ABSTRACT

CIMET, CHARLES MORGAN. The Effects of Atmospheric Uncertainty on Mars Entry, Descent, and Landing. (Under the direction of Dr. Robert Tolson.)

Data from the Mars Mesoscale Model (MMM5) was analyzed and the results were used to build a statistically correlated atmospheric model with density profiles based on the hydrostatic assumption. Dust storm data from the mesoscale model provided a means of modeling local storm effects on the atmosphere. The atmospheric model was used in a Mars entry, descent, and landing (EDL) Monte Carlo analysis to study the impact of atmospheric uncertainty on landing accuracy. The Monte Carlo study used a limited six degree of freedom simulation based on a ballistic entry architecture. Landing dispersion sensitivity to atmospheric parameter variability in different phases of EDL was determined to identify critical measurement needs on future robotic missions. Landing dispersions from trajectories flown through a local dust storm model were compared to dust-free results to determine if local storms could be a contributor to the consistent landing location overshoot observed in previous missions such as the Mars Exploration Rovers (MER) and Phoenix. Local storm effects were compared to other sources of EDL uncertainty to determine its relative importance in achieving precision landing.

Statistical studies of the mesoscale data yielded a highly correlated atmosphere at lower altitudes with increasing statistical independence at higher altitudes. Landing dispersion sensitivities were higher for atmospheric variability in the region of maximum entry vehicle dynamic pressure, which was expected based on previous work. Simulated local dust storms yielded an overshoot bias that was small compared to other sources of landing error, but still significant for future human missions when considering precision landing within 1km of a target. If mesoscale variability is a reasonable predictor of the atmosphere, landing error due to atmospheric uncertainty is an engineering problem.
that requires knowledge of atmospheric parameter distributions and higher fidelity time history modeling for weekly, seasonal, and annual time spans.
Biography

Charles Morgan Cimet was born in New York, New York on March 26th, 1987 to Leonard and Ann Marie Cimet. He attended public school in Brooklyn and East Setauket, New York before being accepted to the Schreyer Honors College at The Pennsylvania State University in 2005. At Penn State he earned a Bachelor of Science in Aerospace Engineering with a Liberal Arts minor in Business in 2009. During his undergraduate studies, he was a member of the Penn State Fencing team, which won two team NCAA National Championships in 2007 and 2009, and a member of the Phi Kappa Phi, Sigma Gamma Tau, and Tau Beta Pi Honors societies. He also co-authored a conference paper titled *Trade Study of Earth to Pluto Trajectories Utilizing a Jovian Gravitational Assist* and presented at the 2009 Aerospace Sciences Meeting in Orlando, Florida.

Upon graduation from Penn State he was accepted into the graduate school at North Carolina State University as a graduate research assistant at the National Institute of Aerospace in Hampton, Virginia. He studied for a Master of Science Degree from 2009 to 2011, investigating the cause of density reconstruction anomalies in Mars Reconnaissance Orbiter aerobraking data before completing his Master’s thesis research with the Atmospheric Flight and Entry Systems Branch at NASA Langley Research Center. After graduation he will reside in Boston, where he has accepted a position at MIT Lincoln Laboratories.
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I owe untold thanks to my parents, Leonard and Ann Marie, as their never-ending love and support over the years has shaped my character and provided me with the abilities to pursue my dreams. Finally, I would like to thank my friends, wherever they may be, for their inspiration, laughter, and good company along the winding road of life.
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1 Introduction

Mars has been the center of intrigue and scientific interest ever since its surface features were first observed by telescope in the 19th century. The possibility of life on our planetary neighbor has driven scientific investigation, along with a push for eventual colonization after the successes of the Apollo era. Robotic missions have been sent into orbit around Mars and to the surface of the planet to collect atmospheric and surface composition data. Landing payloads on the surface has proven to be a very difficult engineering problem that will only get more challenging if humans are to be sent to the Red Planet. Further improvements in atmospheric measurement and modeling capabilities will play a key role in achieving the goal of planning a successful human mission to Mars.

1.1 Motivation

Since the dawn of the space age, one of NASA’s main goals has been the exploration of Mars. Successfully reaching the Red Planet, however, has proven to be a difficult task; more specifically, landing a payload on the surface has been particularly treacherous. Out of the thirteen attempts by world governments to autonomously land on the surface, only seven of those missions have successfully reached their destination and transmitted a signal back to Earth.

NASA has a better track record with a total of six successes in seven attempts, but despite this, still lacks the ability to achieve precision landing for human mission sized payloads [1]. A common measure of landing accuracy is the $3\sigma$ landing ellipse; the ellipse major axis is generally a measure of downrange error and the minor axis is generally a measure of crossrange error. For Pathfinder and MER, the landing ellipse major axes
were 200km and 80km, respectively. Furthermore, each of these missions had a landed mass below 1 metric ton (1000 kg) and the next mission Mars Science Laboratory (MSL) will land around 2 metric tons (t). It is estimated that manned missions will require anywhere from 40-80 t, with the added constraint of precision landing within 1km of the intended ground target.

In the absence of any major technological breakthroughs, future missions will likely be more complex than smaller robotic missions such as MER, which already require an extensive methodology to achieve landing, as seen in Figure 1 [2]. This will necessitate an improved understanding of entry vehicle dynamics and control and the ability to accurately predict the atmosphere. On the atmospheric side, one critical step to achieving these goals is improved knowledge of how atmospheric variability and local weather phenomena, such as dust storms, impact the EDL process.

![Figure 1: MER EDL Concept of Operations](image)
1.2 Literature Review

Two consequences of improved knowledge of the Martian atmosphere are increased model fidelity and real-time prediction capability. Over time, atmospheric models have been refined using data collected by orbiting vehicles and during successive landing attempts. A majority of this data, however, has been collected in the upper atmosphere to improve the process of aerobraking, which uses atmospheric drag to reduce the fuel required to put a spacecraft in Mars orbit [3]. Critical altitudes for EDL are in the lower atmosphere between 0-50km [4] where there is not a lot of data for model validation. Preliminary design for future mission architectures has indicated that larger payloads deliverable to the surface will require greater aerodynamic deceleration in the 0-50km altitude region above the surface [5]. Identifying key atmospheric parameters to measure in this region will help define requirements for data collection on future robotic missions to increase model fidelity. Figure 2 shows the general regions of interest for two random atmospheres produced by the global climate model MarsGRAM, one with low dust levels and the other with high dust levels simulating global dust storm conditions [4]. Within the 0-50km altitude band, density, temperature, winds, and dust levels are important parameters to predict for successful EDL design and control [6].

In the case of density, the limited measurements available for critical EDL altitudes come from orbiter based radio occultation readings, infrared interferometer measurements, and reconstructions based upon entry capsule dynamics and onboard instrumentation. The few density reconstructions available from actual mission data suggested that current atmospheric models over predict the mean density profile; this was further supported by a consistent overshoot observed in actual landing location when compared to expected dispersions [7]. Recent publications have found that Phoenix and MER
entry capsules had a higher than expected trim angle of attack, affecting aerodynamic force and moment coefficients along the trajectory and thus the accuracy of the density reconstruction due to vehicle dynamics [8]. The increase in lift caused by angle of attack was shown to have a larger influence on vehicle overshoot when compared to atmospheric error from inaccurate density reconstructions; density prediction inaccuracies, still had a significant effect on landing dispersion when considering the accuracy required for precision landing, albeit smaller than the entry vehicle lift uncertainty. Furthermore, the potential influence of density variability due to local weather phenomena on the EDL landing accuracy is still an open research topic, as atmospheric models up until MSL were primarily based on global conditions and phenomena.

Early lander missions such as Viking 1 and 2 and even more recent missions such as the Mars Exploration Rovers (MER) relied on global models of the atmosphere for both planning and on-board prediction. Global models such as MarsGRAM are still used today for these purposes; although suitable for less stringent landing requirements, they fail to predict small-scale variations in weather such as local wind conditions and local
dust storms.

For tasks such as landing site selection, mesoscale models are currently used to predict the atmosphere at a higher resolution than global climate models such as MarsGRAM. Two notable mesoscale models are the Mars Regional Atmospheric Modeling System (MRAMS) [9], created by Scot Rafkin at the Southwest Research Institute, and the Mars Mesoscale Model (OSU MMM5) [10], created by Dan Tyler at Oregon State. Currently, mission planners for Mars Science Laboratory (MSL) have used the mesoscale models in conjunction with MarsGRAM and the Program to Optimize Simulated Trajectories (POST) for the purpose of landing site selection and uncertainty analysis. Variability within the mesoscale models is considered to be too low to represent the anticipated modeling and prediction errors during mission planning, so additional engineering uncertainty is added by mission planners to account for these issues on top of the modeled atmospheric variability.

Despite the fact that mesoscale models face high uncertainties at low altitudes due to a lack of climatological data, they allow for the simulation of weather events such as local or regional dust storms, which exhibit different characteristics than the global scale dust events modeled in tools such as MarsGram. Figure 3 shows an example of a local dust storm produced by the OSU MMM5 mesoscale model. The storm's opacity decreases radially from the center and it has a diameter of 750km, which is large enough to fully encompass the ground track projection of an entry trajectory. Dust storm opacity is directly related to storm intensity, which is roughly a measure of its effect on atmospheric variability and mean conditions in the affected region.
1.3 Research Goals

Uncertainty in atmospheric modeling and prediction is an issue at the systems level of EDL design. Based on current design practices and the need for a robust system to ensure mission success, higher atmospheric uncertainty leads to higher landing mass margins and landing dispersions. In the case of controlled entry, these uncertainties lead to downtrack trajectory prediction errors, increasing the required complexity of the on-board control system.

