Abstract

JI, LEI. Dynamic Comparative Advantage and Trade under Schumpeterian Growth. (Under the direction of John Seater.)

We study the effects of trade on economic growth in a Schumpeterian framework. The model excludes scale effects and technology transfer, the two usual channels in the literature through which trade affects growth, leaving only comparative advantage. Comparative advantage and the trading pattern are determined endogenously. Endogeneity of production and trading patterns leads to results quite different from those found in most of the related literature. Trade need not increase initial output of either country because of an externality absent from static models. Irrespective of what happens to initial output, trade may increase the balanced growth rate but also may decrease it. Our model has tractable transition dynamics, which we describe completely. We show that trade leads to a stable world income distribution in some cases, but in other cases leads to an unstable and perhaps even degenerate distribution. In some cases, trade’s effect on a country’s growth rate is the same as if that country had adopted its trading partner’s R&D technology, even though no technology transfer ever occurs. Trade could have negative effect on welfare. All taxes and tariff have negative effect on welfare, even under the case that the government uses the tax revenue of corporate profits to subsidize the incumbent R&D.
Dynamic Comparative Advantage and Trade under Schumpeterian Growth

by
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A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Economics

Raleigh, North Carolina

2010

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Dedication

To Love, and My Grandfather
Biography

Lei Ji was born to Liucheng Ji and Xiying Ren, in Guangzhou, China. She is the first generation of children born under China's one-child family policy, so she enjoys talking to herself very much. In 1998 she was accepted to Sun Yat-sen University and obtained Bachelor degree of international finance. She achieved many scholarship during the years in college. In 2001 she did internship in American International Assurance Company as an actuary analyst. Then she joined Southern Metropolis Daily as a journalist, writing articles to try to make the voice of minorities heard. She was confused by the effects of globalization on the developing countries so she went to NC State University for Ph.D in economics. She has been doing research about the effect of international trade on long run growth and got BB&T research fellowship in 2009. Ph.D study makes her clear in certain social issues, but more confused in many others. She thinks Socrates is right -- The more you learn, the more you realize how little you know.
Acknowledgments

My deepest thankfulness hereby goes to my advisor Dr. John Seater. Without his guidance, none of the following chapters would be presented as they are. I will never forget his patience and encouragement during the hundreds of afternoons when I was struggling with the model set up. During the six years at NC State, I gradually learned from him not only the knowledge, but his philosophy of research and life. He and his wife Susan Seater have also been the major advisors for me, an international student, on all sorts of American regulations from a proper noun to certain custom of this society, oh, and plants although I never remember the names they told me.

My second deepest thankfulness goes to other professors and graduate students who have offered absolutely necessary and selfless help to my research. Without their assistance there would have been no possibility to finish my study. I’d like to express my appreciation to: Dr. Peretto for his sound suggestions and straightforwardness to the model; Dr. Lapp for his patience towards my tons of questions in not only academic but also teachings; Dr. Inoue for his help and humor in econometrics; Dr. Leblebicioglu and Dr. Pearce for their creative questions for the model; Dr. McElroy for his guidance in connecting economic theories with real life social issues; Dr. Flath, the former director, for his kind helps. My special thank to my office-mates Jieyuan Zhao, Maliha and KS, who provided numerous help and laughter that makes our office the most enjoyable place to be.
I am also quite grateful to have spent my PhD years in this friendly department filled with not only knowledgeable and respectable professors, but also lovely and careful staffs. Ms. Robin Carpenter, Carolyn Smith, and Mr. David Strickland are the sweetest persons I have ever met in this country. In my personal life, I spent most of the weekends with fashion show teammates, hiking partners and friends. Xiaoyu Sun, Qihong Zhu, Guanglin Dai, Liping Li, Jingyi Cao, Lily Pam, Ricky Lam, Shen Dong, Qiong Wang, Chun-Hung Kuo, Chien-yu Huang…There are just too many names I would like to mention here to express my heartfelt appreciation for the time you spent with me ad the joy you brought that will always keep shimmery in my memory.

Last but not the least; I would like to acknowledge my family members for their ultimate support. There is no way to fully express my feelings to my dearest family for their mental supports as always. It was the long term support from my parents, JI Liucheng and REN Xiying, who encouraged me to independently go through all the tortuosities in my life; it was the faith, understanding and patience from my husband LIU Fude that accompanied me through all these years.
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Chapter 1

Introduction

In classic static models of international trade, trade increases the level of income by allowing countries to exploit their comparative advantages. It does not work through aggregate scale effects or technology transfer. The situation is quite the reverse in most of the endogenous growth literature, where, with few exceptions, trade affects growth through aggregate scale effects and technology transfer but not through comparative advantage. The situation is somewhat strange. It is well known that aggregate scale effects are rejected by the data, so they are an inappropriate channel for trade to affect growth. Technology transfer is a legitimate channel and even seems to be important, but it is not trade. It may be facilitated by trade, and indeed the evidence suggests that it is, but it is not trade. There is almost no discussion of the relation between the classic trade mechanism of comparative advantage and economic growth, and what little there is takes place in the context of first-generation growth models where the aggregate scale effect is present and mediates the effects of trade and comparative advantage. In this paper, we study the effects of trade on growth in a second-generation growth model whose structure excludes the aggregate scale
effect and in which we have ruled out technology transfer by assumption. We find that trade affects growth solely through comparative advantage but in ways different from the previous small literature on this topic.

The existing literature links international trade to the growth rate primarily through two channels: the aggregate scale effect and technology transfer. In models with aggregate scale effects (which includes all first generation endogenous growth models), opening countries to trade increases the scale of the economies. The details of the scale increase vary with the specific structure of the model, but one way or another trade makes more productive resources available to every economy, effectively increasing broadly-defined total factor productivity and so raising the growth rate. See, for example, Barro and Sala-I-Martin (1997), Connolly (2000), and all other first-generation growth models with trade. In those models, it is not trade per se that matters, only the size of the economy. Anything that increases that size raises the growth rate. Trade does that, so it also increases the growth rate. It is well known, however, that aggregate scale effects are inconsistent with the data. Backus, Kehoe, and Kehoe (1992) is the classic reference. The aggregate scale effect apparently is an inappropriate channel for trade to affect growth. In models of technology transfer, trade is a vehicle for the exchange of technological know-how. Trade opens countries to each other’s knowledge, helping each country learn and adopt the production techniques of its trading partners and so increasing its stock of knowledge. Because the rate of knowledge accumulation is treated as proportional to the stock of existing knowledge, the technology transfer enabled by trade raises the growth rates of all trading partners. Rivera-Batiz and Romer (1991), Howitt (2000) and Peretto (2003) present this mechanism, and Coe and Helpman (1995, 2008) provide evidence that
it is a significant phenomenon. However, trade is not a necessary element in this channel. We can always have technology transfer without trade, as shown by Rivera-Batiz and Romer (1991). Technology transfer can foster growth, trade may facilitate technology transfer, but trade is not a necessary element.

A small literature uses a first-generation growth framework to examine comparative advantage as a channel through which trade can affect economic growth. The main result is that trade may raise or lower the world growth rate depending on the reallocation of resources that trade induces. Suppose, as in Grossman and Helpman (1990), that each country \( i \) produces a set of goods \( G_i \) comprising a single consumption good \( Y_i \) unique to country \( i \) and an infinite variety of intermediate goods \( X_{1i}, X_{2i}, \ldots \), also unique to country \( i \): \( G_i = \{Y_i, X_{1i}, X_{2i}, \ldots\} \). The intersection of any two sets \( G_i \) and \( G_j \) is empty. At any moment, country \( i \) knows how to pr.

Appendix A

If the fixed operating cost not only depends on the average quality level, but also depends on the quality of the individual firm, then the expression of fixed operating cost could be the same as Peretto (2007), i.e. \( \theta Z_i^\delta Z^{1-\delta} \). By the assumption of zero entry/exit cost, this set up doesn’t allow a positive growth rate. The following is the detail: produce \( n_i \) of the intermediate goods in \( G_i \). Country \( i \) does R&D to expand the number of varieties of intermediate goods that it can produce. Intermediate goods are inputs only for the production of final goods. Labor is an input in all sectors and is the only input in the production of intermediate goods and R&D. Intermediate good \( X_{ji} \) requires \( a_{X_{ji}} \) units of labor per unit of \( X_{ji} \) produced. R&D requires \( a_{I_i} \) units of labor per unit of R&D undertaken. The ratio \( a_{I_i}/a_{X_{ji}} = b_i \) is central to the effects of trade on growth. Trade is introduced by allowing the two countries to trade consumption goods and intermediate goods. R&D output is not tradeable. Varieties of goods
discovered by a country can be produced only in that country. Define comparative advantage in R&D as the relation between the two countries’ values of \( b \). A country 1 has a comparative advantage in R&D if and only if \( b_1 > b_2 \). Trade raises the world growth rate if it induces the country with the comparative advantage in R&D to move resources from the final goods and/or intermediate goods sectors into R&D. That need not be the effect of trade, however. It also is possible for trade to raise the demand of a country’s final goods or intermediate goods and thereby induce it to move resources out of R&D and into manufacturing. If that country happens to be the one with the comparative advantage in R&D, the world growth rate falls. See Grossman and Helpman (1991, 1995), Young (1991), and Galor and Mountford (2006) for extensions and variations. These results are interesting, but they suffer from three limitations. First, the patterns of production and trade do not depend on comparative advantage. Each country is endowed with a set of goods that it may develop and produce. No country may develop and produce goods that have been endowed upon another country. Every country trades every type of good it produces, irrespective of its comparative advantage relative to any other country. The meaning, then, of the term "comparative advantage" is very different from the standard usage. In fact, an exogenous pattern of production and trade is common to almost all the literature on trade and growth. Second, in this

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literature comparative advantage is defined in terms of goods that are assumed to be non-tradeable: the fruit of R&D, which is an increase in the number of varieties that a country has learned to produce. Third, all comparative advantage effects work through the same channel as the scale effect. Consequently, changing the model to eliminate the scale effect also may eliminate or at least alter the effects of trade on growth. That is wAppendix A

If the fixed operating cost not only depends on the average quality level, but also depends on the quality of the individual firm, then the expression of fixed operating cost could be the same as Peretto(2007), i.e. $\theta Z_1^{\delta} Z^{1-\delta}$. By the assumption of zero entry/exit cost, this set up doesn’t allow a positive growth rate. The following is the detail:hat happens with taxation, so there is good reason to believe it may happen with trade. In fact, as we shall see, it does happen in some cases.

Seater(2007), Arabshahi(2008), and Yenokyan(2009) provide a partial resolution of the problems with the original contributions on trade and growth by endogenizing the pattern of trade and making it a function of comparative advantage. They use a standard first-generation two-sector model (Barro and Sala-I-Martín, 2004, chapter 5) extended to two countries. Two goods are produced by two types of reproducible factors in Cobb-Douglas production functions. One sector produces a good $Y$ that can be used as consumption $C$ or as an investment good $K$ and the other sector produces another type of capital good $H$ that augments labor. Both $Y$ and $H$ are tradeable. Comparative advantage is defined in terms of the relative prices of $Y$ and

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1Stokey and Rebelo (1995) criticize first-generation growth models because they predict large negative effects of income taxes that are not observed in the data. Peretto (2007) shows that a second-generation growth model, by eliminating the scale effect, also drastically changes the predictions concerning taxes, with some types of taxes predicted to increase the growth rate, some to have ambiguous effects, and some to reduce the growth rate.
$H$, and comparative advantage determines the pattern of trade, that is, which goods a country exports and which it imports. Trade never reduces growth rates and in most cases raises them. The model can be reduced to one with only one reproducible factor, $H$, by dropping $K$ and making $Y$ be only consumption. It then can be shown that the effect of trade on a country’s growth rate depends on the type of good the country imports. If a country imports the consumption good $C$, its growth rate is the same under trade and autarky. In contrast, if a country imports the factor of production $H$, then its growth rate is higher with trade. When a pair of trading partners exchange two factors of production, such as $K$ and $H$, both have higher growth rates than under autarky. The result is intuitively sensible. Endogenous growth is driven by increasing the amounts of reproducible factors of production. If a country is less efficient at producing a factor of production than another country, then the first country can raise its growth rate by reducing its production of that factor and trading for it on the world market. Note that in this approach $H$ is not treated as human capital, which is not tradeable, but rather as a type of physical capital. The idea is that many kinds of machines have embodied in them characteristics that replace skill. The original textile machines of the Industrial Revolution, for example, apparently were like that:

\[\text{With the marvelously perfect and self-acting machinery of today no spe-}\]

\footnote{See Bond and Trask (1997) and Bond, Trask, and Wang (2003) for important early contributions that use the two-sector model to analyze trade and growth but that treat $H$ as non-tradeable human capital. They consequently obtain the same kind of one-sided effects of trade on growth as in Seater (2007), Arabshahi (2008), and Yenokyan (2009) for the case where only $C$ and $H$ are tradeable. Also, Bond and Trask (1997) and Bond, Trask, and Wang (2003) restrict attention to a small open economy that takes world prices as given, whereas Seater (2007), Arabshahi (2008), and Yenokyan (2009) generalize the analysis to countries that may be large relative to each other and thus may affect world prices.}
cial skill is required on the part of the attendant. *The machinery itself supplies the intelligence.*

Quoted by Clark (2007), emphasis added.

This argument is reasonable, but the mathematical treatment is a something of a shortcut and a weakness of the analysis. It would be desirable to distinguish between the physical machine itself and the qualities embodied in it. One would expect the quantity of capital to enter the production function in the usual way and the amount of embodied quality to enter separately as a factor augmenting labor in exactly the same way as the human capital it replaces. The other weakness of the analysis is that the model has an aggregate scale effect. Even though it is not the scale effect that causes the effects of trade on growth, mathematically trade is working through the same part of the model structure that gives rise to the scale effect. Consequently, reformulating the model to eliminate the scale effect may well change the way that trade affects growth.

We retain the ideas proposed by Seater (2007), Arabshahi (2008), and Yenokyan (2009), but we embed them in a second-generation endogenous growth model, thus eliminating the scale effect. We also distinguish between physical intermediate goods and the quality embodied in them. The goods are tradeable, and buying one of them also buys its labor-augmenting quality. Countries differ in their efficiency at producing goods and at doing R&D. The trade pattern is decided endogenously by the quality-adjusted price ratio of the tradeable goods. Opening a closed economy to trade can have surprising results...Trade always has a direct effect of increasing the output levels of the trading partners at the moment that trade opens. However, there is an
indirect effect arising from an externality that in principle could be strong enough that trade actually reduces the output level of one or both partners. Irrespective of trade’s effect on current output levels, it may increase or reduce the growth rate of output, a result reminiscent of Grossman and Helpman (1990) but arising through a completely different mechanism. The effect of trade on the home country’s growth rate depends on its trading partner’s R&D efficiency in quality improvement of the good that the home country imports. The home country’s R&D efficiency in the good it exports is irrelevant to its growth rate.

In some cases, trade guarantees world balanced growth and hence a stable world income distribution. In other cases, however, growth rates differ across countries at least temporarily and possibly permanently. In those cases, the world income distribution is unstable. It is even possible for the world income distribution to degenerate because one country always has a higher growth rate than the other and the difference between the growth rates is asymptotically constant. In that case, the faster growing country’s share of world income goes to 1 asymptotically, not because the other country is shrinking but rather because the other country grows at a slower rate forever. Many of the results on world income distribution differ markedly from those presented by Acemoglu and Ventura (2002). We show that Acemoglu and Ventura’s analysis amounts to a special restricted case of ours.

We also show that trade may mimic technology transfer in its effect on growth. A country that imports an intermediate good that has high quality embodied in it may appear to be acquiring the technology of the country that manufactured the imported good, but in fact all the recipient country is acquiring is the quality embodied in the good. That distinction has not been made in the empirical literature, which as a
result suffers from an "omitted variables" problem and attributes too much effect to technology transfer.

In chapter 2, we first analyze the closed economy and then in chapter 3, we extend the analysis to an open economy to derive the effects of trade on growth. In chapter 4, we do welfare and tariff analysis based on the model, then in the last chapter, we give out the conclusions and some possible extensions.
Chapter 2

Closed Economy

This chapter presents the model in autarky. Firstly, I want to present a simpler version with only one industry, which was revised from Peretto (1996), (1997) and (2007). Then I will extend it into a two-industry autarky case, combining two industries by a Cobb-Douglas production function. With two industries, we are able to discuss more about trade in the following chapters.

2.1 Preliminary: One-Industry Case\textsuperscript{1}

Before talking about the model with asymmetric industries, I present an one-industry version first. I consider a two-sector economy here. One competitive representative firm produces final goods $Y$ (i.e. services) with a variety of intermediate goods $G_i$ and labor $L$. As long as he uses the intermediate good $G_i$, he directly get the quality $Z_i$ of it. So $Z_i$ augments the bunch of labors who are using $G_i$ to produce the final good. The final goods can be consumed, used to produce intermediate goods,

\textsuperscript{1}Based On Peretto (1996), (1997) and (2007).
invested in R&D that rises the quality of existing intermediate goods, or invested in
the creation of new intermediate firms.

Competitive monopolistic firms produce intermediate goods $G_i$ (i.e. manufactures) with the resources from final goods. These firms undertake in-house research and development (R&D) to improve the qualities $Z_i$ of their products, in order to have more market size demanded by the final good producer.

2.1.1 Final Good Sector

One representive firm produces and sells a homogenous final good, $Y$, in a competitive market. The production function is defined as below:

$$Y = \int_0^N G_i^\lambda [Z_i^\delta (Z)^{1-\delta} l_i]^{1-\lambda} di, \quad 0 < \lambda, \delta < 1$$

(2.1)

where $N$ is the number of varieties of non-durable intermediate goods. These goods are vertically differential according to their quality, and later I will proof the quality levels of different intermediates are symmetric in equilibrium. The productivity of workers $l_i$ using $G_i$ unites of intermediate goods depends on the good’s quality, $Z_i$ and the spillover, which is the average quality of all goods $Z = \frac{1}{N} \int_0^N Z_j dj$. Notice that the quality $Z_i$ is embodied in $G_i$ but augments labors who are using $G_i$ in final good production sector.

According to the property of Cobb-Douglas production function, the final good producer pays compensation $\lambda Y$ and $(1 - \lambda)Y$ to intermediate producers and labor respectively, i.e. $\int_0^N G_i P_i di = \lambda Y$, $wL = (1 - \lambda)Y$. Note that quality $Z_i$ dose not get paid directly from final good sector. It’s the quality inside $G_i$. By increasing quality
we can see the demand for $G_i$ increases from the demand function (2.2).

Set the price of final good as numeraire, $P_Y = 1$. Define $P_i$ as the price of intermediate good $i$; and $W$ as the wage. Maximize the profit of the firm in final good sector, we can get the demand functions of intermediate goods and labors according to the first order conditions.

$$\pi_Y = Y - \int_0^N P_i G_i di - \int_0^N W l_i di$$

$$G_i = \left( \frac{\lambda}{P_i} \right)^{\frac{1}{1-\lambda}} Z_i^\delta Z^{1-\delta} l_i$$  \hspace{1cm} (2.2)

$$l_i = \left( \frac{1 - \lambda}{W} \right)^{\frac{1}{1-\lambda}} G_i (Z_i^\delta Z^{1-\delta})^{\frac{1-\lambda}{1-\lambda}}$$  \hspace{1cm} (2.3)

### 2.1.2 Intermediate Good Sector

Intermediate good producers behave non-cooperatively. The task of this section is to construct an equilibrium with free entry and free exit for the intermediate good sector. All firms face identical production, R&D production and demand function. I proof all firms make the same decisions on prices and the investments in R&D, thus all firms are symmetric. To simplify the analysis, I assume the entry and exit involve zero costs\(^2\). Thus the number of firms is free to jump to its equilibrium level. As

\(^2\)This is the assumption in Peretto(1996) and (1999). In his current papers, he usually assume there’s a sunk cost for entry, which gives a richer transitional dynamic. However, this complicates the calculation a lot, and cannot generate a explicit solution for my two-industry model, hence cannot further the discussion about trade issue. So in this paper, I will focus on the simpler case,
Peretto (2006, 2009), I construct an equilibrium where at time $t$ firms commit to time-path strategies, while simultaneously free entry and exit determine the number of firms in the market. So there’s only one decision point in time, where time-path of market structure and economic growth are simultaneously determined, and the model is indeed a one-shot game, without transitional dynamics. 

I construct this equilibrium in three steps. First, I focus on the determination of the strategies (price and investment in R&D) of the firms that are already active in the market (incumbent). Next, I focus on the free entry and exit decisions and the determination of the number of firms in the market. Finally, I combine two sets of results to describe the equilibrium of the intermediate good sector.

2.1.2.1 Incumbent Firms

The typical intermediate firm produces the differential good with a technology that require one unit of final output per unit of intermediate good, and a fixed operating cost $\phi Z$, where $Z = \frac{1}{N} \int_{0}^{N} Z_i \, di$.  

The firm can invest units of final output to increase quality according to the technology

$$\dot{Z}_i = R_i$$

where the entry and exit costs are zero.

3 In the case with two asymmetric industries, the model is not a one-shot game, thus in that case, there’s transition dynamics.

4 Peretto (2007) sets the fixed operating cost as $\theta Z_i^\lambda Z^{1-\lambda}$. I use the different setting because the other setting generates a negative growth rate, which is conflict with the reality. See the detail in Appendix (A).
where $R_i$ is the firm’s R&D investment in units of final output.

The firm’s gross cash flow, which is revenues minus production costs is

$$F_i = G_i(P_i - 1) - \theta Z$$

(2.5)

So the firm’s profit is

$$\Pi_i = F_i - R_i$$

(2.6)

the firm takes average quality $Z$ as given.

The typical incumbent maximizes the present discounted value of net cash flow,

$$V_i(t) = \int_t^\infty e^{-\int_t^\tau r(s) ds} \Pi_i d\tau$$

$$= \int_t^\infty e^{-\int_t^\tau r(s) ds} [G_i(P_i - 1) - \theta Z - R_i] d\tau$$

(2.7)

where $V_i(t)$ is the present discounted value of net cash flow. With perfect foresight, $V_i$ is the stock market value of the firm, the price of the ownership share of an equity holder.

The firm chooses the time path of its product’s price and R&D expenditure in order to maximize (2.7) subject to the demand function (2.2) and R&D production function (2.4). The firm takes average quality, $Z$ in (2.2) and (2.7) as given. So the Current Value Hamiltonian is

$$CVH_i = G_i(P_i - 1) - \theta Z - R_i + q_i R_i$$

(2.8)
where the costate variable is $q_i$, which measures the value of the marginal unit of quality, and the state variable is $Z_i$. The firm has power to set up its own optimal price $P_i$ and decide how much devoted into research, $R_i$.

The transversality condition is $\lim_{t \to \infty} e^{-\int_s^t r(s) ds} q_i(t) Z_i(t) = 0$.

Taking the first order derivative subject to $P_i$, the optimal price of this firm is:

$$P_i = \frac{1}{\lambda}$$ (2.9)

Thus every intermediate good firms set up the same price $P_i = \frac{1}{\lambda}$ in monopolistic competition. Before go further, I want to show the symmetry property for this model. Plug the demand function of intermediate good $i$, (2.2) into the demand of labor, (2.3), and arrange it we get

$$W_i = (1 - \lambda) \left( \frac{\lambda}{P_i} \right)^{1-\delta} Z_i^\delta Z_i^{1-\delta}$$

All firms set up the same price $P_i = \frac{1}{\lambda}$ in monopolistic competition. And if one firm takes off the whole market and becomes monopoly, it still sets the price as $\frac{1}{\lambda}$. This is because the demand function (2.2) has a constant elasticity. For the constant elasticity demand function, price is a constant markup over marginal cost, with the amount of the markup depending on the elasticity of demand, $\lambda$. Thus the only difference on wage is the quality level, $Z_i$. If a firm has a higher quality, $Z_i > Z_j$, then $W_i > W_j$, it will take over the whole market and becomes a monopoly and earn a positive profit. But the profit is incipient. Since we assume entry and exit costs are

$^5$Constant elasticity demand function is one of the two special cases for monopoly behavior. Detail see Microeconomic Analysis (Third Edition) by Hal R. Varian, Section 14.1.
zero, section (2.1.2.2) shows that firms enter the market instantaneously to eliminate the incipient profits, and firms enter with the average quality levels of incumbents. Now all incumbents have the same quality levels, so the wages equal again, and labors allocate to different intermediate goods equally, which means $W_i = W_j$ and $l_i = l_j$. Thus by the two key assumptions which are zero entry/exit costs and firms enter with the average quality level of incumbents, all firms in the market have the same quality levels, and no firm could keep the monopolistic power.

Since firms are symmetric, I can drop the subscript $i$ in the following analysis.

The Hamiltonian is linear in R&D investment. The optimal investment policy is

$$
R = \begin{cases} 
\infty & \text{if } 1 > q \\
> 0 & \text{if } 1 = q \\
0 & \text{if } 1 < q 
\end{cases}
$$

The former case violate the general equilibrium condition and is ruled out. The first order conditions for the interior solution $R > 0$ are given by the equality between the marginal revenue from R&D ($q$ units of final good) and its marginal cost ($1$ unit of final good). So for interior solution, we have

$$1 = q$$

(2.10)

Differentiating the costate variable $q$ will get
\[ r = \frac{\partial F}{\partial Z_i} q + \frac{\dot{q}}{q} \] (2.11)

I keep a subscript \( i \) in \( Z_i \) because I want to highlight the difference between \( Z_i \) which is individual quality level and \( Z \) which is the average quality level in the industry and taken given by individual firm. (2.11) defines the rate of return to R&D as the ratio between revenue from one R&D project and the shadow price of the project, plus the change rate of the shadow price. Since \( 1 = q_i \), the transversality condition is satisfied in the steady state with constant growth if quality does not grow at a rate higher than the interest rate. Combine equation (2.2), (2.5) and (2.10) into (2.11), and symmetric equilibrium, we get the rate of return for quality innovation.

\[ r = \frac{\partial F}{\partial Z_i} = \delta \frac{\lambda}{1-\lambda} \frac{\lambda^{\frac{1-\alpha}{\lambda}}}{l} (\frac{Z}{Z_i})^{1-\delta} \] (2.12)
\[ = \delta \frac{\lambda}{1-\lambda} \frac{\lambda^{\frac{1-\alpha}{\lambda}}}{l} \] (2.13)

This is a perfect-foresight, no-arbitrage condition: the rate of return to R&D must be equal to the cost of the R&D project financed by borrowing at the rate \( r \) (direct cost of R&D); and this must be equal to the return from a riskless loan at rate \( r \) of the resources required for the R&D project (opportunity cost of R&D).
2.1.2.2 Entry and Exit

The value of the firm $V_i$ defined by equation (2.7) and the optimal price and investment strategies, described by (2.9) and (2.10) give the value of incumbent. To determine the entry and exit of the firm, this value $V_i$ has to be compared with the cost of entry and exit. Assume the cost of entry and exit is zero, firms enter if $V_i > 0$; firms exit if $V_i < 0$. Since the entry and exit cost is zero, the number of firms $N$ is a jumping variable, and, at all the time, free entry and exit makes $V_i = 0$.

Differentiating with respect to time of Eq.(2.7), get the firm’s rate of return to equity is

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V_i}}{V_i}$$

(2.14)

Where $V_i$ is the price of firm $i$’s shares. This is a perfect-forsight, no-arbitrage condition for the equilibrium of the capital market. It requires the return to firm ownership be equal to the rate of return to a riskless loan of size $V_i$. The return to firm ownership is given by the ratio between profit ($\Pi_i$) and the firm’s stock market value ($V_i$), plus the capital gain (loss) from the stock appreciation (depreciation).

Eq. (2.14) can also be written as $rV_i = \Pi_i + \dot{V_i}$. Since zero cost entry/exit makes $V_i = 0$ all the time, it means for all value of interest rate $r$, the LHS is zero; and the RHS should also be zero. This implies zero profit condit, $\Pi_i = 0$. Since $\Pi_i = F_i - R_i$.

---

6This is a strong assumption and requires that workers hired away from the other firms can blend together in a new firm at zero cost. Costly entry has been analyzed by other papers of Peretto. I tried to combined a costly entry in this model too, but it complicates the model a lot without too much more insides. So I focus on zero entry condition here.
the level of R&D expenditure can be written as,

\[ R_i = F_i \]

\[ = \frac{1 - \lambda}{\lambda} \lambda \lambda^{1-\delta} Z_i^\delta Z^{1-\delta} l - \theta Z \]  

(2.15)

(2.16)

So the growth rate of quality innovation should be,

\[ \dot{Z}_i = \frac{R_i}{Z_i} = \frac{1 - \lambda}{\lambda} \lambda \lambda^{1-\delta} Z_i^{\delta-1} Z^{1-\delta} l - \frac{\theta Z}{Z_i} \]

\[ = \frac{1 - \lambda}{\lambda} \lambda^{1-\delta} l - \theta \]  

(2.17)

(2.18)

2.1.3 Households

The economy is populated by representative households who supply labor inelastically in perfect competitive market, and purchase assets (corporate equity). Assume there's no population growth. The utility function of the representative household is

\[ U(t) = \int_t^{\infty} \log(c) e^{-\rho t} \]  

(2.19)

where \( c = \frac{C}{L} \). \( c \) is the consumption per capita, \( C \) is the aggregate consumption for the economy, and \( \rho \) is the individual time preference discount rate.

The only assets that the household can accumulate are firms that it owns. The
household’s lifetime budget constraint therefore is

\[ 0 = \int_0^\infty \left( \int_0^N \Pi_i dt + wL - C \right) e^{-\int_t^{\infty} r(s) ds} dt \]  (2.20)

where \( C \) is aggregate consumption and \( L \) is population. The intertemporal consumption plan that maximizes discounted utility (2.19) is given by the consumption Euler equation, which as usual can be written as

\[ r = \rho + \frac{\dot{C}}{C} \]  (2.21)

### 2.1.4 General Equilibrium

In this section, I am going to construct the general equilibrium of the economy. By imposing symmetry of the intermediate good firms, combine eq. (2.2) into eq. (2.1) to eliminate \( G_i \), we get

\[ Y = \kappa Z L, \quad \kappa \equiv \lambda^{1-\lambda} \]  (2.22)

This is essentially an AK model, which generates a balance growth path. So the growth rate of output per capita equals the growth rate of innovation; so is the growth rate of aggregate output, since we assume no population growth.

From household budget constraint (2.20) combing with zero profit condition, \( \pi_i = 0 \), we get \( WL = C \). We have already known \( WL = (1 - \lambda)Y \) from the cobb-douglas final good production function, so the ratio between consumption and output
is constant, i.e. $\frac{C}{Y} = 1 - \lambda$. So we have the growth rates all equal as

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{Z}}{Z} = \frac{\dot{W}}{W} \quad (2.23)$$

Let’s see the growth rate. The supply of credit (saving) is decided by Euler Equation (2.21), and the demand of credit is decided by the firm’s investment decisions, which are eq.(2.12), and eq. (2.17) derived from zero profit condition. By using these three equations, we solve the balanced growth rate of the economy $g^*$, also the number of firms $N^*$.

$$g^* = \frac{\theta - \rho}{1 - \lambda} = \theta \quad (2.24)$$

$$N^* = \frac{\theta - \rho}{(1 - \lambda)\frac{1-\lambda}{\lambda}^{\frac{2}{1+\lambda}}} \quad (2.25)$$

where requires $\theta > \rho$ and $\frac{\theta - \rho}{1 - \lambda} > \theta$.

There’s no transition dynamics by this setting, since $N$ jumps to steady state level simultaneously because of zero entry cost. Thus the differential equation of $Z$, eq.(2.17) can be written as $\dot{Z} = constant \cdot Z$. So this one-industry model is essentially an $AK$ like setting.

Next we will see the formal model – two-industry model. The setting is similar to one-industry case, and the change in the quality ratio between the two industries generate a transition dynamics. Through this two-industry model, we are able to discuss further issues about trade.
2.2 Two-Industry Model in Autarky

Now I extend the symmetric model into the model with two asymmetric industries.

There are three productive sectors: final goods, processed goods, and intermediate goods. Processed goods are combined to produce final goods. Intermediate goods are combined with labor to produce the processed goods. Final goods are used for consumption and as an input into the production of intermediate goods and for investment in research to improve the quality of the intermediate goods. Models of this type usually have only two sectors as section (2.1), one for final goods and one for intermediate goods. Adding the third sector facilitates the discussion of trade, as we discuss later.

2.2.1 Final Good Sector

Identical competitive firms produce a single homogeneous final good $Y$ using two non-durable processed goods $X_1$ and $X_2$ as inputs. The production function for the representative firm is Cobb-Douglas:

$$ Y = X_1^\epsilon X_2^{1-\epsilon} \quad (2.26) $$

We take the final good as the numeraire, so $P_Y = 1$. The representative firm’s profit is

$$ \pi_Y = Y - P_{X_1}X_1 - P_{X_2}X_2 \quad (2.27) $$
from which we obtain the indirect demand functions

\[ P_{X_1} = \epsilon (X_2/X_1)^{1-\epsilon} \]  \hspace{1cm} (2.28) \\
\[ P_{X_2} = \epsilon (X_1/X_2)^{\epsilon} \] \hspace{1cm} (2.29)

where \( P_{X_1} \) and \( P_{X_2} \) are the prices of \( X_1 \) and \( X_2 \). See the Appendix (B.1) for the complete derivation.

### 2.2.2 Processed Goods Sector

The processed goods sector comprises two industries, each producing a single homogeneous good. Both industries are competitive in all markets. The representative firms in the two industries use non-durable intermediate goods and labor to produce their respective processed goods. Their production functions are:

\[ X_1 = \int_0^{N_1} G_{1j}^\lambda \left( Z_{1j}^\delta Z_{1j}^\gamma Z_2^{1-(\delta+\gamma)}l_{1j} \right)^{1-\lambda} dj, \quad 0 < \lambda, \gamma, \delta < 1 \]  \hspace{1cm} (2.30) \\
\[ X_2 = \int_0^{N_2} G_{2j}^\lambda \left( Z_{2j}^\delta Z_{2j}^\gamma Z_1^{1-(\delta+\gamma)}l_{2j} \right)^{1-\lambda} dj, \quad 0 < \lambda, \gamma, \delta < 1 \] \hspace{1cm} (2.31)

where \( G_{ij} \) are intermediate goods, \( Z_{ij} \) is the quality of good \( G_{ij} \), \( Z_i \equiv (1/N_i) \int_0^{N_i} Z_{ij} dj \) is the average quality of class-\( i \) intermediate goods (explained momentarily), \( l_{ij} \) is the amount of labor (number of workers) working with intermediate good \( G_{ij} \), and \( N_i \) is the number of varieties of intermediate goods used in each industry. There are two classes of intermediate goods, \( \{G_{1j}\}_{j=0}^{N_1} \) and \( \{G_{2j}\}_{j=0}^{N_2} \), with one class providing inputs for the \( X_1 \) industry and the other class providing inputs for the \( X_2 \) industry.
Each intermediate good is in one and only one class, so the sets of intermediate goods used by the $X_1$ and $X_2$ industries are disjoint and generally have different numbers of elements (i.e., in general $N_1 \neq N_2$). Each intermediate good $G_{ij}$ has its own quality $Z_{ij}$, determined by the R&D that has been done by the firm that produces $G_{ij}$.

We discuss the industrial structure and R&D of the intermediate goods sector in the next section. Labor productivity depends on the quality of the intermediate good it works with. To allow for knowledge spillovers, we let labor productivity in industry $X_1$ also depend on both the average quality $Z_1 = (1/N_1) \int_{0}^{N_1} Z_{1j} dj$ of the $\{G_{1j}\}$ goods used in industry $X_1$, and $Z_2 = (1/N_2) \int_{0}^{N_2} Z_{2j} dj$ of the $\{G_{2j}\}$ goods used in industry $X_2$. Industry $X_2$'s situation is symmetric. The importance of knowledge spillovers is governed by the magnitude of the parameter $\delta$ and $\gamma$. Setting $\delta + \gamma = 1$ would exclude knowledge spillovers across industries. As discussed in the Introduction, we have assumed here that the quality $Z_{ij}$ of intermediate good $G_{ij}$ is embodied in the good itself but augments the workers that use that good.

An example of this production system is the Shanghai Metro. Metro services are the final good $Y$, produced by combining a control system $X_1$ and an operation system $X_2$. The control system ($X_1$) comprises many different types of computers ($G_{1j}$) that are used by different workers $l_{1j}$. The workers’ productivity depends on qualities $Z_{1j}$ of the computers, a spillover from other qualities in control system, and also possibly a spillover from the operation system $Z_2$. The structure of the operation system $X_2$ is similar.

Processed goods firms choose quantities of intermediate goods and labor to max-
imize their profit:

$$\max_{\{G_{ij}, l_{ij}\}} \pi_{X_i} = P_{X_i}X_i - \int_0^{N_i} P_{G_{ij}}G_{ij}dj - \int_0^{N_i} w l_{ij}dj \quad (2.32)$$

where $P_{G_{ij}}$ is the price of $G_{ij}$, $w$ is the wage rate, and the firm takes all prices as given. The demand functions for intermediate goods and labor are\(^7\)

$$G_{1j} = \left(\frac{\lambda P_{X_1}}{P_{G_{1j}}}\right)^{\frac{1}{\lambda}} Z_{1j}^\delta Z_1^\gamma Z_2^1(l_{1j}^{\lambda+\gamma}) \quad (2.33)$$

$$G_{2j} = \left(\frac{\lambda P_{X_2}}{P_{G_{2j}}}\right)^{\frac{1}{\lambda}} Z_{2j}^\delta Z_2^\gamma Z_1^1(l_{2j}^{\lambda+\gamma}) \quad (2.34)$$

$$l_{1j} = \left(\frac{P_{X_1}}{w} - \frac{\lambda}{\lambda+\gamma}\right) G_{1j}(Z_{1j}^\delta Z_1^\gamma Z_2^1)^{\frac{1}{\lambda}} \quad (2.35)$$

$$l_{2j} = \left(\frac{P_{X_2}}{w} - \frac{\lambda}{\lambda+\gamma}\right) G_{2j}(Z_{2j}^\delta Z_2^\gamma Z_1^1)^{\frac{1}{\lambda}} \quad (2.36)$$

### 2.2.3 Intermediate Goods Sector

The intermediate goods sector, like the processed goods sector, comprises two industries distinguished by which processed goods industry buys their products, as explained above. We divide the discussion of the intermediate goods industries into two parts, the first describing the behavior of incumbents and the second the behavior of entrants.

\(^7\)See derivations in Appendix (B.2).
2.2.3.1 Incumbents

Each intermediate goods industry comprises a continuum of monopolistically competitive firms, each of which produces a single intermediate good \( G_{ij} \) and also undertakes research and development (R&D) to improve the quality \( Z_{ij} \) of the good it produces. An increase in quality raises the demand for the good, as shown above, and thus raises profit.

Production, technologies, R&D technologies, and costs are the same for all firms within a given industry but differ across industries. The industrial structure thus is one of symmetry within each intermediate goods industry but asymmetry across the two industries. The element of asymmetry is unusual in Schumpeterian growth models. It seems realistic and is important in discussing international trade. All firms in industry \( i \) have a linear technology that converts \( A_i \) units of the final good into one unit of intermediate good \( G_{ij} \):

\[
A_i G_{ij} = Y_{ij} \tag{2.37}
\]

where \( Y_{ij} \) is the amount of the final good used by firm \( j \) in industry \( i \). Similarly, the R&D production functions are the same within an industry but differ across them:

\[
\dot{Z}_{ij} = \alpha_i R_{ij} \tag{2.38}
\]

where \( R_{ij} \) is amount of the final good \( Y \) spent on R&D. The firm obtains the resources for \( R \) from retained earnings.\(^8\) Firms face a fixed operating cost \( \phi_{ij} \) that depends on

\(^8\)It would be slightly more precise to distinguish between investment \( I \) and retained earnings \( R \)
the average quality of the firm’s own industry, $Z_i$ and of other industry $Z_k$. There are two channels of influence. First, the operating cost depends positively on own industry quality. A more sophisticated industry is more complex and requires more sophisticated inputs, so the demand for operating cost inputs is increasing in industry quality. On the reasonable assumption, commonly made in the literature, that the cost of producing those inputs rises with their sophistication, higher industry quality then implies a higher price for the factors that are used to run the firms’ operations. We borrow a page from the adjustment cost literature and assume that fixed operating costs are convex in the level of industry sophistication. Second, operating costs are reduced by advances in knowledge, which in our model is captured by quality. We suppose that all knowledge is useful in reducing operating costs, that is, that knowledge spillovers from both an intermediate goods firm’s own industry and also from the other industry lower operating costs. Thus both own industry knowledge $Z_i$ and other industry knowledge $Z_j$ help reduce costs. The general form of the operating cost function is thus $\phi_{ij} = \Phi_{ij} (Z_i; Z_i, Z_k)$ with $\Phi'_1 > 0$, $\Phi'_{11} > 0$, $\Phi'_2 < 0$, and $\Phi'_3 < 0$. To keep the analysis tractable, we assume that all firms in a given industry have the same cost function, which takes the analytically convenient form

$$\Phi_{ij} (Z_i; Z_i, Z_k) = \theta_i \frac{Z_i^3}{Z_i Z_k} = \theta_i \frac{Z_i^2}{Z_k}$$

The cubic term in the numerator captures the convexity of cost in complexity, and the two terms in the denominator capture the effect of knowledge in reducing costs.

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because in principle the two need not be the same. However, the requirements of general equilibrium will make them the same, so we keep the notation simple by imposing $I = R$. 

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Thus we have
\[
\phi_{1j} = \theta_1 \frac{Z_1^2}{Z_2} \equiv \phi_1; \quad \phi_{2j} = \theta_2 \frac{Z_2^2}{Z_1} \equiv \phi_2
\] (2.39)
Dependence of cost on only industry averages and not the firm’s own quality level is not restrictive because as we show later firms within a given industry behave symmetrically, so that each firm’s quality equals the average quality of the industry.

The intermediate goods firm’s profit is revenue less production costs:
\[
F_{ij} = P_{G_{ij}} G_{ij} - A_i G_{ij} - \phi_i
\] (2.40)
The firm retains some amount \( R_{ij} \) of its profit for investment purposes and distributes the rest to its owners. Profit net of retained earnings is
\[
\Pi_{ij} = F_{ij} - R_{ij}
\] (2.41)
The present discounted value \( V_{ij}(t) \) of this net profit is
\[
V_{ij}(t) = \int_t^\infty \Pi_{ij} e^{-\int_t^\tau r(s) ds} d\tau
\]
\[
= \int_t^\infty [G_{ij} (P_{G_{ij}} - A_i) - \phi_i - R_{ij}] e^{-\int_t^\tau r(s) ds} d\tau
\] (2.42)
The firm chooses the paths of its product price \( P_{G_{ij}} \) and its R&D expenditure \( R_{ij} \) to maximize (2.42) subject to the demand function (2.33), the R&D production function (2.38), and the average qualities, \( Z_1 \) and \( Z_2 \), which the firm takes as given.

Differentiating Eq.(2.42) with respect to time gives the firm’s rate of return to
equity (i.e., entry):

\[ r_{ij}^E = \frac{\Pi_{ij}}{V_{ij}} + \frac{\dot{V}_{ij}}{V_{ij}} \]  

(2.43)

which is the usual profit rate plus the capital gain rate.

2.2.3.2 Entrants

We assume that entry and exit are costless. For simplicity, we refer only to entry, even though exit also always is possible. We explored an extension of the model with costly entry, but we were unable to obtain closed-form solutions.\(^9\)

Costless entry imply that \( N_i \) is a jumping variable. Whenever the net present value of a new firm \( V \) differs from the entry cost of zero, new firms jump in or out to restore equality between the value of the firm and the entry cost. We thus have at all times

\[ V_{ij} = 0 \]  

(2.44)

As a result, we also have \( \dot{V} = 0 \). Multiplying both sides of eq. (2.43) by \( V \) and imposing \( V = 0 \) and \( \dot{V} = 0 \) implies that

\[ \Pi_{ij} = 0 \]  

(2.45)

as in Peretto (Oct 1999). We make the usual assumption that new entrants arrive with the average level of quality in their industry. This assumption is not restrictive because, as we show below, the firms within each industry behave symmetrically and

\(^9\)See Peretto (2007) for discussions of costly entry in a framework similar to ours.
always have the same level of quality.

2.2.4 Households

The economy is populated by a representative household that supplies labor inelastically in a perfect competitive market, and purchases assets (corporate equity). We assume for simplicity that there is no population growth. The utility function of the representative household is

$$U(t) = \int_t^\infty \log(c) e^{-\rho t}$$

where $c$ is consumption per capita and $\rho$ is the rate of time preference.

