ABSTRACT

NAMATOVU, WINNIFRED KIWANUKA. A Middle School Math Teacher’s Facilitation of Mathematical Discourse in Two Different Level Math Classes: An Action Research Study. (Under the direction of Dr. Allison McCulloch).

This paper is an action research study of how a 6th grade math teacher facilitates mathematical discourse in her different level math classes. The study was set in 2 sixth grade math classes, one regular and one advanced. The data included video recordings and audio recordings of lessons in each class over four consecutive days. Data was analyzed using the “Math-Talk Learning Community” framework (Hufferd-Ackles, Fuson, & Sherin, 2004). In the framework, there are four components: questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning. The tool focuses on the teacher’s actions in eliciting mathematical thinking from students. The findings from the study indicate that the teacher facilitated mathematical discourse differently between the average and advanced math classes. While students in both classes did have rich discussions about the mathematical concepts presented in the tasks, students in the advanced class were asked more in depth questions that elicited their mathematical thinking while students in the average class were asked questions that were short and direct. Furthermore, more needs to be done in both classes so that students can continuously engage in deep discussions about the concepts without a lot of prompting. Overall, all students can engage in mathematical discourse but the manner in which this teacher facilitates mathematical discourse in different level classes was different.
A Middle School Math Teacher’s Facilitation of Mathematical Discourse in Two Different Level Math Classes: An Action Research Study

by
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To James, Dorothy, Peter, Paul, and Philip.
BIOGRAPHY

Winnifred Kiwanuka Namatovu was born on March, 5, 1986 in Kampala, Uganda. Winnie lived in Uganda until the age of 8 before moving with her 3 brothers to Storrs, Connecticut to join their parents. At the age of 16, her family moved to Raleigh, North Carolina after her father received a position as a professor. After graduating from Sanderson High in 2003, Winnie enrolled in North Carolina State University. She graduated Cum Laude with a Bachelor of Science in Middle Grades Math Education in 2007. Winnie began teaching 6th grade math in Wake County, NC immediately after graduating. During the 2008-2009 school year, Winnie decided to enroll in a few courses at North Carolina State University. After being accepted to the Master’s program in Middle Grades Math Education in early 2009, Winnie formally began working on a Master’s program part-time in the fall of 2009. After receiving her Master’s Degree, Winnie plans to continue teaching middle school math.
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CHAPTER 1

INTRODUCTION

The National Council of Teachers of Mathematics (NCTM, 2000) explains discourse in a mathematics classroom as a way “of representing, thinking, talking, agreeing, and disagreeing; the way ideas are exchanged and what the ideas entail; and as being shaped by the tasks in which students engage as well as by the nature of the learning environment” (Garcia, 2011, p.1). Recently, the Common Core State Standards (CCSS) were released and they also stress the importance of communication in the math classroom. CCSS suggest that students should be able to make connections between conceptual understanding and procedures “through the communication students have around concepts, strategies, and representations” (Garcia, 2011, p.1). These documents point out that it is important for teachers to provide students with opportunities to explore, justify, and build on their mathematical ideas through discussions in the classroom. The teacher’s main role is to be the facilitator and provide students with a classroom environment that encourages them to openly share ideas, question the ideas of their peers, and work together to deepen their mathematical understanding.

I have been a 6th grade math teacher for four years. During my second year as a teacher, I decided to enter the Master’s program in Middle Grades Math Education at North Carolina State University so that I could learn how to become a better teacher. During my 3rd year in the program, I read a few articles about discourse as part of an assignment for one of my classes. As I completed the assignment and engaged in a discussion about discourse with my classmates, I became interested in learning more about discourse, particularly,
mathematical discourse. Over the years, I have tried different techniques to encourage my students to share their mathematical thinking and have found that the manner in which students in each of my four classes engage in the discussions varies. After reading about discourse, I wondered whether the manner in which I facilitate mathematical discourse in each of my classes also varies and might explain my students’ engagement. In order to explore this concern, I decided to complete a self-study of how I facilitate mathematical discourse in different level classes.

Since the goal of this study was to examine the ways in which I facilitate mathematical discourse and my purpose was to reflect on my actions and improve my practice I chose an action research design. Peterat (1997) defines action research as “a path of research practice that can contribute to wisdom of practice as we work with people in practical actions which impact their daily lives” (p. 122). The purpose of the research is to build knowledge about the topic of interest in order to use it to evaluate whether changes in classroom practice need to be made. In this case, the topic of interest is the facilitation of mathematical discourse.

Kemmis (2010) proposes that there are three kinds of understandings that guide action research: theoretical understanding, interpretive understanding, and self-understanding. Theoretical understanding helps us to understand our own practices through the application of different theories. Interpretive understanding helps us understand our own circumstances by looking at what how others in the same situations were affected. Self-understanding is learning about ourselves and our own unique struggles. As I planned the implementation of my study, I had to first read more about mathematical discourse so that I
could gain a better understanding of its meaning. As I was reading about the meaning of mathematical discourse, I also looked at a few studies that had already been done that involved mathematical discourse. I used the information that I gathered to identify my own definition of mathematical discourse as well to decide how to investigate my current facilitation of mathematical discourse.

As I read more about discourse and how teachers can facilitate mathematical discourse in the classroom, I realized that discourse is more than providing students with an opportunity to solve word problems in cooperative learning groups. Discourse involves teachers providing students with opportunities to engage in conversation about their mathematical thinking as they begin to take responsibility of their own learning. The teacher facilitates the discussion that takes places whether it is through questioning or encouraging students to share their ideas with their peers. In order to learn how I currently facilitate mathematical discourse and how I can improve, I decided to study my own teaching in two different math courses, a regular 6th grade math course and an advanced 6th grade math course. Bennett (1994) points out that action research provides a teacher with the opportunity to investigate an area of interest in order to improve on the classroom practices. I want to learn how I can best facilitate mathematical discourse in my classes to ensure my students are making conjectures, sharing ideas, questioning the ideas of their peers, and working together to build on their mathematical knowledge.

Peterat (1997) recommends that the researcher keep the following questions in mind when doing any action research: “whose action is being researched? Whose theories are employed? Is the purpose to develop grounded theory or informed practice? And how should
one position oneself in the research and in the report-writing in relation to others (pg. 121)?”

During my study, I had to continuously remind myself to focus on the fact that I was studying how I facilitate mathematical discourse in my classroom and not on my students. When analyzing my data I also had to remember to focus on my facilitation of mathematical discourse with individual students as well as the whole class. Furthermore, I had to think about how I would use my findings in my future practice as a teacher.

This study specifically addresses the following questions:

1. How do I facilitate mathematical discourse in an average math class?
2. How do I facilitate mathematical discourse in an advanced math class?
3. In what ways does my facilitation of mathematical discourse differ when teaching average and advanced 6th grade math classes?

The following chapters will describe the study in detail. In chapter two, definitions of discourse and mathematical discourse are presented in addition to a review of the literature related to teacher’s facilitation of mathematical discourse. Chapter three details the methodology. Chapter four presents the findings of the study. Finally, chapter five presents a discussion of the findings, a reflection on the findings, and suggestions for teachers (including myself) and future research are offered.
CHAPTER 2

LITERATURE REVIEW

The purpose of this chapter is to present a review of the research on discourse. It begins by offering definitions of discourse and mathematical discourse. Next, information about how mathematical discourse affects learning and how teachers facilitate mathematical discourse are presented. My goal is to provide a background on why students must be encouraged to develop, discuss, and justify their ideas through mathematical discourse in the classroom and justification for this study.

Defining Discourse and Mathematical Discourse

Vygotsky (1978) believed that when humans master how to communicate with each other, then they can transform their thinking from a basic level to a higher level (i.e. they learn). That communication, both verbal and non-verbal, is often called discourse. Piccolo, Harbaugh, Carter, Capraro, and Capraro (2008) point out that in its simplest form, discourse is when “the teacher tells and students respond” while in the most complex form, discourse is when “both stakeholders ask questions of each other” (p. 376-378).

In order to achieve learning, Vygotsky states that a human being transitions through various stages in the zone of proximal development. Within the zone of proximal development, a child learns how to solve problems on their own with the help of an adult or through collaboration with “more capable peers” (p.86). Through more research, other theorists have related parts of Vygotsky’s theory to instruction in the classroom. Singer (2007) claims that, discourse “relies heavily on the ‘peers and instructional environment’ components of Vygotsky’s zone and far less on the ‘support of a teacher’ component” (p. 2).
Furthermore, during discourse “the ownership of learning is placed heavily on the students while the teacher becomes the facilitator of thinking” (p. 2).

However, Zolkower and Shreyar (2007) assert that discourse is more than the negotiation of who controls the classroom discussion. Integrating Vygotsky’s theory with their own idea, they summarize discourse as whole-class discussions that are “teacher-guided meaning-making experiences that can serve as interpersonal gateways for students to appropriate meanings” (p. 178).

Williams and Baxter (1996) define discourse as “the establishment of social norms that empower students to discuss” various concepts and a tool that helps students produce useful knowledge. With this definition in mind, in order for discourse to be effective, appropriate discourse needs to be clearly defined and expectations must be set so that students feel comfortable to share their ideas. By establishing guidelines and setting an open atmosphere, discourse becomes a tool that students use as they work together to build on their current knowledge.

Antón (1999) describes two different forms of discourse that can exist in a classroom: learner-centered discourse and teachers-centered discourse. During learner-centered discourse, the teacher encourages students to share their ideas and strategies with each other. In teacher-centered discourse, the teacher is the only person that delivers information to the students; there is no “explicit dialogue” between the students. Learner-centered discourse allows students to make decisions about the form, content, and classroom behaviors while teacher-centered discourse does not provide as many opportunities for negotiations.
Since the introduction of the mathematics reform movement in the early 1990s, teachers have been encouraged to “aim for mathematical understanding in their instruction; that teachers conduct their classes so that student students develop conceptual knowledge” (Sullivan, p. 3, 1993). The reform movement encourages teachers to move from the “traditional” form of mathematics instruction to a “contemporary” form of mathematics instruction. Chazan and Ball (1995), describe a traditional math classroom as one in which the teacher begins by reviewing the homework assignment from the previous night, then introduces new material, and concludes with teachers assigning guided practice. However, in a reform math classroom, students are given more opportunities to “engage with complex, open-ended problems, in small groups and as a whole class” (p. 23) while the teacher shows and tells less. This type of discourse is referred to as mathematical discourse.

