ABSTRACT

SMITH, JULIA HATTIE. Hearing Their Voices: The Effect of Proof Writing Modifications on English Language Learners. (Under the direction of Dr. Karen Hollebrands).

There has been significant research conducted related to the use of modifications for English Language Learners (ELL). Also a significant number of studies have been conducted to investigate the proof-writing abilities of high school Geometry students. However, there is a gap in the research that connects the two topics. Given the requirements of the No Child Left Behind Act, in which all schools must test every subgroup and provide reasonable accommodations for the students in that subgroup, more research is needed. In this qualitative study, six high school Geometry students were asked to construct four different proofs (two warm-up proofs, one proof without a list of reasons, and one proof with a list of reasons). The researcher’s main goal was to compare the progress each student made on the four proofs and to answer the question: In what ways do ELL students use a list of reasons as they are writing proofs in Geometry? The results of the study show that students who gained some ability to reason logically in their Geometry class used the sheet of reasons to support their statements. However, students that were unable to form a logical argument did not benefit from using a sheet of reasons as they construct proofs.
DEDICATION

With love and gratitude, I dedicate this study to all students for whom math is a struggle, my husband Brett for his unconditional love and support, and my parents Cathy and Bob for expecting nothing but the best and instilling in me the drive to succeed.
BIOGRAPHY

Julia Hattie Smith was born in Bartlett, Illinois May 14, 1981. She is the second of four daughters. In 1984, her family moved from Illinois to Cary, North Carolina and this is where she was raised.

In eighth grade she met a math teacher that changed her life. Miss. G loved math and through her love of math, she learned to love math. She had finally found her special talent. Julia’s good fortune with math teachers followed her throughout high school, which helped her build her confidence and find her niche. Julia always knew she wanted to be a teacher. From eighth grade on she had refined that statement to: “I want to be a math teacher.”

Julia graduated from Appalachian State University in May 2003 and began teaching at Cary High School in August 2003. In July 2004, she married Brett Smith and moved into a house in Holly Springs. Julia has completed 8 years of teaching at Cary High School, and has taught every subject from Pre-Algebra to Advanced Functions and Modeling. Most recently, her teaching focus has been in Geometry. In the fall of 2009, Julia returned to Graduate School and in November 2010, she received her National Board Certification.
ACKNOWLEDGMENTS

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Dr. Karen Keene, a member of my committee. She is optimistic and energetic. Her excitement and enthusiasm are contagious.

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had major projects/tests, supported my desire to complete a thesis, and worked with me to make sure I got all the data I needed.

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CHAPTER 1
INTRODUCTION

Many consider math to be a universal language. In part, this is true. In Algebra, students see numbers and many cultures use Arabic numerals, which makes numbers easy for students to recognize and interpret. In some cases, numbers are the only common thread between cultures. However, math is much more than numbers and operations. It is reasoning and logic and understanding of mathematical terms and definitions is crucial to the learning of mathematics. The introduction of new terminology such as *quotient* and *exponent* can confuse English Language Learners (ELL) because they are words that ELL students do not come across on a daily basis (Cavanagh, 2005). The inclusion of word problems in Algebra makes studying math much more complicated for many groups of students such as exceptional students (students with disabilities), ELL students, or students who have difficulty with reading comprehension.

Geometry requires students to think in a more abstract and theoretical way than Algebra. In fact, there are levels of geometric thinking, the van Hiele levels, named after the husband and wife team who created them (Mason, 1998). Regarding these levels, the van Hiele’s believe that students progress through them sequentially and that instruction can help students progress from one level to the next. However, if the instruction is delivered at a higher level than a student has reached, the student is unlikely to understand and make progress through the levels (Usiskin, 1982; Burger & Shaughnessy, 1986; Senk, 1989; Mayberry, 1983; Clements & Battista, 1992; van Hiele, 1986). These levels could be one reason why students struggle with proof. If a student has not received instruction in reasoning
and proof, he is unlikely to understand a teacher who teaches proof in Geometry (Usiskin, 1982). Relating this to an ELL student, it is possible that in a move from the students’ country of origin to the US, some of this important instruction may be missed. Just like a native English speaker, if an ELL student enters a classroom where the teacher teaches at a level higher than one the student is able to comprehend, that student may struggle for the rest of the course or try to rely on memorization as a strategy for success (Mayberry, 1983; Mason, 1998).

A Perspective on Challenges Facing English Language Learners in the Classroom

In 1965, the Elementary and Secondary Education Act was passed. Though testing students from low-income families was mentioned as a requirement for receiving Title I funding, ELL students were not specifically mentioned until the No Child Left Behind Act of 2002 (NCLB) which required schools to test ELL students, report the scores, and make adequate yearly progress with an ultimate goal of 100% proficient by 2014 (Center on Education Policy, 2010; Willner, Rivera, & Acosta, 2009; Alford, 1965). Ultimately, monitoring growth and making adequate yearly progress was not a priority for school systems prior to the introduction of NCLB (Abedi, Hofstetter, & Lord, 2004).

The implementation of the NCLB act required school systems to hold ELL students to the same standards as native English speakers. School systems work hard at providing a fair and equitable learning environment in which ELL students can achieve the same success as native English speakers. One approach to a level playing field is through the addition of testing modifications.
The ELL population is an unstable one (Abedi & Dietel, 2004). The number of ELL students varies from state to state and even from district to district and school to school. In addition to this, ELL students are at various levels in the program and each year there is a group of students that move out of the English as a Second Language (ESL) program and a new group of students that join the program. In a system where educators need to show consistent and adequate growth in their students each year, growth is slow and exceptionally difficult for ELL students (Abedi, Hofstetter, & Lord, 2004). This continuous change could make it difficult to track, alter, and implement modifications for students who are learning English.

Another factor that affects student achievement is students’ background and culture. Studies have shown that students with parents who have achieved higher levels of education perform better on end of course tests than students with parents that have not completed high school or college. This could be due to the fact that students with struggles and difficulties at home (for example, low-income families who are unable to provide resources or parents that do not stress the importance of education to their children and simply do not provide resources for school), are more likely to allow those stresses from home to creep into their thoughts as they solve problems which in turn can impede their progress and ability to solve problems correctly (Solano-Flores & Trumball, 2003; Abedi, Leon, & Mirocha, 2003). This correlates to ELL students drive to learn English and their parents’ level of education. Parents with more than a high school education have students that learn English faster than students of parents without a high school education.
A third struggle for English Language Learners is being placed in classrooms with teachers that have little to no training working with students whose native language is not English. There are an increasing number of students whose native language is not English entering the public schools and yet only English as a Second Language teachers or Bilingual teachers have special training that assists them in working with English Language Learners to ensure their success in the classroom. However, there are not many of these teachers within a school and many languages spoken by students – the English as a Second Language teachers are not necessarily fluent in every language at the school. States need to implement programs to aid teachers in providing a fair and equal education to English Language Learners (Dong, 2004).

**Geometry**

Geometry rarely deals only with numbers. Geometry is part of the curriculum for every grade level. According to NCTM, (2000):

Instructional programs from prekindergarten through grade 12 should enable all students to-

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

High school Geometry teaches students how to think and how to construct a logical argument. Geometry is the avenue for teaching deductive thinking, and this is typically achieved by teaching proof. In fact, Usiskin (2007) goes so far as to say that the most important concept in Geometry is proof.

A Perspective on the Importance of Proof Writing

High school Geometry teaches students how to think and how to construct a logical argument through proof. According to the Principles and Standards of School Mathematics (2000), “Mathematics should make sense to students; they should see it as reasoned and reasonable” (p. 342). The National Council of Teachers of Mathematics (2000) supports the belief in teaching logic and reasoning through proof. According to their website:

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof. (56)

However, the National Council of Teachers of Mathematics is not the only organization that supports the teaching of proof. Many mathematicians believe that it is important to teach proof, to help students build the skills necessary to construct a formal
proof, and believe that “proof is alive and well in mathematical practice and that it continues to deserve a prominent place in the mathematics curriculum” (Hanna, 2000, p. 5).

Why do we teach proof? Proof teaches students to connect mathematical topics and curricula. For example, in Geometry students learn to connect theorems, properties, definitions, and postulates that reference angles and segments and how those topics connect in order to prove that triangles are congruent or similar. Geometry is also the study of logic and relationships. Writing proofs in Geometry, teaches students how to develop skills that make them mathematically literate, which could ultimately lead to a deeper understanding and an appreciation for proofs and for mathematics in general (Waring, 2001; Hanna, 2000).

Proof is essential to the discipline. Mathematicians use proof to verify the validity of a statement. Teachers use proof to allow their students the freedom to discover properties, theorems, and to test the validity of a statement (Knuth, 2002b). As mathematicians, and in keeping with the beliefs of the National Council of Teachers of Mathematics, we should capitalize on the inquisitive nature of children. Allowing children to express themselves at a young age provides them with the freedom to discover proof and therein to use that inquisitive nature, to develop thinking and reasoning skills that will aid them in their learning throughout their career. Years ago, educators only taught proof to students that might achieve the highest level of education possible. More recently, and especially with the addition of the standards by the National Council of Teachers of Mathematics, educators teach reasoning and proof to all students, grades K - 12, regardless of future endeavors. For the 21st Century where so many careers depend on a strong mathematical foundation, students need to know how to construct proofs to be successful in college mathematics courses that require students
to demonstrate their understanding of mathematics by constructing proofs (Waring, 2001; Weber, 2001).

The Purpose of this Study

The purpose of this study is to investigate how ELL students develop their ability to write proofs in Geometry and how they overcome their struggles with the English Language to be successful in a Geometry classroom filled with English speaking students. ELL students have a unique struggle in every classroom. They must overcome a significant language barrier in order to stay competitive with their peers. In the study, the researcher offered a strategy to help the ELL students write their proofs. The researcher provided each student with a list of reasons that accompany one of the proofs the researcher asked them to complete. The intent of this list was to assist students in forming their logical argument and to take some of the stress off the students.
CHAPTER 2

REVIEW OF THE LITERATURE

Historical Perspective

Prior to 1994, much of the research focused on the impact that the level of second language acquisition (ability of students to acquire a second language) had on a student’s ability to learn mathematics even though ELL students were severely under-tested and underserved in the public schools (Gordon, 1981; Labov, 1970; MacGregor & Price, 1999; Willner, Rivera, & Acosta, 2009). In 1994, states were required to develop tests that were fair for all students and in 2002, Congress passed the No Child Left Behind Act which laid out specific guidelines for state testing and included testing of ELL students (Willner, Rivera, & Acosta, 2009). Therefore, the majority of testing modifications and data collection began after the No Child Left Behind Act of 2002. According to Abedi, Hofstetter, & Lord (2004), the No Child Left Behind Act requires that all Limited English Proficient (LEP) students be included in state testing and that states use appropriate modifications for the testing.

In a state in the southeast, the percent of ELL students that pass the End of Course exam from the 2002-2003 school year to the 2008-2009 school year both increased and decreased over that span of time. The percent of students scoring a level 3 or 4 on End of Course tests from 2002-2009 is summarized in Table 1. A level 3 or 4 is considered to be proficient or passing. These are not high, stable scores. These scores show the need for additional research and implementation of modifications for ELL students.
Table 1  End of Course Testing Data

<table>
<thead>
<tr>
<th>School Year</th>
<th>Dogwood High School</th>
<th>Data from an Eastern County</th>
<th>Data from a Southeast State</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002-2003</td>
<td>63.2%</td>
<td>53.7%</td>
<td>46.5%</td>
</tr>
<tr>
<td>2003-2004</td>
<td>75%</td>
<td>56.3%</td>
<td>59.2%</td>
</tr>
<tr>
<td>2004-2005</td>
<td>58.9%</td>
<td>50.3%</td>
<td>46.5%</td>
</tr>
<tr>
<td>2005-2006</td>
<td>46.6%</td>
<td>46.3%</td>
<td>42.5%</td>
</tr>
<tr>
<td>2006-2007</td>
<td>38.4%</td>
<td>42.8%</td>
<td>38.7%</td>
</tr>
<tr>
<td>2007-2008</td>
<td>51%</td>
<td>49.6%</td>
<td>45.4%</td>
</tr>
<tr>
<td>2008-2009</td>
<td>60.6%</td>
<td>57.1%</td>
<td>52.1%</td>
</tr>
<tr>
<td>2009-2010</td>
<td>59.5%</td>
<td>52.8%</td>
<td>55.8%</td>
</tr>
</tbody>
</table>

North Carolina Department of Public Instruction, 2010

In developing modifications and conducting research on ELL students and their progress in school, one important development was to determine exactly who is an ELL student. The No Child Left Behind Act does not provide a clear definition as to how to classify a student as ELL (Solano-Flores, 2010). As a result, different states use many criteria to identify ELL students (Abedi, Hofstetter, & Lord, 2004; Solano-Flores, 2010). Examples of these criteria include, Home Language Surveys and English Proficiency (Kindler, 2002). Parents could be concerned that if they report that they do not speak English at home, there could be negative consequences. For example, if a family is living in the United States illegally, they may have to return to their native country. Therefore, results of the Home Language Surveys may not be valid. Most states use a type of language proficiency test. According to Kindler (2002), the use of Home Language Surveys and proficiency testing in most states represents a clear consistency among the states in identifying ELL students.

Achievement Gap

Many researchers have investigated and reported on the significant achievement gap between ELL students and native English speakers. In 2004, Abedi & Dietel investigated the
challenges facing English Language Learners and their teachers who are trying to meet the stringent requirements set by the No Child Left Behind Act. The No Child Left Behind Act requires that by 2014, all subgroups will reach 100% proficiency in English language arts and mathematics. Data from the county in which the study was conducted shows that ELL students are not progressing at a pace that indicates the 100% proficient goal is a reasonable one by the year 2014. While the numbers fluctuate from year to year in the county in which the study was conducted, the highest percent proficient ELL students have ever been in the last nine years is 57.1% during the 2008-2009. In a southeastern state, ELL students have performed no higher than 59.2% proficient during the 2003-2004 school year (North Carolina Department of Public Instruction, 2010).

However, the state in which this study was conducted is not the only state struggling with raising ELL performance on state tests. Abedi & Dietel (2004) collected data from the Massachusetts Comprehensive Assessment System and found that in 1998 (the first year of a six year study) only 8% of ELL students achieved the level of proficient in tenth grade English language arts where all students overall for the state were at 38% proficient. Five years later in 2003, both numbers increased to 12% of the ELL population classified as proficient on the test and an overall student performance was 61% proficient. Unfortunately, this just increased the gap between ELL performance and the state performance and makes the goal of reaching 100% proficient in 2014 seem unreasonable and unattainable (Garcia, Lawton, & Diniz de Figueiredo, 2010; Kim & Herman, 2009).

Cavanagh investigated the claim that mathematics is the universal language in 2005. In his article, he reminds us that language is a critical component to mathematics and that
students are learning English at the same time that they are learning mathematics. The research supports that the language barrier is a significant reason for the achievement gap. In 2008, the U.S. Department of Education investigated the achievement gap in elementary school students and discovered that regardless of race, parent’s education, or poverty status, ELL students scored lower than native English speaking students (Abedi, 2002).

**Instability in the subgroup**

One reason why the ELL subgroup never performs as well as native speakers is that there is significant instability in this subgroup. This subgroup is considered unstable because students are regularly “exited” from the English as a second language program and new students move to the United States and enter the schools regularly (yearly, monthly, and sometimes weekly). It is important to remember that once a student is deemed “English proficient,” he or she is removed from the ELL subgroup (Abedi & Dietel, 2004). In their article Abedi & Dietel (2004) address the US Department of Education’s decision to allow students to remain in the subgroup for two years after being reclassified as “English Proficient.” While this would help in the short term, it just delays the inevitable for two years until those students are moved out and the risk of lower test scores lingers again.

ELL students, who know little to no English but must still take all required state tests, are expected to perform at the same level as their native English-speaking counterparts. Students who are just beginning to experience the English language are not as prepared to take and perform well on these state mandated tests (Abedi & Dietel, 2004). Modifications to testing and instruction, the changing ELL population, and the No Child Left Behind Act naturally lead to an inspection of graduation rates for this subgroup. The county in which the
study was conducted defines graduation rate for 2007-2008 as: “The percentage of students entering the 9th grade for the first time during the 2004-2005 school year who earned a diploma by or prior to the spring of 2008” (Haynie, 2008, p. 1-2). The county in which the study was conducted has investigated four-year graduation rates of students and has published these rates. Table 2, 3, and 4 provide a summary of graduation rates for the state, county, school, race, and subgroups.

<table>
<thead>
<tr>
<th>Year</th>
<th>State Graduation Rates</th>
<th>County Graduation Rates</th>
<th>School Graduation Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-2006</td>
<td>68.3%</td>
<td>82.6%</td>
<td>&gt;95%</td>
</tr>
<tr>
<td>2006-2007</td>
<td>69.5%</td>
<td>79.3%</td>
<td>79.7%</td>
</tr>
<tr>
<td>2007-2008</td>
<td>70.3%</td>
<td>78.8%</td>
<td>85.3%</td>
</tr>
<tr>
<td>2008-2009</td>
<td>71.8%</td>
<td>78.4%</td>
<td>78.5%</td>
</tr>
<tr>
<td>2009-2010</td>
<td>74.2%</td>
<td>78.2%</td>
<td>80.9%</td>
</tr>
</tbody>
</table>

Haynie & Regan, 2011, p. 5; North Carolina Department of Public Instruction, 2010

In Table 2, the school consistently outperformed the county and the state; the county consistently outperformed the state. This is a very high achieving area. But even so, graduation rates are only in the high 70’s and low 80’s. Even this county and this school have room to improve and room to grow. The school’s graduation rate goes up and down, but the county graduation rate has pretty steadily decreased after a sharp initial decrease. In contrast to this, the state graduation rate has steadily increased. The school and the county may find their status slipping away if the county meets and eventually surpasses their graduation rate.
The authors point out that the variability in the American Indian subgroup fluctuates because the actual population of American Indian students is very small. However, the lowest performing race is the Hispanic population followed by the Black population but there is generally more than a difference of ten percentage points (Table 3). The indication here is that we are not meeting the needs of Black and Hispanic students as they are not achieving the same level of success as their White and Asian colleagues. The graduation rates for both White and Asian students are relatively stable each year.

Looking at only the subgroups (Table 4), the Limited English Proficient subgroup is the lowest and is rapidly decreasing each year. Last year, only 36.7% of these students graduated on time whereas more than 50% of both the Free or Reduced-Price Lunch and Students with Disabilities subgroup graduated on time. These startling statistics indicate that not only are we not meeting the needs of each of these three subgroups, but we are allowing students in
the Limited English Proficient subgroup to slip through the cracks and they are not graduating from high school on time.

**Effects of Background Culture**

Background culture refers to the student’s home life (such as but not limited to, whether or not the student works to support the family, takes care of siblings so parents can work, or is in the US while the parent is still in the student’s country of origin), socioeconomic status, country of origin (the view of education, role of men, and role of women in that country), parental involvement, and immigration status. Background culture plays an important role in the performance of ELL students. Students in mathematics classrooms could have trouble solving word problems based solely on the translation of a word. For example, in an Algebra I class taught by the researcher, an ELL student did not know what bowling was. In order for the student to be able to solve the problem and understand the results, the researcher had to explain what it meant to go bowling and why one might rent shoes at a bowling alley. As another example, a student whose family is struggling financially may find it difficult to solve word problems dealing with money. In the following example, Solano-Flores & Trumball (2003) show the affect that socioeconomic status can have on students’ ability to solve word problems involving money. They conducted a formal test of background culture where they tested three groups of students with very different backgrounds: a) White suburban students from high-income households, and b) American Indian rural students from low-income households and c) African American inner city students from low-income households. All the students were asked the same math question which stated,
“Sam can purchase lunch at school. Each day he wants to have juice that costs 50¢, a sandwich that costs 90¢, and a fruit that costs 35¢. His mother has only $1.00 bills. What is the least number of $1.00 bills that his mother should give him so he will have enough money to buy lunch for 5 days?” (Solano-Flores & Trumball, 2003, p. 5)

The issue with this problem lies in the third sentence – specifically with the word “only”. Solano-Flores & Trumball found “10 and 18%, respectively of the American Indian and African-American students interpreted the word only as restricting the number of dollars” (2003, p. 5). This means that they thought the sentence meant that Sam’s mother only had $1. This is not an observation that Solano-Flores & Trumball made when White students answered this question (2003). Providing tests without the stress of language factors provides a picture of how background culture can affect a student’s performance (Solano-Flores & Trumball, 2003).

According to Campbell (2009), the amount of time that teachers invest in learning about students’ backgrounds and cultures can influence their performance. Teachers who are from typically underrepresented groups report that cultural differences are important, but can provide conflict between non-minority teachers and minority students. However, non-minority teachers report no problems developing relationships with students of different backgrounds and culture. Some students are self-motivated to succeed in their studies and a relationship with the teacher is secondary to that. Other students need the relationship with the teacher in order to succeed in class. These students may be dealing with at least one of the struggles listed above. For these students, having a relationship with a teacher who understands those struggles and can provide guidance regarding how to deal with outside
stresses and still perform well in school could make the difference between passing and failing a course. Teachers need to work to develop relationships with all of their students and especially to reach out to those students who need the relationship in order to find the motivation to succeed.

Support from home is a significant factor in achievement in education. The goal of every educator is to involve students in their instruction and to deliver relevant material that ties to the curriculum in a way that all students will be successful. Some students are not motivated to participate in class. Research has shown that when it comes to ELL students, well-educated parents are more likely to push their children to work hard, study, and even become proficient in the English language in a timely manner whereas students whose parents do not have education past high school, may not push their children to become proficient in the language quickly which as a result, could affect success in the classroom. Abedi & Dietel support this claim in their 2004 article where they reported that students whose parents had some postgraduate work scored 15 percentiles higher than students whose parents did not graduate from high school.

**Instruction for English Language Learners**

English Language Learners come to the United States at all ages and from many countries throughout the world. As educators, we need to make these students feel welcome in the classroom and include them in the regular classroom instruction. There is a significant amount of research into instruction and assessment for English Language Learners. This research includes clarification of many misconceptions that teachers form when ELL
students enter their classroom, strategies for teaching ELL students, and the challenges that face ELL students in the mathematics classroom.

Clarification of Misconceptions

Many teachers believe that since students were able to acquire a first language, that second language acquisition is similar to first language acquisition. This in fact is not true. There are many factors that play a role in second language acquisition including motivation, culture, and level of first language acquisition. Studies have shown that students who are more advanced in their first language are more likely to acquire their second language at a faster rate. However, a student that is not motivated to acquire his second language or who has little support from home is unlikely to successfully develop his second language at the same rate as a student willing to work at it. Harper & de Jong (2004) made educators aware of the misconceptions surrounding ELL students’ ability to learn English and provide recommendations regarding how to provide the best educational setting for these students.

Students learning English as a second language have a tendency to switch back and forth between their first and second language as they solve math problems. It does not mean that the students do not understand the mathematics or the language; it is actually a “complex language practice” (Moschkovich, 2003, p. 13). ELL students use each language for many different reasons, in different environments, and around different people. Some carry out operations in their native language and then translate their answer to English. There is no research to support the belief that ELL students are any more or less able to reason mathematically; however it may appear that they struggle with the mathematics as ELL students could take longer to respond (Moschkovich, 2010).
The same strategies that work for teaching students whose native language is English, do not necessarily work for ELL students (Harper & de Jong, 2004). It is important to “use visuals, demonstrations, and graphic organizers” in the classroom as a means to make content more accessible to ELL students (Bauer & Manyak, 2008, p. 176; Manyak & Bauer, 2009). For example, a math teacher in a Geometry class might use a picture in addition to a written problem in an effort to level the playing field between ELL students and non-ELL students.

Including English Language Learners in instruction is crucial to their success and self-esteem. Bauer & Manyak state that teachers can include students in classroom instruction by avoiding the use of idioms, providing sufficient wait time, and allowing students to practice both English and students’ native language (2008, p. 177). Harper and de Jong (2004) point out that teachers should not simply redesign instruction so that ELL students are completely nonverbal. While it is acceptable to redesign lessons to meet their needs and to simplify some of the language and language requirements on ELL students, learning to both read and speak English is important for their growth and teachers that do not require any verbal input from ELLs hinder that learning experience (Harper & de Jong, 2004).

