ABSTRACT

MOJARRAD, HAMED. Three Dimensional Nonlinear Finite Element Analysis of Airport Pavements. (Under the direction of Dr. A. A. Tayebali, and Dr. M. S. Rahman.)

The mechanistic-empirical flexible pavement design requires evaluation of pavement responses, i.e., stress, strain, and deformation of pavements. In case of airport pavements, where high aircraft loads with complex tire assemblies are applied to the pavement, the model used to calculate the pavement responses should be developed with additional considerations. In this study, the effects of parameters such as the stress dependency of the aggregate base course (ABC) layer, tire imprint, landing gear configuration, and shear stresses due to tire loading are considered to evaluate the pavement response that governs the pavement design.

In this study the stress dependency of the aggregate base course (ABC) layer which is believed to be the principal structural component of the flexible pavement is modeled using user material subroutines (UMAT) in ABAQUS FEM software where the resilient modulus is assumed to be a function of bulk stress. The resilient modulus is calculated at any load increment for all integration points based on the calculated strains at the beginning of each load increment and will be used to calculate the stresses. In order to have the stress dependent material parameters for the ABC layer, Discrete Element Modeling (DEM) is used and effect of different parameters such as grain size distribution and stress state on the resilient behavior of the ABC layer materials is studied.
The DEM simulations performed in this study conclude that coarser materials tend to have higher resilient modulus values, and the resilient modulus of granular materials increases with increase in bulk stress. The DEM simulation results were calibrated using laboratory test results reported for similar material in previous studies.

The pavement response calculated from the 3D FEM models were used to estimate the maximum allowable number of load repetitions for a given pavement section that was compared with the maximum allowable number of load repetitions calculated using the FAA flexible airport pavement design procedure. The stress dependency of the ABC layer was found to influence the maximum allowable number of load repetitions.

For the design of airport pavements, literature review suggests the use of elliptical tire imprint instead of circular tire imprint for design. In this study, it was observed that using elliptical tire imprint will result in lower allowable number of load repetitions for fatigue cracking \( (N_f) \) which turned out to be more significant in case of full depth asphalt pavement section where the fatigue cracking was the critical pavement distress controlling the total allowable number of load repetitions.

Aircrafts will apply significant shear loads on airport pavements. In this study, lower allowable number of load repetitions was observed for the pavement in presence of shear loads. Although the mentioned observation was captured more clearly for circular tire imprint, but in case of full depth asphalt pavement section, presence of shear loads impacted the pavement response for both circular and elliptical tire imprints.
Three Dimensional Nonlinear Finite Element Analysis of Airport Pavements

by
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DEDICATION

I would like to dedicate this thesis to my family. Without their patience, understanding, and support the completion of this work would not have been possible.
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First and foremost, I would like to express sincere gratitude to my advisors Dr. A. A. Tayebali and Dr. M. S. Rahman for their encouragement, support, and guidance. I wish to express sincere gratitude to Dr. T. M. Evans under whose supervision the DEM modeling section of my thesis was conducted. Last but not least, I would like to thank the faculty members, staff, and all my friends at NCSU for making NCSU a wonderful and memorable place for me.
Hamed Mojarrad was born in 1982, in Ray, Tehran, Iran. In 2001, he entered the civil engineering department at Sharif University of Technology (SUT), the first ranked university in Iran, and received his BS in civil engineering in 2005.

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For his MSc thesis he performed SCPTU soundings on clay core of Masjed Soleyman rock fill dam in Iran. While he was doing his graduate studies at SUT, he was also involved in seismic rehabilitation of existing structures by performing geotechnical investigations as well as performance based analysis of structures.

In 2008, he was admitted to the Ph.D. program at civil, construction and environmental engineering at North Carolina State University (NCSU). He worked as a research assistant on a project studying the effect of gradation on resilient behavior of aggregate base course (ABC) Materials. He conducted the numerical study portion of the project using Discrete Element Modeling (DEM) simulations. For his Ph.D. thesis he used his findings from the DEM simulations in stress dependent 3D Finite Element Modeling (FEM) simulations to evaluate the response of airport flexible pavements under aircraft loads.

During his graduate studies at NCSU he also worked as a teaching assistant for different civil engineering undergraduate courses.
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Chapter 1

Introduction

1-1- Introduction

Mechanistic-empirical flexible pavement design is based on the traffic load and environment induced pavement stresses and strains with empirical models of pavement distress evolution and performance. The pavement response under applied stresses and strains, such as the maximum compressive strain at top of the subgrade ($\varepsilon_c$), the maximum tensile strain at the bottom of the asphalt layer ($\varepsilon_t$) and also the shear stress transferred between different layers of the pavement (Qingxia, 2004), are the key inputs to the pavement performance models. A better understanding of the pavement response by incorporating more realistic material behavior model will lead to a better design of the pavement structure. In case of flexible airport pavement design, due to presence of high traffic loads, large stresses and strains are induced in the pavement causing nonlinear response of pavement. Furthermore, considering parameters such as elliptical load imprint, multiple wheel loads, and presence of shear loads will result in non-axisymmetric condition which will require three dimensional models to more accurately evaluate the stress-strain response of the pavement.

It is not possible to assess every conceivable field condition in the laboratory, and most laboratory tests provide only empirical results relating to material response at specimen boundaries. Furthermore, while conducting a laboratory test, it will not be possible to control
many different parameters that may impact the test results in order to consider their effects on the test. In addition, it is often difficult to know why a specimen behaves in a certain manner because of the lack of information at the particle scale (i.e., the scale that basically governs laboratory and field-scale behavior). We can infer certain causal factors based on our understanding of theoretical soil mechanics, but there will still be gaps in our knowledge. Researchers are increasingly using numerical methods and specially discrete element method (DEM) modeling to bridge those gaps by studying the root causes of changes in material behavior. By better understanding the underlying mechanisms that lead to an observed material response, researchers can better answer questions about the behavior of materials that are not identical to those tested in the laboratory.

1-2- Background

Structural classification of airfield pavements has always been among the most interesting tasks of airport pavement engineering (Momberger, 1988) to ensure the safe utilization of both the aircrafts and the pavement to the maximum extent (Sandhawalia, 1988). It is necessary to have a reliable pavement analysis system to calculate the load bearing capacity of the pavements and the impact of the aircraft load on the pavement to study the performance and maintenance needs of the pavement (Loizos and Charonitis, 2004).

The pavement deformations under the repeated traffic loads are mostly recoverable and can be considered elastic due to the material shakedown occurring after a certain number of load repetitions, resulting in the permanent deformation in each load cycle decreasing to a
minimum. In this case the resilient Modulus (M_R) is used as the elastic modulus of the pavement material. Although Pavement materials demonstrate non-linear response, in design practice, for convenience M_R is used as a primary parameter. In order to more realistically model the nonlinear behavior of the pavement under repeated traffic loads, the resilient modulus and different parameters affecting it are studied.

There are several methods to analyze the stress-strain response of pavements under traffic load. The layered elastic approach is the simplest procedure in which, the system is divided into an arbitrary number of horizontal layers which may have different thicknesses and material properties, but in each layer the material is assumed to be homogeneous and linearly elastic. This method has severe limitations as materials must be homogenous and linearly elastic within each layer, and the wheel loads applied on the surface must be axi-symmetric. A better approach will be 2D finite element analysis, which can be used for plane strain or axis-symmetric conditions and it can rigorously handle material anisotropy, material nonlinearity, and a variety of boundary conditions. 2D models can not accurately capture non-uniform tire contact pressure and multiple wheel loads, so 3D finite element models are required for unsymmetrical loads or combination of loads. Among all the existing methods of evaluating the pavement response, three dimensional finite element method (FEM) has become a powerful tool in analyzing the stresses and strains in pavements especially with its ability to simulate nonlinear material responses in three dimensional problems. Availability of several well verified general purpose commercial FEM softwares (ABAQUS, ANSYS, ADINA) also motivates the researchers to choose advanced method of analyzing the pavement response.
The built-in nonlinear constitutive models in general purpose FEM softwares have not been readily applicable to nonlinear pavement structural analyses since these constitutive models often define material behavior as a function of strain state which does not properly represent deformation characteristics under the applied wheel loads which is stress dependant. In this research, nonlinear resilient response models are programmed as a function of applied stress state through user material subroutine (UMAT) in ABAQUS to predict flexible pavement critical responses (i.e. stresses, strains, and deformations in the pavement structure) that will be used in mechanistic-empirical pavement design using distress models or transfer functions.

1-3- Selected Approach

The main objective of this study is to analyze the airport pavement response under different aircraft loadings in order to be used in the empirical-mechanistic pavement design while considering different parameters such as load imprint (circular v.s. elliptical), wheel configurations, pavement material behavior (stress-dependent v.s. elastic), and presence of shear loads. ABAQUS software is selected for this task and the required user material subroutines (UMAT) to implement the stress dependent behavior of the pavement materials are prepared after thoroughly studying the fundamentals of FEM and understanding the process implementing them in ABAQUS software.

Since field investigations and large scale laboratory resilient triaxial tests are expensive and time consuming and they require specific equipments and experience which may not be available to everyone at all times, DEM is used to calibrate the stress dependent material
behavior model as a function of grain size distribution and stress state. It should be noted that DEM simulations will require simple mechanical properties of the aggregates and the gradation of aggregates for this task, which can be provided by means of very basic laboratory tests. Furthermore, DEM simulations are calibrated with laboratory experiment results from the previous studies and impact of different various parameters on the simulation results can also be studied, if required in future.

Figure 1.1 shows the general approach that is followed in this study. DEM models are simulated by having simple mechanical properties such as particle stiffness and particle friction and grain size distribution. Micro and macro-mechanical response of the assembly of particles are provided by DEM simulations results. Material parameters obtained from DEM simulation results may be used in the empirical pavement design methods. Furthermore, DEM simulation results are used to implement stress dependent 3D FEM models to evaluate the pavement response (stress, strain, and deformation) under applied loads in order to be used in empirical-mechanistic pavement design methods.

Figure 1.1: The Proposed Study; The approach.
1-4- Thesis Outline

Chapter 2: Literature Review

In this chapter the literature related to problem is reviewed and different existing methods of airport pavement design are briefly introduced and their advantages and disadvantages are described. History of different methods of stress-strain analysis of pavements used in mechanistic-empirical pavement design methods are also summarized in this chapter. The DEM method is also introduced in this chapter and its mathematical formulation is presented. At the end, resilient behavior of granular materials and the parameters affecting the resilient modulus are studied and different models predicting the resilient modulus of granular material as functions of different parameters are summarized.

Chapter 3: DEM Simulations

In this chapter two set of DEM simulations (Biaxial Test and Resilient Modulus test) used in this study are described and micro/macro scale analysis and discussions are presented on the results of the DEM simulations. At the end of the chapter the DEM simulations are calibrated with the laboratory test results which were performed on materials with similar gradations in previous studies.

Chapter 4: FEM Simulations

This chapter presents the Finite Element Method (FEM) and introduces ABAQUS software that is used for FEM analysis. The user defined material feature of ABAQUS (UMAT) and its mathematical formulation are also explained in this chapter.
Different ABAQUS FEM models are validated with existing analytical methods, other FEM softwares, and field data in order to confirm the accuracy of the 3D nonlinear FEM models to be used in mechanistic-empirical flexible airport pavement design. The pavement response calculated from the 3D FEM models were used to estimate the maximum allowable number of load repetitions for a given pavement section that was compared with the maximum allowable number of load repetitions calculated using the FAA flexible airport pavement design procedure. The impact of different parameters on the flexible airport pavement response is also studied at the end of this chapter.

Chapter 5: Summary and Conclusions

In this chapter a summary of the performed research is provided and the main conclusions and contributions of the performed research are presented. Recommendations for future research are also provided in this chapter.
REFERENCES


Chapter 2
Literature Review

2-1- Flexible Airport Pavement Design

2-1-1- Introduction

The airport pavement design is very similar to regular highway pavement design in nature. Similar methodologies and procedures are used, but due to the more complex nature of the aircraft loading compared to those of the cars and trucks (Figure 2.1), some differences arise from parameters such as the followings (Yoder and Witczak, 1975):

magnitude of loads
Load magnitude or tire pressure is much higher in case of airport pavement and the rapid advances in the aircraft industry generates even higher tire pressures to be applied on the airport pavement.

load shape
Due to aircrafts tire size and pressure the load cannot be simplified to circular shape and more accurate load shapes (i.e. elliptical) should be considered while analyzing the aircraft loads.

load direction
Aircrafts induce significantly higher shear stresses to the pavement especially at the time of landing while shear stresses do not usually control the highway pavement design.
**Tire configuration**

Due to the size and weight of aircrafts their balance requires more number of tires and more complex configurations compared to cars and trucks.

**Pavement Segment for Analysis**

Previously mentioned parameters will require a 3D pavement segment of specific size compared to the axissymetric models used for highway pavements.

**Fuel Spillage**

Fuel spilled on the pavement occurs in larger volumes in airports and on the other hand, the type of fuel material used for airplanes will have a more important effect on the quality of surface layer in airports so material softening due to fuel spillage is more probable in airports. To avoid this, special maintenance procedures should be followed for airport pavements.

**Other Considerations**

The aircraft engine induces much larger temperature gradients within the airport pavement. Besides the aircraft engine blast will also affect the pavement which should be considered in pavement analysis.

Because of the higher sensitivity of aircrafts, the effects of the pavement on the aircraft should be studied carefully. This implies more conservative failure criteria to be considered for airport pavements. Besides, aircraft engines may get damaged by pavement debris sucked into them so airport pavements and especially their shoulders should be made more resistant to erosion.
In this chapter the most commonly used airport pavement design methods are briefly introduced. These methods are summarized as, but not limited to (i) Corps of Engineers (CBR) Method, (ii) the Federal Aviation Administration (FAA) Method, (iii) the Canadian Department of Transportation (CDOT) Procedure, (iv) the Asphalt Institute Method, (v) The
Subgrade Stress Ratio (SSR) Method, (vi) Aircraft Pavement Structural Design System (APSDS), and (vii) the International Civil Aviation Organization (ICAO) system for aircraft load classification.

(i) Corps of Engineers (CBR) Method
This method is a very simple method based on CBR values of different layers of the pavement and the design charts are generated using empirical correlations. In this method an Equivalent Single Wheel Load (ESWL) is selected based on the equal deflections between the ESWL and the multiple gear (Yoder and Witczak, 1975). Different categories of design curves are provided for light (25 kips), medium (100 kips), and heavy (265 kips) loads based on the design gear tire pressure (=ESWL/Area). These design curves will give the required thickness of pavement above each layer using the CBR values of each layer, wheel loads, tire pressures, gear configurations, and traffic areas. Minimum required thicknesses for each layer are also provided. Knowing the total required thickness and the minimum thicknesses of different layers, the actual thicknesses of all layers can be calculated. Figure 2.2 shows a sample of Corps of Engineering Design charts for heavy load pavements.

(ii) The Federal Aviation Administration (FAA) Method
This method is also based on CBR method of design. Although this method is also empirical, but greater amount of research has been done by FAA to develop more reliable correlations. They have implemented theoretical concepts as well as empirically developed data in their design charts (FAA, 1995). FAA provides separate design charts for airports serving aircrafts with gross weights of more than 30,000 pounds (13,000 kg) and less than 30,000 pounds
(13,000 kg). Similar to Corps of Engineers Method, using the CBR value of each layer, gross aircraft weight, and annual departures the design curves will provide the required total thickness of pavement on top of that layer to avoid shear failure occurring in that layer. By knowing required pavement thickness on top of each layer and also the minimum required thicknesses for different layers, each layer’s design thicknesses can be calculated.

FAA provides minimum specifications for materials that can be used in different pavement layers. They also have considerations regarding drainage conditions and frost problems. Figure 2.3 shows a sample FAA design curve for dual wheel gear.
Figure 2.2: Corps of Engineers flexible pavement design curves for heavy load pavement-twin-twin wheels 37-62-37 inches spacing, $A_e = 267$ in$^2$. (From TM 5-824 2/AFM 88-6 Chapter 2)
Figure 2.3: FAA flexible pavement design curves for dual gear. (From FAA AC 150/5320-6D)
(iii) The Canadian Department of Transportation (CDOT) Procedure

This method is based on the load carrying capacity of the Canadian airport runways using plate bearing tests correlated with cone bearing, penetrometer, CBR, and triaxial tests (Yoder and Witczak, 1975). Since this method is implemented in Canada which has a predominant cold weather, the effect of frost is considered in the analysis. In this method ability of the pavement to operate under 5,000 coverages of the design aircraft is referred to as the capacity of the designed pavement. Figure 2.4 shows a design curve for single wheel loads.

(iv) The Asphalt Institute (AI) Method

This method was is a theoretically oriented design and it was introduced in 1973 for full depth asphalt airfield pavements under gross weights greater than 60 kip (AI, 1973). This method follows the same traffic analysis concepts as highway design methods as it uses a standard aircraft and provides relative effects of different aircraft types.

Similar to AI highway pavement design method, the maximum horizontal tensile strain at the bottom of the asphalt layer ($\varepsilon_h$) and the maximum vertical compressive strain at the top of subgrade ($\varepsilon_c$) are used to calculate the allowable number of load repetitions for fatigue cracking and rutting, respectively. Two set of design curves are provided to calculate the pavement thickness for fatigue and rutting criteria using subgrade modulus, number of repetitions, and mean annual air temperature. Figure 2.5 shows design curves for fatigue criteria.
Figure 2.4: CDOT Flexible pavement design curves for single wheel. (From McLeod, 1956)
(v) The Subgrade Stress Ratio (SSR) Method

This method uses subgrade stress ratio (SSR) which is the ratio of subgrade deviator stress to subgrade compressive strength (Bejarano and Thompson, 2001) to evaluate subgrade rutting in airport flexible pavements. This method estimates the permanent deformation of the subgrade and checks if it is within acceptable range and if it is acceptable, it will be added to the permanent deformation of the pavement layers and the total permanent deformation is then compared with the surface permanent deformation criteria. Figure 2.6 shows the relationship of subgrade permanent deformation versus SSRs.