Piccolo, Harbaugh, Carter, Capraro, and Capraro (2008), define mathematical discourse as “interactive and sustained discourses aligned to the content of the lesson that addresses specific student learning issues” (p.378). Mathematical discourse is more than “talking, acting, interacting, thinking, believing, reading, writing,” it also includes “mathematical values, beliefs, and points of view” (Moschkovich, p. 326). While it is difficult to know whether a student’s ability to communicate mathematically develops from their everyday experiences or school experiences, mathematical discourse serves to enhance the mathematical competence that students develop (Moschkovich, 2003). Through oral and written communication, students are able to explore new ideas, justify their findings, and share their conclusions with other students.
Berger (2005) points out that Vygotsky believed that tools such as words, graphs, algebra symbols, or other physical tools played an important role as mediators of discourse. He states that these tools “do not just facilitate activity; they define and shape inner processes. Thus, Vygotsky saw action mediated by signs as the fundamental mechanism which links the external social world to internal human mental processes” (p. 156). Consequently, students can use words, visuals, or gestures to help them explain and justify their ideas to their peers, this is part of their mathematical discourse.

Hansen-Thomas (2009) present mathematical discourse as three parts, “one part of mathematical discourse in the reform classroom is rooted in social language involving problem solving and negotiation, another part involves the academic register used to interact with the teacher and texts, and yet another part entails content-specific language-a much greater part includes knowing when and how to employ the components” (p. 92). In other words, mathematical discourse involves students being able to manipulate mathematical language as they transition through various situations. Students should be able to understand and apply formal mathematical terms in the classroom as well as figure out how to relate it to informal settings or other classrooms. Furthermore, students should be able to know how one word’s meaning changes when used in different situations. An example would be the “table”; its meaning in everyday situations is different from that used in the math classroom.

Through mathematical discourse, students are allowed to share, defend, and transform mathematical ideas so that they are able to develop argumentative literacy (Singer, 2007). In order to support the student learning, two types of discourse must take place: cognitive discourse and motivational discourse (Stein, 2007). In the course of cognitive discourse, the
teacher can provide students with cues that help students arrive at understanding of the task. For example, a teacher can ask a student to explain how they knew what operation to use when solving the problem or whether or not they can think of another way to solve a problem. Motivational discourse takes place when the teacher provides supportive or non-supportive statements to the students. For example, a teacher can ask a student to explain their mathematical thinking to the class so that other students can help the student figure out a mistake. The types of statements that teachers use with students can serve to either promote mathematical discourse in the classroom or depress mathematical discourse in the classroom. For example, a teacher can encourage a student by saying “You are on the right track, look back at the problem to see if there is any additional information that might help you solve the problem” while a statement like “No, you are not doing it correctly” can discourage a student from exploring and discussing their mathematical thinking.

Like Piccolo, Harbaugh, Capraro, and Capraro, I believe mathematical discourse is the interactive and sustained communication aligned to the content of the math lesson. Additionally, I believe students use various tools to model their mathematical thinking. Models include diagrams, words, gestures, mathematical equations, and manipulatives. In order to be able to represent their mathematical thinking, students need to be provided with open-ended problems or problems that require students to do more than recalling previously taught procedures.

Mathematical Discourse and Student Learning

Discourse as suggested by NCTM is mathematical discourse. NCTM emphasizes that “the communication process requires students to reach agreement about the meanings of
words and to recognize the crucial importance of commonly shared definitions. Opportunities to explain, conjecture, and defend one’s ideas orally and in writing can stimulate deeper understandings of concepts and principles...In the process of discussing mathematical concepts and symbols, students become aware of the connections between them” (Williams and Baxter, 1996). The use of mathematical discourse in the classroom is “diametrically opposed” to the traditional “univocal” or lecture method that is used in most math classrooms. The “dialogic discourse” that takes place during mathematical discourse allows teachers and students to have open discussions about mathematical ideas. This is unlike a classroom that involves “univocal” dialogue because the focus is not on the teacher telling students what they should know and how they should respond (McGuire & Harshman, 2002). Singer (2007) adds that when students are given an opportunity to “read, write, and argue with numbers, or more important, mathematical concepts and ideas,” they are then more likely to become “numerically literate.”

Berger (2005) asserts that the use of words without developing a deep understanding of the word was first introduced by Vygotsky as the use of “pseudoconcepts.” She clarifies that “the use of pseudoconcepts enables children to communicate effectively with adults and that this communication (the intermental aspect) is necessary for the transformation of the complex into a genuine concept (the intramental aspect) for the learner” (p. 159). Thus, Vygotsky believed that in order for a child to enrich his or her understanding of a word or a concept, they used it in conversation with someone that could help them gain a better understanding of the term. During discourse, students can practice correctly using mathematical terms as they discuss concepts with their peers and teachers. Mathematical
discourse is important for ensuring student comfort with questioning and explaining their mathematical thinking, students understanding that there are multiple solution methods, students becoming independent learners, and students getting more comfortable with using mathematical terms (Hufferd-Ackles, Fuson, & Sherin, 2004; Way, 2008; Clement, 1997; Piccolo et al, 2008). According to Martino and Maher (1994) in order to be ready to engage in mathematical discourse, students go through three stages. Students begin by creating their own interpretation or representation of the problem. Students then solve the problem and move to sharing their ideas with their classmates. It is not until students have developed their own conclusion or understanding of the problem that students become comfortable and ready to engage in a conversation about the problem.

Martino and Maher (1994) found that when teachers provide opportunities for students to share ideas, students sometimes gain a better understanding of interpretation or explanations when presented by their peers. In one case, the researcher focused on how the teacher’s questioning helped a student make a generalization about a problem. The researcher found that through questioning, the student was able to come up with a hypothesis, test the hypothesis, and then use her results to make generalizations about similar problems. In that same study, the researcher observed a 4th grade teacher’s interaction with a student as he worked to make connections between different problems. Again, the researchers found that through questioning, the student was able to “stimulate” the student’s mathematical thinking so that he could make connections to previous problems.

The open dialogue that takes place during mathematical discourse provides students with an opportunity to also connect personal experience to mathematical concepts. Harshman
and McGuire (2002) investigated how tasks affect mathematical discourse and students’ retention of mathematical concepts about probability. The fourth and fifth grade students that were studied were able to engage in dialogic discourse as they worked to make connections between real-world situations and the mathematical concepts presented in the tasks. The penny activities and spinner activities were effective tools for introducing students to prediction and probability. Thus, through “well-constructed” activities and conversations, students are provided with a foundation needed to help them build connections that lead to “long-term retention of the concepts” (Harshman and McGuire, 2002).

While it is important for students to be able to correctly interpret the ideas of others, students need to be able to correctly interpret textual information. This information can be that presented in textbooks, word problems, or other written material. When students are not able to correctly interpret textual information, this causes textual interference (Kotsopoulos, 2007). During her observation of transcripts from a 9th grade math classroom, Kotsopoulos found students can have different responses when asked to apply words like factor or cancel which can cause problems with the correct mathematical application of the terms. As students become numerically literate, they must learn how to decode information offered in different texts as they apply that in their own understandings and interpretations of the mathematical concepts. It is not until students are able to break down information presented in various texts that they can talk about how to explore mathematical ideas, building on mathematical ideas, and share their mathematical ideas with their peers.

Moschkovich (2003) found that through discourse, students are able to search for certainty, abstract meanings, and make generalizations which are “highly valued practices” in
a math classroom. When observing a lesson in a 3rd grade urban classroom in California, she noticed that the teacher’s definition of a trapezoid differed from the students’ definition of a trapezoid. While the students were able to come up with a “working definition” of a trapezoid, the teacher was not satisfied with that response. She concludes that while it is important to teach students formal definitions, it is also important to encourage students to begin with a working definition and use that to extend to formal mathematical definitions.

As stated earlier, univocal and dialogic discourse both play an important role in the classroom. Students should be given an opportunity to do more than receive, encode, and store textual information. Blanton et al (2001) observed and interviewed a student teacher’s facilitation of mathematical discourse. During their first observation, they found that the teacher was the main questioner and transmitter of mathematical information. The teacher asked questions that guided students to what she was looking for, was not open to alternative student solutions or strategies, and was quick to jump in to help students when they were struggling to solve a problem. After talking to Blanton and her colleagues, the student teacher decided to let students discuss and explore mathematical problems for a longer period of time. Instead of telling students how to solve the problem, the teacher encouraged students to talk to each other about how to solve the problem. Instead of focusing on her own interpretation of the problem, the teacher asked students guiding questions to help students find and correct their mistakes. By the end of the study, they found that both the student teacher and the students became more comfortable in sharing responsibility in the explanation of mathematical thinking. The teacher was no longer the main source of ideas and the students were able to independently explore, discuss, and create new ideas.
As students work to become more comfortable interpreting a task and forming their own conclusions about the task, they ask questions that help them develop a deeper understanding of the concepts in the task. Hufferd-Ackles et al (2004) point out that students can use the responses of their peers to decide whether they agree or disagree with the work and responses of other students so that they can shift from the role of critic to helper to supporter. This is critical in ensuring that students understand why and how mathematical processes work when solving tasks. In order for students to be able to justify their ideas, they must develop a deep understanding of the mathematical terms, mathematical language, and be able to make connections among different mathematical concepts.

After reviewing 300 minutes of classroom transcriptions, Kotsopoulos (2007) observed that students struggle with understanding the difference between everyday language and mathematical language. For example, one student in the lesson was asked to explain how to solve a problem in which she had to multiply two monomials. When describing the use of the distributive property, the student referred to it as the “rainbow thing.” While the student was able to solve the problem, a closer examination of the student’s strategy shows that due to the student’s minimal understanding of the mathematical language, she could not mathematically explain her strategy. Thus, when students are not able to differentiate between everyday language and mathematical language, then students are not able to take their basic understanding of a mathematical term to conceptual understanding. Students must be familiar with the mathematical language used in the task as well as in the discussion so that they are able to describe their own mathematical thinking to other students. Moreover,
when students are able to connect mathematical terms and concepts to other ideas, students are able to extend their own mathematical thinking.