Given the benefits of reducing atmospheric uncertainties, the next logical question is how can this be accomplished? Clearly, accurate atmospheric measurements must be taken, but to what spatial resolution? how far in advance of landing? is higher accuracy required for different stages of EDL? what role do local weather events play? One method for answering these questions is a statistical analysis.
This study uses Monte Carlo simulations to determine the sensitivity of atmospheric uncertainties on the EDL process. An atmospheric model was built to capture the effects of uncertainty on an EDL trajectory. Spatial and temporal correlations were calculated using mesoscale data to create non-uniform atmospheres that represent local variability while not requiring an excessive amount of computational power. ‘Dust-free’ and local ‘dust storm’ mesoscale data was used to statistically characterize different weather conditions in the Monte Carlo atmospheric model and study the effects of local storms on EDL. Note that directly using the mesoscale data would have been too computationally demanding given the resources available for the study. Downtrack landing dispersions was used as a metric to measure the effects of atmospheric variability on the EDL architecture.

Dynamic simulation of EDL was achieved using a 4th order predictor, 5th order corrector method of numerical integration (ode 45 in MATLAB). A zero-lift ballistic entry architecture was assumed so that results could be compared to previous missions such as MER and Pathfinder. Figure 4 provides an artist’s conception of a ballistic entry capsule decelerating in the upper atmosphere. Zero cross-track motion was assumed to simplify the dynamics and EDL events such as parachute deployment were simulated discretely, with state variables and time constraints used as triggers. Entry capsule aerodynamics were simulated using a polynomial approximation based on wind tunnel derived table look-ups for MER provided by NASA.
2 Modeling and Simulation

To study the effects of atmospheric uncertainty on Mars EDL, a parameterized atmospheric model was built to perform a sensitivity analysis on landing dispersion. Mesoscale model outputs were used to validate the parameterized model and to statistically characterize the variability of the atmosphere. Statistical variance and spatial correlations from the mesoscale data were analyzed and incorporated in the parameterized model. An EDL simulation was built based on ballistic entry architectures from previously flown missions and aerodynamic data provided by NASA. A uniform atmosphere was used to simulate a nominal trajectory that served as a baseline for Monte Carlo landing dispersion results.

2.1 Atmospheric Model

Since the goal of the study is to analyze the effects of atmospheric variations on the entry, descent, and landing (EDL) process for Mars, a base atmospheric model and a method of applying variations must be determined. Planning for early Mars missions relied only on global atmospheric models that did not capture local effects. Recent missions have
leveraged mesoscale models for their higher resolution and their ability to model local variability, two requirements necessary for this study.

Given resource and time constraints, it was therefore decided that mesoscale model data would serve as a baseline for comparison with the end goal of building a simpler model that suited the needs of this study. Mesoscale data was taken from the OSU MMM5 model used by Dan Tyler. The mesoscale data in this study was taken in the vicinity of Holden Crater, along a path designated as a potential Mars Science Laboratory (MSL) trajectory ground track. Eighteen days of data between 312° to 328° longitude and -25° to -29° latitude during the winter season for the southern hemisphere ($L_s = 154^\circ$) were available for simulation in this region.

2.1.1 Mesoscale Density and Temperature Profiles

Before building a model of the atmosphere for simulation purposes, density and temperature profiles from the mesoscale data were analyzed to gain a better understanding of the physics driving the data. Profile variations throughout the day and behavior near the atmospheric boundary layer altitude were of particular interest. The existence of a distinct boundary layer near the surface of the planet has been well documented, but boundary layer effects on temperature and density within the mesoscale data are worth analyzing, especially as the boundary layer changes in altitudinal thickness throughout the Martian day. Note these boundary layer effects are characterized in the data by a distinct change in temperature lapse rate variation within a band of altitudes usually below 8km.

Based on previous studies and the limited data available, daily temperature variations were expected to be noticeable, especially near the surface. The temperature profiles seen
in Figure 5 verify these expectations, notably in the case of the negative lapse rate at 0200-0300 local solar time (LST).

![Mesoscale data temperature profiles for various LST](image)

Figure 5: Mesoscale data temperature profiles for various LST

Given that boundary layer effects are evident in the mesoscale data temperature profiles, it is worthwhile to look at the corresponding effects on the density and pressure profiles. Figure 6 shows the corresponding density profiles. Although the daily changes in the density above 10km are minimal, there are differences near the surface where hourly temperature changes were observed.

Note that as temperature decreases density increases, as predicted by the inverse relationship in the ideal gas law, assuming pressure remains roughly constant. Figure 7 shows that the pressure profiles indeed change very little with time of day.

As expected, the ideal gas law assumption holds in the mesoscale data at a given location above the surface and was incorporated into the parameterized model for this
Figure 6: Mesoscale data density profiles for various LST

study:

\[ P = \rho RT \]  \hspace{1cm} (1)

where the gas constant \( R \) for Mars is 189 J \( / \) kg \( / \) K.
2.1.2 Hydrostatic Tendencies within the Mesoscale Model

When looking to build an empirical model to represent the atmospheric physics seen in the mesoscale data, it would be convenient if mesoscale profiles could be predicted by the hydrostatic relationship:

\[
\frac{dP}{dh} = -\rho g
\]  \hspace{1cm} (2)

This would allow for a relatively simple model to be built around the hydrostatic assumption for the purpose of a computationally efficient EDL simulation. The hydrostatic assumption would also provide a set of convenient atmospheric parameters to use in an EDL sensitivity analysis of atmospheric variability. Note global models assume hydro-
static equilibrium, whereas mesoscale models include local weather effects that result in a violation of this assumption.

To determine whether the hydrostatic equation would provide reasonable model accuracy, mesoscale data was compared to a density profile generated by numerical integration of the hydrostatic equation. Since a main driver of this study is the prediction of atmospheric density for Mars EDL, the hydrostatic relationship for modeling the profiles was written in terms of density. Using the ideal gas law, the relation can be rewritten as the following:

$$\frac{d\rho}{dh} = -\frac{\rho g}{RT} - \frac{\rho \, dT}{T \, dh} - \frac{\rho \, dR}{R \, dh}$$

(3)

If the composition of the atmosphere is constant with altitude, then the $\frac{dR}{dh}$ term can be ignored. For the purpose of EDL analysis starting at the common definition for atmospheric entry altitude (125km) this is a reasonable assumption. The expression can therefore be simplified to:

$$\frac{d\rho}{dh} = -\frac{\rho g}{RT} - \frac{\rho \, dT}{T \, dh}$$

(4)

This expression for $\frac{d\rho}{dh}$ was used in the numerical integration along with temperature profile data and an initial condition surface density from the mesoscale model. Figure 8 shows the agreement between the numerically integrated density profiles and the mesoscale data for six different local times. The numerical integration was based on a known surface density and temperature profile. It can therefore be concluded that the mesoscale data up to 40km could be accurately approximated with the hydrostatic relationship. Since most of the aerodynamic deceleration for current EDL concepts occurs
in the lower 40km of the atmosphere, an accurate approximation of density in these altitudes will provide more accurate landing results.

**Figure 8:** Mesoscale data density profiles for various LST

In addition to hydrostatic validation, the numerical integration also showed that density can be accurately predicted from a temperature profile and surface density measurement. Infrared spectrometers measure thermal radiation in the infrared band at a resolution of half the scale height, thereby determining temperature profiles. Therefore, if we assume the mesoscale data is a strong indicator of the Mars atmosphere, infrared spectrometers could be used to determine density profiles for EDL, given a sur-
face density measurement. Density profiles could also be directly measured using LIDAR instrumentation, which determines gas concentrations, namely CO$_2$ in the case of Mars, by measuring an intensity ratio of two different emitted wavelengths along the path of the laser beam. Since CO$_2$ makes up 97% of the Martian atmosphere, this is a reasonable measure of density. A study of instrument measurement error versus EDL model accuracy would provide insight into which method is best applied to a given mission.

2.1.3 The Hydrostatic Relationship

Since mesoscale data at a given location can be represented by the hydrostatic relationship, this was taken as the starting point when deriving the atmospheric model for EDL simulation. For this model the atmosphere was assumed to be an ideal gas and temperature was assumed to vary with altitude by a constant lapse rate ($\alpha$). The basis for assuming a constant lapse rate is seen in Figure 5, where temperature variation can be reasonably approximated by a constant lapse rate, or slope, from the upper atmosphere to an altitude near the surface. Note the slope is nearly constant all the way to the surface for local solar times near midday, which is when EDL typically takes place. Starting with equation 4 a relationship between density, surface temperature, and height above the surface can then be written as:

$$\frac{d\rho}{\rho} = \frac{-(g + \alpha R)}{R[T_0 + \alpha(h - h_0)]} dh$$

(5)

Where $R$ is the universal gas constant for the atmosphere, $g_0$ is the gravitational constant for Mars ($g_0 \sim 3.72{\text{m/s}}^2$), and $T_0, h_0$ are temperature and altitude values at the surface. Although assuming a constant temperature lapse rate can introduce another potential source of error, it allows for a simpler parameterization of the atmosphere.
during simulation and sensitivity analysis. Integrating equation 5 yields the expression:

$$\rho(h) = \rho_0 \left( \frac{T_0}{T_0 + \alpha(h - h_0)} \right)^\frac{g + \alpha R}{\alpha R}$$  \hspace{1cm} (6)$$

This hydrostatic relationship can be compared to mesoscale density data using temperature values from the mesoscale model to calculate lapse rate. Note that as expected, in the limit as $\alpha \to 0$ the hydrostatic expression becomes the isothermal density relationship where $\frac{RT_0}{g}$ is the scale height:

$$\rho(h) = \rho_0 \exp \frac{-g(h - h_0)}{RT_0}$$  \hspace{1cm} (7)$$

Assuming the mesoscale model is the best representation of the Martian atmosphere, a comparison between the hydrostatic representation and mesoscale data served to test the viability of using the hydrostatic relationship while studying variational atmospheric effects on the EDL process.