The only assets that the household can accumulate are firms that it owns. The household’s lifetime budget constraint therefore is

$$0 = \int_0^\infty \left( \int_0^{N_1} \Pi_{1j} \, dj + \int_0^{N_2} \Pi_{2j} \, dj + wL - C \right) e^{-\int_t^s r(s) \, ds} \, dt$$

where $C$ is aggregate consumption and $L$ is population. The intertemporal consumption plan that maximizes discounted utility (2.46) is given by the consumption Euler

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10Positive population growth would induce perpetual entry at the rate of population growth. Extra population would give rise to incipient profit from entry and so induce the entry necessary to keep the rate of return to entry at zero. No important results obtained below would change. In particular, there would be no scale effect because our model’s second-generation structure (simultaneous quality-improvement and variety expansion) nullifies it.
equation, which as usual can be written as\textsuperscript{11}

\[ r = \rho + \frac{\dot{C}}{C} \]  

(2.48)

2.2.5 General Equilibrium

We start with the intermediate goods firm. Condition (2.45), that instantaneous profit is zero, implies that firms pay no dividends but instead retain all earnings for investment in R&D. The household owners of the firm reap their return in the form of increasing consumption as R&D delivers higher quality and raises output. The optimal values for the prices \( P_{G_{ij}} \) and retained earnings \( R_{ij} \) are slightly different for the two intermediated goods industries. The derivations are straightforward manipulations of the first-order conditions for the firm’s maximization problem and so are relegated to the Appendix (B.3). The solutions for the prices are mark-ups over variable cost:

\[
P_{G_{1j}} = \frac{A_1}{\lambda} \equiv P_{G1} \]  

(2.49)

\[
P_{G_{2j}} = \frac{A_2}{\lambda} \equiv P_{G2} \]  

(2.50)

The intermediate goods producers in a given industry all charge the same price, so we can drop the firm subscript \( j \) and write the price for all intermediate goods in industry \( i \) as \( P_{Gi} \). Equality of prices implies from the demand function (2.33) that the quantities sold are a linear function (which is the same for all firms in industry

\textsuperscript{11}See Appendix (B.4.3) for derivation.
i) of the amount of labor \( l_{ij} \) that the processed goods firms allocate to intermediate good \( G_{ij} \). The firm’s current-value Hamiltonian is

\[
CVH_{ij} = G_i(P_{Gi} - A_i) - \phi_i - R_{ij} + q_{ij}(\alpha_i R_{ij})
\]

where \( q_{ij} \) is the co-state variable. The Hamiltonian is linear in R&D expenditure, so the solution for investment expenditure \( R_{ij} \) is bang-bang:

\[
R_{ij} = \begin{cases} 
\infty & \text{if } 1/\alpha > q_{ij} \\
> 0 & \text{if } 1/\alpha = q_{ij} \\
= 0 & \text{if } 1/\alpha < q_{ij}
\end{cases}
\]

We rule out the first possibility of \( R_{ij} = \infty \) because it is inconsistent with market equilibrium. We also rule out the other corner solution, \( 1/\alpha < q_{ij} \), because it implies no economic growth, and we are interested here in the case where perpetual growth occurs. We thus have the interior solution

\[
\frac{1}{\alpha_i} = q_{ij} \tag{2.51}
\]

The left side of eq. (2.51) is the same for all \( j \), so all firms in industry \( i \) choose the same level of R&D, which we denote \( R_i \). That in turn implies from (2.40) that all firms in the industry have the same profit.

The Maximum Principle gives the necessary condition for the evolution of the
co-state variable \( q_1 \), which we can rearrange as

\[
\frac{r_{ij}}{q_{ij}} = \frac{\partial F_{ij}}{\partial Z_{ij}} + \frac{\dot{q}_{ij}}{q_{ij}} \tag{2.52}
\]

This equation defines the rate of return to R&D (i.e., to quality innovation) \( r_{ij} \) as the percentage marginal revenue from R&D plus the capital gain (percentage change in the shadow price). Because \( 1/\alpha_i = q_{ij} \), we also have \( \dot{q}_{ij}/q_{ij} = 0 \). As with intermediate goods prices, the expressions for the rates of return differ across the two industries. The rate of return for industry 1 is obtained by substituting (2.33), (2.40), (2.49), and (2.51) into (2.52):

\[
r_{1j} = \delta \alpha_1 A_1 \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{X_1}}{A_1/\lambda} \right)^{\frac{1}{1-\lambda}} \left( \frac{Z_{1j}}{Z_2} \right)^{(\delta+\gamma)-1} l_{1j} \tag{2.53}
\]

Following the same steps as in industry 1, we get the rate of return to R&D in industry 2:

\[
r_{2j} = \delta \alpha_2 A_2 \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{X_2}}{A_2/\lambda} \right)^{\frac{1}{1-\lambda}} \left( \frac{Z_{2j}}{Z_1} \right)^{(\delta+\gamma)-1} l_{2j} \tag{2.54}
\]

At this point, we follow Peretto (Oct. 1999) and impose a simplification that avoids technical complications that have no effect on the analysis or results. We assume that entrants enter with the average quality level of the industry and entry cost is zero. These two assumptions lead directly to an equilibrium that is symmetric within each industry\(^{12}\), with all firms within an industry always making the same decisions regarding pricing, R&D expenditures, and facing the same market size. We

\(^{12}\)Symmetry in the model with one industry see section (2.1.2.1). More detail about this specific two-industry model see Appendix (B.3)
thus have symmetry among all firms in industry $i$: all firms choose the same levels of the two things they control, output price and R&D spending, they sell the same quantity of goods, they have the same levels of quality, and they all have the same amount of labor allocated to their goods in the processed goods sectors’ production functions. We henceforth drop the firm subscript except where clarity demands otherwise. Our model is not entirely symmetric. Firms in industry 1 are all alike, firms in industry 2 are all alike, but firms in industry 1 differ from firms in industry 2. This element of asymmetry increases the realism of the model in our opinion and also plays an important role in determining the effect of international trade on economic growth. Thus we also have the simplified rate of return expressions

$$r_{1j} = r_1 = \delta \alpha A_1 \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{X_1}}{A_1/\lambda} \right)^{\frac{1}{1-\lambda}} \left( \frac{Z_1}{Z_2} \right)^{(\delta+\gamma)-1} l_1 \quad (2.55)$$

$$r_{2j} = r_2 = \delta \alpha A_2 \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{X_2}}{A_2/\lambda} \right)^{\frac{1}{1-\lambda}} \left( \frac{Z_2}{Z_1} \right)^{(\delta+\gamma)-1} l_2 \quad (2.56)$$

To determine the value of $l_i$, we use the facts that the final goods and processed goods sectors both are competitive in all markets and have Cobb-Douglas production. The final goods sector pays a total compensation of $\epsilon Y$ to the $X_1$ sector and $(1 - \epsilon) Y$ to the $X_2$ sector. The $X_1$ and $X_2$ sectors in turn pay their workers $(1 - \lambda)$ of their product, that is, $(1 - \lambda) \epsilon Y$ and $(1 - \lambda) (1 - \epsilon) Y$, respectively. However, we also can write total labor compensation in the industries as

$$\int_0^{N_i} w l_{ij} dj = w L_i$$
where
\[ L_i = \int_0^{N_i} l_{ij} dj \]

Then take the ratio of total compensation to labor\textsuperscript{13}
\[
\frac{wL_1}{wL_2} = \frac{L_1}{L_2} = \frac{(1 - \lambda) \epsilon Y}{(1 - \lambda) (1 - \epsilon) Y} = \frac{\epsilon}{1 - \epsilon}
\]

(2.57)

Then substituting \( L_2 = L - L_1 \) and solving for \( L_1 \) gives
\[
L_1 = \epsilon L \\
L_2 = (1 - \epsilon) L
\]

Finally, symmetry of employment implies that
\[
l_1 = \frac{L_1}{N_1} = \epsilon L/N_1 \tag{2.58}
\]
\[
l_2 = \frac{L_2}{N_2} = (1 - \epsilon) L/N_2 \tag{2.59}
\]

As noted above, entry drives instantaneous profit to zero and then keeps it there by forcing intermediate goods firms to retain all earnings and sink them into R&D. As a result, entry lasts only an instant and does not continue through time. Variety expansion thus is not a source of persistent growth in this model. In that regard, the model is similar to Peretto (Oct 1999).

\textsuperscript{13}See derivation in Appendix (B.4.1).
The growth rates of quality innovation are

\[ \frac{\dot{Z}_1}{Z_1} = \frac{\alpha_1 R_1}{Z_1} = \frac{\alpha_1 F_1}{Z_1} \]  \hfill (2.60)

\[ \frac{\dot{Z}_2}{Z_2} = \frac{\alpha_2 R_2}{Z_2} = \frac{\alpha_2 F_2}{Z_2} \]  \hfill (2.61)

These growth rates depend positively on \( L_i/N_i \), which is individual firm size, not with \( L_i \) itself, which is related to population size. That distinction is one of the main differences between second-generation growth models like ours and first-generation models. In our model, an increase in \( L_i \) raises demand by the processed goods sector for intermediate goods and thereby raises profit of the existing intermediate goods firms. The increase in profit induces entry of new firms and raises \( N_i \) to keep \( L_i/N_i \) constant. An increase in population will not cause an increase in growth rate. The scale effect of first-generation models is absent here. That is why we could make population growth positive without affecting anything important.

With the solutions for the intermediate goods sector in hand, we can solve the rest of the model. As noted earlier, the prices \( P_{Gi} \) determine the quantities \( G_i \) from the demand equations (2.33) and (2.34). The values of the \( Z_i \) are the solutions to the differential equations (2.60) and (2.61) subject to the initial values of the \( Z_i \). We describe those solutions below. We also have derived the labor allocations

\(^{14}\) See derivations in Appendix (B.3)
\( l_{ij} = L_i/N_i \). We use those solutions to solve the processed goods sector’s production functions (2.30) and (2.31) for \( X_1 \) and \( X_2 \). We then substitute the solutions for \( X_1 \) and \( X_2 \) into the final goods sector’s production function (2.26) to get \( Y \) and into the indirect demand functions for processed goods (2.28) and (2.29) to get the prices \( P_{X_1} \) and \( P_{X_2} \). Using the solutions for \( P_{X_1}, P_{X_2}, X_1, \) and \( X_2 \), we can write the rates of return in (2.55), (2.56), (2.60) and (2.61) entirely as functions of parameters, the state variables \( Z_1 \) and \( Z_2 \), and the number of firms in each intermediate goods industry \( N_1 \) and \( N_2 \).\(^{15}\)

\[
r_1 = \delta \alpha_1 A_1 \frac{1 - \lambda}{\lambda} \left[ \frac{\lambda^2 \epsilon}{A_1} \right]^{\frac{1}{\gamma_x}} \left( \frac{\epsilon}{1 - \epsilon} \right) \left[ \frac{A_1}{A_2} \right]^{\frac{\lambda (\epsilon - 1)}{\gamma_x}} \left( \frac{Z_1}{Z_2} \right)^{\Gamma - 1} \frac{\epsilon L}{N_1} \tag{2.62}
\]

\[
r_2 = \delta \alpha_2 A_2 \frac{1 - \lambda}{\lambda} \left[ \frac{\lambda^2 (1 - \epsilon)}{A_2} \right]^{\frac{1}{\gamma_x}} \left( \frac{\epsilon}{1 - \epsilon} \right) \left[ \frac{A_1}{A_2} \right]^{\frac{\lambda \epsilon}{\gamma_x}} \left( \frac{Z_1}{Z_2} \right)^{\Gamma} \frac{(1 - \epsilon) L}{N_2} \tag{2.63}
\]

where \( \Gamma = 1 + 2(\delta + \gamma)\epsilon - (\delta + \gamma) - \epsilon \in (0, 1) \)

We have two remaining unknowns: \( N_1 \) and \( N_2 \). We get one equation for determining them by imposing the no arbitrage condition that rates of return must be equal, which allows us to set the two expressions on the right sides of (2.62) and (2.63) equal to each other. The remaining equation is the Euler equation, but to use it we first must look at the amounts of income earned in each sector.

Competitive firms and Cobb-Douglas production together imply that processed goods industries 1 and 2 receive payments of \( \epsilon Y \) and \( (1 - \epsilon) Y \), respectively, so

\(^{15}\)See derivations in Appendix (B.3).
$P_{X_1}X_1 = eY$ and $P_{X_2}X_2 = (1 - \epsilon)Y$, where $P_{Y_i}$ is the price of good $Y_i$ in terms of the final good. The processed goods industries in turn are competitive with Cobb-Douglas production. The intermediate goods firms in class 1 (i.e., those in the set $\{G_{1j}\}$) together receive a total payment of $\lambda P_{X_1}X_1 = \lambda \epsilon Y$, and the workers in processed goods industry 1 receive total compensation of $wL_1 = (1 - \lambda) P_{X_1}X_1 = (1 - \lambda) \epsilon Y$. Similarly, the payments to intermediates and workers employed in processed goods industry 2 are $\lambda P_{X_2}X_2 = \lambda (1 - \epsilon)Y$ and $wL_2 = (1 - \lambda) P_{X_2}X_2 = (1 - \lambda)(1 - \epsilon)Y$. Total compensation paid to intermediate good producers and labor is $N_1G_1P_{G_1} + N_2G_2P_{G_2} = \lambda Y$ and $w(L_1 + L_2) = (1 - \lambda)Y$. Quality $Z_{ij}$ does not get paid directly from the final good sector. The return to $Z_i$ is generated indirectly by increasing the demand for the intermediate $G_{ij}$ in which it is embodied, as shown in equations (2.33) and (2.34).

The household budget constraint (2.47) together with the zero profit condition (2.45) $\Pi_i = 0$ implies that $wL = C$. Thus households consume all their wage income and save (through retained earnings) all their dividend income. We have just shown that $wL = (1 - \lambda)Y$, so the ratio between consumption and output is constant as in a Solow model, i.e. $C/Y = 1 - \lambda$. That means that consumption $C$ always grows at the same rate as income $Y$. Combining the demand functions of intermediate goods (2.33) and (2.34) into the final good production (2.26) to eliminate $G_i$, we get

$$Y = \kappa Z_i^\Gamma Z_2^{1-\Gamma} L$$

where $\kappa = \lambda^{\frac{\lambda}{\lambda - \epsilon}} (1 - \epsilon) \frac{\lambda(1 - \epsilon)}{1 - \lambda} \frac{\lambda}{1 - \lambda} \frac{P_{G_1}^{\frac{\lambda}{1 - \lambda}} P_{G_2}^{\frac{\lambda}{1 - \lambda}}}{\lambda^{\frac{\lambda}{\lambda - \epsilon}}} \epsilon(1 - \epsilon)^{(1 - \epsilon)}$, and $\Gamma = 2(\delta + \gamma)\epsilon - (\delta + \gamma)\epsilon$. See derivation in Appendix (B.4.2).
\( \gamma - \epsilon + 1 \in (0, 1) \). We thus see that the growth rate of \( Y \) is a weighted average of the growth rates of the \( Z_i \):

\[
\frac{\dot{Y}}{Y} = \Gamma g_1 + (1 - \Gamma) g_2
\]

(2.65)

The growth rates \( g_1 \) and \( g_2 \) are given by equations (2.60) and (2.61) and are functions of parameters, the current values of the state variables \( Z_1 \) and \( Z_2 \), and of \( N_1 \) and \( N_2 \). Thus the Euler equation also provides an equation in the two unknowns \( N_1 \) and \( N_2 \), giving us the second equation that we need.

### 2.2.6 Balanced Growth Path

On the balanced growth path, the growth rates of \( Z_1 \) and \( Z_2 \) are equal, which implies that the ratio \( Z_1/Z_2 \) is constant. Then the following growth rates are equal:

\[
g^* = \frac{\dot{Z}_1}{Z_1} = \frac{\dot{Z}_2}{Z_2} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{X}_1}{X_1} = \frac{\dot{X}_2}{X_2} = \frac{\dot{G}_1}{G_1} = \frac{\dot{G}_2}{G_2} = \frac{\dot{w}}{w}
\]

(2.66)

We get the quality ratio on the balanced growth path, \((\frac{Z_1}{Z_2})^*\) by noting that \( g_1 = g_2 = g^* \), \( r_1 = r_2 \equiv r \), and from the Euler equation \( r = g^* + \rho \). From those relations, we obtain (see the Appendix (B.5) for derivations) the following quadratic:

\[
\alpha_1 \theta_1 (\frac{Z_1}{Z_2})^2 - \alpha_2 \theta_2 = 0
\]

(2.67)
The solutions for the quadratic function are:

\[
\left( \frac{Z_1}{Z_2} \right)^* = \sqrt{\frac{\alpha_2\theta_2}{\alpha_1\theta_1}} > 0 \\
\left( \frac{Z_1}{Z_2} \right)^* = -\sqrt{\frac{\alpha_2\theta_2}{\alpha_1\theta_1}} < 0
\]

We discard the negative solution because it is economically meaningless to get the balanced growth rate\(^{17}\):

\[
g^* = \frac{\delta}{1-\delta} \sqrt{\alpha_1\theta_1\alpha_2\theta_2} - \frac{1}{1-\delta}\rho \quad (2.68)
\]

The growth rate depends positively on the R&D productivities \(\alpha_1\) and \(\alpha_2\) and on the fixed operating cost parameters \(\theta_1\) and \(\theta_2\). The higher the productivity of R&D, the higher the return to R&D, which implies a higher growth rate. The higher the fixed operating cost, the lower the profit for incumbents and thus the smaller the number of firms in the market, which drives up the market size of firms \(L_i/N_i\). From eqs. (2.53) and (2.54) we see that the larger the market size, the higher return in R&D. Consequently, the growth rate is positively related with fixed operating cost. This result is the same result obtained by Peretto (Oct 1999) for the same reason.

Define \(\alpha_1\theta_1\) and \(\alpha_2\theta_2\) as “R&D ability” for industries 1 and 2, respectively. The economy’s growth rate is positively related to the R&D abilities of the two industries.

The growth rate is unrelated to unit costs of production, \(A_1\) and \(A_2\). A change in

\(^{17}\)See Appendix (B.5) for derivations
unit costs has two opposite effects that exactly cancel. One effect is a positive “direct
effect”. Eq ((2.62)) shows that a decrease in unit costs directly causes an increase in
the return of R&D and hence also in the growth rate. The other effect is a negative
“indirect effect”. A decrease in unit cost causes a higher incipient profit and induces
entry. A higher number of firms in the market causes a decrease in firm size \( L_i/N_i \)
and hence reduces the return to R&D according to (2.62). These two effects cancel,
so the growth rate is not affected by unit costs. The fact that unit costs do not
affect growth rates is important later in understanding why trade does not guarantee
a higher growth rate. The trade pattern is determined by the quality-adjusted price
ratio, which depends on unit costs, but the growth rate depends only on R&D ability,
which does not depend on unit costs. A country may end up importing a good with
a very low unit cost but also with a low associated R&D ability, thus leading to a
decrease in the country’s growth rate.

2.2.7 Transition Dynamics

Our model permits a full characterization of the economy’s transition dynamics.
The no-arbitrage condition requires the jumping variables \( N_1 \) and \( N_2 \) to adjust in-
stantly to equalize the returns to R&D across the two processed goods industries.
Thus the right sides of (2.55) and (2.56) are always equal. Combining the growth
rates of qualities (2.60) and (2.61) gives

\[
\frac{\dot{Z}_1}{Z_1} - \frac{\dot{Z}_2}{Z_2} = g_1 - g_2 = \frac{r_1}{\delta} - \alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right) - \left[ \frac{r_2}{\delta} - \alpha_2 \theta_2 \left( \frac{Z_2}{Z_1} \right) \right]
\]  

(2.69)
Impose equality of rates of return and multiplying through by \( Z_1/Z_2 \) gives

\[
(Z_1' / Z_2) = -\alpha_1 \theta_1 (Z_1 / Z_2)^2 + \alpha_2 \theta_2
\]

The steady state is the value of \( Z_1/Z_2 \) that makes \( (Z_1' / Z_2) = 0 \) \( \iff (Z_1' / Z_2) / (Z_1 / Z_2) = 0 \) \( \iff \hat{Z}_1/Z_1 = \hat{Z}_2/Z_2 \). Setting \( (Z_1' / Z_2) = 0 \) in (2.70) and rearranging gives the quadratic form

\[
(Z_1/Z_2)^2 = \frac{\alpha_2 \theta_2}{\alpha_1 \theta_1}
\]

One root is \( \sqrt{\alpha_2 \theta_2/\alpha_1 \theta_1} > 0 \), which is stable. When \( Z_1/Z_2 > \sqrt{\alpha_2 \theta_2/\alpha_1 \theta_1} \), eq. (2.70) implies \( (Z_1' / Z_2) < 0 \), so \( Z_1 \) decreases relative to \( Z_2 \), and \( Z_1/Z_2 \) returns to \( \sqrt{\alpha_2 \theta_2/\alpha_1 \theta_1} \). When \( 0 < Z_1/Z_2 < \alpha_2 \theta_2/\alpha_1 \theta_1 \), eq. (2.70) implies \( (Z_1' / Z_2) > 0 \) so that \( Z_1 \) rises relative to \( Z_2 \) and again \( Z_1/Z_2 \) returns to \( \sqrt{\alpha_2 \theta_2/\alpha_1 \theta_1} \). See Figure 1. The other root is \( -\sqrt{\alpha_2 \theta_2/\alpha_1 \theta_1} < 0 \). Being negative, it is of no direct economic interest because \( Z_1/Z_2 \) cannot be negative. However, the fact that the negative root is unstable does imply that at points above it (i.e., less negative) the economy moves even farther above, that is, toward the positive quadrant. Again, see Figure 1. Thus the positive quadrant is globally stable, and there is a unique globally stable value for the quality ratio \( Z_1/Z_2 \). At that equilibrium ratio, \( Z_1 \) and \( Z_2 \) grow at the same rate, so \( \hat{Z}_1/Z_1 = \hat{Z}_2/Z_2 = g^* \) in equilibrium. From (2.65), \( \hat{Y}/Y = g^* \). One can work through the rest of the model’s equations to establish that (2.68) holds, so the stable equilibrium point where \( \hat{Z}_1/Z_1 = \hat{Z}_2/Z_2 \) is indeed the balanced growth path (steady state) of the economy is stable. Figure 1 shows the dynamics of \( Z_1/Z_2 \).

During the transition dynamics, the change of all other variables depend on the
change of $Z_1$ and $Z_2$ only. See Appendix (B.4.2) for derivations.

\[
\frac{\dot{G}_1}{G_1} = \Gamma \frac{\dot{Z}_1}{Z_1} + (1 - \Gamma) \frac{\dot{Z}_2}{Z_2} \tag{2.71}
\]

\[
\frac{\dot{G}_2}{G_2} = \Gamma \frac{\dot{Z}_1}{Z_1} + (1 - \Gamma) \frac{\dot{Z}_2}{Z_2} \tag{2.72}
\]

\[
\frac{\dot{X}_1}{X_1} = \vartheta_1 \frac{\dot{Z}_1}{Z_1} + (1 - \vartheta_1) \frac{\dot{Z}_2}{Z_2} \tag{2.73}
\]

\[
\frac{\dot{X}_2}{X_2} = \vartheta_2 \frac{\dot{Z}_1}{Z_1} + (1 - \vartheta_2) \frac{\dot{Z}_2}{Z_2} \tag{2.74}
\]
\[ \frac{\dot{C}}{C} = \frac{\dot{w}}{w} = \frac{\dot{Y}}{Y} = \Gamma \frac{Z_1}{Z_1} + (1 - \Gamma) \frac{Z_2}{Z_2} \quad (2.75) \]

where \( \Gamma = \epsilon(\delta + \gamma) + (1 - \epsilon)[1 - (\delta + \gamma)] \); \( \vartheta_1 = \delta + \gamma + [1 - 2(\delta + \gamma)]\lambda(1 - \epsilon) \); \( \vartheta_2 = 1 - (\delta + \gamma) - [1 - 2(\delta + \gamma)]\lambda\epsilon \); and \( \Gamma = \epsilon\vartheta_1 + (1 - \epsilon)\vartheta_2. \)
Chapter 3

Dynamic Comparative Advantage and Trade

We now introduce international trade. There are two countries, home and foreign. They have the same production functions for the final good Y and for the processed goods X_1 and X_2. They also have the same utility functions. The countries differ in their intermediate goods sectors, having different production productivities A_1 and A_2, R&D productivities α_1 and α_2, and fixed operating costs φ_1 and φ_2. We assume that only intermediate goods G_{ij} are tradeable. The two countries are “large” in the sense that their actions affect world prices. The more common small open economy assumption then becomes a special case in which certain effects disappear because prices do not respond to a country’s actions. To avoid complications arising from strategic behavior, we suppose that neither economy is able to exercise monopoly power. That assumption is consistent with a model in which each country comprises a multitude of agents who are not able to form a cartel to act as monopolists or
monopsonists.\textsuperscript{1} There is no technology transfer, and for simplicity there also is no foreign investment.

As before, we have two classes of intermediate goods, $\{G_{1j}\}$ and $\{G_{2j}\}$. To keep the model tractable, we assume that a country can produce all of one class of intermediates or import all of that class. It cannot produce some varieties within a class and import others. However, which if either class it chooses to import and which to produce at home and thus to export will be determined by comparative advantage, not imposed \textit{a priori}. In that regard, the model differs from most of the small literature on trade and growth in which comparative advantage plays a role, where countries always trade all varieties they produce and where the allocation of the sets of varieties each country produces is exogenous. See Grossman and Helpman (1990) or Acemoglu and Ventura (2002), for example.

A crucial element of any growth model is that it must have a way either of augmenting non-reproducible factors such as land or labor or of eliminating them. The usual approach is augmentation, and that is the approach followed in the present model. The quality $Z$ of the intermediate goods $G$ augments labor. For trade to affect growth, it somehow must a country’s rate of improvement in quality. The interesting element of the model is that quality is embodied in the intermediate goods and thus cannot be obtained separately from them. In a sense, the intermediate goods in this model are a joint product containing the intermediate goods themselves and of that class. It cannot produce some varieties within a class and import others.

\textsuperscript{1}A serious limitation of the the small open economy model is precisely that it is a model of an atomistic agent and ignores general equilibrium responses, in exactly the same way that the life cycle model of the atomistic household does. Our large open economy assumption is an easy way to incorporate the constraints of general equilibrium.
However, which if either class it chooses to import and which to produce at home and thus to export will be determined by comparative advantage, not imposed *a priori*. In that regard, the model differs from most of the small literature on trade and growth in which comparative advantage plays a role, where countries always trade all varieties they produce and where the allocation of the sets of varieties each country produces is exogenous. See Grossman and Helpman (1990) or Acemoglu and Ventura (2002), for example, also their quality. The two parts enter the production functions for processed goods $X_i$ as independent terms, so in a very real sense they are two different goods tied together in a joint product. As explained in the Introduction, this description seems applicable to much technical progress, that is, as something embodied in capital or intermediate goods even though it augments labor. To get the quality, then, a country must buy joint product. That fact leads to some very interesting and novel results on the effects of trade on growth. First, there is an externality that may cause trade to reduce total output at the moment trade opens between two countries. Second, even if trade raises initial income in both countries, it may raise or lower the growth rate of output of one or both countries. Third, even without any technology transfer, trading in goods leads to outcomes that are identical to those that would emerge from technology transfer.
3.1 Trade Patterns

When the home country is open, processed goods firms have more options for obtaining intermediate goods. They can buy domestic goods, foreign goods, or both. Consider the production function for processed good $X_1$ produced at home:

$$X_{H1} = \int_0^{N_{H1}} (G_{H1j} - G_{E\,H1j})^\lambda \left[ Z_{H1j}^\delta Z_{H1}^\gamma \left( \frac{Z_{H2}}{l_{H1}} \right)^{(1-(\delta+\gamma))} l_{H1H} \right]^{1-\lambda} dj$$

$$+ \int_0^{N_{F1}} (G_{F1k}^I)^\lambda \left[ Z_{F1k}^\delta Z_{F1}^\gamma \left( \frac{Z_{H2}}{l_{H1F}} \right)^{(1-(\delta+\gamma))} l_{H1F} \right]^{1-\lambda} dk$$

The subscripts $H$ and $F$ denote the home and foreign countries, respectively. $X_{H1}$ is output of type-1 processed goods by the home country. $G_{H1j}$ is the quantity of intermediate good $1j$ produced in the home country, $G_{E\,H1j}$ is the amount of $G_{H1j}$ exported, and $G_{H1j} - G_{E\,H1j}$ is the amount of $G_{H1j}$ not exported and used by the home country processed goods firms. $G_{F1k}^I$ is the amount of intermediate good $1k$ imported from the foreign country. $Z_{H1}$ is the average quality of domestic intermediate goods in class 1. As in the closed economy, symmetry within the industry ensures that all individual quality levels are the same and so equal to the average level. $Z_{F1}$ is the quality of foreign in class 2. $N_{H1}$ and $N_{F1}$ are the numbers of firms in industry 1 of home and foreign countries. Finally, $l_{H1H}$ and $l_{H1F}$ are the quantities of labor assigned to work with the domestic and foreign intermediate goods. Note that only one of $G_{E\,H1j}$ and $G_{F1k}^I$ can be positive at any time because we are restricting a trading
country either to export all of one class of intermediate goods or to import it. If, for example, \( G^E_{H1j} \) is positive, then \( G^I_{F1k} \) is zero, \( G^E_{H2j} \) also is zero, and \( G^I_{F2k} \) is positive.

A crucial element of any growth model is that it must have a way either of augmenting non-reproducible factors such as land or labor or of eliminating them. The usual approach is augmentation, and that is the approach followed in the present model. The quality \( Z \) of the intermediate goods \( G \) augments labor. For trade to affect growth, it somehow must a country's rate of improvement in quality. The interesting element of the model is that quality is embodied in the intermediate goods and thus cannot be obtained separately from them. In a sense, the intermediate goods in this model are a joint product containing the intermediate goods themselves and also their quality. The two parts enter the production functions for processed goods \( X_i \) as independent terms, so in a very real sense they are two different goods tied together in a joint product. As explained in the Introduction, this description seems applicable to much technical progress, that is, as something embodied in capital or intermediate goods even though it augments labor. To get the quality, then, a country must buy joint product. That fact leads to some very interesting and novel results on the effects of trade on growth. First, there is an externality that may cause trade to reduce total output at the moment trade opens between two countries. Second, even if trade raises initial income in both countries, it may raise or lower the growth rate of output of one or both countries. Third, even without any technology transfer, trading in goods leads to outcomes that are identical to those that would emerge from technology transfer. Intermediate goods of class 1.

As before, \( \widehat{Z}_{H2} \) is the knowledge spillover from the class-2 intermediates. The
difference now is that the class-2 goods $G_2$ may be of either domestic or foreign origin, so we must keep track of which quality is in play. We assume that $\hat{Z}_{H2}$ equals the quality $Z_{H2}$ of home-produced $G_2$ goods if the home country’s $X_2$ industry uses only domestically produced types of $G_2$, that $\hat{Z}_{H2}$ equals the quality $Z_{F2}$ of foreign-produced $G_2$ goods if the home country’s $X_2$ industry uses only foreign produced types of $G_2$, and that $\hat{Z}_{H2}$ equals the weighting average of domestic and foreign qualities $Z_{H2}$ and $Z_{F2}$ if the home country’s $X_2$ industry uses both domestically produced and foreign produced types of $G_2$:

$$\hat{Z}_{H2} = \begin{cases} Z_{H2} & \text{only home-produced } G_2 \text{ used} \\ Z_{H2}^{\eta}Z_{F2}^{1-\eta} & \text{both home- and foreign-produced } G_2 \text{ used} \\ Z_{F2} & \text{only foreign-produced } G_2 \text{ used} \end{cases}$$

Processed goods firms in industry 1 in the home country choose the combination of intermediate good 1 to buy from domestic firms and foreign firms to maximize profit:

$$\max \pi_{X_1} = P_{XH1}X_{H1} - \int_{0}^{N_1} P_{G_{H1j}} (G_{H1j} - \hat{G}_{H1j}) \, dj - \int_{0}^{N_F} P_{G_{H1Fj}}G_{F1k}^I dk - \int_{0}^{N_1} \hat{w}_{H1H} dj - \int_{0}^{N_1} \hat{w}_{H1F} dk$$
where the choice variables are $G_{H1j} - G_{E1j}^H$, $G_{F1k}^I$, $l_{H1H}$, and $l_{H1F}$, and choice is subject to the trade balance constraint mentioned above. Solving for labor demand and plugging into the first-order conditions for $G_{H1j} - G_{E1j}^H$ and $G_{F1k}^I$ yields a bang-bang solution in which processed goods firms buy only $G_{H1j} - G_{E1j}^H$ or only $G_{F1k}^I$.\(^2\)

The processed good firm buys $G_{H1j} - G_{E1j}^H$ or $G_{F1k}^I$ according to which has the lower quality adjusted price: $P_{G_{H1j}/Z_{H1}^{(\delta+\gamma)(1-\lambda)/\lambda}}$ and $P_{G_{F1j}/Z_{F1}^{(\delta+\gamma)(1-\lambda)/\lambda}}$, respectively. The situation for producers of $X_2$ is similar and need not be discussed.

International trade takes place if each country has a comparative advantage in selling a good. In our model, comparative advantage means that each country has a lower (or equal) quality-adjusted price in one class of intermediate goods, i.e.,

\[
\frac{P_{G_{H1j}}}{Z_{H1}^{(\delta+\gamma)(1-\lambda)/\lambda}} \leq \frac{P_{G_{F1j}}}{Z_{F1}^{(\delta+\gamma)(1-\lambda)/\lambda}} \quad \text{and} \quad \frac{P_{G_{H2j}}}{Z_{H2}^{(\delta+\gamma)(1-\lambda)/\lambda}} \geq \frac{P_{G_{F2j}}}{Z_{F2}^{(\delta+\gamma)(1-\lambda)/\lambda}} \quad (3.1)
\]

or the reverse. The direction of the inequalities determines which goods are exported and imported. That direction is inconsequential to our results, so we suppose hereafter that the equalities are in the direction shown in eqn. (3.1). Because the quality adjusted price does not depend on the quantity bought, a country will buy all of an intermediate good from whoever has the lower quality adjusted price. That means that if the home country decides to buy a class of intermediate goods from the foreign country, the home country will stop producing that class of intermediate goods and so will specialize in production. Countries specialize completely if strict inequality

\(^2\)See the Appendix (C.1) for details.
holds in eqn. (3.1), which means under our imposed direction of the inequalities that
the home country specializes in intermediate good 1 and the foreign country specializes
in good 2. Each country stops producing one class of intermediate goods and
devotes all its energy to producing the other class. With weak inequality, a country
may not fully specialize, meaning that it may import a good but also continue to
make it at home. We discuss complete and incomplete specialization in more detail
below.

Let the final good in the home country be the numeraire. Then $P_{Y_H} \equiv 1$.
The price of the final good in the foreign country is $P_{Y_F}$. Recall that price of
intermediate good equals to monopolistic markup times unit cost, so $P_{G_{H1}} = \frac{A_{H1}}{\lambda}$,
$P_{G_{H2}} = \frac{A_{H2}}{\lambda}$, $P_{G_{F1}} = \frac{P_{Y_F} A_{F1}}{\lambda}$, and $P_{G_{F2}} = \frac{P_{Y_F} A_{F2}}{\lambda}$. Using these facts, we see
that equation (3.1) is equivalent to

$$\frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right) \left( \frac{A_{H1}}{A_{F1}} \right) \geq P_{Y_F} \geq \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right) \left( \frac{A_{H2}}{A_{F2}} \right) \left( \frac{Z_{F2}}{Z_{H2}} \right)$$

(3.2)

The price $P_{Y_F}$ must be in this closed interval. Otherwise both countries would try
to export the same good, implying a market disequilibrium. If we ignore the quality
ratios and look at the unit cost ratio only, then this is a typical trade condition for
Ricardian-type model. In basic static Ricardian model, labor is the only factor of
production for tradeable goods, and the relative wage across countries must be inside
an interval of the unit cost ratios. In our model, it is not final goods but rather
intermediate goods that are traded, which are produced from the final goods. So
the factor price ratio (final good price ratio) needs to be inside the interval of the
productivity ratios. The difference between eqn. (3.2) and the standard expression is that, in traditional static Ricardian model, the productivity ratios are in terms of the unit cost ratios only, which are constant, whereas the productivity ratios in our model include not only the unit cost ratios but also the quality ratios. Appendix eqn. (C.7) gives a detail comparison between our definition of comparative advantage and that in Dornbush, Fisher and Samuelson (1977).

We can arrange the two interval of ((3.2)) into another typical version of the statement of comparative advantage\(^3\):

\[
\begin{bmatrix}
\frac{A_{H1}}{(\delta + \gamma)(1 - \lambda)} \\
\frac{Z_{H1}}{A_{F1}}
\end{bmatrix}
\leq
\begin{bmatrix}
\frac{A_{H2}}{(\delta + \gamma)(1 - \lambda)} \\
\frac{Z_{H2}}{A_{F2}}
\end{bmatrix}
\]

The home country has a lower quality-adjusted price ratio (intermediate good 1 over good 2), so it specializes in intermediate good 1.

Notice that our model differs from most others that have studied the effect of trade on growth through the channel of comparative advantage in that the pattern of trade (which goods a country imports and exports) is determined endogenously rather than being imposed \textit{a priori}. See Grossman and Helpman (1990) and Acemoglu and Ventura (2002) for examples.

\(^3\)See the derivations in Appendix (C.2)
3.2 Complete Specialization

When eqn. (3.2) holds with strict inequality, the home country has a strictly lower quality-adjusted price in intermediate good 1, and foreign country has a strictly lower price in good 2. Both countries completely specialize in the class of goods in which they have a comparative advantage. The final goods industry is competitive and has a Cobb-Douglas production function, so the final good producer in the home country pays compensation $(1 - \epsilon)\lambda Y_H$ to the producers of intermediate industry 2, which are foreign firms. Similarly, the final good industry in the foreign country pays compensation $\epsilon \lambda P_{Y_F} Y_F$, measured with the final good price from the home country, to the intermediate producers of industry 1, which are firms in the home country. Trade balance thus requires $1 \cdot Y_H (1 - \epsilon) \lambda = P_{Y_F} \cdot Y_F \epsilon \lambda$. 4 This condition can be rewritten as $P_{Y_F} = [(1 - \epsilon) L_H/\epsilon L_F]^{1-\lambda}$, which indicates the relative value of all goods imported over the value of those exported by the home country. We therefore can write the condition for complete specialization as, Appendix A

If the fixed operating cost not only depends on the average quality level, but also depends on the quality of the individual firm, then the expression of fixed operating cost could be the same as Peretto(2007), i.e. $\theta Z^\delta Z^{1-\delta}$. By the assumption of zero entry/exit cost, this set up doesn’t allow a positive growth rate. The following is the detail:

$$\frac{A_{H2}}{A_{F2}} \left(\frac{Z_{F2}}{Z_{H2}}\right)^{(\delta+\gamma)(1-\lambda)} > \left[\frac{(1 - \epsilon)L_H}{\epsilon L_F}\right]^{1-\lambda} > \frac{A_{H1}}{A_{F1}} \left(\frac{Z_{F1}}{Z_{H1}}\right)^{(\delta+\gamma)(1-\lambda)}$$ (3.4)

This expression means the relative population size must be inside a certain interval,

\[\text{see the Appendix (C.3)}\]
which depends on the initial quality ratios at the moment of opening to trade, and the unit costs. If the relative population size is too high or too low, then complete specialization does not occur. Intuitively, one country is too small to service all of the other country’s needs. Note that it is not just the population sizes that matter but rather the population sizes weighted by the elasticities $\epsilon$ and $(1 - \epsilon)$ of final good output with respect to the two processed goods inputs and by the elasticity $(1 - \lambda)$ of processed goods with respect to augmented labor. We discuss the interpretation of the expression $[(1 - \epsilon) L_H/\epsilon L_F]^{1-\lambda}$ below when we turn to incomplete specialization.

### 3.2.1 Model Set Up for Complete Specialization

#### 3.2.1.1 Home country: Final good and processed good Sectors

The final good production function is the same as autarky, i.e. $Y_H = X_{H1}^{1-\epsilon}$. But the processed good producers now can choose either domestic or foreign intermediates. By equation (3.1), home country produces class-1 intermediates, and imports class-2 intermediates.

$$ X_{H1} = \int_0^{N_{H1}} (G_{H1j} - G_{H1j}^E) \lambda [Z_{H1}^{(\delta + \gamma)} Z_{F2}^{1-(\delta + \gamma)} l_{H1}]^{1-\lambda} dj; \text{ the amount of intermediate good 1 that used by home is } G_{H1j} - G_{H1j}^E; G_{H1j}^E \text{ is the amount that exports to foreign country, which is the demand of } G_1 \text{ from foreign country. } Z_{F2} \text{ is the spillover from intermediate good 2, which are imported from foreign country.} $$

$$ X_{H2} = \int_0^{N_{F2}} (G_{H2j}^I) \lambda [Z_{F2}^{(\delta + \gamma)} Z_{H1}^{1-(\delta + \gamma)} l_{H2}]^{1-\lambda} dj; \text{ home imports intermediate good from foreign country, and the amount of variety is } N_{F2}, \text{ which are the total varieties}$$
that foreign country produces in industry 2.

Intermediate good firms are symmetric among the same industry, so we drop the subscript $i$. Set the final good of home is the numeraire, and following exactly the same steps in closed economy, we get the demand functions:

$$G_{H1j} - G_{H1j}^E = \left[ \frac{\lambda \epsilon (X_{H1} / X_{H2})^{\epsilon - 1}}{P_{G_{H1}}} \right]^{1-\lambda} Z_{H1j}^\delta Z_{H1}^{\gamma} (Z_{F2})^{1-\gamma} l_{H1} \tag{3.5}$$

$$G_{H2j}^I = \left[ \frac{\lambda (1 - \epsilon) (X_{H1} / X_{H2})^{\epsilon}}{P_{G_{F2}}} \right]^{1-\lambda} Z_{F2j}^\delta Z_{F2}^{\gamma} (Z_{H1})^{1-(\delta + \gamma)} l_{H2} \tag{3.6}$$

And the trade balance condition is $N_{H1} P_{G_{H1}} G_{H1j}^E = N_{F2} P_{G_{F2}} G_{H2j}^I$

### 3.2.1.2 Foreign country: Final good and processed good Sectors

The numeraire is the final good of home, so define the price of final good in foreign country as $P_{Y_F}$. We need to rescale the prices in this country by $P_{Y_F}$. Foreign country completely specializes in $G_2$, and import $G_1$.

$$Y_F = X_{F1}^\epsilon X_{F2}^{1-\epsilon} \tag{3.7}$$

$$X_{F1j} = \int_{0}^{N_{H1}} (G_{F1j}^I)^\lambda [Z_{H1j}^\delta Z_{H1}^{\gamma} Z_{F2}^{1-(\delta + \gamma)} l_{F1}]^{1-\lambda} dj \tag{3.8}$$
where the amount of varieties of class-1 intermediates in this country is $N_{H1}$, which is decided by home country.

$$X_{F2j} = \int_0^{N_{F2}} (G_{F2j} - G^E_{F2j})^\lambda [Z_{F2j}^\delta Z_{F2}^\gamma Z_{H1}^{1-(\delta+\gamma)} l_{F2}]^{1-\lambda} dj$$  \hspace{1cm} (3.9)

Intermediate good firms are symmetric among the same industry, so we drop the subscript $i$. Note that we’ve rescale the whole maximization problem by the final good price, $P_Y$. Following exactly the same steps, we get the demands as,

$$G^I_{F1j} = \left[ \frac{\lambda \epsilon (X_{F1})^{\epsilon-1}}{P_{G_{H1}} / P_Y} \right]^{\frac{1}{\epsilon}} Z_{H1}^\delta Z_{F2}^\gamma (Z_{F2} - Z_{H1})^{1-(\delta+\gamma)} l_{F1}$$  \hspace{1cm} (3.10)

$$G_{F2j} - G^E_{F2j} = \left[ \frac{\lambda (1 - \epsilon) (X_{F2})^{\epsilon}}{P_{G_{F2}} / P_Y} \right]^{\frac{1}{\epsilon}} Z_{F2j}^\delta Z_{F2}^\gamma (Z_{F2} - Z_{H1})^{1-(\delta+\gamma)} l_{H2}$$  \hspace{1cm} (3.11)

And the trade balance condition is

$$N_{H1} P_{G_{H1}} G^E_{H1} = N_{F2} P_{G_{F2}} G^I_{H2}$$  \hspace{1cm} (3.12)

And, $G^I_{F1} = G^E_{H1}; G^E_{F2} = G^I_{H2}$. This is because intermediate good firms are symmetric among the same industry. In industry 1 of home country, each firm faces the same amount of demand from foreign country, and the amount of exports by home equal the amount of imports by foreign country, which are the same for each intermediate good firm.
Add (3.5) and (3.10), we get the total Demand for $G_{H1}$ is,

$$G_{H1j} = (G_{H1j} - G_{H1j}^E) + G_{F1j}^I$$

$$= (G_{H1j} - G_{H1j}^E) + G_{H1j}^E$$

$$= \left[ \lambda \epsilon \left( X_{H1} X_{H2} \right)^{1-\epsilon} \right]^{1-\epsilon} Z_{H1j} Z_{H1}(Z_{F2})^{1-(\delta+\gamma)}(l_{H1} + P_{Y_F}^{1-\gamma} l_{F1})$$

(3.15)

Add (3.6) and (3.11), we get the total demand for $G_{F2}$ is,

$$G_{F2j} = \left[ \frac{\lambda \epsilon (X_{F1})^{1-\epsilon}}{P_{G_{F2}}} \right]^{1-\epsilon} Z_{F2j} Z_{F2}^\delta (Z_{H1})^{1-(\delta+\gamma)}(l_{H2} + P_{Y_F}^{1-\gamma} l_{F2})$$

(3.16)

Following exactly the same steps as the closed economy, we also get, $\frac{X_{H1}}{X_{H2}} = \frac{X_{F1}}{X_{F2}} = \frac{\epsilon}{1-\epsilon} \left( \frac{P_{G_{F2}}}{P_{G_{F2}}} \right)^{-\lambda} \left( \frac{Z_{H1}}{Z_{F2}} \right)^{-[1-2(\delta+\gamma)](1-\lambda)}$. And according to trade balance condition in section (3.2), we get $P_{Y_F} = \left[ \frac{(1-\epsilon) L_{H}}{\epsilon F} \right]^{1-\lambda}$. You can also get $P_{Y_F}$ from eqn. (3.12), which is much more complicated and omitted here.

Compare the demand under trade, eqn. (3.15), (3.16) with the demand under autarky, eqn. (B.13) and (B.14), we see now each firm not only faces the domestic market, but also the international market. In the first generation growth model, a higher population means a higher market size, and hence induces firms to do R&D; here each firm faces the individual market size, which is the population size divided by the number of firms. The number of firms in this model change endogenously to eliminate incipient profit as discussed in closed economy. So the increase in total
market size due to trade will not cause a higher growth rate.

3.2.1.3 Intermediate Good Firms in Both Countries

Given demand function (3.15) and (3.16), and the R&D ability, \( Z_{H1} = \alpha_{H1} R_{H1} \); \( Z_{F2} = \alpha_{F2} R_{F2} \); we follow EXACTLY the same steps as those in closed economy.

