Abstract

BALU, ADITYA SRINIVAS. A Superposition Theory for Efficient Computation of Approximate Lift Distributions on Wind Turbine Rotor Blades. (Under the direction of Dr. Ashok Gopalarathnam.)

Recent work at NC State has shown how superposition of additional and basic lift distributions, applied to aircraft wings, can be used to advantage in optimization of adaptive wings and static aeroelasticity. In this thesis, the theory of superposition of basic and additional lift distributions, which is well-known for wings, is applied to wind turbine rotor blades. The theory of superposition, applied to wind turbine rotor blades, enables efficient and rapid calculation of approximate loadings on the blades at any condition. Because loading on the blade is described using a simple, linear superposition of pre-computed lift distributions, the effect of changes in pitch, wind speed, rotor rpm, blade twist, etc. can all be determined very quickly. Thus, the approach can be used in closed-form solutions of problems in design optimization, aeroelasticity, and optimization of smart blades (with flaps or other effectors). Because superposition is expressed as a matrix equation, the approach allows for easy coupling between aerodynamics and structures for static aeroelasticity problems.
A Superposition Theory For Efficient Computation Of Approximate Lift Distributions On Wind Turbine Rotor Blades

by

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Biography

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Nomenclature

$\alpha$  angle of attack

$\alpha_i$  induced angle

$\delta_f$  flap angle

$\delta_t$  change in twist

$\Gamma$  spanwise distribution of bound circulation

$\Gamma_a$  additional spanwise distribution of bound circulation

$\Gamma_b$  basic spanwise distribution of bound circulation

$\Omega$  angular velocity

$\rho$  air density

$a$  axial induction factor

$a'$  tangential induction factor

$B$  number of blades

$b$  wing-span

$c$  chord distribution
$C_d$  coefficient of drag for the airfoil

$C_L$  total configuration lift coefficient

$C_l$  sectional lift coefficient

$C_T$  rotor coefficient of thrust

$C_t$  sectional coefficient of thrust

$C_{D,ind}$  total induced drag coefficient

$C_{la}$  coefficient of lift distribution due to additional loading

$C_{lb}$  coefficient of lift distribution due to basic loading

$C_{tang}$  coefficient of the tangential force component

$dM$  elemental moment

$L'$  lift per unit length

$P$  power

$r$  radial location

$r/R$  non-dimensional radius

$t$  blade twist

$TSR$  tip-speed ratio

$u$  free stream velocity

$V_0$  wind speed

$V_\infty$  free stream velocity far upstream
$V_{rel}$ relative wind velocity
Chapter 1

Introduction

1.1 Historical Importance

The energy stored in the wind is tremendous and this potential source of free energy was to be tapped sooner or later. The sheer power of a gust of wind is felt during a storm or cyclone. Wind as a source of energy has been used for thousands of years. Initially used for various agricultural applications, such as for water extraction from wells, the modern wind mill has come a long way in its development and use. In the modern era, wind is primarily used for electricity generation.

As described by author Thomas E. Kissell [1], there is plenty of evidence to prove that wind energy has been used independently in different parts of the world by ancient humans. They used this wind power to propel ships to explore new lands in different parts of the Earth. Explorers undertook long and dangerous missions over large expanses of water with the help of the wind. Ancient Persians dating back to 200 BC, used wind power for pumping water and grinding grain. By the fourteenth century, the Dutch on the other hand, harnessed this energy to pump out water from low-lying areas to prevent flooding and increase areas for farm lands. It was only later, around the mid 1800’s, that the windmill was first used by
U.S farmers for all its benefits. Electricity came into use in the early 20th century, and man then discovered that wind power could be used effectively for generation of electricity. This helped in the transition of the wind mill to the wind turbine. Over the next few years, diesel engines and gasoline started being predominantly used for electricity generation. Only when fuel prices went up, and the availability of economical gasoline went down did the need for wind power grow up gradually. The dependence on wind as a power source led to the design and invention of the modern day wind turbine. As a result, substantial research was done on the design of modern-day wind turbine blades.

1.2 Blade Design

Blade design is one of the most important and complicated aspects of efficient wind turbine design. The blades play a very important part as they extract power from the wind and rotate the shaft of the generator. Historical wind mills used paddle-like wooden blades. Then the use of steel to design blades made them more efficient as they were lighter and easier to shape. Modern wind turbine blades are made out of composite materials which are structurally far superior, highly efficient and durable. Wind turbines also need to be affordable and quiet. Many parameters like blade number, pitch, weight, materials, twist, taper, etc play a vital role in determining how well the blade performs.

