ABSTRACT

KUSTUSCH, MARY BRIDGET. Student Understanding of Cross Product Direction and Use of Right-hand Rules: An Exploration of Representation and Context-dependence. (Under the direction of Robert J. Beichner.)

Students in introductory physics struggle with vector algebra and with cross product direction in particular. Some have suggested that this may be due to misapplied right-hand rules, but there are few studies that have had the resolution to explore this. Additionally, previous research on student understanding has noted several kinds of representation-dependence of student performance with vector algebra in both physics and non-physics (or math) contexts (*e.g.* Hawkins et al., 2009; Van Deventer, 2008). Yet with few exceptions (*e.g.* Scaife and Heckler, 2010), these findings have not been applied to cross product direction questions or the use of right-hand rules. Also, the extensive work in spatial cognition is particularly applicable to cross product direction due to the spatial and kinesthetic nature of the right-hand rule.

A synthesis of the literature from these various fields reveals four categories of problem features likely to impact the understanding of cross product direction: (1) the type of reasoning required, (2) the orientation of the vectors, (3) the need for parallel transport, and (4) the physics context and features (or lack thereof). These features formed the basis of the present effort to systematically explore the context-dependence and representation-dependence of student performance on cross product direction questions.

This study used a mix of qualitative and quantitative techniques to analyze twenty-seven individual think-aloud interviews. During these interviews, second semester introductory physics students answered 80-100 cross product direction questions in different contexts and with varying problem features. These features were then used as the predictors in regression analyses for correctness and response time. In addition, each problem was coded for the methods used and the errors made to gain a deeper understanding of student behavior and the impact of these features. The results revealed a wide variety of methods (including six different right-hand rules), many different types of errors, and significant context-dependence and representation-dependence for the features mentioned above.

Problems that required reasoning backward to find $\vec{A}$ (for $\vec{C} = \vec{A} \times \vec{B}$) presented the biggest challenge for students. Participants who recognized the non-commutativity of the cross product would often reverse the order ($\vec{B} \times \vec{A}$) on these problems. Also, this error occurred less frequently when a Guess and Check method was used in addition to the right-hand rule.

Three different aspects of orientation had a significant impact on performance: (1) the physical discomfort of using a right-hand rule, (2) the plane of the given vectors, and to a lesser extent, (3) the angle between the vectors. One participant was more likely to switch the order of the vectors for the physically awkward orientations than for the physically easy orientations; and there was evidence that some of the difficulty with vector orientations that were not in the xy-plane was due to misinterpretations of the into and out of the page symbols. The impact of both physical discomfort and the plane of the vectors was reduced when participants rotated the paper. Unlike other problem features, the issue of parallel transport did not appear to be nearly as prevalent for cross product direction as it is for vector addition and subtraction.

In addition to these findings, this study confirmed earlier findings regarding physics difficulties with magnetic field and magnetic force, such as differences in performance based on the representation of magnetic field (Scaife and Heckler, 2010) and confusion between electric and magnetic fields (Maloney et al., 2001). It also provided evidence of physics difficulties with magnetic field and magnetic force that have been suspected but never explored, specifically the impact of the sign of the charge and the observation location.

This study demonstrated that student difficulty with cross product direction is not as simple as misapplied right-hand rules, although this is an issue. Student behavior on cross product direction questions is significantly dependent on both the context of the question and the representation of various problem features. Although more research is necessary, particularly in regard to individual differences, this study represents a significant step forward in our understanding of student difficulties with cross product direction.

Student Understanding of Cross Product Direction and Use of Right-hand Rules:
An Exploration of Representation and Context-dependence

by
Mary Bridget Kustusch

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Physics

Raleigh, North Carolina

2011

APPROVED BY:

Jason Swarts	John Risley
Ruth Chabay	Robert J. Beichner Chair of Advisory Committee

DEDICATION

To Larry Martin and Peggy Condon,
whose lives and deaths helped me to discover my passion for teaching;
and to my family: past, present, and future.

BIOGRAPHY

Mary Bridget Kustusch grew up on the north side of Chicago and from her earliest memory, she was encouraged to view all of life as a learning experience. This perspective was reinforced throughout her schooling: Montessori training and homeschooling until college. During high school Mary Bridget discovered two passions: physics and American Sign Language (ASL); she also learned some of her earliest lessons as a teacher while teaching mathematics to her younger sisters.

After graduating from high school, Mary Bridget attended North Park University. While there, she had the opportunity to explore her interest in physics and also discovered a love of teaching. She received her B.S. in Physics in 2004 and had the tremendous opportunity to teach at North Park the following year.

Since coming to NC State in 2005, Mary Bridget has expanded her teaching experience. Two of the most influential experiences were the opportunity to work with Dr. Beichner in a SCALE-UP classroom and to teach a section of Modern Physics – through the Preparing the Professoriate Program. Her work in Physics Education Research has only fueled her interest in education.

This dissertation blends Mary Bridget's passions for physics and teaching; it even manages to include her love of ASL through the spatial and gestural aspects of the right-hand rule. It is her hope that this work will assist physics teachers as they face the task of helping their students to use cross products and right-hand rules as tools to understand the world.

ACKNOWLEDGEMENTS

Throughout the process of completing this dissertation, I have often been overwhelmed by the magnitude of the project. Yet I have been equally as amazed by the love and support of the people around me. There are so many people without whom I would never have gotten this far.

First of all, the NCSU STEM Initiative and the National Science Foundation (DUE–0618504) made it possible for me to accomplish this project and still pay by bills–for which I am grateful. Also, this dissertation has seen several rounds of editing by many different people – Chris Kustusch, Susan Meier, Leigh Winfrey, Amy Gaffney, Bob Kustusch, and Elizabeth Kustusch – and I deeply grateful to them all.

Each of my committee members – Bob Beichner, Ruth Chabay, Jason Swarts, and John Risley – encouraged and challenged me in different ways: to take risks, to look at the big picture; to think about the details of the design and analysis; and to consider the practical applications of my work. Having each of these perspectives made this a better project. I want thank Bob in particular for his faith in me and for forcing me to make my own decisions. He told me at the beginning that his goal was for me to become an independent researcher and I have – mostly thanks to him.

The past and present graduate students in the Physics Education Research group have been mentors, colleagues, and dear friends. Their insight, advice, encouragement, and even commiseration have helped me to not only survive graduate school, but to succeed as well. I especially want to thank Bin Xiao for his incredible patience and willingness to put up with so many rounds of inter-rater reliability and for his help with coding.

On the personal front, there are several girlfriends – Leigh Winfrey, Marie Panepinto, Meghan West, Julie Beier, Amy Gaffney, and Marianna Sousa – who provided wake-up calls, study dates, sanity checks, and in general, kept me from falling off an emotional cliff. We have walked through so much together and each and every one of them will always be dear to my heart.

Finally, I am who I am today because of the incredible love, support, and prayers that I have received from each and every one of my family: Mom, Dad, Chris, Colleen, Dan, Katie, Elizabeth, Michaela, and Daniel. I am blessed and honored to be a part of this family and I look forward to passing on that love to my new godchild and to my own children.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

Introduction and Overview

1.1 Introduction

The use of physical mnemonics to remember the direction of the magnetic field of a current-carrying wire date back to the late 1800s (see Figure 1.1 for an early example reproduced from Greenslade, 1980). Since most of these mnemonics involve the use of one's right hand, they are aptly called "right-hand rules" and are most often used to determine the direction of a vector (or cross) product. Right-hand rules have changed and accumulated over the years, culminating in several that are used in most introductory physics courses.

By the end of most introductory calculus-based university physics sequences, students are expected to be conversant with the basics of vector algebra including cross products. Students are usually introduced to right-hand rules when learning about rotational dynamics, since torque and angular momentum are defined in terms of a cross product. Right-hand rules become even more prominent in the study of magnetism, where cross products are used to describe numerous phenomena, such as the force on a charge moving in a magnetic field. Figure 1.2 shows two common right-hand rules used in introductory physics.

There is evidence that students struggle with the concepts in magnetism and perform poorly on cross product questions. Additionally, the same research suggests that students struggle with understanding cross products outside of any physics context, as well

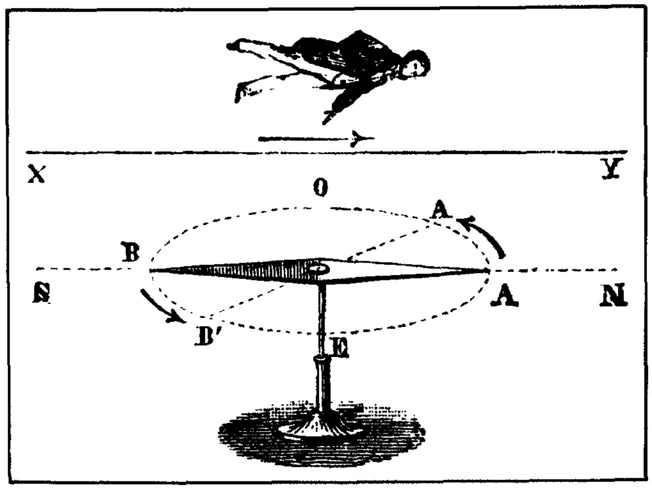


Figure 1.1: Ampére's rule: "Let the experimenter imagine himself swimming head-formost in the direction of the current, and with his face toward the magnetic needle: then the deflection of the needle will, in all cases, be such that the north pole moves toward his left hand." (Lardner, 1874, p.142, in Greenslade, 1980)

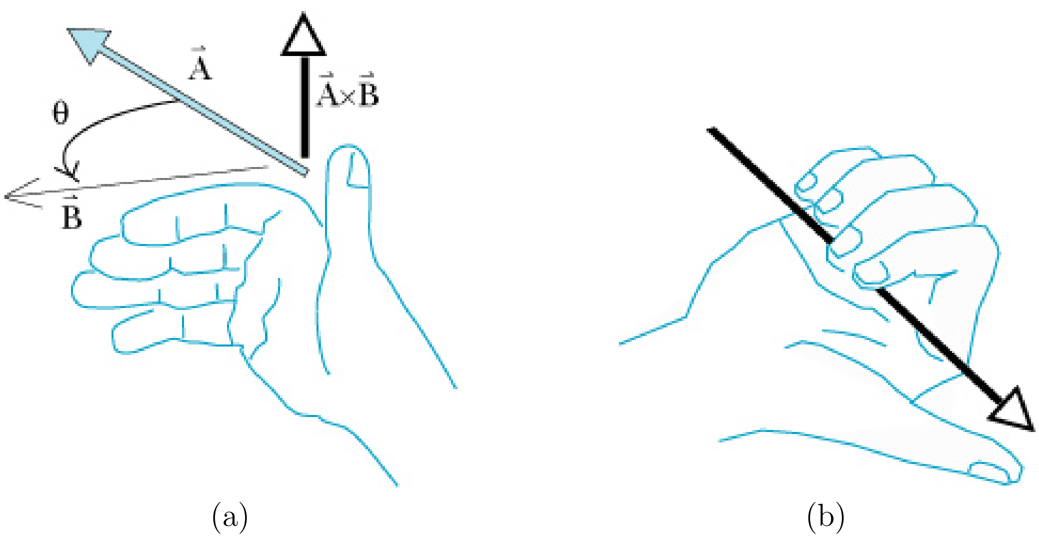


Figure 1.2: The two right-hand rule types presented in *Matter and Interactions* (Chabay and Sherwood, 2009).

as in the context of magnetism and in other physics areas such as torque(*e.g.* Knight, 1995; Scaife and Heckler, 2010; Van Deventer, 2008). Since the right-hand rule is one of the most common methods for finding the direction of a cross product, many researchers have speculated that poor performance on cross product problems may be due to either an inappropriately applied right-hand rule or the failure to use such a rule. There is little evidence to support this conclusion since few studies have had the resolution to observe the use of right-hand rules.

As Chapter 2 will show, there is research from several disciplines suggesting that student difficulties with cross product direction and the use of right-hand rules may be dependent on the context, as well as the representation of the question. Thus, the present study explores the context-dependence and representation dependence of student use of right-hand rules in an effort increase the understanding of student difficulties with cross products.

To provide a foundation for this work, the remainder of this chapter will explore the many ways that cross products and right-hand rules are taught to undergraduates. Chapter 2 will discuss the research on which this study was based. It will also provide a synthesis of this literature, including several categories of problem features that formed the basis of the study design. The study design will be addressed in Chapter 3 and Chapters 4–6 will present the analysis, results, and discussion for each piece of the project. Finally, Chapter 7 will summarize the findings and discuss some implications for instruction and future work.

1.2 Teaching cross products

There are two common methods that introductory math and physics textbooks use to define the cross product between two vectors – most texts employ elements of both (e.g. Chabay and Sherwood, 2007; Giancoli, 2000; Stewart, 1995). One method is geometric and the other algebraic; each can be derived from the other and there is a great deal of debate about which is the fundamental definition and which is merely an "interpretation."

The geometric method defines the cross product as the oriented, or directed, area of the parallelogram formed by the vectors $\vec{A}$ and $\vec{B}$ (Azad, 2001; Shaw, 1985; Dray and Manogue, 2003, 2006). The direction of the resultant vector is typically found through a right-hand rule; this aspect of cross products will be discussed in section 1.3. Calculating the area of the parallelogram yields the magnitude of the cross product:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \theta$$

where θ is the smallest angle between the two vectors. While some texts make reference to the geometric origin of this equation (Stewart, 1995), most present the equation as the definition itself or as derived from the algebraic method discussed below.

The algebraic method relates the components of $\vec{A}$ and $\vec{B}$ using a "dyadic" product, which is the sum of all permutations of pairs of components (Elk, 1997). Again, this origin is not usually presented to students. Instead, they are given the result of this definition in the orthogonal, three-dimensional, Cartesian coordinate basis:

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$

The determinant of the following matrix is often used to remember this relationship:

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

As mentioned above, there is debate about which is the more fundamental definition. One reason for the debate is that the result of the cross product is actually a pseudo-vector. It does not transform under reflection like other vectors and is also not generalizable to higher dimensions, a fact which is not typically presented to students (Bobillo-Ares and Fernandez-Nunez, 1990; Elk, 1997; O'Keeffe, 1981). The language of "geometric algebra," advocated by Hestenes (2003), avoids this issue by defining the cross product as the dual of the outer product. This outer product ($a \wedge b$) is still an oriented area, but oriented in the plane of $\vec{A}$ and $\vec{B}$ instead of perpendicular to that plane. Similarly, Roche (1997) used imaginary vectors to represent motion around the axis, instead of along it.

According to some, there are pedagogical reasons to adopt a geometric approach. For example, Azad (2001) argued that a geometric approach to scalar and vector products is necessary if one wants to focus on geometric and conceptual understanding in calculus. Similarly, Shaw (1985) advocated drawing on geometric intuition about area and volume to stress the simplicity of certain aspects of vector algebra, such as multi-linearity. There is no research to validate these arguments, however, Dray and Manogue (2003) did see evidence from student evaluations suggesting that an emphasis on geometric reasoning may aid in helping students feel more comfortable with the material. While the debate continues, most instructors employ a mix of the definitions presented above, including the right-hand rule as a means of determining direction.

1.3 Right hand rules

One of the problems with using geometric definitions is the difficulty of putting the spatial ideas into writing (Dray and Manogue, 2006). The various right hand rules are, in a way, the translation from the printed page back into the three-dimensional space of the geometric cross product. The right-hand rule in Figure 1.2.a is usually given with these, or similar, instructions (taken directly from Chabay and Sherwood, 2007, p. 335):

- Point fingers of right hand in direction of first vector $\vec{A}$
- Rotate wrist, if necessary, to make it possible to
  - Bend fingers of right hand toward second vector $\vec{B}$ through an angle θ less than 180°
  - Stick out thumb, which points in direction of cross product $\vec{A} \times \vec{B}$

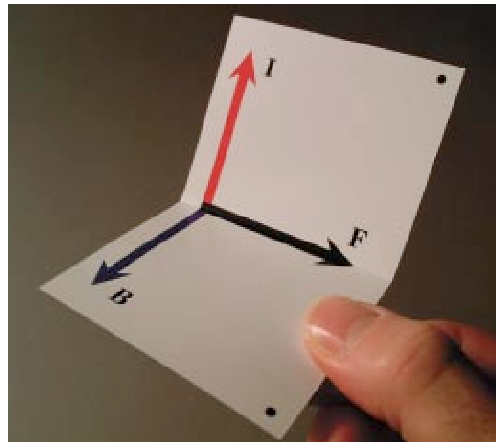
The right-hand rule in Figure 1.2.b is usually presented in a specific physics context. For example, if you point your thumb in the direction of conventional current, your fingers will curl in the direction of the magnetic field.

Figure 1.3: Photo of students attempting to use right-hand rules on an exam (reproduced from Wieberdink, 2002)

Many teachers and researchers suggest that difficulties with implementing a right hand rule are mainly kinesthetic: the accidental use of one's left hand (as seen in Figure 1.3, reproduced from Wieberdink, 2002); losing track by becoming so focused on the manipulation of the hand (Nguyen and Meltzer, 2005); the physical difficulty of aligning one's body in the proper orientation (Klatzky and Wu, 2008); and in some cases, the inability to use one's right hand at all (Van Domelen, 1999). While all of these authors assume that physical discomfort contributes to difficulty with the right-hand rule, none provide experimental evidence to support this assumption.

There have been several instructional interventions designed to address the physical difficulty of using the right-hand rule, usually through the creation of external devices and visual aids. Two such devices are shown in Figure 1.4: Figure 1.4.a is a plastic box

(a)



(b)

Figure 1.4: (a) Artificial right-hand rule device. Colored arrows indicate vector directions; purple dotted arrow (on bottom) shows direction of curl (reproduced from Van Domelen, 1999) (b) Current-Magnetic Field-Force ["I-B-F"] card (reproduced from Nguyen and Meltzer, 2005)

with three colored vectors and a dotted arrow showing the direction to curl (Van Domelen, 1999); and Figure 1.4.b is a bendable card labeled with current, magnetic field, and force vectors Nguyen and Meltzer (2005). The first was designed to meet several criteria: visibility, stability, portability, durability, an obvious application of the right-hand rule, no words to give an advantage over someone using their hand, and finally, reasonable design and cost. The second tool was designed to address the physical issues of using a right-hand rules, but also to help students to recognize which angle to use for the magnitude (*i.e.* the card should never bend backward). Unfortunately, neither of these authors provided any assessment of their tools' effect on student performance.

Klatzky and Wu (2008) did assess the impact of a revised set of instructions for the right-hand rule, reproduced in Figure 1.5. They saw significant improvement on an in-class quiz for students who received these instructions compared to students who received

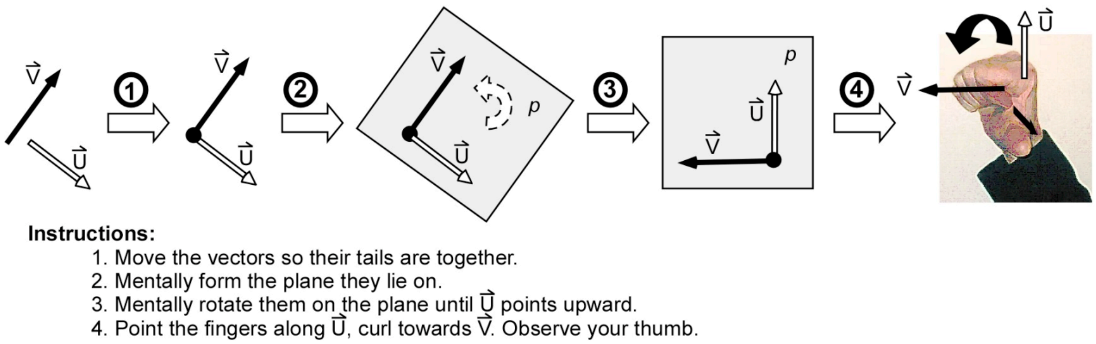


Figure 1.5: Revised instructions for performing the right-hand rule (reproduced from Klatzky and Wu, 2008)

more traditional instructions for the right-hand rule. However, Klatzky and Wu argued that even with revised instructions, a physical right hand rule is unnecessarily difficult. Instead, they advocated using a rule which does not require a physical act: if the rotation from $\vec{A}$ to $\vec{B}$ is clockwise, the cross product is into the plane and if it is counter-clockwise, the result is out of the plane. The cognitive motivation for this rule is discussed in more detail in Section 2.3.

With the exception of the in-class assessment used by Klatzky and Wu (2008), there has been no research on the impact of right-hand rule instruction on performance; and while this topic is not within the scope of this study, these results naturally suggest directions for future study.

Chapter 2

Review of the literature

There are several areas of research that have implications for understanding potential areas of difficulty with the use of the right-hand rule: research on student conceptual understanding of magnetism (Section 2.1), research on student understanding of vector algebra (Section 2.2), and research on spatial cognition (Section 2.3). This chapter will discuss the relevant literature from these disciplines and end with a synthesis of this research that forms the groundwork of the current study (Section 2.4).

2.1 Student conceptual understanding of magnetism

In the second semester of the introductory physics sequence, students encounter, or re-encounter, cross products with the concept of magnetic field. The magnetic field ($\vec{B}$) is usually defined using the Biot-Savart Law for a point particle:

$$\vec{B} \equiv \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{|\vec{r}|^2}$$

where $\frac{\mu_0}{4\pi}$ is a constant, q is the charge on the particle, $\vec{v}$ is the velocity of the particle, $\vec{r}$ is the position vector from the particle to the location where the magnetic field is measured, and $\hat{r}$ is the unit vector in the direction of $\vec{r}$. The direction of the magnetic field at the observation location for a negatively-charged particle is opposite the direction for a positively-charged particle. Students are usually taught to find the direction using the right-hand rule as if it were a positive charge and then reverse the direction at the end.

Since the magnetic field is generated from moving charges, a modified Biot-Savart Law for a short, thin, current-carrying wire is often used to describe the system:

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l} \times \hat{r}}{|\vec{r}|^2}$$

where I is the conventional current, $\Delta\vec{l}$ is a unit of length that points in the direction of the conventional current, and $\vec{r}$ is the position vector that points from the source to the observation location. This problem configuration is an example of a right-hand rule that applies only to magnetic fields, and one of these rules (*c.f* Figure 1.2.b) shows the pattern of the magnetic field around a current-carrying wire.

The force on a charged particle due to an applied magnetic field is also defined using a cross product. The Lorentz force is defined as $\vec{F} \equiv q\vec{E} + q\vec{v} \times \vec{B}$, where $\vec{F}$ is the force on the particle, q is the charge on the particle, $\vec{v}$ is the velocity of the particle, $\vec{E}$ is the applied electric field at the location of the particle, and $\vec{B}$ is the applied magnetic field at the location of the particle. The Lorentz force is usually introduced separately as electric and magnetic forces:

$$\vec{F}_{electric} \equiv q\vec{E}$$

$$\vec{F}_{magnetic} \equiv q\vec{v} \times \vec{B}$$

Once again, the sign of the charge determines whether the magnetic force will be in the same direction as $\vec{v} \times \vec{B}$ or opposite that direction.

For a short length of current-carrying wire, the students are presented an analogous definition for the force:

$$\Delta\vec{F}_{magnetic} = I\Delta\vec{l} \times \vec{B}$$

where I is the conventional current, $\Delta\vec{l}$ is a unit of length that points in the direction of the conventional current, and $\vec{B}$ is the applied magnetic field.

The electric and magnetic forces are actually two different components of the same phenomenon (the electromagnetic force) and there are several analogies between electricity and magnetism. However, the cross products in the definitions of magnetic field and magnetic force introduce some important differences that prove to be major stumbling blocks for students. The electric field points radially outward from a charged particle, whereas the magnetic field points *around* the path of the charged particle. Additionally, if the particle is not moving or does not have a component of its motion perpendicular to the position vector (i.e. it is moving directly toward or away from the observation location), the magnetic field will be zero. Similarly, there must be a component of the velocity of a charged particle perpendicular to the applied magnetic field in order for the magnetic force on that charge to be non-zero. Also, unlike the electric force, the magnetic force does not point in the direction of the field that causes it. As mentioned above, these differences are areas of difficulty for students.

There have been several attempts in recent years to develop and validate a conceptual survey of electromagnetic concepts analogous to the Force Concept Inventory (FCI) for force concepts (Hestenes et al., 1992). These efforts include the Diagnostic Exam for Introductory Electricity and Magnetism (DEEM) (Marx, 1998), the Conceptual Survey of Electricity and Magnetism (CSEM) (Maloney et al., 2001), and the Brief Electricity and Magnetism Assessment (BEMA) (Ding et al., 2006; Kohlmyer et al., 2009).

In addition to the development of these surveys, a great deal of research has attempted to identify areas of difficulty with electromagnetic concepts (e.g. Sağlam and Millar, 2006). There are also studies that strive to describe the mental models employed by students to reason about these magnetism concepts (e.g. Borges et al., 1998); and to understand how students' knowledge structure of these concepts impacts performance on problem solving (e.g Bagno and Eylon, 1997; Greca and Moreira, 1997). The following sections discuss the results of research student understanding of magnetic field (Section 2.1.1) and magnetic force (Section 2.1.2).

2.1.1 Magnetic field

Borges et al. (1998) identified five models of magnetism employed by students and professionals with a variety of educational backgrounds. The vocabulary and constructs used by their participants showed the effects of instruction, where those with a higher educational background tended to hold the more sophisticated models. Many of these models are consistent with the categories of conceptions identified by Guisasola et al. (2004) and the difficulties noted in some of the other studies.

The first model views magnetism as “pulling,” similar to the ”inherent nature” category of Guisasola et al. (2004). In this model, magnetism is a property of the magnets. According to Borges et al. (1998), this model was most common among those who had not studied the subject except in elementary school. For these subjects, the everyday applications of magnets were more important than the mechanism.

A perception of magnetism as “a cloud” was the second model noted by Borges et al. (1998). This model, which was found to some extent with all of the student groups, treats magnetism as a “sphere of influence” around a magnet. Borges et al. suggest that the participants holding this model equate magnetic action with gravitational action and this idea is also consistent with the science fiction notion of a “force field.”

Borges et al.’s 1998 third and fourth model, which treat magnetism as “electricity” and “electric polarization,” are reflected in the “electrical” category of Guisasola et al. (2004). Both of these models appear to be an overgeneralization of the idea that “likes repel and opposites attract” and they seem to be primarily concerned with accounting for the existence and behavior of magnetic poles. The electric polarization model is slightly more sophisticated, in that it attempts to account for magnetism on the microscopic level (Borges et al. point out that it is similar to Coulomb’s model of magnetism). However, both models seem to view stationary electric charges as the source of magnetic field and field lines as pointing from “positive charges” (or the north pole of a magnet) to “negative charges” (or the south pole of a magnet).

These fundamental differences between electric and magnetic fields are the ones that students struggle with the most. Guisasola et al. (2004) found that half of their participants (including third-year physics majors) used reasoning consistent with the view that charges at rest produce magnetic fields, although the majority correctly identified moving charges as the source of magnetic fields when asked explicitly. Similarly, Allen (2001) found that before instruction about 60% chose a radial option for the magnetic field due to a current-carrying wire (reduced to 20% after instruction). Both types of “electrical” reasoning were also seen in other studies (Maloney et al., 2001; Sağlam and Millar, 2006).

Borges et al.'s 1998 fifth and last model is a "field model" that treats the macroscopic interactions as the result of a field-like relationship due to a microscopic phenomenon. They identify three views on the source of magnetism within this model: micro-currents inside magnets (the Ampere model and Guisasola et al.'s 2004 "Amperian"), elemental magnets (or permanent magnetic dipoles), and contribution from both spin and orbital magnetic moments. The participants exhibiting this model often spoke using the language of the previous three models (magnetism as cloud, as electricity, and as electric polarization). However, this model was distinguished from the previous by the recognition that "magnetism" at the macroscopic level is actually a microscopic property of all matter. Not surprisingly, this model was most common with the engineers and physics teachers.

Guisasola et al. (2004) identified an additional category, which they entitled "ingenuous realistic." It is characterized by a tendency to treat field lines as physical entities. This treatment of field lines sees magnetic interactions as due to the "attraction" or "repulsion" of the field lines themselves. This view was also noted by Pocovi and Finley (2002), who point out that this ontological perspective of field lines is actually consistent with some of Faraday's own statements. Additionally, there are many physicists who argue that field lines are actually physical entities (Harpaz, 2002).

Each of these models lends itself to certain errors when solving problems involving magnetic field. However, the most prevalent difficulty, as well as the most relevant to this study, is the one that equates magnetic and electric fields. Students who hold this view would likely not even recognize the need to use the right-hand rule when trying to determine the magnetic field due to a particle.

2.1.2 Magnetic force

The confusion between electric and magnetic fields noted above continues to plague students as they begin to study forces, and this confusion manifests primarily in two ways. While the force due to an applied electric field is parallel (or anti-parallel) to that field, the magnetic force is perpendicular to the magnetic field. The idea that applying a field would produce a force perpendicular to that field is both unexpected and counterintuitive, especially when compared to electric force. Thus, when asked about the direction of magnetic force on a moving charge, many students respond in way that is consistent with electric force–with the the force parallel (or anti-parallel) to the field (Guisasola et al., 2004; Maloney et al., 2001). Scaife and Heckler (2007; 2010) found that issue was dependent on the representation of the magnetic field. They compared the performance of two groups of students who were given questions where the magnetic field was represented using either field lines or magnetic poles. While many students in both groups responded with the force in the direction of the field, this answer was significantly more common for students who received the magnetic poles representation.

In addition to this confusion between electric and magnetic forces, Scaife and Heckler (2010) found that on the questions about the direction of magnetic force, the most common incorrect response involved a sign error. The force would be perpendicular to the field and the velocity of the charge, but opposite the correct direction. This error, too, was dependent on the representation of the field. For those given a problem with magnetic poles, this sign error was at least partly due to reversing the direction of the magnetic field (from south to north instead of north to south). However, Scaife and Heckler (2010) found that these sign errors were not systematic, indicating that there are multiple sources for this incorrect response. In follow up experiments, they identified two additional sources of error: inconsistent execution of the right-hand rule and an unawareness of the non-commutative nature of the cross product. Students who did not recognize this property of the cross product were more likely to make a sign error by reversing the order of the vectors than those who did understand this property.

2.1.3 Summary of research on magnetism

This literature indicates several key areas where students struggle with the fundamental concepts of magnetism. They confuse electric and magnetic concepts on both magnetic field and magnetic force problems. For magnetic force problems, this error was much more common when the field was represented by magnetic poles than when it was represented with field lines. The magnetic poles representation also led to a sign error resulting from reversing the direction of the magnetic field due to those poles. Other sources of sign errors include the misapplication of a right-hand rule and an unawareness of the non-commutative nature of the cross product.

The current study saw evidence of all of these errors (*c.f.* Chapters 5 and 6), as well as identified other sources of physics errors not discussed in this literature: difficulties in accounting for the sign of the charge (for both magnetic field and magnetic force questions) and dealing with the observation location (for magnetic field questions).

2.2 Student understanding of vector algebra

Understanding vector cross products involves also understanding basic vector concepts, such as magnitude and direction. Student difficulties with these concepts, as well as with other vector operations (such as addition and subtraction), can provide insight into factors that may contribute to problems with cross products. This section deals with the literature on student understanding of vector concepts, with the aim of identifying any issues that may contribute to difficulties with cross product direction.

Over the years, the focus of studies on vectors has changed and broadened. Earlier studies (e.g. Aguirre and Erickson, 1984) tended to focus on secondary students and their "conceptions" about vector properties. In more recent work, focus has shifted to college students in introductory physics courses of various types. These studies have also moved from explicit attention on misconceptions to other areas. Some of these areas are the design and assessment of diagnostic tools to assess the level of student ability to use vectors (e.g. Knight, 1995), exploring the sources of student difficulties with vectors (e.g. Nguyen and Meltzer, 2003), and comparing performance on vector tasks in different domains (e.g. Van Deventer and Wittmann, 2007) or with different interventions (e.g. Flores et al., 2004). With few exceptions (e.g. Gagatsis and Demetriadou, 2001), studies on student understanding of vectors are usually found in the Physics Education Research or Science Education Research communities. The contributions from the Mathematics Education Research community tend to focus on teaching approaches (such as the debate presented in Section 1.2).

This dissertation focuses on undergraduates in the calculus-based introductory physics sequence. Thus, Section 2.2.1 will discuss the literature examining the experience that students have with vectors when they enter their first college physics course. A discussion of the research on student difficulties will follow, starting with basic vector concepts (Section 2.2.2), moving to vector multiplication (Section 2.2.3), and concluding with a summary of how all of these results apply to the current study (Section 2.2.4).

2.2.1 College students' previous experience with vector algebra

Most students entering college physics reported experience with vectors in high school math or physics courses or in college math courses (Knight, 1995; Nguyen and Meltzer, 2003; Van Deventer and Wittmann, 2007). The area where the reported level of exposure was lowest was vector multiplication. According to Knight, at the beginning of a calculus based introductory physics course, less than 25% of the class indicated previous experience with vector multiplication. Few students could correctly evaluate a vector cross product and only 5% could correctly evaluate a scalar or dot product. Similarly, Van Deventer (2008) found that fewer than 15% of students were able to correctly answer questions on vector products at the beginning of college physics.

Although performance on basic vector concepts was better than that on vector multiplication, students seem to have an inflated sense of their knowledge about vectors overall, as evidenced by the disconnect between experience and performance (Knight, 1995). According to Knight only one third of the students entering a calculus based introductory physics class had sufficient vector knowledge for mechanics and 50% had no useful vector knowledge. However, Nguyen and Meltzer (2003) saw evidence that errors on basic vector operations were due, not to ignorance, but rather imprecise execution or confusion. They suggest that students have good intuition, but don't know how to apply their knowledge.

As one might expect, Nguyen and Meltzer (2003) found evidence of variation in performance for different populations; students in calculus-based courses performed better than those in algebra-based courses and those in second-semester physics courses better than those in first-semester physics courses. They also saw surprising differences between performance for the same course taught in the spring and in the fall, but did not have the data to speculate about the causes of these differences. While the picture of incoming students' experience is complex, it is clear that for introductory physics students in college, the cross product is a mathematical tool with which they have little experience and great difficulty.

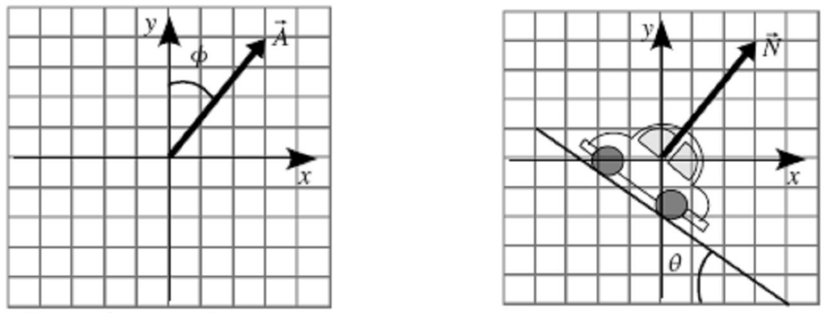
2.2.2 Student understanding of basic vector concepts

The majority of research into student understanding of vectors focuses on difficulties with the more basic forms of vector manipulation. The questions used to probe these concepts include graphical and algebraic representations for components, magnitude, and direction, as well as one- and two-dimensional addition and subtraction. Each of these topics and the difficulties associated with them will be discussed in this section.

Components

Aguirre's work (Aguirre and Erickson, 1984; Aguirre, 1988) focused mainly on secondary student conceptions of "implicit vector characteristics"–those considered too obvious to cover in class. These findings indicate that student understanding of vector components are close to those a physicist might use. Similarly, Van Deventer (2008) found that before instruction, the majority of first-semester college physics students could correctly identify the graphical x and y-components of a vector in both a math and a physics context.

However, Van Deventer (2008) also found that algebraically identifying components was more difficult in a physics context than in a math (non-physics) context. He attributed this to a seemingly slight deviation from isomorphism between the two questions. In the non-physics version (Figure 2.2.a), the angle was given between the vector and the y-axis, whereas in the physics version, it was given as the angle of the hill in the problem statement (Figure 2.2.b). When faced with this fairly standard practice in physics, participants often re-labeled the angle as that between the vector and the x-axis (instead of the y-axis). The majority of incorrect responses in the physics context were due to this confusion (Van Deventer and Wittmann, 2007). These findings indicate that students do not consider the applicability conditions of the recalled trigonometric formula (Van Deventer, 2008). Some instructors and texts address this problem by em-



(a) (b)

Figure 2.1: Diagrams given to students on the (a) non-physics and (b) physics questions in Van Deventer (2008) for questions on vector components

ploying direction cosines to shift the focus from the recalled formula to determining the appropriate angle (Chabay and Sherwood, 2003; Roche, 1997).

The inability to identify the appropriate angle (or appropriate trigonometric function for a component) could also impact students' performance on cross product questions. When finding the magnitude of the cross product, the incorrect angle will obviously produce an error and could also lead to a sign error when using the right-hand rule.

Magnitude and direction

A vector is usually defined in introductory courses as a quantity with both magnitude and direction; and students have difficulties with both of these aspects. Some researchers argue that the directional information gives them more trouble (Knight, 1995), yet the ways in which students struggle with magnitude and direction are often connected. For example, many students treat vectors with equal magnitude and opposite direction as if they were equivalent (Gagatsis and Demetriadou, 2001; Nguyen and Meltzer, 2003); and many believe that in order to have equal magnitude, two vectors must be parallel or anti-parallel (Nguyen and Meltzer, 2003).

These findings could represent a lack of understanding of how to operationally determine the direction of a vector or an inability to recognize parallel vectors (Nguyen and Meltzer, 2003). However, Barniol and Zavala (2009) showed that some of these issues may be due to differences in the definition of the word "direction." In some places outside of the US, two properties of direction–the line of action (sometimes called "orientation") and the sense ("which way it points")–are treated separately (Gagatsis and Demetriadou, 2001; Roche, 1997; Barniol and Zavala, 2009). While this distinction is not explicitly made in US courses, this loose definition of direction may account for the results that Nguyen and Meltzer reported.

Just as students have difficulty determining the direction of a vector, they also sometimes have trouble finding the magnitude of a vector from its components (Knight, 1995). At the end of a semester of mechanics, however, most students could correctly apply the Pythagorean Theorem to find the magnitude of a vector (Van Deventer, 2008). In a different context (vector addition), others have noted a misapplication of this theorem; it is unclear whether this misapplication was due to a difficulty with concept of magnitude or with the addition operation (Flores et al., 2004; Nguyen and Meltzer, 2003).

Even with the difficulties noted above, performance on questions dealing with magnitude and direction are still fairly high compared to questions on vector operations (Beichner, 1994; Nguyen and Meltzer, 2003; Van Deventer and Wittmann, 2007). This discrepancy indicates that there are difficulties with vector operations that are not due to a misunderstanding of the basic vector properties of magnitude and direction.

Vector addition and subtraction

It is clear from many of the studies cited in this section that vector addition and subtraction are more difficult for students than basic concepts such as magnitude and direction. For example, on both vector addition and subtraction questions, Flores et al. (2004) found that less than 30% of students were able to correctly identify the magnitude and the direction of the resultant with correct reasoning; although, this improved to more than than half for modified courses designed to specifically address these issues.

One of the most common errors that students make when adding and subtracting vectors is to treat a vector as a scalar, by failing to take directional information into account (Aguirre and Erickson, 1984; Flores et al., 2004; Gagatsis and Demetriadou, 2001). Although secondary students ranked distinguishing a vector from a scalar as fairly easy (Johnstone and Mughol, 1976), at the end of a semester of physics, Van Deventer (2008) saw as many as 16% of students treat vectors as scalars.

One of the methods for graphical vector addition is to place the tail of the second vector to the tip of the first vector and connecting the tail of the first to the tip of the second. However, students tend to treat vectors as fixed in space and not recognize the need for parallel transport (Nguyen and Meltzer, 2003; Van Deventer, 2008). There also seems to be a representation-dependence associated with this error. Hawkins et al. (2009) found that when the vectors in a were given with tails together, performance was much lower (30-40%) than when the vectors were separated in space (above 70%). A similar problem could impact students solving cross products problems when the vectors are not presented tail-to-tail.

The majority of the other errors in adding and subtracting vectors were due to inappropriate or misapplied algorithms. Van Deventer (2008) identified several of these "tools" or algorithms: tip-to-tail, tail-to-tip, tip-to-tip, minus-means-flip, make a triangle (or close the loop), connecting vector, equilibrium vector, and parallelogram. Some of these tools are appropriate for either vector addition or for vector subtraction, but students have a tendency to use an inappropriate algorithm or to combine algorithms in an inappropriate way (Van Deventer, 2008; Nguyen and Meltzer, 2003; Knight, 1995; Flores et al., 2004). In addition to correct algorithms used inappropriately, there were also incorrect algorithms used. For example, there were many occurrences of the "two-headed" arrow used in one-dimensional addition (Nguyen and Meltzer, 2003).

While these studies demonstrate the diversity of methods used by students to solve vector addition and subtraction problems, there is evidence that this variety is between, but not within subjects (Hawkins et al., 2009). In interviews designed to elicit different methods using representational cues, Hawkins et al. found that while different students chose different methods, all but one student used the same method on every problem. A similar effect was seen with students solving cross product questions (Kustusch et al., 2008). Two possible explanations, proposed by Hawkins et al., are that students lack available tools or do not recognize the advantages of using multiple methods. They suggest that stronger representational cuing might overcome this tendency to employ a single method.

While students can more easily determine components graphically than algebraically, the opposite is true for vector addition; students have more difficulty with graphical addition of vectors than with algebraic addition (Flores et al., 2004; Knight, 1995). At the end of a semester, Van Deventer (2008) found that 40% of students still could not graphically add two vectors. There are also differences in performance between physics and non-physics (math) contexts on vector addition tasks (Van Deventer, 2008). Before instruction, there was no significant difference between performance on questions in a non-physics context and isomorphic questions in a physics context. Just after instruction, the percentage of correct answers on the non-physics questions increased, but there was a decrease in the performance on the physics questions (consistent with Flores et al., 2004). At the end of the semester, the performance on the physics questions improved to the level of the performance on the non-physics version.

In comparing performance on vector addition and vector subtraction, students did more poorly on one-dimensional vector subtraction than on one-dimensional vector addition (Nguyen and Meltzer, 2003; Van Deventer, 2008).Also, students seem equally incapable of solving one-dimensional vector subtraction problems before and just after instruction regardless of context (Van Deventer, 2008). However, at the end of the semester, performance on questions in a non-physics context had improved over performance earlier in the semester.

Somewhat surprisingly, students perform much better on two-dimensional subtraction than on one-dimensional subtraction regardless of context; although performance on two-dimensional subtractions questions in a physics context is consistently below two-dimensional subtractions questions in a non-physics context (Van Deventer, 2008). The differences between one-dimensional and two-dimensional vector subtraction and the differences between contexts are consistent with Shaffer and McDermott (2005).

It is clear that difficulties with vector addition and subtraction are not due solely to problems with vector magnitude, direction, or components. However, issues with basic vector concepts do contribute to difficulties in vector addition and subtraction. Several trends are of particular interest due to the implications for vector multiplication: the inclination to ignore directional information; the tendency to use the incorrect angle or trigonometric function; the representation-dependence and difficulties with parallel transport; the context-dependence of performance on vector addition and subtraction in physics and non-physics domains; and the tendency of individual students to use the same algorithms for all vector addition problems.

2.2.3 Student understanding of vector multiplication

Students have more difficulty with vector multiplication than with the vector concepts already discussed. For example, Van Deventer (2008) found that fewer than 30% of students were able to correctly answer questions on vector products even after a semester of physics and responses were inconsistent between physics and non-physics contexts. This low performance and inconsistency is a indication that the students are guessing. On the other hand, Christensen et al. (2004) found that more than 50% of students were able to answer both the dot and cross product questions correctly after one semester of physics. A potential bias in their sample could account for the discrepancy with Van Deventer's results. Christensen et al. found that the overall course grades for those that took one of the quizzes were statistically higher than those that did not, which implies that their results can be viewed as an upper limit for student performance on cross product problems.

