, and Runge-Kutta Methods)

Modeling and Applications (simple engineering problems that use differential equations in order to derive a final solution)

Combining Methods and Graphs to analyze Dynamical Systems

5. Do the following misconceptions cause problems in the engineering coursework that you teach?

a. not knowing that the solution of a differential equation is the function in the original differential equation
yes/no
b. in separation of variables, thinking that it is okay to separate dy and dt

because they think it is like a fraction
yes/no
c. not knowing that equilibrium solutions exist in separable differential equations but not linear differential equations
yes/no
d. not understanding that approximation methods do not follow the exact solution curve
yes/no

6. Which of the following would you prefer?

1. A student who can't remember the formula for a numerical technique, but has a general idea of why that technique works. This student has an incorrect solution.

2. A student who remembers the formula for the technique, but does not know why this method works. This student has a correct solution.

7. Has technology altered the math curricula that your engineers need or the content that you teach? If so, how?

8. Has technology affected the way you approach engineering problems? If so, how?

9. Extra explanations or comments about any of the survey items:

10. Would you be willing to be contacted for clarification on any of these survey items? If so, please include your email address and/or phone number.

Appendix C: Example Email

Dr. (INSERT NAME)-

My name is Morgan Early and I'm a Masters student in Math education at NC State University. I am writing to you because I am doing research that I believe may be of interest to you.

I am looking for various engineering professors' opinions about topics in a typical differential equations course and how those topics are currently taught. If you would be willing to take the survey, please visit this website:

http://ceres.cals.ncsu.edu/surveybuilder/Form.cfm?TestID=8762

The survey should take 10-20 minutes to complete, depending on how much you wish to say. If you have any questions, please feel free to contact me. Also, if you have any contacts at other universities in the U.S. that you think may be helpful, you may add these in this section as well.

Thank you so much for all you do!

Morgan Early,
M.S. Candidate, Math Education

Appendix D: Follow-Up Email Survey

I contacted you in the fall semester and you completed a study on engineering professors' opinions of differential equations. Thank you very much for completing this survey!

You indicated on the survey that you would be available for follow-up questions on this survey. If you are interested, please respond to the follow-up questions below:

1. You were given 2 scenarios.

A: A student who can't remember the formula for a numerical technique, but has a general idea of why that technique works. This student has an incorrect solution.

B: A student who remembers the formula for the technique, but does not know why this method works. This student has a correct solution.

Preliminary results of this study have shown that engineers preferred scenario A with statistical significance. Yet, there is insufficient evidence that engineering professors will answer yes or no to the following misconceptions being problematic to their engineering courses:

-not knowing that the solution to a differential equation is a function

-splitting dy/dt because they think it is a fraction

-not knowing the equilbrium solutions exist for separable differential equations but

not linear differential equations

-not understanding that approximation methods do not follow the exact solution

curve

Please share your thoughts on why you believe these specific misconceptions are of less importance than the first scenario.

2. What misconceptions (misunderstandings) in differential equations do you find most prevalent and/or troublesome?

3. Matrix methods were ranked very high in importance among all engineering professors. Why do you believe that matrix methods were listed as being so

important and do you believe they are underrepresented in the ordinary differential equations curricula?

4. If you are willing to participate in a phone interview, please list a phone number and times that you might be available.

Thanks so much for your time!

Morgan Early,
North Carolina State University
M.S. Candidate, Math Education

Appendix E: First Phone Interview Protocol

You noted that you have 1 year of experience in a non-academic engineering setting. Tell me about this experience.

I would like to talk about some misconceptions in DE classes.

DE solution being a function: You said that it depends on "how [students] understand the physics of the solution"

Splitting dy/dt: You said it's ok to treat it like a fraction. In separation of variables, splitting like a fraction will give you a correct solution. Do you think that this notion causes problems with students in other situations that may not involve DEs? Do they try to separate it when they shouldn't?

Have him compare/contrast delta y, dy, and dy/dt.

Clarify that the survey meant to refer to sep. and linear "in general." Thus, you can have equilibrium solutions to a DE that is linear and separable. Then ask if he thinks this is an important misconception. If not, ask if there's anything about equilibrium solutions that is problematic.

You also added some important misconceptions. Tell me more about why initial and boundary value problems are important.

You also said you wanted students to draw information directly from the DE system at hand. Can you provide of an example of when you have them do this?

I don't know anything about a zero state solution. Can you briefly explain this and how it's used in your course? Is the important misconception determining the difference between particular vs. homogeneous solutions?

You suggested having engineering courses take transform methods. How do you feel about engineers teaching the whole introductory course to DEs?

How often do you use power series?

What technology software packages do your students use the most to compute DEs?

You mentioned that technology enables students to visualize more. Can you explain how this is especially important for your courses?

What math courses do you think are most crucial for electrical engineers?

In your first survey you said, "Perhaps if more calculus profs knew a few engineering applications, even reading a few engineering books, students would be able to nail things down sooner." Can you give 1 or 2 examples of applications that could be presented in a DE course?

Appendix F: Second Phone Interview Protocol

You noted that you are a consultant. Tell me a little bit about what this entails.

You said that students don't get that a solution of a DE is another equation. Do you make a distinction between function and equation? If no, ask how he would explain the difference, if there is one.

In the survey you noted that all the misconceptions were problems in your classes. (Read them all again.) Can you give some scenarios where these misconceptions have come into play and been problematic to your course material?

You noted in your survey that DE knowledge reduces memorization in your classes. Is this analytical knowledge that is needed or would familiarity with technology that computes DEs be sufficient?

You said that matrix methods are not part of the DE course at Louisiana Tech. Would you like for them to be? If yes, ask why.

What kinds of problems would you like to see in a DE course that would utilize matrix methods?

How do you feel about engineers teaching the whole introductory course to DEs?

What technology software packages do your students use the most to compute and visualize DE systems?

What math courses do you think are most crucial for civil engineers?

Appendix G: IRB Approval

NC STATE UNIVERSITY

North Carolina State University is a land-grant university and a constituent institution of the University of North Carolina

Research and Graduate Studies
Division of Research Administration

Campus Box 7514
Raleigh, North Carolina 27695-7514
919.515.2444 (phone)
919.515.7721 (fax)

From: Debra A. Paxton, Regulatory Compliance Administrator
North Carolina State University
Institutional Review Board

Date: October 19, 2009

Project Title: Differential Equations Course Preferences for Engineering Professors

IRB#: 1149-09-10

Dear Dr. Keene and Dr. Early:

The research proposal named above has received administrative review and has been approved as exempt from the policy as outlined in the Code of Federal Regulations (Exemption: 46.101.b.2). Provided that the only participation of the subjects is as described in the proposal narrative, this project is exempt from further review.

NOTE:

1. This committee complies with requirements found in Title 45 part 46 of The Code of Federal Regulations. For NCSU projects, the Assurance Number is: FWA00003429.
2. Any changes to the research must be submitted and approved by the IRB prior to implementation.
3. If any unanticipated problems occur, they must be reported to the IRB office within 5 business days.

Thank you.

Sincerely,

Debra A. Paxton, NCSU IRB