In that same study, Kotsopoulos (2007) asked students to explain words from the “mathematical register.” Students could use pictures, numbers, or words when explaining the given words. When asked to explain the meaning of “expand,” one student said “Expand? Expand, expand, what does it mean? I don’t know. Um, not really, expand. Expand, make it bigger, stretch it, expand it. That’s probably what it means in math” (p. 304). In this situation, the student should have been able to explain that expand meant to utilize the distributive property by multiplying all terms inside the grouping symbols by all the terms outside the grouping symbols. This example shows that while students are able to use everyday language to describe mathematical thinking, they are not able to correctly use mathematical language to describe their mathematical thinking and strategies. If students are not given the opportunity to learn how to appropriately use mathematical language, then it becomes difficult for them to grow in their learning of new concepts (Kotsopoulos, 2007). As students learn to extend and build their mathematical language, teachers need to provide them with opportunities to discuss different concepts as well as support to correctly apply the mathematical language to different tasks. Thus, opportunities in which students engage in mathematical discourse allow students to talk about their mathematical thinking and apply mathematical language in their conversations so that they can grow in their understanding of the terms as well as build connections between the different terms.

During her study of 3rd grade math teachers in a Spanish/English community, Hansen-Thomas (2009) found that the teachers used certain instructional practices in order to
get students to participate in mathematical discourse. Some of these practices included reading the problems out loud, showing students how to solve the problems orally, and repeating and emphasizing algorithms, concepts, and definitions. These practices were important models of mathematical discourse so that students could be encouraged to verbally interact with each other. Furthermore, teachers can elicit student thinking by repeating student explanations, using cues to direct student thinking or encouraging students to participate. Additionally, one of the teachers who was a Spanish speaker explained some of the mathematical procedures and definitions in Spanish as a way to help students better understand his oral math discourse. Essentially, mathematical discourse allows students to learn how to verbally interact with each other, extend their mathematical thinking and reasoning, and encourages students to talk about their mathematical thinking.

Facilitation of Mathematical Discourse

While students experience some form of informal mathematical discourse in their classrooms, it is important to ensure that quality mathematical discourse is facilitated in all classrooms (Williams & Baxter, 1996; Singer, 2007). Quality mathematical discourse, “encourages pursuit of inquiry…and is characterized by the teacher rephrasing a student’s response for clarity, using student responses to generate new meaning, and using utterances as thinking devices” (McGuire & Harshman, p. 4, 2002). The manner in which a teacher facilitates mathematical discourse is important in supporting students as they discuss and explore mathematical concepts in order to be able to make generalizations and build on prior knowledge.
Williams and Baxter (1996) spent 3 years studying the classroom of a middle school math teacher who was successful at facilitating mathematical discourse. As they examined the teacher’s classroom, they found that the classroom environment encompassed the components of an atmosphere that encouraged deep discussions about mathematical concepts presented. They noted that the teacher’s classroom had visual models and manipulatives, provided students with opportunities to work together in groups as they discussed mathematical procedures, and provided students with opportunities to explore open-ended questions that required more than 1 class period. They also found that over the three years, the teacher made changes according to students’ needs. So if a student needed support in understanding mathematical ideas, the teacher provided scaffolding to help better understand the concepts but if the student needed reminders about the appropriate behavior to exhibit during mathematical discourse, then the teacher provided support in that area. Overall, while the teacher had set up a classroom environment to promote mathematical discourse, she was willing to make changes in her classroom according to the needs of her students.

Williams and Baxter (1996) offer two ideas of how teachers can scaffold or support students during mathematical discourse: analytic scaffolding and social scaffolding. Analytic scaffolding is when the teacher helps students structure their mathematical ideas while social scaffolding is when the teacher establishes norms (1996). An example of analytic scaffolding is when the teacher provides students with several opportunities to share their mathematical ideas. An example of social scaffolding is when a teacher encourages a student to explain their mathematical thinking as a way to get the student to appreciate the importance of talking about mathematical ideas. In their study, Williams and Baxter (1996) found that
while there was some frustration on both the teacher and students’ parts, the discourse was important in providing students with an opportunity to scaffold each other as they work to develop a conceptual understanding of the mathematics. It is important for the teacher to remember that there must be a balance between social scaffolding and analytic scaffolding.

Martino and Maher (1994) studied a 3rd grade classroom, 4th grade classroom, and 5th grade classroom. Their focus was on how teacher questioning helped students in each classroom provide great justifications for their answers, make connections between different problems, and understand the mathematical approaches of their peers. In the 3rd grade classroom, a student was struggling to make a connection between two different problems. In order to help the student extend her mathematical thinking, the teacher provided the student with some manipulatives so that she could display her mathematical thinking and probed the student with questions as she worked through the problem. By scaffolding the student, the student was able to find and correct her mistake while extending her mathematical thinking. The questions that the teacher asked served as a trigger to help the student make connections to previous concepts as well.

Falle (2004) investigated a model of an after-school tutorial program in a rural town. The program included high school students that felt they were unprepared to do well in the “NSW Higher School Certificate.” She focused on how the students’ actions and conversations affected the direction of the discussions that students were having. She found that when she listened carefully to the conversations that students were having, the mistakes that students were making revealed information about students’ current understanding of the material. She points that instead of teachers listening just to correct students, teachers should
use that information to find ways to scaffold students to the correct understanding of the concepts and application of mathematical terms. Furthermore, teacher should model the appropriate use of mathematical language and encourage students to correctly incorporate mathematical language in their discussion. In order to be able to do that, teachers need to be aware of how their use of mathematical language affects their students’ use and understanding of mathematical terms.

Kotsopoulos (2007) found that students often feel like hearing mathematical language is “like hearing a foreign language” (p. 301). She found that when students were not able to make the transition from the use of mathematical language in their everyday lives to the mathematical classroom, it hindered student learning. She suggests that this struggle to correctly understand and apply mathematical language is due to two factors: teacher-talk interference and student-talk interference. Teacher-talk interference is described as when the teacher primarily uses mathematical terms and vocabulary when teaching while student-talk interference is when students primarily use everyday language when talking to each other. When students don’t have a clear understanding of the mathematical concepts and are not able to correctly apply everyday language when describing mathematical concepts, it becomes difficult to break the student-talk interference. Like in the study by Falle, Kotsopoulos (2007) found that language serves a big role in guiding a discussion. She suggests that teachers tape-record themselves and use that as a reflection tool for understanding why students think that mathematics sounds like a foreign language so that they can support students as they work to become comfortable using and applying mathematical language. In addition, she suggests that teachers provide students opportunities
to practice correctly using the mathematical language as well as apply it to different mathematical tasks and situations.

Blanton et al. (2001) studied a student teacher’s “mediation” of mathematical discourse and the traditional method of lecturing. Specifically, they looked at how to help teachers balance a univocal method of teaching with a dialogic method of teaching. The student teacher in this study was not familiar with facilitating mathematical discourse. The students also were not prepared to discuss, explore and share ideas about the mathematical concepts that were presented. Thus, both teacher and student had to change their previous thought of how mathematics is taught and learned. When students were asked to work through a problem with their peers, students were opposed to that idea at first. The teacher continuously had to ask the students to share their ideas with their peers and eventually students were able to come up with solutions and strategies that surprised even the teacher. During whole class discussions, the teacher found that she had to “relinquish” her own strategy and be more open-minded to the strategies that students shared. Based on these findings the researchers suggest that a teacher’s reflections on how they teach mathematics can help them establish social behaviors in a mathematical discourse community. While the control of the lesson can shift between teacher and student throughout the lesson, the goal is to ensure that students take responsibility for themselves and each other as they explore the mathematical concepts.

In their study of a 3rd grade teacher teaching in an urban Latino school, Hufferd-Ackles et al. (2004) found that students take more responsibility for themselves and others when teachers monitor whether students are offering ideas that build on the ideas shared by
others in the classroom, and choose the ordering of sharing of ideas accordingly. Similarly, Piccolo et al (2008) recommend the use of open-ended questions as a way to cue students, improve on students’ weak responses and develop an environment that allows for mathematical curiosity and inquiry.

While much of the research on mathematical discourse is focused on teacher questioning, simply improving questioning does not assure productive mathematical discourse. For example, Kawanaka and Stigler (2000) found that when teachers were encouraged to ask questions that elicited analysis, synthesis, justification, evaluation, and conjecture, also known as “high order” questions, it did not necessarily produce student learning. They reported that students must be encouraged and provided with opportunities to build on their prior knowledge through reflection and not “merely retrieving isolated concepts or processing pieces of information” (p. 278). Moreover asking students to describe or explain their solution does not necessarily mean that the question is a “higher order” question. They suggest that “Higher order” questions should require students to build their mathematical knowledge and develop students’ mathematical reflective thinking skills. Teachers should ensure that the questions they ask serve to both guide and stimulate the mathematical thinking of their students. This is especially important when one considers that teachers’ questions can also serve as models for proper questions for students to ask of each other (Way, 2008).

Through listening to student responses and explanations, teachers might also be able to learn from students. Axiak (2004) found that in her studies of student teachers’ use of questioning as a way to explore middle school students’ reasoning, the student teachers were
able to learn of different methods for solving the problems as well as think about other possible solutions for the problem once they listened to students’ responses and explanations. Thus, it is important for the teacher to play the role of listener as well as initiator in order for the conversational dialogue that takes place during mathematical discourse to be successful.

In a case study of one university math instructor’s facilitation of mathematical discourse Clements (1997) describes how a teacher’s bias about their mathematical explanation of a concept can greatly impact a student’s willingness to participate in mathematical discourse. The teacher’s own interpretation of how she wanted students to interpret the problem and solution prevented the teacher from exploring the student’s interpretation of the problem so that she could assist the student in correcting her mistake. As a result Clements suggests that in order to know how to properly assist students, teachers must examine their own mathematical reasoning and underlying calculations so that they can have a better vision of what their students should gain out of a problem. If students offer a different solution, teachers should use that as an opportunity to explore students’ cognitive development and abilities. When teachers ask students about their mathematical thinking and procedure, they can help students correct their own mistakes without telling them how to do it.