2.1.4 Hydrostatic Relationship vs. Mesoscale Data

The simplest comparison between the hydrostatic density model and the mesoscale data assumes an isothermal atmosphere. Since previous missions have indicated appreciable temperature changes from the surface to the edge of the boundary layer during the Martian day, this case was not considered. The simplest method to account for this variation, was to assume a constant temperature lapse rate from the surface to a specified altitude. Hydrostatic model deviations from the mesoscale model were normalized by the average mesoscale density at each altitude location to visualize the error as a percent of the average. Figure 9 shows the average and 1-σ error for a sampling at 1200 and 1300
LST using the constant lapse rate model and a temperature measurement at 30km. The dotted blue lines are the individual samples included for reference. Note the average error can be attributed to lapse rate approximation error and altitudinally varying winds in the mesoscale model breaking down the hydrostatic equilibrium assumption in the absence of a known temperature profile. In Figure 9, the majority of the error is caused by the lapse rate approximation, as the magnitude and direction vary significantly near the surface with time of day.

![Figure 9: Constant lapse rate vs. mesoscale data for various LST](image)

The constant lapse rate model yielded up to 5% error below 30 km. Note that when extended beyond 30km, the model error reaches 15% at 60km. Although the errors below 30km are fairly reasonable, they can be improved by introducing a known temperature profile from the mesoscale data. Intermediate lapse rates can then be calculated to
better fit the hydrostatic model to the mesoscale values at each successive altitude step. Figure 10 shows the change in normalized error with altitude for the introduction of intermediate lapse rates. The normalized error has been significantly reduced at all altitudes but displays a consistent bias that suggests the intermediate lapse rate model is over-predicting the temperature profile.

![Figure 10: Piecewise lapse rate vs. mesoscale data for various LST](image)

The small, consistently positive bias in the normalized error can be attributed to winds disturbing the hydrostatic equilibrium assumption. Since lapse rate approximation error has decreased significantly with the intermediate rates, it is the resulting change in pressure gradient due to winds that yields the small, consistent bias seen in the Figure 10 temperature error profiles.

Note the hydrostatic model with intermediate lapse rates can not only be used in the
EDL design process, but also provides a viable option for actively measuring atmospheric density before a mission payload enters the Martian atmosphere. Surface density, surface temperature, and a temperature profile along the nominal entry trajectory could be used to provide density measurements just before entry. This could be achieved by determining temperature and pressure from on-orbit infrared spectroscopy and relaying the data to the entry vehicle before it reaches atmospheric interface.

For the purposes of this study, a hydrostatic model with constant lapse rate was used as the base model for the atmosphere. Since the primary goal of the investigation is to characterize the effects of atmospheric uncertainty, the characterization of the deviations from the base model is more important than the accuracy of the base model itself. This choice significantly improved computational efficiency and provided a manageable set of variable parameters for the process of Monte Carlo analysis.

2.2 Atmospheric Uncertainty

Given the decision to parameterize atmospheric profiles using a constant lapse rate, the next phase of the study was to analyze the spatial variability of the atmosphere. A 4-parameter atmosphere was chosen and statistical properties such as distribution and correlation were analyzed along a nominal MSL trajectory ground track in the vicinity of Holden Crater. This process was completed for both ‘dust-free’ and local ‘dust storm’ data from the mesoscale model. Parameter correlations and cross-correlations are discussed in terms of their physical implications to the ‘dust-free’ and ‘dust-storm’ atmospheres.
2.2.1 Atmospheric Correlation Assumptions

To create a more realistic representation of the Martian atmosphere and capture spatial characteristics of dust storms, parameter correlations from the mesoscale model were calculated and applied to the simulated atmosphere used in this study. This was done by generating a random 4-parameter \((\rho_0, T_0, T_{iso}, h_{iso})\), hydrostatic profile at the atmospheric entry interface ground track location, then using the correlations to calculate the conditional probability for random atmospheres along the downtrack. Note \(h_{iso}\) was chosen to be 57km, so henceforth the atmospheric parameters will be referred to as \(\rho_0, T_0, T_{57}\). Implementation of the multiple correlation method to accomplish this task is further discussed in Section 2.3.6. The mesoscale data was used to determine the atmospheric parameter variance and statistical distribution along a sample ground track for application in calculating the statistically correlated atmosphere.

Data from the mesoscale model was sampled over Local Solar Times (LST) of 1200-1700 hours for an 18 day period. The data sample represents a commonly desired mid-afternoon entry time over an 18 day arrival window. It was assumed that spatial correlations, namely how a change in the atmosphere at one location affects an adjacent location, could be calculated using this data without introducing a significant time bias due to hourly or daily variation.

2.2.2 Parameter Distributions Along Trajectory Downtrack

Many atmospheric models assume a normal distribution when generating parameters for random atmospheres, with beta and gamma distributions also being used in some applications. Assuming the mesoscale data provided an accurate representation of the atmosphere, distributions for key atmospheric parameters were calculated along a nomi-
nal trajectory, as seen in Figures 11, 12, and 13 for $\rho_0$, $T_0$, and $T_{57}$ respectively.

The y-axis shows the number of samples that fell within each bin of values and a normal distribution has been fit to the mean and standard deviation of the data as a basis of comparison. Shown along with the distributions are normal probability plots, which provide a visual measure of whether the data is normally distributed; if the data appears as a straight line it is normally distributed, while any deviations from a linear trend indicate non-normal characteristics. A 95% confidence interval for the mean and standard deviation of the normal fit is provided on the normal probability plots.

Looking at the normal fit for Figures 11, 12 and 13 it is apparent the data is not normally distributed. This is confirmed by the normal probability plots, as for each parameter the curve is nonlinear at the edges of the sample range, with a downward skew at lower values and upward skew at higher values. Such a non-linearity indicates the
Figure 12: Parameter distribution and normal probability for $T_0$

Figure 13: Parameter distribution and normal probability for $T_{57}$
distribution has a higher percentage of samples towards the edge of the range, otherwise
known as a long tail or a distribution with higher than expected variability.

Bumps in the normal probability plot indicate a skew in a particular direction. Figure 11
provides a clear example of this, in which a bump above the line towards the lower end
of the sample range indicates a leftward skew in the data; the skew is seen in the Figure 11
histogram, where the data is uniform to the left of the mean instead of decreasing along
the normal fit. These findings are further supported by the confidence intervals on the
standard deviation for the normal fit, as seen on each normal probability plot; the
confidence interval would be a single value if the data were normally distributed. The
certainty interval range on both the mean and standard deviation in Figures 11, 12
and 13 is a further indication of skew in the data, as it indicates the tail length, or
variability of the data, is different on either side of the mean.

An analysis of the parameter distributions was also performed on mesoscale dust
storm data, with similar results. The normal probability plots in Figures 14, 15 and 16
once again show a non-linear skew indicating a higher than expected variability in the
data distribution. The confidence intervals and bump toward higher densities in the
Figure 14 normal probability plot indicate a skew in the \( \rho_0 \) data, which can be seen in
the accompanying histogram.

It is evident that the parameter distributions in the dust-free and dust storm data
are not normal. Note the distributions are based upon mesoscale model assumptions
which in turn are based on a limited set of available data. A more detailed atmospheric
model for entry trajectory prediction and mission planning should consider fitting beta
or gamma distributions to future data sets as they become available to more accurately
model landing dispersion statistics. Note, if non-Gaussian distributions can be shown
Figure 14: Dust storm parameter distribution and normal probability for $\rho_0$

to be a good fit for Mars atmospheric data, the distributions could result in an appreciable landing bias that leads to overshoot or undershoot of the nominal location. For the purposes of this study, normal distributions were chosen to avoid introducing an unproven bias and to leverage simpler statistical prediction methods for building a random atmosphere, in this case the multiple correlation model.
Figure 15: Dust storm parameter distribution and normal probability for $T_0$

Figure 16: Dust storm parameter distribution and normal probability for $T_{57}$
2.2.3 Correlation Analysis Along Trajectory Downtrack

The mesoscale correlations used in the atmospheric model were taken along the downtrack of a nominal Mars Science Laboratory (MSL) trajectory provided by NASA. Latitude and longitude locations were converted to downtrack distance from the landing site and correlations were determined as a function of downtrack for both the dust-free and dust storm mesoscale atmospheres.

Correlations and cross-correlations for the atmospheric parameters $\rho_0$, $T_0$, and $T_{57}$ were analyzed to gain an understanding of the mechanisms driving the mesoscale atmosphere and to eventually build covariance matrices for the multiple correlation method of finding conditional probability. Note there are two covariance matrices of interest: the initial condition covariance ($\Gamma_{IC}$) and the downtrack covariance ($\Gamma_{DT}$). The initial condition covariance was used to generate uniform atmospheric profiles and the initial condition profile at atmospheric entry interface in the case of the correlated atmosphere. The downtrack covariance was used to generate profiles downtrack of atmospheric interface in the correlated atmospheric model. Parameter variance and variation of the time-sampled mean along the downtrack were also calculated for the same purposes.

