ABSTRACT

HU, HAO. Development of a New Constitutive Model for FRP-And-Steel-Confined Concrete. (Under the direction of Dr. Rudolf Seracino.)

In the past 20 years, fiber reinforced polymer materials have been widely applied in construction and structural rehabilitation due to their high strength and stiffness-to-weight ratio and high corrosion resistance. One important application is in strengthening or repair of existing reinforced concrete columns by using FRP as a confining material. This thesis presents a new passive confinement model (including three alternatives) for concrete confined by both internal transverse steel reinforcement and external FRP wrap with a circular section based on the modification of existing FRP-confined concrete models. The modifications include a new lateral-to-axial strain relationship and failure surface function, to improve the prediction of the general stress-strain behavior and ultimate condition. Through comparisons with published test data and other models, the accuracy of the proposed model (Combined Failure Surface Function Method) is shown to be superior to existing models.
Development of a New Constitutive Model for FRP-And-Steel-Confined Concrete

by
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1 INTRODUCTION

1.1 Overview

Fiber-reinforced polymers (FRP) are a composite material made of a polymer matrix and fibers. There are different kinds of fibers commonly used in civil engineering application including glass, carbon and aramid. Because of advantages such as high strength and stiffness to weight ratio, corrosion resistance, high fatigue strength, and non-magnetic properties, FRP has been applied in many areas such as aerospace, defense, marine, corrosion resistant equipment, and automotive sectors. FRP composites have been used as structural materials since World War II, when they were first used in the construction of British Spitfires (Mirmiran et al. 2003). FRP material is linear elastic and ruptures suddenly.

In the past 20 years, FRP material has been widely applied in construction and structural rehabilitation because of the properties already listed as well as the ability to form and shape the FRP to the existing structure. As existing structures deteriorate over time, or their intended purpose changes, strengthening or repair is often necessary. Typical applications range from wrapping of columns or piers, and external bonding to flexural members in buildings and bridges to enhance the shear or flexural resistance.

Columns are a critical component of buildings and bridges, because local failure or damage can lead to disproportionate collapse of the structure. Typical damage in columns during an earthquake includes buckling of the longitudinal reinforcing and shear failure, as shown in Figure 1.1, which is often due to inadequate transverse reinforcement.
There are many methods to repair damaged columns. One of them is to use the addition of heavily reinforced concrete jackets to confine the core of column which has been applied in many projects. Steel jackets can also be used. However, these have many disadvantages. Concrete jackets increase the weight of the structure and the section size, and require significant time and effort to cast and allow strength gain. Steel jacketing requires welding and depends on the skill level of the laborers. However, using FRP materials to wrap columns can overcome these disadvantages: the light weight will not increase the self-weight of structure; the small volume of material required will not increase the section size; and the application process is relatively fast and easy. Thus,
using FRP material to strengthen or repair columns is one of the most effective methods. Figure 1.2 shows examples of seismic retrofit of columns using FRP.

Figure 1.2 Examples of seismic retrofit of columns using FRP (left to right): wet lay-up; adhesively bonded prefabricated jackets; and winding of tow (“Building materials for the renewal of civil infrastructure,” 2005)

1.2 Overall objective of this research

It is typically necessary to predict the response of FRP retrofitted reinforced concrete columns. There are several numerical tools available to simulate columns after repair, for example, moment-curvature analyse can be conducted by “Cumbia”, developed by Montejo (2007), or dynamic analyse of in-elastic structures can be processed by “Ruaumoko” and “OpenSees”, developed by Carr (2001) and McKenna (1997), respectively. However, the accuracy of the numerical simulations is directly related to the robustness of the constitutive models used.
To fulfill this need, the purpose of this research is to develop an appropriate and accurate constitutive model for reinforced concrete confined with both FRP and transverse steel. At this time, the research is restricted to circular sections.

1.3 Thesis content

This thesis consists of seven chapters. A brief description of each follows.

Chapter 2 is a review of the current state of knowledge regarding steel-confined concrete, FRP-confined concrete, and steel-and-FRP-confined concrete. In the literature review, general behaviors and related stress-strain models are presented for each. The justification for this research is included at the end of the chapter.

Chapter 3 expands on the literature review where a detailed assessment of existing models for steel-and-FRP-confined concrete is presented.

Chapter 4 presents the development of three alternate passive confinement models for steel-and-FRP-confined concrete, based on different assumptions. The solution procedure for each approach is also described.

Chapter 5 contains the assessment of the new models compared to actual test results and the other existing models.

Chapter 6 presents an example application of the proposed new passive confinement model. A parametric study is conducted regarding the thickness of the applied FRP wrap.

Chapter 7 summarizes the current research which includes conclusions and recommendations drawn from the previous chapters.
Appendices are included at the end of the thesis which contains the table of nomenclature, and the test databases collected for assessment purposes.
2 LITERATURE REVIEW

2.1 Introduction

This Chapter provides a comprehensive review regarding steel-confined, FRP-confined, and steel-and-FRP-confined concrete with a circular section. Detailed description of general stress-strain behavior and existing models are presented.

For steel-confined concrete, both active and passive confinement models are briefly introduced. For FRP-confined concrete, only passive confinement models are introduced. Specific comparison of different failure surface equations and lateral-to-axial strain relationships are given, which leads to the best FRP-confined concrete model for analysis purposes. For steel-and-FRP-confined concrete, observations of experimental phenomenon and analysis of test results are conducted. The introduction and critical assessment of four existing models is also presented, which identifies the research gap and the purpose of this research.

2.2 Steel-confined concrete

2.2.1 General behavior of steel-confined concrete

Reinforced concrete columns or cylinders confined by transverse steel reinforcement, including spiral, hoop, steel wire cable and steel jacket, typically fail by concrete crushing as shown in Figure 2.1. For reinforced concrete columns confined with transverse steel, the failure mode may also be governed by rupture or buckling of the longitudinal reinforcing. Since steel-confined concrete models relate primarily to the behavior of concrete, this review focuses on steel-confined concrete cylinder tests.
Figure 2.1 Steel-confined concrete failure of (a) column (SJSU Earthquake Engineering Research Institute, Column tested in compression test, photo taken in Berkeley Richmond Earthquake Test Facility) and (b) cylinder (Campione 2002)

Lateral confinement improves the concrete strength and ductility by restraining the dilation of the concrete. The typical stress-strain curve of steel-confined concrete includes an ascending branch and a descending branch, as shown in Figure 2.2 compared to unconfined concrete. In either a RC column or cylinder, the cover concrete becomes ineffective once $f_{co}$ is reached, which is the compressive strength of the unconfined concrete, since it is not confined by the transverse reinforcement. After reaching $f_{co}$, the cover concrete starts to crack and spall off (Li et al. 2005). Meanwhile the core concrete continues to carry stresses at high strains beyond $\varepsilon'_{cc}$ until the axial strain reaches the strain limit $\varepsilon_{cu}$. However, this descending portion of the stress-strain curve after the peak $f'_{cc}$ is highly variable and depends strongly on the test procedures. The ultimate concrete strain $\varepsilon_{cu}$ is strongly dependent on the type of specimen, type of
loading, and rate of testing, and researchers have not reached consensus on an unique expression for $\varepsilon_{cu}$. To calculate $\varepsilon_{cu}$, Mander et al (1988) proposed Equation 2.9, accounting for the influence from material property. Cusson and Paultre (1995) terminate their stress-strain curve when concrete stress drop to 50% of the maximum confined compressive strength $f'_{cc}$. Earlier models like Hognestad (1951) end the stress-strain curve when concrete stress drops to 85% of the compressive strength. Different from this unsolved issue in steel-confined concrete, the ultimate condition in FRP-confined concrete and steel-and-FRP-confined concrete occurs when the FRP wrap ruptures, which is discussed in Sections 2.3 and 2.4. Thus the issue of ultimate strain $\varepsilon_{cu}$ in steel-confined concrete model is not an issue for the current research.

![Figure 2.2 Stress-strain curve of steel-confined concrete](image)

**Figure 2.2 Stress-strain curve of steel-confined concrete**
2.2.2 Stress-strain model of steel-confined concrete

Steel-confined concrete has been widely studied for many years. Basically two types of models have emerged based on the assumptions made, namely: active confinement, and passive confinement models. Some models of each type are shown in Table 2.1.

<table>
<thead>
<tr>
<th>Active confined</th>
<th>Passive confined</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With incremental iterative procedure</td>
</tr>
<tr>
<td></td>
<td>Without incremental iterative procedure</td>
</tr>
<tr>
<td>Mander and Priestley</td>
<td>Madas and Elnashai (1992)</td>
</tr>
</tbody>
</table>

2.2.2.1 Active confinement model

The model proposed by Mander and Priestley (1988) has been widely adopted in many practical applications, and is also employed by many passive FRP-confined concrete models, which is discussed in later Section 2.3. Therefore only Mander’s model is presented here to explain active confinement models.

Mander’s model is based on the assumption of active confinement, which means the concrete core is confined by a constant and uniform confining pressure as shown in Figure 2.3.
Figure 2.3 Free body diagram of half section of steel-confined concrete

The lateral confining pressure $f_i'$ can be derived from the force equilibrium of the free body diagram in Figure 2.3 given by Equation 2.1.

$$f_i' \cdot s \cdot d_c = 2f_{sy} \cdot A_{sp}$$  \hspace{1cm} \text{Equation 2.1}

where $f_{sy} = \text{steel yield stress}$; $A_{sp} = \text{area of transverse steel cross section}$; $s = \text{center to center spacing of the spiral or circular hoop}$; $d_c = \text{diameter of concrete core, measured center to center of the spiral or circular hoop}$.

Due to the fact that the concrete core is not fully confined between adjacent steel hoops or spirals as shown in Figure 2.4, Mander proposed the confinement effectiveness coefficient $k_e$ given by Equation 2.2 or Equation 2.3 to calculate the actual confining pressure.
Figure 2.4 Effectively confined core for circular hoop reinforcement (Mander and Priestley 1988)

For circular hoops

\[ k_e = \left( \frac{1 - \frac{\gamma}{2d_c}}{1 - \rho_{cc}} \right)^2 \]  

Equation 2.2

For circular spirals

\[ k_e = \frac{1 - \frac{s'}{2d_c}}{1 - \rho_{cc}} \]  

Equation 2.3

where \( s' \) = clear vertical spacing between spiral or hoop bars; \( \rho_{cc} \) = ratio of area of longitudinal reinforcement to area of core concrete.

Thus the final effective confining pressure is given by

\[ f_t = f_t' \cdot k_e \]  

Equation 2.4
In Mander’s model, the stress-strain relationship of confined concrete is described following that proposed by Popovics (1973).

\[
\frac{f_c}{f'_{cc}} = \frac{(\varepsilon_c/\varepsilon'_{cc})^r}{r-1+(\varepsilon_c/\varepsilon'_{cc})}
\]

Equation 2.5

where \( f_c \) and \( \varepsilon_c \) = axial stress and axial strain of concrete, respectively; \( f'_{cc} \) and \( \varepsilon'_{cc} \) = the peak axial stress and corresponding axial strain of concrete, respectively; \( r = \frac{E_c}{E_c-f'_{cc}/\varepsilon'_{cc}} \) accounting for the brittleness of concrete, defined by Carreira and Chu (1985); and \( E_c \) = elastic modulus of concrete.

The peak confined concrete stress is a function of confining pressure, defined by the following failure surface.

\[
f'_{cc} = f_{co} \left(2.254 \sqrt{1 + 7.94 \frac{f_l}{f_{co}} - 2 \frac{f_l}{f_{co}} - 1.254}\right)
\]

Equation 2.6

Equation 2.6 is proposed by Mander and Priestley (1988) to predict the peak stress \( f'_{cc} \).

While Equation 2.7 describes the relationship between the maximum confined compressive strength and corresponding strain, first proposed by Richart et al. (1929)

\[
\frac{\varepsilon'_{cc}}{\varepsilon_{co}} = 1 + 5 \left(\frac{f'_{cc}}{f_{co}} - 1\right)
\]

Equation 2.7

Richart et al (1929) also proposed the failure surface function using concrete specimens confined with active hydrostatic fluid pressure, given by

\[
f'_{cc} = f_{co} + k_1 \cdot f_l
\]

Equation 2.8

where \( k_1 = 4.1 \).
The stress-strain curve ends when the axial strain reaches the ultimate strain $\varepsilon_{cu}$ which is given by the following proposed by Priestley et al. (1996)

$$\varepsilon_{cu} = 0.004 + \frac{1.4\rho_s f_{sy} \varepsilon_{su}}{f'_{cc}}$$

Equation 2.9

where $\rho_s$ = the ratio of the volume of confinement reinforcement to the volume of confined concrete core; and $\varepsilon_{su}$ = the steel strain of the steel at maximum tensile stress.

Mander’s model does not have an explicit expression for concrete lateral strain-axial strain. However it should be noted that Equation 2.9 correlates ultimate concrete axial strain with strain in the confinement reinforcement. It is easy to see that Mander’s model contains four parts, as summarized in Table 2.2.

**Table 2.2  Mander’s model**

<table>
<thead>
<tr>
<th>Axial Stress-strain relationship</th>
<th>Failure surface</th>
<th>Ultimate condition</th>
<th>Lateral strain-axial strain relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c = \frac{f_{cc}}{f'_{cc}} \frac{\varepsilon_c}{\varepsilon_c^<em>} \cdot \frac{r}{r - 1 + (\varepsilon_c/\varepsilon_c^</em>)^r}$</td>
<td>$f'<em>{cc} = f</em>{cc} \left( 2.254 \sqrt{1 + 7.94 \frac{f'<em>l}{f</em>{co}} - 2 \frac{f'<em>l}{f</em>{co}} - 1.254} \right)$</td>
<td>$\varepsilon_{cu} = 0.004 + \frac{1.4\rho_s f_{sy} \varepsilon_{su}}{f'_{cc}}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\varepsilon_c^*}{\varepsilon_{co}} = 1 + 5 \left( \frac{f'<em>{cc}}{f</em>{co}} - 1 \right)$</td>
<td>Equation 2.5</td>
<td>Equation 2.6 and Equation 2.7</td>
<td>Equation 2.9</td>
</tr>
</tbody>
</table>
2.2.2.2 Passive confinement model

In reality, the passive confining pressure from transverse steel is uniform and constant only after steel yields. Normally, the confining pressure exerted by the confinement reinforcement depends on the lateral expansion of the concrete, which is referred to passive confinement.

In passive confinement models the lateral confining pressure gradually increases from zero up to the value given by Equation 2.4 rather than a constant and uniform confining pressure as in active confinement models. Based on whether the incremental iterative approach is used, there are generally two types of passive confinement models.

Particularly, the incremental iterative type of passive confinement models are based on the assumption that the axial stress and axial strain of concrete confined by steel at a given dilation state (a fixed axial strain or lateral strain) are the same as concrete actively confined with a constant confining pressure from steel confinement.

That is, the passive confinement curve is determined by a family of active confinement curves, as shown in Figure 2.5. In other words, this incremental iterative type of passive confinement model employs an active confinement model to generate its own stress-strain curve. For example, the passive confinement model proposed by Ahmad and Shah (1982b) employs the active confinement model, which was previously established by Ahmad and Shah (1982a). Madas and Elnashai (1992) model utilizes the active model proposed by Cedolin et al (1977). This concept is also widely used in many passive FRP-confined concrete models.
Figure 2.5 Passive confinement model using active confinement curves in an incremental approach

The second type of passive confinement model, without an incremental iterative procedure, can account for the progressive nature of the passive confinement by defining a particular confined concrete stress-strain curve (Cusson and Paultre (1995)). Afterwards, Légeron and Paultre (2003) developed Cusson’s model into a unique model for normal- and high-strength concrete, either with circular or rectangular section.

The difference between passive confinement models is that different stress-strain relationships, failure surface functions, and lateral-to-axial strain relationships are employed in each model. Specific details of passive confinement models are not introduced here since they are not directly related to this research project.

Mander’s active confinement model is most widely used in engineering practice, but passive confinement models more closely simulate the nature of confinement. A
comparison done by Madas and Elnashai (1992) shows how passive confinement models might predict more accurate stress-strain responses compared to Mander’s model, as shown in Figure 2.6 and summarized in Table A.2 in the Appendix.