(vi) Aircraft Pavement Structural Design System (APSDS)

This method is an Australian layered elastic approach for aircraft pavement design (Mincad, 2000) which is the airfield pavement version of the previously introduced highway pavement design tool Circly (Mincad, 1999) including the effect of aircraft wander (Wardle, et al; 2001). APSDS uses Cumulative Damage Factor (CDF), which is the sum of the damage factors for all loadings, to evaluate the pavement. Whenever the CDF reaches 1.00 at any point the pavement is reached its design life.
Figure 2.5: AI flexible pavement design curves – Fatigue Criteria. (From McLeod, 1956)
The International Civil Aviation Organization (ICAO) Method

This method which is more a classification procedure was introduced by the International Civil Aviation Organization in 1981 (ICAO, 1983) for reporting airfield pavement strength. According to this method, the ACN computed by the aircraft manufacturers and the PCN evaluated by airport authorities should be compatible. Aircraft Classification Number (ACN)
is a number showing the effect of aircraft loading on pavement which is a function of landing gear configuration, tire pressure, and gear load for a specified subgrade strength (Stet and Beuving, 1993) and Pavement Classification Number (PCN) is a number showing the pavement’s bearing strength for unrestricted operations (10,000) on the pavement without damaging the pavement. Figure 2.7 is a sample graph to calculate PCN of different pavements assuming (Loizos and Charonitis, 2000). Equations 2.1 and 2.2 are also an alternate method of evaluating ACN and PCN numbers, respectively (Perez and Sotolongo, 2008).

\[ ACN = \frac{2}{1000} \times (CRSE) \]  

(2.1)

Where CRSE is equivalent single wheel load which depends on the distribution of wheels.

\[ PCN = \frac{1}{500} \times \left( \frac{e^2}{1 - \frac{0.025}{0.57 \times CBR}} \right) \]  

(2.2)

Where e is equivalent thickness in cm, from converting pavement to an equivalent homogeneous material with E = 500 MPa, and CBR is for the subgrade soil in percents.
2-2- Stress-Strain Analysis of Pavements Under Traffic Loads

Stresses and strains distribution within the pavement layers is required to model the response of a pavement structure to traffic loading and evaluating the structural and functional failure. These failure distresses will control the pavement design. Having a more economic pavement design with the ever ending increase in the size of trucks, which leads to increase in tire loading and inflation pressures, requires a better understanding of the pavement stress-strain behavior. The more precise the stress-strain behavior of the pavement due to the applied traffic loads is modeled, the design of the pavement will be more reliable.

Figure 2.7: Pavement Classification Number Evaluation Chart Based on the B767-300 Aircraft. (From Loizos, Charonitis, 2000)
2-2-1- Analytical Methods

2-2-1-1. Linear Elastic Analysis on a Half-Space

The simplest elastic pavement analyses is using Boussinesq (1885) theory which was offered for concentrated loads applied on an elastic half-space and integrating the responses for circular load. Foster and Ahlvin (1954), presented a series of charts for determining different stress components and vertical deflection due to a circular load applied on an incompressible half space (i.e. Poisson ratio = 0.5). Ahlvin and Ulery (1962) then extended their work for different Poisson ratios (Table 2.1).

2-2-1-2. Linear Elastic Analysis on Layered Systems

Pavements are composed of different layers and the stiffness of the layers is decreased with depth which results in having lower stresses, strains compared to the one layer elastic method (Yoder and Witczak, 1975).

The simplest layered approach in analyzing the pavement is the two layer method introduced by Burmeister (1943) (figure 2.8).
### Table 2.1: Summary of One-Layer Elastic Equations (from Ahlvin and Ulery, 1962)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>General Case</th>
<th>Special Case ($\mu = 0.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical stress</td>
<td>$\sigma_v = p[A + B]$</td>
<td>(same)</td>
</tr>
<tr>
<td>Radial horizontal stress</td>
<td>$\sigma_r = p[2\mu A + C + (1 - 2\mu)F]$</td>
<td>$\sigma_r = p[A + C]$</td>
</tr>
<tr>
<td>Tangential horizontal stress</td>
<td>$\sigma_t = p[2\mu A - D + (1 - 2\mu)E]$</td>
<td>$\sigma_t = p[A - D]$</td>
</tr>
<tr>
<td>Vertical radial shear stress</td>
<td>$\tau_{vr} = \tau_{mr} = pG$</td>
<td>(same)</td>
</tr>
<tr>
<td>Vertical strain</td>
<td>$\epsilon_v = \frac{p(1 + \mu)}{E_1}[(1 - 2\mu)A + B]$</td>
<td>$\epsilon_v = \frac{1.5p}{E_1}B$</td>
</tr>
<tr>
<td>Radial horizontal strain</td>
<td>$\epsilon_r = \frac{p(1 + \mu)}{E_1}[(1 - 2\mu)F + C]$</td>
<td>$\epsilon_r = \frac{1.5p}{E_1}C$</td>
</tr>
<tr>
<td>Tangential horizontal strain</td>
<td>$\epsilon_t = \frac{p(1 + \mu)}{E_1}[(1 - 2\mu)E - D)]$</td>
<td>$\epsilon_t = -\frac{1.5p}{E_1}D$</td>
</tr>
<tr>
<td>Vertical deflection</td>
<td>$\Delta_v = \frac{p(1 + \mu)a \left[ \frac{E}{a} A + (1 - \mu)H \right]}{E}$</td>
<td>$\Delta_v = \frac{1.5pa}{E} \left( a + \frac{H}{2} \right)$</td>
</tr>
<tr>
<td>Bulk stress</td>
<td>$\theta = \sigma_v + \sigma_r + \sigma_t$</td>
<td></td>
</tr>
<tr>
<td>Bulk strain</td>
<td>$\epsilon_v = \epsilon_v + \epsilon_r + \epsilon_t$</td>
<td></td>
</tr>
<tr>
<td>Vertical tangential shear stress</td>
<td>$\tau_{vr} = \tau_{mr} = 0$; $[\sigma_v(\epsilon_v)$ is principal stress (strain)]</td>
<td></td>
</tr>
<tr>
<td>Principal stresses</td>
<td>$\sigma_{1,2,3} = \frac{(\sigma_v + \sigma_r) \pm \sqrt{(\sigma_v - \sigma_r)^2 + (2\tau_{mr})^2}}{2}$</td>
<td></td>
</tr>
<tr>
<td>Maximum shear stress</td>
<td>$\tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2}$</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 2.8](Figure.png)

**Figure 2.8:** Burmister’s Two Layer System (from Burmeister, 1943)
Burmister made the following assumptions in his models:

- Each layer is homogeneous, isotropic, and linearly elastic with an elastic modulus and Poisson’s ratio (Hooke’s law).
- Each layer has a uniform thickness and infinite dimensions in all horizontal directions, resting on a semi-infinite elastic half-space.
- Before the application of external loads, the pavement system is free of stresses and deformations.
- All the layers are assumed to be weightless.
- The dynamic effects are assumed to be negligible.

For three layer systems under uniform circular loads, at the intersection of the axis of symmetry with the interfaces Acum and Fox (1951) presented an extensive tabular summary of normal and radial stresses. Assuming Poisson ratio of 0.5 and for concentrated normal force or a uniformly distributed normal load. Peattie (1962), and Jones (1962) came up with more general solutions for vertical and horizontal stresses, respectively. But their work was still not able to take into account the tangential loads and non-uniform loads. In the same year Schiffman (1962) introduced a general analytical solution to the analysis of stresses and displacements of a multi-layer elastic system subjected to non-uniform normal surface loads, tangential surface loads, rigid, semi-rigid and slightly inclined plate bearing loads. Schiffman (1962) used an asymmetric cylindrical coordinate system (Figure 2.9) and separate properties each layer, including elastic modulus \(E_i\), Poisson’s ratio \(\nu_i\), and thickness \(h_i\) (Figure 2.10).
Figure 2.9: Element of Stress in a Multi-layer Elastic System (from Schiffman, 1962)

Figure 2.10: N-layer Elastic System (from Schiffman, 1962).
Many computer programs have been developed based on Burmister's layered elastic theory. Chevron Research Company (Warren and Dieckmann, 1963) developed CHEV which can only be applied to linear elastic materials. Hwang and Witczak (1981) modified CHEV for nonlinear material and introduced DAMA. De Jong et al. (1973) developed BISAR, which takes into account both vertical loads and horizontal loads, but can only model and analyze three-layer systems. Kopperman et al. (1986) also developed ELSYM5 at the University of California, Berkley which was able to analyze the elastic five-layer system under multiple vertical wheel loads, but cannot be used to analyze the horizontal loads. At University of Kentucky in 1985, another program named KENLAYER was developed which could give out the solution for an elastic multiple-layered system under single, dual, dual-tandem circular wheel loads with layers having linearly elastic, non-linearly elastic or visco-elastic material.

The DAMA computer program was introduced to analyze multiple-layered elastic pavements with a maximum number of layers of five, under single or dual wheel loads while the sub-grade and the asphalt layers are considered to be linearly elastic and the untreated sub-base in considered to be non-linear.

2-2-2- Finite Element Method

Finite Element Method (FEM) is a valuable tool for many engineering problems having the ability to model different materials with complex constitutive models as well as complex geometry and different boundary conditions.
FEM is an approximate numerical approximate solution to partial differential equations (PDE) or integral equations that model a system. This approximate equations should be numerically stable (i.e. the errors do not accumulate toward a meaningless answer).

Mesh discretization of a continuous domain into discrete sub-domains (elements) was first introduced by Alexander Hrennikoff (1941) and Richard Courant (1942). The present FEM was first developed by Turner et al. (1956) at Boeing air firm since at that time only large industrial companies and some government agencies were able to afford computers. Later in the 1960s John Argyris at the University of Stuttgart and also Ray W. Clough at Berkeley implemented the use of FEM in civil engineering.

The finite element method works by discretizing a domain into a number of smaller elements which are responsible for capturing variations in displacements, strains, and stresses over their area (2D) or volume (3D). Equation 2.3 gives the relationship between element nodal displacements and strains:

$$\mathbf{\varepsilon} = S\mathbf{U}$$ \hspace{1cm} (2.3)

where, $\mathbf{\varepsilon}$ is the strain vector; $S$ is a suitable linear operator and $\mathbf{U}$ is nodal displacement vector. The element stiffness matrices are computed using:

$$K^e = \int B^T D B \, dv$$ \hspace{1cm} (2.4)

Where $B$ is a matrix of linear operators and $D$ is the constitutive matrix. The element stiffness matrices are assembled into a system stiffness matrix, and the resulting system of
linear equations in is solved for unknown nodal displacements. Strains and stresses can then be recovered from the nodal displacements.

2-2-2-1. Finite Layer Method (Linear Elastic Approach)

Finite Layer Method is a semi-analytical Finite Element Method which enables us to solve a 3D problem in a similar way to 2D problem which reduces the data input and computer storage requirements (Cheung, 1976).

In problems dealing with soils and pavements in which the domain consists of horizontal layering it is not efficient to use analytical or FEM 3D solutions. In these problems Finite Layer Method offers a more efficient solution. Furthermore orthogonal series and integral transforms such as Fourier transform and Henkel transform can be used to reduce a 3D/2D problem to a 2D/1D problem in order to have an easier and more feasible solution. The Accuracy of the Finite Layer Method depends on the mesh and type of element used, the number of harmonic terms in orthogonal series or integral transforms considered, and the volume of materials contained within the boundaries (Cheung and Fang, 1979).

Many researchers studied the pavement response using Finite Layer method. Cheung and Fan (1979) did some parametric studies by modeled half-space under square load and also two layered foundation under circular load and showed that finite layer method is simple and sufficiently accurate. Small and Booker (1984a) presented equations for Finite Layer Analysis of pavements under general 3D loading and then they narrowed their formulation down to strip loading. Small and Booker (1984b) used Hankel transforms instead of Fourier
transforms to formulate the Finite Layer Analysis of pavements under circular loads and then confirmed their Finite Layer formulations by solving examples of half-space and also layered pavements under vertical loads and comparing the results with existing analytical solutions. Booker and Small (1985a) studied consolidation settlement of a layered pavement by using a flexibility matrix and taking the node plane tractions as primary quantities. Small and Brown (1988) solved problems involving sub-surface circular loaded areas. Booker and Small (1988) studied the response of layered pavements due to horizontal surface loads and concluded that horizontal tire forces should not be neglected in pavement design.

Finite Layer Method is also applied to time dependent problems in which loading, boundary conditions or material properties vary with time such as consolidation, groundwater extraction, visco-elasticity, thermo-elasticity, and etc.. Booker and Small (1985b) presented a Finite Layer method to calculate the settlement behavior of circular or general loadings on horizontally layered soils which undergo secondary or creep consolidation. Booker and Small (1987) presented a Finite Layer Method to calculate the consolidation behavior of layered soil subjected to strip, circular, or rectangular surface loading, or subjected to fluid withdrawal due to pumping and confirmed their method by applying it to simple existing examples such as consolidation under uniform strip loading applied to the surface of an infinite half space (Schiffman et. al., 1969), consolidation of a uniform finite soil layer under a circular loaded area at the surface (Gibson et. al., 1970; Booker, 1974), or time settlement behavior of rectangular loaded areas (Gibson and McNamee, 1957). They have also used their method to calculate the consolidation of a layered soil under circular uniform load at the surface and compared their results with Small et al. (1984) method. Small and Booker (1986)
presented a method to calculate the thermo elastic stresses induced in layered soils due to existence of a heat source in the soil and showed that the heat source in the soil can cause stresses in the surrounding material.

2-2-2. General Finite Element Method (Stress Dependent Approach)

Most of the elastic layered Analysis’s are performed assuming linear elastic behavior for the subsurface pavement layers while it is found in the lab that materials used in base/subbase and subgrade have nonlinear, stress-dependent behavior under repeated traffic loading (Brown and Pappin 1981; Uzan 1985; Thompson and Elliott 1985; Tutumluer 1995; Rowshanzamir 1995). The Finite Layer method is also based on assuming similar material properties in layers which is not always true especially in stress dependent materials since the stresses will not necessarily be the same throughout the layers. Three dimensional FEM, on the other hand, is the most commonly used approach in nonlinear analysis of the pavement due to its ability to deal with different material properties as well as different load and boundary conditions.

Several axisymmetric pavement FE analysis models were developed. ILLI_PAVE was developed at university of Illinois at Urbana-Champaign (Raad and Figueroa, 1980) and treats the pavement system as an axisymmetric solid domain.

The MICH_PAVE (Harichandran et al, 1990) computer program is very similar to ILLI_PAVE but yields more reasonable results using the same methods to model granular
materials and soils and the Mohr-Coulomb failure criteria. MICH_PAVE is capable of performing linear and non-linear finite element analysis of flexible pavements assuming axisymmetric loading conditions and equivalent resilient modulus for each pavement layer. With the advances in computer technologies and numerical simulations and existence of several well known general FEM programs (ex. ABAQUS, ANSYS, ADINA, … ) many researchers studied different pavements under various pavement geometry and loading conditions (Hjelmstad and Taciroglu 2000; Schwartz 2002; Sukumaran et al. 2004; Saad et al. 2005).

One of the earliest computer programs used for pavement analysis was the CHEVRON program developed by the Chevron Research company (Warren and Dieckmann, 1963) which was later modified by Hwang and Witczak (1979) to account for the nonlinear elastic granular base behavior. BISAR, was another computer program which was developed by Shell researchers for calculating the response of multi-layer structures with linearly elastic material behavior (De Jong et al., 1973) which was able to analyzes more than one circular load applied on the pavement. ELSYM5 was another computer program which was developed at the University of California, Berkeley (Kopperman et al., 1986 ) and due to its easy use in routine flexible pavement design has become very popular in the US especially among the state transportation agencies. At the University of Kentucky Huang (1993) presented the computer program KENLAYER for the analysis of elastic and viscoelastic layered systems when each layer in the system may behave linear elastic, nonlinear elastic, or viscoelastic.
the pavement deformations under the repeated traffic loads are mostly recoverable and can be considered elastic because due to the material shakedown occurring after a certain number of load repetitions, the permanent deformation in each load cycle decreases to a minimum. In this case the resilient Modulus ($M_R$) is used for the elastic modulus of the pavement material. In order to more realistically model the nonlinear behavior of the pavement under repeated traffic loads, the resilient modulus and different parameters affecting it are studied and a well representative model for the resilient modulus of pavement material to be used in FEM models is chosen.

2-2-3- Discrete Element Modeling

2-2-3-1. Introduction

It is not typically possible to assess every conceivable field condition in the laboratory, and most laboratory tests provide only empirical results relating to material response at specimen boundaries. Often, it is difficult to know why a specimen behaves in a certain manner because of the lack of information at the particle scale (i.e., the scale that governs laboratory and field-scale behavior). We can infer certain causal factors based on our understanding of theoretical soil mechanics, but there will still be gaps in our knowledge. Researchers are increasingly using discrete element method (DEM) modeling to bridge those gaps by studying the root causes of changes in material behavior. By better understanding the underlying mechanisms that lead to an observed material response, researchers can better
answer questions about the behavior of materials that are not identical to those tested in the laboratory.

The DEM was first introduced by Cundall and Strack (1979) and attempts to model the behavior of a soil mass by repeatedly solving Newton's second law for an assembly of densely packed particles using a simple force-displacement law at particle contacts (Cundall and Strack, 1979).

The Discrete Element Method (DEM) was used for simulating the Biaxial Compressive Tests. The specific implementation of the DEM used herein is PFC2D v 3.1 : Particle Flow Code in Two Dimensions v 3.1 (Itasca, 2006), which has well verified performance, complete text documentation, and more flexibility and ease of use comparing to other existing codes (e.g., Kuhn 2003).

2-2-3-2. Mathematical Formulation of DEM

The DEM’s calculations are based on application of the Newton’s second law to the particles to determine the motion of each particle and a force-displacement law at the contacts to update the contact forces. Since the wall motion is specified by the user, Newton’s second law is not applied to walls, but the force-displacement law account for ball-wall contacts. Some important assumptions are necessary for solution of these equations using the DEM (Evans, 2005):

- Particles may be represented as rigid disks or balls.

- Contacts (ball-ball and ball-wall) occur at a point.
- Particles overlap is allowed at contacts but these overlaps are small relative to particle size.

The calculation cycle is simply repeated application of the law of motion to each particle, a force-displacement law to each contact, and a constant updating of wall positions (Figure 2.11).