2.2.4 Variability: ‘Dust-Free’ Atmosphere

The downtrack correlations and variance for each of the atmospheric parameters in the dust-free atmosphere are shown in Figure 17 and the cross-correlations appear in Figure 18. Each of the correlations on the plots are with respect to the parameter at a reference point 520km downrange from the landing site; at this location the correlation is one. It is evident from these plots that the surface temperature and density are highly correlated while temperature at 57km displays more statistical independence along the
Looking at the $\rho_0$ plot in Figure 17, the correlation is roughly 0.95 at the location 350km from the landing site in comparison to the reference point. Thus, if the density at the reference point increases by 2%, the density at 350km from the landing site will increase by a value of nearly 2%, depending on the standard deviation. In terms of building a correlated atmosphere, a high correlation serves to conditionally adjust the mean and variance at that location, based on the random value chosen at the reference point. The same cannot be said for $T_{57}$, as the correlation drops to around 0.3 at 350km
from the landing site. Note a correlation of zero would indicate variability at that location is fully independent of the conditions at the reference point.

Another interesting feature is the correlations associated with surface values are relatively constant along the downtrack. If this phenomena from the mesoscale model can be shown to hold true for the Martian atmosphere, a lower spatial resolution would be required on future orbiter missions to characterize the atmosphere. The quicker the correlations drop off with distance, the closer together measurements must be to properly characterize the variability between measurement locations.

For the plots in Figure 18, the values again represent the correlation of a downrange parameter to the reference parameter at the location 520km from the landing site. For instance, the top left plot shows correlations of $T_0$ with respect to $\rho_0$ at the reference location; note the correlation at 520km is not one, as this refers to the correlation of $T_0$ with respect to $\rho_0$ at the same location.

The cross-correlations associated with surface values are also constant along the downtrack, while those describing the relationship of $T_{57}$ to the other parameters are variable in downtrack and tend toward oscillations about zero. Analyzing the cross-correlations provides some worthwhile insight into the mechanisms that drive the mesoscale model. The high negative correlation between surface temperature and density ($T_0$ and $\rho_0$) shows the effect of the ideal gas law when pressure is constant. The negative correlation between $T_0$ and $T_{57}$ suggests as $T_0$ increases, $T_{57}$ has a tendency to decrease. This suggests a possible conservation of energy ($C_pT$) mechanism from the surface to 57km driven by a convection cycle similar to Earth’s atmosphere. Although the correlations cannot prove energy is conserved, they provide evidence suggesting future missions should collect data to confirm or disprove the hypothesis for the purpose of modeling and predicting the
Figure 18: Dust-free cross-correlations along nominal downtrack Mars atmosphere.

Figure 19 shows the percent variation profiles of the parameter means along the downtrack. Values derived from the mean variation were used to set parameter standard deviations ($\sigma$) in the atmospheric model.
Figure 19: Percent deviation of the parameter means along the downtrack

2.2.5 Variability: Mesoscale Prescribed Dust Storms

The correlations and variance for the dust storm case along the downtrack can be seen in Figure 20 and the cross-correlations are plotted in Figure 21. Similarly to the dust-free case, the surface parameter spatial correlations are nearly one and remain so along the downtrack. Temperature correlations at 57km decrease with distance but at a slower, steady rate, suggesting a consistent weather pattern at high altitudes during the storm.
Cross-correlations between $T_0$ and $\rho_0$, seen in the top-left of Figure 21, are again highly negative and constant, indicative of the ideal gas law and constant surface pressure. Unlike the dust-free case, cross-correlations between $T_{57}$ and $T_0$ are instead positive, with some statistical dependence. This phenomena can be explained by a dust storm formation mechanism described by Rafkin [11], where dust kicked up near the surface absorbs solar radiation, heating the adjacent air molecules. The heated, less dense air then rises, carrying the dust with it and creating a low pressure region near the surface. As pressure equalizes near the surface via winds, more dust is picked up into the air,
Figure 21: Dust storm cross-correlations along nominal downtrack

causing a positive feedback mechanism and a local storm to form. Since heat is carried from the lower atmosphere to the upper atmosphere by dusty air, a positive correlation between $T_{57}$ and $T_0$ can be seen in the storm data. This phenomenon is analogous to
hurricane formation on Earth.

The percent variation of the parameter means along the downtrack are plotted in Figure 22. The variability of the means is lower throughout the altitude profile compared to the dust-free case, suggesting the dust storm has a more predictable, albeit different structure along a projected downtrack.

![Figure 22: Percent deviation of the parameter means along the downtrack](image-url)
2.2.6 Mesoscale Variability vs. Atmospheric Uncertainty

It is important to distinguish between the variability from the mesoscale model and variability used in mission analysis to fully represent atmospheric uncertainty. Using only mesoscale variability for EDL analysis assumes nearly perfect atmospheric measurement and prediction capability on the order of Earth weather prediction. Since there are no full climatological models of the Martian atmosphere, this is not the case and additional uncertainty is added by Mars mission planners to account for mesoscale variability uncertainty and unpredictable annual and seasonal variability. Section 3.5 will address the implications of these issues, along with the effects of entry state uncertainty, in a complete analysis of EDL landing dispersions. Note for now the analysis in Section 2.2 has been based solely on mesoscale data.

2.3 Monte Carlo Simulation

An EDL simulation was built to conduct Monte Carlo analysis using the 4-parameter atmosphere and statistical properties of the mesoscale data to build random atmospheres. A ballistic entry architecture was assumed with events such as parachute deployment that represent previously flown missions. The six degree of freedom (6-DOF) equations of motion were restricted to eliminate crosstrack motion and several assumptions such as a constant gravity field and flat surface were assumed to simplify the dynamics. Aerodynamics for the entry body were modeled using data provided by NASA. The required covariance matrices for the correlated atmosphere were built and the atmosphere was tested for the ‘dust-free’ and ‘dust storm’ cases. A nominal trajectory was run in a uniform atmosphere to use as a basis of comparison for Monte Carlo results.
2.3.1 EDL Architecture

The concept of operations (ConOps) for the simulation used in this study can be seen in Figure 23, starting with atmospheric interface and ballistic deceleration and ending with lander touchdown. The phases shown were initiated and terminated using event triggers, so that relevant entry vehicle properties could be discretely altered to reflect each stage. The primary trigger used for parachute deployment was dynamic pressure, denoted $q_{\text{deploy}}$. Note the trigger was constrained to values where $\frac{dq_{\text{deploy}}}{dt}$ was negative to ensure the parachute was deployed after peak aerodynamic acceleration and not during initial descent. An alternative Mach number trigger of 1.8 was applied to altitudes below 5.5km to create a lower bound for deployment based on realistic system limitations.

![Figure 23: Concept of operations for EDL simulation](image)

Heat shield jettison was given a minimum Mach number trigger ($M_{\text{hsdeploy}}$) of 0.7 and
a required time of 20 seconds \( (t_{hs}) \) after parachute deployment. Backshell release was triggered at an altitude of 150 meters \( (h_{release}) \) and the powered constant rate of descent was triggered at either 10 meters altitude \( (h_{const}) \) or a speed of 5 m/s \( (V_{const}) \). Touchdown signaled the end of the trajectory and likewise the end of a simulation run. A summary of these events and triggers is provided in Table 1.

### Table 1: Event Triggers

<table>
<thead>
<tr>
<th>Event</th>
<th>Trigger</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parachute Deployment</td>
<td>( q_{\text{ddep}} \leq q_{\text{deploy}} ) or ( dq_{\text{ddep}}/dt \leq 0 ) or if ( h \leq 5.5 \text{km} ): ( M_{\text{ddep}} \leq M_{\text{deploy}} )</td>
<td>( q_{\text{deploy}} = 800 \text{ kg/m-s}^2 ) ( M_{\text{deploy}} = 1.8 )</td>
</tr>
<tr>
<td>Heat Shield Jettison</td>
<td>( M_{\text{ddep}} \leq M_{\text{ddeploy}} ) &amp; ( t_{hs} \geq 20 )</td>
<td>( M_{\text{ddeploy}} = 0.7 )</td>
</tr>
<tr>
<td>Backshell Release and Engine Start-up</td>
<td>( h \leq h_{\text{release}} )</td>
<td>( h_{\text{release}} = 150 \text{ m} )</td>
</tr>
<tr>
<td>Propulsive Descent</td>
<td>( h \leq h_{\text{const}} ) or ( V \leq V_{\text{const}} )</td>
<td>( h_{\text{const}} = 10 \text{ m} ) ( V_{\text{const}} = 5 \text{ m/s} )</td>
</tr>
<tr>
<td>Touchdown</td>
<td>( h = 0 )</td>
<td></td>
</tr>
</tbody>
</table>
2.3.2 Assumptions

The first major assumption made in the simulation software was to constrain the six degree of freedom (6-DOF) vehicle dynamics to ignore crosstrack motion in the trajectory. This was done by setting the state variable derivative for crosstrack velocity to zero. A flat non-rotating Mars was also assumed, given the relatively short distance traveled ($\sim 10^\circ$ across the surface) and short time duration during EDL ($\sim 6$-7 minutes). Gravitational acceleration was assumed to be constant with altitude and entry vehicle inertias throughout ballistic entry were assumed to be invariable.

Aerodynamic coefficients for the entry vehicle were calculated using a Mars Exploration Rover (MER) aerodynamic database provided by NASA and the implementation of these values in the aerodynamic model will be discussed further in Section 2.3.5. Entry states were assumed to have zero uncertainty except where specified, at which point an initial state covariance was introduced in the analysis.

