ABSTRACT

EDSON, DAVID MICHAEL. Viscous Rivulet Wetting of a Trench: Near Perfect-Wetting versus Partial-Wetting Fluids. (Under the direction of Thomas Ward.)

The dynamic interfacial behavior of a rivulet flowing over a trench is studied experimentally. The trenches are of square dimensions with depths that vary from slightly smaller than the capillary length to slightly larger. The problem is parameterized using capillary, Reynolds, and Bond numbers. The fluids used in the experiments are a glycerol, glycerol/water mixtures, and a silicone oil with the former two fluids representing a partially wetting fluid and the latter a nearly complete wetting fluid, respectively. Images of the rivulet front, and downstream steady film thickness, are analyzed for the local film height as a function of time and compared with the theory of Gramlich et al. (Phys. Fluid, 2004) for dynamic trench wetting behavior. The results from these experiments suggest that the glycerol and the mixtures tended to roll down the surface of the apparatus rather than slide like the silicone oils. A rich variety of phenomenon is observed within these two trench depths using the two fluids, including the idea that the glycerol and the mixtures tend to roll down the surface of the apparatus rather than slide like the silicone oils, suggesting that trench wetting behavior is greatly affected by a combination of geometry and fluid wetting behavior.
Viscous Rivulet Wetting of a Trench: Near Perfect-Wetting versus Partial-Wetting Fluids

by
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NOMENCLATURE

γ - dynamic surface tension

γ₀ - static surface tension

ρ - density

g - gravity

dᵢ - trench depth (either 2 or 4 mm)

U - velocity of rivulet front

hᵢ - steady film thickness

hₘₐₓ₀ - initial maximum film thickness (x-direction)

hₘₐₓᵧ - maximum film thickness in the y-direction

Δhₘₐₓᵧ - change in maximum film thickness (y-direction)

hₘₐₓₓ - maximum film thickness in the x-direction

hᵢₘₐₓ, Cᵢ, and βᵢ - constants of best fit line in Fig. in 3.7

hₘₐₓₘₐₓ, Cₘₐₓ, and βₘₐₓ - constants of best fit lines in Fig. in 3.8

µ - dynamic viscosity

αₚ - dynamic contact angle

αᵪ - static contact angle

xₙ - rivulet front outside the trench

yₙ - rivulet front inside the trench

Capillary number (Ca) = \( \frac{U}{v} \)

Hydrostatic pressure (ΔP) = \( g\rho h \)

Bond number (Bo) = \( (\frac{d_i}{\ell_c})^2 \)

Reynolds number (Re) = \( \frac{ρ\ell_c U}{\mu} \)

Capillary length (ℓₖ) = \( \sqrt{\frac{γ}{ρg}} \)
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Chapter 1

Introduction

In this manuscript, the dynamic wetting of a trench by a viscous rivulet is experimentally studied. The problem has broad application in many manufacturing and naturally occurring processes [1, 2]. Our main interest here is to compare our results with previously published theoretical and computational analysis, since there are few, if any, experiments for such systems. Here, we examine the difference in thin film formation, and rivulet dynamics between nearly perfect-wetting and partial-wetting fluids [3, 4]. The wettability of a fluid depends on the ratio of the adhesive forces between the liquid and solid, and the cohesive forces within the liquid. If the adhesive forces dominate, then the fluid is considered a wetting fluid. If the cohesive forces dominate, then the liquid is said to be non-wetting. The quantitative measure of wettability for a fluid-solid system is typically measured using two values, the static contact angle $\alpha_0$, or the three phase contact where the liquid-solid-gas meet, and the surface tension, $\gamma$. The dynamics in the vicinity of the three phase region defining the contact angle are complex and are not discussed in detail in this manuscript as it has been thoroughly discussed in others. However, for the purpose of this manuscript, we consider dynamic contact angles, $\alpha_U$, to be dynamic as
defined in [5]. This angle is not where the solid, liquid, and air interface meet as it is commonly defined. Instead, it is a slightly higher up on the leading edge of the liquid-air interface. For a static fluid, perfect wetting fluids have $\alpha_0 = 0$, for non-wetting fluids $\alpha_0 = 180^\circ$ and partial wetting fluids make up the region between these two limits. The other quantitative measure for wettability is the surface tension where the values can range from 10-100 nM/m with smaller values typical for oils on organic surfaces and larger values for liquid metals on glass [6]. In this manuscript, silicone oil is the wetting fluid (nearly-perfect wetting) with $\alpha_0 \approx 0$ and $\gamma_0 = 25$ mN/m and glycerol the partial-wetting fluid with $\alpha_0 = 50^\circ$.

1.1 Literature Review

In general, all theories and experiments suggest that steady film thicknesses always scale like $h_\infty = C_0 Ca^n$ where $Ca = \frac{U\mu}{\gamma}$ is the capillary number based on a steady velocity $U$, liquid viscosity $\mu$, and surface tension $\gamma$. The steady film thickness is $h_\infty$, and the power $n$, and the coefficient $C_0$ are determined for a systems geometry and range of capillary numbers. For extremely small capillary numbers $Ca < 1 \times 10^{-3}$, the power is $n = 2/3$, as was determined theoretically and experimentally by F. P. Bretherton for the film that forms when a bubble penetrates liquid in a tube [7]. For small capillary numbers $1 \times 10^{-3} < Ca < 1 \times 10^{-2}$, the exponent power is slightly larger at $n = 1/2$ which was determined theoretically and experimentally for the liquid film that forms in the same experiments [8, 9, 10]. Another analysis of the bubble in a tube experiment was presented by G. I. Taylor in 1961, who showed that there is an asymptotic limit of $h_\infty$ that is approached for $Ca \geq 1$ [9]. A power of $n = 2/3$ was also determined for
the problem of thin film formation due to displacement of a more viscous fluid by a less viscous fluid in a two dimensional geometry [11, 12, 13]. In this manuscript, we propose a model similar to that of G. I Taylor suggesting that there is an asymptotic limit for the film steady film thickness, of the form $h_\infty(Ca) \propto h_{\infty_{max}}(1 - Ce^{-\beta Ca})$. For this model the three parameters $C$, $\beta$ and $h_{\infty_{max}}$ will be determined from curve fits to experimental data.