Figure 2.6 Experimental and analytical stress-strain relationships for specimen RC-1 and specimen RC-2 (Madas and Elnashai 1992)
2.3 FRP-confined concrete

2.3.1 General behavior of FRP-confined concrete

Since FRP is a linear elastic material, the confining pressure provided by the FRP wrap continuously increases with increasing concrete lateral strain, which relates to the passive confinement model. Failure of FRP-confined concrete is governed by FRP rupture in the hoop direction, which has been observed by many researchers of FRP-confined circular concrete cylinders (e.g. Karbhari and Gao (1997), and Xiao and Wu (2000)). Figure 2.7 shows examples of typical failure in Xiao and Wu’s tests.

![Image](image.png)

**Figure 2.7 Typical Example of Failure (Xiao and Wu 2000)**

The stress-strain behavior of FRP-confined concrete is remarkably different from that of steel-confined concrete, as observed from extensive test results (e.g. Mirmiran and Shahway (1997), Xiao and Wu (2000), Lam and Teng (2003)). The stress-strain curve of FRP-confined concrete generally contains a nonlinear ascending branch followed by a nearly linear ascending branch, characteristic of the bilinear shape shown in Figure 2.8.
However the bilinear trend for FRP-confined concrete shown in Figure 2.8 is not always the case. Teng et al (2004) concluded that if the amount of FRP is smaller than a threshold value, a descending branch rather than a linearly ascending branch would occur. This has been observed in FRP-confined circular concrete cylinder tests (e.g. Xiao and Wu 2000; Jiang and Teng 2007a).

Figure 2.9 (Jiang and Teng 2007a) shows three FRP-confined concrete response classified by the extent of confinement. Specimen 24 with moderate confinement and specimen 34 with heavy confinement have a similar bilinear shape. However, with the low level of confinement, specimen 28 shows a descending branch.
2.3.2 Stress-strain model of FRP-confined concrete

FRP-confined concrete has been extensively studied. Many confinement models have been proposed by investigators. Teng and Lam (2004) provide a critical review of existing studies and exhaustive model assessment. According to their research, FRP-confined models can be classified into two categories: (1) design-oriented models, and (2) analysis-oriented models. Stress-strain models in design-oriented models have closed-form expressions; while analysis-oriented models require an incremental procedure. In the author’s opinions, both types of models have advantages and disadvantages, as summarized in Table 2.3.
<table>
<thead>
<tr>
<th>Advantages</th>
<th>Design-oriented models</th>
<th>Analysis-oriented models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Suitable for direct application in design by hand or spreadsheet</td>
<td>• Most models are able to predict both ascending and descending types of stress-strain curve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• All models can predict both the axial stress-strain behavior and the axial stress-lateral strain behavior</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Ultimate condition is defined as FRP rupture in hoop direction</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disadvantages</th>
<th>Design-oriented models</th>
<th>Analysis-oriented models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Most models only predict ascending stress-strain curves including the model adopted by ACI 440.2R-08</td>
<td>Inconvenient for design due to incremental iterative procedure</td>
</tr>
<tr>
<td></td>
<td>• Incomplete dilation property owing to inability to predict axial stress-lateral strain relationship in most models</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Ultimate condition is not directly related to failure mode (FRP rupture)</td>
<td></td>
</tr>
</tbody>
</table>

As the names indicate, design-oriented models are suitable for design purposes owing to its simple explicit expression; while analysis-oriented models are more utilized for
research purposes due to its robust theoretical foundation. Also some analysis-oriented models may be used in design with advanced tool based in Matlab, for example.

Teng and Lam (2004) concluded that analysis-oriented models utilized four different approaches. For example, the model proposed by Harmon et al. (1998) is based on the concept of crack slip and separation in the concrete. Becque et al. (2003) proposed a model based on Gerstle’s (1981a,b) octahedral stress-strain models, and the model proposed by Karabinis and Rousakis (2002) use the plasticity approach which is more complicated than other analysis-oriented models. Besides these three approaches, the majority of the models are based on active confinement models, including those of Mirmiran and Shahawy (1997), Spoelstra and Monti (1999), Fam and Rizkalla (2001), Chun and Park (2002), Harries and Kharel (2002), Marques et al. (2004), Binici (2005), and Teng et al (2007). To be consistent with what has been introduced in Section 2.2.2.2, this type of model is also referred to as a passive confinement model. According to the critical assessment in Jiang and Teng (2007) of those passive confinement models mentioned above, the model proposed by Teng et al. (2007) is considered to be the best model for its most accurate prediction in ultimate stress and strain, the shape of the lateral-to-axial strain curve, and the axial stress-strain curve.

The concept that a passive confinement curve is determined by a family of curves of active confinement (see Figure 2.5) has been widely adopted for FRP-confined concrete (e.g. those listed previously), as shown in Figure 2.10. These models all adopt Mander’s model as the active confinement model. As Teng and Lam (2004) explained, these models are all based on the assumption that the axial stress and axial strain of FRP-
confined concrete at a given lateral strain are the same as those of the same concrete confined with a constant confining pressure provided by FRP at that given lateral strain.

![Diagram](image)

**Figure 2.10 Passive analysis-oriented models based on active confinement model**

*(Jiang and Teng 2007)*

Therefore the development of passive FRP-confined concrete models generally follows the following steps (specific detail is presented in Chapter 4):

1. A concrete lateral strain, $\varepsilon_l$, is selected.

2. The corresponding confining pressure, $f_i$, is determined.

3. The axial strain, $\varepsilon_a$, is determined via a lateral-axial strain relationship.

4. The concrete stress, $f_c$, is obtained from an active confinement model for concrete, meaning a certain axial stress-strain equation and failure surface function (e.g. Mander’s model).
5. The concrete lateral strain is increased incrementally until it reaches the ultimate concrete lateral strain and steps 2 to steps 4 are repeated, arriving at a passive confinement stress-strain curve for concrete.

As introduced in Section 2.2.2.1, Mander’s model contains four parts: the axial stress-strain equation; peak axial stress and strain prediction of the active confinement model (also called the failure surface function); the lateral-to-axial strain relationship; and the ultimate condition. In passive confinement models, the axial stress-strain equation and failure surface function are used in step 4, while the lateral-to-axial strain relationship is used in step 3. All passive confinement models use same stress-strain equation proposed by Popovics (1973), given by Equation 2.5, and all the ultimate conditions are when the FRP ruptures in the hoop direction. However, different passive confinement models utilize different failure surface functions and lateral-to-axial strain relationships. This conclusion is indicated by Jiang and Teng (2007), and introduced in the following sections.

2.3.3 Failure surface function of FRP-confined concrete

In passive FRP-confined concrete models, failure surface functions predict the peak axial stress $f'_{cc}$ and corresponding axial strain $\varepsilon'_{cc}$ on each stress-strain curve of actively confined concrete. The failure surface function is employed in step 4 and Table 2.4 contains different failure surface functions of all passive FRP-confined concrete models mentioned in Section 2.3.2.
### Table 2.4 Summary of failure surface functions

<table>
<thead>
<tr>
<th>Model</th>
<th>Peak stress</th>
<th>Peak strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirmiran and Shahway (1997)</td>
<td>$f'<em>{cc} = f</em>{co} \left( 2.254 \sqrt{1 + 7.94 \frac{f_i}{f_{co}} - 2 \frac{f_i}{f_{co}} - 1.254} \right)$</td>
<td>$\varepsilon'<em>{cc} = \varepsilon</em>{co} \left[ 1 + 5 \left( \frac{f'<em>{cc}}{f</em>{co}} - 1 \right) \right]$</td>
</tr>
<tr>
<td>Spoelstra and Monti (1999)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fam and Rizkalla (2001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chun and Park (2002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harries and Kharel (2002)</td>
<td>$f'<em>{cc} = f</em>{co} + 4.269 f_i^{0.587}$</td>
<td>$\varepsilon'<em>{cc} = \varepsilon</em>{co} \left[ 1 + 5 \left( \frac{f'<em>{cc}}{f</em>{co}} - 1 \right) \right]$</td>
</tr>
<tr>
<td></td>
<td>Equation 2.6</td>
<td>Equation 2.7</td>
</tr>
<tr>
<td>Marques et al. (2004)</td>
<td>$f'<em>{cc} = f</em>{co} + 6.7 f_i^{0.33}$</td>
<td></td>
</tr>
<tr>
<td>Binici (2005)</td>
<td>$f'<em>{cc} = f</em>{co} \left( 1 + 9.9 \frac{f_i}{f_{co}} \right)$</td>
<td></td>
</tr>
<tr>
<td>Teng et al. (2007a)</td>
<td>$f'<em>{cc} = f</em>{co} + 3.5 f_i$</td>
<td>$\varepsilon'<em>{cc} = \varepsilon</em>{co} \left( 1 + 17.5 \frac{f_i}{f_{co}} \right)$</td>
</tr>
<tr>
<td></td>
<td>Equation 2.12</td>
<td>Equation 2.14</td>
</tr>
</tbody>
</table>

The comparison between different predictions of peak axial stress and Teng’s test results is shown in Figure 2.11. Teng’s equation exhibits better accuracy than other equations when comparing the prediction to the test results.
As introduced in Equation 2.8, Richard et al. (1929) suggested $k_1 = 4.1$ for actively confined and steel confined concrete. However due to some uncertainty and scatter, $k_1$ has a wide range of values (see Candappa et al. 2001). A value of 3.5 proposed by Teng et al (2007) in Equation 2.13 is within this range.

It should be noted that the test results in Figure 2.11 are not recorded test data, but values deduced using Equation 2.5 and Equation 2.7 with data of axial stress and axial strain from FRP-confined concrete cylinder tests from Teng et al. (2006). This deduction method was first proposed by Teng et al. (2007a), and $k_1 = 3.5$ is found to match well with the deduced results. They also realized that there should be a larger test database
of actively-confined concrete to give Equation 2.13 a more robust basis. Equation 2.14 is a simple expression when $f'_{cc}$ in Equation 2.7 is substituted by Equation 2.13.

2.3.4 Lateral-to-axial strain relationship of FRP-confined concrete

The lateral-to-axial strain relationship connects the response of concrete axial strain with concrete lateral strain. In these passive FRP-confined concrete models, compatibility between the very outside layer of concrete and the FRP wrap in the hoop direction is assumed, which means that the lateral concrete strain and the FRP hoop strain are equal. Lateral-to-axial strain relationships of the passive confinement models mentioned in Section 2.3.2 are briefly summarized in Table 2.5. These relationships are either explicitly stated or implicitly given according to Teng et al (2007b), depending on whether $\varepsilon_c$ can be written as an explicit function of $\varepsilon_l$.

A critical assessment of these equations was conducted by Jiang and Teng (2007). As shown in , Teng’s equation matches better with test results compared to other relationships for three levels of confinement.
Table 2.5 Summary of lateral-to-axial strain relationship

<table>
<thead>
<tr>
<th>Model</th>
<th>lateral-to-axial strain relationship</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirmiran and Shahway (1997)</td>
<td>( \frac{d\varepsilon_c}{d\varepsilon_c} = v_0 \left[ 1 + 2.7526 \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right) - 16.08 \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^2 + 34.344 \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^3 \right] )</td>
<td>Equation 2.15 Explicit</td>
</tr>
<tr>
<td>Spoelstra and Monti (1999)</td>
<td>( \varepsilon_t = \frac{E_c \cdot \varepsilon_c - f_c}{2 \beta \cdot f_c} ) where ( \beta = \frac{5700}{\sqrt{f_c}} - 500 )</td>
<td>Equation 2.16 Implicit</td>
</tr>
<tr>
<td>Fam and Rizkalla (2001)</td>
<td>( \frac{\varepsilon_t}{\varepsilon_c} = v_0 \left[ 1.914 \left( \frac{f_t}{f_{cc}} \right) + 0.719 \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right) + 1 \right] ) where ( \varepsilon_{cc}' ) is determined via Equation 2.6 and Equation 2.7.</td>
<td>Equation 2.17 Implicit</td>
</tr>
<tr>
<td>Chun and Park (2002)</td>
<td>( \frac{\varepsilon_t}{\varepsilon_c} = v_0 \left[ 1 + 1.3763 \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right) - 5.36 \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^2 + 8.586 \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^3 \right] )</td>
<td>Equation 2.18 Implicit</td>
</tr>
<tr>
<td>Marques et al. (2004)</td>
<td>( \varepsilon_t = \begin{cases} \frac{v \varepsilon_c}{\varepsilon_{cc}} &amp; \text{for } \varepsilon_c \leq \varepsilon_{cc}^{lim} \ \frac{1 - 2 \nu}{2} \alpha \varepsilon_{co} \left( \frac{\varepsilon_c - \varepsilon_{cc}^{lim}}{\varepsilon_{cc} - \varepsilon_{cc}^{lim}} \right)^2 &amp; \text{for } \varepsilon_c &gt; \varepsilon_{cc}^{lim} \end{cases} )</td>
<td>Equation 2.19 Implicit</td>
</tr>
<tr>
<td>Binici (2005)</td>
<td>( \varepsilon_t = \frac{v_t = v_p \frac{1}{(f_t f_{co} + 0.85)^4}}{1 - \frac{v_t}{v_p}} ) where ( \Delta = \varepsilon_c - \varepsilon_{cc} ) ( \beta = \frac{v_t - v_p}{v_t - v_o} ) ( \varepsilon_{cc} ) is axial strain quantifying elastic limit.</td>
<td>Equation 2.20 Implicit</td>
</tr>
<tr>
<td>Teng et al. (2007a)</td>
<td>( \varepsilon_{cc} = 0.85 \left( 1 + 8 \left( \frac{f_t}{f_{cc}} \right) \right) \left[ 1 + 0.75 \left( \frac{\varepsilon_t}{\varepsilon_{cc}} \right)^{0.7} - \exp \left[ -7 \left( \frac{\varepsilon_t}{\varepsilon_{cc}} \right) \right] \right] )</td>
<td>Equation 2.21 Explicit</td>
</tr>
</tbody>
</table>
Figure 2.12 Comparison of different lateral-to-axial strain relationships (Jiang and Teng 2007)
(a) Weakly-confined concrete
(b) Moderately-confined concrete

(c) Heavily-confined concrete
Although some equations, such as Equation 2.15 and Equation 2.20, also match the results reasonably well for three different confinement levels, Teng’s equation is considered to be the most appropriate one to use based on following reasons.

First of all, Teng’s equation is most accurate one in this assessment. It is an explicit expression which is much simple to use and apply than others. And perhaps the most important consideration is the structure of the equation, which is presented in Section 4.2.1, allowing easy modification for other confinement materials, or combinations of confinement materials.

2.4 Steel-and-FRP-confined concrete

2.4.1 General behavior of steel-and-FRP-confined concrete

In Steel-and-FRP-confined concrete, the core is confined simultaneously by the internal steel spiral and the external FRP wrap, which represents the case when repairing damaged or strengthening undamaged reinforced concrete columns.

Figure 2.14 compares Teng et al (2007) FRP-confined concrete model against three specimens from Lee et al (2010), where Table 2.6 summarized the spiral and FRP detailing, more specific specimen material and geometry information can be found in Table A.3 in the Appendix.
### Table 2.6 Parameters of Lee’s specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Spiral pitch (mm)</th>
<th>FRP layers (number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0F2</td>
<td>No spiral</td>
<td>2</td>
</tr>
<tr>
<td>S4F2</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>S2F2</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>S2F0</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>S2F1</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>S2F2</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>S2F3</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>S2F4</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>S2F5</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>S0F5</td>
<td>No spiral</td>
<td>5</td>
</tr>
<tr>
<td>S6F5</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>S4F5</td>
<td>40</td>
<td>5</td>
</tr>
<tr>
<td>S2F5</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

* Each layer is nominally 0.11 mm thick.

To illustrate the variation in confining material properties, Figure 2.13 shows typical stress-strain curves for common materials. Therefore, depending on the confining material, the confining will be different.
As shown in Figure 2.14, Teng’s model matches well with the result of specimen S0F2 confined with FRP only. However, FRP-confined model always underestimates the strength and ductility of steel-and-FRP-confined concrete particularly when the transverse steel volume ratio is high.
Figure 2.14  Comparison of Teng et al 2007 FRP model and concrete cylinder tests from Lee et al 2010

Figure 2.15 contains four columns of steel-and-FRP-confined concrete from Chastre and Silva (2010) where failure is governed by rupture of the FRP confinement in the hoop direction.