**Home Country** Home country only produces class-1 intermediates. The intermediate goods firm’s profit is revenue less production costs:

\[
F_{H1} = P_{G_{H1}} G_{H1} - A_{H1} G_{H1} - \theta_{H1} \frac{Z_{H1}^2}{Z_{F2}^2}
\]

Incumbents in industry 1 maximize the present discounted value of net profit,

\[
V_{H1}(t) = \int_t^{\infty} \left[ G_{H1} (P_{G_{H1}} - A_i) - \theta_{H1} \frac{Z_{H1}^2}{Z_{F2}} - R_{H1} \right] e^{-\int_t^s r(s) d\tau} d\tau
\]

s.t. \( \dot{Z}_{H1} = \alpha_{H1} R_{H1} \) and demand function (3.15).

The current value Hamiltonian is

\[
CV_{H1} = G_{H1} (P_{G_{H1}} - A_{H1}) - \theta_{H1} \frac{Z_{H1}^2}{Z_{F2}} - R_{H1} + q_{H1}(\alpha_{H1} R_{H1})
\]

The necessary conditions are\(^5\):

\(^5\)Detail can see Appendix (B.3). That’s the derivations for closed economy. But under complete specialization, the derivations are exactly the same as closed economy.
\[ \dot{Z}_{H1} = \alpha_{H1} R_{H1} \]

\[ \frac{\partial CV_{H1}}{\partial P_{G_{H1}}} = 0 \Rightarrow P_{G_{H1}} = A_{H1}/\lambda \]

\[ \frac{\partial CV_{H1}}{\partial R_{H1}} = -1 + q_{H1}\alpha_{H1} \Rightarrow \frac{1}{\alpha_{H1}} = q_{H1} \text{ for interior solution} \]

\[ q_{H1} = -r_{H1} q_{H1} + \frac{\partial CV_{H1}}{\partial Z_{H1}} \]

\[ \Rightarrow r_{H1} = \frac{\partial F_{H1}}{\partial Z_{H1}} \alpha_{H1} \quad (3.17) \]

where \( \frac{\partial F_{H1}}{\partial Z_{H1}} = \delta A_{H1}^{1-\lambda} \frac{\lambda^2}{\lambda - 1} (\frac{\epsilon}{1-\epsilon})^{\frac{\lambda}{1-\lambda}} \left( \frac{P_{G_{H1}}}{P_{G_{F2}}} \right)^{\frac{\lambda(\epsilon-1)}{1-\lambda}} \left( \frac{Z_{H1}}{Z_{F2}} \right)^{-1} (l_{H1} + P_{Y_{F}}^{\frac{1}{1-\lambda}} l_{F1}) \), and

\[ \Gamma = 2(\delta + \gamma)\epsilon - (\delta + \gamma) - \epsilon + 1 \in (0, 1). \]

- \( Z_{1j,t=0} \) is given
- \( \lim_{t \to \infty} e^{-\int_{t}^{\infty} r(s) ds} q_{1j}(t) Z_{1j}(t) = 0 \)

We assume a zero entry/exit cost. As described in closed economy, firms jump into the market to make the profit zero.

\[ g_{H1} = \frac{\dot{Z}_{H1}}{Z_{H1}} = \frac{\alpha_{H1} R_{H1}}{Z_{H1}} = \frac{\alpha_{H1} F_{H1}}{Z_{H1}} \]

\[ = \alpha_{H1} \left\{ A_{H1} \frac{1-\lambda}{\lambda} \left[ \frac{\lambda^2}{A} \right]^{1-\lambda} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{\lambda}{1-\lambda}} \left( \frac{P_{G_{H1}}}{P_{G_{F2}}} \right)^{-\frac{\lambda(\epsilon-1)}{1-\lambda}} \left( \frac{Z_{H1}}{Z_{F2}} \right)^{-1} (l_{H1} + P_{Y_{F}}^{\frac{1}{1-\lambda}} l_{F1}) - \theta_{F1} \frac{Z_{H1}}{Z_{F2}} \right\} \]
**Foreign Country**  Foreign country only produces class-2 intermediates. The intermediate goods firm’s profit is revenue less production costs:

\[ F_{F2} = P_{GF2} G_{F2} - A_{F2} G_{F2} - \theta_{F2} \frac{Z_{F2}^2}{Z_{H1}} \]

Incumbents in industry 2 maximize the present discounted value of net profit,

\[ V_{F2}(t) = \int_t^\infty \left[ G_{F2} (P_{GF2} - A_{F2}) - \theta_{F2} \frac{Z_{F2}^2}{Z_{H1}} - R_{F2} \right] e^{-\int_t^\tau r(s) ds} d\tau \]

s.t. demand function (3.16); and \( \dot{Z}_{F2} = \alpha_{F2} R_{F2} \);  

The current value Hamiltonian is\(^6\)

\[ CVH_{F2} = G_{F2} (P_{GF2} - P_{Y_{F2}} A_{F2}) - P_{Y_{F2}} \theta_{F2} \frac{Z_{F2}^2}{Z_{H1}} - P_{Y_{F}} R_{F2} + q_{F2} (\alpha_{F2} R_{F2}) \]

The necessary conditions are,

- \[ \dot{Z}_{H1} = \alpha_{H1} R_{H1} \]

- \[ \frac{\partial CVH_{F2}}{\partial P_{GF2}} = 0 \Rightarrow P_{GF2} = P_{Y_{F2}} A_{F2}/\lambda \]

\(^6\)Note the rescale of \( P_{Y_{F2}} \) in foreign country.
\[
\frac{\partial CVH_{F_2}}{\partial R_{F_2}} = -P_{Y_F} + q_{2F} \alpha_{F_2} \Rightarrow q_{2F} = \frac{P_{Y_2}}{\alpha_{F_2}} \text{ for interior solution}
\]

\[
q_{F_2} = -r_{F_2} q_{F_2} + \frac{\partial CVH_{F_2}}{\partial Z_{F_2}}
\]

\[
\Rightarrow r_{F_2} = \frac{\partial F_{F_2}}{\partial Z_{F_2}} / q_{F_2}, \text{ and } q_{F_2} = \frac{P_{Y_2}}{\alpha_{F_2}}
\] (3.18)

where \( \frac{\partial F_{F_2}}{\partial Z_{F_2}} = P_{Y_F} A_{F_2} \frac{1-\lambda}{\lambda} \frac{[\lambda^2(1-\epsilon)]^{\frac{1}{\lambda}}}{A_{F_2}} \frac{1}{\epsilon} \frac{1}{1-\epsilon} \frac{P_{G_{H_1}}}{P_{Q_{F_2}}} \frac{1}{\lambda} \frac{1}{l_{H_2} P_{Y_F} \frac{1}{Z_{H_1}}} \) and \( \Gamma = 2(\delta + \gamma) \epsilon - (\delta + \gamma) - \epsilon + 1 \in (0, 1) \).

We assume a zero entry/exit cost. As described in closed economy, firms jump into the market to make the profit zero.

\[
g_{F_2} = \frac{\dot{Z}_{F_2}}{Z_{F_2}} = \frac{\alpha_{F_2} R_{F_2}}{Z_{F_2}} = \frac{\alpha_{F_2} F_{F_2} / P_{Y_F}}{Z_{F_2}} = \alpha_{F_2} \{ A_{F_2} \frac{1-\lambda}{\lambda} \frac{[\lambda^2(1-\epsilon)]^{\frac{1}{\lambda}}}{A_{F_2}} \frac{1}{\epsilon} \frac{1}{1-\epsilon} \frac{P_{G_{H_1}}}{P_{Q_{F_2}}} \frac{1}{\lambda} \frac{1}{l_{H_2} P_{Y_F} \frac{1}{Z_{H_1}}} - \theta_{F_2} \frac{Z_{F_2}}{Z_{H_1}} \}
\]

\[^7\text{Note that in } r_{F_2}, \text{ the market size by each firm we express as } l_{H_2} P_{Y_F} \frac{1}{Z_{H_1}} + l_{F_2}, \text{ but in (3.16), it was } l_{H_2} + l_{F_2} P_{Y_F} \frac{1}{Z_{H_1}}. \text{ It’s because when we caculate } \frac{\partial F_{F_2}}{\partial Z_{F_2}}, \text{ we rescale } P_{Y_F}\text{back. And the following is the detail}
\]

\[
F_{F_2} = G_{F_2} \left( P_{G_{F_2}} - P_{Y_F} A_{F_2} \right) - \theta_{F_2} \frac{Z_{H_1} + Z_{F_2}}{Z_{H_1}} \text{ and plug (3.16) in it, we get}
\]

\[
F_{F_{2j}} = P_{Y_F} A_{F_2} (1 - \epsilon) \left\{ \frac{1}{\lambda} \frac{[\lambda^2(1-\epsilon)]^{\frac{1}{\lambda}}}{A_{F_2}} \frac{1}{\epsilon} \frac{1}{1-\epsilon} \frac{P_{G_{H_1}}}{P_{Q_{F_2}}} \frac{1}{\lambda} \frac{1}{l_{H_2} P_{Y_F} \frac{1}{Z_{H_1}}} - \theta_{F_2} \frac{Z_{F_2}}{Z_{H_1}} \right\} - \theta_{F_2} \frac{Z_{F_2}}{Z_{H_1}} \right) - \frac{P_{Y_F} \frac{1}{Z_{H_1}} l_{F_2}}{l_{F_2}} \}
\]

\[
= P_{Y_F} \left\{ A_{F_2} \frac{1-\lambda}{\lambda} \frac{[\lambda^2(1-\epsilon)]^{\frac{1}{\lambda}}}{A_{F_2}} \frac{1}{\epsilon} \frac{1}{1-\epsilon} \frac{P_{G_{H_1}}}{P_{Q_{F_2}}} \frac{1}{\lambda} \frac{1}{l_{H_2} P_{Y_F} \frac{1}{Z_{H_1}}} \right\} - \theta_{F_2} \frac{Z_{F_2}}{Z_{H_1}} \}
\]

\[
= P_{Y_F} \left\{ A_{F_2} \frac{1-\lambda}{\lambda} \frac{[\lambda^2(1-\epsilon)]^{\frac{1}{\lambda}}}{A_{F_2}} \frac{1}{\epsilon} \frac{1}{1-\epsilon} \frac{P_{G_{H_1}}}{P_{Q_{F_2}}} \frac{1}{\lambda} \frac{1}{l_{H_2} P_{Y_F} \frac{1}{Z_{H_1}}} \right\} - \theta_{F_2} \frac{Z_{F_2}}{Z_{H_1}} \}
\]

\[
= P_{Y_F} A_{F_2} \frac{1-\lambda}{\lambda} \frac{[\lambda^2(1-\epsilon)]^{\frac{1}{\lambda}}}{A_{F_2}} \frac{1}{\epsilon} \frac{1}{1-\epsilon} \frac{P_{G_{H_1}}}{P_{Q_{F_2}}} \frac{1}{\lambda} \frac{1}{l_{H_2} P_{Y_F} \frac{1}{Z_{H_1}}} \text{ for (3.16) }
\]

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3.2.2 Level Effect under Complete Specialization

Trade affects both the level and growth rate of final output. We discuss the level effect first and then turn to the growth rate effect below. Under complete specialization, the home country abandons in industry 2 of the intermediate goods sector, and the foreign country abandons industry 1 of that sector. Once production of a line of intermediate goods has been stopped, there is no value in doing R&D to improve its quality, so R&D as well as production cease. As a result, $Z_{H2}$ and $Z_{F1}$ stop growing, but $Z_{H1}$ and $Z_{F2}$ continue to grow. That widens the price interval within which complete specialization occurs, by eqn. (3.4). So, if the world economy starts in a position of complete specialization, it stays there forever.

This mechanism gives rise to the possibility that a situation of incomplete specialization may be transformed into one of complete specialization. If $P_{Y_F}$ is at the boundary of the permissible interval, one country will be specialized and the other will not. Changes in qualities moves the boundaries, possibly changing incomplete specialization into complete specialization. We discuss that possibility below in the next chapter.

Complete specialization is equivalent to an integrated economy with $G_1$ produced by the technologies from the home country and $G_2$ produced by the technologies from foreign country. The home country produces only intermediate good 1 and sells it at the price $P_{G_{H1}} = \frac{A_{H1}}{\lambda}$, and it imports intermediate good 2 at the price $P_{Y_F} \frac{A_{F2}}{\lambda}$.

Following similar steps as in the closed economy, we get the home country’s final
output under autarky and trade (see the Appendix (C.4)):

\[ Y_{\text{Autarky}} = \kappa_H^\prime \left[ \frac{Z_{H1}^{(\delta+\gamma)}}{P_{GH1}^{\lambda}} \right] Z_{H2}^{1-(\delta+\gamma)} (\epsilon L_H) \left[ \frac{Z_{H2}^{(\delta+\gamma)}}{P_{GH2}^{\lambda}} \right] Z_{H1}^{1-(\delta+\gamma)} ((1 - \epsilon) L_H) \right]^{1-\epsilon} \quad (3.19) \]

\[ Y_{\text{Trade}} = \kappa_H^\prime \left[ \frac{Z_{H1}^{(\delta+\gamma)}}{P_{GH1}^{\lambda}} \right] Z_{F2}^{1-(\delta+\gamma)} (\epsilon L_H) \left[ \frac{Z_{F2}^{(\delta+\gamma)}}{P_{GF2}^{\lambda}} \right] Z_{H1}^{1-(\delta+\gamma)} ((1 - \epsilon) L_H) \right]^{1-\epsilon} \quad (3.20) \]

where \( \kappa_H^\prime = \lambda^{\frac{\lambda}{1-x}} (1 - \epsilon)^{\frac{\lambda(1-\epsilon)}{1-x}} \epsilon^{\frac{\lambda}{1-x}} \). From the derivation of \( Y \) in the closed economy, we can see that \( \left[ \frac{Z_{H2}^{(\delta+\gamma)}}{P_{GH2}^{\lambda}} \right] Z_{H1}^{1-(\delta+\gamma)} (\epsilon L_H) \] is the contribution from \( X_{H1} \) to \( Y_H \), in which \( Z_{H2} \) is the spillover from \( X_{H2} \) and \( \epsilon L_H \) is labor employed in industry 1.

\( \left[ \frac{Z_{H2}^{(\delta+\gamma)}}{P_{GF2}^{\lambda}} \right] Z_{H1}^{1-(\delta+\gamma)} ((1 - \epsilon) L_H) \] is the contribution from \( X_{H2} \) to \( Y_H \) in autarky, in which \( Z_{H1} \) is the spillover from \( X_{H1} \) and \((1 - \epsilon) L_H \) is labor employed in industry 2.

If we compare eqn. (3.19) and (3.20), we see that trade affects final output through imports of intermediate good 2. There are two channels. One is the quality-adjusted price, which we define as \( \frac{P_{GH2}^{\lambda}}{Z_{H2}^{(\delta+\gamma)}} \).\(^8\) Imports have a lower quality-adjusted price, so the quality per price is higher, i.e. \( \frac{Z_{F2}^{(\delta+\gamma)}}{P_{GF2}^{\lambda}} > \frac{Z_{H2}^{(\delta+\gamma)}}{P_{GH2}^{\lambda}} \). This channel has a positive effect on output. The other channel is the spillover to industry 1, \( Z_{F2}^{1-(\delta+\gamma)} \), which is an externality. Firms in a processed goods industry base their decision on which intermediate goods to buy on the quality-adjusted prices. What they ignore is that their decision of which good to buy determines the knowledge spillover on the other processed goods industry. Consider what happens when a firm in the \( X_2 \) industry

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\(^8\)Which equals \( \frac{P_{GH2}^{\lambda}}{Z_{H2}^{(\delta+\gamma)}} \) in (3.1).
decides to use foreign intermediate goods because they have a lower quality adjusted price than domestic goods. The knowledge spillover to the \( X_1 \) industry is \( Z_{H2} = Z_{F2} \). That spillover is an externality, and it can be positive or negative, depending on whether \( Z_{F2} > Z_{H2} \) or \( Z_{F2} < Z_{H2} \). The fact that the foreign goods have a lower quality adjusted price does not imply that they have a higher quality level than the domestic good. The quality level \( Z_{F2} \) of the foreign good can be below the quality level \( Z_{H2} \) of the domestic good and the quality adjusted price of the foreign good can still exceed the quality adjusted price of the domestic good if the foreign good’s unadjusted price \( P_{GF1} \) is sufficiently below the unadjusted price \( P_{GH1} \) of the domestic good. Trade does not change the intermediate goods prices, which still are given by the mark-up rules \( P_{GF1} = A_{F1}/\lambda \) and \( P_{GH1} = A_{H1}/\lambda \). Thus a sufficiently large difference between \( A_{H1} \) and \( A_{F1} \) can make the quality adjusted price for foreign goods lower than the quality adjusted price for domestic goods even if the absolute quality level of the foreign goods is lower than the quality of the domestic goods. Thus the externality can be either positive or negative depending on whether \( Z_{F2} > Z_{H2} \) or \( Z_{F2} < Z_{H2} \). Note that the externality happens across different industries in a given country. There is no externality across countries. This externality comes into play the moment that trade starts. If it is sufficiently strong, output of the processed goods industry affected by it may fall in response to the economy opening to trade, and that in turn could lead to a fall in the output of final goods \( Y \). Also, there is no reason that both processed goods industries cannot experience a negative externality strong enough to reduce output. That would guarantee that output of final goods \( Y \) unambiguously falls. There also is nothing to prevent both countries from experiencing a negative externality.
We thus have the surprising conclusion that trade may reduce the level of income in one or both countries.

Under the trade pattern in eqn. (3.1), which also means \( \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} \frac{\lambda}{(\delta+\gamma)} > \frac{Z_{H2}}{Z_{F2}} \), we have the following results\(^9\):

- The necessary and sufficient conditions for trade to decrease initial output level are \( Z_{H2} > Z_{F2} \) and \( \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} \frac{\lambda}{(\delta+\gamma)} > \frac{Z_{H2}}{Z_{F2}} > \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} (\epsilon(\delta+\gamma) - 2\epsilon(\delta+\gamma)) > 1 \)\(^10\). It means the imports have such a low quality that the externality is high enough to reduce the final output. If \( (\delta+\gamma) \to 1 \), which means the spillover effect from the other industry, \([1 - (\delta + \gamma)]\), is close to 0, then \( \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} (\epsilon(\delta+\gamma) - 2\epsilon(\delta+\gamma)) \to \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} (\delta+\gamma) \), thus the necessary and sufficient condition is hard to hold.

- The sufficient condition for trade to increase initial output level is \( Z_{H2} < Z_{F2} \), which means the quality of imports is higher than quality of the domestic goods.

- The necessary conditions for trade to increase initial output level are

\[-Z_{H2} < Z_{F2} \]

or,

\[-Z_{H2} > Z_{F2} \]

and \( \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} \frac{\lambda}{(\delta+\gamma)} > \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} (\epsilon(\delta+\gamma) - 2\epsilon(\delta+\gamma)) > \frac{Z_{H2}}{Z_{F2}} > 1 \), where \( Z_{F2} \) is not low enough to make the negative externality large enough to dominate the direct positive level effect.

---

\(^9\) See derivation in Appendix (C.4.2).

\(^10\) If \( Z_{H1} > Z_{F2} \) \( \Rightarrow \frac{Z_{H2}}{Z_{F2}} > 1 \), according to trade pattern condition, \( \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} \frac{\lambda}{(\delta+\gamma)} > \frac{Z_{H2}}{Z_{F2}} > 1 \), Appendix (5.4.2) shows \( \left( \frac{P_{GH2}}{P_{GF2}} \right)^{1-\lambda} (\delta+\gamma) > 0 \) by assumption.
3.2.3 Growth Rate Effect Under Complete Specialization

The growth rate of the world economy is derived by the same steps as for the closed economy and so is not discussed here. The world balanced growth rate has exactly the same structure as the closed economy (see Appendix (C.5)):

\[ g_H^* = g_F^* = \frac{\delta}{1 - \delta} \sqrt{\alpha_{H1}\theta_{H1}\alpha_{F2}\theta_{F2}} - \frac{1}{1 - \delta}\rho \]  

(3.21)

We thus have a world balanced growth rate. The derivation follows the same steps as for the closed economy, so we relegate it to the Appendix. We already know that the growth rate is positively related to R&D abilities, \( \alpha_{i1}\theta_{i1} \) and \( \alpha_{i2}\theta_{i2} \), where \( i \) is the country. After complete specialization, the home country abandons production of the whole set of domestic intermediate goods 2 and imports them from the foreign country. Domestic R&D for improving the quality the \( G_2 \) goods ends when production ends, and all quality improvement in the \( G_2 \) goods takes place abroad.

After trade, the number of firms in each industry also changes. Both the changes in the total market size for the whole industry and the changes in unit costs enduce entry thus change the number of firms. However, none of these changes show up in the world balanced growth rate because entry of new firms absorbs them. For example, after trade, the firm size in industry 1 in home country is (see the Appendix
\[(l^*_{H1})^{\text{Trade}} = \frac{L_H}{N^*_{H1}} = \frac{\frac{\delta}{1-\delta} \sqrt{\alpha_{H1} \theta_{H1} \alpha_{F2} \theta_{F2}}} {\alpha_{H1} \delta A_{H1} \left[ \frac{\lambda^2 \epsilon}{A_{H1}} \right]^{\frac{1}{1-\lambda}} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{1}{1-\lambda}} \left( \frac{A_{H2}}{A_{F2}} \right)^{\frac{\delta(1-\epsilon)}{1-\lambda}}} - \frac{\delta}{1-\delta} \rho}{\Gamma - 1} \]

In autarky, the firm size is

\[(l^*_{H1})^{\text{Autarky}} = \frac{\epsilon L_H}{N^*_{H1}} = \frac{\frac{\delta}{1-\delta} \sqrt{\alpha_{H1} \theta_{H1} \alpha_{H2} \theta_{H2}}} {\alpha_{H1} \delta A_{H1} \left[ \frac{\lambda^2 \epsilon}{A_{H1}} \right]^{\frac{1}{1-\lambda}} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{1}{1-\lambda}} \left( \frac{A_{H1}}{A_{H2}} \right)^{\frac{\delta(1-\epsilon)}{1-\lambda}}} - \frac{\delta}{1-\delta} \rho}{\Gamma - 1} \]

When the home country opens to trade, intermediate good producers in industry 1 face not only the domestic market but also the foreign market. So the total market size increases from \( \epsilon L_H \) to \( L_H \). If the number of firm did not change, then the market size of incumbent firms would increase and so would incipient profit. Thus zero-cost entry instantaneously causes \( N_{H1} \) to increase to prevent the incipient profit from being realized. If the RHS is the same as for the closed economy (which means no change in unit costs and R&D abilities after trade), then the individual market size in autarky and trade are the same even though the total market size is larger. So a larger total market size will not affect the return to R&D and hence will not affect the growth rate, either. This is how endogenous entry eliminates the scale effect.

After trade, the home country imports intermediate good 2 from foreign country, so the unit cost of good 2 change to that of the foreign country. This effect also is absorbed by entry, as described above for the closed economy. Consequently, trade
does not affect growth through a change in unit cost. The only channel through which trade affects the growth rate is R&D ability.

Comparative advantage, which is decided by quality-adjusted price ratio, does not guarantee that the home country imports the good with a higher R&D ability. The eqn. (3.4) shows that the trade pattern is decided only by unit costs and the initial value of qualities at the moment that trade opens. Neither unit costs nor initial qualities are related to the growth rate, which depends only on R&D abilities. It is possible that, when trade opens, the home country imports the good with a lower quality-adjusted price and a higher initial quality but also with a lower R&D ability. In that case, trade increases the initial level of output but decreases its growth rate, which means a decrease in the future output relative to what would have happened with no trade.

As mentioned above, when trade opens, a country might import a good with a worse quality than the domestic product but also with a much lower unit cost, so the quality-adjusted price of imports still beats that of the domestic product. As a result, trade could decrease the initial output level. However, that does not necessarily decrease the balanced growth rate because the “worse quality” is only the initial quality level at the moment that trade starts. As long as the country is importing a good with a higher R&D ability, it gets a higher balanced growth rate. R&D ability depends only on R&D productivity $\alpha_{ij}$ and the fixed operating cost parameters $\theta_1$ and $\theta_2$, not the initial level of quality.

In light of the foregoing results, it obviously is even possible for trade to make a country worse off in both its initial output level and its growth rate. If the home
country imports a good with a sufficiently low initial quality level, then trade decreases initial output. If the home country’s trade partner has a lower R&D ability in producing the good that the home imports, then the home country also suffers a lower growth rate.

3.2.4 World Transition Dynamics Under Complete Specialization

When the two countries satisfy the condition for complete specialization but are not on the balanced growth path, the growth rates of their incomes are equal:

\[
g = \frac{\dot{Y}_i}{Y_i} = \Gamma \frac{\dot{Z}_{H1}}{Z_{H1}} + (1 - \Gamma) \frac{\dot{Z}_{F2}}{Z_{F2}}; \text{ where } i = H, F
\]

where the constant \( \Gamma = (2\epsilon - 1)(\delta + \gamma) + 1 - \epsilon \). The formal derivation is similar to the Appendix (B.4.2), but the intuition is straightforward. When the two countries are completely specialized, each does R&D to improve the qualities of one of the two sets of intermediate goods. Each country imports the good that it does not produce. Consequently, each country uses the same sets of intermediate goods, one made at home and one made abroad, and so enjoys the same quality improvements. As a result, their growth rates are the same weighted average of the growth rates of the two qualities \( Z_{H1} \) and \( Z_{F2} \). We will see later that this property does not hold in the
case of incomplete specialization.

We can see how the growth rate of income converges to its balanced growth path by examining the time path of the quality ratio $Z_{H1}/Z_{F2}$, whose growth rate is:

$$\frac{(Z_{H1}/Z_{F2})}{Z_{H1}/Z_{F2}} = g_{H1} - g_{F2} = \frac{r_{H1}}{\delta} - \alpha_{H1}\theta_{H2} \frac{Z_{H1}}{Z_{F2}} - \left(\frac{r_{F2}}{\delta} - \alpha_{F2}\theta_{F2} \frac{Z_{F2}}{Z_{H1}}\right)$$

which is similar to the growth rate of the closed economy’s quality ratio $Z_1/Z_2$, given by equation (2.69). The foregoing expression contains the rates of return $r_{H1}$ and $r_{F2}$. Our model does not include international investment, so the no-arbitrage condition doesn’t hold across countries. Nonetheless, the rate of return to R&D is the same in the two countries. Using the same reasoning as for the closed economy, we can show that each country’s growth rate of consumption equals its growth rate of income. We have just seen that the latter growth rate is the same in the two countries, so the two countries also have the same growth rate of consumption. Then from the Euler equation for each country, we obtain the result that the rates of return $r_{H1}$ and $r_{F2}$ are equal:

$$g = \frac{\dot{c}_i}{c_i} = r_i - \rho; \text{ where } i = H, F$$

The transition dynamics of the qualities under complete specialization thus are similar to the case under the closed economy and we have

$$(Z_{H1}/Z_{F2}) = -\alpha_{H1}\theta_{H1}(\frac{Z_{H1}}{Z_{F2}})^2 + \alpha_{F2}\theta_{F2} \quad (3.23)$$
The positive root $\sqrt{\alpha_{F_2} \theta_{F_2}/\alpha_{H_1} \theta_{H_1}}$ is stable, and the economy converges monotonically to its balanced growth path. Along the transition path, various growth rates are changing but the growth rates of income in the two countries always equal each other.

In our model, the growth rate effect does not depend on cross-country technology spillovers or technology transfers, which are ruled out by construction. Countries cannot learn the technology of their trading partners even with trade. In effect, we are assuming that reverse engineering and is not possible and that firms are able to keep their production techniques secret. A country can get the “quality” but not the “know-how” from importing a good. Nonetheless, the growth rate is affected by trade in the same way as if the country actually had adopted the trading partner’s R&D ability for producing the good at home that is in fact being imported from the trading partner. If its trading partner has higher ability in R&D, then trade will increase this country’s growth rate as if technology transfer had occurred. Otherwise, trade will decrease the growth rate.

Trade with complete specialization leads to equal growth rates, even if the growth rates were unequal before trade. Consequently, trade with complete specialization always generates a balanced world income distribution even without international investment. This result is a generalization of that presented by Acemoglu and Ventura (2002), who studied a world economy in which trading patterns were fixed \textit{a priori}. 

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3.3 Incomplete Specialization

Balance of trade requires that the relative price $P_Y$ (actually, $P_Y/1$) to be inside the closed interval given in condition ((3.2)) because otherwise both countries would try to export the same good. Condition (3.4) shows that when the quantity $[(1 - \epsilon) L_H / \epsilon L_F]^{1-\lambda}$ is inside that interval, $P_Y$ will be equal to it. However, there is no reason that $[(1 - \epsilon) L_H / \epsilon L_F]^{1-\lambda}$ need be inside the interval. If it is outside the interval, then $P_Y$ cannot equal it and will be at whichever boundary is closest to $[(1 - \epsilon) L_H / \epsilon L_F]^{1-\lambda}$. In that case, we have a corner solution. One of the two countries will be completely specialized, producing only one class of intermediates and trading for the other. In contrast, its trading partner will not be completely specialized but instead will produce both types of intermediate goods. Which country specializes depends on which boundary has been hit. Without loss of generality, we assume that

$$\left[\frac{(1 - \epsilon) L_H}{\epsilon L_F}\right]^{1-\lambda} > A_{H2} \left(\frac{Z_{F2}}{Z_{H2}}\right)^{\frac{(\delta + \gamma)(1-\lambda)}{\lambda}} > \frac{A_{H1}}{A_{F1}} \left(\frac{Z_{F1}}{Z_{H1}}\right)^{\frac{(\delta + \gamma)(1-\lambda)}{\lambda}}$$

(3.24)

In that case, $P_Y$ "tries" to equal $[(1 - \epsilon) L_H / \epsilon L_F]^{1-\lambda}$ and so hits the upper bound of the interval:

$$\left[\frac{(1 - \epsilon) L_H}{\epsilon L_F}\right]^{1-\lambda} > \frac{A_{H2}}{A_{F2}} \left(\frac{Z_{F2}}{Z_{H2}}\right)^{\frac{(\delta + \gamma)(1-\lambda)}{\lambda}} = P_Y > \frac{A_{H1}}{A_{F1}} \left(\frac{Z_{F1}}{Z_{H1}}\right)^{\frac{(\delta + \gamma)(1-\lambda)}{\lambda}}$$

(3.25)
We can see what that implies by rearranging the terms in the second part of the inequality as

\[
\frac{A_{H2}}{A_{H1}} \left( \frac{Z_{H1}}{Z_{H2}} \right)^{\frac{(\delta+\gamma)(1-\lambda)}{\lambda}} = P_{Y_F} > \frac{A_{F2}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{F2}} \right)^{\frac{(\delta+\gamma)(1-\lambda)}{\lambda}}
\]

The expressions at the left and right extremes are the relative prices of the two classes of intermediate goods that would prevail under autarky in the home and foreign country, respectively. When \(P_{Y_F}\) equals the left term, the world price of the intermediates equals the autarkic home price, indicating that the home country derives no price advantage from importing either good from abroad. In contrast, \(P_{Y_F}\) is larger than the autarkic foreign price, so the foreign country still finds it advantageous to specialize in intermediate good 2 and import good 1 from the home country.

The equilibrium is one in which the foreign country completely specializes in class-2 intermediates, and the home country produces both goods but also imports class-2 intermediates. In effect, the foreign country is not “big” enough to satisfy the home country’s requirement for class-2 intermediates, i.e., \((1-\epsilon) L_H/\epsilon L_F\) is too high relative to \((A_{H2}/A_{F2}) (Z_{F2}/Z_{H2})^{(\delta+\gamma)(1-\lambda)/\lambda}\) at the moment that trade opens. The term \((1-\epsilon) L_H/\epsilon L_F\) can be large for two reasons. First, the home country’s population can be large relative to that of the foreign country. This is a straightforward relative size effect. As mentioned earlier, it means that the foreign country is simply too small to meet the demands of the home country, so that the home country finds it worthwhile to continue producing intermediate good 2. Note that this is not a scale effect. Increasing the size of the two countries’ populations equiproporionally leaves
everything unchanged, whereas shifting population from one country to another can
move the world from the interior of the critical interval to the boundary (or vice
versa) even if world population as a whole is unchanged. What matters is the rela-
tive size of the home country, not the absolute size. The effect is akin to, though
different from, the "market size effect" that Acemoglu and Zilibotti (2001) discuss
in relation to cross-country productivity differences. Second, the elasticities \( \epsilon \) and
\( 1 - \epsilon \) also play a role in determining whether the world is inside the critical interval
or at the boundary. Intuitively, if intermediate good 2 gets a heavy weight (i.e., has
a high value of \( 1 - \epsilon \)) in the final good production function, the home country may
find it worthwhile to continue producing that good even though the foreign country
has specialized in it. The final determination of whether the world is in the interior
of the critical region or on its boundary depends on the interaction of the relative
population sizes and the production elasticities.

3.3.1 Model Set Up for Incomplete Specialization

3.3.1.1 Home Country: Final Good and Processed Good

In this section, we show the demand functions for processed goods, intermediate
goods and labors. We also show the labor allocation under incomplete specialization
is somehow similar as the case under autarky and complete specialization.

Home country produces both types of intermediate goods, and also imports type-2
intermediates from foreign country. Set the numeraire as the final good in home, then
the price of foreign final good is $P_{Y_F}$.

Final Good production function of home country is,

$$Y_H = X_{H1}^{\epsilon}X_{H2}^{1-\epsilon}$$

And the processed goods production functions are,

$$X_{H1} = \int_0^{N_{H1}} (G_{H1} - G_{H1}^E)^\lambda [Z_{H1}^{(\delta+\gamma)}(Z_{H2}^\gamma l_{H1})^{1-(\delta+\gamma)}l_{H1}]^{1-\lambda} dj \quad (3.26)$$

$$X_{H2} = \int_0^{N_{H2}} G_{H2}^\lambda (Z_{H2}^{(\delta+\gamma)}Z_{H1}^{1-(\delta+\gamma)}l_{H2H})^{1-\lambda} dj + \int_0^{N_{F2}} (G_{F2}^I)^\lambda (Z_{F2}^{(\delta+\gamma)}Z_{H1}^{1-(\delta+\gamma)}l_{H2F})^{1-\lambda} dj \quad (3.27)$$

Home country exports class-1 intermediates, and the amount is $G_{H1}^E$, which is decided by the foreign country. And home uses both domestic and foreign goods in industry 2. The domestic class-2 intermediates are $G_{H2}$, and the imported class-2 intermediates are $G_{F2}^I$. $l_{H2H}$ are the amount of labors who are using domestic goods, and $l_{H2F}$ are the labors who are using foreign goods.

Following similar steps as in closed economy, we easily get the inversed demand for $X_{H1}$ and $X_{H2}$ by taking the first order condition for the maximizing profit problem of the final good producers,
\[ P_{XH1} = \epsilon X_{H1}^{\epsilon-1} X_{H2}^{1-\epsilon} \] (3.28)

\[ P_{XH2} = (1 - \epsilon) X_{H1}^{\epsilon} X_{H2}^{\epsilon} \] (3.29)

Following the similar steps as in closed economy, and applying symmetry among the same industry, we get the demand functions for the intermediate goods and labors by maximizing profit problem of processed good producers,

\[ G_{H1} - G_{H1}^E = \left( \frac{\lambda P_{XH1}}{P_{Gh1}} \right) Z_{H1j}^{\delta} Z_{H1}^{\gamma} (Z_{H2}^{\delta})^{1-(\delta+\gamma)} l_{H1} \] (3.30)

\[ G_{H2} = \left( \frac{\lambda P_{XH2}}{P_{Gh2}} \right) Z_{H2j}^{\delta} Z_{H2}^{\gamma} Z_{H1}^{1-(\delta+\gamma)} l_{H1H} \] (3.31)

\[ G_{F2} = \left( \frac{\lambda P_{XH2}}{P_{Gf2}} \right) Z_{F2j}^{\delta} Z_{F2}^{\gamma} Z_{H1}^{1-(\delta+\gamma)} l_{H1F} \] (3.32)

\[ l_{H1} = (P_{XH1} \frac{1 - \lambda}{W_H})^{\frac{1}{\lambda}} (G_{H1} - G_{H1}^E) [Z_{H1}^{(\delta+\gamma)} (Z_{H2}^{\delta})^{1-(\delta+\gamma)}]^{\frac{1-\lambda}{1}} \] (3.33)

\[ l_{H2H} = (P_{XH2} \frac{1 - \lambda}{W_H})^{\frac{1}{\lambda}} G_{H2} [Z_{H2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)}]^{\frac{1-\lambda}{1}} \] (3.34)
\[
l_{H2F} = (P_{X_{H2}} \frac{1 - \lambda}{W_H})^\lambda G_{F2}^I [Z_{F2}^{(\delta + \gamma)} Z_{H1}^{1-(\delta + \gamma)}]^{1-\lambda}
\] (3.35)

Marginal product of labor must be equal for \(l_{H1}, l_{H2H}\) and \(l_{H2F}\). Thus \(MP l_{H1} = MP l_{H2H} = MP l_{H2F}\).

\[
MP l_{H2H} = MP l_{H2F}
\]
is exactly our trade pattern condition given \(P_{H2} = \frac{A_{H2}}{\lambda}\) and \(P_{F2} = \frac{P_{YF} A_{F2}}{\lambda}\).

\[
P_{H2}/Z_{H2}^{(\delta + \gamma)(1-\lambda)} = P_{F2}/Z_{F2}^{(\delta + \gamma)(1-\lambda)} \iff \frac{A_{H2}/Z_{H2}^{(\delta + \gamma)(1-\lambda)}}{A_{F2}/Z_{F2}^{(\delta + \gamma)(1-\lambda)}} = P_{YF}
\]

It means both domestic goods and foreign goods in industry 2 give the labors the same marginal product, so processed good producers of industry 2 in home country are indifferent for both goods.

Marginal product of labor in industry 1 is, \(MP l_{H1} = P_{X_{H1}} \frac{\partial X_{H1}}{\partial l_{H1}} = P_{X_{H1}} (1 - \lambda)(G_{H1} - G_{H1}^E) \lambda [Z_{H1}^{(\delta + \gamma)} (\hat{Z}_{H2})^{1-(\delta + \gamma)} l_{H1}]^{-\lambda} Z_{H1}^{(\delta + \gamma)} (\hat{Z}_{H2})^{1-(\delta + \gamma)}\), and given the demand function (3.30), we get

\[
MP l_{H1} = P_{X_{H1}} (1 - \lambda) [\lambda P_{X_{H1}} \frac{1}{P_{G_{H1}}} Z_{H1}^{(\delta + \gamma)} (\hat{Z}_{H2})^{1-(\delta + \gamma)} l_{H1}]^\lambda
\]

\[
\cdot [Z_{H1}^{(\delta + \gamma)} (\hat{Z}_{H2})^{1-(\delta + \gamma)} l_{H1}]^{-\lambda} Z_{H1}^{(\delta + \gamma)} (\hat{Z}_{H2})^{1-(\delta + \gamma)}
\]

\[
= P_{X_{H1}} (1 - \lambda) [\lambda P_{X_{H1}} \frac{1}{P_{G_{H1}}} Z_{H1}^{(\delta + \gamma)} (\hat{Z}_{H2})^{1-(\delta + \gamma)}]
\]

Similarly, we get the marginal products of labors in industry 2 as,

\[
MP l_{H2H} = P_{X_{H2}} (1 - \lambda) [\lambda P_{X_{H2}} \frac{1}{P_{G_{H2}}} Z_{H2}^{1-(\delta + \gamma)}]
\]

\[
MP l_{H2F} = P_{X_{H2}} (1 - \lambda) [\lambda P_{X_{H2}} \frac{1}{P_{G_{F2}}} Z_{F2}^{1-(\delta + \gamma)}]
\]

\[
MP l_{H1} = MP l_{H2H} = MP l_{H2F}
\] implies
\[
\left(\frac{P_{X_{H1}}}{P_{X_{H2}}}\right)^{1/\lambda} = \left(\frac{P_{G_{H1}}}{P_{G_{H2}}}\right)^{1/\lambda} \frac{Z^{(\delta+\gamma)} H_2^1 Z^{1-(\delta+\gamma)} H_1}{Z^{(\delta+\gamma)} H_1 \left(Z^{(\delta+\gamma)} H_2^1\right)^{1-(\delta+\gamma)}} = \left(\frac{P_{G_{H1}}}{P_{G_{F2}}}\right)^{1/\lambda} \frac{Z^{(\delta+\gamma)} H_2^1 Z^{1-(\delta+\gamma)} H_1}{Z^{(\delta+\gamma)} H_1 \left(Z^{(\delta+\gamma)} H_2^1\right)^{1-(\delta+\gamma)}}
\]

(3.36)

Now we want to use \(P'_{G_{ij}}\)s and \(Z'_{ij}\)s to express \(P_{X_{H1}}\) and \(P_{X_{H2}}\). First, from the inversed demand functions (3.28) and (3.29), we get \(X_{H1}\) and \(X_{H2}\) as a function of \(P_{X_{H1}}\) and \(P_{X_{H2}}\), then we plug (3.36) in to \(X_{H1}\) and \(X_{H2}\). Last we plug \(X_{H1}\) and \(X_{H2}\) back to the inversed demand to get \(P_{X_{H1}}\) and \(P_{X_{H2}}\).

(3.28) divided by (3.29) is

\[
\frac{P_{X_{H1}}}{P_{X_{H2}}} = \frac{\epsilon}{1-\epsilon} \left(\frac{X_{H1}}{X_{H2}}\right)^{-1} \Rightarrow X_{H1} = \frac{\epsilon}{1-\epsilon} \left(\frac{P_{X_{H1}}}{P_{X_{H2}}}\right)^{-1}
\]

Plug (3.36) inside, we get

\[
\frac{X_{H1}}{X_{H2}} = \frac{\epsilon}{1-\epsilon} \frac{\left(\frac{P_{G_{H1}}}{P_{G_{H2}}}\right)^{1/\lambda} \frac{Z^{(\delta+\gamma)} H_2^1 Z^{1-(\delta+\gamma)} H_1}{Z^{(\delta+\gamma)} H_1 \left(Z^{(\delta+\gamma)} H_2^1\right)^{1-(\delta+\gamma)}}}{Z^{(\delta+\gamma)} H_2^1 \left(Z^{(\delta+\gamma)} H_2^1\right)^{1-(\delta+\gamma)}}
\]

so \(P_{X_{H1}}\) and \(P_{X_{H2}}\) are

\[
P_{X_{H2}} = \left(1 - \epsilon\right) \frac{\epsilon}{1-\epsilon} \left[\frac{P_{G_{H2}}}{P_{G_{H1}}} \right]^{1/\lambda} \frac{Z^{(\delta+\gamma)} H_2^1 Z^{1-(\delta+\gamma)} H_1}{Z^{(\delta+\gamma)} H_1 \left(Z^{(\delta+\gamma)} H_2^1\right)^{1-(\delta+\gamma)}}
\]

(3.37)

\[
P_{X_{H2}} = \left(1 - \epsilon\right) \frac{\epsilon}{1-\epsilon} \left[\frac{P_{G_{F2}}}{P_{G_{H1}}} \right]^{1/\lambda} \frac{Z^{(\delta+\gamma)} H_2^1 Z^{1-(\delta+\gamma)} H_1}{Z^{(\delta+\gamma)} H_1 \left(Z^{(\delta+\gamma)} H_2^1\right)^{1-(\delta+\gamma)}}
\]

(3.38)

and,

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\[ P_{X_{H1}} = \epsilon \left\{ \frac{\epsilon}{1 - \epsilon} \left[ \frac{P_{G_{H1}}}{P_{G_{H2}}} \right] - \frac{Z_{H2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)}}{Z_{H1}^{(\delta+\gamma)} (Z_{H2})^{1-(\delta+\gamma)}} \right\}^{1-\lambda} \epsilon^{-1} \tag{3.39} \]

since \( P_{H2}/Z_{H2}^{(\delta+\gamma)(1-\lambda)} = P_{F2}/Z_{F2}^{(\delta+\gamma)(1-\lambda)} \), so \( P_{X_{H1}} \) also equals the following,

\[ P_{X_{H1}} = \epsilon \left\{ \frac{\epsilon}{1 - \epsilon} \left[ \frac{P_{G_{H1}}}{P_{G_{F2}}} \right] - \frac{Z_{F2}^{(\delta+\gamma)} Z_{F1}^{1-(\delta+\gamma)}}{Z_{F1}^{(\delta+\gamma)} (Z_{F2})^{1-(\delta+\gamma)}} \right\}^{1-\lambda} \epsilon^{-1} \tag{3.40} \]

Now we want to see the labor allocation in home country. Recall that under autarky or complete specialization, the total amount of labor in industry 1 over that in industry 2 is \( \frac{\epsilon}{1-\epsilon} \) according to the property of Cobb-Douglas production function. We want to see if it still holds under incomplete specialization.

Processed good producers in industry 2 of home country use both domestic and foreign good,

\[ X_{H2} = \int_0^{N_{H2}} G_{H2}^\lambda (Z_{H2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} l_{H2H})^{1-\lambda} dj \tag{3.41} \]

\[ + \int_0^{N_{F2}} (G_{F2}^I)^\lambda (Z_{F2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} l_{H2F})^{1-\lambda} dj \tag{3.42} \]

Denote \( X_{H2H} \equiv \int_0^{N_{H2}} G_{H2}^\lambda (Z_{H2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} l_{H2H})^{1-\lambda} dj \),

and \( X_{H2F} \equiv \int_0^{N_{F2}} (G_{F2}^I)^\lambda (Z_{F2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} l_{H2F})^{1-\lambda} dj \). Plug demand functions (3.31) and (3.32) inside and impose symmetry we get
\[ X_{H2H} = N_H2 \left( \frac{{\lambda P_{X_{H2}}}}{P_{GH2}} \right)^\frac{1}{\lambda} (Z_{H2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} l_{H2H})^\lambda (Z_{H2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} l_{H2H})^{1-\lambda} \]

\[ = \left( \frac{{\lambda P_{X_{H2}}}}{P_{GH2}} \right)^\frac{1}{\lambda} Z_{H2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} L_{H2H} \]  

(3.43)

where \( L_{H2H} \equiv N_{H2} l_{H2H} \)

\[ X_{H2F} = \left( \frac{{\lambda P_{X_{H2}}}}{P_{GF2}} \right)^\frac{1}{\lambda} Z_{F2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} L_{H2F} \]  

(3.44)

where \( L_{H2F} \equiv N_{F2} l_{H2F} \)

Given the trade pattern condition that in industry 2, foreign and domestic intermediate goods have the same quality-adjusted prices, i.e., \( \frac{Z_{H2}^{(\delta+\gamma)}}{P_{GH2}} = \frac{Z_{F2}^{(\delta+\gamma)}}{P_{GF2}} \), take a ratio of (3.43) and (3.44), we have,

\[ \frac{X_{H2H}}{X_{H2F}} = \frac{L_{H2H}}{L_{H2F}} \]  

(3.45)

Recall that \( L_{H2H} + L_{H2F} = L_{H2} \), which is the total amount of labor in industry 2. And \( X_{H2H} + X_{H2F} = X_{H2} \), thus, \( X_{H2H} = \frac{L_{H2H}}{L_{H2}} X_{H2} \) and \( X_{H2F} = \frac{L_{H2F}}{L_{H2}} X_{H2} \). Both \( X_{H2H} \) and \( X_{H2F} \) devote \( \lambda \) proportion resource to pay for the labor, \( L_{H2H} \) and \( L_{H2F} \). Thus \( X_{H2} \) devotes \( \lambda \) proportion resource to compensate the labor \( L_{H2} = L_{H2H} + L_{H2F} \).