In regards to fluid mechanics in the vicinity of and away from a moving contact line, there have been several noteworthy studies. Some of these studies were experimental [14, 15, 3, 16, 17] while others were theoretical and/or computational [18, 19, 20, 21, 22, 23, 24, 25, 26, 27], and almost all study thin film flow. The most relevant published work to the local trench dynamics experiments that are also discussed in this manuscript was written by C. M. Gramlich et al. [28]. In their computational paper, the authors simulations concentrated on the Stokes solution for two-dimensional thin film motion over a trench. There are a few differences that are irreconcilable, and therefore, should be noted. For one, the authors of [28] perform computations for a two-dimensional thin film geometry, whereas our experiments produce a three-dimensional rivulet. This is significant, for example, because a rivulet cannot trap air in the trench, whereas Gramlich et al. observed this phenomenon in their simulations. Consequently, we experience other modes of wetting, such that it is still possible to quantify successful wetting for comparison with the two-dimensional geometry studied by [28]. Another difference between this manuscript and [28] is that the curvature of their trench corners is an input variable and are purposely rounded, whereas our are relatively sharp i.e. essentially square. This significantly affects how quickly, and easily, the rivulet front can flow past the top edge of the trench (see Fig. 1.1 for problem schematic). It should be noted that the inaugural paper of flow over topographical features was written by S. Kalliadasis et al. [29] where
the authors looked at the steady shape of a thin film over trenches or mounds under an external body force using the lubrication approximation. There have been others that have looked at simpler external body forces [30], film flowing under the influence of gravity down an inclined periodic wall with transverse rectangular corrugations [31], and non-Newtonian fluids flowing down inclined planes [32].

In our experiments, we visualize a steady rivulet as it flows down the surface of the apparatus. Once the bulb of the rivulet reaches the top edge of the trench, it stops where the bulb thickness grows until it has enough hydrostatic pressure to move the contact line past the edge of the trench. Once the bulb has reached a certain-critical thickness, it will take one of several different paths. For high viscosity wetting fluids, the thickness of the bulb needed to flow past the edge of the trench is slightly larger than the capillary length. If the hydrostatic pressure is large enough, the bulb will flow right past the trench opening and never enter it. Clearly, this path fails to wet the inside surfaces of the trench. If the hydrostatic pressure is less than that, it will drop into the trench and could take one of a couple of different paths. If the bulb volume is too large, it will hit the bottom of the trench before it has time to reach the left side of the wall (see Fig. 1.1 for orientation). If the pressure is too small, the bulb will stall while traveling across the top surface of the trench and the bulb will eventually drop down and hit the bottom of the trench. If the pressure is in the right range, the bulb will drop into the trench, travel across the top surface of the trench, wet the left wall, and then the bottom surface of the trench. This is what we define as a successful wetting process. Depending on the hydrostatic pressure, the bulb could enter in the trench but break off and drop down to the bottom of the trench before reaching the left wall of the trench. This path also fails to wet the inside surfaces of the trench.
In the next chapter, the experimental setup, the procedure, and the operating parameters are discussed. The materials and equipment are also described there. After this chapter, the results of the experiments are described. Since the geometry is different between inside and outside the trench, the results have been split into two chapters. Chapter 3: Outside the trench, velocity and film thickness are measured as a function of the hydrostatic pressure and the Capillary and Reynolds numbers, respectively. Chapter 4: Film thickness and trench wetting quality are discussed for inside of the trench. Discussion on wetting versus non-wetting fluids follows the results chapters in chapter 5. Finally, conclusions are made concerning the experiments in the last part of chapter 5.
rivulet analysis

local trench analysis

Figure 1.1: Problem schematic for the study of a rivulet flowing down a wall (left panel) and over a trench (right panel).
Chapter 2

Experiments

2.1 The Setup

The experimental apparatus was made of acrylic with dimensions approximately 300 mm tall, 100 mm wide, and 10.5 mm deep. A 25 mm hole was tapped almost nearly through one side of the top of the apparatus from where the fluid flowed. On one side of the apparatus, the hole had threads where a tube was attached and the other side was drilled through to about 1 mm in diameter. This part of the apparatus was attached to an acrylic base by two bolts. The base was 110 mm wide and 150 mm long and kept the apparatus upright and contained a reservoir for any fluid that flowed down. A square trench of either \( d_1 = 2 \) or \( d_2 = 4 \) mm was milled across the complete width of the acrylic flow surface. A reservoir was used for the fluid. It had an open top end while the bottom end had an outlet where fluid flowed out into a tube connected to the apparatus. The reservoir was mounted to a ring stand with a clamp that securely held the reservoir and allowed the fluid height to be adjusted as needed.
Figure 2.1: Components used in experiment
Glycerol and silicone oil were the two fluids used. Three different viscosities were used for each. The silicone oil viscosities were 100, 500, and 1000 cSt. Pure glycerol was used along with glycerol/water mixtures of ratios 95:5 and 90:10 to produce equivalent kinematic viscosities of 714, 283, and 122 cSt, respectively, for comparison with the silicone oil [33, 34]. The surface tension of glycerol and silicone oil on acrylic are 55 and 20 mN/m, respectively, based on previously published data [14]. The temperature in the room was assumed constant although there were variations of a few degrees from day to day, but the difference in temperature was small enough that the authors believe it did not significantly influence the results of the experiments.

