ABSTRACT

KIM, JIHWAN. Liquid Crystal Geometric Phase Holograms for Efficient Beam Steering and Imaging Spectropolarimetry. (Under the direction of Dr. Michael J. Escuti.)

Efficient control and detection of light properties are important in modern Electro-Optic (EO) systems. One of the demands of the systems is directional control of light propagation for wide-angle with high efficiency. The ability to steer light source over a large field-of-regard is however difficult to achieve with high accuracy in mechanical platforms. Another important demand for the EO systems is a snapshot acquisition of light/scene with hyper-spectral and full polarization information. However, conventional technologies based on time-sequential measurements are not appropriate in high speed applications. Therefore, efficient light control and detection is essential technologies for the modern EO systems.

We have investigated and applied Geometric Phase Holograms (GPHs) as simple, compact, low-cost, light-weight, highly efficient alternatives to conventional optical beam steering and hyperspectral polarization imaging technologies. In this work, we have proposed novel polarization holography methods to record arbitrary phase profiles as diffractive GPHs based on revised Michelson and Mach-Zehnder interferometric setups. We have fabricated and experimentally demonstrated GPHs as liquid crystal diffractive optics with desirable optical properties that manifest nearly 100% efficiency, high polarization selectivity, and fast electro-optical switching. Moreover, we have introduced and demonstrated new polarization holography techniques that could record scalable period polarization holograms and arbitrarily changed polarization fields that produced various types of GPHs. We have also demonstrated a novel method that offered proximate lithography with easily tunable-periods with multi-axis polarization gratings fabrication method that could fabricate multiple diffraction at different azimuthal angles.

With the unique diffraction properties of the GPHs, we have achieved high throughput, wide-angle, nonmechanical laser beam steering utilizing stacked polarization gratings as one of GPHs. We have suggested and experimentally demonstrated four different steering designs and suggested optimum designs, which manifest their own advantages for various steering parameters and requirements. In this work, we have suggested and evaluated important nonmechanical designs, and derived governing theory of operation. We have also demonstrated a new way to accomplish continuous mechanical steering
with high throughput and wide-angle utilizing a pair of rotating PGs. Moreover, we have demonstrated reduced chromatic dispersion of broadband source steering as utilizing compensation PGs that can adjust the color separation of PG diffraction.

Next novel technology, hyperspectral polarization imaging, using the unique dispersion properties of the GPHs has been introduced and experimentally demonstrated. GPHs have been used to implement snapshot hyperspectral polarization imagers that can provide simultaneous acquisition of both spectral and polarization information at a higher resolution and in a simpler way. We have developed the system matrix of the imaging system, and the matrix can be extendable to other GPH-based imaging systems. We have used numerical simulation to assess reconstruction performance, and showed a favorable comparison with prior work. We have also found that several potentially optimum GPH dispersion patterns, and identified favorable characteristics. Moreover, we have demonstrated working principle of the hyperspectral polarization imaging system with a derivation of governing operation theory and design principles.

In the final portion of this dissertation, we have evaluated the work, summarized the contributions, and made suggestions for how future researchers can take our work and contributions to new application spaces and new research areas.
Liquid Crystal Geometric Phase Holograms for Efficient Beam Steering and Imaging Spectropolarimetry

by

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DEDICATION

To my beloved wife, Meeok, and to our precious son, Aiden, who is the joy of our lives.
BIOGRAPHY

Jihwan Kim was born to Dong-Kwon Kim and In-suk Kim in 1979 in Seoul, Korea. Author’s given name is Ji-Hwan meaning Shining Knowledge or (author) Knows Light. Grandfather, I thank you. I am working in Optics!

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Chapter 1

Introduction

1.1 Light Control: Optical Beam Steering

Modern defense systems utilize electro-optics devices (e.g. LIDARs) for applications such as remote sensing, target detection, etc. One of the requirements of such systems is to have directional control with high efficiency. However, the ability to scan highly directive optical systems over a large field of regard with high accuracy is a difficult mechanical problem, which is normally resolved by utilizing gimbals. An optical gimbal’s size, weight and power (SWaP) characteristics and cost make deployment unattractive for many types of platforms. The deployment of gimbal-based systems is especially difficult for small platforms such as satellites and unmanned airborne vehicles (UAVs). To reduce SWaP, mounting requirements and cost, nonmechanical steering techniques are being considered. With nonmechanical steering, the need to apply mechanical force to move massive optical elements is eliminated, reducing mechanical inaccuracies such as overshoot and ringing. Thus, nonmechanical beam control is expected to be less complicated and more reliable. These nonmechanical benefits are especially desirable if other parameters such as aperture size, efficiency and scanning range are not to be sacrificed. Other optical sensor systems have similar needs, but different attributes enter into the equation. Passive mid-wave infrared sensors for threat warning and target detection require broadband steering. For laser communications, the beam director has to provide continuous fine-angle tracking with good mechanical stability. Whereas, a laser weapon system needs to steer beams with high average power. The common link is that all of these applications require the ability to steer over a large field of regard with good precision.
1.2 Light Detection: Imaging Spectropolarimetry

Hyperspectral polarization imaging in particular is becoming increasingly important as the successful identification of particular signatures, military geological or otherwise, often requires analysis full Stokes parameters over tens to hundreds of separate spectral channels with spectral resolutions on the order of 10 nm. With the advances in computer processing power and detector technology, the field of hyperspectral polarization imaging is continually developing. There are several types of technologies being developed for the measurement, including grating-based spectrometers, filter-based spectrometers, Fourier transform imaging spectrometers, slit spectrometers and whisk-broom scanners. Conventional technologies based on time-sequential measurements or scanning are often inappropriate in high-speed applications where the characteristics of the scene can change before the measurement series is complete (such as in the characterization of the fireball from a chemical explosion). Grating spectrometers are high-speed, particularly if multiple detectors are used, but are usually non-imaging. In addition these traditional approaches to hyperspectral polarization imaging require bulky optics with a large form factor and are expensive. For military applications in the field it is highly desirable to acquire a lower cost imaging system, which has a compact footprint and can be manufactured in high volume numbers due to reduced cost.

1.3 Dissertation Approach and Research Contributions

The overarching theme of this dissertation is to investigate and apply Geometric Phase Holograms as simple, compact, low-cost, light-weight, highly efficient alternatives to conventional optical beam steering and imaging spectropolarimetry technologies. We approach this goal with individual research hypotheses and questions, which will be shown in each chapter. Moreover, we try to meet the research goal in both theory and experimental demonstration.

Chapter 2 begins this effort as providing some necessary background information, theory, and important principles. We will briefly summarize material properties of liquid crystal polymers and reactive mesogens. Then there will be an explanation of properties of the Polarization Grating (PG) as one of the geometric phase holograms, which work
as highly efficient polarization sensitive diffractive optics. The chapter will cover a discussion of light properties and its polarization, and how to describe its behavior using mathematical tools such as Jones Calculus. Finally, the polarization grating fabrication process via polarization holography with LCs will be included.

Chapter 3 addresses the following hypothesis: **We can record arbitrary phase profiles as diffractive Geometric Phase Holograms (GPHs) via interferometric polarization holography.** We utilized Michelson and Mach-Zehnder interferometer setups to fabricate the GPHs, and the same setup can be used to fabricate various GPHs showing various diffraction properties as changing the position of conventional optics used to record GPHs. The GPH fabrication results in high efficiency (93% ∼ 99%), and the diffraction property of the GPH scales with recording and replaying wavelengths ($\lambda_{\text{replay}}/\lambda_{\text{recording}}$). This chapter addresses additional question: **What novel techniques can enable easier fabrication of GPHs?** Compared to conventional polarization holography setup, the interferometric approaches provide easier way to tune periods with smaller working space (more than 5 times less). Moreover, a novel fabrication method, proximate lithography called Risley-Mask also has order of magnitude ability for tuning periods of GPHs with extremely small working space.

The primary scientific contribution within Chapter 3 is demonstration of various GPHs showing arbitrary phase profiles (i.e., lens, micro-lens, and azimuthal profiles), which were not realized in prior art. We also derived governing theory for GPH fabrication, and evaluated two interferometric polarization holography setups: (a) revised Michelson setup that can fabricate polarization lens with simpler method, and (b) revised Mach-zehnder setup that can realize polarization micro-lens and axicons.

Chapter 4 addresses the following question: **Can we achieve high throughput, wide-angle laser beam steering using stacked polarization gratings and polarization selectors? And if so, what are the optimum designs to achieve lowest loss?** We identified two families of beam steering: nonmechanical and mechanical. First, in nonmechanical steering, we found an exponential increase in steering angles as steering stages are added. From the study of steering design, we found that it would be preferable to choose (a) Quasi-Ternary design if number of steering angles is most important, and (b) Supra-Binary design if throughput, ease of fabrication, and reliability at large field-of-regard are most essential. In mechanical steering, we found that Risley-PGs act similar to Risley Prisms with much smaller SWaP. We also built prototypes of nonmechanical and mechanical beam steering systems to various independent evaluators.
The main contribution within Chapter 4 is the demonstration of discrete nonmechanical steering with high throughput and wide-angle with three steering designs: Ternary, Quasi-Ternary, and Supra-Binary. In this work, we suggested and evaluated important non-mechanical designs, and derived governing theory of operation. We also demonstrated a new way to accomplish continuous mechanical steering with high throughput and wide-angle utilizing a pair of rotating PGs. Moreover, we demonstrated reduced chromatic dispersion of broadband source steering as utilizing compensation PGs that can adjust the color separation of diffraction.

Chapter 5 addresses the following hypothesis: PGs can be used to implement snapshot hyperspectral polarization imagers with several performance advantages. We used numerical simulation to assess reconstruction performance, and showed a favorable comparison with prior work. We also found that several potentially optimum PG dispersion patterns, to be evaluated further, and identified favorable characteristics. Moreover, we found algorithmic technique to vastly improve SNR iterative convergence.

The primary scientific contribution within Chapter 5 is demonstration of working principle of the hyperspectral polarization imaging with two PGs for lab-scale and natural objects. We also derived governing theory of operation and design principles, and developed an optimum calibration approach that is simple and generic, applicable to any PG-based imaging polarimeter. Although direct comparison of prior and our imaging system is nearly impossible, we showed bench top results that suggest substantially more accurate reconstruction spectrally and spatially for the full Stokes vector.

Chapter 6 summarizes the research results of this dissertation and ties the contributions together, and also topics for further study are suggested.

1.4 Related Publications

The research presented in this document has been published in 2 peer-reviewed journals, 10 peer-reviewed conference papers, and 2 US patent applications with 6 additional journal submissions in preparation:

• J. Kim and M. J. Escuti, “Tunable-focus Fresnel Zone Plate based on liquid crystal
diffraction optics,” in preparation.

• J. Kim and M. J. Escuti, “Multiple diffraction polarization holograms,”, in prepa-
ration.

• J. Kim, R. K. Komanduri, and M. J. Escuti, “Proximate lithography technique for
tunable period polarization holograms,” in preparation.

• J. Kim and M. J. Escuti, “Scalable period polarization holograms via revised
Michelson interferometric setup,” in preparation.

• S. Serati, J. Kim, and M. J. Escuti, “Non-mechanical conformal beam steering
system with an 80 degree x 80 degree field of regard,” in preparation.

• Y. Li, J. Kim, and M. J. Escuti, “Experimental realization of high efficiency switch-

• J. Kim, M. N. Miskiewicz, S. Serati, and M. J. Escuti, “Demonstration of large-
gle nonmechanical laser beam steering based polymer polarization gratings,”

• M. J. Escuti, J. Kim, et al., “LC polarization gratings: performance review and

• E. Seo, H. C. Kee, Y. Kim, S. Jeong, H. Choi, S. Lee, J. Kim, R. K. Komanduri,
and M. J. Escuti, “Polarization conversion system using a polymer polarization

• J. Kim, C. Oh, S. Serati, and M. J. Escuti, “Wide-angle, nonmechanical beam steer-
ing with optically controlled polarization modulation by liquid crystal polarization

• J. Kim, M. N. Miskiewicz, S. Serati, and M. J. Escuti, “High efficiency quasi-
ternary design for nonmechanical beam-steering utilizing polarization gratings,”

• Y. Li, J. Kim, and M. J. Escuti, “Controlling Orbital angular momentum using


Chapter 2

Optical and Physical Fundamentals and Liquid Crystal Based Optical Elements

2.1 Liquid crystal polymers and Reactive mesogens

A liquid crystal [9, 10, 11] is a phase of soft condensed matter that simultaneously exhibits characteristics of both an isotropic liquid and a crystalline solid. Since the first discovery of cholesteric liquid crystals (LCs) by Reinitzer in 1888, LCs and related technologies have been of great interest to diverse communities in both science and engineering.

The anisotropy in dielectric and optical properties of LCs is a key to enable LC-based technologies. Consider a nematic molecule with the rod-like shape as illustrated in Figs. 2.1(a) and 2.1(b). The nematic molecule exhibits the dielectric anisotropy (expressed as $\Delta \varepsilon = \varepsilon_{\parallel} - \varepsilon_{\perp}$). When an electric field $\mathbf{E}$ is applied, the field tends to polarize the free charges within the molecule and leads to a dipole moment. Due to the dielectric anisotropy, this induced dipole moment makes the nematic molecule reorient either parallel ($\Delta \varepsilon > 0$) or perpendicular ($\Delta \varepsilon < 0$) to the applied electric field. At optical frequencies, the nematic LC also shows the optical anisotropy, conveniently quantified by the birefringence $\Delta n = \Delta n_{\parallel} - \Delta n_{\perp}$ and light passing through the LC medium may experience phase retardation. The ability of the electro-optic control makes liquid crystals uniquely useful for many applications.

Although the nematic director is free to point in any direction, LCs typically exhibit a
preferred orientation with respect to a surface due to chemical and microscopic structural interactions [12]. Rubbing is the most conventional LC alignment process. When a thin polymer film (i.e., polyimide) is rubbed using a cloth, LCs tend to be aligned by surface interactions. A number of non-rubbing alignment techniques have been developed and explored including photoalignment, chemically treated surfaces, and oblique evaporation.

Reactive mesogens (RMs) are liquid crystalline materials with polymerizable end groups [13, 14]. Polymerization of RMs with two or more polymerizable groups leads to densely crosslinked networks in which the liquid crystalline order is permanently fixed. Fig. 2.1(c) shows the two main kinds of reactive mesogens: main-chain and side-chain RMs. Many interesting applications using these RM materials have been suggested [15, 16, 17]: for example, optical compensation films for LCDs, reflective color filters, and micro-actuators for biomedical applications. When reactive mesogens are used to form the polarization grating, the grating structure can be permanently fixed in a thin polymer film via photo-polymerization.

## 2.2 Polarization Gratings

Polarization gratings (PGs) locally modify the polarization state of transmitted light, which is achieved by spatially varying birefringence and/or dichroism. Among many possible types and creation methods, in this work, we utilize the “circular PGs” created by polarization holography and recorded on polymer or LC materials. [18]

The local optical axis orientation at a point \((x, y, z)\) in a PG follows \(\alpha = -\pi x / \Lambda + \alpha_0\).
Figure 2.2: Structure and diffraction properties of the Polarization Grating (PG): (a) Top-view and side-view geometry of the continuous, in-plane configuration of the nematic LC with a periodic linear birefringence; (b) Polarization diffraction from a PG when input is unpolarized light; Diffraction behavior with (c) RCP and (d) LCP polarized incident light; (e) Incident light transmits on-axis when a PG is on-state (with applied voltage). This forms a periodic structure in the \( x \) dimension, which is homogeneous in the other two dimensions. In our work, we use nematic LC materials with positive birefringence, and take \( \alpha_0 = \pi/2 \), as shown in Fig. 2.2(a). The the transfer matrix is then

\[
T(\vec{x}) = \cos \frac{\zeta}{2} I + i \sin \frac{\zeta}{2} \begin{bmatrix} 
\cos -2\alpha & \sin -2\alpha \\
\sin -2\alpha & -\cos -2\alpha
\end{bmatrix} = \cos \frac{\zeta}{2} I + i \sin \frac{\zeta}{2} \begin{bmatrix} 
\cos \frac{2\pi x}{\Lambda} & -\sin \frac{2\pi x}{\Lambda} \\
-\sin \frac{2\pi x}{\Lambda} & -\cos \frac{2\pi x}{\Lambda}
\end{bmatrix}
\] (2.1)

where \( \zeta \) is the relative phase retardation due to the birefringence. The electric field of the \( m^{th} \) order diffraction is

\[
D_m = \frac{1}{\Lambda} \int_0^\Lambda T(x) E_{in} e^{-i \frac{2\pi m x}{\Lambda}} dx
\] (2.2)

\[
D_m = \Gamma_m E_{in}, \quad \Gamma_m = \frac{1}{\Lambda} \int_0^\Lambda T(x) e^{-i \frac{2\pi m x}{\Lambda}} dx
\] (2.3)

Doing the integral in Equation (2.3), we get \( \Gamma_0 = \cos \zeta I, \Gamma_{\pm 1} = \frac{1}{2} \sin \frac{\zeta}{2} \begin{bmatrix} 1 & \mp 1 \\
\mp 1 & -i \end{bmatrix} \), and
Figure 2.3: (a) Photograph of transmissive PG showing the first- and zero-order diffraction images of a lamp arranged behind it, (b) similar to (a), but the camera is focused on the lamp and images are better resolved, (c) photograph of lamp alone, without PG.

$\Gamma_m = 0$ ($m \neq 0, \pm 1$). Thus, a PG has only three orders of diffraction, and the relative intensity depends on the polarization of the input lightwave. If we carefully select the PG thickness so that the effective retardation is half wave, then 100% diffraction efficiency into the first order(s) can be achieved, as shown in Fig. 2.2(c,d). The diffraction angles are determined by the well-known grating equation, since the LCPG is merely a birefringent grating.

$$\sin \theta_m = \left( m \frac{\lambda}{\Lambda} \right) + \sin \theta_{in},$$
(2.4)

where $\theta_{in}$ is the incident angle, $\theta_m$ is the angle of diffraction of transmitted light, and $m = \{-1, 0, +1\}$ is the diffraction order. For the non-diffracting case (Fig. 2.2(e)), an applied voltage much greater than a voltage threshold will reorient the LC director out of the plane and reduce the effective birefringence toward zero ($\Delta n \to 0$). Because of the properties described above, PGs can efficiently diffract circularly polarized light to either zero or first orders, based on the polarization handedness of the input light and applied voltage.

Fig. 2.3 shows the diffraction property of a PG fabricated on a transmissive substrate. When the incident light is broadband and unpolarized, the PG makes three diffraction orders with chromatic dispersion on the two first orders. The structure of a PG is comprised of an in-plane, uniaxial birefringence that varies with position (i.e. $n(x) = [\sin(\pi x/\Lambda), \cos(\pi x/\Lambda), 0]$), where $\Lambda$ is the grating period.
2.3 Jones Calculus

Light propagation in a birefringent medium can be described as a linear superposition of two normal modes (i.e., ordinary and extraordinary waves), that are defined by phase velocities and directions of polarization [19]. In 1941, R. C. Jones invented a simple way, *Jones calculus* [20], to describe polarized light by a $2 \times 1$ Jones vector ($\mathbf{E}$) and linear optical elements by $2 \times 2$ Jones matrices ($\mathbf{J}$). The polarization of the emerging light from an optical element can be found by taking the product of these two vectors. We will apply the Jones calculus to find the resulting polarization of light passing through three different polarization elements: linear polarizers, waveplates, and polarization gratings.

We consider incident light with a polarization state described by the Jones vector $\mathbf{E}_{in} = [E_x, E_y]^T = [E_{0x}\exp(i\delta_x), E_{0y}\exp(i\delta_y)]$ where $E_{0x}$ and $E_{0y}$ are the amplitudes and $\delta_x$ and $\delta_y$ are the phases. The intensity of light is calculated as $I = \mathbf{E} \cdot \mathbf{E}^*$. The Jones matrix for a linear polarizer having its easy axis at $\phi$ is given by

$$
\mathbf{J} = \mathbf{R}(-\phi) \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \mathbf{R}(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}
$$

(2.5)

where $\mathbf{R}$ is a rotation matrix. Similarly, we can write the Jones matrix for an arbitrary waveplate as follows

$$
\mathbf{J} = \mathbf{R}(-\phi) \cdot \begin{bmatrix} \exp(-i\Gamma) & 0 \\ 0 & \exp(i\Gamma) \end{bmatrix} \cdot \mathbf{R}(\phi)
$$

(2.6)

where $\Gamma = \pi \Delta nd/\lambda$ is the retardation of the waveplate ($\Delta n$ - birefringence, $d$ - thickness). The polarization state of light passing through a stack of multiple elements also can be found by a simple multiplication of the individual Jones matrices: $\mathbf{E} = \mathbf{J}_N \cdot \mathbf{J}_{N-1} \cdots \mathbf{J}_1 \cdot \mathbf{E}_{in} = \mathbf{J} \cdot \mathbf{E}_{in}$.

Finally, we will find the analytic solutions for the polarization gratings. The polarization grating can be treated as a planar waveplate which optic axis is periodically varying along the grating direction. Under the paraxial (small angle) approximation, the Jones
matrix for such a medium is given by

\[
\mathbf{J}(x) = \mathbf{R}(-\theta) \cdot \begin{bmatrix}
\exp(-i\Gamma) & 0 \\
0 & \exp(i\Gamma)
\end{bmatrix} \cdot \mathbf{R}(\theta)
\]

\[
= \cos \Gamma \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} - i \sin \Gamma \begin{bmatrix}
\cos 2\theta & \sin 2\theta \\
\sin 2\theta & -\cos 2\theta
\end{bmatrix}
\]

(2.7)

where the azimuthal angle of the optic axis is defined as \( \theta = \theta(x) = \pi x / \Lambda \).

The polarization of the emerging light can be found as \( \mathbf{E}(x) = \mathbf{J}(x) \cdot \mathbf{E}_m \) for \( \mathbf{E}_m = [E_x, E_y]^T \). We can express the far-field electric field of the diffraction order \( m \) as follows

\[
\mathbf{D}_m = \frac{1}{\Lambda} \int_{0}^{\Lambda} \mathbf{E}(x) \exp \left( \frac{-i2\pi mx}{\Lambda} \right) dx 
\]

(2.8)

We can now solve for the diffraction efficiency as \( \eta_m = |\mathbf{D}_m|^2/|\mathbf{E}_m|^2 \):

\[
\eta_0 = \cos^2(\Gamma) 
\]

(2.9a)

\[
\eta_{\pm 1} = \frac{1 \mp S_3'}{2} \sin^2(\Gamma) 
\]

(2.9b)

### 2.4 Fabrication of Liquid Crystal Polarization Gratings

We have successfully fabricated polarization gratings that manifest ~100\% efficiency, high polarization sensitivity, and low incoherent scattering by adopting polarization holography and photo-alignment techniques. In this section, we will review the fabrication details and material-related issues.

Holography is a photographic technique to create an interference pattern using multiple beams of coherent light [21]. While most conventional holography uses an intensity modulation, polarization holography involves a modulation of the polarization state that interference of light with different polarization. The spatially varying birefringence of the PG can be interpreted as the interference of two beams with orthogonal (left- and
right-handed) circular polarization (Fig. 2.4):

\[
E_L = \frac{E_0}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \exp \left(\frac{i\pi x}{\Lambda}\right) \quad (2.10a)
\]

\[
E_R = \frac{E_0}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} \exp \left(-\frac{i\pi x}{\Lambda}\right) \quad (2.10b)
\]

where \(\Lambda = \lambda_R/(2\sin \theta)\) is the modulation period, \(\lambda_R\) is the wavelength of the recording beams, and \(\pm \theta\) is the incident angle of the recording beams. The Jones vector of the result polarization is given by \(\mathbf{E} = \mathbf{E}_L + \mathbf{E}_R = \sqrt{2}E_0[\cos \left(\frac{\pi x}{\Lambda}\right), \sin \left(\frac{\pi x}{\Lambda}\right)]^T\). To capture the polarization hologram, we use thin films of photo-alignment materials that transfer their anisotropy to a liquid crystal layer via surface alignment of the LCs.

We utilize a linear photopolymerizable polymer (LPP) that presents a strong orientational photo-chemical reaction in response to the local direction of linearly polarized UV light [22, 23]. When the LPP is exposed with a linearly polarized UV light, it manifests intermolecular reaction and anisotropic molecular configuration. This alignment technique allows to generate high resolution azimuthal LC director patterns. A proper choice of the exposure fluence \((J/cm^2)\) is necessary to obtain the strong alignment condition that is generally dependent on the surface pattern and LC material properties.

We have developed fabrication processes for high-quality PGs for switchable LC-cells...
and polymer-films as shown in Figs. 2.5(a) and 2.5(b). First, we prepare a thin film of LPP on a substrate by spin-coating. Then, the substrate is exposed to a polarization hologram from a UV laser (i.e., HeCd, 325 nm) with orthogonal circular polarizations (RCP, LCP) as shown in Fig. 2.4.

For switchable LC-cells (LCPGs), we prepare two ITO-substrates coated with a thin LPP layer and make a cell with a uniform gap (a few \( \mu m \)). After UV exposure to the polarization hologram, we fill the cell with liquid crystals, typically above the isotropic temperature. The elastic and electro-optical properties of LCPGs have been investigated [24]. We also use a polymerizable liquid crystal (reactive mesogen, RM) to form polymer-PGs (RMPGs). We coat a thin layer of RM on a LPP substrate that is exposed to the polarization hologram, and we photo-polymerize the RM layer with a blanket UV exposure to fix the anisotropic patterns. Multiple layers of thin RM films can be added on the top until it reaches a desired thickness.

Fig. 2.6(a) shows an experimental setup for PG characterization. The polarization state of incoming light (1550 nm) is changed from linear polarized state to circularly polarized state after passing through the quarter-wave plate. To select the direction of the first orders, a half-wave plate can be used for changing the handedness of circular polarization. An IR detector measures the far-field intensity at each diffraction order to calculate the efficiencies and transmittance of PGs. Fig. 2.6(b) shows dynamic response of an experimental LCPG. RMS voltage is creased from approximately 0V to 10V, and a
Figure 2.6: (a) Photograph of the characterization setup to measure the power of diffraction orders. Note that the input beam is linearly polarized, and after passing through quarter-wave plate, the beam is changed to circularly polarized light (RCP or LCP). Depending on an applied voltage on a LCPG (On/Off), the beam can be diffracted to zero- or first-order; (b) The dynamic response of an experimental LCPG, showing response times less than 2 msec. (from Ref. [3, 1])

switching time of both field On and Off conditions are recorded. The response time was less than 2 msec.

Table 2.1 shows characterization data of PGs [1]. We utilized RM mixture RMS03-001C (Merck, Δn=0.14 at 1550 nm) for RMPGs, and nematic-LC MDA-06-177 (Merck, Δn=0.13 at 1550 nm) for LCPGs. The thickness of the LCPGs is ~6 µm for half-wave effective PG retardation, Δnd = λ/2. \(I_{in}\) is the input power and \(I_m\) is the diffracted power of order \(m\). \(T_m = I_m/I_{in}\) is the transmittance of order \(m\) and \(I_{ref}\) is a reference power of substrate (or cell).\(η^a_m = I_m/I_{ref}\) is the absolute diffraction efficiency, and \(η^i_m = I_m/(I_{-1} + I_0 + I_{+1})\) is the intrinsic diffraction efficiency.

<table>
<thead>
<tr>
<th>Type of PGs</th>
<th>Diff. Angle (deg)</th>
<th>(I_{in}) (mW)</th>
<th>(I_{ref}) (mW)</th>
<th>(I_{+1}) (mW)</th>
<th>(I_0) (mW)</th>
<th>(I_{-1}) (mW)</th>
<th>(T_{+1}) (%)</th>
<th>(η^a_{+1}) (%)</th>
<th>(η^i_{+1}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMPG</td>
<td>±5</td>
<td>41.32</td>
<td>37.91</td>
<td>37.83</td>
<td>0.07</td>
<td>0.01</td>
<td>91.4</td>
<td>99.6</td>
<td>99.8</td>
</tr>
<tr>
<td>RMPG</td>
<td>±10</td>
<td>41.32</td>
<td>37.91</td>
<td>37.44</td>
<td>0.07</td>
<td>0.01</td>
<td>91.1</td>
<td>99.3</td>
<td>99.8</td>
</tr>
<tr>
<td>LCPG</td>
<td>±5</td>
<td>41.71</td>
<td>35.68</td>
<td>35.63</td>
<td>0.08</td>
<td>0.01</td>
<td>85.3</td>
<td>99.7</td>
<td>99.8</td>
</tr>
<tr>
<td>LCPG</td>
<td>±10</td>
<td>41.82</td>
<td>35.68</td>
<td>33.71</td>
<td>0.11</td>
<td>0.005</td>
<td>80.6</td>
<td>94.5</td>
<td>99.7</td>
</tr>
</tbody>
</table>
Chapter 3

Advanced Polarization Holography for Geometric Phase Holograms

Geometric Phase Holograms (GPHs), including polarization gratings, can be fabricated by photo-alignment technique and polarization holography. The polarization holography produces spatially varying linear polarization fields with a constant intensity using two laser-beams interference of orthogonal circular polarization. Conventional holography recording setups require comparably large area and are not commercially viable for creating gradual changes of the linear polarization fields, which are required to fabricate very large grating periods for the polarization gratings. Moreover, there was no practical way to realize and control arbitrary change of the linear polarization fields that leads a freedom of diffraction on the optical elements.