Dot product

Two of the trends noted at the end of section 2.2.2 show up again when students try to solve problems involving dot products: an inability to recognize the need for parallel vector transport (Christensen et al., 2004) and a tendency to use the incorrect angle (Gagatsis and Demetriadou, 2001). A related issues specific to the dot product is that many students do not understand that perpendicular vectors will give a zero result for the dot product or that vectors with a greater than 90 degree angle will yield a negative dot product (Christensen et al., 2004).

In contrast to the tendency to ignore directional information on vector addition and subtraction problems, Van Deventer and Wittmann (2007) found that the majority of students believe that a dot product has a direction. Interviews, as well as the response distributions from the multiple choice quiz, indicated that students tried to use vector addition tools to find this "direction" for the dot product.

Students using a component method were more often able to correctly identify the relative magnitudes of the dot product than those using a more geometric equation (Christensen et al., 2004). However, this effect may be dependent on representation, as students who were given diagrams with both axes and a grid outperformed students with only axes or only a grid (Sayre and Heckler, 2009). Like Hawkins et al. (2009), Sayre and Heckler speculated that different representations may encourage students to use different methods to solve the problems. However, they were not able to examine the methods used by students due to the type of data collected.

Cross product

One of the most common incorrect answers for questions on cross product magnitude was the use of cosine in the expression for the magnitude instead of sine, which could be accounted for by a confusion between the dot product and the cross product (Christensen et al., 2004; Van Deventer, 2008). There are also some indications that students may confuse the component expressions for the dot product and cross product (Kustusch et al., 2008). The confusion between dot and cross products could also account for a drop in performance on cross product problems when students reviewed the dot product while learning Gauss' Law (Sayre and Heckler, 2009).

Using the incorrect trigonometric function for cross product magnitude could also lead students to make mistakes on cross product direction questions; when the vectors are perpendicular, the use of cosine produces a cross product of zero. Christensen et al. (2004) found that more than 15% of students responded in this way when the question was in a non-physics context, but this error was not seen at all when the same question was placed in the context of magnetic fields. Van Deventer (2008), however, found that student were equally incapable of correctly finding the magnitude of the cross product in both physics and non-physics contexts.

As noted in Section 2.1.2, the most common incorrect answer on cross product direction questions was the direction opposite the correct response–a sign error. Christensen et al. (2004) interpreted this as a misapplication of the right-hand rule and Scaife and Heckler (2010) did see some evidence of this. Scaife and Heckler also found that an unawareness of the non-commutative property of the cross product contributed to these sign errors; and Dray and Manogue (2006) suggested that the use of minors with matrix determinants could also produce this error.

The representation-dependence of cross product direction questions has already been mentioned in Section 2.1.2, where Scaife and Heckler (2010) found that students were less likely to make a sign error when the magnetic field was presented as field lines than when it was displayed as magnetic poles. Van Deventer (2008) also noted a context-dependence for cross product direction questions similar to that for vector addition. Before instruction, performance in the physics context (torque) was significantly better than in the non-physics context. Yet, just after instruction, this trend switched as performance on the physics version decreased and performance on non-physics version increased. At the end of the semester, performance on physics version improved such that performance in both contexts were approximately equal.

Van Deventer (2007; 2008) suggested that another break from isomorphism between the physics and non-physics context could account for a difference in the response distribution between contexts. In the physics version, the vectors were not tail to tail at the origin as they were for the non-physics version (Figure 2.2). For the non-physics version, the most common response was a vector in the direction of the vector sum, which Van Deventer attributed to a missing or inappropriate use of a right-hand rule to find the direction. Yet, for the physics version, the most common response was the "direction" of the rotation of the pulley (counter-clockwise). This finding is consistent with Ortiz (2001), who found that many students stated that the direction of net torque would be not along the axis of rotation, but in the direction of rotation (*i.e.* clockwise or counter-clockwise).

In students solving cross product problems, Kustusch et al. (2008) saw a pattern of method use similar to that described by Hawkins et al. (2009) with vector addition problems. Although there were differences between students in their choice of method, each

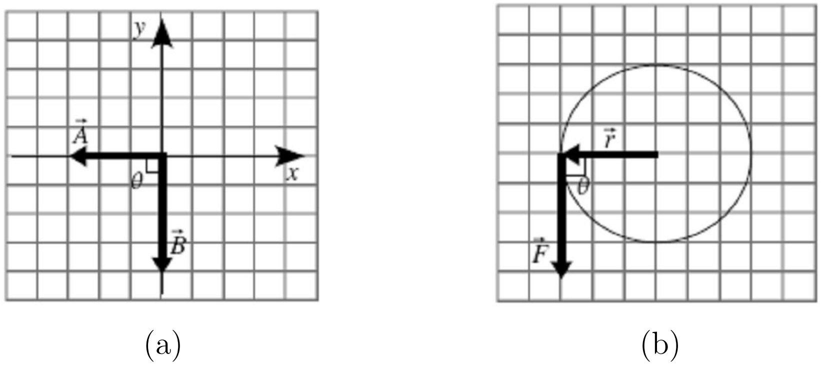


Figure 2.2: Diagrams given to students on the (a) non-physics and (b) physics questions in Van Deventer (2008) for questions on cross product direction

student tended to choose one method to use for all of the cross product questions. Kustusch et al. (2008) also saw some evidence that students have an preference for using the algebraic approach as opposed to the geometric methods to calculate the cross product.

Several of the patterns mentioned in Section 2.2.2 have also been seen in research on student understanding of vector multiplication: difficulties with directional information, the use of the incorrect angle or inappropriate trigonometric function, difficulties with parallel transport, and individual differences in choice of method. The following section will summarize how the current study can build on the research on student understanding on vector algebra and extend it to better understand student difficulties with cross product direction and specifically, the use of right-hand rules.

2.2.4 Application of vector research to right-hand rules

This section outlined the extensive research on student understanding of vector algebra. While there are several studies that have looked at student understanding of cross products, few have focused on cross product direction and only two (Kustusch et al., 2008; Scaife and Heckler, 2010) have had the resolution to observe the use of right-hand rules. However, this section has highlighted some trends seen throughout the literature and some of these have direct implications for the right-hand rule. Students have a tendency to ignore directional information and also seem to have preference for algebraic methods.

One way to address these issues and to probe cross product direction directly, is to provide no numeric information for questions.

The context-dependence and representation-dependence of performance on vector problems has been noted throughout this section; and these dependencies can be probed for right-hand rules as well. Varying the spatial relationship between vectors (*e.g.* head-to-tail or tail-to-tail) and angle between the vectors will address the difficulties with parallel transport and with using an appropriate angle, respectively. Using questions in different physics contexts and with different physics features (such as the representation of magnetic field), also provides the opportunity to verify and extend the results discussed in this and the previous section.

2.3 Research on spatial cognition

Regardless of whether one starts with a geometric or algebraic definition of a cross product, the result is inherently three-dimensional. As a result, dealing with cross products often requires spatial reasoning analogous to mental rotation. The use of a physical right-hand rule introduces a kinesthetic element to this reasoning, but does not eliminate the spatial nature of the cognition required. There is a large body of literature that deals with spatial cognition and this research gives some indications of what factors might influence the difficulty of using a right-hand rule. Due to the breadth of this field, this discussion is restricted to a few key studies that have a direct bearing on the use of right-hand rules.

According to Klatzky and Wu (2008), the primary cognitive issue in using a right-hand rule is one of coordinating and aligning reference frames. Alignment is often explored within the context of sensitivity to orientation; and most studies approach this issue either from the perspective of form perception, such as in Rock's (1973; 1983; 1986) work, or with imagined rotation and translation, such as in Klatzky et al. (1998) and Shelton and McNamara (2001). While right-hand rules do not fall into either of these categories, the results of these studies, and the methodologies they employ, offer insight into how the issue of alignment could contribute to difficulty in the use of a right-hand rule. This section will discuss some key studies from this research on alignment (Section

2.3.1) and also discuss the implications of this research for understanding the use of the right-hand rule, including a task analysis and insights on methodology (section 2.3.2).

2.3.1 Alignment of reference frames

In most of the literature on alignment of reference frames, there is a primary distinction between two types of frames (Klatzky, 1998). In the first, locations are measured with respect to the person ("egocentric" frame) and the second is a frame external to the person ("allocentric" or environmental reference frame). There may be several allocentric frames associated with an action (*e.g.* a "global" frame defined by the walls of a room versus the "local" frame of an object in that room). Many actions require the alignment of these various frames of reference.

Aligning reference frames involves two processes: parameter remapping and coordinate transformation (Klatzky and Wu, 2008). In order to align frames, one must remap space such that the two frames break it up in the same way; and one must be able to move between the coordinates of one frame and the other. If the response requires an action, these processes might need to be used again if the frame used for computing the response is not the same as the frame required for the response. Each of these steps required for alignment (parameter remapping, coordinate transformation, and response computation) can cause failures and errors.

Using several studies as a framework for the discussion, Klatzky and Wu (2008) explored the process of aligning an egocentric frame with an allocentric frame and the cognitive load that comes with this process. In particular, they identify two factors which impact the difficulty of alignment: allocentric layer and obliqueness (see Figure 2.3). The allocentric layer can be thought of how far removed from the body the task-defined frame is (such as whether it is the geometry that surrounds the body, a virtual world, or an imagined world). The more removed the two frames are from each other, the more difficult parameter remapping becomes. Obliqueness describes the degree of angular disparity between the body frame and the task-defined frame. The greater the disparity between reference frames, the more difficult the process of coordinate transformation.

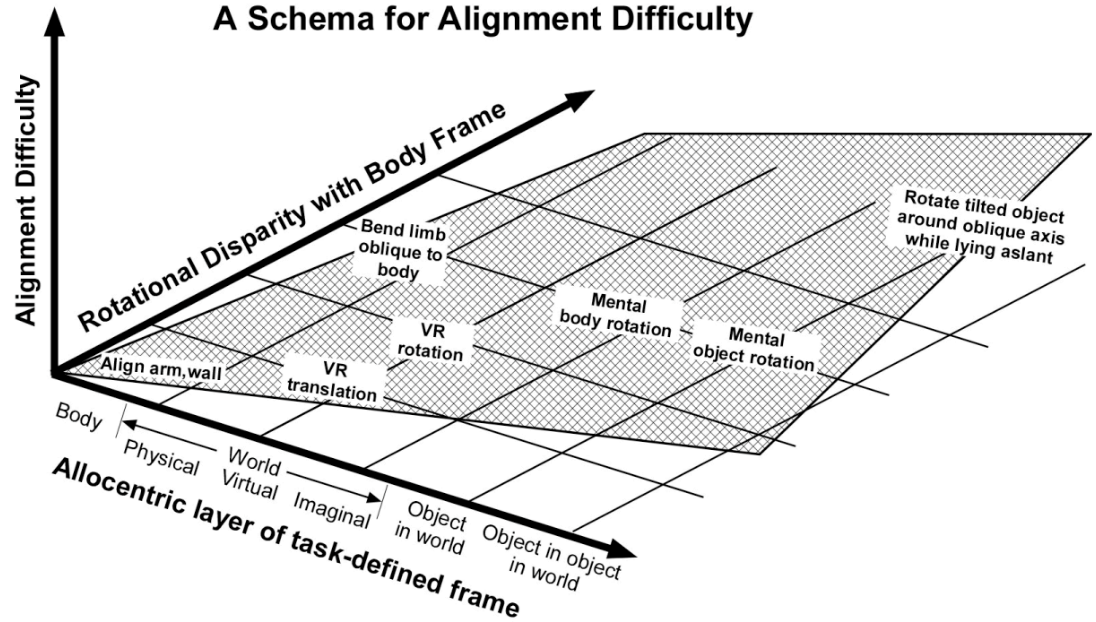


Figure 2.3: Alignment difficulty ordering. The difficulty of a task corresponds to its height in the plane in the two-factor space. (reproduced from Klatzky and Wu, 2008)

Most right-hand rule tasks are in the same allocentric layer, so this aspect of alignment load is not as relevant to this study. However, the issue of obliqueness has a great deal of impact on our understanding of the issues that could influence the difficulty of using a right-hand rule.

Obliqueness

Research from a variety of areas in spatial cognition have identified a sensitivity to orientation. Rock's (1973; 1983; 1986) work strove to understand why form perception changed with orientation in the frontal plane, such as why a face isn't recognized when upside-down. He proposed that the assignment of direction (defining a "top," "bottom," "right," and "left" to the image) influences perception. While Rock did not distinguish between the assignment of "top" and "bottom," further studies have shown that the "top" of an image, as well as the vertical direction, have a special place in human perception and the ability to perform mental rotations. For example, when asked to learn a layout of objects and perform tasks based on that layout, participants interpreted the layout in a uni-directional reference system, *i.e.* with a "top," but not necessarily a "bottom" (Shelton and McNamara, 2001). Similarly, Pani and associates (Pani, 1993; Pani and Dupree, 1994) found that the vertical axis of rotation was the only direction in which participants could consistently perform mental rotations accurately. This would imply that a cross product with the axis of rotation in the vertical (*i.e.* the given vectors are in the plane of the page) will be easier than one with the axis of rotation in another direction.

Pani et al. (1996) also identified alignment in terms of the parallel and perpendicular and presented a theory to explain the cognitive benefit of frames aligned in this way. They showed that parallel and perpendicular conditions have a spatial redundancy (reflection and translational symmetry); and this redundancy, combined with the absence of distractions in mis-aligned systems, accounts for better performance on rotation tasks in aligned frames. Based on this framework, one would expect the horizontal direction to be as important as the vertical direction; yet, although there are situations where the horizontal direction does play a significant role, it is not as vital as the vertical direction. One study that did show the importance of the horizontal asked participants to align a cylinder with a certain direction while in a variety of different arm positions (Flanders and Soechting, 1995). For this task, the horizontal and the vertical orientations were both easier than orientations aligned at $45°$ to the ground or aligned to the forearm.

Assignment of direction is key to alignment in our perception, as well as in which types of mental rotations and spatial reasoning we are capable of performing. There are several things which can influence this assignment of direction, such as the direction one is facing (egocentric bearing), the visual frame, or gravity. There are several studies that have attempted to identify which frames are important for alignment (Pani, 1993; Pani and Dupree, 1994; Pani et al., 1996; Shelton and McNamara, 2001). While the results of these studies reveal complex interactions, there are some trends that emerge.

Two studies (Pani, 1993; Pani and Dupree, 1994) used a square affixed to a rod and measured participants' ability to imagine a rotation with the rod as the axis of rotation. Pani (1993) found that participants were always accurate in the simple case: a vertical axis of rotation, with the object normal to axis of rotation. Performance was still good if either of these orientations was maintained; however, combining an axis of rotation oblique to the environment with an object oblique to the axis of rotation left people stumped. Thus, task difficulty increased with the number of frames needing alignment.

These results were expanded in Pani and Dupree (1994), which assessed the relative importance of three frames of reference for mental rotations: the permanent environmental frame, a local external frame, and the egocentric frame. Rotations aligned with none of these frames proved difficult. However, performance on rotations about an aligned vertical rod were superior to any other orientation of the rod to the environment, regardless of viewer orientation. While the vertical was superior to the horizontal, the horizontal was also superior to other orientations. Additionally, the use of a local reference frame did improve performance in some conditions (errors were smaller in magnitude and of a different type). However, the use of the local frame did not bring performance up to the level of a vertically aligned rod without the local reference system. Pani et al. (1996) saw similar results on tasks involving perception of forms. These findings are consistent with Shelton and McNamara (2001), who found that salient environmental properties (such as walls) strongly influenced the mental representation of the space, but this effect was not present when the external reference system was not salient (*e.g.* in a round room). It is clear from these studies that alignment (or lack thereof) with a local or global environmental reference frames, and the number of frames which must be aligned, can influence the difficulty of many spatial tasks.

2.3.2 Application of spatial cognition research to the use of right-hand rules

Klatzky and Wu (2008) identified several reference frames that must be aligned in order to appropriately use the right-hand rule–those defined by the vectors, by the page, and by the hand relative to the body. From this perspective of coordinating reference frames, Klatzky and Wu also provided a task analysis for determining the direction of the cross product of two vectors $\vec{U} \times \vec{V}$:

> "The performer must (a) create a reference frame in which $\vec{U}$ and $\vec{V}$ lie on a common plane at an arbitrary location in self-defined space, (b) treat the hand

> as an external object within the vector-defined frame and map hand-centered coordinates into that frame so that the wrist lies at the vector intersection, the hand's cross-section is aligned with $\vec{U}$, and $\vec{V}$ is closer to the palm than the back of the hand; (c) observe the direction of the thumb in space and align it with the third dimension in the vector-defined coordinate system." (Klatzky and Wu, 2008, p. 171)

The primary difficulties with this task are in creating the frame from the vectors and in getting one's hand into the appropriate position in that frame. Klatzky and Wu (2008) argued the spatial reasoning required for some of the more intricate postures is correspondingly complex and that these unusual positions could increase the cognitive difficulty. They also suggested several other possible difficulties: recognizing that the vectors form a plane; the need for parallel transport if the vectors are not already presented with the tails together (*c.f* Section 2.2); understanding that it is a binary distinction (into or out of the plane formed by the vectors); recognizing that the magnitude of the vectors does not impact the direction; and possibly confusing the cross product in the plane defined by the vectors with its orientation relative to the egocentric frame.

While the right-hand rule is not properly a mental rotation or the perception of an image, many of the same principles apply. According to the research on alignment, the orientation of the initial vectors is likely to have an effect on the difficulty of using a right-hand rule. One would expect that a right-hand rule with vectors in the xy-plane would be easier than those in the xz-plane or the yz-plane. Also, if the vectors are aligned with the principle axes of the page (the local environmental reference system), a right-hand rule would be easier than if they are off-axis. Additionally, we must account for individual differences in spatial ability (Hegarty and Waller, 2005). These differences can have a significant impact on performance on tasks that require spatial reasoning (Kozhevnikov et al., 2002). Thus, one would expect that those with higher spatial ability would perform better with the right-hand rule, while still acknowledging the cognitive costs of alignment. By administering a test of spatial ability, such as the Cube Comparison Test (Ekstrom et al., 1976) or the Shepard-Metzler Mental Rotations Task (Shepard and Metzler, 1971), this relationship can be explored.

As mentioned in the discussion of right-hand rules in Section 1.2, a revised set of instructions for the right-hand rule based on this task analysis(*c.f.* Figure 1.5) significantly improved performance on an in-class quiz (Klatzky and Wu, 2008). Based on the success of this instruction, it is clear that the issues related to alignment do have an impact on student ability to correctly perform right-hand rules. However, the lack of details in this study make it unclear which of the issues addressed was responsible for the improvement. Additionally, in individual interviews with students solving magnetic force problems, Scaife and Heckler (2010) did not see any impact due to variations in vector orientation.

As noted above, there are several possible issues associated with alignment: physical discomfort, the plane of the given vectors, and the alignment (or mis-alignment) of the given vectors with the local reference frame. By addressing these issues separately, this study aims to resolve the differences between the results of Klatzky and Wu (2008) and Scaife and Heckler (2010).

Most of the studies discussed in this section make the assumption that the process of alignment comes with a measurable cost: the more mis-aligned the frames, the less accurate the response and the longer the response time. Thus, the major dependent variables were response time (or latency) and error (usually measured in angles from the direction of the correct response). This assumption was validated, with the exception of a speed-accuracy tradeoff noted in Pani et al. (1996). Since the right-hand rule requires a process of alignment, this study has also used (and validated) the assumption that accuracy and response time are correlated measures of difficulty (*c.f.* Section 4.2).

While the use of response time and accuracy are important tools when comparing the cognitive difficulty of tasks, it is also important to gain insight into the reasoning that students employ while trying to solve cross product problems. One way to do this is by using a think-aloud protocol and another is by looking at how the participants use their hands. Alibali (2005) presented a review of the literature addressing the roles that gestures play in three spatial cognitive activities: expressing, communicating about, and thinking about spatial information. Although it is not strictly a gesture, the physical nature of the right-hand rule serves some of the same purposes.

In particular, there is also evidence that the use of gestures influences the gesturer's thought processes and mental representations about spatial information: by "off-loading" some thought to one's hands in order to free up their working memory (Scherr, 2008); and by maintaining spatial information in memory, allowing one to remember spatial information for longer (Alibali, 2005). Both of these gestural functions are consistent with the role of right-hand rules as mnemonics to remember the spatial relationships among various vector quantities.

One possible mechanism for the effects of gestures on spatial thinking is the kinesthetic feedback from the actions of the body Scherr (2008). However, Klatzky et al. (1998) and Avraamides et al. (2004) found evidence that a kinesthetic response for task requiring alignment is more difficult than a verbal response. Participants in these studies were asked to imagine walking a path which contained a turn (Turn 1). Then, they were instructed to turn to face home as if they had actually performed the walk. When this response was a physical turn, participants turned as if they had never made Turn 1. However, this error disappeared when the response was a verbal description of how many degrees they would turn. This implies that the kinesthetic response made the task more difficult. However, when participants actually performed the walk or physically made the first turn (instead of just imagining it), this also eliminated the error. With cross product problems, the response usually involves relating the result from the right-hand rule to the local coordinate system, which could cause additional alignment issues. Thus, it will be important to distinguish the gestural response from the verbal response.

2.4 Synthesis of previous research

As noted in previous sections, educators and researchers assume that student difficulties with cross product direction is due in part to an inability to appropriately use right-hand rules; and there is speculation about the possible causes of this difficulty. Yet, there is no research that has explored this issue in depth. In addition, the literature presented here indicates that there are several ways in which the use of right-hand rules is likely to be both context-dependent and representation-dependent. These dependencies can be explored by varying the features of the tasks given to students. By combining the literature from these different domains, several categories of features emerge as likely to impact performance on cross product direction questions. These categories and features are described below and presented in Figure 2.4. Since these features were used to design the present study, they will be described in more detail in Chapter 3.

Kinds of questions

The literature presented here indicates that students struggle with cross product direction more when it is in a physics context than when it is in a non-physics context (*e.g.* Van Deventer, 2008). Additionally, all of the studies studies discussed here only gave questions that required working forward from the given vectors to the cross product. However, some physics texts (*e.g.* Chabay and Sherwood, 2009) also ask students to reason backward from the cross product to one of the initial vectors–a task which is usually considered more difficult. Thus, the type of reasoning required is another way in which the kind of question could influence performance on cross product direction questions.

Orientation

As noted above (Section 2.3), there are primarily three ways in which the orientation of the vectors is likely to influence the difficulty of using a right-hand rule: physical discomfort, the plane of the given vectors, and the angle between the vectors (*e.g.* Klatzky and Wu, 2008). By addressing these aspects of orientation as separate features, the relative impact of each can be evaluated. Additionally, orientation issues could also be related to individual differences in spatial ability. One would expect that those who have higher spatial ability are less likely to be affected by differences in orientation.

Parallel transport

Much of this literature suggests that questions that require parallel transport are more difficult than those that do not (*e.g.* Hawkins et al., 2009; Nguyen and Meltzer, 2003). There are two ways in which parallel transport would be necessary to perform a right-hand rule: when the vectors are not tail-to-tail and when the vectors are separate in space.

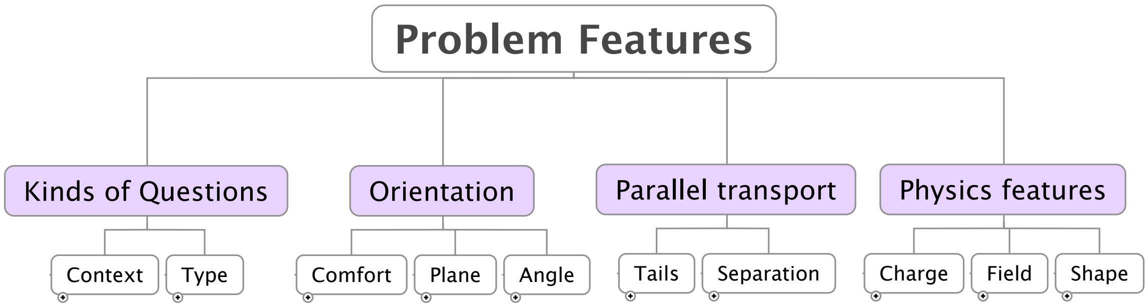


Figure 2.4: Categories and features used as the basis for the study design

Physics features

Based on this literature and conventional wisdom, there are certain aspects of the physical situation that one would expect to impact performance on cross product questions in magnetism contexts. The most obvious is the impact of the sign of the charge on the cross product resultant. The necessity of reversing the direction of the cross product for a negative charge adds an additional layer of difficulty. Additionally, there is evidence that performance on magnetic force questions is dependent on the representation of the magnetic field (Scaife and Heckler, 2010). Finally, for magnetic field questions, there are many different right-hand rules for different current distributions or shapes (*e.g.* a straight wire versus a loop of wire). Each of these physics features–charge, field representation, and current shape–could have a significant impact on performance.

The current study was designed to explore how all of these features–kind of questions, orientation, parallel transport, and physics features–impact student performance on cross product direction questions and the use of right-hand rules in order to provide a foundation to help us reassess our instruction.

Chapter 3

Methodology

This chapter will describe the study design, including the participants (Section 3.1), the tasks (Section 3.2), and an overview of the analysis (Section 3.3).

3.1 Participants

3.1.1 Pilot study

During the fall of 2009, a pilot study was conducted with two groups of students in order to gather data to inform the main study. One group was composed of four students from a second-semester, electricity and magnetism, (E&M) introductory physics course. Each student participated in a one-hour interview within a week of completing the magnetic force unit in class. The other group was composed of ten students from three different sections of a first-semester mechanics introductory physics course. These students were interviewed just after the unit on rotational dynamics. Compensation included $15.00 and the offer of tutoring. While an in depth analysis of the pilot data was not conducted, responses of the students in each group was examined in order to choose a group of students that would most benefit the main study. Each group of students, E&M and Mechanics, demonstrated a behavior that influenced the selection of participants for the main study.

Physics questions given to the second-semester students covered both the generation of a magnetic field from moving point charges (referred to hereafter as "magnetic field" questions) and the magnetic force on moving charge(s) due to an external magnetic field (referred to hereafter as "magnetic force" questions). The sequence of questions alternated between both magnetic field and magnetic force questions. Three of the four E&M students struggled with the magnetic field problems at the beginning of the interview. One continued to struggle on these problems through out the test, while the other two remembered or "relearnt" the magnetic field problems as they progressed. All three made reference to "not having a force"–indicating that their recent work on magnetic force was foremost in their minds. This is similar to the interference between dot products and cross products noted in Sayre and Heckler (2009). To address this issue in the main study (Section 3.1.2), two different groups were interviewed, one after the unit on magnetic fields and the other after the unit on magnetic force. One would expect that even with interference between magnetic field and magnetic force, the second group would perform better overall since they have had more practice with cross products.

First-semester students encountered a separate difficulty during the interview. All ten attempted to use some kind of right-hand rule on the physics questions, but when they were given cross product questions without a physics context, four of the ten participants reverted to methods consistent with vector addition and/or subtraction (described in Van Deventer, 2008). This indicated that they were not familiar enough with cross products to recognize (outside of a physics context) that they could/should use a right-hand rule. Additionally, the number of students attempting to use vector addition or subtraction methods was significantly less than in the pilot study. Since the focus of this work is on student difficulties with right-hand rules, the main study (Section 3.1.2) drew students only from the second-semester and excluded first-semester students in order to avoid bias from the lack of knowledge about cross products.

Both of these issues–the relearning of magnetic fields and attempting to use vector addition and subtraction methods in a non-physics context–could prove to be valuable avenues of future work (Section 7.3).

3.1.2 Main study

During the Spring 2010 Semester, participants were recruited from three sections of the second semester introductory physics course on electricity and magnetism. Each section was taught by a different professor, but all used the third edition of the *Matter and Interactions* textbook (Chabay and Sherwood, 2009) and were taught in an interactive lecture format with studio-style labs taught by graduate students. Recruitment emphasized that not only would students be compensated monetarily ($15.00), but tutoring would also be provided. This was done in an effort to ensure that the study included at least some of the students who were struggling with this material. At the beginning and end of each interview, each participant was asked about his/her confidence and comfort with using the right-hand rule in order to identify those that had selected themselves for assistance.

Fifteen students were interviewed within two weeks of finishing the chapter on magnetic field in class. Two of these participated in the pilot study during their mechanics course. After finishing the chapter on magnetic force, an additional twelve students were interviewed, one of whom had participated in the pilot study. Each participant selected a pseudonym for himself or herself from a list of possibilities and was referenced by that pseudonym in all subsequent data collection and analysis.

3.2 Tasks

There were two major tasks given to the students who participated in the main study. The first task was the Cube Comparison Test, which was used to provide a measure of spatial ability. The second task involved a think-aloud protocol for a set of cross product direction questions in physics and non-physics contexts. Every interview was video- and audio-recorded from two different views (an overhead view as well as a side view) that were later synced and combined. These video recordings, along with the participants' written work on the Cube Comparison Test, comprise the sources of data for this study.

Section 3.2.1 will discuss each of these tasks in the context of the interview, including differences between the pilot study and main study. Section 3.2.2 will address the unexpected gender difference in the Cube Comparison Test scores that was mentioned above and section 3.2.3 will describe the problem features that were varied in the cross product questions.

3.2.1 Interview procedure

Cube Comparison Test

As mentioned in Section 2.3, individual differences in spatial ability could account for some of the variance in student performance on right-hand rule problems. Therefore, it is helpful to have a measure of that ability to indicate the degree to which individual differences in spatial ability account for performance on right-hand rule questions. There were several criteria used to determine an appropriate task:

- established reliability and validity,
- appropriate spread to observe individual differences,
- assesses ability to do mental rotations in 3D instead of 2D,
- less than 15 minutes to allow time for primary tasks, and
- administered in the same format as primary tasks (*i.e.* paper and pencil).

The Cube Comparison Test (Ekstrom et al., 1976) fulfilled all of these requirements and thus, at the beginning of the interview, each participant took this test to provide a baseline for their ability to perform mental rotations.

The Cube Comparison Test is a two-part timed test where the participant is given 3 minutes for each part. Each part is comprised of 21 questions. Each question consists of a diagram of two cubes with alpha-numeric symbols on the showing faces. The task is to identify by the symbols on the faces of the cubes whether the two cubes could be (1) the same cube rotated into a different orientation or (2) whether they must be different cubes (see Figure 3.1 for an example). The score for the test is calculated by taking the number answered correctly and subtracting the number answered incorrectly. Thus, scores can range from -42 to +42, although the mean for college students is typically around +21 (Ekstrom et al., 1976). The data from this study is consistent with this mean, as will be discussed in section 3.2.2).

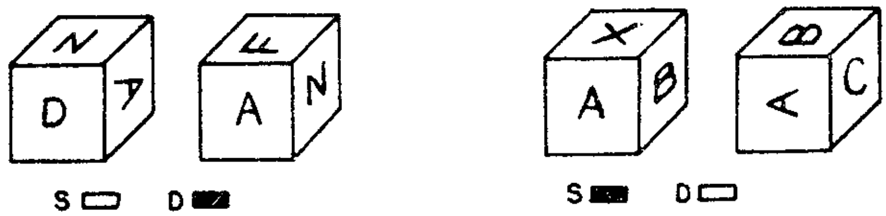


Figure 3.1: An example of a question from the Cube Comparison Test. The participant indicates whether the two blocks could be the same block (S) or if they must be different blocks (D). For the pair of cubes on the left the two blocks must be different cubes because the ‘N’ would be hidden if the right cube was rotated such that the ‘A’ was upright and facing you. However, the pair of cubes on the right could be the same cube because it maintains the orientation of ‘A’ and ‘B’ when it is rotated. (Ekstrom et al., 1976)

Cross product directions questions

Previous research (Chapter 2) has demonstrated that student performance on vector algebra questions is context-dependent(*e.g.* Van Deventer and Wittmann, 2007), as well as representation dependent(*e.g.* Scaife and Heckler, 2010). To address these issues, participants answered questions in two different physics contexts (magnetic field and magnetic force) and a non-physics context. Other problem features (Section 3.2.3) were varied within each of these contexts. The choices for which of the problem features to vary were anchored in published research and anecdotal evidence (*c.f.* Section 2.4).

In order to better understand what the students were thinking, the interviewer asked participants to ”think-aloud” while working on problems. After the Cube Comparison Test, the interviewer administered several warm-up exercises to familiarize the participant with this process. The interviewer also provided feedback after each exercise until the participant had reached a reasonable level of verbalization.

There were two groups of participants: one group was interviewed after the unit on magnetic fields and the other after the unit on magnetic force. The first group of participants (‘Magnetic Field’ group) only answered questions that dealt with magnetic field. The second group (‘Magnetic Force’ group) was given the same set of magnetic field questions and then an additional set of questions dealing with magnetic force, including two questions that required two steps of reasoning. For both groups, the interviewer asked, "What, if anything did you find difficult about this problem?" after each question. After each section, the interviewer occasionally asked the participant to redo problems where the participant's thinking was unclear.

Like the pilot study, after the physics questions were answered, all of the participants answered a set of non-physics cross product directions questions and were asked to do as many as possible in a certain amount of time. There were 50 problems that each asked "What is the direction of $\vec{A} \times \vec{B}$?" ('Working forward' problems) and 20 problems that assumed $\vec{C} = \vec{A} \times \vec{B}$ and asked for either $\vec{A}$ or $\vec{B}$ given the other two vectors ('Working backward' problems). The first group of problems was administered in two sets of 25 questions each and students were given 10 minutes for each set. For the second group, participants were given 6 minutes each of two sets of 10 questions. These changes were made in an effort to ease participant fatigue.

Follow-up questions

After answering the cross product direction questions, the interviewer asked a series of follow up questions:

- Describe the right-hand rule(s) that you use

  Responses to this question were used to create the codes for right-hand rule types (*c.f.* Chapter 5).

- How does the direction of $\vec{A} \times \vec{B}$ compare to the direction of $\vec{B} \times \vec{A}$?

  Many students reverse the order of vectors when performing a right-hand rule; this question was used to determine whether they were aware of the non-commutative nature of the cross product.

- On a scale of one to ten, what is your comfort level or confidence with using the right-hand rule?

  Responses to this question could be compared to the responses to the same question asked at the beginning of the interview.

- Is there anything else that you found difficult about these problems that you have not already mentioned?

  A few of the participants took this opportunity to expound on the issues that they felt they had with using right-hand rules.

Although not analyzed in detail, the responses to these questions were used to supplemental the qualitative analysis (*c.f.* Chapter 6).

3.2.2 Gender differences on the cube comparison test

The initial analysis of the scores on the Cube Comparison Test revealed a gender difference for the Magnetic Field group of students; the mean score for the women was statistically higher than the mean score for the men in that group ($p = 0.0246$ for two-sample t test). Most research on mental rotations shows no difference between gender or shows males outperforming females (Linn and Petersen, 1985). For the Magnetic Force group of students, the difference is in the same direction, but is not statistically significant ($p = 0.233$). This indicates that there may be a self-selection gender bias for those who chose to participate in the study.

In order to ascertain whether this was an artifact of the sample or an artifact of the course, the Cube Comparison Test was administered during almost all lab sections (it was not administered in one section due to scheduling difficulties). From these lab sections, 187 students volunteered to take the test. The only identifying data collected was gender, lecture instructor, and whether they had been interviewed previously for this study. Sixteen of the tests were eliminated: two students started early, one marked 'S' for all questions indicating that he did not take the test seriously, and thirteen came in for interviews and had taken the test already. Including the original scores of the 27 students who were interviewed and the additional 171 tests from lab, there was no statistical difference between the mean score for men and women ($p = 0.6613$). Figure 3.2 and Table 3.1 summarize these results. Although gender may be a factor in student use of right-hand rules, given the apparent self-selection bias, this study will not address the relationship between gender and performance on cross product direction questions.

When gender is ignored (see Figure 3.3 and Table 3.2), there was no statistical difference between the mean score for the class as a whole and the mean score for each group of

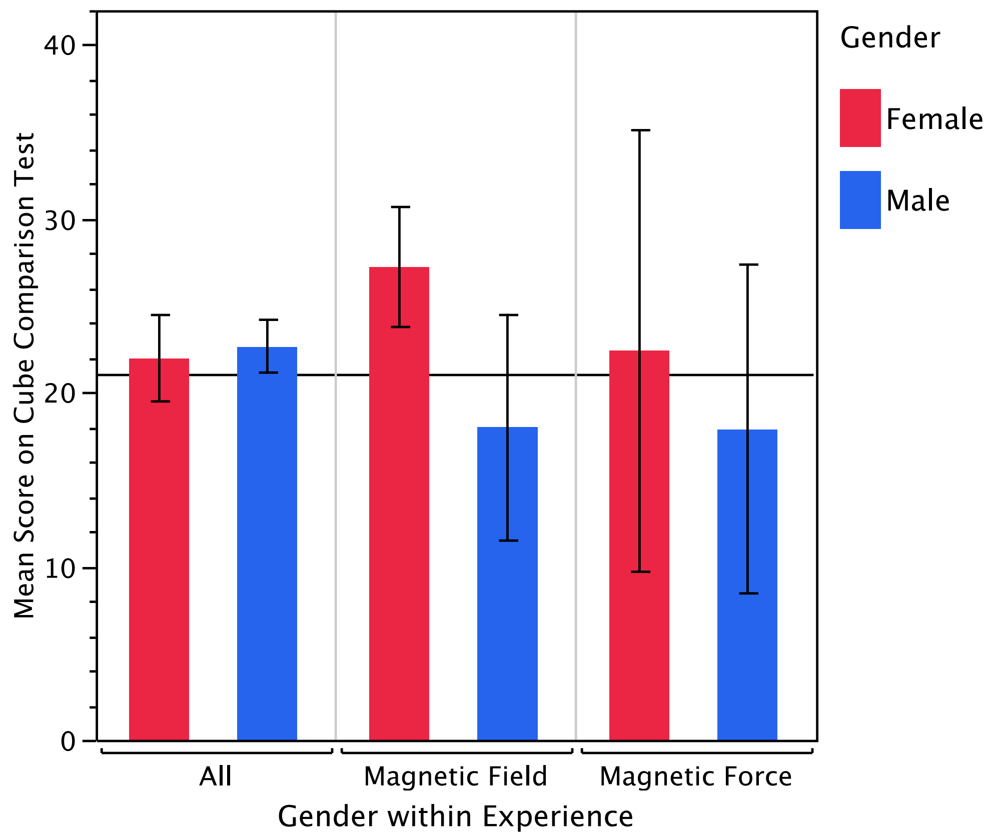


Figure 3.2: Mean score on Cube Comparison Test by Gender for the Magnetic Field group, for the Magnetic Force group and for the class as a whole (All). Error bars were constructed using 95% Confidence Intervals.

interviewed students. Therefore, this study will use these two groups to begin to explore how performance is related to spatial ability ('Spatial') and at what point the students came in for the interview ('Experience'). However, the small sample size and gender bias of this study suggests that a logical extension of this study would be to further explore how these, and other elements of student background, impact the use and understanding of cross products and right-hand rules.

Table 3.1: Cube Comparison Test score by gender for the Magnetic Field group, for the Magnetic Force group and for the class as a whole (All).

	Males			Females			Differences	
	N	Mean	SD	N	Mean	SD	M-F	p value
Magnetic Field group	10	18.0	2.45	5	27.2	3.46	- 9.2	0.0128
Magnetic Force group	7	17.9	3.87	5	22.4	4.58	-4.5	0.469
Whole Class	152	22.6	0.758	46	22.0	1.38	0.6	0.659

Table 3.2: Oneway analysis of variance (ANOVA) of Cube Comparison Test score for the Magnetic Field group, for the Magnetic Force group and for the class as a whole (All).

	N	Mean	SD
Magnetic Field group	15	21.1	2.41
Magnetic Force group	12	19.8	2.69
Whole Class	198	22.5	0.66
p value	0.549		

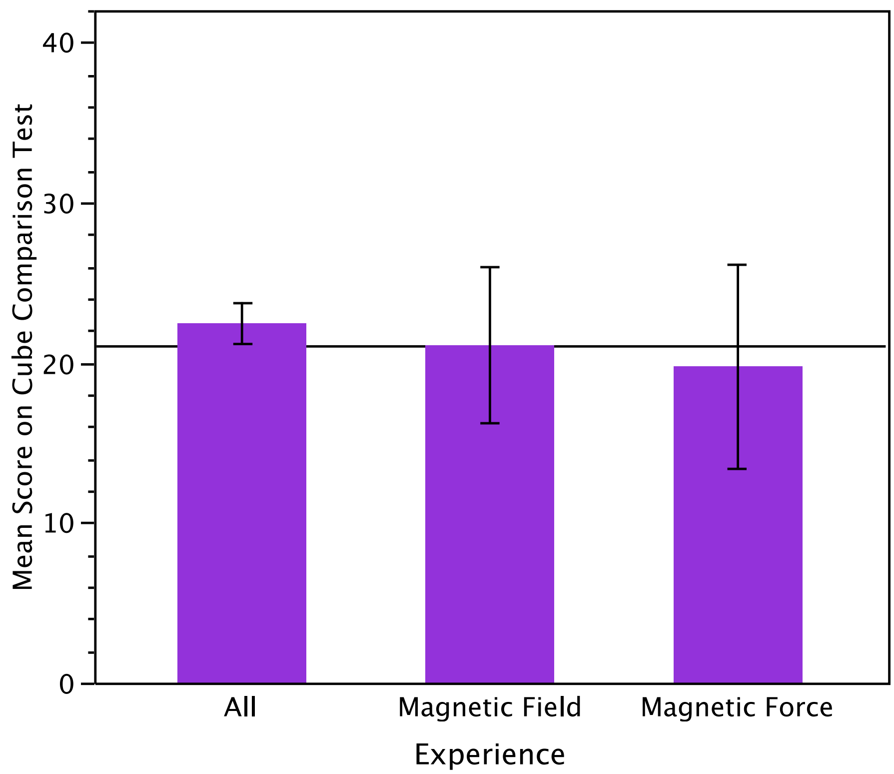


Figure 3.3: Mean score on Cube Comparison Test for the Magnetic Field group, for the Magnetic Force group and for the class as a whole (All). Error bars were constructed using 95% Confidence Intervals.

3.2.3 Problem features

This section will provide a description of the problem features that were varied, as well as which features were varied in which contexts. The justification for the choice to vary these problem features was discussed in Section 2.4. Figure 3.4 provides an overview of all of the problem features and Appendix A contains the complete set of problems in the order they were given to the students.

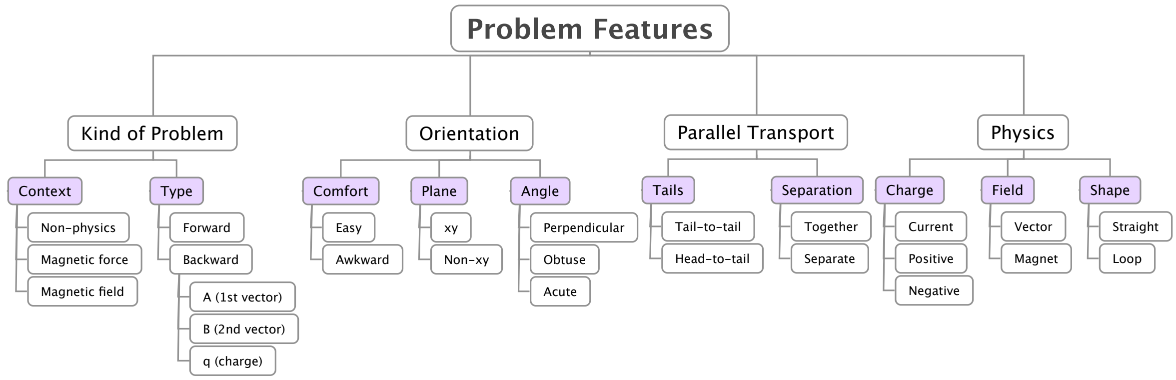


Figure 3.4: Overview of all of the problem features

Kinds of questions: Context and Type

As mentioned above, questions were given in three different contexts: Magnetic Field, Magnetic Force, and Non-physics. In each context, there were questions that involved two types of reasoning: Forward and Backward. Forward reasoning involved determining the cross product from the initial vectors, while Backward reasoning questions required finding one of the initial vectors given the cross product and the other initial vector. Additionally, there were three different kinds of Backward problems: those that asked for the first vector ($\vec{A}$), those that asked for the second vector ($\vec{A}$), and in the Magnetic Field context, those that asked for the sign of the charge. Figure 3.5 shows examples of Forward and Backward problems for all three contexts.