Figure 2.15  Typical failure of steel-and-FRP-confined concrete columns (Chastre and Silva 2010)
The stress-strain response is also influenced by both confinement materials. The shape of this stress-strain curve depends on the dominant confinement material. That is to say, the percent ratio of FRP and steel lateral confining pressure influence the enhancement of concrete compressive strength and strain. The maximum lateral confining pressure is used here to calculate the ratio, given by

\[ f_{l,\text{steel}} = \frac{2f_{y}A_{sp}}{s-d c} \]  
Equation 2.22

\[ f_{l,\text{FRP}} = \frac{2E_{f}f_{u}^{*}\varepsilon_{fu,a}}{d} \]  
Equation 2.23

where \( \varepsilon_{fu,a} \) = rupture strain of the FRP wrap in the hoop direction.

It should be pointed out that Lee et al (2010) calculate the maximum lateral confining pressure using the tensile strength of the FRP sheet, which cannot be reached in practice. Current data shows that the rupture strain of the FRP wrap in the hoop direction is lower than the rupture strain obtained in straight coupon tests (for example, Saaman et al (1998), Matthys et al (2000), and Lam and Teng (2004)). More details are provided in Section 4.3.1 regarding this ultimate condition.

All specimens in Figure 2.16 (a) contains the same amount of confinement steel, with a spiral pitch equal to 20 mm, but varying number of FRP plies from zero (representing only steel confinement) to five. With less FRP, the stress-strain curve of steel-and-FRP-confined concrete (S2F1) behaves more like that of steel-confined concrete (S2F0). Although this steel-confined concrete curve is not typical since high strength transverse steel was used. When the amount of FRP increases, the stress-strain curves exhibit different bilinear trends (but all ascending branches) with increasing slopes, similar to
the curve of FRP moderately-confined concrete shown in Figure 2.9. The strength and ductility also increase with increasing FRP confinement.

In Figure 2.16 (b), all curves exhibit bilinear trend since the FRP provides the dominant confinement. Increasing the amount of steel does not change the general behavior, however the strength and ductility is somewhat enhanced.
Figure 2.16 Experimental stress-strain curves (Lee et al. 2010)

(a) 20 mm spiral pitch, with varying number of FRP layers

(b) 5 FRP layers, with varying spiral pitch
From previous discussions, it is clear that steel and FRP both contribute to the enhancement of strength and ductility. While due to the difference of material properties, the contribution cannot be quantified in the same manner. Even though the total confining pressure may be similar, different relative material portions may result in different stress-strain curves, influencing both the strength and lateral strain response, as shown in Figure 2.17. The material confining pressure ratio of each specimen is summarized in Table 2.7.
(a) Stress-strain curves of specimen “S0F4” and “S6F2”

(b) Stress-strain curves of specimen “C2N1P2N” and “C4NP4C”

Figure 2.17 Experimental stress-strain curve
Table 2.7 Lateral pressure percent ratio of specimens with similar total confining pressure

<table>
<thead>
<tr>
<th>Source</th>
<th>Specimen name</th>
<th>Total confining pressure (MPa)</th>
<th>FRP percent ratio</th>
<th>Steel percent ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lee et al (2010)</td>
<td>S0F4</td>
<td>8.80</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>S6F2</td>
<td>9.46</td>
<td>51%</td>
<td>49%</td>
</tr>
<tr>
<td>Eid and Paultre (2008)</td>
<td>C2N1P2N</td>
<td>9.25</td>
<td>48%</td>
<td>52%</td>
</tr>
<tr>
<td></td>
<td>C4NP4C</td>
<td>9.51</td>
<td>72%</td>
<td>28%</td>
</tr>
</tbody>
</table>

To look deeper into the behavior, let’s introduce a ratio, $\frac{\varepsilon_c}{\varepsilon_l}$, to consider the effect of confinement, or stiffness in the lateral direction. The physical interpretation of this ratio is such that poor confinement is unable to efficiently restrain the dilation of the core concrete, leading to large values of lateral strain $\varepsilon_l$ and therefore small values of $\frac{\varepsilon_c}{\varepsilon_l}$.

Usually this ratio is calculated when the FRP wrap ruptures, as $\frac{\varepsilon_{cu}}{\varepsilon_{lu}}$. The magnitude of this ratio for specimens from three sources is summarized in Table 2.8.
For specimen 28, 24, and 34, \( \frac{\varepsilon_{cu}}{\varepsilon_{lu}} \) clearly illustrates the effect of confinement. From Table 2.7 and Table 2.8, it could be concluded that with more confining pressure from FRP, columns tend to be stiffer and better confined since \( \frac{\varepsilon_{cu}}{\varepsilon_{lu}} \) is larger. This phenomenon is likely due to the material property differences shown in Figure 2.13. After steel yield, the secant modulus of the steel confinement is actually decreasing, in other words, the stiffness is decreasing, while the FRP provides increasing confining pressure until its rupture. Based on this, one might conclude that: with more FRP confinement, the column tends to be stiffer even if the total confining pressure is the same. However, this is just a physical interpretation, which needs more experiments and observations to be validated. Besides, since the specimen parameters and test set-up from different researchers are quite different, it is not appropriate to directly compare the response.
ratio from one experiment to another. For example, the strain measurements from Eid and Paultre (2008) and Lee et al (2010) are different.

As shown in Figure 2.17, the relative proportions of different materials also influence the strength of the confined concrete. With more FRP, the strength tends to be larger. However, as will be seen in the following sections, none of the existing models for steel-and-FRP-confined concrete consider the influence of different material proportion as discussed above.

2.4.2 Stress-strain models for steel-and-FRP-confined concrete

2.4.2.1 Introduction

With most, if not all, FRP repair and strengthening, a steel-and-FRP-confined concrete model is required rather than simply a FRP-confined concrete model, ignoring the contribution of steel confinement.

However, only four models have been proposed based on concrete cylinder and/or column compression tests, as summarized in Table 2.9.
Table 2.9 Model general information

<table>
<thead>
<tr>
<th>Model</th>
<th>Category</th>
<th>Database</th>
<th>Test description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Specimen type</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Confinement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Toutanji (1999)</td>
<td>Column</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Teng and Lam (2002)</td>
<td>Cylinder</td>
</tr>
<tr>
<td></td>
<td>oriented</td>
<td>Xiao and Wu (2000)</td>
<td>Cylinder</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lam and Teng (2004)</td>
<td>Cylinder</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Eid et al (2006)</td>
<td>Large scale column</td>
</tr>
</tbody>
</table>

Harajli (2006) model is a unified model for steel-and-FRP confined concrete and FRP-confined concrete with either circular or rectangular sections. Eid and Paultre (2008) model is a unified model for steel-and-FRP confined concrete and FRP-confined concrete with circular sections. Lee et al (2010) model is developed for steel-and-FRP confined concrete only with circular section. This model can only predict axial stress-axial strain response of steel-and-FRP confined concrete. It is unable to predict the lateral strain response and cannot be reduced to an FRP-confined model or steel-confined model since it is purely empirical. Chastre and Silva (2010) model is also developed for steel-and-FRP confined concrete only with circular section.
All of the models, except Harajli (2006), are classified as design-oriented models for their closed form expressions of axial strain-axial stress relationship and with no incremental procedures. However, Eid and Paultre’s model is more complicated than others design-oriented models. This model inherited the work done by Cusson and Paultre (1995) and Legeron and Paultre (2003) in passive steel-confined concrete, as mentioned in Table 2.1, which avoid an incremental iterative analysis by defining a particular confined concrete stress-strain relationship that can account for the progressive nature of the passive confinement. Eid and Paultre’s model is the only one that can predict the axial-lateral strain relationship, and is a passive confinement model. The details of these four models are briefly introduced in following sections.

2.4.2.2 Harajli (2006)

Harajli divides the stress-strain response into two stages, as shown in Figure 2.18. Due to the two stage definition and the expressions used, the stress-strain curve is discontinuous and has a kink at the intersection point, which conflicts with actual test results.
Figure 2.18  Axial stress-axial strain relationship of Harajli (2006)

The first stage is from zero up to intersection point \((\varepsilon_i, f_i)\) and can be described by the following parabolic expression proposed by Scott et al (1982) for the ascending branch of unconfined or steel-confined concrete.

\[
f_c = f_i \left[ \frac{2\varepsilon_c}{\varepsilon_i} - \left( \frac{\varepsilon_c}{\varepsilon_i} \right)^2 \right] \quad \text{for } \varepsilon_c \leq \varepsilon_i \quad \text{Equation 2.24}
\]

The second stage does not have explicit expression, which is the reason why it is classified as analysis-oriented model. Inspired by Richart et al’s failure surface function shown in Equation 2.8, the axial stress of concrete confined with external FRP wraps and internal transverse steel is expressed as

\[
f_c = f_{co} + k_1 f_{lf} + k_1 f_{is} \frac{A_{cc}}{A_g} \quad \text{Equation 2.25}
\]

where \(k_1 = 4.1; A_{cc}\) = the area of the concrete core confined with internal steel ties, taken from the centerline of the steel ties; \(A_g\) = the gross area of the column section; \(f_{lf} = \)
lateral passive confining pressure provide by the FRP, $f_{ls}$ = lateral passive confining pressure provided by transverse steel, given by

$$f_{lf} = \left( \frac{k_f \rho_f \varepsilon_f}{2} \right) \varepsilon_i$$  \hspace{1cm} \text{Equation 2.26}$$

$$f_{ls} = \left( \frac{k_s k_c \rho_{st} \varepsilon_s}{2} \right) \varepsilon_i \leq \left( \frac{k_s k_c \rho_{st}}{2} \right) f_y$$  \hspace{1cm} \text{Equation 2.27}$$

where $\rho_f$ = the volumetric ratio of the FRP sheets; $k_c$ is calculated by Equation 2.2 and Equation 2.3; $\rho_{st}$ = the volumetric ratio of the transverse steel ties or hoops; $k_f$ and $k_s$ are effective confinement factors that account for different section shape, both equal to one for circular sections.

Combining Equation 2.25, Equation 2.26 and Equation 2.27, the axial stress in second stage is linear monotonic function of lateral concrete strain. The axial stress-axial strain is indirectly given in following expression used by Toutanji (1999), which also gives the relationship of axial strain and lateral strain.

$$\varepsilon_c = \varepsilon_{co} \left[ 1 + (310.57 \varepsilon_l + 1.9) \left( \frac{f_c}{f_{co}} - 1 \right) \right]$$  \hspace{1cm} \text{Equation 2.28}$$

Failure is governed by FRP rupture and the ultimate condition is when the lateral concrete strain reaches the FRP maximum strain in the hoop direction.

Harajli believes that the intersection of both stages occurs when the transverse steel yields, that is $\varepsilon_l = \varepsilon_y$. By combining Equation 2.25 and Equation 2.28 and substituting axial strain, axial stress, and lateral strain with $f_l$, $\varepsilon_l$, $\varepsilon_y$ respectively, the intersection point is obtained as follows:
\[ f_t = f_{co} + k_1 \varepsilon_y \left( \frac{p_{fs} f_t}{2} + \frac{k_{es} \varepsilon_{es}}{2} \left( \frac{A_{cc}}{A_g} \right) \right) \]  \hspace{1cm} \text{Equation 2.29}

\[ \varepsilon_t = \varepsilon_{co} \left[ 1 + \left( 310.57 \varepsilon_y + 1.9 \right) \left( \frac{f_t}{f_{co}} - 1 \right) \right] \]  \hspace{1cm} \text{Equation 2.30}

Note that Equation 2.25 is not a failure surface function since the left side is concrete axial stress rather than the maximum confined compressive strength, or ultimate stress. This equation represents an assumption inspired by the form of failure surface functions, which has not been validated yet. However, it represents a good first attempt to consider confinement from both external FRP wraps and internal transverse steel simultaneously.

Previous discussion on Figure 2.16 and Figure 2.17 indicated that the stress-strain curve is not only influenced by the total amount of confining pressure, but also by the relative confining pressure from each of the two confinement materials. With Equation 2.25 and Equation 2.28, Harajli’s model theoretically can predict the stress-strain response with a certain amount of total confining pressure, \( f_{total} \), which equals \( f_{f} + f_{ls} \frac{A_{cc}}{A_g} \). However, this model cannot discriminate between the confining pressures, and is unable to account for the observation of Figure 2.17, mainly because the same \( k_1 \) factor is used in Equation 2.25.

### 2.4.2.3 Eid and Paultre (2008)

Eid’s model is the most complicated among the four models with nearly forty equations used. But it is a design-oriented model since an explicit expression of the stress-strain curve is presented and no incremental iterative procedures are needed. In this section,
many details are not presented here for the sake of brevity and simplicity. Only the major equations are given to help understand and explain their method.

The stress-strain curve shown in Figure 2.19, is divided into two ascending branches. In Eid’s model, it should be noted that the peak point \( (\varepsilon_{cc}', f_{cc}') \) in these curves represents the transition point between the first and second ascending branches.

**Figure 2.19  Axial and lateral stress-strain curves (Eid and Paultre 2008)**

Equation 2.31 represents the first branch, first proposed by Sargin (1971) and later modified by Eid

\[
f_c = \frac{a \varepsilon_c}{1 + b \varepsilon_c + z \varepsilon_c^2} \quad \text{for} \quad \varepsilon_c \leq \varepsilon_{cc}'
\]

Equation 2.31

where a, b, and z are constants that control the initial slope and the curvature of the first ascending branch.
The second branch is given by

\[ f_c = f_{cc}' \exp[k_1(e_c - e_{cc}')k_2] + E_{cu}(e_c - e_{cc}') \quad \text{for} \quad e_{cu} \geq e_c > e_{cc}' \]  

Equation 2.32

where \( k_1 \) and \( k_2 \) are parameters controlling the shape of the second branch; and \( E_{cu} = \) slope of the second branch.

Eid modified the failure surface function proposed by Lam and Teng (2003) for the ultimate strength, \( f_{cu} \), and strain, \( \varepsilon_{cu} \)

\[ \frac{f_{cu}}{f_{co}} = 1 + 3.3 \left( \frac{f_{isu} + f_{ifu}}{f_{co}} \right) \geq \frac{f_{cc}'}{f_{co}} \]  

Equation 2.33

\[ \frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1.56 + 12 \left( \frac{f_{isu} + f_{ifu}}{f_{co}} \right) \geq \frac{\varepsilon_{cc}'}{f_{co}} \]  

Equation 2.34

where \( f_{isu} \) and \( f_{ifu} \) are the same as Equation 2.26 and Equation 2.27.

Eid’s model establishes the lateral-to-axial relationship from Hooke’s law, as follows

\[ \varepsilon_l = \frac{v_c \varepsilon_c}{1 + (m_{si} + m_{fi})(1 - v_c + 2v_c^2)} \]  

Equation 2.35

where \( m_{si} \) and \( m_{fi} \) is the modulus ratio of transverse steel and concrete and the modulus ratio of FRP and concrete, respectively; \( v_c \) = the concrete secant Poisson’s ratio and is a complicated function of axial strain.

Eid’s model also has two stages, but unlike Harajli’s curve, Eid’s stress-strain curve is continuous because the expression for the second branch is not a linear function. As a design-oriented model, Eid’s model is not readily applicable for design because of the many complicated equations used and various empirical calibrated factors. This is
because of the particular expression used to avoid the incremental iterative analysis (see Section 2.4.2.1).

Among the four models, only Eid proposed an explicit lateral-to-axial strain relationship. Similar to other researchers, except Lee et al, Eid simply sums up the confining pressure from the two confinement materials as the total confining pressure and then applies it to predict the ultimate strength and strain.