2.3.3 Reference Frames

Three reference frames were used in the Monte Carlo simulation: the aerodynamic frame, body frame, and downrange inertial frame. Aerodynamic data was provided in the aerodynamic frame, entry vehicle angular rates were calculated in the body frame, and numerical integration of the equations of motion was performed in the downrange inertial frame. Figure 24 shows each reference frame and the rotation angles between them. The vehicle-carried frame was an intermediate used in the analysis but serves as a convenient reference.
2.3.4 Equations of Motion

The equations of motion in the simulation were solved in terms of downrange inertial state variables for position, velocity, and orientation. Quaternions were used to describe entry vehicle orientation and angular rates tracked the change in orientation. Note the force coefficients listed in the following equations of motion are in the downrange inertial frame and were calculated using the aerodynamic coefficients extracted from the MER database. Including the full set of state equations for completeness, the equations of motion are:

\[
\begin{bmatrix}
\dot{V}_x \\
\dot{V}_y \\
\dot{V}_z 
\end{bmatrix} = F_A \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}
\]  

(8)

\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{yx} & I_{yy} & -I_{yz} \\
-I_{zx} & -I_{zy} & I_{zz}
\end{bmatrix}^{-1} \left( F_A \begin{bmatrix} C_p \\ C_q \\ C_r \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{yx} & I_{yy} & -I_{yz} \\
-I_{zx} & -I_{zy} & I_{zz}
\end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right)
\]  

(9)
\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 \\
\dot{q}_4 \\
\end{bmatrix}
= \frac{1}{2} \begin{bmatrix}
0 & r & -q & p \\
-r & 0 & p & q \\
q & -p & 0 & r \\
-p & -q & -r & 0 \\
\end{bmatrix} \begin{bmatrix}
qu_1 \\
qu_2 \\
qu_3 \\
qu_4 \\
\end{bmatrix}
\] (10)

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z} \\
\end{bmatrix}
= \begin{bmatrix}
V_x \\
V_y \\
V_z \\
\end{bmatrix}
\] (11)

where \( F_A = \frac{1}{2} \rho v^2 S \).

### 2.3.5 Aerodynamic Model

A parameterized model was used to approximate aerodynamic coefficients based on the MER database provided by NASA. The database was constructed using wind-tunnel tests of the MER entry capsule geometry and provides total force and moment coefficients \( C_{AT}, C_{NT}, \) and \( C_{MT} \) at different total angle of attack (\( \alpha_T \)) and Mach number (\( M \)) conditions. Note the coefficients provided were based on \( \alpha_T \) and had to be converted to the orthogonal values \( C_A, C_N, \) and \( C_M \) using the clock angle \( \phi = \arctan(\frac{v}{w}) + \pi \). A least squares method using functions of \( \alpha_T \) and \( M \) formed the equations for approximating the database with a parameter-defined surface as seen in Figure 25. The parametric equations used to approximate the database can be found in Appendix B.

One further assumption for the ballistic phase had to be made based on early testing of the nominal trajectory. It is known that aerodynamic instabilities exist in the pitch stability coefficient \( C_{ma} \) during hypersonic flight in EDL for the 70° cone configuration [12], even at low angles of attack (\( \alpha \)); the instability can lead to sudden growth in
α oscillations that are large enough to exceed normal flight conditions, such as those from the NASA database parameterized by $\alpha_T$ and $M$ in this study. The parameter-defined surface used to approximate the data yielded a sufficient fit of the region defined by the database, but did not realistically represent the vehicle response outside of that region ($\alpha_T > 20$ deg).

As a result, when hypersonic instabilities drove the simulation to a region of high $\alpha_T$, it trimmed at an unrealistically high $\alpha_T$ based upon the parameterized surface, rather than returning to its original state. After researching possible solutions to the issue, it was determined the moment coefficient $C_m$ would be approximated simply as:
Parachute aerodynamics were modeled parametrically assuming zero force and moment coefficients other than $C_d$. The parametric equations, provided by NASA [13], were Mach number dependent and can be found in their entirety in Appendix B.

2.3.6 Atmospheric Model

A few different atmospheric models were used in this study to achieve specific goals. The nominal trajectory was run through a 4-parameter, hydrostatic atmosphere that was uniform along the downtrack, and served as the baseline for the rest of the investigation. The parameter values chosen for the nominal atmosphere were influenced by mesoscale data averages and tuned to yield a reasonable nominal trajectory given the simplifications and assumptions made when building the simulation. All nominal values are detailed in Section 2.3.7. Landing dispersion sensitivity to atmospheric parameter variations at different stages of EDL was determined by adding a randomly generated region to the nominal atmosphere associated with the stage of interest. These random regions use the initial condition covariance ($\Gamma_{IC}$) to generate a single random profile throughout the region of interest for each Monte Carlo run. The resulting landing dispersion sensitivity can thus be characterized as a deviation from the nominal due to variability in a particular atmospheric parameter.

Another model of the Mars atmosphere was used to better capture the variability characteristics in both dust-free and dust storm scenarios. The correlated atmospheric model generates a random, 4-parameter profile as an initial condition at the atmospheric entry interface ground track location, using $\Gamma_{IC}$. Downtrack atmospheres are randomly
generated using $\Gamma_{DT}$ and the multiple correlation method for calculating conditional probability. The correlation matrices $\Sigma_{IC}$ and $\Sigma_{DT}$ for the dust-free case and $\Sigma_{IC,\text{storm}}$ and $\Sigma_{DT,\text{storm}}$ for the dust storm case are shown below for reference.

$$\Sigma_{IC} = \begin{bmatrix} 1.0 & -0.9 & 0.1 \\ -0.9 & 1.0 & -0.25 \\ 0.1 & -0.25 & 1.0 \end{bmatrix} \quad \Sigma_{DT} = \begin{bmatrix} 0.99 & -0.9 & 0.1 \\ -0.9 & 0.99 & -0.2 \\ 0.1 & -0.3 & 0.85 \end{bmatrix}$$

(13)

$$\Sigma_{IC,\text{storm}} = \begin{bmatrix} 1.0 & -0.95 & -0.5 \\ -0.95 & 1.0 & 0.4 \\ -0.5 & 0.4 & 1.0 \end{bmatrix} \quad \Sigma_{DT,\text{storm}} = \begin{bmatrix} 0.99 & -0.95 & -0.5 \\ -0.95 & 0.99 & 0.4 \\ -0.5 & 0.4 & 0.95 \end{bmatrix}$$

Note that initial condition diagonal correlations were assumed to be 1, while initial condition cross-correlations were taken from the data sample. The parameter values at entry interface serve as initial conditions to calculate the next downtrack atmosphere ‘patch’ and this process is repeated until the full atmosphere is generated, as seen in Figure 26. Appendix A details the multiple correlation method used to calculate the conditional probability in the model.

Spacing for patches within the correlated atmosphere was chosen based on the highest controllable signal frequency for MSL. According to Wilson [14] this frequency is 0.2Hz, translating to roughly 30km measurement intervals if a 6km/s entry velocity is assumed. Based on this analysis, a conservative value of 50km was chosen for the size of correlated atmosphere patches. Figure 27 compares the density and temperature structure of random profiles from a correlated atmosphere against the nominal atmosphere.
Random Number

\[ \mu_i \Gamma_{IC} \]

Atmosphere at entry:
\[ \rho_{0,i}, T_{0,i}, T_{57,i} \]

\[ \mu_{i+1} (\rho_{0,i}, T_{0,i}, T_{57,i}) \]

Next ‘patch’ Atmosphere
\[ \rho_{0,i+1}, T_{0,i+1}, T_{57,i+1} \]

\[ \Gamma_{DT} \]

Figure 26: Flow chart for generation of the correlated atmosphere

Figure 27: Correlated atmosphere vs. nominal density and temperature profiles

Each profile shown is for one 50km ‘patch’ of atmosphere along the downtrack. The change in variability with altitude has a similar ‘tornado’ structure in both the density and temperature plots, with a fairly symmetric shape at values above and below the
nominal profile. Note the percent density variation is larger at the surface and higher altitudes in comparison to the percent temperature variation. The covariance matrices included in the atmospheric model introduce the variability to both the temperature and density profiles along the trajectory. This provides the ability to simulate weather patterns in the dust-free and dust storm environments, as opposed to representing them with random uniform atmospheres.

The variation of the mean over the 18 sampled days in the dust-free and dust storm scenarios was used to model the change in average temperature and density along the downtrack for the local dust storm. An average percent difference was calculated for the atmospheric parameters and statistically incorporated in the atmospheric model as a dust storm bias. Figure 28 shows plots of density and temperature percent difference from the correlated dust-free atmosphere mean, for locations along the downtrack, in both the dust-free and dust storm scenarios. Each set of profiles represents the conditions above ground track locations for one atmosphere and the referenced mean is an average profile of the dust-free case depicted in the figure.

The plots in Figure 28 show a dust storm atmosphere with a lower density near the surface and at higher altitudes when compared to dust-free, but a higher density at intermediate altitudes, roughly corresponding to a 20-40km range. The surface density bias can be seen clearly in the top plot, as the dust storm values are roughly 4-5% lower than the dust-free mean at the surface. The $T_0$ and $T_{57}$ biases are seen in the bottom plot, as dust storm temperatures are 2-4% higher at the surface, and up to 14% lower at 57km. The percent deviation in the dust storm temperature structure results in a different temperature lapse rate, which in turn affects the density profile. This is apparent in Figure 28 as the dust storm density and temperature percent deviation curves
both crossover the mean at roughly 20km altitude. A discussion of the density deviation crossover point and its affect on landing dispersion can be found in Section 3.4.

Again, the models shown use standard deviations based on mesoscale variation alone, assuming no uncertainty in the mesoscale variability and no prediction error. Additional variability and an entry covariance were added in later analyses of the trajectory and the implications are discussed.

2.3.7 Nominal Outputs

The key outputs from the simulation are the trajectory states, more specifically position and velocity. Given landing dispersion is a common method for characterizing EDL variability, position at touchdown was considered the most important output. Before conducting Monte Carlo analysis with the simulation, a nominal trajectory based on

Figure 28: Percent deviation of sample atmospheres from the nominal
MER was used to verify the program models and fine tune the code. The results from the nominal trajectory are shown in Figure 29.