Many analysis methods have been developed over the years to predict the performance characteristics of a blade when certain geometric parameters are changed. The most important of them is the Blade Element Momentum Theory. The BEM theory, even though efficient had certain limitations with respect to yawing, 3-D wake effects, etc. Inflow models were then incorporated, but they weren’t effective in determining blade loads over a range of operating conditions as described by Fingersh et al [2]. This led to the development of the vortex methods. Even though the free vortex methods are predictive methods, the prescribed
vortex methods; Kocurek [3], Robison et al. [4], Coton & Wang [5] are considered a post-
ddictive method as they use experimental data for formulation, thereby leading this method
to be limited by the range of experimental conditions for which the formulation needed to
be done. The number of discretized elements used in the free vortex method is substantial
enough, thereby making the process more intensive and computationally demanding, slowing
the process down. The cost factor also played an important role as more experiments needed
to be done to help design a better model or develop a better model to compare against
existing data.

Thus a new method needed to be formulated for rapid and more efficient design of wind
turbine blades. This was the major inspiration which led us to explore the concept of super-
position in determining the characteristics of the blade for a range of operating conditions.

1.3 Methodology of superposition

In this effort, the theory of superposition of basic and additional lift distributions, which
is well-known for wings, is applied to wind turbine rotor blades. With this approach, the
loading on the blade is described using a simple, linear superposition of pre-computed lift
distributions. As a result, the effects of changes in pitch, blade twist, etc., can all be deter-
mined very quickly using the stored information, without the need to use an analysis method
beyond that needed for generating the stored information. Thus, the approach can be used in
closed-form solutions of problems in design optimization, aeroelasticity, and optimization of
smart blades (with flaps or other effectors). Because superposition is expressed as a matrix
equation, the approach allows for easy coupling between aerodynamics and structures for
static aeroelasticity problems. Recent work at NC State has shown how the superposition
theory, applied to aircraft wings, can be used to advantage in optimization of adaptive wings
[6, 7, 8], and in the prediction of elastic torsional deformation and static aeroelasticity [9]. In
this thesis, background material is first presented in chapter 2 for the well-known application of superposition to aircraft wings. Chapter 3 discusses how the theory can be extended to rotor blades along with results which are presented to demonstrate that the theory is successful in predicting the loading on a wind turbine blade with an arbitrary twist distribution. Finally in chapter 4, the example benefits of superposition are shown to illustrate its use in power calculation and inverse design.
Chapter 2

Superposition Approach for Wings

Achieving near-optimum spanwise lift distributions for current-day aircraft wings has been a goal for researchers over the past decade. Spanwise camber variation helps in wing-shape adaptation and reduces profile and induced drag. It also helps in alleviating the structural-load and reducing wing weight. The advantages to this are reduced fuel burn and lower emissions.

The use of trailing-edge (TE) flaps helps in accomplishing spanwise camber change. High performance sailplanes use this concept to reduce drag and increase roll control. Research done by Spillman [10] indicated that the use of TE flaps significantly reduced fuel usage and operating costs when compared to fixed-geometry wings.

Motivated by the remarkable success of the use of TE flaps, this led to the development of a generalized approach which was built on the concept of superposition of additional and basic lift distributions to determine the ideal lift distributions and flap angles for different flight conditions, when applied to an adaptive wing [6].

The concept of basic and additional distribution has been in use for over half a century. First developed in around 1936 [11], it was used to complement the swept-wing lifting line method developed by Weissinger [12]. It has been used effectively for performance predic-
tion for aircraft wings. It was used to calculate span loading for wings with arbitrary plan forms. The original lifting line theory of Prandtl proved inadequate when used to predict the characteristics of wings having appreciable angles of sweep and/or very low aspect ratio. Lifting-surface theories, in contrast, made satisfactory predictions of the characteristics of these wings although the extent of the computing labor involved prevented the undertaking of any general study. This approach of basic and additional loading enabled rapid and satisfactory predictions of incompressible flow characteristics of wings having swept and/or low aspect ratio plan forms and conventional planforms [13]. This has been effectively shown to work for finite wings in several references [11, 14, 15]. So it was evident, that this method enabled the determination of optimum flap angles using a simple, semi analytical approach. The concept is briefly reviewed in the following sections.