**Instruction for ELL Students**

ELL students need to be placed in classes that are equal to their academic level prior to coming to the United States. Lewis-Moreno (2007) points out that it does not help ELL students learn English to be stuck in classes “remediating skills they already have in a language they have yet to acquire” (p. 773). Instead, she suggests placing these students in honors and AP classes, which will honor the skills they bring with them (Lewis-Moreno,
In addition to placing them in classes appropriate to their skill level, these students need scaffolding designed to help students learn English (Lewis-Moreno, 2007). Taking her suggestions could help to eliminate some of the trouble that states are facing in preparing ELL students to attend college. Finkelstein, Huang, & Fong (2009) studied the course patterns that ELL students take and the need for remediation courses in college. They found that not only is there a small population of ELL students who attend college, but those that do often need to take remediation courses because of the courses they took in high school. Poor performance on end-of-course tests resulted in students taking low-level courses, which in turn does not prepare them for college level courses (Finkelstein, Huang, & Fong, 2009).

English as a Second Language teachers should be resources on campus to help teachers best determine the modifications for each ELL student in the classroom. All teachers share the responsibility to educate ELL students and to incorporate any modifications necessary to level the playing field between ELL students and native English speakers. The best way to accomplish this goal is for teachers to have training in teaching English as a Second Language to help them prepare for teaching these students with special needs. An example of that training is the Sheltered Instruction Observation Protocol (SIOP). SIOP began as a teacher observation tool and developed into a method that teachers use to plan lessons to improve instruction for ELL students (Short & Echevarria, 1999).

In younger grades, many teachers use a “word wall” as a way to connect students’ native language and English as a means of helping students develop their English speaking/reading skills, which in turn make students more successful in the classroom (Bauer & Manyak, 2008, p. 177). In high school, scaffolding can be accomplished by providing
partially completed graphic organizers or provide materials in a student’s native language and in English.

Currently, many of our classrooms do not provide the scaffolding needed for ELL students to be successful. Many teachers, especially in mathematics, design their classrooms so that the teacher lectures and the students listen and copy notes. These are not strategies that aid in English language learning – in fact they impede it. It is crucial for students to be involved in the lesson each day (Campbell, Adams, & Davis, 2007). Some suggestions for inclusion of all students in daily lessons are using Think-Pair-Share activities, peer review of papers in English class, pretests such as a People Hunt or an Anticipation Guide, an Open Word Sort helps students familiarize themselves with vocabulary they can expect to see in the next unit (Lewis-Moreno, 2007; Lenski, Ehlers-Zavla, Daniel, & Sun-Irminger, 2006). In mathematics classes teachers involve students in learning by posing questions, evaluating hypotheses, checking the accuracy of work, and focusing students on understanding the problem as a means for solving difficult mathematics problems (Campbell, Adams, & Davis, 2007). These activities are designed to increase student buy-in in the classroom, develop vocabulary, increase student success, and make all students feel equal in the classroom.

**Challenges in the Mathematics Classroom**

ELL students in the mathematics classroom have a double responsibility: to learn English and to learn mathematics. It is the job of the mathematics teacher to teach both the vocabulary necessary to learn mathematics and to teach mathematics. This becomes increasingly difficult with words such as *function* or *translation* that have a meaning in the mathematics classroom and a meaning out of the mathematics classroom. Mathematics
teachers also have to teach language in a mathematics classroom. But for ELL students, learning that language and making it usable in the mathematics classroom is crucial for success in the mathematics classroom (Cuevas, 1984; Harper & de Jong, 2004; Campbell, Adams, & Davis, 2007). MacGregor & Price (1999) support the claims that language plays a part in mathematics achievement. They conducted a study in which they found a positive correlation between language skills and mathematics achievement. They make the claim that before a student can be successful at a test of their mathematical ability; they must first sort through the language. They support this claim with the results of their study in which students with low English ability scored low mathematically and students with high English ability scored high mathematically. This is why teachers need to provide intense, focused vocabulary instruction and to integrate that instruction into the classroom each day (Manyak & Bauer, 2009).

**Programs to Support ELL Students**

ELL students need specialized instruction in a small group setting upon entering US schools before they can be expected to join their native English speaking counterparts. There are many opinions and suggestions regarding how to accomplish this. The driving force behind this specialized instruction is the No Child Left Behind Act that requires 100% proficiency in all areas by all subgroups by the year 2014 (Abedi & Dietel, 2004).

One suggestion is to use Sheltered Instruction Observation Protocol (SIOP) for ELL students. SIOP was originally designed as an observation evaluation tool. However, the teachers involved in the original study decided that since they were going to be evaluated based on this method, they should be able to use the observation tool to plan their lessons.
When a teacher is trained in the SIOP program he should walk away with various skills, strategies, and modifications to help ELL students be more successful in their classes (Short & Echevarria, 1999). The “sheltered instruction” aspect of this model is designed to provide high quality instruction to non-native English speakers which will in turn make ELL students more successful in the classroom (Short & Echevarria, 1999). Many school systems use a type of team teaching in the classroom to deliver sheltered instruction for ELL students in an effort to educate students and close the achievement gap between ELL students and native English speakers.

However, through English as a Second Language (ESL) programs and sheltered instruction, ELL students may not receive the same rigorous education as their native English speaking peers. According to Callahan, Wilkinson, & Muller (2009) and Finkelstein, Huang, & Fong (2009) students placed in ESL programs are often not placed in college preparatory classes and many of them are not prepared to attend a four year university upon graduating from college. Not only that, many of these students are unaware that they are being steered away from these courses. We must place ELL students into the correct classes and provide rigor in these courses so that they can have the same opportunities as their classmates. Instead of putting ELL students into classes below their academic level, they should be placed in higher level courses and modifications should be provided to decrease the language barrier (Callahan, Wilkinson, & Muller, 2009; Callahan, 2005; Finkelstein, Huang, & Fong, 2009). Educators should be cautious of integrating ELL students into the classroom with native English speakers because very low English-proficient students stand a greater chance of being left behind as the remainder of the class moves forward. In addition, lack of funding
and support from school personnel create undesirable learning conditions for ELL students (Platt, Harper, & Mendoza, 2003).

Another suggestion for using sheltered instruction is to use an online course called Help with English Language Proficiency (HELP) along with digital learning tools to include ELL students in the learning process and provide extended time. The advantage to online courses is that students can work at their own pace through the material. The hope for this type of course is that if students have worked hard and put forth the necessary effort, they have learned the material by the end of the course. HELP math curriculum is an integration of sheltered instruction and online instruction for ELL students. Research has shown that both ELL and non-ELL students are beginning to benefit from this program (Freeman & Crawford, 2008). In 2010, López studied the effect of integrating technology into the classroom. In particular, he studied a school district that implemented a “Digital Learning Classroom project, an initiative focused on the improvement of [ELL students] learning using interactive whiteboard (IWB) technology” (López, 2010, p. 901). The description of the IWB technology that follows has capabilities similar to a SMARTBoard. In addition to the IWB, each classroom has an “electronic slate … [that is] a wireless, fully integrated, mini-board that is small enough to sit on a student’s desk and be moved around the classroom” (López, 2010, p. 901-902). His study showed that ELL students that take part in a digital learning classroom score higher than ELL students that participate in a traditional classroom setting. This is one way that teachers can modify curriculum to meet the needs of their students.
**Modifications to Date**

A modification is a type of alteration made to instruction, assessments, or the classroom environment designed to help students that might be at a disadvantage achieve the same level of success as students who are not faced with the same disadvantages. Abedi, Hofstetter, & Lord (2004) investigated multiple types of modifications and developed a list of criteria that the modifications should meet in order to be an appropriate modification. His research included providing tests in a student’s native language, rewording test questions to remove extraneous phrases, providing extra time on assessments, and allowing students to use a dictionary or customized glossary.

Through their research, Abedi & Dietel (2004) discovered that providing tests in a student’s native language does not necessarily increase test scores. They concluded that the issue was not with the test but that the students’ language skills caused a downward effect on their test taking ability. Solano-Flores & Li (2009) conducted a study in which they administered a test to a group of ELL students in both English and Spanish. They then administered the same test in a traditional Spanish dialect and a local dialect of Spanish. In each case, there was a large variance in test scores (between 41% and 48%). Similar to Abedi (2002), they concluded that each student brings their own unique strengths and weaknesses in both their native language and in their English ability.

The results of the research by Abedi, Hofstetter, & Lord (2004) prove that a customized glossary coupled with extended time is the most effective modification based on his research up to 2004. Because ELL students do not often receive instruction in their native language, it is not helpful for them to take tests in their native language, as they cannot
express what they have learned as appropriately as they otherwise could (Butler & Stevens, 1997).

There have been many studies conducted regarding rewording questions (to make the language easier for all students) without providing an advantage to ELL students (Solano-Flores, 2010). In particular, questions are reworded, simplified, and in some cases shortened a bit to retain the skill level but alter the language demand on the student (Abedi, Hofstetter, & Lord, 2004; Abedi & Dietel, 2004; Shaftel, Belton-Kocher, Glasnapp, & Poggio, 2006; Solano-Flores, 2010; Rivera & Stansfield, 2001). The results are overall relatively inconclusive. Some studies show that when questions are reworded, shortened, and language is simplified, many ELL students tend to score better on the test (Abedi, Lord, & Hofstetter, 1998; Shaftel et. al., 2006; Martiniello, 2009; Solano-Flores, 2010). In addition, younger students have a greater tendency to be affected by language on test items (Shaftel et. al., 2006). However, shortening and rewording questions does not aid in comprehension but does affect the students’ attitude toward taking the test because they expect the simplified text to be easier to understand (Lotherington-Woloszyn, 1993). In another study, there was no noticeable difference in the students with modified tests and those without (Brown, 1999). Using computer testing with a pop up glossary did prove effective in leveling the playing field between ELL students and native English speaking students (Abedi, 2009).

However, one could argue that using language that ELL students understand could make assessments more valid (Solano-Flores, 2010). One challenge facing educators and researchers these days is the reliability of tests for ELL students. The goal of the modifications for assessments is to measure the students understanding of the content in the
course absent from the demands of the English language. Assessments must be constructed so that all students have equal opportunity for success (Solano-Flores, 2010). It is impossible to compare native English speakers to ELL students if the language is incomprehensible for them (Wolf & Leon, 2009; Martiniello, 2009) and it is important to have a measure of mathematical ability that is not affected by how much English an ELL student knows.

Extended time is one of the most commonly used and accepted accommodations (Abedi, Hofstetter, & Lord, 2004). Extended time is a simple modification that results in no changes to the test itself. However, studies done to track its effectiveness have proven to be inconclusive (Abedi, Lord, Hofstetter, & Baker, 2000). Using published glossaries was not an effective strategy and the glossaries were difficult to use. (Abedi, Courtney, Mirocha, Leon, & Goldberg, 2001). However, allowing students to make their own glossary and providing extra time was the most effective strategy. Students scored the highest when tested using both accommodations (Abedi, Lord, Hofstetter, & Baker, 2000).

Similar to the modifications addressed above, Lenski, Ehlers-Zavala, Daniel, & Sun-Irminger (2006), suggested multiple methods for assessing ELL students. Their focus was on alternative testing strategies. While they did make conventional suggestions such as simplifying the language and using the students’ native language, their focus was on a collection of assessments which highlight what students can do. Equally important in assessing students is understanding the background of each ELL student and using that information to analyze assessments. Lenski, et. al. (2006) suggests testing through the use of Venn diagrams, charts, PowerPoint slides, and student self-assessments. They also suggest modifications to traditional assessments. Teachers are more likely to employ modifications to
traditional assessments because most of our students have to take standardized tests at the end of each course. There is a limit to how much these tests can actually be modified for ELL students. This will cause teachers to be cautious in their modifications because students need to learn the information and prepare for the test over which the teachers have no control.

**Language and Mathematics**

Schleppegrell writes, “Every school subject is constructed in language” (2010, p. 74). Specifically for mathematics, there is a verbal language, which translates to a visual language (graphs, diagrams, and pictures) and a symbolic language (variables, statistical symbols, and geometric symbols). Geometry is one specific example of a mathematics course in which language is important to student understanding.

Specifically in learning how to write proofs, when students do not understand the language this could lead to not understanding the concepts, not being able to state definitions, or generate and use examples. Lack of any one or all of these skills could affect student’s abilities to write complete and accurate proofs (Moore, 1994).

Teachers need to make both language and content accessible to ELL students and to do so in a way that is respectful of all cultures and backgrounds. Sometimes teachers can inadvertently put up walls that prevent them from effectively teaching ELL students. For instance, a teacher may think that a student that does not have complete command of the English language may not be able to learn the necessary language of mathematics and therefore as a result will be unable to learn mathematics (Gorgorió & Planas, 2001).

In reality, if the teacher works to form a relationship with the ESL department as opposed to relying on the ESL department to teach mathematical language, together they can
help ELL students achieve success in a mathematics classroom (Lager, 2006). In this cooperative team with the ESL department, it is important for all teachers to remember that whether a student is an ELL student or a non-ELL student, no two cultures or students are alike. For example, different cultures may have different ways to construct a mathematical statement (Schleppegrell, 2010). This could present challenges in the mathematics classroom. In addition to background, culture, parent involvement, and modifications, students that are on grade level, attend school regularly, and have good command of the English language can have a positive effect students’ ability to pick up the mathematical language and apply it in class (Lager, 2006; Schleppegrell, 2010; Gorgorió & Planas, 2001).

**Deficit Model**

Gutiérrez & Rogoff (2003) define the deficit model as a method “in which cultural ways that differ from the practices of dominant groups are judged to be less adequate without examining them from the perspective of the community’s participants” (p. 19). The deficit model comes directly from making assumptions about students based on characteristics that are often negative in nature (Moschkovich, 2010).

Unfortunately, it is extremely easy to make assumptions about students without stepping back to see the whole picture. Moschkovich (2010) offers three suggestions to researchers and educators as they try to move away from the deficit model. The first is to stop comparing English Language Learners to the native English-speaking counterparts. In order to help English Language Learners be more successful in the classroom and to study ways to improve instruction for them, the focus needs to be solely on ELL students and not on how ELL students compare to native English-speaking students. Second, research needs
to move away from the mathematics that ELL students do not know and should focus more on the mathematics knowledge that ELL students do have. Third, research has shown that when parents are more educated, they are more involved in their students education and as a result the students are more likely to be successful in class. According to Moshkovich (2010), parents of ELL students are just as involved (if not more involved) as native English-speaking students so this subgroup should not be dismissed based on their ethnicity.

**Introduction to Geometry**

The research has shown that many ELL students struggle with mathematics. Command of the English language is critically important in mathematics classes. One subject in particular in which ELL students struggle, is in Geometry. Geometry is a subject that uses a lot of words and places importance on sentence structure which can be a significant challenge for ELL students. However, teaching Geometry is important. According to Usiskin (2007): “1. Geometry *uniquely* connects mathematics with the physical world. 2. Geometry *uniquely* enables ideas from other areas of mathematics to be pictured. 3. Geometry *nonuniquely* provides an example of a mathematical system.” (p. 72) The preceding research focused on the challenges facing ELL students in mathematics classrooms. The following research sheds light on the importance of proof in Geometry and its place within a full mathematics curriculum. According to Usiskin (2007), “for most teachers, proof is the most important concept…of the geometry course” (p. 73).

**Definitions of Proof**

According to Waring (2001) the definition of mathematical proof varies depending on the level of mathematics. Experts in the field of mathematics might regard proof as “a
complex, rigorous, sometimes lengthy, argument” (p. 4). However, for students in high school Geometry, proof may not be complex, lengthy, or rigorous. Porteous (1994) defines proof for the beginning mathematician as “any adequate expression of the necessity of its truth” (p. 5). This definition of proof provides much more realistic expectations of high school Geometry students. Many teachers define proof in a similar fashion.

Proof has meaning both within the walls of a mathematics classroom and in the surrounding world. The meanings are developed from contexts such as daily life, experimental sciences, professional mathematics, and logic and foundations of mathematics and can impact students’ ability to construct arguments within a mathematics classroom (Recio & Godino, 2001). In daily life absolute truth is not a necessity. If a situation does not follow the rule, it is seen as an exception to the rule—it does not make the rule less valid (Recio & Godino, 2001). Scientific argumentation has a truth value. There is an aspect of absolute truth and verification. The mathematical proof is used to verify the truth of scientific ideas based on a theory or hypothesis (Recio & Godino, 2001). Professional mathematics is concerned with deductive reasoning and informal proof while logic and foundations of mathematics links proof to “deductive reasoning and formal systems” (Recio, & Godino, 2001, p. 94).

The Importance of Teaching Proof

A student once asked, “Who made up math?” Standing in front of the class, the teachers mouth dropped open and she stared in amazement at the student. After a minute, she replied, “Nobody made up math. Math is proven.” This interaction demonstrates the importance of teaching proof from a young age. Based on the definitions of proof from the
research, proof does not have to be formal; it does not have to be complex, or difficult (Waring, 2001). But proof must be taught. The National Council of Teachers of Mathematics (NCTM) supports teaching justification and proof at all levels (2000). Fawcett presents the question, “What is the unique contribution which demonstrative Geometry makes to the general education of young people in our secondary schools?” (1938, p. 3). Fawcett goes on to cite responses to his question (1938). According to Reeve (as cited in Fawcett, 1938):

The purpose of geometry is to make clear to the pupil the meaning of demonstration, the meaning of mathematical precision, and the pleasure of discovering absolute truth. If demonstrative geometry is not taught in order to enable the pupil to have the satisfaction of proving something, to train him in deductive thinking, to give him the power to prove his own statements, then it is not worth teaching at all. (p. 3-4)

Similarly, Keyser summarizes the purpose of the tenth grade year as “to teach the nature of deductive proof and to furnish pupils with a model of all their life thinking” (in Fawcett, 1938, p. 5). Knuth (2002a) points out that recommendations by NCTM “indicate that proof is expected to play a much more significant role in school mathematics than it has in the past” (p. 486). Proof can highlight the connections between mathematical topics. It can also serve to connect new information to previous knowledge (Waring, 2001). Proof can take on various forms in the mathematics classroom. While one major role of proof is to establish truth, it can also be used to explain and understand mathematics (Knuth, 2002a; Knuth, 2002b). De Villiers (2004) provides reasons why proof is important:

- Explanation (providing insight into why it is true)
- Discovery (the discovery or invention of new results)
• Intellectual challenge (the self-realization/fulfillment derived from constructing a proof)

• Systematization (the organization of various results into a deductive system of axioms, concepts and theorems (p. 704).

According to Usiskin (2007), there is no other topic in mathematics that provides the subject with as much variety as proof. It is easy to teach the rules and procedures of mathematics all day long. Where it is easy to fall short sometimes is teaching why and how math is so connected and explaining to students why certain statements are true. Sometimes students are willing to accept what the teacher says at face value and occasionally teachers do not push students to explain why. They are much more content to perceive school as “the providers of knowledge which pupils must absorb in order to pass examinations” (Waring, 2001, p. 5). In fact, some students often see Geometry as the math they have to pass so they can take Algebra II because in Algebra there is very little emphasis on proof (Sriraman, 2003).

In addition to understanding how mathematics is connected, teaching proof helps students develop their logical thinking abilities (Herbst, 2002). The researcher’s students often ask why they have to learn how to prove. She tells them that if they ever have to argue a case or prove their point so they can win a debate, they need to have the ability to connect facts in a way that makes logical sense. Epp (2003) investigated the importance of logic and claims it assists in providing “a language for instructors to explain why mathematicians do the things they do when they prove and disprove mathematical statements and to communicate with their students when the make mistakes” (p. 895). In fact, if deductive
reasoning is not taught and students are asked to explain their answers though a larger emphasis is actually placed on algebraic manipulation of rules and facts, the interconnectedness and logical reasoning aspect of mathematics will be lost (Waring, 2001). Thus, proof can show students how interconnected mathematics is and why a statement is true (Hanna, 2000).

The History of Proof in Schools

Proof has been part of the high school Geometry curriculum since the 19th Century when the Committee of Ten and the Mathematics Conference determined that the best place for proof was in the high school Geometry curriculum. Over the years, proof has evolved from paragraph form with little reasons to support it to the two-column proofs commonly taught in Geometry classrooms today. González & Herbst (2006) studied 20th Century Geometry in the public schools and define “modal argument” as “central tendencies around which the opinion of various individuals could converge” (p. 13). González & Herbst (2006) presented four modal arguments, each of which contains an aspect of proof, justifying the Geometry course:

“Formal Argument: Geometry Teaches to Use Logical Reasoning” (p. 13),

“Utilitarian Argument: Geometry Prepares Students for the Workplace” (p. 16),

“A Mathematical Argument: Geometry for the Experience and Ideas of Mathematicians” (p. 18), and

“An Intuitive Argument: Geometric Expression Helps Students Interpret their Experiences in the World” (p. 20).
The formal argument not only teaches logical reasoning but “a major step from the publication of the Committee of Ten…was the articulation of ways to enact the idea of teaching for transfer (González & Herbst, 2006, p. 13). The view of proof by those who supported a formal argument was that “proofs give opportunities to practice deductive reasoning detached from geometric concepts” (González & Herbst, 2006, p. 23).

Those that supported the utilitarian argument believed that the purpose of Geometry was to prepare students for their future careers which meant that “proofs are not as important as problems that apply geometry to future jobs” (González & Herbst, 2006, p. 23). Because the goal of the mathematical argument was to provide experiences similar to that of mathematicians, González & Herbst (2006) found that there was a mix of beliefs in the research. Some researchers believed that Euclidean Geometry was satisfactory while others believed that in order for students to have experiences similar to mathematicians, they should experience other Geometries as well (González & Herbst, 2006). With regard to proof, these mathematicians felt that “proofs of original problems provide opportunities to experience the activity of mathematicians” (González & Herbst, 2006, p. 23).

Those in favor of the intuitive argument “made a case for geometry as a unique opportunity for students to apply the intuition of the geometric objects to describing the world” (González & Herbst, 2006, p. 20) and believe that “proofs follow informal appreciation of geometric concepts, blurring differences between definitions, and postulates and theorems” (González & Herbst, 2006, p. 23).

With the discussion of proof in high school classroom comes the notion of authentic mathematics. There are four notions of proof that fall under the category of authentic
mathematics: real-world contexts, the nature of the discipline, practices of mathematicians, and “student as mathematician” (Weiss, Herbst, & Chen, 2008, p. 278). These four notions are all ways in which teachers have emphasized proof in the classroom over the years. Recently the focus has been on relating proof to real-world contexts such as “a lawyer trying to persuade a jury with evidence, a doctor trying to reach a conclusive diagnosis of a patient’s ailment, a teenager trying to persuade her parents that they ought to buy her a car” (Weiss, Herbst, & Chen, 2008, p. 279).

The two-column proof is used more as a teaching method than a way to record findings in an orderly manner which have led to calls to scale back the use of the two-column proof in high school but with the absence of two-column proof also comes elimination of proof altogether (Weiss, Herbst, & Chen, 2008). Despite the dislike of the use of two-column proofs, it will continue to have a secure home in the high school Geometry curriculum (Weiss, Herbst, & Chen, 2008).

Also, the need for Geometry concepts to be taught prior to high school Geometry caused a course in Geometry to be added to the elementary school curriculum as well (Herbst & Brach, 2006). This indicates how important the Committee of Ten feels prior knowledge is. The same can be said today. Because mathematics builds on itself and is so connected, teachers count on their students bringing with them a certain amount of prior knowledge.

However, in 1989 when NCTM first introduced their standards, the importance of proof began to dwindle. In fact, two-column proof was suggested to receive decreased emphasis. Proof was not explicitly mentioned in these standards at all. Types of proof or justification and explanation were mentioned in the standards but emphasis on two-column
proof had disappeared. In 2000 NCTM has published new standards indicating that proof should be taught in every grade beginning with Kindergarten (Hanna, 2000). According to Hanna (2000), there is significantly less emphasis placed on proof now in the UK than in years past.

**Teaching Proof**

Keeping in mind that proof is the basis of mathematics, it should be incorporated into all grade levels and all areas of mathematics (Johnson, Thompson, & Senk, 2010). Proof can be taught through the exploration of patterns and to justify new mathematical ideas. Proof can also be incorporated in non-geometry subjects through the textbook.