In this chapter, we introduce and demonstrate new polarization holography techniques that create scalable period polarization holograms and arbitrarily changed polarization fields that produce various types of GPHs. Then, we show a novel method that offers proximate lithography with easily tunable-periods. Finally, we demonstrate multi-axis polarization gratings that make multiple diffraction at different azimuthal angles.
3.1 Scalable Period Polarization Holograms via Michelson Interferometric Setup

Polarization Gratings have been fabricated by the interference of two orthogonally polarized beams. An active area of PG fabricated depends on the size of the beam, and the polarizing optics. With conventional approaches, the relative distance between these optics increases as larger active area is needed, that limits the range of grating period achievable, and is not commercially viable. We introduce a new holographic technique for producing polarization gratings utilizing modified Michelson interferometric configuration. The approach can be scaled for different beam-sizes and grating-periods while maintaining compact size of the setup.

Fig. 3.1(a) describes a schematic illustration of the conventional polarization holography setup, and Fig. 3.1(b) shows actual optics setup of the approach. First, a linearly polarized UV laser beam is split by the Non-Polarizing Beam Splitter (NPBS) and the split beams are redirected by Mirrors (M). After the beams pass through the Quarter-Wave Plates (QWP) with +45° and −45° axis, the polarization state of the beams changes to orthogonally (left- and right-handed) circular polarization. These two beams are overlapped on the sample and make an interference pattern that is used to create holographic grating structures on the sample as described in Chapter 2. When the beams are placed

Figure 3.1: (a) Conventional polarization holography setup: (a) a schematic view of the UV holography setup; (b) a picture of the actual optics setup in the lab. The UV laser is split by a Non-Polarizing Beam Splitter (NPBS) and each beam is redirected by a Mirror (M). After passing through a Quarter-Wave Plate (QWP), the polarization state of the beam changes to circular polarization.
Table 3.1: The recording length (L) of the conventional holographic setup for various diffraction angles and grating periods of PGs when the recording beam’s wavelength (λ) is 0.325 µm and the distance (D) between two redirect mirrors is 100 mm.

<table>
<thead>
<tr>
<th>Diffraction angle of PGs (at 1550 nm)</th>
<th>Grating Periods of PGs (Λ)</th>
<th>Recording length required (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°</td>
<td>4.5 µm</td>
<td>1.4 m</td>
</tr>
<tr>
<td>10°</td>
<td>8.9 µm</td>
<td>2.7 m</td>
</tr>
<tr>
<td>2°</td>
<td>44.4 µm</td>
<td>13.7 m</td>
</tr>
<tr>
<td>1°</td>
<td>88.8 µm</td>
<td>27.3 m</td>
</tr>
</tbody>
</table>

in the x, y-plane and projected on the sample with an angle of $\theta_R$ and $-\theta_R$ from the z-axis, they generate spatially varying linear polarization fields in the x-direction with a periodicity determined by Bragg’s Law:

$$\Lambda = \frac{\lambda}{2\sin(\theta_R)},$$

(3.1)

where $\Lambda$ is the grating period and $\lambda$ is the recording beam’s wavelength. We used HeCd UV-laser ($\lambda = 325$ nm) and UV-AR coated optics having 75∼100 mm (3∼4 inch) diameter suitable for making up to 50∼75 mm (2∼3 inch) polarization holograms. The conventional setup can make various grating periods as changing the angle $\theta_R$ of the beams, and practically the angle should be controlled by the position of the sample that is related to the recording length (L) in the schematic illustration. The distance (D) between two mirrors limits the maximum achievable active area of the polarization hologram and the distance cannot be smaller than the optics diameter used in the setup, and therefore here we set the distance (D) as 100 mm. In this case, the recording angle $\theta_R$ is only governed by the recording distance when,

$$\theta_R = \tan\frac{D}{2L}.$$  

(3.2)

Table. 3.1 shows the estimation of the recording length (L) for various grating period (Λ) of PGs when $D = 100$ mm. Here we note several PGs having 1∼20° diffraction angles at 1550 nm and the grating periods of the PGs are in the range of 5∼90 µm that are used
for various PG-based applications shown in Chapters 4 and 5.

The longer recording length ($L$) is necessary to make larger grating period ($\Lambda$), but it is not practically feasible to set the setup for very large grating period holograms. For example, to create >$100$ $\mu$m grating period of PGs, the setup requires >$30$ m recording length that is hard to get on the setup optical table. In order to get that long recording length, we can place several mirrors on the setup table to redirect the both recording beams, but it is also not practically good way because >$10$ mirrors are required to get the length, >$30$ m.

3.1.1 Modified Michelson Interferometer Setup

To overcome the above limitation, we proposed a new holography approach that is scalable, and can reduce the exposure distance significantly by utilizing Michelson interferometric configuration as shown in Fig. 3.2(a). The holographic method utilizes an NPBS to control the angle between the two recording beams on the sample. This is illustrated in Fig. 3.2(b) where the rotation of the NPBS by an angle $\theta_{BS}$ causes the incident angles of both recording beams to change by $2\theta_{BS}$ and $-2\theta_{BS}$. In this case, therefore, the recording angle $\theta_R$ of the polarization hologram will be $2\theta_{BS}$, and the grating period $\Lambda$.

![Figure 3.2: (a) New polarization holography setup: (a) a schematic view of the Michelson interferometric holography setup; (b) schematic illustration of the incoming and outgoing beams in the NPBS where $\theta_{BS}$ is a rotation angle of NPBS and $\theta_R$ is the recording angle.](image-url)

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Table 3.2: Various grating periods of PGs that can be created by new interferometric holography setup when the recording beam’s wavelength ($\lambda$) is 0.325 $\mu$m.

<table>
<thead>
<tr>
<th>NPBS Angles ($\theta_{BS}$)</th>
<th>Recording Angle of New Setup ($\theta_R$)</th>
<th>Grating Periods of PGs ($\Lambda$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5°</td>
<td>3°</td>
<td>3 $\mu$m</td>
</tr>
<tr>
<td>0.5°</td>
<td>1°</td>
<td>9 $\mu$m</td>
</tr>
<tr>
<td>0.25°</td>
<td>0.5°</td>
<td>18 $\mu$m</td>
</tr>
<tr>
<td>0.05°</td>
<td>0.1°</td>
<td>90 $\mu$m</td>
</tr>
</tbody>
</table>

of the polarization holograms can be derived as below.

$$\Lambda = \frac{\lambda}{2 \sin(2\theta_{BS})},$$

where $\lambda$ is the wavelength of the recording beam and $\theta_{BS}$ is the rotation angle of the NPBS. Table. 3.2 shows examples of the grating periods obtained by the new holography setup. Here we shows <100 $\mu$m grating period examples but far longer period (e.g. few cm) can be obtained by the new setup.

### 3.1.2 Fabrication and Demonstration

This method allows for easy adjustment of holograms for various grating periods without modifying any position of optics. We experimentally fabricated PGs having larger than 100 $\mu$m grating period within 0.3 m $\times$ 0.3 m space. This physical dimension of the new setup is extremely smaller than the required dimension of the conventional setup. Moreover, large active area of holograms can be obtained without increasing the distance between optics. This new setup only needs to use larger size optics to get larger active area of holograms, but the conventional setup demands more room (e.g. longer distance D in Fig. 3.1(a)) as utilizing larger size optics.

Fig. 3.3(a) shows a picture of the Michelson interferometric setup illustrating polarization states of the beams on the path of the setup. An input light source, HeCd UV laser, is Linear Vertical Polarized (LVP) and it is converted to RCP after passing through a QWP with fast axis oriented at $+45^\circ$. Then, the beam is split by the NPBS.
Figure 3.3: (a) A picture of Michelson interferometric holographic setup; (b) polarization microscope images of PG samples: $\Lambda=10\mu$m (upper sample), $\Lambda=50\mu$m (bottom sample).

and one beam (red) reserves its polarization state when it passes through the NPBS, while the other beam (blue) has orthogonal polarization state when it is reflected by the NPBS. After both beams are reflected by mirrors, they are recombined on the sample area by the same NPBS, but the red beam experiences a QWP twice upon reflection from the mirror and it (red) shows opposite circular polarization states from the other beam (blue). Therefore, the new holography setup can produce polarization holograms with the orthogonally polarized beams overlapped on the sample area. Fig. 3.3(b) shows sample pictures taken by polarization microscope. Those samples can be fabricated with simple adjustment of the rotation angle of the NPBS.

We demonstrated the new polarization holography technique based on Michelson interferometer configuration. The setup can create scalable period polarization holograms while maintaining compact size of the setup. Moreover, the setup can produce large size holograms as utilizing large size optics without increase the distance between the optics. We fabricated several PG samples having different grating periods within $0.3\,\text{m} \times 0.3\,\text{m}$ space. Specially, the technique can make it possible to fabricate very large grating period polarization holograms with a small optics setup.
3.2 Arbitrary Phase Polarization Holograms based on Interferometric Holography Methods

When the polarization of an electromagnetic wave experiences a continuous sequence of transformations via a closed loop, the wave achieves a relative phase shift, known as a geometric phase, that depends on the geometry of the polarization path [25, 26]. When a wave is subjected to transversely inhomogeneous polarization transformations, the associated inhomogeneous geometrical phases will induce a wave front reshaping. This wave front control over the optical geometric phase is different way of beam shaping compared to a spatial control of classical diffractive optics based on the optical path difference. Optical diffractive elements based on spatial control of the geometric phase have been studied but previous studies were limited by the fact there was no practical way to realize and control the spatially varying polarization elements [27, 28]. Here we suggest the effective method to generate Geometric Phase Holograms (GPHs) that can modulate arbitrary changes of local polarization utilizing interferometric holography technique and liquid crystal materials. We fabricated and demonstrated GPHs as liquid crystal diffractive optics working as polarization-sensitive diffractive gratings, lenses, micro-lenses, axicons and azimuthal waveplates with excellent optical quality that manifest nearly 100% efficiency, polarization-selectivity, and fast electro-optical switching (an order of milliseconds).

3.2.1 Polarization Holography Setup

The spatial control of light wavefront has been done by locally varying its optical path length, which is governed by the scalar diffraction. Control over vector properties of light (i.e., polarization) can lead to another degree of freedom in beam shaping. The optical geometric phase, often called the Pancharatnam-Berry phase, is one way of changing light phase by polarization modulation but with no optical path difference in a sense of the classical wave theory. Since the geometric phase is induced by the difference in polarization paths on Poincare sphere, the wavefront uniformly propagates in time while experiencing phase variation.

To realize the arbitrary phase GPHs, it is essential to be able to pattern polarization elements with arbitrary orientations. Several different fabrication methods, primarily based on micro-lithographic techniques, were suggested to use subwavelength
features [29] and liquid crystal materials [30]. These approaches, however, suffer from fabrication difficulties and poor optical performance. Our approach is to use polarization holograms to pattern polarization elements as shown in Fig. 3.4. We [18, 3] and two other groups [31, 32] reported high-quality polarization gratings that manifest nearly ideal diffraction properties by using the two-beam interference of orthogonal polarizations. Any given wavefront can be effectively recorded as a polarization hologram when the wave is superimposed with a plane wave reference with the same intensity and orthogonal circular polarizations.

We employ a holography platform based on a Michelson interferometer setup (Fig. 3.4) to create in-line polarization holograms of an object. The modified Michelson interferometer setup allows two separate light paths with orthogonal circular polarization beams. When an object (e.g., a volume optics) is placed in one of the light paths, the phase difference ($\Delta \phi$) between two recording beams leads to variation in the linear polarization orientation ($\Delta \phi / 2$) as a result of the polarization interference. To capture the polarization interference pattern, we utilized a linear-polarization photopolymerizable polymer (LPP) [23] material that undergoes anisotropic cross-linking with linear polarization illumination. The exposed LPP was then used as a surface alignment layer for liquid crystals to actually form the device in volume. The use of liquid crystal materials along with LPP
Table 3.3: Transformation of Conventional Optics to GPHs

<table>
<thead>
<tr>
<th>Conventional Optics</th>
<th>Spatial Phase Profile $\gamma(X)$</th>
<th>GPHs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism</td>
<td>$\pi x/\Lambda$</td>
<td>Polarization Grating</td>
</tr>
<tr>
<td>Lens</td>
<td>$\pi r/\Lambda(r)$</td>
<td>Polarization Lens</td>
</tr>
<tr>
<td>Axicon</td>
<td>$\pi r/\Lambda$</td>
<td>Polarization Axicon</td>
</tr>
<tr>
<td>Spiral Phase Plate</td>
<td>$l\phi + \alpha_0$</td>
<td>Azimuthal Waveplate</td>
</tr>
<tr>
<td>Prism+Spiral Phase Plate</td>
<td>$\pi x/\Lambda + l\phi$</td>
<td>Forked Polarization Grating</td>
</tr>
</tbody>
</table>

provides a unique and effective way to develop the polarization hologram created from superimposed the recording/reference beams with orthogonal circular polarizations.

The proposed holography setup allows a direct transformation of a conventional optics to a geometric phase optics based on the geometric phase. We list examples of the transformation of conventional optics to GPHs with corresponding spatial phase profiles in Table 3.3. The phase profile $\gamma(X)$ is created by a phase difference between the reference beam (plane wave) and the recording beam (refracted by a conventional optics), and the GPHs can be described by the space-variant Jones Matrix $M(X)$ [33] as shown in Eq. 3.4.

$$M(X) = \begin{bmatrix} \cos \gamma(X) & \sin \gamma(X) \\ \sin \gamma(X) & -\cos \gamma(X) \end{bmatrix}$$ (3.4)

3.2.2 Polarization Lens Fabrication

Polarization grating (PG) can be formed when the two circularly polarized recording beams interfere with an angle by a prism. A grating period $\Lambda = \lambda_R/\sin \theta$ is determined by a split angle ($\theta$) of the prism when $\lambda_R$ is a recording beam wavelength. In this case, the angle $\theta$ of the prism is consistent over the area interfered so the phase profile ($\pi x/\Lambda$) of the PG changes linearly on x-axis (i.e., grating vector). Polarization Lens (PL) is formed by a hologram of a conventional lens manifesting the phase profile, $\pi r/\Lambda(r)$, where $r$ is a distance from a center of the hologram. The period varies according to the distance $r$; $\Lambda(r) = \lambda_R\sqrt{FL^2 + r^2}/r$ where $FL$ is a Focal Length of the lens. In this case, the phase profile of the PL is gradually changed from the center, and the same phase can be
Figure 3.5: Phase variation of (a) Polarization Grating (PG) and (b) Polarization Lens (PL); (c) and (d) are the phase profiles redrawn within \( \{0 - \pi\} \) phase; (e) and (f) are simulated intensity profiles of the PG and the PL between crossed polarizers respectively.

found at the position having the same distance from the center. We simulated the phase profiles of the PG and the PL over the spatial variation as shown in Figs. 3.5(a) and (b). To express local optical-axis profiles of the elements along the spatial variation (e.g., x-axis), we redrew the phase variations within \( \{0 - \pi\} \) as shown in Figs. 3.5(c) and (d).

Based on the redrawn phase profiles, intensity profiles of the elements between crossed

Figure 3.6: Lens behavior of polarization lens (a) converging RCP input and (b) diverging LCP input.
polarizers can be illustrated as Figs. 3.5(e) and (f).

The intensity profile of a PG is the same as what we have shown in previous section. In order to verify the intensity profile of a PL, we fabricated the PBOE with two different conventional lenses using the interferometric holography setup. The fabrication process includes the following three steps: (i) LPP-coating on a substrate; (ii) hologram exposure; (iii) Spin-coating Liquid Crystal Polymers. We first prepared a thin (~70 nm) layer of a LPP material (ROP103/2CP, Rolic Technologies Ltd.) on a cleaned glass substrates (VIS-AR glass, PG&O) by spin coating at 3000 rpm for 30 seconds with subsequent pre-baking at 130°C for 10 minutes. Then, the sample was exposed at ~5J/cm² with two recording beams from a UV laser (HeCd, λ = 325 nm) using the modified Michelson interferometer setup. Volume optics (e.g. lens) were placed in the recording beam path (UV-graded plano-convex lenses with f = 100 mm and 300 mm, and D = 25 mm). As the last step, the exposed sample was coated with LCP (RMS09-025, Merck, Δn ≃0.33 at 589 nm) using spin-coater (Laurell Technologies Corp.) at the room temperature. Then the LCP layer is polymerized with UV light (UV-wand with 350 nm peak, Edmund Optics) for 3 minutes. In order to get proper thickness (i.e., retardation) we can coat several more layers on the sample. We fabricated two PLs, and their polarizing microscope images are shown in Figs. 3.7(a) and (b). The polarizing microscope images show the intensity profiles of the PLs placed between two crossed linear polarizers, which present a sinusoidal zone plate profile showing confocal rings.

The LC alignment profile of the PL can be checked with polarizing microscope im-

![Figure 3.7](image-url)

**Figure 3.7:** Polarizing microscope image of GPHs placed between two crossed linear polarizers. The samples was made by convex lenses having (a) 100 mm and (b) 300 mm focal lengths.
ages. We changed optical axis of the top polarizer with different angles $0^\circ$, $45^\circ$, $90^\circ$, and $135^\circ$, while the optical axis of the bottom polarizer is fixed. The corresponding microscope images are shown in Figs. 3.8(a)-(d). The pictures show continuous change of the sinusoidal intensity profile of the LC alignment, which presents a periodic change of the image profile by an angle $\pi$ ($180^\circ$). We could not find any significant misalignment or any defect from the intensity profile and this means the PL has nearly ideal alignment pattern that was exposed by polarization hologram with LC alignment technology.

The polarization lens can be considered as the polarization hologram of volume optics while a sinusoidal zone plate is equivalent to the transmission hologram. Although both diffractive optics have the same focal length, their diffraction properties quite differ from each other. Unlike classical zone plates, the PLs diffract only in the first orders. When the PL function as lens, one of the first orders has focusing property while the other defocusing (i.e., $+1$-order: focusing, $-1$-order: diverging). The diffraction efficiencies are determined by the input polarization and the retardation of the medium as described in Chapter. 2. It must be noted that the PL can act as a converging (diverging) lens depending on the input circular handedness when the retardation corresponds to a half wave. We also note that the first orders have orthogonal circular polarization states regardless the input polarization.

The diffraction property of the PL is shown in Fig. 3.9. The input source was white LED (ocean optics) and it was collimated after passing through several lenses. The polarization state of the beam was one of the (right- and left-handed) circular polarization, which was controlled by a linear polarizer (Edmunds Optics) and an achromatic quarter wave plate (ColorLink Japan). Figs. 3.9(a) and (b) show the output beams of the PL,
Figure 3.9: Polarization-selective lensing property of the PBOE sample (PL). The PL works as both (a) positive and (b) negative lens for two different (right- and left-handed) circular polarization input. The (c) focused and (d) diverged input light are captured at the same distance from the PL.

which show a positive and a negative lens property. We took the pictures of the output beam on a screen placed parallel to the beam direction. For right-handed circular polarization input, the beam is focused as shown in Fig. 3.9(a), while the beam is diverged when the input is left-handed circular polarization as shown in Fig. 3.9(b). The Figs. 3.9(c) and (d) are the focused and diverged beams captured at the same distance from the PL. We measured the PL’s focal length \( f \approx 50 \text{ mm} \) with a red laser (633 nm wavelength). High diffraction efficiency (\( \geq 98 \% \)) of the focused output was calculated by measuring intensity at the focus, and normalized by the substrate absorption. We note that there was no significant high orders and scattering on the output beam.

We also fabricated another type of PL using an axicon which is a specialized type of lens having a conical surface. The axicon can be used to convert a parallel laser beam into a ring, focus a parallel beam into long focus depth, or to turn a Gaussian beam into a non diffractive Bessel beam. The axicon-PL was formed by a hologram of conventional axicon showing the phase profile, \( \pi r/\Lambda \). The phase profile of the axicon-PL is similar to the one of the PL but the phase changes linearly from the center since the period \( \Lambda \) is constant in this case. The period \( \Lambda = \lambda_R/\sin \theta \) is governed by a recording angle \( \theta \) when \( \lambda_R \) is a recording beam wavelength. We used a UV-grade plano-convex axicon (Magic Photonics) shown in Fig. 3.10, which has 25 mm diameter and 178° apex angle producing 0.7° recording angle with the reference beam (plane wave). We followed the same fabrication process of the other GPHs for the axicon-PL, but the thickness (retardation) of LC layers is optimized.
Figure 3.10: (a) a picture of axicon used to make a GPH (axicon-version PL). (b) polarizing microscope image of the axicon-PL placed between two linear polarizers; overall pattern looks like confocal rings having uniform gap. (c) output beam (633 nm, circularly polarized) of the axicon-PL, which was captured by a screen.

for a red laser (half-wave at 633 nm). The polarizing microscope image of the axicon-PL is shown in Fig. 3.10(b); The intensity profile is similar to the one of the PL but the distance (i.e., period) between nearest two lines is constant for the all region of the element. The measured period of the axicon-PL is roughly 13 µm and this is what we can expect from the recording angle 0.7° of the conventional axicon. When the input beam (633 nm laser) passes through the center of the axicon-PL and the input polarization is one of the circular polarization, the beam is diffracted with the same angle and makes a single ring pattern as shown in Fig. 3.10(c). In this case, the input beam’s diameter was nearly 1 mm and the diameter of output ring was 40 mm when the beam traveled nearly 1 m.

### 3.2.3 Polarization Micro-Lens Fabrication

We showed the modified Michelson interferometer holography setup to fabricate several GPHs such as polarization grating, polarization lens, and polarization axicon. Polarization holograms modulated by conventional optics (e.g., prism, lens, axicon) could be recorded on the sample area, when the modified recording beam could be projected with a reasonable distance between the optics and the sample; the recorded area (sample size) changes according to the distance between the optics and the sample. Therefore, the modified Michelson setup cannot project the modulated hologram with 1:1 scale, the setup may not reserve the recording angle information if a conventional optics makes
large variation of the refraction (e.g., hologram of micro-lens arrays having very small focal length). In order to resolve these problems, we propose another polarization holography setup that is based on modified Mach-Zehnder interferometer setup (MZ setup) as shown in Fig. 3.11. The MZ setup controls two beam paths (reference and recording) independently using two QWPs, and the refraction angle information of a conventional optics can be reserved by placing three identical lenses on the both beam paths. The linear polarized input light is split by the first NPBS, and both beams experience different optical elements causing different wavefront and polarization state. After passing through QWPs having $+45^\circ$ and $-45^\circ$ axis, the polarization states of the beams change to LCP and RCP respectively. A recording beam’s wavefront is particularly changed by an object (e.g., micro-lens arrays), and the wavefront information can be reserved and projected on the sample area as the light is guided by two identical lenses (FL = 100 mm, Thorlabs) which focal points are located at the same position. The other beam (reference beam) also travels two identical lenses like the recording beam, and the beam can keep and expose input wavefront (plane wave) on the sample area.

The MZ setup was used to fabricate a GPH formed by a conventional micro-lens arrays, and we call the GPH as a Polarization Micro-Lens (PML). The PML shows the same phase profile of the polarization lens, $\pi r/\Lambda(r)$, periodically. The period of PML also varies depending on the distance $r$. We used a $10\times10$ mm micro-lens arrays (Edmund Optics) with 300 $\mu$m pitch and 1.6° divergence angle. The Fig. 3.12(a) shows
Figure 3.12: Fabrication of a Polarization Micro-Lens arrays (PML); (a) microscope image of a micro-lens arrays showing grids of lens arrays, and SEM image of the lens arrays (sub-figure, provided by Edmund Optics). (b) polarizing microscope image of the PML placed between two crossed polarizers.

A microscope image showing grids of lens arrays (back-side) and a sub-figure represents a SEM image of the micro-lens arrays. The micro-lens was placed at the object position of the MZ setup, and we followed the same fabrication process of the other GPHs. As we can check the polarizing microscope image of the PML shown in Fig. 3.12(b), overall scale of the PML is matched to the dimension of the micro-lens arrays, and the intensity profiles of each lens array are almost the same each other.

The LC alignment profile of the PML was checked with polarizing microscope images as shown in Figs. 3.13(a)-(d). The microscope images were taken with different optical axis of the top polarizer (0°, 45°, 90°, and 135°), while the optical axis of the bottom polarizer is fixed. We can check a continuous change of the sinusoidal intensity profile of the LC alignment, and the pattern shows a periodic change of the image profile by an angle $\pi$ (180°). The PML intensity profile does not show any significant defect but the pattern has some misalignments mainly at the edge of each lens array. These misalignments may come from a multiple reflection of the lens arrays, which is caused by non-AR coating of the micro-lens, and the reflection can effect the hologram itself.

Similar to the property of PLs, the PMLs function as polarization sensitive micro-lens, which can focus and defocus an input beam depending on the handedness of circular polarization of the input light. Ideally the diffraction efficiency of the elements reaches nearly 100% when the retardation of the LC layers corresponds to a half wave for the input wavelength.
Figure 3.13: Polarizing microscope images of a PML placed between two linear polarizers; one polarizer (Blue arrow) is fixed and the other (Red arrow) has different optical axis of 0°, 45°, 90°, and 135°.

We fabricated GPHs as electrically switchable optical devices utilizing nematic liquid crystal for LC cells. The fabrication process of the switchable devices includes the following three steps: (i) cell assembly of two LPP-coated substrates; (ii) hologram exposure; (iii) LC filling. We first prepared a thin (∼70 nm) layer of a LPP material (ROP103/2CP, Rolic Technologies Ltd.) on two cleaned ITO-coated glass substrates (CB-90IN-0111, Delta Technologies Ltd.) by spin coating at 3000 rpm for 30 seconds with subsequent pre-baking at 130°C for 10 minutes. We assembled the two LPP substrates (facing to each other) with 1.7 μm spacers (Dana Enterprises Inc.) and optical glue (NOA63, Norland Products Inc.). The sample was then exposed at ∼5 J/cm² with a polarization hologram from a UV laser (HeCd, λ = 325 nm) using the modified interferometric holography setups. Volume optics were placed in one of the beam paths (recording beam): lens (a UV-graded plano-convex lens with f = 100 mm and D = 25 mm) and micro-lens array (a plano-convex microlens array with f = 5.1 mm and 300 μm pitch). As the last step, the exposed sample was filled with a nematic liquid crystal (MLD-03-874, Merck) slightly above its clearing temperature (at ∼130°C). We note that the cell thickness (d) was chosen for the maximum first-order efficiency at a green wavelength (i.e., d = 0.5λΔn⁻¹ ≈ 1.7μm for Δn = 0.155 and λ = 535 nm).
Figure 3.14: Polarizing microscope images of an electrically switchable PML placed between crossed polarizers with (a) 0 V (Off-state) and (b) 10 V (On-state) applied voltage on the PML.

Owing to dielectric anisotropy and elastic properties of liquid crystals, electro-optical switching of the GPHs is possible when an electric field (beyond a threshold value) is applied across the samples. As LC molecules are reoriented along the direction of the electric field, the effective retardation varies and the lensing effect can vanish with sufficiently strong applied fields. Fig. 3.14 shows polarization microscope images of the switchable PML placed between crossed polarizers with (a) 0 V-rms and (b) 10 V-rms ($V > V_{th}$) applied voltages across the sample. Fast switching times of nearly 8 ms were measured at the operation voltage, which are remarkably better than other LC-based lenses. It should be noted that the active area can be scaled with keeping the same electro-dynamic properties since the thickness remains same for a half wave retardation. We also note that the effective retardation can be tuned for the maximum efficiency at different wavelengths.

Since the GPHs are merely birefringent gratings, the diffraction angles are determined by the well-known grating equation, $\sin \theta_m = m\lambda/\Lambda$. If the GPHs are utilized in different wavelength of recording beams, local diffraction angles are changed and it leads overall change of GPH properties. In order to examine diffraction properties of GPHs in detecting wavelength ($\lambda_D$) when the GPHs are fabricated by recording wavelength ($\lambda_R$), simple relationship between recording and detecting wavelengths can be derived as below.

$$\frac{\sin \theta_R}{\sin \theta_D} = \frac{\lambda_R}{\lambda_D}, \quad (3.5)$$
where $\theta_R$ and $\theta_D$ are recording angles of hologram and local diffraction angle of GPHs at detecting wavelength respectively. If GPHs were used at higher wavelength than recording wavelength, their local diffraction angles would be larger than the recording angle. Therefore the GPHs can work as a prism showing larger splitting angle or a lens having larger F/# when the GPHs were used at higher wavelength than recording wavelength. For example, when a UV-grade prism having $3^\circ$ divergence angle is used to make a GPH with 325 nm Laser, the GPH would show nearly $10^\circ$ diffraction angle at 633 nm wavelength. With the consideration of the equation, we can fabricate a GPH having $\theta_D$ diffraction angle at $\lambda_D$ wavelength with proper conventional optics showing $\theta_R$ refraction angle at $\lambda_R$ wavelength.

So far, we have discussed different types of GPHs that can be fabricated by the proposed interferometric holography setups. Chromatic sensitivity of the optical linear retardation is considered as one major limitation of the GPHs. A pure geometric phase is achieved only when the initial and final polarization states are same everywhere, which only happens between orthogonal circular polarizations due to their singularity natures. Therefore, the retardation across the GPH should be equivalent to a half wave. Fortunately, there is at least one way to circumvent/reduce this limitation by retardation compensation techniques. We demonstrated achromatic polarization gratings with a significantly improved bandwidth (by a factor of $\sim 4$ to $\sim 7$) using twisted LC geometries [34] and stacked LC layers with relative spatial offsets [35]. The same designs can be applied to make achromatic GPHs that can operate with a broadband light source.