Stein (2007) points out that promoting and implementing mathematical discourse in the classroom is difficult but essential in encouraging students to “make conjectures, talk, question, and agree or disagree” about tasks that help them discover important mathematical ideas. She found that when teachers show enthusiasm for learning, set expectations, and establish a classroom environment that fosters positive relationships among the students as
well as the teacher, mathematical discourse can take place. Martino and Maher (1994) add that if students disagree about a solution, the mathematical discourse that takes place gives students an opportunity to re-evaluate their mathematical thinking or find ways to convince their peers that their solution is justified.

Facilitation of mathematical discourse is not easy and research has found that there are many constraints that keep teachers from engaging their students in it. To better understand the transition that teachers and their students go through when trying to move from more traditional to reform mathematics teaching Hufferd-Ackles et al (2004) conducted a case study of one teacher over the course of a year as she tried to move toward reform practices, including mathematical discourse. Through this intense case study they identified a developmental trajectory. The framework served as a tool for the teacher to listen to students’ ideas, elicit students’ mathematical thinking, and encourage students to listen to the ideas of their peers. The framework also guided the teacher’s facilitation of mathematical discourse. The teacher was able to examine the current level of her students and use it to continuously make changes that would help her students move from a level 1 to 3 in each of the four components. This framework shows promise for helping teachers identify strengths and weaknesses in their facilitation of discourse and developing their skills toward more productive and student centered discursive practices. It is for these reasons that I chose to use this as a conceptual framework to guide the analysis in this action research study. I offer more detailed information on how this framework was applied in chapter 3.

Research has shown that students engaging in mathematical discourse are able to discuss and explore mathematical concepts in order to be able to extend their mathematical
thinking to build new ideas. In addition, the field is beginning to build an understanding of how teachers facilitate mathematical discourse, including some of the constraints they face. However, most of these studies are in single classrooms and do not consider how the same teacher facilitates discourse in different settings. For example, how does a teacher handle the differences of working in average level and advanced level math classes? Is there a difference in the way discourse is facilitated? Does the teacher encounter similar or different obstacles when trying to move toward a more reform oriented practice in these different contexts? This is an important idea to explore because students in different level math classes have different needs, therefore the teacher might need to use different strategies to facilitate mathematical discourse.
CHAPTER 3

METHODS

This study was an action research project designed to investigate and compare how I facilitate mathematical discourse in two types of mathematics classes, average and advanced. The purpose of this chapter is to describe the study methodology including: the design, the participants, setting, data sources and methods of analysis. Each is detailed in the sections that follow.

Research Design

The focus of this action research study is my facilitation of mathematical discourse in two different contexts, an average 6th grade math class and an advanced 6th grade math class. Gall et al (2007) assert that a researcher looks at multiple cases or contexts because he or she believes that the results will be the same in all studies or that they will differ. Before beginning my study, I hypothesized that the manner in which I facilitate mathematical discourse in each level of math class is different because of the mathematical ability and achievement levels of the students in each class. In the following sections I detail the design of the study including the participants, data sources, and data analysis.

Participants

The focus within each class will be on me, the teacher. I have been teaching for 4 years, all in 6th grade at the same school. I am certified in Middle Grades Math from 6th to 9th grade and am currently finishing up my masters of Science degree in mathematics education. It was through a course in my masters program on interactions in the mathematics classroom
(EMS 592; Interactions in the Mathematics Classroom) that I began to think about how I facilitate discourse.

I teach in a large county school district in a Southeastern state in the United States. The school is made up of about 1200 students. Class A was an average 6th grade Math class with 24 students, 10 boys and 14 girls. While half of the class is of high ability level, the other half is of low ability level. Students in that class are easily distracted and tend to need a lot of reminders to remain on-task and treat each other with respect. Class B was an Advanced 6th grade Math class with 24 students, 14 boys and 10 girls. Students in that class are eager to ask questions and explain their mathematical thinking about different problems but are easily distracted. 8 students in class A had an IEP while 1 student in class B had an IEP.

Before beginning my study, I sent out a letter to each participant’s parents/guardians explaining the purpose of the study and asking for permission to video and audio tape their child. Students who did not get permission to participate in the study were placed out of camera shot and were not audio taped. Students could drop out of the study at any time.

Sources of Data

In order to capture as much of the discourse as possible and get a sense of my facilitation patterns in each of the two classes, each was videotaped for four consecutive days of instruction (the mathematical context is presented in the next section). Each class was videotaped for 55-60 minutes from March 30th to April 4th. The video recorder was placed on a tripod at the back of the classroom and was aimed at the front of the room. It was unmanned, so it was not possible to roam during class instruction. Since the camera was not
roaming additional audio data was captured to supplement the video data. I carried a digital audio recorder to capture my communication as a back up to the video data. The unit that was taught during this study was focused on probability, specifically permutations and combinations. Details of the lessons appear in Appendix A.

**Data Analysis**

Classroom video data was analyzed in 4 steps. I began by using an adaptation of Powell, Francisco, & Maher’s (2003) framework for analyzing classroom video. First I watched each of the 16 lessons and described them. When describing the lessons, I made notes in 5 minute increments. The notes included information about what the students were doing, what I was doing, questions that students asked, questions that I asked, and responses when students explained their mathematical thinking. Once I described each lesson, I transcribed parts of the lesson when there was interaction between myself and a student or myself and the whole class. While creating each transcript, if a segment was inaudible I went to the audio recording that was attached to me to try to determine what was said. If the audio was still inaudible, I made a note of that in the transcript.

Once the transcriptions were complete and the classroom discourse had been reconstructed (using the video and audio data together), I coded each lesson using the “Math-talk community” framework. Each lesson was coded using the “Math-Talk Community” framework (Hufferd-Ackles et al, 2004). There are multiple tools available for studying mathematical discourse in classroom settings. For example, the Oregon Mathematics Leadership Institute (OMLI) Classroom Observation Protocol (Weaver, 2005) is an instrument that was developed to keep a record of the mathematical discourse that takes place
in a classroom. It is typically used with classroom observations with a focus on student
interactions with those around them (i.e. teachers, students). However, since the focus of this
study is not on students but on the teacher’s facilitation of mathematical discourse, I have
chosen to use the “Math-Talk Community” framework for this study. A copy of this
framework appears in Table 1.
Table 1: Levels of Math-Talk Learning Community: Action Trajectories for Teacher and Student

| Levels of the Math-Talk Learning Community: Action Trajectories for Teacher and Student |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Overview of Shift over Levels 0-3: The classroom community grows to support students acting in central or leading roles and shifts from a focus on answers to a focus on mathematical thinking. |
| Questioning                      | Explaining mathematical thinking | Source of mathematical ideas    | Responsibility for learning     |
| Shift from teacher as questioner to students and teacher as questioners. | Students increasingly explain and articulate their math ideas. | Shift form teacher as the source of all math ideas to students’ ideas also influencing direction of lesson. | Students increasingly take responsibility for learning and evaluation of others and self. Math sense becomes the criterion for evaluation. |

*Level 0: Traditional teacher-directed classroom with brief answer responses from students.*

| Teacher is the only questioner. Short frequent questions function to keep students listening and paying attention to the teacher. | No or minimal teacher elicitation of student thinking, strategies, or explanations; teacher expects answer-focused responses. Teacher may tell answers. | Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math. | Teacher repeats student responses (originally directed to her) for the class. Teacher responds to students’ answers by verifying the correct answer or showing the correct method. |
| Students give short answers and respond to the teacher only. No student-to-student math talk. | No student thinking or strategy-focused explanation of work. Only answers are given. | Students respond to math presented by the teacher. They do not offer their own math ideas. | Students are passive listeners; they attempt to imitate the teacher and do not take responsibility for the learning of their peers or themselves. |
Table 1 Continued

| Level 1: Teacher beginning to pursue student mathematical thinking. Teacher plays central role in the math-talk community. |
| --- | --- | --- | --- |
| Teacher questions begin to focus on student thinking and focus less on answers. Teacher begins to ask follow-up questions about student methods and answers. Teacher is still the only questioner. | Teacher probes student thinking somewhat. One of two strategies may be elicited. Teacher may fill in explanations herself. | Teacher is still the main source of ideas, though she elicits some student ideas. Teacher does some probing to access student ideas. | Teacher begins to set up structures to facilitate students listening to and helping other students. The teacher alone gives feedback. |
| As a student answers a question, other students listen passively or wait for their turn. | Students give information about their math thinking usually as it is probed by the teacher (minimal volunteering of thoughts). They provide *brief descriptions* of their thinking. | Some student ideas are raised in discussions, but are not explored. | Students become more engaged by repeating what other students say or by helping another student at the teacher’s request. This helping mostly involves students showing how *they* solved a problem. |

*Level 2: Teacher modeling and helping students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases. Teacher physically begins to move to side or back of the room.*
Table 1 Continued

<table>
<thead>
<tr>
<th>Teacher continues to ask probing questions and also asks more open questions. She also facilitates student-to-student talk, e.g. by asking students to be prepared to ask questions about other students’ work.</th>
<th>Teacher probes more deeply to learn about student thinking and supports detailed descriptions from students. Teacher open to and elicits multiple strategies.</th>
<th>Teacher follows up on explanations and builds on them by asking students to compare and contrast them. Teacher is comfortable using student errors as opportunities for learning.</th>
<th>Teacher encourages student responsibility for understanding the mathematical ideas of others. Teacher asks other students questions about student work and whether they agree or disagree and why.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students ask questions of one another’s work on the board, often at the prompting of the teacher. Students listen to one another so they do not repeat questions.</td>
<td>Students usually give information as it is probed by the teacher with some volunteering of thoughts. They begin to stake a position and articulate more information in response to probes. They explain steps in their thinking by providing <strong>fuller descriptions</strong> and <strong>begin to defend</strong> their answers and methods. Other students listen supportively.</td>
<td>Students exhibit confidence about their ideas and share their won thinking and strategies even if they are different from others. Student ideas sometimes guide the direction of the math lesson.</td>
<td>Students begin to listen to understand one another. When the teacher requests, they explain other student’ ideas in their own words. Helping involves clarifying other students’ ideas for themselves and others. Students imitate and model teacher’s probing in pair work and in whole-class discussions.</td>
</tr>
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Table 1 Continued

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<tr>
<th>Level 3: Teacher as co-teacher and co-learner. Teacher monitors all that occurs, still fully engaged. Teacher is ready to assist, but now in more peripheral and monitoring role (coach and assister).</th>
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<tr>
<td>Teacher expects students to ask one another questions about their work. The teacher’s questions still may guide the discourse.</td>
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<tr>
<td>Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions and listen to responses. Many questions are “Why?” questions that require justification from the person answering. Students repeat their own or other’s questions until satisfied with answers.</td>
</tr>
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The “Math-Talk Learning Community” framework includes four components: questioning, explaining math thinking, source of mathematical ideas, and responsibility for learning. In each of the categories, there are four levels (0-3). The levels serve to monitor the nature of mathematical discourse. Within the questioning component, the framework is designed to qualitatively codify the nature of the questioning taking place in the class. The explaining mathematical thinking component allows the researcher to code who was doing the explaining and articulating of mathematical thinking. The source of mathematical ideas component focuses on who possesses mathematical authority in the classroom. Finally, the responsibility for learning centers on how students begin to take more responsibility for their learning and other students. The “Math-Talk Community” framework is appropriate for analyzing data related to teachers’ facilitation of mathematical discourse. It doesn’t look at individual students; rather it focuses on the class as a whole and the teacher’s role within.