Orientation: Comfort, Plane, and Angle

As discussed in Section 2.4 there are three ways that the orientation of the vectors could contribute to difficulty: physical discomfort, the plane of the vectors and the angle between the vectors. There are 24 different orientations of perpendicular, on-axis vectors in canonical coordinate space $(\hat{x}, \hat{y}, \hat{z})$. In order to restrict the space and to quantify the level of physical discomfort for various orientations, all combinations of perpendicular, on-axis cross products ($\vec{A} \times \vec{B}$) were given to three physics graduate students and they were asked to rate them on a 5-point scale, with 1 being physically easy and 5 being physically awkward. The rankings were averaged and the five orientations with the highest

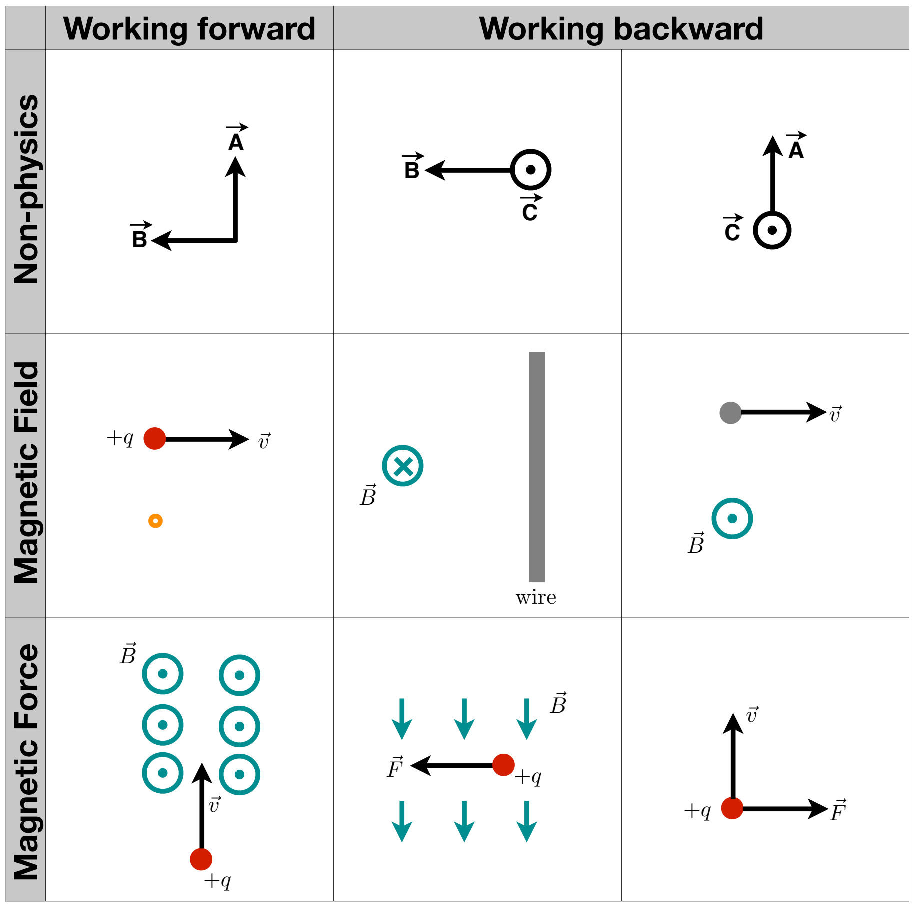


Figure 3.5: Examples of non-physics, magnetic field, and magnetic force questions when the problem involves working forward and working backward.

rank and the five orientations with the lowest rank were identified (Figure 3.6). These ten orientations were then used as a basis set for all questions.

It is important to note that whether an orientation is Easy or Awkward is dependent on both the method used and the errors made. Different versions of the right-hand rule require different hand positions and there are additional complications for physics questions, such as whether one flips for an electron at the beginning or end of the problem. There are also numerous errors, such as reversing the order of the cross product ($\vec{B} \times \vec{A}$ instead of $\vec{A} \times \vec{B}$), that change the awkwardness of the problem. However, using these ten orientations restricted the space to a more manageable number of orientations to explore. This allows for a comparison, to a first-order approximation, of the differences in performance on physically easy and physically awkward problems (*c.f.* Chapter 4). An in-depth look at methods and errors addressed the more subtle differences (*c.f.* Chapters 5 and 6).

Among the ten orientations identified as Easy or Awkward, there were physically easy orientations in the xy and yz planes and physically awkward orientations in the xy, yz, and xz planes. Thus, combining the yz and xz orientations into "non-xy" orientations allows for a comparison of the relative effect of plane and physical discomfort on performance, as show in Figure 3.7.

In order to limit the interview to one hour, the angle between the vectors was only varied for the Non-physics Forward questions. Also, in order to keep the orientations similar to those that students might see in physics problems, only orientations in xy plane were varied with respect to angle. There was a mix of Acute and Obtuse angles to compare to the majority of problems which were Perpendicular (see Figure 3.7).

Parallel Transport: Tails and Separation

In order to assess the degree to which the need for parallel transport was contributing to student difficulty, the non-physics problems were also varied in terms of the representations of the vectors in space. This included whether the vectors were presented as Tail-to-tail or Head-to-tail as well as whether they were Together or Separate in space. Figure 3.8 shows examples of the various possible combinations of these two features for

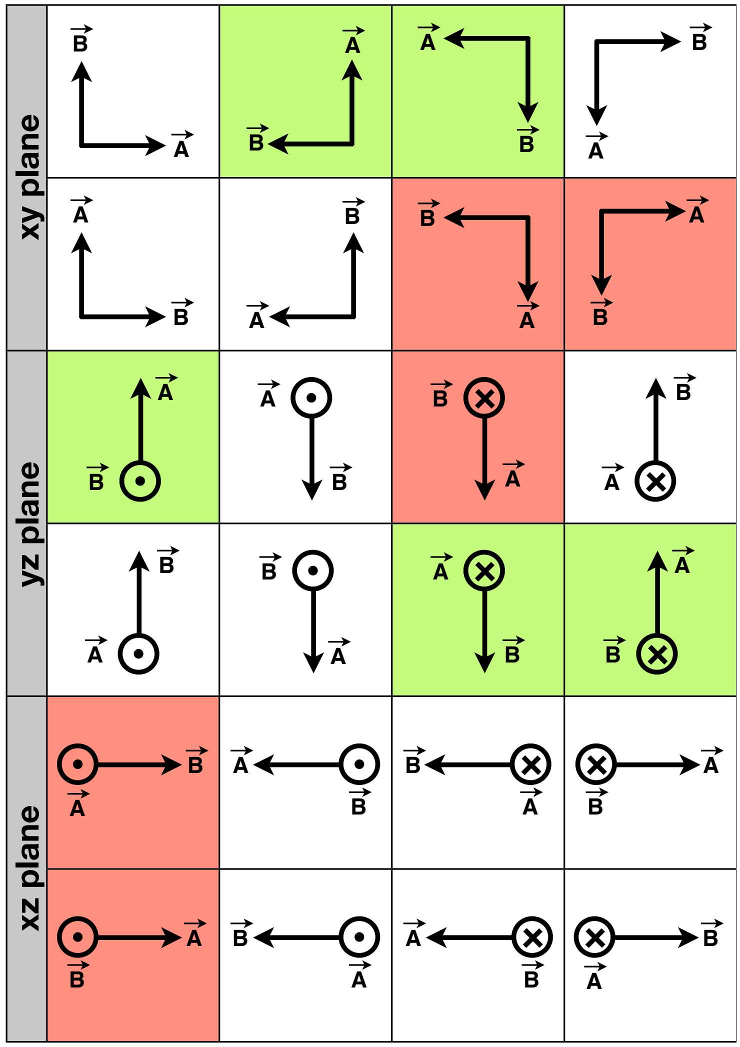


Figure 3.6: All possible cross product combinations for on-axis, perpendicular vectors ($\vec{A} \times \vec{B}$). The green orientations were the five orientations with the lowest average ranking for physical discomfort and the red orientations were those with the highest average ranking for physical discomfort.

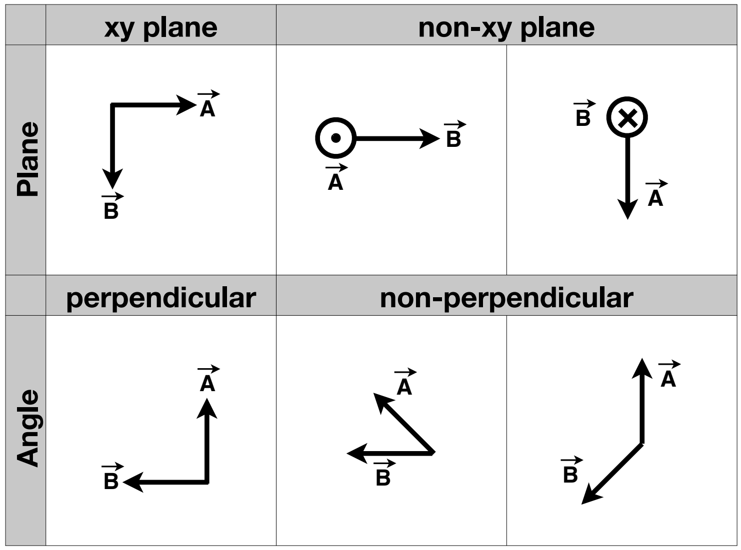


Figure 3.7: The first row shows examples of physically awkward orientations in the xy plane and non-xy plane. The second row shows examples of one orientation with the vectors presented as perpendicular and non-perpendicular (with both acute and obtuse angles).

one orientation. The non-physics problems included all of these combinations. However, the physics problems included only those that were consistent with how these problems are typically presented. Additionally, for the Magnetic Field problems, the second vector was not provided explicitly (*e.g.* the observation location was indicated, but the position vector was not).

Physics features: Charge, Shape, and Field

The three main physics features–the sign of the charge, the shape of the current-carrying wire, and the representation of the magnetic field–were discussed in Section 2.4. The variations in charge included a positive point charge (Positive), a negative point charge (Negative), and a current-carrying wire with the direction of conventional current indicated (Current). For the Magnetic Field problems, the current-carrying wires were either Straight or a Loop. The Magnetic Force problems included problems that represented the magnetic fields using either Vectors or Magnets, analogous to the field lines and magnetic poles in (Scaife and Heckler, 2010). Figure 3.9 shows examples of each feature.

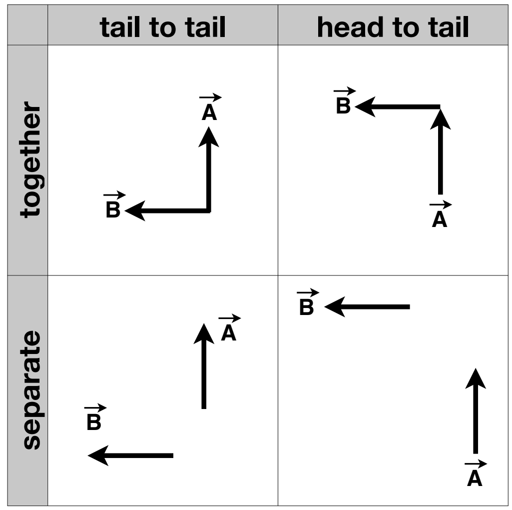


Figure 3.8: One cross product orientation with the vectors Head-to-tail, Tail-to-tail, Together, and Separate.

3.3 Overview of analysis

The analysis will be discussed in detail in the following chapters. This section will present a brief overview of the analysis. Due to the nature of the data, this analysis will include a quantitative and a qualitative component that will inform each other. For the sake of clarity, the qualitative and quantitative aspects will be discussed separately; so there will be some redundancy.

3.3.1 Quantitative analysis: response time and correctness

The primary goal of this study was to determine the relative impact that the features discussed above have on student performance on cross product direction questions. Two ways of measuring performance are response time and accuracy. Thus, Chapter 4 will present a regression analysis for each of these measures using problem features and select participant characteristics (Spatial and Experience) as predictors. The hypotheses for how these contrasts will impact performance are drawn from published research, as well as conventional wisdom and will be presented in detail in Chapter 4.

There are several aspects of this data that restrict the generalizability of a quantitative analysis. The most obvious is the small number of participants and the lack of background information available. Also, as noted in 3.2.2, there was a gender difference in spatial ability that indicated a self-selection bias for the participants. The only background characteristics that used for this analysis were spatial ability (as measured by the Cube Comparison Test) and experience (based on the point during the semester at which the participant was interviewed). Since there are other aspects of participant background that may be of interest, a more in-depth study of the impact of participant background should be explored in the future (*c.f* Section 7.3).

While the number of participants was small, the total number of problems was large enough to perform regression analysis. However, there were limits here as well. All of the problem features could not be varied the same way across all contexts (*e.g. physics features*), to do so would have lengthened the task to an unreasonable length(*e.g.* angle). Thus, the regression analysis could not include all problem features.

The large number of problems also raises the possibility of the data being skewed because of improvement throughout the task (practice effects). In order to isolate these practice effects from the impact of the problem features, the problems within each context were presented in a random order and the problem order was used as a predictor for regression. While the intention was to give each participant the questions in the same randomized order, there were two instances where one of the physics problems was given in a slightly different place (Barry and Lorenzo) and two instances when the second set of non-physics backward problems was given in the reverse order (Humberto and Tanya). Additionally, one student (Nestor) did not complete all of the non-physics problems due to time constraints. These anomalies create additional difficulties for generalizability.

A quantitative analysis of this kind does not give any information as to the methods the students used or the sources of error. This lack of resolution was one of the drawbacks of most of the previous studies. This lack of detail presents additional difficulty because many of the hypotheses are based on the assumption that participants both understand the task and are actually using a right-hand rule. The qualitative analysis revealed that this was not always the case. To address this issue, the qualitative analysis was used to restrict the data for the quantitative analysis to only the appropriate problems.

This method of quantitative analysis was able to paint an picture of trends and patterns, as well as provide an estimate of the relative impact of problem features on student performance. This overall picture then informed and guided the qualitative analysis (Chapters 5 and 6).

3.3.2 Qualitative analysis: methods and errors

There is a wealth of information that is not accessible through a quantitative analysis and much of this information can be gathered through a qualitative analysis of participant behavior. There were two key aspects to this behavior that had a bearing on the goals of this study: the methods that participants use to solve these problems and the errors that they make while solving them.

In order to analyze these behaviors, two sets of codes were created, revised, and validated using inter-rater reliability. The initial set of codes were derived using content logs for interviews from both the pilot study and the main study. These codes were anchored in published research, but were revised by a constant comparison with the data to determine if the codes fit the data. Once this revision process reached a point where it appeared that the codes were an accurate reflection of the data, a second coder was brought in to determine the reliability of the code definitions. This process led to further revisions to clarify definitions. The inter-rater reliability of the final codes was established using Cohen's kappa. The details of the development for each coding scheme, as well as the final definitions, will be discussed in Chapter 5.

The codes provide one of the most important outcomes of this analysis. Every physics teacher has his/her own stories/opinions of what students do and why. This intuition has often been built up over many years, but without the data to support it, this "wisdom" remains merely anecdotal. These codes provide concrete evidence of many of the issues. In addition, by using the quantitative analysis as a guide, these codes shed light on the subtleties of how problem features impact student behavior. This is particularly appropriate for the physics features that could not be adequately included in the quantitative analysis.

However, just as there were constraints on the quantitative analysis, there were also constraints for the qualitative analysis. Although it is a small number of participants for a quantitative analysis, it is a fairly large number of participants for a qualitative study and so, the quantitative analysis was necessary to focus the qualitative analysis on the most relevant issues.

3.3.3 Summary

While there are difficulties in implementing a mixed-method approach, the complementary nature of the quantitative and qualitative analyses best addresses the goals of this research and provides new insight into questions that have arisen in the study of cross product use. The depth of the qualitative data was necessary to provide the resolution that is missing in previous studies. Exploring the impacts of representation and context-dependence with a broader view through quantitative analysis provides a solid basis for conclusions drawn about right-hand rule usage. The detail from the qualitative analysis and the global patterns from the quantitative analysis are necessary in order to provide the foundation to reassess instruction of cross products and magnetism.

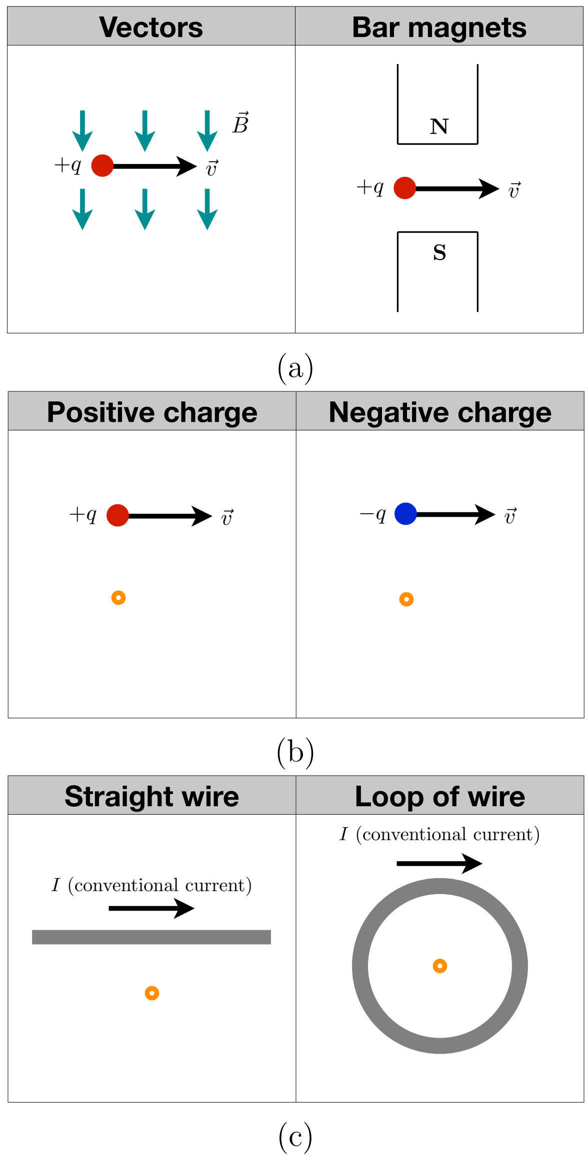


Figure 3.9: Examples of the variation in physics features: (a) the sign of the charge, (c) the shape of the current-carrying wire, and (c) the representation of magnetic field.

Chapter 4

Quantitative analysis: results and discussion

There are many problem features that could contribute to difficulty with right-hand rules in introductory physics. The primary goal of the present study is to compare the impact of these features on performance. One way to do this is to use these features as predictors in regression analysis, using accuracy and response time as measures of performance. This chapter will present and discuss the results of this analysis.

Section 4.1 will present an overview of the regression techniques used–how response time and accuracy were measured, justification for which problems were included, and overall results. These results will then be explored more fully in Section 4.2, using the categories presented in Section 2.4

- Background characteristics and practice effects: Spatial, Experience, Order (4.2.1)
- Kinds of questions: Context and Type (4.2.2)
- Orientation: Comfort, Plane, Angle (4.2.3)
- Parallel transport: Tails, Separation (4.2.4)
- Physics features: Charge, Field Representation (4.2.5)

Following these results, a summary and discussion of this analysis using the same categories will be presented in Section 4.3.

4.1 Response time and accuracy

Most spatial cognition studies (*e.g.* Avraamides et al., 2004) measure accuracy as the angular difference between the correct and the given response; thus, accuracy is a continuous variable. For this study, a categorical response was necessary, where the final response to the problem was categorized as Correct, Incorrect, or No Response (if the participant did not provide a final response). By eliminating the problems coded as No Response ($N = 33$), accuracy was reduced to a binary response (Correctness) and logistic regression was used with correctness as the dependent variable. One of the primary assumptions of this analysis is that response time and accuracy are correlated measures of difficulty. This assumption has been validated in much of the spatial cognition literature that forms the basis of this study. In order to validate this assumption, response time was used as a predictor in a logistic regression for correctness.

Unlike correctness, response time is a continuous variable, which allows for a standard least squares regression analysis. Using the logarithm of response time, Log (T), as the dependent variable instead of the raw response time provides a more normal distribution and a wider spread that is easier to interpret. Thus, any reference to time as a variable refers to the logarithm of response time. As an additional check on the assumption mentioned above, correctness was used as a predictor in the response time regression.

All of the data was video-recorded and the response time (measured in seconds) was determined using time stamps. This was accomplished by subtracting the begin time from the end time for each problem. However, determining the begin time and end time for each problem required some consideration. The begin time was operationally defined as the moment the participant first saw the question. This was done to address the fact that some participants would read the entire problem statement before trying to solve it, while others would jump into the problem and refer back to the problem statement only as needed. This definition of begin time could indicate an inflation of the response time for the physics questions compared to the non-physics questions due to the amount of additional reading; this inflation should be considered when evaluating results (see Section 4.2.2). The operational definition for the end time of a problem was the first instance of the participant's final response. This was to account for two issues: changing his/her response throughout the problem and reiterating the response once s/he had found it.

Both regressions used the features discussed above as predictors, or contrasts. Hypotheses for how each of these contrasts would impact response time and accuracy were drawn from published research, as well as conventional wisdom and are presented in Table 4.1 (including references to where each hypothesis is discussed). The hypotheses were predicated on the assumption that the right-hand rule was the method of problem solving. However, as will be discussed in Chapter 5, this was not necessarily the case for all problems. Using the qualitative coding of methods discussed in that chapter, any problem where a right-hand rule was not used was eliminated, as were the problems where a primary method other than a right-hand rule was used. The problems involving two reasoning steps were not included in the regression. After all of these problems had been eliminated, $N = 2119$ problems remained to be used for regression.

The goal of this quantitative analysis was to describe the overall patterns of the data and to verify the hypotheses about the impact of each contrast on performance. Thus, the regression models only considered main effects. Several features were not considered in the regression analysis, None of the physics features (Charge, Field, and Shape) were included since they were only present on the physics problems. Also, since Angle was only varied for non-physics, working-forward problems in the xy-plane, it was also not included in the regression models. However, an analysis of variance (ANOVA) for response time and a χ^2 test for correctness were conducted for each of these features for the relevant problems. The results of these tests will be discussed in the appropriate sections below.

4.1.1 Least squares regression for response time

The least squares regression for response time is presented in Table 4.2, which includes model level statistics and parameter estimates. This model accounted for 71% of the variance in response time. Although it is difficult to strictly interpret significance due to the limitations of the data, the results show that all of the main effects impacted the response time in a direction consistent with the hypotheses and all were significant at the $p < 0.05$ level except for Tails.

Table 4.1: Hypothesized impact of contrasts on performance, where better performance is operationally defined as faster response time and greater correctness.

Contrast	Hypothesis of effect of contrast on performance
Spatial	Spatial ability will be positively correlated with correctness and negatively correlated with response time (Section 2.3.2).
Order	The order of the questions will be positively correlated with correctness and negatively correlated with response time due to practice effects (Section 4.2.1).
Experience	Participants with more experience will perform better than participants with less experience (Section 3.1.1).
Context	Participants will perform better on non-physics questions than on magnetic field or magnetic force questions (Section 2.2.3).
Type	Participants will perform better on questions that ask for the cross product result (forward reasoning) than on those that ask for one of the given vectors (backward reasoning) (Section 2.4).
Comfort	Participants will perform better on questions with physically easy orientations than on those with awkward orientations (Section 2.3.2).
Plane	Participants will perform better on questions in the xy-plane than on those in either an xz-plane or yz-plane (non-xy) (Section 2.3.2).
Angle	Participants will perform better on questions with perpendicular vectors than on those with acute or obtuse vectors (Section 2.3.2).
Tails	Participants will perform better on questions with vectors presented tail-to-tail than on those presented head-to-tail (Section 2.4).
Separation	Participants will perform better on questions where the vectors are together than on those separate in space (Section 2.4).
Charge	Participants will perform better on questions with positive charge or current-carrying wire than on those with negative charge (Section 2.4).
Field	Participants will perform better on questions where the magnetic field is represented by vectors than on those with magnets (Section 2.1.2).

Table 4.2: Results for least squares regression for Log (T)

Model level statistics	
R^2 (adjusted)	0.7080
Observations (N)	2119
F statistic p value	0.0000

Term	Estimate	p value
Intercept	+3.5	0.0000
Correctness [correct]	−0.087	< 0.0001
Spatial	−0.013	< 0.0001
Order	−0.013	< 0.0001
Experience [magnetic field]	+0.10	< 0.0001
Context [magnetic field]	+0.50	< 0.0001
Context [magnetic force]	+0.48	< 0.0001
Type [backward]	+0.18	< 0.0001
Comfort [awkward]	+0.032	0.0060
Plane [non-xy]	+0.094	< 0.0001
Tails [head]	+0.012	0.4246
Separation [separate]	+0.032	0.0381

4.1.2 Logistic regression for correctness

The logistic regression for correctness is presented in Table 4.3, including model level statistics and odds ratios. For logistic regression, R^2 or U is the proportion of the uncertainty for the Whole Model Fit. For this model, there is only 12% uncertainty attributed to the fit. Odds ratios are included since parameter estimates are not as easily interpreted for logistic regression; odds > 1 indicate a positive impact on correctness and odds < 1 indicate a negative impact on correctness. For continuous contrasts, the unit odds represent the impact on correctness per unit change in the contrast, while the range odds indicate the effect on correctness per change in the contrast over the entire range. For categorical contrasts, the odds specify the probability of getting a correct response with the first level of the contrast over the probability of getting a correct response with the second level of the contrast.

Of the categorical values, only Context had more than two levels (non-physics, magnetic field, and magnetic force). Since there was no significant difference in correctness between magnetic field and magnetic force questions, these were collapsed into one category (physics) in order to reduce Context to a two-level code (non-physics and physics). The impact of Context on correctness will be explored in more detail in Section 4.2.2.

As with the least squares regression for the response time, the odds for all contrasts were in a direction consistent with the hypotheses. However, the impact of many of these contrasts was not as significant for correctness as it was for response time. These differences will be addressed in the following sections. Including these differences, the two regression models summarized above provide clear evidence that for these participants, performance on cross product direction questions is dependent on some key features of those problems.

4.2 Quantitative results by category

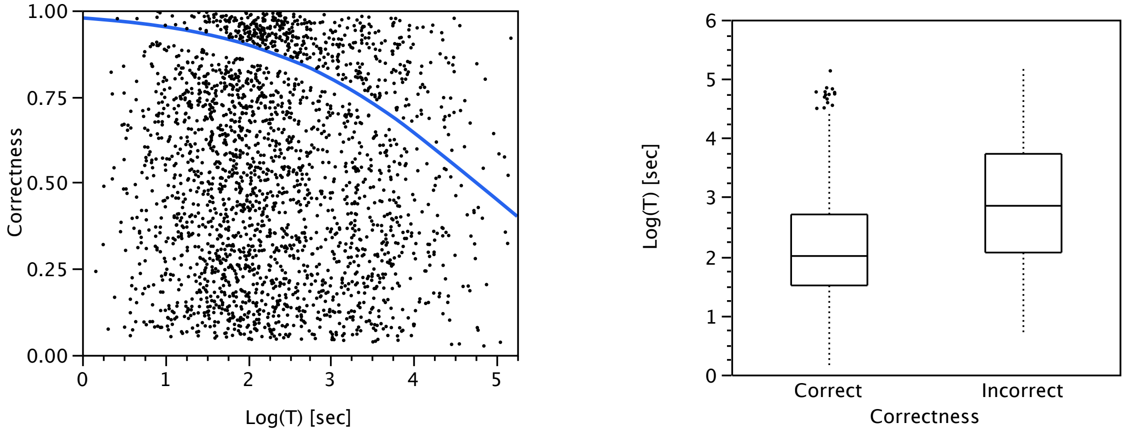
As mentioned above, one of the major assumptions of this analysis was that there would be a positive correlation between greater accuracy and faster response time. Figure 4.1.a shows the probability of obtaining a correct response as Log(T) varies and Figure 4.1.b

Table 4.3: Results for logistic regression for correctness. The odds are for correct versus incorrect, where odds > 1 indicate a positive impact on correctness and odds < 1 indicate a negative impact on correctness. For continuous contrasts, the unit odds are per unit change in contrast and the range odds are per change in contrast over entire range. For categorical contrasts, odds are for Level 1/Level 2.

Model level statistics	
R^2 (U)	0.1254
Observations (N)	2119
χ^2 statistic p value	< 0.0001

Continuous terms	Unit Odds	Range Odds	p value
Log (T)	0.53	0.042	< 0.0001
Spatial	1.0	4.6	< 0.0001
Order	1.0	1.3	0.46

Categorical terms	Level 1	Level 2	Odds	p value
Experience	magnetic force	magnetic field	1.2	0.12
Context	physics	non-physics	0.96	0.86
Type	forward	backward	1.6	0.0047
Comfort	easy	awkward	1.5	0.0070
Plane	xy	non-xy	1.5	0.0046
Tails	tail	head	1.0	0.84
Separation	together	separate	1.1	0.72



(a) (b)

Figure 4.1: Graphs for (a) Correctness by Log (T) (b) Log (T) by Correctness, where the blue line in (a) indicates the probability of obtaining a correct response as Log(T) varies

shows the mean and quartiles for the Log(T) for Correct and Incorrect responses. These figures demonstrate that for this data, correctness and response time can be treated as correlated measures of performance–*e.g.* shorter response time is more likely to be correct. The results for each of the other predictors will be discussed in the relevant sections below.

4.2.1 Background characteristics and practice effects

For the reasons outlined in Chapter 3, only two background characteristics were included: Spatial and Experience. Spatial was participant spatial ability measured by the Cube Comparison Test. Experience differentiated between those who were interviewed earlier in the semester (Magnetic Field group) and those interviewed later in the semester (Magnetic Force group). Although the order of the problems for each section of the interview was random, there was still the possibility for a practice effect from doing so many similar problems in one sitting. Thus, the order of the problems was also included as a predictor (Order).

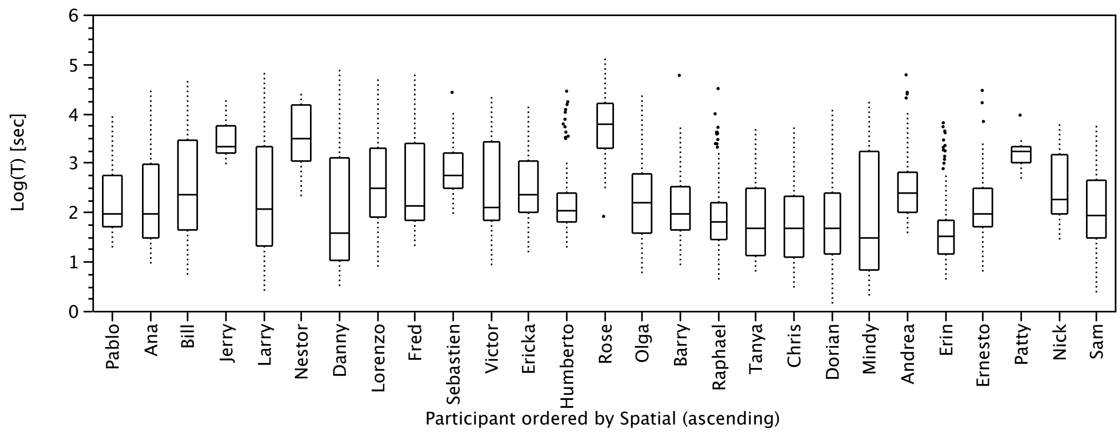


Figure 4.2: Graph of Log(T) by Participant (ordering according to Spatial)

According to the regression analyses, spatial ability does have a significant positive impact on performance on cross product direction questions, for both response time ($p < 0.0001$) and correctness ($p < 0.0001$). With such a small sample, it is difficult to determine the exact nature of the relationship between spatial ability and performance on these questions. This is demonstrated in Figure 4.2, which shows the response time for each participant, with the participants ordered according to their score on the Cube Comparison Test. Despite this complexity, the results of the regression analyses indicate that there is a connection between spatial ability and performance with right-hand rules. Exploring this relationship in more depth for a larger sample could be a fruitful avenue of future study (*c.f.* Section 7.3).

Although both previous experience and the order of the problems had a significant positive impact on response time ($p < 0.0001$ for both), the impact on correctness was not significant for either ($p = 0.12$ for Experience and $p = 0.46$ for Order). Participants were not given feedback on correctness until after the interview to avoid influencing the results. However, given the improvement in response time, one possible avenue for instruction would be a similar task with more feedback (*c.f.* Section 7.2).

For response time, there also appears to be a difference in performance based on context for both Experience and Order. Figure 4.3 and Table 4.4 show the response time

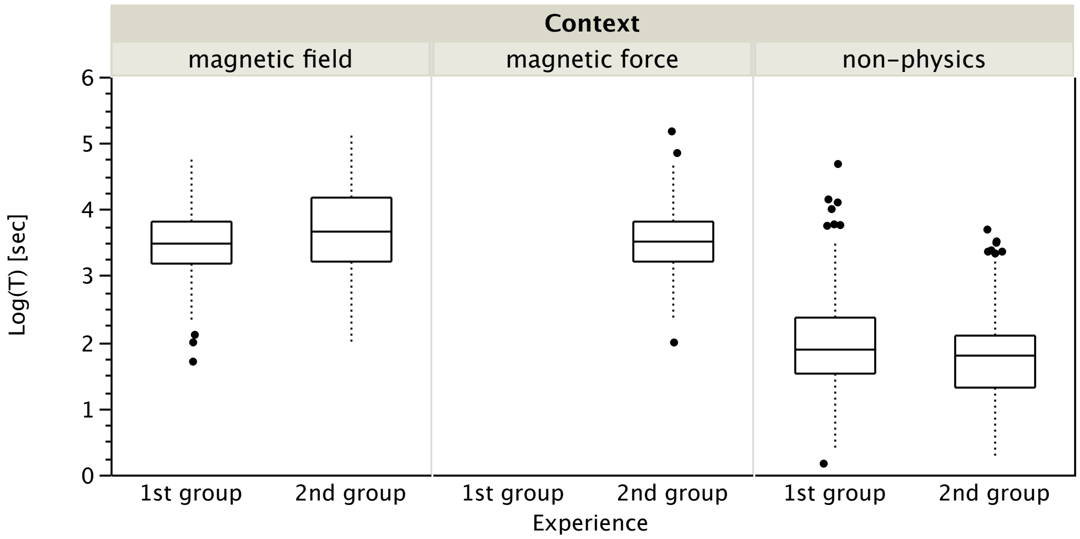


Figure 4.3: Graph of Log(T) vs. Experience by Context, where the missing data is due to the fact that the 1st group (Magnetic Field) did not answer magnetic force questions.

for the two groups for the different contexts. On the magnetic field problems, the mean response time for the Magnetic Field group is faster than for the Magnetic Force group ($p = 0.037$). It is possible that this is related to interference from their recent instruction on magnetic force (*c.f.* Section 3.1.1). This possibility is supported by the fact that the mean for the Magnetic Force group on the magnetic force problems is not significantly different than the mean for the Magnetic Field group on the magnetic field problems. The area where the second group is faster than the first is on the non-physics problems ($p < 0.0001$). It would be interesting to explore in the future how performance might change throughout the semester, such as when they encounter dot products with Gauss' Law (*c.f.* Sayre and Heckler, 2009).

When response time for the order of problems was compared across context, a similar distinction was apparent; the decrease in response time is more pronounced for the non-physics problems than for the physics problems, as demonstrated in Figure 4.4. It is possible that this difference is due to the fact that students were asked to do the non-physics problems as quickly as possible.

Table 4.4: Significance tests for Log (T) by Experience and Context

	Magnetic Field group			Magnetic Force group			Differences	
Type of question	N	Mean	SD	N	Mean	SD	2nd-1st	p value
Magnetic field	210	3.52	0.042	188	3.65	0.044	0.13	0.037
Magnetic force				186	3.54	0.037	0.006	
Non-physics	833	1.96	0.021	760	1.74	0.022	-0.22	< 0.0001

The effects of Experience and Order indicate that practice, without feedback on correctness, has more of an impact on response time than on correctness. The impact on response time is also more pronounced on non-physics problems than on physics problems.

4.2.2 Kinds of questions

The context of the problems had a considerable impact on the response time. In fact, this contrast was the most significant factor in predicting response time ($p = 0.0000$) and Figure 4.5.a shows this difference. This significance may be inflated due to the way in which response time was measured (*c.f.* Section 4.1).

The percentage of questions answered correctly is approximately 15% higher for non-physics problems than for physics problems, as shown in Figure 4.5.b. This result is consistent with the data presented in Table 4.5, which gives the fraction of problems answered correctly by each participant in each context (including problems eliminated from the regression). Figures 4.6 and 4.7 show the distributions of participant percentages for each context. For the non-physics problems, the distribution is negatively-skewed, which means fewer participants with low values. This skew is consistent with the hypothesis that non-physics cross product problems are easier for students than physics cross product problems; the distribution for the physics problems is much closer to normal. In the logistic regression, however, Context was the least significant of the predictors ($p = 0.86$). This highlights once again the need for qualitative analysis for a sample this size.

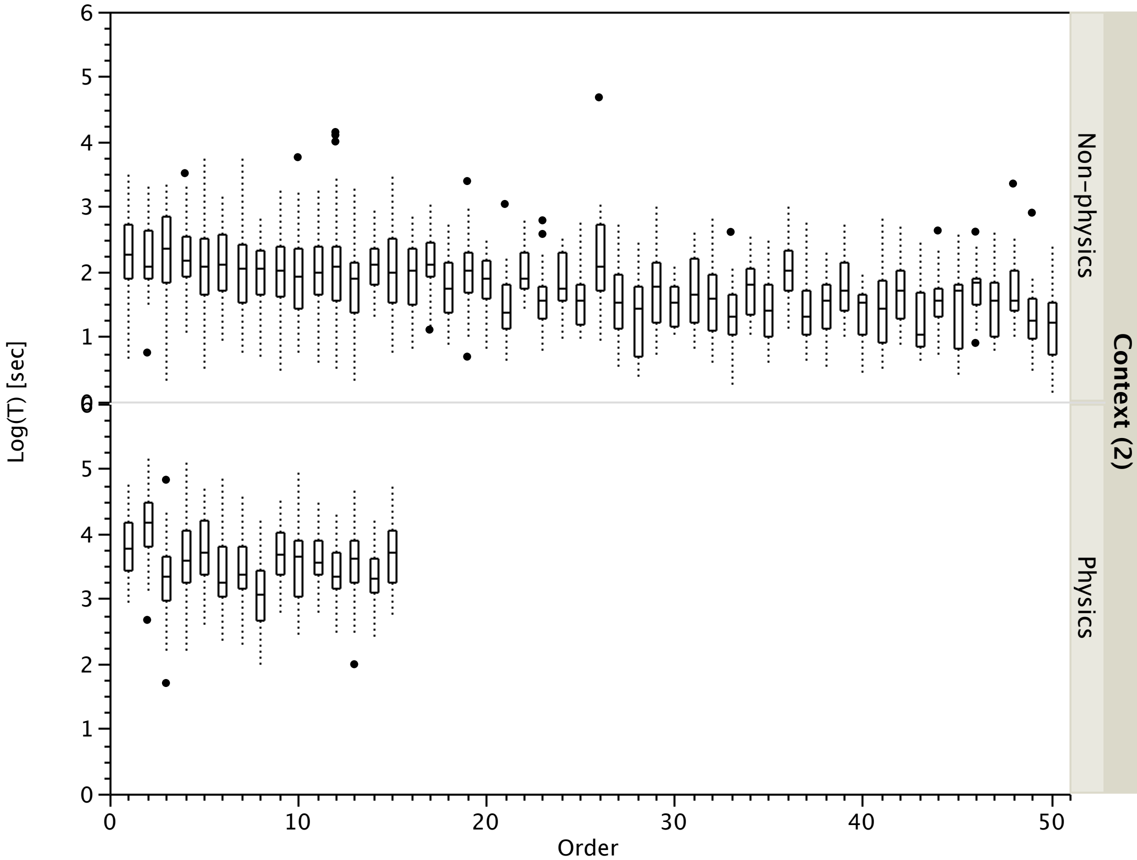
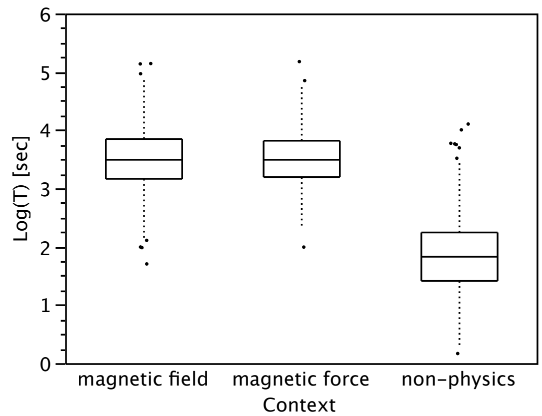
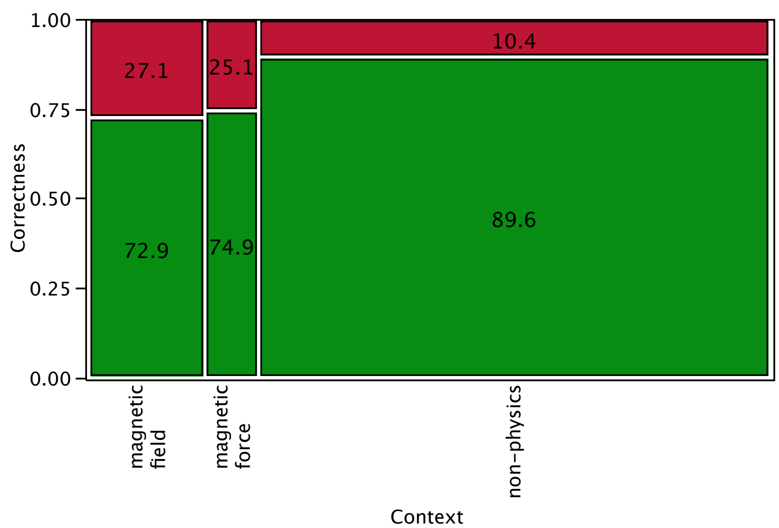


Figure 4.4: Graph of Log(T) by Order and Context



(a)



(b)

Figure 4.5: Graphs of (a) Log(T) by Context and (b) Correctness by Context

Table 4.5: Fraction of problems answered correctly by each participant in each context

		Total		Field		Force		Non-physics	
Participant	Spatial	N	Correct	N	Correct	N	Correct	N	Correct
Ana	8	101	0.73	14	0.43	17	0.18	70	0.93
Andrea	28	84	0.55	14	0.36			70	0.59
Barry	26	84	0.99	14	1.00			70	0.99
Bill	8	101	0.83	14	0.64	17	0.47	70	0.96
Chris	27	101	0.95	14	0.93	17	0.82	70	0.99
Danny	11	101	0.64	14	0.50	17	0.59	70	0.69
Dorian	27	84	0.88	14	0.71			70	0.91
Ericka	20	101	0.56	14	0.07	17	0.59	70	0.66
Erin	29	84	0.99	14	1.00			70	0.99
Ernesto	29	84	0.93	14	0.71			70	0.97
Fred	15	101	0.73	14	0.71	17	0.59	70	0.77
Humberto	21	84	0.89	14	0.50			70	0.97
Jerry	8	84	0.14	14	0.36			70	0.10
Larry	10	101	0.94	14	1.00	17	0.76	70	0.97
Lorenzo	11	84	0.81	14	0.50			70	0.87
Mindy	27	101	0.93	14	1.00	17	0.94	70	0.91
Nestor	10	51	0.84	14	0.86			37	0.84
Nick	36	101	0.94	14	1.00	17	1.00	70	0.91
Olga	23	84	0.93	14	0.86			70	0.94
Pablo	4	84	0.56	14	0.57			70	0.56
Patty	30	84	0.60	14	0.79			70	0.56
Raphael	26	84	0.98	14	1.00			70	0.97
Rose	21	101	0.48	14	0.79	17	0.76	70	0.34
Sam	36	101	0.88	14	0.93	17	0.76	70	0.90
Sebastien	18	84	0.77	14	0.21			70	0.89
Tanya	26	84	0.93	14	0.71			70	0.97
Victor	18	101	0.95	14	0.93	17	0.94	70	0.96
Mean	20.48		0.79		0.71		0.70		0.82
Median	21.00		0.88		0.71		0.76		0.91
Std Deviation	9.16		0.20		0.27		0.23		0.22