Conventional optics materials such as glass are not transparent outside of the visible range of the electromagnetic spectrum, and therefore optics cannot be fabricated easily (e.g., crystalline optics are opaque at $>20 \ \mu$m). Moreover, at many wavelength ranges that there are no materials with an index of refraction significantly larger than one. So our GPHs based on liquid crystal can solve this problem since it can be transparent, diffractive, and manufactured easily for almost every range of the spectrum. For example, GPHs (e.g., PL and PML) will focus/diffract light of many wavelengths to different focal lengths/angles, which means they can also be used to filter out unwanted wavelengths while focusing/diffracting the light of interest with proper angles.
3.2.4 Azimuthal Waveplate Fabrication

Now we discuss the last two GPHs listed in the Table. 3.3. Azimuthal Waveplate (AW) can be generated by a hologram of a spiral phase plate, which phase \((l\phi + \alpha_0)\) shows an azimuthal angular dependence, \(\phi = \arctan(y/x)\) when \(l\) is a charge number and \(\alpha_0\) represents a zero phase. The Forked Polarization Grating (FPG) can be produced by a polarization hologram that is generated by placing two conventional optics (prism and spiral phase plate) on the MZ setup at the same time. The FPG contains the property of two GPHs produced by the two conventional optics. A spatial phase profile of the FPG is also a combination of two phase profiles \((PG + AW)\), \(\pi x/\Lambda + l\phi\). The FPG can control an Orbital Angular Momentum (OAM) of lightwaves with high efficiency and better flexibility than current methods. The FPG can work as polarization-controlled OAM state ladder operators that raise or lower the OAM states (charge \(l\)) of incident lightwaves, by the topological charge \(l_q\) on the FPGs, to new OAM states (charge \(l \pm l_q\)). This feature allows us to generate, detect, and modify the OAM state with an arbitrary and controllable charge. An important application of FPGs are the essential state controlling elements in quantum systems based on OAM eigenstates, which may enable extreme high capacity quantum computation and communication. The main prospective utility includes particle manipulation [36] and quantum communication [37, 38, 39], however, the generation, manipulation, and detection of the OAM states is still fairly challenging in most cases.

An azimuthal waveplate is a birefringent plate that introduces a homogeneous half-

![Figure 3.15: Anisotropic structure of AWs: (a) \(q = +1/2\); (b) \(q = +1\); (c) \(q = +2\). Arrows illustrate the space-varying in-plane local optical axis.](image-url)
wave retardation to the light propagating through, with inhomogeneous optical axis in the transverse plane as shown in Fig. 3.15. It can work as an OAM generator for circular polarized light. The local in-plane optical axis follows $\alpha = q\phi + \alpha_0$, under half-wave retardation condition, omitting the imaginary unit,

$$T(\bar{x}) = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & -\cos 2\alpha \end{bmatrix}$$

(3.6)

for instance, if incident light is circular polarized (LCP/RCP) $E_{in} = E_0 \left(1 \pm i\right)$, then output is

$$E_{out} = T(\bar{x})E_{in} = E_0 e^{\mp i2\alpha} \left(1 \pm i\right)$$

(3.7)

which is also circular polarized (RCP/LCP) wave with an azimuthal-angle-dependent relative phase retardation $e^{\mp i2\alpha} = e^{\mp i2q\phi}$. That is, a $q$-charge AW can introduce a charge $\pm 2q$ phase singularity to the beam propagating through it.

Since the singularities of the beams that write the FPGs are introduced by AWs, the quality of the FPGs are highly dependent on the quality of the AWs. The azimuthal-varying photo-alignment is realized by relative rotation between the sample and the light polarization orientation. [40, 41] The linear polarized light beam from UV-laser (HeCd, 325 nm) is focused by a cylindrical lens into a slim strip on the sample as shown in Fig. 3.16. By continuously rotating the cylindrical lens and the substrate at respective speeds during the exposure, we can modulate the local photo-alignment direction on the

![Cylindrical Lens](image1.png)

**Figure 3.16:** Photograph of the optical setup to fabricate AWs.

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substrate as a function of azimuthal location, following Eq. 3.6. A specific charge AW can be fabricated by setting the rotations according to \( q = \frac{\omega_\text{s}}{\omega_\text{s} - \omega_\text{l}} \), where \( \omega_\text{s} \) and \( \omega_\text{l} \) are the rotation speeds of the sample and the cylindrical lens, respectively. Some samples we made are shown in Fig. 3.17. Because they are essentially wave plates with space-varying optical axis, they are best viewed between two crossed polarizers with a backlight. Every dark-bright-dark transition indicates a \( \pi/2 \) variation of local optical axis, thus, the charge 0.5 \( q \)-plate has two intensity modulations, the charge 1 has four, and the same for the others.

### 3.2.5 Forked Polarization Grating Fabrication

A widely studied optical element to generate a helical wavefront is an isotropic diffraction grating with singularities. These gratings are special isotropic gratings with well-designed fork-shape dislocations at their center. The forked patterns are usually calculated by computer and then made as Computer Generated Holograms (CGHs) with either transmittance modulation [42, 43] or blazed phase modulation [44]. These forked CGHs can be characterized by their topological charge at the dislocation. Whereas a normal 1D PG has periodic anisotropy along one dimension and is uniform along the others, the “forked
Figure 3.18: Fabrication of FPG: (a) thin layer of photo-sensitive polymer is exposed to UV hologram wrote by orthogonal circular polarized lights with OAM charge, which in introduced by AWs; (b) optical setup for the polarization hologram of the FPGs.

PG” we utilize in this work has similar properties, but has a fork-shape dislocation at the center, like the Forked CGHs. With both periodic anisotropy over one dimension and a singularity at the center, FPGs are like a combination of PGs and AWs, or a polarization-modulating version of Forked CGHs.

We fabricate the FPGs by writing a polarization hologram into photo-alignment materials: Two coherent beams with orthogonal circular polarization from a HeCd laser (325nm) superpose with a small angle between them, creates a spatial polarization modulation with constant intensity. To record FPGs, we make the two writing beams now carry OAM charges, which are introduced by the AW(s) in the optical path(s). Fig. 3.18(a) illustrates exposure, the key step in FPG fabrication. In the figure, the photo-sensitive polymer will align to form a FPG with charge $l_g = 2 \times q$. Note that the two writing beams are from the same laser and are separated by beam splitter with equal power. The half-wave plate could be spared if the two beams are originally set to the same handedness polarization as shown in Fig. 3.18(b). It also could be substituted by another AW, in which case the result FPG will have charge $l_g = 2 \times q_1 - 2 \times q_2$, where $q_1$ and $q_2$ are the charges of the two AWs.

Fig. 3.19 shows the polarization-microscope pictures of two FPGs. We made these two FPGs by using the AWs shown in Fig. 3.17(a) and (b), respectively. As derived in Eq. (3.7), charge-0.5 AW introduces charge $l = +1$ OAM to the beam. Together with the other Gaussian beam (equivalent to OAM charge $l = 0$), it writes a charge $l_g = +1$ FPG
as shown in Fig. 3.19(a). In the case of charge-1 AW, the output beam carries charge $l = +2$ OAM, thus the resultant FPG has charge $l_g = +2$ as shown in Fig. 3.19(b). In both cases, the pictures are centered at the singularity to show the variation. In sample areas other than the center, the alignment is uniform in one dimension with a period of 16 $\mu$m in the other dimension, just like a PG as shown in Fig. 3.19(c). This is coherent with both theory and simulation.

We characterized the FPGs by examining the diffractive properties of the FPGs with a laser. We used a collimated beam from HeCd laser (UV, 325nm) as the normally incident light, and detected the diffractions by UV-photodetector (Newport). Most transmitted light ($\sim 94.5\%$) is diffracted to the lowest three orders, among which the zeroth order was much weaker than the first orders. Less than 1% of the light goes to higher orders, the remaining power ($\sim 4.5\%$) is lost by scattering. The power distribution between the two first orders is dependent on the polarization of the incident light as shown in Table. 3.4. Right-handed circular polarized light is mostly diffracted to the $+1$st order, whereas left-handed circular polarized light is diffracted to the $-1$st order. When the incident light is linear polarized or unpolarized, which is equivalent to equally composed right and left circular polarization, the total power is equally distributed into the two first orders. Thus, the FPGs are demonstrated to have the same polarization sensitivity as PGs, as predicted by theory and simulation.

FPGs with good quality have been fabricated. Two key points make our fabrication approach successful. The first one is adapting the classic PG fabrication method for FPGs. As a result, our FPGs keep all of the good diffraction properties of PGs. The
Table 3.4: Characterization results of FPG diffraction: Measured intrinsic diffraction efficiency ($I_m/\sum I_m$) for input light with linear (or unpolarized), left-handed circular, and right-handed circular polarization.

<table>
<thead>
<tr>
<th>Input Light Polarization States</th>
<th>Intrinsic Diffraction Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$+1$st order</td>
</tr>
<tr>
<td>Linear (or Unpolarized)</td>
<td>48.0%</td>
</tr>
<tr>
<td>Left-handed Circular</td>
<td>96.3%</td>
</tr>
<tr>
<td>Right-handed Circular</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

second one is using a AW to introduce OAM singularity. Unlike other OAM elements, such as forked CGHs, AWs are transmissive thin wave plates, which can be integrated into the holographic setup with great flexibility and relative simplicity.
3.3 Proximate Lithography Technique for Tunable Period Polarization Holograms

We propose a new technique creating polarization holograms, Risley-Mask, that can easily tune grating period of recorded holograms via two polarization gratings. With a conventional holography setup, tuning a grating period takes time and effort, and requires many optics that need to be properly aligned. However, the Risley-Mask approach can tune grating period as easily rotating polarization gratings (called Mask-PGs) while maintaining compact size of the setup. The setup does not require any optics (e.g. beam-splitter, wave plate, or mirror), and it can pattern very large grating periods, which are generally hard to get via a conventional holography setup.

3.3.1 Operation Principle of the Risley-Mask

The Risley-Mask employs two advantageous properties of PGs: a replication possibility of PGs by a mask PG, and a continuous beam steering by two rotating PGs. The replication property of PGs was realized with a single mask PG [32]. When a single laser beam is passing through a mask PG, the PG spits the beam into right- and left-handed circular beams. If the beams are overlapped on the sample area, they can create spatially varying linear polarization pattern, which is similar to the pattern of the mask PG. In this case, the mask PG should have a peak wavelength at the laser beam wavelength in order to

Figure 3.20: Concept of the Risley-Mask consisting of two Mask PGs; when the input is Linear Vertical Polarized (LVP), the two mask PGs make two diffracted beams having LCP and RCP.
maximize the intensity of the first order diffractions. The first orders are only required to make the polarization hologram. The second property, a control of output beam angle, was demonstrated utilizing two rotating PGs [45]. When the two PGs are cascaded and their peak wavelengths equal to the recording beam wavelength, a circularly polarized input beam can be diffracted to a certain angle depending on a relative rotation angle of the two PGs. As rotating the azimuthal angle of the two PGs, we can continuously change the polar angle of the output beam. The maximum polar angle is double of the diffraction angle of the PGs.

Fig. 3.20 shows a conceptual design of the Risley-Mask consisting of two mask PGs. The mask PGs should have the same grating period (i.e., the same diffraction angle), and show the same retardation, which is a half-wave at recording wavelength to diffract the input beam into the first orders. The spectrum of the first order efficiency at UV-range (300 nm – 380 nm) is shown in Fig. 3.21 illustrating nearly 100% efficiency at the recording wavelength (325 nm). The Risley-Mask can achieve easy tuning of polarization hologram grating periods described as follows. A linearly- or un-polarized, collimated, narrowband beam at normal angle to the first mask PG is diffracted into two first-orders with orthogonal circular polarization (RCP and LCP). The beam directions are determined by the grating orientation of the first mask PG. Then, the second mask PG receives the circularly polarized beams and diffracts the beams into the final steering angle, which is determined by the relative grating orientations of the mask PGs. Note that the circular polarization is flipped to the opposite handedness upon the second mask

Figure 3.21: The first order efficiency of the mask PG manifesting nearly 100% at 325 nm (recording wavelength).
PG diffraction, and the overlapped beams on the sample area create spatially varying linear polarization pattern, which is similar to the pattern of the mask PGs. In this case, the relative grating orientation and the diffraction angle of the mask PGs determine the net polar angle between the output beams, which consequently decides the grating period of the polarization holograms. For example, when two mask-PGs are aligned close to parallel, the angle between output beams would be very small (close to zero), which can create very large grating period. In another way, when the mask PGs are aligned anti-parallel, the output beams make the maximum angle, which is calculated as $2 \sin^{-1}(2 \sin \theta)$, when $\theta$ is diffraction angle of the mask PG. This angle is close to the double of the diffraction angle, and the output beams can make the smallest grating period that the setup can create. In this manner, the Risley-Mask setup can easily tune various grating periods of polarization holograms as controlling the relative grating orientation of the two mask PGs.

The diffraction angle of a single mask PG at normal incidence is determined only by the grating period $\Lambda$ and the wavelength $\lambda$, which follows the grating equation ($\theta_g = \sin^{-1}(\lambda/\Lambda)$). Diffraction from multiple gratings with different relative orienta-

![Figure 3.22: Operation principle of the Risley-Mask with the vector representation of the two mask PG diffractions in the direction-cosine space [4]: (a) two first-order diffractions of the first mask-PG are described as vector $G^R_1$ and vector $G^L_1$ when they are orthogonally circularly polarized; (b) two first-order diffractions of the second mask-PG are described as vector $G^R_2$ and vector $G^L_2$; (c) the total diffraction directions can be described simply by a sum of the diffraction vectors as $G^R = G^R_1 + G^R_2$, and $G^L = G^L_1 + G^L_2$. When the azimuthal angles of the mask PGs are $\phi$ and $\pi - \phi$, the vector sum varies on $\alpha$-axis. The red dots show a change of vector sum as altering the azimuthal angle $\phi$ of the mask PGs.]

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tions, however, is somewhat more complicated because the angle relationship is nonlinear. Therefore, it is often convenient to introduce the direction cosine space where diffraction can be represented as a simple, linear vector [4]. Figure 3.22 shows the operation principle of the Risley-Mask with the vector representation. The first mask PG makes diffraction with a vector $G^R_1$ and $G^L_1$ for the first mask PG oriented at $\phi$. Then, the beam is diffracted again by the second mask PG oriented at $\pi - \phi$, and the diffraction is represented by another vector $G^R_2$ and $G^L_2$. The final beam direction can be described as a simple vector sum of two diffraction vectors as $G^R = G^R_1 + G^R_2$ and $G^L = G^L_1 + G^L_2$. Note that the length of the final vectors, $G^R$ and $G^L$ is the same each other, and they point opposite direction. The direction cosines of the steered beam are given by

$$\alpha = \sin(\theta) \left[ \cos(\phi_1) - \cos(\phi_2) \right]$$  \hspace{1cm} (3.8a)

$$\beta = \sin(\theta) \left[ \sin(\phi_1) - \sin(\phi_2) \right]$$  \hspace{1cm} (3.8b)

$$\gamma = \sqrt{1 - \alpha^2 - \beta^2},$$  \hspace{1cm} (3.8c)

where $\theta$ is the diffraction angle of the mask PGs, and $\phi_{1,2}$ are the grating orientations of the first and second PGs, respectively. We should note that Eqs. 3.8(a)–(c) are valid only

![Figure 3.23: Calculated net polar angles and recorded grating periods for different grating orientation angle of mask PGs, when the diffraction angle of the PGs is 2° at the recording wavelength (325 nm). The examples of the calculated grating period according to the grating orientation of the mask PGs are shown in the Table. 3.5.](image-url)
where \( \alpha^2 + \beta^2 < 1 \). When the grating orientation of the PGs are \( \phi \) and \( \pi - \phi \), the net polar angle (\( \Theta \)) between the output beams can be obtained from the direction cosines as follows

\[
\Theta = \cos^{-1}(\gamma),
\]

\[
= \cos^{-1}\left(\sqrt{1 - (2 \sin \theta \sin \phi)^2}\right).
\]

Consequently, the grating period of copied PG is determined by the net polar angle (\( \Theta \)) as follows.

\[
\Lambda_{\text{copy}} = \frac{\lambda}{2 \sin \Theta},
\]

where \( \lambda \) is the recording wavelength.

We graph the net polar angles (\( \Theta \)) and the recorded grating periods (\( \Lambda_{\text{copy}} \)) for different grating orientation (\( \phi \)) of the mask PGs in Fig. 3.23. We assume the diffraction angle (\( \theta \)) of the PGs is \( 2^\circ \) at the recording wavelength. Note that the maximum net polar angle is double of the diffraction angle (4\(^\circ\)) when the two mask PGs are aligned to parallel. The net angle decreases as the grating orientation angle is increased, and closes to zero when the PGs are aligned to anti-parallel. Moreover, the grating period of the patterned holograms is increased as the grating orientation angle is increased as shown in the same plot. Here we assume the recording wavelength is 325 nm.

Table 3.5: Net polar angles and corresponding grating periods, which can be created by different grating orientations of the mask PGs (\( \theta_{\text{mask}}=2.5^\circ \) at 325 nm, \( \Lambda_{\text{mask}}=7.45 \mu \text{m} \)).

<table>
<thead>
<tr>
<th>Mask PG</th>
<th>Mask PG Grating Orientation (( \phi ))</th>
<th>Net Polar Angle (( \Theta ))</th>
<th>Recorded Period (( \Lambda_{\text{copy}} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(^\circ)</td>
<td>5.0(^\circ)</td>
<td>1.86 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>30(^\circ)</td>
<td>4.3(^\circ)</td>
<td>2.15 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>60(^\circ)</td>
<td>2.5(^\circ)</td>
<td>3.73 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>75(^\circ)</td>
<td>1.3(^\circ)</td>
<td>7.20 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>85(^\circ)</td>
<td>0.4(^\circ)</td>
<td>21.4 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>89(^\circ)</td>
<td>0.1(^\circ)</td>
<td>107 ( \mu \text{m} )</td>
<td></td>
</tr>
</tbody>
</table>

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3.3.2 Fabrication and Demonstration

A picture of the Risley-Mask setup is shown in Fig. 3.24. HeCd UV laser that is linearly polarized and operates at 325 nm wavelength was used for a recording source, and the beam was expanded and collimated to 25 mm diameter after passing through several optics (not shown in the figure). After reflected by a UV-mirror tilted at 45°, the recording beam passed through the two mask PGs placed in the two rotation mounts (Newport). The azimuthal orientation of the mask PGs (φ and π − φ) can be easily adjusted with the rotation mounts. The overlapped area of the recording beams can be maximized as placing the sample holder close to the mask PGs. The setup is compact compared to other polarization holography setups, and the setup is very stable because all optics and elements are mounted in cage system (Thorlabs).

We experimentally demonstrated the Risley-Mask setup as making several PG samples that have different grating periods. We used two Mask-PGs whose grating period is 5 µm and diffraction angle at 325 nm is 3.7°. The Mask-PGs were cascaded with different grating orientations {86.4°, 89.2°, 89.6°} in order to generate corresponding grating periods {20µm, 90 µm, 180 µm}. Then, we followed general fabrication steps that we discussed in the previous sections. Fig. 3.25 shows polarizing microscope images of the fabricated PGs showing the intended grating periods.

![Figure 3.24: A picture of the Risley-Mask setup utilizing a cage system: two mask PGs were placed inside of the rotation mounts, and a UV-detect card was placed on the sample holder to visualize the recording beams.](image-url)
Figure 3.25: Polarizing optical microscope images of fabricated PGs placed with quarter-wave plate. The PGs were fabricated with various grating orientations of the Risley-Mask setup for (a) 20 µm, (b) 85 µm, and (c) 180 µm.

3.4 Multiple Diffraction Polarization Holograms

As discussed in Section 2.4, spatially varying LC structures of PGs can be created by a polarization hologram. In Chapters 2 and 3, we discussed several holography exposure methods generating the polarization hologram with an interference of orthogonally circularly polarized (RCP and LCP) recording beams. The PGs produced by the two recording beams show their diffractions on the same axis following the hologram exposure direction. Let us call the conventional PGs showing their diffractions on the same axis as single-axis PGs. In this section, we will realize and demonstrate high-efficient multi-axis PGs that make multiple diffraction at various orientations. The multi-axis PGs can be generated by overwriting the polarization holograms for single-axis PG for different orientations. The exposure orientation determines an axis-orientation of the PG diffraction pattern, and the number of hologram exposures decides the number of diffractions as Ono et. al [46] has reported the property of polymer liquid crystal with multiple diffraction. The multi-axis PGs can separate/steer an input beam in multiple directions and convert the polarization state of the beam at the same time. Therefore, the multi-axis PGs would be used to various applications in optical telecommunication networks.

3.4.1 Overview of Multi-exposure

PGs have spatially varying patterns that can be created by polarization hologram involving a modulation of the polarization state of recording beams. When the recording
beams are orthogonally circularly polarized and have the same intensity, the beams can make a proper modulation for the PG patterns. A grating direction (grating vector) of the PGs can be determined by an azimuthal angle ($\phi$) of the two recording beams, and a grating period ($\Lambda = \lambda/2 \sin \theta$) is governed by a polar angle ($\theta$) with a wavelength ($\lambda$) of the beams. Fig 3.26(a) illustrates the angle information of the recording beams for single-axis PG. One pair of Right-handed (R) and Left-handed (L) circularly polarized beams is used when $\phi = 0^\circ$, so there would be only one diffraction axis following the horizontal axis. Figs 3.26(b) and (c) show the angle information of the exposures for multi-axis PGs where $R_n$ and $L_n$ indicate RCP and LCP recording beams of $n$th exposure. The polarization holograms of multi-axis PGs would have multiple diffractions, and each diffraction orientation would follow the azimuthal angle ($\phi$) of each exposure.

### 3.4.2 Fabrication and Demonstration

We fabricated the single- and multi-axis PGs formed by liquid crystal polymer using polarization holography and photo-alignment materials. First, we coated a thin layer of LPP material (ROP-103/2CP, Rolic Technologies Ltd.) on a cleaned glass substrate by spin coater. After baking the material at $130^\circ$ for 1 minute, we exposed two recording beams (HeCd, 325 nm) on the sample using polarization holography setup. In order to make a $N$-axis PG with the setup, the sample should be exposed $N$-times with different
azimuthal angle of the recording beams. Instead of rotating the azimuthal angle of the recording beams, here we used a motorized rotation stage (Thorlabs) that can rotate the sample itself. This technique of rotating the sample can easily keep the polarization state of the recording beams, and it can hold the polar angle of the recording beams, which is necessary to achieve the same grating period for all diffraction axes. Then, in order to obtain proper retardation, we coated several layers of polymerizable liquid crystal (RMS10-025, Merck, \( \Delta n \approx 0.16 \) at 589 nm) on the exposed sample; here, the processing results in half-wave retardation at 633 nm, which minimizes the zero-order leakage at 633 nm.

Based on the fabrication method, we made three different PGs and their polarizing microscope images are shown in Fig 3.27. The polarizing microscope images show the intensity profiles of the PGs placed between two crossed linear polarizer with quarter-wave plate. The intensity profiles manifest anisotropic LC alignment of the samples. Those PGs were exposed by different polarization holograms with various polar angle (\( \theta \)) and azimuthal angle (\( \phi \)) information.

The diffraction property of the multi-axis PGs is shown in Fig 3.28. The input is red-laser (HeNe, 633 nm), which is collimated and expanded by VIS beam expander (Thorlabs). Figs 3.28(a-c) are output diffraction patterns of two-axis PG created by two polarization hologram exposures with \( \phi = \{0^\circ \text{ and } 90^\circ\} \), and Figs 3.28(d-f) are output beams of six-axis PG exposed by six individual exposures with different azimuthal an-
Figure 3.28: Captured output diffracted beams of (a-c) two-axis PG and (d-f) six-axis PG when input is collimated red-laser (HeNe, 633 nm): photographs of the output beams projected on a white board in dark room when the input polarization is (a,d) linear, (b,e) right-handed circular, and (c,f) left-handed circular polarization.

Angles, \( \phi = \{0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, \text{and } 150^\circ\} \). We used a quarter-wave plate with +45° or −45° axis to convert the polarization state of the linearly polarized input beam (laser) into circular polarization. Figs 3.28(a) and (d) are output diffractions when input beam was linearly polarized. The patterns show first orders along the all diffraction axes of the multi-axis PGs. Output diffraction patterns of RCP input are shown in Figs 3.28(b) and (e), and output diffraction patterns of LCP input are shown in Figs 3.28(c) and (d). The circularly polarized input makes only half of the full diffraction patterns, and the direction of diffractions (i.e., positive or negative) can be flipped as converting the handedness of circular polarization. We note that the polarization state of output beams is changed to orthogonal state when the input is circularly polarized beam (i.e., RCP→LCP, LCP→RCP). From the multi-axis PGs, high diffraction efficiency (> 93%) was measured by detecting the intensity of a reference and diffractions. Zero order leakage was very small (< 0.3%), and note that there was no significant high orders and scattering on the output beam.

We demonstrated the multi-axis PGs created by overwriting the polarization holograms patterned using photo-alignment materials and polymer LCs. The PGs showed
fairly high diffraction efficiency without any significant sidelobes. The unique properties of the multi-axis PGs controlling the beam direction and polarization state can be used to fiber- and tele-communication applications that require high-efficient beam steering/separating.
Chapter 4

Optical Beam Steering using Polarization Gratings

Laser radar, laser communications, laser countermeasures and laser weapons are in various stages of development for future deployment. One difficulty to deployment of these advanced laser systems is the ability to accurately and efficiently steer optical beams over large angles without interfering with other platform functions. The beam control technology should provide a compact, lightweight, and conformal means of efficiently steering laser beams over a large area as advanced opto-electronics systems. In this chapter, we introduce and demonstrate the use of polarization-sensitive diffractive elements, polarization gratings, to steer beams based on mechanical and non-mechanical systems. First, we show mechanical beam-steering system utilizing two rotating polarization gratings; this results in continuous steering of a laser beam with large field-of-regard and high throughput. We suggest several non-mechanical beam steering designs that are very compact, lightweight, and efficient; there are based on electrically switchable polarization gratings. The non-mechanical beam steering systems are characterized and compared with each other. Finally, we suggest the way to reduce chromatic dispersion of polarization gratings to steer broadband light sources.
4.1 Continuous Beam Steering

Precision beam pointing is a requirement for optical systems where beam alignment and target tracking are required, such as free-space optical communications, countermeasure, laser weapons, and fiber-optic switches. Optical turrets (gimbals) can be used for the precision beam steering but their placement is very limited on high performance and small electro-optical systems such as stealth and unmanned aerial vehicles, since they can increase air resistance and observability. As this context often demands compact, robust, and cost-effective devices for beam steering, Risley Prisms [47, 48, 49, 50, 51, 52, 53], comprising two or more wedged prisms, have long been used for its high degree of accuracy and stability. Their utility, however, is often limited by small deflection angles and poor size scaling properties due to bulky prismatic elements where wide angles and modest/large apertures are required.

We introduce the Risley grating, a grating version of the Risley prism, which can perform a continuous beam steering using a pair of polarization gratings (PGs) in independent rotation stages. By replacing wedged prisms with PGs formed in a thin Liquid Crystal (LC) layer, ultra-compact beam steering devices can be designed for virtually any size of beam. The polarization diffraction properties of PGs provide unique opportunities for beam steering with high throughputs and low levels of side lobes. Several LC grating structures (i.e., blazed or binary types) were proposed as a beam steering element [54, 55, 56]. The practical use of such LC gratings, however, is limited by their poor angle performance, limited peak efficiency, and low transmittance, and they are not applicable for the Risley gratings. A similar steering operation by rotating two PGs was reported in Ref. [35], but the Risley grating concept was not yet captured. Since two PGs can be placed with a close proximity, the beam walk-off is not an issue and multiple stages can be stacked without increasing the volume.

Here we will show the basic concept and its operation principles of this new beam steering device based on PGs and demonstrate a Risley grating that performs continuous steering of a laser beam (at 1550 nm) with the field-of-regard (FOR) 62° and 89 – 92% throughput. The angle of the emerging beam from the Risley grating is described in the direction cosine space and confirmed by experimental results. Several scanning patterns and side-lobes are also discussed.
4.1.1 Operation Principle of Two Rotating PGs

Here we consider two identical PGs aligned inline but rotated independently as shown in Fig. 4.1. By controlling the relative orientations of the PGs, continuous steering can be achieved described as follows. A circularly polarized, collimated, narrowband beam at normal angle to the first PG is diffracted into one of the first orders depending on the handedness of the circular polarization. As shown in Fig. 4.2, the beam direction is determined by the diffraction angle $\theta_g$ and the grating orientation $\phi_1$. Note that the circular polarization is flipped to the opposite handedness upon the PG diffraction. The second PG (with the same diffraction angle $\theta_g$ and the azimuth angle $\phi_2$) receives this beam and redirects it to the final steering angle, which is determined by the relative grating orientations. As rotating the PGs, the emerging beam points with angles within the field-of-regard, defined by an angle $2\Omega$.