In 2010, Johnson, Thompson, & Senk investigated twenty Algebra I, Algebra II, and Precalculus textbooks. They used NCTM Reasoning and Proof Standard (2000) as the basis for their research (Johnson, Thompson, & Senk, 2010). Johnson, Thompson, & Senk (2010) did find evidence of problems requiring students to use mathematical proof in answering a question. According to Johnson, Thompson, & Senk (2010), “proof-related reasoning includes reading arguments, finding counterexamples, making and investigating conjectures, developing and evaluating arguments, and correcting mistakes in arguments” (p. 416). They point out that students can engage in proof-related activities without having to create a formal proof and with appropriate guidance and instruction from the teacher, will be more prepared and fluent in proof by the time they reach geometry (Johnson, Thompson, & Senk, 2010). However, in order for this to happen, the teacher has to actually assign those problems and hold students accountable for completing them and providing sufficient answers. Epp (2003) states, “Many students arrive in a course emphasizing proof never having had to write a
complete sentence in a mathematics course” (p. 892). If a student had never been required to justify a response by writing a complete sentence in mathematics or if a student had been told to write a complete sentence but was never held accountable for the content or structure of his sentence the geometry teacher would be faced with integrating the importance of correct sentence structure and content into the course along with making sure the student is able to write a proof by the end of the geometry course.

Recio & Godino (2001) studied a group of beginning University students in Spain. Based on previous research and their own findings, they outlined definitions of proof that develop from different contexts that students bring with them to the mathematics classroom: daily life, empirical sciences, professional mathematics, and logic and foundations of mathematics (Recio & Godino, 2001). They believe that teachers must take into consideration both these definitions and students proof schemes (discussed in detail in a later section) in order to structure a class and write lessons that will benefit every student (Recio & Godino, 2001).

Fawcett successfully taught Geometry to a group of students by introducing real scenarios to develop the importance of defining (1938). His class decided that:

1. Definition is helpful in all cases where precise thinking is to be done.

2. Conclusions seem to depend on assumptions but often the assumptions are not recognized.

3. It is difficult to agree on definitions and assumptions in situations which cause one to become excited. (Fawcett, 1938 p. 34)
His class discussions developed from the presentation of problems from which students could develop definitions and make assumptions (Fawcett, 1938). By the end of his course, his students raved in their evaluations. Many of them did not believe they would enjoy Geometry but by the end of the course, most of them raved about the experience. One student reported that he “wouldn’t have missed the course for anything” (Fawcett, 1938, p. 112).

Waring (2001) suggests using a combination of proof activities and proof discussions to stimulate students at varying levels of reasoning and to provide support so that students learn to create their own proofs. Knuth (2002b) suggests that teachers draw on their own definitions of proof as a means to teach proof in the classroom. However, this can have both good and bad effects on teaching proof to students. Knuth also suggests that teachers continue to challenge their own beliefs and definitions of proof to improve their ability to teach proof to high school students (2002b).

An example of this can be found in a study conducted by Martin, McCrone, Bower, & Dindyal (2005), who studied a high school Honors Geometry teacher and his interactions with his students as he taught them how to prove. In this class, students were able to have discussions with their teacher and through this coaching, his students were able to develop and justify their arguments which therefore led them to more success in writing proofs (Martin, McCrone, Bower, & Dindyal, 2005). This is an example of how mathematical proof can be used to convey and promote mathematical understanding in which Hanna (2000) describes the goal of teaching proof in the classroom is to enhance students’ mathematical knowledge and help them expand that knowledge as well as connect it to previous learning.
Waring’s (2001) first phase, learning about proof addresses students need to understand that proof is everywhere in mathematics. NCTM (2000) recommends, students should experience proof through patterns and by justifying their mathematics from kindergarten and up. It is not simply acceptable for a student to only know what a proof is but to use proof to help them understand mathematics. In his study, Weber found that just because a student knows what a proof is and what it looks like, does not guarantee that the student can create a formal proof (2001). One major concern discovered by Stylianides & Stylianides (2009) is that some teachers are unable to tell the difference between empirical evidence and proof. Some teachers believe that simply trying examples is proof enough for elementary school. Research from Stylianides (2007) suggests that students should learn proof in a coherent manner throughout their schooling and teachers should not accept empirical arguments as proof (Stylianides & Ball, 2008). Teachers should not accept empirical arguments as proof because they contain “incomplete evidence” (Stylianides & Ball, 2008, p. 310). Therefore, if teachers are unaware of what is acceptable proof for elementary school children, they may not be able to push their students to a higher level of proving which could have adverse effects on their ability to construct a proof throughout their education.

Using proof to understand mathematics is a definition of proof that teachers left out of the list when interviewed by Knuth (2002b). Teachers are responsible for “helping students construct mathematical ways of knowing that are compatible with those of wider society” (Stylianides, 2007). Teachers must first be able to judge an argument as a proof or simply empirical evidence and then use that to shape learning in the classroom by teaching students
how to evaluate proofs as well (Styliandides, 2007). If this is effective, we can hope that by the time a student reaches Euclidean Geometry in high school, he or she will be ready for phase two – learning to prove. This is the phase where formal proof is introduced and both formal and informal proofs are used to justify mathematics. It is important in this phase not to only use proofs that both students and teachers already know to be true and can intuitively see but to challenge the students to construct new proofs that will make the mathematics more personal for them (Knuth, 2002a). The final phase of teaching proof according to Waring (2001) is to improve proof skills. This phase is meant for higher level more complicated mathematics and may not apply to students in Euclidean Geometry. These three phases should provide the support to create mathematically literate students who are able to not only prove but to reason logically (Waring, 2001).

**Van Hiele Levels and Proof**

“A husband-and-wife team of Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof” developed levels for Geometric reasoning after noticing the trouble their own students had in learning Geometry (Mason, 1998, p. 4). The van Hiele’s initially labeled the levels 0-4 however some researchers (including Pierre van Hiele) have relabeled the levels 1-5 (Mason, 1998; van Hiele, 1986). The labeling of 1-5 is an American label where the labeling of 0-4 is European (Mason, 1998). In addition, the labeling of 1-5 allows for an additional label of 0 for what Clements and Battista call pre-recognition (Clements & Battista, 1992). Clements & Battista (1992) define this “pre-recognition” level as:

children perceive geometric shapes, but perhaps because of a deficiency in perceptual activity, may attend to only a subset of a shape’s visual characteristics. They are
able to identify many common shapes … they may differentiate between a square
and circle, but not between a square and a triangle … Thus, students at this level may
be unable to identify common shapes because they lack the ability to form requisite
visual images. (p. 429)

Clements & Battista (1992) coined this level based on “the bulk of the evidence from van
Hiele-based research, along with research from the Piagetian perspective” (p. 429). This
research indicated that there exists a level that is “more primitive than and probably
prerequisite to van Hiele’s Level 1” (Clements & Battista, 1992, p. 429).

The van Hiele’s believe that children progress logically through the levels of
Geometric reasoning (Usiskin, 1982; Burger & Shaughnessy, 1986; Senk, 1989; Mayberry,

In 1986, Pierre van Hiele wrote a book in which he provides insight into the theory
behind the van Hiele levels and reinforces the purpose of the van Hiele levels. In his book
van Hiele outlines the levels he and his wife developed. He renumbered the levels as 1-5
instead of the original 0-4. According to van Hiele (1986), the van Hiele levels are titled as
follows:

- **First Level**: the visual level
- **Second Level**: the descriptive level
- **Third Level**: the theoretical level; with logical relations, geometry generated
  according to Euclid
- **Fourth Level**: formal logic; a study of the laws of logic
- **Fifth Level**: the nature of logical laws (p. 53)
Van Hiele (1986) points out that the transition from one level to the next is “not a natural process” and that movement to the next level “takes place under influence of a teaching-learning program” (p. 50). With this in mind, van Hiele (1986) outlines five stages that each student progresses through as he transitions to the next level:

1. In the first stage, that of *information*, pupils get acquainted with the working domain.

2. In the second stage, that of *guided orientation*, they are guided by tasks (given by the teacher or made by themselves) with different relations of the network that has to be formed.

3. In the third stage, that of *explicitation*, they become conscious of the relations, they try to express them in words, they learn the technical language accompanying the subject matter.

4. In the fourth stage, that of *free orientation*, they learn by general tasks to find their own way in the network of relations.

5. In the fifth stage, that of *integration*, they build an overview of all they have learned of the subject, of the newly formed network of relations now at their disposal.

(p. 53-54)

Many researchers have used the van Hiele levels to conduct research. Burger & Shaughnessy (1986) found that the van Hiele levels are useful in “describing students thinking processes on polygon tasks” (p. 46; Clements & Battista, 1992). As a result, their study can be extended to other Geometry concepts “such as measurement, transformations, congruence, and similarity” (Burger & Shaughnessy, 1986, p. 46).
However, most commonly, researchers find that one reason students struggle in Geometry is due to the fact that they are receiving instruction at a higher van Hiele level than they have reached (Usiskin, 1982; Mason, 1998; Senk, 1989; Mayberry, 1983; van Hiele, 1986; Clements & Battista, 1992). If a student is pushed to operate at a level that he has not naturally risen to, it is possible that the student will rely on basic memorization and algorithms in an attempt to survive the course (Mayberry, 1983; Mason, 1998). According to Usiskin (1982), a student who is able to understand a proof when the instructor does them in class, but is unable to create his own proofs at home is operating at level 3 while the instructor is operating at level 4. Mayberry (1983) studied a group of pre-service teachers. In her study, she discovered that “there was a difference in levels achieved by those students who had taken high school geometry and those who had not” (p. 67). She goes on to assert that if a student has not received enough Geometry instruction to reach at least level II prior to the start of a course involving deductive reasoning, they “may not benefit from a course in formal geometry because their knowledge and the content of the textbook are organized differently” (p. 68).

Usiskin (1982) studied 2,699 students from 13 schools. The students were given two tests upon entering Geometry (a van Hiele level test and an entering Geometry test) and three tests near the end (a van Hiele level test, a Geometry posttest, and a proof test). Usiskin (1982) used the results of these tests to draw many conclusions.

First, in examining how instruction affects van Hiele level, he realized “there is great variability in the change in van Hiele level from fall to spring” as approximately one third of the students remained at their current level or decreased one level, one third of the students
moved up one level, and one third of the students moved up more than one level (Usiskin, 1982, p. 81).

Second, based on increasing positive correlations on standardized Geometry tests and proof tests, Usiskin (1982) asserts:

van Hiele level is a good predictor of concurrent performance on multiple-choice tests of standard geometry content. Van Hiele level is also a very good predictor of concurrent performance on a proof test, but a content test correlates even higher with proof. (p. 82)

Third, Usiskin (1982) investigated the correlation between students’ van Hiele level and ability to write proofs. In his article, Usiskin (1982) investigated two different criterion: 3 questions correct out of 5 questions and 4 questions correct out of 5 questions. “The 3 of 5 criterion minimizes the chance of missing a student and yields an optimistic picture of students’ levels; the tougher 4 of 5 criterion minimizes the chance of a student being at a level by guessing” (Usiskin, 1982, p. 80). In studying the relationship between van Hiele levels and proof, there is one level in particular that showed a significant difference in students’ ability to write proofs (Usiskin, 1982). For the 3 of 5 criteria, students who are at van Hiele level 3 are more likely to be successful at writing proofs. For the 4 of 5 criteria, students who are at van Hiele level 2 are more likely to be successful at writing proofs. Similarly, Usiskin (1982) concludes that “Above these levels, success in proof is likely. Below these levels, failure in proof is just as likely” (p. 82).

In agreement with Usiskin’s (1982) second and third conclusion, Senk (1989) investigated the link between students’ van Hiele levels and proof-writing ability. She
provided students with a pretest and a posttest of van Hiele levels (in the fall and spring), knowledge of Geometry in the fall, and proof-writing ability in the spring. She found that there exists a positive correlation between students’ Van Hiele level and proof-writing ability. More specifically, she writes that students “at Level 3 or higher substantially outperformed students at Level 2 or below in writing proofs” (p. 319).

Usiskin (1982) goes on to use van Hiele levels to discuss students’ ability to perform well in Geometry classes. Overall, he found that van Hiele levels in the fall are too low to give students even a 2 in 5 chance of being successful writing proofs, approximately half of the students are placed into courses where they have only a 50% chance of being successful at writing proofs, and many students leave “junior high” with little to no knowledge of Geometry making the transition to high school Geometry even more difficult for both students and their teachers (Usiskin, 1982, p. 86). Sadly, Usiskin (1982) reports that the Geometry course simply does not work for many students which is unfortunate when one considers the fact that currently every student wishing to graduate from high school must take Geometry and according to the standards set by the state in which this study was conducted, that Geometry course must contain proofs. However, these conclusions taken together reveal the need for teachers of all grade levels to take into account students reasoning ability and build a course tailored to meet those needs.

**Students Perceptions about Proof**

Many high school students do not see the need for proofs in real life. In fact, when asked what they dislike about Geometry, “there is only one strong answer: proof” (Usiskin, 2007, p. 73). However, on the flip side, teachers view proof as the most important concept in
mathematics (Usiskin, 2007). These beliefs regarding proof are very common. The students in Chazan’s 1993 study felt the same way about proof that my own students do. In fact, one student in his study said,

I have no idea! [why use deductive proofs] … I don’t like ‘em … I don’t think I’ll ever need it so I’m just one of those people who don’t really care that much, about ‘em! … I just didn’t understand why we had to do these ever. ‘Cause I know I’m not going to use it later on. So, I’m not going to go around measuring triangles. People who are going to become geometry teachers and stuff like that, are going to use it but… (p. 380)

Evidence is Proof vs. Proof is Evidence

The purpose of Chazan’s (1993) study was to determine student conceptions of proof. He divided his results into two categories: evidence is proof and proof is evidence. Evidence is proof refers to the belief that a series of examples constitute a proof. Proof is evidence refers to the feeling that a counterexample may still exist and that the proof is only valid for that one specific case but not all general cases of the same nature. Of the 17 students that Chazan (1993) interviewed, 8 definitely agreed that evidence is proof. Chazan (1993) reported some reasons these 8 students gave for believing that evidence is proof which follow: if all of the types of triangles were tested (“acute, obtuse, right, equilateral, and isosceles”), then the statement must work for all triangles (p. 369), if you try special triangles (acute triangles that are also isosceles) and it works, then you must have proven the statement, and proof by exhaustion. The 9 students that did not agree that evidence is proof gave the following reasons: existence of counter examples, uniqueness of examples, and the
inaccuracy of measurement (Chazan, 1993). Johnson, Thompson, & Senk (2010) found that “about 46 percent of the exercises involving proof-related reasoning were based on justifications using a specific case. Without proper guidance from teachers, these specific justifications may reinforce the misconception that examples constitute proof” (p. 416). Hollebrands, Conner, & Smith (2010) found similar results with college students studying non-Euclidean Geometry in which it appeared that students were “unsure about the ways in which data generated from the technology could be used in creating a formal proof or justification” (p. 347).

Regarding the question of proof as evidence, 4 of the students were sure that proof is just a type of evidence and 7 believed that “deductive proofs were meant to be general” (Chazan, 1993, p. 377). The students that viewed proof as evidence did not believe that they were safe from counterexamples, were only single diagrams, and were based on assumptions (Chazan, 1993). All students that believed that proof was not merely evidence believed that “Deductive proofs prove for all cases satisfying the givens” (Chazan, 1993, p. 374). However, some of these students still believed that a counterexample may exist even though they also believe in their argument for any “diagram satisfying the givens” (Chazan, 1993, p. 377). Chazan (1993) believes that teachers can build on this skepticism in class and develop a “valuable and potentially fruitful” conversation from it (Chazan, 1993, p. 384).

**Proof Schemes**

Harel and Sowder (1998) use the term “proof scheme” to refer to “what convinces a person, and to what the person offers to convince others” (p. 275). In their research, Harel & Sowder (1998) and Recio & Godino (2001) both found that even university undergraduate
students did not understand mathematical proof. This is not surprising based on Chazan’s (1993) results of high school students. Harel & Sowder (1998) state that generally students fell into one of three proof schemes: external conviction, empirical, and analytical. Recio & Godino (2001) believe that students fall into one of four proof schemes: explanatory argumentative schemes, empirical-inductive proof scheme, informal deductive proof scheme, and formal deductive proof scheme.

Regarding Harel & Sowder’s (1998) proof schemes, students exhibiting an external conviction or empirical proof scheme would rely on an authority (such as a teacher or textbook) or similar to the students in Chazan’s (1993) study would rely on examples as proof. Harel & Sowder (1998) recommend working to bring students to the level of analytical proof though they insist that the other proof schemes have merit. Usiskin (2007) reports that proofs for mathematicians develop from a given and are completed after “some amount of exploration” (p. 74). Unfortunately, Usiskin (2007) adds that:

The student in geometry learns, usually by default, that all proof in mathematics is like that in elementary geometry, usually written in two columns (which it isn’t), usually written with even obvious statements given (which they aren’t), and usually involving repeating the same sequences of steps again and again (which is almost never done). (p. 74)

The idea of students exhibiting an external conviction proof scheme is supported by Usiskin (2007) who reports that students “seldom explore and are almost always told what they should prove” (p. 73). He recommends doing away with the proofs of “obvious” statements
and is in favor of allowing proof to develop out of exploration as it does with mathematicians (p. 73).

Recio & Godino (2001) developed their own proof schemes based on Harel & Sowder’s (1998) work with proof schemes, the answers to the questionnaire they gave students entering the University in Spain, and the definitions of proof that the students hold at the beginning of their educational career at the University level. When students answered a question with the intent to explain their answer as opposed to prove their answer, they were listed in the *explanatory argumentative proof scheme* (Recio & Godino, 2001). Verification without attempt to prove or verify validity in a general sense would result in an *empirical-inductive proof scheme* (Recio & Godino, 2001). “Informal logical approaches, based on the use of analogies, graphical tools, etc” are classified as *informal deductive proof schemes* (Recio & Godino, 2001). *Formal deductive proof schemes* follow a more formal and logical pattern of proof (Recio & Godino, 2001). These are not necessarily rigorous proofs or answers to questions that are difficult in nature, but they begin to follow the form of formal proof (Recio & Godino, 2001).

Knuth (2002a) considered the body of research regarding students’ ability to construct proofs and described an approach to teaching proof that he believes can begin to change the way students perform in Geometry class with regard to proof. He considered the “explanatory nature of proof” (Knuth, 2002a, p. 486). Knuth (2002a) states, mathematicians recognize that a primary role of proof in mathematics is to establish the truth of a result; yet perhaps more important, particularly from an educational
perspective, is their recognition of its role in fostering understanding of the underlying mathematics. (p. 487)

Knuth (2002a) recommends that teachers should present students with the opportunity to experience multiple types of proof which he believes will lead “not only to a deeper understanding of proof but also a deeper understanding of the underlying mathematics” (p. 489). He recommends accomplishing this through class presentations and whole class discussion. Knuth (2002a) is not the only person who believes that students learn more by making presentations and discussing problems. Fawcett (1938) designed his Geometry course around discussion and problem solving and he found great success in that classroom. Recently, in the Principles and Standards for School Mathematics, NCTM (2000) has also shown support for the idea of group discussion and presentation.

**Instructional Techniques**

Though some students rely on textbooks (Harel & Sowder, 1998), Chazan (1993) noted that the textbooks do not help the proving situation as some textbook proofs are full of holes that need to be filled in. This again could lead to interesting discussion in the classroom. In line with this statement regarding textbooks, the researcher does not allow her students to use a partial or full proof from their textbook ever. She makes them take the given and prove and write their own. Plus, she has found that her own students thrive more in an environment where they create their own proofs instead of filling in the blanks on a proof someone else started or taking steps someone else wrote and placing them in the correct order. However, other teachers (including the one that participated in this study) believe that it is necessary to scaffold proof-writing for students to help them build an understanding of
what a proof is and what one might look like. However, without a transition to writing proofs without scaffolding, students might know what a proof looks like but be unable to write one.

McCrone & Martin (2004) developed a “proof construction assessment” (p. 239). The results indicate that students scored lowest when they were required to construct an original proof (McCrone & Martin, 2004). However, it should be noted that student scores were below 50% “for just about all items on the ‘proof construction assessment,’ even when they were provided with an outline of the proof” (McCrone & Martin, 2004, p. 239). Ultimately, McCrone & Martin’s (2004) study supported Chazan’s (1993) study. They concluded that “students were convinced of the truth of a statement supported by a finite number of examples, particularly in instances when those examples were illustrative of special classes of figures” (McCrone & Martin, 2004, p. 240). Healy & Hoyles (2000) had similar results in their assessment of proof conceptions in Algebra. Again, students were willing to accept empirical arguments as proof (Healy & Hoyles, 2000). These conceptions of proof may have an effect on students’ motivation to “construct proofs” (McCrone & Martin, 2004, p. 240). In addition to testing student conceptions of proof, McCrone & Martin (2004) also studied students’ ability to construct proofs. They found that while students were able to identify the hypothesis and conclusion of a statement, they were not able to “transfer this ability to a chain of reasoning that depended on an ordered set of linked data” (p. 240). Unfortunately, this means that students are largely unable to write proofs (McCrone & Martin, 2004). The study conducted by McCrone & Martin (2004) mimicked results found by Senk in 1982 and by Healy & Hoyles in 2000. Senk (1982) studied 1,520 geometry students and discovered
that after a “full year of a geometry course with proof, only about half the students can do any more than simple proofs” (p. 8).

Jones (2000a) suggests teaching proof by exploring the idea of explanatory proof researched by Recio & Godino (2001) and Knuth (2002a). He claims that utilizing this type of proof “should help teachers connect with students’ reasoning and guard against the students experiencing learning to prove as no more than a ritual determined by the teacher” (p. 83).

**Factors Affecting Success**

Battista (1990) studied how “spatial visualization, logical reasoning, and the discrepancy between them” affects a students’ ability to succeed in Geometry. At the same time, he investigated the effect that gender and instruction have on performance in Geometry (Battista, 1990). He was able to conclude from this study that “spatial visualization and logical reasoning were important factors in geometry achievement and geometric problem solving” for both genders (Battista, 1990, p. 56). Battista found his results aligned with the idea behind the van Hiele levels; students at a low van Hiele level are poor performers in Geometry (Battista, 1990). Regarding gender differences, Battista (1990) only found a difference in “spatial visualization and their performance in high school geometry” (p. 57). Finally, Battista (1990) found that the males in a classroom where spatial visualization was stressed and sometimes required scored “much higher” than the females in the same classroom (p. 56). Thus, teacher interactions can both help and hinder students Geometry achievement.
Logic in Geometry

Epp (2003) studied the role of logic in learning proof. She identified key areas where students struggle to understand logic: the logical equivalence of p only if q and if p then q, the negation of if-then statements, and negation of conjunctions and disjunctions. Epp (2003) argues that regarding the logical equivalence of p only if q and if p then q, is not applicable to all real world situations. For example, “'If it rains, then I won’t go,’ would be equivalent to ‘It rains only if I won’t go,’ which is gibberish” (Epp, 2003, p. 889). Students will inevitably find a statement such as this and confuse themselves thus making the job of the Geometry teacher that much more challenging.

Students have a difficult time negating if-then statements because there are numerous ways to negate a statement (Epp, 2003). For instance, “Imagine that a friend states ‘If I were Ann, I wouldn’t do what she did’ and we disagree. We might well say, ‘No, if you were Ann, you would do exactly what she did’” (Epp, 2003, p. 889). With multiple ways to negate a statement in the real-world and one way to negate a statement in Geometry, this causes some confusion for students in the classroom as well.

Negations of conjunctions and disjunctions can also pose some difficulty for students in Geometry. For example, given the statement “John is tall and thin” a person might write “John is not tall and thin” as its negation which is correct. However, given the statement, “John is tall and John is thin” a person may write “John is not tall and John is not thin” (Epp, 2003, p. 890). This is not a correct negation of the statement. Notice that by simply altering the original statement, the response of the negation changes drastically. Students may not see the need to use DeMorgan’s Laws to negate the second statement.
Summary

In order to successfully write proofs, students need to have reached a certain level of reasoning ability. One major issue we see is that by the time students reach high school, they do not have the tools necessary to reason logically through a proof and therefore are not able to create a formal proof by connecting mathematical statements (Battista & Clements, 1995). Even more upsetting than the inability to create a formal proof is the fact that students have not developed the ability to determine the truth of a statement in Geometry (Clements & Battista, 1992). Students are not able to develop the level of reasoning needed to achieve success in Geometry in that one course in high school (Harel & Sowder, 1998).