**Figure 2.11: Calculation Cycle of DEM (Itasca 2006)**
The following derivation is taken from Itasca 2006 (See figures 2.12 and 2.13 for notations):
To start the unit normals of the contacts in the assembly should be determined:

\[ n_i = \frac{x_i^{[B]} - x_i^{[A]}}{d} \]  

(2.5)

where \( n_i \) is the normal vector, \( x_i^{[A]} \) and \( x_i^{[B]} \) are locations of particle centers for particles A and B, respectively, and \( d \) is the distance between particle centers or the shortest path from the particle center to the wall for ball-wall contacts. The overlap at contacts, \( U^n \), defined as the relative contact displacement in the normal direction is calculated by:

\[
U^n = \begin{cases} 
R^{[A]} + R^{[B]} - d(ball-ball) \\
R^{[B]} - d(ball-wall) 
\end{cases}
\]  

(2.6)

Where \( R^{[\Phi]} \) is the radius of ball \([\Phi]\). The location of the contact point, \( X_i^{[c]} \), is calculated by:

\[
X_i^{[c]} = \begin{cases} 
X_i^{[A]} + \left( R^{[A]} - \frac{1}{2} U^n \right) n_i, (ball-ball) \\
X_i^{[B]} + \left( R^{[B]} - \frac{1}{2} U^n \right) n_i, (ball-wall) 
\end{cases}
\]  

(2.7)

Now the contact force vector, \( F_i \), can be resolved into normal and shear components with respect to the contact plane using the constitutive relation for contact theory (Linear or Hertz-Mindlin):

\[ F_i = F_i^n + F_i^s \]  

(2.8)

Where \( F_i^n \) and \( F_i^s \) are the normal and shear components, respectively. The magnitude of force in the normal direction is then calculated by:
\[ \mathbf{F}^n = \mathbf{K}^n \mathbf{U}^n \]  

(2.9)

Where \( \mathbf{K}^n \) is the contact normal stiffness (secant modulus).

The shear contact velocity, \( V^s \), which is required to calculate shear forces is calculated by:

\[
V^s = \left( \frac{d}{dt} X_{i}^{[\Phi^i_j]} - \frac{d}{dt} X_{i}^{[\Phi^i]} \right) t_i - \omega_3^{[\Phi^i_j]} \left| X_{k}^{[c]} - X_{k}^{[\Phi^i_j]} \right| - \omega_3^{[\Phi^i_j]} \left| X_{k}^{[c]} - X_{k}^{[\Phi^i]} \right|
\]  

(2.10)

Where \( \omega_3 \) is rotational velocity and \([\Phi^i]\) is used to denote either a wall or ball at the given contact as follows:

\[
(\Phi^1, \Phi^2) = \begin{cases} (A, B), (ball - ball) \\ (b, w), (ball - wall) \end{cases}
\]  

(2.11)

Knowing the shear contact velocity, \( V^s \), The incremental shear force \( (\Delta \mathbf{F}^s) \) over a time increment \( (\Delta t) \) can be calculated by:

\[
\Delta \mathbf{F}^s = -k^s V^s \Delta t
\]  

(2.12)

Where \( k^s \) is the tangent shear stiffness for the contact.

The new forces and moments for the particles are found by summing the old ones existing at the start of the time step with the increment forces and moments as follows:
\[ F^s \leftarrow F^s + \Delta F^s \leq \mu F^n \]
\[ F_i^e = F^n_i n_i + F^s_i t_i \]
\[ F_i^{(\Phi_1)} \leftarrow F_i^{(\Phi_1)} - F_i \]
\[ F_i^{(\Phi_2)} \leftarrow F_i^{(\Phi_2)} - F_i \]
\[ M_3^{(\Phi_1)} \leftarrow M_3^{(\Phi_1)} - e_{3jk} \left( X_j^{[c]} - X_j^{(\Phi_1)} \right) F_k \]
\[ M_3^{(\Phi_2)} \leftarrow M_3^{(\Phi_2)} + e_{3jk} \left( X_j^{[c]} - X_j^{(\Phi_2)} \right) F_k \]

(2.13)

Where \( \mu \) is the friction coefficient at the contact, \( M_3 \) is moment, and \( e_{3jk} \) is the permutation operator. Note that there is only one rotational degree of freedom in two dimensions.

Knowing the new particle forces and moments, the acceleration and angular momentum at each time step is calculated by:

\[ F_i = m_i \left( \frac{d^2}{dt^2} X_i - g_i \right) \]
\[ M_3 = \beta m R^2 \frac{d}{dt} \omega_3 \]

(2.14)

Where \( m \) is particle mass, \( g \) is acceleration due to gravity, and \( \beta \) is 0.4 for spheres and 0.5 for disks. The equations of motion (2.14) are discretized and solved using central finite differences:

\[ \frac{d^2}{dt^2} X_i^{[t]} = \frac{1}{\Delta t} \left( \frac{d}{dt} X_i^{[t+0.5\Delta t]} - \frac{d}{dt} X_i^{[t-0.5\Delta t]} \right) \]
\[ \frac{d}{dt} \omega_3^{[t]} = \frac{1}{\Delta t} \left( \omega_3^{[t+0.5\Delta t]} - \omega_3^{[t-0.5\Delta t]} \right) \]
\[ \frac{d}{dt} X_i^{[t+0.5\Delta t]} = \frac{d}{dt} X_i^{[t-0.5\Delta t]} + \left( \frac{F_i}{m + g_i} \right) \Delta t \]

(2.15)
\[
\omega_j^{[r+0.5\Delta t]} = \omega_j^{[r-0.5\Delta t]} + \left( \frac{M_j^{[r]}}{\beta m R^2} \right) \Delta t
\]

\[
X_i^{[r+\Delta t]} = X_i^{[r]} + \left( \frac{d}{dt} X_i^{[r+0.5\Delta t]} \right) \Delta t
\]

The computed solution will remain stable only for time steps not exceeding a critical time step which depends on the minimum eigenperiod of the entire system (Itasca, 2006). Calculating this value is computationally expensive, so PFC2D uses a simplified spring and mass logic to calculate the time step at the start of each cycle. The actual time step used in any cycle is taken as a fraction of this estimated critical value by applying a safety factor to it.

\[
t_{crit} = \begin{cases} \sqrt{\frac{m}{k_{tran}}} , (\text{translational - motion}) \\ \sqrt{\frac{I}{k_{rot}}} , (\text{rotational - motion}) \end{cases}
\]

Where \(k_{\text{tran}}\) and \(k_{\text{rot}}\) are the translational and rotational stiffnesses, respectively, and \(I\) is the moment of inertia of the particle.

Each particle in the assembly of particles modeled in PFC2D may have a different mass and stiffness, so various critical time steps may be found for each different particle in the assembly by applying Eq. (2.17) separately to each degree of freedom, and assuming that the degrees of freedom are uncoupled. The final critical time step is taken to be the minimum of all critical time steps computed for all degrees of freedom of all particles.
2-3- Resilient Modulus of granular Materials

2-3-1- Introduction

The concept of resilient modulus was first introduced by Seed et al. (1962) and quickly gained popularity in the pavement community. The resilient modulus of unbound aggregate material has been used by the American Association of State Highway and Transportation Officials (AASHTO) in 1986 to account for the response of unbound aggregate material to traffic and environmental loads in flexible pavement design (AASHTO, 1986). The resilient modulus \( M_R \) is a measure of the elastic modulus of a material at a given stress state. As shown in figure 2.14, the resilient modulus is defined as the applied deviatoric stress \( \sigma_d \) divided by the recoverable strain \( \varepsilon_r \) in a repeated triaxial compression test wherein the deviatoric stress does not exceed the shear strength of the material:

\[
M_R = \frac{\sigma_d}{\varepsilon_r}
\]  

(2.18)

Recent studies have employed new laboratory techniques such as bender elements (Baig and Nazarian, 1995) or \textit{in-situ} methods such as dynamic cone penetrometer (DCP; Hassan, 1996; Chen et al., 1999), light falling weight deflectometer (LFWD; Nazzal, 2003), or the GeoGauge (Chen et al., 1999) ), or numerical DEM methods (Zeghal, 2001; Zeghal, 2003; Zeghal, 2007; Uthus, 2007; Uthus et al., 2008) to infer (if not directly measure) the resilient characteristics of subgrade and base soils.
2-3-2-Parameters Affecting Resilient Modulus

2-3-2-1. Grain Size, Gradation, and Particle Shape

Parameters such as the inter particle friction and the mechanical interlock between particles which affect the resilient behavior of aggregates are both functions of gradation and particle shape. Hodek and Mayrberger (2007) have shown that ABC materials with coarser grain size distributions (GSDs) have higher resilient modulus than finer specimens under the same conditions. Saleh (2008) found the grain size distribution to be the most significant factor affecting the resilient modulus of HMA. Similar to Hodek and Mayrberger (2007) findings for ABC materials, he observed that coarser gradation will provide significantly
higher Resilient Modulus compared to a finer gradation. As it is shown in figure 2.15, Barksdale and Itani (1989) also found that finer gradation tends to show greater plastic deformation and smaller resilient modulus. Barksdale and Itani (1989) found that the coarse portion of the aggregates is the dominant factor in the amount of plastic deformation while the fine portion has a relatively small effect. Hicks and Monismith (1971) showed that over the range of confining stresses encountered in field pavements (i.e. 0-10 psi), increasing the fines content will result in decreasing the resilient modulus of partially crushed aggregates while it will result in increasing the resilient modulus of crushed aggregates.

For soils with similar grain size distributions and the same fines content Kolisoja (1997) showed that increasing maximum particle size corresponds to increasing resilient modulus due to stiffer response of the assembly.

![Figure 2.15: Effect of gradation and stress state on resilient modulus of ABC (Barksdale and Itani, 1989)](image)

Several researchers have shown that materials with angular particles have a higher resilient modulus than materials with rounded particles (Hicks and Monismith, 1971; Allen and
Thompson, 1974; Barksdale and Itani, 1989). Increased particle angularity causes higher interlocking forces between particles, and hence, higher shear strength and lower plastic deformation. The morphological properties discussed here are visualized in figure 2.16.

![Diagram of particle properties](image)

**Figure 2.16: Key morphological properties of an aggregate particle (Uthus, 2007)**

2-3-2-2. Stress History

Researchers showed that although the plastic deformation and the resilient modulus of soils depend on the stress history of the soil. A material that previously has experienced higher stress levels show significantly higher resistance to permanent deformations and higher resilient modulus due to densification and reorientation of particles. However, applying the same stress amplitude for several cycles will end up the sample in some sort of a steady state (Hicks and Monismith, 1971) where the resilient modulus will remain constant (figure 2.17).
Brown (1975) reported that by keeping the applied stresses low enough to avoid significant plastic deformations in materials, the effect of the stress history on resilient behavior of materials can be neglected and hence several resilient tests can be performed on a single soil sample. As long as the test specimen is in a stress state close to the elastic mobilized strength line in the elasto-plastic zone (figure 2.18), it will not undergo significant plastic deformations and much less reorientations of particles will occur and hence the specimen response will not be affected much by its stress history. On the other hand when the sample is loaded to a stress state far from the elastic mobilized strength line it will undergo more significant plastic strain as the stress state gets closer to the failure envelope.
Many researchers showed that as long as shear failure does not occur, resilient modulus increases with an increase in confining stress (Hicks and Monismith, 1971; Thomson and Robnett 1979; Pezo et al., 1992; Mohammad and Puppala, 1995; Mohammad et al., 1999). More tightly confined materials undergo less permanent deformations and exhibit stiffer response with loading which will result in higher resilient modulus. This increase in resilient modulus is more significant for granular materials (Thomson and Robnett 1979, Pezo and Hudson 1994). As it is shown in figure 2.19, increasing
the confining pressure will move the stress state toward the linear zone which will result in less amount of permanent deformation and a stiffer response and hence higher resilient modulus.

![Stress-Deformation Diagram](image)

**Figure 2.19: Effect of confining pressure on permanent deformation and resilient modulus.**

**2-3-2-4. Deviatoric Stress**

In fine-grained soils, increasing the deviatoric stress will cause the resilient modulus to decrease significantly at small deviatoric stresses while at larger deviatoric stresses the resilient modulus will either decrease slightly or stay constant due to an increase in the deviatoric stress (Thompson and Robnett, 1979; Boateng-Poku and Drumm, 1989; Drumm et al., 1990; Santa, 1994; Mohammad and Puppala, 1995). This can be described by figure 2.20. As it is shown in figure 2.20, the stress state will start in the linear zone and with increasing
the deviatoric stress it will first march in the elastic zone toward elastic mobilized strength line with no significant permanent deformation and little increase in resilient modulus due to stiffer response under higher mean stresses. Further increasing the deviatoric stress will result in marching in elasto-plastic zone toward failure envelope which results in more permanent deformation as the mohr circle grows larger, reducing the stiffness and hence the resilient modulus.

Figure 2.20: Effect of deviatoric stress on permanent deformation and resilient modulus.
2-3-2-5. Density

Studies on the effect of dry density showed that resilient modulus increased with an increase in dry density (Al-Refeai and Al-Suhaibani, 2002). Barksdale and Itani (1989) showed that this is true only at small mean stresses and the increase in Resilient Modulus with increase in density will be less significant at higher stresses. Uthus (2007) found out that under dry conditions the dry density of a material with a high amount of fines seems to be an important parameter affecting the permanent deformations, but under wet conditions the degree of saturation seems to override the effect of the dry density of the samples. This is more significant as the amount of fines increases.

2-3-2-6. Moisture Content

Drumm et al. (1997) performed resilient modulus tests on different soils at optimum moisture content and higher moisture contents and observed that increasing the moisture content results in a decrease in resilient modulus but that the magnitude of the decrease depends on soil type. They then proposed a method of correction for resilient modulus for increasing moisture content (Drumm et al., 1997).

Gupta et al. (2007) proposed a modification to the five-parameter log-log Resilient Modulus model recommended by NCHRP 1-28A (2003) to account for unsaturated soil response. They used a modification similar to that used by Vanapalli et al. (1996) to modify the Fredlund and Rahardjo (1993) equation for unsaturated shear strength by multiplying a
power term of suction to the NCHRP 1-28A (2003) model. Their model shows that increased suction due to decreasing the moisture content in unsaturated soils results in increasing resilient modulus. Uthus (2007) found out that water content is more important in well-graded materials with a high fines content as their resilient modulus and the resistance to permanent deformation is the highest value under dry conditions, but adding water content more than a certain limit significantly decreases the resilient modulus and the resistance to permanent deformations.

2-3-2-7. Freeze-Thaw

Frozen soils are stiffer and hence they have significantly lower permanent deformations and significantly higher Resilient Modulus values. This can cause large decrease in the Resilient Modulus when the soil is thawed. In this case the Resilient Modulus will even be smaller than the unfrozen soil state (Cole et al. 1981).

2-3-3- Existing Models to Predict the Resilient Modulus

Different researchers proposed several models to estimate the Resilient Modulus based on mentioned parameters in sections 2-3-2-1 to 2-3-2-7. Some of these models are presented in table 2.2.

As it can be seen in table 2.2, stress state and specially the confining pressure which is the most important parameter affecting the resilient modulus is seen almost among all of the proposed equations.
Equations in table 2.2 which is the most complete equation combines both the stiffening effect of bulk stress ($\theta$) and the softening effect of shear stress ($\tau_{oct}$). The familiar two-parameter bulk stress model ($k$-$\theta$ model) for granular materials and its companion two-parameter shear stress model for cohesive soils, can be recovered by choosing appropriate parameters in this equation. For the purposes of the present study, only the bulk stress stiffening term in Equation e is considered which simplifies to:

$$M_R = k_1P_0\left(\frac{\theta}{P_0}\right)^{k_2}$$ (2.19)

Which is similar to the $k$-$\theta$ model:

$$M_R = k_1\theta^{k_2}$$ (2.20)

### Table 2.2: Different Models to estimate Resilient Modulus of Unbound Granular Material (Konrad, 2006)

<table>
<thead>
<tr>
<th>Model</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>[a] $M_i = k_1\sigma^2$</td>
<td>Dunlap 1963; Monismith et al. 1967</td>
</tr>
<tr>
<td>[b] $M_i = k_1\sigma^3$</td>
<td>Brown and Pell 1967; Seed et al. 1967</td>
</tr>
<tr>
<td>[c] $M_i = k_1p_0\left(\frac{\theta}{P_0}\right)^{k_6}$$left(\frac{q}{P_0}\right)^{k_7}$</td>
<td>Uzan 1985</td>
</tr>
<tr>
<td>[d] $M_i = k_1p_0\left(\frac{\theta}{P_0}\right)^{k_6}$$left(\frac{\tau_{oct}}{P_0}\right)^{k_8}$</td>
<td>Wütczak and Uzan 1988</td>
</tr>
<tr>
<td>[e] $M_i = k_1p_0\left(\frac{\theta - 3k_{4k}}{P_0}\right)^{k_10}$$left(\frac{\tau_{oct}}{P_0} + k_{15}\right)^{k_{11}}$</td>
<td>Andrei 1999</td>
</tr>
<tr>
<td>[f] $M_i = k_1p_0\left(\frac{q}{P_0}\right)^{k_{12}}$</td>
<td>Tam and Brown 1988</td>
</tr>
<tr>
<td>[g] $M_i = k_1\left(\frac{J_2}{\tau_{oct}}\right)^{k_{15}}$</td>
<td>Johnson et al. 1986</td>
</tr>
<tr>
<td>[h] $M_i = k_1p_0^2q^{k_{17}}\sigma_{oct}^{k_{18}}$</td>
<td>Itani 1990</td>
</tr>
<tr>
<td>[i] $M_i = k_1p_0^2q^{k_{17}}\sigma_{oct}^{k_{18}}$</td>
<td>Pesso and Hudson 1994; Garg and Thompson 1997</td>
</tr>
</tbody>
</table>

**Note:** $k_n$ material parameters for each model.
Table 2.3 shows the typical values for the \(K_1\) and \(K_2\) parameters in Equation (2.20)
summarizes by Huang (1993).