2.4.2.4 Lee et al (2010)

Lee et al (2010) proposed an empirical equation for concrete confined with both steel spirals and FRP wraps with circular section, based on his own cylinder compression tests including steel-confined, FRP-confined, and steel-and-FRP confined cylinders. The proposed equations are given as follows, dividing the stress-strain curve into three segments, as shown in Figure 2.20.

\[ f_c = E_c \cdot \varepsilon_c + (f_{co} - E_c \cdot \varepsilon_{co}) \left( \frac{\varepsilon_c}{\varepsilon_{co}} \right)^2 \quad \text{for} \quad 0 < \varepsilon_c \leq \varepsilon_{co} \quad \text{Equation 2.36} \]

\[ f_c = f_{co} + (f_{cs} - f_{co}) \left( \frac{\varepsilon_{co}}{\varepsilon_{cs}} \right)^{0.7} \quad \text{for} \quad \varepsilon_{co} < \varepsilon_c \leq \varepsilon_{cs} \quad \text{Equation 2.37} \]

\[ f_c = f_{cs} + (f_{cu} - f_{cs}) \left( \frac{\varepsilon_{cs}}{\varepsilon_{cu}} \right)^{0.7} \quad \text{for} \quad \varepsilon_{cs} < \varepsilon_c \leq \varepsilon_{cu} \quad \text{Equation 2.38} \]

where \( \varepsilon_{cs} = \varepsilon_{cu} \left[ 0.85 + 0.03 \cdot \left( \frac{f_{fu}}{f_{fisu}} \right) \right] \) and \( f_{cs} = 0.95 \cdot f_{cu} \quad \text{for} \quad f_{fisu} \geq f_{fisu}; \)

\( \varepsilon_{cs} = 0.7 \cdot \varepsilon_{cu} \quad \text{and} \quad f_{cs} = (\varepsilon_{cs}/\varepsilon_{cu})^{0.40} \cdot f_{cu} \quad \text{for} \quad f_{fisu} \leq f_{fisu}; \quad E_c = 4700 \sqrt{f_{co}} \quad \text{(MPa)}. \)
To estimate the maximum confined compressive strength, $f_{cu}$, Equation 2.39 proposed by Lam and Teng (2003) is adopted.

$$f_{cu} = f_{co} \left(1 + 2 \frac{f_{isu} + f_{ifu}}{f_{co}} \right)$$  \hspace{1cm} \text{Equation 2.39}

For ultimate concrete strain, $\varepsilon_{cu}$, Lee et al calibrate the following equation by introducing two parameters, $k_s$ and $k_f$ (different from Harajli’s model) based on their test results and Lam and Teng’s original model (2003).

$$\varepsilon_{cu} = \varepsilon_{co} \left(1.75 + 5.25 \left(\frac{k_f f_{ifu} + k_s f_{isu}}{f_{co}}\right) \left(\frac{f_{isu}}{\varepsilon_{isu}}\right)^{0.45}\right)$$  \hspace{1cm} \text{Equation 2.40}

where $k_s = \left(2 - f_{ifu}/f_{isu}\right)$ and $k_f = 1$ for $f_{ifu} \leq f_{isu}$; $k_s = 1$ and $k_f = 1$ for $f_{ifu} > f_{isu}$;

$\varepsilon_{f,rup}$ = rupture strain of FRP wrap; $f_{isu}$ = confining pressure from transverse steel when
it yields; \( f_{fu} \) = confining pressure from FRP wrapping when it ruptures in the hoop direction.

Lee et al’s model is based on a regression analysis of their own test results, thus the proposed equation may not be accurate for other tests. Besides, as a design-oriented model, Lee et al’s model is unable to predict the lateral strain-axial stress response. However Lee et al make a good effort to explain the observations in Figure 2.16 and Figure 2.17, by introducing parameters, \( k_s \) and \( k_f \) to represent confinement effect of different materials. Further, the expression \( k_s = (2 - f_{fu}/f_{isu}) \) is key to interpret the phenomenon in Figure 2.17 that a higher FRP confining pressure relative to the total pressure the better confinement effect. Recall the discussion on the ratio \( \frac{\varepsilon_c}{\varepsilon_i} \) in Section 2.4.1.

It must also be noted that Lee et al’s assumption that \( k_s = 1 \) and \( k_f = 1 \) is not appropriate when \( f_{fu} > f_{isu} \) since it conflicts with the discussion above. Even the relationship \( k_s = (2 - f_{fu}/f_{isu}) \) and \( k_f = 1 \) when \( f_{fu} \leq f_{isu} \) might not be appropriate since \( f_{fu} \) and \( f_{isu} \) do not directly relate to material properties.

Therefore a better method is required to account for the influence of the axial strain response for different confinement materials, since Equation 2.39 predicts the same ultimate strength for the same total confining pressure, which also conflicts with Figure 2.17. A better method is also required to quantify the contributions to ultimate strength from the two different confining materials.
2.4.2.5 Chastre and Silva (2010)

Chastre and Silva’s model is a unified model for concrete of circular section confined with steel, FRP, or both. It is a design-oriented model calibrated with their own monotonic axial compression column test results. The model is generally sketched in Figure 2.21.

![Figure 2.21 Axial stress-axial strain relationship of Chastre and Silva (2010)](image)

The axial stress-axial strain relationship is based on an expression of four parameters \((E_1, E_2, f_0, n)\) first proposed by Richard and Abbott (1975):

\[
f_c = \frac{(E_1 - E_2)\varepsilon_c}{1 + \left(\frac{(E_1 - E_2)\varepsilon_c}{f_0}\right)^n} + E_2\varepsilon_c \leq f_{cc}
\]

Equation 2.41

where \(E_1\) and \(E_2\) are the slopes of first and second branches respectively; \(n = 2\) through calibration of curves with the experimental tests; \(f_0\) is the y-intercept, \(f_0 = f_D + 1.28 f_{tu}\); \(f_D\) is compressive strength of the unconfined large-scale concrete column; \(f_{tu}\) is the
maximum total confining pressure from steel hoops, $f_{isu}$, and FRP wrap, $f_{ifw}$, given as

$$f_{tu} = f_{isu} + f_{ifw}.$$ 

The ultimate condition is given as follows:

$$f_{cu} = f_D + k_1 f_{tu} \quad \text{Equation 2.42}$$

$$\varepsilon_{cu} = k_2 \varepsilon_{co} \left(\frac{f_{tu}}{f_D}\right)^{0.7} \quad \text{Equation 2.43}$$

where $k_1 = 5.29$; $k_2 = 17.65$; $\varepsilon_{co}$ = unconfined concrete strain at peak stress.

Chastre and Silva’s model can predict the axial stress-lateral strain curve, which has a similar form as Equation 2.44

$$f_c = \frac{(E_{1l}-E_{2l})\varepsilon_l}{1 + \left(\frac{(E_{1l}-E_{2l})n_l}{f_{ot}}\right)^{n_l}} + E_{2l}\varepsilon_l \leq f_{cc} \quad \text{Equation 2.44}$$

The curve is terminated at FRP wrap rupture, $\varepsilon_l = \varepsilon_{f,u,a}$, same as terminated at $\varepsilon_{cu}$.

Although Chastre and Silva’s model can predict both stress-axial strain and stress-lateral strain responses, there is no lateral-to-axial strain relationship, which means axial strain and lateral strain are isolated and cannot be known simultaneously. Similar with design-oriented FRP-confined concrete models, Chastre and Silva’s model can only predict an ascending stress-strain response because of the expression used. Equation 2.42 and Equation 2.43 show that the compressive strength and strain is only determined by the total amount of confining pressure. Unlike Lee et al’s model, the confining pressure ratio is not considered and will not influence the confinement effect.
2.4.2.6 Summary of existing models

Equations of these four existing models for steel-and-FRP-confined concrete are summarized in Table 2.10.

**Table 2.10 Summary of existing models equation**

<table>
<thead>
<tr>
<th>Model</th>
<th>$f_c - \varepsilon_c$</th>
<th>$f_{cu}, \varepsilon_{cu}$</th>
<th>$\varepsilon_c - \varepsilon_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harajli (2006)</td>
<td>Equation 2.24</td>
<td>Naturally *</td>
<td>Equation 2.28</td>
</tr>
<tr>
<td></td>
<td>Equation 2.25</td>
<td>Equation 2.24, Equation 2.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Equation 2.40</td>
<td></td>
</tr>
<tr>
<td>Chastre and Silva (2010)</td>
<td>Equation 2.41</td>
<td>Equation 2.42, Equation 2.43</td>
<td>unavailable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Equation 2.43</td>
<td></td>
</tr>
<tr>
<td>Eid et al (2008)</td>
<td>Equation 2.31</td>
<td>Equation 2.33, Equation 2.34</td>
<td>Equation 2.35</td>
</tr>
<tr>
<td></td>
<td>Equation 2.32</td>
<td>Equation 2.34</td>
<td></td>
</tr>
</tbody>
</table>

* Substitute FRP rupture strain into Equation 2.24 and Equation 2.25, leading to value of ultimate strength and strain.

These investigators all make good efforts to establish stress-strain curves for steel-and-FRP-confined concrete, using their own methods. However, they also have some limitations, as summarized in the following.

Harajli assumes that axial stress is a bilinear function of confining pressure from FRP and confining pressure from steel, as in Equation 2.25. It is an easy way to quantify the contribution from both confinement materials, but this assumption has not been validated by others and is not robust enough. By dividing the stress-strain curve into two stages with one parabolic function and one linear function, Harajli’s curve has a discontinuity, which does not fit reality. However, the biggest limitation is that, Harajli simply sums the two confining pressures. The model does not realize the confinement
effect could be influenced by the confining pressure ratio. That is to say, the axial strain response is different with various confining pressure ratio even if the total confining pressure is same.

Lee et al proposed empirical equations based on regression analysis of their own test results. Their research is less theoretical and may not be able to predict the stress-strain curve from other test results properly. But one feature that is better than the other three models is that Lee et al correlates confinement pressure ratio to $k_s$ and $k_f$ in Equation 2.40. Although these two expressions are not perfect, it is still an attempt to try to account for the phenomenon observed in Figure 2.17.

Chastre and Silva’s model is also empirical. Since the expression of four parameters is used, this model is not much different from some FRP-confined models based on similar expression as Equation 2.41 (Samaan et al (1998); Cheng et al (2002); Xiao and Wu (2003)) except that they use total confining pressure instead of FRP confining pressure. As a design-oriented model, it can only predict ascending type stress-strain curves that is mentioned in Section 2.3.1. Chastre and Silva also proposed a bi-linear curve for axial stress-lateral strain also based on the versatile expression. However, the lateral-to-axial strain relationship is still unknown and there is no way to obtain axial strain and its corresponding lateral strain or dilation except at the ultimate point.

Eid’s model is too complicated for design as mentioned before, although it is classified as design-oriented model. This steel-and-FRP confined concrete model inherits the methods from Cusson and Paultre (1995) and Legeron and Paultre (2003) for steel-confined concrete, which avoid the incremental iterative procedures by defining a
special curve shape. This is reason why nearly forty equations are required to get the stress-strain curve.

2.4.3 Conclusion

Compared to the passive FRP-confined models in Section 2.3.2, these three steel-and-FRP-confined concrete models (excluding Eid’s model) are much less theoretical. This is due to their design-oriented nature. What needs to be developed is an analysis-oriented model that can account for the passive nature of confinement. Eid et al avoids the incremental iterative procedures, but the trade-off is a complex set of approximate 40 equations and lots of restrictions being added.

On the analogy of passive FRP-confined concrete models which use incremental procedures, a passive confinement model for steel-and-FRP-confined concrete may be developed. Since steel-confined concrete models and FRP-confined concrete models have a longer research history with more robust models, it would be appropriate to consider modifying existing steel-confined or FRP-confined models to fit the steel-and-FRP-confined concrete stress-strain behavior. Figure 2.22 tends to indicate that steel-and-FRP-confined concrete behaves similarly to FRP-confined concrete. Thus, the primary research objective is to modify an appropriate passive FRP-confined concrete model such that it is appropriate for steel-and-FRP-confined concrete.
Such a model is expected to account for both confinement contributions on strength and strain, also should solve the problem that a higher FRP confining pressure ratio relative to the total pressure leads to a stiffer behavior, as observed in Figure 2.17. A more detailed assessment and comparison of existing models is presented in the following Chapter.
3 ASSESSMENT OF EXISTING MODELS

3.1 Introduction

In this chapter, four existing steel-and-FRP-confined concrete models (including Harajli (2006), Eid and Paultre (2008), Chastre and Silva (2010), and Lee et al (2010)) are compared against the stress-strain response in pure compression from test results, summarized in Table 3.1. The experimental results used for comparison were digitized from figures presented in their papers where corresponding parameters of the test specimens are also provided, and summarized in Table A.3 in the Appendix.

Table 3.1 Summary of test databases for comparison

<table>
<thead>
<tr>
<th>Source</th>
<th>Specimen type</th>
<th>Confinement material</th>
<th>Number of Specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xiao and Wu (2000)</td>
<td>Cylinder</td>
<td>FRP</td>
<td>8</td>
</tr>
<tr>
<td>Lee et al. (2010)</td>
<td>Cylinder</td>
<td>FRP and steel</td>
<td>14</td>
</tr>
<tr>
<td>Eid and Paultre (2008)</td>
<td>Large-scale column</td>
<td>FRP and steel</td>
<td>5</td>
</tr>
<tr>
<td>Chastre and Silva (2010)</td>
<td>Large-scale column</td>
<td>FRP and steel</td>
<td>4</td>
</tr>
<tr>
<td>Demers and Neale (1999)</td>
<td>Large-scale column</td>
<td>FRP and steel</td>
<td>2</td>
</tr>
</tbody>
</table>
3.2 Assessment of existing models

3.2.1 General behavior of steel-confined concrete

Only selected comparisons between the four models and the experimental results are presented here, including different levels of the confining pressure ratio $f_{1s}/f_{1f}$, which is calculated and labeled on each subsequent figure, as summarized in Table 3.2. $f_{1s}$ and $f_{1f}$ are confining pressure from transverse steel and FRP wrap, respectively, given by

\[
f_{1s} = \begin{cases} 
\frac{2k_c E_c \varepsilon_l \varepsilon_l A_{sp}}{s d_c} & \text{if } \varepsilon_l < \varepsilon_y \\
\frac{2k_c f_{sy} A_{sp}}{s d_c} & \text{if } \varepsilon_l \geq \varepsilon_y 
\end{cases} \quad \text{Equation 3.1}
\]

\[
f_{1f} = \frac{2E_t f_{t} \varepsilon_l \varepsilon_l s}{s d} \quad \text{Equation 3.2}
\]
### Table 3.2 Confining pressure ratio of selected specimens

<table>
<thead>
<tr>
<th>Source</th>
<th>Specimen type</th>
<th>Specimen number</th>
<th>$f_{ts}/f_{tf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lee et al. (2010)</td>
<td>Cylinders</td>
<td>10</td>
<td>0.9643</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>1.8340</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>2.4765</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19</td>
<td>3.7391</td>
</tr>
<tr>
<td>Chastre and Silva (2010)</td>
<td>Large-scale columns</td>
<td>30</td>
<td>0.0911</td>
</tr>
<tr>
<td>Chastre and Silva (2010)</td>
<td>Large-scale columns</td>
<td>28</td>
<td>0.2506</td>
</tr>
<tr>
<td>Demers and Neale (1999)</td>
<td>Large-scale columns</td>
<td>32</td>
<td>0.3908</td>
</tr>
<tr>
<td>Eid and Paultre (2008)</td>
<td>Large-scale columns</td>
<td>27</td>
<td>0.9529</td>
</tr>
<tr>
<td>Eid and Paultre (2008)</td>
<td>Large-scale columns</td>
<td>23</td>
<td>1.2186</td>
</tr>
</tbody>
</table>

Figure 3.1 - Figure 3.9 show that the prediction of the four compared models are inconsistent, meaning that a model works much better in predicting the general behavior of specimens from their own experiments compared to other specimens. For example, Lee’s model does not work well in predicting specimen 32, 27, and 23, and Chastre’s model does not work well for all specimens except his own (specimen 30 and 28).
Figure 3.1 – Comparison of models with specimen 10 (Lee et al 2010)

Figure 3.2 – Comparison of models with specimen 14 (Lee et al 2010)
Figure 3.3 – Comparison of models with specimen 20 (Lee et al. 2010)

Figure 3.4 – Comparison of models with specimen 19 (Lee et al. 2010)
Figure 3.5 – Comparison of models with specimen 30 (Chastre and Silva 2010)

Figure 3.6 – Comparison of models with specimen 28 (Chastre and Silva 2010)
Figure 3.7 – Comparison of models with specimen 32 (Demers and Neale 1999)

Figure 3.8 – Comparison of models with specimen 27 (Eid and Paultre 2008)
Figure 3.9 – Comparison of models with specimen 23 (Eid and Paultre 2008)
3.2.2 Ultimate conditions \( f_{cu} \) and \( \varepsilon_{cu} \)