By resource allocation of Cobb-Douglass production, we get the same result as under complete specialization, that the labor allocation across two industries is,
\[
\frac{L_{H1}}{L_{H2H} + L_{H2F}} = \frac{\epsilon}{1 - \epsilon} \tag{3.46}
\]

Given the trade pattern condition, \( P_{H2}/Z_{H2}^{(\delta + \gamma)(1 - \lambda)} = P_{F2}/Z_{F2}^{(\delta + \gamma)(1 - \lambda)} \), and apply symmetry among the same industry, we can also write (3.41) as,

\[
X_{H2} = \left( \frac{\lambda P_{X_{H2}}}{P_{G_{H2}}} \right)^{\frac{1}{(\delta + \gamma)}} Z_{H2}^{(\delta + \gamma)} Z_{H1}^{1-(\delta + \gamma)} (L_{H2H} + L_{H2F}) \tag{3.47}
\]

\[
= \left( \frac{\lambda P_{X_{H2}}}{P_{G_{H2}}} \right)^{\frac{1}{(\delta + \gamma)}} Z_{F2}^{(\delta + \gamma)} Z_{H1}^{1-(\delta + \gamma)} (L_{H2H} + L_{H2F}) \tag{3.48}
\]

In this section, we solve the reversed demand for processed good \( X_{H1} \) and \( X_{H2} \). We show the demand functions for intermediate goods and labors. We also express the prices of processed good, \( P_{X_{H1}} \) and \( P_{X_{H2}} \) only on the ratio of intermediate-good prices and the ratio of quality levels across industries. At the end we derive the labor allocation across industries, which are the same as under autarky and complete specialization.

3.3.1.2 Foreign Country: Final Good and Processed Good

Foreign country completely specializes in class-2 intermediates, and imports class-1 intermediates. So the model set-up of this country is similar to the case under
complete specialization. We show the demand functions, and also the labor allocations in this country.

The final good production function is the same as home country.

\[ Y_F = X_{F1}^{1-\epsilon} \]

The numeraire is the final good in home country, so the price of final good in foreign country, \(Y_F\) is \(P_{Y_F}\). The production function of processed goods are,

\[
X_{F1} = \int_0^{NH1} (G_{F1}^l)^{\lambda}[Z_{H1}^{(\delta+\gamma)} Z_{F2}^{1-(\delta+\gamma)} l_{F1}]^{1-\lambda} \, dj
\]

\[
X_{F2} = \int_0^{NF2} (G_{F2} - G_{F2}^E)^{\lambda}[Z_{F2}^{(\delta+\gamma)} Z_{H1}^{1-(\delta+\gamma)} l_{F2}]^{1-\lambda} \, dj
\]

Foreign country imports \(G_{F1}^l\) from home country, and \(G_{F1}^l = G_{H1}^E\). Foreign country export \(G_{F2}^E\) to home country, and \(G_{F2}^E = G_{F2}^l\).

Inverse demand for \(X_{F1}\) and \(X_{F2}\) are (rescaled by \(P_{Y_F}\))

\[
P_{X_{F1}} = P_{Y_F} \epsilon X_{F1}^{\epsilon-1} X_{F2}^{1-\epsilon} \quad (3.49)
\]

\[
P_{X_{F2}} = P_{Y_F} (1 - \epsilon) X_{F1}^{\epsilon} X_{F2}^{\epsilon} \quad (3.50)
\]

Demands for intermediate goods are
\[ G_{H1}^E = G_{F1}^l = \left[ \frac{\lambda P_{X1}}{P_{G1}} \right]^{\frac{1}{\lambda + \gamma}} Z_{H1}^{(\delta + \gamma)} (Z_{F2})^{1 - (\delta + \gamma)} l_{F1} \]  \hspace{1cm} (3.51)

\[ G_{F2} - G_{F2}^E = \left[ \frac{\lambda P_{X2}}{P_{G2}} \right]^{\frac{1}{\lambda + \gamma}} Z_{F2}^{(\delta + \gamma)} (Z_{H1})^{1 - (\delta + \gamma)} l_{H2} \]  \hspace{1cm} (3.52)

From \( MPL_{F1} = MPL_{F2} \), following the same steps as (3.36), we get

\[ \left( \frac{P_{X1}}{P_{X2}} \right)^{\frac{1}{\lambda + \gamma}} = \left( \frac{P_{G1}}{P_{G2}} \right)^{\frac{\lambda + \gamma}{\lambda + \gamma}} \frac{Z_{F2}^{(\delta + \gamma)}}{Z_{H1}} \frac{Z_{H1}^{1 - (\delta + \gamma)}}{Z_{F2}^{1 - (\delta + \gamma)}} \]  \hspace{1cm} (3.53)

Thus following the similar steps as the derivation of (3.39) and (3.37), we get

\[ P_{X1} = \epsilon \left\{ \frac{\epsilon}{1 - \epsilon} \left[ \left( \frac{P_{G1}}{P_{G2}} \right)^{\frac{1}{\lambda + \gamma}} \frac{Z_{F2}^{(\delta + \gamma)}}{Z_{H1}} \frac{Z_{H1}^{1 - (\delta + \gamma)}}{Z_{F2}^{1 - (\delta + \gamma)}} \right]^{1 - (1 - \lambda)} \right\}^{\epsilon - 1} \]  \hspace{1cm} (3.54)

\[ P_{X2} = (1 - \epsilon) \left\{ \frac{\epsilon}{1 - \epsilon} \left[ \left( \frac{P_{G1}}{P_{G2}} \right)^{\frac{1}{\lambda + \gamma}} \frac{Z_{F2}^{(\delta + \gamma)}}{Z_{H1}} \frac{Z_{H1}^{1 - (\delta + \gamma)}}{Z_{F2}^{1 - (\delta + \gamma)}} \right]^{1 - (1 - \lambda)} \right\}^{\epsilon} \]  \hspace{1cm} (3.55)

The labor allocation is exactly the same as under complete specialization,

\[ \frac{N_{H1} l_{F1}}{N_{F2} l_{F2}} = \frac{L_{F1}}{L_{F2}} = \frac{\epsilon}{1 - \epsilon} \]  \hspace{1cm} (3.56)
3.3.1.3 Home Country: Class-1 Intermediate Good Firms

Class-1 intermediate good firms are facing not only domestic demand, but also foreign demand. Combine (3.30) and (3.51), the total demand is,

\[ G_{H1} = (G_{H1} - G_{H1}^E) + G_{F1}^I \]

\[ = \left( \frac{\lambda P_X}{P_G H_1} \right) \frac{1}{1-\delta} Z^{(\delta+\gamma)}(Z_{H2})^{1-(\delta+\gamma)} l_{H1} + \left( \frac{\lambda P_X}{P_G H_1} \right) \frac{1}{1-\delta} Z^{(\delta+\gamma)}(Z_{F2})^{1-(\delta+\gamma)} l_{F1} \]

\[ \Rightarrow G_{H1} = P_{G_{H1}}^{-1} \left[ (\lambda P_X H_1) \frac{1}{1-\delta} Z^{(\delta+\gamma)}(Z_{H2})^{1-(\delta+\gamma)} l_{H1} + (\lambda P_X F_1) \frac{1}{1-\delta} Z^{(\delta+\gamma)}(Z_{F2})^{1-(\delta+\gamma)} l_{F1} \right] \]

\[ (3.57) \]

The cash flow is

\[ F_{H1} = G_{H1}(P_{G_{H1}} - A_{H1}) - \theta_{H1} \frac{Z_{H1}^2}{Z_{H2}} \]

Maximize current value hamiltonian is

\[ CV_{H1} = G_{H1}(P_{G_{H1}} - A_{H1}) - \theta_{H1} \frac{Z_{H1}^2}{Z_{H2}} - R_{H1} + q_{H1}(\alpha_{H1} R_{H1}) \]

s.t. Demand of \( G_{H1}, (3.57) \); and \( \dot{Z}_{H1} = \alpha_{H1} R_{H1} \)

Following the similar steps as under complete specialization, we get the necessary conditions are:

- \( \dot{Z}_{H1} = \alpha_{H1} R_{H1} \)

- \( \frac{\partial CV_{H1}}{\partial P_{G_{H1}}} = 0 \Rightarrow P_{G_{H1}} = \frac{A_{H1}}{\lambda} \)
\( \partial CV_{H1} \over \partial R_{H1} = -1 + q_{H1} \alpha_{H1} \Rightarrow \begin{cases} 
 q_{H1} < \frac{1}{\alpha_{H1}}, & R_{H1} = 0 \\
 q_{H1} = \frac{1}{\alpha_{H1}}, & R_{H1} > 0 \text{(we focus on this)} \\
 q_{H1} > \frac{1}{\alpha_{H1}}, & R_{H1} = \infty 
\end{cases} \)

\( Z_{H1,t=0} \) is given

\( \lim_{t \to \infty} e^{-\int_0^t r(s) ds} q_{H1}(t) Z_{H1}(t) = 0 \)

\( q_{ij} = -r_{ij} q_{ij} + \partial CV_{H1j} \over \partial Z_{ij} \Rightarrow r_{H1} = \partial F_{H1} \over \partial Z_{H1} \over q_{H1} + \dot{q}_{H1} \)

\[ \implies r_{H1} = \delta f_{H1}(Z_{H1} \over Z_{H2}, Z_{H1} \over Z_{F2}, N_{H1}) \tag{3.58} \]

\( f_{H1} = \alpha_{H1} \lambda \over 1 - \lambda A_{H1} Z_{H1}^{(\delta+\gamma)-1} \{ (\lambda P_{XH1} \over A_{H1} \lambda )^{1-\gamma} (Z_{H2})^{1-(\delta+\gamma)} l_{H1} + (\lambda P_{XF1} \over A_{H1} \lambda )^{1-\gamma} (Z_{F2})^{1-(\delta+\gamma)} l_{F1} \} \)

and \( l_{H1} = \frac{e L_{H1}}{N_{H1}}, \ l_{F1} = \frac{e L_{F1}}{N_{H1}} \).

\( P_{XH1} \) is a function of unit costs and \( Z_{H1} \over Z_{H2}, Z_{H1} \over Z_{F2} \), see (3.39);

\( P_{XF1} \) is a function of unit costs and \( Z_{H1} \over Z_{F2} \), see (3.54).

Zero entry/exit cost ensures zero profit condition, so

\[ g_{H1} = \frac{\alpha_{H1} R_{H1}}{Z_{H1}} = \frac{\alpha_{H1} F_{H1}}{Z_{H1}} = f_{H1}(Z_{H1} \over Z_{H2}, Z_{H1} \over Z_{F2}, N_{H1}) - \alpha_{H1} \theta_{H1} \over Z_{H2} \tag{3.59} \]

where \( l_{H1} = \frac{e L_{H1}}{N_{H1}}, \ l_{F1} = \frac{e L_{F1}}{N_{H1}}, \) and
\[ f_{H1} = \alpha_{H1} \frac{1 - \lambda}{\lambda} A_{H1} Z_{H1}^{(\delta + \gamma) - 1} \left\{ (\frac{\lambda P_{X_{H1}}}{A_{H1}/\lambda})^{\frac{1}{1-\lambda}} (Z_{H2})^{1-(\delta + \gamma)} l_{H1} + (\frac{\lambda P_{X_{F1}}}{A_{H1}/\lambda})^{\frac{1}{1-\lambda}} (Z_{F2})^{1-(\delta + \gamma)} l_{F1} \right\}. \]

### 3.3.1.4 Home Country: Class-2 Intermediate Good Firms

Intermediate good firms in industry 2 of home country produces the goods that is only used by home country.

The cash flow is \( F_{H2} = G_{H2}(P_{G_{H2}} - A_{H2}) - \theta_{H2} \frac{Z_{H2}^2}{Z_{H1}} \)

Maximize current value hamiltonian is

\[
CVH_{H2} = G_{H2}(P_{G_{H2}} - A_{H2}) - \theta_{H2} \frac{Z_{H2}^2}{Z_{H1}} - R_{H2} + q_{H2}(\alpha_{H2} R_{H2})
\]

s.t. Demand of \( G_{H2}, (3.31) \), and \( \dot{Z}_{H2} = \alpha_{H2} R_{H2} \)

Following similar steps as under complete specialization, we get the necessary conditions are:

- \( \dot{Z}_{H2} = \alpha_{H2} R_{H2} \)
- \( \frac{\partial CVH_{H2}}{\partial P_{G_{H2}}} = 0 \Rightarrow P_{G_{H2}} = \frac{A_{H2}}{\lambda} \)
- \( \frac{\partial CVH_{H2}}{\partial R_{H2}} = -1 + q_{H2} \alpha_{H2} \Rightarrow \begin{cases} q_{H2} < \frac{1}{\alpha_{H2}}, & R_{H2} = 0 \\ q_{H2} = \frac{1}{\alpha_{H2}}, & R_{H2} > 0 \text{ (we focus on this)} \\ q_{H2} > \frac{1}{\alpha_{H2}}, & R_{H2} = \infty \end{cases} \)
- \( Z_{H2,t=0} \) is given
\[ \lim_{t \to \infty} e^{-\int_0^t r(s) \, ds} q_{H2}(t) Z_{H2}(t) = 0 \]

- where \( f_{H2} = \alpha_{H2} \lambda A_{H2} \left( \frac{\lambda^2 P_{XH2}}{A_{H2}} \right)^{-\frac{1}{\lambda}} \left( \frac{Z_{H2}}{Z_{H1}} \right)^{(\delta + \gamma) - 1} l_{H2H} \),

- \( l_{H2H} = \frac{L_{H2H}}{N_{H2}} \), and \( L_{H2H} = (1 - \epsilon)L_H - L_{H2F} \). \( L_{H2F} \) is a function of \( \frac{Z_{F2}}{Z_{H2}} \)
  by trade balance condition, see (3.71).

- \( P_{XH2} \) is a function of \( \frac{Z_{H1}}{Z_{F1}} \) and \( \frac{Z_{H1}}{Z_{F2}} \) in (3.37).

Zero entry/exit cost implies zero profit condition, thus

\[ g_{H2} = \frac{\alpha_{H2} R_{H2}}{Z_{H2}} = \frac{\alpha_{H2} F_{H2}}{Z_{H2}} = f_{H2} \left( \frac{Z_{H1}}{Z_{H2}}, \frac{Z_{H1}}{Z_{F2}}, \frac{Z_{F2}}{Z_{H2}}, N_{H2} \right) - \alpha_{H2} \theta_{H2} \frac{Z_{H2}}{Z_{H1}} \]  (3.61)

### 3.3.1.5 Foreign Country: Class-2 Intermediate Good Firms

Firms in industry 2 of foreign country produces the goods that are used by both home and foreign countries. Combine (3.32) and (3.52), the total demand is
\[ G_{F_2} = \left[ \lambda \frac{1}{P_{G_{F_2}}} \right]^{\frac{1}{1-\delta}} Z_{F_2}^2 (Z_{H_1})^{1-\delta} \left( P_{X_{H_2}F_2}^{1-\delta} + P_{Y_{H_2}F_2}^{1-\delta} \right) \] (3.62)

The cash flow is \( F_{F_2} = G_{F_2} (P_{G_{F_2}} - P_{Y_{F_2}A_{F_2}}) - P_{Y_{F_2}} P_{F_2} \frac{Z_{H_2}^2}{Z_{H_1}^2} \), and current value Hamiltonian is

\[
CVH_{F_2} = G_{F_2} (P_{G_{F_2}} - P_{Y_{F_2}A_{F_2}}) - P_{Y_{F_2}} P_{F_2} \frac{Z_{H_2}^2}{Z_{H_1}^2} - P_{Y_{F_2}} R_{F_2} + q_{F_2} (\alpha_{F_2} R_{F_2})
\]

s.t. demand function (3.62); and \( \dot{Z}_{F_2} = \alpha_{F_2} R_{F_2} \)

The necessary conditions are,

- \( \dot{Z}_{F_2} = \alpha_{F_2} R_{F_2} \);
- \( \frac{\partial CVH_{F_2}}{\partial P_{G_{F_2}}} = 0 \Rightarrow P_{G_{F_2}} = P_{Y_{F_2} A_{F_2}} \);
- \( \frac{\partial CVH_{F_2}}{\partial R_{F_2}} = -P_{Y_{F_2}} + q_{F_2} \alpha_{F_2} \Rightarrow \text{Interior solution : } q_{2F} = \frac{P_{Y_{F_2}}}{\alpha_{F_2}} \);
- \( Z_{F_2,t=0} \) is given
- \( \lim_{t \to \infty} e^{-\int_0^t r(s) ds} q_{F_2j}(t) Z_{F_2j}(t) = 0 \)
- \( q_{F_2} = -r_{F_2} q_{F_2} + \frac{\partial CVH_{F_2}}{\partial Z_{F_2}} \Rightarrow r_{F_2} = \frac{\partial F_{F_2}}{\partial Z_{F_2}} / q_{F_2} + q_{F_2} \)

\[ r_{F_2} = \delta f_{F_2} \left( \frac{Z_{H_1}}{Z_{F_2}}, \frac{Z_{H_2}}{Z_{F_2}}, \frac{Z_{H_1}}{Z_{F_1}}, N_{F_2} \right) \] (3.63)
where \( f_{F2} = \alpha F_2 A_{F2} \frac{1-\lambda}{\lambda} \left( \frac{\lambda^2}{P_{YF} A_{F2}} \right)^{1/\lambda} \left( \frac{Z_{F2}}{Z_{H1}} \right)^{(\delta+\gamma)-1} (P_{X_{F2}} l_{F2} + P_{X_{H2}} l_{H2F}); \)

- \( P_{X_{F2}} \) is a function of \( \frac{Z_{H1}}{Z_{F2}} \) in (3.55);
- \( P_{X_{H2}} \) is a function of \( \frac{Z_{H1}}{Z_{H2}} \) and \( \frac{Z_{H1}}{Z_{F2}} \) in (3.37);
- \( P_{Y_{F}} \) is a function of \( \frac{Z_{H2}}{Z_{F2}} \) by trade pattern condition; \( l_{F2} = \frac{(1-\epsilon)L_{F}}{N_{F2}} \); \( l_{H2F} = \frac{L_{H2F}}{N_{F2}} \), where \( L_{H2F} \) is a function of \( \frac{Z_{H2}}{Z_{F2}} \) from (3.71).

Free entry/exit implies zero profit condition, so

\[
g_{F2} = \frac{\alpha F_2 R_{F2}}{Z_{F2}} = \frac{\alpha F_2 F_{F2}}{Z_{F2}} = f_{F2} \left( \frac{Z_{H1}}{Z_{F2}}, \frac{Z_{H2}}{Z_{F2}}, \frac{Z_{H1}}{Z_{F1}}, N_{F2} \right) - \alpha F_2 \theta_{F2} \frac{Z_{F2}}{Z_{H1}} (3.64)
\]

### 3.3.1.6 Trade balance condition

Trade balance condition is \( N_{H1} G^{E}_{H1} P_{G_{H1}} = N_{F2} G^{I}_{F2} P_{G_{F2}} \). The value of exports of home country equals the value of imports of home country. We can also see this in another way. The total value of home country’s export is the value of imports of foreign country. Foreign country gives \( P_{Y_{F}} Y_{F} \epsilon \lambda \) resource to compensate \( N_{H1} G^{E}_{H1} P_{G_{H1}} \). The total value of home country’s import is in industry 2, and the total resource compensated is \( Y_{H}(1-\epsilon) \lambda \frac{L_{H2F}}{L_{H2}} \).

So the trade balance condition can be written as,

\[
P_{Y_{F}} Y_{F} \epsilon \lambda = Y_{H}(1 - \epsilon) \lambda \frac{L_{H2F}}{L_{H2}} \quad (3.65)
\]
We know \( P_{Y_F} = \frac{A_{F2}/Z_{F2}}{A_{F2}/Z_{F2}}^{(\delta+\gamma)(1-\lambda)} \) and \( Z_{F2}^{(\delta+\gamma)} = \frac{Z_{H2}}{P_{G_F2}^{\lambda \frac{1}{1-\lambda}}} \) from (3.24), and following the similar steps as derivation for the final output under autarky, equation (C.17) in Appendix, we get the final output for both home and foreign countries.

\[
Y_{H}^{\text{Trade}} = \kappa' \left[ \left( \frac{Z_{H1}^{\delta+\gamma}}{P_{G_{H1}}^{\frac{\lambda}{1-\lambda}}} \right) \left( \frac{Z_{H2}^{1-(\delta+\gamma)}}{P_{Y_F}^{\frac{\lambda}{1-\lambda}}} \right) (\epsilon L_H) \right]^\lambda \left[ \left( \frac{Z_{F2}^{\delta+\gamma}}{P_{G_{F2}}^{\frac{\lambda}{1-\lambda}}} \right) Z_{H1}^{1-(\delta+\gamma)} ((1 - \epsilon)L_H) \right]^{1-\epsilon} \quad (3.66)
\]

\[
Y_{F}^{\text{Trade}} = \kappa' \left[ \left( \frac{Z_{H1}^{\delta+\gamma}}{P_{G_{H1}}^{\frac{\lambda}{1-\lambda}}} \right) Z_{F2}^{1-(\delta+\gamma)} P_{Y_F}^{\frac{\lambda}{1-\lambda}} (\epsilon L_F) \right]^\lambda \left[ \left( \frac{Z_{F2}^{\delta+\gamma}}{P_{G_{F2}}^{\frac{\lambda}{1-\lambda}}} \right) Z_{F1}^{1-(\delta+\gamma)} P_{Y_F}^{\frac{\lambda}{1-\lambda}} ((1 - \epsilon)L_F) \right]^{1-\epsilon} \quad (3.67)
\]

where \( \kappa' = \lambda^{\frac{\lambda}{1-\lambda}} (1 - \epsilon)^{\frac{\lambda(1-\epsilon)}{1-\lambda}} \epsilon^{\frac{\lambda}{1-\lambda}} \)

Plug (3.66), (3.67) and \( P_{Y_F} = \frac{A_{H2}/Z_{H2}^{\lambda \frac{1}{1-\lambda}}}{A_{F2}/Z_{F2}^{\lambda \frac{1}{1-\lambda}}} \) into (3.65), then
$$L_{H2F} = \frac{\epsilon}{1 - \epsilon} P_{Y_F} Y_H L_{H2}$$

(3.68)

$$= \frac{\epsilon}{1 - \epsilon} P_{Y_F} P_{Y_F}^{\lambda} \left[ \left( \frac{Z_{H2}}{Z_{F2}} \right)^{1-(\delta+\gamma)} \right] \epsilon \frac{L_{F}}{L_{H}} (1 - \epsilon)L_{H}$$

(3.69)

$$= \epsilon L_{F} P_{Y_F}^{\frac{1}{1-\lambda}} \left[ \frac{Z_{H2}^{1-(\delta+\gamma)}}{Z_{F2}^{1-(\delta+\gamma)}} \right]$$

(3.70)

$$= \epsilon L_{F} \left( \frac{A_{H2}}{A_{F2}} \right)^{\frac{1}{1-\lambda}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^{\frac{\delta+\gamma}{\lambda}} \left[ \frac{Z_{H2}^{1-(\delta+\gamma)}}{Z_{F2}^{1-(\delta+\gamma)}} \right]$$

(3.71)

$$= \epsilon L_{F} \left( \frac{A_{H2}}{A_{F2}} \right)^{\frac{1}{1-\lambda}} \left( \frac{Z_{H2}}{Z_{F2}} \right)^{-(\delta+\gamma)} + \epsilon \eta [1-(\delta+\gamma)]$$

(3.72)

So $L_{H2F}$ is a function of $\frac{Z_{H2}}{Z_{F2}}$. As $\frac{Z_{H2}}{Z_{F2}}$ changes, the amount of labors who are using foreign goods in industry 2 of home country also changes to keep the trade balanced.

Notice that $L_{H2F} \subseteq [0, (1 - \epsilon)L_{H}]$, so $L_{H2F}$ cannot goes to infinity. When $L_{H2F} = (1 - \epsilon)L_{H}$, which according to above derivations, means $\epsilon L_{F} \left( \frac{A_{H2}}{A_{F2}} \right)^{\frac{1}{1-\lambda}} \left( \frac{Z_{H2}}{Z_{F2}} \right)^{-(\delta+\gamma)} + \epsilon \eta [1-(\delta+\gamma)] = (1 - \epsilon)L_{H}$. When $\eta = 0$, which means the spillover of domestic good is 0, it is equivalent to the boundary condition of equation (3.24), i.e. $\left[ \frac{1-(1-\epsilon)L_{H}}{\epsilon L_{F}} \right]^{1-\lambda} = \frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^{\frac{\delta+\gamma}{1-\lambda}}$. In this case, we can say home country is also under complete specialization, so all workers in industry 2 are using the foreign goods, thus $\eta = 0$ and $L_{H2F} = (1 - \epsilon)L_{H} \equiv L_{H2}$.
3.3.2 Level Effect under Incomplete Specialization

At the moment trade opens, for home country, the output level effect is similar, but not exactly the same as that under complete specialization. The home country imports class-2 intermediates and hence gets $Z_{F2}$ from the foreign country, which causes an externality to the home country’s processed goods industry $X_1$. Because the home country uses two types of intermediate good 2, the spillover to industry 1 is a combination of the two quality levels $Z_{H2}$ and $Z_{F2}$. As mentioned previously, we assume the combination is the weighting average $\hat{Z}_{H2} = Z_{H2}^{\eta}Z_{F2}^{1-\eta}$. Home country’s final output under autarky and trade are\(^{11}\),

$$Y_{H}^{\text{Trade}} = \kappa' \left[ \left( \frac{Z_{H1}^{(\delta+\gamma)}}{P_{G_{H1}}^{1-\lambda}} \right) \left( Z_{H2}^{1-(\delta+\gamma)}(\epsilon L_H) \right)^{\epsilon} \left( \frac{Z_{F2}^{(\delta+\gamma)}}{P_{G_{F2}}^{1-\lambda}} \right) Z_{H1}^{1-(\delta+\gamma)}((1-\epsilon)L_H) \right]^{1-\epsilon}$$

$$Y_{H}^{\text{Autarky}} = \kappa' \left[ \left( \frac{Z_{H1}^{(\delta+\gamma)}}{P_{G_{H1}}^{1-\lambda}} \right) (Z_{H2}^{1-(\delta+\gamma)}(\epsilon L_H)) \right]^{\epsilon} \left[ \left( \frac{Z_{H2}^{(\delta+\gamma)}}{P_{G_{F2}}^{1-\lambda}} \right) Z_{H1}^{1-(\delta+\gamma)}((1-\epsilon)L_H) \right]^{1-\epsilon}$$

where $\kappa' = \lambda^{\frac{\lambda}{1-\lambda}}(1-\epsilon)^{\frac{(1-\epsilon)}{1-\lambda}}\epsilon^{\frac{\lambda}{1-\lambda}}$. Trade condition requires the domestic and foreign class-2 intermediates have the same quality-adjusted prices, $\frac{Z_{H2}^{(\delta+\gamma)}}{P_{G_{H2}}^{1-\lambda}} = \frac{Z_{F2}^{(\delta+\gamma)}}{P_{G_{F2}}^{1-\lambda}}$. Thus at the moment that trade happens, the second term in the final output are the

\(^{11}\)See similar derivations in Appendix (B.4).
same for home country. The difference is the first term. In autarky, the spillover from industry 2 to industry 1 is \((Z_{H2})^{1-(\delta+\gamma)}\). After trade, the spillover is \( (Z_{H2}^\eta Z_{F2}^{1-\eta})^{1-(\delta+\gamma)} \). Thus the sufficient and necessary condition for trade to increase initial output level for home is \( Z_{F2} > Z_{H2} \). The sufficient and necessary condition for trade to decrease initial output is \( Z_{F2} < Z_{H2} \). The level effect for home country only depends on the externality, since home imports the goods with the same quality adjusted price as the domestic goods.

Foreign country completely specializes in class-2 intermediates and imports good 1 from home. Thus the level effect is exactly like the case under complete specialization. In one hand, comparative advantage ensures a lower quality-adjusted price for the imports; in the other hand, the quality of the imports could have negative externality to the other industry. The sufficient and necessary conditions for the changes in final output are exactly the same as under the complete specialization.

### 3.3.3 Growth Rate Effect under Incomplete Specialization

Following the similar way as closed economy but more complicated, we get the growth rates of outputs for both countries under incomplete specialization are both a certain weighted average of growth rate of \( Z_{H1} \), \( Z_{H2} \) and \( Z_{F2} \):

\[ 12 \text{ See Appendix (D.2) for derivations.} \]
\[
\frac{Y_{H}^{\text{Trade}}}{Y_{H}^{\text{Trade}}} = \Gamma \frac{Z_{H1}}{Z_{H1}} + \{\eta[1-(\delta+\gamma)]\epsilon + (\delta+\gamma)(1-\epsilon)\} \frac{Z_{H2}}{Z_{H2}} + \{(1-\eta)[1-(\delta+\gamma)]\epsilon\} \frac{Z_{F2}}{Z_{F2}} \quad (3.73)
\]

where \( \Gamma = 2(\delta+\gamma)\epsilon + 1 - \epsilon - (\delta+\gamma) \); and \( \Gamma + \{\eta[1-(\delta+\gamma)]\epsilon + (\delta+\gamma)(1-\epsilon)\} + \{(1-\eta)[1-(\delta+\gamma)]\epsilon\} = 1 \)

\[
\frac{Y_{F}^{\text{Trade}}}{Y_{F}^{\text{Trade}}} = \Gamma \frac{Z_{H1}}{Z_{H1}} - (\delta+\gamma)\epsilon \frac{Z_{H2}}{Z_{H2}} + \{(1-(\delta+\gamma))\epsilon + \delta\} \frac{Z_{F2}}{Z_{F2}} \quad (3.74)
\]

where \( \Gamma + [- (\delta+\gamma)\epsilon] + \{(1-(\delta+\gamma))\epsilon + \delta\} = 1 \)

Qualities \( Z_{H1} \), \( Z_{H2} \) and \( Z_{F2} \) enter the the growth rates of final good productions by different ways for different countries under incomplete specialization, so the combinations of the weights of \( Z_{H1} \), \( Z_{H2} \) and \( Z_{F2} \) are different for growth rates of \( Y_{H} \) and \( Y_{F} \). For example, \( Z_{H2} \) enters the growth rate of \( Y_{H} \) because home produces class-2 intermediates by itself. And \( Z_{H2} \) enters the growth rate of \( Y_{F} \) even foreign country does not import class-2 intermediates from home. Because \( Z_{H2} \) affects the accumulation of \( Z_{H1} \) back in home country, and \( Z_{H1} \) affects the output of foreign country through foreign country’s imports.

Combine with the households’ choice, on balanced growth path:

\[
\frac{Y_{H}^{\text{Trade}}}{Y_{H}^{\text{Trade}}} = \frac{Y_{F}^{\text{Trade}}}{Y_{F}^{\text{Trade}}} = \frac{Z_{H1}}{Z_{H1}} = \frac{Z_{H2}}{Z_{H2}} = \frac{Z_{F2}}{Z_{F2}} = \frac{\dot{C}_{H}}{C_{H}} = \frac{\dot{C}_{F}}{C_{F}} = r - \rho \quad (3.75)
\]

95
and,

\[
\frac{g^\text{Trade}_H}{g^\text{Trade}_F} = \frac{1}{1-\delta} \sqrt{\alpha_{H1}\theta_{H1}(\alpha_{H2}\theta_{H2})^\eta(\alpha_{F2}\theta_{F2})^{1-\eta} - \rho - \frac{\eta}{1-\delta}} \quad (3.76)
\]

Along the balanced growth path, the two quality ratios \(Z_{H1}/Z_{H2}\) and \(Z_{H2}/Z_{F2}\) are

\[
\left(\frac{Z_{H1}}{Z_{H2}}\right)^* = \left(\frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}}\right)^{-1} \sqrt{\frac{\alpha_{F2}\theta_{F2}}{\alpha_{H1}\theta_{H1}} \left(\frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}}\right)^\eta} \quad (3.77)
\]

\[
\left(\frac{Z_{H2}}{Z_{F2}}\right)^* = \frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}} \quad (3.78)
\]

Recall that \(\eta\) and \((1-\eta)\) are the weights of \(Z_{H2}\) and \(Z_{F2}\) in the spillover from industry 2 to industry 1, which is \(\hat{Z}_{H2} = Z_{H2}^\eta Z_{F2}^{1-\eta}\).

Comparing equation (3.76) with the autarky growth rate given in equation (2.68), we see the following effects of trade under incomplete specialization:

1. The home country’s growth rate increases if the foreign country has a higher R&D ability in the good that the home country imports, i.e., if \(\alpha_{F2}\theta_{F2} > \alpha_{H2}\theta_{H2}\). The effect of \(\alpha_{F2}\theta_{F2}\) is small if \((1-\eta)\) is small.

2. The foreign country’s growth rate increases if the home country has a higher R&D ability in the good that the foreign country imports, i.e., if \(\alpha_{H1}\theta_{H1} > \alpha_{F1}\theta_{F1}\) and/or \(\alpha_{H2}\theta_{H2} > \alpha_{F2}\theta_{F2}\). The home country’s R&D ability in class-2 intermediates enters the growth rate of the foreign country, even though the foreign country does
not import that good. The reason is that \( Z_{H2} \) affects the accumulation of \( Z_{H1} \) which is embodied in the \( G_{H1} \) good that the foreign country does import.

### 3.3.4 Transition Dynamics under incomplete specialization\(^{13}\)

As in the case of complete specialization, the growth rates of home and foreign income along the transition path are weighted averages of the quality level growth rates:

\[
\frac{\dot{Y}^\text{Trade}_H}{Y^\text{Trade}_H} = \Gamma \frac{\dot{Z}_{H1}}{Z_{H1}} + \eta \left[ 1 - (\delta + \gamma) \right] \epsilon + (\delta + \gamma) (1 - \epsilon) \frac{\dot{Z}_{H2}}{Z_{H2}} + \left( (1 - \eta) \left[ 1 - (\delta + \gamma) \right] \epsilon \right) \frac{\dot{Z}_{F2}}{Z_{F2}}
\]

(3.79)

\[
\frac{\dot{Y}^\text{Trade}_F}{Y^\text{Trade}_F} = \Gamma \frac{\dot{Z}_{H1}}{Z_{H1}} - (\delta + \gamma) \epsilon \frac{\dot{Z}_{H2}}{Z_{H2}} + \left( [1 - (\delta + \gamma)] \epsilon + \delta \right) \frac{\dot{Z}_{F2}}{Z_{F2}}
\]

(3.80)

where as before \( \Gamma = (2\epsilon - 1)(\delta + \gamma) + 1 - \epsilon \). Comparing these growth rates with the corresponding growth rates under complete specialization given in equation (3.22) reveals two notable differences. First, the income growth rates under incomplete specialization are weighted averages of the three quality growth rates \( \dot{Z}_{H1}/Z_{H1}, \dot{Z}_{H2}/Z_{H2} \), and \( \dot{Z}_{F2}/Z_{F2} \), whereas under complete specialization they are weighted averages of just two, \( \dot{Z}_{H1}/Z_{H1} \) and \( \dot{Z}_{F2}/Z_{F2} \). Second, under incomplete specialization the income growth rates are different weighted averages of the quality growth rates, whereas under complete specialization they are the same. Consequently, the transitional income growth rates under incomplete specialization generally differ from each other, whereas

\(^{13}\)The derivations see Appendix (D.3).
they are the same under complete specialization. Using eqn. (3.79) and (3.80), we get the difference between the two income growth rates:

\[
\frac{\dot{Y}^{Trade}}{Y_H^{Trade}} - \frac{\dot{Y}^{Trade}}{Y_F^{Trade}} = [\eta \epsilon - \eta (\delta + \gamma) \epsilon + (\delta + \gamma)] \left( \frac{\dot{Z}_H^{2}}{Z_H^{2}} - \frac{\dot{Z}_F^{2}}{Z_F^{2}} \right) \tag{3.81}
\]

where \( \eta \epsilon - \eta (\delta + \gamma) \epsilon + (\delta + \gamma) > 0 \). The income growth rates differ whenever the growth rates of \( Z_H^{2} \) and \( Z_F^{2} \) differ.

To study the behavior of the economy on the transition path, we need the equations for the growth rates of the quality levels \( Z_{H1}, Z_{H2}, \) and \( Z_{F2} \). As before, those can be expressed as functions of the respective rates of return to R&D \( r_{H1}, r_{H2}, \) and \( r_{F2} \):

\[
g_{H1} \equiv \frac{\dot{Z}_{H1}}{Z_{H1}} = \frac{r_{H1}}{\delta} - \alpha_{H1} \theta_{H1} \frac{Z_{H1}}{Z_H^{1-\eta}} \tag{3.82}
\]

\[
g_{H2} \equiv \frac{\dot{Z}_{H2}}{Z_{H2}} = \frac{r_{H2}}{\delta} - \alpha_{H2} \theta_{H2} \frac{Z_{H2}}{Z_{H1}} \tag{3.83}
\]

\[
g_{F2} \equiv \frac{\dot{Z}_{F2}}{Z_{F2}} = \frac{r_{F2}}{\delta} - \alpha_{F2} \theta_{F2} \frac{Z_{F2}}{Z_{H1}} \tag{3.84}
\]
Combine eqn. (3.81) with the two countries’ Euler equations to get

\[ r_{H1} - r_{F2} = \left[ \eta \epsilon - \eta (\delta + \gamma) \epsilon + (\delta + \gamma) \right] \left( \frac{Z_{H2}}{Z_{H2}} - \frac{Z_{F2}}{Z_{F2}} \right) \]  

(3.85)

Notice that under incomplete specialization, in sharp contrast to the situation under complete specialization, \( r_{H1} \) generally does not equal \( r_{F2} \) because the growth rates of income in the two countries generally are different. It is true, however, that the no-arbitrage condition holds within countries, implying that \( r_{H1} = r_{H2} \). Using that fact together with eqn. (D.15) and (D.17), we get the growth rate of \( Z_{H1}/Z_{H2} \) as

\[ \frac{\dot{Z}_{H1}/Z_{H2}}{Z_{H1}/Z_{H2}} = -\alpha_{H1} \theta_{H1} \left( \frac{Z_{H1}}{Z_{H2}} \right)^{\eta} \left( \frac{Z_{H1}}{Z_{F2}} \right)^{1-\eta} + \alpha_{H2} \theta_{H2} \frac{Z_{H2}}{Z_{H1}} \]  

(3.86)

Combining eqn. (D.20) with (D.17) and (D.18) gives the growth rate of \( Z_{H2}/Z_{F2} \):

\[ \frac{\dot{Z}_{H2}/Z_{F2}}{Z_{H2}/Z_{F2}} = \frac{\delta}{-\eta \epsilon (1-\delta-\gamma) - \gamma} \left( -\alpha_{H2} \theta_{H2} \frac{Z_{H2}}{Z_{H1}} + \alpha_{F2} \theta_{F2} \frac{Z_{F2}}{Z_{H1}} \right) \]  

(3.87)

To simplify notation, define \( u \equiv Z_{H1}/Z_{H2}, \ v \equiv Z_{H1}/Z_{F2}, \) and \( w \equiv Z_{H2}/Z_{F2} \). Note that \( v = u \cdot w \), so the evolution of the world economy is described by the evolution of \( u \) and \( w \). Taking time derivatives of \( u \) and \( w \) and using eqn. (D.22) and (3.87) gives

\[ \dot{u} = -\alpha_{H1} \theta_{H1} u^{2} w^{1-\eta} + \alpha_{H2} \theta_{H2} \]  

(3.88)
These differential equations are non-linear, so we linearize them by Taylor expansion around the steady state values $u^*$ and $w^*$ to obtain

$$
\dot{u} = -2\alpha H_1 \theta H_1 u^* (w^*)^{1-\eta} (u - u^*) - [\alpha H_1 \theta H_1 (1 - \eta) (u^*)^2 (w^*)^{-\eta}] (w - w^*)
$$

(3.90)

$$
\dot{w} = \frac{\delta}{\eta \epsilon (1 - \delta - \gamma) + \gamma} \left( \alpha H_2 \theta H_2 \frac{w}{u} - \alpha F_2 \theta F_2 \frac{1}{u} \right)
$$

(3.89)

These equations are the equilibrium loci $\dot{u} = 0$ and $\dot{w} = 0$. The equilibrium loci $\dot{u} = 0$ and $\dot{w} = 0$ are:

$$
u = -\left( \frac{1 - \eta}{2} \frac{u^*}{w^*} \right) w + (3 - \eta) u^*
$$

(3.92)

$$w = w^*
$$

(3.93)

With the previous results in hand, we can analyze the stability of the world economy under incomplete specialization. The crucial variable turns out to be $w$. Recall
that the trade pattern condition under incomplete specialization is given by eqn. (3.25). Given \( w \equiv Z_{H2}/Z_{F2} \), under incomplete specialization the initial value of \( w \) must satisfy

\[
w > \left\{ \left[ \frac{(1 - \epsilon) L_H}{\epsilon L_Y} \right]^{1-\lambda} \frac{A_{H2}}{A_{F2}} \right\}^{\frac{\lambda}{\lambda + \gamma (1 - \lambda)}}
\]

(3.94)

The evolution of the world economy depends on the relation between the initial value of \( w \) and the steady state value \( w^* \).

Suppose for the moment that \( w^* \) also is larger than the right side of (3.94). Then there are three possible cases.

(1) If \( w = w^* \), then \( w \) is on its equilibrium locus and does not change, and \( u \) converges to \( u^* \). The world economy converges to a balanced growth path with perpetual incomplete specialization.

(2) If \( w < w^* \), then \( \dot{w} < 0 \). At some finite time, \( w \) falls below the right side of (3.94). At that point, the economy switches to complete specialization. Its dynamics cease to be governed by (3.79)-(D.18) but instead become governed by (3.22)-(3.23) discussed earlier. We already have seen that the regime of complete specialization has an asymptotically stable balanced growth path, so once the economy crosses from incomplete to complete specialization, it remains completely specialized.

(3) If \( w > w^* \), then \( \dot{w} > 0 \), and the world economy remains incompletely specialized forever. It also diverges from the balanced growth path of the incomplete specialization regime. The difference of the growth rates of two countries converges
Figure 3.1: Dynamics under Incomplete Specialization

\[ \left( \frac{\dot{Y}_H}{Y_H} - \frac{\dot{Y}_F}{Y_F} \right) \rightarrow \left( \delta + \frac{\delta^2}{\eta \epsilon (1 - \delta - \gamma) + \gamma} \right) \alpha_H^2 \theta_H^2 \frac{1}{w^*} \]

The home country’s growth rate is perpetually above that of the foreign country, and the difference is bounded away from zero. Figure 2 shows the phase diagram for the world economy under incomplete specialization when \( w^* \) is larger than the right side of (3.94).

Finally, it may be that the steady state \( w^* \) is below the right side of eqn. (3.94). Then, if the world finds itself in a state of incomplete specialization when trade opens, it necessarily will be in case (3) above because incomplete specialization requires that

\[^{14}\text{See derivations in Appendix (D.4).} \]
Figure 3.2: Region of Specialization

(3.94) be satisfied.

Figure 3 shows another way to visualize the dynamic behavior of the economy.

The horizontal axis is divided into three sections. The middle section is the region of complete specialization, denoted CS in the Figure, in which the home country does R&D on quality $Z_{H1}$ and the foreign country does R&D on quality $Z_{F2}$. Outside the CS region are the two regions of incomplete specialization, denoted IS in the Figure. In the IS region to the left of CS the home country completely specializes and does R&D on quality $Z_{H1}$, whereas the foreign country remains unspecialized and does R&D on both the qualities $Z_{F1}$ and $Z_{F2}$. In the IS region to the right of CS, the home country is unspecialized and does R&D on $Z_{H1}$ and $Z_{H2}$, whereas the foreign country is specialized and does R&D on $Z_{F2}$. It is the latter IS region that we have analyzed above. The boundaries of the regions depend on the quality ratios $Z_{F1}/Z_{H1}$ and $Z_{F2}/Z_{H2}$. When the quantity $[(1 - \epsilon) L_{H1}/\epsilon L_{F1}]^{1-\lambda}$ is inside the CS region, as at point 1, the two quality levels $Z_{H1}$ and $Z_{F2}$ grow through R&D, and the two quality levels $Z_{H2}$ and $Z_{F1}$ are constant (because no R&D is done on
them). Consequently, the quality ratio $Z_{F1}/Z_{H1}$, and the quality ratio $Z_{F2}/Z_{H2}$ rises, causing the CS boundaries to spread farther apart. As a result, the quantity $\left[(1 - \epsilon) \frac{L_H}{\epsilon L_F}\right]^{1-\lambda}$ remains inside the CS region forever, and the two economies remain completely specialized forever. Behavior is different when $\left[(1 - \epsilon) \frac{L_H}{\epsilon L_F}\right]^{1-\lambda}$ is in one of the IS regions. For example, point 2 corresponds to the case analyzed above, where the home country is not specialized and the foreign country specializes in producing good $G_{F2}$. In that case, R&D is active for the three quality levels $Z_{H1}$, $Z_{H2}$, and $Z_{F2}$, so they all grow, and no R&D is performed on $Z_{F1}$, which therefore is constant. The lower (left) boundary of the CS region moves ever lower as time passes, but the movement of the upper (right) boundary depends on whether $w < w^*$ or $w > w^*$ (assuming for expository ease that $w^*$ itself is in the right IS region). If $w < w^*$, $Z_{F2}$ grows faster than $Z_{H2}$, and the upper boundary of the CS region increases over time, eventually passing point 1 and bringing the world into complete specialization. If $w > w^*$, $Z_{F2}$ grows more slowly than $Z_{H2}$, and the upper boundary of the CS region decreases over time, moving away from point 2 and leaving the world farther and farther inside the IS region.