**Table 2.3: typical values for k-0 model parameters (from Huang, 1993).**

<table>
<thead>
<tr>
<th>Material</th>
<th>(K_1) (psi)</th>
<th>(K_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty sand</td>
<td>1620</td>
<td>0.62</td>
</tr>
<tr>
<td>Sand-gravel</td>
<td>4480</td>
<td>0.53</td>
</tr>
<tr>
<td>Sand-aggregate</td>
<td>4350</td>
<td>0.59</td>
</tr>
<tr>
<td>Crushed stone</td>
<td>7210</td>
<td>0.45</td>
</tr>
</tbody>
</table>
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Chapter 3
DEM Simulation Results and Discussions

3-1- Implementing the Actual Grain Size Distribution in DEM Models

DEM simulations were previously performed mostly on monodisperse assemblies of particles or on assemblies of particles containing a few numbers of particle sizes or on assemblies of particles with uniform size distribution over a narrow range. In the current study a method to simulate the actual grain size distributions in DEM is introduced and DEM simulations were performed on four different gradations (i.e. coarse, baseline, fine, and linear or uniform) using PFC\textsuperscript{2D} v3.1 (Itasca, 2006)

2 set of simulations were performed in this study: 1- Biaxial Compression Test simulations.
2- Resilient Modulus Test simulations. Both of these simulations were performed on different grain size distributions to study the effect of gradation on simulations.

Generating assemblies of particles with desired grain size distributions was performed for the first time in DEM simulations. The total number of particles to generate the actual grain size distributions was unreasonably high (>5x10\textsuperscript{5}) which would lead to unaffordable simulation time as the number of particles is the key parameter controlling the DEM simulation time.

Sample size (a function of the maximum particle size) and the minimum particle size were the two parameters affecting the total number of particles in the assembly, so to have a more reasonable number of particles in the assembly and hence much faster simulations, the
particle size bins were mapped into smaller margins as it is shown in figure 3.1 to have analogous grain size distributions to desired grain size distributions.

Figure 3.1: Original and mapped particle sizes for DEM simulations.

The final grain size distributions used in the DEM simulations are shown in figures 3.2 and 3.3. The fine, baseline, and coarse distributions were selected to be generally representative of typical upper, middle, and lower bound distributions of ABC material and linear distribution is symmetric about the \( d_{50} \) of the baseline distribution and is representative of the typically narrow distributions used for DEM simulations (i.e., a uniform sand).
Figure 3.2: Four distinct grain size distribution curves for simulation

Figure 3.3: Numerical particle assemblies prior to shear
3-2- DEM Simulations

According to ASTM recommendations for triaxial shear testing, specimen widths were six times the width of the maximum particle diameter (i.e., $6 \times 1.5$ cm) and had a 2:1 height (18 cm) to width (9 cm) ratio. As it can be seen in figure 3.3 each single particle consists of two overlapped circular particles to reduce the particle rotations (Bardet, 1994).

All samples are consolidated to desired confining pressures using servo-controlled walls and low coefficient of friction for the particles prior to the actual test. After consolidation of the sample the actual coefficient of friction is applied to the particles and the sample is equilibrated using “quiet” function. In both biaxial compression test and resilient modulus test simulations, the confining pressure is applied using servo-controlled lateral walls. In simulating the biaxial compression test, the sample is sheared by applying a constant velocity to the top and bottom walls (platens). For the resilient modulus test simulation, to be able to apply desired deviatoric stresses to the sample, since the PFC 2D does not allow us to apply forces on the walls, the top and bottom walls are replaced with two chains of balls which are fixed together against rotation and differential movement (rigid platens) and particles are added to or removed from the platens to maintain a predefined gap between the platens and the confining walls. The applied force to each particle in these platens is calculated based on the number of particles in the platens and the width of the sample to produce the deviatoric stresses recommended by the AASHTO standards (figure 3.4).
3-2-1. Biaxial Compression Test Simulation

Figure 3.5 shows deviatoric stress versus global axial strain during the shearing. As it can be seen in figure 3.5 graded specimens are stiffer in small strains than the uniform (linear) specimen and they show more decrease in deviatoric stress immediately after peak stress as well. This is due to more interlocking occurring while shearing the graded specimens since they have wider range of particle sizes. Comparing plots for different grain size distributions show that coarser specimens show higher peak strengths which is due to the particle chains which are formed to transfer loads with less number of particles due to the larger particle sizes in coarser specimens. All the specimens will end up in similar values of deviatoric stresses at high global axial strains which show that steady-state deviatoric stress is more a function of particle parameters rather than grain size distribution in two-dimensional simulations.
Figure 3.6 shows volumetric strain versus global axial strain during the shearing. In well graded specimens coarser specimens show more dilation which is due to the translation of larger particles which will result in formation of larger voids to be filled with neighbor particles. As it can be seen in figure 3.6 uniform specimen is more dilatant at high axial strains which is due to the rotational and translational frustration occurring in uniform specimen.

Figure 3.5: Stress-strain response of DEM simulations
Figure 3. 6: Volume change response of DEM simulations

Figure 3.7 shows the Mohr circles corresponding to peak deviatoric stress and also critical state for all grain size distributions. It can be seen that the peak friction angle is larger for coarser specimens and the linear specimen which has particles of nearly the same size as mean particle size of the baseline grain size distribution has almost the same friction angle as the baseline grain size distribution. All specimens has similar critical state conditions and just one Mohr circle for the critical state was drawn based on the average critical state deviatoric stress for all GSDs. Comparing these graphs dilation angles of 7.8, 8.48, and 9.55 degrees are calculated for fine, baseline, and coarse GSDs.
Figure 3.8 shows mean void ratio changes with global axial strain for different specimens. In consistency with figure 3.6 it can be seen that for all specimens the mean void ratio is increasing with increase in axial strain (dilation). This increase in void ratio is more apparent for uniform specimen in high global axial strains.
Figure 3.9 shows the force chains while shearing in different specimens. As it can be seen in figure 3.9 the applied force is transferred through particle chains dominantly passing through the largest particles especially in the well-graded specimens, and a more homogenous force distribution is formed in the uniformly graded specimen. Figure 3.10 shows average contact forces versus particle size which confirms findings from figure 3.9 as relatively larger forces are endured by contacts formed by larger particles.

![Graph: Void Ratio V.S. Global Axial Strain](image)

*Figure 3.8: Void Ratio V.S. Global Axial Strain.*
Figure 3.9: Force chains as a function of global axial strain.
Figure 3.10: Average contact force as a function of strain for each of the GSD’s (note change of scale on x-axis of linear distribution)

Figure 3.11 shows particle rotations while shearing in different specimens. It can be seen in this figure that specimens which have larger fraction of fines in them will have more apparent shear bands as the fine specimen has four distinctive shear bands and the baseline specimen has at least one clear shear bands while in coarse specimen it is hard to identify any significant shear band. This is due to the larger amount of small particles which will somehow flow into the voids formed due to reorientation of larger particles and get rotationally frustrated due to large contact forces.
Figure 3.11: Particle Rotations as a function of global axial strain.
Micromechanical discrete quantities are illustrated in Figure 3.12. Assuming particles contact at a point, the load transfer between particles can be described by a contact force $f_c$. Anisotropy in microstructure can be related to statistical distributions of quantities $n_c$ and $l_c$ denoting the unit vector orthogonal to the contact tangent plane, contact normal, and a contact vector describing the line drawn from the centroids of a contacting particle and the contact point (Bathurst and Rothenburg 1990)

The distribution of contact normal orientations is (Bathurst and Rothenburg, 1990):

$$E(\theta) = \frac{1}{J} \{1 + a_c \cos(2(\theta - \theta_c))\} \quad (3.1)$$

Where $E(\theta)$ is the polar distribution of contact normal orientations and $J$ is the number of intervals used to discretize in the polar direction and $a_c$ is the invariant quantity describing
second-order anisotropy in the distribution of contact normal orientations and $\theta_c$ represent principal direction of contact normal anisotropy.

The average contact force acting at contacts with orientation $\theta$ can be decomposed into an average normal force component $f_n^c(\theta)$, and an average shear force component $f_t^c(\theta)$ (Bathurst and Rothenburg, 1990):

$$f_n^c(\theta) = f_n^0(\theta)n_i + f_t^c(\theta)t_i$$  (3.2)

Rothenburg (1980) has proposed that distributions for average contact force components in two dimensional assemblies may be represented by truncated fourier series expressions:

$$f_n^c(\theta) = f_n^0(\theta)\{1+a_n\cos(2(\theta-\theta_n))\}$$  (3.3)
$$f_t^c(\theta) = f_n^0(\theta)\{a_w - a_t\sin(2(\theta-\theta_t))\}$$  (3.4)

Where $f_n^0(\theta)$ is the average normal contact force when all groups are given equal weight and $a_n, a_t, a_w$ are non dimensional coefficients of contact force anisotropy. Similar to $\theta_c$ in (3.1), $\theta_n, \theta_t$ are the major principal directions of contact force anisotropy.

According to Fourier’s theorem any periodic function, whether natural or artificial of period $2\pi$ can be expressed by the series:

$$f(t) = \frac{a_o}{2} + a_1 \cos t + a_2 \cos 2t + a_3 \cos 3t + ... + b_1 \sin t + b_2 \sin 2t + b_3 \sin 3t + ...$$  (3.5)
Where:

\[ a_n = \frac{1}{\pi} \int_{0}^{\pi} f(t) \cos nt \, dt \]  \hspace{1cm} (3.6)

\[ b_n = \frac{1}{\pi} \int_{0}^{\pi} f(t) \sin nt \, dt \]  \hspace{1cm} (3.7)

Equations (3.1), (3.3) and (3.4) are truncated Fourier series having no more than a second order term and can be rewritten as:

\[ E(\theta) = (1/J) \{1 + [a_c \cos(2\theta_c)] \cos(2\theta) + [a_c \sin(2\theta_c)] \sin(2\theta)\} \]  \hspace{1cm} (3.8)

\[ f_n(\theta)/f_n^0(\theta) = 1 + [a_n \cos(2\theta_n)] \cos(2\theta) + [a_n \sin(2\theta_n)] \sin(2\theta) \]  \hspace{1cm} (3.9)

\[ f_t(\theta)/f_n^0(\theta) = a_w + [a_t \sin(2\theta_t)] \cos(2\theta) - [a_t \cos(2\theta_t)] \sin(2\theta) \]  \hspace{1cm} (3.10)

Comparing (3.8) with (3.5) and using (3.6) and (3.7) will result in:

\[ a_2 = \frac{1}{\pi} \int_{0}^{2\pi} E(\theta) \cos n\theta \, d\theta = a_c \cos(2\theta_c) \]  \hspace{1cm} (3.11)

\[ a_2 = \frac{1}{\pi} \int_{0}^{2\pi} E(\theta) \cos n\theta \, d\theta = a_c \cos(2\theta_c) \]  \hspace{1cm} (3.12)

Calculating the integration by discretizing the domain into J polar intervals will result in:
\[
\frac{1}{\pi} \sum_i E(\theta_i) \cos n \theta_i \cdot \frac{2\pi}{J} = a_c \cos(2\theta_c)
\]
\[
\frac{1}{\pi} \sum_i E(\theta_i) \sin n \theta_i \cdot \frac{2\pi}{J} = a_c \sin(2\theta_c)
\]

(3.13)

\[a_c \text{ and } \theta_c \text{ can now be calculated using (3.11) and numerical simulations data.}\]

Similarly by comparing equations (3.9) and (3.10) with (3.5) and using (3.6) and (3.7) \(a_n, a_t, \theta_n, \theta_t\) can be calculated by:

\[
\frac{2}{J} \sum_i f_n^c(\theta_i) \cos n \theta_i = a_n \cos(2\theta_n)
\]
\[
\frac{2}{J} \sum_i f_n^c(\theta_i) \sin n \theta_i = a_n \sin(2\theta_n)
\]

(3.14)

\[
\frac{2}{J} \sum_i f_t^c(\theta_i) \cos n \theta_i = a_t \sin(2\theta_t)
\]
\[
\frac{2}{J} \sum_i f_t^c(\theta_i) \sin n \theta_i = -a_t \cos(2\theta_t)
\]

(3.15)

Figure 3.13 shows the contact histograms for the sample with baseline GSD during the shearing. As it can be seen in this figure, we start with homogenously distributed contacts due to the homogenous consolidation of the sample prior to the shearing which results in nearly circular distribution of contacts with similar normal contact forces. As the shearing
starts the more contacts are formed in vertical direction where the platens are moving toward each other and the contact forces also increase with shearing. A decrease in the contacts forces specially shear contact forces is observed after 1% global axial strain which corresponds to the post peak response of the material and is in complete agreement with stress strain plot shown in figure 3.5.
Figure 3.13: Contacts Histograms for sample with Baseline GSD while shearing.
Similar figures to figure 3.13 can be generated for different samples. Figure 3.14 is a comparison of such figures for 4% global axial strain. As it can be seen in this figure, at this stage of the simulation the linear GSD corresponds to higher contact forces which can also be confirmed by looking at figure 3.5 where it can be seen that at 6% global axial strain the linear GSD has higher deviatoric forces and hence higher contact forces. Shear forces directions are perpendicular to the contacts directions, so figure 3.13 and 3.14 are in good agreement with the shear bands shown in figure 3.12 as shear bands in all samples are formed in maximum shear forces directions.
Figure 3.14: Contacts Histograms for different samples at 4% axial strain.
Another important microstructural parameter is the coordination number (i.e. number of contacts per particle). Figure 3.15 shows the mean coordination number variations while shearing for different samples. It was observed that the mean coordination number will decrease with shearing for all samples and reaches a steady state value at high global axial strains in the samples that reach a steady state volumetric strain in figure 3.6.

![Figure 3.15: Mean Coordination Number V.S. Global Axial Strain.](image)
Figures 3.16 to 3.18 show the particle orientation histograms during the biaxial compression test for different GSDs. As it can be seen in these figures, in all specimens the particles are reorienting toward horizontal line which is a more stable position for these particles as more contacts will be formed to endure the applied vertical force. This can be seen more clearly in figures 3.16 and 3.17 for fine and baseline GSDs as the histograms become thinner and more horizontal with global axial strain. As stated earlier the applied vertical forces are transferred through the chains of large particles, figure 3.18 shows that since the large particles which are enduring the vertical forces by forming particle chains in the coarse assembly are not able to reorient as easily as in finer assemblies, less reorientation of particles are observed for the coarse assembly during the test.

![Particle orientation histograms for different strains](image)

Figure 3.16: Particle orientation as a function of strain for fine GSD.
Figure 3.17: Particle orientation as a function of strain for baseline GSD

Figure 3.18: Particle orientation as a function of strain for coarse GSD
3-2-2. Resilient Modulus Test Simulation

In recent years, researchers have attempted to use the DEM to simulate resilient modulus tests on granular materials (Zeghal, 2001; Zeghal, 2003; Zeghal, 2007; Uthus, 2007; Uthus et al., 2008). These efforts showed the capability of the DEM to simulate resilient modulus tests. Zeghal (2001) studied the effect of particle shape on resilient modulus values by doing parallel simulations of resilient modulus test using individual particles and triangular clumps of three particles and showed that the resilient modulus value from the simulation using clumps is significantly higher. This can be due to less rotation of clumped particles and also greater interlocking of particles in this assembly which results in a stiffer response of the assembly. Zeghal (2003) studied the effect of sample density on resilient modulus of granular soils and showed that denser samples have higher resilient modulus values.

Zeghal (2007) studied the effect of stress state on the resilient behavior of granular materials and observed that resilient modulus increases with increase in confining stress while deviatoric stress only has an effect at low confining pressures. Uthus et al. (2008) studied the effects of membrane flexibility, interparticle friction angle, confining stress, and the interparticle contact law (i.e. linear vs. Hertzian) on the resilient modulus of granular materials. Uthus et al. (2008) showed that although the membrane flexibility does not have a significant effect on the resilient modulus values at low deviatoric stresses, but using flexible membrane will result in values closer to the experimental values and it will also help the stability of the assembly at higher deviatoric stress. Uthus et al. (2008) showed that an increase in interparticle friction angle, will result in an increase in the resilient modulus and
also observed that an increase in confining pressure increases the resilient modulus, in agreement with Zeghal’s (2007) findings. Furthermore, Uthus et al. (2008) observed that although resilient modulus is more dependent on confining stress, it is also decreasing with an increase in deviatoric stress. Comparing the DEM results to the representative laboratory tests, Uthus et al. (2008) showed that although the linear contact model shows results that are logical comparing to the previous findings, simulations using the Hertzian contact model have a better agreement with their laboratory results.

In the current study, to study the effect of different gradations on the resilient modulus test simulations, the resilient modulus test DEM simulations are performed on similar coarse, baseline, fine, and linear gradations used for biaxial compression test simulations (figures 3.1, 3.2) and macro and micro scale analysis are performed on the simulation results. In order to prevent the overestimation of particle rotations often associated with simulation of spherical particles (Bardet, 1994; Evans, 2005; Frost and Evans, 2007), individual particles were generated as an agglomeration of two identical spheres with a 50% overlap. Thus, particle aspect ratios were 1.5:1 and sieve diameters were equivalent to the diameter of the individual spheres used to generate the agglomerates.

The resilient modulus test was simulated according to AASHTO T292-91 and T307-99. Both standards follow nearly the same procedures until conditioning of the sample and the difference between the two methods comes from the sequence of load application where in AASHTO T292-91 loading starts with higher confining pressures and at each confining pressure deviatoric stress is increased at each loading sequence while in AASHTO T307-99
sample is loaded with lower confining pressures first, and at each confinement the deviatoric stress is increased at different loading sequences.

Brown (1975) reported that by keeping the applied stresses low enough to avoid significant plastic deformations in materials, the effect of the stress history on resilient behavior of materials can be neglected and hence several resilient tests can be performed on a single soil sample. As long as the test specimen is in a stress state close to the elastic mobilized strength line in the elastoplastic zone (figure 3.19), it will not undergo significant plastic deformations and much less reorientations of particles will occur and hence the specimen response will not be affected much by its stress history. On the other hand when the sample is loaded to a stress state far from the elastic mobilized strength line it will undergo more significant plastic strain as the stress state gets closer to the failure envelope.

![Elastic and Plastic Stress State Limits](image)

**Figure 3.19: Elastic and plastic stress state limits.**

Depending on the applied stresses and also the shear strength of the specimens (Figure 3.20), three different cases were observed in resilient modulus test simulations Figure 3.21 shows stress-strain responses corresponding to different simulations showing elastic response,
elastoplastic response, and failure. It can be seen in figure 3.21 that in Case I the sample responds elastically and all the loading cycles fall on the same path. For Case II, the sample undergoes some plastic deformation but after several repetitions of deviatoric load the reorientation of particles and contact chains will lead to a stronger assembly and hence lower plastic deformation to a point that the cycles fall on a single path. Figure 3.21(c) shows that under some load combinations the sample undergoes significant plastic deformations which lead to failure of the sample.