Starting at an altitude of 125km, the vehicle takes approximately 340 seconds to reach touchdown at a downrange distance of around 642km from atmospheric entry. Velocity increases during the start of the ballistic phase in the upper atmosphere, but begins to drop off quickly at around a 50km altitude. Maximum dynamic pressure is reached at roughly 30km, which corresponds to 100 seconds after entry. A discontinuity can be seen in the altitude vs. velocity plot near the landing site where parachute deployment occurs and aerodynamic deceleration increases.

The angle of attack and pitch rate, as seen in Figure 30, oscillate throughout the
ballistic phase, with the magnitude of those oscillations increasing as the dynamic pressure drops off from its maximum value at around 100 seconds. Oscillations in $\alpha_T$ and the angular rates cease at around 175 seconds where parachute deployment occurs and a simple drag model for parachute dynamics is introduced.

Figure 30: Nominal trajectory results: angular rates and $\alpha_T$

The EDL simulation developed was simplified to ease interpretation of the results and to allow for a broad examination of the effects of atmospheric variability. A 6-DOF simulation with less simplifications would be used to study these phenomena for the purposes of mission analysis and onboard navigation development.
3 Results

The results are based upon Monte Carlo analysis of the variable atmospheric inputs and their effect on landing accuracy and other phases of entry, descent, and landing (EDL). The Monte Carlo runs use 1,000 cases to form the results presented. Landing dispersion sensitivities to uncertainty in atmospheric parameters from different phases of EDL are examined to determine which measurements are most critical to landing accuracy. The bias effect of the simulated dust storm on landing dispersion is discussed, as well as the implications of adding additional engineering uncertainty beyond what is seen in the mesoscale model. Entry state uncertainty is applied in the form of a state covariance matrix to perform a basic mission analysis using the EDL simulator and correlated atmosphere; this was done to compare the landing dispersion results to previous work and gauge the impact of local storms compared to other sources of dispersion. The results from this study are compared to MER and Phoenix reconstructions and discussed in terms of persistent overshoot.

3.1 Uniform Atmosphere

Monte Carlo analysis using random atmospheres that are uniform along the trajectory serves as a basis of comparison for dispersion characteristics and provides some insight into the effect of total variance, or model error, on landing accuracy. Standard deviations based on mesoscale data for the atmospheric parameters are listed in Table 2 and were used to generate both uniform and correlated random atmospheres for Monte Carlo analysis.

A downtrack landing dispersion histogram for the random uniform atmospheres is
shown in Figure 31. The mean for the dispersion (642.1 km) matches the downtrack landing location of the nominal trajectory and the distribution is normal with a standard deviation of 1.3 km. The value of sigma is small compared to other EDL analyses, but this is expected, given the variability in the model is based only on mesoscale variations and not on an applied engineering uncertainty or entry covariance. Section 3.5 will discuss the result of adding additional variability to the model to simulate ‘engineering uncertainty’ and the landing dispersions commonly seen in mission analysis studies.
3.2 Correlated Atmosphere and Density Reconstruction

Given the covariance matrix for the trajectory downtrack ($\Gamma_{DT}$), Monte Carlo analysis was performed on random correlated atmospheres to examine the landing dispersions and test the hydrostatic assumption along the trajectory. Figure 32 shows the dispersion histogram for the correlated atmosphere. The mean is 0.1 km downtrack of the nominal trajectory with a sigma of 1.5 km, slightly larger than the random uniform atmospheres. The distribution is normal but slightly skewed towards downtrack landings when compared to the uniform case.

Although it is certain that the hydrostatic assumption holds for each downtrack atmospheric profile in the simulation, this is not necessarily the case for the atmosphere
Figure 32: Landing dispersions: correlated atmosphere

experienced along the trajectory. To test whether the assumption is held, density was reconstructed along the trajectory and compared to a hydrostatic fit of the values for the calculated parameters $\rho_0$, $T_0$, and $T_{57}$. Density was reconstructed from vehicle dynamics using the following expression:

$$\rho = \frac{2ma_x}{V^2SC_x}$$  \hspace{1cm} (14)$$

where $m$ is vehicle mass, $a_x$ is the inertial acceleration in the downtrack direction, $V$ is the magnitude of the vehicle velocity, $S$ is the cross-sectional area, and $C_x$ is the inertial force coefficient in the downtrack direction. As a basis of comparison, the density was reconstructed for both a uniform and correlated atmosphere. Figure 33 shows the reconstructions along with their hydrostatic fit.
Figure 33: Trajectory reconstruction through simulated atmosphere

As expected, the hydrostatic fit is an exact match for the uniform atmosphere trajectory reconstruction. The correlated atmosphere reconstruction is well predicted by the hydrostatic fit except at high altitudes where there is some noticeable error. Recall these plots are based on only two temperature measurements, one at the surface and one at 57km, which provide a single temperature lapse rate. Given intermediate temperature measurements between the surface and 57km altitude, a more accurate density profile could be constructed using the changing lapse rates.

Assuming some knowledge of atmospheric temperature or pressure, the hydrostatic relationship is a valid means for on-board entry vehicle systems to predict downtrack density given current temperature measurements as it enters the upper atmosphere. Note, the argument holds given the assumptions of the simulation and could be used to validate the hydrostatic assumption in actual onboard systems if the correlated atmosphere can be proven to behave like the actual Martian atmosphere with enough certainty.
### 3.3 Dispersion Sensitivities: Dust-Free

Landing dispersion sensitivity to variability within each EDL phase provides an understanding of which entry phase has the largest effect on targeting error. To examine EDL phase based sensitivities, a uniform atmosphere was generated and a single atmospheric parameter was varied within a given phase of the architecture to analyze the impact of its variability on landing dispersions. Figure 34 shows how the EDL trajectory was divided into four phases of interest based on the nominal output: hypersonic upper atmosphere (50 – 125 km), hypersonic max dynamic pressure (20 – 50 km), supersonic ballistic (8 – 20 km), and parachute deploy and propulsive landing (0 – 8 km).

![Figure 34: EDL Phases for Sensitivity Analysis](image)

The hypersonic region was broken into two phases to account for relative density.
changes resulting in higher dynamic pressures and deceleration at lower altitudes. Supersonic ballistic and parachute deployment were separated to account for changing vehicle aerodynamics due to event triggers. A 2% variability was applied to one nominal atmospheric parameter in a given phase and a Monte Carlo analysis was performed. This process was repeated for each of the atmospheric parameters in each of the four phases. Figure 35, Figure 36, and Figure 37 show the results.

![Figure 35: Downrange $\rho_0$ sensitivity by EDL phase](image)

It is immediately clear from a comparison of the dispersion variances that a change in $T_0$ during the hypersonic max dynamic pressure phase, seen in the top-right plot of Figure 36 has the largest effect on landing dispersion. This result makes sense, as $T_0$
dictates low altitude temperature lapse rate and thus the density rate of change from the surface. Landing dispersion due to $\rho_0$ in the same phase, seen in the top-right of Figure 35, has the second largest effect, as a change in density at the surface with a constant lapse rate will shift the density profile to either higher or lower values. Note the two largest sensitivities are for parameters in the region of maximum dynamic pressure. Since the majority of aerodynamic deceleration occurs where dynamic pressure is the highest, it was expected that a density change in this region of the atmosphere would have a greater effect on landing accuracy.
It is clear the lowest sensitivities occur for parameters in the parachute deployment and propulsive landing phase, which are seen in the bottom-right of Figures 35, 36, and 37 and those associated with $T_{57}$ seen in Figure 37. The low speeds, steep descent angle, and thus short distance traveled after parachute deployment attribute to the low dispersion sensitivities, as any density or temperature variation will thus have less impact on horizontal velocity. Note sensitivity to wind uncertainty in the parachute deployment and propulsive landing phase would have been more significant, but wind variability was not included in this study. Since variability in $T_{57}$ influences density at higher altitudes
where the atmosphere is thinner and less deceleration occurs, it has a lower effect on landing precision.

Intermediate sensitivities exist for $\rho_0$ and $T_0$ during the supersonic phase, seen in the bottom-left of Figures 35 and 36, and for $\rho_0$ and $T_0$ during hypersonic upper atmosphere flight, seen in the top-left of Figures 35 and 36. Supersonic atmospheric sensitivities are lower than the region of maximum dynamic pressure simply because the velocity experienced is lower, reducing vehicle deceleration. The effect of $T_0$ is somewhat substantial because of the constant percent variation used to test parameter effects in the analysis. One would expect the effect of $T_{57}$ to be larger at higher altitudes and this is in fact the case when talking about absolute temperature variation in Kelvin. Since this analysis varies both values by 2% the lower magnitude of $T_{57}$ results in a lower impact. In addition, absolute daily variability of $T_{57}$ is slightly higher than $T_0$ so in this sense its expected variability would have a larger impact on landing dispersion for the hypersonic upper atmosphere phase.

### 3.4 Dust Storm Landing Dispersion Bias

Based on statistical analysis of the dust-free and dust storm mesoscale data, it was evident that not only the downtrack correlations were different, but also the mean atmospheric profile. A mean atmospheric profile was calculated for the dust-free and dust storm atmospheres to determine an initial condition bias for the Monte Carlo simulation of the dust storm correlated atmosphere. Table 3 shows the bias values that were applied to the parameters based on the analysis and Figure 38 shows the Monte Carlo landing dispersion.