Lastly, recall that the model is completely symmetric. In eqn. (3.25), we assumed for sake of clarity that $P_{Y_F}$ was at the upper boundary of the interval defining the region of complete specialization. Had it been at the lower boundary, the whole discussion would have been in converse, with the home country incompletely specialized and the foreign country incompletely specialized.
3.4 Conclusion

In this chapter, we study the effects of trade on economic growth in a Schumpete-rian framework. The model excludes scale effects and technology transfer, the two usual channels in the literature through which trade affects growth, leaving only comparative advantage. Comparative advantage and the trading pattern are determined endogenously. Endogeneity of production and trading patterns leads to results quite different from those found in most of the related literature. Trade need not increase initial output of either country because of an externality absent from static models. Irrespective of what happens to initial output, trade may increase the balanced growth rate but also may decrease it. Our model has tractable transition dynamics, which we describe completely. We show that trade leads to a stable world income distribution in some cases, but in other cases leads to an unstable and perhaps even degenerate distribution. In some cases, trade’s effect on a country’s growth rate is the same as if that country had adopted its trading partner’s R&D technology, even though no technology transfer ever occurs.
Chapter 4

Welfare and Tariff Analysis

4.1 Introduction

There’re several possible extensions for the main model. For example, the closed economy model can be modified to discuss about directed technical change. This second-generation Schumpeterian growth model could change certain main results in Acemoglu (2002), which we will discuss more in the next chapter. We can also use this model to give a new look at the long-run implications of taxation and tariffs. Stokey and Rebelo (1995) criticize first-generation growth models because they predict large negative effects of income taxes that are not observed in the data. Peretto (2007) shows that a second-generation growth model, by eliminating the scale effect, also drastically changes the predictions concerning taxes, with some types of taxes
predicted to increase the growth rate, some to have ambiguous effects, and some to reduce the growth rate. Our model adopts certain properties of Peretto (2007), thus generates similar results. We modify the model by adding the labor choice into Utility function, in order to talk about labor income tax later. We will discuss the long run effects of labor income tax, consumption tax, corporate profit tax, and the tariff on the imports. We do not discuss dividend tax, and the tax on capital gain here since a zero-profit condition makes the tax revenues of those always equal to zero. Similar to Peretto (2007), our main results concern equilibrium under the assumptions that government cannot borrow or access to lump-sum taxes, and has to balance the budget all the time. We do two exercises. One is to assume a vector of fixed tax and tariff rates, and government uses all tax revenue for unproductive projects. This exercise allows us to study the distortionary role of tax and tariff rates in isolation. The other exercise is to assume government uses profit tax to subsidize incumbent R&D. This exercise focuses on the trade off between taxes and subsidy. Our results show that all taxes and tariff reduce welfare. Most taxes and tariff only affect level output, and only the tax on profits and R&D subsidy have growth effects. The negative growth effect of profit tax dominates the positive effect of R&D subsidy.  

In this chapter, we first do a welfare analysis pre- and after- trade; then we discuss the long run effects of different taxes and tariff.

\footnote{Peretto (2007) shows a slightly different result that profit tax has an ambiguous effect on growth. This is because he allows costly entry and positive incumbent profits, so a profit tax has two opposite effects on growth. One is the negative effect on the firms’ internal R&D decisions. The other one is that it affects the wedge between pre- and after- tax returns to equity, which affects entry thus the individual market size and return in R&D. Our model assumes zero costs for entry, so the returns to equity of firms are always zero. So the second effect does not present in our model. We will discuss it in detail in section (4.3).}
4.2 Welfare Analysis

4.2.1 Model Set Up

All setup are identical to the case under closed economy in chapter 2, except for the utility function. We add the labor-leisure choice in order to discuss labor income tax later. The economy is populated by a representative household who supplies labor and purchases financial assets in competitive labor market and asset market. The representative household is endowed with one unit of time, and the utility function is,

\[ U(t) = \int_t^\infty e^{-\rho \log u(s)} ds, \quad \rho > 0 \]  

(4.1)

where \( \log u = \log c + \nu \log (1 - l) \);

Denote \( L \) is the total population, and \( C \) is aggregate consumption. Then \( c \) is the consumption per capita, and \( c \equiv \frac{C}{L} \). \( l \) is the fraction of time per capita allocated to work, thus \( (1 - l) \) is the leisure per capita, and \( lL \) is the total working hour in the economy. \( \nu \) measures preference for leisure. \( \rho \) is the individual discount rate.

The household faces the flow budget constraint,

\[ \dot{S} = rS + wl - c \]  

(4.2)

\( S \) is the asset holding, \( r \) is the return on assets. We can thus write the current value Hamiltonian for households as
\[ CVH = \log c + \nu \log (1 - l) + \psi (rS + wl - c) \] (4.3)

The optimal plan for this setup is well known, and the derivation is in Appendix (E.1). The household saves and supplies labor according to

\[ \frac{\dot{c}}{c} = r - \rho \] (4.4)

\[ l = 1 - \frac{\nu c}{w} \] (4.5)

And the Euler equation is \( \frac{\dot{c}}{c} = r - \rho \).

Recall that under the assumption of zero entry/exit cost, zero profit condition requires the consumption equal to wage income, which is a certain proportion of final output in Cobb-Dougalus production, \( cL = wlL = (1 - \lambda)Y \). So both the fraction of time allocated to work and consumption ratio jump to a constant, \(^2\)

\[ l^* = \frac{1}{1 + \nu} \] (4.6)

\[ \frac{cL}{Y} = 1 - \lambda \] (4.7)

In this economy, the total supply of labor is fixed,

\(^2\)See Appendix (E.1) for derivations.
The change in utility function (add the labor-leisure choice) doesn’t affect consumption decision, which is always $(1 - \lambda)Y$. But it affects labor decision. People leave some time for leisure. Recall that $\epsilon$ proportion of labor is allocated to industry 1, and $(1 - \epsilon)$ proportion of labor is allocated to industry 2. So the total labor in industry 1 is $\frac{\epsilon}{1+\nu}L$, and in industry 2 is $\frac{1-\epsilon}{1+\nu}L$. As we mentioned in the closed economy, the change in total market size doesn’t affect growth rate, and only affects the number of incumbents and the final output. Thus the transitional dynamic and the growth rate on balanced growth path keep the same as section (2.2.7) and equation (2.68). Recall that

\[
g^* = \delta \frac{\alpha_1^{\rho_2} - \alpha_2^{\rho_1}}{1 - \delta} \quad (4.9)
\]

But the level output changes from equation (3.19) to a lower amount, since the total labor time decreases.

\[
Y = \kappa'[(\frac{Z_1^{(\delta+\gamma)}}{P_{G_1}^{\frac{1}{\alpha}}})Z_2^{1-(\delta+\gamma)}(\epsilon L \frac{1}{1+\nu})][((\frac{Z_2^{(\delta+\gamma)}}{P_{G_2}^{\frac{1}{\alpha}}})Z_1^{1-(\delta+\gamma)}((1-\epsilon)L \frac{1}{1+\nu})]^{1-\epsilon} \quad (4.10)
\]

where $\kappa' = \lambda^{\frac{1}{1-\gamma}}(1-\epsilon)^{\frac{1-\epsilon}{1-\gamma}}\epsilon^{\frac{\lambda}{1-\gamma}}$. 

\[
l^* L = \frac{1}{1+\nu} L \quad (4.8)
\]
4.2.2 The Changes in Welfare After Trade

This model provides a simple analysis of welfare. Let 0 be an arbitrary starting date. Combine (4.10), at the time \( t > 0 \),

\[
\log Y(t) = \log \Omega l^* + \epsilon \log \left[ \frac{Z_1^{\delta+\gamma}(0)}{P_{G_1}} \right] + (1 - \epsilon) \log \left[ \frac{Z_2^{\delta+\gamma}(0)}{P_{G_2}} \right] \\
+ \epsilon (1 - \delta - \gamma) \log Z_2(0) + (1 - \epsilon) (1 - \delta - \gamma) \log Z_1(0) \\
+ \Gamma \int_0^t g_1(s) ds + (1 - \Gamma) \int_0^t g_2(s) ds
\]

where \( \Omega = \kappa \epsilon (1 - \epsilon)^{1-\epsilon} \), \( \Gamma = 2(\delta + \gamma)\epsilon - (\delta + \gamma) - \epsilon + 1 \in (0, 1) \), \( l^* = \frac{1}{1+\nu} \), \( g_i = \frac{\dot{Z}_i}{Z_i} \).

Using this expression, taking into account that the consumption and employment ratios jump to the steady state level, \( \frac{c_i}{Y} = 1 - \lambda \) and \( l^* = \frac{1}{1+\nu} \), we can write the flow of utility inside the welfare function as
\[ \log u(t) = \log c(t) + \nu \log(1 - l(t)) \]
\[ = \log Y(t) + \log \frac{c(t)}{Y(t)} + \nu \log(1 - l^*) \]
\[ = \log \Omega l^* + \epsilon \log \left[ \frac{Z_1^{1+\gamma}(0)}{P_{G_1}^{1-\lambda}} \right] + (1 - \epsilon) \log \left[ \frac{Z_2^{1+\gamma}(0)}{P_{G_2}^{1-\lambda}} \right] \]
\[ + \epsilon (1 - \delta - \gamma) \log Z_2(0) + (1 - \epsilon) (1 - \delta - \gamma) \log Z_1(0) \]
\[ + \Gamma \int_0^t g_1(s) ds + (1 - \Gamma) \int_0^t g_2(s) ds \]
\[ + \log \left( \frac{c}{Y} \right)^* + \nu \log(1 - l^*) \] (4.11)

Please see the derivations in Appendix (E.2).

One can see, the number of firms in each industry, \( N_1 \) and \( N_2 \) do not have a direct effect on final output, \( Y \), the consumption ratio, \( \frac{c}{Y} \), or the employment ratio, \( l \). The reason why the number of firms matters is that given aggregate market size, it determines firm level market size and thus the return in R&D, which can be seen in equation (2.53). But \( N_i \) does not enter either the level output or the growth rate. We have discussed the details in the previous chapters. \( N_i \) does not enter the growth rate because the number of firms has the opposite effects with the change in unit cost, and the total market size, so the growth rate doesn’t depend on either number for firms, unit costs or the total market size. \( N_i \) does not enter the level output either. The level final output is the sum of all individual output, so the total output depends on the total market size, but not the individual market size, thus \( Y \) does not depend
on the number of firms.

Now let $0$ be the time when trade opens. Under the assumptions of complete specialization in chapter 3, home completely specializes in class-1 intermediate goods, and imports class-2 intermediate goods. Assume before trade, home is on balanced growth path. At time $t = 0$ (the moment trade happens), the change in welfare is

$$\Delta U_{Ho} = \int_0^\infty e^{-\rho t} \log \frac{u_H(t)}{u_H^A(t)} dt$$

(4.12)

where flow utility relative to the initial steady state is,

$$\log \frac{u_H(t)}{u_H^A(t)} = \log \frac{Y_H(t)}{Y_H^A} = \log Y_H(t) - \log Y_H^A$$

$$= (1 - \epsilon) \log \frac{Z_{F_2}^{\delta+\gamma}(0)}{P_{G_{F_2}}^{1/\xi}} - \frac{Z_{H_2}^{\delta+\gamma}(0)}{P_{G_{J_2}}^{1/\xi}}$$

$$+ \epsilon(1 - \delta - \gamma) \log [Z_{F_2}(0) - Z_{H_2}(0)]$$

$$+ \Gamma \int_0^\infty [g_1(s) - g_1^A] ds + (1 - \Gamma) \int_0^\infty [g_2(s) - g_2^A] ds$$

(4.13)

(4.14)

The first term is always positive because comparative advantage guarantee that class-2 intermediates are cheaper in term of quality-adjusted price for foreign good at the moment that trade happens, so $\frac{Z_{F_2}^{\delta+\gamma}(0)}{P_{G_{F_2}}^{1/\xi}} > \frac{Z_{H_2}^{\delta+\gamma}(0)}{P_{G_{J_2}}^{1/\xi}}$. The second term is ambiguous.
Home could import a good with lower quality, but still with a lower quality-adjusted price. As discussed in section (3.2.2), the first two terms together are the level effect after trade. The necessary and sufficient conditions under which the first two terms are positive are discussed in that section. The last term is the growth rate effect.

At any time \( s \) after trade, the growth rates \( g_1(s) \) and \( g_2(s) \) could be either higher or lower than the balanced growth rate under autarky \( g_A^d \). The transition dynamic under complete specialization is discussed in section (3.2.3). On world balanced growth path, whether home country enjoys a higher growth rate depends on whether home imports the good with a higher R&D ability, i.e. \( \alpha_{F2}\theta_{F2} > \alpha_{H2}\theta_{H2} \).

Under the assumptions of incomplete specialization discussed in chapter 3, home produces both goods and imports class-2 intermediates. The first term in (4.13) is zero, since the quality-adjusted prices of class-2 intermediates are equal for foreign and domestic goods. The second terms is ambiguous as we discussed. As the last term, see section (3.35) and (3.36) for the changes in \( g_1(s) \) and \( g_2(s) \).

So the effect of trade on the welfare depends on the level and growth rate effects. Welfare needs not increase after trade.

### 4.3 Tariff and Taxes

We focus on the tariff and taxes analysis for home country under complete specialization. Recall that home country specializes in type-1 intermediates, and imports type-2 intermediates. The analysis is similar to the foreign country and also the case under incomplete specialization. What we discuss here is profit tax \( t_\pi \), consumption
tax $t_C$, labor income tax, $t_L$, and tariff on imports, $\tau$. \(^3\) We assume the government has no access to lump sum tax or public debt, and holds a constant fraction of GDP allocated to government expenditure. The government has to balance the budget all the time.

The basic model set up is the same as in section (4.2.1). In final good sectors for both countries, maximization problem gives us the indirect demand functions for $X_{i1}$ and $X_{i2}$, where $i = H, F$, which are equation (2.28) and (2.29). In processed good sector, maximization problem gives the demand functions for intermediate goods, and labors, which are similar to equation (3.5), (3.6), (3.10), and (3.11). The only differences are, firstly, home country imposes a tariff $\tau$ on the imports, which is type-2 intermediates in this case, while we assume foreign country does not impose tariff for simplicity. So tariff rate $\tau$ will enter the demand function of type-2 intermediates in home. Secondly, we add the labor-leisure choice into the utility, so the optimal fraction of time allocated to work, $l_{i}^{*}$ where $i = H, F$, enters the demand functions. We also already know that the labor allocation to each industry are $\epsilon L_{i}$ and $(1-\epsilon)L_{i}$ where $i = H, F$. We also already proof symmetry among the same industry. Thus the demands of both intermediates for both countries become,

$$G_{H1j} - G_{H1j}^E = \left[ \frac{\lambda \epsilon (X_{H1})^{\rho-1}}{P G_{H1}} \right]^{1/\gamma} Z_{H1}^{\delta+\gamma} (Z_{F2})^{1-(\delta+\gamma)} \frac{\epsilon \epsilon' \epsilon' H_{N} H_{N}}{N_{H1}}$$

\(^3\)Peretto (2007) also discuss the tax on distributed dividends and the tax on capital gain. But these two taxes has no effects in our model, since th dividend and capital gain are both zero due to zero entry/exit cost.
We need to do two exercises. First, I consider a vector of fixed tax and tariff rates, and the government use the revenue for unproductive consumption. The second exercise is to assume government uses profit tax to subsidize incumbent R&D, and other tax revenue for unproductive consumption. We will see the detail set up for intermediate good sectors in section (4.3.1) and (4.3.2).

To decide the optimal time for work, $l^*_H$ and $l^*_F$, we need to see the maximization problem of representative household. It’s identical to the problem in section (4.2.1), with consumption tax $t_C$, and labor income tax, $t_L$ in the budget constraint. So the budget constraint becomes $\dot{S} = rS + (1 - t_L)wl - (1 + t_c)c$. The derivations are identical to Appendix (E.1) with tax rates in it. And the main results are the Euler equation, consumption ratio and optimal labor-leisure allocation.

$$\frac{\dot{c}_i}{c_i} = r_i - \rho; \text{ where } i = H, F$$
\[
\frac{c_H L_H}{Y_H} = (1 - \lambda) \frac{1 - t_L}{1 + t_c}
\]

\[
l^*_i = \frac{1}{1 + \nu} \quad \text{where} \ i = H, F
\]

Taxes on labor income and consumption makes the consumption ratio less, but they do not change the time allocated to labor. This is because the substitution and income effects of the taxes on wage cancel out with each other in this set up, leaving the labor-leisure choice unchanged. Tax on profit does not enter the consumption ratio and labor supply. The reason is that \( t_\pi \) governs the wedge between pre- and after- tax return to saving whose effect on saving and labor supply is sterilized by the assumption of log utility. Pretto (2007) relaxes that assumption of log utility, and he shows it does not affect the results. Tariff also does not affect consumption ratio and labor supply. Imposing a tariff is equivalent to increase the unit cost of intermediates. We’ve shown that, in this model, any changes in unit cost are absorbed by entry, leaving the profit unchanged, hence the other choices.

4.3.1 Exercise One

We first see the case under which home government use all the tax revenue in unproductive projects.

In the intermediate-good sector, every set up is the same as section (4.2). After trade, home country only produces type-1 intermediates and imports type-2 inter-
mediates. The retaining profit of the intermediate firms in type-1 industry of home is,

\[ F_{H1} = (P_{G_{H1}} - A_{H1})G_{H1} - \theta_{H1} \frac{Z_{H1}^2}{Z_{F2}} \]

So the firm’s pre-tax profit is,

\[ \pi_{H1} = F_{H1} - R_{H1} \]

Let \( \sigma \) be the fraction of R&D expenditures that the firm is allowed to subtract from the gross cash flow to determine taxable income. The firm pays total tax as,

\[ t_\pi (F_{H1} - \sigma R_{H1}) \]

Recall that free entry assumption requires zero profit condition, so both the capital gain and the after-tax flow of dividends are zero. And this is the reason why in this set up, we cannot discuss dividend tax and capital gain tax as Peretto (2007) did.

\[ D_{H1} = (1 - t_\pi)F_{H1} - (1 - \sigma t_\pi)R_{H1} = 0. \]

Following similar steps as in section (3.2.1.3), we solve the current value Hamiltonian for home producers,

\[ CV H_{H1} = (1 - t_\pi)F_{H1} - (1 - \sigma t_\pi)R_{H1} + q_{H1}(\alpha_{H1} R_{H1}) \]

s.t. \( G_{H1} = (4.15) + (4.16); \quad Z_{H1} = \alpha_{H1} R_{H1} \).

The taxes do not affect the price of intermediate good 1, so \( P_{G_{H1}} = \frac{\lambda \mu_1}{\lambda} \). The shadow price \( q_{H1} = \frac{1 - \sigma t_\pi}{\alpha_{H1}} \) is less than the case without taxes, which is \( \frac{1}{\alpha_{H1}} \). And the return in R&D becomes,

\[ r_{H1} = \frac{\partial (1-t_\pi)F_{H1}}{\partial Z_{H1}} / q_{H1} = q_{H1} \frac{\alpha_{H1}(1-t_\pi)}{1-\sigma t_\pi} \delta f_{H1} \]

where \( f_{H1} \equiv A_{H1} \frac{1 - \lambda}{\lambda} \left[ \frac{\lambda^2 \epsilon}{A_{H1}} \right]^{\frac{1}{1-\lambda}} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{1}{1-\lambda}} \left( \frac{P_{C_{H1}}}{P_{G_{F2}}} \right)^{\frac{\lambda(\epsilon-1)}{1-\lambda}} \left( \frac{Z_{H1}}{Z_{F2}} \right)^{\Gamma-1} \frac{\epsilon L_{H2} + \epsilon L_F}{N_{H1} I^*_F} \].

And zero profit condition due to free entry requires \( R_{H1} = \frac{(1-t_\pi)F_{H1}}{1-\sigma t_\pi} \). So the growth rate of \( Z_{H1} \) can be written as,
Recall that the foreign country only produces type-2 intermediates. They are facing the tariff \( \tau \) from the home country, which means the unit cost is increased to \((1+\tau)A_{F2}P_{Y_F}\) in the market of the home country, but still keeps the same in the market of the foreign country itself. So they charge the same price in their own country, but a higher price (recalled with \(1+\tau\)) in the home country. Following the similar steps as section (3.2.1.3), the return in \(Z_{F2}\) and the growth rate of \(Z_{F2}\) are,

\[
\begin{align*}
    r_{F2} &= \alpha_{F2}\delta f_{F2}, \quad \text{and,} \quad \dot{z}_{F2} = \alpha_{F2}f_{F2} - \alpha_{F2}Z_{H1}, \\
    &\text{where} \quad f_{F2} = A_{F2}\frac{1-\lambda}{1-\epsilon} \left( \frac{\lambda^2(1-\epsilon)}{A_{F2}} \right)^{1-\lambda} \left( \frac{\epsilon}{1-\epsilon} \right)^{1-\lambda} \left( \frac{P_{H1}}{P_{F2}} \right)^{1-\lambda} \left( \frac{Z_{F2}}{Z_{H1}} \right)^{-1} \cdot \frac{1}{N_{F2}} \left[ (1-\epsilon)^{(1+\tau)} L_H + L_F P_{Y_F}^{\epsilon} \right].
\end{align*}
\]

Assume no borrowing, so the government budget constraint in the home is,

\[
G_H = t_L w_H (L_H l_H^\star) + t_c c_H L_H + t_\pi (F_{H1} - \sigma R_{H1}) N_{H1} + \tau P_{G_{F2}} C_{F2}^\epsilon N_{F2}
\]

Trade balance condition is similar to section (3.2) except that here we have tariff, so \(P_{Y_F} = \left[ \frac{(1-\epsilon) L_H}{\epsilon L_F} \right]^{-1-\lambda} \left( 1+\tau \right)^{-\lambda(1-\epsilon)} \). Recall that all taxes do not affect labor-leisure choice in the home country in this model, so \(l_H^\star\) does not show up here because they are identical for both countries and cancelled out. The same reason for why other tax rates do not affect the balance of trade.

Following the same steps as section (3.2), we get the steady state for quality ratio is \((\frac{Z_{H1}}{Z_{F2}})^\star = \pm \sqrt{\frac{\alpha_{F2} \theta_{F2} (1-\sigma t_\pi)}{\alpha_{H1} \theta_{H1} (1-t_\pi)}}\). And as the case without tariff and taxes, the positive steady state is stable. The transition dynamics is the same as in section (3.2). Thus the balanced growth rate for the world is,
\[ g^* = \frac{\delta}{1 - \delta} \sqrt{\alpha H_1 \theta H_1 \alpha F_2 \theta F_2 \frac{1 - t_{\pi}}{1 - \sigma t_{\pi}}} - \frac{1}{1 - \delta} B \]  

(4.17)

And the output level can be written as,

\[ Y_H = \kappa' [\left( \frac{Z^{(\delta + \gamma)} H_1}{P^{\lambda - \delta}_{G_{H_1}}} \right) Z_{F_2}^{1-(\delta + \gamma)} \left( \epsilon L_H \frac{1}{1 + \nu} \right)]^\epsilon \left[ \left( \frac{Z^{(\delta + \gamma)} F_2}{[(1 + \tau) P_{G_{F_2}}]^{\lambda - \delta}} \right) Z_{H_1}^{1-(\delta + \gamma)} \left( (1 - \epsilon)L_H \frac{1}{1 + \nu} \right) \right]^{1-\epsilon} \]

Wage tax and tariff only have effects on output level. Consumption tax does not affect either output level or the growth rate, but later you will see it enters the welfare through utility. Only profit tax enters the growth rate. Appendix (E.3) derives that \( \frac{\partial g^*}{\partial t_{\pi}} \) \( \begin{cases} < 0 & \text{for } \sigma \in [0, 1) \\ = 0 & \text{for } \sigma = 1 \end{cases} \). If R&D is full expensible, then \( t_{\pi} \) has no effect on the growth rate. The reason is, when \( \sigma = 1 \), after-tax divident is \( D_{H_1} = (1 - t_{\pi}) F_{H_1} - (1 - t_{\pi}) R_{H_1} \). \( t_{\pi} \) is identical to dividend tax. Zero entry cost guarantees zero profit condition, so \( (1 - t_{\pi})(F_{H_1} - R_{H_1}) = 0 \). Any rate of profit tax has no effect on after-tax profit hence the return in R&D and the growth rate. Peretto (2007) has a different result in this case. He assumes a positive entry cost, so value of firm equals the setup cost in the equilibrium of positive entry. Thus according to the return to equity, \( r = \frac{D_i}{V_i} + \frac{\check{V}}{V_i} \), the dividend is positive, i.e. \( D_i > 0 \). In this case, the effects of \( t_{\pi} \) depend on two forces. One is the distortion of the internal return on R&D, which is the negative effect that also shows up in our model. The other one is the distortion on the dividends. A higher \( t_{\pi} \) makes a lower return on dividends, thus fewer firms
enter the market leaving each incumbent a higher market size and a higher return
in R&D. This has a positive effect on the growth rate. So in Peretto (2007), \( t_\pi \) has
two opposite effects on growth rate, and the total effect is ambiguous. When \( \sigma = 1 \),
the first effect disappears, leaving only a positive effect of \( t_\pi \). In our model, based
on a different assumption that the entry cost is zero, \( t_\pi \) does not have the second
effect (the positive effect). So the effect of profit tax is negative on growth rate when
\( \sigma \in [0, 1) \), and zero with full expensibility of R&D taxation.

The taxes and tariff change the flow of utility inside welfare function of home
country from (4.11) to

\[
\log u_H(t) = \log \Omega \hat{t}_H^* + \epsilon \log \left[ \frac{Z_{H1}^{\delta+\gamma}(0)}{P_{H1}^{\lambda \epsilon}} \right] + (1 - \epsilon) \log \left[ \frac{Z_{F2}^{\delta+\gamma}(0)}{[(1 + \tau)P_{F2}^\lambda]^{1 - \epsilon}} \right]
+ \epsilon (1 - \delta - \gamma) \log Z_{F2}(0) + (1 - \epsilon) (1 - \delta - \gamma) \log Z_{H1}(0)
+ \Gamma \int_0^t g_{H1}(s) ds + (1 - \Gamma) \int_0^t g_{F2}(s) ds
+ \log \left( \frac{C_H}{Y_H} \right)^* + \nu \log (1 - \hat{t}_H^*)
\]

where \( \Omega = \kappa^\epsilon (1 - \epsilon)^{1 - \epsilon} \), \( \Gamma = 2(\delta + \gamma) \epsilon - (\delta + \gamma) - \epsilon + 1 \in (0, 1) \).

If we compare the flow of utility before and after the taxes and tariff, we have the
following results. \( \hat{t}_H^* \) does not change after imposing the taxes, because the income
effect and substitution effect cancel each other out. \( \left( \frac{C_H}{Y_H} \right)^* = (1 - \lambda) \frac{1 - \hat{t}_H}{1 + \hat{t}_c} / L_H \) is less
than the case without distortions, where \((\frac{\mu}{Y_H})^* = (1 - \lambda)/L_H\). \( \log \left[ \frac{Z_{F_2}^{\delta+\gamma}(0)}{(1+\tau)P_{G_F}^2} \right] \) decreases comparing with \( \log \left[ \frac{Z_{F_2}^{\delta+\gamma}(0)}{P_{G_F}^2} \right] \) due to the tariff. And the growth rates go down due to the profit tax. So the welfare decreases because of the distortions.

### 4.3.2 Exercise Two

In the second exercise, we assume government in home use the tax revenue from corporate profits to subsidize in-house R&D, and use all other revenues into unproductive projects. Set \( \zeta \) as the subsidy rate, and the government budget constraint becomes

\[ \zeta R_{H1} N_{H1} = t_\pi (F_{H1} - \sigma R_{H1}) N_{H1} \]  \tag{4.18}

where \( F_{H1} = (P_{G_{H1}} - A_{H1})G_{H1} - \theta_{H1} \frac{Z_{H1}}{Z_{F2}} \).

With subsidy in R&D, the current value Hamiltonian of the intermediate firms in industry 1 of home becomes

\[ CV_{H1} = (1 - t_\pi) F_{H1} - (1 - \sigma t_\pi)(1 - \zeta) R_{H1} + q_{H1}(\alpha_{H1} R_{H1}) \]

s.t. \( G_{H1} = (4.15) + (4.16); \dot{Z}_{H1} = \alpha_{H1} R_{H1} \).

The first order conditions are similar to exercise one, except a subsidy rate. Subsidy does not affect the price decision, but it does increase the return of R&D as

\[ r_{H1} = \frac{\partial(1-t_\pi)F_{H1}}{\partial Z_{H1}}/q_{H1} + \frac{\partial \alpha_{H1}}{\partial Z_{H1}} = \frac{\alpha_{H1}(1-t_\pi)}{(1-\sigma t_\pi)(1-\zeta)} \delta f_{H1} \]

where \( f_{H1} \equiv A_{H1} \frac{1-\lambda}{\lambda} \left[ \frac{\lambda^2 \epsilon}{A_{H1}} \right]^{\frac{\gamma-1}{\gamma}} \left( \frac{Z_{H1}}{Z_{F2}} \right)^{-\frac{\lambda - 1}{\gamma}} \frac{P_{G_{H1}}}{P_{G_F}^2} \left( \frac{Z_{H1}}{Z_{F2}} \right)^{\gamma-1} \frac{L_{H} + \epsilon P_{I_F}^\gamma}{N_{H1}} l_{H}^{\ast} \).

From the zero profit condition, \( (1 - \sigma t_\pi)(1 - \zeta) R_{H1} = (1 - t_\pi) F_{H1} \), we can get the
growth rate of $Z_{H1}$ is higher,

$$\frac{Z_{H1}}{Z_{H1}} = \frac{\alpha_{H1}R_{H1}}{Z_{H1}} = \frac{\alpha_{H1}(1-t_\pi)}{(1-\sigma t_\pi)(1-\varsigma)} f_{H1} - \frac{\alpha_{H1}(1-t_\pi)}{(1-\sigma t_\pi)(1-\varsigma)} \theta_{H1} Z_{H1} Z_{H2}$$

The maximization problem of the firms in foreign country is identical as the first exercise, so is the trade balance condition. Repeating the same steps, we get the steady state of quality ratio as $(Z_{H1}/Z_{F2})^* = \pm \sqrt{\frac{\alpha_{F2}\theta_{F2}(1-\sigma t_\pi)(1-\varsigma)}{\alpha_{H1}\theta_{H1}(1-t_\pi)}}$, and the positive steady state is stable. Thus the world balanced growth rate is,

$$g^* = \frac{\delta}{1-\delta} \sqrt{\frac{\alpha_{H1}\theta_{H1}\alpha_{F2}\theta_{F2}}{(1-\sigma t_\pi)(1-\varsigma)}} \frac{1-t_\pi}{1-\delta\rho}$$

We only discuss the case when $\sigma \in [0, 1)$. Because when $\sigma = 1$, as discussed in exercise 1, the tax revenue of corporate profit is zero, so no fund can be used to subsidize R&D based on our assumption, shown by equation (4.18). The effect of $t_\pi$ on the growth rate is negative under $\sigma \in [0, 1)$, and the effect of $\varsigma$ is positive. Please notice the subsidy is funded by the tax revenue. To keep the budget balance, (4.18) shows that $(\varsigma+\sigma t_\pi)R_{H1} = t_\pi F_{H1}$. Zero profit condition requires $(1-\sigma t_\pi)(1-\varsigma)R_{H1} = (1-t_\pi)F_{H1}$. Combine these two conditions, we can get the relation between the profit tax rate and the subsidy rate as$^4$,

$$\varsigma = \frac{t_\pi(1-\sigma)}{1-\sigma t^2_\pi} (4.19)$$

By observation, it’s easy to see that $\frac{d\varsigma}{dt_\pi} > 0$. So an increase of $t_\pi$ directly decreases the growth rate, but in the other hand, it increases the growth through the subsidy effect. Appendix (E.4) also shows the negative effect dominates. So imposing profit

$^4$Derivations in Appendix (E.4).
tax always decreases the growth rate under \( \sigma \in [0, 1) \), even if the government uses the tax revenue for R&D subsidy. And this exercise also decreases welfare.

This chapter we show that trade need not increase welfare because trade need not increase either the level output or the growth rate. All taxes and tariff reduce welfare based on this framework, even if the government use profit tax to subsidize in-house R&D. Only profit tax has effect on growth rate, other taxes and tariff only have level effect.
Chapter 5

Implications (Extensions) and Conclusions

5.1 Implications and Possible Extensions

5.1.1 Directed Technical Change

Whether technical change is biased towards particular factor is of central importance for many problems in macroeconomics, development economics and international trade. According to Acemoglu (2002), the definition of biased technical change is the following. Assume there are two non-reproducible factors, \( L \) and \( H \), technical change is \( L \)-biased if the it increases the marginal product of \( L \) more than that of \( H \). Acemoglu (2002) builds up a framework to analyse the forces that shape these biases.
There are two goods, $Y_L$ and $Y_H$. Consumption, investment and R&D expenditure come out of an aggregate output produced from $Y_L$ and $Y_H$ by CES function. And $Y_L$ and $Y_H$ are produced by different kinds of intermediate goods and non-producible factors. The growth is driven by variety expansion. The paper presents two main results. The first one is a “weak induced-biased hypothesis” : an increase in the relative abundance of a factor causes certain amount of technical change biased towards that factor for any elasticity of substitution between factors. And the second main results is a “strong induced-bias hypothesis”: with a sufficiently high elasticity of substitution, directed technical change can make the long run relative demand curve of $H$ and $L$ slope up. The main forces for these two results are the price effect and the market size effect. The price effect means there is a greater incentive to invest technologies producing more expensive goods. The market size effect means a larger market for the technology leads to more innovation because of the underlying “increasing returns to scale” in the R&D process, which means a new machine is non-rival. So scale effect plays an essential role in this model. If remove the scale effect, market size effect might not exist any more and the main results might change. Acemoglu (2002) also mentions this problem by himself, and he tries to use semi-endogenous model of Jones (1995) to correct the issue. He argues his main results are not affected by using the semi-endogenous model. But semi-endogenous model is weak theoretically and fails empirically. And we want to see if there are any main differences if directed technical change is discussed under second-generation growth model of Howitt-Peretto.

A slight modification of the closed economy in our model gives a possible framework. We can use CES function instead of Cobb-Douglas to connect two processed goods as in Acemoglu (2002), and revise the production function of processed good
to shut down the spillover across industries to match the model in Acemoglu (2002). We predict that the market size effect totally disappears by the endogenous entry. The change in the size of $H$ and $L$ does not affect technology accumulations. The invalidity of market size effect makes the two main hypotheses also invalid, and we need to have a new look at the cross-country income difference and many other issues based on directed technical change.

5.1.2 World Income Distribution

In an important article, Acemoglu and Ventura (2002) argued that trade stabilizes the world income distribution. In their analysis, countries are endowed with non-intersecting sets of intermediate goods that they can produce and sell on the world market. World prices move to equalize rates of return, and the movements of world prices equalize the growth rates of all countries’ incomes. The role of price movements in equalizing growth rates is an important insight. Naturally, as with any model, Acemoglu and Ventura’s model has some limitations, and one would like to know if their major conclusion generalizes to environments that relax their model’s limitations. One limitation of Acemoglu and Ventura’s analysis is that it is carried out in a first-generation endogenous growth model that has a scale effect, and some of their analysis works through the scale effect. Another limitation is that comparative advantage plays no role in determining the patterns of production or trade. Countries are endowed exogenously with non-overlapping sets of goods to produce, and they produce those and only those with no choice in the matter. Finally, production is AK, so there
are no transition dynamics. Our analysis relaxes all these restrictions.

Acemoglu and Ventura’s main result on the stabilization of the world income distribution survives some of the generalizations but not others. Elimination of the scale effect in itself does not alter the conclusion that trade can stabilize the world income distribution. There is a world balanced growth path inside the region of complete specialization on which all countries’ incomes grow at the same rate. In fact, Acemoglu and Ventura’s assumption that countries specialize because of their endowment puts all countries in the world’s region of complete specialization by construction, so in a way it is not surprising that the balanced growth behavior for the case of complete specialization in our model resembles that of Acemoglu and Ventura’s analysis. Our model strengthens their conclusion on income distribution stability in one way because it has transition dynamics. We found that all countries grow at the same rate anywhere inside the region of complete specialization, not just on the balanced growth path, so the world income distribution is stable along transition paths, too, provided the world is inside the region of complete specialization. This last proviso leads to the part of our analysis in which Acemoglu and Ventura’s conclusion does not hold, namely, outside the region of complete specialization. We have shown that in the region of incomplete specialization, except at the unstable balanced growth path, countries’ growth rates differ, which means the world income distribution must be changing. Furthermore, in part of the region of incomplete specialization, countries’ growth rates go asymptotically to a constant difference with one country perpetually growing at a faster rate than the other. In that case, the world income distribution degenerates, with the faster growing economy’s share of world income going asymptotically to 1. Note that the slower growing country does
not disappear. In fact, it always grows. It just grows more slowly than the other country and so vanishes relative to the faster growing country. This result may correspond to the history of sub-Saharan Africa, whose growth rate usually has been positive but also has lagged behind that of the rest of the world for at least 200 years. It also is interesting to note that very small countries have a tendency to end up with relatively low growth rates. One of the factors determining whether the world is in the region of incomplete specialization, where countries have different growth rates, is population size. If a country is very small relative to its trading partners, it has a tendency to end up growing more slowly than its partners. The reason Acemoglu and Ventura do not obtain results like these is that their analysis by construction excludes the possibility of incomplete specialization.

One important empirical implication of Acemoglu and Ventura is the terms-of-trade effect. They predict that a greater relative income between a country to the world, a lower terms of trade for this country. However, their own empirical work shows no relationship between the difference of growth rates of a country and the world, and the changes in terms of trade of that country. Our model provides another prediction that favors their empirical result. In our model, under complete specialization, the growth rate of both countries are always the same, so the relative income between a specific country and the world is constant. The terms of trade, however, change as the ratio of quality levels changes during transition dynamics. So we predict no relationship between these two terms. A further research could be done for this issue.
5.1.3 Technology Transfer

Our analysis also has an implication for econometric analysis of technology transfer. There is a literature, started by Coe and Helpman (1995, 2008), that attempts to measure the amount of technology transfer by matching volume of trade with technical sophistication among trading partners. The general finding is a country that trades with technologically advanced partners has a higher growth rate than a country that trades with less advanced partners. The usual conclusion is that trade facilitates technology transfer. Our analysis suggests that such a conclusion may be unwarranted because trade itself can lead to growth rate results that resemble the effects of technology transfer even when such transfers do not occur.

In our model, trade’s growth rate effect does not work through cross-country technology spillovers or technology transfers, which are ruled out by construction. Countries are not permitted to learn the technology of their trading partners even with trade. In effect, we are assuming that reverse engineering is not possible and that firms are able to keep their production techniques secret. A country can get the “quality” but not the “know-how” from importing a good. Nonetheless, the growth rate can be affected by trade in the same way as if the country actually had adopted the trading partner’s R&D ability for producing the good at home that it imports from the trading partner. If the home country’s trading partner is more efficient at producing a good and also has a higher ability in R&D related to that good, the solution for the growth rate with trade is exactly the same as the solution that would have emerged if the home country had learned the trading partner’s technology and produced the good itself instead of importing it. One would see, then, that countries whose trading
partners are sophisticated have higher growth rates that other countries, but the higher growth rates arise from importing sophisticated goods, not from importing the technology to make those goods domestically. This result casts a shadow on the standard interpretations of the data.

It also is interesting that the growth rate under trade need not be the same as if technology transfer occurred. Suppose the foreign country's quality-adjusted price is lower than the domestic quality-adjusted price for some good. The home country then imports that good. Suppose, however, that the foreign country is relatively inefficient at R&D. When technology is imported in the form of embodied quality, as our model, the result would be a lower growth rate for the home country, as we have discussed above. With technology transfer, in contrast, the home country would merely copy the better production methods of the foreign country and would then pair those with its own relatively efficient R&D. Thus acquiring technology indirectly by buying it as an embodied quality in a traded good is not the same as learning to produce the better quality good yourself.

5.2 Conclusions

We study the effects of trade on economic growth in a Schumpeterian framework. The model excludes scale effects and technology transfer, the two usual channels in the literature through which trade affects growth, leaving only comparative advantage. Comparative advantage and the trading pattern are determined endogenously. Endogeneity of production and trading patterns leads to results quite different from
those found in most of the related literature. Trade need not increase initial output of either country because of an externality absent from static models. Irrespective of what happens to initial output, trade may increase the balanced growth rate but also may decrease it. Our model has tractable transition dynamics, which we describe completely. We show that trade leads to a stable world income distribution in some cases, but in other cases leads to an unstable and perhaps even degenerate distribution. In some cases, trade’s effect on a country’s growth rate is the same as if that country had adopted its trading partner’s R&D technology, even though no technology transfer ever occurs. Trade could have negative effect on welfare. All taxes and tariff have negative effect on welfare, even under the case that the government uses the revenue of corporate profits to subsidize the incumbent R&D.

We have constructed an asymmetric Schumpeterian growth model without scale effects or technology transfer to discuss the impact of trade on both output levels and growth rates through the channel of comparative advantage alone. The model offers several theoretical advances over the existing literature: the asymmetric element, the absence of a scale effect, tractable transition dynamics, and, most important, the endogeneity of trade patterns. The usual assumption in the pertinent literature is that countries trade everything they produce and the division among countries of what is produced is determined exogenously. In our model, countries may choose to trade nothing. If they do choose to trade, which goods they export and which they import are determined endogenously.

We have shown that trade can have surprising effects on both the level and growth rate of output. In contrast to the usual result that trade raises incomes of both trading partners, we obtain the result that trade may reduce the initial income of one
or both trading partners. The mechanism for this unusual result is an externality that is missing from static models. R&D-driven quality improvement is the engine of growth, but relative cross-country efficiency in doing R&D has nothing to do with the determination of comparative advantage. Comparative advantage in turn is the sole determinant of trade patterns but has nothing to do with the determination of economic growth. It thus is possible for a country to import a good that is lower in quality than the domestic good it replaces because that good is produced so much more cheaply by the trading partner. The lower quality, however, reduces output in other industries through a knowledge spillover effect. We do not by any means expect this mechanism to be the norm, but it is a possibility. In the absence of a large negative externality of this type, trade increases the output of both trading partners.

Irrespective of trade's effect on the initial level of income, trade may raise or lower growth rates. Again, the issue is that R&D efficiency determines growth rates but is not taken into account when firms decide whether to import a good. What firms care about is the quality-adjusted price that they are offered today. Thus a cheap good, possibly of high current quality, may replace another good that has associated with it a more efficient R&D program. Replacement of the latter good thus has the unintended side effect of lowering the growth rate.

Under complete specialization and in some cases of incomplete specialization, trade equalizes growth rates at least eventually and thus stabilizes the world income distribution. In other cases of incomplete specialization, the growth rate of the country that is not completely specialized is always higher than the growth rate of its completely specialized trading partner, and the difference between them asyntoti-
cally converges to a constant. We also see trade can yield growth outcomes that mimic those arising from technology transfers.