The performance of all existing models in predicting the ultimate condition is shown in Figure 3.10 to Figure 3.13. The predicted ultimate axial strain and axial stress are normalized by the unconfined concrete properties and divided by the normalized test values, as given in Table A.3 in Appendix. For the purpose of comparison, the mean (\( \mu \)) and the standard deviation (\( \sigma \)) of this ratio are calculated for each model and summarized in Table 3.3. Take \( \frac{\varepsilon_{cu_{\text{predict}}}}{\varepsilon_{cu}} / \frac{\varepsilon_{cu_{\text{test}}}}{\varepsilon_{cu}} \) as an example, \( \mu \) and \( \sigma \) are calculated by following

\[
\mu = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\varepsilon_{cu_{\text{predict}}}}{\varepsilon_{cu}} / \frac{\varepsilon_{cu_{\text{test}}}}{\varepsilon_{cu}} \right) \quad \text{Equation 3.3}
\]

\[
\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \left( \frac{\varepsilon_{cu_{\text{predict}}}}{\varepsilon_{cu}} / \frac{\varepsilon_{cu_{\text{test}}}}{\varepsilon_{cu}} - \mu \right)^2 \right) } \quad \text{Equation 3.4}
\]

The variance (\( \text{Var} \)) of predicted normalized value to test normalized value is also calculated to show the accuracy of prediction, given as following

\[
\text{Var}_{\frac{\varepsilon_{cu}}{\varepsilon_{cu}}} = \sum_{i=1}^{n} \left[ \left( \frac{\varepsilon_{cu_{\text{predict}}}}{\varepsilon_{cu}} / \frac{\varepsilon_{cu_{\text{test}}}}{\varepsilon_{cu}} \right) - \mu \right]^2 \quad \text{Equation 3.5}
\]

\[
\text{Var}_{\frac{f_{cu}}{f_{cu}}} = \sum_{i=1}^{n} \left[ \left( \frac{f_{cu_{\text{predict}}}}{f_{cu}} - f_{cu_{\text{test}}} \right) \right]^2 \quad \text{Equation 3.6}
\]
Table 3.3 Statics analysis of different models in predicting ultimate condition

<table>
<thead>
<tr>
<th>Model</th>
<th>$\frac{\epsilon_{cu,predict}}{\epsilon_{co}}$</th>
<th>$\frac{\epsilon_{cu,test}}{\epsilon_{co}}$</th>
<th>$\frac{f_{cu,predict}}{f_{co}}$</th>
<th>$\frac{f_{cu,test}}{f_{co}}$</th>
<th>$\text{Var}(\frac{\epsilon_{cu}}{\epsilon_{co}})$</th>
<th>$\text{Var}(\frac{f_{cu}}{f_{co}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harajli (2006)</td>
<td>0.694</td>
<td>0.179</td>
<td>1.015</td>
<td>0.114</td>
<td>596.03</td>
<td>2.27</td>
</tr>
<tr>
<td>Eid and Paultre (2008)</td>
<td>0.948</td>
<td>0.223</td>
<td>0.943</td>
<td>0.135</td>
<td>267.13</td>
<td>4.90</td>
</tr>
<tr>
<td>Lee et al (2010)</td>
<td>1.219</td>
<td>1.255</td>
<td>1.129</td>
<td>0.227</td>
<td>524.79</td>
<td>5.44</td>
</tr>
<tr>
<td>Chastre and Silva (2010)</td>
<td>0.811</td>
<td>0.153</td>
<td>1.260</td>
<td>0.235</td>
<td>294.69</td>
<td>24.05</td>
</tr>
</tbody>
</table>

All 33 specimens in the test database are used in this comparison. The comparison is presented here in according to model.

(1) Harajli’s model

Harajli’s model gives the best prediction of ultimate axial stress, as shown in Figure 3.10 (b) and Table 3.3. However, the axial strain prediction in Figure 3.10 (a) is worse than any other model mainly due to the scatters from Lee et al (2010) specimens.
Figure 3.10 – Performance of Harajli’s model in predicting ultimate condition
(2) Eid’s model

Although the axial stress prediction of Harajli’ model is slightly better than Eid’s model, considering both ultimate axial strain and stress, overall it is suggested that Eid’s model performs better than all other models, as seen in Figure 3.11 and Table 3.3.
Figure 3.11 – Performance of Eid’ model in predicting ultimate condition

(a) Ultimate axial strain

(b) Ultimate axial stress
(3) Lee’s model

Based on some equations used (for example, confining pressure used in the denominator, Equation 2.38), Lee’s model cannot be reduced to the FRP-confined concrete case only. Thus specimens from Xiao and Wu (2000) are not used in this comparison.

Figure 3.12 shows that Lee’s model works well for predicting ultimate axial stress. However, axial strain predictions are much worse. Lee’s model is unable to predict axial strain well, even against their own experimental results.
Figure 3.12 – Performance of Lee’s model in predicting ultimate condition
(4) Chastre’ model

Compared to the other models, Chastre’s model highly over estimates the ultimate axial stress, except for his own experimental results, as seen in Figure 3.13 (b).
Figure 3.13 – Performance of Chastre’s model in predicting Ultimate condition
3.3 Conclusion

From the above discussion, Eid and Paultre (2008) is considered to be the best model so far, considering both general behavior and ultimate condition. For the prediction of ultimate axial stress, all models except Chastre and Silva (2010) match well with test results. However, none of them predict the ultimate axial strain particularly well, which indicates that a new model needs to be developed to bridge this research gap and establish a more accurate lateral-to-axial strain relationship.
4 DEVELOPMENT OF PASSIVE CONFINEMENT MODEL

4.1 Introduction

To bridge the research gap concluded in Chapter 2 and demonstrated in Chapter 3, three alternatives are presented to develop a passive confinement model for steel-and-FRP-confined concrete. Fundamentally, all three alternatives are based on the FRP-confined concrete model developed by Teng (2007a). Hence, the Teng (2007a) model is described in detail in Section 4.2.

As mentioned in Section 2.3.2, passive confinement models for FRP-confined concrete, which is the major type of analysis-oriented model, all employ the Mander et al. (1988) model for steel-confined concrete as the basic active confinement model. For a given axial strain or lateral strain, the corresponding axial stress of FRP-confined concrete can be evaluated using an active confinement model. The general procedure of a passive confinement model may be illustrated schematically as shown in Figure 4.1.

![General analysis procedure for passive confinement models](image)

Figure 4.1 – General analysis procedure for passive confinement models
By providing a family of stress-strain active confinement curves, the stress-strain curve for FRP-confinement can be obtained using a step-by-step incremental procedure, which will be present in Section 4.2.1.

The three alternatives developed here all inherit the concept of passive confinement and the incremental procedure. But different modifications have been applied to make the model more and more appropriate for steel-and-FRP-confined concrete. These modifications relate to confining pressure, lateral-to-axial strain relationship, and failure surface function. These changes are made based on the conclusions derived from the literature review in Chapter 2 and the assessment of existing models from Chapter 3.

The first attempt simply sums the confining pressure from both materials and use Teng’s model. Consequently, this approach is referred to as the “Confining Pressure Summation Method”.

In the second approach, a new lateral-to-axial strain relationship is developed considering the influence of material confining pressure ratio on the lateral strain response, as mentioned in Chapter 2. Other aspects of this approach are same as Confining Pressure Summation Method. Consequently, this approach is identified as the “Modified Lateral Strain Response Method”.

However, the material confining pressure ratio not only affects the lateral strain response, but also the ultimate strength. Hence, approach three uses a different failure surface function to account for the contribution from two confining materials on the peak stress $f_{cc}'$. Other aspects of this approach are the same as the Modified Lateral Strain
Response Method. This third approach is named the “Combined Failure Surface Function Method”.

Details of these three approaches are provided in Sections 4.3, 4.4, and 4.5, respectively.

4.2 Teng’s FRP-confined concrete model

Based on the independent variable we selected, Teng’s FRP-confined concrete model can be used in two ways: (1) take the lateral strain $\varepsilon_l$ as the independent variable to obtain axial stress and axial strain via the incremental procedure avoiding iteration, and (2) take axial strain $\varepsilon_c$ as the independent variable through an iterative incremental procedure to obtain the lateral strain and axial stress, as introduced in Sections 4.2.1 and 4.2.2, respectively.

4.2.1 Application of Teng’s model starting with lateral strain

Step (1): Choose $\varepsilon_l$ then determine $f_l$

Choose any value of concrete lateral strain $\varepsilon_l \geq 0$ to obtain the corresponding confining pressure $f_l$, given by the following equilibrium equation for the free-body diagram in Figure 4.2.

$$f_l = \frac{2E_f t_f \varepsilon_f}{d}$$

Equation 4.1

where $d =$ diameter of the confined concrete core $= 2R$; and $\varepsilon_f =$ FRP hoop strain $= \varepsilon_l$. 
Step (2): Predict $\varepsilon_c$ using an appropriate $\varepsilon_1 - \varepsilon_c$ relationship

Based on a careful calibration and interpretation of test results of unconfined concrete cylinders, FRP-confined concrete cylinders, and actively confined concrete cylinders, a suitably accurate lateral-to-axial strain relationship for FRP-confined concrete was established by Teng et al (2007), given by

$$\frac{\varepsilon_c}{\varepsilon_{co}} = 0.85 \left[ 1 + 0.75 \left( \frac{\varepsilon_1}{\varepsilon_{co}} \right) \right]^{0.7} - \exp \left[ \left( -7 \frac{\varepsilon_1}{\varepsilon_{co}} \right) \right] \cdot \left( \alpha + \beta \frac{f_l}{f_{co}} \right) \quad \text{Equation 4.2}$$

where for actively confined concrete, $f_l = \text{constant value throughout the loading history}$, but for FRP-confined concrete $f_l = \text{current confining pressure given by Equation 4.1}$; and $\alpha$ and $\beta$ are constants, with $\alpha = 1$, $\beta = 0$ for unconfined concrete, or $\alpha = 1$, $\beta = 8$ for FRP-confined concrete.
For unconfined concrete, Equation 4.2 can be reduced to the following expression which describes the lateral strain-axial strain relationship of unconfined concrete

\[ \frac{\varepsilon_c}{\varepsilon_{co}} = 0.85 \left\{ \left[ 1 + 0.75 \left( \frac{\varepsilon_l}{\varepsilon_{co}} \right) \right]^{0.7} - \exp \left( -7 \frac{\varepsilon_l}{\varepsilon_{co}} \right) \right\} \]

Equation 4.3

Hence, the following linear term from Equation 4.2 is used to quantify the effect of lateral confinement

\[ \eta = \alpha + \beta \frac{f_l}{f_{co}} \]

Equation 4.4

**Step (3): Determine \( f'_{cc} \), and corresponding \( \varepsilon'_{cc} \) using a failure surface function**

As already presented in Section 2.3.3, Teng et al (2007) proposed the following equations to predicate the peak stress and corresponding strain \((f'_{cc}, \varepsilon'_{cc})\) on each actively confined concrete stress-strain curve, as shown in Figure 4.3.

\[ \frac{f'_{cc}}{f_{co}} = 1 + 3.5 \frac{f_l}{f_{co}} \]

Equation 4.5

\[ \frac{\varepsilon'_{cc}}{\varepsilon_{co}} = 1 + 17.5 \frac{f_l}{f_{co}} \]

Equation 4.6

Recalling Table 2.4, Equation 4.6 is same as Equation 2.7.
Step (4): Use an axial stress-axial strain equation to predict stress $f_c$

The following axial stress-axial strain equation has been adopted by Teng and many other investigators (as mentioned in Section 2.3.2), which was originally proposed by Popvics (1973) and used in the steel-confined concrete model by Mander et al. (1988):

$$\frac{f_c}{f'_{cc}} = \frac{(\varepsilon_c/\varepsilon_{cc})^r}{r-1+(\varepsilon_c/\varepsilon_{cc})^r} \quad \text{Equation 4.7}$$

where $r = \frac{E_c}{E_c-\varepsilon_{cc}E'_c}$.

Step (5): Generate the stress-strain curve for FRP-confined concrete

The general procedure is illustrated in Figure 4.3. Curve Active-1 in Figure 4.3 (a) is generated from the previous four steps corresponding to the lateral strain chosen at step (1). Axial strain and axial stress are obtained from step (2) and step (4), respectively, labeled as $(f_c, \varepsilon_c)$ on curve Active-1. Increasing lateral strain and repeating step (1) through step (4), another axial strain and axial stress can be obtained; such as curves Active-2 and Active-3 in Figure 4.3 (b). Repeating this procedure from zero lateral strain to the ultimate lateral strain when the FRP confinement ruptures, a family of actively confined concrete curves is obtained and the FRP-confined concrete curve is defined by connecting all the $(f_c, \varepsilon_c)$ coordinates as shown by the passive curve in Figure 4.3 (c). Coordinate $(f_{cu}, \varepsilon_{cu})$ defines the termination point, as shown in Figure 4.3 (d).
4.2.2 Application of Teng's model starting with axial strain

Step (1): Choose $\varepsilon_c$ and determine $\varepsilon_t$ using $\varepsilon_t - \varepsilon_c$ relationship

For any selected value of axial strain $\varepsilon_c$, from zero to $\varepsilon_{cu}$, a corresponding $\varepsilon_t$ can be iteratively determined using Equation 4.1 and Equation 4.2. Then, $\varepsilon_{cu}$ is directly obtained by setting $\varepsilon_t$ equal to the FRP rupture strain in Equation 4.2.

Step (2): Recalculate $f_t$

From the iteratively determined $\varepsilon_t$, $f_t$ should be recalculated using Equation 4.1.
Step (3): Determine $f'_{cc}$, and corresponding $\varepsilon'_{cc}$ using a failure surface function

Same as previous application.

Step (4): Use an axial stress-axial strain equation to predict stress $f_c$

Same as previous application.

Step (5): Generate the stress-strain curve for FRP-confined concrete

Basically the same procedure described in the application except the independent variable is $\varepsilon_c$. Thus, $\varepsilon_c$ is increased for every increment.

4.3 Confining pressure summation method

This section introduces the first approach to generate a stress-strain curve for steel-and-FRP-confined concrete. It must be noted that in all three approaches the independent variable may be taken as $\varepsilon_I$ or $\varepsilon_C$, which is same as Teng’s FRP-confined concrete model. However to simplify the explanation of these approaches, only the application starting with variable $\varepsilon_I$ is presented herein. In this approach the confining pressure from both materials is summed and the basic assumptions and detailed procedures are provided in Sections 4.3.1 and 4.3.2, respectively.

4.3.1 Basic assumptions

(1) Average axial stress

When concrete cylinders or columns are confined with both FRP and transverse steel reinforcement, its sectional area can be divided into two parts as shown in Figure 4.4.
The concrete core of diameter $d_c$ defined by the centerline of the steel reinforcement is confined by FRP and steel, while the concrete cover is confined by FRP only.

\[ f_c = \frac{f_{c,\text{core}} A_{\text{core}} + f_{c,\text{cover}} A_{\text{cover}}}{A_{\text{net}}} \quad \text{Equation 4.8} \]

Since the axial stress is experimentally obtained from the axial force divided by the net area of concrete section, Equation 4.8 is compatible with the experimental data.

(2) Confinement material hoop strain compatibility

It is assumed that the lateral strain in concrete, and the tensile strain in the transverse steel and FRP wrap in the hoop direction are all approximately equal

\[ \varepsilon_l = \varepsilon_s = \varepsilon_f \quad \text{Equation 4.9} \]
The compatibility of concrete lateral strain and FRP tensile strain ($\varepsilon_t = \varepsilon_f$) has been widely adopted in many FRP-confined concrete models, particularly the passive confinement models presented in Section 2.3.2.

Further, Eid and Paultre (2008) assume that $\varepsilon_f$ and $\varepsilon_s$ are approximately equal based on the observation of experimental results (Eid et al. 2006; Demers and Neale 1999) and analysis work by Eid and Paultre (2007), shown in Figure 4.5. Both finite element and the plasticity-based model of steel-and-FRP-confined concrete show that $\varepsilon_f$ and $\varepsilon_s$ are approximately equal for a relatively small concrete cover ($R_c / R \geq 0.9$).

![Figure 4.5 – Strain ratio ($\varepsilon_s/\varepsilon_f$) versus radius ratio ($R_c/R$) (Eid and Paultre 2007)](image)

(3) Fundamental active confinement model

As indicated in Chapter 2, active confinement models include three major parts: failure surface function, lateral-to-axial strain relationship, and axial stress-strain relationship. In
this Confining Pressure Summation Method, based on the second assumption, the sum of the confining pressure from the two confinement materials may be taken as the total confining pressure acting on the concrete core. And all equations from Teng’s FRP-confined concrete model can be adopted for this steel-and-FRP-confined scenario. In particular, Teng’s failure surface function as mentioned in Section 2.3.3, which is originally calibrated from cylinder tests of FRP-confined concrete, since every point in the curve represents an active confinement state.