A PixeLINK (model PL-B771G-R) camera mounted with a 10X c-mount camera lens (Computar) was used to capture images of the flowing rivulet. The experimentally best distance determined for the lens from the apparatus was 150 mm. To provide the camera with adequate light, a lamp was used. It was determined that a fluorescent lamp worked better than incandescent bulb for this experiment. The lamp power was only 23 watt, but provided equivalent lighting of a 100 watt incandescent bulb. It was placed behind the apparatus facing the camera lens. This setup allowed the rivulet to be captured by the camera as a silhouette, which worked well for the computer analysis. A ruler was used to help measure how many pixels there were in each millimeter distance. An image was taken with the ruler up against the surface of the apparatus and aligned with the hole in the apparatus. From examination of the image, it was determined how many pixels there were in each millimeter distance. This was necessary to calculate certain quantities such as the rivulet’s velocity. An in-house code written with MATLAB and was used to determine quantities like velocities and film thickness, using image contrast. For example, film thicknesses were calculated by counting how many black pixels there
were from the apparatus wall to a white pixel away from the surface. Similarly, the velocity of the rivulet front was calculated by tracking the lowest black pixel on the moving rivulet front. The camera speed varied from 40 to 64 frames per second (fps) and the value depended on certain parameters like image size, quality of image, etc. However, the speed was sufficient because the rivulets speed was relatively slow. The typical image always had 1024 pixels in each column (height). In contrast, the number of pixels in each row (width) was in the range 344-600.

2.2 Procedure

The first step was to secure the reservoir. Then one end of the tube (initially unattached and clamped) was attached to the reservoir outlet and the other to the apparatus. Next the reservoir was filled with the particular fluid. The top surface of the fluid in the reservoir was set to 292.1 mm (11.5 inches) from the top of the table by adjusting the height of the reservoir clamp. Once this was done, the fluid flowed from the reservoir to the apparatus. As soon as the rivulet was seen at the very top of the camera viewing window, the camera started capturing images. Once the rivulet was out of the view of the camera window, the camera was stopped and the reservoir was lowered below the height of the apparatus hole so the fluid flowed back some in the opposite direction through the tube. The tube was then disconnected from apparatus and the end of the tube was then attached to the clamp, making sure the end of tube was higher than the fluid in the reservoir.

The apparatus was cleaned thoroughly after each completed experiment. Dish wash-
ing soap was applied directly to the apparatus surface where the fluid had been. After rubbing the soap over the surface, all the soap and fluid were thoroughly rinsed off. It was very important to make sure the apparatus hole did not have any soap still in it after rinsing. The apparatus was then wiped dry with a drying cloth. An air compressor was then used to dry the apparatus hole and to clear away any minute particles left by the drying cloth that may influence the results. After increasing the height of the reservoir by 25.4 mm (1 inch), the procedure was repeated. At most, 13 experiments were run to a height of 596.9 mm (23.5 inches) for a given fluid. Depending on the viscosity of the fluid, 292.1 mm was sometimes too low to apply adequate hydrostatic pressure for the fluid to flow out of the apparatus hole.

2.3 Operating Parameters

The capillary numbers for the silicone oils range from $0.0054 < Ca < 0.012$. This is a smaller range than different mixtures of glycerol and water which range from $0.0054 < Ca < 0.097$. Whereas the silicone oils saw a general-uniform increase in capillary number from one fluid viscosity to another as the hydrostatic pressure increased, the glycerol/water mixtures had no such trend. The equation for the $Ca$ number was developed from the normal stress balance equation and describes the relative importance of the viscous stresses to surface tension. Hydrostatic pressure ranged from $2.7 \text{ KPa} < \Delta P < 5.6 \text{ KPa}$ for silicone oil and from $3.6 \text{ KPa} < \Delta P < 7.4 \text{ KPa}$ for the glycerol and glycerol mixtures. The Bond number values are 1.51 for silicone oil 2 mm trench, 6.02 for silicone oil 4 mm trench, 0.83 for glycerol 2 mm trench, and 3.31 for glycerol 4 mm trench. The equation for the $Bo$ number also comes from the normal stress balance.
equation and describes the relative importance of gravity to surface tension. Reynolds number ranged from $0.0012 < Re < 0.18$ for the silicone oils and from $0.0029 < Re < 0.16$ for the pure glycerol and glycerol/water mixtures. The equation for $Re$ number came from the nondimensionalized form of the Navier-Stokes equations. Lastly, capillary length had one value each for both fluids. Glycerol and the glycerol/water mixtures had a capillary length 2.20 mm while the silicone oils had a capillary length of 1.63 mm. The equation for capillary length was developed from the Reynolds number. The equations for Bond number, Reynolds number, Capillary number, capillary length, and hydrostatic pressure can all be seen in the nomenclature on page v.
Chapter 3

Results from Image Analysis - Rivulet analysis

The experimental results are presented in this chapter and the next. This chapter deals with the experiments for the rivulet velocity and dynamics, including steady film thicknesses and maximum film thicknesses. The next deals with the experiments for the local trench dynamics including the transient maximum film thickness at the sharp corner and the dynamic rivulet front thickness inside the trench.