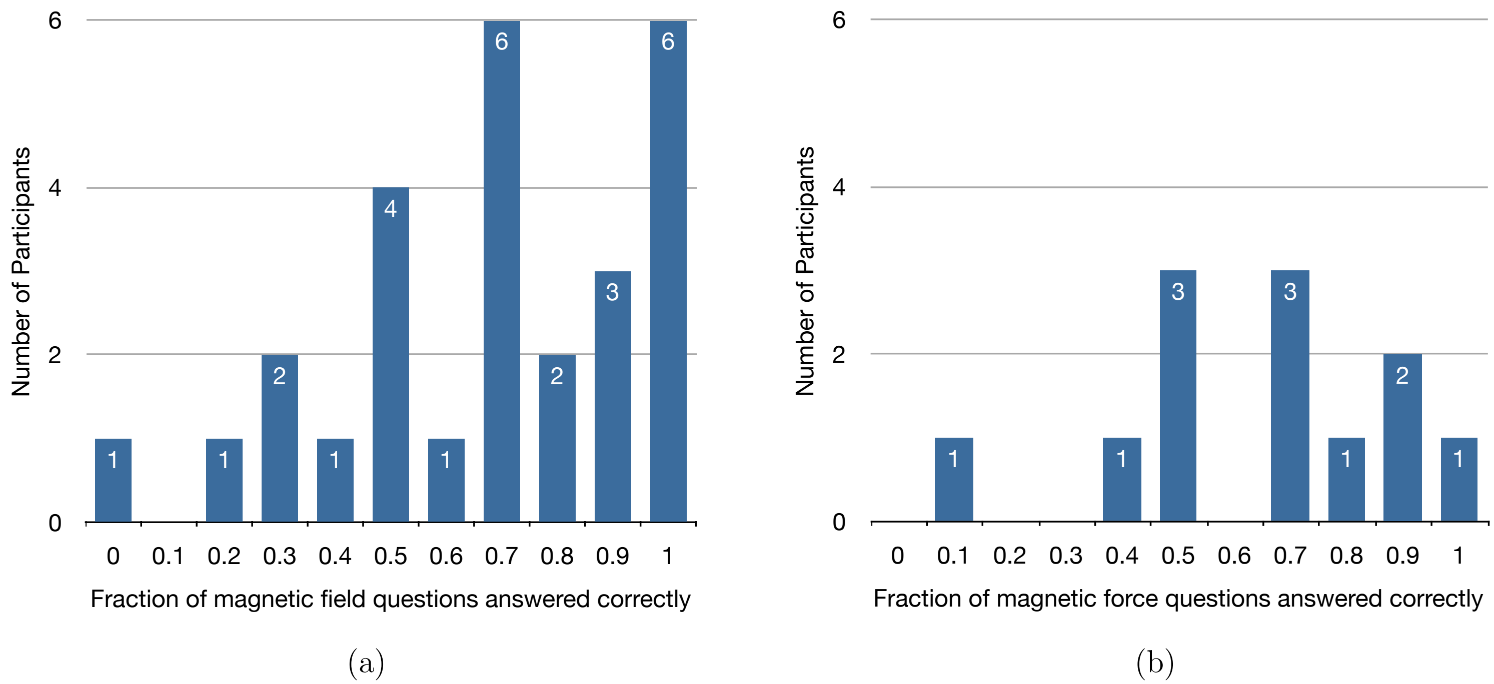
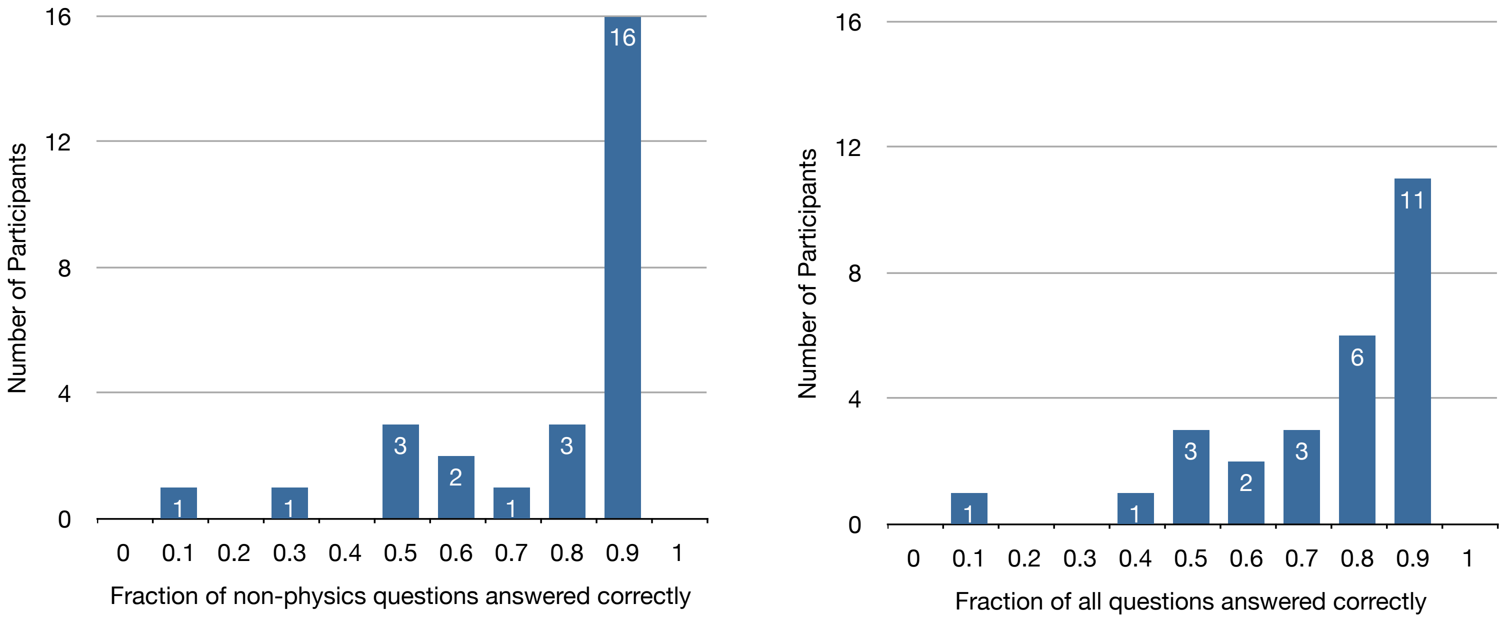
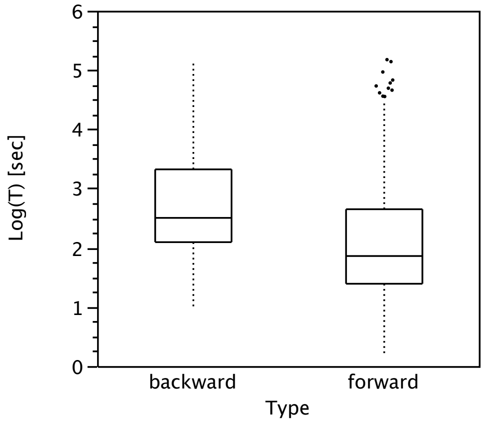


Figure 4.6: The distribution of participant percentages for (a) magnetic field problems and (b) magnetic force problems

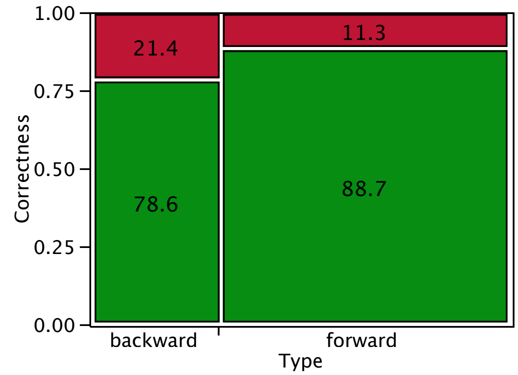


(a) (b)

Figure 4.7: The distribution of participant percentages for (a) non-physics problems and (b) all problems



(a)



(b)

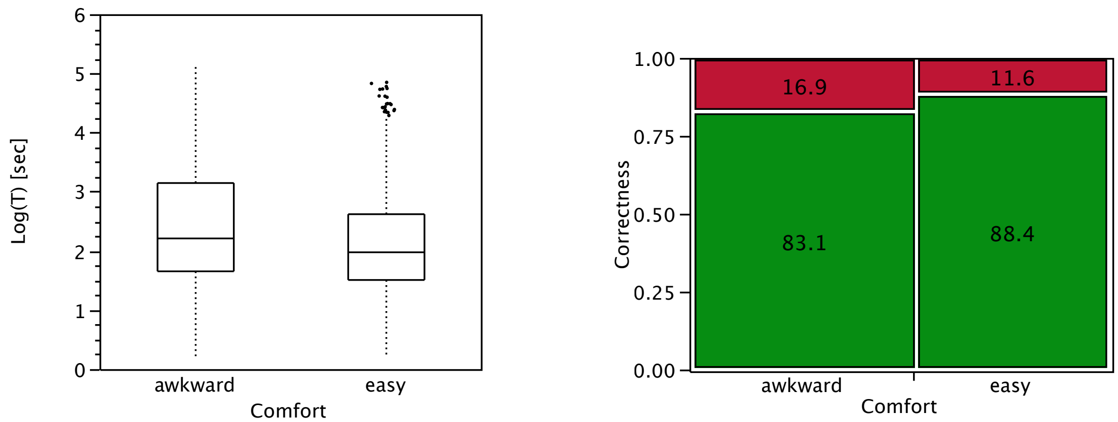
Figure 4.8: Graphs for (a) Log(T) by Type and (b) Correctness by Type

Unlike the context of the question, the type of reasoning had a significant positive effect on both response time ($p < 0.0001$) and correctness ($p = 0.0047$) in the logistic regression, as demonstrated in Figures 4.8.a and 4.8.b, respectively. It is clear that problems which require reasoning backward are significantly more difficult than those that require forward reasoning. Chapter 6 will show that there are also qualitative differences between the different types of backward reasoning.

4.2.3 Orientation features

Research in spatial cognition (*e.g.* Klatzky and Wu, 2008) strongly suggests that the orientation of the vectors should impact difficulty when using a right-hand rule, for kinesthetic reasons as well as issues related to the alignment of reference frames. This study examines several different aspects of orientation in order to determine the relative effect on performance: the level of physical discomfort to perform a right-hand rule, the plane of the given vectors, and the angle between the given vectors.

This study used the most physically easy and the most physically awkward orientations (*c.f.* Section 3.2.3) in order to determine the kinesthetic effect on performance. As shown in Figures 4.9.a and 4.9.b, performance on the physically easy orientations was



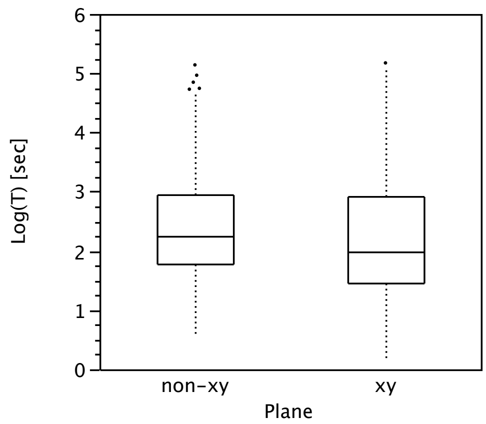
(a) (b)

Figure 4.9: Graphs for (a) Log(T) by Comfort (b) Correctness by Comfort

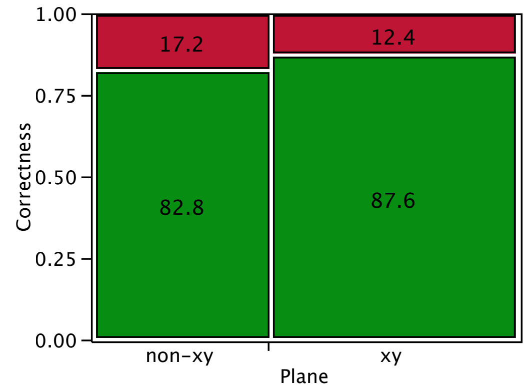
significantly better than on the physically awkward orientations. This was true for both response time ($p = 0.0060$) and for correctness ($p = 0.0070$). However, since only the extremes for physical discomfort were used, it is possible that this effect is inflated and should be considered an upper limit.

The impact of the plane of the vectors was even stronger than that of physical discomfort. This impact was consistent with the hypothesis that participants would perform better on problems where the initial vectors ($\vec{A}$ and $\vec{B}$) were in the xy-plane than on problems where one of these vectors was along the z-axis. This difference was significant for both response time ($p < 0.0001$) and for correctness ($p = 0.0046$), demonstrated in Figures 4.10.a and 4.10.b, respectively.

As discussed above, the angle between the vectors was only varied for non-physics working-forward problems in the xy-plane, so this contrast was not included in the regression analysis. However, using only the relevant problems, Table 4.6 presents the results of an analysis of variance (ANOVA) F test for response time and a Likelihood Ratio χ^2 test for correctness. There was no significant difference in response time ($p = 0.14$), but the correctness on the perpendicular orientations was significantly higher than on either the acute or the obtuse orientations ($p = 0.041$). The high level of correctness for all



(a)



(b)

Figure 4.10: Graphs for (a) Log(T) by Plane and (b) Correctness by Plane

of the problems used for this analysis is consistent with the results discussed above that indicate non-physics working-forward problems in the xy-plane are among the easiest for these participants. Since there is a difference in correctness for these easy problems, it is possible that on more difficult problems, angle would have an even stronger effect. Additionally, angle plays a key role in geometric methods for finding the magnitude of a cross product and thus, this contrast should be considered in any study examining student performance on cross product problems.

4.2.4 Parallel transport features

When the vectors being operated on are not tail-to-tail or are separated in space, parallel transport is required for the cross product. The literature on student understanding of vector algebra indicates that students struggle when the vector operations require moving the vectors (*e.g.* Hawkins et al., 2009). Thus, one would expect performance on cross product problems that require parallel transport to be worse than performance on those that do not. However, of all of the contrasts included in the regression models, Tails and Separation had the least impact. The difference between performance on problems where the vectors were head-to-tail and those with vectors tail-to-tail was not significant either for response time ($p = 0.42$) or for correctness ($p = 0.84$). There was a significant

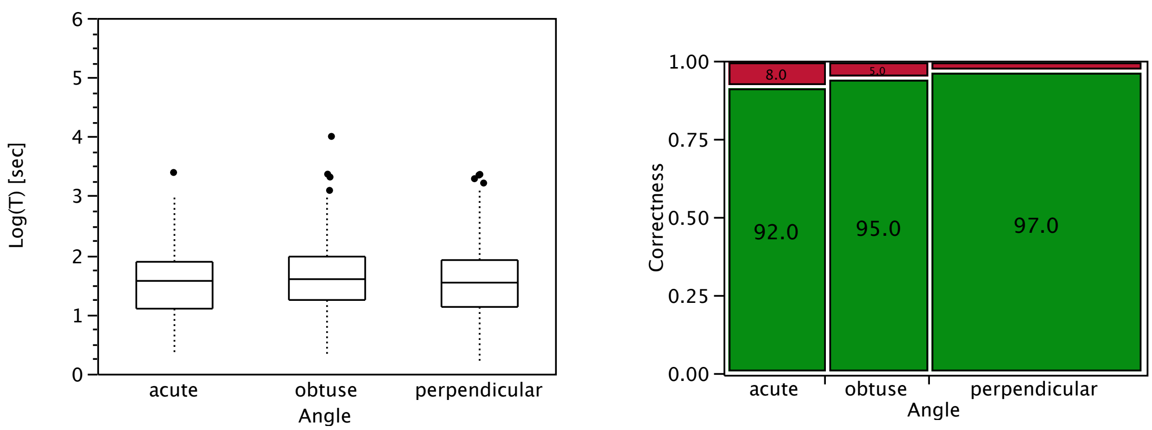
Table 4.6: (a) Analysis of variance (ANOVA) for Log(T) by Angle and (b) Likelihood Ratio Test for Correctness by Angle for non-physics working-forward problems in the xy-plane.

	N	Mean	SD
Acute	175	1.55	0.045
Obtuse	179	1.66	0.045
Perpendicular	369	1.57	0.031
p value for F statistic			0.14

(a)

	N	DF	χ^2	p value
Likelihood Ratio	723	2	6.37	0.041

(b)



(a) (b)

Figure 4.11: Graphs for (a) Log (T) by Angle and (b) Correctness by Angle for non-physics working-forward problems in the xy-plane

positive impact on response time for vectors that were together compared to those that were separate ($p = 0.038$), but no difference in correctness ($p = 0.72$). The combination of head-to-tail and separate in space does seem to negatively impact performance, but only slightly. The results of the qualitative analysis suggest that it is still a problem for some students (*c.f.* Section 6.2.3). Considering those results, it seems likely that parallel transport may not be as much of an issue for cross product direction as it is for vector addition and subtraction.

4.2.5 Physics features

The physics features, like the angle between the vectors, were not included in the regression analysis since they were only varied on the physics problems. For completeness, ANOVA and χ^2 tests were conducted and analyzed for each of the physics features for the relevant problems.

All of the physics problems involved a moving positive point charge, a moving negative point charge, or a current-carrying wire of some shape with the direction of conventional current provided. Since a negative charge changes the sign of the cross product for both magnetic field and magnetic force problems, it was expected that performance on problems with a negative point charge would be worse than the performance on problems with either a positive charge or conventional current. While there was no significant difference in response time for these problems ($p = 0.071$), the correctness for problems with a negative point charge was significantly lower than for the other problems, as seen in Figures 4.12.a and 4.12.b and in Table 4.7.

The shape of the current-carrying wire was expected to impact the type of right-hand rule used, not necessarily impact the correctness or the response time. As expected, there was no significant difference between a straight wire and a loop of wire on correctness ($p = 0.3273$) or response time ($p = 0.2694$).

On the other hand, Scaife and Heckler (2010) clearly demonstrated that the representation of the magnetic field can have a significant impact on performance for magnetic force problems. When the magnetic field is represented as a vector, students are more

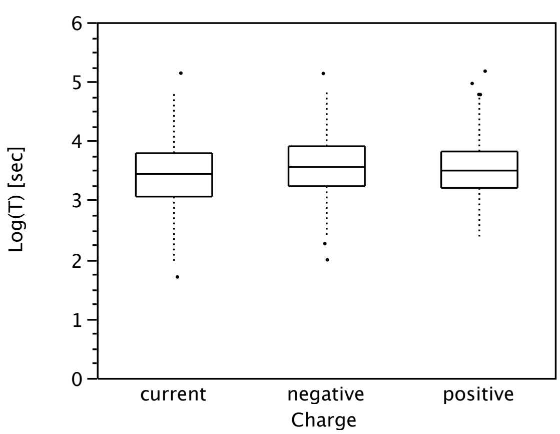
Table 4.7: (a) Analysis of variance (ANOVA) for Log(T) by Charge and (b) Likelihood Ratio Test for Correctness by Charge

	N	Mean	SD
Current	208	3.47	0.039
Negative	164	3.60	0.044
Positive	161	3.55	0.045
p value for F statistic			0.071

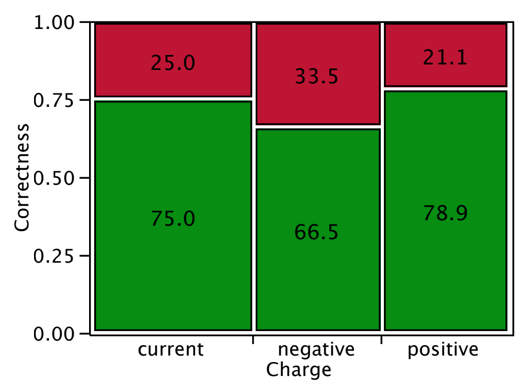
(a)

	N	DF	χ^2	p value
Likelihood Ratio	533	2	6.72	0.035

(b)



(a)



(b)

Figure 4.12: Graphs for (a) Log(T) by Charge and (b) Correctness by Charge for all physics problems

Table 4.8: (a) t Test for Log(T) by Field and (b) Likelihood Ratio Test for Correctness by Field for magnetic force problems with a given magnetic field

	N	Mean	SD
Magnet	34	3.57	0.094
Vector	71	3.46	0.065
p value for t statistic			0.34

(a)

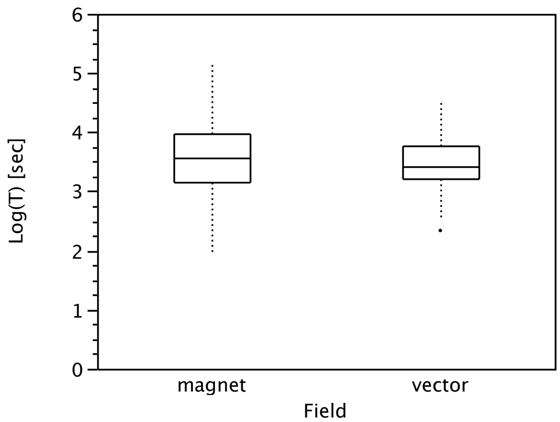
	N	DF	χ^2	p value
Likelihood Ratio	105	1	9.14	0.0025

(b)

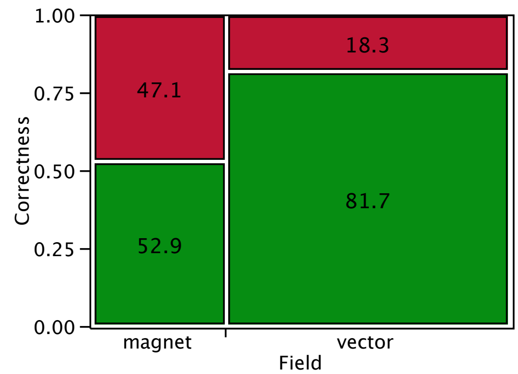
likely to answer correctly than if the magnetic field is represented using magnets. The results of the regression analysis are consistent with this finding, as demonstrated in Figure 4.13.b and in Table 4.8.b, which show that the difference in correctness is significant at the $p = 0.0025$ level. Like Charge and Angle, the difference for response time, shown in Figure 4.13.a and Table 4.8.a, is not significant ($p = 0.34$).

4.3 Discussion of quantitative results

The primary goal of the quantitative analysis was to provide a first-order look at the relationship between the features identified in the literature and the difficulty of using a right-hand rule on a cross product direction question, as measured by correctness and response time. While each contrast impacted performance in a direction consistent with the hypotheses, not all effects were statistically significant. Given the small sample size, it is difficult to strictly interpret the significance of the data, yet it can provide an overall picture.



(a)



(b)

Figure 4.13: Graphs for (a) Log(T) by Field and (b) Correctness by Field for magnetic force problems with a given magnetic field

4.3.1 Background characteristics and practice effects

The ability to perform mental rotations (Spatial) does seem to be positively correlated with performance. Although with a closer look, it is difficult to determine the exact nature of the relationship given the small number of participants and the possibility of influence from other aspects of participant background that could not be explored in this study. As noted above, this could prove a fruitful avenue for future research.

Practice effects, represented by Experience and Order, have a stronger impact on response time than on correctness. However, there also appears to be an interaction with Context for both. While not surprising given the structure of the study, these interactions are something to be explored in the future.

4.3.2 Problem features

Kinds of questions

As expected, there is a large difference between physics and non-physics problems; this is more of a factor for response time than for correctness. The type of reasoning, on the other hand, is one of the strongest predictor among the problem features for both correctness and response time. Problems requiring backward reasoning were a significant challenge for these participants and this is an area that will be explored in much greater depth in the following chapters.

Orientation features

All three features related to orientation impacted performance in a manner consistent with literature on spatial cognition. The need for alignment of reference frames, exemplified in the differences in the plane of the given vectors, appears to be the strongest of these influences. Although, the qualitative analysis suggests that this issue may be due in part to a misinterpretation of symbols (*c.f.* Section 6.4).

The physical discomfort also had a negative impact on performance. As noted above, however, this study used orientations at the two extremes of physical comfort and discomfort. Thus, the effect seen here is likely a stronger one than might be noted with a broader range of orientations. In fact, this could account for why Scaife and Heckler (2010) did not find a measurable difference in orientation.

Although only varied on the most straightforward problems, the angle of the given vectors did have a significant impact on correctness. This should be taken into account for any study looking at cross products, especially given the importance of angle for determining cross product magnitude.

Parallel transport features

One of the most surprising outcomes of this analysis is the fact that difficulties with parallel transport, exemplified through the Tails and Separation features, do not seem to be as much of issue for cross products as they are for vector addition and subtraction. Although we should not discount these difficulties altogether (as will be discussed in the following chapters), this analysis indicates that there are other aspects of cross product problems that are more difficult for students than parallel transport.

Physics features

Finally, by examining the impact of physics features on correctness, this analysis has verified the results of Scaife and Heckler (2010) regarding the impact of field representation on student performance on magnetic force problems. In addition, this analysis has provided clear evidence that the sign of the charge is also an issue for students – something that has been only anecdotal until now.

4.3.3 Summary

The regression analyses presented in this chapter are not intended to provide a complete or final model, especially given the small sample size and asymmetric design. Additionally, this data hints at interactions (such as between Order and Context) that were not included in this analysis. These regressions have provided compelling evidence that performance on cross product direction questions is dependent on several different problem features and has verified previous results. The following chapters will build on this foundation through a qualitative look at the methods which students use to solve these problems and the types of errors that they make.

Chapter 5

Qualitative analysis: coding

The hypotheses that were tested in Chapter 4 assumed the use of a right-hand rule and so, required some knowledge of the methods that participants were using. The results of the quantitative analysis provided a broad look at patterns of representation and context-dependence. In addition to the perspective provided by the quantitative analysis, a deeper look into possible sources of student difficulty will add to our understanding of students' uses and difficulties with right-hand rules. This is accomplished through qualitative categorization of methods and errors, or coding. This chapter will discuss this process of categorization and the following chapter will present and discuss the results of applying these codes to the data. Specifically, Section 5.1 will discuss the basics of coding development used for both method and error codes. Sections 5.2 and 5.3 discuss the method and error codes respectively, including brief descriptions of the final codes. For more complete definitions with examples of the codes at each stage of the process, see Appendix B. The following chapter will discuss how these methods and errors are related to each other and to the problem features.

5.1 Coding development

Using an approach based on Grounded Theory (Corbin and Strauss, 1990), preliminary codes were developed using interviewer notes, as well as "content logs" created from watching the interviews and noting what students were doing and saying. These preliminary codes were then compared to the data and revised to create new code definitions.

Once this revision process reached a point where the codes appeared to be an accurate reflection of the data, a second coder was brought in to code a selection of data. Agreement between the two coders (or raters) was used to establish the reliability of the code definitions. There are numerous ways to calculate interrater agreement (Banerjee et al., 1999); for example, simple agreement is calculated by counting the number of times both raters assigned an item to the same category and dividing by the total number of item.

$$\text{Agreement} = \frac{\#\text{ of coding agreements}}{\#\text{ of coding opportunities}}$$

However, there is the possibility that some of this agreement is merely due to chance. Cohen's kappa statistic is one way to account for this possibility and to provide a more accurate measure of reliability for a set of mutually exclusive codes.

As outlined in Banerjee et al. (1999), in order to calculate Cohen's kappa, one must first find the proportion of agreement that would be expected due to chance. If p_{ij} is the proportion of items assigned to the ith category by the first rater and to the jth category by the second rater, then $p_{i\cdot} = \sum_{j=1}^{m} p_{ij}$ is the proportion of items the first coder assigned to the ith category and $p_{\cdot j} = \sum_{i=1}^{m} p_{ij}$ is the proportion of items the second coder assigned to the jth category. Therefore, the proportion of agreement expected by chance is $p_c = \sum_{i=1}^{m} p_{i\cdot} p_{\cdot i}$ and the observed proportion of agreement is $p_o = \sum_{i=1}^{m} p_{ii}$. Cohen's kappa is then found by:

$$\hat{\kappa} = \frac{p_o - p_c}{1 - p_c}$$

By taking into account the proportion of agreement due to chance, Cohen's kappa is a more valid measurement of reliability; and it has a value that is always less than the value for simple agreement.

It is standard to use 10% of a data set for interrater reliability. There were over 2400 questions to code, so each round of interrater reliability used only 3-8% of the relevant question. To compensate for the smaller data set and to provide for more robust agreement, the problems were chosen to reflect the most difficult questions and the widest variety of codes.

Table 5.1: One scale for the strength of agreement based on Cohen's kappa (reproduced from Landis and Koch, 1977)

Kappa Statistic	Strength of Agreement
<0.00	Poor
0.00-0.20	Slight
0.21-0.40	Fair
0.41-0.60	Moderate
0.61-0.80	Substantial
0.81-1.00	Almost Perfect

The results of the two coders were compared and revised until the interrater reliability reached an acceptable level. Although there is not a universally established standard for acceptable kappa values, one of the most common scales is that proposed by Landis and Koch (1977), reproduced in Table 5.1. The iterative process of code revision and testing for interrater reliability was repeated until all of the codes reached the level of "substantial" agreement using this scale.

5.2 Method code definitions

The methods used by participants were not mutually exclusive, so each problem was coded once for each method that was used during that problem. Thus, Cohen's kappa was calculated for each method using the presence and absence of that method as the codes to be compared.

In creating the preliminary codes, there were indications of two distinct families of methods. The first are Primary methods–methods that on their own can yield a singular, non-zero answer. This code family includes the more formal means of solving cross product problems (right-hand rules and the use of matrices), and the use of vector addition and subtraction methods. The second group is composed of tools, techniques, and strategies that the participants usually used in conjunction with these primary methods. These are Supplemental methods because when used on their own, cannot yield a singular, non-zero answer. In addition to Primary and Supplemental methods, there were some problems where there was not enough gestural or verbal evidence to determine the method used. These problems were coded as Unclear and eliminated from further analysis. They accounted for only 2% of the total number of problems. The final codes for the Primary and Supplemental methods will be discussed in Sections 5.2.1 and 5.2.2, respectively. After presenting these codes, Section 5.2.3 will discuss the major changes between the initial and final code definitions.

5.2.1 Primary methods

As discussed above, primary methods are those that can yield a singular, non-zero answer when used alone. There were three primary methods: Addition, Matrix, and Right-hand rule (RHR).

Addition

During the pilot study (3.1.1), four out of ten of the first-semester students attempted to solve the non-physics cross product problems with methods consistent with vector addition or vector subtraction. While these methods were not as common in the main study, two of the participants, Patty and Rose, did attempt to use vector addition on some of the non-physics problems. Unlike those students in the pilot study, both Patty and Rose did not use this method exclusively on the non-physics problems. Patty recognized her error after several problems and switched to the use of matrices as demonstrated in the following excerpts from two different problems:

> Non-physics Forward Problem 1: "So, you would go from tip to tail, put that there and so it would be that direction...I'm doing this wrong. That is wrong, I can't remember how to do it though."

> Non-physics Forward Problem 5: "I'm thinking, I know when you do the cross product of something, it's when you have, when it's only in two planes, I guess, or axis, on two axis, it's usually, the cross product is contained in the third, no, is that the dot product? That's the dot product, never mind. The

> cross product is, I'm trying to think what the cross product is and calc three is failing me right now, um, very lost, **I know i'm not doing it right with the orientations of just, I'm adding vectors when I do it that way**. Um, A cross B, when you have that, you set it up like a matrix..."

Rose started out by correctly using the right-hand rule, but became confused when she identified them as "normal" vectors; she then began using addition techniques. Throughout the interview, Rose would shift between the use of a right-hand rule and addition, sometimes within the same problem. Approximately halfway through the non-physics problems, she stated, "So, in general, I'm thinking that I don't know how to do normal vectors." Her confusion was more typical of the four participants from the pilot study.

Matrix

The determinant of a matrix is often used to determine the cross product (consistent with the algebraic method of teaching cross products, *c.f* Section 1.2), but it is usually used when values for the vectors are given in component form (*e.g.* $\langle 1, 0, 0 \rangle$). None of the problems given to the participants included values for the components of the vectors. However, two students, Patty and Nestor, still chose to use this method to solve many of the non-physics problems. As noted above, Patty used matrices after realizing that the addition was not correct. Nestor, on the other hand, used matrices in conjunction with right-hand rules, often using one method to check the other. When the results of the two methods disagreed, sometimes he would try to reconcile the two methods. Other times he would simply choose one of the results, as demonstrated in the following excerpts:

> Non-physics Forward 26: "...this [right-hand] rule says it should be negative y. But I have positive y for determinant. A is in the positive x, zero, zero, B is in the zero, zero, positive z, so positive y. I'm going to with the determinant."

> Non-physics Backward 15: "...I must've messed up on the right-hand rule...if A is in the positive z, that's coming out and if B is in the negative y, it's going down [does RHR], why am I getting a positive x direction? Oh, because I messed up on my determinant [corrects matrix], right-hand rule was right."

Right-Hand Rule

As Greenslade (1980) pointed out, there are numerous physical mnemonics and these vary by the hand used, as well as which part of the hand corresponds to which part of the cross product. For the purpose of this study, all such mnemonics have been grouped together as 'Right-hand rules (RHR)' and identified with three sub-codes: the hand that was used (Left or Right), the frame of reference with which the hand is aligned (Table or Other), and the type (Type) of rule used. If any of these sub-codes changed during the problem, it was considered a different use of the right-hand rule. Thus, the right-hand rule was the only method code used more than once per problem. Cohen's kappa was calculated for each set of sub-codes for the right hand rule: Hand, Frame, and Type.

Each question was given in a reference frame defined by the table and the paper (Figure 5.1). A participant could choose to map her/his hand onto this frame. However, s/he could keep the hand in the ego-centric frame and map the answer onto the frame of the table instead. One could also rotate the frame in any other direction to ease physical discomfort. While each of these situations might use the same hand and type of right-hand rule, they differ in the alignment that is required (*c.f* Section 2.3.1). Since the frame of the table was the given reference frame and the one in which the answer was expected, the Frame sub-code categorized right-hand rules based on whether the hand was mapped onto the table frame (Table) or a frame other than the table frame (Other).

At the end of each interview, the participants were asked to describe the right-hand rule(s) they used. These descriptions were analyzed and grouped into six distinct types of right-hand rules, described below using participant descriptions.

Standard

> "By convention, placing your hand in the direction of the first vector and curling your fingers in the direction of the second vector tells you that your thumb is pointing in the direction of the third vector." Barry

> "If you have two vectors, A and B, and you want to find their cross product, you're gonna take A and you're gonna use your hand and put it along the A

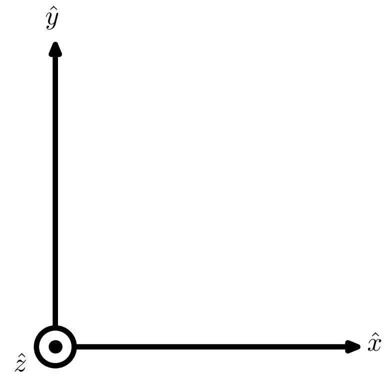


Figure 5.1: Reference frame defined by the table and paper, where $+\hat{x}$ was to the right of the participant, $+\hat{y}$ pointed away from the participant, and $+\hat{z}$ was vertically upward, out of the page (represented by the $\odot$ symbol)

> and curl it into the B in the shortest direction possible and whichever way your thumb points is the direction of the vector C." Sebastien

This type of right-hand rule (*c.f.* Figure 1.2.a) was used by all but two of the participants: Andrea, who used the 3 finger type, and Humberto, who used the Knuckles type. It was used on 80% of the non-physics problems, 80% of the magnetic force problems, and 40% of the magnetic field problems. The name used for this type (Standard) reflects the prevalence of this method.

Current

> "So, for current, thumb always points in the direction of conventional current and then your hand curls around in the direction of magnetic field. So...if the point was under the wire, it would be pointing out at you if the current was going that way [moves thumb leftward]." Nestor

> "You point your thumb in the direction of conventional current and...whatever direction your fingers point at that location is magnetic field." Victor

After Standard, the Current type (*c.f.* Figure 1.2.b) was the most common method. These two methods were the primary right-hand rules presented in the class from which the participants were drawn. Every participant except Patty used this type at some point. Most used this right-hand rule only to find the magnetic field for a straight current-carrying wire, but some also used it to find the magnetic field due to a moving point charge. There were also a few participants who attempted to use this rule when asked for the magnetic force on a current-carrying wire.

Loop

> "...when you have a loop of current...you curl your fingers...along the conventional current and then your thumb points in the direction of the magnetic field in the center of that loop." Erin

> "...with loops, you just curve your fingers along the direction of the conventional current, so that tells you the direction of the magnetic field." Olga

Although one could argue that the Loop rule is simply a variation of the Standard type, the participants clearly distinguished between these two types. This rule was used by 21 of the participants exclusively on magnetic field problems. With a few exceptions, it was only used on problems involving either a square or circular loop of current-carrying wire and was used more often on these problems than any other type of right-hand rule.

3 finger

> "Ok, you're gonna use A as your index finger, B is your middle finger, C is the product of the cross product and then you just maneuver them and if you have to turn them, that's it." Andrea

> "Well, I use this one [3 fingers] when I know velocity, is my pointer finger...and then my middle finger is magnetic field and my thumb is the force. I use this, my right hand for positive charges and my left hand for negative charges." Danny

This method was used by only three participants: Andrea, Danny, and Fred. Danny clearly defines this rule as applicable only to problems involving force, which causes him difficulty when he tries to apply it to magnetic field problems. While Fred is not as explicit in the connection to force, he applies it in a manner similar to Danny. Unlike these two, Andrea, on the other hand, used this method almost exclusively. She also had a great deal of difficulty with it, as she volunteered before answering any questions:

> "It's kind of confusing cause I always forget, like, what finger stands for what thing. Like, I understand...A [index] and B [middle] and this [thumb] is...cross product C, but...when you have to assign that, like, physics stuff like force and um, your velocity or your magnetic field,...that always gets a little bit confusing, especially since I don't know there's so many different ways to do 'em."

Knuckles

> "Well if you just have the vectors then you point A in the direction of your knuckles and you point B, the second vector, you point your fingers in that direction and whichever way your thumb is pointing would be the direction of the cross product." Humberto

Humberto was the only participant to use the Knuckles right-hand rule and he used this rule on all problems where he used a right-hand rule.

Palm

> "I do it with, put my thumb along the first vector, and my fingers along the second...and then the result is out of my palm." Nick

Nick was the only participant who used the Palm right-hand rule. He clearly distinguished between the Palm and Standard types and used both. Nick stated that he was more comfortable with the Palm type since he learned it first, but found that it does not work as well for problems where the vectors are non-perpendicular.

5.2.2 Supplemental methods

Supplemental methods cannot yield a singular, non-zero answer unless used with another method. Occasionally, someone would use only a supplemental method and give a nonsingular (Orthogonality) or zero (Physics Knowledge) response. However, these methods were more often used in conjunction with another supplemental method or with a primary method. The majority of the revisions to the coding involved the Supplemental methods. After all revisions, there were seven Supplemental methods: Physics Knowledge, Rotation, Orthogonality, Guess and Check, Parallel Transport, Diagram, and Multiplication.

Physics Knowledge

At some point during the interview, every participant explicitly drew on physics information that was not provided in the problem statement. This could be as simple as stating the equation they were using (*e.g.* $\vec{F} = q\vec{v} \times \vec{B}$) or indicating the effect of the sign of the charge on the cross product. It could also be an attempt to reason from an analogy to electric field, as Lorenzo does here:

> Magnetic field 1: "So, we have a positive charge going this way and we're looking at it from here. Now...that's positive and it's always going away from the negative, away from the negative into the positive...Uh, let's say that if it's going out of the negative and into the positive, let me think about a dipole. How does a dipole work? Negative, positive, that way, that way, out of the positive into the negative, so I'd say that the direction is straight up...I'd say it's either positive straight up on the y axis or it is diagonal to where the positive charge is moving. It's one of those two, um, I think it's just positive straight up."

Rotation

More than half of the participants ($N = 16$) rotated the paper occasionally, often before or while doing a right-hand rule. Several participants were asked at the end of the interview why they rotated the paper:

> "...I didn't want to get the axis mixed up, sort of like a visual thing I guess."
> Ana

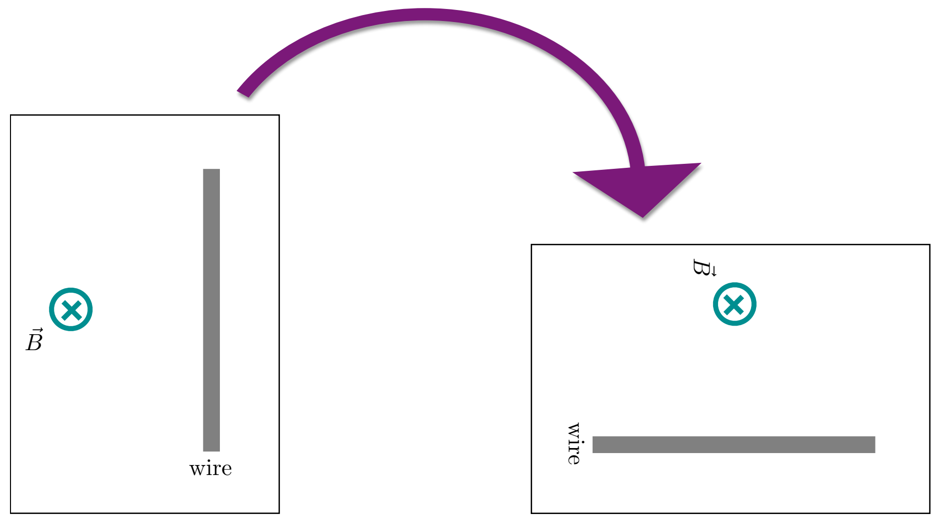


Figure 5.2: An example of a Rotation designed to make the use of a right-hand rule less awkward.

> "It helps me visualize it better, if I can...put my thumb in the direction,...if it's in different planes,...if the paper's flat and your hand's up here, they're like in different areas, so if you move it together, I can visualize it easier."
> Sebastien

> "...some of the ones where it's really unnatural to bend your hand that way, you have to rotate the page and then the coordinate system gets confusing."
> Chris

It is clear from these responses that there may be a connection between the use of rotation and certain problem features, like physical discomfort or the plane of the given vectors. Figure 5.2 shows an example of a possible rotation designed to make the right-hand rule less awkward. The Rotation method was used on approximately 20% of all problems, which is more often than any other supplemental method, except Physics Knowledge.

Orthogonality

Klatzky and Wu's (2008) task analysis for the right-hand rule states that one must create a reference frame where $\vec{A}$ and $\vec{B}$ form a plane, map the hand onto that reference frame and perform a right-hand rule, and then align the result with the third dimension of that frame (*c.f.* 2.3.2). Half of the participants ($N = 14$) utilized a method that reduces cognitive load by switching the last two steps of this process. The cross product is, by definition, perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$, so if one can identify the axis that is perpendicular to that plane, the set of possible answers is reduced to two: positive or negative.

Guess and Check

When trying to reason backward, it is sometimes easier to "guess" a direction and then "check" that guess with a primary method, usually a right-hand rule. More than half of the participants ($N = 15$) used this method at some point. For non-physics problems, this method was used almost exclusively on working backward problems and for physics problems it was used three times as often on working-backward problems as it was on working-forward problems. This method could significantly reduce the cognitive difficulty of working backward problems.

Diagram

Although all of the problems used in this study contained a diagram, five participants supplemented the given diagram with one of their own. Three participants did so on only a few problems: Danny redrew the given diagram once; Lorenzo drew coordinate axes on one problem and on another tried to draw the field of a dipole; Victor used diagrams on all three problems involving a bar magnet representation of magnetic field and both problems involving two steps of reasoning.