The diffraction angle of a single PG at normal incidence is determined only by the grating period $\Lambda$ and the wavelength $\lambda$, which follows the grating equation ($\theta_g = \sin^{-1}(\lambda/\Lambda)$). Diffraction from multiple gratings with different relative orientations, however, is somewhat more complicated because the angle relationship is nonlinear. Therefore, it is often convenient to introduce the direction cosine space where diffraction can be represented as a simple, linear vector [57]. Fig. 4.3(a) shows the diffraction vector $G_1$ for the first PG oriented at $\phi_1$. The beam is diffracted again by the second PG of which diffraction is represented by another vector $G_2$. The final beam direction can be described as a simple vector sum of two diffraction vectors $G = G_1 + G_2$ as shown in

![Figure 4.1: Continuous beam steering by two rotating polarization gratings (PGs), named the Risley grating: (a) the single-order diffraction from a PG; (b) double diffraction from two PGs with relative orientations.](image-url)
Figure 4.2: Steering coverage and beam tracks of the Risley grating: (a) the field of regard of the Risley grating defined by the maximum diffraction angle \( \Omega = \sin^{-1}(2\lambda/\Lambda) \), where \( \lambda \) is the wavelength and \( \Lambda \) is the grating period of the PG. Note that the input beam is circularly polarized and the grating is optimized for a half-wave retardation for 100% efficiency (\( \Delta nd = \lambda/2 \)); (b) steering with two inline PGs with orientations \( \{ \phi_1, \phi_2 \} \) having values of (i) \( \{0^\circ, 180^\circ\} \), (ii) \( \{0^\circ, 0^\circ\} \), and (iii) \( \{180^\circ, 0^\circ\} \); and (c) continuous scanning patterns as orientations are varied as (iv) \( \{\phi, 0^\circ\} \), (v) \( \{\phi + 180^\circ, \phi\} \), (vi) \( \{\phi - 90^\circ, -\phi + 90^\circ\} \), and (vii) \( \{\phi, -\phi + 180^\circ\} \).

Fig. 4.3(b). The direction cosines of the steered beam are discussed in Section 3 but we shown again to derive the output beam direction.

\[
\begin{align*}
\alpha &= \sin(\theta_g) \left[ \cos(\phi_1) - \cos(\phi_2) \right] \quad (4.1a) \\
\beta &= \sin(\theta_g) \left[ \sin(\phi_1) - \sin(\phi_2) \right] \quad (4.1b) \\
\gamma &= \sqrt{1 - \alpha^2 - \beta^2}, \quad (4.1c)
\end{align*}
\]

where \( \phi_{1,2} \) are the grating orientations of the first and second PGs, respectively. We should note that Eqs. (4.1a)–(4.1c) are valid only where \( \alpha^2 + \beta^2 < 1 \). The azimuthal and
polar angles of the beam direction can be obtained from the direction cosines as follows

\[
\phi = \tan^{-1} \left( \frac{\beta}{\alpha} \right) \quad (4.2a)
\]
\[
\theta = \cos^{-1} (\gamma) \quad (4.2b)
\]

Steering at the maximum angle \( \Omega = \sin^{-1}(2\lambda/\Lambda) \) occurs when \( \phi_2 = \phi_1 + \pi \) or two PGs are aligned in an antiparallel orientation. The field of regard (FOR) is defined as \( 2\Omega \), and the steered beam always points a direction within the solid angle of \( \pm\Omega \).

### 4.1.2 Fabrication and Demonstration

We demonstrated the Risley grating beam steering with 62° FOR at 1550 nm using a pair of PGs formed as liquid crystal cells (\( \Lambda = 6 \mu \text{m} \) and \( \theta_g = 15^\circ \) at 1550 nm). As shown in Fig. 4.4, the two PGs were separately mounted in rotation stages and their orientations were controlled to achieve steering of a collimated beam from an infrared laser with circular polarization. We confirmed the steering angle for different grating orientations and measured the beam powers in the forwarding direction.

Defect-free PG samples, formed as liquid crystal cells, were fabricated using polarization holography and photo-alignment materials. In particular, we utilized a linear-
photopolymerizable polymer (LPP) ROP-103/2CP (from Rolic) as a photo-alignment material and nematic liquid crystal LCMS-102 (from Boulder Nonlinear Systems, $\Delta n = 0.31$ at 1550 nm). Two glass substrates were coated with LPP and then assembled to make a cell with 2.5 $\mu$m thickness. This cell was exposed with two orthogonal, circularly polarized beams from a HeCd laser (at 325 nm) to record the PG pattern onto the LPP layers. After the UV exposure, the LC material was filled in the isotropic state (at 150°C). The samples exhibit nearly ideal PG diffraction with $> 98\%$ first-order efficiency and no observable higher order. Note that both glass surfaces were laminated with anti-reflection coatings to minimize reflection losses.

Continuous steering of the beam within $\Omega = \pm 31^\circ$ was achieved by rotating two PGs with high optical throughput. The angles of the steered beam were measured on an IR sensitive detecting screen with different PG orientations. The direction cosines corresponding to these angles were calculated as shown in Fig. 4.5(a), which are well matched to calculated angles from Eqs. (4.1a)–(4.1c). We also show several simple scanning patterns (circles) of the steered beam in Fig. 4.5(b). The sets of pictures that are superimposed each other are shown in Figs. 4.5(c) and (d), which illustrate continuous steering following crossed lines and two circles respectively.
Figure 4.5: Demonstration of the Risley grating with 62° FOR using two PGs (Λ = 6 µm): (a) scanning patterns (with fixed first-PG orientations and rotating second-PG) of the steered IR laser beam (1550 nm) on an IR sensitive detecting screen; (b) the direction cosines calculated from the measured steering spots (circles), which well matched to the analytical solutions (solid lines) from Eqs. (4.1a) and (4.1b); (c) beam scanning demonstration shown in Fig. 4.1(e). Note that individual pictures of the steered beam incident on an IR sensitive detecting screen were captured and superimpose. Two PGs were optimized for the maximum efficiency (> 98%) at 1550 nm (Δnd = λ/2, where Δn = 0.31 and d ≈ 2.5 µm) and the IR laser was circularly polarized before the first PG.

We measured the transmitted power for different steering angles and confirmed high throughputs up to 92% transmittance as shown in Fig. 4.6(a) (i.e., transmittance defined as $T = \frac{P_{\text{out}}}{P_{\text{in}}}$, where $P_{\text{in}}$ is the input power and $P_{\text{out}}$ is the power in the steered direction). We also calculated the diffraction efficiency (Fig. 4.6(a)) as a normalization that removes the effect of the substrates, defined as $\eta = \frac{P_{\text{out}}}{P_{\text{tot}}}$, where $P_{\text{tot}}$ is the total transmitted power in the forwarding direction. The results show high transmittance from 89% to 92% (efficiency from 92% to 96%) for all steering angles, with some dependency on
Figure 4.6: High throughput from 89% to 92% (efficiency from 92% to 96%) of the Risley grating for all steering angles between $\pm 31^\circ$: (a) measured transmittance and efficiency for different steering angles; (b)–(d) power scanning across $\pm 40^\circ$ for three different steering angles of $-30^\circ$, 0, and $30^\circ$. Sidelobes in the range of 1% to 6% of the input power were observed at angles that are multiples of the diffraction angle ($\theta_g$), which are diffraction leakages primarily due to oblique incidence to the second PG.

the steering angle. We estimate roughly 4% of the transmittance loss due to Fresnel-type reflections at the glass/LC interfaces. The remaining loss is due to diffraction leakages that appear in sidelobes.

To characterize sidelobes, we measured the transmitted power by scanning observation
angles ($\pm 40^\circ$) in the forwarding direction. Figs. 4.6(b)–(d) show measured transmittance for three different steering angles ($-31^\circ$, $0^\circ$, $+31^\circ$). The level of sidelobes varies in the range of 1% to 6% of the input power, again also depending on the steering angle. Note that, in all cases, the sidelobe was observed at angles that are multiples of $\theta_g$. These leakages primarily result from oblique incidence to the second PG, and can be reduced by the use of higher birefringence LC materials and additional retardation compensation techniques.

We introduced the new beam steering device based on two rotating PGs, named the Risley grating, that can perform continuous steering with a wide FOR and high throughputs. We demonstrated a prototype Rislay grating with $62^\circ$ FOR at 1550 nm wavelength using two identical PGs ($\Lambda = 6 \mu m$, $> 98\%$ first-order efficiency). The Risley grating manifests excellent steering operation with high optical throughput ($89\%$ to $92\%$). Since the PGs are formed in thin liquid crystal layers (a few $\mu m$ thickness), the Risley grating can be scaled to almost arbitrarily large areas with an ultra-compact and lightweight form factor. Larger steering angles, further loss reduction, and implementation at other wavelengths are certainly possible through continued optimization of substrates and PG materials, as discussed.
4.2 Non-Mechanical Beam Steering

Non-mechanical beam steering promises substantial benefits to optoelectronic systems such as free-space laser communications, laser weapons, laser remote sensing, and fiber-optics; however, any viable solution must offer some or all of the following [58]: high throughput, rapid pointing ability, robustness to high intensity, and compact size. In order to achieve non-mechanical beam control, diverse approaches have been explored including: micro-lens arrays, electro-optic prisms, holographic glasses, and diffractive acousto-optic techniques [59, 60, 61, 62, 63]. All of these approaches are, however, plagued by one or more of the following limitations: low throughput, scattering, small steering angle/aperture, and large physical size.

One of the more promising approaches is termed volume holography [64], that can diffract an incident beam into the direction of a reference beam. This approach contains multiple holographic gratings [65, 66] within each glass substrate (usually two or less, in order to minimize scattering and other losses). While efficiency of individual gratings can be quite high, the two inherent limitations of this approach are that (1) two fine-angle steering stages are necessary, and more crucially, that (2) the number of gratings necessary is linearly proportional to the steering angular range. For the sake of discussion, to achieve a range of $\pm 40^\circ$ in one dimension and a resolution of $1.25^\circ$, a minimum of 32 glass substrates are required (if each has two multiplexed gratings). This leads to thick systems, and allows the losses of each stage to compound.

Another approach uses multiple stages of birefringent prisms [67, 68] to steer the beam. A series of prisms can steer to one of two states depending on the polarization of input beam, and active wave plates are placed to switch the polarization state before each prism. Each prism adds incremental angular deflections by altering the polarization states at the input of each birefringent prism. But thick prisms are needed for large steering angles, causing significant walk-off, and leading to very long systems where the length is necessarily many times larger than the beam diameter.

A recent approach is termed V-COPA [69], which contains a vertically aligned, continuous phase optical phased array that uses a negative dielectric anisotropic LC material. As controlling voltages applied to electrodes pattern on substrates, a steering angle of V-COPA device is variable. The angle tunability is based on Pancharatnam’s idea. High steering efficiency is demonstrated by modeling the LC director field, but the device requires complicated electrode patterns to control the in-plane director configuration.
through the use of voltage offsets and is not suited for wide-angle steering. This leads to comparably high fabrication costs and requires even more complicated patterns to get wide steering angles.

In this chapter, we introduce a novel beam steering device based on the polarization sensitive properties of liquid crystal Polarization Gratings (PGs) [70, 71]. This single device is capable of diffracting incident light into one of three possible diffracted orders (0th and ±1st) according to the input polarization and the applied voltage. Based on steering designs of multiple stacked stages, we show that it is practical to achieve a wide range of discrete steering angles, and these exhibit high throughput (e.g., > 90%) and wide field-of-regard (e.g., 90°) compared to prior non-mechanical steering systems. Since the PGs can be formed as thin polymer films (or within thin LC cells) and they are scalable to large areas without increasing their thickness, the device provides dramatic aspect ratio. Moreover, our device can be tailored to operate at nearly any wavelength from visible to midwave–infrared. We show the beam steering device that performs non-mechanical, wide-angle, discrete steering of a laser beam. Our discrete beam steering technology can be combined with a fine-angle continuous steering for wide-angle continuous steering as shown in Fig. 4.7. First, we show the beam steering properties of PGs and single steering stage which will be interlaced for coarse steerer. Then we will look at several possible design options for coarse steering technique and compare their respective efficiencies with experimental data. Finally, we will discuss a choice of steering design among the possible coarse steering designs.

Figure 4.7: Conceptual view of our beam steering system including fine and coarse steering modules, which performs full 80° × 80° field-of-regard steering.
4.2.1 Fine and Coarse Steering Modules

The fine angle steering module is constructed by using two 1×12,288 element Optical Phased Arrays (OPAs) [72]. We have used two sets of OPAs, implemented at Boulder Nonlinear Systems (BNS), each with a 2 cm × 2 cm aperture as shown in Fig. 4.8. The 2D fine angle steering system is expected to provide ~90% of throughput: the ratio of output intensity to input intensity. This steering is designed to cover a ±3.125° range in both the horizontal and vertical dimensions, and expands the steered beam by a factor of 2.5, thus reducing the steer angle by the same amount. For example, an incident beam with ±3.125° coverage and 2 cm diameter, will be expanded to a 5 cm diameter beam with a ±1.25° steering range. In order to achieve wide-angle (e.g., ±40° range in both dimensions), continuous beam steering, the expanded output beam of the fine angle module passes through the next module, coarse angle steering module, which cover ±40° field-of-regard with a steering resolution of 1.25° (discrete).

The essence of the coarse angle steering module is a use of polarization gratings. Unlike amplitude and phase gratings, PGs operate by modulating the polarization of light, and therefore PGs can be used for constructing polarization sensitive steering device, capable of highly efficient wide-angle operation. As we discussed the basic of the polarization gratings in Chapter 2, the ideal diffraction efficiency at normal incidence can be derived by Jones calculus as following the Eq. 2.9. If the thickness of the LC layer is chosen as \( d = \lambda/2\Delta n \) (half-wave retardation), then all of light will deflect to the first orders \( \sum \eta_{\pm1} = 1 \), and the portion of the light diffracted into one of the first orders

Figure 4.8: Fine steering module pictures: (a) 1×12,288 OPA coarse beam steering system (provided by BNS); (b) Optical head of the system.
is determined by an ellipticity of the incident light polarization ($S'_3$ parameter). For example, when the incident light is Left-handed Circular Polarization (LCP), $S'_3 = -1$, diffraction efficiency will be $\eta_{+1} = 1$ and $\eta_{-1} = 0$. This means all of the light passing through the PG is diffracted into the positive first order as shown in Fig. 4.9(a). In the opposite case, if the light is Right-handed Circular Polarization (RCP), $S'_3 = +1$, all of the light is diffracted into the negative first order ($\eta_{-1} = 1$) as Fig. 4.9(b). Moreover, after passing through the PG, the handedness of circular polarized light will be changed to the opposite state since the light experiences a relative phase shift due to the LC layer. As shown in Figs. 4.9(a) and (b), once the light of LCP passes through the PG, the polarization state would be changed to the RCP. Alternatively, when RCP light goes through the PG, passed light would be LCP state. These diffraction properties are the same for switchable PGs [3, 73, 74] and polymer PGs [18], but the non-diffraction case shown in Fig. 4.9(c) is only for a switchable PG case. When the applied voltage on the switchable PG much greater than a voltage threshold, the LC director will be out of the plane and the effective birefringence will be reduced toward zero ($\Delta n \rightarrow 0$). By effectively erasing the grating of the active PG, incident light can pass directly through the PG without any change of polarization state.

The diffraction angles are determined by the well-known grating equation, since the PG is merely a birefringent grating. The output angle of the PG can be calculated as follow.

$$\sin \theta_m = \left( m \frac{\lambda}{\Lambda} \right) + \sin \theta_i,$$

(4.3)
where $\theta_{in}$ is the incident angle, $\theta_m$ is the angle of diffraction of transmitted light, and $m = \{-1, 0, +1\}$ is the diffraction order. For instance, when the grating period of the PG is 9 $\mu$m, normal incident light will be diffracted with $10^\circ$ angle at 1550 nm wavelength. Diffraction angle change for different grating period and input wavelength is shown in Fig. 4.10. In recent study [71], we found that PGs can retain high diffraction efficiencies for modest incident angles ($\leq 20^\circ$).

Fig. 4.11(a) shows the spectral response of the zero-order transmittance for different values of applied voltage. If the active PG is initially designed that no zero order is present in the communication wavelength (e.g., 1550 nm), an applied voltage which is higher than threshold voltage can erase the gratings and make $\sim$100% zero order transmittance. Fig. 4.11(b) and 4.11(c) show polarizing optical microscope images of the PGs with different applied voltages [3]. In these images, the PG is located between crossed polarizers and the black fringes correspond to areas where the LC molecules are oriented to the one of polarizers’ axis. As the voltage is increased, the structure of the PG loses its periodic nature.

From these properties above, the PG can efficiently diffract circularly polarized light to either zero or first orders, based on the polarization handedness of the input light and applied voltage. Moreover, PG’s thickness is independent of the aperture size and deflection angle, so wide-angle steering with large apertures can be allowed with our device. Since PGs have their own deflection angles inherently according to the grating period, each grating stage can be stacked to increase the maximum steering angle in one
Figure 4.11: Experimental results: (a) transmittance spectra of active PG with various applied voltages; (b) polarizing microscope image without applied voltage (0V) and (c) with applied voltage (2V, 10V) higher than threshold.

dimension without major efficiency reductions.

PGs can function as highly efficient beam steering elements, by deflecting all of the incident light into one of the three diffraction orders. We have identified several combinations of PGs and liquid crystal wave plates, that can perform the three angle steering. These designs can be implemented with active or passive PGs, with each approach having its own advantages and disadvantages. Fig. 4.12(a) describes an active PG beam steering stage, which contains an active PG and an LC half-wave plate. In this scheme, the incident light is assumed to be circularly polarized. Ideally when no voltage is applied, the LC wave plate switches the handedness of the incident light (i.e. RCP → LCP or LCP → RCP), but under external applied voltage it allows the incoming light to pass through without changing its polarization state. When the PG shows half-wave retardation on the input beam wavelength, the PG diffracts RCP and LCP beams into the +1 and −1 orders respectively, with nearly 100% efficiency. Hence the polarization sensitive diffraction of PGs can be used to select the steering direction into one of the first orders, by simply switching the LC half-wave plate. Moreover, active PGs can steer all of the incident light into the zero-order under an applied voltage. Therefore, the active PG steering stage can provide three unique steering directions corresponding to the three diffraction orders, by simply switching the voltage across both the elements. Since this approach involves two switchable LC devices, it requires 4 glass substrates and
Figure 4.12: Primary configurations of a single PG steering stage: (a) "Active PG" stage, with one switchable PG and one variable LC half-wave plate; (b) "Passive PG" stage with two polymer PGs and two variable LC half-wave plates.

4 transparent-conducting-electrodes.

In Fig. 4.12(b), a passive PG steering stage is shown, which is able to steer the incident light to three different directions. The stage includes two passive PGs and two LC half-wave plates to essentially perform in the same manner as the active PG steering stage. The number of active elements is still the same as the active stage, but the passive stage uses the two passive PGs, relying on two active LC wave plates for switching the polarization state. Therefore in this stage, the steering directions are determined by the voltage-states of the two LC wave plates, positioned in front of the two passive PGs. When both wave plates are in same state (i.e., both ON or both OFF), the diffraction from the first PG is compensated by the second one. But when wave plates are in the opposite states, the incident beam can be diffracted into the positive or negative order (i.e., RCP → + order, LCP → − order). So even in this case, three steering directions are possible, and moreover, the passive PGs are firm and thin (polymerized low-molecular weight LC molecules [14]) and able to make small grating period, which makes bigger deflection angle. Since the passive stage approach involves two switchable LC devices plus two polymer films likely on additional substrates, it requires 6 glass substrates and 4 transparent-conducting-electrodes. Therefore, while the passive PG approach may enable larger angles to be reached, each stage is necessarily thicker and has more interfaces.

Before cascading the single stages to achieve wide-angle beam steering, we need to characterize parameters of each single stage. First, we measured three steered beams
of single steering stages with 1550 nm wavelength laser. The stages can steer the input beam into ±5° and ±10° angles. Photographs of each diffraction order are achieved with IR sensitive detecting card which is 30 cm off from the PG. Fig. 4.13(a) shows the detected three orders of the stages. Here we check the different diffraction angles indicated with different intervals between the orders; the longer grating period makes the smaller diffraction angle. In Fig. 4.13(b), the aggregate transmittances (throughputs) are shown for different states of the single steering stage. Three different conditions make three diffraction orders denoted by different color bars. Note that the used substrates are not optimized, so aggregate transmittance is lower than what we expect. However, diffraction efficiency is very good; all diffracted beams can be detected at the proposed orders.

4.2.2 Binary Steering Design and Demonstration

Both the passive and active PG stages discussed in the previous section can steer the incident light into three different directions, according to the voltages applied on their active elements. Several units like these can also be stacked or cascaded to implement a coarse, wide-angle beam steering system with an increased operation range, by arranging each single individual unit appropriately. Such a coarse beam steerer can be engineered to provide wide-angle coverage of 80°×80° with a resolution of about 1.25°×1.25°.

The coarse beam steerer design in Fig. 4.14 is based on the binary steering de-
Figure 4.14: Binary coarse steerer design: (a) overall construction with 10 single stages (5 for AZ, 5 for EL); each stage can be implemented with (b) active PGs, or (c) passive PGs.

design, and follows a geometric progression. The diffraction angle of the first stage determines the resolution of coarse beam steerer, and the following stage’s diffraction angle is double of previous stage’s diffraction angle (e.g. $1.25^\circ$, $2.5^\circ$, $5^\circ$, etc). For a coarse beam steerer consisting of $N$ single stages, a total of $2^{N+1}$ distinct steering angles are theoretically possible. Therefore, in order to provide $\pm 40^\circ$ coverage with $1.25^\circ$ resolution, five single stages (i.e. when $N=5$, $2^{N+1}=64$, which also equals the required resolution $2\times(40^\circ/1.25^\circ)=64$) are needed. As shown in Fig. 4.14, these single stages ($1.25^\circ$, $2.5^\circ$, $5^\circ$, $10^\circ$ and $20^\circ$) are cascaded in different directions to provide two-dimensional steering (AZ, EL). For example, in order to steer the incident light into $22.5^\circ$ in azimuthal and $-16.25^\circ$ in elevation angles respectively, each stage selectively generates the following states to steer the incident beam in this direction: $2.5^\circ_{(stage2)}+20^\circ_{(stage5)}=22.5^\circ$ in AZ and $-1.25^\circ_{(stage1)}-5^\circ_{(stage3)}-10^\circ_{(stage4)}=-16.25^\circ$ in EL. Fig. 4.15 shows the state (ON/OFF) of applied voltage on each element and describes the final steering angles from the active binary coarse steerer. Full table is too long so only part of table is presented. This design can work the same as binary design, while this design has less elements. We can expect the increased throughput as well as reduced system losses because fewer elements are used. In this binary steering design, the geometric progression is critical. Only one additional steering stage or element is necessary in order to double the field-of-regard in any one dimension. Contrarily, in the holographic
<table>
<thead>
<tr>
<th>Elements</th>
<th>State of Applied Voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCWP</td>
<td></td>
</tr>
<tr>
<td>PG1 (1.25°)</td>
<td>+1.25° +1.25° +1.25° +1.25° +1.25° +1.25°</td>
</tr>
<tr>
<td>LCWP</td>
<td></td>
</tr>
<tr>
<td>PG2 (2.50°)</td>
<td>+2.50° +2.50° +2.50° +2.50° +2.50° +2.50°</td>
</tr>
<tr>
<td>LCWP</td>
<td></td>
</tr>
<tr>
<td>PG3 (5.00°)</td>
<td>+5.00° +5.00° +5.00° +5.00° +5.00° +5.00°</td>
</tr>
<tr>
<td>LCWP</td>
<td></td>
</tr>
<tr>
<td>PG4 (10.00°)</td>
<td>+10.00° +10.00° +10.00° +10.00° +10.00° +10.00°</td>
</tr>
<tr>
<td>LCWP</td>
<td></td>
</tr>
<tr>
<td>PG5 (20.00°)</td>
<td></td>
</tr>
<tr>
<td>Steering Angle</td>
<td>0° 1.25° 2.5° 3.75° 5° 6.25° 7.5° 8.75° 10° 11.25° 12.5° 13.75° 15°</td>
</tr>
</tbody>
</table>

Figure 4.15: Applied voltages on each element for binary discrete coarse steerer design. Note that only a subset (0° to +15°) of the ±40° range is shown.

glass approach [64], a doubling of the field-of-regard requires a doubling of the number of stages.

Having different beam steer designs, we need to estimate the performance between these designs. In order to compare them, we first need to define expected efficiency and losses from each element. There are four main parameters, and for the sake of discussion, we list the nominal best-case-value of each (based in part on our experimental work below):

1. Intrinsic Diffraction Efficiency ($\eta$) - fraction of received light steered into the intended order.

2. Diffuse Scattering Loss ($D$) - loss from light randomly scattered by the PGs away from the steered direction.

3. Fresnel Reflectance Loss ($R$) - loss from reflections at interfaces of differing refractive indices.

4. Absorption Loss ($A$) - loss due to absorption within the transparent-conducting-electrode layers.
If we assume that the PG efficiency and losses of each stage are the same, we can approximate the overall system transmittance $T$ in the following way:

$$T = (\eta)^N (1 - D)^N (1 - R)^M (1 - A)^M,$$

(4.4)

where $N$ is the number of PGs, and $M$ is the number of LC cells of a steerer. In Fig. 4.16, expected efficiency and losses are shown for passive and active binary coarse steerer designs. Here we assumed the best values for the parameters as $\eta > 99.5\%$, $D < 0.3\%$, $R < 0.1\%$, and $A < 0.2\%$. From a loss analysis standpoint, the design with the fewest number of elements and transparent-conducting-electrode layers would be the best option. The $80^\circ \times 80^\circ$ improved discrete beam steerer with $1.25^\circ$ resolution will operate with a total transmittance approximately $87\%$. Then, as combining with a fine angle steerer whose transmittance is around $\sim 90\%$, the overall transmittance is expected to $\sim 78\%$.

We stacked two single active stages of the binary design to demonstrate seven different steering angles. Using the PGs having $\pm 5^\circ$ and $\pm 10^\circ$ diffraction angles, the stage can selectively control seven steering angles from $-15^\circ$ to $+15^\circ$ with $5^\circ$ steps. Fig. 4.17(a) shows the concept of multi-stage achieving more than three steering angles. The multi-stage contains two active PGs and two variable LC half-wave plates, each active element is electrically controlled (switched) with a voltage through the wires. Fig. 4.17(b) presents seven diffracted beams from the multi-stage steerer, which cover $\pm 15^\circ$ with $5^\circ$ steps. Spots are photographed with an IR sensitive detecting card located 40 cm off from the

![Table: Intrinsic Efficiency and Losses](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Individual value</th>
<th>Component</th>
<th>Transmittance</th>
<th>Component</th>
<th>Transmittance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intrinsic Efficiency</td>
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<td>20</td>
<td>90.5%</td>
<td>10</td>
<td>95.1%</td>
</tr>
<tr>
<td>Fresnel Loss</td>
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<td>40</td>
<td>96.1%</td>
<td>20</td>
<td>98.0%</td>
</tr>
<tr>
<td>Absorption</td>
<td>0.2%</td>
<td>40</td>
<td>92.3%</td>
<td>40</td>
<td>92.3%</td>
</tr>
<tr>
<td>Scatter</td>
<td>0.3%</td>
<td>20</td>
<td>94.2%</td>
<td>10</td>
<td>97.0%</td>
</tr>
<tr>
<td>Total Transmittance (T)</td>
<td></td>
<td>75.6%</td>
<td></td>
<td>83.4%</td>
<td></td>
</tr>
</tbody>
</table>
stage. All diffraction spots are well aligned each other and no significant walkoff was observed. Experimentally demonstrated intrinsic efficiency (steering efficiency) ranges from 99.4% to 99.6% for all seven diffraction beams, and the case of 15° diffraction angle is shown in Fig. 4.17(c).

4.2.3 Ternary Steering Design and Demonstration

In this section, we introduce and demonstrate multi-stage beam steerer, which is based on unique three-way (ternary) steering design. The ternary design is based on active PG stages containing switchable LC half-wave plate and active PG. A diffraction angle of the first stage determines a resolution of the ternary steerer, and the diffraction angle of the following stage is three times larger than the diffraction angle of prior stage (e.g. 1°, 3°, 9°, etc). Each steering stage can provide three unique steering directions corresponding to the three diffraction orders by simply switching the voltage across both the elements. For the coarse beam steerer based on ternary design consisting of \( N \) single stages, a total of \( 3^N \) distinct steering angles are theoretically possible. Since this approach involves two switchable LC devices for each stage, it requires \( 2N \) glass substrates and \( 2N \) transparent-conducting-electrodes for the \( N \)-stage steerer.

Fig. 4.18(a) shows a single steering stage that comprises a switchable Wave Plate (WP) and a switchable PG resulting in three-way steering. The WP ensures that the input to the PG is either of the two orthogonal (Left/Right) circular polarization states.
Depending on the handedness of polarization, the PG diffracts the beam into one of the first orders (+θ or −θ) and flips its handedness. When voltage is applied on the PG (V >> V_{th}, on-state PG), the grating profile is effectively erased (ie, Δn ≈ 0) and the incident beam passes through the PG preserving its polarization state. The diffraction angle is established by the classic grating equation, \( \theta_{\text{out}} = \sin^{-1}(m\lambda/\Lambda + \sin \theta_{\text{in}}) \), where the order \( m \) depends on the incident polarization and \( \Lambda \) is the grating period. Our ternary PG beam steering concept comprises multiple stacked stages of this WP/PG assembly. Fig. 4.19(b) shows the ternary design (with \( N = 3 \) stages), where each stage has a different grating period and can access a different set of angles. The most novel feature of this beam steerer is fact of three possible steering directions of every stage, and the ability of each stage to add and subtract from the input angle (or leave it unchanged). Prior multistage diffractive approaches [55, 75] comprised stages with only two output states (deflecting or not).