3.1 Qualitative Analysis

The first set of images, Fig. 3.1, display the rivulet as it flows down a flat surface. The top row of images, Figs. 3.1(a)-3.1(c) show results of 90:10, 95:5 glycerol/water mixtures and pure glycerol, respectively, at a pressure ratio of $0.003 < \frac{\Delta P}{P} < 0.04$. The second row, Figs. 3.1(d)-3.1(f), show results for silicone oil at viscosities of 100, 500 and 1000 cSt,
respectively, at a pressure ratio of $0.001 < \frac{\Delta P}{P} < 0.03$. The leading edge of the images in the top row, Figs. 3.1(a)-3.1(c), appear to show a dynamic contact angle that is greater than $90^\circ$ indicating a non-wetting fluid, and the magnitude of the angle also varies with the fluid viscosity in this set of images. The same trend, though, is not observed in the silicone oil images, where the contact angle appears nearly constant across the bottom row, Figs. 3.1(d)-3.1(f), of images. Figure 3.2 i-iv, show results of the measured front position $x_N$ as a function of elapsed time $\Delta t$ for 100 cSt silicone oil, 90:10 glycerol/water, 1000 cSt silicone oil and pure glycerol respectively, at dimensionless pressures as listed in the caption. The linear slopes of the lines suggest constant velocities.

### 3.2 Quantitative Analysis

For all of the following quantitative analysis, the first image was analyzed after the rivulet front had reached the 300th pixel from the top, while the last image analyzed was either the one where the rivulet had reached the left side of the trench or when the rivulet front had reached the bottom side/edge of the trench. The first two figures (3.3 and 3.4) for the data set show log-log plots of the measured capillary number and velocity as functions of the hydrostatic pressure normalized by the atmospheric pressure. In the first figure, the range of capillary number span from a minimum of approximately 0.004 to a maximum of 0.1. The glycerol/water mixtures span the complete range with the pure glycerol reaching the maximum while the 90:10 water/glycerol mixture reaches the minimum in the range of capillary numbers. The silicone oil is in a much narrower range of values and show a trend where the 500 cSt silicone oil actually has the largest capillary number versus the other silicone oils for a given pressure ratio. The next figure shows
Figure 3.1: Images of a rivulet flowing down the acrylic wall with fluid properties a) 90:10 glycerol/water, b) 95:5 glycerol/water, c) pure glycerol, d) 100 cSt silicone oil, e) 500 cSt silicone oil and f) 1000 cSt silicone oil. A line normal to the acrylic substrate and located approximately at the triple phase contact is drawn in panels a) and d) illustrating the dynamic contact angle behavior.
Figure 3.2: Position ($x_N$) vs. time ($\Delta t$) for: i) 100 cSt, $\Delta P/P_{atm} = 0.066$ ii) 90:10 glycerol, $\Delta P/P_{atm} = 0.044$ iii) 1000 cSt, $\Delta P/P_{atm} = 0.043$ iv) Pure glycerol, $\Delta P/P_{atm} = 0.041$
the velocity results and trends that are noticeably different than the capillary number data. The range of velocities range from 0.02-10 mm/s. In this data set, the silicone oil actually spans the complete range with the 100 cSt silicone oil possessing the largest velocity values and the 1000 cSt silicone oil the smallest for a given pressure ratio. The glycerol data is in a narrow range in this data set with the 90:10 glycerol/water data possessing the largest velocity values. There is little difference in the velocity for the 95:5 glycerol/water and pure glycerol data. The next two figures are the estimates of the dynamic constant angle, $\alpha_U$, versus capillary number for the silicone oil, Fig. 3.5, and glycerol, Fig. 3.6. The estimates seem to suggest that the silicone oil starts off at dynamic angles that are less than 90° but transition to values that are greater than this at higher capillary numbers. Also, several of the data points overlap in the silicone oil data. The glycerol data trends are different where the values are always greater than or equal to 90° and do not overlap when plotted versus capillary number.

The next two figures are the steady and initial maximum film measurements, each normalized by the capillary length, as a function of the capillary number. Fig. 3.7 shows the steady film as a function of the capillary number where all of the data fits into a narrow range. The silicone oil data steady film thickness range is extremely narrow with several data points overlapping one another for a given capillary number, regardless of the fluid viscosity. The same is not true for the glycerol and glycerol/water mixtures data where the points do reside in a narrow rage but few points overlap. Fig. 3.8 shows the normalized initial maximum film thickness versus capillary number where the data sets for a given fluid do overlap but each fluid data set does not. The range of maximum film thickness ranges from 0.4 to approximately 1 for the silicone oil while the glycerol data range has a minimum of unity and a maximum of approximately 1.4. The silicone
oil data points overlap and so do the glycerol and glycerol/water mixtures data in this particular graph.
Figure 3.3: $Ca$ versus hydrostatic pressure, where the hydrostatic pressure pressure is normalized by atmospheric pressure. The symbols shown in the legend correspond to the rivulet fluid.
Figure 3.4: Rivulet velocity versus hydrostatic pressure, where the hydrostatic pressure is normalized by atmospheric pressure. The symbols shown in the legend correspond to the rivulet fluid.
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Figure 3.6: Semi-log plots of the measured dynamic contact angle $\alpha_U$ versus the capillary number for glycerol and glycerol/water mixtures. At viscosities or water concentration as listed in the legend.
Figure 3.7: Steady rivulet film thickness scaled by the capillary length $h_\infty/\ell_c$. The symbols shown in the legend correspond to the rivulet fluid.
Figure 3.8: Initial maximum rivulet film thickness scaled by the capillary length $h_{\text{max}}/\ell_c$. The symbols shown in the legend correspond to the rivulet fluid.
Chapter 4