Figure 3.22 shows the time histories corresponding to one load sequence. It can be seen in this figure that the sample is in elastic phase and no significant plastic deformation is observed. Figure 3.23 shows that the recoverable strain does not change significantly with time in this case, resulting in minimal changes in resilient modulus values.

Figures 3.24 and 3.25 show the results of a resilient modulus test simulation located in the elastoplastic region and as it can be seen in this figure, the specimen will undergo plastic deformation as well as elastic deformation and the particles will reorient to form a stronger assembly, but the plastic deformation is small enough so that the sample does not fail and the calculated resilient modulus will converge to a single value after some cycles of applying deviatoric stress and shakedown of the specimen (Garcia-Rojo and Hermann, 2005).

For very high deviatoric loads, the specimen is loaded to failure and deformations do not converge to a constant value (i.e., no shakedown with cycling), as shown in figure 3.26. Lack of convergence of deformation with time results in a constantly changing resilient modulus (Figure 3.27).
Figure 3.20: Mohr Circles for different GSDs.
Figure 3.21: (a) Case I – Elastic; (b) Case II – Elastoplastic; (c) Case III – Failure stress-strain responses for different simulations (note different axis scales to highlight material response).
Figure 3.22: Resilient Modulus Test, (a) deviatoric stress and (b),(c) strain time histories (Elastic Response).
Figure 3.23: Resilient modulus time history (Elastic Response).
Figure 3. 24: Resilient Modulus Test, (a)deviatoric stress and (b),(c)strain time histories (Elastoplastic Response).
Figure 3.25: Resilient modulus time history (Elastoplastic Response).

Figure 3.26: Resilient Modulus Test, Strain time histories (Plastic Response).

Figure 3.27: Resilient modulus time history (Plastic Response).
Figure 3.28 shows the results of resilient modulus simulations performed on different grain size distributions. In this figure the resilient modulus is plotted versus the bulk stress ($\theta = \sigma_1 + 2\sigma_3 = \sigma_d + 3\sigma_3$). This figure shows that coarser grain size distributions have higher resilient modulus values and the resilient modulus will increase with increasing mean stress which is in perfect agreement with the laboratory research performed by Barksdale and Itani (1989). In AASHTO T307-99 the assembly is loaded from lower confining stresses and, hence, higher over consolidation ratios relative to AASHTO T292-91 and also a stiffer response is expected. Thus, the slopes of the lines in Figure 29 are lower for AASHTO T307-99 simulations. Resilient modulus values corresponding to simulations performed according to AASHTO T307-99 tend to have similar slopes for different GSDs and the main difference is the initial resilient modulus for each GSD.

![Resilient modulus simulation results](image-url)

Figure 3. 28: Resilient modulus simulation results.
Table 3.1 and figures 3.29 and 3.30 show the k-θ model parameters fitted to DEM simulation results for different gradations. It can be seen in these figures that baseline GSD will have the highest $K_1$ values and the lowest $K_2$ values. It can also be seen in these figures that AASHTO T307-99 will have bigger $K_1$ values and smaller $K_2$ values comparing to AASHTO T292-91 for all gradations.

**Table 3.1: k-θ model fitting parameters for different gradations.**

<table>
<thead>
<tr>
<th>Grain Size Distribution</th>
<th>AASHTO Standard</th>
<th>$K_1$</th>
<th>$K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine</td>
<td>T 307-99</td>
<td>1750</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>T 292-91</td>
<td>994</td>
<td>0.301</td>
</tr>
<tr>
<td>Baseline</td>
<td>T 307-99</td>
<td>3692</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td>T 292-91</td>
<td>2927</td>
<td>0.136</td>
</tr>
<tr>
<td>Coarse</td>
<td>T 307-99</td>
<td>2438</td>
<td>0.229</td>
</tr>
<tr>
<td></td>
<td>T 292-91</td>
<td>562</td>
<td>0.455</td>
</tr>
</tbody>
</table>

**Figure 3.29: $K_1$ Variations with gradation.**
Figure 3.30: $K_2$ Variations with gradation.

In figure 3.28 the effect of mean stresses on the resilient modulus was presented. To have a better understanding of the effect of each stress component on the resilient modulus test results, some additional simulations were performed to obtain figures 3.31 and 3.32. In these figures the effects of confining pressure and deviatoric stress on resilient modulus test results were presented, respectively. As it can be seen in figure 3.31 the resilient modulus values tend to increase significantly with increase in confinement. This is due to the stiffer response of the specimen in higher confinements which leads to smaller recoverable strains and hence larger resilient modulus values. As it is shown in figure 3.31, the deviatoric stress, on the other hand does not have a significant effect on the resilient modulus test results. With increase in the deviatoric stress the resilient modulus value tends to increase at small deviatoric stresses and then decrease at higher deviatoric stresses. Comparing figures 3.31 and 3.32 it can be seen that confining pressure has more significant effect on the resilient
modulus test results than deviatoric stress. This can be seen better in comparing the linear fits to both set of data.

**Figure 3.31:** The effect of confining pressure on resilient modulus.

**Figure 3.32:** The effect of deviatoric stress on resilient modulus.
Figure 3.28 shows that especially for simulation using AASHTO T307-99, resilient modulus is increasing with increase in mean stress with similar rate, for all grain size distributions. So knowing one point (e.g. the initial points) on each line corresponding to each grain size distribution will enable us to have a good prediction of the resilient behavior of the material. Figure 3.33 shows the variation of the initial resilient modulus with the average coordination number. This figure does not provide a confident response of the material and more data points are required to validate the observed trend.

**Figure 3.33: Initial resilient modulus versus average coordination number.**

Figure 3.34 which shows the variation of the initial resilient modulus with friction angle provide us with a more confident trend than figure 3.34 as all data points are lied around the same line and shows that larger resilient modulus is observed in specimens with higher friction angle. Using the linear fit equation shown in figure 3.34, for a given soil sample with
known friction angle, we can have a good starting estimate for the resilient modulus of the sample using lines parallel to linear fittings provided in figure 3.28 for different GSDs (AASHTO T307-99).

![Graph showing initial resilient modulus versus friction angle](image)

**Figure 3.34: Initial resilient modulus versus sample friction angle.**

Figure 3.35 shows the variation of the fraction of shear contacts with cycling for loading and rest cycles. No significant changes in fraction of shear contacts were observed during the simulation and it can be seen that due to higher deviatoric stresses applied during load cycles comparing to rest cycles, significantly more shear contacts (i.e. more than 8 times) exist in the loaded sample for all grain size distributions.
Figure 3.36 shows the variation of the average coordination number with cycling for loading and rest cycles. A relatively small decrease in average coordination number toward a steady state value is observed in all samples. It can be seen that in all samples the coordination number is higher during loading cycles due to larger vertical stress applied causing the assembly to become denser and more contacts to be formed. In general lower coordination number is observed for coarser material except for the fine grain size distribution which has lower coordination number comparing to the baseline grain size distribution. This is due to the large number of small particles floating in the voids between larger particles in fine grain size distribution.

![Graph showing Sfracti on during load and rest cycles.](image)

**Figure 3.35: Sfracti on during load and rest cycles.**
Since the applied stresses in the resilient modulus test is further less than the shear strength of the sample, the contacts forces histograms will not undergo any significant changes during the test and similar histograms for each load combination was observed throughout the test. Figure 3.37 shows the contacts forces histograms at different steps of one load combination applied to the baseline grain size distribution. It can be seen in this figure that this histograms are not sensitive to cycling and they will remain similar during the test. During the rest cycles since the sample is homogenously loaded in both directions, the contacts are homogenously orientated and the shear forces are low while during the loading cycles when the deviatoric stress is applied, the contacts are reoriented toward the direction of application of deviatoric stresses and more significant shear forces are transferred in contacts.
Figure 3.38 shows the contacts forces histograms at different load combinations for baseline GSD. It can be seen that the histograms vary depending on the applied stress state. Looking at this figure together with figure 3.39 it can be seen that load combinations causing higher mobilized friction angle have larger histograms, especially larger shear forces.

Figure 3.40 shows the particle rotations at different stages of the resilient modulus test on baseline simulation. Since the specimen does not undergo significant deformations during the test so very little particle orientations and hence little ability of particles to rotate occurs and very minimal changes in particles rotations is observed. If we look deeper into the specimen as it is shown in figure 3.40 it can be seen that most of the particle reorientations is occurring around the smaller particles which are more freely rotating in the voids between the larger particles which are enduring the applied vertical forces by forming particle chains by contacts with high forces.
Figure 3.37: Contacts forces histograms at different steps (baseline GSD).
Conditioning ($\sigma_3 = 103.4$ kPa, $\sigma_d = 93.1$ kPa)

Rest

Load

Loading Phase 1 ($\sigma_3 = 20.7$ kPa, $\sigma_d = 18.6$ kPa)

Rest

Load

Loading Phase 2 ($\sigma_3 = 20.7$ kPa, $\sigma_d = 37.3$ kPa)

Rest

Load

Figure 3.38: Contacts forces histograms at different loading phases (baseline GSD).
Figure 3.39: Mobilized friction angle at different loading combinations.
Figure 3.40: Particle rotations for baseline GSD.
In order to be able to use the resilient modulus model parameters estimated from the results of the DEM simulations we need to calibrate them with some lab experiments results. Allen (1973) in a study at University of Illinois at Urbana-Champagn performed a series of resilient modulus tests on different granular materials with different densities and estimated the model parameters for different cases. Allen (1973)’s test specimen properties are summarized in table 3.2. The grain size distributions of the specimens are shown in figure 3.41.

Table 3.2: Resilient Modulus Test Specimens (from Allen, 1973)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Material</th>
<th>Density, lb/ft$^3$</th>
<th>% Moisture</th>
<th>% Saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD-1</td>
<td>Crushed Stone</td>
<td>138.0 High</td>
<td>5.7</td>
<td>78</td>
</tr>
<tr>
<td>MD-1</td>
<td>Crushed Stone</td>
<td>134.0 Intermediate</td>
<td>6.3</td>
<td>73</td>
</tr>
<tr>
<td>LD-1</td>
<td>Crushed Stone</td>
<td>130.0 Low</td>
<td>7.0</td>
<td>70</td>
</tr>
<tr>
<td>HD-2</td>
<td>Gravel</td>
<td>139.4 High</td>
<td>6.3</td>
<td>82</td>
</tr>
<tr>
<td>MD-2</td>
<td>Gravel</td>
<td>134.0 Intermediate</td>
<td>6.5</td>
<td>74</td>
</tr>
<tr>
<td>LD-2</td>
<td>Gravel</td>
<td>131.0 Low</td>
<td>6.7</td>
<td>69</td>
</tr>
<tr>
<td>HD-3</td>
<td>Blend</td>
<td>139.5 High</td>
<td>6.3</td>
<td>88</td>
</tr>
<tr>
<td>MD-3</td>
<td>Blend</td>
<td>134.5 Intermediate</td>
<td>6.8</td>
<td>78</td>
</tr>
<tr>
<td>LD-3</td>
<td>Blend</td>
<td>131.0 Low</td>
<td>7.2</td>
<td>74</td>
</tr>
</tbody>
</table>

Taciroglu (1998) fitted different material models to Allen (1983) test results and summarized his findings in a tabular format (Table 3.3).

In order to have the best calibration of the DEM simulation results, a similar process using the same fitting functions and procedures to what was applied to the DEM results were performed on Allen (1973)’s test results and the k-$\theta$ model parameters were estimated for different specimens. Figures 3.42 to 3.44 illustrate the curve fitting process.
Figure 3.41: Grain Size Distribution of Resilient Modulus Test Specimens (from Allen, 1973)
Table 3.3: Material constants for various models (from Taciroglu, 1998)

<table>
<thead>
<tr>
<th>Materials</th>
<th>Linear Elasticity</th>
<th>Linear–0 Model</th>
<th>Uzan–Witzeak Model</th>
<th>Coupled Hyperelastic Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E$ (ksi)</td>
<td>$v$</td>
<td>$k$ (ksi)</td>
<td>$r$</td>
</tr>
<tr>
<td>HD1</td>
<td>34.9</td>
<td>0.34</td>
<td>14.2</td>
<td>0.33</td>
</tr>
<tr>
<td>HD2</td>
<td>28.2</td>
<td>0.38</td>
<td>22.2</td>
<td>0.43</td>
</tr>
<tr>
<td>HD3</td>
<td>44.4</td>
<td>0.40</td>
<td>6.3</td>
<td>0.41</td>
</tr>
<tr>
<td>LD1</td>
<td>31.0</td>
<td>0.36</td>
<td>3.7</td>
<td>0.36</td>
</tr>
<tr>
<td>LD2</td>
<td>26.0</td>
<td>0.36</td>
<td>6.0</td>
<td>0.36</td>
</tr>
<tr>
<td>LD3</td>
<td>27.5</td>
<td>0.36</td>
<td>8.3</td>
<td>0.36</td>
</tr>
<tr>
<td>MT1</td>
<td>33.9</td>
<td>0.38</td>
<td>3.8</td>
<td>0.39</td>
</tr>
<tr>
<td>MT2</td>
<td>28.8</td>
<td>0.40</td>
<td>13.7</td>
<td>0.29</td>
</tr>
<tr>
<td>MT3</td>
<td>29.8</td>
<td>0.40</td>
<td>3.6</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Figure 3.42: k-θ Model Curve fittings for LD Specimens.
Figure 3.43: k-θ Model Curve fittings for MD Specimens.

Figure 3.44 k-θ Model Curve fittings for HD Specimens.
The $k$ and $\theta$ parameters for different specimens are summarized in table 3.4. This model parameters are later used in chapter 4 in FEM models.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$K$ [Pa]</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD1</td>
<td>81670</td>
<td>0.663</td>
</tr>
<tr>
<td>LD2</td>
<td>54820</td>
<td>0.686</td>
</tr>
<tr>
<td>LD3</td>
<td>1148414</td>
<td>0.428</td>
</tr>
<tr>
<td>MD1</td>
<td>60284</td>
<td>0.691</td>
</tr>
<tr>
<td>MD2</td>
<td>3617013</td>
<td>0.335</td>
</tr>
<tr>
<td>MD3</td>
<td>4187548</td>
<td>0.329</td>
</tr>
<tr>
<td>HD1</td>
<td>2358409</td>
<td>0.388</td>
</tr>
<tr>
<td>HD2</td>
<td>6267609</td>
<td>0.292</td>
</tr>
<tr>
<td>HD3</td>
<td>1447221</td>
<td>0.451</td>
</tr>
</tbody>
</table>

Comparing the Allen (1973) model parameters with the DEM simulation results shows that the HD2 specimen has a similar response to the DEM simulation results. Comparing the grain size distributions of Allen (1973)’s specimens with the grain size distributions of DEM specimens shows that HD2 specimen (also other Allen (1973)’s specimens) has similar gradation to the DEM baseline grain size distribution (figure 3.45) so a calibration factor of 4.5 is used which makes the DEM baseline $k$-$\theta$ model to fall on the material model fitted to HD2 specimen test results. Figures 3.46 and 3.47 compare the $k$-$\theta$ model fitted to the HD2
specimen test results and the baseline DEM simulation results before and after calibration, respectively.

Figure 3.45: GSDs for Allen (1973) specimens and the DEM specimens.

Figure 3.46: k-0 model fitted to HD2 specimen and the DEM specimens.
Figure 3.47: Calibrating the DEM k-θ models.
REFERENCES


Chapter 4

FEM Simulations Results and Discussions

4.1- Introduction

In this chapter ABAQUS 3D FEM models are used to evaluate the effects of different parameters such as material nonlinearity, tire imprint, landing gear configuration, presence of shear loads, aircraft size, and etc. on airfield flexible pavement response as well as the design outcome. Model response (strains) are used to estimate the allowable number of load applications on the pavement before major distresses such as fatigue cracking and rutting occur. Stress dependent ABC material model (k-θ model) is implemented in the model using UMAT subroutines provided by ABAQUS software. Besides, the 3 dimensionality of the model enables us to model any desired landing gear configuration and different tire imprints. Different validation simulations were performed to ensure the accuracy of the ABAQUS 3D FEM model responses including confirming the ABAQUS simulation results with simple axisymmetric semi-infinite problems where analytical solutions are available, layered elastic problems (Huang, 2003b), existing nonlinear models (Sivaneswaran, et. Al; 1999), and field or large scale studies (NAPTF).
4-2- User Defined Materials in ABAQUS (UMAT)

Main reason for choosing ABAQUS software over other existing FEM softwares in this study is the ability of ABAQUS to model stress or strain dependent materials by means of the UMAT subroutines implemented in the software which makes it possible to define any constitutive model of arbitrary complexity. Although it should be mentioned that the 3D capabilities, user friendly graphical interface, ease of use, powerful data processing tools, batch processing options, and availability of the software at Civil and Environmental Engineering Department at North Carolina State University were also some of the advantages of ABAQUS over other existing FEM softwares.

UMAT subroutines can be used to define the mechanical constitutive behavior of a material. They will be called at all material calculation points of elements at all iterations. UMAT will get the stresses, strains, and state variables at the beginning of each time increment and uses this information and the material constitutive models to update the stress and state variables to the values at the end of the time increment and provides the material Jacobian stiffness which is defined as follows:

$$ C = \frac{\partial \Delta \sigma}{\partial \Delta \varepsilon} \quad (4.1) $$

Where $\Delta \sigma$ is the increment in (Cauchy) stress and $\Delta \varepsilon$ is the increment in strain. It should be noted that this matrix can be nonsymmetric as a result of constitutive equation or the integration process. Jacobian is easily calculated for forward integration methods and if large
deformations/ volume changes are considered, the exact form of the consistent Jacobian should be used for faster convergence:

\[ C = \frac{1}{J} \frac{\partial \Delta(J\sigma)}{\partial \Delta \varepsilon} \]  \hspace{1cm} (4.2)

Where \( J \) is the determinant of the deformation gradient.

Figure 4.1 shows the flow diagram of how UMAT subroutines are linked to the ABAQUS analysis and figure 4.2 shows the steps performed in a UMAT subroutine.
Figure 4.1: Flow Diagram of Nonlinear ABAQUS Analysis (from Kim, 2005)
Figure 4.2: Flow Diagram of UMAT Subroutine in ABAQUS (from Kim, 2005)
4-3- Mathematical Formulation Used in UMAT

In the displacement based finite element analysis where a profile of the displacement is assumed to determine the final displacement field by satisfying the governing partial differential equations (Liu and Quek, 2003). After estimating the new state of displacement, strain in each element can be evaluated. Constitutive equations are then used to find the stress state that corresponds with the evaluated strain state.