Finally, we see trade need not increase welfare because trade need not increase either the level output or the growth rate. All taxes and tariff reduce welfare based on this framework, even if the government use profit tax to subsidize in-house R&D.
Bibliography


Appendices
Appendix A

Discussion on Fixed Operating Cost

If the fixed operating cost not only depends on the average quality level, but also depends on the quality of the individual firm, then the expression of fixed operating cost could be the same as Peretto(2007), i.e. \( \theta Z_i^{\delta} Z^{1-\delta} \). By the assumption of zero entry/exit cost, this set up doesn’t allow a positive growth rate. The following is the detail:

Now cash flow changes from (2.5) to

\[
F_i = G_i(P_i - 1) - \theta Z_i^{\delta} Z^{1-\delta} \tag{A.1}
\]

Solve for current value Hamiltonian,

\[
CVH_i = G_i(P_i - 1) - \theta Z_i^{\delta} Z^{1-\delta} - R_i + q_i R_i \tag{A.2}
\]
We get the same expression as (2.11),

\[ r = \frac{\partial F}{\partial Z_i} q + \dot{q} \]

(A.3)

and the same as before, \( q = 1 \). But in this set up, \( \frac{\partial F}{\partial Z_i} \) includes

\[ \delta \theta Z_i^{1-\delta} Z_1^{1-\delta} = \delta \theta \]

thus the return of R&D is less than (2.12) by \( \delta \theta \)

\[ r = \delta \frac{1 - \lambda}{\lambda} \lambda^{2-\lambda} l - \delta \theta \]

(A.4)

\[ = \delta \left( \frac{1 - \lambda}{\lambda} \lambda^{2-\lambda} l - \theta \right) \]

(A.5)

Zero entry/exist condition requires zero profit condition, and the growth rate of quality would be the same as (2.17),

\[ \frac{\hat{Z}_i}{Z_i} = \left( \frac{1 - \lambda}{\lambda} \lambda^{2-\lambda} l - \theta \right) \]

(A.6)

(A.6) and (A.5) shows that

\[ \frac{\hat{Z}_i}{Z_i} = \frac{r}{\delta} \]

(A.7)

Euler equation connect (A.5) and (A.6) by

\[ \frac{\dot{c}}{c} = \frac{\hat{Z}_i}{Z_i} = r - \rho \]

(A.8)

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(A.8) is saving (supply of credit) and (A.7) is investment (demand of credit). Solve this two equations,

\begin{align*}
    r &= \rho \frac{\delta - 1}{\delta} \quad \text{(A.9)} \\
    g &= -\frac{\rho}{\delta} \quad \text{(A.10)}
\end{align*}

They are both negative given that \( \delta < 1 \). This is because entry is too cheap, so too many firms enter to drive down the the return in R&D, thus (A.5) shows return in R&D is too low. If we make fixed operating cost only depends on average quality level, then the return of R&D is higher, and we can get a positive return and growth rate.
Appendix B

Closed Economy with two industries

B.1 Final Good Sector in Closed Economy

Final-good sector is perfect competition, and the production Function is,

\[ Y = X_1^{1-\epsilon} \]  \hspace{1cm} (B.1)

Final-good producers maximize profit by choosing the amount of processed good. Final-good sector is prefectly competitive, so final-good firms take the price of processed good as given. Set the price of \( Y \) as numeraire, so the lagrange function of maximize profit for final-good producers is,
\[ \pi_Y = Y - P_{X_1}X_1 - P_{X_2}X_2 \]  
\hspace{1cm} (B.2)

The first order conditions are,

\[ \frac{\partial \pi_Y}{\partial X_1} = \epsilon X_1^{\epsilon - 1} X_2^{1-\epsilon} - P_{X_1} = 0 \]  
\hspace{1cm} (B.3)

\[ \frac{\partial \pi_Y}{\partial X_2} = (1 - \epsilon) X_1^{\epsilon} X_2^{-\epsilon} - P_{X_2} = 0 \]  
\hspace{1cm} (B.4)

So the demand for \( X_i' \)s are

\[ X_1 = (\frac{\epsilon}{P_{X_1}})^{\frac{1}{1-\epsilon}} X_2 \]  
\hspace{1cm} (B.5)

\[ X_2 = (\frac{\epsilon}{P_{X_2}})^{\frac{1}{1-\epsilon}} X_1 \]  
\hspace{1cm} (B.6)

So the inverse demand functions for \( X_i' \)s are,

\[ P_{X_1} = \epsilon X_1^{\epsilon - 1} X_2^{1-\epsilon} \]  
\hspace{1cm} (B.7)

\[ P_{X_2} = (1 - \epsilon) X_1^{\epsilon} X_2^{-\epsilon} \]  
\hspace{1cm} (B.8)
And,

\[
\frac{P_{X_1}}{P_{X_2}} = \frac{\epsilon}{1 - \epsilon} \left( \frac{X_1}{X_2} \right)^{-1}
\]

(B.9)

The competitive final-good producer pays compensation \( \epsilon Y \) and \((1 - \epsilon)Y\) to the processed-good 1 and processed-good 2. So we get \( \epsilon Y = P_{X_1}X_1 \) and \((1 - \epsilon)Y = P_{X_2}X_2\).

### B.2 Processed-Good Sector in Closed Economy

Process-good sector is perfect competition and the production functions is,

\[
X_1 = \int_0^{N_1} \hat{G}_{1j}^\lambda \left( Z_{1j}^{\delta} Z_1^\gamma Z_1^{1-(\delta+\gamma)} l_{1j} \right)^{1-\lambda} dj, \quad 0 < \lambda, \delta < 1
\]

\[
X_2 = \int_0^{N_2} \hat{G}_{2j}^\lambda \left( Z_{2j}^{\delta} Z_2^\gamma Z_1^{1-(\delta+\gamma)} l_{2j} \right)^{1-\lambda} dj, \quad 0 < \lambda, \delta < 1
\]

(B.10)

Where \( Z_1 = (1/N_1) \int_0^{N_1} Z_{1j} dj \), and \( Z_2 = (1/N_1) \int_0^{N_2} Z_{2j} dj \). Assume labors are free to flow across firms and industries, so the wage must be equal. Taken the prices of intermediate goods and wage as given, firms in the first industry, who produce \( X_1 \), choose the amounts of \( G_{1j}'s \) and labors to maximize profit,

\[
\pi_{X_1} = P_{X_1}X_1 - \int_0^{N_1} P_{G_{1j}} G_{1j} dj - \int_0^{N_1} w l_{1j} dj
\]

(B.11)

The First Order Conditions are,
\[
\frac{\partial \pi}{\partial G_{1j}} = P_{X_1} \left( \frac{\partial X_1}{\partial G_{1j}} \right) - P_{G_{1j}} = 0
\]

so \( P_{X_1} \lambda G_{ij}^{1-1} (Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)} l_{1j})^{1-\lambda} = P_{G_{1j}} \Rightarrow G_{1j}^{1-\lambda} = \frac{P_{X_1} \lambda (Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)} l_{1j})^{1-\lambda}}{P_{G_{1j}}} \Rightarrow \)

\[
G_{1j} = \left[ \frac{\lambda P_{X_1}}{P_{G_{1j}}} \right]^{1-x} Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)} l_{1j}
\]

\[
\frac{\partial \pi}{\partial l_{1j}} = P_{X_1} \left( \frac{\partial X_1}{\partial l_{1j}} \right) - w_{1j} = 0
\]

so \( P_{X_1} G_{1j}^\lambda (Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)} l_{1j})^{-\lambda} (1 - \lambda) Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)} l_{1j} = w_{1j} \)

\[
\Rightarrow \frac{P_{X_1} (1-\lambda) G_{1j}^\lambda}{w_{1j}} (Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)} l_{1j})^{1-\lambda} = l_{1j}^\lambda
\]

\[
\Rightarrow l_{1j} = (P_{X_1} \frac{1-\lambda}{w}) \frac{1}{\lambda} G_{1j} (Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)})^{\frac{1-\lambda}{\lambda}}
\]

The same for industry two,

\[
\pi_{X_2} = P_{X_2} X_2 - \int_0^{N_2} P_{G_{2j}} G_{2j} dj - \int_0^{N_2} w l_{2j} dj
\]

(B.12)

And the FOC’s are,

\[
\frac{\partial \pi}{\partial G_{2j}} = P_{X_2} \left( \frac{\partial X_2}{\partial G_{2j}} \right) - P_{G_{2j}} = 0
\]

\[
\frac{\partial \pi}{\partial l_{1j}} = P_{X_2} \left( \frac{\partial X_1}{\partial l_{1j}} \right) - w_{2j} = 0
\]

So the demand functions for \( G_{1j}'s \) and \( l_{1j}'s \) are the following,

\[
G_{1j} = \left[ \frac{\lambda P_{X_1}}{P_{G_{1j}}} \right]^{1-x} Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)} l_{1j}
\]

(B.13)

\[
G_{2j} = \left[ \frac{\lambda P_{X_2}}{P_{G_{2j}}} \right]^{1-x} Z_{ij}^\delta Z_2^\gamma Z_1^{1-(\delta+\gamma)} l_{1j}
\]

(B.14)

\[
l_{1j} = (P_{X_1} \frac{1-\lambda}{w}) \frac{1}{\lambda} G_{1j} (Z_{ij}^\delta Z_1^\gamma Z_2^{1-(\delta+\gamma)})^{\frac{1-\lambda}{\lambda}}
\]

(B.15)
\[ l_{2j} = (P_{X2} \frac{1 - \lambda}{w})^{\frac{1}{\lambda}} P_{G2j} (Z_{2j}^\delta Z_2^\gamma Z_1^{1-(\delta+\gamma)})^{\frac{1-\lambda}{\lambda}} \]  

(B.16)

Plug the inverse demand for \( X_1 \) and \( X_2 \), which are (B.7) and (B.8) into (B.13) and (B.14), we get the demand functions of \( G'_{ij} \)s as,

\[ G_{1j} = \left[ \frac{\lambda \epsilon \left( \frac{X_1}{X_2} \right)^{\epsilon-1}}{P_{G_{1j}}} \right]^{\frac{1}{1+\gamma}} Z_{1j}^\delta Z_1^{\gamma} Z_2^{1-(\delta+\gamma)} l_{1j} \]  

(B.17)

\[ G_{2j} = \left[ \frac{\lambda(1-\epsilon) \left( \frac{X_1}{X_2} \right)^{\epsilon}}{P_{G_{2j}}} \right]^{\frac{1}{1+\gamma}} Z_{2j}^\delta Z_2^{\gamma} Z_1^{1-(\delta+\gamma)} l_{2j} \]  

(B.18)

The competitive processed-good producer pays \( \lambda P_{X_i} X_i \) to intermediate good producers, so \( \lambda P_{X_1} X_1 = \int_0^{N_1} G_{1j} P_{1j} dj \) and \( \lambda P_{X_2} X_2 = \int_0^{N_2} G_{2j} P_{2j} dj \). And they pay \((1-\lambda)P_{X_i} X_i\) to labors, so \((1-\lambda)P_{X_1} X_1 = \int_0^{N_1} w l_{1j} dj\) and \((1-\lambda)P_{X_2} X_2 = \int_0^{N_2} w l_{2j} dj\).

And the profits of processed goods are zero, which means,

\[ P_{X_1} X_1 = \int_0^{N_1} (P_{G_{1j}} G_{1j} + w l_{1j}) dj \]  

(B.19)

\[ P_{X_2} X_2 = \int_0^{N_2} (P_{G_{2j}} G_{2j} + w l_{2j}) dj \]  

(B.20)

The resource allocation are \( \lambda \epsilon Y = \int_0^{N_1} G_{1j} P_{1j} dj; \lambda(1-\epsilon)Y = \int_0^{N_2} G_{2j} P_{2j} dj; (1-\lambda)\epsilon Y = \int_0^{N_1} w l_{1j} dj\) and \((1-\lambda)(1-\epsilon)Y = \int_0^{N_2} w l_{2j} dj\). Define \( \int_0^{N_1} l_{1j} dj = L_1 \), which is the total amount of labor in industry 1; and \( \int_0^{N_2} l_{2j} dj = L_2 \), we get the labor
allocation is
\[
\frac{\int_0^{N_1} w_{L_1} dj}{\int_0^{N_2} w_{L_2} dj} = \frac{\int_0^{N_1} l_{L_1} dj}{\int_0^{N_2} l_{L_2} dj} = \frac{(1-\lambda)\epsilon Y}{(1-\lambda)(1-\epsilon)Y} = \frac{\epsilon}{1-\epsilon}, \quad \text{so}
\]
\[
\frac{L_1}{L_2} = \frac{\epsilon}{1-\epsilon}
\]
We can also get above equation by setting \(MPl_{1j} = MPl_{2j} \equiv w\).

**B.2.1 Derivation of Processed-good Ratio**

\[
X_1 = \int_0^{N_1} G_{l_{1j}}^\lambda \left( Z_{1j}^\delta Z_{1j}^{1-(\delta+\gamma)} l_{1j} \right)^{1-\lambda} dj
\]
\[
= \int_0^{N_1} \left[ \frac{\lambda e^{(X_1/X_2)}^\epsilon - 1}{P_{G_{l_{1j}}}} \right]^{\frac{1-\lambda}{\epsilon}} (Z_{1j}^\delta Z_{1j}^{1-(\delta+\gamma)} l_{1j})^\lambda (Z_{1j}^\delta Z_{1j}^{1-(\delta+\gamma)} l_{1j})^{1-\lambda} \quad (B.22)
\]
\[
= \int_0^{N_1} \left[ \frac{\lambda e^{(X_1/X_2)}^\epsilon - 1}{P_{G_{l_{1j}}}} \right]^{\frac{1-\lambda}{\epsilon}} Z_{1j}^\delta Z_{1j}^{1-(\delta+\gamma)} l_{1j} dj
\]

Similar, we get

\[
X_2 = \int_0^{N_2} \left[ \frac{\lambda (1-\epsilon) (X_1/X_2)^\epsilon}{P_{G_{l_{2j}}}} \right]^{\frac{1-\lambda}{\epsilon}} (Z_{2j}^\delta Z_{2j}^{1-(\delta+\gamma)} l_{2j}) dj
\]

Take a ratio between (B.22) and (B.23), we get
\[
\frac{X_1}{X_2} = \left( \frac{\epsilon}{1-\epsilon} \right)^{1-\gamma} \left( \frac{Z_1}{Z_2} \right)^{(\delta+\gamma)-1+\gamma} \frac{\int_{N_1}^N Z_{1j}^{\lambda} I_{1j} P_{G_{1j}}^{-\lambda \gamma} \, dj}{\int_{N_2}^N Z_{2j}^{\lambda} I_{2j} P_{G_{2j}}^{-\lambda \gamma} \, dj} \\
\Rightarrow \left( \frac{X_1}{X_2} \right)^{1-\gamma} = \left( \frac{\epsilon}{1-\epsilon} \right)^{1-\gamma} \left( \frac{Z_1}{Z_2} \right)^{\delta+2\gamma-1} \frac{\int_{N_1}^N I_{1j} P_{G_{1j}}^{-\lambda \gamma} Z_{1j}^{\delta} \, dj}{\int_{N_2}^N I_{2j} P_{G_{2j}}^{-\lambda \gamma} Z_{2j}^{\delta} \, dj} \\
\Rightarrow \frac{X_1}{X_2} = \left( \frac{\epsilon}{1-\epsilon} \right)^{1-\gamma} \left( \frac{Z_1}{Z_2} \right)^{(\delta+2\gamma-1)(1-\lambda)} \frac{\int_{N_1}^N I_{1j} P_{G_{1j}}^{-\lambda \gamma} Z_{1j}^{\delta} \, dj}{\int_{N_2}^N I_{2j} P_{G_{2j}}^{-\lambda \gamma} Z_{2j}^{\delta} \, dj} (B.24)
\]

### B.3 Intermediate Good Sector in Closed Economy

#### B.3.1 Incumbents in industry 1

Intermediate-good sector is monopolistic competition. So each incumbent has power to set up its own price. The fixed operation cost is the same across the same industry, \( \theta_1 \frac{Z_1}{Z_2} \), where \( Z_1 \) and \( Z_2 \) are the average level of qualities, and taken as given for individual firms. The fixed operation cost can be interpreted by two parts, \( Z_1 \) and \( \frac{Z_1}{Z_2} \). Fixed operation costs depend on how complex your operation is. The more technically advanced the operation is, the costlier it is to run, thus \( Z_1 \) is positively related with the operation cost. Cost is driven by the difference between the complexity of your operation \( Z_1 \) and the complexity of the rest of the economy \( Z_2 \). If you are way out of line with a relatively high \( Z_1 \), which means \( \frac{Z_1}{Z_2} \) is high, then you have high costs. If you are way out of line with a relatively low \( Z_1 \), which means \( \frac{Z_1}{Z_2} \) is low, then you have low costs. However, if you stay in line with everybody else, your costs stay constant. In this formulation, cost stays constant if \( \frac{Z_1}{Z_2} \) stays constant, meaning that
the cost imposed by increases in your own complexity are completely offset by the saving induced by spillovers from other parts of the economy. The first $Z_1$ captures the idea that you have higher cost the more complex your operation is. The second term, $\frac{Z_1}{Z_2}$ captures the idea that cost partly depends on your complexity compared to the complexity of the rest of the economy. In this formulation, spillovers do not fully offset the costs associated with greater complexity.

$A_1$ is the unit cost for intermediate good, i.e. 1 unit of intermediate good requires $A_1$ units of final good. The profit of firm is,

$$F_{1j} = G_{1j} (P_{G_{1j}} - A_1) - \theta_1 \frac{Z_1^2}{Z_2}$$

(B.25)

Given the demand, each firm chooses price and R&D activity to maximize the present value of profit, so the current value of Hamiltonian is,

$$CVH_{1j} = G_{1j} (P_{G_{1j}} - A_1) - \theta_1 \frac{Z_1^2}{Z_2} - R_{1j} + q_{1j}(\alpha_1 I_{1j})$$

(B.26)

s.t. $G_{1j} = [\lambda \epsilon(\frac{X_1}{X_2})^{\epsilon-1}]^{1-\gamma} Z_1^\delta Z_1^1 Z_2^{1-(\delta+\gamma)}l_{1j}$; and $\dot{Z}_{1j} = \alpha_1 I_{1j}$; Note that the $Z_1$ and $Z_2$ inside the demand function is the average quality, and each firm choose the quality level to contribute to it. While the fixed operation cost is taken as given for each firm.

Define $r_1$ as the return in R&D in industry 1, and $r_2$ as the return in R&D in industry 2, we get the definition for $I_1$ as
\[
\begin{cases}
I_{1j} = 0 & \text{if } r_1 < r_2 \\
I_{1j} = R_{1j} & \text{if } r_1 = r_2 \\
I_{2j} = R_{1j} + \frac{N_2}{N_1} R_{2j} & \text{if } r_1 > r_2
\end{cases}
\]

Later we will see the number of firms in each industry \(N_1\) and \(N_2\) always jump to make \(r_1 = r_2\), thus \(I_{1j} = R_{1j}\).

All the necessary conditions are:

- \(\dot{Z}_{1j} = \alpha_1 I_{1j}\);
- \(\frac{\partial CVH_{1j}}{\partial P_{G1j}} = 0\);
- \(\frac{\partial CVH_{1j}}{\partial R_{1j}} = -1 + q_{1j} \alpha_1\);
- \(q_{1j} = -r_{1j} q_{1j} + \frac{\partial CVH_{1j}}{\partial Z_{1j}}\);
- \(Z_{1j,t=0}\) is given
- \(\lim_{t \to \infty} e^{-\int_0^t r(s) ds} q_{1j}(t) Z_{1j}(t) = 0\)

The following are the detail:

Optimal Condition (1) \(\frac{\partial CVH_{1j}}{\partial P_{G1j}} = 0\)

\[G_{1j} = \left[ \frac{\lambda P_{X1}}{P_{G1j}} \right] \frac{1}{\lambda} Z_{1j}^{\lambda} Z_{2j}^{\gamma} \left( 1 - (\delta + \gamma) \right) I_{1j},\]

\[\frac{\partial CVH_{1j}}{\partial P_{G1j}} = 0 \Rightarrow \frac{\partial (P_{G1j} - A_1 G_{1j})}{\partial P_{G1j}} = 0, \text{ since all other parts in } CVH_{1j} \text{ do not include } P_{G1j},\]

\[P_{G1j}\]

\[0 = \frac{\partial (P_{G1j} - A_1 P_{G1j})}{\partial P_{G1j}} \Rightarrow 0 = \frac{\lambda}{1 - \lambda} P_{G1j}^{-\frac{1}{\lambda}} + \frac{1}{1 - \lambda} A_1 P_{G1j}^{-\frac{1}{\lambda} - 1} = 0 \]

\[\Rightarrow P_{G1j} = \frac{A_1}{\lambda} \quad (B.27)\]
Note that since each firm in industry 1 face the same unit cost, i.e., $A_1$ units of $Y$ to produce 1 unit of $G_{1j}$, so the prices of all intermediate-good firms inside the same industry are the same, which is the same unit cost, times the same monopoly markup, which is equation (B.27). In this case, from (B.24) combined with (B.21) the expression of $\frac{X_1}{X_2}$ can be written as,

$$\frac{X_1}{X_2} = \left(\frac{\epsilon}{1-\epsilon}\right)^\lambda \left(\frac{Z_1}{Z_2}\right)^{(\delta+2\gamma-1)(1-\lambda)} \left(\frac{\int_0^{N_1} l_{1j} P_{G_{1j}}^{1-\lambda} Z_{1j}^\delta dj}{\int_0^{N_2} l_{2j} P_{G_{2j}}^{1-\lambda} Z_{2j}^\delta dj}\right)^{1-\lambda}$$

(B.28)

$$= \left(\frac{\epsilon}{1-\epsilon}\right)^\lambda \left(\frac{Z_1}{Z_2}\right)^{(\delta+2\gamma-1)(1-\lambda)} \left(\frac{P_{G_1}}{P_{G_2}}\right)^{1-\lambda} \left(\frac{\int_0^{N_1} l_{1j} P_{G_{1j}} Z_{1j}^\delta dj}{\int_0^{N_2} l_{2j} Z_{2j}^\delta dj}\right)^{1-\lambda}$$

(B.29)

Optimal Condition (2) \[\frac{\partial C V H_{1j}}{\partial R_{1j}} = -1 + q_{1j} \alpha_1\]

$$\implies \begin{cases} R_{1j} = \infty & \text{if } \frac{1}{\alpha_1} > q_{1j} \\ R_{1j} > 0 & \text{if } \frac{1}{\alpha_1} = q_{1j} \\ R_{1j} = 0 & \text{if } \frac{1}{\alpha_1} < q_{1j} \end{cases}$$

The interior solution, $\frac{1}{\alpha_1} = q_{1j}$, is the same for all $j$, implying that in the interior all firms in industry 1 choose the same level of R&D, which we denote $R_1$.

Optimal Condition (3):
\begin{align*}
q_{ij} & = -r_{1j}q_{ij} + \frac{\partial CVH_{1j}}{\partial Z_{1j}} \quad \text{(B.30)} \\
\Rightarrow -\frac{\partial CVH_{1j}}{\partial Z_{1j}} & = -r_{1j}q_{ij} + \dot{q}_{ij} \quad \text{(B.31)} \\
\Rightarrow \frac{\partial F_{1j}}{\partial Z_{1j}} & = -r_{1j}q_{ij} + \dot{q}_{ij} \quad \text{(B.32)} \\
\Rightarrow r_{1j} & = \frac{\partial F_{1j}}{\partial Z_{1j}} / q_{ij} + \frac{\dot{q}_{ij}}{q_{ij}} \quad \text{(B.33)}
\end{align*}

The transversality condition \( \lim_{t \to \infty} e^{-\int_{t}^{\infty} r(s) ds} q_{1j}(t) Z_{1j}(t) = 0 \)

Plug demand function B.13 into the cash ow (B.25), we get

\begin{align*}
F_{1j} & = A_{1}(\frac{1}{\lambda} - 1)\left\{ \frac{\lambda P_{X_{1}}}{P_{G_{i}}} \right\}^{\frac{1}{1-\gamma}}(Z_{1j}^{\delta} Z_{2}^{\gamma} Z_{2}^{1-(\delta+\gamma)l_{1j}}) - \theta_{1} \frac{Z_{1}^{2}}{Z_{2}} \\
& = A_{1}(\frac{1 - \lambda}{\lambda})(\frac{\lambda P_{X_{1}}}{A_{1}/\lambda})^{\frac{1}{1-\gamma}}(Z_{1j}^{\delta} Z_{2}^{\gamma} Z_{2}^{1-(\delta+\gamma)l_{1j}}) - \theta_{1} \frac{Z_{1}^{2}}{Z_{2}} \quad \text{(B.34)}
\end{align*}

Note that the fixed operation cost is taken as given for each firm, so the \( Z_{1} \) and \( Z_{2} \) there is taken as given. In order to see the difference between the endogenous R&D decision, and given average industry level of quality, I denote them as \( Z_{1j} \) and \( Z_{1} \) in (B.34).

So

\[ \frac{\partial F_{1j}}{\partial Z_{1j}} = [\delta] A_{1} \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{X_{1}}}{A_{1}/\lambda} \right)^{\frac{1}{1-\gamma}} Z_{1j}^{\delta} Z_{2}^{\gamma} Z_{2}^{1-(\delta+\gamma)l_{1j}} \quad \text{(B.35)} \]
Given that \( r_{1j} = \frac{\partial F_{1j}}{\partial Z_{1j}} \frac{1}{q_{1j}} + \frac{\dot{q}_{1j}}{q_{1j}} \), combine (B.35), we can get the return in R&D in industry 1 as

\[
\begin{align*}
    r_{1j} &= \frac{\partial F_{1j}}{\partial Z_{1j}} \frac{1}{q_{1j}} + \frac{\dot{q}_{1j}}{q_{1j}} = \frac{\partial F_{1j}}{\partial Z_{1j}} \frac{1}{q_{1j}} + 0 = \frac{\partial F_{1j}}{\partial Z_{1j}} \alpha_1 \tag{B.36} \\
    &= \alpha_1 \delta A \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{X1}}{A_{1j} / \lambda} \right)^{1-\gamma} Z_{1j}^{1-\gamma} Z_{1j}^{1-\gamma} Z_{1j}^{1-\gamma} Z_{1j}^{1-\gamma} \tag{B.37}
\end{align*}
\]

Now we want to prove symmetry among the same industry. Plug demand of \( G_{1j} \) into the demand of \( l_{1j} \), which means plug (B.13) into (B.15), we get

\[
    w = (1 - \lambda) P_{X1} \left( \frac{\lambda P_{X1}}{P_{G_{1j}}} \right)^{1-\gamma} Z_{1j}^{\delta-1} Z_{1j}^{\gamma} Z_{1j}^{1-\gamma} Z_{1j}^{1-\gamma} Z_{1j}^{1-\gamma}. 
\]

In the same industry, say industry 1, all firms face the same spillovers, \( Z_1 \) and \( Z_2 \), and the price of processed good, \( P_{X1} \), and also they set up the same \( P_{G_{1j}} \) according to (B.27). If \( Z_{1j} > Z_{1k} \), then all labors move to use \( G_{1j} \) get paid a higher wage, so they all move to use \( G_{1j} \) and leave no demand for \( G_{1k} \). \( G_{1j} \) takes the whole industry 1 as a monoply.\(^1\) And the processed good production function (B.10) becomes \( X_1 = G_1^{A_1} \left( Z_{1j}^{\delta+\gamma} Z_{1j}^{1-\gamma} L_1 \right)^{1-\lambda} \). Where \( L_1 = \epsilon L \) based on (B.21). And the return in R&D (B.36) becomes,

\[
    r_1 = \alpha_1 (\delta + \gamma) A \frac{1-\lambda}{\lambda} \left( \frac{\lambda P_{X1}}{A_{1j} / \lambda} \right)^{1-\gamma} Z_{1j}^{(\delta+\gamma)-1} Z_{1j}^{1-(\delta+\gamma)} (\epsilon L) 
\]

We assume zero cost entry/exit condition. If outsiders observe incipient profit,

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\(^1\)Note that at this time, the monopolistic firm still set price as B.27, even not it is facing a larger market size. This is because the price elasticity of the demand is constant. This is the one of the two special case in monopoly.
then they enter industry 1 with a new variety without any costs. And they enter with the average level of quality \( Z_1 \). The incumbent here is a monopoly, thus the average quality level equals the monopoly’s quality. With the same quality, they share industry 1 again. So the assumption that “entrants enter at the average quality” ensures the symmetry quality among the same industry. So all \( Z_{1j} = Z_{1k} = Z_1 \). Thus (B.29) becomes

\[
\frac{X_1}{X_2} = (\frac{\epsilon}{1-\epsilon})^\lambda (\frac{Z_1}{Z_2})^{(\delta + 2\gamma - 1)(1-\lambda)} (\frac{P_{G_1}}{P_{G_2}})^{-\lambda} (\frac{\sum_{j}^{N_1} l_{1j} d_{1j}}{\sum_{j}^{N_2} l_{2j} d_{2j}})^{1-\lambda}
\]

\[
= (\frac{\epsilon}{1-\epsilon})^\lambda (\frac{Z_1}{Z_2})^{(\delta + 2\gamma - 1)(1-\lambda)} (\frac{P_{G_1}}{P_{G_2}})^{-\lambda} (\frac{L_1}{L_2})^{1-\lambda} = (\frac{\epsilon}{1-\epsilon}) (\frac{Z_1}{Z_2})^{(\delta + 2\gamma - 1)(1-\lambda)} (\frac{P_{G_1}}{P_{G_2}})^{-\lambda}
\]

(B.38)

Plug above back to the inverse demand function (B.7) to get \( P_{X_1} \), and then back to (B.36), we get

\[
P_{X_1} = \epsilon X_1^{1-\epsilon} X_2^{1-\epsilon} = \epsilon (\frac{\epsilon}{1-\epsilon})^{(1-\epsilon)} (\frac{Z_1}{Z_2})^{(\delta + 2\gamma - 1)(1-\epsilon)} (\frac{P_{G_1}}{P_{G_2}})^{-\lambda (1-\epsilon)}
\]

We need to proof \( \Gamma - 1 < 0 \), which means \( \Gamma = \delta + \gamma + [2(\delta + \gamma) - 1](\epsilon - 1) = 1 + 2(\delta + \gamma)\epsilon - (\delta + \gamma) - \epsilon < 1 \)

We can proof \( [\delta + \gamma + (2\delta + 2\gamma - 1)(\epsilon - 1)] \in (0, 1) \) by two steps:

Firstly, \( [\delta + \gamma + (2\delta + 2\gamma - 1)(\epsilon - 1)] = \delta + \gamma + (1 - 2(\delta + \gamma))(1 - \epsilon) > 0; \)
Secondly, \( \delta + \gamma + [2(\delta + \gamma) - 1](\epsilon - 1) < 1 \iff \delta + \gamma + [2(\delta + \gamma) - 1](\epsilon - 1) - 1 < 0 \)

And \( \delta + \gamma + [2(\delta + \gamma) - 1](\epsilon - 1) - 1 = 2(\delta + \gamma)\epsilon - (\delta + \gamma) - \epsilon = (\delta + \gamma)(\epsilon - 1) + \epsilon(\delta + \gamma - 1) < 0 \),

Given that \( \delta + \gamma < 1 \) and \( \epsilon \in (0, 1) \).

From Section (2.3) we will see a zero cost in entry/exit causes firms jump into the market if observe incipient profits, thus distributed profit is zero, i.e.

\[
0 = \pi_{1j} = F_{1j} - R_1 = G_{1j} \left( P_{G_{1j}} - A_1 \right) - \theta_1 \frac{Z_2^2}{Z_2} - R_1
\]

\[
= A_1 \frac{1-\lambda}{\epsilon} \left[ \frac{X_s}{A_1} \right] \frac{1}{\Gamma(1-\epsilon)} \left( \frac{P_{G_{11}}}{P_{G_{21}}} \right)^{\lambda(\epsilon-1)} Z_1^1 Z_2^{1-1} l_{1j} - \theta_1 \frac{Z_2^2}{Z_2} - R_1
\]

From \( \frac{1}{\alpha_1} = q_{1j} \) thus all \( R_{1j} = R_1 \); and \( P_{G_{1j}} = \frac{A_1}{\lambda} \) we see that, all firms choose the same levels of the two things they control, output price and R&D spending. \( Z_1 \) and \( Z_2 \) are the average quality levels, so the same for all firms. \( \int_0^{N_1} l_{1j} = L_1 \). From (B.21) and \( L_1 + L_2 = L \), we know that the total market size for industry 1, \( L_1 \) is fixed. Thus entry makes \( l_{1j} \) equal among the same industry from above equation. Because all firms in the industry make the same choices, we henceforth drop the firm subscript. Note that symmetry also implies that individual firm quality \( Z_{1j} \) equals the industry average quality \( Z_1 \). And \( l_1 = \frac{L_1}{N_1} \).

### B.3.2 Incumbents in industry 2

We’ve already prooved symmetry among the same industry. Following the similar steps we get the results in industry 2.
\[ CVH_2 = G_2(P_{G_2} - A_2) - \theta_2 \frac{Z_2^2}{Z_1} - R_2 + q_2(\alpha_2 I_2) \quad (B.40) \]

s.t. \( G_2 = \left[ \frac{\lambda P_{X_2}}{P_{G_2}} \right]^{\frac{1}{1-\lambda}} Z_2^{(\delta+\gamma)} Z_1^{1-(\delta+\gamma)} \lambda_2 \), and \( \dot{Z}_2 = \alpha_2 R_2; I_2 = R_2 \) on BGP.

FOC(A) \( \frac{\partial CVH_2}{\partial P_{G_2}} = 0 \)

\[ \Rightarrow P_{G_2} = \frac{A_2}{\lambda} \quad (B.41) \]

FOC(2) \( \frac{\partial CVH_2}{\partial R_2} = -1 + q_2 \alpha_2 \)

\[ \begin{cases} 
R_2 = \infty & \text{if } \frac{1}{\alpha_2} > q_2 \\
R_2 > 0 & \text{if } \frac{1}{\alpha_2} = q_2 \\
R_2 = 0 & \text{if } \frac{1}{\alpha_2} < q_2 
\end{cases} \]

The transversality condition \( \lim_{t \to \infty} e^{-\int_0^t r(s) ds} q_{2j}(t) Z_{2j}(t) = 0 \)

\( F_2 = G_2(P_{G_2} - A_2) - \theta_2 \frac{Z_2^2}{Z_1} \), where \( Z_1 \) and \( Z_2 \) in the fixed operation cost are the average levels of the industries, and taken as given for every individual firm.

\( F_2 = G_2(P_{G_2} - A_2) - \theta_2 \frac{Z_2^2}{Z_1} \) and plug demand function and price inside, we get

\[ F_2 = A_2 \left( 1 - \frac{1}{\lambda} \right) \left( \frac{\lambda P_{X_2}}{P_{G_2}} \right)^{\frac{1}{1-\lambda}} (Z_2^{(\delta+\gamma)} Z_1^{1-(\delta+\gamma)} \lambda_2) - \theta_2 \frac{Z_2^2}{Z_1} \]

\[ = A_2 \frac{1}{\lambda} \left( \frac{\lambda P_{X_2}}{A_2/\lambda} \right)^{\frac{1}{1-\lambda}} (Z_2^{(\delta+\gamma)} Z_1^{1-(\delta+\gamma)} \lambda_2) - \theta_2 \frac{Z_2^2}{Z_1} \quad (B.42) \]

where \( P_{X_2} = (1 - \epsilon)(\frac{X_1}{X_2})^\epsilon = (1 - \epsilon)(\frac{1}{1-\epsilon})^\epsilon \left( \frac{Z_1}{Z_2} \right)^{(2\delta+2\gamma-1)(1-\lambda)} \epsilon \left( \frac{P_{G_1}}{P_{G_2}} \right)^{-\lambda} \)

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In order to see the difference between the endogenous R&D decision, and given average industry level of quality, I denote them as $Z_{2j}$ and $Z_2$.

So

$$\frac{\partial F_2}{\partial Z_{2j}} = \delta A_2 \frac{1 - \lambda}{\lambda} \left[ \frac{\lambda P_{X_2}}{A_2/\lambda} \right]^{1/\lambda} (Z_2^{\delta+\gamma-1} Z_1^{1-(\delta+\gamma)}) l_2$$

(B.43)

$$r_2 = \frac{\partial F_2}{\partial Z_2} \frac{1}{q_2} + \frac{\dot{q}_2}{q_2} = \frac{\partial F_2}{\partial Z_2} \frac{1}{q_2} + 0 = \frac{\partial F_2}{\partial Z_{2j}} \alpha_2$$

(B.45)

$$= \delta \alpha_2 A_2 \frac{1 - \lambda}{\lambda} \left[ \frac{\lambda^2 (1 - \epsilon)}{A_2} \right]^{1/\lambda} \left( \frac{\epsilon}{1 - \epsilon} \right)^{1/\lambda} \left( \frac{Z_1}{Z_2} \right)^{(1 - 2\delta - 2\gamma)\epsilon} (P_{G_1}^{\gamma})^{1/\lambda} (Z_2^{\delta-1} Z_1^{1-\delta}) l_2$$

(B.46)

Since $1 > \Gamma > 0$, so $-\Gamma < 0$, thus there’s a diminishing return to $Z_2$ giving $l_2$ and others constant.

### B.3.3 Zero entry and exit

Zero entry cost implies zero profit condition (see the paper for more explanation). Since $\Pi_i = F_i - R_i$, so zero profit condition means $R_i = F_i$. And we also assume $\dot{Z}_1 = \alpha_1 R_1$ and $\dot{Z}_2 = \alpha_2 R_2$, combine them with zero profit condition, and (B.34), (B.42), we get the growth rate of $Z_1$ and $Z_2$ as,
\[ g_1 \equiv \frac{\dot{Z}_1}{Z_1} = \frac{\alpha_1 R_1}{Z_1} = \frac{\alpha_1 F_1}{Z_1} = \alpha_1 \left\{ A_1 \frac{1 - \lambda}{\lambda} \frac{\lambda^2 \epsilon}{A_1} \frac{1 - \epsilon}{1 - \epsilon} \frac{L_1}{N_1} \right\} \frac{Z_1}{Z_2} \Gamma^{-1} - \theta_1 \frac{Z_1}{Z_2} \] (B.47)

\[ g_2 \equiv \frac{\dot{Z}_2}{Z_2} = \frac{\alpha_2 R_2}{Z_2} = \frac{\alpha_2 F_2}{Z_2} = \alpha_2 \left\{ A_2 \frac{1 - \lambda}{\lambda} \frac{\lambda^2 (1 - \epsilon)}{A_2} \frac{1 - \epsilon}{1 - \epsilon} \frac{L_2}{N_2} \right\} \frac{Z_2}{Z_1} \Gamma^{-1} - \theta_2 \frac{Z_2}{Z_1} \] (B.48)

\section*{B.4 General Equilibrium in Closed Economy}

\subsection*{B.4.1 Market clear conditions}

\subsubsection*{B.4.1.1 Labor market clear conditions}

Labor Supply:

\[ L = L_1 + L_2 = \int_0^{N_1} l_{1j} dj + \int_0^{N_2} l_{2j} dj \] (B.49)
Labor Demand is equation (B.15) and (B.16).

and from (B.21) we know

$$\frac{L_1}{L_2} = \frac{\epsilon}{1 - \epsilon}$$

so

$$L_1 = \epsilon L$$  \hspace{1cm} (B.50)$$

$$L_2 = (1 - \epsilon)L$$  \hspace{1cm} (B.51)

B.4.1.2 good market clear conditions

$$Y = \int_0^{N_1} A_1 G_1 d\gamma + \int_0^{N_1} R_1 d\gamma + \int_0^{N_1} \phi_{1j} d\gamma + \int_0^{N_2} A_2 G_2 d\gamma + \int_0^{N_2} R_2 d\gamma + \int_0^{N_2} \phi_{2j} d\gamma + C$$  \hspace{1cm} (B.52)

where \( \phi_{1j} = \theta_1 \frac{Z_1}{Z_2} \), \( \phi_{2j} = \theta_2 \frac{Z_2}{Z_1} \).

The factor payments is the following: \( \epsilon \cdot Y \) pays to \( P_{X_1}, X_1 \), and \( (1 - \epsilon) \cdot Y \) pays to \( P_{X_2}, X_2 \); \( \lambda P_{X_1}, X_1 \) pays to \( \int_0^{N_1} P_{G_1} G_1 d\gamma \), and \( (1 - \lambda) P_{X_1}, X_1 \) pays to \( \int_0^{N_1} w_1 d\gamma \); \( \lambda P_{X_2}, X_2 \) pays to \( \int_0^{N_2} P_{G_2} G_2 d\gamma \), and \( (1 - \lambda) P_{X_2}, X_2 \) pays to \( \int_0^{N_2} w_2 d\gamma \). Sum them up we can write as the following,
\[ 1 \cdot Y = P_{X_1}X_1 + P_{X_2}X_2 \]

\[ = \int_0^{N_1} P_{G_1j}G_{1j}dj + \int_0^{N_1} w_{1j}dj + \int_0^{N_2} P_{G_2j}G_{2j}dj + \int_0^{N_2} w_{2j}dj \]  \( (B.53) \)

And zero profit condition requires \( P_{G_1j}G_{1j} = A_1G_{1j} + R_{1j} + \phi_{1j} \); and \( P_{G_2j}G_{2j} = A_2G_{2j} + R_{2j} + \phi_{2j} \). So \( (B.53) \) can be written as,

\[ Y = \int_0^{N_1} A_1G_{1j}dj + \int_0^{N_1} R_{1j}dj + \int_0^{N_1} \phi_{1j}dj + \int_0^{N_2} A_1G_{2j}dj + \int_0^{N_2} R_{2j}dj + \int_0^{N_2} \phi_{2j}dj + wL \]  \( (B.54) \)

Compare \( (B.54) \) and \( (B.52) \), we get

\[ C = wL = (1 - \lambda)(P_{X_1}X_1 + P_{X_2}X_2) \]  \( (B.55) \)

\[ = (1 - \lambda)Y \]  \( (B.56) \)

so

\[ \frac{wL}{Y} = \frac{C}{Y} = 1 - \lambda \]  \( (B.57) \)
so
\[
\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{w}}{w}
\]

(B.58)

B.4.1.3 credit market equilibrium

In equilibrium, no arbitrage condition requires that \( r_1 = r_2 \), so (B.36) = (2.63). If one return is higher, then all resource goes to accumulate that quality. We’ve already seen there’s diminishing return to R&D given the spillover from the other industry is fixed, so two returns eventually equal. See detail in the paper.

B.4.2 Derivations of Growth Rates

Combine the inverse demand for \( X_1 \) (B.5) and (B.38) into demand for \( G_1 \), (B.13),

Write it out and rearrange it as,

\[
G_1 = \left( \frac{\lambda \epsilon}{G_1} \right)^{\frac{1}{1-\epsilon}} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{\epsilon-1}{\epsilon-1}} \left( \frac{P_{G1}}{P_{G2}} \right)^{-\frac{\lambda(\epsilon-1)}{\epsilon-1}} \left( \frac{Z_1}{Z_2} \right)^{-(1-2(\delta+\gamma))(1-\epsilon)} \left( Z_1^{\delta+\gamma} Z_2^{1-(\delta+\gamma)} \right) \frac{L_1}{N_1}
\]

(B.59)

\[
= \xi_1 Z_1^{[1-2(\delta+\gamma)](1-\epsilon) + (\delta+\gamma)} Z_2^{[1-2(\delta+\gamma)](1-\epsilon)+1-\delta-\gamma} \frac{L_1}{N_1}
\]

(B.60)

where \( \xi_1 = \left( \frac{\lambda \epsilon}{P_{G1}} \right)^{\frac{1}{1-\epsilon}} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{\epsilon-1}{\epsilon-1}} \left( \frac{P_{G1}}{P_{G2}} \right)^{-\frac{\lambda(\epsilon-1)}{\epsilon-1}} \).

We see that \( P_{G1} \) and \( P_{G2} \) are constant, which equal the unit cost times monopolistic markup. \( N_1 \) is a jumping variable. As soon as the outsiders observe a incipient profit,
they will enter the market instantaneously with a zero entry cost. \( L_1 = \epsilon L \). So the only factors that affect the growth rate of \( G_1 \) is the state variables, \( Z_1 \) and \( Z_2 \). So,

\[
\frac{\dot{G}_1}{G_1} = \Gamma \frac{\dot{Z}_1}{Z_1} + (1 - \Gamma) \frac{\dot{Z}_2}{Z_2}
\]  

(B.61)

where \( \Gamma \equiv [1 - 2(\delta + \gamma)](1 - \epsilon) + \delta = 1 - \epsilon - (\delta + \gamma) + 2(\delta + \gamma)\epsilon \).

Similar derivation for \( G_2 \), we get the expression for \( G_2 \) as,

\[
G_2 = \xi_2 Z_1^{1 - \epsilon - (\delta + \gamma) + 2(\delta + \gamma)\epsilon} Z_2^{(\delta + \gamma) - 2(\delta + \gamma)\epsilon} \frac{L_2}{N_2}
\]  

(B.62)

where \( \xi_2 = \left( \frac{\lambda(1 - \epsilon)\gamma}{P_{G_2}} \right)^{\frac{1}{1 - \epsilon}} \left( \frac{\epsilon}{1 - \epsilon} \right)^{\frac{1}{1 - \epsilon}} \left( \frac{P_{G_1}}{P_{G_2}} \right)^{\frac{\lambda}{1 - \epsilon}} \).

So the growth rate of \( G_2 \) is,

\[
\frac{\dot{G}_2}{G_2} = \Gamma \frac{\dot{Z}_1}{Z_1} + (1 - \Gamma) \frac{\dot{Z}_2}{Z_2}
\]  

(B.63)

where \( \Gamma = 1 - \epsilon - (\delta + \gamma) + 2(\delta + \gamma)\epsilon \).

Now let’s get the growth rate of \( X_1 \). First of all, we need to write down the expression of \( X_1 \) in terms of qualities. Plug the demand function of \( G_1 \), which is (B.13) into (B.10), we get (B.22). Combine (B.38) and (B.50) with (B.22), and rearrange it as the following,
\[ X_1 = \left[ \frac{\lambda \epsilon \left( \frac{X_1}{X_2} \right)^{\epsilon - 1}}{P_{G_1}} \right]^{\frac{1}{1 - \epsilon}} Z_1^{\delta + \gamma} Z_2^{1 - (\delta + \gamma)} L_1 \]

\[ = \frac{\lambda \epsilon \left( \frac{P_{G_1}}{P_{G_2}} \right)^{-\lambda} \left( \frac{Z_1}{Z_2} \right)^{-1 + 2(\delta + \gamma)(1 - \lambda)}^{\epsilon - 1}}{P_{G_1}} \]

\[ = \frac{(\lambda \epsilon)^{\frac{1}{1 - \epsilon}} \left( \frac{\epsilon}{1 - \epsilon} \right)^{\frac{\lambda(\epsilon - 1)}{1 - \lambda}} \left( \frac{P_{G_1}}{P_{G_2}} \right)^{-\frac{\lambda^2(\epsilon - 1)}{1 - \lambda}}}{P_{G_1}^{\frac{1}{1 - \epsilon}}} \left( \frac{Z_1}{Z_2} \right)^{1 - 2(\delta + \gamma)(1 - \lambda)} \]

So we can arrange the terms and get,

\[ X_1 = \zeta_1 Z_1^{\delta + \gamma + (1 - 2(\delta + \gamma))(1 - \lambda)} Z_2^{1 - (\delta + \gamma) - 1 - 2(\delta + \gamma)(1 - \lambda)} (\epsilon L) \tag{B.64} \]

where \( \zeta_1 = (\lambda \epsilon)^{\frac{1}{1 - \epsilon}} \left( \frac{\epsilon}{1 - \epsilon} \right)^{\frac{\lambda(\epsilon - 1)}{1 - \lambda}} \left( \frac{P_{G_1}}{P_{G_2}} \right)^{-\frac{\lambda^2(\epsilon - 1)}{1 - \lambda}} \)

And we get easily get the growth rate of \( X_1 \) from (B.64) as,

\[ \frac{\dot{X}_1}{X_1} = \vartheta_1 \frac{\dot{Z}_1}{Z_1} + (1 - \vartheta_1) \frac{\dot{Z}_2}{Z_2} \tag{B.65} \]

where \( \vartheta_1 = \delta + \gamma + [1 - 2(\delta + \gamma)] \lambda (1 - \epsilon) \).