Local Trench Analysis - Trench dynamics

4.1 Qualitative Analysis

Images of the trench wetting process are analyzed in this section. The purpose is to qualitatively examine features that are readily distinguishable from visual analysis of the images. There are four different paths that the flowing rivulet may take based on this analysis with the examples shown in Figs. 4.1 - 4.4. In Fig. 4.1 and 4.2, the fluids are the 90:10 glycerol/water mixture with trench dimensions of 2 and 4 mm, respectively, and in Fig. 4.3 and 4.4, they are a 95:5 glycerol/water mixture and 500 cSt silicone oil with trench dimensions of 4 and 2 mm, respectively. The capillary numbers are listed in the figure caption.

In Fig. 4.1, the bulb flows down the surface of the apparatus and reaches the top edge of the trench. At this point, the bulb stops moving down the surface and grows
Figure 4.1: 90:10 glycerol/water mixture, Bo = 0.83, Ca = 0.018, ℓ_c = 2.20 mm
Figure 4.2: 90:10 glycerol/water mixture, $Bo = 3.31$, $Ca = 0.0054$, $\ell_c = 2.20$ mm
Figure 4.3: 95:5 glycerol/water mixture, $\text{Bo} = 3.31$, $Ca = 0.011$, $\ell_c = 2.20$ mm
Figure 4.4: 1000 cSt silicone oil, Bo = 1.51, $Ca = 0.034$, $\ell_c = 1.63$ mm
in size until it reaches a certain-critical volume. Once this volume is achieved, the bulb drops down, and past, the entire opening of the trench forming a liquid bridge until the bridge becomes thin enough so that, finally, the bulb breaks off. This process is repeated indefinitely i.e. again, the bulb at the top edge of the trench begins growing in size until it reaches a certain volume, forms a liquid bridge that spans the trench and breaks off. It has been observed that this situation, where the bulb drops past the trench opening without entering, only occurs for the 2 mm trenches. In Fig. 4.2, the sequence of images show the rivulet front flowing until it reaches the trench edge. Once the critical-volume is reached, the rivulet front moves past the top edge of the trench but also starts moving horizontally into the trench. The rivulet front falls to the bottom surface before it reaches the left side of the trench. The rivulet front breaks away from the rest of the rivulet and forms a droplet on the bottom of the trench.

In Fig. 4.3, the rivulet front flows down the surface of the apparatus and reaches the top edge of the trench and grows in size until it reaches a critical-volume. Once this volume is reached, the rivulet front drops down past the top edge and begins to move horizontally across the top side of the trench. Just like in Fig. 4.2, the bulb drops to the bottom side of the trench before reaching the left side. However, this time the rivulet front does not break away from the rivulet to form a drop. Instead, the fluid continues to move across both the top and bottom sides of the trench until it reaches the left side. Once the fluid reaches the left side by moving along the bottom wall, it wets that surface while also completing the wetting on the other two surfaces inside the trench as well. This is considered a successful wetting. In Fig. 4.4, a similar process occurs where once the critical thickness is reached at the trench edge, the rivulet front moves past the edge and starts to move horizontally into the trench. The rivulet front continues to move
across the top side of the trench while also moving vertically lower and grows vertically in thickness, closer to the bottom side of the trench, without any liquid touching the bottom during this stage of the wetting process. As the bulb reaches the left side of the trench it quickly wets the rest of the inside surfaces, yielding a successful wetting.

### 4.2 Quantitative Analysis

As in the previous chapter, the first image was analyzed after the rivulet front had reached the 300th pixel from the top, while the last image analyzed was either the one where the rivulet had reached the left side of the trench or when the rivulet front had reached the bottom side/edge of the trench. The series of results in Figs. 4.5 - 4.8 show plots of the thickest part of the rivulet as a function of time outside the trench. The max film thickness as the rivulet is flowing down the side of the wall before reaching the edge of the trench is subtracted from the plot so that the level part of the plots are zero. Recall that once the rivulet reaches the edge of the trench, the film thickness grows until it reaches a size large enough to move the contact line around the top edge of the trench. If the rivulet goes into the trench, the film thickness outside of the trench can drop below zero as can be seen in several experiments according to the y-axis in the figures. While Figs. 4.5 - 4.8 do show quantitative results that are influenced by the trench, the figures do not give any direct indication of successful trench wetting versus unsuccessful wetting.