Our ternary PG design therefore enables far more angles to be steered by the same number of stages. The total number of steering angles \( S \) and angle resolution \( r \) are determined by number of stages \( N \):

\[
S = 3^N \quad \text{(4.5a)}
\]

\[
r = \frac{\text{FOR}}{(3^N - 1)} \quad \text{(4.5b)}
\]

where \( \text{FOR} \) is the field-of-regard. We highlight the exponential increase of \( S \) in Fig. 4.18(b).
The diffraction angle $\theta_l$ and grating period $\Lambda_l$ of each stage number $l$ is

\[
\sin \theta_l = \sin((3^l - 1)r/2) - \sin((3^{l-1} - 1)r/2) \quad (4.6a)
\]
\[
\Lambda_l = \lambda / \sin \theta_l, \quad (4.6b)
\]

where $\lambda$ is the wavelength of incident beam. The overall output angle $\Theta$ can be expressed as

\[
\sin \Theta = \sum_{l=1}^{N} (-1)^{V_{WP}^l} V_{PG}^l \sin \theta_l \quad (4.7)
\]

where $V_{WP}^l$ is the state of the $l$th WP (0 or 1 when the WP output is LCP or RCP, respectively), and $V_{PG}^l$ is the state of $l$th PG (0 or 1 when $V >> V_{th}$ or $V = 0$, respectively). If we assume that the PG efficiency and losses of each stage are the same, we can approximate the overall system transmittance $T$ in the following way:

\[
T = (\eta_{+1})^N (1 - D)^N (1 - R)^{2N} (1 - A)^{2N}, \quad (4.8)
\]

where $\eta_{+1}$ is the experimental intrinsic diffraction efficiency of each PG, $D$ is the diffuse scattering of each PG, and where $R$ and $A$ are the Fresnel reflectance and absorption losses, respectively, of each LC cell.

We graph $T$ for three cases in Fig. 4.19. Case (i) corresponds to the parameters we

![Figure 4.19: Ternary PG beam steering design: number of steering angles and calculated transmittance vs number of stages. Three cases of $\{\eta_{+1}, D, R, A\}$ in Eq. 4.8 are shown, as described in the text.](image)
Table 4.1: Individual active PG characterization data.

<table>
<thead>
<tr>
<th>l (stage)</th>
<th>$\theta_l$ (deg)</th>
<th>$\Lambda_l$ (µm)</th>
<th>$D_l$ (%)</th>
<th>$\eta_{+1}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.7</td>
<td>52.6</td>
<td>0.0</td>
<td>99.9</td>
</tr>
<tr>
<td>2</td>
<td>5.1</td>
<td>17.5</td>
<td>0.3</td>
<td>99.8</td>
</tr>
<tr>
<td>3</td>
<td>14.9</td>
<td>6.0</td>
<td>1.6</td>
<td>99.7</td>
</tr>
</tbody>
</table>

were able to experimentally demonstrate in this work (ie, $\eta_{+1} \approx 99.6\%$, $D = 0.3\%$, $R = 1.2\%$, and $A = 1.0\%$), where simple indium-tin-oxide (ITO) electrodes were implemented. Case (ii) corresponds to the case when commercially available index-matched ITO are used, to reduce $R = 0.1\%$. Case (iii) corresponds to the best-case scenario, where low loss transparent conductors are employed to reach $A = 0.2\%$, the performance expected from Transcon 1000 Ω/sq conductors [58]. In all cases, we observe a roughly linear decrease in $T$ as $N$ increases. However, we simultaneously observe an exponential increase in $S$, a very favorable scaling behavior. Since most useful steering applications require $25 \leq M \leq 100$, we predict $72\% \leq T \leq 94\%$, primarily depending on the sophistication of the electrodes.

To demonstrate this steering concept, we prepared three PGs using polarization holography [1], commercial materials, and standard LC cell processing. We utilized a linear photopolymerizable polymer (LPP) (ROP-103/2CP, from Rolic) as the photoalignment material and orthogonally circularly polarized beams from a He-Cd laser (325 nm) to record the PG pattern. A $\sim 2.5$ µm cell thickness was achieved with glass spheres (Dana Enterprises) to set half-wave effective retardation with the nematic LC (LCMS-102, from Boulder Nonlinear Systems, $\Delta n = 0.31$ measured [76] at 1550 nm). To minimize reflection loss, all LC elements were laminated to each other with optical glue (NOA-63, Norland), and glass with anti-reflection coatings (PG&O) were glued to the front and back faces. The three WP were fabricated in a similar fashion, but with uniformly aligned LPP layers.

The parameters $\Lambda_l$ and $\theta_l$ of each PG were chosen according to Eqs. 4.6, and are listed in Table 4.1, which shows characterization data of the individual PGs with an infrared laser (1550 nm, 40 mW). In order to obtain an experimental quantity $\eta_{+1}$ comparable to Eq. (4.8), we define the intrinsic diffraction efficiency of order $m$ as $\eta_m = P_m/(P_{-1} + P_0 + P_{+1})$ where $P_m$ is the measured power of the $m^{th}$ diffraction order, when the input is circularly polarized. We define the scattering loss $D$ as the fraction of transmitted light.
Figure 4.20: Captured output beams of the $N = 3$ ternary beam steerer operating at 1550 nm: composite image of photographs of the 27 steered beams on an IR viewing-card. Three-digit number next to the output beam picture indicates the diffraction state of the three steering stages. For example, $\{-, 0, +\}$ denotes \{subtracting, passing, adding\} angle state of the \{3rd, 2nd, 1st\} stage.

(measured using an integrating sphere) that does not appear within one of the three diffraction orders. These PGs exhibit nearly ideal diffraction properties that $\geq 99.7\%$ fraction of light is steered into the intended direction without observable higher orders ($\eta_0 \leq 0.2\%$, $\eta_{m\geq 2} < 0.05\%$).

We assembled a prototype ternary beam steerer containing three steering stages, which covered $44^\circ$ $FOR$ with $1.7^\circ$ resolution at 1550 nm wavelength. Photographs of the captured output beams of the ternary beam steerer are shown in Fig. 4.20. The images include the 27 steered beams on an IR viewing-card. The signs next the images indicate angle state of each stage.

In Fig. 4.21(a), we show the relative transmitted power across the observed output angle range $\pm 30^\circ$ for two steered direction settings: $-22^\circ$ (all PGs diffracting) and $0^\circ$ (all PGs non-diffracting). The only significant sidelobe ($\sim 0.8\%$) appears at $-6.8^\circ$ for
Figure 4.21: Measured output of the $N = 3$ Ternary PG steerer operating at 1550 nm: (a) relative power (i.e., transmittance) at all output angles for two intended mainlobe steering angles ($-22^\circ$ and $0^\circ$); and (b) transmittance and diffraction efficiency of the mainlobe for all steering angles.

diffracting case. Very low sidelobes are generally observed throughout the steering range. The measured diffraction efficiency and transmittance of the mainlobe are shown in Fig. 4.21(b). The measured transmittance, comparable to Eq. 4.8, is calculated as $T = P_{\text{main}}/P_{\text{in}}$, where $P_{\text{main}}$ is the mainlobe power and $P_{\text{in}}$ is the input power. The efficiency, a normalization that removes the effect of the substrates to reveal the aggregate effect of the diffractive PGs, is defined as $\eta = P_{\text{main}}/P_{\text{tot}}$, where $P_{\text{tot}}$ is the total transmitted power into the exit hemisphere, measured with an integrating sphere. For all steering angles, solid transmittance ($78 - 83\%$) was observed while, along with high diffraction efficiency ($94 - 99\%$). This confirms that losses in this demonstration are predominantly related to the substrate absorption and reflection, and that the PGs are fairly efficient at redirecting light as expected even when the incidence angle is far from the normal direction.
4.2.4 Quasi-Ternary Steering Design and Demonstration

In the section, we introduce and demonstrate a quasi-ternary design, which is an alternative design that utilized the fact that each PG switches the handedness of a beam passing through it, resulting in a fewer number of total elements in the steerer. This is important to make the system compact and to reduce losses from these individual units such as absorption from glass, transparent-conducting-electrodes, fresnel reflection losses at different interfaces and scattering from the gratings. For this reason, the quasi-ternary design may be superior to the ternary design. The reduced number of elements (cells) in the quasi-ternary design directly translates to higher steering efficiency and transmittance. Different from the ternary design, the quasi-ternary design only requires one LC cell (LCPG) for each stage, except the first stage that has two LC cells (LCWP+LCPG). The quasi-ternary design uses sequentially additive and subtractive approaches; that is to say the steering angle is determined by alternatively adding and subtracting the diffraction angles of each stage in turn. This is possible because the sign of diffraction angle depends on the handedness of the incident polarization, and the handedness would change to the opposite state when the beam passes through a PG having half-wave retardation. In this way, when circularly polarized light passes through the multiple PGs, the direction of diffraction angles is alternately positive and negative. Then, by controlling the applied voltage on each PG, we can selectively accumulate positive or negative diffraction angles.

Fig. 4.22 shows the quasi-ternary coarse steerer design implemented with Liquid Crysl-
tal Wave Plate (LCWP) and Liquid Crystal Polarization Grating (LCPG). The quasiternary design consists of only one LCWP and several LCPGs. The front LCWP can change the handedness of the incident polarization, and the following LCPGs are responsible for accumulating the steering angle by directing the beam to either the zero, positive first, or negative first diffraction order. An applied voltage will select the zero-order, while the handedness of the incident polarization selects either the positive or negative first-order.

As we discussed in previous section, the handedness of circular polarization would be changed to opposite state when the input beam passes through the LCPG having half-wave retardation. If the output beam passes through another LCPG, the direction of diffraction angle would be opposite from direction of diffraction angle from the previous LCPG. In this way, when circularly polarized light travels the multiple LCPGs, the directions of diffraction angle are to be positive and negative repeatedly. Then, as controlling the applied voltage on each LCPG, we can accumulate positive or negative diffraction angle by turns.

In Fig. 4.23, we describe the state (ON/OFF) of applied voltage of each element for different steering angles. The number (angle) in the block denotes the diffraction angle made by each LCPG, and the final steering angle can be calculated as adding them all. Here we assume five-stage quasi-ternary steerer that consists of one LCWP and five LCPGs having 1.25°, 3.75°, 8.75°, 18.75° and 20° diffraction angles. The front LCWP

<table>
<thead>
<tr>
<th>Elements</th>
<th>State of Applied Voltages</th>
</tr>
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<tbody>
<tr>
<td>LCWP</td>
<td></td>
</tr>
<tr>
<td>PG1 (1.25°)</td>
<td>+1.25° -1.25° +1.25° -1.25° +1.25° -1.25°</td>
</tr>
<tr>
<td>PG2 (3.75°)</td>
<td>+3.75° +3.75° -3.75° -3.75° +3.75° +3.75°</td>
</tr>
<tr>
<td>PG3 (8.75°)</td>
<td>+8.75° +8.75° +8.75° +8.75° +8.75° +8.75°</td>
</tr>
<tr>
<td>PG4 (18.75°)</td>
<td>+18.75° +18.75° +18.75° +18.75° +18.75° +18.75°</td>
</tr>
<tr>
<td>PG5 (20.00°)</td>
<td></td>
</tr>
</tbody>
</table>

Non-diffraction (Voltage ON) Diffraction (Voltage OFF)

Figure 4.23: Applied voltages on each element for quasi-ternary steerer design. This five-stage steerer can cover ±40° FOR with 1.25° steps. Note that only a subset (0° to +15°) of the ±40° range is shown.
may change the handedness of incident polarization, and following LCPGs control the
diffraction orders to the zero- or first-order according to the voltages applied to them. That
means, LCPGs may pass the beam without changing the angle of beam, or it may deflect
the beam to one of the first orders depending on the handedness of incident polarization.

For the quasi-ternary design, the first stage’s diffraction angle (e.g. \( r \)) determines the
resolution of the coarse beam steerer. The steering design containing \( N \) single stages (i.e.,
\( N \) LCPGs) can have a total of \( 2^{N+1} - 1 \) distinct steering angles theoretically. Therefore,
the quasi-ternary design enables far more angles to be steered by the same number of
stages. The total number of steering angles \( S \) and angle resolution \( r \) are determined by
number of stages \( N \):

\[
S = 2^{N+1} - 1 \tag{4.9a}
\]

\[
r = \frac{\text{FOR}}{(2^{N+1} - 2)} \tag{4.9b}
\]

where FOR is the field-of-regard. We highlight the exponential increase of \( S \) in Fig. 4.22.
The diffraction angle \( \theta_l \) and grating period \( \Lambda_l \) of each stage number \( l \) is

\[
\sin \theta_l = \sin((2^l - 1)r) \tag{4.10a}
\]

\[
\Lambda_l = \frac{\lambda}{\sin \theta_l} \tag{4.10b}
\]

where \( \lambda \) is the wavelength of incident beam. The overall output angle \( \Theta \) can be expressed
as

\[
\sin \Theta = \sum_{l=1}^{N} (-1)^{P_l^{PG}} V_l^{PG} \sin \theta_l \tag{4.11}
\]

where \( P_l^{PG} \) is the input polarization state of the \( l^{th} \) PG (0 or 1 when the input polarization
is LCP or RCP, respectively), and \( V_l^{PG} \) is the state of \( l^{th} \) PG (0 or 1 when \( V \gg V_{th} \) or
\( V = 0 \), respectively).

The diffraction angle \( \theta_l \) of the stage \( l \) is exponentially increased with the factor of
2, and then the last PG angle should be fairly large to cover large FOR. However, very
large diffraction angle of PG is not suitable for the steerer since the diffraction efficiency
would be decreasing as the angle of PG is increasing. For example, to cover 80° FOR
with 5-stage quasi-ternary steerer, the last PG angle should be 40°, which is not feasible
for fabrication. One possible way to make the same FOR with smaller PG angle is the
use of antiparallel arranged PG. When the last active PG is interlaced as an antiparallel
arrangement comparing with the other PGs, the last PG has its anisotropic profile of 
\( n(x) = [\sin(\pi x/\Lambda), -\cos(\pi x/\Lambda), 0] \), where \( n \) is a unity vector describing the orientation of the linear birefringence. Therefore after passing through the last PG, LCP light is diffracted to negative direction and RCP light diffracted to positive direction in contrast to other PGs. In this case, the steerer can cover the same FOR with smaller last PG angle \( \sim 18^\circ \). The last PG angle is determined by the equation below,

\[
\sin \theta_{N-1} + \sin \theta_N = \sin(FOR/2) \\
\sin((2^N-1)r) + \sin \theta_N = \sin(FOR/2).
\]

(4.12a) \hspace{1cm} (4.12b)

Therefore, we can derive the equation for the last PG angle \( \theta_N \) as below,

\[
\theta_N = \arcsin(\sin(FOR/2) - \sin((2^{N-1} - 1)r)).
\]

(4.13)

Having different beam steerer designs, we need to estimate the performance between these designs. Here we consider three different designs: binary [1], ternary [77], and quasi-ternary. The binary design consists of active LC cells (LCPG and LCWP) and is based on additive approach; it accumulates the diffraction angles of each stage as the sign of the direction of angles are the same; therefore the configuration gives a total of \( 2^{N+1} - 1 \) distinct steering angles with \( N \) single stages. In this section, we will compare the three designs in terms of the number of steering angles and expected transmittance.

The total number of steering angles is determined by number of stages \( N \) of each steering design. Since the LC cells are the main cause of system losses, a smaller number of LC cells should be used for better steering performance. Therefore, the total number of steering angles should be considered with the same number of LC cells. In Table 4.2, the numbers of steering angles of the three designs are compared with the number of

<table>
<thead>
<tr>
<th>Steering Design Option</th>
<th># of Steering Angles (N stages)</th>
<th># of LC Cells (M cells)</th>
<th># of Steering Angles (M cells)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary [1]</td>
<td>( 2^{N+1} - 1 )</td>
<td>( M=2N )</td>
<td>( 2\sqrt{2}^M - 1 )</td>
</tr>
<tr>
<td>Ternary [77]</td>
<td>( 3^N )</td>
<td>( M=2N )</td>
<td>( \sqrt{3}^M )</td>
</tr>
<tr>
<td>Quasi-Ternary</td>
<td>( 2^{N+1} - 1 )</td>
<td>( M=N + 1 )</td>
<td>( 2^M - 1 )</td>
</tr>
</tbody>
</table>
LC cells $M$. The ternary design enables $3^N$ angles to be steered with $N$ steering stages and each stage consists of two LC cells. While the quasi-ternary design can only steer $2^{N+1} - 1$ angles with $N$ steering stages, each stage only requires one additional LCPG (except the first stage that also requires one LCWP). Therefore, the quasi-ternary design gives far more angles using the same number of LC cells as shown in Fig. 4.24.

In order to compare performance of the designs, the expected PG efficiency and losses were defined based on the nominal best-case-value of our experimental work listed in Sec. 4.2.2. When the PG efficiency and losses of each stage are the same, the overall system transmittance $T$ can be estimated as below:

$$T = (\eta_{+1})^N (1 - D)^N (1 - R)^{N+1} (1 - A)^{N+1},$$

where $\eta_{+1}$ is the experimental intrinsic diffraction efficiency of each PG, $D$ is the diffuse scattering of each PG, and where $R$ and $A$ are the Fresnel reflectance and absorption losses, respectively, of each LC cell. We graph the transmittance estimation for each of the three designs in Fig. 4.25, based on our experimental data of the LCPGs and LCWPs. In all designs, transmittance is decreased as the number of steering angles increases. However, we observe that the transmittance of the quasi-ternary design is slowly decreased in a very favorable scale behavior. We predict $82\% \leq T \leq 88\%$ for the most useful steering applications that require $25 \sim 100$ number of steering angles for the quasi-ternary design option.
We expect the quasi-ternary steerer can show fairly high performance, but the estimation is based on our sample data, which can be improved with higher quality LC elements and substrates. Here, we show expected parameter and transmittance of quasi-ternary design for three different cases in Table 4.3. Case (i) corresponds to the parameters we were able to experimentally demonstrate in this work where simple indium-tin-oxide (ITO) electrodes were implemented. Case (ii) corresponds to the case with lower scattering LCPG ($D = 0.5\%$) when commercially available index-matched ITO are used, to reduce $R=0.1\%$. Case (iii) corresponds to the best-case scenario, where low loss transparent conductors are employed to reach $A=0.2\%$, the performance expected from Transcon 1000 Ω/sq conductors [58].

In order to demonstrate the quasi-ternary design, we prepared four LCPGs that are used for each stage of the steerer ($N = 4$). The LCPGs utilized polarization holography, commercial materials, and standard LC cell processing. We utilized a linear photo-

Table 4.3: Estimated parameters and transmittance of four-stage quasi-ternary steerer.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\eta$</th>
<th>$D$</th>
<th>$R$</th>
<th>$A$</th>
<th>Transmittance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>99.5%</td>
<td>1.0%</td>
<td>1.5%</td>
<td>1.0%</td>
<td>83.0%</td>
</tr>
<tr>
<td>(ii)</td>
<td>99.5%</td>
<td>0.5%</td>
<td>0.1%</td>
<td>1.0%</td>
<td>90.9%</td>
</tr>
<tr>
<td>(iii)</td>
<td>99.5%</td>
<td>0.3%</td>
<td>0.1%</td>
<td>0.2%</td>
<td>95.4%</td>
</tr>
</tbody>
</table>
Table 4.4: Characterization data of LCPGs having different diffraction angles ($\lambda = 1550$ nm).

<table>
<thead>
<tr>
<th>$\theta_l$ (deg)</th>
<th>$I_{in}$ (mW)</th>
<th>$I_{ref}$ (mW)</th>
<th>$I_{+1}$ (mW)</th>
<th>$I_0$ (mW)</th>
<th>$I_{-1}$ (mW)</th>
<th>$T_{+1}$ (%)</th>
<th>$\eta_{+1}^a$ (%)</th>
<th>$\eta_{+1}^i$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>54.40</td>
<td>49.10</td>
<td>48.70</td>
<td>0.22</td>
<td>0.01</td>
<td>89.54</td>
<td>99.21</td>
<td>99.55</td>
</tr>
<tr>
<td>5.1</td>
<td>54.40</td>
<td>49.10</td>
<td>48.75</td>
<td>0.10</td>
<td>0.01</td>
<td>89.63</td>
<td>99.31</td>
<td>99.80</td>
</tr>
<tr>
<td>11.9</td>
<td>54.30</td>
<td>48.70</td>
<td>48.08</td>
<td>0.15</td>
<td>0.02</td>
<td>88.58</td>
<td>98.77</td>
<td>99.69</td>
</tr>
<tr>
<td>13.6</td>
<td>54.30</td>
<td>49.10</td>
<td>48.40</td>
<td>0.20</td>
<td>0.03</td>
<td>89.19</td>
<td>98.64</td>
<td>99.59</td>
</tr>
</tbody>
</table>

polymerizable polymer ($LPP$) (ROP-103/2CP, from Rolic) as the photo-alignment material and orthogonally circularly polarized beams from a HeCd laser (325 nm) to record the PG pattern. A $\sim 2.5\mu m$ cell thickness was achieved with glass spheres (Dana Enterprises) to set the half-wave effective retardation with a nematic LC (LCMS-102, from Boulder Nonlinear Systems, $\Delta n = 0.31$ measured [76] at 1550 nm). We show individual characterization data of the LCPGs in Table 4.4. We used an infrared laser (1550 nm, 40 mW), and measured three possible diffraction orders, $I_m$, that corresponds to the diffracted power of order $m$. $I_{in}$ is the input power, $T_m = I_m/I_{in}$ is the transmittance of order $m$, $I_{ref}$ is the transmitted power of substrate/cell filled with glue (used for reference), $\eta_{m}^a = I_m/I_{ref}$ is the absolute diffraction efficiency of the grating, and $\eta_m^i = I_m/(I_{-1} + I_0 + I_{+1})$ is the intrinsic diffraction efficiency.

Intrinsic efficiency quantifies the inherent diffraction efficiency of the grating alone, normalizing out the effects of the substrates and any scattering. From the data, all LCPGs exhibit nearly ideal diffraction properties that $> 99.5\%$ fraction of light is steered into the intended direction without observable other orders. Absolute diffraction efficiency ($> 98.6\%$) includes the scattering loss of LCPGs, but still normalizes out the losses of substrates. The transmittance of LCPGs in the measurement was between 88.6 – 89.6%, and most of losses were primarily in the interface of elements. The reference intensity can be measured with the same element which has no grating; the measured value was almost 90% of the input beam due to the Fresnel and absorption losses driving from the unmatched indices and the transparent-conducting-electrode layers of the substrates. The expected throughput will be obtained with the optimized substrates that can reduce the losses with anti-reflection coating and index-matched transparent-conducting-electrode layers.

We assembled a prototype quasi-ternary beam steerer containing four steering stages
Figure 4.26: Experimental setup to measure power of the steering angles; $P_{in}$ is the input power and $P_{\text{main}}$ is the mainlobe power of diffracted beam.

(one LCWP + four LCPGs listed in Table 4.4). This steerer can cover 52° \textit{FOR} with 1.7° resolution at 1550 nm wavelength. In order to minimize reflection loss, all LC cells were laminated to each other with optical glue (NOA-63, Norland), and glass with anti-reflection coating (PG&O) were glued to the front and back faces. The LCWP was fabricated in a similar fashion, but with uniformly aligned LPP layers.

Fig. 4.26 shows experimental setup to characterize diffracted beams. Note that initially the laser is linearly polarized, but after passing through Quarter-Wave Plate (QWP) whose axis is at 45°, the beam is changed to circularly polarized light. Passing through the elastically controlled LC cells, the beam is then diffracted to the desired angle. To control the steering angle, a custom controller was built. The controller consists of a connection box, a data output board (NI-DAQ 6722) connected to a computer, and a custom software application. Both electrodes on every LC cell were connected to individual terminals in the connection box, which was connected to the output board. The software then determines which cells to activate to achieve the desired steering angle and sends appropriate voltages to each cell. A cell that needs to be in the diffractive state is given 0V. A cell that needs to be in the transparent state is given a 10V P-P square wave at 1 kHz. Using these parameters, the controller can switch between any two steering angles in about 25 msec (40 Hz).

We used IR power detector with integrating sphere to measure the power of diffracted beams for all steering angles. In order to observe the diffracted power distribution, we show the relative transmitted power across the observed output angle range ±35° for
two steered directions: $+26^\circ$ (maximum steering angle, two LCPGs diffracting) and $0^\circ$ (minimum steering angle, all LCPGs non-diffracting). In case of maximum angle ($+26^\circ$), small sidelobes ($\sim 0.9\%$) were detected at the angle of $7^\circ$ and $-1^\circ$ and very low sidelobes are generally observed throughout the steering angles, as shown in Fig. 4.27. In case of minimum angle ($0^\circ$), there was no significant sidelobe across the observed angle range, and most of power passed through each LC cells that were on-state (no grating).

The measured diffraction efficiency and transmittance of the mainlobe for each steering angle are shown in Fig. 4.28. Following Eq. 4.14, the transmittance $T$ is calculated as $T = P_{\text{main}}/P_{\text{in}}$, where $P_{\text{main}}$ is the mainlobe power and $P_{\text{in}}$ is the input power. The efficiency, a normalization that removes the effect of the substrates to reveal the aggregate effect of the diffractive PGs, is defined as $\eta = P_{\text{main}}/P_{\text{tot}}$, where $P_{\text{tot}}$ is the total transmitted power into the exit hemisphere, measured with an integrating sphere. For every steering angle, a transmittance ($77 - 84\%$) and high diffraction efficiency ($92 - 99\%$) was observed. Also, it is worth noting that the steerer is still relatively efficient even when the incidence angle is far from normal. The difference between transmittance and efficiency confirms that losses in this demonstration are predominantly related to the substrate absorption and reflection, and that the LCPGs are fairly efficient at redirecting light as expected. The reflectance and absorption is primarily due to the electrode material and the various interfaces, while the LC itself has comparatively very low absorption.
Figure 4.28: Characterization data of the quasi-ternary steerer operating at 1550 nm: transmittance and diffraction efficiency of the mainlobe for all possible steering angles.

We have demonstrated the quasi-ternary steering design, which use a single wave plate and \( N \) PGs to generate \( 2^{(N+1)} - 1 \) steering angles. When compared to binary and ternary liquid crystal PG steering designs, this technique uses fewer elements arranged in a simpler configuration to obtain the same number of steering angles. This advantageous property can be achieved by selecting proper diffraction angles and alignment of the PGs. Due to fewer elements per stage, losses due to electrode absorption and Fresnel reflections are reduced, thereby increasing the overall steering efficiency. Using the approach, we demonstrated a four-stage \( (N = 4) \) quasi-ternary beam steering device that achieves 52° FOR with 1.7° resolution (31 steering angles) at 1550 nm wavelength. The device shows high optical throughput (77 – 84%) that is mainly limited by the losses from the substrates and electrode materials, which can be further improved.
4.2.5 Supra-Binary Steering Design and Demonstration

We have suggested and demonstrated beam-steering devices that are based on active liquid crystal PGs in Sec. 4.2.3 and 4.2.4. The LCPGs can diffract the input beam to three different orders depending on input polarization states. However, scattering loss of the LCPG increases as the diffraction angle of LCPG increases. According to our previous experiments [1], the diffraction angle of LCPGs is limited (normally < 15°) when the PGs are fabricated with LCMS-102 (Dn 0.31 at 1550nm, BNS). Here we suggest to use passive polymer-PGs for new steering design, supra-binary configuration. Compared to other binary steering design using passive PGs discussed in Sec. 4.2.2, the supra-binary design requires fewer LC-elements arranged in a simpler configuration to achieve the same number of steering angles. Moreover, the design can be adopted to wide-angle steering application since the polymer-PGs perform large angle steering without significant loss.

Fig. 4.29(a) shows a single stage that comprises a polarization selector (e.g. LC half-wave plate) and a polymer-PG resulting in two-way steering. The polarization selector ensures that the input to the polymer-PG is either of the two orthogonal (right or left) circular polarization states. Depending on the handedness of polarization, the polymer-PG diffracts the beam into one of the first orders (+θ or −θ). Since the polymer-PG is a passive element, there is no zero-order on the stage. Fig. 4.29(b) shows three-stage supra-binary design, where each stage can access a different set of diffraction angles. The diffraction angle (θ_l) of each stage (l) is nearly r(2^{l-1}), when r is the smallest

![Figure 4.29: Supra-binary design beam steering: (a) Beam steering in single steering stage containing a PG and a Polymer-PG; (b) Beam steering in three-stage supra-binary steerer. Θ is steering angle when Ω_l = sin θ_l (θ_l is the diffraction angle of each stage number l).](image)
diffraction angle of the polymer-PGs. Each single stage has two steering options (positive or negative) and final steering angle is determined by adding and subtracting diffraction angles of the stages.

The supra-binary design enables large steering angles without significant loss from PGs. The total number of steering angles $S$ and angle resolution $r$ are determined by number of stages $N$:

$$S = 2^N$$

$$r = \frac{\text{FOR}}{(2^N - 1)},$$

(4.15a)

(4.15b)

where $\text{FOR}$ is the field-of-regard. We show the increase of $S$ in Fig. 4.29(b). The diffraction angle $\theta_l$ and grating period $\Lambda_l$ of each stage number $l$ is

$$\sin \theta_l = \sin((2^l - 1)r/2) - \sin((2^l - 1 - 1)r/2)$$

$$\Lambda_l = \frac{\lambda}{\sin \theta_l},$$

(4.16a)

(4.16b)

where $\lambda$ is the wavelength of incident beam. The overall output angle $\Theta$ can be expressed as

$$\sin \Theta = \sum_{l=1}^{N} (-1)^{V_l^{\text{WP}}} \sin \theta_l$$

(4.17)

where $V_l^{\text{WP}}$ is the state of the $l^{th}$ WP (0 or 1 when the WP output is LCP or RCP, respectively). If we assume that the polymer-PG efficiency and losses of each stage are the same, we can approximate the overall system transmittance $T$ in the following way:

$$T = (\eta_{+1})^N (1 - D)^N (1 - R)^{2N} (1 - A)^N,$$

(4.18)

where $\eta_{+1}$ is the experimental intrinsic diffraction efficiency of each polymer-PG, $D$ is the diffuse scattering of each PG, and where $R$ is the Fresnel reflectance of each element (LC wave plate + polymer PG) and $A$ is the absorption losses of each LC cell (LC wave plate only).