Within the assumption of linear aerodynamics (linear $C_l - \alpha$ variation and linear $C_l - \Gamma$ relationship), the spanwise distribution of bound circulation, denoted by $\Gamma$ (or alternatively, lift distribution), over a wing can be expressed as a sum of two contributions:

i) basic distribution, $\Gamma_b(y)$, and ii) additional distribution, $\Gamma_a(y)$:

$$\Gamma(y) = \Gamma_b(y) + \Gamma_a(y)$$

2.1 Methodology

2.1.1 Basic Loading

Basic loading is that existing with zero net lift on the wing and is therefore due to twist or effective twist (e.g., spanwise change in camber) of the wing chord plane. The addition of load due to uniform spanwise angle of attack change does not have any effect on the basic loading. Moreover the basic loading is equal at all angles of attack, to that found for zero net
lift on the wing. In other words, the basic distribution $\Gamma_b$, is the $\Gamma$ distribution at $C_l = 0$, and is solely the result of spanwise variations in geometric twist, aerodynamic twist due to camber, and flap deflections. Furthermore, the $\Gamma_b$ distributions due to twist, camber, or flap deflections, scale linearly with that particular parameter, and individual $\Gamma_b$ distributions can be added to obtain the total $\Gamma_b$ distribution. For example, the total $\Gamma_b$ due to wing twist, spanwise camber variation, and flap-angle variation is simply the sum of the individual $\Gamma_b$ distributions:

$$\Gamma_b(y) = \Gamma_{b,\text{twist}}(y) + \Gamma_{b,\text{camber}}(y) + \Gamma_{b,\text{flap}}(y)$$

### 2.1.2 Additional Loading

Additional loading is that due to equal geometric angle-of-attack change at each section of the wing. The distribution of additional loading is a function of only wing planform and is thus independent of any basic loading due to geometric and aerodynamic twist existing on the wing. The magnitude of the additional loading is a function only of angle of attack of the wing and thus each equal increment of angle of attack will give the same increase and distribution of additional loading irrespective of the total loading on the wing. That is, the additional $\Gamma$ distribution, $\Gamma_a$, depends only on angle of attack and scales with wing $C_L$. Thus, the additional $\Gamma$ distribution for $C_L$, written as $\Gamma_{a,1}$, can be precomputed for a wing and used to compute the $\Gamma_a$ for any $C_L$, as follows:

$$\Gamma_a(y) = C_L \Gamma_{a,1}(y)$$
2.1.3 Superposition

If the component of the induced velocities along the freestream direction, $u$, are assumed to be small compared to $V_\infty$, then the magnitude of the local loading is proportional to the local $\Gamma$ distribution:

$$L'(y) = \rho(V_\infty + u(y))\Gamma(y) \approx \rho V_\infty \Gamma(y)$$

which results in the following linear relationship between $\Gamma$ and the local lift coefficient, $C_l$:

$$C_l = \frac{2\Gamma(y)}{c(y)V_\infty}$$

It is, therefore, possible to write the spanwise $C_l$ distribution using superposition as follows:

$$C_l(y) = C_{lb,\text{twist}}(y) + C_{lb,\text{camber}}(y) + C_{lb,\text{flap}}(y) + C_L C_{b,1}(y) \quad (2.1)$$
2.2 Example Application: A Wing with Multiple TE Flaps

To illustrate the benefit of superposition, consider a wing with multiple trailing edge (TE) flaps, as shown in Fig. 2.1. The five flaps can be independently adjusted in flight, allowing for the wing to be adapted to suit different flight conditions.

The advantage of using the superposition concept is that the net $C_L$ distribution for a particular wing $C_L$ can be posed in terms of the unknown flap angles. Assuming $N$ flaps on the wing, the expression for the net $C_L$ distribution is:

$$C_l = C_L C_{l_{0}}, 0 + C_{lb, 1} \delta_{f, 1} + C_{lb, 2} \delta_{f, 2} + ... + C_{lb, N} \delta_{f, N}$$  \hspace{1cm} (2.2)

where $C_{lb, 0}$ is the zero-flap basic $C_l$ distribution due to geometric twist, and aerodynamic twist resulting from spanwise changes to the wing airfoil. The increment in basic $C_l$ distribution due to a unit flap deflection for flap $i$ is denoted by $C_{lb, i}$.

The total induced drag coefficient, $C_{D, ind}$, can be expressed using all the combinations of

Figure 2.1: Multiple TE flaps on a swept wing.
pairs formed by the different elementary lift distributions as follows:

\[ C_{D_{\text{ind}}} = C_{Daa}C_L^2 + C_{D00} + (C_{Da0} + C_{D0a})C_L + \sum_{j=1}^{N} (C_{Daj}C_L + C_{Dja}C_L + C_{D0j} + C_{Dj0})\delta_{f,j} + \ldots \]

\[ \sum_{i=1}^{N} \sum_{j=1}^{N} C_{Di,j}\delta_{f,i}\delta_{f,j} \tag{2.3} \]

which can be written compactly in matrix notation as:

\[ C_{D_{\text{ind}}} = f^T D f \tag{2.4} \]

For illustration of the concept, an example wing with a vertical winglet of height \(0.125b/2\) is considered [6]. This wing was analyzed using the AVL (Athena Vortex Lattice) code [16]. Four TE flaps are located along the span of the planar portion of the wing, and one on the winglet, with each flap having the same span and flap-to-chord ratio of 0.2. In this particular example, the wing is assumed to have zero geometric twist and zero section camber. In Fig. 2.2(a), \(C_{la,1}\) for the wing, computed using AVL, is shown. Also shown are the \(C_{lb}\) distributions (scaled by 10 times, for clarity) from AVL for 1-deg deflections of (i) flap 3 and (ii) flap 5, located on the winglet. Only the right side of the wing geometry is shown because of symmetry.