In Fig. 4.5, the fluid is 1000 cSt silicone oil and the elapsed time range is between 10 seconds and 20 seconds. The difference in the film thickness, and maximum thickness, varies by about 0.2 mm for $0.029 < Ca < 0.057$. In Fig. 4.6, the 100 cSt oil has a range
Figure 4.5: Outside the trench plots of film thickness ($\Delta h_{\text{max}_y}$) versus elapsed time ($\Delta t$) for 1000 cSt silicone oil, $Bo = 6.02$
Figure 4.6: Outside the trench plots of film thickness ($\Delta h_{\text{max}_y}$) versus elapsed time ($\Delta t$) for 100 cSt silicone oil, $Bo = 1.51$
Figure 4.7: Outside the trench plots of film thickness ($\Delta h_{\text{max}_y}$) versus elapsed time ($\Delta t$) for Glycerol, $Bo = 3.31$
Figure 4.8: Outside the trench plots of film thickness ($\Delta h_{max_y}$) versus elapsed time ($\Delta t$) for 95:5 glycerol/water mixture, $Bo = 0.83$
Figure 4.9: Gramlich et al. - Outside the trench plot of film thickness versus elapsed time
between less than 1 second to just over 6 seconds, for the smallest and largest elapsed times. The difference in the film thickness, and maximum thickness, varies by 0.3 mm, with $0.0054 < Ca < 0.043$, for this experiment. In Fig. 4.7, the fluid is pure glycerol and the smallest elapsed time is around 6 seconds while the largest is around 18 seconds. (There is no clear trend when it comes to the rivulet front thickness with the capillary number in this experiment.) The difference in the film thickness, and maximum rivulet front film thickness, varies by about 0.5 mm for $0.020 < Ca < 0.037$. In Fig. 4.8, the fluid is 95:5 glycerol/water mixture and its elapsed time range is 4 to 17 seconds. As can be seen from the plot, depending on the capillary number, the difference in the film thickness once the rivulet has reached the edge of the trench and has increased to its maximum thickness varies by about 0.6 mm for $0.0084 < Ca < 0.023$. Note, that the plots abruptly end right after reaching a peak film thickness because the trench is 2 mm i.e. as rivulet front drops past the top edge, and consequently the rivulet never enters into the trench in this set of experiments (see Fig. 4.1). These figures can be compared to the computational work done by Gramlich et al. [28] in Fig. 4.9. In this figure, the authors have not only subtracted away the initial maximum film thickness like we did, but they also removed all the data leading up to the thin film reaching the pressure point at the top edge of the trench. Therefore, what is being plotted is the maximum film thickness outside the trench starting at the point where thin film front has already reached the pressure point. Comparison of these results are discussed in chapter 5.

The next series of results, Figs. 4.10 - 4.13, show plots of the rivulet front thickness as a function of time inside the trench. The plots are normalized by the trench depth so the y-axis range is 0 to 1. The curves in the plot that go to 1 on the y-axis are examples where the rivulet front reaches the bottom of the trench before reaching the left side of
Figure 4.10: Inside the trench plots of film thickness versus time for 1000 cSt silicone oil, $Bo = 6.00$
Figure 4.11: Inside the trench plots of film thickness versus time for 500 cSt silicone oil, $Bo = 1.51$
Figure 4.12: Inside the trench plots of film thickness versus time for Glycerol, $Bo = 0.83$
Figure 4.13: Inside the trench plots of film thickness versus time for 95:5 glycerol/water mixture, $Bo = 3.28$
Figure 4.14: Gramlich et al. - Inside the trench plot of film thickness versus elapsed time
the trench. If the curve does not reach a value of 1, then the rivulet reached the left side and wetted the trench successfully. These plots reveal details of the dynamics of the rivulet front after overcoming the pressure point at the top edge of the trench (rotating the contact line), where the rivulet front flows into the trench. In this particular experiment, there are two trends that are exhibited: either the rivulet front flows into the trench and vertically grows in thickness until it reaches the bottom of the trench or the front slides horizontally across the top side of the trench before dropping down to the bottom. In Fig. 4.10, the results for 1000 cSt silicone oil are plotted, where all of the rivulet fronts have the same trend of reaching the left side before dropping to the bottom side of the trench, where \(0.029 < Ca < 0.057\). The times for the rivulet to reach the bottom or left side of the trench range about 4.5 second to about 6.5 seconds. In Fig. 4.11, the results for 500 cSt silicone oil are plotted, where there is a mixture of results with some of the rivulets reaching the left side before dropping to the bottom side of the trench and others that do not. In general, higher capillary numbers result in successful wetting as compared to the lower capillary numbers for this particular fluid and trench size as shown in Fig. 4.11, where \(0.030 < Ca < 0.12\). The times for the rivulet to reach the bottom or left side of the trench range from about 0.3 seconds to 1.7 seconds. Fig. 4.12 shows the results for pure glycerol, where every rivulet front follows the same trend i.e. all of them drop straight down to the bottom edge of the trench. Also, none of the rivulets reach the left side or travel far into the 2 mm trench in these experiments (see Fig. 4.1) where \(0.015 < Ca < 0.031\). The times for the rivulet to reach the bottom edge of the trench range from about 1 second to a little more than 7 seconds. In Fig. 4.13, the fluid is glycerol with \(0.0075 < Ca < 0.018\) where no experiments reach the left side of the trench. The times for the rivulet to reach the bottom of the trench range from 1 second to just over 4.25 seconds. Fig. 4.14 is a plot from the work of Gramlich et al.
that is similar to figures 4.10 - 4.13 and is discussed in chapter 5.