When using the framework to code the transcribed data I began by highlighting parts of the transcript that dealt with questioning. They could have been questions that I asked or questions that students asked. Once I highlighted all the questioning throughout the lesson, I used the framework to decide what level each question was. If the questions I asked were short and answer focused, then the question was assigned a level 0. If the questions I asked began to focus on student thinking and less on answers, then the question was assigned a level 1. If the questions I asked were open-ended or if I encouraged students to ask each other questions about thinking then it was assigned a level 2. If the question was asked by a student or if I encouraged students to ask each other questions until they are satisfied with their peer’s response then it was assigned a level 3. Once I assigned each question a level, I looked
at how many level 0, level 1, level 2, and level 3 questions there were in the lesson. The level of questioning that appeared the most (the mode) was used to assign the overall level of questioning of the lesson. For example, if most of the questions were at a level 1, the whole lesson was assigned a level 1. I continued this process for each of the lessons. Once all the lessons were assigned a level, I looked at what level appeared the most in order to assign a level of questioning to each class.

Next I went through the transcript again and highlighted all episodes in which the teacher was eliciting student thinking. Again, I used the “Math Talk Community” framework to assign each highlighted excerpt a code of level 0, 1, 2, or 3. A level 0 was assigned when students didn’t explain their thinking or strategies and were not elicited to do so. A level 1 was assigned when the teacher elicited some explanation of student thinking or strategies but students only gave explanations if they were asked to do so by the teacher. In a level 2, the teacher began to ask for more detailed explanations and strategies but the students were still offering their responses only when asked to do so by the teacher; there were a few voluntary responses. Finally a level 3 was assigned when the teacher motivated students to explain their mathematical thinking and make it more complete by defending and justifying their work and strategies. Once I assigned each excerpt a level, I looked at how many level 0, level 1, level 2, and level 3 responses were in the lesson. The level of responses that appeared the most, determined the overall level of explaining mathematical thinking of the lesson. Once each lesson was assigned a level, I looked what level appeared the most in all the lessons and assigned that level for the set of lessons for class as a whole.
Next, I coded the transcript for the sharing of mathematical ideas assigning each highlighted excerpt at level 0, 1, 2, or 3. I referred back to all the questions and explanations of mathematical thinking that I had highlighted as well as the levels of each item. If I was the main questioner and the main source of mathematical ideas, then the lesson received a level 0. If I was the main questioner and students shared their own ideas about concepts then the lesson was assigned a level 1. If I was the main questioner but I let the ideas shared by students guide the discussion or course of the lesson then the lesson was assigned a level 2. If the lesson was student led with very little input from me, then the lesson was assigned a level 3. I continued this process for each lesson. Once each lesson was assigned a level, I looked at the level that appeared the most in each class’ lessons. If most or all the lessons received a level 1, then the class was assigned a level 1 for source of mathematical ideas.

Lastly, I highlighted statements that represented responsibility for learning. In order to earn a level 0, the lesson was made up of mostly teacher repetition of student responses, teacher verification of student responses, and teacher explanation of student responses. In level 1, the lesson is still mostly made up of teacher feedback but students are encouraged to help each other or to repeat their explanations so that other students can learn how to solve a problem. During a level 2 lesson, the teacher motivates students to take responsibility for their own understanding of mathematical ideas presented by other students and also asks students to give their opinion about the work of their peers. In a level 3 lesson, the teacher facilitates the discussion as students question and clarify the work of their peers. I assigned each statement a level 0, 1, 2, or 3. I looked at what level appeared the most in the lesson and assigned the overall lesson using that level. For example, if most of the statements were at a
level 1, then the overall lesson was assigned a level 1. I continued this process for each lesson. Once each lesson was assigned a level, I looked at what level appeared the most in all the lessons and assigned that level for the class. For example, if most or all of the lessons were at a level 0, then the class was assigned a level 0 for responsibility for learning.

To ensure that I was applying the codes within the Math-talk Community framework consistently and appropriately, I had a colleague use the framework to code a sample lesson and compared this to my coding. We agreed on the coding 100%. Once we agreed on the use of codes, I coded the remaining lessons myself. This process helped to ensure the validity of my findings and reliability of my coding.
CHAPTER 4
FINDINGS

In this chapter the intention is to present what I learned about my facilitation of discourse in two different levels of mathematics classes based on my analysis. I will begin by first presenting the findings related to class 1, the facilitation of mathematical discourse in the average math class, then I will discuss the findings in class 2, the facilitation of mathematical discourse in the advanced math class, and finally I present a comparison of the two classes. Within each class and the cross comparison I will present the results according to the four categories in the “Math-Talk Learning Community” framework: questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility.

Facilitation of discourse in an average math class

*Questioning*

The overall questioning in this class was at a level 0. At a level 0, the teacher is the main questioner and short questions are asked frequently in order to keep students focused and listening to the teacher. I asked questions such as “So we want to take the probability of H and multiply it by something. Gina?”, “If we multiply $3/6$ times what?” and “So I’m going to do $3.99$ minus what?” When students didn’t seem to remember how to solve a problem, I asked questions like “Probability of even…so when you look at that first spinner, what is the probability of getting an even number?”, “What’s the probability of an odd number?”, and “What is the probability that Rachel will pick an even numbered chip and then an odd number WITHOUT replacing them? That means she’s not putting them back and she’s not putting in a new one….so probability of even times the probability of odd?” Those questions were asked to help remind students of the procedures that we had talked about in the previous
unit when finding the probability of compound events. When students didn’t understand how to apply a vocabulary word, I asked questions like “What is the theoretical probability, any time you roll a number cube, of getting a 6? What is the theoretical probability of getting a 6?” In response to the questions asked, students answered with responses that were short and not detailed.

While the overall lessons were at a level 0, there were some moments that were at a level 1. These questions were meant to focus on student thinking and less on the students’ answers. When Luke and Rita were struggling with how to solve a problem, I asked “What is the question asking you?” Luke responded with “How much money? That’s subtracting…that’s what I said in the beginning.” I reminded Luke that “Just because you’re talking about money, doesn’t mean you are necessarily going to subtract” so then Luke asked “Add?” Before walking away I reminded them not to guess and asked them to think about “Why, are you subtracting first of all? You have to figure that part out first. Why are you subtracting?” I also asked follow-up questions such as “Where’s the $2.50 coming from? Where did we get that number from?”, “Stella’s group, why do you think that it is true?”, “Why is it 11 over 27 not 12 over 27?”, and “$2/13$, where did you get the 2 from?” While I was still the main questioner, I used the level 1 questions as opportunities to encourage students to explain their methods and answers. Since I asked questions of this type less than 50% of the time, I rated the lesson at a 0 instead of a level 1.

**Explaining Mathematical Thinking**

The overall explaining mathematical thinking in this class was at a level 0. At a level 0, there is little teacher elicitation of student mathematical thinking. Questions that the
teacher asks are “answer-focused” and there are moments when the teacher may tell students the answers. As mentioned before, I asked students questions that elicited student thinking, strategies, and explanations less than half the time during the lesson. Students responded with answers that were short and focused. For example, when I asked Stella “Alright, so how many did you come up with Stella?”, she responded with “I came up with 12.” When I asked Bonnie “What did you start your tree diagram with?”, she responded with “Red, green, and purple.” When asked “If the vampire is first, who can go second?” and students responded with “The ghost.” Other examples of short responses without detailed explanations are “$\frac{3}{6}$ which equals $\frac{1}{2}$”, “12”, “2R, 2B, 2P, 3P, 3B, 3R, 4B, 4P, 4R” and “J and that one had K, A.”

When I probed students for their thinking behind their strategies, student responses were at a level 1. Although the explanations were brief, they provided descriptions and gave information about students’ thinking. When asked “So, where did you get your numbers from, Stella?”, Stella responded with “I got the numbers from the spinner.” When Dan was asked to explain why he couldn’t decide if he agreed with the advertisement claim in the problem, he offered that “Well, first off, I put yes because I did get $1…over $1 back. Second off, I said no because they said there are over 21 unique combinations, there’s only 18.” When asked “Why is it 11 over 27 not 12 over 27?”, Stella responded with “Because you didn’t put the other black sock back.”

Students were also given an opportunity to offer multiple strategies for solving a problem, which demonstrated a level 1 of explaining mathematical thinking. After creating multiple tree diagrams and lists, Rita pointed out that “Can’t you just like…when you get…since you know you have 4 numbers and 3 colors, can’t you multiply 4 by 3 and that
would be…” While this idea was offered before I was ready to introduce the fundamental counting principle, students were able to pick up on the pattern as they listed all the possible combinations using tree diagrams and discussing their answers with their peers. Another example of when a student offered a different strategy for solving a problem was when Luke said “And then I put, since I didn’t simplify I put \( \frac{12}{60} \) and then I simplified that and I got \( \frac{1}{5} \). I got \( \frac{1}{5} \)” when talking about how he found the probability of a compound event.