According to Harel & Sowder (1998), there are a variety of proof schemes: external conviction (where the reliance is on a teacher or authority), empirical proof schemes (reliance on examples), and analytical proof schemes (encompasses mathematical proof and focuses on students thinking). It may be important for teachers to recognize each proof scheme and help students develop their abilities even further in high school Geometry classes. Since students do not always come to high school with the tools necessary to write a formal proof, teachers must be able to recognize when the student has the right idea and is simply having difficulty expressing that idea (Harel & Sowder, 1998). However, if Fischbein’s (1982) recommendations are followed and proof is taught initially by building on students natural inclinations and incorporated with the recommendations of NCTM (2000) to teach proof at all levels of mathematics, by the time a student reaches high school Geometry, each student should have a firm foundation in mathematical proof and be ready to build on that foundation.
Summary

This review of the literature covered two distinct topics: English Language Learners and Proofs in high school Geometry. Though these topics may not seem related, they are in fact related to each other. Test results from the state, county, and school level show that ELL students are not performing at the same level as their native English speaking colleagues (North Carolina Department of Public Instruction, 2010).

Research in the area of ELL students focused on how to accommodate instruction and assessments to best meet the needs of this subgroup, the effects of background culture on their ability to perform at the same level as their peers, opportunities for teachers to become more educated in the ELL discipline, and the struggles and challenges facing ELL students in the classroom each and every day. The results indicate the need for additional research into how to best serve ELL students in our classrooms and sheds light on many of the challenges of this group. These challenges alone are enough to frustrate these students, but to then take students already struggling in an English speaking environment and throw them into Geometry (a course heavy in English reading and writing) could be detrimental without proper scaffolding.

The research into proof yielded results that (as a Geometry teacher) the researcher was not surprised to read. Students do not know how to write proofs. Some believe that this is caused by an inability to form a logical argument (Epp, 2003) or an inability to determine mathematical truth (Clements & Battista, 1992). Others associate students reasoning and proof-writing ability with the van Hiele levels (Usiskin, 1982; Mason, 1998; Senk, 1989; Mayberry, 1983; van Hiele, 1986; Clements & Battista, 1992; Burger & Shaughnessy, 1986).
Even though in 2000, NCTM indicated that reasoning and proof should be taught in each grade level, students still enter tenth grade Geometry unable to write a proof. Harel & Sowder (1998) and Recio & Godino, (2001) developed proof schemes that describe students reasoning ability. Knuth (2002a & 2002b) and Fawcett (1938) investigated ways for teachers to implement instruction to help students understand the importance of proofs and definitions in Geometry.

Given the difficulty that so many students have in Geometry and with writing proofs, concern for ELL students and their ability to be successful in this challenging environment grows even deeper. Out of this concern for the progress and equality for ELL students comes the purpose of this study.

The purpose of this study is to investigate how ELL students develop their ability to write proofs in Geometry and how they overcome their struggles with the English Language to be successful in a Geometry classroom filled with English speaking students. ELL students have a unique struggle in every classroom. They must overcome a significant language barrier in order to stay competitive with their peers. In the study, the researcher offered a strategy to help the ELL students write their proofs. The researcher provided each student with a list of reasons that accompany one of the proofs the researcher asked them to complete. The intent of this list was to assist students in forming their logical argument and to take some of the stress off the students.
CHAPTER 3

METHODOLOGY

The Participants

The six participants are high school English Language Learners (ELL) taking regular Geometry and have the same instructor (who is not the researcher). All six students were enrolled in the course during the 2010-2011 school year. Table 5 outlines the demographics of the students who participated in the study.

<table>
<thead>
<tr>
<th>Name</th>
<th>Grade</th>
<th>Culture</th>
<th>Length of Time in United States</th>
<th>Semester Taking Geometry</th>
<th>Consult or Direct Services</th>
<th>Foundations of Geometry?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beth</td>
<td>10</td>
<td>Hispanic</td>
<td>Most of her life</td>
<td>Fall</td>
<td>Consult</td>
<td>No</td>
</tr>
<tr>
<td>David</td>
<td>10</td>
<td>Hispanic</td>
<td>10 years</td>
<td>Fall &amp; Spring</td>
<td>Consult</td>
<td>No</td>
</tr>
<tr>
<td>Linda</td>
<td>11</td>
<td>Chinese</td>
<td>2 years</td>
<td>Spring</td>
<td>Direct Services</td>
<td>No</td>
</tr>
<tr>
<td>Susan</td>
<td>11</td>
<td>Chinese</td>
<td>18 months</td>
<td>Spring</td>
<td>Direct Services</td>
<td>Yes</td>
</tr>
<tr>
<td>Arnold</td>
<td>10</td>
<td>Turkish</td>
<td>9 months</td>
<td>Spring</td>
<td>Direct Services</td>
<td>No</td>
</tr>
<tr>
<td>Joan</td>
<td>10</td>
<td>Hispanic</td>
<td>Most of her life</td>
<td>Spring</td>
<td>Consult</td>
<td>Yes</td>
</tr>
</tbody>
</table>

In the English as a Second Language (ESL) department, the students fall into one of two categories: consult or direct services. Students who are on consult are still on the caseload of an ESL teacher but do not take the ESL class. Their case manager checks in on them and regular education teachers provide progress reports to the ESL teacher periodically throughout the year. There are no specific dates or timelines for progress reports. Typically, just once or twice a semester the ESL teacher and the regular education teacher touch base to
discuss the progress of ELL students. In some cases, the only reason that a student is still on the ESL caseload is that the student has not been able to pass the ESL exit test.

Students who receive direct services from the ESL department have lower English proficiency than those students who are on consult. For these students, one of their four periods a day is devoted to an ESL class. During this class, students learn English and receive tutoring. The tutoring portion of the class consists of students working on their homework from other classes. During this time, teachers and students from local universities work with them one on one.

The county in which the researcher conducted the study offers a course prior to Geometry for students who struggle in mathematics called Foundations of Geometry. The Foundations of Geometry course focuses on strengthening Algebra skills while introducing Geometry concepts. For example, the unit on congruent triangles would not focus on proving that two triangles are congruent or proving that corresponding parts are congruent. Instead, it would require students to set up and solve an equation to find the lengths of the missing sides given expressions for the lengths of sides of congruent triangles.

The school at which the study was conducted, Dogwood High School (a pseudonym), is a multicultural school located in a large town neighboring the state capital. The town, Dogwood, is also culturally diverse. The reason for the diversity in the school is a direct result of the school system’s diversity policy in which students from low-income areas attend a variety of schools throughout the county.

The purpose of the diversity policy is to limit the number of inner-city schools and to afford every student equal opportunity for success. Dogwood High School has 2,250
students. 28.9% of the student body receives free and reduced lunch, 7% are classified as Limited English proficient, and 3.2% are learning English as a Second Language (Wake County Public School System, 2011). The other schools in the system have similar statistics with a few schools having significantly more and others having significantly less.

Methods

Each student completed four proofs. The proofs align with the curriculum of the state. Specifically, the state standard states, “The learner will use geometric and algebraic properties of figures to solve problems and write proofs” (North Carolina Department of Public Instruction, 2003, p. 48). The objectives for this standard require students to:

- Use logic and deductive reasoning to draw conclusions and solve problems.
- Apply properties, definitions, and theorems of angles and lines to solve problems and write proofs.
- Apply properties, definitions, and theorems of two-dimensional figures to solve problems and write proofs (North Carolina Department of Public Instruction, 2003, p. 48).

The goal of the study is to measure each student’s ability to create a logical argument with and without a list of reasons. The proofs for the study were chosen based on the Geometry curriculum as dictated by the state and implemented by the school. They were difficult proofs that contained numerous Geometry concepts. The original plan for the study was to observe how the list of reasons affected student achievement throughout the semester in Geometry. However, due to restrictions from the county, the researcher was unable to implement her study in this way. Therefore, she wrote proofs that contained a number of
Geometry concepts including proof patterns, congruent triangles, and isosceles triangles. The researcher compared how an individual student performed on the first proof and the second proof to identify similarities and differences in each student’s conception and approaches. The researcher analyzed each student’s ability to reason logically based on the conversations the researcher had with each student while they were completing their proofs. The researcher took into account what the students were able to verbalize to her as their reasons and what they were able to write on paper.

**Procedure**

The study occurred over the course of one school year (August 2010 – June 2011). During that period, the researcher contacted all six students’ mid-way through their study of proofs. The researcher designed the proofs to address isosceles triangles and to bring in prior proof patterns such as the definition of between and angle addition/subtraction. The researcher’s goal was to meet with all six students near the time that they studied isosceles triangles and wrote isosceles triangle proofs so that the material would still be fresh in their minds as they worked through the proofs the researcher gave them.

For the most part, the students completed their proofs in one sitting. In order to get the students to participate and feel comfortable doing so, the researcher needed to work with their schedules. Due to circumstances in their personal lives, many of them could not stay after school. Fortunately, the researcher could meet with half of them (Linda, Arnold, and Susan) during the tutoring part of their English as a Second Language class. This was enough time for these three students to complete their proofs in one sitting. Beth was able to meet with the researcher after school since she did not have any responsibilities at home and the
school offers transportation home later in the afternoon two days a week for students that stay after school to work with teachers. This is the only way that she was able to stay after school and work on her proofs. David had to meet during lunch because he could not stay after school at all. Fortunately, he and the researcher had the same lunch. However, this process took three days to complete because he needed to have time to get some food from the cafeteria and come to work on proofs for a period of time and the school only gives 40 minutes for lunch (including travel time to the next class). Joan and the researcher met during Joan’s Geometry class. She missed a lot of school and was not able to stay after because she had to take care of younger children at home. Sometimes this responsibility kept her from attending school. She and the researcher did not have the same lunch and so meeting during that time was not possible. However, in half of a Geometry class period, she was able to complete all four proofs. Each student was able to take as long as they needed to complete their proofs.

The researcher began with two “warm-up” proofs (Appendix A). The reason for the warm-up was to review two major proof patterns that would be useful in the two later proofs. The first warm-up proof was a basic definition-of-between proof. The students only needed to employ the proof pattern for segment addition and subtraction using the reflexive property of equality to complete this proof. The second warm-up proof was an angle pattern proof. The students had to use the proof pattern for angle subtraction again using the reflexive property of equality to complete this proof. With both warm-ups, the researcher answered any questions they had, reminded them of any missing steps from the pattern, encouraged
them to use the tools their teacher taught them, and made sure that they understood and remembered the proof patterns prior to starting the other two proofs.

After completing the two warm-up proofs, the researcher gave each student a proof (Appendix B) and told them that they would not receive any help while they completed this proof. They needed to do their very best, but they should not worry about their proof being perfect. The researcher stressed that if they do not know, it is perfectly fine. The researcher wrote this proof to include many topics from Geometry. The students needed to draw on their previous knowledge of the between pattern, isosceles triangle theorem, base angle theorem, and congruent triangles. In addition, the students had to be able to identify a pair of congruent overlapping triangles.

For the final proof (Appendix C), the researcher did provide some assistance for the students. The researcher gave each student a sheet of reasons (Appendix D) both in English and in the students’ native language to use as they wrote their proof. The sheet of reasons contained all of the necessary reasons to support the statements in the two-column proof along with some additional reasons that the students would not need in order to complete the proof. The researcher placed the reasons on the sheet in no particular order. The researcher offered the sheet to the students in both English and in their native language and allowed them to choose one or both. The researcher wrote this problem again to incorporate many concepts from Geometry. For example, in this proof the students had to be able to use the angle pattern, isosceles triangle theorem, and congruent triangles. One major difference between the two proofs is that students did not have to identify overlapping triangles but they
did have to show that two triangles were congruent and then use those congruent triangles to identify a third triangle as isosceles.

The list of reasons contained definitions, theorems, properties, and postulates in “If..., then...” form. The researcher considered using only the name of the definitions, theorems, properties, and postulates but the Geometry professional learning team (PLT) at the school agreed that students should be able to write the definitions out as an extension of learning how to link conditional statements from the logic unit. The researcher also contemplated including both the name and the “If..., then...” form of the definition, postulate, theorem, and property. Again, because the Geometry PLT at the school decided to focus on having students know and be able to write out the reasons in their proofs, the researcher did not provide names for the reasons on the sheet.

Analysis

On Choosing Qualitative Research

Qualitative research occurs in a natural state unaffected by the researcher. It is descriptive in nature (Bogdan & Biklen, 2003; Wolcott, 2001). In this study, the researcher observed the students in a setting as close to their natural classroom setting as possible. Due to limitations by the school system, the researcher was unable to observe the students completing proofs as a part of their normal classroom requirements. Instead, each student completed four proofs outside of their regular classroom but still in a math classroom at Dogwood High School. Bogdan & Biklen (2003) regard qualitative research as “soft” which, according to them, means “rich in description of people, places, and conversations and not easily handled by statistical procedures” (p. 2). Geometric proofs fit well into this category
because the evaluation and analysis of Geometric proofs focus on a student’s ability to form a logical argument not on a student’s ability to perform significant calculations.

Qualitative researchers can alter their research question as their study progresses and as they collect data. Many researchers start out with one particular idea in mind but as they collect data and do research, they begin to define their exact questions. During this process, researchers must be careful that they do not only choose participants and research that will support the claim they aim to make (Bogdan & Biklen, 2003; Johnson, 2008; Wolcott, 2001).

In the case of this study, the researcher began with the idea that the researcher would essentially follow the ELL students through their Geometry course and evaluate their progress on unit assessments. However, restrictions from the county would not allow this to take place and as a result, the researcher had to alter her methodology thereby only getting a onetime snapshot of each students’ ability to write a proof.

**Purpose of the Study**

The purpose of this study is to investigate how ELL students develop their ability to write proofs in Geometry and how they overcome their struggles with the English Language to be successful in a Geometry classroom filled with English speaking students. ELL students have a unique struggle in every classroom. They must overcome a significant language barrier in order to stay competitive with their peers. In the study, the researcher offered a modification to help the ELL students write their proofs. The researcher provided each student with a list of reasons that accompany one of the proofs the researcher asked them to complete. The intent of this list was to assist students in forming their logical argument and to take some of the stress off the students.
Research Question

In the researcher’s experience as a Geometry teacher, she noticed that her ELL students work harder and spend more hours on their schoolwork than non-ELL students spend. This caused the researcher to consider developing a modification for ELL students. Through this study the researcher aimed to answer the following question: In what ways do ELL students use a list of reasons as they are writing proofs in Geometry?
CHAPTER 4

FINDINGS

Introduction

The findings will describe each student’s ability to reason logically as they complete four proofs. The researcher will focus on each student’s ability to construct a logical argument, gaps in their logical argument, their ability to support their statements with reasons, and the extent to which having a list of reasons assisted the students in filling in the gaps and supporting statements with reasons.

Beth

Introduction

Beth is the only student in the study that took and passed Geometry during the fall semester of the 2010-2011 school year. Beth is Hispanic and the United States is the only country she remembers living in. She speaks Spanish at home but English at school. Her English is very good. Beth’s teacher reports that she really struggled with the proof writing portion of the Geometry course but that her redeeming feature was her ability to use the definitions from the course to set up and solve algebraic equations. However, she had a very difficult time producing a firm and logical argument in her proofs. For all of Beth’s work, see Appendix E.

Warm-up Proofs

The researcher gave Beth the first proof (which is a definition-of-between proof) and gave her some time to work on it. The researcher stressed that if she needed help with these
warm-up proofs, they could talk about them. The purpose of these proofs was to remind her of these patterns, which would be useful in the later proofs.

It did not take long to realize that these proofs would be difficult for Beth. She immediately marked her picture (Figure 1). This means that she was given that two segments were congruent and she marked those in her picture using her pencil. She was also told to prove that two different segments were congruent and she marked those as well.

After she sat there for a few minutes quietly, the researcher asked what she was thinking about in this proof. Her response was that she knew she was supposed to “subtract something, because [MT and AH] overlap here [Beth points to AT] but I’m trying to figure out how I’m going to do that.” Beth was able to represent her problem graphically on the diagram given. She knew generally the process she should take to write the proof and could even point out the piece that needed to be subtracted but she could not remember how to write it. Ultimately, the researcher taught Beth the steps for completing this pattern proof.

Along the way, there were some moments of struggle and moments of success, that indicated both the difficulty Beth has in forming her logical argument but at the same time, reaffirms her ability to use some of the information taught in the course. As Beth and the researcher started to work together to complete the definition of between proof, the researcher asked Beth to think about part of the given statement MT. The researcher asked, “You want to get from MT to what?” She responded “AH”. The researcher was concerned
by this answer because \( \overline{AH} \) is part of the given. This meant that either Beth did not understand what the researcher was asking or she did not understand the entire purpose of using what she was given to show what she was trying to prove. At this point, the researcher backed up to reinforce the information that she was given in the proof and then to emphasize what she was trying to show. The researcher asked her what she would need to take away from \( \overline{MT} \) to get down to \( \overline{MA} \). Beth responded, “\( \overline{AT} \)”. The researcher asked the same question using the other segment from the given, what would she need to take away from \( \overline{AH} \) to get down to \( \overline{TH} \). She responded again, “\( \overline{AT} \)”. 

Based on her answers to the researchers’ questions, it was clear that Beth could see the big picture but was unable to produce the steps in a logical order. Beth was unable on her own to write the second statement or the second reason in which we simply rewrite our given in terms of equals, instead of congruence. However, when the researcher started the statement, “if two segments are congruent, then” Beth was able to jump in and finish the statement “they are equal”. This led into a discussion of the reflexive property. The researcher noticed too that Beth had a difficult time remembering the properties (both the reflexive property and the substitution property of equality).

Beth continued to struggle throughout the proof. She confused the subtraction property of equality and the definition of between. She had a hard time comprehending the fact that they were two different steps, each needed their own statement and reason, and she could not keep track of which statement went with the definition of between and which statement went with the subtraction property of equality. Not only did she have trouble with the definition of between and subtraction property of equality, but she also did not write the
segments in the correct order. For example, she wrote “MA – AT = MT and AT – AH = TH” instead of MT – AT = MA and AH – AT = TH. Beth and the researcher finished this proof together. She really struggled through it. Based on the statements that she verbalized to the researcher, it was clear that Beth had a general idea of how to prove that $\overline{MA} \cong \overline{TH}$ but had trouble with recall and forming a logical argument.

As she began the second warm-up proof, the researcher reminded her that the second proof is very similar to the first proof with some slight differences. One of those differences is that it is an angle pattern proof. Beth had very little trouble writing this proof. However, the process was almost identical to the first warm-up proof. Because she had so much trouble with the first proof, the researcher placed the first proof on Beth’s desk for reference as she was completing the second proof. She did not use it right away but, early on in the proof, she lined up the two papers side by side. As she completed the proof, she asked the researcher if she was right before she wrote a statement or a reason.

The major issues Beth had with this proof was that she used the angle addition postulate before the reflexive property and she wanted to jump to proving that $m\angle IFS = m\angle UFN$ before she used the addition property of equality. She was not able to write the correct statement for the addition property of equality for this proof without help from the researcher. When Beth completed the proof, the researcher noticed that she had mixed up the given and prove. The researcher told her that had those been switched, her proof would have been perfect. She had to be careful because she wrote it backward. These warm-up proofs showed Beth’s inability to construct a logical argument on her own even with the standard patterns that her teacher taught this semester in Geometry.
**Proof without a list of Reasons Provided**

Beth was not given any extra assistance on this proof (i.e. a list of reasons to help her construct the proof). She began by marking the picture (Figure 2) and immediately wrote the given as her first step. Notice in figure 2 below, that Beth put what the researcher refers to as congruent slashes on $\overline{AC}$, $\overline{CF}$, $\overline{DF}$, and $\overline{BD}$ indicating that all four segments are congruent. These four slashes indicated to the researcher that she may have thought that C and D are midpoints. This was not stated in the given so it was information she was inferring from the picture. She also marked $\angle ACB \cong \angle BDA$ in blue and green, respectively. This was what she was trying to prove. After all the marking, Beth spent a considerable length of time looking at this proof and finally wrote $\overline{AC} = \overline{BD}$. Again, she spent a long time looking at her markings, the given, and her statement ($\overline{AC} = \overline{BD}$) before she finally said, “I don’t know how to do it”. Beth used the given information to mark the picture and then the researcher inferred tried to use the picture to construct her argument.

![Figure 2: Beth’s picture for the proof without reasons. Note the congruent slashes on $\overline{AC}$, $\overline{CF}$, $\overline{BD}$, & $\overline{DF}$.](image)

The researcher asked if she had any thoughts about the proof that maybe she was unable to write down. Beth indicated that the four segments that she marked with congruent
slashes (\( \overline{AC}, \overline{CF}, \overline{BD}, \text{and} \overline{DF} \)) she thought were equal because “if [\( \overline{CF} \) and \( \overline{DF} \) are equal], then [\( \overline{BD} \)] is half of \( \overline{BF} \) and [\( \overline{AC} \)] is half of [\( \overline{AF} \)].” The researcher asked if that was going to be her reason to show that \( AC = BD \). She responded “yeah”. The researcher asked where she planned on going from there and she had no idea. She only used one piece of information from her given (\( CF = DF \)), had some major errors in her ability to reason logically, and was unable to find connections between the given information and the picture to form a string of logical statements.

In this proof, Beth would need to be able to show that \( \triangle ABD \cong \triangle BAC \). To do this, knowing that \( AC = BD \) would be an acceptable statement for this proof. Where Beth fell short was that she was unable to connect those segments as part of the overlapping triangles that would ultimately be congruent. In a way Beth implied that \( AF = BF \) because she made all four segments (\( \overline{AC}, \overline{CF}, \overline{BD}, \text{and} \overline{DF} \)) equal. The problem with this was that she used the definition of midpoint based on how the picture looked instead of using the fact that the triangle was isosceles (from the given). If Beth did not realize she should have used the definition of an isosceles triangle in this proof, she had no hope of being able to use the base angle theorem to say that \( \angle CAB \cong \angle DBA \). Overall, Beth is missing a significant amount of logical reasoning.

**Proof with a list of Reasons Provided**

On this proof, the researcher gave Beth a list of reasons both in Spanish and English (see Appendix D). Unfortunately, Beth was unable to complete the proof even with a list of reasons. She did spend considerable time consulting the list of reasons before giving up. She again marked the given information in the picture and highlighted the triangle she was
supposed to show was isosceles in green (Figure 3). Marking her picture and writing the
given is always the first thing Beth does with a proof. After consulting the list, she put
congruent slashes on $\overline{MU}$ and $\overline{PU}$ followed by writing $m\angle SUP = m\angle TUM$ but quickly
erased it.

![Figure 3](image)

**Figure 3:** Beth’s markings on the figure corresponding to the proof with reasons.

When the researcher asked Beth if she had any thoughts about this proof, she said,

“Well, it said it is an isosceles triangle, so I made these two the same” and pointed to $\overline{MU}$
and $\overline{PU}$. The problem with this statement is that $\overline{MU}$ and $\overline{PU}$ are the two sides that she
would need to show are congruent in order to prove that $\triangle MUP$ is isosceles. Then she
followed her statement about an isosceles triangle by saying, “I don’t know why”. She
clearly did know the definition of isosceles triangle, but the researcher did not think she
realized that that she was using the definition. There was a connection in her mind between
isosceles triangles and congruent sides but she was lacking the piece that connects two
congruent sides as the definition. Next she stated, “I just kind of got the feeling that it would
be the same because um, sup and tum overlap right here” and she pointed to $\angle MUP$. “So I
figured these two would be the same because they overlapped” and she pointed to $\angle MUS$
and $\angle$PUT. She was correct, those two angles would be congruent, but she needed to use the angle pattern from the warm-up proof which she also had a difficult time with.

These statements made it clear to the researcher that Beth lacked a firm foundation in triangles and in writing proofs. She knew the definition of isosceles triangle but did not realize that she was using it. At the same time, she knew that if she took away the overlap, the result was a pair of smaller congruent angles. Again, even with a list of reasons, she was not able to write out the angle pattern. Beth had some good ideas in her proof but was unable to connect them.

**Summary**

Beth was not able to write a proof with or without a list of reasons. She even struggled when the researcher walked her through every step of the patterns. Beth showed no growth or ability to apply knowledge from one proof to the next and while she consulted the sheet of reasons, they did not prove to be helpful to her while she was trying to write her proofs. Based on the observations of Beth while she completed these proofs, her general method of attack was to use the given to mark the picture, and use the picture to form an argument. With the addition of the list of reasons, she used the given information to mark the picture, used the picture to try to find something useful on the sheet of reasons and then attempted to construct her proof from there which did not prove to be a successful method.

As the researcher reviewed the tape of Beth completing her proof, she noticed that Beth labeled her columns “given” and “prove” instead of “statements” and “reasons”. This supplements the claim that Beth struggled with forming a logical argument. From a discussion the researcher had with Beth, she was not even sure if she was going to pass
Geometry. The researcher followed up with her teacher who said that Beth was strong enough algebraically and could usually piece an idea together to earn some points for proofs, and would pass Geometry. However, her teacher did agree that Beth struggled with the proof writing aspect of the course.