The total number of steering angles is determined by number of stages $N$ of each steering design. Since the LC cells are the main cause of system losses, a smaller number of LC cells should be used for better steering performance. Therefore, the total number of steering angles should be considered with the same number of LC cells: LCPG and
Table 4.5: Total number of steering angles of beam steering designs.

<table>
<thead>
<tr>
<th>Steering Design Option</th>
<th># of Steering Angles $(N$ stages$)$</th>
<th># of LC Cells $(M$ cells$)$</th>
<th># of Steering Angles $(M$ cells$)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary</td>
<td>$2^{N+1} - 1$</td>
<td>$M=2N$</td>
<td>$2\sqrt{2}^M - 1$</td>
</tr>
<tr>
<td>Ternary</td>
<td>$3^N$</td>
<td>$M=2N$</td>
<td>$\sqrt{3}^M$</td>
</tr>
<tr>
<td>Quasi-Ternary</td>
<td>$2^{N+1} - 1$</td>
<td>$M=N+1$</td>
<td>$2^M - 1$</td>
</tr>
<tr>
<td>Supra-Binary</td>
<td>$2^N$</td>
<td>$M=N$</td>
<td>$2^M$</td>
</tr>
</tbody>
</table>

liquid crystal wave plate (LCWP). In Table 4.5, the numbers of steering angles of the four designs are compared with the number of LC cells $M$. First, the binary design [1] needs two LC cells for each steering stage and the design can achieve $2^{N+1} - 1$ steering angles with $N$ stages. The ternary design [77] also requires two LC cells for each stage, but the design enables $3^N$ angles to be steered with $N$ steering stages. The quasi-ternary design [78] can steer only $2^{N+1} - 1$ angles with $N$ steering stages, but each stage consists of only one LCPG (except the first stage that also requires one LCWP). So the design can steer $2^M - 1$ angles with $M$ LC cells. The new design, supra-binary requires only one LC cell on each stage and covers $2^N$ total steering angles, therefore $2^M$ steering angles can be accomplished with $M$ LC cells. The supra-binary gives far more angles than other designs using the equivalent number of cells, which leads to better steering performance. We graph the number of steering angles for each of the four steering design in Fig. 4.30.

Figure 4.30: Number of LC cells vs Number of steering angles for the three designs.
When comparing steering designs, we also need to consider the diffraction angles of the PGs used in each design. Normally, a PG with a larger diffraction angle shows more scattering loss and more zero-order leakage, and each steering design requires a different set of PGs with various diffraction angles. In our designs, the PG diffracting the largest angle is placed in the last stage, so we can compare the diffraction angle of the last PG in the steering designs. Since the quasi-ternary and supra-binary designs show almost the same performance in terms of the number of steering angles, we compare the PG diffraction angles of the two designs. In Table 4.6, diffraction angles of each PG are shown when five LC cells are used for both steering designs. In this case, the quasi-ternary design has one less steering stage than the supra-binary design since the quasi-ternary requires an additional LC cell (LCWP) at the first steering stage.

These two steering designs consist of the same number of LC cells ($M = 5$), and their total field-or-regard (FOR) and steering resolution are almost the same. For example, if the resolution of the designs is set as $1^\circ$, they can cover nearly $30^\circ$ in one dimension. However, when comparing the largest diffraction angle of PG of the designs, the supra-binary can perform the same steering FOR and resolution with smaller diffraction angle PGs. For the supra-binary, the largest diffraction angle is $\sim \text{FOR}/2$, while the quasi-ternary requires a PG with $\text{FOR}$ diffraction angle. In this case, when the resolution is $1^\circ$, the quasi-ternary should have an active PG having $15^\circ$ diffraction angle, which may affect the total throughput. Moreover, when compared to steering designs based on active PGs (e.g. ternary and quasi-ternary designs), the supra-binary can cover larger steering angles since the design only involves polymer PGs that can be fabricated for large steering angles (e.g. $> 15^\circ$) without significant scattering loss. Because of these reasons, the supra-binary design may be better for performing large steering angle (FOR) as maintaining high throughput.
It is important to note that being a passive approach, the PGs in the supra-binary design are “always on.” This is opposed to the active designs where the PGs are toggled on and off. This means that in the supra-binary design, any losses from a PG will be present in every steering angle, as opposed to the active designs where the losses only show up when the PG is in the on state. The total effect is that the PG quality is more important in the supra-binary method.

Here, we aim to verify this beam steering concept, and evaluate limitations and the non-idealities. Fig. 4.31 shows a design configuration of 5-stage supra-binary coarse steerer that can steer $80^\circ$ with $\sim 2.6^\circ$ steps. Here, we just consider horizontal steering assembly including five LCWP and LC polymer PGs. Detailed grating periods and corresponding diffraction angles of the LC polymer PGs are shown in Table 4.7.

The fabrication process includes the following three steps: (i) LPP-coating on a substrate; (ii) holographic exposure; (iii) Spin-coating LC polymers. We first prepared a thin

Table 4.7: Diffraction angles at 1064 nm and grating periods of polymer-PGs for supra-binary design.

<table>
<thead>
<tr>
<th>Stage ($l$)</th>
<th>Resolution</th>
<th>FOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.58</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>2.58</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.13</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10.04</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>18.14</td>
<td></td>
</tr>
<tr>
<td>Diffraction Angles (deg)</td>
<td>1.29</td>
<td>2.58</td>
</tr>
<tr>
<td>Grating Periods (µm)</td>
<td>47.26</td>
<td>23.64</td>
</tr>
</tbody>
</table>
(\sim 70 \text{ nm}) layer of a LPP material (ROP103/2CP, Rolic Technologies Ltd.) on a cleaned glass substrate by spin coating at 3000 rpm for 30 seconds with subsequent pre-baking at 130^\circ C for 10 minutes. The sample was exposed with \sim 5 J/cm^2 with two recording beams from a UV laser (HeCd, \lambda = 325 \text{ nm}) using a polarization holography [79, 1]. Next, the exposed substrate was coated with an LC prepolymer mixture (RMS09-025, Merck, \Delta n \sim 0.33 [76] at 589 nm) using a spin-coater (Laurell Technologies Corp.) at the room temperature. Lastly, this layer was polymerized with UV light (roughly 0.4 J/cm^2 around 365 nm) for 3 minutes. In order to get proper thickness (i.e. Retardation) with best alignment and optical behavior, multiple LC layers can be coated on the sample.

We fabricated five polymer PGs whose grating periods are indicated in Table 4.7. As a result of the coating process, the samples had proper thickness showing half-wave
Table 4.8: Characterization data of polymer PGs used for each steering stage (stage #1 − #5): \( \theta \) is diffraction angle at 1064 nm, and \( \Lambda \) is grating period of the polymer PGs. \( T_{+1} = I_{+1}/I_{in} \) is the transmittance of the first order, when \( I_{+1} \) and \( I_{in} \) indicate the first order power and an input power respectively. \( \eta^a_{+1} = I_{+1}/I_{ref} \) is the absolute diffraction efficiency, and \( \eta^i_{+1} = I_{+1}/(I_{-1} + I_0 + I_{+1}) \) is the intrinsic diffraction efficiency, when \( I_{ref} \) is the transmitted power of substrate (used for reference).

<table>
<thead>
<tr>
<th>PG</th>
<th>( \theta ) (deg)</th>
<th>( \Lambda ) (( \mu )m)</th>
<th>( T_{+1} ) (%)</th>
<th>( \eta^a_{+1} ) (%)</th>
<th>( \eta^i_{+1} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>1.3</td>
<td>47.3</td>
<td>89.7</td>
<td>99.5</td>
<td>99.6</td>
</tr>
<tr>
<td>#2</td>
<td>2.6</td>
<td>23.6</td>
<td>89.6</td>
<td>99.4</td>
<td>99.7</td>
</tr>
<tr>
<td>#3</td>
<td>5.1</td>
<td>11.9</td>
<td>89.6</td>
<td>99.3</td>
<td>99.6</td>
</tr>
<tr>
<td>#4</td>
<td>10.0</td>
<td>6.1</td>
<td>88.5</td>
<td>98.2</td>
<td>98.7</td>
</tr>
<tr>
<td>#5</td>
<td>18.1</td>
<td>3.4</td>
<td>87.7</td>
<td>97.2</td>
<td>97.5</td>
</tr>
</tbody>
</table>

retardation at 1064 nm, which is our target wavelength. Fig. 4.32 shows the first order efficiency of the sample showing nearly 100% efficiency at 1064 nm.

Fig. 4.33 shows polarizing microscope images of the samples having different grating periods. The polarizing microscope images show the intensity profiles of the PGs placed between two crossed linear polarizers, which present grating profiles.

Figure 4.34: Laser (1064 nm) beam profiles of (a) input and output passing through different PG samples: (b) PG#1 (1.3°), (c) PG#3 (5.1°), and (d) PG#5 (18.1°).
The transmittance of the polymer PGs in the measurement was between 87.7 – 89.7%, and most of losses were primarily in the interface of elements. Absolute diffraction efficiency (> 97.2%) includes the scattering loss of the PGs, but still normalizes out the losses of substrates. Intrinsic efficiency quantifies the inherent diffraction efficiency of the grating alone, normalizing out the effects of the substrates and any scattering. From the data, all polymer PGs exhibit nearly ideal diffraction properties, that is, most of light is steered into the intended direction without observable other orders.

Fig. 4.34 illustrate the beam profiles of the input beam and steered beams made by three different PGs. We used a 1064 nm laser that is circularly polarized as an input, and the output beam was captured by an optical beam profiler (Thorlabs). The beam was diffracted following the horizontal direction (i.e., x-axis) and diffraction angles at 1064 nm were 1.3°, 5.1°, and 18.1°. For all cases, we could not observe noticeable deformation of the beam profile including the peak intensity value.

As the single steering stages were assembled and integrated for a coarse steering module, several measurements were conducted. Fig. 4.35 shows experimental setup to characterize steered beams. Note that initially the laser is linearly polarized, but after passing through Quarter-Wave Plate (QWP) whose axis is at 45°, the beam is changed to circularly polarized light. Passing through the set of single steering stages, the beam is then diffracted to the desired angle. To control the steering angle, a custom controller

![Supra-binary design beam steering](image)

**Figure 4.35:** Supra-binary design beam steering: (a) Beam steering in single steering stage containing a PG and a Polymer-PG; (b) Beam steering in three-stage supra-binary steerer. \( \Theta \) is steering angle when \( \Omega_l = \sin \theta_l \) (\( \theta_l \) is the diffraction angle of each stage number \( l \)).
was built including a data output board (NI-DAQ 6722) connected to a computer and a custom software application. Both electrodes on every LC half-wave plates were connected to individual terminals in the connection box, which was connected to the output board. The software then determines which LC cells to activate to achieve the desired steering angle and sends appropriate voltages to each cell. A waveplate cell in the on state (active) is given 0V. A waveplate cell in the off state is given a 10V P-P square wave at 1kHz.

We assembled a prototype supra-binary beam steerer consisting of five steering stages (five LCWPs + five LC polymer PGs listed in Table 4.8). This steerer can cover 80° FOR with 2.6° resolution at 1064 nm wavelength. In order to minimize reflection loss, all LC elements were laminated to each other with optical glue (NOA-63, Norland), and anti-reflection coating (PG&O) substrates were glued to the front and back faces.

Fig. 4.36 shows the result concerning a steering accuracy for all positive steering angle (0° − 40°). There was less than 0.1° angle difference for small and large steering angles, but the angle difference increases to 0.8° for mid-range steering angles. The offset is caused by the non-linear diffraction property of the PGs since the input angle affects the

![Figure 4.36: Desired angles vs measured angles for all steering angles (positive angle only).](image)

96
output angle slightly as shown in Eq. 4.3. These static offsets are what we are expected and can be compensated as placing additional active grating if it is necessary.

The photographs of the steered beams captured by an IR viewing-card are shown in Fig. 4.37. We fixed the position of the camera and took each steered beam projected on the card that was 20 cm away from the input source. To measure the power of steered beams, we used IR power detector with integrating sphere (Newport). The measured diffraction efficiency and transmittance of the mainlobe for each steering angle are shown in Fig. 4.38. The transmittance $T$ is calculated as $T = \frac{P_{\text{main}}}{P_{\text{in}}}$, where $P_{\text{main}}$ is the mainlobe power and $P_{\text{in}}$ is the input power. The efficiency, a normalization that removes the effect of the substrates to reveal the aggregate effect of the diffractive PGs, is defined as $\eta = \frac{P_{\text{main}}}{P_{\text{tot}}}$, where $P_{\text{tot}}$ is the total transmitted power into the exit hemisphere, measured with an integrating sphere. For every steering angle, a transmittance (66–70%) and high diffraction efficiency (88–91%) was observed. Also, it is worth noting that the

![Figure 4.37: Photographs of the captured steered beams on an IR viewing-card: composite image of the 16 steered beams of the supra-binary beam steerer (half of the full FOR, 80°).](image-url)
steerer is still relatively efficient even when the incidence angle is far from normal. The difference between transmittance and efficiency confirms that losses in this demonstration are predominantly related to the substrate absorption and reflection from the LC half-wave plates, and that the LC polymer PGs are fairly efficient at redirecting light as expected. The reflectance and absorption is primarily due to the various interfaces and electrode material that is not optimized for index matching (nearly 6% loss from each LC cells), while the LC itself has comparatively very low absorption.

In order to observe the diffracted power distribution, we show the relative transmitted power across the observed output angle range ±45° for maximum steering angle: +40°. In this case, small sidelobes (0.9% and 2.1%) were detected at the angle of −8.8° and +19.3° and very low sidelobes are generally observed throughout the steering angles, as shown in Fig. 4.39.

We have demonstrated an LC polymer PG based, wide-angle, nonmechanical beam steerer having 80° FOR with 2.6° resolution (totally, 32 steering angles) at 1064 nm and have described its design principles. The device shows high efficiency (88 – 91%) with 66–70% throughput that is mainly limited by the reflectance and absorption losses due to the various interfaces and electrode materials. The throughput can be further improved with higher quality electrodes (lower IR absorption) and substrates index matched to the polymer films. The supra-binary beam steerer offers more steering angles with the same
Figure 4.39: Characterization data of the supra-binary steerer operating at 1064 nm: relative power (transmittance) distribution at all output angles for maximum steering angle (+40°).

number of LC cells (and therefore lower loss) compared with other steering designs, can achieve very large steering angles with low sidelobes, and supports comparatively large beam diameters paired with a very thin assembly and low beam walk-off. Moreover, the switching time of the steerer depends on the switching time of polarization selectors (e.g. LC half-wave plates), and therefore the steerer can be made very fast using fast polarization selectors (e.g. ferroelectric material). Therefore, high-speed steering can be realized with the supra-binary design. Note that while the design described here is limited to steering in one dimension, two-dimensional steering can also be implemented by arranging two of these PG steering assemblies sequentially.
4.3 Chromatic Dispersion Reduction for Broadband Beam Steering

In previous section, we demonstrated non-mechanical, wide-angle, high-efficiency steering capability of PG-based beam steering devices using collimated and single wavelength light source (e.g., laser). The diffractive elements, PGs, have favorable characteristics and provide effective solution for steering diverging and broadband light sources (e.g., LED) with no moving parts. However, the diffractive elements have a common problem of chromatic dispersion for the broadband light because of their fundamental diffraction phenomena. As we discussed in Sec. 4.2.2, the color separation would be worse in small grating period elements. For instance, 2 \( \mu \text{m} \) period PG makes nearly 6° color separation with Red, Green, and Blue (RGB) components: diffraction angle at 630 nm (R) and 430 nm (B) would be 18.4° and 12.4° respectively. Here, we propose a method to correct the color separation problem using additional grating that compensates and balance out the color separation. As shown in Fig. 4.40, the technique uses a set of LCPGs that can steer broadband light with high efficiency, and the additional LCPG compensates the direction of red light without affecting the direction of blue light. The combination of LCPGs can result in a compact, self-contained steering device that can be easily integrated into other systems.

We first show a simple example of the color dispersion reduction with two PGs having different diffraction angle and efficiency at visible range. Table. 4.9 shows angle parame-

![Figure 4.40](image-url)

Figure 4.40: (a) Chromatic dispersion problem of the PG-based beam steerer for broadband input; (b) proposed technique with additional PG that compensates angle separation.
Table 4.9: Diffraction angles of PG#1 and PG#2 and compensated steering angles at three different wavelengths.

<table>
<thead>
<tr>
<th>Colors (wavelength)</th>
<th>PG#1 Diff.Angle</th>
<th>PG#2 Diff.Angle</th>
<th>PG#2 Diff.Efficiency</th>
<th>Steering Angle (percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red (630 nm)</td>
<td>10.3°</td>
<td>−3.3°</td>
<td>97%</td>
<td>7°(97%), 10.3°(3%)</td>
</tr>
<tr>
<td>Green (530 nm)</td>
<td>8.6°</td>
<td>−2.8°</td>
<td>66%</td>
<td>5.8°(66%), 8.6°(34%)</td>
</tr>
<tr>
<td>Blue (430 nm)</td>
<td>7.0°</td>
<td>−2.2°</td>
<td>3%</td>
<td>7°(97%), 4.8°(3%)</td>
</tr>
</tbody>
</table>

ters and efficiencies of the two PGs at three different wavelengths (R, G, and B). When the PG#1 diffracts the broadband input (RGB) with high efficiency, there would be 3.3° color dispersion within visible range. If the PG#2 diffracts the output beam of the PG#1 with the efficiency denoted in the table, the red light would be compensated without affecting the angle of the blue light. Consequently, the most of red and blue components would be steered into the target angle (7°), and green component would direct around the target angle (5.8° − 8.6°).

In order to demonstrate the example of dispersion reduction, we fabricated the two PGs with different recipes. First, the PGs were patterned for different grating periods as 3.5 µm for PG#1 and 11.2 µm for PG#2. Both samples were processed with the same LC material (RMS03-001C, ∆n=0.16, Merck), but coated for different thickness: 1.7 µm

Figure 4.41: The first-order diffraction efficiency of two PGs with different thickness: 1.65 µm (PG#1) and 2.3 µm (PG#2).
Figure 4.42: Chromatic dispersion reduction with two PGs: (a) Projection of the collimated broadband light (input) on a black board; (b) diffraction of the input light with the PG#1 only; (c) dispersion compensation by PG#1 and PG#2. The horizontal line denotes steering angle with values of (i) \(10.3^\circ\), (ii) \(8.6^\circ\), (iii) \(7.0^\circ\), and (iv) \(0^\circ\).

(PG#1) and 2.3 \(\mu\)m (PG#2), and therefore they show different peak wavelength: 530 nm (PG#1) and 630 nm (PG#2). The first-order diffraction efficiency of the two PGs is shown in Fig. 4.41, which illustrates sinusoidal changes of the efficiency over the visible range. As shown in the spectra, the PG#1 diffracts most of visible light (RGB) with fairly high efficiency, while the PG#2 mainly diffracts the red component (\(~630\) nm). As we discussed above, the PG#2 can compensate the angle difference between red and blue components, but the PG#2 also partially affects other wavelength (e.g. green) with various efficiency, so we need to consider the diffraction angles and efficiencies at the other wavelengths between red and blue components.

We used fairly collimated broadband light source (LED) that is circularly polarized. Fig. 4.42 shows the projected beam of the input light, steered light by PG#1, and compensated light by PG#1 and PG#2. The experimental results obviously show that the compensating PG works properly to reduce the color separation caused by PG diffraction property, and most of input light is steered into the target angle.

Fig. 4.43 shows the design of the PG-based beam steerer containing Compensating PG (CPG) at the end of the steering stages. The design configuration is based on the quasi-ternary steering design that consists of only one LC wave plate and cascaded several LCPGs. The front Polarization Selector (PS, e.g., LC half-wave plate) determines the handedness of the incident polarization as RCP or LCP, and the following LCPGs accumulate the diffraction angles of the LCPGs. The input beam can be directed into either the zero, positive first, or negative first diffraction order depending on the handedness of the input polarization and the state of applied voltage on each LCPG. The
Figure 4.43: Diffraction property of PGs and the design of broadband light steerer: (a) Diffraction behavior of PGs (#1, #2, #4) and CPG, and (b) PG#3 that has anti-parallel arrangement; (c) four-stage beam steerer for broadband light with Compensating Polarization Grating (CPG).

The stack of the elements from PS to PG#3 follows the angle design of quasi-ternary as detailed in Table 4.10. Note that we use anti-parallel arranged PG for PG#3 in order to minimize the color dispersion, which would be worse in the PG having large diffraction angle. As shown in Figs. 4.43(a) and (b), once the LCP light passes through the PGs, the anti-paralleled PG diffracts the input beam to negative direction in contrast to other PGs. The last PG (PG#4) just works as adding the diffraction angle to the positive or negative direction according to the input polarization state, so total FOR could be increased by the angle of the PG. Consequently, the four-stage design can cover 30° FOR with 1.36° steps, which steers totally 23 steering angles.

Table 4.10 shows the diffraction angles of the PGs at 530 nm (green), which is center wavelength for broadband beam steering. The compensating PG has 10.1 µm grating period that makes 3.58° diffraction angle at 630 nm (red) wavelength. Since the

<table>
<thead>
<tr>
<th>PGs</th>
<th>Grating Periods (µm)</th>
<th>Diffraction Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>22.3</td>
<td>1.36° (at 530 nm)</td>
</tr>
<tr>
<td>#2</td>
<td>7.4</td>
<td>4.09° (at 530 nm)</td>
</tr>
<tr>
<td>#3</td>
<td>5.6</td>
<td>5.45° (at 530 nm)</td>
</tr>
<tr>
<td>#4</td>
<td>5.6</td>
<td>5.45° (at 530 nm)</td>
</tr>
<tr>
<td>Comp</td>
<td>10.1</td>
<td>3.58° (at 630 nm)</td>
</tr>
</tbody>
</table>
input polarization on the CPG is RCP for positive steering case and LCP for negative steering case, the CPG can automatically reduce the color dispersion for both cases and compensate the angle of red component. The CPG works mainly for large steering angle (> ±10°) and the detailed steering angle information is described in Fig. 4.44. The input state is LCP or RCP selected by PS, and the input beam can be diffracted on each PG based on the state of applied voltage: ON (red block) and OFF (white block with steering angle). The output angle denotes the final steering angle at the center wavelength (green), and only largely steered angles are redirected by the CPG, which works mainly for red component without affecting blue component.

The broadband beam steerer was simulated with a realistic input, which is circularly polarized and diverging as shown in Fig. 4.45. The input has bell-shaped angular profile and its spectrum is continuous over the visible range. The simulated result shows the

<table>
<thead>
<tr>
<th>Input(P)</th>
<th>PG#1</th>
<th>PG#2</th>
<th>PG#3</th>
<th>PG#4</th>
<th>Output(˚)</th>
<th>Output(P)</th>
<th>CPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCP</td>
<td>[ON state]</td>
<td>4.09</td>
<td>5.45</td>
<td>5.45</td>
<td>15.00</td>
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<td>- comp</td>
</tr>
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<td>4.09</td>
<td>5.45</td>
<td>5.45</td>
<td>13.64</td>
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<td>- comp</td>
</tr>
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<td>5.45</td>
<td>5.45</td>
<td>12.27</td>
<td>RCP</td>
<td>- comp</td>
</tr>
<tr>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>10.91</td>
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<td>- comp</td>
</tr>
<tr>
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<td>5.45</td>
<td>5.45</td>
<td>9.55</td>
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<td></td>
</tr>
<tr>
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<td>5.45</td>
<td>5.45</td>
<td>8.18</td>
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<tr>
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<td>5.45</td>
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<td>6.82</td>
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</tr>
<tr>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>LCP</td>
<td></td>
</tr>
<tr>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>4.09</td>
<td>RCP</td>
<td></td>
</tr>
<tr>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>2.73</td>
<td>RCP</td>
<td></td>
</tr>
<tr>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
<td>RCP</td>
<td></td>
</tr>
<tr>
<td>RCP/LCP</td>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>0.00</td>
<td>RCP/LCP</td>
<td></td>
</tr>
<tr>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
<td>LCP</td>
<td></td>
</tr>
<tr>
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<td>-1.36</td>
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<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
<td>LCP</td>
<td></td>
</tr>
<tr>
<td>RCP</td>
<td>1.36</td>
<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
<td>LCP</td>
<td></td>
</tr>
<tr>
<td>LCP</td>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
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<tr>
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<td>5.45</td>
<td>5.45</td>
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<td>5.45</td>
<td>5.45</td>
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<td></td>
</tr>
<tr>
<td>RCP</td>
<td>1.36</td>
<td>5.45</td>
<td>5.45</td>
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<td>RCP</td>
<td></td>
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<tr>
<td>RCP</td>
<td>-1.36</td>
<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
<td>RCP</td>
<td></td>
</tr>
<tr>
<td>RCP</td>
<td>1.36</td>
<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
<td>RCP</td>
<td></td>
</tr>
<tr>
<td>RCP</td>
<td>-1.36</td>
<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
<td>RCP</td>
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<td>RCP</td>
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<td>1.36</td>
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<td>5.45</td>
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<td>1.36</td>
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<td>5.45</td>
<td>5.45</td>
<td>5.45</td>
<td>1.36</td>
<td>RCP</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.44: Lookup table for electrical control of broadband beam steerer; the red block indicates the ON-state of PGs with applied voltage (V > Vth), and the number in the white block denotes the diffraction angle at 530 nm, which occurs at the element.
Figure 4.45: Simulation results of the dispersion reduction: (a) Bell-shaped angular profile of the input beam; (b) reduced dispersion result with $-11^\circ$ steering.

The steered beam seems to have almost same angular profile without showing significant color dispersion over the diverging angle. Only the edge part has regions of the color separation, but the regions could be negligible compared to the other regions.

We assembled a prototype broadband beam steerer containing the PGs described in Table 4.10. The prototype can steer $\pm 15^\circ$ FOR with 1.36° resolutions, which has 23 steering angles. Fig 4.46 shows the steered beams with $\pm 3^\circ$ and $\pm 12^\circ$ steering angles on white board. Since the spectrum of the input LED is white (not a strictly RGB), the steered beam could widen as it steers to larger steering angles but low sidelobes and color dispersion are generally observed throughout the steering ranges as shown in the picture of Fig 4.46. Here we used only one compensating PG for large steering angles, but extra PG can be used for more detailed adjustment for small steering angles and larger steering angles.
Figure 4.46: Captured output beams of the broadband beam steerer with the dispersion compensation: photographs of the steered beams on a white board. (a) and (b) shows ±3° broadband steering, and (c) and (d) shows ±12° steering, which is compensated by CPG.
Chapter 5

Hyper-spectral imaging polarimetry

Measurements of complete polarization and spectral content across a broad wavelength range of a scene are used in various fields including astronomy, remote sensing, and target detection. Most current methods to acquire spectral and polarimetric information need moving parts or modulation processes which lead to significant complexity or reduce sampling resolution. Here we present a novel snapshot imaging spectropolarimeter based on anisotropic diffraction gratings known as polarization gratings (PGs). Using multiple PGs and waveplates, we can simultaneously acquire both spectrally dispersed and highly polarized diffractions of a scene on a single focal plane array. The PGs create chromatic dispersion (spectral information) patterns are linearly proportional to Stokes vectors (polarization information) embedded in a scene. PGs uniquely produce only three diffracted orders (0 and ±1), polarization independent zeroth-order, polarization sensitive first-orders that depend linearly with the Stokes parameters, and easily fabricated as polymer films suitable for visible to infrared wavelength operation. We develop and test a system matrix for reconstructing the object information from this diffraction pattern. This matrix can be extended to various configurations containing several PGs. From a matrix representation, a continuous model can be converted to spatially and spectrally quantized model in discrete manner. Moreover, our imaging spectropolarimeter is calibrated with single wavelength light source (i.e. laser) and narrow-band color filters, and is experimentally demonstrated with scenes having spectral and polarization variation. This demonstrates an imaging spectropolarimeter approach, that reconstructs both screen generated scenes and outdoor objects. Reconstructed objects are sampled at $100 \times 100 \times 51$ (x, y, λ) with 4 nm spectral resolution. The most significant advantage
of our spectropolarimeter over other snapshot imaging systems is its capability to provide simultaneous acquisition of both spectral and polarization information at a higher resolution, and in a simpler and more compact way.

5.1 Introduction

5.1.1 Objective and Motivation

The overall purpose of the research is to make an in depth study of digital imaging systems as building and developing an imaging hyper-spectral polarimeter based on novel liquid crystal diffraction grating. The imaging system will allow us to measure spectral and polarization information of dynamic events with no moving or tunable part by using the liquid crystal diffraction gratings, also known as polarization gratings (PGs). Spatially varying birefringence patterns of PGs lead to the imaging spectrum (red to violet) from the angular dispersion and polarization separation (full Stokes parameters) from the order of diffractions. By using the multiple PGs with proper configurations, we can achieve full spectral and polarization separation on a single detection arrays such as CCD or CMOS.

There are different types of information that can be extracted from a scene. These include spatial information captured by detector (ex. normal cameras), spectral information retrieved by spectrometers, and polarization states achieved by polarimeters. Each one of these instruments shows a different way of identifying objects. For example, the spectral information specifies material components in a scene, whereas the polarization information tells us the surface feature, shape, and roughness with vector contents of the optical field. Measurements of the spectral and polarization information of the image are used in a various fields such as astronomy, remote sensing, atomic and molecular physics, and material characterization [80, 81, 82, 83].