When student explanations lacked detail I sometimes filled in with more detail. For example, when I asked Thomas “Why is it 19 not 20?” Thomas answered with “We took one away” and I added that “We took one away. We did not put it back. We didn’t put a new one in there. If it says without replacement, your total changes.” Similarly, when students were asked if a restaurant’s advertisement claim for 4 types of pancakes, 3 types of eggs, 3 kinds of cereal, and 3 kinds of juice offered more than 100 combinations was true. Students said that it was true and I added the explanation that “…because they told us that you could eat for 100 days and have a different combination each day, the advertisement is true because we ended up with 108 combinations which is more than 100.”

*Source of Mathematical Ideas*

The overall source of mathematical ideas was at a level 1. At a level 1, the teacher is still the main source of ideas but student ideas are elicited through questioning. While I was still the main source of information, students were asked to offer ideas on how to solve problems as well as offer some explanations on strategies used. As mentioned above, I usually had to probe students for their ideas through questioning. When asked “How would you simplify \( \frac{4}{12} \)?”, Thomas answered with “Divide 4 by 4 and 12 divide by 4.” When I
asked students “Can we leave it like that?”, Luke responded with “You can simplify by 2.”

When I asked Dan “…we want to know how much you save so how do I figure that part out?”, he said that “You do $3.99 minus $2.50.” When discussing whether the advertisement claim was true, students offered various ideas “It’s not true because…when Suzie got…she did pizza, corn, and milk, she only saved $0.40…Which is a lot less… it’s not even half of a dollar.”, “Because if you add…if you buy everything separately, it is more money.” and “Well, I saved $0.94 and that’s pretty much the same thing as $1…it’s only $0.06 lower.”

At a level 1, students offer ideas during discussions but these ideas are sometimes not explored. Sometimes students introduced an idea that I purposely chose not to explore. For example, students pointed out that you could multiply the number of items in each category as we continued to create tree diagrams and lists but I did not want to introduce them to the fundamental counting principle, so I didn’t explore those ideas. Thus, when Rita raised the question “can’t you multiply 4 by 3 and that would be…”, I didn’t focus on that idea at that moment because I wanted other students to continue exploring the relationship between combinations and the fundamental counting principle.

Responsibility for Learning

The responsibility for learning in this class was at a level 0. While students were willing to participate, I continuously had to encourage students to take responsibility for their learning and their peers. For example, when discussing whether the advertisement claim was true, Amanda was willing to offer her explanation, “It doesn’t always save you $1 or more” but I had to encourage John to offer his own input by saying “Okay, John you need to offer input.” During that same lesson, I asked other students to talk to their group members by
saying “Laura and Chris, you need to talk about it, please. Dan and Terry, you need to talk about it, please. Talk about your responses.” In order to encourage students to make sure that they fully understood the concepts, I continuously asked “Are there any questions?” during all four lessons. Typically students did not ask any questions or responded with “No, ma’am.”

There were two moments during the four days that students were more engaged in helping their peers understand the material being covered and it was during those moments that a level 1 of responsibility for learning was demonstrated. During day 3, students were asked to order from a menu, find the total cost of their meal, and compare it to the price of the special, $2.50. Since they were talking about food and money, students were able to relate those ideas to their real life. In the first part of the lesson, students offered different ideas on whether the advertisement claim was true. Anna said “Because I saved more than $1.” Stella offered that “…if you buy everything separately, it is more money.” Kelly responded with “Umm, it’s no because I paid $3.25 and I only saved $0.75.” During that same conversation, students went back and forth about whether a penny matters. Thomas said “If you have…you’re just adding a penny.” When I asked “Okay, so what if you buy 200 things?”, some students said that they would let the cashier keep the change. Dan exclaimed “With $0.99, I mean you paid $1, what’s $0.01 going to get you, a piece of gum?” It was during that conversation that students gave each other feedback and explored the ideas offered by their peers. Additionally, since students value money, students were eager to talk about the value of a penny and it was a way to present their own argument for how valuable a penny is.
Facilitation of discourse in an advanced math class

*Questioning*

The questioning across the 4 days in this class ranged from a level 0 to level 2, overall the questioning in class was at a level 1. During the four lessons, I was the main questioner but I asked questions that focused students on their mathematical thinking and less on their answers. I asked questions such as “How many main dishes, side dishes, and/or beverages do I need? Please explain how you figured this out so I can do it on my own.”, and “How did you get it?” I also asked follow-up questions like “where are you getting the 75 from?” “What do your numbers represent to you? You told me 1H, 1S, 2H, 2S, and so on. What do your numbers represent?” , and “Okay, so the theoretical probability of getting a what? What is that based on?” I also asked questions like “Now, where does that 12 come from? Where did the 12 come from?” and “…what does the example show?” so that students could focus on explaining their methods as well as answers.

There were moments when I asked questions that were at a level 0 and did not push students to explain their mathematical thinking. During those moments in the lessons, I asked questions like “So if she rolled the number cube 13 times, gets a 5 twice, what is the theoretical probability?”, “What’s another one?”, and “How many of them were actual English words?” When asked those questions, students tended to respond with short, quick responses without much detail. There were also moments in the lessons, where the questioning was at a level 2. During the four days of the study, students asked questions about the work of the peers at least 2-3 times during the lessons. For example on day 1, George asked, “Can we just write the number?” On day 2, Alex asked, “If they just wrote less than, is it right?” This occurred more when students were grading the work of their
peers. In some cases, students took it as an opportunity to explain their mathematical thinking to their peers. For example, when Henry didn’t understand Robert’s work, Robert pointed out that “Oh no, I meant, hold on, my bad…because it said…it said that…it said the odd and even numbers. There’s like 1…there’s like 2…” As students continued to create tree diagrams and lists, they began to ask questions such as “Could you have done 4 times 3?” Students were beginning to make connections with their work and discussions with their peers as they found patterns about the fundamental counting principle.

**Explaining Mathematical Thinking**

The explaining mathematical thinking was a level 1. Students offered information about their mathematical thinking voluntarily. For example, Hank said “We do $\frac{6}{14}$ times $\frac{5}{13}$ and you get 30 over 182. You can divide that by 2 and you get 15 over 91” and in response to a question about creating a list of all the possibilities given 3 events (location, audience size, and audience reaction), Derek said, “It’s just like this last one, instead of having another added on. It’s just 3 instead of 2.” Students also provided brief descriptions about their strategies. When explaining how he solved a problem, Robert said “Because it said…it said that…it said the odd and even numbers. There’s like 1…there’s like 2…” When asked to explain how she knew that she had listed all the possible combinations, Mary said “I said that you multiplied…you just count the list and multiply.” Eric pointed out that in order to check if he had all the possible combinations, “You go back and see if you can do any other ones.”

There were also moments when I had to probe students for more detailed explanations about their mathematical thinking. For example, when Jackson offered an answer, I asked him to explain where he got the 75% from. He explained that “Because this
is 3 out of 4. It’s 15 out of 20 but if you simplify it…if you divide both by 5 you get 3 over 4 and that equals 75%.” Another example of a student elaborating on their thinking was when Robert explained his work by saying that “And then $\frac{4}{7}$ because she didn’t put it back in. Then I got 16 over 48…I mean 56, and then I divided that by 8, I got 2 over 7.”

While students may offer information about their mathematical thinking, teachers may also have to fill in explanations for student thinking. During day 2, when I asked Jennifer where the number 12 came from, she responded with “Well, it’s the total, like, times to get it.” I added that “It’s the total times he performed the experiment. So you can either count up the rows or it told you at the top that he performed the experiment 12 times. So either one of those would have worked. So then you figure out well he got a 1, 3 times out of 12. So 3 out of 12 would be the experimental probability of getting a 1 because it is based on the results.” Another example was during day 4 when Alex answered that the experimental probability was bigger than the theoretical probability, I elaborated by saying “So experimental probability is greater than theoretical probability. Experimental is based on your results. They got a 6, 4 times out of the 12 that the experiment was performed. The theoretical was $\frac{1}{6}$ you expect to get it 2 times out of 12 which is $\frac{1}{6}$

Source of Mathematical Ideas

The overall source of mathematical ideas was at a level 1. I was the main source of ideas but I did ask for students to share their mathematical ideas and strategies. On day 2, when students were asked to explain how they knew that they had listed all the possible combinations with two events (i.e. location and audience reaction), Mary said “I said that you multiplied…you just count the list and multiply” and Eric said “If there’s 1 happy and 1 sad
for each of them and there’s 6 events, I mean 6 locations, then you get a total of 12.” On day 3 when I introduced the permutation theorem, Eric realized that “When there’s always 3, there’s always 6. When there’s always 4, I think it’s like 25 or 26”. He was starting to notice a strategy for remembering the number of permutations or orderings given a certain number of objects to arrange. On day 4, Mike said “The probability of getting less than 3 is ½ and the probability of red is” and Henry said “Umm, I did $\frac{12}{28}$ times $\frac{11}{27}$ and I got 132 over 756 and then I…”

As mentioned earlier, students were able to find patterns between the number of items used to create each tree diagram and the total number of combinations listed. Through questioning each other and discussing different questions posed on the worksheets, students voluntarily offered their own ideas of how to find the total number of combinations without creating tree diagrams and lists. On day 2, Robert asked “Could you have done 4 times 3?” On day 3, Peter asked “Why don’t you just times that by 6?” While those were the connections that I wanted students to make between their number of items in each category and the total number of combinations, I continued to encourage students to create tree diagrams and lists before introducing them to the fundamental counting principle.

Responsibility for Learning

The overall responsibility for learning throughout the four lessons was at a level 1. At a level 1, the teacher facilitates conversations that encourage students to listen to each other as well as help each other to better understand the mathematical concepts. Through this action, students become more engaged in the material and are eager to show other students how to solve a problem. In this class, students did ask questions about their peers’ work. For
example, when going over a question in which students were asked to compare the theoretical probability of a single event to the experimental probability of that same event, a student asked “Umm…so he put $T$ equals $2 \frac{1}{6}$ and then he put $E$ equals 2. That’s wrong, right?”

When going over a question that asked students about the probability of getting a prime number when rolling a number cube, Robert asked a question about why his work was wrong. As Robert explained his work, students listened to his explanation and Henry explained to Robert that “1 isn’t a prime number” when Robert included the number 1 in his list of prime numbers. As the conversation continued, students pointed out that the number 1 is not prime because it only had 1 factor.