David

*Introduction*

David is originally from Mexico and has lived in the US for ten years. David says that his parents speak Spanish. At home he speaks Spanish but says that he has to think about what he wants to say in Spanish before he can say it. David also does not remember living in Mexico. This is David’s second time taking Geometry. He took the course in the fall semester 2010 and did not pass. He took the course a second time in the spring semester 2011 with the same teacher. The second time, David did much better in the course though the researcher realized that David lacked confidence in his ability to write proofs. He also did really well with the Algebra aspect of Geometry, but struggled in writing complete proofs perhaps because of his lack of confidence. A copy of David’s work can be found in Appendix F.

*Warm-Up Proofs*

When David began the first proof, he went immediately from the given to the definition of between. He wrote this step perfectly and his ability to understand the process of this proof was evident in the conversation between David and the researcher throughout the proof. After writing his first two steps (the given and the definition of between) David said he was not sure what to do next. He and the researcher backtracked to discuss changing from
congruent segments to equal lengths first. David was very timid throughout our conversation here. When the researcher asked David to supply the reason that supports the change from congruent segments to equal lengths, he thought about it for a while and then ultimately said, “I don’t know”. At this point, the researcher began to lead David through this proof re-teaching the between pattern. However, the following transcript from David’s conversation with the researcher indicates that David knew and understood what segment he was supposed to subtract from each side and the need for the reflexive property:

R: What exactly is it that we are doing? Where are we starting, where are we trying to go? So, if we mark on here with markers [researcher marks the picture], what we are given in pink and we are given that this whole segment [MT] is congruent to this whole segment [AH]. … What we want to get to, we are going to mark in purple [researcher marks the picture]. So what we want to show is that just this piece from here [M] to A is congruent to from H to T. Ok? So looking at that to get from here [MT] to here [MA], what am I going to take away?

S: AT.

R: AT. To get from here [AH] to here [HT], what am I going to take away?

S: AT.

R: AT. Alright, so when we see that when we are taking away the exact same thing from both pieces,

S: Reflexive
After jumping in with the reflexive property, David was able to write the statement and reason without additional help. Based on our previous conversation, David understood that he needed to subtract to get from each larger segment to the smaller segment.

One major concern that came up as David was working through this proof was that without the researcher forcing it on him, he did not see the need for the subtraction property of equality. In an effort to provide some scaffolding for David, the researcher wrote out the subtraction property of equality and pointed to the segments that each piece of the subtraction property of equality represented. Even with this support, David just kept trying to go right for the definition of between and had the researcher let him, would have completely skipped the subtraction property of equality. After a long conversation about the subtraction property of equality, David finally asked if his statement \((MT – AT = MA)\) was correct. The researcher responded that the left side was correct, but the right side was not. At this point David shouted, “Oh! I see!” and fixed his statement to represent the subtraction property of equality \((MT – AT = AH – AT)\).

At the beginning of the proof, David wanted to write the definition of between and he did it correctly. However, following what was a long discussion to get to the definition of between; David had a difficult time recalling the correct statement. When the researcher said it was time for the definition of between, David said, “\(MA + AT = MT\)”. At this point, the researcher reminded David that he was starting with \(MT\) and that he was taking away \(AT\). When asked what he would be left with he responded, “\(MA\)”. From here, he was able to write the entire statement and reason himself.
At this point, he was ready to skip ahead to the statement he was trying to prove so again the researcher stepped in and redirected him. The researcher explained that he needed to use the substitution property of equality by referring back to statements 4 and 5 (the subtraction property of equality and the definition of between). The researcher only had to remind David that since $MA - AT = MA$ (statement 5), $MA$ could replace $MA - AT$ in statement 4. David then quickly said $MA = TH$ which was perfect substitution. Finally, David made the equal segments congruent to finish the proof. The researcher mentioned to David as he was finishing the proof that the reason for the last statement was the converse of “If two segments are congruent, then they have equal length.” David had no problem verbalizing the converse of the statement and writing it down.

In the first proof, the researcher marked the picture for David to help him visualize where we started and where we hoped to end up. In the second proof, David chose to mark the proof himself (Figure 4).

![Warm-Up Problem #2](Image)

**Figure 4:** David’s markings on the second warm-up proof.
The researcher asked David if he noticed any differences in the two proofs. He pointed out that in the second proof, the angles were already equal so he did not need to have an extra step in which he made the angles equal. David had little trouble with this proof. He mixed up a few angles as he was working through the proof, but he found his own mistake and fixed it without help from the researcher. David struggled with the reasons in this proof. For example, he wanted to use the definition of between instead of the subtraction property of equality (again). This is the same problem he ran into in the previous warm-up proof. The researcher stopped him to discuss the differences in the subtraction property of equality and the definition of between. In addition, the researcher pointed out that we do not use the definition of between in angle proofs; we use the angle subtraction postulate.

David did well with the statement side of this proof but struggled forming the correct reasons. The fact that David could write the statement side of the proof indicated to the researcher that in general, he was able to apply the knowledge from the first warm-up proof to the second (because they were so similar). The reasons that were a struggle for him before continued to be a struggle, and the reasons that were different were also a struggle. David showed signs of having trouble extending a concept. After David had completed the proof, the researcher reviewed a few errors in his logic. For example, David forgot to use the reflexive property which was surprising because he did not forget in the first warm-up proof. Also, he wrote the definition of subtraction instead of subtraction property of equality. Thinking about the big picture, David did well. He was able to form the overall argument with only a few pieces missing.
**Proof without a list of Reasons Provided**

Because David had to complete these over the course of a few days at the beginning of each day, the researcher allowed David to look at the warm-up proofs. During this proof, David was not given a list of reasons from which to choose as he constructed his proof, however the researcher did leave the warm-up proofs on his desk for reference as she did with each student. David began his proof by writing the given. David then used the definition of an isosceles triangle to write that AF = BF. The researcher asked David to state the definition of isosceles triangle. He replied that it has “2 equal sides”. The researcher repeated and corrected him by saying “at least 2 congruent sides”. After this David changed his statement from AF = BF to “AF ≅ BF”. Even though the researcher corrected his definition, when asked, David still verbalized that isosceles meant that two sides were equal. With the encouragement of the researcher, David marked his picture. The researcher also encouraged him to identify another piece of information from his given. David said, CF = DF but did not do anything with it. He just stated it.

David was quiet for a while so the researcher (in an attempt to keep him moving on the proof) asked David what he was trying to show. He responded appropriately. The researcher then encouraged David to mark the angles David was trying to show were congruent. This is so that David could get a better picture of what he is trying to show. In addition, the researcher suggested that David mark $\overline{CF} \cong \overline{DF}$. The researcher was trying to get David to continue his chain of reasoning (see the need for the between pattern and find the congruent, overlapping triangles). Figure 5 is David’s picture with all the markings on it.
David wanted to use the definition of isosceles triangle to say $\overline{CF} \cong \overline{DF}$. The researcher said that $\overline{CF}$ was only part of $\overline{AF}$ and $\overline{DF}$ was only part of $\overline{BF}$ so he should consider whether or not the definition of isosceles triangle would work. David instead wanted to write down $\overline{CF} = \overline{DF}$ by the definition of congruent. However, $\overline{CF} = \overline{DF}$ was in his given so the researcher continued a conversation with him implying that if he wanted to use those segments, he could essentially make them congruent. Unfortunately, this just confused him and it was a completely unnecessary step. The researcher pointed out that $\overline{CF} = \overline{DF}$ was part of the given to help David visualize and it spiraled out of control. David wanted to use it and rather than tell David that he should not do anything with it, the researcher tried to support David’s thought process. It is possible that if the researcher had just left David alone, this confusion could have been avoided.

David did not willingly write statements for his proof nor did he mark his picture without encouragement from the researcher. In fact, he stared at his proof until a thought finally popped into his mind. Therefore, the researcher continuously pushed David to look at...
his given, look at the picture, and look at his previous steps to see what can be connected.

This did work for a while. David eventually discovered that there were two congruent triangles and he decided that if he could show that the triangles were congruent, he could prove $\angle ACB \cong \angle BDA$. However, the problem with David’s work was in getting to this point. David was able to use the base angle theorem to show that $\angle BAC \cong \angle ABD$. In fact, David’s reason was, “Because in an isosceles triangle, the angles have to be congruent”. One issue with the base angle theorem is that David originally wrote $\angle BAC \cong \angle ABC$ and the researcher began to mark these angles with a green marker. The researcher believed that David did not mean to write $\angle ABC$ because he was able to verbalize the base angle theorem and the definition of an isosceles triangle. David quickly changed his statement to $\angle BAC \cong \angle ABD$.

David had the ability to write this entire proof. He saw the big picture and some of the details it took to construct the proof but he lacked confidence in proof writing. Even though he was able to verbalize the need for the base angle theorem to the researcher, when it was time to write it down, David proceeded to say, “I don’t know why”. The researcher responded, “You just told me why”. He then said to the researcher, I know how to say it, but I don’t know how to write it”. The researcher told him to write it however he would say it and that would be acceptable.

With the encouragement of the researcher to look at his previous steps, picture, and the given, David then said he thought “$\triangle EAC = \triangle EBD$”. The researcher asked how he knew that. David was quiet for a while and the researcher said, “It would be great if we knew that and thinking along the lines of congruent triangles is a good thing”. David responded
immediately with “so how about $\Delta ABC \cong \Delta BCD$”. The researcher suggested that he draw them out on his paper since they were lying on top of each other. That way we could see exactly what he was talking about. When David drew out the triangles, he actually drew $\Delta ABC$ and $\Delta BAD$ which are the two overlapping triangles that David needed to show were congruent. However, even though David was able to see the big picture – he found the triangles and agreed that if the triangles were congruent, he could prove that $\angle BDA \cong \angle ACB$ - he was missing many of the details he needed to get there. From his original picture, he was able to use the fact that $\angle BAC \cong \angle ABC$ but he thought he could immediately show that $\angle ABC \cong \angle BAC$. However when the researcher asked him why he did not have a reason, David and the researcher agreed that since he did not have reason, he should table the idea until he had one.

David was able to find the reflexive segment on his own stating that “segment AB is the same length as segment AB”. This was actually an improvement from the warm-up proofs. David never remembered the reflexive property in his warm-up proofs. However, in this instance the reflexive property was being used in a slightly different way in this proof from the warm-up proof. David’s biggest error in this proof was that it did not occur to David to use the between pattern to show that $\overline{AC} \cong \overline{BD}$. He tried to find other congruent angles that unfortunately did not work out.

David had a good general idea of how to complete this proof but forgot some details. The researcher helped David with the details of the between pattern again so that she could see how he would handle showing that the triangles were congruent. Near the end of the proof, David wanted to prove that $\Delta ABC \cong \Delta BAD$ because “If congruent, then =”. This is
evidence of a large gap in David’s recall ability. David also wanted to make his triangles equal instead of congruent and it is as if he could not recall any of the postulates and theorems used to prove triangles are congruent. The researcher provided David with a list and he was able to choose correctly from that list but was unable to come up with the correct reason on his own. However, he remembered that the angles were congruent because of “CPCTC”.

David saw the big picture but there were holes in his logic that without a push from an authority figure, David did not think about. These holes caused him to give up. David’s proof would have been over very early on if the researcher had not pushed him to look back at the picture, given, and previous statements.

**Proof with a list of Reasons Provided**

As David began this proof, the researcher offered him a list of reasons that were in English and in Spanish that he could use while he was completing this final proof. In addition, the researcher laid out the warm-up proofs that David had completed two days before and told David that if they were helpful he could use them as well.

In this problem, David did not use the sheet of reasons. He did mark his picture with more than he was able to write down (Figure 6). In addition, he was able to verbalize the reason for marking his picture the way he did. David was able to use his given to include the following two statements: \( \overline{SR} \cong \overline{RT} \) (supported by the reason “If a triangle is isosceles, then it has 2 congruent sides”) and \( SU = UT \) (supported by the reason “Definition of Midpoint”). These were David’s two successes in this proof. Otherwise, David struggled to form a logical argument. He did not even seem to talk about the big picture of the proof (gave no indication
of triangles that should be proven congruent). He seemed to be stuck on using segments which did not get him anywhere. He should have focused his attention on angles. In fact, he has the angles from the given marked in his picture but he never did anything with them. In addition, David marked \( \angle S \cong \angle T \) but he did not ever use that in his proof either.

![Figure 6: David’s marks on his final proof.](image)

Because David could not come up with a reason to support his statements about segments, he ultimately got stuck in this proof. The researcher encouraged him to read over the list of reasons and try to find something on the list to help him make some progress. This was where he found the definition of midpoint. He actually wrote the reason and then backtracked to write the statement after he had written the definition.

This proof was a struggle for David. Once he marked \( \angle S \cong \angle T \) he could not come up with a reason to support it. He tried to say that the reason to support \( \angle S \cong \angle T \) was ASA. The researcher tried to use the same tactic as in the previous proof with David. She suggested that he start with what he was trying to prove. David verbalized that he knew that in order to show that the triangle was isosceles; he would have to show that two sides were congruent. David’s ability to reason logically broke down on both sides of proving that the triangles
were congruent. Working from the statement he was trying to prove towards what he was given, David was able to identify that he wanted to show that $\overline{MU} \cong \overline{PU}$ but he was unable to identify two triangles that could be proven congruent. Working from the given to the statement he was trying to prove, David marked two of the three necessary pieces of information. He marked $\angle S \cong \angle T$ but he could never provide a reason for it. He also marked $SU = UT$ by the definition of midpoint but he was unable to use the angle subtraction pattern which would allow him to obtain a second pair of congruent angles which would lead him to identify and prove that triangles were congruent.

The researcher pushed David to express what he was thinking. David started to talk about how if we had parallel lines (pointing to $\overline{MP}$ and $\overline{SU}$) then we could conclude that alternate interior angles were congruent (pointing to $\angle PMU$ and $\angle SUM$). David pointed out that we do not really know that we have parallel lines, it just looks that way. Based on his statements about parallel lines, it was clear towards the end of this proof that David was not even thinking about proving triangles congruent which was why he was having such a hard time forming the argument. After the parallel lines conversation, the researcher asked David what else he was thinking about in this proof. David said that he knew that if “two angles of one triangle were congruent, then the sides opposite them were congruent” but he did not know how to use that. He pointed to $\angle MSU$ and $\angle MUS$ but agreed he could not use it because we only have one angle with a congruent mark in that triangle. The researcher continued to push David to make some progress on this proof by asking him, “What else are you thinking about” and the last time David responded he said, “$\triangle MSU \cong \triangle PTU$” but when the researcher asked how he knew that, David said, “I don’t…yet”. The researcher tried to
push David on how to prove that ΔUMP is isosceles if we knew that the triangles were congruent and David responded by saying we needed to show that “MP was between SR and MP is between RT” but he had no way of connecting that statement to an isosceles triangle.

**Summary**

David has gaps in his Geometry skills. These gaps keep him from being able to form a logical argument. In addition, they can lead him down the wrong path and make it difficult, if not impossible, for him to redirect himself. David tried to look at the last proof multiple ways but had so many gaps he was not making any progress. Even if David had been able to prove that the triangles were congruent, he did not see how the corresponding parts related to the isosceles triangle and ultimately made very little progress. While there is a lot of Geometry information in David’s brain, he struggled with using only what was on the page and he allowed himself to get distracted by things that he thought looked true which unfortunately he could not use.

**Linda**

**Introduction**

Linda was originally placed in an Algebra I Part B class but it was clear to her teacher that she was very good at Algebra and bored in the class and so she was moved to a Geometry class four weeks into the school year. At four weeks, the Geometry classes at Dogwood High School have already started the proof patterns and so Linda had to play catch up on four weeks of school plus complete all the work her class was currently working on as she played catch up. Linda is from China and has been in the country for two years and her English proficiency level is low. Based on a conversation the researcher had with her teacher,
Linda does struggle to understand English and her teacher also noticed that Linda struggles to write down the reasons that support her statements in her proofs. Linda does not have her own English/Chinese translator (an electronic dictionary that converts English to Chinese) but another student who is also from China sits next to her and does have one and they share.

**Warm-up Proofs**

Linda did well on her warm-up proofs. On both proofs she struggled to recall the reasons for each statement but generally the statement side of her proofs was very good. For all of Linda’s work, see Appendix G. She made a similar mistake on both warm-up proofs and the first time the researcher addressed the problem with her she seemed receptive. The second time, she insisted that she was doing this as she had been taught. The researcher decided that it was not likely that she would get Linda to see her error without a great struggle and so she let the issue go. Figures 7 and 8 are samples of Linda’s work on warm-up proofs 1 and 2 respectively.

Figure 7: Linda’s first warm-up proof.
In both instances the proofs required subtraction of segments (first warm-up proof) and angles (second warm-up proof) and she used the subtraction property of equality perfectly. Prior to the subtraction property of equality, she used the reflexive property (in both warm-up proofs). Prior to the reflexive property of equality she used the definition of between and added segments: $MA + AT = MT$ and $AT + TH = AH$ when she should have subtracted segments: $MT - AT = MA$ and $AH - AT = TH$ instead. In addition, she put the definition of between in the wrong place (it belongs after the subtraction property of equality). In the second warm-up proof, Linda used the angle addition postulate before the reflexive property instead of using the angle subtraction postulate and placing it after the subtraction property of equality.

As stated before, Linda struggled with the reasons for these proofs. In the first warm-up proof, Linda was unable to supply the “definition of between” as a reason to support adding the segments, she wrote “Ref…” next to the step where she used the reflexive
property, and “sub…” next to the step in which she used the subtraction property of equality. She tried really hard to write substitution but she could not spell it correctly. When the researcher was helping Linda with the reasons for the statements and Linda was trying to recall the subtraction property of equality, she said to the researcher, “I know it in Chinese”. This statement is exactly the type of difficulty that English Language Learners face in the classroom. Linda knew the reason but was completely unable to communicate the reason to the researcher in English. This is why modifications need to be made to tests and instruction and scaffolding must be provided to ensure success in an English-only environment. Linda struggled again to support her statements with reasons for the second proof as well. This time she was able to write “reflex” for the reflexive property of equality. The researcher told her that the reflexive property of equality is “a = a” and sometimes people find it easier to write “a = a” instead of writing the word reflexive. Other than the given, Linda was unable to support any other statements with reasons in the second proof.

**Proof without a list of Reasons Provided**

Linda initially looked at this proof with which the researcher did not give her any assistance (Linda did not receive a list of reasons she could use in her proof) and said, “I know this one”. Her comment sounded like she regarded all proofs as patterns and she just had to recall which one it was. In a way, in regular Geometry at Dogwood High School proofs are presented in a pattern. Because most of the curriculum focuses on triangles, many of the proofs at Dogwood High School involve proving triangles are congruent and then draw some conclusion from there.
One noticeable flaw in Linda’s ability to write proofs was that she could not write the reasons for the statements. Figure 9 is a sample of Linda’s work from the proof without reasons. Notice that this proof is essentially just the statement side of a two column proof. The only reason she attempted to give is near the bottom of the left column. She wrote “verte” and said out loud that she had located a pair of vertical angles. The struggle that Linda had writing the reasons was evident on her warm-up proofs as well as when she made the comment, “I know it in Chinese” which relates back to the purpose of this study and the idea of providing appropriate accommodations for ELL students so that they have the opportunity to be as successful as their native English speaking colleagues.

\[ \triangle AEF \text{ is isosceles with base } AB. \]

\[ AF = BF. \]

\[ EC = DF. \text{ \hspace{1cm} Given} \]

\[ \triangle AEF - CF = AC. \]

\[ BF - DF = BD. \]

\[ \angle AEC \cong \angle BFD. \text{ \hspace{1cm} Vert} \]

\[ \angle AFB = \angle ABD. \]

\[ \angle BAC = \angle ABC. \]

Figure 9: Linda’s attempt at the proof without reasons. 
Notice that includes only “verte” and “give” as reasons.

In analyzing Linda’s proof, the researcher noticed that there were some gaps in her logic. In general, Linda’s approach was to write her proof from the information in the given though she did incorporate the picture to extend her statements. For instance, Linda used the definition of an isosceles triangle to indicate that \( AF = BF \) (though she should have
concluded that $\overline{AF} \cong \overline{BF}$ and then made the congruent segments equal). She used her given to state $CF = DF$ and then used the picture to subtract $CF$ from $AF$ and $DF$ from $BF$ which allowed her to state $AC = BD$ (though she did not draw this conclusion for a while after doing the subtraction). In this line of reasoning, she also did not use the between pattern for subtraction correctly. She used the definition of between perfectly (which she did not do in the warm-up proofs) but forgot the subtraction property of equality (which was not an issue in the warm-up proofs). In addition, she did not finish the pattern right away. Linda was a very quiet worker – she mumbled to herself and worked so quickly the researcher had to stop her to discuss her thought process as she worked.

When the researcher stopped Linda to ask about her thought process, Linda had just finished using the definition of between and was focused on trying to show that $\triangle AEC$ and $\triangle BDE$ were congruent. She found a pair of vertical angles and wrote that they were congruent. She tried to do this by focusing on things that she “thought she saw” in the picture. For instance, she tried to say that $\overline{AD}$ and $\overline{BC}$ bisect each other which would allow her to conclude that segments are congruent. Then she tried to use the base angle theorem and the definition of bisect to conclude that $\angle CAE = \angle DBE$. What ensued was a rather confusing conversation between Linda and the researcher in which there was serious miscommunication. The researcher ultimately challenged Linda to review information that she knew was correct and be wary of information that she was trying to infer from the picture. Linda thought and looked at her picture and had a few “Oh I have it! No never mind” moments and finally realized that she was looking at the wrong two triangles.
Once Linda found the correct triangles, she was able to pull some pieces together. For instance, she noticed that they shared a side (\overline{AB}) , she finished the between pattern she had started and concluded that \( AC = BD \). She used the base angle theorem to conclude “\( \angle BAC = \angle ABD \)” though she was unable to write reasons for any of the statements. She said, “I have two sides and one angle” but she did not write a statement for the congruent triangles. She highlighted them on her paper but did not write them down or provide a reason for being able to conclude that the triangles were congruent. In addition, she discussed how to show the angles were congruent and wanted to use hypotenuse leg. Again, she could not write it down. After saying hypotenuse leg, she said that her teacher had given her a long name but also a short name that meant the same thing. The researcher asked her if she was thinking of “Corresponding Parts of Congruent Triangles are Congruent (CPCTC)?” She replied that she was. Linda was ultimately able to write this entire proof with some holes in her logic all the way to the congruent triangles. Based on the markings in her picture, she knew the congruent triangles but was unable to transfer the markings from her picture to the paper though she could verbalize her thoughts well.

Proof with a list of Reasons Provided

Linda began this problem by marking the information she needed to prove in her picture and then used the given to find the two congruent sides of the isosceles triangle. Linda’s statement for this was “\( RS = RT \)” (using equal lengths again instead of congruent segments). She then immediately moved on to look at what she was going to write for her next statement until the researcher stopped her and requested she fill in the reason. This is significant because Linda does not write reasons. In this proof, the researcher handed Linda a
list of reasons to use and told her she had to write reasons in this proof. Linda indicated that she wanted to write all the statements first and then go back to the reasons. This was probably because Linda was much more comfortable with the left column of the proof than with the right column. Also, to many students the statements column makes more sense as it applies directly to the picture and the given information and so students find it easier to follow that chain of reasoning before going back and including the reasons for their statements. The researcher said that was fine but Linda still at that point looked for the reason for her statement. Interestingly enough, she looked down the list only until she found one that looked right and she chose the converse of the correct reason. The reason Linda actually needed was the last one on the sheet but she did not look at the entire list.

Linda could recall a lot of information from her Geometry class. Unfortunately, the information got tangled and she was confused either because of the language barrier or because some of it was so similar. Through the researcher’s interaction with Linda, she could not decipher where her problems came from. From what the researcher could understand, Linda wanted to use \( \triangle MSU \cong \triangle PTU \) to conclude that \( MU = PU \). Linda spent a long time caught up in trying to show sets of angles were congruent. Even when she would indicate something that was right and the researcher would say, “Ok. So, write that” Linda backed away or marked something else in her picture (for instance after the angles, she marked \( SU = UT \)).

In a couple of minutes, Linda verbalized that she knew \( U \) was the midpoint of \( ST \) so \( SU = UT \) and she talked through the angle subtraction pattern out loud which allowed her to conclude that “\( \angle MUS \cong \angle PUT \)”. The researcher told Linda that she had to write down
what she was verbalizing. She was able to do most of this well. Figure 10 is the result of Linda trying to write what she had just said out loud to the researcher.