Fig. 5.1 shows one example of the ability of polarization to defeat spectral camouflage. From the previous research we could know that manmade objects are source of emitted and reflected polarization while natural backgrounds are predominantly unpolarized. Even when the target is not spectrally camouflaged, polarization is used to improve target contrast. Applications of imaging spectropolarimetry are employed in several fields including geology, where they are used in remote sensing to locate and identify Al, Cu, Fe, Pb and quartz based on their polarized reflection spectrum [84]. Astronomy is possibly the most prevalent application of spectropolarimetry. Spectral contents are used
to classify the spectral signature and locations of distant stars, study the composition of planets, and characterize other celestial objects by their emitted and reflected spectra, while the polarization contents are used to determine the structure of various types of nebulae and galaxy [85].

5.1.2 Background of Imaging Spectropolarimetry

Imaging Spectrometers generate a two dimensional spatial image with one spectral dimension information of a scene using a two dimensional detector array. The spectrum measurement of different position of a scene is shown in Fig. 5.2. The 3D data set is called the object cube, which contains two-spatial \((x, y)\) and one-spectral \((\lambda)\) information. Most conventional spectrometers are unable to image the two spatial dimensions and one spectral dimension information of the object cube at a same time. For example, a camera with a narrow band filter could be used to obtain spatial information with a fixed wavelength. This method need to scan some range of the wavelength in order to measure the entire object cube [86, 87]. In this case, moving parts are generally employed, and these are undesirable from the standpoint of reliability and a relatively long time is required for the capture of a complete data set since multiple exposures are made sequentially in time. Moreover, if the target scene is changed during the scanning, it may make artifacts in the results. In that sense, the scanning techniques thus have limited
Figure 5.2: The concept of imaging spectroscopy is shown with a spectrum measured for each spatial element in an image (Green et al. 1998) [5].

ability to obtain target data set on the dynamic scenes such as moving targets or targets from the moving system [88]. In order to escape these drawbacks, simultaneous detection of both spatial dimensions and spectral dimension is suggested by a Computed Tomography Imaging Spectrometer (CTIS) [89, 90] that utilizes three crossed normal gratings. The CTIS has its advantage in collection data on dynamic scene at high speed with a robust process. In a CTIS, the object cube is projected onto a two dimensional detector array by passing through a computer-generated holography (CGH) disperser [6]. The effect of the dispersion pattern with the CGH disperser can be represented as generating several projections which have various angles onto the two dimensional detector array, as illustrated in Fig. 5.3.

While the spectral information can tell us the material properties, the polarization information can be used to detect roughness as well as shape of the material. In the semiconductor industry, polarization information is commonly used for characterizing thin films. The process of measuring the polarization properties of the image over some defined spectral region is called imaging spectropolarimetry. To measure the polarization contents of the image, several different methods are used. One commonly used
Figure 5.3: Dispersion of the CGH disperser causes the separation of diffraction orders to increase with wavelength, resulting in several projections of the data cube on the detector array (Descour et al. 1997) [6].

The technique of polarimetry contains some moving parts such as rotating polarization elements to modulate incident light of the image [91]. This technique is used comparably easily and quite straightforward, but it cannot be used for a dynamic circumstance and it can cause registration problems if there is any wobble during the detection process. Another suggested polarimetry technique is using multi-detectors to collect each modulated light from the image [92, 7]. The technique needs several detectors which require large space and increase system cost. Further, special care must be taken for the alignment of each detector and different aberrations of each detected image. Recent polarimetry using Fourier transform modulation to exploit the wavelength dependence of the retardance was introduced [93]. The set of spectra which can be measured from the modulated light depends on the polarization state. Therefore, spectrum measurement is important for this technique. By means of the method incorporating this polarimetry into an imaging CTIS spectrometry, imaging spectropolarimeter, which is called Computed Tomographic Imaging Channeled sSpectropolarimeter (CTICS), can be realized with only one detector and detection (snapshot) [94]. Fig. 5.4 illustrates this combined technique which can detect spectral and polarization information with snapshot detection. This imaging spectropolarimetry has advantages on snapshot capability for both spectral and polariza-
tion contents, however, it has several tradeoffs introduced by the modulation technique. First, the resolution in the Stokes spectra is at least seven times worse than that of the measured spectrum since the spectrum modulation process needs at least seven channels dividing up the bandwidth of the spectrum measurement. Moreover, this technique basically uses domain conversion - Fourier transform that may create aliasing problems that can increase overall system error. Another possible error comes from the system instability derived from the thermal sensitivity of the optics; this system uses two thick waveplates to modulate the spectrum of the light. To overcome these drawbacks of the imaging spectropolarimeter, we need to find new approach which does not need to use the spectrum modulation and domain conversion process to find polarization state of each pixel. And the new approach should not contain thick waveplates which are sensitive to the thermal effect. Therefore, we propose the novel material, polarization grating which permits us to measure spectral and polarization information simultaneously without any moving parts and additional modulation process.
Fig. 5.5 illustrates object cubes that conceptually represent high dimensional information of a scene. The detected image from the Focal Plane Array (FPA) can be extended with spectral information and shows the range of spectral contents in terms of a height of the object cube, but it contains no information concerning the polarization state. The rightmost four cubes indicate the object cube with full Stokes parameters, representing the polarization state of the contents of each object cube. To obtain these multi-dimensional full Stokes object cubes from the two-dimensional image we apply our imaging spectropolarimeter, named as Polarization Grating Imaging Spectropolarimeter (PGIS). Fig. 5.6 shows the chromatic dispersion of color image from a PG captured by a digital camera. The image shown is displayed as a wallpaper over a dark background on an LCD monitor, and an achromatic PG with 6 µm grating pitch was placed in front of it. The camera was positioned behind the PG and focused on the diffracted orders, with the distance between the PG and the image adjusted to provide sufficient angular dispersion. Achromatic PG of 6 µm grating period was located in front of the camera following the x-axis of image but we could make different angular dispersion as simply changing the optical axis of PG. Light coming from the monitor is linearly polarized (no circular polarization component) and it leads almost same diffraction intensity of the first orders in Fig. 5.6. It looks like there are only three-color chromatic dispersions since the
Figure 5.6: Dispersion pattern of the image as seen through achromatic PG having 6 \( \mu m \) grating period. The zeroth-order (0) pattern reserves the same image of input (no dispersion) with the same polarization state of input polarization while the first-orders (+1 and −1) are dispersed by wavelength (color-separation), and the intensity of the patterns are sensitive to circular-polarization state \((S_3)\) of the input light.

back-light of LCD monitor may contain three color sources (R,G,B).

5.2 System Design and Mathematical Model

5.2.1 Conceptual Design of Imaging System

In the previous section, we learned that PG can separate polarization information on its unique three orders while the orders contain dispersed spectral information. In this section, as showing the conceptual design of our imaging system, we can find how the system makes its unique patterns including separated spectral and polarization information and how each pattern contains their unique information.

Escuti et. al [18] has derived the properties of PG diffraction based on Jones matrix reasoning [95, 96, 97, 73]. When the local electric field \( E_m \) passes the anisotropic grating \( T(x) \), the far-field electric field \( D_m \) of the diffraction order \( m \) can be expressed as below:

\[
D_m = \frac{1}{\lambda} \int_0^\Lambda T(x) E_m \exp(-j2\pi mx/\Lambda) \, dx.
\]  

(5.1)

where \( \Lambda \) is the grating period of the PG. The transfer matrix of PG is
\[ T(x) = R(-\pi x / \Lambda) \begin{bmatrix} \exp(-j\Gamma) & 0 \\ 0 & \exp(j\Gamma) \end{bmatrix} R(-\pi x / \Lambda) \]  

(5.2)

where \( R \) is the rotation matrix, \( \Gamma = \pi \Delta n d / \lambda \) is the normalized retardation, \( \Delta n \) is the birefringence of the LC, \( d \) is the thickness of the LC layer, and \( \lambda \) is the wavelength of incident light. If we assume the the incident electric field is uniform in the \( x \) (does not depend on \( x \)), Eq 5.1 can be rewritten as follows

\[ D_m = T_m E_{in} \]  

(5.3)

where the transfer matrix is defined as \( T_m = \Lambda^{-1} \int_0^\Lambda T(x) \exp(-j2\pi mx / \Lambda) \, dx \). Since the grating transfer matrix of PG has non-zero solutions only for +1, 0, and −1, we can rewrite the Eq 5.3 as

\[ D_0 = E_{in} \cos \Gamma \]  

(5.4a)

\[ D_{\pm 1} = \frac{1}{2} E_{in} \sin \Gamma \begin{bmatrix} -j & \mp 1 \\ \mp 1 & j \end{bmatrix} \]  

(5.4b)

Then, the diffraction efficiency of PG can be solved as the ratio of output to input intensity as \( \eta_m = |D_m|^2 / |E_{in}|^2 \),

\[ \eta_0 = \cos^2 \Gamma \]  

(5.5a)

\[ \eta_{\pm 1} = \frac{1}{2} (1 \mp S'_3) \sin^2 \Gamma \]  

(5.5b)

where \( S'_3 = S_3 / S_0 \) is the normalized Stokes parameter related to ellipticity of the incident light. Since we can expect the normalized retardation (\( \Gamma \)), Stokes parameter \( S_0 \) and \( S_3 \) at wavelength \( \lambda \) can be calculated from the first order intensity values \( I_{+1}(\lambda) \) and \( I_{-1}(\lambda) \) that are generated by a PG only.

\[ S_0 = \frac{I_{+1}^{PG\text{only}}(\lambda) + I_{-1}^{PG\text{only}}(\lambda)}{\sin^2 \Gamma} \]  

(5.6a)

\[ S_3 = \frac{I_{-1}^{PG\text{only}}(\lambda) - I_{+1}^{PG\text{only}}(\lambda)}{\sin^2 \Gamma} \]  

(5.6b)
As modulating input light with achromatic quarter-wave plates having axis of 0° and 45° to the grating vector of a PG, we can also find other Stokes parameters $S_1$ and $S_2$ [18]. When quarter-wave plate (0°) is placed with a PG, the far-field electric field of the first order diffraction can be written as follows

$$
D_{\pm 1} = \frac{1}{2\sqrt{2}} E_m \sin \Gamma \left[ \begin{array}{cc} 1 - j & \mp(1 + j) \\ \mp(1 + j) & 1 + j \end{array} \right] \quad (5.7)
$$

Then, the diffraction efficiency of the quarter-wave plate (0°) and PG can be solved as

$$\eta_{\pm 1} = \frac{1}{2}(1 \pm S'_2) \sin^2 \Gamma \quad (5.8)$$

where $S'_2 = S_2/S_0$ is the normalized Stokes parameter related to the component of incident light linearly polarized in the ±45° directions. As we show in Eq. 5.6, $S_2$ at wavelength $\lambda$ can be calculated from the first order intensity values that are generated by the quarter-wave plate (0°) and PG.

$$S_2 = \frac{I^{QWP(0°)+PG}_{\pm 1}(\lambda) - I^{QWP(0°)+PG}_{-1}(\lambda)}{\sin^2 \Gamma} \quad (5.9)$$

In order to obtain $S_1$ Stokes parameter, a quarter-wave plate (45°) should be placed with a PG. In this case, the far-field electric field of the first order diffraction can be written as follows

$$
D_{\pm 1} = \frac{1}{2\sqrt{2}} E_m \sin \Gamma \left[ \begin{array}{cc} -j \pm j & -1 \mp 1 \\ 1 \mp 1 & j \mp j \end{array} \right] \quad (5.10)
$$

Then, the diffraction efficiency of the quarter-wave plate (45°) and PG can be solved as

$$\eta_{\pm 1} = \frac{1}{2}(1 \mp S'_1) \sin^2 \Gamma \quad (5.11)$$

where $S'_1 = S_1/S_0$ is the normalized Stokes parameter related to the component of incident light linearly polarized in the horizontal and vertical directions. We can discover $S_1$ parameter at wavelength $\lambda$ from the first order intensity values that are generated by the quarter-wave plate (45°) and PG.

$$S_1 = \frac{I^{QWP(45°)+PG}_{-1}(\lambda) - I^{QWP(45°)+PG}_{1}(\lambda)}{\sin^2 \Gamma} \quad (5.12)$$
Therefore, full Stokes parameter ($S_0-3$) of input light can be determined by the intensities of the zero- and first-order diffractions that are generated by different combination of a quarter-wave plate and PG. As we use a stack of two quarter-wave plates ($0^\circ$, $45^\circ$) and three PGs having different axis of diffraction vector, for example, an input light (or image) can be separated after passing through the stack and makes diffractions showing different intensities that depend on combination of the elements. Fig. 5.7 shows a conceptual design of PGIS that generates three pairs of diffraction orders that have different sensitivity to the Stokes parameter. When the input is an image having spatial ($x, y$) information, the diffraction orders are prismatic dispersed patterns that are projected by an object cube - conceptual information cube adding wavelength dimension. The dispersed patterns can be projected and detected by image sensor (e.g. CCD) but it is challenging to extract spectral and polarization information of image since the spectral images (red to violet) are overlapped each other. Each pixel of an image sensor detects sum of intensities of overlapped spectral images so the imaging system requires alternative way to estimate the spectral information of the dispersed patterns. After estimating the spectral information that is sensitive to specific polarization states, another step of estimation is necessary to reconstruct polarization information ($S_0-3$) of the image. We will continue this discussion in the next section.
A conceptual optical layout for PGIS is shown in Fig. 5.8. Light from a scene is imaged onto a field stop by object lens, and the spatial extent of the scene can be limited by placing a field stop at a conjugate plane to the FPA. A collimating lens that is simply located in front of the FPA provides a collimated ray that illuminates the PGIS. The incoming ray is separated to the three orders, and a diffraction axis is determined by the grating vector of the PGs. The lengths of generated diffraction patterns are governed by the distance between PGIS and FPA and diffraction angle from PGIS, which is related to the grating period of PGs.

PGIS gains many benefits from using the main element, PGs, which have their own attractive properties. Unlike conventional phase or amplitude gratings, the PG operates on modulating local polarization states of incoming light and it leads to remarkable optical performance that includes $\sim 100\%$ diffraction efficiency into a single diffraction order and good angle response. The PGs are fabricated with liquid crystal (LC) and photo-alignment [79] materials patterned by polarization holography for various grating periods within the visible, near-infrared, and midwave-infrared ranges. Since the intensity of the diffraction patterns from PGs is linearly proportional to the Stokes parameters of a scene, the reconstruction process can be done without any post-processing that limits the spectral resolution and detection speed, such as Fourier transform. In order to make various diffraction angles and patterns that are essential to CT reconstruction, PGs can be easily stacked with proper parameters, as we have demonstrated with a prototype nonmechanical beam steering system [1] based on stacked PGs and WPs. In addition, the robust, compact, and lightweight thin-film assembly design of PGIS can be adopted with conventional imaging systems.
5.2.2 Mathematical Model

The imaging system, PGIS, is a snapshot imaging spectropolarimeter that needs no moving parts or scanning processes. In this system, the spatial, spectral, and polarization information of the scene is allowed to overlap in two-dimensional projections and collected in a single FPA. The continuous model of the PGIS can be rewritten as:

\[
g(r') = \sum_{j=1}^{3} \sum_{i=0}^{3} \int \int f(r, \lambda, i) h(r, \lambda, i, j : r') \, d^2r \, d\lambda + n(r') \quad (5.13)
\]

where the projection and object cube are defined as \( g(r') \) and \( f(r, \lambda, i) \) respectively. They are continuous functions of their arguments, spatial vector \( r \) and wavelength \( \lambda \). Variable \( i \) denotes the polarization information of object that corresponds to \( S_0-3 \) object cubes in Fig. 5.9, and \( j \) describes polarization sensitivity of diffraction patterns on the FPA; for example, \( j = 3 \) pattern denotes the projections made by a PG alone, and the pattern is related to \( S_3 \) Stokes parameter as derived in Eq. 5.5. The variable \( j \) of the patterns can be various depending on a stack of the PGs and quarter-wave plates, and their orientation. Transfer function \( h(r, \lambda, i, j : r') \) contains geometric mapping information with spectral and polarimetric response between the object cube \( f \) and projection \( g \) when \( r \) is spatial vector in object space and \( r' \) represents spatial vector in image space. The last term \( n \) defines the system noise of the model.

The Fig. 5.9 illustrates the simple geometric mapping between the object cube and the diffracted patterns on the FPA. The diffraction angle \( \theta_m \) is determined by the grating equation as follows

\[
\sin \theta_m = \left( \frac{m \lambda}{\Lambda} \right) + \sin \theta_{in} \quad (5.14)
\]

where \( \theta_{in} \) is the incident angle on the PG and \( m = \{0, \pm1\} \) is the diffraction order. When a collimated light comes to a PG and the diffraction angle is \( \theta_m \), a displacement length can be derived as below

\[
d(\lambda) = L \cdot \tan \theta_m(\lambda), \quad (5.15)
\]

where the \( L \) is a normal distance between the object cube (e.g. PG assembly) and the FPA, and \( \theta_m(\lambda)=\sin^{-1}(m\lambda/\Lambda) \). Then, a displacement vector on the imaging space can now be described with a unit vector \( \hat{r}' \), which defines the azimuth angle of the diffraction.

\[
d(\lambda) = d(\lambda) \cdot \hat{r}', \quad (5.16)
\]
Assuming that collimated light comes to the system, the diffraction displacement vector on the projection can be described as follows

$$d(\lambda) = L \cdot \tan \left(\sin^{-1}(m\lambda/\Lambda)\right) \cdot \hat{r}'$$  \hspace{1cm} (5.17)

Using additional Dirac-$\delta$ functions [98], the transfer function can be derived as

$$h(r, \lambda, i, j : r') = \delta\{r' - r - d(\lambda)\} \cdot S(i, j) \cdot \eta(\lambda)$$  \hspace{1cm} (5.18)

where the first term indicates geometric mapping information between object and image space, and the second term represents the polarization sensitivity of each projected pattern. If $i = 0$, $S(i, j) = 1/2$ and if $i \neq 0$, $S(i, j) = (m/2) \delta(i - j)$. The last term $\eta(\lambda)$ denotes the spectral transmittance, which is dominated by a spectral efficiency of the PGs and that of the camera. Finally, the system function can now be described as:

$$g(r') = \sum_{j=1}^{3} \sum_{i=0}^{3} \int \int \int f(r, \lambda, i) \cdot \delta\{r' - r - d(\lambda)\} \cdot S(i, j) \cdot \eta(\lambda) \cdot d^2r \cdot d\lambda + n(r')$$  \hspace{1cm} (5.19)
This continuous function provides the analytic relationship between multi-dimensional object cube and projection on the FPA.

While the system function can be described in a continuous manner, the object cubes and projections should be considered discrete, since all projection data is from the discrete sensor arrays, and is related to the corresponding discrete volumetric units of the object cubes. Analogous to a pixel being the smallest data unit on a sensor array, a volumetric data unit (voxel) can be considered in object cubes. The relationship between an object vector $f$ having voxels and $g$ containing pixels can be described by the system matrix $H$, which contains all of the geometric mapping information, along with spectral and polarimetric responses. The system function is now transformed to the following, with the noise-free assumption for simplicity:

$$g = Hf,$$

(5.20a)

$$g = [g_1 \ g_2 \ g_3]^T,$$

(5.20b)

$$f = [f_0 \ f_1 \ f_2 \ f_3]^T.$$

(5.20c)

Here $g_j$ is the projection vector of pattern $j$ that is sensitive to $S_j(j = 1, 2, 3)$ polarization information and the size of the vector $g_j$ is $M_j \times 1$ when $M_j$ is the number of pixels on the pattern $j$; for example, when the number of pixels of each diffracted pattern is $400 \times 100$ ($x, y$), $M_j$ is $4 \cdot 10^4 \times 1$ and the size of the vector $g$ is $12 \cdot 10^4 \times 1$. $f_i$ is the object vector of the $S_i(i = 0, 1, 2, 3)$ Stokes parameter object cube. The size of the vector $f_i$ is $N \times 1$ when $N$ is the number of voxels in a single object cube; for example, $N = 10^6$ when object cube is sampled at $100 \times 100 \times 100$ ($x, y, \lambda$) and the size of vector $f$ is $4 \cdot 10^6 \times 1$. Therefore, the size of system matrix $H$ is $12 \cdot 10^4 \times 4 \cdot 10^6$ in this case. As we can check from this example, it is challenging to estimate object $f$ (unknown) from the projection $g$ (known) since the unknown information is much bigger than known information. In a linear algebra, more than $N$ number of equations (known) is required to find $N$ unknown variables but here we have much fewer number of known information, so special reconstruction algorithm should be applied. We will continue the reconstruction technique in the next section.

In order to represent the system matrix $H$, a response mapping matrix of pattern $j$,
\( W_j \), can be defined as:

\[
W_j = \begin{bmatrix}
w_{1,1} & w_{1,2} & \cdots & w_{1,N} \\
w_{2,1} & w_{2,2} & \cdots & w_{2,N} \\
\vdots & \vdots & \ddots & \vdots \\
w_{M_j,1} & w_{M_j,2} & \cdots & w_{M_j,N}
\end{bmatrix},
\]

(5.21a)

\[
Z_j = \mathbf{0}_{M_j,N}
\]

(5.21b)

where \( W_j \) is the \( M_j \times N \) matrix that represents the linear mapping from voxels in the object cube \( f \) to the projection \( g_j \), and entry \( w_{a,b} \) is spectral response between the voxel \((b)\) in object cubes and the pixel \((a)\) on a sensor array. \( Z_j \) is a zero matrix with all entries being zero, and the size of \( Z_j \) is the same as \( W_j \) \((M_j \times N)\). Then, the system matrix \( H \) can be written with block partitioned matrices, \( W_j \) and \( Z_j \). The arrangement of the response mapping matrixes is determined by the entree of object vector \( f \).

\[
H = \begin{bmatrix}
W_1 & W_1 & Z_1 & Z_1 \\
W_2 & Z_2 & W_2 & Z_2 \\
W_3 & Z_3 & Z_3 & W_3
\end{bmatrix},
\]

(5.22)

\( H \) is a \( M \)-by-\( 4N \) matrix \((M=\sum_{j=1}^{3} M_j)\) whose number of rows represents the total number of pixels on the FPA and the number of columns indicates total number of voxels in object cubes.

\[
\begin{bmatrix}
g_1 \\
g_2 \\
g_3
\end{bmatrix} = \begin{bmatrix}
W_1 & W_1 & Z_1 & Z_1 \\
W_2 & Z_2 & W_2 & Z_2 \\
W_3 & Z_3 & Z_3 & W_3
\end{bmatrix} \begin{bmatrix}
f_0 \\
f_1 \\
f_2 \\
f_3
\end{bmatrix}
\]

(5.23)

From the practical point of view, the length of columns \( g \) and \( H \) can be extended according to the size of projection patterns; for example, if the object were estimated with doubled diffraction patterns on the FPA, the length of the vector \( g \) and matrix \( H \) would be doubled and it generally improves the estimation of the unknown vector \( f \).

### 5.2.3 Reconstruction Algorithm

Imaging spectropolarimeters extract high dimensional information (spatial, spectral and polarization) from a scene using a low dimensional detector array (1D or 2D). To find high
dimensional information from the low dimensional known, most conventional systems generally employ moving parts or multiple detections which lead to undesirable reliability issues. Especially, to extract the information from the moving target, simultaneous detection of a scene is essential to prevent the artifacts of the results. In order to avoid the problem, Okamoto suggested a technique [99] based on the computed tomography (CT) [100, 101, 102] that allows simultaneous acquisition of spectral image information with transmission gratings and monochromatic camera. Since CT is an indirect imaging system where projected images from a object scene are overlapped on the same detector, special techniques are required to reconstruct information in higher dimensions (i.e., 2D spatial images across many wavelengths).

One of the technique for this problem is iterative reconstruction approach. For ease of illustration, we show simple example in Fig. 5.10(a). There are four unknown blocks and four measurements, \( p_1, p_2, p_3, \) and \( p_4 \), are taken in the horizontal and vertical directions. These measurements are not linearly independent so it may require additional measurement value such as diagonal sum of the blocks to ensure their orthogonality. One possible solution is iterative method that needs multiple steps and estimations.

Fig. 5.10(b) shows values of each block and corresponding projection measurements. First, we assume it is homogeneous since we have no apriori knowledge of the object, so we can assign the average of the projection measurements: 2.5 (3 + 7 = 10 or 4 + 6 = 10, then 10/4 = 2.5) as Fig. 5.10(c). With the comparison of estimated projections (5 and 5) with the measured values (3 and 7), we can figure out that the top row is overestimated by 2 and the bottom row is underestimated by 2. Here we assume the difference between

\[
\begin{align*}
\text{(a)} & \quad p_1 = a + b \\
\text{c} & \quad p_2 = c + d \\
\text{d} & \quad p_3 = a + c \\
\text{b} & \quad p_4 = b + d \\
\text{(b)} & \quad 1 \quad 2 \quad 3 \\
\text{c} & \quad 3 \quad 4 \quad 7 \\
\text{d} & \quad 25 \quad 25 \quad 5 \\
\text{b} & \quad 1.5 \quad 15 \quad 5 \\
\text{a} & \quad 1 \quad 2 \quad 3 \\
\text{4} & \quad 6 \quad 6 \\
\text{4} & \quad 5 \quad 5 \\
\end{align*}
\]

Figure 5.10: A simple example of iterative reconstruction approach [8]. (a) Block variables (object) and their projections, (b) Original object and projections, (c) initial estimation of object, (d) the second estimation of object, and (d) final estimation of object and projections.
Figure 5.11: Examples of expected disperse patterns including concentric, conical, and nonlinear patterns. In the all figures, PG\# pattern corresponds to the diffracted pattern that is sensitive to $S_{\#}$ Stokes parameters. The center arrows in the third patterns indicate a diffraction direction of object.

estimated and measured values need to be split evenly to the all blocks, and therefore we can re-estimate the values as adding and subtracting the rows by 1, as shown in Fig. 5.10(d). Again, we can follow the same process for the vertical projections, and then the columns are increased and decreased by 0.5, as shown in Fig. 5.10(e). Now the estimated values are matched to measured projections, so the reconstruction process stops [8]. Our imaging system is described by $g = Hf$ when multi-dimensional object $f$ can be projected as a dispersed images $g$ on a two-dimensional space by a system with its system matrix $H$ [103]. Once the projected dispersion $g$ is obtained from a detector, the actual object cube $f$ can be estimated by iterative algorithms as we discuss above. One of the iterative algorithm is multiplicative algebraic reconstruction techniques (MART) [104, 105] that we use for our imaging system.

MART : $\hat{f}_n^{(k+1)} = \hat{f}_n^{(k)} \frac{(H^T g)_n}{(H^T H \hat{f}^{(k)})_n}$ (5.24)

where $\hat{f}$ is an estimate of the object data. The iterative algorithms require more computational time to acquire relatively higher resolution, however, they are still attractive methods for adopting the snapshot imaging systems.

To design a PGIS system, we need to consider the dispersed diffraction patterns to enhance the system performance and efficiency. Fig. 5.11 shows pattern examples that
can be obtained according to the configuration of PGs and optical elements in the system. Since the dimension of detection array (e.g. CCD/CMOS) is limited, the patterns should be dispersed and filled the array as much as possible to process system reconstruction with fully separated dispersion patterns. As we use the new polarization holography techniques introduced in Section 3, we may fabricate two-dimensional spatial varying PG patterns and the technique will make new dispersed diffraction patterns such as helical or conical patterns. In any case, the system should have at least three PGs to achieve full Stokes parameter (full polarization information).

5.3 Simulation Results

The proposed imaging system is simulated based on the optical property of PGs and geometric information of the system. It is assumed that the system is noise-free and has perfect alignment for all optical elements. Virtually created object contains spectral and polarization information, and a set of images presents different spectral information of the object. In the $S_0$ object cube, a grayscale of the image is used for representing the brightness of the image at different wavelength. In the $S_1$-$S_3$ object cubes, the grayscale of the image indicates the polarization state of the image; for example, the white in $S_3$ object cube denotes right-handed circularly polarized ($S_3 = +1$), and black in $S_3$ object cube means left-handed circularly polarized ($S_3 = -1$).

5.3.1 Spectral Object Reconstruction

As a simple version of simulation, we create a virtual object that contains only the spatial and spectral information. In this case, the object has spatially uniform intensity profile on each wavelength image and different intensity values are registered on the different wavelength images. As shown in Fig. 5.12, original object has the uniform pattern and each wavelength image has different intensity values that can be distinguished by a brightness of the images. A spatial resolution of the original object is $32 \times 32$ and the intensity values are from 0 (black) to 1 (white). A spectral resolution is 1 nm and 400 nm - 406 nm wavelength range is simulated with seven images having different intensity values. It is assumed that three PGs are used to create diffraction patterns on the FPA. Using the system matrix and the iterative reconstruction approach shown previous sections, the object is reconstructed and estimated as shown in Fig. 5.12(bottom row).
To evaluate the reconstruction performance, the reconstructed object is compared with the original object and the normalized root mean square error (RMSE) is calculated:

$$\text{RMSE} = \left( \frac{1}{M} \sum_{m=1}^{M} (f_m - \hat{f}_m)^2 \right)^{1/2},$$

(5.25)

where $M$ is the number of unit object (number of entry in vector $f$) and $f_{\text{mean}}$ is the mean value for unit object. Calculated RMSE of the reconstructed images is 0.129 and overall the result appears that the system works reasonably well for this simple object having only spatial and spectral information.