For the growth rate of \( X_2 \), it’s very similar as that of \( X_1 \). Combine (B.29), and (B.51) into (B.23), we get,
\[ X_2 = N_2 \left( \frac{\lambda (1 - \epsilon)(X_2^*)^\epsilon}{P_{G_2}} \right) \frac{1}{1 - \epsilon} \left( Z_2^{\delta + \gamma} Z_1^{1 - (\delta + \gamma)} L_2 \right) \]

\[ = \left\{ \frac{\lambda (1 - \epsilon)(Z_2^*)^{1 - 2(\delta + \gamma)(1 - \lambda)}^{\epsilon}}{P_{G_2}} \right\} \frac{1}{1 - \epsilon} Z_2^{\delta + \gamma} Z_1^{1 - (\delta + \gamma)} L_2 \]

\[ = \frac{[\lambda (1 - \epsilon)]^{1 - \lambda}}{P_{G_2}^{1 - \lambda}} (\epsilon)^{1 - \lambda} \left( \frac{P_{G_1}}{P_{G_2}} \right)^{1 - \lambda} (Z_1^*)^{1 - 2(\delta + \gamma)\lambda} Z_2^{\delta + \gamma} Z_1^{1 - (\delta + \gamma)} (1 - \epsilon) L \]

Rearrange the terms we get,

\[ X_2 = \zeta_2 Z_1^{1 - (\delta + \gamma) - [1 - 2(\delta + \gamma)]\lambda} Z_2^{\delta + \gamma + [1 - 2(\delta + \gamma)]\lambda} (1 - \epsilon) L \quad (B.66) \]

where \( \zeta_2 = [\lambda (1 - \epsilon)]^{1 - \lambda} (\epsilon)^{1 - \lambda} \left( \frac{P_{G_1}}{P_{G_2}} \right)^{1 - \lambda} (Z_1^*)^{1 - 2(\delta + \gamma)\lambda}. \)

So the growth rate of \( X_2 \) only depends on the growth rate of \( Z_1 \) and \( Z_2 \). We can easily get it from (B.66),

\[ \frac{\dot{X}_2}{X_2} = \vartheta_2 \frac{\dot{Z}_1}{Z_1} + (1 - \vartheta_2) \frac{\dot{Z}_2}{Z_2} \quad (B.67) \]

where \( \vartheta_2 = 1 - (\delta + \gamma) - [1 - 2(\delta + \gamma)]\lambda \epsilon \).

Now let’s see the growth rate of \( Y \), where from (B.1) we know that \( Y = X_1^\epsilon X_2^{1 - \epsilon} \). Combine (B.64) and (B.66) with the (B.1), we get
\[ Y = X_1^\epsilon X_2^{1-\epsilon} \]
\[ = [\zeta_1 Z_1^{(\delta+\gamma)+[1-2(\delta+\gamma)]\lambda(1-\epsilon)} Z_2^{1-(\delta+\gamma)-[1-2(\delta+\gamma)]\lambda(1-\epsilon)} (\epsilon L)]^{1-\epsilon} \]
\[ \cdot [\zeta_2 Z_1^{-(\delta+\gamma)-[1-2(\delta+\gamma)]\lambda \epsilon} Z_2^{\delta+\gamma+[1-2(\delta+\gamma)]\lambda \epsilon} (1-\epsilon) L]^{1-\epsilon} \]
\[ = (\zeta_1 \zeta_2^{1-\epsilon})^{\epsilon L} (1-\epsilon)^{1-\epsilon} \cdot Z_1^{(\delta+\gamma)\epsilon}[1-(\delta+\gamma)(1-\epsilon)]+1-2(\delta+\gamma)]\lambda(1-\epsilon)\epsilon-1-2(\delta+\gamma)]\lambda(1-\epsilon)\epsilon \]
\[ \cdot Z_2^{1-(\delta+\gamma)\epsilon+(\delta+\gamma)(1-\epsilon)]+1-2(\delta+\gamma)]\lambda(1-\epsilon)\epsilon-1-2(\delta+\gamma)]\lambda(1-\epsilon)\epsilon \]
\[ = \kappa Z_1^{(\delta+\gamma)\epsilon}[1-(\delta+\gamma)(1-\epsilon)] Z_2^{1-(\delta+\gamma)\epsilon+(\delta+\gamma)(1-\epsilon)] L \]

where \( \kappa = (\zeta_1 \zeta_2^{1-\epsilon})^{\epsilon L} (1-\epsilon)^{1-\epsilon} \lambda^{\frac{\lambda}{1-\lambda}} (1-\epsilon) \frac{\lambda(1-\epsilon)}{1-\lambda} P_{G_1} \frac{\lambda \epsilon}{1-\lambda} P_{G_2} \frac{\lambda(1-\epsilon)}{1-\lambda} \epsilon \epsilon^2 (1-\epsilon)^{(1-\epsilon)}; \)

\( (\delta + \gamma)\epsilon + [1 - (\delta + \gamma)](1-\epsilon) = 2(\delta + \gamma)\epsilon - (\delta + \gamma) - \epsilon + 1 \)

(B.68) shows that the growth rate of \( Y \) only depends on \( Z_1 \) and \( Z_2 \),

\[ \frac{\dot{Y}}{Y} = \frac{\dot{w}}{w} = \frac{\dot{C}}{C} = \Gamma \frac{\dot{Z}_1}{Z_1} + (1 - \Gamma) \frac{\dot{Z}_2}{Z_2} \]

(B.69)

where \( \Gamma = \epsilon(\delta + \gamma) + (1-\epsilon)[1-(\delta + \gamma)] = \epsilon \theta_1 + (1-\epsilon) \theta_2. \)

From (B.58) we get,

\[ \frac{\dot{C}}{C} = \frac{\dot{w}}{w} = \frac{\dot{Y}}{Y} = \Gamma \frac{\dot{Z}_1}{Z_1} + (1 - \Gamma) \frac{\dot{Z}_2}{Z_2} \]

(B.70)
B.4.3 Euler Equation

Maximize $U(t) = \int_{t}^{\infty} \log(c) e^{-\rho t}$, where $c = \frac{C}{t}$; consumption per capita.

s.t. $\dot{S} = rS + wL - cL$

where $S$ is assets holding and $r$ is the rate of return on assets.

The current value hamiltonian is

$H = \log(c_t) + \psi_t(rS + wL - cL); \text{ where } \psi_t = e^{\rho t} \lambda_t$

All the necessary conditions are:

$\dot{S} = rS + wL - cL$

$\dot{\psi} = \rho \psi_t - \frac{\partial H}{\partial S_t} = (\rho - r)\psi_t$

$\frac{\partial H}{\partial c_t} = 0 \implies \frac{1}{c_t} = \psi_t$

$S_0$ is given; $\lim_{t \to \infty} e^{-\rho T} \psi_T S_T = 0$

Plug $\frac{1}{c_t} = \psi_t$ into $\dot{\psi} = (\rho - r)\psi_t$, we get

$\frac{1}{c_t} = (\rho - r)\frac{1}{c_t} \implies -\frac{\dot{c_t}}{c_t} = (\rho - r)\frac{1}{c_t} \implies -\frac{\dot{c_t}}{c_t} = (\rho - r)$

$\implies \frac{\dot{c_t}}{c_t} = r - \rho \quad \text{(B.71)}$

And since $cL = C$, combine (B.70), on BGP,

$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} = \frac{\dot{w}}{w} = \frac{\dot{Y}}{Y} = \frac{\dot{Z}_1}{Z_1} = \frac{\dot{Z}_2}{Z_2} = r - \rho \quad \text{(B.72)}$
B.5 Balanced Growth Path in Closed Economy

B.5.1 List the functions

To get the balanced growth path, we have 9 unknowns, which are

\[ g_1, g_2, r_1, r_2, L_1, L_2, \frac{Z_1}{Z_2}, N_1, N_2 \]

And on balance growth path, we have 9 equations, which are the following 9:

1. \[ g = g_1 = g_1 \] (B.73)

2. \[ r = r_1 = r_2 \] (B.74)

3. \[ L_1 + L_2 = L \] (B.75)

4. \[ \frac{L_1}{L_2} = \frac{\epsilon}{1 - \epsilon} \] (B.76)

\[ \text{rewrite (B.36): } r_1 = \delta f_1 \left( \frac{L_1}{N_1}, \frac{Z_1}{Z_2} \right) \] (B.77)

\[ \text{rewrite (B.47),} \]
\begin{align*}
g_1 &= f_1\left(\frac{L_1}{N_1}, \frac{Z_1}{Z_2}\right) - \alpha_1 \theta_1\left(\frac{Z_1}{Z_2}\right) \quad (B.78) \\
\text{rewrite (B.45)}: \quad r_2 &= \delta f_2\left(\frac{L_2}{N_2}, \frac{Z_1}{Z_2}\right) \quad (B.79) \\
\text{rewrite (B.48),} \\
g_2 &= f_2\left(\frac{L_2}{N_2}, \frac{Z_1}{Z_2}\right) - \alpha_2 \theta_2\left(\frac{Z_2}{Z_1}\right) \quad (B.80) \\
\text{Euler Equation}: \quad g = r - \rho \quad (B.81)
\end{align*}

B.5.2 Solve for Steady States

From (B.47) and (B.36) we can see they are very similar in structure, and we can rewrite them as,

\begin{align*}
r_1 &= \delta f_1\left(\frac{L_1}{N_1}, \frac{Z_1}{Z_2}\right) \quad (B.82)
\end{align*}
and

\[ g_1 = f_1 \left( \frac{L_1}{N_1}, \frac{Z_1}{Z_2} \right) - \alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right) \]  \hspace{1cm} (B.83)

\[ \Rightarrow f_1 = g_1 + \alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right) \]  \hspace{1cm} (B.84)

where

\[ f_1 = A_1 \frac{1 - \lambda \left( \frac{Z_2}{Z_1} \right)}{\lambda} \left( \frac{\epsilon_1}{1 - \epsilon_1} \right)^{\frac{1}{1 - \gamma}} \left( \frac{P_0}{P_0^g} \right)^{\frac{\lambda(\epsilon_1 - 1)}{\lambda - 1}} \frac{L_1}{N_1} \left( \frac{Z_1}{Z_2} \right)^{\Gamma - 1} \] \[ \text{and } \Gamma = (\delta + \gamma) - [1 - 2(\delta + \gamma)](\epsilon - 1) \in [0, 1]. \]

Plug (B.84) into (B.82), we get

\[ r_1 = \delta [g_1 + \alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right)] \]  \hspace{1cm} (B.85)

Combine the previous equation with Euler equation, which is \( r_1 = g_1 + \rho \), we get

\[ g_1 + \rho = \delta [g_1 + \alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right)] \], so

\[ g_1^* \equiv \frac{\dot{Z}_1}{Z_1} = \frac{\delta}{1 - \delta} \alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right)^* - \frac{1}{1 - \delta} \rho \]  \hspace{1cm} (B.86)

\[ r_1^* = g_1^* + \rho = \frac{\delta}{1 - \delta} \alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right)^* - \frac{1}{1 - \delta} \rho + \rho \]  \hspace{1cm} (B.87)

Similarly, combine (2.63), (B.48), and Euler equation, we get

\[ g_2^* \equiv \frac{\dot{Z}_2}{Z_2} = \frac{\delta}{1 - \delta} \alpha_2 \theta_2 \left( \frac{Z_2}{Z_1} \right)^* - \frac{1}{1 - \delta} \rho \]  \hspace{1cm} (B.88)
\[ r^*_2 = g^*_2 + \rho = \frac{\delta}{1 - \delta} \alpha_2 \theta_2 \left( \frac{Z^*_2}{Z^*_1} \right)^* - \frac{1}{1 - \delta} \rho + \rho \]  \hspace{1cm} (B.89)

We set \( \frac{\dot{Z}_1}{Z_1} = \frac{\dot{Z}_2}{Z_2} \), which means (B.86)=(B.88), and we can get the steady state of \((\frac{Z^*_1}{Z^*_2})^*\). At that ratio, the whole economy is on balanced growth path, and \( g_1 = g_2 = g^* \).

\[ \frac{\delta}{1 - \delta} \alpha_1 \theta_1 \left( \frac{Z^*_1}{Z^*_2} \right)^* - \frac{1}{1 - \delta} \rho = \frac{\delta}{1 - \delta} \alpha_2 \theta_2 \left( \frac{Z^*_2}{Z^*_1} \right)^* - \frac{1}{1 - \delta} \rho \]

\[ \Rightarrow \alpha_1 \theta_1 \left( \frac{Z^*_1}{Z^*_2} \right)^* = \alpha_2 \theta_2 \left( \frac{Z^*_2}{Z^*_1} \right)^* \]

\[ \Rightarrow \alpha_1 \theta_1 (\frac{Z^*_1}{Z^*_2})^2 - \alpha_2 \theta_2 = 0 \]  \hspace{1cm} (B.90)

Solve (B.90) by regular way, we get two solutions

\[ (\frac{Z^*_1}{Z^*_2})^*_1 = \sqrt{\frac{\alpha_2 \theta_2}{\alpha_1 \theta_1}} > 0 \]  \hspace{1cm} (B.91)

\[ (\frac{Z^*_1}{Z^*_2})^*_2 = -\sqrt{\frac{\alpha_2 \theta_2}{\alpha_1 \theta_1}} < 0 \]

We rule out the negative number, so

\[ (\frac{Z^*_1}{Z^*_2})^* = \sqrt{\frac{\alpha_2 \theta_2}{\alpha_1 \theta_1}} > 0 \]  \hspace{1cm} (B.92)

We plug (B.92) into (B.86) and (B.87), we get the balanced growth rate as,
\[ g^* = g^*_1 = \frac{\delta}{1-\delta} \sqrt{\alpha_1 \theta_1 \alpha_2 \theta_2} - \frac{1}{1-\delta} \rho \]  
\[ = g^*_2 = \frac{\delta}{1-\delta} \sqrt{\alpha_1 \theta_1 \alpha_2 \theta_2} - \frac{1}{1-\delta} \rho \]  
(B.93)

\[ \frac{\partial g^*}{\partial (\alpha_1 \theta_1)} = \frac{\delta}{1-\delta} \frac{1}{2} (\alpha_1 \theta_1 \alpha_2 \theta_2)^{-\frac{1}{2}} \alpha_2 \theta_2 > 0 \]  
(B.95)

\[ \frac{\partial g^*}{\partial (\alpha_2 \theta_2)} = \frac{\delta}{1-\delta} \frac{1}{2} (\alpha_1 \theta_1 \alpha_2 \theta_2)^{-\frac{1}{2}} \alpha_1 \theta_1 > 0 \]  
(B.96)

### B.5.3 Solve for Firm Sizes

From (B.21) and (B.49) we get

\[ L_1 = \epsilon L \]  
(B.97)

\[ L_2 = (1 - \epsilon) L \]  
(B.98)

The labor allocation is decided by the Cobb-Douglas factor payments (see deriv-
tion part in equation (B.21), so the previous conditions hold all the time. \( \epsilon L \) is the TOTAL market size for industry 1, and \((1 - \epsilon)L\) is the total market size for industry 2.

To get \( N_1 \), we need to use (B.82). See (B.82), what we have got is \((Z_1 Z_2)^*\), which is (B.92); and \( r^* \), which is \( r^* = g^* + \rho \), where \( g^* \) is expression (B.93). So combine all these equations together, we have \((\frac{L_1}{N_1})^*\), where \( L_1 = \epsilon L \). And the detail is the following,

\[
\alpha_1 \delta A_1 \frac{1-\lambda}{\lambda} \frac{Z}{A_1} \frac{1}{1-\epsilon} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{1}{1-\lambda}} \left( \frac{A_1}{A_2} \right)^{\frac{(\epsilon-1)}{(1-\lambda)}} \left( \frac{Z_2}{Z_2} \right)^{\frac{1}{1-\lambda}} \Gamma^{-1} \frac{L_1}{N_1} = \frac{\delta}{1-\delta} \sqrt{\alpha_1 \theta_1 \alpha_2 \theta_2} - \frac{\delta}{1-\delta} \rho + \rho = \frac{\delta}{1-\delta} \sqrt{\alpha_1 \theta_1 \alpha_2 \theta_2} - \frac{\delta}{1-\delta} \rho
\]

where \( \Gamma \equiv [(\delta + \gamma) - (1 - 2(\delta + \gamma))(\epsilon - 1)] \)

so,

\[
l^* = \frac{L_1}{N_1} = \frac{\epsilon L}{N_1} = \frac{\frac{\delta}{1-\delta} \sqrt{\alpha_1 \theta_1 \alpha_2 \theta_2} - \frac{\delta}{1-\delta} \rho}{\alpha_1 \delta A_1 \frac{1-\lambda}{\lambda} \frac{Z}{A_1} \frac{1}{1-\epsilon} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{1}{1-\lambda}} \left( \frac{A_1}{A_2} \right)^{\frac{(\epsilon-1)}{(1-\lambda)}} \left( \frac{Z_2}{Z_2} \right)^{\frac{1}{1-\lambda}} \Gamma^{-1}}
\]

We can write it as \( \frac{\epsilon L}{N_1} = \frac{\frac{\delta}{1-\delta} \sqrt{\alpha_1 \theta_1 \alpha_2 \theta_2} - \frac{\delta}{1-\delta} \rho}{\alpha_1 \delta A_1 \frac{1-\lambda}{\lambda} \frac{Z}{A_1} \frac{1}{1-\epsilon} \left( \frac{\epsilon}{1-\epsilon} \right)^{\frac{1}{1-\lambda}} \left( \frac{A_1}{A_2} \right)^{\frac{(\epsilon-1)}{(1-\lambda)}} \left( \frac{Z_2}{Z_2} \right)^{\frac{1}{1-\lambda}}} \). So \( \frac{\partial l^*}{\partial A_1} > 0; \frac{\partial l^*}{\partial A_2} > 0; \)

We can see the firm size \( \frac{L_1}{N_1} \) is constant on balanced growth path, given \((\frac{Z_1}{Z_2})^* = \frac{\alpha_1 \theta_1}{\alpha_2 \theta_2} \). If \( L_1 \) increases for some reason, then \( N_1 \) jumps (zero entry cost) to a value to keep the ratio constant.

From (B.99) and (B.93) we also see that the unit cost \( A_1 \) and \( A_2 \) do not enter the growth rate, and all the differences in unit costs enter to market size. For example, if there’s an increase in unit cost, \( A_1 \), then it cause the incipient profit decrease, and this
has a negative effect on $r_1$ according to (B.39). At the same time, a lower incipient profit enduces new firms to leave instantaneously without any cost, $\frac{\partial N_1}{\partial A_1} < 0$; which increases market size for each firm according to zero profit condition, $\frac{\partial l^*_1}{\partial A_1} > 0$; and increase $r_1$. Both these changes are canceled out by the each other, so unit costs do not affect the return in R&D.

$\frac{\partial r_1}{\partial A_2} > 0$ is because, when $A_2$ increases, $P_{X_1}$ decreases according to inverse demand function for $X_1$, (B.7) and (B.24). Thus demand for $G_{1j}$ decrease according to demand function of $G_{1j}$, (B.13). Thus a lower profit and firms leave. So $\frac{\partial N_1}{\partial A_2} < 0$ and $\frac{\partial r_1}{\partial A_2} > 0$.

A lower profit makes $r_1$ lower; and fewer incumbents makes market size higher and $r_1$ higher. These two opposite effects cancel out hence $A_2$ doesn’t enter $r_1$.

$$\frac{L_2}{N_2} = \frac{(1-\epsilon)L}{N_2} = \frac{\delta}{\alpha_2 \delta A_2} \frac{\lambda_2^2 (1-\epsilon)}{\lambda_2 (1-\epsilon)} \frac{\frac{\delta}{\lambda_2}}{\frac{\delta}{\lambda_2}} \frac{\lambda_2^2 (1-\epsilon)}{\lambda_2 (1-\epsilon)} \frac{\lambda_2^2 (1-\epsilon)}{\lambda_2 (1-\epsilon)} \frac{\lambda_2^2 (1-\epsilon)}{\lambda_2 (1-\epsilon)} \frac{\lambda_2^2 (1-\epsilon)}{\lambda_2 (1-\epsilon)} ; \text{ similar analysis.}$$
Appendix C

Trade Under Complete Specialization

C.1 Define Trade Pattern

Final good production function in home country is, \( Y_H = X^\epsilon_H X^{1-\epsilon}_H \)

And the production function of processed good 1 is

\[
X_{H1} = \int^{N_{H1}}_0 (G_{H1j} - G^E_{H1j}) \lambda [Z_{H1}^{(\delta+\gamma)} (\hat{Z}_{H2})^{1-(\delta+\gamma)} l_{H1H}]^{1-\lambda} dj
+ \int^{N_{F1}}_0 (G'_{F1j}) \lambda [Z_{F1}^{(\delta+\gamma)} (\hat{Z}_{H2})^{1-(\delta+\gamma)} l_{H1F}]^{1-\lambda} dj
\]

where \( \hat{Z}_{H2} = \begin{cases} 
Z_{H2} & \text{if industry 2 chooses domestic good} \\
Z_{H2} Z_{F2}^{1-\eta} & \text{if industry 2 chooses both goods} \\
Z_{F2} & \text{if industry 2 chooses foreign good} 
\end{cases} \)

Maximize the profit of processed-good producer \( X_{H1} \) given the prices of intermedi-
ate goods and wage, we can get the demand function for domestic good, \((G_{H1j}^H - G_{H1j}^E)\), and the foreign good, \(G_{F1j}^I\).

Set the final good of home country as numerical, and price of domestic good as \(P_{G_{H1}}\), and price for foreign good as \(P_{G_{F1}}\). Maximize profit, given the prices of intermediate goods and wages, we get
\[
P_{X_{H1}}[\lambda(G_{H1j} - G_{H1j}^E)^{\lambda-1}(Z_{H1}^{(\delta+\gamma)}(Z_{H2}^{)}l_{H1H})^{1-\lambda}] = P_{G_{H1}},\]
so the demand function for domestic good is
\[
G_{H1j} - G_{H1j}^E = \left\{\frac{\lambda P_{X_{H1}}}{P_{G_{H1}}^H}\right\}^{\lambda-1} Z_{H1}^{(\delta+\gamma)}(Z_{H2}^{)}l_{H1H}^{1-\lambda} \tag{C.1}
\]

Similarly, we can get the demand for foreign good as,
\[
G_{F1j}^I = \left\{\frac{\lambda P_{X_{H1}}}{P_{G_{H1}^F}}\right\}^{\lambda-1} Z_{F1}^{(\delta+\gamma)}(Z_{H2}^{)}l_{H1F}^{1-\lambda} \tag{C.2}
\]

And the demands for labors are,
\[
l_{H1H} = (P_{X_{H1}} \frac{1-\lambda}{w})^{\frac{1}{\lambda}} (G_{H1j} - G_{H1j}^E)(Z_{H1j}^\delta Z_{H1}^\gamma (Z_{H2}^\gamma))^1 \tag{C.3}
\]
\[
l_{H1F} = (P_{X_{H1}} \frac{1-\lambda}{w})^{\frac{1}{\lambda}} G_{F1j}^I (Z_{F1j}^\delta Z_{F1}^\gamma (Z_{H2}^\gamma))^1 \tag{C.4}
\]

In the demand functions for intermediate goods, (C.1) and (C.2), we see the difference are the prices of intermediate goods, \(P_{G_{H1}}\) and \(P_{G_{H1}^F}\); the quality of the goods, \(Z_{H1}\) and \(Z_{F1}\); also the labors who are using the goods, \(l_{H1H}\) and \(l_{H1F}\). If plug
the demands of labor (C.4) into (C.2), you will see $G_i^e$'s in both sides cancelled, i.e.

$$G_{F1j}^I = \frac{\lambda P_{X_{H1}}}{P_{H1j}} \lambda X F_{H2}^{\gamma} Z_{F1}^{\delta + \gamma} (Z_{H2}^{\delta + \gamma})^{1 - (\delta + \gamma)} l_{H1F}$$

$$\Rightarrow G_{F1j}^I = \frac{\lambda P_{X_{H1}}}{P_{H1j}} \lambda X F_{H2}^{\gamma} Z_{F1}^{\delta + \gamma} (P_{X_{H1}} \frac{1 - \lambda}{w}) \lambda G_{F1j}^I (Z_{F1j}^{\delta} Z_{F1}^{\gamma} (Z_{H2}^{\delta + \gamma})^{1 - (\delta + \gamma)} \lambda X F_{H2}^{\gamma} Z_{F1}^{\delta + \gamma} (Z_{H2}^{\delta + \gamma})^{1 - (\delta + \gamma)}$$

$$\Rightarrow 1 = \frac{\lambda P_{X_{H1}}}{P_{H1j}} \lambda X F_{H2}^{\gamma} Z_{F1}^{\delta + \gamma} (P_{X_{H1}} \frac{1 - \lambda}{w}) \lambda Z_{F1}^{\delta + \gamma}$$

$$\Rightarrow w = P_{X_{H1}} \lambda P_{X_{H1}} \lambda X F_{H2}^{\gamma} (1 - \lambda) Z_{F1}^{\delta + \gamma} Z_{H2}^{\delta + \gamma}$$

(C.5)

$w$ above is the wage the labors can get if the process-good firms use $G_{F1j}^I$.

If plug C.3 into C.2, $G_{H1j}^I - G_{H1j}^E$ cancelled by both sides, and we get,

$$w = P_{X_{H1}} \lambda P_{X_{H1}} \lambda X F_{H2}^{\gamma} (1 - \lambda) Z_{F1}^{\delta + \gamma} Z_{H2}^{\delta + \gamma}$$

(C.6)

Rearrange (C.5) and (C.6), we get,

$$w_{H1H} = \gamma (\frac{Z_{H1}^{\delta + \gamma} (1 - \lambda)}{P_{H1H}}) \lambda X F_{H2}^{\gamma}$$; 
$$w_{H1F} = \gamma (\frac{Z_{F1}^{\delta + \gamma} (1 - \lambda)}{P_{F1}}) \lambda X F_{H2}^{\gamma}$$

(C.7)

where $\gamma = P_{X_{H1}} \lambda P_{X_{H1}} \lambda X F_{H2}^{\gamma} (1 - \lambda) Z_{H2}^{\delta + \gamma}$.

From (C.7), given the spillover level $Z_{H2}$, processed-good producers in industry 1 chooses the intermediate good 1 which gives the labor a higher marginal product.

Processed-good producers $X_2$ do exactly the same thing. Given the spillover from industry 1, which is $Z_{H1}$, firms choose the intermediate good 2 which give the labor
who are using them a higher marginal product. When $X_2$ make decisions about $G_2$, what they care is quality-adjusted price, and they don’t care the externality they bring to industry 1, which is $\widehat{Z_{H2}}$. This externality will be discussed in more detail in section (5.5).

Note that the prices that intermediate good firms set up, $P_{GH1}$ and $P_{GF1}$ always equal to unit cost divided by a markup, and has nothing to do with market size, $l$. This is a constant elasticity demand function, which is one of the two special cases for monopoly behavior. Detail see Microeconomic Analysis (Third Edition) by Hal R. Varian, Section 14.1.

Recall that in closed economy, we also use this way to proof symmetry among the same industry before. Wage is $W_i = (1 - \lambda)(\frac{\lambda}{P_i})^{\frac{1}{\lambda - 1}} Z_i^{\delta} Z^{1 - \delta}$. Inside the same industry, every firm sets price as $\frac{1}{\lambda}$, thus the only difference on wage is the quality level, $Z_i$. If a firm has a higher quality, $Z_i > Z_j$, then $W_i > W_j$, it will take over the whole market and becomes a monopoly and earn a positive profit. But zero entry/exit condition allows firms enter instantaneously, and with the average quality of incumbent (by assumption), so the equilibrium is all firms in the same industry have the same quality level.

### C.2 Comparative Advantage

International trade happens if each country has a lower (or equal) quality-adjusted price at one intermediate good, i.e.
\[
\frac{P_{G_1}}{Z_{H_1}} \leq \frac{P_{G_1}}{Z_{F_1}}; \text{ and } \frac{P_{G_2}}{Z_{H_2}} \geq \frac{P_{G_2}}{Z_{F_2}} \tag{C.8}
\]

Set the numeraire as the final good in home, so the price of the final good in foreign country is \(P_Y\). Thus above condition becomes:

\[
\frac{A_{H1}/\lambda}{Z_{H1}} \leq \frac{P_{Y}}{A_{H1}/\lambda} \quad \text{and} \quad \frac{A_{H2}/\lambda}{Z_{H2}} \geq \frac{P_{Y}}{A_{H2}/\lambda}
\]

\[
\Rightarrow \frac{A_{H1}/\lambda}{Z_{H1}} \leq \frac{P_{Y}}{A_{H1}/\lambda} \quad \Rightarrow \frac{A_{H2}/\lambda}{Z_{H2}} \geq \frac{P_{Y}}{A_{H2}/\lambda}
\]

When strict inequality applies, the economy is complete specialization.

### C.3 Trade balance condition under complete specialization

Home Country imports intermediate good 2 from foreign country, and foreign country imports intermediate good 1 from home.

Set the price of final good in home country as numeraire, so the price of final good of foreign country is \(P_Y\). Home’s final good production is \(Y_H = X_{H1} X_{H2}^{1-\epsilon}\). By Cobb-Douglas production property, the final good producer in home country pays compensation \((1 - \epsilon)Y_H\) to the intermediate producers of industry 2, which are the foreign firms. Similarly, the final good producer in foreign country pays compensation \(\epsilon Y_Y \cdot Y_F\), measured with the final good price from home country, to the intermediate
producers of industry 1, which are the firms from home country. So Trade balance condition requires:

\[(1 - \epsilon)\lambda Y_H = \epsilon \lambda P_{Y_F} Y_F\]  
(C.9)

The final good production of home is the same as (B.68)

\[Y_H = \kappa_H Z_H^\Gamma Z_{F2}^{1-\Gamma} L_H\]  
(C.10)

where

\[\kappa_H = \lambda^{1-\lambda}(1 - \epsilon)^{1-\lambda} \epsilon^\lambda (\frac{\epsilon}{1-\epsilon})^{2\lambda-\lambda+\epsilon} P_{G_{H1}}^\Gamma P_{G_{F2}}^{-\lambda(1-\epsilon)}\]

Final good production of foreign country is a little different since we need to rescale with the relative final good price, \(P_{Y_F}\). Follow the same step as we derive (B.68), and combine (3.10), (3.11), (3.8), (3.9) into the production function of foreign final good (3.7), we get

\[Y_F = X_{F1}^{\epsilon} X_{F2}^{1-\epsilon}\]  
(C.11)

\[= [\xi F_1 P_{Y_F}^{\lambda} Z_{H1}^{\lambda(\delta+\gamma) + [1-2(\delta+\gamma)]\lambda(1-\epsilon)} Z_{F2}^{1-(\delta+\gamma) - [1-2(\delta+\gamma)]\lambda(1-\epsilon)} (\epsilon L_F)]^\epsilon\]  
(C.12)

\[\cdot [\xi F_2 P_{Y_F}^{\lambda} Z_{H1}^{1-(\delta+\gamma) - [1-2(\delta+\gamma)]\lambda(1-\epsilon)} Z_{F2}^{\lambda(\delta+\gamma) + [1-2(\delta+\gamma)]\lambda(1-\epsilon)} (1 - \epsilon) L_F]^{1-\epsilon}\]  
(C.13)

\[= \kappa_F Z_H^\Gamma Z_{F2}^{1-\Gamma} (L_F P_{Y_F}^{1-\lambda})\]  
(C.14)

where

\[\kappa_F = \lambda^{1-\lambda}(1 - \epsilon)^{1-\lambda} \epsilon^{1-\lambda} (\frac{\epsilon}{1-\epsilon})^{2\lambda-\lambda+\epsilon} P_{G_{H1}}^\Gamma P_{G_{F2}}^{-\lambda(1-\epsilon)} = \kappa_H; \text{ and } P_{G_{F2}} = P_{Y_F} A_{F2}/\lambda\]
So plug (C.14) and (C.10) into (C.9), we get \((1 - \epsilon)L_H = \epsilon P_Y L_F P_{Y_F}^{\frac{\lambda}{1-\lambda}}\), and

\[
P_{Y_F} = \frac{(1 - \epsilon)L_H}{\epsilon L_F} [1 - \lambda]
\]

We can also get (C.15) by calculating (C.39), which is much more complicated, and omitted here.

\section*{C.4 Discussion about the level effect under complete specialization}

\subsection*{C.4.1 Externality}

Same as the paper, we assume \((\frac{Z_{H1}}{P_{G_H1}})^{\frac{\lambda}{1-\lambda}} \geq (\frac{Z_{F1}}{P_{G_F1}})^{\frac{\lambda}{1-\lambda}}\), and \((\frac{Z_{H2}}{P_{G_H2}})^{\frac{\lambda}{1-\lambda}} \leq (\frac{Z_{F2}}{P_{G_F2}})^{\frac{\lambda}{1-\lambda}}\), which means \(\frac{P_{G_H1}}{Z_{H1}^{\lambda}} \leq \frac{P_{G_F1}}{Z_{F1}^{\lambda}}\) and \(\frac{P_{G_H2}}{Z_{H2}^{\lambda}} \geq \frac{P_{G_F2}}{Z_{F2}^{\lambda}}\) in the paper. This means home country chooses domestic intermediate good 1, and imports intermediate good 2. From section (5.1), if \(X_2\) chooses the foreign intermediate good 2, then there’s a spillover to the first industry, which is \(\hat{Z}_{H2} = Z_{F2}\). If \(Z_{F2} > Z_{H2}\), where \(Z_{H2}\) is the autarky quality level in intermediate good 2, then importing the foreign intermediate good 2 is a positive externality from industry 2 to industry 1; if \(Z_{F2} < Z_{H2}\), but since quality-adjusted price of foreign good is still lower due to a even lower price, then
getting the foreign good in industry 2 will have a negative externality to industry 1.

Rewrite (B.68), we can show the production in autarky of home country as,

\[
Y_{H}^{\text{Autarky}} = \kappa' Z_{H1}^{(\delta+\gamma)\kappa+1-(\delta+\gamma)](1-\epsilon)} Z_{H2}^{1-(\delta+\gamma)(1-\epsilon)} P_{G_{H1}}^{-\lambda} P_{G_{H2}}^{-\lambda(1-\epsilon)} (\epsilon L_H)^{\epsilon} \left[1-(\epsilon)\lambda H\right]
\]

\[
= \kappa' \left[\left(\frac{Z_{H1}^{(\delta+\gamma)}}{P_{G_{H1}}}\right) Z_{H2}^{1-(\delta+\gamma)} (\epsilon L_H)\right]^\epsilon \left[\left(\frac{Z_{H2}^{(\delta+\gamma)}}{P_{G_{H2}}}\right) Z_{H1}^{1-(\delta+\gamma)} ((1-\epsilon)L_H)\right]^{1-\epsilon} \quad \text{(C.17)}
\]

where \(\kappa' = \lambda (1-\epsilon) \frac{\lambda(1-\epsilon)}{1-\lambda} \epsilon \frac{\lambda}{1-\lambda}\)

From the derivation of B.68, we can see that \([\left(\frac{Z_{H1}^{(\delta+\gamma)}}{P_{G_{H1}}}\right) Z_{H2}^{1-(\delta+\gamma)} (\epsilon L_H)]^\epsilon\) is the contribution from \(X_{H1}\) to \(Y_H\), in which \(Z_{H2}\) is the spillover from \(X_{H2}\), and \(\epsilon L_H\) is the total amount of population that work in industry 1. \([\left(\frac{Z_{H2}^{(\delta+\gamma)}}{P_{G_{H2}}}\right) Z_{H1}^{1-(\delta+\gamma)} ((1-\epsilon)L_H)]^{1-\epsilon}\) is the contribution from \(X_{H2}\) to \(Y_H\), in which \(Z_{H1}\) is the spillover from \(X_{H1}\), and \((1-\epsilon)L_H\) is the total amount of population that work in industry 2.

At the moment of trade, home country keep the domestic intermediate good 1, but imports intermediate good 2, and the final good production becomes,

\[
Y_{H}^{\text{Trade}} = \kappa' \left[\left(\frac{Z_{H1}^{(\delta+\gamma)}}{P_{G_{H1}}}\right) Z_{F2}^{1-(\delta+\gamma)} (\epsilon L_H)\right]^\epsilon \left[\left(\frac{Z_{F2}^{(\delta+\gamma)}}{P_{G_{F2}}}\right) Z_{H1}^{1-(\delta+\gamma)} ((1-\epsilon)L_H)\right]^{1-\epsilon} \quad \text{(C.18)}
\]

Compare (C.18) with (C.17), we see that imports affect the final output through 2
channels. One is the quality-adjusted price, which we define as $P_{GH}^{1-\lambda} = \left[ \frac{P_{GH}^{1-\lambda}}{Z_{H2}^{1-(\delta+\gamma)}} \right]^{1-\lambda}$. Imports have a lower quality-adjusted price, so the quality per price is higher, i.e. $\frac{Z_{F2}^{1-(\delta+\gamma)}}{P_{GH}^{1-\lambda}} > \frac{Z_{H2}^{1-(\delta+\gamma)}}{P_{GH}^{1-\lambda}}$. This channel has a positive effect on output. The other channel is the spillover to industry 1, $Z_{F2}^{1-(\delta+\gamma)}$. If $Z_{F2} > Z_{H2}$, then it’s a positive externality, which increases the first term in (C.18), $\left[ Z_{F2}^{1-(\delta+\gamma)}(\epsilon L_H) \right]$, thus the final output; while if $Z_{F2} < Z_{H2}$, it is a negative externality. When the processed-good firms in industry 2 choose the intermediate goods, they don’t take the externality in account. They only care which goods give them a higher marginal product to labor, which is a lower quality-adjusted price. So it’s possible for them to come up to import a good with a worse quality than domestic good, i.e. $Z_{F2} < Z_{H2}$, while the price of that good is so low, so still satisfy $\frac{Z_{F2}^{1-(\delta+\gamma)}}{P_{GH}^{1-\lambda}} > \frac{Z_{H2}^{1-(\delta+\gamma)}}{P_{GH}^{1-\lambda}}$, so they still import the good, and benefit from a higher marginal product to labors who are using that good. And they do not take account that the lower $Z_{F2}$ enters industry 1 as a negative externality. So the total final output might decrease due to this negative externality.

### C.4.2 Parameter Conditions

It’s easy to see the sufficient condition for trade to increase output is $Z_{F2} > Z_{H2}$. When the foreign good 2 has a lower quality, it’s possible to decrease output. Now we discuss the parameter conditions of it.

What we have here are (1) Home imports intermediate good 2 (so foreign good 2 has a lower quality-adjusted price); (2) the import goods have a lower quality than domestic good; (3) output level decreases after trade. And express those in mathe
are the following,

\[
\frac{Z^\delta \gamma}{P_G^{\lambda}} > \frac{Z^\delta \gamma}{P_{G_H}^{\lambda}} \quad \text{(C.19)}
\]

\[
\Leftrightarrow \frac{P_{G_H}^{\lambda}}{P_{G_H}^{\lambda-\lambda}} > \frac{Z^\delta \gamma}{Z_{H2}^{\lambda}} \quad \text{(C.20)}
\]

\[
\Leftrightarrow \left( \frac{P_{G_H}^{\lambda}}{P_{G_F}^{\lambda}} \right)^{1-\lambda(\delta+\gamma)} > \left( \frac{Z_{H2}^{\lambda}}{Z_{F2}^{\lambda}} \right) \quad \text{(C.21)}
\]

\[
\Leftrightarrow \left( \frac{P_{G_F}^{\lambda}}{P_{G_H}^{\lambda}} \right)^{1-\lambda(\delta+\gamma)} < \left( \frac{Z_{F2}^{\lambda}}{Z_{H2}^{\lambda}} \right) \quad \text{(C.22)}
\]

\[
Z_{F2} < Z_{H2} \quad \text{(C.23)}
\]
\[
\frac{Z_{F2}}{P_{G_{F2}}} < \frac{Z_{H2}}{P_{G_{H2}}} < \frac{Z_{F2}}{P_{G_{F2}}} < \frac{Z_{H2}}{P_{G_{H2}}},
\]

Combine (C.21) and (C.30), we get

\[
\left(\frac{P_{G_{H2}}}{P_{G_{F2}}}\right)^{\frac{\lambda}{1-\lambda}(\delta+\gamma)} \frac{Z_{H2}}{Z_{F2}} \frac{P_{G_{H2}}}{P_{G_{F2}}} \left(\frac{P_{G_{H2}}}{P_{G_{F2}}}\right)^{\frac{\lambda(1-\epsilon)}{(1-\lambda)(\delta+\gamma) + 2(\delta+\gamma)}}
\]

So the sufficient and necessary condition for trade to decrease output level at this specific trade pattern (Home imports good 2) is, \( \frac{P_{G_{H2}}}{P_{G_{F2}}} \left(1 - \frac{\lambda}{1-\lambda}(\delta+\gamma)\right) > \frac{Z_{H2}}{Z_{F2}} > \frac{P_{G_{H2}}}{P_{G_{F2}}} \left(1 - \frac{\lambda}{1-\lambda}(\delta+\gamma) + 2(\delta+\gamma)\right) \); and \( Z_{H2} > Z_{F2} \).\(^1\) If \( \frac{P_{G_{H2}}}{P_{G_{F2}}} \left(1 - \frac{\lambda}{1-\lambda}(\delta+\gamma) + 2(\delta+\gamma)\right) < \frac{Z_{H2}}{Z_{F2}} \), then it violates the trade pattern condition, eqn (C.21); if \( \frac{Z_{H2}}{Z_{F2}} < \frac{P_{G_{H2}}}{P_{G_{F2}}} \left(1 - \frac{\lambda}{1-\lambda}(\delta+\gamma) + 2(\delta+\gamma)\right) \), then it violates (C.30).

\(^1\)From (C.21) and (C.23), we know \( \frac{P_{G_{H2}}}{P_{G_{F2}}} \left(1 - \frac{\lambda}{1-\lambda}(\delta+\gamma)\right) < \frac{Z_{F2}}{Z_{H2}} < 1 \Rightarrow P_{G_{F2}} < P_{G_{H2}} \Rightarrow \frac{P_{G_{H2}}}{P_{G_{F2}}} > 1. \) So
Summarize it, we get the following result:

Under the assumption that home country completely specialize in intermediate good 1, and imports intermediate good 2, which means,

\[
\left(\frac{\delta+\gamma}{\lambda} \right) \frac{1}{1-\lambda} \geq \left(\frac{\delta+\gamma}{\lambda} \right) \frac{1}{1-\lambda} \text{; and } \left(\frac{\delta+\gamma}{\lambda} \right) \frac{1}{1-\lambda} \leq \left(\frac{\delta+\gamma}{\lambda} \right) \frac{1}{1-\lambda}
\]

- The sufficient and necessary condition for trade to decrease output level at the moment of opening are,

\[
\left(\frac{P_{G_{H_2}}}{P_{G_{F_2}}} \right)^{\lambda(1-\lambda)(\delta+\gamma)} > \frac{Z_{H_2}}{Z_{F_2}} > \left(\frac{P_{G_{H_2}}}{P_{G_{F_2}}} \right)^{\lambda(1-\lambda)(\delta+\gamma)} \quad \text{and } Z_{H_2} > Z_{F_2}.
\]

- The sufficient condition for trade to increase output level at the moment of opening is,

\[
Z_{H_2} < Z_{F_2}
\]

- The necessary conditions for trade to increase output level at the moment of opening are,

\[
- \quad Z_{H_2} < Z_{F_2}
\]

\[
\text{or,}
\]

\[
- \quad Z_{H_2} > Z_{F_2} \quad \text{, } \left(\frac{P_{G_{H_2}}}{P_{G_{F_2}}} \right)^{\lambda(1-\lambda)(\delta+\gamma)} > \frac{Z_{H_2}}{Z_{F_2}} \quad \text{(so comparative advantage condition satisfied), and } Z_{H_2} < \left(\frac{P_{G_{H_2}}}{P_{G_{F_2}}} \right)^{\lambda(1-\lambda)(\delta+\gamma)} \quad \text{; where } Z_{F_2} \text{ is not low enough}
\]

to satisfy \(\left(\frac{P_{G_{H_2}}}{P_{G_{F_2}}} \right)^{\lambda(1-\lambda)(\delta+\gamma)} > \left(\frac{P_{G_{H_2}}}{P_{G_{F_2}}} \right)^{\lambda(1-\lambda)(\delta+\gamma)}\) in (C.31), we need

\[
\frac{1}{\delta} > \frac{(1-\epsilon)}{(\epsilon+\delta-2\epsilon\delta)} \quad \text{(C.32)}
\]

\[
\Leftrightarrow \delta(1-\epsilon) < \epsilon(1-\delta) + \delta(1-\epsilon) \quad \text{(C.33)}
\]

\[
\Leftrightarrow 0 < \epsilon(1-\delta) \quad \text{(C.34)}
\]

Since \(0 < \epsilon(1-\delta)\) is our pre-assumption about parameters, So as long as \(P_{G_{F_2}} < P_{G_{H_2}}\), we have

\[
\left(\frac{P_{G_{H_2}}}{P_{G_{F_2}}} \right)^{\lambda(1-\lambda)(\delta+\gamma)} > \left(\frac{P_{G_{H_2}}}{P_{G_{F_2}}} \right)^{\lambda(1-\lambda)(\delta+\gamma)}.
\]
to make the negative externality large enough to dominate the other positive level effect.

C.5 Derivations for world balanced growth rate

This is a complete specialization case, so the world economy is as if two country intergrate together, with home country produces only industry 1, and foreign country produces only industry 2. Similar to closed economy in section 4, now we have an intergrated economy with 9 unknowns and 9 equations like section (4.1.1); PLUS a world trade balance condition, which is solved in section (5.3) to get \( P_{Y_F} \).