The \(C_l\) distribution for the wing at \(C_L = 0.5\) with flap 3 deflected to \(\delta_f = 5\) deg, and flap 5 deflected to \(\delta_f = -10\) deg, was then computed using the AVL code. This distribution from direct analysis is compared in Fig. 2.2(b) to the distribution achieved by superposition of (i) \(C_{la,1}\) for flap 3 scaled by 5, and (iii) \(C_{lb}\) for flap 5 scaled by \(-10\). The \(C_{D_{\text{ind}}}\) for this configuration from AVL is 0.00838 and is 0.00842 from superposition (Eqn. 2.4). The excellent comparison for the \(C_l\) distribution and the \(C_{D_{\text{ind}}}\) value illustrates the effectiveness of superposition of lift distributions, even when there are rapid changes in the spanwise loading.
and associated strong trailing vortices at the flap tips.

Figure 2.2: Illustration of superposition: (a) basic and additional $C_l$ distributions and (b) $C_l$ due to superposition compared with $C_l$ from direct analysis. Inset shows right side of geometry.
Chapter 3

Superposition Theory for Wind Turbine Rotor Blades

3.1 Methodology

In this section, we describe how the superposition theory for infinite wings is adapted to wind turbine rotor blades. The PROPID code [17] has been used in the analysis mode to generate the basic and additional loadings for this illustration for verification of the superposition approach.

3.1.1 Additional Load Distribution

In a finite wing, the additional loading is caused solely due to changes in $\alpha$ on a wing with zero twist or flap deflection or spanwise airfoil-camber change. Such a wing will have uniformly zero spanwise lift distribution when the wing $C_L$ is zero. (In contrast, a twisted wing at $C_L=0$ will have non-zero lift distribution, with some portions of the wing having positive lift and others negative.) This property of additional load distribution can be used to determine the requirement for rotor blades. To generate additional loading, a rotor blade
of given chord distribution will need to be twisted in such a way, so that, at some pitch angle (referred to in this paper as 'the zero-thrust pitch angle'), the blade will have uniformly zero loading from hub to tip. To better understand how this can be achieved, consider the familiar velocity diagram for a section of the blade shown below in Fig. 3.1. In this diagram, the induced inflow velocities are omitted for clarity.

![Figure 3.1: Velocity diagram for a section of the blade.](image)

From inspection of the figure, it is clear that, for zero lift on the section, the zero-lift line of the section has to have a twist, \( t(r) \), given by:

\[
t(r) = \tan^{-1}\left(\frac{V}{r\Omega}\right) = \tan^{-1}\left(\frac{1}{TSR(r/R)}\right)
\]

(3.1)

where, \( V \) is the wind speed (freestream), \( \Omega \) is the angular velocity of rotation, and \( r \) is the radial location. Clearly, if all sections of the blade are twisted in this manner, then at the zero-thrust pitch angle, the entire blade will have zero loading. This twist distribution will be referred to as base twist in this paper. It is noted that the base twist distribution for
For wings, additional loadings due to angle-of-attack changes scale nearly perfectly with wing lift coefficient. If the scaling were to work just as well for rotor blades, then for a rotor blade having base twist, the $C_l(r)$ distribution should scale perfectly with the rotor thrust coefficient, $C_T$, when the blade pitch is changed. That, however, works only for small pitch-angle changes. Figure 3.3 shows the $C_l(r/R)$ for the example rotor blade with base twist for four pitch angles over a large range from $= -20$ to $+4$ deg. It is clear that the distributions do not scale perfectly. To study the scaling effectiveness with $C_T$, Fig. 3.4 shows the $C_l$ distribution for each pitch angle scaled by the ratio of $C_T$ for $-20$ pitch case to the $C_T$ of that pitch angle. Perfect scaling of additional loading with $C_T$ would have resulted in all the lines coinciding. We see, however, that while the scaling works well for the outer half of the blade, the distributions do not scale well on the inner half of the blade. The imperfect scaling could be due to the following reasons: (i) on wind turbine rotor blades, the direction of section lift varies along the blade radius and is not the same as that of the rotor thrust, (ii) the component of the section lift along the thrust direction is sensitive to the induced angle
(\alpha_i), especially on the inboard portion of the rotor and (iii) the conditions studied could be outside the range of validity for the blade-element momentum theory. Scaling on wings works much better because the direction of section lift is largely the same along the wing span and this direction is coincident with the direction of wing lift. Because the scaling of additional loading on rotor blades works well only for small pitch changes, it is necessary to generate and store loading data for a range of pitch angles. The additional loading for a particular case can then be scaled from the stored condition with the closest value of $C_T$.