The generalized Hooke’s law can be represented by:

\[
S = \frac{E}{1+v}(\alpha \varepsilon I + E)
\]  \hspace{1cm} (4.3)

Where:

\( E \) is the material Elastic Modulus.

\( v \) is the Poisson’s ratio

\( \alpha \) is a parameter that depends on the Poisson’s ratio:

\[
\alpha = \frac{v}{1-2v}
\]  \hspace{1cm} (4.4)

\( S \) is the stress tensor:

\[
S = \begin{bmatrix}
\sigma_{11} & \tau_{12} & \tau_{13} \\
\tau_{12} & \sigma_{22} & \tau_{23} \\
\tau_{13} & \tau_{23} & \sigma_{33}
\end{bmatrix}
\]  \hspace{1cm} (4.5)

\( E \) is the strain tensor:

\[
E = \begin{bmatrix}
\varepsilon_{11} & \gamma_{12} & \gamma_{13} \\
\gamma_{12} & \varepsilon_{22} & \gamma_{23} \\
\gamma_{13} & \gamma_{23} & \varepsilon_{33}
\end{bmatrix}
\]  \hspace{1cm} (4.6)

\( \varepsilon \) is the invariant of the strain tensor (change in volume for small strains) which is equal to the trace of the strain tensor:

\[
\varepsilon = \text{tr}(E) = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}
\]  \hspace{1cm} (4.7)
I is the identity tensor.

Considering the resilient behavior of the base materials used in pavement construction, the Elastic modulus in equation (4.3) can be replaced with the resilient modulus which can be a function of stresses:

\[ S = \frac{M_r(S)}{1+\nu} (\alpha e I + E) \]  

(4.8)

Letting \( C(S) = \frac{M_r(S)}{1+\nu} \) equation (4.8) can be written as:

\[ S = C(S)(\alpha e I + E) \]  

(4.9)

It should be noted that although both Poisson’s ratio and modulus are stress dependent, but compared to the modulus, Poisson’s ratio is not significantly stress dependent and hence is assumed to be constant (Kim, 2005). Referring to the above mentioned procedure for the displacement based finite element analysis, the stress state is determined by finding the roots of the following equation:

\[ g(S) = S - C(S)(\alpha e I + E) = 0 \]  

(4.10)

In order to evaluating the stress state from equation (4.10) it is better to express resilient modulus (i.e. \( C(S) \)) as a function of strain rather than stress. This can be done by decomposing equation (4.10) into bulk and deviatoric parts by letting \( \bar{g} = g - \frac{1}{3} (tr(g)) I \) and observing both \( tr(g) \) and \( \bar{g} \) equal to zero:

\[ \bar{g} = g - \frac{1}{3} (tr(g)) I = 0 \]  

(4.11)

Substituting equation (4.10) into equation (4.11) will result in:

\[ S - C(S)(\alpha e I + E) - \frac{1}{3} tr[S - C(S)(\alpha e I + E)] I = 0 \]  

(4.11)
Which can be simplified to:

\[ S - C(S)\alpha \epsilon l - C(S)E - \frac{1}{3} \text{tr}(S)l + C(S)\alpha \epsilon l + \frac{1}{3} C(S)\epsilon l = 0 \]  
(4.12)

Letting \( \sigma = \frac{1}{3} \text{tr}(S) \) and simplifying more, equation (4.12) can be rewritten as:

\[ S - C(S)E - \sigma l + \frac{1}{3} C(S)\epsilon l = 0 \]  
(4.13)

Letting \( \bar{S} = S - \sigma l \) and \( C(S)E - \frac{1}{3} C(S)\epsilon l = C(S) \left[ E - \frac{1}{3} C(S)\epsilon l \right] = C(S)\bar{E} \) will result in:

\[ \bar{g} = \bar{S} - C(S)\bar{E} \quad \text{or} \quad \bar{S} = C(S)\bar{E} \]  
(4.14)

On the other hand observing \( \text{tr}(g) \) equal to zero will result in:

\[ \text{tr}[S - C(S)(\alpha \epsilon l + E)] = 0 \]  
(4.15)

Hence:

\[ \text{tr}(S) - C(S)(3\alpha \epsilon + \epsilon) = 0 \]  
(4.16)

Hence:

\[ 3\sigma - 3C(S)\epsilon \left( \alpha + \frac{1}{3} \right) = 0 \]  
(4.17)

Hence:

\[ \sigma = C(S) \left( \alpha + \frac{1}{3} \right) \epsilon = 0 \]  
(4.18)

Knowing \( \theta = |\sigma| \) and letting \( q = |\epsilon| \) and \( \bar{a} = \alpha + \frac{1}{3} \) will simplify equation (4.18) to:

\[ \theta = \bar{a}C(S)q \]  
(4.19)

Replacing the \( M_r = K\theta^n \) model \ (Hicks and Monismith, 1971) into equation 4.19 will give:

\[ \theta = \bar{a} \frac{K\theta^n}{\nu} q \]  
(4.20)

Letting \( \kappa = \frac{K}{1+\nu} \) and solving for \( \theta \) results in:
\[ \theta = (\kappa \bar{\alpha} q)^{\frac{1}{1-n}} \]  

(4.21)

Substituting equation 4.21 into \( C(S) \) will transfer it into \( \hat{C}(\varrho) \) as follows:

\[ \hat{C}(\varrho) = \kappa \left[ (\kappa \bar{\alpha} q)^{\frac{1}{1-n}} \right]^n \]

(4.22)

The constitutive equation (4.9) now can be rewritten in terms of strains as follows:

\[ S = \hat{C}(\varrho)(\alpha \varepsilon + E) \]  

(4.23)

With equation (4.23) the strain based constitutive equations in the strain based finite element analysis in order to find the stiffness matrix can be solved. Two different methods will be discussed, Tangent stiffness and secant stiffness which will be used in the simulations based on the problem and the convergence of the model.

**4-3-1- Tangent Stiffness**

Tangent Stiffness matrix is calculated by:

\[ K_T = \int B^T CB \, dv \]  

(4.24)

Where \( C \) is the material tangent stiffness matrix which can be computed directly by differentiating the matrix \( S \) in equation (4.23) as follows:

\[ C = \frac{\partial \Delta \sigma}{\partial \Delta \varepsilon} = \frac{\partial S}{\partial E} = \hat{C}(\varrho)(\mathbf{1} + \alpha \varepsilon \otimes \varepsilon) + (\alpha \varepsilon + E) \otimes \nabla_E \hat{C}(\varrho) \]  

(4.25)

Where \( \nabla_E \hat{C}(\varrho) \) can be calculated as:

\[ \nabla_E \hat{C}(\varrho) = \frac{\partial \hat{C}(\varrho)}{\partial \varrho} \frac{\partial \varrho}{\partial E} \]  

(4.26)
Knowing:
\[
\frac{\partial q}{\partial E} = \frac{\partial q}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial E} = s\text{gn}(\varepsilon)I \quad (4.27)
\]

And:
\[
\frac{\partial \hat{c}(q)}{\partial \varrho} = \left( \kappa (\kappa \alpha \beta)^{\frac{n}{1-n}} \left( \frac{n}{1-n} \right)^{\frac{n}{1-n}-1} \right) \hat{\alpha} (\varrho)^{\frac{n}{1-n}-1} \quad (4.28)
\]

Now letting \( \beta = \left( \kappa (\kappa \alpha \beta)^{\frac{n}{1-n}} \left( \frac{n}{1-n} \right)^{\frac{n}{1-n}-1} \right) \) and noting \( s\text{gn}(\varepsilon) = \frac{\varepsilon}{|\varepsilon|} = \frac{\varepsilon}{\varrho} \) will result in:
\[
C = \frac{\partial s}{\partial E} = \hat{c}(\varrho) (1 + \alpha l \otimes l) + \left( \alpha \beta \varepsilon l + E \beta \frac{\varepsilon}{\varrho} \right) \otimes l \quad (4.29)
\]

Similarly the material stiffness matrix for the \( M_r = K_1 \theta^n \tau^m \) model (Witczak and Uzan, 1988) is calculated by:
\[
C = \hat{c}(\varrho) \left[ 1 + (\mu n \bar{\alpha} + \alpha) l \otimes l + \frac{\mu m}{3} N \otimes N + \frac{\mu \alpha m}{3 \gamma} I \otimes N + \frac{\mu n}{\varepsilon} N \otimes I \right] \quad (4.30)
\]

4-3-2- Secant Stiffness

Due to the stress dependent nature of k-\( \theta \) model and hardening of resilient modulus of unbound granular materials while stress increases, tangent stiffness matrix cannot be used for this material model (Tutumluer, 1995) as convergence is not easily achieved. An alternate method is to use secant stiffness matrix which uses the slope of the secant line connecting the origin to each load displacement state instead of the slope of the load displacement curve at that state (Figure 4.1):
\[
K_S = \int B^T DB \, dv \quad (4.31)
\]

Where \( D \) is the material secant stiffness matrix which is defined as:
\[
D = \hat{c}(1 + \alpha l \otimes l) \quad (4.32)
\]
In order to improve the convergence properties of the secant method, some researchers (Tutumluer, 1995; Brown and Pappin, 1981) proposed a damped secant method ($\tilde{C}$) in which a secant modulus is estimated from the current and the previous increment:

$$\tilde{C} = \beta \hat{C}(U^i) + (1 - \beta) \hat{C}(U^{i-1})$$

(4.33)

Where the damping factor $\beta$ is a number between 0 and 1 and Tutumluer (1995) observed that $\beta = 0.8$ shows good convergence.
Figure 4.3: Direct Secant Stiffness method (from Tutumluer, 1995)
4-4- FEM Simulations

4-4-1- Model Validations

In this section a series of ABAQUS models are developed and verified with some previously confirmed solutions. This is to confirm the accuracy of the ABAQUS simulations whose results will be used in the process of pavement design in the next sections. Furthermore, it should be noted that after validating the accuracy of ABAQUS models, they can be used in performing different parametric studies. These model validations include comparison of Semi-Infinite 3D elastic ABAQUS model with axissymetric Analytical solution, comparison of 3D layered elastic model with layered elastic KENPAVE model (Huang, 2003b), comparison of 3D nonlinear model with layered nonlinear EVERSTRESS model (Sivaneswaran, et. Al; 1999), and comparison of 3D nonlinear models with Existing field data (NAPTM).

4-4-1-1. Comparison of Semi-Infinite 3D Elastic Model with Axissymetric Analytical Method

As the first step in validating ABAQUS models a homogenous 3D elastic model with the same material properties throughout the media, representing a semi-infinite elastic problem, is selected. A circular load is applied to the model acting in the Z direction and the stress and displacement responses due to this circular load are compared with the ones calculated by Boussinesq (1885) method. Equations (4.34) and (4.35) were proposed by Boussinesq (1885) to calculate the vertical stress and vertical displacement distributions with depth (z) in a semi-infinite elastic media due to a circular load (q) under the load center.
\[ \sigma_z = q \left[ 1 - \left( \frac{1}{a^2 + \left( \frac{a}{z} \right)^2} \right)^2 \right] \]  

(4.34)

\[ w_z = \frac{q}{E} \left[ 2(1 - \nu^2)(a^2 + z^2)^{\frac{1}{2}} - \frac{(1+\nu)z^2}{(a^2+2z^2)^{\frac{1}{2}}} + (\nu + 2\nu^2 - 1)z \right] \]  

(4.35)

Where \( a \) is the radius of the circularly loaded area.

All ABAQUS models used in this study are consisting of an initial step where the gravity is applied to the model and the traffic loadings are applied after this initial step. In order to avoid shear locking in the model (Sun, 2006) 20-node quadratic brick elements (C3D20R) were used. Figure 4.4 shows the ABAQUS model used for semi-infinite elastic analysis. The dimensions of the model are \( x=16 \text{m} \), \( y=16 \text{m} \), and \( z=9 \text{m} \). The elastic modulus and Poisson’s ratio used in this model are 517 MPa (75ksi) and 0.35, respectively. A 300 kPa (45 psi) load is applied on a circular surface with radius of 25 cm (9.85in).

Figure 4.5 and 4.6 are showing contours of vertical stress and vertical displacement, respectively. Note that the contours are plotted on a half cut section in order to show the stress and displacement distributions under the center of the applied circular load. It should also be noted that these contour plots show the effect of both gravity and the applied load together.

Figure 4.7 illustrates the vertical stress and displacement distributions under the center of the loaded area, respectively. In case of vertical stress distribution, as illustrated in figure 4.7, the ABAQUS model shows good match with the Boussinesq (1885) equation. The vertical displacement distribution with depth from the ABAQUS model shows good agreement with that of the Boussinesq (1885) estimation. The two graphs will have a relatively small
difference at higher depths which is due to the fact that the ABAQUS model has a limited depth and is fixed against vertical movement at the bottom.

Figure 4.4: ABAQUS Model Used for Semi Infinite Elastic Analysis
Figure 4.5: Vertical stress distribution contour plot.

Figure 4.6: Vertical displacement distribution contour plot.
Figure 4. 7: Vertical stress/displacement distribution with depth, ABAQUS model v.s. Boussinesq.
4-4-1-2. Comparison of 3D Layered Elastic Model with Layered Elastic KENPAVE Model

The model used for this part of the study is shown in figure 4.8. As it is shown in figure 4.8, the model consists of a layered system with three layers: a 15.2 cm asphalt concrete (AC) layer with elastic modulus of 1380 MPa and the Poisson’s ratio of 0.35, resting on top of a 71.1 cm base course layer with elastic modulus of 240 MPa and the Poisson’s ratio of 0.33, on top of a subgrade with elastic modulus of 50 MPa and Poisson’s ratio of 0.45. A 500 kPa load is applied to the model on a 25 cm radius circle.

![Figure 4.8: The Pavement Model used for comparison between ABAQUS and KENPAVE models.](image)
Figure 4.9 shows the ABAQUS model used in this section. The same model parameters (i.e. layer thicknesses, material properties, and load) as the KENPAVE model are used to obtain comparable results. As it can be seen in figure 4.9, finer mesh distribution is used in the center core of the model where the load is applied, to have more accurate results.

Figures 4.10 to 4.12 show the vertical strain, radial strain, and vertical stress distributions with depth, respectively. As it can be seen in these figures, ABAQUS and KENPAVE models used are in good agreement. In order to be able to confidently use ABAQUS models in the pavement design process in the next sections, the two important parameters in pavement design, radial tensile strain distribution at the bottom of the AC layer ($\varepsilon_t$) and the vertical compressive strain distribution at top of subgrade ($\varepsilon_c$), estimated from the two models were also compared (Figures 4.13 to 4.16).
Figure 4.9: ABAQUS Model used for comparison with KENPAVE.
Figure 4.10: Vertical Strain Distribution with Depth (ABAQUS v.s. KENPAVE).
Figure 4.11: Radial Strain Distribution with Depth (ABAQUS v.s. KENPAVE).
Figure 4.12: Vertical Stress Distribution with Depth (ABAQUS v.s. KENPAVE).

Figures 4.13 and 4.14 illustrate vertical stress and vertical strain distributions at top of subgrade, respectively. As it can be seen in these figures, both ABAQUS and KENPAVE models are in a good agreement.
Figure 4.13: Vertical Stress Distribution at Top of subgrade (ABAQUS v.s. KENPAVE).

Figure 4.14: Vertical Strain Distribution at Top of subgrade (ABAQUS v.s. KENPAVE).
Figures 4.15 and 4.16 illustrate radial stress and radial strain distributions at the bottom of AC layer, respectively. As it can be seen in these figures, both ABAQUS and KENPAVE models are in a good agreement.

![Radial Stress Distribution](image1)

**Figure 4.15: Radial Stress Distribution at the Bottom of Subgrade (ABAQUS v.s. KENPAVE).**

![Radial Strain Distribution](image2)

**Figure 4.16: Radial Strain Distribution at the Bottom of Subgrade (ABAQUS v.s. KENPAVE).**
4-4-1-3. Comparison of 3D Nonlinear Model with Nonlinear EVERSTRESS Model

After confirming ABAQUS models with analytical solutions and KENPAVE linear elastic model, the nonlinear feature of the ABAQUS models should be tested against some existing nonlinear model. For this goal EVERSTRESS (Sivaneswaran, et. Al; 1999) software is chosen and similar model was generated in both ABAQUS and EVERSTRESS software to be analyzed and compared. Two models are studied in this section, the same model used in section 4-4-1-2 with the same layer thicknesses and elastic material properties and a similar model with the only difference of using stress dependent base material and a 200 kPa load in order to have an easier convergence. K and n parameters used for the base layer in the nonlinear model were, 170 MPa and 0.29, respectively.

Figures 4.17 to 4.19 show the elastic analysis results from both ABAQUS and EVERSTRESS models. Figure 4.17 shows the vertical stress distribution with depth, Figure 4.18 shows the vertical strain distribution with depth, and figure 4.19 shows the horizontal strain distribution with depth. As it can be seen in these figures, ABAQUS and EVERSTRESS have a perfect match in the elastic analysis which confirms the comparison study performed using KENPAVE software.

After confirming the elastic model with KENPAVE and EVERSTRESS softwares, a comparison was performed on ABAQUS and EVERSTRESS stress dependent models. The results are illustrated in figures 4.20 to 4.22. As it can be seen in these figures ABAQUS and EVERSTRESS are in good agreement in their stress dependent analysis which makes the
author confident to use ABAQUS to analyze more complex stress dependent models in the next sections.

Figure 4.17: Vertical Stress Distribution with Depth (ABAQUS v.s. EVERSTRESS).

Figure 4.18: Vertical Strain Distribution with Depth (ABAQUS v.s. EVERSTRESS).
Figure 4. 19: Horizontal Strain Distribution with Depth (ABAQUS v.s. EVERSTRESS).

Figure 4. 20: Vertical Stress Distribution with Depth (ABAQUS v.s. EVERSTRESS).
Figure 4. 21: Vertical Strain Distribution with Depth (ABAQUS v.s. EVERSTRESS).

Figure 4. 22: Horizontal Strain Distribution with Depth (ABAQUS v.s. EVERSTRESS).
4-4-1-4. Comparison of 3D Nonlinear Model with Existing Large Scale Laboratory Test Data

In this section more capabilities of ABAQUS models such as material nonlinearity, multiple wheel loads, wheel load interaction effects, load geometry, and many more features are validated by comparing the ABAQUS model responses with the large scale laboratory measured responses at the National Airport Pavement Test Facility (NAPTF).