The whole dynamic system reduces to (B.47), (B.48), (B.36), (B.45), with all \( \alpha_1 \theta_1 \) as \( \alpha_{H1} \theta_{H1} \); all \( \alpha_2 \theta_2 \) as \( \alpha_{F2} \theta_{F2} \). And the price of \( G_1 \) is the price of domestic-produced \( G_1 \), which is similar to (B.27), \( P_{G_{H1}} = \frac{A_{H1}}{\lambda} \). The price of \( G_2 \) is the price of foreign produced good, which is a rescale of (B.41), \( P_{G_{F2}} = P_{Y_F} \frac{A_{F2}}{\lambda} \). As we see in the closed economy, prices do not enter the growth rate, so the changes in parameters in the prices do no matter in our transition dynamic. Euler equation is the same for identical preference across countries. Follow EXACTLY the same steps as Section (4), and just change \( \alpha_1 \theta_1 \) and \( \alpha_2 \theta_2 \) to the country paramters accordingly, we get the balanced growth rate as,
\[ g_H^* = \frac{\delta}{1 - \delta} \sqrt{\alpha_{H1} \theta_{H1} \alpha_{F2} \theta_{F2}} - \frac{1}{1 - \delta} \rho \] 

\[ = g_F^* \quad \text{(C.35)} \]

where \((\frac{z_{H1}}{z_{F2}})^*\) is exactly (B.92), with \(\alpha_1 \theta_1 = \alpha_{H1} \theta_{H1}\) and \(\alpha_2 \theta_2 = \alpha_{F2} \theta_{F2}\), which is \((??)=0\).

### C.6 Changes in Market Size before and after trade

Exactly like section (4.1.3), we get

\[ L_{H1} = \epsilon L_H; \quad L_{H2} = (1 - \epsilon) L_H \] 

\[ L_{F1} = \epsilon L_F; \quad L_{F2} = (1 - \epsilon) L_F \] 

\[ \text{(C.36)} \]

\[ \text{(C.37)} \]

so \(L_{H1} + \frac{1}{D_{YF}^{1-\lambda}} L_{F1} = \epsilon L_H + \{(\frac{1-\epsilon}{\epsilon L_F})^{1-\lambda}\} \frac{1}{1-\lambda} \epsilon L_F = \epsilon L_H + (1 - \epsilon) L_H = L_H\), which is the TOTAL market size the intermediate good firms in industry 1 in home country face. Trade make the total size for industry 1 in home increases from \(\epsilon L_H\) to the whole home population.
\[ P_{Y_f}^{\frac{1}{1-\lambda}} L_{H2} + L_{F2} = \left\{ \left[ \frac{(1-\epsilon)\bar{L}_H}{\epsilon L_F} \right]^{\frac{1}{1-\lambda}} (1-\epsilon) L_H + (1-\epsilon) L_F \right\} e^{\lambda} (1-\epsilon) L_F + (1-\epsilon) L_F = \bar{L}_F, \]

which is the TOTAL market size the intermediate good firms in industry 2 in foreign country face. Trade makes the total market size they face increases from \((1-\epsilon) L_F\) to the whole foreign population.

And still just like section (4.1.3), we plug (C.35) and Euler equation back to (3.58), we get the market size for each firm in home country as,

\[ l^*_H = \frac{L_H}{N^*_H} = \frac{\delta^{\frac{1}{\alpha H1}} \eta H1 \alpha F2 \theta F2^{-\frac{\delta}{\alpha F2}}}{A_H1 A_F2^{\frac{1}{\lambda H1}} \theta H1^{1-\lambda} \left( \frac{\alpha H1 \theta H1}{\alpha F2 \theta F2} \right)^{1-\lambda}}. \]

And recall autarky market size for each firm is (B.99),

\[ l^*_{H1} = \frac{\epsilon L_H}{N^*_H} = \frac{\delta^{\frac{1}{\alpha H1}} \eta H1 \alpha F2 \theta F2^{-\frac{\delta}{\alpha F2}}}{A_H1 A_F2^{\frac{1}{\lambda H1}} \theta H1^{1-\lambda} \left( \frac{\alpha H1 \theta H1}{\alpha F2 \theta F2} \right)^{1-\lambda}}. \]

Compare the two equations — on the RHS, what change are (1) steady state value of quality ratio, and the partial effect to market size per firm is mixed; (2) the unit cost of intermediate good firms in the second industry, \(A_2\), and the partial effect to market size per firm is positive. The reason see the closed economy, section

Look at the LHS, trade makes the total market size for the whole industry increase, just like what we states before.

Combine both sides, trade effect on the number of firms \(N_{H1}\) is mixed.

However, we are sure about the most important thing: any changes in unit costs, and total market size are absorbed by firm entry, so in our model, scale effect doesn’t play a role.
C.7 Discussion about comparative advantage, and compare with that in the classical Ricardian model

We can compare our comparative advantage result with that in the classical Ricardian model, which is Dornbush, Fisher and Samuelson (1977).

First of all, the classical Ricardian model doesn’t have the externality effect that we discussed in section (5.4). So trade always has a positive effect on output level in those classical Ricardian model. However, in our model, externality makes the level effect uncertain. We’ve already derived the necessary and sufficient conditions for how trade affect the level output at the moment of opening.

Secondly, our way to define comparative advantage are similar for these two models. DFS (1977) is a multiple good model, and we can set $i = 2$ to enforce a two-good trade model. Labor is the only factor to produce trading goods, and the cost of labor is wage. There’s two countries, home and foreign countries. The wage in home is $w_H$ and wage in foreign country is $w_F$.\footnote{DFS (1977) uses $w$ and $w^*$ as home and foreign wages. We change a little bit here to make it more consistent with our notation.} The requirement of labors in good 1 in home country is $A_{H1}$, and that in foreign country is $A_{F1}$. The requirement of labors in good 2 is $A_{H2}$ for home, and $A_{F2}$ for foreign country.\footnote{We change the notation again to be consistent with our model.} Home country produces the good with lower domestic labor cost. They assume perfect competition, so the cost equals the price. Home produces the good 1 if
\[ w_H A_{H1} < w_F A_{F1} \]  \hspace{1cm} (C.38)

(C.38) is equivalent to equation (2) in DFS(1977).

And trade requires

\[ w_H A_{H1} < w_F A_{F1} \text{ and } w_H A_{H2} > w_F A_{F2} \]  \hspace{1cm} (C.39)

or the opposite situation. Here we just focus the condition in (C.39). (C.39) can be rewritten as \( \frac{w_H}{w_F} < \frac{A_{F1}}{A_{H1}} \) and \( \frac{A_{F2}}{A_{H2}} < \frac{w_H}{w_F} \), which can be combined as,

\[ \frac{A_{F2}}{A_{H2}} < \frac{w_H}{w_F} < \frac{A_{F1}}{A_{H1}} \]  \hspace{1cm} (C.40)

The previous condition means the factor price ratio must be in the interval of unit-cost ratios. DSF(1977) didn’t discuss corner solution.

DSF(1977) assumes the same preference across countries, and the trade balance condition requires \( \frac{w_H}{w_F} = func(\frac{L_H}{L_F}) \), which is their equation (10) and (11). Combine this with (C.40), we see that the population ratio must be inside the interval of the ratios of unit costs for complete specialization.

\[ \frac{A_{F2}}{A_{H2}} < func(\frac{L_H}{L_F}) < \frac{A_{F1}}{A_{H1}} \]  \hspace{1cm} (C.41)

What if the population ratio is outside the interval of (C.41). It will be an incomplete specialization. But DSF(1977) and most other Ricardian models did not
state that by formal mathes.

From (C.40) we can also get the comparative advantage expression, which is \( \frac{A_{H1}}{A_{H2}} < \frac{A_{F1}}{A_{F}} \).

In our model, we get similar stuff, except that our trading goods are produces under monopolistic competition, not perfect competition; our prices is quality-adjusted; our factors for trading goods are accumulative which is the final goods, but not labor; our model is dynamic; and we also talk about the corner solution.

In our model, home country chooses the good with lower quality-adjusted price. So the condition for trade is \( \frac{P_{GH1}}{Z_{H1}^{(\delta+\gamma)(1-\lambda)}} \leq \frac{P_{GF1}}{Z_{F1}^{(\delta+\gamma)(1-\lambda)}} \); and \( \frac{P_{GH2}}{Z_{H2}^{(\delta+\gamma)(1-\lambda)}} \geq \frac{P_{GF2}}{Z_{F2}^{(\delta+\gamma)(1-\lambda)}} \), which is similar to the trade condition (C.39) in Ricardian model. And complete specialization requires that \( \frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^{(\delta+\gamma)(1-\lambda)} > \left[ \frac{(1-\epsilon)L_{H}}{cL_{F}} \right]^{1-\lambda} \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right)^{(\delta+\gamma)(1-\lambda)} \), which is similar to (C.41) in Ricardian model. And the comparative advantage here is the ratios of unit costs which are adjusted by qualities,

\[
\left( \frac{\frac{A_{H1}}{Z_{H1}^{(\delta+\gamma)(1-\lambda)}}}{\frac{A_{H2}}{Z_{H2}^{(\delta+\gamma)(1-\lambda)}}} \right) \leq \left( \frac{\frac{A_{F1}}{Z_{F1}^{(\delta+\gamma)(1-\lambda)}}}{\frac{A_{F2}}{Z_{F2}^{(\delta+\gamma)(1-\lambda)}}} \right) \] (C.42)

And we also talk about the incomplete specialization case while the population ratio is outside the interval.
Appendix D

Trade Under Incomplete Specialization

D.1 Level effect

- Home Country

\[ Y_H^{\text{Trade}} = \kappa' \left[ \left( \frac{Z_{H1}}{P_{C_H1}} \right)^{1-(\delta+\gamma)} (\epsilon L_H) \right] \left[ \left( \frac{Z_{H2}}{P_{C_H2}} \right)^{1-(\delta+\gamma)} ((1-\epsilon) L_H) \right]^{1-\epsilon} \text{ (from 3.66)} \]

\[ Y_H^{\text{Autarky}} = \kappa' \left[ \left( \frac{Z_{H1}}{P_{C_H1}} \right)^{1-(\delta+\gamma)} (\epsilon L_H) \right] \left[ \left( \frac{Z_{H2}}{P_{C_H2}} \right)^{1-(\delta+\gamma)} ((1-\epsilon) L_H) \right]^{1-\epsilon} \]

(-following similar steps as section (5.4))

- where \( \kappa' = \lambda^{1-\epsilon} (1-\epsilon)^{\frac{\lambda(1-\epsilon)}{1-\lambda}} L_H^{\frac{\lambda}{1-\lambda}} \) and \( Z_{F1}^{(\delta+\gamma)} = \frac{Z_{F2}^{(\delta+\gamma)}}{Z_{C_H1}^{(\delta+\gamma)}} \)

- After trade, nothing changed in the second term \( \left[ \left( \frac{Z_{F2}}{P_{C_H2}} \right)^{1-(\delta+\gamma)} ((1-\epsilon) L_H) \right]^{1-\epsilon} \)
\[ L_H \right)^{1-\epsilon}, \text{since} \frac{Z_{F_2}^{(\delta+\gamma)}}{P_{G_{F_2}}^{1-\lambda}} = \frac{Z_{H_2}^{(\delta+\gamma)}}{P_{G_{H_2}}^{1-\lambda}}. \]

- The spillover from industry 2 to industry 1 is not the domestic quality \( Z_{H_2} \) any more. Instead, it’s a combination of domestic good and foreign good, \( \widehat{Z_{H_2}} = Z_{H_2}^{\eta}Z_{F_2}^{1-\eta} \).

- Thus the sufficient and necessary condition for trade to increase level output at the moment of trade, i.e. \( Y_{H}^{\text{Trade}} > Y_{H}^{\text{Autarky}} \) is \( Z_{F_2} > Z_{H_2} \). The imports in industry 2 have a positive externality to industry 1.

- Foreign Country

\[
- Y_{F}^{\text{Trade}} = \kappa' \left[ \frac{Z_{H_1}^{(\delta+\gamma)}}{P_{G_{H_1}}^{1-\lambda}} \right] \frac{Z_{F_2}^{1-(\delta+\gamma)}}{P_{G_{F_2}}^{1-\lambda}} \left[ (\epsilon L_F) \right]^{\epsilon} \left[ \frac{Z_{F_2}^{(\delta+\gamma)}}{P_{G_{F_2}}^{1-\lambda}} \right] \left[ (1-\epsilon) L_F \right]^{1-\epsilon} \\
- Y_{F}^{\text{Autarky}} = \kappa' \left[ \frac{Z_{F_2}^{(\delta+\gamma)}}{P_{G_{F_2}}^{1-\lambda}} \right] \frac{Z_{F_2}^{1-(\delta+\gamma)}}{P_{G_{F_2}}^{1-\lambda}} \left[ (\epsilon L_F) \right]^{\epsilon} \left[ \frac{Z_{F_2}^{(\delta+\gamma)}}{P_{G_{F_2}}^{1-\lambda}} \right] \left[ (1-\epsilon) L_F \right]^{1-\epsilon} \\
- \text{where} \; \kappa' = \lambda^{1-\lambda} (1-\epsilon)^{\lambda(1-\epsilon)} \epsilon^{1-\lambda} \\
- \text{Foreign country is completely specialization. So the level effects at the moment when trade happens, the necessary and sufficient conditions are exactly like section (5.4).}

D.2 Growth rate effect

\[
Y_{H}^{\text{Trade}} = \kappa' \left[ \frac{Z_{H_1}^{(\delta+\gamma)}}{P_{G_{H_1}}^{1-\lambda}} \right] \left( \widehat{Z_{H_2}}^{1-\delta} \right) \left[ (\epsilon L_H) \right]^{\epsilon} \left[ \frac{Z_{F_2}^{(\delta+\gamma)}}{P_{G_{F_2}}^{1-\lambda}} \right] \left[ (1-\epsilon) L_H \right]^{1-\epsilon}; \; \text{where} \; \frac{Z_{H_2}^{(\delta+\gamma)}}{P_{G_{H_2}}^{1-\lambda}} = \\
\frac{Z_{F_2}^{(\delta+\gamma)}}{P_{G_{F_2}}^{1-\lambda}}
\]

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\[ Y^{\text{Trade}}_H = \kappa' \left[ \frac{Z^{(\delta + \gamma)}_{H1}}{P^{1 - \lambda}_{C H1}} \right] \left[ \frac{Z^{(\delta + \gamma)}_{H2}}{P^{1 - \lambda}_{C H2}} \right] \left[ \frac{Z^{1-(\delta + \gamma)}(\epsilon L_H)}{\epsilon} \right] \left[ \frac{Z^{1-(\delta + \gamma)}((1 - \epsilon)L_H)}{1-\epsilon} \right] \]

\[ = \text{constant} \cdot Z^{(\delta + \gamma)\epsilon + [1-(\delta + \gamma)][1-\epsilon]}_{H1} \left[ \frac{Z^{(\delta + \gamma)}_{H2}}{P^{1 - \lambda}_{C H2}} \right] \left[ 1-(\delta + \gamma) \right] \left[ 1-\epsilon \right] Z^{(\delta + \gamma)(1-\epsilon)}_{H2} \]

\[ = \text{constant} \cdot Z^{\gamma} Z_{H1}^\eta \left[ 1-(\delta + \gamma) \right] \epsilon \left[ 1-(\eta)(1-(\delta + \gamma)) \right] \left[ 1-(\delta + \gamma) \right] \epsilon \]

\[ = \text{constant} \cdot Z^{\gamma} Z_{H1}^\eta \left[ 1-(\delta + \gamma) \right] \epsilon + \left( \delta + \gamma \right) \epsilon \left[ 1-(\eta)(1-(\delta + \gamma)) \right] \epsilon \]

\[ \left( \frac{Y^{\text{Trade}}_H}{Y^{\text{Trade}}_F} \right) = \Gamma \left( \frac{Z^{\gamma}_{H1}}{Z_{H1}} \right) + \left\{ \eta \left[ 1 - (\delta + \gamma) \right] \epsilon + (\delta + \gamma)(1-\epsilon) \right\} \left( \frac{Z^{\gamma}_{H2}}{Z_{H2}} \right) \]

\[ + \left\{ (1-\eta) \left[ 1 - (\delta + \gamma) \right] \epsilon \right\} \left( \frac{Z^{\gamma}_{F2}}{Z_{F2}} \right) \]

where \( \Gamma = (\delta + \gamma)\epsilon + [1 - (\delta + \gamma)](1-\epsilon) = 2(\delta + \gamma)\epsilon + 1 - \epsilon - (\delta + \gamma); \)

and \( \Gamma + [\eta(1 - (\delta + \gamma))\epsilon + (\delta + \gamma)(1-\epsilon)] + [(1-\eta)(1-(\delta + \gamma))\epsilon] \)

\[ = (\delta + \gamma)\epsilon + 1 - \epsilon - (\delta + \gamma) + (\delta + \gamma)\epsilon + \epsilon - \epsilon(\delta + \gamma) + (\delta + \gamma) - (\delta + \gamma)\epsilon = 1 \]

\[ Y^{\text{Trade}}_F = \kappa' \left[ \frac{Z^{(\delta + \gamma)}_{F2}}{P^{1 - \lambda}_{C F2}} \right] P^{1 - \lambda}_{Y_F} (\epsilon L_F) \left[ \frac{Z^{(\delta + \gamma)}_{F2}}{P^{1 - \lambda}_{C F2}} \right] \left[ (\epsilon L_F) \right] \left[ (1 - \epsilon L_F) \right] \]

where \( \frac{Z^{(\delta + \gamma)}_{F2}}{P^{1 - \lambda}_{C F2}} = \frac{Z^{(\delta + \gamma)}_{H2}}{P^{1 - \lambda}_{C H2}} \) and \( P_{Y_F} = \frac{A_{H2}/Z_{H2}}{A_{F2}/Z_{F2}} \left( \frac{\lambda}{(\delta + \gamma)(1-\lambda)} \right). \)

so, \( Y^{\text{Trade}}_F = \kappa' \left[ \frac{Z^{(\delta + \gamma)}_{F2}}{P^{1 - \lambda}_{C F2}} \right] P^{1 - \lambda}_{Y_F} (\epsilon L_F) \left[ \frac{Z^{(\delta + \gamma)}_{F2}}{P^{1 - \lambda}_{C F2}} \right] \left[ (\epsilon L_F) \right] \left[ (1 - \epsilon L_F) \right] \]

\[ = \text{constant} \cdot Z^{(\delta + \gamma)\epsilon + (1-(\delta + \gamma))(1-\epsilon)}_{H1} \left( \frac{Z^{(\delta + \gamma)}_{F2}}{Z^{(\delta + \gamma)}_{H2}} \right) \left( \frac{Z^{(\delta + \gamma)}_{F2}}{Z^{(\delta + \gamma)}_{H2}} \right) \left( 1-\epsilon \right) Z^{(\delta + \gamma)(1-\epsilon)}_{H2} \]

\[ = \text{constant} \cdot Z^{\gamma} Z_{H1}^{\eta} \left[ 1-(\delta + \gamma) \right] \epsilon + (\delta + \gamma) \]

\[ \left( \frac{Y^{\text{Trade}}_F}{Y^{\text{Trade}}_H} \right) = \Gamma \left( \frac{Z^{\gamma}_{H1}}{Z_{H1}} \right) - (\delta + \gamma)\epsilon \left( \frac{Z^{\gamma}_{H2}}{Z_{H2}} \right) + \left\{ [1 - (\delta + \gamma)]\epsilon + (\delta + \gamma) \right\} \left( \frac{Z^{\gamma}_{F2}}{Z_{F2}} \right) \]

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where $\Gamma = (\delta + \gamma)\epsilon + [1 - (\delta + \gamma)](1 - \epsilon) = 2(\delta + \gamma)\epsilon + 1 - \epsilon - (\delta + \gamma)$,
and $\Gamma + [-(\delta + \gamma)\epsilon + [(1 - (\delta + \gamma))\epsilon + (\delta + \gamma)]]$

$$ = 2(\delta + \gamma)\epsilon + 1 - \epsilon - (\delta + \gamma) - (\delta + \gamma)\epsilon + \epsilon - (\delta + \gamma)\epsilon + (\delta + \gamma) = 1$$

On Balance Growth Path, recall in section (3) that $\frac{\dot{y}_i}{y_i}$ is constant, thus

$$\frac{Y_{H}^{\text{Trade}}}{Y_{H}^{\text{Trade}}} = \frac{Y_{F}^{\text{Trade}}}{Y_{F}^{\text{Trade}}} = \frac{Z_{H}^{\dot{1}}}{Z_{H}^{1}} = \frac{Z_{F}^{\dot{2}}}{Z_{F}^{2}} = \frac{\dot{C}_{H}}{C_{H}} = \frac{\dot{C}_{F}}{C_{F}} = r - \rho \quad (D.4)$$

Combine (3.58), (3.59), (3.60), (3.61), (3.63), (3.64) and Euler equation applies to every industry on BGP, then we have

$$g_{H1} = \frac{\delta}{1 - \delta} \alpha_{H1} \theta_{H1} \frac{Z_{H1}^{\dot{1}}}{Z_{H2}^{1}} - \frac{\rho}{1 - \delta} = \frac{\delta}{1 - \delta} \alpha_{H1} \theta_{H1} (\frac{Z_{H1}^{\dot{1}}}{Z_{H2}^{1}})^{\eta} \frac{(Z_{H1}^{\dot{1}}/Z_{F2}^{2})^{1-\eta}}{1-\delta}$$ \quad (D.5)

$$g_{H2} = \frac{\delta}{1 - \delta} \alpha_{H2} \theta_{H2} \frac{Z_{H2}^{\dot{2}}}{Z_{H1}^{1}} - \frac{\rho}{1 - \delta} \quad (D.6)$$

$$g_{F2} = \frac{\delta}{1 - \delta} \alpha_{F2} \theta_{F2} \frac{Z_{F2}^{\dot{2}}}{Z_{H1}^{1}} - \frac{\rho}{1 - \delta} \quad (D.7)$$

$$g_{H1} = g_{H2} = g_{F2} \quad (D.8)$$

(D.6)=(D.7) we get $\frac{Z_{H2}^{\dot{2}}}{Z_{H1}^{1}} = \frac{\alpha_{F2} \theta_{F2} Z_{F2}^{\dot{2}}}{\alpha_{H2} \theta_{H2} Z_{H1}^{1}}$. Plug it into (D.5), and set (D.5)=(D.7),
then

\[ \alpha_{H1\theta H1} \left( \frac{\alpha_{F2\theta F2}}{\alpha_{H2\theta H2}} \frac{Z_{F2}}{Z_{H1}} \right)^{-\eta} \left( \frac{Z_{H1}}{Z_{F2}} \right)^{1-\eta} = \frac{\alpha_{F2\theta F2}}{\alpha_{H1\theta H1}} \frac{Z_{F2}}{Z_{H1}} \]  \hspace{1cm} (D.9)

\[ \alpha_{H1\theta H1} \left( \frac{\alpha_{F2\theta F2}}{\alpha_{H2\theta H2}} \right)^{-\eta} \left( \frac{Z_{H1}}{Z_{F2}} \right) = \frac{\alpha_{F2\theta F2}}{\alpha_{H1\theta H1}} \frac{Z_{F2}}{Z_{H1}} \]  \hspace{1cm} (D.10)

\[ \left( \frac{Z_{H1}}{Z_{F2}} \right)^2 = \frac{\alpha_{F2\theta F2}}{\alpha_{H1\theta H1}} \frac{\alpha_{F2\theta F2}}{\alpha_{H2\theta H2}} \]  \hspace{1cm} (D.11)

\[ \left( \frac{Z_{H1}}{Z_{F2}} \right)^* = \sqrt{\frac{\alpha_{F2\theta F2}}{\alpha_{H1\theta H1}} \frac{\alpha_{F2\theta F2}}{\alpha_{H2\theta H2}}} \]  \hspace{1cm} (D.12)

\[ \frac{Z_{H2}}{Z_{H1}} = \frac{\alpha_{F2\theta F2}}{\alpha_{H2\theta H2}} \left( \frac{Z_{F2}}{Z_{H1}} \right)^* = \frac{\alpha_{F2\theta F2}}{\alpha_{H2\theta H2}} \sqrt{\frac{\alpha_{F2\theta F2}}{\alpha_{H1\theta H1}} \frac{\alpha_{F2\theta F2}}{\alpha_{H2\theta H2}}}^{-1} \]  \hspace{1cm} (D.13)

Thus

\[ g_{\text{Trade}^*} = \frac{\delta}{1-\delta} \sqrt{\alpha_{H1\theta H1} (\alpha_{H2\theta H2})^\eta (\alpha_{F2\theta F2})^{1-\eta}} - \frac{\rho}{1-\delta} \]  \hspace{1cm} (D.14)

**D.3 Transition Dynamics**

Recall that from (3.58), (3.59), (3.60), (3.61), (3.63) and (3.64), we have the following results:
Inside home country, no-arbitrage condition requires that \( r_{H1} = r_{H2} \), as the case in closed economy. And we can get the relation between \( r_{H1} \) and \( r_{F2} \) by a similar way that we did under complete specialization.

Using the growth rates of final goods in both countries, (D.1) and (D.3), we get the difference between them:

\[
\frac{\dot{Y}_{H1 \text{trade}}}{Y_{H1 \text{trade}}} - \frac{\dot{Y}_{F2 \text{trade}}}{Y_{F2 \text{trade}}} = \{\eta[1 - (\delta + \gamma)]\epsilon + (\delta + \gamma)(1 - \epsilon) + (\delta + \gamma)\epsilon\} \frac{\dot{Z}_{H2}}{Z_{H2}} + \{(1 - \eta)[1 - (\delta + \gamma)]\epsilon - [1 - (\delta + \gamma)]\epsilon - (\delta + \gamma)\} \frac{\dot{Z}_{F2}}{Z_{F2}}
\]

thus,

\[
g_{H1} = \frac{r_{H1}}{\delta} - \alpha_{H1} \theta_{H1} \frac{Z_{H1}}{Z_{H2}}; \quad \text{ (D.15)}
\]

where \( \hat{Z}_{H2} = Z_{H2}^{\eta}Z_{F2}^{1-\eta} \) \hspace{1cm} \text{ (D.16)}

\[
g_{H2} = \frac{r_{H2}}{\delta} - \alpha_{H2} \theta_{H2} \frac{Z_{H2}}{Z_{H1}} \quad \text{ (D.17)}
\]

\[
g_{F2} = \frac{r_{F2}}{\delta} - \alpha_{F2} \theta_{F2} \frac{Z_{F2}}{Z_{H1}} \quad \text{ (D.18)}
\]
Combine (D.17) and (D.18),

\[
\frac{\dot{Y}_H^{\text{Trade}}}{Y_H^{\text{Trade}}} = \frac{\dot{Y}_F^{\text{Trade}}}{Y_F^{\text{Trade}}} = \{\eta - \eta(\delta + \gamma)\} \left( \frac{\dot{Z}_{H2}}{Z_{H2}} - \frac{\dot{Z}_{F2}}{Z_{F2}} \right) \tag{D.19}
\]

where \(\{\eta - \eta(\delta + \gamma)\} = \eta[1 - (\delta + \gamma)] + (\delta + \gamma) > 0\) given that \(0 < (\eta + \gamma) < 1\).

Recall that in equilibrium, by Euler equation, \(\frac{\dot{Y}_H^{\text{Trade}}}{Y_H^{\text{Trade}}} = \frac{\dot{c}_H}{c_H} = r_{H1} - \rho = r_{H2} - \rho\) and \(\frac{\dot{Y}_F^{\text{Trade}}}{Y_F^{\text{Trade}}} = \frac{\dot{c}_F}{c_F} = r_{F2} - \rho\). According to (D.19),

\[
r_{H1} - r_{F2} = \{\eta - \eta(\delta + \gamma)\} \left( \frac{\dot{Z}_{H2}}{Z_{H2}} - \frac{\dot{Z}_{F2}}{Z_{F2}} \right) \tag{D.20}
\]

Combine \(r_{H1} = r_{H2}\) with (D.15) and (D.17),

\[
\frac{(Z_{H1}/Z_{H2})}{Z_{H1}/Z_{H2}} = g_{H1} - g_{H2} \tag{D.21}
\]

\[
= -\alpha_{H1}\theta_{H1} (\frac{Z_{H1}}{Z_{H2}})^\eta (\frac{Z_{H1}}{Z_{F2}})^{1-\eta} + \alpha_{H2}\theta_{H2} \frac{Z_{H2}}{Z_{H1}} \tag{D.22}
\]

Combine (D.20) with (D.17) and (D.18),

\[
\frac{(Z_{H2}/Z_{F2})}{Z_{H2}/Z_{F2}} = g_{H2} - g_{F2} = \frac{r_{H2} - r_{F2}}{\delta} - \alpha_{H2}\theta_{H2} \frac{Z_{H2}}{Z_{H1}} + \alpha_{F2}\theta_{F2} \frac{Z_{F2}}{Z_{H1}}
\]

\[
= \frac{\{\eta - \eta(\delta + \gamma)\} \left( \frac{Z_{H2}}{Z_{H1}} - \frac{Z_{F2}}{Z_{F2}} \right) - \alpha_{H2}\theta_{H2} \frac{Z_{H2}}{Z_{H1}} + \alpha_{F2}\theta_{F2} \frac{Z_{F2}}{Z_{H1}}}{\delta}
\]

\[
\Rightarrow \{1 - \frac{\{\eta - \eta(\delta + \gamma)\} \left( \frac{Z_{H2}/Z_{F2}}{Z_{H2}/Z_{F2}} \right)}{\delta} = \frac{\{\eta - \eta(\delta + \gamma)\} \left( \frac{Z_{H2}/Z_{F2}}{Z_{H2}/Z_{F2}} \right)}{\delta} - \alpha_{H2}\theta_{H2} Z_{H1} + \alpha_{F2}\theta_{F2} Z_{H1} \Rightarrow \frac{(Z_{H2}/Z_{F2})}{Z_{H2}/Z_{F2}}
\]

\[
= -\alpha_{H2}\theta_{H2} Z_{H1} + \alpha_{F2}\theta_{F2} Z_{H1} \Rightarrow \frac{(Z_{H2}/Z_{F2})}{Z_{H2}/Z_{F2}}
\]

\[
= -\alpha_{H2}\theta_{H2} Z_{H1} + \alpha_{F2}\theta_{F2} Z_{H1} \Rightarrow \frac{(Z_{H2}/Z_{F2})}{Z_{H2}/Z_{F2}} = \frac{\delta}{\eta(\delta + \gamma)} \left[ -\alpha_{H2}\theta_{H2} Z_{H1} + \alpha_{F2}\theta_{F2} Z_{H1} \right]
\]
Denote \( \frac{Z_{H1}}{Z_{H2}} \equiv u; \frac{Z_{H1}}{Z_{F2}} = v; \frac{Z_{H1}}{Z_{F2}} = w \); note that \( v = u \cdot w \). Rewrite (D.22) and (3.87) as,

\[
\dot{u} = -\alpha_{H1} \theta_{H1} u^n v^{1-\eta} + \alpha_{H2} \theta_{H2} u^{-1}
\]

\[
= -\alpha_{H1} \theta_{H1} u^n (uw)^{1-\eta} + \alpha_{H2} \theta_{H2} u^{-1}
\]

\[
= -\alpha_{H1} \theta_{H1} u w^{1-\eta} + \alpha_{H2} \theta_{H2} u^{-1}
\]

\[
\dot{w} = -\frac{\alpha_{H1} \theta_{H1} u^2 w^{1-\eta} + \alpha_{H2} \theta_{H2}}{\eta (1-\delta - \gamma)} (\alpha F_2 \theta F_2 v^{-1})
\]

\[
= \frac{\alpha_{H2} \theta_{H2}}{\eta (1-\delta - \gamma) + \gamma} \left[ \alpha F_2 \theta F_2 v^{-1} - \alpha_{F2} \theta_{F2} (uw)^{-1} \right]
\]

Thus (D.23) and (D.24) are differential equations for \( u \) and \( w \) that only depend on \( u \) and \( w \). They’re non-linear, so we linearize them by Taylor Expansion around steady state \( u^* \) and \( w^* \). From section (6.5), we get the value of steady states are:

\[
u^* \equiv \left( \frac{Z_{H1}}{Z_{H2}} \right)^* = \left( \frac{\alpha F_2 \theta F_2}{\alpha_{H2} \theta_{H2}} \right)^{-1} \sqrt{\frac{\alpha F_2 \theta F_2}{\alpha_{H1} \theta_{H1} \alpha_{H2} \theta_{H2}}} \]

\[
w^* = \left( \frac{Z_{H2}}{Z_{F2}} \right)^* = \frac{\alpha F_2 \theta F_2}{\alpha_{H2} \theta_{H2}} \]

The Taylor expansion for (D.23) is:
\[
\dot{u} = -2\alpha_{H1}\theta_{H1}u^*(w^*)^{1-\eta}(u - u^*) - \alpha_{H1}\theta_{H1}(1 - \eta)(u^*)^2(w^*)^{-\eta}(w - w^*) \tag{D.27}
\]

Thus the locus of \( \dot{u} = 0 \) is

\[
u = \left(-\frac{1 - \eta}{2}\frac{u^*}{w^*}\right)w + (3 - \eta)u^* \tag{D.28}
\]

The Taylor expansion for (D.24) is:

\[
\dot{w} = \frac{\delta}{\eta(1 - \delta - \gamma) + \gamma} \alpha_{H2}\theta_{H2} w - w^* + \frac{\delta}{\eta(1 - \delta - \gamma) + \gamma} (\alpha_{H2}\theta_{H2}w^* - \alpha_{F2}\theta_{F2})(-1)(u^*)^{-2}(u - \dot{u})
\]

Since \( w^* = (\frac{Z_{H2}}{Z_{F2}})^* = \frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}} \), so \( \alpha_{H2}\theta_{H2}w^* - \alpha_{F2}\theta_{F2} = 0 \) and the second term of previous equation is 0. Thus,

\[
\dot{w} = \frac{\delta}{\eta(1 - \delta - \gamma) + \gamma} \alpha_{H2}\theta_{H2} \frac{w - w^*}{u^*} \tag{D.29}
\]

So the locus of \( \dot{w} = 0 \) is

\[
w = w^* \tag{D.30}
\]
D.4 Stability of incomplete specialization

If $w > w^*$, then $\dot{w} > 0$. World economy stays on incomplete specialization, but never converges to BGP. In this case, the difference of the growth rates of two countries converges to a constant, $(\delta + \frac{\delta^2}{\eta(1-\delta-\gamma)} + \gamma) \alpha_{H2}\theta_{H2} \frac{1}{w}$, where $u^* = (\frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}})^{-1} \sqrt{\frac{\alpha_{F2}\theta_{F2}}{\alpha_{H1}\theta_{H1}} \frac{\alpha_{F2}\theta_{F2}}{\alpha_{H2}\theta_{H2}}} \eta$

The proof is as below,

From (D.19) and (D.24)

$\frac{Y^T_{i,ade}}{Y^F_{i,ade}} - \frac{Y^T_{i,ade}}{Y^F_{i,ade}} = \{\eta \epsilon - \eta (\delta + \gamma) \epsilon + (\delta + \gamma)\} \left( \frac{Z_{H2}}{Z_{H2}} - \frac{Z_{F2}}{Z_{F2}} \right) = \left\{ \eta \epsilon - \eta (\delta + \gamma) \epsilon + (\delta + \gamma) \right\} \frac{w}{w^*} \rightarrow \frac{\delta}{\eta(1-\delta-\gamma)+\gamma} \alpha_{H2}\theta_{H2} \frac{w-w^*}{u^*} / w$

$= (\delta + \frac{\delta^2}{\eta(1-\delta-\gamma)+\gamma}) \alpha_{H2}\theta_{H2} \frac{1}{w^*} - (\delta + \frac{\delta^2}{\eta(1-\delta-\gamma)+\gamma}) \alpha_{H2}\theta_{H2} \frac{w^*}{w}$

$\rightarrow (\delta + \frac{\delta^2}{\eta(1-\delta-\gamma)+\gamma}) \alpha_{H2}\theta_{H2} \frac{1}{w^*}$ as $w \rightarrow \infty$. 
Appendix E

Welfare and Tariff Analysis

E.1 Current Value Hamiltonian for Household

\[ CVH = \log c + \nu \log (1 - l) + \psi (rS + wl - c) \]

Necessary Conditions are:

\[ \frac{\partial CVH}{\partial c} = \frac{1}{c} - \psi = 0 \]

\[ \frac{1}{c} = \psi \]  \hspace{1cm} (E.1)
\[
\frac{\partial CVH}{\partial l} = 0 \Rightarrow \frac{\nu}{1-l} + \lambda w = 0
\]

\[
\frac{\nu}{1-l} = \psi w \quad \text{(E.2)}
\]

\[
\dot{S} = \frac{\partial CVH}{\partial \psi} = rS + wl - c
\]

\[
\dot{\psi} = \rho \psi - \frac{\partial CVH}{\partial S} = (\rho - r)\psi \quad \text{(E.3)}
\]

\[
S_0 \text{ is given;}
\]

\[
\lim_{T \to \infty} e^{-\rho T} \psi_T S_T = 0
\]

Combine (E.1) and (E.3), we get, \(\frac{1}{c} = (\rho - r)\frac{1}{c} \Rightarrow -\frac{\dot{c}}{c} = (\rho - r)\frac{1}{c}
\]

\[
\Rightarrow \frac{\dot{c}}{c} = r - \rho \quad \text{(E.4)}
\]

Combine (E.2) and (E.3), we get, \(\frac{\nu}{1-l} = \frac{w}{c}
\]

\[
\Rightarrow l = 1 - \frac{\nu c}{w} \quad \text{(E.5)}
\]

Recall that we assume zero entry/exit cost, so the number of firms always jump to make profit zero. The budget constraint can be written as

\[
0 = \int_0^\infty \left( \int_0^{N_1} \Pi_{1j} dj + \int_0^{N_2} \Pi_{2j} dj + \omega L - C \right) e^{-\int_0^T r(s)ds} dt \text{ with } \Pi_{ij} = 0.
\]

So \(\omega L = C \Rightarrow \omega L = cL\). Recall that by the property of Cobb-Douglas function, \(\omega L = (1-\lambda)Y\), thus \(\omega L = cL = (1-\lambda)Y\). Plug this condition back to (E.5),

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\[ l = 1 - \frac{\nu c}{w} \Rightarrow ll = L - \frac{\nu(cL/Y)}{w/Y} \Rightarrow lL = L - \frac{\nu((1-\lambda)l)}{wL/Y} = 1 - \frac{\nu((1-\lambda)}{(1-\lambda)} = \frac{1}{1 - \nu} \]

\[ \Rightarrow l^* = \frac{1}{1 + \nu} \] (E.6)

### E.2 Welfare

\[
\log Y(t) = \log [\kappa' \left( \frac{c}{1+\nu} \right)^{\epsilon(1-\epsilon)(1-\nu)}] + \epsilon\log \left[ \frac{Z^{\delta+\gamma}(t)}{P_{G_1}} \right] + (1 - \delta - \gamma)\log Z_2(t) + (1 - \epsilon)\log \left[ \frac{Z^{\delta+\gamma}(0)}{P_{G_1}} \right]
\]

And \( Z_1(t) = Z_1(0) \cdot e^{\int g_1(s)ds} \); \( Z_2(t) = Z_2(0) \cdot e^{\int g_2(s)ds} \); where \( g_1(t) \) and \( g_2(t) \) are the growth rates for quality 1 and 2 at time \( t \), thus,

\[
\log Y(t) = \log [\kappa' \epsilon^{(1 - \epsilon)(1-\nu)}] + \epsilon\log \left[ \frac{Z^{\delta+\gamma}(0)}{P_{G_1}} \right] + \epsilon(\gamma + \delta) \int_0^t g_1(s)ds + \epsilon(1 - \delta - \gamma)\log Z_2(0) + (1 - \epsilon)\int_0^t g_2(s)ds
\]

\[
+(1 - \epsilon)\log \left[ \frac{Z^{\delta+\gamma}(0)}{P_{G_1}} \right] + (1 - \epsilon)(\delta + \gamma) \int_0^t g_2(s)ds + (1 - \epsilon)(1 - \delta - \gamma)\log Z_1(0) + (1 - \epsilon)(1 - \delta - \gamma) \int_0^t g_1(s)ds
\]

\[
= \log \Omega l^* + \epsilon\log \left[ \frac{Z^{\delta+\gamma}(0)}{P_{G_1}} \right] + \epsilon(1 - \delta - \gamma)\log Z_2(0) + (1 - \epsilon)(1 - \delta - \gamma)\log Z_1(0) + \Gamma \int_0^t g_1(s)ds + (1 - \Gamma) \int_0^t g_2(s)ds
\]

where \( \Gamma = \epsilon(\gamma + \delta) + (1 - \epsilon)(1 - \delta - \gamma) = 2(\delta + \gamma)\epsilon - (\delta + \gamma) - \epsilon + 1 \)

\[
\log u(t) = \log c(t) + \nu\log(1 - l(t)) = \log Y(t) \cdot \frac{c(t)}{Y(t)} + \nu\log(1 - l^*)
\]

\[
= \log Y(t) + \log \left[ \frac{c(t)}{Y(t)} \right] + \nu\log(1 - l^*)
\]

where \( \frac{c}{Y} = \frac{cL}{YL} = (1 - \lambda) \frac{1}{L} \).
E.3  The Effect of Profit Tax in the first exercise

\[ \frac{\partial g^*}{\partial t_\pi} = \frac{\delta}{1-\delta} \left( \alpha H_1 \theta H_1 \alpha F_2 \theta F_2 \frac{1-t_\pi}{1-\sigma t_\pi} \right)^{-\frac{1}{2}} \cdot \frac{d^{\frac{1-t_\pi}{\sigma t_\pi}}}{dt_\pi}, \text{ and} \]

\[ \frac{d^{\frac{1-t_\pi}{\sigma t_\pi}}}{dt_\pi} = \frac{-(1-\sigma t_\pi) - (1-t_\pi)(-\sigma)}{(1-\sigma t_\pi)^2} = \frac{-1+\sigma t_\pi + \sigma - \sigma t_\pi}{(1-\sigma t_\pi)^2} = \frac{-1+\sigma}{(1-\sigma t_\pi)^2} \begin{cases} < 0 & \text{for } \sigma \in [0, 1) \\ = 0 & \text{for } \sigma = 1 \end{cases} \]

so \[ \frac{\partial g^*}{\partial t_\pi} \begin{cases} < 0 & \text{for } \sigma \in [0, 1) \\ = 0 & \text{for } \sigma = 1 \end{cases} \]

E.4  The Effect of Profit Tax in the second exercise

Zero profit condition is,

\[ (1-\sigma t_\pi)(1-\varsigma)R_{H1} = F_{H1}(1-t_\pi) \implies R_{H1} = \frac{(1-t_\pi)}{(1-\sigma t_\pi)(1-\varsigma)} F_{H1} \]

Plug the previous equation back to government budget constraint,

\[ \varsigma R_{H1} N_{H1} = t_\pi (F_{H1} - \sigma R_{H1}) N_{H1} \implies (\varsigma + \sigma t_\pi) R_{H1} = t_\pi F_{H1} \]

\[ \implies (\varsigma + \sigma t_\pi) \frac{(1-t_\pi)}{(1-\sigma t_\pi)(1-\varsigma)} F_{H1} = t_\pi F_{H1} \]

\[ \implies (\varsigma + \sigma t_\pi) \frac{(1-t_\pi)}{(1-\sigma t_\pi)(1-\varsigma)} = t_\pi \]

\[ \implies \varsigma(1-t_\pi) + \sigma t_\pi (1-t_\pi) = t_\pi (1-\sigma t_\pi) - \varsigma t_\pi (1-\sigma t_\pi) \]

\[ \implies \varsigma(1-t_\pi + t_\pi - \sigma t_\pi^2) = t_\pi (1-\sigma t_\pi - \sigma + \sigma t_\pi) \]

\[ \implies \varsigma(1-\sigma t_\pi^2) = t_\pi (1-\sigma) \]

\[ \implies \varsigma = \frac{t_\pi (1-\sigma)}{1-\sigma t_\pi^2} \]

Plug \[ \varsigma = \frac{t_\pi (1-\sigma)}{1-\sigma t_\pi^2} \text{ back into balanced growth rate, then it becomes} \]

\[ g^* = \frac{\delta}{1-\delta} \sqrt{\alpha H_1 \theta H_1 \alpha F_2 \theta F_2 \frac{1-t_\pi}{1-\sigma t_\pi} \frac{1-t_\pi (1-\sigma)}{(1-\sigma t_\pi)(1-\sigma t_\pi^2)}} - \frac{1}{1-\delta} \rho \]
Denote \( \frac{1-t_\sigma}{(1-\sigma t_\pi)[1-\frac{1-t_\pi}{1-\sigma t_\pi}]} \equiv \nabla \), and \( \frac{\partial g^*}{\partial t_\pi} = \frac{\partial g^*}{\partial \nabla} \frac{d\nabla}{dt_\pi} \). By observation, \( \frac{\partial g^*}{\partial \nabla} > 0 \), and we need to know the sign of \( \frac{d\nabla}{dt_\pi} \).

\[
\nabla = \frac{1-t_\sigma}{(1-\sigma t_\pi)[1-\frac{1-t_\pi}{1-\sigma t_\pi}]} = \frac{1-t_\sigma}{(1-\sigma t_\pi)} \frac{1-\sigma t_\pi + \sigma t_\pi}{1-\sigma t_\pi^2} = \frac{1-t_\sigma}{1-\sigma t_\pi^2} = \frac{1-\sigma t_\pi^2}{1-\sigma t_\pi^2} \nabla
\]

\[
\frac{d\nabla}{dt_\pi} = \frac{(1-\sigma^2 t_\pi^2)(-2\sigma t_\pi)-(1-\sigma t_\pi^2)(-\sigma^2 2 t_\pi)}{(1-\sigma^4 t_\pi^2)^2} = \frac{-2\sigma t_\pi + 2\sigma^3 t_\pi^3 + \sigma^2 2 t_\pi - 2\sigma^3 t_\pi^3}{(1-\sigma^2 t_\pi^2)^2} \frac{2\sigma t_\pi(1-\sigma)}{(1-\sigma^2 t_\pi^2)^2} < 0 \text{ for } \sigma \in [0, 1). \text{ So } \frac{\partial g^*}{\partial t_\pi} = \frac{\partial g^*}{\partial \nabla} \frac{d\nabla}{dt_\pi} < 0.
\]

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