(4) Axial strain compatibility

The assumption of full compatibility of axial strain between the concrete core and concrete cover is made, which gives a unique value for the whole concrete section.

(5) Ultimate condition by FRP rupture

The limit of the axial stress-axial strain curve occurs when the FRP wrap ruptures in the hoop direction. Most published experimental evidence (for example, Matthys et al (2000), and Lam and Teng (2004)) shows that the FRP wrap rupture strain is smaller than ultimate tensile strain obtained from flat coupon tests or as given by the FRP manufacturer.

Matthys et al (2000) give the ratio of actual rupture strain to the ultimate strain from coupon tests at approximately 0.6, which is within the typical range of 0.58-0.91, given by Lam and Teng (2004). Chen et al (2009) identify at least 15 factors relating to geometry, material and loading contributing to the observed reduction of FRP rupture strain. These include geometrical discontinuities at the ends of the FRP wrap, FRP
overlap zone, properties of the adhesive, and the three dimensional stress states in the FRP.

The rupture strain of wrapped FRP is a complicated and ongoing research issue beyond the scope of this research. Thus, for the purpose of this model, the FRP rupture strains from the actual tests of each specimen are used when generating the stress-strain curve in all three alternatives, and also in the Chapter 5 model evaluation when comparing to other models. 67% of the FRP rupture strain from coupon tests is used to illustrate the practical design example in Chapter 6.

4.3.2 Analysis procedure

Step (1): Choose \( \varepsilon_1 \) then determine \( f_l \)

Recalling Mander’s model (see Section 2.2.2.1) and Teng’s model, we have the confining pressure for single materials

\[
 f_{ls} = \begin{cases} 
 \frac{2k_eE_s\varepsilon_1 A_{sp}}{s'd_c} & \text{if } \varepsilon_1 < \varepsilon_{sy} \\
 \frac{2k_e f_{sy} A_{sp}}{s'd_c} & \text{if } \varepsilon_1 \geq \varepsilon_{sy} 
\end{cases} \quad \text{Equation 4.10}
\]

\[
 f_{lf} = \frac{2E_f f_{ls} \varepsilon_{ls}}{s'd} \quad \text{Equation 4.11}
\]

The first step is based on all the assumptions made above. Cover concrete is confined by the FRP wrap only and, according to force equilibrium of the half FRP wrap shown in Figure 4.6, the cover concrete confining pressure is simply given by

\[
 f_{l, cover} = f_{ls} \quad \text{Equation 4.12}
\]
Core concrete is confined by both the FRP wrap and transverse steel. The confining pressure can be derived via the force equilibrium of the half section shown in Figure 4.7. The total confining pressure is the summation of the confining pressure from each material, given by

\[ f_{l,core} = f_{ls} + f_{f} \]  

Equation 4.13
Step (2): Predict $\varepsilon_c$ using an appropriate $\varepsilon_l - \varepsilon_c$ relationship

In the Confining Pressure Summation Method, according to assumption 3 all equations used in Teng’s model can be utilized. Based on assumption 4, the division of area is not a concern for predicting axial strain. Equation 4.2 predicts concrete axial strain, with $\alpha = 1, \beta = 8$, specifically given by

$$\frac{\varepsilon_c}{\varepsilon_{co}} = 0.85 \left\{ \left[ 1 + 0.75 \left( \frac{\varepsilon_l}{\varepsilon_{co}} \right) \right]^{0.7} - \exp \left[ \left( \frac{-7 \cdot \varepsilon_l}{\varepsilon_{co}} \right) \right] \right\} \cdot \left( 1 + 8 \cdot \frac{f_{ls} + f_{tf}}{f_{co}} \right)$$  \hspace{1cm} \text{Equation 4.14}

Step (3): Determine $f'_{cc}$, and corresponding $\varepsilon'_{cc}$ using a failure surface function

Equation 4.5 and Equation 4.6 also can be utilized to determine $f'_{cc}$, and the corresponding $\varepsilon'_{cc}$. Specific equations are given by the following

for cover concrete

$$\frac{f'_{cc\text{ cover}}}{f_{co}} = 1 + 3.5 \frac{f_{l\text{ cover}}}{f_{co}}$$  \hspace{1cm} \text{Equation 4.15}

$$\frac{\varepsilon'_{cc\text{ cover}}}{\varepsilon_{co}} = 1 + 17.5 \frac{f_{l\text{ cover}}}{f_{co}}$$  \hspace{1cm} \text{Equation 4.16}

and for core concrete

$$\frac{f'_{cc\text{ core}}}{f_{co}} = 1 + 3.5 \frac{f_{l\text{ core}}}{f_{co}}$$  \hspace{1cm} \text{Equation 4.17}

$$\frac{\varepsilon'_{cc\text{ core}}}{\varepsilon_{co}} = 1 + 17.5 \frac{f_{l\text{ core}}}{f_{co}}$$  \hspace{1cm} \text{Equation 4.18}
Step (4): Use an axial stress-axial strain equation to predict stress $f_c$

According to assumption 3, Equation 4.7 can also be used to predict stress $f_c$ of both cover concrete and core concrete. The equations are given by following

For cover concrete

$$\frac{f_{c, cover}}{f'_{cc, cover}} = \frac{(\varepsilon_c/\varepsilon'_{cc, cover}) r_{cover}}{r_{cover} - 1 + (\varepsilon_c/\varepsilon'_{cc, cover}) r_{cover}}$$

Equation 4.19

where $r_{cover} = \frac{E_c}{E_c - f'_{cc, cover}/\varepsilon'_{cc, cover}}$; and $E_c = 4730 \sqrt{f_{co}}$ (MPa) according to ACI. 318-08.

and for cover concrete

$$\frac{f_{c, core}}{f'_{cc, core}} = \frac{(\varepsilon_c/\varepsilon'_{cc, core}) r_{core}}{r_{core} - 1 + (\varepsilon_c/\varepsilon'_{cc, core}) r_{core}}$$

Equation 4.20

where $r_{core} = \frac{E_c}{E_c - f'_{cc, core}/\varepsilon'_{cc, core}}$.

Then based on assumption 1, Equation 4.8 is used to obtain the axial stress $f_c$.


The general procedure is the same as described in Figure 4.3. Increase the lateral strain from zero to the FRP rupture strain and repeat steps (1) to (4) to generate the full stress-strain curve. As indicated in assumption 5, the actual FRP wrap rupture strain is used to terminate the stress-strain curve.
4.4 Modified lateral strain response method

Compared to the Confining Pressure Summation Method, the only difference in this approach is a new lateral-to-axial strain relationship. Basic assumptions and detailed procedure are provided in Sections 4.4.1 and 4.4.2, respectively.

4.4.1 Basic assumptions

With the exception of the third assumption, the same assumptions described in first approach (see Section 4.3) apply and hence, are not repeated again here.

(3) Fundamental active confinement model

The same fundamental active confinement model is used except for a new lateral-to-axial strain relationship, which can explain the observation discussed in Section 2.4.1 that the lateral response is influenced by the material confining pressure ratio.

In order to calibrate a new $\varepsilon_l - \varepsilon_c$ relationship, Equation 4.4 has to be modified as as given by the following to quantify the new effect of lateral confinement

$$\eta = \alpha + \beta_{FRP} \frac{f_{fr}}{f_{co}} + \beta_{TRS} \frac{f_{us}}{f_{co}}$$  \hspace{1cm} \text{Equation 4.21}

where $\alpha = 1$ and $\beta_{FRP} = 8$, which makes sure the new lateral-to-axial strain relationship can be reduced to FRP-confined concrete; $\beta_{TRS}$ is the factor that needs to be determined.

Then, the new lateral-to-axial strain relationship is given by

$$\frac{\varepsilon_c}{\varepsilon_{co}} = 0.85 \left[1 + 0.75 \left(\frac{\varepsilon_l}{\varepsilon_{co}}\right)^{0.7}\exp\left[-7 \frac{\varepsilon_l}{\varepsilon_{co}}\right]\right] \cdot \left(\alpha + \beta_{FRP} \frac{f_{fr}}{f_{co}} + \beta_{TRS} \frac{f_{us}}{f_{co}}\right)$$  \hspace{1cm} \text{Equation 4.22}
Recalling that the response ratio $\frac{\varepsilon_{cu}}{\varepsilon_{tu}}$ mentioned in Section 2.4.1, represents the confinement effect such that larger the ratio, the stiffer system behaves. Referring to Equation 4.22, $\eta$ (or $\beta_{TRS}$) is positively correlated to $\frac{\varepsilon_{cu}}{\varepsilon_{tu}}$.

A new factor called relative stiffness, $R_s$ is introduced here, which is the ratio of axial rigidity of transverse steel (before yield) to the axial rigidity of the FRP wrap, given by

$$R_s = \frac{E_{\text{eff, steel}}}{E_{\text{eff, FRP}}}$$  \hspace{1cm} \text{Equation 4.23}

$$E_{\text{eff, steel}} = \frac{E_{\text{steel}}}{\Delta_{\text{steel}}} = \frac{2 \cdot E_s \cdot \varepsilon_{i} \cdot A_p}{2\pi \cdot d_c \cdot \varepsilon_{l}} = \frac{E_s \cdot A_p}{\pi \cdot d_c}$$  \hspace{1cm} \text{Equation 4.24}

$$E_{\text{eff, FRP}} = \frac{F_{\text{FRP}}}{\Delta_{\text{FRP}}} = \frac{2 \cdot E_f \cdot t_f \cdot \varepsilon_{l}s}{2\pi \cdot d \cdot \varepsilon_{l}} = \frac{E_f \cdot t_f \cdot s}{\pi \cdot d}$$  \hspace{1cm} \text{Equation 4.25}

According to the conclusion in Section 2.4.1, that more transverse steel leads to more ductile behavior, $\beta_{TRS}$ might be expected to be a decreasing function of $R_s$. This can be proved by following regression analysis.

For each specimen in Table 3.1, $\beta_{TRS}$ can be calculated using Equation 4.22, given corresponding $\varepsilon_{cu}$ and $\varepsilon_{tu}$ (FRP rupture strain in hoop direction), and $R_s$ is easily calculated by its definition in Equation 4.23. Both factors are dimensionless and are given in Table 4.1 and plotted for each specimen in Figure 4.8.
<table>
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Figure 4.8 – Curve regression of data calculated in Table 3.1

From the curve regression analysis in Figure 4.8, $\beta_{TRS}$ is a decreasing function of $R_s$, simplified for design purpose, given by

$$\beta_{TRS} = 23.43R_s^{-0.50}$$

$R^2 = 0.515$

4.4.2 Analysis procedure

With the exception of the step (2), the same steps described in first approach (Section 4.3) apply and hence, not repeated again here.

**Step (2): Predict $\varepsilon_c$ with appropriate $\varepsilon_l - \varepsilon_c$ relationship**

Equation 4.22 and Equation 4.26 are utilized here to predict axial strain $\varepsilon_c$. 
4.5 Combined failure surface function method

Compared to the Modified Lateral Strain Response Method, the only difference is that no unique failure surface function is used to predict $f'_c$. Mander’s failure surface function and Teng’s failure surface function are used to quantify the contribution from steel and FRP, respectively. The basic assumptions and detailed procedure are provided in Sections 4.5.1 and 4.5.2 respectively.

4.5.1 Basic assumptions

With the exception of the third assumption, the same assumptions described in the first approach (Section 4.3.1) apply and hence, are not repeated again here.

(3) Fundamental active confinement model

In the fundamental active confinement model, the axial stress-strain relationship and lateral-to-axial strain relationship are the same as the second approach. What different is that two failure surface functions are combined to predict the peak stress $f'_c$. More specifically, the peak stress increment from each confinement material is summed up. This modification is also based on the observation in Section 2.4.1, that the ultimate strength of confined concrete is influenced by the material confining pressure ratio. And Teng’s original failure surface might not work well for steel-confined concrete, which is assumed functionable in the second approach.

4.5.2 Analysis procedure

With the exception of the step (3), the same steps described in the second approach (Section 4.4) apply and hence, are not repeated again here.
Step (3): Determine $f'_{cc}$, and corresponding $\varepsilon'_{cc}$ using a failure surface function

For cover concrete confined by FRP only, Equation 4.15 and Equation 4.16 can be utilized to determine peak value, $f'_{cc\_cover}$, and $\varepsilon'_{cc\_cover}$. For core concrete confined by FRP wrapping and transverse steel, following equations are used to obtain peak value, $f'_{cc\_core}$, $\varepsilon'_{cc\_core}$.

The increment of peak stress is defined as

$$\Delta f'_{cc} = f'_{cc} - f_{co}$$  \hspace{1cm} \text{Equation 4.27}

The contribution of the increment of peak stress $\Delta f'_{cc}$ from the FRP wrap is obtained from Teng’s failure surface equation (see Equation 2.13), given by

$$\Delta f'_{cc\_FRP} = 3.5 f_{l\_core\_FRP}$$  \hspace{1cm} \text{Equation 4.28}

where $f_{l\_core\_FRP} = f_{lf}$, given in Equation 4.11.

The contribution of the increment of peak stress $\Delta f'_{cc}$ from the transverse steel is obtained from Mander’s failure surface equation (see Equation 2.6), given by

$$\Delta f'_{cc\_TRS} = f_{co} \left( 2.254 \sqrt{1 + 7.94 \frac{f_{l\_core\_TRS}}{f_{co}} - 2 \frac{f_{l\_core\_TRS}}{f_{co}} - 2.254} \right)$$  \hspace{1cm} \text{Equation 4.29}

where $f_{l\_core\_TRS} = f_{ls}$, given in Equation 4.10.
So, for the core concrete, the peak stress is obtained by summing the contributions from both the FRP wrap and transverse steel. The reason why the sum can be used is because of assumption 2 (Section 4.3), where the confining pressures on the right side of Equation 4.28 and Equation 4.29 both correspond to the same lateral strain. Then $f'_{cc\_core}$ is given by

$$f'_{cc\_core} = f'_{co} + \Delta f'_{cc\_FRP} + \Delta f'_{cc\_TRS}$$

Equation 4.30

And for the corresponding strain at peak stress, $\varepsilon'_{cc\_core}$, the equation proposed by Richart et al. (1929) is used, which naturally is the same as Equation 4.6

$$\varepsilon'_{cc\_core} = 1 + 5 \left( \frac{f'_{cc\_core}}{f_{co}} - 1 \right)$$

Equation 4.31
4.6 Summary

The major differences between the approaches presented in this chapter are the different $\varepsilon_l - \varepsilon_c$ relationships and failure surface functions utilized. The major modifications are summarized in Table 4.2. The three approaches are evaluated using the available experimental database in the next chapter.

<table>
<thead>
<tr>
<th>Term</th>
<th>Teng's model</th>
<th>Confining Pressure Summation Method</th>
<th>Modified Lateral Strain Response Method</th>
<th>Combined Failure Surface Function Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>$1 + 8 \cdot f_{1\text{FRP}}$</td>
<td>$1 + 8 \cdot f_{1\text{total}}$</td>
<td>$1 + 8 \cdot f_{l\text{frp}} + \beta_{\text{TRS}} \cdot f_{is}$</td>
<td>$f_{c} = f_{co} + \Delta f_{c\text{FRP}} + \Delta f_{c\text{TRS}}$</td>
</tr>
<tr>
<td>$f'_{cc}$</td>
<td>$f'<em>{cc} = f</em>{co} + 3.5 f_{i}$</td>
<td>$f'<em>{cc} = f</em>{co} + 3.5 f_{i\text{total}}$</td>
<td>$f'<em>{cc} = f</em>{co} + 3.5 f_{i\text{total}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\beta_{\text{TRS}} = \frac{23.5}{\sqrt{R}}$</td>
<td>$\beta_{\text{TRS}} = \frac{23.5}{\sqrt{R}}$</td>
</tr>
</tbody>
</table>
5 EVALUATION OF NEW MODEL

5.1 Introduction

In this chapter, the evaluation of the three alternatives developed in Chapter 4 is conducted by comparing the predicted stress-strain response with the pure compression test results from the test database summarized in Table 3.1. The

In particular, the Modified Lateral Strain Response Method and Combined Failure Surface Function Method are compared to the four existing models discussed in Chapter 3 including Harajli (2006), Eid and Paultre (2008), Chastre and Silva (2010), and Lee et al (2010).