Figure 3.3: Additional $C_l$ distributions for four pitch angles for a TSR = 5.

Figure 3.4: Illustration of scaling of additional $C_l$ distributions for different pitch angles with thrust coefficient.
To study the validity of blade-element momentum theory for the four pitch angles used in Figs. 3.3 and 3.4, the variation of $a$ is plotted in Fig. 3.5. It is seen that the values of the axial induction factor, $a$, for a pitch of $-20$ go beyond 0.4. This is indicative of the blade-element momentum theory operating in regions outside validity bounds giving incorrect scaling. So, as long as the values of the axial induction factor are kept less than 0.4, it was observed that scaling could be done by storing only one value. This also improved the scaling of the additional load distribution as it meant that the reference pitch angle needn’t be far away from the analysis pitch angle. Thus loadings need not be stored for a wide range of pitch angles.

The next objective was to find the pitch angles under which the superposition theory would be effective and that the axial induction factor, $a$, wouldn’t go beyond 0.4. The range of pitch angles for effective superposition was found to be from $-8$ to $+8$ for the range of TSR involved. Two TSR values are chosen for illustration: TSR of 3 and 6. Figures 3.6, 3.7 and 3.8 show the $C_l$ distributions, scaled $C_l$ distributions, and $a$ distributions for a TSR of 3 for pitch angles of $-6$, $-2$ and 6 degrees. It is seen that the values of $a$ are less than 0.4, indicating that the blades are operating within the validity range of the blade-element
momentum theory. The scaling of $C_l$ in Fig. 3.7 is also seen to be excellent. The same is repeated in Figures 3.9, 3.10 and 3.11 for a TSR of 6 for pitch angles $-6$, $-2$ and 6 degrees. Fig. 3.11 clearly shows the value of the axial induction factor, $a$, for this case to be well below 0.4 for all the pitch angles, indicating that the blade-element momentum theory condition is satisfied and that the blades are operating within the validity range. Also seen in Fig. 3.10 is that the $C_l$ scaling is accurate.

If analysis is to be carried out for a different TSR, the additional and basic loadings for each TSR have to be computed and stored. While most wind turbines work under the TSR range from 1 to 8, high efficiency turbines are designed for a TSR of around 6 to 7. Thus additional and basic loadings were stored for a small range of TSR ranging from 1 to 8.

Figure 3.6: Additional $C_l$ distributions for 3 pitch angles for a TSR = 3.

Figure 3.7: Illustration of scaling of additional $C_l$ distributions for different pitch angles with thrust coefficient.
Figure 3.8: Axial induction factor for different pitches at TSR = 3.

Figure 3.9: Additional $C_l$ distributions for 3 pitch angles for a TSR = 6.

Figure 3.10: Illustration of scaling of additional $C_l$ distributions for different pitch angles with thrust coefficient.
This constraint for the axial induction factor, $a$, also plays a vital role in accurate calculation of power for a given turbine running at a certain pitch angle at a given TSR. This is explained in section 4.1.

### 3.1.2 Basic Load Distribution

Basic loading on rotor blades is caused by twist deviations from the base twist. If we consider the blade to be defined using several segments, then we can examine the loading due to twist of each of those segments. The blade pitch in each case is adjusted to result in zero $C_T$. For illustration, we consider the example rotor blade to be divided into 20 segments. Fig. 3.12 shows the basic $C_T(r/R)$ distributions for 1-deg deviation from the base twist for radial sections 5, 10, and 15. The additional loading for the 0-deg pitch case is also shown for comparison.
Figure 3.12: Basic loadings for unit twist applied to three radial locations along the blade for a TSR = 6.