Two features of the dispersion are noteworthy when compared to the results for
Table 3: Dust storm mean value biases

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean value bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$</td>
<td>-1.5% (-2.58E-4 kg/m$^3$)</td>
</tr>
<tr>
<td>$T_0$</td>
<td>+1.49 K</td>
</tr>
<tr>
<td>$T_{57}$</td>
<td>-1.26 K</td>
</tr>
</tbody>
</table>

Figure 38: Landing dispersions: dust storm atmosphere

correlated, random atmospheres with no dust. First, the variability is unchanged from the dust-free case. Based on the assumptions made in this study, we can say the different downtrack correlations in the dust storm case have very little effect on variability. It is therefore the variance of the atmospheric parameters that has the largest impact on dispersion variability.
Secondly, the mean of the dispersion is 0.4 km downrange of the dust-free case. This result is a direct consequence of the initial condition bias applied for the dust storm data; an understanding of the mechanisms driving the overshoot in the dispersion mean can be derived from an analysis of density percent difference between average atmospheric profiles for dust-free and dust storm scenarios. Figure 39 shows two plots, each with a comparison of randomly generated atmospheres along a trajectory and their deviation from the dust-free average.

![Dust storm density and temperature deviation: increasing $\mu_{T_0}$](image)

The left-hand plot, created using the dust storm bias mentioned in Section 3.3, shows a dust storm having higher density between roughly 15 and 30 km altitude, with lower comparable density elsewhere along the profile. The right-hand plot has a slightly higher $T_0$ bias and a more negative $\rho_0$ bias; as a result the dust storm has a higher density between 25 and 40 km altitude, with lower density at other points in the profile. Since most of the aerodynamic deceleration and highest atmospheric parameter sensitivities on landing accuracy occur in the 20-50 km altitude band, it follows that the overshoot bias would be less for the atmosphere on the right; this is indeed the case as seen in Figure 40, which
shows downrange landing dispersions based on the dust storm atmospheres in Figure 39. The distribution shapes are very similar, but the mean downrange landing location for the left-hand plot is 642.6km, while the right-hand plot has a mean of 642.3km. Each of these values indicate an overshoot bias when compared to the nominal landing location of 642.1km downrange. Thus, the plot on the right has a lower overshoot, as expected from the atmosphere with higher density in the 20-50km altitude range.

![Figure 40: Dust storm landing dispersion comparison: increasing $\mu T_0$](image)

Note the percent difference profiles in Figure 39 are normalized around the dust-free mean to more clearly show the percent difference in density. If the dust-free and dust storm atmospheres were averaged and each average profile compared to the nominal profile, the difference would be very subtle and hard to observe on a plot.

A case study was performed to understand the sources of varying dust storm structure and how it impacts the altitudes at which dust storm density transitions from being lower than dust-free to greater than dust-free and vice versa; this altitude is referred to as the dust density percent difference crossover point, or simply crossover point, in the discussion. Using a script provided by Dan Tyler to generate atmospheres that mimic the
mesoscale model, temperature profiles and surface pressure were varied independently to
determine how they impact the crossover point. Recall that a higher density in the 20-50
km range would likely yield an undershoot, while a lower density in that range would
yield an overshoot.

Figure 41 shows a set of three atmospheres, in which simulated dust storms are com-
pared against dust-free nominal atmospheres to investigate the factors that affect the
crossover point. The dust storm atmospheres in Figure 41 were created by introducing a
different temperature profile while holding surface pressure ($P_0$) constant. The temper-
ature profiles in the first two plots (referenced from top-left going clockwise) are taken
directly from the mesoscale model while the last plot uses smoothed curves. Note that
temperature lapse rate below 30km decreases for each atmosphere starting in the top-left
and going clockwise.

It is clear from these plots that the difference in temperature lapse rate between
the ‘dusty’ and ‘nominal’ cases and the density difference at the surface determine the
location of the crossover point on the percentage density difference plot. If the dust storm
temperature lapse rate is less than the nominal, the rate of change for the percentage
density difference is negative at that altitude and vice versa for a positive lapse rate.
The rate of change, or derivative, referred to in this case is with respect to an increase
in altitude.
Figure 41: Effect of varying temperature lapse rate on crossover point

Figure 42 shows two atmospheres in which the prescribed temperature profiles are held constant while $P_0$ is decreased from the first to the second ‘dusty’ profile. In the bottom plot, the density percent difference profile is shifted to the left as a result of decreasing $P_0$, while the slope remains constant. This is an expected result based on the application of the ideal gas law at the surface and the fact that temperature lapse rates are left unchanged.

Based on this case study, temperature lapse rate and surface pressure are reasonable indicators of how a dust storm will bias the landing dispersion of an entry trajectory.
Figure 42: Effect of varying surface pressure on crossover point

Once again, this analysis assumed the hydrostatic relationship was a reasonable predictor of density given a surface pressure and temperature profile.
3.5 Effects of Adding Additional Uncertainty and Entry Covariance

Upon first glance, the Monte Carlo landing dispersions that have thus far been discussed might appear too small when compared to mission analysis studies of other trajectories. The 3-σ landing ellipse major axis from MER mission analysis was 80 kilometers, whereas downtrack dispersions to this point in the study have been around 10km. The dispersions have been small because atmospheric uncertainty from the mesoscale data has been the only variability included; this is synonymous to assuming no uncertainty in mesoscale variability, advanced knowledge of Mars weather cycles, and perfect knowledge of the atmospheric entry states. In mission analysis, an entry state covariance is calculated based upon orbit determination for the last five days prior to Mars entry. Uncertainty in model variability and weather cycle prediction are taken into account by applying additional engineering uncertainty to the atmosphere, in the absence of a climatological model for annual and seasonal variation. For example, roughly 10% additional engineering uncertainty was applied to mesoscale model variability while performing landing site selection analysis for MSL. One question this raises is the accuracy of the statistical distributions used and whether they truly represent the variability of the atmosphere.

To properly compare results from the EDL simulator used in this study to previous mission analyses, a set of random states at Mars B-Plane intersection were provided by NASA to create an entry covariance matrix and an additional 10% variability was added to the atmosphere to represent the engineering uncertainty normally applied. Note the random states were provided in the Mars Mean Equator coordinate system at around 9 minutes from entry interface; as a result, they had to be propagated to entry interface
and converted to the downrange inertial coordinate system for the simulation to calculate the entry state covariance matrix. A Monte Carlo analysis was performed using the entry state covariance and an invariant uniform atmosphere to better understand the contribution of entry uncertainty to landing dispersion. Figure 43 shows the results of this analysis, where entry uncertainty yielded a 20km downrange dispersion, or a standard deviation of 4.1km, which is much larger than the mesoscale variation.

![Landing dispersion for entry uncertainty in a uniform atmosphere](image)

**Figure 43:** Landing dispersion for entry uncertainty in a uniform atmosphere

A Monte Carlo analysis with additional engineering uncertainty added to the mesoscale atmosphere was also performed assuming zero uncertainty in the entry states. Recall this study has thus far assumed that mesoscale model variability is an accurate representation of atmospheric uncertainty. Figure 44 shows the landing dispersion with the additional uncertainty applied to the atmosphere, representing an equivalent result to Mars EDL
mission analysis common practice. Since space agencies are unable to predict day-to-day conditions due to a lack of data, the artificial engineering uncertainty applied to the EDL analysis is a conservative representation of these unknowns.

Figure 44: Landing dispersion engineering atmospheric uncertainty included

A comparison of Figure 43 to Figure 44 provides a relative understanding of how entry errors and atmospheric errors effect the magnitude of the landing dispersion. Based on the comparison, atmospheric uncertainty has a larger effect than the entry covariance, but only because of the engineering variability applied. This suggests that a climatological model of Mars would greatly improve landing precision for future missions, even more so than higher fidelity entry state determination. The ability to predict the atmosphere over long time spans (seasonal/annual) will be important for mission planners and over shorter time spans (∼ 5-10 days) for onboard trajectory correction.
3.6 A Comparison to MER and Phoenix Reconstructions

Although previous missions such as MER and Phoenix have landed within the predicted landing ellipse, they have consistently touched down beyond the expected mean location. The consistent overshoot for previous NASA missions has been identified as a potential bias in EDL analysis by Desai [7] and it was earlier suggested the root cause was the over-prediction of density. Even with updated atmospheric measurements a few days prior to landing, Figure 45 shows the predicted density to be at least 10% greater than the reconstructed atmosphere from onboard measurements taken during entry [7]. Note the density profile denoted 'Dec 27' is based upon measurements taken only a few days before entry and 'tau=1' is a predicted worst case high dust content atmosphere.

As previously stated, recent studies have shown that angle of attack (α) was larger than expected during entry, causing a subsequent increase in lift and side force. Although increased lift has been identified as a major contributor to landing overshoot and under-
prediction of density from the entry trajectory reconstruction, the contribution of the atmosphere is still worth investigating for future missions. This is especially the case since weather phenomena such as local dust storms were not included in the analysis.