For both figures, 4.15 and 4.16, the initial or constant film thickness was subtracted off for an accurate comparison of the change in max film thickness vs. Ca number. For Fig 4.15, the different viscosities of silicone oil were compared. The Ca numbers ranged from about 0.020 to 0.012 and the max film thickness values ranged from 0.010 to 0.40. In the case of silicone oil, there does appear to be a trend or relation between the change in max film thickness and Ca. As the Ca number increases, the change in max film thickness has a general decreasing trend. In Fig 4.16, only the pure glycerol and glycerol/water mixtures were compared. The Ca numbers ranged from 0.0050 to 0.036 and the change in max film thickness values ranged from 0.057 to 0.42. The y-axis is made dimensionless by dividing the max film thickness by the capillary length. It is difficult to conclude if there is any trend between max film thickness and Ca for pure glycerol. For the glycerol/water mixture there is a trend as the capillary number increases.
Figure 4.15: Change in maximum film thickness normalized by capillary length \( \frac{\Delta h_{\text{max}, y}}{l_c} \) vs. \( Ca \) for Silicone oils with viscosities as indicated in the legend.
Figure 4.16: Change in maximum film thickness normalized by capillary length \( \Delta \frac{h_{\text{max}}}{\ell_c} \) vs. \( Ca \) for Glycerol water mixtures with viscosities as indicated in the legend.
Chapter 5

Discussion and Conclusion

5.1 Discussion

In this section, we combine the qualitative and quantitative analysis of both the rivulet measurements and local trench analysis to comprehend general trends that appear in these results and to provide some physical insight as to their existence. The viscosity range of the three silicone oils and the glycerol and glycerol/water mixtures is similar suggesting that the relative surface energies are responsible for much of the difference seen in the experiments.

Best fit data

Plotted in Figs 3.7 and 3.8 are the best fit data for the steady film thickness versus capillary number, and the initial maximum film thickness versus capillary number, respectively. The best fit lines are fits to the function,

\[
\frac{h_\infty}{\ell_c} = h_{\infty,\text{max}}(1 - C_\infty e^{\beta_\infty Ca})
\]  

(5.1)
and a similar expression for the maximum film thickness,

$$\frac{h_{\text{max}}}{\ell_c} = h_{\text{max, max}} (1 - C_{\text{max}} e^{\beta_{\text{max}} C_a}).$$

(5.2)

For the steady film thickness the constants are $h_{\infty, \text{max}} = 0.6$, $C_{\infty} = 1.3$ and $\beta_{\infty} = 49$. The small inset in Fig. 3.7 show a more detailed analysis of the curve fitting where the $\ln |1 - h_{\infty}/h_{\infty, \text{max}}|$ is plotted versus $\ln C_a$. Most of the data appears to fall on the line shown in the inset, where the glycerol data has points that are both above and below. The glycerol data may have a slightly different functional form if taken separately, where it would have a larger value for $C_{\infty}$ than what is used. However, the values for $h_{\infty, \text{max}}$ and $\beta_{\infty}$ are within a fairly small range suggesting that their values are indeed similar.

For the initial maximum film thickness, the constants are $h_{\text{max, max}} = 1.2$, $C_{\text{max}} = 0.9$ and $\beta_{\text{max}} = 33$, for the silicone oil and $h_{\text{max, max}} = 1.4$, $C_{\text{max}} = 0.3$ and $\beta_{\text{max}} = 21$, for the pure glycerol and glycerol/water mixtures. Given that the glycerol and silicone oil have different surface tension and contact angle values, but similar viscosities in these experiments, then the difference in the constant is primarily due to the difference in wettability between these two fluids. The glycerol maximum film thickness is never less than the capillary length for our range of capillary numbers, resulting in a larger value for the maximum $h_{\text{max, max}}$. The capillary number coefficient $\beta_{\text{max}}$ is smaller for the glycerol data. These two parameters seems to be affected the most by the wettability. The other parameter, $C_{\text{max}}$ determines how shallow the best fit curves appear in Fig. 3.8 with a smaller value representing a relatively shallower curve. Glycerol clearly has the much shallower curve and hence the smaller value for $C_{\text{max}}$. 

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Rivulet dynamics: rolling versus sliding

The authors believe the differences in liquid transport are due to 1) the surface tension, where the glycerol and glycerol/water mixtures have a surface tension approximately twice that of the silicone oil, and 2) the contact angle where the glycerol and glycerol/water mixture static contact angles are large (> 50°) and the dynamic contact angles appear to be even larger (> 90°). In fact, the dynamic contact angle for the glycerol is never less than (90°). This combination for glycerol leads to an interesting observation when looking at, for example, the rivulet velocity versus hydrostatic pressure head, where the velocity of pure glycerol and an 95:5 water mixture, which have a viscosity that is approximately 1/5 the former, have similar velocity. Part of this is due to the high surface tension, which leads to non-uniform motion of the pure glycerol, which resembles stick-slip motion although there is not enough information to draw a firm conclusion as to the phenomena source. The silicone oil, on the other hand, readily wets the acrylic surface i.e. it has a very small contact angle and, thus, little surface tension. This leads to a result that rivulets that flow down an inclined plane due to a hydrostatic pressure head will produce velocities that are proportional to only the viscosity of the fluid. This is seen in Fig. 3.4 where the rivulet velocity is proportional to the fluid viscosity suggesting that the low surface tension leads to negligible contact line effects. Also, we note that the slope of the velocity, or capillary number, versus dimensionless pressure shown in Fig. 3.4 are all similar suggesting that the change in the rates due to a change in pressure are similar even though contact line affects produce different transport rates.