The National Airport Pavement Test Facility (NAPTF) was constructed in April 1999 at the William J. Hughes Technical Center near Atlantic City, NJ by Federal Aviation Administration (FAA). NAPTF provides an estimated 25 gigabytes of test data per year (Teubert, et. Al, 2002) from rigid and flexible pavements subjected to simulated aircraft traffic (Figure 4.23).

Figure 4. 23: The National Airport Pavement Test Vehicle (from NAPTF website).
NAPTF can carry up to 75000 pounds per wheel independently and it has up to 20 test wheels which can represent two complete loading gears such as single, dual, dual-tandem, dual-tridem gears. NAPTF has a 900 feet long by 60 feet wide fully instrumented test track equipped with computerized data acquisition system. The test vehicle is programmed for a controlled aircraft wander simulation up to a speed of up to 15 miles per hour (limited to 5 miles per hour).

NAPTF reported 5 different groups of test sections named as Construction Cycle 1 (CC1) to Construction Cycle 5 (CC5). For our study CC1 data is selected which includes nine test pavements composed of six flexible and three rigid sections. Three different subgrade materials (i.e. low, medium, and high density) and two base sections (i.e. conventional and stabilized bases) are used in CC1 sections as shown in figure 4.24.

Figure 4.25 shows the loading gear configuration used in CC1 test sections. As it can be seen in figure 4.25, a dual-tridem gear is used for the first carriage and a dual-tandem configuration is used for the second carriage.

In order to measure the pavement response, several sensors including Multi-Depth Deflectometers (MDD), Pressure Cells (PC), and Asphalt Strain Gauges (ASG) are installed within the CC1 sections. Figure 4.26 shows the vertical locations of the MDD (Multi-Depth Deflectometer) sensors in MFC (Medium Strength Flexible Pavement Section with Conventional Base) and LFC (Low Strength Flexible Pavement Section with Conventional Base) sections and figure 4.27 shows the vertical locations of PCs (Pressure Cells).
Figure 4.24: Cross Section of NAPTF Pavement Test Sections (from Kim, 2005).

Figure 4.25: CC1 Test Section Loading Gear Configuration (from NAPTF website).
Figure 4.26: The Vertical Locations of the MDD Sensors (from Kim, 2005).
Two CC1 sections (MFC, LFC) are selected for the field validation of the ABAQUS simulations. The CC1 sections information and their measured responses are well published in the literature as well as the NAPTF website and the information for the section properties and the material properties and even different material test results performed on different materials are available. Besides, some researchers performed back calculation analysis to obtain material parameters as well (Gopalakrishnan, 2004; and Gomez-Ramirez, 2002). For this study the k-θ model parameters for base (P-209) and subbase (P-154) materials are estimated from existing resilient modulus test data from the NAPTF website using similar curve fitting procedure to what is performed in chapter 3 (figure 4.28). The subgrade modulus values for MFC and LFC sections are taken from McQueen et al. (2001). The AC Layer modulus values for MFC and LFC section are taken from Gopalakrishnan (2004). The
material properties and the layer thicknesses used in ABAQUS 3D models for MFC and LFC sections are summarized in table 4.1 and Figure 4.29.

A dual-tridem load gear configuration is used similar to that of Boeing 77 aircraft with 1372 mm wheel spacing and 1448 mm axle spacing (Figure 4.30) and a tire pressure of 1.3 MPa was applied on circles with radius of 222 mm (Gopalakrishnan, 2004).

![Figure 4.28: Estimating k-θ Model Parameters for Base and Subbase Materials.](image-url)
Figure 4.29: MFC and LFC Sections Material Properties and Layer Thicknesses (from Gopalakrishnan, 2004)

Table 4.1: Material Properties and Layer Thicknesses Used in ABAQUS 3D Models

<table>
<thead>
<tr>
<th>Material</th>
<th>Section</th>
<th>Thickness (mm)</th>
<th>Unit Weight [kg/m³]</th>
<th>v</th>
<th>Modulus Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC (P-401)</td>
<td>MFC</td>
<td>130</td>
<td>2400</td>
<td>0.35</td>
<td>E = 8,268,000 kPa</td>
</tr>
<tr>
<td></td>
<td>LFC</td>
<td>127</td>
<td>2400</td>
<td>0.35</td>
<td>E = 7,579,000 kPa</td>
</tr>
<tr>
<td>Base (P-209)</td>
<td>MFC</td>
<td>201</td>
<td>2470</td>
<td>0.38</td>
<td>K = 273131 Pa, n = 0.599</td>
</tr>
<tr>
<td></td>
<td>LFC</td>
<td>197</td>
<td>2470</td>
<td>0.38</td>
<td>K = 273131 Pa, n = 0.599</td>
</tr>
<tr>
<td>Subbase (P-154)</td>
<td>MFC</td>
<td>307</td>
<td>2111</td>
<td>0.38</td>
<td>K = 1181111 Pa, n = 0.647</td>
</tr>
<tr>
<td></td>
<td>LFC</td>
<td>925</td>
<td>2111</td>
<td>0.38</td>
<td>K = 1181111 Pa, n = 0.647</td>
</tr>
<tr>
<td>Subgrade</td>
<td>MFC</td>
<td>2408</td>
<td>1500</td>
<td>0.40</td>
<td>E = 101,000 kPa</td>
</tr>
<tr>
<td></td>
<td>LFC</td>
<td>2405</td>
<td>1500</td>
<td>0.40</td>
<td>E = 75,000 kPa</td>
</tr>
</tbody>
</table>
Figure 4.30: Dual-Tridem Gear Configuration Used in ABAQUS Model.

Figure 4.31 and 4.32 show the ABAQUS models used in this section for MFC and LFC sections, respectively. As illustrated in figure 4.31 and 4.32, in order to have finer mesh in the neighborhood of the loads and reduce the simulation time, the symmetry of the problem is considered and only a quarter model is analyzed. The model symmetry planes are fixed against moving normal to the planes.

Figures 4.33 to 4.35 illustrate the comparison between measured (NAPTF) and predicted (ABAQUS) pavement responses for MFC section.

Figure 4.33 shows the vertical subgrade stress distribution in X direction (figure 4.30), figure 4.34 shows the vertical subgrade displacement distribution in Y direction (figure 4.30), and 4.35 shows the vertical surface displacement distribution in Y direction (figure 4.30).

Figures 4.36 to 4.38 illustrate the comparison between measured (NAPTF) and predicted (ABAQUS) pavement responses for LFC section.
Figure 4.36 shows the vertical subgrade stress distribution in X direction, figure 4.37 shows the vertical subgrade displacement distribution in Y direction, and figure 4.38 shows the vertical surface displacement distribution in Y direction.

Figure 4.31: ABAQUS Model used for comparison with NAPTF Data for MFC section.
Figure 4.32: ABAQUS Model used for comparison with NAPTF Data for LFC section.
Figure 4.33: Vertical Stress Distribution at Subgrade for MFC Section.

Figure 4.34: Vertical Displacement Distribution at Subgrade for MFC Section.

Figure 4.35: Vertical Displacement Distribution at Surface for MFC Section.
Figure 4.36: Vertical Stress Distribution at Subgrade for LFC Section.

Figure 4.37: Vertical Displacement Distribution at Subgrade for LFC Section.

Figure 4.38: Vertical Displacement Distribution at Surface for LFC Section.
Figures 4.33 to 4.38 illustrate a reasonable agreement between NAPTF data and ABAQUS simulation for MFC and LFC section. The Minor differences found in these figures between the predicted response of the pavement by the model and the measured response of the MFC and LFC test sections may be due to the dynamic nature of moving wheel loads (Kim, 2005), presence of shear loads due to the movement of the wheels, the inaccuracies in estimating the material properties used in model, and the differences between the boundary conditions of the model and those of the test sections.

Figures 4.39 to 4.41 compare the ABAQUS model responses for MFC section at the surface and on top of subgrade. Similarly Figures 4.42 to 4.44 compare the ABAQUS model responses for LFC section at the surface and on top of subgrade.

Figures 4.45 to 4.47 compare the ABAQUS model responses for LFC and MFC sections at different locations.
Figure 4.39: ABAQUS Vertical Stress and Deformation Distributions for MFC Section.
Figure 4.40: ABAQUS Horizontal Strain Distributions for MFC Section.
Figure 4.41: ABAQUS Vertical Strain Distributions for MFC Section.
Figure 4.42: ABAQUS Vertical Stress and Deformation Distributions for LFC Section.
Figure 4.43: ABAQUS Horizontal Strain Distributions for LFC Section.
Figure 4.44: ABAQUS Vertical Strain Distributions for LFC Section.
Figure 4.45: ABAQUS Vertical Stress and Deformation Distributions.
Figure 4. 46: ABAQUS Horizontal Strain Distributions.
Figure 4.47: ABAQUS Vertical Strain Distributions.
Results

4-4-2-1. Introduction

In the mechanistic-empirical flexible pavement design method, the pavement distresses (rutting, fatigue cracking) are predicted from the pavement responses (stresses, strains) determined from mechanistic structural models. In this section the effects of different parameters on the airport pavement design are studied. It should be noted that most of the parameters studied in this section were mostly not taken into account in the pavement design and analysis models in the past.

The Federal Aviation Administration (FAA) airport pavement design procedure (FAA, 1995) is used for the initial design and then the designed section is analyzed with ABAQUS Nonlinear 3D FEM models to check its life time with the annual number of departures that is used for the design.

Two different landing gear configurations (i.e. dual and dual tandem) are selected for this study. The pavement section is designed assuming the following parameters:

Subgrade Elastic Modulus: 50 MPa

Gross Aircraft Weight: 200,000 lb

Annual Departures: 25000

Since FAA (1995) design charts are based on the subgrade CBR value, the given elastic modulus should be converted to CBR value using a valid correlation equation. AASHTO design guide suggests using equation 4.36 (Heukelom and Klomp, 1962):

\[ E(\text{psi}) = 1,500 \ CBR \]  
\[ \text{(4.36)} \]
Sukumara et al. (2002) showed that equation 4.36 over predicts the elastic modulus, so for the current study, the Transportation and Road Research Laboratory (TRRL) correlation is used to convert the subgrade elastic modulus to CBR value (from Sukumara, et al., 2002):

\[ E(\text{psi}) = 2,555 \ CBR^{0.64} \]  

(4.36)

Plugging \( E = 50 \text{MPa} \) in equation 4.36 results in CBR value of 5.1. Using this CBR value and the mentioned traffic parameters and figures 4.48 and 4.49 will give the total required pavement thickness on top of the 50MPa subgrade with the given traffic parameters for Dual and Dual Tandem gear configurations.

Figure 4.48 shows the pavement design procedure for Dual gear configuration. As it is shown in this figure a total pavement thickness of 45in is required. If a pavement section having a crushed aggregate base course (P-209) and a hot mix asphalt (HMA) layer is assumed, according to FAA (1995) design procedure the 45in total thickness is divided into a 4in HMA layer and a 41in P-209 Base Course.

Figure 4.49 shows the pavement design procedure for Dual Tandem gear configuration. As it is shown in this figure a total pavement thickness of 39in is required. Similar to the Dual gear, if a pavement section having a crushed aggregate base course (P-209) and a hot mix asphalt (HMA) layer is assumed, according to FAA (1995) design procedure the 39in total thickness is divided into a 4in HMA layer and a 35in P-209 Base Course.

ABAQUS models are generated with the same thicknesses for comparison studies. Material properties and layer thicknesses used in ABAQUS 3D models are illustrated in table 4.2 and 4.3 for Dual and Dual Tandem wheel gears, respectively.
Figure 4.48: FAA Design Chart for Dual Wheel Gear.
Figure 4.49: FAA Design Chart for Dual-Tandem Wheel Gear.
Table 4.2: Material Properties and Layer Thicknesses Used in ABAQUS 3D Models (Dual Gear)

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness (mm)</th>
<th>Unit Weight [kg/m³]</th>
<th>ν</th>
<th>Modulus Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC (P-401)</td>
<td>102</td>
<td>2400</td>
<td>0.35</td>
<td>E = 1,380,000 kPa</td>
</tr>
<tr>
<td>Base (P-209)</td>
<td>1041</td>
<td>2470</td>
<td>0.38</td>
<td>K = 273131 Pa, n = 0.599</td>
</tr>
<tr>
<td>Subgrade</td>
<td>5000</td>
<td>1500</td>
<td>0.40</td>
<td>E = 50,000 kPa</td>
</tr>
</tbody>
</table>

Table 4.3: Material Properties and Layer Thicknesses Used in ABAQUS 3D Models (Dual-Tandem Gear)

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness (mm)</th>
<th>Unit Weight [kg/m³]</th>
<th>ν</th>
<th>Modulus Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC (P-401)</td>
<td>102</td>
<td>2400</td>
<td>0.35</td>
<td>E = 1,380,000 kPa</td>
</tr>
<tr>
<td>Base (P-209)</td>
<td>897</td>
<td>2470</td>
<td>0.38</td>
<td>K = 273131 Pa, n = 0.599</td>
</tr>
<tr>
<td>Subgrade</td>
<td>5000</td>
<td>1500</td>
<td>0.40</td>
<td>E = 50,000 kPa</td>
</tr>
</tbody>
</table>

The tire pressures and tire spacing are selected from tables 4.4 and 4.5 for dual and dual-tandem wheel loads, respectively. As it can be seen in tables 4.4 and 4.5 for 200,000 lb gross weight airplane tire pressure is 200 psi and 160 psi for dual and dual tandem wheel configurations, respectively.

Since it is assumed that 95% of the gross weight is applied to the main landing gear assembly and 5% of the gross weight is applied to the nose gear assembly (FAA, 1995), 200,000lb × 0.95 is the load that is applied to each landing gear configuration. There is 2×2 tire pressure
in a dual assembly and 2×4 tire pressure in a dual tandem assembly, so the load carried by each tire is \( \frac{200,000 \text{lb} \times 0.95}{4} \) for dual wheel and \( \frac{200,000 \text{lb} \times 0.95}{8} \) for dual tandem wheel configuration. It is assumed that the tire loads are applied either to a circular area with radius \( R \) or to a 2:1 elliptical area with small radius of \( a \) and the large radius of 2\( a \). Knowing \( \text{Pressure} = \frac{\text{Load}}{\text{Area}} \) and also \( \text{Area}_{\text{circle}} = \pi R^2 \) and \( \text{Area}_{\text{ellipse}} = \pi (a) \left( \frac{a}{2} \right) \) the dimensions of circular and elliptical tire imprint for dual and dual tandem wheel configurations are calculated and summarized in table 4.6 and figures 4.50 to 4.53.

Table 4.4: Tire Pressure and Spacing for Dual Assembly (from FAA, 1995)

<table>
<thead>
<tr>
<th>Gross Weight (lbs)</th>
<th>Tire Pressure (psi)</th>
<th>Dual Spacing (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000 (22,700)</td>
<td>80</td>
<td>20 (51)</td>
</tr>
<tr>
<td>75,000 (34,000)</td>
<td>110</td>
<td>21 (53)</td>
</tr>
<tr>
<td>100,000 (45,400)</td>
<td>140</td>
<td>23 (58)</td>
</tr>
<tr>
<td>150,000 (68,000)</td>
<td>160</td>
<td>30 (76)</td>
</tr>
<tr>
<td>200,000 (90,700)</td>
<td>200</td>
<td>34 (86)</td>
</tr>
</tbody>
</table>

Table 4.5: Tire Pressure and Spacing for Dual-Tandem Assembly (from FAA, 1995)

<table>
<thead>
<tr>
<th>Gross Weight (lbs)</th>
<th>Tire Pressure (psi)</th>
<th>Dual Spacing (in)</th>
<th>Tandem Spacing (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000 (45,400)</td>
<td>120</td>
<td>20 (51)</td>
<td>45 (114)</td>
</tr>
<tr>
<td>150,000 (68,000)</td>
<td>140</td>
<td>20 (51)</td>
<td>45 (114)</td>
</tr>
<tr>
<td>200,000 (90,700)</td>
<td>160</td>
<td>21 (53)</td>
<td>46 (117)</td>
</tr>
<tr>
<td>300,000 (136,100)</td>
<td>180</td>
<td>26 (66)</td>
<td>51 (130)</td>
</tr>
<tr>
<td>400,000 (181,400)</td>
<td>200</td>
<td>30 (76)</td>
<td>55 (140)</td>
</tr>
</tbody>
</table>
Table 4.6: Wheel Load Properties Used in ABAQUS 3D Models

<table>
<thead>
<tr>
<th>Wheel Gear</th>
<th>Tire Imprint</th>
<th>Load Dimension (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dual</td>
<td>Circular</td>
<td>r = 221</td>
</tr>
<tr>
<td></td>
<td>Elliptical</td>
<td>a = 312</td>
</tr>
<tr>
<td>Dual-Tandem</td>
<td>Circular</td>
<td>r = 175</td>
</tr>
<tr>
<td></td>
<td>Elliptical</td>
<td>a = 247</td>
</tr>
</tbody>
</table>

Figure 4.50: Dual Gear Configuration with Circular Tire imprint.

Figure 4.51: Dual Gear Configuration with Elliptical Tire imprint.
The mentioned layer properties and landing gear configurations was used to build four ABAQUS models for more detailed study:

1- Circular imprint, Dual Wheel
2- Elliptical imprint, Dual Wheel
3- Circular imprint, Dual Tandem Wheel
4- Elliptical imprint, Dual Tandem Wheel
As mentioned earlier, in order to have finer mesh around the tires and reduce the simulation time, the symmetry of the models is considered and quarter models are analyzed instead of whole models. Figures 4.54 to 4.57 show different ABAQUS models used for this part of study.

As mentioned in the literature review chapter, the maximum horizontal tensile strain at the bottom of the asphalt layer ($\varepsilon_t$) and the maximum vertical compressive strain at the top of subgrade ($\varepsilon_c$) are used to calculate the allowable number of load repetitions for fatigue cracking and rutting, respectively. In order to find the maximum strain values, the strain distributions at the desired locations in X and Y directions (Figures 50 to 53) are plotted and the maximums in both directions are found and the one with a bigger magnitude is selected. There are various equations to estimate the number of allowable load repetitions from $\varepsilon_t$ and $\varepsilon_c$. Equations 4.37 and 4.38 are provided by the Asphalt Institute (AI) to calculate the number of load repetitions corresponding to fatigue cracking and rutting in highway pavements, respectively (from Huang, 2003):

$$N_f = 0.0796(\varepsilon_t)^{-3.291}|E^*|^{-0.854}$$  \hspace{1cm} (4.37)

$$N_d = 1.365 \times 10^{-9}(\varepsilon_c)^{-4.477}$$  \hspace{1cm} (4.38)

Where:

$N_f$ is the number of allowable load repetitions until fatigue cracking occurs.