Though few, there were some moments when the lessons were at a level 2. At a level 2, the teacher encourages students to take responsibility for understanding the mathematical ideas of other students. For example, when creating a list of all the possible two event combinations that could be created with 6 concert locations and 2 audience reactions, I asked “Brandon, do you understand what he did?” and he responded with “No” so I told Alex “Okay, can you make sure that he understands what you are doing?” As I walked away, I heard Alex explaining to the rest of his groups his thought process for creating his list as well as how he knew that he had all the possible combinations listed. In response to the notes explaining when to use the fundamental counting principle and when to create a list of all the possible combinations, Sam said “I don’t see another one but I have a question…how do you know if you’re like done?” In this example, Sam was taking responsibility for his learning so that he could get clarification on the concept being discussed.
Cross Class Comparison

In this section, I compare the levels of each of the four categories between the average class and advanced class. As noted above, the manner in which I posed questions and facilitated the discussions in the two classes differs in several ways. The manner in which students respond to questions asked also differs between the two classes. I also intend to point out similarities between the two classes.

Questioning

The questioning levels between the two classes varied by 1-2 levels. While the questioning in my average class tended to be at a level 0 with some exceptions, my advanced class was at a level 1 with a few moments that were more consistent with level 2. In both classes, I was the main questioner but the manner in which the questions were asked differed between the two classes. The average class was asked questions that were meant to keep them listening, focused on the answer, and guide them through solving the problems. I also would stop every 5-10 minutes to have a whole class discussion about questions presented in the activity before asking students to continue on to a new section. This was another way to keep the students focused and engaged in the lesson. The advanced class was asked questions that were meant to both focus students on the answer as well elicit students’ mathematical thinking. When students only offered answers, there were moments when I asked them questions that encouraged them to explain their strategies. I also let students in this class work at their own pace but I walked around to listen to students’ conversations as well as ask students questions that helped them think through their strategies. While students in the average class tended not to ask questions about the ideas of their peers or concepts that I
presented, students in the advanced class were will willing to do so. Thus, the role of questioner in the advanced class shifted between teacher and students while it did not do so in the average class.

*Explaining Mathematical Thinking*

The level of explaining mathematical thinking differed between the two classes as well. The level of explaining mathematical thinking was at a level 0 with some exceptions in the average class while it was at a level 1 with some exceptions in the advanced math class. Students in the average class offered answers that were short and direct. Students only offered responses that explained their mathematical thinking or strategies when they were asked to do so by the teacher. While their explanations were brief, they did provide descriptions and information about students’ mathematical thinking. Students’ explanations in advanced class were also brief but students in that class offered explanations for their mathematical thinking and strategies more frequently without teacher prompting. In cases when students were asked to explain their mathematical thinking by their peers, students openly shared their strategies and solutions. Students in the advanced class were also eager to present strategies that related back to concepts that their previous teacher had taught them.

In both classes, as students continued to create tree diagrams and lists for finding the total number of combinations, students were able to offer other strategies for finding the total number of possible combinations given any set of objects. While there were moments in both class when I had to probe students for more detailed descriptions, I spent more time probing for more in-depth explanations in my advanced class because of students’ responses to my questioning and prompts.
**Source of Mathematical Ideas**

In both classes, the source of mathematical ideas was at a level 1. In the average class, students offered ideas with the help of guiding questions from me. Through my questioning and prompting, students in the average class were able to offer solutions to problems and offer new ideas for finding the total number of combinations possible with any set of objects. Students in this class were also more willing to offer their own ideas when it came to concepts that they could relate to. For example, when talking about saving money when ordering food, students were eager to offer their strategies for comparing prices. When talking about the value of a penny, students were eager to offer their ideas about how valuable a penny is. In the advanced class, students were willing to share ideas openly with minimal prompting. Students in the advanced class also referred back to lessons from previous units as well as elementary school as they made connections to new concepts, rather than their out of school experiences. In both classes, students shared patterns that they noticed between the objects placed on their tree diagrams and the total number of combinations that they found. Students found that if you multiplied the number of objects in each category, you could find the number of possible combinations. In the advanced class, once I introduced the permutation theorem, students were able to relate the algorithm to the number of arrangements that they created with Christina, Hank, and Mike. Therefore, in both classes students shifted between questioner and source of mathematical ideas depending on the context of the problem as well as students’ understanding of the problem.
Responsibility for Learning

Responsibility for learning in the average class was mainly at a level 0 while it was at a level 1 in the advanced class. As the main questioner and source of ideas in the average class, I did not ask questions or use prompts that were appropriate for encouraging students to take ownership of their learning. Students did not ask questions about the concepts covered or ideas that were shared. Students also did not question the work or solutions of their peers. Students often offered short responses unless probed for more detailed explanations. In the advanced class, I was still the main questioner and source of ideas but students in this class were more eager to show and explain how they solved a problem. I did not have to prompt students to ask questions about the work of their peers or for clarification about ideas that I presented. In both classes, while I set up structures that encouraged students to take responsibility for understanding the new concepts and asking questions about the strategies offered, I did not have to do much to encourage students in the advanced math class to ask questions or share ideas.
CHAPTER 5
DISCUSSION

This study was focused on examining how the manner in which I facilitate mathematical discourse differs when teaching regular and advanced 6th grade math classes. Specifically, I wanted to investigate how the classroom environment that I created affected the following: questioning, explanation of mathematical thinking, source of mathematical ideas, and responsibility for learning. The purpose of this section is to discuss my findings, reflect on my findings, and presents ideas on what I will do to improve the way that I facilitate mathematical discourse in both levels of classes.

The manner in which I facilitated mathematical discourse in the average math class was different than how I facilitated mathematical discourse in the advanced math class. While students in both classes were encouraged to share ideas and ask questions, the manner in which I prompted the students differed between the two classes. Students in the average class needed more guiding questions, hints, and prompting in order to build on prior knowledge, make connections, and explain their mathematical thinking. Students in the advanced did not need as much prompting to share ideas and explain their mathematical thinking. As a result there were differences in the levels between the classes in the categories of questioning, explaining mathematical ideas, and responsibility for learning. Only the source of mathematical ideas category was at the same level in both classes. Furthermore, though the levels were higher on three categories for the advanced math class, I did not facilitate discourse at a high level in either class.
I chose to use a unit on probability for the context of this study due to the nature of the tasks that I planned to incorporate. The tasks were open ended so I thought that they would encourage students to apply concepts from previous units to the new concepts. The tasks also allowed students to explore patterns and use them to make generalizations about the mathematical procedure for finding the total number of possible combinations given a set of items.

Research has found that students work to better understand and apply formal mathematical terms, they also need to be able to relate them to informal settings outside of the classroom (Hansen-Thomas, 2009). Since the tasks and materials that students worked with were relatable to the students, students in both classes were willing to complete the tasks and discuss the questions in the tasks. In addition, since students had been introduced to experimental and theoretical probability in the previous unit, students could relate their data results to each of those terms as well explain factors that affected their results. Throughout the year, I struggled to encourage students in the average class to engage in mathematical discussions about the topics covered but during these tasks, they were more eager to engage in the discussion because the tasks incorporated topics that they could easily relate to (i.e. money, food, clothes). Based on the discussions that students engaged in around these tasks, I will continue to incorporate open ended tasks that relate to real-world situations.

However, though the tasks were open ended, that alone did not lead to high levels of discourse in either of my classes. This finding is consistent with Kawanaka and Stigler (2000), who found that “not all questions that request descriptions or explanations can be called higher order questions” (p. 277). In order to get students to think on a higher level,
teachers need to present tasks that find ways to encourage students to apply mathematical concepts to different problems instead of just regurgitating material. When students are not encouraged to extend their mathematical thinking, then they struggle to move beyond their basic understanding of the problem.

I found that when students in the average class struggled with solving a problem, I was quick to step in to help them instead of letting them struggle a bit. Martino and Maher (1994) point out that when students have shared ideas and extended their thoughts as far as they can, then the teacher’s role becomes critical in helping students to extend their ideas, justify their ideas, and provide generalizations about their ideas. Students in my average class were not given many opportunities to explore, discuss, and justify their mathematical ideas with their peers because I was afraid of them shutting down and not continuing to search for solutions to the problems presented. Instead of jumping in when students were stuck or frustrated with the problems, I should have let the students continue to struggle with the problem on their own while providing them scaffolding to answer the questions through guiding questions. After looking at my results, I think that I need to offer those students more opportunities to explore mathematical ideas on their own so that students can become comfortable struggling with mathematics and discussing their ideas with their peers. I think that this will help shift both explaining mathematical thinking and source of mathematical ideas toward the students and away from me.

In both classes, I could have improved my implementation of the tasks (in this case the experiments) by extending them and asking the students to change them and analyze how the changes affected the results. Students could have also been given other problems that
incorporated all the strategies that students used in one problem. For example, students could have been asked to find the number of ways to arrange 5 students if 2 students don’t want to sit next to each other. Students would have to draw a picture and look at how to relate that to the permutation theorem that they had covered previously. As discussed by Williams and Baxter (1996), mathematical discourse gives students an opportunity to make connections between mathematical concepts and symbols through oral and written communication throughout the year, I need to offer students opportunities to apply prior knowledge and incorporate multiple strategies to solve problems through both oral and written assignments.

In addition to task selection, the type of questions that the teachers asks are critical in helping students find the mistakes that are contributing to their difficulty in solving a problem, helping students extend their current mathematical thinking about a problem, and helping students make connections between their ideas and those of their peers as they work to find a solution or common understanding of a problem. The questioning that I used with students in the average class was different than the questioning that was used in the advanced math class. Students in the average class were asked questions that were meant to focus them on the answer and very few questions that elicited their mathematical thinking. I would like to see students in the average class at a level 2. In order to do this, I need to ask questions that encourage students to offer more in-depth responses without always being probed to do so.