Next simulation also contains only spatial and spectral information but the object looks more complex and the range of wavelength is larger than previous simulation. Alphabet letters (N, C, S, and U) are used to represent different spectral information of object and totally 20 images cover the spectral range (450 nm to 640 nm) with 10 nm resolution. Fig. 5.13(b) shows the original object and each image denote the intensity profile of the object at the wavelength interested. We simulate the object and dispersed patterns generated by 4 PGs aligned with different orientation angles ($0^\circ$, $45^\circ$, $90^\circ$, and $135^\circ$). The dispersed patterns are shown in Fig. 5.13(a) and the $x$- and $y$-axis of the patterns correspond pixel numbers of a detection array. In the dispersed patterns, the $0_{th}$ order pattern is what we can see in real, since the $0_{th}$ order does not have dispersion and contains all overlapped spectral images. This object only has spectral information (no polarization information) so all first-order dispersion patterns are spread with the same intensity profile ($I_{+1}=I_{-1}$). The dispersed patterns are used to reconstruct the object via iterative technique, MART (Sec 5.2.3) and the iterative process of MART may
Figure 5.13: (a) Simulated dispersion pattern (3X3 pattern); (b) Original spectral images that correspond different spectral information from 450 nm to 640 nm with 10 nm resolution; Reconstructed spectral images with (c) 5-iteration and (d) 100-iteration via MART.

Table 5.1: Image difference and RMS of reconstructed object processed with different iteration numbers.

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image Difference</td>
<td>0.15</td>
<td>0.11</td>
<td>0.09</td>
<td>0.08</td>
<td>0.07</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>RMS</td>
<td>0.40</td>
<td>0.31</td>
<td>0.27</td>
<td>0.23</td>
<td>0.21</td>
<td>0.18</td>
<td>0.16</td>
</tr>
</tbody>
</table>
sult on the reconstructed objects in the Fig. 5.13(c) and (d). Table 5.1 shows a decreasing tendency of error (RMS and Image Difference) as increasing the number of iteration. We also simulate the reconstruction with an extremely large number of iteration but only small change is observed.

5.3.2 Spectral and polarimetric Object Reconstruction

In the second simulation, we use a virtual object which has both spectral and polarization variation. To simulate the polarization variation of the object, we manually produce normalized $S_3$ parameter information on each wavelength image and the information is added on the spectral object comprised of alphabet letters (i.e., the object in Fig. 5.13). Fig. 5.14 shows the examples of simulated dispersed patterns generated by different letter sets and polarization information. Because of the polarization information of the object, the dispersed patterns spread with different angle show various intensity; intensity ratio of the patterns is determined by the $S_3$ state of the object. Fig. 5.14(a) and (b) illustrate $3 \times 3$ dispersion patterns, and (c) and (d) show $5 \times 5$ dispersion patterns, and the high-order patterns show long dispersion caused by large diffraction angle of the PGs used.

Fig. 5.15 shows the reconstruction performance comparison among the different dispersion patterns. For these simulation, we fix the same iteration number, 20, and each pattern comes from various object information. RMS and image difference values of each simulation appear almost same tendency and the performance is getting worse as increasing complexity of object. In the Fig. 5.15, for example, pattern (a) shows lower reconstruction errors than the result of pattern (b) having more complex object, and more

![Figure 5.14: Different dispersion patterns for S0 and S3 Stokes parameters: (a) 3X3 N-pattern, (b) 3x3 NCSU-pattern, (c) 5x5 NC-pattern, and (d) 5x5 NCSU-pattern.](image-url)
Figure 5.15: Comparison of simulation results: Image difference and RMS of reconstructed objects estimated by different dispersion patterns: (a) $3 \times 3$ N-pattern for $S_0$, (b) $3 \times 3$ NCSU-pattern for $S_0$, (c) $3 \times 3$ NCSU-pattern for $S_0$ and $S_3$, (d) $5 \times 5$ NC-pattern for $S_0$ and $S_3$, and (e) $5 \times 5$ NCSU-pattern for $S_0$ and $S_3$.

Errors can be found in the result of pattern (c) that includes spectral and polarization information in the object. In the plot, we can check that the reconstruction performance is also dependent to the sort of dispersion patterns. For instance, the pattern (c) and (d) use the same object information having $S_0$ and $S_3$ contents, but small improvement of reconstruction can be found on the result of (d) that is estimated by the complex pattern ($5 \times 5$) having more number of dispersion patterns. The simulation shows a trend of performance, but the result values may not be a general case, and it should be treated as guidelines for design of imaging systems.

Next simulation object contains more complex spectral information with the full Stokes parameters (full polarization contents). In order to imitate a real situation of spectral information nature of an object, which changes smoothly we use motion images (i.e., the face of the tiger) that vary a bit among the images. Moreover, we assume different regional polarization sensitivity of the images, and manipulate each spectral image as adding the polarization information on the spectral images. For example, the light from the nose of the tiger may be partially linearly polarized in the horizontal and vertical direction ($S_1$), and the light corresponding to the tongue of the tiger can be linearly polarized in the diagonal direction ($S_2$). We also assume that the light coming from the background (i.e., grass) is partially polarized to the $S_3$ parameter corresponding to circularly polarized light as shown in the Fig. 5.16(a). For this simulation, we suppose that the object is dispersed by the combination of the PGs oriented with different angles;
totally 6 PGs are used and every two PGs are sensitive to the one of the polarization information ($S_{1-3}$). Those PGs are aligned on the same axis, so the PGs make only one 0th order with 12 diffraction patterns that are dispersed by spectral information. Fig. 5.16(b) shows the dispersion pattern we simulate using the assumed object and a set of PGs. Since the different parts of the object is sensitive to the different polarization and each dispersed pattern is relevant to one polarization state, the dispersed patterns show their unique sensitivity of polarization. The image resolution is 32×32 pixels and 9 images are used to represent different wavelength images with spectral resolution of 1 nm.

We use the same reconstruction technique with 10 iterations. The estimated images of the object are shown in Fig. 5.17. To simulate spectral information, we use 9 different spectral images but here we only show 4 images of them. Each column corresponds different spectral information, and the upper row shows original object and the reconstructed object is shown in lower row. The first two rows of Fig. 5.17 represent $S_0$ information that is about a brightness (illumination) of the object at the specific wavelength and the information can be detected by normal VIS-camera (also by human eye). As we can see the reconstructed images, most of $S_0$ information is well reconstructed over the all spectral range. There are some peak errors on the reconstruction images, but all results generally follow the same values of the original images and those error pixels can be improved by post-processing. The original and reconstructed images of linear polarization
Figure 5.17: $S_0$ information reconstruction; (a) Original objects. (b) Reconstructed image; $S_1/S_0$ information reconstruction: (c) Original objects, (d) Reconstructed image; $S_2/S_0$ information reconstruction: (e) Original objects, (f) Reconstructed image; $S_3/S_0$ information reconstruction: (g) Original objects, (h) Reconstructed image.
Table 5.2: Image difference and RMS of reconstructed object processed with 20 iteration numbers.

<table>
<thead>
<tr>
<th>Object Cube</th>
<th>$S_0$</th>
<th>$S_1/S_0$</th>
<th>$S_2/S_0$</th>
<th>$S_3/S_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image Difference</td>
<td>0.044</td>
<td>0.103</td>
<td>0.067</td>
<td>0.089</td>
</tr>
<tr>
<td>RMS</td>
<td>0.059</td>
<td>0.175</td>
<td>0.162</td>
<td>0.152</td>
</tr>
</tbody>
</table>

(horizontal and vertical) contents are shown in the row (c) and (d). The normalized parameter ($S_1/S_0$) is used for the polarization information, and the parameter has the value between −1 and +1. Active part of the image (nose of tiger) has the polarization value of 0.8, 0.8, 0.4, and 0.4 respectively, and higher value looks brighter than the lower value. The linear polarization (diagonal) contents are simulated in the third set of images: row (e) and row (f). The normalized $S_2$ parameter ($S_2/S_0$) is used for the value of images, and the same value (0.8) is used to present the polarized state of the active part (tongue of tiger). In the same way, normalized circularly polarized contents ($S_3/S_0$) are simulated in the last set of images, and the row (g) and (h) illustrate the original and reconstructed images respectively. We assume that mainly the background (grass) is sensitive to the circular polarization and the background part of the all spectral images has 0.6 value. As we can see the reconstructed images, most of the polarized regions are recognizable and distinguishable even though the reconstructed images show peak errors over the whole images. Some estimated images show reconstruction artifacts (error) in the part of other polarization contents but the error may be improved as increasing the complexity of the dispersion patterns and iteration number of the reconstruction algorithm. The table 5.2 show the detailed error values of each reconstructed object ($S_{0-3}$).

5.4 Demonstration of PG-based Imaging Spectropolarimeter

5.4.1 System Calibration

The system matrix $H$ contains the spectral response and geometric mapping information between a voxel in the object cubes and a pixel on the FPA. Since we assumed a linear system, this information can be estimated based on the system parameters such as a
focal length of the imaging lens and the diffraction properties of the PGs. However, in real situations, the geometric mapping may not be easily calculated. For example, the misalignment of lenses can cause dislocation of the patterns, and the aberration of lenses may cause further distortions. In order to calibrate the geometric mapping information of the system, projection distance $d(\lambda)$ of selected voxels is measured with various light sources; we utilize single wavelength (HeNe 633 nm, Nd:YAG 532 nm) light and narrowband (FWHM < 10 nm, 400 nm ~ 700 nm) sources to calibrate the geometric mapping information.

Fig. 5.18(a) shows a sample of calibration image containing the position of diffracted beams of two single wavelength sources: 633 nm and 532 nm lasers in Fig. 5.18(b). The spots are collected at the center of the FPA and denote the relative distances between the zero and first diffraction orders. System matrix $H$ can be revised based on this mapping

Figure 5.18: (a) Projection image on a FPA with two different light sources (532nm and 633nm); (b) Calibration setup with the lasers: 633 nm and 532 nm.

Figure 5.19: Spectra of the beams passing through color filters: (a) centered at 450 nm, (b) 500 nm, (c) 550 nm, and (d) 600 nm (only 4 of 8 spectra are shown).
distance, and several sample spots were used to make it more reliable. Both projections of the lasers can provide geometric references to set the system matrix $\mathbf{H}$ for two single wavelength information but for wavelengths where information is difficult to acquire, data from neighboring wavelengths can be interpolated. We also use bandpass color filters to make narrow band light sources to figure out good estimation of geometric and spectral response information of the visual light. We use 8 different color filters that have less than 10 nm FWHM centered from 400 nm to 750 nm and Fig. 5.19 shows the spectra of the filters have different center wavelengths. Using an integrating sphere (Newport), spectra of the beams passing through the filters can detected, and the measured spectral response is adopted in the system matrix $\mathbf{H}$.

The Fig. 5.20(a) shows 3-dimensional schematic of two PGs and one QWP that make totally 9 dispersion patterns ($3 \times 3$) on the FPA. When the PG#1 generates three patterns along the $x$-axis ($0^\circ$), the first orders are diffracted with the circularly polarized contents of the input light (one is for the right-handed and the other is for the left-handed circular polarization), while the zero order just passes through the PG and it reserves the same polarization content of the input light. Next, the three orders pass through the QWP oriented with $0^\circ$ on $x$-axis, and the QWP makes the first two orders change to the linearly polarized light (one is $+45^\circ$ and the other is $-45^\circ$ direction) without changing the polarization state of the zero order. Then, the second polarization grating, PG#2 that is oriented $90^\circ$ to the $x$-axis, makes another three diffractions of each diffraction order.

![Diagram](image)

Figure 5.20: (a) 3-dimensional schematic of the PG#1 and PG#2 in addition to the quarter-wave plate (QWP). (b) Expected dispersion patterns on the FPA. (c) Zero-order efficiency of the PG#1 and PG#2 over the visible range.
made by PG\#1. Therefore totally, 9 diffractions can be generated and their dispersion patterns are projected on the FPA as shown in Fig. 5.20(b). Each pattern has their own polarization sensitivity that depends on the order of diffraction and axis of the optical elements. For example, the pattern (i) is the global zero order that contains the same polarization contents of the input light so the pattern represents the $S_0$ polarization information of the input light. The pattern (ii) and (iii) are generated as the zero order of PG\#1 passes through the QWP ($0^\circ$) and the second grating, PG\#2, that makes the diffractions toward the $90^\circ$ on $x$-axis. According to the Eq. 5.8, the combination of the elements makes the diffraction whose efficiency is proportional to the $S_2$ Stokes parameter of the input light: linear polarized light in the $+45^\circ$ and $-45^\circ$ directions. The other patterns, (iv) – (ix), are created by the PG\#1 and PG\#2, but their polarization dependency is only determined by the PG\#1 that separates circular polarization contents of the input light. The second grating, PG\#2, merely makes additional dispersion and separation of the first order of PG\#1. The efficiency of the patterns (iv) – (ix) is governed by the Eq. 5.5, so the $S_3$ Stokes parameter can be extracted from the dispersion patterns.

The set of elements including two PGs and one QWP can separate polarization contents related to three Stokes parameters ($S_0, S_2$, and $S_3$) as shown above. The design of the set is one of the possible configuration to separate polarization contents of the input light, and additional elements can be added to get more complex dispersion patterns. Moreover, in order to estimate $S_1$ Stokes parameter, additional PG and QWP($45^\circ$) should be stacked on the set of elements. The total set of elements (three PGs + two QWPs) is the minimum requirement for separating full Stokes parameters of the light.

The intensity of the dispersion patterns is determined by the diffraction efficiency of the PGs. In order to spread the input light to all of the diffraction patterns, and avoid any saturated or empty pixel on the FPA, here we use the achromatic polarization gratings [34] that shows chromatically uniform diffraction efficiency. The Fig. 5.20(c) shows the measured zero order efficiency of the achromatic PG in the visible wavelength range ($\eta_0 \approx 30\%$). The diffraction efficiency of the zero and first order can be rewritten with the measured value as follows.

\[
\eta_0 = K \quad (5.26a)
\]
\[
\eta_{\pm 1} = \left(\frac{1}{2} + \frac{S_3}{2S_0}\right) (1 - K) \quad (5.26b)
\]
where $K$ is a factor of the zero order efficiency measured.

Based on the Eqs. 5.5-5.12, the intensity of the dispersion patterns shown in Fig. 5.20(b) can be described as below.

\begin{align*}
I^{(i)}(\lambda) &= K_1(\lambda) K_2(\lambda) S_0(\lambda), \\
I^{(ii)}(\lambda) &= \frac{1}{2} K_1(\lambda) (1 - K_2(\lambda)) (S_0(\lambda) + S_2(\lambda)), \\
I^{(iii)}(\lambda) &= \frac{1}{2} K_1(\lambda) (1 - K_2(\lambda)) (S_0(\lambda) - S_2(\lambda)), \\
I^{(iv)}(\lambda) &= \frac{1}{2} (1 - K_1(\lambda)) K_2(\lambda) (S_0(\lambda) - S_3(\lambda)), \\
I^{(v)}(\lambda) &= \frac{1}{4} (1 - K_1(\lambda)) (1 - K_2(\lambda)) (S_0(\lambda) - S_3(\lambda)), \\
I^{(vi)}(\lambda) &= \frac{1}{4} (1 - K_1(\lambda)) (1 - K_2(\lambda)) (S_0(\lambda) - S_3(\lambda)), \\
I^{(vii)}(\lambda) &= \frac{1}{2} (1 - K_1(\lambda)) K_2(\lambda) (S_0(\lambda) + S_3(\lambda)), \\
I^{(viii)}(\lambda) &= \frac{1}{4} (1 - K_1(\lambda)) (1 - K_2(\lambda)) (S_0(\lambda) + S_3(\lambda)), \\
I^{(ix)}(\lambda) &= \frac{1}{4} (1 - K_1(\lambda)) (1 - K_2(\lambda)) (S_0(\lambda) + S_3(\lambda))
\end{align*}

where $I^k(\lambda)$ is the intensity of the dispersion pattern $k$ at wavelength $\lambda$, and $K_1(\lambda)$ and $K_2(\lambda)$ are the zero order efficiency of the PG1 and PG2 respectively. As we can check above, the intensity of the pattern varies according to the polarization information of the object. Therefore, we can estimate original polarization state of the object as detecting each dispersion pattern. We have calculated and calibrated the geometric mapping information between the dispersion patterns and the object, and now we know what polarization contents of the object are projected on the dispersion patterns. In the next section, we will show experimental results of our imaging system with the indoor and outdoor test objects.

### 5.4.2 Experimental Setup and Preliminary Test

Fig. 5.21 shows experimental setup for our imaging system. For a robust alignment of elements, we place all optics, camera, and PGs on the rail system (Newport). Color images are used for the object and the images are generated on a flat-panel display (LCD monitor, Dell) and the spectral information of the object can be modified easily as
changing the color of the object. To change the polarization information of the object, we place polarizers and wave plates on the display so the object can have locally modified polarization information. Since the object is located far from the setup, we can assume fairly collimated light is passing through the system. The field stop limits the field of view so the dispersion pattern can have properly overlapped images that are separated spatially. As we discuss in previous section, two PGs and one WP can make $3 \times 3$ dispersion patterns that can separate $S_0$, $S_2$, and $S_3$ polarization contents. The dispersion pattern is focused via image lens and projected on a monochromatic visible CCD camera (DCU223M, Thorlabs). The camera uses 1/3 inch CCD sensor (Sony) that has 8-bit A/D converter resolution that presents 256-level grayscale.

First, rectangular images having different color information (R, G, B, and W) are used for the objects. The four vertically aligned rectangular images are generated on the display and only one PG is placed on the setup. Fig. 5.22 shows the dispersion patterns of the four rectangular images, and obviously they show different dispersion patterns that are spread according to the spectral information of the objects; Red image is dispersed further than other color images (Green and Blue) because of the wavelength dependency of the PG. White image (W) contains all spectral information of the other color images (R, G, and B), so the dispersion region of the white image covers all other dispersion patterns shown right above. Fig. 5.22(b) indicates intensity profile of the four dispersion patterns along the diffraction direction. Here we can check the diffraction property of the PG and the displacement length with the different wavelength images.
Figure 5.22: (a) Dispersion patterns that are captured by camera with one PG; (2) Intensity profile of red, green, blue, and white boxes (processed by high-resolution CCD camera: DCU223M, Thorlab Inc).

5.4.3 Indoor Test

For the indoor test, we use a target image generated by LCD monitor. Fig. 5.23(a) shows the image that includes three circles having different spectral information with different colors: red, green, and yellow. The center spectrum of the circle \((i)\) is 600 nm and \((ii)\) has a peak wavelength at 540 nm. The circle \((iii)\) consists of the both spectra of \((i)\) and \((ii)\), so it has two peak wavelength at 600 nm and 540 nm. In order to manipulate polarization information of the circles, linear polarizers and QWPs are placed on the

Figure 5.23: (a) Target image generated by LCD monitor (100×100 pixels), (b) Projected dispersion patterns on the CCD array of the on FPA (400×400 pixels).
target image; For example, a linear polarizer with axis at $-45^\circ$ is placed on the circle $(i)$ to change the polarization state as $S_2=-1$. On the circle $(ii)$, a polarizer ($-45^\circ$ axis) and a QWP ($0^\circ$ axis) are set to manipulate the output polarization as $S_3=+1$. The output polarization of the circle $(iii)$ is changed by a polarizer ($-45^\circ$ axis) and a QWP ($90^\circ$ axis) as $S_3=-1$. These modifications of spectral and polarization information help to examine the imaging system since we already know the spectral and polarization state of the image.

The dispersion patterns of the target image is captured by the monochromatic camera as shown in Fig. 5.23(b). The panchromatic image in the global zero order is only sensitive to the intensity profile of the target image, while the other dispersions show different polarization sensitivity according to the set of the elements (PGs+QWPs). The spatial resolution of the object is determined by the size of the zero order (100×100 pixels), and the spectral resolution of the system is selected depending upon the dispersion length of the target. Here we set the spectral resolution as 4 nm.

The dispersion pattern is reconstructed using the system matrix that contains spatial mapping information with spectral and polarization response between the object and the dispersion pattern. The reconstructed data are depicted in Fig. 5.24, where a grayscale of the image represents the intensity and normalized polarization state of the object. The reconstructed spectral resolution is 4 nm, so totally 51 spectral band images are estimated within 500 – 700 nm, but only 11 spectral images are shown in the result. Spatial resolution of the result is the same as original object (100×100) that utilizes $10^4$ pixels of the CCD. The first row (a) illustrates the $S_0$ reconstruction while the second and third rows are about the reconstruction of normalized $S_2$ and $S_3$ polarization contents respectively. Here we can see good correlation between the actual object and the reconstructed result; For example, in the result (a) that reveals spectral information of the image, red circle (top left) is displayed only at the range of 600 – 660 nm and green circle (top right) is shown in the reconstructed image within the range of 520 – 580 nm. The yellow circle (bottom) is also properly reconstructed, and we can check the circle shown in 520 – 680 nm wavelength range. For the polarization contents reconstruction, the result (b) illustrates a circle that is linearly polarized at $-45^\circ$ ($S_2 \simeq -1$), and the result (c) reconstructs two circles that are circularly polarized; upper right circle shows $S_3 \simeq +1$ and bottom circle shows $S_3 \simeq -1$ polarization state.
Figure 5.24: Reconstructed Stokes images in the range of 500nm to 700nm (the figure only shows 11 of 51 spectral band reconstruction - this figure should be separated with 3 sub-figures as tiger figs above).
Figure 5.25: Measured results of the two circles (Red and Green) reconstruction: (a) Reference and reconstructed $S_0$ spectra of the two circles regions. Solid lines denote the reference spectra that measured with a spectrometer. (b) Normalized $S_2$ and $S_3$ spectra of the red circle region. (c) Normalized $S_2$ and $S_3$ spectra of the green circle region.

Fig. 5.25(a) shows the reconstruction result of spectral estimation of $S_0$ for red and green circles. The solid lines denote the actual spectra of the two circles measured by a spectrometer with an integrating sphere, and the dashed lines show the reconstructed spectra of the two regions. The result spectra almost match the original spectral values for the two regions. In Figs. 5.25(b) and 5.25(c), the estimation result of normalized $S_2$ and $S_3$ information for the two circles are shown. Once again we can check the reconstructed results of the polarization information correspond to the original polarization values that we set.
5.4.4 Outdoor Test

Here we try to take an actual object outside and reconstruct spectral and polarization information using the same system setup. A raw image captured by a normal camera is depicted in Fig. 5.26(a). The image was taken on a clear and sunny afternoon and it shows full color information of the image. We focus on the part in the white dashed line of the image, and take the part with our imaging system generating $3 \times 3$ dispersion pattern. The dispersion pattern is shown in Fig. 5.26(b) and the intensity profile of the image can be found on the global zero order pattern. This dispersion pattern is one of the example that can be captured by our imaging system. Since we use the same setup and the system parameters are already calibrated for the indoor test, the same system matrix and reconstruction algorithm can be used for the outdoor test.

Fig. 5.27(a) shows the two target images (vehicles) that we use for the reconstruction of polarization information. Target (i) and (ii) were captured by our imaging system but the background of the target contained quite similar spectral information of the target. Because of the similarity of the spectral information, it was hard to reconstruct the spectral contents and it also make it hard to estimate polarization contents since both contents are projected on the same dispersion pattern. So here we narrowed spectral bandwidth with a narrowband filter (FWHM < 10nm) and estimated polarization infor-

Figure 5.26: An example of target image and its dispersion pattern: (a) Raw image of a moving vehicle captured by normal chromatic camera. (b) Dispersion pattern of the vehicle. The global zero order pattern shows the intensity profile of the target image, and other patterns are created according to the polarization contents of the image.
information of the target from a new dispersion patterns. The filter was placed in front of the CCD and reduced the dispersion of the target image. Figs. 5.27(b) and 5.27(c) illustrate the reconstruction results of the two target images. The first column of the results shows the reconstructed $S_0$ Stokes parameter that is related to the intensity profile of the target. The second column describes the normalized Stokes parameter $S_3/S_0$ that shows circularly polarized parts of the target, and the third column gives the value of Degree Of Circular Polarization (DOCP). Even in this narrow spectral range, especially when we look at the last column, man-made object such as the target appears distinctly different from the background. In fact details that are hidden in the $S_0$ column are revealed in the second and third columns with polarization information. The quality of reconstructed images may be improved by increasing the resolution (and level) of the camera used. The spectral resolution may be improved by using PGs with smaller periods providing more dispersion in the diffraction patterns. For this demonstration, we used a simple imaging system that consisted of only two PGs and one QWP, but several other configuration and combination of PGs and wave plates may offer other advantages, need to be tested further.

We have introduced the concept of Polarization Grating based Snapshot Imaging Spectropolarimeter (PGIS). We employ multiple PGs to create chromatic dispersion patterns that are linearly proportional to Stokes vectors (polarization information) embedded in a target object. We demonstrated an imaging system which produces a 2D projection of an object containing spatial, spectral and polarization information. The multi-dimensional information of an object was successfully reconstructed using the approach. PGIS may have capability to reconstruct the object with higher resolution in all dimensions, since it can estimate 4D object data from the dispersion patterns without Fourier-domain post-processing, and may allow a faster overall reconstruction than current snapshot imaging approaches. We continue to develop a more optimal and practical PGIS approach by studying several other promising configurations.
Figure 5.27: (a) Targets (i) and (ii) in outdoor image for reconstruction. Reconstructed $S_0$, $S_3/S_0$, and DOCP for (b) Target (i), and (c) (ii) respectively. Interference fringes are located in areas of the scene that are linearly polarized. These fringes are particularly evident in the hood of the vehicle. Spatial resolution is 250×250 pixels.
Chapter 6

Discussion and Conclusions

The overarching theme of this dissertation was to investigate and apply Geometric Phase Holograms as simple, compact, low-cost, light-weight, highly efficient alternatives to conventional optical beam steering and hyperspectral polarization imaging technologies. We approached the goal with individual research hypotheses and questions, which were proved and answered in each chapter. Moreover, we tried to fulfill the research goal in both theory and experimental demonstration.

In Chapter 3, we proposed novel polarization holography setups to record arbitrary phase profiles as diffractive Geometric Phase Holograms (GPHs) using revised Michelson and Mach-Zehnder interferometric approaches. We fabricated and demonstrated GPHs as liquid crystal diffractive optics working as polarization-sensitive diffractive gratings, lenses, micro-lenses, axicons and azimuthal waveplates with excellent optical quality that manifests nearly 100% efficiency, polarization-selectivity, and fast electro-optical switching (an order of milliseconds). We fabricated polarization lens GPHs with two different convex lenses having 100 mm and 300 mm focal lengths. Polarization sensitive axicon and micro-lens GPHs were also fabricated utilizing Mach-Zehnder interferometric setup. We expect the polarization axicon GPH can be used to convert a parallel laser beam into a ring, focus a parallel beam into long focus depth, or to turn a Gaussian beam into a non diffractive Bessel beam. Conventional holography recording setups required comparably large area and were not commercially viable for creating gradual changes of the linear polarization fields, which were required to fabricate very large grating periods for the polarization gratings. Moreover, there was no practical way to realize and control arbitrary change of the linear polarization fields that
leads a freedom of diffraction on the optical elements. We introduced and demonstrated new polarization holography techniques that could create scalable period polarization holograms and arbitrarily changed polarization fields that produced various types of GPHs. We demonstrated a novel method that offered proximate lithography with easily tunable-periods with multi-axis polarization gratings fabrication method that could fabricate multiple diffraction at different azimuthal angles.

Chapter 4 addressed the following question: *Can we achieve high throughput, wide-angle laser beam steering using stacked polarization gratings and polarization selectors? And if so, what are the optimum designs to achieve lowest loss?* We identified two families of beam steering: nonmechanical and mechanical. We found an exponential increase in steering angles as steering stages were added for the nonmechanical steering. Moreover, we found that it would be preferable to choose two steering designs selectively: Quasi-Ternary design if number of steering angles is most important; Supra-Binary design if throughput, ease of fabrication, and reliability at large field-of-regard are most essential. In mechanical steering, we found that Risley-PGs act similar to Risley Prisms with much smaller SWaP. We also built prototypes of nonmechanical and mechanical beam steering systems to various independent evaluators. As the main contribution within Chapter 4, we demonstrated discrete nonmechanical steering for high throughput (more than 80%) and wide-angle (up to 80° field-of-regard) with three unique steering designs: Ternary, Quasi-Ternary, and Supra-Binary. In this work, we suggested and evaluated important non-mechanical designs, and derived governing theory of operation. We also demonstrated a new way to accomplish continuous mechanical steering with high throughput and wide-angle utilizing a pair of rotating PGs. Moreover, we demonstrated reduced chromatic dispersion of broadband source steering as utilizing compensation PGs that can adjust the color separation of diffraction.

Chapter 5 addressed the following hypothesis: *PGs can be used to implement snapshot hyperspectral polarization imagers with several performance advantages.* We used numerical simulation to assess reconstruction performance, and showed a favorable comparison with prior work. We also found that several potentially optimum PG dispersion patterns, to be evaluated further, and identified favorable characteristics. Moreover, we found algorithmic technique to vastly improve SNR iterative convergence. As a primary scientific contribution within Chapter 5, we demonstrated working principle of the hyperspectral polarization imaging system containing two PGs for lab-scale and natural objects. We also derived governing theory of operation and design
principles, and developed an optimum calibration approach that is simple and generic, applicable to any PG-based imaging polarimeter. Although direct comparison of prior and our imaging system is nearly impossible, we showed bench top results that suggest substantially more accurate reconstruction spectrally and spatially for the full Stokes vector.
REFERENCES