5.2 Assessment of new models

5.2.1 Comparison of analytical predictions with experimental data of FRP-confined concrete

The Confining Pressure Summation Method, Modified Lateral Strain Response Method, and Combined Failure Surface Function Method can all be reduced to Teng’s FRP-confined concrete model based on the design nature of each alternative.

Figure 5.1 and Figure 5.2 show that Teng’s FRP-confined concrete model (coincident with three alternatives) matches well with test results from Xiao and Wu (2000), which giving confidence in the selected fundamental model used.
Figure 5.1 – Comparison of three alternatives with test results of specimen 1

Figure 5.2 – Comparison of three alternatives with test results of specimen 5
5.2.2 Comparison of analytical predictions with experimental data of FRP-and-steel-confined concrete

To be consistent with Chapter 3, only selected comparisons between the three alternatives and experimental results are presented here, including different levels of confining pressure ratio $f_{cs}/f_{cf}$ (same term as in Chapter 3), which is calculated and labeled on each the following figures, and summarized in Table 3.2.

The comparisons are shown in Figure 5.3 - Figure 5.11 following the specimen order shown in Table 3.2, in which both lateral strain and axial strain are plotted versus axial stress. As expected, the Confining Pressure Summation Method predicts axial strain much worse than the other two approaches that use the calibrated factor $\beta_{TRS}$. It is also observed that the Combined Failure Surface Function Method works better than the Modified Lateral Strain Response Method for all levels of confining pressure ratio.

![Figure 5.3 – Comparison of three alternatives with test results of specimen 10](image)

Figure 5.3 – Comparison of three alternatives with test results of specimen 10
Figure 5.4 – Comparison of three alternatives with test results of specimen 14

Figure 5.5 – Comparison of three alternatives with test results of specimen 20
Figure 5.6 – Comparison of three alternatives with test results of specimen 19

Figure 5.7 – Comparison of three alternatives with test results of specimen 30
Figure 5.8 – Comparison of three alternatives with test results of specimen 28

Figure 5.9 – Comparison of three alternatives with test results of specimen 32
Figure 5.10 – Comparison of three alternatives with test results of specimen 27

Figure 5.11 – Comparison of three alternatives with test results of specimen 23
5.2.3 Comparison of analytical predictions with existing models of FRP-and-steel-confined concrete

The comparisons between the two proposed methods (Modified Lateral Strain Response Method, and Combined Failure Surface Function Method) and the four existing models (Harajli (2006), Eid and Paultre (2008), Lee et al (2010), and Chastre and Silva (2010)) are presented in this section using the same specimens.

Since Lee's model is unable to predict the lateral strain-stress curve, in Figure 5.12 to Figure 5.20 only axial strain is plotted versus axial stress. Further, to ensure a comparison, the actual FRP rupture strain is used for all models, given in Table 2.6 in Appendix. It is clear that both new methods perform better than any of the other existing models in predicting ultimate strength or ultimate strain. As concluded in Chapter 3, all four existing models might perform well in some cases, but generally speaking the new methods behave best.
Figure 5.12 – Comparison of new models with existing models of specimen 10

Figure 5.13 – Comparison of new models with existing models of specimen 14
Figure 5.14 – Comparison of new models with existing models of specimen 20

Figure 5.15 – Comparison of new models with existing models of specimen 19
Figure 5.16 – Comparison of new models with existing models of specimen 30

Figure 5.17 – Comparison of new models with existing models of specimen 28
Figure 5.18 – Comparison of new models with existing models of specimen 32

Figure 5.19 – Comparison of new models with existing models of specimen 27
5.2.4 Ultimate condition

The performance of the two new approaches in predicting the ultimate condition is shown in Figure 5.21 and Figure 5.22. There is no difference between two methods when predicting ultimate axial strain since of the same lateral-to-axial strain relationship is used.

Comparing Figure 5.21 (b) and Figure 5.22 (b), it is clear to see that the prediction of ultimate strength has improved slightly due to the modification applied to the failure surface functions where the FRP and steel confinement are combined. Also, by comparing to the performance of other models in predicting ultimate condition in Chapter 3, it can be concluded these two new methods predict more accurately the
ultimate condition. This conclusion is further demonstrated by the comparison of variance shown in Table 5.1.

**Table 5.1 Statics analysis of different models in predicting ultimate condition**

<table>
<thead>
<tr>
<th>Model</th>
<th>$\frac{\varepsilon_{cu,\text{ predict}}}{\varepsilon_{co}}$</th>
<th>$\frac{\varepsilon_{cu,\text{ test}}}{\varepsilon_{co}}$</th>
<th>$\frac{f_{cu,\text{ predict}}}{f_{co}}$</th>
<th>$\frac{f_{cu,\text{ test}}}{f_{co}}$</th>
<th>$\text{Var}(\frac{\varepsilon_{cu}}{\varepsilon_{co}})$</th>
<th>$\text{Var}(\frac{f_{cu}}{f_{co}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harajli (2006)</td>
<td>0.6940</td>
<td>0.1789</td>
<td>1.0154</td>
<td>0.1136</td>
<td>596.03</td>
<td>2.27</td>
</tr>
<tr>
<td>Eid and Paultre (2008)</td>
<td>0.9479</td>
<td>0.2231</td>
<td>0.9430</td>
<td>0.1350</td>
<td>267.13</td>
<td>4.90</td>
</tr>
<tr>
<td>Lee et al (2010)</td>
<td>1.2192</td>
<td>1.2554</td>
<td>1.1288</td>
<td>0.2266</td>
<td>524.79</td>
<td>5.44</td>
</tr>
<tr>
<td>Chastre and Silva (2010)</td>
<td>0.8114</td>
<td>0.1527</td>
<td>1.2604</td>
<td>0.2352</td>
<td>294.69</td>
<td>24.05</td>
</tr>
<tr>
<td>Modified Lateral Strain Response Method</td>
<td>0.9458</td>
<td>0.1981</td>
<td>0.9152</td>
<td>0.1048</td>
<td>177.41</td>
<td>4.86</td>
</tr>
<tr>
<td>Combined Failure Surface Function Method</td>
<td>0.9458</td>
<td>0.1981</td>
<td>0.9648</td>
<td>0.1070</td>
<td>177.41</td>
<td>3.29</td>
</tr>
</tbody>
</table>
Figure 5.21 – Performance of Modified Lateral Strain Response Method in predicting Ultimate condition

(a) Ultimate axial strain

(b) Ultimate axial stress
Figure 5.22 – Performance of Combined Failure Surface Function Method in predicting Ultimate condition
5.3 Conclusion

From the above Comparison and discussion, the Modified Lateral Strain Response Method and Combined Failure Surface Function Method predict the general behavior and ultimate condition better than any other model. And, the Combined Failure Surface Function Method performs better than Modified Lateral Strain Response Method, which indicates that the calculation of peak stress by separating the contribution from the two confinement materials might be the better approach.
6 FRP STRENGTHENING APPLICATION

6.1 Introduction

In Chapter 4 and Chapter 5 a new constitutive model for concrete confined by both FRP wrap and transverse steel was developed and evaluated. To illustrate the application of this newly developed model, a design example is presented, demonstrating the procedure to obtain the fundamental confined concrete stress-strain curve, moment-curvature response, and axial load-moment interaction diagram.

6.2 RC column

The RC column used in this design example was taken from Wight and MacGregor (2009) and designed according to ACI 318-08. The summary of material and geometric information are shown in Table 6.1 and Figure 6.1, respectively. This column was designed to resist an axial load of 1600 Kips (7112.33 kN) and moment of 150 Kip-ft (203 kN·m).

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$ (mm)</td>
<td>$d_c$ (mm)</td>
<td>$f_{co}$ (MPa)</td>
<td>$f_{sy}$ (MPa)</td>
<td>$f_y$ (MPa)</td>
<td>$A_s$ (mm$^2$)</td>
</tr>
<tr>
<td>660.40</td>
<td>574.68</td>
<td>27.58</td>
<td>413.68</td>
<td>413.68</td>
<td>8190</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A_{sp}$ (mm$^2$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>71</td>
</tr>
</tbody>
</table>
Let’s say that undamaged steel-confined column is to be strengthened with three layers (nominal thickness of 0.165 mm per layer) of Carbon FRP with properties, given in Table 6.2. As discussed in Section 4.3, the FRP rupture strain is smaller than the value obtained from straight coupon tests. The ratio of actual rupture strain to the ultimate strain from coupon test is normally in the range of 0.58-0.91. Hence, a value of $\varepsilon_{frup} = 0.01$ is selected, giving a ratio of 0.67 when $\varepsilon_f = 0.015$ which is consistent with typical data collected (see Table A.3 in Appendix).

<table>
<thead>
<tr>
<th>$t_f$ (mm)</th>
<th>$E_f$ (MPa)</th>
<th>$f_f$ (MPa)</th>
<th>$\varepsilon_f$</th>
<th>$\varepsilon_{frup}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.495</td>
<td>256000</td>
<td>3790</td>
<td>0.015</td>
<td>0.01</td>
</tr>
</tbody>
</table>
6.3 Design example

This example only shows the application of the Combined Failure Surface Function Method to obtain confined concrete stress-strain curve, starting with selected lateral strains.

6.3.1 Application of new passive confinement model

Step (1): Choose $\varepsilon_l$ then determine $f_l$

Choose any value of concrete lateral strain $0 \leq \varepsilon_l \leq \varepsilon_{f.u.a}$ to obtain the corresponding confining pressure for each material, given by

\[
 f_{ls} = \begin{cases} 
 \frac{2k_eE_f\varepsilon_lA_{sp}}{s'd_c} & \text{if } \varepsilon_l < \varepsilon_{sy} \\
 \frac{2k_e\varepsilon_l^2A_{sp}}{s'd_c} & \text{if } \varepsilon_l \geq \varepsilon_{sy} 
\end{cases} 
\]  

Equation 6.1

\[
 f_{lf} = \frac{2E_f t_f \varepsilon_l^2 s}{s'd} 
\]  

Equation 6.2

where $k_e$ is given by Equation 2.2 or Equation 2.3.

for the cover concrete, the confining pressure is simply given by

\[
 f_{l,\text{cover}} = f_{lf} 
\]  

Equation 6.3

and for the core concrete confined by both the FRP wrap and transverse steel, the confining pressure is given by

\[
 f_{l,\text{core}} = f_{ls} + f_{lf} 
\]  

Equation 6.4
Step (2): Predict $\varepsilon_c$ using an appropriate $\varepsilon_l - \varepsilon_c$ relationship

The following lateral-to-axial strain relationship is used to predict $\varepsilon_c$, given by

$$\frac{\varepsilon_c}{\varepsilon_{co}} = 0.85 \left\{1 + 0.75 \left(\frac{\varepsilon_l}{\varepsilon_{co}}\right)^{0.7} - \exp \left[\left(-7\frac{\varepsilon_l}{\varepsilon_{co}}\right)\right]\right\} \cdot \left(\alpha + \beta_{FRP} \frac{f_{lf}}{f_{co}} + \beta_{TRS} \frac{f_{ls}}{f_{co}}\right)$$

Equation 6.5

where $\alpha = 1$, $\beta_{FRP} = 8$, $\beta_{TRS} = \frac{23.5}{\sqrt{R_s}}$, and $R_s = \frac{E_s A_{sp} d}{E_{tf} t_f d_{cc} s}$.

In this example, $\varepsilon_{co}$ is taken as 0.002 which is typical for normal strength concrete.

Step (3): Determine $f'_{cc}$ and corresponding $\varepsilon'_{cc}$ using a failure surface function

For cover concrete confined by FRP only, the peak values, $f'_{cc, cover}$, $\varepsilon'_{cc, cover}$, are determined by the following

$$\frac{f'_{cc, cover}}{f_{co}} = 1 + 3.5 \frac{f_{l, cover}}{f_{co}}$$

Equation 6.6

$$\frac{\varepsilon'_{cc, cover}}{\varepsilon_{co}} = 1 + 17.5 \frac{f_{l, cover}}{f_{co}}$$

Equation 6.7

For core concrete confined by both the FRP and transverse steel, the peak stress, $f'_{cc, core}$, is obtained using the summation of the increment of concrete strength, defined as

$$f'_{cc, core} = f_{co} + \Delta f'_{cc} = f_{co} + \Delta f'_{cc, FRP} + \Delta f'_{cc, TRS}$$

Equation 6.8

where the contribution of the increment of peak stress $\Delta f'_{cc}$ from the FRP wrap is obtained from Teng’s failure surface equation, given by

$$\Delta f'_{cc, FRP} = 3.5 f_{lf}$$

Equation 6.9
and the contribution of the increment of peak stress $\Delta f'_{cc}$ from the transverse steel is obtained from Mander’s failure surface equation, given by

$$\Delta f'_{cc\,TRS} = f_{co} \left( 2.254 \sqrt{1 + 7.94 \frac{f_{is}}{f_{co}} - 2 \frac{f_{is}}{f_{co}} - 2.254} \right)$$  \hspace{1cm} \text{Equation 6.10}

And for the corresponding strain at peak stress, $\varepsilon'_{cc\,core}$, the equation proposed by Richart et al. (1929) is used, given by

$$\varepsilon'_{cc\,core} = 1 + 5 \left( \frac{f'_{cc\,core}}{f_{co}} - 1 \right)$$  \hspace{1cm} \text{Equation 6.11}

**Step (4): Use an axial stress-axial strain relationship to predict stress $f_c$ for varying $\varepsilon_c$**

For cover concrete

$$\frac{f_{c\,cover}}{f'_{cc\,cover}} = \frac{(\varepsilon_c/\varepsilon'_{cc\,cover}) r_{cover}}{r_{cover} - 1 + (\varepsilon_c/\varepsilon'_{cc\,cover}) r_{cover}}$$  \hspace{1cm} \text{Equation 6.12}

where $r_{cover} = \frac{E_c}{E_{c-f'_{cc\,cover}/\varepsilon'_{cc\,cover}}}$; and $E_c = 4730 \sqrt{f_{co}}$ (MPa)

For cover concrete

$$\frac{f_{c\,core}}{f'_{cc\,core}} = \frac{(\varepsilon_c/\varepsilon'_{cc\,core}) r_{core}}{r_{core} - 1 + (\varepsilon_c/\varepsilon'_{cc\,core}) r_{core}}$$  \hspace{1cm} \text{Equation 6.13}

where $r_{core} = \frac{E_c}{E_{c-f'_{cc\,core}/\varepsilon'_{cc\,core}}}$.

The final axial concrete compressive stress is finally given by a weighted average

$$f_c = \frac{f_{c\,core} A_{core} + f_{c\,cover} A_{cover}}{A_{net}}$$  \hspace{1cm} \text{Equation 6.14}
Step (5): Generate the stress-strain curve for steel-and-FRP-confined concrete

Increase lateral strain $\varepsilon_l$ and repeat step (1) to step (4) to generate the full passive stress-strain curve, as shown in Figure 6.2. The point on the active confinement curve $(f_{c}, \varepsilon_{c})$ for a given $\varepsilon_l$ that corresponding to the point on the passive confinement curve being developed is given by $\varepsilon_c$ from Equation 6.5 in step (2). The actual FRP wrap rupture strain $\varepsilon_{fu,a}$ is used to define the end of the stress-strain curve.

![Figure 6.2 – General procedure](image)

6.3.2 Confined concrete stress-strain curve

Using the approach derived in the previous section, the confined concrete stress-strain curve obtained by the Combined Failure Surface Function Method is given in Figure 6.3 which also includes the same for Mander’s model including the effect of the transverse...
steel only, and the ACI 440.2R-08 model (which is the same as Lam and Teng (2003)) and only considers the effect of the FRP wrap. Also shown for reference in Figure 6.3 is the unconfined concrete curve using a simple parabolic curve. It is clear that neither Mander’s model nor ACI 440.2R-08 alone capture the response of the combined confining effect of both the FRP wrap and transverse steel reinforcement.

Figure 6.3 – Model prediction of confined concrete compressive stress-strain curve

6.3.3 Moment-curvature response

Using the confined concrete constitutive model developed in Section 6.3.2 and shown in Figure 6.3, moment-curvature analysis can be conducted using Cumbia (Montejo 2007) the results of which are shown in Figure 6.4 corresponding to an axial compressive load
of 7112 kN. The prediction of the Combined Failure Surface Function Method shows that in this example the moment capacity has been increased 46% by using FRP wrapping technique, compared to the prediction of Mander’s model for the original RC column only. Since the failure of the strengthened column is controlled by rupture of the FRP wrap the ductility is reduced.