Local dust storms are an important atmospheric phenomena to study, as they can form very quickly and influence a density profile prediction taken only a few days before entry. In terms of the hydrostatic relationship, if $T_0$ increases as we have seen thus far in dust storm data, $\rho_0$ should decrease, shifting the density profile to lower values. Clearly, there are more complex mechanisms at work, so in this study the dust storms from the mesoscale model were used to run a Monte Carlo analysis with full entry covariance and additional engineering uncertainty in the atmosphere to look for an overshoot. Figure 46 shows the results of the analysis, in which the mean overshoots the nominal trajectory by 0.7 km. Compared to the dust-free analysis, seen in Figure 47, the dust storm mean only overshoots the dust-free mean by 0.2 km. Note the overshoot in the dust-free case is due to dynamic pressure and Mach number bounds for parachute deployment, which are based on real-world system limitations.
Figure 46: Dust storm dispersion: entry and engineering uncertainty included

Figure 47: Dust-free dispersion: entry and engineering uncertainty included
Based upon these results, local dust storms cause a slight overshoot bias, but it seems unlikely that they are a significant contributor to the large overshoot seen in previous Mars missions. Collecting more atmospheric data at altitudes below 50km would provide a better understanding of the physical mechanisms that drive dust storms and other local weather and clarify the true effects of these phenomena on Mars landing accuracy. More specifically, measurements of local dust storm properties over their life cycle would improve model fidelity and help to differentiate their structure and effects on EDL from those of global storms and other atmospheric phenomena.
4 Conclusions

Understanding the Martian atmosphere is important for the planning and execution of future missions that require precision landing on the surface of the red planet. This study sought to understand how uncertainty in the atmosphere affects the EDL process in terms of both mission planning and onboard operations. Mesoscale model data was used to build a statistically correlated atmosphere based on the hydrostatic assumption that represents trends in spatial variability for different weather conditions such as dust-free and local dust storms.

A sensitivity analysis was performed to determine which atmospheric parameters have the greatest effect on final landing dispersion and thus should be measured with the highest precision. Dust storm data was used to simulate the effects of local dust storms on the EDL process with the intent of addressing the consistent overshoot in previous landed missions such as MER and Phoenix. Additional engineering uncertainty was added to the atmosphere to simulate variability uncertainty and an entry covariance was applied to the states at entry interface to simulate orbit insertion errors. These other mission uncertainties were applied to a Monte Carlo simulation to compare total landing dispersions to previous mission analyses and investigate the possible causes of consistent landing overshoot.

4.1 The Atmosphere and EDL

Statistical analysis of the mesoscale model yielded some interesting insight into how the atmosphere varies spatially with time. It was found that surface density and temperature ($\rho_0$ and $T_0$) were highly correlated over the ground track of a typical entry trajectory while
temperature in the upper atmosphere \( (T_{57}) \) exhibited a lower correlation. If variability in the mesoscale model is representative of Martian atmosphere variability this would suggest it is highly predictable along the trajectory ground track. If this can be proven, a lower spatial resolution would be required on future measurement campaigns and an architecture could be designed around improving vertical and temporal data collection. Future missions should also focus on measuring the statistical distribution of atmospheric parameters over time to improve modeling and determine if the Gaussian assumption is valid or introduces an unwanted landing bias in EDL analysis.

Sensitivity analysis of the atmospheric parameters throughout the stages of EDL showed that \( \rho_0 \) and \( T_0 \) variability in the region of maximum dynamic pressure have the largest impact on landing dispersion, followed by \( \rho_0 \) and \( T_0 \) in the supersonic phase. Variability of \( T_{57} \) had little effect in all phases except the hypersonic upper atmosphere. This suggests future robotic campaigns should focus their precision measurements on the region of expected maximum dynamic pressure, which in the case of ballistic entry proved to be between 20-50km altitude. Roughly speaking, a 2% error in measurement uncertainty near maximum dynamic pressure translates to a \( 1-\sigma \) variance of roughly 1km. Dust storm bias and correlation yielded a slight overshoot in Monte Carlo analysis with variability based solely on the mesoscale data.

### 4.2 EDL and Overshoot

It has been suggested in previous literature that the landing location overshoot experienced by previous Mars missions was due to inaccurate density predictions, as compared to the reconstructions created from entry dynamics data. Given the predictions were based on measurements taken only a few days before entry and that mesoscale model
correlations showed high day-to-day correlations, there were two possible scenarios; either the vehicle dynamics were incorrectly predicted or the atmospheric variability was not being properly modeled.

Previous studies have showed that angle of attack was larger than expected for MER and Phoenix, which introduced a significant overshoot bias in the landing dispersion. This study looked at the contribution of mesoscale model derived local dust storms in a correlated atmosphere to overshoot bias. Results showed a small influence on overshoot due to the simulated storms that was significantly less than the error in angle of attack prediction. Based on this study, local dust storms as represented by current mesoscale models do not have a large impact on ballistic entry trajectories. They may, however, be shown to contribute to overshoot on future architectures that experience more aero-dynamic deceleration below 40km altitude or if current mesoscale models are updated with atmospheric measurements that show a larger impact on density, temperature, and pressure. The largest contributor to landing uncertainty was the engineering uncertainty, applied to increase the variability of the mesoscale atmosphere, in accordance with current mission planning practice. EDL is therefore currently an engineering problem that can be solved by diligently studying the Martian atmosphere to reduce uncertainty in model variability.

It should also be noted if the Mars atmosphere has high spatial correlations as the mesoscale model suggests, this would significantly aid a controlled precision landing. On-board computers would be able to predict density along the trajectory and its downtrack to a fairly high degree of certainty and thus reduce controller error during maneuvers intended to target a landing site. MSL will be the first mission featuring roll control based on entry vehicle bank angle and could help put these results to the test.
4.3 Future Work

It will be very important to characterize atmospheric uncertainty for future missions, especially with the requirement of precision landing for transportation of humans to the surface. Climatological studies with a coarse spatial resolution should be performed to determine if the relatively constant surface parameter correlations observed in the mesoscale model are representative of the Martian atmosphere; the data could then be used to determine if the Mars atmosphere is in fact highly predictable. In addition to general atmospheric measurements, data should be collected on local storms to further improve modeling and gain a better understanding of their life cycle.

Architecture concepts for future missions should be tested in an atmospheric model that simulates both global and local weather phenomena to determine if local dust storms have a greater impact on their landing accuracy. It is expected that lifting architectures and high mass entry vehicles would be more sensitive to the effects of local storms due to increased aerodynamic deceleration, but the magnitude of this effect should be investigated.
References


[7] Prasun N. Desai. All Recent Mars Landers Have Landed Downrange - Are Mars Atmosphere Models Mis-Predicting Density? In *Mars Atmosphere: Modeling and


Appendices
Appendix A: Multiple Correlation Method

For a partitioned vector $W$:

$$W = \begin{bmatrix} Y \\ X \end{bmatrix}$$  \hspace{1cm} (15)$$

where the scalar $Y$ represents the dependent variable. The vector $X$ [3x1] represents the independent variables or in the correlated atmosphere, the initial condition. Assume the partitioned variable $W$ [4x1] has the multinormal density:

$$f(w) = \frac{1}{(2\pi)^{\frac{1}{2}(p+1)}|\Sigma|^{\frac{1}{2}}} \exp[-\frac{1}{2}(w - \mu)'\Sigma^{-1}(w - \mu)]$$  \hspace{1cm} (16)$$

The mean vector and covariance matrix in partitioned form are given as:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12}' \\ \sigma_{12} & \Sigma_{22} \end{bmatrix}$$  \hspace{1cm} (17)$$

Where $\mu_1$ is the scalar mean of the dependent variable and $\mu_2$ [3x1] contains the independent variable means. The covariance matrix $\Sigma$ [4x4] contains $\sigma_{11}$ the scalar variance of the dependent variable, $\sigma_{12}$ [3x1] the covariance of $Y$ with the elements of $X$, and $\Sigma_{22}$ [3x3] the covariance matrix of $X$. The conditional density of $Y$ for $X = x$ is represented as:

$$g(y|x) = \frac{f(y, x)}{f_1(x)}$$  \hspace{1cm} (18)$$

where $f(y, x)$ is the probability density of the partitioned vector and $f_1(x)$ is the marginal density of the variate $X$. We can then write the conditional probability as:
\[ g(y, x) = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_{11.2}^{\frac{1}{2}}} \exp \left[ \frac{-\frac{1}{2}(y - \mu_1 - \beta_1'(x - \mu_2))^2}{\sigma_{11.2}} \right] \] (19)

where \( \beta_1 = \Sigma_{22}^{-1} \sigma_{12} \) [3x1] and \( \sigma_{11.2} = \sigma_{11} - \sigma'_{12} \Sigma_{22}^{-1} \sigma_{12} \). Note \( \sigma_{11.2} \) is a scalar quantity.

Based on this density function, the mean is:

\[ E(Y|x) = \mu_1 + \beta_1'(x - \mu_2) \] (20)

with a variance of:

\[ Var(Y|x) = \sigma_{11.2} = \sigma_{11} - \sigma'_{12} \Sigma_{22}^{-1} \sigma_{12} \] (21)

both of which are scalar quantities [15].
Appendix B: Parametric Equations for Aerodynamic Approximation

MER entry capsule aerodynamics:

\[ C_A = 1.535 + 0.013M + 3.814sin(\alpha_T) - 3.822\alpha_T + 20.772sin^2(\alpha_T) - 19.578\alpha_T^2. \]
\[ C_N = -5.980 + 6.949E^{-4}M + 0.786sin(\alpha_T) - 0.678\alpha_T + 5.978cos(\alpha_T) + 3.126sin^2(\alpha_T) \]
\[ C_M = 49.330 - 4.448E^{-5}M + 15.958sin(\alpha_T) - 16.094\alpha_T - 49.331cos(\alpha_T) - 24.339sin^2(\alpha_T) \]

(22)

Parachute aerodynamics:

\[ C_d(M < 0.6) = 0.613 \]
\[ C_d(0.6 \leq M < 0.95) = 4.47319M^3 - 12.01215M^2 + 9.75865M - 1.88447 \]
\[ C_d(0.95 \leq M < 1.3) = -6.5432M^4 + 36.9234M^3 - 77.4799M^2 + 71.7326M - 24.1678 \]
\[ C_d(1.3 \leq M \leq 2.6) = 0.052378M^3 - 0.388579M^2 + 0.742769M + 0.151879 \]

(23)

Note \( C_d \) for parachute aerodynamics above Mach 2.6 is undefined.