In addition to these three participants, Rose and Larry used diagrams more extensively. Rose applied this technique on more than half of all of the problems and Larry on almost all of the magnetic field problems (11/14), as well as a few magnetic force

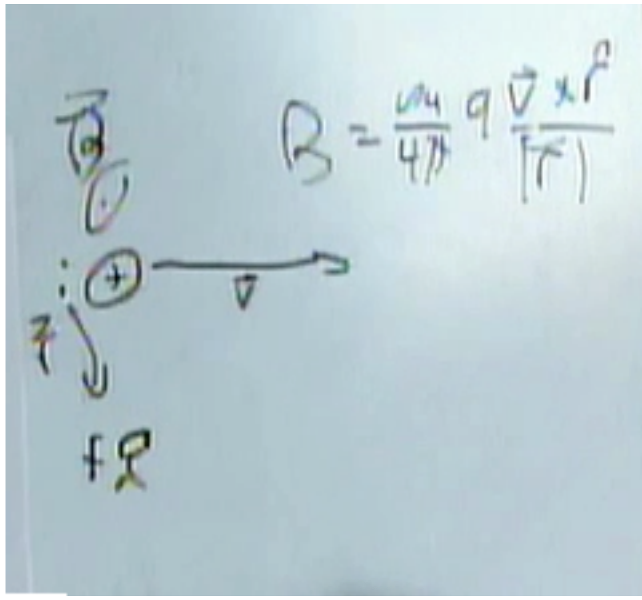


Figure 5.3: An example of the Diagram method–a screenshot of Larry's whiteboard after completing Magnetic Field Problem 1.

problems. Larry was the only participant to explicitly draw the position vector from the source to the observation location, shown in Figure 5.3. Rose was the only participant to draw a diagram for non-physics problems, almost always using this strategy on problems where she also used Addition. Her diagrams primarily consisted of redrawing the diagram, moving the vectors (to be tip-to-tail), and drawing coordinate axes to rotate the problem.

Parallel Transport

Six of the participants explicitly moved the vectors (either by gesture or diagram) before using a primary method. Sebastien, Erin, and Chris each used this method only once when the vectors were given head-to-tail. Rose and Patty used this method predominantly on those problems when they also used Addition. Lorenzo had difficulty remembering whether the vectors were supposed to be tail-to-tail or tip-to-tail:

> "A cross B, tip to tail, so, hopefully, I'm doing this right, tip to tail, cause if it's like it is right now [does RHR], that's negative z direction. But if I do tip to tail [does RHR], then that's positive z, I'm not sure if I have to put 'em

tip to tail because they're all like that. (pause). I'm just going to say the tip to tail thing, I don't need to do that."

Multiplication

Lastly, there were two participants, Patty and Danny, who attempted to use multiplication heuristics on the non-physics problems. Patty used Orthogonality to determine the axis of the cross product and then used the following reasoning to determine the sign:

> "I started to realize that when you had one negative and one positive, you generally had it going in the negative direction...then if you had two that were the same sign, you would generally have a positive."

Although Patty acknowledged the non-commutativity of the cross product when asked directly, she relied on this commutative multiplication heuristic to find the sign. While attempting to solve the non-physics working-backward problems, Danny tried to think of how I can do this as quickly as possible. He proceeded to attempt cyclic permutations of the right hand rule (*e.g.* $\vec{A} = \vec{B} \times \vec{C}$) in order to be able to do a right-hand rule directly for the backward problems. However, after debate, he settled on $\vec{A} = \vec{C} \times \vec{B}$ and $\vec{B} = \vec{A} \times \vec{C}$, which are incorrect and resulted in a sign error for all of his non-physics backward problems.

5.2.3 Initial to final method codes

The initial set of method codes was applied to a small subset of problems (N = 87) from eight different interviews that reflected a variety of methods, as well as the more difficult problems to categorize. All of the final primary methods had high kappa values and the definitions did not change much from the original definitions, but some of the supplemental methods that were more problematic. Four of the initial codes (Guess, Coordinate Reference, Rotation, and Check) were either eliminated or changed significantly and these changes are discussed below.

Originally, there was a Guess code that involved times when the participant would make explicit mention of not knowing the answer, but provides one anyway ("I would say negative. I'm not real sure, it's almost a guess." Lorenzo). However, after the initial coding, it was clear that the Guess category did not provide any additional information and this category was folded into Unclear, since it was unknown how the guess was made.

Coordinate Reference was originally defined as when the participant explicitly referenced (either verbally or gesturally) the instruction sheet or problem statement to verify the direction of coordinates or if they drew coordinate axes on the board. Unfortunately, a verbal or gestural check of direction is often indistinguishable from other reference to the problem statement. For situations when a participant would draw coordinate axes on the board, this was already covered in the Diagram category and the revision of the Rotation code incorporated the times when the axes were drawn at an angle. Thus, the important aspects of Coordinate Reference were covered in other codes and so the category was also eliminated.

The discussion about the two original Rotation codes (Rotation: Vertical and Rotation: Horizontal) made it clear that for the purposes of this study, the direction of the rotation was irrelevant, since it was the fact of the rotation that was interesting. Thus, these codes were combined into one final code: Rotation. There were also other difficulties with the original rotation codes. With vertical rotation, the main point of contention regarded motivation–whether there was a difference between someone who held all of the problems vertically and moved from one to the next (such as Nick) and someone who actively picked up the paper explicitly to help with the problem. Since the main point of the code was to identify whether a rotation happened, regardless of when or why, the definition was revised to reflect this. For horizontal rotation, there were several problems where the rotation was slight and there was debate about whether these should be considered a rotation or just sloppiness, especially in regard to redrawn diagrams. The final code definition states that only rotations of more than $\approx 15°$ should be coded in order to eliminate this issue. This was designed to be a rough marker of intent.

The category of "Check" originally included two distinct behaviors: one where the participant used one primary method to verify the result obtained by a different primary method (such as Nestor's tendency to use Matrix and RHR to check each other) and the other where the participant would make a guess about the solution and then use a primary method to verify that guess. Since every method used on a problem was coded,

Table 5.2: Cohen's kappa for each method code.

Method Code	Kappa	Agreement
Addition	0.931	0.995
Matrix	1.000	1.000
RHR hand	0.853	0.960
RHR frame	0.752	0.874
RHR type	0.898	0.950
Physics	0.731	0.871
Rotation	0.904	0.952
Guess and check	0.672	0.941
Orthogonality	0.723	0.973
Parallel transport	0.792	0.984
Diagram	0.971	0.995
Multiplication	1.000	1.000
Unclear	1.000	1.000

the first type of check provided redundant information. Thus, this aspect was eliminated and the change in focus is reflected in a name change from Check to Guess and Check. The new code makes clear that two elements must be present: a 'guess' and a 'check.' A guess without verification and a check on an answer obtained through another method are therefore explicitly excluded from this code. In the recoding, there was an additional point of discussion regarding the combination of the Guess and Check and Orthogonality codes. When Orthogonality is used to restrict the solution to one axis, the participant may then explicitly choose one of those directions to perform a Guess and Check or try both solutions and decide between them. In both of these cases, the problem would be coded with both Orthogonality and Guess and Check.

With the final code definitions, both coders recoded the initial set of data and the entirety of one additional interview (Victor–chosen for his extensive use of Rotation and Guess and Check). The final kappa values and simple agreement for each method code are presented in Table 5.2. All differences were discussed and resolved and the rest of the interviews were coded using this final set of codes.

5.3 Error code definitions

Errors are also not mutually exclusive. Initially, the errors were coded like the methods, with each problem receiving a code for each relevant error and a No Error code only if no other errors had been made or corrected. However, during the revision process, it became apparent that the number, variety, and overlapping nature of the codes required a different approach.

In the final definitions, each error was revised to be its own code family, which was usually composed of three codes: Error, Corrected, No Error. When a certain type of error contributed to the final response, that type of error was coded as Error. However, there were times when an error was made, but it did not contribute to the final response. This typically occurred because the participant recognized the error and corrected it or because s/he abandoned the approach that led to that error–these situations were coded as Corrected. A code of No Error indicated that there was no evidence for that type of error being committed. Every problem received an error code for each type of error and Cohen's kappa was calculated for each code family. Thus, a problem that received only No Error codes was answered correctly, although the converse is not necessarily true due to corrected errors and multiple errors that cancel each other.

All of the errors can be grouped into the three following categories: errors associated with a primary method (Section 5.3.1), physics errors (Section 5.3.2), and all other types of errors (Section 5.3.3). After presenting these codes, Section 5.3.4 will discuss the major changes between the initial and final code definitions.

5.3.1 Errors associated with primary methods

For each primary method code, there was one associated error code. The focus of this study is on right-hand rules and although errors with Addition or Matrix methods do not have direct bearing on the research question, these methods were often used in conjunction with right-hand rules. Thus, it was important to identify any errors made with these methods.

Addition

The use of addition and subtraction techniques on a cross product problem is incorrect by nature, which meant that this was the only code family that did not have a No Error code. However, as mentioned in Section 5.2.1, both participants who used this method were unsure of its validity and thus, it was important to identify the problems where they chose to abandon this method in favor of a different approach.

Matrix

Errors associated with the Matrix method were typically either problems with setting up the matrix or in making a sign error in finding the determinant. As mentioned in Section 5.2.1, Nestor often used Matrix and RHR in conjunction to provide a check. In cases where the results from each method differed, he would often try to resolve the difference. However, other times, he would simply choose between the two answers.

Right-Hand Rule (RHR)

Since right-hand rules were the focus of this study, this code family was the most complex and was the only code family with more than four codes. Each use of a right-hand rule was given one error code. Thus, it was necessary to revise the initial codes to create a set mutually exclusive codes (this process is discussed in Section 5.3.4).

No Error There were two situations where a right-hand rule could be coded as No Error. One of these was if the right-hand was performed correctly. The other situation was where there was a clear misinterpretation, but the right-hand rule was performed correctly for the misinterpreted situation. For example, the participant clearly stated that a given vector was into the page when it was actually out of the page, but performed the right-hand rule in a manner correctly for a vector into the page.

Inappropriate There were some right-hand rules that were only appropriate for certain situations. For example, the Current right-hand rule was designed to find the magnetic field of moving point charges. However, some participants would use this type of right-hand rule when asked to find the magnetic force on a current-carrying wire. Similarly, Fred and Danny specified that the 3 finger right-hand rule was used only for questions with magnetic force, but both occasionally tried to use it on magnetic field problems. This category also included the inappropriate use of the left hand. If any other right-hand rule error was made (such as Direction or Order errors), the Inappropriate code would take precedence.

Direction One of the ways to achieve a sign error with a right-hand rule is by reversing the direction of one of the vectors. For example, for the problem shown in Figure 5.4, some participants would curl out of the page and toward the left ($-\hat{x}$ direction) instead of toward the right ($+\hat{x}$ direction). During the revision process, this code was clearly distinguished from a Misinterpret code, which required an explicit statement of the wrong direction. For a Direction–Error or Direction–Corrected code, the participant must either not state the direction or explicitly state the correct direction and use the opposite direction. There is some overlap between this code and some of the physics error codes. For example, if a participant implicitly reverses the direction of the current, this would be coded as both RHR: Direction–Error and Physics: Charge–Implicit.

What is the direction of $\vec{A} \times \vec{B}$?

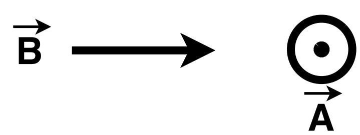


Figure 5.4: Example of a problem where a RHR: Direction–Error might occur.

Order In addition to switching the direction of a vector, another way to get a sign error would be by reversing the order of the vectors, such that the participant was performing $\vec{B} \times \vec{A}$ instead of $\vec{A} \times \vec{B}$. If this reversal was due to an explicitly stated incorrect physics equation (*e.g.* $\vec{F} = \vec{B} \times \vec{v}$), it would be also be coded as Physics: Other–Error. A reversal of order error is closely tied to the non-commutativity of the cross product–an issue raised by Scaife and Heckler (2010). As mentioned in Section 3.2.1, the follow-up questions directly addressed the issue of commutativity. It is interesting to note that there were several students who clearly stated that $\vec{B} \times \vec{A}$ and $\vec{A} \times \vec{B}$ would be in opposite directions and yet still reversed the order of the vectors on some problems. This will be discussed in more detail in Chapter 6.

RHR: Other The three right-hand rule errors mentioned above–Inappropriate, Direction, and Order–comprise the most clearly identifiable errors that participants committed. However, there were other less identifiable errors that were still clearly related to the application (or misapplication) of a right-hand rule. One of the more common of these errors was when a participant using a Current right-hand rule does not curl to the appropriate location. While this was clearly a Physics: Observation Location–Error and related to a misapplication of the Current right-hand rule, the exact nature of the misapplication was unclear. In order to create mutually exclusive RHR codes, this category also included situations where multiple RHR errors are committed, such as both a Direction and an Order error. However, as mentioned above, if there are multiple errors, but one of them is an Inappropriate error, the Inappropriate code takes precedence over the multiple errors.

5.3.2 Errors associated with physics pnowledge

There were three main types of physics errors: Observation Location (Obs Loc), Charge, and Electric Field (Efield). In addition to these error types, other less common physics errors were all grouped together into a Physics: Other category. Every physics problem received a Charge, Efield and Other code and every magnetic field problem received an Obs Loc code.

Observation Location (Obs Loc)

The cross product that is used to find the magnetic field of a moving point charge ($\hat{B} = q\hat{v} \times \vec{r}$) involves the position vector, $\vec{r}$, which points from the source of the field to the location where that field is observed. Most diagrams included in physics problems (including those used in this study) depict the observation location and not the position vector. Thus, compared to both magnetic force problems and non-physics problems, an additional step was needed. When a participant did not appropriately account for the observation location, either implicitly or explicitly, the problem was coded as a Physics: Obs Loc–Error. One of the common ways this error manifested itself was in the use of the Current type right-hand rule when the participant would not curl to the appropriate location. Some participants explicitly stated that they did not know how the observation location would impact the solution. For example, when asked what he found difficult about the first problem, Sebastien responded, "Well, I'm not sure how it would change if the observation location was different."

Charge

One of the many difficulties that is added when moving a cross product problem from a non-physics to a physics context is the effect of charge. Participants struggled with this issue when distinguishing between positive and negative point charges, as well as between conventional and electron current. Errors in dealing with the charge were the most common physics errors. Unlike most other code families, there was a distinction between implicit and explicit errors–making this a 4-level code (Explicit, Implicit, Corrected, No Error). Problems coded as Physics: Charge–Implicit usually involved the participant neglecting to consider the charge issue at all. Explicit errors were typically situations where the participant would "double-correct" for the charge by switching direction at the beginning and at the end of the problem or where the participant explicitly stated that conventional current was the direction of electron flow. Example of this reasoning are shown in the following excerpts:

> Magnetic force 1: "So, conventional current means you have electrons flowing up, so you have to use your left hand. It's out of the page, so that means the force is to the left, or negative x direction." Danny

> Magnetic field 6: "Um, conventional current is to the, plus x direction, so the electron current is in the negative x direction, so the velocity is in the negative x direction..." Humberto

Electric Field (Efield)

Previous research on student understanding of magnetic fields (*c.f.* Section 2.1) found that one of the most common errors involved not distinguishing between electric and magnetic fields (Guisasola et al., 2004; Maloney et al., 2001). While some participants used analogies to electric field appropriately, this confusion between electric and magnetic fields was also evident. Like Charge, Efield was a 4-level code, where Explicit errors involved situations where the participant would explicitly reason from an invalid or inappropriate analogy to electric field. Implicit errors were those when the connection was not explicitly stated, but the response given was consistent with an invalid or inappropriate analogy to electric fields. For example, several participant stated that the magnetic field was zero at the center of a loop of current, but did not provide an explanation for this response. The following excerpts represent an explicit use of the same error:

> Magnetic field 13: "So, conventional current flows clockwise, so, but that, it says what is the direction at the center of the loop. So, it's going in the negative x, but since it's at the center, the only reason why it goes in the negative x at an observation out here is because you have those vectors that, don't complete, they can't, they kind of cancel out, but **you have a resulting vector, but since it's in the center, they're all going to cancel out, so it just has to be, zero magnitude.**" Danny

Physics: Other

As stated above, there were other physics errors which were grouped into a final physics category. One of the most common of these was the explicit use of incorrect or inappropriate equations (*e.g.* $\hat{B} = \hat{r} \times \hat{v}$ or $\vec{F} = q\vec{v} \times \vec{B}$ on a magnetic field problem). A second issue, also noted in Scaife and Heckler (2010), was when the participant would switch the direction of the magnetic field when the field was represented with magnets (see Figure 5.5 for an example of this type of problem).

What is the direction of the force on a positive charge that is moving in the $+x$ direction due to the magnets shown?

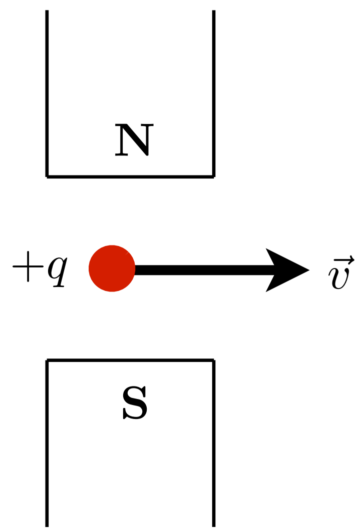


Figure 5.5: Example of a problem with the magnetic field represented using two magnets.

5.3.3 All other types of errors

In addition to physics errors and those associated with primary methods, every problem received error codes for three other error types: Misinterpret, Match, and Other.

Misinterpret

There is always the possibility of misinterpreting the diagram or misreading the problem statement and these situations were coded as Misinterpret–Error or Misinterpret–Corrected. There are numerous ways to misinterpret a problem, but one of the most common was to treat the into the page symbol ($\bigotimes$) as the out of the page symbol ($\bigodot$). This was only coded as Misinterpret if the participant explicitly stated the wrong direction–otherwise, it was coded as a RHR: Direction error. Other misinterpretations involved answering a question other than the one that was asked or treating the vector symbol over $\vec{A}$ as the vector direction. One of the most interesting misinterpretations was explained by Pablo at the end of his interview when he said:

> "I just realized that the previous set, I was confused cause I thought it was like, a non-moving particle or a non-moving vector, that, I forgot that diagram meant into or out to the page."

Match

One of the difficulties in any cognitive activity that requires alignment of reference frames is the possibility of a mismatch. In this case, once a right-hand rule is performed, one must then align the result obtained with the given coordinate system. This can often result in a mismatch between the gestural and the verbal response. For example, after correctly performing a Standard right-hand rule in the Table frame, the participant's thumb points upward, but s/he says positive y instead of positive z. This was one of the most corrected error types. When the participant did not provide a gestural (or diagrammatic) response or did not give a final response, the problem was coded as Match–N/A in order to distinguish it from those problems where a match was required and there was no error. Additionally, a Match–No Error code would be assigned regardless of the correctness of the answer as long as the gestural response matched the verbal response.

Other

Numerous errors have been described above; they do not comprise an exhaustive list and any error that did not fall into one of these categories was coded as Other–Error or Other–Corrected. This category was primarily composed of situations where the participant did not give a final response, gave a guess without any justification, or where there was not enough gestural or verbal evidence to determine the origin of the error.

5.3.4 Initial to final error codes

As noted at the beginning of this section, the primary changes to the error codes was in how the codes were applied and in changing the right-hand rule code family to be one set of mutually exclusive codes. However, the interrater reliability process yielded other substantive changes to the definitions. Throughout the revisions, problems with an Unclear method code tended to shift between various codes (Other, Unclear Error, Unclear). The final code definitions reflect the decision to eliminate these problems from further analysis due to the lack of information they provide.

Another revision was the elimination of the Incomplete code. This code was originally designed to catch those situations where what the participant did was correct, but the problem was not completed. There were two main ways this would happen. The first case was a participant who would solve the first step of 2-step problem, but not complete the problem. Since this was a case of not answering the question that was asked, this piece of the Incomplete code was incorporated into Misinterpret. The second case was when a participant would use Orthogonality to correctly determine the axis of the cross product, but would not choose between the positive and negative. Although Orthogonality was used correctly, the lack of a response indicated that something else was going on. Therefore, the Other category was revised to include any problem where the participant did not give a final response. With the relevant information covered by other Misinterpret and Other, the Incomplete code was eliminated.

One of the biggest discussions early in the revision process was the distinction between Misinterpret and RHR: Direction since some of the most common misinterpretations resulted in the participant reversing the direction of one of the vectors. In order to avoid this overlap, the revised definitions stated that a reversal of direction should be coded only as Misinterpret if the participant explicitly stated the wrong direction. If the participant didn't state a direction or stated the correct direction and used the opposite direction for the right-hand rule, this would be coded as only a Direction error.

A reoccurring theme, particularly in discussions about the Physics codes, was what needed to be explicit and what didn't in order to be coded a certain way. Thus, one of the major changes in the final definitions is the addition of a split code (Explicit/Implicit) for the Charge and Efield error types. This dramatically increased the agreement for these two code families.

The interrater reliability process for the error codes involved multiple sessions of coding and resolving differences. The error coding was based more on gestures than on speech and thus, instead of excerpts from the transcript, descriptive examples were provided. Perhaps one of the most helpful aspects of the interrater reliability process was in fleshing out appropriate and clear examples that also addressed the possibilities for overlap between codes. The problems used for the interrater reliability process were drawn from twelve different interviews and were chosen to reflect the largest variety of situations and some of the most difficult problems to categorize. The final kappa values and simple agreement for each code family are presented in Table 5.3. After achieving an appropriate level of agreement, all of differences were discussed and resolved, and the rest of the interviews were coded using this final set of codes.

5.4 Summary of coding

This chapter represents a systematic approach to categorizing the many types of methods used and errors made by participants solving cross product direction questions. These categories were created by constantly comparing the code definitions to the data. The reliability of the codes was assessed using a second coder and Cohen's kappa as a measure of interrater reliability. The method and error codes are displayed graphically in Figures 5.6 and 5.7, respectively.

Table 5.3: Cohen's kappa for each error code family.

Error Code Family	# Codes	Kappa	Agreement
Addition	2	1.000	1.000
Matrix	3	1.000	1.000
RHR	9	0.710	0.791
Physics: Charge	4	1.000	1.000
Physics: Efield	4	0.816	0.960
Physics: Observation Location	3	0.784	0.969
Physics: Other	3	0.783	0.960
Misinterpret	3	0.797	0.933
Match	4	0.865	0.944
Other	3	0.775	0.956

The number and variety of methods and errors highlights the need for this kind of qualitative analysis to illuminate the quantitative analysis already presented. In particular, the error codes identify the sources of many of the incorrect responses and provide a starting place to think about instructional change. The following chapter will present an analysis of how these codes relate to each other and to various problem features, in order to deepen our understanding of these issues.

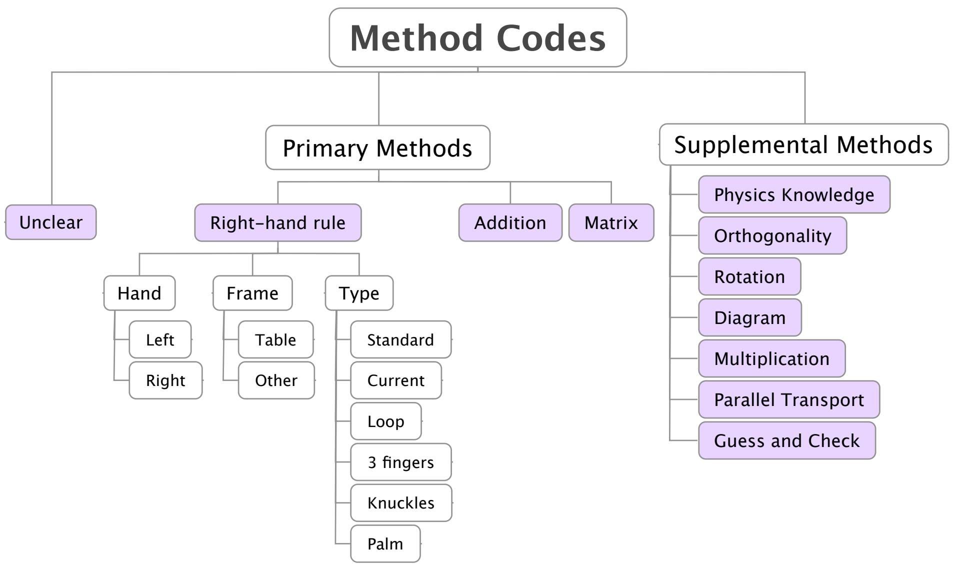


Figure 5.6: Graphical depiction of method codes and sub-codes.

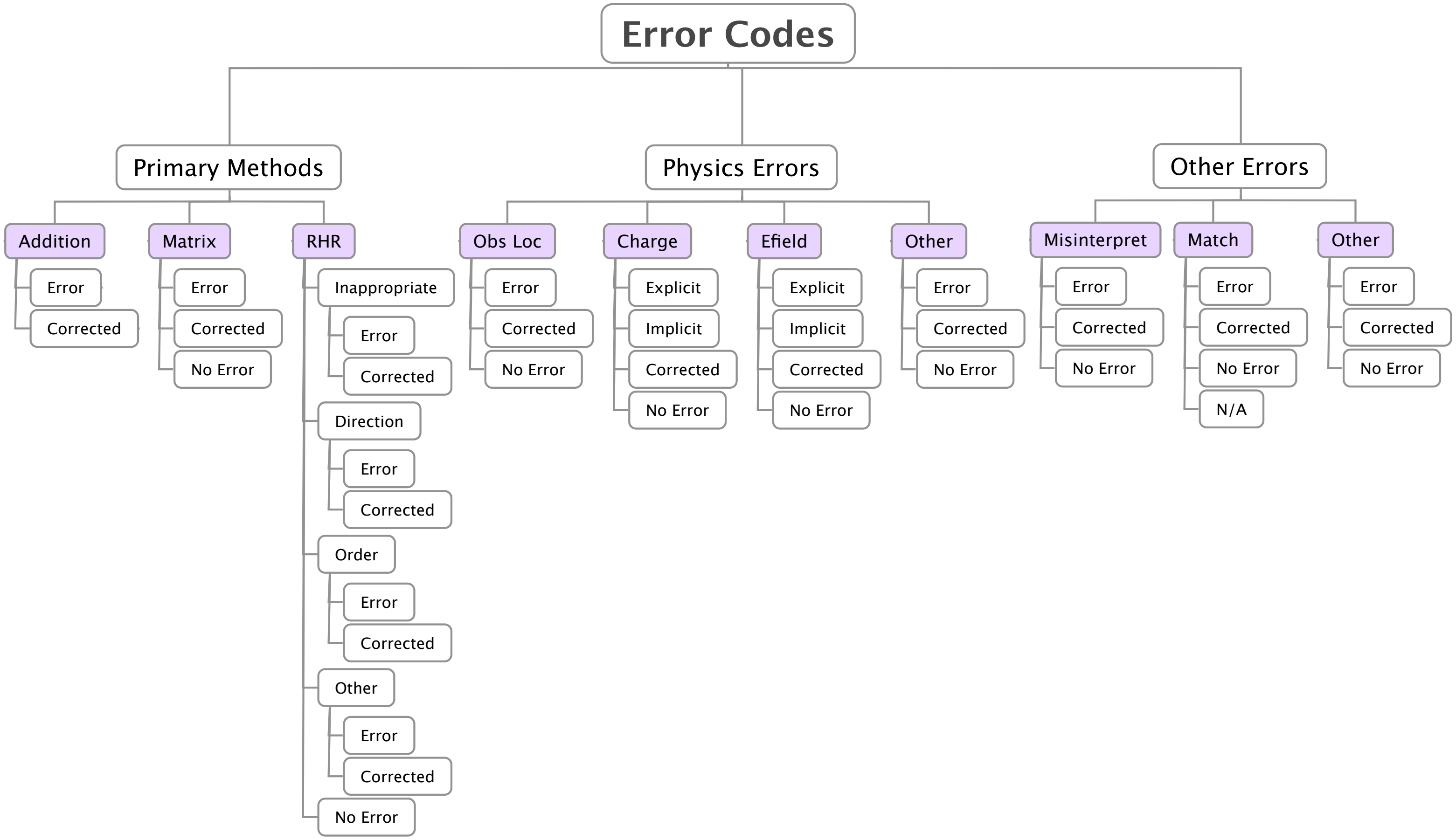


Figure 5.7: Graphical depiction of error codes and sub-codes.

Chapter 6

Qualitative analysis: results and discussion

Chapter 5 presented the process and results of a qualitative coding of the data for the methods participants used and the errors that they made. The identification of these methods and errors provides insight into student understanding of cross product direction. Examining these codes in more depth will reveal some of the individual differences that were obscured in the quantitative analysis presented in Chapter 4.

This chapter will explore several interactions between the methods participants used, the errors they made, and the problem features that were varied. Section 6.1 will provide an overview of how and when participants used the various methods. The following sections will then explore the relationships between:

- Right-hand rule methods and right-hand rule errors (Section 6.2);
- Physics errors and physics problem features (Section6.3; and
- Other errors and problem features 6.4.

Finally, Section 6.5 will discuss how three supplemental methods helped these participants to mitigate some of the difficulties discussed in this chapter.

6.1 Individual differences in the use of methods

The participants in this study used a wide variety of methods, both primary and supplemental, to solve cross product direction questions. There were up to fifteen possible methods (including different right-hand rules); yet, the total number of methods used by each participant ranged from four to twelve (median=6). On any given problem, most participants only used one or two methods; the largest number of method used on a problem was seven.

Every participant explicitly used prior physics knowledge to answer physics problems. Most used Rotation, Orthogonality, or Guess and Check methods as well. These three methods played a role in reducing the difficulties due to some problem features (discussed in Section 6.5). The remaining Supplemental methods (Diagram, Parallel Transport, and Multiplication) were used by few participants and their use has already been discussed in Section 5.2.2.

Right-hand rules were the predominant method for solving the physics problems. Some participants relied primarily on one type of right-hand rule, such as Humberto (Knuckles) or Sebastien (Current). Others used a mix of right-hand rule types, frames, and even hands. For example, on Magnetic Field Problem 5, Danny used three different right-hand rule types and both hands in attempting to solve the problem. The individual differences in method use on physics questions were largely related to the right-hand rule, but this was not the case for the non-physics problems. With few exceptions, if a participant used a right-hand rule on the non-physics problems, s/he used only one type (usually Standard) in one frame (either Table or Other) and with the right hand.

There were four participants whose main method on these problems was not a right-hand rule. Jerry used only Orthogonality to determine the axis of the answer, but gave both the positive and negative direction as his response. At the end of the interview, it was clear through questioning that he did not realize he could use a right-hand rule. When pressed to give a single answer, he used a Matrix method instead. Nestor used the Matrix method as his primary means of answering non-physics problems, although he did occasionally use a right-hand rule to check his response. Patty also used matrices, but only after realizing that in her initial attempts, she was actually adding vectors.

Vector addition was Rose's method of choice for the non-physics questions, although she also occasionally used a right-hand rule and seemed unsure of which method was most appropriate. This use of alternate methods for solving non-physics problems suggests that further exploration into student background is needed (*e.g.* Section 7.3).

6.2 Right-hand rule errors by method

There were six different types of right-hand rules used, with the hand and frame varying; four types of errors were committed or corrected. These methods and errors are depicted graphically in Figure 6.1.

By collapsing all of the right-hand rule error types into Error and Corrected categories, it becomes clear that a relatively small percentage (13%) of the right-hand rules used actually had an error associated with them; almost a third of these did not contribute to the final solution (see Figure 6.2). This indicates that although misapplied right-hand rules do contribute to students answering questions incorrectly, they are not the only factor.

Examining the right-hand rules that had errors associated with them (see Figure 6.3), we see that some types of errors were more common than others, while some were more often corrected than others. The most uncorrected errors involved reversing the order of the vectors, which is consistent with the findings of Scaife and Heckler (2010). On the other hand, the use of an inappropriate right-hand rule was more often than not either corrected or abandoned. These errors will be discussed in more detail below.

6.2.1 Hand

The majority of right-hand rules used did, in fact, use the right hand. The left hand was used on less than 2% of all right-hand rules, and then only on physics problems. As might be expected, half of these "left-hand" rules had associated errors ($N = 21$), primarily Inappropriate errors. There were three participants who used the left hand and they did so in different ways. Fred and Danny both used the left hand in an appropriate

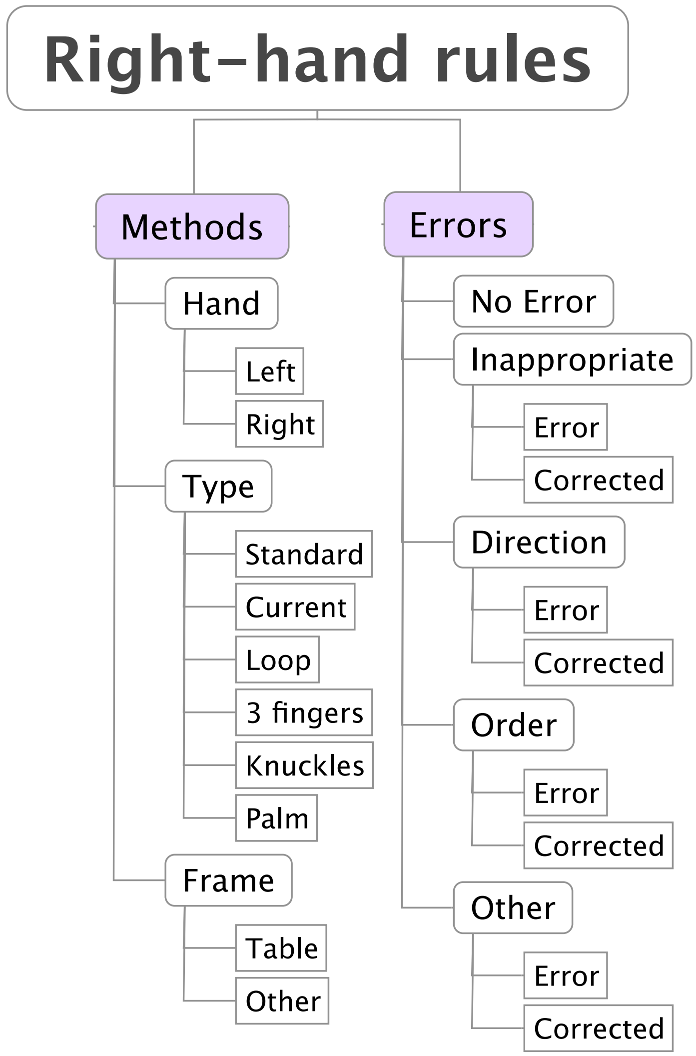


Figure 6.1: Right-hand rule methods and errors

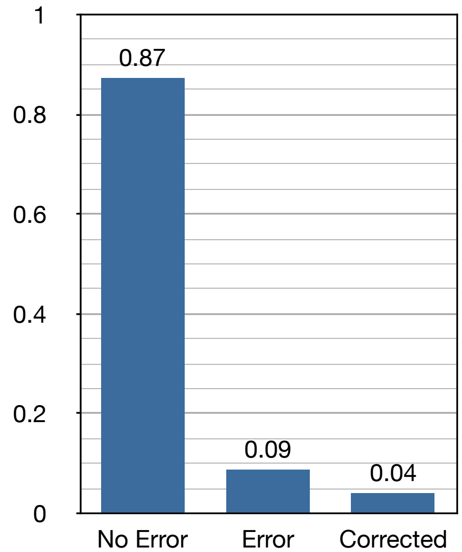


Figure 6.2: Percentage of Right-hand rules coded as No Error, Error, or Corrected, regardless of error type

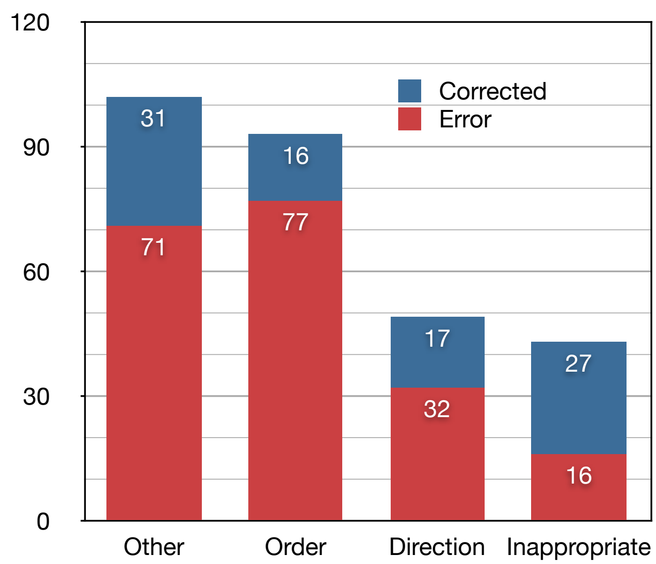


Figure 6.3: Frequency of Right-hand rule error types

manner for negative charges. Also, Danny, and to a lesser extent Fred, had the tendency to try to use $\vec{F} = q\vec{v} \times \vec{B}$ on magnetic field problems. This error accounted for half of the Inappropriate errors ($N = 8$) and all of these errors were corrected. This behavior is consistent with the claim made in Section 4.2.1 that the longer response time for the Magnetic Force group on the magnetic field problems was due to interference from their recent instruction on magnetic force.

All of the uncorrected Inappropriate errors with the left hand were committed by Jerry, who seemed to use his left hand without realizing it. This unintentional use of the left hand is something that many physics teachers see on exams (*c.f.* Figure 1.3). It is possible that this may be due in part to the need to write with the right hand; since this study did not require participants to write, this issue was not present.

6.2.2 Frame

There were more errors made when the right-hand rule was performed in a reference frame other than the table than when performed in the frame of the table (14% for Other, 11% for Table); yet, these errors were also more often corrected. There were some differences in the types of errors for the table and other frames (see Figure 6.4). Since most participants tended to stay within the same frame throughout the interview, it is possible that these differences are due more to individuals than to the use of the frame itself.

In addition to the differences for the right-hand rule types between context that were discussed in Chapter 5, there were also differences in both the number and type of errors. The Knuckles right-hand rule was only used by Humberto and he made no errors (corrected or uncorrected) with his use of this right-hand rule. Similarly, only Nick used the Palm right-hand rule and he made only one Order error and one Other error, which he corrected. For both Humberto and Nick, almost all of their incorrect responses were due to error not associated with a right-hand rule. This was not the case for the other right-hand rule types.

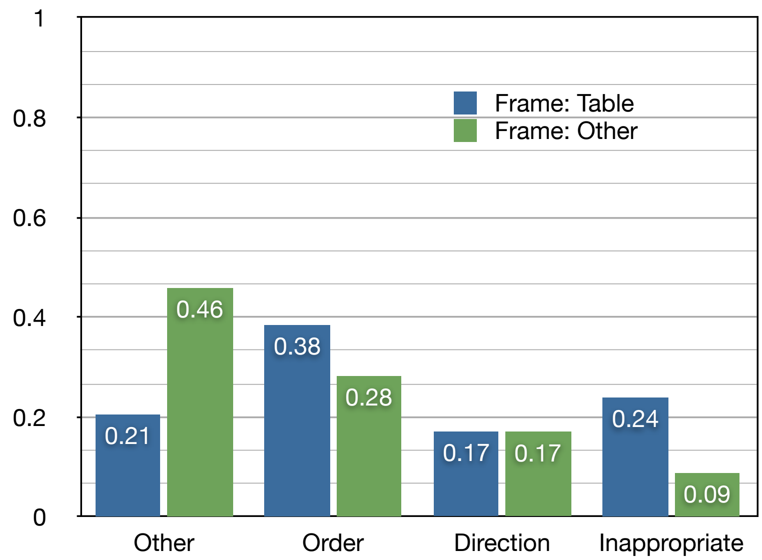


Figure 6.4: Percentage of Right-hand rule errors by error type and Frame

6.2.3 Type

As discussed in Section 5.2, two of the right-hand rule types (Knuckles and Palm) were used by only one participant. Humberto made no errors with the Knuckles right-hand rule and Nick made only two errors with the Palm right-hand rule and he corrected one. Thus, for Humberto and Nick, all of their incorrect responses were due to errors that were not associated with a right-hand rule. This was not the case for the other right-hand rule types.

The 3 fingers right-hand rule was used by three participants: Andrea, Fred, and Danny. Both Fred and Danny only used this right-hand rule for $\vec{F} = q\vec{v} \times \vec{B}$, but they sometimes tried to use this rule on magnetic field problems; this accounts for all of the Inappropriate errors. Andrea used this right-hand rule almost exclusively. All of the other errors with this right-hand rule were hers: a large number of Order errors ($N = 22$), but also some Other errors and one Direction error. Of Andrea's 32 errors (four of which were corrected), 27 of them were on problems with physically awkward orientations. One of the ways she dealt with this awkwardness was to switch her fingers, resulting in a reversal of order. Unlike many of the participants, Andrea was unaware that this would give her a sign error due to the non-commutativity of cross products. Near the end of the interview, the following exchange took place:

> Interviewer: "And if here, you were doing, instead of A cross B, you were doing B cross A?"
> Andrea: "You were doing B cross A. I feel like it would be the same thing."

As Scaife and Heckler (2010) point out, if this were a systematic error, there should be a larger number of Order errors. However, it is clear that for Andrea, reversing the order of the vectors is one of her ways of dealing with the physical discomfort of some orientations.

As discussed in Chapter 5, both the Loop and Current right-hand rules were used almost exclusively on physics problems. The Loop right-hand rule was used only on magnetic field problems and there were only nine errors (15%) associated with the use of this rule; these were committed by seven different participants. The uncorrected errors were predominantly instances where the participant applied the loop rule in a situation where the current was actually straight (*i.e.* Inappropriate error).

Some kind of error was associated with almost a third of the Current right-hand rules used (49 out of 178). More than half of these ($N = 27$) were Other errors. Almost all of these Other errors involved a Physics: Observation Location error. The Direction errors ($N = 4$) were all due to an implicit use of electron current. These will be discussed in more detail in Section 6.3. The remaining errors ($N = 18$) were Inappropriate errors that were predominantly corrected attempts to use the Current right-hand rule on magnetic force problems involving a current-carrying wire.

The Standard right-hand rule was used by all the participants except Humberto and Andrea. It was used on 80% of the non-physics, 80% of the magnetic force problems and 40% of the magnetic field problems. Approximately 10% of the Standard right-hand rules had some kind of error associated with them (188 out of 1733); the distribution of the these errors by type is presented in Figure 6.5.

Between Jerry's use of the left hand and various participants' attempts to use $\vec{F} = q\vec{v} \times \vec{B}$ on magnetic field problems, all of the Inappropriate errors on the Standard right-

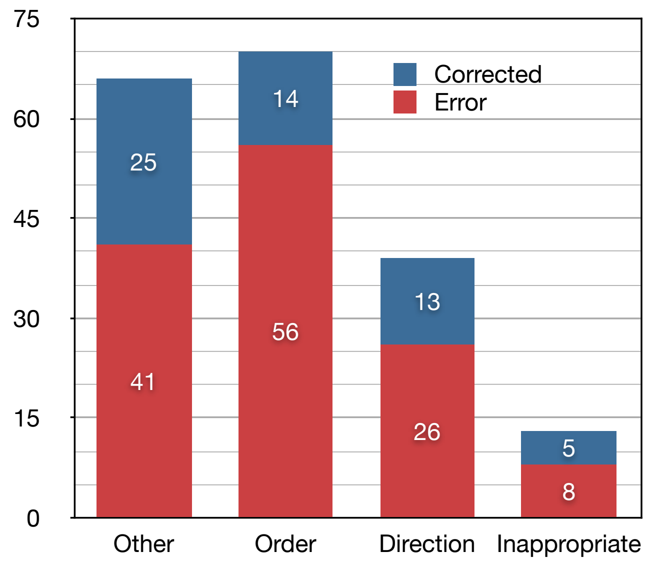


Figure 6.5: Frequency of Standard Right-hand rule errors by error type

hand rule have been accounted for. Many of the Other errors involve multiple errors (a Direction and Order error) or the participant curling his/her hand in an apparently random direction.