![Figure 6.4 – Predicted moment-curvature response](image)

6.3.4 Axial load-moment interaction

The moment-curvature analysis can be repeated for different axial loads to obtain the axial load-moment interaction diagram using the Combined Failure Surface Function Method, Mander’s model, and ACI 440.2R-08, as shown in Figure 6.5. It can be seen that the Combined Failure Surface Function Method considering both FRP and
transverse steel increases the failure envelope everywhere, particularly within the compressive controlled region.

**Figure 6.5 – Axial load-moment interaction diagram**

6.4 Parametric study

The effect of the number of layers of FRP is evaluated in this section using the Combined Failure Surface Function Method using the same concrete section as in the previous design example. The results of varying the number of FRP layers from 1 to 5 include the confined concrete constitutive model, and moment-curvature response in
Figure 6.6 and Figure 6.7, respectively. The results are summarized in Table 6.3, where the offset of increasing the layers of FRP on the ultimate conditions can be clearly seen.

Figure 6.6 – Confined concrete stress-strain curve varying FRP layers
Figure 6.7 – Moment-curvature response varying FRP layers

Table 6.3 – Summary of ultimate values

<table>
<thead>
<tr>
<th>FRP layer number</th>
<th>$f_{cu}$ (MPa)</th>
<th>$\varepsilon_{cu}$</th>
<th>$M_u$ (kN·m)</th>
<th>$\phi_u$ (1/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39.52</td>
<td>0.0105</td>
<td>1411.81</td>
<td>0.0253</td>
</tr>
<tr>
<td>2</td>
<td>44.26</td>
<td>0.0139</td>
<td>1522.39</td>
<td>0.0371</td>
</tr>
<tr>
<td>3</td>
<td>48.91</td>
<td>0.0171</td>
<td>1605.30</td>
<td>0.0492</td>
</tr>
<tr>
<td>4</td>
<td>53.56</td>
<td>0.0202</td>
<td>1707.95</td>
<td>0.0571</td>
</tr>
<tr>
<td>5</td>
<td>58.12</td>
<td>0.0231</td>
<td>1770.72</td>
<td>0.0647</td>
</tr>
</tbody>
</table>
7 CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary of Research

The use of FRP material to strengthen existing, or repair damaged columns is one of the most effective methods. The availability of an accurate confined concrete stress-strain relationship is fundamental in order to evaluate the performance of the column. A test database was established including 33 specimens for the purpose of model assessment. A careful literature review and assessment of existing models was conducted, identifying the research gap that there is currently not a sufficiently accurate theoretical model for steel-and-FRP-confined concrete. Three alternatives have been proposed based on the Teng et al (2007) passive FRP-confined concrete model. The modifications involve a new lateral-to-axial strain relationship and failure surface function. The application of the new passive confinement model (Combined Failure Surface Function Method) is presented in an illustrative example indicating that by repairing columns with FRP wraps, the column can support more axial load and moment. Conclusions and recommendations for future research are presented in the subsequent sections of this chapter.

7.2 Conclusions

A new passive confinement model was developed for steel-and-FRP-confined concrete, including three alternatives: Confining Pressure Summation Method, Modified Lateral Strain Response Method, and Combined Failure Surface Function Method. A critical
assessment of these three alternatives was conducted using all 33 specimens from a collected test database. The following conclusions can be made from the observations and analysis carried out during the assessment.

1. Literature review and assessment of existing models show that they are generally inadequate to predict the general stress-strain behavior of steel-and-FRP-confined concrete and the ultimate stress and strain due to the deficiency of each model as concluded in Chapter 2.

2. Particularly for the modification to the lateral-to-axial strain relationship, a factor $\beta_{TRS}$ is calibrated as a decreasing function of relative stiffness $R_s$ (steel to FRP) given by

$$\beta_{TRS} = \frac{23.5}{\sqrt{R_s}}$$

Equation 7.1

This equation can account for the phenomenon observed in actual test results and coincides with the physical interpretation that more steel confinement leads to ductile behavior in the lateral direction; even if the total confining pressure is the same.

3. The Combined Failure Surface Function Method gives good results compared to the actual test results; better than other two methods and all existing models, both in the prediction of general stress-strain behavior and ultimate condition. This is mainly due to the solution that the contribution from both confining materials are quantified separately with different failure surface functions, and the new calibrated lateral-to-axial strain relationship.
7.3 Recommendations

Based on the assessment and conclusions described before, several recommendations have been made to proceed with future research. Future research may include, but should not be limited to the following:

1. The physical interpretation that with same amount of total confining pressure more transverse steel confinement leads to ductile behavior needs more experimental results to be validated. To-date only two comparisons can support this assumption.

2. With more experimental data, better regression of factor $\beta_{TRS}$ can be conducted to get a more accurate lateral-to-axial strain relationship. The definition of relative stiffness is defined as the ratio of axial rigidity of steel to FRP. But this factor is only calculated before steel yield. After yield, the effective stiffness of the steel decreases continuously. A better term may need to be defined in place of $R_s$. 

References


Available from [www.geocities.com/lumontv/eng](http://www.geocities.com/lumontv/eng)


Appendix
### Table A.1 Definition of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{cc}$</td>
<td>mm$^2$</td>
<td>area of the concrete core confined with internal transverse steel, measured from centerline of transverse steel of one side to the other side</td>
</tr>
<tr>
<td>$A_g$</td>
<td>mm$^2$</td>
<td>the gross area of the column section</td>
</tr>
<tr>
<td>$A_{sp}$</td>
<td>mm$^2$</td>
<td>area of transverse steel cross section</td>
</tr>
<tr>
<td>$c$</td>
<td>-</td>
<td>cover concrete</td>
</tr>
<tr>
<td>$d$</td>
<td>mm</td>
<td>the diameter of whole concrete section</td>
</tr>
<tr>
<td>$d_c$</td>
<td>mm</td>
<td>the diameter of concrete core, measured center to center of the spiral or circular hoop</td>
</tr>
<tr>
<td>$E_c$</td>
<td>MPa</td>
<td>the initial concrete modulus of elasticity</td>
</tr>
<tr>
<td>$E_f$</td>
<td>MPa</td>
<td>FRP modulus of elasticity</td>
</tr>
<tr>
<td>$f_c$</td>
<td>MPa</td>
<td>concrete axial stress</td>
</tr>
<tr>
<td>$f_{cc}$</td>
<td>MPa</td>
<td>the peak stress of steel-confined concrete</td>
</tr>
<tr>
<td>$f_{co}$</td>
<td>MPa</td>
<td>the peak stress of unconfined concrete, obtained from cylinder tests</td>
</tr>
<tr>
<td>$f_{cu}$</td>
<td>MPa</td>
<td>ultimate confined concrete stress</td>
</tr>
<tr>
<td>$f_i$</td>
<td>MPa</td>
<td>confining pressure</td>
</tr>
<tr>
<td>$f_{lf}$</td>
<td>MPa</td>
<td>lateral passive confining pressure provided by FRP</td>
</tr>
<tr>
<td>$f_{ls}$</td>
<td>MPa</td>
<td>lateral passive confining pressure provided by transverse steel</td>
</tr>
<tr>
<td>$f_{sy}$</td>
<td>MPa</td>
<td>transverse steel yield stress</td>
</tr>
<tr>
<td>$k_e$</td>
<td>-</td>
<td>confinement effectiveness coefficient</td>
</tr>
<tr>
<td>$R_s$</td>
<td>-</td>
<td>relative stiffness, the ratio of axial rigidity of transverse steel (before yield) to axial rigidity of FRP wrap</td>
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<tr>
<td>$s$</td>
<td>mm</td>
<td>center to center spacing of the transverse steel spiral or circular hoop</td>
</tr>
<tr>
<td>$s'$</td>
<td>mm</td>
<td>clear vertical spacing between spiral or hoop reinforcement</td>
</tr>
<tr>
<td>$t_f$</td>
<td>mm</td>
<td>thickness of FRP wrap</td>
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<tr>
<td>$\beta_{FRP}$</td>
<td>-</td>
<td>confining factor of FRP wrap in lateral-to-axial strain relationship</td>
</tr>
<tr>
<td>$\beta_{TRS}$</td>
<td>-</td>
<td>confining factor of transverse steel in lateral-to-axial strain</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
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<tr>
<td>$\varepsilon_c$</td>
<td>concrete axial strain</td>
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<tr>
<td>$\varepsilon_{cc}'$</td>
<td>corresponding strain of steel-confined concrete at peak stress</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{co}$</td>
<td>corresponding strain of unconfined concrete at peak stress</td>
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<tr>
<td>$\varepsilon_{cu}$</td>
<td>ultimate confined concrete strain</td>
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<tr>
<td>$\varepsilon_f$</td>
<td>rupture strain of FRP, obtained in flat coupon test</td>
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<tr>
<td>$\varepsilon_{fu,a}$</td>
<td>rupture strain of FRP wrap in hoop direction</td>
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<tr>
<td>$\varepsilon_l$</td>
<td>concrete lateral strain</td>
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<tr>
<td>$\varepsilon_s$</td>
<td>strain of transverse steel</td>
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<tr>
<td>$\varepsilon_{sta}$</td>
<td>strain of the steel at maximum tensile stress</td>
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<tr>
<td>$\eta$</td>
<td>effect of lateral confinement</td>
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<tr>
<td>$\rho_{cc}$</td>
<td>the ratio of area of longitudinal reinforcement to area of core concrete</td>
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<tr>
<td>$\rho_f$</td>
<td>the volumetric ratio of the FRP sheets</td>
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<tr>
<td>$\rho_s$</td>
<td>the ratio of the volume of confinement reinforcement to the volume of confined concrete core</td>
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<tr>
<td>$\rho_{st}$</td>
<td>the volumetric ratio of the transverse steel ties or hoops</td>
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<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
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Table A.2 Comparison of experimental results of steel-confined concrete with the values predicted by four models (Madas and Elnashai 1992)

<table>
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<tr>
<th>Specimen</th>
<th>Experimental values</th>
<th>Active Mander et al</th>
<th>Park et al</th>
<th>Passive Ahmad and Shah</th>
<th>Madas and Elnashai</th>
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<tr>
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<td>$K_s$</td>
<td>$\frac{f_{cc}}{f_{co}}$</td>
<td>$K_s$</td>
<td>%</td>
<td>$\frac{f_{cc}}{f_{co}}$</td>
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<td>II/1-0^2</td>
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<td>1.905</td>
<td>1.574</td>
<td>30.6</td>
<td>3.857</td>
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<td>4.857</td>
<td>2.062</td>
<td>38.7</td>
<td>6.333</td>
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<td>1.459</td>
<td>1.420</td>
<td>27.9</td>
<td>3.091</td>
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<td>1.560</td>
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* % = (1-analytical value/experimental value) × 100, $K_s = \frac{f_{cc}}{f_{co}}$
Table A.3 Specimen Parameters of test database for assessment

| Source            | Specimen name | Specimen number | $f_{co}$ (Mpa) | $E_f$ (Mpa) | $f_f$ (Mpa) | $f_{sy}$ (Mpa) | $E_s$ (Mpa) | $t_f$ (mm) | $A_{sy}$ (mm$^2$) | $s$ (mm) | $A_{cu}$ (mm$^2$) | $h$ (mm) | $d_e$ (mm) | $e_{in}$ | $e_{co}$ | $e_{cu}$ | Steel type |
|-------------------|---------------|----------------|----------------|------------|-------------|---------------|------------|------------|------------------|---------|------------------|--------|----------|---------|----------|
| Xiao and Wu (2000)| LC1L          | 1              | 33.68          | 105000     | 1577        | -             | -          | 0.381      | 0                | -       | 0                | 305    | 152      | 0.0114  | 0.0021   | 0.0138  | Spiral    |
|                   | LC2L          | 2              | 33.68          | 105000     | 1577        | -             | -          | 0.762      | 0                | -       | 0                | 305    | 152      | 0.0099  | 0.0021   | 0.0220  | Spiral    |
|                   | LC3L          | 3              | 33.68          | 105000     | 1577        | -             | -          | 1.143      | 0                | -       | 0                | 305    | 152      | 0.0088  | 0.0021   | 0.0066  | Spiral    |
|                   | MC1L          | 4              | 43.77          | 105000     | 1577        | -             | -          | 0.381      | 0                | -       | 0                | 305    | 152      | 0.0085  | 0.0023   | 0.0104  | Spiral    |
|                   | MC2L          | 5              | 43.77          | 105000     | 1577        | -             | -          | 0.762      | 0                | -       | 0                | 305    | 152      | 0.0090  | 0.0023   | 0.0166  | Spiral    |
|                   | MC3L          | 6              | 43.77          | 105000     | 1577        | -             | -          | 1.143      | 0                | -       | 0                | 305    | 152      | 0.0081  | 0.0023   | 0.0181  | Spiral    |
|                   | HC2L          | 7              | 55.2           | 105000     | 1577        | -             | -          | 0.762      | 0                | -       | 0                | 305    | 152      | 0.0080  | 0.0024   | 0.0150  | Spiral    |
|                   | HC3L          | 8              | 55.2           | 105000     | 1577        | -             | -          | 1.143      | 0                | -       | 0                | 305    | 152      | 0.0076  | 0.0024   | 0.0148  | Spiral    |
| Lee et al. (2010) | S6F1          | 9              | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.11       | 19.63           | 60      | 0                | 300    | 150      | 0.0040  | 0.0024   | 0.0170  | Spiral    |
|                   | S6F2          | 10             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.22       | 19.63           | 60      | 0                | 300    | 150      | 0.0066  | 0.0024   | 0.0250  | Spiral    |
|                   | S6F4          | 11             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.44       | 19.63           | 60      | 0                | 300    | 150      | 0.0054  | 0.0024   | 0.0340  | Spiral    |
|                   | S6F5          | 12             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.55       | 19.63           | 60      | 0                | 300    | 150      | 0.0059  | 0.0024   | 0.0360  | Spiral    |
|                   | S4F1          | 13             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.11       | 19.63           | 40      | 0                | 300    | 150      | 0.0055  | 0.0024   | 0.0190  | Spiral    |
|                   | S4F2          | 14             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.22       | 19.63           | 40      | 0                | 300    | 150      | 0.0057  | 0.0024   | 0.0230  | Spiral    |
|                   | S4F3          | 15             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.33       | 19.63           | 40      | 0                | 300    | 150      | 0.0054  | 0.0024   | 0.0290  | Spiral    |
|                   | S4F4          | 16             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.44       | 19.63           | 40      | 0                | 300    | 150      | 0.0061  | 0.0024   | 0.0300  | Spiral    |
|                   | S4F5          | 17             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.55       | 19.63           | 40      | 0                | 300    | 150      | 0.0060  | 0.0024   | 0.0360  | Spiral    |
|                   | S2F1          | 18             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.11       | 19.63           | 20      | 0                | 300    | 150      | 0.0063  | 0.0024   | 0.0390  | Spiral    |
|                   | S2F2          | 19             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.22       | 19.63           | 20      | 0                | 300    | 150      | 0.0061  | 0.0024   | 0.0360  | Spiral    |
|                   | S2F3          | 20             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.33       | 19.63           | 20      | 0                | 300    | 150      | 0.0061  | 0.0024   | 0.0325  | Spiral    |
|                   | S2F4          | 21             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.44       | 19.63           | 20      | 0                | 300    | 150      | 0.0063  | 0.0024   | 0.0380  | Spiral    |
|                   | S2F5          | 22             | 36.2           | 250000     | 4510        | 1200          | 200000     | 0.55       | 19.63           | 20      | 0                | 300    | 150      | 0.0055  | 0.0024   | 0.0430  | Spiral    |

(Continued)
### Table A.3 Continued

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<th>Specimen number</th>
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<th>f_f (Mpa)</th>
<th>f_{sy} (Mpa)</th>
<th>E_s (Mpa)</th>
<th>t_f (mm)</th>
<th>A_{sp} (mm^2)</th>
<th>s (mm)</th>
<th>A_{s} [^{2}] (mm^2)</th>
<th>h (mm)</th>
<th>d (mm)</th>
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<th>ε_{fu}</th>
<th>ε_{co}</th>
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Note: (1) f_{co} is unconfined concrete strength for cylinder or column. Scale effect has been already considered. (2) Chastre and Silva (2010) did not give the thickness of cover concrete in paper, thus a cover of 20 mm is assumed, in between values of 7.5mm and 25mm of other two databases. (3) A_{s} is total area of longitudinal reinforcement within the concrete section.