![Image of Linda’s attempt at forming a logical argument.](image)

Notice that in this figure, Linda does not support her first statement with a reason, uses the wrong reason to support SU = UT, supports $\angle SUP = \angle TUM$ by writing “Give” which is perfect, and then incorrectly uses the angle pattern (similar to the mistakes she made in the warm-up proof).

Her issues with the angle pattern were that she used the angle addition postulate, reflexive property, and then the angle subtraction postulate. She should have taken out the angle addition postulate and inserted the subtraction property of equality following the reflexive property. Generally, as Linda completed the proof and if she knew a reason, she did not use the sheet to help her at all. If she did not know a reason, she used the sheet. However,
this proved to be difficult because Linda knew the names of the properties such as “Substitution” but did not know the longer property “If \( a = b \), then each may replace each other in any algebraic expression”. Therefore, some reasons she needed she was unable to find on the sheet. Once Linda reached the point in the proof where figure 10 ends, the researcher asked, “Now what”? Linda responded by going back to the picture to find more statements. She did not try to use the information she had just recorded to draw more conclusions.

In the previous proof, Linda really struggled when she reached the point of proving that she had two congruent triangles and inferring information from there to continue her proof. Linda was very focused on trying to use the between pattern to show that \( MS = PT \). But then suddenly, as she was inspecting her picture, she shouted, “Oh! Isosceles angle!” and put congruent marks on \( \triangle RST \) and \( \triangle RTS \). What was interesting was her placement of the statement “\( \angle RST = \angle RTS \)”. She placed it after her second statement. Figure 11 shows the placement of this statement. In addition, she shouted a reason (which was geometrically incorrect) but did not write one. In fact, she did not even look at the list of reasons, she just moved right on to proving that her triangles were equal and then congruent by Angle-Side-Angle. She even circled the two statements and wrote ASA next to them as though they went together (see Appendix G).
Finally, she concluded that MU = PU (which again really should have been congruent since they are corresponding sides of two congruent triangles). She supported it with “If two angles are congruent, then their side are congruent” which is reminiscent of the beginning of the proof in which she kept saying she was trying to show that “∠MSU = ∠STU” so that she could conclude that MU = PU. It appears that she thought she was right and just needed to do a bunch of work to get there which is interesting because the teachers at Dogwood High School always teach that after you prove that two triangles are congruent, you will always use CPCTC as your next step to continue your proof (if it is not complete yet). Finally, since she had her two “equal” sides, she concluded that “MUP is isosceles Δ”. But she did not provide a reason for the last statement. Based on earlier comments Linda made, she was aware that it takes two sides to have equal length in order for a triangle to be isosceles. She misses only the “at least two” and the fact that the definition states “congruent sides”.

**Summary**

Linda is overall very skilled at constructing the left side (statements) of a two column proof. She is very detail-oriented and sometimes misses the big picture. Linda seemed to focus on each step individually instead of looking at the overall goal of the proof. She does always mark the information she is trying to prove but it appears that she sometimes loses
sight of what she is trying to prove in constructing the exact right statement with all of the
necessary detail. It appeared that having the sheet of reasons both helped and hurt Linda in
this process. One way in which it was not beneficial was that she often knew the name of a
property, theorem, definition, or postulate instead of the “if…then… form”. The sheet of
reasons did not have many names of definitions, properties, theorems, and postulates on it
and was designed in the style the professional learning team at Dogwood High School agreed
to teach these important conditionals. In addition, because both a conditional and its converse
could be and in some cases were located on the list of reasons, Linda accidentally chose a
converse over a conditional to support a statement. Finally, because some of the reasons were
extra and not intended to be used, they acted as a distraction and on more than one occasion
Linda used the wrong reason to support her statement even when she could verbalize the
correct one (which she was able to do about 50% of the time). However, the advantage to
having the list of reasons is that Linda actually wrote reasons for her proof whereas the
previous three were just a list of statements with no Geometric support.

Susan

Introduction

Susan took Foundations of Geometry in the fall with the same teacher she had for
regular Geometry in the spring. This teacher felt as though Susan was bored in Foundations
of Geometry and probably could have taken regular Geometry and skipped Foundations. In a
conversation with her Geometry teacher, the researcher learned that Susan moved from
China to the United States eighteen months ago but she works really hard and is very smart.
Her teacher said that when Susan is absent, she still makes an A. Susan has one of the highest
grades in her Geometry class. Her teacher said that the only part of proofs that Susan struggles with is the reasons and because of this her teacher usually allows Susan to use her notes on assignments. In addition Susan has an electronic English/Chinese translator that she uses regularly in class. A copy of Susan’s work is located in Appendix H.

**Warm-up Proofs**

Susan initially stated that she did not know how to do the first warm-up proof. Therefore, the researcher took on the role of the teacher and began to teach Susan the “between” pattern. The researcher asked Susan how she should begin the proof and Susan said with the given. Then Susan proceeded to write the given as her first statement and “MA + AT = MT; AT + TH = AH” as her second statement. The researcher encouraged Susan to mark the picture and began a discussion about whether adding was the appropriate way to approach this proof. Susan said that she could write the statements but not the reasons and the researcher told her to write all that she could. Figure 12 shows the five steps that Susan created. She created these five statements with some help from the researcher.
Figure 12: Susan’s statements for the first warm-up proof.

Susan erased her original second statement and wrote $AT = AT$ after the researcher asked Susan to look at the given segments and look at what she was trying to prove. Then again, for the third statement, Susan wanted to use an addition problem. She said, “$MA + AT = AT + AH$”. The researcher had Susan focus on the fact that she was going from big segments to small segments and Susan quickly said, “Oh! I said it wrong! $MT - AT = AH - AT$”. Then Susan wrote statements three through five on her own (Figure 12). With help from the instructor, Susan was able to create the overall picture of this proof. She missed a couple of details in her logic (forgot to make the congruent segments from her given equal and did not use the definition of between).

Susan said she did not know how to do the reasons yet when the researcher was filling the reasons in for her, Susan could state the subtraction property of equality and “if
two segments are congruent, then they have equal length” (and its converse). She was able to remember a piece of the definition of between (AB + BC = AC) and the name of the substitution property of equality but not the property in “if…then…” form.

On the second proof, Susan did well although she forgot to use the angle subtraction postulate. She did use the subtraction property of equality and called it the “Subtraction Angle Postulate” though. Also on the second proof Susan completed the statement side first and put her pencil down as though she was done. The researcher told her to try to fill in the reasons. She could not remember how to spell or say postulate and she refused to try to write it. The researcher spelled it for her after she tried to pronounce it. The most interesting thing about this second proof is that the researcher left the first warm-up proof out for Susan to look at (which she did not) which would have helped her remember a few of the steps. Susan generally had the right idea but had some gaps in her logic in both proofs. She also does not mark pictures unless explicitly instructed to do so.

**Proof without a list of Reasons Provided**

Susan had a very difficult time developing a plan for this proof. Initially, she could not even find the angles she was trying to show were congruent. Once she found those angles and wrote the given, she just stared at the picture. When the researcher asked her what she was thinking about, Susan replied, CE = ED but she did not have a way to support it. The researcher asked if she was thinking about anything else. Susan said, AC = BD which she supported by using the information she was trying to prove. The researcher reminded her that she was not allowed to use that $\angle ACB \equiv \angle BDA$ since she was trying to show that and asked if she had another way to get AC = BD. She said AF = BF but gave no reason to
support the statement and then realized she could subtract $AC - CF = AC$ and $BF - DF = BD$. The researcher noticed that she managed to remember the statement for the definition of between which she never did in either warm-up proof and forgot the statement for the subtraction property of equality but used it as the reason to support the definition of between.

Even though she said that $AC = BD$, she never stated it in her proof. She had most of the pieces but did not follow it through until the end. Just as with all of the students, Susan had the warm-up proof sitting in front of her to help her make the connection between the proof patterns and the proof she was working on. Figure 13 is a picture of her statements and reasons from this proof. Notice that immediately following the definition of between, she concluded that the angles she was trying to prove were equal and then congruent. When the researcher asked how she went from segments to angles so quickly she had no reason why. She stared at the proof patterns for a long time. The researcher jumped in and explained the remainder of the proof to Susan because she did not have any idea how to complete the proof. She had taken her logic as far as she was able to.
Figure 13: Susan's statements and reasons for the proof without a list of reasons.

It looked as though Susan did not have a plan for this proof. She had one or two ideas but no reasons to support why she thought she should take certain steps and she certainly did not see the triangles or the need for showing that two triangles were congruent. Even as the researcher talked through the remainder of the proof with Susan, there were times when she was confused by the explanation and needed a different explanation.

**Proof with a list of Reasons Provided**

For the final proof, Susan was given a list of reasons and told that she could use that list to help her construct her proof. Just as with the previous students, Susan was warned that some reasons may be used more than once and some not at all, but every reason she needed for her proof was on the sheet. Susan used the list to help her write her proof. She found a reason on the sheet that matched with something she knew and wrote it on the paper.

She still had some holes in her logic. Figure 14 is a picture of the last four statements and reasons for her proof. Notice in statement 3 she combined the angle subtraction postulate...
and the subtraction property of equality and never concluded that $m\angle MUS = m\angle PUT$. She included the reflexive property of equality but placed it after the angle subtraction postulate and subtraction property of equality. She indicated to the researcher that she was trying to figure out how to show $MU = PU$ and felt she could do it because “$\angle mup = \angle pum$”. Her argument began to fall apart at this point. Notice too that her last statement is SAS which would actually be a reason and her last reason is “If $a = b$ and $b = c$, then $a = c$” which has nothing to do with SAS. The researcher asked Susan, where the last statement and reason came from and Susan was only able to say that $\angle MSU \cong \angle PTU$ with no reason to support the statement (and she never put it in her proof). When the researcher asked why, Susan said “$\angle MUS = \angle PUT$”. When the researcher asked why again, Susan said, “$\angle SMU = \angle TPU$”. The researcher asked, “How will knowing these angles are congruent (researcher pointed to $\angle SMU \& \angle TPU$) help us prove this (researcher pointed to the statement to prove)”. Susan said that, $\angle MUS = \angle PUT$ so we know that $MU = PU$. The researcher asked why and Susan said, “I just don’t know”.

\[
\begin{align*}
\angle SUP - \angle SUM & = \angle TUM - \angle TUP \\
\angle SUP - \angle SUM & = \angle mup \\
\angle TUM - \angle TUP & = \angle pum
\end{align*}
\]

\[
\begin{align*}
\angle mup & \cong \angle pum \\
MU & = PU \\
SAS & \text{ If } a = b \text{ and } b = c, \text{ then } a = c
\end{align*}
\]

Figure 14: The last four statements and reasons for Susan’s proof with reasons.

Note step 3 and 6 in particular.
Summary

It seemed that language was a major barrier between Susan and the researcher. Many times, the researcher had to reword a statement or write and speak at the same time. Susan never marked her own pictures as she wrote a proof. The researcher did this for her as she was recapping Susan’s reasoning. In addition to having difficulty understanding English and not marking pictures, Susan had major gaps in her logical reasoning during the last two proofs but no gaps in her reasoning in the proof patterns (the first two proofs). This made it harder for her to construct a chain of logical reasoning that made sense and could be used to construct a proof.

It was clear to the researcher throughout that Susan could recall pieces of information from Geometry, but struggled with some topics. For instance, Susan never once mentioned the definition of isosceles triangle or the base angle theorem. Once Susan was done working on the last proof the researcher brought it up and Susan responded by saying, “Oh yeah! I forgot!” As was discussed earlier, she made a mistake with her letters that may have caused her to be unable to complete her entire proof correctly. In Susan’s case, the researcher felt as though there were many small conceptual errors that stood in her way in addition to the language barrier.

Overall, the list of reasons seemed to have helped Susan create a proof that contained more of a logical argument and more reasons than the proof without a list of reasons. She used the sheet to help her form this argument. Her general pattern was to find a reason on the sheet that corresponded to a statement she knew and then she wrote the two together. Without the list of reasons, it is possible that her proof would have been sparse.
Arnold

Introduction

Arnold has lived in the United States for only 9 months. His family moved here from Turkey for the specific purpose of providing Arnold with the opportunity to attend college and Arnold knows that. Therefore, school is incredibly important to Arnold. He works hard at learning English and earning good grades. Arnold’s teacher reports that he has one of the highest grades in class, excels in the Algebra portion of Geometry, learned the patterns of writing proofs and as a result he improved as the course progressed.

Warm-up Proofs

Arnold had great confidence in his warm-up proofs. Arnolds work can be found in Appendix I. He jumped right into them and with no help from the researcher wrote an almost perfect between pattern and angle pattern proof. In the first proof, he simply forgot to include as his second step, MT = AH and as his fourth step, the definition of between (MT + AT = MA; AH – AT = HT). Arnold listened intently to the comments the researcher made following the first proof and employed them on his second proof, including using the definition of between. Unfortunately, the definition of between is only for segments. The angle pattern uses the angle addition/subtraction postulate. Arnold constructed his logical argument well with only one hole in his reasoning. Then on the second proof he attempted to transfer what he learned on the first proof to the second proof which caused him still to make a mistake but also provided another opportunity for learning. The information could not be transferred directly because the proofs (though both patterns) were not exactly the same.
Arnold had some trouble remembering some of the reasons for the warm-up proofs. For example, when he got to the reflexive property, Arnold said, “It begins with an R. ref…” and the researcher finished the word for him. When he was ready to use the subtraction property of equality, he said, “substraction”. It was clear to the researcher that difficulty with translation caused Arnold to have a difficult time recalling reasons exactly as he learned them. For example, for the subtraction property of equality, he wrote “subs of property” and for substitution, he wrote, “substution”. Finally, the last reason he used on the first proof typically is written if two segments have equal lengths, then they are congruent. He wrote If \(a = b\), then \(a \cong b\) which was not wrong. It was actually exactly what his teacher taught using words in symbolic form. It appeared to the researcher that Arnold’s language barrier was not significant enough to keep him from learning Geometry and being a successful participant in the classroom. However, based on his ambiguous abbreviations that could stand for more than one property, it could be enough to cause him to get a few points taken off his proof if he was not able to recall the correct spelling or abbreviation of a particular term.

**Proof without a list of Reasons Provided**

Arnold felt very confident in his ability to write this proof in which he was not provided a list of reasons to choose from. He took the markers and marked in purple that \(\angle BAC \cong \angle ABD\) and when the researcher asked why he knew that, he said “because of isosceles”. The researcher noticed that he never actually used the definition of isosceles triangle but used the base angle theorem because he knew the triangle was isosceles. This was evidence of some holes in his ability to reason logically. In addition, his notation was less than perfect. For example, after he marked the two angles congruent above, he named...
them as “∠A = ∠B”. The problem with this is that there are three ∠A's and three ∠B's.

Figure 15 is a picture of the proof with Arnold’s marks on it. Notice the purple marks and that there are three different angles by each purple mark. This was why it was so important to name an angle with three letters. In the Geometry classes at Dogwood High School, an entire lesson was devoted to the correct way to name an angle. Unfortunately, it appears that Arnold did not take much away from this lesson.

Arnold processed this proof by examining the given information and looking at the picture. Through this process, he found a pair of vertical angles and then was able to conclude ∠ACE = ∠BDE and then he was done. The researcher asked him to explain what he was thinking about in this proof. He said that because these angles (Arnold pointed to ∠BAC & ∠ABD) and these angles (Arnold pointed to ∠AEC & ∠DEB) he knows that these angles (pointed to ∠ACE & ∠BDE) are equal. When pressed further, Arnold said he

Figure 15: Arnold’s markings on the picture for the proof without reasons.
did not have a reason why they are equal he just knows they are. Arnold was unable to supply a reason for this statement.

Arnold had one or two good ideas for this proof but lacked the big picture and many of the Geometry concepts needed to form the complete logical argument. It appeared that Arnold was able to get one statement using his given (though he used it incorrectly) and one statement using his picture (which was geometrically correct but unnecessary for this proof).

Proof with a list of Reasons Provided

When presented with the list of reasons in English and Turkish, Arnold was the only person who wanted both sheets. Arnold started this proof by marking a pair of angles that he never used again. He never even wrote them in his proof. Arnold placed congruent marks on $\angle MSU$ and $\angle PTU$ but never wrote that they were equal or congruent in his proof. The researcher asked him why he marked them and he replied because the triangle is isosceles. The first set of statements that Arnold used was the angle subtraction pattern. Figure 16 shows Arnolds attempt at using the angle subtraction pattern. Notice that even with the list of reasons, he was unable to support a couple of his statements.

![Figure 16: Arnold's attempt at the angle subtraction pattern for the proof with reasons. Note the missing reasons.](image-url)
In addition, his second and third statements are technically out of order. Arnold again struggled with writing the word subtraction as his reason for the second statement. The researcher asked Arnold if he saw Subtraction (either the word or the symbol for subtraction on the list of reasons) and Arnold pointed to the “Angle Subtraction Postulate”. She asked again if he saw the symbol for subtraction anywhere on the page and he pointed to “If a = b and c = d, then a – c = b – d”. The researcher asked Arnold if that is what he did in his proof and he replied, “This definition of between I can only use for segments”. It appeared to the researcher that there was some level of confusion going on for Arnold. Rather than get into a lesson about the definition of between, the researcher said, “Ok. Keep going.” For the reasons Arnold was unable to fill in, he looked at the sheet to see if he saw a reason on the sheet that fit what he did. The fact that he was not able to find one indicated that he was unsure of many of the terms in Geometry.

However, Arnold diverged in his attempt to complete this proof from what every other student did. Arnold noticed that if he could show that two pairs of angles in two triangles were congruent, then the third pair also had to be congruent and he was able to write and verbalize this to the researcher well. Interestingly enough, he kept saying to the researcher that “∠S = ∠T” but he never included it as a statement in his proof.

Once Arnold reached this point in his proof, he had all three pairs of angles in ΔMUS and ΔPUT. The researcher asked him what it would take for him to prove that ΔMUP is isosceles. He said that he would need to show that ∠UMP = ∠UPM. The researcher asked if that was it and if he knew anything else about isosceles triangles. Arnold responded that “at least two angles must be equal”. There was a little bit right and a lot wrong with Arnold’s
statement. “At least two” is correct but we look for sides to be congruent instead of angles to be equal. Once we knew sides are congruent, we concluded that the angles opposite those sides were also congruent. Arnold’s statement indicated that he did not know the definition of an isosceles triangle and he was either missing the connection between the measure of an angle and the length of the sides or language and the fact that he has only been in the US for 9 months played a role in his ability to construct the argument.

Arnold indicated that he wanted to continue his proof by saying that $\overline{MP} \parallel \overline{ST}$. The researcher asked if he knew that and he responded, “No. But I think they are” which unfortunately was not good enough. Arnold was very interested in finding a way to show that $\overline{MP} \parallel \overline{ST}$. He then asked if “it” was a parallelogram and moved his pen over quadrilateral $MPTU$ but did not specifically say that is the shape he thought was a parallelogram. The researcher asked why he thought it was a parallelogram and he said because it had four sides which he also cannot say without more information. The researcher asked him “If it was a parallelogram, what would you do”. He said, “There’s a ‘z’ so $\angle MPU = \angle PUT$ and $\angle PMU = \angle MUS$. So they are the same and the triangle is isosceles”. The researcher asked Arnold if he could think of anything else. Arnold pointed to $\angle RPM$ and $\angle RPM$ and said that they were the same as $\angle MSU$ and $\angle PTU$. But he was not able to support his statement to show that $\triangle MUP$ was isosceles.

**Summary**

Arnold definitely lacks some ability to reason logically. He thought about his proofs in a different way than his classmates which tended to lead him down a path that others did not think of. Arnold would have been more successful if he had been given more information
Arnold started out strong with the proof patterns. But he was unable to take those proof patterns and insert them into a more difficult proof. He tried in the last proof where he was allowed a sheet of reasons but he missed reasons and wrote a couple in the wrong order. He had a habit of marking things in his picture that he never actually used in his proof as well. This was troubling because it indicated that he knew more than he used in his proof. If he used every tool at his disposal, he may find more success in proof writing.

Arnold never tried to use congruent triangles at any time during the final proof. Nor did he try to use CPCTC to get pieces of congruent triangles congruent. This was likely because he set his mind to another path and really focused on wanting segments to be parallel even though that was not a statement we knew. Because he never brought them up, the researcher assumed he felt more comfortable in working with parallel lines and parallelograms (it is likely these topics were covered in class more recently than congruent triangles) which could have pushed Arnold in a particular direction.

Joan

Introduction

Joan had a significant attendance issue which made it more difficult for her to make the same progress as her classmates in writing proofs. She had responsibilities at home that sometimes kept her from attending school and definitely kept her busy after school which made it difficult for her to get the additional help in Geometry that she needed. Fortunately, she took the Foundations of Geometry course and therefore developed skills in using
Geometry to set up and solve equations. But the biggest challenge in regular Geometry (even for a student that took the foundations course) is proof writing.

Joan’s teacher describes her as quiet – the type of student that you could lose track of if you are not careful. Joan’s teacher has had to work hard to keep Joan from vanishing in the crowd and making sure to keep her involved in the lessons she was present for. Joan’s teacher agreed that her absences caused her to have more trouble writing proofs than her classmates.

**Warm-up Proofs**

Similar to her classmates, Joan is stronger with the statement side of the proofs than with the reasons for the proof. Her approach to proof writing is slightly different than her classmates. Joan uses the subtraction property of equality to help her find the resulting segment (see Appendix J). For example, in Figure 17, notice that on the left side Joan marked out the T’s on the left side of the equation and the H’s on the right side. The left side worked well, Joan wrote “MT – AT” and crossed out the two T’s which left MA. Unfortunately, the right side caused a problem. It appears she wrote “AH – HT” which caused her to cross out the H’s and she was left with AT. As Joan and the researcher discussed where she went wrong at the end, they concluded that she might have written AH – AT and her second A just looks like an H which confused her or she wrote it wrong which caused her to make a mistake at the end. In an earlier statement, she wrote AH – AT = TH. Therefore, it is more likely that she wrote an A and mistook it for an H.
Joan made other mistakes that were similar to her classmates including using the definition of between before the reflexive property, forgetting to make the segments equal before subtracting them, and general inability to complete the reasons in a proof. These errors did not improve from one warm-up proof to another. In fact, in the second proof she also left out the reflexive property altogether.

**Proof without a list of Reasons Provided**

Joan was not provided any additional help on this problem. She was not allowed to have a list of reasons as she completed this proof. She was able to verbalize that two triangles were “the same” (ΔACB and ΔADB) however, it is in the details of this explanation that lacks a firm argument. Joan said that because ΔABF is isosceles and has the same base as ΔACB and ΔADB, that means that ΔACB and ΔADB are “the same”. It was very insightful of Joan to pick out those two triangles from this picture when there are many more triangles that could possibly have distracted her and that had distracted her classmates. In addition, she
picked out the side they share which did not support her statement with the correct reason. After Joan found the triangles that were “the same”, Joan said, “I forgot the second step” which the researcher found interesting because she was able to verbalize what she thought a step in the proof should be. The researcher asked if she skipped the second step, what would come next and Joan responded, “the prove” which was exactly right. The statement Joan was trying to prove would come after proving triangles are “the same”. The researcher asked Joan what she means when she says that triangles are “the same”. Joan said, “equal” but made no mention of congruent triangles. Joan’s biggest setback with this proof was that even though she was able to pick out the two congruent triangles, she skipped every detailed step that would have led her to actually showing that the two triangles were congruent.

Proof with a list of Reasons Provided

Joan struggled on this proof as well. She was not able to progress any further than the given statement though she tried incredibly hard. In her discussion with the researcher, she said that she wanted to use the midpoint to show that ΔMUP was isosceles. When the researcher asked how she could do that, all she said was that “SU and UT form the baseline”. The researcher shifted the focus, asking her what it would take for us to be able to say that ΔUMP is isosceles. Joan responded that she needed UM and MP to be the same. The researcher pressed Joan for a reason why and ultimately Joan said that since ΔRST is isosceles, that meant that MU = MP. Unfortunately, this reasoning needs some work. Joan agreed that if a triangle is isosceles, two sides are congruent. The two sides Joan picked out were not part of the triangle she knew was isosceles. Joan had significant gaps in the Geometry she knew which kept her from being able to produce a logical chain of reasoning.
Unfortunately, even with a list of reasons she was unable to develop anything of substance. She looked at the list (though briefly) and it did not help her which is not surprising since her reasoning was so inaccurate she likely felt as though she did not need it.