The difference between the rivulet dynamics of these two fluids are more than just a difference in contact angle. They seem to represent different modes of transport. The authors believe that the rivulet fronts dominate most of the dynamic measurements that
have been mentioned in this manuscript. The main difference between the two appears to be the difference between rolling versus sliding, where the glycerol rivulet front experiences a rolling motion while the silicone oil rivulet front tends to slide along the surface. This would explain much of the behavioral difference between these two rivulet fronts since rolling would be independent of the fluid viscosity and proportional to the size of the rivulet front. Sliding on the other hand is dominated by viscous forces and hence the scaling for rivulets that execute sliding motion would be the capillary number. The rest of this discussion highlights this observed difference.

The trench dynamics for the silicone oil can also be attributed to its wetting properties. The silicone oil wets the trench for nearly all of the experiments performed with the 4 mm crack and a few of the 2 mm crack experiments. This is also partially due to the small capillary length for the silicone oil (generating Bond numbers that are always greater than unity) but this cannot explain all the behavior since the glycerol capillary length is less than 4 mm too, but does not wet the trench at higher capillary numbers. Once inside the trench, the silicone oil also efficiently spreads and typically-rapidly wets the three trench surfaces before either laterally wetting the trench or, if it has enough momentum, to actually wet all three surfaces then exit the trench. The change in max film thickness outside the trench versus time for the silicone oil has a consistent trend that can be seen in Figs. 4.5 - 4.8. As $Ca$ increases, the change in max film thickness decreases. This can be explained by the increasing momentum of the fluid, which accompanies an increasing $Ca$ as it flows down the apparatus. With increased momentum, the rivulet’s max film thickness does not need to grow as large to overcome the pressure point at the top edge of the trench. These figures can be compared to the computational work done by Gramlich et al [28]. in Fig. 4.9. The most significant similarity between our
results and [28] is that as $Ca$ increases, the change in maximum film thickness decreases. Gramlich et al. also has another plot that can be compared with a set of our plots. Fig. 4.14 is a computational plot produced by [28] showing max film thickness inside the trench versus time. The main difference between the [28] figure and ours is that [28]’s has an infinitely deep trench. The data points in Figs. 4.10 - 4.13 end once the rivulet reaches either the left side of the trench or the bottom side. However, [28]’s plot does not have any such limits. Therefore, at most, only up to time 50 on [28]’s plot can ours be compared to. Within that time, however, similar results can be seen between our plots and [28]’s plot. The plots do not look exactly alike, but enough to compare to each other.

The trench dynamics for glycerol and glycerol/water mixtures can also be explained in terms of their wetting properties for the 4 mm trench. Outside of the trench, the glycerol leading edge grows fairly large leading to a large volume of fluid. This large volume of fluid cannot enter the trench because its cross sectional area grows to be larger than that of the trench itself. The only way to overcome this phenomenon is for the glycerol and glycerol/water mixtures to develop enough momentum to overcome the pinning pressure at the sharp corner. This occurs at higher capillary numbers, but does not lead to successful wetting because glycerol has a relatively high surface tension, so the rivulet front does not move horizontally across the top surface easily or quickly enough before gravity acts on the interface causing a drop to form. Therefore, what is typically observed with glycerol is a flow into the trench, but then it drops down to the bottom side before it reaches the left side of the trench, or it completely overshoots it. For the smaller trench, the glycerol capillary length is always larger than the trench itself so a phenomenon is observed where the rivulet front breaks off and forms a periodic stream of drops. The change in max film thickness outside the trench versus time for the glycerol
and glycerol/water mixtures (see Figs. 4.7 and 4.8) does not have as obvious a trend, if any, as the silicone oil does. The only thing that can really be said about these two plots is that, like silicone oil, the increasing momentum of the fluid helps the rivulet flow past the pressure point at the top edge of the trench more quickly.

5.2 Conclusion

In this manuscript, we present experimental measurements and visual images of a rivulet flowing over a trench (square with depths of either 2 or 4 mm) with surface made of acrylic. The characteristic dimensions, such as the steady, maximum film thickness, along with dynamic properties such as the rivulet velocity are measured for a wide range of parameters. The fluids used in the experiments are a silicone oil and glycerol and glycerol/water mixture of similar viscosity range but differing surface wetting properties. The glycerol is a partially wetting fluid with a static contact angle of approximately 50° and the silicone oil nearly perfectly wets the acrylic surface. The fluids are driven by hydrostatic pressure generated from a reservoir sitting above the experimental apparatus.

The parameters used to characterize the fluid transport behavior are the capillary, Reynolds, and Bond numbers. The Reynolds numbers are small in each experiment, regardless of the fluid or hydrostatic pressure, suggesting that momentum transport is dominated by viscous stresses. The capillary numbers are in the range of $0.0010 < Ca < 0.10$ which is consistent with other studies on measuring film thickness. The Bond numbers are greater than unity in each experiment except for the glycerol fluid with the smallest trench depth of 2 mm. The results show strikingly different behavior for each
fluid, in the range of parameters, suggesting that both rivulet wetting and transport dynamics are strongly influenced by wetting behavior.

In the future, it will be beneficial to perform similar experiments or different surfaces such as metallic, or porous material such as concrete, to determine how much surface properties influence rivulet dynamics. Also, it will be interesting to study similar phenomenon on corrugated, or woven surfaces, with fluids that are wetting, partial wetting and non-wetting. Another set of experiments that would be useful, even with the current setup, would be a comparison of Newtonian versus non-Newtonian fluids to see how shear driven changes in momentum transport affect rivulet flow dynamics, and trench wetting ability.
REFERENCES