$N_d$ is the number of allowable load repetitions until rutting occurs.

$|E^*|$ is the dynamic modulus of the asphalt mixture.
During the case of this study it is observed that equations 4.37 and 4.38 will not have a good estimate of the allowable number of load repetitions for airport pavements under heavy aircraft loads and they will give small number of allowable load repetitions. In order to have a more accurate estimation of allowable load equations that are solely provided for airport pavements are studied (Hayhoe et al., 2004; LEDGA, 1999) and equations 4.39 and 4.40 which illustrate relatively better estimations are selected for this study (LEDGA, 1999):

\[
N_f = 10^{(2.68-5 \times \log(\varepsilon_f)-2.665 \times \log(E_A))} \tag{4.39}
\]

\[
N_d = 10^4 \times \left( \frac{0.001347+0.000061 \times \log \left( \frac{E_{SUB}}{10000} \right)}{\varepsilon_c} \right) \left( 3.662532 \times \left( \frac{E_{SUB}}{10000} \right)^{0.386383} \right) \tag{4.40}
\]

Where \( E_A \) and \( E_{SUB} \) are the elastic modulus of the asphalt layer and subgrade in psi, respectively.
Figure 4.54: ABAQUS Model for Circular Tire Imprint, Dual Wheel Configuration.
Figure 4.55: ABAQUS Model for Elliptical Tire Imprint, Dual Wheel Configuration.
Figure 4.56: ABAQUS Model for Circular Tire Imprint, Dual Tandem Wheel Configuration.
Figure 4. 57: ABAQUS Model for Elliptical Tire Imprint, Dual Tandem Wheel Configuration.
Equation 4.41 is used to calculate the damage ratio of the pavement which is a measure of the life time of the pavement when the damage ratio is equal to one.

\[
\text{Damage Ratio} = \frac{N}{N_F} + \frac{N}{N_d} = 1 \quad (4.41)
\]

Values calculated from equations 4.39 and 4.40 will be plugged into equation 4.41 for damage ratio equal to one to find the total number of allowable load repetitions (N) for different cases:

\[
N = \frac{1}{\frac{1}{N_F} + \frac{1}{N_d}} \quad (4.42)
\]

Equation 4.43 is used to calculate the annual departures from the total number of allowable load repetitions:

\[
\text{Annual Departures} = \frac{PC \times N}{Y} \quad (4.43)
\]

Where PC is the pass to coverage ratio and is selected from table 4.7 and Y is the design life of the pavement which will be the constant number of 20 years according to FAA design manual (FAA, 1995).

In the following sections, the annual departures calculated for different pavements are compared as a measure of pavement response.
Table 4.7: Pass to Coverage Ratio for Different Landing Gear Assemblies (from FAA, 1995)

<table>
<thead>
<tr>
<th>Design Curve</th>
<th>Pass-to-Coverage Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Wheel</td>
<td>5.18</td>
</tr>
<tr>
<td>Dual Wheel</td>
<td>3.48</td>
</tr>
<tr>
<td>Dual Tandem</td>
<td>1.84</td>
</tr>
<tr>
<td>A-300 Model B2</td>
<td>1.76</td>
</tr>
<tr>
<td>A-300 Model B4</td>
<td>1.73</td>
</tr>
<tr>
<td>B-747</td>
<td>1.85</td>
</tr>
<tr>
<td>B-757</td>
<td>1.94</td>
</tr>
<tr>
<td>B-767</td>
<td>1.95</td>
</tr>
<tr>
<td>C-130</td>
<td>2.07</td>
</tr>
<tr>
<td>DC 10-10</td>
<td>1.82</td>
</tr>
<tr>
<td>DC 10-30</td>
<td>1.69</td>
</tr>
<tr>
<td>L-101 1</td>
<td>1.81</td>
</tr>
</tbody>
</table>
4-4-2-2. Effect of Tire Imprint

In this section the effect of the tire imprint, (i.e. circular v.s. elliptical) on pavement response is studied. Figures 4.58 and 4.59 show the pavement responses for the pavement under dual tandem elliptical and circular tire imprints, respectively. The maximum horizontal tensile strain at the bottom of HMA layer and the maximum vertical compressive strain on top of subgrade are calculated in both X and Y directions and the maximum value is used as highlighted in figures 4.58 and 4.59. Figure 4.60 compares the pavement responses for circular and elliptical tire imprints on a single y axis for the responses.

Table 4.8 summarizes the allowable load repetitions for fatigue cracking, rutting, and annual departures for elliptical and circular tire imprints.

As it can be seen in table 4.8 the only parameter that has a considerable difference for circular and elliptical tire imprints is the allowable number of load repetitions for the fatigue cracking ($N_f$) as the elliptical tire imprint applies more damage in one direction causing higher strains and hence smaller $N_f$. Since the value of $N_d$ is smaller for both tire imprints, it will ultimately be controlling the overall allowable load repetitions and since elliptical and circular tire imprints both have similar $N_d$ values, the overall allowable load repetitions ($N$) and hence the annual departures for both cases will be similar.

It should be noted that the pavement section used in this section is selected based on the FAA flexible airport pavement design manual for 50 MPa subgrade and 200,000 lb gross aircraft weight and 25000 annual departures, but table 4.8 shows less number of annual departures. It cannot confidently be concluded that the FAA design is not sufficient as equations 4.39 and
4.40 might not be modeling the pavement response realistically and further research should be performed in this subject.

In order to better illustrate the effect of tire imprint on airport pavement design, since in pavements with thicker HMA layers fatigue cracking may be the critical distress, a full depth asphalt pavement section with 30 cm (12 in) of asphalt layer on top of subgrade layer is analyzed under circular and elliptical tire imprints and the results are summarized in table 4.9. As it can be seen in table 4.9, similar to table 4.8, $N_d$ values for elliptical and circular tire imprints are similar and elliptical tire imprint results in smaller $N_f$ values. In this case $N_f$ is smaller than $N_d$ for elliptical tire imprint and results in a smaller number of allowable load repetitions and hence smaller annual departures for elliptical tire imprint.
Figure 4.58: Pavement Response for Dual Tandem Gear, Elliptical Tire Imprint.
Figure 4.59: Pavement Response for Dual Tandem Landing Gear, Circular Tire Imprint.
Figure 4.60: Pavement Response for Dual Tandem Landing Gear, Circular and Elliptical Tire Imprints.

Table 4.8: Allowable number of load repetitions (Elliptical v.s. Circular Tire Imprints)

<table>
<thead>
<tr>
<th>Tire Imprint</th>
<th>N_f</th>
<th>N_d</th>
<th>N</th>
<th>Annual Departures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical</td>
<td>7891506</td>
<td>217993</td>
<td>210012</td>
<td>19321</td>
</tr>
<tr>
<td>Circular</td>
<td>15979527</td>
<td>217776</td>
<td>212699</td>
<td>19568</td>
</tr>
</tbody>
</table>
Table 4.9: Allowable number of load repetitions (Elliptical v.s. Circular Tire Imprints – Full Depth Asphalt)

<table>
<thead>
<tr>
<th>Tire Imprint</th>
<th>$N_r$</th>
<th>$N_d$</th>
<th>$N$</th>
<th>Annual Departures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical</td>
<td>320246</td>
<td>366120</td>
<td>169117</td>
<td>15559</td>
</tr>
<tr>
<td>Circular</td>
<td>788869</td>
<td>394431</td>
<td>260325</td>
<td>23950</td>
</tr>
</tbody>
</table>

4-4-2-3. Effect of Material Nonlinearity

In this section the effect of material nonlinearity (i.e. elastic base v.s. nonlinear base) on pavement response is studied. In order to have comparable results for the pavement response, both elastic properties and k-θ model parameters of the same base material should be available and used in ABAQUS models. For this reason, HD1 base material from table 3.3 is used in this section. It should be noted that dual load configuration and circular tire imprint is selected for this section. Figures 4.61 and 4.62 show the pavement responses for the pavement section with nonlinear and elastic base materials, respectively. Figure 4.63 compares the pavement responses for nonlinear and elastic bases on a single y axis for the responses. As it can be seen in these figures the pavement section with the nonlinear base material tends to have smaller strains which can be due to the material hardening that occurs with increasing the stress state in the k-θ model.

Table 4.10 summarizes the allowable load repetitions for fatigue cracking, rutting, and annual departures for nonlinear and elastic sections. As it can be seen in table 4.10 in all cases there is a considerable difference between the values calculated for the section with
elastic base and the one with nonlinear base and in all of the cases the section with nonlinear base will have larger values of allowable number of load repetitions which is due to the smaller strains that are observed in figures 4.61 to 4.63. These differences suggest that use of nonlinear base materials will provide a more economic design. However, it should be noted that table 4.10 is generated based on discrete number of FEM simulations and the magnitude of the differences observed in table 4.10 between elastic and nonlinear base material may vary depending on the material properties, layer thicknesses, load configurations and etc.

Figure 4. 61: Pavement Response for Nonlinear Base.
Figure 4.62: Pavement Response for Elastic Base.
Figure 4.63: Pavement Response for Nonlinear and Elastic Base Materials.

Table 4.10: Allowable number of load repetitions (Elastic v.s. k-θ)

<table>
<thead>
<tr>
<th>Base Material</th>
<th>$N_f$</th>
<th>$N_d$</th>
<th>N</th>
<th>Annual Departures</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-θ</td>
<td>1059798</td>
<td>850653</td>
<td>467170</td>
<td>42980</td>
</tr>
<tr>
<td>Elastic</td>
<td>3513</td>
<td>36553</td>
<td>3173</td>
<td>292</td>
</tr>
</tbody>
</table>
4-4-2-4. Effect of Presence of Shear Loads

In this section the effect of presence of shear loads on pavement response is studied. Presence of the shear load will eliminate the symmetry of the model in the direction of the shear load and will not let the use the quarter model in this case and hence half models are used instead. Figures 4.64 and 4.65 show the ABAQUS models used for circular and elliptical tire imprints, respectively.

Figure 4.66 and 4.67 show the pavement responses with and without shear load for the circular and elliptical tire imprints, respectively. As it can be seen in figures 4.68 and 4.69, presence of shear loads will mostly influence the horizontal strain distributions in the direction of shear load application (i.e. X) and will not significantly influence the horizontal strain distributions in Y direction and also the vertical stress/strain distributions in both X and Y directions. Comparing Figures 4.66 and 4.67, the presence of shear loads tends to have a more noticeable effect for the elliptical tire imprint. This can be investigated in more details by comparing the allowable number of load repetitions for both cases. Table 4.1 illustrates the allowable number of load repetitions with and without shear load for both tire imprints.

For circular tire imprint, as it can be seen in table 4.1 the allowable number of load repetitions is almost similar with and without the shear load. On the other hand, for elliptical tire imprint, the allowable number of load repetitions is noticeably lower in presence of shear loads. This can be due to the wider contact area in X direction for elliptical tire imprint and hence more cumulative strains in the pavement section.
Figure 4.64: ABAQUS Model for Dual Wheel Configuration with Shear, Circular Tire Imprint.

Figure 4.65: ABAQUS Model for Dual Wheel Configuration with Shear, Elliptical Tire Imprint.
Figure 4.66: Pavement Response for Dual Tire Configuration with and without Shear Load, Circular Tire Imprint.
Figure 4.67: Pavement Response for Dual Tire Configuration with and without Shear Load, Elliptical Tire Imprint.
Table 4.11: Allowable number of load repetitions (Shear v.s. No Shear)

<table>
<thead>
<tr>
<th>Tire Imprint</th>
<th>Shear</th>
<th>N_f</th>
<th>N_d</th>
<th>N</th>
<th>Annual Departures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical</td>
<td>Yes</td>
<td>3034</td>
<td>38115</td>
<td>2782</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2891</td>
<td>38138</td>
<td>2660</td>
<td>245</td>
</tr>
<tr>
<td>Circular</td>
<td>Yes</td>
<td>4565</td>
<td>42375</td>
<td>4080</td>
<td>375</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>6057</td>
<td>42415</td>
<td>5247</td>
<td>483</td>
</tr>
</tbody>
</table>

The effect of presence of shear loads on pavement response for elliptical tire imprint was not clearly presented in table 4.11. In order to capture the effect of presence of shear loads on pavement response in case of elliptical tire imprint, similar simulations with and without shear loads were performed on full depth asphalt sections with 30 cm (12 in) of asphalt layer resting on subgrade and the results are presented in table 4.12. As it can be seen in table 4.12 presence of shear loads will result in smaller number of allowable load repetitions and annual departures for both elliptical and circular tire imprints.

Table 4.12: Allowable number of load repetitions (Shear v.s. No Shear – Full Depth Asphalt)

<table>
<thead>
<tr>
<th>Tire Imprint</th>
<th>Shear</th>
<th>N_f</th>
<th>N_d</th>
<th>N</th>
<th>Annual Departures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical</td>
<td>Yes</td>
<td>1842</td>
<td>41648</td>
<td>1746</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2218</td>
<td>44356</td>
<td>2091</td>
<td>192</td>
</tr>
<tr>
<td>Circular</td>
<td>Yes</td>
<td>3012</td>
<td>28245</td>
<td>2695</td>
<td>248</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>3987</td>
<td>28336</td>
<td>3460</td>
<td>318</td>
</tr>
</tbody>
</table>
REFERENCES


18- SolidWorks Express, “How to use symmetry and anti-symmetry boundary conditions.”, Tech Tips


Chapter 5

Summary and Conclusions

5-1- Summary

This chapter summarizes the findings and conclusions obtained from this research, the contributions made to the literature, and the author’s suggestions for future research.

This thesis consists of two main chapters: (i) DEM simulations results and discussions, (ii) FEM simulations results and discussions.

The DEM simulations were performed to study the effect of different parameters such as grain size distribution and stress state on the resilient behavior of granular base materials. In addition, micro mechanical particle scale response of the ABC granular assemblies were studied by means of DEM simulations. Furthermore, the actual grain size distribution was modeled for the first time in DEM models. At the end the macro mechanical response from DEM simulations results were calibrated with existing experimental data. Calibrated DEM simulations can be used to investigate the influence of various other parameters on the granular material behavior, if required.

As stated in chapters 1 and 2 the commonly performed FEM modelings in the mechanistic-empirical flexible pavement design process do not take into account the resilient behavior of the aggregate base course layer. Some researchers have previously considered the resilient behavior of the ABC materials in their FEM modelings but they have not studied the effect of parameters such as the tire imprint, complex tire configurations, and presence of shear loads
on their models. Furthermore a side by side comparison of the existing airport pavement
design procedure with the mechanistic-empirical design process using nonlinear 3D FEM
simulations results is also performed in this study.

5-2- DEM Simulation Conclusions

The conclusions gained from the conducted DEM simulations can be summarized as:

- Simulated material response varies across multiple scales as a function of GSD.
- Finer specimens are highly prone to localization, even during hardening.
- Graded specimens tend to have a stiffer response at small strains compared to mono
  disperse specimen.
- Coarser specimens will have higher maximum stress and also higher resilient
  modulus values compared to finer specimens.
- Large particles tend to inhibit shear banding, but can result in large local voids.
- Increase in bulk stress will result in higher resilient modulus values for all specimens.
- Particle-scale response in coarser specimens tends to be more chaotic.
- Particle-scale properties in dynamic simulations shows much less variations after
  some initial load cycles (shakedown).
5-3- FEM Simulation Conclusions

The conclusions gained from the conducted DEM simulations can be summarized as:

- Abaqus models were in reasonable agreement with existing numerical models and full scale lab model.
- Similar response to what was expected from FAA designed pavement sections was observed in nonlinear ABAQUS simulations performed on the same pavement sections.
- Elliptical tire imprint tend to have more fatigue damage on the pavement compared to circular tire imprint. This will have a higher impact in case of full depth asphalt pavement sections with high HMA layer thickness, where fatigue cracking will be the critical distress and will result in lower allowable annual departures.
- Models with nonlinear base material showed greater allowable number of load repetitions compared to models with elastic base materials.
- Considering presence of shear loads in the simulations resulted in more fatigue damage on the pavement (hence smaller allowable number of load repetitions), especially for elliptical tire imprint. The effect of presence of shear loads was more apparent for both elliptical and tire pressures in full depth asphalt model.
5-4. Contributions

The current research’s major contributions to the literature can be summarized as:

- Implementing the actual grain size distribution in Discrete Element Models.
- Implementing Biaxial Compression Test and Resilient Modulus Tests simulating using DEM.
- Studying the effect of gradation on macro/micro scale properties of granular materials.
- Calibrating the stress-dependant resilient model for ABC aggregates using DEM simulations.
- 3D Nonlinear FEM analysis of pavements under heavy aircraft loads.
- Studying the effect of tire imprint on airport pavement response.
- Studying the effect of stress dependent materials on airport pavement response.
- Studying the effect of presence of shear loads on the pavement response.
- Performing mechanistic-empirical airport pavement design considering the mentioned parameters.
5-5. **Future Research**

In order to completely achieve the goals of the performed research, the following tasks are recommended for future research:

1- Conducting 3D DEM simulations to avoid the 2D effects on the simulation results.

2- Running parallel experiments and DEM simulations for different materials to have more reliable material parameters.

3- Conducting more DEM simulations to obtain both elastic and stress dependent material properties for different base materials.

4- Including the viscoelastic behavior of asphalt concrete in the FEM models and study the effects of dynamic nature of moving wheel.

5- Including non-isotropic material behavior of the ABC layer in the FEM models.

6- Performing more research and laboratory tests in order to find a better model to predict the maximum allowable number of load repetitions corresponding to different distresses in the pavement section.

7- Running more FEM simulations on different pavement sections with different landing gear geometries and gross weights in order to obtain airport flexible pavement design charts similar to FAA design charts.

8- Studying the effect of different nonlinear material models on the pavement model response and finding the more realistic model.

9- Performing FEM simulations on pavement sections with different base material properties (from DEM simulations) in order to obtain acceptance criteria for ABC gradations (comparable to NCDOT gradation criteria).