### 3.1.3 Total Load Distribution Using Superposition

For any given turbine with a given twist distribution and a desired thrust coefficient $C_{T,desired}$, the total $C_l$ distribution can be determined by superposition of precomputed and stored basic and additional distributions. For this, we use the stored additional loading, $C_{ladd,ref}$, for that particular TSR. The basic $C_l$ distribution for each segment is scaled by $\delta t$, which is the difference between the actual twist at that segment for the given turbine and the base twist for the turbine at that TSR. The resulting equation is:

$$C_l(r/R) = C_{ladd,ref}(r/R)\left(\frac{C_{T,desired}}{C_{T,ref}}\right) + C_{lb,1}(r/R)\delta t_1 + C_{lb,2}(r/R)\delta t_2 + \cdots + C_{lb,N}(r/R)\delta t_N$$ (3.2)

which can be written in matrix notation as:

$$\{C_l\} = \{C_{ladd,ref}\}\left(\frac{C_{T,desired}}{C_{T,ref}}\right) + \{C_{lb}\}\{\delta t\}$$ (3.3)
3.2 Results

We present the results in two subsections taking two example turbines. In the first, we present an example to show the effectiveness of the current superposition approach in predicting the $C_l$ distribution for the AWT 27 rotor blade. This is followed by the second example which is for the AOC 15/50 rotor blade.

The additional loading is scaled as mentioned before using the ratios of the rotor coefficients of thrust. The basic loading of the analysis TSR is used in the final superposition after it is multiplied by the difference in twist between the analysis and the reference basetwist ($\delta t$).

For final superposition, additional and basic loadings were stored for a pitch angle of $-2$ for TSR ranging from 1 to 8 along with the corresponding basic loadings. Analysis was carried out at different TSR and pitch angles. Pre-computed data from the analysis TSR is used for superposition. The equation used for the final superposition is the one given in Eqn 3.3. Some examples are illustrated to show the concept of superposition.

3.2.1 AWT 27 rotor blade

The first example is the AWT-27, whose variation of airfoil chord and twist with the radial position is shown in Figs 3.13 and 3.14. It is a two bladed turbine with a rotor diameter of 45.1 feet.
Figure 3.13: Chord distribution for the AWT 27 rotor blade.

Figure 3.14: Twist distribution for the AWT 27 rotor blade.
Figure 3.15: Illustration of superposition for the AWT 27 at a TSR of 6 and different pitch angles.

Fig 3.15 shows the $C_l$ distribution calculated using pre-computed additional and basic loading for a TSR of 6 at two different pitch angles of $-6$ and $8$. It is observed that the final scaling attained by the superposition principle is good. The large difference in $C_l$ between the additional and the actual $C_l$ distributions is a result of the large difference in twist between the actual blade and the basetwist.
Figure 3.16: Illustration of superposition for the AWT 27 at a TSR of 8 and different pitch angles.

Fig. 3.16 illustrates the $C_l$ distribution from superposition of stored basic and additional loading for a TSR of 8 for pitch angles $-4$ and $6$. It is seen that the concept of superposition is quite successful in predicting the $C_l$ distribution for the given twist distribution of the rotor blade. The scaling is good even when the analysis pitch is far away from the pitch angle at which the additional and basic loadings are stored for any TSR. It is noted that analysis was carried out assuming constant $C_d$ for the airfoil along the entire blade.
3.2.2 AOC 15/50 rotor blade

The next example illustrated is for the AOC 15/50 turbine. It is a 3 bladed turbine with a rotor diameter of 24.61 feet whose chord and twist distribution are as shown in Figs. 3.17 and 3.18.

![Chord distribution for the AOC 15/50 rotor blade.](image1)

![Twist distribution for the AOC 15/50 rotor blade.](image2)

To show that the concept of superposition can be applied to any turbine, the following figures show the effective calculation of the $C_l$ distribution for the AOC 15/50 turbine blade. Analysis was carried out at two different TSR and pitch angles similar to the previous case. The additional and basic loadings were stored for different TSR at a pitch angle of $-2$. 

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Figure 3.19: Illustration of superposition for the AOC 15/50 at a TSR of 4 and different pitch angles.

Figure 3.19 shows the comparison of $C_l$ distribution from superposition with that from direct analysis (using PROPID) for the example rotor blade at a TSR of 4. Even at low TSR, it is seen that the superposition approach can be successfully used in calculating the $C_l$ distribution for any pitch angle.
Figure 3.20: Illustration of superposition for the AOC 15/50 at a TSR of 7 and different pitch angles.

Figure 3.20 shows the scaled $C_l$ distribution along with the analysis $C_l$ distribution at a TSR of 7 and pitch angles of 6 and 8. As noted, the scaling is good at any TSR, even though the additional $C_l$ distribution is more varied than the actual $C_l$ distribution. Analysis is performed assuming linear section lift curves for the airfoil. It is recognized that the large $C_l$ values over the inboard portions of the blade are well beyond stall; the example, however, is only intended to provide an illustration of the effectiveness of the superposition approach.
Chapter 4

Example benefits of the concept of superposition

4.1 Power calculation

The coefficient of power is an important factor when it comes to the design characteristics of a wind turbine. Power can be calculated using Eqn 4.1.