On physics problems, there were few Direction errors with the Standard right-hand rule; all of these were typically connected to a physics error, such as reversing the direction of the position vector. For the non-physics problems, there seemed to be two groups of participants committing Direction errors. The first group was those that reversed the vector that was along the z-axis. This was probably a misinterpretation of the $\otimes$ and $\odot$ symbols, but since the direction was not clearly stated, it was coded as a Direction error. If these were the only errors, one would expect that there would be more Direction errors for problems in the non-xy planes. However, the second group reversed the direction of a vector in the xy-plane. It is interesting to note for these errors, the vector reversed was usually the one whose head points to the tail of the other. This was the primary way that difficulty with parallel transport manifested itself.

Clearly the most prevalent error for the Standard right-hand rules is the Order error. Unlike Andrea's Order errors with the 3 finger right-hand rule, these were evenly distributed between the physically easy and physically awkward orientations. However, these errors were much more common for working backward problems that asked for the direction of the first vector ($\vec{A}$) than for any other type of problem, as demonstrated in Figure 6.6. One of the more articulate participants noted that these problems were particularly difficult:

> Magnetic Field Problem 2: "It's hard to start without the initial direction,...because what I see on the page is two points and so I'm thinking this [points fingers left, from charge to observation location] automatically, but really, that's the second step in finding a direction [points fingers up and curls to observation location]." Barry

The "automatic" response that Barry mentioned (to point the fingers in the direction given) could indicate why problems asking for the first vector were more likely to result in an Order error even for those participants who know that the cross product is non-commutative.

6.3 Physics errors and features

As mentioned above, some of the right-hand rule errors were directly connected to physics errors; there were other problems where the physics errors were unconnected to the right-hand rule. This section will discuss these physics errors and how they relate to the physics features of the problems.

6.3.1 Observation Location errors

Out of all of the magnetic field problems ($N = 376$), a little less than 10% ($N = 35$, four corrected) had an Observation Location error (four of these were corrected). As noted above, the majority of these problems involved the use of the Current right-hand rule–the participant either did not curl the fingers far enough or did not curl in the right direction. This was a systemic issue for Ericka (11/14) and for Sebastien (8/14) who both used the Current (or Loop) right-hand rule for all of the magnetic field problems. After

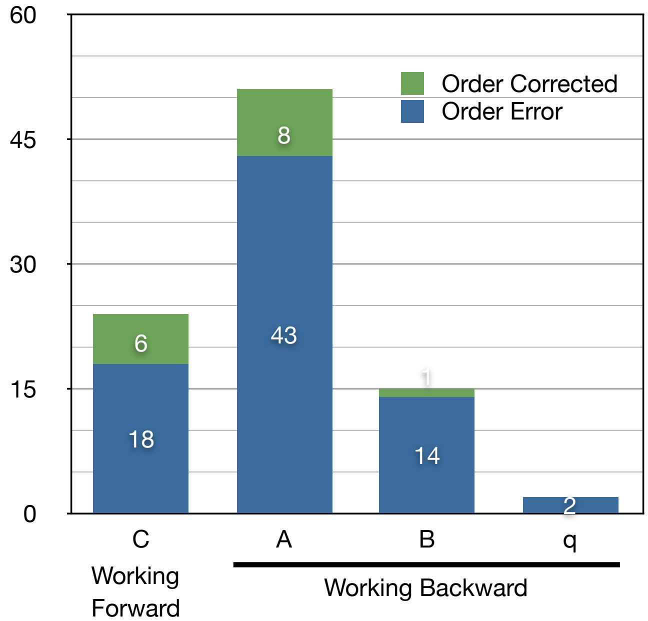


Figure 6.6: Frequency of Right-hand rule Order errors by the type of reasoning required

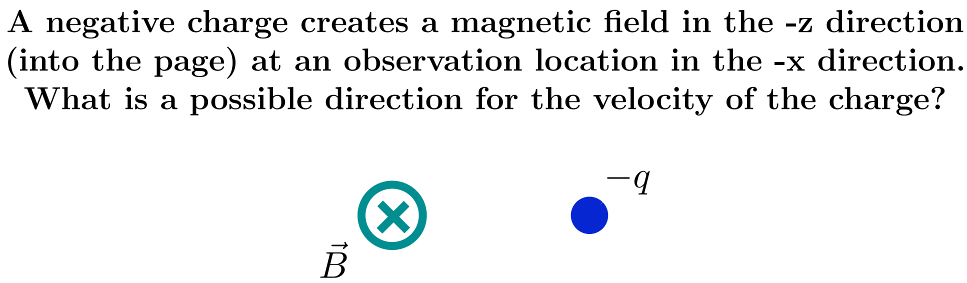


Figure 6.7: Magnetic Field Problem 2

the first problem, Sebastien explicitly stated, “Well, I’m not sure how it would change if the observation location was different.” For the those who made an Observation Location error using a different right-hand rule, the error was often due to a either a reversal of the position vector or was made only on Magnetic Field Problem 2, which had many complicating factors (discussed below).

Magnetic Field Problem 2 (shown in Figure 6.7) was clearly the most difficult in the entire study. There were almost twice as many errors made on this problem as on any other problem ($N = 55$); the only kinds of errors that were not made were Match errors and Electric Field errors. It involved working backward to find the velocity; it had a negative charge and a physically awkward orientation. For those in the Magnetic Force group, the fact that they were given the magnetic field seemed to cue the use of $\vec{F} = \vec{v} \times \vec{B}$. Regarding the Observation Location errors noted above, many participants did not recognize that the magnetic field was given at the observation location. The most common incorrect responses for this problem were velocity in +x or -x direction. This problem provides an excellent example of many of the problem features that cause students difficulty.

6.3.2 Charge errors

More than 10% of all physics problems (63/576) had a charge error and only six of these were corrected. As expected, the majority of these errors ($N = 27$) were on problems with a negative point charge and were implicit errors, where the participant would ignore the charge. The explicit errors for these problems ($N = 9$) were due to situations where the participant would double-count the charge or would explicitly consider the charge, but was unsure how to deal with it. Olga demonstrates this on the following problems:

> Magnetic Field Problem 5: "What is the sign of the charge on the particle? Don't know that it matters..."

> Magnetic Field Problem 7: "...but now I'm questioning the whole negative charge thing...I don't know if it's opposite because it's negative. I don't know if we're always assuming. It's a good question I should get answered at some point. I should find that out [laughs]...but I guess, I'm just gonna stick with the charge doesn't matter..."

In addition to Charge errors on problems with negative point charges, there were also a significant number of errors ($N = 20$) on problems with a current-carrying wire where the direction of the conventional current was given. On these problems, the difficulty lay in the connection between conventional current and electron current, as demonstrated by Danny and Humberto:

> Magnetic Force Problem 1: "So, conventional current means you have electrons flowing up, so you have to use your left hand. It's out of the page, so that means the force is to the left, or negative x direction." Danny

> Magnetic Field Problem 6: "Um, conventional current is to the, plus x direction, so the electron current is in the negative x direction, so the velocity is in the negative x direction..." Humberto

Many traditional physics texts do not address the issue of electron current, so this may be an issue that is more applicable to the Matter &Interactions curriculum.

6.3.3 Electric Field (Efield) errors

There were a variety of ways in which participants reasoned from invalid or inappropriate analogies to electric field or provided an answer consistent with such reasoning. Although it only occurred on 4% of the physics problems ($N = 24$, with two corrected), these errors were committed by 13 of the 27 participants at some point. Eight of these participants stated that the magnetic field at the center of a loop of current would be zero on at least one problem ($N = 11$). This was typically coded an implicit error, unless the analogy was clear, as it was here:

> Magnetic Field Problem 13: "So, conventional current flows clockwise,...but that, it says what is the direction at the center of the loop. So, it's going in the negative x, but since it's at the center, the only reason why it goes in the negative x at an observation out here is because you have those vectors that...don't complete, they can't, they kind of cancel out, but **you have a resulting vector, but since it's in the center, they're all going to cancel out, so it just has to be...zero magnitude.**" Danny

Others found that the force on a moving charge would be parallel to the field when the field was represented with a magnet ($N = 7$).

> Magnetic Force Problem 2: "Direction of a force due, on a positive charge, it's moving in the plus x direction due to the magnets...I would say, the force would be down...negative y, because it's pulled towards,...the south and pushed away from north." Ana

> Magnetic force 2: "So this is north and they'll attract each other, then that would go away...and the south pole will push down, so the direction of force would be in the negative y direction." Bill

The remainder of the Electric Field errors involved trying to relate the magnetic field to the electric field of point charge or a dipole, such as on Magnetic Field Problem 1:

> "Let's say that if it's going out of the negative and into the positive, let me think about a dipole. How does a dipole work? Negative, positive, that way, that way, out of the positive into the negative, so I'd say that the direction is straight up." Lorenzo

> "The magnetic field points away from the positive charge, so I'm going to say that it's moving with the velocity in the positive x direction." Patty

These invalid analogies between electric and magnetic field are seen throughout the literature (*c.f.* Section 2.1). In this study, lthough there are not a large number of Electric Field errors, it is clear that many of them are cued by very specific problem features, such as a loop of current or magnets to represent the magnetic field.

6.3.4 Other physics errors

Among the other physics errors ($N = 29$), there were two main trends. One of these was the use of an invalid or incorrect equation, such as using $\vec{v} \times \hat{B}$ on magnetic field problems or using $\vec{r} \times \hat{v}$ instead of $\vec{v} \times \hat{r}$. The other trend was the reversal of the direction of the magnetic field on problems where the field was represented as a magnet. Rose and Chris both committed this error for all three magnet problems; although Victor initially did the same, he corrected his error.

6.4 Other errors and features

In addition to the right-hand rule errors and physics errors, there were three other types of errors: Misinterpret, Match, and Other – these will be discussed briefly in connection with relevant problem features.

6.4.1 Misinterpret errors

At some point during the interview, most of the participants misinterpreted the diagram or misread the problem statement, though these misinterpretations occurred on less than 5% of all problems ($N = 101$). However, as discussed in Section 6.2.3, some of the Direction errors with the right-hand rule may have been misinterpretations that were not stated explicitly. This kind of direction error that reverses the meaning of the $\otimes$ and $\odot$ symbols was perhaps the most common of the misinterpretations. Another common misinterpretation was to treat the vector symbol over $\vec{A}$ as the direction of the vector itself. Of all the participants, Pablo had the most difficulty with this type of error. He made both errors mentioned above, but he also tried to treat the $\otimes$ and $\odot$ symbols as "a non-moving particle or a non-moving vector."

Significantly more of these misinterpretations involved orientations in the non-xy plane ($N = 63$) than orientations in the xy-plane ($N = 38$). This confusion over symbols contributed to the overall effect due to plane of the vector on the difficulty noted in Chapter 4. This indicates that the effect of plane is not solely due to alignment effects– something that should be further explored (*e.g.* Section 7.3).

6.4.2 Match errors

Match errors differed from most of the other errors discussed here. Although a mismatch occurred on just under 5% of all problems ($N = 110$), more than half of these errors were corrected ($N = 59$). Also, there were only four participants who did not have any match errors: Erin, Jerry, Nestor, and Sebastien. Both Jerry and Nestor had significantly fewer problems with a gestural response than most of the other participants. For the rest of the participants, the number of mismatches ranged from 1 to 16, with a median of 5; yet, there were no particular problems or problem features which seemed to have greater mismatch than any other. As will be discussed in Section 6.5, the use of Orthogonality did have an effect on these errors.

6.4.3 Other errors

The total number of Other errors was slightly more than the Match and Misinterpret errors ($N = 121$); yet, these errors were rarely corrected ($N = 2$). Four participants committed 80% of the Other errors: Jerry ($N = 61$), Patty ($N = 18$), Pablo ($N = 10$), and Ericka ($N = 7$). Both Jerry and Patty used Orthogonality on a significant portion of their non-physics problems. In Jerry's case, he stopped and gave both the positive and negative directions as his response, whereas Patty attempted to use a heuristic drawn from commutative multiplication (discussed in Section 5.2.2). Pablo's way of misinterpreting the $\otimes$ and $\odot$ symbols significantly contributed to these Other errors. Finally, on thirteen of the sixteen non-perpendicular questions, Ericka did not give a response (this includes several Unclear methods). For Ericka, the angle between the vectors is a stumbling block, but one that she did not try to get over. On a similar note, although

Samantha did very well on all of the problems (90% correct), on 3 of the 4 problems that were acute and tail-to-tail, she gave a response of zero, "because they both go the same direction." For both Sam and Ericka, we see evidence of how the angle between the vectors can influence performance.

6.5 Impact of supplemental methods on difficulties

This section has discussed numerous errors committed by participants on cross product problems. As we look toward how to help students with these difficulties, the first place to start is with the students themselves. As noted in Section 6.1, there were three of the Supplemental methods that were used by most of the participants: Rotation, Guess and Check, and Orthogonality. Each one of these suggests an attempt to compensate for a difficult element of the problem and this section will evaluate the impact these methods have on the relevant problem features and errors.

6.5.1 Rotation

Several of the participants were asked why they chose to rotate the paper. Their responses indicate that they are using this method to mitigate the difficulties associated with the orientation of the vectors. In order to determine whether this method was having the desired effect, right-hand rule errors were compared across the plane of the given vectors, both for problems when Rotation was used and problems when Rotation was not used (Figures 6.8). This comparison shows that when Rotation was used, there were a smaller number of errors overall, but also a reduction in the gap between problems in the xy-plane and the non-xy plane. When physical discomfort is compared with and without rotation, as shown in Figures 6.9, as similar reduction occurs. These results demonstrate that the rotation the paper can reduce some of the difficulties associated with orientation and point toward a possible direction for instruction intervention (*c.f* Section 7.2).

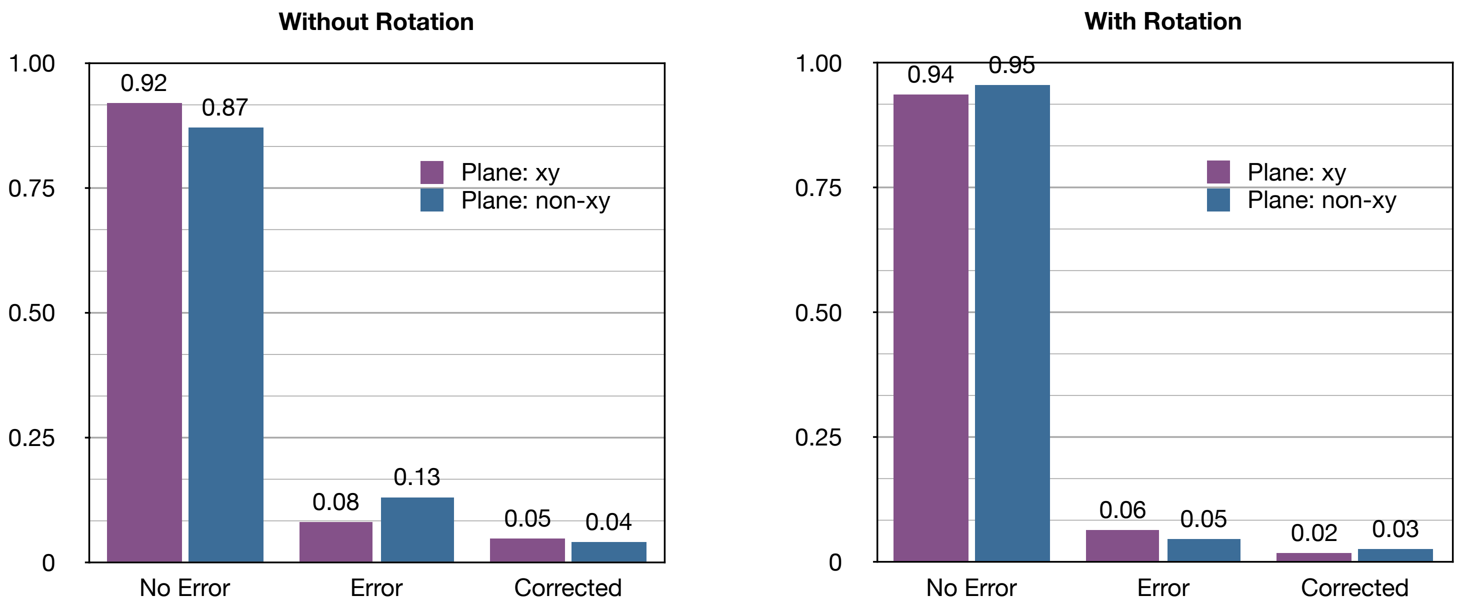


Figure 6.8: Percentage of Right-hand rule errors by Plane with and without Rotation

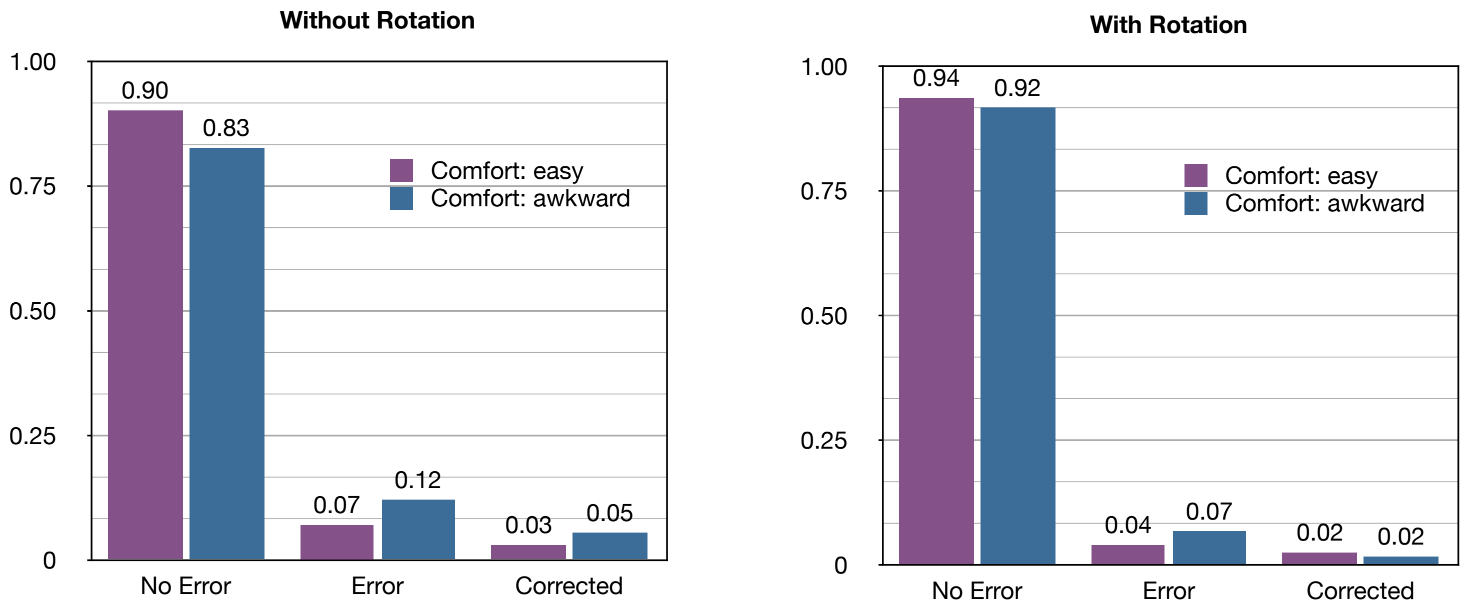


Figure 6.9: Percentage of Right-hand rule errors by Comfort with and without Rotation

6.5.2 Guess and Check

The Guess and Check method was used almost exclusively on the working-backward problems and presumably helped participants with the difficulties associated with reasoning backward. As noted in Section 6.2, there were more Order errors for working backward problems that asked for the direction of the first vector ($\vec{A}$) than for any other type of problem. When Guess and Check was used on these problems, the percentage of Order errors was reduced from 15% (48/324) to 8% (2/24). Also, for the problems asking for the sign of the charge (q), when Guess and Check was not used, there were 15 corrected and uncorrected errors (33%), but there were no errors at all on the eight problems when Guess and Check was used. Although the numbers are small, they do indicate that the use of Guess and Check does help to address the difficulties of working-backward and in particular, the Order errors when trying to find the first vector of the cross product.

6.5.3 Orthogonality

The use of orthogonality not only reduces the possible answers to two (positive or negative), but it also identifies the axis along which the cross product lies. The Match errors discussed above demonstrate a mismatch between the gestural response from the right-hand rule and the verbal response aligned with the frame of the table. By using orthogonality first to determine the axis, one would expect that the likelihood of a mismatch would be lower. Although the percentage of Match errors was fairly low to begin with, it did decrease even further when Orthogonality is used, as shown in Figure 6.10. This indicates that identifying the axis of the response first can help to reduce the gestural/verbal mismatch due to alignment.

6.6 Summary of qualitative analysis

The analysis presented in this chapter reveals many complex connections between the individual participants, the methods they used, the errors they made and features of the problems. For example, in examining right-hand rule errors, it is clear that for Andrea the Order errors stem from physical discomfort and a lack of recognition of the non-commutativity of the cross product. However, for most of the other participants,

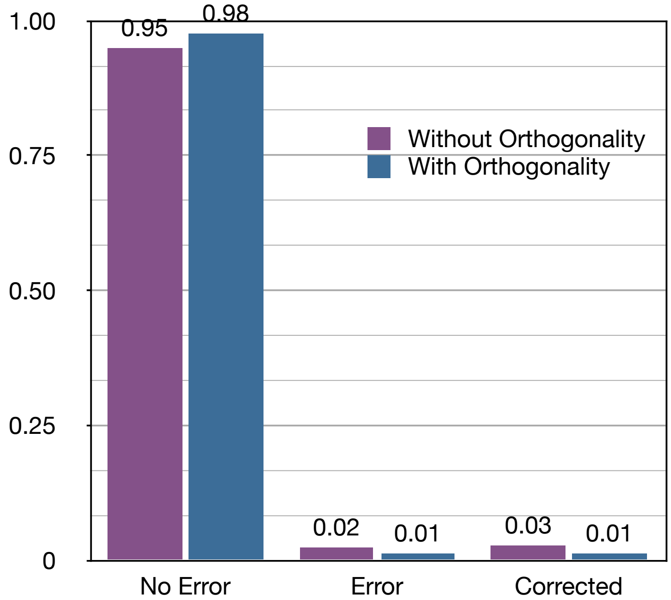


Figure 6.10: Percentage of Match errors with and without Orthogonality

particularly those using a Standard right-hand rule, the Order error is more strongly associated with backward reasoning problems, particularly those asking for $\vec{A}$.

The analysis of physics errors and problem features demonstrated systemic physics issues for some participants that have not been examined in previous literature, such as observation location for Ericka or the use of electron current for Danny. Additionally, on magnetic force problems with the magnetic field represented by magnets, there was evidence of two different physics errors: a reversal of the direction of the magnetic field due to the magnets and a response with the force in the direction of the field. These were the two errors that Scaife and Heckler (2010) found contributed to the representation dependence for these problems. Finally, a look at some of the more frequently used supplemental methods has demonstrated how these might be used as potential starting points when considering instructional change for cross products in physics.

Chapter 7

Conclusions

Students in introductory physics struggle with vector algebra and in particular with cross product direction. Some have suggested that this may be due to misapplied right-hand rules, but there are few studies that have had the resolution to explore this connection. Additionally, previous research on student understanding has noted several kinds of representation-dependence of student performance with vector algebra in both physics and non-physics (or math) contexts (*e.g.* Hawkins et al., 2009; Van Deventer, 2008). Yet, with few exceptions (*e.g.* Scaife and Heckler, 2010), these findings have not been applied to cross product direction questions or the use of right-hand rules. Also, the extensive work in spatial cognition is particularly applicable to cross product direction due to the spatial and kinesthetic nature of the right-hand rule.

This study has built on previous research in several domains–student understanding of magnetism, student understanding of vector algebra, and spatial cognition–to identify the problem features likely to contribute to difficulty with cross product direction. Through the use of quantitative and qualitative techniques, this study has shed light on the representation and context-dependence of student performance on cross product direction questions. This chapter will summarize the major findings of the study (Section 7.1) and discuss the implications of these findings for instruction (Section 7.2) and for future work (Section 7.3).

7.1 Review of findings

This study used a mix of qualitative and quantitative techniques to explore participant performance on cross product questions. The regression analyses presented in Chapter 4 used response time and correctness as correlated measures of performance, with the various problem features drawn from the literature as predictors. The qualitative coding presented in Chapter 5 identified a wide variety of methods and errors, which were used in Chapter 6 to explore individual differences and context dependence in more depth.

7.1.1 Kinds of questions

As noted previously, there are striking differences in how the study participants approach physics and non-physics cross product questions. As expected, the response time is significantly longer on physics questions than on non-physics questions. However, other differences in performance appear to be related to the additional complications of various physics features, possible physics errors, and the type of reasoning required. Of all of the problem features, the type of reasoning had the strongest impact on all aspects of performance: response time, correctness, and types of right-hand rule errors made. Also as expected, backward reasoning problems took longer and were less likely to be correct than problems requiring forward reasoning. When compared across all types of backward questions, more than half of all Order errors were made on problems asking for the first vector. Additionally, these Order errors were committed even by participants who explicitly acknowledged the non-commutativity of the cross product. It is clear that for these participants, questions that require backward reasoning are more difficult than questions that require forward reasoning, but when a participant employed the use of a Guess and Check method on backward reasoning questions, s/he was more likely to answer the question correctly than if this Supplemental method was not used.

7.1.2 Impact of vector orientation on performance

Research in spatial cognition (*e.g.* Klatzky and Wu, 2008) strongly suggests that the orientation of the vectors should impact difficulty when using a right-hand rule. However, **??** did not see any measurable effects on performance for cross product questions due to orientation. One possible reason for this lack of signal was that orientation was treated as one factor. In contrast, this study has drawn on spatial cognition literature to identify three different aspects of vector orientation that could contribute to difficulty: physical discomfort, the plane of the given vectors, and the angle between the vectors. Each of these features had a measurable impact on participant performance consistent with the spatial cognition literature: performance was better for physically easy orientations than for those that were physically awkward, orientations in the x-y plane than those not the in the x-y plane, and for those that were perpendicular than those that were not.

There were some individual differences revealed by the qualitative analysis that contributed additional insight into the understanding of how orientation contributes to performance. The difficulty associated with orientations in non-xy planes is likely due to issues of alignment. The Misinterpret errors suggest that at least some of this difficulty results from misinterpretation of the $\otimes$ and $\odot$ symbols, which is consistent with conventional wisdom. Additionally, this may imply that alignment errors are not as significant to the mis-application of right-hand rules as is suggested in the literature, but are exacerbated by students confusing the mathematical symbols for into and out of the page. As expected, physical discomfort was an issue for participants. One way, which several students, coped with this issue was to rotate the problem sheet. Of particular note, one participant, Andrea, switched the order of the vectors to make the right hand rule less uncomfortable; she did not seem to realize that the order of vectors matters. The impact of the angle between the vectors was not as clear-cut, possibly because it was only varied for non-physics, forward reasoning problems.

7.1.3 Impact of the need for parallel transport on performance

One of the more surprising results of this study was how little the issue of parallel transport seemed to matter. For some participants, a head-to-tail representation caused them to reverse the direction of the vector. However, neither the Tails or Separation features had nearly as strong an impact as was expected. This could indicate that although parallel transport is a serious issue for students learning vector addition and subtraction, it may not be as much of a problem in vector multiplication.

7.1.4 Impact of physics features on performance

As mentioned above, most of the differences between physics and non-physics contexts are related to physics errors and features. This study has identified several significant physics errors and also related these errors to specific physics features. Each of these findings is consistent with previous literature and in some cases (such as the issue of charge) provides evidence for issues that have been only anecdotal until now.

One of the most significant physics errors was not appropriately accounting for the sign of the charge. While most of these errors resulted from participants ignoring or neglecting the issue, some participants explicitly did not understand how the sign of the charge would impact the cross product. On questions involving conventional current in a wire, there were several students who confused conventional and electron current. This data provides evidence that the impact of the charge on magnetic field and magnetic force is indeed a source of difficulty for students, which until this study has only been assumed.

A previously un-explored issue in the correct application of right-hand rules in magnetism is the effect of the observation location on the direction of the magnetic field. Observation location errors were one of the few instances where a specific type of right-hand rule was associated with a particular error. The majority of observation location errors were made when using the Current right-hand rule. For at least two of the participants (Ericka and Sebastien), this was a systemic issue.

The impact of the representation of magnetic field observed by Scaife and Heckler (2010) was also observed here. Participants were more likely to answer incorrectly on questions where the field was represented by magnets than on questions where it was represented using vectors. There were two different physics errors that contributed to this issue and both are consistent with the findings of Scaife and Heckler. The first involved reversing the direction of the magnetic field due to the magnets and the second involved finding the force parallel to the field using an inappropriate analogy to electric field. This confusion between electric and magnetic fields, which has been explored in several previous studies (*e.g* Maloney et al., 2001), was also seen on magnetic field problems where participants would state that the field "points away from the positive and toward the negative" or that the field at the center of a loop was zero.

Categorizing these physics issues and their dependence on problem features helps to illuminate why cross product direction problems are more difficult in a physics context than in a non-physics context. By recognizing where these physics difficulties are for students, we can more appropriately begin to change our instruction.

7.2 Instructional Implications

These results present compelling evidence that there are many factors influencing students' difficulties with cross product direction in introductory Electricity & Magnetism. Some of these factors include the application of various right-hand rules, physics content issues, the orientation of the vectors, the symbols used, and the type of reasoning required. If we want our students to answer our test questions correctly, we can design problems that avoid these issues. If instead we want our students to be able to correctly use the cross product as a tool in a variety of situations, we must help them learn to deal with these issues. This study suggests an initial roadmap for instructional change. It also raises questions for further inquiry which will be discussed in Section 7.3.

Through the coding process described in Chapter 5, this study identified several difficulties with the physics content. Addressing these difficulties is the first place to start in reassessing our instruction. The most common physics error involved inappropriately accounting for the sign of the charge. Most of these errors resulted from participants ignoring or neglecting the charge, but some participants explicitly did not understand how the sign of the charge would impact the cross product. Additionally, on questions involving conventional current in a wire, there were several students who confused conventional and electron current. These results clearly indicate that more explicit emphasis on the impact of charge is needed. One possible solution would be to encourage student to use the right hand for positive charges and the left hand for negative charges, as seen with some of these participants.

The representation dependence on magnetic force problems seen in Scaife and Heckler (2010) was also observed in this study, where participants were more likely to answer incorrectly if the field was represented by magnets than if represented by vectors. There were two different conceptual errors that contributed to this difference. First, some students reversed the direction of the magnetic field due to the magnets (from South to North instead of North to South). The second issue is more fundamental and involved an inappropriate analogy to electric field (*i.e.* North is like a positive and South is like a negative, so the positive particle will be attracted to the South pole). Since both of these representations are useful for certain situations, when assigning problems, one must be aware that the magnet representation requires the extra step of remembering the direction of the magnetic field and is also more likely to cue this analogy to electric field.

In addition to inappropriate analogies between electric and magnetic fields, there was also confusion between magnetic field and magnetic force problems. These results highlight the need to be more aware of what other content might interfere with students' ability to perform tasks successfully.

There was one physics error that was closely tied to a particular type of right-hand rule. Several participants had systemic difficulties with understanding how the observation location impacted the magnetic field; this was most obvious and most severe when the Current right-hand rule was used. Also, while most participants used only one right-hand rule type on the non-physics problems, they used a variety of different types to answer the physics questions. This plethora of right-hand rules, a few of which are only appropriate for some situations, can be confusing to students. These results highlights the need to emphasize the limits and conditions of applicability for each right-hand rule.

In addition to these difficulties with physics concepts, there were also difficulties with the cross product itself and with the use of right-hand rules apart from the physics. The largest number of uncorrected errors with a right-hand rule came from reversing the order of the vectors. Thus, it is important to stress the non-commutative nature of the cross product. Additionally, these errors were were reduced from 15% to 8% when a Guess and Check method was used. Thus, teaching this and other methods that reduce the number of possible solutions could help reduce the difficulty of cross product problems, especially those that require backward reasoning. Another of the methods that reduces the possibilities is the use of Orthogonality to restrict the solution to a single access. This method was particular useful in reducing the number of gestural/verbal mismatches.

Finally, it is necessary to provide ways to deal with the difficulties related to the orientation of the vectors. There were two aspects of orientation that had a significant impact on performance: physical discomfort and the plane of the given vectors. One way that the participants dealt with these issues was by rotating the paper and this helped to reduce the percentage of errors overall. It also helped to reduce the gap between physically easy and physically awkward orientations, and between orientations in the xy plane and the non-xy plane. These findings indicate that manipultatives such as those designed by Van Domelen (1999) and Nguyen and Meltzer (2003) could be effective in addressing some of these issues, although there has not yet been any assessment of these tools. We should also consider revised instructions for the right-hand rule, such as those proposed by Klatzky and Swendson (in Klatzky and Wu, 2008). Some of the difficulties with the plane of the vectors resulted from misinterpretations of the $\odot$ and $\otimes$ symbols. While any labeling system designed to reduce three dimensions to two may have similar issues, it would be wise to revisit the use of this convention.

There are many aspects of this study that should be considered in the reassessment of the use of cross products and right-hand rules in introductory physics. The results outlined above provide some suggestions for starting to think about instructional change. Further research into how these changes impact performance will help us to better understand our students and provide them with appropriate tools that they can use effectively.

7.3 Areas for future inquiry

The conclusions of this study suggest several areas for future inquiry; possible avenues of research are described here. Spatial ability does not predict performance as well as would be expected from the literature. There was also a self selection bias in the data based on gender, and there were numerous individual differences in the participants' choice of methods. There is a need to investigate these and other aspects of student background and their impact on the understanding of cross product direction and the appropriate use of right hand rules.

The results of the pilot study and the tendency of students in the Magnetic Force group to apply an inappropriate right-hand rule on magnetic field questions implies that there maybe an interference effect due to instruction similar to that in Sayre and Heckler (2009); therefore, we should look at how student performance changes over time both within and across semesters.

An environmental complication that has not been addressed here is how the impact of orientation would change for tasks on a computer. The computer reference frame is more consistent with lecturers' use of whiteboards, but exams are usually given on paper requiring different frame alignment. Also, the inability to rotate the computer screen as one can rotate a piece of paper could present challenges, since this study has shown that Rotation reduces errors due to orientation. Exploring these issues is important due to the prevalence of online homework systems (*e.g.* WebAssign).

In addition to these cognitive, instructional, and alignment issues, it has been suggested that the inappropriate use of the left hand may be due to students using their right hands to write. As participants were not required to write in this study, it is impossible to draw conclusions about the effect of writing while solving right-hand rule problems from the data collected here.

Studies to address these areas of student difficulty would be a natural progression of this research and would add to our understanding of students' comprehension of and difficulties with right-hand rules.

REFERENCES

Aguirre, J.M. (1988), Student preconceptions about vector kinematics. *The Physics Teacher*, **26**(4), 212–216.

Aguirre, J.M. and Erickson, G. (1984), Students' conceptions about the vector characteristics of three physics concepts. *Journal of Research in Science Teaching*, **21**(5), 439–457.

Alibali, M. (2005), Gesture in spatial cognition: Expressing, communicating, and thinking about spatial information. *Spatial Cognition and Computation*, **5**(4), 307–331.

Allen, L.D. (2001), *An investigation into student understanding of magnetic induction*. Ph.D. thesis, The Ohio State University.

Avraamides, M.N., Klatzky, R.L., Loomis, J.M., and Golledge, R.G. (2004), Use of cognitive versus perceptual heading during imagined locomotion depends on the response mode. *Psychological Science*, **15**(6), 403–408.

Azad, H. (2001), On the definition of the cross-product. *International Journal of Mathematical Education in Science and Technology*, **32**(4), 585–587.

Bagno, E. and Eylon, B.S. (1997), From problem solving to a knowledge structure: An example from the domain of electromagnetism. *American Journal of Physics*, **65**(8), 726–736.

Bamber, D. (1975), The area above the ordinal dominance graph and the area below the receiver operating characteristic graph. *Journal of Mathemathematical Psychology*, **12**(4), 387–415.

Banerjee, M., Capozzoli, M., McSweeney, L., and Sinha, D. (1999), Beyond kappa: A review of interrater agreement measures. *The Canadian Journal of Statistics*, **27**(1), 3–23.

Barniol, P.J. and Zavala, G. (2009), Investigation of students' preconceptions and difficulties with the vector direction concept at a Mexican university. In M. Sabella, C. Henderson, and C. Singh (Eds.), *Physics Education Research Conference Proceedings*, vol. 1179, (pp. 85–88), Melville, New York: AIP Conference Proceedings.

Beichner, R.J. (1994), Testing student interpretation of kinematics graphs. *American Journal of Physics*, **62**(8), 750–762.

Bobillo-Ares, N. and Fernandez-Nunez, J. (1990), The rationale for the vector product. *International Journal of Mathematical Education in Science and Technology*, **21**(1), 105–109.

Borges, A.T., Tecnico, C., and Gilbert, J.K. (1998), Models of magnetism. *International Journal of Science Education*, **20**(3), 361–378.

Chabay, R. and Sherwood, B. (2003), Modern mechanics. *American Journal of Physics*, **72**(4), 439–445.

Chabay, R. and Sherwood, B. (2007), *Matter & Interactions I. Modern Mechanics.* Hoboken, NJ: John Wiley and Sons, Inc., 2nd ed.

Chabay, R. and Sherwood, B. (2009), *Matter & Interactions II: Electric and Magnetic Interactions.* Hoboken, NJ: John Wiley and Sons, Inc., 3rd ed.

Christensen, W.M., Nguyen, N.L., and Meltzer, D. (2004), Student difficulties with graphical representation of vector products: crossing and dotting beyond t's and i's. Contributed Poster: PERC, Sacramento, CA.

Corbin, J. and Strauss, A. (1990), Grounded theory research: procedures, canons, and evaluative criteria. *Qualitative Sociology*, **13**(1), 3–21.

Ding, L., Chabay, R., Sherwood, B., and Beichner, R. (2006), Evaluating an electricity and magnetism assessment tool: Brief electricity and magnetism assessment. *Physical Review Special Topics: Physics Education Research*, **2**(010105).

Dray, T. and Manogue, C. (2003), Using differentials to bridge the vector calculus gap. *The College Mathematics Journal*, **34**(4), 283–290.

Dray, T. and Manogue, C. (2006), The geometry of dot and cross products. *Journal of Online Mathematics and its Applications*, **6**.

Ekstrom, R.B., French, J.W., Harman, H.H., and Dermen, D. (1976), *Kit of factor referenced cognitive tests.* Princeton, NJ: Educational Testing Service.

Elk, S.B. (1997), The cross product of two vectors is not just another vector – a major misconception being perpetuated in calculus and vector analysis textbooks. *International Journal of Mathematical Education in Science and Technology*, **28**(4), 531–543.

Flanders, M. and Soechting, J.F. (1995), Frames of reference for hand orientation. *Journal of Cognitive Neuroscience*, **7**(2), 182–195.

Flores, S., Kanim, S.E., and Kautz, C.H. (2004), Student use of vectors in introductory mechanics. *American Journal of Physics*, **72**(4), 460–468.

Gagatsis, A. and Demetriadou, H. (2001), Classical versus vector geometry in problem solving. An empirical research among Greek secondary pupils. *International Journal of Mathematical Education in Science and Technology*, **32**(1), 105–125.

Giancoli, D.C. (2000), *Physics for Scientist and Engineers*. Upper Saddle River, NJ: Prentice Hall, 3rd ed.

Greca, I.M. and Moreira, M. (1997), The kinds of mental representations–models, propositions and images–used by college physics students regarding the concept of field. *International Journal of Science Education*, **19**(6), 711–724.

Greenslade, T.B. (1980), Ancestors of the right-hand rule. *The Physics Teacher*, **18**(9), 669–670.

Guisasola, J., Almudí, J.M., and Zubimendi, J.L. (2004), Difficulties in learning the introductory magnetic field theory in the first years of university. *Science Education*, **88**(3), 443–464.

Harpaz, A. (2002), The nature of fields. *European Journal of Physics*, **23**, 263–268.

Hawkins, J.M., Thompson, J.R., and Wittmann, M.C. (2009), Students' consistency of graphical vector addition method on 2-D vector addition tasks. In M. Sabella, C. Henderson, and C. Singh (Eds.), *Physics Education Research Conference Proceedings*, vol. 1179, (pp. 161–164), Melville, New York: AIP Conference Proceedings.

Hegarty, M. and Waller, D.A. (2005), Individual differences in spatial abilities. In *The Cambridge handbook of visuospatial thinking*, chap. 4, (pp. 121–169), Cambridge University.

Hestenes, D. (2003), Oersted Medal Lecture 2002: Reforming the mathematical language of physics. *American Journal of Physics*, **71**(2), 104–121.

Hestenes, D., Wells, M., and Swackhamer, G. (1992), Force concept inventory. *The Physics Teacher*, **30**(3), 141–158.

Johnstone, A.H. and Mughol, A.R. (1976), Concepts of physics at secondary level. *Physics Education*, **11**(7), 466–469.

Klatzky, R.L. (1998), Allocentric and egocentric spatial representations: Definitions, distinctions, and interconnections. In C. Freska, C. Habel, and K.F. Wender (Eds.), *Spatial Cognition: An Interdisciplinary Approach to Representing and Processing Spatial Knowledge*, Lecture Notes in Artificial Intelligence 1404, (pp. 1–17), Springer.

Klatzky, R.L., Loomis, J.M., Beall, A.C., Chance, S.S., and Golledge, R.G. (1998), Spatial updating of self-position and orientation during real, imagined, and virtual locomotion. *Psychological Science*, **9**(4), 293–298.

Klatzky, R.L. and Wu, B. (2008), The embodied actor in multiple frames of reference. In R.L. Klatzky, B. MacWhinney, and M. Behrmann (Eds.), *Embodiment, Ego-Space, and Action*, vol. 34 of *Carnegie Mellon Symposia on Cognition*, chap. 5, (pp. 145–177), New York: Psychology Press.

Knight, R.D. (1995), The vector knowledge of beginning physics students. *The Physics Teacher*, **33**(2), 74–77.

Kohlmyer, M.A., Caballero, M.D., Catrambone, R., Chabay, R.W., Ding, L., Haugan, M.P., Marr, M.J., Sherwood, B.A., and Schatz, M.F. (2009), Tale of two curricula: The performance of 2000 students in introductory electromagnetism. *Physical Review Special Topics: Physics Education Research*, **5**(020105).

Kozhevnikov, M., Hegarty, M., and Mayer, R.E. (2002), Revising the visualizer-verbalizer dimension: evidence for two types of visualizers. *Cognition and Instruction*, **20**(1), 47–77.

Kustusch, M.B., Chabay, R., and Beichner, R. (2008), Solving cross product problems in physics and mathematics: student choices of solution method. Contributed Poster: AAPT, Baltimore, MD.

Landis, J.R. and Koch, G.G. (1977), The Measurement of observer agreement for categorical data. *Biometrics*, **33**(1), 159–174.

Lardner, D. (1874), *Handbook of Natural Philosophy: Electricity, Magnetism and Acoustics*. London: Lockwood & Co.

Linn, M.C. and Petersen, A.C. (1985), Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Child Development*, **56**(6), 1479–1498.

Maloney, D.P., O'Kuma, T.L., Hieggelke, C.J., and Heuvelen, A.V. (2001), Surveying students' conceptual knowledge of electricity and magnetism. *Physics Education Research, American Journal of Physics Supplement*, **69**(7), S12–S23.

Marx, J.D. (1998), *Creation of a diagnostic exam for introductory, undergraduate electricity and magnetism*. Ph.D. thesis, Rensselaer Polytechnic Institute.

Mason, S.J. and Graham, N.E. (2002), Areas beneath the relative operating characteristics (ROC) and relative operating levels (ROL) curves: Statistical significance and interpretation. *Quarterly Journal of the Royal Meteorological Society*, **128**, 2145–2166.