Students in the advanced class were asked questions that focused on the answer, elicited their mathematical thinking and encouraged them to make connections. I would like to see students in the advanced class at a level 2 or 3. Eventually I would like to see all of the
classes at a level 3 but I expect that it will take much more time to get there with the average classes. In order to transition through the levels, I will make small changes in the average classes so that I can help those students get used to the changes until eventually those students get to a level 3 just like their peers. In order to do this, I need to continue to encourage those students to create their own ideas, ask each other questions about their work, and justify their conclusions. While there were moments in both classes where I asked follow-up questions about students’ work, I think that I should be doing more of that to allow students to become more independent learners and more willing to develop their own mathematical ideas.

In order for students to feel comfortable about engaging in mathematical discourse, teachers must play two roles, moderator and observer (Martino & Maher, 1994). While there were moments in both classes when I was the moderator, there were also moments when I had to be the observer. In the average class, I was more of a moderator than an observer because students in that class needed more redirection to focus on discussing the mathematical concepts presented in the activities. In the advanced class, I was more of an observer than a moderator because students in that class tended to be more independent. They were able to work through the activities, discuss possible strategies and solutions, and then pose questions for clarification. Again, I need to continue to encourage all students to think critically, explore their own ideas, and justify their conclusions. In order to do this, I need let all students work through problems independently and only step in to provide students with guiding questions that extend their mathematical thinking or encourage them to examine whether their strategy needs to be revised.
In revisiting my theoretical understanding, interpretive understanding, and self-understanding after completing this action research study, I found that I learned a lot about my facilitation of mathematical discourse. Through this study and the research that I read, I was able to come up with my own theory of what mathematical discourse is and what I want it to look like in my classroom. After reading about studies that other researchers completed, I was able to gain a better understanding of what mathematical discourse is as well as examine similar struggles that others have when facilitating mathematical discourse.

Mathematical discourse is when students explore mathematical ideas, build on existing mathematical ideas, and share their own mathematical ideas with others around them through the use of visuals, words, and written language. After examining my facilitation of mathematical discourse, I would like to see how other teachers facilitate mathematical discourse in different level math classes so that I can develop a better interpretation of my results. Lastly, the study allowed me to develop a better self-understanding of why I facilitate mathematical discourse differently between my different level classes and what I can do to overcome this struggle.

Overall, I was more comfortable allowing students in the advanced classes work through the activities more independently than students in the average class because I was worried about how students would answer the questions. I found that I had to stop between sections in each activity to check for students’ understanding in the average class as well as clarify student misconceptions. I allowed students in the advanced class to ask clarifying questions, correct each other, and freely share ideas whether they were right or wrong. I did not do that in the average class because I felt the need to correct all mistakes. I struggle with
the types of questions to ask so that students in the average class are guided to right ideas as well as when to jump in so that they don’t explore the wrong ideas. In the future I plan to start out the school year by allowing students in the average class to work independently in 5 minute increments, then 10 minute increments, and then 15 minute increments and so on until students in those classes learn how to continuously try different strategies to solve problems even when they get frustrated.

Although I completed this study 6 months ago, I have made some changes in my classroom this year based on these results. I started the year off by regularly encouraging my students to explore their own mathematical ideas share them with their peers rather than waiting to do this during particular units of study. I have also stepped back more and let students struggle with tasks that I assign them before asking them guiding questions to develop their strategies. I encourage all my students to verbalize their mathematical thinking as well as write their mathematical thinking in words. I also encourage all my students to use mathematical terms when sharing their strategies with the class as well as when they are working to complete tasks individually. If I continue to encourage students to create their own mathematical ideas, extend their mathematical thinking, make connections between concepts, and share their findings with others around them, I think that all my classes will reach a level 2 or 3 in each of the four components of the “Math-Talk Community” framework before the end of the year. I can already see a difference compared to my facilitation last year.
Implications for future research

Future research on how teachers facilitate mathematical discourse should focus on questioning because it was questioning that impacted students’ explanation of mathematical thinking, the source of mathematical ideas, and students’ responsibility for learning. While there is a lot of research on teacher questioning, there is little on how teachers adjust their questioning given the students that they are working with. Given what I found about the difference in the way that I questioned the two different level classes and my reasons for doing so, I think this is an area that needs more work. Most importantly, future research should include replications of this study across multiple teachers to help tease out the teacher’s role in mathematical discourse related to average and advanced students.

Through more research on mathematical discourse, teachers can also share best instructional practices with each other. Teachers are able to see the types of tasks that encourage student discussions in both average and advanced classes. Teachers are able to see how to play the role of moderator and observer in both the average and advanced classes. Thus, through professional development, teachers should be encouraged to record their own classrooms as they facilitate mathematical discourse and examine how they currently scaffold students to solving problems. Through these self-assessments, teachers can find ways to improve on ways to encourage students to discuss, explore, and justify their own mathematical ideas. Additionally, more action research is needed in this area so that teachers could learn not only from their own practice, but from the struggles and adjustments of others as they try to improve their practice.
Limitations

One of the limitations in this study was that data was collected for only four consecutive days. Since I was limited on the amount of time that I could implement the study, the variety of lessons was limited and the depth to which the lessons could be explored was limited because I wanted to make sure that there was closure to the unit. More time to implement the study would have also allowed me to present students with lessons that incorporated a variety of concepts, analyze my data, make changes in the manner that I facilitated mathematical discourse, teach additional lessons incorporating changes that I found I need to make based on the analysis, and continue this process throughout the school year.

Another limitation was that the position of the camera in the classroom. Since the camera was in the corner of the classroom, I had to manually move the angle of the shot myself so that I could include more students in the videos. If I were able to move the camera around the classroom then I would be able to get more student-to-student interactions and individual group discussions as students worked through the activities. It would have given me more data on how I could improve on the manner in which I facilitated whole group and small group discussions. While I had an audio recorder attached to me, there were times when I still couldn’t hear the audio when transcribing the lessons. I think this problem could have been solved by either have more audio recorders in the classroom or finding a way to move the camera around the room to hear more of the conversations.
Conclusion

Although there were some limitations in this study, I learned a lot about the manner in which I currently facilitate mathematical discourse in my average and advanced math classes. Through the use of the “Math-Talk Learning Community” framework, I learned that the level of questioning differed a lot between the two classes. In the average class, I did not always ask questions that elicited mathematical thinking so students did not give me explanations that were detailed. Since students in the average class were not used to doing independent work, they depended on me to be the source of mathematical ideas and I had to encourage them to take responsibility for their own learning by asking questions about ideas presented. However, I also found that when given the appropriate tasks, students in the average class are eager to engage in discussions about their mathematical thinking and strategies. With more practice, I can shift from being the main source of ideas and explanations of mathematical thinking to students doing it. All of my students can engage in mathematical discourse when I choose the right task and appropriate questions that elicit mathematical thinking. I just need to be confident in both their abilities and my own.
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APPENDIX
## Appendix A

### Outline of Lessons

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<th>Day</th>
<th>Topic(s):</th>
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<tr>
<td>1</td>
<td>- Theoretical probability. Experimental probability, and Combinations</td>
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**Activity:**

Students were asked to pretend that they were a band going on tour. They had 6 locations, 3 audience sizes, and 2 audience reactions that they could choose from. Before beginning the experiment, students were asked to pick their ideal event (ideal location, ideal audience size, ideal audience reaction). Once students had decided on their ideal event, they were then asked to perform the experiment using the materials provided (6-sided number cube, 3 color tiles, and a coin). Students performed the experiment 12 times, recording their results after each turn. After collecting their data, students used it to answer questions about the theoretical probability and experimental probability of each of their ideal events. Lastly, students created a list of possible 2 event combinations and 3 event combinations then used the lists to answer another question about the theoretical probability of their ideal event using the 3 event combination list that they created. Students were assigned homework practice problems that reviewed the concepts covered in class.

**Extension:**

Students were asked how they would improve the game if they were given three spinners instead of a number cube, color tiles, and a coin. Students labeled the first spinner with 3 different concert locations, the second spinner 2 different modes of transportation, and the third spinner with 2 different outfits. Once students labeled each spinner, they were asked to pick their ideal event, list all the possible combinations with the 3 spinners, find the theoretical probability of getting their ideal event, and explain how they found the theoretical probability of their ideal event.
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<th>Day</th>
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<tr>
<td>2</td>
<td>- Theoretical probability. Experimental probability, and Combinations</td>
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**Activity:**
Students shared their answers, corrected their mistakes, and discussed questions from the previous day. Students were then asked to use the scenario (band on tour) from the previous day to answer a question about the validity of an advertisement claim. Students created a tree diagram to find all the possible combinations at the Unique diner, picked a combination from their tree diagram, found the cost of their meal, and compared it to the cost of the special. Students explained whether the advertisement claim was true before creating a set of tree diagrams to find the number of possible combinations for a set of problems.

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<th>Day</th>
<th>Topics:</th>
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<tr>
<td>3</td>
<td>- Theoretical probability. Experimental probability, and Combinations</td>
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**Activity:**
Students were given time at the beginning of class to finish up the tree diagrams that were assigned on Day 2. Once students completed the questions, there was a whole class discussion about the Unique Diner’s advertisement claim and the number of different combinations that were possible in each of the problem sets. Next, students were asked to check and share answers to the homework from the previous night. As students shared their answers, they found a pattern for finding the number of different combinations in a situation. After going over the homework, I introduced students to the fundamental counting principle and related it to the tree diagrams and lists that were used to find the number of combinations in each of the problems that had been presented during days 1 and 2.

**Extension:**
Students were asked to write a response letter to the owner of the Unique Diner. In the letter, students were asked to write 3-4 sentences explaining how the owner could create a lunch special that offered 50 unique combinations. Students also had to explain how they figured it out so that the owner could do it on his own.
Day 4

Topics:
- Theoretical probability. Experimental probability, and Combinations

Activity:
Students were given time at the beginning of class to practice questions that were similar to what they would see on upcoming assessments. There was a whole class discussion of the answers and work for each question on the review assignment. Next, students were introduced to how to apply a tree diagram to find the number of orderings or permutations in a situation. Once students created the tree diagrams, they created a list, and used that to answer questions about the theoretical probability of various events. After completing a few examples, students were introduced to the permutation theorem. Lastly, students completed some practice problems that incorporated each of the following methods: tree diagram, list, and the permutation theorem.

Extension:
Students were asked to find the number of possible permutations with the letters: a, i, l, and r. Once students created a complete list or tree diagram of all the possible orderings, students had to list the permutations that represented English words.