\[ dP = \Omega \cdot dM \quad (4.1) \]

where \( dM \) is the elemental moment (torque about the rotor shaft) and \( \Omega \) is the angular velocity. The expression for \( dM \) is given by:

\[ dM = \frac{0.5 \rho BV_0 (1 - a) \Omega r (1 + a') c_{tang} r dr}{(\sin \phi \cos \phi)} \quad (4.2) \]

Since, \( dP = \Omega \cdot dM \),

\[ (a) \]
\[
dP = \frac{0.5\rho BV_0(1-a)\Omega^2r(1+a')cC_{\text{tang}}rdr}{(\sin \phi \cos \phi)} \tag{4.3}
\]

Substituting \( C_{\text{tang}} = \left( L \sin \phi - D \cos \phi \right) / (0.5\rho V_{\text{rel}}^2c) \), \( \sin \phi = V_0(1-a)/V_{\text{rel}} \), \( \cos \phi = r\Omega/V_{\text{rel}} \)
and assuming \( a' = 0 \) we get:

\[
dP = 0.5\rho Bcr^2\Omega^2drV_0(1-a)\left( \frac{C_{\text{i}}V_{\text{rel}}}{r\Omega} - \frac{C_{\text{d}}V_{\text{rel}}}{V_0(1-a)} \right) \tag{4.4}
\]

From Eqn 4.4 one can see that by knowing the superposed lift distribution and the axial induction factor, the approximate elemental power output of a turbine at any given condition can be calculated effectively. It is further summed to get the total power output. The \( C_t \) distribution is the superposed \( C_t \) as calculated in the previous chapter. The \( a \) or the axial induction factor for any condition can be found out using the elemental coefficient of thrust distribution. Thus knowing \( C_t \), \( a \) can be calculated accurately.

For one particular TSR, the total \( C_t \) distribution can be found by knowing the rotor coefficient of thrust, or \( C_T \), and the difference in twist from the reference basetwist for that condition, \( \delta t \). The additional \( C_t \) distribution along with the basic \( C_{tb} \) distribution is pre-stored for one particular pitch for each TSR. These pre-stored values are used in calculating the total \( C_t \) distribution for any condition at any TSR, similar to the calculation of the \( C_t \) distribution. The additional \( C_t \) distribution is found by Eqn 4.5:

\[
C_{t,a} = C_{t,\text{ref}} \frac{C_{T,\text{anal}}}{C_{T,\text{ref}}} \tag{4.5}
\]

and the basic \( C_t \) distribution is calculated using Eqn 4.6:

\[
C_{t,b} = C_{t,\text{basic}} \delta t \tag{4.6}
\]

Thus the final elemental coefficient of thrust distribution is the linear sum of the two...
contributions of additional and basic $C_t$ distributions:

$$C_t = C_{t,a} + C_{t,b}$$  \hspace{1cm} (4.7)

Using $C_t$ derived from Eqn 4.7, the axial induction factor or $a$ can be calculated using Eqn 4.8:

$$a = \frac{1 - \sqrt{1 - C_t^2}}{2}$$  \hspace{1cm} (4.8)

For accurate calculation of power, it is seen that the $C_t$ calculation has to be accurate. This in turn helps in calculating the correct axial induction factor. It is imperative that the calculated value of $a$ does not go beyond 0.4. As mentioned before in section 3.1.1, additional and basic loadings are stored for a range of pitch angles where $a$ does not exceed 0.4. It was found that for a given TSR, the $C_t/C_T$ curves for different pitch angles mostly collapsed to a single curve for the small range of pitch angles. This behavior led to accurate estimation of $C_{t,a}$ according to Eqn 4.5. The following plots in Figs 4.1 and 4.2 show the coincident $C_t/C_T$ ratios for different pitch angles at two different TSR values.

![Figure 4.1: $C_t/C_T$ ratios for a TSR = 8.](image-url)
Examples illustrating the effective calculation of $C_t$ are shown below. Fig. 4.3 shows the $C_t$ distribution calculated from superposition along with the distribution obtained from PROPID for the AWT 27 turbine at a TSR of 3 and a pitch of $-4$ deg.

Fig. 4.4 shows the $C_t$ distribution from analysis and superposition for the AOC 15/50 turbine run at a TSR of 2 and a pitch of $-2$. 

Figure 4.2: $C_t/C_T$ ratios for a TSR = 6.

Figure 4.3: Comparison of $C_t$ from analysis and superposition for the AWT 27.

Figure 4.4: Comparison of $C_t$ from analysis and superposition for the AOC 15/50.
Figure 4.4: Comparison of $C_t$ from analysis and superposition for the AOC 15/50.

The $C_t$ values obtained from superposition match well with that obtained from analysis. The slight error can be attributed to the assumption that the $\frac{C_t}{C_P}$ ratios collapse exactly onto one line, along with the fact that the difference in twist in the inner part of the blade is large.