Summary

Joan clearly struggled with writing proofs and forming a logical argument which are of importance in Geometry. Unfortunately, this could be due to the fact that attendance for Joan was an issue and as a result, she spent most of her time playing catch up in class. She only consulted the sheet but it did not improve her ability to write a proof. She did relatively well with the patterns (those consisting mainly of memorization) but Joan did not see the need to take those patterns and insert them into the more difficult proofs. She appeared to have forgotten basic geometric information taught in all Geometry classes at Dogwood High School. For instance, Joan’s Geometry teacher taught her class to look for the “three amigos” when proving that triangles were congruent and Joan could not remember basic properties she would have learned in Algebra I, Foundations of Geometry, and then reviewed again in her regular Geometry class. Joan succeeded in writing statements for the warm-up proofs and recalling those patterns not perfectly but well and she was able to immediately pick out the pair of overlapping triangles that she needed to show were congruent. The problem was that she missed all of the details it would take to achieve that goal.

Summary

Chapter 4 explored each student’s attempt at proof writing and to evaluate the effectiveness of the addition of a list of reasons as the students completed four proofs. There were many similarities among the students as they completed the proof. These similarities
include the use of equal instead of congruent, using information they thought they saw in the picture over what was given to them, difficulty remembering the definition of between and subtraction property of equality, and logical gaps.

**Summary of Student Work**

The idea of using equal instead of congruent was a big issue in every proof. The students that were able to prove that triangles were congruent actually proved them equal instead of congruent. In their Geometry class it was common to hear segments referred to as congruent or having equal length. Similarly, angles can be congruent or have equal measure, but triangles are proven to be congruent to each other – not equal to each other. In addition, the corresponding parts used to prove that triangles are congruent are stated as congruent so there is no basis for proving triangles are equal and yet most of the students proved that their triangles were equal. However, proving triangles congruent was not the only time that the students struggled with the difference in the two terms. Most students felt that the definition of isosceles meant that two sides were “the same”. When the researcher inquired about the use of the term “the same” the students responded by saying they meant equal. In addition, the statement they wrote in their proofs indicated “the same” meant equal. In the realm of writing a proof for a tenth grade Geometry student, this is certainly not a logical error and the student can still construct a proof that has meaning even with these mistakes in it. Using equals instead of congruent is the equivalent of spelling a word wrong or making a grammatical error in an English paper. It would not be the end of the world, but you want to try to avoid it.
A much larger problem in proof writing was using information students believed they saw in the picture over information that was actually given to them. The students wanted to use parallel lines, midpoint, bisect, and properties of parallelograms but none of that information was given to them. A drawback to the implementation of this project was that the students were near the end of their Geometry course and as a result, they had learned a lot more Geometry than they needed to employ in these proofs. This could be why they tried to infer information from the picture. However, attempting to use information from the picture is a common mistake that many Geometry students make. Unfortunately, it was also a costly mistake as the students that attempted this path often missed the big picture and as a result were unsuccessful in constructing a full logical argument.

Given the warm-up proofs, many students struggled with the necessity of the definition of between and the subtraction property of equality. In general, the students either forgot what they were, mixed them up, used one instead of the other, or used them inconsistently (added instead of subtracted). The researcher noticed that even with help, the students that struggled with the definition of between and subtraction property of equality in the first warm-up proof were equally as unsuccessful in writing the angle subtraction postulate and the subtraction property of equality in the second warm-up proof. An issue that comes with the inability to recall the proof patterns was that the students then were less likely to take the proof patterns and insert them into the more complicated proofs. This snowball effect kept many of the students from achieving more success on the proofs in this study.

The final mistake that many students made was including gaps in their arguments. Not one student was able to construct an airtight logical argument. Each student had some
trouble with a concept whether it was the patterns, isosceles triangles, or congruent triangles. One unfortunate aspect to Geometry is that when a student struggles with a concept, unless the student is remediated, that struggle is going to follow that student for the remainder of the course. The struggle then will impede the student from achieving his or her maximum potential.

Effectiveness of the List of Reasons

There were three types of students in this study: a) students who noticed both the details and the big picture, b) those who did not, and c) students that noticed either the big picture or the details but not both. The students that saw the big picture and many of the details required to construct the picture, found some success with the list of reasons. However, these students already knew many of the reasons they simply had trouble remembering them in English. This was where the list of reasons was actually helpful. For example, during her warm-up proofs, Linda was trying to remember a reason and said, “I know it in Chinese”. The list of reasons would have been helpful for her in English and Chinese for that proof. Arnold was the only student who wanted the sheet in English and Turkish (his native language). Having the list of reasons in both languages would make translating easier and allow him to focus on actually writing the proof. These students did not need the list to help them write the proof. They needed the list to help them support what they already knew. Unfortunately, the other side of this was the students who could not recall enough of the details from their Geometry course to see the big picture and ultimately construct a proof. For this student the list of reasons was not helpful because this student could not even begin the proof. Ultimately, the list of reasons is not a cure for every ELL
student in Geometry. The student must be able to form at least part (typically the left side) of a two column proof.
CHAPTER 5

CONCLUSION

Purpose of the Study

The purpose of this study was to explore how English Language Learners (ELL) made use of a list of theorems and definitions provided for them to use as they proved theorems. According to Schleppegrell (2010), students use language to build mathematics knowledge and if teachers do not provide adequate resources for ELL students, they may struggle to build the same mathematics knowledge as their native English speaking counterparts. Because every subject is rooted in language, ELL students have difficulty in courses in the US which is due mainly to the fact that they do not know English (Schleppegrell, 2010). Researchers have suggested ways to assist ELL students in achieving their potential in the classroom. Some of these suggestions include a customized glossary, providing tests in a student’s native language, providing extended time, and lowering the difficulty of the language of the test (Abedi, Hofstetter, & Lord, 2004). Prior to The No Child Left Behind Act of 2002, schools did not have to test ELL students in the same way they tested native English speakers in order to receive Title I funding (Alford, 1965). However, the addition of this Act in 2002 requires schools to not only test ELL students just as they do native English speakers, but to provide appropriate accommodations for those students. Schools are also expected to make adequate yearly progress each year culminating in 100% of students proficient in their end of course testing by the year 2014 (Willner, Acosta, & Rivera, 2009, Abedi & Dietl, 2004, Abedi, Hofstetter, & Lord, 2004).
Kindler (2002) studied the growth of the ELL population and found that the number of Limited English Proficient (LEP) students has grown by 105% since 1990 while the overall general school population has only grown by 12%. In addition, the ELL population is one that constantly changes and is often described as instable. Each year, new students move to the US and are enrolled in schools as ELL students while others that have been in the program for years, pass the exit test and leave the program. In a setting where this occurs each year, it is hard to measure positive growth (Abedi & Dietl, 2004). Proof of this statement lies in testing data from the state and county in which the study was conducted (North Carolina Department of Public Instruction, 2011).

In addition to providing support to ELL students in their courses, this study focuses specifically on proof-writing performance of ELL students in their Geometry classes. The state in which the study was conducted regards proof writing as an important aspect of the Geometry course, including it in their standard course of study. The National Council of Teachers of Mathematics include proof-writing in their Principles and Standards of School Mathematics and most recently, the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) developed a group of standards called the Common Core Standards. Similar to NCTM, these standards stress the importance of reasoning and proof at all grade levels and also incorporate a series of mathematical proficiency standards to which each student should be held accountable (North Carolina Department of Public Instruction, 2003; NCTM, 2009, Common Core State Standards Initiative, 2011). Some researchers regard proof writing as the most important aspect of Geometry because it teaches students to think logically, highlights the connections
between mathematical topics, provides variety in mathematics, and allows teachers the opportunity to provide challenges for their students (Hanna, 2000; Knuth, 2002b; Waring, 2001; Weber, 2001; Fawcett, 1938; Usiskin, 2007; DeVilliers, 1994).

**Summary of Findings**

This study showed some effectiveness with the list of reasons. Ultimately, the students fell into one of two groups: Those that had some ability to reason logically and those that did not. The students that had some ability to reason logically, struggled to form the reasons that belong in the right column of their two-column proof. The addition of the list of reasons helped these students develop a more complete proof. For example, Linda was consistently able to write a list of reasons that was almost complete in the warm-up proofs and the proof without a list of reasons provided. Figures 7, 8, and 9 are samples of Linda’s work on proofs in which she was not provided a list of reasons. Figure 7 and 8, the researcher helped Linda write the appropriate reasons after she was able to create the statements. Then Figure 10 shows a sample of Linda’s work on a proof in which she was provided a list of reasons. Even though Linda struggled with this proof, she was able to use the list of reasons to support some of the statements that she did write. Linda’s ability to include reasons in her proof increased from using partial words to finding 2 or 3 full reasons from the sheet to support her statements. In addition, Susan struggled to support her statements with reasons. However, once she was allowed a list of reasons, she was able to find the appropriate reasons on the sheet to fit her statements. Though Susan does have some ability to reason logically, she struggled with proof-writing more than Linda did. The researcher noticed that the list of reasons did not help any student write a perfect proof. However for the group of students that
had little ability to reason logically, the list of reasons was not helpful. Beth, David, and Joan were the trio that struggled to construct a logical argument with and without a list of reasons. All three of these students needed significant help from the researcher on the warm-up proofs and because of this difficulty; they struggled to create a logical stream of statements in their proofs. Unfortunately, because they could not create a logical argument, they could not support these statements with reasons. R.C. Moore (1994) developed a mapping that associates student difficulty in forming proofs. Among the difficulties he outlined, is the effect that language can have on a student’s ability to understand the concepts, definitions, and create examples, which can all lead to the inability to successfully write a proof. Specifically, Beth was unable to write any statements on their own for both the proof without a list of reasons provided and the proof with a list of reasons provided. Joan was able to create an argument with some gaps for the warm-up proofs but in the proof without a list of reasons provided and the proof with a list of reasons provided, Joan was unable to write a logical argument of statements which prevented her from supporting those statements with reasons. David could have done better if he had had more confidence. David had the ability to reason logically but lacked the confidence to write proofs without assistance from an authority. These students were unable to form a logical argument without the researcher leading them through the proof. Arnold was the closest of anyone to creating a good proof with and without a list of reasons. Arnold was able to reason logically (with some gaps) with and without the list of reasons and was able to provide some support for his statements with and without the list of reasons. From her interaction with Arnold, the researcher believes that the list of reasons helped him slightly but that he was able to write reasons on his own.
In general, though the list of reasons did help a couple of students slightly, it was not as effective as the researcher had anticipated. There are many possible causes for the ineffectiveness of the list of reasons. First, R.C. Moore (1994) documented the struggle students have with proof. It is possible that because the students have not mastered the English Language, they have more difficulty understanding the concepts, definitions, and examples which could cause them to struggle with proof writing. In addition, the students all had the same Geometry teacher. She chose to teach proofs differently than the Professional Learning Team had agreed to. As a result, it is possible that the list of reasons did not conform to the way the students had been taught to write reasons which would cause it to not be helpful. Also, the researcher had created the list of reasons for the students. Therefore, they were not familiar with the reasons on the sheet or the order they were in. Had the students been able to write their own list of reasons, they might have used the sheet more because it would have been tailored to meet their individual needs. Finally, because of her schedule and theirs, the researcher met with some of the students long after they had completed isosceles triangle proofs and many of them knew other information that they wanted to use (parallel lines, properties of parallelograms). However, none of this was in the given and none of it was on the sheet. Since some students wanted to use information that was not provided, the list of reasons was not helpful for them.

The only student who verbalized a plan prior to starting the proof was Linda. When she looked at the proof without a list of reasons provided, she shouted, “I know how to do this one!” as though she regarded every proof as a pattern and it was just a matter of recalling the exact steps. This could have caused some difficulty for Linda. According to R.C. Moore
(1994) when students view proof as procedures and explanation, lacking rigor, they could struggle with the writing a proof because they do not understand the basics behind proof writing (definitions and concepts). The other students did not have a plan as they started. Some of the students focused on trying to use the given information to develop statements, some used the given and the picture together, and some just stared at the proof until a random thought occurred to them. There was not one common approach that they had learned in class that they all used successfully. The lack of a plan could also indicate a lack of understanding of the concepts and definitions of Geometry (Moore, 1994).

Discussion

Teaching Proof and the Effectiveness of the Modification

One way that the students could have performed better in writing proofs is if their teachers had incorporated proof into courses prior to Geometry. Johnson, Thompson, & Senk (2010) found that many textbooks present problems in which students are expected to prove, conjecture, or explain some mathematical fact as part of the exercises. These types of exercises take away from the rote problem solving that many students can do in their sleep and allow the variety of mathematics and proof writing to bleed into other courses. The results of the study could have been affected by the fact that these students have diverse educational and socioeconomic backgrounds and may not have been in a course that emphasized proof prior to the high school Geometry course (Epp, 2003). In addition, these students come from various countries some within the last few years and others have lived in the United States for most of their life. In Euclidean Geometry at this high school, there is great emphasis placed on the details in proof writing that may not be considered to be as
important in other countries and even at other schools or in other grade levels. Thus, in trying to write proofs, the students may have lost sight of the big picture as they were trying to remember the details (i.e. changing from congruent segments to equal lengths before using the addition property of equality).

**Advantages to the Modification vs. Disadvantages to the Modification**

In some ways, the modification provided both advantages and disadvantages to students’ progress as they wrote their proof. For instance, without the list of modifications, Linda and Susan would never have written a complete reason on their own. An advantage to the list of reasons is that it helped them to actually attempt to support their statements but at the same time, a disadvantage is that it halted their progress because they had an idea of what they were looking for and would sometimes pick the converse of the statement or just pick and incorrect statement altogether.

For Arnold, David, Joan, and Beth the list of reasons provided fewer advantages. For Arnold, the list of reasons did not apply to the approach he wanted to use and as a result, the list of reasons was not helpful to him. David, Joan, and Beth all struggled to even form a logical argument. Because they could not get much past the given, the list of reasons was not helpful to them as well. Their struggles seemed to mimic Moore’s diagram of students’ inability to construct proofs stemming from lack of understanding of the language and leading into a lack of understanding of concepts and definitions.

**Van Hiele Levels and the Effectiveness of the Modification**

When a student is not exposed to proof writing prior to high school Geometry, it is possible that he might have a difficult time understanding what is going on in the
mathematics classroom. For example, a student who struggles to understand the Geometry in
the classroom may not have had a teacher who emphasized proof-writing, making
conjectures, or explaining results in their previous math classes. As a result, the student was
not able to advance to a Van Hiele level that would allow him to understand and be
successful in proof writing in Geometry (Usiskin, 1982; Mason, 1998; Senk, 1989;
Mayberry, 1983; van Hiele, 1986; Clements & Battista, 1992). For the students in this study
who struggled with proof writing, this is a possible explanation of their educational
experience.

Students Understanding of Proof

The teacher in this study takes into account varying levels of geometric reasoning
ability by scaffolding all proofs at first and slowly weaning the students away from this “fill-
in-the-blank” type proof. Research suggests that students do not see a need for proof in the
“real world” (Usiskin, 2007). This could result in students setting up a wall which could
prevent them from successfully writing proofs. In this study, no student openly expressed the
opinion that proof was unimportant to their life. The researcher asked each student to define
proof. The general consensus among the students was that proof verifies the truth of a
statement or that proof allows you to show that two things are congruent. David claimed
repeatedly that he was bad at proofs and said the teacher goes too fast, but everybody tried
the proofs (some with more enthusiasm than others). The students also varied in their
individual proof schemes as well. For example, some students refused to write anything
down without asking the researcher if they were right first. This is an example of an external
proof scheme in which the student relies on the teacher in proof-writing (Harel & Sowder,
1998). Other students were able to draw on the knowledge their teacher had imparted through classroom lessons and explorations. These students are exemplifying what Recio & Godino (2001) coin formal deductive proof schemes in which the form begins to follow that of a formal proof.

**Recommendations**

For this particular study, the researcher was only able to meet with the students in Geometry near the end of their course instead of throughout the course. The original method of implementation was to build the list throughout the course as a testing modification. However due to restrictions from the county in which the study was conducted, all work had to be completed outside the regular classroom and could not affect student grades in any way. The researcher recommends that this accommodation be used during the development of proof-writing and that the list be built as the course progresses. While it is not likely to have drastic results in most ELL students, it decreases the stress of balancing learning English in an environment where all the instruction and assessments are provided in English.

This study can be expanded to include students with disabilities. The researcher has known many special programs students who struggle in recalling many of the theorems, postulates, and definitions and as a result, have a difficult time forming a logical argument. Special programs students often receive modifications similar to the modifications that ELL students receive such as extended time, modified assignments, and decreasing the linguistic difficulty in word problems. When providing modifications for students, they must be reasonable modifications. All of these students (both special programs and ELL students) will take a final exam (either a teacher made exam or the state end of course test).
particular, with the state end of course test there are only so many modifications that are reasonably allowed. Throughout the year in the classroom, teachers must make sure that the modifications they are providing for students do not give the student an advantage that he would not have on a state end of course test. In the case of this particular modification, it does not provide an extreme advantage. In fact, in a way, this modification can be likened to that of a multiple choice test which is exactly the format of a state end of course exam.

**Limitations**

The original intent of this study was to progressively analyze ELL students’ ability to form logical arguments as they learn to write proofs. The goal of the original idea was to track them throughout their Geometry course and gather an overall picture of how these students process learning how to write proofs and whether the addition of a testing modification (list of reasons) affects their ability to write proofs. In the original idea for the study, there were to be two groups (control and experimental) in which one group got the strategy and the other did not. The students were going to be assessed based on their ability to write proofs on their tests (which would be a common measure among the Geometry classes at Dogwood High School). However, the county placed restrictions on the researcher and would not allow her to gather any data from the classroom. Therefore, all work had to be done outside the classroom and could not be attached to anything the students earned a grade on.

The effect of these regulations meant that the researcher had less time to collect data which resulted in fewer students participating in the study. Instead of having three or four semesters in which to collect data, the researcher only had two semesters. This did not allow
for a control group and experimental group. It also did not allow for a progressive view of the development of proof-writing ability. Because the researcher had to meet with each student outside of class time, she only had enough time for one meeting per student in which all four proofs were administered at once. She was not able to meet with each student during each unit that involved proof-writing to assess the progress continuously. Had she been able to collect data straight from the classroom, she could have collected data progressively throughout the semester. In addition, by the time the researcher met with the students, they had learned a lot of Geometry. As a result, they wanted to use some of the most recent information they had learned (parallel lines and quadrilaterals) instead of the information given in the proof. Had the researcher been able to make this an exercise that examined proof-writing ability throughout the course, the later information along with information from the beginning and middle of the course would have been relevant. Instead, it just stood in the way of the students’ ability to write proofs.

The teacher involved in the study approached the course in a different manner than the rest of the Geometry professional learning team. The effect this had on the students could have made a difference in the usefulness of the list of reasons. For example, the Geometry professional learning team decided as a group that they would not allow students to use the names of definitions, theorems, and postulates. They indicated that they would have students memorize the full “If…then…” statement. Therefore, the list of reasons was set up in that manner. However, the teacher of the students that participated in the study allowed her students to use the names of the theorems, definitions, and postulates. As a result, the students were unable to find the reasons, even though they were included on the sheet. This
made the list of reasons useless to them. One way to fix this for future studies is to include the name along with the full “If…then…” statement that accompanies the theorem, definition, or postulate. Then, no matter what learning environment the students have been in, they all have an equal chance of using the list to its full potential.

The researcher had never conducted a research study prior to this one. She is a classroom teacher and has been for a number of years. As a classroom teacher, her inclination is to help when she sees a student floundering. The idea of allowing a student to flounder and not being able to say to the student, “no that’s wrong look at this instead” broke her heart and she struggled with the fine line between revealing too much and too little. For the researcher, it was a battle to the end to remain completely unbiased and not want to help the students. Ultimately, nothing the researcher said gave a student enough ammunition to complete a proof perfectly and so the researcher did not have a costly affect on her study.

Throughout the study, the students’ use of notation was relatively incorrect and inconsistent. However, this was not an aspect of their proof-writing ability that the researcher was able to investigate or report on because in her haste to prepare the problems for the students, she left out important notation. This could have affected the students’ ability to use correct notation in their own proofs. But, even with the notation issue the researcher was able to assess each student’s ability to construct a logical argument and the detail and manner in which the student used the list of reasons to help write a formal proof. This provided insight into students reasoning ability. In future studies, the researcher should make sure that she has no notation errors and thus she would be able to analyze student work through that avenue as well.
These findings as they exist apply only to English Language Learners taking Geometry. As stated earlier, this study could be extended to students with disabilities. In addition, if all students not just those with disabilities and English Language Learners were offered this modification, it might help all students to support their statements with reasons. Because proof-writing is not as heavily emphasized in Algebra, this modification is one that is pertinent only to Geometry – extending it to Algebra would not yield functional results for those courses.

**Summary**

This study investigated the effect of a list of reasons on English Language Learners ability to write proofs in High School Geometry. The researcher used qualitative methods to analyze the logical reasoning ability of each student and to analyze their thought process by their verbal statements on film and symbolic statements on paper in the proof. The findings agreed with the research that students’ ability to construct a formal proof is tied to their ability to reason logically. Through the study, the researcher found that the students who could reason logically were more likely to have positive results stem from using the list of reasons than those who are unable to reason logically. Though no student was able to create a perfect formal proof at any time throughout the process, some came very close missing only a few statements and reasons.

In this case, the modification was not used as part of a regular class and therefore, could have affected the outcome of the study simply by not providing as much support as some students may need and we cannot see the full effects of the modification. In addition, this study can be extended to other groups of students that receive modifications on
assessments such as special programs students. These students may benefit from this type of modification as well as they learn to write proofs.
REFERENCES


Usiskin, Z. (2007). What should not be in the algebra and geometry curricula of average college-bound students? *Mathematics Teacher* 100(Special Issue), 68-77.


APPENDICES
APPENDIX A

STUDENT WARM-UP PROOFS

Warm – Up Problem #1

Given:

Prove:

Warm – Up Problem #2

Given: $m\angle IFU = m\angle SFN$

Prove: $m\angle IFS = m\angle UFN$
APPENDIX B

PROOF WITHOUT A LIST OF REASONS PROVIDED

Given: ΔABF is isosceles with base CF = DF

Prove: \( \angle ACB \cong \angle BDA \)
Given: ΔRST is isosceles with base \(TU\);
\[ m\angle SUP = m\angle TUM; \]
U is the midpoint of \(TU\).

Prove: ΔUMP is isosceles
APPENDIX D
LIST OF REASONS IN ENGLISH, SPANISH, CHINESE, & TURKISH

English Reasons

Some of these may be used more than once and some may not be used at all.

If two angles of one triangle are , then the sides opposite them are .

If a = b, then each may replace the other in any algebraic expression.

If two segments have equal length, then they are .

a ≅ a

CPCTC

Angle Subtraction Postulate

Definition of Vertical Angles

Given

a = a

Side Angle Side Postulate

If a = b and b = c, then a = c

If a triangle has at least two sides, then it is isosceles.

Angle Addition Postulate

If two angles are vertical angles, then they are congruent.

If two sides of one triangle are , then the angles opposite those sides are also .

Angle Side Angle Theorem

Definition of Midpoint

If a = b, then each may replace the other in any algebraic expression.
Side Side Side Postulate

Definition of Between

If \( a = b \) and \( c = d \), then \( a - c = b - d \)

If a triangle is isosceles, then it has at least two sides.
Spanish Reasons

Algunos de estos pueden ser utilizados más de una vez y otros no pueden utilizarse en absoluto.

Si dos ángulos de un triángulo son $\cong$, entonces los lados opuestos son $\cong$.

Si $a = b$, entonces cada uno puede reemplazar al otro en cualquier expresión algebraica.

Si dos segmentos tienen la misma longitud, entonces son $\cong$.

$a \cong a$

CPCTC

Ángulo de Sustracción Postulado

Significado de <Angulos Verticales>

Teniendo en cuenta /Dado

$a = a$

Lado lateral del Ángulo Postulado

Si $a = b$ y $b = c$, entonces $a = c$

Si un triángulo tiene por lo menos dos lados $\cong$, es isósceles.

Ángulo de Adición Postulado

Si dos ángulos son opuestos por el vértice, entonces son congruentes.

Si dos lados de un triángulo son $\cong$, entonces los ángulos opuestos a los lados también son $\cong$.

Lado lateral del Ángulo de Teorema

Significado de <Punto Medio>

Si $a = b$, entonces cada uno puede reemplazar al otro en cualquier expresión algebraica.