Nguyen, N.L. and Meltzer, D. (2003), Initial understanding of vector concepts among students in introductory physics courses. *American Journal of Physics*, **71**(6), 630–638.

Nguyen, N.L. and Meltzer, D. (2005), Visualization tool for 3-D relationships and the right-hand rule. *The Physics Teacher*, **43**(3), 155–157.

O'Keeffe, J.D. (1981), A generalization of the vector product. *International Journal of Mathematical Education in Science and Technology*, **12**(5), 541–547.

Ortiz, L.G. (2001), *Identifying and addressing student difficulties with rotational dynamics*. Ph.D. thesis, University of Washington.

Pani, J.R. (1993), Limits on the comprehension of rotational motion: mental imagery of rotations with oblique components. *Perception*, **22**(7), 785–808.

Pani, J.R. and Dupree, D. (1994), Spatial reference systems in the comprehension of rotational motion. *Perception*, **23**(8), 929–946.

Pani, J.R., Jeffres, J.A., Shippey, G.T., and Schwartz, K.J. (1996), Imagining projective transformations: Aligned orientations in spatial organization. *Cognitive Psychology*, **31**(2), 125–167.

Pocovi, M.C. and Finley, F. (2002), Lines of force: Faraday's and students' views. *Science and Education*, **11**, 459–474.

Roche, J. (1997), NEW APPROACHES: Introducing vectors. *Physics Education*, **32**(5), 339–345.

Rock, I. (1973), Orientation and form. *Cognitive Psychology*, **31**(2), 125–167.

Rock, I. (1983), *Logic of perception*. Cambridge, Massachusetts: The MIT Press.

Rock, I. (1986), The description and analysis of object and event perception. In K.R. Boff, L. Kaufman, and J.P. Thomas (Eds.), *Handbook of perception and human performance*, vol. II, New York: John Wiley and Sons.

Sağlam, M. and Millar, R. (2006), Upper high school student' understanding of electromagnetism. *International Journal of Science Education*, **28**(5), 543–566.

Sayre, E.C. and Heckler, A.F. (2009), Changes in student understanding of vector products. Contributed Poster: AAPT, Ann Arbor, MI.

Scaife, T.M. and Heckler, A.F. (2007), The effect of field representation on student responses to magnetic force questions. In L. Hsu, C. Henderson, and L. McCullough (Eds.), *Physics Education Research Conference Proceedings*, vol. 951 of *AIP Conference Proceedings*, (pp. 180–183), Melville, New York.

Scaife, T.M. and Heckler, A.F. (2010), Student understanding of the direction of the magnetic force on a charged particle. *American Journal of Physics*, **78**(8), 869–876.

Scherr, R.E. (2008), Gesture analysis for physics education researchers. *Phys. Rev. ST Phys. Educ. Res.*, **4**(1), 010101.

Shaffer, P.S. and McDermott, L.C. (2005), A research-based approach to improving student understanding of the vector nature of kinematical concepts. *American Journal of Physics*, **73**(10), 921–931.

Shaw, R. (1985), The teaching of vector algebra. *International Journal of Mathematical Education in Science and Technology*, **16**(5), 593–602.

Shelton, A.L. and McNamara, T.P. (2001), Systems of spatial reference in human memory. *Cognitive Psychology*, **43**(4), 274–310.

Shepard, R.N. and Metzler, J. (1971), Mental Rotation of Three-Dimensional Objects. *Science*, **171**(3972), 701–703.

Stewart, J. (1995), *Calculus: Early Transcendentals*. Pacific Grove, CA: Brooks/Cole Publishing Company, 3rd ed.

Van Deventer, J. (2008), *Comparing student performance on isomorphic math and physics vector representations*. Master of science in teaching thesis, University of Maine.

Van Deventer, J. and Wittmann, M.C. (2007), Comparing student use of mathematical and physical vector representations. In L. Hsu, C. Henderson, and L. McCullough (Eds.), *Physics Education Research Conference Proceedings*, vol. 951, (pp. 208–211), Melville, New York: AIP Conference Proceedings.

Van Domelen, D.J. (1999), Artificial right-hand rule device. *The Physics Teacher*, **37**(8), 500–501.

Wieberdink, T. (2002), Photo of the Month. *The Physics Teacher*, **40**(2), 115.

APPENDICES

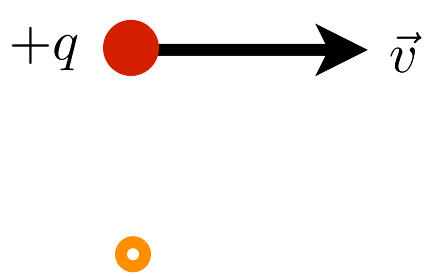
Appendix A

Cross product direction questions

A.1 Magnetic field problems

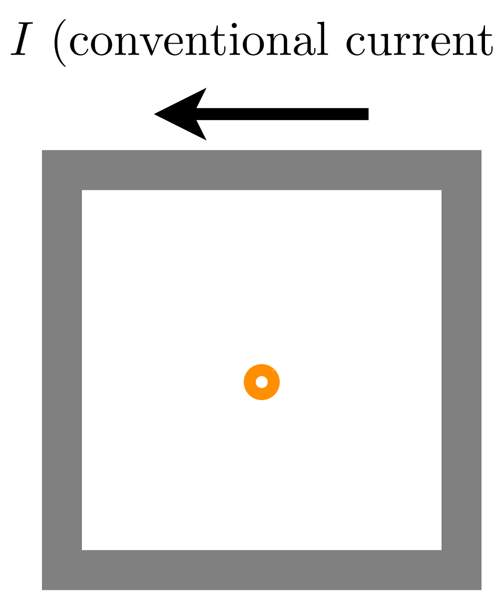
The following are the problems in the magnetic field context that were administered to all participants in the order in which they were given. The problems were administered one at a time with each problem on a single sheet of paper.

1. A positive charge is moving in the $+x$ direction, as shown below. What is the direction of the magnetic field at the observation location represented by the small orange circle?

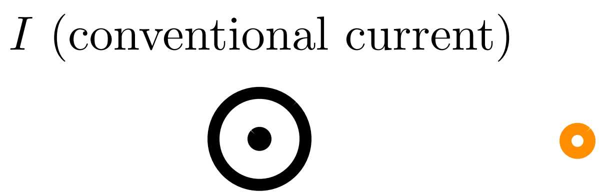


2. A negative charge creates a magnetic field in the $-z$ direction (into the page) at an observation location in the $-x$ direction. What is a possible direction for the velocity of the charge?

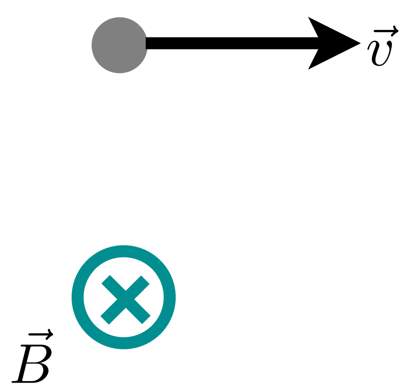
3. The conventional current flows counter-clockwise in the loop below. What is the direction of the magnetic field at the center of the loop (the small orange circle)?



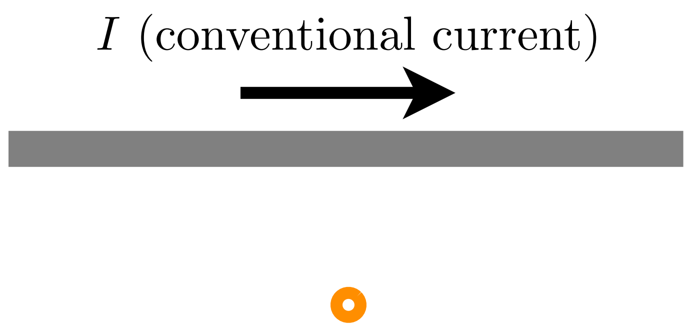
4. The conventional current in a long, straight wire flows in the $+z$ direction (out of the page). What is the direction of the magnetic field at the observation location represented by the small orange circle?



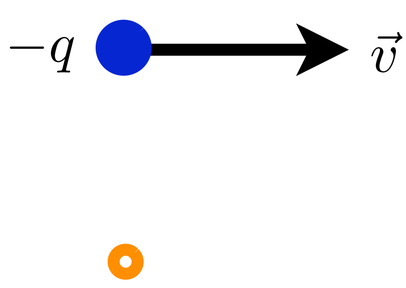
5. A charged particle moving in the $+x$ direction creates a magnetic field in the $-z$ direction (into the page) at an observation location in the $-y$ direction. What is sign of the charge on the particle?



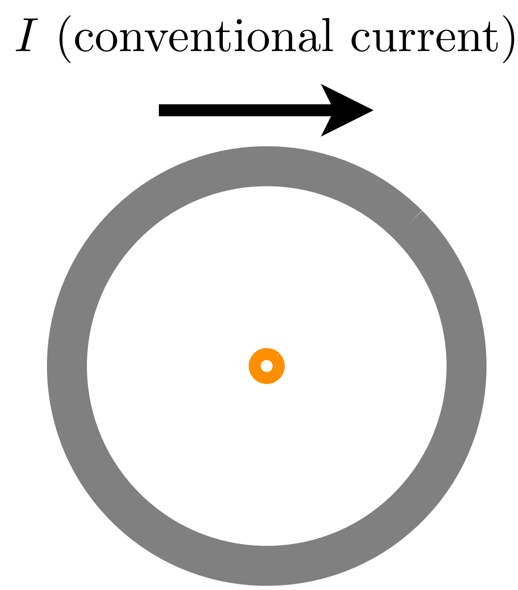
6. The conventional current in a long, straight wire flows in the $+x$ direction, as shown below. What is the direction of the magnetic field at the observation location represented by the small orange circle?



7. A negative charge is moving in the $+x$ direction, as shown below. What is the direction of the magnetic field at the observation location represented by the small orange circle?



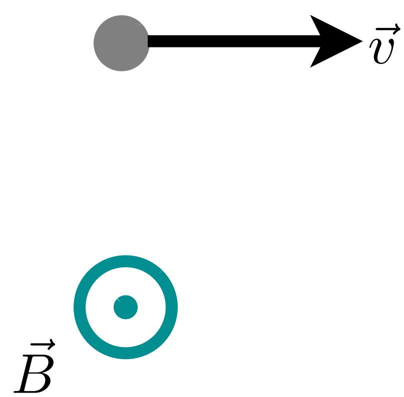
8. The conventional current flows clockwise in the loop below. What is the direction of the magnetic field at the center of the loop (the small orange circle)?



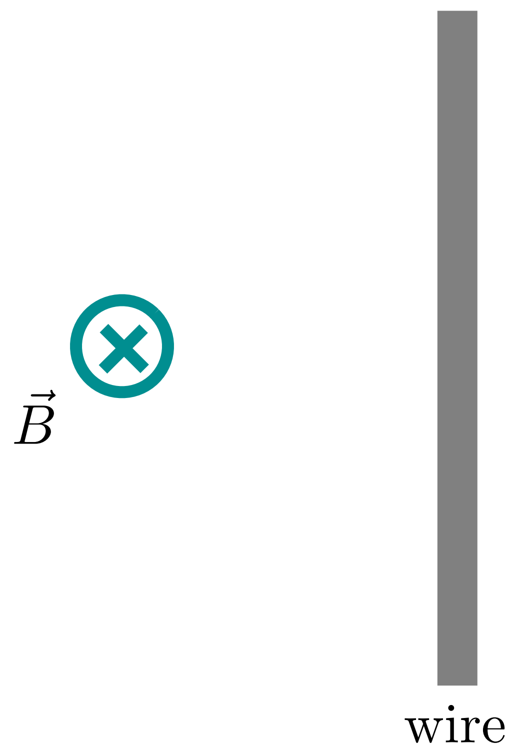
9. A positive charge creates a magnetic field in the $-z$ direction (into the page) at an observation location in the $-x$ direction. What is a possible direction for the velocity of the charge?

10. A positive charge is moving in the $+z$ direction (out of the page). What is the direction of the magnetic field at the observation location represented by the small orange circle?

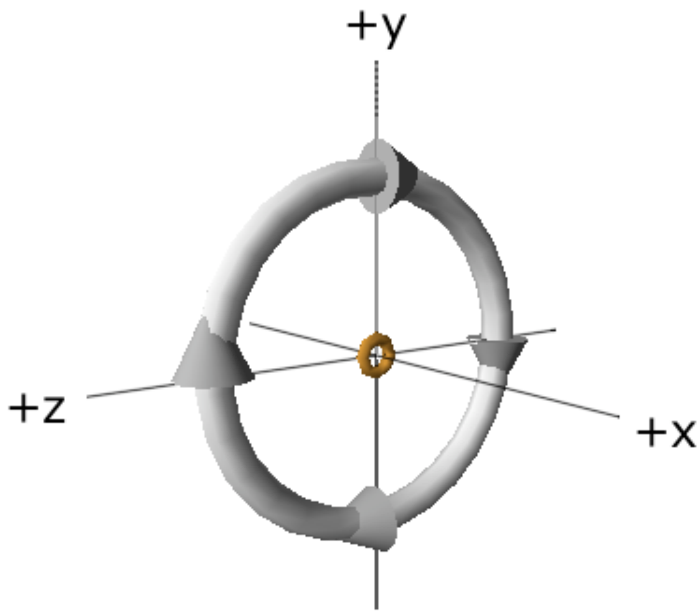
11. A charged particle moving in the $+x$ direction creates a magnetic field in the $+z$ direction (out of the page) at an observation location in the $-y$ direction. What is sign of the charge on the particle?

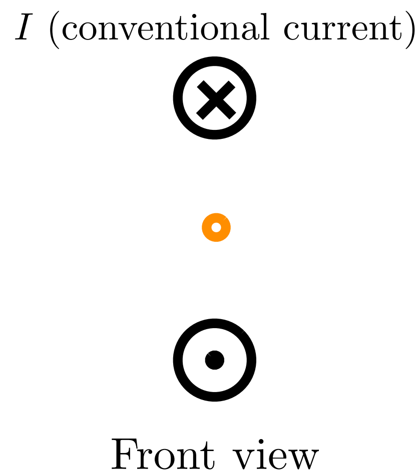


12. A long, straight wire creates a magnetic field in the $-z$ direction (into the page) at an observation location in the $-x$ direction. What is the direction of the conventional current in the wire?



13. When viewed from the $+x$ axis, the conventional current in the loop below flows clockwise (see side view and front view below). What is the direction of the magnetic field at the center of the loop (represented by the small orange circle)?



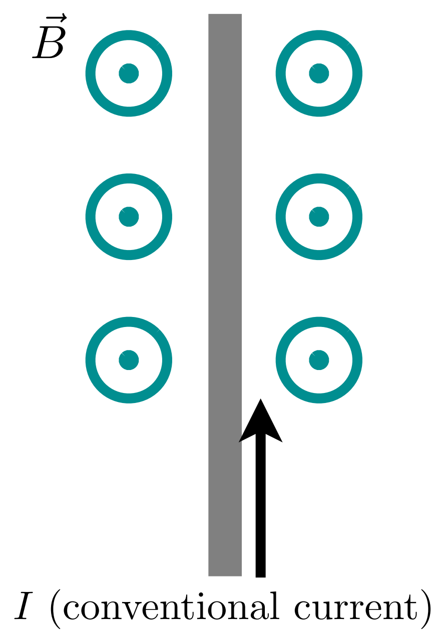


14. A negative charge is moving in the $+z$ direction (out of the page). What is the direction of the magnetic field at the observation location represented by the small orange circle?

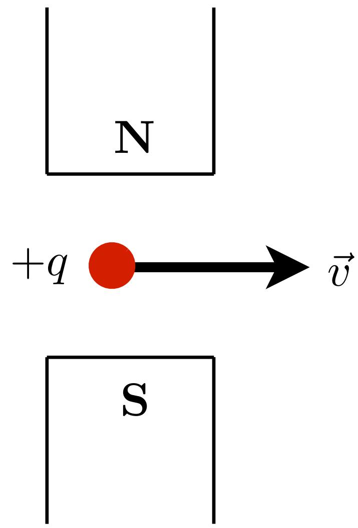
A.2 Magnetic force problems

The following are the problems in the magnetic force context that were administered to the second group of participants after the magnetic field problems in the order in which they were given. The problems were administered one at a time with each problem on a single sheet of paper.

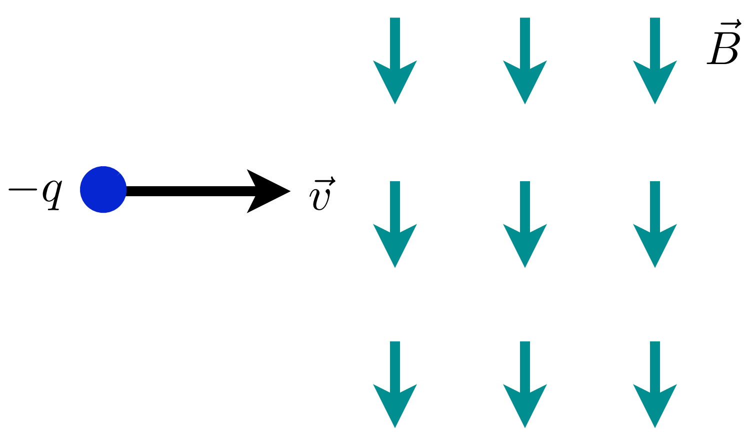
1. A current carrying wire with conventional current in the $+y$ direction experiences a force due to a uniform magnetic field. The magnetic field is in the $+z$ direction (out of the page). What is the direction of the force on the wire? The source of the magnetic field is not shown.



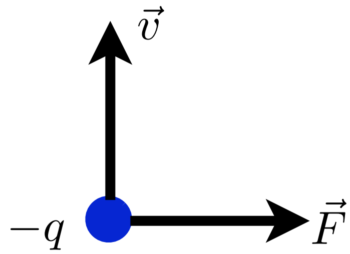
2. What is the direction of the force on a positive charge that is moving in the $+x$ direction due to the magnets shown?



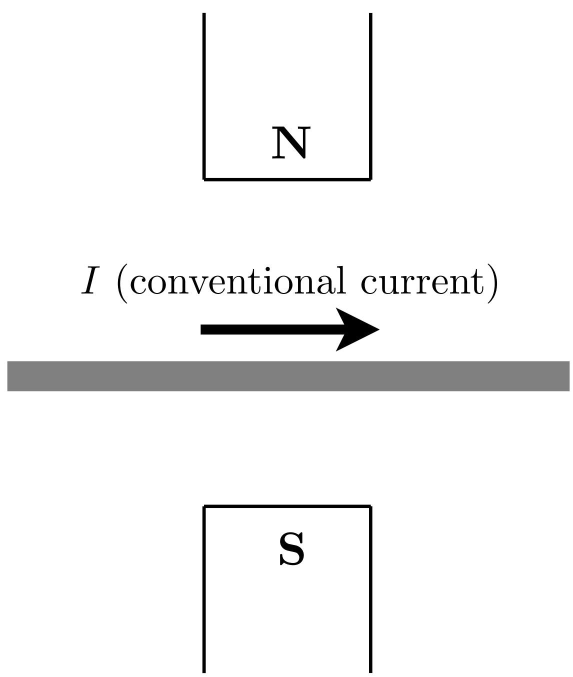
3. A negative charge moving in the $+x$ direction enters an area of uniform magnetic field in the $-y$ direction (the source of the magnetic field is not shown). What is the direction of the force on the charge?



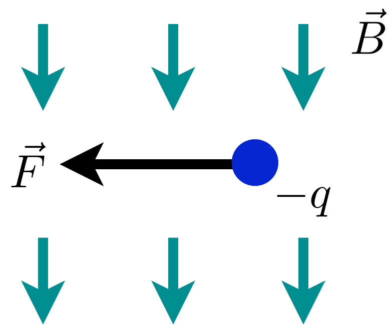
4. A negative charge moving in the $+y$ direction experiences a force in the $+x$ direction due to a magnetic field (not shown). What is a possible direction for the magnetic field?



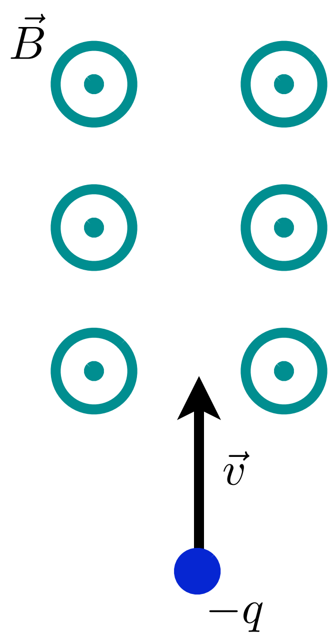
5. What is the direction of the force on a current carrying wire with conventional current in the $+x$ direction due to the magnets shown below?



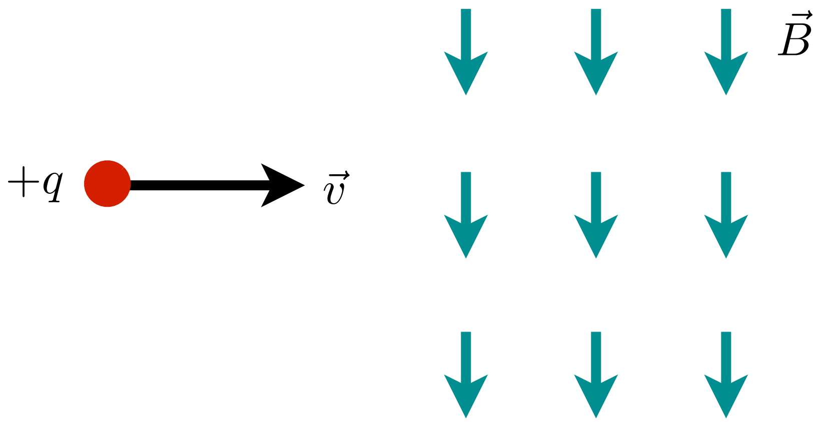
6. A negatively charged particle is in a uniform magnetic field that points in the $-y$ direction (the source of the magnetic field is not shown). The force on the charge is in the $-x$ direction. What is a possible direction for the velocity of the particle?



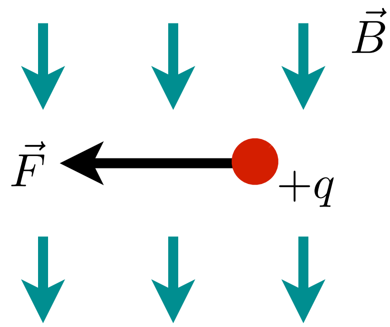
7. A negative charge moving in the $+y$ direction enters an area of uniform magnetic field in the $+z$ direction (out of the page). The source of the magnetic field is not shown. What is the direction of the force on the charge?



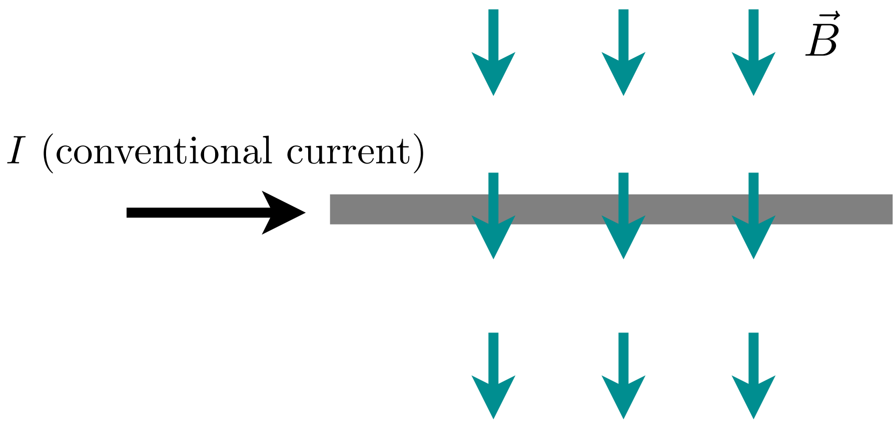
8. A positive charge moving in the $+x$ direction enters an area of uniform magnetic field in the $-y$ direction (the source of the magnetic field is not shown). What is the direction of the force on the charge?



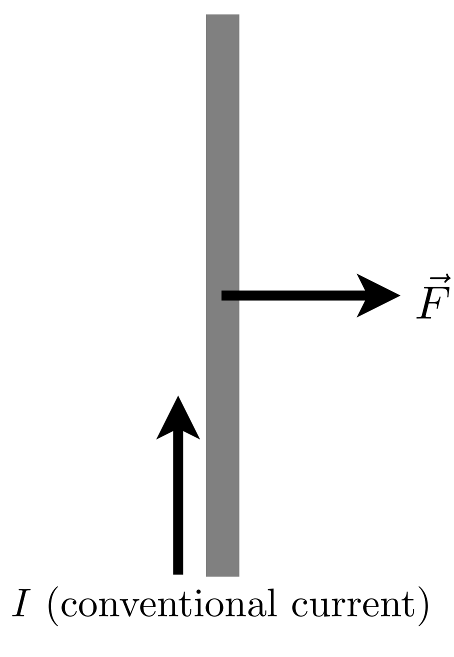
9. A positively charged particle is in a uniform magnetic field that points in the $-y$ direction (the source of the magnetic field is not shown). The force on the charge is in the $-x$ direction. What is a possible direction for the velocity of the particle?



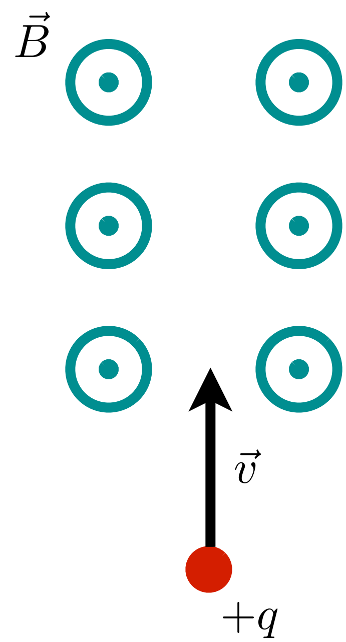
10. A current carrying wire with conventional current in the $+x$ direction experiences a force due to a uniform magnetic field. The magnetic field is in the $-y$ direction (the source of the magnetic field is not shown). What is the direction of the force on wire?



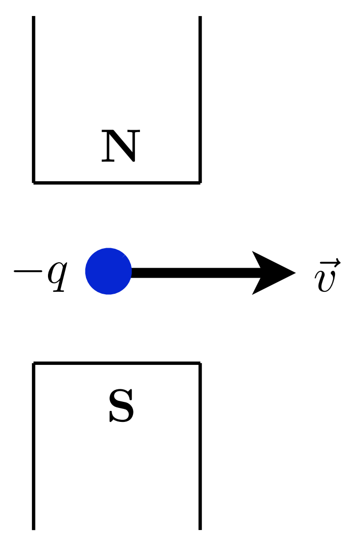
11. A current carrying wire with conventional current in the $+y$ direction experiences a force due to a magnetic field (not shown). The force is in the $+x$ direction. What is a possible direction for the magnetic field?



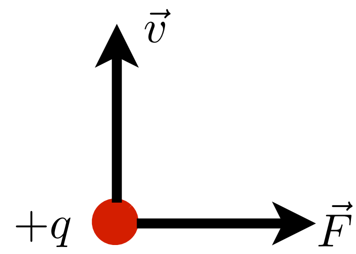
12. A positive charge moving in the $+y$ direction enters an area of uniform magnetic field in the $+z$ direction (out of the page). What is the direction of the force on the charge? The source of the magnetic field is not shown.



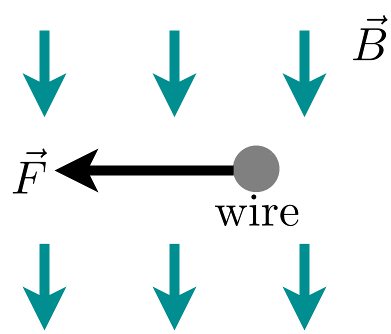
13. What is the direction of the force on a negative charge moving in the $+x$ direction due to the magnets shown?



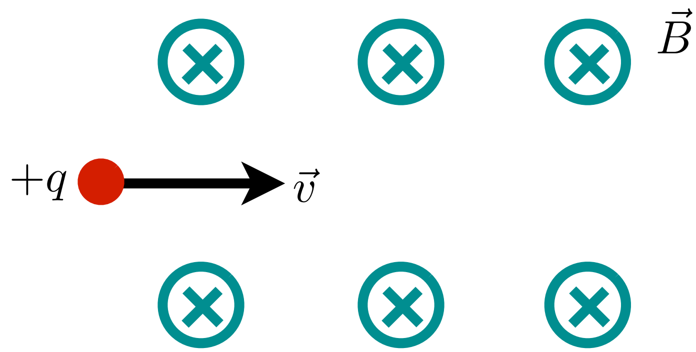
14. A positive charge moving in the $+y$ direction experiences a force due to a magnetic field (not shown). The force is in the $+x$ direction. What is a possible direction for the magnetic field?



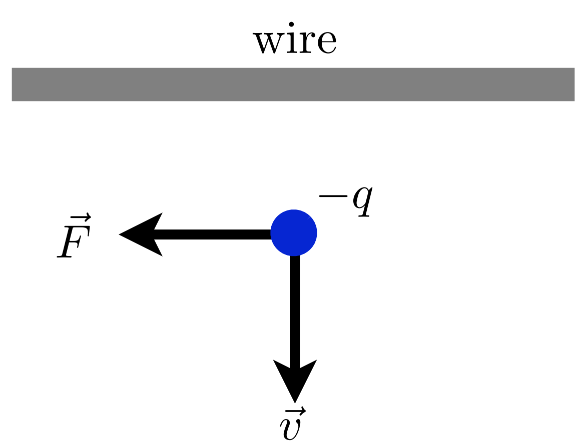
15. A wire poking out of the page (represented by the gray circle) is in a uniform magnetic field that points in the $-y$ direction. The source of the magnetic field is not shown. The force on the wire is in the $-x$ direction. What is the direction of the conventional current in the wire?



16. A positively charged particle moving in the $+x$ direction enters a uniform electric and magnetic field, where the magnetic field in the $-z$ direction (into the page). As the particle moves through the region, it remains undeflected. What is the direction of the electric field in this region?



17. A negatively charged particle is moving in the $-y$ direction relative to a current carrying wire. The charged particle experiences a force due to the wire in the $-x$ direction. What is the direction of the conventional current in the wire?



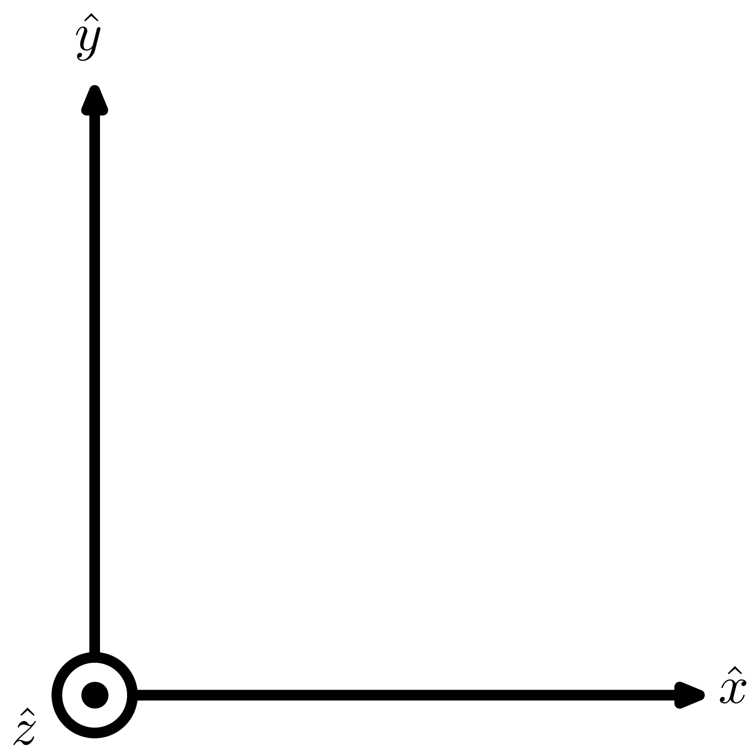
A.3 Non-physics working forward problems

The following are the instructions and non-physics working forward problems that were administered to all participants in the order in which they were given. The problems were administered one at a time with each problem on a single sheet of paper, as demonstrated with the first problem.

For each of the following diagrams, you will be asked to determine the cross product of two vectors:

$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$$

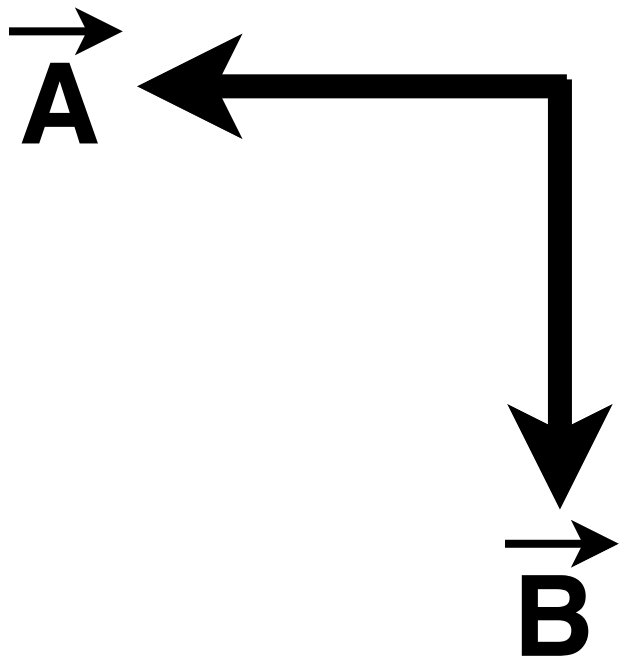
Assume the following coordinate system:



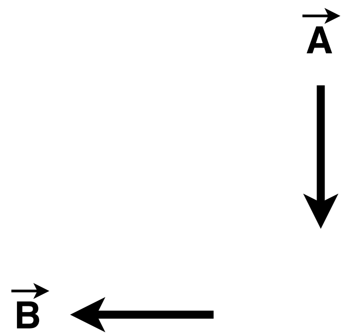
$\odot$ $+\hat{\boldsymbol{z}}$ (out of the page)

$\otimes$ $-\hat{\boldsymbol{z}}$ (into the page)

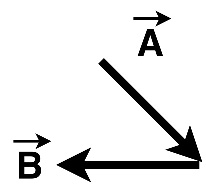
1. What is the direction of $\vec{A} \times \vec{B}$?



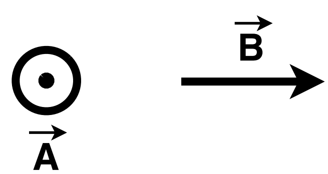
2. What is the direction of $\vec{A} \times \vec{B}$?



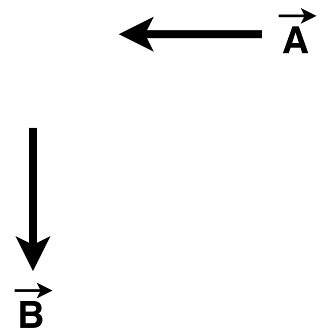
3. What is the direction of $\vec{A} \times \vec{B}$?



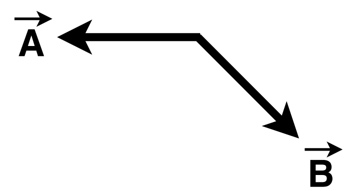
4. What is the direction of $\vec{A} \times \vec{B}$?



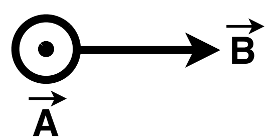
5. What is the direction of $\vec{A} \times \vec{B}$?



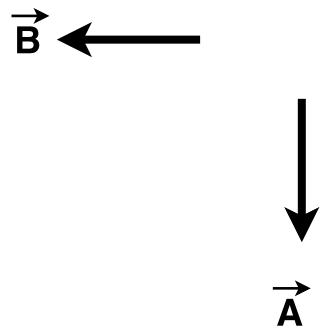
6. What is the direction of $\vec{A} \times \vec{B}$?



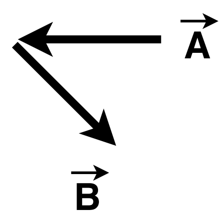
7. What is the direction of $\vec{A} \times \vec{B}$?



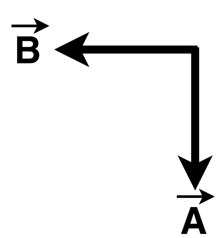
8. **What is the direction of $\vec{A} \times \vec{B}$?**



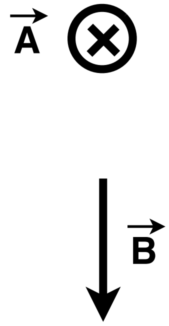
9. **What is the direction of $\vec{A} \times \vec{B}$?**



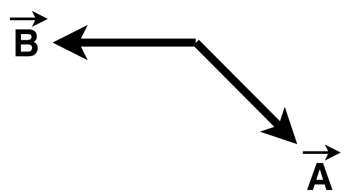
10. **What is the direction of $\vec{A} \times \vec{B}$?**



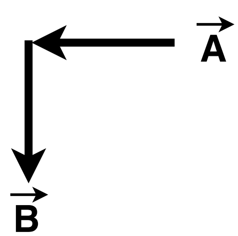
11. What is the direction of $\vec{A} \times \vec{B}$?



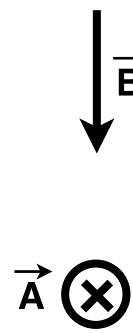
12. What is the direction of $\vec{A} \times \vec{B}$?



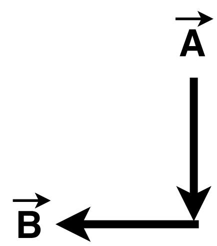
13. What is the direction of $\vec{A} \times \vec{B}$?



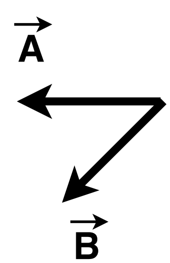
14. What is the direction of $\vec{A} \times \vec{B}$?



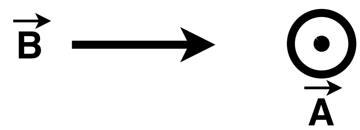
15. What is the direction of $\vec{A} \times \vec{B}$?



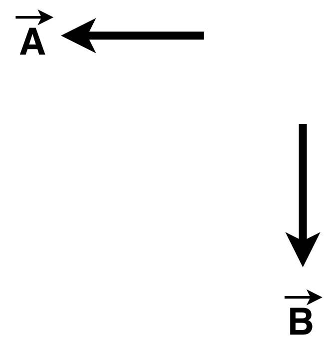
16. What is the direction of $\vec{A} \times \vec{B}$?



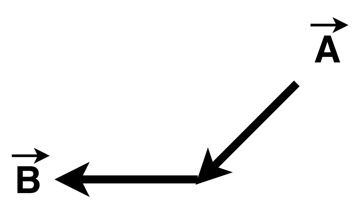
17. **What is the direction of $\vec{A} \times \vec{B}$?**



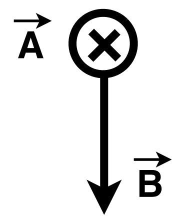
18. **What is the direction of $\vec{A} \times \vec{B}$?**



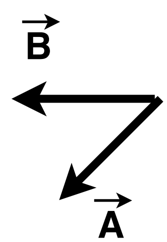
19. **What is the direction of $\vec{A} \times \vec{B}$?**



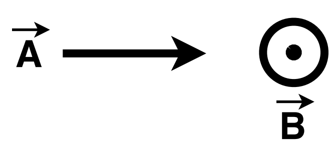
20. What is the direction of $\vec{A} \times \vec{B}$?



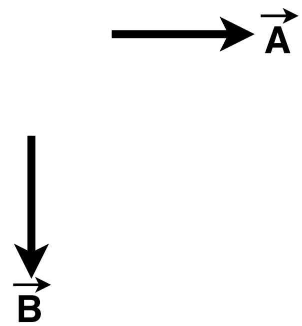
21. What is the direction of $\vec{A} \times \vec{B}$?



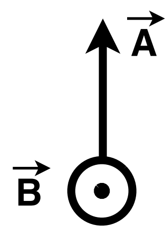
22. What is the direction of $\vec{A} \times \vec{B}$?



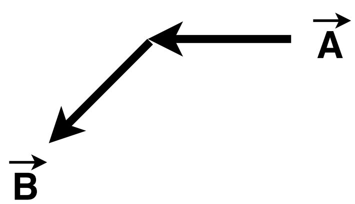
23. What is the direction of $\vec{A} \times \vec{B}$?



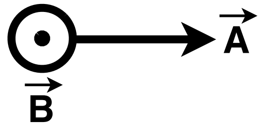
24. What is the direction of $\vec{A} \times \vec{B}$?



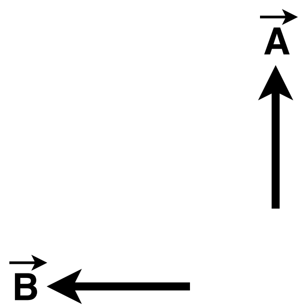
25. What is the direction of $\vec{A} \times \vec{B}$?



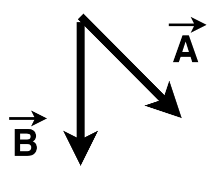
26. What is the direction of $\vec{A} \times \vec{B}$?



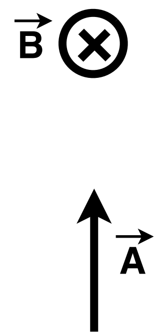
27. What is the direction of $\vec{A} \times \vec{B}$?



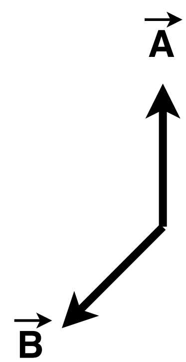
28. What is the direction of $\vec{A} \times \vec{B}$?



29. What is the direction of $\vec{A} \times \vec{B}$?

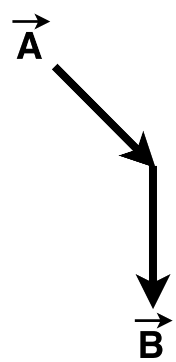


30. What is the direction of $\vec{A} \times \vec{B}$?

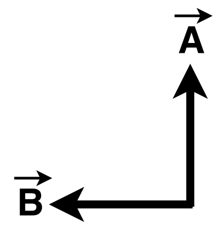


31. What is the direction of $\vec{A} \times \vec{B}$?

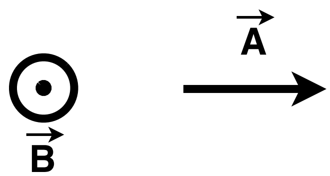
32. What is the direction of $\vec{A} \times \vec{B}$?



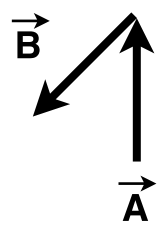
33. **What is the direction of $\vec{A} \times \vec{B}$?**



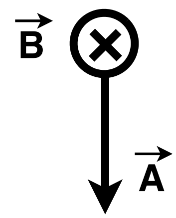
34. **What is the direction of $\vec{A} \times \vec{B}$?**



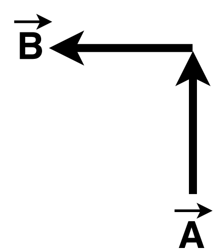
35. **What is the direction of $\vec{A} \times \vec{B}$?**



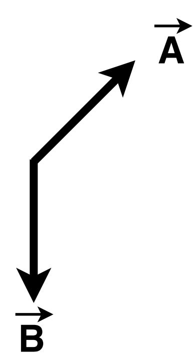
36. What is the direction of $\vec{A} \times \vec{B}$?



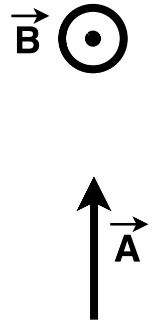
37. What is the direction of $\vec{A} \times \vec{B}$?



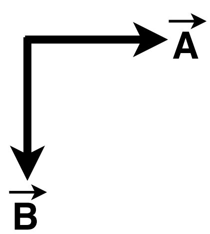
38. What is the direction of $\vec{A} \times \vec{B}$?