The power and finally the coefficient of power or $C_P$ can thus be calculated. Figs. 4.5 and 4.6 show the comparison of $C_P$ calculated from superposition to that calculated from analysis (PROPID).
Figure 4.5: Comparison of $C_P$ distribution for AWT 27 at TSR = 6 for pitch from $-6$ to $+6$.

Figure 4.6: Comparison of $C_P$ distribution for AOC 15/50 at TSR = 2 for pitch from $-6$ to $+6$.

As one can see, the superposition principle is seen to work well in predicting power using the stored basic and additional loadings. Even though the match is not perfect, the trends are captured very well. The benefit of this is that the value of coefficient of power can be calculated efficiently for any given TSR. Using pre-computed data makes the calculation much faster without the need for running an external code or software.

4.2 Inverse Design

4.2.1 Methodology

In this example, we consider the use of stored basic and additional loadings for an inverse design demonstration. As before, two example turbines are used to show the effective use of superposition for this approach. The objective of the inverse problem considered here is to find the twist distribution that will result in a target $C_l$ distribution at a TSR. As illustration, the target is selected as a constant $C_l$ over the whole blade. For this, the matrix equation 3.3 is recast so that $\delta t$ is the unknown:
\[ [C_{lb}] \delta t = C_{l,\text{target}} - C_{l,\text{add,ref}} \left( \frac{C_{T,\text{desired}}}{C_{T,\text{ref}}} \right) \]  \hspace{1cm} (4.9)

To use this matrix equation, the \( C_{T,\text{desired}} \) must be calculated. For this example, the desired \( C_T \) has been estimated by integrating the desired \( C_l \) distribution along the radius using standard procedure, while assuming zero \( \alpha_i \). Once calculated, it is then incorporated in Eqn. 4.9 to calculate the required basetwist for achieving the target \( C_l \) distribution.

### 4.2.2 Results

The first example is for a target \( C_l \) distribution of 1.5 for the AWT 27 rotor blade for which inverse design was used at a TSR of 3. The following figures show the twist distribution (including blade pitch) required for achieving the target \( C_l \) distribution. For comparison, the baseline twist distribution is also shown in Fig. 4.7. Figure 4.8 compares the resulting \( C_l \) distribution achieved by the inverse approach with the baseline and the target. It is seen that superposition method has been used successfully in calculating the target \( C_l \) distribution.

![Figure 4.7: Twist distribution from inverse design compared to the analysis.](image)

![Figure 4.8: \( C_l \) distribution from inverse design compared to target and analysis.](image)

The next example is for a target \( C_l \) distribution of 0.8 for the AOC 15/50 rotor blade for
which inverse design was used at a TSR of 2.

Figure 4.9: Twist distribution from inverse design compared to the analysis.

Figure 4.10: $C_l$ distribution from inverse design compared to target and analysis.

Fig. 4.9 shows the comparison between the baseline twist distribution and the inverse twist distribution. It is evident from Fig. 4.10 that the inverse design method is remarkably successful in achieving the desired $C_l$ specification. Even with such a large difference between the two twists the results are good. Also noted is the large change in the $C_l$ distribution from the analysis and the $C_l$ distribution achieved by inverse design. These results are a very promising sign for the use of superposition to be effectively used for any turbine at any given TSR, in the rapid and efficient computation for inverse design.
Chapter 5

Conclusions

In the current research, a superposition theory for finite wings has been adapted to predict the loading on wind turbine rotor blades. The theory allows for use of pre-computed basic and additional loadings to be superposed to efficiently and rapidly predict the loading for an arbitrary twist. The advantages are that (i) the resulting closed-form matrix expressions can be used in design optimization, inverse problems, and aeroelasticity and (ii) once the basic and additional loadings are pre-computed, they can be used in an “off-line” mode - without the need for the analysis method (such as PROPID [17] or WTPERF [18]). The advantage of such an off-line analysis is that, in solving coupled problems, such as aero-structure or aero-control problems, the superposition approach can be used without the need to integrate a legacy aerodynamics prediction code. Also this method is much faster compared to current CFD methods which require high levels of computation. While the approach is successful in predicting the loading on a blade for any twist distribution or in determining the twist distribution required for achieving a desired loading, a current limitation is that no post-stall analysis has been done with regard to superposition. Also further work needs to be done to couple aerodynamics with structures and prove that this approach will be successful in aero-elastic and aero-control problems.
Motivated by the successful use in finite-wing aerodynamics, this approach has been successfully demonstrated for wind turbine rotor blades. This provides confidence that the approach can be adapted to other fields of aerodynamics such as helicopters.
Bibliography