Lado al otro lado del Lado Postulado
Significado de <Entre >

Si a = b y c = d, entonces a - c = b - d

Si un triángulo es isosceles, entonces tiene al menos dos lados que son .
Chinese Reasons

如果两个角全等的话，那它们的对边也全等。

如果 \( \alpha = \beta \)，那么它们其中一个可以在等式里替换另一个。

如果两条线段有相同的长度，那么它们全等。

\[ \alpha \equiv \alpha \]

全等三角形对应的部位全等。

角的减法

直角的定义

给出条件

\[ \alpha = \alpha \]

边角边公理

如果 \( \alpha = \beta, b = c \)，那么 \( \alpha = c \)

如果一个三角形至少有两条全等的边，那么这个三角形是等腰三角形。

角的加法
如果两个角都是直角，那么它们全等。

如果一个三角形里有两条边全等，那么它们所对应的角也全等。

边边角定理

中点的定义

如果 \( a = b \)，那么它们其中一个可以在等式中替换另一个。

边边边公理

中间的定义

如果 \( a = b, c = d \)，那么 \( a - c = b - d \)

如果一个三角形是等腰三角形，那么它至少有两条全等的边。
1) Eğer iki açıların bitanesi eşit değişse ozaman ters kenarlar eşit değildir.
2) Eğer a eşitse b ye ozaman her hangi birisi diğerinin yerine geçebilir cebirsel ifadede
3) Eğer iki parça eşit uzunluğu sahipse ozaman onlar eşittir
4) A eşittir a ya
5) CPCTC
6) Açı çıkarma postulat
7) Verilen
8) A eşittir a ya
9) Kenar açı kenar postulat
10) Eğer a eşitse b ve b eşitse c ye ozaman a eşittir c ye
11) Eğer bir üçgenin en az iki kenarı eşitse ozaman o üçgen ikizkenardır.
12) Açı toplama postulat
13) Eğer iki açları dikey açılarsa ozaman onlar eşittir
14) Eğer bir üçgenin iki yan eşitse ozaman o yanlara ters olan açılar ayrıca eşittir
15) Açı yan açı teorem
16) Orta naktanın tanımı
17) Eğer a eşitse b ye ozaman herhangi birisi diğerlerinin yerini alabilir cebirsel deyimin içinde.
18) Yan yan yan postulat
19) Ortanın tanımı
20) Eğer a eşitse b ye ve c eşitse d ye ozaman a eksi c eşittir b eksi d ye
21) Eğer bir üçgen ikizkenarsa ozaman en az iki eşit kenarları vardır

Turkish Reasons
APPENDIX E
COPY OF BETH'S WORK

Warm-Up Proofs

Warm-Up Problem #1

Given: MT ≅ AH
Prove: MA ≅ TH

Warm-Up Problem #2

Given: m∠IFU = m∠SFN
Prove: m∠IFS = m∠UFN
Proof Without a List of Reasons Provided

Given: ΔABF is isosceles with base AB
      CF = DF

Prove: ∠ACB ≅ ∠BDA

Given:
1) ΔABF is isosceles with base AB
   CF = DF
2) AC = BD

Proof With a List of Reasons Provided

Given: ΔRST is isosceles with base ST;
      ∠SUP = ∠TUM; U is the midpoint of ST

Prove: ΔUMP is isosceles

Given:
1) ΔRST is isosceles with base ST;
   ∠SUP = ∠TUM; U is the midpoint of ST
2) AS = TS
3) US = TS
4) U is the midpoint of ST
APPENDIX F
COPY OF DAVID'S WORK

Warm-Up Proofs

1. Given
2. If \( \equiv \), then =
3. Reflexive
4. If \( a = b \) and \( c = d \), then \( a - c = b - d \)
5. Def. of between
6. Substitution
7. If \( = \), then it is \( \equiv \)
Proof Without a List of Reasons Provided

Given: $\triangle ABC$ is isosceles with base $AB$
$CF = DF$

Prove: $\triangle ACB \cong \triangle BDA$

1. $\triangle ABC$ is isosceles with base $AB$  
   1. given
2. $BC = AF$
3. $CF = DF$
4. $\angle BAC = \angle ABD$
5. $\overline{AB} = \overline{AB}$
6. $AF - FC = BF - FD$
7. $AF - FC = AC$
   $BC - FD = BD$
8. $AC = BD$
9. $\triangle ACB \cong \triangle BDA$
10. $\angle ACB \cong \angle BDA$
11. $\text{CPCTC}$
Proof With a List of Reasons Provided

Given: $\triangle RST$ is isosceles with base $ST$; 
$\angle SU = \angle TUM$; $U$ is the midpoint of $ST$

Prove: $\triangle UMP$ is isosceles

1. $\triangle RST$ is isosceles with base $ST$; $\angle SU = \angle TUM$; $U$ is the midpoint of $ST$
2. $SR \cong UT$
3. $SU + UT = ST$
4. $SM + MP = EP + PT$
5. $SU = UT$
6. 

1. Given
2. If a triangle has two equal sides, then it is isosceles.
3. 1
4. $U$
5. Def. of midpoint.
APPENDIX G

COPY OF LINDA'S WORK

Warm-Up Proofs

Warm-Up Problem #1
Given: \( MT \cong AH \)
Prove: \( MA \cong TH \)

Given: \( MT \cong AH \)
Prove: \( MA \cong TH \)

\( \angle MA + AT = MT \)

\( \angle AT + TH = AH \)

Subtraction property of =

\( MA = TH \)

Warm-Up Problem #2
Given: \( m\angle IFU = m\angle SFN \)
Prove: \( m\angle IFS = m\angle UFN \)

\( m\angle IFU = m\angle SFN \) 
Given

\( m\angle IFS + m\angle SFU = m\angle IFU \)

\( m\angle SFU + m\angle UFN = m\angle SFN \)

Add. Postulate

\( m\angle SFU = m\angle SFU \) 
Reflex

\( m\angle IFU - m\angle SFU = m\angle SFN - m\angle SFU \)

Sub
Proof Without a List of Reasons Provided

Given: \( \triangle ABF \) is isosceles with base \( AB \)
\( CF = DF \)

Prove: \( \angle ACB \cong \angle BDA \)

\( \triangle ABF \) is isosceles with base \( AB \)

\[ AF = BF \]
\[ CF = DF \quad \text{Given} \]
\[ AF - CF = AC \]
\[ BF - DF = BD \]

\( \angle AEC \cong \angle BED \quad \text{Verte} \)

\( AB = AB \)
\( AC = BD \)
\( \angle BAC = \angle ABD \)
Proof With a List of Reasons Provided

Given: \( \triangle RST \) is isosceles with base \( ST \);
\( \angle SUP = \angle TUM \); \( U \) is the midpoint of \( ST \)

Prove: \( \triangle UMP \) is isosceles

- \( RST \) is isosceles with base \( ST \).
- \( RS = RT \); if a triangle has at least two \( \approx \) side, then it's isosceles.
- \( U \) is the midpoint of \( ST \).
- \( SU = UT \); definition of between.

\( \angle SUP = \angle TUM \); given.

\( \angle SUP + \angle UMP = \angle SUP \); angle addition postulate.

\( \angle TUR + \angle MUP = \angle TUM \).

\( \angle MUP = \angle MUP \); reflex.

\( \angle SUP - \angle MUP = \angle SUM \).

\( \angle TUR - \angle MUP = \angle TUR \); angle subtraction postulate.

\( \angle SUM = \angle TUR \); substitute.

\( \triangle MSU \cong \triangle PUT \); ASA.

\( \triangle MSU \cong \triangle PNU \).

\( MU = PU \); two angles and a side are \( \approx \) then the sides are \( \approx \).
APPENDIX H

COPY OF SUSAN'S WORK

Warm-Up Proofs

Warm-Up Problem #1

Given: \( MT \cong AH \)
Prove: \( MA \cong TH \)

1. Given
2. \( MT = AH \)
3. \( MT = AT \) (Given)
4. Subtracting property of =
   If \( a = b + c = d \), then \( a - c = b - d \).
5. If \( B \) is between \( A \) and \( C \), then \( AB + BC = AC = A, B, C \) are collinear
6. Substitution: If \( a = b \), then each can replace the other in any algebraic expression.

Warm-Up Problem #2

Given: \( m\angle IFU = m\angle SFN \)
Prove: \( m\angle IFS = m\angle UFN \)

1. Given
2. \( m\angle IFU = m\angle SFN \)
3. Subtracting property of =
4. Substitution
5. \( m\angle IFS = m\angle UFN \)
Proof Without a List of Reasons Provided

Given: \( \triangle ADF \) is isosceles with base AB
\[ CF = DF \]

Prove: \( \angle ACB \cong \angle BDA \)

1. \( \triangle ADF \) is isosceles with base AB
   \[ CF = DF \]  (1) Given

2. \( AF = BF \)

3. \( AF - CF = AC \)
   \[ BF - DF = BD \]  (3) Subtraction property

4. \( \angle ACB \cong \angle BDA \)

5. \( \angle ACB \cong \angle BDA \)
Proof With a List of Reasons Provided

Given: \( \triangle RST \) is isosceles with base \( ST \);
\( \angle SUP = \angle TUM \); \( U \) is the midpoint of \( ST \)

Prove: \( \triangle UMP \) is isosceles

1. \( \triangle RST \) is isosceles with base \( ST \).  
   \( \angle SUP = \angle TUM \); \( U \) is the midpoint of \( ST \)
   \( \text{Given} \)

2. \( SU = UT \)  
   \( \text{Definition of Midpoint} \)

3. \( \angle SUP - \angle SUM = \angle TUM - \angle TUP \)  
   \( \angle SUP - \angle SUM = \angle MUP \)  
   \( \angle TUM - \angle TUP = \angle MUP \)  
   \( \text{Subtraction postulate} \)

4. \( \angle MUP = \angle MUP \)  
   \( \theta \) \( a = a \)

5. \( MU = PU \)

6. \( SAS \)  
   \( \text{If } a = b \text{ and } b = c, \text{ then } a = c \)
APPENDIX I

COPY OF ARNOLD'S WORK

Warm-Up Proofs

Warm-Up Problem #1

\[ \begin{align*}
\text{Given: } & MT \cong AH \\
\text{Prove: } & MA \cong TH \\
\end{align*} \]

1. \( \overline{MT} \cong \overline{AH} \) \quad 1. Given
2. \( AT = AT \) \quad 2. Reflexive
3. \( MT - AT = AH - AT \) \quad 3. Subs. of property
4. \( MT - AT = MA; AH - AT = HT \) \quad 4. Substitution
5. \( MA = TH \) \quad 5. If \( a = b \), then \( a \cong b \)
6. \( \overline{MA} \cong \overline{TH} \)

Warm-Up Problem #2

\[ \begin{align*}
\text{Given: } & m\angle TFU = m\angle SFN \\
\text{Prove: } & m\angle IFS = m\angle UFN \\
\end{align*} \]

1. \( m\angle IFU = m\angle SFN \) \quad 1. Given
2. \( m\angle SFU = m\angle SFU \) \quad 2. Reflexive
3. \( m\angle IFU - m\angle SFU = m\angle SFN - m\angle SFU \) \quad 3. Subs. of property
4. \( m\angle IFU - m\angle SFU = m\angle IFS \) \quad 4. Definition \( \angle \)-between
   \( m\angle SFN - m\angle SFU = m\angle IFS \) \quad 5. Subtraction post.
5. \( m\angle IFS = m\angle UFN \) \quad 5. Substitution.
Proof Without a List of Reasons Provided

Given: \( \triangle ABF \) is isosceles with base \( \overline{AB} \)
\( CF = DF \)

Prove: \( \angle ACB \cong \angle BDA \)

1. \( \triangle ABF \) is isosceles with base \( \overline{AB} \)
\( CF = DF \)

2. \( \angle A = \angle B \)

3. \( \angle AEC = \angle BDE \)

4. \( \angle ACE = \angle BDE \)
Proof With a List of Reasons Provided

**Given:** $\triangle RST$ is isosceles with base $\overline{ST}$; 
$m\angle SUP = m\angle TUM$; $U$ is the midpoint of $\overline{ST}$

**Prove:** $\triangle UMP$ is isosceles

1. $\triangle RST$ is isosceles with base $\overline{ST}$
   
   $m\angle SUP = m\angle TUM$; $U$ is the midpoint of $\overline{ST}$

2. $m\angle SUP - m\angle MUP = m\angle TUM - m\angle MUP$

3. $m\angle MUP = m\angle MUP$

4. $m\angle SUP - m\angle MUP = m\angle TUM - m\angle MUP = m\angle TUP$

5. $m\angle SUP = m\angle TUP$

6. $m\angle SMU = m\angle TPU$

7. $m\angle SMU = m\angle TPU$

8. $m\angle SMU = m\angle TPU = \text{to each other}$

9. $2\angle's$ are
   
   to each other

10. $3\angle's$ are =
    
    to each other.
APPENDIX J

COPY OF JOAN'S WORK

Warm-Up Proofs

**Warm-Up Problem #1**

Given: $MT \cong AH$

Prove: $MA \cong TH$

1) $MT \cong AH$
2) $MT - AT = MA$
3) $AH - AT = TH$
4) $MT - AT = MA$
5) $MA = TA$

**Warm-Up Problem #2**

Given: $m\angle IFU = m\angle SFN$

Prove: $m\angle IFS = m\angle UFN$

1) $m\angle IFU = m\angle SFN$
2) $m\angle SFU = m\angle SFU$
3) $m\angle IFS = m\angle SFN - m\angle SFU$
4) $m\angle IFS = m\angle UFN$
Proof Without a List of Reasons Provided

Given: \(\triangle ABF\) is isosceles with base \(AB\)
\[CF = DF\]

Prove: \(\angle ACB \cong \angle BDA\)

1) \(\triangle ABF\) is isosceles with base \(AB\)
\[CF = DF\]

2)

Proof With a List of Reasons Provided

Given: \(\triangle RST\) is isosceles with base \(ST\);
\[\angle SUP \cong \angle TUM;\] U is the midpoint of ST

Prove: \(\triangle UMP\) is isosceles

1)
2) \(m\angle SUP = m\angle TUM\)
3) \(UM = MP\)
APPENDIX K
COPY OF IRB FORM

North Carolina State University
Institutional Review Board for the Use of Human Subjects in Research
SUBMISSION FOR NEW STUDIES

GENERAL INFORMATION

1. Date Submitted: 10/19/2009
   1a. Revised Date: 
2. Title of Project: Developing Proof Writing Abilities in English Language Learners
3. Principal Investigator: Julia Smith
4. Department: Mathematics Education
5. Campus Box Number: n/a
6. Email: jhsmith4@ncsu.edu
7. Phone Number: 919-337-3948
8. Fax Number: n/a
9. Faculty Sponsor Name and Email Address if Student Submission: Dr. Karen Hollebrands, Karen_Hollebrands@ncsu.edu
10. Source of Funding? (required information): There is no funding necessary.
11. Is this research receiving federal funding?: No
12. If Externally funded, include sponsor name and university account number:
13. RANK:
   - Faculty
   - Student: □ Undergraduate; □ Masters; or □ PhD
   - Other (specify): 

As the principal investigator, my signature testifies that I have read and understood the University Policy and Procedures for the Use of Human Subjects in Research. I assure the Committee that all procedures performed under this project will be conducted exactly as outlined in the Proposal Narrative and that any modification to this protocol will be submitted to the Committee in the form of an amendment for its approval prior to implementation.

Principal Investigator:

Julia H Smith

(typed/printed name) (*) (signature) (date)

As the faculty sponsor, my signature testifies that I have reviewed this application thoroughly and will oversee the research in its entirety. I hereby acknowledge my role as the principal investigator of record.

Faculty Sponsor:

Karen Hollebrands

(typed/printed name) (*)
*Electronic submissions to the IRB are considered signed via an electronic signature. For student submissions this means that the faculty sponsor has reviewed the proposal prior to it being submitted and is copied on the submission.

Please complete this application and email as an attachment to: debra_paxton@ncsu.edu or send by mail to: Institutional Review Board, Box 7514, NCSU Campus (Administrative Services III). Please include consent forms and other study documents with your application and submit as one document.

For SPARCS office use only
Reviewer Decision (Expedited or Exempt Review)
☐ Exempt ☐ Approved ☐ Approved pending modifications ☐ Table

Expedited Review Category: ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8a ☐ 8b ☐ 8c ☐ 9

Reviewer Name ___________________________ Signature ___________________________ Date ___________________________
In your narrative, address each of the topics outlined below. Every application for IRB review must contain a proposal narrative, and failure to follow these directions will result in delays in reviewing/processing the protocol.

A. INTRODUCTION
1. Briefly describe in lay language the purpose of the proposed research and why it is important.
   The purpose of the proposed study research is to investigate methods of developing geometry proof writing abilities in English Language Learners. It is important because it will help the students be more successful in developing logical thinking skills and tactics for writing proofs.

2. If student research, indicate whether for a course, thesis, dissertation, or independent research.
   The research is for a final project for a course which will be developed into a thesis.

B. SUBJECT POPULATION
1. How many subjects will be involved in the research?
   There will be no more than 30 subjects involved in the research (15 control and 15 experimental).

2. Describe how subjects will be recruited. Please provide the IRB with any recruitment materials that will be used.
   Participation in the study is voluntary and the students will be asked if they would like to participate.

3. List specific eligibility requirements for subjects (or describe screening procedures), including those criteria that would exclude otherwise acceptable subjects.
   The students must be English Language Learners taking Geometry.

4. Explain any sampling procedure that might exclude specific populations.
   None

5. Disclose any relationship between researcher and subjects - such as, teacher/student; employer/employee.
   I am the teacher of these students.

6. Check any vulnerable populations included in study:
   X minors (under age 18) - if so, have you included a line on the consent form for the parent/guardian signature (yes)
   □ fetuses
   □ pregnant women
7. If any of the above are used, state the necessity for doing so. Please indicate the approximate age range of the minors to be involved.

Students’ aged 14 – 17 will be involved in the study. These students need to be used because they are enrolled in Geometry and the principal investigator is the only instructor familiar with using the strategy to be used with ELL students in the study.

C. PROCEDURES TO BE FOLLOWED

1. In lay language, describe completely all procedures to be followed during the course of the experimentation. Provide sufficient detail so that the Committee is able to assess potential risks to human subjects. In order for the IRB to completely understand the experience of the subjects in your project, please provide a detailed outline of everything subjects will experience as a result of participating in your project. Please be specific and include information on all aspects of the research, through subject recruitment and ending when the subject’s role in the project is complete. All descriptions should include the informed consent process, interactions between the subjects and the researcher, and any tasks, tests, etc. that involve subjects. If the project involves more than one group of subjects (e.g. teachers and students, employees and supervisors), please make sure to provide descriptions for each subject group.

During high school geometry, the students have to take tests that involve proof writing. English Language Learners (ELLs) are at a disadvantage in developing the skills necessary to write proofs since English is not their native language and there are many theorems to memorize and put to use. During each unit test that involves proof writing; one group of students will be given an additional sheet of paper that contains a list of possible theorems, postulates, definitions, and properties while another group of students will not be given an additional sheet of paper. The students with the list of possible reasons can then use the list to help them develop their proofs. I will gather data from their proof writing ability and compare the proofs from both groups of students. The students’ role in the project will be complete upon successful completion of high school geometry.

2. How much time will be required of each subject?

The students are required to take the geometry test whether or not they participate in the study. Therefore, no additional time is required of each student.

D. POTENTIAL RISKS

1. State the potential risks (physical, psychological, financial, social, legal or other) connected with the proposed procedures and explain the steps taken to minimize these risks.

There are no risks connected with this study.

2. Will there be a request for information that subjects might consider to be personal or sensitive (e.g. private behavior, economic status, sexual issues, religious beliefs, or other matters that if made
public might impair their self-esteem or reputation or could reasonably place the subjects at risk of criminal or civil liability)?

| There will be no request for any information that subjects might consider personal or sensitive. |

a. If yes, please describe and explain the steps taken to minimize these risks.

b. Could any of the study procedures produce stress or anxiety, or be considered offensive, threatening, or degrading? If yes, please describe why they are important and what arrangements have been made for handling an emotional reaction from the subject.

| No |

3. How will data be recorded and stored?

| The data will be recorded using a numbering system. The students' name will be associated with a number and only the number will be used for the analysis. The data will be kept in a locked filing cabinet. |

a. How will identifiers be used in study notes and other materials?

| In study notes and materials, numbers will be used to refer to all participants. |

b. How will reports be written, in aggregate terms, or will individual responses be described?

| Individual responses will not be described. The reports will be an overall analysis of the benefit of using the instructional strategy to help English Language Learners to develop (write) proofs. |

4. If audio or videotaping is done how will the tapes be stored and how/when will the tapes be destroyed at the conclusion of the study.

| There will be no audio or videotaping done. |

5. Is there any deception of the human subjects involved in this study? If yes, please describe why it is necessary and describe the debriefing procedures that have been arranged.

| There is no deception of the subjects involved. |

E. POTENTIAL BENEFITS

This does not include any form of compensation for participation.

1. What, if any, direct benefit is to be gained by the subject? If no direct benefit is expected, but indirect benefit may be expected (knowledge may be gained that could help others), please explain.

| The benefit for the subject is a better understanding of developing proof, better understanding of the English language, and more developed logical reasoning skills. |
F. COMPENSATION

Please keep in mind that the logistics of providing compensation to your subjects (e.g., if your business office requires names of subjects who received compensation) may compromise anonymity or complicate confidentiality protections. If, while arranging for subject compensation, you must make changes to the anonymity or confidentiality provisions for your research, you must contact the IRB office prior to implementing those changes.

1. Describe compensation
   There is no compensation.

2. Explain compensation provisions if the subject withdraws prior to completion of the study.
   There is no compensation.

3. If class credit will be given, list the amount and alternative ways to earn the same amount of credit.
   There is no credit given for participating in the study.

G. COLLABORATORS

1. If you anticipate that additional investigators (other than those named on Cover Page) may be involved in this research, list them here indicating their institution, department and phone number.
   No additional collaborators.

2. Will anyone besides the PI or the research team have access to the data (including completed surveys) from the moment they are collected until they are destroyed.
   No.

H. CONFLICT OF INTEREST

1. Do you have a significant financial interest or other conflict of interest in the sponsor of this project?
   No

2. Does your current conflicts of interest management plan include this relationship and is it being properly followed? There is no conflict of interest in this study.

I. ADDITIONAL INFORMATION

1. If a questionnaire, survey or interview instrument is to be used, attach a copy to this proposal.

2. Attach a copy of the informed consent form to this proposal.

3. Please provide any additional materials that may aid the IRB in making its decision.

J. HUMAN SUBJECT ETHICS TRAINING

*Please consider taking the Collaborative Institutional Training Initiative (CITI), a free, comprehensive ethics training program for researchers conducting research with human subjects. Just click on the underlined link.
APPENDIX L
IRB CONSENT FORM

North Carolina State University
INFORMED CONSENT FORM for RESEARCH
Developing Proof Writing Abilities In English Language Learners

Julia Smith
Karen Hollebrands

What are some general things you should know about research studies?
You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate or to stop participating at any time without penalty. The purpose of research studies is to gain a better understanding of a certain topic or issue. You are not guaranteed any personal benefits from being in a study. Research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher(s) named above.

What is the purpose of this study?
The purpose of the proposed study research is to investigate methods of developing geometry proof writing abilities in English Language Learners. It is important because it will help the students be more successful in developing logical thinking skills and tactics for writing proofs.

What will happen if you take part in the study?
If you agree to participate in this study, you will be asked to complete two additional proofs outside of your regular class time. During this time, you will complete two proofs and participate in a short interview. On one proof, you will be given additional information to assist you with proof writing. When I assess your proofs for the study, I will be looking for how you formed your logical argument and if the extra information made a difference in your ability to form a logical proof.

Risks
There are no risks involved in participating in this study.

Benefits
The benefit in participating in the study is the possibility of being able to develop a logical argument when writing your proof which in turn will help your grade in your mathematics class.

Confidentiality
The information in the study records will be kept confidential. Data will be stored securely in a locked filing cabinet. No reference will be made in oral or written reports which could link you to the study.

Compensation
You will not receive anything for participating.

What if you have questions about this study?
If you have questions at any time about the study or the procedures, you may contact the researcher, Julia Smith, at 638 Walnut St., or 919-460-3549.
What if you have questions about your rights as a research participant?
If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

Consent To Participate
“I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to participate or to stop participating at any time without penalty or loss of benefits to which I am otherwise entitled.”

Subject's signature_______________________________________ Date ______________

Parent’s signature _______________________________________ Date ______________

Investigator’s signature___________________________________ Date ______________