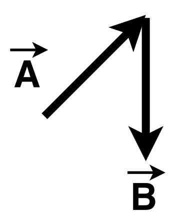
39. What is the direction of $\vec{A} \times \vec{B}$?



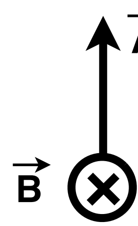
40. What is the direction of $\vec{A} \times \vec{B}$?



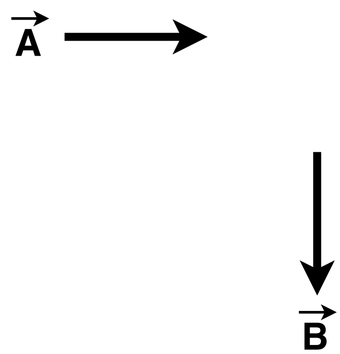
41. What is the direction of $\vec{A} \times \vec{B}$?



42. What is the direction of $\vec{A} \times \vec{B}$?

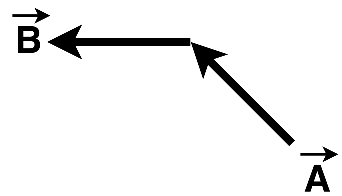


43. What is the direction of $\vec{A} \times \vec{B}$?

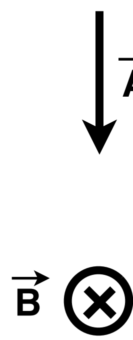


44. What is the direction of $\vec{A} \times \vec{B}$?

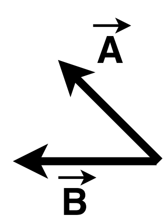
45. **What is the direction of $\vec{A} \times \vec{B}$?**



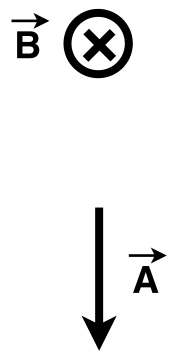
46. **What is the direction of $\vec{A} \times \vec{B}$?**



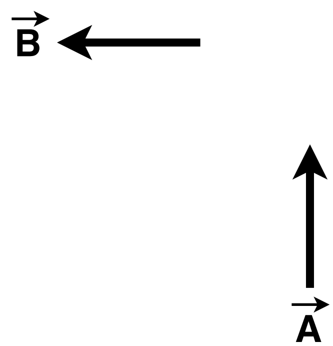
47. **What is the direction of $\vec{A} \times \vec{B}$?**



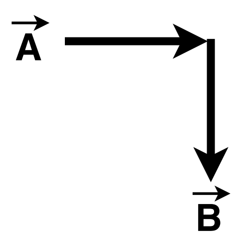
48. What is the direction of $\vec{A} \times \vec{B}$?



49. What is the direction of $\vec{A} \times \vec{B}$?



50. What is the direction of $\vec{A} \times \vec{B}$?



A.4 Non-physics working backward problems

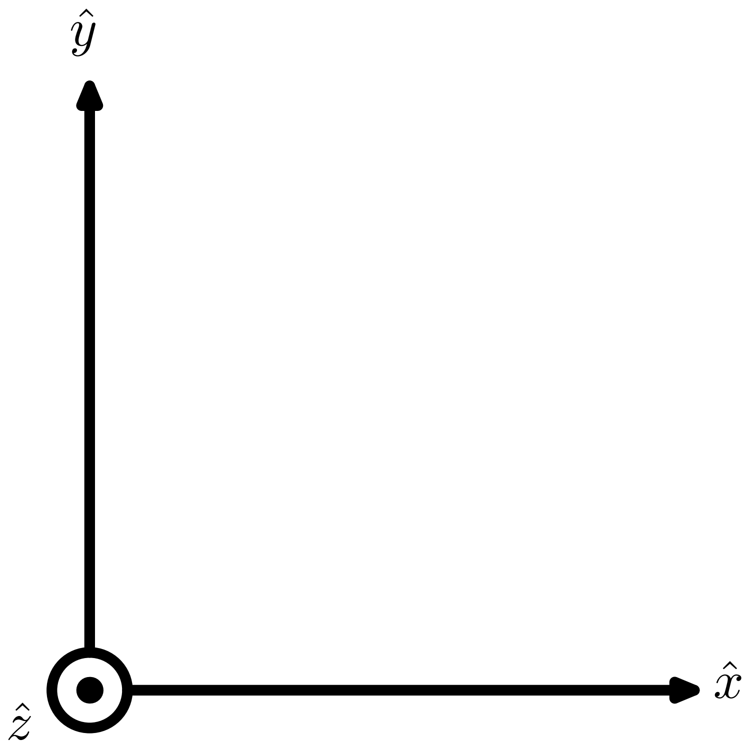
The following are the instructions for the non-physics working forward problems that were administered to all participants in the order in which they were given. The problems were administered one at a time with each problem on a single sheet of paper, as demonstrated with the first problem.

For each of the following diagrams,
assume three perpendicular vectors such that:

$$\vec{C} = \vec{A} \times \vec{B}$$

You will be given $\vec{C}$ and either $\vec{A}$ or $\vec{B}$ and asked to determine the third vector.

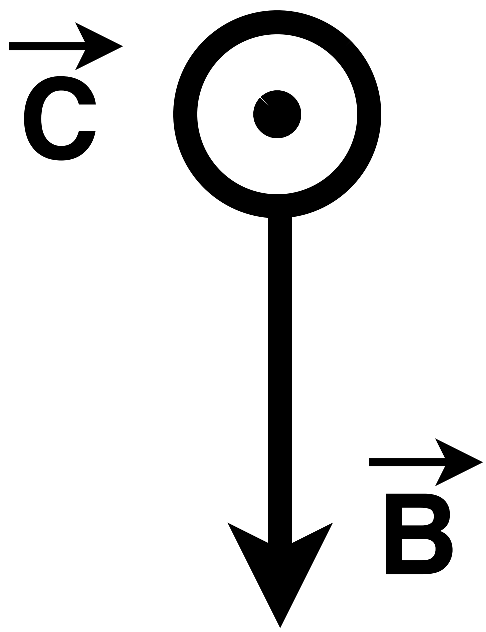
Assume the following coordinate system:



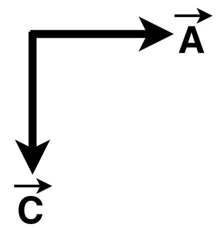
$\odot$ $+\hat{\boldsymbol{z}}$ (out of the page)

$\otimes$ $-\hat{\boldsymbol{z}}$ (into the page)

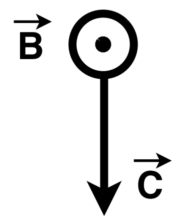
1. What is the direction of $\vec{A}$?



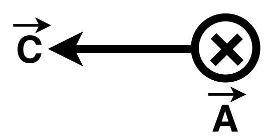
2. **What is the direction of $\vec{B}$?**



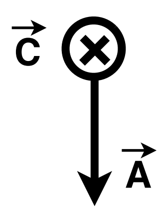
3. **What is the direction of $\vec{A}$?**



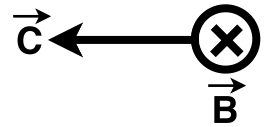
4. **What is the direction of $\vec{B}$?**



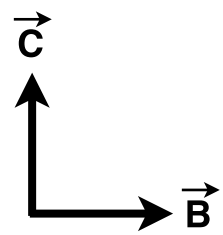
5. **What is the direction of $\vec{B}$?**



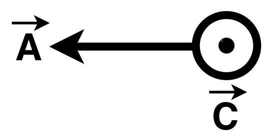
6. What is the direction of $\vec{A}$?



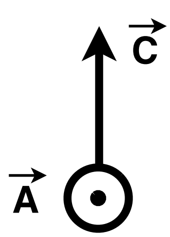
7. What is the direction of $\vec{A}$?



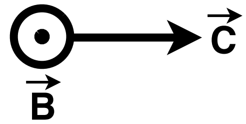
8. What is the direction of $\vec{B}$?



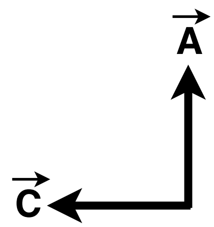
9. What is the direction of $\vec{B}$?



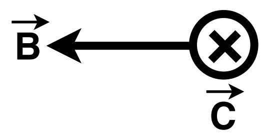
10. What is the direction of $\vec{A}$?



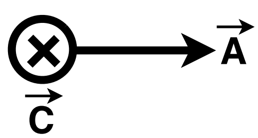
11. What is the direction of $\vec{B}$?



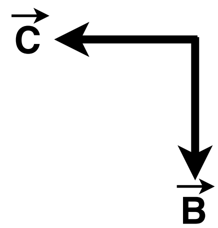
12. What is the direction of $\vec{A}$?



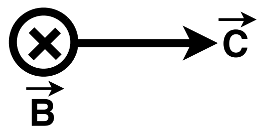
13. What is the direction of $\vec{B}$?



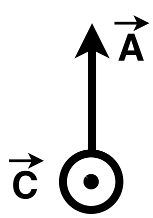
14. What is the direction of $\vec{A}$?



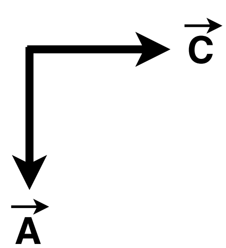
15. What is the direction of $\vec{A}$?



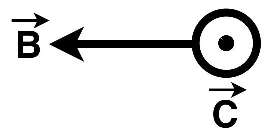
16. What is the direction of $\vec{B}$?



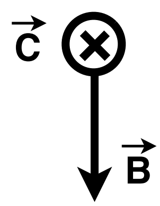
17. What is the direction of $\vec{B}$?



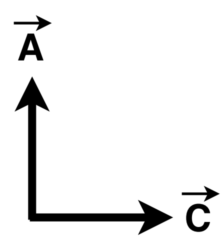
18. What is the direction of $\vec{A}$?



19. What is the direction of $\vec{A}$?



20. What is the direction of $\vec{B}$?



Appendix B

Qualitative code definitions

B.1 Method code definitions

This section includes the preliminary, initial and final code definitions for the methods used. Where appropriate, descriptive examples or transcript excerpts are also provided. The definitions for what constitutes a primary or supplemental method did not change from initial to final and the sub-codes for the right-hand rule did not change throughout the process. Thus, only the final definitions for these codes are given.

Table B.1: Preliminary definitions for methods

<table>
<tr><th>Code Name</th><th>Code Definition</th></tr>
<tr><td>Right-hand rule</td><td>Participant uses a physical technique with a hand to find the direction of the cross product.</td></tr>
<tr><td>Matrix</td><td>Participant makes use of matrices and determinants in finding the direction of the cross product.</td></tr>
<tr><td>Addition</td><td>Participant uses methods consistent with vector addition or subtraction (e.g completing the triangle).</td></tr>
<tr><td>Efield</td><td>Participant uses an implicit or explicit connection to electric field. For example:
field at the center of the loop is zero
field points away from positive and towards negative
field of a dipole looks like this, so it would be in this direction</td></tr>
<tr><td>Supplemental</td><td>Participant uses a method that would not provide an answer on it's own. These methods can be used either on their own (without a final answer) or in conjunction with another method.</td></tr>
<tr><td>Guess</td><td>Participant makes explicit mention of not knowing the answer, but provides one anyway (e.g. "i don't know, but I'll go with...").</td></tr>
<tr><td>Unclear</td><td>There is not enough verbal or gestural evidence to determine what methods the participant is using to solve the problem.</td></tr>
</table>

Table B.2: Preliminary definitions for supplemental sub-codes

Code Name	Code Definition
Orthogonality	Participant uses the plane of the given vectors to determine the correct orientation of the cross product. For example: • I have an x and a y, so it has to be a z • A is in the z, B is in the y, so C is in the x
Rotation	Participant rotates the paper, either in the plane of the table or such that the paper is vertical.
Multiplication	Participant uses either commutative or non-commutative multiplication rules. For example: • two negatives make a positive (*e.g.* A is in the negative x and B is in the negative z, so C has to be in the positive y) • a Levi-Civitas permutation (*e.g.* C cross A equals B)
Poles	Participant explicitly uses the magnetic poles to find the magnetic field in order to determine the magnetic force.
Guess and check	Participant chooses a direction and then uses another method to determine if that is the correct direction (*e.g.* I'll assume it's positive and see whether that works).

Table B.3: Initial definitions for primary methods

Code Name	Code Definition
Right-hand rule	Participant uses a physical technique (either a right-hand or left-hand rule) to find the direction of the cross product.
Addition	Participant uses methods consistent with vector addition or subtraction. For example: • "Going this way [points from tail of A to tip of B] from A to B."
Matrix	Participant makes use of matrices and determinants in finding the direction of the cross product.
Guess	Participant makes explicit mention of not knowing the answer, but provides one anyway. For example: • "I would say negative. I'm not real sure, it's almost a guess." • "Ah, man, this one's confusing me, into the page, B is going to the left, B is going to the left [does rhr], negative y [shakes head and shrugs], i don't know."
Unclear	There is not enough verbal or gestural evidence to determine what methods participant is using to solve the problem. For example: • "Um, so, in a loop, you would follow the conventional current, uh, therefore, you would get a positive z magnetic field." • "I'm not completely sure how to solve this problem. I guess, I don't really uh, like, I haven't really known the rules of um, of a ring yet. So, I'm not exactly sure how to solve this problem."

Table B.4: Initial definitions for supplemental methods

Code Name	Code Definition
Check	Participant explicitly chooses a direction (or sign) and uses another method to determine if that will produce the desired result or uses one method to determine a direction and then uses a different method to check the answer. For example: • "if it was that way, it would be out, so..." • "...you don't want that way, so..." • "A needs to be y, so A [does rhr], no opposite" • "So, plug that in and see ' • "Check myself..." • "Let's try it out..." • "cause if it's like it is right now, that's negative z direction. But if I do tip to tail, then that's positive z," • "Actually, now i'm gonna segment it into a bunch of straight wires and see if I can find a general direction." • "if the right hand rule uh, works out, without me changing anything, then that would mean the sign of the charge is positive and if the B field is in the opposite direction of where my thumb is pointing, that would mean the charge is negative."
Coordinate Reference	Participant explicitly references (either verbally or gesturally) the instruction sheet or problem statement to verify direction of coordinates or draws coordinate axes on the board. For example: • "wait, out of the page, actually it would be negative..." • "positive z, positive z, wait [looks at instruction sheet], out of the page, yeah."

Table B.4: (continued)

Code Name	Code Definition
Diagram	Participant draws a diagram on the whiteboard. For example: • redrawing the diagram given in the problem statement • drawing an explicit position vector from the charge to the observation location • drawing a set of coordinate axes • drawing vectors tip-to-tail or tail-to-tail • NOT writing equations (*i.e.* "Physics knowledge") or numbers (*i.e.* "Matrix")
Multiplication	Participant uses either commutative or non-commutative multiplication rules outside of an explicit calculation (i.e. "Matrix"). For example: • two negatives make a positive: "I started to realize that when you had one negative and one positive, you generally had it going in the negative direction...then if you had two that were the same sign, you would generally have a positive." • a Levi-Civitas permutation: "So, A to C would be the same and C to B would give you A."
Parallel Transport	Participant explicitly mentions moving the vectors or moves the vectors when drawing a diagram. For example: • "you need to put 'em together, tip to tail" • "Put two vectors together [points from tail of B to tail of A]"

Table B.4: (continued)

Code Name	Code Definition
Orthogonality	Participant restricts the solution to one axis either with no justification or with a justification other than that provided by another explicit method (such as "Matrix" or "Physics Knowledge").For example: • "it needs to be a y at least" • "it's going to have to be an x here" • "A needs to be in or out of the page" • "when you do the cross product of something, it's when you have, when it's only in two planes, I guess, or axis, on two axis, it's usually, the cross product is contained in the third" • "you generally had it going in the negative direction of the opposite, of the field that wasn't present, or the axis that wasn't present."
Rotation: Horizontal	Participant rotates the paper in the plane of the table or draws diagram at an angle to the board.
Rotation: Vertical	Participant rotates the paper vertically or draws a diagram that is rotated vertically (*e.g.* redraws diagram such that the z axis is toward the top of the board).

Table B.4: (continued)

Code Name	Code Definition
Physics Knowledge	Participant explicitly mentions physics not present in the problem statement or uses physics information from the problem statement to reason about the problem (does NOT include restating the problem statement). For example: • "the direction of the magnetic, magnetic field in a loop is zero." • "the only reason why it goes in the negative x at an observation out here is because you have those vectors that, um, don't complete, they can't, they kind of cancel out, but you have a resulting vector, but since it's in the center, they're all going to cancel out, so it just has to be um, zero magnitude." • "so you're gonna have to do v cross B" • "you have negative charge, so I'm gonna use my left hand" • "since it's negative, whatever the outcome is, is going to have to be switched" • "magnetic field, there's no force cause it would be moving down, or up if there were a force" • "I guess if you treat this as, kind of like conventional current if it were flowing through a wire, I guess you can use that as kind of the same thing" • "A long straight wire, so I think with a wire, there's gonna be a magnetic field's going around it." • "Conventional current, conventional current is in the opposite way of um, it's the uh, magnetic current, er, electric current." • "Let's say that if it's going out of the negative and into the positive, let me think about a dipole. How does a dipole work?"

Table B.5: Final definitions for method types

Code Name	Code Definition
Primary	A method that on it's own could always provide a singular, non-zero answer. These methods can be used on their own or in conjunction with another method or methods.
Supplemental	A method that on it's own cannot always provide a singular, non-zero answer. These methods can be used on their own (usually without a singular, non-zero answer) or in conjunction with another method or methods.

Table B.6: Final definitions for primary methods

Code Name	Code Definition
Right-hand rule	Participant uses a physical technique (either a right-hand or left-hand rule) to find the direction of the cross product.
Addition	Participant uses methods consistent with vector addition or subtraction. For example: • "Going this way [points from tail of A to tip of B] from A to B."
Matrix	Participant makes use of matrices and determinants in finding the direction of the cross product.
Unclear	There is not enough verbal or gestural evidence to determine what methods participant is using to solve the problem. For example: • "Um, so, in a loop, you would follow the conventional current, uh, therefore, you would get a positive z magnetic field." • "I'm not completely sure how to solve this problem. I guess, I don't really uh, like, I haven't really known the rules of um, of a ring yet. So, I'm not exactly sure how to solve this problem."

Table B.7: Final definitions for right-hand rule sub-codes

Code Name		Code Definition
Hand	Right	Participant uses the RIGHT hand.
	Left	Participant uses the LEFT hand.
Frame	Table	Right-hand rule is performed in the plane of the table. • $+\hat{x}$ is to the right of the participant • $+\hat{y}$ is away from the participant • $+\hat{z}$ is vertically upward
	Other	Right-hand rule is performed in a plane other than that of the table. For example: • $+\hat{z}$ is toward participant instead of vertically upward • $+\hat{x}$ is vertically upward instead of to the right
Type	Standard	The fingers point in the direction of the first vector (A), curl in the direction of the second vector (B) and the thumb points in the direction of A cross B.
	Current	Thumb points in the direction of the conventional current and the fingers curl in the direction of the magnetic field, such that at a given observation location, the fingers point in the direction of the magnetic field at that location.
	Loop	Fingers curl around a loop of wire and thumb gives the direction of the magnetic field at the center of the loop.
	3 Fingers	The index finger points in the direction of the first vector (A), the middle finger points in the direction of the second vector (B) and the thumb points in the direction of A cross B.
	Knuckles	The knuckles point in the direction of the first vector (A), fingers in the direction of the second vector (B) and the thumb points in the direction of A cross B.
	Palm	The thumb points in the direction of the first vector (A), fingers in the direction of the second vector (B) and the palm points in the direction of A cross B.

Table B.8: Final definitions for supplemental methods

Code Name	Code Definition
Guess& Check	Participant explicitly "guesses" an answer or makes an assumption (*e.g.* that the particle has positive charge or is moving in a specific direction) and then uses a primary method to determine whether that assumption is consistent with the given information. For example: • "So, plug that in and see" • "Actually, now i'm gonna segment it into a bunch of straight wires and see if I can find a general direction." • "I need an x here, so it's coming out of the page, A cross [does rhr] that way. You don't want that way. So, A cross [does rhr] that way. So, A would be in the plus x direction." • if the right hand rule uh, works out, without me changing anything, then that would mean the sign of the charge is positive and if the B field is in the opposite direction of where my thumb is pointing, that would mean the charge is negative."
Rotation	The given frame of reference is rotated by more than 15 degrees using either the paper or a diagram.
Diagram	Participant draws a diagram on the whiteboard. For example: • redrawing the diagram given in the problem statement • drawing an explicit position vector from the charge to the observation location • drawing a set of coordinate axes • drawing vectors tip-to-tail or tail-to-tail • NOT writing equations (*i.e.* "Physics knowledge") or numbers (*i.e.* "Matrix")

Table B.8: (continued)

<table>
<tr><th>Code Name</th><th>Code Definition</th></tr>
<tr><td>Multiplication</td><td>Participant uses either commutative or non-commutative multiplication rules outside of an explicit calculation (i.e. “Matrix”). For example:
two negatives make a positive: “I started to realize that when you had one negative and one positive, you generally had it going in the negative direction...then if you had two that were the same sign, you would generally have a positive.”
a Levi-Civitas permutation: “So, A to C would be the same and C to B would give you A.”</td></tr>
<tr><td>Parallel Transport</td><td>Participant explicitly mentions (or gestures) moving the vectors or moves the vectors when drawing a diagram. For example:
“you need to put ’em together, tip to tail”
“Put two vectors together [points from tail of B to tail of A]”</td></tr>
<tr><td>Orthogonality</td><td>Participant restricts the solution to one axis either with no justification or with a justification other than that provided by another explicit method (such as “Matrix” or “Physics Knowledge”).For example:
“it needs to be a y at least”
“it’s going to have to be an x here”
“A needs to be in or out of the page”
“when you do the cross product of something, it’s when you have, when it’s only in two planes, I guess, or axis, on two axis, it’s usually, the cross product is contained in the third”
“you generally had it going in the negative direction of the opposite, of the field that wasn’t present, or the axis that wasn’t present.”</td></tr>
</table>

Table B.8: (continued)

Code Name	Code Definition
Physics Knowledge	Participant explicitly mentions physics not present in the problem statement or uses physics information from the problem statement to reason about the problem (does NOT include restating the problem statement). For example: • "the direction of the magnetic, magnetic field in a loop is zero." • "the only reason why it goes in the negative x at an observation out here is because you have those vectors that, um, don't complete, they can't, they kind of cancel out, but you have a resulting vector, but since it's in the center, they're all going to cancel out, so it just has to be um, zero magnitude." • "so you're gonna have to do v cross B" • "you have negative charge, so I'm gonna use my left hand" • "since it's negative, whatever the outcome is, is going to have to be switched" • "magnetic field, there's no force cause it would be moving down, or up if there were a force" • "I guess if you treat this as, kind of like conventional current if it were flowing through a wire, I guess you can use that as kind of the same thing" • "A long straight wire, so I think with a wire, there's gonna be a magnetic field's going around it." • "Conventional current, conventional current is in the opposite way of um, it's the uh, magnetic current, er, electric current." • "Let's say that if it's going out of the negative and into the positive, let me think about a dipole. How does a dipole work?"

B.2 Error code definitions

This section includes the preliminary, initial, revised, and final code definitions for the errors made. Since most of these errors involved gestures, descriptive examplesinstead of transcript excerpts are provided where appropriate. The directions for which code families should be applied to which problems is outlined in Table B.13.

Table B.9: Preliminary definitions for errors

Table B.10: Initial definitions for errors

Table B.11: Revised definitions for errors

Table B.12: Final definitions for errors

Table B.13: Coding directions

Table B.9: Preliminary definitions for errors

Code Name	Code Definition
No Error	Participant makes no errors, corrected or uncorrected, and obtains the correct response from clear methods.
Addition [Corrected]	Participant attempts to use vector addition or subtraction method [and corrects the error or abandons the approach].
Matrix Error [Corrected]	Participant makes an error in his/her use of matrices [and corrects the error or abandons the approach].
Misinterpret [Corrected]	Participant gives a verbal solution that does not match his/her gestural response [and corrects the error]. For example: • treats the vector symbol over A as the vector direction • interprets observation location symbol as out of the page symbol • treating into/out of the page symbols as "stationary" • answers a question other than the one asked

Table B.9: (continued)

Code Name	Code Definition
Mismatch [Corrected]	Participant gives a verbal solution that does not match his/her gestural response [and corrects the error]. For example: • does rhr in the frame of the table and thumb points upward, but s/he says positive y instead of positive z • says negative z when thumb is pointing upward
Incomplete	Participant makes no errors (or corrects all errors) but doesn't obtain correct response because problem is not completed. For example: • reduces problem to a single axis, but doesn't find a final response. • correctly solves one part of two-step problem, but does not complete the second step (also a Misinterpret error).

Table B.9: (continued)

Code Name		Code Definition
Other		There is at least one error whose origin is unclear or does not fit into the categories provided. For example: • method is unclear regardless of whether the response is correct (includes no response if method is unclear) • other errors do not account fully for the final response • guesses without any justification
RHR	Inappropriate [Corrected]	Participant attempts to make use of a right hand rule that is inappropriate for the given situation [and corrects the error or abandons the approach]. For example: • inappropriately uses the left hand • current rule when given a magnetic force problem • loop rule when current is a straight wire • v cross B for a magnetic field problem

Table B.9: (continued)

Code Name		Code Definition
RHR	Order Reversal [Corrected]	Participant does the right hand rule in such a way that the order of the vectors is reversed (B cross A) [and corrects the error or abandons the approach]. For example: • when solving for A, curls from the given B into A, instead of finding an A that will curl into B • r cross v or B cross v (also a Physics: Other error if explicit).
RHR	Direction Reversal [Corrected]	Participant inappropriately reverses the direction of one of the vectors [and corrects the error or abandons the approach]. For example: • curls into the page instead of out of the page or vice versa • a vector points left, but is positioned to the right and participant treats it as if it points right • uses r pointing from observation location to the particle • switches direction of magnetic field due to poles (also a Physics: Other error if stated explicitly)

Table B.9: (continued)

Code Name		Code Definition
RHR	Other [Corrected]	Participant misapplies a right-hand rule such that it is unclear what error(s) are made or it is an error that does not fall into one of the other RHR categories [and corrects the error or abandons the approach].
Physics	Charge [Corrected]	Participant neglects or incorrectly accounts for the effect of the sign of the charge or the distinction between conventional and electron current and corrects the error or abandons the approach. For example: • forgets to account for the sign of the charge • counts the sign of the charge twice (*e.g.* at beginning and end) • uses electron current as if it were conventional current
Physics	Obs Location [Corrected]	Participant doesn't appropriately account for effect of observation location [and corrects the error or abandons the approach]. For example: • this is usually when using a current right-hand rule and means that they don't curl enough or do not curl in the right direction. • also usually involves an inappropriately applied right-hand rule

Table B.9: (continued)

Code Name		Code Definition
Physics	Efield [Corrected]	Participant explicitly attempts to reason from, or provides an answer consistent with, an invalid or incorrect electric field analogy [and corrects the error or abandons the approach]. For example: • field of a point charge or dipole • force parallel to field • field of loop is zero
Physics	Other [Corrected]	Participant explicitly invokes incorrect or inappropriate physics that is not covered in the above categories [and corrects the error or abandons the approach]. For example: • wrong equation • wrong direction of magnetic field from the poles of a magnetic

Table B.10: Initial definitions for errors

Code Name	Code Definition
No Error	Participant makes no errors, corrected or uncorrected, and obtains the correct response from clear methods.
Addition [Corrected]	Participant uses vector addition or subtraction method [and corrects the error or abandons the approach in favor of a different approach].
Matrix Error [Corrected]	Participant makes an error in his/her use of matrices [and corrects the error or abandons the approach in favor of a different approach].
Misinterpret [Corrected]	Participant misinterprets the diagram or misreads the problems statement [and corrects the error]. For example: • treats the vector symbol over A as the vector direction • interprets observation location symbol as out of the page symbol • treating into/out of the page symbols as "stationary" • answers a question other than the one asked

Table B.10: (continued)

Code Name	Code Definition
Mismatch [Corrected]	Participant gives a verbal solution that does not match the gestural response [and corrects the error]. For example: • does right-hand rule in the frame of the table and thumb points upward, but says positive y instead of positive z • says negative z when thumb is pointing upward
Incomplete	Participant makes no errors (or corrects all errors) but doesn't obtain correct response because problem is not completed. For example: • reduces problem to a single axis, but does not find a final response. • correctly solves one part of two-step problem, but does not complete the second step (also a Misinterpret error).
Other	There is at least one error whose origin is unclear or does not fit into the categories provided. For example: • other errors do not account fully for final response • guesses without any justification

Table B.10: (continued)

Code Name		Code Definition
Unclear		If the method is unclear regardless of whether the response is correct, incorrect, or no response.
RHR	Inappropriate [Corrected]	Participant attempts to make use of a right hand rule that is inappropriate for the given situation [and corrects the error or abandons the approach in favor of a different approach]. For example: • inappropriately uses the left hand • current rule when given a magnetic force problem • loop rule when current is a straight wire • v x B for a magnetic field problem (also Physics: Other if explicit)
RHR	Order Reversal [Corrected]	Participant does the right hand rule in such a way that the order of the vectors is reversed (B cross A) [and corrects the error or abandons the approach in favor of a different approach]. For example: • when solving for A, curls from the given B into A, instead of finding an A that will curl into B • r cross v or B cross v (also a Physics: Other error if explicit).

Table B.10: (continued)

Code Name		Code Definition
RHR	Direction Reversal [Corrected]	Participant inappropriately reverses the direction of one of the given vectors [and corrects the error or abandons the approach in favor of a different approach]. For example: • curls into the page instead of out of the page • vector points left, but on the right and treated as if it points right • uses r pointing from observation location to the particle • switches direction of magnetic field due to poles (also a Physics: Other error if stated explicitly)
RHR	Other [Corrected]	Participant uses a right-hand rule such that it is unclear what error(s) they are making or the error does not fall into one of the other right-hand rule categories [and corrects the error or abandons the approach in favor of a different approach]. For example: • mixes up the order of vectors (beyond an RHR: Order Reversal error) without explanation (*e.g.* r cross B to find I) • curls in a random direction

Table B.10: (continued)

Code Name		Code Definition
Physics	Obs Location [Corrected]	Participant does not appropriately account for the observation location [and corrects the error or abandons the approach in favor of a different approach]. For example: • when using a current rhr, participant doesn't curl enough or does not curl in the right direction.
Physics	Charge [Corrected]	Participant neglects or incorrectly accounts for the effect of the sign of the charge or the distinction between conventional and electron current [and corrects the error or abandons the approach in favor of a different approach]. For example: • forgets to account for the sign of the charge • counts the sign of the charge twice (*e.g.* at beginning and end) • explicitly uses electron current as if it were conventional current (also a RHR: Direction Reversal)

Table B.10: (continued)

Code Name		Code Definition
Physics	Efield [Corrected]	Participant explicitly attempts to reason from, or provides an answer consistent with, an invalid or incorrect electric field analogy [and corrects the error or abandons the approach in favor of a different approach]. For example: • field of a point charge or dipole • force parallel to field • field of loop is zero
Physics	Other [Corrected]	Participant explicitly invokes incorrect or inappropriate physics that is not covered in the above categories [and corrects the error or abandons the approach in favor of a different approach]. For example: • incorrect or inappropriate equation • wrong direction of magnetic field from the poles of a magnetic

Table B.11: Revised definitions for errors

Code Name	Code Definition
No Error	Participant makes no errors, corrected or uncorrected, and obtains the correct response from clear methods.
Addition [Corrected]	Participant uses vector addition or subtraction method [and corrects the error or abandons the approach in favor of a different approach].
Matrix Error [Corrected]	Participant makes an error in his/her use of matrices [and corrects the error or abandons the approach in favor of a different approach].
Misinterpret [Corrected]	Participant misinterprets the diagram or misreads the problems statement [and corrects the error]. For example: • treats the vector symbol over A as the vector direction • interprets observation location symbol as out of the page symbol • treating into/out of the page symbols as "stationary" • answers a question other than the one asked **Exception:** if participant misinterprets the same way on question(s) immediately following without restating, still code as Misinterpret.

Table B.11: (continued)

Code Name	Code Definition
Mismatch [Corrected]	Participant gives a verbal solution that does not match the gestural response [and corrects the error]. For example: • does right-hand rule in the frame of the table and thumb points upward, but says positive y instead of positive z • says negative z when thumb is pointing upward
Other	There is at least one error whose origin is unclear or does not fit into the categories provided. For example: • other errors do not account fully for final response • guesses without any justification • reduces problem to a single axis, but doesn't give a final response • doesn't reach a conclusion • the method is unclear, but the conclusion is wrong
Unclear Error	Participant uses an unclear method, but the response is correct.

Table B.11: (continued)

Code Name		Code Definition
RHR	Inappropriate [Corrected]	Participant attempts to make use of a right hand rule that is inappropriate for the given situation [and corrects the error or abandons the approach in favor of a different approach]. For example: • inappropriately uses the left hand • current rule when given a magnetic force problem • loop rule when current is a straight wire • explicitly $\vec{F} = \vec{v} \times \vec{B}$ for magnetic field (also Physics: Other)
RHR	Order Reversal [Corrected]	Participant does the right hand rule in such a way that the order of the vectors is reversed and they are doing B cross A [and corrects the error or abandons the approach for another approach]. For example: • B cross A • when solving for A, curls from the given B into A, instead of finding an A that will curl into B • r cross v or B cross v (also a Physics: Other error if explicit).

Table B.11: (continued)

Code Name		Code Definition
RHR	Direction Reversal [Corrected]	Participant inappropriately reverses the direction of one of the given vectors by explicitly stating correct direction and using the wrong direction or by using the wrong direction without stating it. [and corrects the error or abandons the approach in favor of a different approach]. For example: • treats into the page as out of the page without mention of the direction • states vector is in positive x, but curls towards negative x.
RHR	Other [Corrected]	Participant uses a right-hand rule such that it is unclear what error(s) they are making or the error does not fall into one of the other right-hand rule categories [and corrects the error or abandons the approach in favor of a different approach]. For example: • r = v cross B or v = r cross B • v = B cross F • curls in a random direction

Table B.11: (continued)

Code Name		Code Definition
Physics	Obs Location [Corrected]	Participant does not appropriately account for the observation location [and corrects the error or abandons the approach in favor of a different approach]. For example: • doesn't curl enough • doesn't curl in the right direction
Physics	Charge [Corrected]	Participant neglects or incorrectly accounts for the effect of the sign of the charge or the distinction between conventional and electron current [and corrects the error or abandons the approach in favor of a different approach]. For example: • forgets to account for the sign of the charge • counts the sign of the charge twice (*e.g.* at the beginning and the end) • uses electron current as if it were conventional current

Table B.11: (continued)

Code Name		Code Definition
Physics	Efield [Corrected]	Participant reasons from an invalid or inappropriate analogy to electric field [and corrects the error or abandons the approach in favor of a different approach]. For example: • field of a point charge or dipole • force parallel to field • field of loop is zero
Physics	Other [Corrected]	Participant explicitly invokes incorrect or inappropriate physics that is not covered in the above categories [and corrects the error or abandons the approach for another approach]. For example: • incorrect or inappropriate equation • wrong direction of magnetic field from the poles of a magnetic **Exception to explicit:** if participant uses the same wrong physics on the question(s) immediately following without restating the physics, still code as Physics:Other.

Table B.12: Final definitions for errors

Code Family		Code Definition
Addition	Error	Participant uses a vector addition or subtraction method (*e.g.* connects tip to tail or tip to tip).
	Corrected	Participant abandons the addition method in favor of a different approach or a non-response.
Matrix	Error	Participant makes at least one error with the use of matrices that contributes to the final response.
	Corrected	Participant corrects all errors from use of matrices or abandons matrices in favor of another method or a non-response.
	No Error	Participant makes no errors with the use of matrices regardless of whether the final response includes an error based on another method.
RHR	No Error	Participant makes no errors with this RHR regardless of whether the final response includes an error based on another method. (*e.g.* performs the RHR correctly or misinterprets the diagram, but does RHR correctly for that situation (also Misinterpret–Error))

Table B.12: (continued)

Code Family			Code Definition
RHR	Inappropriate	Error	Participant attempts to make use of a right-hand rule that is inappropriate for the given situation. If any other error (e.g. Direction or Order) is made, this code takes precedence. For example: • inappropriately uses the left hand (does NOT include for electrons or explicitly electron current) • explicitly does current rule for electron current instead of conventional current • current rule when given a magnetic force problem • loop rule when current is a straight wire • explicitly $\vec{F} = \vec{v} \times \vec{B}$ for a magnetic field problem (also Physics: Other–Error) • use a 3 finger right-hand rule on magnetic field problem (for Fred and Danny only)
		Corrected	Participant abandons the inappropriate right-hand rule for a different method (including another right-hand rule) or a non-response.

Table B.12: (continued)

Code Family			Code Definition
RHR	Direction	Error	Participant inappropriately reverses the direction of one of the given vectors by explicitly stating correct direction and using the wrong direction or by using the wrong direction without stating it. For example: • treats into the page as out of the page • states the vector is in positive x, but curls towards negative x. • treats r as the vector from the observation location to charge (also a Physics: Obs Loc–Error) • reverses direction of current without stating reason (also a Physics: Charge–Implicit)”
		Corrected	Participant corrects the direction or abandons the right-hand rule for a different method (including another right-hand rule) or a non-response.

Table B.12: (continued)

Code Family			Code Definition
RHR	Order	Error	Participant does the right hand rule in such a way that the order of the vectors is reversed and they are performing $\vec{B} \times \vec{A}$ instead of $\vec{A} \times \vec{B}$. For example: • when solving for A, curls from the given B into A, instead of finding an A that will curl into B • $\vec{r} \times \vec{v}$ or $\vec{B} \times \vec{v}$ (also a Physics: Other–Error if stated explicitly).
		Corrected	Participant corrects the order or abandons the right-hand rule for a different method (including another right-hand rule) or a non-response.

Table B.12: (continued)

Code Family			Code Definition
RHR	Other	Error	Participant uses a right-hand rule such that it is unclear what error(s) they are making or the error does not fall into one of the other right-hand rule categories or they make multiple errors. For example: • $\hat{r} = \hat{v} \times \hat{B}$ or $\hat{v} = \hat{r} \times \hat{B}$ • $\hat{v} = \hat{B} \times \hat{F}$ • curls in a random direction • doesn't curl enough (also a Physics: Obs Loc–Error) • both a RHR: Direction–Error and an RHR: Order–Error • does NOT include a RHR: Inappropriate–Error and any other error (code as RHR: Inappropriate–Error)
		Corrected	Participant corrects all errors or abandons the right-hand rule for a different method (including another right-hand rule) or a non-response.

Table B.12: (continued)

Code Family			Code Definition
Physics	Obs Loc	Error	Participant does not appropriately account for the observation location, either implicitly or explicitly. For example: • doesn't curl enough (also RHR: Other–Error) • curls in a random direction (also RHR: Other–Error) • misinterprets the observation location on diagram • treats r as the vector from the observation location to charge (also a RHR: Direction–Error)"
		Corrected	Participant corrects all errors regarding the observation location.
		No Error	Participant correctly accounts for the observation location (explicitly or implicitly).

Table B.12: (continued)

Code Family			Code Definition
Physics	Charge	Explicit	Participant explicitly accounts incorrectly for the sign of the charge or distinction between conventional and electron current. For example: • counts the sign of the charge twice • mentions switching direction (for charge), but doesn't actually perform the switch • explicitly uses the flow of electrons as conventional current
		Implicit	Participant does not appropriately account for the sign of the charge or type of current, but does not state explicitly. For example: • neglects to consider the effect of a negative charge • implicitly uses electron current as if were conventional current (also a RHR: Direction–Error)
		Corrected	Participant corrects all errors with sign of charge or type of current.
		No Error	Participant correctly accounts for the sign of the charge or type of current (explicitly or implicitly).

Table B.12: (continued)

Code Family			Code Definition
Physics	Efield	Explicit	Participant explicitly reasons from an invalid or inappropriate analogy to electric field. For example: • field of a point charge (away from + and toward -) • field of a dipole as analogy for field of a magnet • force parallel to field • field of loop is zero
		Implicit	Participant provides an answer consistent with an invalid or inappropriate analogy to electric field, but does not explicitly state the connection.
		Corrected	Participant corrects all errors regarding analogies to electric field or the analogy does not contribute to the final response.
		No Error	Participant either does not use an analogy to electric or uses it correctly.

Table B.12: (continued)

Code Family			Code Definition
Physics	Other	Error	Participant explicitly invokes incorrect or inappropriate physics not covered in the other "Physics" categories or uses the same wrong physics on question(s) immediately following an explicit error without restating the physics. For example: • incorrect equation (*e.g.* $\vec{B} = \hat{r} \times \vec{v}$) • inappropriate equation (*e.g.* explicit $\vec{F} = \vec{v} \times \vec{B}$ on a magnetic field problem) • wrong direction for magnetic field from the magnets • force in direction of velocity (can be implicit)
		Corrected	Participant corrects all physics errors or they do not contribute to the final response.
		No Error	Participant doesn't make any physics errors other than those in the other "Physics" categories.

Table B.12: (continued)

Code Family		Code Definition
Misinterpret	Error	Participant misinterprets the diagram or misreads the problems statement in a way that contributes to the final response. For example: • answers a question other than the one asked • treats the symbol over $\vec{A}$ as the vector direction • treating into/out of the page symbols as "stationary" (*e.g.* Pablo) • explicitly states wrong direction (*e.g.* into the page when symbol is out of the page or $+\hat{x}$ when it is $-\hat{x}$) **Note:** code if participant uses the same wrong direction on the question(s) immediately following even if the direction isn't stated explicitly.
	Corrected	Participant misinterprets the diagram or misreads the problems statement, but corrects the error either explicitly or implicitly.
	No Error	There is no evidence that the participant misinterpreted the diagram or misread the problem.

Table B.12: (continued)

Code Family		Code Definition
Match	Error	Participant gives a verbal solution that does not match the gestural (or diagrammatic) response. For example: • does right-hand rule in the frame of the table and thumb points upward, but says positive y instead of positive z • says negative z when thumb is pointing upward
	Corrected	Participant gives a verbal solution that does not match the gestural (or diagrammatic) response, but corrects the error regardless of whether the response is correct.
	No Error	Participant gives a verbal solution that matches the gestural (or diagrammatic) response regardless of whether the response is correct.
	N/A	Participant does not provide a gestural (or diagrammatic) response or does not give a final response.

Table B.12: (continued)

Code Family		Code Definition
Other	Error	There is at least one error that contributes to the final response which is not due to a primary method, physics, misinterpretation or a gestural mismatch. For example: • other errors do not account fully for final response • the origin of an error is unclear • reduces the problem to a single axis, but does not find a final response • guesses without any justification • gives a non-response
	Corrected	All “Other” errors are corrected or do not contribute to the final response.
	No Error	There is no evidence of errors other than those from primary methods, physics, misinterpretation or a gestural mismatch.

Table B.13: Coding directions

	Code Family	Code Definition
Method	Unclear	Every problem with an Unclear method code should not be coded for errors.
	Addition	Every problem with an Addition method code should have one Addition error code.
	Matrix	Every problem with an Matrix method code should have one Matrix error code.
	Right-hand rule (RHR)	Every RHR method should have one associated RHR error code. Each error code should be based on errors made with that RHR that contributed to a response (*i.e.* do NOT count initial attempts to orient the hand). If there is no response, coding should be based on the last attempt made with that RHR.
Physics	Observation Location	Every magnetic field problem should have one Physics: Obs Loc error code.
	Charge	Every physics problem should have one Physics: Charge error code.
	Electric Field	Every physics problem should have one Physics: Efield error code.
	Other	Every physics problem should have one Physics: Other error code.
	Misinterpret	Every problem should have one Misinterpret error code.
	Mismatch	Every problem should have one Mismatch error code.
	Other	Every problem should have one Other error code.